Assignment 3 -- Evan Waldmann -- S3620596

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1.

```
clc
% part a.
type RKStage5.m

% part b.
format compact
format long

exactS = 20-(385)^(1/2)
% Experimental Determination of Order
a=1; b=5;
pts=zeros(4,2);
for ic=1:4
    n=10*(2^(ic-1)); h=(b-a)/n;
    out=RKStage5(@(Y) [1,(4*Y(2))/(Y(2)-4*Y(1))],[1,3],h,n,n)
    err=exactS-out(2,2);
    pts(ic,:)=[log(h),log(abs(err))];
    disp(sprintf('steps = %2d   h = %6.4f  t = %3.1f  u =
 %12.10g', ...
        n,h,out(2,1),out(2,2)));
    disp(sprintf('error = %8.4e  log(h) = %8.4e  log(err) = %8.4e
\n',...
        err, pts(ic,:)));
end;

plot(pts(:,1),pts(:,2),'-ob'),...
    xlabel('log(h)'),ylabel('log|error|'),...
    title('log|error| versus log(h)'),daspect([0.2,1,1]);

X = [ones(4,1)  pts(:,1)];
a = X\pts(:,2);
slope=a(2)
slope2=(pts(4,2)-pts(3,2))/(pts(4,1)-pts(3,1))


function pts=RKStage5(flist,init,h,steps,freq)
Y=init;
```

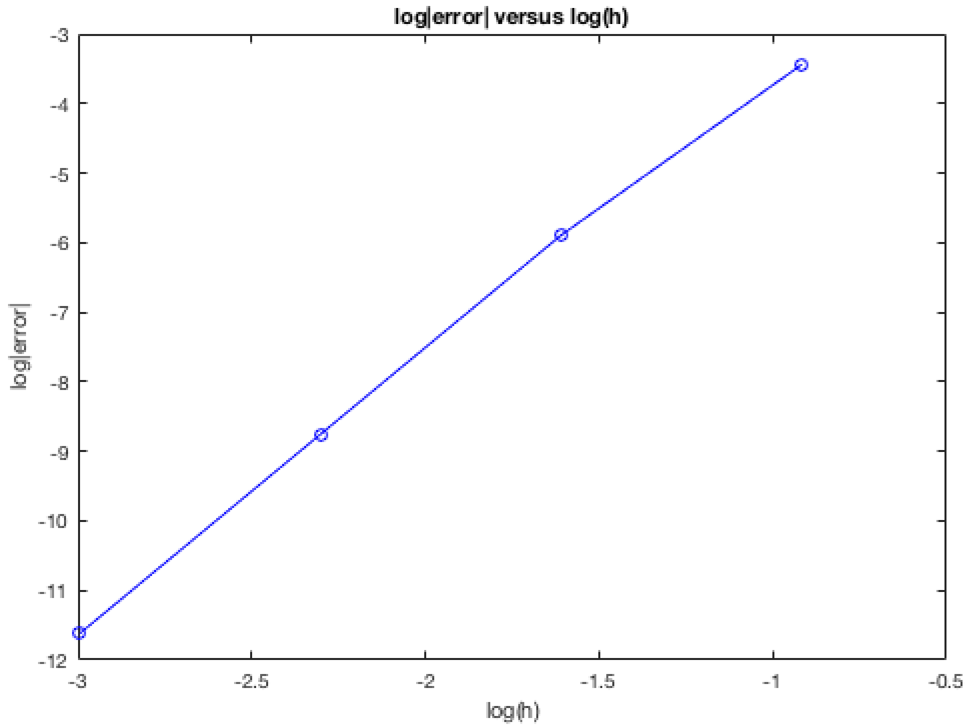
```
pts=zeros(1+fix(steps/freq),length(init));
pts(1,:)=Y; ic=1;
for i=1:steps
    k1=h*flist(Y);
    k2=h*flist(Y+0.25*k1);
    k3=h*flist(Y+0.5*k2);
    k4=h*flist(Y+1/2*k1 - k2 + 5/4*k3);
    k5=h*flist(Y +2/3*k2+1/3*k4);
    Y=Y+(3*k1+4.0*k2+10*k3+4*k4+3*k5)/24.0;
    if rem(i,freq)==0 ic=ic+1; pts(ic,:)=Y; end;
end;
exactS =
   0.378583129651417
out =
   1.000000000000000   3.000000000000000
   5.000000000000001   0.346250179709179
steps = 10   h = 0.4000  t = 5.0  u = 0.3462501797
error = 3.2333e-02  log(h) = -9.1629e-01  log(err) = -3.4317e+00

out =
   1.000000000000000   3.000000000000000
   5.000000000000002   0.375816429307679
steps = 20   h = 0.2000  t = 5.0  u = 0.3758164293
error = 2.7667e-03  log(h) = -1.6094e+00  log(err) = -5.8901e+00

out =
   1.000000000000000   3.000000000000000
   4.999999999999999   0.378426156982120
steps = 40   h = 0.1000  t = 5.0  u =  0.378426157
error = 1.5697e-04  log(h) = -2.3026e+00  log(err) = -8.7594e+00

out =
   1.000000000000000   3.000000000000000
   4.999999999999990   0.378574220717495
steps = 80   h = 0.0500  t = 5.0  u = 0.3785742207
error = 8.9089e-06  log(h) = -2.9957e+00  log(err) = -1.1628e+01

slope =
   3.961597524731721
slope2 =
   4.139116777793334
```




```
Use your plot to determine a relationship between the error and h.
How accurate is the method? Is this what you would expect? Please discuss.

The method is 4th order, which we know from the slope of the graph.

From the notes we have,
"In general, a Runge-Kutta method which has order p for a single
equation also has order p for a first order system of equations
if p ? 4; for p > 4 the order for a system may be less than p."

Thus the Runge-Kutta stage 5 method being 4th order is expected.
```

2.

```
clear;
b= 4*10^-6;
h=2*10^-6;
n=.1;
% numbers based on second last digit (9)
a=.03;
c=200;
m=700;
k=.009;

years= 10000;
To = a/b; %7500
```

```
Eo = 1;
out=RKStage5(@(Y)[Y(1)*(a-b*Y(1))-(c*Y(1))/(m+Y(1))*Y(2), Y(2)*(-k
+(h*Y(1))/(Y(2)+n))],...
    [To,Eo/710],1,years,1);
plot(out(:,1),out(:,2),'b'),...
    xlabel('Trees'),ylabel('Elephants'), title('1) To=a/b -- Eo=1
 (10000 years)');
snapnow

years = 2100;
Eo = 142;
To = 710;
out=RKStage5(@(Y)[Y(1)*(a-b*Y(1))-((c*Y(1))/(m+Y(1)))*Y(2), Y(2)*(-k
+(h*Y(1))/(Y(2)+n))],...
    [To,Eo/710],1,years,1);
plot(out(:,1),out(:,2),'b'),...
    xlabel('Trees'),ylabel('Elephants'), title('2) To=710 -- Eo=142
 (2100 years)');
snapnow

years = 700;
Eo = 142;
To = 4600;
out=RKStage5(@(Y)[Y(1)*(a-b*Y(1))-((c*Y(1))/(m+Y(1)))*Y(2), Y(2)*(-k
+(h*Y(1))/(Y(2)+n))],...
    [To,Eo/710],1,years,1);
plot(out(:,1),out(:,2),'b'),...
    xlabel('Trees'),ylabel('Elephants'), title('3) To=4600 --
 Eo=142/710 (700 years)');
snapnow
```

Does it matter what the starting values are?

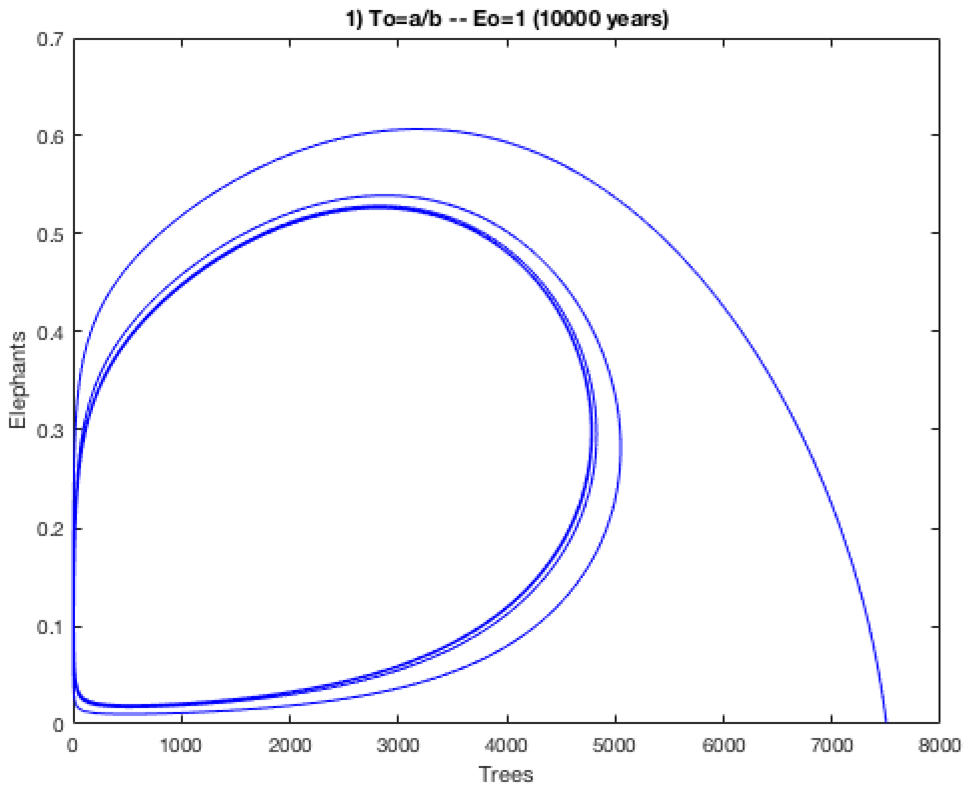
The starting values only effect how long it takes to get into the limit cycle behavior. If the values are far from the values that the cycle takes, the point will take a longer time to converge to the limit cycle. Over a long period of time the starting values do not matter.

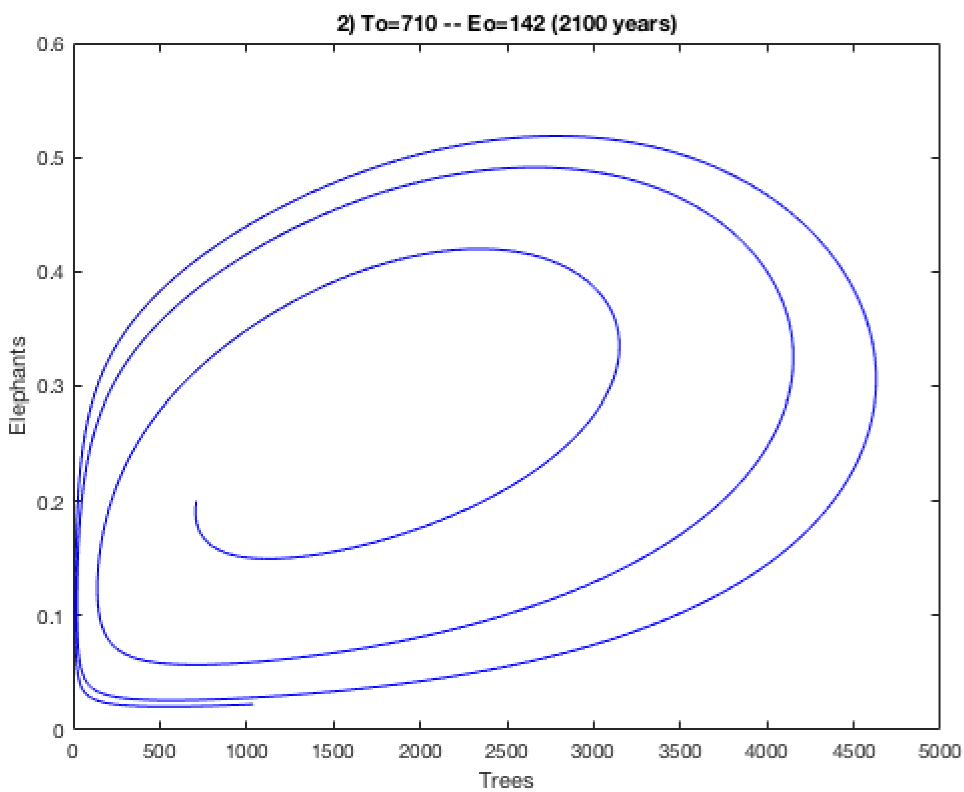
Do you obtain limit cycle behavior for different starting values?

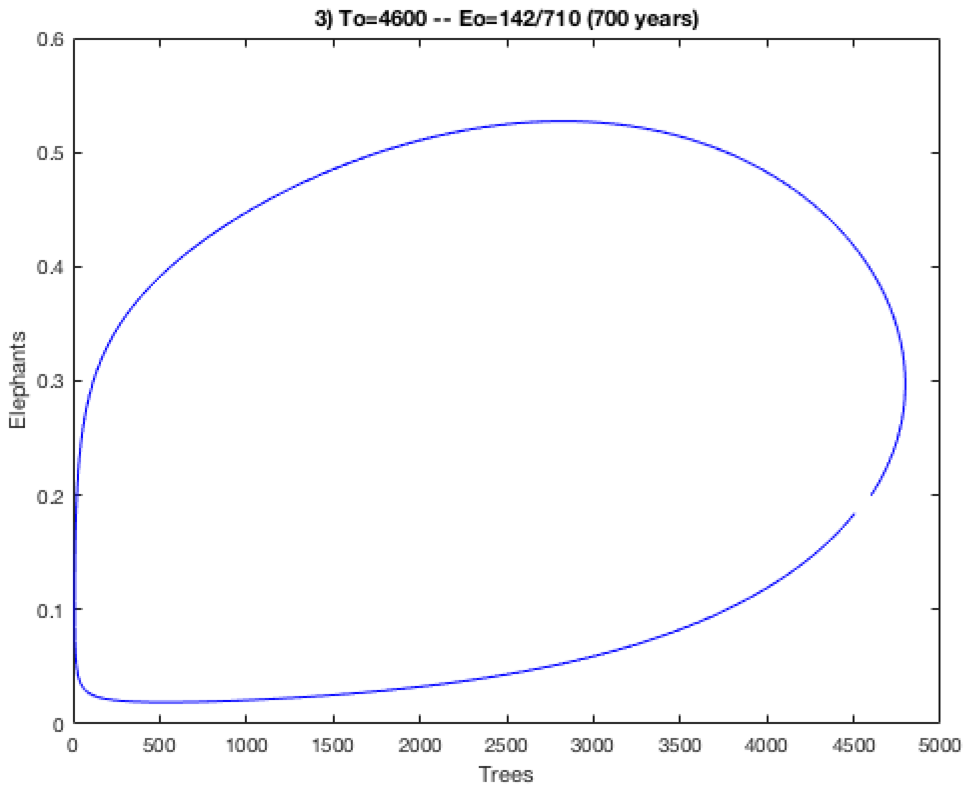
As long as you have sufficient time, you obtain a limit cyle behavior.

If starting values are chosen inside the limit cycle does the solution spiral outwards towards the limit cycle?

Yes. You can see this in the graph labled "2) To=710 -- Eo=142 (2100 years


1) To=a/b -- Eo=1 (10000 years)
Elephants
Trees
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
1000
2000
3000
4000
5000
6000
7000
8000


2) To=710 -- Eo=142 (2100 years)
Elephants
Trees
0
0.1
0.2
0.3
0.4
0.5
0.6
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000



3.

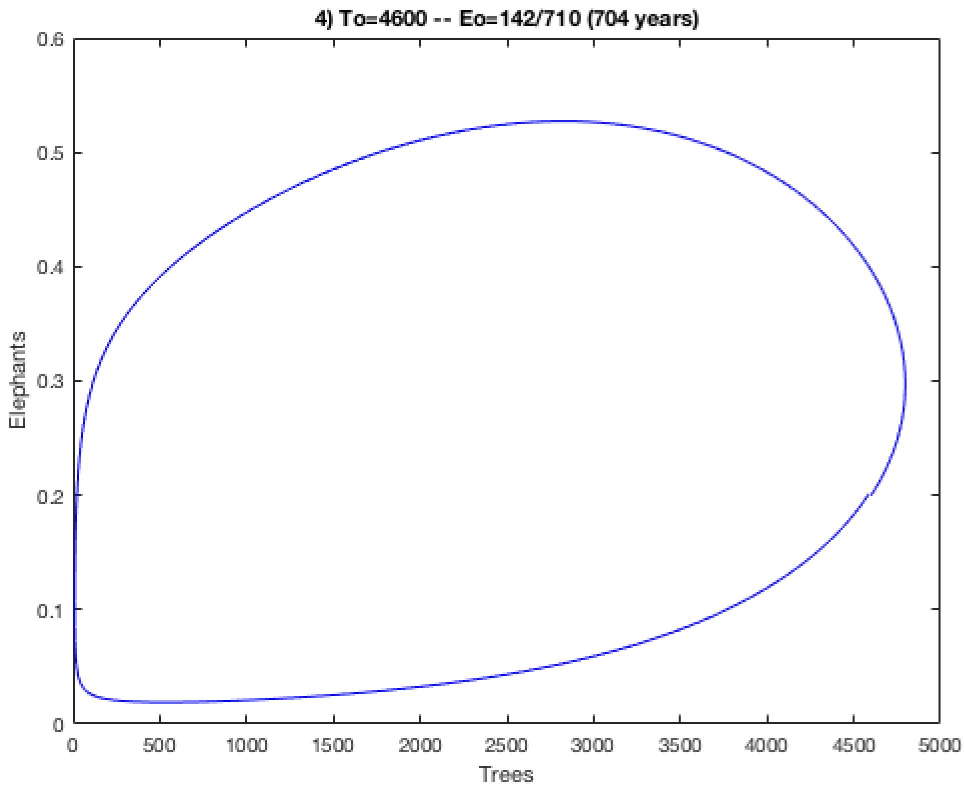
```
years = 704;
Eo = 142;
To = 4600;
out=RKStage5(@(Y)[Y(1)*(a-b*Y(1))-((c*Y(1))/(m+Y(1)))*Y(2), Y(2)*(-k
+(h*Y(1))/(Y(2)+n))],...
    [To,Eo/710],1,years,1);
plot(out(:,1),out(:,2),'b'),...
    xlabel('Trees'),ylabel('Elephants'), title('4) To=4600 --
 Eo=142/710 (704 years)');
snapnow
```

Determine the period of the limit cycle.

The period of the limit cycle is about 704 years, which is shown in the fourth graph.

Considering the length of the limit cycle period (in years) is it feasible to confirm the limit cycle behavior by observation?

It it is feasible to approximate the period of the limit cycle behavior



4.

```
clear;
type culling.m

b= 4*10^-6;
h=2*10^-6;
n=.1;
% numbers based on second last digit (9)
a=.03;
c=200;
m=700;
k=.009;

Eo = 337;
To =5100;

out=RKStage5(@(Y)[1,Y(2)*(a-b*Y(2))-((c*Y(2))/(m+Y(2)))*Y(3),...
    Y(3)*(-k+(h*Y(2))/(Y(3)+n))-culling(Y(1))/710],...
    [0,To,Eo/710],1,50,1);
plot(out(:,2),out(:,3),'b'),...
    xlabel('Trees'),ylabel('Elephants'), title('5) Culling Graph 50
 years');
fifty = [out(50,1),out(50,2),out(50,3)]
xlim([0, 8000]);
```

```
ylim([0, .6 ]);
snapnow

out=RKStage5(@(Y)[1,Y(2)*(a-b*Y(2))-((c*Y(2))/(m+Y(2)))*Y(3),...
    Y(3)*(-k+(h*Y(2))/(Y(3)+n))-culling(Y(1))/710],...
    [0,To,Eo/710],1,1000,1);
plot(out(:,2),out(:,3),'r'),...
    xlabel('Trees'),ylabel('Elephants'), title('6) Culling Graph 1000
 years');
xlim([0, 8000]);
ylim([0, .6 ]);
thousand = [out(1000,1),out(1000,2),out(1000,3)]
snapnow
```

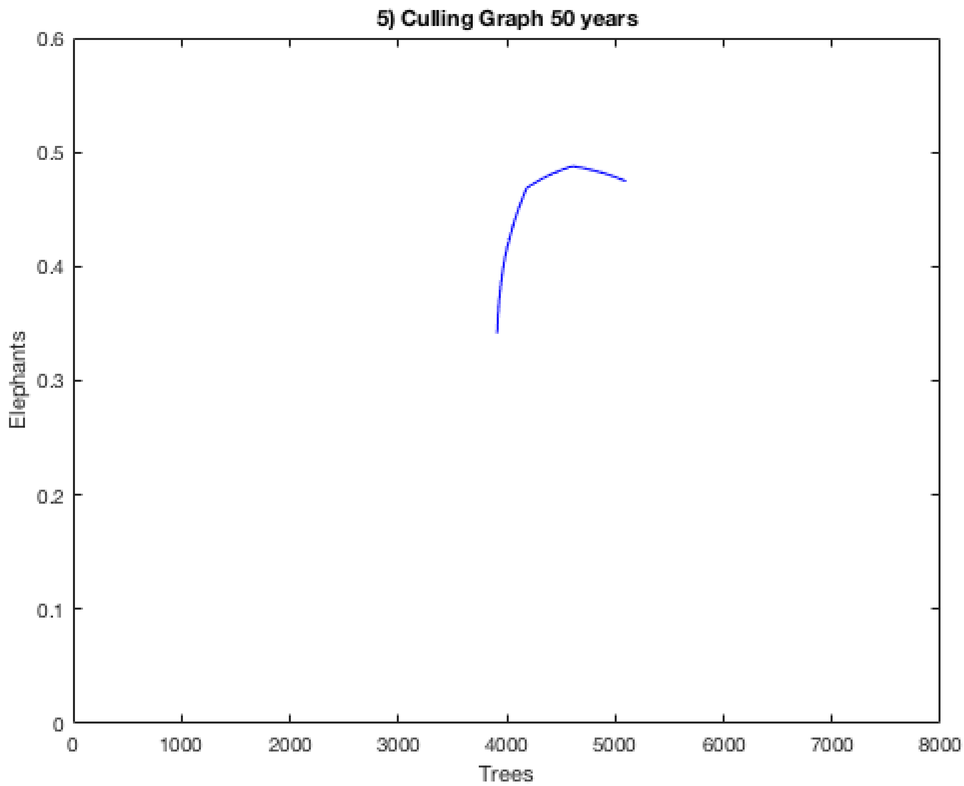
Estimate the number of elephants and the tree density when the system is i

From the graphs, the tree density and and elephant density appears to be about 3900 for trees and .33 for elephants. Looking at the data after 50 years the tree density is 3915 and the elephany density is .348. After the system reaches its stable point at a thousand years the tree density is 3777 and elephant density is .333

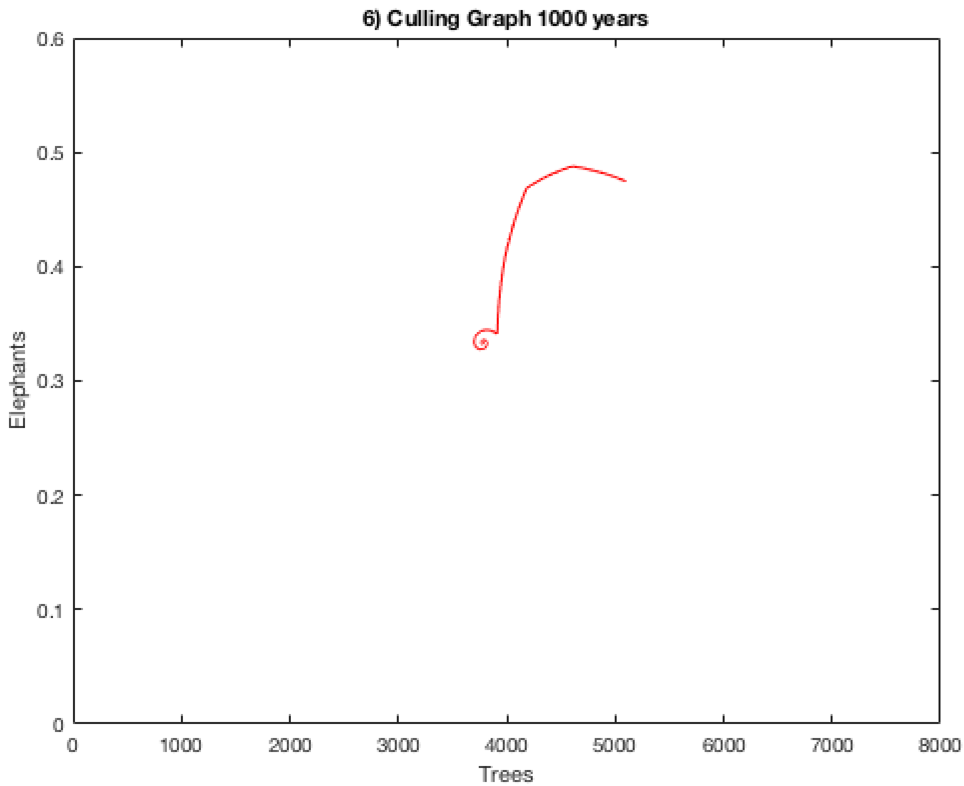
```
function [p] = culling(t)

if (t>50)
    p = 2;
elseif (t>40)
    p =7;
elseif (t>30)
    p = 6;
elseif (t>15)
    p = 3;
elseif (t>=0)
    p = 2;
else
    display('error')
end


end
fifty =
   1.0e+03 *
   0.049000000000000   3.915032459409618   0.000348489388366
```




```
thousand =
   1.0e+03 *
   0.999000000000000   3.776729627626820   0.000333258707815
```



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