5. Implement Euler and Runge-Kutta Methods

Consider the initial value problem

$$y' = \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}' = \begin{pmatrix} y_1 \\ -y_0 \end{pmatrix} \qquad\qquad y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

This is a first order equivalent for the ODE $x'' + x = 0$.

1. Solve the ODE with the Euler method, improved Euler method and the classical Runge-Kutta method in the time interval $[0, 2\pi]$ with $n = 10, 20, \ldots, 100$ steps. Make a solution plot only for the last number of steps.

2. Make a log-log plot of n versus the error of $y_0(2\pi)$ of the first component at the last time-step for all three methods. The correct solution is $y_0(t) = \cos(t)$.