Waldmann

(5.4)

1.) $\frac{1}{h}\left(\frac{3}{2}u(t) - 2u(t-h) + \frac{1}{2}u(t-2h)\right) + O(h^2)$ =

$$u(t-2h) = u(\bar{t}) - 2h\,u'(\bar{t}) + \frac{4h^2}{2}u''(\bar{t}) + O(h^3).$$

$$u(t-h) = u(\bar{t}) - h\,u'(\bar{t}) + \frac{1}{2}h^2 u''(\bar{t}) + O(h^3)$$

$$\frac{1}{h}\left(\frac{3}{2}u(t) - 2u(t) + 2h\,u'(t) - (2)\frac{1}{2}h^2u''(t) + \frac{1}{2}u(t) - h u'(t) + \frac{2}{2}h^2u''(t) + O(h^3)\right) + O(h^2)$$

$$= \frac{1}{h}\left(\left[\frac{3}{2}u(t) - 2u(t) + \frac{1}{2}u(t)\right] + \left[2hu'(t) - hu'(t)\right] + h^2u''(t) - h^2u''(t)\right) + O(h)^2$$

$$= \frac{1}{h}\left(h\,u'(t) + \right) + O(h^2)$$

$$= u'(t) + O(h^2). \checkmark$$

2.) $y' = \frac{1}{h}\left(\frac{3}{2}y(t) - 2y(t-h) + \frac{1}{2}y(t-2h)\right) + O(h^2) = f(y,t).$

$$y(t) = \left(hf(y,t) + 2y(t-h) - \frac{1}{2}y(t-2h)\right)\frac{2}{3}$$

$$u^K = \frac{2}{3}\left(hf + 2u^{K-1} - \frac{1}{2}u^{K-2}\right)$$

3. $O(h^2)$, from part 1 $\tau = \left|u'(t) - (u'(t) + O(h))\right|$

$$= O(h^2)$$