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WAVES—I

16-1 WHAT IS PHYSICS?

One of the primary subjects of physics is waves. To see how important waves are in the modern world, just consider the music industry. Every piece of music you hear, from some retro-punk band playing in a campus dive to the most eloquent concerto playing on the Web, depends on performers producing waves and your detecting those waves. In between production and detection, the information carried by the waves might need to be transmitted (as in a live performance on the Web) or recorded and then reproduced (as with CDs, DVDs, or the other devices currently being developed in engineering labs worldwide). The financial importance of controlling music waves is staggering, and the rewards to engineers who develop new control techniques can be rich.

This chapter focuses on waves traveling along a stretched string, such as on a guitar. The next chapter focuses on sound waves, such as those produced by a guitar string being played. Before we do all this, though, our first job is to classify the countless waves of the everyday world into basic types.

16-2 Types of Waves

Waves are of three main types:

1. **Mechanical waves.** These waves are most familiar because we encounter them almost constantly; common examples include water waves, sound waves, and seismic waves. All these waves have two central features: They are governed by Newton's laws, and they can exist only within a material medium, such as water, air, and rock.
2. **Electromagnetic waves.** These waves are less familiar, but you use them constantly; common examples include visible and ultraviolet light, radio and television waves, microwaves, x rays, and radar waves. These waves require no material medium to exist. Light waves from stars, for example, travel through the vacuum of space to reach us. All electromagnetic waves travel through a vacuum at the same speed $c = 299\,792\,458$ m/s.
3. **Matter waves.** Although these waves are commonly used in modern technology, they are probably very unfamiliar to you. These waves are associated with electrons, protons, and other fundamental particles, and even atoms and molecules. Because we commonly think of these particles as constituting matter, such waves are called matter waves.

Much of what we discuss in this chapter applies to waves of all kinds. However, for specific examples we shall refer to mechanical waves.

16-3 Transverse and Longitudinal Waves

A wave sent along a stretched, taut string is the simplest mechanical wave. If you give one end of a stretched string a single up-and-down jerk, a wave in the form of a single *pulse* travels along the string. This pulse and its motion can occur

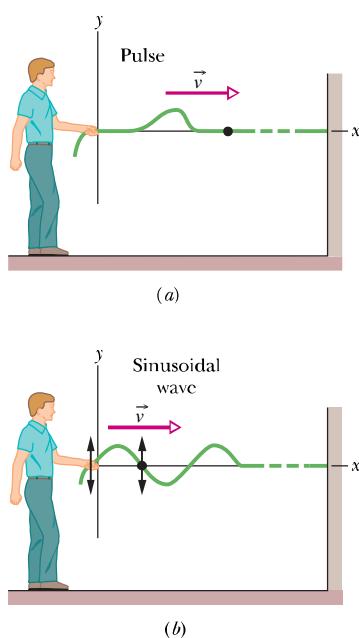


Fig. 16-1 (a) A single pulse is sent along a stretched string. A typical string element (marked with a dot) moves up once and then down as the pulse passes. The element's motion is perpendicular to the wave's direction of travel, so the pulse is a **transverse wave**. (b) A sinusoidal wave is sent along the string. A typical string element moves up and down continuously as the wave passes. This too is a transverse wave.

because the string is under tension. When you pull your end of the string upward, it begins to pull upward on the adjacent section of the string via tension between the two sections. As the adjacent section moves upward, it begins to pull the next section upward, and so on. Meanwhile, you have pulled down on your end of the string. As each section moves upward in turn, it begins to be pulled back downward by neighboring sections that are already on the way down. The net result is that a distortion in the string's shape (a pulse, as in Fig. 16-1a) moves along the string at some velocity \vec{v} .

If you move your hand up and down in continuous simple harmonic motion, a continuous wave travels along the string at velocity \vec{v} . Because the motion of your hand is a sinusoidal function of time, the wave has a sinusoidal shape at any given instant, as in Fig. 16-1b; that is, the wave has the shape of a sine curve or a cosine curve.

We consider here only an “ideal” string, in which no friction-like forces within the string cause the wave to die out as it travels along the string. In addition, we assume that the string is so long that we need not consider a wave rebounding from the far end.

One way to study the waves of Fig. 16-1 is to monitor the **wave forms** (shapes of the waves) as they move to the right. Alternatively, we could monitor the motion of an element of the string as the element oscillates up and down while a wave passes through it. We would find that the displacement of every such oscillating string element is *perpendicular* to the direction of travel of the wave, as indicated in Fig. 16-1b. This motion is said to be **transverse**, and the wave is said to be a **transverse wave**.

Figure 16-2 shows how a sound wave can be produced by a piston in a long, air-filled pipe. If you suddenly move the piston rightward and then leftward, you can send a pulse of sound along the pipe. The rightward motion of the piston moves the elements of air next to it rightward, changing the air pressure there. The increased air pressure then pushes rightward on the elements of air somewhat farther along the pipe. Moving the piston leftward reduces the air pressure next to it. As a result, first the elements nearest the piston and then farther elements move leftward. Thus, the motion of the air and the change in air pressure travel rightward along the pipe as a pulse.

If you push and pull on the piston in simple harmonic motion, as is being done in Fig. 16-2, a sinusoidal wave travels along the pipe. Because the motion of the elements of air is parallel to the direction of the wave's travel, the motion is said to be **longitudinal**, and the wave is said to be a **longitudinal wave**. In this chapter we focus on transverse waves, and string waves in particular; in Chapter 17 we focus on longitudinal waves, and sound waves in particular.

Both a transverse wave and a longitudinal wave are said to be **traveling waves** because they both travel from one point to another, as from one end of the string to the other end in Fig. 16-1 and from one end of the pipe to the other end in Fig. 16-2. Note that it is the wave that moves from end to end, not the material (string or air) through which the wave moves.

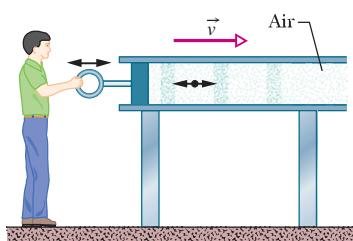


Fig. 16-2 A sound wave is set up in an air-filled pipe by moving a piston back and forth. Because the oscillations of an element of the air (represented by the dot) are parallel to the direction in which the wave travels, the wave is a **longitudinal wave**.

16-4 Wavelength and Frequency

To completely describe a wave on a string (and the motion of any element along its length), we need a function that gives the shape of the wave. This means that we need a relation in the form

$$y = h(x, t), \quad (16-1)$$

in which y is the transverse displacement of any string element as a function h of the time t and the position x of the element along the string. In general, a sinu-

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soidal shape like the wave in Fig. 16-1b can be described with h being either a sine or cosine function; both give the same general shape for the wave. In this chapter we use the sine function.

Imagine a sinusoidal wave like that of Fig. 16-1b traveling in the positive direction of an x axis. As the wave sweeps through succeeding elements (that is, very short sections) of the string, the elements oscillate parallel to the y axis. At time t , the displacement y of the element located at position x is given by

$$y(x, t) = y_m \sin(kx - \omega t). \quad (16-2)$$

Because this equation is written in terms of position x , it can be used to find the displacements of all the elements of the string as a function of time. Thus, it can tell us the shape of the wave at any given time and how that shape changes as the wave moves along the string.

The names of the quantities in Eq. 16-2 are displayed in Fig. 16-3 and defined next. Before we discuss them, however, let us examine Fig. 16-4, which shows five “snapshots” of a sinusoidal wave traveling in the positive direction of an x axis. The movement of the wave is indicated by the rightward progress of the short arrow pointing to a high point of the wave. From snapshot to snapshot, the short arrow moves to the right with the wave shape, but the string moves *only* parallel to the y axis. To see that, let us follow the motion of the red-dyed string element at $x = 0$. In the first snapshot (Fig. 16-4a), this element is at displacement $y = 0$. In the next snapshot, it is at its extreme downward displacement because a *valley* (or extreme low point) of the wave is passing through it. It then moves back up through $y = 0$. In the fourth snapshot, it is at its extreme upward displacement because a *peak* (or extreme high point) of the wave is passing through it. In the fifth snapshot, it is again at $y = 0$, having completed one full oscillation.

Amplitude and Phase

The **amplitude** y_m of a wave, such as that in Fig. 16-4, is the magnitude of the maximum displacement of the elements from their equilibrium positions as the wave passes through them. (The subscript m stands for maximum.) Because y_m is a magnitude, it is always a positive quantity, even if it is measured downward instead of upward as drawn in Fig. 16-4a.

The **phase** of the wave is the *argument* $kx - \omega t$ of the sine in Eq. 16-2. As the wave sweeps through a string element at a particular position x , the phase changes linearly with time t . This means that the sine also changes, oscillating between +1 and -1. Its extreme positive value (+1) corresponds to a peak of the wave moving through the element; at that instant the value of y at position x is y_m . Its extreme negative value (-1) corresponds to a valley of the wave moving through the element; at that instant the value of y at position x is $-y_m$. Thus, the sine function and the time-dependent phase of a wave correspond to the oscillation of a string element, and the amplitude of the wave determines the extremes of the element's displacement.

Wavelength and Angular Wave Number

The **wavelength** λ of a wave is the distance (parallel to the direction of the wave's travel) between repetitions of the shape of the wave (or *wave shape*). A typical wavelength is marked in Fig. 16-4a, which is a snapshot of the wave at time $t = 0$. At that time, Eq. 16-2 gives, for the description of the wave shape,

$$y(x, 0) = y_m \sin kx. \quad (16-3)$$

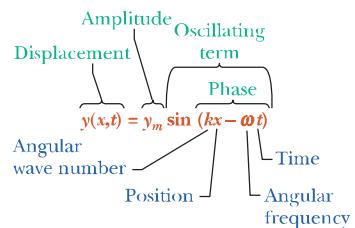


Fig. 16-3 The names of the quantities in Eq. 16-2, for a transverse sinusoidal wave.

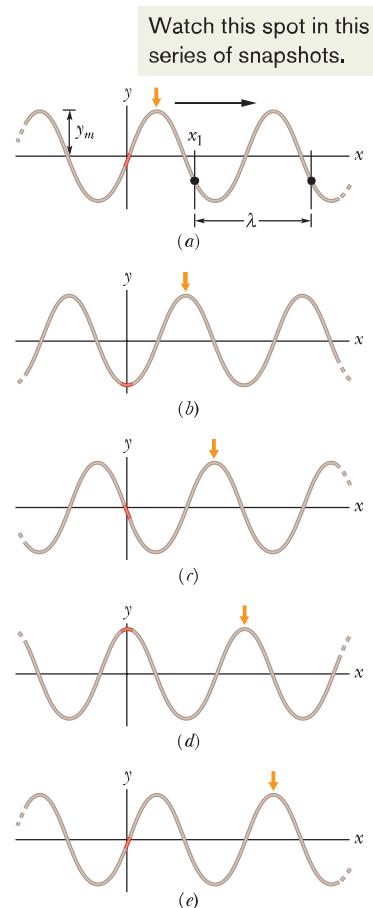


Fig. 16-4 Five “snapshots” of a string wave traveling in the positive direction of an x axis. The amplitude y_m is indicated. A typical wavelength λ , measured from an arbitrary position x_1 , is also indicated.

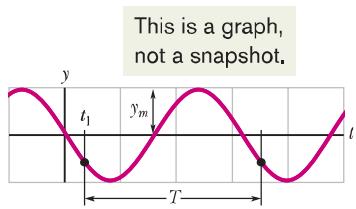


Fig. 16-5 A graph of the displacement of the string element at $x = 0$ as a function of time, as the sinusoidal wave of Fig. 16-4 passes through the element. The amplitude y_m is indicated. A typical period T , measured from an arbitrary time t_1 , is also indicated.

By definition, the displacement y is the same at both ends of this wavelength—that is, at $x = x_1$ and $x = x_1 + \lambda$. Thus, by Eq. 16-3,

$$\begin{aligned} y_m \sin kx_1 &= y_m \sin k(x_1 + \lambda) \\ &= y_m \sin(kx_1 + k\lambda). \end{aligned} \quad (16-4)$$

A sine function begins to repeat itself when its angle (or argument) is increased by 2π rad, so in Eq. 16-4 we must have $k\lambda = 2\pi$, or

$$k = \frac{2\pi}{\lambda} \quad (\text{angular wave number}). \quad (16-5)$$

We call k the **angular wave number** of the wave; its SI unit is the radian per meter, or the inverse meter. (Note that the symbol k here does *not* represent a spring constant as previously.)

Notice that the wave in Fig. 16-4 moves to the right by $\frac{1}{4}\lambda$ from one snapshot to the next. Thus, by the fifth snapshot, it has moved to the right by 1λ .

Period, Angular Frequency, and Frequency

Figure 16-5 shows a graph of the displacement y of Eq. 16-2 versus time t at a certain position along the string, taken to be $x = 0$. If you were to monitor the string, you would see that the single element of the string at that position moves up and down in simple harmonic motion given by Eq. 16-2 with $x = 0$:

$$\begin{aligned} y(0, t) &= y_m \sin(-\omega t) \\ &= -y_m \sin \omega t \quad (x = 0). \end{aligned} \quad (16-6)$$

Here we have made use of the fact that $\sin(-\alpha) = -\sin \alpha$, where α is any angle. Figure 16-5 is a graph of this equation, with displacement plotted versus time; it *does not* show the shape of the wave.

We define the **period** of oscillation T of a wave to be the time any string element takes to move through one full oscillation. A typical period is marked on the graph of Fig. 16-5. Applying Eq. 16-6 to both ends of this time interval and equating the results yield

$$\begin{aligned} -y_m \sin \omega t_1 &= -y_m \sin \omega(t_1 + T) \\ &= -y_m \sin(\omega t_1 + \omega T). \end{aligned} \quad (16-7)$$

This can be true only if $\omega T = 2\pi$, or if

$$\omega = \frac{2\pi}{T} \quad (\text{angular frequency}). \quad (16-8)$$

We call ω the **angular frequency** of the wave; its SI unit is the radian per second.

Look back at the five snapshots of a traveling wave in Fig. 16-4. The time between snapshots is $\frac{1}{4}T$. Thus, by the fifth snapshot, every string element has made one full oscillation.

The **frequency** f of a wave is defined as $1/T$ and is related to the angular frequency ω by

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \quad (\text{frequency}). \quad (16-9)$$

Like the frequency of simple harmonic motion in Chapter 15, this frequency f is a number of oscillations per unit time—here, the number made by a string element as the wave moves through it. As in Chapter 15, f is usually measured in hertz or its multiples, such as kilohertz.

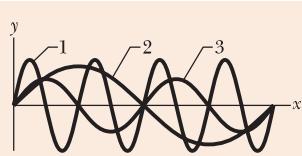
16-5 THE SPEED OF A TRAVELING WAVE

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CHECKPOINT 1

The figure is a composite of three snapshots, each of a wave traveling along a particular string. The phases for the waves are given by (a) $2x - 4t$, (b) $4x - 8t$, and (c) $8x - 16t$. Which phase corresponds to which wave in the figure?



Phase Constant

When a sinusoidal traveling wave is given by the wave function of Eq. 16-2, the wave near $x = 0$ looks like Fig. 16-6a when $t = 0$. Note that at $x = 0$, the displacement is $y = 0$ and the slope is at its maximum positive value. We can generalize Eq. 16-2 by inserting a **phase constant** ϕ in the wave function:

$$y = y_m \sin(kx - \omega t + \phi). \quad (16-10)$$

The value of ϕ can be chosen so that the function gives some other displacement and slope at $x = 0$ when $t = 0$. For example, a choice of $\phi = +\pi/5$ rad gives the displacement and slope shown in Fig. 16-6b when $t = 0$. The wave is still sinusoidal with the same values of y_m , k , and ω , but it is now shifted from what you see in Fig. 16-6a (where $\phi = 0$).

The effect of the phase constant ϕ is to shift the wave.

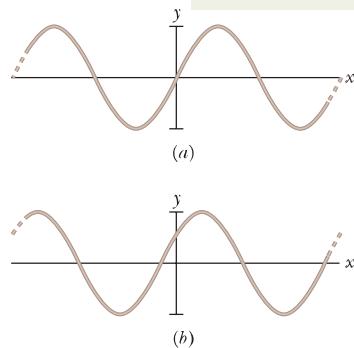


Fig. 16-6 A sinusoidal traveling wave at $t = 0$ with a phase constant ϕ of (a) 0 and (b) $\pi/5$ rad.

16-5 The Speed of a Traveling Wave

Figure 16-7 shows two snapshots of the wave of Eq. 16-2, taken a small time interval Δt apart. The wave is traveling in the positive direction of x (to the right in Fig. 16-7), the entire wave pattern moving a distance Δx in that direction during the interval Δt . The ratio $\Delta x/\Delta t$ (or, in the differential limit, dx/dt) is the **wave speed** v . How can we find its value?

As the wave in Fig. 16-7 moves, each point of the moving wave form, such as point A marked on a peak, retains its displacement y . (Points on the string do not retain their displacement, but points on the wave *form* do.) If point A retains its displacement as it moves, the phase in Eq. 16-2 giving it that displacement must remain a constant:

$$kx - \omega t = \text{a constant.} \quad (16-11)$$

Note that although this argument is constant, both x and t are changing. In fact, as t increases, x must also, to keep the argument constant. This confirms that the wave pattern is moving in the positive direction of x .

To find the wave speed v , we take the derivative of Eq. 16-11, getting

$$\begin{aligned} k \frac{dx}{dt} - \omega &= 0 \\ \frac{dx}{dt} &= v = \frac{\omega}{k}. \end{aligned} \quad (16-12)$$

Using Eq. 16-5 ($k = 2\pi/\lambda$) and Eq. 16-8 ($\omega = 2\pi/T$), we can rewrite the wave speed as

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \quad (\text{wave speed}). \quad (16-13)$$

The equation $v = \lambda/T$ tells us that the wave speed is one wavelength per period; the wave moves a distance of one wavelength in one period of oscillation.

Equation 16-2 describes a wave moving in the positive direction of x . We can find the equation of a wave traveling in the opposite direction by replacing t in

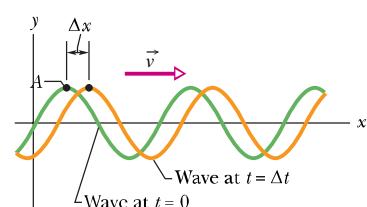


Fig. 16-7 Two snapshots of the wave of Fig. 16-4, at time $t = 0$ and then at time $t = \Delta t$. As the wave moves to the right at velocity \vec{v} , the entire curve shifts a distance Δx during Δt . Point A “rides” with the wave form, but the string elements move only up and down.

Eq. 16-2 with $-t$. This corresponds to the condition

$$kx + \omega t = \text{a constant}, \quad (16-14)$$

which (compare Eq. 16-11) requires that x decrease with time. Thus, a wave traveling in the negative direction of x is described by the equation

$$y(x, t) = y_m \sin(kx + \omega t). \quad (16-15)$$

If you analyze the wave of Eq. 16-15 as we have just done for the wave of Eq. 16-2, you will find for its velocity

$$\frac{dx}{dt} = -\frac{\omega}{k}. \quad (16-16)$$

The minus sign (compare Eq. 16-12) verifies that the wave is indeed moving in the negative direction of x and justifies our switching the sign of the time variable.

Consider now a wave of arbitrary shape, given by

$$y(x, t) = h(kx \pm \omega t), \quad (16-17)$$

where h represents *any* function, the sine function being one possibility. Our previous analysis shows that all waves in which the variables x and t enter into the combination $kx \pm \omega t$ are traveling waves. Furthermore, all traveling waves *must* be of the form of Eq. 16-17. Thus, $y(x, t) = \sqrt{ax + bt}$ represents a possible (though perhaps physically a little bizarre) traveling wave. The function $y(x, t) = \sin(ax^2 - bt)$, on the other hand, does *not* represent a traveling wave.

CHECKPOINT 2

Here are the equations of three waves:

$$(1) \ y(x, t) = 2 \sin(4x - 2t), \quad (2) \ y(x, t) = \sin(3x - 4t), \quad (3) \ y(x, t) = 2 \sin(3x - 3t).$$

Rank the waves according to their (a) wave speed and (b) maximum speed perpendicular to the wave's direction of travel (the transverse speed), greatest first.

Sample Problem

Transverse wave, amplitude, wavelength, period, velocity

A wave traveling along a string is described by

$$y(x, t) = 0.00327 \sin(72.1x - 2.72t), \quad (16-18)$$

in which the numerical constants are in SI units (0.00327 m, 72.1 rad/m, and 2.72 rad/s).

(a) What is the amplitude of this wave?

KEY IDEA

Equation 16-18 is of the same form as Eq. 16-2,

$$y = y_m \sin(kx - \omega t), \quad (16-19)$$

so we have a sinusoidal wave. By comparing the two equations, we can find the amplitude.

Calculation: We see that

$$y_m = 0.00327 \text{ m} = 3.27 \text{ mm.} \quad (\text{Answer})$$

(b) What are the wavelength, period, and frequency of this wave?

Calculations: By comparing Eqs. 16-18 and 16-19, we see that the angular wave number and angular frequency are

$$k = 72.1 \text{ rad/m} \quad \text{and} \quad \omega = 2.72 \text{ rad/s.}$$

We then relate wavelength λ to k via Eq. 16-5:

$$\begin{aligned} \lambda &= \frac{2\pi}{k} = \frac{2\pi \text{ rad}}{72.1 \text{ rad/m}} \\ &= 0.0871 \text{ m} = 8.71 \text{ cm.} \end{aligned} \quad (\text{Answer})$$

Next, we relate T to ω with Eq. 16-8:

$$T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{2.72 \text{ rad/s}} = 2.31 \text{ s,} \quad (\text{Answer})$$

and from Eq. 16-9 we have

$$f = \frac{1}{T} = \frac{1}{2.31 \text{ s}} = 0.433 \text{ Hz.} \quad (\text{Answer})$$

(c) What is the velocity of this wave?

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Calculation: The speed of the wave is given by Eq. 16-13:

$$v = \frac{\omega}{k} = \frac{2.72 \text{ rad/s}}{72.1 \text{ rad/m}} = 0.0377 \text{ m/s}$$

$$= 3.77 \text{ cm/s.} \quad (\text{Answer})$$

Because the phase in Eq. 16-18 contains the position variable x , the wave is moving along the x axis. Also, because the wave equation is written in the form of Eq. 16-2, the *minus* sign in front of the ωt term indicates that the wave is moving in the *positive* direction of the x axis. (Note that the quantities calculated in (b) and (c) are independent of the amplitude of the wave.)

- (d) What is the displacement y of the string at $x = 22.5$ cm and $t = 18.9$ s?

Calculation: Equation 16-18 gives the displacement as a function of position x and time t . Substituting the given values into the equation yields

$$y = 0.00327 \sin(72.1 \times 0.225 - 2.72 \times 18.9)$$

$$= (0.00327 \text{ m}) \sin(-35.1855 \text{ rad})$$

$$= (0.00327 \text{ m})(0.588)$$

$$= 0.00192 \text{ m} = 1.92 \text{ mm.} \quad (\text{Answer})$$

Thus, the displacement is positive. (Be sure to change your calculator mode to radians before evaluating the sine. Also, note that we do *not* round off the sine's argument before evaluating the sine. Also note that both terms in the argument are properly in radians, a dimensionless quantity.)

Sample Problem

Transverse wave, transverse velocity, transverse acceleration

In the preceding sample problem, we showed that at $t = 18.9$ s the transverse displacement y of the element of the string at $x = 22.5$ cm due to the wave of Eq. 16-18 is 1.92 mm.

- (a) What is u , the transverse velocity of the same element of the string, at that time? (This velocity, which is associated with the transverse oscillation of an element of the string, is in the y direction. Do not confuse it with v , the constant velocity at which the *wave form* travels along the x axis.)

KEY IDEAS

The transverse velocity u is the rate at which the displacement y of the element is changing. In general, that displacement is given by

$$y(x, t) = y_m \sin(kx - \omega t). \quad (16-20)$$

For an element at a certain location x , we find the rate of change of y by taking the derivative of Eq. 16-20 with respect to t while treating x as a constant. A derivative taken while one (or more) of the variables is treated as a constant is called a *partial derivative* and is represented by the symbol $\partial/\partial x$ rather than d/dx .

Calculations: Here we have

$$u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t). \quad (16-21)$$

Next, substituting numerical values from the preceding sample problem, we obtain

$$u = (-2.72 \text{ rad/s})(3.27 \text{ mm}) \cos(-35.1855 \text{ rad})$$

$$= 7.20 \text{ mm/s.} \quad (\text{Answer})$$

Thus, at $t = 18.9$ s, the element of the string at $x = 22.5$ cm is moving in the positive direction of y with a speed of 7.20 mm/s.

- (b) What is the transverse acceleration a_y of the same element at that time?

KEY IDEA

The transverse acceleration a_y is the rate at which the transverse velocity of the element is changing.

Calculations: From Eq. 16-21, again treating x as a constant but allowing t to vary, we find

$$a_y = \frac{\partial u}{\partial t} = -\omega^2 y_m \sin(kx - \omega t).$$

Comparison with Eq. 16-20 shows that we can write this as

$$a_y = -\omega^2 y.$$

We see that the transverse acceleration of an oscillating string element is proportional to its transverse displacement but opposite in sign. This is completely consistent with the action of the element itself—namely, that it is moving transversely in simple harmonic motion. Substituting numerical values yields

$$a_y = -(2.72 \text{ rad/s})^2 (1.92 \text{ mm})$$

$$= -14.2 \text{ mm/s}^2. \quad (\text{Answer})$$

Thus, at $t = 18.9$ s, the element of string at $x = 22.5$ cm is displaced from its equilibrium position by 1.92 mm in the positive y direction and has an acceleration of magnitude 14.2 mm/s² in the negative y direction.



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16-6 Wave Speed on a Stretched String

The speed of a wave is related to the wave's wavelength and frequency by Eq. 16-13, but *it is set by the properties of the medium*. If a wave is to travel through a medium such as water, air, steel, or a stretched string, it must cause the particles of that medium to oscillate as it passes, which requires both mass (for kinetic energy) and elasticity (for potential energy). Thus, the mass and elasticity determine how fast the wave can travel. Here, we find the wave speed through a medium in terms of these properties in two ways.

Dimensional Analysis

In dimensional analysis we carefully examine the dimensions of all the physical quantities that enter into a given situation to determine the quantities they produce. In this case, we examine mass and elasticity to find a speed v , which has the dimension of length divided by time, or LT^{-1} .

For the mass, we use the mass of a string element, which is the mass m of the string divided by the length l of the string. We call this ratio the *linear density* μ of the string. Thus, $\mu = m/l$, its dimension being mass divided by length, ML^{-1} .

You cannot send a wave along a string unless the string is under tension, which means that it has been stretched and pulled taut by forces at its two ends. The tension τ in the string is equal to the common magnitude of those two forces. As a wave travels along the string, it displaces elements of the string by causing additional stretching, with adjacent sections of string pulling on each other because of the tension. Thus, we can associate the tension in the string with the stretching (elasticity) of the string. The tension and the stretching forces it produces have the dimension of a force—namely, MLT^{-2} (from $F = ma$).

We need to combine μ (dimension ML^{-1}) and τ (dimension MLT^{-2}) to get v (dimension LT^{-1}). A little juggling of various combinations suggests

$$v = C \sqrt{\frac{\tau}{\mu}}, \quad (16-22)$$

in which C is a dimensionless constant that cannot be determined with dimensional analysis. In our second approach to determining wave speed, you will see that Eq. 16-22 is indeed correct and that $C = 1$.

Derivation from Newton's Second Law

Instead of the sinusoidal wave of Fig. 16-1b, let us consider a single symmetrical pulse such as that of Fig. 16-8, moving from left to right along a string with speed v . For convenience, we choose a reference frame in which the pulse remains stationary; that is, we run along with the pulse, keeping it constantly in view. In this frame, the string appears to move past us, from right to left in Fig. 16-8, with speed v .

Consider a small string element of length Δl within the pulse, an element that forms an arc of a circle of radius R and subtending an angle 2θ at the center of that circle. A force $\vec{\tau}$ with a magnitude equal to the tension in the string pulls tangentially on this element at each end. The horizontal components of these forces cancel, but the vertical components add to form a radial restoring force \vec{F} . In magnitude,

$$F = 2(\tau \sin \theta) \approx \tau(2\theta) = \tau \frac{\Delta l}{R} \quad (\text{force}), \quad (16-23)$$

where we have approximated $\sin \theta$ as θ for the small angles θ in Fig. 16-8. From that figure, we have also used $2\theta = \Delta l/R$. The mass of the element is given by

$$\Delta m = \mu \Delta l \quad (\text{mass}), \quad (16-24)$$

where μ is the string's linear density.

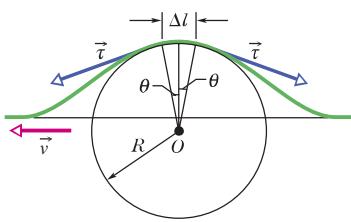


Fig. 16-8 A symmetrical pulse, viewed from a reference frame in which the pulse is stationary and the string appears to move right to left with speed v . We find speed v by applying Newton's second law to a string element of length Δl , located at the top of the pulse.

16-7 ENERGY AND POWER OF A WAVE TRAVELING ALONG A STRING

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At the moment shown in Fig. 16-8, the string element Δl is moving in an arc of a circle. Thus, it has a centripetal acceleration toward the center of that circle, given by

$$a = \frac{v^2}{R} \quad (\text{acceleration}). \quad (16-25)$$

Equations 16-23, 16-24, and 16-25 contain the elements of Newton's second law. Combining them in the form

$$\text{force} = \text{mass} \times \text{acceleration}$$

gives

$$\frac{\tau \Delta l}{R} = (\mu \Delta l) \frac{v^2}{R}.$$

Solving this equation for the speed v yields

$$v = \sqrt{\frac{\tau}{\mu}} \quad (\text{speed}), \quad (16-26)$$

in exact agreement with Eq. 16-22 if the constant C in that equation is given the value unity. Equation 16-26 gives the speed of the pulse in Fig. 16-8 and the speed of *any* other wave on the same string under the same tension.

Equation 16-26 tells us:



The speed of a wave along a stretched ideal string depends only on the tension and linear density of the string and not on the frequency of the wave.

The *frequency* of the wave is fixed entirely by whatever generates the wave (for example, the person in Fig. 16-1b). The *wavelength* of the wave is then fixed by Eq. 16-13 in the form $\lambda = v/f$.



CHECKPOINT 3

You send a traveling wave along a particular string by oscillating one end. If you increase the frequency of the oscillations, do (a) the speed of the wave and (b) the wavelength of the wave increase, decrease, or remain the same? If, instead, you increase the tension in the string, do (c) the speed of the wave and (d) the wavelength of the wave increase, decrease, or remain the same?

16-7 Energy and Power of a Wave Traveling Along a String

When we set up a wave on a stretched string, we provide energy for the motion of the string. As the wave moves away from us, it transports that energy as both kinetic energy and elastic potential energy. Let us consider each form in turn.

Kinetic Energy

A string element of mass dm , oscillating transversely in simple harmonic motion as the wave passes through it, has kinetic energy associated with its transverse velocity \vec{u} . When the element is rushing through its $y = 0$ position (element *b* in Fig. 16-9), its transverse velocity—and thus its kinetic energy—is a maximum. When the element is at its extreme position $y = y_m$ (as is element *a*), its transverse velocity—and thus its kinetic energy—is zero.

Elastic Potential Energy

To send a sinusoidal wave along a previously straight string, the wave must necessarily stretch the string. As a string element of length dx oscillates transversely, its

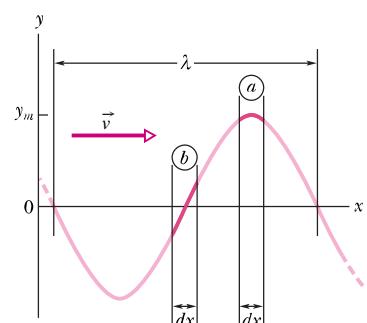


Fig. 16-9 A snapshot of a traveling wave on a string at time $t = 0$. String element *a* is at displacement $y = y_m$, and string element *b* is at displacement $y = 0$. The kinetic energy of the string element at each position depends on the transverse velocity of the element. The potential energy depends on the amount by which the string element is stretched as the wave passes through it.

length must increase and decrease in a periodic way if the string element is to fit the sinusoidal wave form. Elastic potential energy is associated with these length changes, just as for a spring.

When the string element is at its $y = y_m$ position (element *a* in Fig. 16-9), its length has its normal undisturbed value dx , so its elastic potential energy is zero. However, when the element is rushing through its $y = 0$ position, it has maximum stretch and thus maximum elastic potential energy.

Energy Transport

The oscillating string element thus has both its maximum kinetic energy and its maximum elastic potential energy at $y = 0$. In the snapshot of Fig. 16-9, the regions of the string at maximum displacement have no energy, and the regions at zero displacement have maximum energy. As the wave travels along the string, forces due to the tension in the string continuously do work to transfer energy from regions with energy to regions with no energy.

Suppose we set up a wave on a string stretched along a horizontal x axis so that Eq. 16-2 describes the string's displacement. We might send a wave along the string by continuously oscillating one end of the string, as in Fig. 16-1*b*. In doing so, we continuously provide energy for the motion and stretching of the string—as the string sections oscillate perpendicularly to the x axis, they have kinetic energy and elastic potential energy. As the wave moves into sections that were previously at rest, energy is transferred into those new sections. Thus, we say that the wave *transports* the energy along the string.

The Rate of Energy Transmission

The kinetic energy dK associated with a string element of mass dm is given by

$$dK = \frac{1}{2} dm u^2, \quad (16-27)$$

where u is the transverse speed of the oscillating string element. To find u , we differentiate Eq. 16-2 with respect to time while holding x constant:

$$u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t). \quad (16-28)$$

Using this relation and putting $dm = \mu dx$, we rewrite Eq. 16-27 as

$$dK = \frac{1}{2}(\mu dx)(-\omega y_m)^2 \cos^2(kx - \omega t). \quad (16-29)$$

Dividing Eq. 16-29 by dt gives the rate at which kinetic energy passes through a string element, and thus the rate at which kinetic energy is carried along by the wave. The ratio dx/dt that then appears on the right of Eq. 16-29 is the wave speed v , so we obtain

$$\frac{dK}{dt} = \frac{1}{2}\mu v \omega^2 y_m^2 \cos^2(kx - \omega t). \quad (16-30)$$

The *average* rate at which kinetic energy is transported is

$$\begin{aligned} \left(\frac{dK}{dt} \right)_{\text{avg}} &= \frac{1}{2}\mu v \omega^2 y_m^2 [\cos^2(kx - \omega t)]_{\text{avg}} \\ &= \frac{1}{4}\mu v \omega^2 y_m^2. \end{aligned} \quad (16-31)$$

Here we have taken the average over an integer number of wavelengths and have used the fact that the average value of the square of a cosine function over an integer number of periods is $\frac{1}{2}$.

Elastic potential energy is also carried along with the wave, and at the same average rate given by Eq. 16-31. Although we shall not examine the proof, you

should recall that, in an oscillating system such as a pendulum or a spring-block system, the average kinetic energy and the average potential energy are equal.

The **average power**, which is the average rate at which energy of both kinds is transmitted by the wave, is then

$$P_{\text{avg}} = 2 \left(\frac{dK}{dt} \right)_{\text{avg}} \quad (16-32)$$

or, from Eq. 16-31,

$$P_{\text{avg}} = \frac{1}{2} \mu v \omega^2 y_m^2 \quad (\text{average power}). \quad (16-33)$$

The factors μ and v in this equation depend on the material and tension of the string. The factors ω and y_m depend on the process that generates the wave. The dependence of the average power of a wave on the square of its amplitude and also on the square of its angular frequency is a general result, true for waves of all types.

Sample Problem

Average power of a transverse wave

A string has linear density $\mu = 525 \text{ g/m}$ and is under tension $\tau = 45 \text{ N}$. We send a sinusoidal wave with frequency $f = 120 \text{ Hz}$ and amplitude $y_m = 8.5 \text{ mm}$ along the string. At what average rate does the wave transport energy?

KEY IDEA

The average rate of energy transport is the average power P_{avg} as given by Eq. 16-33.

Calculations: To use Eq. 16-33, we first must calculate an-

gular frequency ω and wave speed v . From Eq. 16-9,
 $\omega = 2\pi f = (2\pi)(120 \text{ Hz}) = 754 \text{ rad/s}$.

From Eq. 16-26 we have

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{45 \text{ N}}{0.525 \text{ kg/m}}} = 9.26 \text{ m/s.}$$

Equation 16-33 then yields

$$\begin{aligned} P_{\text{avg}} &= \frac{1}{2} \mu v \omega^2 y_m^2 \\ &= \left(\frac{1}{2}\right)(0.525 \text{ kg/m})(9.26 \text{ m/s})(754 \text{ rad/s})^2(0.0085 \text{ m})^2 \\ &\approx 100 \text{ W.} \end{aligned} \quad (\text{Answer})$$



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16-8 The Wave Equation

As a wave passes through any element on a stretched string, the element moves perpendicularly to the wave's direction of travel. By applying Newton's second law to the element's motion, we can derive a general differential equation, called the *wave equation*, that governs the travel of waves of any type.

Figure 16-10a shows a snapshot of a string element of mass dm and length ℓ as a wave travels along a string of linear density μ that is stretched along a horizontal x axis. Let us assume that the wave amplitude is small so that the element can be tilted only slightly from the x axis as the wave passes. The force \vec{F}_2 on the right end of the element has a magnitude equal to tension τ in the string and is directed slightly upward. The force \vec{F}_1 on the left end of the element also has a magnitude equal to the tension τ but is directed slightly downward. Because of the slight curvature of the element, these two forces produce a net force that causes the element to have an upward acceleration a_y . Newton's second law written for y components ($F_{\text{net},y} = ma_y$) gives us

$$F_{2y} - F_{1y} = dm a_y. \quad (16-34)$$

Let's analyze this equation in parts.

Mass. The element's mass dm can be written in terms of the string's linear density μ and the element's length ℓ as $dm = \mu\ell$. Because the element can have

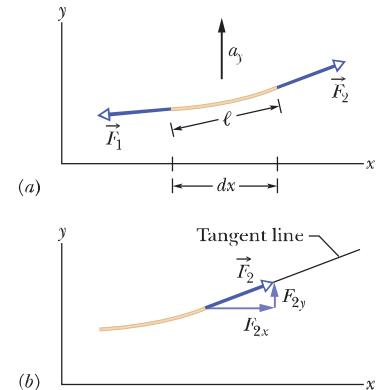


Fig. 16-10 (a) A string element as a sinusoidal transverse wave travels on a stretched string. Forces \vec{F}_1 and \vec{F}_2 act at the left and right ends, producing acceleration \vec{a} having a vertical component a_y . (b) The force at the element's right end is directed along a tangent to the element's right side.

only a slight tilt, $\ell \approx dx$ (Fig. 16-10a) and we have the approximation

$$dm = \mu dx. \quad (16-35)$$

Acceleration. The acceleration a_y in Eq. 16-34 is the second derivative of the displacement y with respect to time:

$$a_y = \frac{d^2y}{dt^2}. \quad (16-36)$$

Forces. Figure 16-10b shows that \vec{F}_2 is tangent to the string at the right end of the string element. Thus we can relate the components of the force to the string slope S_2 at the right end as

$$\frac{F_{2y}}{F_{2x}} = S_2. \quad (16-37)$$

We can also relate the components to the magnitude $F_2 (= \tau)$ with

$$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2}$$

or

$$\tau = \sqrt{F_{2x}^2 + F_{2y}^2}. \quad (16-38)$$

However, because we assume that the element is only slightly tilted, $F_{2y} \ll F_{2x}$ and therefore we can rewrite Eq. 16-38 as

$$\tau = F_{2x}. \quad (16-39)$$

Substituting this into Eq. 16-37 and solving for F_{2y} yield

$$F_{2y} = \tau S_2. \quad (16-40)$$

Similar analysis at the left end of the string element gives us

$$F_{1y} = \tau S_1. \quad (16-41)$$

We can now substitute Eqs. 16-35, 16-36, 16-40, and 16-41 into Eq. 16-34 to write

$$\tau S_2 - \tau S_1 = (\mu dx) \frac{d^2y}{dt^2},$$

or

$$\frac{S_2 - S_1}{dx} = \frac{\mu}{\tau} \frac{d^2y}{dt^2}. \quad (16-42)$$

Because the string element is short, slopes S_2 and S_1 differ by only a differential amount dS , where S is the slope at any point:

$$S = \frac{dy}{dx}. \quad (16-43)$$

First replacing $S_2 - S_1$ in Eq. 16-42 with dS and then using Eq. 16-43 to substitute dy/dx for S , we find

$$\frac{dS}{dx} = \frac{\mu}{\tau} \frac{d^2y}{dt^2},$$

$$\frac{d(dy/dx)}{dx} = \frac{\mu}{\tau} \frac{d^2y}{dt^2},$$

$$\text{and} \quad \frac{\partial^2y}{\partial x^2} = \frac{\mu}{\tau} \frac{\partial^2y}{\partial t^2}. \quad (16-44)$$

In the last step, we switched to the notation of partial derivatives because on the left we differentiate only with respect to x and on the right we differentiate only with respect to t . Finally, substituting from Eq. 16-26 ($v = \sqrt{\tau/\mu}$), we find

$$\frac{\partial^2y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2y}{\partial t^2} \quad (\text{wave equation}). \quad (16-45)$$

This is the general differential equation that governs the travel of waves of all types.

16-9 The Principle of Superposition for Waves

It often happens that two or more waves pass simultaneously through the same region. When we listen to a concert, for example, sound waves from many instruments fall simultaneously on our eardrums. The electrons in the antennas of our radio and television receivers are set in motion by the net effect of many electromagnetic waves from many different broadcasting centers. The water of a lake or harbor may be churned up by waves in the wakes of many boats.

Suppose that two waves travel simultaneously along the same stretched string. Let $y_1(x, t)$ and $y_2(x, t)$ be the displacements that the string would experience if each wave traveled alone. The displacement of the string when the waves overlap is then the algebraic sum

$$y'(x, t) = y_1(x, t) + y_2(x, t). \quad (16-46)$$

This summation of displacements along the string means that

 Overlapping waves algebraically add to produce a **resultant wave** (or **net wave**).

This is another example of the **principle of superposition**, which says that when several effects occur simultaneously, their net effect is the sum of the individual effects.

Figure 16-11 shows a sequence of snapshots of two pulses traveling in opposite directions on the same stretched string. When the pulses overlap, the resultant pulse is their sum. Moreover,

 Overlapping waves do not in any way alter the travel of each other.

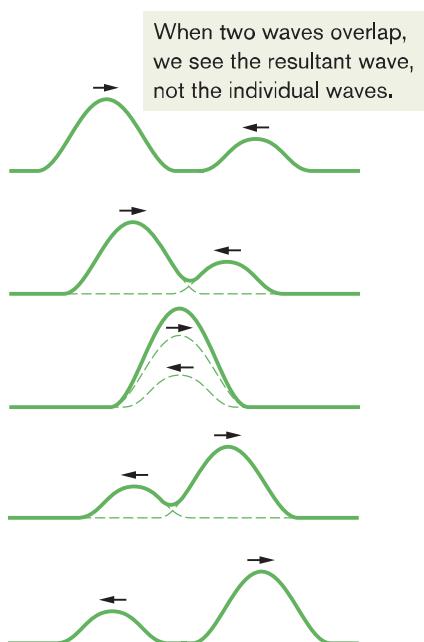


Fig. 16-11 A series of snapshots that show two pulses traveling in opposite directions along a stretched string. The superposition principle applies as the pulses move through each other.

16-10 Interference of Waves

Suppose we send two sinusoidal waves of the same wavelength and amplitude in the same direction along a stretched string. The superposition principle applies. What resultant wave does it predict for the string?

The resultant wave depends on the extent to which the waves are *in phase* (in step) with respect to each other—that is, how much one wave form is shifted from the other wave form. If the waves are exactly in phase (so that the peaks and valleys of one are exactly aligned with those of the other), they combine to double the displacement of either wave acting alone. If they are exactly out of phase (the peaks of one are exactly aligned with the valleys of the other), they combine to cancel everywhere, and the string remains straight. We call this phenomenon of combining waves **interference**, and the waves are said to **interfere**. (These terms refer only to the wave displacements; the travel of the waves is unaffected.)

Let one wave traveling along a stretched string be given by

$$y_1(x, t) = y_m \sin(kx - \omega t) \quad (16-47)$$

and another, shifted from the first, by

$$y_2(x, t) = y_m \sin(kx - \omega t + \phi). \quad (16-48)$$

These waves have the same angular frequency ω (and thus the same frequency f), the same angular wave number k (and thus the same wavelength λ), and the same amplitude y_m . They both travel in the positive direction of the x axis, with the same speed, given by Eq. 16-26. They differ only by a constant angle ϕ , the phase constant. These waves are said to be *out of phase* by ϕ or to have a *phase difference* of ϕ , or one wave is said to be *phase-shifted* from the other by ϕ .

$$\overbrace{y'(x,t)}^{\text{Displacement}} = \underbrace{[2y_m \cos \frac{1}{2}\phi]}_{\substack{\text{Magnitude} \\ \text{gives} \\ \text{amplitude}}} \underbrace{\sin(kx - \omega t + \frac{1}{2}\phi)}_{\text{Oscillating term}}$$

Fig. 16-12 The resultant wave of Eq. 16-51, due to the interference of two sinusoidal transverse waves, is also a sinusoidal transverse wave, with an amplitude and an oscillating term.

From the principle of superposition (Eq. 16-46), the resultant wave is the algebraic sum of the two interfering waves and has displacement

$$y'(x,t) = y_1(x,t) + y_2(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi). \quad (16-49)$$

In Appendix E we see that we can write the sum of the sines of two angles α and β as

$$\sin \alpha + \sin \beta = 2 \sin \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta). \quad (16-50)$$

Applying this relation to Eq. 16-49 leads to

$$y'(x,t) = [2y_m \cos \frac{1}{2}\phi] \sin(kx - \omega t + \frac{1}{2}\phi). \quad (16-51)$$

As Fig. 16-12 shows, the resultant wave is also a sinusoidal wave traveling in the direction of increasing x . It is the only wave you would actually see on the string (you would *not* see the two interfering waves of Eqs. 16-47 and 16-48).



If two sinusoidal waves of the same amplitude and wavelength travel in the *same* direction along a stretched string, they interfere to produce a resultant sinusoidal wave traveling in that direction.

The resultant wave differs from the interfering waves in two respects: (1) its phase constant is $\frac{1}{2}\phi$, and (2) its amplitude y'_m is the magnitude of the quantity in the brackets in Eq. 16-51:

$$y'_m = |2y_m \cos \frac{1}{2}\phi| \quad (\text{amplitude}). \quad (16-52)$$

If $\phi = 0$ rad (or 0°), the two interfering waves are exactly in phase, as in Fig. 16-13a. Then Eq. 16-51 reduces to

$$y'(x,t) = 2y_m \sin(kx - \omega t) \quad (\phi = 0). \quad (16-53)$$

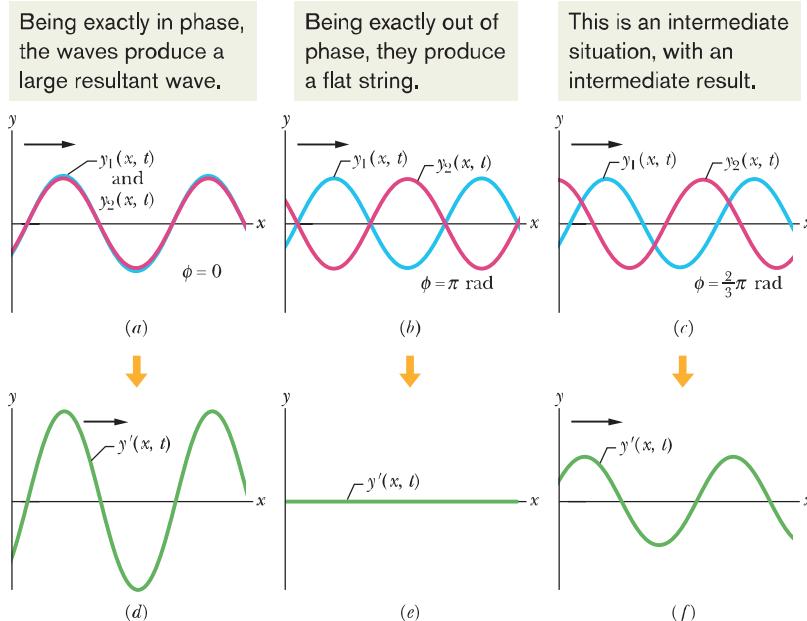


Fig. 16-13 Two identical sinusoidal waves, $y_1(x,t)$ and $y_2(x,t)$, travel along a string in the positive direction of an x axis. They interfere to give a resultant wave $y'(x,t)$. The resultant wave is what is actually seen on the string. The phase difference ϕ between the two interfering waves is (a) 0 rad or 0° , (b) π rad or 180° , and (c) $\frac{2}{3}\pi$ rad or 120° . The corresponding resultant waves are shown in (d), (e), and (f).

This resultant wave is plotted in Fig. 16-13d. Note from both that figure and Eq. 16-53 that the amplitude of the resultant wave is twice the amplitude of either interfering wave. That is the greatest amplitude the resultant wave can have, because the cosine term in Eqs. 16-51 and 16-52 has its greatest value (unity) when $\phi = 0$. Interference that produces the greatest possible amplitude is called *fully constructive interference*.

If $\phi = \pi$ rad (or 180°), the interfering waves are exactly out of phase as in Fig. 16-13b. Then $\cos \frac{1}{2}\phi$ becomes $\cos \pi/2 = 0$, and the amplitude of the resultant wave as given by Eq. 16-52 is zero. We then have, for all values of x and t ,

$$y'(x, t) = 0 \quad (\phi = \pi \text{ rad}). \quad (16-54)$$

The resultant wave is plotted in Fig. 16-13e. Although we sent two waves along the string, we see no motion of the string. This type of interference is called *fully destructive interference*.

Because a sinusoidal wave repeats its shape every 2π rad, a phase difference of $\phi = 2\pi$ rad (or 360°) corresponds to a shift of one wave relative to the other wave by a distance equivalent to one wavelength. Thus, phase differences can be described in terms of wavelengths as well as angles. For example, in Fig. 16-13b the waves may be said to be 0.50 wavelength out of phase. Table 16-1 shows some other examples of phase differences and the interference they produce. Note that when interference is neither fully constructive nor fully destructive, it is called *intermediate interference*. The amplitude of the resultant wave is then intermediate between 0 and $2y_m$. For example, from Table 16-1, if the interfering waves have a phase difference of 120° ($\phi = \frac{2}{3}\pi$ rad = 0.33 wavelength), then the resultant wave has an amplitude of y_m , the same as that of the interfering waves (see Figs. 16-13c and f).

Two waves with the same wavelength are in phase if their phase difference is zero or any integer number of wavelengths. Thus, the integer part of any phase difference *expressed in wavelengths* may be discarded. For example, a phase difference of 0.40 wavelength (an intermediate interference, close to fully destructive interference) is equivalent in every way to one of 2.40 wavelengths, and so the simpler of the two numbers can be used in computations.

Table 16-1

Phase Difference and Resulting Interference Types^a

Phase Difference, in			Amplitude of Resultant Wave	Type of Interference
Degrees	Radians	Wavelengths		
0	0	0	$2y_m$	Fully constructive
120	$\frac{2}{3}\pi$	0.33	y_m	Intermediate
180	π	0.50	0	Fully destructive
240	$\frac{4}{3}\pi$	0.67	y_m	Intermediate
360	2π	1.00	$2y_m$	Fully constructive
865	15.1	2.40	$0.60y_m$	Intermediate

^aThe phase difference is between two otherwise identical waves, with amplitude y_m , moving in the same direction.



CHECKPOINT 4

Here are four possible phase differences between two identical waves, expressed in wavelengths: 0.20, 0.45, 0.60, and 0.80. Rank them according to the amplitude of the resultant wave, greatest first.

Sample Problem**Interference of two waves, same direction, same amplitude**

Two identical sinusoidal waves, moving in the same direction along a stretched string, interfere with each other. The amplitude y_m of each wave is 9.8 mm, and the phase difference ϕ between them is 100° .

- (a) What is the amplitude y'_m of the resultant wave due to the interference, and what is the type of this interference?

KEY IDEA

These are identical sinusoidal waves traveling in the *same direction* along a string, so they interfere to produce a sinusoidal traveling wave.

Calculations: Because they are identical, the waves have the *same amplitude*. Thus, the amplitude y'_m of the resultant wave is given by Eq. 16-52:

$$\begin{aligned} y'_m &= |2y_m \cos \frac{1}{2}\phi| = |(2)(9.8 \text{ mm}) \cos(100^\circ/2)| \\ &= 13 \text{ mm}. \end{aligned} \quad (\text{Answer})$$

We can tell that the interference is *intermediate* in two ways. The phase difference is between 0 and 180° , and, correspondingly, the amplitude y'_m is between 0 and $2y_m$ ($= 19.6 \text{ mm}$).

- (b) What phase difference, in radians and wavelengths, will give the resultant wave an amplitude of 4.9 mm?

Calculations: Now we are given y'_m and seek ϕ . From Eq. 16-52,

$$y'_m = |2y_m \cos \frac{1}{2}\phi|,$$

we now have

$$4.9 \text{ mm} = (2)(9.8 \text{ mm}) \cos \frac{1}{2}\phi,$$

which gives us (with a calculator in the radian mode)

$$\begin{aligned} \phi &= 2 \cos^{-1} \frac{4.9 \text{ mm}}{(2)(9.8 \text{ mm})} \\ &= \pm 2.636 \text{ rad} \approx \pm 2.6 \text{ rad}. \end{aligned} \quad (\text{Answer})$$

There are two solutions because we can obtain the same resultant wave by letting the first wave *lead* (travel ahead of) or *lag* (travel behind) the second wave by 2.6 rad . In wavelengths, the phase difference is

$$\begin{aligned} \frac{\phi}{2\pi \text{ rad/wavelength}} &= \frac{\pm 2.636 \text{ rad}}{2\pi \text{ rad/wavelength}} \\ &= \pm 0.42 \text{ wavelength}. \end{aligned} \quad (\text{Answer})$$



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16-11 Phasors

We can represent a string wave (or any other type of wave) vectorially with a **phasor**. In essence, a phasor is a vector that has a magnitude equal to the amplitude of the wave and that rotates around an origin; the angular speed of the phasor is equal to the angular frequency ω of the wave. For example, the wave

$$y_1(x, t) = y_{m1} \sin(kx - \omega t) \quad (16-55)$$

is represented by the phasor shown in Figs. 16-14a to d. The magnitude of the phasor is the amplitude y_{m1} of the wave. As the phasor rotates around the origin at angular speed ω , its projection y_1 on the vertical axis varies sinusoidally, from a maximum of y_{m1} through zero to a minimum of $-y_{m1}$ and then back to y_{m1} . This variation corresponds to the sinusoidal variation in the displacement y_1 of any point along the string as the wave passes through that point.

When two waves travel along the same string in the same direction, we can represent them and their resultant wave in a *phasor diagram*. The phasors in Fig. 16-14e represent the wave of Eq. 16-55 and a second wave given by

$$y_2(x, t) = y_{m2} \sin(kx - \omega t + \phi). \quad (16-56)$$

This second wave is phase-shifted from the first wave by phase constant ϕ . Because the phasors rotate at the same angular speed ω , the angle between the two phasors is always ϕ . If ϕ is a *positive* quantity, then the phasor for wave 2 *lags* the phasor for wave 1 as they rotate, as drawn in Fig. 16-14e. If ϕ is a negative quantity, then the phasor for wave 2 *leads* the phasor for wave 1.

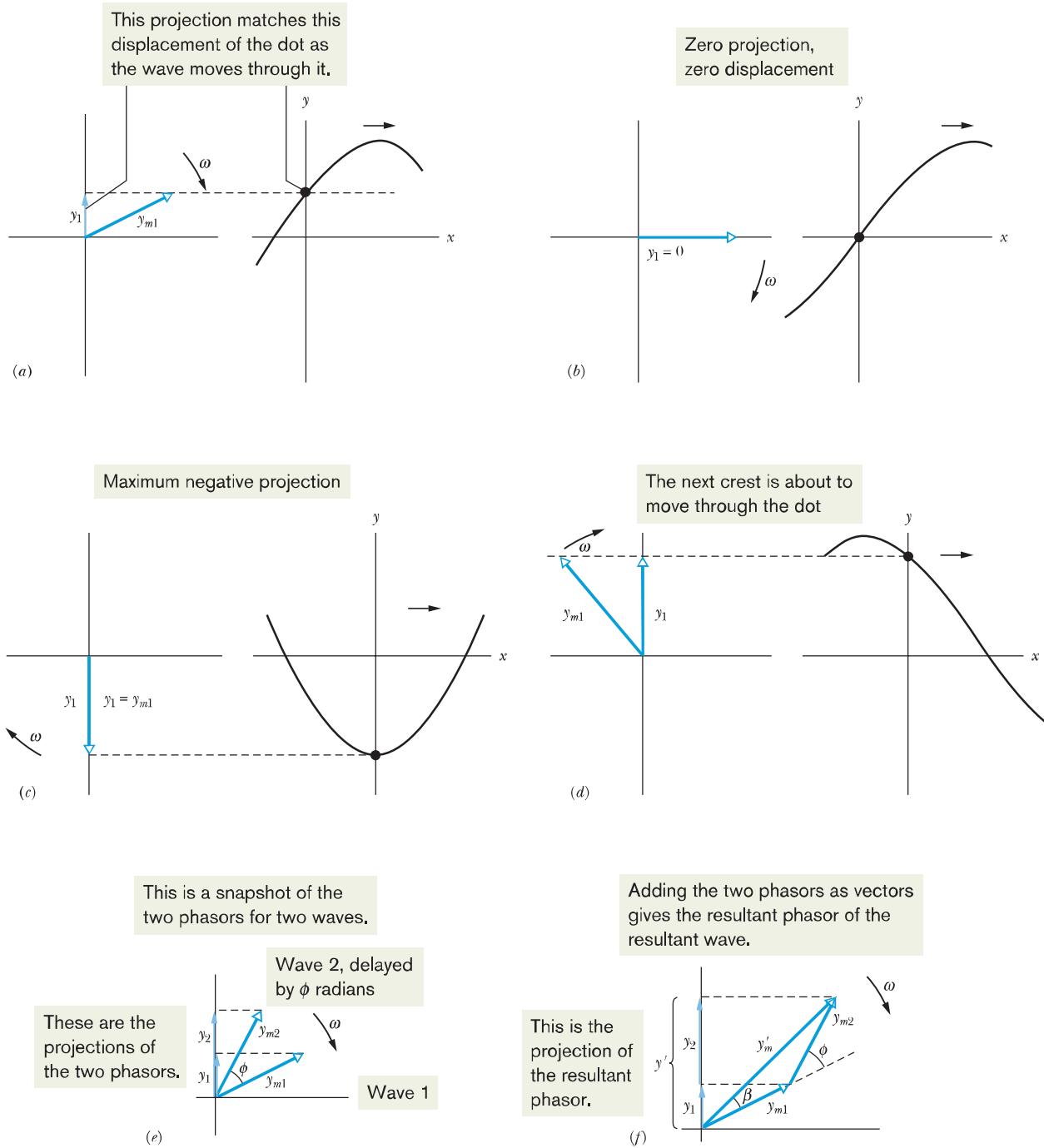


Fig. 16-14 (a)-(d) A phasor of magnitude y_{m1} rotating about an origin at angular speed ω represents a sinusoidal wave. The phasor's projection y_1 on the vertical axis represents the displacement of a point through which the wave passes. (e) A second phasor, also of angular speed ω but of magnitude y_{m2} and rotating at a constant angle ϕ from the first phasor, represents a second wave, with a phase constant ϕ . (f) The resultant wave is represented by the vector sum y'_m of the two phasors.

Because waves y_1 and y_2 have the same angular wave number k and angular frequency ω , we know from Eqs. 16-51 and 16-52 that their resultant is of the form

$$y'(x, t) = y'_m \sin(kx - \omega t + \beta), \quad (16-57)$$

where y'_m is the amplitude of the resultant wave and β is its phase constant. To find the values of y'_m and β , we would have to sum the two combining waves, as we did to obtain Eq. 16-51. To do this on a phasor diagram, we vectorially add the two phasors at any instant during their rotation, as in Fig. 16-14f where phasor y_{m2} has been shifted to the head of phasor y_{m1} . The magnitude of the vector sum equals the amplitude y'_m in Eq. 16-57. The angle between the vector sum and the phasor for y_1 equals the phase constant β in Eq. 16-57.

Note that, in contrast to the method of Section 16-10:



We can use phasors to combine waves even if their amplitudes are different.

Sample Problem

Interference of two waves, same direction, phasors, any amplitudes

Two sinusoidal waves $y_1(x, t)$ and $y_2(x, t)$ have the same wavelength and travel together in the same direction along a string. Their amplitudes are $y_{m1} = 4.0$ mm and $y_{m2} = 3.0$ mm, and their phase constants are 0 and $\pi/3$ rad, respectively. What are the amplitude y'_m and phase constant β of the resultant wave? Write the resultant wave in the form of Eq. 16-57.

KEY IDEAS

(1) The two waves have a number of properties in common: Because they travel along the same string, they must have the same speed v , as set by the tension and linear density of the string according to Eq. 16-26. With the same wavelength λ , they have the same angular wave number k ($= 2\pi/\lambda$). Also, with the same wave number k and speed v , they must have the same angular frequency ω ($= kv$).

(2) The waves (call them waves 1 and 2) can be represented by phasors rotating at the same angular speed ω about an origin. Because the phase constant for wave 2 is greater than that for wave 1 by $\pi/3$, phasor 2 must lag phasor 1 by $\pi/3$ rad in their clockwise rotation, as shown in Fig. 16-15a. The resultant wave due to the interference of waves 1 and 2 can then be represented by a phasor that is the vector sum of phasors 1 and 2.

Calculations: To simplify the vector summation, we drew phasors 1 and 2 in Fig. 16-15a at the instant when phasor 1 lies along the horizontal axis. We then drew lagging phasor 2 at positive angle $\pi/3$ rad. In Fig. 16-15b we shifted phasor 2 so its tail is at the head of phasor 1. Then we can draw the phasor y'_m of the resultant wave from the tail of phasor 1 to the head of phasor 2. The phase constant β is the angle phasor y'_m makes with phasor 1.

To find values for y'_m and β , we can sum phasors 1 and 2 as vectors on a calculator. Here we shall sum them by components. For the horizontal components we have

Add the phasors as vectors.

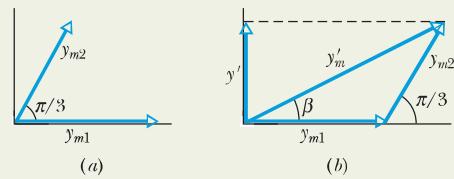


Fig. 16-15 (a) Two phasors of magnitudes y_{m1} and y_{m2} and with phase difference $\pi/3$. (b) Vector addition of these phasors at any instant during their rotation gives the magnitude y'_m of the phasor for the resultant wave.

$$\begin{aligned} y'_{mh} &= y_{m1} \cos 0 + y_{m2} \cos \pi/3 \\ &= 4.0 \text{ mm} + (3.0 \text{ mm}) \cos \pi/3 = 5.50 \text{ mm}. \end{aligned}$$

For the vertical components we have

$$\begin{aligned} y'_{mv} &= y_{m1} \sin 0 + y_{m2} \sin \pi/3 \\ &= 0 + (3.0 \text{ mm}) \sin \pi/3 = 2.60 \text{ mm}. \end{aligned}$$

Thus, the resultant wave has an amplitude of

$$\begin{aligned} y'_m &= \sqrt{(5.50 \text{ mm})^2 + (2.60 \text{ mm})^2} \\ &= 6.1 \text{ mm} \end{aligned} \quad (\text{Answer})$$

and a phase constant of

$$\beta = \tan^{-1} \frac{2.60 \text{ mm}}{5.50 \text{ mm}} = 0.44 \text{ rad.} \quad (\text{Answer})$$

From Fig. 16-15b, phase constant β is a *positive* angle relative to phasor 1. Thus, the resultant wave lags wave 1 in their travel by phase constant $\beta = +0.44$ rad. From Eq. 16-57, we can write the resultant wave as

$$y'(x, t) = (6.1 \text{ mm}) \sin(kx - \omega t + 0.44 \text{ rad}). \quad (\text{Answer})$$

16-12 Standing Waves

In Section 16-10, we discussed two sinusoidal waves of the same wavelength and amplitude traveling *in the same direction* along a stretched string. What if they travel in opposite directions? We can again find the resultant wave by applying the superposition principle.

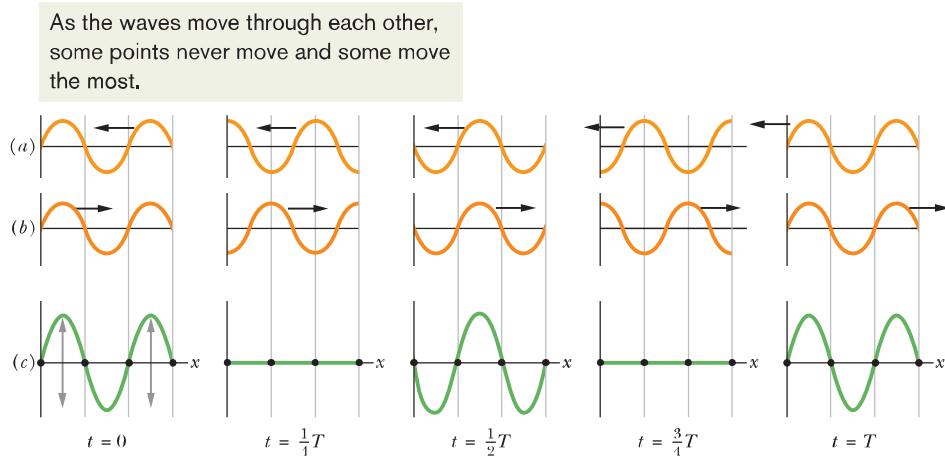


Fig. 16-16 (a) Five snapshots of a wave traveling to the left, at the times t indicated below part (c) (T is the period of oscillation). (b) Five snapshots of a wave identical to that in (a) but traveling to the right, at the same times t . (c) Corresponding snapshots for the superposition of the two waves on the same string. At $t = 0, \frac{1}{2}T$, and T , fully constructive interference occurs because of the alignment of peaks with peaks and valleys with valleys. At $t = \frac{1}{4}T$ and $\frac{3}{4}T$, fully destructive interference occurs because of the alignment of peaks with valleys. Some points (the nodes, marked with dots) never oscillate; some points (the antinodes) oscillate the most.

Figure 16-16 suggests the situation graphically. It shows the two combining waves, one traveling to the left in Fig. 16-16a, the other to the right in Fig. 16-16b. Figure 16-16c shows their sum, obtained by applying the superposition principle graphically. The outstanding feature of the resultant wave is that there are places along the string, called **nodes**, where the string never moves. Four such nodes are marked by dots in Fig. 16-16c. Halfway between adjacent nodes are **antinodes**, where the amplitude of the resultant wave is a maximum. Wave patterns such as that of Fig. 16-16c are called **standing waves** because the wave patterns do not move left or right; the locations of the maxima and minima do not change.



If two sinusoidal waves of the same amplitude and wavelength travel in *opposite* directions along a stretched string, their interference with each other produces a standing wave.

To analyze a standing wave, we represent the two combining waves with the equations

$$y_1(x, t) = y_m \sin(kx - \omega t) \quad (16-58)$$

and

$$y_2(x, t) = y_m \sin(kx + \omega t). \quad (16-59)$$

The principle of superposition gives, for the combined wave,

$$y'(x, t) = y_1(x, t) + y_2(x, t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t).$$

Applying the trigonometric relation of Eq. 16-50 leads to Fig. 16-17 and

$$y'(x, t) = [2y_m \sin kx] \cos \omega t. \quad (16-60)$$

Displacement

$$y'(x, t) = \underbrace{[2y_m \sin kx]}_{\text{Magnitude}} \underbrace{\cos \omega t}_{\text{Oscillating term}}$$

Magnitude gives amplitude at position x

Fig. 16-17 The resultant wave of Eq. 16-60 is a standing wave and is due to the interference of two sinusoidal waves of the same amplitude and wavelength that travel in opposite directions.

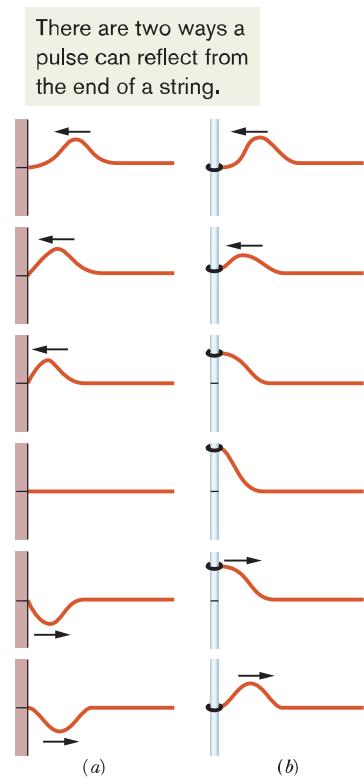


Fig. 16-18 (a) A pulse incident from the right is reflected at the left end of the string, which is tied to a wall. Note that the reflected pulse is inverted from the incident pulse. (b) Here the left end of the string is tied to a ring that can slide without friction up and down the rod. Now the pulse is not inverted by the reflection.

This equation does not describe a traveling wave because it is not of the form of Eq. 16-17. Instead, it describes a standing wave.

The quantity $2y_m \sin kx$ in the brackets of Eq. 16-60 can be viewed as the amplitude of oscillation of the string element that is located at position x . However, since an amplitude is always positive and $\sin kx$ can be negative, we take the absolute value of the quantity $2y_m \sin kx$ to be the amplitude at x .

In a traveling sinusoidal wave, the amplitude of the wave is the same for all string elements. That is not true for a standing wave, in which the amplitude *varies with position*. In the standing wave of Eq. 16-60, for example, the amplitude is zero for values of kx that give $\sin kx = 0$. Those values are

$$kx = n\pi, \quad \text{for } n = 0, 1, 2, \dots \quad (16-61)$$

Substituting $k = 2\pi/\lambda$ in this equation and rearranging, we get

$$x = n \frac{\lambda}{2}, \quad \text{for } n = 0, 1, 2, \dots \quad (\text{nodes}), \quad (16-62)$$

as the positions of zero amplitude—the nodes—for the standing wave of Eq. 16-60. Note that adjacent nodes are separated by $\lambda/2$, half a wavelength.

The amplitude of the standing wave of Eq. 16-60 has a maximum value of $2y_m$, which occurs for values of kx that give $|\sin kx| = 1$. Those values are

$$\begin{aligned} kx &= \frac{1}{2}\pi, \frac{3}{2}\pi, \frac{5}{2}\pi, \dots \\ &= (n + \frac{1}{2})\pi, \quad \text{for } n = 0, 1, 2, \dots \end{aligned} \quad (16-63)$$

Substituting $k = 2\pi/\lambda$ in Eq. 16-63 and rearranging, we get

$$x = \left(n + \frac{1}{2}\right) \frac{\lambda}{2}, \quad \text{for } n = 0, 1, 2, \dots \quad (\text{antinodes}), \quad (16-64)$$

as the positions of maximum amplitude—the antinodes—of the standing wave of Eq. 16-60. The antinodes are separated by $\lambda/2$ and are located halfway between pairs of nodes.

Reflections at a Boundary

We can set up a standing wave in a stretched string by allowing a traveling wave to be reflected from the far end of the string so that the wave travels back through itself. The incident (original) wave and the reflected wave can then be described by Eqs. 16-58 and 16-59, respectively, and they can combine to form a pattern of standing waves.

In Fig. 16-18, we use a single pulse to show how such reflections take place. In Fig. 16-18a, the string is fixed at its left end. When the pulse arrives at that end, it exerts an upward force on the support (the wall). By Newton's third law, the support exerts an opposite force of equal magnitude on the string. This second force generates a pulse at the support, which travels back along the string in the direction opposite that of the incident pulse. In a “hard” reflection of this kind, there must be a node at the support because the string is fixed there. The reflected and incident pulses must have opposite signs, so as to cancel each other at that point.

In Fig. 16-18b, the left end of the string is fastened to a light ring that is free to slide without friction along a rod. When the incident pulse arrives, the ring moves up the rod. As the ring moves, it pulls on the string, stretching the string and producing a reflected pulse with the same sign and amplitude as the incident pulse. Thus, in such a “soft” reflection, the incident and reflected pulses reinforce each other, creating an antinode at the end of the string; the maximum displacement of the ring is twice the amplitude of either of these two pulses.



CHECKPOINT 5

Two waves with the same amplitude and wavelength interfere in three different situations to produce resultant waves with the following equations:

$$(1) y'(x, t) = 4 \sin(5x - 4t)$$

$$(2) y'(x, t) = 4 \sin(5x) \cos(4t)$$

$$(3) y'(x, t) = 4 \sin(5x + 4t)$$

In which situation are the two combining waves traveling (a) toward positive x , (b) toward negative x , and (c) in opposite directions?

16-13 Standing Waves and Resonance

Consider a string, such as a guitar string, that is stretched between two clamps. Suppose we send a continuous sinusoidal wave of a certain frequency along the string, say, toward the right. When the wave reaches the right end, it reflects and begins to travel back to the left. That left-going wave then overlaps the wave that is still traveling to the right. When the left-going wave reaches the left end, it reflects again and the newly reflected wave begins to travel to the right, overlapping the left-going and right-going waves. In short, we very soon have many overlapping traveling waves, which interfere with one another.

For certain frequencies, the interference produces a standing wave pattern (or **oscillation mode**) with nodes and large antinodes like those in Fig. 16-19. Such a standing wave is said to be produced at **resonance**, and the string is said to *resonate* at these certain frequencies, called **resonant frequencies**. If the string is oscillated at some frequency other than a resonant frequency, a standing wave is not set up. Then the interference of the right-going and left-going traveling waves results in only small (perhaps imperceptible) oscillations of the string.

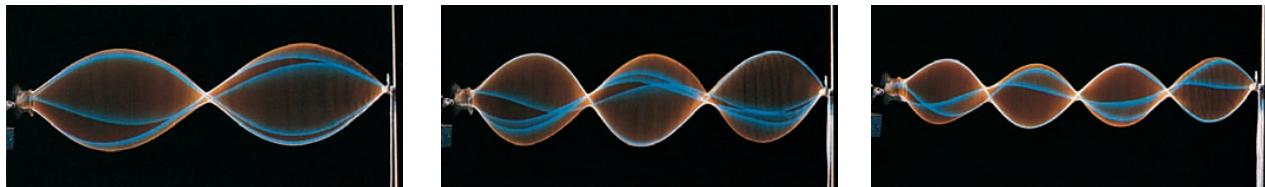


Fig. 16-19 Stroboscopic photographs reveal (imperfect) standing wave patterns on a string being made to oscillate by an oscillator at the left end. The patterns occur at certain frequencies of oscillation. (Richard Megna/Fundamental Photographs)

Let a string be stretched between two clamps separated by a fixed distance L . To find expressions for the resonant frequencies of the string, we note that a node must exist at each of its ends, because each end is fixed and cannot oscillate. The simplest pattern that meets this key requirement is that in Fig. 16-20a, which shows the string at both its extreme displacements (one solid and one dashed, together forming a single “loop”). There is only one antinode, which is at the center of the string. Note that half a wavelength spans the length L , which we take to be the string’s length. Thus, for this pattern, $\lambda/2 = L$. This condition tells us that if the left-going and right-going traveling waves are to set up this pattern by their interference, they must have the wavelength $\lambda = 2L$.

A second simple pattern meeting the requirement of nodes at the fixed ends is shown in Fig. 16-20b. This pattern has three nodes and two antinodes and is said

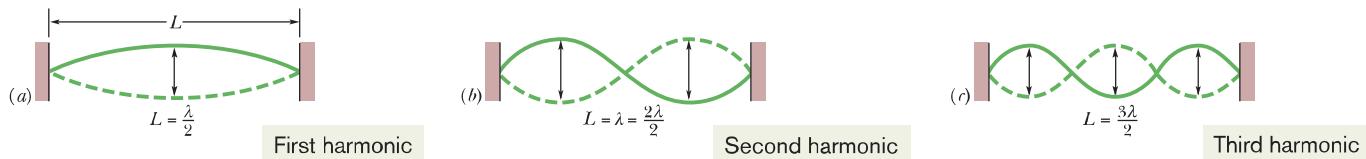


Fig. 16-20 A string, stretched between two clamps, is made to oscillate in standing wave patterns. (a) The simplest possible pattern consists of one *loop*, which refers to the composite shape formed by the string in its extreme displacements (the solid and dashed lines). (b) The next simplest pattern has two loops. (c) The next has three loops.



Fig. 16-21 One of many possible standing wave patterns for a kettledrum head, made visible by dark powder sprinkled on the drumhead. As the head is set into oscillation at a single frequency by a mechanical oscillator at the upper left of the photograph, the powder collects at the nodes, which are circles and straight lines in this two-dimensional example. (*Courtesy Thomas D. Rossing, Northern Illinois University*)

to be a two-loop pattern. For the left-going and right-going waves to set it up, they must have a wavelength $\lambda = L$. A third pattern is shown in Fig. 16-20c. It has four nodes, three antinodes, and three loops, and the wavelength is $\lambda = \frac{2}{3}L$. We could continue this progression by drawing increasingly more complicated patterns. In each step of the progression, the pattern would have one more node and one more antinode than the preceding step, and an additional $\lambda/2$ would be fitted into the distance L .

Thus, a standing wave can be set up on a string of length L by a wave with a wavelength equal to one of the values

$$\lambda = \frac{2L}{n}, \quad \text{for } n = 1, 2, 3, \dots \quad (16-65)$$

The resonant frequencies that correspond to these wavelengths follow from Eq. 16-13:

$$f = \frac{v}{\lambda} = n \frac{v}{2L}, \quad \text{for } n = 1, 2, 3, \dots \quad (16-66)$$

Here v is the speed of traveling waves on the string.

Equation 16-66 tells us that the resonant frequencies are integer multiples of the lowest resonant frequency, $f = v/2L$, which corresponds to $n = 1$. The oscillation mode with that lowest frequency is called the *fundamental mode* or the *first harmonic*. The *second harmonic* is the oscillation mode with $n = 2$, the *third harmonic* is that with $n = 3$, and so on. The frequencies associated with these modes are often labeled f_1, f_2, f_3 , and so on. The collection of all possible oscillation modes is called the **harmonic series**, and n is called the **harmonic number** of the n th harmonic.

For a given string under a given tension, each resonant frequency corresponds to a particular oscillation pattern. Thus, if the frequency is in the audible range, you can hear the shape of the string. Resonance can also occur in two dimensions (such as on the surface of the kettledrum in Fig. 16-21) and in three dimensions (such as in the wind-induced swaying and twisting of a tall building).

✓ CHECKPOINT 6

In the following series of resonant frequencies, one frequency (lower than 400 Hz) is missing: 150, 225, 300, 375 Hz. (a) What is the missing frequency? (b) What is the frequency of the seventh harmonic?

Sample Problem

Resonance of transverse waves, standing waves, harmonics

Figure 16-22 shows a pattern of resonant oscillation of a string of mass $m = 2.500 \text{ g}$ and length $L = 0.800 \text{ m}$ and that is under tension $\tau = 325.0 \text{ N}$. What is the wavelength λ of the transverse waves producing the standing-wave pattern, and what is the harmonic number n ? What is the frequency f of the transverse waves and of the oscillations of the moving string elements? What is the maximum magnitude of the transverse velocity u_m of the element oscillating at coordinate $x = 0.180 \text{ m}$ (note the x axis in the figure)? At what point

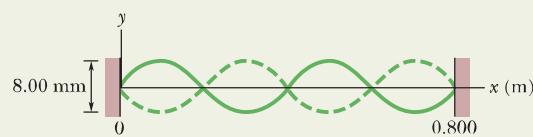


Fig. 16-22 Resonant oscillation of a string under tension.

during the element's oscillation is the transverse velocity maximum?

KEY IDEAS

(1) The traverse waves that produce a standing-wave pattern must have a wavelength such that an integer number n of half-wavelengths fit into the length L of the string. (2) The frequency of those waves and of the oscillations of the string elements is given by Eq. 16-66 ($f = nv/2L$). (3) The displacement of a string element as a function of position x and time t is given by Eq. 16-60:

$$y'(x, t) = [2y_m \sin kx] \cos \omega t. \quad (16-67)$$

Wavelength and harmonic number: In Fig. 16-22, the solid line, which is effectively a snapshot (or freeze frame) of the oscillations, reveals that 2 full wavelengths fit into the length $L = 0.800\text{ m}$ of the string. Thus, we have

$$\begin{aligned} 2\lambda &= L, \\ \text{or } \lambda &= \frac{L}{2}. \quad (16-68) \\ &= \frac{0.800\text{ m}}{2} = 0.400\text{ m}. \quad (\text{Answer}) \end{aligned}$$

By counting the number of loops (or half-wavelengths) in Fig. 16-22, we see that the harmonic number is

$$n = 4. \quad (\text{Answer})$$

We reach the same conclusion by comparing Eqs. 16-68 and 16-65 ($\lambda = 2L/n$). Thus, the string is oscillating in its fourth harmonic.

Frequency: We can get the frequency f of the transverse waves from Eq. 16-13 ($v = \lambda f$) if we first find the speed v of the waves. That speed is given by Eq. 16-26, but we must substitute m/L for the unknown linear density μ . We obtain

$$\begin{aligned} v &= \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\tau}{m/L}} = \sqrt{\frac{\tau L}{m}} \\ &= \sqrt{\frac{(325\text{ N})(0.800\text{ m})}{2.50 \times 10^{-3}\text{ kg}}} = 322.49\text{ m/s.} \end{aligned}$$

After rearranging Eq. 16-13, we write

$$\begin{aligned} f &= \frac{v}{\lambda} = \frac{322.49\text{ m/s}}{0.400\text{ m}} \\ &= 806.2\text{ Hz} \approx 806\text{ Hz.} \quad (\text{Answer}) \end{aligned}$$

Note that we get the same answer by substituting into Eq. 16-66:

$$\begin{aligned} f &= n \frac{v}{2L} = 4 \frac{322.49\text{ m/s}}{2(0.800\text{ m})} \\ &= 806\text{ Hz.} \quad (\text{Answer}) \end{aligned}$$

Now note that this 806 Hz is not only the frequency of the waves producing the fourth harmonic but also it is said to be the fourth harmonic, as in the statement, “The fourth harmonic of this oscillating string is 806 Hz.” It is also the frequency of the string elements as they oscillate vertically in the figure in simple harmonic motion, just as a block on a vertical spring would oscillate in simple harmonic motion. Finally, it is also the frequency of the sound you would hear from the string as the oscillating string elements periodically push against the air, sending out sound waves.

Transverse velocity: The displacement y' of the string element located at coordinate x is given by Eq. 16-67 as a function of time t . The term $\cos \omega t$ contains the dependence on time and thus provides the “motion” of the standing wave. The term $2y_m \sin kx$ sets the extent of the motion—that is, the amplitude. The greatest amplitude occurs at an antinode, where $\sin kx$ is +1 or -1 and thus the greatest amplitude is $2y_m$. From Fig. 16-22, we see that $2y_m = 4.00\text{ mm}$, which tells us that $y_m = 2.00\text{ mm}$.

We want the transverse velocity—the velocity of a string element parallel to the y axis. To find it, we take the time derivative of Eq. 16-67:

$$\begin{aligned} u(x, t) &= \frac{\partial y'}{\partial t} = \frac{\partial}{\partial t} [(2y_m \sin kx) \cos \omega t] \\ &= [-2y_m \omega \sin kx] \sin \omega t. \quad (16-69) \end{aligned}$$

Here the term $\sin \omega t$ provides the variation with time and the term $-2y_m \omega \sin kx$ provides the extent of that variation. We want the absolute magnitude of that extent:

$$u_m = |-2y_m \omega \sin kx|.$$

To evaluate this for the element at $x = 0.180\text{ m}$, we first note that $y_m = 2.00\text{ mm}$, $k = 2\pi/\lambda = 2\pi/(0.400\text{ m})$, and $\omega = 2\pi f = 2\pi(806.2\text{ Hz})$. Then the maximum speed of the element at $x = 0.180\text{ m}$ is

$$\begin{aligned} u_m &= \left| -2(2.00 \times 10^{-3}\text{ m})(2\pi)(806.2\text{ Hz}) \right. \\ &\quad \left. \times \sin\left(\frac{2\pi}{0.400\text{ m}}(0.180\text{ m})\right) \right| \\ &= 6.26\text{ m/s.} \quad (\text{Answer}) \end{aligned}$$

To determine when the string element has this maximum speed, we could investigate Eq. 16-69. However, a little thought can save a lot of work. The element is undergoing simple harmonic motion and must come to a momentary stop at its extreme upward position and extreme downward position. It has the greatest speed as it zips through the midpoint of its oscillation, just as a block does in a block-spring oscillator.



Additional examples, video, and practice available at WileyPLUS

REVIEW & SUMMARY

Transverse and Longitudinal Waves Mechanical waves can exist only in material media and are governed by Newton's laws. **Transverse** mechanical waves, like those on a stretched string, are waves in which the particles of the medium oscillate perpendicular to the wave's direction of travel. Waves in which the particles of the medium oscillate parallel to the wave's direction of travel are **longitudinal** waves.

Sinusoidal Waves A sinusoidal wave moving in the positive direction of an x axis has the mathematical form

$$y(x, t) = y_m \sin(kx - \omega t), \quad (16-2)$$

where y_m is the **amplitude** of the wave, k is the **angular wave number**, ω is the **angular frequency**, and $kx - \omega t$ is the **phase**. The **wavelength** λ is related to k by

$$k = \frac{2\pi}{\lambda}. \quad (16-5)$$

The **period** T and **frequency** f of the wave are related to ω by

$$\frac{\omega}{2\pi} = f = \frac{1}{T}. \quad (16-9)$$

Finally, the **wave speed** v is related to these other parameters by

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f. \quad (16-13)$$

Equation of a Traveling Wave Any function of the form

$$y(x, t) = h(kx \pm \omega t) \quad (16-17)$$

can represent a **traveling wave** with a wave speed given by Eq. 16-13 and a wave shape given by the mathematical form of h . The plus sign denotes a wave traveling in the negative direction of the x axis, and the minus sign a wave traveling in the positive direction.

Wave Speed on Stretched String The speed of a wave on a stretched string is set by properties of the string. The speed on a string with tension τ and linear density μ is

$$v = \sqrt{\frac{\tau}{\mu}}. \quad (16-26)$$

Power The **average power** of, or average rate at which energy is transmitted by, a sinusoidal wave on a stretched string is given by

$$P_{\text{avg}} = \frac{1}{2} \mu v \omega^2 y_m^2. \quad (16-33)$$

QUESTIONS

1 The following four waves are sent along strings with the same linear densities (x is in meters and t is in seconds). Rank the waves according to (a) their wave speed and (b) the tension in the strings along which they travel, greatest first:

- (1) $y_1 = (3 \text{ mm}) \sin(x - 3t)$, (3) $y_3 = (1 \text{ mm}) \sin(4x - t)$,
 (2) $y_2 = (6 \text{ mm}) \sin(2x - t)$, (4) $y_4 = (2 \text{ mm}) \sin(x - 2t)$.

Superposition of Waves When two or more waves traverse the same medium, the displacement of any particle of the medium is the sum of the displacements that the individual waves would give it.

Interference of Waves Two sinusoidal waves on the same string exhibit **interference**, adding or canceling according to the principle of superposition. If the two are traveling in the same direction and have the same amplitude y_m and frequency (hence the same wavelength) but differ in phase by a **phase constant** ϕ , the result is a single wave with this same frequency:

$$y'(x, t) = [2y_m \cos \frac{1}{2}\phi] \sin(kx - \omega t + \frac{1}{2}\phi). \quad (16-51)$$

If $\phi = 0$, the waves are exactly in phase and their interference is fully constructive; if $\phi = \pi$ rad, they are exactly out of phase and their interference is fully destructive.

Phasors A wave $y(x, t)$ can be represented with a **phasor**. This is a vector that has a magnitude equal to the amplitude y_m of the wave and that rotates about an origin with an angular speed equal to the angular frequency ω of the wave. The projection of the rotating phasor on a vertical axis gives the displacement y of a point along the wave's travel.

Standing Waves The interference of two identical sinusoidal waves moving in opposite directions produces **standing waves**. For a string with fixed ends, the standing wave is given by

$$y'(x, t) = [2y_m \sin kx] \cos \omega t. \quad (16-60)$$

Standing waves are characterized by fixed locations of zero displacement called **nodes** and fixed locations of maximum displacement called **antinodes**.

Resonance Standing waves on a string can be set up by reflection of traveling waves from the ends of the string. If an end is fixed, it must be the position of a node. This limits the frequencies at which standing waves will occur on a given string. Each possible frequency is a **resonant frequency**, and the corresponding standing wave pattern is an **oscillation mode**. For a stretched string of length L with fixed ends, the resonant frequencies are

$$f = \frac{v}{\lambda} = n \frac{v}{2L}, \quad \text{for } n = 1, 2, 3, \dots \quad (16-66)$$

The oscillation mode corresponding to $n = 1$ is called the **fundamental mode** or the **first harmonic**; the mode corresponding to $n = 2$ is the **second harmonic**; and so on.

2 In Fig. 16-23, wave 1 consists of a rectangular peak of height 4 units and width d , and a rectangular valley of depth 2 units and width d . The wave travels rightward along an x axis. Choices 2, 3, and 4 are similar waves, with the same heights, depths, and widths, that will travel leftward along that axis and through wave 1. Right-going wave 1 and one of the left-going waves will interfere as they

pass through each other. With which left-going wave will the interference give, for an instant, (a) the deepest valley, (b) a flat line, and (c) a flat peak $2d$ wide?

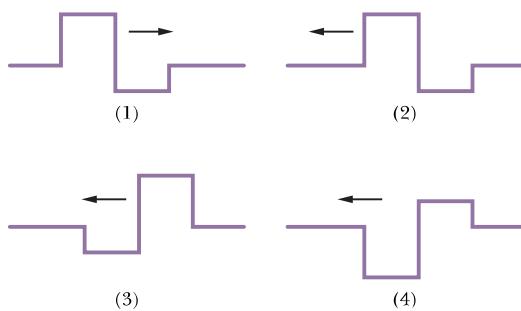


Fig. 16-23 Question 2.

3 Figure 16-24a gives a snapshot of a wave traveling in the direction of positive x along a string under tension. Four string elements are indicated by the lettered points. For each of those elements, determine whether, at the instant of the snapshot, the element is moving upward or downward or is momentarily at rest. (*Hint:* Imagine the wave as it moves through the four string elements, as if you were watching a video of the wave as it traveled rightward.)

Figure 16-24b gives the displacement of a string element located at, say, $x = 0$ as a function of time. At the lettered times, is the element moving upward or downward or is it momentarily at rest?

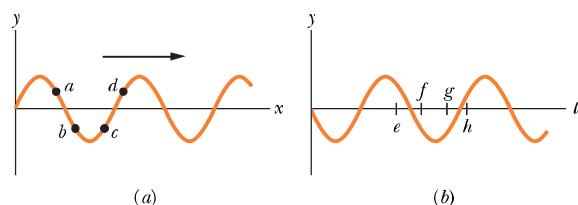


Fig. 16-24 Question 3.

4 Figure 16-25 shows three waves that are *separately* sent along a string that is stretched under a certain tension along an x axis. Rank the waves according to their (a) wavelengths, (b) speeds, and (c) angular frequencies, greatest first.

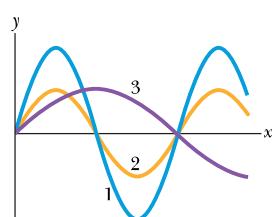


Fig. 16-25 Question 4.

5 If you start with two sinusoidal waves of the same amplitude traveling in phase on a string and then somehow phase-shift one of them by 5.4 wavelengths, what type of interference will occur on the string?

6 The amplitudes and phase differences for four pairs of waves of equal wavelengths are (a) 2 mm , 6 mm , and $\pi \text{ rad}$; (b) 3 mm , 5 mm , and $\pi \text{ rad}$; (c) 7 mm , 9 mm , and $\pi \text{ rad}$; (d) 2 mm , 2

mm, and 0 rad . Each pair travels in the same direction along the same string. Without written calculation, rank the four pairs according to the amplitude of their resultant wave, greatest first. (*Hint:* Construct phasor diagrams.)

7 A sinusoidal wave is sent along a cord under tension, transporting energy at the average rate of $P_{\text{avg},1}$. Two waves, identical to that first one, are then to be sent along the cord with a phase difference ϕ of either 0 , 0.2 wavelength, or 0.5 wavelength. (a) With only mental calculation, rank those choices of ϕ according to the average rate at which the waves will transport energy, greatest first. (b) For the first choice of ϕ , what is the average rate in terms of $P_{\text{avg},1}$?

8 (a) If a standing wave on a string is given by

$$y'(t) = (3 \text{ mm}) \sin(5x) \cos(4t),$$

is there a node or an antinode of the oscillations of the string at $x = 0$? (b) If the standing wave is given by

$$y''(t) = (3 \text{ mm}) \sin(5x + \pi/2) \cos(4t),$$

is there a node or an antinode at $x = 0$?

9 Strings *A* and *B* have identical lengths and linear densities, but string *B* is under greater tension than string *A*. Figure 16-26 shows four situations, (a) through (d), in which standing wave patterns exist on the two strings. In which situations is there the possibility that strings *A* and *B* are oscillating at the same resonant frequency?

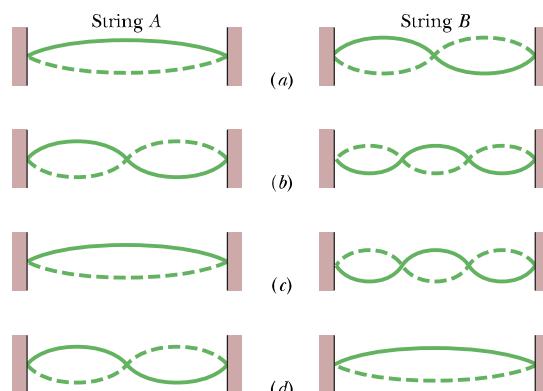


Fig. 16-26 Question 9.

10 If you set up the seventh harmonic on a string, (a) how many nodes are present, and (b) is there a node, antinode, or some intermediate state at the midpoint? If you next set up the sixth harmonic, (c) is its resonant wavelength longer or shorter than that for the seventh harmonic, and (d) is the resonant frequency higher or lower?

11 Figure 16-27 shows phasor diagrams for three situations in which two waves travel along the same string. All six waves have the same amplitude. Rank the situations according to the amplitude of the net wave on the string, greatest first.

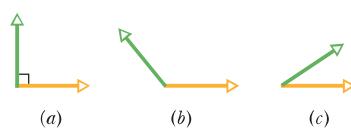


Fig. 16-27 Question 11.

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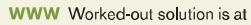
Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual



Number of dots indicates level of problem difficulty

Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

Worked-out solution is at

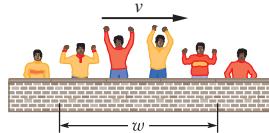


Interactive solution is at

<http://www.wiley.com/college/halliday>**sec. 16-5 The Speed of a Traveling Wave**

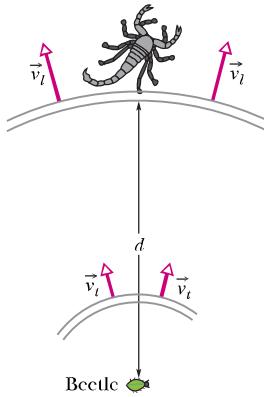
- 1** If a wave $y(x, t) = (6.0 \text{ mm}) \sin(kx + (600 \text{ rad/s})t + \phi)$ travels along a string, how much time does any given point on the string take to move between displacements $y = +2.0 \text{ mm}$ and $y = -2.0 \text{ mm}$?

- 2** A *human wave*. During sporting events within large, densely packed stadiums, spectators will send a wave (or pulse) around the stadium (Fig. 16-28). As the wave reaches a group of spectators, they stand with a cheer and then sit. At any instant, the width w of the wave is the distance from the leading edge (people are just about to stand) to the trailing edge (people have just sat down). Suppose a human wave travels a distance of 853 seats around a stadium in 39 s, with spectators requiring about 1.8 s to respond to the wave's passage by standing and then sitting. What are (a) the wave speed v (in seats per second) and (b) width w (in number of seats)?

**Fig. 16-28** Problem 2.

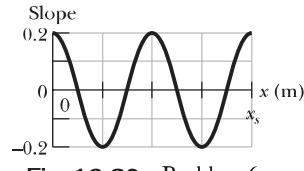
- 3** A wave has an angular frequency of 110 rad/s and a wavelength of 1.80 m . Calculate (a) the angular wave number and (b) the speed of the wave.

- 4** A sand scorpion can detect the motion of a nearby beetle (its prey) by the waves the motion sends along the sand surface (Fig. 16-29). The waves are of two types: transverse waves traveling at $v_t = 50 \text{ m/s}$ and longitudinal waves traveling at $v_l = 150 \text{ m/s}$. If a sudden motion sends out such waves, a scorpion can tell the distance of the beetle from the difference Δt in the arrival times of the waves at its leg nearest the beetle. If $\Delta t = 4.0 \text{ ms}$, what is the beetle's distance?

**Fig. 16-29** Problem 4.

- 5** A sinusoidal wave travels along a string. The time for a particular point to move from maximum displacement to zero is 0.170 s . What are the (a) period and (b) frequency? (c) The wavelength is 1.40 m ; what is the wave speed?

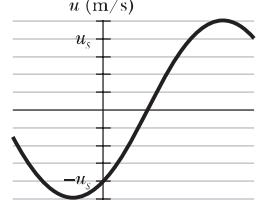
- 6** A sinusoidal wave travels along a string under tension. Figure 16-30 gives the slopes along the string at time $t = 0$. The scale of the x axis is set by $x_s = 0.80 \text{ m}$. What is the amplitude of the wave?

**Fig. 16-30** Problem 6.

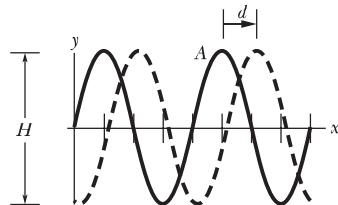
- 7** A transverse sinusoidal wave is moving along a string in the positive direction of an x axis with a speed of 80 m/s . At $t = 0$, the

string particle at $x = 0$ has a transverse displacement of 4.0 cm from its equilibrium position and is not moving. The maximum transverse speed of the string particle at $x = 0$ is 16 m/s . (a) What is the frequency of the wave? (b) What is the wavelength of the wave? If $y(x, t) = y_m \sin(kx \pm \omega t + \phi)$ is the form of the wave equation, what are (c) y_m , (d) k , (e) ω , (f) ϕ , and (g) the correct choice of sign in front of ω ?

- 8** Figure 16-31 shows the transverse velocity u versus time t of the point on a string at $x = 0$, as a wave passes through it. The scale on the vertical axis is set by $u_s = 4.0 \text{ m/s}$. The wave has the form $y(x, t) = y_m \sin(kx - \omega t + \phi)$. What is ϕ ? (Caution: A calculator does not always give the proper inverse trig function, so check your answer by substituting it and an assumed value of ω into $y(x, t)$ and then plotting the function.)

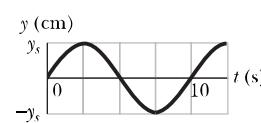
**Fig. 16-31** Problem 8.

- 9** A sinusoidal wave moving along a string is shown twice in Fig. 16-32, as crest A travels in the positive direction of an x axis by distance $d = 6.0 \text{ cm}$ in 4.0 ms . The tick marks along the axis are separated by 10 cm ; height $H = 6.00 \text{ mm}$. If the wave equation is of the form $y(x, t) = y_m \sin(kx \pm \omega t)$, what are (a) y_m , (b) k , (c) ω , and (d) the correct choice of sign in front of ω ?

**Fig. 16-32** Problem 9.

- 10** The equation of a transverse wave traveling along a very long string is $y = 6.0 \sin(0.020\pi x + 4.0\pi t)$, where x and y are expressed in centimeters and t is in seconds. Determine (a) the amplitude, (b) the wavelength, (c) the frequency, (d) the speed, (e) the direction of propagation of the wave, and (f) the maximum transverse speed of a particle in the string. (g) What is the transverse displacement at $x = 3.5 \text{ cm}$ when $t = 0.26 \text{ s}$?

- 11** A sinusoidal transverse wave of wavelength 20 cm travels along a string in the positive direction of an x axis. The displacement y of the string particle at $x = 0$ is given in Fig. 16-33 as a function of time t . The scale of the vertical axis is set by $y_s = 4.0 \text{ cm}$. The wave equation is to be in the form $y(x, t) =$

**Fig. 16-33** Problem 11.

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••11 $y_m \sin(kx \pm \omega t + \phi)$. (a) At $t = 0$, is a plot of y versus x in the shape of a positive sine function or a negative sine function? What are (b) y_m , (c) k , (d) ω , (e) ϕ , (f) the sign in front of ω , and (g) the speed of the wave? (h) What is the transverse velocity of the particle at $x = 0$ when $t = 5.0$ s?

••12 The function $y(x, t) = (15.0 \text{ cm}) \cos(\pi x - 15\pi t)$, with x in meters and t in seconds, describes a wave on a taut string. What is the transverse speed for a point on the string at an instant when that point has the displacement $y = +12.0 \text{ cm}$?

••13 **ILW** A sinusoidal wave of frequency 500 Hz has a speed of 350 m/s. (a) How far apart are two points that differ in phase by $\pi/3$ rad? (b) What is the phase difference between two displacements at a certain point at times 1.00 ms apart?

sec. 16-6 Wave Speed on a Stretched String

••14 The equation of a transverse wave on a string is

$$y = (2.0 \text{ mm}) \sin[(20 \text{ m}^{-1})x - (600 \text{ s}^{-1})t].$$

The tension in the string is 15 N. (a) What is the wave speed? (b) Find the linear density of this string in grams per meter.

••15 **SSM** **WWW** A stretched string has a mass per unit length of 5.00 g/cm and a tension of 10.0 N. A sinusoidal wave on this string has an amplitude of 0.12 mm and a frequency of 100 Hz and is traveling in the negative direction of an x axis. If the wave equation is of the form $y(x, t) = y_m \sin(kx \pm \omega t)$, what are (a) y_m , (b) k , (c) ω , and (d) the correct choice of sign in front of ω ?

••16 The speed of a transverse wave on a string is 170 m/s when the string tension is 120 N. To what value must the tension be changed to raise the wave speed to 180 m/s?

••17 The linear density of a string is $1.6 \times 10^{-4} \text{ kg/m}$. A transverse wave on the string is described by the equation

$$y = (0.021 \text{ m}) \sin[(2.0 \text{ m}^{-1})x + (30 \text{ s}^{-1})t].$$

What are (a) the wave speed and (b) the tension in the string?

••18 The heaviest and lightest strings on a certain violin have linear densities of 3.0 and 0.29 g/m. What is the ratio of the diameter of the heaviest string to that of the lightest string, assuming that the strings are of the same material?

••19 **SSM** What is the speed of a transverse wave in a rope of length 2.00 m and mass 60.0 g under a tension of 500 N?

••20 The tension in a wire clamped at both ends is doubled without appreciably changing the wire's length between the clamps. What is the ratio of the new to the old wave speed for transverse waves traveling along this wire?

••21 **ILW** A 100 g wire is held under a tension of 250 N with one end at $x = 0$ and the other at $x = 10.0 \text{ m}$. At time $t = 0$, pulse 1 is sent along the wire from the end at $x = 10.0 \text{ m}$. At time $t = 30.0 \text{ ms}$, pulse 2 is sent along the wire from the end at $x = 0$. At what position x do the pulses begin to meet?

••22 A sinusoidal wave is traveling on a string with speed 40 cm/s. The displacement of the particles of the string at $x = 10 \text{ cm}$ varies with time according to $y = (5.0 \text{ cm}) \sin[1.0 - (4.0 \text{ s}^{-1})t]$. The linear density of the string is 4.0 g/cm. What are (a) the frequency and (b) the wavelength of the wave? If the wave equation is of the form $y(x, t) = y_m \sin(kx \pm \omega t)$, what are (c) y_m , (d) k , (e) ω , and (f) the correct choice of sign in front of ω ? (g) What is the tension in the string?

••23 **SSM** **ILW** A sinusoidal transverse wave is traveling along a string in the negative direction of an x axis. Figure 16-34 shows a plot of the displacement as a function of position at time $t = 0$; the scale of the y axis is set by $y_s = 4.0 \text{ cm}$. The string tension is 3.6 N, and its linear density is 25 g/m. Find the (a) amplitude, (b) wavelength, (c) wave speed, and (d) period of the wave. (e) Find the maximum transverse speed of a particle in the string. If the wave is of the form $y(x, t) = y_m \sin(kx \pm \omega t + \phi)$, what are (f) k , (g) ω , (h) ϕ , and (i) the correct choice of sign in front of ω ?

••24 In Fig. 16-35a, string 1 has a linear density of 3.00 g/m, and string 2 has a linear density of 5.00 g/m. They are under tension due to the hanging block of mass $M = 500 \text{ g}$. Calculate the wave speed on (a) string 1 and (b) string 2. (*Hint:* When a string loops halfway around a pulley, it pulls on the pulley with a net force that is twice the tension in the string.) Next the block is divided into two blocks (with $M_1 + M_2 = M$) and the apparatus is rearranged as shown in Fig. 16-35b. Find (c) M_1 and (d) M_2 such that the wave speeds in the two strings are equal.

••25 A uniform rope of mass m and length L hangs from a ceiling. (a) Show that the speed of a transverse wave on the rope is a function of y , the distance from the lower end, and is given by $v = \sqrt{gy}$. (b) Show that the time a transverse wave takes to travel the length of the rope is given by $t = 2\sqrt{L/g}$.

sec. 16-7 Energy and Power of a Wave Traveling Along a String

••26 A string along which waves can travel is 2.70 m long and has a mass of 260 g. The tension in the string is 36.0 N. What must be the frequency of traveling waves of amplitude 7.70 mm for the average power to be 85.0 W?

••27 **GO** A sinusoidal wave is sent along a string with a linear density of 2.0 g/m. As it travels, the kinetic energies of the mass elements along the string vary. Figure 16-36a gives the

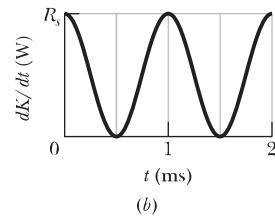
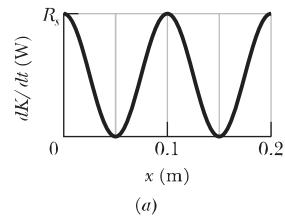


Fig. 16-36 Problem 27.

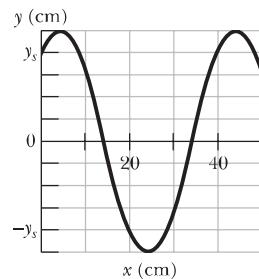


Fig. 16-34 Problem 23.

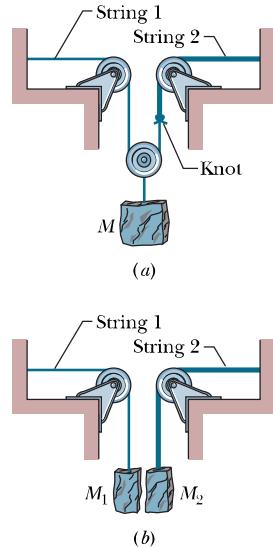


Fig. 16-35 Problem 24.

rate dK/dt at which kinetic energy passes through the string elements at a particular instant, plotted as a function of distance x along the string. Figure 16-36b is similar except that it gives the rate at which kinetic energy passes through a particular mass element (at a particular location), plotted as a function of time t . For both figures, the scale on the vertical (rate) axis is set by $R_s = 10 \text{ W}$. What is the amplitude of the wave?

sec. 16-8 The Wave Equation

- 28 Use the wave equation to find the speed of a wave given by

$$y(x, t) = (3.00 \text{ mm}) \sin[(4.00 \text{ m}^{-1})x - (7.00 \text{ s}^{-1})t].$$

- 29 Use the wave equation to find the speed of a wave given by

$$y(x, t) = (2.00 \text{ mm})[(20 \text{ m}^{-1})x - (4.0 \text{ s}^{-1})t]^{0.5}.$$

- 30 Use the wave equation to find the speed of a wave given in terms of the general function $h(x, t)$:

$$y(x, t) = (4.00 \text{ mm}) h[(30 \text{ m}^{-1})x + (6.0 \text{ s}^{-1})t].$$

sec. 16-10 Interference of Waves

- 31 **SSM** Two identical traveling waves, moving in the same direction, are out of phase by $\pi/2 \text{ rad}$. What is the amplitude of the resultant wave in terms of the common amplitude y_m of the two combining waves?

- 32 What phase difference between two identical traveling waves, moving in the same direction along a stretched string, results in the combined wave having an amplitude 1.50 times that of the common amplitude of the two combining waves? Express your answer in (a) degrees, (b) radians, and (c) wavelengths.

- 33 Two sinusoidal waves with the same amplitude of 9.00 mm and the same wavelength travel together along a string that is stretched along an x axis. Their resultant wave is shown twice in Fig. 16-37, as valley A travels in the negative direction of the x axis by distance $d = 56.0 \text{ cm}$ in 8.0 ms. The tick marks along the axis are separated by 10 cm, and height H is 8.0 mm. Let the equation for one wave be of the form $y(x, t) = y_m \sin(kx \pm \omega t + \phi_1)$, where $\phi_1 = 0$ and you must choose the correct sign in front of ω . For the equation for the other wave, what are (a) y_m , (b) k , (c) ω , (d) ϕ_2 , and (e) the sign in front of ω ?

- 34 A sinusoidal wave of angular frequency 1200 rad/s and amplitude 3.00 mm is sent along a cord with linear density 2.00 g/m and tension 1200 N. (a) What is the average rate at which energy is transported by the wave to the opposite end of the cord? (b) If, simultaneously, an identical wave travels along an adjacent, identical cord, what is the total average rate at which energy is transported to the opposite ends of the two cords by the waves? If, instead, those two waves are sent along the *same* cord simultaneously, what is the total average rate at which they transport energy when their phase difference is (c) 0, (d) $0.4\pi \text{ rad}$, and (e) $\pi \text{ rad}$?

sec. 16-11 Phasors

- 35 **SSM** Two sinusoidal waves of the same frequency travel in the same direction along a string. If $y_{m1} = 3.0 \text{ cm}$, $y_{m2} = 4.0 \text{ cm}$, $\phi_1 = 0$, and $\phi_2 = \pi/2 \text{ rad}$, what is the amplitude of the resultant wave?

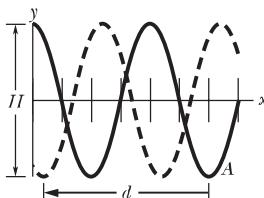


Fig. 16-37 Problem 33.

- 36 Four waves are to be sent along the same string, in the same direction:

$$y_1(x, t) = (4.00 \text{ mm}) \sin(2\pi x - 400\pi t)$$

$$y_2(x, t) = (4.00 \text{ mm}) \sin(2\pi x - 400\pi t + 0.7\pi)$$

$$y_3(x, t) = (4.00 \text{ mm}) \sin(2\pi x - 400\pi t + \pi)$$

$$y_4(x, t) = (4.00 \text{ mm}) \sin(2\pi x - 400\pi t + 1.7\pi).$$

What is the amplitude of the resultant wave?

- 37 **GO** These two waves travel along the same string:

$$y_1(x, t) = (4.60 \text{ mm}) \sin(2\pi x - 400\pi t)$$

$$y_2(x, t) = (5.60 \text{ mm}) \sin(2\pi x - 400\pi t + 0.80\pi \text{ rad}).$$

What are (a) the amplitude and (b) the phase angle (relative to wave 1) of the resultant wave? (c) If a third wave of amplitude 5.00 mm is also to be sent along the string in the same direction as the first two waves, what should be its phase angle in order to maximize the amplitude of the new resultant wave?

- 38 Two sinusoidal waves of the same frequency are to be sent in the same direction along a taut string. One wave has an amplitude of 5.0 mm, the other 8.0 mm. (a) What phase difference ϕ_1 between the two waves results in the smallest amplitude of the resultant wave? (b) What is that smallest amplitude? (c) What phase difference ϕ_2 results in the largest amplitude of the resultant wave? (d) What is that largest amplitude? (e) What is the resultant amplitude if the phase angle is $(\phi_1 - \phi_2)/2$?

- 39 Two sinusoidal waves of the same period, with amplitudes of 5.0 and 7.0 mm, travel in the same direction along a stretched string; they produce a resultant wave with an amplitude of 9.0 mm. The phase constant of the 5.0 mm wave is 0. What is the phase constant of the 7.0 mm wave?

sec. 16-13 Standing Waves and Resonance

- 40 Two sinusoidal waves with identical wavelengths and amplitudes travel in opposite directions along a string with a speed of 10 cm/s. If the time interval between instants when the string is flat is 0.50 s, what is the wavelength of the waves?

- 41 **SSM** A string fixed at both ends is 8.40 m long and has a mass of 0.120 kg. It is subjected to a tension of 96.0 N and set oscillating. (a) What is the speed of the waves on the string? (b) What is the longest possible wavelength for a standing wave? (c) Give the frequency of that wave.

- 42 A string under tension τ_i oscillates in the third harmonic at frequency f_3 , and the waves on the string have wavelength λ_3 . If the tension is increased to $\tau_f = 4\tau_i$ and the string is again made to oscillate in the third harmonic, what then are (a) the frequency of oscillation in terms of f_3 and (b) the wavelength of the waves in terms of λ_3 ?

- 43 **SSM WWW** What are (a) the lowest frequency, (b) the second lowest frequency, and (c) the third lowest frequency for standing waves on a wire that is 10.0 m long, has a mass of 100 g, and is stretched under a tension of 250 N?

- 44 A 125 cm length of string has mass 2.00 g and tension 7.00 N. (a) What is the wave speed for this string? (b) What is the lowest resonant frequency of this string?

- 45 **SSM ILW** A string that is stretched between fixed supports separated by 75.0 cm has resonant frequencies of 420 and 315 Hz, with no intermediate resonant frequencies. What are (a) the lowest resonant frequency and (b) the wave speed?

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- 46** String *A* is stretched between two clamps separated by distance *L*. String *B*, with the same linear density and under the same tension as string *A*, is stretched between two clamps separated by distance *4L*. Consider the first eight harmonics of string *B*. For which of these eight harmonics of *B* (if any) does the frequency match the frequency of (a) *A*'s first harmonic, (b) *A*'s second harmonic, and (c) *A*'s third harmonic?

- 47** One of the harmonic frequencies for a particular string under tension is 325 Hz. The next higher harmonic frequency is 390 Hz. What harmonic frequency is next higher after the harmonic frequency 195 Hz?

- 48**  If a transmission line in a cold climate collects ice, the increased diameter tends to cause vortex formation in a passing wind. The air pressure variations in the vortexes tend to cause the line to oscillate (*gallop*), especially if the frequency of the variations matches a resonant frequency of the line. In long lines, the resonant frequencies are so close that almost any wind speed can set up a resonant mode vigorous enough to pull down support towers or cause the line to *short out* with an adjacent line. If a transmission line has a length of 347 m, a linear density of 3.35 kg/m, and a tension of 65.2 MN, what are (a) the frequency of the fundamental mode and (b) the frequency difference between successive modes?

- 49**  A nylon guitar string has a linear density of 7.20 g/m and is under a tension of 150 N. The fixed supports are distance *D* = 90.0 cm apart. The string is oscillating in the standing wave pattern shown in Fig. 16-38. Calculate the (a) speed, (b) wavelength, and (c) frequency of the traveling waves whose superposition gives this standing wave.

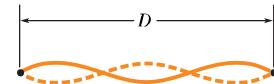


Fig. 16-38 Problem 49.

- 50** For a certain transverse standing wave on a long string, an antinode is at *x* = 0 and an adjacent node is at *x* = 0.10 m. The displacement *y*(*t*) of the string particle at *x* = 0 is shown in Fig. 16-39, where the scale of the *y* axis is set by *y*_s = 4.0 cm. When *t* = 0.50 s, what is the displacement of the string particle at (a) *x* = 0.20 m and (b) *x* = 0.30 m? What is the transverse velocity of the string particle at *x* = 0.20 m at (c) *t* = 0.50 s and (d) *t* = 1.0 s? (e) Sketch the standing wave at *t* = 0.50 s for the range *x* = 0 to *x* = 0.40 m.

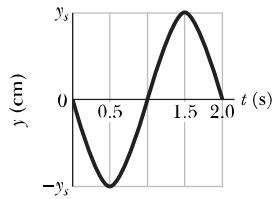


Fig. 16-39 Problem 50.

- 51**  Two waves are generated on a string of length 3.0 m to produce a three-loop standing wave with an amplitude of 1.0 cm. The wave speed is 100 m/s. Let the equation for one of the waves be of the form *y*(*x*, *t*) = *y*_m sin(*kx* + *ωt*). In the equation for the other wave, what are (a) *y*_m, (b) *k*, (c) *ω*, and (d) the sign in front of *ω*?

- 52** A rope, under a tension of 200 N and fixed at both ends, oscillates in a second-harmonic standing wave pattern. The displacement of the rope is given by

$$y = (0.10 \text{ m}) (\sin \pi x / 2) \sin 12\pi t,$$

- where *x* = 0 at one end of the rope, *x* is in meters, and *t* is in seconds. What are (a) the length of the rope, (b) the speed of the waves on the rope, and (c) the mass of the rope? (d) If the rope oscillates in a third-harmonic standing wave pattern, what will be the period of oscillation?

- 53** A string oscillates according to the equation

$$y' = (0.50 \text{ cm}) \sin \left[\left(\frac{\pi}{3} \text{ cm}^{-1} \right) x \right] \cos [(40\pi \text{ s}^{-1})t].$$

- What are the (a) amplitude and (b) speed of the two waves (identical except for direction of travel) whose superposition gives this oscillation? (c) What is the distance between nodes? (d) What is the transverse speed of a particle of the string at the position *x* = 1.5 cm when *t* = $\frac{9}{8}$ s?

- 54**  Two sinusoidal waves with the same amplitude and wavelength travel through each other along a string that is stretched along an *x* axis. Their resultant wave is shown twice in Fig. 16-40, as the antinode *A* travels from an extreme upward displacement to an extreme downward displacement in 6.0 ms. The tick marks along the axis are separated by 10 cm; height *H* is 1.80 cm. Let the equation for one of the two waves be of the form *y*(*x*, *t*) = *y*_m sin(*kx* + *ωt*). In the equation for the other wave, what are (a) *y*_m, (b) *k*, (c) *ω*, and (d) the sign in front of *ω*?

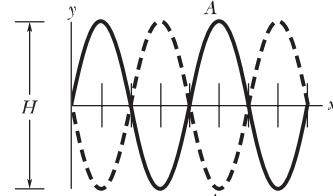


Fig. 16-40 Problem 54.

- 55** The following two waves are sent in opposite directions on a horizontal string so as to create a standing wave in a vertical plane:

$$y_1(x, t) = (6.00 \text{ mm}) \sin(4.00\pi x - 400\pi t)$$

$$y_2(x, t) = (6.00 \text{ mm}) \sin(4.00\pi x + 400\pi t),$$

- with *x* in meters and *t* in seconds. An antinode is located at point *A*. In the time interval that point takes to move from maximum upward displacement to maximum downward displacement, how far does each wave move along the string?

- 56** A standing wave pattern on a string is described by

$$y(x, t) = 0.040 (\sin 5\pi x)(\cos 40\pi t),$$

- where *x* and *y* are in meters and *t* is in seconds. For *x* ≥ 0, what is the location of the node with the (a) smallest, (b) second smallest, and (c) third smallest value of *x*? (d) What is the period of the oscillatory motion of any (nonnode) point? What are the (e) speed and (f) amplitude of the two traveling waves that interfere to produce this wave? For *t* ≥ 0, what are the (g) first, (h) second, and (i) third time that all points on the string have zero transverse velocity?

- 57** A generator at one end of a very long string creates a wave given by

$$y = (6.0 \text{ cm}) \cos \frac{\pi}{2} [(2.00 \text{ m}^{-1})x + (8.00 \text{ s}^{-1})t],$$

- and a generator at the other end creates the wave

$$y = (6.0 \text{ cm}) \cos \frac{\pi}{2} [(2.00 \text{ m}^{-1})x - (8.00 \text{ s}^{-1})t].$$

- Calculate the (a) frequency, (b) wavelength, and (c) speed of each wave. For *x* ≥ 0, what is the location of the node having the (d) smallest, (e) second smallest, and (f) third smallest value of *x*? For *x* ≥ 0, what is the location of the antinode having the (g) smallest, (h) second smallest, and (i) third smallest value of *x*?

- 58 GO** In Fig. 16-41, a string, tied to a sinusoidal oscillator at *P* and running over a support at *Q*, is stretched by a block of mass *m*. Separation *L* = 1.20 m, linear density μ = 1.6 g/m, and the oscillator frequency *f* = 120 Hz. The amplitude of the motion at *P* is small enough for that point to be considered a node. A node also exists at *Q*. (a) What mass *m* allows the oscillator to set up the fourth harmonic on the string? (b) What standing wave mode, if any, can be set up if *m* = 1.00 kg?

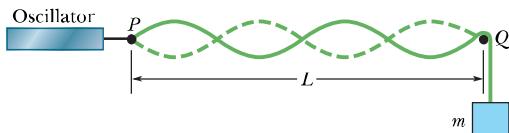


Fig. 16-41 Problems 58 and 60.

- 59** In Fig. 16-42, an aluminum wire, of length L_1 = 60.0 cm, cross-sectional area 1.00×10^{-2} cm 2 , and density 2.60 g/cm 3 , is joined to a steel wire, of density 7.80 g/cm 3 and the same cross-sectional area. The compound wire, loaded with a block of mass *m* = 10.0 kg, is arranged so that the distance L_2 from the joint to the supporting pulley is 86.6 cm. Transverse waves are set up on the wire by an external source of variable frequency; a node is located at the pulley. (a) Find the lowest frequency that generates a standing wave having the joint as one of the nodes. (b) How many nodes are observed at this frequency?

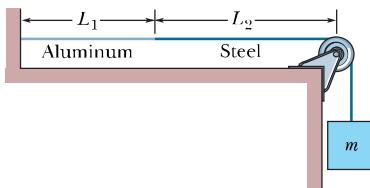


Fig. 16-42 Problem 59.

- 60 GO** In Fig. 16-41, a string, tied to a sinusoidal oscillator at *P* and running over a support at *Q*, is stretched by a block of mass *m*. The separation *L* between *P* and *Q* is 1.20 m, and the frequency *f* of the oscillator is fixed at 120 Hz. The amplitude of the motion at *P* is small enough for that point to be considered a node. A node also exists at *Q*. A standing wave appears when the mass of the hanging block is 286.1 g or 447.0 g, but not for any intermediate mass. What is the linear density of the string?

Additional Problems

- 61** In an experiment on standing waves, a string 90 cm long is attached to the prong of an electrically driven tuning fork that oscillates perpendicular to the length of the string at a frequency of 60 Hz. The mass of the string is 0.044 kg. What tension must the string be under (weights are attached to the other end) if it is to oscillate in four loops?

- 62** A sinusoidal transverse wave traveling in the positive direction of an *x* axis has an amplitude of 2.0 cm, a wavelength of 10 cm, and a frequency of 400 Hz. If the wave equation is of the form $y(x, t) = y_m \sin(kx \pm \omega t)$, what are (a) y_m , (b) k , (c) ω , and (d) the correct choice of sign in front of ω ? What are (e) the maximum transverse speed of a point on the cord and (f) the speed of the wave?

- 63** A wave has a speed of 240 m/s and a wavelength of 3.2 m. What are the (a) frequency and (b) period of the wave?

- 64** The equation of a transverse wave traveling along a string is

$$y = 0.15 \sin(0.79x - 13t),$$

in which *x* and *y* are in meters and *t* is in seconds. (a) What is the displacement *y* at *x* = 2.3 m, *t* = 0.16 s? A second wave is to be added to the first wave to produce standing waves on the string. If the wave equation for the second wave is of the form $y(x, t) = y_m \sin(kx \pm \omega t)$, what are (b) y_m , (c) k , (d) ω , and (e) the correct choice of sign in front of ω for this second wave? (f) What is the displacement of the resultant standing wave at *x* = 2.3 m, *t* = 0.16 s?

- 65** The equation of a transverse wave traveling along a string is

$$y = (2.0 \text{ mm}) \sin[(20 \text{ m}^{-1})x - (600 \text{ s}^{-1})t].$$

Find the (a) amplitude, (b) frequency, (c) velocity (including sign), and (d) wavelength of the wave. (e) Find the maximum transverse speed of a particle in the string.

- 66** Figure 16-43 shows the displacement *y* versus time *t* of the point on a string at *x* = 0, as a wave passes through that point. The scale of the *y* axis is set by y_s = 6.0 mm. The wave is given by $y(x, t) = y_m \sin(kx - \omega t + \phi)$. What is ϕ ? (Caution: A calculator does not always give the proper inverse trig function, so check your answer by substituting it and an assumed value of ω into $y(x, t)$ and then plotting the function.)

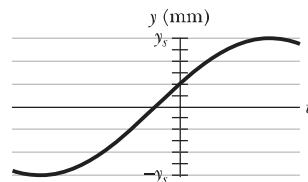


Fig. 16-43 Problem 66.

- 67** Two sinusoidal waves, identical except for phase, travel in the same direction along a string, producing the net wave $y'(x, t) = (3.0 \text{ mm}) \sin(20x - 4.0t + 0.820 \text{ rad})$, with *x* in meters and *t* in seconds. What are (a) the wavelength λ of the two waves, (b) the phase difference between them, and (c) their amplitude y_m ?

- 68** A single pulse, given by $h(x - 5.0t)$, is shown in Fig. 16-44 for *t* = 0. The scale of the vertical axis is set by h_s = 2. Here *x* is in centimeters and *t* is in seconds. What are the (a) speed and (b) direction of travel of the pulse? (c) Plot $h(x - 5t)$ as a function of *x* for *t* = 2 s. (d) Plot $h(x - 5t)$ as a function of *t* for *x* = 10 cm.

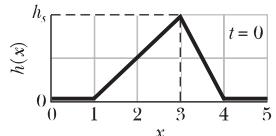


Fig. 16-44 Problem 68.

- 69 SSM** Three sinusoidal waves of the same frequency travel along a string in the positive direction of an *x* axis. Their amplitudes are y_1 , $y_1/2$, and $y_1/3$, and their phase constants are 0, $\pi/2$, and π , respectively. What are the (a) amplitude and (b) phase constant of the resultant wave? (c) Plot the wave form of the resultant wave at *t* = 0, and discuss its behavior as *t* increases.

- 70 GO** Figure 16-45 shows transverse acceleration a_y versus time *t* of the point on a string at *x* = 0, as a wave in the form of $y(x, t) = y_m \sin(kx - \omega t + \phi)$ passes through that point. The scale of the vertical axis is set by a_s = 400 m/s 2 . What is ϕ ? (Caution: A calculator does not

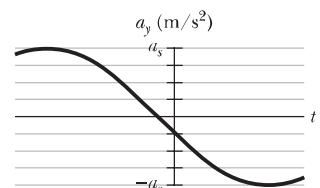


Fig. 16-45 Problem 70.

always give the proper inverse trig function, so check your answer by substituting it and an assumed value of ω into $y(x, t)$ and then plotting the function.)

- 71** A transverse sinusoidal wave is generated at one end of a long, horizontal string by a bar that moves up and down through a distance of 1.00 cm. The motion is continuous and is repeated regularly 120 times per second. The string has linear density 120 g/m and is kept under a tension of 90.0 N. Find the maximum value of (a) the transverse speed u and (b) the transverse component of the tension τ .

(c) Show that the two maximum values calculated above occur at the same phase values for the wave. What is the transverse displacement y of the string at these phases? (d) What is the maximum rate of energy transfer along the string? (e) What is the transverse displacement y when this maximum transfer occurs? (f) What is the minimum rate of energy transfer along the string? (g) What is the transverse displacement y when this minimum transfer occurs?

- 72** Two sinusoidal 120 Hz waves, of the same frequency and amplitude, are to be sent in the positive direction of an x axis that is directed along a cord under tension. The waves can be sent in phase, or they can be phase-shifted. Figure 16-46 shows the amplitude y' of the resulting wave versus the distance of the shift (how far one wave is shifted from the other wave). The scale of the vertical axis is set by $y'_s = 6.0$ mm. If the equations for the two waves are of the form $y(x, t) = y_m \sin(kx \pm \omega t)$, what are (a) y_m , (b) k , (c) ω , and (d) the correct choice of sign in front of ω ?

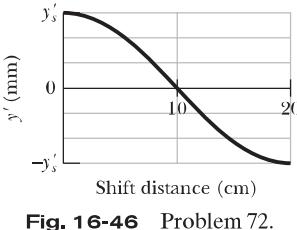


Fig. 16-46 Problem 72.

- 73** At time $t = 0$ and at position $x = 0$ m along a string, a traveling sinusoidal wave with an angular frequency of 440 rad/s has displacement $y = +4.5$ mm and transverse velocity $u = -0.75$ m/s. If the wave has the general form $y(x, t) = y_m \sin(kx - \omega t + \phi)$, what is phase constant ϕ ?

- 74** Energy is transmitted at rate P_1 by a wave of frequency f_1 on a string under tension τ_1 . What is the new energy transmission rate P_2 in terms of P_1 (a) if the tension is increased to $\tau_2 = 4\tau_1$ and (b) if, instead, the frequency is decreased to $f_2 = f_1/2$?

- 75** (a) What is the fastest transverse wave that can be sent along a steel wire? For safety reasons, the maximum tensile stress to which steel wires should be subjected is 7.00×10^8 N/m². The density of steel is 7800 kg/m³. (b) Does your answer depend on the diameter of the wire?

- 76** A standing wave results from the sum of two transverse traveling waves given by

$$y_1 = 0.050 \cos(\pi x - 4\pi t)$$

and

$$y_2 = 0.050 \cos(\pi x + 4\pi t).$$

- where x , y_1 , and y_2 are in meters and t is in seconds. (a) What is the smallest positive value of x that corresponds to a node? Beginning at $t = 0$, what is the value of the (b) first, (c) second, and (d) third time the particle at $x = 0$ has zero velocity?

- 77 SSM** The type of rubber band used inside some baseballs and golf balls obeys Hooke's law over a wide range of elongation of the band. A segment of this material has an unstretched length ℓ and a mass m . When a force F is applied, the band stretches an additional

length $\Delta\ell$. (a) What is the speed (in terms of m , $\Delta\ell$, and the spring constant k) of transverse waves on this stretched rubber band? (b) Using your answer to (a), show that the time required for a transverse pulse to travel the length of the rubber band is proportional to $1/\sqrt{\Delta\ell}$ if $\Delta\ell \ll \ell$ and is constant if $\Delta\ell \gg \ell$.

- 78** The speed of electromagnetic waves (which include visible light, radio, and x rays) in vacuum is 3.0×10^8 m/s. (a) Wavelengths of visible light waves range from about 400 nm in the violet to about 700 nm in the red. What is the range of frequencies of these waves? (b) The range of frequencies for shortwave radio (for example, FM radio and VHF television) is 1.5 to 300 MHz. What is the corresponding wavelength range? (c) X-ray wavelengths range from about 5.0 nm to about 1.0×10^{-2} nm. What is the frequency range for x rays?

- 79 SSM** A 1.50 m wire has a mass of 8.70 g and is under a tension of 120 N. The wire is held rigidly at both ends and set into oscillation. (a) What is the speed of waves on the wire? What is the wavelength of the waves that produce (b) one-loop and (c) two-loop standing waves? What is the frequency of the waves that produce (d) one-loop and (e) two-loop standing waves?

- 80** When played in a certain manner, the lowest resonant frequency of a certain violin string is concert A (440 Hz). What is the frequency of the (a) second and (b) third harmonic of the string?

- 81** A sinusoidal transverse wave traveling in the negative direction of an x axis has an amplitude of 1.00 cm, a frequency of 550 Hz, and a speed of 330 m/s. If the wave equation is of the form $y(x, t) = y_m \sin(kx \pm \omega t)$, what are (a) y_m , (b) ω , (c) k , and (d) the correct choice of sign in front of ω ?

- 82** Two sinusoidal waves of the same wavelength travel in the same direction along a stretched string. For wave 1, $y_m = 3.0$ mm and $\phi = 0$; for wave 2, $y_m = 5.0$ mm and $\phi = 70^\circ$. What are the (a) amplitude and (b) phase constant of the resultant wave?

- 83 SSM** A sinusoidal transverse wave of amplitude y_m and wavelength λ travels on a stretched cord. (a) Find the ratio of the maximum particle speed (the speed with which a single particle in the cord moves transverse to the wave) to the wave speed. (b) Does this ratio depend on the material of which the cord is made?

- 84** Oscillation of a 600 Hz tuning fork sets up standing waves in a string clamped at both ends. The wave speed for the string is 400 m/s. The standing wave has four loops and an amplitude of 2.0 mm. (a) What is the length of the string? (b) Write an equation for the displacement of the string as a function of position and time.

- 85** A 120 cm length of string is stretched between fixed supports. What are the (a) longest, (b) second longest, and (c) third longest wavelength for waves traveling on the string if standing waves are to be set up? (d) Sketch those standing waves.

- 86** (a) Write an equation describing a sinusoidal transverse wave traveling on a cord in the positive direction of a y axis with an angular wave number of 60 cm^{-1} , a period of 0.20 s, and an amplitude of 3.0 mm. Take the transverse direction to be the z direction. (b) What is the maximum transverse speed of a point on the cord?

- 87** A wave on a string is described by

$$y(x, t) = 15.0 \sin(\pi x/8 - 4\pi t),$$

- where x and y are in centimeters and t is in seconds. (a) What is the transverse speed for a point on the string at $x = 6.00$ cm when $t = 0.250$ s? (b) What is the maximum transverse speed of

any point on the string? (c) What is the magnitude of the transverse acceleration for a point on the string at $x = 6.00$ cm when $t = 0.250$ s? (d) What is the magnitude of the maximum transverse acceleration for any point on the string?

88 **Body armor.** When a high-speed projectile such as a bullet or bomb fragment strikes modern body armor, the fabric of the armor stops the projectile and prevents penetration by quickly spreading the projectile's energy over a large area. This spreading is done by longitudinal and transverse pulses that move *radially* from the impact point, where the projectile pushes a cone-shaped dent into the fabric. The longitudinal pulse, racing along the fibers of the fabric at speed v_l ahead of the denting, causes the fibers to thin and stretch, with material flowing radially inward into the dent. One such radial fiber is shown in Fig. 16-47a. Part of the projectile's energy goes into this motion and stretching. The transverse pulse, moving at a slower speed v_t , is due to the denting. As the projectile increases the dent's depth, the dent increases in radius, causing the material in the fibers to move in the same direction as the projectile (perpendicular to the transverse pulse's direction of travel). The rest of the projectile's energy goes into this motion. All the energy that does not eventually go into permanently deforming the fibers ends up as thermal energy.

Figure 16-47b is a graph of speed v versus time t for a bullet of mass 10.2 g fired from a .38 Special revolver directly into body armor. The scales of the vertical and horizontal axes are set by $v_s = 300$ m/s and $t_s = 40.0 \mu\text{s}$. Take $v_l = 2000$ m/s, and assume that the half-angle θ of the conical dent is 60° . At the end of the collision, what are the radii of (a) the thinned region and (b) the dent (assuming that the person wearing the armor remains stationary)?

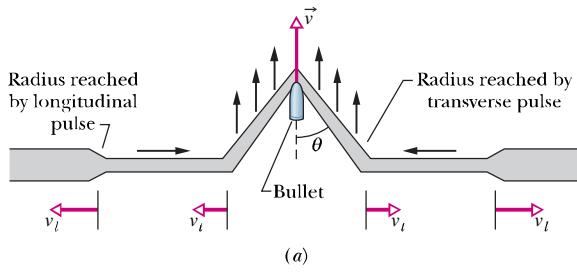


Fig. 16-47 Problem 88.

89 Two waves are described by

$$y_1 = 0.30 \sin[\pi(5x - 200)t]$$

$$\text{and} \quad y_2 = 0.30 \sin[\pi(5x - 200t) + \pi/3],$$

where y_1, y_2 , and x are in meters and t is in seconds. When these two waves are combined, a traveling wave is produced. What are the (a) amplitude, (b) wave speed, and (c) wavelength of that traveling wave?

90 A certain transverse sinusoidal wave of wavelength 20 cm is moving in the positive direction of an x axis. The transverse velocity of the particle at $x = 0$ as a function of time is shown in Fig. 16-48, where the scale of the vertical axis is set by $u_s = 5.0$ cm/s. What are the (a) wave speed, (b) amplitude, and (c) frequency? (d) Sketch the wave between $x = 0$ and $x = 20$ cm at $t = 2.0$ s.

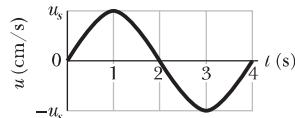


Fig. 16-48 Problem 90.

91 SSM In a demonstration, a 1.2 kg horizontal rope is fixed in place at its two ends ($x = 0$ and $x = 2.0$ m) and made to oscillate up and down in the fundamental mode, at frequency 5.0 Hz. At $t = 0$, the point at $x = 1.0$ m has zero displacement and is moving upward in the positive direction of a y axis with a transverse velocity of 5.0 m/s. What are (a) the amplitude of the motion of that point and (b) the tension in the rope? (c) Write the standing wave equation for the fundamental mode.

92 Two waves,

$$y_1 = (2.50 \text{ mm}) \sin[(25.1 \text{ rad/m})x - (440 \text{ rad/s})t]$$

$$\text{and} \quad y_2 = (1.50 \text{ mm}) \sin[(25.1 \text{ rad/m})x + (440 \text{ rad/s})t],$$

travel along a stretched string. (a) Plot the resultant wave as a function of t for $x = 0, \lambda/8, \lambda/4, 3\lambda/8$, and $\lambda/2$, where λ is the wavelength. The graphs should extend from $t = 0$ to a little over one period. (b) The resultant wave is the superposition of a standing wave and a traveling wave. In which direction does the traveling wave move? (c) How can you change the original waves so the resultant wave is the superposition of standing and traveling waves with the same amplitudes as before but with the traveling wave moving in the opposite direction? Next, use your graphs to find the place at which the oscillation amplitude is (d) maximum and (e) minimum. (f) How is the maximum amplitude related to the amplitudes of the original two waves? (g) How is the minimum amplitude related to the amplitudes of the original two waves?

93 A traveling wave on a string is described by

$$y = 2.0 \sin\left[2\pi\left(\frac{t}{0.40} + \frac{x}{80}\right)\right],$$

where x and y are in centimeters and t is in seconds. (a) For $t = 0$, plot y as a function of x for $0 \leq x \leq 160$ cm. (b) Repeat (a) for $t = 0.05$ s and $t = 0.10$ s. From your graphs, determine (c) the wave speed and (d) the direction in which the wave is traveling.