load. For brittle materials, we will ensure that the stress levels are well below the ultimate tensile strength. We will also analyze deflection where it is critical to safety or performance of a part.

Two other failure modes that apply to machine members are fatigue and wear. *Fatigue* is the response of a part subjected to repeated loads (see Chapter 5). *Wear* is discussed within the chapters devoted to the machine elements, such as gears, bearings, and chains, for which it is a major concern.

3–3 REPRESENTING STRESSES ON A STRESS ELEMENT

One major goal of stress analysis is to determine *the point* within a load-carrying member that is subjected to the highest stress level. You should develop the ability to visualize a *stress element*, a single, infinitesimally small cube from the member in a highly stressed area, and to show vectors that represent the kind of stresses that exist on that element. The orientation of the stress element is critical, and it must be aligned with specified axes on the member, typically called x, y, and z.

Figure 3–5 shows three examples of stress elements with two basic fundamental kinds of stress: Normal (tensile and compressive) and shear. Both the complete three-dimensional cube and the simplified, two-dimensional square forms for the stress elements are shown. The square is one face of the cube in a selected plane. The sides of the square represent the projections of the faces of the cube that are perpendicular to the selected plane. It is recommended that you visualize the cube form first and then represent a square stress element showing stresses on a particular plane of interest in a given problem. In some problems with more general states of stress, two or three square stress elements may be required to depict the complete stress condition.

Tensile and compressive stresses, called *normal stresses*, are shown acting perpendicular to opposite faces of the stress element. Tensile stresses tend to pull on the element, whereas compressive stresses tend to crush it.

Shear stresses are created by direct shear, vertical shear in beams, or torsion. In each case, the action on an element subjected to shear is a tendency to *cut* the element by exerting a stress downward on one face while simultaneously exerting a stress upward on the opposite, parallel face. This action is that of a simple pair of shears or scissors. But note that if only one pair of shear stresses acts on a stress element, it will not be in equilibrium. Rather, it will tend to spin because the pair of shear stresses forms a couple. To produce equilibrium, a second pair of shear stresses on the other two faces of the element must exist, acting in a direction that opposes the first pair.

In summary, shear stresses on an element will always be shown as two pairs of equal stresses acting on (parallel to) the four sides of the element. Figure 3–5(c) shows an example.

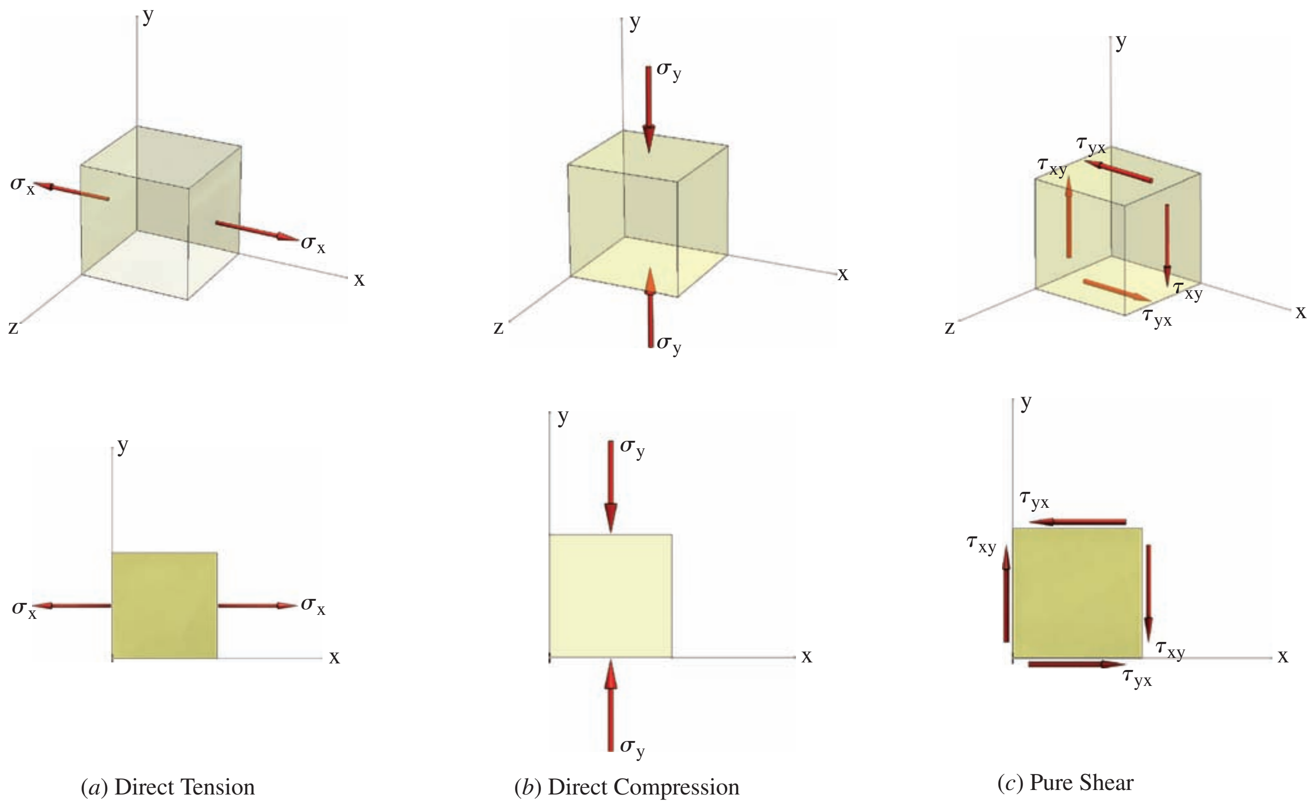

FIGURE 3–5 Stress elements for normal and shear stresses

Sign Convention for Shear Stresses

This book adopts the following convention:

> ***Positive shear stresses tend to rotate the element in a clockwise direction.***
>
> ***Negative shear stresses tend to rotate the element in a counterclockwise direction.***

A double subscript notation is used to denote shear stresses in a plane. For example, in Figure 3–5(c), drawn for the x–y plane, the pair of shear stresses, τ_{xy}, indicates a shear stress acting on the element face that is perpendicular to the x-axis and parallel to the y-axis. Then τ_{yx} acts on the face that is perpendicular to the y-axis and parallel to the x-axis. In this example, τ_{xy} is positive and τ_{yx} is negative.

3–4 NORMAL STRESSES DUE TO DIRECT AXIAL LOAD

Stress can be defined as the internal resistance offered by a unit area of a material to an externally applied load. *Normal stresses* (σ) are either *tensile* (positive) or *compressive* (negative).

For a load-carrying member in which the external load is uniformly distributed across the cross-sectional area of the member, the magnitude of the stress can be calculated from the direct stress formula:

➪ Direct Tensile or Compressive Stress

$$\sigma = \text{force/area} = F/A \tag{3–1}$$

The units for stress are always *force per unit area*, as is evident from Equation (3–1). Common units in the U.S. Customary system and the SI metric system follow.

U.S. Customary Units	*SI Metric Units*
lb/in^2 = psi	N/m^2 = pascal = Pa
kips/in^2 = ksi	N/mm^2 = megapascal
Note: 1.0 kip = 1000 lb	= 10^6 Pa = MPa
1.0 ksi = 1000 psi	

The conditions on the use of Equation (3–1) are as follows:

1. The load-carrying member must be straight.
2. The line of action of the load must pass through the centroid of the cross section of the member.
3. The member must be of uniform cross section near where the stress is being computed.
4. The material must be homogeneous and isotropic.
5. In the case of compression members, the member must be short to prevent buckling. The conditions under which buckling is expected are discussed in Chapter 6.

Example Problem 3–1 A tensile force of 9500 N is applied to a 12-mm-diameter round bar, as shown in Figure 3–6. Compute the direct tensile stress in the bar.

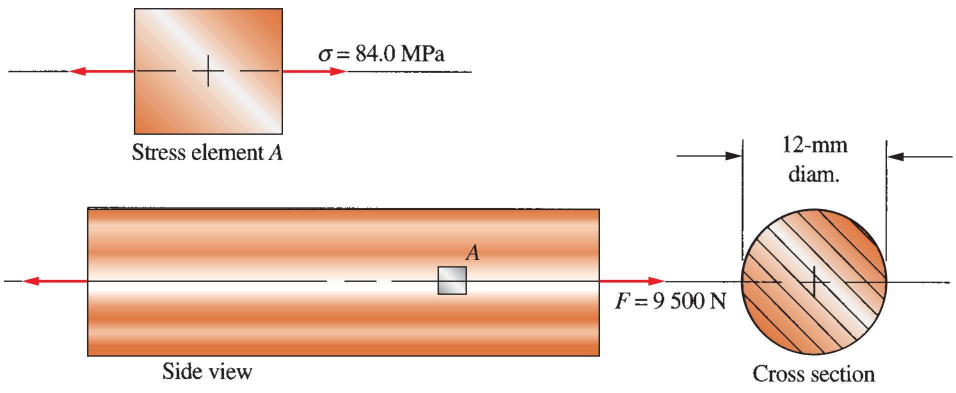

FIGURE 3–6 Tensile stress in a round bar

Solution

Objective: Compute the tensile stress in the round bar.

Given: Force = F = 9500 N; diameter = D = 12 mm.

Analysis: Use the direct tensile stress formula, Equation (3–1): $\sigma = F/A$. Compute the cross-sectional area from $A = \pi D^2/4$.

Results:

$$A = \pi D^2/4 = \pi(12\text{ mm})^2/4 = 113\text{ mm}^2$$

$$\sigma = F/A = (9500\text{ N})/(113\text{ mm}^2) = 84.0\text{ N/mm}^2 = 84.0\text{ MPa}$$

Comment: The results are shown on stress element A in Figure 3–6, which can be taken to be anywhere within the bar because, ideally, the stress is uniform on any cross section. The cube form of the element is as shown in Figure 3–5 (a).

Example Problem 3–2 For the round bar subjected to the tensile load shown in Figure 3–6, compute the total deformation if the original length of the bar is 3600 mm. The bar is made from a steel having a modulus of elasticity of 207 GPa.

Solution

Objective: Compute the deformation of the bar.

Given: Force $= F = 9500$ N; diameter $= D = 12$ mm.
Length $= L = 3600$ mm; $E = 207$ GPa

Analysis: From Example Problem 3–1, we found that $\sigma = 84.0$ MPa. Use Equation (3–3).

Results:
$$\delta = \frac{\sigma L}{E} = \frac{(84.0 \times 10^6\ \text{N/m}^2)(3600\ \text{mm})}{(207 \times 10^9\ \text{N/m}^2)} = 1.46\ \text{mm}$$

3–5 DEFORMATION UNDER DIRECT AXIAL LOAD

The following formula computes the stretch due to a direct axial tensile load or the shortening due to a direct axial compressive load:

$$\delta = FL/EA \tag{3–2}$$

➩ Deformation Due to Direct Axial Load

where δ = total deformation of the member carrying the axial load
F = direct axial load
L = original total length of the member
E = modulus of elasticity of the material
A = cross-sectional area of the member

Noting that $\sigma = F/A$, we can also compute the deformation from

$$\delta = \sigma L/E \tag{3–3}$$

3–6 SHEAR STRESS DUE TO DIRECT SHEAR LOAD

Direct shear stress occurs when the applied force tends to cut through the member as scissors or shears do or when a punch and a die are used to punch a slug of material from a sheet. Another important example of direct shear in machine design is the tendency for a key to be sheared off at the section between the shaft and the hub of a machine element when transmitting torque. Figure 3–7 shows the action.

The method of computing direct shear stress is similar to that used for computing direct tensile stress because the applied force is assumed to be uniformly distributed across the cross section of the part that is resisting the force. But the kind of stress is *shear stress* rather than *normal stress*. The symbol used for shear stress is the Greek letter tau (τ). The formula for direct shear stress can thus be written

➩ Direct Shear Stress

$$\tau = \text{shearing force/area in shear} = F/A_s \tag{3–4}$$

This stress is more properly called the *average shearing* stress, but we will make the simplifying assumption that the stress is uniformly distributed across the shear area.

3–7 TORSIONAL LOAD—TORQUE, ROTATIONAL SPEED, AND POWER

The relationship among the power *(P)*, the rotational speed *(n)*, and the torque *(T)* in a shaft is described by the equation

➩ Power–Torque–Speed Relationship

$$T = P/n \tag{3–5}$$

In SI units, power is expressed in the unit of *watt* (W) or its equivalent, *newton meter per second* (N·m/s), and the rotational speed is in *radians per second* (rad/s).

In the U.S. Customary Unit System, power is typically expressed as *horsepower*, equal to 550 ft·lb/s. The typical unit for rotational speed is rpm, or revolutions per minute. But the most convenient unit for torque is the pound-inch (lb·in). Considering all of these quantities and making the necessary conversions of units, we use the following formula to compute the torque (in lb·in) in a shaft carrying a certain power P (in hp) while rotating at a speed of n rpm.

➩ *P–T–n* Relationship for U.S. Customary Units

$$T = 63\,000\ P/n \tag{3–6}$$

The resulting torque will be in lb·in. You should verify the value of the constant, 63 000.