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1. $-\varepsilon u'' - u' = 0 \qquad u(0) = 0, \quad u(1) = 1$

$$-\varepsilon \lambda^2 e^{\lambda x} - \lambda e^{\lambda x} = 0$$

Solve characteristic eqn.

$$(-\varepsilon \lambda^2 - \lambda)\, e^{-\lambda x} = 0$$

$\lambda = 0$; $\lambda = -\frac{1}{\varepsilon}$

$u_1(x) = c_1$; $u_2(x) = c_2 e^{-x/\varepsilon}$

General form of solution.

$$u(x) = c_1 + c_2 e^{-x/\varepsilon}$$

Initial conditions.

$u(0) = 0$; $u(1) = 1$

$c_1 + c_2 = 0$; $c_1 + c_2^{-x/\varepsilon} = 1$

$\Rightarrow$

Solve system

$c_1 = -c_2$

$-c_2 + c_2 e^{-1/\varepsilon} = 1$

$-c_2 (1 - e^{-1/\varepsilon}) = 1$

$c_2 = \frac{-1}{1 - e^{-1/\varepsilon}}$

$c_1 = \frac{1}{1 - e^{1/\varepsilon}}$

Plug in to $u(x)$.

$$\frac{1}{1 - e^{-1/\varepsilon}} - \frac{1}{1 - e^{-1/\varepsilon}} \left(e^{-x/\varepsilon}\right) = u(x)$$

$$\boxed{\frac{1 - e^{-x/\varepsilon}}{1 - e^{-1/\varepsilon}} = u(x)}$$