Example Problem 3–3

Figure 3–7 shows a shaft carrying two sprockets for synchronous belt drives that are keyed to the shaft. Figure 3–7 (b) shows that a force F is transmitted from the shaft to the hub of the sprocket through a square key. The shaft has a diameter of 2.25 in and transmits a torque of 14 063 lb·in. The key has a square cross section, 0.50 in on a side, and a length of 1.75 in. Compute the force on the key and the shear stress caused by this force.

Solution

Objective: Compute the force on the key and the shear stress.

Given: Layout of shaft, key, and hub shown in Figure 3–7.
Torque $= T = 14\,063$ lb·in; key dimensions $= 0.5$ in $\times$ 0.5 in $\times$ 1.75 in.
Shaft diameter $= D = 2.25$ in; radius $= R = D/2 = 1.125$ in.

Analysis: Torque $T =$ force $F \times$ radius R. Then $F = T/R$.
Use equation (3–4) to compute shearing stress: $\tau = F/A_s$.
Shear area is the cross section of the key at the interface between the shaft and the hub: $A_s = bL$.

Results:
$F = T/R = (14\,063 \text{ lb·in})/(1.125 \text{ in}) = 12\,500$ lb
$A_s = bL = (0.50 \text{ in})(1.75 \text{ in}) = 0.875 \text{ in}^2$
$\tau = F/A = (12\,500 \text{ lb})/(0.875 \text{ in}^2) = 14\,300 \text{ lb/in}^2$

Comment: This level of shearing stress will be uniform on all parts of the cross section of the key.

Example Problem 3–4

Compute the torque on a shaft transmitting 750 W of power while rotating at 183 rad/s. (*Note:* This is equivalent to the output of a 1.0-hp, 4-pole electric motor, operating at its rated speed of 1750 rpm. See Chapter 21.)

Solution

Objective: Compute the torque T on the shaft.

Given: Power $= P = 750$ W $= 750$ N·m/s.
Rotational speed $= n = 183$ rad/s.

Analysis: Use Equation (3–5).

Results:
$T = P/n = (750 \text{ N·m/s})/(183 \text{ rad/s})$
$T = 4.10 \text{ N·m/rad} = 4.10$ N·m

Comments: In such calculations, the unit of N·m/rad is dimensionally correct, and some advocate its use. Most, however, consider the radian to be dimensionless, and thus torque is expressed in N·m or other familiar units of force times distance.

Example Problem 3–5

Compute the torque on a shaft transmitting 1.0 hp while rotating at 1750 rpm. Note that these conditions are approximately the same as those for which the torque was computed in Example Problem 3–4 using SI units.

Solution

Objective: Compute the torque on the shaft.

Given: $P = 1.0$ hp; $n = 1750$ rpm.

Analysis: Use Equation (3–6).

Results: $T = 63\,000\, P/n = [63\,000(1.0)]/1750 = 36.0$ lb·in

3–8 SHEAR STRESS DUE TO TORSIONAL LOAD

When a *torque*, or twisting moment, is applied to a member, it tends to deform by twisting, causing a rotation of one part of the member relative to another. Such twisting causes a shear stress in the member. For a small element of the member, the nature of the stress is the same as that experienced under direct shear stress. However, in *torsional shear*, the distribution of stress is not uniform across the cross section.

The most frequent case of torsional shear in machine design is that of a round circular shaft transmitting power. Chapter 12 covers shaft design.