FE630 Portfolio Theory and Applications - Assignment #2

Deadline for submission: August 23, 2021

Prof. Papa Momar Ndiaye

July 27, 2021

Instructions

- **Read these instructions and follow them precisely.**
- **Independence:** All students must work independently.
- **Submission:** Submit your answer document via Canvas.
- **Answer Document:** Your answer document MUST be in the form of a single pdf file that contains all of your answers including code printouts and graphs. Do not submit your answer document in any format other than pdf. Any answer document that does not comprise a single pdf file complete with all answers will receive a grade of zero.
- **Cover Sheet:** Your answer document must include a cover sheet that states the course name, the homework number, the date, and your name.
- **Legibility and Logical Presentation:** Answer documents that are not easily legible, or not logically presented, or have a non-professional appearance will not be graded.
- **Source Code Requirement:** Your submission **must** also contain a separate set of source code files for all of your solutions. I may run your source code to ensure that it provides the results that you claim.
- **Permissible Computer Languages:** You can use any matrix-oriented computer programming language (R, Matlab or Python with Pandas, for example), but do not use any spreadsheets. Problems solved with spreadsheets will receive no credit.
- **Late Submission Policy:** If answer documents and source code files are not submitted by the due date and time, there will be a penalty of 20% if the submission occurs within the next 5 days. Assignments received after one week will not be graded. Only in extreme circumstances will exceptions will be made to this late submission policy. Technical, network, or computer problems are not considered extreme circumstances.

Important note: **All answers need to be justified.** You will not receive credit for answers provided without justification or copied from a textbook.
Handwritten submissions will not be graded. Submit via Canvas a PDF file with your answers and also a zip file with the code used for computations and all graphical outputs.
Note: The allocation of points for this assignment is as follows: 25 points for Problem 1, 25 points for Problem 2 and 50 points for the Problem 3.

Reading (no submission required)

All material and knowledge needed for this assignment have been covered in class and in the slides. Suggested additional reading is chap. 6 and 7 of *Modern Portfolio Theory* by Francis and Kim.

1 Problem 1: Markowitz Mean-Variance with Risky Securities (25 points)

Consider an Investment Universe made of 3 securities S_1, S_2 and S_3 with the following characteristics:

$$\text{Covariance matrix: } \Sigma = \begin{pmatrix} 10\% & 2\% & 1\% \\ 2\% & 11\% & 3\% \\ 1\% & 3\% & 20\% \end{pmatrix}; \text{ Expected Return vector: } \rho = \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix} = \begin{pmatrix} 4.5\% \\ 0.30\% \\ 2.85\% \end{pmatrix}.$$

1. (5 points) Using the method of Lagrange multipliers, find (P_*), the Global Minimal Variance Portfolio, which is the solution of

$$\begin{cases} \min\limits_{\omega \in \mathbb{R}^3} & \omega^T \Sigma \omega \\ \text{s.t.} & \mathbf{e}^T \omega = 1. \text{ where } \mathbf{e} = (1,1,1)^T \end{cases} \tag{1}$$

2. (5 points) Using the method of Lagrange multipliers, find (P_1), the Markowitz Mean-Variance Portfolio with Expected Return equal to $\frac{\rho_1}{3}$. Find also (P_2), the Markowitz Mean-Variance Portfolio with Expected Return equal to $\rho_2 + \rho_3$.

3. (5 points) Using the Portfolios (P_1) and (P_2) previously found, apply the Two-fund Theorem to find (P_3), the Markowitz Mean-Variance Portfolio with Expected Return equal to $\frac{\rho_1+\rho_2+\rho_3}{3}$.

4. (10 points) Write a computer program in R, Matlab or Python to solve the 3 previous questions using a quadratic optimizer. Apply the Two-fund Theorem to generate and plot the efficient frontier. Add the asymptotes to the graph. Add the Portfolio (P_3) to the graph (the graph should also display the Expected Returns and volatility of each of the securities S_1, S_2 and S_3).

2 Problem 2: Markowitz with Risky and Risk-Free securities (25 points)

Assume now that we add a Risk-free security S_0 with return $\rho_0 = 0.6\%$.

1. (10 points) We denote the Tangent Portfolio by (P_T). Derive the expressions of its weights (P_T), its Expected Return (ρ_T), its volatility (σ_T) .

2. (5 points) Add the tangent Portfolio (P_T) and the new Efficient Frontier to the graph generated in question 1.4. Does that efficient frontier intersect with the one previously obtained (when all the assets were risky)? Explain why.

3. (5 points) **Building a Target Return Portfolio** Using the One-fund Theorem, find (P_4), the Efficient Portfolio with target Expected Return equal to 5%.

4. (5 points) **Building a Target Risk Portfolio** Using the One-fund Theorem, find (P_5), the Efficient Portfolio with with target volatility equal to 16%.

5. (5 points) Add the Portfolios (P_4) and (P_5) to the graph from question 2.

Important note: All answers need to be justified. You will not receive credit for answers provided (or copied from a textbook) without justification.

3 Problem 3: Designing Target Beta Portfolios (50 points)

An investor wishes to design a hedge basket to offset of a position she holds in stocks of Apple (Ticker: AAPL) by taking position in a Mean-variance portfolios with well-chosen target betas. The market (S&P-500 index) will be represented by the SPY ETF and the investment universe she is considering is a set 10 ETFs with tickers below:

$$\mathcal{U} = \{\text{FXE}, \text{EWJ}, \text{GLD}, \text{QQQ}, \text{SHV}, \text{DBA}, \text{USO}, \text{XBI}, \text{ILF}, \text{EPP}, \text{FEZ}\}.$$

1. (5 points) Download the historical data for the ticker AAPL, the SPY and the tickers in the set $\mathcal{U}$ for the period starting in Jan. 1^{st}, 2020 and ending on Mar. 30^{th}, 2021. Remove all missing entries. Compute and store the daily returns of all securities in a dataFrame.

2. (15 points) **Two-fund Theorem with Target Beta.** In this question, you will assume that the investment decision is taken on January 2nd, 2021 and you will use 1 year of historical data to compute the Expected Returns and Covariance needed for the optimization.

   - The set $\mathcal{U}$ being considered as your investment universe, find $\text{P}_{mV}(\beta_T)$, the minimum variance portfolio with Beta equal to β_T under the sole constraint of being totally invested, in other words, the solution of

   $$\begin{cases} \min\limits_{\omega} & \omega^T \Sigma \omega \\ \text{s.t.} & \beta^T \omega = \beta_T \\ & \mathbf{e}^T \omega = 1, \text{ where } \mathbf{e} = (1, 1, \ldots, 1)^T \end{cases} \tag{2}$$

   - Given $\beta_1 = 0.5$, $\beta_2 = 1.5$ and $\beta_3 = 1$, apply the 2 fund-theorem to derive the portfolio $\text{P}_{mV}(\beta_3)$ from the two portfolios $\text{P}_{mV}(\beta_1)$ and $\text{P}_{mV}(\beta_2)$.

3. (15 points) **Hedging a Long position in Apple.** In this question, you will assume that the investment decision is taken on January 2nd, 2021 and you will use 1 year of historical data to compute the Expected Returns, Covariance and Betas needed for the optimization.

   (a) Compute the betas of the securities with respect to the S&P-500 index. Compute β_A, the Beta of the stock Apple with respect to the S&P-500 index.

   (b) **Beta Hedging Portfolio.** Find the portfolio $\text{P}_{mV}(\beta_A)$. How would you use that portfolio to hedge your position in Apple? What is the expecting return of your hedged strategy?

   (c) **Beta Neutral Portfolio.** Considering the enlarged investment universe $\mathcal{V} = \mathcal{U} \cup \{AAPL\}$, find in that universe the Minimum Variance Portfolio with Target Beta equal to 0. (Beta neutral Portfolio). Compare that portfolio with the aggregated position you built in the question 3(b). How do they compare in terms of Risk, Expected Return and Beta?

4. (15 points) **Comparison of the 2 Hedging strategies**

   (a) Compute the realized daily returns of the Portfolios built in questions 3(a) and 3(b) for the period starting in January 3rd, 2021 and ending on March 30th 2021.

   (b) Compare the time series of the returns of the 2 hedging strategies (probability distribution graphs, Expected Returns, volatility, 95%-Value-at-Risk, Skewness, Kurtosis and Beta)

Submit all code in a zip file with comments and a separate written report presenting your findings. Do not include code printout in that report. If you choose to submit a single Jupyter Notebook, that notebook should imperatively contain all detailed mathematical proofs and justifications of your methodology.