

5.3

Waldmann

$$(1) \quad y^{n+1} = y_n + \frac{1}{4}h f(y_n, t_n) + \frac{3}{4}h f\left(y_n + \frac{2}{3}hf(y_n, t_n), t_n + \frac{2}{3}h\right)$$

$$K_1 = f(y_n, t_n)$$

$$K_2 = f\left(y_n + \frac{2}{3}hK_1, t_n + \frac{2}{3}h\right)$$

$$y^{n+1} = y_n + \frac{1}{4}hK_1 + \frac{3}{4}hK_2$$

0	0	
$\frac{2}{3}$	$\frac{2}{3}$	0
	$\frac{1}{4}$	$\frac{3}{4}$

(2)

0				
$\frac{1}{3}$	$\frac{1}{3}$			
$\frac{2}{3}$	$-\frac{1}{3}$	1		
1	1	-1	1	
	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$K_1 = f(y_n, t_n)$$

$$K_2 = f\left(y_n + \frac{1}{3}hK_1, t_n + \frac{1}{3}h\right)$$

$$K_3 = f\left(y_n - \frac{1}{3}hK_1 + hK_2, t_n + \frac{2}{3}h\right)$$

$$K_4 = f\left(y_n + hK_1 - hK_2 + hK_3, t_n + h\right)$$

$$y^{n+1} = y_n + \frac{h}{8}K_1 + \frac{3h}{8}K_2 + \frac{3h}{8}K_3 + \frac{h}{8}K_4$$