

# **Session 5: loglinear regression part 2**

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CUNY SPH Biostatistics 2

# Learning objectives and outline

# Learning objectives

- 1 Define and identify over-dispersion in count data
- 2 Define the negative binomial (NB) distribution and identify applications for it
- 3 Define zero-inflated count models
- 4 Fit and interpret Poisson and NB, with and without zero inflation

- ① Review of log-linear Poisson glm
- ② Review of diagnostics and interpretation of coefficients
- ③ Over-dispersion
  - Negative Binomial distribution
- ④ Zero-inflated models

## Resources:

- Vittinghoff section 8.1-8.3
- Short tutorials on regression in R (and Stata, SAS, SPSS, Mplus)
  - <https://stats.idre.ucla.edu/other/dae/>

**Session 5:  
loglinear  
regression  
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# Review

# Components of GLM

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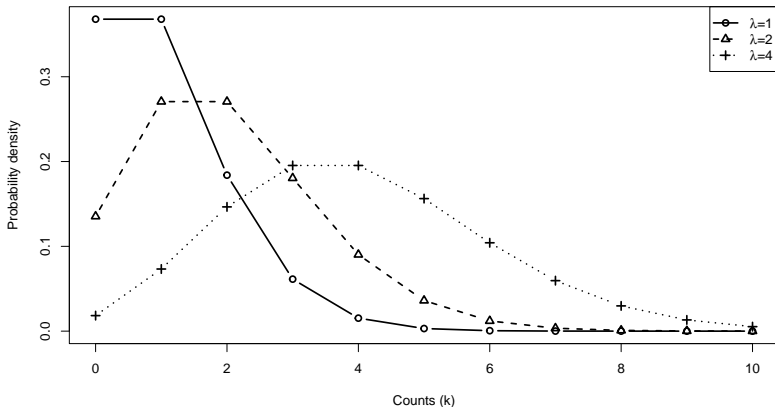
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- **Random component** specifies the conditional distribution for the response variable - it doesn't have to be normal but can be any distribution that belongs to the "exponential" family of distributions
- **Systematic component** specifies linear function of predictors (linear predictor)
- **Link** [denoted by  $g(\cdot)$ ] specifies the relationship between the expected value of the random component and the systematic component, can be linear or nonlinear

# Motivating example: Choice of Distribution

- Count data are often modeled as Poisson distributed:
  - mean  $\lambda$  is greater than 0
  - variance is also  $\lambda$
  - Probability density  $P(k, \lambda) = \frac{\lambda^k}{k!} e^{-\lambda}$



# Poisson model: the GLM

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The **systematic part** of the GLM is:

$$\log(\lambda_i) = \beta_0 + \beta_1 \text{RACE}_i + \beta_2 \text{TRT}_i + \beta_3 \text{ALCH}_i + \beta_4 \text{DRUG}_i$$

Or alternatively:

$$\lambda_i = \exp(\beta_0 + \beta_1 \text{RACE}_i + \beta_2 \text{TRT}_i + \beta_3 \text{ALCH}_i + \beta_4 \text{DRUG}_i)$$

The **random part** is (Recall the  $\lambda_i$  is both the mean and variance of a Poisson distribution):

$$y_i \sim \text{Poisson}(\lambda_i)$$



## Example: Risky Drug Use Behavior

- Outcome is # times the drug user shared a syringe in the past month (`shared_syr`)
- Predictors: `sex`, `ethn`, `homeless`
  - filtered to `sex` "M" or "F", `ethn` "White", "AA", "Hispanic"

```
summary(needledat2$shared_syr)
```

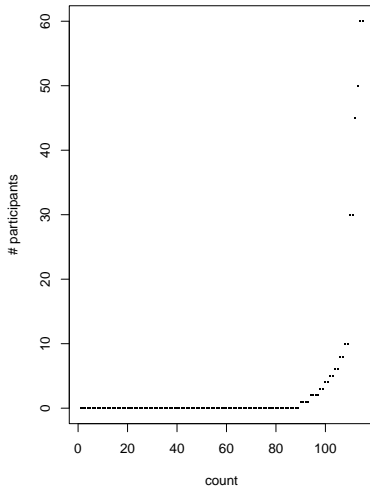
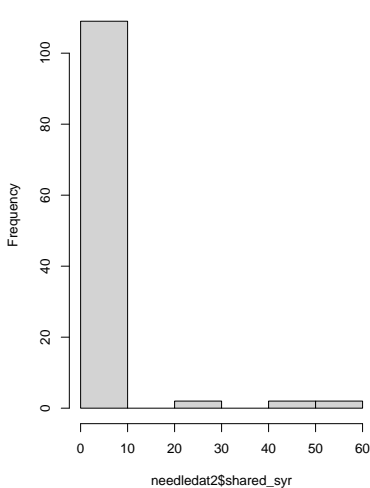
```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.     NA's  
##      0.000   0.000   0.000   3.122   0.000   60.000         2
```

```
var(needledat2$shared_syr, na.rm = TRUE)
```

```
## [1] 113.371
```

# Example: Risky Drug Use Behavior

## Exploratory plots

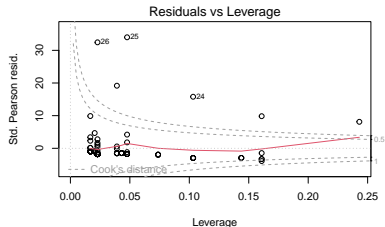
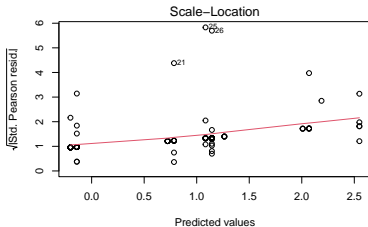
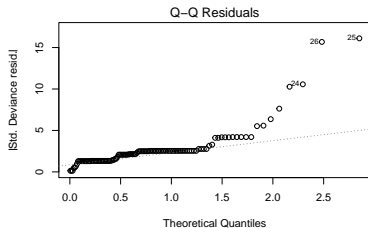
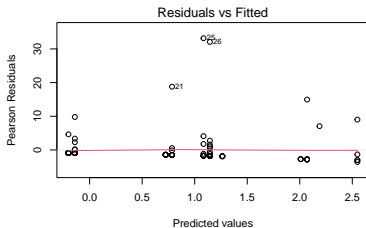


- There are a *lot* of zeros and variance is much greater than

# Fitting a Poisson model

```
fit.pois <- glm(shared_syr ~ sex + ethn + homeless,  
                 data = needledat2,  
                 family = poisson(link = "log"))
```

# Residuals plots



\* Poisson model is definitely not a good fit.

# Over-dispersion

# When the Poisson model doesn't fit

- 1 Variance  $>$  mean (over-dispersion)
  - Negative binomial distribution
- 2 Excess zeros (zero inflation)
  - Can introduce zero-inflation

# Negative binomial distribution

- The binomial distribution is the number of successes in  $n$  trials:
  - Roll a die ten times, how many times do you see a 6?
- The negative binomial distribution is the number of successes it takes to observe  $r$  failures:
  - How many times do you have to roll the die to see a 6 ten times?
  - Note that the number of rolls is no longer fixed.
  - In this example,  $p=5/6$  and a 6 is a “failure”

## Negative binomial GLM

One way to parametrize a NB model is with a **systematic part** equivalent to the Poisson model:

$$\log(\lambda_i) = \beta_0 + \beta_1 \text{RACE}_i + \beta_2 \text{TRT}_i + \beta_3 \text{ALCH}_i + \beta_4 \text{DRUG}_i$$

Or:

$$\lambda_i = \exp(\beta_0 + \beta_1 \text{RACE}_i + \beta_2 \text{TRT}_i + \beta_3 \text{ALCH}_i + \beta_4 \text{DRUG}_i)$$

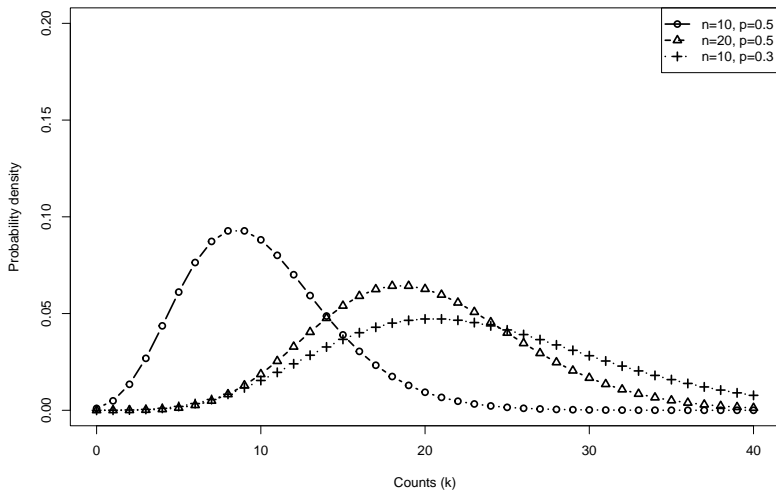
And a **random part**:

$$y_i \sim NB(\lambda_i, \theta)$$

- $\theta$  is a **dispersion parameter** that is estimated
- When  $\theta = 0$  it is equivalent to Poisson model
- `MASS::glm.nb()` uses this parametrization, `dnbinom()` does not
- The Poisson model can be considered **nested** within the Negative Binomial model

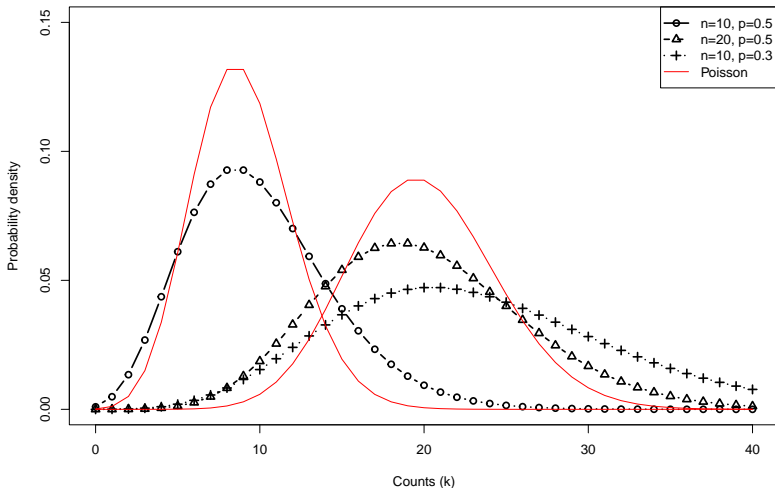


# Negative Binomial Random Distribution



# Compare Poisson vs. Negative Binomial

Negative Binomial Distribution has two parameters: # of trials  $n$ , and probability of success  $p$



# Negative Binomial Regression

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```
library(MASS)
fit.negbin <- MASS::glm.nb(shared_syr ~ sex +
                             ethn + homeless,
                             data = needledat2)
```

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```
summary(fit.negbin)
```

```
##
## Call:
## MASS::glm.nb(formula = shared_syr ~ sex + ethn + homeless, data = needledat2,
##   init.theta = 0.07743871374, link = log)
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   0.4641     0.8559   0.542  0.5876
## sexM          -1.0148     0.8294  -1.224  0.2211
## ethnHispanic   1.3424     1.3201   1.017  0.3092
## ethnWhite      0.2429     0.7765   0.313  0.7544
## homelessyes    1.6445     0.7073   2.325  0.0201 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for Negative Binomial(0.0774) family taken to be 1)
##
##    Null deviance: 62.365  on 114  degrees of freedom
## Residual deviance: 56.232  on 110  degrees of freedom
## (2 observations deleted due to missingness)
## AIC: 306.26
##
## Number of Fisher Scoring iterations: 1
##
##              Theta:  0.0774
##             Std. Err.: 0.0184
##
## 2 x log-likelihood: -294.2550
```

# Likelihood ratio test

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Basis: Under  $H_0$ : no improvement in fit by more complex model, difference in model residual deviances is  $\chi^2$ -distributed.

Deviance:  $\Delta(D) = -2 * \Delta(\log \text{likelihood})$

```
(ll.negbin <- logLik(fit.negbin))
```

```
## 'log Lik.' -147.1277 (df=6)
```

```
(ll.pois <- logLik(fit.pois))
```

```
## 'log Lik.' -730.0133 (df=5)
```

```
pchisq(2 * (ll.negbin - ll.pois), df=1,  
       lower.tail=FALSE)
```

```
## 'log Lik.' 1.675949e-255 (df=6)
```

# NB regression residuals plots

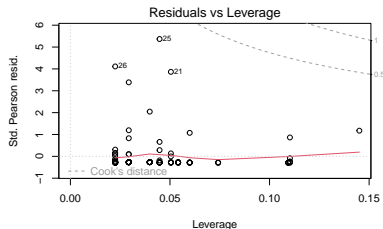
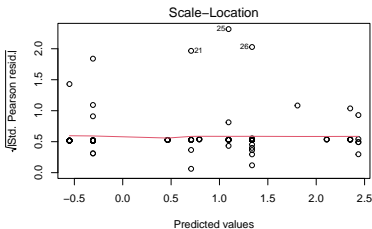
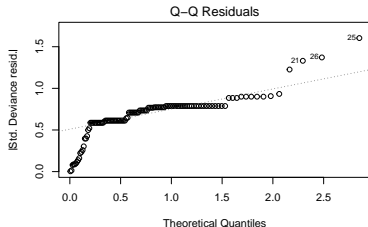
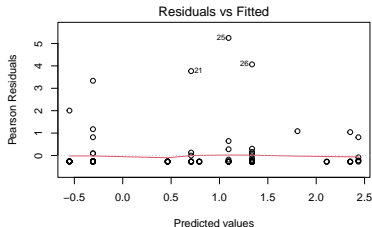
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# Zero Inflation

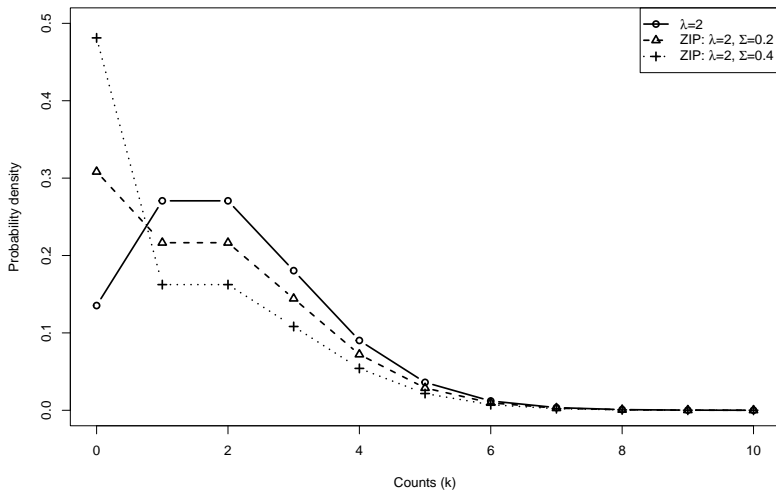
# Zero inflated “two-step” models

**Step 1:** logistic model to determine whether count is zero or Poisson/NB

**Step 2:** Poisson or NB regression distribution for  $y_i$  not set to zero by 1.



# Poisson Distribution with Zero Inflation



# Zero-inflated Poisson regression

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```
library(pscl)
fit.ZIpois <-
  pscl::zeroinfl(shared_syr~sex+ethn+homeless,
                  dist = "poisson",
                  data = needledat2)
```

```
summary(fit.ZIpois)
```

```
##
## Call:
## pscl::zeroinfl(formula = shared_syr ~ sex + ethn + homeless, data = needledat2,
##   dist = "poisson")
##
## Pearson residuals:
##      Min       1Q   Median       3Q      Max
## -1.0761 -0.5784 -0.4030 -0.3341 10.6835
##
## Count model coefficients (poisson with log link):
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   3.2169    0.1796  17.909 < 2e-16 ***
## sexM          -1.4725    0.1442 -10.212 < 2e-16 ***
## ethnHispanic  -0.1525    0.1576  -0.968 0.333223
## ethnWhite     -0.5236    0.1464  -3.577 0.000347 ***
## homelessyes   1.2034    0.1455   8.268 < 2e-16 ***
##
## Zero-inflation model coefficients (binomial with logit link):
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   2.06262    0.65227   3.162 0.00157 **
## sexM          -0.05067    0.58252  -0.087 0.93068
## ethnHispanic  -1.76120    0.81177  -2.170 0.03004 *
## ethnWhite     -0.50187    0.56919  -0.882 0.37792
## homelessyes   -0.53013    0.48108  -1.102 0.27048
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Number of iterations in BFGS optimization: 18
## Log-likelihood: -299.8 on 10 Df
```

# Zero-inflated Negative Binomial regression

```
fit.ZInegbin <-  
  pscl::zeroinfl(shared_syr~sex+ethn+homeless,  
                  dist = "negbin",  
                  data = needledat2)
```

- *NOTE*: zero-inflation model can include any of your variables as predictors
- *WARNING* Default in `zeroinfl()` function is to use *all* variables as predictors in logistic model

```
summary(fit.ZInegbin)
```

```
##
## Call:
## pscl::zeroinfl(formula = shared_syr ~ sex + ethn + homeless, data = needledat2,
##   dist = "negbin")
##
## Pearson residuals:
##      Min       1Q   Median       3Q      Max
## -0.5401 -0.3255 -0.2715 -0.1926  5.1489
##
## Count model coefficients (negbin with log link):
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    2.8401     1.1845   2.398  0.01649 *
## sexM           -2.2278     0.9350  -2.382  0.01720 *
## ethnHispanic  -0.4116     0.9832  -0.419  0.67545
## ethnWhite     -0.4294     0.8647  -0.497  0.61949
## homelessyes    1.9461     0.7103   2.740  0.00615 **
## Log(theta)    -1.1972     0.5159  -2.320  0.02032 *
##
## Zero-inflation model coefficients (binomial with logit link):
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    1.6863     0.8466   1.992  0.0464 *
## sexM           -0.9919     0.8016  -1.237  0.2159
## ethnHispanic  -11.3556    112.8675  -0.101  0.9199
## ethnWhite     -0.7452     0.7304  -1.020  0.3076
## homelessyes    0.3555     0.7397   0.481  0.6308
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Theta = 0.302
## Number of iterations in BFGS optimization: 37
## Log-likelihood: -142.8 on 11 Df
```

# Zero-inflated NB - simplified

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- Model is much more interpretable if the exposure of interest is *not* included in the zero-inflation model.
- E.g. with HIV status as the only predictor in zero-inflation model:

```
fit.ZInb2 <- pscl::zeroinfl(shared_syr ~ sex + ethn +  
                             homeless + hiv | hiv,  
                             dist = "negbin",  
                             data = needledat2)
```

```
summary(fit.ZInb2)
```

```
##
## Call:
## pscl::zeroinfl(formula = shared_syr ~ sex + ethn + homeless + hiv | hiv,
##   data = needledat2, dist = "negbin")
##
## Pearson residuals:
##      Min       1Q   Median       3Q      Max
## -0.4299 -0.3646 -0.3559 -0.3299  6.3053
##
## Count model coefficients (negbin with log link):
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   3.6685     0.9470   3.874 0.000107 ***
## sexM          -1.7648     0.6205  -2.844 0.004454 **
## ethnHispanic  -1.5808     0.7446  -2.123 0.033757 *
## ethnWhite     -1.1268     0.6924  -1.627 0.103662
## homelessyes    1.0313     0.5692   1.812 0.070025 .
## hivpositive   -1.0820     1.0167  -1.064 0.287245
## hivyes        2.3723     0.7829   3.030 0.002443 **
## Log(theta)    0.1396     0.4647   0.300 0.763941
##
## Zero-inflation model coefficients (binomial with logit link):
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   1.2163     0.2851   4.266 1.99e-05 ***
## hivpositive   -0.3493     0.9389  -0.372 0.710
## hivyes       -17.9654    3065.6162 -0.006 0.995
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Theta = 1.1498
## Number of iterations in BFGS optimization: 59
## Log-likelihood: -122.5 on 11 Df
```

# Intercept-only ZI model

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```
fit.ZInb3 <-  
  pscl::zeroinfl(shared_syr~sex+ethn+homeless|1,  
    dist = "negbin",  
    data = needledat2)
```



```
summary(fit.ZInb3)
```

```
##
## Call:
## pscl::zeroinfl(formula = shared_syr ~ sex + ethn + homeless | 1, data = needledat2,
##   dist = "negbin")
##
## Pearson residuals:
##      Min      1Q  Median      3Q      Max
## -0.3159 -0.3123 -0.3040 -0.2953  5.2941
##
## Count model coefficients (negbin with log link):
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  2.08551    1.42665   1.462  0.1438
## sexM         -1.43812    0.89188  -1.612  0.1069
## ethnHispanic  0.48126    1.16639   0.413  0.6799
## ethnWhite    -0.07421    0.81066  -0.092  0.9271
## homelessyes   1.62076    0.67705   2.394  0.0167 *
## Log(theta)   -1.12533    0.89365  -1.259  0.2079
##
## Zero-inflation model coefficients (binomial with logit link):
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.5211     0.7599   0.686   0.493
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Theta = 0.3245
## Number of iterations in BFGS optimization: 37
## Log-likelihood: -146.8 on 7 Df
```

# Confidence intervals

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Use the `confint()` function for all these models (don't try to specify which package `confint` comes from). E.g.:

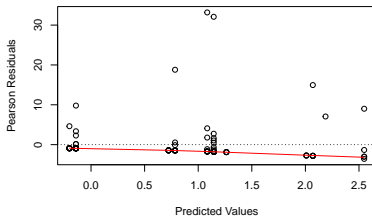
```
confint(fit.ZInb3)
```

##		2.5 %	97.5 %
##	count_(Intercept)	-0.7106815	4.8816948
##	count_sexM	-3.1861734	0.3099239
##	count_ethnHispanic	-1.8048157	2.7673339
##	count_ethnWhite	-1.6630653	1.5146489
##	count_homelessyes	0.2937556	2.9477604
##	zero_(Intercept)	-0.9683324	2.0105711

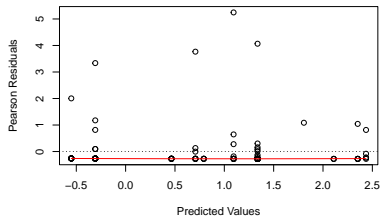
# Residuals vs. fitted values

I invisibly define functions `plotpanel1` and `plotpanel2` that will work for all types of models (see lab). These use Pearson residuals.

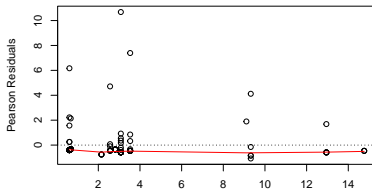
Residuals vs. Fitted  
Poisson



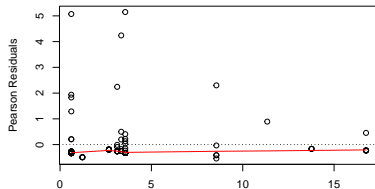
Residuals vs. Fitted  
Negative Binomial



Residuals vs. Fitted  
Zero-inflated Poisson

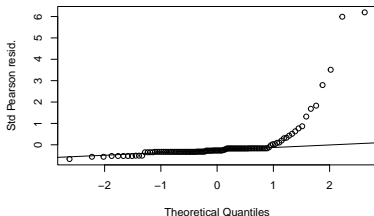


Residuals vs. Fitted  
Zero-inflated Negative Binomial

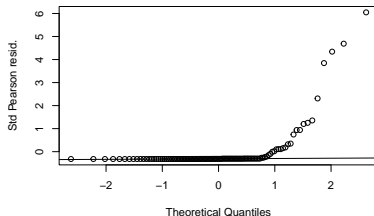


# Quantile-quantile plots for residuals

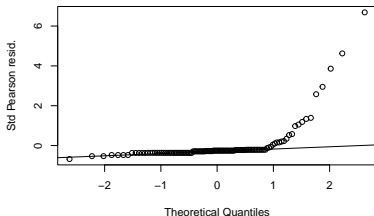
Normal Q-Q Plot  
Poisson



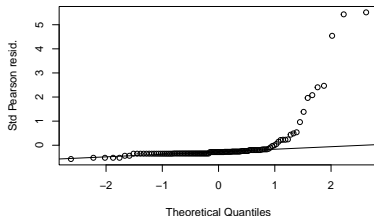
Normal Q-Q Plot  
Negative Binomial



Normal Q-Q Plot  
Zero-inflated Poisson



Normal Q-Q Plot  
Zero-inflated Negative Binomial



*still* over-dispersed - ideas?

# Summary / Conclusions

- These are multiplicative models
- Fitting zero-inflated models can be problematic (convergence, over-complicated default models), especially for small samples
- Use QQ and residuals plots to assess model fit
- Can use LRT to compare nested models