
HAPPY NEW YEAR
2020

Review 2019

INTERACTION

Topics in Data Science – chapter 5.1 to 5.3 – Hanna Braun – 08.01.2020

Single intervention

heart transplant:
A = 1

$Y^{a=1}$

no heart transplant:
A = 0

$Y^{a=0}$

Joint interventions

	heart transplant: **A = 1**	**no heart transplant:** **A = 0**
vitamins **E = 1**	$A = 1, E = 1$ $Y^{a=1,e=1}$	$A = 0, E = 1$ $Y^{a=0,e=1}$
no vitamins **E = 0**	$A = 1, E = 0$ $Y^{a=1,e=0}$	$A = 0, E = 0$ $Y^{a=0,e=0}$

interventions on two or more treatments = joint interventions

Interaction – formal definition

There is interaction between two treatments A and E if the causal effect of A on Y after a joint intervention that set E to 1 differs from the causal effect of A on Y after a joint intervention that set E to 0.

Interaction between A and E on the additive scale (risk difference):

$$\underbrace{\Pr[Y^{a=1,e=1}=1] - \Pr[Y^{a=0,e=1}=1]}_{0.1} \neq \underbrace{\Pr[Y^{a=1,e=0}=1] - \Pr[Y^{a=0,e=0}=1]}_{0.2}$$

$$0.1 < 0.2$$

➡ there is interaction between A and E

Interaction – formal definition

There is interaction between two treatments A and E if the causal effect of A on Y after a joint intervention that set E to 1 differs from the causal effect of A on Y after a joint intervention that set E to 0, and vice versa.

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$$\Pr[Y^{a=1,e=1}=1] - \Pr[Y^{a=1,e=0}=1] \neq \Pr[Y^{a=0,e=1}=1] - \Pr[Y^{a=0,e=0}=1]$$

Interaction vs. effect modification

Effect modification

„a variable V is a modifier of the effect of A on Y when the average causal effect of A on Y varies across levels of V“

- refers to causal effect of A, not of V
- A and V are **not** of equal status

→ only involves counterfactual outcomes Y^a **not** $Y^{a,v}$

Interaction

„there is interaction between two treatments A and E if the causal effect of A on Y after a joint intervention that set E to 1 differs from the causal effect of A on Y after a joint intervention that set E to 0“

- refers to causal effect of A and the causal effect of E on Y
- A and E **are** of equal status

→ involves counterfactual outcomes $Y^{a,e}$

Identifying interactions

Identifiability conditions

1) exchangeability
2) positivity
3) consistency

→ required for **both** treatments (A **and** E)

Identifying interactions

1) random and unconditional assignment of $\boldsymbol{E}$

identifiability conditions ✓

→ exchangeability: treated E = 1 and untreated E = 0 are exchangeable

→ marginal risk $\Pr[Y^{a=1,e=1} = 1]$ is equal to conditional risk $\Pr[Y^{a=1} = 1 | E = 1]$

→ definition of interaction between A and E on the additive scale:

$$\Pr[Y^{a=1} = 1 | E = 1] - \Pr[Y^{a=0} = 1 | E = 1]$$
$$\neq \Pr[Y^{a=1} = 1 | E = 0] - \Pr[Y^{a=0} = 1 | E = 0]$$

➡ the concepts of interaction and effect modification coincide

Identifying interactions

2) no random assignment of $\boldsymbol{E}$

a. marginal risk

- 4 marginal risks $\Pr[Y^{a,e} = 1]$ can be computed by standardization or inverse probability weighting (under identifiability assumptions)

b. $\boldsymbol{AE}$ as combined treatment

- four levels (11, 01, 10, 00)
- identification of interaction is not different from identification of causal effect of one treatment (under identifiability assumptions)

Identifying interactions

3) (conditional) exchangeability for $\boldsymbol{A}$ but not for $\boldsymbol{E}$

e.g., when estimating the causal effect of A in a subgroup defined by E in a randomized experiment

⇒ Interaction cannot be identified, but presence of effect modification by E

Response types (one dichotomous treatment)

Table 1.1

	$Y^{a=0}$	$Y^{a=1}$
Rheia	0	1
Kronos	1	0
Demeter	0	0
Hades	0	0
Hestia	0	0
Poseidon	1	0
Hera	0	0
Zeus	0	1
Artemis	1	1
Apollo	1	0
Leto	0	1
Ares	1	1
Athena	1	1
Hephaestus	0	1
Aphrodite	0	1
Cyclope	0	1
Persephone	1	1
Hermes	1	0
Hebe	1	0
Dionysus	1	0

Table 5.1

Type	$Y^{a=0}$	$Y^{a=1}$
Doomed	1	1
Helped	1	0
Hurt	0	1
Immune	0	0

Response types (one dichotomous treatment)

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Artemis	1	1
Apollo	1	0
Leto	0	1
Ares	1	1
Athena	1	1
Hephaestus	0	1
Aphrodite	0	1
Cyclope	0	1
Persephone	1	1
Hermes	1	0
Hebe	1	0
Dionysus	1	0

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Hermes	1	0
Hebe	1	0
Dionysus	1	0

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Aphrodite	0	1
Cyclope	0	1
Persephone	1	1
Hermes	1	0
Hebe	1	0
Dionysus	1	0

Table 5.1

Type	$Y^{a=0}$	$Y^{a=1}$
Doomed	1	1
Helped	1	0
Hurt	0	1
Immune	0	0

Response types (two dichotomous treatments)

Table 5.2

	$Y^{a,e}$ for each a, e value			
Type	$1,1$	$0,1$	$1,0$	$0,0$
1	1	1	1	1
2	1	1	1	0
3	1	1	0	1
4	1	1	0	0
5	1	0	1	1
6	1	0	1	0
7	1	0	0	1
8	1	0	0	0
9	0	1	1	1
10	0	1	1	0
11	0	1	0	1
12	0	1	0	0
13	0	0	1	1
14	0	0	1	0
15	0	0	0	1
16	0	0	0	0

1,1: $Y^{a=1,e=1} = 1$  

0,1: $Y^{a=0,e=1} = 1$ 

1,0: $Y^{a=1,e=0} = 1$ 

0,0: $Y^{a=0,e=0} = 1$

Response types & interaction

Table 5.2

	$Y^{a,e}$ for each a, e value			
Type	$1,1$	$0,1$	$1,0$	$0,0$
1	1	1	1	1
2	1	1	1	0
3	1	1	0	1
4	1	1	0	0
5	1	0	1	1
6	1	0	1	0
7	1	0	0	1
8	1	0	0	0
9	0	1	1	1
10	0	1	1	0
11	0	1	0	1
12	0	1	0	0
13	0	0	1	1
14	0	0	1	0
15	0	0	0	1
16	0	0	0	0

die regardless of the treatment

only die if treated with vitamins, independent of transplant

only die if treated with transplant, independent of vitamins

only die if not treated with transplant, independent of vitamins

only die if not treated with vitamins, independent of transplant

live regardless of the treatment

Response types & interaction

Table 5.2

	$Y^{a,e}$ for each a, e value			
Type	$1,1$	$0,1$	$1,0$	$0,0$
1	1	1	1	1
2	1	1	1	0
3	1	1	0	1
4	1	1	0	0
5	1	0	1	1
6	1	0	1	0
7	1	0	0	1
8	1	0	0	0
9	0	1	1	1
10	0	1	1	0
11	0	1	0	1
12	0	1	0	0
13	0	0	1	1
14	0	0	1	0
15	0	0	0	1
16	0	0	0	0

For an individual with one of those response types, the causal effect of treatment A on the outcome Y is the same regardless of the value of the treatment E, and vice versa.

→ no interaction

Response types & interaction

Table 5.2

	$Y^{a,e}$ for each a, e value			
Type	$1,1$	$0,1$	$1,0$	$0,0$
1	1	1	1	1
2	1	1	1	0
3	1	1	0	1
4	1	1	0	0
5	1	0	1	1
6	1	0	1	0
7	1	0	0	1
8	1	0	0	0
9	0	1	1	1
10	0	1	1	0
11	0	1	0	1
12	0	1	0	0
13	0	0	1	1
14	0	0	1	0
15	0	0	0	1
16	0	0	0	0

The presence of additive interaction between A and E implies that, for some individuals in the population, the value of their two counterfactual outcomes under $A = a$ cannot be determined without knowledge of the value E. That is, there must be individuals in at least one of the following classes:

1) those who would develop the outcome under only one of the four treatment combinations (8, 12, 14, and 15)
2) those who would develop the outcome under two treatment combinations, effect of each treatment is exactly the opposite under each level of the other treatment (7 and 10)
3) those who would develop the outcome under three of the four treatment combinations (2, 3, 5, and 9)

Summary

What are interactions?

- "There is interaction between two treatments A and E if the causal effect of A on Y is dependent on the level of E, and vice versa"
- related to but not the same as effect modifications

How to identify interactions?

- dependent on study design
- identifiability conditions have to be met for both treatments (if only for one treatment → effect modification)

Response types & interactions?

- are different combinations of counterfactual outcomes that characterize individuals
- give insights in whether an interaction is present or not

Discussion „starting points“

- Does an absence of additive interaction between A and E mean that there is no individual in the population that belongs to latter response types?
- How many treatments can we possibly combine? Can there be too many?
- Are there practical implications of the response type?
- Interactions on multiplicative scale (Technical point 5.1)?