ASSIGNMENT III OPTICAL FLOW & VIDEO STABILIZATION.

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1. Camera attached to a moving robotic system which is observing a far scene.

Assuming (u,v) motion for all pixel (x,y) between correspondence between each pairs of image in sequence.

Affine model parameters $\theta = (a\ b\ c\ d\ e\ f)^T$

$$u = ax + by + c$$
$$v = dx + cy + f.$$

Show optical flow constraint $I_x u + I_y v + I_t = 0$ is also a solution of $\underset{n\times 6}{A}\,\underset{6\times 1}{\theta} = \underset{n\times 1}{b}$. matrices.

$$I_x = \frac{\delta I}{\delta x} = \frac{\delta I}{\delta u}\cdot\frac{\delta u}{\delta x} + \frac{\delta I}{\delta v}\cdot\frac{\delta v}{\delta x}$$
$$= \frac{\delta I}{\delta u}\cdot a + \frac{\delta I}{\delta v}\cdot b.$$

$$I_x = \frac{\delta I}{\delta y} = \frac{\delta I}{\delta u}\cdot\frac{\delta u}{\delta y} + \frac{\delta I}{\delta v}\cdot\frac{\delta v}{\delta y}$$
$$= \frac{\delta I}{\delta u}\cdot c + \frac{\delta I}{\delta v}\cdot d.$$

$\frac{\delta I}{\delta u}$ and $\frac{\delta I}{\delta v}$ are partial derivatives.

so $$\frac{\delta I}{\delta u}(ax + by + e) + \frac{\delta I}{\delta v}(dx + dy + f) + \frac{\delta I}{\delta t} = 0$$

Expand:
$$a\left(\frac{\delta I}{\delta u}\right)u + b\left(\frac{\delta I}{\delta u}\right)y + e\left(\frac{\delta I}{\delta u}\right) + c\left(\frac{\delta I}{\delta v}\right)x + d\left(\frac{\delta I}{\delta v}\right)y + f\left(\frac{\delta I}{\delta v}\right)f + \frac{\delta I}{\delta t} = 0.$$

Express in matrix form:

$$\begin{bmatrix} \frac{\delta I}{\delta u}x & \frac{\delta I}{\delta u}y + \frac{\delta I}{\delta v}x + \frac{\delta I}{\delta v}y & \frac{\delta I}{\delta u} & \frac{\delta I}{\delta v} \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} + \frac{\delta I}{\delta t} = 0$$

$\equiv A\theta = b$ Any solution of OFCE is also a solution of $A\theta = b$.

2. Propose a method by LUKAS-KANADE (LK) optimal flow algorithm to estimate θ using all image pixels.

- The least square solution of $\underset{n\times 6}{A}\ \underset{6\times 1}{\theta} = \underset{n\times 1}{b}$

$$\theta = (A^T A)^{-1} A^T b$$

if $\underset{25\times 2}{A}\ \underset{2\times 1}{\delta} = \underset{25\times 1}{b}$

we minimize $\|Ad - b\|^2$. so

Solving the least square problem. of.

$$\begin{bmatrix} I_x(P_1)[0] & I_y(P_1)[0] \\ I_x(P_1)[1] & I_y[P_1][1] \\ \vdots & \\ I_x(P_5)[0] & I_y(P_5)[0] \\ I_x(P_5)[1] & I_y[P_5][1] \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(P_1)[0] \\ I_t(P_1)[0] \\ \vdots \\ I_t(P_5)[0] \\ I_t(P_5)[1] \end{bmatrix}$$

OR

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

so $\theta = (A^T A)^{-1} A^T b$.