Normal_cor_simulation

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Contents

1 Motivation

Some theoretical results suggest that if the covariates follows normal distributions, then even they are not indepedent to each, we still can use the proposed method to estimate the interaction effect unbaisly.

1.1 Model

$$y_i = \alpha + \sum_{j=1}^{p} x_j \beta_j + \sum_{j \neq k} \gamma_{ji} x_j x_k + \epsilon_i.$$

Matrix form

$$y_i = X_i^T \beta + X_i^T \Gamma X_i + \epsilon_i,$$

Where

- $X_i = (x_{i1}, \dots x_{ip})^T$, in our case assumpe $X \sim N(0, \Sigma_p)$

- $\epsilon \perp\!\!\!\perp x_{ji}$

- $\beta = (\beta_1, \dots, \beta_p)^T$ is fixed

- Γ is a $p \times p$ matrix with diagonal elements equal to 0.

1.2 Goal of the study

$$\begin{aligned} Var(Y_i) &= Var(X_i^T \beta + X_i^T \Gamma X_i) + Var(\epsilon_i) \\ &= Var(X_i^T \beta) + Var(X_i^T \Gamma X_i) + Cov(X_i^T \beta, X_i^T \Gamma X_i) + Var(\epsilon_i) \end{aligned}$$

$$\begin{aligned}
Cov(X_i^T\beta, X_i^T\Gamma X_i) &= E[(X_i^T\beta - E(X_i^T\beta))(X_i^T\Gamma X_i - E(X_i^T\Gamma X_i))] \\
&= E[X_i^T\beta(X_i^T\Gamma X_i - E(X_i^T\Gamma X_i))] \\
&= E[X_i^T\beta(X_i^T\Gamma X_i - trace(\Gamma\Sigma_p))] \\
&= E[X_i^T\beta \cdot X_i^T\Gamma X_i] \\
&= E[(\sum_m (x_{im}\beta_m))(\sum_j \sum_k \gamma_{jk}x_{ij}x_{ik})] \\
&= 0 \qquad\qquad\qquad\qquad\qquad\qquad \text{Note that } \gamma_{jj} = 0
\end{aligned}$$

2 Simulation result

2.1 Setup

$$X \sim N(0, \Sigma_p)$$

Where

$$\Sigma_p = \begin{bmatrix} 1 & \cdots & \rho \\ \vdots & \ddots & \vdots \\ \rho & \cdots & 1 \end{bmatrix}$$

$\rho = \{0.1, \ldots, 0.9\}$

2.1.1 Averaged estimation Correlated-Normal

Table 1: correlation-0.1-null

Main			Interaction		
true_main	GCTA_main	prop_main	true_interaction	GCTA_interaction	prop_interaction
3.088373	2.2629134	2.6810971	6.445714	7.7078883	6.3421346
0.000000	0.5095913	0.4890747	0.000000	1.4190317	0.9831575
3.712037	3.2604039	4.0312605	4.752567	5.3313724	5.6001181
0.000000	0.5476864	0.5132892	0.000000	1.0871311	1.0439581
2.681057	2.1712492	2.9792021	5.469751	3.6457060	6.1875434
0.000000	0.3667526	0.4521873	0.000000	0.7242043	0.9909888

Table 2: correlation-0.2-null

Main			Interaction		
true_main	GCTA_main	prop_main	true_interaction	GCTA_interaction	prop_interaction
3.053130	2.2170802	2.6678900	6.214107	6.4985646	5.6397438
0.000000	0.5944800	0.4480933	0.000000	1.4905036	1.0466755
3.553199	1.8826877	3.4683129	4.475504	1.1963946	4.0876341
0.000000	0.4518005	0.5279246	0.000000	0.3304510	0.8316342
2.346156	0.9602659	2.4056761	5.452543	1.5006302	5.3123168
0.000000	0.2237377	0.4383947	0.000000	0.2875944	0.8173162

Table 3: correlation-0.3-null

Main			Interaction		
true_main	GCTA_main	prop_main	true_interaction	GCTA_interaction	prop_interaction
2.923335	3.1116123	3.0790003	7.493621	9.9872651	6.7525273
0.000000	0.7168690	0.4625207	0.000000	1.5570798	1.1331375
3.177118	1.4836731	2.9450186	4.135163	0.5568038	3.6233722
0.000000	0.5075044	0.4244663	0.000000	0.2191097	0.7248550
2.082130	0.6638717	1.8373159	4.419051	0.8571361	3.5220102
0.000000	0.2459053	0.3655716	0.000000	0.3173453	0.7818311

Table 4: correlation-0.4-null

Main			Interaction		
true_main	GCTA_main	prop_main	true_interaction	GCTA_interaction	prop_interaction
2.967121	3.1046591	3.6375580	8.101689	10.4165112	6.5820007
0.000000	0.7063818	0.5403943	0.000000	1.5469132	0.9358535
2.900933	1.2218489	2.9546162	3.773409	0.2746577	3.3916797
0.000000	0.4011364	0.4510457	0.000000	0.0878513	0.7950213
1.843065	0.4277090	1.9797686	3.880287	0.2210265	3.1506855
0.000000	0.1895094	0.3919166	0.000000	0.0959511	0.6900077

Table 5: correlation-0.5-null

Main			Interaction		
true_main	GCTA_main	prop_main	true_interaction	GCTA_interaction	prop_interaction
2.945555	2.1497417	2.7830068	9.512110	8.6159597	7.4123267
0.000000	0.4807449	0.4245746	0.000000	1.1259187	1.0134897
2.704770	1.7836148	2.9072267	3.141388	0.1504048	3.0310716
0.000000	0.4878225	0.5350919	0.000000	0.1054500	0.8478031
1.684237	0.5280842	1.5990481	3.550564	0.1247029	2.8454342
0.000000	0.2146278	0.3474024	0.000000	0.0435080	0.7950938

Table 6: correlation-0.6-null

Main			Interaction		
true_main	GCTA_main	prop_main	true_interaction	GCTA_interaction	prop_interaction
2.840187	3.4255557	3.6075327	13.183739	11.3110130	12.6503986
0.000000	0.6164349	0.5497321	0.000000	0.9272107	1.2512915
2.620650	1.5387039	2.5066288	3.032240	0.1204276	3.0714704
0.000000	0.3709169	0.3816089	0.000000	0.0789471	0.8263944
1.362578	0.5149362	1.2847950	3.172002	0.1199249	2.8407743
0.000000	0.2235774	0.3522378	0.000000	0.0850224	0.6474535

Table 7: correlation-0.7-null

Main			Interaction		
true_main	GCTA_main	prop_main	true_interaction	GCTA_interaction	prop_interaction
2.921671	2.3387362	2.8414534	14.876244	11.7346465	14.3432345
0.000000	0.4934600	0.4930917	0.000000	0.8654897	1.1592587
2.202138	1.1621946	1.9412529	2.563225	0.0337007	2.3039060
0.000000	0.3361885	0.3905409	0.000000	0.0318753	0.8395119
1.205999	0.5551601	1.1075252	3.059272	0.1667304	3.0455072
0.000000	0.2216231	0.3112669	0.000000	0.0908275	0.8467851

Table 8: correlation-0.8-null

Main			Interaction		
true_main	GCTA_main	prop_main	true_interaction	GCTA_interaction	prop_interaction
2.6688253	3.5039081	3.9745800	17.532693	11.7821539	18.3254286
0.0000000	0.5739670	0.6092979	0.000000	0.7556400	1.6198094
1.9782238	1.5076674	2.1516555	1.362698	0.0071326	1.3309489
0.0000000	0.3669671	0.4215761	0.000000	0.0231179	0.7670574
0.8447751	0.3164986	0.7481253	1.934832	0.0885135	1.7759901
0.0000000	0.1632832	0.2611656	0.000000	0.0668263	0.6656269

Table 9: correlation-0.9-null

Main			Interaction		
true_main	GCTA_main	prop_main	true_interaction	GCTA_interaction	prop_interaction
2.6570991	1.7946098	2.1760449	23.6527893	12.1641929	25.8535477
0.0000000	0.3427883	0.4199620	0.0000000	0.6071203	1.6459841
1.6713933	1.2925165	1.4944993	0.8595516	0.0311803	0.8747202
0.0000000	0.2978465	0.3466506	0.0000000	0.0332608	0.6366171
0.6484439	0.4155443	0.6954721	1.1294039	0.1208959	1.0610190
0.0000000	0.1935329	0.2713971	0.0000000	0.0702455	0.6781311

2.1.2 Histgram of 100 iterations Correlated-Normal
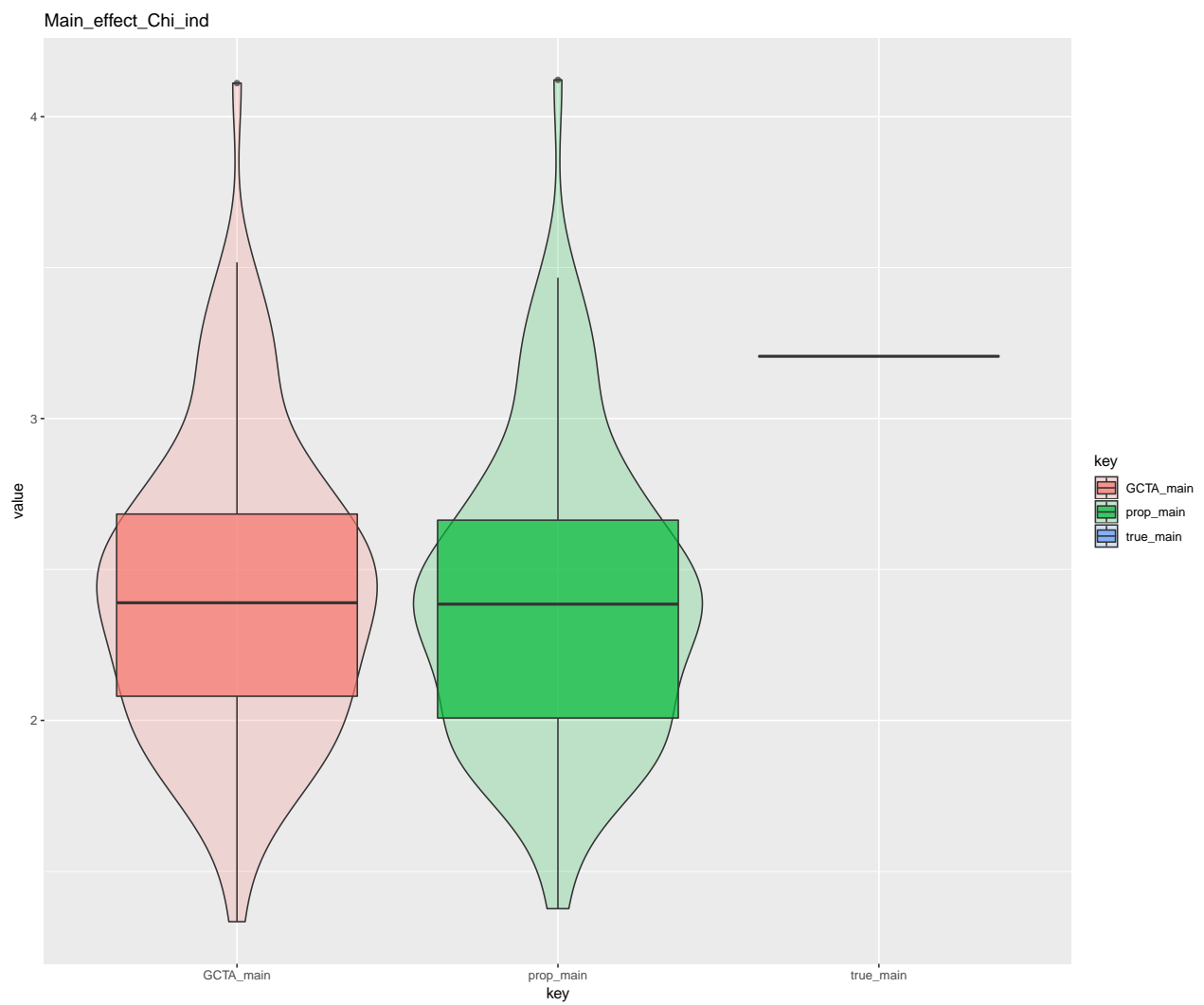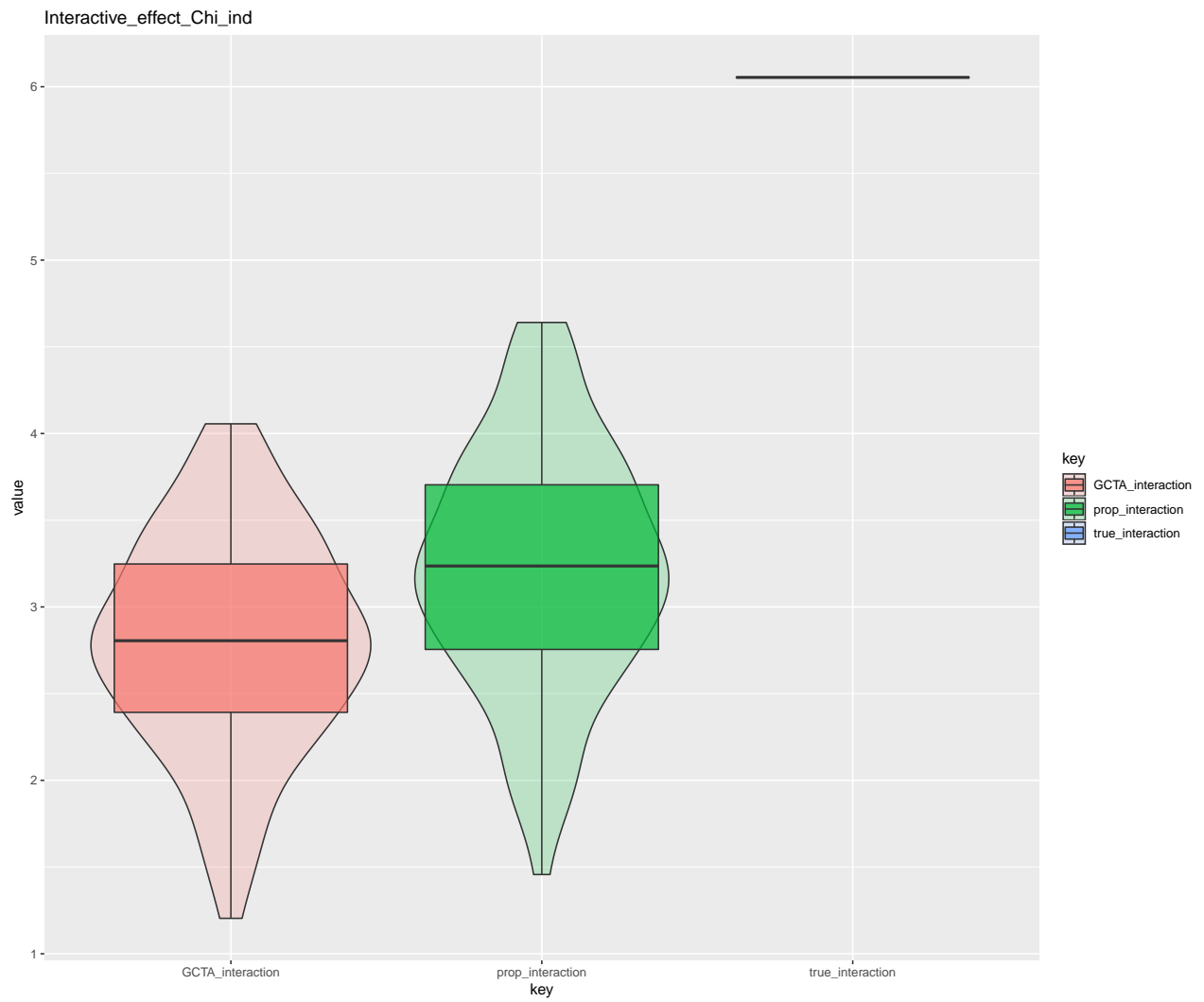
Main_effect_Normal_cor
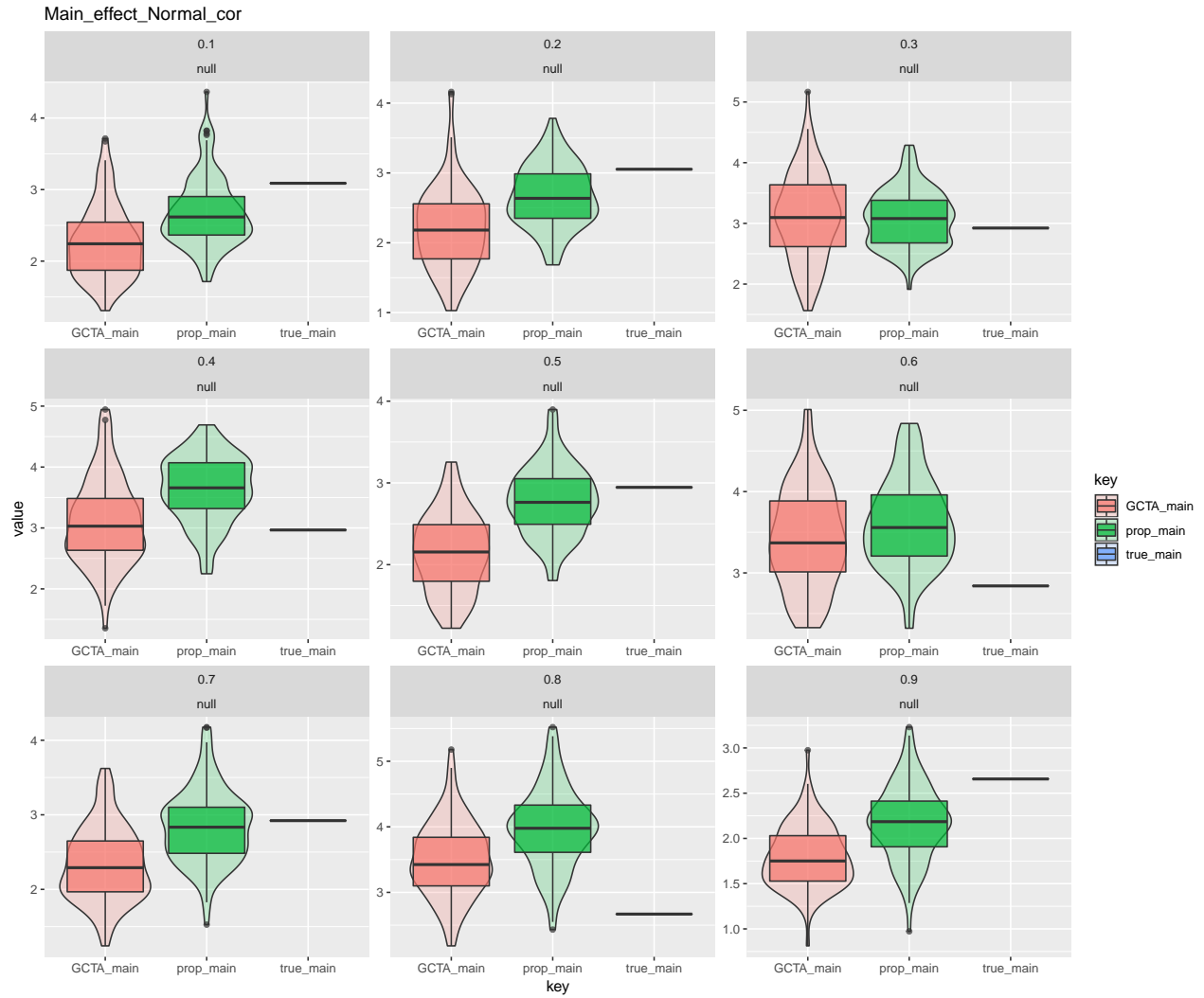
Interactive_effect_Normal_cor
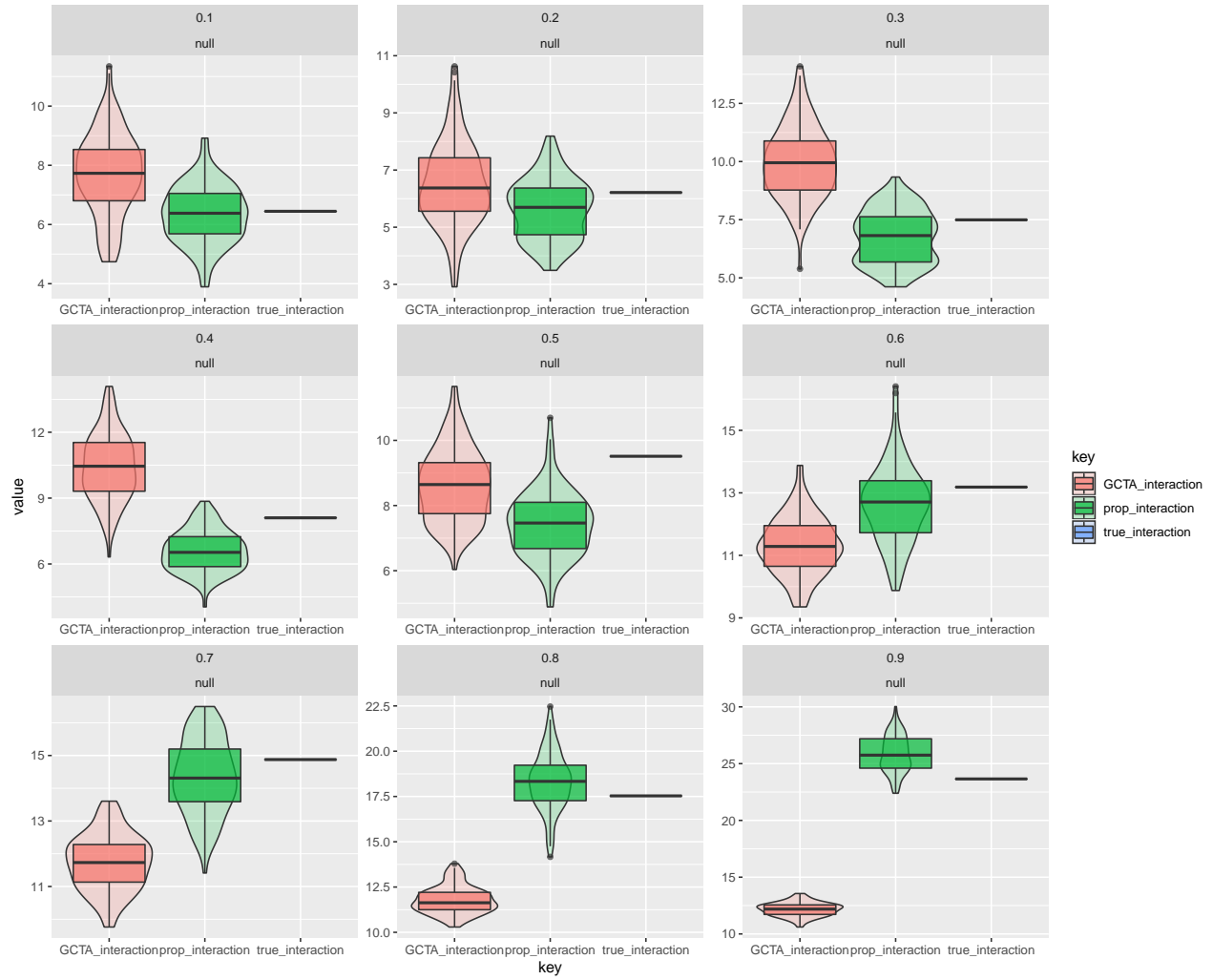
2.1.3 Averaged estimation Indepdent Chi-squared

Table 10: -null

Main			Interaction		
true_main	GCTA_main	prop_main	true_interaction	GCTA_interaction	prop_interaction
3.206260	2.4278846	2.3987387	6.053616	2.7948026	3.2127886
0.000000	0.4877009	0.4836191	0.000000	0.6241777	0.6946283
4.184742	3.7079258	3.6457226	5.662183	5.0952825	5.4150564
0.000000	0.6460932	0.6383936	0.000000	1.0122512	1.0203281
2.751369	2.0729482	2.2093535	5.967688	3.0214403	2.9723835
0.000000	0.4635415	0.4757792	0.000000	0.6891246	0.7034145

2.1.4   Histgram of 100 iterations Indepdent Chi-squared

Main_effect_Chi_ind

Interactive_effect_Chi_ind

3 Conclusion

4 Further work