

# **Session 1: Multiple linear regression review**

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# Learning objectives and outline

# Learning objectives

- 1 identify systematic and random components of a multiple linear regression model
- 2 define terminology used in a multiple linear regression model
- 3 define and explain the use of dummy variables
- 4 interpret multiple linear regression coefficients for continuous and categorical variables
- 5 use model formulae to multiple linear models
- 6 define and interpret interactions between variables
- 7 interpret ANOVA tables

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Learning  
objectives and  
outline

Multiple  
Linear  
Regression

Interaction  
(effect  
modification)

Analysis of  
Variance

Model  
formulae

- 1 multiple regression terminology and notation
- 2 continuous & categorical predictors
- 3 interactions
- 4 ANOVA tables
- 5 Model formulae

# Multiple Linear Regression

# Systematic part of model

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For more detail: Vittinghoff section 4.2

$$E[y|x] = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$$

- $E[y|x]$  is the expected value of  $y$  given  $x$
- $y$  is the outcome, response, or dependent variable
- $x$  is the vector of predictors / independent variables
- $x_p$  are the individual predictors or independent variables
- $\beta_p$  are the regression coefficients

# Random part of model

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$$y_i = E[y_i|x_i] + \epsilon_i$$

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \dots + \beta_p x_{pi} + \epsilon_i$$

- $x_{ji}$  is the value of predictor  $x_j$  for observation  $i$

Assumption:  $\epsilon_i \stackrel{iid}{\sim} N(0, \sigma_\epsilon^2)$

- Normal distribution
- Mean zero at every value of predictors
- Constant variance at every value of predictors
- Values that are statistically independent

# Continuous predictors

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- **Coding:** as-is, or may be scaled to unit variance (which results in *adjusted* regression coefficients)
- **Interpretation for linear regression:** An increase of one unit of the predictor results in this much difference in the continuous outcome variable
  - *additive model*



# Binary predictors (2 levels)

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- **Coding:** indicator or dummy variable (0-1 coding)
- **Interpretation for linear regression:** the increase or decrease in average outcome levels in the group coded “1”, compared to the reference category (“0”)
  - e.g.  $E(y|x) = \beta_0 + \beta_1 x$
  - where  $x = \{ 1 \text{ if male, } 0 \text{ if female } \}$

# Multilevel Categorical Predictors (Ordinal or Nominal)

- **Coding:**  $K - 1$  dummy variables for  $K$ -level categorical variables \*
- **Interpretation for linear regression:** as above, the comparisons are done with respect to the reference category
- Testing significance of multilevel categorical predictor: partial F-test, a.k.a. nested ANOVA

\* STATA and R code dummy variables automatically, behind-the-scenes

# Inference from multiple linear regression

- Coefficients are t-distributed when assumptions are correct
- Variance in the estimates of each coefficient can be calculated
- The t-test of the null hypothesis  $H_0 : \beta_1 = 0$  and from confidence intervals tests whether  $x_1$  predicts  $y$ , *holding other predictors constant*
  - often used in causal inference to control for confounding: see section 4.4

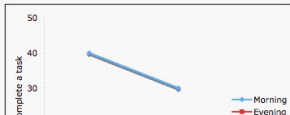
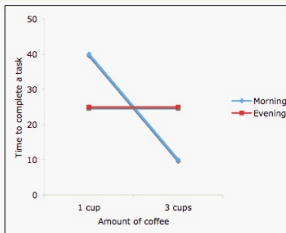
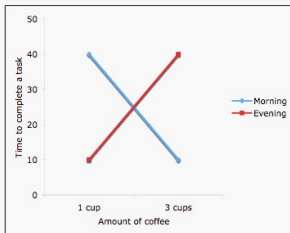
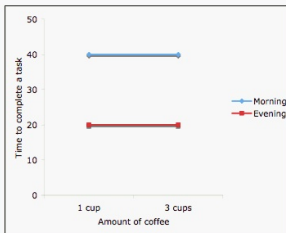
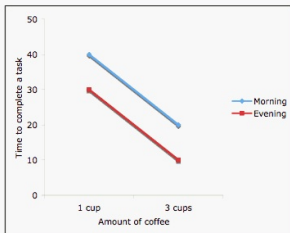
# Interaction (effect modification)

# How is interaction / effect modification modeled?

Interaction is modeled as the product of two covariates:

$$E[y|x] = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 * x_2$$

# What is interaction / effect modification?



# Analysis of Variance

# Review of the ANOVA table

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Source of Variation	Sum Sq	Deg Fr	Mean Sq	F
Model	MSS	k	MSS/k	(MSS/k)/
Residual	RSS	n-(k-1)	RSS/(n-k-1)	
Total	TSS	n-1		

- $k$  = Model degrees of freedom = coefficients - 1
- $n$  = Number of observations
- **F** is F-distributed with  $k$  numerator and  $n - (k - 1)$  denominator degrees of freedom



# Model formulae

# What are model formulae?

## Model formulae tutorial

- Model formulae are shortcuts to defining linear models in R
- Regression functions in R such as `aov()`, `lm()`, `glm()`, and `coxph()` all accept the “model formula” interface.
- The formula determines the model that will be built (and tested) by the R procedure. The basic format is:  
*response variable ~ explanatory variables*
- The tilde means “is modeled by” or “is modeled as a function of.”

# Model formula for simple linear regression

$$y \sim x$$

- where “x” is the explanatory (independent) variable
- “y” is the response (dependent) variable.

# Model formula for multiple linear regression

Additional explanatory variables would be added as follows:

$$y \sim x + z$$

Note that “+” does not have its usual meaning, which would be achieved by:

$$y \sim l(x + z)$$

# Types of standard linear models

`lm( y ~ u + v)`

u and v factors: **ANOVA**

u and v numeric: **multiple regression**

one factor, one numeric: **ANCOVA**

# Model formulae cheatsheet

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symbol	example	meaning
+	+ x	include this variable
-	- x	delete this variable
:	x : z	include the interaction
	x * z	include these variables and their interaction
/	x / z	nesting: include z nested within x
	x   z	conditioning: include x given z
^	(u + v + w)^3	include these variables and all interactions up to three way
1	-1	intercept: delete the intercept

# Model formulae comprehension Q&A #1

How to interpret the following model formulae?

$$y \sim u + v + w + u:v + u:w + v:w$$

$$y \sim u * v * w - u:v:w$$

$$y \sim (u + v + w)^2$$

# Model formulae comprehension Q&A #2

How to interpret the following model formulae?

$$y \sim u + v + w + u:v + u:w + v:w + u:v:w$$

$$y \sim u * v * w$$

$$y \sim (u + v + w)^3$$