ASSIGNMENT II : SIMULTANEOUS LOCALIZATION AND MAPPING (SLAM)

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A 2D mobile robot navigating a straight path from A to B in a room containing Landmarks, its mounted with a 2D Lidar sensor and odometer.

1. Considering a differential drive robot, the dynamic model equation of the evolution model to recursively estimate the current pose (X_t, Y_t, θ_t) and the center position (X_{L1}, Y_{L1}), (X_{L2}, Y_{L2}), $(X_{L3}\ Y_{L3})$ of the 3 Landmarks.

State $S_t = [x_t\ y_t\ \theta_t\ L_1^T\ L_2^T\ L_3^T]^T$ & differential drive:

input $u_t = [V_t\ \omega_t]^T$

$V_L = (W - H/2)\omega$

$V_r = (W + H/2)\omega$

$V_x = (V_L + V_r)/2$

Process model :

$$\begin{bmatrix} X_{t-1} \\ y_{t-1} \\ \theta_{t-1} \\ X_{L1} \\ Y_{L1} \\ X_{L2} \\ Y_{L2} \\ X_{L3} \\ Y_3 \end{bmatrix} + \begin{bmatrix} \Delta t\left(\frac{V_{Lt-1} + V_{Rt-1}}{2}\right)\cos(\theta_{t-1}) \\ \Delta t\left(\frac{V_{Lt-1} + V_{rt-1}}{2}\right)\sin(\theta_{t-1}) \\ \Delta t\left(\frac{V_r - V_L}{L}\right) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} W_x \\ W_y \\ W_\theta \\ W_{xL1} \\ W_{yL1} \\ W_{xL2} \\ W_{yL2} \\ W_{xL3} \\ W_{yL3} \end{bmatrix}$$

2. Observation model :

$$h_i(s_t, v_t) = \begin{bmatrix} \sqrt{(X_t - X_{Li})^2 + (Y_t - Y_{Li})^2} \\ \tan^{-1}\left(\frac{Y_t - Y_{Li}}{X_t - X_{Li}}\right) - \theta_t \end{bmatrix} + \begin{bmatrix} V_r \\ V_b \end{bmatrix}$$

$$\begin{bmatrix} r_1 \\ \alpha_1 \\ r_2 \\ \alpha_2 \\ r_3 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} \sqrt{(X_t - X_{Li})^2 + (Y_t - Y_{Li})^2} \\ \tan^{-1}\left(\frac{y_t - y_{L1}}{X_t - X_{L1}}\right) - \theta_t \\ \sqrt{(X_t - X_{L2})^2 + (Y_t - Y_{L2})^2} \\ \tan^{-1}\left(\frac{Y_t - Y_{L2}}{X_t - X_{L2}}\right) - \theta_t \\ \sqrt{(X_t - X_{L3})^2 + (y_t - Y_{L3})^2} \\ \tan^{-1}\left(\frac{Y_t - Y_{L3}}{X_t - X_{L3}}\right) - \theta_t \end{bmatrix} + \begin{bmatrix} V_{r_1} \\ V_{\alpha_1} \\ V_{r_2} \\ V_{\alpha_2} \\ V_{r_3} \\ V_{\alpha_3} \end{bmatrix}$$

3:
(i) Process Jacobian :

$$\begin{bmatrix} 1 & 0 & -\delta t\left[\frac{V_{Lt-1}+V_{rt-1}}{2}\right]\sin[\theta_{t-1}) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \delta t\left(\frac{V_{Lt-1}+V_{r_{t-1}}}{2}\right) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(ii) Observation (sensor) model Jacobian matrix: is a 6x9 matrix which is a concatenation of Jacobians of each Landmarks.

$H = [H_{l1} \; H_{l2} \; H_{l3}]$ where H_{l1}, H_{l2} & H_{l3} is a 2x3 Jacobian of each land mark.

eg $H_{l1} = [\;\;]$

4.) Assuming pairing of each observation (r, α) and its Land mark is giving by an oracle; How would you initialize the positions and the uncertainties of the Landmark.?

If given by oracle :

Landmark l $L_1(0,0)$

$L_2(10,0)$

$L_3(10,8)$

We could assume fairly confident about the approximate location of the landmarks, and believe they are within 1 meter of thier true positions, so we set the mean and Covariance position of each Landmarks

mean $L_1(1,1)$ $\quad \Sigma_{L1} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$ $\quad \Sigma_{L2} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$ $\quad \Sigma_{L3} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$

$L_2(9,1)$

$L_3(9,7)$

5] In a condition where the associations ~~of~~ the landmarks and observations are not given.

To associate one measurement (r, α) with one of the three Landmarks, is to use the Nearest neighboor approach based on the Euclidean distance between the expected measurement and the actual measurement.

Given measurement (r, α) at time t, we calculate Expected measurement for each landmarks (L_1, L_2, L_3) using thier current estimate of robot position and landmark position. We then compute the Euclidean distance of each Expected measurements and the actual measurement
Then the measurement with the smallest distance is then associated with the Landmark.

We can use the Mahalonobis distance metric instead, to account for data uncertainties.

so;

1. Calculate Expected measurement using current estimate of robot's position and land marks' (L_1, L_2, L_3) position.
2. Calculate Mahalanobis distance between actual measurement and each of the Expected measurement using measurement error covariance matrix
3. Associate the measurement with the landmark with the smallest Mahalanobi's distance.