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23-1 WHAT IS PHYSICS?

One of the primary goals of physics is to find simple ways of solving seemingly complex problems. One of the main tools of physics in attaining this goal is the use of symmetry. For example, in finding the electric field \vec{E} of the charged ring of Fig. 22-10 and the charged rod of Fig. 22-11, we considered the fields $d\vec{E} (= k dq/r^2)$ of charge elements in the ring and rod. Then we simplified the calculation of \vec{E} by using symmetry to discard the perpendicular components of the $d\vec{E}$ vectors. That saved us some work.

For certain charge distributions involving symmetry, we can save far more work by using a law called Gauss' law, developed by German mathematician and physicist Carl Friedrich Gauss (1777–1855). Instead of considering the fields $d\vec{E}$ of charge elements in a given charge distribution, Gauss' law considers a hypothetical (imaginary) closed surface enclosing the charge distribution. This **Gaussian surface**, as it is called, can have any shape, but the shape that minimizes our calculations of the electric field is one that mimics the symmetry of the charge distribution. For example, if the charge is spread uniformly over a sphere, we enclose the sphere with a spherical Gaussian surface, such as the one in Fig. 23-1, and then, as we discuss in this chapter, find the electric field on the surface by using the fact that



Gauss' law relates the electric fields at points on a (closed) Gaussian surface to the net charge enclosed by that surface.

We can also use Gauss' law in reverse: If we know the electric field on a Gaussian surface, we can find the net charge enclosed by the surface. As a limited example, suppose that the electric field vectors in Fig. 23-1 all point radially outward from the center of the sphere and have equal magnitude. Gauss' law immediately tells us that the spherical surface must enclose a net positive charge that is either a particle or distributed spherically. However, to calculate how *much* charge is enclosed, we need a way of calculating how much electric field is intercepted by the Gaussian surface in Fig. 23-1. This measure of intercepted field is called *flux*, which we discuss next.

23-2 Flux

Suppose that, as in Fig. 23-2a, you aim a wide airstream of uniform velocity \vec{v} at a small square loop of area A . Let Φ represent the *volume flow rate* (volume per unit time) at which air flows through the loop. This rate depends on the angle between \vec{v} and the plane of the loop. If \vec{v} is perpendicular to the plane, the rate Φ is equal to vA .

If \vec{v} is parallel to the plane of the loop, no air moves through the loop, so Φ is zero. For an intermediate angle θ , the rate Φ depends on the component of \vec{v} normal to the plane (Fig. 23-2b). Since that component is $v \cos \theta$, the rate of volume flow through the loop is

$$\Phi = (v \cos \theta)A. \quad (23-1)$$

This rate of flow through an area is an example of a **flux**—a *volume flux* in this situation.

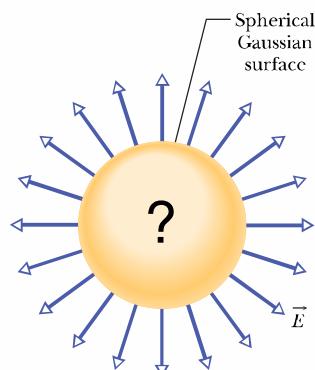
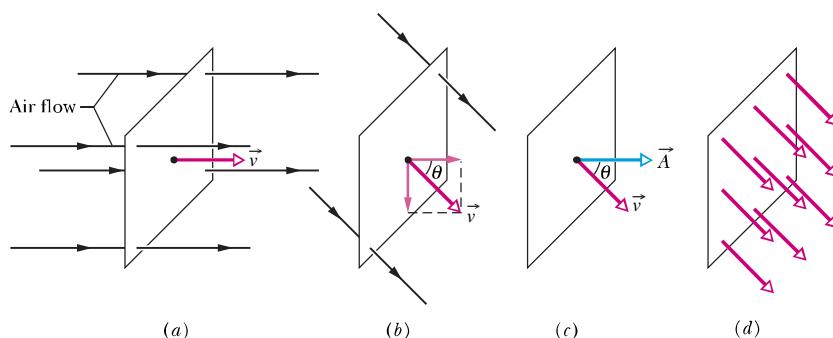


Fig. 23-1 A spherical Gaussian surface. If the electric field vectors are of uniform magnitude and point radially outward at all surface points, you can conclude that a net positive distribution of charge must lie within the surface and have spherical symmetry.

Fig. 23-2 (a) A uniform airstream of velocity \vec{v} is perpendicular to the plane of a square loop of area A . (b) The component of \vec{v} perpendicular to the plane of the loop is $v \cos \theta$, where θ is the angle between \vec{v} and a normal to the plane. (c) The area vector \vec{A} is perpendicular to the plane of the loop and makes an angle θ with \vec{v} . (d) The velocity field intercepted by the area of the loop.



Before we discuss a flux involved in electrostatics, we need to rewrite Eq. 23-1 in terms of vectors. To do this, we first define an *area vector* \vec{A} as being a vector whose magnitude is equal to an area (here the area of the loop) and whose direction is normal to the plane of the area (Fig. 23-2c). We then rewrite Eq. 23-1 as the scalar (or dot) product of the velocity vector \vec{v} of the airstream and the area vector \vec{A} of the loop:

$$\Phi = vA \cos \theta = \vec{v} \cdot \vec{A}, \quad (23-2)$$

where θ is the angle between \vec{v} and \vec{A} .

The word “flux” comes from the Latin word meaning “to flow.” That meaning makes sense if we talk about the flow of air volume through the loop. However, Eq. 23-2 can be regarded in a more abstract way. To see this different way, note that we can assign a velocity vector to each point in the airstream passing through the loop (Fig. 23-2d). Because the composite of all those vectors is a *velocity field*, we can interpret Eq. 23-2 as giving the *flux of the velocity field through the loop*. With this interpretation, flux no longer means the actual flow of something through an area—rather it means the product of an area and the field across that area.

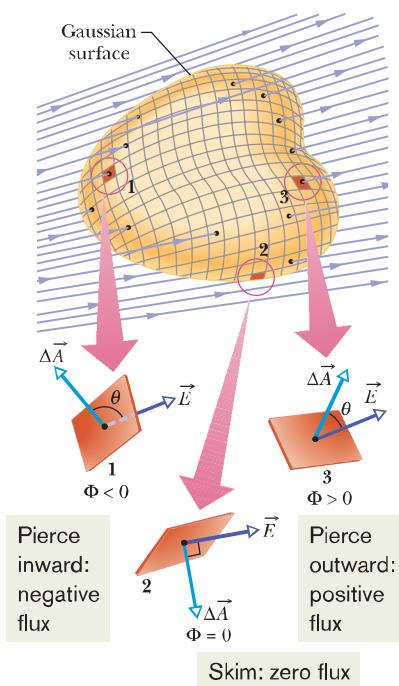


Fig. 23-3 A Gaussian surface of arbitrary shape immersed in an electric field. The surface is divided into small squares of area ΔA . The electric field vectors \vec{E} and the area vectors $\vec{\Delta A}$ for three representative squares, marked 1, 2, and 3, are shown.

23-3 Flux of an Electric Field

To define the flux of an electric field, consider Fig. 23-3, which shows an arbitrary (asymmetric) Gaussian surface immersed in a nonuniform electric field. Let us divide the surface into small squares of area ΔA , each square being small enough to permit us to neglect any curvature and to consider the individual square to be flat. We represent each such element of area with an area vector $\vec{\Delta A}$, whose magnitude is the area ΔA . Each vector $\vec{\Delta A}$ is perpendicular to the Gaussian surface and directed away from the interior of the surface.

Because the squares have been taken to be arbitrarily small, the electric field \vec{E} may be taken as constant over any given square. The vectors $\vec{\Delta A}$ and \vec{E} for each square then make some angle θ with each other. Figure 23-3 shows an enlarged view of three squares on the Gaussian surface and the angle θ for each.

A provisional definition for the flux of the electric field for the Gaussian surface of Fig. 23-3 is

$$\Phi = \sum \vec{E} \cdot \vec{\Delta A}. \quad (23-3)$$

This equation instructs us to visit each square on the Gaussian surface, evaluate the scalar product $\vec{E} \cdot \vec{\Delta A}$ for the two vectors \vec{E} and $\vec{\Delta A}$ we find there, and sum the results algebraically (that is, with signs included) for all the squares that make up the surface. The value of each scalar product (positive, negative, or zero) determines whether the flux through its square is positive, negative, or zero. Squares like square 1 in Fig. 23-3, in which \vec{E} points inward, make a negative contribution to the sum of Eq. 23-3. Squares like 2, in which \vec{E} lies in the surface, make zero contribution. Squares like 3, in which \vec{E} points outward, make a positive contribution.

23-3 FLUX OF AN ELECTRIC FIELD

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The exact definition of the flux of the electric field through a closed surface is found by allowing the area of the squares shown in Fig. 23-3 to become smaller and smaller, approaching a differential limit dA . The area vectors then approach a differential limit $d\vec{A}$. The sum of Eq. 23-3 then becomes an integral:

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad (\text{electric flux through a Gaussian surface}). \quad (23-4)$$

The loop on the integral sign indicates that the integration is to be taken over the entire (closed) surface. The flux of the electric field is a scalar, and its SI unit is the newton-square-meter per coulomb ($N \cdot m^2/C$).

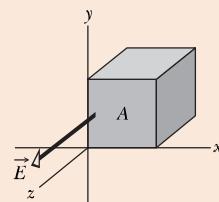
We can interpret Eq. 23-4 in the following way: First recall that we can use the density of electric field lines passing through an area as a proportional measure of the magnitude of the electric field \vec{E} there. Specifically, the magnitude E is proportional to the number of electric field lines per unit area. Thus, the scalar product $\vec{E} \cdot d\vec{A}$ in Eq. 23-4 is proportional to the number of electric field lines passing through area $d\vec{A}$. Then, because the integration in Eq. 23-4 is carried out over a Gaussian surface, which is closed, we see that

 The electric flux Φ through a Gaussian surface is proportional to the net number of electric field lines passing through that surface.



CHECKPOINT 1

The figure here shows a Gaussian cube of face area A immersed in a uniform electric field \vec{E} that has the positive direction of the z axis. In terms of E and A , what is the flux through (a) the front face (which is in the xy plane), (b) the rear face, (c) the top face, and (d) the whole cube?



Sample Problem

Flux through a closed cylinder, uniform field

Figure 23-4 shows a Gaussian surface in the form of a cylinder of radius R immersed in a uniform electric field \vec{E} , with the cylinder axis parallel to the field. What is the flux Φ of the electric field through this closed surface?

KEY IDEA

We can find the flux Φ through the Gaussian surface by integrating the scalar product $\vec{E} \cdot d\vec{A}$ over that surface.

Calculations: We can do the integration by writing the flux as the sum of three terms: integrals over the left cylinder cap a , the cylindrical surface b , and the right cap c . Thus, from Eq. 23-4,

$$\begin{aligned} \Phi &= \oint \vec{E} \cdot d\vec{A} \\ &= \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A}. \end{aligned} \quad (23-5)$$

For all points on the left cap, the angle θ between \vec{E} and $d\vec{A}$ is 180° and the magnitude E of the field is uniform. Thus,

$$\int_a \vec{E} \cdot d\vec{A} = \int E(\cos 180^\circ) dA = -E \int dA = -EA,$$

where $\int dA$ gives the cap's area A ($= \pi R^2$). Similarly, for the

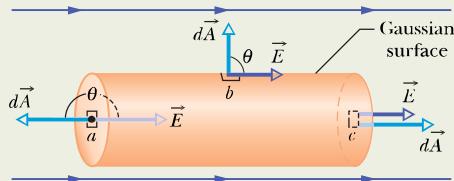


Fig. 23-4 A cylindrical Gaussian surface, closed by end caps, is immersed in a uniform electric field. The cylinder axis is parallel to the field direction.

right cap, where $\theta = 0$ for all points,

$$\int_c \vec{E} \cdot d\vec{A} = \int E(\cos 0^\circ) dA = EA.$$

Finally, for the cylindrical surface, where the angle θ is 90° at all points,

$$\int_b \vec{E} \cdot d\vec{A} = \int E(\cos 90^\circ) dA = 0.$$

Substituting these results into Eq. 23-5 leads us to

$$\Phi = -EA + 0 + EA = 0. \quad (\text{Answer})$$

The net flux is zero because the field lines that represent the electric field all pass entirely through the Gaussian surface, from the left to the right.



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Sample Problem**Flux through a closed cube, nonuniform field**

A *nonuniform* electric field given by $\vec{E} = 3.0\hat{x} + 4.0\hat{y}$ pierces the Gaussian cube shown in Fig. 23-5a. (E is in newtons per coulomb and x is in meters.) What is the electric flux through the right face, the left face, and the top face? (We consider the other faces in another sample problem.)

KEY IDEA

We can find the flux Φ through the surface by integrating the scalar product $\vec{E} \cdot d\vec{A}$ over each face.

Right face: An area vector \vec{A} is always perpendicular to its surface and always points away from the interior of a Gaussian surface. Thus, the vector $d\vec{A}$ for any area element (small section) on the right face of the cube must point in the positive direction of the x axis. An example of such an element is shown in Figs. 23-5b and c, but we would have an identical vector for any other choice of an area element on that face. The most convenient way to express the vector is in unit-vector notation,

$$d\vec{A} = dA\hat{i}.$$

From Eq. 23-4, the flux Φ_r through the right face is then

$$\begin{aligned}\Phi_r &= \int \vec{E} \cdot d\vec{A} = \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{i}) \\ &= \int [(3.0x)(dA)\hat{i} \cdot \hat{i} + (4.0)(dA)\hat{j} \cdot \hat{i}] \\ &= \int (3.0x dA + 0) = 3.0 \int x dA.\end{aligned}$$

We are about to integrate over the right face, but we note that x has the same value everywhere on that face—namely, $x = 3.0$ m. This means we can substitute that constant value

for x . This can be a confusing argument. Although x is certainly a variable as we move left to right across the figure, because the right face is perpendicular to the x axis, every point on the face has the same x coordinate. (The y and z coordinates do not matter in our integral.) Thus, we have

$$\Phi_r = 3.0 \int (3.0) dA = 9.0 \int dA.$$

The integral $\int dA$ merely gives us the area $A = 4.0$ m² of the right face; so

$$\Phi_r = (9.0 \text{ N/C})(4.0 \text{ m}^2) = 36 \text{ N} \cdot \text{m}^2/\text{C}. \quad (\text{Answer})$$

Left face: The procedure for finding the flux through the left face is the same as that for the right face. However, two factors change. (1) The differential area vector $d\vec{A}$ points in the negative direction of the x axis, and thus $d\vec{A} = -dA\hat{i}$ (Fig. 23-5d). (2) The term x again appears in our integration, and it is again constant over the face being considered. However, on the left face, $x = 1.0$ m. With these two changes, we find that the flux Φ_l through the left face is

$$\Phi_l = -12 \text{ N} \cdot \text{m}^2/\text{C}. \quad (\text{Answer})$$

Top face: The differential area vector $d\vec{A}$ points in the positive direction of the y axis, and thus $d\vec{A} = dA\hat{j}$ (Fig. 23-5e). The flux Φ_t through the top face is then

$$\begin{aligned}\Phi_t &= \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{j}) \\ &= \int [(3.0x)(dA)\hat{i} \cdot \hat{j} + (4.0)(dA)\hat{j} \cdot \hat{j}] \\ &= \int (0 + 4.0 dA) = 4.0 \int dA \\ &= 16 \text{ N} \cdot \text{m}^2/\text{C}. \quad (\text{Answer})\end{aligned}$$



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23-4 Gauss' Law

Gauss' law relates the net flux Φ of an electric field through a closed surface (a Gaussian surface) to the *net* charge q_{enc} that is *enclosed* by that surface. It tells us that

$$\epsilon_0 \Phi = q_{\text{enc}} \quad (\text{Gauss' law}). \quad (23-6)$$

By substituting Eq. 23-4, the definition of flux, we can also write Gauss' law as

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \quad (\text{Gauss' law}). \quad (23-7)$$

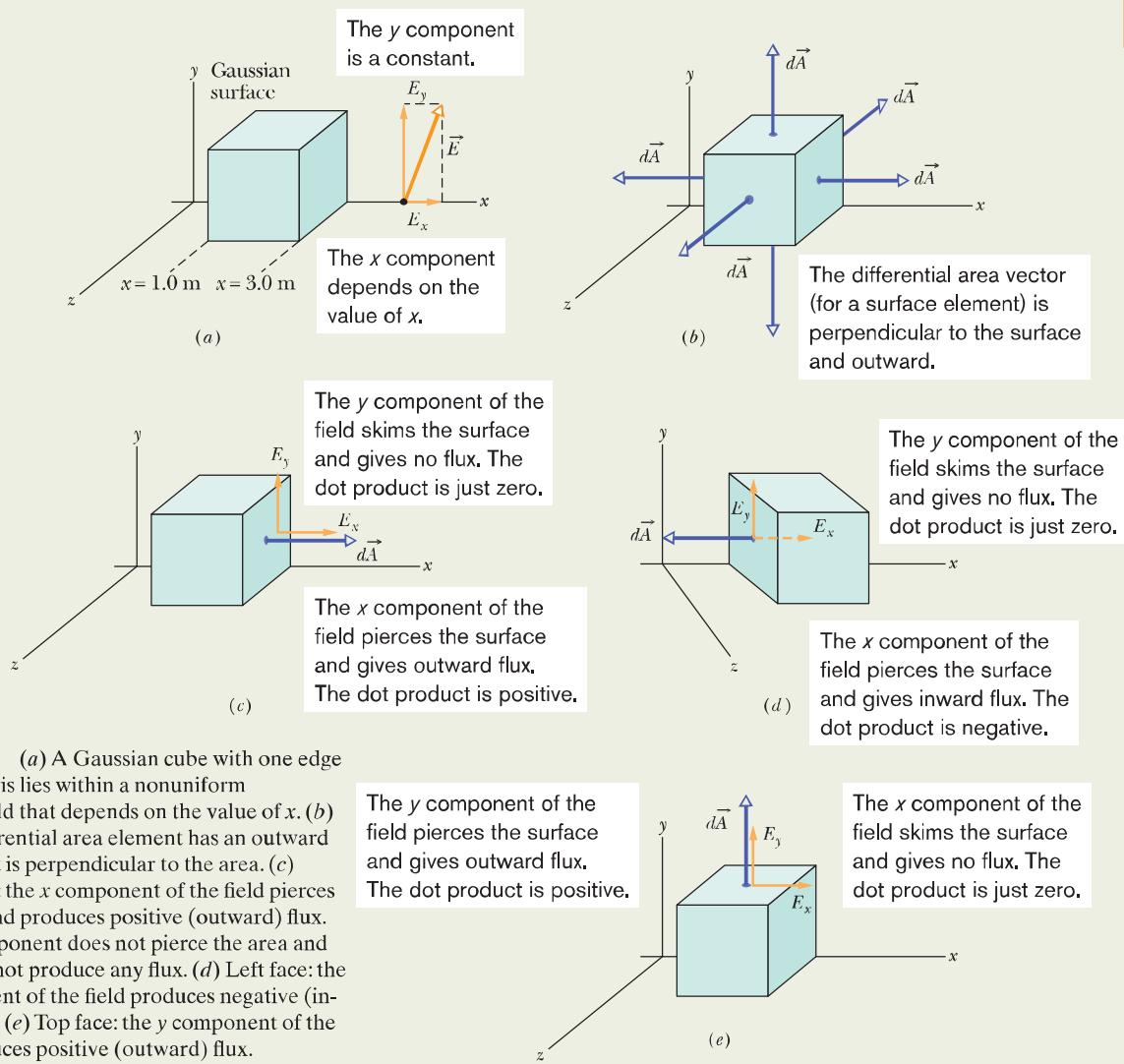


Fig. 23-5 (a) A Gaussian cube with one edge on the x axis lies within a nonuniform electric field that depends on the value of x . (b) Each differential area element has an outward vector that is perpendicular to the area. (c) Right face: the x component of the field pierces the surface and produces positive (outward) flux. The y component does not pierce the area and thus does not produce any flux. (d) Left face: the x component of the field produces negative (inward) flux. (e) Top face: the y component of the field produces positive (outward) flux.

Equations 23-6 and 23-7 hold only when the net charge is located in a vacuum or (what is the same for most practical purposes) in air. In Chapter 25, we modify Gauss' law to include situations in which a material such as mica, oil, or glass is present.

In Eqs. 23-6 and 23-7, the net charge q_{enc} is the algebraic sum of all the *enclosed* positive and negative charges, and it can be positive, negative, or zero. We include the sign, rather than just use the magnitude of the enclosed charge, because the sign tells us something about the net flux through the Gaussian surface: If q_{enc} is positive, the net flux is *outward*; if q_{enc} is negative, the net flux is *inward*.

Charge outside the surface, no matter how large or how close it may be, is not included in the term q_{enc} in Gauss' law. The exact form and location of the charges inside the Gaussian surface are also of no concern; the only things that

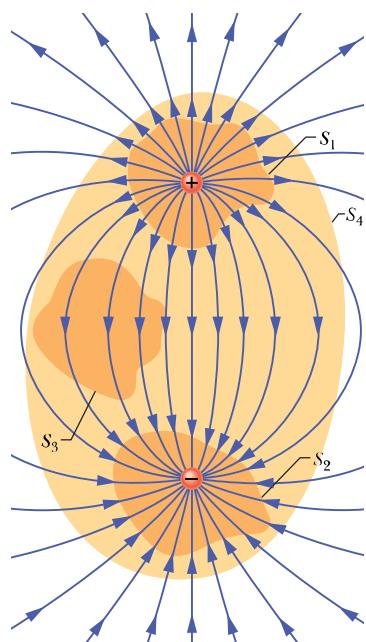


Fig. 23-6 Two point charges, equal in magnitude but opposite in sign, and the field lines that represent their net electric field. Four Gaussian surfaces are shown in cross section. Surface S_1 encloses the positive charge. Surface S_2 encloses the negative charge. Surface S_3 encloses no charge. Surface S_4 encloses both charges and thus no net charge.

matter on the right side of Eqs. 23-6 and 23-7 are the magnitude and sign of the net enclosed charge. The quantity \vec{E} on the left side of Eq. 23-7, however, is the electric field resulting from *all* charges, both those inside and those outside the Gaussian surface. This statement may seem to be inconsistent, but keep this in mind: The electric field due to a charge outside the Gaussian surface contributes zero net flux *through* the surface, because as many field lines due to that charge enter the surface as leave it.

Let us apply these ideas to Fig. 23-6, which shows two point charges, equal in magnitude but opposite in sign, and the field lines describing the electric fields the charges set up in the surrounding space. Four Gaussian surfaces are also shown, in cross section. Let us consider each in turn.

Surface S_1 . The electric field is outward for all points on this surface. Thus, the flux of the electric field through this surface is positive, and so is the net charge within the surface, as Gauss' law requires. (That is, in Eq. 23-6, if Φ is positive, q_{enc} must be also.)

Surface S_2 . The electric field is inward for all points on this surface. Thus, the flux of the electric field through this surface is negative and so is the enclosed charge, as Gauss' law requires.

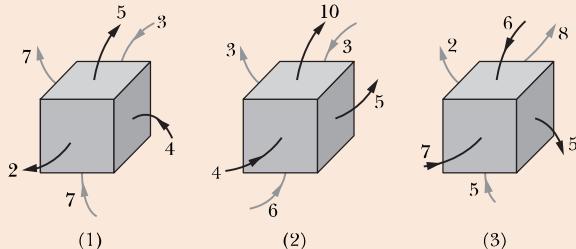
Surface S_3 . This surface encloses no charge, and thus $q_{\text{enc}} = 0$. Gauss' law (Eq. 23-6) requires that the net flux of the electric field through this surface be zero. That is reasonable because all the field lines pass entirely through the surface, entering it at the top and leaving at the bottom.

Surface S_4 . This surface encloses no *net* charge, because the enclosed positive and negative charges have equal magnitudes. Gauss' law requires that the net flux of the electric field through this surface be zero. That is reasonable because there are as many field lines leaving surface S_4 as entering it.

What would happen if we were to bring an enormous charge Q up close to surface S_4 in Fig. 23-6? The pattern of the field lines would certainly change, but the net flux for each of the four Gaussian surfaces would not change. We can understand this because the field lines associated with the added Q would pass entirely through each of the four Gaussian surfaces, making no contribution to the net flux through any of them. The value of Q would not enter Gauss' law in any way, because Q lies outside all four of the Gaussian surfaces that we are considering.

CHECKPOINT 2

The figure shows three situations in which a Gaussian cube sits in an electric field. The arrows and the values indicate the directions of the field lines and the magnitudes (in $\text{N} \cdot \text{m}^2/\text{C}$) of the flux through the six sides of each cube. (The lighter arrows are for the hidden faces.) In which situation does the cube enclose (a) a positive net charge, (b) a negative net charge, and (c) zero net charge?



Sample Problem

Relating the net enclosed charge and the net flux

Figure 23-7 shows five charged lumps of plastic and an electrically neutral coin. The cross section of a Gaussian surface S is indicated. What is the net electric flux through the surface if $q_1 = q_4 = +3.1 \text{ nC}$, $q_2 = q_5 = -5.9 \text{ nC}$, and $q_3 = -3.1 \text{ nC}$?

KEY IDEA

The *net* flux Φ through the surface depends on the *net* charge q_{enc} enclosed by surface S .

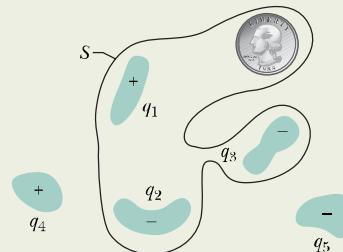
Calculation: The coin does not contribute to Φ because it is neutral and thus contains equal amounts of positive and negative charge. We could include those equal amounts, but they would simply sum to be zero when we calculate the *net* charge enclosed by the surface. So, let's not bother. Charges q_4 and q_5 do not contribute because they are outside surface S . They certainly send electric field lines

Fig. 23-7 Five plastic objects, each with an electric charge, and a coin, which has no net charge. A Gaussian surface, shown in cross section, encloses three of the plastic objects and the coin.

through the surface, but as much enters as leaves and no net flux is contributed. Thus, q_{enc} is only the sum $q_1 + q_2 + q_3$ and Eq. 23-6 gives us

$$\begin{aligned}\Phi &= \frac{q_{\text{enc}}}{\epsilon_0} = \frac{q_1 + q_2 + q_3}{\epsilon_0} \\ &= \frac{+3.1 \times 10^{-9} \text{ C} - 5.9 \times 10^{-9} \text{ C} - 3.1 \times 10^{-9} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} \\ &= -670 \text{ N} \cdot \text{m}^2/\text{C}. \quad (\text{Answer})\end{aligned}$$

The minus sign shows that the net flux through the surface is inward and thus that the net charge within the surface is negative.



Sample Problem

Enclosed charge in a nonuniform field

What is the net charge enclosed by the Gaussian cube of Fig. 23-5, which lies in the electric field $\vec{E} = 3.0\hat{x} + 4.0\hat{y}$? (E is in newtons per coulomb and x is in meters.)

KEY IDEA

The net charge enclosed by a (real or mathematical) closed surface is related to the total electric flux through the surface by Gauss' law as given by Eq. 23-6 ($\epsilon_0\Phi = q_{\text{enc}}$).

Flux: To use Eq. 23-6, we need to know the flux through all six faces of the cube. We already know the flux through the right face ($\Phi_r = 36 \text{ N} \cdot \text{m}^2/\text{C}$), the left face ($\Phi_l = -12 \text{ N} \cdot \text{m}^2/\text{C}$), and the top face ($\Phi_t = 16 \text{ N} \cdot \text{m}^2/\text{C}$).

For the bottom face, our calculation is just like that for the top face *except* that the differential area vector $d\vec{A}$ is now directed downward along the y axis (recall, it must be *outward* from the Gaussian enclosure). Thus, we have

$d\vec{A} = -dA\hat{j}$, and we find

$$\Phi_b = -16 \text{ N} \cdot \text{m}^2/\text{C}.$$

For the front face we have $d\vec{A} = dA\hat{k}$, and for the back face, $d\vec{A} = -dA\hat{k}$. When we take the dot product of the given electric field $\vec{E} = 3.0\hat{x} + 4.0\hat{y}$ with either of these expressions for $d\vec{A}$, we get 0 and thus there is no flux through those faces. We can now find the total flux through the six sides of the cube:

$$\begin{aligned}\Phi &= (36 - 12 + 16 - 16 + 0 + 0) \text{ N} \cdot \text{m}^2/\text{C} \\ &= 24 \text{ N} \cdot \text{m}^2/\text{C}.\end{aligned}$$

Enclosed charge: Next, we use Gauss' law to find the charge q_{enc} enclosed by the cube:

$$\begin{aligned}q_{\text{enc}} &= \epsilon_0\Phi = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(24 \text{ N} \cdot \text{m}^2/\text{C}) \\ &= 2.1 \times 10^{-10} \text{ C}. \quad (\text{Answer})\end{aligned}$$

Thus, the cube encloses a *net* positive charge.



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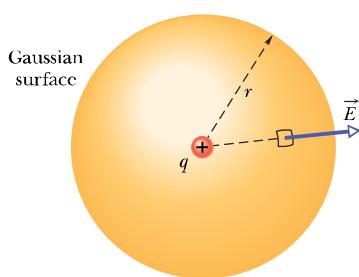


Fig. 23-8 A spherical Gaussian surface centered on a point charge q .

23-5 Gauss' Law and Coulomb's Law

Because Gauss' law and Coulomb's law are different ways of describing the relation between electric charge and electric field in static situations, we should be able to derive each from the other. Here we derive Coulomb's law from Gauss' law and some symmetry considerations.

Figure 23-8 shows a positive point charge q , around which we have drawn a concentric spherical Gaussian surface of radius r . Let us divide this surface into differential areas dA . By definition, the area vector $d\vec{A}$ at any point is perpendicular to the surface and directed outward from the interior. From the symmetry of the situation, we know that at any point the electric field \vec{E} is also perpendicular to the surface and directed outward from the interior. Thus, since the angle θ between \vec{E} and $d\vec{A}$ is zero, we can rewrite Eq. 23-7 for Gauss' law as

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = \epsilon_0 \oint E dA = q_{\text{enc}}. \quad (23-8)$$

Here $q_{\text{enc}} = q$. Although E varies radially with distance from q , it has the same value everywhere on the spherical surface. Since the integral in Eq. 23-8 is taken over that surface, E is a constant in the integration and can be brought out in front of the integral sign. That gives us

$$\epsilon_0 E \oint dA = q. \quad (23-9)$$

The integral is now merely the sum of all the differential areas dA on the sphere and thus is just the surface area, $4\pi r^2$. Substituting this, we have

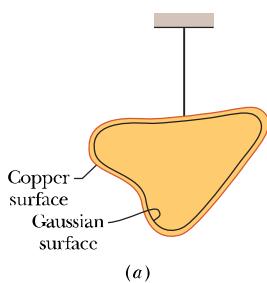
$$\epsilon_0 E (4\pi r^2) = q$$

$$\text{or} \quad E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}. \quad (23-10)$$

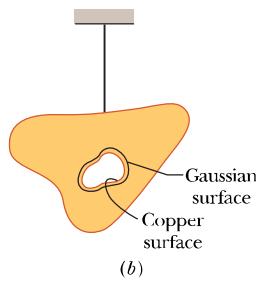
This is exactly Eq. 22-3, which we found using Coulomb's law.

CHECKPOINT 3

There is a certain net flux Φ_i through a Gaussian sphere of radius r enclosing an isolated charged particle. Suppose the enclosing Gaussian surface is changed to (a) a larger Gaussian sphere, (b) a Gaussian cube with edge length equal to r , and (c) a Gaussian cube with edge length equal to $2r$. In each case, is the net flux through the new Gaussian surface greater than, less than, or equal to Φ_i ?



(a)



(b)

Fig. 23-9 (a) A lump of copper with a charge q hangs from an insulating thread. A Gaussian surface is placed within the metal, just inside the actual surface. (b) The lump of copper now has a cavity within it. A Gaussian surface lies within the metal, close to the cavity surface.

23-6 A Charged Isolated Conductor

Gauss' law permits us to prove an important theorem about conductors:

If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.

This might seem reasonable, considering that charges with the same sign repel one another. You might imagine that, by moving to the surface, the added charges are getting as far away from one another as they can. We turn to Gauss' law for verification of this speculation.

Figure 23-9a shows, in cross section, an isolated lump of copper hanging from an insulating thread and having an excess charge q . We place a Gaussian surface just inside the actual surface of the conductor.

The electric field inside this conductor must be zero. If this were not so, the field would exert forces on the conduction (free) electrons, which are always present in a conductor, and thus current would always exist within a conductor. (That is, charge would flow from place to place within the conductor.) Of course, there is no such perpetual current in an isolated conductor, and so the internal electric field is zero.

(An internal electric field *does* appear as a conductor is being charged. However, the added charge quickly distributes itself in such a way that the net internal electric field—the vector sum of the electric fields due to all the charges, both inside and outside—is zero. The movement of charge then ceases, because the net force on each charge is zero; the charges are then in *electrostatic equilibrium*.)

If \vec{E} is zero everywhere inside our copper conductor, it must be zero for all points on the Gaussian surface because that surface, though close to the surface of the conductor, is definitely inside the conductor. This means that the flux through the Gaussian surface must be zero. Gauss' law then tells us that the net charge inside the Gaussian surface must also be zero. Then because the excess charge is not inside the Gaussian surface, it must be outside that surface, which means it must lie on the actual surface of the conductor.

An Isolated Conductor with a Cavity

Figure 23-9b shows the same hanging conductor, but now with a cavity that is totally within the conductor. It is perhaps reasonable to suppose that when we scoop out the electrically neutral material to form the cavity, we do not change the distribution of charge or the pattern of the electric field that exists in Fig. 23-9a. Again, we must turn to Gauss' law for a quantitative proof.

We draw a Gaussian surface surrounding the cavity, close to its surface but inside the conducting body. Because $\vec{E} = 0$ inside the conductor, there can be no flux through this new Gaussian surface. Therefore, from Gauss' law, that surface can enclose no net charge. We conclude that there is no net charge on the cavity walls; all the excess charge remains on the outer surface of the conductor, as in Fig. 23-9a.

The Conductor Removed

Suppose that, by some magic, the excess charges could be “frozen” into position on the conductor's surface, perhaps by embedding them in a thin plastic coating, and suppose that then the conductor could be removed completely. This is equivalent to enlarging the cavity of Fig. 23-9b until it consumes the entire conductor, leaving only the charges. The electric field would not change at all; it would remain zero inside the thin shell of charge and would remain unchanged for all external points. This shows us that the electric field is set up by the charges and not by the conductor. The conductor simply provides an initial pathway for the charges to take up their positions.

The External Electric Field

You have seen that the excess charge on an isolated conductor moves entirely to the conductor's surface. However, unless the conductor is spherical, the charge does not distribute itself uniformly. Put another way, the surface charge density σ (charge per unit area) varies over the surface of any nonspherical conductor. Generally, this variation makes the determination of the electric field set up by the surface charges very difficult.

However, the electric field just outside the surface of a conductor is easy to determine using Gauss' law. To do this, we consider a section of the surface that is small enough to permit us to neglect any curvature and thus to take the section

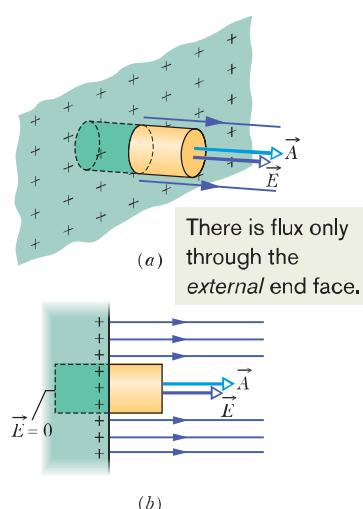


Fig. 23-10 (a) Perspective view and (b) side view of a tiny portion of a large, isolated conductor with excess positive charge on its surface. A (closed) cylindrical Gaussian surface, embedded perpendicularly in the conductor, encloses some of the charge. Electric field lines pierce the external end cap of the cylinder, but not the internal end cap. The external end cap has area A and area vector \vec{A} .

to be flat. We then imagine a tiny cylindrical Gaussian surface to be embedded in the section as in Fig. 23-10: One end cap is fully inside the conductor, the other is fully outside, and the cylinder is perpendicular to the conductor's surface.

The electric field \vec{E} at and just outside the conductor's surface must also be perpendicular to that surface. If it were not, then it would have a component along the conductor's surface that would exert forces on the surface charges, causing them to move. However, such motion would violate our implicit assumption that we are dealing with electrostatic equilibrium. Therefore, \vec{E} is perpendicular to the conductor's surface.

We now sum the flux through the Gaussian surface. There is no flux through the internal end cap, because the electric field within the conductor is zero. There is no flux through the curved surface of the cylinder, because internally (in the conductor) there is no electric field and externally the electric field is parallel to the curved portion of the Gaussian surface. The only flux through the Gaussian surface is that through the external end cap, where \vec{E} is perpendicular to the plane of the cap. We assume that the cap area A is small enough that the field magnitude E is constant over the cap. Then the flux through the cap is EA , and that is the net flux Φ through the Gaussian surface.

The charge q_{enc} enclosed by the Gaussian surface lies on the conductor's surface in an area A . If σ is the charge per unit area, then q_{enc} is equal to σA . When we substitute σA for q_{enc} and EA for Φ , Gauss' law (Eq. 23-6) becomes

$$\epsilon_0 EA = \sigma A,$$

from which we find

$$E = \frac{\sigma}{\epsilon_0} \quad (\text{conducting surface}). \quad (23-11)$$

Thus, the magnitude of the electric field just outside a conductor is proportional to the surface charge density on the conductor. If the charge on the conductor is positive, the electric field is directed away from the conductor as in Fig. 23-10. It is directed toward the conductor if the charge is negative.

The field lines in Fig. 23-10 must terminate on negative charges somewhere in the environment. If we bring those charges near the conductor, the charge density at any given location on the conductor's surface changes, and so does the magnitude of the electric field. However, the relation between σ and E is still given by Eq. 23-11.

Sample Problem

Spherical metal shell, electric field and enclosed charge

Figure 23-11a shows a cross section of a spherical metal shell of inner radius R . A point charge of $-5.0 \mu\text{C}$ is located at a distance $R/2$ from the center of the shell. If the shell is electrically neutral, what are the (induced) charges on its inner and outer surfaces? Are those charges uniformly distributed? What is the field pattern inside and outside the shell?

KEY IDEAS

Figure 23-11b shows a cross section of a spherical Gaussian surface within the metal, just outside the inner wall of the shell. The electric field must be zero inside the metal (and thus on the Gaussian surface inside the metal). This means that the electric flux through the Gaussian surface must also

be zero. Gauss' law then tells us that the *net* charge enclosed by the Gaussian surface must be zero.

Reasoning: With a point charge of $-5.0 \mu\text{C}$ within the shell, a charge of $+5.0 \mu\text{C}$ must lie on the inner wall of the shell in order that the net enclosed charge be zero. If the point charge were centered, this positive charge would be uniformly distributed along the inner wall. However, since the point charge is off-center, the distribution of positive charge is skewed, as suggested by Fig. 23-11b, because the positive charge tends to collect on the section of the inner wall nearest the (negative) point charge.

Because the shell is electrically neutral, its inner wall can have a charge of $+5.0 \mu\text{C}$ only if electrons, with a total

23-7 APPLYING GAUSS' LAW: CYLINDRICAL SYMMETRY

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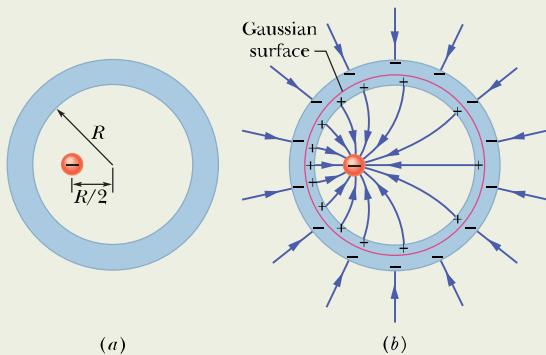


Fig. 23-11 (a) A negative point charge is located within a spherical metal shell that is electrically neutral. (b) As a result, positive charge is nonuniformly distributed on the inner wall of the shell, and an equal amount of negative charge is uniformly distributed on the outer wall.



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charge of $-5.0 \mu\text{C}$, leave the inner wall and move to the outer wall. There they spread out uniformly, as is also suggested by Fig. 23-11b. This distribution of negative charge is uniform because the shell is spherical and because the skewed distribution of positive charge on the inner wall cannot produce an electric field in the shell to affect the distribution of charge on the outer wall. Furthermore, these negative charges repel one another.

The field lines inside and outside the shell are shown approximately in Fig. 23-11b. All the field lines intersect the shell and the point charge perpendicularly. Inside the shell the pattern of field lines is skewed because of the skew of the positive charge distribution. Outside the shell the pattern is the same as if the point charge were centered and the shell were missing. In fact, this would be true no matter where inside the shell the point charge happened to be located.

23-7 Applying Gauss' Law: Cylindrical Symmetry

Figure 23-12 shows a section of an infinitely long cylindrical plastic rod with a uniform positive linear charge density λ . Let us find an expression for the magnitude of the electric field \vec{E} at a distance r from the axis of the rod.

Our Gaussian surface should match the symmetry of the problem, which is cylindrical. We choose a circular cylinder of radius r and length h , coaxial with the rod. Because the Gaussian surface must be closed, we include two end caps as part of the surface.

Imagine now that, while you are not watching, someone rotates the plastic rod about its longitudinal axis or turns it end for end. When you look again at the rod, you will not be able to detect any change. We conclude from this symmetry that the only uniquely specified direction in this problem is along a radial line. Thus, at every point on the cylindrical part of the Gaussian surface, \vec{E} must have the same magnitude E and (for a positively charged rod) must be directed radially outward.

Since $2\pi r$ is the cylinder's circumference and h is its height, the area A of the cylindrical surface is $2\pi rh$. The flux of \vec{E} through this cylindrical surface is then

$$\Phi = EA \cos \theta = E(2\pi rh) \cos 0 = E(2\pi rh).$$

There is no flux through the end caps because \vec{E} , being radially directed, is parallel to the end caps at every point.

The charge enclosed by the surface is λh , which means Gauss' law,

$$\epsilon_0 \Phi = q_{\text{enc}},$$

reduces to

$$\epsilon_0 E(2\pi rh) = \lambda h,$$

yielding

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \quad (\text{line of charge}). \quad (23-12)$$

This is the electric field due to an infinitely long, straight line of charge, at a point that is a radial distance r from the line. The direction of \vec{E} is radially outward from the line of charge if the charge is positive, and radially inward if it is negative. Equation 23-12 also approximates the field of a *finite* line of charge at points that are not too near the ends (compared with the distance from the line).

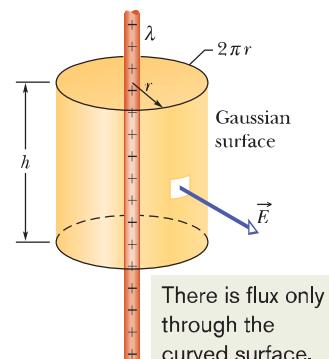


Fig. 23-12 A Gaussian surface in the form of a closed cylinder surrounds a section of a very long, uniformly charged, cylindrical plastic rod.

Sample Problem

Gauss' law and an upward streamer in a lightning storm

Upward streamer in a lightning storm. The woman in Fig. 23-13 was standing on a lookout platform in the Sequoia National Park when a large storm cloud moved overhead. Some of the conduction electrons in her body were driven into the ground by the cloud's negatively charged base (Fig. 23-14a), leaving her positively charged. You can tell she was highly charged because her hair strands repelled one another and extended away from her along the electric field lines produced by the charge on her.

Lightning did not strike the woman, but she was in extreme danger because that electric field was on the verge of causing electrical breakdown in the surrounding air. Such a breakdown would have occurred along a path extending away from her in what is called an *upward streamer*. An upward streamer is dangerous because the resulting ionization of molecules in the air suddenly frees a tremendous number of electrons from those molecules. Had the woman in Fig. 23-13 developed an upward streamer, the free electrons in the air would have moved to neutralize her (Fig. 23-14b), producing a large, perhaps fatal, charge flow through her body. That charge flow is dangerous because it could have interfered with or even stopped her breathing (which is obviously necessary for oxygen) and the steady beat of her heart (which is obvi-

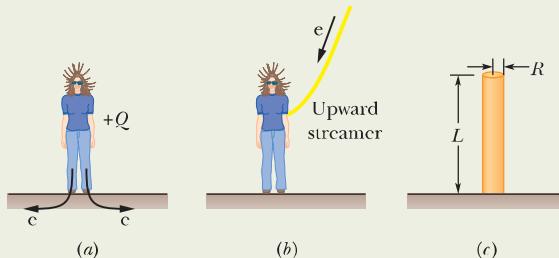


Fig. 23-14 (a) Some of the conduction electrons in the woman's body are driven into the ground, leaving her positively charged. (b) An upward streamer develops if the air undergoes electrical breakdown, which provides a path for electrons freed from molecules in the air to move to the woman. (c) A cylinder represents the woman.

ously necessary for the blood flow that carries the oxygen). The charge flow could also have caused burns.

Let's model her body as a narrow vertical cylinder of height $L = 1.8 \text{ m}$ and radius $R = 0.10 \text{ m}$ (Fig. 23-14c). Assume that charge Q was uniformly distributed along the cylinder and that electrical breakdown would have occurred if the electric field magnitude along her body had exceeded the critical value $E_c = 2.4 \text{ MN/C}$. What value of Q would have put the air along her body on the verge of breakdown?

KEY IDEA

Because $R \ll L$, we can approximate the charge distribution as a long line of charge. Further, because we assume that the charge is uniformly distributed along this line, we can approximate the magnitude of the electric field along the side of her body with Eq. 23-12 ($E = \lambda/2\pi\epsilon_0 r$).

Calculations: Substituting the critical value E_c for E , the cylinder radius R for radial distance r , and the ratio Q/L for linear charge density λ , we have

$$E_c = \frac{Q/L}{2\pi\epsilon_0 R},$$

or $Q = 2\pi\epsilon_0 R L E_c$.

Substituting given data then gives us

$$\begin{aligned} Q &= (2\pi)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.10 \text{ m}) \\ &\quad \times (1.8 \text{ m})(2.4 \times 10^6 \text{ N/C}) \\ &= 2.402 \times 10^{-5} \text{ C} \approx 24 \mu\text{C}. \end{aligned} \quad (\text{Answer})$$



Fig. 23-13 This woman has become positively charged by an overhead storm cloud. (Courtesy NOAA)



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23-8 APPLYING GAUSS' LAW: PLANAR SYMMETRY

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23-8 Applying Gauss' Law: Planar Symmetry

Nonconducting Sheet

Figure 23-15 shows a portion of a thin, infinite, nonconducting sheet with a uniform (positive) surface charge density σ . A sheet of thin plastic wrap, uniformly charged on one side, can serve as a simple model. Let us find the electric field \vec{E} at a distance r in front of the sheet.

A useful Gaussian surface is a closed cylinder with end caps of area A , arranged to pierce the sheet perpendicularly as shown. From symmetry, \vec{E} must be perpendicular to the sheet and hence to the end caps. Furthermore, since the charge is positive, \vec{E} is directed away from the sheet, and thus the electric field lines pierce the two Gaussian end caps in an outward direction. Because the field lines do not pierce the curved surface, there is no flux through this portion of the Gaussian surface. Thus $\vec{E} \cdot d\vec{A}$ is simply $E dA$; then Gauss' law,

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}},$$

becomes

$$\epsilon_0 (EA + EA) = \sigma A,$$

where σA is the charge enclosed by the Gaussian surface. This gives

$$E = \frac{\sigma}{2\epsilon_0} \quad (\text{sheet of charge}). \quad (23-13)$$

Since we are considering an infinite sheet with uniform charge density, this result holds for any point at a finite distance from the sheet. Equation 23-13 agrees with Eq. 22-27, which we found by integration of electric field components.

Two Conducting Plates

Figure 23-16a shows a cross section of a thin, infinite conducting plate with excess positive charge. From Section 23-6 we know that this excess charge lies on the surface of the plate. Since the plate is thin and very large, we can assume that essentially all the excess charge is on the two large faces of the plate.

If there is no external electric field to force the positive charge into some particular distribution, it will spread out on the two faces with a uniform surface charge density of magnitude σ_1 . From Eq. 23-11 we know that just outside the plate this charge sets up an electric field of magnitude $E = \sigma_1/\epsilon_0$. Because the excess charge is positive, the field is directed away from the plate.

Figure 23-16b shows an identical plate with excess negative charge having the same magnitude of surface charge density σ_1 . The only difference is that now the electric field is directed toward the plate.

Suppose we arrange for the plates of Figs. 23-16a and b to be close to each other and parallel (Fig. 23-16c). Since the plates are conductors, when we bring them into this arrangement, the excess charge on one plate attracts the excess charge on the other plate, and all the excess charge moves onto the inner faces of the plates as in Fig. 23-16c. With twice as much charge now on each inner face, the new surface charge density (call it σ) on each inner face is twice σ_1 . Thus, the electric field at any point between the plates has the magnitude

$$E = \frac{2\sigma_1}{\epsilon_0} = \frac{\sigma}{\epsilon_0}. \quad (23-14)$$

This field is directed away from the positively charged plate and toward the negatively charged plate. Since no excess charge is left on the outer faces, the electric field to the left and right of the plates is zero.

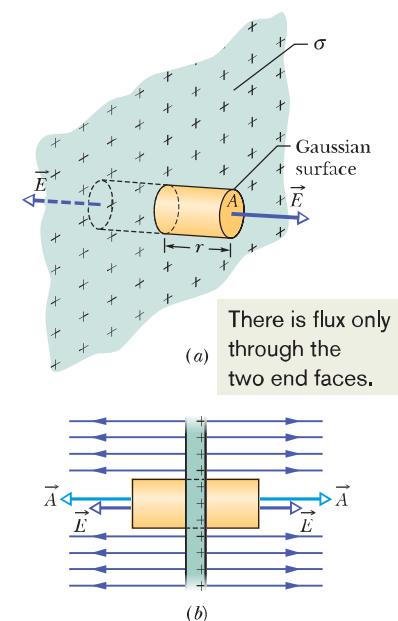


Fig. 23-15 (a) Perspective view and (b) side view of a portion of a very large, thin plastic sheet, uniformly charged on one side to surface charge density σ . A closed cylindrical Gaussian surface passes through the sheet and is perpendicular to it.

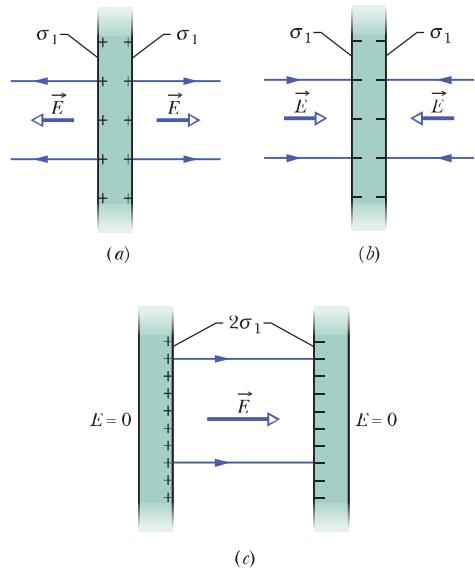


Fig. 23-16 (a) A thin, very large conducting plate with excess positive charge. (b) An identical plate with excess negative charge. (c) The two plates arranged so they are parallel and close.

Because the charges on the plates moved when we brought the plates close to each other, Fig. 23-16c is *not* the superposition of Figs. 23-16a and b; that is, the charge distribution of the two-plate system is not merely the sum of the charge distributions of the individual plates.

You may wonder why we discuss such seemingly unrealistic situations as the field set up by an infinite line of charge, an infinite sheet of charge, or a pair of infinite plates of charge. One reason is that analyzing such situations with Gauss' law is easy. More important is that analyses for "infinite" situations yield good approximations to many real-world problems. Thus, Eq. 23-13 holds well for a finite nonconducting sheet as long as we are dealing with points close to the sheet and not too near its edges. Equation 23-14 holds well for a pair of finite conducting plates as long as we consider points that are not too close to their edges.

The trouble with the edges of a sheet or a plate, and the reason we take care not to deal with them, is that near an edge we can no longer use planar symmetry to find expressions for the fields. In fact, the field lines there are curved (said to be an *edge effect* or *fringing*), and the fields can be very difficult to express algebraically.

Sample Problem

Electric field near two parallel charged metal plates

Figure 23-17a shows portions of two large, parallel, nonconducting sheets, each with a fixed uniform charge on one side. The magnitudes of the surface charge densities are $\sigma_{(+)} = 6.8 \mu\text{C}/\text{m}^2$ for the positively charged sheet and $\sigma_{(-)} = 4.3 \mu\text{C}/\text{m}^2$ for the negatively charged sheet.

Find the electric field \vec{E} (a) to the left of the sheets, (b) between the sheets, and (c) to the right of the sheets.

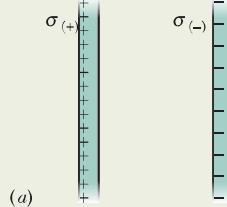
KEY IDEA

With the charges fixed in place (they are on nonconductors), we can find the electric field of the sheets in Fig. 23-17a by (1) finding the field of each sheet as if that sheet were isolated and (2) algebraically adding the fields of the isolated sheets via the superposition principle. (We can add the fields algebraically because they are parallel to each other.)

Calculations: At any point, the electric field $\vec{E}_{(+)}$ due to the positive sheet is directed *away* from the sheet and, from Eq. 23-13, has the magnitude

$$\begin{aligned} E_{(+)} &= \frac{\sigma_{(+)}}{2\epsilon_0} = \frac{6.8 \times 10^{-6} \text{ C/m}^2}{(2)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} \\ &= 3.84 \times 10^5 \text{ N/C.} \end{aligned}$$

Fig. 23-17 (a) Two large, parallel sheets, uniformly charged on one side. (b) The individual electric fields resulting from the two charged sheets. (c) The net field due to both charged sheets, found by superposition.



Similarly, at any point, the electric field $\vec{E}_{(-)}$ due to the negative sheet is directed *toward* that sheet and has the magnitude

$$\begin{aligned} E_{(-)} &= \frac{\sigma_{(-)}}{2\epsilon_0} = \frac{4.3 \times 10^{-6} \text{ C/m}^2}{(2)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} \\ &= 2.43 \times 10^5 \text{ N/C.} \end{aligned}$$

Figure 23-17b shows the fields set up by the sheets to the left of the sheets (*L*), between them (*B*), and to their right (*R*).

The resultant fields in these three regions follow from the superposition principle. To the left, the field magnitude is

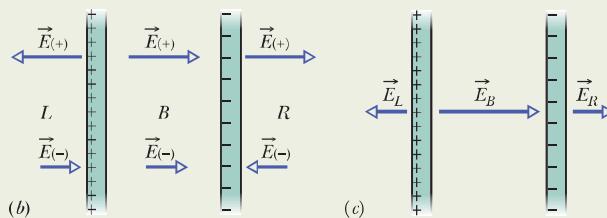
$$\begin{aligned} E_L &= E_{(+)} - E_{(-)} \\ &= 3.84 \times 10^5 \text{ N/C} - 2.43 \times 10^5 \text{ N/C} \\ &= 1.4 \times 10^5 \text{ N/C.} \end{aligned} \quad (\text{Answer})$$

Because $E_{(+)}$ is larger than $E_{(-)}$, the net electric field \vec{E}_L in this region is directed to the left, as Fig. 23-17c shows. To the right of the sheets, the electric field has the same magnitude but is directed to the right, as Fig. 23-17c shows.

Between the sheets, the two fields add and we have

$$\begin{aligned} E_B &= E_{(+)} + E_{(-)} \\ &= 3.84 \times 10^5 \text{ N/C} + 2.43 \times 10^5 \text{ N/C} \\ &= 6.3 \times 10^5 \text{ N/C.} \end{aligned} \quad (\text{Answer})$$

The electric field \vec{E}_B is directed to the right.



23-9 APPLYING GAUSS' LAW: SPHERICAL SYMMETRY

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23-9 Applying Gauss' Law: Spherical Symmetry

Here we use Gauss' law to prove the two shell theorems presented without proof in Section 21-4:



A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.



If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.

Figure 23-18 shows a charged spherical shell of total charge q and radius R and two concentric spherical Gaussian surfaces, S_1 and S_2 . If we followed the procedure of Section 23-5 as we applied Gauss' law to surface S_2 , for which $r \geq R$, we would find that

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad (\text{spherical shell, field at } r \geq R). \quad (23-15)$$

This field is the same as one set up by a point charge q at the center of the shell of charge. Thus, the force produced by a shell of charge q on a charged particle placed outside the shell is the same as the force produced by a point charge q located at the center of the shell. This proves the first shell theorem.

Applying Gauss' law to surface S_1 , for which $r < R$, leads directly to

$$E = 0 \quad (\text{spherical shell, field at } r < R), \quad (23-16)$$

because this Gaussian surface encloses no charge. Thus, if a charged particle were enclosed by the shell, the shell would exert no net electrostatic force on the particle. This proves the second shell theorem.

Any spherically symmetric charge distribution, such as that of Fig. 23-19, can be constructed with a nest of concentric spherical shells. For purposes of applying the two shell theorems, the volume charge density ρ should have a single value for each shell but need not be the same from shell to shell. Thus, for the charge distribution as a whole, ρ can vary, but only with r , the radial distance from the center. We can then examine the effect of the charge distribution "shell by shell."

In Fig. 23-19a, the entire charge lies within a Gaussian surface with $r > R$. The charge produces an electric field on the Gaussian surface as if the charge were a point charge located at the center, and Eq. 23-15 holds.

Figure 23-19b shows a Gaussian surface with $r < R$. To find the electric field at points on this Gaussian surface, we consider two sets of charged shells—one set inside the Gaussian surface and one set outside. Equation 23-16 says that the charge lying *outside* the Gaussian surface does not set up a net electric field on the Gaussian surface. Equation 23-15 says that the charge *enclosed* by the surface sets up an electric field as if that enclosed charge were concentrated at the center. Letting q' represent that enclosed charge, we can then rewrite Eq. 23-15 as

$$E = \frac{1}{4\pi\epsilon_0} \frac{q'}{r^2} \quad (\text{spherical distribution, field at } r \leq R). \quad (23-17)$$

If the full charge q enclosed within radius R is uniform, then q' enclosed within radius r in Fig. 23-19b is proportional to q :

$$\frac{\left(\text{charge enclosed by}\right)}{\left(\text{sphere of radius } r\right)} = \frac{\text{full charge}}{\left(\text{volume enclosed by}\right)}$$

$$\frac{\left(\text{sphere of radius } r\right)}{\left(\text{volume enclosed by}\right)} = \frac{\text{full volume}}$$

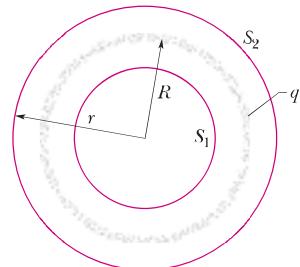
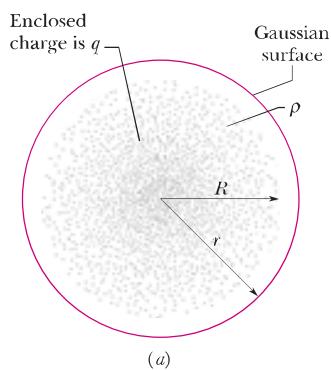
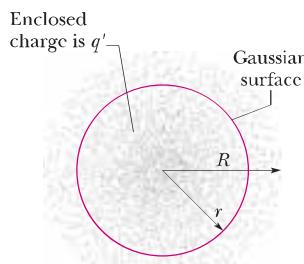


Fig. 23-18 A thin, uniformly charged, spherical shell with total charge q , in cross section. Two Gaussian surfaces S_1 and S_2 are also shown in cross section. Surface S_2 encloses the shell, and S_1 encloses only the empty interior of the shell.



(a)



(b)

The flux through the surface depends on only the *enclosed* charge.

Fig. 23-19 The dots represent a spherically symmetric distribution of charge of radius R , whose volume charge density ρ is a function only of distance from the center. The charged object is not a conductor, and therefore the charge is assumed to be fixed in position. A concentric spherical Gaussian surface with $r > R$ is shown in (a). A similar Gaussian surface with $r < R$ is shown in (b).

or

$$\frac{q'}{\frac{4}{3}\pi r^3} = \frac{q}{\frac{4}{3}\pi R^3}. \quad (23-18)$$

This gives us

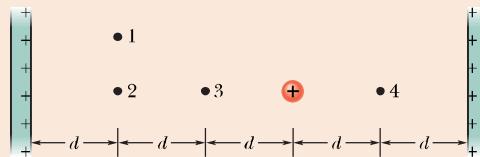
$$q' = q \frac{r^3}{R^3}. \quad (23-19)$$

Substituting this into Eq. 23-17 yields

$$E = \left(\frac{q}{4\pi\epsilon_0 R^3} \right) r \quad (\text{uniform charge, field at } r \leq R). \quad (23-20)$$

CHECKPOINT 4

The figure shows two large, parallel, nonconducting sheets with identical (positive) uniform surface charge densities, and a sphere with a uniform (positive) volume charge density. Rank the four numbered points according to the magnitude of the net electric field there, greatest first.



REVIEW & SUMMARY

Gauss' Law Gauss' law and Coulomb's law are different ways of describing the relation between charge and electric field in static situations. Gauss' law is

$$\epsilon_0 \Phi = q_{\text{enc}} \quad (\text{Gauss' law}), \quad (23-6)$$

in which q_{enc} is the net charge inside an imaginary closed surface (a *Gaussian surface*) and Φ is the net flux of the electric field through the surface:

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad (\text{electric flux through a Gaussian surface}). \quad (23-4)$$

Coulomb's law can be derived from Gauss' law.

Applications of Gauss' Law Using Gauss' law and, in some cases, symmetry arguments, we can derive several important results in electrostatic situations. Among these are:

1. An excess charge on an isolated *conductor* is located entirely on the outer surface of the conductor.
2. The external electric field near the *surface of a charged conductor* is perpendicular to the surface and has magnitude

$$E = \frac{\sigma}{\epsilon_0} \quad (\text{conducting surface}). \quad (23-11)$$

Within the conductor, $E = 0$.

3. The electric field at any point due to an infinite *line of charge* with uniform linear charge density λ is perpendicular to the line of charge and has magnitude

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \quad (\text{line of charge}), \quad (23-12)$$

where r is the perpendicular distance from the line of charge to the point.

4. The electric field due to an *infinite nonconducting sheet* with uniform surface charge density σ is perpendicular to the plane of the sheet and has magnitude

$$E = \frac{\sigma}{2\epsilon_0} \quad (\text{sheet of charge}). \quad (23-13)$$

5. The electric field *outside a spherical shell of charge* with radius R and total charge q is directed radially and has magnitude

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad (\text{spherical shell, for } r \geq R). \quad (23-15)$$

Here r is the distance from the center of the shell to the point at which E is measured. (The charge behaves, for external points, as if it were all located at the center of the sphere.) The field *inside a uniform spherical shell of charge* is exactly zero:

$$E = 0 \quad (\text{spherical shell, for } r < R). \quad (23-16)$$

6. The electric field *inside a uniform sphere of charge* is directed radially and has magnitude

$$E = \left(\frac{q}{4\pi\epsilon_0 R^3} \right) r. \quad (23-20)$$

QUESTIONS

- 1 A surface has the area vector $\vec{A} = (2\hat{i} + 3\hat{j}) \text{ m}^2$. What is the flux of a uniform electric field through the area if the field is (a) $\vec{E} = 4\hat{i} \text{ N/C}$ and (b) $\vec{E} = 4\hat{k} \text{ N/C}$?

- 2 Figure 23-20 shows, in cross section, three solid cylinders, each of length L and uniform charge Q . Concentric with each cylinder is a cylindrical Gaussian surface, with all three surfaces having the same radius. Rank the Gaussian surfaces according to the electric field at any point on the surface, greatest first.

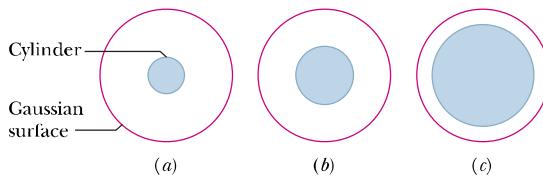


Fig. 23-20 Question 2.

- 3 Figure 23-21 shows, in cross section, a central metal ball, two spherical metal shells, and three spherical Gaussian surfaces of radii R , $2R$, and $3R$, all with the same center. The uniform charges on the three objects are: ball, Q ; smaller shell, $3Q$; larger shell, $5Q$. Rank the Gaussian surfaces according to the magnitude of the electric field at any point on the surface, greatest first.

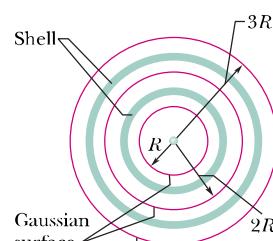


Fig. 23-21 Question 3.

- 4 Figure 23-22 shows, in cross section, two Gaussian spheres and two Gaussian cubes that are centered on a positively charged particle. (a) Rank the net flux through the four Gaussian surfaces, greatest first. (b) Rank the magnitudes of the electric fields on the surfaces, greatest first, and indicate whether the magnitudes are uniform or variable along each surface.

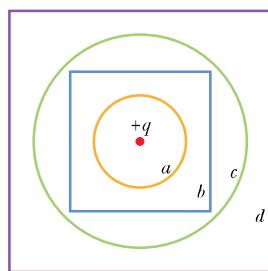


Fig. 23-22 Question 4.

- 5 In Fig. 23-23, an electron is released between two infinite nonconducting sheets that are horizontal and have uniform surface charge densities $\sigma_{(+)}$ and $\sigma_{(-)}$, as indicated. The electron is subjected to the following three situations involving surface charge densities and sheet separations. Rank the magnitudes of the electron's acceleration, greatest first.

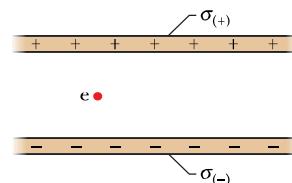


Fig. 23-23 Question 5.

| Situation | $\sigma_{(+)}$ | $\sigma_{(-)}$ | Separation |
|-----------|----------------|----------------|------------|
| 1 | $+4\sigma$ | -4σ | d |
| 2 | $+7\sigma$ | $-\sigma$ | $4d$ |
| 3 | $+3\sigma$ | -5σ | $9d$ |

- 6 Three infinite nonconducting sheets, with uniform positive surface charge densities σ , 2σ , and 3σ , are arranged to be parallel like the two sheets in Fig. 23-17a. What is their order, from left to right, if the electric field \vec{E} produced by the arrangement has magnitude $E = 0$ in one region and $E = 2\sigma/\epsilon_0$ in another region?

- 7 Figure 23-24 shows four situations in which four very long rods extend into and out of the page (we see only their cross sections). The value below each cross section gives that particular rod's uniform charge density in microcoulombs per meter. The rods are separated by either d or $2d$ as drawn, and a central point is shown midway between the inner rods. Rank the situations according to the magnitude of the net electric field at that central point, greatest first.

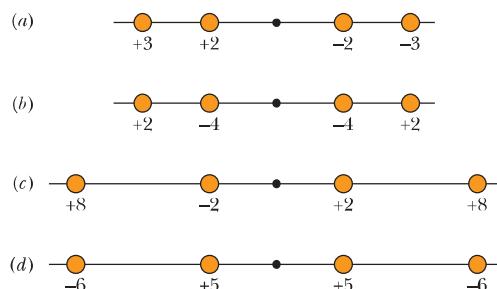


Fig. 23-24 Question 7.

- 8 Figure 23-25 shows four solid spheres, each with charge Q uniformly distributed through its volume. (a) Rank the spheres according to their volume charge density, greatest first. The figure also shows a point P for each sphere, all at the same distance from the center of the sphere. (b) Rank the spheres according to the magnitude of the electric field they produce at point P , greatest first.

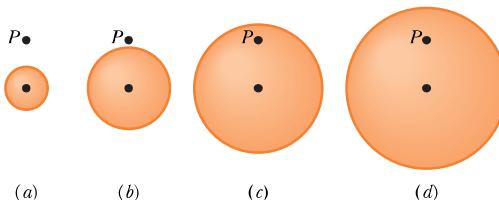


Fig. 23-25 Question 8.

- 9 A small charged ball lies within the hollow of a metallic spherical shell of radius R . For three situations, the net charges on the ball and shell, respectively, are (1) $+4q$, 0 ; (2) $-6q$, $+10q$; (3) $+16q$, $-12q$. Rank the situations according to the charge on (a) the inner surface of the shell and (b) the outer surface, most positive first.

- 10 Rank the situations of Question 9 according to the magnitude of the electric field (a) halfway through the shell and (b) at a point $2R$ from the center of the shell, greatest first.

PROBLEMS



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual



Number of dots indicates level of problem difficulty

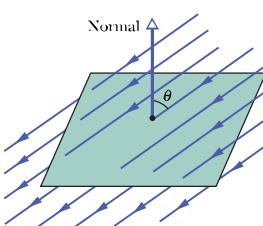
Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

WWW Worked-out solution is at

ILW Interactive solution is at

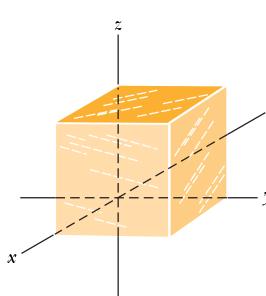
<http://www.wiley.com/college/halliday>**sec. 23-3 Flux of an Electric Field**

- 1 SSM** The square surface shown in Fig. 23-26 measures 3.2 mm on each side. It is immersed in a uniform electric field with magnitude $E = 1800 \text{ N/C}$ and with field lines at an angle of $\theta = 35^\circ$ with a normal to the surface, as shown. Take that normal to be directed "outward," as though the surface were one face of a box. Calculate the electric flux through the surface.

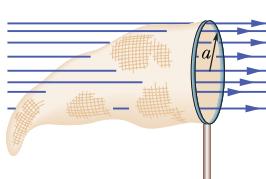
**Fig. 23-26** Problem 1.

- 2** An electric field given by $\vec{E} = 4.0\hat{i} - 3.0(y^2 + 2.0)\hat{j}$ pierces Gaussian cube of edge length 2.0 m and positioned as shown in Fig. 23-5. (The magnitude E is in newtons per coulomb and the position x is in meters.) What is the electric flux through the (a) top face, (b) bottom face, (c) left face, and (d) back face? (e) What is the net electric flux through the cube?

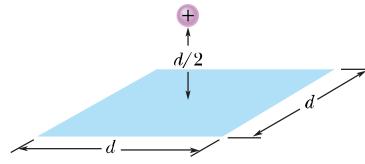
- 3** The cube in Fig. 23-27 has edge length 1.40 m and is oriented as shown in a region of uniform electric field. Find the electric flux through the right face if the electric field, in newtons per coulomb, is given by (a) $6.00\hat{i}$, (b) $-2.00\hat{j}$, and (c) $-3.00\hat{i} + 4.00\hat{k}$. (d) What is the total flux through the cube for each field?

**Fig. 23-27** Problems 3, 6, and 9.**sec. 23-4 Gauss' Law**

- 4** In Fig. 23-28, a butterfly net is in a uniform electric field of magnitude $E = 3.0 \text{ mN/C}$. The rim, a circle of radius $a = 11 \text{ cm}$, is aligned perpendicular to the field. The net contains no net charge. Find the electric flux through the netting.

**Fig. 23-28** Problem 4.

- 5** In Fig. 23-29, a proton is a distance $d/2$ directly above the center of a square of side d . What is the magnitude of the electric flux through the square? (*Hint:* Think of the square as one face of a cube with edge d .)

**Fig. 23-29** Problem 5.

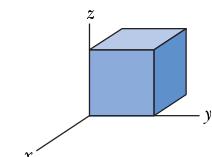
- 6** At each point on the surface of the cube shown in Fig. 23-27, the electric field is parallel to the z axis. The length of each edge of the cube is 3.0 m. On the top face of the cube the field is $\vec{E} = -34\hat{k} \text{ N/C}$, and on the bottom face it is $\vec{E} = +20\hat{k} \text{ N/C}$. Determine the net charge contained within the cube.

- 7** A point charge of $1.8 \mu\text{C}$ is at the center of a Gaussian cube 55 cm on edge. What is the net electric flux through the surface?

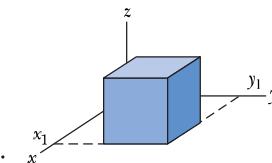
- 8** When a shower is turned on in a closed bathroom, the splashing of the water on the bare tub can fill the room's air with negatively charged ions and produce an electric field in the air as great as 1000 N/C . Consider a bathroom with dimensions $2.5 \text{ m} \times 3.0 \text{ m} \times 2.0 \text{ m}$. Along the ceiling, floor, and four walls, approximate the electric field in the air as being directed perpendicular to the surface and as having a uniform magnitude of 600 N/C . Also, treat those surfaces as forming a closed Gaussian surface around the room's air. What are (a) the volume charge density ρ and (b) the number of excess elementary charges e per cubic meter in the room's air?

- 9** **ILW** Fig. 23-27 shows a Gaussian surface in the shape of a cube with edge length 1.40 m. What are (a) the net flux Φ through the surface and (b) the net charge q_{enc} enclosed by the surface if $\vec{E} = (3.00y\hat{j}) \text{ N/C}$, with y in meters? What are (c) Φ and (d) q_{enc} if $\vec{E} = [-4.00\hat{i} + (6.00 + 3.00y)\hat{j}] \text{ N/C}$?

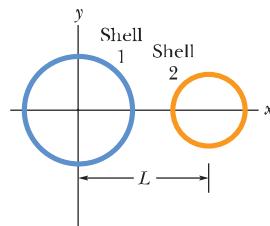
- 10** Figure 23-30 shows a closed Gaussian surface in the shape of a cube of edge length 2.00 m. It lies in a region where the nonuniform electric field is given by $\vec{E} = (3.00x + 4.00)\hat{i} + 6.00\hat{j} + 7.00\hat{k} \text{ N/C}$, with x in meters. What is the net charge contained by the cube?

**Fig. 23-30** Problem 10.

- 11** **SSM** Figure 23-31 shows a closed Gaussian surface in the shape of a cube of edge length 2.00 m, with one corner at $x_1 = 5.00 \text{ m}$, $y_1 = 4.00 \text{ m}$. The cube lies in a region where the electric field vector is given by $\vec{E} = -3.00\hat{i} - 4.00y^2\hat{j} + 3.00\hat{k} \text{ N/C}$, with y in meters. What is the net charge contained by the cube?

**Fig. 23-31** Problem 11.

- 12** Figure 23-32 shows two nonconducting spherical shells fixed in place. Shell 1 has uniform surface charge density $+6.0 \mu\text{C/m}^2$ on its outer surface and radius 3.0 cm; shell 2 has uniform surface charge density $+4.0 \mu\text{C/m}^2$ on its outer surface and radius 2.0 cm; the shell centers are separated by $L = 10 \text{ cm}$. In unit-vector notation, what is the net electric field at $x = 2.0 \text{ cm}$?

**Fig. 23-32** Problem 12.

- 13** **SSM** The electric field in a certain region of Earth's atmosphere is directed vertically down. At an altitude of 300 m the field

has magnitude 60.0 N/C; at an altitude of 200 m, the magnitude is 100 N/C. Find the net amount of charge contained in a cube 100 m on edge, with horizontal faces at altitudes of 200 and 300 m.

••14 Flux and nonconducting shells. A charged particle is suspended at the center of two concentric spherical shells that are very thin and made of nonconducting material. Figure 23-33a shows a cross section. Figure 23-33b gives the net flux Φ through a Gaussian sphere centered on the particle, as a function of the radius r of the sphere. The scale of the vertical axis is set by $\Phi_s = 5.0 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$. (a) What is the charge of the central particle? What are the net charges of (b) shell A and (c) shell B?

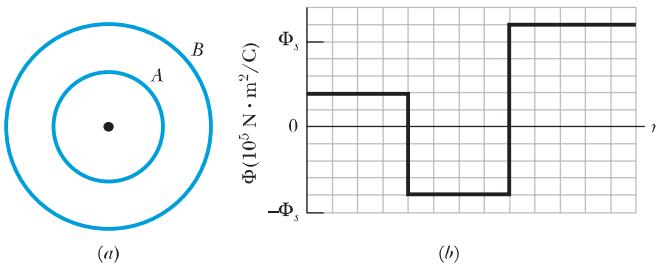


Fig. 23-33 Problem 14.

••15 A particle of charge $+q$ is placed at one corner of a Gaussian cube. What multiple of q/ϵ_0 gives the flux through (a) each cube face forming that corner and (b) each of the other cube faces?

••16 The box-like Gaussian surface shown in Fig. 23-34 encloses a net charge of $+24.0\epsilon_0 \text{ C}$ and lies in an electric field given by $\vec{E} = [(10.0 + 2.00x)\hat{i} - 3.00\hat{j} + bz\hat{k}] \text{ N/C}$, with x and z in meters and b a constant. The bottom face is in the xz plane; the top face is in the horizontal plane passing through $y_2 = 1.00 \text{ m}$. For $x_1 = 1.00 \text{ m}$, $x_2 = 4.00 \text{ m}$, $z_1 = 1.00 \text{ m}$, and $z_2 = 3.00 \text{ m}$, what is b ?

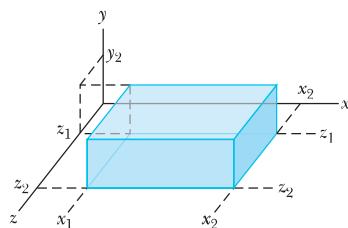


Fig. 23-34 Problem 16.

sec. 23-6 A Charged Isolated Conductor

•17 SSM A uniformly charged conducting sphere of 1.2 m diameter has a surface charge density of $8.1 \mu\text{C}/\text{m}^2$. (a) Find the net charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

•18 The electric field just above the surface of the charged conducting drum of a photocopying machine has a magnitude E of $2.3 \times 10^5 \text{ N/C}$. What is the surface charge density on the drum?

•19 Space vehicles traveling through Earth's radiation belts can intercept a significant number of electrons. The resulting charge buildup can damage electronic components and disrupt operations. Suppose a spherical metal satellite 1.3 m in diameter accumulates $2.4 \mu\text{C}$ of charge in one orbital revolution. (a) Find the resulting surface charge density. (b) Calculate the magnitude of the electric field just outside the surface of the satellite, due to the surface charge.

•20 Flux and conducting shells. A charged particle is held at the center of two concentric conducting spherical shells. Figure 23-35a shows a cross section. Figure 23-35b gives the net flux Φ through a Gaussian sphere centered on the particle, as a function of the radius r of the sphere. The scale of the vertical axis is set by $\Phi_s = 5.0 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$. What are (a) the charge of the central particle and the net charges of (b) shell A and (c) shell B?

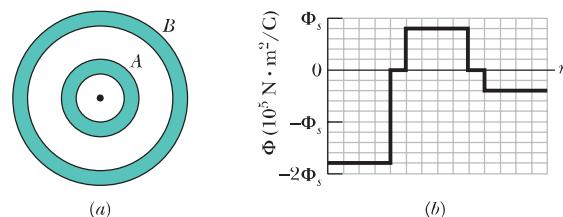


Fig. 23-35 Problem 20.

•21 An isolated conductor has net charge $+10 \times 10^{-6} \text{ C}$ and a cavity with a point charge $q = +3.0 \times 10^{-6} \text{ C}$. What is the charge on (a) the cavity wall and (b) the outer surface?

sec. 23-7 Applying Gauss' Law: Cylindrical Symmetry

•22 An electron is released 9.0 cm from a very long nonconducting rod with a uniform $6.0 \mu\text{C}/\text{m}$. What is the magnitude of the electron's initial acceleration?

•23 (a) The drum of a photocopying machine has a length of 42 cm and a diameter of 12 cm. The electric field just above the drum's surface is $2.3 \times 10^5 \text{ N/C}$. What is the total charge on the drum? (b) The manufacturer wishes to produce a desktop version of the machine. This requires reducing the drum length to 28 cm and the diameter to 8.0 cm. The electric field at the drum surface must not change. What must be the charge on this new drum?

•24 Figure 23-36 shows a section of a long, thin-walled metal tube of radius $R = 3.00 \text{ cm}$, with a charge per unit length of $\lambda = 2.00 \times 10^{-8} \text{ C/m}$. What is the magnitude E of the electric field at radial distance (a) $r = R/2.00$ and (b) $r = 2.00R$? (c) Graph E versus r for the range $r = 0$ to $2.00R$.

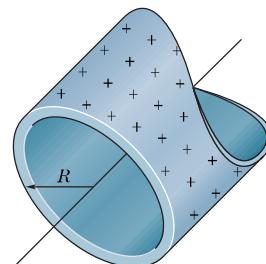


Fig. 23-36 Problem 24.

•25 SSM An infinite line of charge produces a field of magnitude $4.5 \times 10^4 \text{ N/C}$ at distance 2.0 m. Find the linear charge density.

•26 Figure 23-37a shows a narrow charged solid cylinder that is coaxial with a larger charged cylindrical shell. Both are noncon-

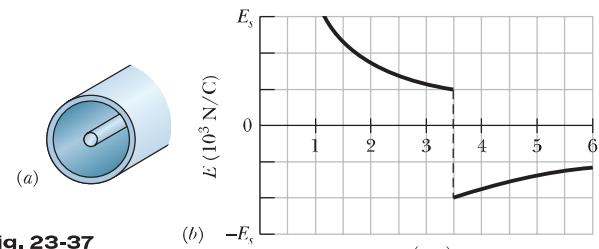


Fig. 23-37 Problem 26.

ducting and thin and have uniform surface charge densities on their outer surfaces. Figure 23-37b gives the radial component E of the electric field versus radial distance r from the common axis, and $E_s = 3.0 \times 10^3 \text{ N/C}$. What is the shell's linear charge density?

••27 A long, straight wire has fixed negative charge with a linear charge density of magnitude 3.6 nC/m . The wire is to be enclosed by a coaxial, thin-walled nonconducting cylindrical shell of radius 1.5 cm . The shell is to have positive charge on its outside surface with a surface charge density σ that makes the net external electric field zero. Calculate σ .

••28 A charge of uniform linear density 2.0 nC/m is distributed along a long, thin, nonconducting rod. The rod is coaxial with a long conducting cylindrical shell (inner radius = 5.0 cm , outer radius = 10 cm). The net charge on the shell is zero. (a) What is the magnitude of the electric field 15 cm from the axis of the shell? What is the surface charge density on the (b) inner and (c) outer surface of the shell?

••29 SSM WWW Figure 23-38 is a section of a conducting rod of radius $R_1 = 1.30 \text{ mm}$ and length $L = 11.00 \text{ m}$ inside a thin-walled coaxial conducting cylindrical shell of radius $R_2 = 10.0R_1$ and the (same) length L . The net charge on the rod is $Q_1 = +3.40 \times 10^{-12} \text{ C}$; that on the shell is $Q_2 = -2.00Q_1$. What are the (a) magnitude E and (b) direction (radially inward or outward) of the electric field at radial distance $r = 2.00R_2$? What are (c) E and (d) the direction at $r = 5.00R_1$? What is the charge on the (e) interior and (f) exterior surface of the shell?

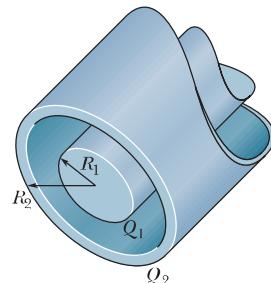


Fig. 23-38 Problem 29.

••30 In Fig. 23-39, short sections of two very long parallel lines of charge are shown, fixed in place, separated by $L = 8.0 \text{ cm}$. The uniform linear charge densities are $+6.0 \mu\text{C/m}$ for line 1 and $-2.0 \mu\text{C/m}$ for line 2. Where along the x axis shown is the net electric field from the two lines zero?

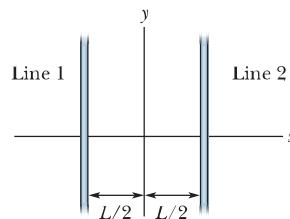


Fig. 23-39 Problem 30.

••31 ILW Two long, charged, thin-walled, concentric cylindrical shells have radii of 3.0 and 6.0 cm . The charge per unit length is $5.0 \times 10^{-6} \text{ C/m}$ on the inner shell and $-7.0 \times 10^{-6} \text{ C/m}$ on the outer shell. What are the (a) magnitude E and (b) direction (radially inward or outward) of the electric field at radial distance $r = 4.0 \text{ cm}$? What are (c) E and (d) the direction at $r = 8.0 \text{ cm}$?

••32 A long, nonconducting, solid cylinder of radius 4.0 cm has a nonuniform volume charge density ρ that is a function of radial distance r from the cylinder axis: $\rho = Ar^2$. For $A = 2.5 \mu\text{C/m}^5$, what is the magnitude of the electric field at (a) $r = 3.0 \text{ cm}$ and (b) $r = 5.0 \text{ cm}$?

sec. 23-8 Applying Gauss' Law: Planar Symmetry

••33 In Fig. 23-40, two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have excess surface charge

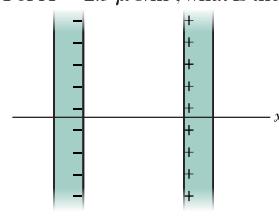


Fig. 23-40 Problem 33.

densities of opposite signs and magnitude $7.00 \times 10^{-22} \text{ C/m}^2$. In unit-vector notation, what is the electric field at points (a) to the left of the plates, (b) to the right of them, and (c) between them?

••34 In Fig. 23-41, a small circular hole of radius $R = 1.80 \text{ cm}$ has been cut in the middle of an infinite, flat, nonconducting surface that has uniform charge density $\sigma = 4.50 \text{ pC/m}^2$. A z axis, with its origin at the hole's center, is perpendicular to the surface. In unit-vector notation, what is the electric field at point P at $z = 2.56 \text{ cm}$? (Hint: See Eq. 22-26 and use superposition.)

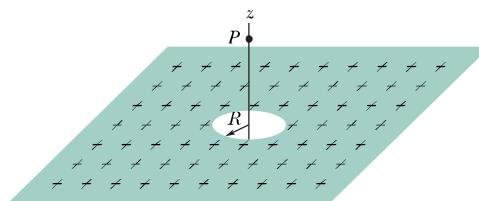


Fig. 23-41 Problem 34.

••35 Figure 23-42a shows three plastic sheets that are large, parallel, and uniformly charged. Figure 23-42b gives the component of the net electric field along an x axis through the sheets. The scale of the vertical axis is set by $E_s = 6.0 \times 10^5 \text{ N/C}$. What is the ratio of the charge density on sheet 3 to that on sheet 2?

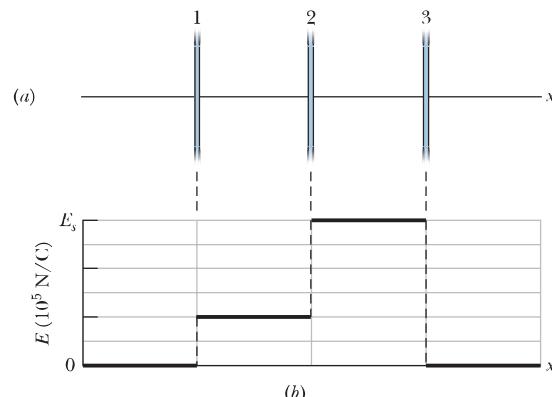


Fig. 23-42 Problem 35.

••36 Figure 23-43 shows cross sections through two large, parallel, nonconducting sheets with identical distributions of positive charge with surface charge density $\sigma = 1.77 \times 10^{-22} \text{ C/m}^2$. In unit-vector notation, what is \vec{E} at points (a) above the sheets, (b) between them, and (c) below them?

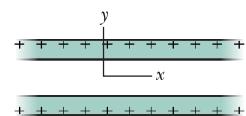


Fig. 23-43
Problem 36.

••37 SSM WWW A square metal plate of edge length 8.0 cm and negligible thickness has a total charge of $6.0 \times 10^{-6} \text{ C}$. (a) Estimate the magnitude E of the electric field just off the center of the plate (at, say, a distance of 0.50 mm from the center) by assuming that the charge is spread uniformly over the two faces of the plate. (b) Estimate E at a distance of 30 m (large relative to the plate size) by assuming that the plate is a point charge.

••38 In Fig. 23-44a, an electron is shot directly away from a uniformly charged plastic sheet, at speed $v_s = 2.0 \times 10^5 \text{ m/s}$. The sheet is

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nonconducting, flat, and very large. Figure 23-44b gives the electron's vertical velocity component v versus time t until the return to the launch point. What is the sheet's surface charge density?

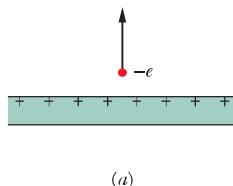
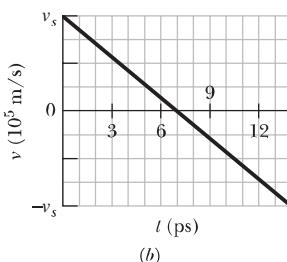


Fig. 23-44 Problem 38.



(a)

(b)

••39 **SSM** In Fig. 23-45, a small, nonconducting ball of mass $m = 1.0 \text{ mg}$ and charge $q = 2.0 \times 10^{-8} \text{ C}$ (distributed uniformly through its volume) hangs from an insulating thread that makes an angle $\theta = 30^\circ$ with a vertical, uniformly charged nonconducting sheet (shown in cross section). Considering the gravitational force on the ball and assuming the sheet extends far vertically and into and out of the page, calculate the surface charge density σ of the sheet.

••40 Figure 23-46 shows a very large nonconducting sheet that has a uniform surface charge density of $\sigma = -2.00 \mu\text{C}/\text{m}^2$; it also shows a particle of charge $Q = 6.00 \mu\text{C}$, at distance d from the sheet. Both are fixed in place. If $d = 0.200 \text{ m}$, at what (a) positive and (b) negative coordinate on the x axis (other than infinity) is the net electric field \vec{E}_{net} of the sheet and particle zero? (c) If $d = 0.800 \text{ m}$, at what coordinate on the x axis is $\vec{E}_{\text{net}} = 0$?

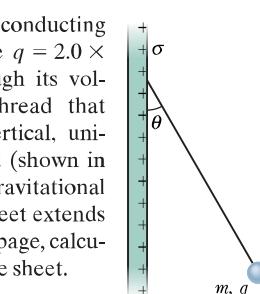


Fig. 23-45 Problem 39.

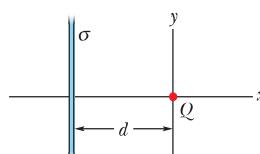


Fig. 23-46 Problem 40.

••41 **GO** An electron is shot directly toward the center of a large metal plate that has surface charge density $-2.0 \times 10^{-6} \text{ C/m}^2$. If the initial kinetic energy of the electron is $1.60 \times 10^{-17} \text{ J}$ and if the electron is to stop (due to electrostatic repulsion from the plate) just as it reaches the plate, how far from the plate must the launch point be?

••42 Two large metal plates of area 1.0 m^2 face each other, 5.0 cm apart, with equal charge magnitudes $|q|$ but opposite signs. The field magnitude E between them (neglect fringing) is 55 N/C . Find $|q|$.

••43 Figure 23-47 shows a cross section through a very large nonconducting slab of thickness $d = 9.40 \text{ mm}$ and uniform volume charge density $\rho = 5.80 \text{ fC/m}^3$. The origin of an x axis is at the slab's center. What is the magnitude of the slab's electric field at an x coordinate of (a) 0, (b) 2.00 mm , (c) 4.70 mm , and (d) 26.0 mm ?

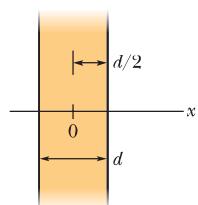


Fig. 23-47 Problem 43.

sec. 23-9 Applying Gauss' Law: Spherical Symmetry

••44 Figure 23-48 gives the magnitude of the electric field inside and outside a sphere with a positive charge distributed uniformly through-

out its volume. The scale of the vertical axis is set by $E_s = 5.0 \times 10^7 \text{ N/C}$. What is the charge on the sphere?

••45 Two charged concentric spherical shells have radii 10.0 cm and 15.0 cm . The charge on the inner shell is $4.00 \times 10^{-8} \text{ C}$, and that on the outer shell is $2.00 \times 10^{-8} \text{ C}$. Find the electric field (a) at $r = 12.0 \text{ cm}$ and (b) at $r = 20.0 \text{ cm}$.

••46 A point charge causes an electric flux of $-750 \text{ N}\cdot\text{m}^2/\text{C}$ to pass through a spherical Gaussian surface of 10.0 cm radius centered on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (b) What is the value of the point charge?

••47 **SSM** An unknown charge sits on a conducting solid sphere of radius 10 cm . If the electric field 15 cm from the center of the sphere has the magnitude $3.0 \times 10^3 \text{ N/C}$ and is directed radially inward, what is the net charge on the sphere?

••48 A charged particle is held at the center of a spherical shell. Figure 23-49 gives the magnitude E of the electric field versus radial distance r . The scale of the vertical axis is set by $E_s = 10.0 \times 10^7 \text{ N/C}$. Approximately, what is the net charge on the shell?

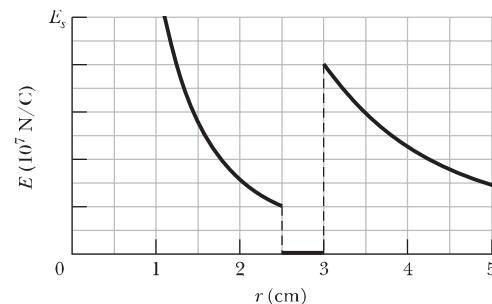


Fig. 23-49 Problem 48.

••49 In Fig. 23-50, a solid sphere of radius $a = 2.00 \text{ cm}$ is concentric with a spherical conducting shell of inner radius $b = 2.00a$ and outer radius $c = 2.40a$. The sphere has a net uniform charge $q_1 = +5.00 \text{ fC}$; the shell has a net charge $q_2 = -q_1$. What is the magnitude of the electric field at radial distances (a) $r = 0$, (b) $r = a/2.00$, (c) $r = a$, (d) $r = 1.50a$, (e) $r = 2.30a$, and (f) $r = 3.50a$? What is the net charge on the (g) inner and (h) outer surface of the shell?

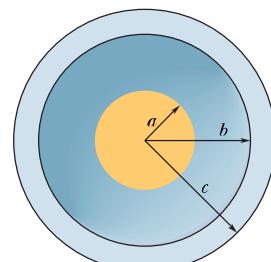


Fig. 23-50 Problem 49.

••50 **GO** Figure 23-51 shows two nonconducting spherical shells fixed in place on an x axis. Shell 1 has uniform surface charge density $+4.0 \mu\text{C}/\text{m}^2$ on its outer surface and radius 0.50 cm , and shell 2 has uniform surface charge density $-2.0 \mu\text{C}/\text{m}^2$ on its outer surface and radius 2.0 cm ; the centers are separated by $L = 6.0 \text{ cm}$. Other than at $x = \infty$, where on the x axis is the net electric field equal to zero?

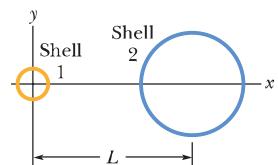


Fig. 23-51 Problem 50.

- 51 [SSM WWW]** In Fig. 23-52, a nonconducting spherical shell of inner radius $a = 2.00\text{ cm}$ and outer radius $b = 2.40\text{ cm}$ has (within its thickness) a positive volume charge density $\rho = A/r$, where A is a constant and r is the distance from the center of the shell. In addition, a small ball of charge $q = 45.0\text{ fC}$ is located at that center. What value should A have if the electric field in the shell ($a \leq r \leq b$) is to be uniform?

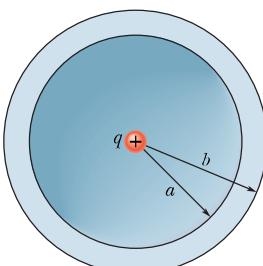


Fig. 23-52 Problem 51.

- 52** Figure 23-53 shows a spherical shell with uniform volume charge density $\rho = 1.84\text{ nC/m}^3$, inner radius $a = 10.0\text{ cm}$, and outer radius $b = 2.00a$. What is the magnitude of the electric field at radial distances (a) $r = 0$; (b) $r = a/2.00$, (c) $r = a$, (d) $r = 1.50a$, (e) $r = b$, and (f) $r = 3.00b$?

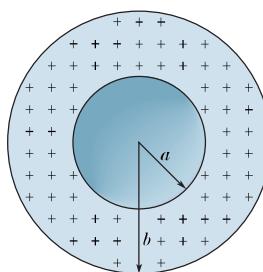


Fig. 23-53 Problem 52.

- 53 [ILW]** The volume charge density of a solid nonconducting sphere of radius $R = 5.60\text{ cm}$ varies with radial distance r as given by $\rho = (14.1\text{ pC/m}^3)r/R$. (a) What is the sphere's total charge? What is the field magnitude E at (b) $r = 0$, (c) $r = R/2.00$, and (d) $r = R$? (e) Graph E versus r .

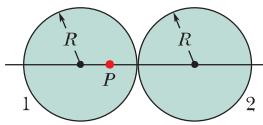


Fig. 23-54 Problem 54.

- 54** Figure 23-54 shows, in cross section, two solid spheres with uniformly distributed charge throughout their volumes. Each has radius R . Point P lies on a line connecting the centers of the spheres, at radial distance $R/2.00$ from the center of sphere 1. If the net electric field at point P is zero, what is the ratio q_2/q_1 of the total charges?

- 55** A charge distribution that is spherically symmetric but not uniform radially produces an electric field of magnitude $E = Kr^4$, directed radially outward from the center of the sphere. Here r is the radial distance from that center, and K is a constant. What is the volume density ρ of the charge distribution?

Additional Problems

- 56** The electric field in a particular space is $\vec{E} = (x + 2)\hat{i}\text{ N/C}$, with x in meters. Consider a cylindrical Gaussian surface of radius 20 cm that is coaxial with the x axis. One end of the cylinder is at $x = 0$. (a) What is the magnitude of the electric flux through the other end of the cylinder at $x = 2.0\text{ m}$? (b) What net charge is enclosed within the cylinder?

- 57** A thin-walled metal spherical shell has radius 25.0 cm and charge $2.00 \times 10^{-7}\text{ C}$. Find E for a point (a) inside the shell, (b) just outside it, and (c) 3.00 m from the center.

- 58** A uniform surface charge of density 8.0 nC/m^2 is distributed over the entire xy plane. What is the electric flux through a spherical Gaussian surface centered on the origin and having a radius of 5.0 cm?

- 59** Charge of uniform volume density $\rho = 1.2\text{ nC/m}^3$ fills an infinite slab between $x = -5.0\text{ cm}$ and $x = +5.0\text{ cm}$. What is the magnitude of the electric field at any point with the coordinate (a) $x = 4.0\text{ cm}$ and (b) $x = 6.0\text{ cm}$?

- 60** *The chocolate crumb mystery.* Explosions ignited by electrostatic discharges (sparks) constitute a serious danger in facilities handling grain or powder. Such an explosion occurred in chocolate crumb powder at a biscuit factory in the 1970s. Workers usually emptied newly delivered sacks of the powder into a loading bin, from which it was blown through electrically grounded plastic pipes to a silo for storage. Somewhere along this route, two conditions for an explosion were met: (1) The magnitude of an electric field became $3.0 \times 10^6\text{ N/C}$ or greater, so that electrical breakdown and thus sparking could occur. (2) The energy of a spark was 150 mJ or greater so that it could ignite the powder explosively. Let us check for the first condition in the powder flow through the plastic pipes.

Suppose a stream of *negatively* charged powder was blown through a cylindrical pipe of radius $R = 5.0\text{ cm}$. Assume that the powder and its charge were spread uniformly through the pipe with a volume charge density ρ . (a) Using Gauss' law, find an expression for the magnitude of the electric field \vec{E} in the pipe as a function of radial distance r from the pipe center. (b) Does E increase or decrease with increasing r ? (c) Is \vec{E} directed radially inward or outward? (d) For $\rho = 1.1 \times 10^{-3}\text{ C/m}^3$ (a typical value at the factory), find the maximum E and determine where that maximum field occurs. (e) Could sparking occur, and if so, where? (The story continues with Problem 70 in Chapter 24.)

- 61 [SSM]** A thin-walled metal spherical shell of radius a has a charge q_a . Concentric with it is a thin-walled metal spherical shell of radius $b > a$ and charge q_b . Find the electric field at points a distance r from the common center, where (a) $r < a$, (b) $a < r < b$, and (c) $r > b$. (d) Discuss the criterion you would use to determine how the charges are distributed on the inner and outer surfaces of the shells.

- 62** A point charge $q = 1.0 \times 10^{-7}\text{ C}$ is at the center of a spherical cavity of radius 3.0 cm in a chunk of metal. Find the electric field (a) 1.5 cm from the cavity center and (b) anywhere in the metal.

- 63** A proton at speed $v = 3.00 \times 10^5\text{ m/s}$ orbits at radius $r = 1.00\text{ cm}$ outside a charged sphere. Find the sphere's charge.

- 64** Equation 23-11 ($E = \sigma/\epsilon_0$) gives the electric field at points near a charged conducting surface. Apply this equation to a conducting sphere of radius r and charge q , and show that the electric field outside the sphere is the same as the field of a point charge located at the center of the sphere.

- 65** Charge Q is uniformly distributed in a sphere of radius R . (a) What fraction of the charge is contained within the radius $r = R/2.00$? (b) What is the ratio of the electric field magnitude at $r = R/2.00$ to that on the surface of the sphere?

- 66** Assume that a ball of charged particles has a uniformly distributed negative charge density except for a narrow radial tunnel through its center, from the surface on one side to the surface on the opposite side. Also assume that we can position a proton anywhere along the tunnel or outside the ball. Let F_R be the magnitude of the electrostatic force on the proton when it is located at the ball's surface, at radius R . As a multiple of R , how far from the surface is there a point where the force magnitude is $0.50F_R$ if we move the proton (a) away from the ball and (b) into the tunnel?

- 67 [SSM]** The electric field at point P just outside the outer surface of a hollow spherical conductor of inner radius 10 cm and outer radius 20 cm has magnitude 450 N/C and is directed outward. When an unknown point charge Q is introduced into the center of the sphere, the electric field at P is still directed outward but is now 180 N/C. (a) What was the net charge enclosed by the

outer surface before Q was introduced? (b) What is charge Q ? After Q is introduced, what is the charge on the (c) inner and (d) outer surface of the conductor?

68 The net electric flux through each face of a die (singular of dice) has a magnitude in units of $10^5 \text{ N} \cdot \text{m}^2/\text{C}$ that is exactly equal to the number of spots N on the face (1 through 6). The flux is inward for N odd and outward for N even. What is the net charge inside the die?

69 Figure 23-55 shows, in cross section, three infinitely large nonconducting sheets on which charge is uniformly spread. The surface charge densities are $\sigma_1 = +2.00 \mu\text{C}/\text{m}^2$, $\sigma_2 = +4.00 \mu\text{C}/\text{m}^2$, and $\sigma_3 = -5.00 \mu\text{C}/\text{m}^2$, and distance $L = 1.50 \text{ cm}$. In unit-vector notation, what is the net electric field at point P ?

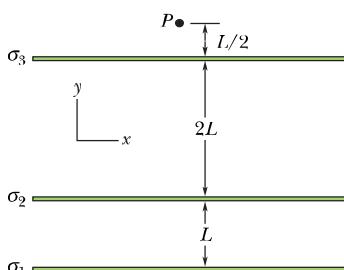


Fig. 23-55 Problem 69.

70 Charge of uniform volume density $\rho = 3.2 \mu\text{C}/\text{m}^3$ fills a nonconducting solid sphere of radius 5.0 cm . What is the magnitude of the electric field (a) 3.5 cm and (b) 8.0 cm from the sphere's center?

71 A Gaussian surface in the form of a hemisphere of radius $R = 5.68 \text{ cm}$ lies in a uniform electric field of magnitude $E = 2.50 \text{ N/C}$. The surface encloses no net charge. At the (flat) base of the surface, the field is perpendicular to the surface and directed into the surface. What is the flux through (a) the base and (b) the curved portion of the surface?

72 What net charge is enclosed by the Gaussian cube of Problem 2?

73 A nonconducting solid sphere has a uniform volume charge density ρ . Let \vec{r} be the vector from the center of the sphere to a general point P within the sphere. (a) Show that the electric field at P is given by $\vec{E} = \rho \vec{r}/3\epsilon_0$. (Note that the result is independent of the radius of the sphere.) (b) A spherical cavity is hollowed out of the sphere, as shown in Fig. 23-56. Using superposition concepts, show that the electric field at all points within the cavity is uniform and equal to $\vec{E} = \rho \vec{a}/3\epsilon_0$, where \vec{a} is the position vector from the center of the sphere to the center of the cavity.



Fig. 23-56
Problem 73.

74 A uniform charge density of 500 nC/m^3 is distributed throughout a spherical volume of radius 6.00 cm . Consider a cubical Gaussian surface with its center at the center of the sphere. What is the electric flux through this cubical surface if its edge length is (a) 4.00 cm and (b) 14.0 cm ?

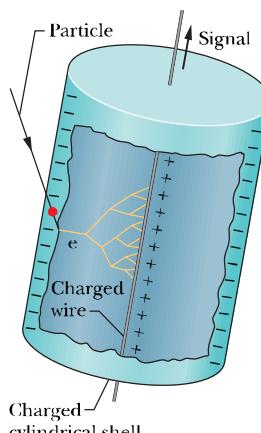


Fig. 23-57 Problem 75.

75 Figure 23-57 shows a Geiger counter, a device used to detect ionizing radiation, which causes ionization of atoms. A thin, positively

charged central wire is surrounded by a concentric, circular, conducting cylindrical shell with an equal negative charge, creating a strong radial electric field. The shell contains a low-pressure inert gas. A particle of radiation entering the device through the shell wall ionizes a few of the gas atoms. The resulting free electrons (e) are drawn to the positive wire. However, the electric field is so intense that, between collisions with gas atoms, the free electrons gain energy sufficient to ionize these atoms also. More free electrons are thereby created, and the process is repeated until the electrons reach the wire. The resulting "avalanche" of electrons is collected by the wire, generating a signal that is used to record the passage of the original particle of radiation. Suppose that the radius of the central wire is $25 \mu\text{m}$, the inner radius of the shell 1.4 cm , and the length of the shell 16 cm . If the electric field at the shell's inner wall is $2.9 \times 10^4 \text{ N/C}$, what is the total positive charge on the central wire?

76 Charge is distributed uniformly throughout the volume of an infinitely long solid cylinder of radius R . (a) Show that, at a distance $r < R$ from the cylinder axis,

$$E = \frac{\rho r}{2\epsilon_0},$$

where ρ is the volume charge density. (b) Write an expression for E when $r > R$.

77 SSM A spherical conducting shell has a charge of $-14 \mu\text{C}$ on its outer surface and a charged particle in its hollow. If the net charge on the shell is $-10 \mu\text{C}$, what is the charge (a) on the inner surface of the shell and (b) of the particle?

78 A charge of 6.00 pC is spread uniformly throughout the volume of a sphere of radius $r = 4.00 \text{ cm}$. What is the magnitude of the electric field at a radial distance of (a) 6.00 cm and (b) 3.00 cm ?

79 Water in an irrigation ditch of width $w = 3.22 \text{ m}$ and depth $d = 1.04 \text{ m}$ flows with a speed of 0.207 m/s . The mass flux of the flowing water through an imaginary surface is the product of the water's density (1000 kg/m^3) and its volume flux through that surface. Find the mass flux through the following imaginary surfaces: (a) a surface of area wd , entirely in the water, perpendicular to the flow; (b) a surface with area $3wd/2$, of which wd is in the water, perpendicular to the flow; (c) a surface of area $wd/2$, entirely in the water, perpendicular to the flow; (d) a surface of area wd , half in the water and half out, perpendicular to the flow; (e) a surface of area wd , entirely in the water, with its normal 34.0° from the direction of flow.

80 Charge of uniform surface density 8.00 nC/m^2 is distributed over an entire xy plane; charge of uniform surface density 3.00 nC/m^2 is distributed over the parallel plane defined by $z = 2.00 \text{ m}$. Determine the magnitude of the electric field at any point having a z coordinate of (a) 1.00 m and (b) 3.00 m .

81 A spherical ball of charged particles has a uniform charge density. In terms of the ball's radius R , at what radial distances (a) inside and (b) outside the ball is the magnitude of the ball's electric field equal to $\frac{1}{4}$ of the maximum magnitude of that field?

82 SSM A free electron is placed between two large, parallel, nonconducting plates that are horizontal and 2.3 cm apart. One plate has a uniform positive charge; the other has an equal amount of uniform negative charge. The force on the electron due to the electric field \vec{E} between the plates balances the gravitational force on the electron. What are (a) the magnitude of the surface charge density on the plates and (b) the direction (up or down) of \vec{E} ?