3. Coding: Finite Differences

Consider the derivative $u'(\bar{x})$ for $u(x) = \sin(x)$ and $\bar{x} = 1.0$. Calculate the derivative exactly and approximate it by the difference quotients $D_+u(1.0)$, $D_-u(1.0)$ and $D_0u(1.0)$.

1. Make a log-log plot of the absolute errors $|u'(\bar{x}) - D_\square u(\bar{x})|$ with $\square \in \{+, -, 0\}$ and $h = \exp(-n)$ with $n = 0, \ldots, 40$.

   Hint: Use `matplotlib.pyplot.loglog`

2. For a Numpy array `a`, the command `np.finfo(a.dtype).eps` returns the maximal (relative) round off error of the floating point numbers used in the array. Compare this number to the errors you calculated in part (a). What do you observe?