MMAE-540 Introduction to Robotics - Fall 2009 Homework 3 Solutions

1. Consider the three-link planar manipulator shown below. Derive the forward kinematics equations using the DH convention. The DH parameters are given as:

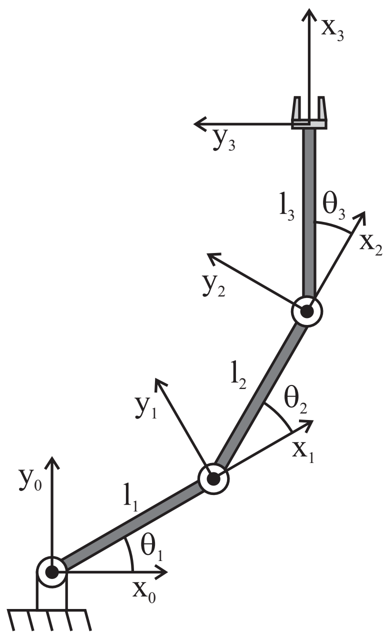

Figure 1: Manipulator for Problem 1

Link	a_i	α_i	d_i	θ_i
1	l_1	0	0	θ_1
2	l_2	0	0	θ_2
3	l_3	0	0	θ_3

The transformation matrices are given as:

$$^0\mathbf{T}^1 = \begin{bmatrix} c_1 & -s_1 & 0 & l_1 s_1 \\ s_1 & c_1 & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$^1\mathbf{T}^2 = \begin{bmatrix} c_2 & -s_2 & 0 & l_2 s_2 \\ s_2 & c_2 & 0 & l_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$^2\mathbf{T}^3 = \begin{bmatrix} c_3 & -s_3 & 0 & l_3 s_3 \\ s_3 & c_3 & 0 & l_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$^0\mathbf{T}^3 = {^0\mathbf{T}^1} * {^1\mathbf{T}^2} * {^2\mathbf{T}^3} = \begin{bmatrix} c_{123} & -s_{123} & 0 & l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ s_{123} & c_{123} & 0 & l_1 s_1 + l_2 s_{12} + l_3 s_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

The forward kinematics are given in the last column of ${}^0\mathbf{T}^3$:

$$x = l_1 c_1 + l_2 c_{12} + l_3 c_{123}$$
$$y = l_1 s_1 + l_2 s_{12} + l_3 s_{123}$$

2. Consider the two-link planar manipulator show below with a revolute joint and a prismatic joint. Derive the forward kinematic equations using the DH convention.

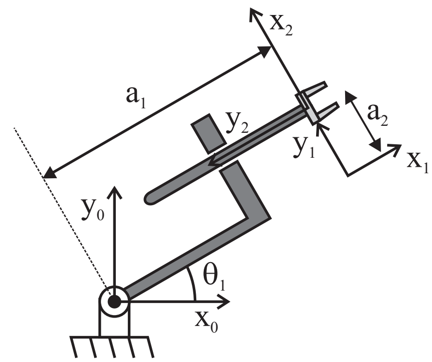

Figure 2: Manipulator for Problem 2

The DH parameters are given as:

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	$\pi/2$

The forward kinematics are the last column of the transformation matrix:

$$ {}^0\mathbf{T}^2 = \begin{bmatrix} -s_1 & -c_1 & 0 & a_1c_1 - a_2s_1 \\ c_1 & -s_1 & 0 & a_2c_1 + a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

such that:

$$x = a_1c_1 - a_2s_1$$
$$y = a_2c_1 + a_1s_1$$

3. Consider the 3D manipulator shown below. Draw a schematic of the manipulator, label the diagram with the appropriate coordinate frames and derive the forward kinematic equations using the DH convention.

There are several configurations you could have chosen as the “base” configuration of the robot. This one was easy to keep the X-axes pointing in the same direction, which makes it easy to visualize the link twists. The axes I choose are shown in Fig 4:

The DH parameters are:

Link	a_i	α_i	d_i	θ_i
1	0	90	d_1	θ_1
2	a_2	90	0	θ_2
3	0	0	d_3	0

The forward kinematics of the manipulator are:

$$ {}^0\mathbf{T}^1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

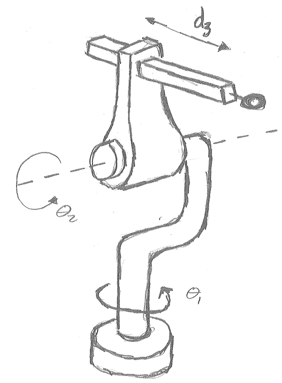

Figure 3: Manipulator for Problem 3.

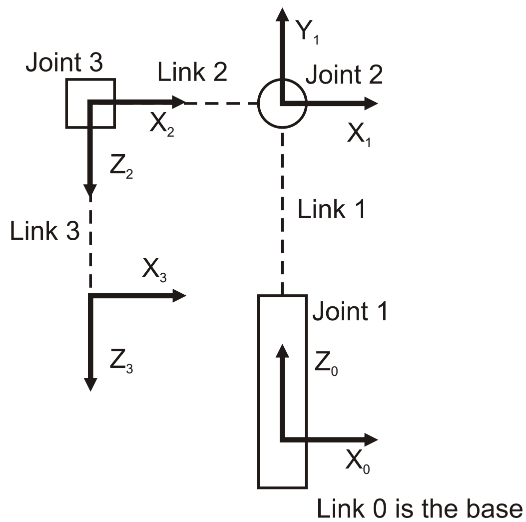

Figure 4: Solution for manipulator for problem 3

$$
{}^{1}\mathbf{T}^{2} = \begin{bmatrix} c_2 & 0 & s_2 & a_2c_2 \\ s_2 & 0 & -c_2 & a_2s_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}\mathbf{T}^{3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}\mathbf{T}^{3} = {}^{0}\mathbf{T}^{1} * {}^{1}\mathbf{T}^{2} * {}^{2}\mathbf{T}^{3} = \begin{bmatrix} c_1c_2 & s_1 & c_1s_2 & a_2c_1c_2 + d_3c_1s_2 \\ s_1c_2 & -c_1 & s_1s_2 & a_2c_2s_1 + d_3s_1s_2 \\ s_2 & 0 & -c_2 & a_2s_2 - d_3c_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

4. Consider the 3D manipulator shown below. Draw a schematic of the manipulator, label the diagram with the appropriate coordinate frames and derive the forward kinematic equations using the DH convention.

   The DH parameters are:

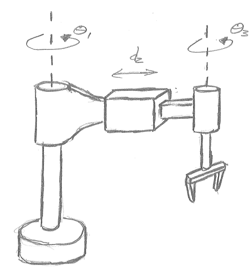

Figure 5: Manipulator for Problem 4.

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	θ_1
2	0	-90	d_2	0
3	0	0	d_3	θ_3

5. Consider the 3D Cartesian manipulator shown below. Solve the ***inverse*** position kinematics.

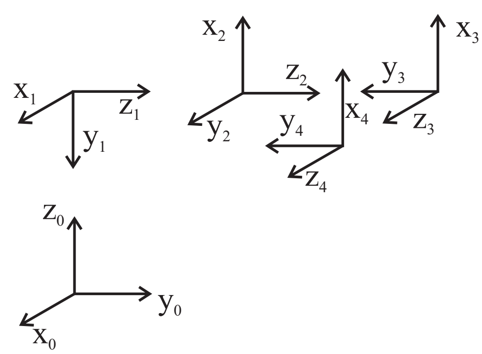

Figure 6: Reference frames for cartesian manipulator for Problem 5.

In this problem its helpful to add in an additional reference frame.

Link	a_i	α_i	d_i	θ_i
1	0	-90	d_1	0
2	0	0	d_2	-90
3	0	-90	0	0
4	0	0	d_3	0

The resulting DH Matrix is given as:

$$ {}^{0}\mathbf{T}^{3} = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & d_2 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6} $$

As you can see, the inverse problem is very simple for this example since

$$ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_3 \\ d_2 \\ d_1 \end{bmatrix} \tag{7} $$