Simulation of block decorrelation on chi-square data

Xuelong Wang

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Contents

1 Motivation

We used resampling method to evaluate the proposed method performance. However, there is an drawback of this resampling method That is the decorrelation step is highly affected the sample size, especially when n < p. More specifically, when re-sampling size is too small, the estimated inverse covariance matrix is way off the target. Thus, that will affect the final result of the proposed method.

On possible solution is to modify the process of estimation of the covariance matrix. For instance, we could only decorrelate part of the covariate matrix i.e. the main effect part or the interactive effect part. we call it decorrelation by block. this process eventually will reduce the number of parameters we are doing to estimate of the covaraince matrix.

2 Decorrelation by Block

Let X_t be the total covariate,

$$X_t = \begin{bmatrix} X_m \\ X_i \end{bmatrix}$$

Then the covariance matrix of X_t is

$$Var(X_t) = \begin{bmatrix} Var(X_m) & Cov(X_m, X_i) \\ Cov(X_i, X_m) & Var(X_i) \end{bmatrix}.$$

Let A be the decorrelation matrix, we have

$$A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix},$$

where $A_1 = \Sigma_{x_m}^{-1/2}$, and $A_2 = \Sigma_{x_i}^{-1/2}$

To decorrelate a vector, we could left multiple A and X_t,

$$AX_t = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} * \begin{bmatrix} X_m \\ X_i \end{bmatrix} = \begin{bmatrix} A_1 X_m \\ A_2 X_i \end{bmatrix}.$$

The covariance matrix of AX_t is

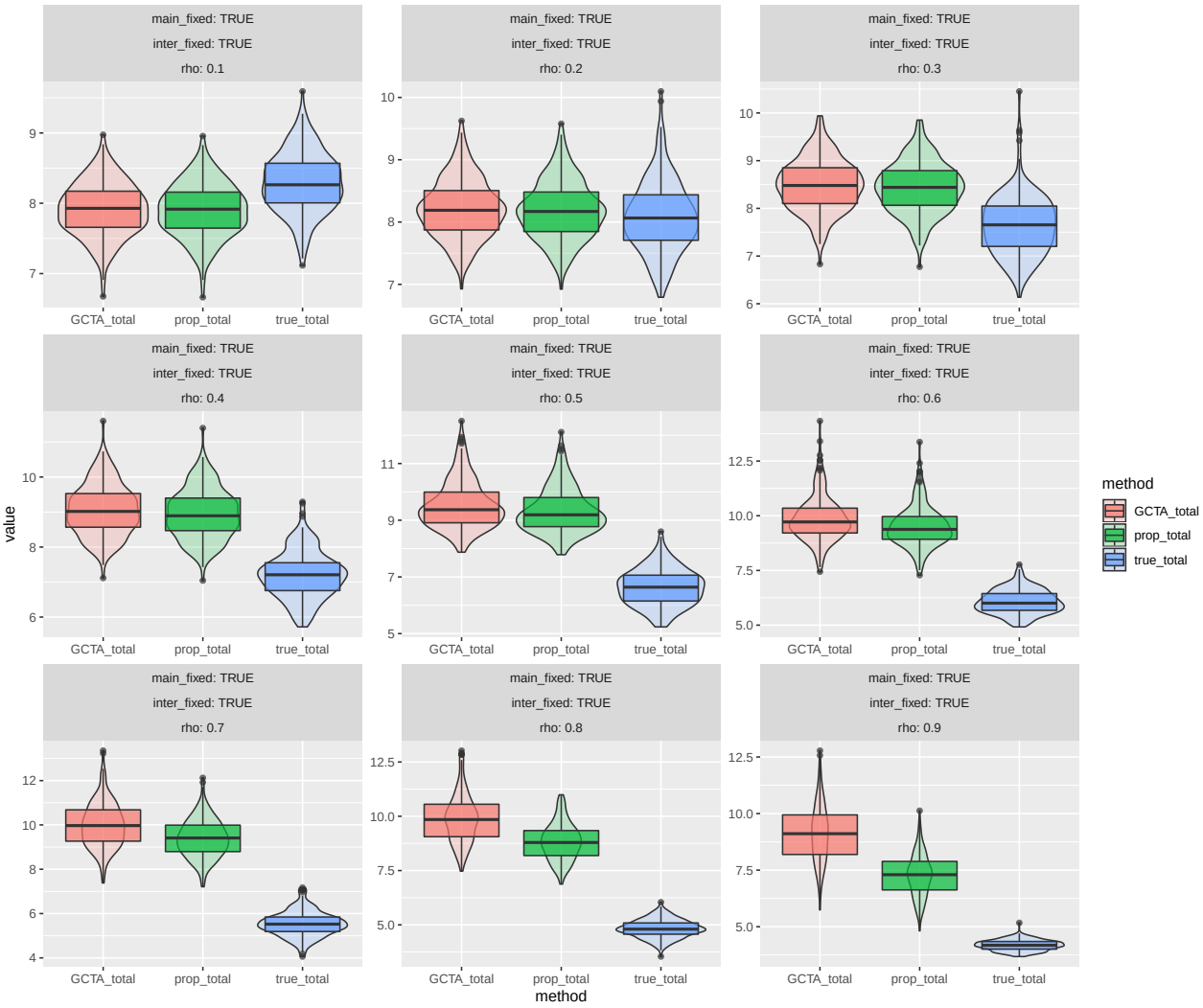
$$Var(X_t) = Var\left(\begin{bmatrix} A_1 X_m \\ A_2 X_i \end{bmatrix}\right) = \begin{bmatrix} A_1 \Sigma_{x_m} A_1^T & A_1 Cov(X_m, X_i) A_2^T \\ A_2 Cov(X_i, X_m) A_1^2 & A_2 \Sigma_{x_i} A_2^T \end{bmatrix} = \begin{bmatrix} I_m & A_1 Cov(X_m, X_i) A_2^T \\ A_2 Cov(X_i, X_m) A_1^2 & I_i \end{bmatrix}.$$

We could choose to only decorrelate the main effect or interactive effect or both. The advantage is we don't care about the off diagonal values, which will reduce the sensitivity of the process on the sample size.

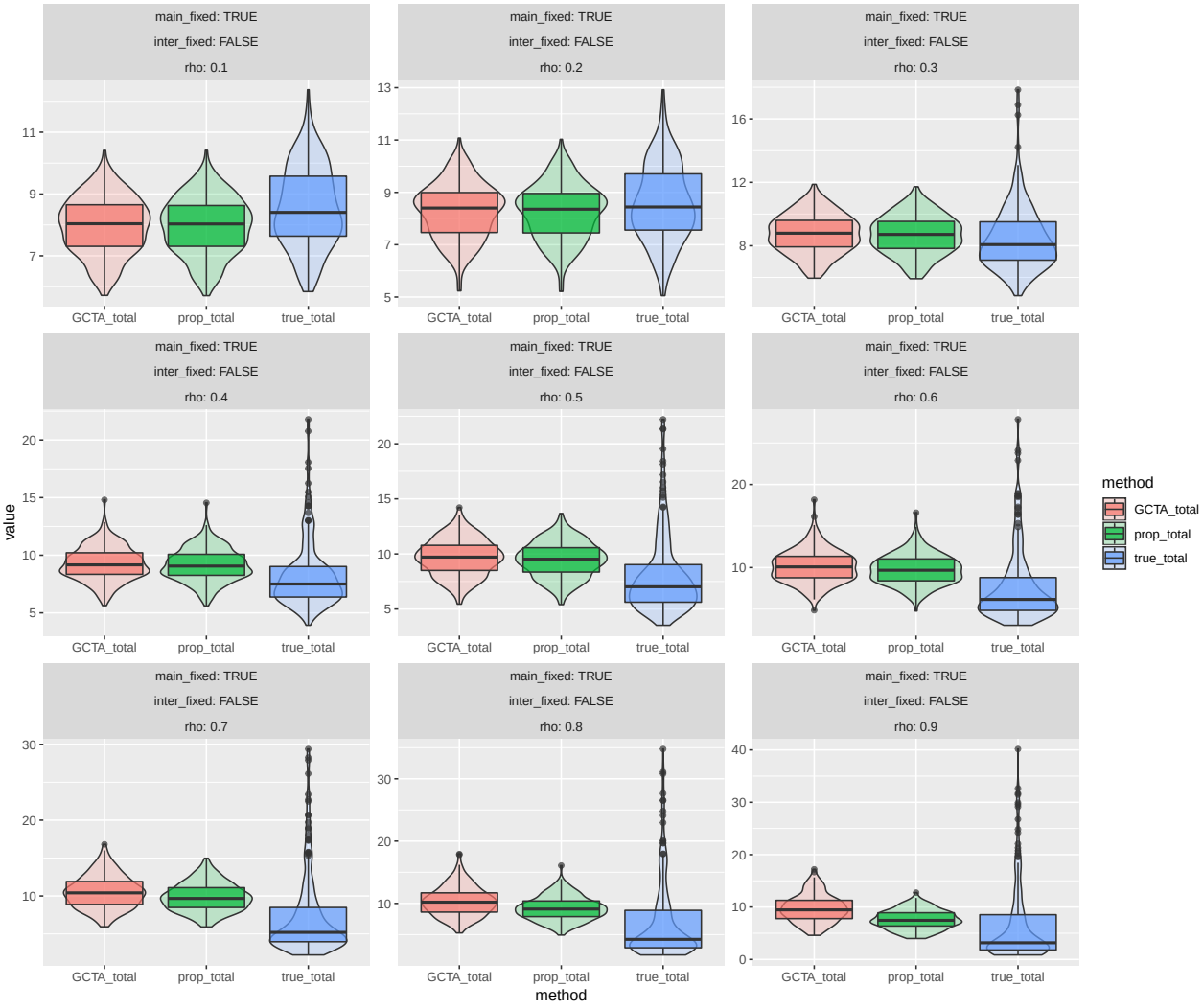
3 Simulation result

3.1 Chi-square with only main effects decorrelated

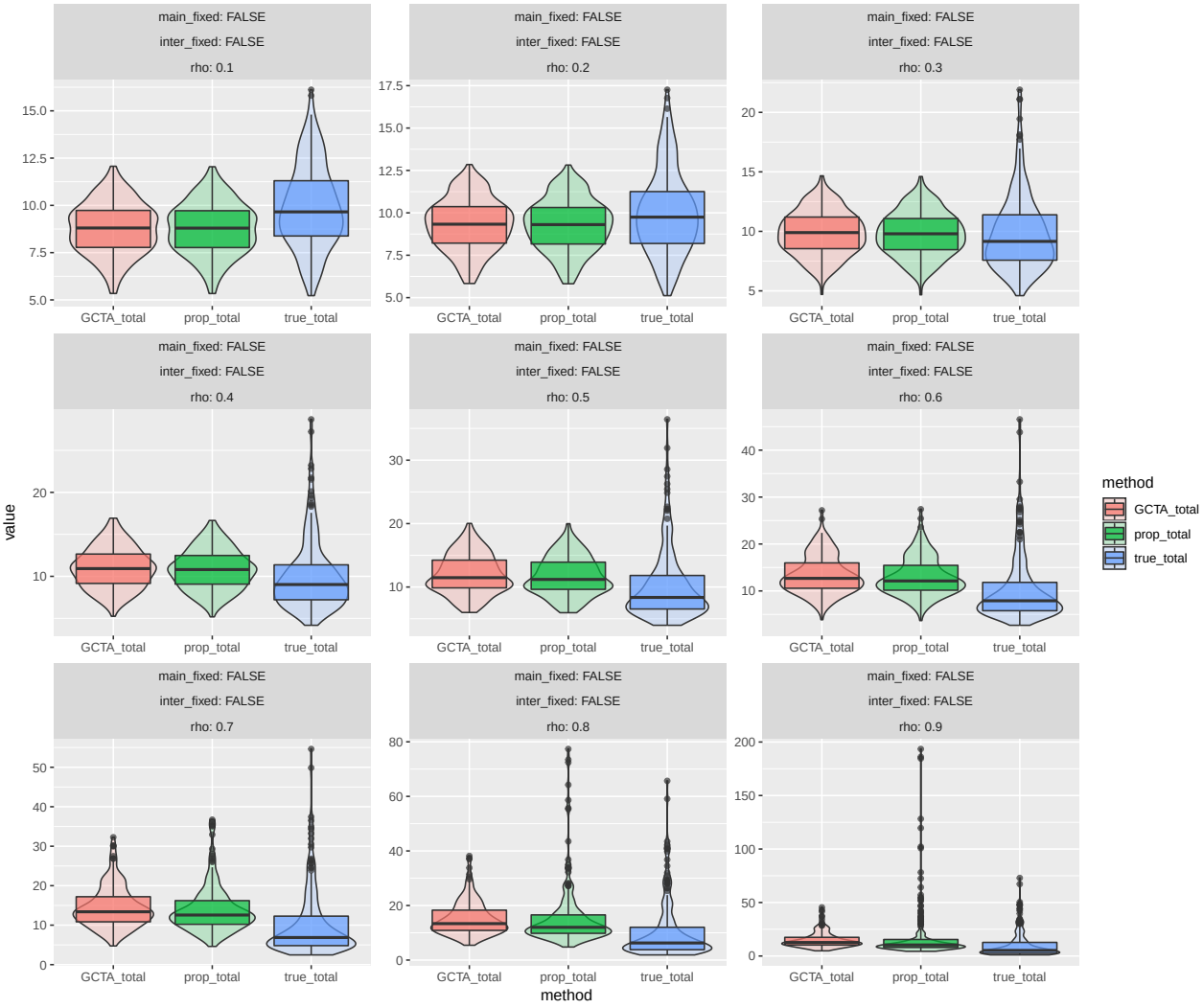
3.1.1 fixed main and fixed interactive effect



3.1.2 fixed main and random interactive effect



3.1.3 random main and random interactive effect



3.2 Chi-square with both main effects and inter effects decorrelated

3.2.1 fixed main and fixed interactive effect