

Simulation Fixed Effect

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1 Motivation

Based on the previous simulation results, the interaction effect estimation is biased. One possible reason is that we consider the β 's as random effects, which will affect the covariance structure of the signals. More specifically, If β is fixed, then we have following

$$Var(X\beta) = \beta^T \Sigma_X \beta,$$

Where $\Sigma_X = Var(X)$. We can see that the X's covariance has an affect on the signal's variance.

However, if we consider β as an random effect, then $Var(X\beta)$ is not only related with X but also with β 's covariance. Therefore, in the following simulation the β is fixed and there is no average out summary across different values of β .

2 Simulation result

2.1 Setup

In the following simulation, I just randomly generated 3 different sets of values for β (main and interaction) . Give each set of β 's, the values of y is generated 100 times and thus 100 estimations of variance of main effects and interaction effects.

2.2 Full data

2.2.1 Averaged estimation

Table 1: null-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
3.2158113	49.40766	4.3544031	9.8020884	6.3131046	19.9637333
0.0000000	0.00000	1.7708630	0.3238356	1.3104548	0.5987790
1.4348505	13.20353	4.2029131	0.5565153	8.8826209	1.8856730
0.0000000	0.00000	0.7651304	0.0759305	0.7295852	0.2124003
0.8500824	29.33039	9.0731883	3.6950261	11.2518946	3.9226269
0.0000000	0.00000	1.3603169	0.2463424	1.1417417	0.2855682

Table 2: rank-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.6873762	4.291104	2.8521057	5.6127113	2.9875348	0.8293864
0.0000000	0.000000	0.5219216	0.9289016	0.4989542	0.4139479
1.5461695	1.903794	0.7875891	0.8296039	1.8003643	0.2571868
0.0000000	0.000000	0.2677982	0.2455065	0.3886742	0.2667834
0.4947499	5.449791	-0.0100967	4.7920416	1.1578751	0.4965576
0.0000000	0.000000	0.0475158	0.6945106	0.2787832	0.2714290

Table 3: quantile-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.808878	6.248359	4.2725694	7.1142656	5.1323716	2.7324792
0.000000	0.000000	0.6741903	0.8282731	0.6488152	0.5126386
1.493517	2.945494	1.3331202	0.8386872	2.5937373	0.6850581
0.000000	0.000000	0.3637601	0.2097083	0.4655614	0.3297226
0.577709	8.209279	0.1982071	4.9213341	2.5304851	1.5261274
0.000000	0.000000	0.2193605	0.6382106	0.4427720	0.3743509

Table 4: log-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.6937805	6.107954	5.0208023	7.7236308	5.5848707	1.3456888
0.0000000	0.000000	0.7177300	0.9656956	0.6280478	0.3237447
1.6151267	2.360681	0.5707424	0.8223152	2.0697999	0.3163978
0.0000000	0.000000	0.2375397	0.2171377	0.3980136	0.2521233
0.5805219	6.281979	0.0926633	3.3529820	1.9173150	0.4860522
0.0000000	0.000000	0.1422832	0.5058683	0.3574811	0.2421702

Table 5: sqrt-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.8923511	16.154550	5.8498863	11.1069955	11.1032750	17.2475250
0.0000000	0.000000	1.0780650	0.6580931	0.9545424	1.0366619
1.3593689	4.740357	1.6579450	0.7002400	3.6527756	2.2664512
0.0000000	0.000000	0.4476342	0.1572187	0.5130434	0.5033004
0.6224476	11.267165	0.5178294	3.2208614	3.7014124	2.2444958
0.0000000	0.000000	0.4421995	0.3660503	0.5833727	0.2835238

Table 6: cat-10-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.6921130	4.090163	3.2023234	5.3127400	3.0219314	2.3507786
0.0000000	0.000000	0.5573301	0.9149534	0.4715617	0.6328867
1.4802190	2.017695	0.7938607	1.0346787	1.7375762	0.9872728
0.0000000	0.000000	0.2702883	0.2869367	0.3890169	0.4708470
0.5241891	5.244097	-0.0565079	4.7258258	0.9887204	1.3254654
0.0000000	0.000000	0.0360078	0.6983307	0.2675379	0.4672848

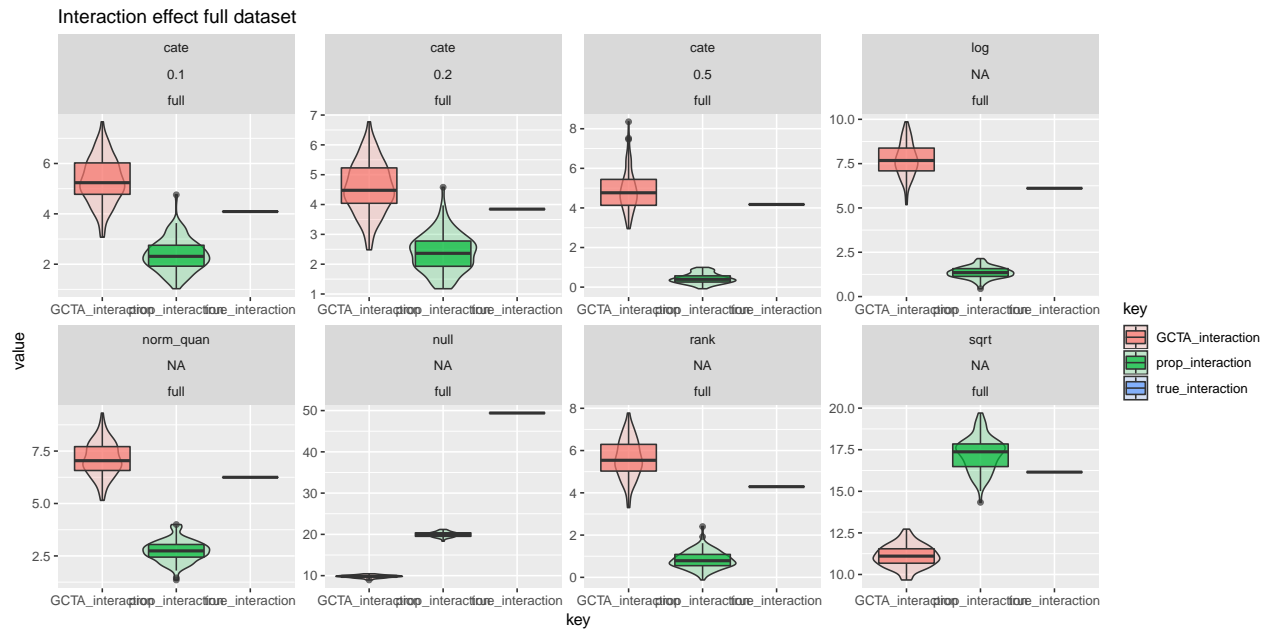
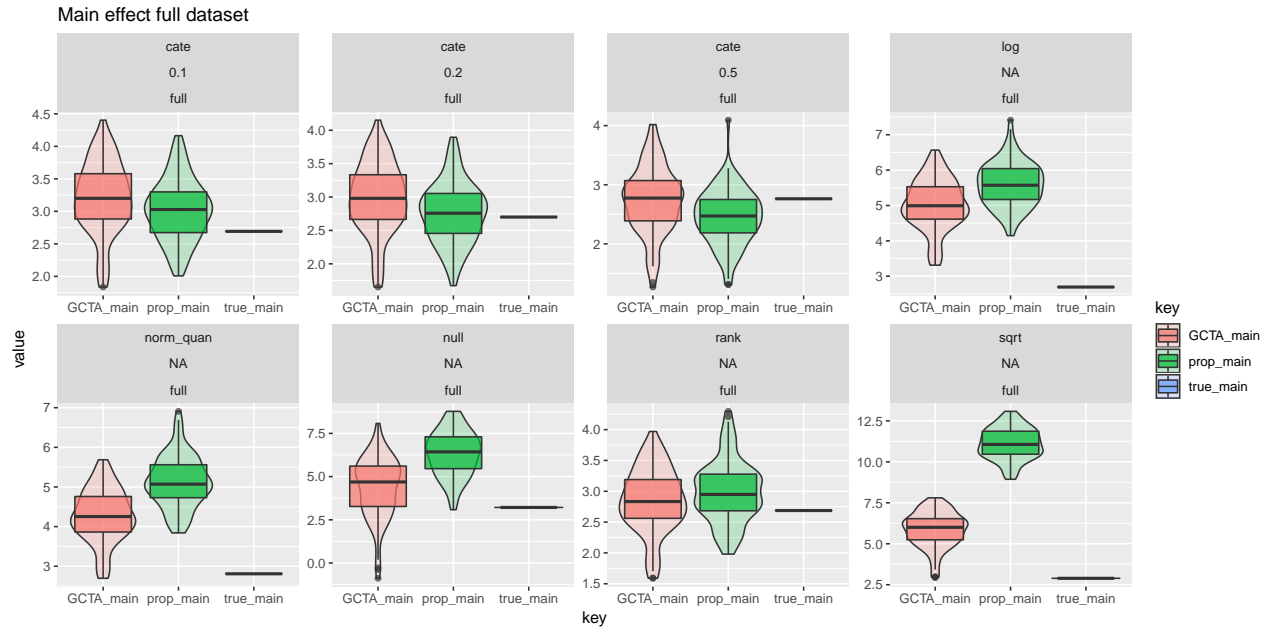
Table 7: cat-5-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.7000186	3.845413	2.9814892	4.5664152	2.7577871	2.3837907
0.0000000	0.000000	0.5389448	0.8707940	0.4572031	0.6435033
1.5233407	2.095205	0.6560493	1.1843935	1.6552562	1.4631640
0.0000000	0.000000	0.2460147	0.3280028	0.3682952	0.5492955
0.5844635	5.039599	-0.0089763	4.9405562	0.9122658	1.7972413
0.0000000	0.000000	0.0597922	0.7460484	0.2607959	0.5506845

Table 8: cat-2-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.7625360	4.174994	2.7357569	4.8980288	2.4478650	0.4305927
0.0000000	0.000000	0.5691043	1.0556699	0.4673456	0.2555173
2.0988911	2.415461	0.6686098	1.4511920	2.0919194	0.2109776
0.0000000	0.000000	0.2497561	0.4816661	0.4485330	0.1846766
0.8243926	4.359497	0.0476966	7.2911735	0.5193437	0.6674842
0.0000000	0.000000	0.0733865	1.3249693	0.2321657	0.2455678

2.2.2 Histogram of 100 iterations



2.3 Remove 7 subset data

2.3.1 Averaged estimation

Table 9: null-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.326047	11.04140	2.9874170	1.1487788	8.6919450	1.4596337
0.000000	0.00000	0.8428856	0.1439049	0.7281087	0.1605680
1.201223	43.48742	-0.7370419	7.7927874	14.1694376	5.0495201
0.000000	0.00000	1.4242304	0.3720183	1.0371484	0.2705839
1.012090	10.01595	1.2742855	1.4205766	5.6320471	1.8196181
0.000000	0.00000	0.6075589	0.1547535	0.7061331	0.1748635

Table 10: rank-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.3586213	1.7099875	3.4010825	0.5564496	2.9142539	0.1923090
0.0000000	0.0000000	0.5509228	0.1968368	0.4829440	0.2634942
0.8726516	4.8404613	0.4423640	6.9471978	0.8599029	0.2515139
0.0000000	0.0000000	0.1999645	0.9871361	0.3064646	0.1832223
0.9815092	0.6539307	0.9953976	0.0356594	1.1013735	-0.0422894
0.0000000	0.0000000	0.2466602	0.0546254	0.2439542	0.1668013

Table 11: quantile-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.3824357	2.1391474	3.8844804	0.6773023	3.4683656	0.2292941
0.0000000	0.0000000	0.5892784	0.2351244	0.5273831	0.2151068
0.9031948	8.2892384	-0.1832356	8.8315811	2.2951339	1.0422520
0.0000000	0.0000000	0.1090191	0.9395930	0.4331523	0.3285844
0.9728945	0.9235943	1.0575944	0.0864704	1.2314344	0.0207744
0.0000000	0.0000000	0.2574253	0.1056672	0.2871658	0.1485006

Table 12: log-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.3753731	2.1161346	3.4175941	0.5725289	3.1828043	-0.0139009
0.0000000	0.0000000	0.5604288	0.1949268	0.4740381	0.1380879
1.0225968	7.1152262	-0.1619669	6.8799631	0.6129037	0.4824211
0.0000000	0.0000000	0.0915385	0.8938767	0.2883702	0.2353967
0.9626496	0.9838793	1.1858619	0.0188071	1.3573222	-0.0626020
0.0000000	0.0000000	0.2710320	0.0669914	0.2651128	0.1017684

Table 13: sqrt-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.3533282	4.161740	3.9640479	1.2051462	5.1160571	1.1605548
0.0000000	0.000000	0.6546046	0.2669356	0.5408662	0.1912425
1.0021315	17.942518	-1.3056548	8.5014977	4.9446045	3.4550858
0.0000000	0.000000	0.6491687	0.6615934	0.5798050	0.3046042
0.9796173	1.796764	1.6992198	0.4067622	2.3081148	0.6194889
0.0000000	0.000000	0.3404351	0.1760265	0.3708196	0.1935866

Table 14: cat-10-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.3457539	1.7426962	3.3972965	0.6514204	2.8515749	0.7056741
0.0000000	0.0000000	0.5548752	0.2188062	0.4614171	0.4096501
0.8910848	4.5900370	0.3960131	6.5894462	0.8452006	0.8720525
0.0000000	0.0000000	0.1896358	0.9805470	0.2703615	0.3231284
0.9856932	0.6969061	1.0796944	0.0418221	1.1523672	0.1498357
0.0000000	0.0000000	0.2573330	0.0588064	0.2616573	0.3030559

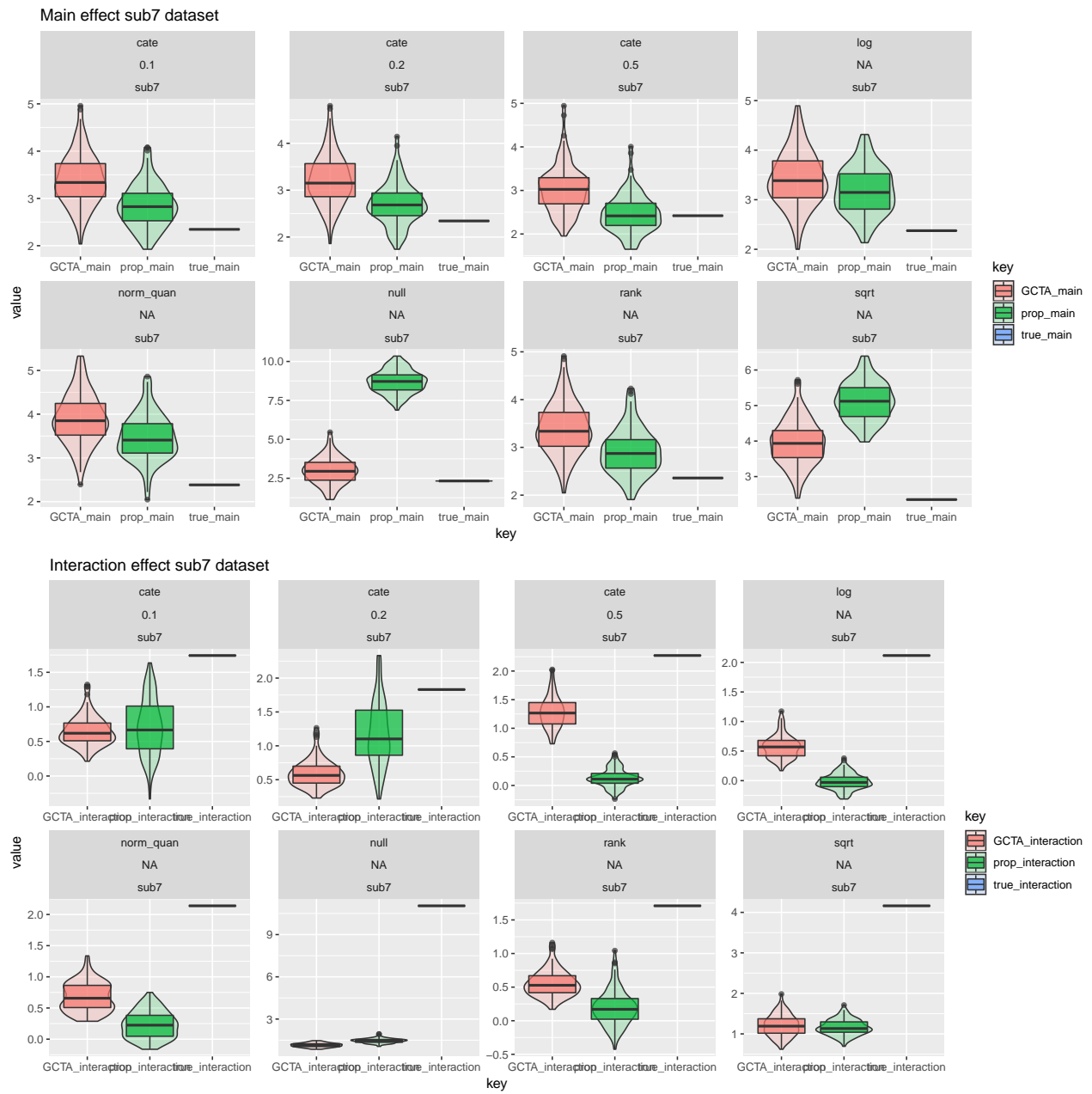
Table 15: cat-5-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.3430307	1.8309328	3.2338604	0.5909319	2.7097974	1.1819612
0.0000000	0.0000000	0.5517777	0.2007778	0.4455850	0.4850347
0.9577299	4.1246182	0.4700609	6.0277129	0.7653511	1.4748726
0.0000000	0.0000000	0.2049841	0.9805762	0.2545628	0.4718127
0.9904863	0.7478858	1.0610726	0.0538243	1.0722304	0.3622345
0.0000000	0.0000000	0.2581064	0.0648982	0.2624449	0.3666438

Table 16: cat-2-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.419468	2.271585	3.0376944	1.2854136	2.4576324	0.1298660
0.000000	0.000000	0.5485878	0.2819082	0.4326550	0.1626354
1.154539	2.698032	0.4646134	3.9490138	1.1831423	0.9354951
0.000000	0.000000	0.2102208	0.9285393	0.3262500	0.2752220
1.020003	1.312834	1.0350381	0.2530629	1.0065813	0.2144515
0.000000	0.000000	0.2848867	0.1596577	0.2784719	0.1572541

2.3.2 Histogram of 100 iterations



2.4 Remove 11 subset data

2.4.1 Averaged estimation

Table 17: null-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.7276371	7.708914	4.4996190	0.4411341	5.8478080	0.3475610
0.0000000	0.000000	0.6803805	0.0948647	0.7255906	0.0896674
0.6782210	42.661785	-12.8748992	7.1828950	6.7056205	6.4783280
0.0000000	0.000000	0.9794912	0.3166469	1.1283617	0.2888135
0.2546506	6.389698	0.7262481	0.3570885	1.4882557	0.4238215
0.0000000	0.000000	0.3645878	0.0613997	0.3866327	0.0865650

Table 18: rank-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.7824456	4.265218	3.2448657	5.7896516	3.3443064	0.3522793
0.0000000	0.000000	0.4651041	0.8349188	0.4655195	0.3027861
0.5280393	7.799071	0.1915601	12.8500545	1.0191591	0.8875884
0.0000000	0.000000	0.1012524	1.3903298	0.3209896	0.2369106
0.2060431	2.062459	0.0258525	2.1557215	0.3198292	0.3661274
0.0000000	0.000000	0.0471755	0.4058309	0.2069629	0.1993032

Table 19: quantile-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.7681842	4.328651	3.4609212	3.1788078	3.7258411	0.3012130
0.0000000	0.000000	0.4966609	0.4563100	0.4782636	0.2437290
0.5720811	8.861475	-0.0018297	7.9277357	1.7294550	1.0911548
0.0000000	0.000000	0.0888270	0.7577563	0.3681743	0.3112944
0.2102370	2.610060	-0.0241759	1.7490392	0.5688029	0.3255822
0.0000000	0.000000	0.0369378	0.3481334	0.2419591	0.1781135

Table 20: log-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.8015527	4.236281	3.0436241	3.1225664	2.9212645	-0.0994888
0.0000000	0.000000	0.4760496	0.4733557	0.4227479	0.1129165
0.6880008	9.662155	-0.2681637	10.3492981	1.2794970	0.6664158
0.0000000	0.000000	0.1165141	0.9915211	0.3792009	0.2268278
0.2162960	2.792241	0.0495233	2.1310087	0.8529974	0.1647751
0.0000000	0.000000	0.0943554	0.4004703	0.3055376	0.1289602

Table 21: sqrt-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.7496796	4.991774	3.0698755	1.5439183	3.6669050	0.7706162
0.0000000	0.000000	0.5303200	0.2727694	0.5097033	0.2278629
0.5875359	16.125727	-3.4030933	7.2550194	3.5858543	3.8049015
0.0000000	0.000000	0.4246016	0.5041032	0.6744699	0.4693367
0.2177059	4.100766	0.1425676	1.0639445	1.0052705	0.4198141
0.0000000	0.000000	0.2658448	0.1919450	0.2954893	0.1160203

Table 22: cat-10-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.7746561	4.240363	3.2331377	6.0088024	3.1935942	1.3455091
0.0000000	0.000000	0.4643888	0.8805231	0.4701595	0.4992688
0.5420920	7.563943	0.2166931	13.1758133	0.8792574	2.4572655
0.0000000	0.000000	0.1080425	1.4595610	0.3151668	0.4405500
0.2080076	2.064543	0.0303424	2.2531848	0.2375642	0.9637317
0.0000000	0.000000	0.0489175	0.4187737	0.1802348	0.3818249

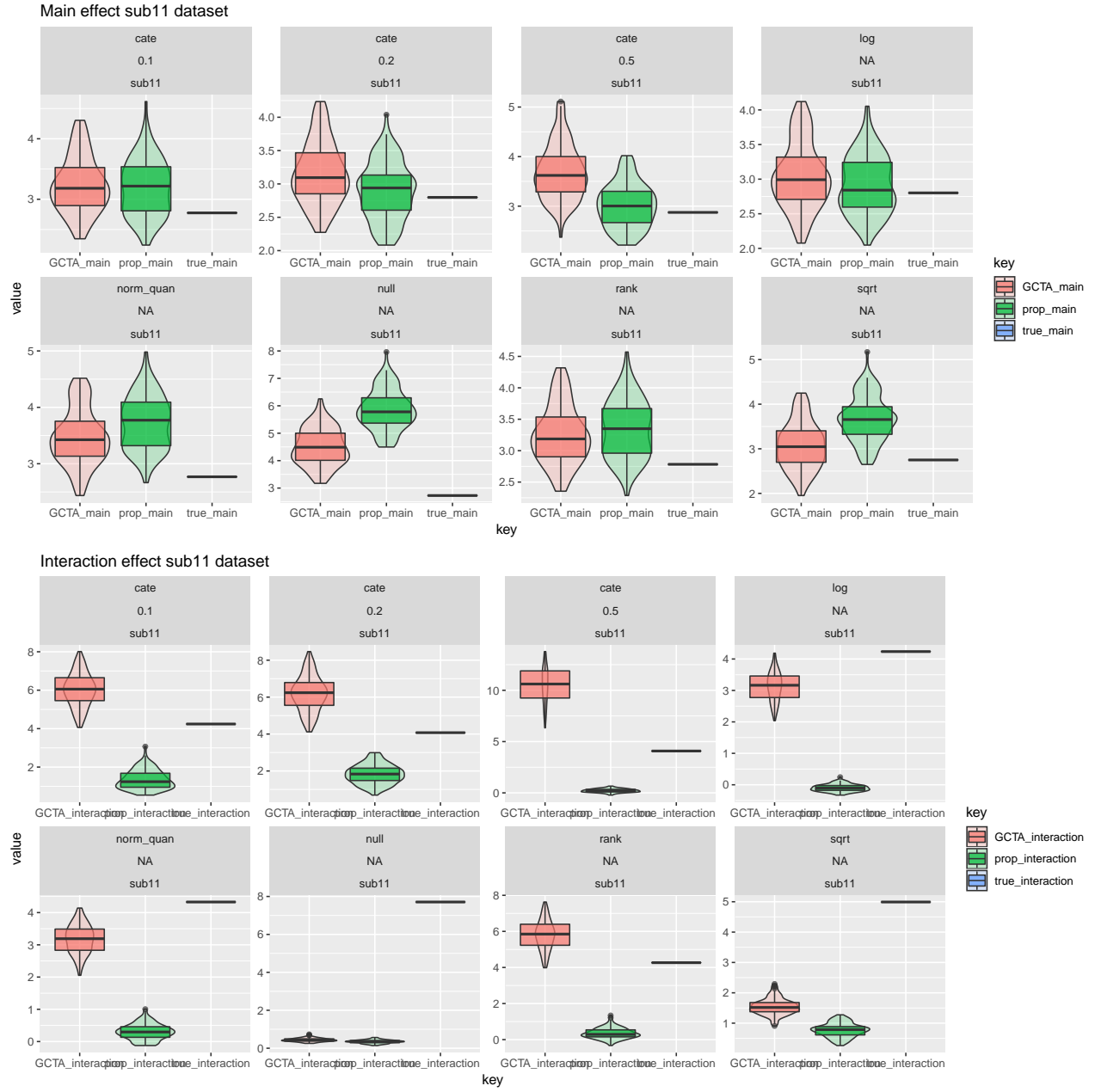
Table 23: cat-5-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.7983825	4.074101	3.1560364	6.1755836	2.8901981	1.8244826
0.0000000	0.000000	0.4657834	0.9608235	0.4291065	0.5208825
0.5726266	7.321955	0.2078162	14.0873765	0.6228696	3.5184265
0.0000000	0.000000	0.1087906	1.6044594	0.2741144	0.5669474
0.2206165	2.011403	0.0396105	2.3074825	0.2352814	1.3355904
0.0000000	0.000000	0.0539100	0.4253049	0.1751183	0.4619231

Table 24: cat-2-full

true_main	true_interaction	GCTA_main	GCTA_interaction	prop_main	prop_interaction
2.8738907	4.081361	3.6856333	10.4923095	3.0103562	0.2178511
0.0000000	0.000000	0.5611821	1.7717503	0.4416661	0.1725719
0.7660228	6.637190	0.2339092	19.1607206	0.7280669	0.7893286
0.0000000	0.000000	0.1074537	2.4910043	0.2514666	0.1600060
0.2473301	1.650984	0.0654467	2.1605270	0.1597307	0.6293876
0.0000000	0.000000	0.0702851	0.6144138	0.1620340	0.2556775

2.4.2 Histogram of 100 iterations



3 Conclusion

4 Further work