

**Numerical Modelling of Tidally-Driven Fractures
Interacting with Meltwater Lenses on the Surface of
Europa**

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Key Points:

- Three-dimensional simulations of the Europan surface systematically quantify fracturing with varying location, orientation, and latitude
- The growth of fractures is quantified for homogeneous domains and for domains that contain subsurface meltwater lenses
- Subsurface meltwater lenses create mechanical conditions leading to fracture and fragmentation, conducive to chaos terrain formation

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13 **Abstract**

14 The ice crust of Jupiter's Galilean satellite Europa exhibits a number of large-scale lineae features, as well as numerous regions with smaller-scale lineae patterns and chaos features. In this work, a three-dimensional finite element simulator is used to model lineae as fractures in the crust that nucleate, grow, and interact. The growth of fractures is modelled geometrically as a function of multi-modal stress intensity factors computed at the fracture tips, and the tidal forces that drive this fracturing process are computed according to a closed-form satellite tidal stress model. Fracture growth is modelled across the span of the satellite, with emphasis on analysing fracture statistics in the tropical region around 30° North at various longitudes. Specific focus is given to fracture behaviour in the presence of subsurface meltwater lenses. The effect that these lenses have upon fracture development is systematically investigated in order to determine whether a subsurface lens may catalyse the development of chaos terrain on the surface. The results show a dependence on longitude, with the most pronounced effects seen around the sub-jovian point (0°), 90°E, 180°E and 270°E, which correspond to the regions of greatest change in the stress field. In most regions, the observed behaviour of fractures in proximity to a lens correlates with the hypothesis that lenses lead to chaos formation.

30 **Plain Language Summary**

31 Europa is the fourth largest moon of Jupiter, and it is encased within an ice crust approximately 25 km thick. Between the rock and ice is believed to be a liquid water ocean approximately 100 km deep. This ocean is one of the prime candidates for possible extraterrestrial life within the solar system, and therefore the ocean and Europa as a whole are of great interest to the scientific community. The outer surface of the ice also shows a number of linear features called lineae, believed to be either fractures or former fractures, as well as rugged and broken up areas called chaos. These fractures and their behaviour, especially in the context of chaos, are the focus of research for this work. The work consists of calculating tidal stresses imparted on Europa by Jupiter using a satellite stress model, and then using these stresses to drive fracture behaviour within the ice crust, modelled using a numerical simulator. Some simulations also contain a subsurface meltwater lens. The results from the simulations are analysed in the context of how fracturing and lineae in the vicinity of a subsurface meltwater lens behave. The results indicate that subsurfaces lenses may contribute to the formation of chaos terrain.

45 1 Introduction

46 The four largest moons of Jupiter were first observed by Galileo in 1610, and there-
 47 after have been known as the Galilean moons. The smallest of the Galilean moons, Eu-
 48 ropa, became the subject of scientific curiosity when modern imaging revealed that it
 49 was entirely encased within an ice crust estimated to be between 1 and 30 km thick (Bills,
 50 2005). The lower bound on the thickness is determined by mechanical flexure analyses
 51 (Figueroedo et al., 2002; Nimmo et al., 2007), and the upper bound from thermodynamic
 52 analyses (Hussmann et al., 2002; McKinnon, 1999; Ojakangas & Stevenson, 1989b), with
 53 impact cratering analyses yielding estimates between these bounds (Greeley et al., 1998;
 54 Moore et al., 1998; Schenk, 2002).

55 Interest in Europa grew when considerations of tidal heating implied the existence
 56 of a vast subsurface ocean between the silicate core and icy crust (Cassen et al., 1979),
 57 with this conclusion supported by the results from the Galileo mission (Carr et al., 1998).
 58 This ocean, thought to be approximately 100 km thick (Pappalardo et al., 1999), may
 59 be the prime candidate for extra-terrestrial life within the solar system (Schulze-Makuch
 60 & Irwin, 2001; Kargel et al., 2000; Greenberg et al., 2000) and therefore understanding
 61 the ocean-crust system of Europa is of importance.

62 Current observational knowledge of Europa is limited due to its distance from Earth.
 63 The planned Europa Clipper (Bayer et al., 2019) and JUICE (Witasse, 2020) missions
 64 are expected to significantly improve the quality and quantity of Europan observational
 65 data, but in their absence the dataset is limited to the images and data yielded by NASA
 66 missions Pioneer 10 and 11, Voyager 1 and 2, Galileo, New Horizons, and Juno. The re-
 67 turned images of the visible ice crust reveal many types of geomorphological feature, of
 68 which two broad categories are investigated in this work; regions of broken and jagged
 69 terrain called *chaos* (Collins & Nimmo, 2009), and *lineae*, line-like features that exist across
 70 many length-scales and are thought to be either active or healed fractures in the ice crust
 71 (Helfenstein & Parmentier, 1983).

72 The presence of fractures in the form of *lineae* is thought to be possible due to the
 73 presence of the subsurface ocean (Pappalardo et al., 1999; Carr et al., 1998), where the
 74 main driving stress on the ice crust causing the fracturing is the tidal influence from the
 75 interaction of Europa with its parent body, Jupiter (Greenberg et al., 1998; Hoppa et
 76 al., 1999). The study performed by Mondro et al. (2023) categorised a subset of the Eu-
 77 ropan *lineae* showing that their lengths can be described by a power law, although the
 78 present imaging quality is insufficient to resolve the smaller fractures in most regions,
 79 as only 32% of the surface is imaged with a quality of 1 km pixel⁻¹ or better (Bland et
 80 al., 2021; Greeley et al., 2000), and therefore the power law is extracted from the larger
 81 observed *lineae*. A computational study of these fractures would aid with understand-
 82 ing the presently unseen behaviour of the ice crust.

83 Previously, Linear Elastic Fracture Mechanics (LEFM) methods have been used
 84 to model fracturing in the Europan crust, for example Marshall and Kattenhorn (2005),
 85 Lee et al. (2005), Craft et al. (2016) and Poinelli et al. (2019). The work undertaken by
 86 Marshall and Kattenhorn (2005) and Poinelli et al. (2019) concern cycloidal *lineae*; Marshall
 87 and Kattenhorn (2005) showed that these features are driven by diurnal tidal stressors
 88 while emphasising the role shear stresses play in cycloid formation while Poinelli et al.
 89 (2019) concludes that dormant growth periods may account for cycloid development. Craft
 90 et al. (2016) considered the formation of subsurface water sills as a result of surface and
 91 basal fracture propagation, taking into account shell cooling and thickening as shell stress-
 92 sors in two dimensions, showing that horizontal stresses appear to be insufficient to turn
 93 fractures to form sills.

94 Lee et al. (2005) investigated the vertical propagation of surface fractures, conclud-
 95 ing that, assuming the presence of a subsurface ocean, surface tidally driven fractures

may penetrate the entirety of the Europan crust. This conclusion was however disputed by Qin et al. (2007), where it was posited that normal stresses acting upon the fractures were not considered and in this revised formulation the fractures may no longer penetrate to the ocean (this in turn was replied to by Lee et al. (2007)). Further work on vertical propagation was conducted by Rudolph and Manga (2009) and Walker et al. (2021) using a boundary element method and theoretical quasi-static LEFM respectively. Both predict that it is unlikely that fractures would penetrate the whole crust due to lithostatic overburden, Rudolph and Manga (2009) stating that penetration is only likely if the crust is 2.5 km thick or thinner, which is less than most current estimates.

A theory pertaining to the formation of chaos morphologies on the surface is that they are the consequence of subsurface meltwater sills or lenses (Collins et al., 2000; O'Brien et al., 2002; Schmidt et al., 2011). Pockets of fluids are expected to form directly beneath the ice surface, these meltwater pockets are the result of localised melting due to tidal flexing. Numerical models have suggested that these isolated meltwater pockets are likely to develop in depths of up to 5km, in particular for a hypothesis of ice shelf thickness between 15–35 km (Vilella et al., 2020). This is further supported by a hypothesis of the presence of brine on Europa's surface (Sieme & Rezaei-Ghaleh, 2024), which would lower the freezing point of water, facilitating the formation of lenses. The proposed mechanism for chaos formation relies on fracturing from the shallow lens to the surface to allow for water to float the observed ice “rafts” in the terrain (Becker et al., 2023). Such lenses may be observable by the upcoming Europa Clipper and JUICE missions (Heggy et al., 2017; Di Paolo et al., 2022), though in the interim the distribution of these saltwater lenses is not well known. Alternative formation mechanisms have been proposed, for example by Howell et al. (2021) where dry porous compaction is used to explain chaos formation, and therefore eliminating the need for subsurface meltwater. However, in all proposed mechanisms, there must be more contribution than solely tidal forcing due to the irregular distribution of chaos across the surface, as opposed to the regular predictable concentrations if tidal stresses alone were sufficient.

The present research seeks to build on these works by increasing the computational detail applied to the fractures and the domain, using an existing geologic fracture simulator. The primary objective is to systematically study the behaviour of fractures on the surface and more specifically to investigate the interaction of fractures with subsurface meltwater lenses, and whether this interaction may contribute to the formation and evolution of chaos terrain. This study is executed through modelling of the growth of Europan surface lineae as a result of tidal forcing from Jupiter, assuming that the lineae can be described by a first principles discrete fracture mechanics growth model that combines damage and energy-based modelling of fracture growth.

2 Methodology

In this work the term “fractures” refers in the most generic sense to discontinuities in the solid mass of the Europan crust, which may be open or closed, and may progressively accrue surface area in response to mechanical effects.

2.1 Tidal Stress Model

It is believed that the principal driving force for the development of lineae on the Europan surface are the tidal stresses exerted by Europa's interaction with Jupiter (Helfenstein & Parmentier, 1983), and are therefore implemented as the driving factor behind fracture growth in this work, where the primary tidal stressors are diurnal stresses and Non-Synchronous Rotation (NSR) stresses. While further sources of stress have been theorised, such as orbital obliquity by Bills (2005) or librational tides by Van Hoolst et al. (2008), these are taken to be second order effects (Wahr et al., 2009) and not considered.

145 Diurnal stresses are resultant from the eccentricity of Europa's orbit about Jupiter,
 146 yielding a stress field on the order of 100 kPa that fluctuates over the course of a Eu-
 147 ropan day (3.5 Earth days), where the rate of rotation is referred to as n . NSR arises
 148 due to the mechanical decoupling of the ice crust from the silicate core as a result of the
 149 ocean, allowing the ice crust to rotate at a different rate than that of the tidally-locked
 150 core (Geissler et al., 1998). The additional rotation due to NSR is b , such that the over-
 151 all rotation of the moon is $n + b$. Due to the lack of tidal lock, the tidal bulge moves
 152 with respect to the ice crust therefore exerting stress upon the ice. The magnitude of
 153 the NSR stress depends on both the viscosity of ice and the NSR period, taking values
 154 from 0 MPa to 3 MPa. Further detail on the NSR is given in Section 3.1.

155 2.1.1 Tidal Stress Model Implementation

156 The tidal stresses are calculated using a model based upon the theory developed
 157 by Wahr et al. (2009). This model starts from the fundamental gravitational potential
 158 to derive analytical expressions for the tidal stress exerted on the surface. The expres-
 159 sions are parametrised by the location and time, with time measured from Europen pe-
 160 riapsis (the closest approach of Europa to Jupiter). The gravitational potential, V_T , is
 161 given by

$$V_T(r, \theta, \phi, t) = Z \left(\frac{r}{R_s} \right)^2 [T_* + T_0 + T_1 + T_2] \quad (1)$$

162 with

$$T_* = \frac{1}{6}(1 - 3\cos^2\theta), \quad (2)$$

$$T_0 = \frac{1}{2}\sin^2\theta\cos(2\phi + 2bt), \quad (3)$$

$$T_1 = \frac{\epsilon}{2}(1 - 3\cos^2\theta)\cos(nt), \quad (4)$$

$$T_2 = \frac{\epsilon}{2}\sin^2\theta[3\cos(2\phi)\cos(nt) + 4\sin(2\phi)\sin(nt)], \quad (5)$$

166 where (r, θ, ϕ, t) are the time-dependent spherical polar coordinates (θ given as co-latitude),
 167 which will rotate compared to stationary reference frame but are fixed with regard to
 168 Europa itself, ϵ is the Europen orbital eccentricity, and Z is a constant given by

$$Z = \frac{3Gm^*R_s^2}{2a^{*3}} \quad (6)$$

169 where G is the gravitational constant, m^* is the mass of Jupiter, R_s is the radius of Eu-
 170 ropa and a^* is the semi-major axis of the Europen orbit.

171 In order to derive surface stresses from the gravitational potential, the model pre-
 172 sumes a four-layered Maxwell solid structure for Europa, as described by Jaeger et al.
 173 (2009). This structure includes a silicate core, an oceanic layer of liquid water, and an
 174 ice crust. The ice crust is further divided into an inner, warm, viscoelastic, and ductile
 175 layer surrounded by an outer, cold, brittle layer. The model calculates the stress ten-
 176 sor, τ , although due to context there are no stresses in the r direction, leaving only $\tau_{\theta\theta}$,
 177 $\tau_{\phi\phi}$, and $\tau_{\theta\phi} = \tau_{\phi\theta}$. This is shown as follows, where a tilde denotes a complex quan-
 178 tity:

$$\begin{aligned} \tau_{\theta\theta} = \frac{Z}{2gR_s} Re & \left[\left(\tilde{\beta}_1(2b) - \tilde{\gamma}_1(2b)\cos(2\theta) \right) e^{i(2\phi+2bt)} \right. \\ & + 3\epsilon \left(\tilde{\beta}_1(n) - \tilde{\gamma}_1(n)\cos(2\theta) \right) e^{int}\cos(2\phi) \\ & - \epsilon \left(\tilde{\beta}_1(n) + 3\tilde{\gamma}_1(n)\cos(2\theta) \right) e^{int} \\ & \left. - 4\epsilon \left(\tilde{\beta}_1(n) - \tilde{\gamma}_1(n)\cos(2\theta) \right) ie^{int}\sin(2\phi) \right], \end{aligned} \quad (7)$$

$$\begin{aligned} \tau_{\phi\phi} = \frac{Z}{2gR_s} Re & \left[\left(\tilde{\beta}_2(2b) - \tilde{\gamma}_2(2b)\cos(2\theta) \right) e^{i(2\phi+2bt)} \right. \\ & + 3\epsilon \left(\tilde{\beta}_2(n) - \tilde{\gamma}_2(n)\cos(2\theta) \right) e^{int}\cos(2\phi) \\ & - \epsilon \left(\tilde{\beta}_2(n) + 3\tilde{\gamma}_2(n)\cos(2\theta) \right) e^{int} \\ & \left. - 4\epsilon \left(\tilde{\beta}_2(n) - \tilde{\gamma}_2(n)\cos(2\theta) \right) ie^{int}\sin(2\phi) \right], \end{aligned} \quad (8)$$

$$\begin{aligned} \tau_{\phi\theta} = \tau_{\theta\phi} = \frac{2Z}{gR_s} Re & \left[\tilde{\Gamma}(2b)ie^{i(2\phi+2bt)}\cos\theta \right. \\ & - 4\epsilon\tilde{\Gamma}(n)ie^{int}\cos\theta\cos(2\phi) \\ & \left. - 3\epsilon\tilde{\Gamma}(n)e^{int}\cos\theta\sin(2\phi) \right] \end{aligned} \quad (9)$$

¹⁷⁹ where

$$\tilde{\beta}_1 = \tilde{\mu} \left[\tilde{\alpha}(\tilde{h} + 3\tilde{l}) \right] \quad (10)$$

$$\tilde{\gamma}_1 = \tilde{\mu} \left[\tilde{\alpha}(\tilde{h} - \tilde{l}) \right] \quad (11)$$

$$\tilde{\beta}_2 = \tilde{\mu} \left[\tilde{\alpha}(\tilde{h} - 3\tilde{l}) \right] \quad (12)$$

$$\tilde{\gamma}_2 = \tilde{\mu} \left[\tilde{\alpha}(\tilde{h} + \tilde{l}) \right] \quad (13)$$

$$\tilde{\alpha} = \frac{3\tilde{\lambda} + 2\tilde{\mu}}{\tilde{\lambda} + 2\tilde{\mu}} \quad (14)$$

$$\tilde{\Gamma} = \tilde{\mu}\tilde{l} \quad (15)$$

¹⁸⁰ where \tilde{h} and \tilde{l} are the complex Love numbers (discussed further in Section 3.1), and $\tilde{\lambda}$
¹⁸¹ and $\tilde{\mu}$ are the complex Lamé parameters given by

$$\tilde{\mu}(\omega) = \mu \left(\frac{1}{1 - i\Delta} \right) \quad (16)$$

$$\tilde{\lambda}(\omega) = \lambda \left(\frac{1 - i\Delta \left(\frac{2\mu+3\lambda}{3\lambda} \right)}{1 - i\Delta} \right) \quad (17)$$

where ω is the rotation rate of the tidal effect, $\Delta = \frac{\mu}{\eta\omega}$, and η is the viscosity. See Walding et al. (2024) for validation against Wahr et al. (2009). The resulting stress field is shown in Fig. 1.

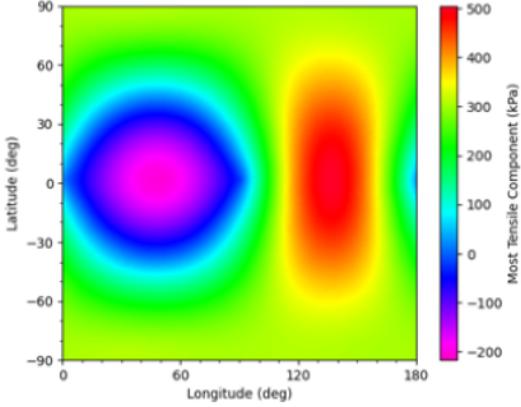


Figure 1: The calculated stress field across the Europa surface. The heatmap is only given up to 180° East as the stress field is symmetric across the Eastern and Western hemispheres.

These equations only pertain to surface stresses however, while the present implementation requires stresses at depth. These subsurface stresses are determined by a simple heuristic relation;

$$\sigma(r, \theta, \phi) = \left(\frac{r}{R_E} \right)^2 \cdot \sigma(\theta, \phi) \quad (18)$$

where R_E is the radius of Europa.

2.2 Deformation and Fracture Growth

The modelling procedure consists of numerically solving mechanical deformation partial differential equations on a discretised domain representing the Europa ice crust, where the domain is assumed to be homogeneous (aside from inserted features), isotropic and linear elastic. The three-dimensional geometry is meshed automatically and the deformation and damage to the domain is computed across the surface, before inserting mechanically active fractures. Each fracture then becomes part of the domain and influences the stress field in its vicinity. Meshes are composed of isoparametric quadratic triangles and tetrahedra. Around fracture tips, isoparametric quadratic quarter-point triangles and tetrahedra are inserted to reduce numerical errors in the displacement field. At each step, stress intensity factors are computed ahead of each fracture tip location to control changes to the fracture geometry. As growth progresses, the geometry evolves and the domain is progressively remeshed to capture changes due to the fractures' evolution. This operation is described in Algorithm 1.

Algorithm 1 Deformation of the domain and fracture growth

```

while Current growth step < Maximum growth steps & State of model has changed do
    Define initial state
    Discretise domain (create mesh)
    Map variables (displacement and damage)
    Apply boundary conditions and stresses
    Compute deformation
    for each fracture do
        for each fracture tip do
            Compute propagation criterion (Equation 20)
            if Propagation criterion fulfilled then
                Compute extension criterion (Equations 21 and 22)
                Compute angle criterion (Equation 23)
                Compute new tip location
            end if
            Resolve intersections
            Update fracture geometry
        end for
    end for
end while

```

203 ***2.2.1 Mechanical Deformation***

204 A three-dimensional volumetric finite element method is used to solve for the me-
 205 chanical deformation of the system, computed as elastic deformation (Cook et al., 2007),
 206 in response to the tidal stresses, and thus determining the distribution of the displace-
 207 ment, stress and strain fields across the ice crust. This assumes that the outer layer of
 208 ice is in a brittle and linear elastic state, which given the very low temperatures of the
 209 European surface (110 K in the warmest equatorial region, down to 50 K in the polar vicin-
 210 ity (Ashkenazy, 2019)) is a reasonable assumption.

211 ***2.2.2 Fracture Growth***

212 Fracture growth assumes a quasi-static regime and is modelled using Stress Inten-
 213 sity Factors (SIFs). The opening deformation (K_I), in-plane deformation (K_{II}) and out-
 214 of-plane deformation (K_{III}) SIFs are calculated for every fracture's tips using the dis-
 215 placement correlation method described by Nejati et al. (2015). The SIFs then deter-
 216 mine whether a given fracture tip will extend, and the magnitude of extension, by means
 217 of growth criteria (Paluszny & Zimmerman, 2013).

218 The mixed-mode propagation criterion developed by Richard et al. (2014) is used
 219 to determine whether or not a fracture tip will extend. The evaluated SIFs at a fracture
 220 tip are used to compute a derived quantity K_V , as follows;

$$K_V = \frac{1}{2}K_I + \frac{1}{2}\sqrt{{K_I}^2 + 4(\alpha_1 K_{II})^2 + 4(\alpha_2 K_{III})^2} \quad (19)$$

221 where α_1 is the ratio of mode I (opening) fracture toughness to mode II (in-plane shear),
 222 and α_2 the same ratio for mode III (out-of-plane shear); both empirical parameters of
 223 value 1.3268^{-1} and 0.6297^{-1} respectively. K_V is then compared to the critical stress in-
 224 tensity factor, K_{IC} ,

$$K_V \geq K_{IC} \quad (20)$$

For fracture tips where this criterion is fulfilled, the extension magnitude and orientation are then determined by further criteria, drawn from Richard et al. (2005) and Thomas et al. (2020).

The extension criterion given by Thomas et al. (2020) is a two-step Paris-type (Paris & Erdogan, 1963) law governed by the strain energy release rate at each fracture tip, \mathcal{G} , calculated with an I -integral detailed in Nejati et al. (2015). The criterion is controlled by three parameters: Δa_v , β_f and β_n , where the β parameters, or Paris Exponents, are empirical quantities given in Table 1 and Δa_v is the original radius of a given fracture. The first step of the equation is as follows:

$$\Delta a_f = \Delta a_v \left(\frac{\mathcal{G}_f}{\mathcal{G}_v} \right)^{\beta_f} \quad (21)$$

where Δa_f indicates the maximum allowable extension for any tip within a fracture. \mathcal{G}_f represents the peak value of the energy release rate \mathcal{G} observed at any fracture tip, while \mathcal{G}_v denotes the maximum energy release rate throughout the entire volume of the material. The extension for a specific fracture tip, denoted as Δa_n , is calculated based on these values, reflecting the localised response of the material at that particular tip, given as

$$\Delta a_n = \Delta a_f \left(\frac{\mathcal{G}_n}{\mathcal{G}_f} \right)^{\beta_n} \quad (22)$$

where \mathcal{G}_n is \mathcal{G} at the tip. Each tip that passes the propagation criterion will then extend by Δa_n .

The criterion for propagation angle is adapted from Richard et al. (2005), which therein calculates two angles - perpendicular and tangential to the fracture front. In the ICGT the tangent angle is excluded from consideration in service of model simplicity. The perpendicular, or deflection, angle φ is calculated as follows;

$$\varphi = \mp \left[A \frac{|K_{II}|}{K_I + |K_{II}| + |K_{III}|} + B \left(\frac{|K_{II}|}{K_I + |K_{II}| + |K_{III}|} \right)^2 \right] \quad (23)$$

where $A = 140^\circ$ and $B = -70^\circ$. Further details on this methodology can be found in Paluszny and Zimmerman (2013) and Thomas et al. (2020).

2.3 Quantification

The quantification strategy for this work is a single probe approach. Given a homogeneous domain, a probe fracture consisting of a planar disc with twenty fracture tips is inserted into the domain at various locations. Each fracture has a radius of 1 km and is inserted at a depth of 1.5 km. The simulation then progresses for two growth steps. This is to allow the stress fields and geometry to settle around the probe fracture (this will not be a steady state, however, as fractures at this stage in almost all configurations will continue to grow). The following parameters are then extracted: K_I , K_{II} , K_{III} , and the growth vector for the next step. The derived quantity K_V is then calculated with Eqn. 19.

The growth vectors of all of the tips in a fracture are examined collectively to determine the tip that is to most extend in the next step, and the tip to extend the least. The fracture tips' locations are used to calculate the centroid of the fracture, and then the vectors from the centroid to the most and least extending tips are calculated. The

normal of these two vectors is taken, which is then used to calculate with simple trigonometry the in-plane and out-of-plane deflection, ϕ_{frac} and ψ_{frac} respectively, of the fracture body. The arbitrary reference plane is taken as the x - y plane, and the reference direction is the x -axis. This thus yields the five tip-based parameters and four fracture-based parameters used for quantification in this work. This process is also applied to the growth vector of greatest extension to calculate ϕ and ψ for the direction of greatest extension.

In the case with a subsurface meltwater lens, the probe location is varied to quantify the impact of the lens as its presence significantly affects the spatial dependence of the fields. The number and layout of probe locations differs depending on the simulation purpose. In general, each simulation considers a single fracture probe which changes location over the structure of a grid, in order to capture for individual fractures how the meltwater lens affects their growth. Growth is quantified independently from fracture interaction which would incorporate further complexity due to the overlapping of stress concentration regions. The result is a distribution of growth behaviour quantified over a grid on the x - y plane overlying the lens, parallel to the surface.

Simulations consider three initial orientations of the fracture probe, namely horizontal, vertical and diagonal (Fig. 2), with normal vectors $(0, 0, 1)$, $(1, 0, 0)$ and $(1, 1, 1)$ respectively. These investigate the effect of initial conditions on the evolution of fracture geometry, as well as the effect on stress concentrations.

3 Simulation Setup

3.1 Physical Parameters and Love Numbers

The physical quantities that parameterize the system along with their taken values are detailed in Table 1. The Love numbers h , k , and l are dimensionless, complex parameters that describe how a body responds to tidal forces. The Love number k is not employed in this work. The overall model incorporates effects from layers not directly simulated, such as the silicate core, through their influence on these Love numbers. The Love numbers as used in this work are given in Table 2. For further detail on the selection and use of the physical parameters and Love numbers in this work, see Walding et al. (2024).

3.2 Geometry of the Domain

The geometry used in this work is cuboidal, of dimension $200 \text{ km} \times 200 \text{ km} \times 20 \text{ km}$, although only the interior $100 \text{ km} \times 100 \text{ km}$ is available for fracture placement. This is such that the lens and the domain boundary do not interact and create edge effects. The 20 km side is in the core-ward z -direction and the top of the slab is taken to be the surface of Europa. At this length scale, the surface of Europa can be approximated to be flat. These domains are shown in context in Fig. 3.

There still exists uncertainty and debate as to the thickness of the solid ice layer, but there is some agreement on overall water layer thickness (both oceanic and ice) at close to 120 km (Billings & Kattenhorn, 2005; McKinnon, 2004). The oceanic layer is then estimated to be 100 km thick (Pappalardo et al., 1999), thus leaving 20 km for the total ice layer (Lucchitta & Soderblom, 1982; Schenk, 2002; Turtle & Ivanov, 2002) (McKinnon, 1999; Ojakangas & Stevenson, 1989a). This therefore is the value taken for the 20 km core-ward side length of the domain.

A Dirichlet boundary condition fixes the bottom of the domain (defined as the inner face toward the core), the outer surface (the top) is a free surface, with the other faces having symmetric boundary conditions. Stresses are applied as triaxial loads, although in the present work there is never a non-zero stress component in the z -direction. Load

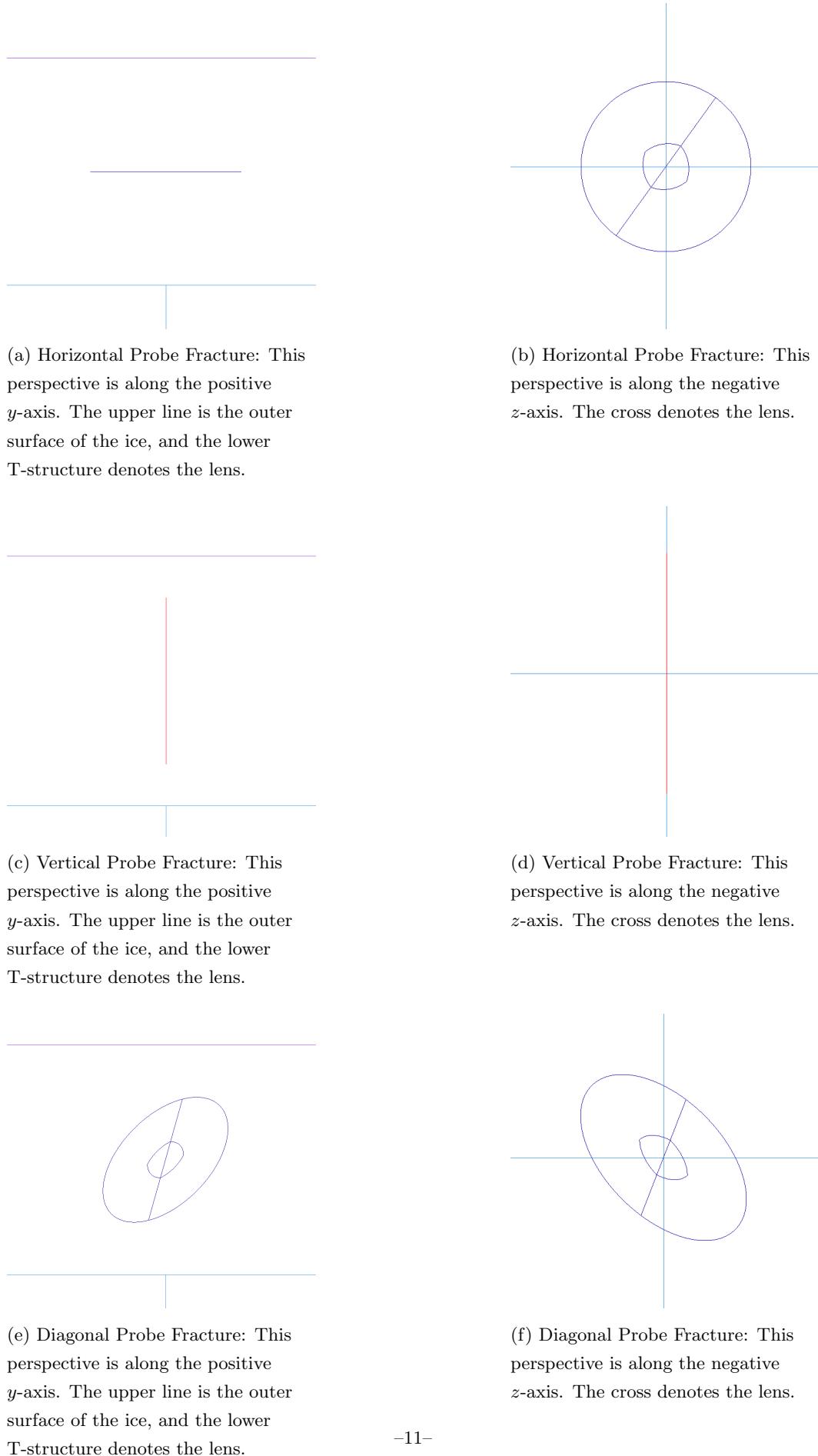


Figure 2: Schematic representations of probe fractures. The fractures are the central elements.

Table 1: Summary of physical parameters.

Physical Parameter	Symbol	Value	Unit	Reference
Young's Modulus of Ice	E	9	GPa	(Nimmo, 2004)
Fracture Toughness of Ice	K_{IC}	108	$\text{kPa} \cdot \text{m}^{\frac{1}{2}}$	(Goodman, 1980)
Poisson's Ratio of Ice	σ	0.33	-	(Sinha, 1989)
Tensile Strength of Ice	UTS	2	MPa	(Petrovic, 2003)
Diurnal Angular Rate	$f_{Diurnal}$	2.0485×10^{-5}	$\text{rad} \cdot \text{s}^{-1}$	(Burns et al., 1986)
NSR Angular Rate	f_{NSR}	1.7434×10^{-14}	$\text{rad} \cdot \text{s}^{-1}$	(Geissler et al., 1998)
Orbital Eccentricity	ϵ	0.0094	-	(Cassen et al., 1979)
Jovian Mass	$M_{Jupiter}$	1.898×10^{27}	kg	(Jacobson et al., 2000)
Europan Mass	M_{Europa}	4.799×10^{22}	kg	(Jacobson et al., 2000)
Europan Radius	R_{Europa}	1.561×10^3	km	(Archinal et al., 2018)
Jovian Semi-Major Axis	$a_{Jupiter}$	7.78479×10^8	km	(Office et al., 2016)
Europan Semi-Major Axis	a_{Europa}	6.71×10^5	km	(Cassen et al., 1979)
Europan Surface Gravity	g	1.315	$\text{m} \cdot \text{s}^{-2}$	<i>derived</i>
Lamé μ	μ	3.487	GPa	(Gammon et al., 1983)
Lamé κ	κ	9.3	GPa	(Gammon et al., 1983)
Brittle Ice Viscosity	$\nu_{Brittle}$	1×10^{22}	$\text{Pa} \cdot \text{s}$	(Wahr et al., 2009)
Paris Exponent f	β_f	0.35	-	(Paris & Erdogan, 1963)
Paris Exponent n	β_n	1	-	(Paris & Erdogan, 1963)

Table 2: Model Love numbers, drawn from Wahr et al. (2009).

Love Number	Symbol	Real Component	Imaginary Component
Diurnal Love Number h	$h_{Diurnal}$	1.192	-6.293×10^{-5}
NSR Love Number h	h_{NSR}	1.813	-4.186×10^{-3}
Diurnal Love Number l	$l_{Diurnal}$	0.3094	-2.903×10^{-5}
NSR Love Number l	l_{NSR}	0.4748	-2.766×10^{-3}

310 stresses are calculated by the tidal stress model at each mesh node within the domain,
 311 and are applied as *in situ* stresses at each location.

3.2.1 Melt Water Lenses

313 Melt water lenses are included in simulations as directly modelled heterogeneities
 314 within the ice. Taking the example of the *Thera Macula*, a region of chaos terrain ap-
 315 proximately 80 km in diameter, used in Schmidt et al. (2011) as representative of this
 316 phenomenon, the lenses are modelled with a volume of 20,000 km³ and at a depth of 3
 317 km, which is also supported by recent research by Vilella et al. (2020). The heterogeneities
 318 are geometrically represented as spherical caps of height one twentieth of the radius of
 319 the source sphere, yielding lenses of height 6.87 km and flat-face radius of 42.9 km (and
 320 therefore diameter 85.8 km, close correspondence to the *Thera Macula*). Finally, the bound-
 321 ing circle constraining the cap is chamfered to yield a softer edge as shown in Fig. 4. This
 322 is to prevent unnatural stress concentration at the edge due to precise geometry. As the
 323 lens is a melt water pocket, the lens is modelled as being soft yet unable to be fractured.

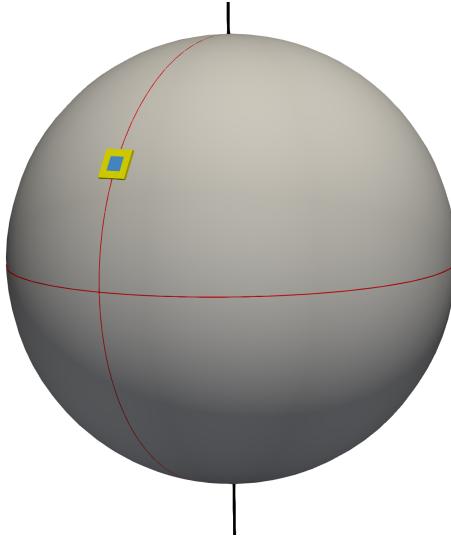


Figure 3: The surface ice of Europa is shown in light gray with a radius of 1561 km, with the equator and subjovian meridian in red, and the rotation axis in black. An example slab domain of dimension 200 km \times 200 km \times 20 km centred at 30°N 0° is shown in yellow, with the interior 100 km \times 100 km active simulation region in blue.

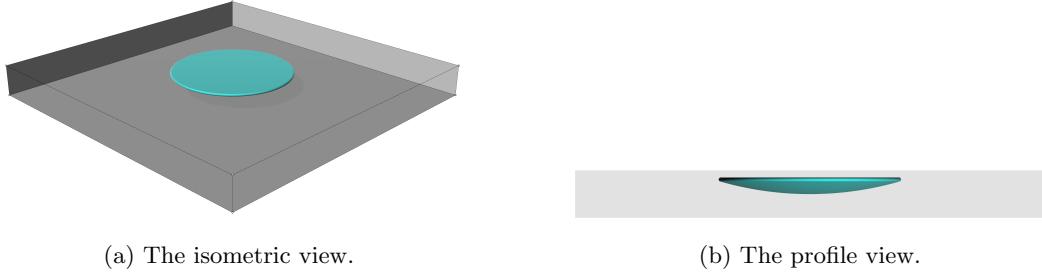


Figure 4: The lens geometry for the slab domain.

4 Results

The visualisation heatmaps presented in this section are the individual fracture probe results in the same domain interpolated with the Python Scipy library's `griddata` function, to create the gradients and contours. Each individual probe simulation models the ice crust with a discretised domain comprised of approximately 43,000 nodes and 35,000 elements. Each heatmap evaluates a potential scenario of fracture growth, given a set of initial geometric conditions for the fractures and a set of external boundary conditions, depending on the longitude of the lens.

The results presented in Fig. 5 pertain to simulations run in a homogeneous domain without a lens, to act as a base case for comparison with the lens cases. In Fig. 5a, showing a heatmap for K_V with vertical probe fractures, there is a smooth transition from West to East of low stress to high stress, whereas the heatmap of the same with horizontal probe fractures (Fig. 5b) still shows a very weak West-East trend though with distinct irregularities that are not present in the vertical case. The quiver plot for ϕ (the deflection within the x - y plane away from the line $x = 0$) with vertical probe fractures

339 presented in Fig. 5c shows total uniformity with no deflection. By contrast the ϕ quiver
 340 plot for horizontal probes in Fig. 5d shows no ordered structure with completely arbit-
 341 trary directionality.

342 The following results are drawn from quantification runs executed in the vicinity
 343 of a subsurface meltwater lens, designed to study the effect that the lens has on the spa-
 344 tial distribution of stresses and propagation angles, as well as how these parameters change
 345 with regard to the longitude of the test domain. The fractures are distributed in a 13
 346 by 13 grid across the domain, with a simulation for each of the three orientations run
 347 at each grid location, and the domain run at 0°E, 60°E and 120°E, for a total of 1,521
 348 simulations.

349 The first set of results in Fig. 6 are heatmaps of the respective parameters at 30°N
 350 0°E with diagonal test fractures. Figs. 6a and 6b for K_I and K_{II} show strong spatial
 351 dependence, with K_I having a stress concentration in the South-West quarter peaking
 352 at around $30 \text{ MPa} \cdot \text{m}^{\frac{1}{2}}$, and K_{II} with a mirror concentration in the North-East quar-
 353 ter at a lower around $20 \text{ MPa} \cdot \text{m}^{\frac{1}{2}}$. This is contrasted against the largely symmetric ring
 354 formation exhibited by the K_{III} heatmap, seen in Fig. 6c, peaking at around 10 MPa
 355 $\cdot \text{m}^{\frac{1}{2}}$. The combination of the three SIFs, where K_I and K_{II} have opposite formations
 356 and K_{III} is symmetric, leads to a largely symmetric K_V (Fig. 6d) of maximum magni-
 357 tude around $35 \text{ MPa} \cdot \text{m}^{\frac{1}{2}}$.

358 Figs. 7 and 8 are quiver plots of both extension direction and fracture body ori-
 359 entation. Fig. 7a shows on the whole expected behaviour, as the probe fractures in this
 360 case were diagonally oriented and this is largely maintained, though there is some irreg-
 361 ularity in the upper right region, in close correspondence to the region where K_{II} is high-
 362 est. In Fig. 7b it can be seen that the out-of-plane deflection leads to a bistable config-
 363 uration, where a fracture will settle into a 30° tilt in either direction. Curvature is seen
 364 in the interior lens region of Fig. 8a, where the fractures are growing perpendicularly to
 365 the edge of the lens. This excludes a diagonal region from the top left to the bottom right,
 366 bisecting the lens. The overall growth on different sides of this divide are in different di-
 367 rections. Of more interest in this case however, is Fig. 8b, where it can be seen that the
 368 vertical deflection of fractures across the tested domain is very limited.

369 Fig. 9 shows K_I heatmaps with all three probe fracture orientations and at differ-
 370 ent longitudes (0°E, 60°E and 120°E). The colourbar is normalised across all five sub-
 371 figures. The results from the horizontal and vertical orientations are largely similar in
 372 form, with stress concentration in a ring following the edge of the lens, though the ver-
 373 tical probe fracture exhibits heightened stresses in the bottom half. The diagonal and
 374 combined orientations however are quite divergent, with the diagonal probe fracture show-
 375 ing a large stress increase in the lower left quadrant. These figures together show that
 376 the initial condition of the fracture is quite influential on the stress conditions, which is
 377 expected in the earliest stages of a simulation. Significant change is observed by longi-
 378 tude, with the heatmap at 60°E showing only very weak stress concentration following
 379 the edge of the lens, and the heatmap at 120°E being the opposite of that seen at 0°E,
 380 with the heightened stresses now in the upper right. This longitudinal variation is ob-
 381 served across all measured parameters to some degree, as seen in the following.

382 The maximum, minimum and average values of each parameter by longitude are
 383 shown in Figs. 10 and 11. There were simulations run for every 5° of longitude, where
 384 at each longitude the fractures were in a 7 by 7 grid across the domain, and each loca-
 385 tion was run with each of the three orientations for a total of 5,439 simulations. The light
 386 grey lines show a randomly selected quarter of the individual simulations where possi-
 387 ble.

388 The results presented in Figs. 12 and 13 are of different form, and are qualitative
 389 views of dense fracture networks to show visually how the fractures grow. The network

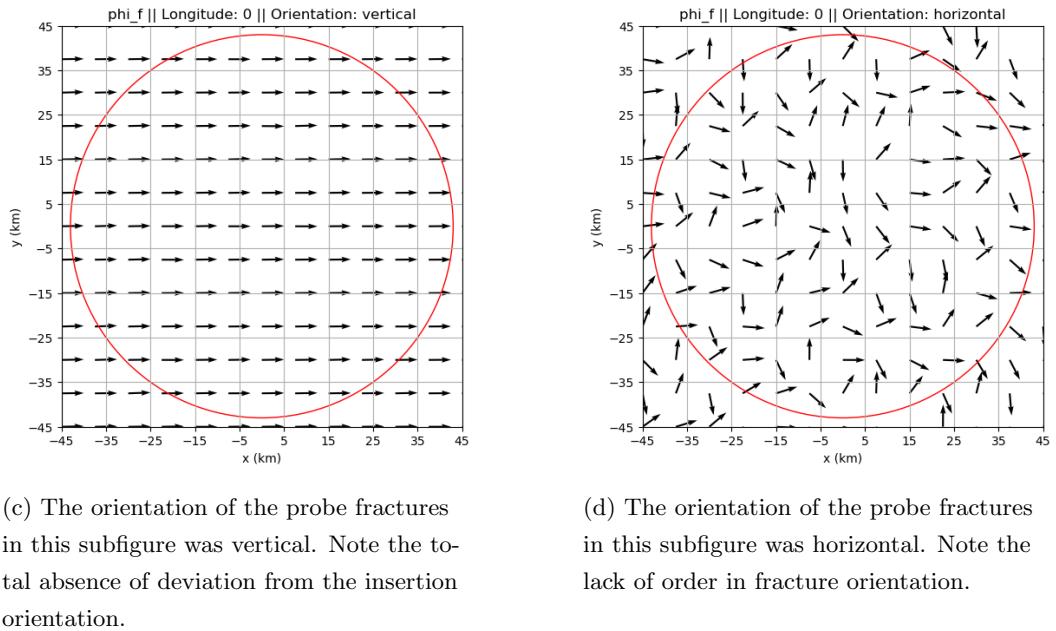
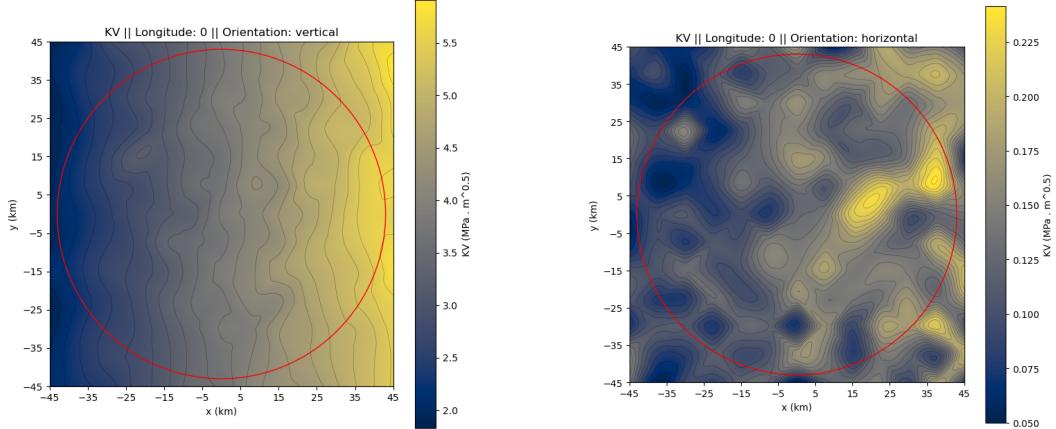
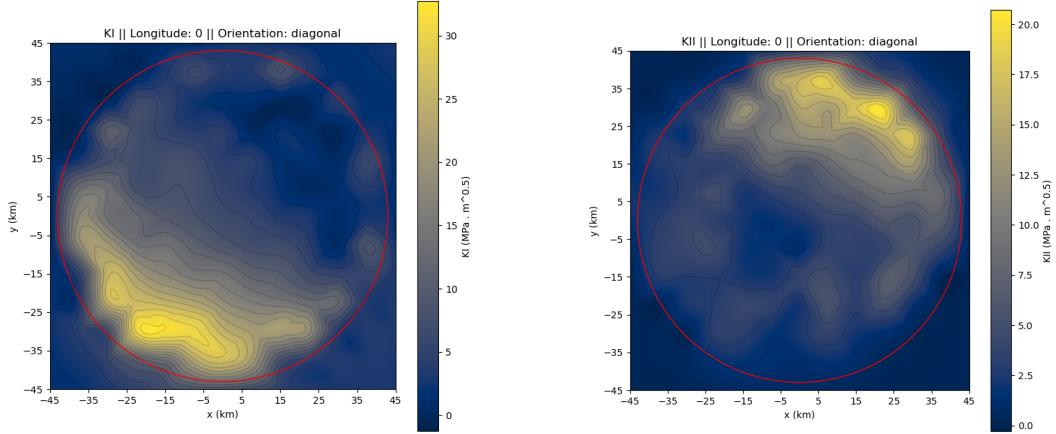
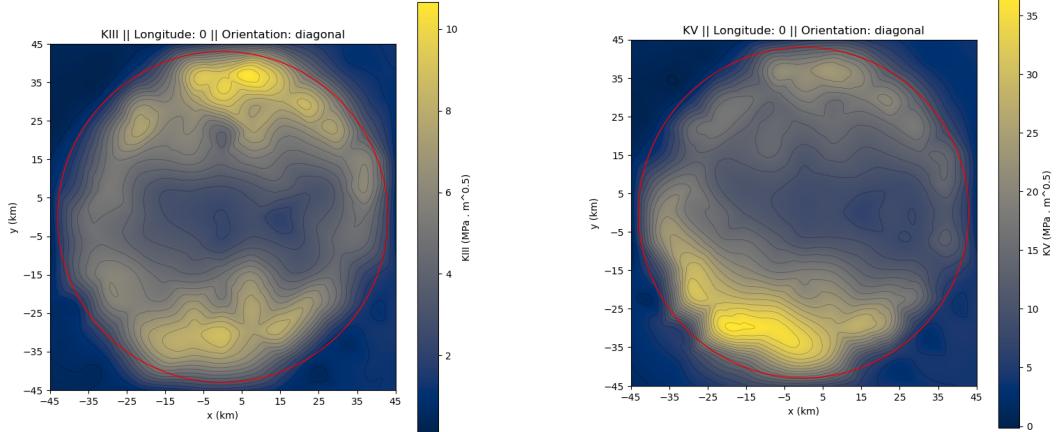


Figure 5: Homogeneous domain, no lenses. The red circles denote where the subsurface meltwater lens would be were it present for comparison with the later simulations. Subfigures (a) and (b) give the values of K_V across a slab that does not contain a subsurface lens. Subfigures (c) and (d) give the values of ϕ for fracture bodies (the deflection within the x - y plane away from the line $x = 0$) across a slab that does not contain a subsurface lens, where the lines are the normals to the plane formed from the most and least extending fracture tips' vectors from the fracture centroid. Location: 30°N 0°E.



(a) K_I : Note the large asymmetry of the stress field.

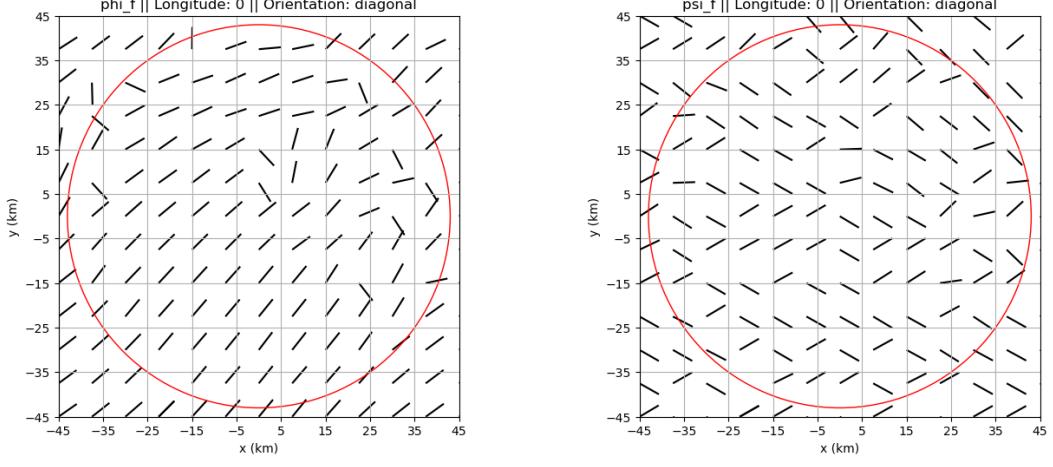
(b) K_{II} : Note again the similarly large asymmetry of the stress field as compared to K_I , though opposite and also slightly lesser in magnitude.



(c) K_{III} : In this case the stress field is symmetric about the line $y = 0$, though of substantially smaller magnitude than for K_I and K_{II} .

(d) K_V : The distribution is a combination of that of K_I , K_{II} and K_{III} , though with the peak seen in the K_I distribution dominating.

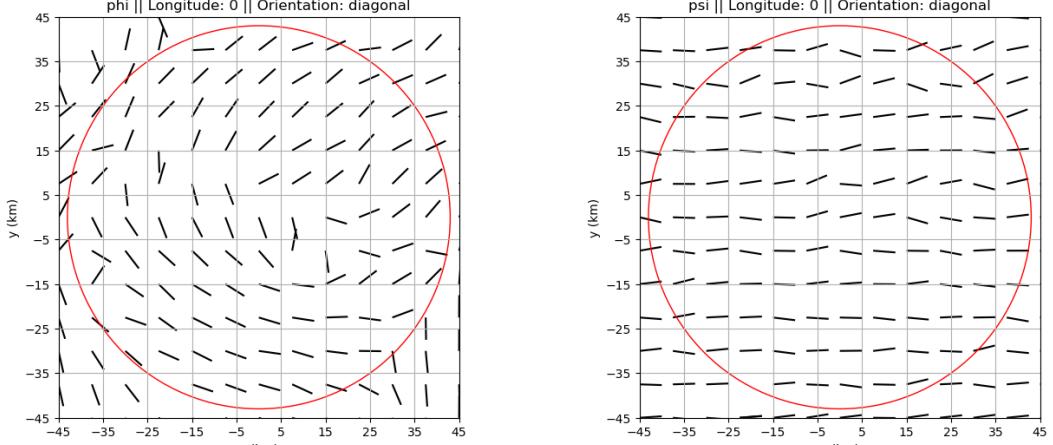
Figure 6: The values of modal SIFs presented across a domain that contains a subsurface lens, the edge of which is denoted by the red line. The orientation of the probe fractures was diagonal. Each subfigure has 507 constituent simulations. Location: 30°N 0°E.



(a) ϕ : The deflection within the x - y plane away from the line $x = 0$.

(b) ψ : The deflection away from the x - y plane.

Figure 7: The values of ϕ and ψ for fracture bodies across a slab that contains a sub-surface lens, the edge of which is denoted by the red line. The orientation of the probe fractures was diagonal. The lines are the normals to the plane formed from the most and least extending fracture tips' vectors from the fracture centroid. Each subfigure has 507 constituent simulations. Location: 30°N 0°E.



(a) ϕ : The deflection within the x - y plane away from the line $x = 0$.

(b) ψ : The deflection away from the x - y plane.

Figure 8: The values of ϕ and ψ for fracture extension across a slab that contains a sub-surface lens, the edge of which is denoted by the red line. The orientation of the probe fractures was diagonal. The lines are the normals to the plane formed from the most and least extending fracture tips' vectors. Each subfigure has 507 constituent simulations. Location: 30°N 0°E.

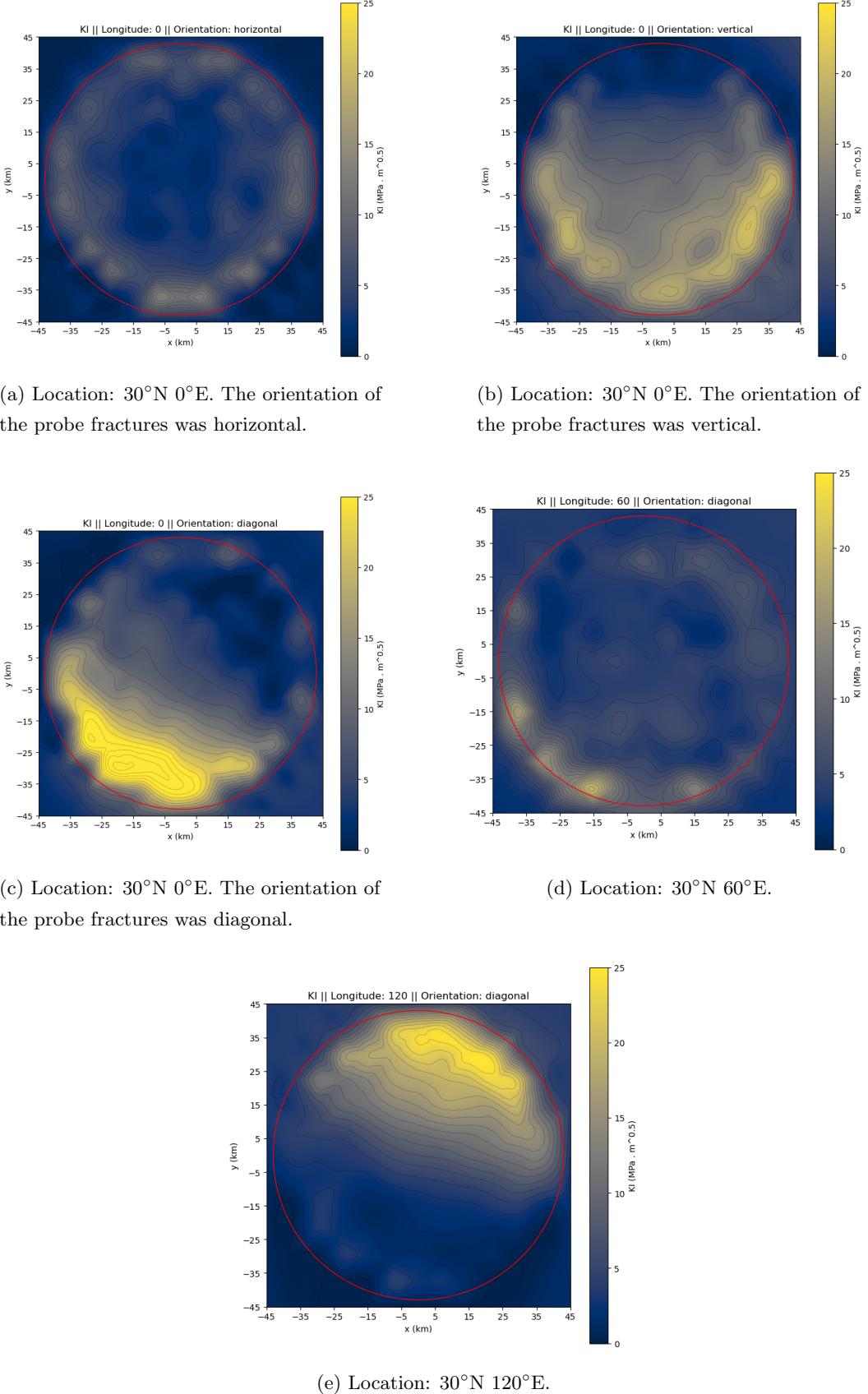
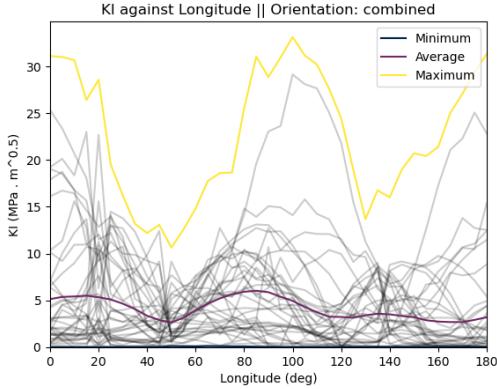
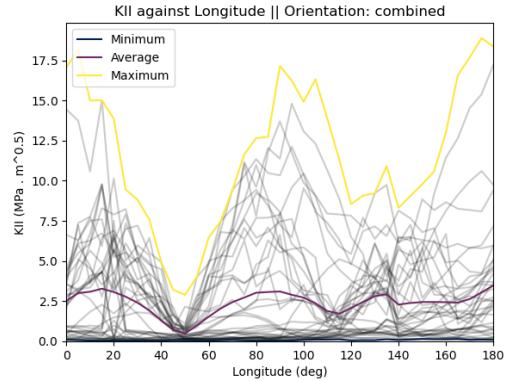


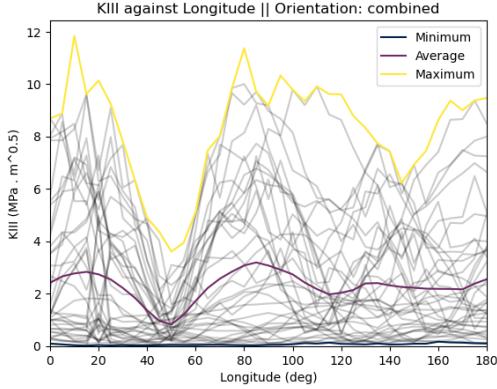
Figure 9: The values of K_I across slabs that contains a subsurface lens, the edge of which is denoted by the red line. Subfigures (a), (b) and (c) show different probe fracture orientations. (c), (d) and (e) show different locations. Each subfigure has 507 constituent simulations.



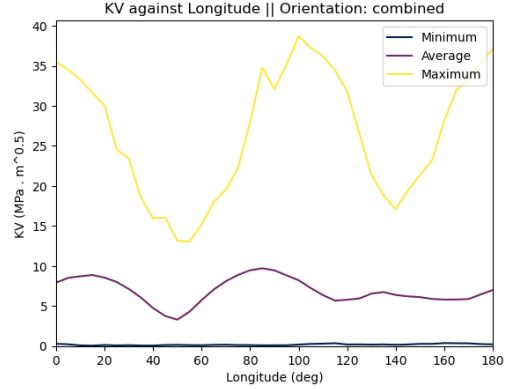
(a) K_I : There are distinct sharp peaks in the maximum line around 0° and 100° . In the average line there are smooth peaks between 0° and 120° , with a close to monotonically decreasing line beyond.



(b) K_{II} : There are peaks in the same areas as with Fig. 10a for the maximum line, though the valley at 50° is deeper and sharper, with this mirrored in the average line.

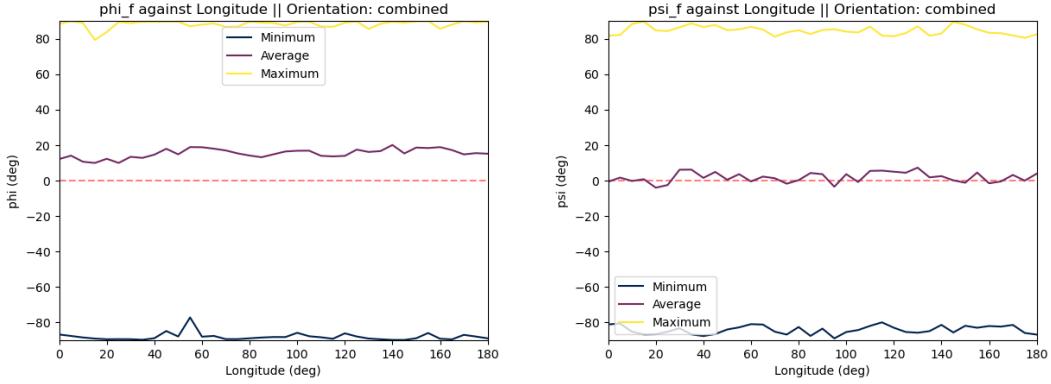


(c) K_{III} : The average line follows the same trend as with K_I and K_{II} , though the maximum line has a significantly less pronounced valley around 140° .



(d) K_V : The maximum line has smooth and close to symmetric peaks around 0° and 100° , similar to K_I . The average line again shows smooth peaks, though they are closer together at 20° and 80° , and the region after the second peak is flat.

Figure 10: The measured K parameters as a function of longitude, giving their respective maximum, minimum and average values. The light grey lines, where present, show a random quarter of the simulation runs used to calculate those values. The difference between the first half of the average lines and the second half corresponds to the difference in form between the stress concentrations in Fig. 1. These figures are the result of 5,439 simulations.



(a) ϕ : The deflection within the x - y plane away from the line $x = 0$. The maximum and minimum lines are in the vicinity of 90° , though the average line is consistently around 10° to 20° .

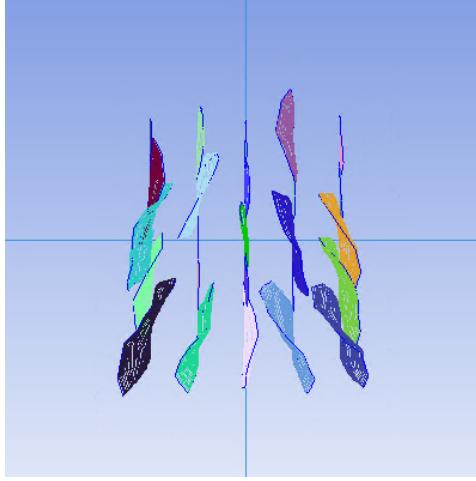
(b) ψ : The deflection away from the x - y plane. The maximum and minimum lines are again in the vicinity of 90° , except in this case the average line remains at 0° .

Figure 11: The measured angle parameters as a function of longitude, giving their respective maximum, minimum and average values. These figures are the result of 5,439 simulations.

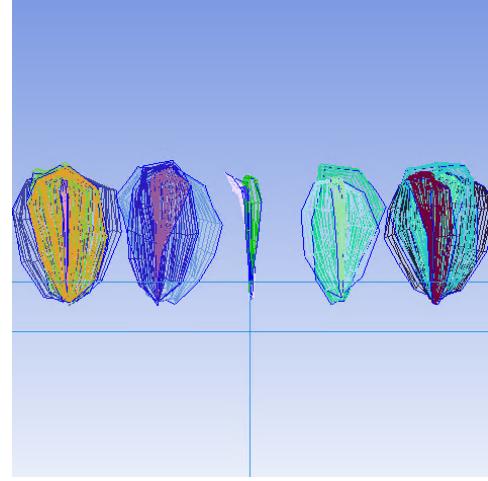
begins with a grid of 25 fractures of initial radius 800 m and separation 2 km, and then the fractures are allowed to grow to steady state as opposed to terminating after two growth steps as is the case from the quantification simulations. These models are significantly denser and more complex, with approximately 110,000 nodes and 90,000 elements, over double the complexity of the probe simulations. In Figs. 12a and 12b show the final state of a simulation with vertical fracture nuclei over the centre of the lens. The fractures show little deflection and mostly grew vertically, with some deflection that was mirrored East-to-West. The same simulation but with the network centered over the edge of the lens is presented in Figs. 12c and 12d, where increased growth is seen. The above view illustrates that one row of the network has been effectively shielded from stress by the neighbouring fractures, while the side view shows one of the outer fractures has grown significantly more than its neighbours, as stress can concentrate into one fracture's growth. Finally, Figs. 13a and 13b show the same simulation but with horizontal fracture nuclei. This leads to an increase in overlapping fractures, as well as still further East-West coverage due to more growth. The fracture growth in this case is also more uniform than with vertical fractures, with no outlier fractures.

5 Discussion

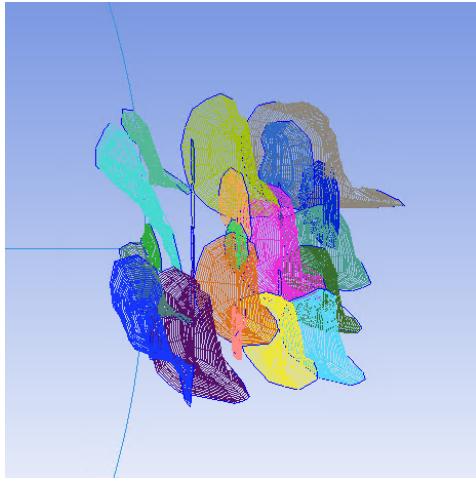
Heatmaps presented in Figs. 5a and 5b show that in a homogeneous domain, with no meltwater lens present, fractures will not preferentially grow in a horizontal orientation, and will instead tend to grow perpendicularly to the Europa surface. This result is further corroborated by the in-plane deflection quiver plots shown in Figs. 5c and 5d, where the horizontal fractures show no structure due to the lack of a conducive environment for fracture development. This is consistent with the assumption that linea are fractures within the ice that intersect the Europa surface (Helfenstein & Parmentier, 1983).



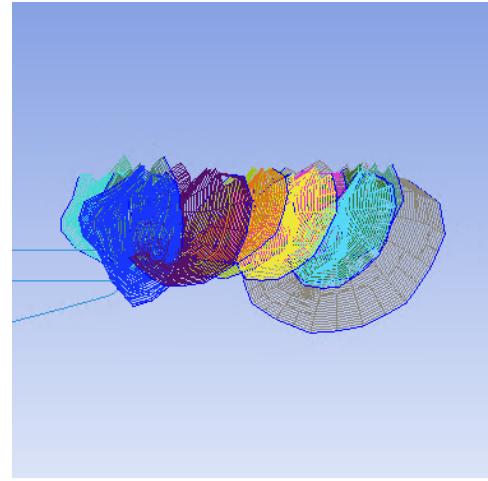
(a) Vertical fracture nuclei at the centre of the lens from above. Note the lack of significant deflection, though what deflection there is increased in the South and is East-West symmetric.



(b) Vertical fracture nuclei at the centre of the lens from the side. Again note the limited deflection.



(c) Vertical fracture nuclei at the edge of the lens from above. There is significantly increased fracture growth compared to in the centre.



(d) Vertical fracture nuclei at the edge of the lens from the side. Note the pronounced lean of the fractures that aligns with the lens, as well as the dominant growth of one fracture.

Figure 12: Networks of 25 fractures with an initial radius of 800m and a centre-to-centre separation of 2 km with varying initial orientations and locations with respect to the lens. The lens is centred at 30°N 100°E, a location of heightened stress.

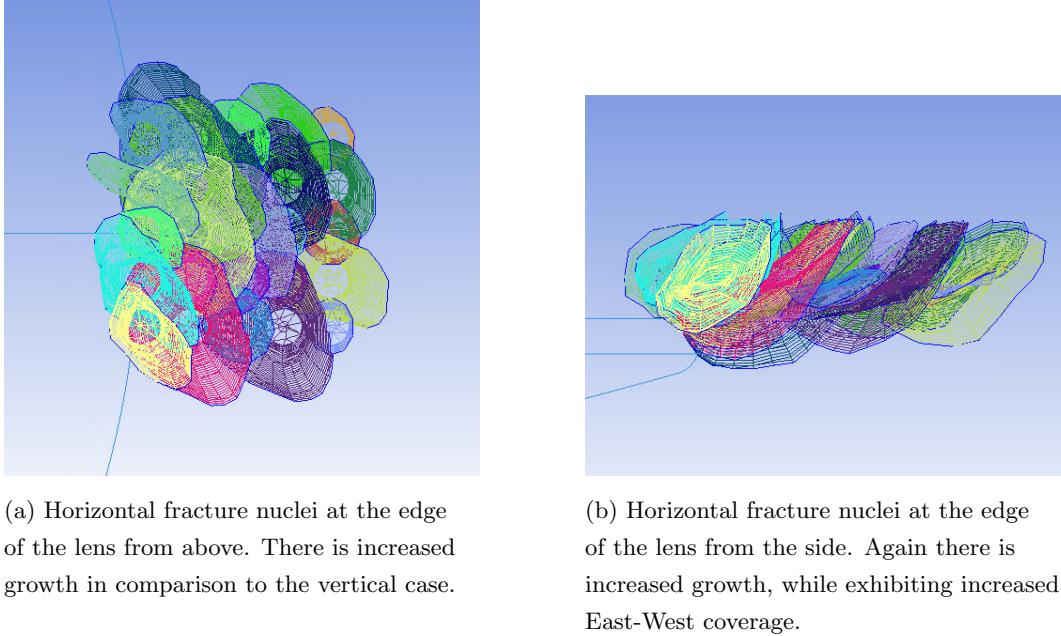


Figure 13: Networks of 25 fractures with an initial radius of 800m and a centre-to-centre separation of 2 km with varying initial orientations and locations with respect to the lens. The lens is centred at 30°N 100°E, a location of heightened stress.

Results in Fig. 8b imply that in the vicinity of a lens, fractures will preferentially grow horizontally, within the ice, as opposed to towards the surface, as observed in the homogeneous domain simulations. This supports the earlier results presented by Walding et al. (2024), further contributing to the hypothesis that the presence of these parallel fractures in proximity to a lens may weaken the ice matrix above, leading to fragmentation.

The longitudinal plots for the two angles, ϕ and ψ , exhibit no visible structured behaviour with a varying longitude, as seen in Figs. 11a and 11b. In both cases, the maximum and minimum lines remain close to $\pm 90^\circ$ which is the maximum allowed deflection, indicating that there will always be at least one fracture with a high deflection. However, the average ϕ deflection hovers around 10° to 20° , this shows unified in-plane deflection, where fractures align with the overall stress field. The average ψ deflection is less pronounced and is in the vicinity of 0° , showing that the bulk of fractures remain horizontal. The remaining parameters all yield substantial peaks around 0° E and 100° E, with K_I among the SIFs being both the highest value overall and also exhibiting the most pronounced peak. Around 40 - 60° E and 140 - 160° E, the three SIFs get the closest in magnitude, indicating increased fragmentation and fracture network complexity in these regions. These peaks and valleys directly correlate with the regions between the stress concentrations seen in Fig. 1, whereas the areas of closest magnitude correlate to the stress concentrations themselves.

Across the presented heatmaps, there is a clear increase in stress around the rim of the subsurface lens, therefore leading to increased fracture growth in these regions. This may lead to separation of the ice overlaying the lens from the overall ice matrix, resulting in something similar to a calving event for a terrestrial ice sheet (Benn & Åström, 2018). Some preliminary further work exploring this effect is presented in Fig. 14, where

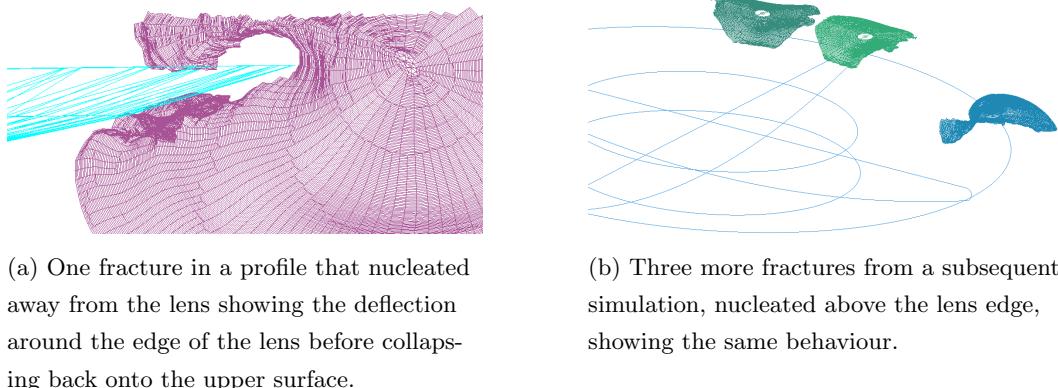


Figure 14: Two simulations both showing fractures growing around a well-defined subsurface meltwater lens.

individual fractures can be seen curving over the lens edge, while both touching the outer Europa surface as well as touching the lens body. This constitutes total mechanical separation in this region.

Figs. 12 and 13 qualitatively show the additional fracture growth that occurs at the edge of the lens. The final overall form of the fracture networks is largely independent of the initial fracture orientation for the shown cases, consistent with simulations of fracture growth in rocks (Paluszny & Matthäi, 2009). This indicates that, for sufficiently small initial fracture radii, initial fracture orientation is qualitatively of insubstantial impact on the final results. These factors combined lend strong support to the theory that the presence of subsurface meltwater lenses is likely to lead to the nucleation of chaos terrain on the surface.

The results seen in Fig. 10 show that longitude has a first order effect on the stress concentration felt by fractures, with strong spatial heterogeneity. This is conducive to fracture growth, and would also lead to differences in fracture networks across the surface. The form of the parameters against longitude further imply that there would be greater chaos formation around each quarter turn around the moon, starting from the subjovian point, although the current understanding of meltwater lens distribution is not yet refined enough to make this assertion.

6 Conclusions

The simulations presented in this work have investigated fracture formation in the ice on the Jovian moon Europa, specifically how fractures behave in the vicinity of subsurface meltwater lenses. This investigation has used a tidal stress model paired with a three-dimensional finite element method fracture simulator in order to model fracture development at a high degree of resolution.

The authors contend that the scientific implications of these simulations are that subsurface meltwater lenses are likely to be the nuclei above which chaos terrain forms. However, further study of individual fractures at the edge of the lens that exhibit the lipping phenomenon is warranted, as these fractures may potentially play a significant role in initial chaos formation. Work on this topic is ongoing.

469 **7 Open Research**

470 The software used in this work consists of the Imperial College Geomechanics Toolkit
 471 (Paluszny & Matthäi, 2009), as well as a tidal stress model based upon SatStress (Wahr
 472 et al., 2009). The executable is available at https://github.com/waldingjc/Europa_meltwater_lenses.
 473 The batch files to run the simulator, the raw output data used in this work and the vi-
 474 sualisation scripts are also available at the same repository. The data contained therein
 475 will be archived in the National Geoscience Data Centre (NGDC) (<https://www.bgs.ac.uk/geological->
 476 [data/national-geoscience-data-centre/](https://www.bgs.ac.uk/geological-data-national-geoscience-data-centre/)) operated by the British Geological Survey upon
 477 paper acceptance, with the associated DOI and relevant information shared here. This
 478 work also uses ANSYS ICEM CFD 2022 R2 (ANSYS Inc., 2022) for meshing and Rhinoceros3D
 479 (McNeel et al., 2010) for visualisation and geometry construction.

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