6. Let us now consider a more refined argument why the upwind-scheme performs better that the central scheme for u'. Recall that we proved stability for the Poisson equation via a discrete maximum principle. We say that the grid function U has a strict local maximum at i, if $U_i > U_{i-1}$ and $U_i > U_{i+1}$. Show that

   (a) U cannot have a strict local maximum if $h < 2\epsilon$ if U is the solution of the central scheme.

   (b) U cannot have a strict local maximum for all h and ϵ if U is the solution of the upwind scheme.

   What is the implication for the two numerical methods? Which solutions can oscillate and which cannot? Compare this to the behaviour of the correct solution.

   Hint: The proofs are similar to the proof of the discrete maximum principle in class.