An Introduction to Camera Self-Calibration

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CONTENTS

Calibration

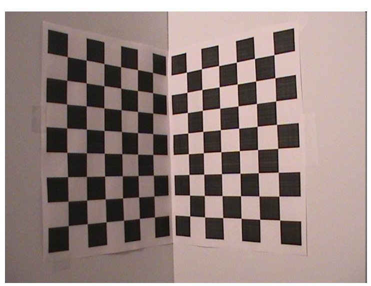
Camera calibration is the process of finding the « true » parameters of the camera that took your photographs. Some of these parameters are focal length, format size, principal point and lens distortion.

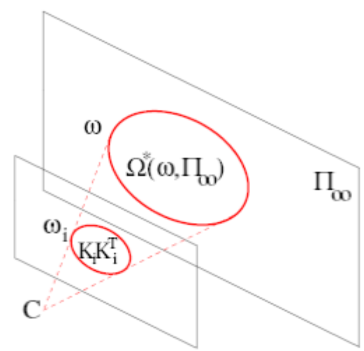
- Accuracy
- Metric reconstruction at scale

Self-Calibration

Self-calibration is calibration with no user intervention nor calibration pattern, performed on-line during the acquisition process.

- Varying parameters
- Automatic process





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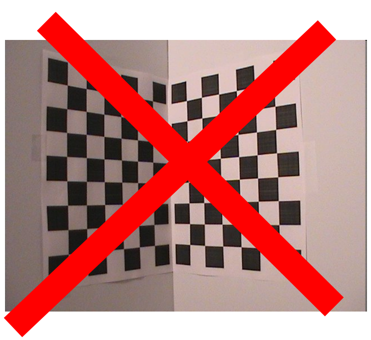

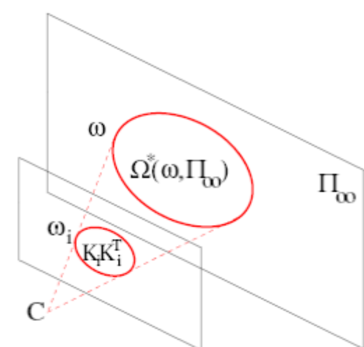

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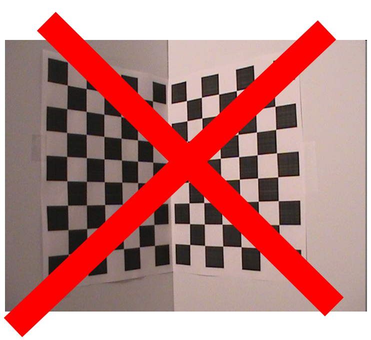

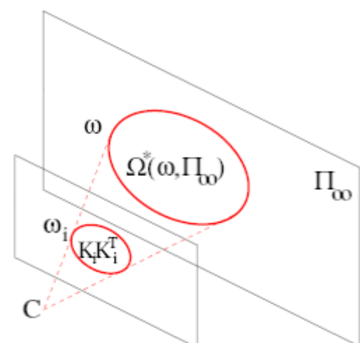

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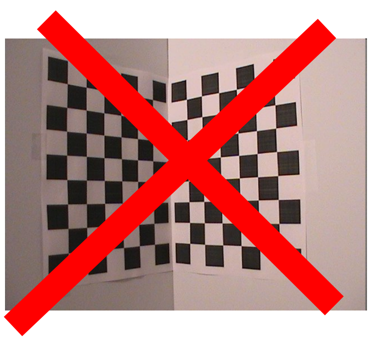
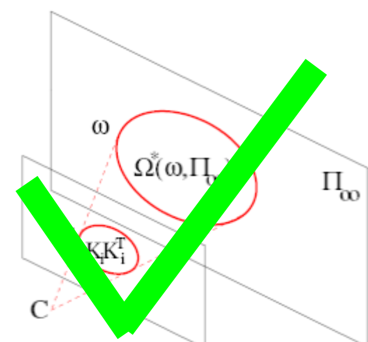
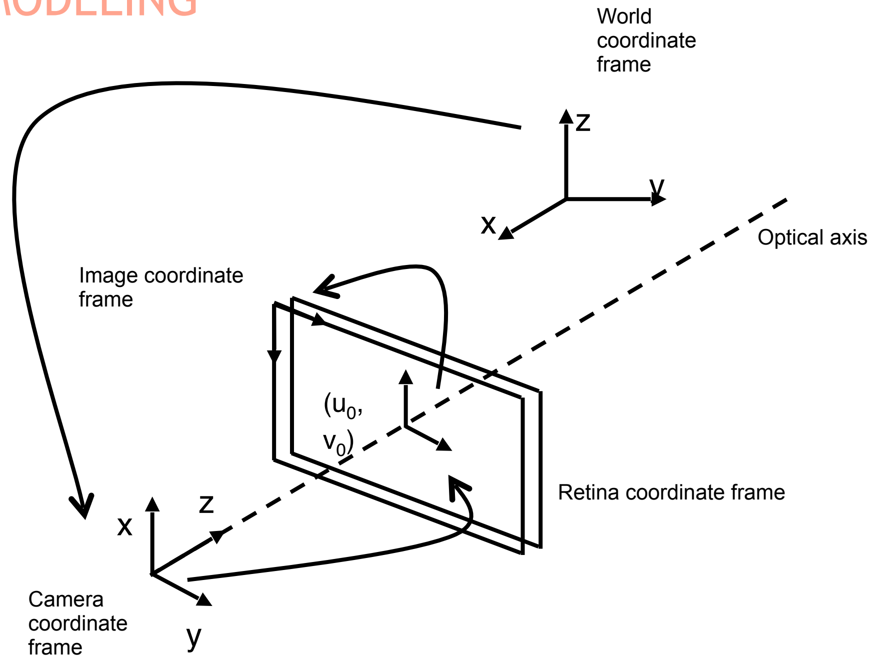
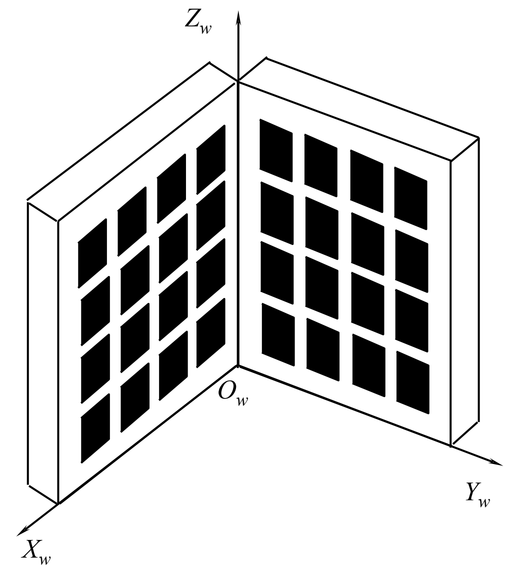








❶

$$ {}^{c}\mathbf{T}_{w} = \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ 0 & 1 \end{bmatrix} $$

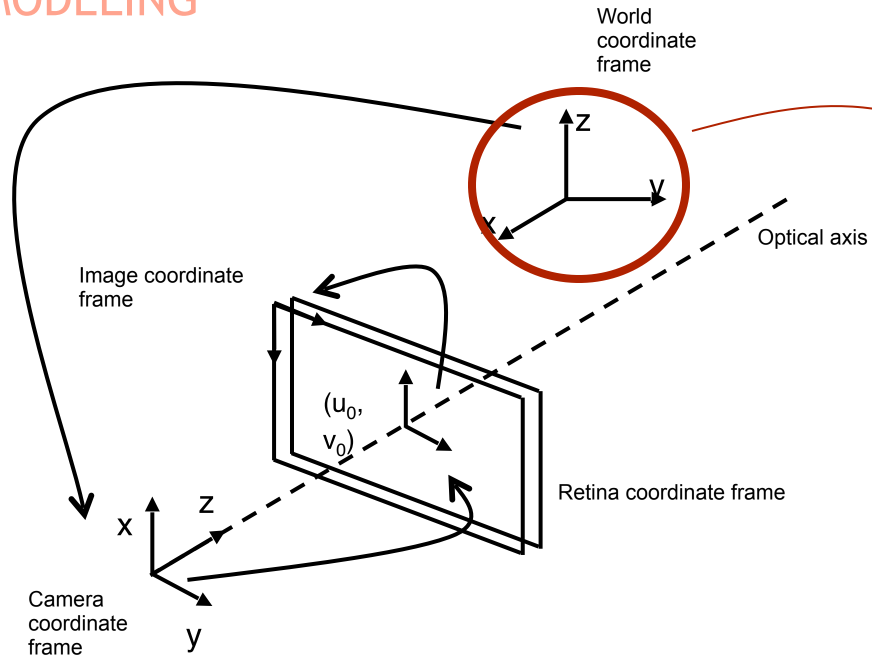
❷

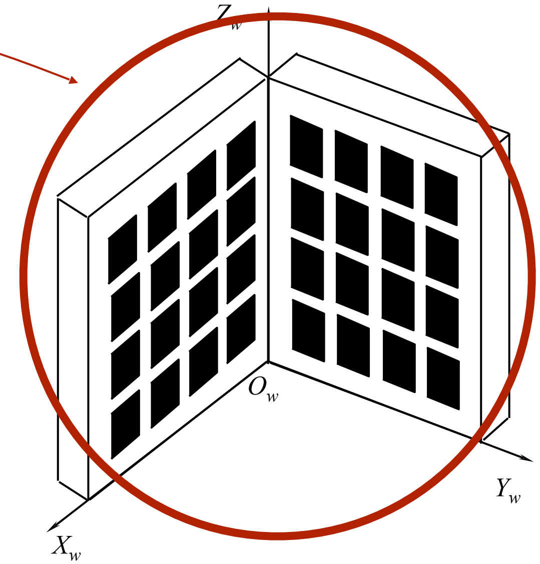
$$ {}^{r}\mathbf{T}_{c} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} $$

❸

$$ \mathbf{K} = {}^{i}\mathbf{T}_{r} = \begin{bmatrix} \alpha_u & \theta & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} $$

$$ \lambda \cdot \mathbf{m} = \mathbf{P} \cdot \mathbf{M} \qquad \Big| \qquad \mathbf{P} = {}^{i}\mathbf{T}_{r} \cdot {}^{r}\mathbf{T}_{c} \cdot {}^{c}\mathbf{T}_{w} $$





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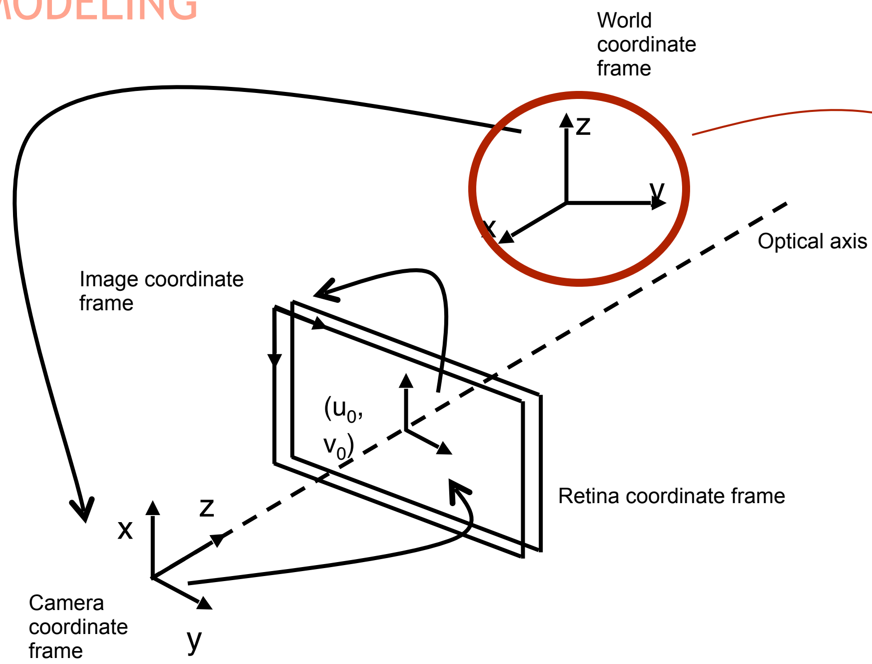

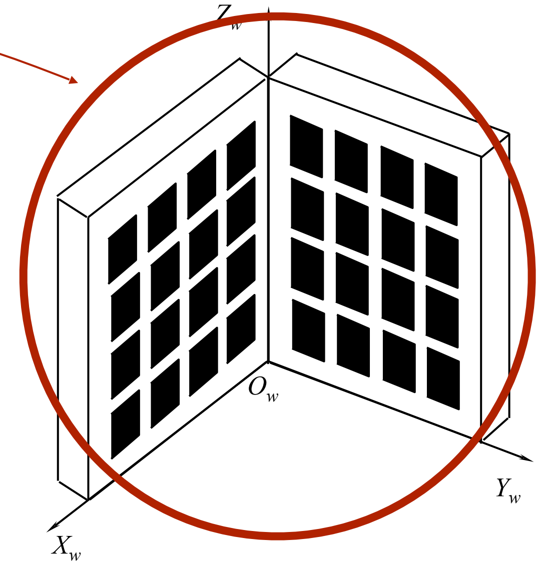

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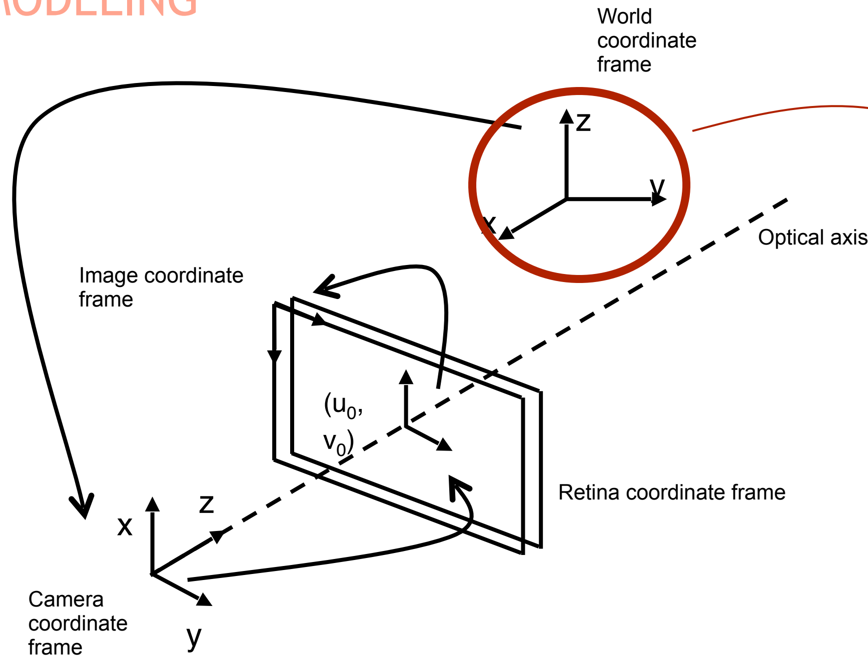

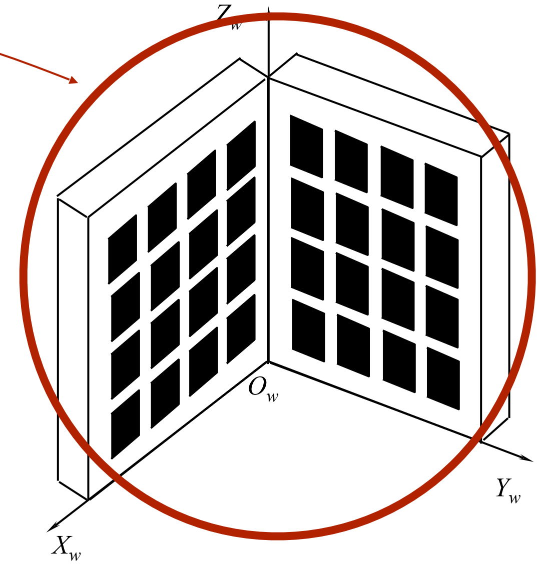

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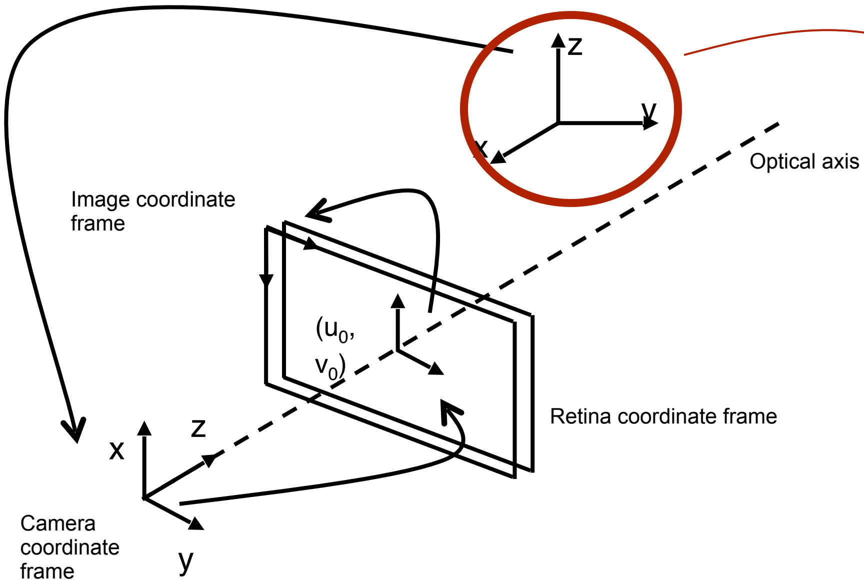
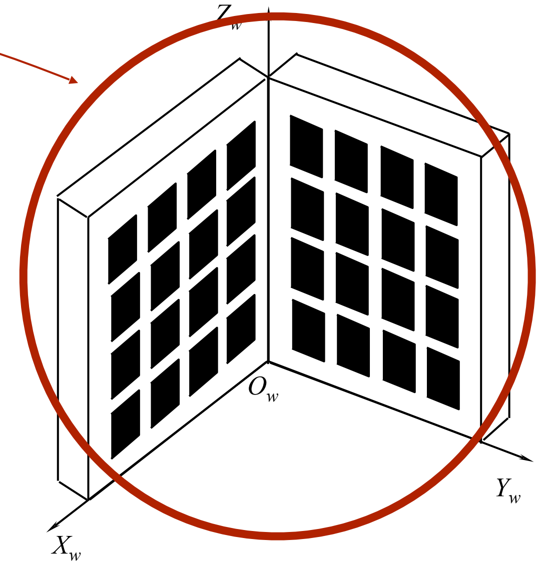
BASIC PRINCIPLES





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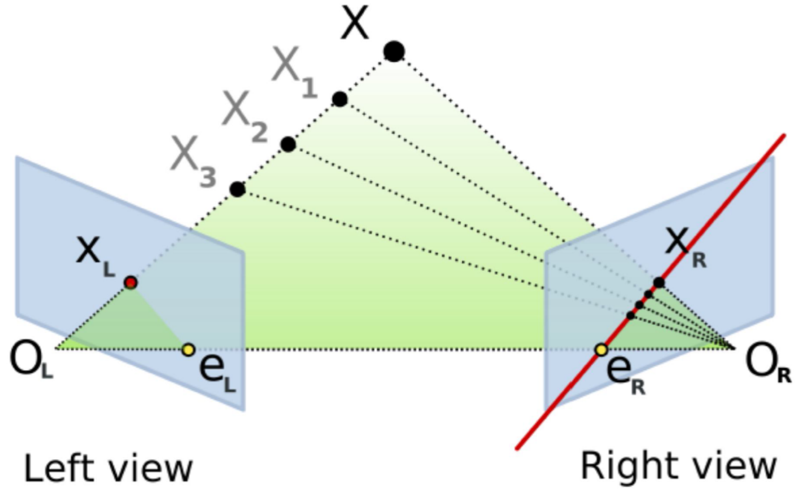


$$\mathbf{x}_R^T \mathbf{F} \mathbf{x}_L = 0 \qquad \mathbf{F} = \mathbf{K}_R^{-T} [\mathbf{t}]_\times \mathbf{R} \mathbf{K}_L^{-1}$$

$$\left| \begin{array}{l} \mathbf{F}\mathbf{e}_L = 0 \\ \mathbf{l}_R = \mathbf{F}\mathbf{x}_L \end{array} \right.$$

$$\left| \; \mathbf{t} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \Rightarrow [\mathbf{t}]_\times = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \right.$$

If Ks are known (calibrated cameras):

$$\mathbf{E} = \mathbf{K}_R^T \mathbf{F} \mathbf{K}_L = [\mathbf{t}]_\times \mathbf{R}$$

BASIC PRINCIPLES

Rough approach: $\arg\min \left\| \lambda \cdot \mathbf{m} - \mathbf{PM} \right\|^2$

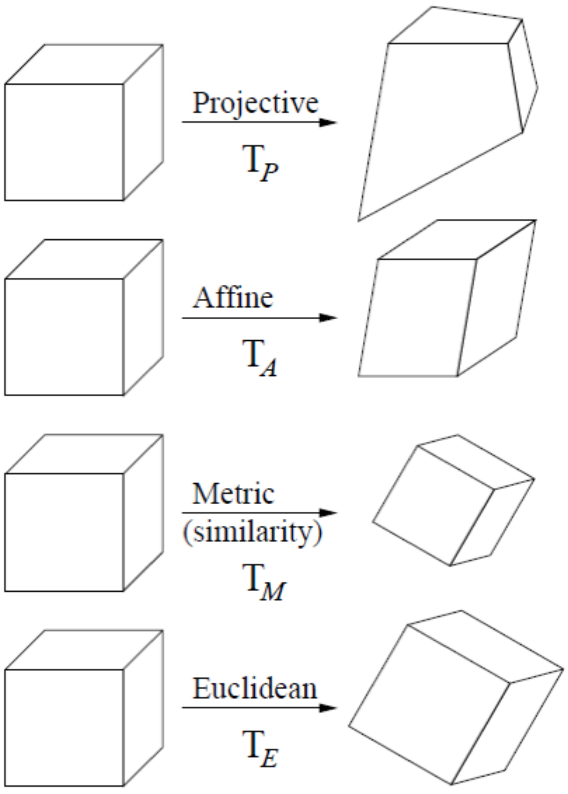
$$\lambda \cdot \mathbf{m} = \mathbf{PM}$$

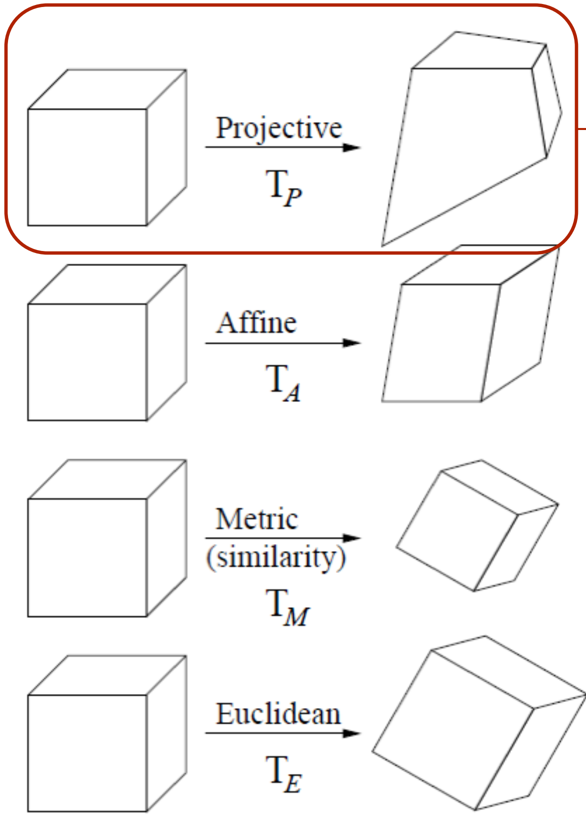
$$\lambda \cdot \mathbf{m} = \mathbf{P}\left(\mathbf{W}\mathbf{W}^{-1}\right)\mathbf{M}$$

$$\lambda \cdot \mathbf{m} = \left(\mathbf{PW}\right)\left(\mathbf{W}^{-1}\mathbf{M}\right)$$

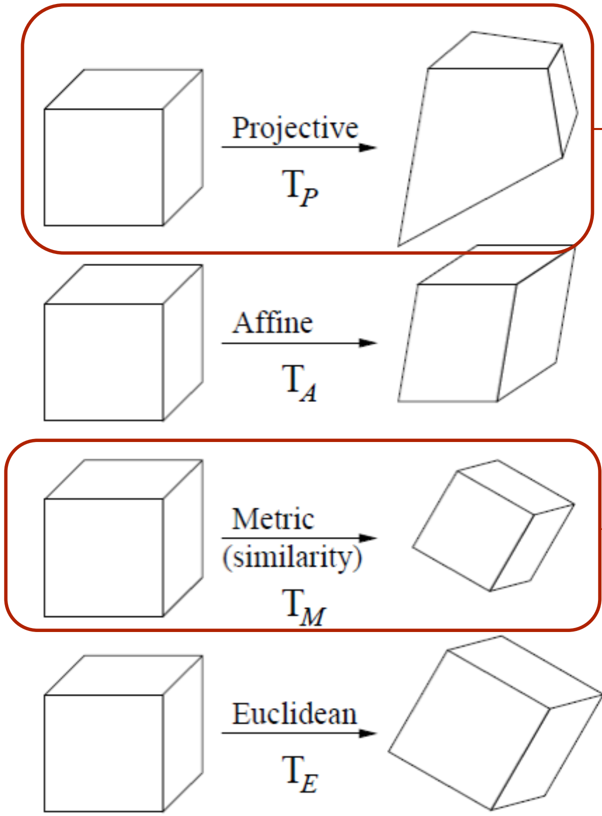
If **P** and **M** are solutions, then **PW** and $\mathbf{W}^{-1}\mathbf{M}$ are also solutions

W is a 4x4 invertible matrix with no particular structure:
it represents a projective transformation!


Projective
T_P
Affine
T_A
Metric
(similarity)
T_M
Euclidean
T_E



- Distance, orthogonality and parallelism are not preserved under projective transformation.
- Colinearities, coplanarities and the cross-ratio are preserved under projective transformation.



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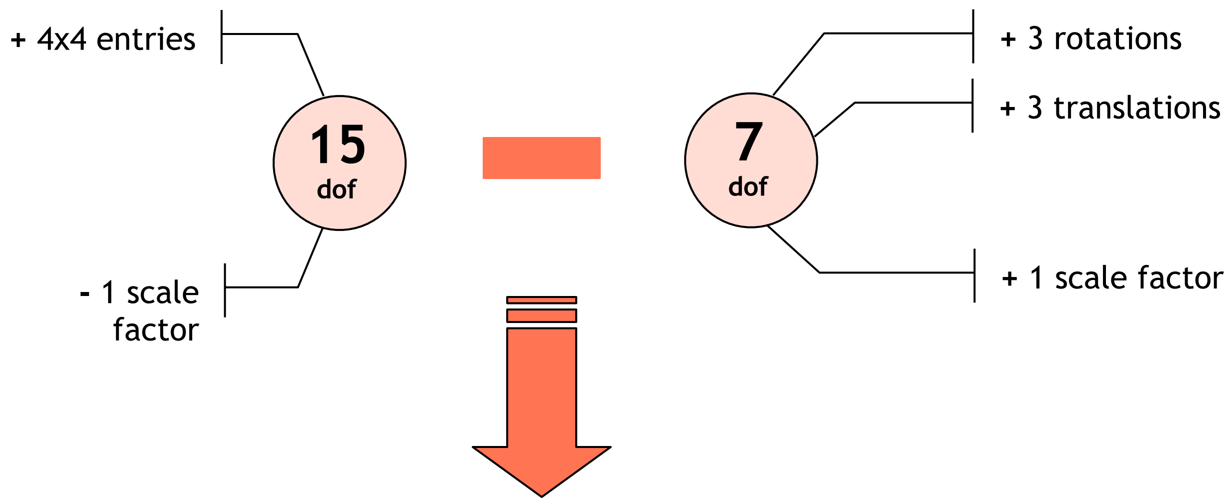
With self-calibration, you can only do this…

From a projective space to a metric space…



8 constraints are required!!

Knowing an intrinsic camera parameter for *n* views gives *n* constraints, fixing one yields only *n* - 1 constraints.

BASIC PRINCIPLES

$$n \times (\#\,known) + (n-1) \times (\#\,fixed) \geq 8$$

Table 1. A few examples of minimum sequence length required to allow self-calibration.

Constraints	Known	Fixed	Min no. of images
No skew	s		8
Fixed aspect ratio and absence of skew	s	$\frac{f_y}{f_x}$	5
Known aspect ratio and absence of skew	$s, \frac{f_y}{f_x}$		4
Only focal length is unknown	$s, \frac{f_y}{f_x}, ux, uy$		2
Standard self-calibration problem		f_x, f_y, ux, uy, s	3

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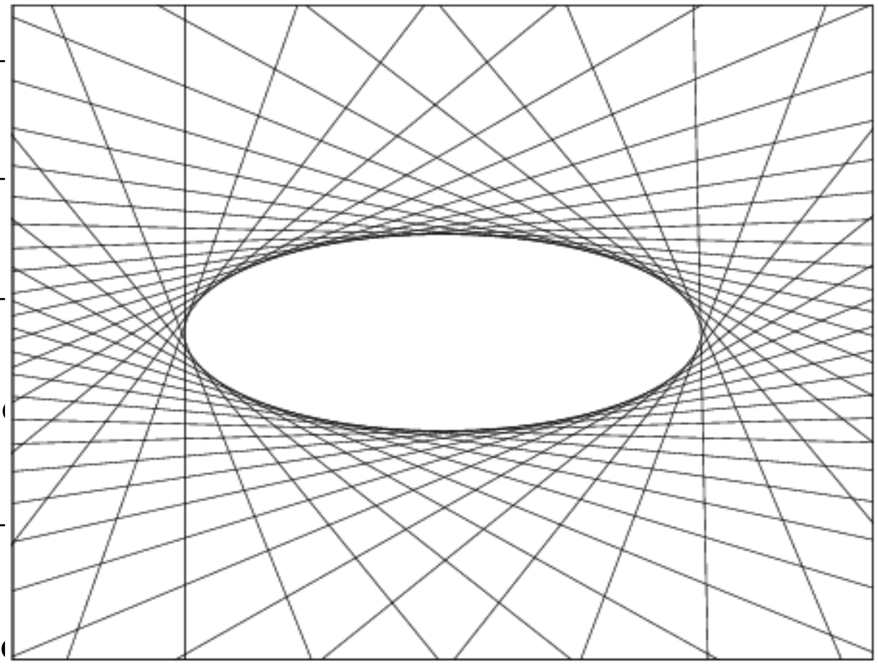
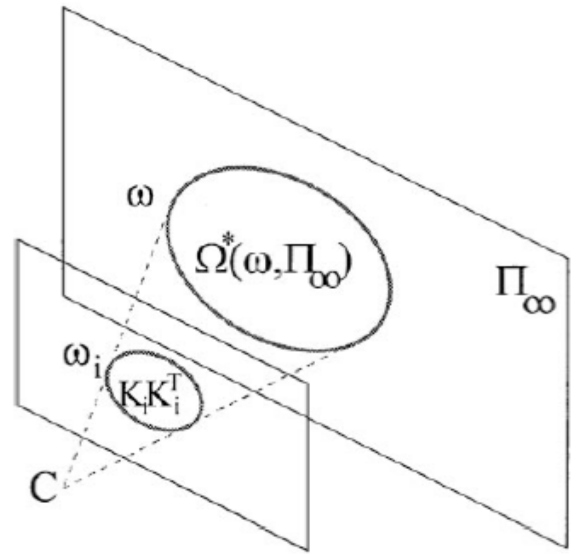
2D points	$\mathbf{m} = \begin{bmatrix} u & v & s \end{bmatrix}^T$		$\mathbf{m} \rightarrow \mathbf{Tm}$
3D points	$\mathbf{M} = \begin{bmatrix} X & Y & Z & T \end{bmatrix}^T$		$\mathbf{M} \rightarrow \mathbf{TM}$
Lines	$\mathbf{l} = \begin{bmatrix} a & b & c \end{bmatrix}^T$	$\mathbf{lm} = 0$	$\mathbf{l} \rightarrow \mathbf{T}^{-T}\mathbf{l}$
Planes	$\boldsymbol{\pi} = \begin{bmatrix} a & b & c & d \end{bmatrix}^T$	$\boldsymbol{\pi}\mathbf{M} = 0$	$\boldsymbol{\pi} \rightarrow \mathbf{T}^{-T}\boldsymbol{\pi}$
Conics	$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{12} & c_{22} & c_{23} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}$	$\mathbf{m}^T\mathbf{Cm} = 0$	$\mathbf{C} \rightarrow \mathbf{T}^{-T}\mathbf{CT}^{-1}$
		$\mathbf{l}^T\mathbf{C}^*\mathbf{l} = 0$	$\mathbf{C}^* \rightarrow \mathbf{TC}^*\mathbf{T}^T$
Quadrics	$\mathbf{Q} = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ q_{14} & q_{24} & q_{34} & q_{44} \end{bmatrix}$	$\mathbf{M}^T\mathbf{QM} = 0$	$\mathbf{Q} \rightarrow \mathbf{T}^{-T}\mathbf{QT}^{-1}$
		$\boldsymbol{\pi}^T\mathbf{Q}^*\boldsymbol{\pi} = 0$	$\mathbf{Q}^* \rightarrow \mathbf{TQ}^*\mathbf{T}^T$

2D points	$\mathbf{m} = \begin{bmatrix} u & v & s \end{bmatrix}^T$		$\mathbf{m} \rightarrow \mathbf{T}\mathbf{m}$
3D points			$\mathbf{M} \rightarrow \mathbf{T}\mathbf{M}$
Lines			$\mathbf{l} \rightarrow \mathbf{T}^{-T}\mathbf{l}$
Planes			$\boldsymbol{\pi} \rightarrow \mathbf{T}^{-T}\boldsymbol{\pi}$
Conics			$\mathbf{C} \rightarrow \mathbf{T}^{-T}\mathbf{C}\mathbf{T}^{-1}$ $\mathbf{C}^* \rightarrow \mathbf{T}\mathbf{C}^*\mathbf{T}^T$
Quadrics	$\begin{matrix} q_{13} & q_{23} & q_{33} & q_{34} \\ q_{14} & q_{24} & q_{34} & q_{44} \end{matrix}$	$\boldsymbol{\pi}^T\mathbf{Q}^*\boldsymbol{\pi} = 0$	$\mathbf{Q} \rightarrow \mathbf{T}^{-T}\mathbf{Q}\mathbf{T}^{-1}$ $\mathbf{Q}^* \rightarrow \mathbf{T}\mathbf{Q}^*\mathbf{T}^T$



Absolute Conic

The absolute conic ω is invariant under Euclidean transformation (e.g. displacement). Thus, its image depends only upon the intrinsic parameters...



$$\omega : x^2 + y^2 + z^2 = 0 \quad \text{and} \quad t = 0$$

It is more convenient to use the **absolute dual quadric** which encodes both the plane at infinity and the absolute conic...

$$\left.\begin{aligned} \mathbf{\Omega}^* &= \begin{bmatrix} \mathbf{I} & 0 \\ 0 & 0 \end{bmatrix} \\ \mathbf{T} &= \begin{bmatrix} s\mathbf{R} & \mathbf{T} \\ 0 & 1 \end{bmatrix} \end{aligned}\right\} \quad \mathbf{\Omega}^* \rightarrow \mathbf{T}\mathbf{\Omega}^*\mathbf{T}^T \rightarrow \mathbf{\Omega}^*$$

Similarity transformation

$$\boldsymbol{\omega}_i^* = \mathbf{K}_i\mathbf{K}_i^T \approx \mathbf{P}_i\mathbf{\Omega}^*\mathbf{P}_i^T$$

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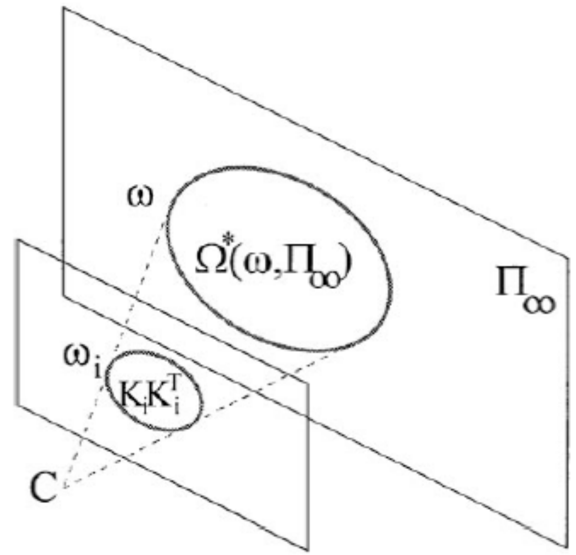

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$$\Omega^* = \begin{bmatrix} \mathbf{I} & 0 \\ 0 & 0 \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} s\mathbf{R} & \mathbf{T} \\ 0 & 1 \end{bmatrix}$$

$$\Omega^* \rightarrow \mathbf{T}\Omega^*\mathbf{T}^T \rightarrow \Omega^*$$

Similarity transformation

$$\boxed{\omega_i^* = \mathbf{K}_i\mathbf{K}_i^T \approx \mathbf{P}_i\Omega^*\mathbf{P}_i^T}$$

A possible parameterization for the **absolute dual quadric…**

$$\Omega^* = \begin{bmatrix} \mathbf{K}\mathbf{K}^\top & -\mathbf{K}\mathbf{K}^\top a \\ -a^\top \mathbf{K}\mathbf{K}^\top & a^\top \mathbf{K}\mathbf{K}^\top a \end{bmatrix}$$

$$\Pi_\infty = [\, a^\top \;\; 1]^\top$$

which simplifies the transformation from projective to metric…

$$\mathbf{T}_{\mathcal{P}\rightarrow\mathcal{M}} = \begin{bmatrix} \mathbf{K}^{-1} & 0 \\ a^\top & 1 \end{bmatrix}$$

i.e. to transform Ω* from a generic position to its canonical position

$$\mathbf{K}_i \mathbf{K}_i^\top \propto \mathbf{P}_i \begin{bmatrix} \mathbf{K}_1 \mathbf{K}_1^\top & -\mathbf{K}_1 \mathbf{K}_1^\top a^\top \\ -a \mathbf{K}_1 \mathbf{K}_1^\top & a \mathbf{K}_1 \mathbf{K}_1^\top a^\top \end{bmatrix} \mathbf{P}_i^\top$$

$$\mathbf{K}_i = \begin{bmatrix} f & 0 & 0 \\ & f & 0 \\ & & 1 \end{bmatrix} \quad \Longrightarrow \quad \lambda \begin{bmatrix} f_i^2 & 0 & 0 \\ 0 & f_i^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathbf{P}_i \begin{bmatrix} b_1 & 0 & 0 & b_2 \\ 0 & b_1 & 0 & b_3 \\ 0 & 0 & 1 & b_4 \\ b_2 & b_3 & b_4 & b_5 \end{bmatrix} \mathbf{P}_i^\top$$

When only two views are available the solution is only determined up to a one parameter family : **this results in up to 4 possible solutions**.

INTRINSIC CONSTRAINTS

Summarizing...

$$\arg\min \left\|\lambda \cdot \mathbf{m} - \mathbf{PM}\right\|^2$$

❶ Computing a projective reconstruction → P_i

$$\boldsymbol{\omega}_i^* = \mathbf{K}_i\mathbf{K}_i^T \approx \mathbf{P}_i\boldsymbol{\Omega}^*\mathbf{P}_i^T$$

❷ Computing the absolute dual quadric → Ω^*

❸ Retrieving the absolute dual conic → ω^*_i

$$\mathbf{K}_i = \begin{bmatrix} f & 0 & 0 \\ & f & 0 \\ & & 1 \end{bmatrix}$$

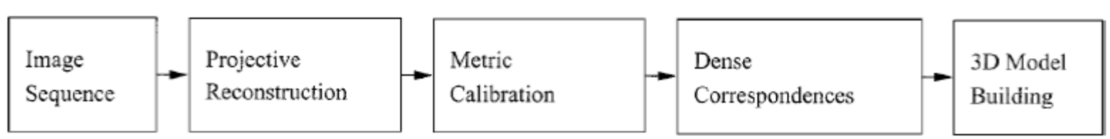
❹ Extracting the intrinsic parameters and the plane at infinity → K, a

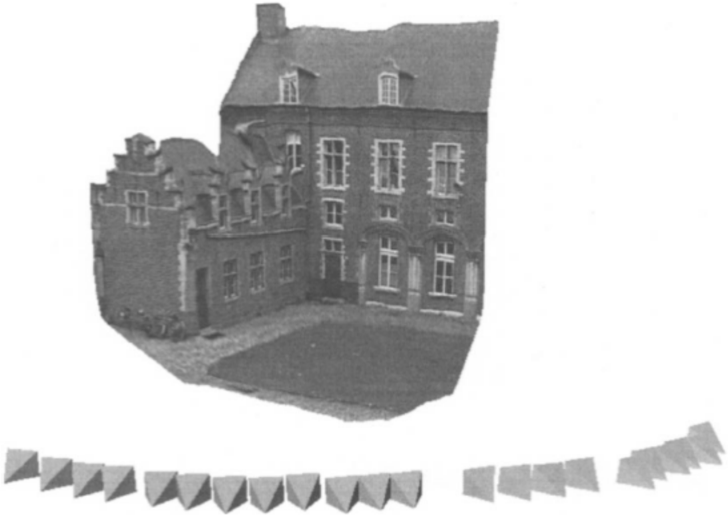
❺ Upgrading the projective reconstruction to metric

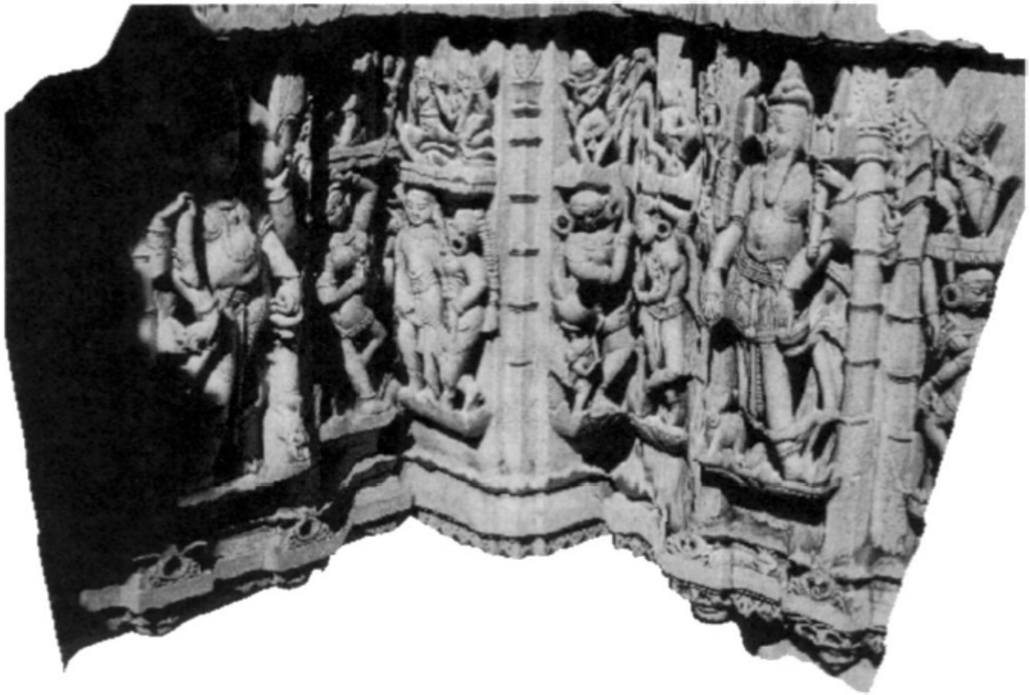
$$\mathbf{T}_{\mathcal{P}\to\mathcal{M}} = \begin{bmatrix} \mathbf{K}^{-1} & 0 \\ a^\top & 1 \end{bmatrix}$$

❻ Performing bundle adjustment

$$\mathcal{C}_{ML}(\mathtt{M}_l, \mathbf{K}_i, \mathbf{R}_i, \mathtt{t}_i) = \sum_{i=1}^{n}\sum_{l\in I_i}\left((x_{li} - \frac{\mathtt{P}_{i1}\mathtt{M}_l}{\mathtt{P}_{i3}\mathtt{M}_l})^2 + (y_{li} - \frac{\mathtt{P}_{i2}\mathtt{M}_l}{\mathtt{P}_{i3}\mathtt{M}_l})^2\right)$$







Metric transformations are a sub-group of projective transformations…

A projective transformation **W** exists that upgrades a projective reconstruction into a metric reconstruction!

$$\mathbf{P}_M = \mathbf{W}\mathbf{P}_P$$

W is a 4x4 invertible matrix with 15 degrees of freedom…

15 independent metric constraints are necessary to compute W!

(Problem: how to extract them from a priorily unknown scene?)

SCENE CONSTRAINTS

Alignments, parallelisms, distances, orthogonalities…

$$w_{31}x_{p'} + w_{32}y_{p'} + w_{33}z_{p'} + w_{34}t_{p'} = 0$$

$$w_{41}x_{A'} + w_{42}y_{A'} + w_{43}z_{A'} + w_{44}t_{A'}$$

$$- \frac{w_{31}x_{B'} + w_{32}y_{B'} + w_{33}z_{B'} + w_{34}t_{B'}}{w_{41}x_{B'} + w_{42}y_{B'} + w_{43}z_{B'} + w_{44}t_{B'}} \Bigg)^2 = d^2$$

$$z_p = \frac{w_{31}x_{p'} + w_{32}y_{p'} + w_{33}z_{p'} + w_{34}t_{p'}}{w_{41}x_{p'} + w_{42}y_{p'} + w_{43}z_{p'} + w_{44}t_{p'}}$$

Fixing a distance

Fixing the origin

SCENE CONSTRAINTS

Alignments, parallelisms, distances, orthogonalities…

$$\left(\frac{w_{11}x_{A'}+w_{12}y_{A'}+w_{13}z_{A'}+w_{14}t_{A'}}{w_{41}x_{A'}+w_{42}y_{A'}+w_{43}z_{A'}+w_{44}t_{A'}} - \frac{w_{11}x_{B'}+w_{12}y_{B'}+w_{13}z_{B'}+w_{14}t_{B'}}{w_{41}x_{B'}+w_{42}y_{B'}+w_{43}z_{B'}+w_{44}t_{B'}}\right)^2 + \left(\frac{w_{21}x_{A'}+w_{22}y_{A'}+w_{23}z_{A'}+w_{24}t_{A'}}{w_{41}x_{A'}+w_{42}y_{A'}+w_{43}z_{A'}+w_{44}t_{A'}} - \frac{w_{21}x_{B'}+w_{22}y_{B'}+w_{23}z_{B'}+w_{24}t_{B'}}{w_{41}x_{B'}+w_{42}y_{B'}+w_{43}z_{B'}+w_{44}t_{B'}}\right)^2 + \left(\frac{w_{31}x_{A'}+w_{32}y_{A'}+w_{33}z_{A'}+w_{34}t_{A'}}{w_{41}x_{A'}+w_{42}y_{A'}+w_{43}z_{A'}+w_{44}t_{A'}} - \frac{w_{31}x_{B'}+w_{32}y_{B'}+w_{33}z_{B'}+w_{34}t_{B'}}{w_{41}x_{B'}+w_{42}y_{B'}+w_{43}z_{B'}+w_{44}t_{B'}}\right)^2 = d^2$$

Fixing a distance

$$w_{31}x_{p'} + w_{32}y_{p'} + w_{33}z_{p'} + w_{34}t_{p'} = 0$$

Point lying on the horizontal plane

$$x_p = \frac{w_{11}x_{p'}+w_{12}y_{p'}+w_{13}z_{p'}+w_{14}t_{p'}}{w_{41}x_{p'}+w_{42}y_{p'}+w_{43}z_{p'}+w_{44}t_{p'}}$$

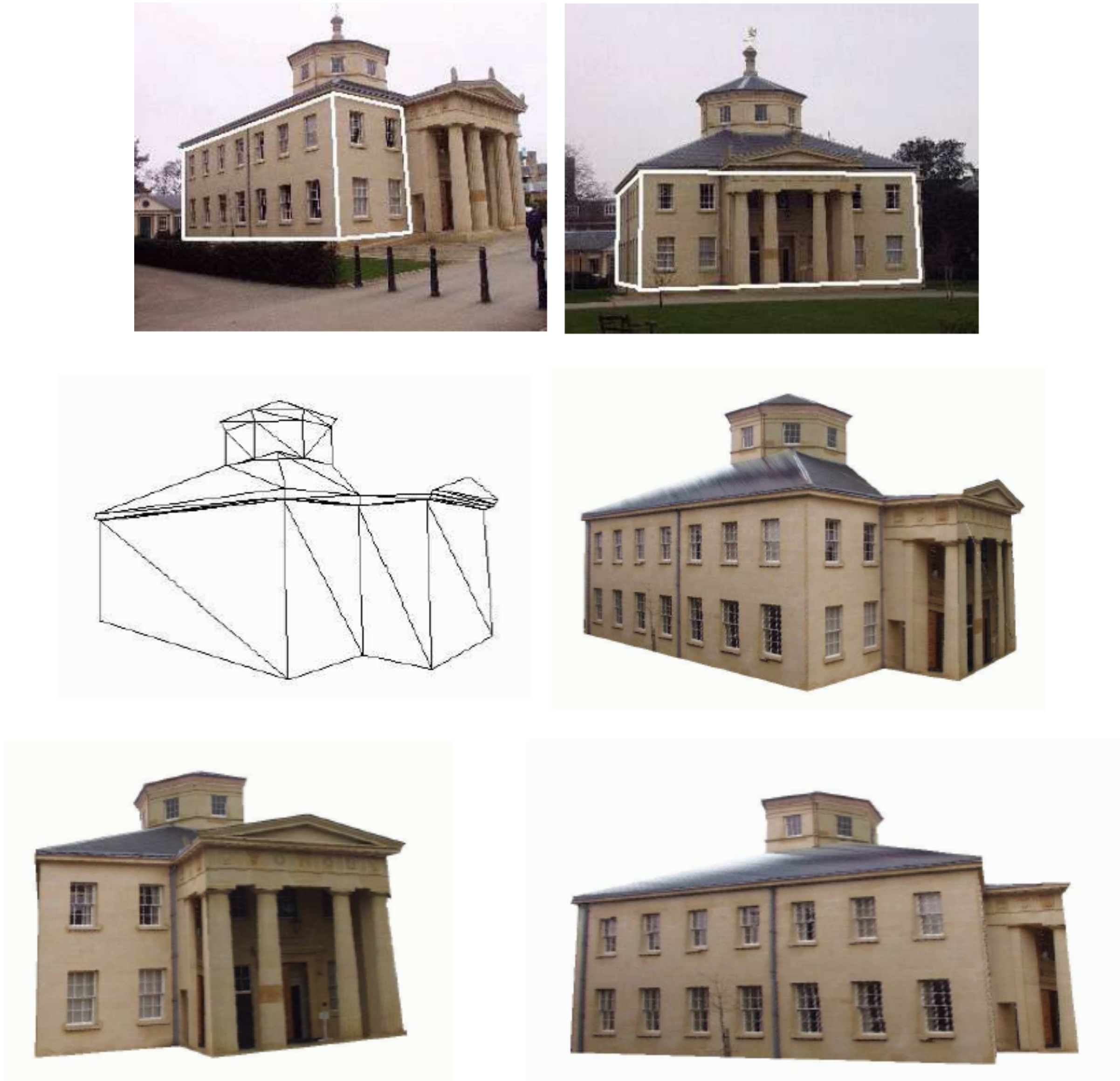
$$y_p = \frac{w_{21}x_{p'}+w_{22}y_{p'}+w_{23}z_{p'}+w_{24}t_{p'}}{w_{41}x_{p'}+w_{42}y_{p'}+w_{43}z_{p'}+w_{44}t_{p'}}$$

$$z_p = \frac{w_{31}x_{p'}+w_{32}y_{p'}+w_{33}z_{p'}+w_{34}t_{p'}}{w_{41}x_{p'}+w_{42}y_{p'}+w_{43}z_{p'}+w_{44}t_{p'}}$$

Fixing the origin

4.3 RESULTS

SCENE CONSTRAINTS



SCENE CONSTRAINTS

SCENE CONSTRAINTS

QUESTIONS

QUESTIONS

QUESTIONS

QUESTIONS

QUESTIONS