MATLAB Tutorial and Weekly Test 8

The parts of this tutorial that are flagged in the section header for submission are to be used for assessment purposes. The code used to generate the input for that exercise must be included in the submitted work along with the output and handed in to be marked. You may cut and paste the command lines from the command window and function files into a word document, then annotate this and send this to me via my email: andy.eberhard@rmit.edu.au.

Any script or function code that *you have modified or written* must also be so included. Figures may be saved and imported into the word file. This exercise is due on the Friday of the following week (I will return them marked each fortnight during the next lab class that is assessed).

0.1 Solving Linear Programming Problems

A furniture company manufactures tables and chairs. The question arises as to how many of each should be made in each month in order to best utilize the available material resources? These products use two different types of timber which are delivered to the company in standard cross sections. The company has on hand:

1700	meters of softwood
1700	metres of hardwood

To make one table the carpenter uses

5	meters of softwood
2	metres of hardwood

and to make a set of four chairs the carpenters use

2	meters of softwood
6	metres of hardwood

The company has 8 full-time carpenters (for 7 hours a day) and one part-time carpenter (for 4 hours a day) and one part-time painter (for 5 hours a day). There are 20 working days in the month. Each chair set requires 3 hours of carpentry and 20 minutes of painting. Each table requires 3 hours of carpentry and 10 minutes of painting before it is made. In general they not painted until the day after it has been assembled (in order for the glue to dry).

The company sells a table for \$200 and a set of four chairs for \$160. How many tables and chairs should be made each in order to maximize potential profit. Assume all finished product is sold.

0.2 Formulation of the Model:

First one must decided on decision variables. these must characterise the physical quantities we need to determine in order to make a plan for the month. In this case let x_1 denote the number of tables to be made in a given month and x_2 the number of sets of chairs to be made in a given month. The objective is to maximize potential receipts so we want to

$$\max_{(x_1,x_2)} \quad 200x_1 + 160x_2$$

subject to not running out of raw materials and violating other procedural rules we have described. First the hardwood and softwood constraints:

$$\begin{aligned} 5x_1 + 2x_2 &\leq 1700 \quad \text{(can't exceed the supply of softwood)} \\ 2x_1 + 6x_2 &\leq 1700 \quad \text{(can't exceed the supply of hardwood)} \end{aligned}$$

Now we need to ensure that we allocate the available time. There are 8 carpenters working 7 hours a day for 20 days per month which is $(8 \times 7 \times 20)$ hours available per month plus one part time carpenter working 4 hours a day giving (4×20) hours available per month. This is a total of 1200 hours per month of carpentry time. As the carpenters require 3 hours per table and 3 hours per set of chairs we have

$$3x_1 + 3x_2 \leq 1200.$$

For the painter working 5 hours per day for 20 days per month (a total of $5 \times 20 = 100$ hours), since 10 minutes is required per table and 20 minutes per set of chairs, then either

$$\begin{aligned} \frac{1}{6}x_1 + \frac{1}{3}x_2 &\leq 100 \quad \text{hours or} \\ 10x_1 + 20x_2 &\leq 6000 \quad \text{minutes.} \end{aligned}$$

This does not take into account the requirement that a day must elapse before a piece of furniture is painted. If this is a continuous process then we may carry over partly made items to the next month and then this constraint does not affect the solution. If we stop at the end of the month or have to start up this is going to change the formulation. Now this is only an approximate formulation since only wholly completed tables and chairs can be sold. Thus x_1 and x_2 should be integers!

0.3 Solving the Model

In summary

$$\begin{aligned}
&\max \quad 200x_1 + 160x_2 \\
&\text{Subject to} \\
&\qquad 5x_1 + 2x_2 \le 1700 \\
&\qquad 2x_1 + 6x_2 \le 1700 \\
&\qquad 3x_1 + 3x_2 \le 1200 \\
&\qquad \frac{1}{6}x_1 + \frac{1}{3}x_2 \le 100 \\
&\qquad x_1,\, x_2 \ge 0.
\end{aligned} \tag{A.1}$$

We may solve Linear Programs (LP) using MATLAB by invoking the `linprog` command. Depending on the options used it will run an interior point method (the Mehrotra algorithm) of the Simplex algorithm. The syntax is:

```
>> [x,fval,exitflag,output,lambda] = ...
.            linprog(f,A,b,Aeq,beq,lb,ub,options);
```

This solves the *minimization* problem

$$\begin{aligned}
&\min \quad f^T x \\
&\text{Subject to} \\
&\qquad Ax \le b \\
&\qquad A_{eq}x = b_{eq} \\
&\qquad lb \le x \le ub.
\end{aligned}$$

The outputs are `x` the optimal solution, `fval` the optimal objective value, `exitflag` that describes the exit condition, `output` that contains information about the optimization and `lambda` the optimal Lagrange multiplier. Set `Aeq = []` and `beq = []` with empty place holders if no equalities exist etc. The starting point to `x0` option is only available with the 'interior point algorithm'. The default is the large-scale interior point algorithm. The simplex algorithm ignores any starting point and need to be chosen . Use `optimoptions` settings. In order to use the simplex algorithm you need to set the options via the following command:

```
>> options=optimoptions(@linprog,'Algorithm', 'dual-simplex');
```

Use

```
>> help linprog
```

to find out more detail of the variables `exitflag,output,lambda`.

In order to use an interior point method use:

```
>> options = optimoptions(@linprog,'Algorithm', 'interior-point')
```

but this being the default (actually a legacy version interior-point" method is now the default) it will be used unless we have reset the choice algorithm in the current session. Use the simplex method to solve the problem (P1).

0.4 A More Detailed Model

Now suppose we want to consider the introduction of kitchen dressers into our product range. The company is consider either a Colonial style dresser of a Victoria style. The resources required to make each item and the receipts and costs are in the next table:

						Receipts	Costs
Item	Softwood	hardwood	Carpentry	Painting	Glass		
Table	5 m	2 m	3 hours	10 min	nil	200	160
Chair set	2 m	6 m	3 hours	20 min	nil	160	110
Colonial Dresser	3 m	2 m	4 hours	15 min	nil	255	200
Victoria Dresser	2 m	3 m	3 hours	15 min	2 m^2	300	250
Available	1700	1700	1200	100	50 m^2		

A similar analysis to that used before gives the following problem. We denote the number of Colonial dressers made per month by x_3 and Victoria Dresser by x_4 to get:

$$\begin{aligned} \max \quad & 40x_1 + 50x_2 + 55x_3 + 50x_4 \\ \text{Subject to} \quad & \\ & 5x_1 + 2x_2 + 3x_3 + 2x_4 \leq 1700 \\ & 2x_1 + 6x_2 + 2x_3 + 4x_4 \leq 1700 \\ & 3x_1 + 3x_2 + 4x_3 + 3x_4 \leq 1200 \\ & \frac{1}{6}x_1 + \frac{1}{3}x_2 + \frac{1}{4}x_3 + \frac{1}{4}x_4 \leq 100 \\ & \qquad\qquad\qquad\qquad 2x_4 \leq 50 \\ & x_1,\, x_2,\, x_3,\, x_4 \geq 0. \end{aligned} \tag{A.2}$$

Solve this problem using the linear programming solver `linprog` with

```
>> options=optimoptions(@linprog,'Algorithm', 'dual-simplex')
>> [x1, fVal1, ExitFlag1, Out1, Lambda1] = linprog(c, A ,b, [], [],...
                    lb, ub, options);
```

Which constraints are active? Check to see which constraints hold with equality? Try adding one unit to each of the right hand side of the each active constraint in turn and resolving. Save the previous solution and hence optimal function value so you can calculate the next change in optimal function value obtained in each case. By our theory these should be negative the associated Lagrange multiplier. You can access the value of the (say) third multiplier for the inequality constraints using:

```
>> Lambda1.ineqlin(3)
```

On increasing the right hand side of the inequality constraints by one and resolving using

```
>> [x2, fVal2, ExitFlag2, Out2, Lambda2] = linprog(c, A ,b, [], [],...
                           lb, ub, options);
```

you can find the marginal cost change using

```
>> marginal3 = fVal1 - fVal2
```

0.5 The Insurance Problem (the Dual)

For insurance reasons the company wants to value the worth of its resources in the case they are destroyed by fire. The firm wants to minimize the amount of insurance it pays which is equivalent to minimizing the valuation subject to the requirement that future profits are covered by this evaluation. Denote

y_1	be the value in \$ of a meter of softwood
y_2	be the value in \$ of a meter of hardwood
y_3	be the value in \$ of an hour of carpentry time
y_4	be the value in \$ of an hour of painting time
y_5	Be the value in \$ of a m^2 of glass

Then the objective is to minimize the net evaluation

$$\min \quad \text{total evaluation} = 1700y_1 + 1700y_2 + 1200y_3 + 100y_4 + 50y_5$$

but this has to be subject to the use of the material to generate the profit that would be lost because they cannot sell the constructed items. For table the valuation of the material used would be

$$5y_1 + 2y_2 + 3y_3 + \frac{1}{6}y_4$$

which must cover the profit per table of \$40. Thus

$$5y_1 + 2y_2 + 3y_3 + \frac{1}{6}y_4 \geq 40.$$

For chairs we need

$$2y_1 + 6y_2 + 3y_3 + \frac{1}{3}y_4 \geq 50.$$

In summary for all items we have the problem

$$
\begin{aligned}
\min \quad & 1700y_1 + 1700y_2 + 1200y_3 + 100y_4 \\
\text{Subject to} & \\
& 5y_1 + 2y_2 + 3y_3 + \frac{1}{6}y_4 \geq 40 \\
& 2y_1 + 6y_2 + 3y_3 + \frac{1}{3}y_4 \geq 50 \\
& 3y_1 + 2y_2 + 4y_3 + \frac{1}{4}y_4 \geq 55 \\
& 2y_1 + 4y_2 + 3y_3 + \frac{1}{4}y_4 + 2y_5 \geq 50 \\
& y_1,\ y_2,\ y_3,\ y_4,\ y_5 \geq 0
\end{aligned}
$$

Solve this problem using `linprog` and check that the values generated correspond to the Lagrange multipliers of the original problem.

0.6 Linear Programming (to be handed in)

Example 1 *A whitegoods firm makes stoves, washing machines, dishwasher machines and clothes dryers. The major manufacturing departments are stamping department, the motor and transmission departments and the washing and dryer assembly departments, all other departments have excess capacity. Monthly departmental capacities and contributions to profit per item is given in the following table:*

	Stoves	*Washers*	*Dishwashers*	*Dryers*
Stamping Dept.	*15,000*	*10,000*	*16,000*	*12,000*
Motor & trans. Dept.	-	*15,000*	*12,500*	*20,000*
Washer Assembly	-	*8,000*	*16,000*	-
Dryer Assembly	-	-	-	*7,000*
Profit per item ($)	*110*	*220*	*250*	*140*

Let

x_1- *Number of stoves made*

x_2- *Number of Washers made*

x_3- *Number of Dishwashers made*

x_4- *Number of Dryers made*

The capacities for each department should be interpreted as a maximum capacities if all resources were concentrated on one product only. For instance, the washer assembly department can make either 8000 washers or 16,000 dishwashers. So if a combination *of both are made we need to use the fact that assembling a washer takes twice as long as assembling a dishwasher (we only can assemble 8,000 washers while in the same time we can assemble 16,000 dishwasher.) Thus we obtain a constraint for the washer assembly department:*

$$x_2 + \frac{1}{2}x_3 \leq 8,000.$$

This constraint is normalised relative to the variable x_2. Suppose also that at least 1000 *stoves,* 2000 *washing machines,* 1000 *dishwashers and* 2500 *dryers must be made to satisfy current customer orders.*

1. *Write down "normalised" constraints for the capacities of the other departments, using x_2 as the reference variable.*
2. *Formulate a linear program with an objective to maximize profit. Recall that all quantities are positive.*
3. *Solve this model and detail the solution by including total profit, the values of all variables and slack variables, along with dual variables ("opportunity costs") for each constraint.*
4. *By how much would profit increase if the stamping department had the capacity for washers increased by one extra unit in a given month?*
5. *If we relaxed the target for stoves form 1000 to 999 stoves, by how much would profit be increased?*