

5.2 Higher order to first order

$$y^{(4)} = (y'')^2 + \sin y - \cos t$$

$$v^{(4)} = (v^{(2)})^2 + \sin(v) - \cos(t)$$

$$u_1(t) = v(t)$$

$$u_2(t) = v'(t) = u_1'(t)$$

$$u_3(t) = v''(t) = u_2'(t)$$

$$u_4(t) = v'''(t) = u_3'(t)$$

$$u_5(t) = t$$

$$u_5'(t) = 1$$

$$t_0 = 0, \quad u_5(t_0) = t_0 = 0$$

$$u_1'(t) = u_2(t)$$

$$u_2'(t) = u_3(t)$$

$$u_3'(t) = u_4(t)$$

$$u_4'(t) = u_3^2(t) + \sin(u_1(t)) - \cos(u_5(t))$$

Then we have $u'(t) = f(u(t))$.

$$f(u) = \begin{bmatrix} u_2 \\ u_3 \\ u_4 \\ u_3^2 + \sin(u_1) - \cos(u_5) \\ 1 \end{bmatrix}$$

$$\text{and } u(t_0) = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ 0 \end{bmatrix}$$

$$u_1'(t) = u_2(t)$$

$$u_2'(t) = u_3(t)$$

$$u_3'(t) = u_4(t)$$

$$u_4'(t) = u_3^2(t) + \sin(u_1(t)) - \cos(u_5(t))$$