Graphing and Equations

Plotting Functions (Graphing)

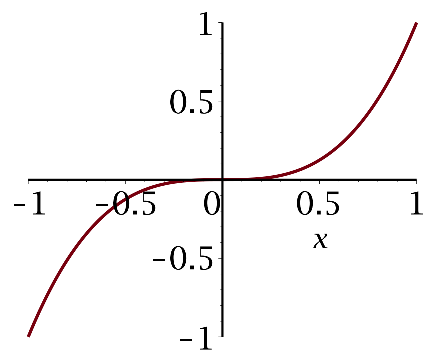
Let's see how to plot the graphs of functions. If we want to graph the function f(x) on the interval [a,b] then we type in:

plot(f(x), x=a..b)

That second part is x equals a period period b. So, the number a immediately followed by two periods and then the number b. It goes without saying that all decimal numbers need leading zeros!

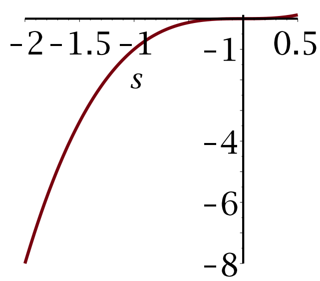
Let's graph x^3 on -1 to 1.

$plot(x^3,\ x=-1..1)$



If we wanted to use a different interval, we simply change it:

$plot(s^3,\ s=-2..0.5)$



- **Note:** The variable doesn't matter as long as the variable for the function and for the interval match.

Resizing Graphs:

The default size of graphs is great for the screen, but too big to fit on a page. You can easily resize them by clicking on them and then grabing a corner and draging to resize.

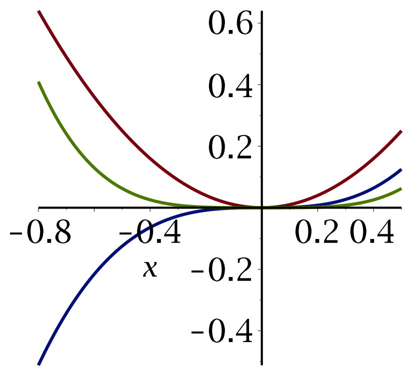
Multiple Functions on one Graph:

To plot more than one function, say f1(x), f2(x),..., fn(x) on [a,b] then we type in:

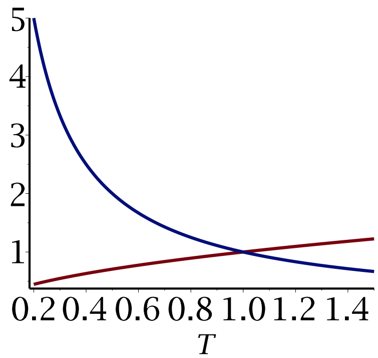
plot([f1(x),f2(x),...,fn(x)],x=a..b)

So for example:

$plot([x^2, x^3, x^4], x=-0.8..0.5)$



$plot\left(\left[\text{sqrt}(T), \frac{1}{T}\right], T=0.2..1.5\right)$



What if we had defined several functions? Could we use those? Sure.

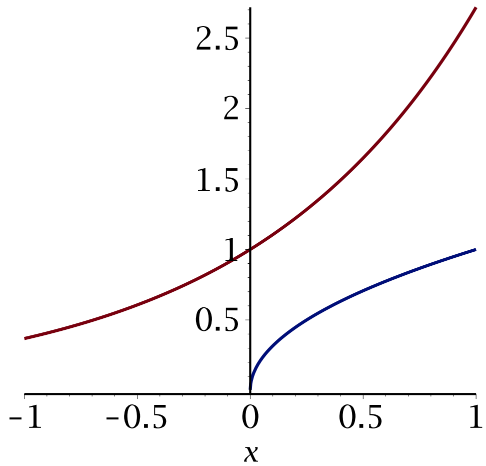
$f := x \rightarrow \exp(x);\ g := x \rightarrow \text{sqrt}(x);\ h := x \rightarrow \sin(x)$

$$x \to e^x$$

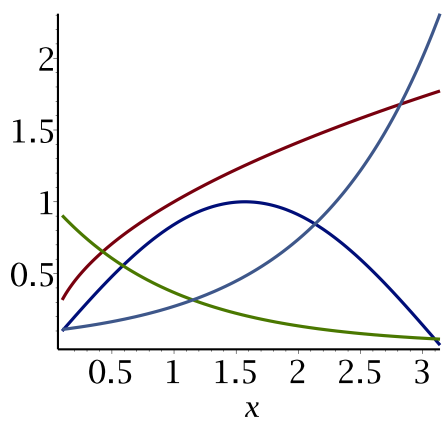
$$x \to \sqrt{x}$$

$$x \to \sin(x) \tag{1.2.1}$$

$plot([f(x), g(x)], x=-1..1)$



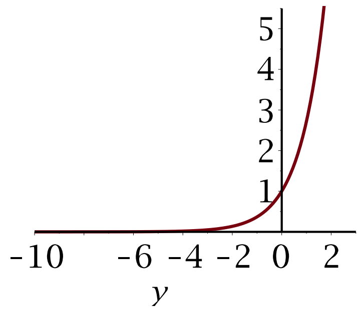
$plot\left(\left[g(x), h(x), \frac{1}{f(x)}, \frac{1}{10} \cdot f(x)\right], x = 0.1..3.14\right)$



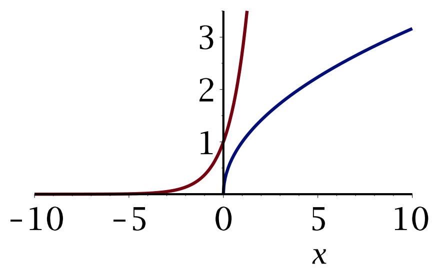
What if you leave out the x=a..b part?

If you leave out the ends of the interval, then Maple does its best to try to guess at them for you:

$plot(f(y))$



$plot([f(x), g(x)])$

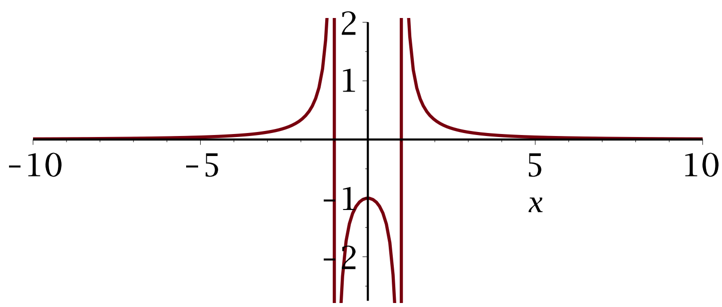


It does o.k., and is a good idea to try if you don't know what your interval should be.

How does Maple handle asymptotes?

The short answer is: it doesn't. Like, not at all. For example:

$$plot\left(\frac{1}{(x-1)\ (x+1)}\right)$$



Note that the asymptotes of the function are represented by vertical lines passing through -1 and 1. Actually Maple thinks they *are* part of the graph. This is because, roughly, Maple is sampling a large number of x values in between -10 and 10 and then connecting the outputs of the function on those values with straight lines. The sampling number is so high that a) the tiny straight lines join together to look curved to our naked eyes, and b) Those "straight line" asymptotes are *very nearly* straight lines connecting huge positive and huge negative values just outside of x=-1 and values just outside of x=1. **You must be very careful when looking at graphs that may/may not have asymptotes in Maple. Vertical looking lines are a good indicator of an asymptote, but they are not a 100% confirmation of asymptotes.**

Adding in a legend

If we are plotting multiple functions on the same axes, it would be helpful to have a marker to say which function was which. We do this with the legend option in the plot() command. It works like this:

$f := x \to x^2;\ g := x \to \frac{f(x)}{2+f(x)};\ h := f(x) + \frac{1}{f(x)+1}$

$$x \to x^2$$

$$x \to \frac{f(x)}{2+f(x)}$$

$$x^2 + \frac{1}{x^2+1} \qquad \textbf{(1.5.1)}$$

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plot([f(x), g(x), h(x)], x=-2..2, legend=["f(x)", "g(x)",
  "h(x)"])
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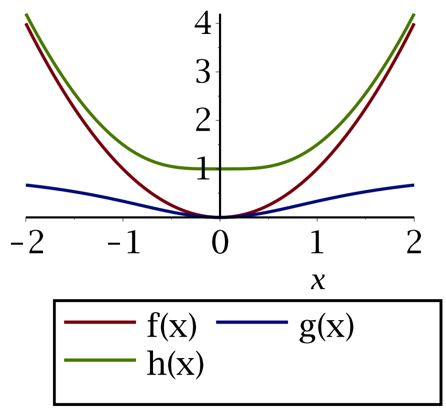


And now we have a nice key to let us know which curve is which!

Let's do one more example of this. Let's say we have two functions:

$P := q \to -q^2 + q + 1$

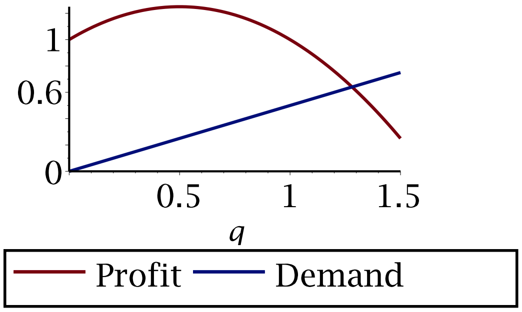
$$q \to -q^2 + q + 1 \qquad \textbf{(1.5.2)}$$

$Q := q \to 0.5\, q$

$$q \to 0.5\ q \qquad \textbf{(1.5.3)}$$

Where P(q) represents profit and Q(q) represents the demand for a given item. Then we could do:

$plot([P(q), Q(q)], q = 0..1.5, legend = [\text{"Profit"}, \text{"Demand"}])$



Exercises 1:

Open a new document and title it **Plotting Functions.** Put your name underneath. Skip a few lines and do the following:

1. Define the functions $f(x) = 1 - 2x + x^3$, $g(x) = \dfrac{x^2}{2x^3 + 1}$.

2. Plot f(x). Let Maple decide its x range.
3. Plot f(x) over the interval [-5,5]
4. Plot f(x) and g(x) on the same graph over the interval [-2,2]. Include a legend in your graph that labels the curves "f(x)" and "g(x)".
5. On a seperate sheet of paper, answer the following: Does it look like g(x) has an asymptote? Use algebra and sided limits to show that it does. [Hint: set the denominator equal to zero and solve. It doesn't factor, but you can take a cube root]

Save your file, print it. Attach it to your other sheet.

Solving Equations

Let's consider the functions:

$f := x \to x^2$

$$x \to x^2 \qquad \textbf{(2.1)}$$

$g := x \to 1 - 2\ x^2$

$$x \to 1 - 2\ x^2 \qquad \textbf{(2.2)}$$

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plot([f(x), g(x)], x=-1.05..0.75, legend=['f','g'])
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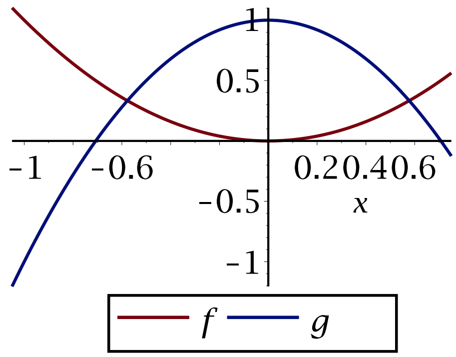

It looks like f(x) and g(x) cross near x=-0.5, and x=0.5. How can we find this intersection exactly? We solve: f(x)=g(x) ! Well, in Maple, it's that easy:

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solve(f(x) = g(x))
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$$\frac{1}{3}\sqrt{3}, -\frac{1}{3}\sqrt{3} \qquad \textbf{(2.3)}$$

Neat right?! What if we wanted to find the x-intercepts of g(x)? We solve g(x) = 0.

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solve(g(x) = 0)
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$$-\frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{2} \qquad \textbf{(2.4)}$$

Lets use this to solve other equations:

$solve(x^2 - 2\ x + 1 = 0)$

$$1, 1 \qquad \textbf{(2.5)}$$

$solve\left(\frac{1}{g(x)} - 1 = f(x)\right)$

$$0, 0, \frac{1}{2}\, I\sqrt{2}, -\frac{1}{2}\, I\sqrt{2} \qquad \textbf{(2.6)}$$

$solve(x^3 = -16\ x)$

$$0, 4\ I, -4\ I \qquad \textbf{(2.7)}$$

Remember: I is Maple's way of saying sqrt(-1).

Let's try two more:

$solve(x^3 + 2\ x^2 - 3\ x = x + 5)$

$$-1, -\frac{1}{2} - \frac{1}{2}\sqrt{21}, -\frac{1}{2} + \frac{1}{2}\sqrt{21} \qquad \textbf{(2.8)}$$

$solve(x^3 = -2\ x^2 + 5)$

$$\frac{1}{6}\left(476+12\sqrt{1545}\right)^{1/3}+\frac{8}{3\left(476+12\sqrt{1545}\right)^{1/3}}-\frac{2}{3},$$
$$-\frac{1}{12}\left(476+12\sqrt{1545}\right)^{1/3}-\frac{4}{3\left(476+12\sqrt{1545}\right)^{1/3}}-\frac{2}{3}+\frac{1}{2}I\sqrt{3}\left(\frac{1}{6}\left(476+12\sqrt{1545}\right)^{1/3}-\frac{8}{3\left(476+12\sqrt{1545}\right)^{1/3}}\right),$$
$$-\frac{1}{12}\left(476+12\sqrt{1545}\right)^{1/3}-\frac{4}{3\left(476+12\sqrt{1545}\right)^{1/3}}-\frac{2}{3}-\frac{1}{2}I\sqrt{3}\left(\frac{1}{6}\left(476+12\sqrt{1545}\right)^{1/3}-\frac{8}{3\left(476+12\sqrt{1545}\right)^{1/3}}\right) \quad \textbf{(2.9)}$$

Ack! That's horrifying! Maple is a CAS and so it's doing its best to give perfect algebraic results. But the above result is so convoluted that it's even hard to read. Did you notice that there were 3 solutions? Did you notice that the last two of them were imaginary and only the first one was real? What if we don't care about a perfect answer? What if we just want that one real numerical solution? The answer is to use: **fsolve().**

$fsolve(x^3=-2\ x^2+5)$

$$1.241896563 \quad \textbf{(2.10)}$$

This gives the one real solution to a very good degree of precision. Let's take a look at another one of the previous solutions using fsolve().

$fsolve(x^3+2\ x^2-3\ x=x+5)$

$$-2.791287847,\ -1.,\ 1.791287847 \quad \textbf{(2.11)}$$

If the equation we are trying to solve has no real solutions, then fsolve() returns nothing:

$fsolve(x^2=-1\ )$

[There was nothing returned in the above, not even a return number].

Exercises 2:

Open a new document and title it **Solving Equations**. Put your name under it and skip a few lines.

1. Define two functions $f(x)=x-1$ and $g(x)=2\,x^3$.
2. Plot both f(x) and g(x) on the same axes. Include a legend and let Maple

decide your interval for x. Try to grab your graph and resize it so that it is reasonable in size.
3. Looks pretty bad right? If you look closely, you can see that f(x) is smashed up against the x-axis. Let's fix this by plotting f(x) and g(x) again, this time over the interval [-3,3].
4. Plot both f(x) and g(x) on the same axes one last time. You pick a smaller interval that contains the place where f(x) and g(x) might have an intersection.
5. Use solve() to find the exact value for x where f(x) and g(x) intersect.
6. Find the x-intercepts of $h(x) = 1 - 2x + x^2$ by solving h(x) = 0.
7. Solve: $e^x = 6x^4 + 2x^2 + 12x$ using fsolve()
8. Use fsolve() to find the numerical solutions to: $x^3 - 8x = x^5 + 19x^2 + 3$
9. Use fsolve() to find the numerical values for x where the graphs of $y = 3x^3 - 2x^2 + 1$ and $y = 2x^3 + x^2 + x + 5$.

Save and print.