Low level of covariance simulation

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1 Motivation

Based on the investigation of the covariance matrix patter of the PCBs from 1999 - 2004, we found some patterns that appears among the covariance matrix of PCBs from different years. Besides, we also use the historical data to estimate the covariance matrix, then use the sample covariance matrix to decorrelate the PCBs for each year. For most years, the decorrelation procedure can reduce the correlation, e.g 1999-2001, but there are some years which the correlations still are high after decorrelation, e.g 2005-2006.

Since we are trying to borrow information of historical dataset, the decorrelated data will probably not be perfert uncorrelted. In other words, we will end up with data with low correlations among their coveriates. So the goal of this report is to see if the GCTA and EigenPrism method could work well under the low correlation setup.

2 setup

2.1 Sample covariance matrix

I use two different sample covariane matrices: 1999-2001 and 2007 -2008.

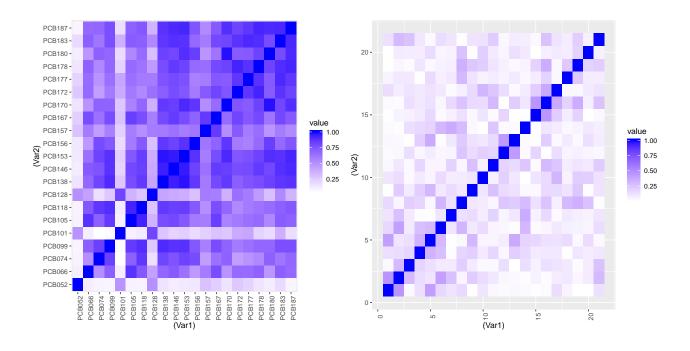


Figure 1: 1999-2000

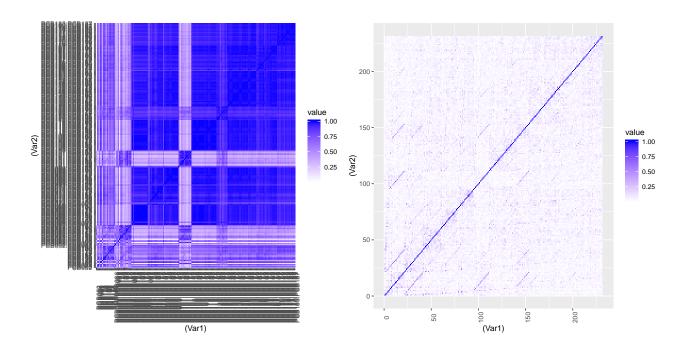
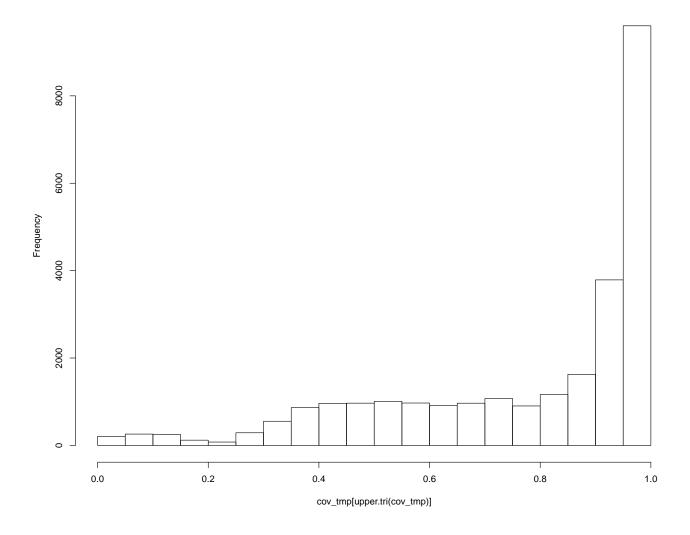
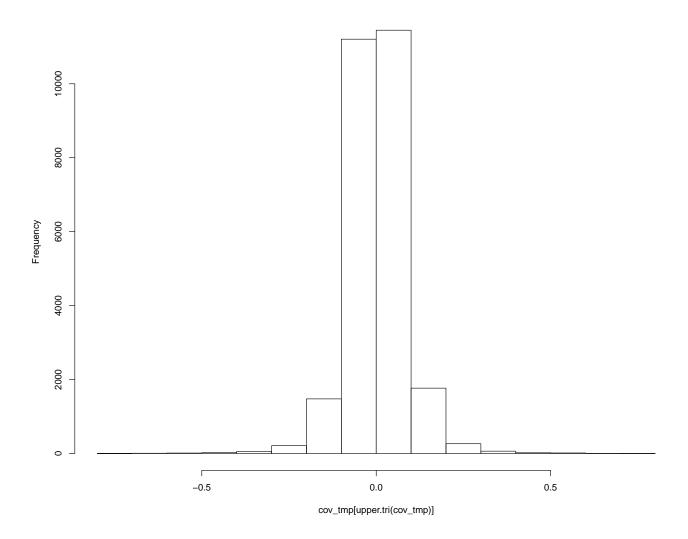


Figure 2: Combined main and interaction 1999-2000

correlation of main and inter



decorrelated correlation of main and inter



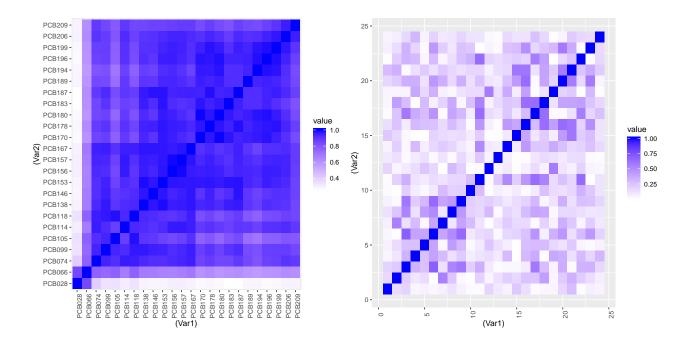


Figure 3: 2005-2006

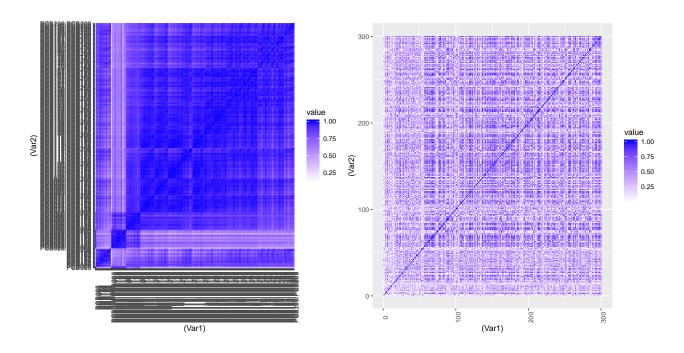
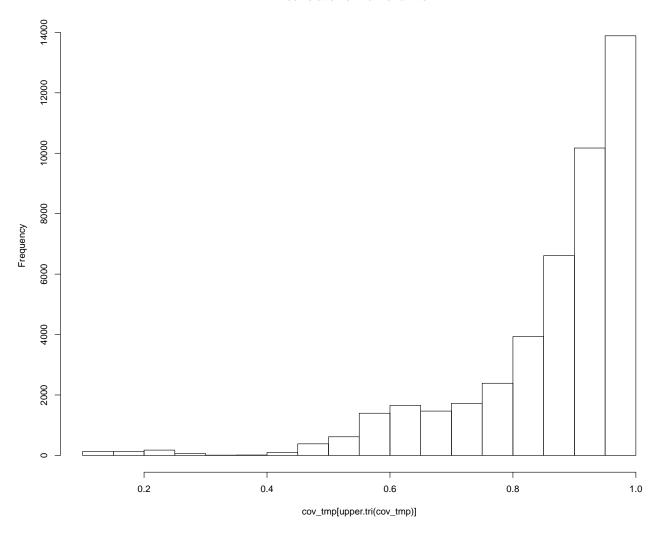
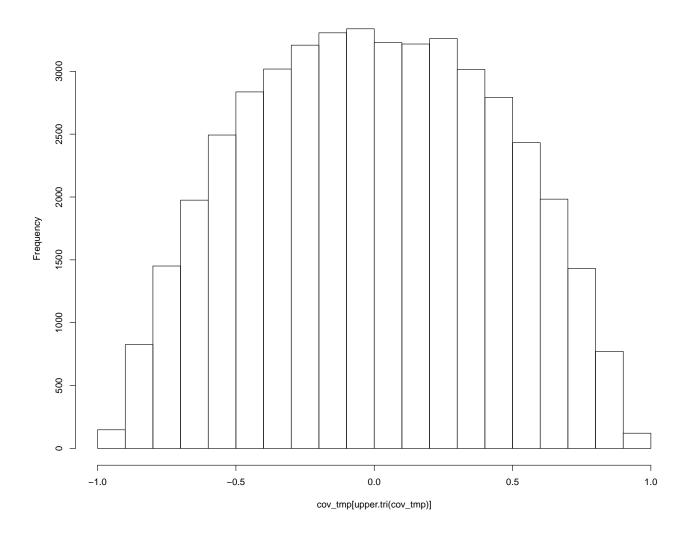


Figure 4: Combined main and interaction 2005-2006

correlation of main and inter



decorrelated correlation of main and inter



2.2 Simulation setup

- 1. p = 231 for 1999 or 300 for 2005
- 2. $n \in \{100, 150, p\}$
- 3. the simulated covariates will follow Normal or Chi with $\mathrm{d} f = 1$
- 4. Variance estimation method will be GCTA and EigenPrism method
- 5. Iteration time is 100

3 Result

3.1 1999

	n	MSE	est_var	est_mean	${\tt NA_total}$	method	var_main_effect	x_dist
1:	100	626	138	32	9	${\tt EigenPrism}$	10	chi
2:	150	790	140	36	0	${\tt EigenPrism}$	10	chi
3:	231	740	61	36	0	${\tt EigenPrism}$	10	chi
4:	100	852	242	35	0	GCTA	10	chi
5:	150	848	191	36	0	GCTA	10	chi
6:	231	702	66	35	0	GCTA	10	chi
7:	100	504	100	30	4	${\tt EigenPrism}$	10	normal
8:	150	604	74	33	0	${\tt EigenPrism}$	10	normal
9:	231	700	70	35	0	${\tt EigenPrism}$	10	normal
10:	100	649	173	32	0	GCTA	10	normal
11:	150	656	99	34	0	GCTA	10	normal
12:	231	705	101	35	0	GCTA	10	normal
	n	MSE	est_var	est_mear	n NA_total	L method	l var_main_effect	x_dist
1:		MSE 16.9				l method DeigenPrism		_
			16.9	9.7	7 (n 10	chi
2:	100	16.9	16.9 9.5	9.7 10.0	, () EigenPrism	n 10	chi chi
2:	100 150 231	16.9 9.4	16.9 9.5 5.2	9.7 10.0 10.2	7 () (<u>2</u> () EigenPrism) EigenPrism) EigenPrism	10 n 10 n 10	chi chi chi
2: 3: 4:	100 150 231	16.9 9.4 5.2	16.9 9.5 5.2 18.5	9.7 10.0 10.2 9.1	7 () () () (D EigenPrism D EigenPrism D EigenPrism D GCTA	1 10 10 10 10 10 10 10 10 10 10 10 10 10	chi chi chi chi chi
2: 3: 4: 5:	100 150 231 100	16.9 9.4 5.2 19.0	16.9 9.5 5.2 18.5 8.6	9.7 10.0 10.2 9.1 9.9	7 (C)	D EigenPrism D EigenPrism D EigenPrism D GCTA D GCTA	10 10 10 10 10 10 10 10 10 10 10 10 10 1	chi chi chi chi chi chi chi chi
2: 3: 4: 5:	100 150 231 100 150 231	16.9 9.4 5.2 19.0 8.5	16.9 9.5 5.2 18.5 8.6 4.9	9.7 10.0 10.2 9.1 9.9 10.1	7 (C)	D EigenPrism D EigenPrism D EigenPrism D GCTA D GCTA	1 10 10 10 10 10 10 10 10 10 10 10 10 10	chi chi chi chi chi chi chi chi
2: 3: 4: 5: 6: 7:	100 150 231 100 150 231 100	16.9 9.4 5.2 19.0 8.5 4.8	16.9 9.5 5.2 18.5 8.6 4.9	9.7 10.0 10.2 9.1 9.9 10.1 9.9	7 () () () () () () () () ()	D EigenPrism D EigenPrism D EigenPrism D GCTA D GCTA D GCTA	10 10 10 10 10 10 10 10 10 10 10 10 10 1	chi chi chi chi chi chi chi chi chi
2: 3: 4: 5: 6: 7:	100 150 231 100 150 231 100	16.9 9.4 5.2 19.0 8.5 4.8 19.2	16.9 9.5 5.2 18.5 8.6 4.9 19.4 13.2	9.7 10.0 10.2 9.1 9.9 10.1 9.9	7 (0) (0) (2) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1	D EigenPrism D EigenPrism D EigenPrism D GCTA D GCTA D EigenPrism	10 10 10 10 10 10 10 10 10 10 10 10 10 1	chi chi chi chi chi chi chi chi normal
2: 3: 4: 5: 6: 7:	100 150 231 100 150 231 100 150 231	16.9 9.4 5.2 19.0 8.5 4.8 19.2	16.9 9.5 5.2 18.5 8.6 4.9 19.4 13.2 4.6	9.7 10.0 10.2 9.1 9.9 10.1 9.9 10.5	7 (0) (2) (2) (3) (4) (4) (5) (6) (6)	D EigenPrism D EigenPrism D EigenPrism D GCTA D GCTA D EigenPrism D EigenPrism D EigenPrism D EigenPrism	1 10 10 10 10 10 10 10 10 10 10 10 10 10	chi chi chi chi chi chi chi chi chi normal
2: 3: 4: 5: 6: 7: 8: 9:	100 150 231 100 150 231 100 150 231 100	16.9 9.4 5.2 19.0 8.5 4.8 19.2 13.3	16.9 9.5 5.2 18.5 8.6 4.9 19.4 13.2 4.6 22.1	9.7 10.0 10.2 9.1 9.9 10.1 9.9 10.5 10.6 9.6	7 (0) (0) (1) (2) (1) (1) (1) (1) (1) (1) (1) (1) (1) (1	EigenPrism EigenPrism Corr Corr Corr Corr Corr Corr Corr Cor	10 10 10 10 10 10 10 10 10 10 10 10 10 1	chi chi chi chi chi chi chi chi normal

3.2 2005

	n	MSE	est_var	est_mean	NA_total	method	var_main_effect	x_{dist}
1:	100	897	132	38	6	${\tt EigenPrism}$	10	chi
2:	150	1131	140	41	0	${\tt EigenPrism}$	10	chi
3:	300	1574	109	48	44	${\tt EigenPrism}$	10	chi
4:	100	1274	287	41	0	GCTA	10	chi
5:	150	1673	327	47	0	GCTA	10	chi
6:	300	2090	213	53	0	GCTA	10	chi
7:	100	1250	173	43	6	${\tt EigenPrism}$	10	normal
8:	150	1162	129	42	0	${\tt EigenPrism}$	10	normal
9:	300	1404	104	46	45	${\tt EigenPrism}$	10	normal
10:	100	1789	366	48	0	GCTA	10	normal
11:	150	1570	266	46	0	GCTA	10	normal
12:	300	2069	257	53	0	GCTA	10	normal
	n	MSE	est_var	est_mean	NA_total	method	var_main_effect	x_dist
1:		MSE 16.4	est_var 15.8	est_mean 10.9	_	method EigenPrism	var_main_effect 10	x_dist chi
1: 2:	100		_	_	_			_
	100	16.4	15.8	10.9	0	EigenPrism	10	chi
2:	100 150 300	16.4 13.3	15.8 11.6	10.9 11.3	0	EigenPrism EigenPrism	10 10	chi chi
2: 3:	100 150 300 100	16.4 13.3 4.5	15.8 11.6 4.6	10.9 11.3 9.9	0 0 0	EigenPrism EigenPrism EigenPrism	10 10 10	chi chi chi
2: 3: 4:	100 150 300 100	16.4 13.3 4.5 12.3	15.8 11.6 4.6 11.2	10.9 11.3 9.9 11.1	0 0 0 0	EigenPrism EigenPrism EigenPrism GCTA	10 10 10 10	chi chi chi chi
2: 3: 4: 5:	100 150 300 100 150 300	16.4 13.3 4.5 12.3 11.0	15.8 11.6 4.6 11.2 7.7	10.9 11.3 9.9 11.1 11.8	0 0 0 0	EigenPrism EigenPrism EigenPrism GCTA GCTA	10 10 10 10 10	chi chi chi chi chi
2: 3: 4: 5: 6:	100 150 300 100 150 300 100	16.4 13.3 4.5 12.3 11.0 3.7	15.8 11.6 4.6 11.2 7.7 3.0	10.9 11.3 9.9 11.1 11.8 10.9	0 0 0 0 0	EigenPrism EigenPrism EigenPrism GCTA GCTA	10 10 10 10 10 10	chi chi chi chi chi
2: 3: 4: 5: 6: 7:	100 150 300 100 150 300 100	16.4 13.3 4.5 12.3 11.0 3.7 15.1	15.8 11.6 4.6 11.2 7.7 3.0 13.6	10.9 11.3 9.9 11.1 11.8 10.9 11.3	0 0 0 0 0	EigenPrism EigenPrism EigenPrism GCTA GCTA GCTA EigenPrism	10 10 10 10 10 10 10	chi chi chi chi chi chi
2: 3: 4: 5: 6: 7:	100 150 300 100 150 300 100 150 300	16.4 13.3 4.5 12.3 11.0 3.7 15.1 11.4	15.8 11.6 4.6 11.2 7.7 3.0 13.6 10.6	10.9 11.3 9.9 11.1 11.8 10.9 11.3 11.0	0 0 0 0 0 0	EigenPrism EigenPrism EigenPrism GCTA GCTA GCTA EigenPrism EigenPrism	10 10 10 10 10 10 10 10	chi chi chi chi chi chi normal
2: 3: 4: 5: 6: 7: 8:	100 150 300 100 150 300 100 150 300	16.4 13.3 4.5 12.3 11.0 3.7 15.1 11.4 6.7	15.8 11.6 4.6 11.2 7.7 3.0 13.6 10.6 6.7	10.9 11.3 9.9 11.1 11.8 10.9 11.3 11.0	0 0 0 0 0 0 0	EigenPrism EigenPrism GCTA GCTA GCTA GCTA EigenPrism EigenPrism EigenPrism	10 10 10 10 10 10 10 10 10	chi chi chi chi chi chi normal