Biot-Savart Law

- Jean-Baptiste Biot and Felix Savart also studied the forces on wires around the same time as Ampere developed his theories.
- To summarise their observations, at a point a distance *r* from the current *qu*,

$$\boxed{B = \frac{\mu qu \sin\theta}{4\pi r^2}}$$

- *Θ* is the angle between the direction of the current and the point at which the magnetic field is being obtained.

Biot-Savart Law

- There is a simple analogy here between **electric** and **magnetic** fluxes:

$$\vec{E} = \frac{q}{4\pi\varepsilon_0\varepsilon_r r^2}\hat{r}$$

$$B = \frac{\mu q u \sin\theta}{4\pi r^2}$$

- We can compare the source of electric field being the charge with the source of magnetic field being a ***moving charge***.

Biot-Savart Law

- The above expression for *B* is not the standard way of viewing the Biot-Savart Law.
- Instead we base it upon a *current element*.

- The charge passing a point in unit time is the current,
  - $qu=idl$

Biot-Savart Law

- So

$$B = \frac{\mu q u \sin\theta}{4\pi r^2}$$

becomes

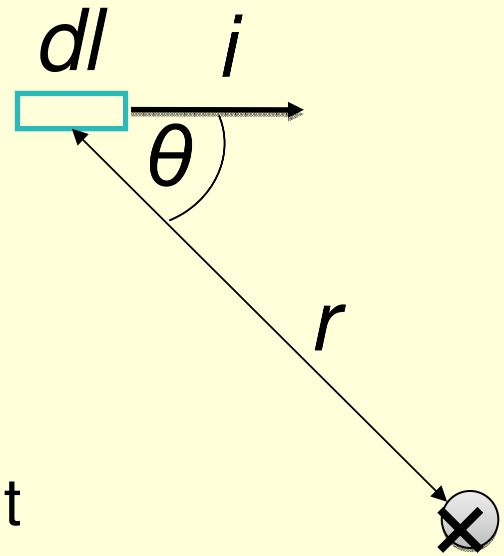
$$\boxed{dB = \frac{\mu i \sin\theta \, dl}{4\pi r^2}}$$

with *dB* being a small contribution to the total magnetic flux density arising from the small part of the current in the wire element "*idl*".

Biot-Savart Law

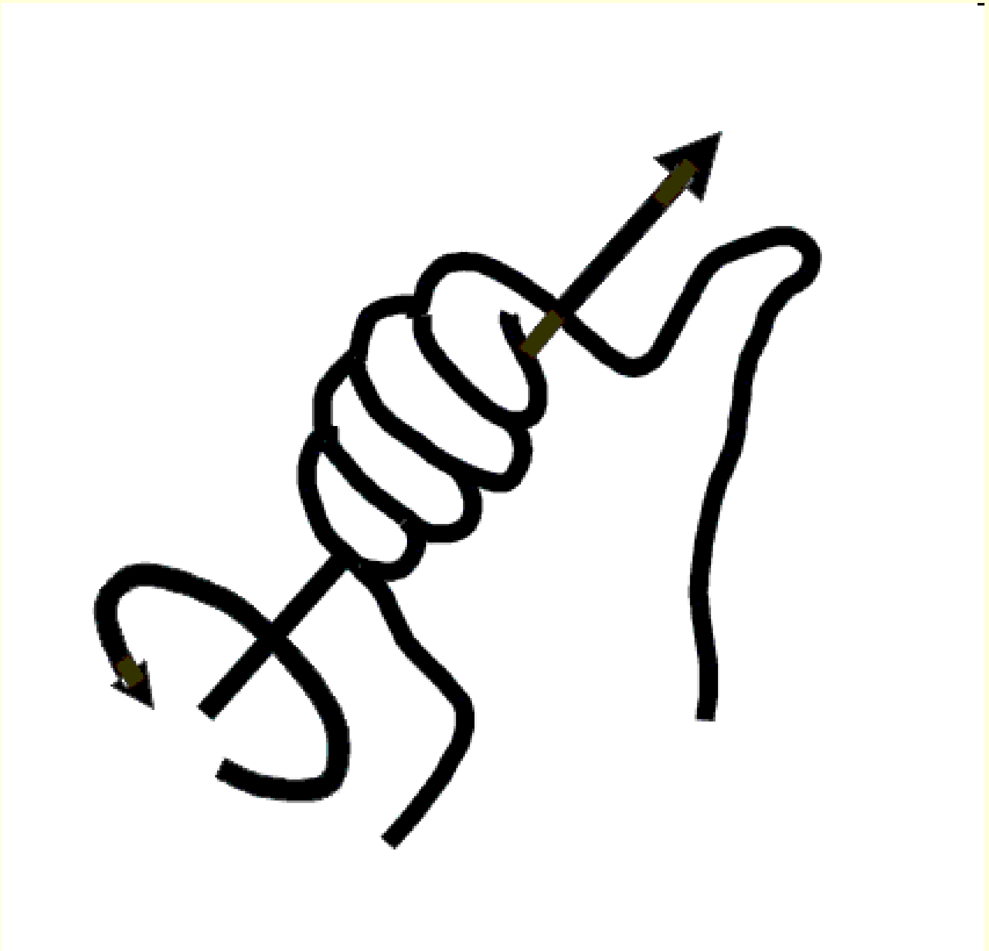
- We can eliminate *B* in favour of *H*:

$$\boxed{dH = \frac{i \sin\theta \, dl}{4\pi r^2}}$$



- The ***direction*** of the magnetic field is into the plane of the screen at *r*
  – This can be obtained using the "right hand screw rule".

Right hand screw rule



Biot-Savart Law

- It would be a mistake to interpret the Biot-Savart law as stated as indicating that there is a small amount of magnetic field coming from each element.
- The law is only complete in the ***integral*** form:

$$\boxed{\int dH = H = \int \frac{i \sin\theta \, dl}{4\pi r^2}}$$

Biot-Savart Law

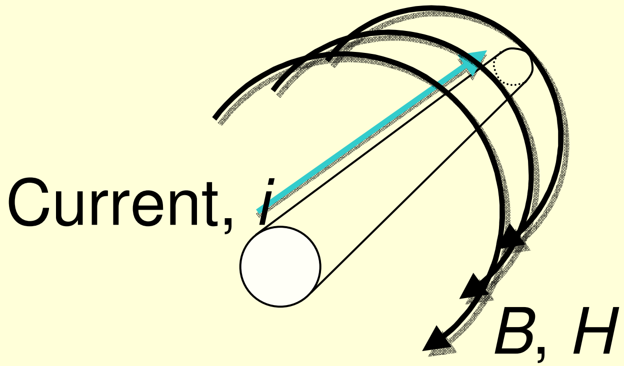
$$\vec{H} = \int \frac{i\, \vec{dl} \times \vec{r}}{4\pi r^3}$$

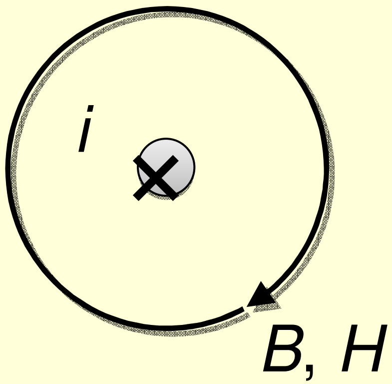
Application of the Biot-Savart Law

$$\int dH = H = \int \frac{i \sin\theta \, dl}{4\pi r^2}$$

- The Biot-Savart law in this form can be used to obtain the magnetic field due to some important systems.
- We shall now look at the example of the field about a long straight wire…

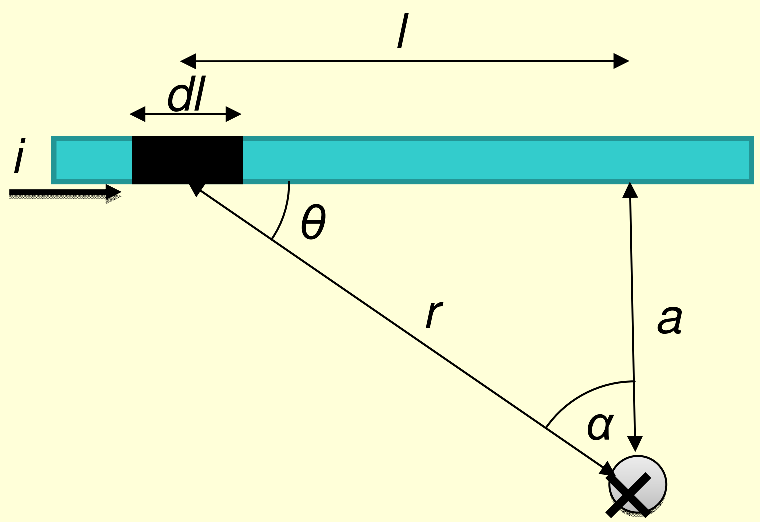
Application of the Biot-Savart Law





- The application requires us to evaluate the integral for a point relative to the wire, effectively summing over all current elements along an infinite wire.

Application of the Biot-Savart Law

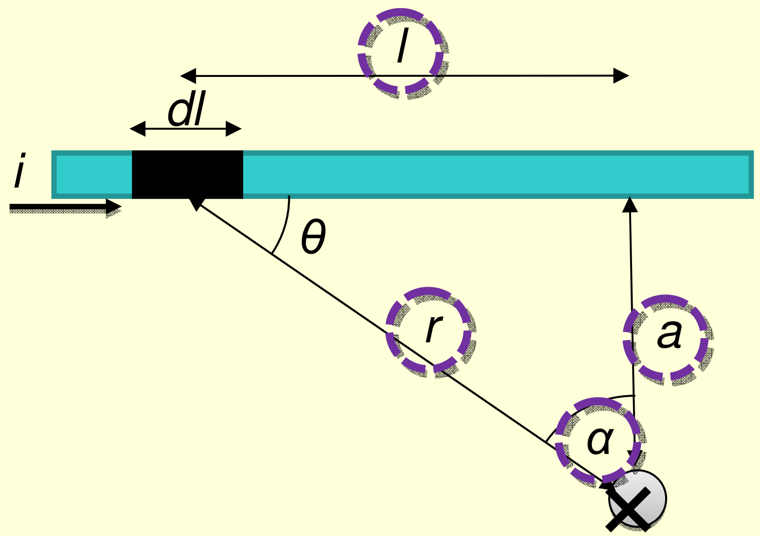


- The sum is over elements of this type, so that

$$H = \int_{-\infty}^{\infty} \frac{i \sin \theta \, dl}{4\pi r^2}$$

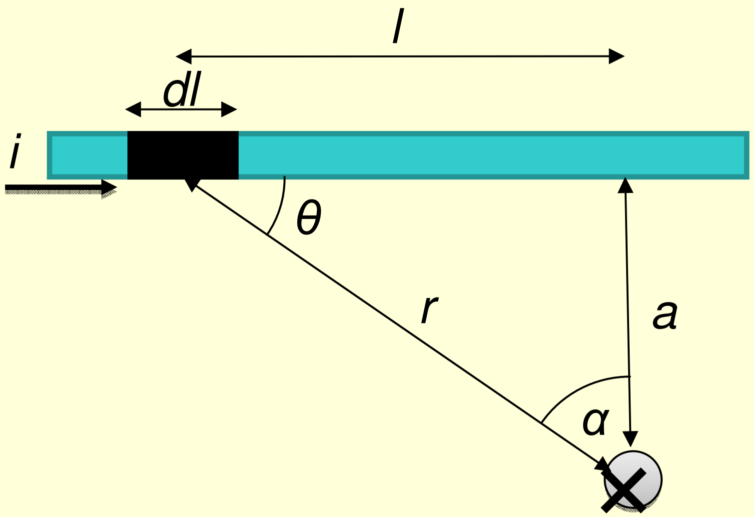
$$H = \int_{-\infty}^{\infty} \frac{i \cos \alpha \, dl}{4\pi r^2}$$

Application of the Biot-Savart Law



- To evaluate the integral, we need eliminate the interdependence of terms such as *r* and α.
- We can express *l* and *r* in terms of α.

Application of the Biot-Savart Law



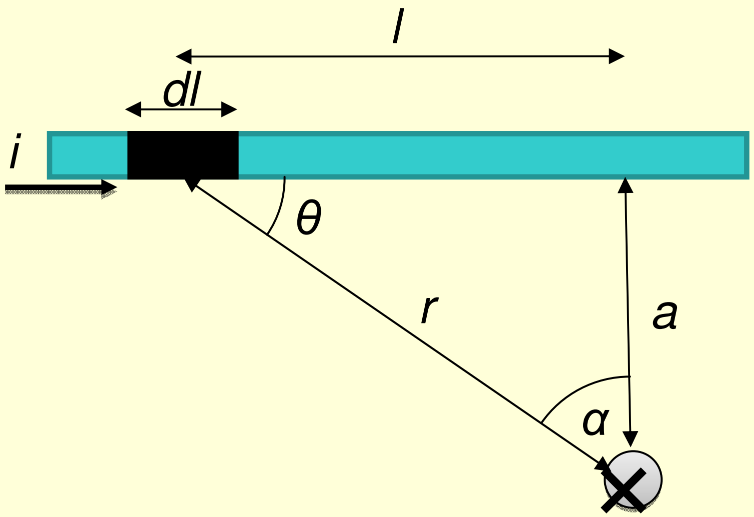
1. $a \tan \alpha = l$
2. $r \cos \theta = a$

- Differentiating 1 gives
  - $dl = a \sec^2\alpha \, d\alpha$

$$H = \int_{-\pi/2}^{\pi/2} \frac{i \cos \alpha \, a \, d\alpha}{4\pi r^2 \cos^2\alpha}$$

Note, when $l=-\infty$, $\alpha=-\pi/2$,
and when $l=\infty$, $\alpha=\pi/2$

Application of the Biot-Savart Law

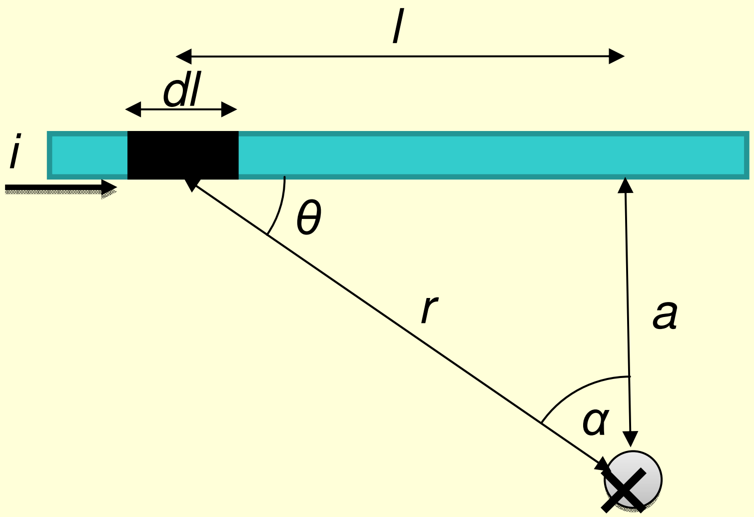


1. $a \tan \alpha = l$
2. $r \cos \theta = a$

- Substituting 2 gives

$$H = \int_{-\pi/2}^{\pi/2} \frac{i \cos \alpha \; a \; d\alpha}{4\pi a^2}$$

Application of the Biot-Savart Law

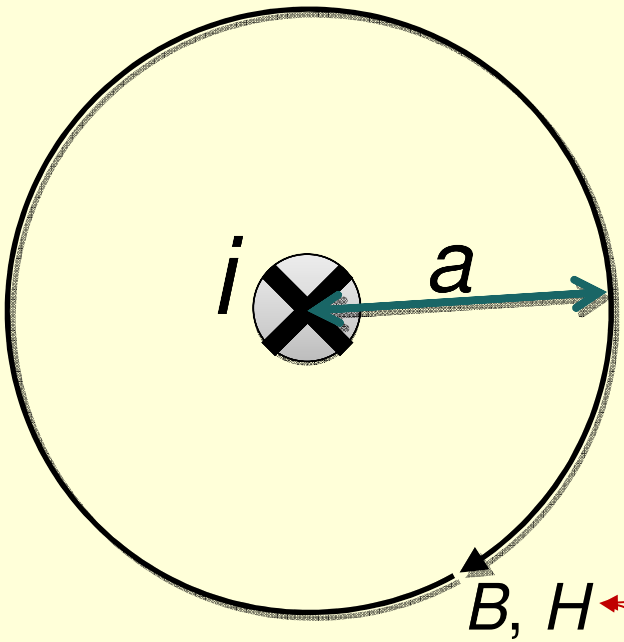


$$H = \int_{-\pi/2}^{\pi/2} \frac{i \cos\alpha \, a \, d\alpha}{4\pi a^2}$$

can be integrated readily, yielding:

$$\boxed{H = i/2\pi a}$$

The infinite, straight, uniform wire



Geometrically, the magnetic field strength, *H*, is equal to the current divided by the loop length, $2\pi a$.

$$H = \frac{i}{2\pi a}$$

$$B = \frac{\mu i}{2\pi a}$$

The field for a ***finite*** wire…

- What happens at the ends of a wire?
  - Hint: consider a *semi-infinite* wire.

Ampere's Law

- We can consider the generality of the observed geometrical interpretation of the result for the infinite wire.

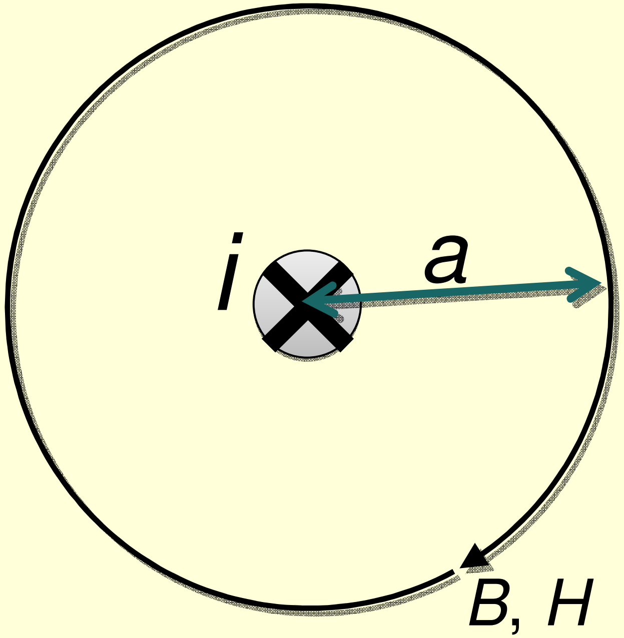
$$\boxed{H = \frac{i}{2\pi a}}$$

- We integrate the magnetic field around a closed loop enclosing the current…

$$\oint H.dl$$

Ampere's Law

- Since *H* and *dl* are **always parallel**, the scalar product is simply the product of the magnitudes:



$$\oint H.dl = \int_0^{2\pi a} \frac{i\,dl}{2\pi a} = i$$

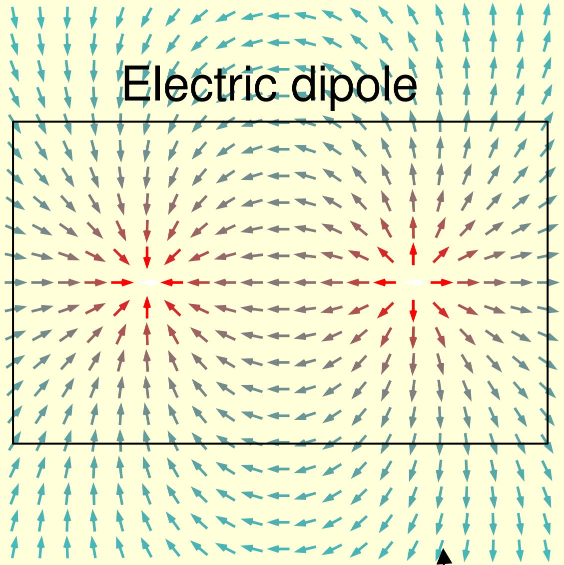
Ampere's Law

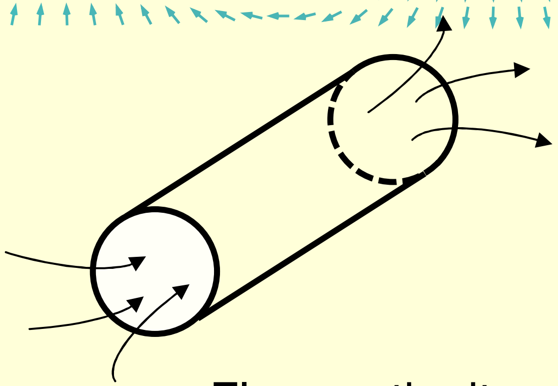
- Ampere showed that this is generally true, so that

$$\oint H.dl = i$$

- This is ***Ampere's Law***.
- In words, the law states that "The line integral of *H* around any closed path is equal to the current enclosed".

Gauss' Law





Flux continuity

- The circulation of magnetic field lines leads to an important result.
  - This is in stark contrast to electrostatics where electrical charge is a source of diverging or converging field lines
- The fact that magnetic flux circulates means that for any closed surface, as much flux must enter as leave
  - This is a conservation or continuity law
- There are no isolated magnetic flux sources or sinks

Gauss' Law

$$\oint_s B.ds = 0$$

- This is Gauss' Law of magnetic fields.
  - Do not confuse this with Gauss' law for electrostatics
- There is no **divergence** of *B*
  - *This refers to a derivative form of the law.*

Faraday's Law – the beginnings of electromagnetic machines

- In 1831, Michael Faraday established the principles which lay the foundations of most modern electrical machines.
  - Up to this time, all electricity came from chemical cells (batteries).
- He inverted the idea
  - *If an electrical current can produce a magnetic field, then the opposite should also be true: a magnetic field should be able to generate a current in a wire.*

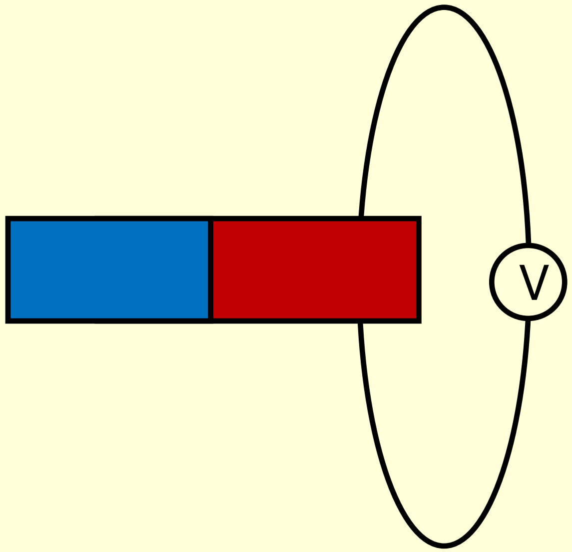
Faraday

Faraday’s Law – the beginnings of electromagnetic machines

- He spend a long time trying to show this idea in practice, placing magnets in the vicinity of wires and looking for in induced current or voltage – there was none.
- However, eventually it was found that a ***moving magnet*** next to a wire ***did*** induce an e.m.f.

Faraday

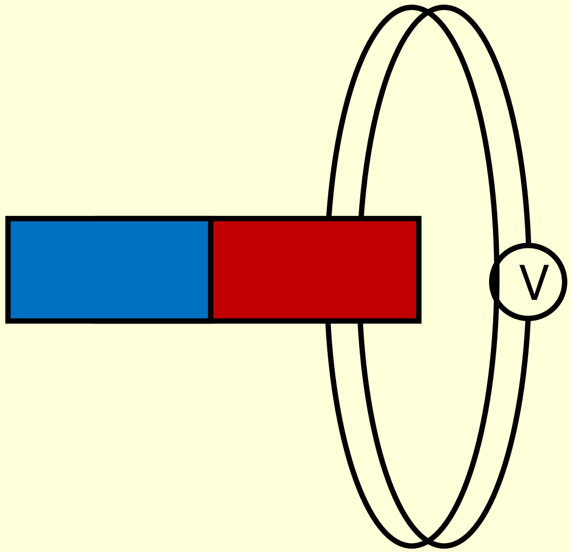
Faraday's Law – the beginnings of electromagnetic machines



- This is a simple arrangement that demonstrates the effect.
- Some of the magnet flux, ψ, from the permanent magnet links the coil.
- As the magnet moves the amount of flux linking the coil changes.
- The voltage induced in the coil is

$$V = \frac{d\psi}{dt}$$

Question

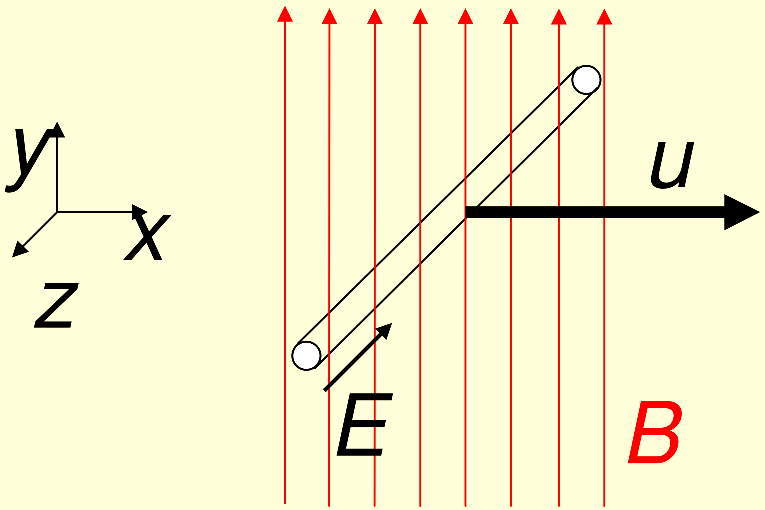


- What would happen if we added a second turn to the coil?

Application of Faraday's Law

Motional EMF

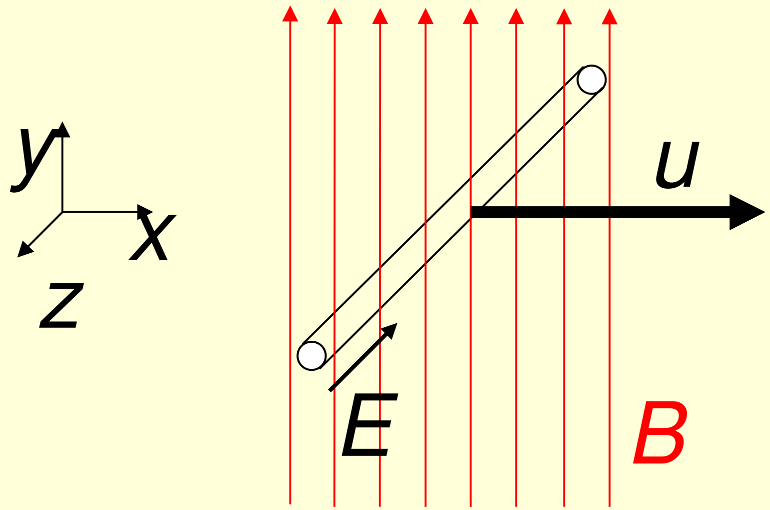
- Another view of the same effect can be obtained by viewing the impact upon a moving metal rod passing through a magnetic field of flux density, *B*.



$$F = q\,\vec{u} \times \vec{B}$$

Motional EMF

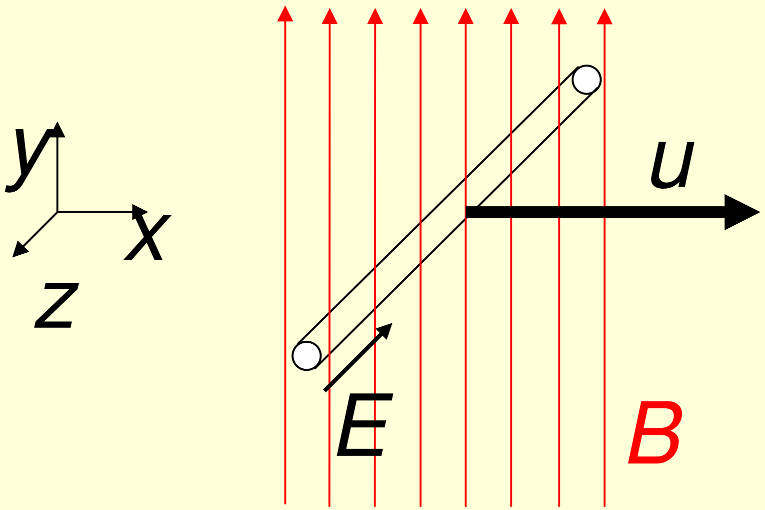
- For movement in the *x*-direction, and *B* in the y-direction, the **right-hand-rule** for the cross-product directs the force in the positive *z*-direction.
- The force acts on the electrons with charge -*e*.



$$F_z = -e\,u\,B$$

Motional EMF

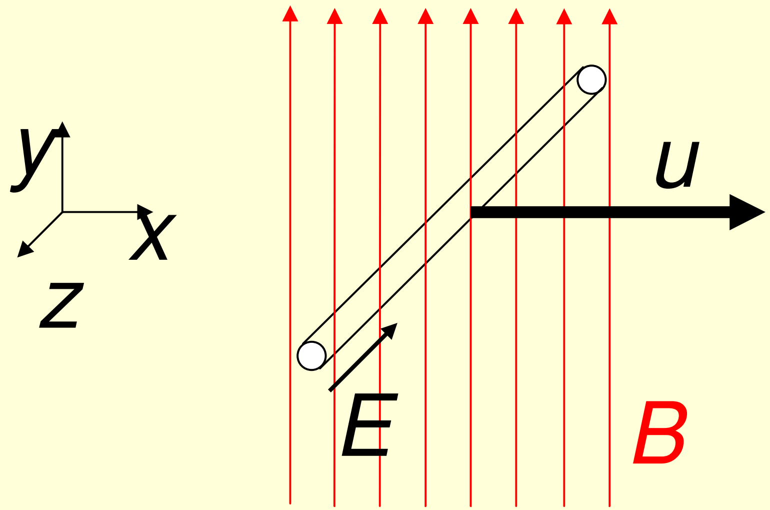
- The force on the electrons move the electrons in the negative *z*-direction.
- This creates a space charge, and hence an **electric field**, *E*, and a force given by
  - $F=qE=-eE$
  - E directed in the negative *z*-direction, so F is in the positive *z*-direction
    - C.f. Lenz's Law



$$\boxed{F_z = -e\,u\,B}$$

Motional EMF

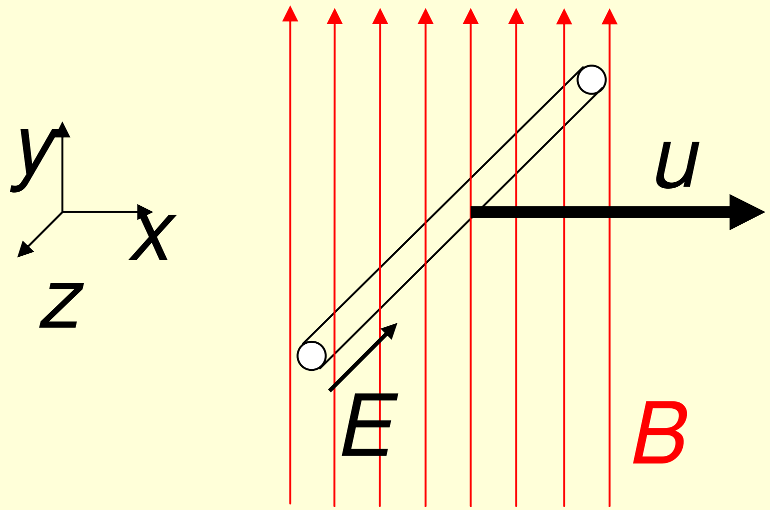
- In ***equilibrium***, the forces balance.
  - $E=uB$
- In terms of the vector quantities,
  - $\vec{E}=\vec{B} \times \vec{u}$



$$\boxed{e\,E = e\,u\,B}$$

Motional EMF

- If the rod is of length *L*, and moves in a direction perpendicular to the magnetic field, then the induced voltage is given by the integral of the electric field along the length of the rod.
- This is the same effect as the Faraday Law we saw earlier.



$$V = \int_0^L E\,dl = B\,L\,u$$

Comparison of induced voltages

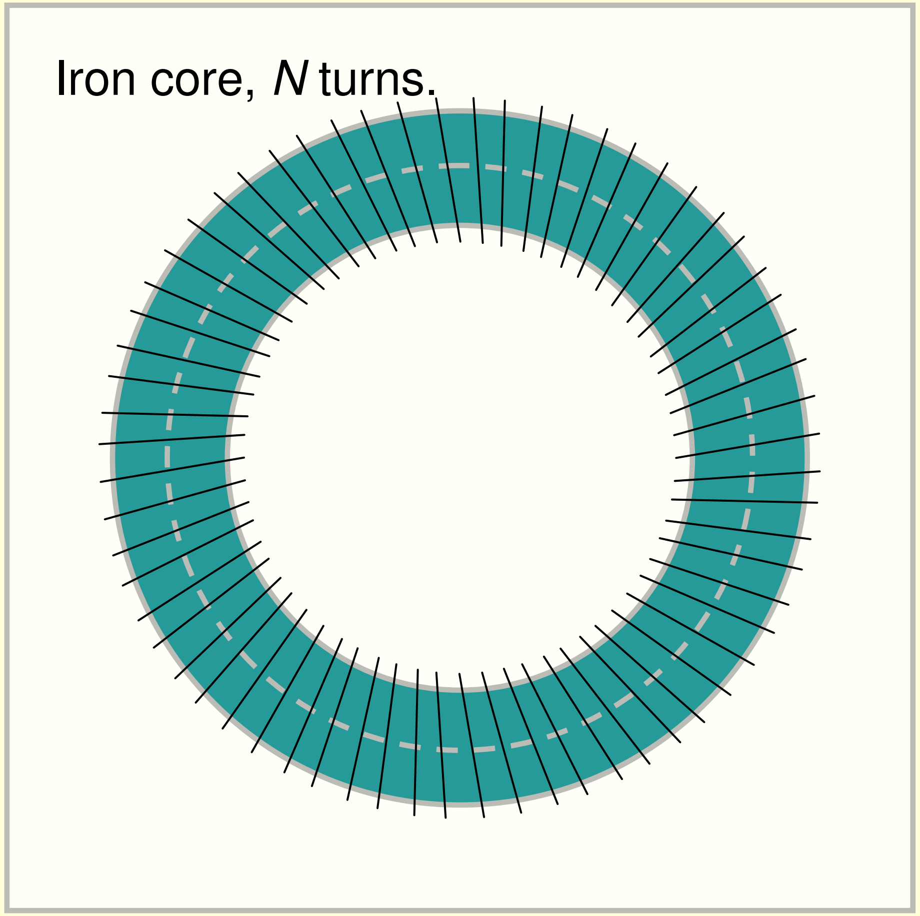
- Faraday's Law

$$V = \frac{d\psi}{dt}$$

- Lorentz force

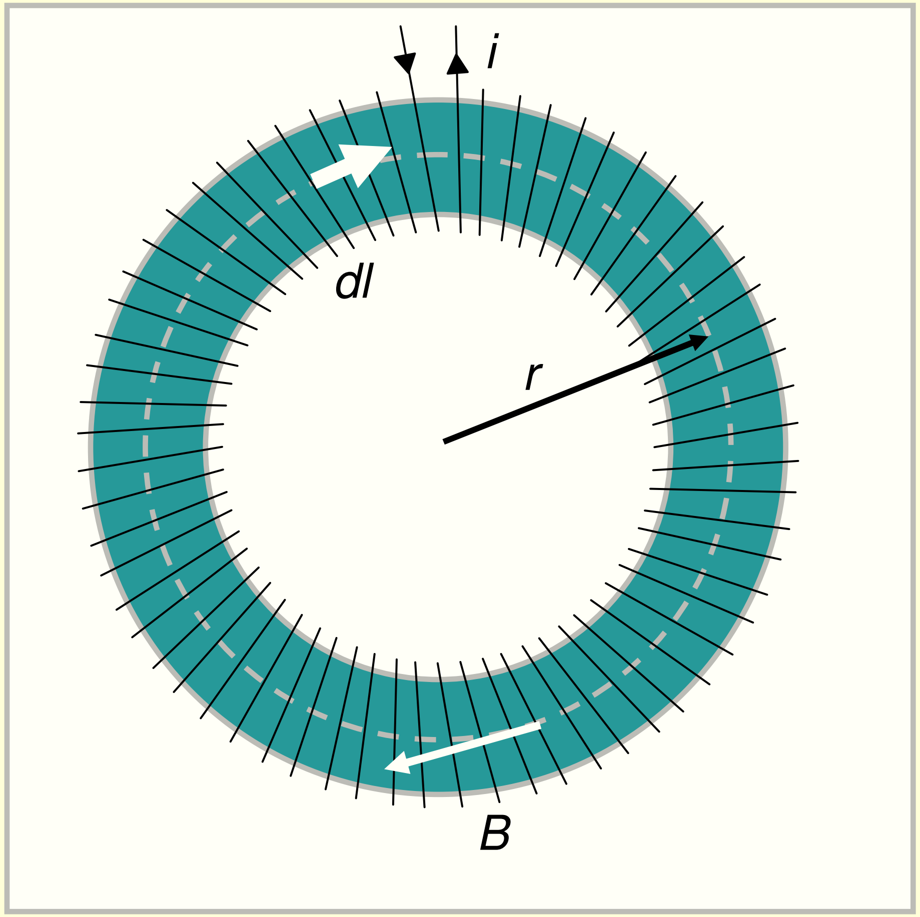
$$V = \int_0^L E\,dl = B\,L\,u$$

Application of Ampere's Law
Field in a solenoid



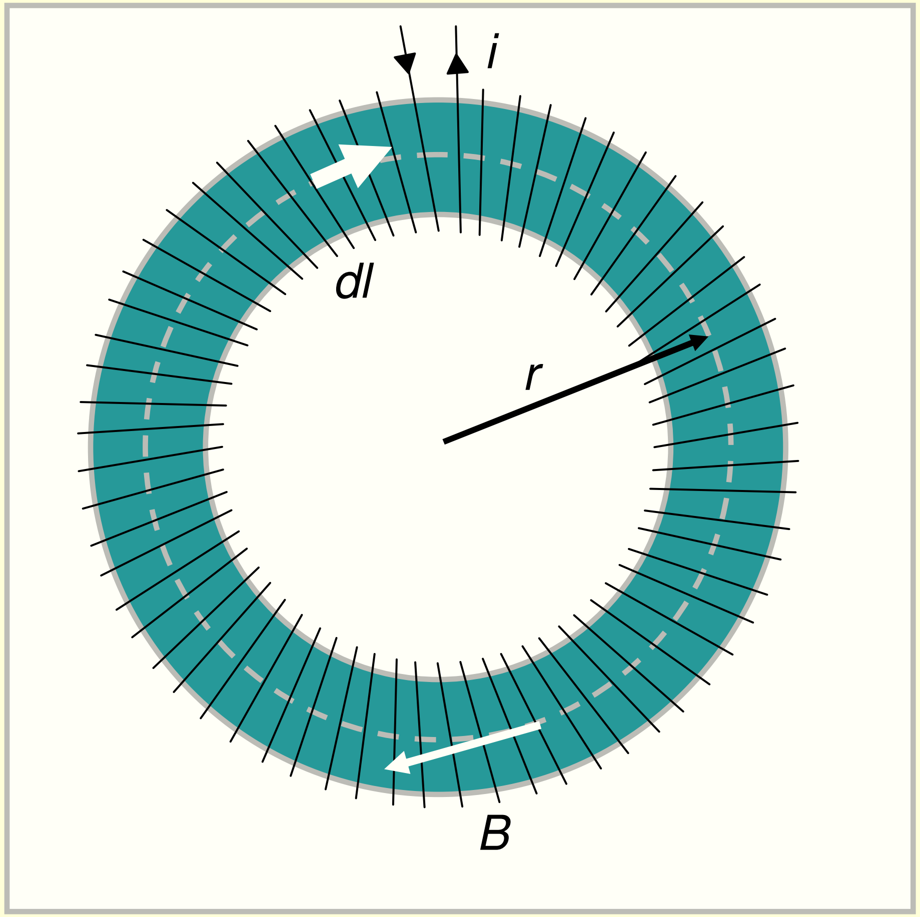
- We imagine an ideal toroidal solenoid to be uniform turn density, where all turns are perpendicular to the axis of the ring and the radius of the ring large enough for variations in the field with radius to be negligible.

Application of Ampere's Law
Field in a solenoid



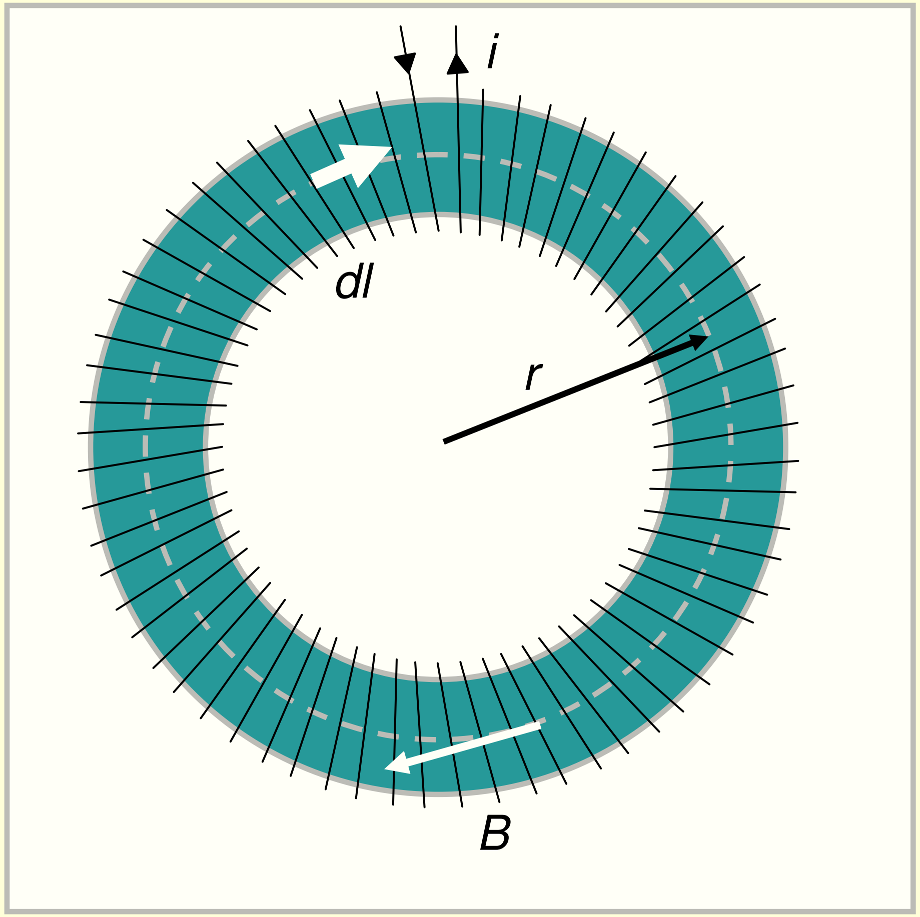
- The current enters and leaves the circuit at (approximately) the same place.
- We draw a circuit for Ampere's law through the axis of the ring.

Application of Ampere's Law
Field in a solenoid



- We have to integrage *B.ds* over the circle of radius *r*.
- By **symmetry** *H* and *B* are constant in magnitude on this circle.
- *B* and *H* are **tangential** to the circle.

Application of Ampere's Law
Field in a solenoid

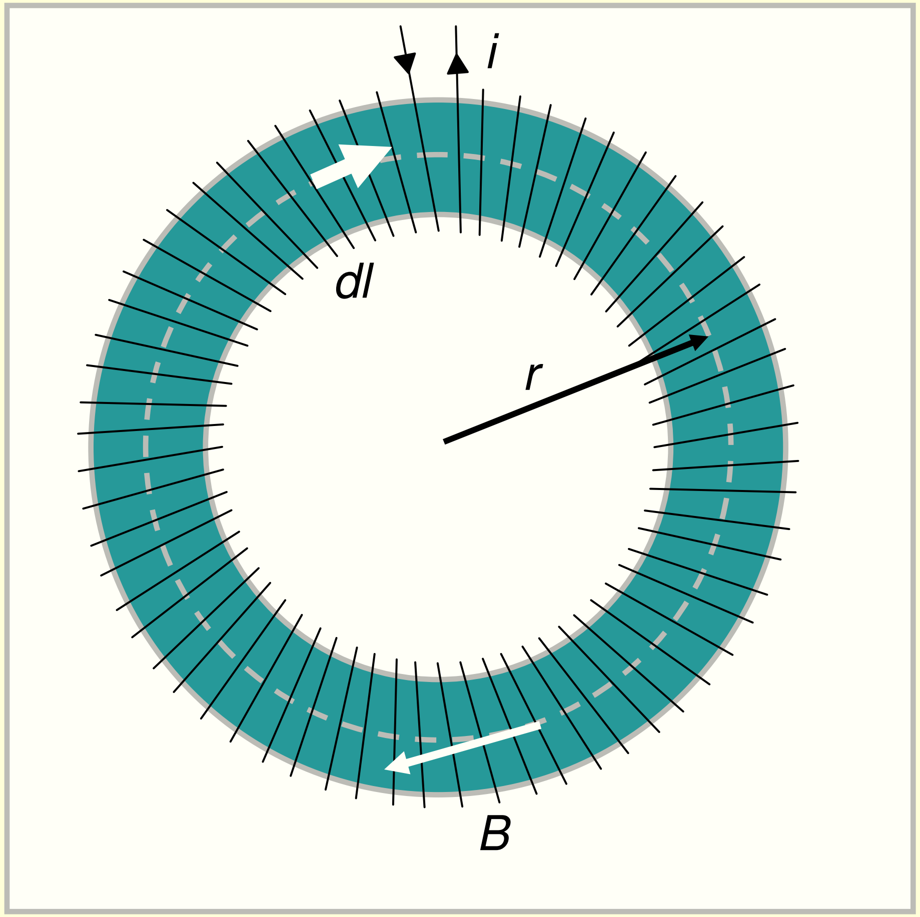


- We now apply Ampere's law:

$$\oint H.dl = i$$

- Since *H* and *dl* are **parallel**, the scalar product *H.dl* is just *Hdl.*

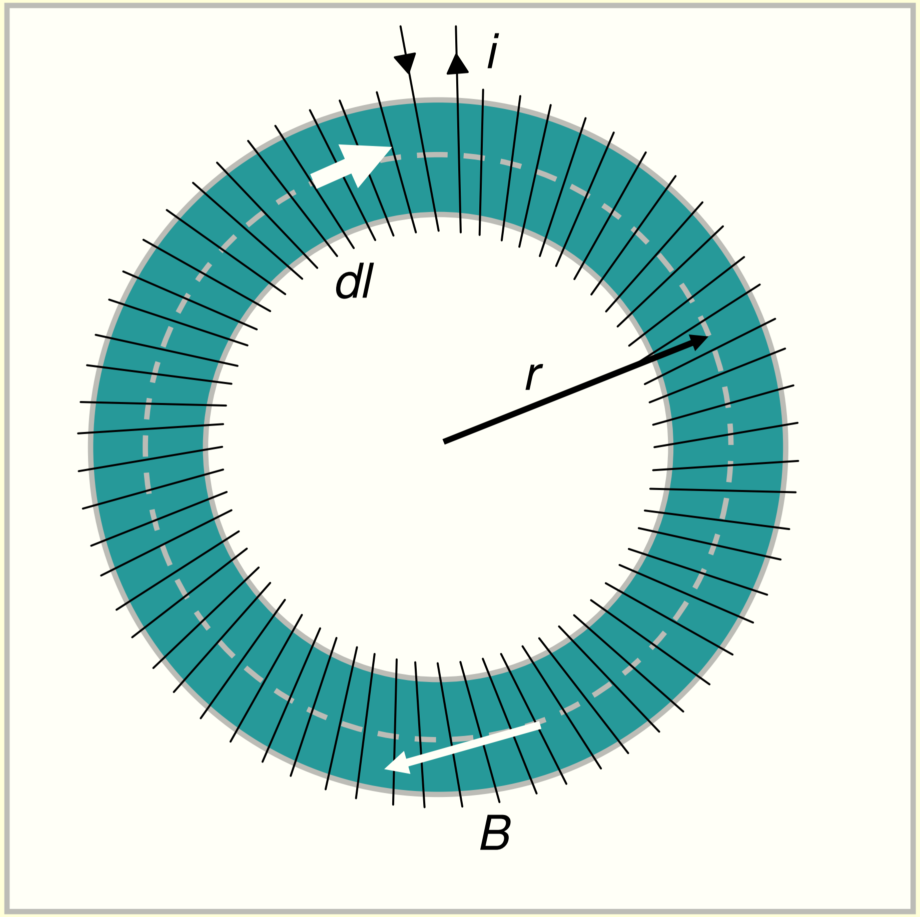
Application of Ampere's Law Field in a solenoid



- The path length is $2\pi r$.
- Since the path threads N loops carrying current i, the right hand side is Ni:

$$H \oint dl = H 2\pi r = Ni$$

Application of Ampere's Law Field in a solenoid



- We can note the MMF is equal to the integral, so is Ni.

$$H\oint dl = H2\pi r = Ni$$

- The magnetc flux density can be obtained from $B=\mu H$.

Application of Ampere's Law Field in a solenoid

- What is the field outside the coil?
- Is the answer to this subject to any assumptions and/or approximations in the derivation?

Example

- A coil of 1000 turns is wrapped around an iron ring of diameter 20mm, and cross-section $100mm^2$. The ring has a relative permeability of 120. If the coil carries a d.c. current of 2.0A, calculate the following:

1. The MMF of the coil
2. The H-field in the coil
3. The B-field
4. The flux which flows
5. The flux linkage of the coil

Example

1. If we now apply an a.c. current of 50Hz and peak amplitude 2.0A, calculate the voltage at the coil terminals
2. Can you work out the coil inductance?