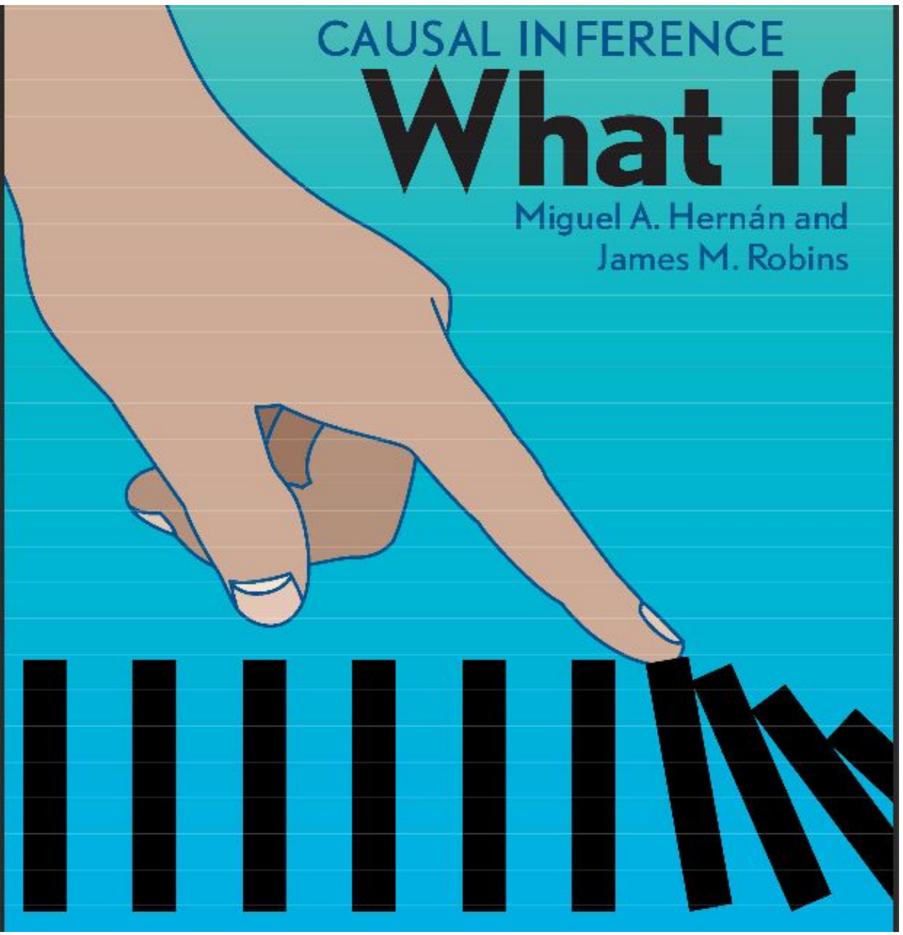
book club – chapter 7.5 and 7.6



> surrogate confounders

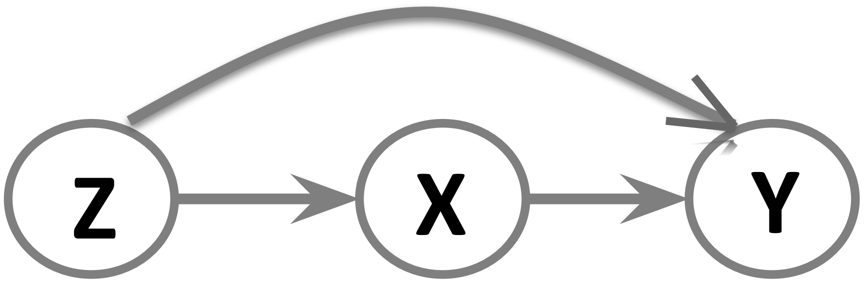
> negative outcome controls

> front-door adjustment

> single world intervention graphs

Fine Point 7.3 **Surrogate confounders**

case I: measured confounder (Z)



covariate: disease severity

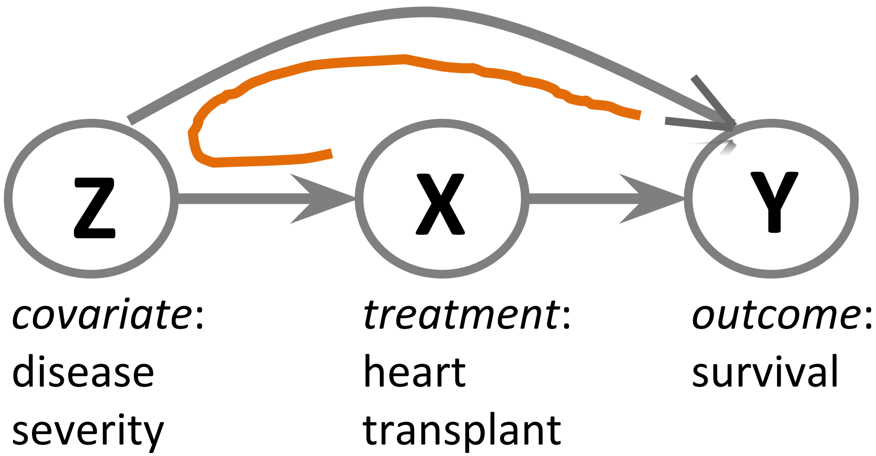
treatment: heart transplant

outcome: survival

Source: Causal Inference; Hernan and Robins; 2019

Fine Point 7.3 **Surrogate confounders**

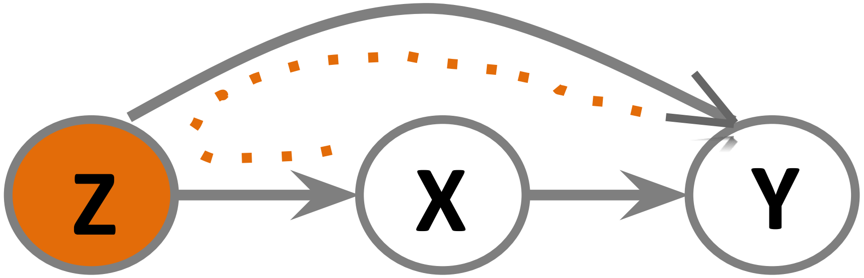
case I: measured confounder (Z), open backdoor path: Y ⟵ Z⟶ X



Source: Causal Inference; Hernan and Robins; 2019

Fine Point 7.3 **Surrogate** confounders

case I: measured confounder (Z), **blocked** backdoor path: Y ... Z→ X



covariate: disease severity

treatment: heart transplant

outcome: survival

conditional exchangeability:

$$Y \perp\!\!\!\perp X \mid Z$$

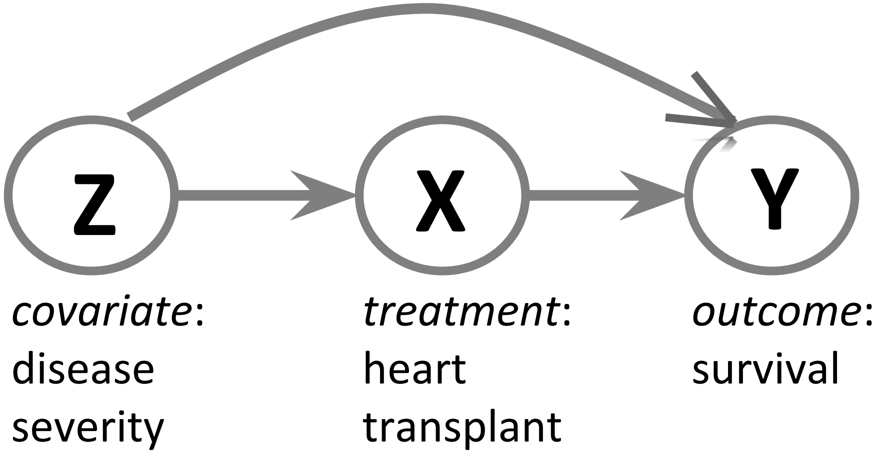
back-door adjustment**:**

$$P(y|\hat{x}) = \sum_{z} P(y|x,z)P(z)$$

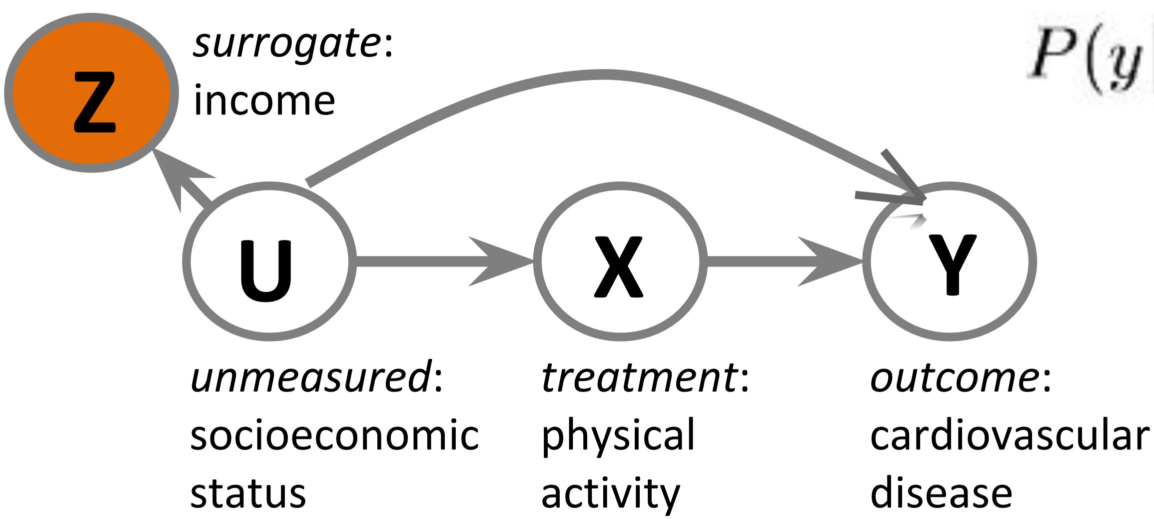
Source: Causal Inference; Hernan and Robins; 2019

Fine Point 7.3 **Surrogate confounders**

case I: measured confounder (Z)



case II: unmeasured confounder (U)



back-door adjustment:

$$P(y|\hat{x}) = \sum_z P(y|x,z)P(z)$$

if: $Z \not\perp\!\!\!\perp U$

Source: Causal Inference; Hernan and Robins; 2019

7.6 **Confounding adjustment**

methods to adjust for confounders

- G-methods: **generalized** forms of standardization, IP weighting
  $\longrightarrow$ favorable for time varying treatment and confounders
  (Chapter 20)

- **stratification** based methods: stratification & matching,
  propensity scores (Chapter 15)

conditional exchangeability given L is required

Source: Causal Inference; Hernan and Robins; 2019

7.6 **Confounding adjustment**

methods to adjust for confounders

- G-methods: **generalized** forms of standardization, IP weighting
  $\longrightarrow$ favorable for time varying treatment and confounders
  (Chapter 20)

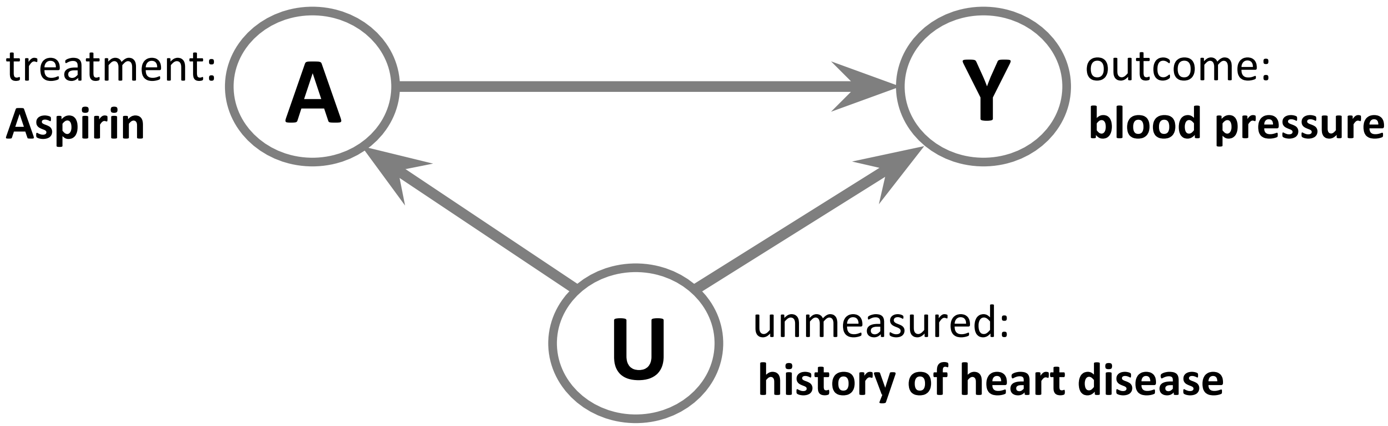
- **stratification** based methods: stratification & matching,
  propensity scores (Chapter 15)

conditional exchangeability given L is required

methods which **do not demand** for conditional exchangeability:
- front-door criterion / adjustment (Technical Point 7.4)
- difference-in-differences / negative outcome controls
  (Technical Point 7.3)
- instrumental variable estimation (Chapter 16)

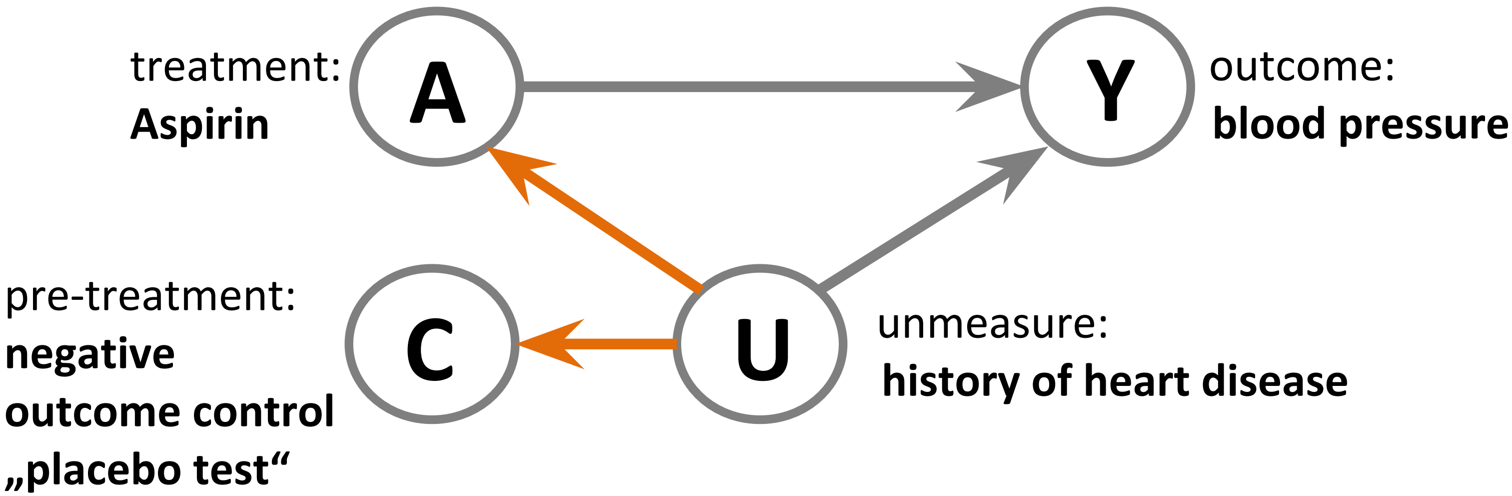
Source: Causal Inference; Hernan and Robins; 2019

Technical Point 7.3 **Negative outcome controls**



Source: Causal Inference; Hernan and Robins; 2019

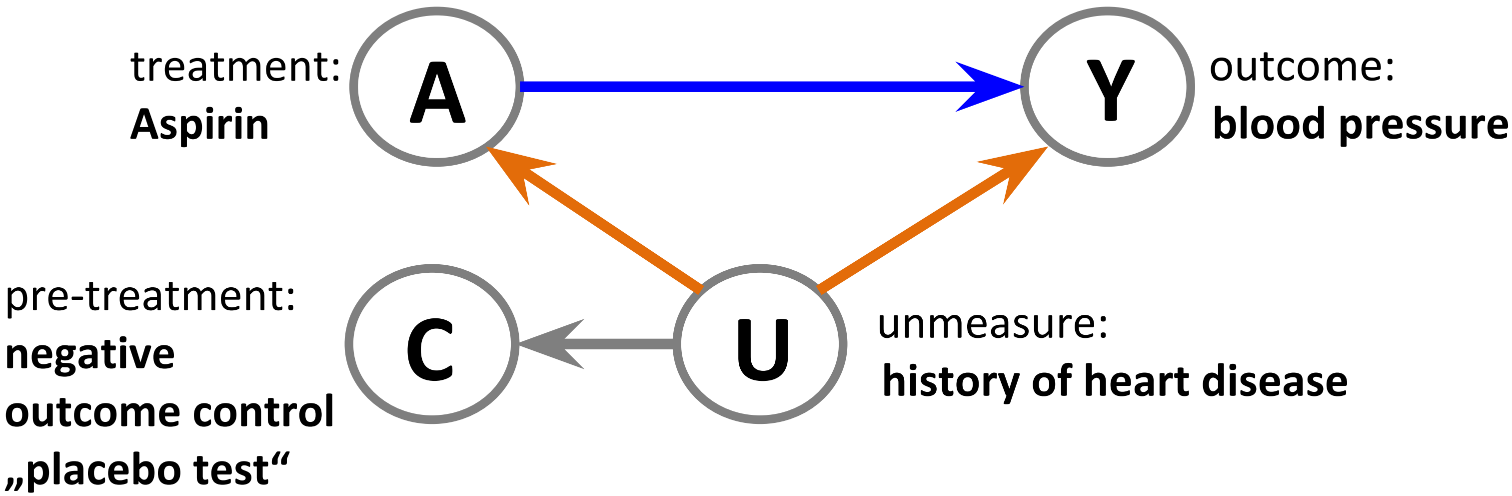
Technical Point 7.3 **Negative outcome controls**



$$0 \neq E[C|A=1] - E[C|A=0]$$

Source: Causal Inference; Hernan and Robins; 2019

Technical Point 7.3 **Negative outcome controls**



$0 \neq E[C|A=1] - E[C|A=0]$

additive eqi-confounding:

$E[Y^0|A=1] - E[Y^0|A=0] = E[C|A=1] - E[C|A=0]$

*the effect in the treated (*causal effect in the exposed*):*

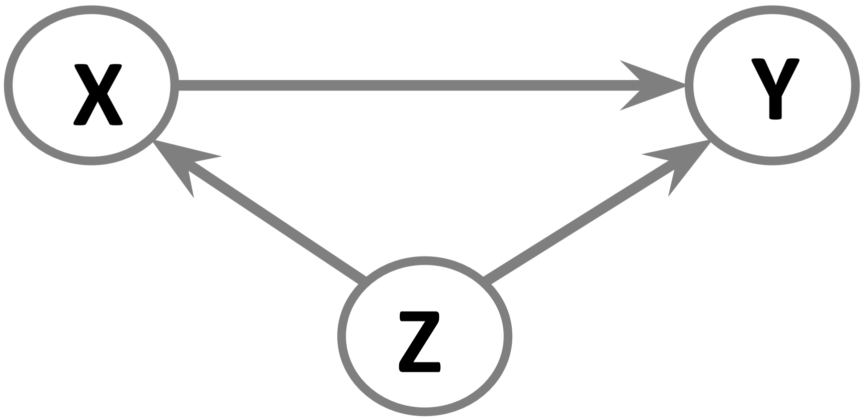
$E[Y^1 - Y^0 |A=1] = (E[Y|A=1] - E[Y|A=0]) - (E[C|A=1] - E[C|A=0])$

requires pre-and post-exposure outcome measurements

Source: Causal Inference; Hernan and Robins; 2019

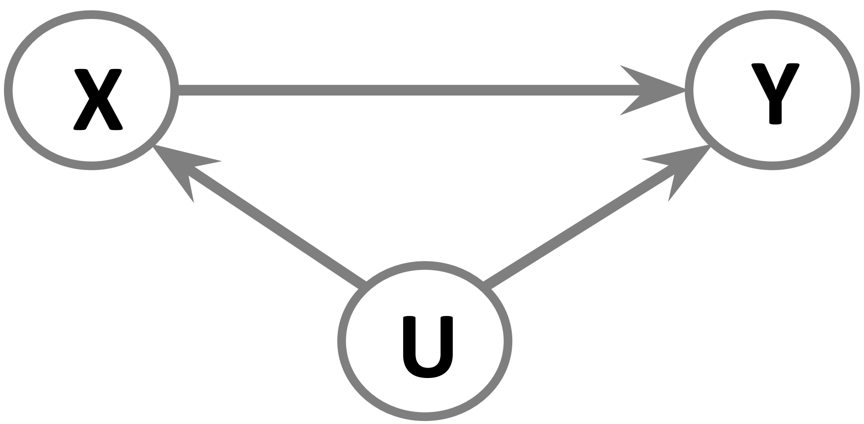
Technical Point 7.4 **Front-door adjustment**

back-door adjustment:



$$P(y|\hat{x}) = \sum_z P(y|x,z)P(z).$$

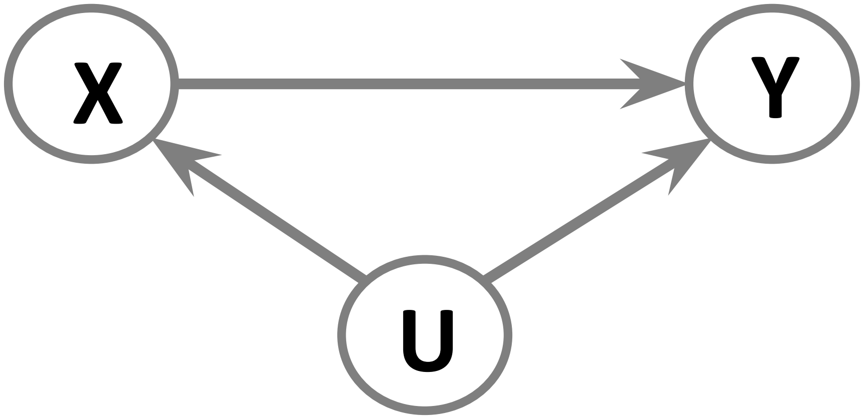
what if confounder is unmeasured (U) ??



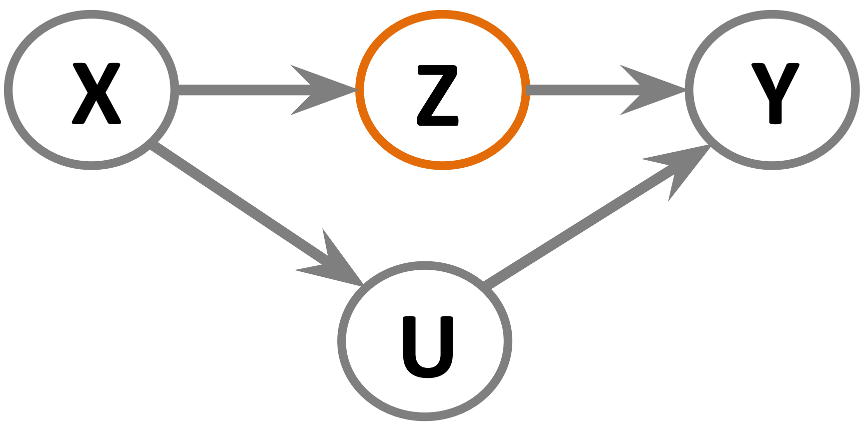
Source: Causal Inference; Hernan and Robins; 2019

Technical Point 7.4 **Front-door adjustment**

what if confounder is unmeasured (U) ??



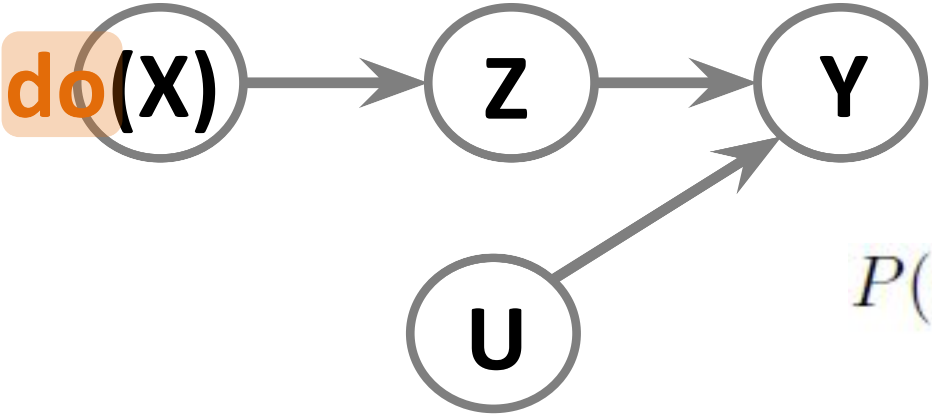
front-door criterion: measured shielding mediator (Z)



Source: Causal Inference; Hernan and Robins; 2019

Technical Point 7.4 **Front-door adjustment**

intervention: do(x) closes backdoor path



$$P(y|\hat{x}) = \sum_z P(y|z,\hat{x})P(z|\hat{x})$$

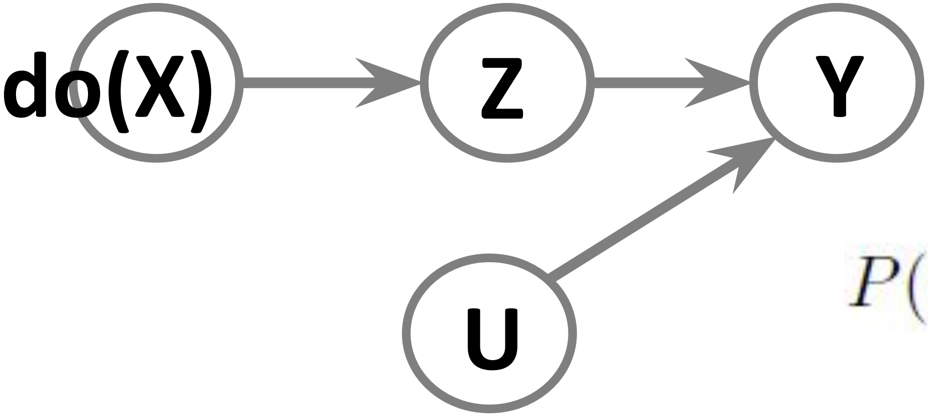
do-calculus: replace do(x) with observational data

$$P(y|\hat{x}) = \sum_z P(z|x) \sum_{x'} P(y|x',z)P(x').$$

Source: „Causality: Models, Reasoning, and Inference"; Judea Pearl; 2000 (http://bayes.cs.ucla.edu/BOOK-2K/ch3-3.pdf)

Technical Point 7.4 **Front-door adjustment**

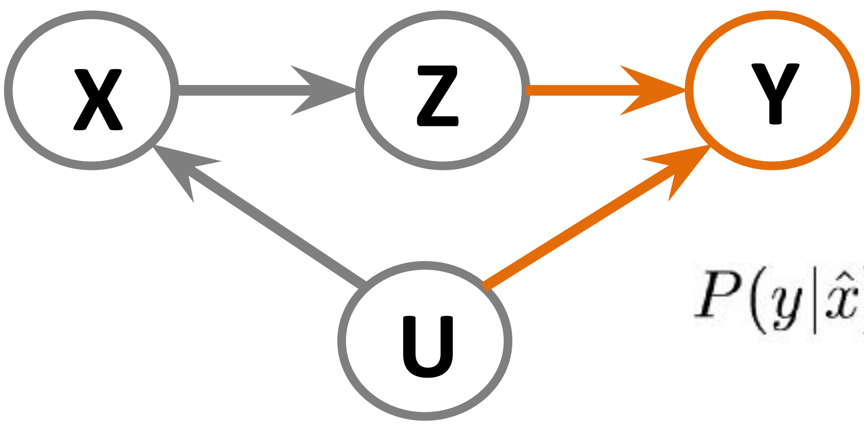
intervention: do(x) closes backdoor path



$$P(y|\hat{x}) = \sum_z P(y|z,\hat{x})P(z|\hat{x})$$

do-calculus: replace do(x) with observational data

e.g P(z|do(x)) = P(z|x)



$$P(y|\hat{x}) = \sum_z P(z|x) \sum_{x'} P(y|x', z)P(x').$$

Source: „Causality: Models, Reasoning, and Inference"; Judea Pearl; 2000 (http://bayes.cs.ucla.edu/BOOK-2K/ch3-3.pdf)
)

Technical Point 7.4 **Front-door adjustment**

The following three rules are valid for every interventional distribution compatible with G.

Rule 1 (Insertion/deletion of observations):

$$P(y|do(x), z, w) = P(y|do(x), w) \quad \text{if } (Y \perp\!\!\!\perp Z|X, W)_{G_{\overline{X}}} \tag{3}$$

Rule 2 (Action/observation exchange):

$$P(y|do(x), do(z), w) = P(y|do(x), z, w) \quad \text{if } (Y \perp\!\!\!\perp Z|X, W)_{G_{\overline{X}\underline{Z}}} \tag{4}$$

Rule 3 (Insertion/deletion of actions):

$$P(y|do(x), do(z), w) = P(y|do(x), w) \quad \text{if } (Y \perp\!\!\!\perp Z|X, W)_{\overline{X}\,\overline{Z(W)}}, \tag{5}$$

where $Z(W)$ is the set of Z-nodes that are not ancestors of any W-node in $G_{\overline{X}}$.

Source: „Causality: Models, Reasoning, and Inference"; Judea Pearl; 2000 (http://bayes.cs.ucla.edu/BOOK-2K/ch3-3.pdf)
)

smoking & cancer: **front-door adjustment**

example: does smoking cause cancer?

	Group Type	$P(x, z)$ Group Size (% of Population)
$X = 0,\ Z = 0$	Non-smokers, No tar	47.5
$X = 1,\ Z = 0$	Smokers, No tar	2.5
$X = 0,\ Z = 1$	Non-smokers, Tar	2.5
$X = 1,\ Z = 1$	Smokers, Tar	47.5

47.5/(2.5+47.5) = 95% of smokers have developed high levels of tar in their lungs
2.5/(2.5+47.5) = 5% of non-smokers have high levels of tar

Source: „Causality: Models, Reasoning, and Inference"; Judea Pearl; 2000 (http://bayes.cs.ucla.edu/BOOK-2K/ch3-3.pdf)

smoking & cancer: **front-door adjustment**

example: does smoking cause cancer?

	Group Type	$P(x,z)$ Group Size (% of Population)	$P(Y=1\|x,z)$ % of Cancer Cases in Group
$X=0,\ Z=0$	Non-smokers, No tar	47.5	10
$X=1,\ Z=0$	Smokers, No tar	2.5	90
$X=0,\ Z=1$	Non-smokers, Tar	2.5	5
$X=1,\ Z=1$	Smokers, Tar	47.5	85

47.5/(2.5+47.5) = 95% of smokers have developed high levels of tar in their lungs
2.5/(2.5+47.5) = 5% of non-smokers have high levels of tar

(2.5*0.05 + 47.5*0.85)/(2.5+47.5) = 81% of subjects with tar have developed lung cancer
(47.5*0.10 + 2.5*0.90)/(47.5+2.5) = 14% among subjects with no tar deposits have cancer

interpretation I: smoking causes cancer...

Source: „Causality: Models, Reasoning, and Inference"; Judea Pearl; 2000 (http://bayes.cs.ucla.edu/BOOK-2K/ch3-3.pdf)

smoking & cancer: **front-door adjustment**

example: does smoking cause cancer?

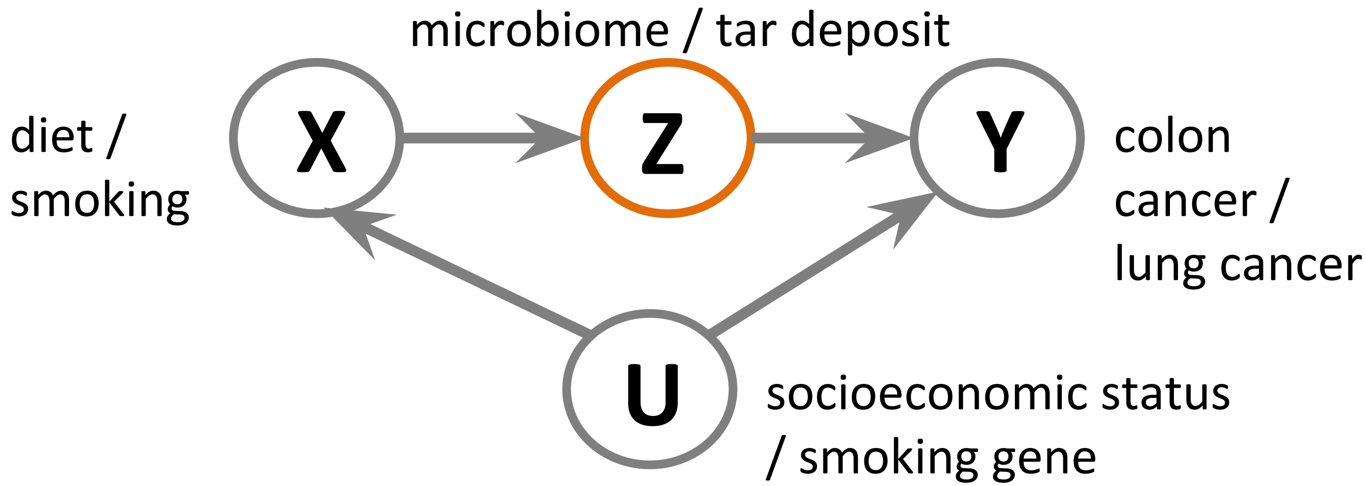
	Group Type	$P(x,z)$ Group Size (% of Population)	$P(Y=1\|x,z)$ % of Cancer Cases in Group
$X=0,\ Z=0$	Non-smokers, No tar	47.5	10
$X=1,\ Z=0$	Smokers, No tar	2.5	90
$X=0,\ Z=1$	Non-smokers, Tar	2.5	5
$X=1,\ Z=1$	Smokers, Tar	47.5	85

	Group Type	$P(x,z)$ Group Size (% of Population)	$P(Y=1\|x,z)$ % of Cancer Cases in Group
$X=0,\ Z=0$	Non-smokers, No tar	47.5	10
$X=1,\ Z=0$	Smokers, No tar	2.5	90
$X=0,\ Z=1$	Non-smokers, Tar	2.5	5
$X=1,\ Z=1$	Smokers, Tar	47.5	85

interpretation II: smoking promotes tar and tar deposits reduces risk of cancer...

Source: „Causality: Models, Reasoning, and Inference"; Judea Pearl; 2000 (http://bayes.cs.ucla.edu/BOOK-2K/ch3-3.pdf)

smoking & cancer: **front-door adjustment**



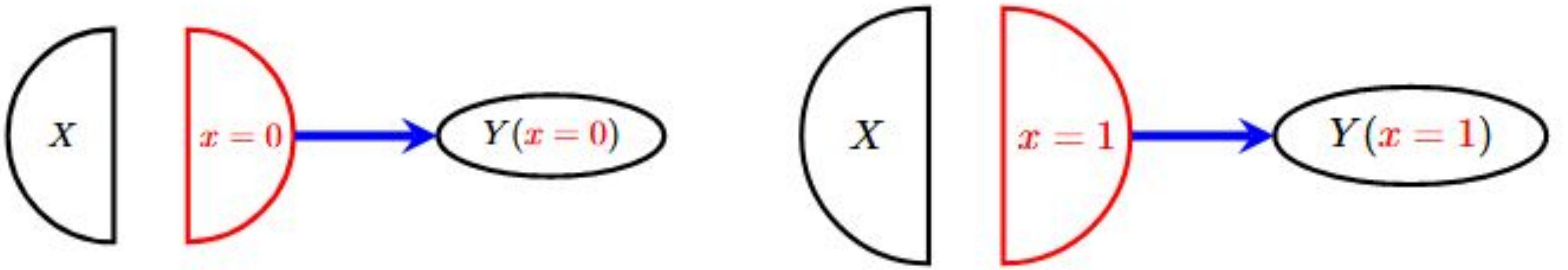
$$P(y|\hat{x}) = \sum_z P(z|x) \sum_{x'} P(y|x', z) P(x').$$

$$\begin{aligned}
P(Y=1|do(X=1)) &= .05(.10 \times .50 + .90 \times .50) \\
&\quad +.95(.05 \times .50 + .85 \times .50) \\
&= .05 \times .50 + .95 \times .45 = .4525, \\
P(Y=1|do(X=0)) &= .95(.10 \times .50 + .90 \times .50) \\
&\quad +.05(.05 \times .50 + .85 \times .50) \\
&= .95 \times .50 + .05 \times .45 = .4975.
\end{aligned}$$

Source: „Causality: Models, Reasoning, and Inference"; Judea Pearl; 2000 (http://bayes.cs.ucla.edu/BOOK-2K/ch3-3.pdf))

7.5 Single world intervention graphs (**SWIGs**)

"SWIG encodes the counterfactual independences associated with a specific hypothetical intervention on the set of treatment variables." (Richardson & Robins)



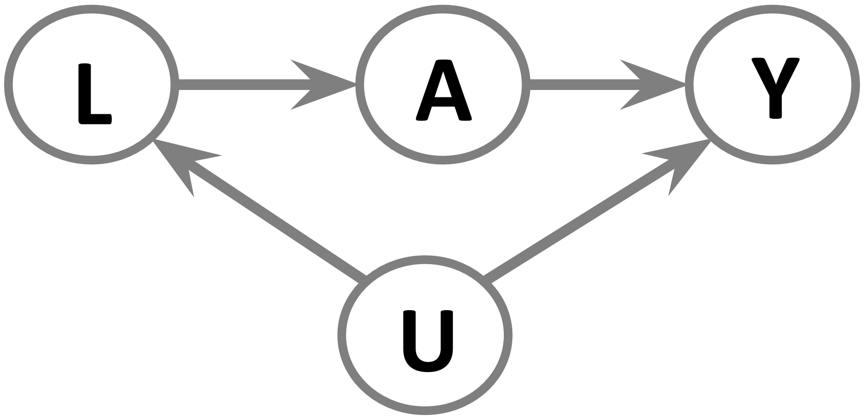
$$X \perp\!\!\!\perp Y(x=0) \quad \text{and} \quad X \perp\!\!\!\perp Y(x=1)$$

$$Y^x \perp\!\!\!\perp X$$

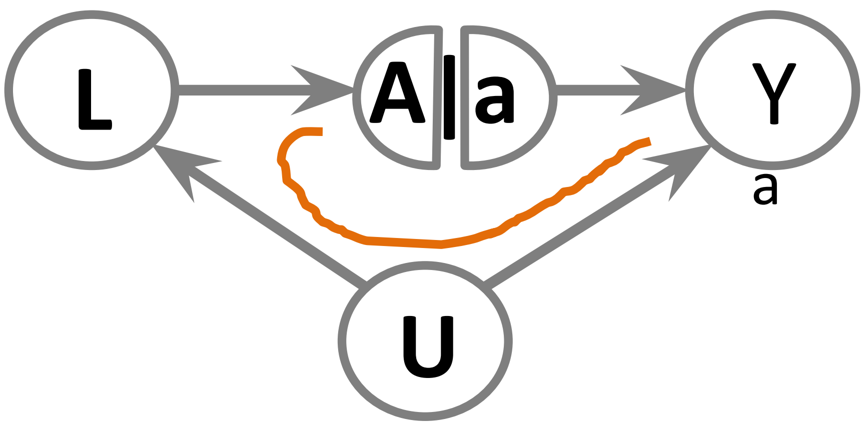
Source: „Single World Intervention Graphs: A Primer" ; Richardson & Robins; (http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.644.1881&rep=rep1&type=pdf)

7.5 Single World Intervention Graphs (**SWIGs**)

causal DAG:



SWIG:

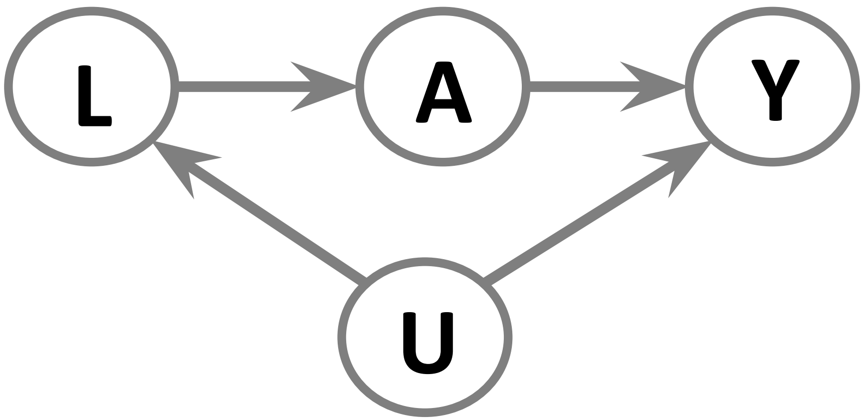


$$Y^a \perp\!\!\!\perp A \mid L$$

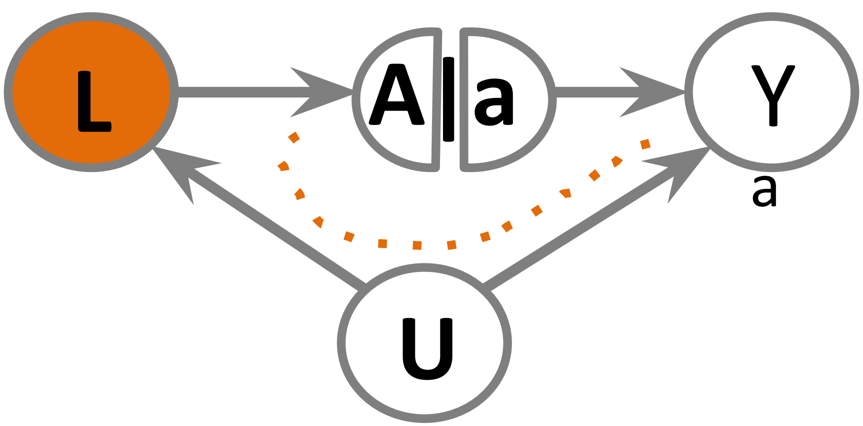
Source: Causal Inference; Hernan and Robins; 2019

7.5 Single World Intervention Graphs (**SWIGs**)

causal DAG:



SWIG:



$$Y^a \perp\!\!\!\perp A \mid L$$

Source: Causal Inference; Hernan and Robins; 2019

7.5 Single World Intervention Graphs (**SWIGs**)

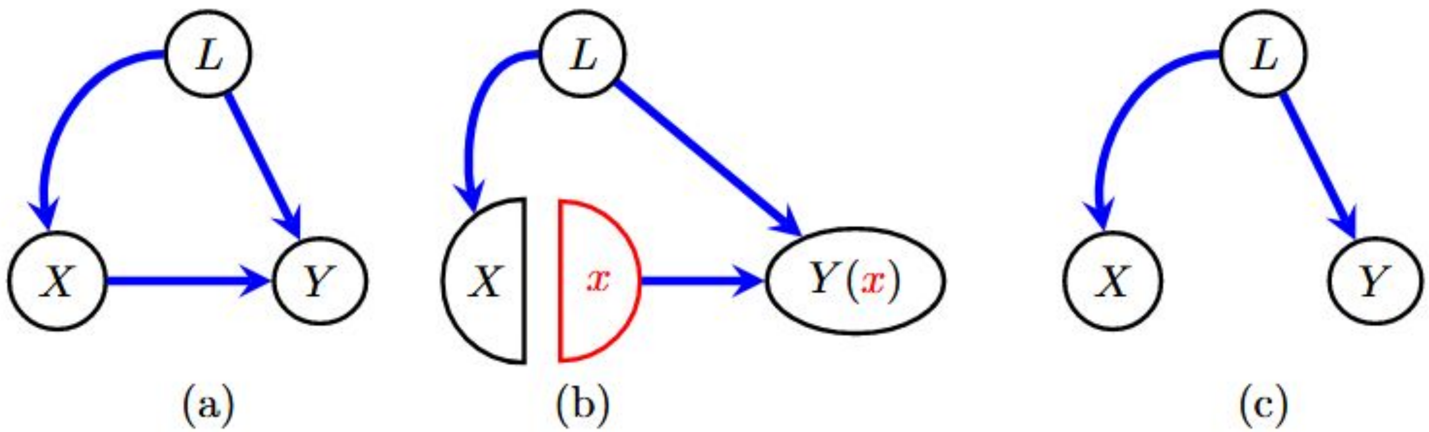


Figure 5: Adjusting for confounding. (a) The original causal graph. (b) The template $\mathcal{G}(x)$, which shows that $Y(x) \perp\!\!\!\perp X \mid L$. (c) The DAG $\mathcal{G}_{\underline{X}}$ obtained by removing edges from X advocated in Pearl (1995, 2000, 2009) to check his 'backdoor condition'.

→ Graph $G_{\underline{x}}$ **only** represents the **null hypothesis** that X does not causally affect Y.

Source: „Single World Intervention Graphs: A Primer" ; Richardson & Robins; (http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.644.1881&rep=rep1&type=pdf)

7.5 Single World Intervention Graphs (**SWIGs**)

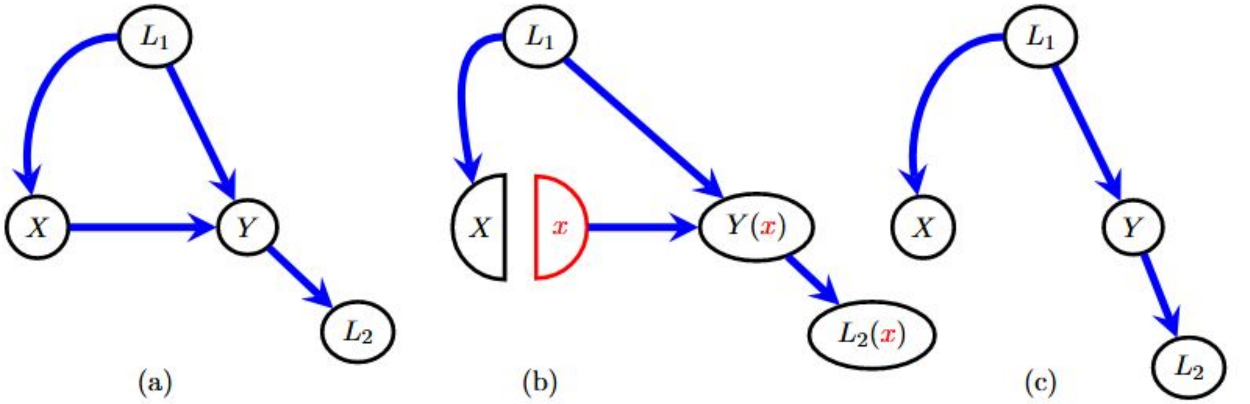
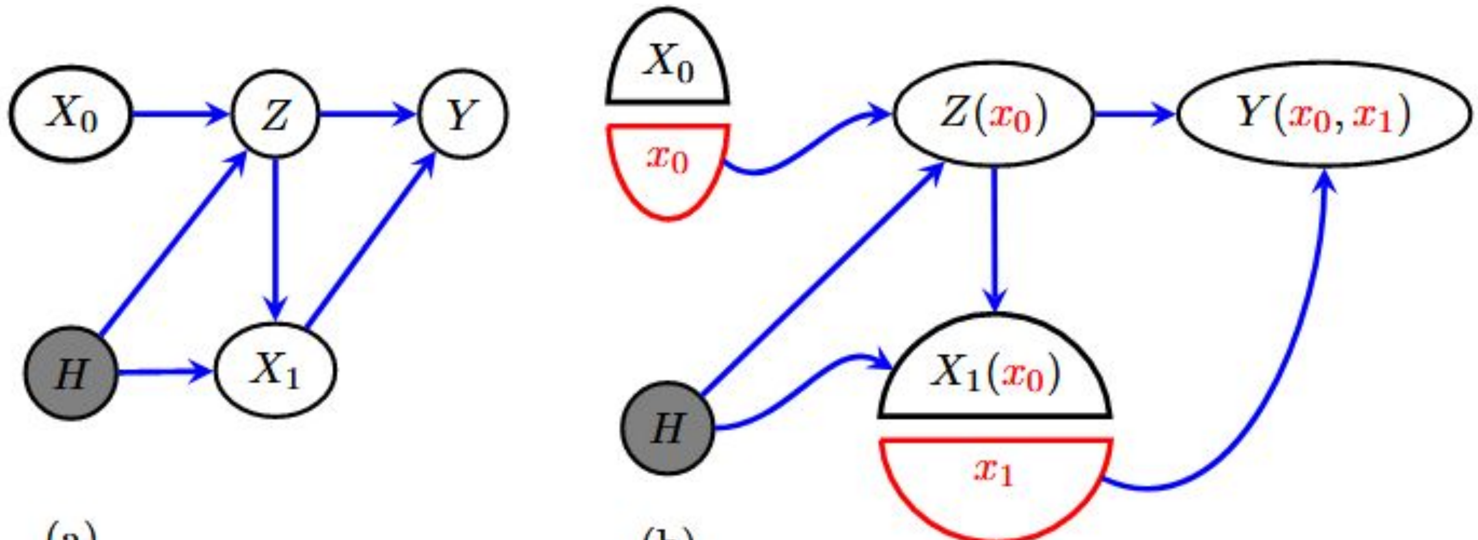


Figure 7: Simplification of the backdoor criterion. (a) The original causal graph $\mathcal{G}$. (b) The template $\mathcal{G}(x)$, which shows that $Y(x) \perp\!\!\!\perp X \mid L_1$, but does not imply $Y(x) \perp\!\!\!\perp X \mid \{L_1, L_2\}$ when there exists an arrow from X to Y, i.e. the null hypothesis is false. (c) The DAG $\mathcal{G}_{\underline{X}}$ obtained by removing edges from X advocated in Pearl (2000, 2009).

Source: „Single World Intervention Graphs: A Primer" ; Richardson & Robins; (http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.644.1881&rep=rep1&type=pdf)

7.5 Single World Intervention Graphs (**SWIGs**)

Figure 8: (a) The DAG $\mathcal{G}$, Ex. 11.3.3, Fig. 11.12 in Pearl (2009, p.353); H is unobserved ; (b) the template $\mathcal{G}(x_0, x_1)$.



(a) (b)

Pearl (2009) in Example 11.3.3 claims that under the NPSEM associated with the causal DAG in Figure 8(a) the following conditional independence does not hold:

$$Y(x_0, x_1) \perp\!\!\!\perp X_1 \mid Z, X_0 = x_0. \tag{9}$$

Pearl concludes from this that a claim of Robins is false because if it were true then (9) would hold.

in Figure 8(b) shows that (9) is indeed true under this NPSEM, and that Pearl is thus incorrect. Specifically, we see by examining the template $\mathcal{G}(x_0, x_1)$ shown in Figure 8(b), that:

$$Y(x_0, x_1) \perp\!\!\!\perp X_1(x_0) \mid Z(x_0), X_0, \tag{10}$$

from which it follows that

$$Y(x_0, x_1) \perp\!\!\!\perp X_1(x_0) \mid Z(x_0), X_0 = x_0. \tag{11}$$

Source: „Single World Intervention Graphs: A Primer" ; Richardson & Robins; (http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.644.1881&rep=rep1&type=pdf)