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from LinkedBinaryTree import LinkedBinaryTree
class ExpressionTree(LinkedBinaryTree):
    """An arithmetic expression tree."""
    def __init__(self, token, left=None, right=None):
        """Create an expression tree.

        In a single parameter form, token should be a leaf value (e.g., '42'),
        and the expression tree will have that value at an isolated node.

        In a three-parameter version, token should be an operator,
        and left and right should be existing ExpressionTree instances
        that become the operands for the binary operator.
        """
        super().__init__( ) # LinkedBinaryTree initialization
        if not isinstance(token, str):
            raise TypeError('Token must be a string')
        self._add_root(token) # use inherited, nonpublic method
        if left is not None: # presumably three-parameter form
            if token not in '+-*x/':
                raise ValueError('token must be valid operator')
            self._attach(self.root( ), left, right) # use inherited, nonpublic method

    def __str__(self):
        """Return string representation of the expression."""
        pieces = [ ] # sequence of piecewise strings to compose
        self._parenthesize_recur(self.root( ), pieces)
        return ".join(pieces)

    def _parenthesize_recur(self, p, result):
        """Append piecewise representation of p's subtree to resulting list."""
        if self.is_leaf(p):
            result.append(str(p.element( ))) # leaf value as a string
        else:
            result.append('(') # opening parenthesis
            self._parenthesize_recur(self.left(p), result) # left subtree
            result.append(p.element()) # operator
            self._parenthesize_recur(self.right(p), result) # right subtree
            result.append(')') # closing parenthesis

    def evaluate(self):
        """Return the numeric result of the expression."""
        return self._evaluate_recur(self.root( ))

    def _evaluate_recur(self, p):
        """Return the numeric result of subtree rooted at p."""
        if self.is_leaf(p):
            return float(p.element( )) # we assume element is numeric
        else:
            op = p.element( )
            left_val = self._evaluate_recur(self.left(p))

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right_val = self._evaluate_recur(self.right(p))
if op == '+':
    return left_val + right_val
elif op == '-':
    return left_val - right_val
elif op == '/':
    return left_val / right_val
else:
    return left_val * right_val # treat 'x' or '*' as multiplication

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def build_expression_tree(tokens):
    """Returns an ExpressionTree based upon a tokenized expression."""
    S = [] # we use Python list as stack
    for t in tokens:
        if t in '+-x*/': # t is an operator symbol
            S.append(t) # push the operator symbol
        elif t not in '()': # consider t to be a literal
            S.append(ExpressionTree(t)) # push trivial tree storing value
        elif t == ')': # compose a new tree from three constituent parts
            right = S.pop() # right subtree as per LIFO
            op = S.pop() # operator symbol
            left = S.pop() # left subtree
            S.append(ExpressionTree(op, left, right)) # repush tree
        # we ignore a left parenthesis
    return S.pop()

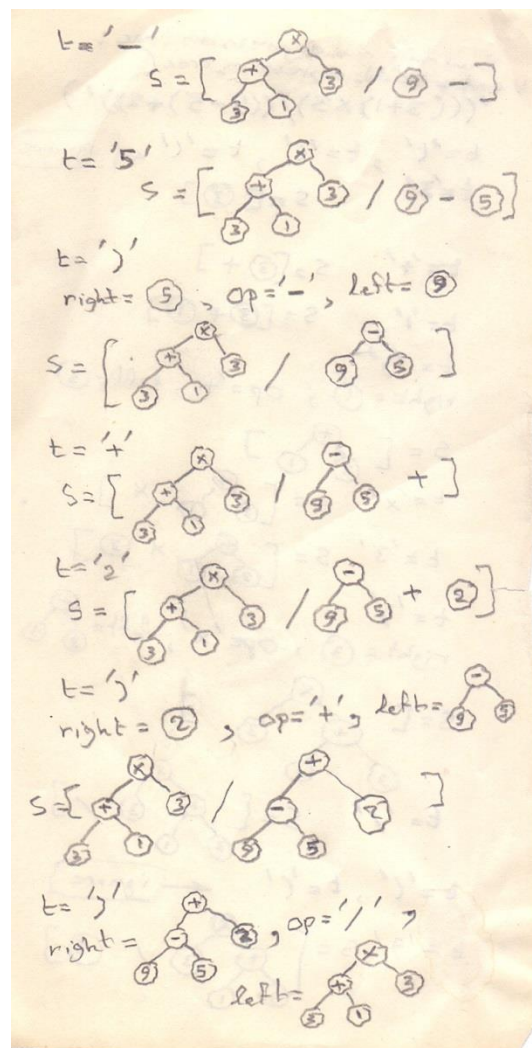
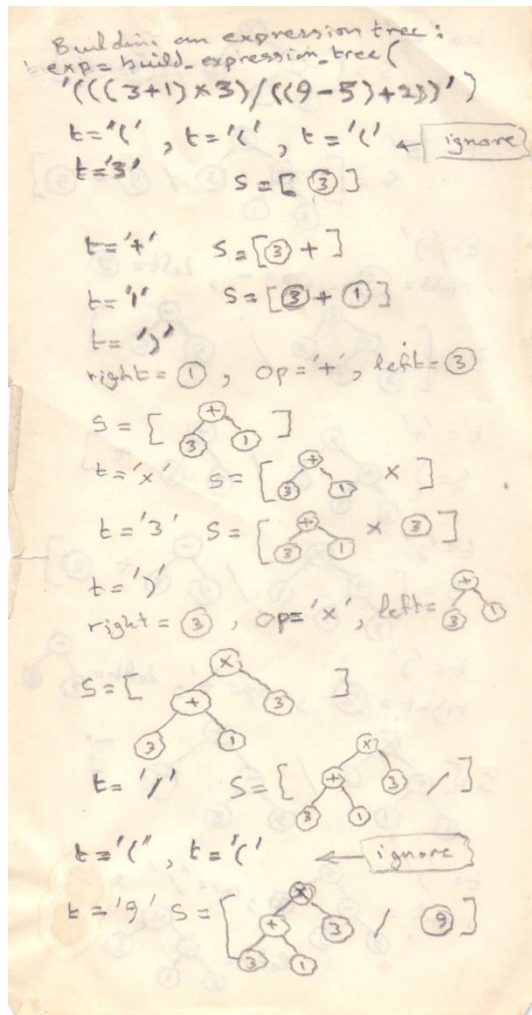
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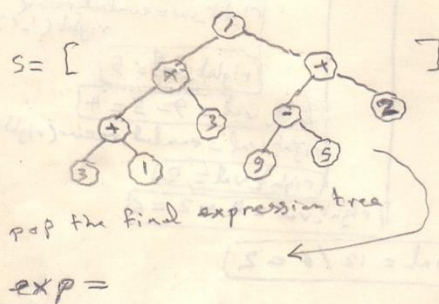
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S = '(((30+20)x40)/((90-50)+10))'
print(tokenize(S))
exp = build_expression_tree(tokenize(S))
print(exp)
print(exp.evaluate())

```

Build Expression Tree Example





$val = exp \cdot evaluate()$
 $evaluate_recur(root)$

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OP = '/'
left_val = evaluate_recur(left('/'))
    OP = 'x'
    left_val = evaluate_recur(left('x'))
        OP = '+'
        left_val = evaluate_recur(left('+'))
            left_val = 3
            right_val = evaluate_recur(right('+'))
                right_val = 1
            left_val = 3 + 1 = 4
        right_val = evaluate_recur(right('x'))
            right_val = 3
        left_val = 4 x 3 = 12
    right_val = evaluate_recur(right('/'))
        OP = '+'
        left_val = evaluate_recur(left('+'))
            OP = '-'
            left_val = evaluate_recur(left('-'))
                left_val = 9
                right_val = evaluate_recur(right('-'))
                    right_val = 5
                left_val = 9 - 5 = 4
            right_val = evaluate_recur(right('+'))
                right_val = 2
            left_val = 4 + 2 = 6
        right_val = 12 / 6 = 2
    
```

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    left_val = 9
    right_val = evaluate_recur(right('-'))
        right_val = 5
    left_val = 9 - 5 = 4
    right_val = evaluate_recur(right('+'))
        right_val = 2
    left_val = 4 + 2 = 6
    right_val = 12 / 6 = 2
    
```