

4. Higher order method via higher order finite difference formula

1. Prove the finite difference formula

$$u'(t) = \frac{1}{h} \left(\frac{3}{2}u(t) - 2u(t-h) + \frac{1}{2}u(t-2h) \right) + \mathcal{O}(h^2).$$

2. Use this finite difference formula to derive a numerical method to solve the ODE

$$y' = f(y, t), \quad y(0) = y_0.$$

3. What is the local truncation error of this method?