1. Order

Consider the boundary value problem

$$-u_i'' = f_i, \qquad u(0) = 0, \qquad u(1) = 1,$$

$i \in \{0, 1\}$ with right hand sides and corresponding solutions

$$u_0 = \sin(\pi x) \qquad f_0 = \pi^2 \sin(\pi x)$$

and

$$u_1 = \begin{cases} \sin(\pi x) & x \leq 0.5 \\ 4x(1-x) & x > 0.5 \end{cases} \qquad f_1 = \begin{cases} \pi^2 \sin(\pi x) & x \leq 0.5 \\ 8 & x > 0.5 \end{cases}$$

1. Implement the second order finite difference method and the fourth order finite difference method from Section 2.20.1.
2. For $m \in \{10, 20, 30, \ldots, 200\}$ grid points, make loglog plots of the errors for each method and each problem in the maximum norm.
3. Why does the result for the fourth order method applied to f_1 not contradict the theoretical convergence bounds?