BIOS 621/821 Session 1 - multiple linear regression review

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Outline - session 1

- multiple regression terminology and notation
  - continuous & categorical predictors
  - interactions
  - ANOVA tables
  - Model formulae

Multiple Linear Regression Model (sec. 4.2)

Systematic part of model:

$$E[y|x] = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_p x_p$$

- $E[y|x]$ is the expected value of y given x
- y is the outcome, response, or dependent variable
- x is the vector of predictors / independent variables
- x_p are the individual predictors or independent variables
- β_p are the regression coefficients

Multiple Linear Regression Model (cont'd)

Random part of model:

$y_i = E[y_i|x_i] + \epsilon_i$

$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + ... + \beta_p x_{pi} + \epsilon_i$

- x_{ji} is the value of predictor x_j for observation i

Assumption: $\epsilon_i \overset{iid}{\sim} N(0, \sigma_\epsilon^2)$

- Normal distribution
- Mean zero at every value of predictors
- Constant variance at every value of predictors
- Values that are statistically independent

Continuous predictors

- **Coding:** as-is, or may be scaled to unit variance (which results in *adjusted* regression coefficients)
- **Interpretation for linear regression:** An increase of one unit of the predictor results in this much difference in the continuous outcome variable
  - *additive model*

Binary predictors (2 levels)

- **Coding:** indicator or dummy variable (0-1 coding)
- **Interpretation for linear regression:** the increase or decrease in average outcome levels in the group coded "1", compared to the reference category ("0")
  - *e.g.* $E(y|x) = \beta_0 + \beta_1 x$
  - where x=$\{$ 1 if male, 0 if female $\}$

Multilevel Categorical Predictors (Ordinal or Nominal)

- **Coding:** $K - 1$ dummy variables for K-level categorical variables *
- **Interpretation for linear regression:** as above, the comparisons are done with respect to the reference category
- Testing significance of multilevel categorical predictor: partial F-test, a.k.a. nested ANOVA

* STATA and R code dummy variables automatically, behind-the-scenes

Inference from multiple linear regression

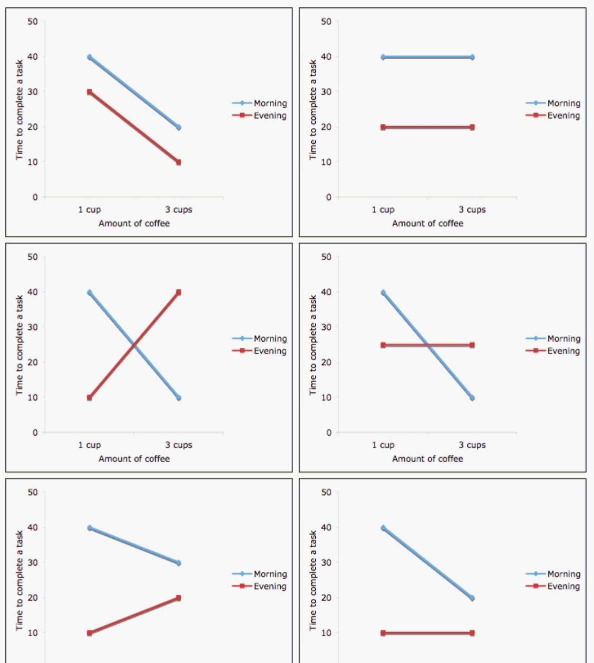
- Coefficients are t-distributed when assumptions are correct
- Variance in the estimates of each coefficient can be calculated
- The t-test of the null hypothesis $H_0 : \beta_1 = 0$ and from confidence intervals tests whether x_1 predicts y, *holding other predictors constant*
    - often used in causal inference to control for confounding: see section 4.4

Interaction (effect modification)

Interaction is modeled as the product of two covariates:

$$E[y|x] = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 * x_2$$

Interaction (effect modification)



ANOVA table

Source of Variation	Sum Sq	Deg Fr	Mean Sq	F
Model	MSS	k	MSS/k	(MSS/k)/MSE
Residual	RSS	n-(k-1)	RSS/(n-k-1)	
Total	TSS	n-1		

- k = Model degrees of freedom = coefficients - 1
- n = Number of observations
- **F** is F-distributed with k numerator and $n-(k-1)$ denominator degrees of freedom

Regression in R: model formulae

Model formulae tutorial

- regression functions in R such as `aov()`, `lm()`, `glm()`, and `coxph()` use a "model formula" interface.
- The formula determines the model that will be built (and tested) by the R procedure. The basic format is:

  response variable $\sim$ *explanatory variables*

- The tilde means "is modeled by" or "is modeled as a function of."

Model formulae (cont'd)

Model formula for simple linear regression:

$$y \sim x$$

- where "x" is the explanatory (independent) variable
- "y" is the response (dependent) variable.

Model formulae (cont'd)

Additional explanatory variables would be added as follows:

$$y \sim x + z$$

Note that "+" does not have its usual meaning, which would be achieved by:

$$y \sim I(x + z)$$

Types of standard linear models

`lm( y ~ u + v)`

`u` and `v` factors: **ANOVA**
`u` and `v` numeric: **multiple regression**
one factor, one numeric: **ANCOVA**

Model formulae (cont'd)

<table>
<tr><th>symbol</th><th>example</th><th>meaning</th></tr>
<tr><td>+</td><td>+ x</td><td>include this variable</td></tr>
<tr><td>-</td><td>- x</td><td>delete this variable</td></tr>
<tr><td>:</td><td>x : z</td><td>include the interaction</td></tr>
<tr><td></td><td>x * z</td><td>include these variables and their interactions</td></tr>
<tr><td>/</td><td>x / z</td><td>nesting: include z nested within x</td></tr>
<tr><td>|</td><td>x | z</td><td>conditioning: include x given z</td></tr>
<tr><td>^</td><td>(u + v + w)^3</td><td>include these variables and
all interactions up to three way</td></tr>
<tr><td>1</td><td>-1</td><td>intercept: delete the intercept</td></tr>
</table>

Model formulae (cont'd)

How to interpret the following model formulae?

y ~ u + v + w + u:v + u:w + v:w
y ~ u * v * w - u:v:w
y ~ (u + v + w)^2

Model formulae (cont'd)

How to interpret the following model formulae?

$y \sim u + v + w + u{:}v + u{:}w + v{:}w + u{:}v{:}w$

$y \sim u * v * w$

$y \sim (u + v + w)\hat{}3$