

Python version = 2.7.11|Anaconda4.0.0(32-bit)|(default, Mar42016, 15 : 18 : 41)[MSCv.150032bit(Intel)]

$$\mathbf{a} = a^t \mathbf{e}_t + a^x \mathbf{e}_x + a^y \mathbf{e}_y + a^z \mathbf{e}_z$$

$$\mathbf{M} = M$$

$$\begin{aligned} & + M^t \mathbf{e}_t + M^x \mathbf{e}_x + M^y \mathbf{e}_y + M^z \mathbf{e}_z \\ & + M^{tx} \mathbf{e}_t \wedge \mathbf{e}_x + M^{ty} \mathbf{e}_t \wedge \mathbf{e}_y + M^{tz} \mathbf{e}_t \wedge \mathbf{e}_z + M^{xy} \mathbf{e}_x \wedge \mathbf{e}_y + M^{xz} \mathbf{e}_x \wedge \mathbf{e}_z + M^{yz} \mathbf{e}_y \wedge \mathbf{e}_z \\ & + M^{txy} \mathbf{e}_t \wedge \mathbf{e}_x \wedge \mathbf{e}_y + M^{txz} \mathbf{e}_t \wedge \mathbf{e}_x \wedge \mathbf{e}_z + M^{tyz} \mathbf{e}_t \wedge \mathbf{e}_y \wedge \mathbf{e}_z + M^{xyz} \mathbf{e}_x \wedge \mathbf{e}_y \wedge \mathbf{e}_z \\ & + M^{txyz} \mathbf{e}_t \wedge \mathbf{e}_x \wedge \mathbf{e}_y \wedge \mathbf{e}_z \end{aligned}$$

$$\mathbf{a}\mathbf{a} = (a^t)^2 - (a^x)^2 - (a^y)^2 - (a^z)^2$$

$$\begin{aligned} \mathbf{a}^{-1} = & \frac{a^t}{(a^t)^2 - (a^x)^2 - (a^y)^2 - (a^z)^2} \mathbf{e}_t \\ & + \frac{a^x}{(a^t)^2 - (a^x)^2 - (a^y)^2 - (a^z)^2} \mathbf{e}_x \\ & + \frac{a^y}{(a^t)^2 - (a^x)^2 - (a^y)^2 - (a^z)^2} \mathbf{e}_y \\ & + \frac{a^z}{(a^t)^2 - (a^x)^2 - (a^y)^2 - (a^z)^2} \mathbf{e}_z \end{aligned}$$

$$\langle \mathbf{M} \rangle_1 \langle \mathbf{M} \rangle_1 = (M^t)^2 - (M^x)^2 - (M^y)^2 - (M^z)^2$$

$$\begin{aligned} \langle \mathbf{M} \rangle_1^{-1} = & \frac{M^t}{(M^t)^2 - (M^x)^2 - (M^y)^2 - (M^z)^2} \mathbf{e}_t \\ & + \frac{M^x}{(M^t)^2 - (M^x)^2 - (M^y)^2 - (M^z)^2} \mathbf{e}_x \\ & + \frac{M^y}{(M^t)^2 - (M^x)^2 - (M^y)^2 - (M^z)^2} \mathbf{e}_y \\ & + \frac{M^z}{(M^t)^2 - (M^x)^2 - (M^y)^2 - (M^z)^2} \mathbf{e}_z \end{aligned}$$

$$\langle \mathbf{M} \rangle_3 = M^{txy} \mathbf{e}_t \wedge \mathbf{e}_x \wedge \mathbf{e}_y + M^{txz} \mathbf{e}_t \wedge \mathbf{e}_x \wedge \mathbf{e}_z + M^{tyz} \mathbf{e}_t \wedge \mathbf{e}_y \wedge \mathbf{e}_z + M^{xyz} \mathbf{e}_x \wedge \mathbf{e}_y \wedge \mathbf{e}_z$$

$$\langle \mathbf{M} \rangle_3 \langle \mathbf{M} \rangle_3 = -(M^{txy})^2 - (M^{txz})^2 - (M^{tyz})^2 + (M^{xyz})^2$$

$$\begin{aligned} \langle \mathbf{M} \rangle_3^{-1} = & - \frac{M^{txy}}{(M^{txy})^2 + (M^{txz})^2 + (M^{tyz})^2 - (M^{xyz})^2} \mathbf{e}_t \wedge \mathbf{e}_x \wedge \mathbf{e}_y \\ & - \frac{M^{txz}}{(M^{txy})^2 + (M^{txz})^2 + (M^{tyz})^2 - (M^{xyz})^2} \mathbf{e}_t \wedge \mathbf{e}_x \wedge \mathbf{e}_z \\ & - \frac{M^{tyz}}{(M^{txy})^2 + (M^{txz})^2 + (M^{tyz})^2 - (M^{xyz})^2} \mathbf{e}_t \wedge \mathbf{e}_y \wedge \mathbf{e}_z \\ & - \frac{M^{xyz}}{(M^{txy})^2 + (M^{txz})^2 + (M^{tyz})^2 - (M^{xyz})^2} \mathbf{e}_x \wedge \mathbf{e}_y \wedge \mathbf{e}_z \end{aligned}$$