CHAPTER

24 ELECTRIC POTENTIAL

24-1 WHAT IS PHYSICS?

One goal of physics is to identify basic forces in our world, such as the electric force we discussed in Chapter 21. A related goal is to determine whether a force is conservative—that is, whether a potential energy can be associated with it. The motivation for associating a potential energy with a force is that we can then apply the principle of the conservation of mechanical energy to closed systems involving the force. This extremely powerful principle allows us to calculate the results of experiments for which force calculations alone would be very difficult. Experimentally, physicists and engineers discovered that the electric force is conservative and thus has an associated electric potential energy. In this chapter we first define this type of potential energy and then put it to use.

24-2 Electric Potential Energy

When an electrostatic force acts between two or more charged particles within a system of particles, we can assign an **electric potential energy** U to the system. If the system changes its configuration from an initial state i to a different final state f, the electrostatic force does work W on the particles. From Eq. 8-1, we then know that the resulting change ΔU in the potential energy of the system is

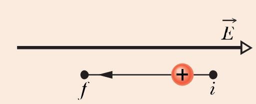
$$\Delta U = U_f - U_i = -W. \tag{24-1}$$

As with other conservative forces, the work done by the electrostatic force is *path independent.* Suppose a charged particle within the system moves from point i to point f while an electrostatic force between it and the rest of the system acts on it. Provided the rest of the system does not change, the work W done by the force on the particle is the same for *all* paths between points i and f.

For convenience, we usually take the *reference configuration* of a system of charged particles to be that in which the particles are all infinitely separated from one another. Also, we usually set the corresponding *reference potential energy* to be zero. Suppose that several charged particles come together from initially infinite separations (state i) to form a system of neighboring particles (state f). Let the initial potential energy U_i be zero, and let W_∞ represent the work done by the electrostatic forces between the particles during the move in from infinity. Then from Eq. 24-1, the final potential energy U of the system is

$$U = -W_\infty. \tag{24-2}$$

CHECKPOINT 1



In the figure, a proton moves from point i to point f in a uniform electric field directed as shown. (a) Does the electric field do positive or negative work on the proton? (b) Does the electric potential energy of the proton increase or decrease?

Sample Problem

Work and potential energy in an electric field

Electrons are continually being knocked out of air molecules in the atmosphere by cosmic-ray particles coming in from space. Once released, each electron experiences an electrostatic force $\vec{F}$ due to the electric field $\vec{E}$ that is produced in the atmosphere by charged particles already on Earth. Near Earth's surface the electric field has the magnitude $E = 150$ N/C and is directed downward. What is the change ΔU in the electric potential energy of a released electron when the electrostatic force causes it to move vertically upward through a distance $d = 520$ m (Fig. 24-1)?

KEY IDEAS

(1) The change ΔU in the electric potential energy of the electron is related to the work W done on the electron by the electric field. Equation 24-1 ($\Delta U = -W$) gives the relation.

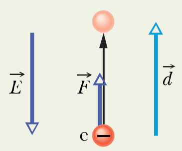


Fig. 24-1 An electron in the atmosphere is moved upward through displacement $\vec{d}$ by an electrostatic force $\vec{F}$ due to an electric field $\vec{E}$.

(2) The work done by a constant force $\vec{F}$ on a particle undergoing a displacement $\vec{d}$ is

$$W = \vec{F} \cdot \vec{d}. \tag{24-3}$$

(3) The electrostatic force and the electric field are related by the force equation $\vec{F} = q\vec{E}$, where here q is the charge of an electron ($= -1.6 \times 10^{-19}$ C).

Calculations: Substituting for $\vec{F}$ in Eq. 24-3 and taking the dot product yield

$$W = q\vec{E} \cdot \vec{d} = qEd \cos \theta, \tag{24-4}$$

where θ is the angle between the directions of $\vec{E}$ and $\vec{d}$. The field $\vec{E}$ is directed downward and the displacement $\vec{d}$ is directed upward; so $\theta = 180°$. Substituting this and other data into Eq. 24-4, we find

$$W = (-1.6 \times 10^{-19}\text{ C})(150\text{ N/C})(520\text{ m}) \cos 180°$$
$$= 1.2 \times 10^{-14}\text{ J}.$$

Equation 24-1 then yields

$$\Delta U = -W = -1.2 \times 10^{-14}\text{ J}. \qquad \text{(Answer)}$$

This result tells us that during the 520 m ascent, the electric potential energy of the electron *decreases* by 1.2×10^{-14} J.

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24-3 Electric Potential

The potential energy of a charged particle in an electric field depends on the charge magnitude. However, the potential energy *per unit charge* has a unique value at any point in an electric field.

For an example of this, suppose we place a test particle of positive charge 1.60×10^{-19} C at a point in an electric field where the particle has an electric potential energy of 2.40×10^{-17} J. Then the potential energy per unit charge is

$$\frac{2.40 \times 10^{-17}\text{ J}}{1.60 \times 10^{-19}\text{ C}} = 150\text{ J/C}.$$

Next, suppose we replace that test particle with one having twice as much positive charge, 3.20×10^{-19} C. We would find that the second particle has an electric potential energy of 4.80×10^{-17} J, twice that of the first particle. However, the potential energy per unit charge would be the same, still 150 J/C.

Thus, the potential energy per unit charge, which can be symbolized as U/q, is independent of the charge q of the particle we happen to use and is *characteristic only of the electric field* we are investigating. The potential energy per unit charge at a point in an electric field is called the **electric potential** V (or simply the **potential**) at that point. Thus,

$$V = \frac{U}{q}. \tag{24-5}$$

Note that electric potential is a scalar, not a vector.

The *electric potential difference* ΔV between any two points i and f in an electric field is equal to the difference in potential energy per unit charge between the two points:

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}. \tag{24-6}$$

Using Eq. 24-1 to substitute $-W$ for ΔU in Eq. 24-6, we can define the potential difference between points i and f as

$$\Delta V = V_f - V_i = -\frac{W}{q} \quad \text{(potential difference defined).} \tag{24-7}$$

The potential difference between two points is thus the negative of the work done by the electrostatic force to move a unit charge from one point to the other. A potential difference can be positive, negative, or zero, depending on the signs and magnitudes of q and W.

If we set $U_i = 0$ at infinity as our reference potential energy, then by Eq. 24-5, the electric potential V must also be zero there. Then from Eq. 24-7, we can define the electric potential at any point in an electric field to be

$$V = -\frac{W_\infty}{q} \quad \text{(potential defined),} \tag{24-8}$$

where W_∞ is the work done by the electric field on a charged particle as that particle moves in from infinity to point f. A potential V can be positive, negative, or zero, depending on the signs and magnitudes of q and W_∞.

The SI unit for potential that follows from Eq. 24-8 is the joule per coulomb. This combination occurs so often that a special unit, the *volt* (abbreviated V), is used to represent it. Thus,

$$1 \text{ volt} = 1 \text{ joule per coulomb.} \tag{24-9}$$

This new unit allows us to adopt a more conventional unit for the electric field $\vec{E}$, which we have measured up to now in newtons per coulomb. With two unit conversions, we obtain

$$\begin{aligned} 1 \text{ N/C} &= \left(1\frac{\text{N}}{\text{C}}\right)\left(\frac{1 \text{ V}\cdot\text{C}}{1 \text{ J}}\right)\left(\frac{1 \text{ J}}{1 \text{ N}\cdot\text{m}}\right) \\ &= 1 \text{ V/m}. \end{aligned} \tag{24-10}$$

The conversion factor in the second set of parentheses comes from Eq. 24-9; that in the third set of parentheses is derived from the definition of the joule. From now on, we shall express values of the electric field in volts per meter rather than in newtons per coulomb.

Finally, we can now define an energy unit that is a convenient one for energy measurements in the atomic and subatomic domain: One *electron-volt* (eV) is the energy equal to the work required to move a single elementary charge e, such as that of the electron or the proton, through a potential difference of exactly one volt. Equation 24-7 tells us that the magnitude of this work is $q\,\Delta V$; so

$$\begin{aligned} 1 \text{ eV} &= e(1 \text{ V}) \\ &= (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) = 1.60 \times 10^{-19} \text{ J}. \end{aligned}$$

Work Done by an Applied Force

Suppose we move a particle of charge q from point i to point f in an electric field by applying a force to it. During the move, our applied force does work W_{app} on the charge while the electric field does work W on it. By the work–kinetic energy theorem of Eq. 7-10, the change ΔK in the kinetic energy of the particle is

$$\Delta K = K_f - K_i = W_{\text{app}} + W. \tag{24-11}$$

Now suppose the particle is stationary before and after the move. Then K_f and K_i are both zero, and Eq. 24-11 reduces to

$$W_{\text{app}} = -W. \tag{24-12}$$

In words, the work W_{app} done by our applied force during the move is equal to the negative of the work W done by the electric field—provided there is no change in kinetic energy.

By using Eq. 24-12 to substitute W_{app} into Eq. 24-1, we can relate the work done by our applied force to the change in the potential energy of the particle during the move. We find

$$\Delta U = U_f - U_i = W_{\text{app}}. \tag{24-13}$$

By similarly using Eq. 24-12 to substitute W_{app} into Eq. 24-7, we can relate our work W_{app} to the electric potential difference ΔV between the initial and final locations of the particle. We find

$$W_{\text{app}} = q\,\Delta V. \tag{24-14}$$

W_{app} can be positive, negative, or zero depending on the signs and magnitudes of q and ΔV.

CHECKPOINT 2

In the figure of Checkpoint 1, we move the proton from point i to point f in a uniform electric field directed as shown. (a) Does our force do positive or negative work? (b) Does the proton move to a point of higher or lower potential?

24-4 Equipotential Surfaces

Adjacent points that have the same electric potential form an **equipotential surface,** which can be either an imaginary surface or a real, physical surface. No net work W is done on a charged particle by an electric field when the particle moves between two points i and f on the same equipotential surface. This follows from Eq. 24-7, which tells us that W must be zero if $V_f = V_i$. Because of the path independence of work (and thus of potential energy and potential), $W = 0$ for *any* path connecting points i and f on a given equipotential surface regardless of whether that path lies entirely on that surface.

Figure 24-2 shows a *family* of equipotential surfaces associated with the electric field due to some distribution of charges. The work done by the electric field

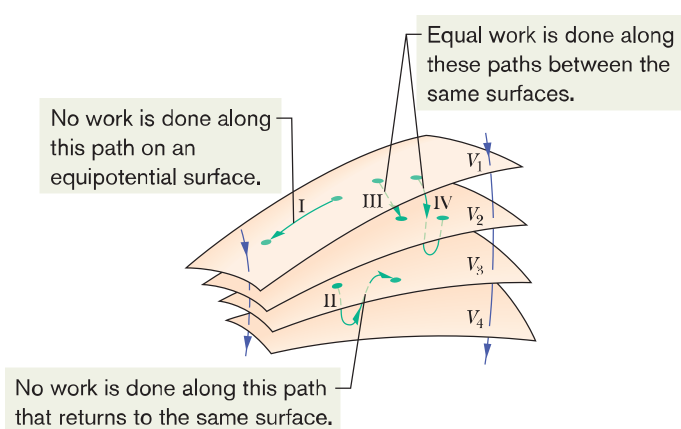


Fig. 24-2 Portions of four equipotential surfaces at electric potentials $V_1 = 100$ V, $V_2 = 80$ V, $V_3 = 60$ V, and $V_4 = 40$ V. Four paths along which a test charge may move are shown. Two electric field lines are also indicated.

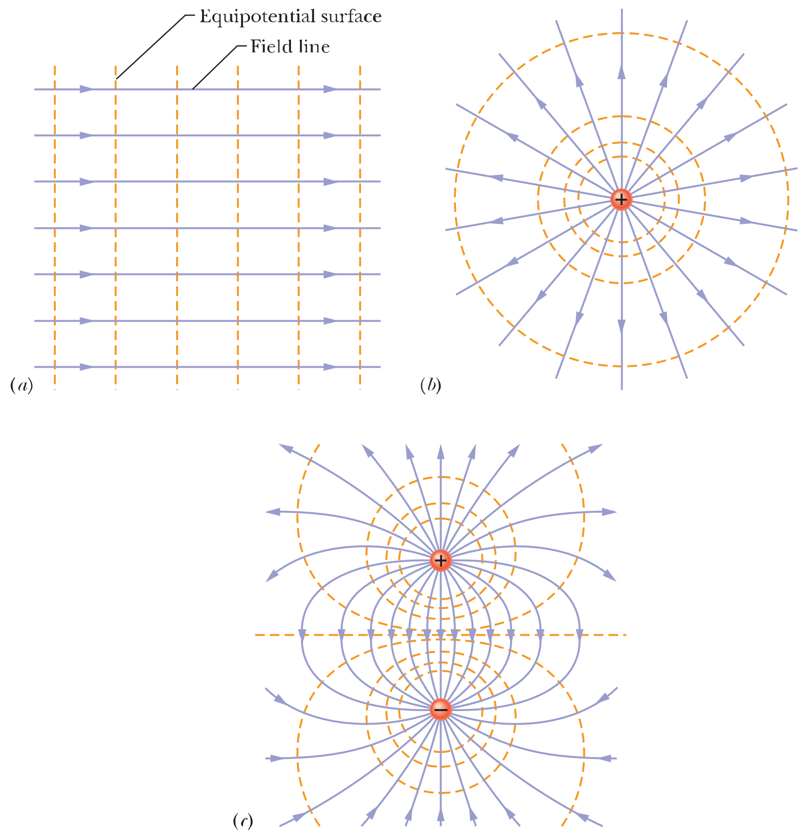


Fig. 24-3 Electric field lines (purple) and cross sections of equipotential surfaces (gold) for (*a*) a uniform electric field, (*b*) the field due to a point charge, and (*c*) the field due to an electric dipole.

on a charged particle as the particle moves from one end to the other of paths I and II is zero because each of these paths begins and ends on the same equipotential surface and thus there is no net change in potential. The work done as the charged particle moves from one end to the other of paths III and IV is not zero but has the same value for both these paths because the initial and final potentials are identical for the two paths; that is, paths III and IV connect the same pair of equipotential surfaces.

From symmetry, the equipotential surfaces produced by a point charge or a spherically symmetrical charge distribution are a family of concentric spheres. For a uniform electric field, the surfaces are a family of planes perpendicular to the field lines. In fact, equipotential surfaces are always perpendicular to electric field lines and thus to $\vec{E}$, which is always tangent to these lines. If $\vec{E}$ were *not* perpendicular to an equipotential surface, it would have a component lying along that surface. This component would then do work on a charged particle as it moved along the surface. However, by Eq. 24-7 work cannot be done if the surface is truly an equipotential surface; the only possible conclusion is that $\vec{E}$ must be everywhere perpendicular to the surface. Figure 24-3 shows electric field lines and cross sections of the equipotential surfaces for a uniform electric field and for the field associated with a point charge and with an electric dipole.

24-5 Calculating the Potential from the Field

We can calculate the potential difference between any two points i and f in an electric field if we know the electric field vector $\vec{E}$ all along any path connecting those points. To make the calculation, we find the work done on a positive test charge by the field as the charge moves from i to f, and then use Eq. 24-7.

Consider an arbitrary electric field, represented by the field lines in Fig. 24-4, and a positive test charge q_0 that moves along the path shown from point i to point f. At any point on the path, an electrostatic force $q_0\vec{E}$ acts on the charge as it moves through a differential displacement $d\vec{s}$. From Chapter 7, we know that the differential work dW done on a particle by a force $\vec{F}$ during a displacement $d\vec{s}$ is given by the dot product of the force and the displacement:

$$dW = \vec{F} \cdot d\vec{s}. \tag{24-15}$$

For the situation of Fig. 24-4, $\vec{F} = q_0\vec{E}$ and Eq. 24-15 becomes

$$dW = q_0\vec{E} \cdot d\vec{s}. \tag{24-16}$$

To find the total work W done on the particle by the field as the particle moves from point i to point f, we sum—via integration—the differential works done on the charge as it moves through all the displacements $d\vec{s}$ along the path:

$$W = q_0 \int_i^f \vec{E} \cdot d\vec{s}. \tag{24-17}$$

If we substitute the total work W from Eq. 24-17 into Eq. 24-7, we find

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}. \tag{24-18}$$

Thus, the potential difference $V_f - V_i$ between any two points i and f in an electric field is equal to the negative of the *line integral* (meaning the integral along a particular path) of $\vec{E} \cdot d\vec{s}$ from i to f. However, because the electrostatic force is conservative, all paths (whether easy or difficult to use) yield the same result.

Equation 24-18 allows us to calculate the difference in potential between any two points in the field. If we set potential $V_i = 0$, then Eq. 24-18 becomes

$$V = -\int_i^f \vec{E} \cdot d\vec{s}, \tag{24-19}$$

in which we have dropped the subscript f on V_f. Equation 24-19 gives us the potential V at any point f in the electric field *relative to the zero potential* at point i. If we let point i be at infinity, then Eq. 24-19 gives us the potential V at any point f relative to the zero potential at infinity.

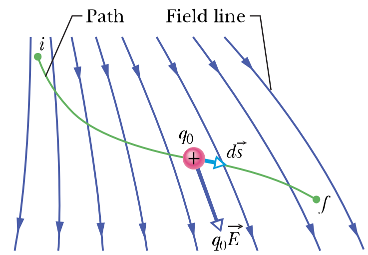
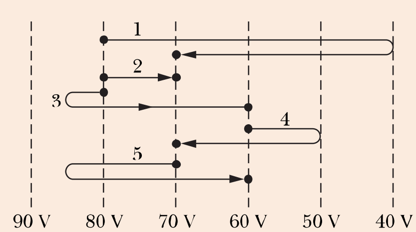


Fig. 24-4 A test charge q_0 moves from point i to point f along the path shown in a nonuniform electric field. During a displacement $d\vec{s}$, an electrostatic force $q_0\vec{E}$ acts on the test charge. This force points in the direction of the field line at the location of the test charge.

CHECKPOINT 3

The figure here shows a family of parallel equipotential surfaces (in cross section) and five paths along which we shall move an electron from one surface to another. (a) What is the direction of the electric field associated with the surfaces? (b) For each path, is the work we do positive, negative, or zero? (c) Rank the paths according to the work we do, greatest first.



Sample Problem

Finding the potential change from the electric field

(a) Figure 24-5*a* shows two points i and f in a uniform electric field $\vec{E}$. The points lie on the same electric field line (not shown) and are separated by a distance d. Find the potential difference $V_f - V_i$ by moving a positive test charge q_0 from i to f along the path shown, which is parallel to the field direction.

KEY IDEA

We can find the potential difference between any two points in an electric field by integrating $\vec{E}\cdot d\vec{s}$ along a path connecting those two points according to Eq. 24-18.

Calculations: We begin by mentally moving a test charge q_0 along that path, from initial point i to final point f. As we move such a test charge along the path in Fig. 24-5*a*, its differential displacement $d\vec{s}$ always has the same direction as $\vec{E}$. Thus, the angle θ between $\vec{E}$ and $d\vec{s}$ is zero and the dot product in Eq. 24-18 is

$$\vec{E}\cdot d\vec{s} = E\,ds\cos\theta = E\,ds. \tag{24-20}$$

Equations 24-18 and 24-20 then give us

$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s} = -\int_i^f E\,ds. \tag{24-21}$$

Since the field is uniform, E is constant over the path and can be moved outside the integral, giving us

$$V_f - V_i = -E\int_i^f ds = -Ed, \qquad \text{(Answer)}$$

in which the integral is simply the length d of the path. The minus sign in the result shows that the potential at point f in Fig. 24-5*a* is lower than the potential at point i. This is a general result: The potential always decreases along a path that extends in the direction of the electric field lines.

(b) Now find the potential difference $V_f - V_i$ by moving the positive test charge q_0 from i to f along the path icf shown in Fig. 24-5*b*.

Calculations: The Key Idea of (a) applies here too, except now we move the test charge along a path that consists of two lines: ic and cf. At all points along line ic, the displacement $d\vec{s}$ of the test charge is perpendicular to $\vec{E}$. Thus, the angle θ between $\vec{E}$ and $d\vec{s}$ is 90°, and the dot product $\vec{E}\cdot d\vec{s}$ is 0. Equation 24-18 then tells us that points i and c are at the same potential: $V_c - V_i = 0$.

For line cf we have $\theta = 45°$ and, from Eq. 24-18,

$$V_f - V_i = -\int_c^f \vec{E}\cdot d\vec{s} = -\int_c^f E(\cos 45°)\,ds$$

$$= -E(\cos 45°)\int_c^f ds.$$

The integral in this equation is just the length of line cf; from Fig. 24-5*b*, that length is $d/\cos 45°$. Thus,

$$V_f - V_i = -E(\cos 45°)\frac{d}{\cos 45°} = -Ed. \qquad \text{(Answer)}$$

This is the same result we obtained in (a), as it must be; the potential difference between two points does not depend on the path connecting them. Moral: When you want to find the potential difference between two points by moving a test charge between them, you can save time and work by choosing a path that simplifies the use of Eq. 24-18.

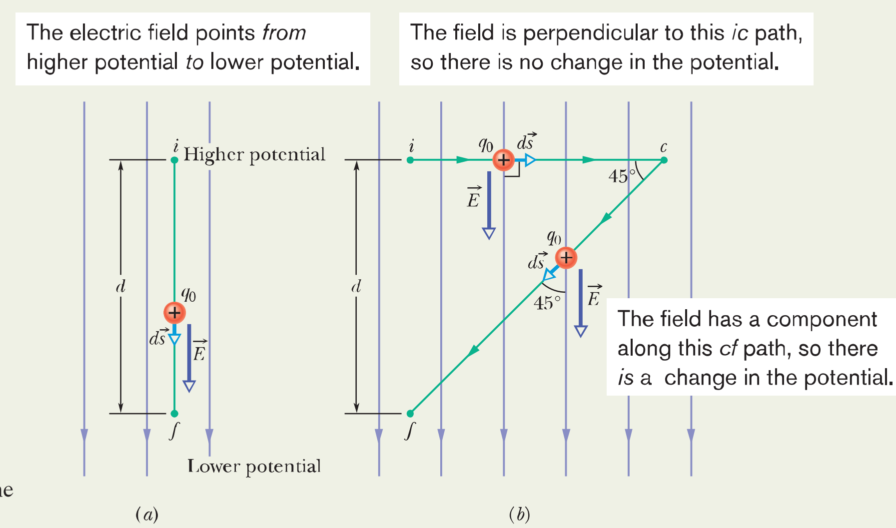


Fig. 24-5 (*a*) A test charge q_0 moves in a straight line from point i to point f, along the direction of a uniform external electric field. (*b*) Charge q_0 moves along path icf in the same electric field.

24-6 Potential Due to a Point Charge

We now use Eq. 24-18 to derive, for the space around a charged particle, an expression for the electric potential V relative to the zero potential at infinity. Consider a point P at distance R from a fixed particle of positive charge q (Fig. 24-6). To use Eq. 24-18, we imagine that we move a positive test charge q_0 from point P to infinity. Because the path we take does not matter, let us choose the simplest one—a line that extends radially from the fixed particle through P to infinity.

To use Eq. 24-18, we must evaluate the dot product

$$\vec{E}\cdot d\vec{s} = E\cos\theta\, ds. \tag{24-22}$$

The electric field $\vec{E}$ in Fig. 24-6 is directed radially outward from the fixed particle. Thus, the differential displacement $d\vec{s}$ of the test particle along its path has the same direction as $\vec{E}$. That means that in Eq. 24-22, angle $\theta = 0$ and $\cos\theta = 1$. Because the path is radial, let us write ds as dr. Then, substituting the limits R and ∞, we can write Eq. 24-18 as

$$V_f - V_i = -\int_R^\infty E\, dr. \tag{24-23}$$

Next, we set $V_f = 0$ (at ∞) and $V_i = V$ (at R). Then, for the magnitude of the electric field at the site of the test charge, we substitute from Eq. 22-3:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}. \tag{24-24}$$

With these changes, Eq. 24-23 then gives us

$$0 - V = -\frac{q}{4\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}\, dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_R^\infty$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q}{R}. \tag{24-25}$$

Solving for V and switching R to r, we then have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \tag{24-26}$$

as the electric potential V due to a particle of charge q at any radial distance r from the particle.

Although we have derived Eq. 24-26 for a positively charged particle, the derivation holds also for a negatively charged particle, in which case, q is a negative quantity. Note that the sign of V is the same as the sign of q:

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

Figure 24-7 shows a computer-generated plot of Eq. 24-26 for a positively charged particle; the magnitude of V is plotted vertically. Note that the magnitude increases as $r \to 0$. In fact, according to Eq. 24-26, V is infinite at $r = 0$, although Fig. 24-7 shows a finite, smoothed-off value there.

Equation 24-26 also gives the electric potential either *outside or on the external surface of* a spherically symmetric charge distribution. We can prove this by using one of the shell theorems of Sections 21-4 and 23-9 to replace the actual spherical charge distribution with an equal charge concentrated at its center. Then the derivation leading to Eq. 24-26 follows, provided we do not consider a point within the actual distribution.

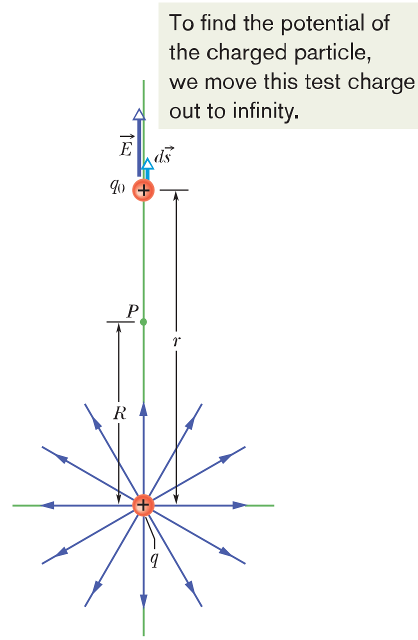


Fig. 24-6 The positive point charge q produces an electric field $\vec{E}$ and an electric potential V at point P. We find the potential by moving a test charge q_0 from P to infinity. The test charge is shown at distance r from the point charge, during differential displacement $d\vec{s}$.

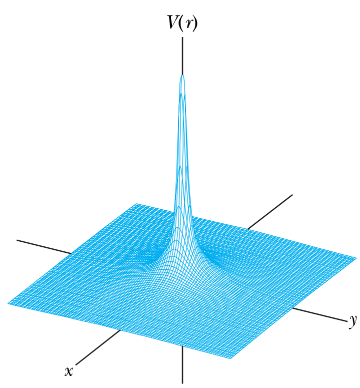


Fig. 24-7 A computer-generated plot of the electric potential $V(r)$ due to a positive point charge located at the origin of an xy plane. The potentials at points in the xy plane are plotted vertically. (Curved lines have been added to help you visualize the plot.) The infinite value of V predicted by Eq. 24-26 for $r = 0$ is not plotted.

24-7 Potential Due to a Group of Point Charges

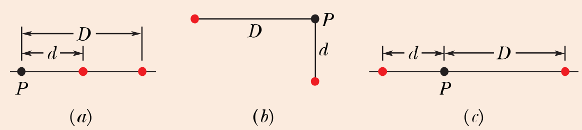
We can find the net potential at a point due to a group of point charges with the help of the superposition principle. Using Eq. 24-26 with the sign of the charge included, we calculate separately the potential resulting from each charge at the given point. Then we sum the potentials. For n charges, the net potential is

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \quad (n \text{ point charges}). \tag{24-27}$$

Here q_i is the value of the ith charge and r_i is the radial distance of the given point from the ith charge. The sum in Eq. 24-27 is an *algebraic sum*, not a vector sum like the sum that would be used to calculate the electric field resulting from a group of point charges. Herein lies an important computational advantage of potential over electric field: It is a lot easier to sum several scalar quantities than to sum several vector quantities whose directions and components must be considered.

CHECKPOINT 4

The figure here shows three arrangements of two protons. Rank the arrangements according to the net electric potential produced at point P by the protons, greatest first.



Sample Problem

Net potential of several charged particles

What is the electric potential at point P, located at the center of the square of point charges shown in Fig. 24-8a? The distance d is 1.3 m, and the charges are

$$q_1 = +12\ \text{nC}, \qquad q_3 = +31\ \text{nC},$$
$$q_2 = -24\ \text{nC}, \qquad q_4 = +17\ \text{nC}.$$

KEY IDEA

The electric potential V at point P is the algebraic sum of the electric potentials contributed by the four point charges.

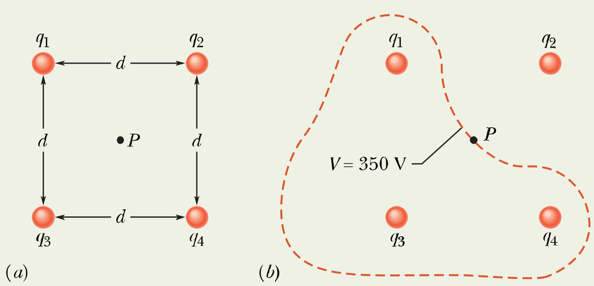


Fig. 24-8 (a) Four point charges are held fixed at the corners of a square. (b) The closed curve is a cross section, in the plane of the figure, of the equipotential surface that contains point P. (The curve is drawn only roughly.)

(Because electric potential is a scalar, the orientations of the point charges do not matter.)

Calculations: From Eq. 24-27, we have

$$V = \sum_{i=1}^{4} V_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right).$$

The distance r is $d/\sqrt{2}$, which is 0.919 m, and the sum of the charges is

$$q_1 + q_2 + q_3 + q_4 = (12 - 24 + 31 + 17) \times 10^{-9}\ \text{C}$$
$$= 36 \times 10^{-9}\ \text{C}.$$

Thus,
$$V = \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(36 \times 10^{-9}\ \text{C})}{0.919\ \text{m}}$$
$$\approx 350\ \text{V}. \qquad \text{(Answer)}$$

Close to any of the three positive charges in Fig. 24-8a, the potential has very large positive values. Close to the single negative charge, the potential has very large negative values. Therefore, there must be points within the square that have the same intermediate potential as that at point P. The curve in Fig. 24-8b shows the intersection of the plane of the figure with the equipotential surface that contains point P. Any point along that curve has the same potential as point P.

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Sample Problem

Potential is not a vector, orientation is irrelevant

(a) In Fig. 24-9*a*, 12 electrons (of charge $-e$) are equally spaced and fixed around a circle of radius R. Relative to $V = 0$ at infinity, what are the electric potential and electric field at the center C of the circle due to these electrons?

KEY IDEAS

(1) The electric potential V at C is the algebraic sum of the electric potentials contributed by all the electrons. (Because electric potential is a scalar, the orientations of the electrons do not matter.) (2) The electric field at C is a vector quantity and thus the orientation of the electrons *is* important.

Calculations: Because the electrons all have the same negative charge $-e$ and are all the same distance R from C, Eq. 24-27 gives us

$$V = -12\frac{1}{4\pi\varepsilon_0}\frac{e}{R}. \qquad \text{(Answer)} \qquad (24\text{-}28)$$

Because of the symmetry of the arrangement in Fig. 24-9*a*, the electric field vector at C due to any given electron is canceled by the field vector due to the electron that is diametrically opposite it. Thus, at C,

$$\vec{E} = 0. \qquad \text{(Answer)}$$

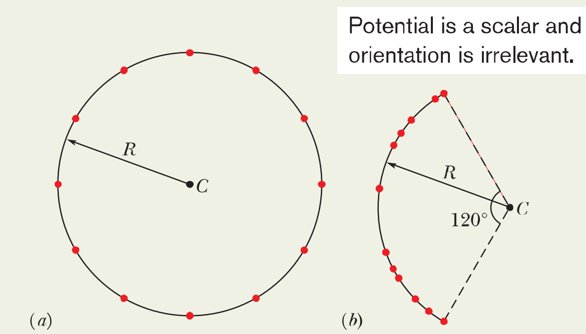


Fig. 24-9 (*a*) Twelve electrons uniformly spaced around a circle. (*b*) The electrons nonuniformly spaced along an arc of the original circle.

(b) If the electrons are moved along the circle until they are nonuniformly spaced over a 120° arc (Fig. 24-9*b*), what then is the potential at C? How does the electric field at C change (if at all)?

Reasoning: The potential is still given by Eq. 24-28, because the distance between C and each electron is unchanged and orientation is irrelevant. The electric field is no longer zero, however, because the arrangement is no longer symmetric. A net field is now directed toward the charge distribution.

24-8 Potential Due to an Electric Dipole

Now let us apply Eq. 24-27 to an electric dipole to find the potential at an arbitrary point P in Fig. 24-10*a*. At P, the positive point charge (at distance $r_{(+)}$) sets up potential $V_{(+)}$ and the negative point charge (at distance $r_{(-)}$) sets up potential $V_{(-)}$. Then the net potential at P is given by Eq. 24-27 as

$$\begin{aligned} V &= \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right) \\ &= \frac{q}{4\pi\varepsilon_0}\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}. \end{aligned} \qquad (24\text{-}29)$$

Naturally occurring dipoles—such as those possessed by many molecules—are quite small; so we are usually interested only in points that are relatively far from the dipole, such that $r \gg d$, where d is the distance between the charges. Under those conditions, the approximations that follow from Fig. 24-10*b* are

$$r_{(-)} - r_{(+)} \approx d\cos\theta \quad \text{and} \quad r_{(-)}r_{(+)} \approx r^2.$$

If we substitute these quantities into Eq. 24-29, we can approximate V to be

$$V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2},$$

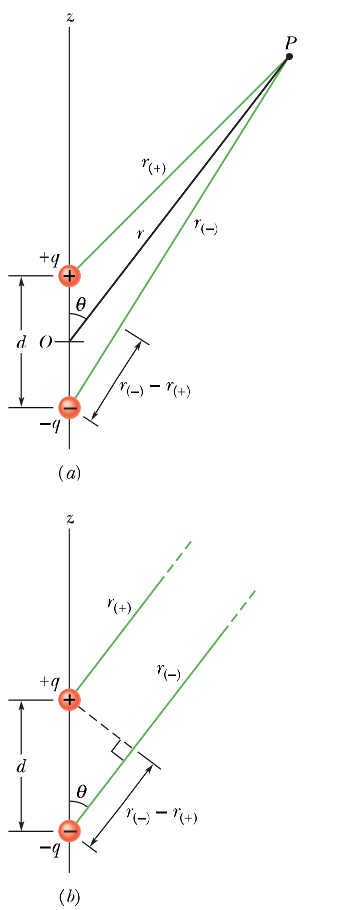


Fig. 24-10 (*a*) Point P is a distance r from the midpoint O of a dipole. The line OP makes an angle θ with the dipole axis. (*b*) If P is far from the dipole, the lines of lengths $r_{(+)}$ and $r_{(-)}$ are approximately parallel to the line of length r, and the dashed black line is approximately perpendicular to the line of length $r_{(-)}$.

where θ is measured from the dipole axis as shown in Fig. 24-10*a*. We can now write V as

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \quad \text{(electric dipole)}, \tag{24-30}$$

in which p ($= qd$) is the magnitude of the electric dipole moment $\vec{p}$ defined in Section 22-5. The vector $\vec{p}$ is directed along the dipole axis, from the negative to the positive charge. (Thus, θ is measured from the direction of $\vec{p}$.) We use this vector to report the orientation of an electric dipole.

CHECKPOINT 5

Suppose that three points are set at equal (large) distances r from the center of the dipole in Fig. 24-10: Point a is on the dipole axis above the positive charge, point b is on the axis below the negative charge, and point c is on a perpendicular bisector through the line connecting the two charges. Rank the points according to the electric potential of the dipole there, greatest (most positive) first.

Induced Dipole Moment

Many molecules, such as water, have *permanent* electric dipole moments. In other molecules (called *nonpolar molecules*) and in every isolated atom, the centers of the positive and negative charges coincide (Fig. 24-11*a*) and thus no dipole moment is set up. However, if we place an atom or a nonpolar molecule in an external electric field, the field distorts the electron orbits and separates the centers of positive and negative charge (Fig. 24-11*b*). Because the electrons are negatively charged, they tend to be shifted in a direction opposite the field. This shift sets up a dipole moment $\vec{p}$ that points in the direction of the field. This dipole moment is said to be *induced* by the field, and the atom or molecule is then said to be *polarized* by the field (that is, it has a positive side and a negative side). When the field is removed, the induced dipole moment and the polarization disappear.

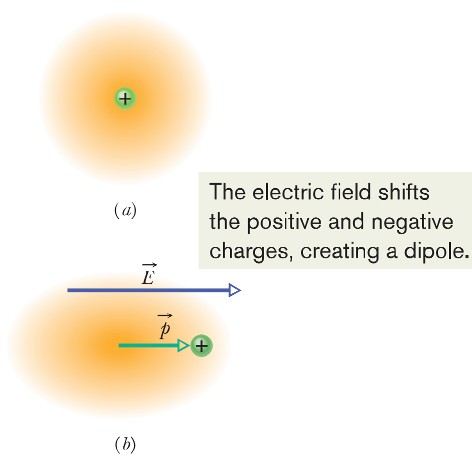


Fig. 24-11 (*a*) An atom, showing the positively charged nucleus (green) and the negatively charged electrons (gold shading). The centers of positive and negative charge coincide. (*b*) If the atom is placed in an external electric field $\vec{E}$, the electron orbits are distorted so that the centers of positive and negative charge no longer coincide. An induced dipole moment $\vec{p}$ appears. The distortion is greatly exaggerated here.

24-9 Potential Due to a Continuous Charge Distribution

When a charge distribution q is continuous (as on a uniformly charged thin rod or disk), we cannot use the summation of Eq. 24-27 to find the potential V at a point P. Instead, we must choose a differential element of charge dq, determine the potential dV at P due to dq, and then integrate over the entire charge distribution.

Let us again take the zero of potential to be at infinity. If we treat the element of charge dq as a point charge, then we can use Eq. 24-26 to express the potential dV at point P due to dq:

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \quad \text{(positive or negative } dq\text{).} \tag{24-31}$$

Here r is the distance between P and dq. To find the total potential V at P, we integrate to sum the potentials due to all the charge elements:

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}. \tag{24-32}$$

The integral must be taken over the entire charge distribution. Note that because the electric potential is a scalar, there are *no vector components* to consider in Eq. 24-32.

We now examine two continuous charge distributions, a line and a disk.

Line of Charge

In Fig. 24-12*a*, a thin nonconducting rod of length L has a positive charge of uniform linear density λ. Let us determine the electric potential V due to the rod at point P, a perpendicular distance d from the left end of the rod.

We consider a differential element dx of the rod as shown in Fig. 24-12*b*. This (or any other) element of the rod has a differential charge of

$$dq = \lambda\, dx. \tag{24-33}$$

This element produces an electric potential dV at point P, which is a distance $r = (x^2 + d^2)^{1/2}$ from the element (Fig. 24-12*c*). Treating the element as a point

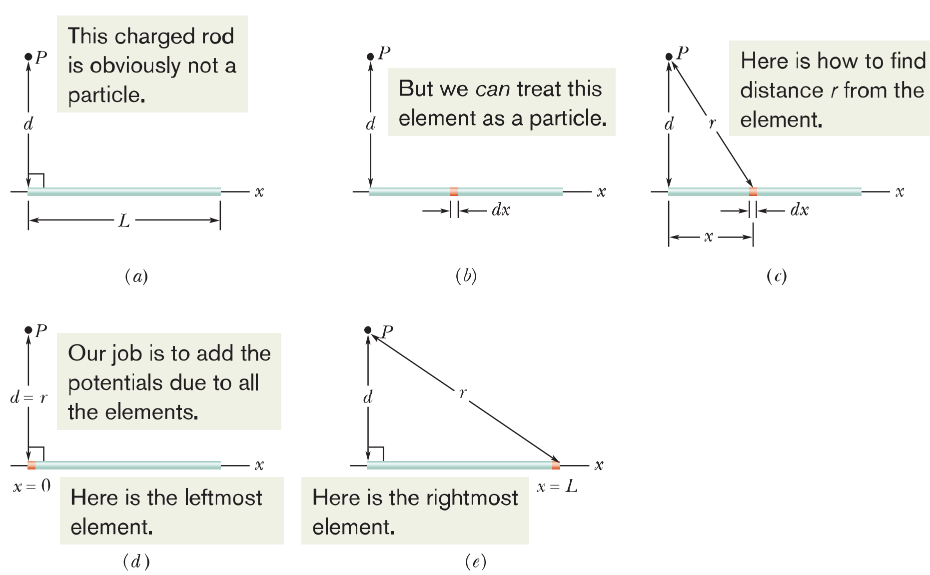


Fig. 24-12 (*a*) A thin, uniformly charged rod produces an electric potential V at point P. (*b*) An element can be treated as a particle. (*c*) The potential at P due to the element depends on the distance r. We need to sum the potentials due to all the elements, from the left side (*d*) to the right side (*e*).

charge, we can use Eq. 24-31 to write the potential dV as

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dx}{(x^2 + d^2)^{1/2}}. \tag{24-34}$$

Since the charge on the rod is positive and we have taken $V = 0$ at infinity, we know from Section 24-6 that dV in Eq. 24-34 must be positive.

We now find the total potential V produced by the rod at point P by integrating Eq. 24-34 along the length of the rod, from $x = 0$ to $x = L$ (Figs. 24-12d and e), using integral 17 in Appendix E. We find

$$\begin{aligned} V &= \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{(x^2 + d^2)^{1/2}}\,dx \\ &= \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(x^2 + d^2)^{1/2}} \\ &= \frac{\lambda}{4\pi\varepsilon_0}\Big[\ln\Big(x + (x^2 + d^2)^{1/2}\Big)\Big]_0^L \\ &= \frac{\lambda}{4\pi\varepsilon_0}\Big[\ln\Big(L + (L^2 + d^2)^{1/2}\Big) - \ln d\Big]. \end{aligned}$$

We can simplify this result by using the general relation $\ln A - \ln B = \ln(A/B)$. We then find

$$V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{L + (L^2 + d^2)^{1/2}}{d}\right]. \tag{24-35}$$

Because V is the sum of positive values of dV, it too is positive, consistent with the logarithm being positive for an argument greater than 1.

Charged Disk

In Section 22-7, we calculated the magnitude of the electric field at points on the central axis of a plastic disk of radius R that has a uniform charge density σ on one surface. Here we derive an expression for $V(z)$, the electric potential at any point on the central axis.

In Fig. 24-13, consider a differential element consisting of a flat ring of radius R' and radial width dR'. Its charge has magnitude

$$dq = \sigma(2\pi R')(dR'),$$

in which $(2\pi R')(dR')$ is the upper surface area of the ring. All parts of this charged element are the same distance r from point P on the disk's axis. With the aid of Fig. 24-13, we can use Eq. 24-31 to write the contribution of this ring to the electric potential at P as

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi R')(dR')}{\sqrt{z^2 + R'^2}}. \tag{24-36}$$

We find the net potential at P by adding (via integration) the contributions of all the rings from $R' = 0$ to $R' = R$:

$$V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{R'\,dR'}{\sqrt{z^2 + R'^2}} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right). \tag{24-37}$$

Note that the variable in the second integral of Eq. 24-37 is R' and not z, which remains constant while the integration over the surface of the disk is carried out. (Note also that, in evaluating the integral, we have assumed that $z \geq 0$.)

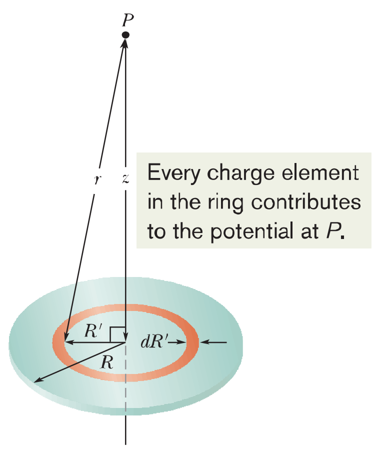


Fig. 24-13 A plastic disk of radius R, charged on its top surface to a uniform surface charge density σ. We wish to find the potential V at point P on the central axis of the disk.

24-10 Calculating the Field from the Potential

In Section 24-5, you saw how to find the potential at a point f if you know the electric field along a path from a reference point to point f. In this section, we propose to go the other way—that is, to find the electric field when we know the potential. As Fig. 24-3 shows, solving this problem graphically is easy: If we know the potential V at all points near an assembly of charges, we can draw in a family of equipotential surfaces. The electric field lines, sketched perpendicular to those surfaces, reveal the variation of $\vec{E}$. What we are seeking here is the mathematical equivalent of this graphical procedure.

Figure 24-14 shows cross sections of a family of closely spaced equipotential surfaces, the potential difference between each pair of adjacent surfaces being dV. As the figure suggests, the field $\vec{E}$ at any point P is perpendicular to the equipotential surface through P.

Suppose that a positive test charge q_0 moves through a displacement $d\vec{s}$ from one equipotential surface to the adjacent surface. From Eq. 24-7, we see that the work the electric field does on the test charge during the move is $-q_0\,dV$. From Eq. 24-16 and Fig. 24-14, we see that the work done by the electric field may also be written as the scalar product $(q_0\vec{E})\cdot d\vec{s}$, or $q_0E(\cos\theta)\,ds$. Equating these two expressions for the work yields

$$-q_0\,dV = q_0E(\cos\theta)\,ds, \tag{24-38}$$

or

$$E\cos\theta = -\frac{dV}{ds}. \tag{24-39}$$

Since $E\cos\theta$ is the component of $\vec{E}$ in the direction of $d\vec{s}$, Eq. 24-39 becomes

$$E_s = -\frac{\partial V}{\partial s}. \tag{24-40}$$

We have added a subscript to E and switched to the partial derivative symbols to emphasize that Eq. 24-40 involves only the variation of V along a specified axis (here called the s axis) and only the component of $\vec{E}$ along that axis. In words, Eq. 24-40 (which is essentially the reverse operation of Eq. 24-18) states:

> The component of $\vec{E}$ in any direction is the negative of the rate at which the electric potential changes with distance in that direction.

If we take the s axis to be, in turn, the x, y, and z axes, we find that the x, y, and z components of $\vec{E}$ at any point are

$$E_x = -\frac{\partial V}{\partial x};\qquad E_y = -\frac{\partial V}{\partial y};\qquad E_z = -\frac{\partial V}{\partial z}. \tag{24-41}$$

Thus, if we know V for all points in the region around a charge distribution—that is, if we know the function $V(x, y, z)$—we can find the components of $\vec{E}$, and thus $\vec{E}$ itself, at any point by taking partial derivatives.

For the simple situation in which the electric field $\vec{E}$ is uniform, Eq. 24-40 becomes

$$E = -\frac{\Delta V}{\Delta s}, \tag{24-42}$$

where s is perpendicular to the equipotential surfaces. The component of the electric field is zero in any direction parallel to the equipotential surfaces because there is no change in potential along the surfaces.

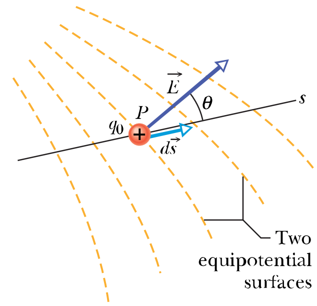
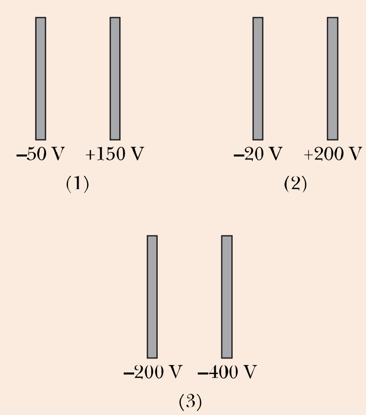


Fig. 24-14 A test charge q_0 moves a distance $d\vec{s}$ from one equipotential surface to another. (The separation between the surfaces has been exaggerated for clarity.) The displacement $d\vec{s}$ makes an angle θ with the direction of the electric field $\vec{E}$.

CHECKPOINT 6

The figure shows three pairs of parallel plates with the same separation, and the electric potential of each plate. The electric field between the plates is uniform and perpendicular to the plates. (a) Rank the pairs according to the magnitude of the electric field between the plates, greatest first. (b) For which pair is the electric field pointing rightward? (c) If an electron is released midway between the third pair of plates, does it remain there, move rightward at constant speed, move leftward at constant speed, accelerate rightward, or accelerate leftward?



Sample Problem

Finding the field from the potential

The electric potential at any point on the central axis of a uniformly charged disk is given by Eq. 24-37,

$$V = \frac{\sigma}{2\varepsilon_0}(\sqrt{z^2 + R^2} - z).$$

Starting with this expression, derive an expression for the electric field at any point on the axis of the disk.

KEY IDEAS

We want the electric field $\vec{E}$ as a function of distance z along the axis of the disk. For any value of z, the direction of $\vec{E}$ must be along that axis because the disk has circular symmetry about that axis. Thus, we want the component E_z of $\vec{E}$ in the direction of z. This component is the negative of the rate at which the electric potential changes with distance z.

Calculation: Thus, from the last of Eqs. 24-41, we can write

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\frac{d}{dz}(\sqrt{z^2 + R^2} - z)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right). \qquad \text{(Answer)}$$

This is the same expression that we derived in Section 22-7 by integration, using Coulomb's law.

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24-11 Electric Potential Energy of a System of Point Charges

In Section 24-2, we discussed the electric potential energy of a charged particle as an electrostatic force does work on it. In that section, we assumed that the charges that produced the force were fixed in place, so that neither the force nor the corresponding electric field could be influenced by the presence of the test charge. In this section we can take a broader view, to find the electric potential energy of a *system* of charges due to the electric field produced *by* those same charges.

For a simple example, suppose you push together two bodies that have charges of the same electrical sign. The work that you must do is stored as electric potential energy in the two-body system (provided the kinetic energy of the bodies does not change). If you later release the charges, you can recover this stored energy, in whole or in part, as kinetic energy of the charged bodies as they rush away from each other.

We define the electric potential energy *of a system of point charges,* held in fixed positions by forces not specified, as follows:

> The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance.

We assume that the charges are stationary both in their initial infinitely distant positions and in their final assembled configuration.

Figure 24-15 shows two point charges q_1 and q_2, separated by a distance r. To find the electric potential energy of this two-charge system, we must mentally build the system, starting with both charges infinitely far away and at rest. When we bring q_1 in from infinity and put it in place, we do no work because no electrostatic force acts on q_1. However, when we next bring q_2 in from infinity and put it in place, we must do work because q_1 exerts an electrostatic force on q_2 during the move.

We can calculate that work with Eq. 24-8 by dropping the minus sign (so that the equation gives the work *we* do rather than the field's work) and substituting q_2 for the general charge q. Our work is then equal to q_2V, where V is the potential that

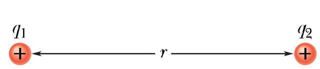

Fig. 24-15 Two charges held a fixed distance r apart.

has been set up by q_1 at the point where we put q_2. From Eq. 24-26, that potential is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}.$$

Thus, from our definition, the electric potential energy of the pair of point charges of Fig. 24-15 is

$$U = W = q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}. \tag{24-43}$$

If the charges have the same sign, we have to do positive work to push them together against their mutual repulsion. Hence, as Eq. 24-43 shows, the potential energy of the system is then positive. If the charges have opposite signs, we have to do negative work against their mutual attraction to bring them together if they are to be stationary. The potential energy of the system is then negative.

Sample Problem

Potential energy of a system of three charged particles

Figure 24-16 shows three point charges held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that $d = 12$ cm and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which $q = 150$ nC.

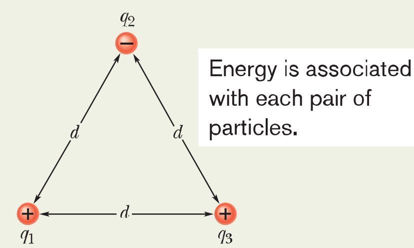

Fig. 24-16 Three charges are fixed at the vertices of an equilateral triangle. What is the electric potential energy of the system?

KEY IDEA

The potential energy U of the system is equal to the work we must do to assemble the system, bringing in each charge from an infinite distance.

Calculations: Let's mentally build the system of Fig. 24-16, starting with one of the point charges, say q_1, in place and the others at infinity. Then we bring another one, say q_2, in from infinity and put it in place. From Eq. 24-43 with d substituted for r, the potential energy U_{12} associated with the pair of point charges q_1 and q_2 is

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d}.$$

We then bring the last point charge q_3 in from infinity and put it in place. The work that we must do in this last step is equal to the sum of the work we must do to bring q_3 near q_1 and the work we must do to bring it near q_2. From Eq. 24-43, with d substituted for r, that sum is

$$W_{13} + W_{23} = U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{d} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{d}.$$

The total potential energy U of the three-charge system is the sum of the potential energies associated with the three pairs of charges. This sum (which is actually independent of the order in which the charges are brought together) is

$$\begin{aligned} U &= U_{12} + U_{13} + U_{23} \\ &= \frac{1}{4\pi\varepsilon_0}\left(\frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d}\right) \\ &= -\frac{10q^2}{4\pi\varepsilon_0 d} \\ &= -\frac{(8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2})(10)(150 \times 10^{-9}\ \mathrm{C})^2}{0.12\ \mathrm{m}} \\ &= -1.7 \times 10^{-2}\ \mathrm{J} = -17\ \mathrm{mJ}. \qquad \text{(Answer)} \end{aligned}$$

The negative potential energy means that negative work would have to be done to assemble this structure, starting with the three charges infinitely separated and at rest. Put another way, an external agent would have to do 17 mJ of work to disassemble the structure completely, ending with the three charges infinitely far apart.

Sample Problem

Conservation of mechanical energy with electric potential energy

An alpha particle (two protons, two neutrons) moves into a stationary gold atom (79 protons, 118 neutrons), passing through the electron region that surrounds the gold nucleus like a shell and headed directly toward the nucleus (Fig. 24-17). The alpha particle slows until it momentarily stops when its center is at radial distance $r = 9.23$ fm from the nuclear center. Then it moves back along its incoming path. (Because the gold nucleus is much more massive than the alpha particle, we can assume the gold nucleus does not move.) What was the kinetic energy K_i of the alpha particle when it was initially far away (hence external to the gold atom)? Assume that the only force acting between the alpha particle and the gold nucleus is the (electrostatic) Coulomb force.

KEY IDEA

During the entire process, the mechanical energy of the *alpha particle + gold atom* system is conserved.

Reasoning: When the alpha particle is outside the atom, the system's initial electric potential energy U_i is zero because the atom has an equal number of electrons and protons, which produce a *net* electric field of zero. However, once the alpha particle passes through the electron region surrounding the nucleus on its way to the nucleus, the electric field due to the electrons goes to zero. The reason is that the electrons act like a closed spherical shell of uniform negative charge and, as discussed in Section 23-9, such a shell produces zero electric field in the space it encloses. The alpha particle still experiences the electric field of the protons in the nucleus, which produces a repulsive force on the protons within the alpha particle.

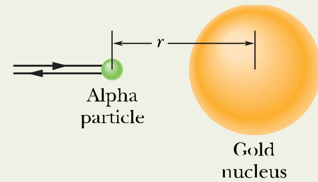

Fig. 24-17 An alpha particle, traveling head-on toward the center of a gold nucleus, comes to a momentary stop (at which time all its kinetic energy has been transferred to electric potential energy) and then reverses its path.

As the incoming alpha particle is slowed by this repulsive force, its kinetic energy is transferred to electric potential energy of the system. The transfer is complete when the alpha particle momentarily stops and the kinetic energy is $K_f = 0$.

Calculations: The principle of conservation of mechanical energy tells us that

$$K_i + U_i = K_f + U_f. \tag{24-44}$$

We know two values: $U_i = 0$ and $K_f = 0$. We also know that the potential energy U_f at the stopping point is given by the right side of Eq. 24-43, with $q_1 = 2e$, $q_2 = 79e$ (in which e is the elementary charge, 1.60×10^{-19} C), and $r = 9.23$ fm. Thus, we can rewrite Eq. 24-44 as

$$K_i = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(79e)}{9.23 \text{ fm}}$$

$$= \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(158)(1.60 \times 10^{-19} \text{ C})^2}{9.23 \times 10^{-15} \text{ m}}$$

$$= 3.94 \times 10^{-12} \text{ J} = 24.6 \text{ MeV}. \qquad \text{(Answer)}$$

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24-12 Potential of a Charged Isolated Conductor

In Section 23-6, we concluded that $\vec{E} = 0$ for all points inside an isolated conductor. We then used Gauss' law to prove that an excess charge placed on an isolated conductor lies entirely on its surface. (This is true even if the conductor has an empty internal cavity.) Here we use the first of these facts to prove an extension of the second:

> An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor—whether on the surface or inside—come to the same potential. This is true even if the conductor has an internal cavity and even if that cavity contains a net charge.

Our proof follows directly from Eq. 24-18, which is

$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}.$$

Since $\vec{E} = 0$ for all points within a conductor, it follows directly that $V_f = V_i$ for all possible pairs of points i and f in the conductor.

Figure 24-18*a* is a plot of potential against radial distance r from the center for an isolated spherical conducting shell of 1.0 m radius, having a charge of 1.0 μC. For points outside the shell, we can calculate $V(r)$ from Eq. 24-26 because the charge q behaves for such external points as if it were concentrated at the center of the shell. That equation holds right up to the surface of the shell. Now let us push a small test charge through the shell—assuming a small hole exists—to its center. No extra work is needed to do this because no net electric force acts on the test charge once it is inside the shell. Thus, the potential at all points inside the shell has the same value as that on the surface, as Fig. 24-18*a* shows.

Figure 24-18*b* shows the variation of electric field with radial distance for the same shell. Note that $E = 0$ everywhere inside the shell. The curves of Fig. 24-18*b* can be derived from the curve of Fig. 24-18*a* by differentiating with respect to r, using Eq. 24-40 (recall that the derivative of any constant is zero). The curve of Fig. 24-18*a* can be derived from the curves of Fig. 24-18*b* by integrating with respect to r, using Eq. 24-19.

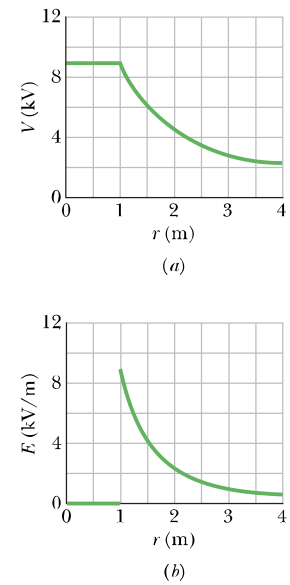

Fig. 24-18 (*a*) A plot of $V(r)$ both inside and outside a charged spherical shell of radius 1.0 m. (*b*) A plot of $E(r)$ for the same shell.

Fig. 24-19 A large spark jumps to a car's body and then exits by moving across the insulating left front tire (note the flash there), leaving the person inside unharmed. *(Courtesy Westinghouse Electric Corporation)*

Spark Discharge from a Charged Conductor

On nonspherical conductors, a surface charge does not distribute itself uniformly over the surface of the conductor. At sharp points or sharp edges, the surface charge density—and thus the external electric field, which is proportional to it—may reach very high values. The air around such sharp points or edges may become ionized, producing the corona discharge that golfers and mountaineers see on the tips of bushes, golf clubs, and rock hammers when thunderstorms threaten. Such corona discharges, like hair that stands on end, are often the precursors of lightning strikes. In such circumstances, it is wise to enclose yourself in a cavity inside a conducting shell, where the electric field is guaranteed to be zero. A car (unless it is a convertible or made with a plastic body) is almost ideal (Fig. 24-19).

Isolated Conductor in an External Electric Field

If an isolated conductor is placed in an *external electric field,* as in Fig. 24-20, all points of the conductor still come to a single potential regardless of whether the conductor has an excess charge. The free conduction electrons distribute themselves on the surface in such a way that the electric field they produce at interior points cancels the external electric field that would otherwise be there. Furthermore, the electron distribution causes the net electric field at all points on the surface to be perpendicular to the surface. If the conductor in Fig. 24-20 could be somehow removed, leaving the surface charges frozen in place, the internal and external electric field would remain absolutely unchanged.

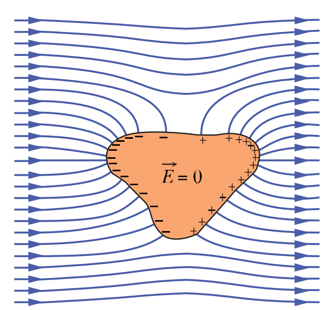

Fig. 24-20 An uncharged conductor is suspended in an external electric field. The free electrons in the conductor distribute themselves on the surface as shown, so as to reduce the net electric field inside the conductor to zero and make the net field at the surface perpendicular to the surface.

REVIEW & SUMMARY

Electric Potential Energy The change ΔU in the electric potential energy U of a point charge as the charge moves from an initial point i to a final point f in an electric field is

$$\Delta U = U_f - U_i = -W, \tag{24-1}$$

where W is the work done by the electrostatic force (due to the external electric field) on the point charge during the move from i to f. If the potential energy is defined to be zero at infinity, the **electric potential energy** U of the point charge at a particular point is

$$U = -W_\infty. \tag{24-2}$$

Here W_∞ is the work done by the electrostatic force on the point charge as the charge moves from infinity to the particular point.

Electric Potential Difference and Electric Potential We define the **potential difference** ΔV between two points i and f in an electric field as

$$\Delta V = V_f - V_i = -\frac{W}{q}, \tag{24-7}$$

where q is the charge of a particle on which work W is done by the electric field as the particle moves from point i to point f. The **potential** at a point is defined as

$$V = -\frac{W_\infty}{q}. \tag{24-8}$$

Here W_∞ is the work done on the particle by the electric field as the particle moves in from infinity to the point. The SI unit of potential is the *volt:* 1 volt = 1 joule per coulomb.

Potential and potential difference can also be written in terms of the electric potential energy U of a particle of charge q in an electric field:

$$V = \frac{U}{q}, \tag{24-5}$$

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}. \tag{24-6}$$

Equipotential Surfaces The points on an **equipotential surface** all have the same electric potential. The work done on a test charge in moving it from one such surface to another is independent of the locations of the initial and final points on these surfaces and of the path that joins the points. The electric field $\vec{E}$ is always directed perpendicularly to corresponding equipotential surfaces.

Finding *V* from $\vec{E}$ The electric potential difference between two points i and f is

$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}, \tag{24-18}$$

where the integral is taken over any path connecting the points. If the integration is difficult along any particular path, we can choose a different path along which the integration might be easier. If we choose $V_i = 0$, we have, for the potential at a particular point,

$$V = -\int_i^f \vec{E}\cdot d\vec{s}. \tag{24-19}$$

Potential Due to Point Charges The electric potential due to a single point charge at a distance r from that point charge is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}, \tag{24-26}$$

where V has the same sign as q. The potential due to a collection of point charges is

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}. \tag{24-27}$$

Potential Due to an Electric Dipole At a distance r from an electric dipole with dipole moment magnitude $p = qd$, the electric potential of the dipole is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \tag{24-30}$$

for $r \gg d$; the angle θ is defined in Fig. 24-10.

Potential Due to a Continuous Charge Distribution For a continuous distribution of charge, Eq. 24-27 becomes

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}, \tag{24-32}$$

in which the integral is taken over the entire distribution.

Calculating $\vec{E}$ from *V* The component of $\vec{E}$ in any direction is the negative of the rate at which the potential changes with distance in that direction:

$$E_s = -\frac{\partial V}{\partial s}. \tag{24-40}$$

The x, y, and z components of $\vec{E}$ may be found from

$$E_x = -\frac{\partial V}{\partial x}; \quad E_y = -\frac{\partial V}{\partial y}; \quad E_z = -\frac{\partial V}{\partial z}. \tag{24-41}$$

When $\vec{E}$ is uniform, Eq. 24-40 reduces to

$$E = -\frac{\Delta V}{\Delta s}, \tag{24-42}$$

where s is perpendicular to the equipotential surfaces. The electric field is zero parallel to an equipotential surface.

Electric Potential Energy of a System of Point Charges The electric potential energy of a system of point charges is equal to the work needed to assemble the system with the charges initially at rest and infinitely distant from each other. For two charges at separation r,

$$U = W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}. \tag{24-43}$$

Potential of a Charged Conductor An excess charge placed on a conductor will, in the equilibrium state, be located entirely on the outer surface of the conductor. The charge will distribute itself so that the following occur: (1) The entire conductor, including interior points, is at a uniform potential. (2) At every internal point, the electric field due to the charge cancels the external electric field that otherwise would have been there. (3) The net electric field at every point on the surface is perpendicular to the surface.

QUESTIONS

1 In Fig. 24-21, eight particles form a square, with distance d between adjacent particles. What is the electric potential at point P at the center of the square if the electric potential is zero at infinity?

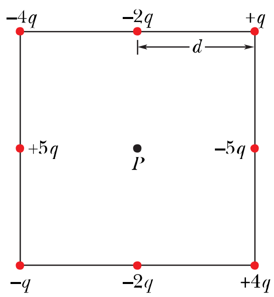

Fig. 24-21 Question 1.

2 Figure 24-22 shows three sets of cross sections of equipotential surfaces; all three cover the same size region of space. (a) Rank the arrangements according to the magnitude of the electric field present in the region, greatest first. (b) In which is the electric field directed down the page?

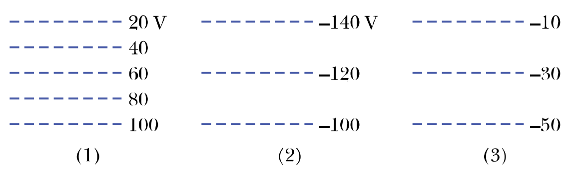

Fig. 24-22 Question 2.

3 Figure 24-23 shows four pairs of charged particles. For each pair, let $V = 0$ at infinity and consider V_{net} at points on the x axis. For which pairs is there a point at which $V_{net} = 0$ (a) between the particles and (b) to the right of the particles? (c) At such a point is $\vec{E}_{net}$ due to the particles equal to zero? (d) For each pair, are there off-axis points (other than at infinity) where $V_{net} = 0$?

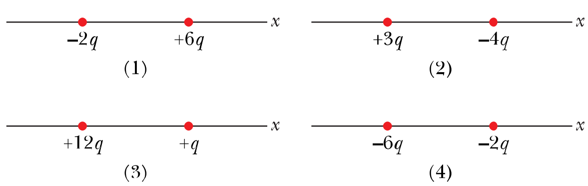

Fig. 24-23 Questions 3 and 9.

4 Figure 24-24 gives the electric potential V as a function of x. (a) Rank the five regions according to the magnitude of the x component of the electric field within them, greatest first. What is the direction of the field along the x axis in (b) region 2 and (c) region 4?

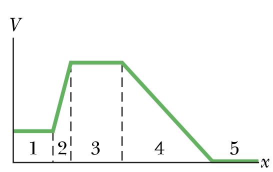

Fig. 24-24 Question 4.

5 Figure 24-25 shows three paths along which we can move the positively charged sphere A closer to positively charged sphere B, which is held fixed in place. (a) Would sphere A be moved to a higher or lower electric potential? Is the work done (b) by our force and (c) by the electric field due to B positive, negative, or zero? (d) Rank the paths according to the work our force does, greatest first.

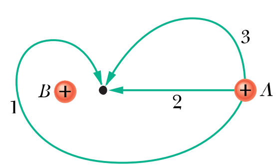

Fig. 24-25 Question 5.

6 Figure 24-26 shows four arrangements of charged particles, all the same distance from the origin. Rank the situations according to the net electric potential at the origin, most positive first. Take the potential to be zero at infinity.

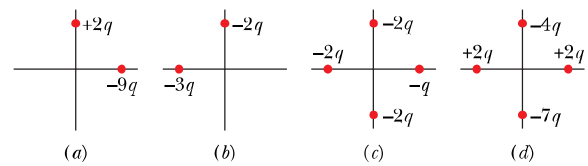

Fig. 24-26 Question 6.

7 Figure 24-27 shows a system of three charged particles. If you move the particle of charge $+q$ from point A to point D, are the following quantities positive, negative, or zero: (a) the change in the electric potential energy of the three-particle system, (b) the work done by the net electrostatic force on the particle you moved (that is, the net force due to the other two particles), and (c) the work done by your force? (d) What are the answers to (a) through (c) if, instead, the particle is moved from B to C?

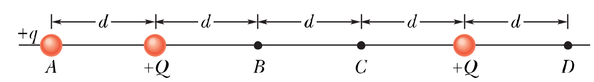

Fig. 24-27 Questions 7 and 8.

8 In the situation of Question 7, is the work done by your force positive, negative, or zero if the particle is moved (a) from A to B, (b) from A to C, and (c) from B to D? (d) Rank those moves according to the magnitude of the work done by your force, greatest first.

9 Figure 24-23 shows four pairs of charged particles with identical separations. (a) Rank the pairs according to their electric potential energy (that is, the energy of the two-particle system), greatest (most positive) first. (b) For each pair, if the separation between the particles is increased, does the potential energy of the pair increase or decrease?

10 (a) In Fig. 24-28a, what is the potential at point P due to charge Q at distance R from P? Set $V = 0$ at infinity. (b) In Fig. 24-28b, the same charge Q has been spread uniformly over a circular arc of radius R and central angle 40°. What is the potential at point P, the center of curvature of the arc? (c) In Fig. 24-28c, the same charge Q has been spread uniformly over a circle of radius R. What is the potential at point P, the center of the circle? (d) Rank the three situations according to the magnitude of the electric field that is set up at P, greatest first.

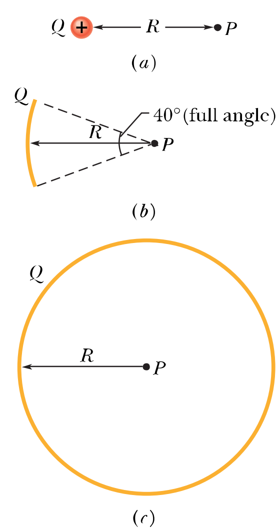

Fig. 24-28 Question 10.

PROBLEMS

GO Tutoring problem available (at instructor's discretion) in *WileyPLUS* and WebAssign
SSM Worked-out solution available in Student Solutions Manual
WWW Worked-out solution is at
• – ••• Number of dots indicates level of problem difficulty
ILW Interactive solution is at
http://www.wiley.com/college/halliday
Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

sec. 24-3 Electric Potential

•1 SSM A particular 12 V car battery can send a total charge of 84 A · h (ampere-hours) through a circuit, from one terminal to the other. (a) How many coulombs of charge does this represent? (*Hint:* See Eq. 21-3.) (b) If this entire charge undergoes a change in electric potential of 12 V, how much energy is involved?

•2 The electric potential difference between the ground and a cloud in a particular thunderstorm is 1.2×10^9 V. In the unit electron-volts, what is the magnitude of the change in the electric potential energy of an electron that moves between the ground and the cloud?

•3 Much of the material making up Saturn's rings is in the form of tiny dust grains having radii on the order of 10^{-6} m. These grains are located in a region containing a dilute ionized gas, and they pick up excess electrons. As an approximation, suppose each grain is spherical, with radius $R = 1.0 \times 10^{-6}$ m. How many electrons would one grain have to pick up to have a potential of −400 V on its surface (taking $V = 0$ at infinity)?

sec. 24-5 Calculating the Potential from the Field

•4 Two large, parallel, conducting plates are 12 cm apart and have charges of equal magnitude and opposite sign on their facing surfaces. An electrostatic force of 3.9×10^{-15} N acts on an electron placed anywhere between the two plates. (Neglect fringing.) (a) Find the electric field at the position of the electron. (b) What is the potential difference between the plates?

•5 SSM An infinite nonconducting sheet has a surface charge density $\sigma = 0.10\ \mu C/m^2$ on one side. How far apart are equipotential surfaces whose potentials differ by 50 V?

•6 When an electron moves from A to B along an electric field line in Fig. 24-29, the electric field does 3.94×10^{-19} J of work on it. What are the electric potential differences (a) $V_B - V_A$, (b) $V_C - V_A$, and (c) $V_C - V_B$?

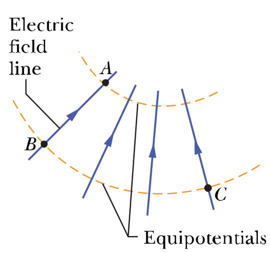

Fig. 24-29 Problem 6.

••7 The electric field in a region of space has the components $E_y = E_z = 0$ and $E_x = (4.00\ N/C)x$. Point A is on the y axis at $y = 3.00$ m, and point B is on the x axis at $x = 4.00$ m. What is the potential difference $V_B - V_A$?

••8 A graph of the x component of the electric field as a function of x in a region of space is shown in Fig. 24-30. The scale of the vertical axis is set by $E_{xs} = 20.0$ N/C. The y and z components of the electric field are zero in this region. If the electric potential at the origin is 10 V, (a) what is the electric potential at $x = 2.0$ m, (b) what is the greatest positive value of the electric potential for points on the x axis for which $0 \le x \le 6.0$ m, and (c) for what value of x is the electric potential zero?

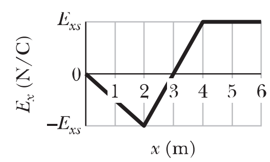

Fig. 24-30 Problem 8.

••9 An infinite nonconducting sheet has a surface charge density $\sigma = +5.80\ pC/m^2$. (a) How much work is done by the electric field due to the sheet if a particle of charge $q = +1.60 \times 10^{-19}$ C is moved from the sheet to a point P at distance $d = 3.56$ cm from the sheet? (b) If the electric potential V is defined to be zero on the sheet, what is V at P?

•••10 Two uniformly charged, infinite, nonconducting planes are parallel to a yz plane and positioned at $x = -50$ cm and $x = +50$ cm. The charge densities on the planes are $-50 nC/m^2$ and $+25\ nC/m^2$, respectively. What is the magnitude of the potential difference between the origin and the point on the x axis at $x = +80$ cm? (*Hint:* Use Gauss' law.)

•••11 A nonconducting sphere has radius $R = 2.31$ cm and uniformly distributed charge $q = +3.50$ fC. Take the electric potential at the sphere's center to be $V_0 = 0$. What is V at radial distance (a) $r = 1.45$ cm and (b) $r = R$. (*Hint:* See Section 23-9.)

sec. 24-7 Potential Due to a Group of Point Charges

•12 As a space shuttle moves through the dilute ionized gas of Earth's ionosphere, the shuttle's potential is typically changed by −1.0 V during one revolution. Assuming the shuttle is a sphere of radius 10 m, estimate the amount of charge it collects.

•13 What are (a) the charge and (b) the charge density on the surface of a conducting sphere of radius 0.15 m whose potential is 200 V (with $V = 0$ at infinity)?

•14 Consider a point charge $q = 1.0\ \mu C$, point A at distance $d_1 = 2.0$ m from q, and point B at distance $d_2 = 1.0$ m. (a) If A and B are diametrically opposite each other, as in Fig. 24-31a, what is the elec-

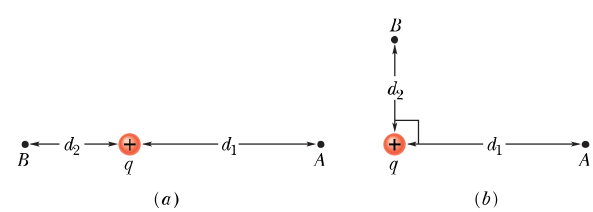

Fig. 24-31 Problem 14.

tric potential difference $V_A - V_B$? (b) What is that electric potential difference if A and B are located as in Fig. 24-31b?

••15 SSM ILW A spherical drop of water carrying a charge of 30 pC has a potential of 500 V at its surface (with $V = 0$ at infinity). (a) What is the radius of the drop? (b) If two such drops of the same charge and radius combine to form a single spherical drop, what is the potential at the surface of the new drop?

••16 GO Figure 24-32 shows a rectangular array of charged particles fixed in place, with distance $a = 39.0$ cm and the charges shown as integer multiples of $q_1 = 3.40$ pC and $q_2 = 6.00$ pC. With $V = 0$ at infinity, what is the net electric potential at the rectangle's center? (*Hint:* Thoughtful examination can reduce the calculation.)

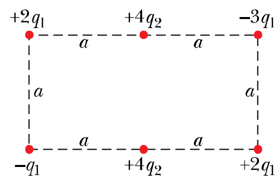

Fig. 24-32 Problem 16.

••17 GO In Fig. 24-33, what is the net electric potential at point P due to the four particles if $V = 0$ at infinity, $q = 5.00$ fC, and $d = 4.00$ cm?

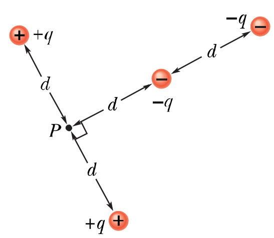

Fig. 24-33 Problem 17.

••18 GO Two charged particles are shown in Fig. 24-34a. Particle 1, with charge q_1, is fixed in place at distance d. Particle 2, with charge q_2, can be moved along the x axis. Figure 24-34b gives the net electric potential V at the origin due to the two particles as a function of the x coordinate of particle 2. The scale of the x axis is set by $x_s = 16.0$ cm. The plot has an asymptote of $V = 5.76 \times 10^{-7}$ V as $x \to \infty$. What is q_2 in terms of e?

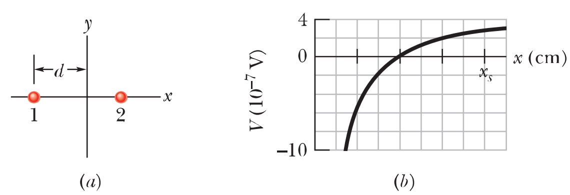

Fig. 24-34 Problem 18.

••19 In Fig. 24-35, particles with the charges $q_1 = +5e$ and $q_2 = -15e$ are fixed in place with a separation of $d = 24.0$ cm. With electric potential defined to be $V = 0$ at infinity, what are the finite (a) positive and (b) negative values of x at which the net electric potential on the x axis is zero?

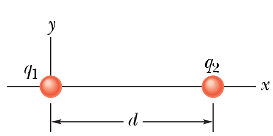

Fig. 24-35 Problems 19, 20, and 97.

••20 Two particles, of charges q_1 and q_2, are separated by distance d in Fig. 24-35. The net electric field due to the particles is zero at $x = d/4$. With $V = 0$ at infinity, locate (in terms of d) any point on the x axis (other than at infinity) at which the electric potential due to the two particles is zero.

sec. 24-8 Potential Due to an Electric Dipole

•21 ILW The ammonia molecule NH_3 has a permanent electric dipole moment equal to 1.47 D, where 1 D = 1 debye unit = 3.34×10^{-30} C·m. Calculate the electric potential due to an ammonia molecule at a point 52.0 nm away along the axis of the dipole. (Set $V = 0$ at infinity.)

••22 In Fig. 24-36a, a particle of elementary charge $+e$ is initially at coordinate $z = 20$ nm on the dipole axis (here a z axis) through an electric dipole, on the positive side of the dipole. (The origin of z is at the center of the dipole.) The particle is then moved along a circular path around the dipole center until it is at coordinate $z = -20$ nm, on the negative side of the dipole axis. Figure 24-36b gives the work W_a done by the force moving the particle versus the angle θ that locates the particle relative to the positive direction of the z axis. The scale of the vertical axis is set by $W_{as} = 4.0 \times 10^{-30}$ J. What is the magnitude of the dipole moment?

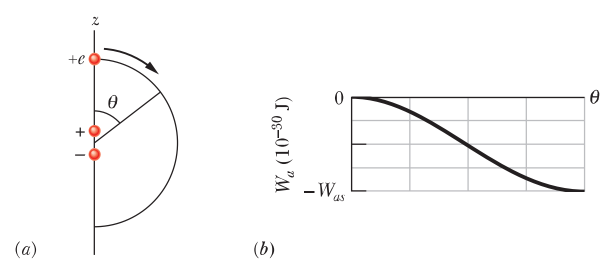

Fig. 24-36 Problem 22.

sec. 24-9 Potential Due to a Continuous Charge Distribution

•23 (a) Figure 24-37a shows a nonconducting rod of length $L = 6.00$ cm and uniform linear charge density $\lambda = +3.68$ pC/m. Assume that the electric potential is defined to be $V = 0$ at infinity. What is V at point P at distance $d = 8.00$ cm along the rod's perpendicular bisector? (b) Figure 24-37b shows an identical rod except that one half is now negatively charged. Both halves have a linear charge density of magnitude 3.68 pC/m. With $V = 0$ at infinity, what is V at P?

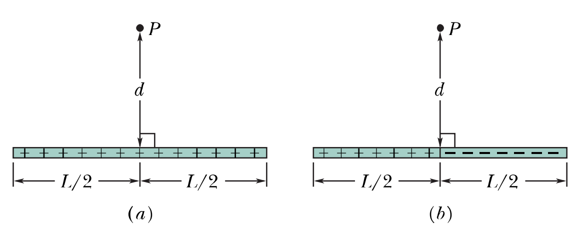

Fig. 24-37 Problem 23.

•24 In Fig. 24-38, a plastic rod having a uniformly distributed charge $Q = -25.6$ pC has been bent into a circular arc of radius $R = 3.71$ cm and central angle $\phi = 120°$. With $V = 0$ at infinity, what is the electric potential at P, the center of curvature of the rod?

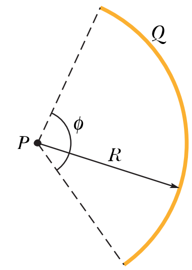

Fig. 24-38 Problem 24.

•25 A plastic rod has been bent into a circle of radius $R = 8.20$ cm. It has a charge $Q_1 = +4.20$ pC uniformly distributed along one-quarter of its circumference and a charge $Q_2 = -6Q_1$ uniformly distributed along the rest of the circumference (Fig. 24-39). With $V = 0$ at infinity, what is the electric potential at (a) the center C of the circle and (b) point P, on the central axis of the circle at distance $D = 6.71$ cm from the center?

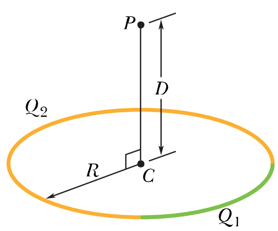

Fig. 24-39 Problem 25.

••26 GO Figure 24-40 shows a thin rod with a uniform charge density of 2.00 μC/m. Evaluate the electric potential at point P if $d = D = L/4.00$.

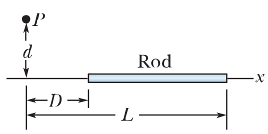

Fig. 24-40 Problem 26.

••27 In Fig. 24-41, three thin plastic rods form quarter-circles with a common center of curvature at the origin. The uniform charges on the rods are $Q_1 = +30$ nC, $Q_2 = +3.0Q_1$, and $Q_3 = -8.0Q_1$. What is the net electric potential at the origin due to the rods?

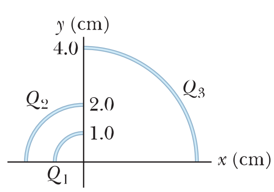

Fig. 24-41 Problem 27.

••28 GO Figure 24-42 shows a thin plastic rod of length $L = 12.0$ cm and uniform positive charge $Q = 56.1$ fC lying on an x axis. With $V = 0$ at infinity, find the electric potential at point P_1 on the axis, at distance $d = 2.50$ cm from one end of the rod.

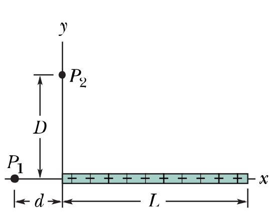

Fig. 24-42 Problems 28, 33, 38, and 40.

••29 In Fig. 24-43, what is the net electric potential at the origin due to the circular arc of charge $Q_1 = +7.21$ pC and the two particles of charges $Q_2 = 4.00Q_1$ and $Q_3 = -2.00Q_1$? The arc's center of curvature is at the origin and its radius is $R = 2.00$ m; the angle indicated is $\theta = 20.0°$.

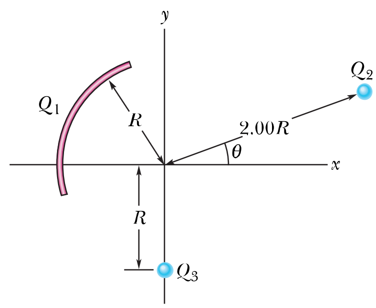

Fig. 24-43 Problem 29.

••30 GO The smiling face of Fig. 24-44 consists of three items:

1. a thin rod of charge -3.0 μC that forms a full circle of radius 6.0 cm;
2. a second thin rod of charge 2.0 μC that forms a circular arc of radius 4.0 cm, subtending an angle of 90° about the center of the full circle;
3. an electric dipole with a dipole moment that is perpendicular to a radial line and has magnitude 1.28×10^{-21} C·m.

What is the net electric potential at the center?

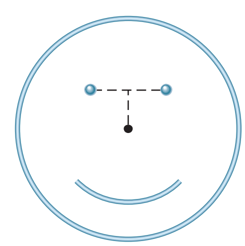

Fig. 24-44 Problem 30.

••31 SSM WWW A plastic disk of radius $R = 64.0$ cm is charged on one side with a uniform surface charge density $\sigma = 7.73$ fC/m^2, and then three quadrants of the disk are removed. The remaining quadrant is shown in Fig. 24-45. With $V = 0$ at infinity, what is the potential due to the remaining quadrant at point P, which is on the central axis of the original disk at distance $D = 25.9$ cm from the original center?

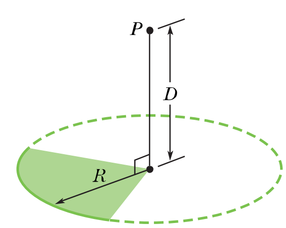

Fig. 24-45 Problem 31.

•••32 A nonuniform linear charge distribution given by $\lambda = bx$, where b is a constant, is located along an x axis from $x = 0$ to $x = 0.20$ m. If $b = 20$ nC/m^2 and $V = 0$ at infinity, what is the electric potential at (a) the origin and (b) the point $y = 0.15$ m on the y axis?

•••33 The thin plastic rod shown in Fig. 24-42 has length $L = 12.0$ cm and a nonuniform linear charge density $\lambda = cx$, where $c = 28.9$

pC/m^2. With $V = 0$ at infinity, find the electric potential at point P_1 on the axis, at distance $d = 3.00$ cm from one end.

sec. 24-10 Calculating the Field from the Potential

•34 Two large parallel metal plates are 1.5 cm apart and have charges of equal magnitudes but opposite signs on their facing surfaces. Take the potential of the negative plate to be zero. If the potential halfway between the plates is then $+5.0$ V, what is the electric field in the region between the plates?

•35 The electric potential at points in an xy plane is given by $V = (2.0\ \text{V/m}^2)x^2 - (3.0\ \text{V/m}^2)y^2$. In unit-vector notation, what is the electric field at the point (3.0 m, 2.0 m)?

•36 The electric potential V in the space between two flat parallel plates 1 and 2 is given (in volts) by $V = 1500x^2$, where x (in meters) is the perpendicular distance from plate 1. At $x = 1.3$ cm, (a) what is the magnitude of the electric field and (b) is the field directed toward or away from plate 1?

••37 SSM What is the magnitude of the electric field at the point $(3.00\hat{i} - 2.00\hat{j} + 4.00\hat{k})$ m if the electric potential is given by $V = 2.00xyz^2$, where V is in volts and x, y, and z are in meters?

••38 Figure 24-42 shows a thin plastic rod of length $L = 13.5$ cm and uniform charge 43.6 fC. (a) In terms of distance d, find an expression for the electric potential at point P_1. (b) Next, substitute variable x for d and find an expression for the magnitude of the component E_x of the electric field at P_1. (c) What is the direction of E_x relative to the positive direction of the x axis? (d) What is the value of E_x at P_1 for $x = d = 6.20$ cm? (e) From the symmetry in Fig. 24-42, determine E_y at P_1.

••39 An electron is placed in an xy plane where the electric potential depends on x and y as shown in Fig. 24-46 (the potential does not depend on z). The scale of the vertical axis is set by $V_s = 500$ V. In unit-vector notation, what is the electric force on the electron?

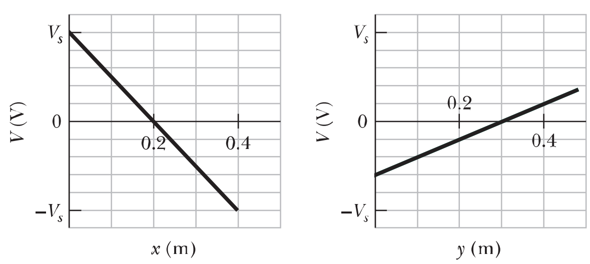

Fig. 24-46 Problem 39.

•••40 The thin plastic rod of length $L = 10.0$ cm in Fig. 24-42 has a nonuniform linear charge density $\lambda = cx$, where $c = 49.9$ pC/m^2. (a) With $V = 0$ at infinity, find the electric potential at point P_2 on the y axis at $y = D = 3.56$ cm. (b) Find the electric field component E_y at P_2. (c) Why cannot the field component E_x at P_2 be found using the result of (a)?

sec. 24-11 Electric Potential Energy of a System of Point Charges

•41 A particle of charge $+7.5\ \mu$C is released from rest at the point $x = 60$ cm on an x axis. The particle begins to move due to the presence of a charge Q that remains fixed at the origin. What is the kinetic energy of the particle at the instant it has moved 40 cm if (a) $Q = +20\ \mu$C and (b) $Q = -20\ \mu$C?

•42 (a) What is the electric potential energy of two electrons separated by 2.00 nm? (b) If the separation increases, does the potential energy increase or decrease?

•43 SSM ILW WWW How much work is required to set up the arrangement of Fig. 24-47 if $q = 2.30$ pC, $a = 64.0$ cm, and the particles are initially infinitely far apart and at rest?

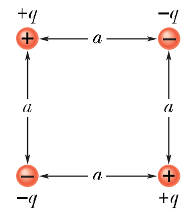

Fig. 24-47 Problem 43.

•44 In Fig. 24-48, seven charged particles are fixed in place to form a square with an edge length of 4.0 cm. How much work must we do to bring a particle of charge $+6e$ initially at rest from an infinite distance to the center of the square?

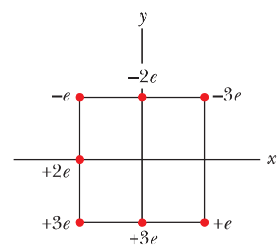

Fig. 24-48 Problem 44.

••45 ILW A particle of charge q is fixed at point P, and a second particle of mass m and the same charge q is initially held a distance r_1 from P. The second particle is then released. Determine its speed when it is a distance r_2 from P. Let $q = 3.1\ \mu$C, $m = 20$ mg, $r_1 = 0.90$ mm, and $r_2 = 2.5$ mm.

••46 A charge of -9.0 nC is uniformly distributed around a thin plastic ring lying in a yz plane with the ring center at the origin. A -6.0 pC point charge is located on the x axis at $x = 3.0$ m. For a ring radius of 1.5 m, how much work must an external force do on the point charge to move it to the origin?

••47 GO What is the *escape speed* for an electron initially at rest on the surface of a sphere with a radius of 1.0 cm and a uniformly distributed charge of 1.6×10^{-15} C? That is, what initial speed must the electron have in order to reach an infinite distance from the sphere and have zero kinetic energy when it gets there?

••48 A thin, spherical, conducting shell of radius R is mounted on an isolating support and charged to a potential of -125 V. An electron is then fired directly toward the center of the shell, from point P at distance r from the center of the shell ($r \gg R$). What initial speed v_0 is needed for the electron to just reach the shell before reversing direction?

••49 GO Two electrons are fixed 2.0 cm apart. Another electron is shot from infinity and stops midway between the two. What is its initial speed?

••50 In Fig. 24-49, how much work must we do to bring a particle, of charge $Q = +16e$ and initially at rest, along the dashed line from infinity to

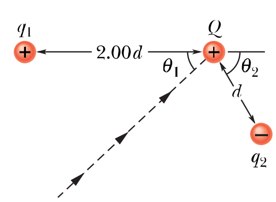

Fig. 24-49 Problem 50.

the indicated point near two fixed particles of charges $q_1 = +4e$ and $q_2 = -q_1/2$? Distance $d = 1.40$ cm, $\theta_1 = 43°$, and $\theta_2 = 60°$.

••51 GO In the rectangle of Fig. 24-50, the sides have lengths 5.0 cm and 15 cm, $q_1 = -5.0\ \mu C$, and $q_2 = +2.0\ \mu C$. With $V = 0$ at infinity, what is the electric potential at (a) corner A and (b) corner B? (c) How much work is required to move a charge $q_3 = +3.0\ \mu C$ from B to A along a diagonal of the rectangle? (d) Does this work increase or decrease the electric potential energy of the three-charge system? Is more, less, or the same work required if q_3 is moved along a path that is (e) inside the rectangle but not on a diagonal and (f) outside the rectangle?

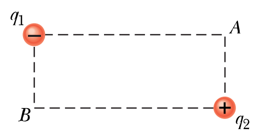

Fig. 24-50 Problem 51.

••52 Figure 24-51a shows an electron moving along an electric dipole axis toward the negative side of the dipole. The dipole is fixed in place. The electron was initially very far from the dipole, with kinetic energy 100 eV. Figure 24-51b gives the kinetic energy K of the electron versus its distance r from the dipole center. The scale of the horizontal axis is set by $r_s = 0.10$ m. What is the magnitude of the dipole moment?

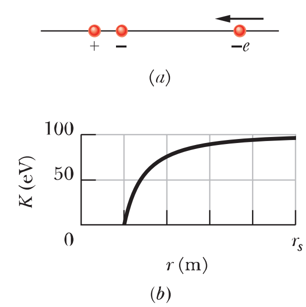

Fig. 24-51 Problem 52.

••53 Two tiny metal spheres A and B, mass $m_A = 5.00$ g and $m_B = 10.0$ g, have equal positive charge $q = 5.00\ \mu C$. The spheres are connected by a massless nonconducting string of length $d = 1.00$ m, which is much greater than the radii of the spheres. (a) What is the electric potential energy of the system? (b) Suppose you cut the string. At that instant, what is the acceleration of each sphere? (c) A long time after you cut the string, what is the speed of each sphere?

••54 A positron (charge $+e$, mass equal to the electron mass) is moving at 1.0×10^7 m/s in the positive direction of an x axis when, at $x = 0$, it encounters an electric field directed along the x axis. The electric potential V associated with the field is given in Fig. 24-52. The scale of the vertical axis is set by $V_s = 500.0$ V. (a) Does the positron emerge from the field at $x = 0$ (which means its motion is reversed) or at $x = 0.50$ m (which means its motion is not reversed)? (b) What is its speed when it emerges?

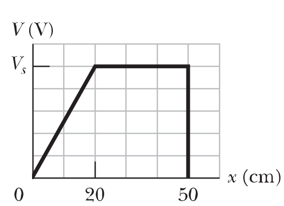

Fig. 24-52 Problem 54.

••55 An electron is projected with an initial speed of 3.2×10^5 m/s directly toward a proton that is fixed in place. If the electron is initially a great distance from the proton, at what distance from the proton is the speed of the electron instantaneously equal to twice the initial value?

••56 Figure 24-53a shows three particles on an x axis. Particle 1 (with a charge of $+5.0\ \mu C$) and particle 2 (with a charge of $+3.0\ \mu C$) are fixed in place with separation $d = 4.0$ cm. Particle 3 can be moved along the x axis to the right of particle 2. Figure 24-53b gives the electric potential energy U of the three-particle system as a function of the x coordinate of particle 3. The scale of the vertical axis is set by $U_s = 5.0$ J. What is the charge of particle 3?

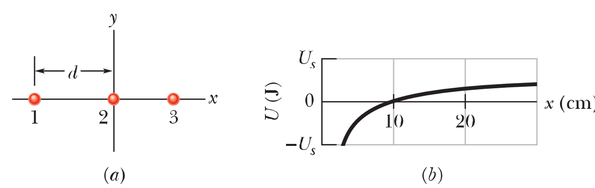

Fig. 24-53 Problem 56.

••57 SSM Identical 50 μC charges are fixed on an x axis at $x = \pm 3.0$ m. A particle of charge $q = -15\ \mu C$ is then released from rest at a point on the positive part of the y axis. Due to the symmetry of the situation, the particle moves along the y axis and has kinetic energy 1.2 J as it passes through the point $x = 0$, $y = 4.0$ m. (a) What is the kinetic energy of the particle as it passes through the origin? (b) At what negative value of y will the particle momentarily stop?

••58 GO *Proton in a well.* Figure 24-54 shows electric potential V along an x axis. The scale of the vertical axis is set by $V_s = 10.0$ V. A proton is to be released at $x = 3.5$ cm with initial kinetic energy 4.00 eV. (a) If it is initially moving in the negative direction of the axis, does it reach a turning point (if so, what is the x coordinate of that point) or does it escape from the plotted region (if so, what is its speed at $x = 0$)? (b) If it is initially moving in the positive direction of the axis, does it reach a turning point (if so, what is the x coordinate of that point) or does it escape from the plotted region (if so, what is its speed at $x = 6.0$ cm)? What are the (c) magnitude F and (d) direction (positive or negative direction of the x axis) of the electric force on the proton if the proton moves just to the left of $x = 3.0$ cm? What are (e) F and (f) the direction if the proton moves just to the right of $x = 5.0$ cm?

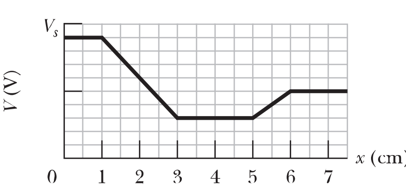

Fig. 24-54 Problem 58.

••59 In Fig. 24-55, a charged particle (either an electron or a proton) is moving rightward between two parallel charged plates separated by distance $d = 2.00$ mm. The plate potentials are $V_1 = -70.0$ V and $V_2 = -50.0$ V. The particle is slowing from an initial speed of 90.0 km/s at the left plate. (a) Is the particle an electron or a proton? (b) What is its speed just as it reaches plate 2?

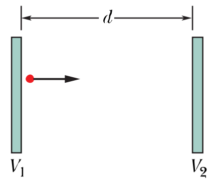

Fig. 24-55 Problem 59.

••60 In Fig. 24-56*a*, we move an electron from an infinite distance to a point at distance $R = 8.00$ cm from a tiny charged ball. The move requires work $W = 2.16 \times 10^{-13}$ J by us. (a) What is the charge Q on the ball? In Fig. 24-56*b*, the ball has been sliced up and the slices spread out so that an equal amount of charge is at the hour positions on a circular clock face of radius $R = 8.00$ cm. Now the electron is brought from an infinite distance to the center of the circle. (b) With that addition of the electron to the system of 12 charged particles, what is the change in the electric potential energy of the system?

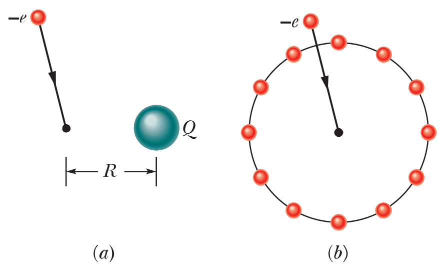

Fig. 24-56 Problem 60.

•••61 Suppose N electrons can be placed in either of two configurations. In configuration 1, they are all placed on the circumference of a narrow ring of radius R and are uniformly distributed so that the distance between adjacent electrons is the same everywhere. In configuration 2, $N - 1$ electrons are uniformly distributed on the ring and one electron is placed in the center of the ring. (a) What is the smallest value of N for which the second configuration is less energetic than the first? (b) For that value of N, consider any one circumference electron—call it e_0. How many other circumference electrons are closer to e_0 than the central electron is?

sec. 24-12 Potential of a Charged Isolated Conductor

•62 Sphere 1 with radius R_1 has positive charge q. Sphere 2 with radius $2.00R_1$ is far from sphere 1 and initially uncharged. After the separated spheres are connected with a wire thin enough to retain only negligible charge, (a) is potential V_1 of sphere 1 greater than, less than, or equal to potential V_2 of sphere 2? What fraction of q ends up on (b) sphere 1 and (c) sphere 2? (d) What is the ratio σ_1/σ_2 of the surface charge densities of the spheres?

•63 SSM WWW Two metal spheres, each of radius 3.0 cm, have a center-to-center separation of 2.0 m. Sphere 1 has charge $+1.0 \times 10^{-8}$ C; sphere 2 has charge -3.0×10^{-8} C. Assume that the separation is large enough for us to say that the charge on each sphere is uniformly distributed (the spheres do not affect each other). With $V = 0$ at infinity, calculate (a) the potential at the point halfway between the centers and the potential on the surface of (b) sphere 1 and (c) sphere 2.

•64 A hollow metal sphere has a potential of +400 V with respect to ground (defined to be at $V = 0$) and a charge of 5.0×10^{-9} C. Find the electric potential at the center of the sphere.

•65 SSM What is the excess charge on a conducting sphere of radius $r = 0.15$ m if the potential of the sphere is 1500 V and $V = 0$ at infinity?

••66 Two isolated, concentric, conducting spherical shells have radii $R_1 = 0.500$ m and $R_2 = 1.00$ m, uniform charges $q_1 = +2.00$ μC and $q_2 = +1.00$ μC, and negligible thicknesses. What is the magnitude of the electric field E at radial distance (a) $r = 4.00$ m, (b) $r = 0.700$ m, and (c) $r = 0.200$ m? With $V = 0$ at infinity, what is V at (d) $r = 4.00$ m, (e) $r = 1.00$ m, (f) $r = 0.700$ m, (g) $r = 0.500$ m, (h) $r = 0.200$ m, and (i) $r = 0$? (j) Sketch $E(r)$ and $V(r)$.

••67 A metal sphere of radius 15 cm has a net charge of 3.0×10^{-8} C. (a) What is the electric field at the sphere's surface? (b) If $V = 0$ at infinity, what is the electric potential at the sphere's surface? (c) At what distance from the sphere's surface has the electric potential decreased by 500 V?

Additional Problems

68 Here are the charges and coordinates of two point charges located in an xy plane: $q_1 = +3.00 \times 10^{-6}$ C, $x = +3.50$ cm, $y = +0.500$ cm and $q_2 = -4.00 \times 10^{-6}$ C, $x = -2.00$ cm, $y = +1.50$ cm. How much work must be done to locate these charges at their given positions, starting from infinite separation?

69 SSM A long, solid, conducting cylinder has a radius of 2.0 cm. The electric field at the surface of the cylinder is 160 N/C, directed radially outward. Let A, B, and C be points that are 1.0 cm, 2.0 cm, and 5.0 cm, respectively, from the central axis of the cylinder. What are (a) the magnitude of the electric field at C and the electric potential differences (b) $V_B - V_C$ and (c) $V_A - V_B$?

70 *The chocolate crumb mystery.* This story begins with Problem 60 in Chapter 23. (a) From the answer to part (a) of that problem, find an expression for the electric potential as a function of the radial distance r from the center of the pipe. (The electric potential is zero on the grounded pipe wall.) (b) For the typical volume charge density $\rho = -1.1 \times 10^{-3}$ C/m^3, what is the difference in the electric potential between the pipe's center and its inside wall? (The story continues with Problem 60 in Chapter 25.)

71 SSM Starting from Eq. 24-30, derive an expression for the electric field due to a dipole at a point on the dipole axis.

72 The magnitude E of an electric field depends on the radial distance r according to $E = A/r^4$, where A is a constant with the unit volt–cubic meter. As a multiple of A, what is the magnitude of the electric potential difference between $r = 2.00$ m and $r = 3.00$ m?

73 (a) If an isolated conducting sphere 10 cm in radius has a net charge of 4.0 μC and if $V = 0$ at infinity, what is the potential on the surface of the sphere? (b) Can this situation actually occur, given that the air around the sphere undergoes electrical breakdown when the field exceeds 3.0 MV/m?

74 Three particles, charge $q_1 = +10$ μC, $q_2 = -20$ μC, and $q_3 = +30\mu$C, are positioned at the vertices of an isosceles triangle as shown in Fig. 24-57. If $a = 10$ cm and $b = 6.0$ cm, how much work must an external agent do to exchange the positions of (a) q_1 and q_3 and, instead, (b) q_1 and q_2?

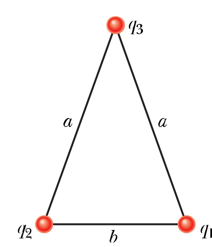

Fig. 24-57 Problem 74.

75 An electric field of approximately 100 V/m is often observed near the surface of Earth. If this were the field over the entire surface, what would be the electric potential of a point on the surface? (Set $V = 0$ at infinity.)

76 A Gaussian sphere of radius 4.00 cm is centered on a ball that has a radius of 1.00 cm and a uniform charge distribution. The total (net) electric flux through the surface of the Gaussian sphere is $+5.60 \times 10^4\ \mathrm{N \cdot m^2/C}$. What is the electric potential 12.0 cm from the center of the ball?

77 In a Millikan oil-drop experiment (Section 22-8), a uniform electric field of 1.92×10^5 N/C is maintained in the region between two plates separated by 1.50 cm. Find the potential difference between the plates.

78 Figure 24-58 shows three circular, nonconducting arcs of radius $R = 8.50$ cm. The charges on the arcs are $q_1 = 4.52$ pC, $q_2 = -2.00q_1$, $q_3 = +3.00q_1$. With $V = 0$ at infinity, what is the net electric potential of the arcs at the common center of curvature?

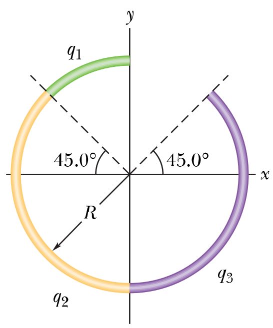

Fig. 24-58 Problem 78.

79 An electron is released from rest on the axis of an electric dipole that has charge e and charge separation $d = 20$ pm and that is fixed in place. The release point is on the positive side of the dipole, at distance $7.0d$ from the dipole center. What is the electron's speed when it reaches a point $5.0d$ from the dipole center?

80 Figure 24-59 shows a ring of outer radius $R = 13.0$ cm, inner radius $r = 0.200R$, and uniform surface charge density $\sigma = 6.20\ \mathrm{pC/m^2}$. With $V = 0$ at infinity, find the electric potential at point P on the central axis of the ring, at distance $z = 2.00R$ from the center of the ring.

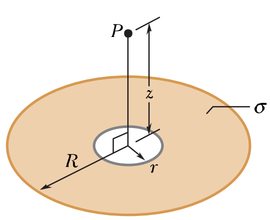

Fig. 24-59 Problem 80.

81 *Electron in a well.* Figure 24-60 shows electric potential V along an x axis. The scale of the vertical axis is set by $V_s = 8.0$ V. An electron is to be released at $x = 4.5$ cm with initial kinetic energy 3.00 eV. (a) If it is initially moving in the negative direction of the axis, does it reach a turning point (if so, what is the x coordinate of that point) or does it escape from the plotted region (if so, what is its speed at $x = 0$)? (b) If it is initially moving in the positive direction of the axis, does it reach a turning point (if so, what is the x coordinate of that point) or does it escape from the plotted region (if so, what is its speed at $x = 7.0$ cm)? What are the (c) magnitude F and (d) direction (positive or negative direction of the x axis) of the electric force on the electron if the electron moves just to the left of $x = 4.0$ cm? What are (e) F and (f) the direction if it moves just to the right of $x = 5.0$ cm?

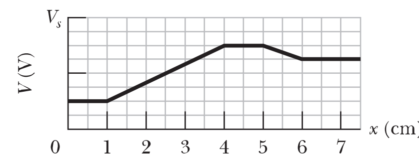

Fig. 24-60 Problem 81.

82 (a) If Earth had a uniform surface charge density of 1.0 electron/m^2 (a very artificial assumption), what would its potential be? (Set $V = 0$ at infinity.) What would be the (b) magnitude and (c) direction (radially inward or outward) of the electric field due to Earth just outside its surface?

83 In Fig. 24-61, point P is at distance $d_1 = 4.00$ m from particle 1 ($q_1 = -2e$) and distance $d_2 = 2.00$ m from particle 2 ($q_2 = +2e$), with both particles fixed in place. (a) With $V = 0$ at infinity, what is V at P? If we bring a particle of charge $q_3 = +2e$ from infinity to P, (b) how much work do we do and (c) what is the potential energy of the three-particle sytem?

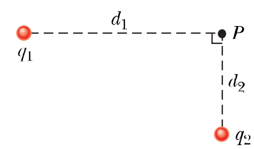

Fig. 24-61 Problem 83.

84 A solid conducting sphere of radius 3.0 cm has a charge of 30 nC distributed uniformly over its surface. Let A be a point 1.0 cm from the center of the sphere, S be a point on the surface of the sphere, and B be a point 5.0 cm from the center of the sphere. What are the electric potential differences (a) $V_S - V_B$ and (b) $V_A - V_B$?

85 In Fig. 24-62, we move a particle of charge $+2e$ in from infinity to the x axis. How much work do we do? Distance D is 4.00 m.

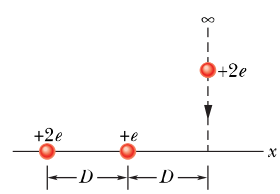

Fig. 24-62 Problem 85.

86 Figure 24-63 shows a hemisphere with a charge of 4.00 μC distributed uniformly through its volume. The hemisphere lies on an xy plane the way half a grapefruit might lie face down on a kitchen table. Point P is located on the plane, along a radial line from the hemisphere's center of curvature, at radial distance 15 cm. What is the electric potential at point P due to the hemisphere?

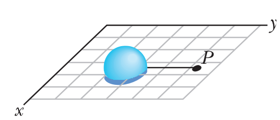

Fig. 24-63 Problem 86.

87 SSM Three +0.12 C charges form an equilateral triangle 1.7 m on a side. Using energy supplied at the rate of 0.83 kW, how many days would be required to move one of the charges to the midpoint of the line joining the other two charges?