TABLE 7–7 8-mm Pitch GT Drive Selection Table

Sprocket combinations			Center distance (inches)															
			920-8MGT P.L. 36.220	960-8MGT P.L. 37.795	1040-8MGT P.L. 40.945	1064-8MGT P.L. 41.890	1120-8MGT P.L. 44.094	1160-8MGT P.L. 45.669	1200-8MGT P.L. 47.244	1224-8MGT P.L. 48.189	1280-8MGT P.L. 50.394	1440-8MGT P.L. 56.693	1512-8MGT P.L. 59.528	1584-8MGT P.L. 62.362	1600-8MGT P.L. 62.992	1760-8MGT P.L. 69.291	1800-8MGT P.L. 70.866	2000-8MGT P.L. 78.740
Driver	Driven	Velocity ratio	115 teeth	120 teeth	130 teeth	133 teeth	140 teeth	145 teeth	150 teeth	153 teeth	160 teeth	180 teeth	189 teeth	198 teeth	200 teeth	220 teeth	225 teeth	250 teeth
22	22	1.000	14.65	15.43	17.01	17.48	18.58	19.37	20.16	20.63	21.73	24.88	26.30	27.72	28.03	31.18	31.97	35.90
24	24	1.000	14.33	15.12	16.69	17.17	18.27	19.06	19.84	20.32	21.42	24.57	25.98	27.40	27.72	30.87	31.65	35.59
26	26	1.000	14.02	14.80	16.38	16.85	17.95	18.74	19.53	20.00	21.10	24.25	25.67	27.09	27.40	30.55	31.34	35.28
28	28	1.000	13.70	14.49	16.06	16.54	17.64	18.43	19.21	19.69	20.79	23.94	25.35	26.77	27.09	30.24	31.02	34.96
30	30	1.000	13.39	14.17	15.75	16.22	17.32	18.11	18.90	19.37	20.47	23.62	25.04	26.46	26.77	29.92	30.71	34.65
32	32	1.000	13.07	13.86	15.43	15.91	17.01	17.80	18.58	19.06	20.16	23.31	24.72	26.14	26.46	29.61	30.39	34.33
34	34	1.000	12.76	13.54	15.12	15.59	16.69	17.48	18.27	18.74	19.84	22.99	24.41	25.83	26.14	29.29	30.08	34.02
36	36	1.000	12.44	13.23	14.80	15.28	16.38	17.17	17.95	18.43	19.53	22.68	24.09	25.51	25.83	28.98	29.76	33.70
38	38	1.000	12.13	12.91	14.49	14.96	16.06	16.85	17.64	18.11	19.21	22.36	23.78	25.20	25.51	28.66	29.45	33.39
40	40	1.000	11.67	12.46	14.03	14.50	15.61	16.39	17.18	17.65	18.76	21.91	23.32	24.74	25.06	28.21	28.99	32.93
44	44	1.000	11.18	11.97	13.54	14.02	15.12	15.91	16.69	17.17	18.27	21.42	22.83	24.25	24.57	27.72	28.50	32.44
48	48	1.000	10.55	11.34	12.91	13.39	14.49	15.28	16.06	16.54	17.64	20.79	22.21	23.62	23.94	27.09	27.87	31.81
56	56	1.000	9.29	10.08	11.65	12.13	13.23	14.02	14.80	15.28	16.38	19.53	20.95	22.36	22.68	25.83	26.61	30.55
64	64	1.000	8.03	8.82	10.39	10.87	11.97	12.76	13.54	14.02	15.12	18.27	19.69	21.10	21.42	24.57	25.35	29.29
72	72	1.000	-	-	9.13	9.61	10.71	11.50	12.28	12.76	13.86	17.01	18.43	19.84	20.16	23.31	24.10	28.03
80	80	1.000	-	-	-	-	9.45	10.24	11.02	11.50	12.60	15.75	17.17	18.58	18.90	22.05	22.84	26.77
24	30	1.250	13.85	14.64	16.22	16.69	17.79	18.58	19.37	19.84	20.94	24.09	25.51	26.93	27.24	30.39	31.18	35.12
32	40	1.250	12.43	13.22	14.80	15.27	16.37	17.16	17.95	18.42	19.52	22.67	24.09	25.51	25.82	28.97	29.76	33.70
64	80	1.250	-	-	9.10	9.57	10.68	11.47	12.26	12.73	13.84	16.99	18.41	19.83	20.14	23.29	24.08	28.02
72	90	1.250	-	-	-	-	9.25	10.04	10.83	11.30	12.41	15.56	16.98	18.40	18.72	21.87	22.66	26.60
24	32	1.333	13.70	14.48	16.06	16.53	17.63	18.42	19.21	19.68	20.78	23.93	25.35	26.77	27.08	30.23	31.02	34.96
30	40	1.333	12.59	13.38	14.95	15.42	16.53	17.32	18.10	18.58	19.68	22.83	24.25	25.66	25.98	29.13	29.92	33.85

(*continued*)

TABLE 7–7 (*continued*)

Sprocket combinations			Center distance (inches)															
			920-8MGT	960-8MGT	1040-8MGT	1064-8MGT	1120-8MGT	1160-8MGT	1200-8MGT	1224-8MGT	1280-8MGT	1440-8MGT	1512-8MGT	1584-8MGT	1600-8MGT	1760-8MGT	1800-8MGT	2000-8MGT
			P.L. 36.220	P.L. 37.795	P.L. 40.945	P.L. 41.890	P.L. 44.094	P.L. 45.669	P.L. 47.244	P.L. 48.189	P.L. 50.394	P.L. 56.693	P.L. 59.528	P.L. 62.362	P.L. 62.992	P.L. 69.291	P.L. 70.866	P.L. 78.740
Driver	Driven	Velocity ratio	115 teeth	120 teeth	130 teeth	133 teeth	140 teeth	145 teeth	150 teeth	153 teeth	160 teeth	180 teeth	189 teeth	198 teeth	200 teeth	220 teeth	225 teeth	250 teeth
36	48	1.333	11.48	12.27	13.85	14.32	15.42	16.21	17.00	17.47	18.57	21.72	23.14	24.56	24.87	28.03	28.81	32.75
48	64	1.333	9.26	10.05	11.63	12.10	13.20	13.99	14.78	15.25	16.36	19.51	20.93	22.35	22.66	25.81	26.60	30.54
24	36	1.500	13.36	14.15	15.73	16.20	17.30	18.09	18.88	19.35	20.46	23.61	25.02	26.44	26.76	29.91	30.70	34.63
32	48	1.500	11.78	12.57	14.15	14.62	15.73	16.52	17.30	17.78	18.88	22.03	23.45	24.87	25.18	28.34	29.12	33.06
48	72	1.500	8.58	9.37	10.96	11.43	12.54	13.33	14.12	14.60	15.70	18.86	20.28	21.70	22.01	25.17	25.96	29.90
22	44	2.000	12.87	13.66	15.24	15.71	16.81	17.60	18.39	18.87	19.97	23.12	24.54	25.96	26.28	29.43	30.22	34.16
24	48	2.000	12.38	13.17	14.75	15.23	16.33	17.12	17.91	18.39	19.49	22.65	24.06	25.48	25.80	28.95	29.74	33.68
28	56	2.000	11.41	12.20	13.79	14.26	15.37	16.16	16.95	17.42	18.53	21.69	23.11	24.53	24.84	28.00	28.79	32.73
32	64	2.000	10.43	11.22	12.81	13.29	14.40	15.19	15.98	16.46	17.56	20.73	22.15	23.57	23.88	27.04	27.83	31.77
36	72	2.000	9.43	10.24	11.83	12.31	13.42	14.22	15.01	15.49	16.60	19.76	21.18	22.61	22.92	26.08	26.87	30.81
40	80	2.000	8.42	9.23	10.84	11.32	12.44	13.23	14.03	14.51	15.62	18.79	20.22	21.64	21.96	25.12	25.91	29.85
56	112	2.000	-	-	-	-	-	9.18	10.00	10.49	11.63	14.85	16.29	17.73	18.05	21.23	22.03	25.99
72	144	2.000	-	-	-	-	-	-	-	-	-	-	12.22	13.70	14.02	17.26	18.06	22.07
32	80	2.500	8.97	9.78	11.40	11.88	13.01	13.81	14.61	15.08	16.20	19.38	20.81	22.23	22.55	25.71	26.51	30.46
36	90	2.500	7.71	8.55	10.19	10.68	11.82	12.62	13.43	13.91	15.03	18.22	19.66	21.09	21.40	24.58	25.37	29.32
24	72	3.000	10.27	11.08	12.69	13.17	14.29	15.08	15.88	16.36	17.47	20.65	22.07	23.50	23.82	26.98	27.77	31.72
30	90	3.000	8.10	8.94	10.60	11.09	12.23	13.04	13.85	14.33	15.46	18.65	20.09	21.52	21.84	25.02	25.81	29.77
48	144	3.000	-	-	-	-	-	-	-	-	-	12.29	13.81	15.31	15.64	18.92	19.73	23.76

TABLE 7–8 Service Factor

DriveN machine	DriveR					
The driveN machines listed below are representative samples only. Select a driveN machine whose load characteristics most closely approximate those of the machine being considered.	AC Motors: Normal Torque, Squirrel Cage, Synchronous, Split Phase, Inverter Controlled DC Motors: Shunt Wound Stepper Motors Engines: Multiple Cylinder Internal Combustion			AC Motors: High Torque, High Slip, Repulsion-Induction, Single Phase, Series Wound, Slip Ring DC Motors: Series Wound, Compound Wound Servo Motors Engines: Single Cylinder Internal Combustion, Line Shafts, Clutches		
	Intermittent Service (Up to 8 Hours Daily or Seasonal)	Normal Service (8–16 Hours Daily)	Continuous Service (16–24 Hours Daily)	Intermittent Service (Up to 8 Hours Daily or Seasonal)	Normal Service (8–16 Hours Daily)	Continuous Service (16–24 Hours Daily)
Display, Dispending Equipment Instrumentation Measuring Equipment Medical Equipment Office, Projection Equipment	1.0	1.2	1.4	1.2	1.4	1.6
Appliances, Sweepers, Sewing Machines Screens, Oven Screens, Drum, Conical Woodworking Equipment (Light): Band Saws, Drills Lathes	1.1	1.3	1.5	1.3	1.5	1.7
Agitators for Liquids Conveyors: Belt, Light Package Drill Press, Lathes, Saws Laundry Machinery Wood Working Equipment (Heavy): Circular Saws, Jointers, Planers	1.2	1.4	1.6	1.6	1.8	2.0
Agitators for Semi-Liquids Compressor: Centrifugal Conveyor Belt: Ore, Coal, Sand Dough Mixers Line Shafts Machine Tools: Grinder, Shaper, Boring Mill, Milling Machines Paper Machinery (except Pulpers): Presses, Punches, Shears Printing Machinery Pumps: Centrifugal, Gear Screens: Revolving, Vibratory	1.3	1.5	1.7	1.6	1.8	2.0
Brick Machinery (except Pug Mills) Conveyor: Apron, Pan, Bucket, Elevator Extractors, Washers Fans, Centrifugal Blowers Generators & Exciters Hoists Rubber Calender, Mills, Extruders	1.4	1.6	1.8	1.8	2.0	2.2
Centrifuges Screw Conveyors Hammer Mills Paper Pulpers Textile Machinery	1.5	1.7	1.9	1.9	2.1	2.3
Blowers: Positive Displacement, Mine Fans Pulvertzers	1.6	1.8	2.0	2.0	2.2	2.4
Compressors: Reciprocating Crushers: Gyratory, Jaw, Rol Mils: Ball, Rod, Pebble, etc. Pumps: Reciprocating Saw Mill Equipment	1.7	1.9	2.1	2.1	2.3	2.5

These service factors are adequate for most belt drive applications. Note that service factors cannot be substituted for good engineering judgment. Service factors may be adjusted based upon an understanding of the severity of actual drive operating conditions.

4. Calculate the design power by multiplying the driver rated power by the service factor.

$$\text{Design power} = P_{des} = P_{rated} \cdot SF$$

5. Determine the required pitch of the belt using the belt pitch selection guide Figure (7–27). The belt pitch is based on the design power and the angular velocity of the faster (smaller) sprocket. The belt pitches available are 5 mm, 8 mm, 14 mm, and 20 mm. The design horsepower is along the x-axis and the rpm of the faster sprocket is along the y-axis. As the design power increases or the smaller sprocket angular velocity decreases, a larger belt pitch would be required. The 14-mm belt pitch is selected for the design power and angular velocity in its shaded area, but would work for any application to the left of its shaded area. This means that the 14-mm belt pitch would work for a point located in the 5-mm and 8-mm belt pitch areas, but would be considered over-designed and not an economical design choice.
6. Calculate the velocity ratio VR between the driver and driven sprockets. Review Section 7–2 for this equation.

$$VR = \frac{\omega_{driving}}{\omega_{driven}} = \frac{PD_{driven}}{PD_{driving}} = \frac{N_{driven}}{N_{driving}}$$

7. Select the candidate combinations using Table 7–7 of the number of teeth in the driver sprocket to that in the driven sprocket that will produce the calculated velocity ratio, VR.
8. Eliminate the sprocket combinations that will not work due to space limitations and shaft diameter requirements. Some of the larger sprockets may interfere with the machine or guarding and can be eliminated due to these space limitations. The shaft diameter will dictate the minimum taper-lock bushing (Figure 7–25) that will fit on the shaft. Once the taper-lock bushing is known the minimum sprocket can be determined. This will eliminate any sprockets smaller than this minimum sprocket.
9. Using the desired range of acceptable center distances, determine a standard belt length that will produce a suitable value. Table 7–7 shows that the center distance is determined by the belt length and sprocket velocity ratio. The available belt lengths are determined by the manufacturer. The belt center distance selection is influenced by the belt drive design center distance. A fixed or adjustable center distance design should be considered when selecting the proper belt length. A belt drive design that has an

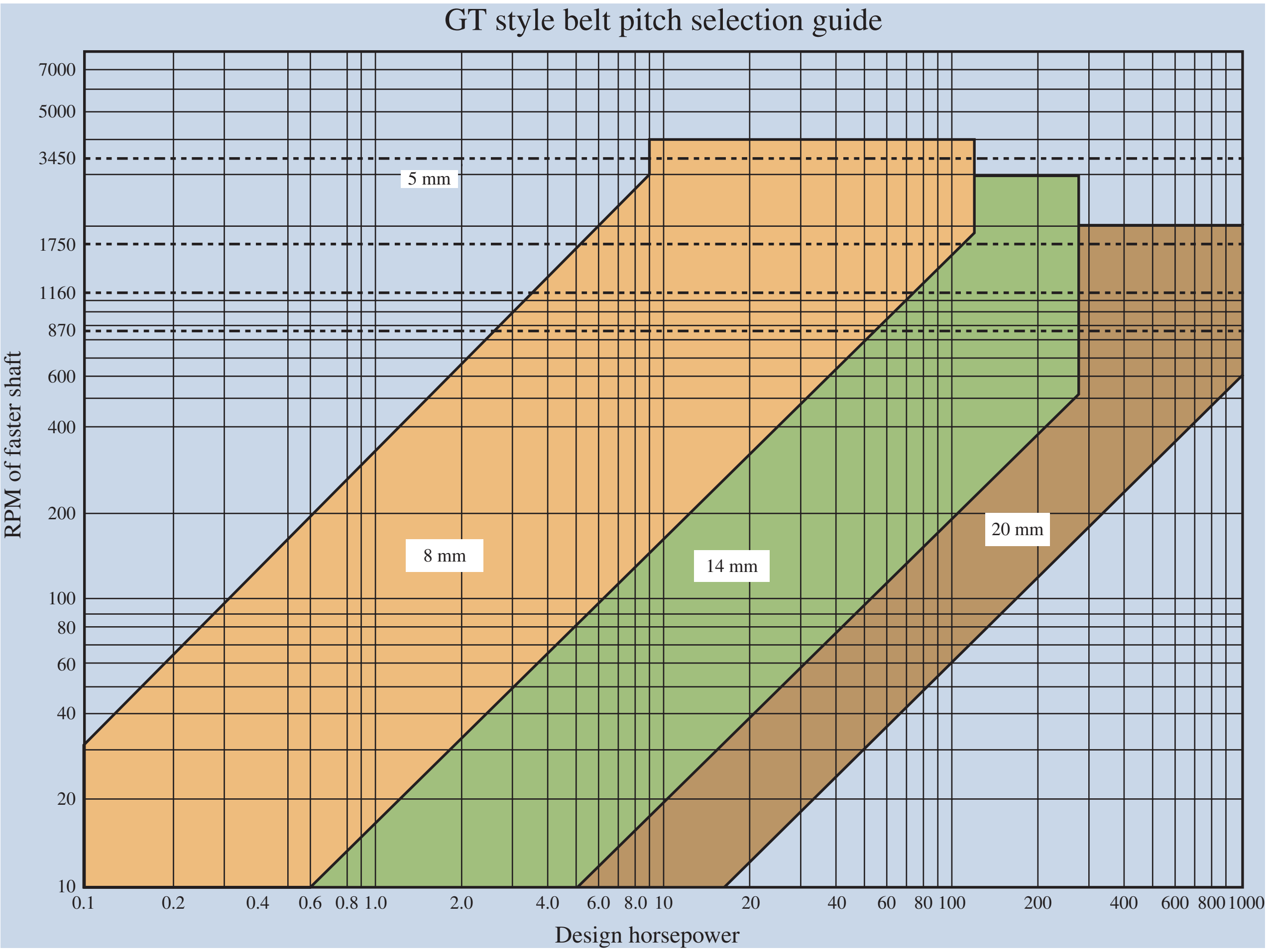


FIGURE 7–27 Belt pitch selection guide for GT style belts

adjustable center distance (Figure 7–26) will require the belt center distance to be within this range. If the belt drive design has a fixed center distance, the belt center distance must be larger than the fixed center distance. This belt drive system will require the use of a tensioner (Figure 7–30, discussed later) to take up the difference in belt lengths. An inside or outside tensioner will be selected, depending on how much the belt center distance exceeds the fixed center distance. This will require a drive belt layout to determine the best available solution.

10. Selection of the width of the belt: Although there are four belt pitches available (5 mm, 8 mm, 14 mm, and 20 mm), we will focus on the 8-mm belt pitch. An 8-mm pitch belt is available in four different widths: 20 mm, 30 mm, 50 mm, and 85 mm. The belt width selection Tables 7–9 and 7–10 are shown for the 30-mm and 50-mm wide belts. The 20-mm and 85-mm wide belt tables can be found in the manufacturer's website. The angular velocity of the faster (smaller) sprocket along with the number of teeth of this smaller sprocket is used to find the base rated horsepower. Let's first look at the 30 mm belt width table. You will notice for a given sprocket size as the speed increases, the power rating of the belt increases. For a given speed of the sprocket, the belt power rating will increase as the size of the sprocket (or number of teeth) increases. The 50 mm

TABLE 7–9 8M GT Style Belt Power Rating Table—30-mm Belt Width

RPM of faster shaft	Base rated horsepower for small sprocket (Number of grooves and pitch diameter, inches)															
	22 2.206	24 2.406	26 2.607	28 2.807	30 3.008	32 3.208	34 3.409	36 3.609	38 3.810	40 4.010	44 4.411	48 4.812	56 5.614	64 6.416	72 7.218	80 8.020
10	0.10	0.12	0.13	0.15	0.16	0.17	0.19	0.20	0.22	0.23	0.26	0.29	0.34	0.40	0.45	0.51
20	0.20	0.22	0.25	0.28	0.31	0.33	0.36	0.39	0.42	0.44	0.50	0.55	0.66	0.76	0.87	0.98
40	0.37	0.43	0.48	0.53	0.59	0.64	0.69	0.75	0.80	0.85	0.96	1.06	1.27	1.47	1.68	1.88
60	0.54	0.62	0.70	0.78	0.86	0.94	1.01	1.09	1.17	1.25	1.40	1.55	1.86	2.16	2.46	2.76
100	0.87	1.00	1.12	1.25	1.38	1.51	1.63	1.76	1.89	2.01	2.26	2.51	3.00	3.49	3.98	4.47
200	1.64	1.89	2.13	2.38	2.63	2.87	3.12	3.36	3.60	3.84	4.33	4.80	5.76	6.70	7.64	8.58
300	2.37	2.74	3.10	3.46	3.82	4.18	4.54	4.90	5.25	5.61	6.32	7.02	8.42	9.80	11.2	12.5
400	3.08	3.56	4.04	4.51	4.99	5.46	5.93	6.40	6.87	7.33	8.26	9.18	11.0	12.8	14.6	16.4
500	3.77	4.36	4.95	5.54	6.13	6.71	7.29	7.87	8.45	9.02	10.2	11.3	13.6	15.8	18.0	20.2
600	4.45	5.15	5.85	6.55	7.25	7.94	8.63	9.31	10.0	10.7	12.0	13.4	16.1	18.7	21.4	24.0
700	5.11	5.93	6.74	7.54	8.35	9.15	9.95	10.7	11.5	12.3	13.9	15.5	18.6	21.6	24.7	27.7
800	5.77	6.69	7.61	8.52	9.44	10.3	11.2	12.1	13.0	13.9	15.7	17.5	21.0	24.5	27.9	31.4
870	6.22	7.22	8.22	9.20	10.2	11.2	12.2	13.1	14.1	15.1	17.0	18.9	22.7	26.5	30.2	33.9
1000	7.05	8.19	9.33	10.5	11.6	12.7	13.8	14.9	16.0	17.1	19.3	21.5	25.8	30.1	34.3	38.5
1160	8.06	9.37	10.7	12.0	13.3	14.5	15.8	17.1	18.4	19.6	22.2	24.7	29.6	34.5	39.4	44.2
1200	8.31	9.66	11.0	12.3	13.7	15.0	16.3	17.6	19.0	20.3	22.9	25.4	30.6	35.6	40.6	45.6
1400	9.54	11.1	12.7	14.2	15.7	17.3	18.8	20.3	21.8	23.3	26.3	29.3	35.2	41.0	46.8	52.4
1600	10.7	12.5	14.3	16.0	17.8	19.5	21.2	23.0	24.7	26.4	29.8	33.1	39.8	46.3	52.8	59.1
1750	11.6	13.6	15.5	17.4	19.3	21.2	23.0	24.9	26.8	28.6	32.3	36.0	43.2	50.3	57.2	64.1
2000	13.1	15.3	17.5	19.6	21.8	23.9	26.0	28.1	30.2	32.3	36.5	40.6	48.7	56.7	64.5	72.1
2400	15.4	18.0	20.5	23.1	25.6	28.1	30.7	33.1	35.6	38.1	43.0	47.8	57.3	66.6	75.6	84.4
2800	17.6	20.6	23.6	26.5	29.4	32.3	35.2	38.0	40.9	43.7	49.3	54.8	65.6	76.1	86.2	96.0
3200	19.8	23.2	26.5	29.8	33.1	36.4	39.6	42.8	46.0	49.2	55.4	61.6	73.6	85.2	96.2	
3450	21.1	24.7	28.3	31.9	35.4	38.9	42.3	45.8	49.2	52.5	59.2	65.7	78.4	90.6	102.2	
4000	24.0	28.1	32.2	36.2	40.3	44.2	48.1	52.0	55.9	59.7	67.1	74.5	88.5			
4500	26.6	31.1	35.6	40.1	44.5	48.9	53.2	57.5	61.7	65.9	74.0	82.0				
5000	29.0	34.0	39.0	43.8	48.7	53.4	58.1	62.8	67.3	71.8	80.6	89.1				
5500	31.4	36.8	42.2	47.5	52.7	57.8	62.9	67.8	72.7	77.5	86.8					

TABLE 7–10 8M GT Style Belt Power Rating Table—50-mm Belt Width

RPM of faster shaft	Base rated horsepower for small sprocket (Number of grooves and pitch diameter, inches) 28 / 2.807	30 / 3.008	32 / 3.208	34 / 3.409	36 / 3.609	38 / 3.810	40 / 4.010	44 / 4.411	48 / 4.812	56 / 5.614	64 / 6.416	72 / 7.218	80 / 8.020
10	0.25	0.28	0.30	0.33	0.35	0.38	0.40	0.45	0.50	0.59	0.69	0.78	0.88
20	0.49	0.53	0.58	0.63	0.68	0.72	0.77	0.86	0.96	1.14	1.33	1.51	1.70
40	0.93	1.02	1.11	1.21	1.30	1.39	1.48	1.66	1.84	2.20	2.56	2.92	3.27
60	1.35	1.49	1.63	1.76	1.90	2.03	2.17	2.43	2.70	3.23	3.75	4.28	4.80
100	2.18	2.40	2.62	2.84	3.06	3.28	3.50	3.93	4.36	5.22	6.08	6.92	7.77
200	4.14	4.57	4.99	5.42	5.84	6.26	6.68	7.52	8.35	10.0	11.7	13.3	14.9
300	6.02	6.65	7.27	7.90	8.52	9.14	9.75	11.0	12.2	14.6	17.0	19.4	21.8
400	7.85	8.67	9.49	10.3	11.1	11.9	12.7	14.4	16.0	19.2	22.3	25.5	28.6
500	9.63	10.7	11.7	12.7	13.7	14.7	15.7	17.7	19.7	23.6	27.5	31.4	35.2
600	11.4	12.6	13.8	15.0	16.2	17.4	18.6	20.9	23.3	28.0	32.6	37.2	41.7
700	13.1	14.5	15.9	17.3	18.7	20.1	21.4	24.2	26.9	32.3	37.6	42.9	48.2
800	14.8	16.4	18.0	19.6	21.1	22.7	24.2	27.3	30.4	36.5	42.6	48.6	54.5
870	16.0	17.7	19.4	21.1	22.8	24.5	26.2	29.5	32.9	39.5	46.0	52.5	58.9
1000	18.2	20.1	22.1	24.0	25.9	27.9	29.8	33.6	37.4	44.9	52.4	59.7	67.0
1160	20.8	23.1	25.3	27.5	29.7	32.0	34.1	38.5	42.9	51.5	60.0	68.5	76.8
1200	21.5	23.8	26.1	28.4	30.7	33.0	35.2	39.8	44.2	53.1	61.9	70.6	79.2
1400	24.7	27.4	30.0	32.7	35.3	38.0	40.6	45.8	51.0	61.2	71.3	81.3	91.2
1600	27.9	30.9	33.9	36.9	39.9	42.9	45.9	51.8	57.6	69.2	80.6	91.8	102.9
1750	30.2	33.5	36.8	40.1	43.3	46.6	49.8	56.2	62.5	75.0	87.4	99.5	111.4
2000	34.1	37.8	41.5	45.2	48.9	52.6	56.2	63.4	70.6	84.7	98.5	112.1	125.4
2400	40.2	44.6	48.9	53.3	57.6	62.0	66.2	74.7	83.1	99.7	115.8	131.5	146.8
2800	46.1	51.2	56.2	61.2	66.2	71.1	76.0	85.7	95.3	114.1	132.3	149.9	166.9
3200	51.9	57.6	63.2	68.9	74.5	80.0	85.5	96.4	107.1	128.0	148.1	167.4	
3450	55.4	61.5	67.6	73.6	79.6	85.5	91.3	102.9	114.3	136.4	157.5	177.7	
4000	63.0	70.0	76.9	83.7	90.4	97.1	103.7	116.8	129.5	154.0			
4500	69.7	77.4	85.0	92.6	100.0	107.3	114.5	128.7	142.5				
5000	76.2	84.7	92.9	101.1	109.1	117.1	124.9	140.1	154.9				
5500	82.5	91.6	100.5	109.3	117.9	126.4	134.7	150.9					

belt width will have a higher power rating than the 30 mm belt width. A larger sprocket will decrease the belt width required and yield a longer service life. The belt width should not exceed the sprocket diameter. This base rated horsepower must be adjusted by the belt length correction factor shown in Table 7–11. Catalog data will show factors less than 1.0 for shorter belt lengths and greater than 1.0 for longer belt lengths. This reflects the frequency with which a given tooth of the belt encounters a high-stress area as it enters the smaller sprocket.

$$\textit{Base Rated Power}_{adjusted} = \textit{Base Rated Power} \cdot \textit{Length Correction Factor}$$

11. Calculate the belt linear velocity. Belt speeds above 3500 fpm increase the noise level of the synchronous belt drive. Also verify that the belt linear velocity does not exceed 6500 fpm, due to the excessive centrifugal forces that are placed on a sprocket.

$$v_{belt} = \frac{PD_1}{2} \cdot \omega_1 = \frac{PD_2}{2} \cdot \omega_2$$

12. Specify the final design details for the belt drive system. This includes all sprockets, type and bore size of taper-lock bushings, belt, and tensioner if required. Summarize the design, check compatibility with other components of the system, and prepare the purchasing documents.

TABLE 7–11 8M GT Style Belt Length Correction Factor

Pitch/Length designation	No. of teeth	Correction factor	Pitch/Length designation	No. of teeth	Correction factor	Pitch/Length designation	No. of teeth	Correction factor	Pitch/Length designation	No. of teeth	Correction factor
384-8MGT	48	0.70	920-8MGT	115	1.00	1440-8MGT	180	1.10	2600-8MGT	325	1.20
480-8MGT	60	0.80	960-8MGT	120	1.00	1512-8MGT	189	1.10	2800-8MGT	350	1.20
560-8MGT	70	0.80	1040-8MGT	130	1.00	1584-8MGT	198	1.10	3048-8MGT	381	1.20
600-8MGT	75	0.80	1064-8MGT	133	1.00	1600-8MGT	200	1.10	3280-8MGT	410	1.20
640-8MGT	80	0.90	1120-8MGT	140	1.00	1760-8MGT	220	1.10	3600-8MGT	450	1.20
720-8MGT	90	0.90	1160-8MGT	145	1.00	1800-8MGT	225	1.20	4400-8MGT	550	1.20
800-8MGT	100	0.90	1200-8MGT	150	1.00	2000-8MGT	250	1.20			
840-8MGT	105	0.90	1224-8MGT	153	1.00	2200-8MGT	275	1.20			
880-8MGT	110	0.90	1280-8MGT	160	1.10	2400-8MGT	300	1.20			

Example Problem 7–3 For the belt drive layout shown in Figure 7–28 perform the following calculations for the kinematics of the drive:

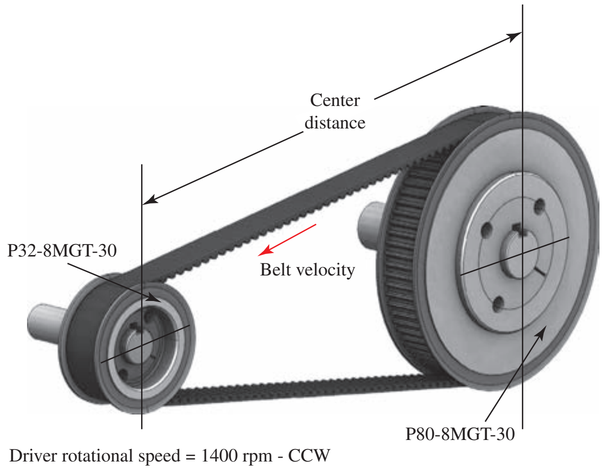


FIGURE 7–28 Synchronous belt drive for Example Problem 7–3

a. Specify the number of teeth and the pitch diameter of each sheave.
b. Calculate the velocity ratio of the belt drive and the angular velocity of the driven sheave.
c. Calculate the belt linear velocity.
d. Determine the center distance using a 1200-8MGT-30 belt.

Solution Given Data in Figure 7–28. Input sprocket speed = 1400 rpm

a. Pitch diameters: From Table 7–4,

P32-8MGT-30 sprocket: 32 teeth; Pitch diameter = 3.208 in
P80-8MGT-30 sprocket: 80 teeth; Pitch diameter = 8.020 in

b. Velocity ratio: Using Equation (7–4) in Section 7–2,

$$VR = \frac{n_{driving}}{n_{driven}} = \frac{N_{driven}}{N_{driving}} = \frac{80}{32} = 2.50$$

Driven speed of the driven sprocket:

$$n_{driven} = \frac{n_{driving}}{VR} = \frac{1400 \text{ rpm}}{2.5} = 560 \text{ rpm}$$

c. Belt speed: Using Equation (7–3) in Section 7–2,

$$v_{belt} = \frac{PD_{driving}}{2} n_{driving} = \frac{3.208 \text{ in}}{2} \cdot 1400 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1.0 \text{ ft}}{12 \text{ in}} = 1175.8 \text{ ft/min}$$

d. Center distance: From Table 7–7, using 32 and 80 teeth and 1200-8MGT-30 belt,

$$CD = 14.61 \text{ in}$$

Example Problem 7–4

Figure 7–29 shows a 20-hp electric motor driving a gear pump using a synchronous belt drive. The normal torque motor speed is 1750 rpm and the gear pump speed is to be 875 rpm. The drive system is on a steel mounting plate with slots permitting an adjustment of the pump position by ± 0.375 in from the nominal center distance of 22.00 in. The motor shaft diameter is 1.625 in and the gear pump shaft diameter is 1.375 in. The sprocket on the pump shaft cannot exceed 14.00 in due to potential interference with the base plate. The pump will operate 16–24 h/day, 5 days per week. Specify the complete drive system.

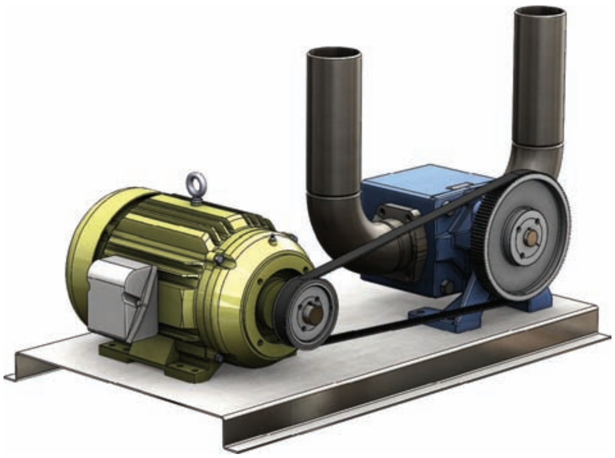


FIGURE 7–29 Gear pump drive system for Example Problem 7–4

Solution Use the General Selection Procedure for Synchronous Belt Drives

Given Motor: 20 hp; 1750 rpm; Normal torque; shaft diameter = 1.625 in

Gear pump: 875 rpm; shaft diameter = 1.375 in; Maximum OD = 14.00 in

Center distance between motor and pump shafts = 22.00 in ± 0.375 in; The range is 21.625 in minimum to 22.375 in maximum

Operating conditions: 16 to 24 h/day, 5 days per week.

Specify the synchronous belt drive system including driving and driven sprockets, bushings, and the belt.

Results

1. Motor speed = 1750 rpm; Pump speed = 875 rpm
2. Rated power = 20 hp
3. Service factor: From Table 7–8, $SF = 1.7$
4. Design power: $P_{des} = P_{rated} \cdot SF = 20 \text{ hp} \cdot 1.7 = 34 \text{ hp}$
5. Required pitch for the belt: Using Figure 7–27, the intersection of 34 hp and 1750 rpm of the driver falls in the range of the 8 mm, 8MGT belt system.
6. Velocity ratio:

$$VR = \frac{n_{driving}}{n_{driven}} = \ = \frac{1750 \text{ rpm}}{875 \text{ rpm}} = 2.00$$

7. Select candidate combinations of driver and driven sprockets: Table 7–7 lists eight possible combinations having $VR = 2.00$. They are

   22:44, 24:48, 28:56, 32:64, 36:72, 40:80, 56:112, 72:144

8. Eliminate the sprocket combinations that are not acceptable because of space limitations and shaft diameter requirements:

   For shaft diameters, the bushing style and the maximum bore sizes must be checked using data in Tables 7–4 and 7–5.

   First check the motor shaft size of 1.625 in.

Sprocket	Bushing no.	Max. Bore (in)	
22	1108	1.00	Not acceptable; too small
24	1108	1.00	
28	1108	1.00	
32	1210	1.25	
36	1610	1.50	
40	1610	1.50	
56	2012	1.875	Acceptable
72	2012	1.875	

Now check the size of the pump sprockets; Maximum OD = 14.00 in. From Table 7–4, all pump sprockets up through 112 teeth have a flange size less than 14.00 in.

For 144 teeth, OD = 14.437 in—Too large.

Therefore, the only combination that satisfies all limitations is 56:112.

For this design: $PD_1 = 5.614$ in, $PD_2 = 11.229$ in

9. Now check the belt length and center distance for this combination:

   From Table 7–7, we find that the 1800-8MGT belt will have a center distance of 22.03 in for a 56-tooth driving sprocket and a 112-tooth driven sprocket. That is within the required range. The 0.75-in long slots in the base plate will permit the installation of the belt and then setting the proper belt tension.

10. Select the belt width: Data in Tables 7–9 to 7–11 can be used. Table 7–9 is for a belt width of 30 mm and for a motor speed of 1750 rpm and 56 grooves in that sprocket, we find the rated power is 43.2 hp. To this value, a belt length correction factor must be applied, found in Table 7–11: For the 1800-8MGT belt, $C_L = 1.20$. Then the adjusted rated power is

$$P_{adjusted} = P_{rated} \cdot C_L = 43.2 \text{ hp} \cdot 1.20 = 51.8 \text{ hp}$$

   This value is well above the design power of 34.0 hp found in Step 4.

11. Calculate the belt speed to ensure that it does not exceed 6500 rpm.

$$V_{belt=} = \frac{PD_1}{2} \cdot \omega_1 = \frac{5.614 \text{ in}}{2} \cdot \frac{1750 \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 2572 \text{ ft/min} = 2572 \text{ fpm}$$

   This is an acceptable belt speed.

12. Specify the final design details: [Quantity (1) each]

   P56-8MGT-30 Sprocket—Motor shaft

   2012–1.625 in diameter bore—Taper-lock bushing

   P112-8MGT-30 Sprocket—Pump shaft

   2517–1.375 in diameter bore—Taper-lock bushing

   1800-8MGT-30 Belt

Alternate Configurations for Synchronous Belt Drives

Idlers and belt tensioners are used to set the correct belt length and take up belt slack if fixed centers are required between the driver and driven sprockets. Idlers do not directly drive any component and are fixed in the belt drive system. A tensioner is an idler that is adjustable to provide the correct belt tension. The location of a tensioner should be on the slack side of the belt span. The tensioner can be located on either the inside or outside of the slack side belt span shown in Figure 7–30. Tensioners located on the inside of the

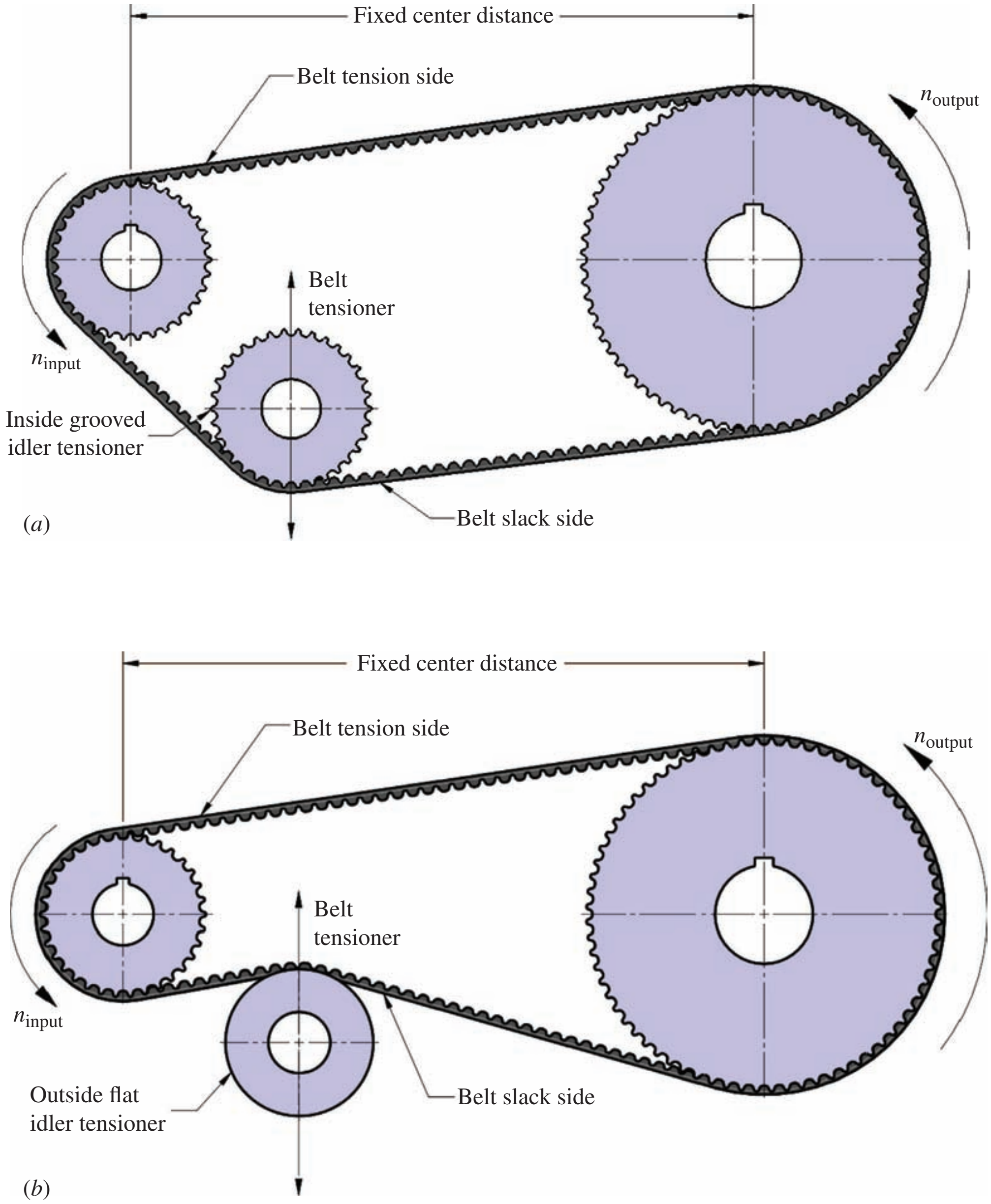


FIGURE 7–30 Belt drives with fixed center distances and added tensioners. (a) Inside grooved-idler tensioner. (b) Outside flat-idler tensioner

belt should use a grooved sprocket and a flat pulley should be used if the idler is located on the outside of the belt. The tensioner may decrease the life of the belt and the belt manufacturer should be consulted.

Belt drives can be used to transmit motion and power reliably and efficiently in a variety of configurations. Figure 7–31 shows two different multiple shaft belt drive configurations. Figure 7–31(a) shows a belt drive that has four sprockets that are the same size. The input sprocket (1) is driving two output sprockets (2 and 3) and an inside tensioner (4) is used to set the length and proper tension of the belt. The sprockets are all rotating in the same direction at the same speed. Figure 7–31(b) shows a belt drive that has an input sprocket (1) that drives two output sprockets (2) and (3). The belt drive also has a flat pulley used as an outside belt tensioner (4). All sprockets are rotating in the same direction. The larger sprocket (2) is rotating slower than the input sprocket. Output sprocket (3) is the same size as the input sprocket and will rotate at the same speed.

Twin power belts shown in Figure 7–32 have teeth on both sides of the belt to provide a positive drive from either side of the belt. Serpentine belt drives allow

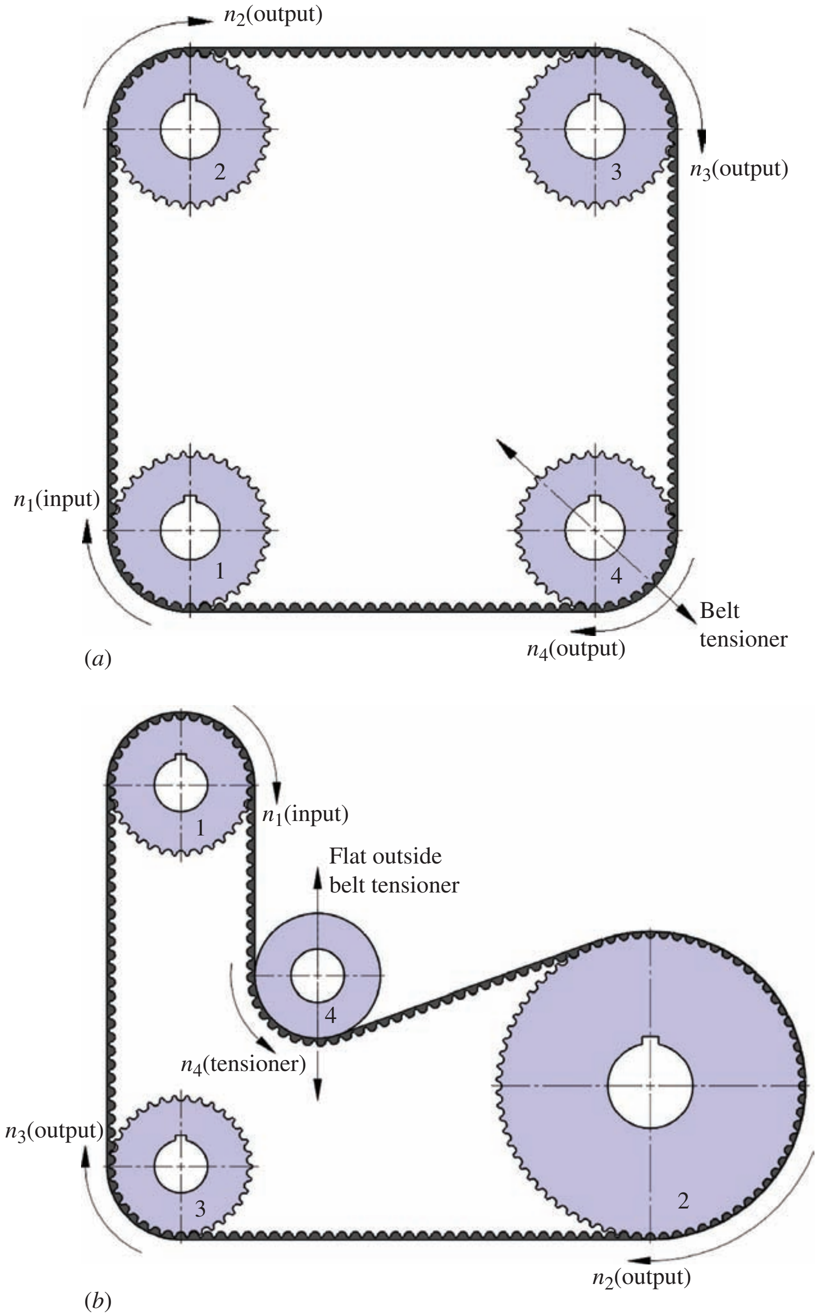


FIGURE 7–31 Multiple shaft drive configurations

designs with multiple drive points to reverse the shaft rotation. Figure 7–32(a) shows a twin tooth serpentine belt drive with the input sprocket (1), output sprocket (2), fixed idler (3), and inside belt tensioner (4). The objective of this design is to have the output sprocket (2) rotate in the opposite direction of the input sprocket (1). The fixed idler (3) does not drive anything, but it is used to wrap the belt around the output sprocket (2) to provide more teeth to carry the belt driving tension. The inside belt tensioner (4) is an idler that is movable and is used to position the belt to wrap the output sprocket (2) and to set proper belt tension. The belt tensioner (4) also does not drive any output directly. Figure 7–31(b) shows a twin tooth serpentine belt drive with an input sprocket (1) and five driven sprockets. The input and output sprockets (2), (4), and (6) have clockwise rotation while the output sprockets (3) and (5) have opposite rotation due to the serpentine belt wrap. The speed of the five output sprockets are dependent on the input and output sprocket ratios.

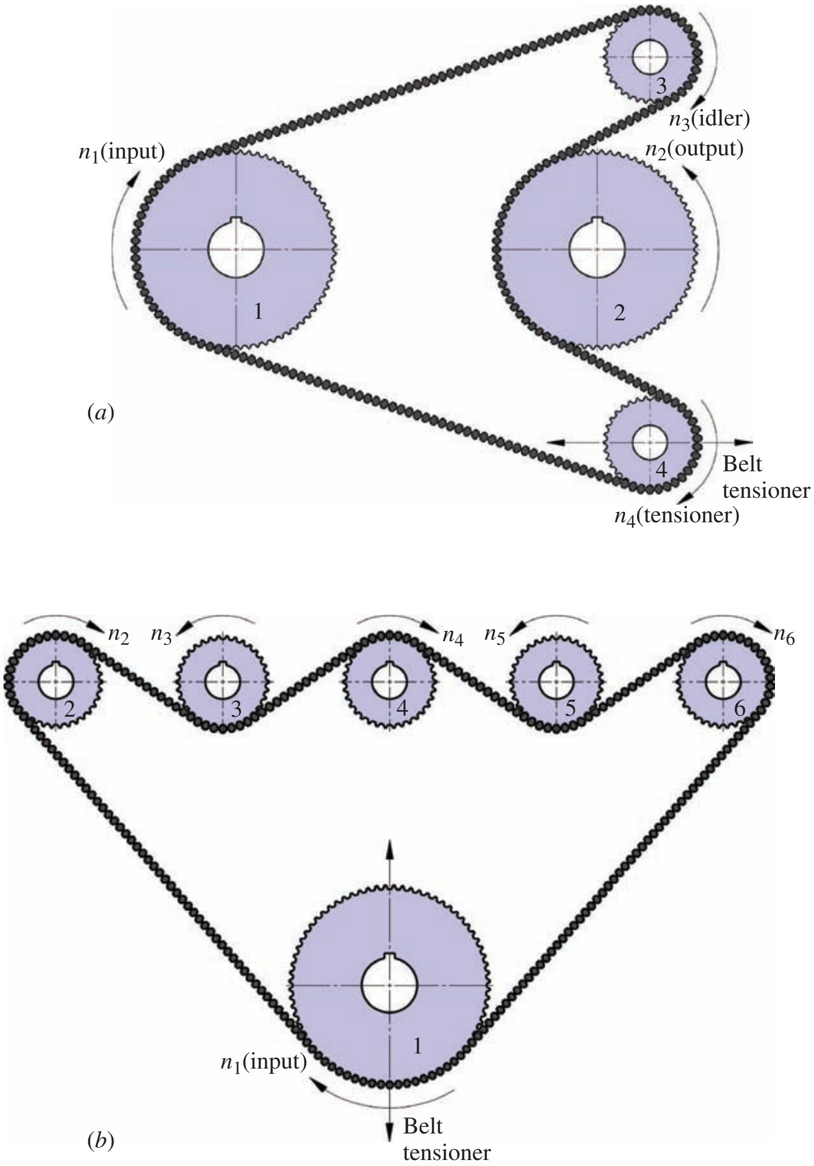


FIGURE 7–32 Serpentine belt drive configurations

7–6 CHAIN DRIVES

A chain is a power transmission element made as a series of pin-connected links. The design provides for flexibility while enabling the chain to transmit large tensile forces. See References 1–3 and Internet sites 1, 4, 6–12, 14, and 15 for more technical information and manufacturers' data.

When transmitting power between rotating shafts, the chain engages mating toothed wheels, called sprockets. Figure 7–33 shows a typical chain drive.

The most common type of chain is the *roller chain,* in which the roller on each pin provides exceptionally low friction between the chain and the sprockets.

Roller chain is classified by its *pitch,* the distance between corresponding parts of adjacent links. The pitch is usually illustrated as the distance between the centers of adjacent pins. U.S. Standard roller chain carries a size designation from 40 to 240, as listed in Table 7–11.

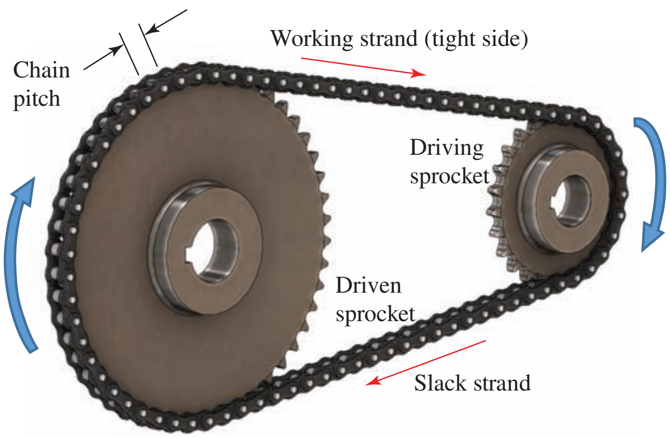


FIGURE 7–33 Basic arrangement of an industrial chain drive

See Reference 2. The digits (other than the final zero) indicate the pitch of the chain in eighths of an inch, as in the table. For example, the no. 100 chain has a pitch of 10/8 or $1\frac{1}{4}$ in. A series of heavy-duty sizes, with the suffix *H* on the designation (60H–240H), has the same basic dimensions as the standard chain of the same number except for thicker side plates. In addition, there are the smaller and lighter sizes: 25, 35, and 41.

The average tensile strengths of the various chain sizes are also listed in Table 7–12. These data can be used for very-low-speed drives or for applications in which the function of the chain is to apply a tensile force or to support a load. It is recommended that only 10% of the average tensile strength be used in such applications. For power transmission, the rating of a given chain size as a function of the speed of rotation must be determined, as explained later in this chapter.

ISO standards define several different chain types, data for three of which are listed in Table 7–13. One commonly used style from ISO-606 has basically the same design dimensions as for many of the standard U.S. roller chains. Then the pitch and dimensions for sprocket features and bore sizes are listed in the metric unit of mm making it more convenient to integrate familiar chain designs into an all-metric piece of equipment. ISO-3512 includes eight sizes of chain used for heavy-duty power transmission and lifting equipment. Some of the pitches for these chains are also equal to common U.S. sizes. Agricultural equipment such as tractor accessories, planters, harvesters, and mowers employ many chain drives to actuate moving systems. ISO-487 defines eight S-designations that cover a wide range of power transmission and tension pull applications. See Reference 3 and Internet sites 7, 9, 14, and 15 for more information on metric-style chains and for manufacturers' data.

Another ISO document that is closely related to U.S. roller chain sizes is ISO 10823 and those designations are shown in Table 7–12. The designations are very similar to those listed in Table 7–13 from ISO 606, except the letter following the number is A instead of B.

Other types of chains include multiple strand designs, heavy series chains, double-pitch chains, and double-pitch conveyor chains as shown on the left side of Figure 7–34. A wide variety of attachments are available to facilitate the application of roller chain to conveying or other material handling uses. Usually in the form of extended plates or tabs with holes provided, the attachments make it easy to connect rods, buckets, parts pushers, part support devices, or conveyor slats to the chain. The right side of Figure 7–34 shows some attachment styles.

Figure 7–35 shows a variety of chain types used especially for conveying and similar applications. Such chain typically has a longer pitch than standard roller chain (usually twice the pitch), and the link plates are heavier. The larger sizes have cast link plates.

TABLE 7–12 U.S. Roller Chain Sizes

Chain number	ISO 10823	Pitch (in)	Average tensile strength (lb)
25	4A	1/4	925
35	6A	3/8	2100
41		1/2	2000
40	8A	1/2	3700
50	10A	5/8	6100
60	12A	3/4	8500
80	18A	1	14 500
100	20A	$1\frac{1}{4}$	24 000
120	24A	$1\frac{1}{2}$	34 000
140	28A	$1\frac{3}{4}$	46 000
160	32A	2	58 000
180	36A	$2\frac{1}{4}$	80 000
200	40A	$2\frac{1}{2}$	95 000
240	48A	3	130 000

Reference: ANS1 Standard B29.1.

Design of Chain Drives

The rating of chain for its power transmission capacity considers three modes of failure: (1) fatigue of the link plates due to the repeated application of the tension in the tight side of the chain, (2) impact of the rollers as they engage the sprocket teeth, and (3) galling between the pins of each link and the bushings on the pins.

The ratings are based on empirical data with a smooth driver and a smooth load (service factor = 1.0) and with a rated life of approximately 15 000 h. The important variables are the pitch of the chain and the size and rotational speed of the smaller sprocket. Lubrication is critical to the satisfactory operation of a chain drive. Manufacturers recommend the type of lubrication method for given combinations of chain size, sprocket size, and speed. Details are discussed later.

Tables 7–14 to 7–16 list the rated power for three sizes of standard chain: no. 40 (1/2 in), no. 60 (3/4 in), and no. 80 (1.00 in). These are typical of the types of data available for all chain sizes in manufacturers' catalogs and can be used for problems in this book. When making final designs and specification, you should consult the catalog data for the particular manufacturer you are using. Notice these features of the data:

1. The ratings are based on the speed of the smaller sprocket and an expected life of approximately 15 000 h.
2. For a given speed, the power capacity increases with the number of teeth on the sprocket. Of course, the larger the number of teeth, the larger the diameter of the sprocket. Note that the use of a chain with a small pitch on a large sprocket produces the quieter drive.

TABLE 7–13 Metric Roller Chain Sizes and Strength Ratings

General-Purpose power transmission

ISO-606

Chain number	Pitch		Breaking strength		Alternate designation
	in	mm	lb	kN	
04B	0.250	6.350	—	—	—
05B	0.315	8.000	989	4.4	—
06B	0.375	9.525	2 001	8.9	Metric 35
08B	0.500	12.700	4 002	17.8	Metric 40
10B	0.625	15.88	4 991	22.2	Metric 50
12B	0.75	19.05	6 497	28.9	Metric 60
16B	1.00	25.40	9 510	42.3	Metric 80
20B	1.25	31.75	14 501	64.5	Metric 100
24B	1.50	38.10	22 010	97.9	Metric 120
28B	1.75	44.45	29 002	129.0	Metric 140
32B	2.00	50.8	37 995	169.0	Metric 160
40B	2.50	63.5	58 993	262.4	Metric 200
48B	3.00	76.2	89 996	400.3	Metric 240
56B	3.50	88.9	122 010	542.7	—
64B	4.00	101.6	160 004	711.7	—
72B	4.50	114.3	202 001	898.5	—

Heavy-Duty power transmission

ISO-3512

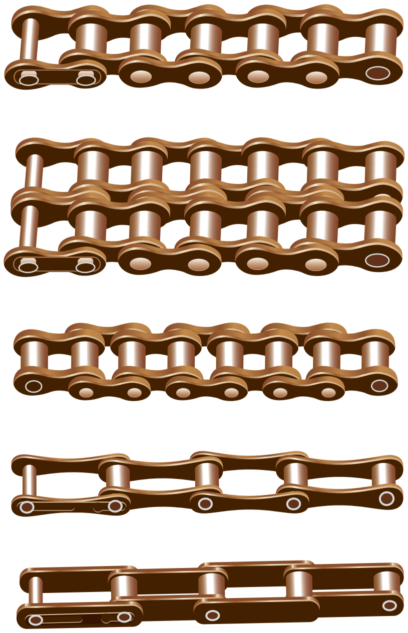
Chain number	Pitch		Breaking strength	
	in	mm	lb	kN
2010	2.500	63.5	58 903	262.0
2512	3.067	77.9	84 982	378
2814	3.500	88.9	116 007	516
3315	4.073	103.5	133 993	596
3618	4.500	114.3	183 004	814
4020	5.000	127.0	236 960	1054
4824	6.000	152.4	341 951	1521
5628	7.000	177.8	464 928	2068

Power transmission for agricultural uses

ISO-487

Chain number	Pitch		Breaking strength	
	in	mm	lb	kN
S32	1.150	29.2	1 799	8.0
S42	1.375	34.9	6 003	26.7
S45	0.843	21.4	4 002	17.8
S52	1.500	38.1	4 002	17.8
S55	1.630	41.4	4 002	17.8
S62	1.650	41.9	6 003	26.7
S77	2.297	58.3	10 004	44.5
S88	2.609	66.3	10 004	44.5

Notes: Not all suppliers offer all sizes.
Breaking strength data must be verified with specific supplier.



Standard roller chain, single strand

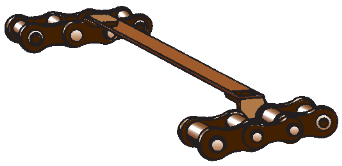
Standard roller chain, two-strand (also available with three and four strands)

Heavy series roller chain

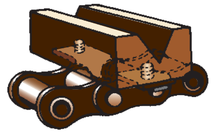
Double-pitch drive chain

Double-pitch conveyor chain

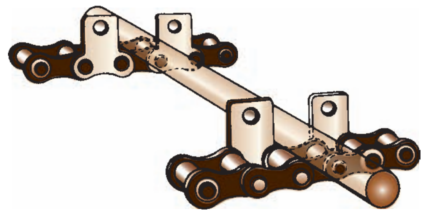
(*a*)



(*a*) Slats assembled to attachments to form a flat conveying surface



(*b*) *V* block assembled to attachments to convey round objects of varying diameters



(*c*) Attachments used as spacers to convey and position long objects

(*b*)

FIGURE 7–34 Other roller chain and examples of attachments

3. For a given sprocket size (a given number of teeth), the power capacity increases with increasing speed up to a point; then it decreases. Fatigue due to the tension in the chain governs at the low to moderate speeds; impact on the sprockets governs at the higher speeds. Each sprocket size has an absolute upper-limit speed due to the onset of galling between the pins and the bushings of the chain. This explains the abrupt drop in power capacity to zero at the limiting speed.
4. The ratings are for a single strand of chain. Although multiple strands do increase the power capacity, they do not provide a direct multiple of the single-strand capacity. Multiply the capacity in the tables by the following factors.

   Two strands: Factor $= 1.7$

   Three strands: Factor $= 2.5$

   Four strands: Factor $= 3.3$

5. The chain manufacturer's ratings are for a service factor of 1.0. Specify a service factor for a given application as shown in Table 7–16. The combination of the nature of the driving member and the driven machine permit the selection of the service factor. Then compute the design power, P_{des}, from,

$$P_{des} = SF(P)$$

where P is the power delivered to the chain drive.

DESIGN GUIDELINES FOR CHAIN DRIVES

The following are general recommendations for designing chain drives:

1. The minimum number of teeth in a sprocket should be 17 unless the drive is operating at a very low speed, under 100 rpm.
2. The maximum speed ratio should be 7.0, although higher ratios are feasible. Two or more stages of reduction can be used to achieve higher ratios.
3. The center distance between the sprocket axes should be approximately 30 to 50 pitches (30–50 times the pitch of the chain).
4. The larger sprocket should normally have no more than 120 teeth.
5. The preferred arrangement for a chain drive is with the centerline of the sprockets horizontal and with the tight side on top.
6. The chain length must be an integral multiple of the pitch, and *an even number of pitches is recommended.* The center distance should be made adjustable to accommodate the chain length and to take up for tolerances and wear. Excessive sag on the slack side should be avoided, especially on drives that are not horizontal. A convenient relation between center distance (CD), chain length (L_c), number of teeth in the small sprocket (N_1), and number of teeth in the large sprocket (N_2), expressed *in pitches*, is

Chain Length in Pitches

$$L_c = 2CD + \frac{N_2 + N_1}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 CD} \quad \textbf{(7–18)}$$

The center distance for a given chain length, again *in pitches,* is

Center Distance in Pitches

$$CD = \frac{1}{4}\left[L_c - \frac{N_2 + N_1}{2} + \sqrt{\left[L_c - \frac{N_2 + N_1}{2}\right]^2 - \frac{8(N_2 - N_1)^2}{4\pi^2}}\right] \quad \textbf{(7–19)}$$

The computed center distance assumes no sag in either the tight or the slack side of the chain, and thus it is a *maximum.* Negative tolerances or adjustment must be provided. Adjustment for wear must also be provided.

7. The pitch diameter of a sprocket with N teeth for a chain with a pitch of p is

Pitch Diameter of Sprocket

$$PD = \frac{p}{\sin(180°/N)} \text{ inches or mm} \quad \textbf{(7–20)}$$

8. The minimum sprocket diameter and therefore the minimum number of teeth in a sprocket are often limited by the size of the shaft on which it is mounted. Check the sprocket catalog.
9. The arc of contact, θ_1, often called the *angle of wrap,* of the chain on the smaller sprocket should be greater than 120°.

Angle of Wrap Smaller Sprocket

$$\theta_1 = 180° - 2\sin^{-1}[(PD_2 - PD_1)/2CD] \quad \textbf{(7–21)}$$

10. For reference, the arc of contact, θ_2, on the larger sprocket is

Angle of Wrap Large Sprocket

$$\theta_2 = 180° + 2\sin^{-1}[(PD_2 - PD_1)/2CD] \quad \textbf{(7–22)}$$

Mill, narrow series
(drive and conveyor sizes)
Offset cast-link chain used primarily in the lumber industry for conveyor applications.

Combination mill
(wide conveyor sizes)
Cast block links and steel sidebar construction for drag conveyor applications.

Heavy-duty drag chain
Cast steel offset block links. Used in ash and clinker conveyors.

Pintle chain
Chain constructed of a series of cast offset links coupled by steel pins or rivets. Suitable for slow-to moderate-speed drive, conveyor and elevator service.

Roller-top transfer
Cast links with top rollers used in several strands to convey material transversely.

Roof-top
Cast roof-shaped links used in several strands on transfer conveyors.

Detachable
Consists of unit links, each with an open-type hook that flexes on the end bar of the adjacent link. Used for slow- to moderate-speed drive and conveyor application.

Drop-forged
Drop-forged inner and outer links coupled by headed pins. Used for trolley, scraper, flight, and similar conveyors.

FIGURE 7–35 Conveyor chains (Rexnord Industries, LLC, Milwaukee, WI)

TABLE 7–14 Horsepower Ratings—Single-Strand Roller Chain No. 40

No. of teeth	0.500-in pitch										Rotational speed of small sprocket, rev/min												
	10	25	50	100	180	200	300	500	700	900	1000	1200	1400	1600	1800	2100	2500	3000	3500	4000	5000	6000	7000	8000	9000
11	0.06	0.14	0.27	0.52	0.91	1.00	1.48	2.42	3.34	4.25	4.70	5.60	6.49	5.57	4.66	3.70	2.85	2.17	1.72	1.41	1.01	0.77	0.61	0.50	0.00
12	0.06	0.15	0.29	0.56	0.99	1.09	1.61	2.64	3.64	4.64	5.13	6.11	7.09	6.34	5.31	4.22	3.25	2.47	1.96	1.60	1.15	0.87	0.69	0.57	0.00
13	0.07	0.16	0.31	0.61	1.07	1.19	1.75	2.86	3.95	5.02	5.56	6.62	7.68	7.15	5.99	4.76	3.66	2.79	2.21	1.81	1.29	0.98	0.78	0.00	
14	0.07	0.17	0.34	0.66	1.15	1.28	1.88	3.08	4.25	5.41	5.98	7.13	8.27	7.99	6.70	5.31	4.09	3.11	2.47	2.02	1.45	1.10	0.87	0.00	
15	0.08	0.19	0.36	0.70	1.24	1.37	2.02	3.30	4.55	5.80	6.41	7.64	8.86	8.86	7.43	5.89	4.54	3.45	2.74	2.24	1.60	1.22	0.97	0.00	
16	0.08	0.20	0.39	0.75	1.32	1.46	2.15	3.52	4.86	6.18	6.84	8.15	9.45	9.76	8.18	6.49	5.00	3.80	3.02	2.47	1.77	1.34	0.00		
17	0.09	0.21	0.41	0.80	1.40	1.55	2.29	3.74	5.16	6.57	7.27	8.66	10.04	10.69	8.96	7.11	5.48	4.17	3.31	2.71	1.94	1.47	0.00		
18	0.09	0.22	0.43	0.84	1.48	1.64	2.42	3.96	5.46	6.95	7.69	9.17	10.63	11.65	9.76	7.75	5.97	4.54	3.60	2.95	2.11	1.60	0.00		
19	0.10	0.24	0.46	0.89	1.57	1.73	2.56	4.18	5.77	7.34	8.12	9.66	11.22	12.64	10.59	8.40	6.47	4.92	3.91	3.20	2.29	0.09	0.00		
20	0.10	0.25	0.48	0.94	1.65	1.82	2.69	4.39	6.07	7.73	8.55	10.18	11.81	13.42	11.44	9.07	6.99	5.31	4.22	3.45	2.47	0.00			
21	0.11	0.26	0.51	0.98	1.73	1.91	2.83	4.61	6.37	8.11	8.98	10.69	12.40	14.10	12.30	9.76	7.52	5.72	4.54	3.71	2.65	0.00			
22	0.11	0.27	0.53	1.03	1.81	2.01	2.96	4.83	6.68	8.50	9.40	11.20	12.99	14.77	13.19	10.47	8.06	6.13	4.87	3.98	2.85	0.00			
23	0.12	0.28	0.56	1.08	1.90	2.10	3.10	5.05	6.98	8.89	9.83	11.71	13.58	15.44	14.10	11.19	8.62	6.55	5.20	4.26	3.05	0.00			
24	0.12	0.30	0.58	1.12	1.98	2.19	3.23	5.27	7.28	9.27	10.26	12.22	14.17	16.11	15.03	11.93	9.18	6.99	5.54	4.54	0.87	0.00			
25	0.13	0.31	0.60	1.17	2.06	2.28	3.36	5.49	7.59	9.66	10.69	12.73	14.76	16.78	15.98	12.68	9.76	7.43	5.89	4.82	0.00				
26	0.13	0.32	0.63	1.22	2.14	2.37	3.50	5.71	7.89	10.04	11.11	13.24	15.35	17.45	16.95	13.45	10.36	7.88	6.25	5.12	0.00				
28	0.14	0.35	0.67	1.31	2.31	2.55	3.77	6.15	8.50	10.82	11.97	14.26	16.53	18.79	18.94	15.03	11.57	8.80	6.99	5.72	0.00				
30	0.15	0.37	0.72	1.41	2.47	2.74	4.04	6.59	9.11	11.59	12.82	15.28	17.71	20.14	21.01	16.67	12.84	9.76	7.75	6.34	0.00				
32	0.16	0.40	0.77	1.50	2.64	2.92	4.31	7.03	9.71	12.38	13.66	16.30	18.89	21.48	23.14	18.37	14.14	10.76	8.54	1.41					
35	0.18	0.43	0.84	1.64	2.88	3.19	4.71	7.69	10.62	13.52	14.96	17.82	20.67	23.49	26.30	21.01	16.17	12.30	9.76	0.00					
40	0.21	0.50	0.96	1.87	3.30	3.65	5.38	8.79	12.14	15.45	17.10	20.37	23.62	26.85	30.06	25.67	19.76	15.03	0.00						
45	0.23	0.56	1.08	2.11	3.71	4.10	6.06	9.89	13.66	17.39	19.24	22.92	26.57	30.20	33.82	30.63	23.58	5.53	0.00						
	Type A				Type B									Type C											

Type A: Manual or drip lubrication
Type B: Bath or disc lubrication
Type C: Oil stream lubrication

TABLE 7–15 Horsepower Ratings—Single-Strand Roller Chain No. 60

No. of teeth	0.750-in pitch				Rotational speed of small sprocket, rev/min																				
	10	25	50	100	120	200	300	400	500	600	800	1000	1200	1400	1600	1800	2000	2500	3000	3500	4000	4500	5000	5500	6000
11	0.19	0.46	0.89	1.72	2.05	3.35	4.95	6.52	8.08	9.63	12.69	15.58	11.85	9.41	7.70	6.45	5.51	3.94	3.00	2.38	1.95	1.63	1.39	1.21	0.00
12	0.21	0.50	0.97	1.88	2.24	3.66	5.40	7.12	8.82	10.51	13.85	17.15	13.51	10.72	8.77	7.35	6.28	4.49	3.42	2.71	2.22	1.86	1.59	1.38	0.00
13	0.22	0.54	1.05	2.04	2.43	3.96	5.85	7.71	9.55	11.38	15.00	18.58	15.23	12.08	9.89	8.29	7.08	5.06	3.85	3.06	2.50	2.10	1.79	0.00	
14	0.24	0.58	1.13	2.19	2.61	4.27	6.30	8.30	10.29	12.26	16.15	20.01	17.02	13.51	11.05	9.26	7.91	5.66	4.31	3.42	2.80	2.34	0.41	0.00	
15	0.26	0.62	1.21	2.35	2.80	4.57	6.75	8.90	11.02	13.13	17.31	21.44	18.87	14.98	12.26	10.27	8.77	6.28	4.77	3.79	3.10	2.60	0.00		
16	0.27	0.66	1.29	2.51	2.99	4.88	7.20	9.49	11.76	14.01	18.46	22.87	20.79	16.50	13.51	11.32	9.66	6.91	5.26	4.17	3.42	1.78	0.00		
17	0.29	0.70	1.37	2.66	3.17	5.18	7.65	10.08	12.49	14.88	19.62	24.30	22.77	18.07	14.79	12.40	10.58	7.57	5.76	4.57	3.74	0.00			
18	0.31	0.75	1.45	2.82	3.36	5.49	8.10	10.68	13.23	15.76	20.77	25.73	24.81	19.69	16.11	13.51	11.53	8.25	6.28	4.98	4.08	0.00			
19	0.33	0.79	1.53	2.98	3.55	5.79	8.55	11.27	13.96	16.63	21.92	27.16	26.91	21.35	17.48	14.65	12.50	8.95	6.81	5.40	0.20	0.00			
20	0.34	0.83	1.61	3.13	3.73	6.10	9.00	11.86	14.70	17.51	23.08	28.59	29.06	23.06	18.87	15.82	13.51	9.66	7.35	5.83	0.00				
21	0.36	0.87	1.69	3.29	3.92	6.40	9.45	12.46	15.43	18.38	24.23	30.02	31.26	24.81	20.31	17.02	14.53	10.40	7.91	6.28	0.00				
22	0.38	0.91	1.77	3.45	4.11	6.71	9.90	13.05	16.17	19.26	25.39	31.45	33.52	26.60	21.77	18.25	15.58	11.15	8.48	0.00					
23	0.40	0.95	1.85	3.61	4.29	7.01	10.35	13.64	16.90	20.13	26.54	32.88	35.84	28.44	23.28	19.51	16.66	11.92	9.07	0.00					
24	0.41	0.99	1.93	3.76	4.48	7.32	10.80	14.24	17.64	21.01	27.69	34.31	38.20	30.31	24.81	20.79	17.75	12.70	9.66	0.00					
25	0.43	1.04	2.01	3.92	4.67	7.62	11.25	14.83	18.37	21.89	28.85	35.74	40.61	32.23	26.38	22.11	18.87	13.51	10.27	0.00					
26	0.45	1.08	2.09	4.08	4.85	7.93	11.70	15.42	19.11	22.76	30.00	37.17	43.07	34.18	27.98	23.44	20.02	14.32	10.90	0.00					
28	0.48	1.16	2.26	4.39	5.23	8.54	12.60	16.61	20.58	24.51	32.31	40.03	47.68	38.20	31.26	26.20	22.37	16.01	0.00						
30	0.52	1.24	2.42	4.70	5.60	9.15	13.50	17.79	22.05	26.26	34.62	42.89	51.09	42.36	34.67	29.06	24.81	17.75	0.00						
32	0.55	1.33	2.58	5.02	5.98	9.76	14.40	18.98	23.52	28.01	36.92	45.75	54.50	46.67	38.20	32.01	27.33	19.56	0.00						
35	0.60	1.45	2.82	5.49	6.54	10.67	15.75	20.76	25.72	30.64	40.39	50.03	59.60	53.38	43.69	36.62	31.26	1.35	0.00						
40	0.69	1.66	3.22	6.27	7.47	12.20	18.00	23.73	29.39	35.02	46.16	57.18	68.12	65.22	53.38	44.74	38.20	0.00							
45	0.77	1.86	3.63	7.05	8.40	13.72	20.25	26.69	33.07	38.39	51.92	64.33	76.63	77.83	63.70	53.38	12.45	0.00							
	Type A				Type B							Type C													

Type A: Manual or drip lubrication

Type B: Bath or disc lubrication

Type C: Oil stream lubrication

TABLE 7–16 Horsepower Ratings—Single-Strand Roller Chain No. 80

No. of teeth	1.000-in pitch											Rotational speed of small sprocket, rev/min											
	10	25	50	75	88	100	200	300	400	500	600	700	800	900	1000	1200	1400	1600	1800	2000	2500	3000	3500	4000	4500
11	0.44	1.06	2.07	3.05	3.56	4.03	7.83	11.56	15.23	18.87	22.48	26.07	27.41	22.97	19.61	14.92	11.84	9.69	8.12	6.83	4.96	3.77	3.00	2.45	0.00
12	0.48	1.16	2.26	3.33	3.88	4.39	8.54	12.61	16.82	20.59	24.53	28.44	31.23	26.17	22.35	17.00	13.49	11.04	9.25	7.90	5.65	4.30	3.41	2.79	0.00
13	0.52	1.26	2.45	3.61	4.21	4.76	9.26	13.66	18.00	22.31	26.57	30.81	35.02	29.51	25.20	19.17	15.21	12.45	10.43	8.91	6.37	4.85	3.85	3.15	
14	0.56	1.35	2.63	3.89	4.53	5.12	9.97	14.71	19.39	24.02	28.62	33.18	37.72	32.98	28.16	21.42	17.00	13.91	11.66	9.96	7.12	5.42	4.30	3.52	
15	0.60	1.45	2.82	4.16	4.86	5.49	10.68	15.76	20.77	25.74	30.66	35.55	40.41	36.58	31.23	23.76	18.85	15.43	12.93	11.04	7.90	6.01	4.77	0.00	
16	0.64	1.55	3.01	4.44	5.18	5.86	11.39	16.81	22.16	27.45	32.70	37.92	43.11	40.30	34.41	26.17	20.77	17.00	14.25	12.16	8.70	6.62	5.25	0.00	
17	0.68	1.64	3.20	4.72	5.50	6.22	12.10	17.86	23.54	29.17	34.75	40.29	45.80	44.13	37.68	28.66	22.75	18.62	15.60	13.32	9.53	7.25	0.00		
18	0.72	1.74	3.39	5.00	5.83	6.59	12.81	18.91	24.93	30.88	36.79	42.66	48.49	48.08	41.05	31.23	24.78	20.29	17.00	14.51	10.39	7.90	0.00		
19	0.76	1.84	3.57	5.28	6.15	6.95	13.53	19.96	26.31	32.60	38.84	45.03	51.19	52.15	44.52	33.87	26.88	22.00	18.44	15.74	11.26	0.36	0.00		
20	0.80	1.93	3.76	5.55	6.47	7.32	14.24	21.01	27.70	34.32	40.88	47.40	53.88	56.32	48.08	36.58	29.03	23.76	19.91	17.00	12.16	0.00			
21	0.84	2.03	3.95	5.83	6.80	7.69	14.95	22.07	29.08	36.03	42.92	49.77	56.58	60.59	51.73	39.36	31.23	25.56	21.42	18.29	13.09	0.00			
22	0.88	2.13	4.14	6.11	7.12	8.05	15.66	23.12	30.47	37.75	44.97	52.14	59.27	64.97	55.47	42.20	33.49	27.41	22.97	19.61	14.03				
23	0.92	2.22	4.33	6.39	7.45	8.42	16.37	24.17	31.85	39.46	47.01	54.51	61.97	69.38	59.30	45.11	35.80	29.30	24.55	20.97	15.00				
24	0.96	2.32	4.52	6.66	7.77	8.78	17.09	25.22	33.24	41.18	49.06	56.88	64.66	72.40	63.21	48.08	38.16	31.23	26.17	22.35	15.99				
25	1.00	2.42	4.70	6.94	8.09	9.15	17.80	26.27	34.62	42.89	51.10	59.25	67.35	75.42	67.20	51.12	40.57	33.20	27.83	23.76	8.16				
26	1.04	2.51	4.89	7.22	8.42	9.52	18.51	27.32	36.01	44.61	53.14	61.62	70.05	78.43	71.27	54.22	43.02	36.22	29.51	25.20	0.00				
28	1.12	2.71	5.27	7.77	9.06	10.25	19.93	29.42	38.78	48.04	57.23	66.36	75.44	84.47	79.65	60.59	48.08	39.36	32.98	28.16	0.00				
30	1.20	2.90	5.64	8.33	9.71	10.98	21.36	31.52	41.55	51.47	61.32	71.10	80.82	90.50	88.33	67.20	53.33	43.65	36.58	31.23					
32	1.28	3.09	6.02	8.89	10.36	11.71	22.78	33.62	44.32	54.91	65.41	75.84	86.21	96.53	97.31	74.03	58.75	48.08	40.30	5.65					
35	1.40	3.38	6.58	9.72	11.33	12.81	24.92	36.78	48.47	60.05	71.54	82.95	94.29	105.58	111.31	84.68	67.20	55.00	28.15	0.00					
40	1.61	3.87	7.53	11.11	12.95	14.64	28.48	42.03	55.40	68.63	81.76	94.80	107.77	120.67	133.51	103.46	82.10	40.16	0.00						
45	1.81	4.35	8.47	12.49	14.57	16.47	32.04	47.28	62.32	77.21	91.98	106.65	121.24	135.75	150.20	123.45	72.28	0.00							
	Type A				Type B								Type C												

Type A: Manual or drip lubrication
Type B: Bath or disc lubrication
Type C: Oil stream lubrication

TABLE 7–17 Service Factors for Chain Drives

Load type	Type of driver: Hydraulic drive	Electric motor or turbine	Internal combustion engine with mechanical drive
Smooth Agitators; fans; generators; grinders; centrifugal pumps; rotary screens; light, uniformly loaded conveyors	1.0	1.0	1.2
Moderate shock Bucket elevators; machine tools; cranes; heavy conveyors; food mixers and grinders; ball mills; reciprocating pumps; woodworking machinery	1.2	1.3	1.4
Heavy shock Punch presses; hammer mills; boat propellers; crushers; reciprocating conveyors; rolling mills; logging hoists; dredges; printing presses	1.4	1.5	1.7

Lubrication

It is essential that adequate lubrication be provided for chain drives. There are numerous moving parts within the chain, along with the interaction between the chain and the sprocket teeth. The designer must define the lubricant properties and the method of lubrication.

Lubricant Properties. Petroleum-based lubricating oil similar to engine oil is recommended. Its viscosity must enable the oil to flow readily between chain surfaces that move relative to each other while providing adequate lubrication action. The oil should be kept clean and free of moisture. Table 7–18 gives the recommended lubricants for different ambient temperatures.

TABLE 7–18 Recommended Lubricant for Chain Drives

Ambient temperature °F	Ambient temperature °C	Recommended lubricant
20 to 40	−7 to 5	SAE 20
40 to 100	5 to 38	SAE 30
100 to 120	38 to 49	SAE 40
120 to 140	49 to 60	SAE 50

Method of Lubrication. The American Chain Association recommends three different types of lubrication depending on the speed of operation and the power being transmitted. See Tables 7–14 to 7–16 or manufacturers' catalogs for recommendations. Refer to the following descriptions of the methods and the illustrations in Figure 7–36.

Type A. *Manual or drip lubrication:* For manual lubrication, oil is applied copiously with a brush or a spout can, at least once every 8 hours of operation. For drip feed lubrication, oil is fed directly onto the link plates of each chain strand.

Type B. *Bath or disc lubrication:* The chain cover provides a sump of oil into which the chain dips continuously. Alternatively, a disc or a slinger can be attached to one of the shafts to lift oil to a trough above the lower strand of chain. The trough then delivers a stream of oil to the chain. The chain itself, then, does not need to dip into the oil.

Type C. *Oil stream lubrication:* An oil pump delivers a continuous stream of oil on the lower part of the chain.

Example Problem 7–5 Figure 7–37 shows a drive for a heavily loaded conveyor for use in the fields of a large commercial produce farm to take heavy containers of potatoes from the field onto trucks that will transport them to the processing plant. The conveyor is to be driven by a gasoline engine delivering 15.0 hp at a speed of 900 rpm. The conveyor pulley speed is to be 230 to 240 rpm. Design the chain drive.

Solution Objective: Design the chain drive.

Given: Power transmitted = 15 hp to a heavily loaded produce conveyor

Speed of motor = 900 rpm; output speed range = 230 to 240 rpm

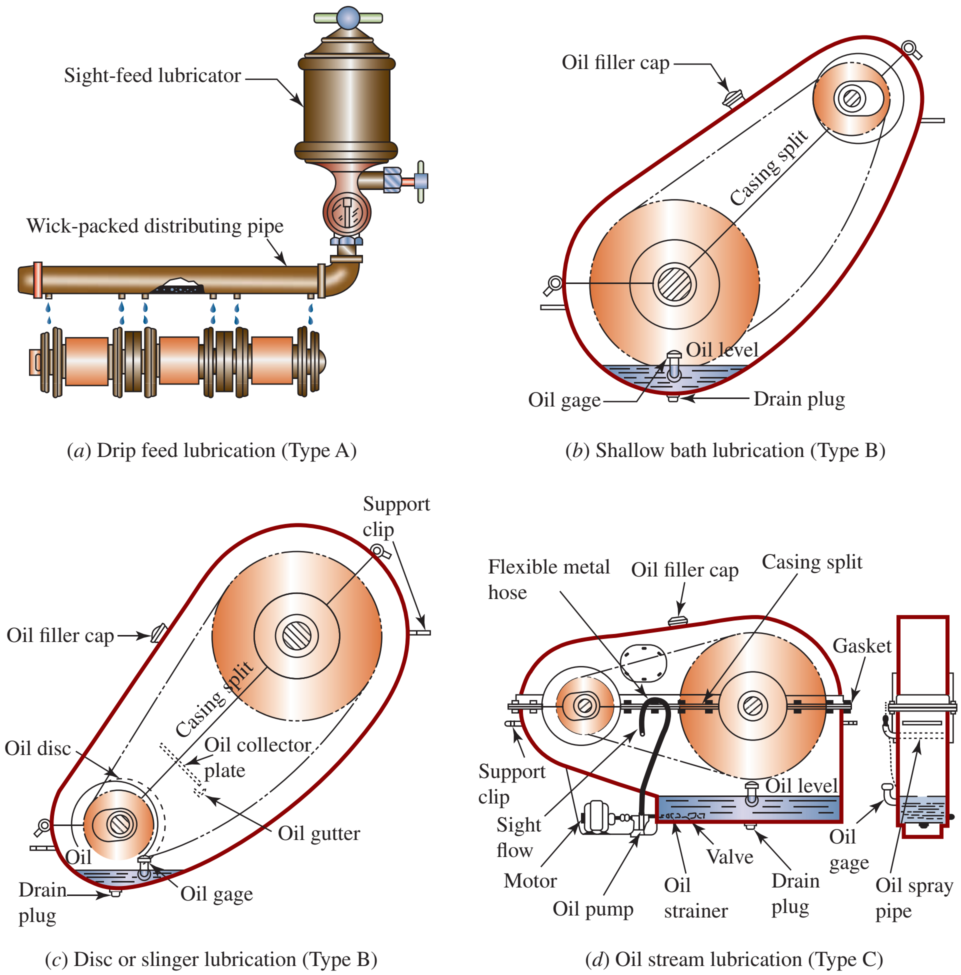


FIGURE 7–36 Lubrication methods

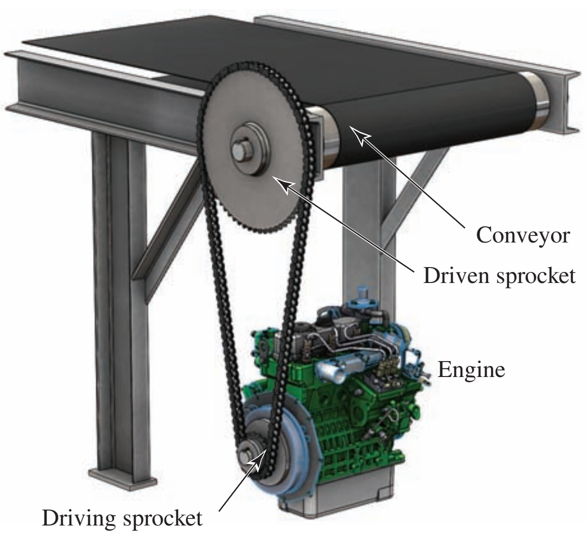


FIGURE 7–37 Chain drive for a heavy-duty conveyor for Example Problem 7–5

Analysis Use the design data presented in this section. The solution procedure is developed within the Results section of the problem solution.

Results ***Step 1.*** Specify a service factor and compute the design power. From Table 7–17, for moderate shock from the loads applied to the heavy-duty conveyor and a gasoline engine drive, use a service factor of $SF = 1.4$. Then,

$$\text{Design power} = P_{des} = SF(P) = 1.4(15.0\text{ hp}) = 21.0\text{ hp}$$

Step 2. Compute the desired ratio. Using the middle of the required range of output speeds, we have

$$\text{Ratio} = (900\text{ rpm})/(235\text{ rpm}) = 3.83$$

Step 3. Refer to the tables for power capacity (Tables 7–14 to 7–16), and select the chain pitch. For a single strand, the no. 60 chain with $p = 3/4$ in seems best. A 17-tooth sprocket is rated at 21.96 hp at 900 rpm by interpolation. At this speed, Type B lubrication (oil bath) is required.

Step 4. Compute the required number of teeth on the large sprocket:

$$N_2 = N_1 \times \text{ratio} = 17(3.83) = 65.11$$

Let's use the integer: 65 teeth.

Step 5. Compute the actual expected output speed:

$n_2 = n_1(N_1/N_2) = 900\text{ rpm}(17/65) = 235.3\text{ rpm}$ (Okay! The speed is within the specified range.)

Step 6. Compute the pitch diameters of the sprockets using Equation (7–20):

$$PD_1 = \frac{p}{\sin(180°/N_1)} = \frac{0.75\text{ in}}{\sin(180°/17)} = 4.082\text{ in}$$

$$PD_2 = \frac{p}{\sin(180°/N_2)} = \frac{0.75\text{ in}}{\sin(180°/65)} = 15.524\text{ in}$$

Step 7. Specify the nominal center distance. Let's use the middle of the recommended range, 40 pitches.

Step 8. Compute the required chain length in pitches from Equation (7–18):

$$L_c = 2CD + \frac{N_2 + N_1}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 CD}$$

$$L_c = 2(40) + \frac{65 + 17}{2} + \frac{(65 - 17)^2}{4\pi^2(40)} = 122.5\text{ pitches}$$

Step 9. Specify an integral number of pitches for the chain length, and compute the actual theoretical center distance. Let's use 122 pitches, an even number. Then, from Equation (7–19),

$$CD = \frac{1}{4}\left[L_c - \frac{N_2 + N_1}{2} + \sqrt{\left[L_c - \frac{N_2 + N_1}{2}\right]^2 - \frac{8(N_2 - N_1)^2}{4\pi^2}}\right]$$

$$CD = \frac{1}{4}\left[122 - \frac{65 + 17}{2} + \sqrt{\left[122 - \frac{65 + 17}{2}\right]^2 - \frac{8(65 - 17)^2}{4\pi^2}}\right]$$

$$CD = 39.766\text{ pitches} = 39.766(0.75\text{ in}) = 29.825\text{ in}$$

Step 10. Compute the angle of wrap of the chain for each sprocket using Equations (7–21) and (7–22). Note that the minimum angle of wrap should be 120°.

For the small sprocket,

$$\theta_1 = 180° - 2\sin^{-1}[(PD_2 - PD_1)/2CD]$$

$$\theta_1 = 180° - 2\sin^{-1}[(15.524 - 4.082)/(2(29.825))] = 158°$$

Because this is greater than 120°, it is acceptable.
For the larger sprocket,

$$\theta_2 = 180° + 2\sin^{-1}[(PD_2 - PD_1)/2CD]$$

$$\theta_2 = 180° + 2\sin^{-1}[(15.524 - 4.082)/(2(29.825))] = 202°$$

Comments **Summary of Design**

Figure 7–38(a) shows a sketch of the design to scale.

Pitch: No. 60 chain, 3/4-in pitch

Length: 122 pitches = 122(0.75) = 91.50 in

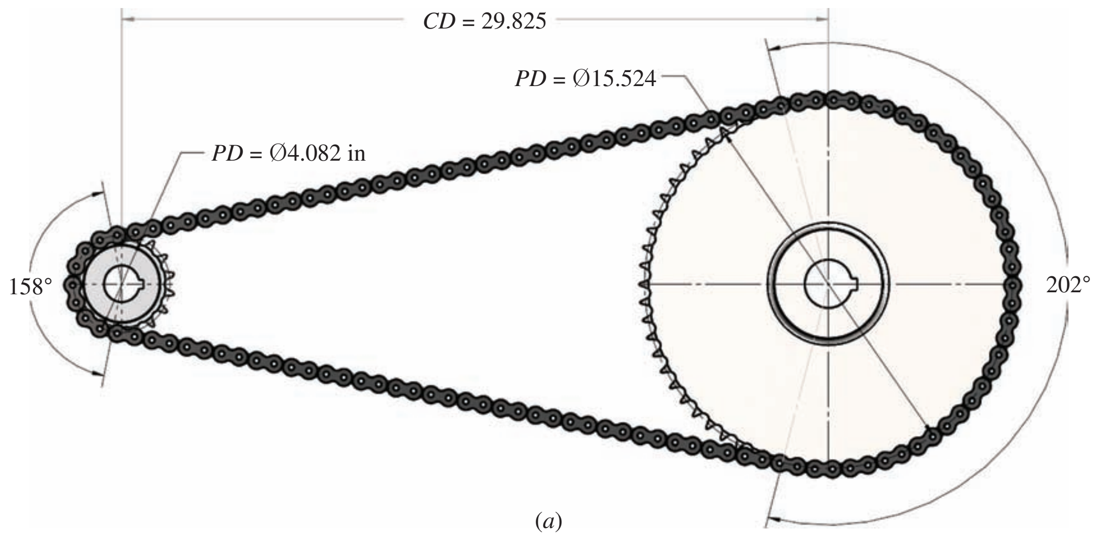
Center distance: CD = 29.825 in (maximum)

Sprockets: Single-strand, no. 60, 3/4-in pitch

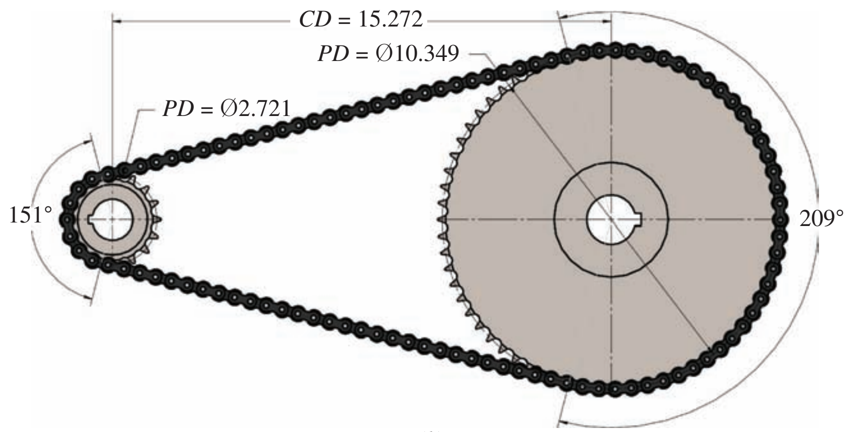
Small: 17 teeth, PD = 4.082 in

Large: 65 teeth, PD = 15.524 in

Type B lubrication is required. The large sprocket can dip into an oil bath.



(a)



(b)

FIGURE 7–38 Scale drawings of layouts for chain drives for Example Problems 7–5 and 7–6

Example Problem 7–6 Create an alternate design for the conditions of Example Problem 7–5 to produce a smaller drive.

Solution

Objective: Design a smaller chain drive for the application in Example Problem 7–5.

Given: Power transmitted = 15 hp to a conveyor

Speed of motor = 900 rpm; output speed range = 230 to 240 rpm

Analysis: Use a multistrand design to permit a smaller-pitch chain to be used to transmit the same design power (21.0 hp) at the same speed (900 rpm). Use the design data presented in this section. The solution procedure is developed within the Results section of the problem solution.

Results: Let's try a four-strand chain for which the power capacity factor is 3.3. Then the required power per strand is

$$P = 21.0/3.3 = 6.36 \text{ hp}$$

From Table 7–14, we find that a no. 40 chain (1/2-in pitch) with a 17-tooth sprocket will be satisfactory. Type B lubrication, oil bath, can be used.

For the required large sprocket,

$$N_2 = N_1 \times \text{ratio} = 17(3.83) = 65.11$$

Let's use $N_2 = 65$ teeth.

The sprocket diameters are

$$PD_1 = \frac{p}{\sin(180°/N_1)} = \frac{0.500 \text{ in}}{\sin(180°/17)} = 2.721 \text{ in}$$

$$PD_2 = \frac{p}{\sin(180°/N_2)} = \frac{0.500 \text{ in}}{\sin(180°/65)} = 10.349 \text{ in}$$

For the center distance, let's try the minimum recommended: $C = 30$ pitches.

$$30(0.50 \text{ in}) = 15.0 \text{ in}$$

The chain length is

$$L_c = 2(30) + \frac{65 + 17}{2} + \frac{(65 - 17)^2}{4\pi^2(30)} = 102.9 \text{ pitches}$$

Specify the integer length, $L_c = 104$ pitches $= 104(0.50) = 52.0$ in. The actual maximum center distance is

$$CD = \frac{1}{4}\left[104 - \frac{65 + 17}{2} + \sqrt{\left[104 - \frac{65 + 17}{2}\right]^2 - \frac{8(65 - 17)^2}{4\pi^2}}\right]$$

$$CD = 30.54 \text{ pitches} = 30.54(0.50) = 15.272 \text{ in}$$

Compute the angle of wrap of the chain for each sprocket using Equations (7–21) and (7–22). Note that the minimum angle of wrap should be 120°.

For the small sprocket,

$$\theta_1 = 180° - 2\sin^{-1}[(PD_2 - PD_1)/2C]$$

$$\theta_1 = 180° - 2\sin^{-1}[(10.349 - 2.721)/(2(15.272))] = 151.1°$$

Because this is greater than 120°, it is acceptable.

For the larger sprocket,

$$\theta_1 = 180° + 2\sin^{-1}[(PD_2 - PD_1)/2C]$$

$$\theta_2 = 180° + 2\sin^{-1}[(10.349 - 2.721)/(2(15.272))] = 208.9°$$

Comments: **Summary**

Figure 7–38(b) shows the new design to the same scale as the first design. The space reduction is significant.

Chain: No. 40, 1/2-in pitch, four-strand, 104 pitches, 52.0 in length

Sprockets: No. 40–4 (four strands), 1/2-in pitch

Small: 17 teeth, $PD_1 = 2.721$ in

Large: 65 teeth, $PD_2 = 10.349$ in

Maximum center distance: 15.272 in

Type B lubrication (oil bath)

Spreadsheet for Chain Design

Figure 7–39 shows a spreadsheet that assists in the design of chain drives using the procedure developed in this section. The user enters data shown in italics in the gray-shaded cells. Refer to Tables 7–11 to 7–17 for required data. Results for Example Problem 7–6 are shown in the figure.

CHAIN DRIVE DESIGN					
Initial Input Data:	Example Problem 7–6—Multiple strands				
Application:	*Coal conveyor*				
Drive/type:	*Engine-mechanical drive*				
Driven machine:	*Heavily loaded conveyor*				
Power input:	*15* hp				
Service factor:	*1.4*	Table 7–17			
Input speed:	*900* rpm				
Desired output speed:	*235* rpm				
Computed Data:					
Design power:	21 hp				
Speed ratio:	3.83				
Design Decisions—Chain Type and Teeth Numbers:					
Number of strands:	*4*	1	2	3	4
Strand factor:	*3.3*	1.0	1.7	2.5	3.3
Required power per strand:	6.36 hp				
Chain number:	*40*	Table 7–14, 7–15, or 7–16			
Chain pitch:	*0.5* in				
Number of teeth—driver sprocket:	*17*				
Computed no. of teeth—driven sprocket:	65.11				
Enter: Chosen number of teeth:	65				
Computed Data:					
Actual output speed:	235.4 rpm				
Pitch diameter—driver sprocket:	2.721 in				
Pitch diameter—driven sprocket:	10.349 in				
Center Distance, Chain Length, and Angle of Wrap:					
Enter: Nominal center distance:	*30* pitches	30 to 50 pitches recommended			
Computed nominal chain length:	102.9 pitches				
Enter: Specified no. of pitches:	*104* pitches	Even number recommended			
Actual chain length:	52.00 in				
Computed actual center distance:	30.545 pitches				
Actual center distance:	15.272 in				
Angle of wrap—driver sprocket:	151.1°	Should be greater than 120°			
Angle of wrap—driven sprocket:	208.9°				

FIGURE 7–39 Spreadsheet for chain design

7–7 WIRE ROPE

Application of Wire Rope

Wire rope is used in many industries such as oil and gas, cranes and lifting, construction, mining, military, and shipping. Many types of machines and structures require wire rope for lifting, moving, and stabilizing bodies and it is used in both dynamic and static systems. Examples of dynamic systems entailing wire ropes are material handling, elevators, shovels, and heavy lifting equipment for mechanical power transmission shown in Figure 7–40. Static wire ropes are used to support towers and create suspension bridges.

Due to wire ropes having a variety of functions, there is a complete range of wire rope sizes, steel grades, finishes, and construction types available. A balance of wire rope properties such as strength, abrasion resistance, crush resistance, bending fatigue resistance, and corrosion resistance are used to optimize the selection of wire rope. Understanding the requirements of these applications and wire rope design will optimize wire rope selection leading to optimal performance. Wire ropes must be properly selected for each application while being periodically inspected and maintained during the life of its operation. This will ensure that the wire rope's service life is longer and safer.

Wire Rope Construction

The design of a typical wire rope is shown in Figure 7–41. It is made from three components: wires, strands, and the core. The various combinations of wire arrangement and size form a strand. These strands are helically laid over the center or core to form the wire rope and most wire ropes are constructed with 6, 7, or 8 strands. The center of the wire rope is called a core and is made from steel or fiber and its function is to provide support for the strands and keep the strands properly positioned as the rope flexes and bends under loaded conditions. Two commonly used core designations are a *fiber core* (FC) and an *independent wire rope core* (IWRC). Fiber cores are manufactured from natural or synthetic fibers and the independent wire rope core is made from steel.

FIGURE 7–40 Overhead crane

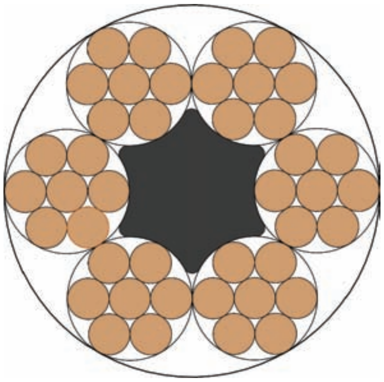


FIGURE 7–41 Wire rope construction

A fiber core provides excellent flexibility while the independent wire rope core provides crush resistance and increased strength. The wire, strands, and core components of the wire rope continuously interact with each other during operation.

The nominal wire rope diameters available are shown in Table 7–19. The wire rope diameter, *d*, is measured at the extreme outer limits of the strand as shown in Figure 7–42. See Internet sites 17-19 for wire rope producers.

Wire Rope Classification

Wire ropes are grouped into standard classifications based on the number of strands and the number of wires per strand, as shown in Table 7–20. A classification

TABLE 7–19 Nominal Wire Rope Diameter

inches	mm	inches	mm
1/4	6.5	2 1/8	54
5/16	8	2 1/4	58
3/8	9.5	2 3/8	60
7/16	11.5	2 1/2	64
1/2	13	2 5/8	67
9/16	14.5	2 3/4	71
5/8	16	2 7/8	74
3/4	19	3	77
7/8	22	3 1/8	80
1	26	3 1/4	83
1 1/8	29	3 3/8	87
1 1/4	32	3 1/2	90
1 3/8	35	3 3/4	96
1 1/2	38	4	103
1 5/8	42	4 1/4	109
1 3/4	45	4 1/2	115
1 7/8	48	4 3/4	122
2	52	5	128

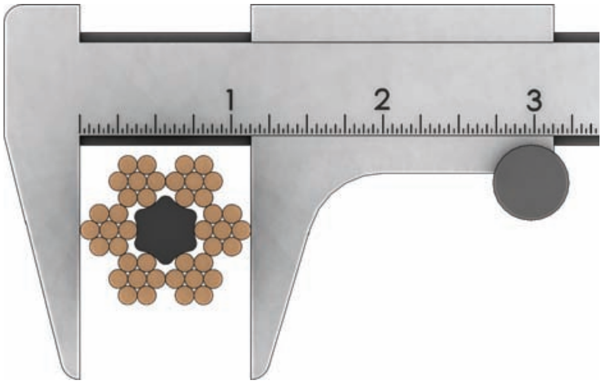


FIGURE 7–42 Correct method of measurement of a wire rope

TABLE 7–20 Wire Rope Classification

Classification	Wires per strand	Maximum number of outer wires in strand
6×7	7 through 15	9
6×19	16 through 26	12
6×36	27 through 49	18
6×61	50 through 74	24

*Classifications are the same in 7 and 8 strand wire ropes

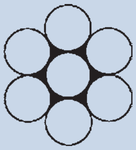
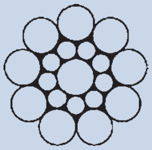
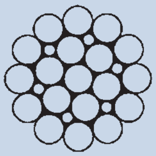
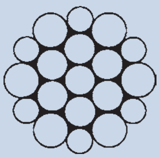
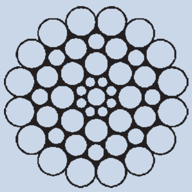
of 6×19 has a wire count of 16 through 26 wires per strand. Wire ropes within a classification have the same strength, weight per foot, and cost.

Basic Strand Construction

Table 7–21 shows the various strands that are constructed from different combinations of wire arrangements and diameters. *Warrington, Seale,* and *Filler Wire* are basic strand constructions discussed in this table. The strand design affects the physical characteristics of the wire rope's resistance to abrasion and fatigue. A strand made up of a large number of smaller wires will be less abrasion resistant and more fatigue resistant compared to a strand made up of small number of large wires which will be more abrasion resistant and less fatigue resistant.

The direction of lay of individual wires within the strand, and the direction of lay of the strand in the wire rope are described next. Figure 7–43 shows the different combinations of how the wires and strands are wound within a wire rope. The terms *Right Lay* and *Left Lay* describe the direction the strands are wound in the wire rope. Figures 7–43(a) and (c) show the strands wound with Right Lay. When looking at the wire rope, the strands are angled to the right as shown. Figures 7–43(b) and (d) show the strands wound with Left Lay. This

TABLE 7–21 Strand Construction

Single layer		Single wire in the center with six wires of the same diameter
Seale		Equal number of wires in each layer All wires in layer are the same diameter Large outer wires rest in the valley between the small inner wires
Filler wire		Inner layer having half the number of wires as outer layer Smaller filler wires equal in number to the inner layer are laid in the valleys of the inner layer
Warrington		One diameter of wire in the inner layer Two diameters of wire alternating large and small in the outer layer The large outer layer wires rest in the valleys The smaller wires rest on the crowns of the inner layer
Combined pattern		Strand is formed using two or more of the above constructions

(*a*) (*b*) (*c*) (*d*)

FIGURE 7–43 Types of lay of wire rope

(a)Right Lay – Regular Lay, (b) Left Lay – Regular Lay, (c) Right Lay – Lang Lay, (d) Left Lay – Lang Lay

shows the strands angled to the left as they are wound around the center core of the wire rope.

The terms *Regular Lay* and *Lang Lay* describe the direction the individual wires are wound in the strand.

- In Regular Lay, the individual wires are wound (lay) in the opposite direction of the strand lay.
- In Lang Lay wire rope, the individual wires have the same lay direction as the strands.

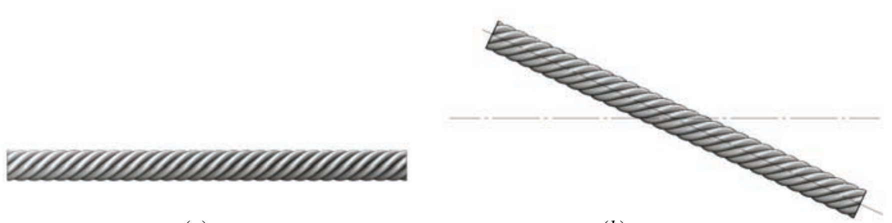
Let's look at the wire rope in Figure 7–43(a) which has *Right Lay – Regular Lay*. The term Right Lay tells us the strand lay is angled to the right in the wire rope. The term Regular Lay tells us the individual wires lay in the opposite direction of the strands, so if the strand is Right Lay the individual wires will lay to the left as shown in Figure 7–44(a). As these strands are wound with Right Lay to form the wire rope, the individual wires in the strands appear to run parallel with the rope length. Figure 7–44(b) shows one strand with Right Lay – Regular Lay and Figure 7–43(a) shows the complete wire rope.

Next, let's look at a wire rope that has *Right Lay – Lang Lay* shown in Figure 7–43(c). The strands in this wire rope also have Right Lay, so the strands will be angled to the right as the previous wire rope. Since this wire rope is Lang Lay, the individual wires are wound in the same direction as the strand. Therefore the individual wires will lay right since the strand is Right Lay as shown in Figure 7–45(a). The individual wires will appear to run perpendicular to the direction of the wire rope as the strands are wound with Right Lay. Figure 7–45(b) shows one strand wound with Right Lay and the complete wire rope is shown in Figure 7–43(c).

Lang Lay wire ropes are more flexible than Regular Lay wire ropes. As the wire rope bends over a sheave, the individual wires in Lang Lay are not bent as sharply as Regular Lay. The individual wires in Regular Lay lie in parallel with the wire rope axis, and the radius of curvature is equal to the radius of the sheave plus the diameter of the wire rope. The individual wires in a Lang Lay wire rope lie at an angle to the wire rope axis and have a larger radius of curvature. This reduces the stresses in the Lang Lay wire rope compared to the Regular Lay wire rope. Lang Lay will have a higher fatigue resistance due to the lower bending stress. Lang Lay also has a higher abrasion resistance than Regular Lay wire rope, since the wear is distributed differently in each of these types of lays. In Lang Lay, the wear is distributed over a longer distance of the wire, whereas in Regular Lay the wear is concentrated over a shorter distance in the wire. This gives Lang Lay a longer period before the individual wires will be worn and break due to bending stresses compared to Regular Lay. Regular Lay is more resistant to crushing which tends to distort the shape of the wire rope. Regular Lay also tends to be more stable. A stable wire rope is resistant to kinking, does not tangle when relaxed, and spools smoothly on and off the drum.

The distance a strand makes as it is wound one complete revolution around the core of the wire rope is called *Lay Length* and is shown in Figure 7–46.

The selection of the proper lay in the winding of wire rope on a smooth or grooved drum is illustrated in Figures 7–47(a) and (b). Right Lay wire rope is used when winding from the left to the right on a drum rotating clockwise (looking from the left end of the drum). Left Lay wire rope is used when winding from the right to the left on a drum rotating clockwise (looking from the left end of the drum). Another way to look at it is if the drum is left hand grooved, use a Right Lay wire rope. A drum that has a right hand groove will use Left Lay wire rope. The correct lay of rope will keep the winding tight if the load is released.



(*a*) (*b*)

FIGURE 7–44 *Right Lay – Regular Lay* wire rope

(a) One strand shown straight with Regular Lay—individual wires lay left, opposite of strand lay

(b) One strand shown with Right Lay—individual wires run parallel with length of wire rope

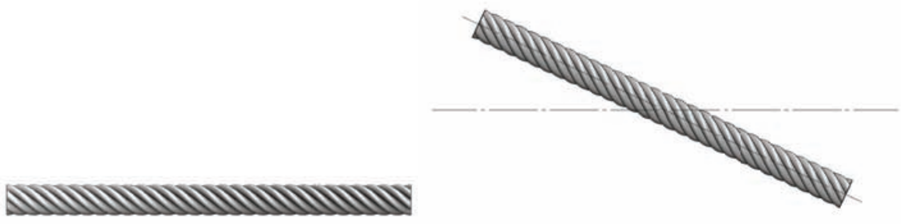


FIGURE 7–45 *Right Lay – Lang Lay* wire rope

(a) One strand shown straight with Lang lay—individual wires lay right, same direction as strand lay
(b) One strand shown with Right Lay—individual wires run perpendicular to the length of wire rope

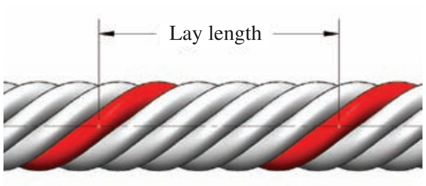


FIGURE 7–46 Lay length

A non-rotating wire rope is designed to be counterbalanced with little tendency to twist in either direction. This is accomplished by having one layer of strands wound with Right Lay and a second layer wound with Left Lay.

Sheave and Drum Design

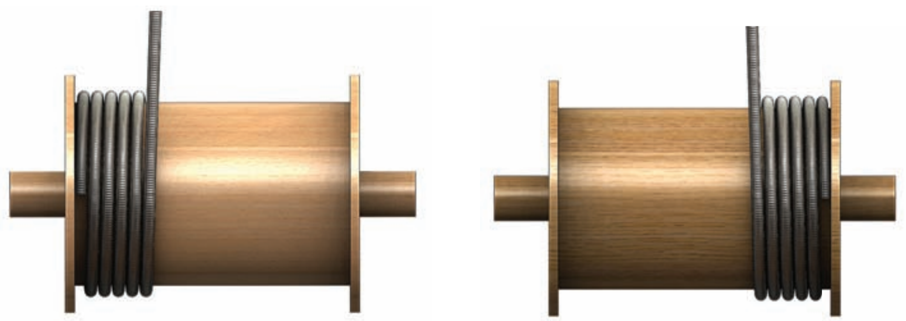
Sheaves, drums, and rollers must be properly designed to optimize the service life of the wire rope. The wire rope strands and wires move relative to one another, as the wire rope travels over a sheave or drum. As the wire rope bends over a sheave, the inside wires would travel a shorter distance than the outside wires.

This distance is compensated by the relative movement of the wires. The wire rope is subjected to bending stresses as it travels over a sheave or drum. This cyclic bending stress will cause fatigue, and eventually failure, of the wire rope. The magnitude of the stresses are related to the load, sheave diameter, and wire rope diameter.

A suggested and minimum diameter (D) of the sheave or drum is selected based on the wire rope diameter (d) and the construction of the wire rope. This diameter is called the *tread diameter* (D) and is where the wire rope rides on the sheave or grooved drum. The D/d ratio measures how tightly the wire rope bends over a sheave. The smaller the ratio, the tighter the bend in the wire rope. The suggested and minimum D/d ratios based on wire rope construction are shown in Table 7–22. Figure 7–48

TABLE 7–22 Sheave and Drum Diameter Factors

Construction	Suggested *D/d* ratio	Minimum *D/d* ratio
6×7	72	42
6×19 Seale	51	34
6×21 Filler Wire	45	30
6×25 Filler Wire	39	26
6×31 Warrington Seale	39	26
6×36 Warrington Seale	35	23
6×41 Seale Filler Wire	20	20
6×41 Warrington Seale	32	21
6×42 Tiller	21	14
8×19 Seale	41	27
8×25 Filler Wire	32	21



(*a*) Right Lay wire rope used on a smooth drum (*b*) Left Lay wire rope used on a smooth drum

FIGURE 7–47 Proper lay of wire rope on a smooth drum

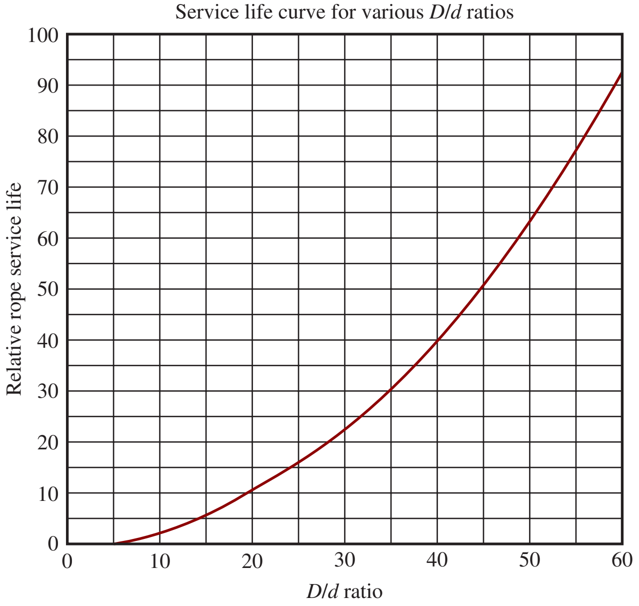


FIGURE 7–48 Service life curve

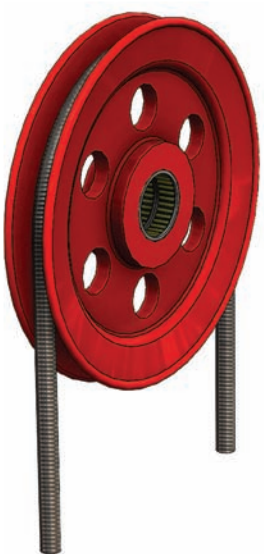
shows the effects of the *D/d* ratio on the service life of the wire rope. A larger *D/d* ratio will extend the life of the wire rope and the sheave and should be based on the practical limit of the machine application, along with cost and weight limitations. Bending a wire rope over a sheave with a small diameter will result in excessive wear in the wire rope and sheave. This is caused by excessive bending and straightening of the wire rope as it enters and exits the sheave. Reverse bending of the wire rope will accelerate wire rope fatigue and should be avoided if possible.

Sheave and Drum Design and Dimensions

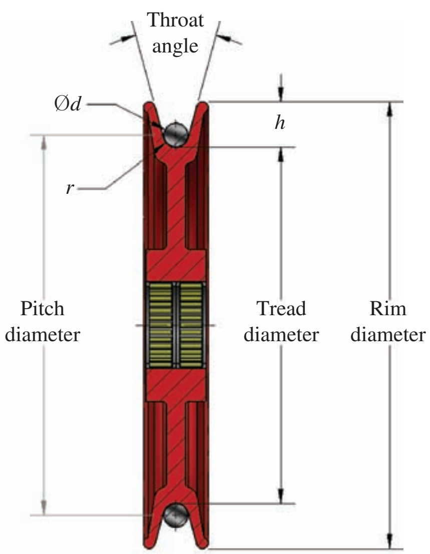
As shown in Figure 7–49(a) a typical sheave design will have a hub and bore that will be used to mount a bearing element. The bearing element can be a ball bearing, roller bearing, or a bronze bushing and is selected based on rotational speed, loading, and operating conditions of the sheave. The sheave has a groove designed in the rim that the wire rope will ride in. The web supports this rim and may have lightening holes in order to decrease the inertia of the sheave. Figure 7–49(b) shows the dimensional specifications and terminology of the sheave. The tread diameter (D) is based on the suggested minimum D/d ratio. The sheave throat angle should be between 35° and 45°. The groove depth of a sheave should be between

$$h_{min} = 1.50 \times d \text{ and } h_{max} = 1.75 \times d \qquad (7\text{–}23)$$

The throat angle, along with the groove depth (h), allows the wire rope to enter and exit the sheave with minimum wear on the wire rope and sheave. The proper groove radius (r) of a sheave will give the maximum support to the wire rope. If the groove radius is tight or small, it will pinch the wire rope and increase the groove pressure along this contact point. If the groove radius is oversized, the wire rope will not be properly supported and will tend to flatten out. This will cause the wire rope to be unbalanced and will lead to wire rope crushing and early fatigue. The correct groove radius and angle of contact will improve wire rope support and maximize the wire rope's lifespan. The groove radius for new sheaves and sheaves that are in operation are given in Table 7–23. The groove radius is shown for the different wire rope diameters. A groove gage is used to measure the size and contour of the groove for both new and used sheaves. The gage should make an angle of contact of 150° with the groove and verify the radius. Scheduled maintenance includes periodic inspections of the sheave groove radius. If the groove radius measures smaller than the value listed as "*Worn*" in Table 7–23, the sheave groove should be re-machined to the "*New*" radius dimension.



(*a*) Sheave with roller bearing



(*b*) Sheave dimensional specifications

FIGURE 7–49 Design of a roller bearing wire rope sheave

TABLE 7–23 Recommended Sheave and Drum Groove Radius

Nominal wire rope diameter		Groove radius			
		New		Worn	
inches	mm	inches	mm	inches	mm
1/4	6.5	0.137	3.48	0.129	3.28
3/8	9.5	0.201	5.11	0.190	4.83
1/2	13	0.271	6.88	0.256	6.50
5/8	16	0.334	8.48	0.320	8.13
3/4	19	0.401	10.19	0.380	9.65
7/8	22	0.468	11.89	0.440	11.18
1	26	0.543	13.79	0.513	13.03
1 1/4	32	0.669	16.99	0.639	16.23
1 1/2	38	0.803	20.40	0.759	19.28
1 3/4	45	0.939	23.85	0.897	22.78
2	52	1.070	27.18	1.019	25.88
2 1/2	64	1.338	33.99	1.279	32.49
3	77	1.607	40.82	1.538	39.07
3 1/2	90	1.869	47.47	1.794	45.57
4	103	2.139	54.33	2.050	52.07
4 1/2	115	2.396	60.86	2.298	58.37
5	128	2.663	67.64	2.557	64.95

Figure 7–50(a) shows the design of a helically grooved drum. The drum will have end flanges and journal ends that will be mounted in bearing units such as a pillow block to allow the drum to rotate. The drum will also require a means of securing the wire rope to the drum. This could be a clamping device that is bolted to the drum. Figure 7–50(b) shows the dimensional specifications of the grooved drum. The groove radius (r) must be proper for the wire rope diameter used. The pitch (p) of the helical groove is the distance from one groove to the next adjacent groove along the pitch diameter of the wire rope. The pitch distance should range between

$$\text{Pitch} = p = 2.065 \times r \text{ and } p = 2.18 \times r \quad (7\text{–}24)$$

The minimum groove depth (h) is the distance from the drum diameter to the tread diameter (D) on a helically grooved drum. This distance is given by

$$h = 0.374 \times d \quad (7\text{–}25)$$

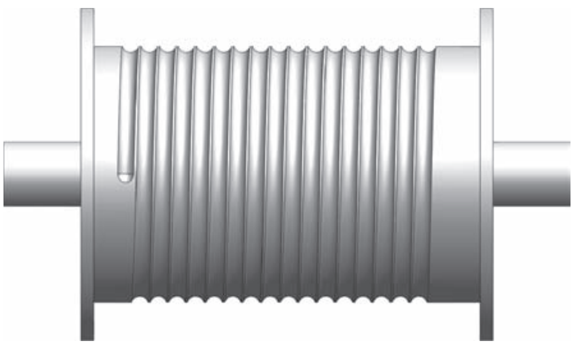
The *fleet angle* is the included angle between the wire rope which extends from the sheave to the drum and the perpendicular line to the axis of the drum. It is desirable to have a small fleet angle to reduce wear between the wire rope and flange of the sheave as the wire rope wraps the drum. The minimum fleet angle should be $\frac{1}{2}°$ to prevent the wire rope from piling up on the drum. The maximum fleet angle should be $1\frac{1}{2}°$ on a smooth drum and 2° on a grooved drum. Figure 7–51 shows the right and left fleet angles between a sheave and a smooth drum.

Wire Material and Grades

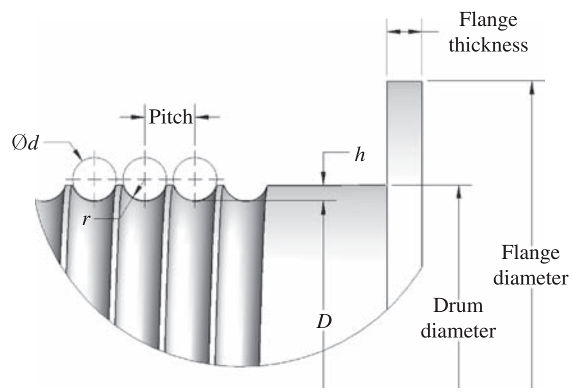
The most common wire material is an uncoated (bright) high-carbon steel. The properties of steel allow it to be drawn into wires with high strength, and good wear and fatigue resistance. Stainless steel or galvanized steel wire are used in applications with corrosive environments.

Plow steel is a term used for wire rope grades. Each different plow steel grade has improved tensile strength as shown in Table 7–24. *Improved plow steel* (IPS) is the most commonly used grade for wire rope, while *Extra improved plow steel* (XIP) is used for special applications.

The hardness of a wire rope can range from 42 to 50 Rc and will wear a sheave or drum if made out of a material that is too soft. A recommended sheave groove hardness should be 250–300 Brinell for a steel material. If a sheave is too soft, the wire rope will score and corrugate the groove of the sheave. A new wire rope will not track in a corrugated sheave and will cause excessive wear in the wire rope.



(*a*) Grooved drum with flanges and journal ends



(*b*) Grooved drum dimensional specifications

FIGURE 7–50 Design details for a grooved wire rope drum

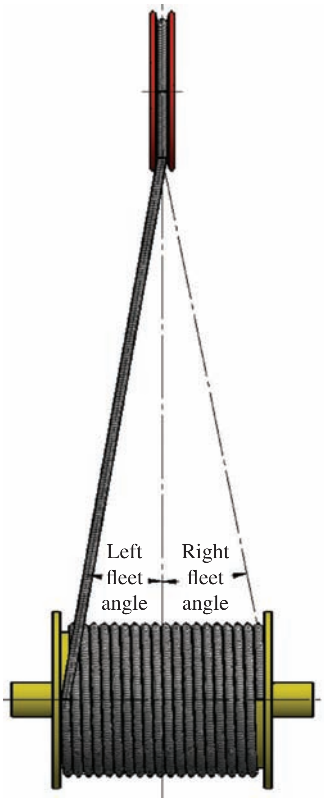


FIGURE 7–51 Fleet angle

TABLE 7–24 Grades of Wire Rope

Grade	Tensile strength
Plow Steel	1570 N/mm^2
Improved Plow Steel	1770 N/mm^2
Extra Improved Plow Steel	1960 N/mm^2

Wire Rope Selection

Wire rope construction refers to the arrangement of wires, strands, and the type of core used. The construction produces specific operating characteristics of the wire rope that will allow it to meet the performance requirements of its application. The attributes of a wire rope include strength, flexibility, resistance to bending fatigue, abrasion resistance, and resistance to crushing. A compromise between these attributes may have to be made in order to select the best wire rope for a specific application.

A wire rope's *strength* is shown as a minimum breaking force and is measured in tons (2000 lb /ton). Its minimum breaking force is determined by placing a new rope in a pull testing machine that applies a constantly increasing force until the wire rope breaks. The strength of the wire rope can be increased by increasing the diameter of the wire rope, using a better grade of steel, using a steel core construction, or increasing the cross-sectional steel content of the wire rope.

Flexibility relates to the wire rope's ability to bend around a sheave or drum. Some applications require the use of more sheaves where the wire rope will have to make a lot of bends or smaller-diameter sheaves where the wire rope will have to make a tighter bend. Flexibility can be increased by increasing the quantity of smaller-diameter wires in a strand. Some wire rope constructions are more flexible than others. The same diameter wire rope that has a strand with 37 smaller-diameter wires instead of 26 larger-diameter wires would improve the flexibility of the wire rope. Using a fiber core instead of a steel core will also improve the flexibility of the wire rope.

Fatigue resistance is required when wire ropes are subjected to repeated bending over sheaves and drums. This phenomenon involves metal fatigue of the individual wires that make up the wire rope. This is similar to taking a paper clip and bending it back and forth until it breaks. The same occurrence takes place in the individual wires as they are repeatedly bent around sheaves and drums. Sharper bends and reverse bending will accelerate the rate of fatigue of the wire rope. Using a larger number of smaller individual wires, while keeping the overall diameter of the wire rope the same, will increase the fatigue resistance of the wire rope. The smaller individual wires have a greater ability to bend than a larger wire. As discussed previously, the bending fatigue can also be reduced by having a large D/d ratio.

Abrasion resistance refers to the actual wearing of material away from the individual wires in a wire rope. The removal of material creates a destructive condition and weakens the wire rope which will lead to eventual failure. Abrasion can occur when the wire rope is dragged through a gritty material or across a stationary object. Abrasion also happens to individual wires within the wire rope as the wires rub against each other when the wire rope is bending and unbending or being loaded and unloaded. Abrasion will also manifest when the wire rope is being wound or unwound from a drum and the wire rope will have rope-to-rope contact. Proper sheave alignment, groove diameter, fleet angle, and drum winding will minimize abrasion of the wire rope. These should be checked during regular scheduled maintenance outages. Large-diameter wires will improve the wire rope's resistance to abrasion.

Crushing resistance is the ability of the wire rope to maintain its cross-sectional shape while under an external pressure. IWRC core, six strand, and regular lay wire rope construction will improve crush resistance. A crushed or distorted wire rope will not allow the individual wires to move relative to one another and operate properly. An IRWC core has a higher crush resistance compared to a fiber core.

As was discussed earlier, using a small number of larger-diameter wires will improve abrasion resistance, but a lower resistance to bending fatigue while using a large number of small-diameter wires will improve the resistance to bending fatigue and reduce the resistance to abrasion. A fiber core is more flexible than an IWRC core but has less crush resistance and strength. Lang lay has a higher abrasion resistance, fatigue resistance, and is more flexible than a regular lay wire rope. Regular lay wire rope is more resistant to crushing and is more stable than Lang lay. Based on the best suitable wire rope design for the application, a compromise between these characteristics of the wire ropes will need to be made.

Several characteristics of the design application should be considered:

- Speed of operation
- Acceleration and deceleration
- Length of rope
- Number, size, and location of sheaves and drums
- Conditions of the environment
- Human and property safety concerns

These design characteristics will be used to select of the best wire rope for the application.

Design Factors and Working Loads

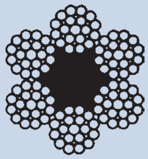
Industry standards and regulations require minimum design factors to be applied when using wire rope. The design factor is applied to the minimum breaking force of the wire rope. A minimum design factor (*SF*) of 5 is used in overhead cranes, gantry cranes, and overhead hoists. The maximum working load of a wire rope can then be determined in the following equation.

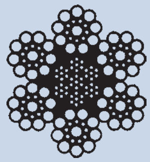
$$\text{Maximum working load} = \text{Minimum breaking force}/SF$$

This allowable working load is the load the wire rope is expected to carry not only due to static loading, but also loading resulting from accelerations and shock loading conditions on a system. Shock loads can be significantly higher than static loads. Shock loading can occur when there is a sudden change or jerking movement of the load and should be avoided.

The technical data are the same for both 6×19 and 6×36 classification of wire rope and are shown in Table 7–25. The minimum breaking force is given for

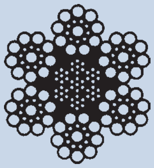
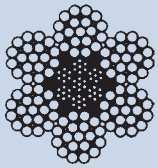
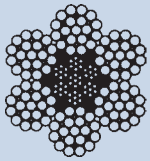
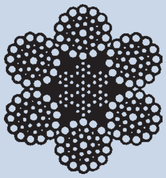
TABLE 7–25 6×19 and 6×36 Classes Technical Data

Fiber core 

IWRC 

Diameter	Fiber core: Weight per foot	Min breaking force IPS	Min breaking force XIP	IWRC: Weight per foot	Min breaking force IPS	Min breaking force XIP
in	lb/ft	tons	tons	lb/ft	tons	tons
1/4	0.105	2.74	3.02	0.116	2.94	3.4
5/16	0.164	4.26	4.69	0.18	4.58	5.27
3/8	0.236	6.1	6.72	0.26	6.56	7.55
7/16	0.32	8.27	9.1	0.35	8.89	10.2
1/2	0.42	10.7	11.8	0.46	11.5	13.3
9/16	0.53	13.5	14.9	0.59	14.5	16.8
5/8	0.66	16.7	18.3	0.72	17.9	20.6
3/4	0.95	23.8	26.2	1.04	25.6	29.4
7/8	1.29	32.2	35.4	1.42	34.6	39.8
1	1.68	41.8	46	1.85	44.9	51.7
1 1/8	2.13	52.6	57.8	2.34	56.5	65
1 1/4	2.63	64.6	71.1	2.89	69.4	79.9
1 3/8	3.18	77.7	85.5	3.5	83.5	96
1 1/2	3.78	92	101	4.16	98.9	114
1 5/8	4.44	107	118	4.88	115	132
1 3/4	5.15	124	137	5.67	133	153
1 7/8	5.91	141	156	6.5	152	174
2	6.72	160	176	7.39	172	198
2 1/8	7.59	179	197	8.35	192	221
2 1/4	8.51	200	220	9.36	215	247

TABLE 7–26 Properties of Standard 6×19 and 6×36 Wire Ropes

6×19S (Seale)		Good resistance to abrasion and crushing Fatigue resistance is less than 6×25
6×25F (Filler Wire)		Higher resistance to fatigue than 6×19 class Best combination of flexibility and wear resistance for 6×19 class Filler wire provides support and stability to the strand
6×26 (Warrington Seale)		High resistance to crushing Good wear resistance Good Flexibility
6×36 (Warrington Seale)		Provides good fatigue resistance without having wires that are too small Most flexible due to large number of small wires Susceptible to crushing, use IWRC to minimize

both improved plow steel and extra improved plow steel with a fiber core and an IWRC core. The weight per foot is shown for both fiber and IWRC cores.

The 6×19 steel core and fiber core classifications of wire rope are the most widely used. The combination of flexibility and wear resistance makes it suitable for diverse types of machinery and equipment. Compared to the 6×19 classification, the 6×36 classification of wire rope is more flexible but less abrasion resistant. Table 7–26 gives the operating properties of some 6×19 and 6×36 wire ropes.

The nomenclature of a wire rope defines the length, size (diameter), direction of lay, grade of rope, finish, construction, and type of core. An example of a complete wire rope designation is

500 ft × 5/8 in diameter 6×19 Filler Wire – Right Lay–Left Lang Improved Plow Steel IWRC

Example Problem 7–7 Recommend the diameter for a grooved drum on which a wire rope with a diameter of 16 mm is to be wound as part of a winch system. The construction of the rope is to be 6×36 Warrington Seale type.

Solution

Given 6×36 Warrington Seale type wire rope with a diameter of $d = 16$ mm

Results Table 7–22 lists the suggested *D/d* ratio for this size and type wire rope to be 35, where the value of D is the tread diameter of the drum. Then,

$$D = 35\,d = 35 \cdot 16 \text{ mm} = 560 \text{ mm}$$

The tread diameter, D, is the measurement between the lowest point of the groove on the top and bottom of the drum, as shown in Figure 7–50(b).

The minimum groove depth, h, can be calculated from Equation (7–20).

$$h = 0.374 \cdot d = 0.374 \cdot 16 \text{ mm} = 6 \text{ mm}$$

Therefore, the drum diameter is

$$\text{Drum diameter} = D + 2 \cdot h = 560 \text{ mm} + 2 \cdot 6 \text{ mm} = 572 \text{ mm}$$

Example Problem 7–8 Recommend the allowable working load for a wire rope having 6×19 construction, a diameter of 1.25 in, and made from XIP steel to be used in a crane application.

Solution Given Wire rope, 6×19 construction, $D = 1.25$ in

Results Table 7–25 lists the minimum breaking load for 1.25-in diameter wire rope made from XIP (Extra improved plow steel) to be 71.1 tons. A minimum service factor (*SF*) of 5 is recommended. Then,

Maximum working load = Breaking force/*SF* = 71.1 tons/5 = 14.22 tons

Converting to pounds force gives,

$$14.22 \text{ tons} \times 2000 \text{ lb/ton} = 28\,440 \text{ lb}$$

REFERENCES

1. American Chain Association. *Chains for Power Transmission and Material Handling*. 2nd ed., Boca Raton, FL: CRC Press Taylor & Francis, 2006.
2. American Society of Mechanical Engineers. *ASME Standard B29.1:2011, Precision Power Transmission Roller Chains, Attachments, and Sprockets*. New York: American Society of Mechanical Engineers, 2011.
3. International Organization for Standardization (ISO). (*Various standards listed below*). Geneva, Switzerland: ISO.
   (a) ISO 2790:2004. *Belt Drives—V-Belts for the Automotive Industry and Corresponding Pulleys—Dimensions*, 2004.
   (b) ISO 5290:2001. *Belt Drives—Grooved Pulleys for Joined Narrow V-Belts—Groove Sections 9N/J, 15N/J, and 25N/J*, 2001.
   (c) ISO 5292:1995. *Belt Drives—V-Belts and V-Ribbed Belts—Calculation of Power Ratings*, 1995.
   (d) ISO 5296:2012. *Synchronous Belt Drives—Belts: Pitch Codes MXL, XL, L, H, XH, and XXH—Metric and Inch Dimensions*, 2012.
   (e) ISO 8419: 2003. *Belt Drives—Narrow V-Belts—Sections 9N/J, 15N/J, and 25N/J (Lengths in the Effective System)*, 2003.
   (f) ISO 9981:1998. *Belt Drives—Pulleys and V-Ribbed Belts for the Automotive Industry—PK Profile: Dimensions*, 1998.
   (g) ISO 9982:1998. *Belt Drives—Pulleys and V-Ribbed Belts for the Industrial Applications—PH, PJ, PK, PL and PM Profiles: Dimensions*, 1998.
   (h) ISO 487:1998. *Steel Roller Chains, Type S and C*, 1998.
   (i) ISO 10823:2004. *Guidelines for the Selection of Roller Chain Drives*, 2004.
   (j) ISO 606:2015. *Short-Pitch Transmission Precision Roller and Bush Chains, Attachments and Associated Chain Sprockets*, 2015.
   (k) ISO 3512:1992. *Heavy-Duty Cranked-Link Transmission Chains*, 1992.
4. Society of Automotive Engineers. *SAE Standard J636—V-Belts and Pulleys*. Warrendale, PA: Society of Automotive Engineers, 2012.
5. Society of Automotive Engineers. *SAE Standard J637—Automotive V-Belt Drives*. Warrendale, PA: Society of Automotive Engineers, 2012.
6. Society of Automotive Engineers. *SAE Standard J1278—SI (Metric) Synchronous Belts and Pulleys*. Warrendale, PA: Society of Automotive Engineers, 2012.
7. Society of Automotive Engineers. *SAE Standard J1313—Automotive Synchronous Belt Drives*. Warrendale, PA: Society of Automotive Engineers, 2012.
8. Society of Automotive Engineers. *SAE Standard J1459—V-Ribbed Belts and Pulleys*. Warrendale, PA: Society of Automotive Engineers, 2009.

INTERNET SITES RELATED TO BELT DRIVES AND CHAIN DRIVES

1. **American Chain Association.** A national trade organization for companies providing products for the chain drive industry. Publishes standards and design aids for designing, applying, and maintaining chain drives and engineering chain conveyor systems.
2. **Dayco Products LLC.** Manufacturer of Dayco industrial and automotive belt drive systems of the poly-v and timing belt designs
3. **Baldor-Dodge.** Manufacturer of numerous power transmission components, including V-belt and synchronous belt drive systems.
4. **Regal Beloit Americas, Inc.** Manufacturer of a variety of power transmission products under several brand names, including Browning V-belt drives and Morse roller-chain drives.
5. **Gates Rubber Company.** Rubber products for the automotive and industrial markets, including V-belt drives and synchronous belt drives.
6. **Grainger Industrial Supply.** A distributor of a wide array of industrial products, including belts, sheaves, chain, and sprockets for power transmission.
7. **International Organization for Standardization.** The premier organization for establishing and promulgating technical standards for worldwide implementation. Search on *Standards* and the subject for which you are seeking ISO standards.
8. **Martin Sprocket and Gear Company.** Manufacturer of a wide range of mechanical power transmission products, including chain sprockets, V-belt sheaves, synchronous belt sprockets, and bushings.

9. **Maryland Metrics.** U.S. based company that supplies metric power transmission products rated by the ISO, BS, and DIN standards.
10. **Power Transmission.** A comprehensive website for companies providing products for the power transmission industry, many of which supply belt and chain drive systems.
11. **Ensinger Precision Components.** Producer of plastic injection–molded mechanical drive components, including plastic chain, sprockets, and synchronous belt pulleys.
12. **Rexnord Corporation.** Manufacturer of power transmission and conveying components, including roller chain drives and engineered chain drive systems.
13. **SAE International.** The Society of Automotive Engineers, the engineering society for advancing mobility on land or sea, in air or space. Offers standards on V-belts, synchronous belts, pulleys, and drives for automotive applications.
14. **SDP/SI.** The Stock Drive Products/Sterling Instruments Company distributes a wide array of mechanical drive components with a heavy emphasis on small, precision mechanical drive components, including synchronous belts, pulleys, chain, and sprockets. Both U.S. Customary and Metric styles of components are included.
15. **Wippermann Company.** Manufacturer of a wide range of chain drive products based in Germany; wide selection of German DIN standard chains and sprockets.
16. **T. B. Wood's Sons Company.** Manufacturer of many mechanical drives products, including V-belt drives, synchronous belt drives, and adjustable speed drives.
17. **Bridon.** Manufacturer of wire rope for the crane, oilfield, surface mining, underground mining, and related industries.
18. **Union - A WireCo(R) WorldGroup Brand.** Manufacturer of wire rope for crane, oil and gas, surface mining, logging, slings, and related products. The site includes a Technical Reference section providing guidance on installation, operation, maintenance, inspection and other topics.
19. **Wirerope Works, Inc.** Manufacturer of Bethlehem Wire Rope(R) for elevators, oilfield and marine applications, logging, mining, ski lifts, bridge suspension, and other applications.

PROBLEMS

V-Belt Drives

1. Specify the standard 3V belt length (from Table 7–2) that would be applied to two sheaves with pitch diameters of 5.25 in and 13.95 in with a center distance of no more than 24.0 in.
2. For the standard belt specified in Problem 1, compute the actual center distance that would result.
3. For the standard belt specified in Problem 1, compute the angle of wrap on both of the sheaves.
4. Specify the standard 5V belt length (from Table 7–2) that would be applied to two sheaves with pitch diameters of 8.4 in and 27.7 in with a center distance of no more than 60.0 in.
5. For the standard belt specified in Problem 4, compute the actual center distance that would result.
6. For the standard belt specified in Problem 4, compute the angle of wrap on both of the sheaves.
7. Specify the standard 8V belt length (from Table 7–2) that would be applied to two sheaves with pitch diameters of 13.8 in and 94.8 in with a center distance of no more than 144 in.
8. For the standard belt specified in Problem 7, compute the actual center distance that would result.
9. For the standard belt specified in Problem 7, compute the angle of wrap on both of the sheaves.
10. If the small sheave of Problem 1 is rotating at 1750 rpm, compute the linear speed of the belt.
11. If the small sheave of Problem 4 is rotating at 1160 rpm, compute the linear speed of the belt.
12. If the small sheave of Problem 7 is rotating at 870 rpm, compute the linear speed of the belt.
13. For the belt drive from Problems 1 and 10, compute the rated power, considering corrections for speed ratio, belt length, and angle of wrap.
14. For the belt drive from Problems 4 and 11, compute the rated power, considering corrections for speed ratio, belt length, and angle of wrap.
15. For the belt drive from Problems 7 and 12, compute the rated power, considering corrections for speed ratio, belt length, and angle of wrap.
16. Describe a standard 15N belt cross section. To what size belt (inches) would it be closest?
17. Describe a standard 17A belt cross section. To what size belt (inches) would it be closest?

For Problems 18–22 (Table 7–27), design a V-belt drive. Specify the belt size, the sheave sizes, the number of belts, the actual output speed, and the center distance.

Roller Chain

23. Describe a standard roller chain, no. 140.
24. Describe a standard roller chain, no. 60.
25. Specify a suitable standard chain to exert a static pulling force of 1250 lb.
26. Roller chain is used in a hydraulic forklift truck to elevate the forks. If two strands support the load equally, which size would you specify for a design load of 5000 lb?
27. List three typical failure modes of roller chain.
28. Determine the power rating of a no. 60 chain, single-strand, operating on a 20-tooth sprocket at 750 rpm. Describe the preferred method of lubrication. The chain connects a hydraulic drive with a meat grinder.
29. For the data of Problem 28, what would be the rating for three strands?
30. Determine the power rating of a no. 40 chain, single-strand, operating on a 12-tooth sprocket at 860 rpm. Describe the preferred method of lubrication. The small

TABLE 7–27 V-Belt Drive Design Problems

Problem number	Driver type	Driven machine	Service (h/day)	Input speed (rpm)	Input power (hp)	Input power (kW)	Nominal output speed (rpm)
18.	AC motor (HT)	Hammer mill	8	870	25	18.6	310
19.	AC motor (NT)	Fan	22	1750	5	3.73	725
20.	6-cylinder engine	Heavy conveyor	16	1500	40	29.8	550
21.	DC motor (compound)	Milling machine	16	1250	20	14.9	695
22.	AC motor (HT)	Rock crusher	8	870	100	74.6	625

Note: NT indicates a normal-torque electric motor. *HT* indicates a high-torque electric motor.

TABLE 7–28 Chain Drive Design Problems

Problem number	Driver type	Driven machine	Input speed (rpm)	Input power (hp)	Input power (kW)	Nominal output speed (rpm)
38.	AC motor	Hammer mill	310	25	18.6	160
39.	AC motor	Agitator	750	5	3.73	325
40.	6-cylinder engine	Heavy conveyor	500	40	29.8	250
41.	Steam turbine	Centrifugal pump	2200	20	14.9	775
42.	Hydraulic drive	Rock crusher	625	100	74.6	225

sprocket is applied to the shaft of an electric motor. The output is to a coal conveyor.

31. For the data of Problem 30, what would be the rating for four strands?

32. Determine the power rating of a no. 80 chain, single-strand, operating on a 32-tooth sprocket at 1160 rpm. Describe the preferred method of lubrication. The input is an internal combustion engine, and the output is to a fluid agitator.

33. For the data of Problem 32, what would be the rating for two strands?

34. Specify the required length of no. 60 chain to mount on sprockets having 15 and 50 teeth with a center distance of no more than 36 in.

35. For the chain specified in Problem 34, compute the actual center distance.

36. Specify the required length of no. 40 chain to mount on sprockets having 11 and 45 teeth with a center distance of no more than 24 in.

37. For the chain specified in Problem 36, compute the actual center distance.

For Problems 38–42 (Table 7–28), design a roller chain drive. Specify the chain size, the sizes and number of teeth in the sprockets, the number of chain pitches, and the center distance.

Synchronous Belts

For any of the sets of problem data in Tables 7–27 and 7–28, use manufacturers' catalogs to specify the belt size and sprocket sizes for synchronous belts. See Internet sites 3–6, 8, 9, 11, 14, and 16 for manufacturers of synchronous belts and sprockets.

43. A synchronous belt drive is to have an input sprocket rotational speed of 800 rpm and an output sprocket speed of 600 rpm.
   (a) List the sprocket combinations that could be used and list the pitch diameters for each sprocket.
   (b) For each sprocket combination from (a) and using a 1440-8MGT belt, compute the center distance and the belt linear velocity.

44. A synchronous belt drive is to have an input sprocket rotational speed of 1200 rpm and an output sprocket speed of 600 rpm.
   (a) List the sprocket combinations with an 8-mm pitch that could be used and list the pitch diameters for each sprocket.
   (b) For each sprocket combination from (a) and using a 1800-8MGT-30 belt, compute the center distance and the belt linear velocity.
   (c) For each sprocket, list the taper-lock bushing required and the minimum and maximum bores.

CHAPTER EIGHT

KINEMATICS OF GEARS

THE BIG PICTURE — Kinematics of Gears

Discussion Map

- Gears are toothed, cylindrical wheels used for transmitting motion and power from one rotating shaft to another.
- Most gear drives cause a change in the speed of the output gear relative to the input gear.
- Some of the most common types of gears are *spur gears*, *helical gears, bevel gears, and worm/wormgear sets.*

Discover

Identify at least two machines or devices that employ gears. Describe the operation of the machines or devices and the appearance of the gears.

This chapter will help you learn about the features of different kinds of gears, the kinematics of a pair of gears operating together, and the operation of gear trains having more than two gears.

Gears are toothed, cylindrical wheels used for transmitting motion and power from one rotating shaft to another. The teeth of a driving gear mesh accurately in the spaces between teeth on the driven gear as shown in Figure 8–1. The driving teeth push on the driven teeth, exerting a force perpendicular to the radius of the gear. Thus, a torque is transmitted, and because the gear is rotating, power is also transmitted.

Speed Reduction Ratio. Often gears are employed to produce a change in the speed of rotation of the driven gear relative to the driving gear. In Figure 8–1, if the smaller top gear, called a *pinion*, is driving the larger lower gear, simply called the *gear*, the larger gear will rotate more slowly. The amount of speed reduction is dependent on the ratio of the number of teeth in the pinion to the number of teeth in the gear according to this relationship:

$$n_P/n_G = N_G/N_P \qquad (8\text{–}1)$$

The basis for this equation will be shown later in this chapter. But to show an example of its application

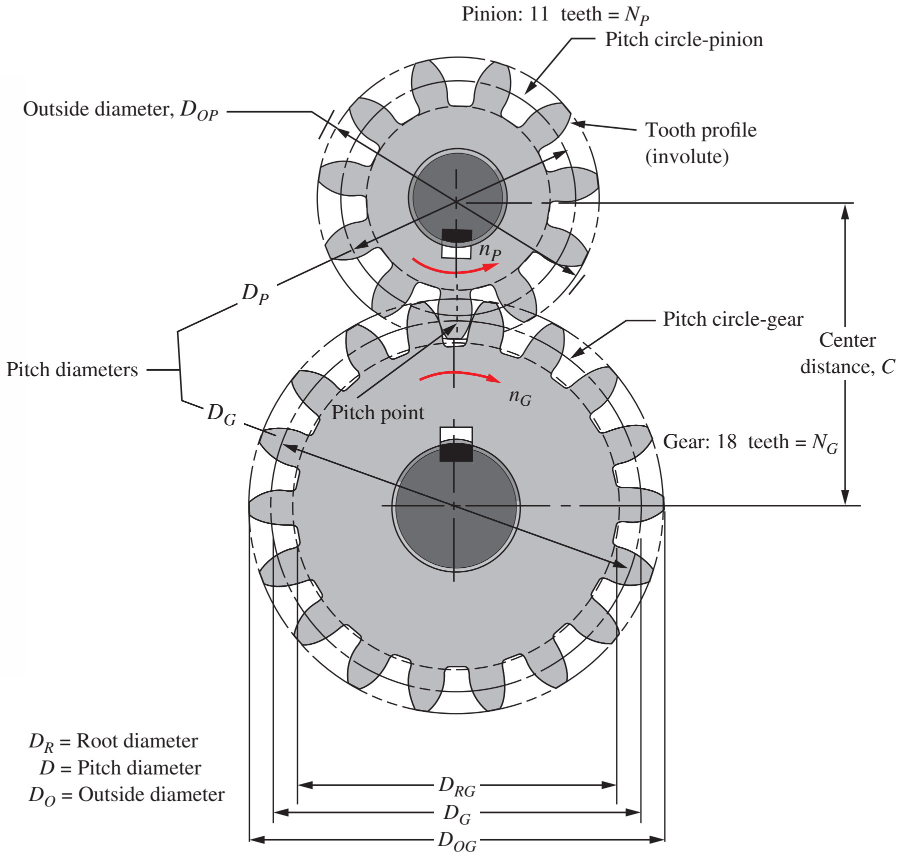


FIGURE 8–1 Pair of spur gears. The pinion drives the gear.

here, consider that the pinion in Figure 8–1 is rotating at 1800 rpm. You can count the number of teeth in the pinion to be 11 and the number of teeth in the gear to be 18. Then we can compute the rotational speed of the gear by solving Equation (8–1) for n_G:

$$n_G = n_P(N_P/N_G) = (1800\text{ rpm})(11/18) = 1100\text{ rpm}$$

When there is a reduction in the speed of rotation of the gear, there is a simultaneous proportional *increase* in the torque transmitted to the shaft carrying the gear. More will be said about this later, also.

Kinds of Gears. Several kinds of gears having different tooth geometries are in common use. To acquaint you with the general appearance of some, their basic descriptions are given here. Later we will describe their geometry more completely.

Figure 8–2 shows a photograph of many kinds of gears. Labels indicate the major types of gears that are discussed in this chapter: *spur gears, helical gears, bevel gears,* and *worm/wormgear sets*. Obviously, the shafts that would carry the gears are not included in this photograph. See References 4, 7, 10, 11–13, and 19 and Internet sites 1, 4, and 5 for more information on gearing.

Spur gears have teeth that are straight and arranged parallel to the axis of the shaft that carries the gear. The curved shape of the faces of the spur gear teeth have a special geometry called an *involute curve,* described later in this chapter. This shape makes it possible for two gears to operate together with smooth, positive transmission of power. Figure 8–1 also shows the side view of spur gear teeth, and the involute curve shape is evident there. The shafts carrying the gears are parallel.

The teeth of *helical gears* are arranged so that they lie at an angle with respect to the axis of the shaft. The angle, called the *helix angle*, can be virtually any angle. Typical helix angles range from approximately 10° to 30°, but angles up to 45° are practical. The helical teeth operate more smoothly than equivalent spur gear teeth, and stresses are lower. Therefore, a smaller helical gear can be designed for a given power-transmitting capacity as compared with spur gears. One disadvantage of helical gears is that an axial force, called a *thrust force,* is generated in addition to the driving force that acts tangent to the basic cylinder on which the teeth are arranged. The designer must consider the thrust force when selecting bearings that will hold the shaft during operation. Shafts carrying helical gears are typically arranged parallel to each other. However, a special design, called *crossed helical* gears, has 45° helix angles, and their shafts operate 90° to each other.

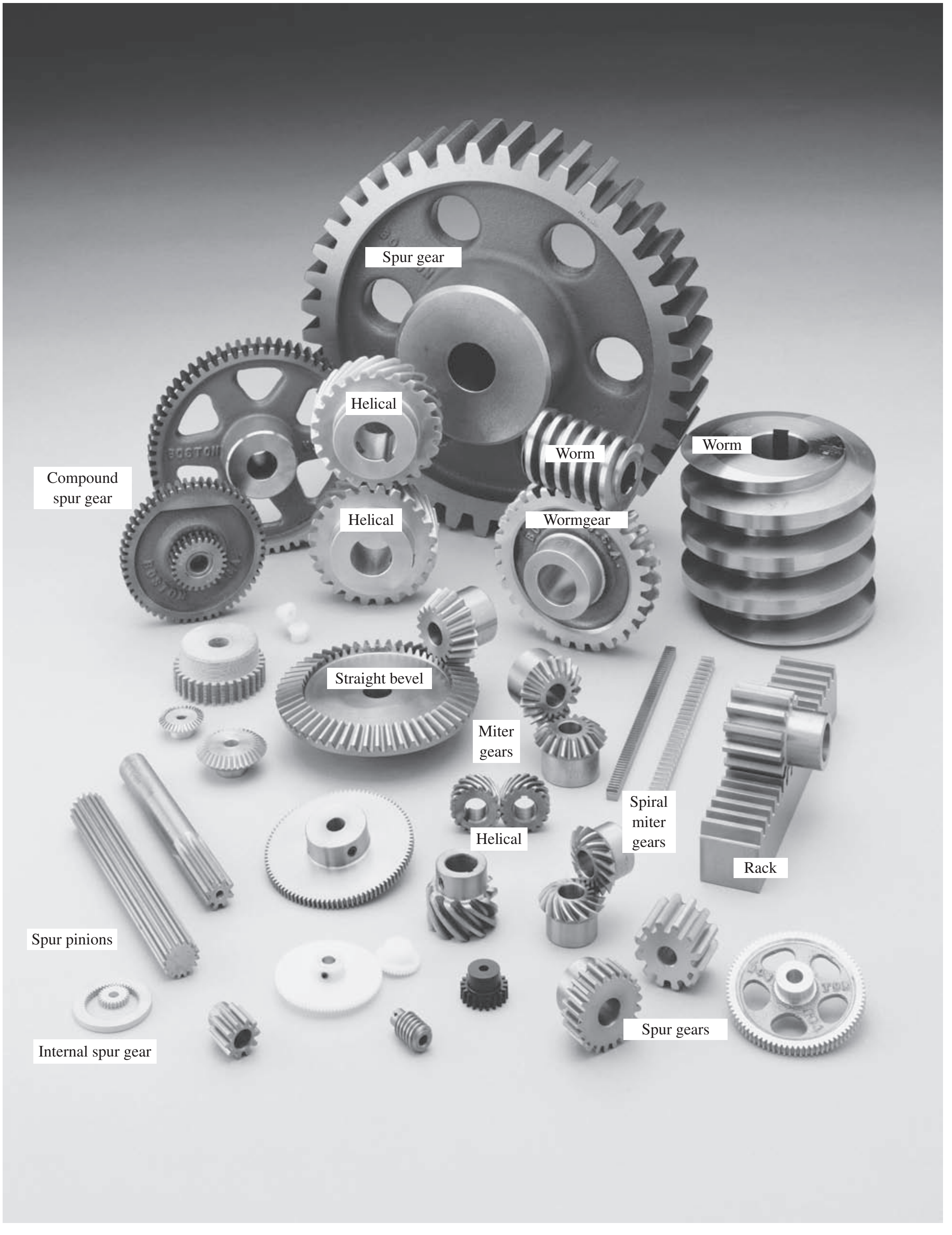


FIGURE 8–2 A variety of gear types
(Courtesy of Boston Gear, an Altra Industrial Motion Company)

Bevel gears have teeth that are arranged as elements on the surface of a cone. The teeth of straight bevel gears appear to be similar to spur gear teeth, but they are tapered, being wider at the outside and narrower at the top of the cone. Bevel gears typically operate on shafts that are 90° to each other. Indeed, this is often the reason for specifying bevel gears in a drive system. Specially designed bevel gears can operate on shafts that are at some angle other than 90°. When bevel gears are made with teeth that form a helix angle similar to that in helical gears, they are called *spiral bevel gears*. They operate more smoothly than straight bevel gears and can be made smaller for a given power transmission capacity. When both bevel gears in a pair have the same number of teeth, they are called *miter gears* and are used only to change the axes of the shafts to 90°. No speed change occurs.

Now look closely at Figure 8–3 that shows an example of a large, commercially available reducer with three stages that employs a combination of bevel, helical, and spur gears that were just described. See Internet site 6. Seeing them in one unit can help you appreciate the similarities and differences among them. Follow the flow of power through the reducer as outlined here:

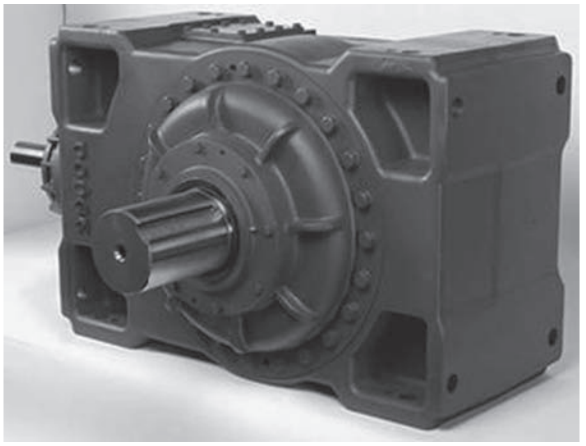
1. The input shaft at the left end carries the spiral bevel pinion for the right angle first stage of reduction.
2. The helical pinion behind the output gear of the bevel gear pair drives the large helical output gear of the second stage of reduction.
3. The output shaft from the helical gear pair carries the spur-type sun gear of a planetary gear train whose output shaft drives the final output shaft projecting from the front of the reducer.

A *rack* is a straight gear that moves linearly instead of rotating. When a circular gear is mated with a rack, as shown toward the right side of Figure 8–2, the combination is called *a rack and pinion drive*. You may have heard that term applied to the steering mechanism of a car or to a part of other machinery. See Section 8–3 for more discussion about a rack.

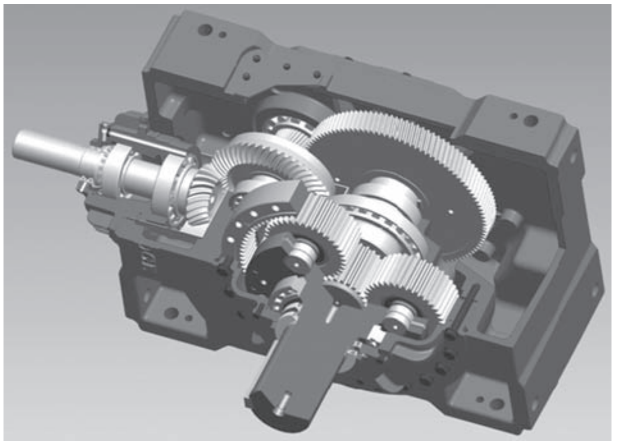
A *worm and its mating wormgear* operate on shafts that are at 90° to each other. They typically accomplish a rather large speed reduction ratio compared with other types of gears. The worm is the driver, and the wormgear is the driven gear. The teeth on the worm appear similar to screw threads, and, indeed, they are often called *threads* rather than *teeth*. The teeth of the wormgear can be straight like spur gear teeth, or they can be helical. Often the shape of the tip of the wormgear teeth is enlarged to partially wrap around the threads of the worm to improve the power transmission capacity of the set. One disadvantage of the worm/wormgear drive is that it has a somewhat lower mechanical efficiency than most other kinds of gears because there is extensive rubbing contact between the surfaces of the worm threads and the sides of the wormgear teeth.

Where Have You Observed Gears? Think of exam-ples where you have seen gears in actual equipment. Describe the operation of the equipment, particularly the power transmission system. Sometimes, of course, the gears and the shafts are enclosed in a housing, making it difficult for you to observe the actual gears. Perhaps you can find a manual for the equipment that shows the drive system. Or look elsewhere in this chapter and in Chapters 9 and 10 for some photographs of commercially available gear reducers. *(**Note:** If the equipment you are observing is operating, be very careful not to come in contact with any moving parts!)* Try to answer these questions:

- What was the source of the power? An electric motor, a gasoline engine, a steam turbine, a hydraulic motor? Or were the gears operated by hand?



(*a*) External view of reducer



(*b*) Cutaway view showing internal components

FIGURE 8–3 Triple reduction gear reducer employing bevel, helical, and spur gears
(Baldor/Dodge, Greenville, SC)

- How were the gears arranged together, and how were they attached to the driving source and the driven machine?
- Was there a speed change? Can you determine how much of a change?
- Were there more than two gears in the drive system?
- What types of gears were used? (You should refer to Figure 8–2.)
- What materials were the gears made from?
- How were the gears attached to the shafts that supported them?
- Were the shafts for mating gears aligned parallel to each other, or were they perpendicular to one another?
- How were the shafts themselves supported?
- Was the gear transmission system enclosed in a housing? If so, describe it.

This chapter will help you learn the basic geometries and kinematics of gears and pairs of gears operating together. You will also learn how to analyze gear trains having more than two gears so that you can describe the motion of each gear. Then you will learn how to devise a gear train to produce a given speed reduction ratio. In later chapters, you will learn how to analyze gears for their power transmission capacity and to design gear trains to transmit a given amount of power at a specified ratio of the speed of the input shaft to the speed of the output shaft.

YOU ARE THE DESIGNER

A gear-type speed reducer was described in Chapter 1, and a sketch of the layout of the gears within the reducer was shown in Figure 1–12. You are advised to review that discussion now because it will help you understand how the present chapter on *gear geometry* and *kinematics* fits into the design of the complete speed reducer.

Assume that you are responsible for the design of a speed reducer that will take the power from the shaft of an electric motor rotating at 1750 rpm and deliver it to a machine that is to operate at approximately 292 rpm. You have decided to use gears to transmit the power, and you are proposing a double-reduction speed reducer like the concept sketch shown in Figure 1–12. This chapter will give you the information you need to define the general nature of the gears, including their arrangement and their relative sizes.

The input shaft (shaft 1) is coupled to the motor shaft. The first gear of the gear train is mounted on this shaft and rotates at the same speed as the motor, 1750 rpm. Gear 1 drives the mating gear 2, which is larger, causing the speed of rotation of shaft 2 to be slower than that of shaft 1. But the speed is not yet down to 292 rpm as desired.

The next step is to mount a third gear (gear 3) on shaft 2 and mate it with gear 4 mounted on the output shaft, shaft 3. With proper sizing of all four gears, you should be able to produce an output speed equal or quite close to the desired speed. This process requires knowledge of the concept of *velocity ratio* and the techniques of designing gear trains as presented in this chapter.

But you will also need to specify the appearance of the gears and the geometry of the several features that make up each gear. Whereas the final specification also requires the information from following chapters, you will learn how to recognize common forms of gears and to compute the dimensions of key features. This will be important when completing the design for strength and wear resistance in later chapters.

Let's say that you have chosen to use spur gears in your design. What design decisions must you make to complete the specification of all four gears? The following list gives some of the important parameters for each gear:

- The number of teeth.
- The form of the teeth.
- The size of the teeth as indicated by the *pitch*.
- The width of the face of the teeth.
- The style and dimensions of the gear blank into which the gear teeth are to be machined.
- The design of the hub for the gear that facilitates its mounting to the shaft.
- The degree of precision of the gear teeth and the corresponding method of manufacture that can produce that precision.
- The means of attaching the gear to its shaft.
- The means of locating the gear axially on the shaft.

To make reliable decisions about these parameters, you must understand the special geometry of spur gears as presented first in this chapter. However, there are other forms of gears that you could choose. Later sections give the special geometry of helical gears, bevel gears, and worm/wormgear sets. The methods of analyzing the forces on these various kinds of gears are described in later chapters, including the stress analysis of the gear teeth and recommendations on material selection to ensure safe operation with long life. ■

8–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will be able to:

1. Recognize and describe the main features of *spur gears, helical gears, bevel gears,* and *worm/wormgear sets.*
2. Describe the important operating characteristics of these various types of gears with regard to the similarities and differences among them and their general advantages and disadvantages.
3. Describe the *involute-tooth form* and discuss its relationship to the *law of gearing.*
4. Describe the basic functions of the American Gear Manufacturers Association (AGMA) and identify pertinent standards developed and published by this organization.

5. Define *velocity ratio* as it pertains to two gears operating together.
6. Specify appropriate numbers of teeth for a mating pair of gears to produce a given velocity ratio.
7. Define *train value* as it pertains to the overall speed ratio between the input and output shafts of a gear-type speed reducer (or speed increaser) that uses more than two gears.

8–2 SPUR GEAR STYLES

Figure 8–4 shows several different styles of commercially available spur gears. When gears are large, the spoked design in Part (a) is often used to save weight. The gear teeth are machined into a relatively thin rim that is held by a set of spokes connecting to the hub. The bore of the hub is typically designed to be a close sliding fit with the shaft that carries the gear. A keyway is usually machined into the bore to allow a key to be inserted for positive transmission of torque. The first illustration does not include a keyway because this gear is sold as a stock item, and the ultimate user finishes the bore to match a given piece of equipment.

The solid hub design in Figure 8–4(b) is typical of smaller spur gears. Here the finished bore with a keyway is visible. The set screw over the keyway allows the locking of the key in place after assembly.

When spur gear teeth are machined into a straight, flat bar, the assembly is called a rack, as shown in Figure 8–4(c). The rack is essentially a spur gear with an infinite radius. In this form, the teeth become straight-sided, rather than the curved, involute form typical of smaller gears.

Gears with diameters between the small solid form [Part (b)] and the larger spoked form [Part (a)] are often produced with a thinned web as shown in Part (d), again to save weight.

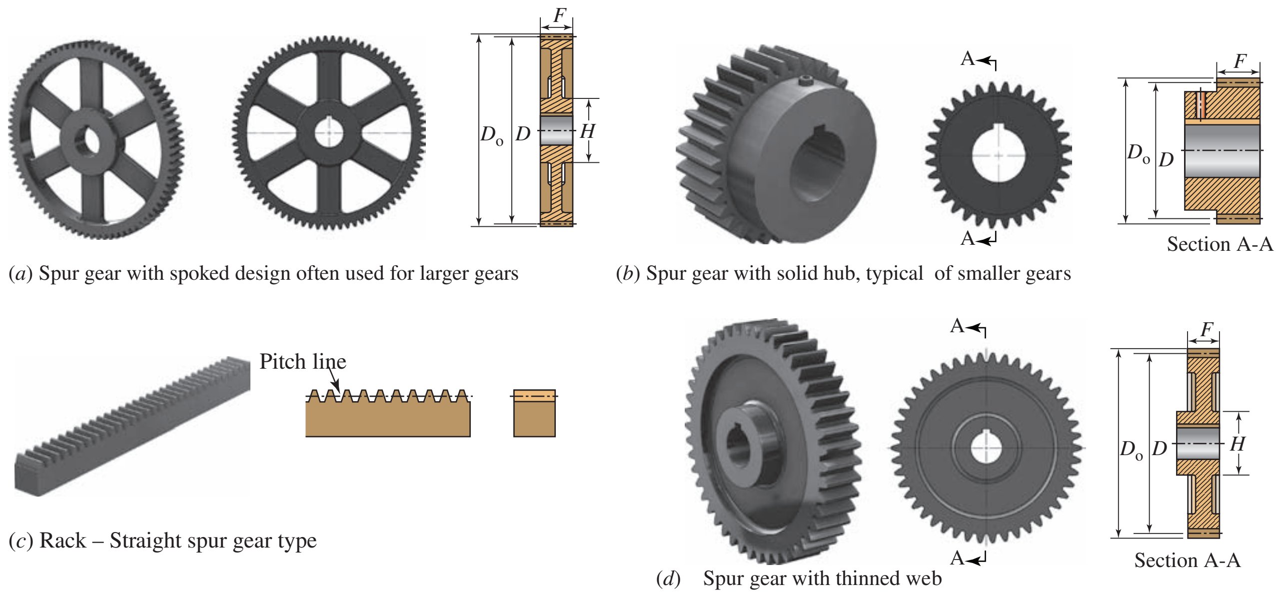
You as a designer may create special designs for gears that you implement into a mechanical device or system. One useful approach is to machine the gear teeth of small pinions directly into the surface of the shaft that carries the gear. This is very often done for the input shaft of gear reducers.

8–3 SPUR GEAR GEOMETRY-INVOLUTE-TOOTH FORM

The most widely used spur gear tooth form is the full-depth involute form. Its characteristic shape is shown in Figure 8–5. See References 10–15 and 18 for more on the kinematics of gearing.

The involute is one of a class of geometric curves called *conjugate curves*. When two such gear teeth are in mesh and rotating, there is a *constant angular velocity* ratio between them: From the moment of initial contact to the moment of disengagement, the speed of the driving gear is in a constant proportion to the speed of the driven gear. The resulting action of the two gears is very smooth. If this were not the case, there would be some speeding up and slowing down during the engagement, with the resulting accelerations causing vibration, noise, and dangerous torsional oscillations in the system.

You can easily visualize an involute curve by taking a cylinder and wrapping a string around its circumference. Tie a pencil to the end of the string. Then start with the pencil tight against the cylinder, and hold the string taut. Move the pencil away from the cylinder while keeping the string taut. The curve that you will draw is an involute. Figure 8–6 is a sketch of the process.



(*a*) Spur gear with spoked design often used for larger gears
(*b*) Spur gear with solid hub, typical of smaller gears
(*c*) Rack – Straight spur gear type
(*d*) Spur gear with thinned web

FIGURE 8–4 Examples of spur gears and a rack

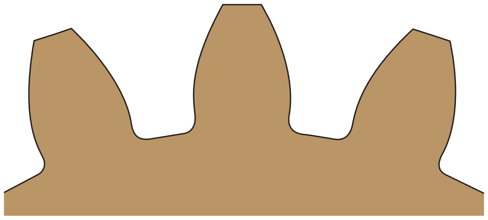


FIGURE 8–5 Involute-tooth form

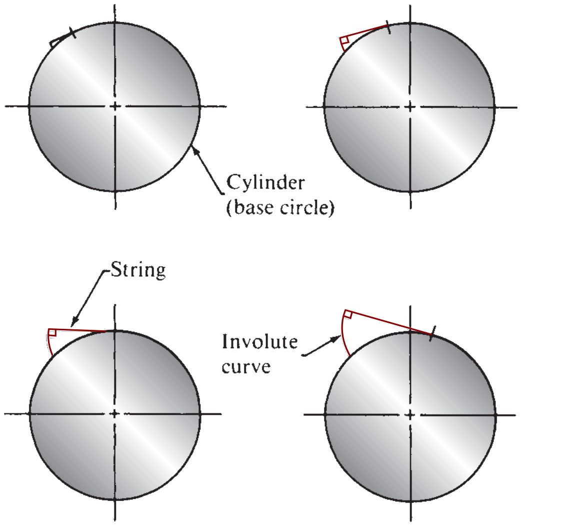


FIGURE 8–6 Graphical presentation of an involute curve

The circle, the end view of the cylinder, is called the *base circle*. Notice that at any position on the curve, the string represents a line tangent to the base circle and, at the same time, perpendicular to the involute. Drawing another base circle along the same centerline in such a position that the resulting involute is tangent to the first one, as shown in Figure 8–7, demonstrates that at the point of contact, the two lines tangent to the base circles are coincident and will stay in the same position as the base circles rotate. This is what happens when two gear teeth are in mesh.

It is a fundamental principle of *kinematics,* the study of motion, that if the line drawn perpendicular to the surfaces of two rotating bodies at their point of contact always crosses the centerline between the two bodies at the same place, the angular velocity ratio of the two bodies will be constant. This is a statement of the *law of gearing*. As demonstrated here, the gear teeth made in the involute-tooth form obey the law.

Of course, only the part of the gear tooth that actually comes into contact with the mating tooth needs to be in the involute form.

Figure 8–8 shows the gear tooth involute profile. The tooth profile is the curve forming the side of the tooth and is bounded by the major diameter and minor diameters of the gear. The active profile is the portion of the gear tooth involute curve that makes contact with the mating gear. The *start of the active profile* (SAP) corresponds to the lowest point on the active profile. The *true involute form* (TIF) diameter is the diameter above which the tooth profile is a true involute, but the mating gear teeth do not contact. The tip chamfer is a tooth modification used to eliminate the sharp edge of the gear tooth flank and the major diameter of the gear.

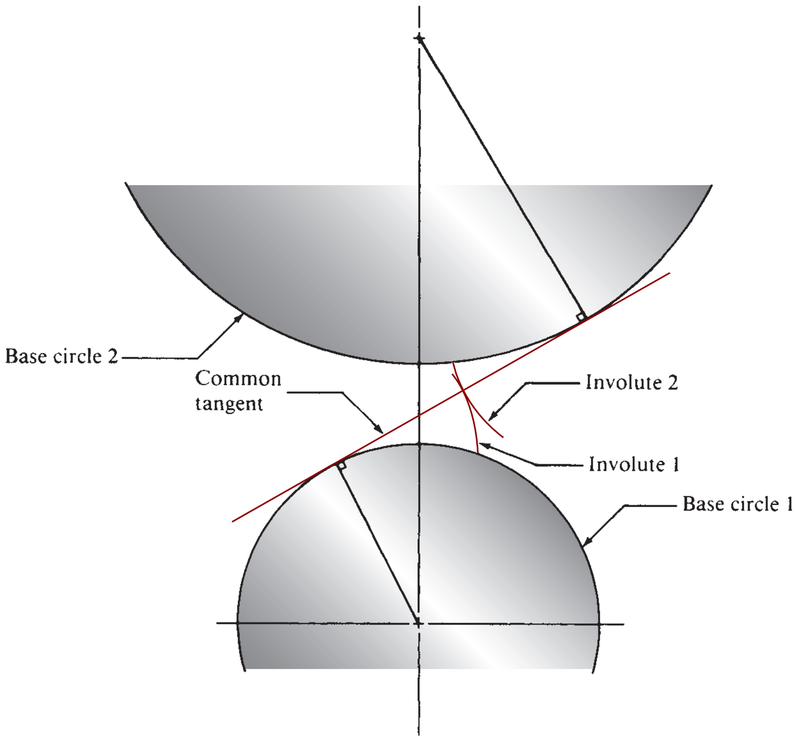


FIGURE 8–7 Mating involutes

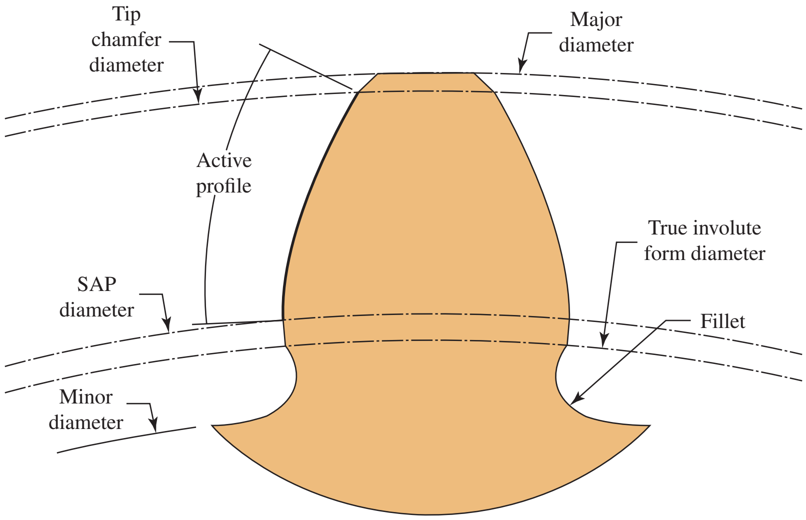


FIGURE 8–8 Gear tooth profile

8–4 SPUR GEAR NOMENCLATURE AND GEAR-TOOTH FEATURES

This section describes several features of individual spur gear teeth, complete gears, and the basic geometry of two mating gears. Terms and symbols used here conform mostly to American Gear Manufacturers Association (AGMA) standards. Because there is variation among the several applicable standards, the primary reference is AGMA 2001-D04 *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*. This standard is the basis for analytical design methods that are described in Chapters 9 and 10 for spur gears and helical gears, respectively. Where appropriate, the terms and symbols used by other AGMA standards and international standards such as ISO, DIN (Germany), and JIS (Japan) are noted. Both the conventional U.S. system of units, called the *Diametral Pitch System*, and the SI metric system, called the *Metric Module System*, are discussed.

Reference is made to several figures and tables that depict the geometry of interest in the design of gear pairs:

1. Figure 8–1 shows two mating spur gears, indicating the dimensions related to diameters and center distance.
2. Figure 8–9 shows details of spur gear teeth with the many terms used to denote specific parts of the teeth and their relationship with the pitch diameter. These terms are defined later in this section.
3. Figure 8–10 shows two gears in mesh with several important diameters, center distance, and other features. See also Internet sites 7 and 8 for animations of teeth engagement.
4. Figure 8–11 shows how spur gear teeth engage as the gears rotate. Gear 1 rotates clockwise and drives gear 2 that rotates counterclockwise. The teeth on gear 1, labeled A_1, B_1, C_1, and D_1, contact the teeth on gear 2, labeled A_2, B_2, C_2, and D_2 respectively. The contact between any two teeth remains along the line of action, until the teeth are no longer engaged.
5. Figures 8–12 and 8–13 show various sizes of gear teeth in both the diametral pitch and metric module systems. Both figures are full size, enabling you to compare physical gears to the drawings to gain an appreciation of gear tooth sizes.
6. Table 8–1 is a composite reference tool for identifying the names, symbols, definitions, units, and formulas related to the several features of gear teeth and mating gears.

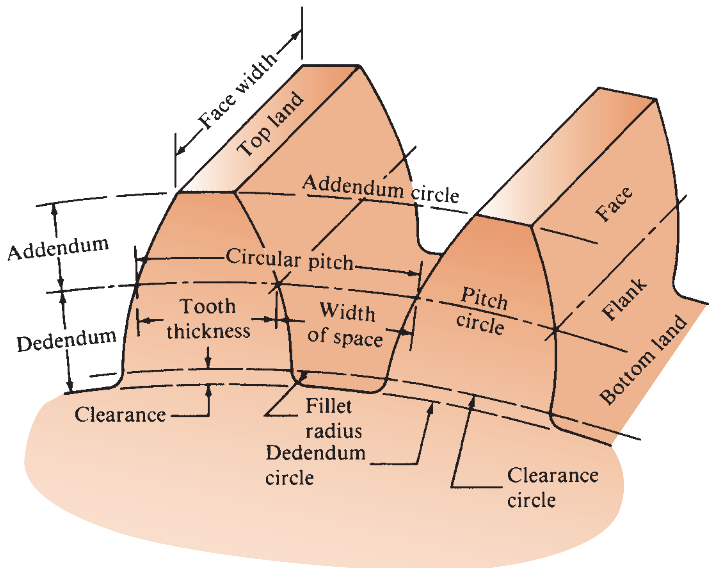


FIGURE 8–9 Spur gear teeth features

A note about accuracy: Gears and gear trains are precision mechanical devices with tolerances on critical dimensions typically in the range of a few ten thousandths of an inch (0.0001 in or about 0.0025 mm). Therefore, it is expected that such dimensions be

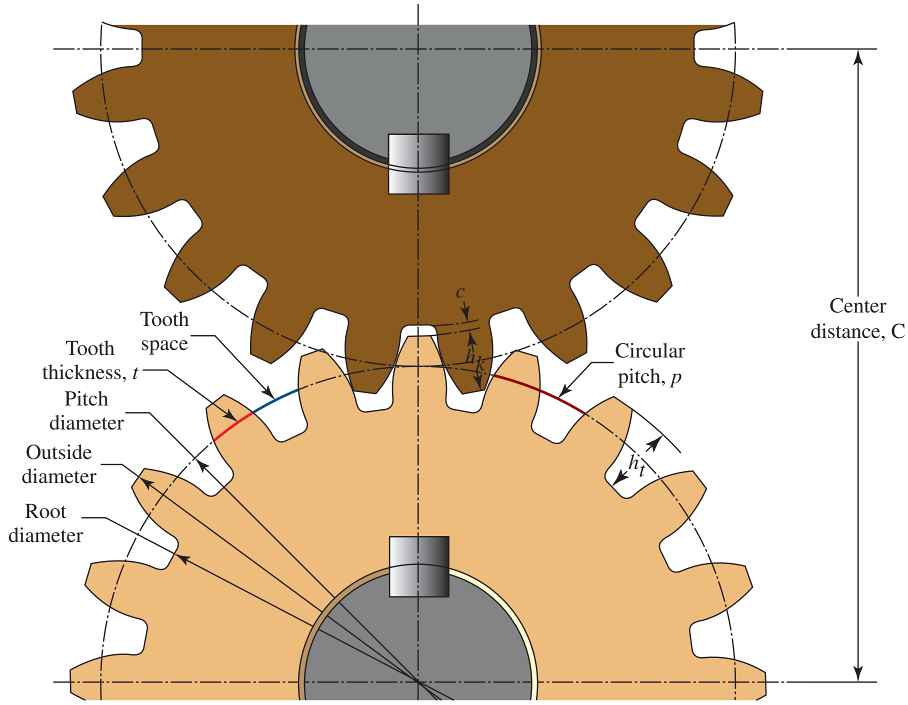
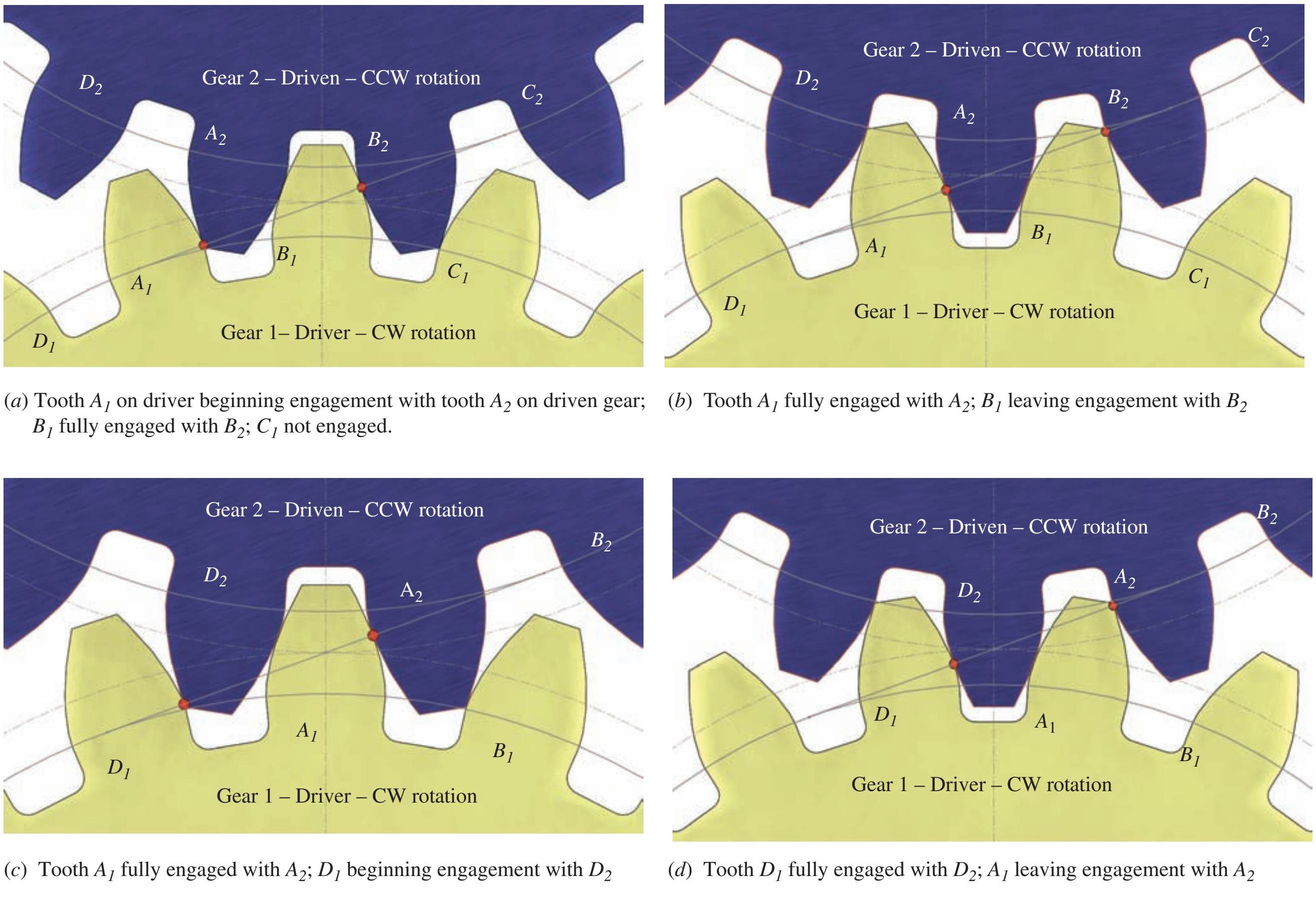


FIGURE 8–10 Details of two meshing spur gears showing several important geometric features.



(*a*) Tooth A_1 on driver beginning engagement with tooth A_2 on driven gear; B_1 fully engaged with B_2; C_1 not engaged.

(*b*) Tooth A_1 fully engaged with A_2; B_1 leaving engagement with B_2

(*c*) Tooth A_1 fully engaged with A_2; D_1 beginning engagement with D_2

(*d*) Tooth D_1 fully engaged with D_2; A_1 leaving engagement with A_2

FIGURE 8–11 Cycle of engagement of gear teeth

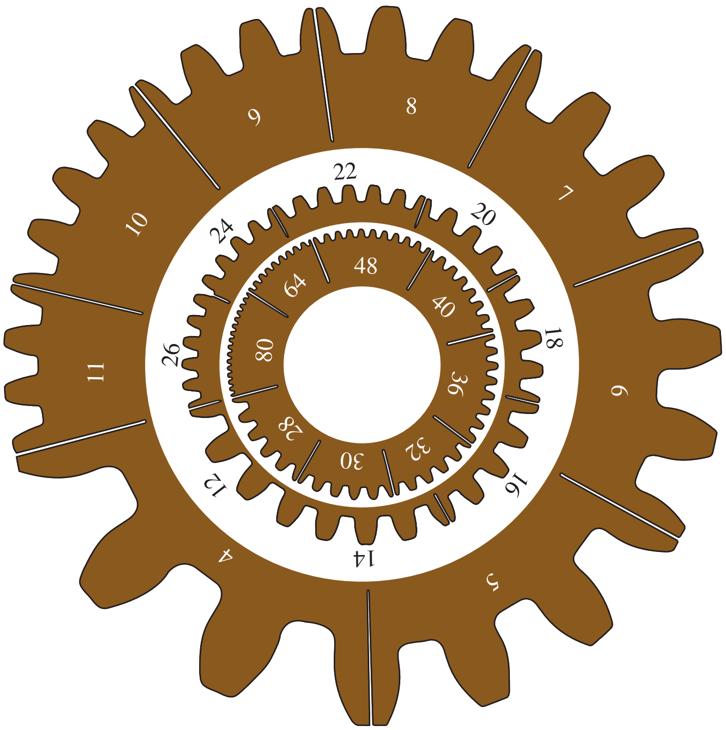


FIGURE 8–12 Gear-tooth size as a function of diametral pitch—actual size

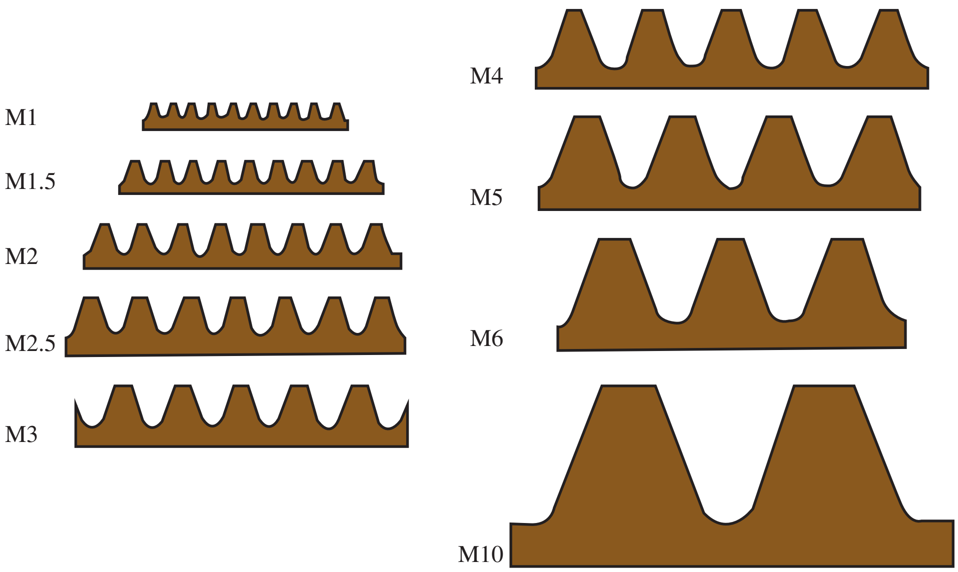


FIGURE 8–13 Selected standard metric modules in rack form–actual size

TABLE 8–1 Gear and Tooth Features, Diameters, Center Distance for a Gear Pair

Number of teeth and Pitches	Symbol	Definition	Typical unit	General formula	Formulas: U.S. Full-depth involute system: Coarse pitch $P_d < 20$ (in)	Fine pitch $P_d \geq 20$ (in)	Metric module system (mm)
Number of teeth	N	Integer count of teeth on a gear					
Circular pitch	p	Arc distance between corresponding points on adjacent teeth	in or mm	$p = \pi D/N$	$p = \pi/P_d$		$p = \pi m$
Diametral pitch	P_d	Number of teeth per inch of pitch diameter	in^{-1}	$P_d = N/D$			
Module	m	Pitch diameter divided by number of teeth	mm	$m = D/N$			$m = 25.4/P_d$
Diameters							
Pitch diameter	D	Kinematic characteristic diameter for a gear; Diameter of the pitch circle	in or mm		$D = N/P_d$		$D = mN$
Outside diameter	D_o	Diameter to the outside surface of the gear teeth	in or mm		$D_o = (N + 2)/P_d$		$D_o = m(N + 2)$
Root diameter	D_R	Diameter to the root circle of the gear at the base of the teeth	in or mm	$D_R = D - 2b$			
Gear Tooth Features							
Addendum	a	Radial distance from pitch circle to outside of tooth	in or mm		$a = 1.00/P_d$		$a = 1.00m$
Dedendum	b	Radial distance from pitch circle to bottom of tooth space	in or mm		$b = 1.25/P_d$	$b = 1.20/P_d + 0.002$	$b = 1.25m$[1]
Clearance	c	Radial distance from top of fully engaged tooth of mating gear to bottom of tooth space	in or mm		$c = 0.25/P_d$	$c = 0.20/P_d + 0.002$	$c = 0.25m$[1]
Whole depth	h_t	Radial distance from top of a tooth to bottom of tooth space	in or mm	$h_t = a + b$	$h_t = 2.25/P_d$	$h_t = 2.20/P_d + 0.002$	$h_t = 2.25m$[1]
Working depth	h_k	Radial distance a gear tooth projects into tooth space of mating gear	in or mm	$h_k = a + a = 2a$	$h_k = 2.00/P_d$	$h_k = 2.00/P_d$	$h_k = 2.00m$[1]
Tooth thickness	t	Theoretical arc distance equal to 1/2 of circular pitch	in or mm	$t = p/2$	$t = \pi/[2(P_d)]$		$t = \pi m/2$
Face width	F	Width of tooth parallel to axis of gear	in or mm	*Design decision*	Approximately $12/P_d$		
Pressure angle	ϕ	Angle between the tangent to the pitch circle and the perpendicular to the gear tooth surface	degrees	*Design decision*	Most common value = 20° Others: 14 1/2°, 25°		
Center Distance	C	Distance from between centerlines of mating gears	in or mm	$C = (D_P + D_G)/2$	$C = (N_P + N_G)/2P_d$		$C = m(N_P + N_G)/2$

Note: [1]Factors in formula for dedendum may vary in metric module system to obtain custom clearance.

reported to at least the nearest ten thousandth of an inch (four decimal places) or the nearest 0.001 mm. Some applications require even more precision. See References 8 and 9 for more on accuracy of gearing. Additional discussion about accuracy is included later in Section 8–12 on Gear Quality.

***Pinion* and *Gear*.** For two gears in mesh, the smaller gear is called the *pinion* and the larger is called, simply, the *gear.*

Number of Teeth, *N*. It is essential that there are an integer number of teeth in any gear. This text uses the symbol N for the number of teeth, with N_P for the pinion and N_G for the gear. These subscripts are applied to other gear features as well. Another commonly used symbol for the number of teeth is z, with similar subscripts or simply called z_1 and z_2.

***Pitch*.** Refer to Figures 8–9 and 8–10. The *pitch* of a gear, in general, is defined as follows:

> ***The pitch of a gear is the arc distance from a point on a tooth at the pitch circle to the corresponding point on the next adjacent tooth, measured along the pitch circle.***

The pitch circle is defined next. It is important to note that the pitch of both mating gears must be identical to ensure the smooth engagement of the teeth as the gears rotate. Standard pitches are defined in three different systems, described later in this section.

***Pitch Circle* and *Pitch Diameter*.** When two gears are in mesh, they behave as if two smooth rollers are rolling on each other without slipping. The surface of each roller defines the *pitch circle* and its diameter is called the *pitch diameter.* The *pitch diameter*, called D in this book, is used as the characteristic size of the gear for calculations of speeds. Note that the pitch diameter for a gear is a theoretical concept and cannot be measured directly. It falls within the gear teeth and is dependent on which standard system for pitch is specified for a particular gear pair. The commonly used units for D are inches (in) for the U.S. system and millimeters (mm) for the SI metric system. Figures 8–1, 8–10, and 8–15 show the pitch diameters on meshing gears.

TABLE 8–2 Standard Circular Pitches (in)

10.0	7.5	5.0
9.5	7.0	4.5
9.0	6.5	4.0
8.5	6.0	3.5
8.0	5.5	

Circular Pitch, p. The pitch corresponding exactly to the basic definition of pitch given above is called the *circular pitch, p*. Some large gears that are made by casting are made to standard sizes of circular pitch such as those listed in Table 8–2. They represent a very small portion of gears in common use. The formula for p comes from dividing the circumference of the pitch circle of the gear into N parts. That is,

➩ Circular Pitch

$$p = \pi D/N \tag{8–2}$$

***Diametral Pitch*, P_d.** The most common pitch system in use in the United States at this time is *diametral pitch system*. We use the symbol, P_d, to denote *diametral pitch*. Note that some references use the term *DP*. The definition of P_d is stated here for either the pinion or the gear and both must be identical.

➩ Diametral Pitch

$$P_d = N_P/D_P = N_G/D_G \tag{8–3}$$

Analysis of units shows that P_d has the unit of in^{-1}, but the unit is rarely reported. It is necessary to not confuse the terms *diametral pitch*, P_d, and *pitch diameter, D*. Note that designers often refer to gears in this system as, for example, 8-pitch for $P_d = 8$ and 20-pitch for $P_d = 20$.

In this book, we use only those values of P_d listed in Table 8–3 because they are the most readily available as stock gears and most gear manufacturers have tooling for these sizes. Smaller pitches have larger teeth; larger pitches have smaller teeth. Note that pitches under 20 are called *coarse*, while those 20 and higher are called

TABLE 8–3 Standard Diametral Pitches (teeth/in)

Coarse pitch ($P_d < 20$)				Fine pitch ($P_d \geq 20$)	
1	2	5	12	20	72
1.25	2.5	6	14	24	80
1.5	3	8	16	32	96
1.75	4	10	18	48	120
				64	

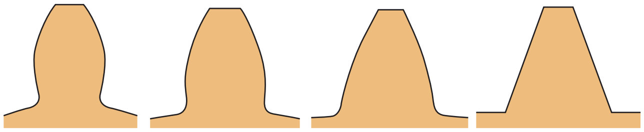


FIGURE 8–14 Involute curve shape for varying numbers of teeth for a diametral pitch of 5

fine. Refer to Figure 8–13 that shows actual sizes of teeth with certain diametral pitches. ***Note that not all listed values of P_d are readily available and those such as 7, 9, 11, 22, and 26 should not be specified.***

Number of Teeth Related to Pitch Diameter

The relationship between the pitch diameter, diametral pitch, and the number of teeth is given by Equation 8–3. As the number of teeth increases for a given diametral pitch, the pitch diameter increases. So as the pitch diameter increases, the base circle diameter increases. A larger base circle will decrease the curvature of the involute curve of the gear tooth. Figure 8–14 shows as the number of teeth increases for a given diametral pitch, the radius of the involute curve of the gear tooth will decrease. Inversely, as the number of teeth decrease the radius of the involute curve of the gear tooth will increase. The teeth in this figure are all drawn with a diametral pitch of 5. The gear rack can be thought of as a gear with an infinitely large diameter, where the involute curve of the tooth profile becomes a straight line.

Metric Module, m. The basic definition of the metric module, m, is given here for both the pinion and the gear and they must be identical.

Metric Module

$$m = D_p/N_p = D_G/N_G \tag{8–4}$$

The unit of mm is typically used. Note that smaller values of m denote smaller teeth and vice versa. Some references and vendors' tables use the symbol, M, for module and write M5 for $m = 5$, for example. Figure 8–13 shows actual sizes of nine standard modules, illustrated as the teeth of *racks*, straight gears with infinite diameters. Many more modules are available and Table 8–4 lists a total of 19 values that cover typical applications featured in this book.

Relation between P_d and m. With globally integrated design and marketing of products and systems, it is likely that the need to convert from one system to another will be needed. Note that the definition of P_d is fundamentally the inverse of the definition of m. That is,

$$m = 1/P_d \quad \text{or} \quad P_d = 1/m$$

However, because different units are employed for each term, a conversion factor of 25.4 mm/in is required, resulting in useful forms of the relationship as

TABLE 8–4 Standard modules

Module (mm)	Equivalent P_d	Closest standard P_d (teeth/in)
0.3	84.667	80
0.4	63.500	64
0.5	50.800	48
0.8	31.750	32
1	25.400	24
1.25	20.320	20
1.5	16.933	16
2	12.700	12
2.5	10.160	10
3	8.466	8
4	6.350	6
5	5.080	5
6	4.233	4
8	3.175	3
10	2.540	2.5
12	2.117	2
16	1.587	1.5
20	1.270	1.25
25	1.016	1

Relation between Module and Diametral Pitch

$$m = 25.4/P_d \quad \text{or} \quad P_d = 25.4/m \tag{8–5}$$

Table 8–4 uses this relationship to compute the equivalent diametral pitch, P_d, for given standard metric modules, m. Note that the conversions do not deliver standard values of P_d such as those listed in Table 8–3. Therefore, we list the nearest standard P_d value in Table 8–4 as an aid to designers who are considering converting a design from one system to the other. We can say, for example, that a module of $m = 1.25$ is a closely similar size to $P_d = 20$. Of course, the design for either system must be completed independently.

Gear Tooth Features. Table 8–1 includes the definitions of several other features of individual teeth that designers must be familiar with. Refer to Figures 8–9 and 8–10 to visualize those features.

Face Width, F. Face width is the width of the gear parallel to the axis of the gear. It is defined by the designer as one of the required *design decisions*. More is said about face width in Chapter 9, where strength of the teeth is considered. For now, we can state that a nominal value for face width is approximately $F \sim 12/P_d$, but a wide range is permitted.

Center Distance, C. One of the most critical dimensions for a gear pair is the *center distance*, defined as the linear distance from the centerline of the pinion to the centerline of the gear as shown in Figure 8–1. The theoretical value is best represented as the sum of the pitch radii of the pinion and the gear. That is,

Pitch radii: $R_P = D_P/2$ and $R_G = D_G/2$

Center distance: $C = R_P + R_G = D_P/2 + D_G/2$

$$C = (D_P + D_G)/2 \quad (8\text{–}6)$$

Other useful equations for C, recommended for use in this book, are developed here.

Diametral pitch system: From Equation (8–3),

➪ Center Distance in terms of N_G, N_P, and P_d

$$D_P = N_P/P_d \quad \text{and} \quad D_G = N_G/P_d$$
$$C = (D_P + D_G)/2 = (N_P/P_d + N_G/P_d)/2$$
$$C = (N_P + N_G)/2P_d \quad (8\text{–}7)$$

Metric module system: From Equation (8–4),

➪ Center Distance in terms of N_G, N_P, and m

$$D_P = mN_P \quad \text{and} \quad D_G = mN_G$$
$$C = (D_P + D_G)/2 = (mN_P + mN_G)/2$$
$$C = m(N_P + N_G)/2 \quad (8\text{–}8)$$

An important advantage of the final forms of the center distance Equation (8–7) and (8–8) is that all numbers on the right side are typically integers or exact fractional dimensions such as 1.25, 1.5, or 2.5. Therefore, the highest level of accuracy is obtained from using those forms. Conversely, some values for pitch diameters are irrational numbers. For example, a 12-pitch gear with 65 teeth has a pitch diameter of

$$D = N/P_d = 65/12 = 5.416666\ldots \text{ in} \quad \text{and}$$
$$R = D/2 = 2.708333\ldots \text{ in}$$

Depending on rounding, an inaccurate calculation for center distance could result from using these values.

Comments on Gear Tooth Features. The definitions and formulas given in Table 8–1 yield the theoretical values and it is typical for gear designers and manufacturers to modify some features to produce preferred performance characteristics. It is important to realize that some of these practices change the fundamental geometry of the gears and they may result in weaker tooth shapes, vibration during operation, and/or increased noise. Some examples are given below:

1. *Addendum modification:* The upper part of the flank of a gear tooth is the first to engage its mating tooth and it penetrates most deeply into the tooth space of the mating gear. Some applications benefit from relieving the true surface of the tooth or from shortening the addendum to promote smooth engagement or to avoid damage.
2. *Dedendum modification:* Some designers prefer greater clearance between the bottom of the tooth space and the fully engaged mating tooth to facilitate lubrication. Using a longer dedendum dimension provides this capability. Plastic gears may also employ dedendum modification to accommodate thermal expansion and/or swelling because of moisture absorption.
3. *Center distance modification:* Some combinations of gear tooth geometry result in very small clearances between teeth and possibly interference between the top of the engaging tooth and the lower part of the flank of the mating tooth. These conditions should be avoided by proper design decisions to avoid the interference and this topic is discussed in Chapter 9. However, some designers remove the interference by expanding the center distance slightly from its theoretical dimension.
4. *Tooth thickness modification:* A tooth thickness defined exactly as $\frac{1}{2}$ of the circular pitch will result in a tight fit of a tooth in the tooth space of the mating gear, possibly causing binding or precluding entry of lubrication at the contact point on the teeth. Using a smaller tooth thickness provides space for lubrication and facilitates assembly. The space created is called *backlash* and it is discussed next.

- ***Backlash:*** To provide backlash, the cutter generating the gear teeth can be fed more deeply into the gear blank than the theoretical value on either or both of the mating gears. Alternatively, backlash can be created by adjusting the center distance to a larger value than the theoretical value.

  The magnitude of backlash depends on the desired precision of the gear pair and on the size and the pitch of the gears. It is actually a design decision, balancing cost of production with desired performance. The American Gear Manufacturers Association (AGMA) provides recommendations for backlash in their standards. (See Reference 2.) Table 8–5 lists examples of recommended ranges for several values of pitch.

Pressure Angle

> ***The* pressure angle *is the angle between the tangent to the pitch circles and the line drawn normal (perpendicular) to the surface of the gear tooth (see Figure 8–15).***

The normal line is sometimes referred to as the *line of action*. When two gear teeth are in mesh and are transmitting power, the force transferred from the driver to the driven gear tooth acts in a direction along the line of action. The *length of action* is the distance AB along

TABLE 8–5 Recommended Minimum Backlash for Coarse Pitch Gears

A. Diametral pitch system (backlash in inches)

	Center distance, *C* (in)				
P_d	2	4	8	16	32
18	0.005	0.006			
12	0.006	0.007	0.009		
8	0.007	0.008	0.010	0.014	
5		0.010	0.012	0.016	
3		0.014	0.016	0.020	0.028
2			0.021	0.025	0.033
1.25				0.034	0.042

B. Metric module system (backlash in millimeters)

	Center distance, *C* (mm)				
Module, *m*	50	100	200	400	800
1.5	0.13	0.16			
2	0.14	0.17	0.22		
3	0.18	0.20	0.25	0.35	
5		0.26	0.31	0.41	
8		0.35	0.40	0.50	0.70
12			0.52	0.62	0.82
18				0.80	1.00

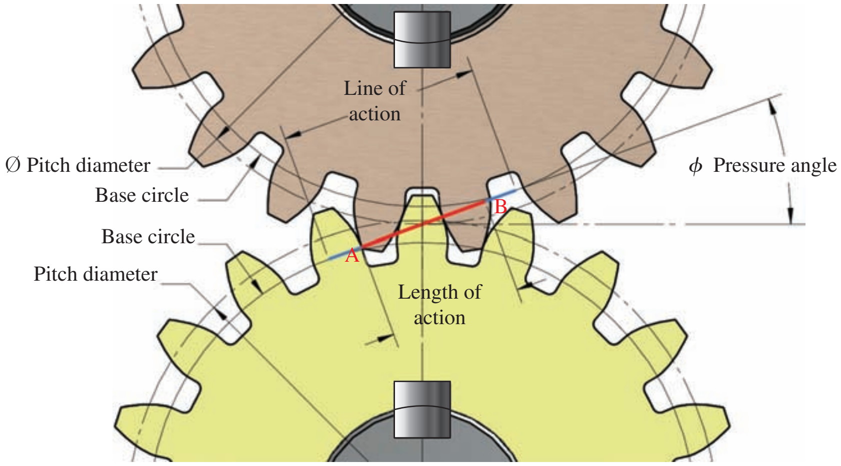


FIGURE 8–15 Two spur gears in mesh showing the pressure angle, line of action, base circles, pitch diameters, and other features

the line of action. This length represents the path of the point of contact during the meshing of the two gears.

Also, the actual shape of the gear tooth depends on the pressure angle, as illustrated in Figure 8–16. The teeth in this figure were drawn according to the proportions for a 20-tooth, 5-pitch gear having a pitch diameter of 4.000 in.

All three teeth have the same tooth thickness because, as stated in Table 8–1 the thickness at the pitch line depends only on the pitch. The difference between the three teeth shown is due to the different pressure angles because the pressure angle determines the size of the base circle. Remember that the base circle is the circle from which the involute is generated. The line of action is always tangent to the base circle. Therefore, the size of the base circle can be found from

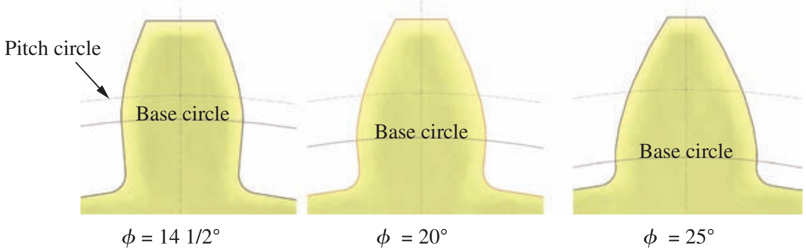


FIGURE 8–16 Illustration of how the shape of gear teeth change as the pressure angle, (phi), changes

Base Circle Diameter

$$D_b = D \cos \phi \tag{8–9}$$

As the pressure angle increases, the thickness at the bottom of the tooth increases. Therefore the larger the pressure angle, the stronger the tooth and the higher the load carrying capacity of the tooth. A higher pressure angle may not run as smoothly or quietly as the smaller pressure angle. As the pressure angle decreases, the top land of the tooth will increase. This can create interference problems when a small number of pinion teeth are required. The topic of gear interference will be covered in more detail in Section 8–5.

Standard values of the pressure angle are established by gear manufacturers, and the pressure angles of two gears in mesh must be the same. Current standard pressure angles are $14\frac{1}{2}°$, 20°, and 25° as illustrated in Figure 8–16. Actually, the $14\frac{1}{2}°$ tooth form is considered obsolete. Although it is still available, it should be avoided for new designs. The 20° tooth form is the most readily available at this time. The advantages and disadvantages of the different values of pressure angle relate to the strength of the teeth, the occurrence of interference, and the magnitude of forces exerted on the shaft. Interference is discussed in Section 8–5. The other points are discussed in a later chapter.

Contact Ratio

When two gears mesh, it is essential for smooth operation that a second tooth begin to make contact before a given tooth disengages. The term *contact ratio* is used to indicate the average number of teeth in contact during the transmission of power. A recommended minimum contact ratio is 1.2 and typical spur gear combinations often have values of 1.5 or higher.

The contact ratio is defined as the ratio of the length of the line-of-action to the base pitch for the gear. The line-of-action is the straight-line path of a tooth from where it encounters the outside diameter of the mating gear to the point where it leaves engagement. The base pitch is the diameter of the base circle divided by the number of teeth in the gear. A convenient formula for computing the contact ratio, m_f, is,

Contact Ratio

$$m_f = \frac{\sqrt{R_{oP}^2 - R_{bP}^2} + \sqrt{R_{oG}^2 - R_{bG}^2} - C \sin \phi}{p \cos \phi} \tag{8–10}$$

where,

ϕ = Pressure angle

R_{oP} = Outside radius of the pinion $= D_{oP}/2 = (N_P + 2)/(2P_d)$

R_{bP} = Radius of the base circle for the pinion $= D_{bP}/2 = (D_P/2) \cos \phi = (N_P/2P_d) \cos \phi$

R_{oG} = Outside radius of the gear $= D_{oG}/2 = (N_G + 2)/(2P_d)$

R_{bG} = Radius of the base circle for the gear $= D_{bG}/2 = (D_G/2) \cos \phi = (N_G/2P_d) \cos \phi$

C = Center distance $= (N_P + N_G)/(2P_d)$

p = Circular pitch $= (\pi D_p/N_p) = \pi/P_d$

For example, consider a pair of gears with the following data:

$$N_P = 18,\ N_G = 64,\ P_d = 8,\ \phi = 20°$$

Then,

$$R_{oP} = (N_P + 2)/(2P_d) = (18 + 2)/[2(8)] = 1.2500 \text{ in}$$

$$R_{bP} = (N_P/2P_d) \cos \phi = 18/[2(8)] \cos 20° = 1.05715 \text{ in}$$

$$R_{oG} = (N_G + 2)/(2P_d) = (64 + 2)/[2(8)] = 4.1250 \text{ in}$$

$$R_{bG} = (N_G/2P_d) \cos \phi = 64/[2(8)] \cos 20° = 3.75877 \text{ in}$$

$$C = (N_P + N_G)/(2P_d) = (18 + 64)/[2(8)] = 5.1250 \text{ in}$$

$$p = \pi/P_d = \pi/8 = 0.392699 \text{ in}$$

Finally, the contact ratio is,

$$m_f = \frac{\sqrt{(1.250)^2 - (1.05715)^2} + \sqrt{(4.125)^2 - (3.75877)^2} - (5.125) \sin 20°}{(0.392699) \cos 20°}$$

$$m_f = 1.66$$

This states that for these two gears in mesh, there will always be one tooth in full contact and for 66% of the time there will be two teeth in contact. The value of $m_f = 1.66$ is comfortably above the recommended minimum value of 1.20.

Similar developments can be used to determine the factors needed to implement the contact ratio calculation in Equation (8–10) with terms expressed in the Metric Module System. Table 8–6 summarizes the relationships for both systems.

TABLE 8–6 Formulas for Use When Implementing Gear Pair Contact Ratio Calculation In U.S. and SI Systems in Terms of Diametral Pitch and Module

	Factors	Diametral Pitch System	Metric Module System
ϕ	Pressure angle		
R_{oP}	Outside radius—pinion	$(N_p + 2)/(2P_d)$	$m(N_p + 2)/2$
R_{bP}	Base circle radius—pinion	$(N_p/2P_d) \cos \phi$	$(mN_p/2) \cos \phi$
R_{oG}	Outside radius—gear	$(N_G + 2)/(2P_d)$	$m(N_G + 2)/2$
R_{bG}	Base circle radius—gear	$(N_G/2P_d) \cos \phi$	$(mN_G/2) \cos \phi$
C	Center distance	$(N_p + N_G)/(2P_d)$	$m(N_p + N_G)/2$
p	Circular pitch	π/P_d	πm

Example Problem 8–1 For the pair of gears shown in Figure 8–1, compute all of the features of the gear teeth described in this section. The gears conform to the standard AGMA form and have a diametral pitch of 12 and a 20° pressure angle.

Solution

Given $P_d = 12$; $N_p = 11$; $N_G = 18$; $\phi = 20°$.

Analysis Unless otherwise noted, we use equations from Table 8–1 to compute the features. Refer to the text in this section for explanation of terms.

Note that results are reported to four decimal places as is typical for precise mechanical devices like gears. A similar level of accuracy is expected for problems in this book.

Results *Pitch Diameters*

For the pinion,

$$D_p = N_p/P_d = 11/12 = 0.9167 \text{ in}$$

For the gear,

$$D_G = N_G/P_d = 18/12 = 1.5000 \text{ in}$$

Circular Pitch

Three different approaches could be used.

$$p = \pi/P_d = \pi/12 = 0.2618 \text{ in}$$

Note that data for either the pinion or the gear data may also be used. For the pinion,

$$p = \pi D_P/N_P = \pi(0.9167 \text{ in})/11 = 0.2618 \text{ in}$$

For the gear,

$$p = \pi D_G/N_G = \pi(1.500 \text{ in})/18 = 0.2618 \text{ in}$$

Addendum

$$a = 1/P_d = 1/12 = 0.8333 \text{ in}$$

Dedendum

Note that the 12-pitch gear is considered to be coarse. Thus,

$$b = 1.25/P_d = 1.25/12 = 0.1042 \text{ in}$$

Clearance

$$c = 0.25/P_d = 0.25/12 = 0.0208 \text{ in}$$

Outside Diameters

For the pinion,

$$D_{oP} = (N_P + 2)/P_d = (11 + 2)/12 = 1.0833 \text{ in}$$

For the gear,

$$D_{oG} = (N_G + 2)/P_d = (18 + 2)/12 = 1.6667 \text{ in}$$

Root Diameters

First, for the pinion,

$$D_{RP} = D_P - 2b = 0.9167 \text{ in} - 2(0.1042 \text{ in}) = 0.7083 \text{ in}$$

For the gear,

$$D_{RG} = D_G - 2b = 1.500 \text{ in} - 2(0.1042 \text{ in}) = 1.2917 \text{ in}$$

Whole Depth

$$h_t = a + b = 0.0833 \text{ in} + 0.1042 \text{ in} = 0.1875 \text{ in}$$

Working Depth

$$h_k = 2a = 2(0.0833 \text{ in}) = 0.1667 \text{ in}$$

Tooth Thickness

$$t = \pi/[2(P_d)] = \pi/[2(12)] = 0.1309 \text{ in}$$

Center Distance

$$C = (N_G + N_P)/(2P_d) = (18 + 11)/[2(12)] = 1.2083 \text{ in}$$

Base Circle Diameter

$$D_{bP} = D_P \cos\phi = (0.9167 \text{ in}) \cos(20°) = 0.8614 \text{ in}$$
$$D_{bG} = D_G \cos\phi = (1.5000 \text{ in}) \cos(20°) = 1.4095 \text{ in}$$

8–5 INTERFERENCE BETWEEN MATING SPUR GEAR TEETH

For certain combinations of numbers of teeth in a gear pair, there is interference between the tip of the teeth on the pinion and the fillet or root of the teeth on the gear. Obviously this cannot be tolerated because the gears simply will not mesh. The probability that interference will occur is greatest when a small pinion drives a large gear, with the worst case being a small pinion driving a rack. A *rack* is a gear with a straight pitch line; it can be thought of as a gear with an infinite pitch diameter [see Figure 8–4(c)].

It is the designer's responsibility to ensure that interference does not occur in a given application. The surest way to do this is to control the minimum number of teeth in the pinion to the limiting values shown on the left side of Table 8–7. With this number of teeth or a greater number, there will be no interference with a rack or with any other gear. A designer who desires to use fewer than the listed number of teeth can use a graphical layout to test the combination of pinion and gear for interference. Texts on kinematics provide the necessary procedure. The right side of Table 8–7 indicates the maximum number of 20° full depth gear teeth that you can use for a given number of pinion teeth to avoid interference. (See References 11 and 17.)

Using the information in Table 8–7, we can draw the following conclusions:

1. If a designer wants to be sure that there will not be interference between any two gears when using the $14\frac{1}{2}°$, full-depth, involute system, the pinion of the gear pair must have no fewer than 32 teeth.
2. For the 20°, full-depth, involute system, using no fewer than 18 teeth will ensure that no interference occurs.
3. For the 25°, full-depth, involute system, using no fewer than 12 teeth will ensure that no interference occurs.
4. If a designer desires to use fewer than 18 teeth in a pinion having 20°, full-depth teeth, there is an upper limit to the number of teeth that can be used on

TABLE 8–7 Number of Pinion Teeth to Ensure No Interference

For a pinion meshing with a rack		For a 20°, full-depth pinion meshing with a gear		
Tooth form	Minimum number of teeth	Number of pinion teeth	Maximum number of gear teeth	Maximum ratio
$14\frac{1}{2}°$, involute, full-depth	32	17	1309	77.00
20°, involute, full-depth	18	16	101	6.31
25°, involute, full-depth	12	15	45	3.00
		14	26	1.85
		13	16	1.23

the mating gear without interference. For 17 teeth in the pinion, any number of teeth on the gear can be used up to 1309, a very high number. Most gear drive systems use no more than about 200 teeth in any gear. But a 17-tooth pinion *would* have interference with a *rack* which is effectively a gear with an infinite number of teeth or an infinite pitch diameter. Similarly, the following requirements apply for 20° full-depth teeth:

A 16-tooth pinion requires a gear having 101 or fewer teeth, producing a maximum velocity ratio of $N_G/N_P = 101/16 = 6.31$.

A 15-tooth pinion requires a gear having 45 or fewer teeth, producing a maximum velocity ratio of $45/15 = 3.00$.

A 14-tooth pinion requires a gear having 26 or fewer teeth, producing a maximum velocity ratio of $26/14 = 1.85$.

A 13-tooth pinion requires a gear having 16 or fewer teeth, producing a maximum velocity ratio of $16/13 = 1.23$.

As noted earlier, the $14\frac{1}{2}°$ system is considered to be obsolete. The data in Table 8–7 indicate one of the main disadvantages with that system: its potential for causing interference.

Overcoming Interference

If a proposed design encounters interference, there are ways to make it work. But caution should be exercised because the tooth form or the alignment of the mating gears is changed, causing the stress and wear analysis to be inaccurate. With this in mind, the designer can provide for undercutting, modification of the addendum on the pinion or the gear, or modification of the center distance:

Undercutting is the process of cutting away the material at the fillet or root of the gear teeth, thus relieving the interference.

Figure 8–17 shows the result of undercutting. It should be obvious that this process weakens the tooth; this point is discussed further in Chapter 9 in the section on stresses in gear teeth.

To alleviate the problem of interference, increase the addendum of the pinion while decreasing the addendum of the gear. The center distance can remain the same as its theoretical value for the number of teeth in the pair. But the resulting gears are, of course, nonstandard. (See Reference 12.) It is possible to make the pinion of a gear pair larger than standard while keeping the gear standard if the center distance for the pair is enlarged. (See Reference 11.)

Also shown in Figure 8–17 is tip relief that modifies the top of the tooth profile to provide clearance when a gear tooth bends under load.

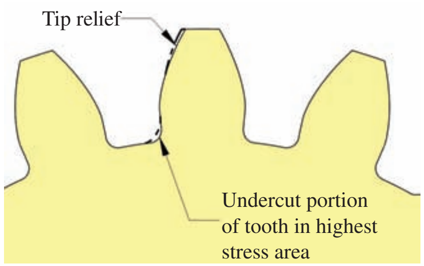


FIGURE 8–17 Undercutting and tip relief of gear teeth

8–6 INTERNAL GEAR GEOMETRY

An* internal gear *is one for which the teeth are machined on the inside of a ring instead of on the outside of a gear blank.

A small internal gear mating with a standard, external pinion is illustrated at the lower left in Figure 8–2, along with a variety of other kinds of gears.

Figure 8–18 is a drawing of an external pinion driving an internal gear. Note the following:

1. The gear rotates in the *same direction* as the pinion. This is different from the case when an external pinion drives an external gear.
2. The center distance is

➩ Center Distance-Internal Gear

$$C = D_G/2 - D_P/2 = (D_G - D_P)/2$$

$$C = (N_G/P_d - N_P/P_d)/2 = (N_G - N_P)/(2P_d) \quad \textbf{(8–10)}$$

The last form is preferred because its factors are all integers for typical gear trains, giving maximum precision in calculations.

3. The descriptions of most other features of internal gears are the same as those for external gears presented earlier. Exceptions for an internal gear are as follows:

The addendum, a, is the radial distance from the pitch circle to the inside of a tooth.

The inside diameter, D_i, is

$$D_i = D - 2a$$

The root diameter, D_R, is

$$D_R = D + 2b$$

where b = dedendum.

Internal gears are used when it is desired to have the same direction of rotation for the input and the output. Also note that less space is taken for an internal gear mating with an external pinion compared with two external gears in mesh.

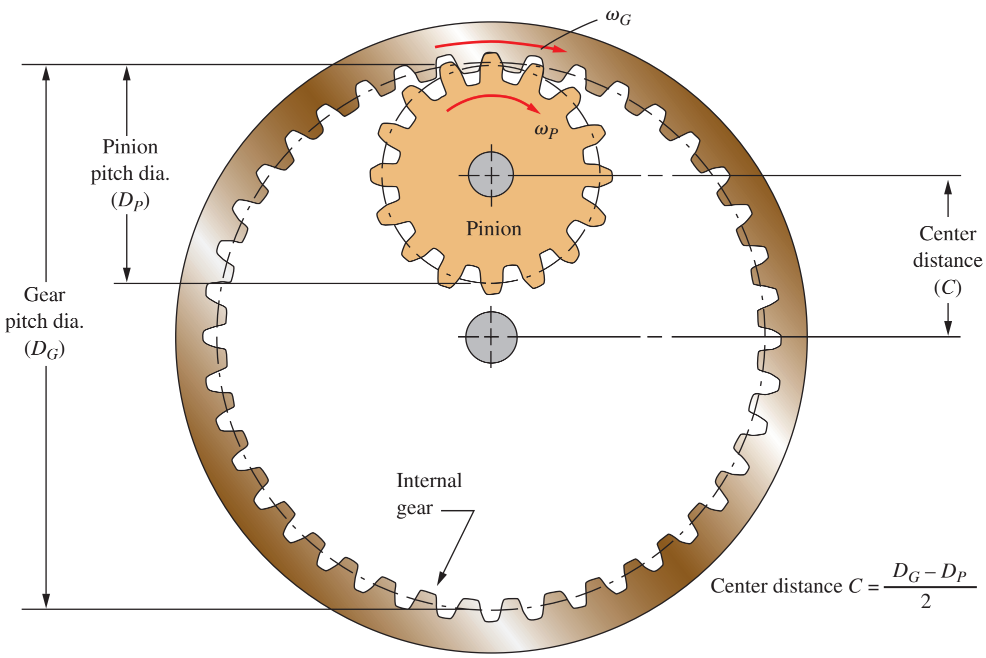


FIGURE 8–18 Internal gear driven by an external pinion

8–7 HELICAL GEAR GEOMETRY

Helical and spur gears are distinguished by the orientation of their teeth. On spur gears, the teeth are straight and are aligned with the axis of the gear. On helical gears, the teeth are inclined at an angle with the axis, that angle being called the *helix angle*. If the gear were very wide, it would appear that the teeth wind around the gear blank in a continuous, helical path. However, practical considerations limit the width of the gears so that the teeth normally appear to be merely inclined with respect to the axis. Figure 8–19 shows two examples of commercially available helical gears.

The forms of helical gear teeth are very similar to those discussed for spur gears. The basic task is to account for the effect of the helix angle.

Helix Angle

The helix for a given gear can be either *left-hand* or *right-hand*. The teeth of a right-hand helical gear would appear to lean to the right when the gear is lying on a flat surface. Conversely, the teeth of a left-hand helical gear would lean to the left. In normal installation, helical gears would be mounted on parallel shafts as shown in Figure 8–19(a). To achieve this arrangement, it is required that one gear be of the right-hand design and that the other be left-hand with an equal helix angle. If both gears in mesh are of the same hand, as shown in Figure 8–19(b), the shafts will be at 90° to each other. Such gears are called *crossed helical gears*.

The parallel shaft arrangement for helical gears is preferred because it results in a much higher powertransmitting capacity for a given size of gear than the crossed helical arrangement. In this book, we will assume that the parallel shaft arrangement is being used unless otherwise stated.

The main advantage of helical gears over spur gears is smoother engagement because a given tooth assumes its load gradually instead of suddenly. Contact starts at one end of a tooth near the tip and progresses across the face in a path downward across the pitch line to the lower flank of the tooth, where it leaves engagement. Simultaneously, other teeth are coming into engagement before a given tooth leaves engagement, with the result that a larger average number of teeth are engaged and are sharing the applied loads compared with a spur gear. The lower average load per tooth allows a greater power transmission capacity for

(*a*) Right hand and left hand helical gears with 20° helix angle and parallel axes

(*b*) Two right handed helical gears with 45° helix angle and crossed axes

FIGURE 8–19 Two varieties of pairs of helical gears a given size of gear, or a smaller gear can be designed to carry the same power. Helical gears also generate less noise and vibration than spur gears due to the smoother tooth engagement.

The main disadvantage of helical gears is that an *axial thrust load* is produced as a natural result of the inclined arrangement of the teeth. The bearings that hold the shaft carrying the helical gear must be capable of reacting against the thrust load.

The helix angle is specified for each given gear design. A balance should be sought to take advantage of the smoother engagement of the gear teeth when the helix angle is high while maintaining a reasonable value of the axial thrust load that increases with increasing helix angle. A typical range of values of helix angles is from 15° to 45°.

Primary Planes for Helical Gears

The three primary planes of a helical gear are illustrated in Figure 8–20(a) for a circular gear and 8–21(b) for a rack.

The *tangential plane* shown in Figure 8–20(b) is tangent to the pitch diameter (pitch circle) of the helical gear and is also called the pitch plane. The *transverse plane* shown in Figure 8–20(c) is perpendicular to the axis of the helical gear and the tangential plane. The transverse plane coincides with the plane of rotation. The *normal plane* shown in Figure 8–20(d) is perpendicular to the tooth and the tangential plane.

Pressure Angles for Helical Gears

In design of a helical gear, there are three angles of interest as illustrated in Figure 8–21(d): (1) the *helix angle*, ψ; (2) the *normal pressure angle*, ϕ_n; and (3) the *transverse pressure angle*, ϕ_t. Designers must specify the helix angle and one of the two pressure angles. The other pressure angle can be computed from the following relationship:

$$\tan \phi_n = \tan \phi_t \cos \psi \qquad \textbf{(8–11)}$$

For example, one manufacturer's catalog offers standard helical gears with a normal pressure angle of $14\frac{1}{2}°$ and a 45° helix angle. Then the transverse pressure angle is found from

$$\tan \phi_n = \tan \phi_t \cos \psi$$
$$\tan \phi_t = \tan \phi_n / \cos \psi = \tan(14.5°)/\cos(45°) = 0.3657$$
$$\phi_t = \tan^{-1}(0.3657) = 20.09°$$

Pitches for Helical Gears

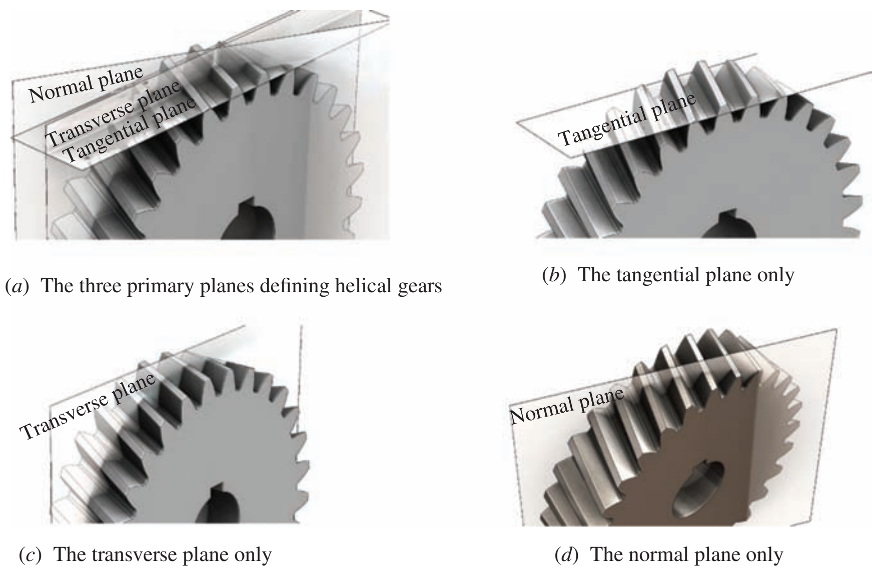
To obtain a clear picture of the geometry of helical gears, you must understand the following five different pitches. The helical rack shown in Figure 8–21 will be used to describe and explain the different pitches of a helical gear.

Transverse circular pitch, p_t. The transverse circular pitch is the distance from a point on one tooth to the corresponding point on the next adjacent tooth, measured at the pitch line in the transverse plane. The transverse circular pitch is the length of line AB shown on the transverse plane and the tangential plane in Figure 8–21(d). This is the same definition used for spur gears. Then

⇨ Transverse Circular Pitch

$$p_t = \pi D/N = \pi/P_d \qquad \textbf{(8–12)}$$

Normal Circular Pitch, p_n. *Normal circular pitch* is the distance between corresponding points on adjacent teeth measured on the pitch surface in the normal direction. The normal circular pitch is the length of the line



(*a*) The three primary planes defining helical gears

(*b*) The tangential plane only

(*c*) The transverse plane only

(*d*) The normal plane only

FIGURE 8–20 Identities of the three primary planes for helical gears

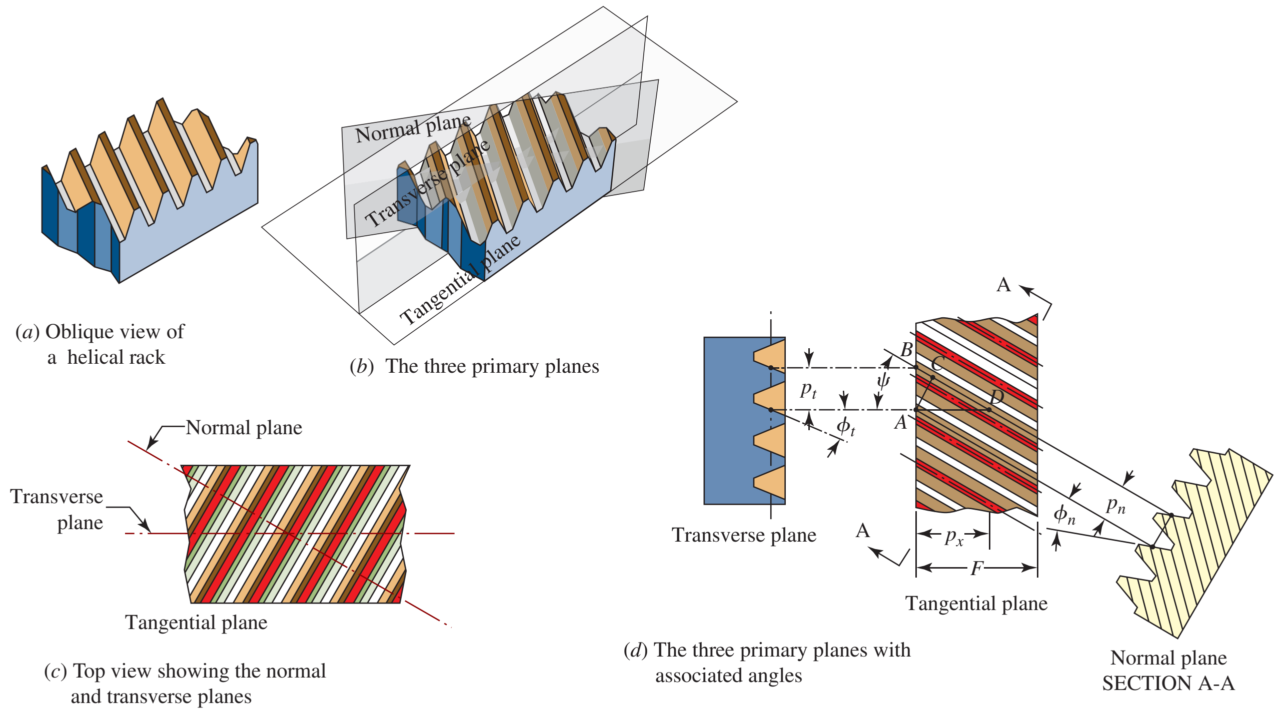


FIGURE 8–21 Identities of the three primary planes and associated angles shown on a helical rack

AC shown on the normal plane in Figure 8–21(d). The triangle ABC is a right triangle with the angle BAC equal to the helix angle. Pitches p_t and p_n are related by the following equation:

➪ Normal Circular Pitch

$$p_n = p_t \cos \psi \tag{8–13}$$

Axial Pitch, p_x. *Axial pitch* is the distance *AD* between corresponding points on adjacent teeth, measured on the pitch surface in the axial direction.

➪ Axial Pitch

$$p_x = p_t/\tan \psi = \pi(P_d/\tan \psi) = \pi m > \tan \psi \tag{8–14}$$

It is necessary to have at least two axial pitches in the face width to have the benefit of full helical action and its smooth transfer of the load from tooth to tooth. A convenient method of checking this concept is to compute the *face contact ratio* defined as:

$$\textit{Face Contact Ratio} = F/P_x$$

The face contact ratio is also called the *helical overlap*. Then we can check a given design to ensure that

$$F/P_x > 2.0$$

Diametral Pitch, P_d. *Diametral pitch* is the ratio of the number of teeth in the gear to the pitch diameter. This is the same definition as the one for spur gears; it applies in considerations of the form of the teeth in the diametral or transverse plane. Thus, this pitch is sometimes called the *transverse diametral pitch:*

➪ Diametral Pitch

$$P_d = N/D \tag{8–15}$$

Normal Diametral Pitch, P_{nd}. Normal diametral pitch is the equivalent diametral pitch in the plane normal to the teeth:

➪ Normal Diametral Pitch

$$P_{nd} = P_d/\cos\psi \tag{8–16}$$

It is helpful to remember these relationships:

$$P_d p = \pi \tag{8–17}$$

$$P_{nd} p_n = \pi \tag{8–18}$$

Metric Module, m. As stated for spur gears, the metric module is essentially the inverse of the diametral pitch with the value reported in mm.

➪ Metric Module

$$m = D/N$$

This applies in the transverse plane of the gear.

Normal Metric Module, m_n. This is the inverse of the normal diametral pitch with the value reported in mm. It is the module in the plane normal to the gear tooth.

➪ Normal Metric Module

$$m_n = 1/P_{nd} = 1/(P_d/\cos \psi) = \cos \psi/P_d$$

$$m_n = \cos \psi/(N/D) = D \cos \psi/N = m \cos \psi \tag{8–19}$$

The use of Equations (8–11) through (8–16) is now illustrated in the following example problem.

Example Problem 8–2

A helical gear has a transverse diametral pitch of 12, a transverse pressure angle of $14\frac{1}{2}°$, 28 teeth, a face width of 1.25 in, and a helix angle of 30°. Compute the transverse circular pitch, normal circular pitch, normal diametral pitch, axial pitch, pitch diameter, and the normal pressure angle. Compute the number of axial pitches in the face width.

Solution

Transverse Circular Pitch
Use Equation (8–12):

$$p_t = \pi/P_d = \pi/12 = 0.262 \text{ in}$$

Normal Circular Pitch
Use Equation (8–13):

$$p_n = p_t \cos\psi = (0.262)\cos(30) = 0.227 \text{ in}$$

Normal Diametral Pitch
Use Equation (8–16)

$$P_{nd} = P_d/\cos\psi = 12/\cos(30) = 13.856$$

Axial Pitch
Use Equation (8–14):

$$p_x = p_t/\tan\psi = 0.262/\tan(30) = 0.453 \text{ in}$$

Pitch Diameter
Use Equation (8–15):

$$D = N/P_d = 28/12 = 2.333 \text{ in}$$

Normal Pressure Angle
Use Equation (8–11):

$$\phi_n = \tan^{-1}(\tan\phi_t \cos\psi)$$

$$\phi_n = \tan^{-1}[\tan(14\tfrac{1}{2})\cos(30)] = 12.62°$$

Number of Axial Pitches in the Face Width

$$F/p_x = 1.25/0.453 = 2.76 \text{ pitches}$$

Since this is greater than 2.0, there will be full helical action.

8–8 BEVEL GEAR GEOMETRY

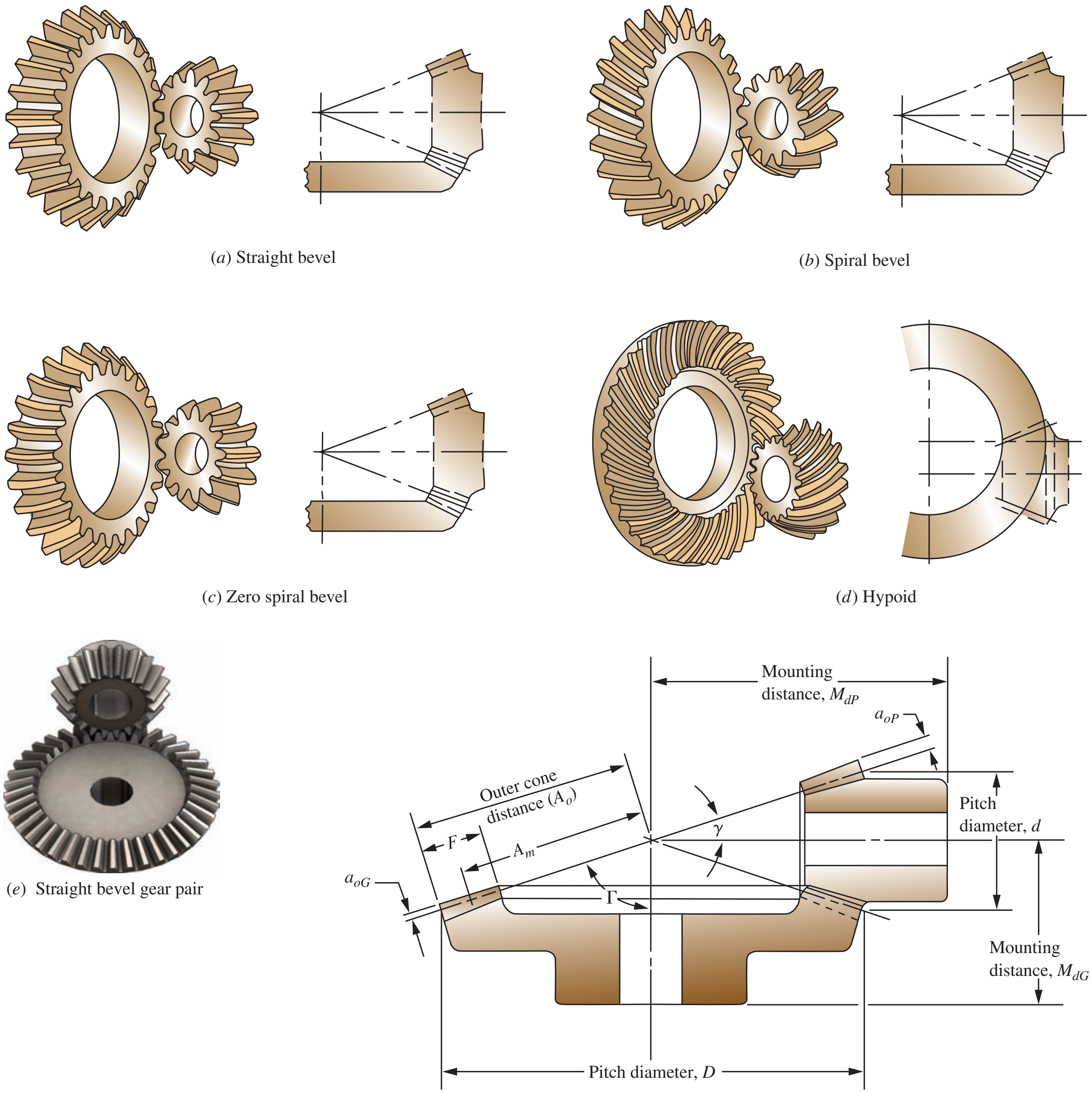
Bevel gears are used to transfer motion between nonparallel shafts, usually at 90° to one another. The four primary styles of bevel gears are straight bevel, spiral bevel, zero spiral bevel, and hypoid. Figure 8–22 shows the general appearance of these four types of bevel gear sets. The surface on which bevel gear teeth are machined is inherently a part of a cone. The differences occur in the specific shape of the teeth and in the orientation of the pinion relative to the gear. (See References 3, 5, 14, and 16.)

Straight Bevel Gears

The teeth of a straight bevel gear are straight and lie along an element of the conical surface. See Figure 8–22 (a), (e) and (f). Lines along the face of the teeth through the pitch circle meet at the apex of the pitch cone. As shown in Figure 8–22(f), the centerlines of both the pinion and the gear also meet at this apex. In the standard configuration, the teeth are tapered toward the center of the cone.

Key dimensions are specified either at the outer end of the teeth or at the mean, midface position. The relationships that control some of these dimensions are listed in Table 8–8 for the case when the shafts are at the 90° angle. The pitch cone angles for the pinion and the gear are determined by the ratio of the number of teeth, as shown in the table. Note that their sum is 90°. Also, for a pair of bevel gears having a ratio of unity, each has a pitch cone angle of 45°. Such gears, called *miter gears,* are used simply to change the direction of the shafts in a machine drive without affecting the speed of rotation.

You should understand that many more features need to be specified before the gears can be produced. Furthermore, many successful, commercially available gears are made in some nonstandard form. For example,



(*a*) Straight bevel

(*b*) Spiral bevel

(*c*) Zero spiral bevel

(*d*) Hypoid

(*e*) Straight bevel gear pair

(*f*) Key dimensions of straight bevel gear pair

FIGURE 8–22 Types of bevel gears [Parts (*a*) through (*d*) extracted from ANSI/AGMA 2005-C96, *Design Manual for Bevel Gears,* with the permission of the publisher

the addendum of the pinion is often made longer than that of the gear. Some manufacturers modify the slope of the root of the teeth to produce a uniform depth, rather than using the standard, tapered form. Reference 5 gives many more data.

The pressure angle, ϕ, is typically 20°, but 22.5° and 25° are often used to avoid interference. The minimum number of teeth for straight bevel gears is typically 12. More is said about the design of straight bevel gears in Chapter 10.

The mounting of bevel gears is critical if satisfactory performance is to be achieved. Most commercial gears have a defined mounting distance similar to that shown in Figure 8–22(f). It is the distance from some reference surface, typically the back of the hub of the gear, to the apex of the pitch cone. Because the pitch cones of the mating gears have coincident apexes, the mounting distance also locates the axis of the mating gear. If the gear is mounted at a distance smaller than the recommended mounting distance, the teeth will

TABLE 8–8 Geometrical Features of Straight Bevel Gears

Given Diametral pitch = $P_d = N_P/d = N_G/D$ or $m = d/N_P = D/N_G$

where N_P = number of teeth in pinion

N_G = number of teeth in gear

Dimension	Formula
Gear ratio	$m_G = N_G/N_P$
Pitch diameters:	
Pinion	$d = N_P/P_d$ or $d = mN_P$
Gear	$D = N_G/P_d$ or $D = mN_G$
Pitch cone angles:	
Pinion	$\gamma = \tan^{-1}(N_P/N_G)$ (lowercase Greek *gamma*)
Gear	$\Gamma = \tan^{-1}(N_G/N_P)$ (uppercase Greek *gamma*)
Outer cone distance	$A_o = 0.5D/\sin(\Gamma)$
Face width must be specified:	$F =$
Nominal face width	$F_{nom} = 0.30A_o$
Maximum face width	$F_{max} = A_o/3$ or $F_{max} = 10/P_d$ or $m/2.54$ (whichever is less)
Mean cone distance	$A_m = A_o - 0.5F$
	(*Note:* A_m is defined for the gear, also called A_{mG}.)
Mean circular pitch	$p_m = (\pi/P_d)(A_m/A_o)$ or $\pi m(A_m/A_o)$
Mean working depth	$h = (2.00/P_d)(A_m/A_o)$ or $2.00\ m(A_m/A_o)$
Clearance	$c = 0.125h$
Mean whole depth	$h_m = h + c$
Mean addendum factor	$c_1 = 0.210 + 0.290/(m_G)^2$
Gear mean addendum	$a_G = c_1 h$
Pinion mean addendum	$a_P = h - a_G$
Gear mean dedendum	$b_G = h_m - a_G$
Pinion mean dedendum	$b_P = h_m - a_P$
Gear dedendum angle	$\delta_G = \tan^{-1}(b_G/A_{mG})$
Pinion dedendum angle	$\delta_P = \tan^{-1}(b_P/A_{mG})$
Gear outer addendum	$a_{oG} = a_G + 0.5F \tan \delta_P$
Pinion outer addendum	$a_{oP} = a_P + 0.5F \tan \delta_G$
Gear outside diameter	$D_o = D + 2a_{oG} \cos \Gamma$
Pinion outside diameter	$d_o = d + 2a_{oP} \cos \gamma$

likely bind. If it is mounted at a greater distance, there will be excessive backlash, causing noisy and rough operation.

Spiral Bevel Gears

The teeth of a spiral bevel gear are curved and sloped with respect to the surface of the pitch cone. See Figure 8–22(b). Spiral angles, ψ, of 20° to 45° are used, with 35° being typical. Contact starts at one end of the teeth and moves along the tooth to its end. For a given tooth form and number of teeth, more teeth are in contact for spiral bevel gears than for straight bevel gears. The gradual transfer of loads and the greater average number of teeth in contact make spiral bevel gears smoother and allow smaller designs than for typical straight bevel gears. Recall that similar advantages were described for a helical gear relative to a spur gear.

The pressure angle, ϕ, is typically 20° for spiral bevel gears, and the minimum number of teeth is typically 12 to avoid interference. But nonstandard spiral gears allow as few as five teeth in the pinion of high-ratio sets if the tips of the teeth are trimmed to avoid interference. The rather high average number of teeth in contact (high contact ratio) for spiral gears makes this approach acceptable and can result in a very compact design. Reference 5 gives the relationships for computing the geometric features of spiral bevel gears that are extensions of those given in Table 8–8.

Zero Spiral Bevel Gears

The teeth of a zero spiral bevel gear are curved somewhat as in a spiral bevel gear, but the spiral angle is zero. See Figure 8–22(c). These gears can be used in the same mountings as straight bevel gears, but they operate more smoothly. They are sometimes called ZEROL® bevel gears.

Hypoid Gears

The major difference between hypoid gears and the others just described is that the centerline of the pinion for a set of hypoid gears is offset either above or below the centerline of the gear. See Figure 8–22(d). The teeth are designed specially for each combination of offset distance and spiral angle of the teeth. A major advantage is the more compact design that results, particularly when applied to vehicle drive trains and machine tools. (See References 5, 14, and 16 for more data.)

The hypoid gear geometry is the most general form of bevel gearing, and the others are special cases. The hypoid gear has an offset axis for the pinion, and its curved teeth are cut at a spiral angle. Then the spiral bevel gear is a hypoid gear with a zero offset distance. A ZEROL® bevel gear is a hypoid gear with a zero offset and a zero spiral angle. A straight bevel gear is a hypoid gear with a zero offset, a zero spiral angle, and straight teeth.

Example Problem 8–3

Compute the values for the geometrical features listed in Table 8–8 for a pair of straight bevel gears having a diametral pitch of 8, a 20° pressure angle, 16 teeth in the pinion, and 48 teeth in the gear. Specify a suitable face width. The shafts are at 90°.

Solution

Given: $P_d = 8$; $N_p = 16$; $N_G = 48$.

Computed Values

Gear Ratio

$$m_G = N_G/N_P = 48/16 = 3.000$$

Pitch Diameter

For the pinion,

$$d = N_P/P_d = 16/8 = 2.000 \text{ in}$$

For the gear,

$$D = N_G/P_d = 48/8 = 6.000 \text{ in}$$

Pitch Cone Angles

For the pinion,

$$\gamma = \tan^{-1}(N_P/N_G) = \tan^{-1}(16/48) = 18.43°$$

For the gear,

$$\Gamma = \tan^{-1}(N_G/N_P) = \tan^{-1}(48/16) = 71.57°$$

Outer Cone Distance

$$A_o = 0.5\ D/\sin(\Gamma) = 0.5(6.00 \text{ in})/\sin(71.57°) = 3.162 \text{ in}$$

Face Width

The face width must be specified based on the following guidelines:

Nominal face width:

$$F_{nom} = 0.30A_o = 0.30(3.162 \text{ in}) = 0.949 \text{ in}$$

Maximum face width:

$$F_{max} = A_o/3 = (3.162 \text{ in})/3 = 1.054 \text{ in}$$

or

$$F_{max} = 10/P_d = 10/8 = 1.25 \text{ in}$$

Therefore the face width should be in the range from 0.949 in to 1.054 in. Let's specify $F = 1.000$ in.

Mean Cone Distance

$$A_m = A_{mG} = A_o - 0.5F = 3.162 \text{ in} - 0.5(1.00 \text{ in}) = 2.662 \text{ in}$$

Ratio $A_m/A_o = 2.662/3.162 = 0.842$ (This ratio occurs in several following calculations.)

Mean Circular Pitch

$$p_m = (\pi/P_d)(A_m/A_o) = (\pi/8)(0.842) = 0.331 \text{ in}$$

Mean Working Depth

$$h = (2.00/P_d)(A_m/A_o) = (2.00/8)(0.842) = 0.210 \text{ in}$$

Clearance

$$c = 0.125h = 0.125(0.210 \text{ in}) = 0.026 \text{ in}$$

Mean Whole Depth

$$h_m = h + c = 0.210 \text{ in} + 0.026 \text{ in} = 0.236 \text{ in}$$

Mean Addendum Factor

$$c_1 = 0.210 + 0.290/(m_G)^2 = 0.210 + 0.290/(3.00)^2 = 0.242$$

Gear Mean Addendum

$$a_G = c_1 h = (0.242)(0.210 \text{ in}) = 0.051 \text{ in}$$

Pinion Mean Addendum

$$a_p = h - a_G = 0.210 \text{ in} - 0.051 \text{ in} = 0.159 \text{ in}$$

Gear Mean Dedendum

$$b_G = h_m - a_G = 0.236 \text{ in} - 0.051 \text{ in} = 0.185 \text{ in}$$

Pinion Mean Dedendum

$$b_P = h_m - a_P = 0.236 \text{ in} - 0.159 \text{ in} = 0.077 \text{ in}$$

Gear Dedendum Angle

$$\delta_G = \tan^{-1}(b_G/A_{mG}) = \tan^{-1}(0.185/2.662) = 3.975°$$

Pinion Dedendum Angle

$$\delta_P = \tan^{-1}(b_P/A_{mG}) = \tan^{-1}(0.077/2.662) = 1.657°$$

Gear Outer Addendum

$$a_oG = a_G + 0.5F\tan\delta_P$$
$$a_{oG} = (0.051 \text{ in}) + (0.5)(1.00 \text{ in})\tan(1.657°) = 0.0655 \text{ in}$$

Pinion Outer Addendum

$$a_oP = a_P + 0.5F\tan\delta_G$$
$$a_oP = (0.159 \text{ in}) + (0.5)(1.00 \text{ in})\tan(3.975°) = 0.1937 \text{ in}$$

Gear Outside Diameter

$$D_o = D + 2a_{oG}\cos\Gamma$$
$$D_o = 6.000 \text{ in} + 2(0.0655 \text{ in})\cos(71.57°) = 6.041 \text{ in}$$

Pinion Outside Diameter

$$d_o = d + 2a_{oP}\cos\gamma$$
$$d_o = 2.000 \text{ in} + 2(0.1937 \text{ in})\cos(18.43°) = 2.368 \text{ in}$$

8–9 TYPES OF WORMGEARING

Wormgearing is used to transmit motion and power between nonintersecting shafts, usually at 90° to each other. The drive consists of a worm on the high-speed shaft which has the general appearance of a power screw thread: a cylindrical, helical thread. The worm drives a wormgear, which has an appearance similar to that of a helical gear. Figures 8–23 shows a typical worm and wormgear set. Sometimes the wormgear is referred to as a *worm wheel* or simply a *wheel* or *gear.* (See Reference 6.)

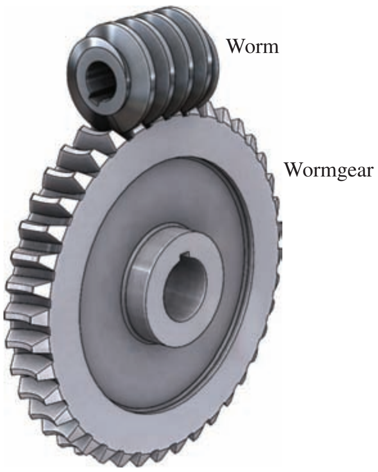


FIGURE 8–23 Worm and wormgear with a single-threaded worm

Worms and wormgears can be provided with either right hand or left hand threads on the worm and correspondingly designed teeth on the wormgear affecting the rotational direction of the wormgear.

Several variations of the geometry of wormgear drives are available. The most common one, shown in Figures 8–23 and 8–24, employs a cylindrical worm mating with a wormgear having teeth that are throated, wrapping partially around the worm. This is called a *single-enveloping type* of wormgear drive. The contact between the threads of the worm and wormgear teeth is along a line, and the power transmission capacity is quite good. Many manufacturers offer this type of wormgear set as a stocked item. Installation of the worm is relatively easy because axial alignment is not very critical. However, the wormgear must be carefully aligned radially in order to achieve the benefit of the enveloping action. Figure 8–25 shows a cutaway of a commercial wormgear reducer.

A simpler form of wormgear drive allows a special cylindrical worm to be used with a standard spur gear or helical gear. Neither the worm nor the gear must be aligned with great accuracy, and the center distance is not critical. However, the contact between the worm threads and the wormgear teeth is theoretically a point, drastically reducing the power transmission capacity of the set. Thus, this type is used mostly for nonprecision positioning applications at low speeds and low power levels.

A third type of wormgear set is the *double-enveloping type* in which the worm is made in an hourglass shape and mates with an enveloping type of wormgear. This results in area contact rather than line or point contact and allows a much smaller system to transmit a given power at a given reduction ratio. However, the worm is more difficult to manufacture, and the alignment of both the worm and the wormgear is very critical.

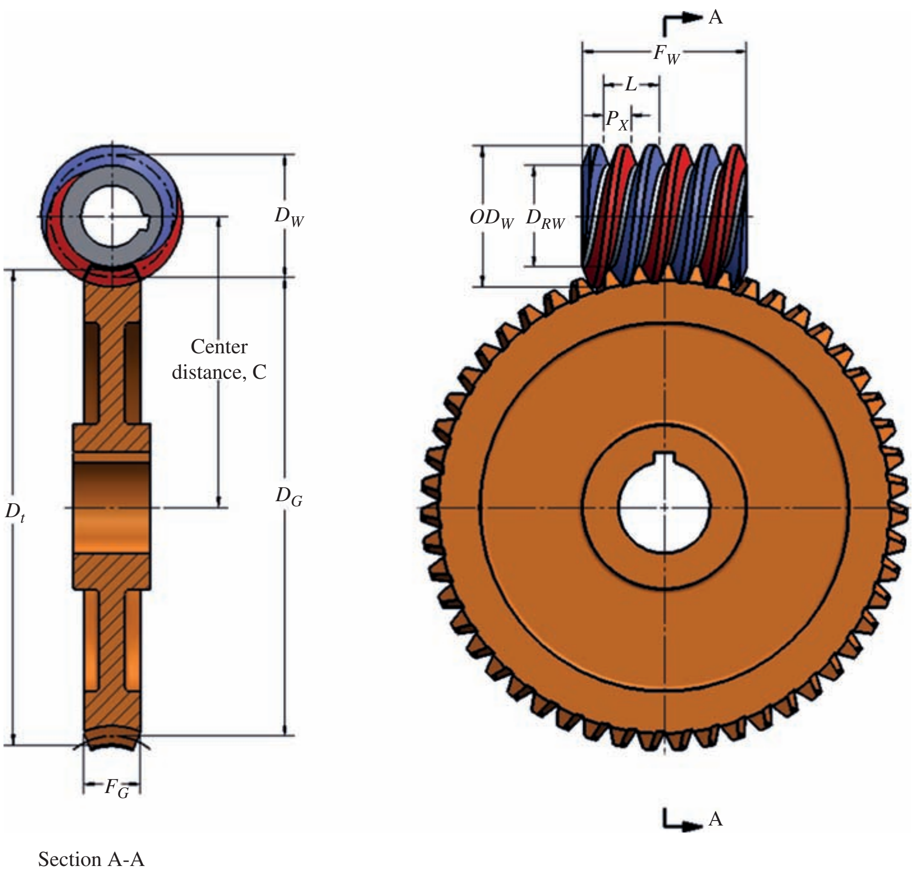


FIGURE 8–24 Single-enveloping wormgear set with a double-threaded worm

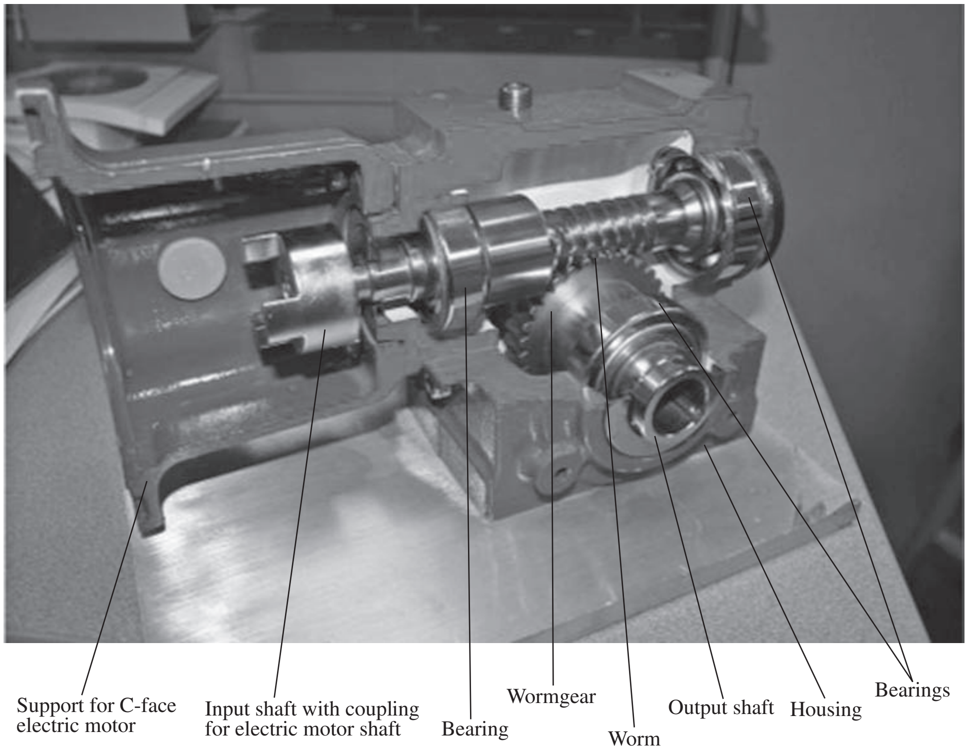


FIGURE 8–25 Cutaway view of a wormgear reducer (Baldor/Dodge, Greenville, SC)

8–10 GEOMETRY OF WORMS AND WORMGEARS

Pitches, p, P_d, and m

A basic requirement of the worm and wormgear set is that the *axial pitch* of the worm must be equal to the *circular pitch* of the wormgear in order for them to mesh. Figure 8–24 shows the basic geometric features of a single-enveloping worm and wormgear set. *Axial pitch,* P_x, is defined as the distance from a point on the worm thread to the corresponding point on the next adjacent thread, measured axially on the pitch cylinder. As before, the circular pitch is defined for the wormgear as the distance from a point on a tooth on the pitch circle of the gear to the corresponding point on the next adjacent tooth, measured along the pitch circle. Thus, the circular pitch is an arc distance that can be calculated from

➪ Circular Pitch

$$p = \pi D_G/N_G \tag{8–20}$$

where D_G = pitch diameter of the gear
N_G = number of teeth in the gear

Some wormgears are made according to the circular pitch convention. But, as noted with spur gears, commercially available wormgear sets are usually made to a diametral pitch convention with the following pitches readily available: 48, 32, 24, 16, 12, 10, 8, 6, 5, 4, and 3. See Internet sites 2, 3, 10, and 11. The diametral pitch is defined for the gear as

➪ Diametral Pitch

$$P_d = N_G/D_G \tag{8–21}$$

The conversion from diametral pitch to circular pitch can be made from the following equation:

$$P_d p = \pi \tag{8–22}$$

Metric module worms and wormgears are available commercially with modules from 0.50 to 6.00. See Internet sites 9 and 10. As for other types of gears, the module is defined for the wormgear as

$$m = D/N$$

Then the pitch of the wormgear is $p = \pi m$ and this is also the axial pitch of the worm.

Number of Worm Threads, N_W

Worms can have a single thread, as in a typical screw, or multiple threads, usually 2 or 4, but sometimes 3, 5, 6, 8, or more. It is common to refer to the number of threads as N_W and then to treat that number as if it were the number of teeth in the worm. The number of threads in the worm is frequently referred to as the number of *starts;* this is convenient because if you look at the end of a worm, you can count the number of threads that start at the end and wind down the cylindrical worm.

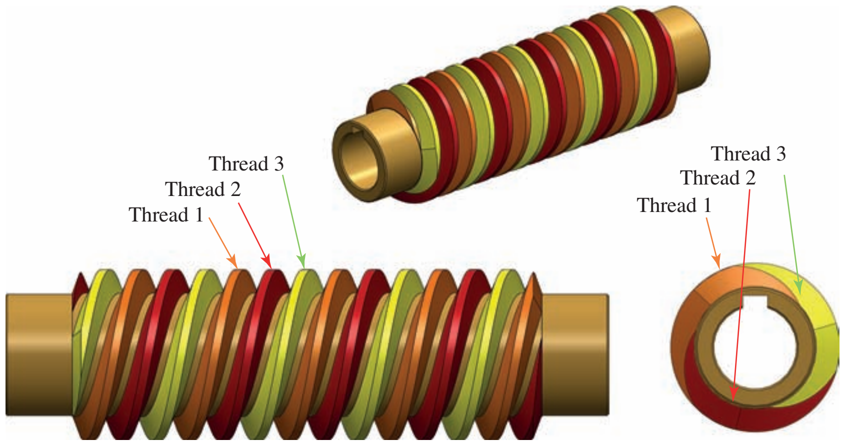


FIGURE 8–26 Worm with triple thread

Figure 8–26 shows a worm with a triple thread, with each thread shown as a different color.

Lead, L

The *lead* of a worm is the axial distance that a point on the worm would move as the worm is rotated one revolution. Lead is related to the axial pitch, P_x, by

➪ Lead

$$L = N_W P_x \tag{8–23}$$

See Figure 8–24 that shows the axial pitch and lead on a double-threaded worm.

Lead Angle, λ

The *lead angle* is the angle between the tangent to the worm thread and the line perpendicular to the axis of the worm. To visualize the method of calculating the lead angle, refer to Figure 8–27, which shows a simple triangle that would be formed if one thread of the worm were unwrapped from the pitch cylinder and laid flat on the paper. The length of the hypotenuse is the length of the thread itself. The horizontal side is the lead, L. The vertical side is the circumference of the pitch cylinder, πD_W, where D_W is the pitch diameter of the worm. Then

➪ Lead Angle

$$\tan \lambda = L/\pi D_W \tag{8–24}$$

Pitch Line Speed, v_t

As before, the pitch line speed is the linear velocity of a point on the pitch line for the worm or the wormgear. For the worm having a pitch diameter D_W in, rotating at n_W rpm,

➪ Pitch Line Speed for worm

$$v_{tW} = \frac{\pi D_W n_W}{12}\ \text{ft/min} \quad \text{or} \quad v_{tW} = \frac{\pi D_W n_W}{60000}\ m/s$$

For the wormgear having a pitch diameter D_G in, rotating at n_G rpm,

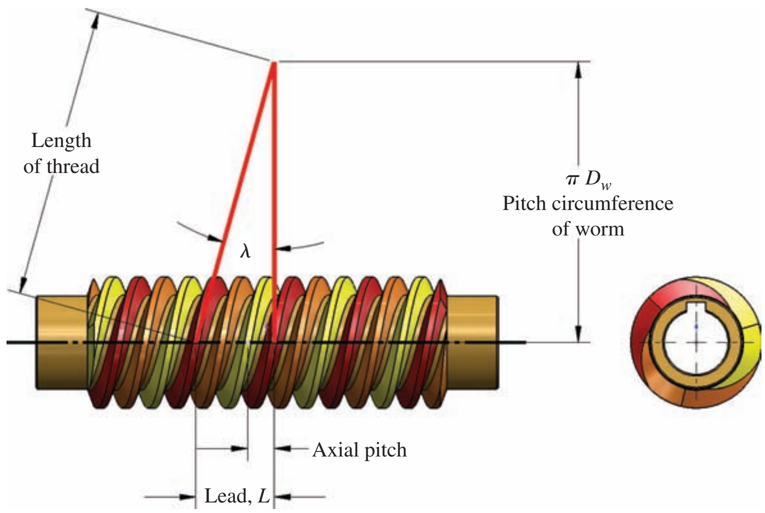


FIGURE 8–27 Lead angle

➪ Pitch Line Speed for Gear

$$v_{tG} = \frac{\pi D_G n_G}{12} \text{ ft/min} \quad \text{or} \quad v_{tG} = \frac{\pi D_G n_G}{60\,000} m/s$$

Note that these two values for pitch line speed are *not* equal.

Velocity Ratio, *VR*

It is most convenient to calculate the velocity ratio of a worm and wormgear set from the ratio of the input rotational speed to the output rotational speed:

➪ Velocity Ratio for Worm/Wormgear Set

$$VR = \frac{\text{speed of worm}}{\text{speed of gear}} = \frac{n_W}{n_G} = \frac{N_G}{N_W} \quad (8\text{–}25)$$

Typical commercially available wormgear drives have ratios of 5, 7.5, 10, 12.5, 15, 20, 25, 30, 35, 40, 45, 50, 60, and 70. Drives may have a single worm/wormgear pair or two or more pairs in series. Of course, specially designed drives can have a virtually unlimited array of ratios within limits of size and practicality.

Example Problem 8–4

A wormgear has 52 teeth and a diametral pitch of 6. It mates with a triple-threaded worm that rotates at 1750 rpm. The pitch diameter of the worm is 2.000 in. Compute the circular pitch, the axial pitch, the lead, the lead angle, the pitch diameter of the wormgear, the center distance, the velocity ratio, and the rotational speed of the wormgear. See Figure 8–28.

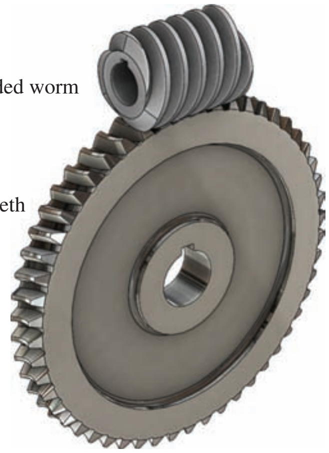


FIGURE 8–28 Worm and wormgear set for Example Problem 8-4

Solution

Circular Pitch

$$p = \pi/P_d = \pi/6 = 0.5236 \text{ in}$$

Axial Pitch

$$P_x = p = 0.5236 \text{ in}$$

Lead

$$L = N_W P_x = (3)(0.5236) = 1.5708 \text{ in}$$

Lead Angle

$$\lambda = \tan^{-1}(L/\pi D_W) = \tan^{-1}(1.5708/\pi 2.000)$$
$$\lambda = 14.04°$$

Gear Pitch Diameter

$$D_G = N_G/P_d = 52/6 = 8.667 \text{ in}$$

Center Distance

$$C = (D_W + D_G)/2 = (2.000 + 8.667)/2 = 5.333 \text{ in}$$

Velocity Ratio

$$VR = N_G/N_W = 52/3 = 17.333$$

Gear rpm

$$n_G = n_W/VR = 1750/17.333 = 101 \text{ rpm}$$

Pressure Angle

Most commercially available wormgears are made with pressure angles of $14\frac{1}{2}°$, 20°, 25°, or 30°. The low pressure angles are used with worms having a low lead angle and/or a low diametral pitch. For example, a $14\frac{1}{2}°$ pressure angle may be used for lead angles up to about 17°. For higher lead angles and with higher diametral pitches (smaller teeth), the 20° or 25° pressure angle is used to eliminate interference without excessive undercutting. The 20° pressure angle is the preferred value for lead angles up to 30°. From 30° to 45° of lead angle, the 25° pressure angle is recommended. Either the normal pressure angle, ϕ_n, or the transverse pressure angle, ϕ_t, may be specified. These are related by

➩ Pressure Angle

$$\tan \phi_n = \tan \phi_t \cos \lambda \tag{8–26}$$

Self-Locking Wormgear Sets

Self-locking is the condition in which the worm drives the wormgear, but if torque is applied to the gear shaft, the worm does not turn. It is locked! This means the wormgear cannot back-drive the worm. The locking action is produced by the friction force between the worm threads and the wormgear teeth, and this is highly dependent on the lead angle. It is recommended that a lead angle no higher than about 5.0° be used in order to ensure that self-locking will occur. This lead angle usually requires the use of a single-threaded worm. Note that the triple-threaded worm in Example Problem 8–4 has a lead angle of 14.04°. It is *not* likely to be self-locking.

Although a worm gear set may be self-locking, they are not meant to be used as a holding device. If vibration and shock are present in the system, the worm gear set could back-drive. A brake should be designed in the system if a wormgear set is required not to backdrive, especially if safety is a concern.

Typical Designs of Wormgear Sets

Considerable latitude is permissible in the design of wormgear sets because the worm and wormgear combination is designed as a unit. However, there are some guidelines as shown next.

GENERAL GUIDELINES FOR WORM AND WORMGEAR DIMENSIONS

Typical Tooth Dimensions

Table 8–9 shows typical values used for the dimensions of worm threads and gear teeth.

TABLE 8–9 Typical Tooth Dimensions for Worms and Wormgears

Dimension	Formula
Addendum	$a = 0.3183P_x = 1/P_d$
Whole depth	$h_t = 0.6866P_x = 2.157/P_d$
Working depth	$h_k = 2a = 0.6366P_x = 2/P_d$
Dedendum	$b = h_t - a = 0.3683P_x = 1.157/P_d$
Root diameter of worm	$D_{rW} = D_W - 2b$
Outside diameter of worm	$D_{OW} = D_W + 2a = D_W + h_k$
Root diameter of gear	$D_{rG} = D_G - 2b$
Throat diameter of gear	$D_t = D_G + 2a$

Worm Diameter

The diameter of the worm affects the lead angle, which in turn affects the efficiency of the set. For this reason, small diameters are desirable. But for practical reasons and proper proportion with respect to the wormgear, it is recommended that the worm diameter be approximately $C^{0.875}/2.2$, where C is the center distance in inches between the worm and the wormgear. Variation of about 30% is allowed. (See Reference 6.) Thus, the worm diameter should fall in the range

➩ Wormgear Drive Center Distance-*in*

$$1.6 < \frac{C^{0.875}}{D_W} < 3.0 \tag{8–27}$$

For metric designs with dimensions in mm,

➩ Wormgear Drive Center Distance, *mm*

$$1.07 < \frac{C^{0.875}}{D_W} < 2.0 \tag{8–27M}$$

The recommended nominal worm diameter is approximately $C^{0.875}/1.54$.

But some commercially available wormgear sets fall outside this range, especially in the smaller sizes. Also, those worms designed to have a through-hole bored in them for installation on a shaft are typically larger than you would find from Equation (8–27). Proper proportion and efficient use of material should be the guide. The worm shaft must also be checked for deflection under operating loads. For worms machined integral with the shaft, the root of the worm threads determines the minimum shaft diameter. For worms having bored holes, sometimes called *shell worms,* care must be exercised to leave sufficient material between the thread root and the keyway in the bore. Figure 8–29 shows the recommended thickness above the keyway to be a minimum of one-half the whole depth of the threads.

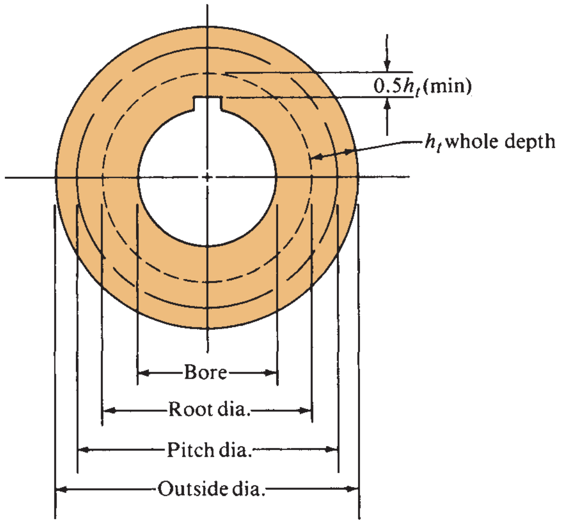


FIGURE 8–29 Shell worm

Wormgear Dimensions

We are concerned here with the single-enveloping type of wormgear, as shown in Figures 8–23 and 8–30. Its addendum, dedendum, and depth dimensions are assumed to be the same as those listed in Table 8–9, measured at the throat of the wormgear teeth. The throat is in line with the vertical centerline of the worm. The recommended face width for the wormgear is

➩ Face Width of Wormgear

$$F_G = (D_{oW}^2 - D_W^2)^{1/2} \tag{8–28}$$

This corresponds to the length of the line tangent to the pitch circle of the worm and limited by the outside diameter of the worm. Any face width beyond this value would not be effective in resisting stress or wear, but a convenient value slightly greater than the minimum should be used. The outer edges of the wormgear teeth should be chamfered approximately as shown in Figure 8–30.

Another recommendation, which is convenient for initial design, is that the face width of the gear should be approximately 2.0 times the circular pitch. Because we are working in the diametral pitch system, we will use

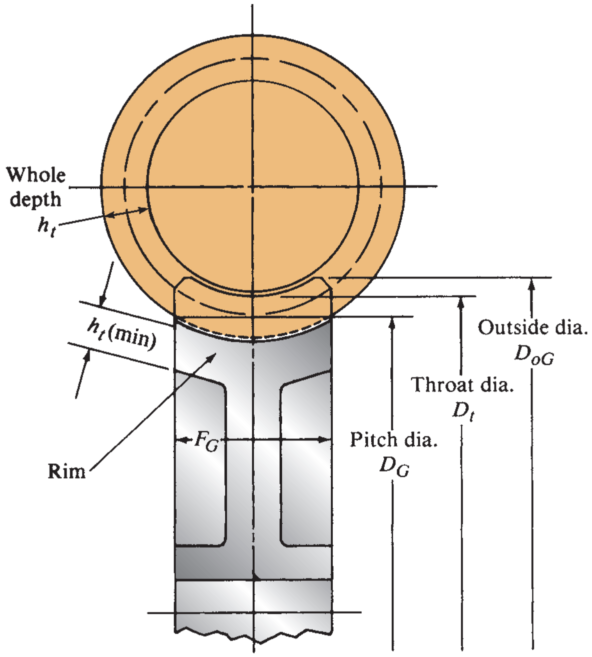


FIGURE 8–30 Wormgear details

$$F_G = 2p = 2\pi/P_d \tag{8–29}$$

However, since this is only approximate and 2π is approximately 6, we will use

$$F_G = 6/P_d \tag{8–30}$$

If the gear web is thinned, a rim thickness at least equal to the whole depth of the teeth should be left.

Face Length of the Worm

For maximum load sharing, the worm face length should extend to at least the point where the outside diameter of the worm intersects the throat diameter of the wormgear. This length is

➩ Face Length of Worm

$$F_w = 2[(D_t/2)^2 - (D_G/2 - a)^2]^{1/2} \tag{8–31}$$

Example Problem 8–5 A worm and wormgear set is to be designed to produce a velocity ratio of 40. It has been proposed that the diametral pitch of the wormgear be 8, based on the torque that must be transmitted. (This will be discussed in Chapter 10.) Using the relationships presented in this section, specify the following:

Worm diameter, D_W

Number of threads in the worm, N_W

Number of teeth in the gear, N_G

Actual center distance, C

Face width of the gear, F_G

Face length of the worm, F_W

Minimum thickness of the rim of the gear

Solution Many design decisions need to be made, and multiple solutions could satisfy the requirements. Presented here is one solution, along with comparisons with the various guidelines discussed in this section. This type of analysis precedes the stress analysis and the determination of the power-transmitting capacity of the worm and wormgear drive which is discussed in Chapter 10.

Trial Design: Let's specify a double-threaded worm: $N_W = 2$. Then there must be 80 teeth in the wormgear to achieve a velocity ratio of 40. That is,

$$VR = N_G/N_W = 80/2 = 40$$

With the known diametral pitch, $P_d = 8$, the pitch diameter of the wormgear is

$$D_G = N_G/P_d = 80/8 = 10.000 \text{ in}$$

An initial estimate for the magnitude of the center distance is approximately $C = 6.50$ in. We know that it will be greater than 5.00 in, the radius of the wormgear. Using Equation (8–27), the recommended minimum size of the worm is

$$D_W = C^{0.875}/3.0 = 1.71 \text{ in}$$

Similarly, the maximum diameter should be

$$D_W = C^{0.875}/1.6 = 3.21 \text{ in}$$

A small worm diameter is desirable. Let's specify $D_W = 2.25$ in. The actual center distance is

$$C = (D_W + D_G)/2 = 6.125 \text{ in}$$

Worm Outside Diameter

$$D_{oW} = D_W + 2a = 2.25 + 2(1/P_d) = 2.25 + 2(1/8) = 2.50 \text{ in}$$

Whole Depth

$$h_t = 2.157/P_d = 2.157/8 = 0.270 \text{ in}$$

Face Width for Gear

Let's use Equation (8–28):

$$F_G = (D_{oW}^2 - D_W^2)^{1/2} = (2.50^2 - 2.25^2)^{1/2} = 1.090 \text{ in}$$

Let's specify $F_G = 1.25$ in.

Addendum

$$a = 1/P_d = 1/8 = 0.125 \text{ in}$$

Throat Diameter of Wormgear

$$D_t = D_G + 2a = 10.000 + 2(0.125) = 10.250 \text{ in}$$

Recommended Minimum Face Length of Worm

$$F_W = 2[(D_t/2)^2 - (D_G/2 - a)^2]^{1/2} = 3.16 \text{ in}$$

Let's specify $F_W = 3.25$ in.

Minimum Thickness of the Rim of the Gear

The rim thickness should be greater than the whole depth:

$$h_t > 0.270 \text{ in}$$

8–11 GEAR MANUFACTURE

The discussion of gear manufacture will begin with the method of producing the gear blank. Small gears are frequently made from wrought plate or bar, with the hub, web, spokes, and rim machined to final or near-final dimensions before the gear teeth are produced. The face width and the outside diameter of the gear teeth are also produced at this stage. Other gear blanks may be forged, sand cast, or die cast to achieve the basic form prior to machining. A few gears in which only moderate precision is required may be die cast with the teeth in virtually final form.

Large gears are frequently fabricated from components. The rim and the portion into which the teeth are machined may be rolled into a ring shape from a flat bar and then welded. The web or spokes and the hub are then welded inside the ring. Very large gears may be made in segments with the final assembly of the segments by welding or by mechanical fasteners.

The popular methods of machining the gear teeth are form milling, shaping, and hobbing. (See References 11, 12 and 19.)

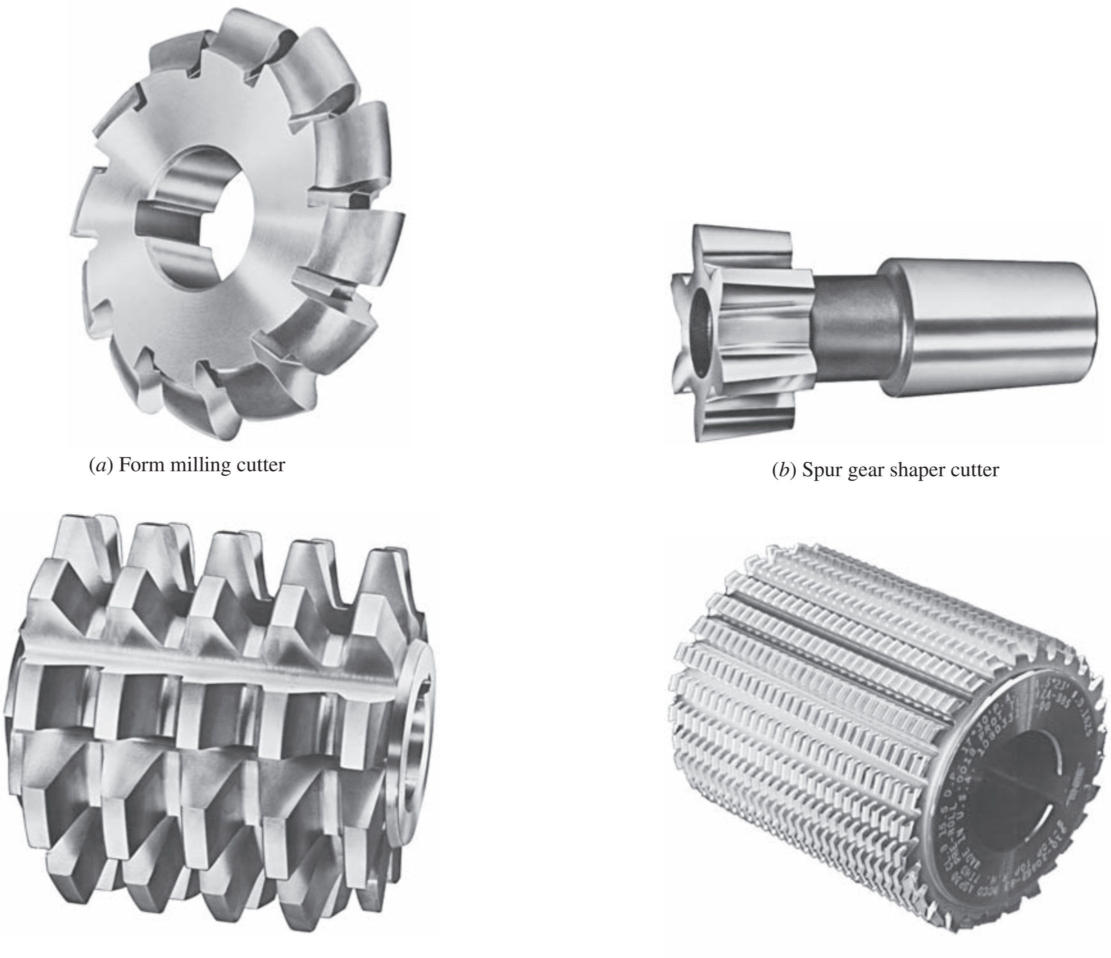
In *form milling* [Figure 8–31(a)], a milling cutter that has the shape of the tooth space is used, and each space is cut completely before the gear blank is indexed to the position of the next adjacent space. This method is used mostly for large gears, and great care is required to achieve accurate results.

Shaping [Figures 8–31(b) and 8–32] is a process in which the cutter reciprocates, usually on a vertical spindle. The shaping cutter rotates as it reciprocates and is fed into the gear blank. Thus, the involute-tooth form is generated gradually. This process is frequently used for internal gears.

Scudding is a process similar to shaping but it is much faster, and closer to hobbing in production rate. The workpiece and the scudding cutter rotate synchronously and the cutter feeds directly through the workpiece. Both internal and external gears can be produced by scudding. See Internet site 12.

Hobbing [Figures 8–31(c) and (d) and 8–33] is a process similar to milling except that the workpiece (the gear blank) and the cutter (the hob) rotate in a coordinated fashion. Here also, the tooth form is generated gradually as the hob is fed into the blank.

The gear teeth are finished to greater precision after form milling, shaping, or hobbing by the processes of grinding, shaving, and honing. Being products of secondary processes, they are expensive and should be used only where the operation requires high accuracy in the tooth form and spacing. Figure 8–34 shows a gear grinding machine.



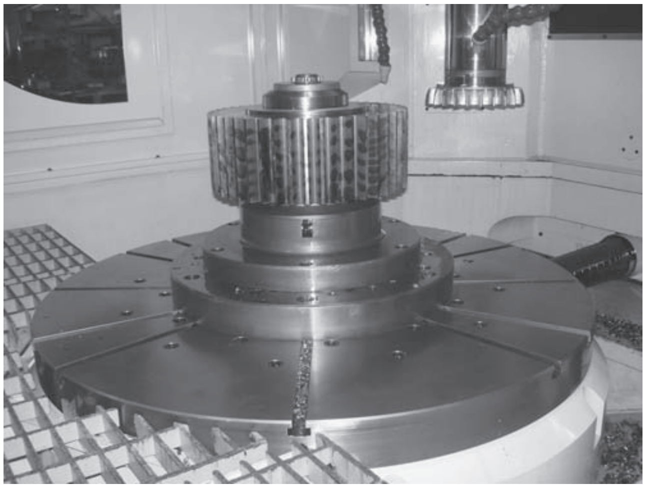
(*a*) Form milling cutter

(*b*) Spur gear shaper cutter

(*c*) Hob for small pitch gears having large teeth

(*d*) Hob for high pitch gears having small teeth

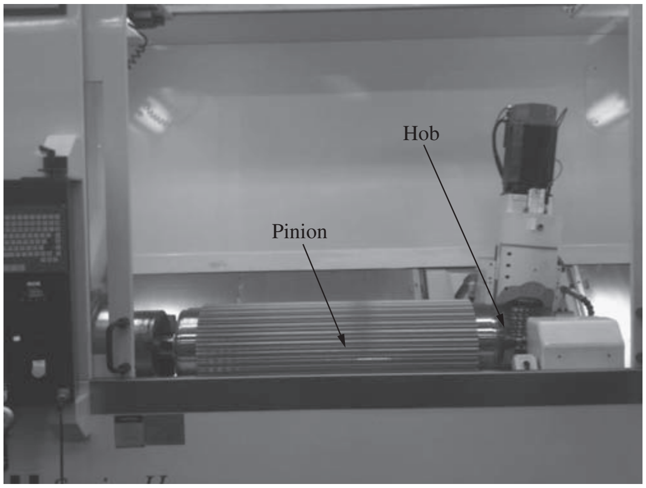
FIGURE 8–31 A variety of gear cutting tools (Courtesy of Gleason Cutting Tools Corporation)



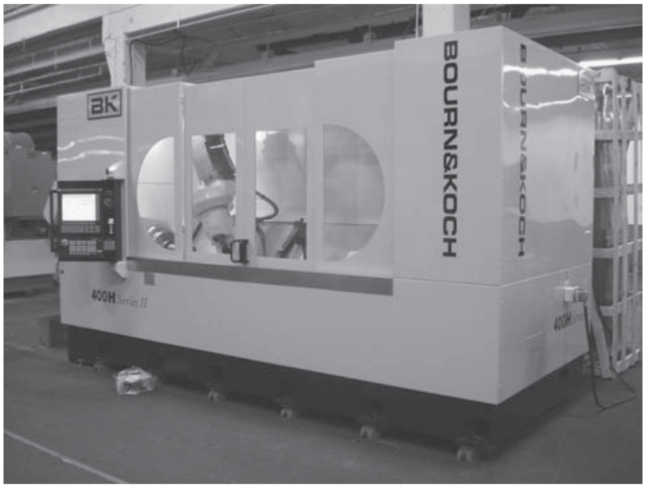
(*a*) Gear being shaped with shaping cutter

(*b*) Gear shaping machine

FIGURE 8–32 Gear shaping operation and shaping machine (Supplied by Bourn & Koch Inc)

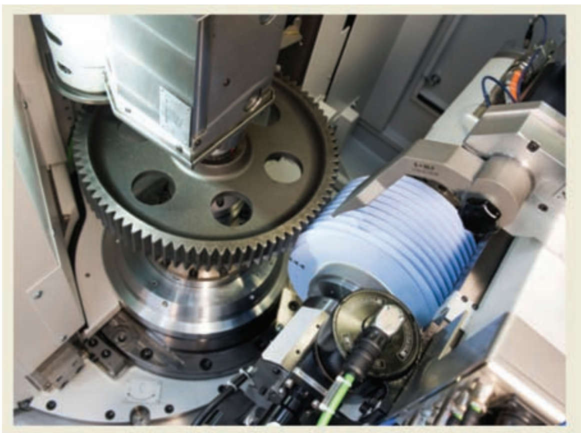


(*a*) Long pinion and a hob in a hobbing machine



(*b*) Large hobbing machine

FIGURE 8–33 Gear hobbing operation and hobbing machine (Supplied by Bourn & Koch Inc)



(*a*) Gear with grinding wheel

(*b*) Gear grinding machine

FIGURE 8–34 Gear grinding operation and grinding machine (Supplied by Bourn & Koch Inc)

8–12 GEAR QUALITY

Ensuring proper dimensional accuracy of power-transmission gears is essential to their suitability for use in machinery. Stresses in gears, smoothness of operation, life, and noise are affected by the degree of precision obtained in manufacturing. References 2, 8, and 12 provide comprehensive treatments of gear quality. Here, we give an overview of the fundamentals and equipment used to measure gear quality.

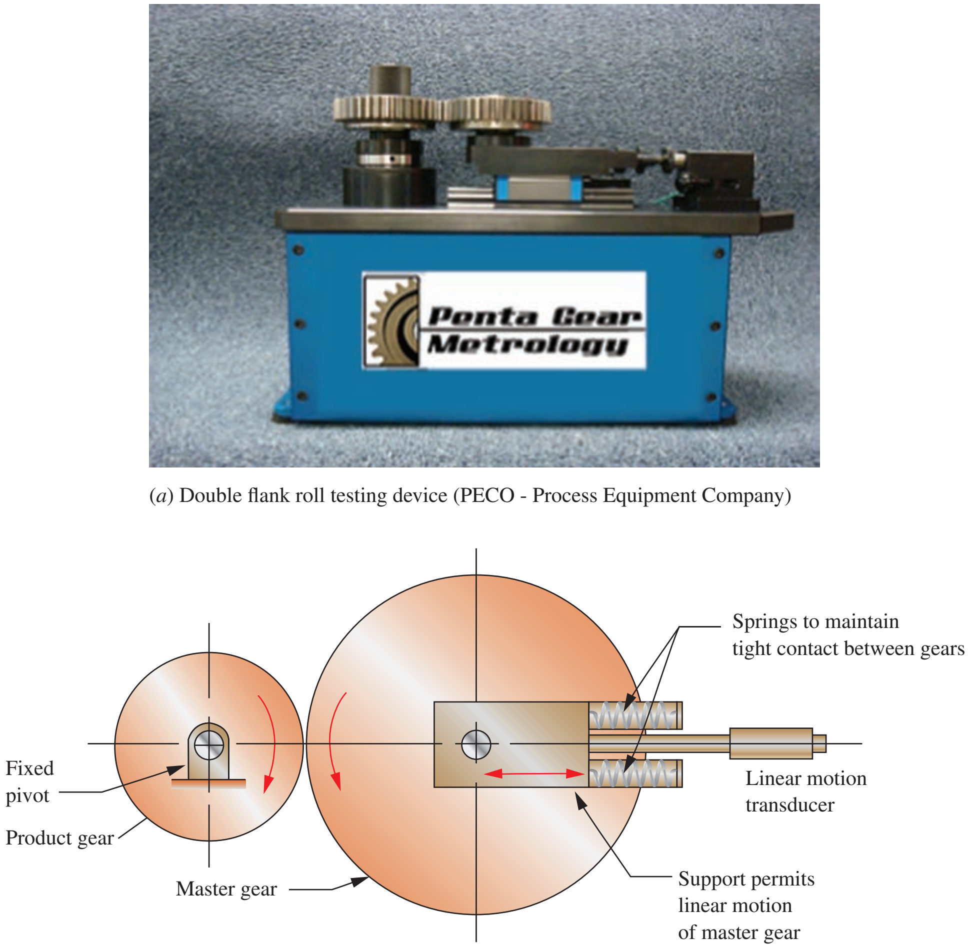
Quality in gearing is indicated by either of two methods: (1) the composite variation of a product test gear rotating in mesh with a precise master gear, called *functional measurement* or (2) the precision of specific features of a single gear, called *analytical measurement*. These two methods are described next.

Functional Measurement

Functional measurement employs a device called a *double flank roll tester* to measure the *total radial composite deviation* and the *tooth-to-tooth radial composite variation*. Figure 8–35 shows a photograph (a) of one commercially available roll tester and a schematic diagram of its construction (b). The primary features of the physical testing device are:

1. Fixed mounting of the axis for the product gear to be tested
2. Mounting for an accurate master gear on a linearly movable slide with means for maintaining full double flank contact between the two gears
3. Measurement device for indicating the movement of the axis for the master gear as the product gear rotates one complete revolution
4. Recorder to display the total excursion of the master gear during the cycle

Figure 8–36 shows an example of a test report for a gear generated by the double-flank roll tester shown in part (a). Shown are the *Total composite error*, the *Tooth to tooth error*, and the *Runout*. These data are sensitive to the amount of radial force exerted on the test gear by the master gear and that force is indicated on the test report; 13 oz in this example.



(*a*) Double flank roll testing device (PECO - Process Equipment Company)

(*b*) Schematic diagram of double flank roll tester

FIGURE 8–35 Radial composite deviation testing of gears "Penta Gear Metrology, Dayton, Ohio"

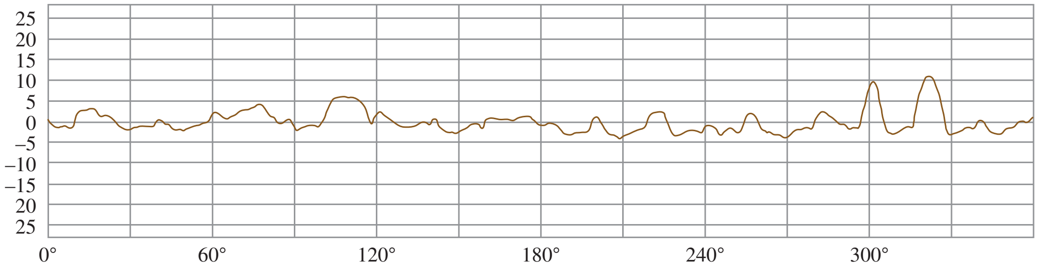
Double flank roll testing, Helical gears

gms 32

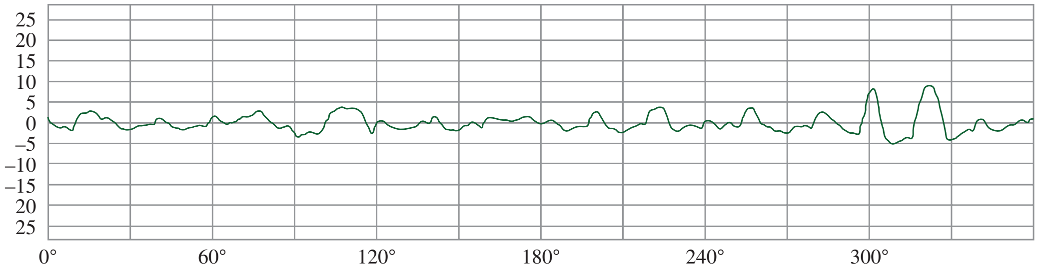
Test gear		222996	Master gear		MG 6907	Part No.	
Number of teeth	z	18	Number of teeth	z	96	Contract No.	
Diametral Pitch	DP	48	Direction of rotation		Left	Machine No.	
Pressure angle	α	14°30'00"	Runout Compensation		---	Date	
Helix angle	β	+00°00'00"	Centre distance	a"		Checked by	Nicholson
Face width	b	0.0000 inch	Measuring force		13 Oz	Note	Part No 2

Fi" - Total composite error, [0.0001 inch]

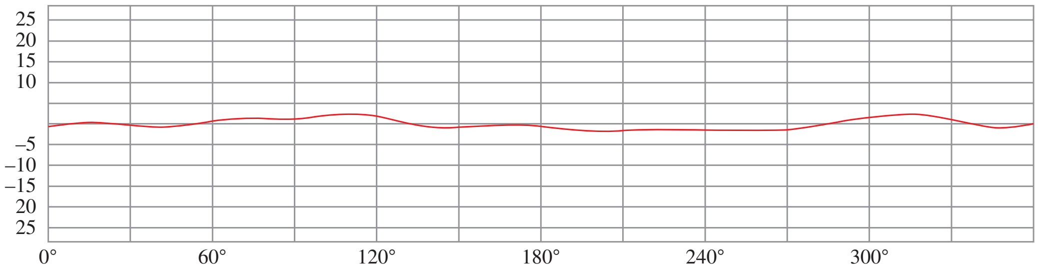
Measuring force is documented



Fi" - Tooth to tooth error, [0.0001 inch]



Fr" - Runout, [0.0001 inch]



Measuring Parameters			Allowed	Measured
Total composite error	F''_i	[inch]	0.0005	0.0015
Tooth to tooth error	f''_i	[inch]	0.0003	0.0014
Mean value	$f''_{i,m}$	[inch]		0.0006
Runout	F''_r	[inch]	0.0002	0.0004
Centre distance deviation	$A''_{ai} \div A''_{ae}$	[inch]		
Backlash	j_n	[inch]		
Measurement over teeth	M_z	[inch]		

FIGURE 8–36 Example of a report from a double flank roll test of a gear.

Analytical Measurement

Analytical measurement measures numerous features of individual teeth using a highly sensitive probe moving over the surface of the tooth under the control of a specially designed coordinate measurement machine (CMM). The computer in the complete system guides the probe over specified trajectories while accurately measuring its position. Figure 8–37(a)shows a commercially available system. Figure 8–37(b)shows a test gear in position for making the measurements using sensitive probes that can be seen at the right of the gear. The probe is guided over a prescribed path, making a complete survey of the geometry of the tooth features described next.

Figure 8–38 shows an example report giving extensive detailed information about the test gear provided by an analytical measurement system. On the left side are graphs for the *Profile* and *Helix* variations on both the right and left sides for selected teeth on the gear.

On the right side, data are reported on *Index Variation, Pitch Variation, Pitch Line Runout*, and *Tooth Thickness*.

Profile: The measurement of the actual profile of the surface of a gear tooth from the point of the start of the active profile to the tip of the tooth. The theoretical profile is a true involute curve. Variations of the actual profile from the theoretical profile cause variations in the instantaneous velocity ratio between the two gears in mesh, affecting the smoothness of the motion and noise.

Helix: The deviation of the actual line on the gear tooth surface at the pitch circle from the theoretical line. Measurements are made across the face from one end to the other. For a spur gear, the theoretical line is straight. For a helical gear, it is a part of a helix. Helix measurement is sometimes called the *tooth alignment* measurement. It is important because excessive misalignment causes nonuniform loading on the gear teeth.

Index variation: The difference between the actual location of a point on the face of a gear tooth at the pitch circle and the corresponding point of a reference tooth measured on the pitch circle. The variation causes inaccuracy in the action of mating gear teeth.

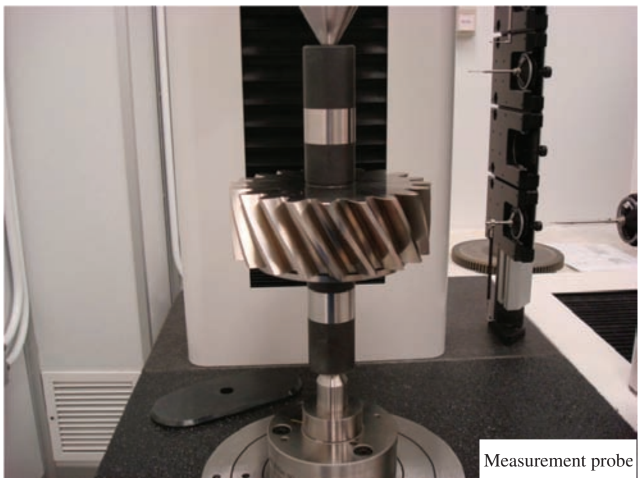
Pitch variation: The measurement of the variation of the pitch between gear teeth through a complete revolution.

Pitch line runout: A measure of the eccentricity and out of roundness of a gear. Excessive runout causes the contact point on mating gear teeth to move radially during each revolution.

Tooth thickness: A measure of the thickness of each tooth on the gear.

As the analytical measurement system completes its tests, comparisons are automatically made with the theoretical tooth forms and with tolerance values from the

(*a*) Analytical gear measurement system



(*b*) Gear installed in an analytical gear measurement system. Note probe at right.

FIGURE 8–37 Analytical measurement system for gear quality "Penta Gear Metrology, Dayton, Ohio" (PECO- Process Equipment Company)

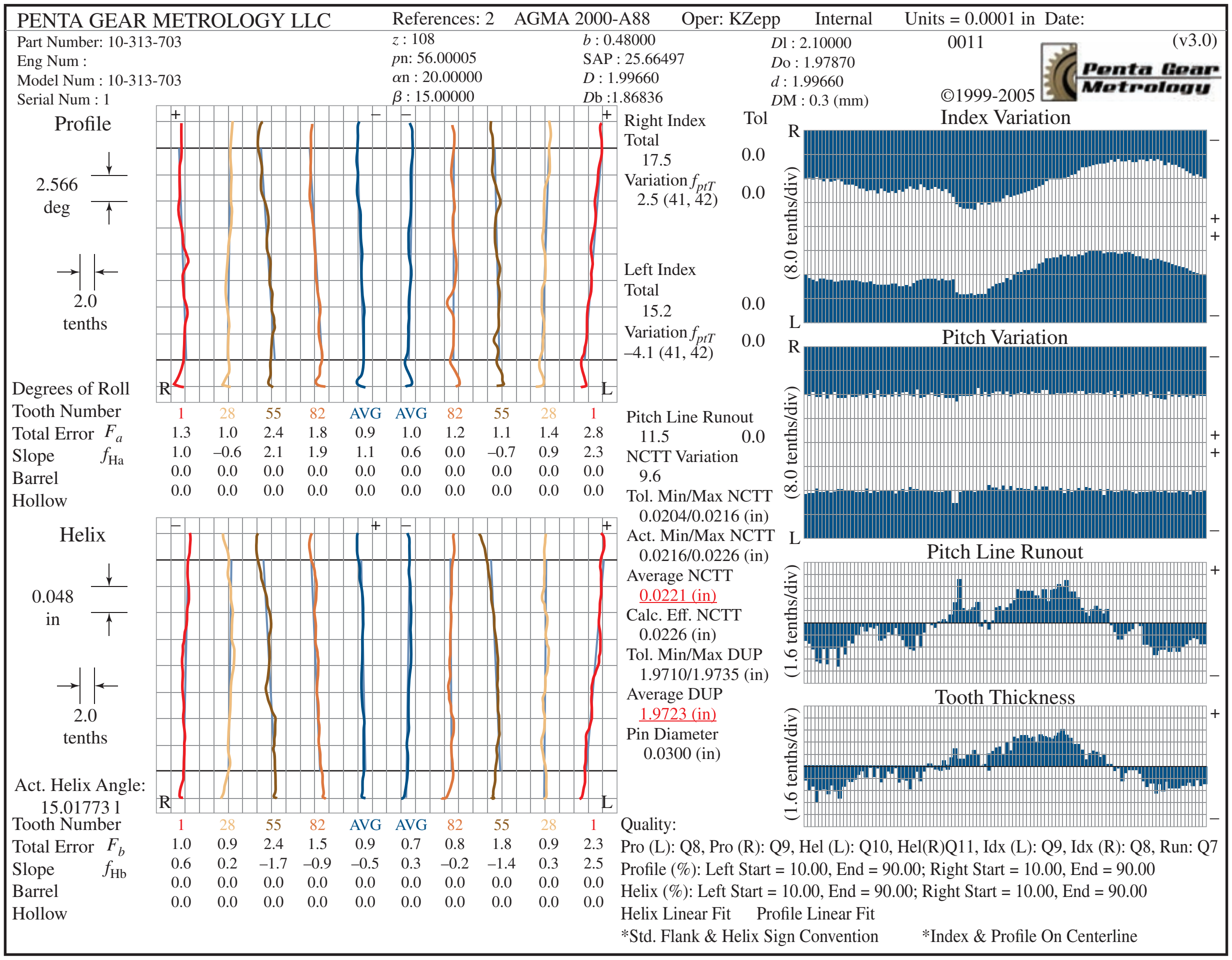


FIGURE 8–38 Example test data report for an analytical measurement of a helical gear "Penta Gear Metrology, Dayton, Ohio"

applicable standard (described in Chapter 9.) to report the resulting *quality number*. The data are also useful to manufacturing staff for making adjustments to gear cutting and grinding machine control settings to improve the accuracy of the total process.

Using the general capabilities of the analytical measurement system, dimensions of features other than those of the gear teeth may also be determined while the gear is in its fixture. For example, when a gear is machined onto a shaft, diameters, keyseats, shoulder fillets, and other geometric features may be checked for dimensions, perpendicularity, parallelism, and concentricity. Gear segments, composite gears having two or more gears on the same shaft, splines, and cam surfaces can also be measured.

8–13 VELOCITY RATIO AND GEAR TRAINS

A* gear train *is one or more pairs of gears operating together to transmit power.

Normally there is a speed change from one gear to the next due to the different sizes of the gears in mesh. The fundamental building block of the total speed change ratio in a gear train is the *velocity ratio* between two gears in a single pair.

Velocity Ratio

The* velocity ratio *(VR) is defined as the ratio of the rotational speed of the input gear to that of the output gear for a single pair of gears.

To develop the equation for computing the velocity ratio, it is helpful to view the action of two gears in mesh, as shown in Figure 8–39. The action is equivalent to the action of two smooth wheels rolling on each other without slipping, with the diameters of the two wheels equal to the pitch diameters of the two gears. Remember that when two gears are in mesh, their pitch circles are tangent, obviously, the gear teeth prohibit any slipping.

As shown in Figure 8–39, without slipping there is no relative motion between the two pitch circles at the pitch point, and therefore the tangential linear velocity of a point on either pitch circle is the same. We will use the symbol v_t for this velocity. The linear velocity of a point that is in rotation at a distance R from its center

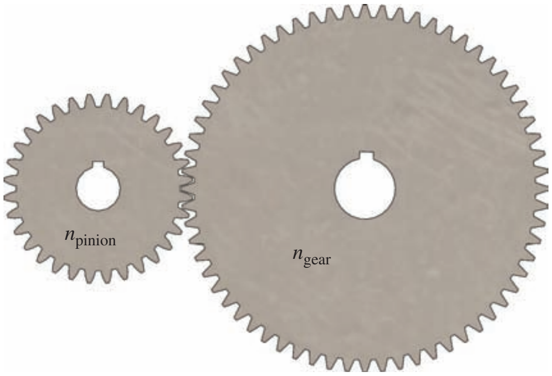


FIGURE 8–39 Two spur gears in mesh with the pinion driving the gear

of rotation and rotating with an angular velocity, ω, is found from

⇨ Pitch Line Speed of a Gear

$$v_t = R\,\omega \tag{8–32}$$

Using the subscript P for the pinion and G for the gear for two gears in mesh, we have

$$v_t = R_P\omega_P \quad \text{and} \quad v_t = R_G\omega_G$$

This set of equations says that the pitch line speeds of the pinion and the gear are the same. Equating these two and solving for ω_P/ω_G gives our definition for the velocity ratio, VR:

$$VR = \omega_P/\omega_G = R_G/R_P$$

In general, it is convenient to express the velocity ratio in terms of the pitch diameters, the rotational speeds, or the numbers of teeth of the two gears in mesh. Remember that

$$R_G = D_G/2$$
$$R_P = D_P/2$$
$$D_G = N_G/P_d$$
$$D_P = N_P/P_d$$
$$n_P = \text{rotational speed of the pinion} \quad \text{(in rpm)}$$
$$n_G = \text{rotational speed of the gear} \quad \text{(in rpm)}$$

The velocity ratio can then be defined in any of the following ways:

⇨ Velocity Ratio for Gear Pair

$$VR = \frac{\omega_P}{\omega_G} = \frac{n_P}{n_G} = \frac{R_G}{R_P} = \frac{D_G}{D_P} = \frac{N_G}{N_P} = \frac{\text{speed}_P}{\text{speed}_G} = \frac{\text{size}_G}{\text{size}_P} \tag{8–33}$$

Another useful form for velocity ratio is shown next and is used in following example problems.

$$VR = \frac{\omega_{\text{input}}}{\omega_{\text{output}}} = \frac{N_{\text{output}}}{N_{\text{input}}}$$

Most gear drives are *speed reducers;* that is, their output speed is lower than their input speed. This results in a velocity ratio greater than 1. If a *speed increaser* is desired, then VR is less than 1. Note that not all books and articles use the same definition for velocity ratio. Some define it as the ratio of the output speed to the input speed, the inverse of our definition. It is thought that the use of VR greater than 1 for the reducer—that is, the majority of the time—is more convenient.

Let's look at examples of both a speed decreaser and a speed increaser.

Example Problem 8–6

a. Figure 8–40 shows a gear pair in which a 20-tooth pinion drives a 40-tooth gear. The pinion has an angular velocity of 1000 rpm. Determine the velocity ratio and the angular velocity of the gear.

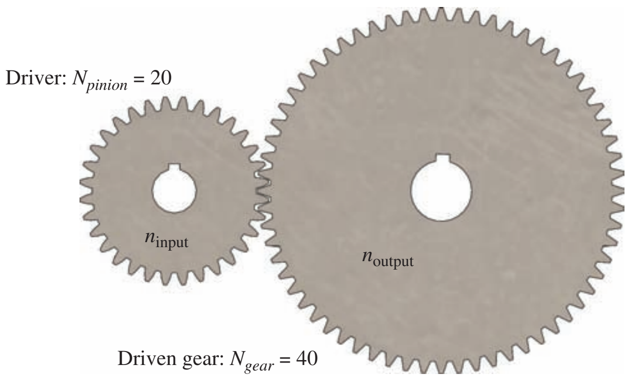


FIGURE 8–40 Speed decreaser gear drive

The pinion is the input that drives the gear. The gear is the output that is driven by the pinion. The velocity ratio is:

$$VR = \frac{\omega_{\text{input}}}{\omega_{\text{output}}} = \frac{N_{\text{output}}}{N_{\text{input}}} = \frac{40}{20} = \frac{2}{1}$$

This gear drive system is a speed reducer where the input angular velocity is twice the output angular velocity. The output angular velocity can be determined by:

$$VR = \frac{\omega_{\text{input}}}{\omega_{\text{output}}}$$

$$\omega_{\text{output}} = \frac{\omega_{\text{input}}}{VR} = \frac{1000 \text{ rpm}}{2} = 500 \text{ rpm}$$

The input angular velocity is 1000 rpm and the velocity ratio is 2 which gives us an output angular velocity of 500 rpm.

b. Let's take the gear and place it on the input drive shaft and place the pinion on the output drive shaft as shown in Figure 8–41. The 40-tooth gear is now the input and drives the 20-tooth pinion which in now the output. Determine the velocity ratio and the output angular velocity if the input shaft rotates at 1000 rpm.

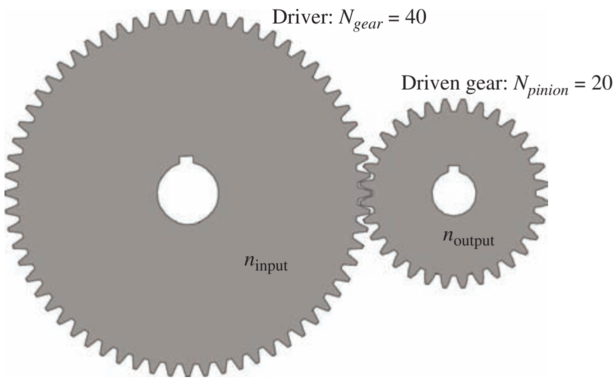


FIGURE 8–41 Speed increaser gear drive

In this example, the gear is the input and drives the pinion. The pinion is the output and is driven by the gear. The velocity ratio is:

$$VR = \frac{\omega_{\text{input}}}{\omega_{\text{output}}} = \frac{N_{\text{output}}}{N_{\text{input}}} = \frac{20}{40} = \frac{1}{2}$$

This gear drive system is a speed increaser. The output angular velocity can be determined:

$$VR = \frac{\omega_{\text{input}}}{\omega_{\text{output}}}$$

$$\omega_{\text{output}} = \frac{\omega_{\text{input}}}{VR} = \frac{1000 \text{ rpm}}{0.5} = 2000 \text{ rpm}$$

Keeping the input angular velocity at 1000 rpm, the velocity ratio is 1/2 which gives us an output angular velocity of 2000 rpm. While most gear drives are speed reducers, some gear drives are speed increasers.

Train Value

When more than two gears are in mesh, the term **train value** *(TV) refers to the ratio of the input speed (for the first gear in the train) to the output speed (for the last gear in the train). By definition the train value is the product of the values of VR for each* **gear pair** *in the train. In this definition, a gear pair is any set of two gears with a driver and a follower (driven) gear.*

Again, *TV* will be greater than 1 for a reducer and less than 1 for an increaser. For example, consider the gear train shown in Figure 8–42. The input is through the

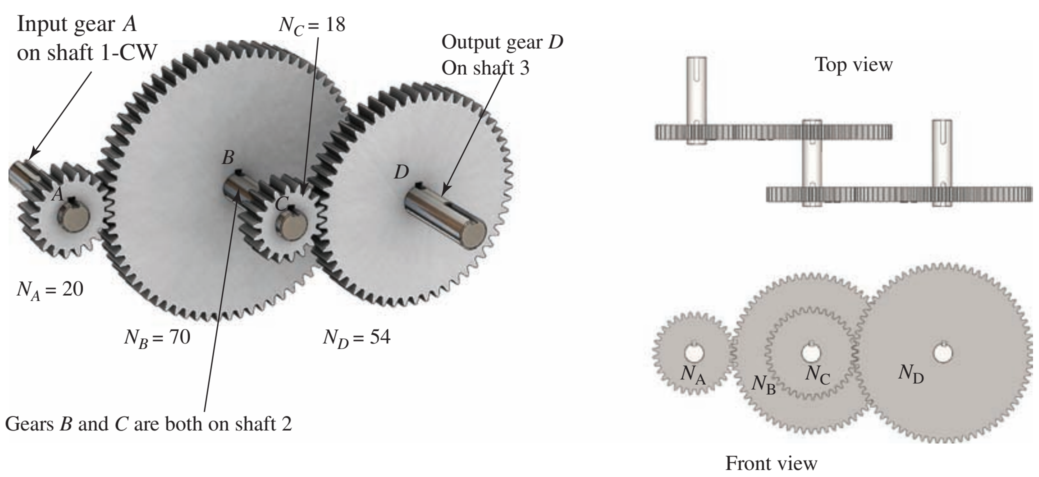


FIGURE 8–42 Double-reduction gear train

shaft carrying gear A. Gear A drives gear B. Gear C is on the same shaft with gear B and rotates at the same speed. Gear C drives gear D, which is connected to the output shaft. Then gears A and B constitute the first gear pair, and gears C and D constitute the second pair. The velocity ratios are

$$VR_1 = n_A/n_B \qquad VR_2 = n_C/n_D$$

The train value is

$$TV = (VR_1)(VR_2) = \frac{n_A\, n_C}{n_B\, n_D}$$

But, because they are on the same shaft, $n_B = n_C$, and the preceding equation reduces to

$$TV = n_A/n_D$$

This is the input speed divided by the output speed, the basic definition of the train value. This process can be expanded to any number of stages of reduction in a gear train.

Remember that any of the forms for velocity ratio shown in Equation (8–33) can be used for computing the train value. In design, it is often most convenient to express the velocity ratio in terms of the numbers of teeth in each gear because they must be integers. Then, once the diametral pitch or module is defined, the values of the diameters or radii can be determined.

The train value of the double-reduction gear train in Figure 8–42 can be expressed in terms of the numbers of teeth in the four gears as follows:

$$VR_1 = N_B/N_A$$

Note that this is the number of teeth in the *driven gear* B divided by the number of teeth in the *driving gear* A. This is the typical format for velocity ratio. Then VR_2 can be found similarly:

$$VR_2 = N_D/N_C$$

Thus, the train value is

$$TV = (VR_1)(VR_2) = (N_B/N_A)(N_D/N_C)$$

This is usually shown in the form

➩ Train Value

$$TV = \frac{N_B\, N_D}{N_A\, N_C}$$

$$TV = \frac{\text{product of number of teeth in the driven gears}}{\text{product of number of teeth in the driving gears}} \qquad \textbf{(8–34)}$$

This is the form for train value that we will use most often.

The direction of rotation can be determined by observation, noting that there is a direction reversal for each pair of external gears.

We will use the term* positive train value *to refer to one in which the input and output gears rotate in the same direction. Conversely, if they rotate in the opposite direction, the train value will be negative.

Example Problem 8–7 For the gear train shown in Figure 8–42, if the input shaft rotates at 1750 rpm clockwise, compute the speed of the output shaft and its direction of rotation.

Solution We can find the output speed if we can determine the train value:

$$TV = n_A/n_D = \text{input speed/output speed}$$

Then

$$n_D = n_A/TV$$

But

$$TV = (VR_1)(VR_2) = \frac{N_B}{N_A}\frac{N_D}{N_C} = \frac{70}{20}\frac{54}{18} = \frac{3.5}{1}\frac{3.0}{1} = \frac{10.5}{1} = 10.5$$

Now

$$n_D = n_A/TV = (1750 \text{ rpm})/10.5 = 166.7 \text{ rpm}$$

Gear *A* rotates clockwise; gear *B* rotates counterclockwise.

Gear *C* rotates counterclockwise; gear *D* rotates clockwise.

Thus, the train in Figure 8–42 is a positive train.

Idler Gear

The gear train shown in Figure 8–42 is sometimes refered to as a compound gear train. A compound gear has two gears mounted on one shaft. Gears B and C are mounted on the same shaft and would be considered compound gears. Since gears B and C are mounted on the same shaft, they also have the same angular velocity. Figure 8–43 shows a gear train that has only one gear mounted on each shaft and is referred to as a simple gear train.

The train value for the simple gear train can be expressed as the ratio of the number of teeth of the output gear to the number of teeth of the input gear. Looking at the first two gears in the drive train, gear A is the driving gear and gear B is the driven gear. The velocity ratio for these two gears in mesh is:

$$VR_1 = \frac{N_B}{N_A}$$

The next set of gears in mesh are gear B which is the driving gear and gear C which is the driven gear. The velocity ratio for this gear set is:

$$VR_2 = \frac{N_C}{N_B}$$

Similarly, the velocity ratios can be determined for each of the remaining gear sets in mesh.

$$VR_3 = \frac{N_D}{N_C}$$

$$VR_4 = \frac{N_E}{N_D}$$

Thus the train value for the simple gear train is:

$$TV = \frac{\omega_A}{\omega_E} = (VR_1)(VR_2)(VR_3)(VR_4)$$

Substituting the number of gear teeth into the equation, the train value is:

$$TV = \frac{\omega_A}{\omega_E} = \frac{N_B}{N_A}\cdot\frac{N_C}{N_B}\cdot\frac{N_D}{N_C}\cdot\frac{N_E}{N_D}$$

This equation can be reduced by cancelling the number of teeth of gears B, C, and D. This leaves the following train value:

$$TV = \frac{\omega_A}{\omega_E} = \frac{N_E}{N_A}$$

The train value or velocity ratio of the entire gear drive system is reduced to the ratio of the number of teeth on the output gear to the number of teeth on the input gear. Gears B, C, and D, do not have any effect on the ratio. These gears are classified as *idler gears*.

The example above introduces the concept of the idler gear, defined as follows:

Any gear in a gear train that performs as both a driving gear and a driven gear is called an idler gear or simply an idler.

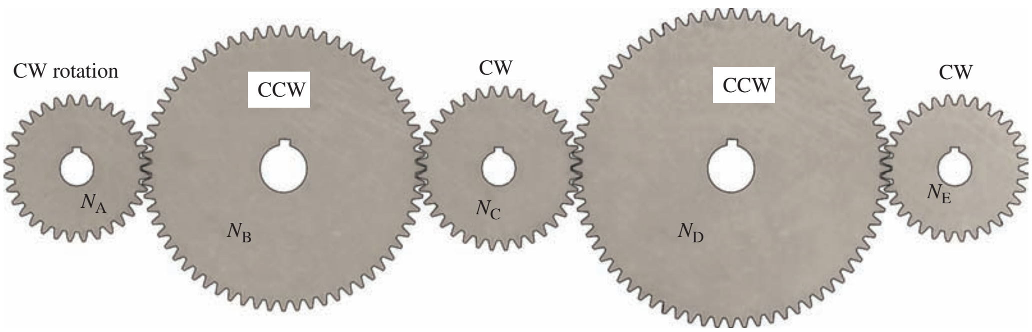


FIGURE 8–43 Simple gear train

The main features of an idler are as follows:

1. An idler does not affect the train value of a gear train because, since it is both a driver and driven gear, its number of teeth appears in both the numerator and denominator of the train value equation, Equation (8–34)
2. Placing an idler in a gear train causes a direction reversal of the output gear. In Figure 8–43, input gear A rotates clockwise and output gear E also rotates clockwise. If any one of the idler gears B, C, or D is removed, the direction of rotation of the output gear E will be counter-clockwise.
3. An idler gear may be used to fill a space between two gears in a gear train when the desired distance between their centers is greater than the center distance for the two gears alone.

Example Problem 8–8 Determine the train value for the train shown in Figure 8–44. If the shaft carrying gear *A* rotates at 1750 rpm clockwise, compute the speed and the direction of the shaft carrying gear *E.*

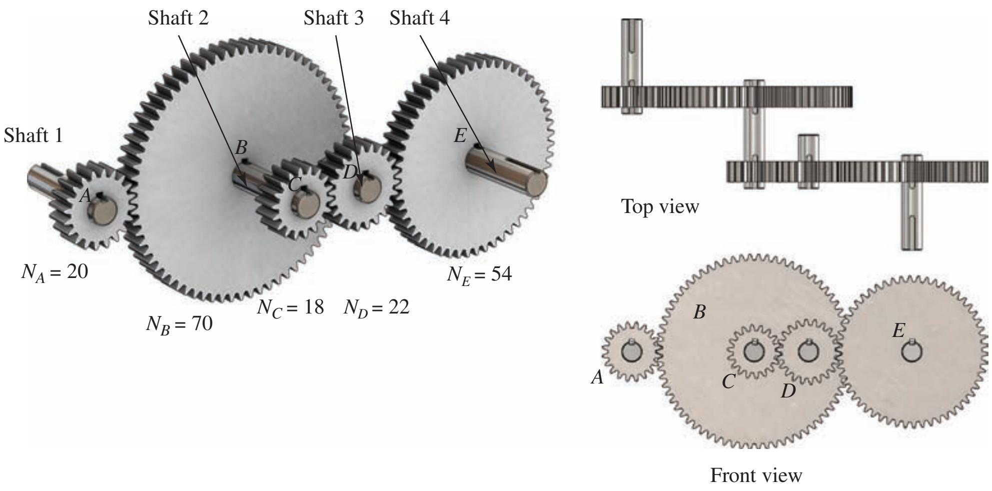


FIGURE 8–44 Double-reduction gear train with an idler. Gear *D* is an idler

Solution Look first at the direction of rotation. Remember that a gear pair is defined as any two gears in mesh (a driver and a follower). There are actually three gear pairs:

Gear *A* drives gear *B: A* rotates clockwise; *B,* counterclockwise.
Gear *C* drives gear *D: C* rotates counterclockwise; *D,* clockwise.
Gear *D* drives gear *E: D* rotates clockwise; *E,* counterclockwise.

Because gears *A* and *E* rotate in opposite directions, the train value is negative. Now

$$TV = -(VR_1)(VR_2)(VR_3)$$

In terms of the number of teeth,

$$TV = -\frac{N_B}{N_A}\frac{N_D}{N_C}\frac{N_E}{N_D}$$

Note that the number of teeth in gear *D* appears in both the numerator and the denominator and thus can be canceled. The train value then becomes

$$TV = -\frac{N_B}{N_A}\cdot\frac{N_E}{N_C} = -\frac{70}{20}\cdot\frac{54}{18} = -\frac{3.5}{1}\frac{3.0}{1} = -10.5$$

Gear *D* is called an *idler.* As demonstrated here, it has no effect on the magnitude of the train value, but it does cause a direction reversal. The output speed is then found from

$$TV = n_A/n_E$$
$$n_E = n_A/TV = (1750 \text{ rpm})/(-10.5) = -166.7 \text{ rpm (counterclockwise)}$$

Velocity of a Rack

Figure 8–45 shows the basic configuration of a *rack-and-pinion* drive. The function of such a drive is to produce a linear motion of the rack from the rotational motion of the driving pinion. The opposite is also true: If the rack is the driver having a linear motion, it produces a rotational motion of the pinion.

The linear velocity of the rack, v_R, must be the same as the pitch line velocity of the pinion, v_t, as defined by Equation (8–32), repeated here. Recall that ω_P is the angular velocity of the pinion:

$$v_R = v_t = R_P\omega_P = (D_P/2)\omega_P$$

Note on Units for Pitch Line Speed. In general, any velocity units may be used for pitch line speed at the discretion of the designer as long as careful manipulation of units is exercised. However, there are certain units that are preferred for later calculations in this book and for gear design in particular. The preferred units for pitch line speed are as follows:

U.S. Standard Diametral Pitch System: ft/min or fpm

Metric Module System: m/s

Certain data, design guidelines, and subsequent calculations are keyed to pitch line speed in these units. Because such calculations appear often in the following chapters, we develop here some unit-specific relationships to facilitate calculations in these units. Other assumptions for typical units for related quantities are included in these developments.

U.S. Standard Diametral Pitch System: Assumptions: Dimensions are in inches; Rotational speed is in rpm

$$v_t = (D/2)\omega = \frac{D(\text{in})}{2}\,\frac{n(\text{rev})}{(\text{min})}\,\frac{2\pi(\text{rad})}{(\text{rev})}\,\frac{1.0(\text{ft})}{12(\text{in})}$$

$$v_t = \frac{\pi D n}{12}\,\text{ft/min} \tag{8–35}$$

Metric Module System: Assumptions: Dimensions are in mm; Rotational speed is in rpm

$$v_t = (D/2)\omega$$

$$v_t = \frac{D\,(\text{mm})}{2}\,\frac{n\,(\text{rev})}{(\text{min})}\,\frac{2\pi\,(\text{rad})}{(\text{rev})}\,\frac{1.0\,(\text{min})}{60\,(\text{s})}\,\frac{1.0\text{ m}}{1000\text{ mm}}$$

$$v_t = \frac{\pi D n}{60\,000}\,\text{m/s} \tag{8–36}$$

Provided that data are supplied in the given units, the final form of Equations (8–35) or (8–36) can be used for problems and designs for this book. We refer to this type of equation as *unit specific*, meaning that it is only valid for input data in the proper units.

The linear displacement or position of the rack can be related to the angular displacement of the pinion, θ, by the following equation:

$$s_{\text{RACK}} = \frac{D_P}{2}\cdot\theta_P \tag{8–37}$$

The concept of center distance does not apply directly for a rack-and-pinion set because the center of the rack is at infinity. But it is critical that the pitch circle of the pinion be tangent to the pitch line of the rack as

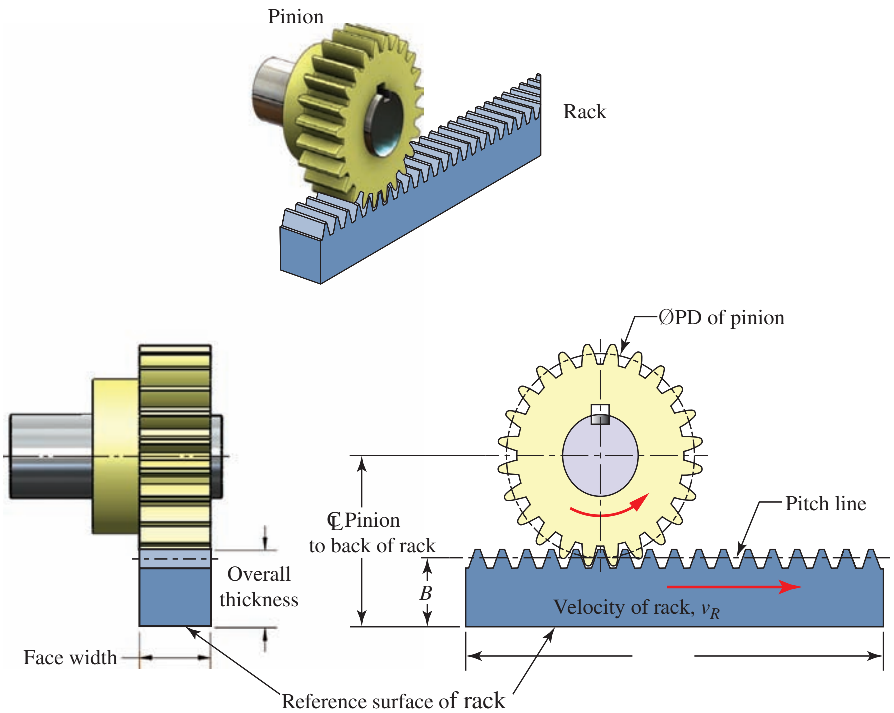


FIGURE 8–45 Rack driven by a pinion

TABLE 8–10 Example rack specifications

Diametral pitch	Pitch line to back (*B*)	Overall thickness	Face width	Nominal length [ft]
64	0.109	0.125	0.125	2
48	0.104	0.125	0.125	2
32	0.156	0.187	0.187	4
24	0.208	0.250	0.25	4
20	0.450	0.500	0.5	6
16	0.688	0.750	0.75	6
12	0.917	1.000	1	6
10	1.150	1.250	1.25	6
8	1.375	1.500	1.5	6
6	1.333	1.500	2	6
5	1.300	1.500	2.5	6
4	1.750	2.000	3.5	6

shown in Figure 8–28. The rack will be machined so that there is a specified dimension between the pitch line and a reference surface, typically the back of the rack. This is dimension B in Figure 8–28. Then the location of the center of the pinion can be computed using the relationships shown in the figure.

Table 8–10 gives examples of the basic rack dimensional information for different diametral pitches that can be used for problems in this book. This includes the pitch line to back of rack dimension, B, which is used to locate the rack from the centerline of the pinion. Data from each specific manufacturer should be used.

Example Problem 8–9 A rack is driven by pinion shown in Figure 8–46. The pinion rotates at 125 rpm, has 24 teeth, and a diametral pitch of 6.

a. Determine the pitch diameter of the pinion.
b. Determine the dimension from the pitch line to the back of the rack, B.
c. Calculate the distance of the pinion center line to the back of the rack.
d. Determine the linear velocity of the rack.
e. How long would it take to move a rack that has a length of 20 ft?
f. How many revolutions would the pinion turn in moving the rack 20 ft?

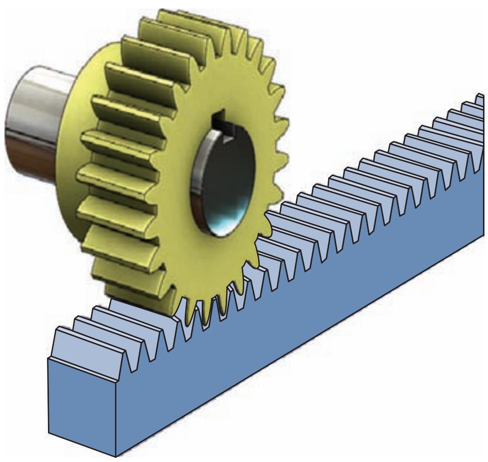


FIGURE 8–46 Pinion and rack for Example Problem 8–9

Solution Given: $n_{pinion} = 125$ rpm; Pinion $P_d = 6$; $N_{pinion} = 24$; Length of rack $= L = 20$ ft

a. $D_p = N_p/P_d = 24/6 = 4.000$ in
b. Distance from the pitch line to the back of the rack: $B = 1.333$ in [From Table 8–10]

c. Distance from back of the rack to the pinion centerline: [Call this value B-C]

$$B-C = B + \text{Pinion radius} = B + D_p/2 = 1.333\text{ in} + (4.000\text{ in})/2 = 3.333\text{ in}$$

d. Linear velocity of the rack $= V_{rack} = r\omega = (D_p/2)(n_p)$

$$V_{rack} = (2.000\text{ in})(125\text{ rev/min})(2\pi\text{ rad/rev})(1.0\text{ ft/12 in}) = 130.9\text{ ft/min} = 130.9\text{ fpm}$$

e. Time to move rack 20 ft: Using $V = s/t$, then $t = s/V$

$$t = \frac{s}{V} = \frac{20\text{ ft}}{130.9\text{ ft/min}} \cdot \frac{60\text{ sec}}{\text{min}} = 9.167\text{ sec}$$

f. Number of pinion revolutions, θ_P, to move rack 20 ft:

From Equation 8–37:

$$s_{RACK} = \frac{D_P}{2} \cdot \theta_P$$

Then

$$\theta_P = \frac{s_{RACK}}{D_P/2} = \frac{20\text{ ft}}{2.0\text{ in}} \cdot \frac{12\text{ in}}{\text{ft}} = 120\text{ rad} \cdot \frac{1\text{ rev}}{2\pi\text{ rad}} = 19.09\text{ rev}$$

8–14 DEVISING GEAR TRAINS

Now we will show several methods for devising gear trains to produce a desired train value. The result will typically be the specification of the number of teeth in each gear and the general arrangement of the gears relative to each other. The determination of the types of gears will generally not be considered except for how they may affect the direction of rotation or the general alignment of the shafts. Additional details can be specified after completion of the study of the design procedures in later chapters.

Hunting Tooth

Some designers recommend that integer velocity ratios be avoided, if possible, because the same two teeth would come into contact frequently and produce uneven wear patterns on the teeth. For example when using a velocity ratio of exactly 2.0, a given tooth on the pinion would contact the same two teeth on the gear with every two revolutions. In Chapter 9 you will learn that the pinion teeth are often made harder than the gear because the pinion experiences higher stresses. As the gears rotate, the pinion teeth tend to smooth any inherent roughness of the gear teeth, a process sometimes called *wearing in.* Each tooth on the pinion then has a slightly different geometry causing unique wear patterns on the few teeth with which it mates.

A more uniform wear pattern will result if the velocity ratio in not an integer. Adding or subtracting one tooth from the number of teeth in the gear produces the result that each pinion tooth would contact a different gear tooth with each revolution and the wear pattern would be more uniform. The added or subtracted tooth is called the *hunting tooth*. Obviously the velocity ratio for the gear pair will be slightly different, but that is often not a concern unless precise timing between the driver and driven gears is required. Consider the following example.

An initial design for a gear pair calls for the pinion to be mounted to the shaft of an electric motor having a nominal speed of 1750 rpm. The pinion has 18 teeth and the gear has 36 teeth, resulting in a velocity ratio of 36/18 or 2.000. The output speed would then be:

Initial design: $n_2 = n_1(N_P/N_G)$
$= 1750\text{ rpm }(18/36) = 875\text{ rpm}$

Now consider adding or subtracting one tooth from the gear. The output speeds would be,

Modified design: $n_2 = n_1(N_P/N_G)$
$= 1750\text{ rpm }(18/35) = 900\text{ rpm}$

Modified design: $n_2 = n_1(N_P/N_G)$
$= 1750\text{ rpm }(18/37) = 851\text{ rpm}$

The output speeds for the modified designs are less than 3.0 percent different from the original design. You would have to decide if that is acceptable in a given design project. However, be aware that the motor speed is typically not exactly 1750 rpm. As discussed in Chapter 21, 1750 rpm is a typical *full load speed* of a four-pole alternating current electric motor. When operating at a torque less than the full load torque the speed would be greater than 1750 rpm. Conversely, a greater torque would result in a slower speed. When precise speeds are required, a variable speed drive that can be adjusted according to actual loads is recommended.

A few general principles that were discussed earlier in this chapter are reviewed next.

GENERAL PRINCIPLES FOR DEVISING GEAR TRAINS

1. The velocity ratio for any pair of gears can be computed in a variety of ways as indicated in Equation (8–33).
2. The number of teeth in any gear must be an integer.
3. Mating gears must have the same tooth form, pressure angle, and pitch.
4. When external gears mesh, there is a direction reversal of their shafts.
5. When an external pinion meshes with an internal gear, their shafts rotate in the same direction.
6. An idler is a gear that performs as both a driver and a driven gear in the same train. Its size and number of teeth have no effect on the magnitude of the train value, but the direction of rotation is changed.
7. Spur and typical helical gears operate on parallel shafts.
8. Bevel gears, crossed helical gears, and a worm/wormgear set operate on shafts perpendicular to each other.
9. The number of teeth in the pinion of a gear pair should not be such that it causes interference with the teeth of its mating gear. Refer to Table 8–7.
10. In general, the number of teeth in the gear should not be larger than about 150. This is somewhat arbitrary, but it is typically more desirable to use a double-reduction gear train rather than a very large, single-reduction gear pair.
11. Any given gear train design problem may require one or more pairs of gears depending on the overall train value required, space available, and practical gear size.
12. It is generally desirable to design the gear train with the fewest practical number of gears. Each added gear requires its own shaft that must be supported by bearings.
13. In general, it is expected that the overall gear train be small in size, compact, and arranged in a manner that facilitates assembly.
14. You should determine the required directions of the driver for the gear train and for the output shaft; at least whether the input and output should rotate in the same direction (a positive train) or the opposite direction (a negative train).
15. The use of the "hunting tooth" concept described above for any gear pair is desirable. However, in this book we will consider any practical ratio for a given gear pair to be acceptable.
16. It will be shown in Chapters 9 and 10 that gear pairs operating at slower speeds will be subjected to higher torque and tooth loads as compared with those operating at higher speed. This results in the slower gear teeth having a larger tooth size and circular pitch for a given velocity ratio. For this reason, it is desirable to allocate a higher percentage of the total train value to the higher speed pairs of gears to obtain a more optimal overall design for the train.

Three different design procedures are outlined next and demonstrated in example problems. It is recommended that all three be studied and understood so you can select the method most suitable to any given design situation. Introductory comments are given here on the types of design situations where each procedure should be applied.

1. **Single pair of gears to produce a desired velocity ratio:** This is the fundamental process required to define the number of teeth in the pinion and the gear to produce a desired ratio.
2. **Residual ratio:** This process is used when two or more pairs of gears in a train are required. It calls for specifying all but one of the required ratios to produce an overall train value. Then you will be able to compute the required value of the final ratio. *This is the most general approach and the most frequently used approach in this book.*
3. **Factoring approach:** When two or more pairs of gears in a train are required and an exact ratio for the overall train value is also required, then the velocity ratio of each gear pair must be a factor of the overall train value. Determining the factors of the desired ratio is a necessary skill to apply this method.

Getting started: For any gear train design problem, first determine the minimum number of pairs of gears necessary to produce the overall train value. Here is an outline of the recommended approach.

1. Determine the overall train value, TV, required from the application data.
2. Determine the maximum velocity ratio, VR_{max}, that can be achieved with one pair of gears, considering a reasonable maximum number of teeth for the gear and a number of teeth for the pinion that will not result in interference according to Table 8–7.
3. If the train value is greater than the ratio found from Step 2, divide TV/VR_{max}. This will aid in determining how many pairs of gears are required.
4. Note that the overall train value is the product of the velocity ratio for each pair of gears. That is,

$$TV = VR_1 \times VR_2 \times VR_3 \ldots$$

5. Determine if an idler gear is necessary to achieve the required output shaft direction.
6. Specify each individual velocity ratio using the guidelines listed above.
7. Specify the numbers of teeth in each gear of each pair of gears.
8. Sketch the arrangement of the gears to show how they are placed on shafts in proper relation to each other. At this stage, a schematic diagram is adequate.

Three methods are given in Example Problems 8–10, 8–11, and 8–12 that give tools that can be used to devise gear trains for a variety of applications.

- Example Problem 8–10 shows a method that is fundamental to the process of deciding on the number of teeth in each of two mating gears that will produce a desired velocity ratio.

- Example Problem 8–11 describes the *Residual Ratio Method*, used when two or more stages of speed reduction are employed. All but one of the component ratios are decided first and then the remaining part, called the residual ratio, is computed.
- The *Factoring Approach* is used when an exact train value is desired that can be achieved by using individual stages of reduction that are factors of the overall train value.

Design of a Single Pair of Gears to Produce a Desired Velocity Ratio

Example Problem 8–10 Devise a gear train to reduce the speed of rotation of a drive from an electric motor shaft operating at 3450 rpm to approximately 650 rpm. Use $N_{max} = 150$ teeth.

Solution First we will compute the nominal train value:

$$TV = (\text{input speed})/(\text{output speed}) = 3450/650 = 5.308$$

If a single pair of gears is used then the train value is equal to the velocity ratio for that pair. That is, $TV = VR = N_G/N_P$.

Let's decide that spur gears having 20°, full-depth, involute teeth are to be used. Then we can refer to Table 8–7 and determine that no fewer than 16 teeth should be used for the pinion in order to avoid interference. We can specify the number of teeth in the pinion and use the velocity ratio to compute the number of teeth in the gear:

$$N_G = (VR)(N_P) = (5.308)(N_P)$$

All possible examples are given in Table 8–11.

Conclusion and Comments The combination of $N_P = 26$ and $N_G = 138$ gives the most ideal result for the output speed. But all of the trial values give output speeds reasonably close to the desired value. Only two are more than 2.0 rpm off the desired value. It remains a design decision as to how close the output speed must be to the stated value of 650 rpm. Note that the input speed is given as 3450 rpm, the full load speed of an electric motor. But how accurate is that? The actual speed of the input will vary depending on the load on the motor. Therefore, it is not likely that the ratio must be precise.

TABLE 8–11 All Possible Values for N_P and N_G to Produce the Desired Velocity Ratio

N_P	Computed $N_G = (5.308)(N_P)$	Nearest integer N_G	Actual VR: $VR = N_G/N_P$	Actual output speed (rpm): $n_G = n_P/VR = n_P(N_P/N_G)$
16	84.92	85	85/16 = 5.31	649.4
17	90.23	90	90/17 = 5.29	651.7
18	95.54	96	96/18 = 5.33	646.9
19	100.85	101	101/19 = 5.32	649.0
20	106.15	106	106/20 = 5.30	650.9
21	111.46	111	111/21 = 5.29	652.7
22	116.77	117	117/22 = 5.32	648.7
23	122.08	122	122/23 = 5.30	650.4
24	127.38	127	127/24 = 5.29	652.0
25	132.69	133	133/25 = 5.32	648.5
26	138.00	138	138/26 = 5.308	650.0 Exact
27	143.31	143	143/27 = 5.30	651.4
28	148.61	149	149/28 = 5.32	648.3
29	153.92	154 **Too large**		

Residual Ratio Method

Example Problem 8–11

Devise a gear train for a conveyor drive. The drive motor rotates at 1150 rpm, and it is desired that the output speed for the shaft that drives the conveyor be in the range of 24 to 28 rpm. Use a double-reduction gear train. Power transmission analysis indicates that it would be desirable for the reduction ratio for the first pair of gears to be somewhat greater than that for the second pair.

Solution

We use the *getting started* recommendations given earlier to start the solution.

Permissible Train Values

First let's compute the nominal train value that will produce an output speed of 26.0 rpm at the middle of the allowable range:

$$TV_{\text{nom}} = (\text{input speed})/(\text{nominal output speed}) = 1150/26 = 44.23$$

Now we can compute the minimum and maximum allowable speed ratio:

$$TV_{\text{min}} = (\text{input speed})/(\text{maximum output speed}) = 1150/28 = 41.07$$

$$TV_{\text{max}} = (\text{input speed})/(\text{minimum output speed}) = 1150/24 = 47.92$$

Possible Ratio for Single Pair

The maximum ratio that any one pair of gears can produce occurs when the gear has 150 teeth and the pinion has 17 teeth (see Table 8–7). Then

$$VR_{\text{max}} = N_G/N_P = 150/17 = 8.82 \text{ (too low)}$$

Possible Train Value for Double-Reduction Train

$$TV = (VR_1)(VR_2)$$

But the maximum value for either VR is 8.82. Then the maximum train value is

$$TV_{\text{max}} = (8.82)(8.82) = (8.82)^2 = 77.9$$

A double-reduction train is practical.

Optional Designs

The general layout of the proposed train is shown in Figure 8–47. Its train value is

$$TV = (VR_1)(VR_2) = (N_B/N_A)(N_D/N_C)$$

We need to specify the number of teeth in each of the four gears to achieve a train value within the range just computed. Our approach is to specify two ratios, VR_1 and VR_2, such that their product is within the desired range. If the two ratios were equal, each would be the square root of the target ratio, 44.23. That is,

$$VR_1 = VR_2 = \sqrt{44.23} = 6.65$$

But, as described in Item 16 of the *General Principles for Devising Gear Trains,* we want the first ratio to be somewhat larger than the second. Let's specify

$$VR_1 = 8.0 = (N_B/N_A)$$

If we let pinion A have 17 teeth, the number of teeth in gear B must be

$$N_B = (N_A)(8) = (17)(8) = 136$$

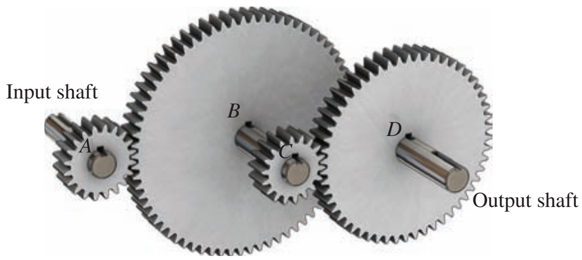


FIGURE 8–47 General layout of the proposed gear train for Example Problem 8–11

Then the second ratio should be approximately

$$VR_2 = TV/(VR_1) = 44.23/8.0 = 5.53$$

This is the *residual ratio* left after the first ratio has been specified. Now if we specify 17 teeth for pinion *C,* gear *D* must be

$$VR_2 = 5.53 = N_D/N_C = N_D/17$$

$$N_D = (5.53)(17) = 94.01$$

Rounding this off to 94 is likely to produce an acceptable result. Finally,

$$N_A = 17 \quad N_B = 136 \quad N_C = 17 \quad N_D = 94$$

We should check the final design:

$$TV = (136/17)(94/17) = 44.235 = n_A/n_D$$

The actual output speed is

$$n_D = n_A/TV = (1150 \text{ rpm})/44.235 = 26.0 \text{ rpm}$$

This is right in the middle of the desired range.

Factoring Approach for Compound Gear Trains

Example Problem 8–12

Devise a gear train for a recorder for a precision measuring instrument. The input is a shaft that rotates at exactly 3600 rpm. The output speed must be exactly 11.25 rpm. Use 20°, full-depth, involute teeth; no fewer than 17 teeth; and no more than 150 teeth in any gear.

Solution

Target TV

$$TV_{\text{nom}} = 3600/11.25 = 320$$

Maximum Single VR

$$VR_{\text{max}} = 150/17 = 8.824$$

Maximum TV for Double Reduction

$$TV_{\text{max}} = (8.824)^2 = 77.8 \text{ (too low)}$$

Maximum TV for Triple Reduction

$$TV_{\text{max}} = (8.824)^3 = 687 \text{ (okay)}$$

Design a triple-reduction gear train such as that shown in Figure 8–48. The train value is the product of the three individual velocity ratios:

$$TV = (VR_1)(VR_2)(VR_3)$$

If we can find three factors of 320 that are within the range of the possible ratio for a single pair of gears, they can be specified for each velocity ratio.

Factors of 320

One method is to divide by the smallest prime numbers that will divide evenly into the given number, typically 2, 3, 5, or 7. For example,

$$320/2 = 160$$
$$160/2 = 80$$
$$80/2 = 40$$
$$40/2 = 20$$
$$20/2 = 10$$
$$10/2 = 5$$

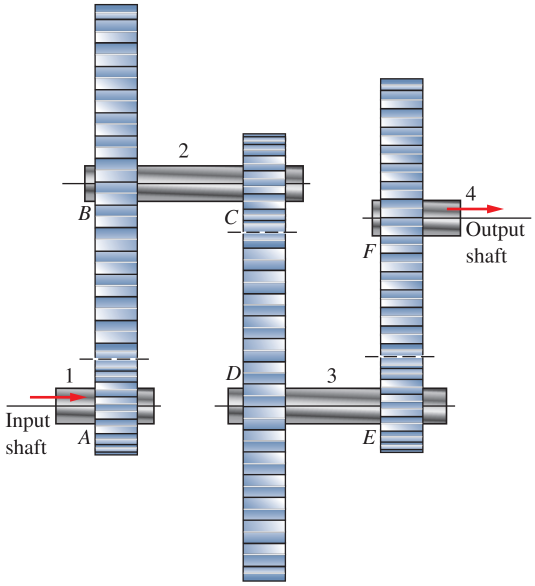


FIGURE 8–48 Triple-reduction gear train

Then the prime factors of 320 are 2, 2, 2, 2, 2, 2, and 5. We desire a set of three factors, which we can find by combining each set of three "2" factors into their product. That is,

$$(2)(2)(2) = 8$$

Then the three factors of 320 are

$$(8)(8)(5) = 320$$

Now let the number of teeth in the pinion of each pair be 17. The number of teeth in the gears will then be (8)(17) = 136 or (5)(17) = 85. Finally, we can specify

$$N_A = 17 \qquad N_C = 17 \qquad N_E = 17$$
$$N_B = 136 \qquad N_D = 136 \qquad N_F = 85$$

REFERENCES

1. American Gear Manufacturers Association. Standard 1012-G05 (R2011). *Gear Nomenclature, Definitions of Terms with Symbols*. Alexandria, VA: American Gear Manufacturers Association, 2011.
2. American Gear Manufacturers Association. Standard 2002-B88 (R2012). *Tooth Thickness Specification and Measurement*. Alexandria, VA: American Gear Manufacturers Association, 2012.
3. American Gear Manufacturers Association. Standard 2008-D11. *Assembling Bevel Gears*. Alexandria, VA: American Gear Manufacturers Association, 2011.
4. American Gear Manufacturers Association. Standard 917-B97. *Design Manual for Parallel Shaft Fine-Pitch Gearing*. Alexandria, VA: American Gear Manufacturers Association, 1997.
5. American Gear Manufacturers Association. Standard 22849-A12. *Design Recommendations for Bevel Gears*. Alexandria, VA: American Gear Manufacturers Association, 2012.
6. American Gear Manufacturers Association. Standard 6022-C93(R2014). *Design Manual for Cylindrical Wormgearing*. Alexandria, VA: American Gear Manufacturers Association, 2014.
7. American Gear Manufacturers Association. Standard 6001-E08 (R2014). *Design and Selection of Components for Enclosed Gear Drives*. Alexandria, VA: American Gear Manufacturers Association, 2014.
8. American Gear Manufacturers Association. Standard 2000-A88. *Gear Classification and Inspection Handbook—Tolerances and Measuring Methods for Unassembled Spur and Helical Gears (Including Metric Equivalents)*. Alexandria, VA: American Gear Manufacturers Association, 1988.
9. American Gear Manufacturers Association. Standard 2015/915-1-A02. *Accuracy Classification System—Tangential Measurement Tolerance Tables for Cylindrical Gears*. Alexandria, VA: American Gear Manufacturers Association, 2002.
10. Dooner, David B., and Ali A. Seireg. *The Kinematic Geometry of Gearing: A Concurrent Engineering Approach*. New York: John Wiley & Sons, 1995.

11. Drago, Raymond J. *Fundamentals of Gear Design*. New York: Elsevier Science & Technology Books, 1988.
12. Radzevich, Stephen. *Dudley's Handbook of Practical Gear Design and Manufacture,* 3rd ed. Boca Raton, FL: CRC Press, 2016.
13. International Standards Organization. Standard ISO 54:1996. *Cylindrical Gears for General Engineering and for Heavy Engineering—Modules*. Geneva, Switzerland: International Standards Organization, 1996.
14. International Standards Organization. Standard ISO 677:1976. *Straight Bevel Gears for General Engineering and Heavy Engineering*. Geneva, Switzerland: International Standards Organization, 1976.
15. International Standards Organization. Standard ISO 1122-1:1998. *Vocabulary of Gear Terms—Part 1: Definitions Related to Geometry*. Geneva, Switzerland: International Standards Organization, 1998.
16. International Standards Organization. Standard ISO 23509:2006. *Bevel and Hypoid Gear Geometry*. Geneva, Switzerland: International Standards Organization, 2006.
17. Lipp, Robert. "Avoiding Tooth Interference in Gears." *Machine Design* 54, no. 1 (January 7, 1982).
18. Litvin, Faydor, and Alfonso Fuentes. *Gear Geometry and Applied Theory,* 2nd ed. Cambridge, UK: Cambridge University Press, 2004.
19. Oberg, Erik, Franklin D. Jones, Holbrook L. Horton, and Henry H. Ryffel. *Machinery's Handbook,* 30th ed. New York: Industrial Press, 2016.

INTERNET SITES RELATED TO KINEMATICS OF GEARS

1. **American Gear Manufacturers Association (AGMA).** Develops and publishes voluntary, consensus standards for gears and gear drives. Some standards are jointly published with the American National Standards Institute (ANSI).
2. **Boston Gear Company.** A manufacturer of gears and complete gear drives. Part of Altra Industrial Motion, Inc. Data provided for spur, helical, bevel, and worm gearing.
3. **Regal-Beloit Corporation.** The Browning and Morse Divisions produce spur, helical, bevel, and worm gearing and complete gear drives.
4. **Gear Industry Home Page.** Information source for many companies that manufacture or use gears or gearing systems. Includes gear machinery, gear cutting tools, gear materials, gear drives, open gearing, tooling & supplies, software, training and education. Publishes *Gear Technology Magazine, The Journal of Gear Manufacturing*.
5. **Power Transmission Home Page.** Clearinghouse on the Internet for buyers, users, and sellers of power transmission-related products and services. Included are gears, gear drives, and gearmotors.
6. **Baldor/Dodge.** Manufacturer of many power transmission components, including complete gear-type speed reducers, bearings, and components such as belt drives, chain drives, clutches, brakes, and couplings.
7. **Gear—Wikipedia site** General discussion of the kinematics and gears, including an animated drawing of meshing gears.
8. **Gear tooth engagement animation** Search the Internet on "gears meshing animation." Several sites appear showing involute gear teeth engaging.
9. **Maryland Metrics Co.** Distributor of a wide variety of metric hardware and power transmission products, including many types of gears. On the home page, search on "Mechanical Power Transmission Products."
10. **Stock Drive Products/Sterling Instruments Co.** Distributor of a wide variety of U.S. styles and metric hardware and mechanical power transmission products, including many types of gears.
11. **W. M. Berg Co.** Distributor of a wide variety of U.S. styles of hardware and power transmission products, including many types of gears.
12. **Profilator.** Germany-based manufacturer of a variety of machine tools for the gear-cutting industry, using several processes, including scudding. U.S. based representation is by the German Machine Tools of America (GMTA).

PROBLEMS

Gear Geometry

1. A gear has 44 teeth of the 20°, full-depth, involute form and a diametral pitch of 12. Compute the following:
(a) Pitch diameter
(b) Circular pitch
(c) Equivalent module
(d) Nearest standard module
(e) Addendum
(f) Dedendum
(g) Clearance
(h) Whole depth
(i) Working depth
(j) Tooth thickness
(k) Outside diameter

Repeat Problem 1 for the following gears:

2. $N = 34; P_d = 24$
3. $N = 45; P_d = 2$
4. $N = 18; P_d = 8$
5. $N = 22; P_d = 1.75$
6. $N = 20; P_d = 64$
7. $N = 180; P_d = 80$
8. $N = 28; P_d = 18$
9. $N = 28; P_d = 20$

For Problems 10–17, repeat Problem 1 for the following gears in the metric module system. Replace Part (c) with equivalent P_d and Part (d) with nearest standard P_d.

10. $N = 34; m = 3$
11. $N = 45; m = 1.25$
12. $N = 18; m = 12$
13. $N = 22; m = 20$
14. $N = 20; m = 1$
15. $N = 180; m = 0.4$
16. $N = 28; m = 1.5$

17. $N = 28$; $m = 0.8$
18. Define *backlash,* and discuss the methods used to produce it.
19. For the gears of Problems 1 and 12, recommend the amount of backlash.

Velocity Ratio

20. An 8-pitch pinion with 18 teeth mates with a gear having 64 teeth as shown in Figure P8-20. The pinion rotates at 2450 rpm. Compute the following:
 (a) Center distance
 (b) Velocity ratio
 (c) Speed of gear
 (d) Pitch line speed

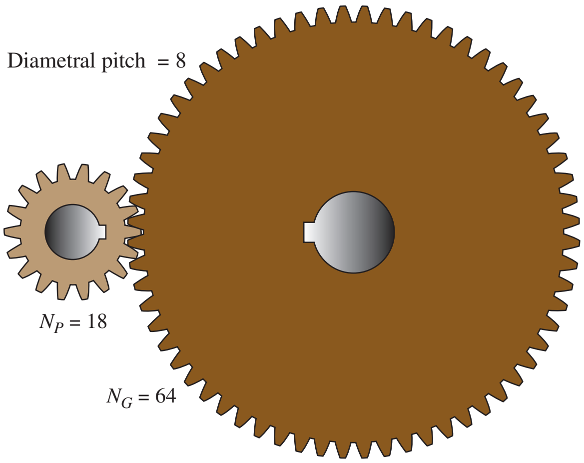


FIGURE P8–20 Gear pair for Problem 8–20

Repeat Problem 20 for the following data:

21. $P_d = 4$; $N_P = 20$; $N_G = 92$; $n_P = 225$ rpm
22. $P_d = 20$; $N_P = 30$; $N_G = 68$; $n_P = 850$ rpm
23. $P_d = 64$; $N_P = 40$; $N_G = 250$; $n_P = 3450$ rpm
24. $P_d = 12$; $N_P = 24$; $N_G = 88$; $n_P = 1750$ rpm
25. $m = 2$; $N_P = 22$; $N_G = 68$; $n_P = 1750$ rpm
26. $m = 0.8$; $N_P = 18$; $N_G = 48$; $n_P = 1150$ rpm
27. $m = 4$; $N_P = 36$; $N_G = 45$; $n_P = 15$ rpm
28. $m = 12$; $N_P = 15$; $N_G = 36$; $n_P = 480$ rpm

For Problems 29–32, all gears are made in standard 20°, full-depth, involute form. Tell what is wrong with the following statements:

29. An 8-pitch pinion having 24 teeth mates with a 10-pitch gear having 88 teeth. The pinion rotates at 1750 rpm, and the gear at approximately 477 rpm. The center distance is 5.900 in.
30. A 6-pitch pinion having 18 teeth mates with a 6-pitch gear having 82 teeth. The pinion rotates at 1750 rpm, and the gear at approximately 384 rpm. The center distance is 8.3 in.
31. A 20-pitch pinion having 12 teeth mates with a 20-pitch gear having 62 teeth. The pinion rotates at 825 rpm, and the gear at approximately 160 rpm. The center distance is 1.850 in.
32. A 16-pitch pinion having 24 teeth mates with a 16-pitch gear having 45 teeth. The outside diameter of the pinion is 1.625 in. The outside diameter of the gear is 2.938 in. The center distance is 2.281 in.

Housing Dimensions

33. The gear pair described in Problem 20 is to be installed in a rectangular housing. Specify the dimensions X and Y as sketched in Figure P8–33 that would provide a minimum clearance of 0.10 in.

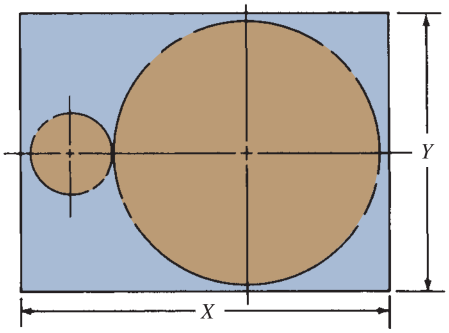


FIGURE P8–33 (Problems 33, 34, 35, and 36)

34. Repeat Problem 33 for the data of Problem 23.
35. Repeat Problem 33 for the data of Problem 26, but make the clearance 2.0 mm.
36. Repeat Problem 33 for the data of Problem 27, but make the clearance 2.0 mm.

Analysis of Simple Gear Trains: Problems 37–40

For the gear trains sketched in the given figures, compute the output speed and the direction of rotation of the output shaft if the input shaft rotates at 1750 rpm clockwise.

37. Use Figure P8–37.
38. Use Figure P8–38.
39. Use Figure P8–39.
40. Use Figure P8–40.

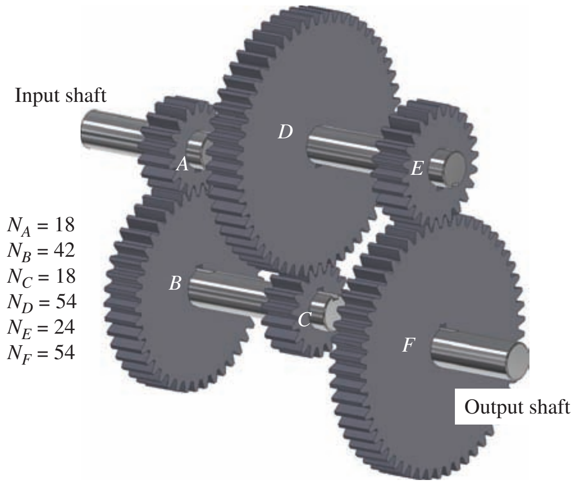


FIGURE P8–37 Gear train layout for Problem 8–37

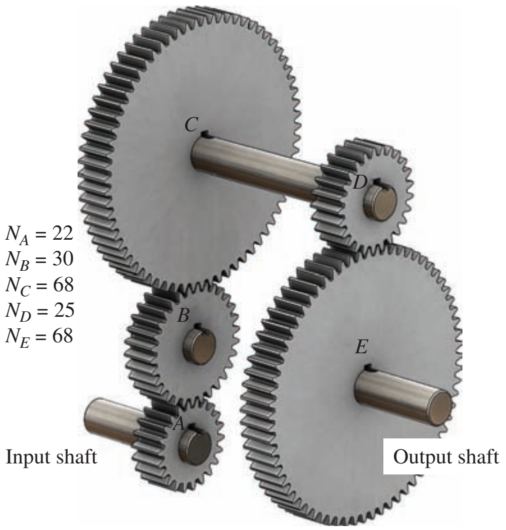


FIGURE P8–38 Gear train layout for Problem 8–38

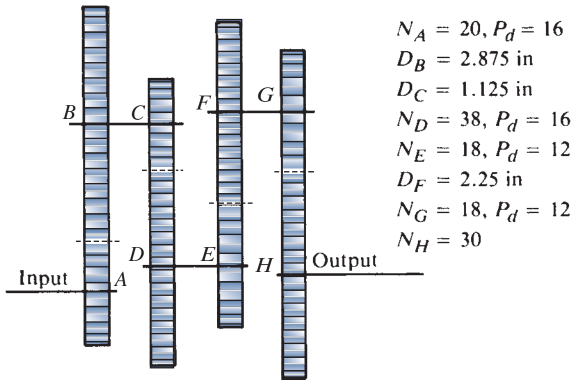


FIGURE P8–39 Gear train layout for Problem 8–39

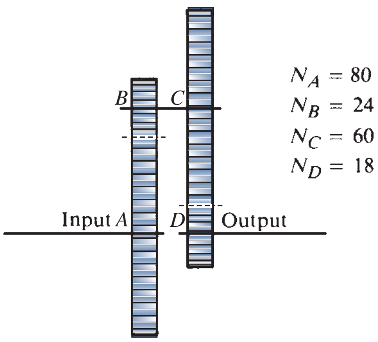


FIGURE P8–40 Gear train layout for Problem 8–40

Helical Gearing

41. A helical gear has a transverse diametral pitch of 8, a transverse pressure angle of $14\frac{1}{2}°$, 45 teeth, a face width of 2.00 in, and a helix angle of 30°. Compute the circular pitch, normal circular pitch, normal diametral pitch, axial pitch, pitch diameter, and normal pressure angle. Then compute the number of axial pitches in the face width.

42. See Figure P8–42. A helical gear has a normal diametral pitch of 12, a normal pressure angle of 20°, 48 teeth, a face width of 1.50 in, and a helix angle of 45°. Compute the circular pitch, normal circular pitch, transverse diametral pitch, axial pitch, pitch diameter, and transverse pressure angle. Then compute the number of axial pitches in the face width.

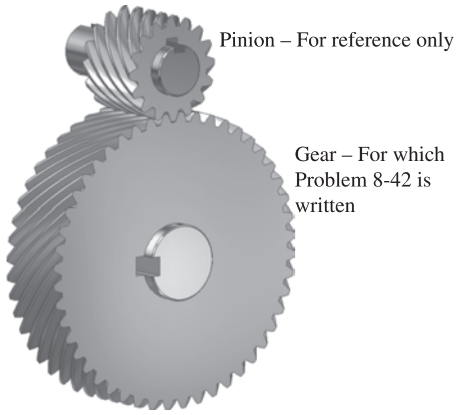


FIGURE P8–42 Helical gear for Problem 8–42

43. A helical gear has a transverse diametral pitch of 6, a transverse pressure angle of $14\frac{1}{2}°$, 36 teeth, a face width of 1.00 in, and a helix angle of 45°. Compute the circular pitch, normal circular pitch, normal diametral pitch, axial pitch, pitch diameter, and normal pressure angle. Then compute the number of axial pitches in the face width.

44. A helical gear has a normal diametral pitch of 24, a normal pressure angle of $14\frac{1}{2}°$, 72 teeth, a face width of 0.25 in, and a helix angle of 45°. Compute the circular pitch, normal circular pitch, transverse diametral pitch, axial pitch, pitch diameter, and transverse pressure angle. Then compute the number of axial pitches in the face width.

Bevel Gears

45. A straight bevel gear pair has the following data: $N_P = 15$; $N_G = 45$; $P_d = 6$; 20° pressure angle. Compute all of the geometric features from Table 8–8.

46. Draw the gear pair of Problem 45 to scale. The following additional dimensions are given (refer to Figure 8–17). Mounting distance (M_{dP}) for the pinion $= 5.250$ in; M_{dG} for the gear $= 3.000$ in; face width $= 1.250$ in. Supply any other needed dimensions.

47. A straight bevel gear pair has the following data: $N_P = 25$; $N_G = 50$; $P_d = 10$; 20° pressure angle. Compute all of the geometric features from Table 8–8.

48. Draw the gear pair of Problem 47 to scale. The following additional dimensions are given (refer to Figure 8–17). Mounting distance (M_{dP}) for the pinion $= 3.375$ in; M_{dG} for the gear $= 2.625$ in; face width $= 0.700$ in. Supply any other needed dimensions.

49. A straight bevel gear pair has the following data: $N_P = 18$; $N_G = 72$; $P_d = 12$; 20° pressure angle. Compute all of the geometric features from Table 8–8.

50. A straight bevel gear pair has the following data: $N_P = 16$; $N_G = 64$; $P_d = 32$; 20° pressure angle. Compute all of the geometric features from Table 8–8.
51. A straight bevel gear pair has the following data: $N_P = 12$; $N_G = 36$; $P_d = 48$; 20° pressure angle. Compute all of the geometric features from Table 8–8.

Wormgearing

52. A wormgear set has a single-thread worm with a pitch diameter of 1.250 in, a diametral pitch of 10, and a normal pressure angle of 14.5°. If the worm meshes with a wormgear having 40 teeth and a face width of 0.625 in, compute the lead, axial pitch, circular pitch, lead angle, addendum, dedendum, worm outside diameter, worm root diameter, gear pitch diameter, center distance, and velocity ratio.
53. Three designs are being considered for a wormgear set to produce a velocity ratio of 20 when the wormgear rotates at 90 rpm. All three have a diametral pitch of 12, a worm pitch diameter of 1.000 in, a gear face width of 0.500 in, and a normal pressure angle of 14.5°. One has a single-thread worm and 20 wormgear teeth; the second has a double-thread worm and 40 wormgear teeth; the third has a four-thread worm and 80 wormgear teeth. For each design, compute the lead, axial pitch, circular pitch, lead angle, gear pitch diameter, and center distance.
54. A wormgear set has a double-threaded worm with a normal pressure angle of 20°, a pitch diameter of 0.625 in, and a diametral pitch of 16. Its mating wormgear has 100 teeth and a face width of 0.3125 in. Compute the lead, axial pitch, circular pitch, lead angle, addendum, dedendum, worm outside diameter, center distance, and velocity ratio.
55. A wormgear set has a four-threaded worm with a normal pressure angle of $14\frac{1}{2}°$, a pitch diameter of 2.000 in, and a diametral pitch of 6. Its mating wormgear has 72 teeth and a face width of 1.000 in. Compute the lead, axial pitch, circular pitch, lead angle, addendum, dedendum, worm outside diameter, center distance, and velocity ratio.
56. A wormgear set has a single-threaded worm with a normal pressure angle of $14\frac{1}{2}°$, a pitch diameter of 4.000 in, and a diametral pitch of 3. Its mating wormgear has 54 teeth and a face width of 2.000 in. Compute the lead, axial pitch, circular pitch, lead angle, addendum, dedendum, worm outside diameter, center distance, and velocity ratio.
57. A wormgear set has a four-threaded worm with a normal pressure angle of 25°, a pitch diameter of 0.333 in, and a diametral pitch of 48. Its mating wormgear has 80 teeth and a face width of 0.156 in. Compute the lead, axial pitch, circular pitch, lead angle, addendum, dedendum, worm outside diameter, center distance, and velocity ratio.

Analysis of Complex Gear Trains

58. The input shaft for the gear train shown in Figure P8–58 rotates at 3450 rpm cw. Compute the rotational speed and direction of the output shaft.
59. The input shaft for the gear train shown in Figure P8–59 rotates at 12 200 rpm. Compute the rotational speed of the output shaft.
60. The input shaft for the gear train shown in Figure P8–60 rotates at 6840 rpm. Compute the rotational speed of the output shaft.

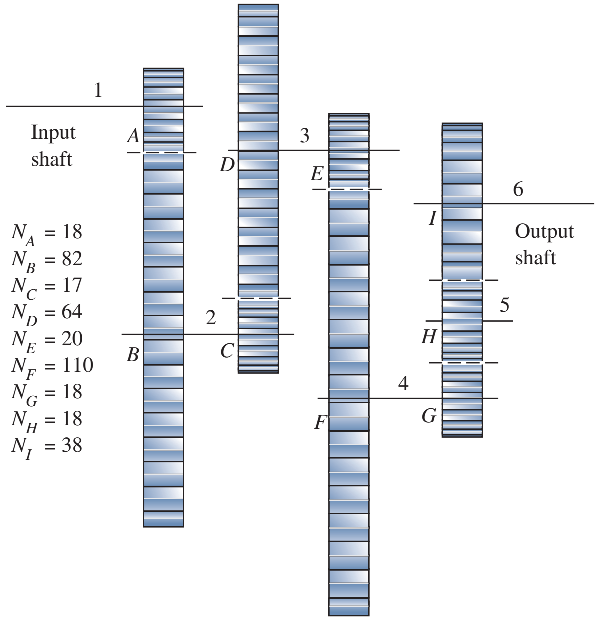


FIGURE P8–58 Gear train for Problem 8–58

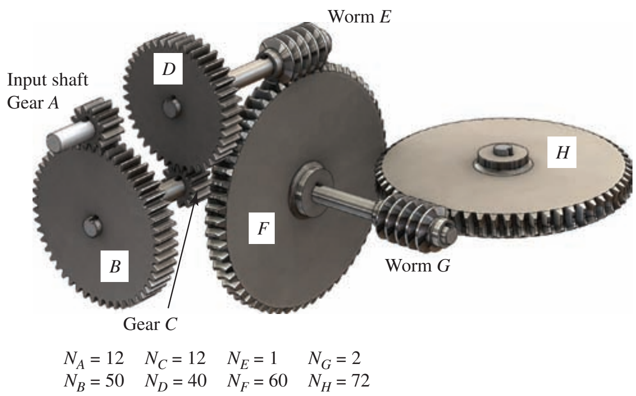


FIGURE P8–59 Gear train for Problem 8–59

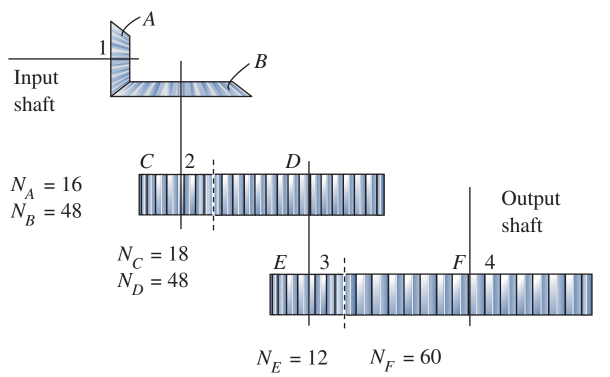


FIGURE P8–60 Gear train for Problem 8–60

61. The input shaft for the gear train shown in Figure P8–61 rotates at 2875 rpm. Compute the rotational speed of the output shaft.

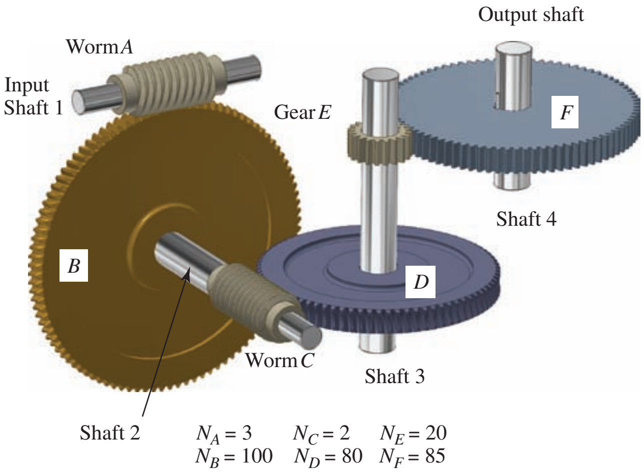


FIGURE P8–61 Gear train for Problem 8–61

Kinematic Design of a Single Gear Pair

62. Specify the numbers of teeth for the pinion and gear of a single gear pair to produce a velocity ratio of π as closely as possible. Use no fewer than 16 teeth nor more than 24 teeth in the pinion.

63. Specify the numbers of teeth for the pinion and gear of a single gear pair to produce a velocity ratio of $\sqrt{3}$ as closely as possible. Use no fewer than 16 teeth nor more than 24 teeth in the pinion.

64. Specify the numbers of teeth for the pinion and gear of a single gear pair to produce a velocity ratio of $\sqrt{38}$ as closely as possible. Use no fewer than 18 teeth nor more than 24 teeth in the pinion.

65. Specify the numbers of teeth for the pinion and gear of a single gear pair to produce a velocity ratio of 7.42 as closely as possible. Use no fewer than 18 teeth nor more than 24 teeth in the pinion.

Kinematic Design of Gear Trains

For Problems 66–75, devise a gear train using all external gears on parallel shafts. Use 20° full-depth involute teeth and no more than 150 teeth in any gear. Ensure that there is no interference. Sketch the layout for your design.

Problem no.	Input speed (rpm)	Output speed range (rpm)
66.	1800	2.0 Exactly
67.	1800	21.0 to 22.0
68.	3360	12.0 Exactly
69.	4200	13.0 to 13.5
70.	5500	221 to 225
71.	5500	13.0 to 14.0
72.	1750	146 to 150
73.	850	40.0 to 44.0
74.	3000	548 to 552 Use two pairs
75.	3600	3.0 to 5.0

For Problems 76–80, devise a gear train using any type of gears. Try for the minimum number of gears while avoiding interference and having no more than 150 teeth in any gear. Sketch your design.

Problem no.	Input speed (rpm)	Output speed (rpm)
76.	3600	3.0 to 5.0
77.	1800	8.0 Exactly
78.	3360	12.0 Exactly
79.	4200	13.0 to 13.5
80.	5500	13.0 to 14.0

Rack and Pinion Analysis

81. In Figure P8–81, the rack is driven by the pinion that has a rotational speed of 50 rpm. The diametral pitch is 12, $N_P = 30$, and the length of the rack is 7.00 ft. Find the following:
- (a) The pitch diameter of the pinion
- (b) The distance from the pitch line to the back of the rack
- (c) The center distance
- (d) The linear velocity of the rack
- (e) The time required to move the rack 7.00 ft.
- (f) The number of revolutions of the pinion as the rack moves 7.00 ft.

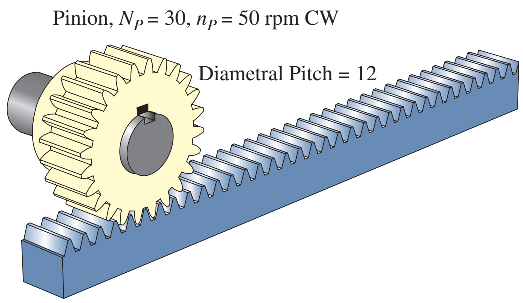


FIGURE P8–81 Rack and pinion for Problem 8–81

CHAPTER NINE

SPUR GEAR DESIGN

THE BIG PICTURE Spur Gear Design

Discussion Map

- A spur gear has involute teeth that are straight and parallel to the axis of the shaft that carries the gear.
- You need to understand how to design spur gears, specifying the form and size of the teeth, the face width, the material and its heat treatment.

Discover

Describe the action of the teeth of the driving gear on those of the driven gear. What kinds of stresses are produced?

How do the geometry of the gear teeth, the materials from which they are made, and the operating conditions affect the stresses and the life of the drive system?

This chapter will help you acquire the skills to perform the necessary analyses and to design safe spur gear drive systems that demonstrate long life.

The goal of this chapter is to help you gain the knowledge and skills necessary to design spur gears to transmit power from a source such as an electric motor, gasoline engine, fluid power motor, turbine, or other prime mover to a driven machine while changing the speed of the input shaft to some other speed at the output shaft. Most gear-type power transmissions are *speed reducers* that deliver the power to the driven machine at a lower speed and a higher torque. Examples of such transmissions are:

1. Electric motor drives through the transmission to a conveyor in a factory.
2. Gasoline engine for a vehicle drives through the transmission to the drive wheels.
3. Fluid power motor drives through the transmission to a winch on a tractor.
4. Water turbine drives through the transmission to an electric generator.
5. Gas turbine (jet) engine drives through the transmission to the rotor of a helicopter.

Although more rare than speed reducers, transmissions may also be used as *speed increasers*. An important example is a wind turbine that rotates relatively

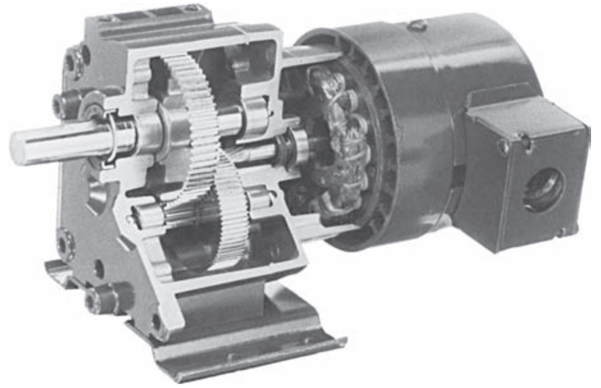


FIGURE 9–1 Double-reduction spur gear reducer (Bison Gear & Engineering Corporation, St. Charles, IL)

slowly, say 20.0 rpm, and the speed must be increased to 1800 rpm to drive an electric generator.

Figure 9–1 shows a commercially available double-reduction, spur gear type speed reducer driven by an electric motor that can be applied to many kinds of applications. This is an example of the type of reducer addressed in this chapter and about which much of design process discussed in Part II of this book (Chapters 7–15) is directed. Make note of how the gears are positioned and how they are mounted on shafts that are then supported by bearings mounted in a rigid housing.

A *spur gear* is one of the most fundamental types of gears. Its teeth are straight and parallel to the axis of the shaft that carries the gear. The teeth have the involute form described in Chapter 8. So, in general, the action of one tooth on a mating tooth is like that of two convex, curved members in contact: As the driving gear rotates, its teeth exert a force on the mating gear that is tangential to the pitch circles of the two gears. Because this force acts at a distance equal to the pitch radius of the gear, a torque is developed in the shaft that carries the gear. When the two gears rotate, they transmit power that is proportional to the torque. Indeed, that is the primary purpose of the spur gear drive system.

Consider the action described in the preceding paragraph:

- How does that action relate to the design of the gear teeth? Look back at Figure 8–1 as you consider this question and those that follow.
- As the force is exerted by the driving tooth on the driven tooth, what kinds of stresses are produced in the teeth? Consider both the point of contact of one tooth on the other and the whole tooth. Where are stresses a maximum?
- How could the teeth fail under the influence of these stresses?
- What material properties are critical to allow the gears to carry such loads safely and with a reasonable life span?
- What important geometric features affect the level of stress produced in the teeth?
- How does the precision of the tooth geometry affect its operation?
- How does the nature of the application affect the gears? What if the machine that the gears drive is a rock crusher that takes large boulders and reduces them to gravel made up of small stones? How would that loading compare with that of a gear system that drives a fan providing ventilation air to a building?
- What is the influence of the driving machine? Would the design be different if an electric motor were the driver or if a gasoline engine were used?
- The gears are typically mounted on shafts that deliver power from the driver to the input gear of a gear train and that take power from the output gear and transmit it to the driven machine. Describe various ways that the gears can be attached to the shafts and located with respect to each other. How can the shafts be supported?

This chapter contains the kinds of information that you can use to answer such questions and to complete the analysis and design of spur gear power transmission systems.

Chapters 10–15 cover similar topics for helical gears, bevel gears, and worm gearing, along with the design and specification of keys, couplings, seals, shafts, and bearings—all of which are needed to design a complete mechanical drive.

YOU ARE THE DESIGNER

You have already made the design decision that a spur gear type of speed reducer is to be used for a particular application. How do you complete the design of the gears themselves?

This is a continuation of a design scenario that was started in Chapter 1 of this book when the original goals were stated and when an overview of the entire book was given. The introduction to Part II continued this theme by indicating that the arrangement of the chapters is aligned with the design process that you could use to complete the design of the speed reducer.

Then in Chapter 8, you, as the designer, dealt with the kinematics of a gear reducer that would take power from the shaft of an electric motor rotating at 1750 rpm and deliver it to a machine that was to operate at approximately 292 rpm. There you limited your interest to the decisions that affected motion and the basic geometry of the gears. It was decided that you would use a double-reduction gear train to reduce the speed of rotation of the drive system in two stages using two pairs of gears in series. You also learned how to specify the layout of the gear train, along with key design decisions such as the numbers of teeth in all of the gears and the relationships among the diametral pitch, the number of teeth in the gears, the pitch diameters, and the distance between the centers of the shafts that carry those gears. For a chosen diametral pitch, you learned how to compute the dimensions of key features of the gear teeth such as the addendum, dedendum, and tooth width.

But the design is not complete until you have specified the material from which the gears are to be made and until you have verified that the gears will withstand the forces exerted on the gears as they transmit power and the corresponding torque. The teeth must not break, and they must have a sufficiently long life to meet the needs of the customer who uses the reducer.

To complete the design, you need more data: How much power is to be transmitted? To what kind of machine is the power from the output of the reducer being delivered? How does that affect the design of the gears? What is the anticipated duty cycle for the reducer in terms of the number of hours per day, days per week, and years of life expected? What options do you have for materials that are suitable for gears? Which material will you specify, and what will be its heat treatment?

You are the designer. The information in this chapter will help you complete the design. ■

9–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will be able to demonstrate the competencies listed below. They are presented in the order that they are covered in this chapter. The primary objectives are numbers 6, 7, and 8, which involve (a) the calculation of the bending strength and the ability of the gear teeth to resist pitting and (b) the design of gears to be safe with regard to both strength and pitting resistance. The competencies are as follows:

1. Compute the forces exerted on gear teeth as they rotate and transmit power.
2. Describe various methods for manufacturing gears and the levels of precision and quality to which they can be produced.
3. Specify a suitable level of quality for gears according to the use to which they are to be put.
4. Describe suitable metallic materials from which to make the gears, in order to provide adequate performance for both strength and pitting resistance.
5. Use the standards of the American Gear Manufacturers Association (AGMA) as the basis for completing the design of the gears.
6. Use appropriate stress analyses to determine the relationships among the applied forces, the geometry of the gear teeth, the precision of the gear teeth, and other factors specific to a given application, in order to make final decisions about those variables.
7. Perform the analysis of the tendency for the contact stresses exerted on the surfaces of the teeth to cause pitting of the teeth, in order to determine an adequate hardness of the gear material that will provide an acceptable level of pitting resistance for the reducer.
8. Complete the design of the gears, taking into consideration both the stress analysis and the analysis of pitting resistance. The result will be a complete specification of the gear geometry, the material for the gear, and the heat treatment of the material.

Outline of the Chapter: The process of designing spur gear drives contains numerous steps and requires the determination of several quantities that affect the performance of the drives. This chapter builds that process within the following sections, all of which contribute to the final design process described in Section 9–9. The list below gives the primary steps in the process for the design of a drive consisting of two steel gears.

1. Define the goals of the gear drive design: power to be transmitted, speed of the input gear, desired speed of the output gear, the kind of driver that provides the power, the kind of driven machine, and any special design features.
2. Determine the forces applied to the gear (Section 9–3).
3. Define the precision required for the gears by specifying the quality number (Section 9–5).
4. Understand the types of steel alloys typically used for gears and the kinds of heat treatment available (Section 9–7).
5. Propose the geometry of the gears including the pitch, number of teeth in each gear, pitch diameter, form of the teeth, and face width (Section 9–9).
6. Determine the expected stress due to bending in the gear teeth. This step requires specification of several factors that are functions of the manner of use and the manufacturing processes that will be used to produce the gears (Section 9–5).
7. Determine the expected actual contact stress experienced by the face of the teeth (Section 9–6).
8. Using the bending stress from Step 6 and the contact stress from Step 7, determine the strength and hardness required for the materials from which the gears are to be made to ensure adequate safety and life and specify the steel alloy and heat treatment that will meet these requirements (Section 9–8).
9. Summarize the design details.

9–2 CONCEPTS FROM PREVIOUS CHAPTERS

As you study this chapter, it is assumed that you are familiar with the geometry of gear features and the kinematics of one gear driving another as presented in Chapter 8.

(See also References 4 and 25.) Key relationships that you should be able to use include the following:

$$\text{Pitch line speed} = v_t = R\omega = (D/2)\omega$$

where R = radius of the pitch circle
D = pitch diameter
ω = angular velocity of the gear

Because the pitch line speed is the same for both the pinion and the gear, values for R, D, and ω can be for either. In the computation of stresses in gear teeth, in the U.S. system, it is usual to express the pitch line speed in the units of ft/min, while the size of the gear is given as its pitch diameter expressed in inches. Speed of rotation is typically given as n rpm—that is, n rev/min. Let's compute the unit-specific equation that gives pitch line speed in ft/min:

Pitch Line Speed U.S. Units

$$v_t = (D/2)\omega = \frac{D \text{ in}}{2} \cdot \frac{n \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1 \text{ ft}}{12 \text{ in}}$$
$$= (\pi D n/12) \text{ ft/min} \quad \textbf{(9–1)}$$

In the SI metric system, diameter is typically expressed in mm and speed in rpm while the pitch line speed is in m/s. Using a process similar to that used above for Equation (9–1), the following unit-specific expression was developed in Chapter 8.

Pitch Line Speed SI Units

$$v_t = (\pi D n/ 60\ 000) \text{ m/ s} \quad \textbf{(9–2)}$$

The velocity ratio can be expressed in many ways. For the particular case of a pinion driving a larger gear,

Velocity Ratio

The general definition of velocity ratio, VR, for gear drives is:

$$VR = (\omega_{\text{input}})/(\omega_{\text{output}}) = N_{output}/N_{input}$$

When the pinion is the driver and the gear is driven, the velocity ratio can be written as:

$$\text{Velocity ratio} = VR = \frac{\omega_P}{\omega_G} = \frac{n_P}{n_G} = \frac{R_G}{R_P} = \frac{D_G}{D_P} = \frac{N_G}{N_P} \quad \textbf{(9–3)}$$

A related ratio, m_G, called the *gear ratio*, is often used in analysis of the performance of gears. It is always defined as the ratio of the number of teeth in the larger gear to the number of teeth in the pinion, regardless of which is the driver. Thus, m_G is always greater than or equal to 1.0. When the pinion is the driver, as it is for a speed reducer, m_G is equal to VR. That is,

Gear Ratio

$$\text{Gear ratio} = m_G = N_G/N_P \geq 1.0 \quad \textbf{(9–4)}$$

In the U.S. system the diametral pitch, P_d, characterizes the physical size of the teeth of a gear. It is related to the pitch diameter and the number of teeth as follows:

Diametral Pitch

$$P_d = N_G/D_G = N_P/D_P \quad \textbf{(9–5)}$$

In the SI metric system, the metric module, m, characterizes the size of the teeth. It is defined as

Metric Module

$$m = D_G/N_G = D_P/N_P \quad \textbf{(9–6)}$$

The pressure angle, ϕ, is an important feature that characterizes the form of the involute curve that makes up the active face of the teeth of standard gears. See Figure 8–13. Also notice in Figure 8–12 that the angle between a normal to the involute curve and the tangent to the pitch circle of a gear is equal to the pressure angle.

9–3 FORCES, TORQUE, AND POWER IN GEARING

To understand the method of computing stresses in gear teeth, consider the way power is transmitted by the gear system shown in Figure 9–2. Part (a) is a perspective view and part (b) is the side view. The electric motor delivers power at the speed of its shaft through a flexible coupling to the input gear. The input gear drives the larger output gear to achieve a speed reduction. The shaft carrying the output gear transmits the power through a second flexible coupling to drive a roller in a printing press. It is important to determine the torque in both the input and output shafts using the following equation:

Torque

$$\text{Torque} = \text{power/rotational speed} = P/n \quad \textbf{(9–7)}$$

Because the speed of the pinion is higher than the speed of the gear, the output torque is higher than the input torque. The power is transmitted from the input shaft to the pinion through a key. The teeth of the pinion drive the teeth of the gear and thus transmit the power to the gear. But again, power transmission actually involves the application of a torque during rotation at a given speed. The torque is the product of the force acting tangent to the pitch circle of the pinion times the pitch radius of the pinion. We will use the symbol W_t to indicate the *tangential force*. As described, W_t is the force exerted *by the pinion teeth on the gear teeth*. This force is used to transmit power from one gear to the other and is referred to as the *driving force*. But if the gears are rotating at a constant speed and are transmitting a uniform level of power, the system is in equilibrium. Therefore, there must be an equal and opposite tangential force exerted by the gear teeth back on the pinion teeth as shown in Figure 9–2(c). This is an application of the principle of action and reaction.

To complete the description of the power flow, the tangential force on the gear teeth produces a torque on the gear equal to the product of W_t times the pitch radius of the gear. Because W_t is the same on the pinion and the gear, but the pitch radius of the gear is larger than that of the pinion, the torque on the gear (the output torque) is greater than the input torque. However, note that the power transmitted to the gear is only slightly less because of the high mechanical inefficiency of the gearing. The power then flows from the gear through the key to the output shaft, the second flexible coupling, and finally to the driven machine.

From this description of power flow, we can see that gears transmit power by exerting a force by the driving teeth on the driven teeth while the reaction force acts back on the teeth of the driving gear. Figure 9–3 shows a single gear tooth with the tangential force W_t acting on it. But this is not the total force on the tooth as shown in Figure 9–2(d). Because of the involute form of the tooth, the total force transferred from one tooth to the mating tooth acts normal to the involute profile. This action is shown as W_n. The tangential force W_t is actually the horizontal component of the total force. To complete the picture, note that there is a vertical component of the total force acting radially on the gear tooth, indicated by W_r. The radial force tends to separate the gear set and is called the *separating force*.

The normal and radial forces on the gears must be resisted by the bearings that support the shafts as shown in Figure 9–2(e). This observation is important for the design of the shafts carrying the gears as will be demonstrated in Chapter 12.

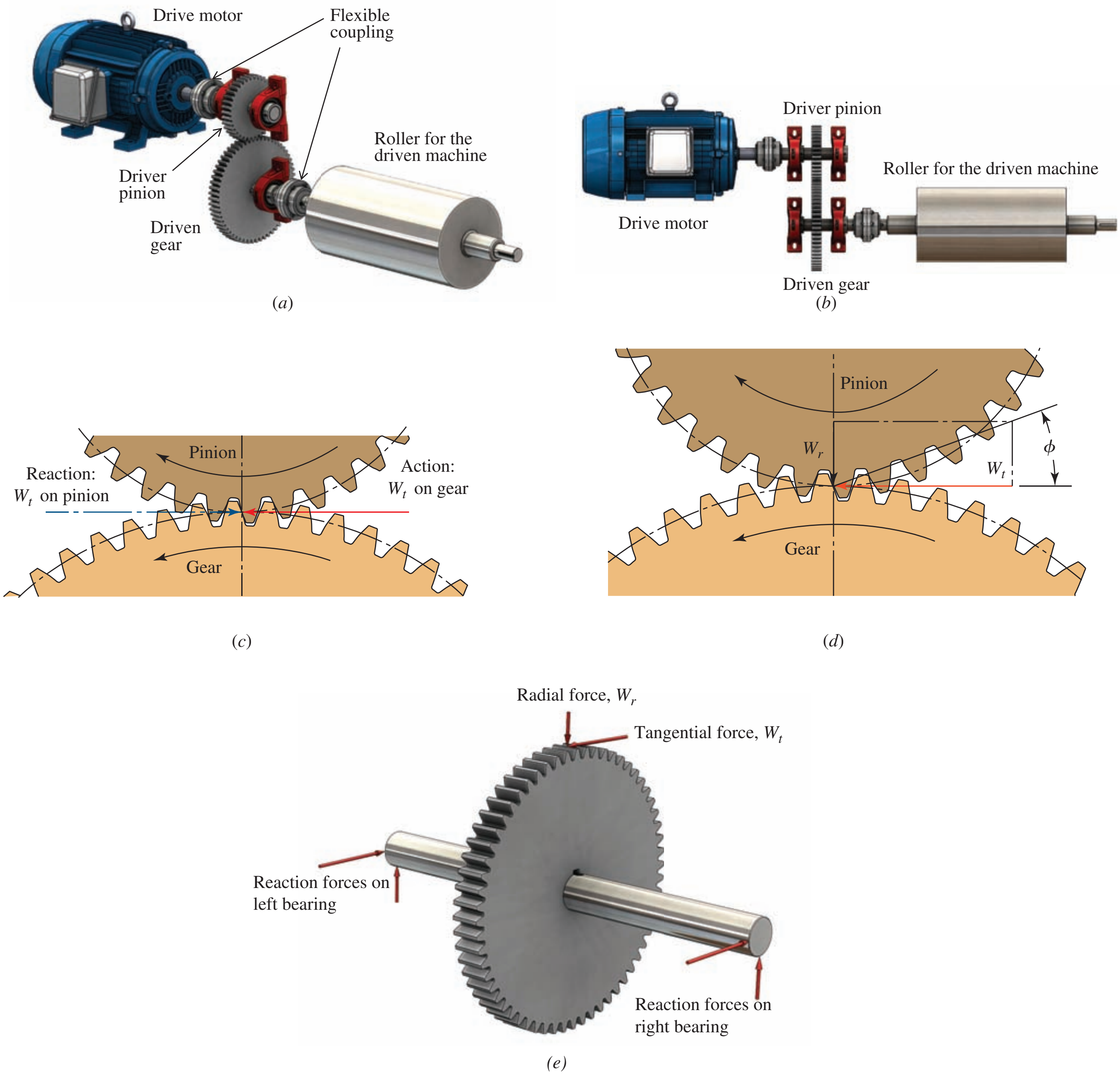


FIGURE 9–2 (*a*) Power flow through a gear pair (*b*) Side view of drive train (*c*) View of gear teeth in mesh showing tangential forces on both gears (*d*) Tangential and radial forces exerted by the pinion tooth on the gear tooth (e) Forces on the shaft carrying the driven gear

The discussion about power flow and forces existing in gears thus far is generic and independent of units. Following discussion is developed primarily for the U.S. unit system based on the diametral pitch, P_d. Later, we adapt the results to the SI metric unit system and module, m.

We will start the computation of forces with the transmitted force, W_t, because its value is based on the given data for power and speed. It is convenient to develop unit-specific equations for W_t because standard practice typically calls for the following units for key quantities pertinent to the analysis of gear sets:

Forces in pounds (lb)

Power in horsepower (hp) (Note that 1.0 hp = 550 lb · ft/s.)

Rotational speed in rpm, that is, rev/min

Pitch line speed in ft/min

Torque in lb · in

The torque exerted on a gear is the product of the transmitted load, W_t, and the pitch radius of the gear. The torque is also equal to the power transmitted divided by the rotational speed. Then

$$T = W_t(R) = W_t(D/2) = P/n$$

Then we can solve for the force, and the units can be adjusted as follows:

➪ Tangential Force

$$W_t = \frac{2P}{Dn} = \frac{2P(\text{hp})}{D(\text{in})\cdot n(\text{rev/min})}\cdot\frac{550\ \text{lb}\cdot\text{ft/s}}{(\text{hp})}\cdot\frac{1.0\ \text{rev}}{2\pi\ \text{rad}}\cdot\frac{60\ \text{s/min}}{}\cdot\frac{12\ \text{in}}{\text{ft}}$$

$$W_t = (126\,000)(P)/(nD)\ \text{lb} \tag{9–8}$$

Data for either the pinion or the gear can be used in this equation. Other relationships are now developed because they are needed in other parts of the process of analyzing the gears or the shafts that carry them.

Power is also the product of transmitted force, W_t, and the pitch line velocity:

$$P = W_t \cdot v_t$$

Then, solving for the force and adjusting units, we have

➪ Tangential Force

$$W_t = \frac{P}{v_t} = \frac{P(\text{hp})}{v_t(\text{ft/min})}\cdot\frac{550\ \text{lb}\cdot\text{ft/s}}{1.0\ \text{hp}}\cdot\frac{60\ \text{s/min}}{}$$

$$= 33\,000\ (P)/(v_t)\ \text{lb} \tag{9–9}$$

We may also need to compute torque in lb · in:

➪ Torque

$$T = \frac{P}{\omega} = \frac{P(\text{hp})}{n(\text{rev/min})}\cdot\frac{550\ \text{lb}\cdot\text{ft/s}}{1.0\ \text{hp}}\cdot\frac{1.0\ \text{rev}}{2\pi\ \text{rad}}\cdot\frac{60\ \text{s/min}}{}\cdot\frac{12\ \text{in}}{\text{ft}}$$

$$T = 63\,000(P)/n\ \text{lb}\cdot\text{in} \tag{9–10}$$

These values can be computed for either the pinion or the gear by appropriate substitutions. Remember that the pitch line speed is the same for the pinion and the gear and that the transmitted loads on the pinion and the gear are the same, except that they act in opposite directions.

The normal force, W_n, and the radial force, W_r, can be computed from the known W_t by using the right triangle relations evident in Figure 9–3:

➪ Radial Force

$$W_r = W_t \tan\phi \tag{9–11}$$

➪ Normal Force

$$W_n = W_t/\cos\phi \tag{9–12}$$

where ϕ = pressure angle of the tooth form

In addition to causing the stresses in the gear teeth, these forces act on the shaft. In order to maintain equilibrium, the bearings that support the shaft must provide the reactions. Figure 9–2(e) shows the free-body diagram of the output shaft of the reducer.

Let's look at an example of a pinion driving a gear to see how the rotational speed, torque, and forces are determined.

Power Flow and Efficiency

The discussion thus far has focused on power, torque, and forces for a single pair of gears. For compound gearing having two or more pairs of gears, the flow of power and the overall efficiency become increasingly important.

Power losses in gear drives made from spur, helical, and bevel gears depend on the action of each tooth on its mating tooth, a combination of rolling and sliding. For accurate, well-lubricated gears, the power loss ranges from 0.5% to 2.0% and is typically taken to be approximately 1.0%. (See Reference 20.) *Because this is quite small, it is customary to neglect it in sizing individual gear pairs; we do that in this book.*

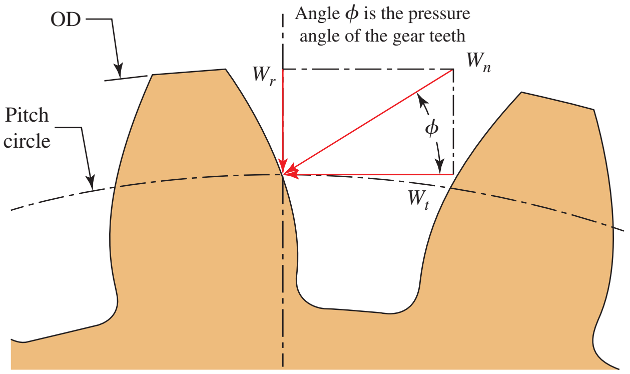


FIGURE 9–3 Forces on gear teeth: Tangential force, W_t; Radial force, W_r; Normal force, W_n

Example Problem 9–1

A pair of spur gears with a 20° pressure angle, full depth, involute teeth transmit 30 hp. The pinion is mounted on a jack shaft assembly that is directly coupled to the shaft of the electric motor operating at 600 rpm as shown in Figure 9–2(a) and (b). The pinion has 36 teeth and a diametral pitch of 5. The gear has 60 teeth and is mounted on a jack shaft assembly that is directly coupled to the output roller. The output roller is part of the driven machinery. Compute the following:

a. The velocity ratio and the gear ratio for the gear pair.
b. The rotational speed of the gear.
c. The pitch diameter of the pinion and the gear.
d. The center distance of the shafts carrying the pinion and the gear.
e. The pitch line speed for both the pinion and the gear.
f. The torque on the pinion shaft and gear shaft.
g. The tangential force and on the teeth of each gear.
h. The radial force acting on the teeth of each gear.
i. The normal force acting on the teeth of each gear.

Solution

a. From Equation (9–3), with the pinion as the driver and the gear being driven, the velocity ratio is:

$$\textit{Velocity ratio} = VR = \frac{\omega_p}{\omega_g} = \frac{N_g}{N_p} = \frac{60}{36} = 1.667$$

b. The velocity ratio can be rewritten to solve for the rotational speed of the gear:

$$\omega_g = \frac{\omega_p}{VR} = \frac{600 \text{ rpm}}{1.667} = 360 \text{ rpm}$$

c. The pitch diameters of the pinion and gear are:

$$D_p = \frac{N_p}{P_d} = \frac{36}{5} = 7.200 \text{ in} \qquad D_g = \frac{N_g}{P_d} = \frac{60}{5} = 12.000 \text{ in}$$

d. The center distance for the pinion and gear:

$$CD = \frac{D_p + D_g}{2} = \frac{7.200 \text{ in} + 12.000 \text{ in}}{2} = 9.600 \text{ in}$$

e. The pitch line speed can be solved using either the pinion or the gear:

$$v_t = \frac{D}{2} \cdot \omega$$

$$v_{tp} = \frac{D_p}{2} \cdot \omega_p = \frac{7.200 \text{ in}}{2} \cdot \frac{600 \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 1130 \text{ fpm}$$

$$v_{tg} = \frac{D_g}{2} \cdot \omega_g = \frac{12.000 \text{ in}}{2} \cdot \frac{360 \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 1130 \text{ fpm}$$

f. The torque on the pinion shaft and gear shaft can be determined by solving the power equation in terms of torque

$$T = \frac{P}{\omega}$$

$$T_p = \frac{P}{\omega_p} = \frac{30 \text{ hp}}{600 \text{ rpm}} \cdot \frac{33000 \frac{\text{lb} \cdot \text{ft}}{\text{min}}}{1 \text{ hp}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 3151.3 \text{ lb} \cdot \text{in}$$

$$T_g = \frac{P}{\omega_g} = \frac{30 \text{ hp}}{360 \text{ rpm}} \cdot \frac{33000 \frac{\text{lb} \cdot \text{ft}}{\text{min}}}{1 \text{ hp}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 5252.1 \text{ lb} \cdot \text{in}$$

You will notice the ratio of the torque on the gear shaft to the torque on the pinion shaft is the same as the gear ratio: The power through each shaft is 30 hp. If the rotational speed of the shaft decreases, the torque will increase proportionally keeping the power constant.

$$m_g = \frac{T_g}{T_p} = \frac{5252.1 \text{ lb}\cdot\text{in}}{3151.3 \text{ lb}\cdot\text{in}} = 1.67$$

g. The tangential force is solved using the pinion torque and the pitch circle radius or the gear torque and gear pitch circle radius. This is the drive force the pinion tooth applies to the gear tooth and equal and opposite reaction force the gear tooth applies to the pinion tooth.

$$W_t = \frac{T_p}{\left(\frac{D_p}{2}\right)} = \frac{3151.3 \text{ lb}\cdot\text{in}}{\left(\frac{7.200 \text{ in}}{2}\right)} = 875.4 \text{ lb}$$

$$W_t = \frac{T_g}{\left(\frac{D_g}{2}\right)} = \frac{5252.1 \text{ lb}\cdot\text{in}}{\left(\frac{12.000 \text{ in}}{2}\right)} = 875.4 \text{ lb}$$

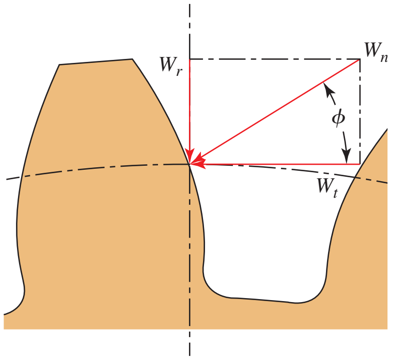


FIGURE 9–4 Forces on the gear tooth

h. The radial force is calculated using the tangential force and the pressure angle. As you can see the direction of the force is toward the centerline of the shaft and tends to separate the gear set.

$$W_r = W_t \cdot \tan(\varphi) = 875.4 \text{ lb} \cdot \tan(20°) = 318.6 \text{ lb}$$

i. The normal force to the tooth profile is solved using Pythagorean theorem. This force is along the line of action and is normal to the tooth profile as shown in Figure 9–4.

$$W_n = \sqrt{(W_t)^2 + (W_r)^2} = \sqrt{(875.4 \text{ lb})^2 + (318.6 \text{ lb})^2} = 931.5 \text{ lb}$$

Example Problem 9–2 follows, in which the power from a drive motor is delivered through a gear train to four separate output shafts, demonstrating the concept of power flow through a gear train. Tracking the power, torque, and speed of each gear in the train is critical to proper analysis.

Example Problem 9–2

Figure 9–5 represents the power flow through a gear drive to multiple output shafts. Gear A is mounted on an electric motor that has a rotational speed of 1500 rpm (CW). Gear A drives a simple gear train consisting of gears B, C, D, and E. Gear A transfers the input power through each gear to the shaft on which it is mounted. The power levels delivered from shafts B, C, D, and E are 3 hp, 8 hp, 3 hp, and 3 hp, respectively. All gears have a diametral pitch of 10 and the following numbers of teeth:

$$N_A = 60 \qquad N_B = 30 \qquad N_C = 90 \qquad N_D = 30 \qquad N_E = 30 \qquad P_d = 8$$

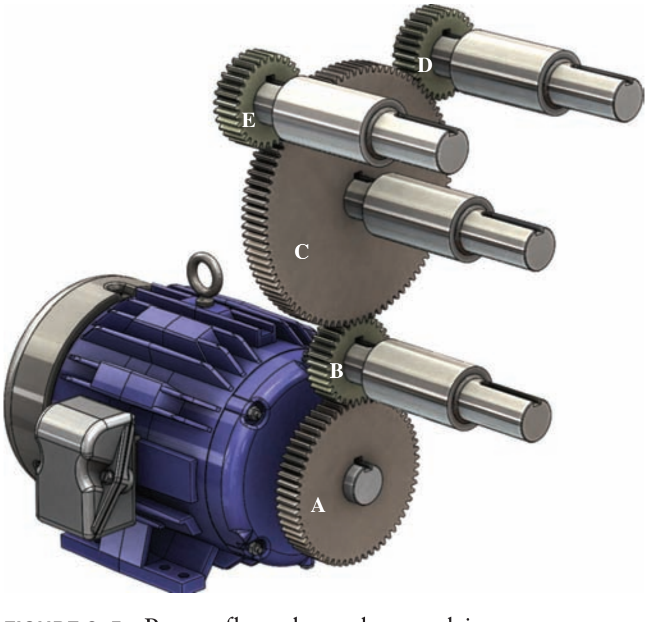


FIGURE 9–5 Power flow through gear drive system

Determine the following data for the drive system:

a. The pitch diameter of each gear
b. The center distance of each gear mesh
c. The rotational speed of each output shaft
d. Neglecting efficiency losses, find the power requirement of the electric motor
e. The torque through each output shaft and the tangential force on the gear teeth

Solution a. Pitch Diameter for each gear:

$$D_A = \frac{N_A}{p_d} = \frac{60}{10} = 6.000 \text{ in} \qquad D_B = \frac{N_B}{p_d} = \frac{30}{10} = 3.000 \text{ in} \qquad D_C = \frac{N_C}{p_d} = \frac{90}{10} = 9.000 \text{ in}$$

$$D_D = \frac{N_D}{p_d} = \frac{30}{10} = 3.000 \text{ in} \qquad D_E = \frac{N_E}{p_d} = \frac{30}{10} = 3.000 \text{ in}$$

b. The center distance for each gear mesh:

$$CD_{A-B} = \frac{D_A + D_B}{2} = \frac{6.0 \text{ in} + 3.0 \text{ in}}{2} = 4.500 \text{ in} \qquad CD_{B-C} = \frac{D_B + D_C}{2} = \frac{3.0 \text{ in} + 9.0 \text{ in}}{2} = 6.000 \text{ in}$$

$$CD_{C-D} = \frac{D_C + D_D}{2} = \frac{9.0 \text{ in} + 3.0 \text{ in}}{2} = 6.000 \text{ in} \qquad CD_{C-E} = \frac{D_C + D_E}{2} = \frac{9.0 \text{ in} + 3.0 \text{ in}}{2} = 6.000 \text{ in}$$

c. The next item we want to solve for is the rotational speed of each gear of the gear train. Let's step through one gear mesh at a time, starting with gear A that drives gear B:

$$VR_{A-B} = \frac{n_A}{n_B} = \frac{N_B}{N_A} = \frac{30}{60} = \frac{1}{2}$$

The equation $VR_{A-B} = \frac{n_A}{n_B}$ is used to solve for the rotational speed of gear B. The velocity ratio gives a speed increase from gear A to gear B:

$$n_B = \frac{n_A}{VR_{A-B}} = \frac{1500 \text{ rpm}}{0.5} = 3000 \text{ rpm}$$

Likewise, considering gear B driving gear C, the velocity ratio is:

$$VR_{B-C} = \frac{n_B}{n_C} = \frac{N_C}{N_B} = \frac{90}{30} = 3$$

The rotation speed of gear C: $n_C = \frac{n_B}{VR_{B-C}} = \frac{3000 \text{ rpm}}{3} = 1000 \text{ rpm}$

It is important to note, however, that using the velocity ratio equation from gear A to gear C would give us the same result:

$$VR_{A-C} = \frac{n_A}{n_C} = \frac{N_B}{N_A} \cdot \frac{N_C}{N_B} = \frac{30}{60} \cdot \frac{90}{30} = 1.5$$

The rotational speed of gear C: $n_C = \frac{n_A}{VR_{A-C}} = \frac{1500 \text{ rpm}}{1.5} = 1000 \text{ rpm}$

The rotational speed of gear D and gear E will be the same since both gears have the same number of teeth and are driven by gear C.

$$VR_{C-D} = \frac{n_C}{n_D} = \frac{N_D}{N_C} = \frac{30}{90} = \frac{1}{3}$$

$$n_E = n_D = \frac{n_C}{VR_{C-D}} = \frac{1000 \text{ rpm}}{0.333} = 3000 \text{ rpm}$$

Again, we could have used the velocity ratio equation from gear A to gear D or E. That would have given us the same results:

$$VR_{A-D} = \frac{n_A}{n_D} = \frac{N_B}{N_A} \cdot \frac{N_C}{N_B} \cdot \frac{N_D}{N_C} = \frac{30}{60} \cdot \frac{90}{30} \cdot \frac{30}{90} = \frac{1}{2}$$

The rotational speed of gear D and gear E $n_D = n_E = \dfrac{n_A}{VR_{A-D}} = \dfrac{1500 \text{ rpm}}{0.5} = 1000 \text{ rpm}$

d. The power required by the motor, assuming no losses in the system, is equal to the sum of the power required by each drive shaft. This can be written as:

$$Power_{in} = Power_{out}$$
$$P_{in} = P_B + P_C + P_D + P_E$$
$$P_{in} = 3 \text{ hp} + 8 \text{ hp} + 3 \text{ hp} + 3 \text{ hp} = 17 \text{ hp}$$

e. The next part of the problem is asking us to find the torque transmitted through each output shaft and the tangential force on the gear teeth. Start with gear E and work through to the drive motor gear A. The torque transmitted through shaft E can be found using the required output power and the rotational speed of shaft.

$$P_{out\,E} = T_E \cdot n_E$$

$$T_E = \frac{P_{out\,E}}{n_E} = \frac{3.0 \text{ hp}}{3000 \text{ rev/min}} \cdot \frac{33000 \cdot \dfrac{\text{lb} \cdot \text{ft}}{\text{min}}}{1 \text{ hp}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 63.0 \text{ lb} \cdot \text{in}$$

The tangential gear force, $W_{t\,C/E}$, is the force gear C applies to gear E to obtain the torque required to transmit the required power through shaft E. The tangential force, $W_{t\,E/C}$ is the equal to but opposite the reaction force gear E applies to gear C. The tangential gear force can be determined:

$$W_{t\,D/C} = W_{t\,E/C} = \frac{T_E}{\left(\dfrac{D_E}{2}\right)} = \frac{63.0 \text{ lb} \cdot \text{in}}{\left(\dfrac{3.0 \text{ in}}{2}\right)} = 42.0 \text{ lb}$$

Since gear E has the same rotational speed, power requirement, and pitch diameter as gear D, the torque through shaft E and the tangential gear tooth force is the same. This is illustrated in Figure 9–6.

$$T_D = T_E = 63.0 \text{ lb} \cdot \text{in}$$
$$W_{t\,C/D} = W_{t\,D/C} = 42.0 \text{ lb}$$

8 hp is transmitted through shaft C at a rotational speed of 1000 rpm. The torque through shaft C can then be determined by rearranging this equation:

$$P_{out\,C} = T_C \cdot n_C$$

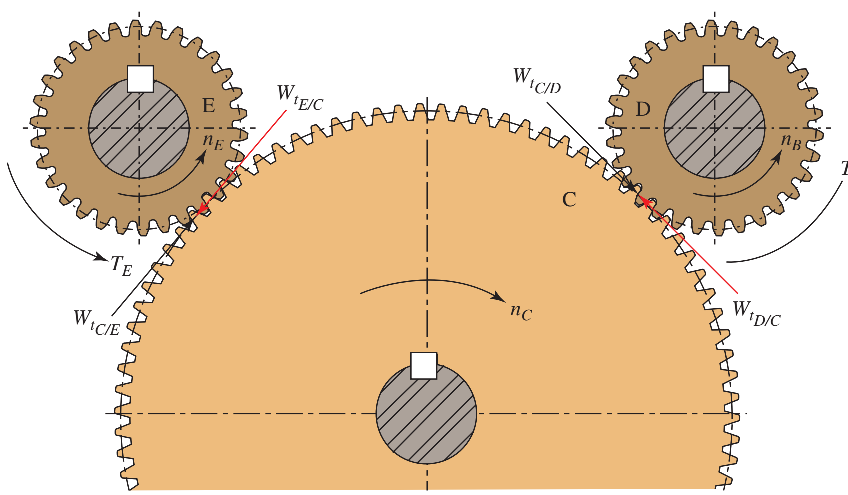


FIGURE 9–6 Torques and tangential gear force shown on gears C, D, and E

$$T_C = \frac{P_{out\,C}}{n_C} = \frac{8.0\text{ hp}}{1000\text{ rev/min}} \cdot \frac{33000\,\frac{\text{lb}\cdot\text{ft}}{\text{min}}}{1\text{ hp}} \cdot \frac{1\text{ rev}}{2\pi\text{ rad}} \cdot \frac{12\text{ in}}{1\text{ ft}} = 504.2\text{ lb}\cdot\text{in}$$

The total torque required to drive gears C, D, and E:

$$T_{C\,total} = T_C + W_{t\,E/C} \cdot \left(\frac{D_C}{2}\right) + W_{t\,D/C} \cdot \left(\frac{D_C}{2}\right)$$

$$T_{C\,total} = 504.2\text{ lb}\cdot\text{in} + 42.0\text{ lb} \cdot \left(\frac{9.0\text{ in}}{2}\right) + 42.0\text{ lb} \cdot \left(\frac{9.0\text{ in}}{2}\right) = 882.3\text{ lb}\cdot\text{in}$$

We can verify this torque, using the sum of the power required by shafts C, D, and E.

$$P_{C\,total} = P_C + P_D + P_E = 8\text{ hp} + 3\text{ hp} + 3\text{ hp} = 14\text{ hp}$$

The total torque required to drive gears C, D, and E:

$$T_{C\,total} = \frac{P_{C\,total}}{n_C} = \frac{14\text{ hp}}{1000\text{ rev/min}} \cdot \frac{33000\,\frac{\text{lb}\cdot\text{ft}}{\text{min}}}{1\text{ hp}} \cdot \frac{1\text{ rev}}{2\pi\text{ rad}} \cdot \frac{12\text{ in}}{1\text{ ft}} = 882.3\text{ lb}\cdot\text{in}$$

We can see this gives us the same value as was shown above. This torque is used to find the tangential force gear C applies to gear B:

$$W_{t\,C/B} = \frac{T_{C\,total}}{\left(\frac{D_C}{2}\right)} = \frac{882.3\text{ lb}\cdot\text{in}}{\left(\frac{9.0\text{ in}}{2}\right)} = 196.0\text{ lb}$$

The tangential force gear C applies to gear B is equal to the reaction force gear B applies to gear C.

$$W_{t\,C/B} = W_{t\,B/C} = 196.0\text{ lb}$$

This is shown in Figure 9–7.

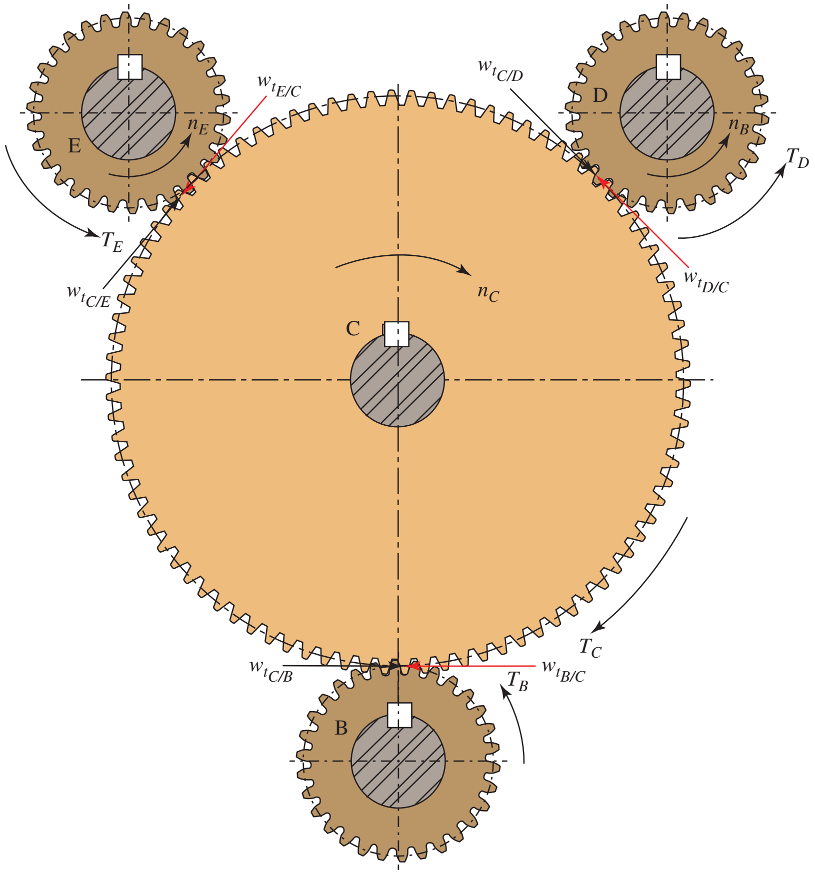


FIGURE 9–7 Tangential gear forces on gears B, C, D, and E

Shaft B transmits 3 hp at a rotational speed of 3000 rpm. The torque applied to shaft B to drive the output:

$$T_B = \frac{P_B}{n_B} = \frac{3.0\ \text{hp}}{3000\ \text{rev/min}} \cdot \frac{33000\ \frac{\text{lb}\cdot\text{ft}}{\text{min}}}{1\ \text{hp}} \cdot \frac{1\ \text{rev}}{2\pi\ \text{rad}} \cdot \frac{12\ \text{in}}{1\ \text{ft}} = 63.0\ \text{lb}\cdot\text{in}$$

The total torque required to drive gears B, C, D, and E

$$T_{B\ total} = T_B + W_{t\ C/B} \cdot \left(\frac{D_B}{2}\right) = 63.0\ \text{lb}\cdot\text{in} + 196.0\ \text{lb} \cdot \left(\frac{3.0\ \text{in}}{2}\right) = 357.1\ \text{lb}\cdot\text{in}$$

We can verify this torque using the sum of the power required by shafts B, C, D, and E.

$$P_{B\ total} = P_B + P_C + P_D + P_E = 3\ \text{hp} + 8\ \text{hp} + 3\ \text{hp} + 3\ \text{hp} = 17\ \text{hp}$$

The total torque required to drive gears B, C, D, and E:

$$T_{B\ total} = \frac{P_{B\ total}}{n_C} = \frac{17\ \text{hp}}{1000\ \text{rev/min}} \cdot \frac{33000\ \frac{\text{lb}\cdot\text{ft}}{\text{min}}}{1\ \text{hp}} \cdot \frac{1\ \text{rev}}{2\pi\ \text{rad}} \cdot \frac{12\ \text{in}}{1\ \text{ft}} = 357.1\ \text{lb}\cdot\text{in}$$

This gives us the same result as was shown above. This torque is used to find the tangential force gear B applies to gear A:

$$W_{t\ B/A} = \frac{T_{B\ total}}{\left(\frac{D_B}{2}\right)} = \frac{357.1\ \text{lb}\cdot\text{in}}{\left(\frac{3.0\ \text{in}}{2}\right)} = 238.1\ \text{lb}$$

The tangential force gear B applies to gear A is equal to the reaction force gear A applies to gear B and is shown in Figure 9–8.

$$W_{t\ B/A} = W_{t\ A/B} = 238.1\ \text{lb}$$

The required torque of the motor is:

$$T_{Motor} = T_A = W_{t\ A/B} \cdot \left(\frac{D_A}{2}\right) = 238.1\ \text{lb}\cdot\text{in} \cdot \left(\frac{6.0\ \text{in}}{2}\right) = 714.3\ \text{lb}\cdot\text{in}$$

The motor torque and rotational speed can be used to verify the input power requirement.

$$P_{Motor} = T_{Motor} \cdot n_{Motor} = 714.3\ \text{lb}\cdot\text{in} \cdot 1500\ \text{rpm} \cdot \frac{1\ \text{hp}}{33000\ \frac{\text{lb}\cdot\text{ft}}{\text{min}}} \cdot \frac{2\pi\ \text{rad}}{1\ \text{rev}} \cdot \frac{1\ \text{ft}}{12\ \text{in}} = 17\ \text{hp}$$

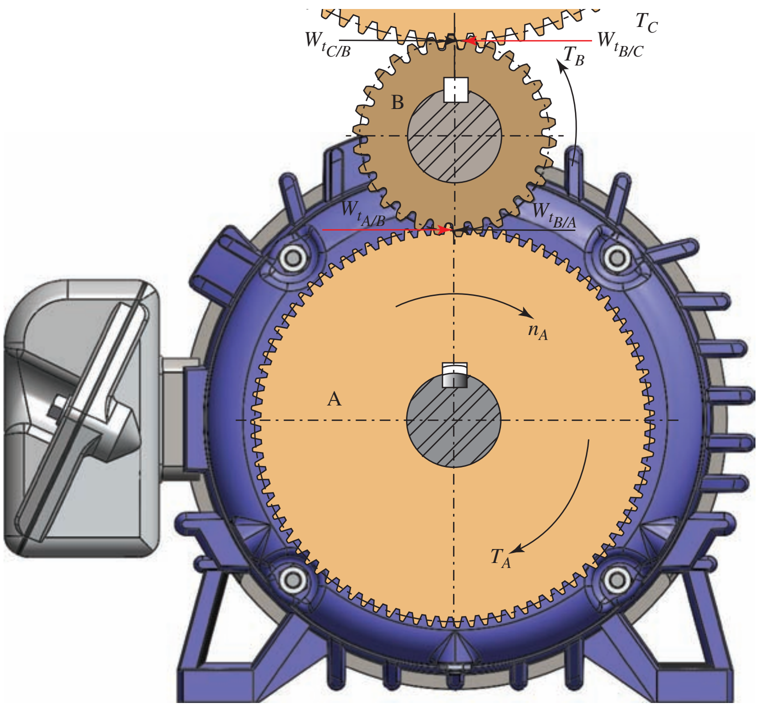


FIGURE 9–8 Tangential gear forces on gears A and B

Compound gear drives employ several pairs of gears in series to produce large speed reduction ratios. With 1.0% power loss in each pair, the accumulated power loss for the system can become significant, and it can affect the size of motor to drive the system or the ultimate power and torque available for use at the output. Furthermore, the power loss is transferred to the environment or into the gear lubricant and, for large power transmissions, the management of the heat generated is critical to the overall performance of the unit. The viscosity and load-carrying ability of lubricants is degraded with increasing temperature.

Tracking power flow in a simple or compound gear train is simple, the power is transferred from one gear pair to the next with only a small power loss at each mesh. More complex designs may split the power flow at some point to two or more paths. This is typical of planetary gear trains. In such cases, you should consider the basic relationship among power, torque, and rotational speed shown in Equation (9–7), $P = T \times n$. We can present this in another form. Let the rotational speed, n, that is typically taken to be in the units of rpm, be the more general term *angular velocity*, ω, in the units of rad/s. Now express the torque in terms of the transmitted forces, W_t, and the pitch radius of the gear, R. That is, $T = W_tR$. Equation (9–7) then becomes,

$$P = T \times n = W_tR\omega$$

But $R\ \omega$ is the pitch line velocity for the gears, v_t. Then,

$$P = W_tR\omega = W_tv_t$$

Knowing how the power splits enables the determination of the transmitted load at each mesh.

9–4 INTRODUCTION TO STRESS ANALYSIS FOR GEARS

Later in this chapter, design procedures are presented in which two forms of gear-tooth failure are considered.

A gear tooth acts like a cantilever beam in resisting the force exerted on it by the mating tooth. The point of highest tensile bending stress is at the root of the tooth where the involute curve blends with the fillet. The AGMA has developed a set of *allowable bending stress numbers*, called s_{at}, which are compared to computed bending stress levels in the tooth to rate the acceptability of a design.

A second, independent form of failure is the pitting of the surface of the teeth, usually near the pitch line, where high contact stresses occur. The transfer of force from the driving to the driven tooth theoretically occurs across a line contact because of the action of two convex curves on each other. Repeated application of these high contact stresses can cause a type of fatigue failure of the surface, resulting in local fractures and an actual loss of material. This is called *pitting*. The AGMA has developed a set of *allowable contact stress numbers*, called s_{ac}, which are compared to computed contact stress levels in the tooth to rate the acceptability of a design.

Both bending stress and contact stress must be at safe levels in any pair of gears. The processes involved in verifying those safe stress levels are developed and demonstrated in the following sections 9–5 through 9–12, as outlined here:

- Section 9–5 describes the method for calculating the bending stress in gear teeth and demonstrates that process in Example Problem 9–3.
- Section 9–6 describes the method for calculating the contact stress in gear teeth and demonstrates that process in Example Problem 9–4.
- Section 9–7 describes typical metallic materials that are used for gearing and provides sample data for allowable stresses that can be used for problem solving in this book. Additional data of this type can be found in References 6, 8, and 9.
- Section 9–8 describes the process for evaluating the safety of materials proposed for application to a particular pair of gears.
    - Example Problem 9–5 demonstrates the evaluation of three proposed materials for a given application with regard to bending stress.
    - Example Problem 9–6 then demonstrates a similar evaluation for contact stress.
    - The comparison of the results for Example Problems 9–3 to 9–6 is then presented and conclusions are drawn on which proposed material is most satisfactory.
- Section 9–9 then presents an alternate approach to the design of gear pairs in which the final step is the specification of the material that will satisfy both the bending stress and the contact stress. Example Problem 9–7 demonstrates that process.
- Section 9–10 modifies the process developed in Section 9–9 for design of gearing in the metric module system. Example Problem 9-8 demonstrates that process.
- Sections 9–11 and 9–12 show examples of the use of spreadsheets to perform the many calculations required for analysis and design of gear pairs, along with the use of the spreadsheets to work toward an optimum design.

9–5 BENDING STRESS IN GEAR TEETH

The stress analysis of gear teeth is facilitated by consideration of the orthogonal force components, W_t and W_r, as shown in Figure 9–3.

The tangential force, W_t, produces a bending moment on the gear tooth similar to that on a cantilever beam. The resulting bending stress is maximum at the base of the tooth in the fillet that joins the involute profile to the bottom of the tooth space. Taking the detailed geometry of the tooth into account, Wilfred Lewis developed the equation for the stress at the base of the involute profile, which is now called the *Lewis equation*:

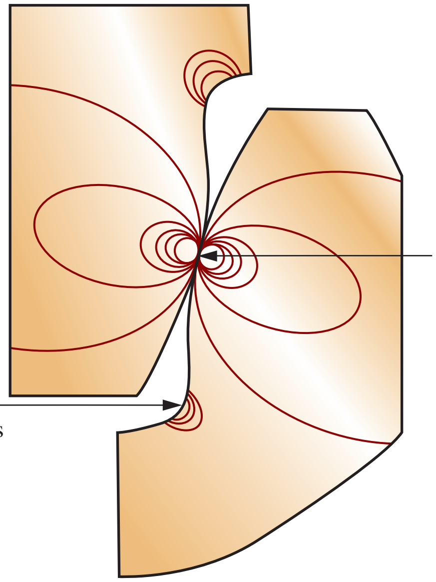


FIGURE 9–9 Photoelastic study of gear teeth under load

➩ Lewis Equation for Bending Stress in Gear Teeth

$$\sigma_t = \frac{W_t P_d}{FY} \tag{9–13}$$

where

W_t = tangential force

P_d = diametral pitch of the tooth

F = face width of the tooth

Y = *Lewis form factor*, which depends on the tooth form, the pressure angle, the diametral pitch, the number of teeth in the gear, and the place where W_t acts

While the theoretical basis for the stress analysis of gear teeth is presented, the Lewis equation must be modified for practical design and analysis. One important limitation is that it does not take into account the stress concentration that exists in the fillet of the tooth. Figure 9–9 is a drawing made from an experimental stress analysis of the stress distribution in gear teeth. It indicates a stress concentration in the fillet at the root of the tooth as well as high contact stresses at the mating surface (contact stress is discussed in the following section). Comparing the actual stress at the root with that predicted by the Lewis equation enables us to determine the stress concentration factor, K_t, for the fillet area. Placing this into Equation (9–13) gives

$$\sigma_t = \frac{W_t P_d K_t}{FY} \tag{9–14}$$

The value of the stress concentration factor is dependent on the form of the tooth, the shape and size of the fillet at the root of the tooth, and the point of application of the force on the tooth. Note that the value of the Lewis form factor, Y, also depends on the tooth geometry. Therefore, the two factors are combined into one term, the *geometry factor, J,* where $J = Y/K_t$. The value of J also, of course, varies with the location of the point of application of the force on the tooth because Y and K_t vary.

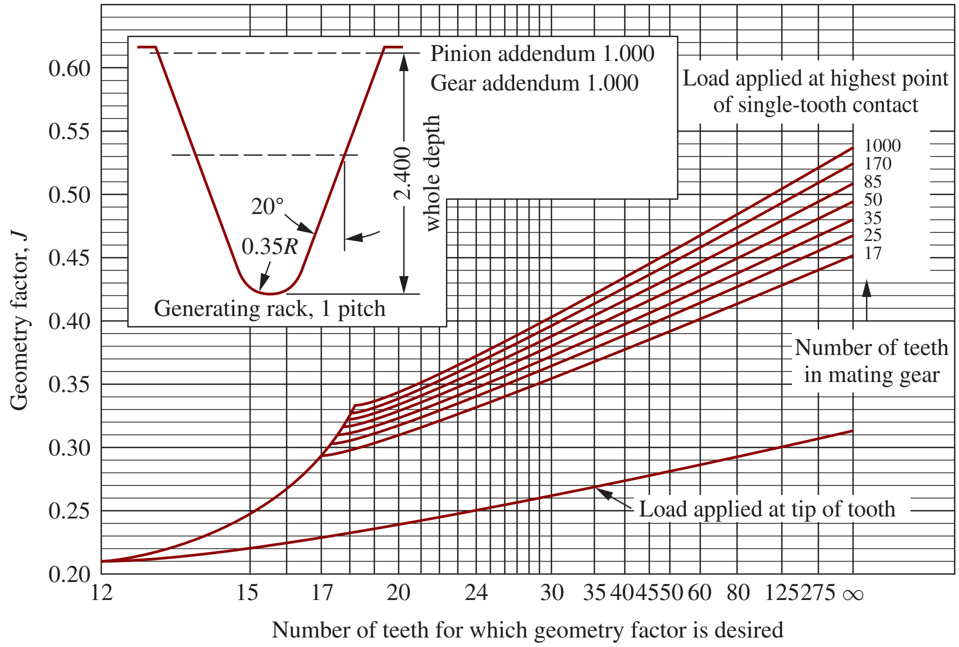
Figure 9–10 shows graphs giving the values for the geometry factor for 20° and 25°, full-depth, involute teeth. The safest value to use is the one for the load applied at the tip of the tooth. However, this value is overly conservative because there is some load sharing by another tooth at the time that the load is initially applied at the tip of a tooth. The critical load on a given tooth occurs when the load is at the highest point of single-tooth contact, when the tooth carries the entire load. The upper curves in Figure 9–10 give the values for J for this condition.

Using the geometry factor, J, in the stress equation gives

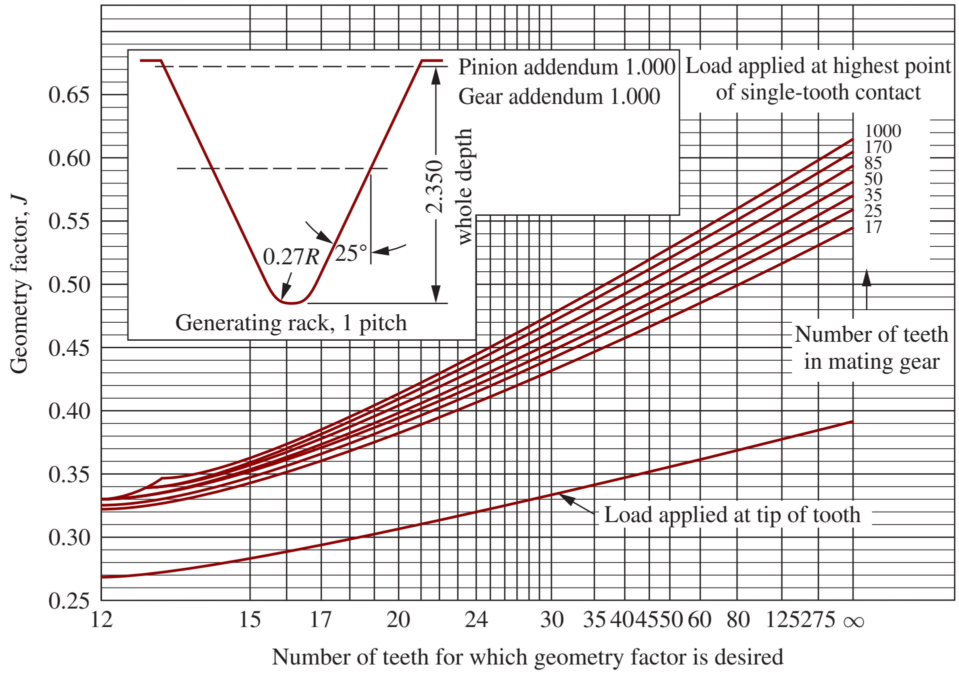
$$\sigma_t = \frac{W_t P_d}{FJ} \tag{9–15}$$

The graphs in Figure 9–10 are taken from the former AGMA Standard 218.01 which has been superseded by the two new standards: AGMA 2001, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, (Reference 6), and AGMA 908 *Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth*, (Reference 3). Standard 908 includes an analytical method for calculating the geometry factor, J. But the values for J are unchanged from those in the former standard. Rather than graphs, the new standard reports values for J for a variety of tooth forms in tables. The graphs from the former standard are shown in Figure 9–10 so that you can visualize the variation of J with the number of teeth in the pinion and the gear.

Note also that J factors for only two tooth forms are included in Figure 9–10 and that the values are valid only for those forms. Designers must ensure that J factors for the tooth form actually used, including the form of the fillet, are included in the stress analysis.



(*a*) 20° spur gear: standard addendum



(*b*) 25° spur gear: standard addendum

FIGURE 9–10 Geometry factor, *J* (Extracted from AGMA 218.01 Standard, *Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314)

Equation (9–15) can be called the *modified Lewis equation*. Other modifications to the equation are recommended by the AGMA in Standard 2001 for practical design to account for the variety of conditions that can be encountered in service.

The approach used by the AGMA is to apply a series of additional modifying factors to the bending stress from the modified Lewis equation to compute a value called the *bending stress number*, s_t. These factors represent the degree to which the actual loading case differs from the theoretical basis of the Lewis equation. The result is a better estimate of the real level of bending stress that is produced in the teeth of the gear and the pinion.

Then, separately, the allowable bending stress number, s_{at}, is modified by a series of factors that affect that value when the environment is different from the nominal situation assumed when the values for s_{at} are set.

The result here is a better estimate of the real level of the bending strength of the material from which the gear or the pinion is made.

The design is completed in a manner that ensures that the bending stress number is less than the modified allowable bending stress number. This process should be completed for both the pinion and the gear of a given pair because materials may be different; the geometry factor, J, is different; and other operating conditions may be different. This is demonstrated in example problems later in this chapter.

Often the major decision to be made is the specification of suitable materials from which to make the pinion and the gear. In such cases, the required basic allowable bending stress number, s_{at}, will be computed. When steel is used, the required hardness of the material is found from the data described in Section 9–7. Finally, the material and its heat treatment are specified to ensure that it will have at least the required hardness.

We proceed now with the discussion of the bending stress number, s_t.

Bending Stress Number, s_t

The design analysis method used here is based primarily on AGMA Standard 2001. However, because values for some of the factors are not included in the standard, data from other sources are added. These data illustrate the kinds of conditions that affect the final design. The designer ultimately has the responsibility for making appropriate design decisions.

The following equation will be used in this book:

➪ Bending Stress Number, s_t

$$s_t = \frac{W_t P_d}{FJ} K_o K_s K_m K_B K_v \qquad \textbf{(9–16)}$$

where W_t = tangential force
P_d = diametral pitch
F = face width
J = geometry factor
K_o = overload factor for bending strength
K_s = size factor for bending strength
K_m = load-distrubution factor for bending strength
K_B = rim thickness factor
K_v = dynamic factor for bending strength

Methods for specifying values for these factors are discussed below.

Tangential Force, W_t,

The tangential force, W_t, is the driving force of the gear mesh. The pinion transmits a force tangent to the pitch circle on the gear. The method for calculating this force is described in detail in Section 9–3.

Diametral Pitch, P_d

The diametral pitch is the size of the gear tooth and is related to the number of teeth and the pitch diameter. If the diametral pitch is not given as a specific value in a problem, use Figure 9–11 to select a trial value. Figure 9–11 provides initial guidance when selecting a diametral pitch for a pair of steel gears. The graph of design power transmitted versus the pinion rotational speed was derived for selected pitches and pinion diameters. Steel that is through hardened to HB 300 is used to generate the graph. Because of the numerous variables involved, the value of P_d read from the figure is only an initial target value. Subsequent iterations may require considering a different value, either higher or lower.

The design power equation is:

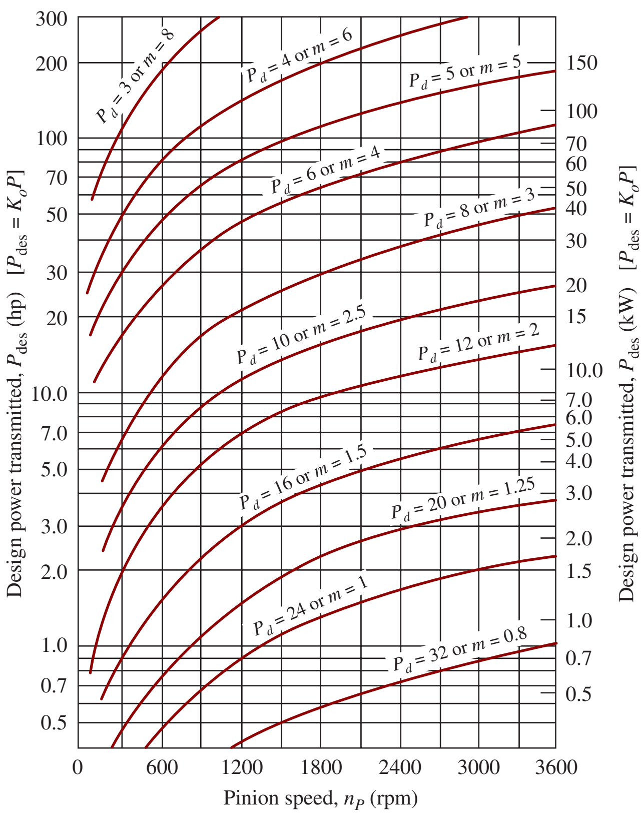
$$P_{design} = K_o \cdot P$$

where

K_o is the overload factor which will be explained in detail in this section

P is the power being transmitted by the gear pair

P_{design} is the power level used for the design of the gear drive system



For all curves: 20° full depth teeth;
$N_P = 24$; $N_G = 96$; $m_G = 4.00$; $F = 12/P_d$; $A_v = 11$
Steel gears, HB 300; $s_{at} = 36000$ psi (250 MPa); $s_{ac} = 126000$ psi (869 MPa)

FIGURE 9–11 Design power transmitted versus pinion speed for spur gears with different pitches and diameters

Face Width, F

The face width of the gear tooth should fall within this range

$$\frac{8}{P_d} < F < \frac{16}{P_d}$$

The upper limit given tends to minimize alignment problems and ensures reasonable loading across the face. When the face width is less than the lower limit, it is probable that a more compact design can be achieved with a smaller diametral pitch. Also, the face width normally is less than twice the pitch diameter of the pinion.

For problems solved in this book, either a face width will be given that falls within the range or the nominal face width value can be calculated using:

$$F = \frac{12}{P_d}$$

Geometry Factor, J

The geometry factor, as discussed in the previous section, is found for both the pinion and gear. Figure 9–10(a) and (b) are used to select the geometry factor, based on the pressure angle and the number of teeth of the meshing gear set. For a 30 tooth pinion and a 50 tooth gear with a pressure angle of 20°, the geometry factor from Figure 9–10(a) would be 0.38 for the pinion and 0.41 for the gear.

Overload Factor, K_o

Overload factors consider the probability that load variations, vibrations, shock, speed changes, and other application-specific conditions may result in peak loads greater than W_t being applied to the gear teeth during operation. A careful analysis of actual conditions should be made, and the AGMA Standard 2001-D04 gives no specific values for K_o. Reference 20 gives some recommended values, and many industries have established suitable values based on experience.

For problem solutions in this book, we will use the values shown in Table 9–1. The primary considerations are the nature of *both* the driving power source and the driven machine. An overload factor of 1.00 would be applied for a perfectly smooth electric motor driving a perfectly smooth generator through a gear-type speed reducer. Any rougher conditions call for a value of K_o greater than 1.00. For power sources we will use the following:

Uniform: Electric motor or constant-speed gas turbine

Light shock: Water turbine, variable-speed drive

Moderate shock: Multicylinder engine

Examples of the roughness of driven machines include the following:

Uniform: Continuous-duty generator, paper, and film winders.

Light shock: Fans and low-speed centrifugal pumps, liquid agitators, variable-duty generators, uniformly loaded conveyors, rotary positive displacement pumps, and metal strip processing.

Moderate shock: High-speed centrifugal pumps, reciprocating pumps and compressors, heavy-duty conveyors, machine tool drives, concrete mixers, textile machinery, meat grinders, saws, bucket elevators, freight elevators, escalators, concrete mixers, plastics molding and processing, sewage disposal equipment, winches, and cable reels.

Heavy shock: Rock crushers, punch press drives, pulverizers, processing mills, tumbling barrels, wood chippers, vibrating screens, railroad car dumpers, log conveyors, lumber handling equipment, metal shears, hammer mills, commercial washers, heavy-duty hoists and cranes, reciprocating feeders, dredges, rubber processing, compactors, and plastics extruders.

Size Factor, K_s

The AGMA indicates that the size factor can be taken to be 1.00 for most gears. But for gears with large-size teeth or large face widths, a value greater than 1.00 is recommended. Reference 20 recommends a value of 1.00 for diametral pitches of 5 or greater or for a metric module of 5 or smaller. For larger teeth, the values shown in Table 9–2 can be used.

Load-Distribution Factor, K_m

The determination of the load-distribution factor is based on many variables in the design of the gears themselves

TABLE 9–1 Suggested Overload Factors, K_o

	Driven Machine			
Power source	Uniform	Light shock	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.50	1.75
Light shock	1.20	1.40	1.75	2.25
Moderate shock	1.30	1.70	2.00	2.75

TABLE 9–2 Suggested Size Factors, K_s

Diametral pitch, P_d	Metric module, m	Size factor, K_s
≥ 5	≤ 5	1.00
4	6	1.05
3	8	1.15
2	12	1.25
1.25	20	1.40

as well as in the shafts, bearings, housings, and the structure in which the gear drive is installed. Therefore, it is one of the most difficult factors to specify. Much analytical and experimental work is continuing on the determination of values for K_m.

If the intensity of loading on all parts of all teeth in contact at any given time were uniform, the value of K_m would be 1.00. However, this is seldom the case. Any of the following factors can cause misalignment of the teeth on the pinion relative to those on the gear:

1. Inaccurate gear teeth.
2. Misalignment of the axes of shafts carrying gears.
3. Elastic deformations of the gears, shafts, bearings, housings, and support structures.
4. Clearances between the shafts and the gears, the shafts and the bearings, or the bearings and the housing.
5. Thermal distortions during operation.
6. Crowning or end relief of gear teeth.

AGMA Standard 2001 presents extensive discussions of methods of determining values for K_m. One is empirical and considers gears up to 40 in (1000 mm) wide. The other method is analytical and considers the stiffness and mass of individual gears and gear teeth and the total mismatch between mating teeth. We will not provide so much detail. However, rough guidelines are given below.

The designer can minimize the load-distribution factor by specifying the following:

1. Accurate teeth (a low quality number from AGMA 2015).
2. Narrow face widths.
3. Gears centered between bearings (straddle mounting).
4. Short shaft spans between bearings.
5. Large shaft diameters (high stiffness).
6. Rigid, stiff housings.
7. High precision and small clearances on all drive components.

You are advised to study the details of AGMA Standard 2001 which covers a wide range of physical sizes for gear systems. But the gear designs discussed in this book are of moderate size, typical of power transmissions in light industrial and vehicular applications. A more limited set of data are reported here to illustrate the concepts that must be considered in gear design.

We will use the following equation for computing the value of the load-distribution factor:

$$K_m = 1.0 + C_{pf} + C_{ma} \qquad \textbf{(9–17)}$$

where C_{pf} = pinion proportion factor (see Figure 9–12)
C_{ma} = mesh alignement factor (see Figure 9–13)

In this book, we are limiting designs to those with face widths of 15 in or less. Wider face widths call for additional factors. Also, some commercially successful designs employ modifications to the basic tooth form to achieve a more uniform meshing of the teeth. Such methods are not discussed in this book.

Figure 9–12 shows that the pinion proportion factor is dependent on the actual face width of the pinion and on the ratio of the face width to the pinion pitch diameter. Figure 9–13 relates the mesh alignment factor to expected accuracy of different methods of applying gears.

- *Open gearing* refers to drive systems in which the shafts are supported in bearings that are mounted on structural elements of the machine with the expectation that relatively large misalignments will result.
- In *commercial-quality enclosed gear units*, the bearings are mounted in a specially designed housing that provides more rigidity than for open gearing, but for which the tolerances on individual dimensions are fairly loose.
- *Precision enclosed gear units* are made to tighter tolerances.
- *Extra-precision enclosed gear* units are made to exacting precision and are often adjusted at assembly to achieve excellent alignment of the gear teeth.

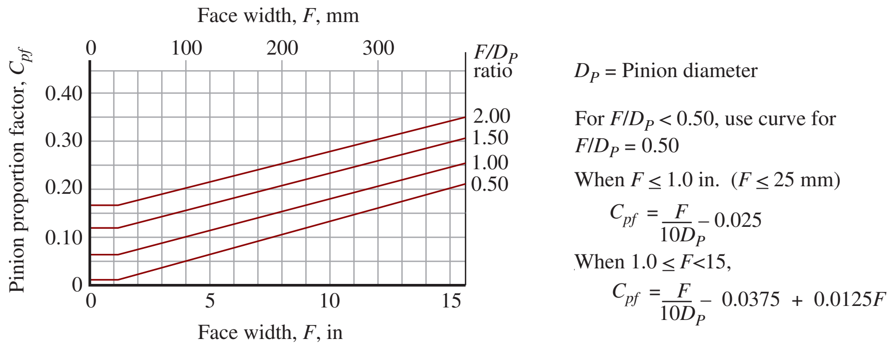


FIGURE 9–12 Pinion proportion factor, C_{pf} (Extracted from AGMA 2001 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth,* with permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314)

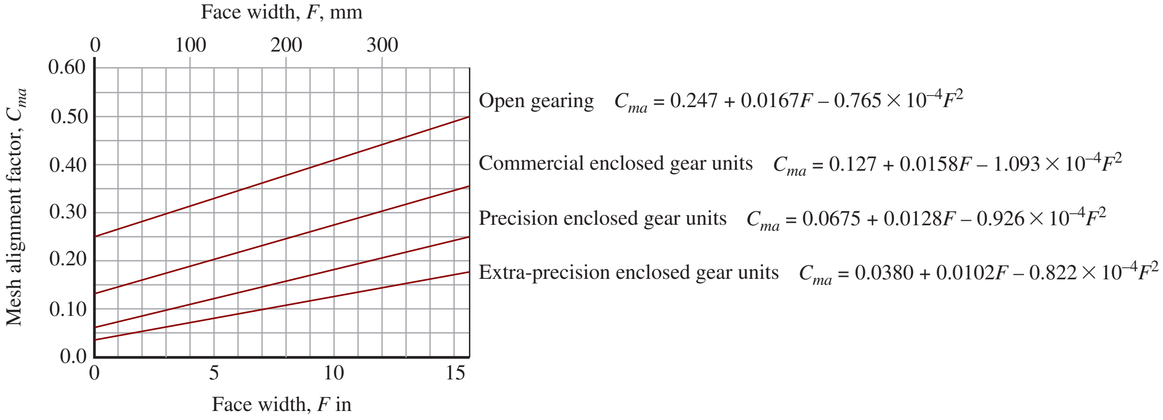


FIGURE 9–13 Mesh alignment factor, C_{ma} (Extracted from AGMA 2001 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314)

Experience with similar units in the field will help you gain better understanding among the different types of designs. *Commercial* or *precision* types are recommended for this book.

Rim Thickness Factor, K_B

The basic analysis used to develop the Lewis equation assumes that the gear tooth behaves as a cantilever attached to a perfectly rigid support structure at its base. This is true if the gear is made from a solid blank as shown in Figure 8–4(b) or with a blank that has a thinned web as shown in Part (d) of that figure. These are typical of small to medium-sized gears. Larger gears are often made with the spoked design shown in Figure 8-4(a) in order to save material and produce a lighter gear. Commercially made spoked gears can be expected to have a well-supported rim. For these kinds of gears, $K_B = 1.0$ can be used.

However, when designing a spoked gear for a special application, care must be exercised that the rim is sufficiently stiff to support the gear teeth without dangerous stresses created in the rim. Figure 9–14 should be used to estimate the influence of rim thickness. The key geometry parameter is called the *backup ratio*, m_B, where

$$m_B = t_R/h_t$$

t_R = rim thickness

h_t = whole depth of the gear tooth

For $m_B > 1.2$, the rim is sufficiently strong and stiff to support the tooth, and $K_B = 1.0$.

Special Case for Small Pinions Mounted to a Shaft with a Keyseat.

A frequent design option is to mount a small pinion onto a shaft as shown in Figure 9–15, with a key used to transmit the torque from the pinion to the shaft, and requiring a keyway to be machined into the bore of the pinion and into the shaft. Care must be taken to ensure that there is sufficient material above the keyway. It is recommended that the condition of $m_B > 1.2$ be applied in such cases where the t_R, dimension is measured above the top of the keyway. Then the factor $K_B = 1.0$ can be used.

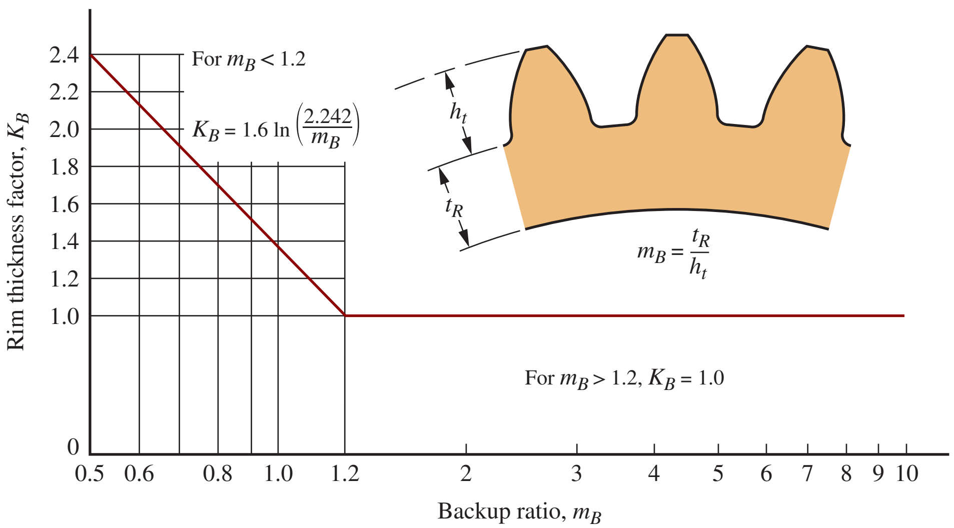


FIGURE 9–14 Rim thickness factor, K_B (Extracted from AGMA 2001 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314)

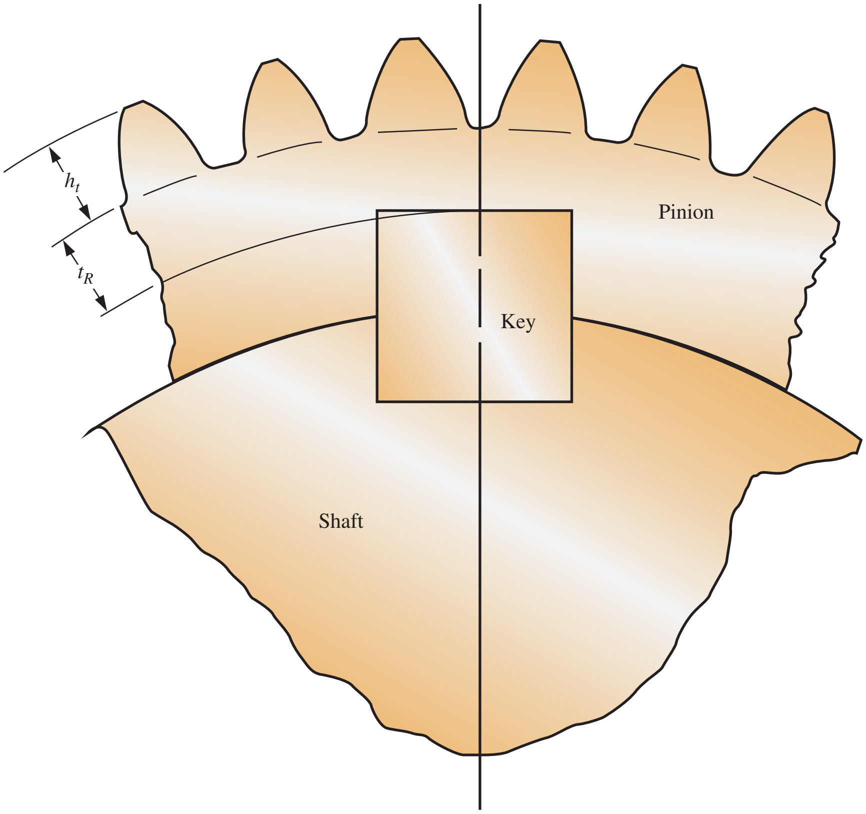


FIGURE 9–15 Pinion mounted on a shaft

If it is impractical to provide $m_B > 1.2$, it is recommended that *the pinion be machined integral with the shaft*, thus eliminating the need for a key and then using $K_B = 1.0$.

Dynamic Factor, K_v

The dynamic factor accounts for the fact that the load is assumed by a tooth with some degree of impact and that the actual load subjected to the tooth is higher than the transmitted load alone. The value of K_v depends on the accuracy of the tooth profile, the elastic properties of the tooth, and the speed with which the teeth come into contact. These factors are quantified by the use of *quality numbers* as described next.

Standards for Gear Quality

Manufacturers, designers, and users of gears must agree on standards used to determine the acceptability of gears produced. Several standard-setting organizations exist, including the AGMA (United States), ISO (International), DIN (German), JIS (Japan), and others. While the standards are similar, they are not identical. Designers and manufacturers should become familiar with the provisions of these standards in order to communicate effectively the intent of their designs, the capability of manufacturing processes, and the acceptability of a given gear.

As indicated in Table 9–3, the current AGMA standard 2015-1-A01 (Reference 15), employs 10 accuracy grades from A2 (most precise) to A11 (least precise). An analytical measurement system [Figure 8-37] performs a total of nine elemental tangential measurements required for high-accuracy gears (A2–A5); five measurements for medium accuracy gears (A6–A9); and three for low accuracy gears (A10–A11).

The AGMA 2015-1-A01 standard replaces standard AGMA 2000-A88 that was used in the previous edition of this book and for which there are many gears in

TABLE 9–3 Accuracy Groups for AGMA 2015 Gear Quality System

Low (L)	A10–A11
Medium (M)	A6–A9
High (H)	A2–A5

TABLE 9–4 General Corelations of AGMA 2000, AGMA 2015, and ISO 1328 Gear Quality Systems

AGMA 2000	AGMA 2015	ISO 1328	AGMA 2000	AGMA 2015	ISO 1328
	—Least precise—				
Q5	—	12	Q11	A6	6
Q6	A11	11	Q12	A5	5
Q7	A10	10	Q13	A4	4
Q8	A9	9	Q14	A3	3
Q9	A8	8	Q15	A2	2
Q10	A7	7		—Most precise—	

service that were made to that standard. It also replaces ISO 1328-1 that had been an interim standard modeled on the ISO methodology. For reference, Table 9–4 shows the rough relationships among the three standards although the actual values of tolerances are not identical. One notable difference is that the former AGMA 2000 standard employed classifications from Q5 (least precise) to Q15 (most precise); the order is opposite from AGMA 2015. A convenient comparison can be made by noting that the sum of the quality numbers from AGMA 2015 and AGMA 2000 is always 17. For example, Q12 and A5 are closely equivalent.

In this book, we use the AGMA 2015-1-A01 standard. Its classifications of A2 to A11 are keyed to a term called the *dynamic factor,* K_v, introduced later in this chapter. Note that the actual tables of data for tolerances are listed in terms of the metric module system with tolerance values given in micrometers (μm). Be aware that the tolerance values are quite small. For tolerance grade A5, tolerances are in the order of around 6.0 μm (0.00024 in) for single pitch deviation for small gears (100 mm; 3.94 in) with small teeth ($m = 1$ mm; $P_d = 25.4$) to about 22 μm (0.00088 in) for larger gears (800 mm; 31.5 in) with large teeth ($m = 50$ mm; $P_d = 0.51$). Gears are *very precise* mechanical components.

For some applications, often for those of the low accuracy types, the parties involved may agree to use another part of the standard, AGMA 2015-2-A06 for radial measurements based on the total radial composite deviation and the tooth-to-tooth radial composite deviation as shown in Figure 8-36. For this standard, nine classifications are called C4 (most precise) through C12 (least precise). The *double flank roll tester*, described earlier, is used to perform the testing.

Recommended Quality Numbers

All gear-type machinery should be manufactured with good levels of precision reflecting the accuracy with which the gears must operate for good performance, long life, smooth operation, and low noise. The design of the entire system, including shafts, bearings, and the housing must be consistent with the expected degree of precision. Of course, the system should not be made more precise than necessary because of cost. For this reason, manufacturers recommend quality numbers that will give satisfactory performance at a reasonable cost for a variety of applications. Table 9–5 lists several examples.

Machine tools, such as lathes, machining centers, and grinders, are included in the lower part of Table 9–5 with the recommended quality numbers keyed to the pitch line speed of the gears, as defined in Equations (9–1) and (9–2). Higher speeds require greater accuracy. ***These values should be used for other kinds of precision industrial equipment not listed in the first part of the table.***

We now continue with the discussion of the dynamic factor, K_v. Figure 9–16 shows a graph of the AGMA-recommended values for K_v, where the A_v numbers are the AGMA-quality numbers described in this section. Gears in typical machine design would fall into the classes represented by curves 8 to 11 which are for gears made by hobbing or shaping with average to good tooling. If the teeth are finish-ground or shaved to improve the accuracy

TABLE 9–5 Recommended AGMA Quality Numbers

Application	Quality number	Application	Quality number
Cement mixer drum drive	A11	Small power drill	A9
Cement kiln	A11	Clothes washing machine	A8
Steel mill drives	A11	Printing press	A7
Grain harvester	A10	Computing mechanism	A6
Cranes	A10	Automotive transmission	A6
Punch press	A10	Radar antenna drive	A5
Mining conveyor	A10	Marine propulsion drive	A5
Paper-box-making machine	A9	Aircraft engine drive	A4
Gas meter mechanism	A9	Gyroscope	A2

Machine tool drives and drives for other high-quality mechanical systems

Pitch line speed (fpm)	Quality number	Pitch line speed (m/s)
0–800	A10	0–4
800–2000	A8	4–11
2000–4000	A6	11–22
Over 4000	A4	Over 22

of the tooth profile and spacing, curves 6 or 7 should be used. Under special conditions where teeth of high precision are used in applications where there is little chance of developing external dynamic loads, the shaded area can be used ($A_v = 2-5$). If the teeth are cut by form milling, curve 12 or higher should be used. Note that the quality 12 gears should not be used at pitch line speeds above 3000 ft/min (15 m/s). Note also that the dynamic factors are approximate. For severe applications, especially those operating above 4000 ft/min (20 m/s), approaches taking into account the material properties, the mass and inertia of the gears, and the actual error in the tooth form should be used to predict the dynamic load. (See References 6, 19, 20, 22, and 24.)

Equations for Dynamic Factor, K_v. Reading values for K_v from either part of Figure 9–16 gives adequate accuracy because the charts are approximate and slight

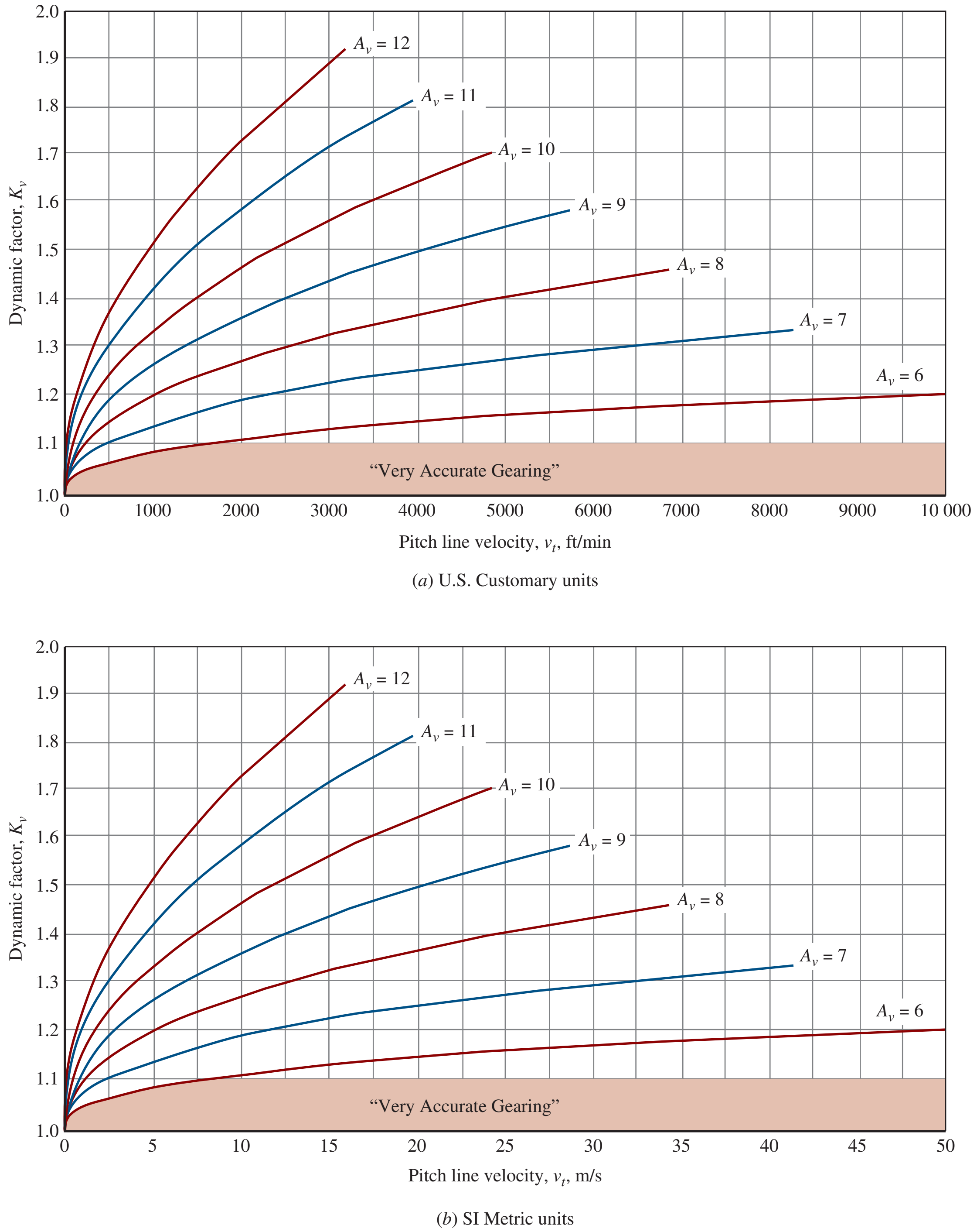


FIGURE 9–16 Dynamic factor, K_v (Adapted from AGMA 2001 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5^{th} floor, Alexandria, VA 22314)

differences in reading should not cause difficulty with a gear design. However, it is recommended that an analytical method be used when performing several designs. Later in this chapter, a computer-aided approach using a spreadsheet is shown with the analytical method embedded. Table 9–6 shows equations adapted from Reference 6 and presented in both the U.S. and SI Metric unit systems. Variables involved are the dimensionless quality number (A_v) and the pitch line velocity in ft/min for the U.S. system and m/s for the SI Metric system. Both of these data are specified or computed in early stages of the gear design process. The method involves the calculation of two intermediate terms, B and C, and then computing K_v.

TABLE 9–6 Analytical Method for Computing Dynamic Factor, K_v

U.S. units		SI Metric units	
Given A_v and v_t (ft/min)		Given A_v and v_t (m/s)	
$B = 0.25(A_v - 5.0)^{0.667}$		$B = 0.25(A_v - 5.0)^{0.667}$	
$C = 50 + 56(1.0 - B)$		$C = 3.5637 + 3.9914(1.0 - B)$	
$K_v = \left[\frac{C}{C + \sqrt{v_t}}\right]^{-B}$		$K_v = \left[\frac{C}{C + \sqrt{v_t}}\right]^{-B}$	
Maximum velocity for a given A_v:		Maximum velocity for a given A_v:	
$v_{t\,max} = [C + (14 - A_v)]^2$		$v_{t\,max} = [C + (14 - A_v)]^2$	
A_v	$v_{t\,max}$ (ft/min)	A_v	$v_{t\,max}$ (m/s)
6	10 000	6	50.8
7	8239	7	41.9
8	6867	8	34.9
9	5731	9	29.1
10	4767	10	24.2
11	3937	11	20.0
12	3219	12	16.4

Completing the Calculation of Bending Stress

At this point, all of the data required for completing the calculation of the bending stress number, s_{at}, using Equation 9–16, have been defined. Example Problem 9–3 demonstrates the final calculation of that value. Note that some of the same data are used for computing the contact stress in the next section.

Example Problem 9–3 Compute the bending stress numbers for the pinion and gear of the pair of gears similar to the pair shown in Figure 9–2(a) and (b). The pinion rotates at 1750 rpm, driven directly by an electrical motor. The driven machine is an industrial saw requiring 25 hp. The gear unit is enclosed and is made to commercial standards. Gears are straddle mounted between bearings. The following gear data apply:

$$N_p = 20 \quad N_g = 70 \quad A_v = 10$$

The gear teeth are 20°, full depth, involute teeth and the gear blanks are solid. The gears will be made from steel, so Figure 9–11 can be used to select an initial value for the diametral pitch.

Solution The equation for the design power is:

$$P_{\text{design}} = K_o \cdot P$$

The required power for the industrial saw is known to be 25 hp. The overload factor is found from Table 9–1. For a smooth, uniform electric motor driving an industrial saw generating moderate shock, a reasonable value would be:

$$K_o = 1.5$$
$$P_{\text{design}} = 1.5 \cdot (25 \text{ hp}) = 37.5 \text{ hp}$$

Since the gears will be made of steel, Figure 9–11 can be used to find an initial diametral pitch based on the design power and the pinion angular velocity, $n_p = 1750$ rpm.

The diametral pitch selected is then $P_d = 6$. Recall that this value has the unit of *teeth/in* or in^{-1}. We will use this form later to ensure proper units in the calculation for stresses in the pinion and the gear.

Data that are useful to visualize the overall size of the gear pair and that serve as input to later decisions are now computed:

Pitch diameters of gear set:

$$D_P = \frac{N_P}{P_d} = \frac{20}{6} = 3.333 \text{ in}$$
$$D_G = \frac{N_G}{P_d} = \frac{70}{6} = 11.667 \text{ in}$$

Center distance of gear set:

$$C = \frac{D_P + D_G}{2} = \frac{3.333 \text{ in} + 11.667 \, in}{2} = 7.500 \text{ in}$$

Velocity ratio of gear set:

$$VR = m_G = \frac{n_P}{n_G} = \frac{N_G}{N_P} = \frac{70}{20} = 3.50$$

The speed of the gear can be found from by rewriting the equation for the velocity ratio:

$$VR = \frac{n_P}{n_G}$$

$$n_G = \frac{n_P}{VR} = \frac{1750 \text{ rpm}}{3.50} = 500 \text{ rpm}$$

The pitch line speed (in ft/min or fpm) can be calculated using the pitch diameter and the angular velocity of the pinion:

$$v_t = \frac{D_p}{2} \cdot n_P = \frac{3.333 \text{ in}}{2} \cdot 1750 \text{ rev/min} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 1527 \text{ fpm}$$

We can also use the gear pitch diameter and angular velocity to calculate the pitch line speed:

$$v_t = \frac{D_G}{2} \cdot n_G = \frac{11.667 \text{ in}}{2} \cdot 500 \text{ rev/min} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 1527 \text{ fpm}$$

We can then use the principles from Section 9–3 to compute the transmitted load on the gear teeth. First find the torque on the pinion and gear:

$$T_P = \frac{Power}{n_P} = \frac{25 \text{ hp}}{1750 \text{ rev/min}} \cdot \frac{33000 \, \frac{\text{lb} \cdot \text{ft}}{\text{min}}}{1 \text{ hp}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 900.4 \text{ lb} \cdot \text{in}$$

$$T_G = \frac{Power}{n_G} = \frac{25 \text{ hp}}{500 \text{ rev/min}} \cdot \frac{33000 \, \frac{\text{lb} \cdot \text{ft}}{\text{min}}}{1 \text{ hp}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 3151.3 \text{ lb} \cdot \text{in}$$

We can see that a smaller torque on the pinion shaft will produce a larger torque on the gear shaft.

$$\frac{T_G}{T_P} = \frac{3151.3 \text{ lb} \cdot \text{in}}{900.4 \text{ lb} \cdot \text{in}} = 3.50$$

You will notice this is the same value as the velocity ratio. The output torque will increase in proportion to the decrease in angular velocity. This means that the motor torque required is 900.4 lb-in for an output torque of 3151.3 lb-in.

We will use the Equation (9–16) to compute the bending stress number:

$$s_t = \frac{W_t P_d}{FJ} K_o K_s K_m K_B K_v$$

Let's go through each term of this equation:

The tangential force (gear driving force) is based on the pinion torque and pinion pitch diameter:

$$W_t = \frac{T_P}{\left(\frac{D_P}{2}\right)} = \frac{900.4 \text{ lb} \cdot \text{in}}{\left(\frac{3.333 \text{ in}}{2}\right)} = 540.3 \text{ lb}$$

The tangential force can also be calculated using the gear torque and gear pitch diameter:

$$W_t = \frac{T_G}{\left(\frac{D_G}{2}\right)} = \frac{3151.3 \text{ lb} \cdot \text{in}}{\left(\frac{10.667 \text{ in}}{2}\right)} = 540.3 \text{ lb}$$

Although the radial and normal gear forces are not required for the bending stress equation, we will calculate all the forces on the spur gear teeth. The radial force (gear separating force) is:

$$W_r = W_t \cdot \tan(\varphi) = 540.3 \text{ lb} \cdot \tan(20°) = 196.6 \text{ lb}$$

The normal force along the line of action is:

$$W_n = \sqrt{W_t^2 + W_r^2} = \sqrt{(540.3 \text{ lb})^2 + (196.6 \text{ lb})^2} = 575 \text{ lb}$$

The nominal face width of the gears has been defined as $F = 12/P_d$ and we know $P_d = 6$.

$$F = \frac{12}{P_d} = \frac{12}{6} = 2.00 \text{ in}$$

The geometry factor, J, will be determined for both the pinion and the gear. The pressure angle is 20° so we use Figure 9–10(a).

$$J_P = 0.335$$
$$J_G = 0.420$$

The overload factor was already determined:

$$K_o = 1.5$$

Teeth with a diametral pitch, $P_d = 6$, are relatively small so, from Table 9–2, the size factor is:

$$K_S = 1$$

The load-distribution factor, K_m, can be found from Equation (9–17) for commercial enclosed gear drives. For this design, $F = 2.00$ in and

$$\frac{F}{D_P} = \frac{2.00 \text{ in}}{3.333 \text{ in}} = 0.60$$

From Figure 9–12 we can find the approximate value for the pinion proportion factor:

$$C_{pf} \approx 0.04$$

To obtain a more precise value for the pinion proportion factor, we can use the equation that represents the curve in Figure 9–12, when $1.0 \leq F < 15$:

$$C_{pf} = \frac{F}{10D_P} - 0.0375 + 0.0125F = \frac{2.00}{10 \cdot 3.333} - 0.0375 + 0.0125 \cdot 2.00 = 0.047$$

The mesh alignment factor, C_{ma}, can be determined from Figure 9–13 using the commercial enclosed gear unit curve:

$$C_{ma} = 0.16$$

We can also use the equation that represents the commercial enclosed gear unit curve to calculate a more precise value:

$$C_{ma} = 0.127 + 0.0158 \cdot F - 1.093 \times 10^{-4} \cdot F^2$$
$$C_{ma} = 0.127 + 0.0158 \cdot 2.00 - 1.093 \times 10^{-4} \cdot (2.00)^2 = 0.158$$

Substitute these two factors into Equation (9–17) to calculate the load distribution factor:

$$K_m = 1.0 + C_{PF} + C_{ma} = 1.0 + 0.047 + 0.158 = 1.21$$

The rim thickness factor, K_B, can be taken as 1.00 because the gears are to be made from solid blanks with no cast or machined rim.

$$K_B = 1.00$$

The dynamic factor, K_v, can be read from Figure 9–16. For a pitch line velocity, $v_t = 1527$ fpm and a gear quality number of, $A_v = 10$, the dynamic factor is:

$$K_v = 1.41$$

The bending stress can now be computed from Equation (9–16). We will compute the bending stress of the pinion first:

$$s_{tP} = \frac{540 \text{ lb} \cdot 6 \text{ in}^{-1}}{2.00 \text{ in} \cdot 0.335} \cdot 1.50 \cdot 1.0 \cdot 1.21 \cdot 1.0 \cdot 1.41 = 12\,376 \text{ psi}$$

Notice all factors in the stress equation are the same for the gear except the value of the geometry factor, J. The bending stress number for the gear is

$$s_{tG} = \frac{540 \text{ lb} \cdot 6 \text{ in}^{-1}}{2.00 \text{ in} \cdot 0.420} \cdot 1.50 \cdot 1.0 \cdot 1.21 \cdot 1.0 \cdot 1.41 = 9871 \text{ psi}$$

The stress in the pinion teeth will always be higher than the stress in the gear teeth because the value of J increases as the number of teeth increases.

9–6 CONTACT STRESS IN GEAR TEETH

In addition to being safe from bending, gear teeth must also be capable of operating for the desired life without significant pitting of the tooth form. *Pitting* is the phenomenon in which small particles are removed from the surface of the tooth faces because of the high contact stresses, causing fatigue. Refer again to Figure 9–9 showing the high, localized contact stresses. Prolonged operation after pitting begins causes the teeth to roughen, and eventually the form is deteriorated. Rapid failure follows. Note that both the driving and driven teeth are subjected to these high contact stresses equally. Reference 11 provides a comprehensive treatment of the wear and failure of gear teeth.

The action at the contact point on gear teeth is that of two externally curved surfaces. If the gear materials were infinitely rigid, the contact would be a simple line. Actually, because of the elasticity of the materials, the tooth shape deforms slightly, resulting in the transmitted force acting on a small rectangular area. The resulting stress is called a *contact stress* or *Hertz stress*. Reference 17 gives the following form of the equation for the Hertz stress,

➩ Hertz Contact Stress on Gear Teeth

$$\sigma_c = \sqrt{\frac{W_c}{F}\,\frac{1}{\pi\{[(1-\nu_1^2)/E_1] + [(1-\nu_2^2)/E_2]\}}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)} \qquad \textbf{(9–18)}$$

where the subscripts 1 and 2 refer to the materials of the two bodies in contact. The tensile modulus of elasticity is E and the Poisson's ratio is ν. W_c is the contact force exerted between the two bodies, and F is the length of the contacting surfaces. The radii of curvature of the two surfaces are called r_1 and r_2.

When Equation (9–18) is applied to gears, F is the face width of the gear teeth and W_c is the normal force delivered by the driving tooth on the driven tooth, found from Equation (9–12) to be,

$$W_N = W_t/\cos\phi \qquad \textbf{(9–19)}$$

The second term in Equation (9–18) (including the square root) can be computed if the elastic properties of the materials for the pinion and gear are known. It is given the name *elastic coefficient*, C_P. That is,

➩ Elastic Coefficient

$$C_P = \sqrt{\frac{1}{\pi\{[(1-\nu_P^2)/E_P] + [(1-\nu_G^2)/E_G]\}}} \qquad \textbf{(9–20)}$$

Table 9–7 gives values for the most common combinations of materials for pinions and gears.

The terms r_1 and r_2 are the radii of curvature of the involute tooth forms of the two mating teeth. These radii change continuously during the meshing cycle as the contact point moves from the top of the tooth through the pitch

TABLE 9–7 Elastic Coefficient, C_p

		Gear material and modulus of elasticity, E_G, lb/in² (MPa)					
Pinion material	Modulus of elasticity, E_P, lb/in² (MPa)	Steel 30×10^6 (2×10^5)	Malleable iron 25×10^6 (1.7×10^5)	Nodular iron 24×10^6 (1.7×10^5)	Cast iron 22×10^6 (1.5×10^5)	Aluminum bronze 17.5×10^6 (1.2×10^5)	Tin bronze 16×10^6 (1.1×10^5)
Steel	30×10^6	2300	2180	2160	2100	1950	1900
	(2×10^5)	(191)	(181)	(179)	(174)	(162)	(158)
Mall. iron	25×10^6	2180	2090	2070	2020	1900	1850
	(1.7×10^5)	(181)	(174)	(172)	(168)	(158)	(154)
Nod. iron	24×10^6	2160	2070	2050	2000	1880	1830
	(1.7×10^5)	(179)	(172)	(170)	(166)	(156)	(152)
Cast iron	22×10^6	2100	2020	2000	1960	1850	1800
	(1.5×10^5)	(174)	(168)	(166)	(163)	(154)	(149)
Al. bronze	1.75×10^6	1950	1900	1880	1850	1750	1700
	(1.2×10^5)	(162)	(158)	(156)	(154)	(145)	(141)
Tin bronze	16×10^6	1900	1850	1830	1800	1700	1650
	(1.1×10^5)	(158)	(154)	(152)	(149)	(141)	(137)

Source: Extracted from AGMA Standard 2001-D04, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314.

Note: Poisson's ratio = 0.30; units for C_p are $(\text{lb/in}^2)^{0.5}$ or $(\text{MPa})^{0.5}$.

circle, and onto the lower flank of the tooth before leaving engagement. We can write the following equations for the radius of curvature when contact is at the pitch point,

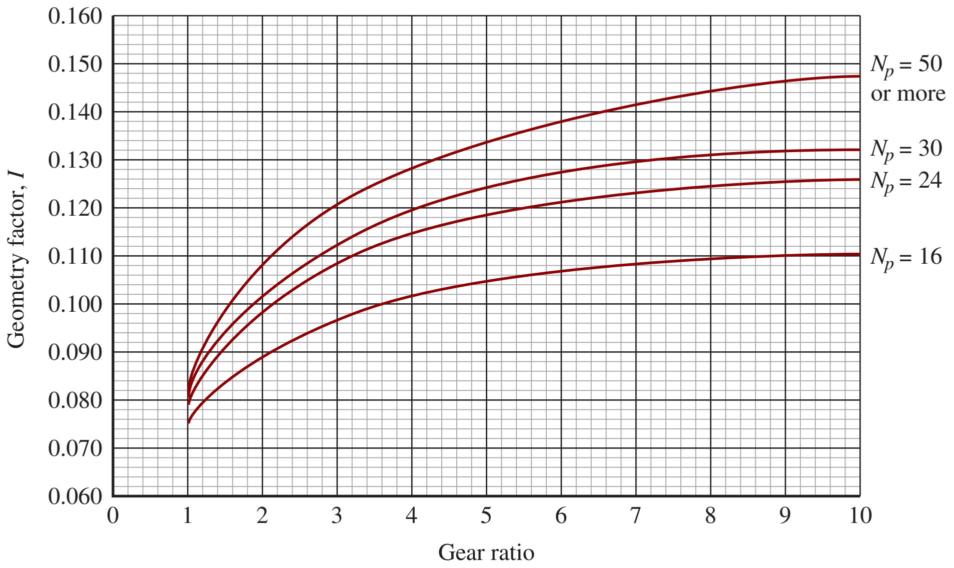
$$r_1 = (D_P/2)\sin\phi \quad \text{and} \quad r_2 = (D_G/2)\sin\phi \qquad \textbf{(9–21)}$$

However, the AGMA calls for the computation of the contact stress to be made at the lowest point of single tooth contact (LPSTC) because above that point, the load is being shared with other teeth. Computation of the radii of curvature for the LPSTC is somewhat complex. A geometry factor for pitting, I, is defined by the AGMA to include the radii of curvature terms and the $\cos\phi$ term in Equation (9–19) because they all involve the specific geometry of the tooth. The variables required to compute I are the pressure angle ϕ, the gear ratio $m_G = N_G/N_P$, and the number of teeth in the pinion N_p.

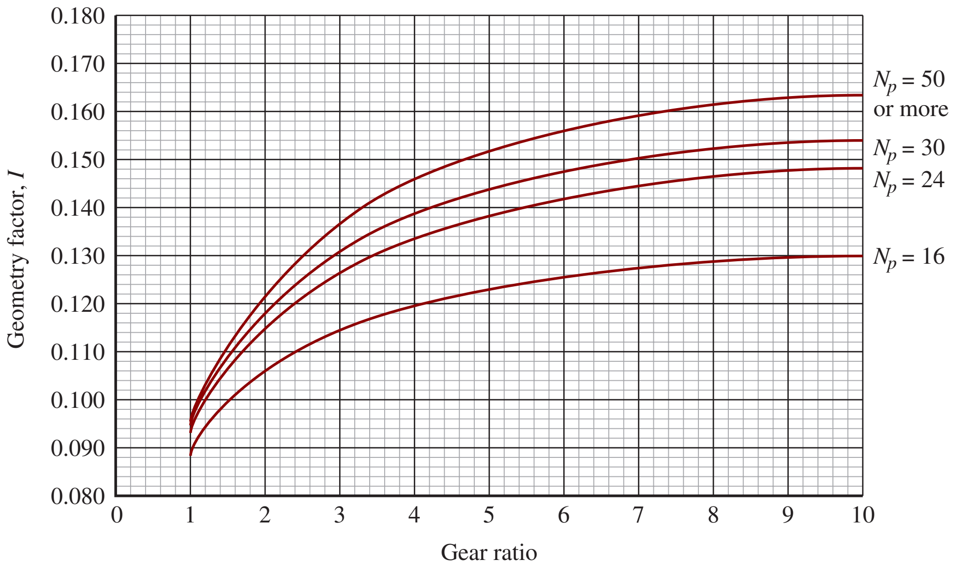
Another factor in the contact stress equation is the pinion diameter that is not included in I. The contact stress equation then becomes,

$$\sigma_c = C_p\sqrt{\frac{W_t}{FD_PI}} \qquad \textbf{(9–22)}$$

Values for the geometry factor, I, for a few common cases are graphed in Figure 9–17 and should be used for



(a) 20° pressure angle, full-depth teeth (standard addendum = $1/P_d$)



(b) 25° pressure angle, full-depth teeth (standard addendum = $1/P_d$)

FIGURE 9–17 External spur pinion geometry factor, I, for standard center distances. All curves are for the lowest point of single-tooth contact on the pinion (Extracted from AGMA Standard 218.01, *Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314) problem solving in this book. Appendix 19 provides an approach to computing the value for I for spur gears as given in Reference 3.

As with the equation for bending stress in gear teeth, several factors are added to the equation for contact stress as shown below. The resulting quantity is called the *contact stress number*, s_c:

➩ Contact Stress Number

$$s_c = C_p\sqrt{\frac{W_t K_o K_s K_m K_v}{F D_P I}} \qquad (9\text{–}23)$$

The values for the overload factor, K_o; the size factor, K_s; the load-distribution factor, K_m; and the dynamic factor, K_v, can be taken to be the same as the corresponding values for the bending stress analysis in the preceding sections.

Equation 9–23 is used to compute the contact stress for both the pinion and the gear; they are equal. It is not correct to use the diameter of the gear, D_G, in this equation.

Example Problem 9–4 follows to illustrate the use of Equation 9–23. Then Section 9–7 presents more information about the types of metallic materials typically used for gears and their heat treatment. Section 9–8 summarizes the process of selecting gear materials, applying information from Sections 9–4 to 9–7.

Example Problem 9–4 Compute the contact stress number for the gear pair described in Example Problem 9–3.

Solution: Data from Example Problem 9–3 are summarized as follows:

$N_P = 20$ $\quad N_G = 70$ $\quad F = 2.00$ in $\quad W_t = 540$ lb $\quad D_P = 3.333$ in

$K_o = 1.50$ $\quad K_s = 1.00$ $\quad K_m = 1.21$ $\quad K_v = 1.41$ $\quad P_d = 6$

Using the gear ratio:

$$m_G = \frac{N_G}{N_P} = \frac{70}{20} = 3.50$$

The number of teeth of the pinion and the pressure angle 20° are used to read an approximate value of pitting resistance geometry factor, I, from Figure 9–17(a):

$$I = 0.108$$

The design analysis for the bending strength indicated that two steel gears should be used. From Table 9–7, we find that:

$$C_P = 2300$$

From Equation (9–23) the contact stress number is:

$$s_C = C_P\sqrt{\frac{W_t K_o K_S K_m K_v}{F D_P I}}$$

$$s_C = 2300\sqrt{\frac{540 \cdot 1.50 \cdot 1.00 \cdot 1.21 \cdot 1.41}{2.00 \cdot 3.333 \cdot 0.108}} = 100\,769 \text{ psi}$$

This value is used for both the pinion and the gear.

9–7 METALLIC GEAR MATERIALS

Gears can be made from a wide variety of materials to achieve properties appropriate to the application. From a mechanical design standpoint, strength and pitting resistance are the most important properties. But, in general, the designer should consider the producibility of the gear, taking into account all of the manufacturing processes involved, from the preparation of the gear blank, through the forming of the gear teeth, to the final assembly of the gear into a machine. Other considerations are weight, appearance, corrosion resistance, noise, and, of course, cost. This section discusses several types of metals used for gears. Plastics are covered in a later section.

Many carbon and alloy steels are used for gears and most are given heat treatments to provide controlled hardness and strength (References 8 and 26). Selection of the alloy depends, in part, on the heat-treatment process used to achieve the final properties. Table 9–8 gives some examples.

TABLE 9–8 Examples of Gear Materials

Heat treatment	Typical alloys (SAE numbers)
Through-hardened or Case-hardened by flame or induction hardening	1045, 4140, 4150, 4340, 4350
Carburizing, case-hardened	1020, 4118, 4320, 4820, 8620, 9310

It is important to recognize that alloys listed for through-hardening, flame, or induction hardening must have good hardenability to achieve the desired hardness levels described in the following section. They must also retain reasonable ductility indicated by the percent elongation in the heat-treated condition. All those listed are medium-carbon steels.

Steels that are to be carburized are typically low carbon steels because the carburization process itself infuses significant amounts of carbon into the case while the steel is at a high temperature. Then the final case hardness is achieved by quenching and tempering, leaving the core of the teeth at a lower strength but with higher ductility.

Steel Gear Materials

Through-Hardened Steels. Gears for machine tool drives and many kinds of medium- to heavy-duty speed reducers and transmissions are typically made from medium-carbon steels. AGMA Standard 2001 (Reference 6) gives data for the allowable bending stress number, s_{at}, and the allowable contact stress number, s_{ac}, for steels in the through-hardened condition. Figures 9–18 and 9–19 are graphs relating the stress numbers to the Brinell hardness number, HB, for the teeth. Notice that only knowledge of the hardness is required because of the direct relationship between hardness and the tensile strength of steels. See Appendix 17 for data that correlate the Brinell hardness number, with the tensile strength of steel in ksi. The range of hardnesses covered by the AGMA data is from 180 to 400 HB, corresponding to a tensile strength of approximately 87 to 200 ksi. ***It is not recommended to use through-hardening above 400 HB because of inconsistent performance of the gears in service.*** Typically, case hardening is used when there is a desire to achieve a surface hardness above 400 HB. This is described later in this section.

The hardness measurement for the allowable bending stress number is to be taken at the root of the teeth because that is where the highest bending stress occurs. The allowable contact stress number is related to the surface hardness on the face of the gear teeth where the mating teeth experience high contact stresses.

When selecting a material for gears, the designer must specify one that can be hardened to the desired hardness. Review Chapter 2 for discussions about heat-treatment techniques. Consult Appendices 3 and 4 for representative data. For the higher hardnesses, say, above 250 HB, a medium-carbon-alloy steel with good hardenability is desirable. Examples are, listed in Table 9–8. Ductility is also rather important because of the numerous cycles of stress experienced by gear teeth and the likelihood of occasional overloads, impact, or shock loading. A percent elongation value of 12% or higher is desired.

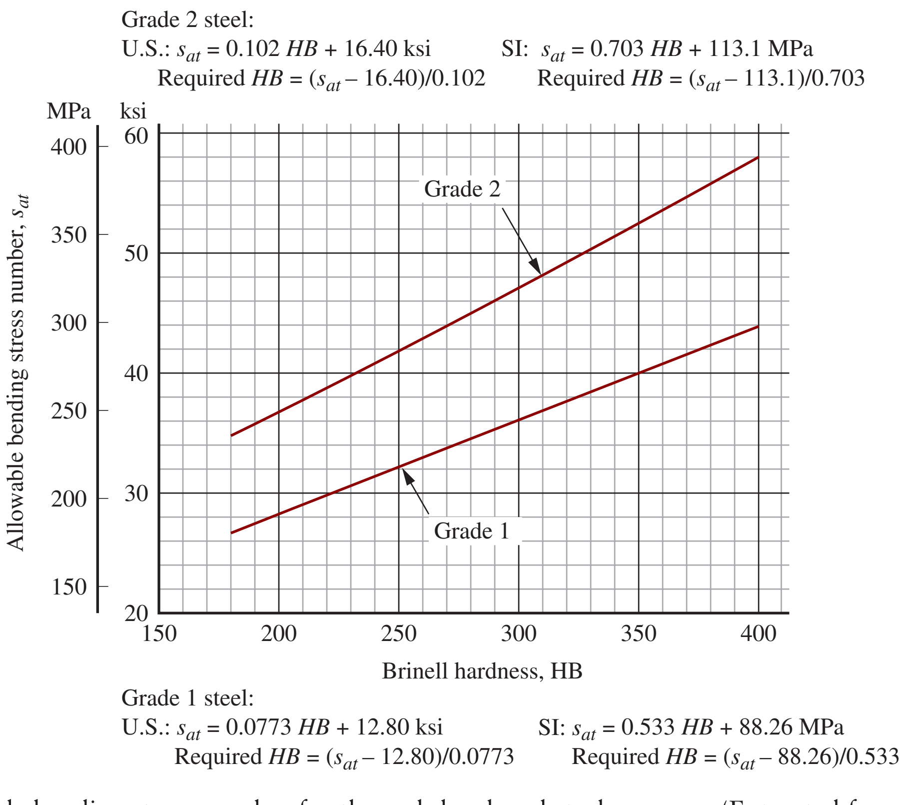


FIGURE 9–18 Allowable bending stress number for through-hardened steel gears, s_{at} (Extracted from AGMA 2001-D04 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314) [Reference 6]

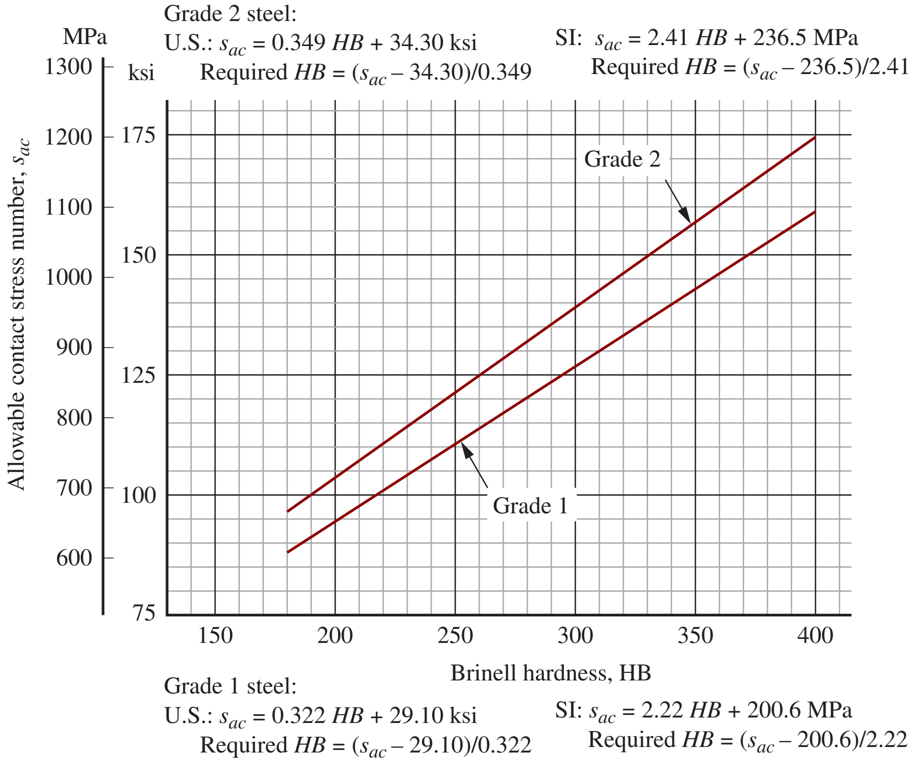


FIGURE 9–19 Allowable contact stress number for through-hardened steel gears, s_{ac} (Extracted from AGMA 2001-D04 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314) [Reference 6]

The curves in Figures 9–18 and 9–19 include two grades of steel: Grade 1 and Grade 2. ***Grade 1 is considered to be the basic standard and will be used for problem solutions in this book.*** Grade 2 requires a higher degree of control of the microstructure, alloy composition, greater cleanliness, prior heat treatment, nondestructive testing performed, core hardness values, and other factors. See AGMA Standard 2001 (Reference 6) for details. Because of these extra requirements, cost is likely to be higher.

Case-Hardened Steels. Flame hardening, induction hardening, carburizing, and nitriding are processes used to produce a high hardness in the surface layer of gear teeth. See the related discussion in Section 2–6. These processes provide surface hardness values from 50 to 64 HRC (Rockwell C) and correspondingly high values of s_{at} and s_{ac}, as shown in Table 9–9. Special discussions are given below for each of the types of case-hardening processes.

Case-hardened steel gears can be produced to Grades 1, 2, and 3 with Grade 1 referring to typically available steels as discussed for through-hardened steels. Because of the special care required in producing Grades 2 and 3, Table 9–9 shows data for only Grade 1 steels as recommended for use in this book. Furthermore, because nitriding can be done in several ways and it is not used as frequently as carburizing, flame-, or induction hardening, data for design values of bending or pitting resistance stresses are not listed here. Consult Reference 6 for the necessary data.

Flame- and Induction-Hardened Gear Teeth. Recall that these processes involve the local heating of the surface of the gear teeth by high-temperature gas flames or electrical induction coils. By controlling the time and energy input, the manufacturer can control the depth of heating and the depth of the resulting case. It is essential that the heating occur around the entire tooth, producing the hard case on the face of the teeth *and in the fillet and root areas*, in order to use the stress values listed in Table 9–9. This may require a special design for the flame shape or the induction heater. Refer to Reference 6.

The specifications for flame- or induction-hardened steel gear teeth call for a resulting hardness of HRC 50 to 54. Because these processes rely on the inherent hardenability of the steels, you must specify a material that can be hardened to these levels. Normally, medium-carbon-alloy steels (approximately 0.40% to 0.60% carbon) are specified. Table 9–8 and Appendices 3 and 4 list some suitable materials.

Carburizing. Carburizing produces surface hardnesses in the range of 55 to 64 HRC. It results in some of the highest strengths in common use for gears. Special

TABLE 9–9 Allowable Stress Numbers for Case-Hardened Grade 1 Steel Materials

	Allowable bending stress number, s_{at}		Allowable contact stress number, s_{ac}	
Hardness at surface	**(ksi)**	**(Mpa)**	**(ksi)**	**(Mpa)**
Flame- or induction-hardened				
50 HRC	45	310	170	1172
54 HRC	45	310	175	1207
Carburized and case-hardened				
55–64 HRC	55	379	180	1241

Source: Extracted from AGMA Standard 2001-D04, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314.

carburizing steels are listed in Appendix 5. Figure 9–20 shows the AGMA recommendation for the thickness of the case for carburized gear teeth. The effective case depth is defined as the depth from the surface to the point where the hardness has reached 50 HRC.

Nitriding. Nitriding produces a very hard *but very thin* case. It is specified for applications in which loads are smooth and well known. Nitriding should be avoided when overloading or shock can be experienced, because the case is not sufficiently strong or well supported to resist such loads. Because of the thin case, the Rockwell 15N scale is used to specify hardness. See References 6 and 8 for design data for nitrided gears.

Iron and Bronze Gear Materials

Cast Irons. Two types of iron used for gears are *gray cast iron* and *ductile* (sometimes called *nodular*) iron. Table 9–10 gives the common ASTM grades used, with their corresponding allowable bending stress numbers and contact stress numbers. Remember that gray cast iron is brittle, so care should be exercised when shock loading is possible. Also, the higher-strength forms of the

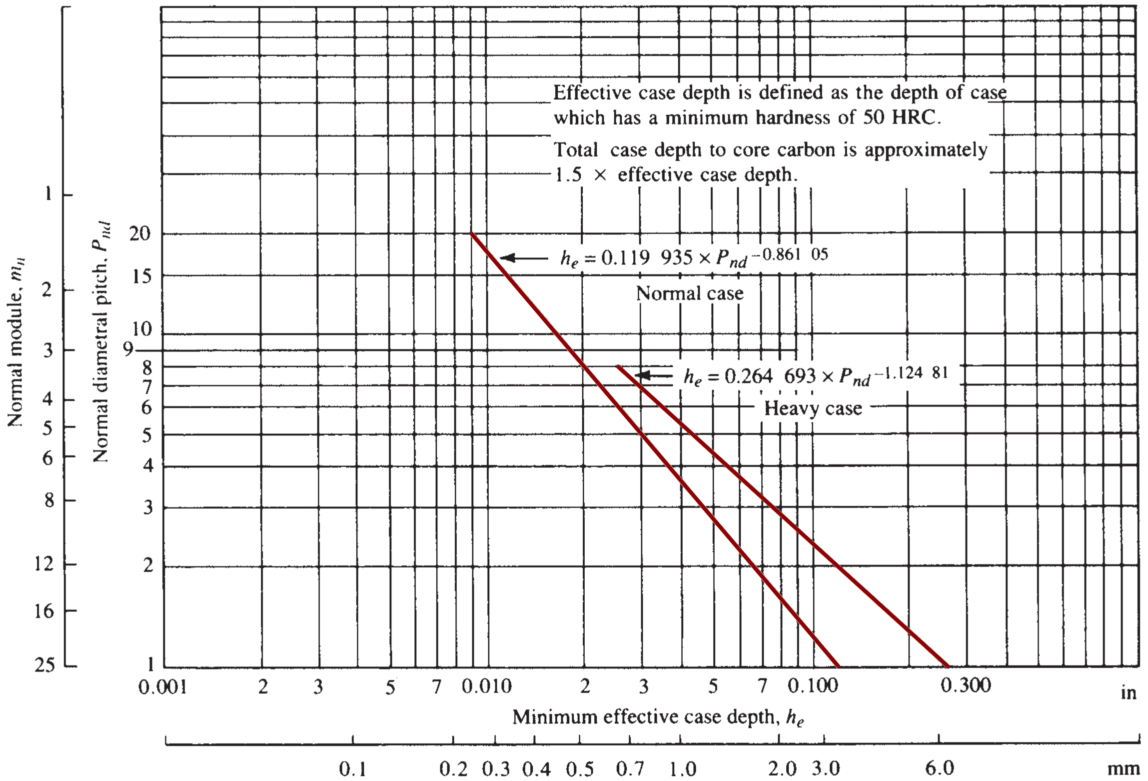


FIGURE 9–20 Effective case depth for carburized gears, h_e (Extracted from AGMA 2001-D04 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314)

TABLE 9–10 Allowable Stress Numbers for Iron and Bronze Gears

Material designation	Minimum hardness at surface (HB)	Allowable bending stress number, s_{at} (ksi)	(MPa)	Allowable contact stress number, s_{ac} (ksi)	(MPa)
Gray cast iron, ASTM A48, as cast					
Class 20		5	35	50	345
Class 30	174	8.5	59	65	448
Class 40	201	13	90	75	517
Ductile (nodular) iron, ASTM A536					
60-40-18 annealed	140	22	152	77	530
80-55-06 quenched and tempered	179	22	152	77	530
100-70-03 quenched and tempered	229	27	186	92	634
120-90-02 quenched and tempered	269	31	214	103	710
Bronze, sand-cast, $s_{u\,min} = 40$ ksi (275 MPa)		5.7	39	30	207
Bronze, heat-treated, $s_{u\,min} = 90$ ksi (620 MPa)		23.6	163	65	448

Source: Extracted from AGMA Standard 2001-D04, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314.

other irons have low ductility. Austempered ductile iron (ADI) is being used in some important automotive applications. However, standardized allowable stress numbers have not yet been specified.

Bronzes. Four families of bronzes are typically used for gears: (1) phosphor or tin bronze, (2) manganese bronze, (3) aluminum bronze, and (4) silicon bronze. Yellow brass is also used. Most bronzes are cast, but some are available in wrought form. Corrosion resistance, good wear properties, and low friction coefficients are some reasons for choosing bronzes for gears. Table 9–10 shows allowable stress numbers for one bronze alloy in two common forms.

9–8 SELECTION OF GEAR MATERIALS

We now use the values of bending stress found in Section 9–5 and contact stress from Section 9–6 to specify a suitable material and its condition that will withstand those stresses without tooth breakage caused by bending stress and with good resistance to pitting on the face of the teeth caused by contact stress. We develop here an approach for specifying steel for the gears. A similar method can be used for other materials such as cast iron and bronze. Later in this chapter, we discuss the design of plastic gears. The goal of the process is to ensure that the predicted stresses are less than the allowable strength of the material. Inputs to the decisions are:

1. Estimated bending stress number, s_t, as found in Section 9–5, Equation (9–16).
2. Estimated contact stress number, s_c, as found in Section 9–6, Equation (9–23).
3. Desired service life and reliability for the gears, discussed in this section.
4. Chosen safety factor, *SF*, discussed in this section.
5. Allowable bending stress number, s_{at}, and allowable contact stress number for the material, s_{ac}, discussed in Section 9–7.

Note about temperature. The methods developed in this section assume that the operating temperature for the gears is less than 250°F (121°C) and above 32°F (0°C) because we rely on published data for the materials obtained within these limits. Higher temperatures can be considered if data are available on how the material strength is affected. One concern is that many steel gears are heat treated to produce desired strength and ductility and higher operating temperatures can reduce the strength due to tempering. Operating below freezing temperature can be done provided that testing is done to verify adequate impact strength using the Charpy or Izod methods discussed in Chapter 2.

General principles for specifying materials. We use the following relationships to guide the process of specifying suitable materials for gears.

$$s_t < s_{at}' \qquad \text{and} \qquad s_c < s_{ac}' \tag{9–24}$$

The "prime" applied to the allowable strengths indicates that adjustments are made to published data for s_{at} and s_{ac} that are presented in Section 9–7. Conditions for those data are:

- Operating temperatures are above 32°F (0°C) and below 250°F (121°C).
- The materials are exposed to 10^7 loading cycles.

- The expected reliability is 99%, less than one failure in 100.
- The safety factor, SF, is 1.00.

The adjustments to s_{at} and s_{ac} considered here take the form

$$s_{at}' = s_{at}\frac{Y_N}{(SF)(K_R)} \quad (9\text{–}25)$$

$$s_{ac}' = s_{ac}\frac{Z_N}{(SF)(K_R)} \quad (9\text{–}26)$$

where SF = safety factor
K_R = reliability factor
Y_N = bending strength stress cycle factor
Z_N = pitting resistance stress cycle factor

Safety factor: The value of SF is typically taken to be 1.0 because most of the uncertainties involved in computing the bending and contact stresses are included in the equations for s_{at} and s_{ac} by the factors K_o, K_s, K_v, and K_m. A design decision can be made for using $SF > 1.00$ for extra safety or for anticipated undesirable conditions beyond these factors. AGMA recommends

$$1.00 \le SF \le 1.50$$

Reliability factor: Table 9–11 shows typical values for K_R and the choice of which to apply in a given project is a design decision.

Desired life of the gear drive: Designers must evaluate each project to specify what the design life is to be. Table 9–12 shows one set of recommended design life values in hours of operation. The range from 20 000 to 30 000 hours for general industrial machines is a reasonable choice unless known conditions of the types listed exist. The same table for design life is used for rolling contact bearings in Chapter 14. Also needed is the *number of load cycles* that can be computed using this equation:

$$N_c = (60)(L)(n)(q) \quad (9\text{–}27)$$

where N_c = expected number of cycles of loading
L = design life in hours
n = rotational speed of the gear in rpm
q = number of load applications per revolution

TABLE 9–11 Reliability Factor, K_R

Reliability	K_R
0.90, one failure in 10	0.85
0.99, one failure in 100	1.00
0.999, one failure in 1000	1.25
0.9999, one failure in 10 000	1.50

The normal number of load applications per revolution for any given tooth is typically, of course, one. But consider the case of an idler gear that serves as both a driven and a driving gear in a gear train. It receives two cycles of load per revolution: one as it receives power from and one as it delivers power to its mating gears. Also, in certain types of gear trains, one gear may deliver power to two or more gears mating with it. Gears in a planetary gear train often have this characteristic.

As an example of the application of Equation (9–27), consider that the pinion in Example Problems 9–3 and 9–4 is designed to have a life of 20 000 h. When rotating at 1750 rpm. Then

$$N_c = (60)(L)(n)(q) = (60)(20\,000)(1750)(1)$$
$$= 2.1 \times 10^9 \text{ cycles}$$

Because this is higher than 10^7, an adjustment in the allowable bending stress number must be made.

Stress Cycle Factors: Figures 9–21 and 9–22 show recommended values of Y_N for bending stress and Z_N for contact stress for steel gears. Features of these graphs include:

- Each is plotted on log–log scales with the *number of load cycles* on the horizontal axis and the *stress cycle factor* on the vertical axis.
- The stress cycle factor is approximately 1.0 for 10^7 load cycles on both graphs.
- The stress cycle factors are less than 1.0 for load cycles higher than 10^7 and the values are independent of the type of steel or its condition. Some designs for critical service applications use values lower than those shown and the AGMA standard provides an added shaded area for use in such conditions.

TABLE 9–12 Recommended Design Life

Application	Design life (h)
Domestic appliances	1000–2000
Aircraft engines	1000–4000
Automotive	1500–5000
Agricultural equipment	3000–6000
Elevators, industrial fans, multipurpose gearing	8000–15 000
Electric motors, industrial blowers, general industrial machines	20 000–30 000
Pumps and compressors	40 000–60 000
Critical equipment in continuous 24-h operation	100 000–200 000

Source: Eugene A. Avallone and Theodore Baumeister III, eds. *Marks' Standard Handbook for Mechanical Engineers.* 9th ed. New York: McGraw-Hill, 1986.

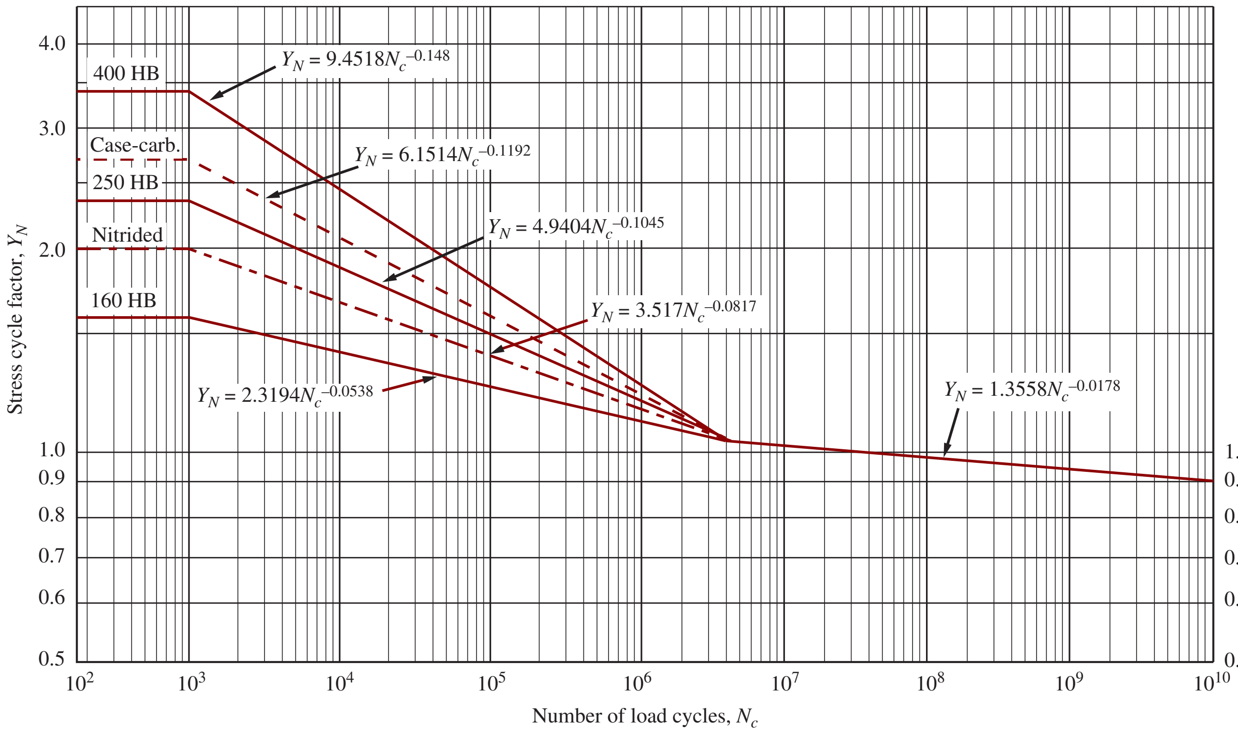


FIGURE 9–21 Bending strength stress cycle factor, Y_N (Adapted from AGMA Standard 2001-D04, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth,* with permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314)

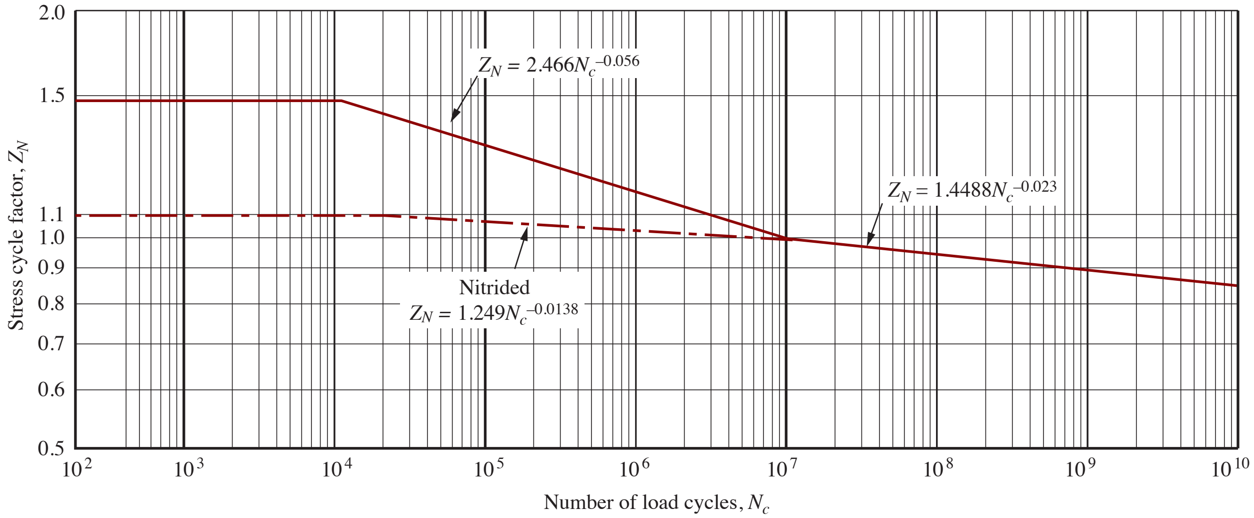


FIGURE 9–22 Pitting Resistance stress cycle factor, Z_N (Adapted from AGMA Standard 2001-D04, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth,* with permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314)

- The stress cycle factors are greater than 1.0 for load cycles lower than 10^7 and the values are dependent on the type of steel and its condition.
- Equations of an exponential form are given in the graphs to facilitate calculation of the values of the stress cycle factors.
- These charts do not apply to cast iron or to nonferrous materials such as bronze. Values of $Y_N = 1.00$ and $Z_N = 1.00$ can be used while ensuring that the actual strengths of materials specified have a modest margin of safety.

We can now describe the general approach to specifying suitable steels or other metallic materials for gears. The goal is to access the data in Section 9–7, including Figures 9–18 and 9–19 and Tables 9–9, and 9–10, that give accepted values of s_{at} and s_{ac} for through-hardened steels, case-hardened steels using flame or induction hardening, steels that are carburized and case hardened, cast irons, and bronzes. To facilitate this approach, we can combine Equations (9–24) to (9–26) and then solve for s_{at} and s_{ac}.

$$s_t < s_{at}' = s_{at}\frac{Y_N}{(SF)(K_R)} \tag{9–28}$$

$$s_c < s_{ac}' = s_{ac}\frac{Z_N}{(SF)(K_R)} \tag{9–29}$$

Then, solving for s_{at} and s_{ac} gives

$$\text{Required } s_{at} > s_t\frac{(SF)(K_R)}{Y_N} \tag{9–30}$$

$$\text{Required } s_{ac} > s_c\frac{(SF)(K_R)}{Z_N} \tag{9–31}$$

Both Equations (9–30) and (9–31) must be satisfied. It should be noted here that for gear drives with relatively long design lives, the design will most often be governed by the pitting resistance and that assumption is made in the following example problems and in more general design procedures.

Both Equations (9–30) and (9–31) must be satisfied and are used when selecting a gear material. If the material is known in the problem, Equations (9–28) and (9–29) are used to solve for the safety factor. The resulting Equations (9–32) and (9–33) are listed next.

For bending stress:

$$SF = \frac{s_{at}}{s_t}\cdot\frac{Y_N}{K_R} \tag{9–32}$$

For contact stress:

$$SF = \frac{s_{ac}}{s_c}\cdot\frac{Y_N}{K_R} \tag{9–33}$$

Example Problem 9–5 Consider three different materials and calculate the safety factor for the pinion and gear using the results from Example Problem 9–3 for the computed bending stresses. Design for a reliability of 0.999, fewer than one failure in 1000. The application is a drive for an industrial saw that will be fully utilized on a normal, one-shift, five-day per-week operation. The materials proposed are:

a. Ductile iron ASTM A536 type 80-55-06 quenched and tempered.
b. Though hardened steel pinion and gear: SAE 4340 OQT 1000.
c. SAE 8620 steel pinion and gear case hardened by carburizing to a hardness of Rockwell C 60-64 with a minimum effective case depth of 0.025 in.

Solution: The results required from Example Problem 9–3 are summarized here:

$N_P = 20$	$N_G = 70$	$P_d = 6$	$F = 2.00$ in
$n_P = 1750$ rpm	$n_G = 500$ rpm	$D_P = 3.333$ in	$D_G = 11.667$ in
$s_{tP} = 12\,376$ psi	$s_{tG} = 9871$ psi		

The equation relating the bending stress number with the adjusted allowable bending stress number is:

$$s_t < s_{at}' = s_{at}\frac{Y_N}{SF\cdot K_R}$$

Solving this equation for the safety factor, SF:

$$SF = \frac{s_{at}}{s_t}\cdot\frac{Y_N}{K_R}$$

Design decisions will be made on the reliability and design life. From Table 9–11, we find $K_R = 1.25$ for the desired reliability of 0.999 as specified in the problem. Because the saw will be fully utilized in an industrial environment, we choose a life of $L = 20\,000$ hours, using Table 9–12 as a guide.

Compute the number of load cycles for the pinion and the gear using Equation (9–27). Each tooth sees one load cycle per revolution, $q = 1$.

$$N_{cP} = (60)\cdot L\cdot n_p\cdot q = 60\cdot 20\,000\cdot 1750\cdot 1 = 2.10\times 10^9 \text{ cycles}$$

$$N_{cG} = (60)\cdot L\cdot n_G\cdot q = 60\cdot 20\,000\cdot 500\cdot 1 = 6.00\times 10^8 \text{ cycles}$$

From Figure 9–21, the bending strength stress cycle factor for the pinion and gear is:

$$Y_{NP} = 0.93$$

$$Y_{NG} = 0.95$$

Part (a): Let's look first at using ductile iron ASTM type A536 80-55-06. Using Table 9–10, the allowable bending stress number for this material is

$$s_{at} = 22\,000 \text{ psi}$$

Solving for the safety factor for the pinion:

$$SF = \frac{s_{at}}{s_{tP}} \cdot \frac{Y_{NP}}{K_R} = \frac{22\,000 \text{ psi}}{12\,376 \text{ psi}} \cdot \frac{0.93}{1.25} = 1.3$$

Using the same equation for the safety factor of the gear:

$$SF = \frac{s_{at}}{s_{tG}} \cdot \frac{Y_{NG}}{K_R} = \frac{22\,000 \text{ psi}}{9871 \text{ psi}} \cdot \frac{0.95}{1.25} = 1.7$$

The minimum safety factor is for the pinion and it is within the range of 1.0 to 1.5, as recommended by AGMA, so this design is considered to be satisfactory for bending stress.

Part (b) considers a through hardened steel material SAE 4340 OQT 1000 for the pinion and gear. Using Appendix 3 the properties of the material are:

$$s_u = 171\,000 \text{ psi}, s_y = 158\,000 \text{ psi, HB} = 363, 16\% \text{ elongation}$$

Using Figure 9–18, along with the material Brinell hardness of HB $= 363$, the allowable bending stress number for the though-hardened steel gear is:

$$s_{at} = 41\,000 \text{ psi}$$

Solving for the safety factor SF of the pinion and the gear:

$$SF_p = \frac{s_{at}}{s_{tP}} \cdot \frac{Y_{NP}}{K_R} = \frac{41\,000 \text{ psi}}{12\,376 \text{ psi}} \cdot \frac{0.93}{1.25} = 2.5$$

$$SF_G = \frac{s_{at}}{s_{tG}} \cdot \frac{Y_{NG}}{K_R} = \frac{41\,000 \text{ psi}}{9871 \text{ psi}} \cdot \frac{0.95}{1.25} = 3.2$$

The safety factor is well over 1.5 for both the pinion and the gear for bending stress with this through-hardened steel.

Part (c) considers a gear material, SAE 8620 case hardened by carburizing to Rockwell C 60-64 with a minimum effective case depth of 0.025 in.
Using Table 9–9, the allowable bending stress number for both the pinion and the gear is:

$$s_{at} = 54\,000 \text{ psi}$$

Solving for the safety factor of the pinion and the gear:

$$SF_P = \frac{s_{at}}{s_{tP}} \cdot \frac{Y_{NP}}{K_R} = \frac{54\,000 \text{ psi}}{12\,376 \text{ psi}} \cdot \frac{0.93}{1.25} = 3.2$$

$$SF_G = \frac{s_{at}}{s_{tG}} \cdot \frac{Y_{NG}}{K_R} = \frac{54\,000 \text{ psi}}{9871 \text{ psi}} \cdot \frac{0.95}{1.25} = 4.2$$

The safety factor is well over 1.5 for both the pinion and the gear for bending stress with a carburized, case hardened steel material.

Recall that it was stated earlier that contact stress is most often the most critical stress in the design and operation of gearing. Therefore, we need to evaluate these proposed materials for the gears for contact stress before judging whether they are satisfactory. We do that in Example Problem 9–6.

Example Problem 9–6

Consider the same three materials used in Example Problem 9–5 and calculate the safety factor for the pinion and gear. Use the results from Example Problems 9–1, 9–2, and 9–3 for the computed contact stress and use the same reliability and number of load cycles as used in Example Problem 9–4.

a. Ductile iron ASTM A536 80-55-06 quenched and tempered
b. Though hardened steel pinion and gear: SAE 4340 OQT 1000
c. SAE 8620 steel, case hardened by carburizing to Rc 60-64 with a minimum effective case depth of 0.025 in.

Solution: The results required from Example Problems 9–1, 9–2, 9–3 are:

$N_P = 20$	$N_G = 70$	$P_d = 6$	$F = 2.00$ in
$n_P = 1750$ rpm	$n_G = 500$ rpm	$D_P = 3.333$ in	$D_G = 11.667$ in
$s_{cP} = s_{cG} = 100\,769$ psi	$K_R = 1.25$	$N_{cP} = 2.1 \times 10^9$ cycles	$N_{cG} = 6.0 \times 10^9$ cycles

The equation relating the contact stress number with the adjusted allowable contact stress number is:

$$s_c < s'_{ac} = s_{at}\frac{Z_N}{SF \cdot K_R}$$

Use this equation to solve for the safety factor, SF:

$$SF = \frac{s_{ac}}{s_c} \cdot \frac{Z_N}{K_R}$$

From Figure 9–22, we find pitting resistance stress cycle factors:

$$Z_{NP} = 0.88$$
$$Z_{NG} = 0.91$$

Part (a) Let's consider the first proposed material, ductile iron ASTM A536 type 80-55-06. Using Table 9–9, the allowable contact stress number for this material is

$$s_{ac} = 77\,000 \text{ psi}$$

Solving for the safety factor SF of the pinion and gear:

$$SF_p = \frac{s_{ac}}{s_{cP}} \cdot \frac{Z_{NP}}{K_R} = \frac{77\,000 \text{ psi}}{100\,769 \text{ psi}} \cdot \frac{0.88}{1.25} = 0.54$$
$$SF_G = \frac{s_{ac}}{s_{cG}} \cdot \frac{Z_{NG}}{K_R} = \frac{77\,000 \text{ psi}}{100\,769 \text{ psi}} \cdot \frac{0.91}{1.25} = 0.56$$

Both safety factors are below the minimum value of 1.00, as recommended by AGMA and are therefore considered to be unsatisfactory for this application.

Part (b) considers a through hardened steel SAE 4340 OQT 1000. Using Appendix 3, the properties of the material are:

$$s_u = 171\,000 \text{ psi}, \ s_y = 158\,000 \text{ psi, HB} = 363, 17\% \text{ elongation}$$

Using Figure 9–19, along with a Brinell hardness of HB = 363 for though-hardened steel gears material, the allowable contact stress number is:

$$s_{ac} = 146\,000 \text{ psi}$$

The safety factors, SF, of the pinion and the gear are:

$$SF_p = \frac{s_{ac}}{s_{cP}} \cdot \frac{Z_{NP}}{K_R} = \frac{146\,000 \text{ psi}}{100\,769 \text{ psi}} \cdot \frac{0.88}{1.25} = 1.0$$
$$SF_G = \frac{s_{ac}}{s_{cG}} \cdot \frac{Z_{NP}}{K_R} = \frac{146\,000 \text{ psi}}{100\,769 \text{ psi}} \cdot \frac{0.91}{1.25} = 1.1$$

The safety factors for the pinion and gear are both within the range of 1.0 to 1.5, as recommended by AGMA, with the pinion's factor at the lowest limit.

Part (c) considers SAE 8620 steel, case hardened by carburizing to a Rc 60-64, with a minimum effective case depth of 0.025 in. Using Table 9–9, the allowable contact stress number is:

$$s_{ac} = 180\,000 \text{ psi}$$

The safety factors SF for the pinion and the gear are:

$$SF = \frac{s_{ac}}{s_{cP}} \cdot \frac{Z_{NP}}{K_R} = \frac{180\,000 \text{ psi}}{100\,769 \text{ psi}} \cdot \frac{0.88}{1.25} = 1.26$$
$$SF = \frac{s_{ac}}{s_{cP}} \cdot \frac{Z_{NP}}{K_R} = \frac{180\,000 \text{ psi}}{100\,769 \text{ psi}} \cdot \frac{0.91}{1.25} = 1.30$$

The safety factors for both the pinion and gear are well within the range of 1.0 to 1.5, as recommended by AGMA.

This completes the development of analyses for both bending and contact stresses and demonstration of how to apply them individually. Here we summarize the results and draw conclusions from them. Example Problems 9–3, 9–4, 9–5, and 9–6 were spread over many pages because the design principles and procedures were developed and demonstrated as those problems were completed. Let's now show the summary of the major design decisions and results of calculations.

Summary of the Results for Example Problems 9–3, 9–4, 9–5, and 9–6

Given data:

Application: Industrial saw operating at 500 rpm, powered by a 25 hp electric motor operating at 1750 rpm, using a gear reducer between them.

Output speed from the gear reducer: 500 rpm; velocity ratio: 3.50

Production operation: fully utilized; design life = 20 000 hours

Enclosed gear reducer made to commercial standards

High reliability for gear design: $R = 0.999$; less than one failure in 1000

Design decisions:

Diametral pitch: 6

Pressure angle: $\phi = 20°$

Face width: $F = 2.00$ in

Numbers of teeth: pinion = 20; gear = 70

Quality number for gear geometry: $A_v = 10$

Calculated results:

Geometry of gear pair and forces

Pitch diameters: $D_P = 3.333$ in; $D_G = 11.667$ in

Center distance: $C = 7.500$ in

Pitch line speed: $v_t = 1527$ ft/min

Transmitted load: $W_t = 540$ lb

Radial force: $W_r = 196.6$ lb

Normal force: $W_n = 575$ lb

Factors in stress analyses:

Overload factor: $K_o = 1.50$

Size factor: $K_s = 1.0$

Load-distribution factor: $K_m = 1.21$

Rim factor $K_B = 1.0$

Dynamic factor: $K_v = 1.41$

Geometry factors: $J_P = 0.335$; $J_G = 0.420$

Geometry factor: $I = 0.108$

Stresses in gear teeth:

Bending stress: pinion $s_{tP} = 12\,376$ psi; gear $s_{tG} = 9871$ psi

Contact stress: Pinion and gear $s_c = 100\,769$ psi

Safety factor for three alternative materials for bending stress:

Ductile iron ASTM A536 80-55-06 quenched and tempered

$$SF_P = 1.30 \qquad SF_G = 1.70$$

Though hardened pinion and gear: SAE 4340 OQT 1000

$$SF_P = 2.5 \qquad SF_G = 3.2$$

SAE 8620 steel, case hardened by carburizing to Rc 60-64; minimum effective case depth of 0.025 in

$$SF_P = 3.20 \qquad SF_G = 4.20$$

Safety factor for three alternative materials for contact stress:

Ductile iron ASTM A536 80-55-06 quenched and tempered

$$SF_P = 0.54 \qquad SF_G = 0.56$$

Though hardened pinion and gear: SAE 4340 OQT 1000

$$SF_P = 1.00 \qquad SF_G = 1.10$$

SAE 8620 steel, case hardened by carburizing to Rc 60-64; minimum effective case depth of 0.025 in

$$SF_P = 1.26 \qquad SF_G = 1.30$$

All three materials satisfy the safety factor requirements of AGMA for bending stress.

For contact stress the ductile iron gear material had a safety factor of 0.54, which does not meet the minimum value of 1.00 recommended by AGMA. The gears made from ductile iron could be redesigned in order to lower the contact stress number. A larger number of teeth and correspondingly larger pitch diameter would lower the tangential force. The face width could also be increased. While these changes would lower the contact stress number, the gears would get larger which would increase the overall gearbox housing and related components. This gives a larger gearbox footprint and will increases the cost of all the components. This may not be a prudent design decision.

The through-hardened and case carburized gear materials have safety factors that fall within the 1.0 to 1.50 range recommended by AGMA for contact stress. The considerations then turn to material cost and availability, along with manufacturing process cost, and size requirements of the gear design. The quality number for this design, A_v 10, is relatively low and can be attained with a hobbing process. The gear mesh combination using the through-hardened steel pinion mating with a through-hardened gear could be manufactured using the hobbing process and would not require a secondary operation such as grinding. This would reduce the manufacturing cost of the gear design.

The two case carburized surface hardened gears are the strongest and most durable gear meshes. The disadvantage of this hardening process is the distortion of the gear teeth after heat treating. This would require a secondary grinding process to remove distortions and finish the teeth within specifications. The secondary process would add cost to the gear manufacturing process. Case carburized gears are typically designed to be smaller in size compared to a through-hardened gear set. This decrease in gear size will reduce the overall footprint size of the gearbox which may offset the extra cost for heat treatment and finish grinding.

Final decisions on which material to use and other details of the gear designs are part of the responsibility of the team creating the complete industrial saw.

9–9 DESIGN OF SPUR GEARS TO SPECIFY SUITABLE MATERIALS FOR THE GEARS

In previous sections, the main objective was the *analysis* of the performance of a gear pair with a specific geometry using three proposed materials and heat treatments. This section approaches the gear design process differently. First the geometric characteristics are specified, and then the required material properties, particularly the hardness of the gear teeth, are computed. Finally, a suitable material is specified that meets those requirements.

In designs involving gear drives, normally the required speeds of rotation of the pinion and the gear and the amount of power that the drive must transmit are known. These factors are determined from the application. Also, the environment and operating conditions to which the drive will be subjected must be understood. It is particularly important to know the type of driving device and the driven machine, in order to judge the proper value for the overload factor.

The designer must decide the type of gears to use; the arrangement of the gears on their shafts; the materials of the gears, including their heat treatment; and the geometry of the gears: numbers of teeth, diametral pitch, pitch diameters, tooth form, face width, and quality numbers.

This section presents a design procedure that accounts for the bending fatigue strength of the gear teeth and the pitting resistance, called *surface durability*. This procedure makes extensive use of the design equations presented in the preceding sections of the chapter and of the tables of material properties in Appendices 3 through 5, 8, and 12.

You should understand that there is no one best solution to a gear design problem; several good designs are possible. Your judgment and creativity and the specific requirements of the application will greatly affect the final design selected. The purpose here is to provide a means of approaching the problem to create a reasonable design.

Design Objectives

Some overall objectives of a design are listed below. The resulting drive should

- Be compact and small
- Operate smoothly and quietly
- Have long life
- Be low in cost
- Be easy to manufacture
- Be compatible with the other elements in the machine, such as bearings, shafts, the housing, the driver, and the driven machine.

The major objective of the design procedure is to define a safe, long-lasting gear drive. General steps and guidelines are outlined here to produce a reasonable initial design. However, because of the numerous variables involved, several iterations are typically made to work toward an optimum design. Details of the procedure are presented in Example Problem 9–7.

Table 9–13 is a combination of a glossary of terms and a reference for finding equations, tables, or figures required to complete a gear design. The procedure given below uses U.S. Customary units and the diametral pitch, P_d, system. Section 9–10 adjusts this procedure using SI metric units and the metric module, m, system. You should refer back to earlier sections of this chapter and to Chapter 8 for details. Several of the unit-specific formulas developed in Section 9–3 are applied in this section to minimize unit manipulations. *You should understand fully the terms used and the how units were handled when these formulas were developed.*

PROCEDURE FOR DESIGNING A SAFE AND LONG-LASTING GEAR DRIVE ▼

1. From the design requirements, identify the input speed of the pinion, n_P, the desired output speed of the gear, n_G, and the power to be transmitted, P.
2. Choose the type of material for the gears, such as steel, cast iron, or bronze.
3. Considering the type of driver and the driven machine, specify the overload factor, K_o, using Table 9–1. The primary concern is the expected level of shock or impact loading.
4. Specify a trial value for the diametral pitch. When steel gears are used, Figure 9–11 provides initial guidance.

TABLE 9–13 Summary and Glossary of Terms Used in Gear Design

Term by cluster	Symbol	Description of use	Reference location
Recommended quality numbers	A_v	Specify value of A_v	Table 9–5
Allowable bending stress number—steel	s_{at}	Specify material	Figure 9–18
Allowable contact stress number—steel	s_{ac}	Specify material	Figure 9–19
Allowable stresses—case-hardened steel	s_{at} and s_{ac}	Specify material	Table 9–9
Allowable stresses—nonferrous	s_{at} and s_{ac}	Specify material	Table 9–10
Geometry factors for bending stress	J_P and J_G	Bending stress	Figure 9–10
Overload factor	K_o	Bending and contact stress	Table 9–1
Size factor	K_s	Bending and contact stress	Table 9–2
Alignment factor: $K_m = 1.0 + C_{pf} + C_{ma}$	K_m	Bending and contact stress	Equation 9–17
Proportion factor:	C_{pf}	Bending and contact stress	Figure 9–12
Mesh alignment factor:	C_{ma}	Bending and contact stress	Figure 9–13
Rim thickness factor	K_B	Bending and contact stress	Figure 9–14
Dynamic factor	K_v	Bending and contact stress	Figure 9–16
Equations for K_v	K_v	Bending and contact stress	Table 9–6
Elastic coefficient	C_P	Contact stress	Table 9–7
Geometry factor for contact stress	I	Contact stress	Figure 9–17
Reliability factor	K_R	Bending and contact stress	Table 9–11
Recommended life in hours	L	Bending and contact stress	Table 9–12
Bending stress cycle factor	Y_N	Bending stress	Figure 9–21
Contact stress cycle factor	Z_N	Contact stress	Figure 9–22
Design pitch selection aid		Specify trial P_d or m	Figure 9–11
		Primary equations used in gear design:	
Bending stress number	S_t	$s_t = \dfrac{W_t P_d}{FJ} K_o K_s K_m K_B K_v$	Equation 9–16
Required allowable bending stress number	S_{at}	Required $s_{at} > s_t \dfrac{(SF)(K_R)}{Y_N}$	Equation 9–28
Contact stress number	S_c	$s_c = C_P \sqrt{\dfrac{W_t K_o K_s K_m K_v}{F D_P I}}$	Equation 9–23
Required allowable contact stress number	S_{ac}	Required $s_{ac} > s_c \dfrac{(SF)(K_R)}{Z_N}$	Equation 9–31

The graph of design power transmitted versus the pinion rotational speed was derived for selected pitches and pinion diameters. Design power, $P_{des} = K_o P$. Steel that is through hardened to HB 300 is used. Because of the numerous variables involved, the value of P_d read from the figure is only an initial target value. Subsequent iterations may require considering a different value, either higher or lower.

5. Specify the face width within the following recommended range for general machine drive gears, described earlier:

➪ Nominal Face Width

$$8/P_d < F < 16/P_d$$

$$\text{Nominal value of } F = 12/P_d$$

6. Specify or compute the following values:
   - Number of teeth in the pinion (N_P) and the gear (N_G) to achieve the desired output speed of the gear.
   - Compute the actual output speed of the gear and ensure that it is satisfactory.
   - Compute key geometric features: pitch diameters D_P, D_G, and center distance, C. Judge that they are acceptable.
   - Compute the pitch line speed, v_t, and the transmitted load, W_t.
   - Determine the geometry factors for bending stress, J_P and J_G, and the geometry factor for contact stress, I.

- Specify the quality number for the teeth of the gears using Table 9–5 as a guide.
- Determine values for all of the factors in Equation (9–16) for bending stress and Equation (9–23) for contact stress.

7. Compute the bending stress and the contact stress on the pinion and gear teeth. Judge whether the stresses are reasonable (neither too low nor too high) in terms of being able to specify a suitable material. If not, select a new pitch or revise the number of teeth, pitch diameter, or face width. Typically the contact stress on the pinion is the limiting value for gears designed for a long life.
8. Iterate the design process to seek more optimum designs. It is not unusual to make several trials before settling on a particular design. Using computer aids, such as the spreadsheets described in Sections 9–11 and 9–12, can make successive trials quickly.

Guidelines for Specifying Metallic Materials for Gears

1. Given the bending stress, s_t, the contact stress, s_c, and the rotational speed of both the pinion, n_P, and the gear, n_G from analyses similar to those in Example Problems 9–3 and 9–4. Note that this also required that the material of choice is to be steel, cast iron, or bronze.
2. Decide on the safety factor; typically $SF = 1.00$ unless unusual conditions exist. AGMA recommends values for SF from 1.0 to 1.5.
3. Decide on the desired reliability and use Table 9–11 to determine K_R; typically use $K_R = 1.00$ for a reliability of 0.99 (one failure in 100).
4. Specify the desired life in hours for the gears using Table 9–12 as a guide,
5. Compute the expected number of load cycles for both the pinion and the gear using Equation (9–27).
6. Use Figure 9–21 to determine Y_N for the pinion and Figure 9–22 to determine Z_N for the gear. The values may be read from the graphs or computed from the equations given in the figures.
7. Use Equation (9–31) to evaluate the required s_{acP} for the pinion because it typically has the most critical value for s_{ac}. Then consult Figure 9–19 to determine if through-hardened Grade 1 steel can be used. Enter the chart from the value of s_{ac} on the vertical axis; project horizontally to the Grade 1 line; then project vertically down to the Brinell hardness axis to read the minimum acceptable HB hardness. Note that the value should be between HB 180 and HB 400. The most desirable portion of this range is about HB 250 to HB 400. If the value is lower, then a more compact gear size can usually be designed. No value greater than HB 400 should be considered for through-hardened steels. Only Grade 1 steels are recommended.
8. If Step 7 produces a reasonable result, proceed to compute s_{acG} for the gear using Equation (9–31) and compute s_{at} for both the pinion and the gear using Equation (9–30). Check that the bending stress numbers are acceptable by consulting Figure 9–18; they will almost always be acceptable.
9. Using the required hardness HB for the pinion computed in Step 7, specify a suitable material and its heat treatment using Appendices 3 and 4. The charts in Appendix 4 are recommended because they give the widest range of tempering temperatures and corresponding hardness values from which to choose. Examine the lower line of data for HB to find a value just larger than the minimum found from Step 7 and specify the tempering temperature that will produce that hardness level. Table 9–8 recommends several steel alloys that are typically used for gears that are through-hardened; SAE 1045 (similar to 1040), 4140, and 4340 are included and charts for these three alloys are shown in Appendix 4.
10. If Step 8 is not successful, either redesign the gears or consider using case-hardened steel with the values for s_{ac} taken from Table 9–9.
11. If the design is using cast iron or bronze, consult Table 9–10 for values for s_{ac}.

Guidelines for Adjustments in Successive Iterations

The following relationships should help you determine what changes in your design assumptions you should make after the first set of calculations to achieve a more optimum design:

- Decreasing the numerical value of the diametral pitch results in larger teeth and generally lower stresses. Also, the lower value of the pitch usually means a larger face width, which decreases stress and increases surface durability.
- Increasing the diameter of the pinion decreases the transmitted load, generally lowers the stresses, and improves the surface durability.
- Increasing the face width lowers the stress and improves the surface durability, but to a generally lesser extent than either the pitch or the pitch diameter changes discussed previously.
- Gears with more and smaller teeth tend to run more smoothly and quietly than gears with fewer and larger teeth.
- Standard values of diametral pitch should be used for ease of manufacture and lower cost (see Table 8-3).
- Using high-alloy steels with high surface hardness results in the most compact system, but the cost is higher.
- Using very accurate gears (with ground or shaved teeth) results in a higher quality number, lower dynamic loads, and consequently lower stresses and improved surface durability, but the cost is higher.
- The number of teeth in the pinion should generally be as small as possible to make the system compact. But the possibility of interference is greater with fewer teeth. Check Table 8–7 to ensure that no interference will occur. (See Reference 23.)

Example Problem 9–7

Design a pair of spur gears to be used as a part of the drive for a chipper to prepare pulpwood for use in a paper mill. Intermittent use is expected. An electric motor transmits 3.0 horsepower to the pinion at 1750 rpm and the gear must rotate between 460 and 465 rpm. A compact design is desired.

Solution and General Design Procedure

Step 1. Considering the transmitted power, P, the pinion speed, n_P, and the application, refer to Figure 9–11 to determine a trial value for the diametral pitch, P_d. The overload factor, K_o, can be determined from Table 9–1, considering both the power source and the driven machine.

For this problem, $P = 3.0$ hp and $n_P = 1750$ rpm, $K_o = 1.75$ (uniform driver, heavy shock driven machine). Then $P_{des} = (1.75)(3.0 \text{ hp}) = 5.25$ hp. Try $P_d = 12$ for the initial design.

Step 2. Specify the number of teeth in the pinion. For small size, use 17 to 20 teeth as a start.

For this problem, let's specify $N_P = 18$.

Step 3. Compute the nominal velocity ratio from $VR = n_P/n_G$.

For this problem, use $n_G = 462.5$ rpm at the middle of the acceptable range.

$$VR = n_P/n_G = 1750/462.5 = 3.78$$

Step 4. Compute the approximate number of teeth in the gear from $N_G = N_P(VR)$.

For this problem, $N_G = N_P(VR) = 18(3.78) = 68.04$ Specify $N_G = 68$.

Step 5. Compute the actual velocity ratio from $VR = N_G/N_P$.

For this problem, $VR = N_G/N_P = 68/18 = 3.778$.

Step 6. Compute the actual output speed from $n_G = n_P(N_P/N_G)$.

For this problem, $n_G = n_P(N_P/N_G) = (1750 \text{ rpm})(18/68) = 463.2$ rpm. OK.

Step 7. Compute the pitch diameters, center distance, pitch line speed, and transmitted load and judge the general acceptability of the results.

For this problem, the pitch diameters are:

$$D_P = N_P/P_d = 18/12 = 1.500 \text{ in}$$
$$D_G = N_G/P_d = 68/12 = 5.667 \text{ in}$$

Center distance:

$$C = (N_P + N_G)/(2P_d) = (18 + 68)/(24) = 3.583 \text{ in}$$

$$\text{Pitch line speed: } v_t = \pi D_P n_P/12 = [\pi(1.500)(1.750)]/12 = 687 \text{ ft/min}$$

$$\text{Transmitted load: } W_t = 33\,000(P)/v_t = 33\,000(3.0)/687 = 144 \text{ lb}$$

These values seem to be acceptable.

Step 8. Specify the face width of the pinion and the gear.

For this problem: Lower limit $= 8/P_d = 8/12 = 0.667$ in.

Upper limit $= 16/P_d = 16/12 = 1.333$ in

Nominal value $= 12/P_d = 12/12 = 1.00$ in. Use this value.

Step 9. Specify the type of material for the gears and determine C_p from Table 9–7.

For this problem, specify two steel gears. $C_p = 2300$.

Step 10. Specify the quality number, A_v, using Table 9–5 as a guide. Determine the dynamic factor from Figure 9–16.

For this problem, specify $A_v = 11$ for a wood chipper. $K_v = 1.35$.

Step 11. Specify the tooth form, the bending geometry factors for the pinion and the gear from Figure 9–10 and the pitting geometry factor from Figure 9–17.

For this problem, specify 20° full depth teeth. $J_P = 0.325$, $J_G = 0.410$, $I = 0.104$.

Step 12. Determine the load-distribution factor, K_m, from Equation (9–17) and Figures 9–12 and 9–13. The precision class of the gear system design must be specified. Values may be computed from equations in the figures or read from the graphs.

For this problem: $F = 1.00$ *in*, $D_P = 1.500$. $F/D_P = 0.667$. Then $C_{pf} = 0.042$.

Specify open gearing for the wood chipper, mounted to the frame. $C_{ma} = 0.264$.

Compute: $K_m = 1.0 + C_{pf} + C_{ma} + 0.042 + 0.264 = 1.31$

Step 13. Specify the size factor, K_s, from Table 9–2.

For this problem, $K_s = 1.00$ for $P_d = 12$.

Step 14. Specify the rim thickness factor, K_B, from Figure 9–14.

For this problem, specify a solid gear blank. $K_B = 1.00$.

Step 15. Specify a service factor, *SF*, typically from 1.00 to 1.50, based on uncertainty of data.

For this problem, there is no unusual uncertainty. Let $SF = 1.00$.

Step 16. Specify a reliability factor using Table 9–11 as a guideline.

For this problem, specify a reliability of 0.99. $K_R = 1.00$.

Step 17. Specify a design life. Compute the number of loading cycles for the pinion and the gear. Determine the stress cycle factors for bending (Y_N) and pitting (Z_N) for the pinion and the gear.

For this problem, intermittent use is expected. Specify the design life to be 3000 hours, similar to agricultural machinery. The numbers of loading cycles are:

$$N_{cP} = (60)(3000\text{ hr})(1750\text{ rpm})(1) = 3.15 \times 10^8\text{ cycles}$$

$$N_{cG} = (60)(3000\text{ hr})(463.2\text{ rpm})(1) = 8.34 \times 10^7\text{ cycles}$$

Then, from Figure 9–21, $Y_{NP} = 0.96$, $Y_{NG} = 0.98$. From Figure 9–22, $Z_{NP} = 0.92$, $Z_{NG} = 0.95$.

Step 18. Compute the expected bending stresses in the pinion and the gear using Equation (9–16).

$$s_{tP} = \frac{W_t P_d}{F J_P} K_o\, K_s\, K_m\, K_B\, K_v = \frac{(144)(12)}{(1.00)(0.325)}(1.75)(1.0)(1.31)(1.0)(1.35) = 16\,455\text{ psi}$$

$$s_{tG} = s_{tP}(J_P/J_G) = (16\,455)(0.325/0.410) = 13\,044\text{ psi}$$

Step 19. Adjust the bending stresses using Equation 9–30.

For this problem, for the pinion:

$$s_{atP} > s_{tP}\frac{K_R(SF)}{Y_{(NP)}} = (16\,455)\frac{(1.00)(1.00)}{0.96} = 17\,141\text{ psi}$$

For the gear:

$$s_{atG} > s_{tG}\frac{K_R(SF)}{Y_{(NG)}} = (13\,044)\frac{(1.00)(1.00)}{0.98} = 13\,310\text{ psi}$$

Step 20. Compute the expected contact stress in the pinion and the gear from Equation (9–23). Note that this value will be the same for both the pinion and the gear.

$$s_c = C_P\sqrt{\frac{W_t\, K_o\, K_s\, K_m\, K_v}{F D_P I}} = 2300\sqrt{\frac{(144)(1.75)(1.0)(1.31)(1.35)}{(1.00)(1.50)(0.104)}} = 122\,933\text{ psi}$$

Step 21. Adjust the contact stresses for the pinion and the gear using Equation (9–31).

$$s_{acP} > s_{cP}\frac{K_R(SF)}{Z_{NP}} = (122\,933)\frac{(1.00)(1.00)}{(0.92)} = 133\,623\text{ psi}$$

For the gear:

$$s_{acG} > s_{cG}\frac{K_R(SF)}{Z_{NG}} = (122\,933)\frac{(1.00)(1.00)}{(0.95)} = 129\,403\text{ psi}$$

Step 22. Specify materials for the pinion and the gear that will have suitable through hardening or case hardening to provide allowable bending and contact stresses greater than those required from Steps 19 and 21. Typically the contact stress in the pinion is the controlling factor. Refer to Figures 9–18 and 9–19 and Tables 9–9 and 9–10 for data on required hardness. Refer to Appendices 3 to 5 for properties of steel to specify a particular alloy and heat treatment.

For this problem, the contact stress for the pinion is the controlling factor, as is often the case. A steel must be specified that is rated to handle approximately $s_{ac} = 133.6$ ksi. First check Figure 9–19 to explore whether or not through-hardened steel is practical. We can use the equation for Grade 1 steel in U.S. units to determine the required Brinell hardness number, *HB*.

$$\text{Reqd. } HB = (s_{ac} - 29.10)/0.322 = (133.6 \text{ ksi} - 29.10)/0.322 = 324$$

This value is well within the recommended hardness for through-hardened steels. Using Appendix 4, we can specify SAE 4140 OQT 1000 steel having $HB = 341$ and 18% elongation indicating good ductility. We can also check the required hardness for the gear that has a required $s_{ac} = 129.4$ ksi.

$$\text{Reqd. } HB = (s_{ac} - 29.10)/0.322 = (129.4 \text{ ksi} - 29.10)/0.322 = 311$$

This value can be met by SAE 4140 OQT 1100 steel having $HB = 311$ and 20% elongation. However, because both the pinion and the gear experience nearly the same contact stress, it may be prudent to specify the same heat treatment for both to permit them to be produced by the same process.

9–10 GEAR DESIGN FOR THE METRIC MODULE SYSTEM

Here we take the principles, guidelines, and design methodology that were initially developed for the U.S. Customary unit system based on the diametral pitch, P_d, and adapt them to the SI Metric unit system based on the metric module, m. The primary variables involved are listed below with units in both systems.

Variables	U.S. units	SI units
Length (D, C, pitch)	in	mm
Force (W_t, W_r)	lb	N
Power, P	hp	watts or kW
Pitch line speed, v_t	ft/min	m/s
Stresses	psi or ksi	MPa

Some of the required adjustments to calculations and equations are listed below, to help you to mentally relate the two systems to each other. In general, we recommend that designs be completed in one system or the other with minimal conversions.

- Refer to Table 8–1 gear teeth and gear pair features.
- Charts, tables, and graphs in this chapter contain both sets of units.
- Face width, F, recommended limits:

  U.S. units (in): $8/P_d < F < 16/P_d$ Nominal: $12/P_d$
  SI units (mm): $8m < F < 16m$ Nominal: $12m$
- Pitch line speed, v_t:

  U.S. units (ft/min): $v_t = \pi D n/12$ ft/min
  [D in inches, n in rpm]

  SI units (m/s): $v_t = \pi D n/60\,000$ m/s
  [D in mm, n in rpm]
- Transmitted load, W_t:

  U.S. units (lb): $W_t = 33\,000(P)/v_t$ [P in hp, v_t in ft/min]
  SI units (m/s): $W_t = 1000(P)/v_t$ [P in kW, v_t in m/s]

The following example problem uses SI units. The procedure will be virtually the same as that used to design with U.S. Customary units in Example Problem 9–7.

Example Problem 9–8 A gear pair is to be designed to transmit 15.0 kilowatts (kW) of power to a large meat grinder in a commercial meat processing plant. The pinion is attached to the shaft of an electric motor rotating at 575 rpm. The gear must operate at 270 to 280 rpm. The gear unit will be enclosed and of commercial quality. Commercially hobbed (quality number A11), 20°, full-depth, involute gears are to be used in the metric module system. The maximum center distance is to be 200 mm. Specify the design of the gears.

Solution The nominal velocity ratio is

$$VR = 575/275 = 2.09$$

Specify an overload factor of $K_o = 1.50$ from Table 9–1 for a uniform power source and moderate shock for the meat grinder. Then compute design power,

$$P_{\text{des}} = K_o P = (1.50)(15 \text{ kW}) = 22.5 \text{ kW}$$

From Figure 9–11, $m = 5$ is a reasonable trial module. Then

$$N_P = 18 \quad \text{(design decision)}$$
$$D_P = N_P m = (18)(5) = 90 \text{ mm}$$
$$N_G = N_P(VR) = (18)(2.09) = 37.6 \quad \text{(Use 38)}$$
$$D_G = N_G m = (38)(5) = 190 \text{ mm}$$
$$\text{Final output speed} = n_G = n_P(N_P/N_G)$$
$$n_G = 575 \text{ rpm} \times (18/38) = 272 \text{ rpm} \quad \text{(OK)}$$

Center distance $= C = (N_P + N_G)m/2$[Table 8–1]

$$C = (18 + 38)(5)/2 = 140 \text{ mm} \quad \text{(OK)}$$

In SI units, the pitch line speed in meters per second (m/s) is

$$v_t = \pi D_P n_P/(60\,000) = [(\pi)(90)(575)]/(60\,000) = 2.71 \text{ m/s}$$

In SI units, the transmitted load, W_t, is in newtons (N). If the power, P, is in kW, and v_t is in m/s,

$$W_t = 1000(P)/v_t = (1000)(15)/(2.71) = 5536 \text{ N}$$

Face width, F: Let's specify the nominal $F = 12m = 12(5) = 60$ mm.

Factors in stress analysis:

$K_o = 1.50$ (found earlier)

$K_s = 1.00$ (Table 9–2; $m = 5$) $K_B = 1.00$ (Use solid gear blanks)

$K_R = 1.00$ (Table 9–11; 0.99 reliability) $SF = 1.00$ (No unusual conditions)

$K_v = 1.31$ (Figure 9–16; $A_v = 11$)

$K_m = 1.21$ (Figures 9–12 and 9–13; $F = 60$ mm; $F/D_P = 60/90 = 0.67$)

$J_P = 0.315$; $J_G = 0.380$ (Figure 9–10; $N_P = 18$, $N_G = 38$)

$C_p = 191$ (Table 9–7) $I = 0.092$ (Figure 9–17)

Pinion contact stress:

(Equation 9–23)

$$S_c = C_P\sqrt{\frac{W_t K_o K_s K_m K_v}{FD_P I}} = 191\sqrt{\frac{(5536)(1.50)(1.0)(1.21)(1.31)}{(60)(90)(0.092)}} = 983 \text{ MPa}$$

Adjustments for number of cycles, from Figures 9–21 and 9–22:

$$Y_{Np} = 0.94 \quad Z_{NP} = 0.91 \quad Y_{NG} = 0.96 \quad Z_{NG} = 0.92$$

Required $s_{acP} = s_c(SF)(K_R)/Z_{NP} = 983$ MPa $(1.0)(1.0)/0.91 = 1080$ MPa

Using $s_{acP} = 1080$ MPa, Figure 9–19 shows the required hardness = HB 396 for through-hardened Grade 1 steel. This is acceptable but near the upper end of recommended range.

Material specification:

From Figure A4–5 (other possibilities exist),

SAE 4340 OQT 800; HB 415; $s_y = 1324$ MPa; $s_u = 1448$ MPa; 12% elongation.

Check other stresses:

The contact stress for the gear and the bending stress for the pinion and the gear are expected to require less material hardness and strength.

Required $s_{acG} = s_c(SF)(K_R)/Z_{NG} = 983$ MPa$(1.0)(1.0)/0.92 = 1068$ MPa

This is slightly lower than for the pinion (OK)

$$S_{tP} = \frac{W_t K_o K_s K_B K_m K_v}{FmJ_P} = \frac{(5536)(1.50)(1)(1)(1.21)(1.31)}{(60)(5)(0.315)} = 139 \text{ MPa}$$

Required $s_{atP} = s_{tP}(SF)(K_R)/Y_{NP} = 139$ MPa$(1.0)(1.0)/0.94 = 148$ MPa

Referring to Figure 9–18, it is obvious that bending stress requires far lower hardness for the gear teeth, less than HB 180. The stress in the gear is always less than that in the pinion so it will obviously be safe as well.

Summary of the Design:

$P = 15.0$ kW from an electric motor to a large meat grinder

Pinion speed: $n_P = 575$ rpm Gear speed: $n_G = 272$ rpm

Number of teeth: $N_P = 18$; $N_G = 38$ Center distance: $C = 140.00$ mm

Module: $m = 5$ mm Diameters: $D_P = 90$ mm; $D_G = 190$ mm

Material: Steel-SAE 4340 OQT 800

Comment:

A redesign may be considered with several possible approaches:

1. Increase the face width, F, to lower the stresses and permit the choice of a material with more moderate required hardness and better ductility. The recommended upper limit of face width is $16m = 16(5) = 80$ mm.
2. Increase the size of pinion and its number of teeth (same module) to lower stresses.

   Possible trial: Module: $m = 5$ mm Number of teeth: $N_P = 22$; $N_G = 46$

   Center distance: $C = 170.00$ mm Diameters: $D_P = 110$ mm; $D_G = 230$ mm
3. Consider case-hardened steel for the initial design, rather than through-hardened steel. A smaller design is possible.

9–11 COMPUTER-AIDED SPUR GEAR DESIGN AND ANALYSIS

This section presents one approach to assisting the gear designer with the many calculations and judgments that must be made to produce an acceptable design. The spreadsheet shown in Figure 9–23 facilitates the completion of a prospective design for a pair of gears in a few minutes by an experienced designer. You must have studied all of the material here and in Chapter 8 in order to understand the data needed in the spreadsheet and to use it effectively.

The recommended use of the spreadsheet is to create a series of design iterations that allow you to progress toward an optimum design in a short amount of time. It follows the process outlined in Section 9–9 up to the point of computing the required allowable bending stress number and the allowable contact stress number for both the pinion and the gear. The designer must use those data to specify suitable materials for the gears and their heat treatments.

Given below is a discussion of the essential features of the spreadsheet. In general, it first calls for the input of basic performance data, allowing a proposed geometry to be specified. The final result is the completion of the stress analyses for bending and pitting resistance for both the pinion and the gear. Equations (9–16) and (9–30) are combined for the bending analysis. The analysis of pitting resistance uses Equations (9–23) and (9–31). The designer must provide data for the several factors in those equations taken from appropriate figures and charts or based on design decisions. Virtually all computations are performed by the spreadsheet, allowing the designer to exercise judgment based on the intermediate results.

The format used for the spreadsheet helps the designer follow the process. After defining the problem at the top of the sheet, the first column at the left calls for several pieces of input data. Any value in italics within a gray-shaded area must be entered by the designer. White areas offer the results of calculations and provide guidance. The upper part of the second column also guides the designer in determining values for the several factors needed to complete the stress analyses for bending and pitting resistance. The area at the lower right of the spreadsheet gives the primary output data for stresses on which the design decisions for materials and heat treatments are based.

The data in Figure 9–23 are taken from Example Problem 9–7 which was completed in the traditional manner in Section 9–9.

Discussion of the Use of the Spur Gear Design Spreadsheet

1. **Describing the application:** In the heading of the sheet, the designer is asked to describe the application for identification purposes and to focus on the basic uses for the gears. Use the nature of the prime mover and the driven machine to specify the overload factor, K_o, using Table 9–1 as a guide.
2. **Initial input data:** It is assumed that designers begin with a knowledge of the power transmission requirement, the rotational speed of the pinion of the gear pair, and the desired output speed. Using the nature of the application and the overload factor, K_o, compute the *design power* from

$$P_{\text{des}} = K_o P$$

Then use Figure 9–11 to determine a trial value of the diametral pitch using the design power and the rotational speed of the pinion. The number of teeth in the pinion is a critical design decision because the size of the system depends on this value. Ensure against interference. An initial trial value of $N_P = 17$ to 20 is often a good choice.
3. **Number of gear teeth:** The spreadsheet computes the approximate number of gear teeth to produce the desired output speed from $N_G = N_P(n_G/n_P)$. But, of course, the number of teeth in any gear must be an integer, and the actual value of N_G is entered by the designer.
4. **Computed data:** The seven values reported in the middle of the first column are all determined from the input data, and they allow the designer to evaluate the suitability of the geometry of the proposed design at this point. Changes to the input data can be made at this time if any value is out of the desired range in the judgment of the designer.
5. **Secondary input data:** When a suitable geometry for the gears is obtained, the designer enters the data called for at the lower part of the first column of the spreadsheet. The locations of data in pertinent tables and figures are listed.
6. **Factors in design analysis:** The stress analysis requires many factors to account for the unique situation of the design being pursued. Again, guidance is offered, but the designer must enter the values of the required factors. Many of the factors can have a value of 1.00 for normal conditions.
7. **Alignment factor:** The alignment factor depends on two other factors: the pinion proportion factor and the mesh alignment factor as shown in Figures 9–12 and 9–13. The suggested values in the white areas are computed from the equations given in the figures. Note the listed value of F/D_P. If $F/D_P < 0.50$, use $F/D_P = 0.50$ to find C_{pf}. The designer must decide on the type of gearing to be used (open or closed) and the degree of precision to be designed into the system. The final result is computed from the input data.
8. **Size, and rim thickness factors:** Consult Table 9–2 along with Figure 9–14. Note that the rim thickness factor can be different for the pinion and the gear. Sometimes the smaller pinion is made from a solid blank while the larger gear can use a rim-and-spoke design.

DESIGN OF SPUR GEARS

APPLICATION:	*Example Problem 9–7*		
Wood Chipper driven by an electric motor			
Initial Input Data:			
Input Power:	$P =$	*3.00* hp	
Input Speed:	$n_P =$	*1750* rpm	
Diametral Pitch:	$P_d =$	*12*	
Number of Pinion Teeth:	$N_P =$	*18*	
Desired Output Speed:	$n_G =$	*462.5* rpm	
Computed number of gear teeth:		68.1	
Enter: Chosen No. of Gear Teeth:	$N_G =$	*68*	
Computed data:			
Actual Output Speed:	$n_G =$	463.2 rpm	
Gear Ratio:	$m_G =$	3.78	
Pitch Diameter—Pinion:	$D_P =$	1.500 in	
Pitch Diameter—Gear:	$D_G =$	5.667 in	
Center Distance:	$C =$	3.583 in	
Pitch Line Speed:	$v_t =$	687 ft/min	
Transmitted Load:	$W_t =$	144 lb	
Secondary Input Data:			
	Min	Nom	Max
Face Width Guidelines (in):	0.667	1.000	1.333
Enter: Face Width:	$F =$	*1.000* in	
Ratio: Face width/pinion diameter:	$F/D_P =$	0.67	
Recommended ratio	$F/D_P <$	2.00	
Enter: Elastic Coefficient:	$Cp =$	*2300*	Table 9–7
Enter: Quality Number:	$A_v =$	*11*	Table 9–5
Enter: Bending Geometry Factors:			
Pinion:	$J_P =$	*0.325*	Figure 9–10
Gear:	$J_G =$	*0.410*	Figure 9–17
Enter: Pitting Geometry Factor:	$I =$	*0.104*	Figure 9–17
REF:	$m_G =$	3.78	

Factors in Design Analysis:				
Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$	If $F < 1.0$	If $F > 1.0$	$F/D_P =$	0.67
Pinion Proportion Factor, $C_{pf} =$	0.042	0.042	$[0.50 < F/D_P < 2.00]$	
Enter: $C_{pf} =$	*0.042*	Figure 9–12		
Type of gearing:	Open	Commer.	Precision	Ex. Prec.
Mesh Alignment Factor, $C_{ma} =$	0.264	0.143	0.080	0.048
Enter: $C_{ma} =$	*0.264*	Figure 9–13		
Alignment Factor: $K_m =$	1.31	[Computed]		
Overload Factor: $K_o =$	*1.75*	Table 9–1		
Size Factor: $K_s =$	*1.00*	Table 9–2: Use 1.00 if $P_d >= 5$		
Pinion Rim Thickness Factor: $K_{BP} =$	*1.00*	Figure 9–14: Use 1.00 if solid blank		
Gear Rim Thickness Factor: $K_{BG} =$	*1.00*	Figure 9–14: Use 1.00 if solid blank		
Dynamic Factor: $K_v =$	1.35	[Computed: See Table 9–16]		
Service Factor: $SF =$	*1.00*	Use 1.00 if no unusual conditions		
Reliability Factor: $K_R =$	*1.00*	Table 9–11 Use 1.00 for $R = .99$		
Enter: Design Life:	*3000*	hours	See Table 9–12	
Pinion - Number of load cycles: $N_P =$	3.15E + 08	Guidelines: Y_N, Z_N		
Gear - Number of load cycles: $N_G =$	8.34E + 07	10^7	$>10^7$	$<10^7$
Bending Stress Cycle Factor: $Y_{NP} =$	*0.96*	1.00	0.96	Figure 9–21
Bending Stress Cycle Factor: $Y_{NG} =$	*0.98*	1.00	0.98	Figure 9–21
Pitting Stress Cycle Factor: $Z_{NP} =$	*0.92*	1.00	0.92	Figure 9–22
Pitting Stress Cycle Factor: $Z_{NG} =$	*0.95*	1.00	0.95	Figure 9–22
Stress Analysis: Bending				
Pinion: Required $s_{at} =$	17,102	psi	See Figure 9–18 or	
Gear: Required $s_{at} =$	13,280	psi	Table 9–9 or 9–10	
Stress Analysis: Pitting				
Pinion: Required $s_{ac} =$	133,471	psi	See Figure 9–19 or	
Gear: Required $s_{ac} =$	129,256	psi	Table 9–9 or 9–10	
Specify materials, alloy and heat treatment, for most severe requirement.				
One possible material specification:				
Pinion: Requires HB > 324; AISI 4140 OQT 1000, HB = 341, s_{ac} = 140,000 psi				
Gear: Requires HB > 311; AISI 4140 OQT 1000, HB = 341, s_{ac} = 140,000 psi				

FIGURE 9–23 Spreadsheet solution for Example Problem 9–9; alternate spur gear design using data for Example Problem 9–7

9. **Dynamic factor:** The spreadsheet uses the equations included in Table 9–6 to compute the dynamic factor using the quality number and pitch line speed found from data in the first column.
10. **Safety factor:** This is a design decision as discussed in Section 9–9. Often a value of 1.00 is used if no unusual conditions are expected that are not already accounted for in other factors. Larger safety factors allow for a higher degree of safety or to account for uncertainties. For extra safety, use *SF* up to 1.50.
11. **Reliability factor:** The designer must select a value from Table 9–11 according to the desired level of reliability.
12. **Stress cycle factors:** Here the designer must specify the design life in hours of operation for the gear pair being designed. Table 9–12 provides suggestions according to the use of the system. The number of cycles of stress is then computed for both the pinion and the gear, assuming the normal case of one cycle of one-direction stress per revolution. If the gears operate in a reversing mode, as idlers, or in planetary gear trains, this calculation must be adjusted to account for the multiple cycles of stress experienced in each revolution. Guidelines recommend factors of 1.00 for 10^7 cycles for which the allowable stress numbers are computed. For a larger number of cycles, equations given in Figures 9–21 and 9–22 are used to compute the recommended factors. Because a variety of data are given for the case of fewer than 10^7 cycles, the designer is referred to the figures to determine the factors. In any case, the user of the spreadsheet must enter the selected values.
13. **Stress analyses for bending and pitting resistance:** Finally, the required allowable bending stress number and the required allowable contact stress number are computed using Equations (9–30) and (9–31), adjusted for the special values of factors for the pinion and the gear.
14. **Specification of the materials and their heat treatment:** The final step is left to the designer to use the computed values from the stress analyses and to specify materials that will provide an adequate strength and surface hardness of the gear teeth. Pertinent data are listed in Figures 9–18 and 9–19 and Tables 9–9 and 9–10. The appendices tables for material properties may also be consulted once the required hardnesses of the materials are determined.

9–12 USE OF THE SPUR GEAR DESIGN SPREADSHEET

The spreadsheet developed in Section 9–11 is a useful tool that aids the designer in the process of completing a design for a pair of gears to be safe with regard to bending stresses in the teeth of the gears and for pitting resistance. The use of the spreadsheet was demonstrated for the data in Example Problem 9–7 as shown in Figure 9–23.

An important use for the spreadsheet is to propose and analyze several design alternatives and to work toward a goal of optimizing the design with regard to size, cost, or other parameters important to a particular design objective.

Designers for typical machine and vehicle drives would plan to use Grade 1 steels and standard quenching and tempering heat treatments. Where small size is critical or where cost is not a major concern, case hardening by carburizing, induction or flame hardening, or nitriding can be used. Use of more high-capacity Grade 2 or Grade 3 steels may also be specified if data are available. Therefore, it is usually desirable to produce several design alternatives that can be analyzed for cost and manufacturability. Then the final selection can be made with assurance that a reasonably optimum design has been identified.

Successive Iterations. We now continue the design process by making carefully selected changes in design decisions using the *Guidelines for Adjustments in Successive Iterations* from Section 9–9, just before Example Problem 9–7.

The design for the wood chipper drive from Example Problem 9–7 was very satisfactory for the goal of having an efficient design using through-hardened steel. Use of SAE 4140 OQT 1000 steel with a hardness of HB 341 is capable of resisting potential pitting caused by the pinion contact stress of approximately 134 ksi.

Now, how can we improve this design? The answer requires judgment about what constitutes improvement. Generally desirable design objectives for a gear drive, stated at the beginning of Section 9–9, were that the drive should:

Be compact and small	Operate smoothly and quietly
Have long life	Be low in cost
Be easy to manufacture	

Be compatible with other elements in the machine, such as bearings, shafts, the housing, the driver, and the driven machine.

In a given project, some of these objectives may be given higher priority than others. Furthermore, some objectives are counter to others; a highly compact and small drive will not likely be the lowest cost or the easiest to manufacture. However, it may be "worth the cost" to produce a smaller drive. More must be known about the application before making such judgments.

Let's proceed with the premise that the smallest practical gear drive for the wood chipper is desired while cost and ease of manufacture are secondary considerations. Referring to the *Guidelines,* we can conclude that the following changes will facilitate the design of a smaller, yet still safe design:

1. Obviously, each gear must be smaller than the initial design from Example Problem 9–7.

2. Using a higher value of diametral pitch with the same number of teeth will result in smaller gears and a correspondingly smaller center distance.
3. Smaller gears typically result in higher bending and contact stresses in the teeth, requiring higher strength materials.
4. The number of teeth (18) cannot be made much smaller without risking interference.
5. The face width can be used to *fine-tune* the design.
6. More accurate teeth (higher value of A_v) can also be used to *fine-tune* the design.

Design decisions for second trial design: The primary change to achieve a smaller design is to raise the value of diametral pitch. We can try $P_d = 16$ instead of the $P_d = 12$ used in Example Problem 9–7. Now we can show the advantage of using a spreadsheet as a calculation aid. Figure 9–24 shows the final result called Example Problem 9–9,—a much smaller design that is still safe. ***Reaching this redesign took only a few minutes.*** The changed data are highlighted within bold boxes. Those that are design decisions are in the gray-shaded boxes while those computed by the spreadsheet have white backgrounds. User-controlled changes are:

1. $P_d = 16$.
2. $F = 1.00$ in [At the maximum end of the recommended range; $F_{max} = 16/P_d$].
3. $A_v = 10$ [Modestly more precise than the original value of $A_v = 11$; note that $K_v = 1.24$ is moderately lower than the value of 1.35 in the initial design].
4. $K_m = 1.33$ [Modestly higher than the value of 1.31; caused by the change in $C_{pf} = 0.064$ from 0.042 for the initial trial because the ratio F/D_P changed. K_v was computed by the spreadsheet after the computed adjustment to C_{pf} was displayed and selected by the user].

Comparison of Results:

	Design 1 Example Problem 9–7	Design 2 Example Problem 9–9
a. Pinion diameter: D_P	1.500 in	1.125 in
b. Gear diameter: D_G	5.667 in	4.250 in
c. Center distance: C	3.583 in	2.688 in
d. Pitch line speed: v_t	687 ft/min	515 ft/min
e. Transmitted load: W_t	144 lb	192 lb
f. Required s_{atP}:	17 102 psi	28 496 psi
g. Required s_{atG}:	13 280 psi	22 127 psi
h. Required s_{acP}:	133 471 psi	172 288 psi
i. Required s_{acPG}:	129 256 psi	166 847 psi
j. Material:	SAE 4140 OQT 1000 Through hardened	SAE 6150 OQT 1200, Case hardened by induction hardening

Notes about the changes and their effects:

(a) Smaller gears do produce higher stresses.

(b) The size of the gears and the overall drive were reduced significantly as can be seen in Figure 9–25 that shows a scale drawing of the two designs. One measure is the set of minimum inside dimensions of the housing to enclose the two gears, shown as x and y.

$x_1 = 7.333$ in $\qquad x_2 = 5.500$ in (25% smaller)
$y_1 = 5.833$ in $\qquad y_2 = 4.375$ in (25% smaller)

Where $x = C + D_{oP}/2 + D_{oG}/2$ and $y = D_{oG}$

(c) The contact stress in the pinion (the governing stress value) increased by 29% for design #2 as compared with design #1. This was the result of the finer teeth and the smaller pitch diameter for the gears.

(d) The value of the contact stress in the pinion of approximately 172 ksi made it impractical to use through-hardened steel. Note that the calculation built into the spreadsheet indicates that a hardness of 445 HB would be required. However, Figure 9–19 indicates that no design should rely on a hardness over 400 HB.

(e) Therefore, resisting $s_{acP} = 172$ ksi requires some kind of case hardening, either *induction hardening*, *flame hardening*, or *case hardening by carburizing* as indicated in Table 9–9.

(f) Use of induction hardening was a design decision but induction is a very frequent choice in the gear industry. See Internet site 19 for one provider of commercially available production-oriented induction hardening systems.

(g) Specification of SAE 6150 steel for the gears was a design decision based on the need for a highly hardenable steel that can be hardened to 54 HRC minimum (as noted in Table 9–9). Referring to Figure A4–6 indicates that this alloy in the as-quenched condition can be in the range of 627 HB, corresponding to approximately 58 HRC (Appendix 17).

(h) Induction hardening is considered a tertiary process for gears because:
   (a) Gears are first machined, typically by hobbing.
   (b) Then the gears are heat-treated to produce desirable properties in the core of the teeth. In this case, by referring to Figure A4–6, we chose to use the OQT 1200 heat treatment that will produce a core hardness of HB 293 and 20% elongation, a highly ductile condition.
   (c) Then the gears are induction hardened for a moderate depth to achieve the high pitting resistance on the faces of the teeth. Well-designed processes will also produce case hardening in the root area where the highest bending stresses occur.

(i) Because the heat treatment may cause distortion, final grinding or other finishing processes may be needed to produce the final gear quality number; in the case $A_v = 11$.

DESIGN OF SPUR GEARS

APPLICATION: *Example Problem 9–9 –Redesign of* Example Problem 9–7

Wood Chipper driven by an electric motor

Initial Input Data:			
Input Power:	$P =$	*3.00*	hp
Input Speed:	$n_P =$	*1750*	rpm
Diametral Pitch:	$P_d =$	*16*	
Number of Pinion Teeth:	$N_P =$	*18*	
Desired Output Speed:	$n_G =$	*462.5*	rpm
Computed number of gear teeth:		68.1	
Enter: Chosen No. of Gear Teeth:	$N_G =$	*68*	
Computed data:			
Actual Output Speed:	$n_G =$	463.2	rpm
Gear Ratio:	$m_G =$	3.78	
Pitch Diameter - Pinion:	$D_P =$	1.125	in
Pitch Diameter - Gear:	$D_G =$	4.250	in
Center Distance:	$C =$	2.688	in
Pitch Line Speed:	$v_t =$	515	ft/min
Transmitted Load:	$W_t =$	192	lb

Secondary Input Data:				
	Min	**Nom**	**Max**	
Face Width Guidelines (in):	0.500	0.750	1.000	
Enter: Face Width:	$F =$	*1.000* in		
Ratio: Face width/pinion diameter:	$F/D_P =$	0.89		
Recommended ratio	$F/D_P <$	2.00		
Enter: Elastic Coefficient:	$Cp =$	2300	Table 9–7	
Enter: Quality Number:	$A_v =$	*10*	Table 9–5	
Enter: Bending Geometry Factors:				
Pinion:	$J_P =$	*0.325*	Fig. 9–10	
Gear:	$J_G =$	*0.410*	Fig. 9–17	
Enter: Pitting Geometry Factor:	$I =$	*0.104*	Fig. 9–17	
REF:	$m_G =$	3.78		

Factors in Design Analysis:

Alignment Factor, $K_m = 1.0 + C_{pf} + C_{ma}$	If $F < 1.0$	If $F > 1.0$	$F/D_P =$	0.89
Pinion Proportion Factor, $C_{pf} =$	0.064	0.064	$[0.50 < F/D_P < 2.00]$	
Enter: $C_{pf} =$	*0.064*	Figure 9–12		
Type of gearing:	Open	Commer.	Precision	Ex. Prec.
Mesh Alignment Factor, $C_{ma} =$	0.264	0.143	0.080	0.048
Enter: $C_{ma} =$	*0.264*	Figure 9–13		
Alignment Factor: $K_m =$	1.33	[Computed]		
Overload Factor: $K_o =$	*1.75*	Table 9–1		
Size Factor: $K_s =$	*1.00*	Table 9–2: Use 1.00 if $P_d >= 5$		
Pinion Rim Thickness Factor: $K_{BP} =$	*1.00*	Fig. 9–14: Use 1.00 if solid blank		
Gear Rim Thickness Factor: $K_{BG} =$	*1.00*	Fig. 9–14: Use 1.00 if solid blank		
Dynamic Factor: $K_v =$	1.24	[Computed: See Table 9–16]		
Service Factor: $SF =$	*1.00*	Use 1.00 if no unusual conditions		
Reliability Factor: $K_R =$	*1.00*	Table 9–11 Use 1.00 for $R = .99$		
Enter: Design Life:	*3000*	hours	See Table 9–12	
Pinion - Number of load cycles: $N_P =$	3.15E+08	Guidelines: Y_N, Z_N		
Gear - Number of load cycles: $N_G =$	8.34E+07	10^7 cycles	$>10^7$	$<10^7$
Bending Stress Cycle Factor: $Y_{NP} =$	*0.96*	1.00	0.96	Fig. 9–21
Bending Stress Cycle Factor: $Y_{NG} =$	*0.98*	1.00	0.98	Fig. 9–21
Pitting Stress Cycle Factor: $Z_{NP} =$	*0.92*	1.00	0.92	Fig. 9–22
Pitting Stress Cycle Factor: $Z_{NG} =$	*0.95*	1.00	0.95	Fig. 9–22

Stress Analysis: Bending

Pinion: Required $s_{at} =$	28,496	psi	See Fig. 9–18 or
Gear: Required $s_{at} =$	22,127	psi	Table 9–9 or 9–10

Stress Analysis: Pitting

Pinion: Required $s_{ac} =$	172,288	psi	See Fig. 9–19 or
Gear: Required $s_{ac} =$	166,847	psi	Table 9–9 or 9–10

Specify materials, alloy and heat treatment, for most severe requirement.

Required hardnesses for pinion and gear are too high for through hardening.

Specifying case hardening by induction to HRC 54 minimum-Both pinion and gear

Pinion and Gear: SAE 6150; Core heat treatment OQT 1200; HB 293.

FIGURE 9–24 Spreadsheet solution for Example Problems 9–9; the alternate spur gear design using data for Example Problem 9–7.

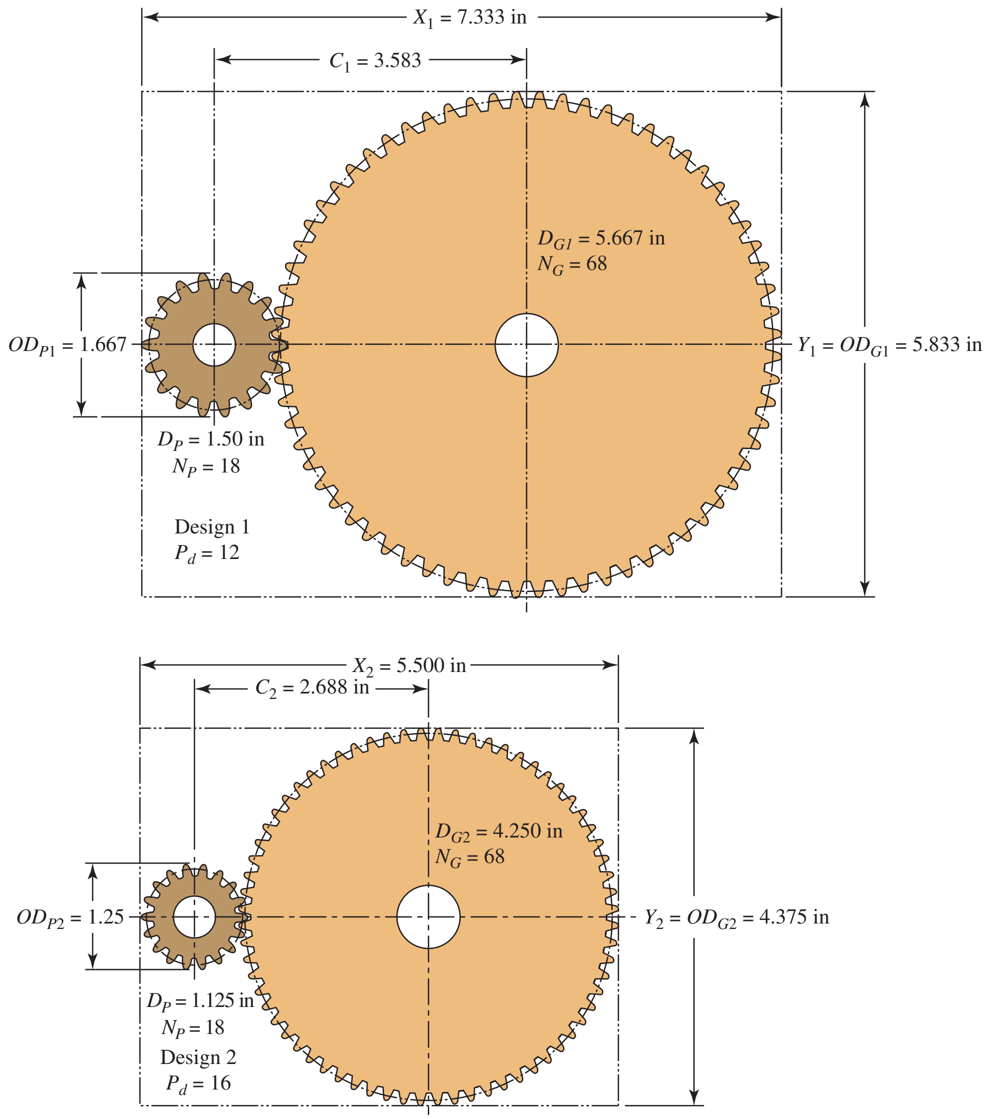


FIGURE 9–25 Comparison of sizes for gear designs for Example Problems 9–7 and 9–9 with minimum inside dimensions for housings

(j) This long list of processing steps makes the choice of case hardening by induction hardening much more costly than basic through hardening. The designer must make the judgment that the value of the benefit of the smaller system exceeds the added costs.

It is expected that readers of this book use similar decision-making processes when designing gear drives and specifying materials and heat treatments in their designs.

9–13 POWER-TRANSMITTING CAPACITY

It is sometimes desirable to compute the amount of power that a gear pair can safely transmit after it has been completely defined. The *power-transmitting capacity* P_{cap} is the capacity when the tangential load causes the expected stress to equal the allowable stress number with all of the modifying factors considered. The capacity should be computed for both bending and pitting resistance and for both the pinion and the gear.

When similar materials are used for both the pinion and the gear, it is likely that the pinion will be critical for bending stress. But the most critical condition is usually pitting resistance. The following relationships can be used to compute the power-transmitting capacity. In this analysis, it is assumed that the operating temperature of the gears and their lubricants is 250°F and that gears are produced with the appropriate surface finish.

Bending

We start with Equations (9–16) and (9–30) in which the computed bending stress number is compared with the modified allowable bending stress number for the gear:

$$s_{at} = s_t\frac{(SF)(KR)}{Y_N} = \frac{W_tP_d}{FJ}K_oK_sK_mK_BK_v \times \frac{(SF)(KR)}{Y_N}$$

But solving for W_t gives

$$W_t = \frac{s_{at} Y_N F J}{(SF) K_R K_o K_s K_m K_B K_v P_d} \quad \textbf{(9–34)}$$

It was shown in Equation (9–8) that

$$W_t = (126\ 000)(P)/(n_P D_P)$$

Then substituting into Equation (9–34) and calling the power P_{cap} gives

$$\frac{(126\ 000)(P_{cap})}{n_P D_P} = \frac{s_{at} Y_N F J}{(SF) K_R K_o K_s K_m K_B K_v P_d}$$

Solving for P_{cap}, we have

$$P_{cap} = \frac{s_{at} Y_N F J n_P D_P}{(126\ 000)(P_d)(SF) K_R K_o K_s K_m K_B K_v} \quad \textbf{(9–35)}$$

This equation should be solved for both the pinion and the gear. Most variables will be the same except for s_{at}, Y_N, J, and possibly K_B.

Pitting Resistance

Here we start with Equations (9–23) and (9–31) in which the computed contact stress number is compared with the modified allowable contact stress number for the gear. Equation (9–26) can be expressed in the form

$$s_{ac} = s_c \frac{(SF)(KR)}{Z_N} = C_P \sqrt{\frac{W_t K_o K_s K_m K_v}{D_P F I}} \times \frac{(SF)(KR)}{Z_N}$$

Squaring both sides of this equation and solving for W_t gives

$$\frac{W_t K_o K_s K_m K_v}{D_P F I} = \left[\frac{s_{ac} Z_N}{(SF) K_R C_P}\right]^2$$

$$W_t = \frac{D_P F I}{K_o K_s K_m K_v}\left[\frac{s_{ac} Z_N}{(SF) K_R C_P}\right]^2 \quad \textbf{(9–36)}$$

Now substituting this into Equation (9–8) and solving for the power P_{cap} gives

$$P_{cap} = \frac{W_t D_P n_P}{126\ 000} = \frac{D_P n_P D_P F I}{126\ 000\ K_o K_s K_m K_v}\left[\frac{s_{ac} Z_N}{(SF) K_R C_P}\right]^2$$

$$P_{cap} = \frac{n_P F I}{126\ 000\ K_o K_s K_m K_v}\left[\frac{s_{ac} D_P Z_N}{(SF) K_R C_P}\right]^2 \quad \textbf{(9–37)}$$

Equations (9–35) and (9–37) should be used to compute the power-transmitting capacity for a pair of gears of known design with particular materials. From the material specification along with its condition (typically a heat treatment or case-hardening process), the limiting values for s_{at} and s_{ac} can be found from Figures 9–18 and 9–19 and from Tables 9–9 and 9–10. Entering those values along with the known data for the proposed gear design permits the calculation of the power-transmitting capacity, P_{cap}. This value can be shared with colleagues and customers to verify the suitability of a design for a particular application.

9–14 PLASTICS GEARING

Plastics are satisfying an important and growing part of the applications for gearing. Some of the numerous advantages of plastics in gearing systems compared with steels and other metals are:

- Lighter weight.
- Lower inertia.
- Possibility of running with little or no external lubrication.
- Quieter operation.
- Low sliding friction, which results in efficient gear meshing.
- Chemical resistance and ability to operate in corrosive environments.
- Ability to operate well under conditions of moderate vibration, shock, and impact.
- Relatively low cost when made in large quantities.
- Ability to combine several features into one part.
- Accommodation of larger tolerances because of resiliency.
- Material properties that can be tailored to meet the needs of the application.
- Less wear among some plastics compared to metals in certain applications.

The advantages must be weighed against disadvantages such as:

- Relatively lower strength of plastics as compared with metals.
- Lower modulus of elasticity.
- Higher coefficients of thermal expansion.
- Difficulty operating at high temperatures.
- Initial high cost for design, development, and mold manufacture.
- Dimensional change with moisture absorption that varies with conditions.
- Wide range of possible material formulations, which makes design more difficult.

Some plastic gears are cut using hobbing or shaping processes similar to those used to cut metallic gears. However, most plastic gears are produced with the injection molding process because of its ability to make large quantities rapidly with low unit cost. Mold design is critical because it must accommodate the shrinking that occurs as the molten plastic solidifies. The typical successful approach accounts for predicted shrinkage by making the die larger than the required finished gear size. However, the allowance is not uniform throughout the gear, and significant amounts of data are required about the material molding properties and the molding process itself to produce plastic gears with high dimensional accuracy. Computer-assisted mold design software that simulates the flow of molten plastic through the mold cavities and the curing process is often used. The gear mold or the gear cutting tools are designed to produce dimensionally accurate gear teeth with tooth thickness controlled to produce a proper amount of backlash during operation. The electrical discharge machining process (EDM) is typically used to produce accurate gear tooth forms in molds made from high-hardness, wear-resistant steels to ensure that large production runs can be made without replacing tooling.

Plastic Materials for Gears

The great variety of plastics available makes material selection difficult, and it is recommended that gear system designers consult with material suppliers, mold designers, and manufacturing staff during the design process. While simulation can aid in reaching a suitable design, it is recommended that testing be done in realistic conditions before committing the design to production. Some of the more popular types of materials used for gears are:

Nylon	Acetal	ABS (acrylonitrile-butadiene-styrene)
Polycarbonate	Polyurethane	Polyester thermoplastic
Polyimide	Phenolic	Polyphenylene sulfide
Polysulfones	Phenylene oxides	Styrene-acrylonitrile (SAN)

Designers must seek a balance of material characteristics appropriate to the application, considering, for example:

- Strength in flexure under fatigue conditions.
- High modulus of elasticity for stiffness.
- Impact strength and toughness.
- Wear and abrasion resistance.
- Dimensional stability under expected temperatures.
- Dimensional stability due to moisture absorption from liquids and humidity.
- Frictional performance and need for lubrication, if any.
- Operation in vibration environments.
- Chemical resistance and compatibility with the operating environment.
- Sensitivity to ultraviolet radiation.
- Creep resistance if operated under load for long periods of time.
- Flame retarding ability.
- Cost.
- Ease of processing and molding.
- Assembly and disassembly considerations.
- Compatibility with mating parts.
- Environmental impact during processing, use, recycling, and disposal.

The basic plastic materials listed previously are typically modified with fillers and additives to produce optimum as-molded properties. Some of these are:

Reinforcements for strength, toughness, moldability, long-term stability, thermal conductivity, and dimensional stability: Long glass fibers, chopped glass fibers, milled glass, woven glass fibers, carbon fibers, glass beads, aluminum flake, mineral, cellulose, rubber modifiers, wood flour, cotton, fabric, mica, talc, and calcium carbonate.

Fillers to improve lubricity and overall frictional performance: PTFE (polytetrafluoroethylene), silicone, carbon fibers, graphite powders, and molybdenum disulfide (MoS_2).

Refer to Section 2-17 for additional discussion about plastic materials, their properties, and special considerations for selecting plastics. See Internet sites 1, 8, 18, and 20.

Design Strength for Plastic Gear Materials

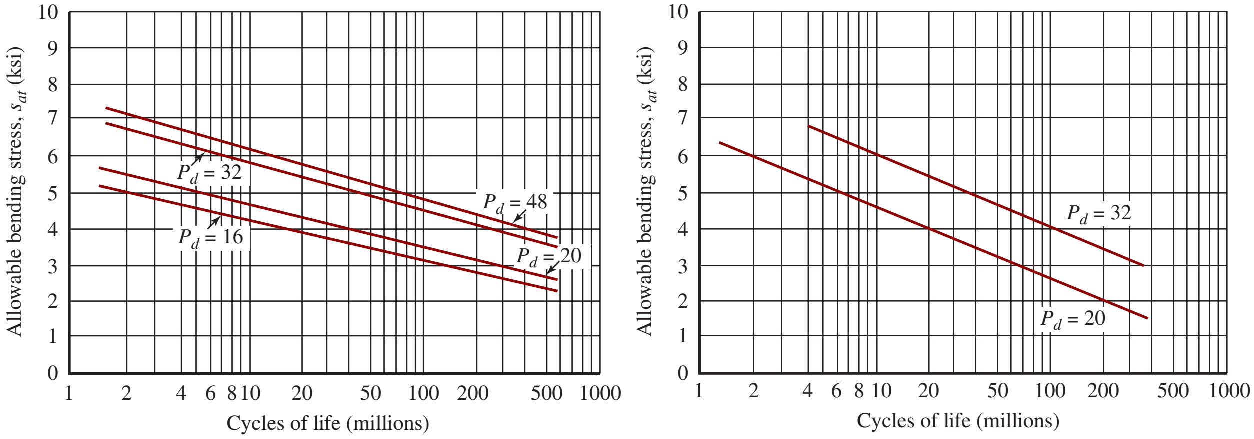
Data are provided here for typical plastic materials used for gears. They can be applied to problem solving in this book. However, verification of properties for materials to be actually used in a commercial application, with due regard for the operating conditions, should be acquired from the material supplier. The effects of temperature on strength, modulus, toughness, chemical stability, and dimensional precision are particularly important. Manufacturing processes must be controlled to ensure that final properties are consistent with prescribed values.

Table 9–14 lists some selected data for allowable tooth bending stress, s_{at}, in plastic gears. Much additional data for other materials can be found in References 19 and 21. Note the significant increase in allowable strength provided by the glass reinforcement. The combination of glass fibers and the basic plastic matrix performs like a composite material with the amount of reinforcement typically ranging from 20% to 50%.

TABLE 9–14 Approximate Allowable Tooth Bending Stress, s_{at}, in Plastic Gears

	Approximate allowable bending stress, s_{at}, psi (MPa)	
Material	**Unfilled**	**Glass-filled**
ABS	3000 (21)	6000 (41)
Acetal	5000 (34)	7000 (48)
Nylon	6000 (41)	12 000 (83)
Polycarbonate	6000 (41)	9000 (62)
Polyester	3500 (24)	8000 (55)
Polyurethane	2500 (17)	

Source: Plastics Gearing. Manchester, CT: ABA/PGT Publishing, 1994.



(*a*) Fatigue life data for DuPont© Zytel nylon resin (*b*) Fatigue life data for DuPont© Delrin acetal resin

FIGURE 9–26 Fatigue life data for two types of plastic materials used for gears

Material suppliers may be able to provide fatigue data for plastics in charts such as those shown in Figure 9–26, showing allowable bending stress versus number of cycles to failure for DuPont Zytel© nylon resin and Delrin© acetal resin. These data are for molded gears operating at room temperature with diametral pitches shown, pitch line velocity below 4000 ft/min, and continuous lubrication. Reductions should be applied for cut gears, higher temperatures, different pitches, and different lubrication conditions. See Reference 21.

Tooth Geometry

In general, standard tooth geometry for plastic gears conforms to the configurations described in Section 8–4. Standard diametral pitches from Table 8–3 and standard metric modules from Table 8–4 should be used unless there are major advantages to using other values. Suppliers' ability to provide nonstandard pitches should be investigated. Pressure angles of $14\frac{1}{2}°$, 20°, and 25° are used, with 20° usually preferred. Standard formulas for addendum, dedendum, and clearance for full-depth involute teeth are listed in Table 8–1. Gear quality values are set similarly to those for metallic gears as discussed in Section 9–9. The typical AGMA quality number produced by injection molding is in range of A11 to A7.

Designers sometimes use special tooth forms to tailor the strength of plastic gear teeth to the demands of particular applications. The 20° stub tooth system provides a shorter, broader tooth than the standard 20° full-depth tooth system, decreasing tooth-bending stress. The Plastics Gearing Technology unit of the ABA-PGT company has developed another system that is finding favor with some designers. See References 1, 2, 12, and 13.

Many designers of plastic gears prefer to use a longer addendum on the pinion and a shorter addendum on the mating gear to produce more favorable operation because of the greater flexibility of plastics as compared with metals. Tooth thickness is typically thinned on either or both of the pinion and gear to provide acceptable backlash and to ensure that mating gears do not bind. Binding may result from deflection of the teeth under load or from expansions due to increased temperature or moisture absorption from exposure to water or high humidity. Enlarging the center distance is another method employed to adjust for backlash. Designers must specify these feature sizes on drawings and in specifications. Consult AGMA Standard 1006 (Reference 12) *Tooth Proportions for Plastic Gears* for details. Reference 2 provides useful tables of formulas and data for adjustments to tooth form and center distance. Reference 21 recommends the range of backlash values shown in Figure 9–27.

Shrinkage

During manufacture of plastic gears using injection molding, enlarging the effective diametral pitch and the pitch diameter of the gear teeth cut into the mold accommodates shrinkage. The pressure angle is also

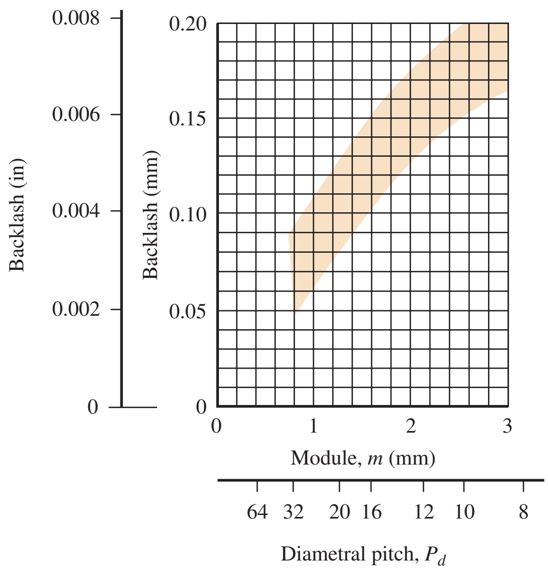


FIGURE 9–27 Recommended backlash for plastic gears

adjusted. The nominal corrections are computed as follows:

$$P_{dc} = \frac{P_d}{(1 + S)} \tag{9–38}$$

$$\cos \phi_1 = \frac{\cos \phi}{(1 + S)} \tag{9–39}$$

$$D_c = N/P_{dc} \tag{9–40}$$

where S = shrinkage of material
P_d = standard diametral pitch for the gear
P_{dc} = modified diametral pitch of the teeth in the mold
ϕ = standard pressure angle for the gear
ϕ_1 = modified pressure angle of the teeth in the mold
N = number of teeth
D_c = modified pitch diameter of teeth in the mold

After molding, the teeth should very nearly conform to standard geometry. Additional adjustments are sometimes made, relieving the tips of the teeth for smoother engagement and increasing the tooth width at the base near the point of highest bending stress.

Stress Analysis

Bending stress analysis for plastic gears relies on the basic Lewis formula introduced in Section 9–5, Equation (9–13). The modifying factors called for by the AGMA standards for steel gears are not specified for plastic gears at this time. We can account for uncertainty or shock loading by inserting a safety factor. The overload factor from Table 9–1 can be used as a guide. Testing of the proposed design in realistic conditions should be completed. The bending stress equation then becomes

$$\sigma_t = \frac{W_t P_d (SF)}{FY} \tag{9–41}$$

Values for the Lewis form factor, Y, shown in Table 9–15, describe the geometry of the involute gear teeth acting as a cantilever beam with the load applied near the pitch point. Thus Equation (9–41) gives the bending stress at the root of the tooth. Most plastic gear designs call for a generous fillet radius between the start of the active involute profile on the flank of the tooth and the root, resulting in little, if any, stress concentration.

TABLE 9–15 Lewis Tooth Form Factor, Y, for Load Near the Pitch Point

	Tooth form		
Number teeth	$14\frac{1}{2}°$ Full depth	20° Full depth	20° Stub
14	–	–	0.540
15	–	–	0.566
16	–	–	0.578
17	–	0.512	0.587
18	–	0.521	0.603
19	–	0.534	0.616
20	–	0.544	0.628
22	–	0.559	0.648
24	0.509	0.572	0.664
26	0.522	0.588	0.678
28	0.535	0.597	0.688
30	0.540	0.606	0.698
34	0.553	0.628	0.714
38	0.566	0.651	0.729
43	0.575	0.672	0.739
50	0.588	0.694	0.758
60	0.604	0.713	0.774
75	0.613	0.735	0.792
100	0.622	0.757	0.808
150	0.635	0.779	0.830
300	0.650	0.801	0.855
Rack	0.660	0.823	0.881

Wear Considerations

Wear of tooth surfaces in plastic gear teeth is a function of the contact stress between mating teeth as it is with metal teeth. Equation (9–18) can be used to compute the contact stress. However, published data are lacking for allowable contact stress values.

In reality, lubrication and the *combination of materials in mating gears* play major roles in the wear life of the pair. Communication with material suppliers and testing of proposed designs are recommended.

Presented here are some general guidelines from References 2 and 21.

- Continuously lubricated gearing promotes the longest life.
- With continuous lubrication and light loads, fatigue resistance, not wear, typically determines life.
- Unlubricated gears tend to fail by wear, not fatigue, provided proper design bending stresses are used.
- When continuous lubrication is not practical, initially lubricating the gearing can aid in the run-in process and add life compared with gears that are never lubricated.
- When continuous lubrication is not practical, the combination of a nylon pinion and an acetal gear exhibits low friction and wear.
- Excellent wear performance for relatively high loads and pitch line speeds can be obtained by using a lubricated pair of a hardened steel pinion (HRC > 50)

mating with a plastic gear made from nylon, acetal, or polyurethane.

- Wear accelerates when operating temperatures rise. Cooling to promote heat dissipation can increase life.

Gear Shapes and Assembly

References 2 and 21 include many recommendations for the geometric design of gears considering strength, inertia, and molding conditions. Many smaller gears are simply made with uniform thickness equal to the face width of the gear teeth. Larger gears often have a rim to support the teeth, a thinned web for lightening and material savings, and a hub to facilitate mounting on a shaft. Figure 9–28 shows recommended proportions. Symmetrical cross sections are preferred, along with balanced section thicknesses to promote good flow of material and to minimize distortion during molding.

Fastening gears to shafts requires careful design. Keys placed in shaft key seats and keyways in the hub of the gear provide reliable transmission of torque. For light torques, setscrews can be used, but slippage and damage of the shaft surface are possible. The bore of the gear hub can be lightly press-fit onto the shaft with care to ensure that a sufficient torque can be transmitted while not overstressing the plastic hub. Knurling the shaft before pressing the gear on increases the torque capability. Some designers prefer to use metal hubs to facilitate the use of keys. Plastic is then molded onto the hub to form the rim and gear teeth.

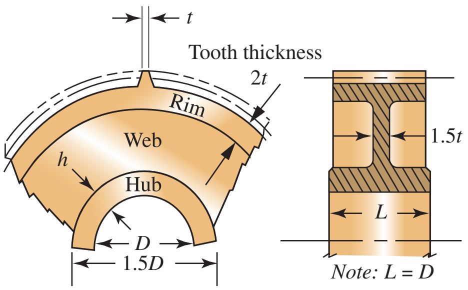


FIGURE 9–28 Suggested plastic gear proportions

Design Procedure

Design of plastic gearing should consider a variety of possibilities, and it is likely to be an iterative process. The following procedure outlines the steps for a given trial using U.S. Customary units for use in this book.

PROCEDURE FOR DESIGNING PLASTIC GEARS

1. Determine the required horsepower, P, to be transmitted and the speed of rotation, n_P, of the pinion in rpm.
2. Specify the number of teeth, N, and select a trial diametral pitch for the pinion.
3. Compute the pinion diameter from $D_P = N_P/P_d$.
4. Compute the transmitted load, W_t (in lb), from Equation (9–8), repeated here.

$$W_t = (126\,000)(P)/(n_P D_P)$$

5. Specify the tooth form and determine the Lewis form factor, Y, from Table 9–15.
6. Specify a safety factor, SF. Refer to Table 9–1 for guidance.
7. Specify the material to be used and determine the allowable stress, s_{at}, from Table 9–15 or Figure 9–31.
8. Solve Equation (9–41) for the face width, F, and compute its value from,

$$F = \frac{W_t P_d (SF)}{s_{at} Y} \quad \textbf{(9–42)}$$

9. Judge the suitability of the computed face width as it relates to the application. Consider its mounting on a shaft, space available in the diametral and axial directions, and whether the general proportions are acceptable for injection molding. See References 2 and 21. No general recommendations are published for the face width of plastic gears and often they are narrower than similar metallic gears.
10. Repeat steps 2 to 9 until a satisfactory design for the pinion is achieved. Specify convenient dimensions for the final value of the face width and other features of the pinion.
11. Considering the desired velocity ratio between the pinion and the gear, compute the required number of teeth in the gear and repeat steps 3 to 9 using the same diametral pitch as the pinion. Using the same face width as for the pinion, the stress in the gear teeth will always be lower than in the pinion because the form factor Y will increase and all other factors will be the same. When the same material is to be used for the gear, it will always be safe. Alternatively, you could compute the bending stress directly from Equation (9–41) and specify a different material for the gear that has a suitable allowable bending stress.

Example Problem 9–10 Design a pair of plastic gears for a paper shredder to transmit 0.25 horsepower at a pinion speed of 1160 rpm. The pinion will be mounted on the shaft of an electric motor that has a diameter of 0.625 in with a keyway for a 3/16 × 3/16 in key. The gear is to rotate approximately 300 rpm.

Given Data $P = 0.25$ hp, $n_P = 1160$ rpm,

Shaft diameter $= D_s = 0.625$ in, Keyway for a 3/16 × 3/16 in key.

Approximate gear speed $= n_G = 300$ rpm

Solution Use the design procedure outlined in this section.

Step 1. Consider the given data.

Step 2. Specify $N_P = 18$ and $P_d = 16$ (Design decisions)

Step 3. $D_P = N_P/P_d = 18/16 = 1.125$ *in*. This seems reasonable for mounting on the 0.625 in motor shaft.

Step 4. Compute the transmitted load,

$$W_t = (126\,000)(P)/(n_P D_P) = (126\,000)(0.25)/[(1160)(1.125)] = 24.1 \text{ lb}$$

Step 5. Specify 20° full-depth teeth. Then $Y = 0.521$ for 18 teeth from Table 9–15.

Step 6. Specify a safety factor, SF. The shredder will likely experience light shock; the preference is to operate the gears without lubrication. Specify $SF = 1.50$ from Table 9–1.

Step 7. Specify unfilled nylon. From Table 9–14, $s_{at} = 6000$ psi.

Step 8. Compute the required face width using Equation (9–42).

$$F = \frac{W_t P_d (SF)}{s_{at} Y} = \frac{(24.1)(16)(1.50)}{(6000)(0.521)} = 0.185 \text{ in}$$

Step 9. The dimensions seem reasonable.

Step 10. Appendix 2 lists a preferred size for the face width of 0.200 in.

Comment: In summary, the proposed pinion has the following features:

$$P_d = 16,\ N_P = 18 \text{ teeth},\ D_P = 1.125 \text{ in},\ F = 0.200 \text{ in, Bore} = 0.625 \text{ in},$$

Keyway for a 3/16 × 3/16 in key. Unfilled nylon material.

Step 11. Gear design: Specify $F = 0.200$ in, $P_d = 16$. Compute the number of teeth in the gear.

$N_G = N_P(n_P/n_G) = 18(1160/300) = 69.6$ teeth

Specify $N_G = 70$ teeth

Pitch diameter of gear $= D_G = N_G/P_d = 70/16 = 4.375$ in

From Table 9–15, $Y_G = 0.728$ by interpolation.

Stress in gear teeth using Equation (9–41):

$$\sigma_t = \frac{W_t P_d (SF)}{FY} = \frac{(24.1)(16)(1.50)}{(0.200)(0.728)} = 3973 \text{ psi}$$

Comment: This stress level is safe for nylon. The gear could also be made from acetal to achieve better wear performance.

9–15 PRACTICAL CONSIDERATIONS FOR GEARS AND INTERFACES WITH OTHER ELEMENTS

It is important to consider the design of the entire gear system when designing the gears because they must work in harmony with the other elements in the system. This section will briefly discuss some of these practical considerations and will show commercially available speed reducers.

Our discussion so far has been concerned primarily with the gear teeth, including the tooth form, pitch, face width, material selection, and heat treatment. Also to be considered is the type of gear blank. Figures 8–2 and 8–4 show several styles of blanks. Smaller gears and lightly loaded gears are typically made in the plain style. Gears with pitch diameters of approximately 5.0 in through 8.0 in are frequently made with thinned webs between the rim and the hub for lightening, with some having holes bored in the webs for additional lightening. Larger gears, typically with pitch diameters greater than 8.0 in, are made from cast blanks with spokes between the rim and the hub.

In many precision special machines and gear systems produced in large quantities, the gears are machined integral with the shaft carrying the gears. This, of course, eliminates some of the problems associated with mounting and location of the gears, but it may complicate the machining operations.

In general machine design, gears are usually mounted on separate shafts, with the torque transmitted from the shaft to the gear through a key. This setup provides a positive means of transmitting the torque while permitting easy assembly and disassembly. The axial location of the gear must be provided by another means, such as a shoulder on the shaft, a retaining ring, or a spacer (see Chapters 11 and 12).

Other considerations include the forces exerted on the shaft and the bearings that are due to the action of the gears. These subjects are discussed in Section 9–3. The housing design must provide adequate support for the bearings and protection of the interior components. Normally, it must also provide a means of lubricating the gears.

See References 9, 19, 20, 22, and 25 for additional practical considerations.

Lubrication

The action of spur gear teeth is a combination of rolling and sliding. Because of the relative motion, and because of the high local forces exerted at the gear faces, adequate lubrication is critical to smoothness of operation and gear life. A continuous supply of oil at the pitch line is desirable for most gears unless they are lightly loaded or operate only intermittently.

In splash-type lubrication, one of the gears in a pair dips into an oil supply sump and carries the oil to the pitch line. At higher speeds, the oil may be thrown onto the inside surfaces of the case; then it flows down, in a controlled fashion, onto the pitch line. Simultaneously, the oil can be directed to the bearings that support the shafts. One difficulty with the splash type of lubrication is that the oil is churned; at high gear speeds, excessive heat can be generated, and foaming can occur.

A positive oil circulation system is used for high-speed and high-capacity systems. A separate pump draws the oil from the sump and delivers it at a controlled rate to the meshing teeth.

The primary functions of gear lubricants are to reduce friction at the mesh and to keep operating temperatures at acceptable levels. It is essential that a continuous film of lubricant be maintained between the mating tooth surfaces of highly loaded gears and that there be a sufficient flow rate and total quantity of oil to maintain cool temperatures. Heat is generated by the meshing gear teeth, by the bearings, and by the churning of the oil. This heat must be dissipated from the oil to the case or to some other external heat-exchange device in order to keep the oil itself below 200°F (approximately 93°C). Above this temperature, the lubricating ability of the oil, as indicated by its viscosity, is severely decreased. Also, chemical changes can be produced in the oil, decreasing its lubricity. Because of the wide variety of lubricants available and the many different conditions under which they must operate, it is recommended that suppliers of lubricants be consulted for proper selection. (See also References 10, 16, 27–29.)

The AGMA, in Reference 10, defines several types of lubricants for use in gear drives.

- ***Rust and oxidation inhibited gear oils*** (called R&O) are petroleum based with chemical additives.
- ***Compounded gear lubricants*** (CP) blend 3% to 10% of fatty oils with petroleum oils.
- ***Extreme pressure lubricants*** (EP) include chemical additives that inhibit scuffing of gear tooth faces.
- ***Synthetic gear lubricants*** (S) are special chemical formulations applied mostly in severe operating conditions.

R&O lubricants are supplied in 10 ISO viscosity grades where the lower numbers refer to the lower viscosities. Similar numbers are used for the other types with modified grade designations carrying suffixes *CP*, *EP*, or *S*. The recommended lubricant grade depends on the ambient temperature around the drive and the pitch line velocity of the lowest speed pair of gears in a reducer. See Tables 9–16 and 9–17. Wormgear drives call for higher viscosity grades.

TABLE 9–16 Recommended Lubricant Viscosity Grades for Enclosed Gear Drives

ISO viscosity grade	Midpoint viscosity at 40°C (mm^2/s)	Former AGMA grade equivalent
ISO VG 32	32	0
ISO VG 46	46	1
ISO VG 68	68	2
ISO VG 100	100	3
ISO VG 150	150	4
ISO VG 220	220	5
ISO VG 320	320	6
ISO VG 460	460	7
ISO VG 680	680	8
ISO VG 1000	1000	8A

Notes:
1. Viscostiy unit of mm^2/s is commonly referred to as centistokes (cSt).
2. ISO standard prescribes minimum and maximum kinematic viscosity limits for each grade.

Adapted from AGMA 6013 (Reference 9) *Standard for Industrial Enclosed Drives*, with permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th Floor, Alexandria, VA

TABLE 9–17 Viscosity Grade Guidelines for Enclosed Gear Drives

Approximate temperature range				Approximate pitch line velocity of final stage of the drive, ft/min (m/s)						
Ambient		Oil sump		<1000	1000–2000	2000–3000	3000–4000	4000–5000	5000–6000	6000–7000
°F	°C	°F	°C	(<5)	(5–10)	(10–15)	(15–20)	(20–25)	(25–30)	(30–35)
−40 to −10	−40 to −23.3	<60	<15.6	68S	68S	46S	46S	46S	32S	32S
−10 to +20	−23.3 to −6.7	60 to 90	15.6 to 32.2	100S	100S	68S	68S	46S	32S	32S
20 to 40	−6.7 to 4.5	90 to 150	32.2 to 65.6	150	150	150	68	68	68	46
40 to 80	4.5 to 26.7	150 to 190	65.6 to 87.8	320	220	220	150	100	100	100
80 to 120	26.7 to 48.9	190 to 210	87.8 to 98.9	460	460	320	320	220	150	100
>120	>48.9	>210	>98.9	Not recommended						

Notes: Viscosity grades are for R&O type, unless Synthetic (S) is specified.

See Table 9–16 for listing of ISO viscosity grades.

Adapted from AGMA 6013 (Reference 9) *Standard for Industrial Enclosed Drives*, with permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th Floor, Alexandria, VA

Commercially Available Gear-Type Speed Reducers

By studying the design of commercially available gear-type speed reducers, you should get a better feel for design details and the relationships among the component parts: the gears, the shafts, the bearings, the housing, the means of providing lubrication, and the coupling to the driving and driven machines.

Figure 9–29 shows a double-reduction spur gear speed reducer with an electric motor rigidly attached. Such a unit is often called a *gear motor*. Figure 9–30 shows a triple-reduction unit with spur gears in the final two stages and helical gears in the first stage (as discussed in Chapter 10). The cross-sectional drawing shown with Figure 9–31 gives a clear picture of the several components of a reducer.

The planetary reducer in Figure 9–32 has quite a different design to accommodate the placement of the sun, planet, and ring gears. Figure 9–33 shows the eight-speed transmission from a large farm tractor and illustrates the high degree of complexity that may be involved in the design of transmissions.

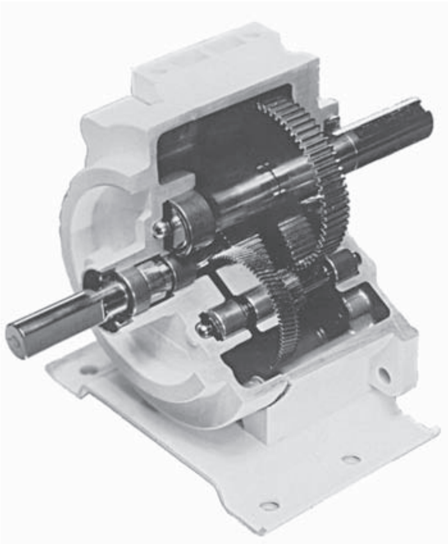


FIGURE 9–30 Triple-reduction gear reducer. Helical gears for stage one; spur gears for stages two and three. The pinion for stage three is on the lower left and not visible. (Bison Gear & Engineering Corporation, St. Charles, IL)

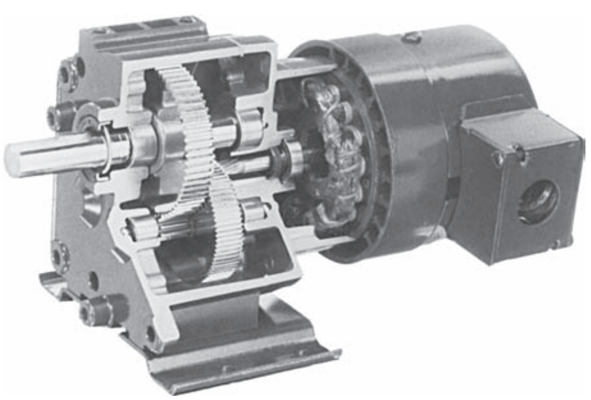
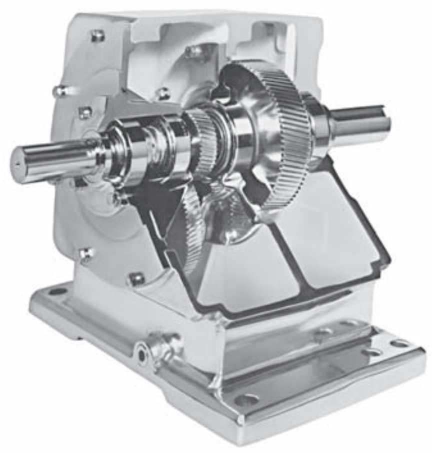
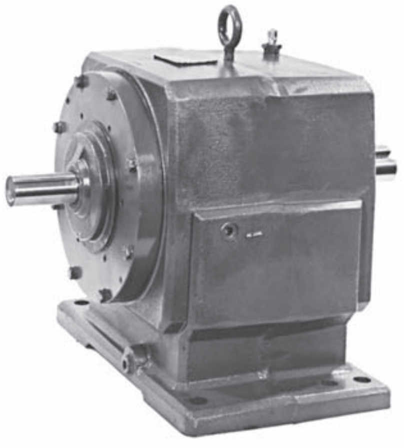


FIGURE 9–29 Double-reduction spur gear reducer (Bison Gear & Engineering Corporation, St. Charles, IL)


(*a*) Cutaway of a concentric helical gear reducer


(*b*) Complete reducer

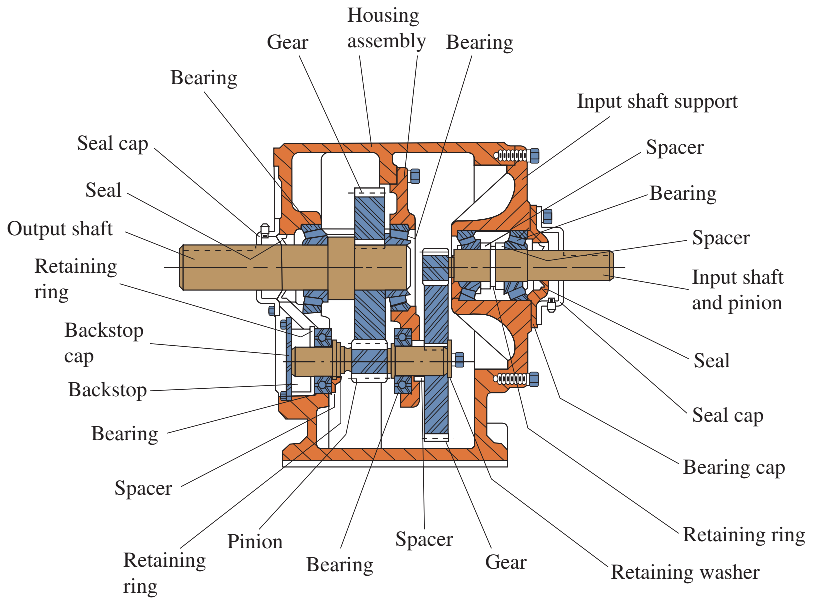
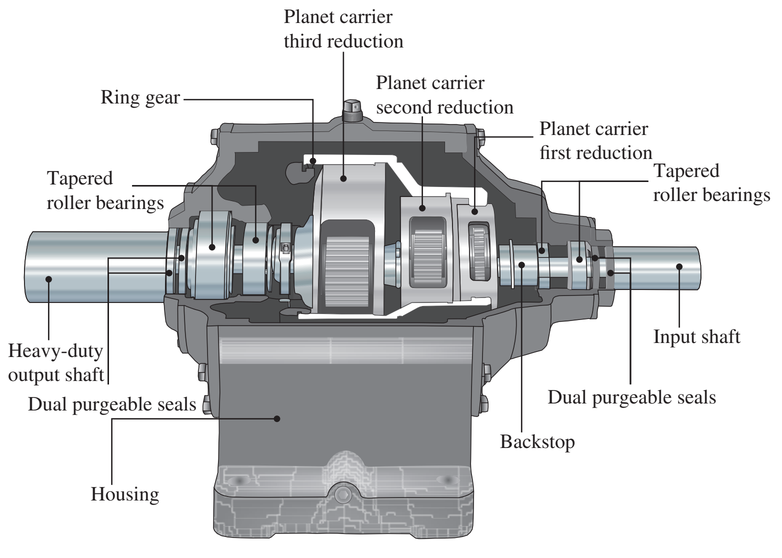
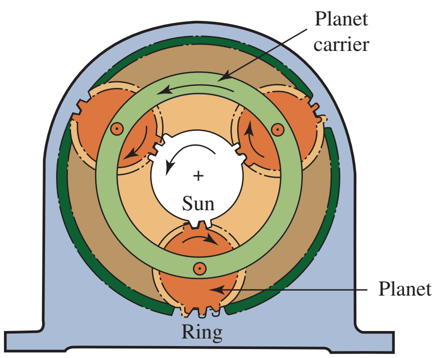


FIGURE 9–31 Concentric helical gear reducer



(*a*) Cutaway model with key features labeled



(*b*) Schematic arrangement of planetary gearing

FIGURE 9–32 Cutaway model of a triple reduction planetary gear reducer

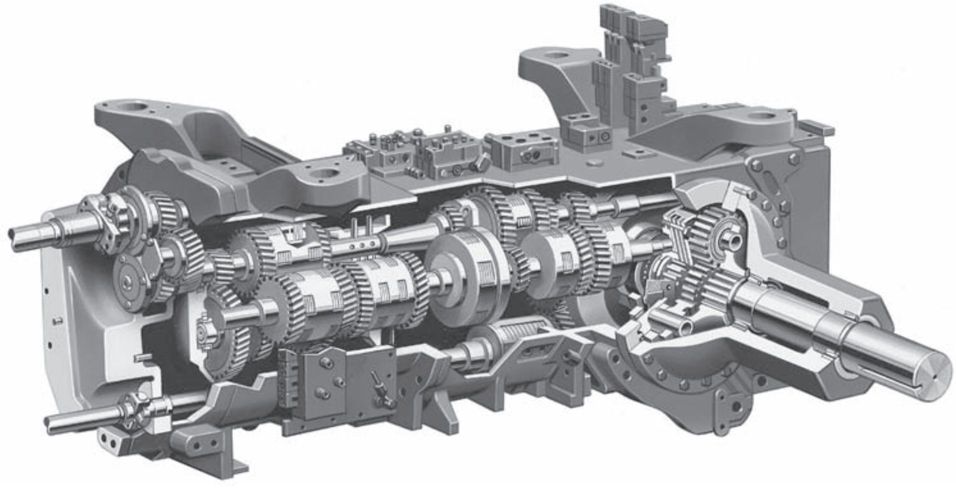


FIGURE 9–33 Eight-speed tractor transmission (Case IH, Racine, WI)

REFERENCES

1. ABA-PGT, Inc. *Plastics Gearing*. Manchester, CT:ABA-PGT Publishing, 1994.
2. Adams, Clifford E. *Plastics Gearing: Selection and Application*. Boca Raton, FL: CRC Press, 1986.
3. American Gear Manufacturers Association. Standard 908-B89 (R2015). *Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical, and Herringbone Gear Teeth*. Alexandria, VA.: American Gear Manufacturers Association, 2015.
4. American Gear Manufacturers Association. Standard 1012-G05 (R2011). *Gear Nomenclature, Definitions of Terms with Symbols*. Alexandria, VA: American Gear Manufacturers Association, 2011.
5. Radzevich, Stephen P. *Gear Cutting Tools—Fundamentals of Design and Computation*. Boca Raton, FL: CRC Press, 2010.
6. American Gear Manufacturers Association. Standards 2001-D04 (R2016) (based on P_d) and 2101-D04 (R2016) (based on Metric Module, m). *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*. American Gear Manufacturers Association. Alexandria, VA: American Gear Manufacturers Association, 2016.
7. American Gear Manufacturers Association. Standard 2002-B88 (R2012). *Tooth Thickness Specification and Measurement*. Alexandria, VA: American Gear Manufacturers Association, 2012.
8. American Gear Manufacturers Association. Standard 2004-C08 (R2014). *Gear Materials, Heat Treatment and Processing Manual*. Alexandria, VA: American Gear Manufacturers Association, 2014.
9. American Gear Manufacturers Association. Standard 6013-A06 (R2011). *Standard for Industrial Enclosed Gear Drives*. Alexandria, VA: American Gear Manufacturers Association, 2011.
10. American Gear Manufacturers Association. Standard 9005-E02 (R2013). *Industrial Gear Lubrication*. Alexandria, VA: American Gear Manufacturers Association, 2013.
11. American Gear Manufacturers Association. Standard 1010-F14. *Appearance of Gear Teeth-Terminology of Wear and Failure*. Alexandria, VA: American Gear Manufacturers Association, 2014.
12. American Gear Manufacturers Association. Standard 1006-A97 (R2009). *Tooth Proportions for Plastic Gears*. Washington, DC:AGMA, 2009.
13. American Gear Manufacturers Association. Standard 1106-A97 (R2009). *Tooth Proportions for Plastic Gears (Metric)*. Washington, DC:AGMA, 2009.
14. American Gear Manufacturers Association. Standard 2000-A88. *Gear Classification and Inspection Handbook—Tolerances and Measuring Methods for Unassembled Spur and Helical Gears (Including Metric Equivalents)*. Alexandria, VA: American Gear Manufacturers Association, 1988.
15. American Gear Manufacturers Association. Standard 2015/915-1-A02. *Accuracy Classification System—Tangential Measurement Tolerance Tables for Cylindrical Gears*. Alexandria, VA: American Gear Manufacturers Association, 2002.
16. Bhushan, Bharat. *Introduction to Tribology*, 2nd ed. New York: Wiley, 2013.
17. Budynas, Richard G., and Keith J. Nisbett. *Shigley's Mechanical Engineering Design*. 10th ed. New York: McGraw-Hill, 2015.
18. Davis, J. R. (Editor). *Gear Materials, Properties and Manufacture*. Materials Park, OH: ASM International, 2005.
19. Drago, Raymond J. *Fundamentals of Gear Design*. New York: Elsevier Science & Technology Books, 1988.
20. Radzevich, Stephen. *Dudley's Handbook of Practical Gear Design and Manufacture*, 3rd ed. Boca Raton, FL: CRC Press, 2016.
21. Du Pont Polymers. *Design Handbook for Du Pont Engineering Polymers, Module I—General Design Principles*. Wilmington, DE: Du Pont Polymers, 2000.
22. International Standards Organization. Standard ISO 54:1996. *Cylindrical Gears for General Engineering and for Heavy Engineering—Modules*. Geneva, Switzerland: International Standards Organization, 1996.

23. Lipp, Robert. "Avoiding Tooth Interference in Gears." *Machine Design* 54, no. 1 (January 7, 1982).
24. Jelaska, Damir. *Gears and Gear Drives*. New York: Wiley, 2012.
25. Oberg, Erik, Franklin D. Jones, Holbrook L. Horton, and Henry H. Ryffel. *Machinery's Handbook*. 30th ed. New York: Industrial Press, 2016.
26. Rakhit, A. K. *Heat Treatment of Gears: A Practical Guide for Engineers*. Materials Park, OH: ASM International, 2000.
27. Totten, George. *Handbook of Lubrication and Tribology: Volume I - Application and Maintenance,* 2nd ed. Boca Raton, FL: CRC Press, 2006.
28. Bruce, Robert. *Handbook of Lubrication and Tribology: Volume II - Theory and Design,* 2nd ed. Boca Raton, FL: CRC Press, 2012.
29. Booser, Richard. *Handbook of Lubrication and Tribology: Volume III - Monitoring, Materials, Synthetic Lubricants, and Applications.* Boca Raton, FL: CRC Press, 2015.

INTERNET SITES RELATED TO SPUR GEAR DESIGN

1. **ABA-PGT, Inc.** The ABA division produces molds for making plastic gears using injection molding; the PGT division is dedicated to plastic gearing technology.
2. **American Gear Manufacturers Association (AGMA).** Develops and publishes voluntary, consensus standards for gears and gear drives.
3. **Baldor/Dodge.** Manufacturer of many power transmission components, including complete gear-type speed reducers, bearings, and components such as belt drives, chain drives, clutches, brakes, and couplings.
4. **Bison Gear, Inc.** Manufacturer of fractional horsepower gear reducers and gear motors. Site includes several videos showing Bison Gear products.
5. **Boston Gear, Company.** Manufacturer of gears and complete gear drives. Part of Altra Industrial Motion, Inc. Data provided for spur, helical, miter, bevel, and worm gearing.
6. **Bourn & Koch, Inc.** Manufacturer of hobbing, grinding, shaping, and other types of machines to produce gears, including the Barber-Colman line and Fellows shapers. Also provides remanufacturing services for a wide variety of existing machine tools.
7. **Drivetrain Technology Center.** Research center for gear-type drivetrain technology. Part of the Applied Research Laboratory of Penn State University.
8. **DuPont Polymers.** Information and data on plastics and their properties. A database by type of plastic or application.
9. **Regal Beloit Corporation.** The Browning and Morse Divisions produce spur, helical, bevel, and wormgearing and complete gear drives. Several other gear drive companies are part of Regal Beloit.
10. **Gear Technology Magazine.** Information source for many companies that manufacture or use gears or gearing systems. Includes gear machinery, gear cutting tools, gear materials, gear drives, open gearing, tooling and supplies, software, training and education.
11. **Gleason Corporation.** Manufacturer of many types of gear cutting machines for hobbing, milling, shaping, and grinding. The Gleason Cutting Tools Corporation manufactures a wide variety of milling cutters, hobs, shaper cutters, shaving cutters, and grinding wheels for gear production equipment. Producer of analytical gear inspection systems.
12. **International Organization for Standardization.** Organization that establishes standards for numerous types of products and devices including gearing. Recognized in most parts of the world. Most standards are presented in SI metric system units.
13. **Peerless-Winsmith, Inc.** Manufacturer of a wide variety of gear reducers and power transmission products, including wormgearing, planetary gearing, and combined helical/wormgearing. Subsidiary of HBD Industries, Inc.
14. **Power Transmission Engineering.** Clearinghouse on the Internet for buyers, users, and sellers of power transmission-related products and services. Included are gears, gear drives, and gear motors. Site includes several videos of power transmission products.
15. **Penta Gear Metrology.** Producer of innovative machines, products, systems, and services, including gear measurement systems. Provides gear inspection services also.
16. **QTC Metric Gears.** Supplier of stock metric gears.
17. **Star-SU, Inc.** Manufacturer of a wide range of gear-production systems including hobbing, grinding, shaping, and shaving, along with cutting tools for the gear industry.
18. **Stock Drive Products—Sterling Instruments.** Manufacturer and distributor of commercial and precision mechanical components, including gear reducers. Site includes an extensive handbook of design and information on metallic and plastic gears.
19. **Eldec Induction USA, Inc.** Producer of complete induction-hardening systems for production operations with special capabilities in gear manufacturing.
20. **Celanese Engineering Polymers.** Producer of plastic gears made from numerous high-performance polymers for many applications such as automotive, HVAC, industrial power tools, electronics components, medical, and home and garden equipment. Search under *Industrial–Gears*.

PROBLEMS

Forces on Spur Gear Teeth

1. A pair of spur gears with 20°, full-depth, involute teeth transmits 7.5 hp. The pinion is mounted on the shaft of an electric motor operating at 1750 rpm. The pinion has 20 teeth and a diametral pitch of 12. The gear has 72 teeth. Compute the following:
   - **a.** The rotational speed of the gear
   - **b.** The velocity ratio and the gear ratio for the gear pair
   - **c.** The pitch diameter of the pinion and the gear
   - **d.** The center distance between the shafts carrying the pinion and the gear
   - **e.** The pitch line speed for both the pinion and the gear
   - **f.** The torque on the pinion shaft and on the gear shaft
   - **g.** The tangential force acting on the teeth of each gear
   - **h.** The radial force acting on the teeth of each gear
   - **i.** The normal force acting on the teeth of each gear

2. A pair of spur gears with 20°, full-depth, involute teeth transmits 50 hp. The pinion is mounted on the shaft of an electric motor operating at 1150 rpm. The pinion has 18 teeth and a diametral pitch of 5. The gear has 68 teeth. Compute the following:
   a. The rotational speed of the gear
   b. The velocity ratio and the gear ratio for the gear pair
   c. The pitch diameter of the pinion and the gear
   d. The center distance between the shafts carrying the pinion and the gear
   e. The pitch line speed for both the pinion and the gear
   f. The torque on the pinion shaft and on the gear shaft
   g. The tangential force acting on the teeth of each gear
   h. The radial force acting on the teeth of each gear
   i. The normal force acting on the teeth of each gear
3. A pair of spur gears with 20°, full-depth, involute teeth transmits 0.75 hp. The pinion is mounted on the shaft of an electric motor operating at 3450 rpm. The pinion has 24 teeth and a diametral pitch of 24. The gear has 110 teeth. Compute the following:
   a. The rotational speed of the gear
   b. The velocity ratio and the gear ratio for the gear pair
   c. The pitch diameter of the pinion and the gear
   d. The center distance between the shafts carrying the pinion and the gear
   e. The pitch line speed for both the pinion and the gear
   f. The torque on the pinion shaft and on the gear shaft
   g. The tangential force acting on the teeth of each gear
   h. The radial force acting on the teeth of each gear
   i. The normal force acting on the teeth of each gear
4. For the data of Problem 1, repeat Parts (g), (h), and (i) if the teeth have 25° full depth instead of 20°.
5. For the data of Problem 2, repeat Parts (g), (h), and (i) if the teeth have 25° full depth instead of 20°.
6. For the data of Problem 3, repeat Parts (g), (h), and (i) if the teeth have 25° full depth instead of 20°.
7. Figure P9–7 shows a drive system in which a 20-hp electric motor drives three separate output shafts. Gear A is mounted on the motor shaft that has a rotational speed of 1750 rpm clockwise. Gear A drives a gear train consisting of gears B, C, and D that deliver power through the shafts on which they are mounted. All gears have a diametral pitch of $P_d = 8$. The following data are given for the gear system:

   Power delivered by gears B, C, and D: P_B = 8 hp, $P_C = 7$ hp, $P_D = 5$ hp

   Numbers of teeth for all gears: N_A = 24, N_B = 48, $N_C = 96$, $N_D = 24$

   Determine the following of the drive system:
   a. The pitch diameter of each gear.
   b. The center distance of each gear mesh.
   c. The rotational speed of each output shaft.
   d. The torque through each output shaft.
   e. The tangential force on the teeth of each gear.

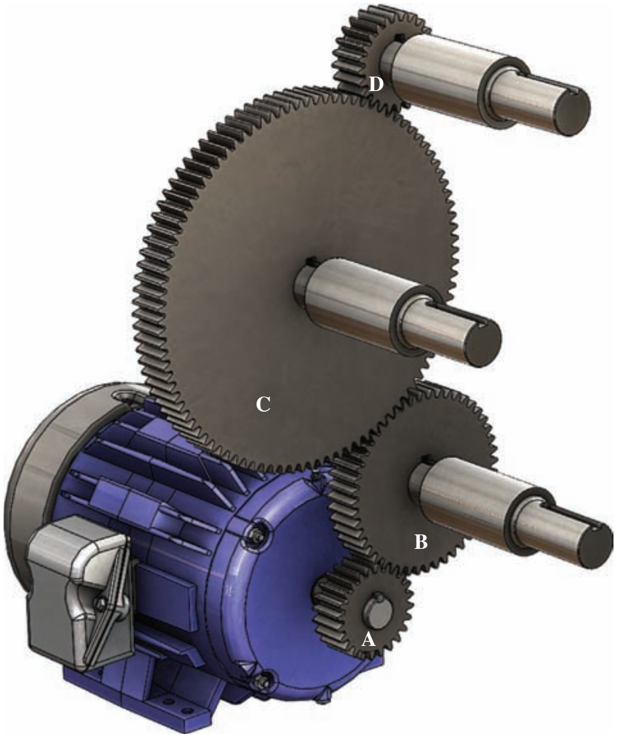


FIGURE P9–7 Gear drive with multiple output shafts

Gear Quality

8. Specify a suitable quality number for the gears in the drive for a grain harvester.
9. Specify a suitable quality number for the gears in the drive for a high-speed printing press.
10. Specify a suitable quality number for the gears in the drive for an automotive transmission.
11. Specify a suitable quality number for the gears in the drive for a gyroscope used in the guidance system for a spacecraft.
12. List five geometric factors measured by analytical gear quality measurement devices.
13. Identify the AGMA standard that is the basis for gear quality measurements and describe the range of quality numbers it includes. Compare that list with the two most recent predecessor standards that had been in use.
14. Specify a suitable quality number for the gears of Problem 1 if the drive is part of a precision machine tool.
15. Specify a suitable quality number for the gears of Problem 2 if the drive is part of a precision machine tool.
16. Specify a suitable quality number for the gears of Problem 3 if the drive is part of a precision machine tool.

Gear Materials

17. Identify the two major types of stresses that are created in gear teeth as they transmit power. Describe how the stresses are produced and where the maximum values of such stresses are expected to occur.
18. Describe the nature of the data contained in AGMA standards that relate to the ability of a given gear tooth to withstand the major types of stresses that it sees in operation.

19. Describe the general nature of steels that are typically used for gears, and list at least five examples of suitable alloys.
20. Describe the range of hardness that can typically be produced by through-hardening techniques and used successfully in steel gears.
21. Describe the general nature of the differences among steels produced as Grade 1, Grade 2, and Grade 3.
22. Suggest at least three applications in which Grade 2 or Grade 3 steel might be appropriate.
23. Describe three methods of producing gear teeth with strengths greater than can be achieved with through-hardening.
24. What AGMA standard should be consulted for data on the allowable stresses for steels used for gears?
25. In the AGMA standard identified in Problem 24, for what other materials besides steels are strength data provided?
26. Determine the allowable bending stress number and the allowable contact stress number for the following materials:
    - **a.** Through-hardened, Grade 1 steel with a hardness of 200 HB
    - **b.** Through-hardened, Grade 1 steel with a hardness of 300 HB
    - **c.** Through-hardened, Grade 1 steel with a hardness of 400 HB
    - **d.** Through-hardened, Grade 1 steel with a hardness of 450 HB
    - **e.** Through-hardened, Grade 2 steel with a hardness of 200 HB
    - **f.** Through-hardened, Grade 2 steel with a hardness of 300 HB
    - **g.** Through-hardened, Grade 2 steel with a hardness of 400 HB
27. If the design of a steel gear indicates that an allowable bending stress number of 36 000 psi is needed, specify a suitable hardness level for Grade 1 steel. What hardness level would be required for Grade 2 steel?
28. What level of hardness can be expected for gear teeth that are case-hardened by carburizing?
29. Name three typical steels that are used in carburizing.
30. What is the level of hardness that can be expected for gear teeth that are case-hardened by flame or induction hardening?
31. Name three typical steels that are used for flame or induction hardening. What is an important property of such steels?
32. State the minimum hardness level at the surface of gear teeth made from ASTM A536 ductile iron, Grade 80-55-06.
33. Determine the allowable bending stress number and the allowable contact stress number for the following materials:
    - **a.** Flame-hardened SAE 4140 steel, Grade 1, with a surface hardness of 50 HRC
    - **b.** Flame-hardened SAE 4140 steel, Grade 1, with a surface hardness of 54 HRC
    - **c.** Carburized and case-hardened SAE 4620 Grade 1 steel, DOQT 300
    - **d.** Carburized and case-hardened SAE 4620 Grade 2 steel, DOQT 300
    - **e.** Carburized and case-hardened SAE 1118 Grade 1 steel, SWQT 350
    - **f.** Gray cast iron, class 20
    - **g.** Gray cast iron, class 40
    - **h.** Ductile iron, 100-70-03
    - **i.** Sand-cast bronze with a minimum tensile strength of 40 ksi (275 MPa)
    - **j.** Heat-treated bronze with a minimum tensile strength of 90 ksi (620 MPa)
    - **k.** Glass-filled nylon
    - **l.** Glass-filled polycarbonate
34. What depth should be specified for the case for a carburized gear tooth having a diametral pitch of 6?
35. What depth should be specified for the case for a carburized gear tooth having a metric module of 6?

Bending Stresses in Gear Teeth

For Problems 36–41, compute the bending stress number, s_t, using Equation (9–16). Assume that the gear blank is solid unless otherwise stated. (*Note that the data in these problems are used in later problems through Problem 59. You are advised to keep solutions to earlier problems accessible so that you can use data and results in later problems. The four problems that are keyed to the same set of data require the analysis of bending stress and contact stress and the corresponding specification of suitable materials based on those stresses. Later design problems, 60–70, use the complete analysis within each problem.*)

36. A pair of gears with 20°, full-depth, involute teeth transmits 10.0 hp while the pinion rotates at 1750 rpm. The diametral pitch is 12, and the quality number is A11. The pinion has 18 teeth, and the gear has 85 teeth. The face width is 1.25 in. The input power is from an electric motor, and the drive is for an industrial conveyor. The drive is a commercial enclosed gear unit.
37. A pair of gears with 20°, full-depth, involute teeth transmits 40 hp while the pinion rotates at 1150 rpm. The diametral pitch is 6, and the quality number is A11. The pinion has 20 teeth, and the gear has 48 teeth. The face width is 2.25 in. The input power is from an electric motor, and the drive is for a cement kiln. The drive is a commercial enclosed gear unit.
38. A pair of gears with 20°, full-depth, involute teeth transmits 0.50 hp while the pinion rotates at 3450 rpm. The diametral pitch is 32, and the quality number is A7. The pinion has 24 teeth, and the gear has 120 teeth. The face width is 0.50 in. The input power is from an electric motor, and the drive is for a small machine tool. The drive is a precision enclosed gear unit.
39. A pair of gears with 25° full-depth, involute teeth transmits 15.0 hp while the pinion rotates at 6500 rpm. The diametral pitch is 10, and the quality number is A5. The pinion has 30 teeth, and the gear has 88 teeth. The face width is 1.50 in. The input power is from a universal electric motor, and the drive is for an actuator on an aircraft. The drive is an extra-precision, enclosed gear unit.
40. A pair of gears with 25°, full-depth, involute teeth transmits 125 hp while the pinion rotates at 2500 rpm. The diametral pitch is 4, and the quality number is A9. The pinion has 32 teeth, and the gear has 76 teeth. The face width is 1.50 in. The input power is from a gasoline engine, and the drive is for a portable industrial water pump. The drive is a commercial enclosed gear unit.

41. A pair of gears with 25°, full-depth, involute teeth transmits 2.50 hp while the pinion rotates at 680 rpm. The diametral pitch is 10, and the quality number is A11. The pinion has 24 teeth, and the gear has 62 teeth. The face width is 1.25 in. The input power is from a vane-type fluid motor, and the drive is for a small lawn and garden tractor. The drive is a commercial enclosed gear unit.

Required Allowable Bending Stress Number

For Problems 42–47, compute the required allowable bending stress number, s_{at}, using Equation (9–30). Assume that no unusual conditions exist unless stated otherwise. That is, use a service factor, *SF*, of 1.00. Then specify a suitable steel and its heat treatment for both the pinion and the gear based on bending stress.

42. Use the data and results from Problem 36. Design for a reliability of 0.99 and a design life of 20 000 h.
43. Use the data and results from Problem 37. Design for a reliability of 0.99 and a design life of 8000 h.
44. Use the data and results from Problem 38. Design for a reliability of 0.9999 and a design life of 12 000 h. Consider that the machine tool is a critical part of a production system calling for a service factor of 1.25 to avoid unexpected down time.
45. Use the data and results from Problem 39. Design for a reliability of 0.9999 and a design life of 4000 h.
46. Use the data and results from Problem 40. Design for a reliability of 0.99 and a design life of 8000 h.
47. Use the data and results from Problem 41. Design for a reliability of 0.90 and a design life of 2000 h. The uncertainty of the actual use of the tractor calls for a service factor of 1.25. Consider using cast iron or bronze if the conditions permit.

Pitting Resistance

For Problems 48–53, compute the expected contact stress number, s_c, using Equation (9–23). Assume that both gears are to be steel unless stated otherwise.

48. Use the data and results from Problems 36 and 42.
49. Use the data and results from Problems 37 and 43.
50. Use the data and results from Problems 38 and 44.
51. Use the data and results from Problems 39 and 45.
52. Use the data and results from Problems 40 and 46.
53. Use the data and results from Problems 41 and 47.

Required Allowable Contact Stress Number

For Problems 54–59, compute the required allowable contact stress number, s_{ac}, using Equation (9–31). Use a service factor, *SF*, of 1.00 unless stated otherwise. Then specify suitable material for the pinion and the gear based on pitting resistance. Use steel unless an earlier decision has been made to use another material. Then evaluate whether the earlier decision is still valid. If not, specify a different material according to the most severe requirement. If no suitable material can be found, consider redesigning the original gears to enable reasonable materials to be used.

54. Use the data and results from Problems 36, 42, and 48.
55. Use the data and results from Problems 37, 43, and 49.
56. Use the data and results from Problems 38, 44, and 50.
57. Use the data and results from Problems 39, 45, and 51.
58. Use the data and results from Problems 40, 46, and 52.
59. Use the data and results from Problems 41, 47, and 53.

Design Problems

Problems 60–70 describe design situations. For each, design a pair of spur gears, specifying (at least) the diametral pitch, the number of teeth in each gear, the pitch diameters of each gear, the center distance, the face width, and the material from which the gears are to be made. Design for recommended life with regard to both strength and pitting resistance. Work toward designs that are compact. Use standard values of diametral pitch, and avoid designs for which interference could occur. See Example Problem 9–7. Assume that the input to the gear pair is from an electric motor unless otherwise stated.

If the data are given in SI units, complete the design in the metric module system with dimensions in millimeters, forces in newtons, and stresses in megapascals. See Example Problem 9–8.

60. A pair of spur gears is to be designed to transmit 5.0 hp while the pinion rotates at 1200 rpm. The gear must rotate between 385 and 390 rpm. The gear drives a reciprocating compressor.
61. A gear pair is to be a part of the drive for a milling machine requiring 20.0 hp with the pinion speed at 550 rpm and the gear speed to be between 180 and 190 rpm.
62. A drive for a punch press requires 50.0 hp with the pinion speed of 900 rpm and the gear speed of 225 to 230 rpm.
63. A single-cylinder gasoline engine has the pinion of a gear pair on its output shaft. The gear is attached to the shaft of a small cement mixer. The mixer requires 2.5 hp while rotating at approximately 75 rpm. The engine is governed to run at approximately 900 rpm.
64. A four-cylinder industrial engine runs at 2200 rpm and delivers 75 hp to the input gear of a drive for a large wood chipper used to prepare pulpwood chips for paper making. The output gear must run between 4500 and 4600 rpm.
65. A small commercial tractor is being designed for chores such as lawn mowing and snow removal. The wheel drive system is to be through a gear pair in which the pinion runs at 600 rpm while the gear, mounted on the hub of the wheel, runs at 170 to 180 rpm. The wheel is 300 mm in diameter. The gasoline engine delivers 3.0 kW of power to the gear pair.
66. A water turbine transmits 75 kW of power to a pair of gears at 4500 rpm. The output of the gear pair must drive an electric power generator at 3600 rpm. The center distance for the gear pair must not exceed 150 mm.

67. A drive system for a large commercial band saw is to be designed to transmit 12.0 hp. The saw will be used to cut steel tubing for automotive exhaust pipes. The pinion rotates at 3450 rpm, while the gear must rotate between 725 and 735 rpm. It has been specified that the gears are to be made from SAE 4340 steel, oil quenched and tempered. Case hardening is *not* to be used.
68. Repeat Problem 67, but consider a case-hardened carburized steel from Appendix 5. Try to achieve the smallest practical design. Compare the result with the design from Problem 67.
69. A gear drive for a special-purpose, dedicated machine tool is being designed to mill a surface on a rough steel casting. The drive must transmit 20 hp with a pinion speed of 650 rpm and an output speed between 110 and 115 rpm. The mill is to be used continuously, two shifts per day, six days per week, for at least five years. Design the drive to be as small as practical to permit its being mounted close to the milling head.
70. A cable drum for a crane is to rotate between 160 and 166 rpm. Design a gear drive for 25 hp in which the input pinion rotates at 925 rpm and the output rotates with the drum. The crane is expected to operate with a 50% duty cycle for 120 hours per week for at least 10 years. The pinion and the gear of the drive must fit within the 24-in inside diameter of the drum, with the gear mounted on the drum shaft.

Power-Transmitting Capacity

71. Determine the power-transmitting capacity for a pair of spur gears having 20°, full-depth teeth, a diametral pitch of 10, a face width of 1.25 in, 25 teeth in the pinion, 60 teeth in the gear, and an AGMA quality class of A9. The pinion is made from SAE 4140 OQT 1000, and the gear is made from SAE 4140 OQT 1100. The pinion will rotate at 1725 rpm on the shaft of an electric motor. The gear will drive a centrifugal pump.
72. Determine the power-transmitting capacity for a pair of spur gears having 20°, full-depth teeth, a diametral pitch of 6, 35 teeth in the pinion, 100 teeth in the gear, a face width of 2.00 in, and an AGMA quality class of A11. A gasoline engine drives the pinion at 1500 rpm. The gear drives a conveyor for crushed rock in a quarry. The pinion is made from SAE 1040 WQT 800. The gear is made from gray cast iron, ASTM A48-83, class 30. Design for 15 000 hr life.
73. It was found that the gear pair described in Problem 72 wore out when driven by a 25-hp engine. Propose a redesign that would be expected to give 15 000 hr life under the conditions described.

Design of Double-Reduction Drives

74. Design a double-reduction gear train that will transmit 10.0 hp from an electric motor running at 1750 rpm to an assembly conveyor whose drive-shaft must rotate between 146 and 150 rpm. Note that this will require the design of two pairs of gears. Sketch the arrangement of the train, and compute the actual output speed.
75. A commercial food waste grinder in which the final shaft rotates at between 40 and 44 rpm is to be designed. The input is from an electric motor running at 850 rpm and delivering 0.50 hp. Design a double-reduction spur gear train for the grinder.
76. A small, powered hand drill is driven by an electric motor running at 3000 rpm. The drill speed is to be approximately 550 rpm. Design the speed reduction for the drill. The power transmitted is 0.25 hp.
77. The output from the drill described in Problem 76 provides the drive for a small bench-scale band saw used in a home shop. The saw blade is to move with a linear velocity of 375 ft/min. The saw blade rides on 9.0-in-diameter wheels. Design a spur gear reduction to drive the band saw. Consider using plastic gears.
78. Design a rack-and-pinion drive to lift a heavy access panel on a furnace. A fluid power motor rotating 1500 rpm will provide 5.0 hp at the input to the drive. The linear speed of the rack is to be at least 2.0 ft/s. The rack moves 6.0 ft each way during the opening and closing of the furnace doors. More than one stage of reduction may be used, but attempt to design with the fewest number of gears. The drive is expected to operate at least six times per hour for three shifts per day, seven days per week, for at least 15 years.
79. Design the gear drive for the wheels of an industrial lift truck. Its top speed is to be 20 mph. It has been decided that the wheels will have a diameter of 12.0 in. A DC motor supplies 20 hp at a speed of 3000 rpm. The design life is 16 hours per day, six days per week, for 20 years.

Plastics Gearing

80. Design a pair of plastic gears to drive a small band saw. The input is from a 0.50 hp electric motor rotating at 860 rpm, and the pinion will be mounted on its 0.75-inch diameter shaft with a keyway for a 0.1875×0.1875 in key. The gear is to rotate between 265 and 267 rpm.
81. Design a pair of plastic gears to drive a paper feed roll for an office printer. The pinion rotates at 88 rpm and the gear must rotate between 20 and 22 rpm. The power required is 0.06 hp. Work toward the smallest practical size.
82. Design a pair of plastic gears to drive the wheels of a small remote control car. The gear is mounted on the axle of the wheel and must rotate between 120 and 122 rpm. The pinion rotates at 430 rpm. The power required is 0.025 hp. Work toward the smallest practical size using unfilled nylon.
83. Design a pair of plastic gears to drive a commercial food-chopping machine. The input is from a 0.65 hp electric motor rotating at 1560 rpm, and the pinion will be mounted on its 0.875-inch diameter shaft with a keyway for a 0.1875×0.1875 in key. The gear is to rotate between 468 and 470 rpm.

CHAPTER TEN

HELICAL GEARS, BEVEL GEARS, AND WORMGEARING

THE BIG PICTURE Helical Gears, Bevel Gears, and Wormgearing

Discussion Map

- The geometry of helical gears, bevel gears, and wormgearing was described in Chapter 8.
- The principles of stress analysis of gears were discussed in Chapter 9 for spur gears. Much of that information is applicable to the types of gears discussed in this chapter.

Discover

Review Chapters 8 and 9 now.

Recall some of the discussion at the beginning of Chapter 8 about uses for gears that you see in your world. Review that information now, and focus your discussion on helical gears, bevel gears, and wormgearing.

In this chapter, you acquire the skills to perform the necessary analyses to design safe gear drives that use helical gears, bevel gears, and wormgearing and that demonstrate long life.

Much was said in Chapter 8 about the geometry of spur gears, helical gears, bevel gears, and wormgearing and the kinematics of a single pair of gears and trains made from two or more pairs of gears. Also, parts of Chapters 8 and 9 discussed the types of metallic materials commonly used for power transmission gears, methods of manufacturing gears, principles of gear quality, and measurements required to ensure that quality. Then you learned how to analyze the bending stress in the fillet at the base of spur gear teeth and the contact stress along the face of the teeth near the pitch line. This was combined with determining the required bending strength of the gear material to avoid fatigue failure and the required hardness of the face of the gear to provide adequate resistance to surface pitting. The result led to a method of designing spur gear drives to achieve satisfactory performance and life. Section 9–14 applied gear design principles to plastics gearing.

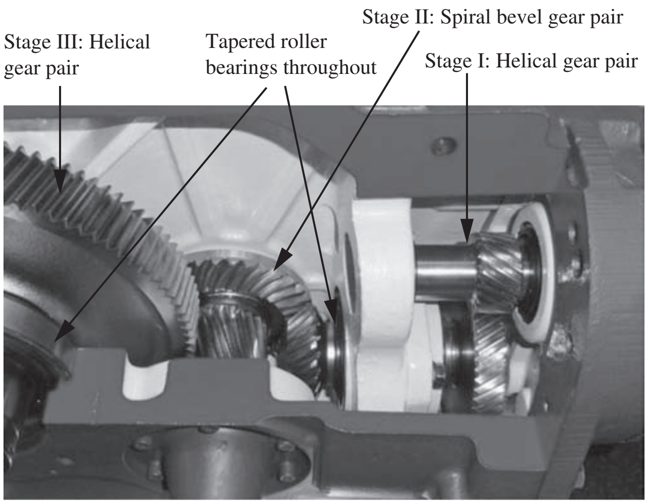
Similar analyses and design approaches are developed in this chapter for helical gears, bevel gears, and wormgearing. You will need to refer back to Chapters 5, 8, and 9 as we proceed. More detailed information can be found in the AGMA standards listed in References 1–14.

Figure 10–1 shows a combination of a gear-type speed reducer driven by a close-coupled electric motor; the combination is often called a *gearmotor.* Part (b) of the figure shows the internal arrangement of the three-stage speed reducer. At the right end, a helical pinion shaft is driven directly by the motor and it then drives its mating gear for the first stage of reduction. The same shaft with the output helical gear carries the bevel pinion for the second stage of reduction. The helical pinion for the third stage is placed close to the output bevel gear on the same shaft. The large output helical gear then delivers the power at the greatly reduced speed and at a correspondingly higher torque to the driven machine. The design of the output shaft employs a hollow shaft with taper-lock bushing that facilitates connection to the input shaft of the driven machine.

Figure 10–2 shows a double-reduction, helical gear reducer that receives power from the electric motor mounted above and through the belt drive that produces an initial speed reduction. Take note of the arrangement of the gears, shafts, and bearings in the housing shown to the left. Tapered roller bearings are used on all shafts to carry the combination of radial and axial forces that are inherently produced by helical gears.

Refer back to Figure 8–25 for an example of a wormgear reducer.

(*a*) Assembly of a gear reducer and a drive motor, called a gearmotor



(*b*) Cutaway showing the internal arrangement of the helical-bevel-helical three-stage reduction

FIGURE 10–1 Gearmotor assembly employing a three-stage gear reducer (Baldor/Dodge, Greenville, SC)

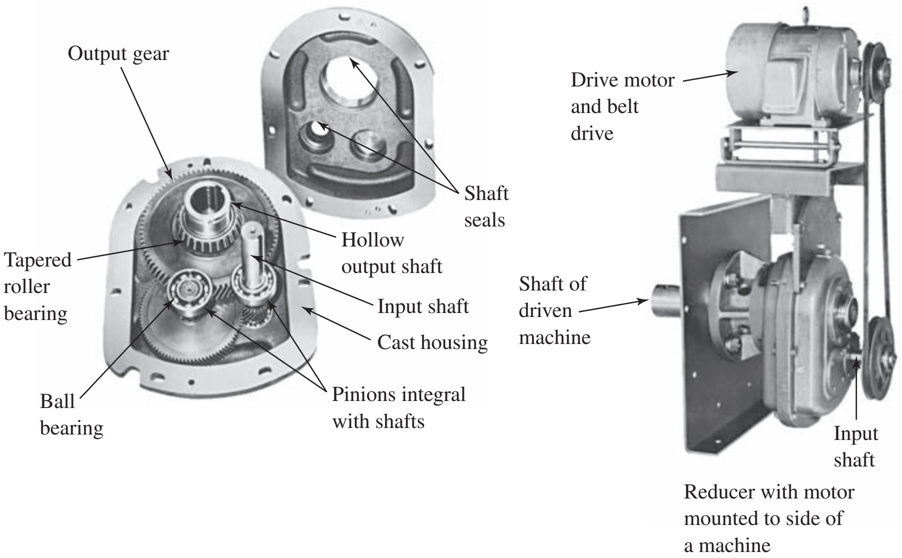


FIGURE 10–2 Helical shaft-mount reducer (Power Transmission Solutions, a business unit of Emerson Industrial Automation)

YOU ARE THE DESIGNER

The gear drives that were designed in Chapter 9 all assumed that spur gears would be used to accomplish the speed reduction or speed increase between the input and the output of the drive. But many other types of gears could have been used. Assume that you are the designer of the drive for the wood chipper described in Example Problem 9–7. How would the design be different if helical gears were used instead of spur gears? What forces would be created and transferred to the shafts carrying the gears and to the bearings carrying the shafts? Would you be able to use smaller gears? How is the geometry of helical gears different from that of spur gears?

Rather than having the input and output shafts parallel as they were in designs up to this time, how can we design drives that deliver power to an output shaft at right angles to the input shaft? What special analysis techniques are applied to bevel gears and wormgearing?

The information in this chapter will help you answer these and other questions. ■

10–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will be able to:

1. Describe the geometry of helical gears and compute the dimensions of key features.
2. Compute the forces exerted by one helical gear on its mating gear.
3. Compute the stress due to bending in helical gear teeth and specify suitable materials to withstand such stresses.
4. Design helical gears for surface durability.
5. Describe the geometry of bevel gears and compute the dimensions of key features.
6. Analyze the forces exerted by one bevel gear on another and show how those forces are transferred to the shafts carrying the gears.
7. Design and analyze bevel gear teeth for strength and surface durability.
8. Describe the geometry of worms and wormgears.
9. Compute the forces created by a wormgear drive system and analyze their effect on the shafts carrying the worm and the wormgear.
10. Compute the efficiency of wormgear drives.
11. Design and analyze wormgear drives to be safe for bending strength and wear.

References at the end of the chapter are recommended for additional information for design and application of helical gears, bevel gears, and wormgearing.

10–2 FORCES ON HELICAL GEAR TEETH

Figure 10–3 shows a drawing of two helical gears in mesh and designed to be mounted on parallel shafts. This is the basic configuration that we analyze in this chapter. Refer to Figure 10–4 for a representation of the force system that acts between the teeth of two helical

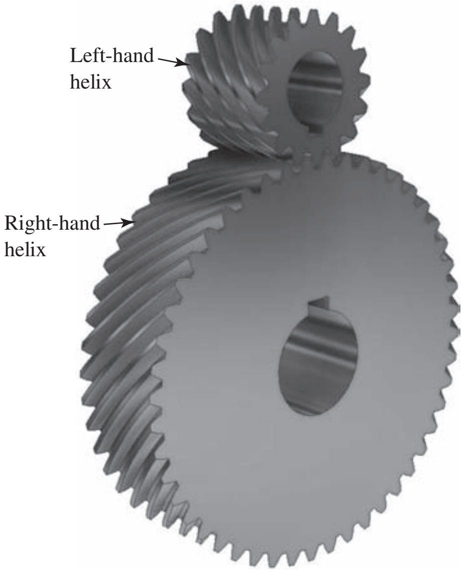


FIGURE 10–3 Helical gears in mesh. These gears have a 45° helix angle

gears in mesh. Also, it is useful to review Section 8–7 in Chapter 8 on *Helical Gear Geometry*.

We now define the array of forces acting on helical gear teeth as shown in Figure 10–4.

- W_N is the *true normal force* that acts perpendicular to the face of the tooth in the plane normal to the surface of the tooth. We seldom need to use the value of W_N because its three orthogonal components, defined next, are used in the analyses performed for helical gears. The values for the orthogonal components depend on the following three angles that help define the geometry of the helical gear teeth:

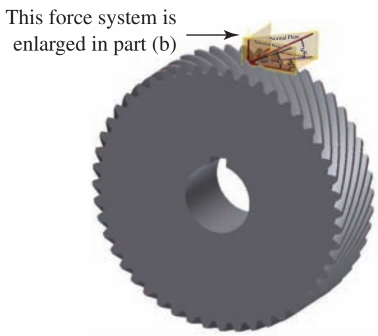
  Normal pressure angle: ϕ_n
  Transverse pressure angle: ϕ_t
  Helix angle: ψ

  For helical gears, the helix angle and one of the other two are specified. The third angle can be computed from

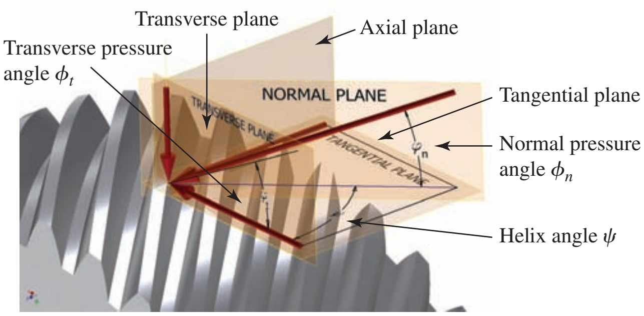
$$\tan \phi_n = \tan \phi_t \cos \psi \qquad \textbf{(10–1)}$$

- W_t is the *tangential force* that acts in the transverse plane and tangent to the pitch circle of the helical gear and that causes the torque to be transmitted from the driver to the driven gear. Therefore, this force is often called the *transmitted force*. It is functionally similar to W_t used in the analysis of spur gears in Chapters 8 and 9. We can compute its value from the same equations, as follows:

  If the torque being transmitted (T) and the size of the gear (D) are known,



(a) System of forces acting on a gear tooth



(b) Normal force and its three components

FIGURE 10–4 Helical gear geometry showing forces acting on gear teeth

➪ Tangential Force

$$W_t = T/(D/2) \qquad \textbf{(10–2)}$$

If the power being transmitted (P) and the rotational speed (n) are known,

$$T = (P/n) \qquad \textbf{(10–3)}$$

Power, torque, and forces for the U.S. unit system: Here we bring concepts and equations from Section 9–3 developed for spur gears and apply them for helical gears. *These are unit-specific equations.*

When power, P, is expressed in hp, rotational speed, n, is in rpm, and diameters, D, are in inches:

Torque: $T = 63\ 000(P)/n$ lb·in **(10–4)**

Pitch line speed: $v_t = \pi Dn/12$ ft/min **(10–5)**

Tangential force: $W_t = (126\ 000)(P)/[(n)(D)]$ lb **(10–6)**

Or: $W_t = (33\ 000)(P)/v_t$ lb **(10–7)**

Power, torque, and forces for the SI metric unit system: Again, bringing concepts and equations from Sections 9–3 and 9–10 developed for spur gears, we apply them for helical gears. *These are unit-specific equations.*

When power, P, is expressed in kW, rotational speed, n, is in rpm, and diameters, D, are in mm:

Torque: $T = 9549(P)/n$ N·m **(10–4M)**

Pitch line speed: $v_t = \pi Dn/60\ 000$ m/s **(10–5M)**

Tangential force: $W_t = (19\ 099)(P)/[(n)(D)]$ N **(10–6M)**

Or: $W_t = (1000)(P)/v_t$ N **(10–7M)**

The value of the tangential load is the most fundamental of the three orthogonal components of the true normal force. The calculation of the bending stress number and the contact stress number of the gear teeth depends on W_t.

- W_r is the *radial force* that acts toward the center of the gear perpendicular to the pitch circle and to the tangential force. It tends to push the two gears apart. As can be seen in Figure 10–4(b)

➪ Radial Force

$$W_r = W_t \tan \phi_t \tag{10–8}$$

- where ϕ_t = transverse pressure angle for the helical teeth
- W_x is the *axial force* that acts parallel to the axis of the gear and causes a thrust load that must be resisted by the bearings carrying the shaft. With the tangential force known, the axial force is computed from

➪ Axial Force

$$W_x = W_t \tan \psi \tag{10–9}$$

Example Problem 10–1 A helical gear has a normal diametral pitch, P_{nd}, of 8, a normal pressure angle of 20°, 32 teeth, a face width of 3.00 in, and a helix angle of 15°. Compute the diametral pitch, the transverse pressure angle, and the pitch diameter. If the gear is rotating at 650 rpm while transmitting 7.50 hp, compute the pitch line speed, the tangential force, the axial force, and the radial force.

Solution *Diametral Pitch:*

$$P_d = P_{nd} \cos \psi = 8 \cos (15°) = 7.727$$

Transverse Pressure Angle: [Equation (10–1)]

$$\phi_t = \tan^{-1}(\tan \phi_n/\cos \psi)$$

$$\phi_t = \tan^{-1}[\tan(20°)/\cos(15°)] = 20.65°$$

Pitch Diameter:

$$D = N/P_d = 32/7.727 = 4.141 \text{ in}$$

Pitch Line Speed, v_t: [Equation (10–5)]

$$v_t = \pi Dn/12 = \pi(4.141)(650)/12 = 704.7 \text{ ft/min}$$

Tangential Force, W_t: [Equation (10–7)]

$$W_t = 33\,000(P)/v_t = 33\,000(7.5)/704.7 = 351 \text{ lb}$$

Axial Force, W_x: [Equation (10–9)]

$$W_x = W_t \tan \psi = 351 \tan(15°) = 94 \text{ lb}$$

Radial Force, W_r: [Equation (10–8)]

$$W_r = W_t \tan \phi_t = 351 \tan(20.65°) = 132 \text{ lb}$$

The following example problem illustrates similar calculations for the SI metric system.

Example Problem 10–2 A helical gear has a normal module of 3 mm, a normal pressure angle of 25°, a helix angle of 22°, 32 teeth, and a face width of 75 mm. Compute the transverse module, the transverse pressure angle, and the pitch diameter. Then, if the gear is rotating at 650 rpm while transmitting 5.0 kW of power, compute the pitch line speed, the tangential force, the axial force, and the radial force.

Solution *Transverse Module:*

$$m_n = m \cos \psi$$

$$m = m_n/\cos \psi = 3.00 \text{ mm}/\cos 22° = 3.236 \text{ mm}$$

Transverse Pressure Angle: [Equation (10–1)]

$$\phi_t = \tan^{-1}(\tan \phi_n/\cos \psi) = \tan^{-1}[\tan(25°)/\cos(22°)] = 23.38°$$

Pitch Diameter: $D = mN = (3.236 \text{ mm})(32) = 103.54 \text{ mm}$

Pitch Line Speed: [Equation (10–5M)]

$$v_t = \pi Dn/(60\,000) = \pi(103.54 \text{ mm})(650 \text{ rpm})/(60\,000) = 3.524 \text{ m/s}$$

Tangential Force: [Equation (10–6M)]

$$W_t = (1000)(P)/v_t = (1000)(5.0 \text{ kW})/(3.524 \text{ m/s}) = 1419 \text{ N}$$

Axial Force: [Equation (10–9)]

$$W_x = W_t \tan\psi = (1419 \text{ N})\tan(22°) = 573.3 \text{ N}$$

Radial Force: [Equation (10–8)]

$$W_r = W_t \tan\phi_t = (1419 \text{ N})\tan(23.38°) = 613.5 \text{ N}$$

10–3 STRESSES IN HELICAL GEAR TEETH

We will use the same basic equation for computing the bending stress number for helical gear teeth as we did for spur gear teeth in Chapter 9, given in Equation (9–16) and repeated here:

$$s_t = \frac{W_t P_d}{FJ} K_o K_s K_m K_B K_v$$

Figures 10–5 to 10–7 show the values for the geometry factor, J, for helical gear teeth with 15°, 20°, and 22° normal pressure angles, respectively.[1] The K factors are the same as those used for spur gears. See References 9 and 18 and the following locations for values:

K_o = overload factor (Table 9–1)

K_s = size factor (Table 9–2)

K_m = load-distribution factor [Figures 9–12 and 9–13 and Equation (9–17)]

K_B = rim thickness factor (Figure 9–14)

K_v = dynamic factor (Figure 9–16)

For design, a material must be specified that has an allowable bending stress number, s_{at}, greater than the computed bending stress number, s_t. Design values of s_{at} can be found in:

Figure 9–18: Steel, through-hardened, Grades 1 and 2

Table 9–9: Case-hardened steels

Table 9–10: Cast iron and bronze

(See also Reference 11 and 17–21.) The data for steel, iron, and bronze apply to a design life of 10^7 cycles at a reliability of 99% (fewer than one failure in 100). If other values for design life or reliability are desired, the allowable stress can be modified using the procedure described in Section 9–8.

10–4 PITTING RESISTANCE FOR HELICAL GEAR TEETH

Pitting resistance for helical gear teeth is evaluated using the same procedure as that discussed in Chapter 9 for spur gears. Equation (9–23) is repeated here:

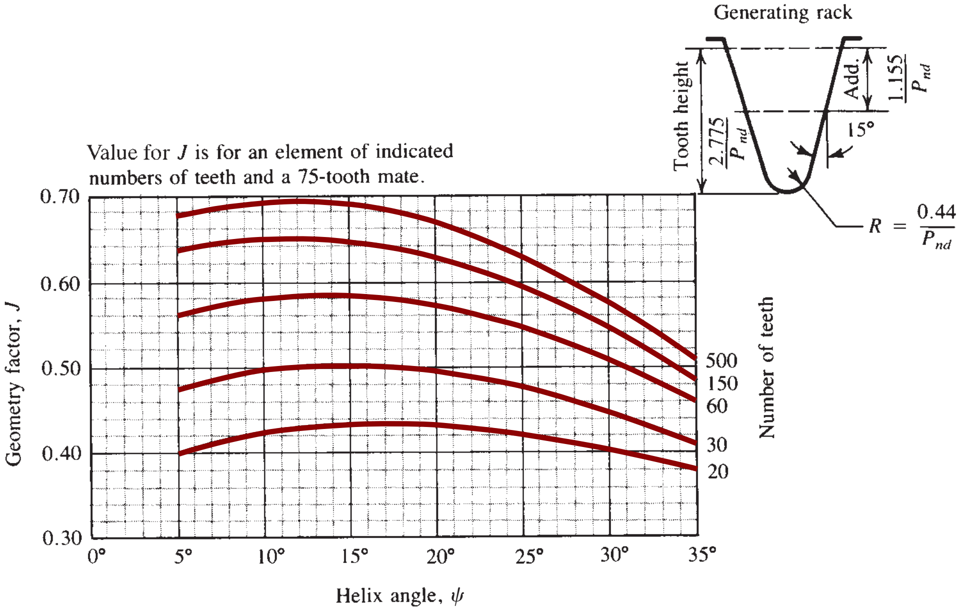
$$s_c = C_p \sqrt{\frac{W_t K_o K_s K_m K_v}{F D_p I}} \qquad (9\text{–}23)$$

All of the factors are the same for helical gears except the geometry factor for pitting resistance, I. The values for C_p are found in Table 9–7. Note that the other K factors have the same values as the K factors discussed and identified in Section 10–3.

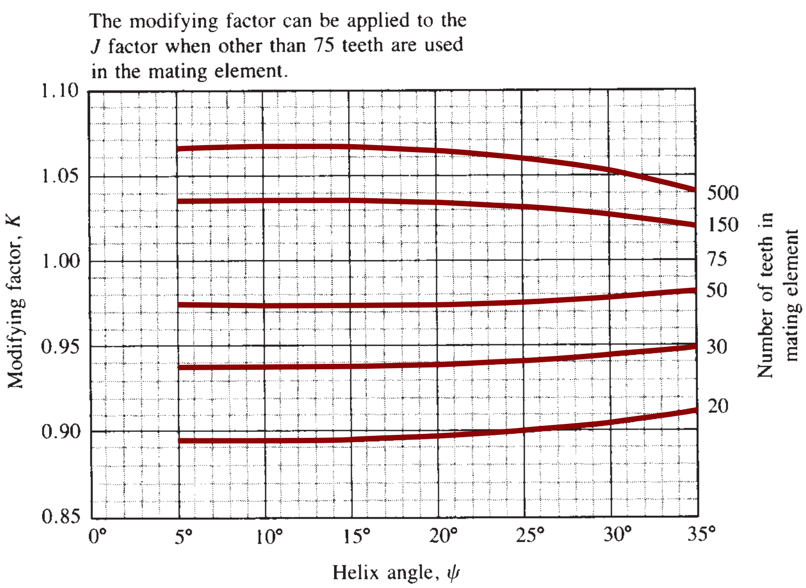
Because of the larger variety of geometric features needed to define the form of helical gears, it is not reasonable to reproduce all of the necessary tables of values or the complete formulas for computing I. Values change with the gear ratio, the number of teeth in the pinion, the tooth form, the helix angle, and the specific values for addendum, whole depth, and fillet radius. See References 6 and 13 for extensive discussions about the procedures. To facilitate problem solving in this book, Tables 10–1 and 10–2 give a few values for I.

For design, when the computed contact stress number is known, a material must be specified that has an

[1] Figures 10–5 to 10–7:
Graphs for the geometry factor, J, for helical gears have been taken from AGMA Standard 218.01-1982, *Standard for Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314. This standard has been superseded by two standards: (1) Standard 908-B89 (R 1999), *Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical, and Herringbone Gear Teeth*, 1999 and (2) Standard 2001-D04, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, 2004. The method of calculating the value for J has not been changed. However, the new standards do not contain the graphs. Users are cautioned to ensure that geometry factors for a given design conform to the specific cutter geometry used to manufacture the gears. Standards 908-B89 (R 1999) and 2001-D04 should be consulted for the details of computing the values for J and for rating the performance of the gear teeth.



(a) Geometry factor (J) for 15° normal pressure angle and indicated addendum



(b) J factor multipliers

FIGURE 10–5 Geometry factor (J) for 15° normal pressure angle

allowable contact stress number, s_{ac}, greater than s_c. Design values for s_{ac} can be found from the following:

Figure 9–19: Steel, through-hardened, Grades 1 and 2

Table 9–9: Steel, case-hardened, Grade 1; flame- or induction-hardened, or carburized

Figure 9–10: Cast iron and bronze

The data from these sources apply to a design life of 10^7 cycles at a reliability of 99% (fewer than one failure in 100). If other values for design life or reliability are desired, or if a service factor is to be applied, the allowable contact stress number can be modified using the procedure described in Section 9–8.

10–5 DESIGN OF HELICAL GEARS

The example problem that follows illustrates the procedure to design helical gears.

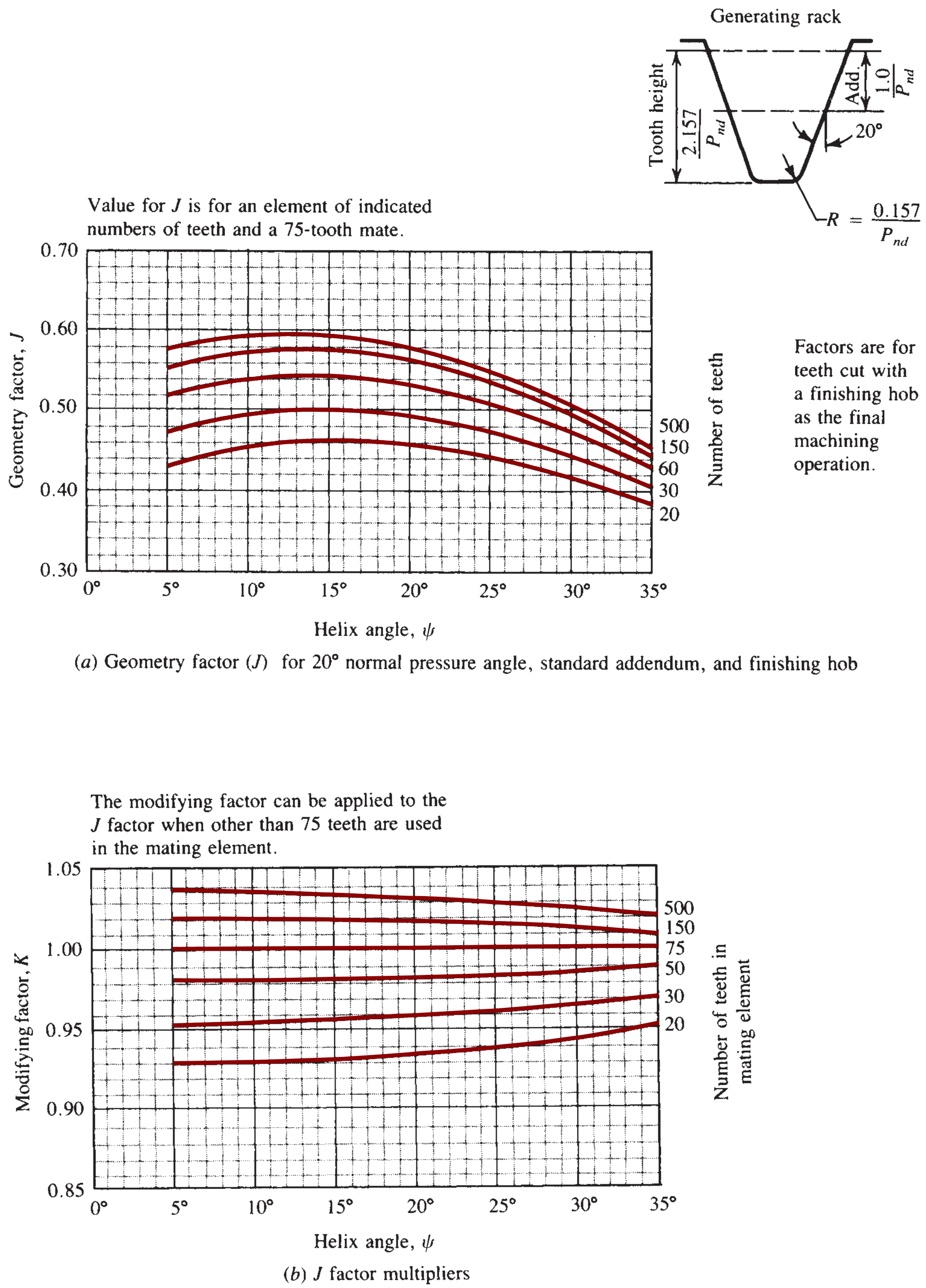


FIGURE 10–6 Geometry factor (J) for 20° normal pressure angle

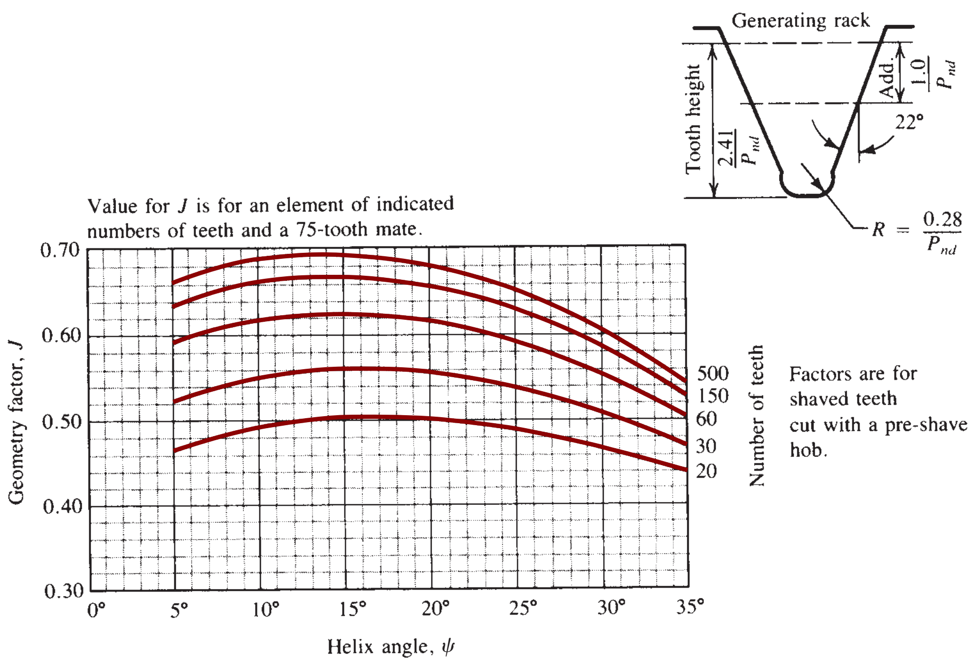
Example Problem 10–3 A pair of helical gears for a milling machine drive is to transmit 65 hp with a pinion speed of 3450 rpm and a gear speed of 1100 rpm. The power is from an electric motor. Design the gears.

Solution Of course, there are several possible solutions. Here is one. Let's try a normal diametral pitch of 12, 24 teeth in the pinion, a helix angle of 15°, a normal pressure angle of 20°, and a quality number of A9.

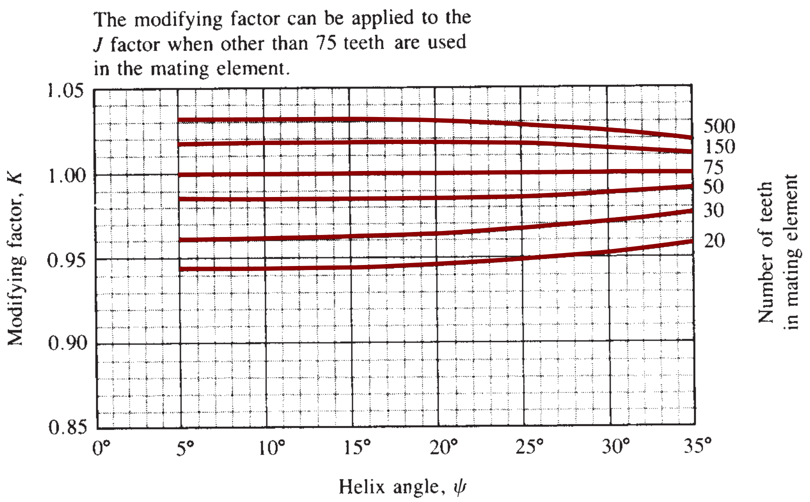
Now compute the transverse diametral pitch, the axial pitch, the transverse pressure angle, and the pitch diameter. Then we will choose a face width that will give at least two axial pitches to ensure true helical action.

$$P_d = P_{dn} \cos \psi = 12 \cos(15°) = 11.59$$

$$P_x = \frac{\pi}{P_d \tan \psi} = \frac{\pi}{11.59 \tan(15°)} = 1.012 \text{ in}$$



(*a*) Geometry factor (J), for 22° normal pressure angle, standard addendum, and pre-shave hob



(*b*) J factor multipliers

FIGURE 10–7 Geometry factor (J) for 22° normal pressure angle

$$\phi_t = \tan^{-1}(\tan \phi_n/\cos \psi) = \tan^{-1}[\tan(20°)/\cos(15°)] = 20.65°$$

$$D_p = N_p/P_d = 24/11.59 = 2.071 \text{ in}$$

$$F = 2P_x = 2(1.012) = 2.024 \text{ in} \quad \text{(nominal face width)}$$

Let's use 2.25 in, a more convenient value. The pitch line speed and the transmitted load are

$$v_t = \pi D_p n/12 = \pi(2.071)(3450)/12 = 1871 \text{ ft/min}$$

$$W_t = 33\,000(\text{hp})/v_t = (33\,000)(65)/1871 = 1146 \text{ lb}$$

Now we can calculate the number of teeth in the gear:

$$VR = N_G/N_P = n_P/n_G = 3450/1100 = 3.14$$

$$N_G = N_P(VR) = 24(3.14) = 75 \text{ teeth} \quad \text{(integer value)}$$

TABLE 10–1 Geometry Factors for Pitting Resistance, *I*, for Helical Gears with 20° Normal Pressure Angle and Standard Addendum

A. Helix angle $\psi = 15.0°$

Gear teeth	Pinion teeth 17	21	26	35	55
17	0.124				
21	0.139	0.128			
26	0.154	0.143	0.132		
35	0.175	0.165	0.154	0.137	
55	0.204	0.196	0.187	0.171	0.143
135	0.244	0.241	0.237	0.229	0.209

B. Helix angle $\psi = 25.0°$

Gear teeth	Pinion teeth 14	17	21	26	35	55
14	0.123					
17	0.137	0.126				
21	0.152	0.142	0.130			
26	0.167	0.157	0.146	0.134		
35	0.187	0.178	0.168	0.156	0.138	
55	0.213	0.207	0.199	0.189	0.173	0.144
135	0.248	0.247	0.244	0.239	0.230	0.210

Source: Extracted from AGMA Standard 908-B89 (R 1999), *Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314.

The values for the factors in Equation (9–6) must now be determined to enable the calculation of the bending stress. The geometry factor for the pinion is found in Figure 10–6 for 24 teeth in the pinion and 75 teeth in the gear: $J_P = 0.48$. The value of J_G will be greater than the value of J_P, resulting in a lower stress in the gear.

The *K* factors are as follows:

K_o = overload factor = 1.5 (moderate shock)
K_s = size factor = 1.0
K_m = load-distribution factor = 1.26 for $F/D_P = 1.09$ and commercial-quality, enclosed gearing
K_B = rim thickness factor = 1.0 (solid gears)
K_v = dynamic factor = 1.35 for $A_v = 9$ and $v_t = 1871$ ft/min

The bending stress in the pinion can now be computed:

$$s_{tP} = \frac{W_t P_d}{F J_P} K_o K_s K_m K_B K_v$$

$$s_{tP} = \frac{(1146)(11.59)}{(2.25)(0.48)}(1.50)(1.0)(1.26)(1.0)(1.35) = 31\ 400 \text{ psi}$$

From Figure 9–18, a Grade 1 steel with a hardness of approximately 250 HB would be required. Let's proceed to the design for pitting resistance.

Use Equation (9–23):

$$s_c = C_p \sqrt{\frac{W_t K_o K_s K_m K_v}{F D_p I}}$$

TABLE 10–2 Geometry Factors for Pitting Resistance, *I*, for Helical Gears with 25° Normal Pressure Angle and Standard Addendum

A. Helix angle $\psi = 15.0°$

Gear teeth	Pinion teeth 14	17	21	26	35	55
14	0.130					
17	0.144	0.133				
21	0.160	0.149	0.137			
26	0.175	0.165	0.153	0.140		
35	0.195	0.186	0.175	0.163	0.143	
55	0.222	0.215	0.206	0.195	0.178	0.148
135	0.257	0.255	0.251	0.246	0.236	0.214

B. Helix angle $\psi = 25.0°$

Gear teeth	Pinion teeth 12	14	17	21	26	35	55
12	0.129						
14	0.141	0.132					
17	0.155	0.146	0.135				
21	0.170	0.162	0.151	0.138			
26	0.185	0.177	0.166	0.154	0.141		
35	0.203	0.197	0.188	0.176	0.163	0.144	
55	0.227	0.223	0.216	0.207	0.196	0.178	0.148
135	0.259	0.258	0.255	0.251	0.246	0.235	0.213

Source: Extracted from AGMA Standard 908-B89, *Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314.

For two steel gears, $C_p = 2300$. Rough interpolation from the data in Table 10–1 for $N_P = 24$ and $N_G = 75$ gives $I = 0.202$. It is recommended that the computational procedure described in the AGMA standards be used to compute a more precise value for critical work. The contact stress is then

$$s_c = 2300\sqrt{\frac{(1146)(1.50)(1.0)(1.26)(1.35)}{(2.25)(2.071)(0.202)}} = 128\,200 \text{ psi}$$

It is obvious that the contact stress governs this design. Let's adjust the solution for a higher reliability and to account for the expected number of cycles of operation. Certain design decisions must be made. For example, consider the following:

Design for a reliability of 0.999 (less than one failure in 1000): $K_R = 1.25$ (Table 9–11). Design life: Let's design for 10 000 h of life as suggested in Table 9–12 for multipurpose gearing. Then, using Equation (9–27), we can compute the number of cycles of loading. For the pinion rotating at 3450 rpm with one cycle of loading per revolution,

$$N_c = (60)(L)(n)(q) = (60)(10\,000)(3450)(1.0) = 2.1 \times 10^9 \text{ cycles}$$

From Figure 9–22, we find that $Z_N = 0.89$.

No unusual conditions seem to exist in this application beyond those already considered in the various *K* factors. Therefore, we use a service factor, *SF,* of 1.00.

We can use Equation (9–27) to apply these factors.

$$\frac{K_R(SF)}{Z_N}s_c = s_{ac} = \frac{(1.25)(1.00)}{(0.89)}(128\,200 \text{ psi}) = 180\,000 \text{ psi}$$

Table 9–9 indicates that Grade 1 steel, case hardened by carburizing, would be suitable. From Appendix 5, let's specify AISI 4320 SOQT 450, having a case hardness of HRC 59 and a core hardness of 415 HB. This should be satisfactory for both bending and pitting resistance. Both the pinion and the gear should be of this material.

10–6 FORCES ON STRAIGHT BEVEL GEARS

Review Section 8–8 and Figure 8–17 for the geometry of bevel gears. Also see References 1, 10, and 12.

Because of the conical shape of bevel gears and because of the involute-tooth form, a three-component set of forces acts on bevel gear teeth. Using notation similar to that for helical gears, we will compute the tangential force, W_t; the radial force, W_r; and the axial force, W_x. It is assumed that the three forces act concurrently at the midface of the teeth and on the pitch cone (see Figure 10–8). Although the actual point of application of the resultant force is a little displaced from the middle, no serious error results.

The tangential force acts tangential to the pitch cone and is the force that produces the torque on the pinion and the gear. The torque can be computed from the known power transmitted and the rotational speed:

$$T = 63\,000\ P/n$$

Then, using the pinion, for example, the transmitted load is

$$W_{tP} = T/r_m \tag{10–10}$$

where r_m = mean radius of the pinion.
The value of r_m can be computed from

$$r_m = d/2 - (F/2)\sin\gamma \tag{10–11}$$

Remember that the pitch diameter, d, is measured to the pitch line of the tooth at its large end. The angle, γ, is the pitch cone angle for the pinion as shown in Figure 10–8(a). The radial load acts toward the center of the pinion, perpendicular to its axis, causing bending of the pinion shaft. Thus,

$$W_{rP} = W_t \tan\phi\cos\gamma \tag{10–12}$$

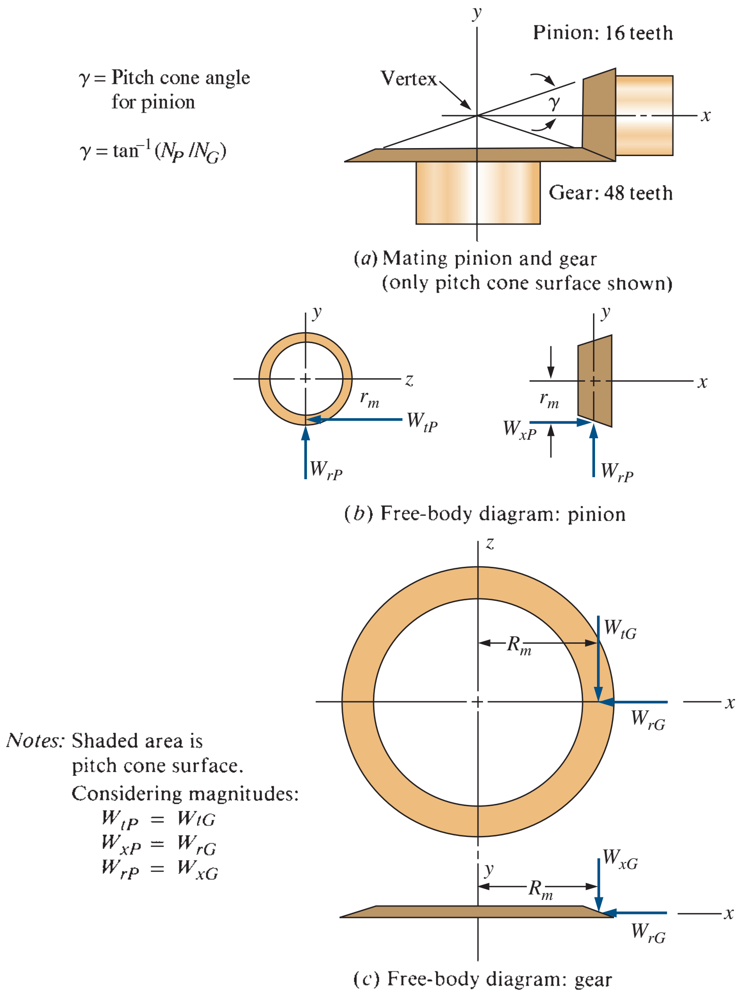


FIGURE 10–8 Forces on bevel gears

The angle, ϕ, is the pressure angle for the teeth.

The axial load acts parallel to the axis of the pinion, tending to push it away from the mating gear. It causes a thrust load on the shaft bearings. It also produces a bending moment on the shaft because it acts at the distance from the axis equal to the mean radius of the gear. Thus,

$$W_{xP} = W_t \tan \phi \sin \gamma \qquad (10\text{–}13)$$

The values for the forces on the gear can be calculated by the same equations shown here for the pinion, if the geometry for the gear is substituted for that of the pinion. Refer to Figure 10–8 for the relationships between the forces on the pinion and the gear in both magnitude and direction.

Example Problem 10–4

For the gear pair described in Example Problem 8–3, calculate the forces on the pinion and the gear if they are transmitting 2.50 hp with a pinion speed of 600 rpm. The geometry factors computed in Example Problem 8–3 apply. The data are summarized here.

Summary of Pertinent Results from Example Problem 8–3 and Given Data

Number of teeth in the pinion: $N_P = 16$

Number of teeth in the gear: $N_G = 48$

Diametral pitch: $P_d = 8$

Pitch diameter of pinion: $d = 2.000$ in

Pressure angle: $\phi = 20°$

Pinion pitch cone angle: $\gamma = 18.43°$

Gear pitch cone angle: $\Gamma = 71.57°$

Face width: $F = 1.00$ in

Rotational speed of pinion: $n_P = 600$ rpm

Power transmitted: $P = 2.50$ hp

Solution

Forces on the pinion are described by the following equations:

$$W_t = T/r_m$$

But

$$T_p = 63\,000(P)/n_P = [63\,000(2.50)]/600 = 263 \text{ lb}\cdot\text{in}$$

$$r_m = d/2 - (F/2)\sin\gamma$$

$$r_m = (2.000/2) - (1.00/2)\sin(18.43°) = 0.84 \text{ in}$$

Then

$$W_t = T_P/r_m = 263 \text{ lb}\cdot\text{in}/0.84 \text{ in} = 313 \text{ lb}$$

$$W_r = W_t \tan\phi \cos\gamma = 313 \text{ lb} \tan(20°)\cos(18.43°) = 108 \text{ lb}$$

$$W_x = W_t \tan\phi \sin\gamma = 313 \text{ lb} \tan(20°)\sin(18.43°) = 36 \text{ lb}$$

To determine the forces on the gear, first let's calculate the rotational speed of the gear:

$$n_G = n_P(N_P/N_G) = 600 \text{ rpm}(16/48) = 200 \text{ rpm}$$

Then

$$T_G = 63\,000(2.50)/200 = 788 \text{ lb}\cdot\text{in}$$

$$R_m = D/2 - (F/2)\sin\Gamma$$

$$R_m = 6.000/2 - (1.00/2)\sin(71.57°) = 2.53 \text{ in}$$

$$W_t = T_G/R_m = (788 \text{ lb}\cdot\text{in})/(2.53 \text{ in}) = 313 \text{ lb}$$

$$W_r = W_t \tan\phi \cos\Gamma = 313 \text{ lb} \tan(20°)\cos(71.57°) = 36 \text{ lb}$$

$$W_x = W_t \tan\phi \sin\Gamma = 313 \text{ lb} \tan(20°)\sin(71.57°) = 108 \text{ lb}$$

Note from Figure 10–8 that the forces on the pinion and the gear form an *action-reaction pair.* That is, the forces on the gear are equal to those on the pinion, but they act in the opposite direction. Also, because of the 90° orientation of the shafts, the radial force on the pinion becomes the axial thrust load on the gear, and the axial thrust load on the pinion becomes the radial load on the gear.

10–7 BEARING FORCES ON SHAFTS CARRYING BEVEL GEARS

Because of the three-dimensional force system that acts on bevel gears, the calculation of the forces on shaft bearings can be cumbersome. An example is worked out here to show the procedure. In order to obtain numerical data, the arrangement shown in Figure 10–9 is proposed for the bevel gear pair that was the subject of Example Problems 8–3 and 10–4. The locations for the bearings are given with respect to the vertex of the two pitch cones where the shaft axes intersect.

Note that both the pinion and the gear are *straddle mounted*; that is, each gear is positioned between the supporting bearings. This is the most preferred arrangement because it usually provides the greatest rigidity and maintains the alignment of the teeth during power transmission. Care should be exercised to provide rigid mountings and stiff shafts when using bevel gears.

The arrangement of Figure 10–9 is designed so that the bearing on the right resists the axial thrust load on the pinion, and the lower bearing resists the axial thrust load on the gear.

Example Problem 10–5 Compute the reaction forces on the bearings that support the shafts carrying the bevel gear pair shown in Figure 10–9. The values of Example Problems 8–3 and 10–4 apply.

Solution Referring to the results of Example Problem 10–4 and Figure 10–8, we have listed the forces acting on the gears:

Force	Pinion	Gear
Tangential	$W_{tP} = 313$ lb	$W_{tG} = 313$ lb
Radial	$W_{rP} = 108$ lb	$W_{rG} = 36$ lb
Axial	$W_{xP} = 36$ lb	$W_{xG} = 108$ lb

It is critical to be able to visualize the directions in which these forces are acting because of the three-dimensional force system. Notice in Figure 10–8 that a rectangular coordinate system has been set up. Figure 10–10 is an isometric sketch of the free-body diagrams of the pinion and the gear, simplified to represent the concurrent forces acting at the pinion/gear interface and at the bearing locations. Although the two free-body diagrams are separated for clarity, notice that you can bring them together by moving the point called *vertex* on each sketch together. This is the point in the actual gear system where the vertices of the two pitch cones lie at the same point. The two pitch points also coincide.

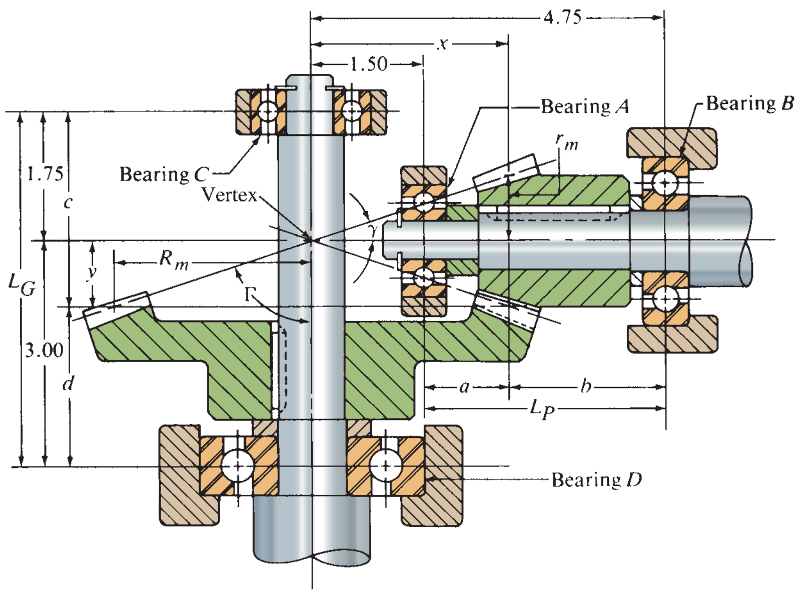
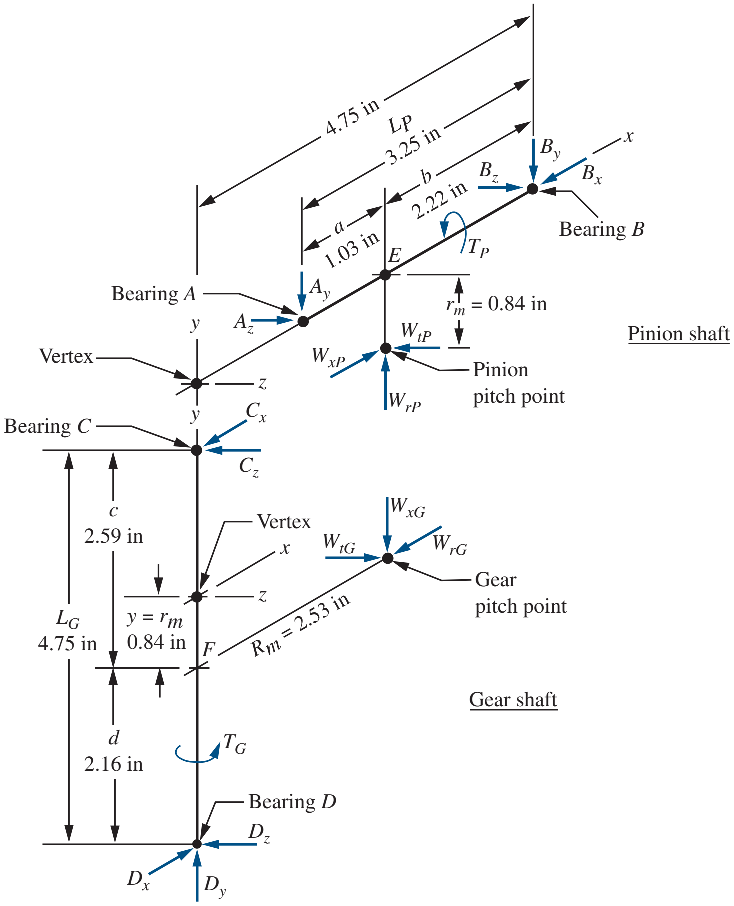


FIGURE 10–9 Layout of bevel gear pair for Example Problem 10–5

FIGURE 10–10 Free-body diagrams for pinion and gear shafts



For setting up the equations of static equilibrium needed to solve for the bearing reactions, the distances *a, b, c, d,* L_P, and L_G are needed, as shown in Figure 10–9. These require the two dimensions labeled *x* and *y*. Note from Example Problem 10–4 that

$$x = R_m = 2.53 \text{ in}$$

$$y = r_m = 0.84 \text{ in}$$

Then

$$a = x - 1.50 = 2.53 - 1.50 = 1.03 \text{ in}$$

$$b = 4.75 - x = 4.75 - 2.53 = 2.22 \text{ in}$$

$$c = 1.75 + y = 1.75 + 0.84 = 2.59 \text{ in}$$

$$d = 3.00 - y = 3.00 - 0.84 = 2.16 \text{ in}$$

$$L_P = 4.75 - 1.50 = 3.25 \text{ in}$$

$$L_G = 1.75 + 3.00 = 4.75 \text{ in}$$

These values are shown in Figure 10–10.

To solve for the reactions, we need to consider the horizontal (*x-z*) and the vertical (*x-y*) planes separately. It may help you to look also at Figure 10–11, which breaks out the forces on the pinion shaft in these two planes. Then we can analyze each plane using the fundamental equations of equilibrium.

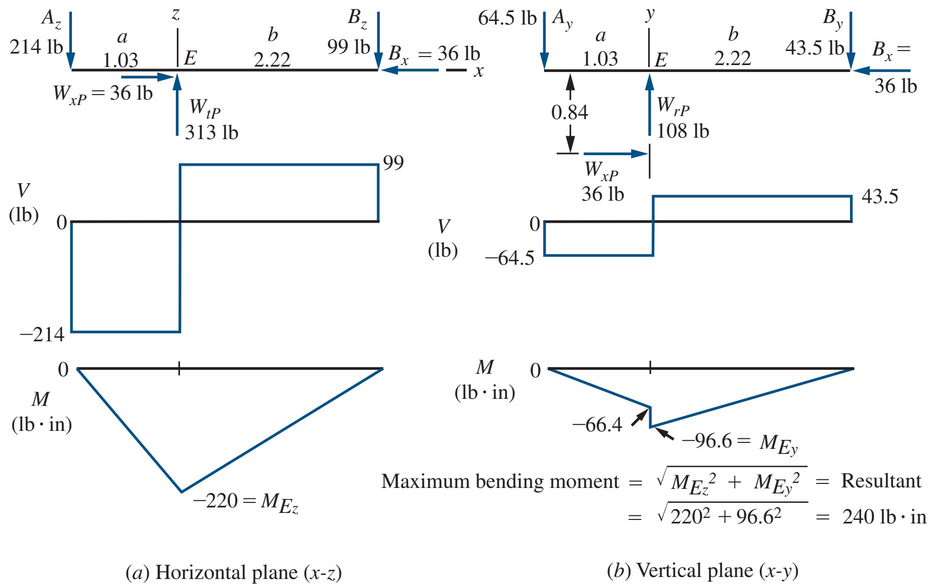
Bearing Reactions, Pinion Shaft: Bearings A and B

Step 1. To find B_z and A_z: In the *x-z* plane, only W_{tP} acts. Summing moments about *A* yields

$$0 = W_{tP}(a) - B_z(L_P) = 313(1.03) - B_z(3.25)$$

$$B_z = 99.2 \text{ lb}$$

FIGURE 10–11 Pinion shaft bending moments



Summing moments about B yields

$$0 = W_{tP}(b) - A_z(L_P) = 313(2.22) - A_z(3.25)$$

$$A_z = 214 \text{ lb}$$

Step 2. To find B_y and A_y: In the x-y plane, both W_{rP} and W_{xP} act. Summing moments about A yields

$$0 = w_{rP}(a) + W_{xP}(r_m) - B_y(L_P)$$

$$0 = 108(1.03) + 36(0.84) - B_y(3.25)$$

$$B_y = 43.5 \text{ lb}$$

Summing moments about B yields

$$0 = W_{rP}(b) + W_{xP}(r_m) - A_y(L_P)$$

$$0 = 108(2.22) - 36(0.84) - A_y(3.25)$$

$$A_y = 64.5 \text{ lb}$$

Step 3. To find B_x: Summing forces in the x-direction yields

$$B_x = W_{xP} = 36 \text{ lb}$$

This is the thrust force on bearing B.

Step 4. To find the total radial force on each bearing: Compute the resultant of the y- and z-components.

$$A = \sqrt{A_y^2 + A_z^2} = \sqrt{64.5^2 + 214^2} = 224 \text{ lb}$$

$$B = \sqrt{B_y^2 + B_z^2} = \sqrt{43.5^2 + 99.2^2} = 108 \text{ lb}$$

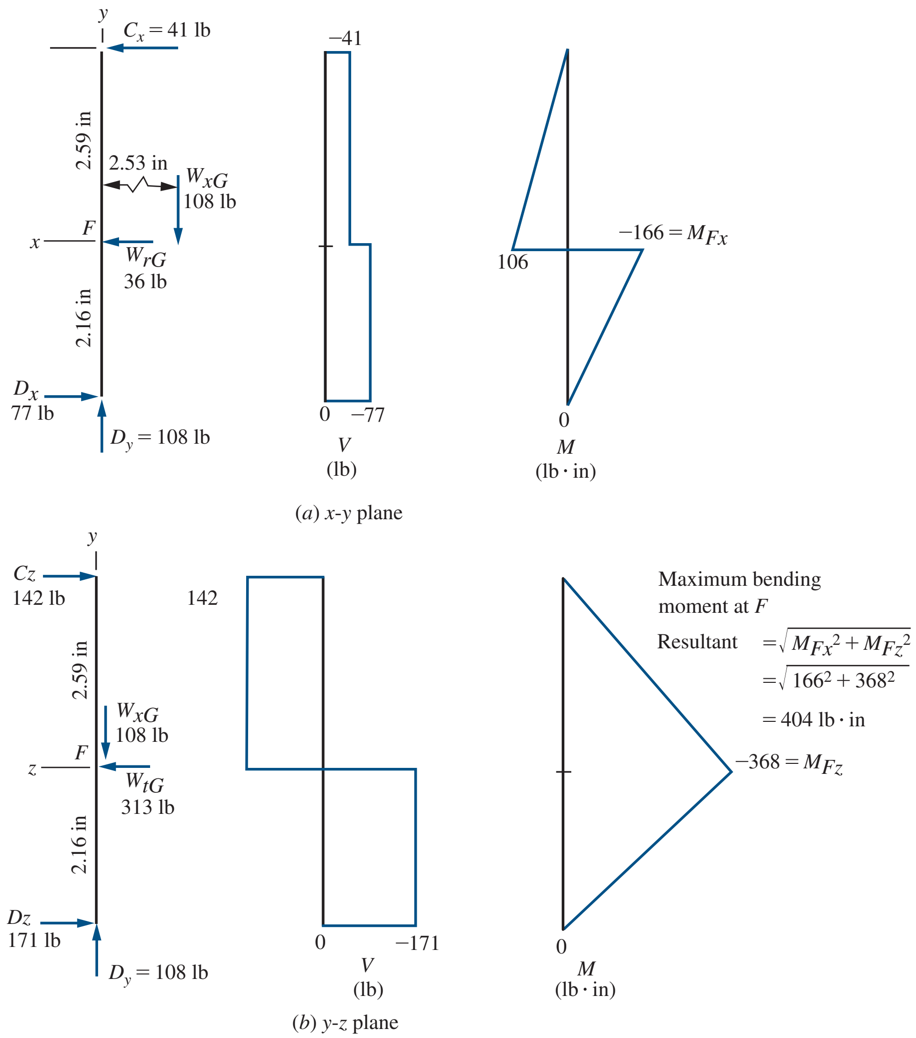
Bearing Reactions, Gear Shaft: Bearings C and D
Using similar methods, we can find the forces in Figure 10–12.

$$\left.\begin{aligned} C_z &= 142 \text{ lb} \\ C_x &= 41.1 \text{ lb} \end{aligned}\right\} C = 148 \text{ lb (radial force on } C)$$

$$\left.\begin{aligned} D_z &= 171 \text{ lb} \\ D_x &= 77.1 \text{ lb} \end{aligned}\right\} D = 188 \text{ lb (radial force on } D)$$

$$D_y = W_{xG} = 108 \text{ lb (thrust force on } D)$$

FIGURE 10–12 Gear shaft bending moments



Summary In selection of the bearings for these shafts, the following capacities are required:

Bearing *A:* 224-lb radial

Bearing *B:* 108-lb radial; 36-lb thrust

Bearing *C:* 148-lb radial

Bearing *D:* 188-lb radial; 108-lb thrust

10–8 BENDING MOMENTS ON SHAFTS CARRYING BEVEL GEARS

Because there are forces acting in two planes on bevel gears, as discussed in the preceding section, there is also bending in two planes. The analysis of the shearing force and bending moment diagrams for the shafts must take this into account.

Figure 10–11 and Figure 10–12 show the resulting diagrams for the pinion and the gear shafts, respectively, for the gear pair used for Example Problems 8–3, 10–4, and 10–5. Notice that the axial thrust load on each gear provides a concentrated moment to the shaft equal to the axial load times the distance that it is offset from the axis of the shaft. Also notice that the maximum bending moment for each shaft is the resultant of the moments in the two planes. On the pinion shaft, the maximum moment is 240 lb · in at *E,* where the lines of action for the radial and tangential forces intersect the shaft. Similarly, on the gear shaft, the maximum moment is 404 lb · in at *F.* These data are used in the shaft design (as discussed in Chapter 12).

10–9 STRESSES IN STRAIGHT BEVEL GEAR TEETH

This section generally follows AGMA Standard 2003 (Reference 10), *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, Zerol Bevel and Spiral Bevel Gear Teeth*, considered to be the primary standard in the United States. However, only straight bevel gears are treated here. The standard presents design analysis in both the U.S. unit system based on diametral pitch, P_d, and the SI Metric unit system based on metric module, m.

This book will maintain similar notations and symbols for gear tooth features, allowable stresses, and modifying factors that were initially presented in Chapter 9 for spur gear design to facilitate the comparison of design approaches. The reader should note that Standard 2003-C10 presents design analysis in SI units using terminology from ISO standards that employ radically different symbol sets. In this book, we maintain similar notations for factors in both systems except for the basic terms, diametral pitch, P_d, and metric module, m.

Furthermore, as introduced in Chapter 9, this book makes the following assumptions:

1. The gears are operating at temperatures between 32°F and 250°F (0°C and 120°C) for which the temperature factor $K_T = 1.0$ and, therefore, it is not written in the equations for stress analysis.
2. Gear teeth are made to normal standards without special modifications. This does, however, assume that crowning of the teeth is included in the manufacturing process, typical of the bevel gear industry.
3. Both the pinion and the gear of the bevel gear set are straddle mounted as shown in Figure 10–9, providing the most rigid arrangement. If either gear is not straddle mounted, the overhung arrangement, being generally less stiff, requires the application of additional modifying factors.
4. The hardness of both the pinion and the gear are nearly equal to enable each to withstand the contact stress without pitting, allowing the use of the hardness factor, $C_H = 1.0$, and not including that factor in the equations. The standard contains significant discussion and data to modify allowable strengths when the pinion is significantly harder than the gear, an approach used by some designers to promote *wearing in* of the gear teeth by the harder pinion. Also included in the standard with the hardness factor are adjustments based on the surface roughness of the teeth that are not included in this book.
5. When case hardening is used, as is frequently done, the case has sufficient depth and the core hardness and strength are sufficiently high to avoid case crushing or subsurface failure of the core due to either bending or contact stresses. The standard includes much discussion of these factors.
6. Residual stresses in gear teeth are not considered in this section. Note that beneficial compressive residual stresses in the root area from peening processes can significantly enhance the life of gears. Conversely, residual tensile stresses can be detrimental.

Following is the discussion of bending stress and contact stress calculations along with the corresponding parameters and modification factors:

Pitch Diameter, D: An important difference in the analysis of bevel gears is the definition of pitch diameter; it is measured at the large (outer) end of the gear rather than at the middle of the teeth as it was for spur and helical gears. This difference is accommodated in the determination of the geometry factors J and I that are shown later.

Pitches: The pitch for bevel gears is defined at the outer pitch diameter and is called the *outer transverse pitch*. Calculation is the same as for spur and helical gears with the exception of the definition of pitch diameter given above.

U.S. Units: Outer transverse diametral pitch $= P_d = N/D$ (units are in^{-1}; rarely reported)

SI Metric Units: Outer transverse metric module $= m = D/N$ mm

Tangential Force, W_t: As in Chapter 9, we will use the following unit-specific equations for torque on a gear, pitch line speed, and the resulting tangential force.

U.S. Units: Power, P, in hp; rotational speed, n, in rpm; diameters, D, in inches

Pitch line speed $= v_t = \pi Dn/12$ ft/min

Torque $= T = 63\,000\ P/n$ lb·in

Tangential force $= W_t = T/(D/2) = 126\,000\,P/(Dn)$ lb

Or, Tangential force $= W_t = 33\,000\ P/v_t$ lb

SI Metric Units: Power, P, in kW; rotational speed, n, in rpm; diameters, D, in mm

Pitch line speed $= v_t = \pi Dn/(60\,000)$ m/s

Torque $= T = 9550\ P/n$ N·m

Tangential force $= W_t = T/(D/2)$
$= 19.1 \times 10^6\ P/(Dn)$ N $= 19\,100\ P/(Dn)$ kN

Or, Tangential force $= W_t = 1000\ P/v_t$ N $= P/v_t$ kN

Bending Stress Number, s_t: The maximum bending stress number occurs in the root area of the teeth as it does in spur and helical gears; the equations are as follows:

U.S. Units: $$s_t = \frac{W_t P_d K_o K_s K_m K_v}{FJ} \text{ psi} \quad \textbf{(10–14)}$$

SI Metric Units: $$s_t = \frac{W_t K_o K_s K_m K_v}{FJm} \text{ MPa} \quad \textbf{(10–14M)}$$

Overload Factor, K_o: Use the same values given in Table 9–1.

Size Factor, K_s for Bending Strength: Use Figure 10–13, adapted from AGMA Standard 2003-C10.

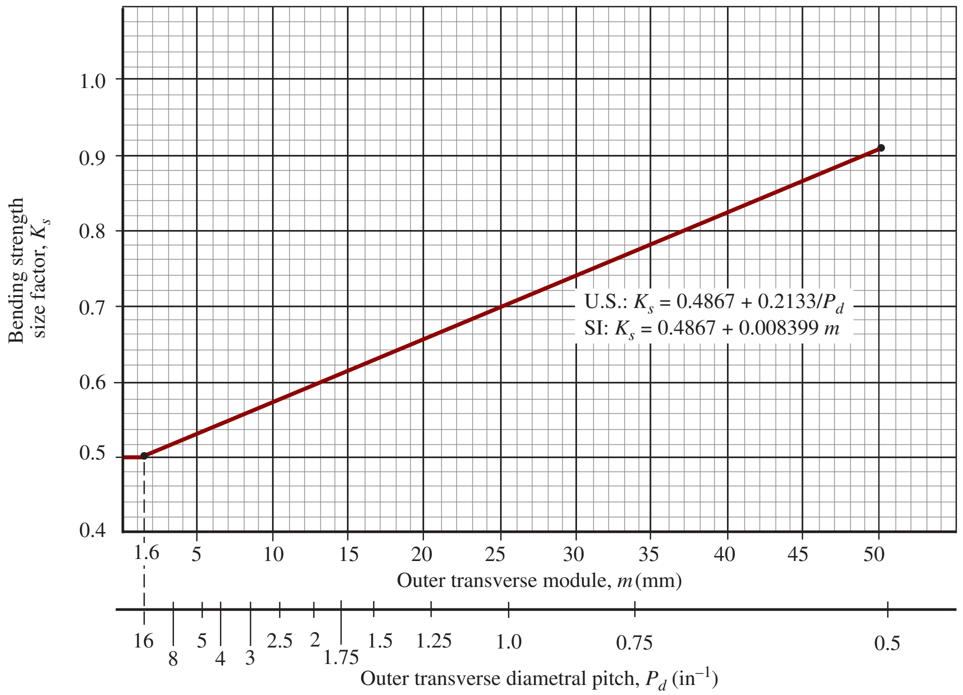


FIGURE 10–13 Size factor for bending stress, K_s, for bevel gears (Adapted from AGMA 2003-C10, *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, Zerol Bevel and Spiral Bevel Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th Floor, Alexandria, VA.)

For $P_d \geq 16\,(m \leq 1.6$ mm) use $K_s = 0.50$. The equation for the sloped portion of Figure 10–13 is

$$K_s = 0.4867 + 0.2133/P_d \qquad \textbf{(10–15)}$$

$$K_s = 0.4867 + 0.008399\,m \qquad \textbf{(10–15M)}$$

Load Distribution Factor, K_m: Typical bevel gear teeth are crowned along the profile and from end to end to ensure smooth engagement under foreseeable conditions of tooth accuracy and deflection under load. During the engagement/disengagement cycle, the contact pattern across the teeth should spread the load over the entire area of the faces. When additional misalignment occurs, this pattern is changed and the effects are more prominent with larger face widths and the manner of mounting. Use Figure 10–14 when discrete analysis of deformations is not practical. The three curves conform to the equations:

U.S. Units: $K_m = K_{mb} + 0.0036\,F^2$ **(10–16)**

SI Metric Units: $K_m = K_{mb} + 5.6 \times 10^{-6}\,F^2$ **(10–16M)**

where,

$K_{mb} = 1.00$ for both gears straddle mounted

$K_{mb} = 1.10$ for one gear straddle mounted

$K_{mb} = 1.25$ for neither gear straddle mounted

Problems in this book can assume that both gears are straddle mounted unless otherwise noted. Designers should take steps to provide a precise and rigid system comprised of the gears, shafts, bearings, and housing.

Dynamic Factor, K_v: Use the same chart shown for spur gears in Figure 9–16, based on the quality system defined in AGMA standards 2015 (Reference 5) and 2001 (Reference 9), in which quality numbers from A11 (least accurate) to A4 (most accurate) are used. Bevel gear standard AGMA 2003 (Reference 10) includes the former chart for K_v based on the Q-system of quality from Q5 (least accurate) to Q11 (most accurate). To a reasonable degree of precision for values of K_v in these two charts are compatible provided that the quality number in the A-system plus the quality number in the Q-system add to 17. For example, Q8 is similar to A9 and Q10 is similar to A7.

Geometry Factor for Bending Strength, J: Use Figure 10–15 for problems in this book. This figure is for straight-tooth bevel gears with a 20° pressure angle and a shaft angle between the shafts of the pinion and the gear of 90°. Standard AGMA 2003 (Reference 10) gives formulas from which the value for other designs can be computed when detailed geometry for the teeth is known.

Example Problem 10–6 illustrates the calculation for bending stress number for straight bevel gear teeth, using the same data from earlier problems. Then we will introduce allowable stresses and material selection to resist bending stresses.

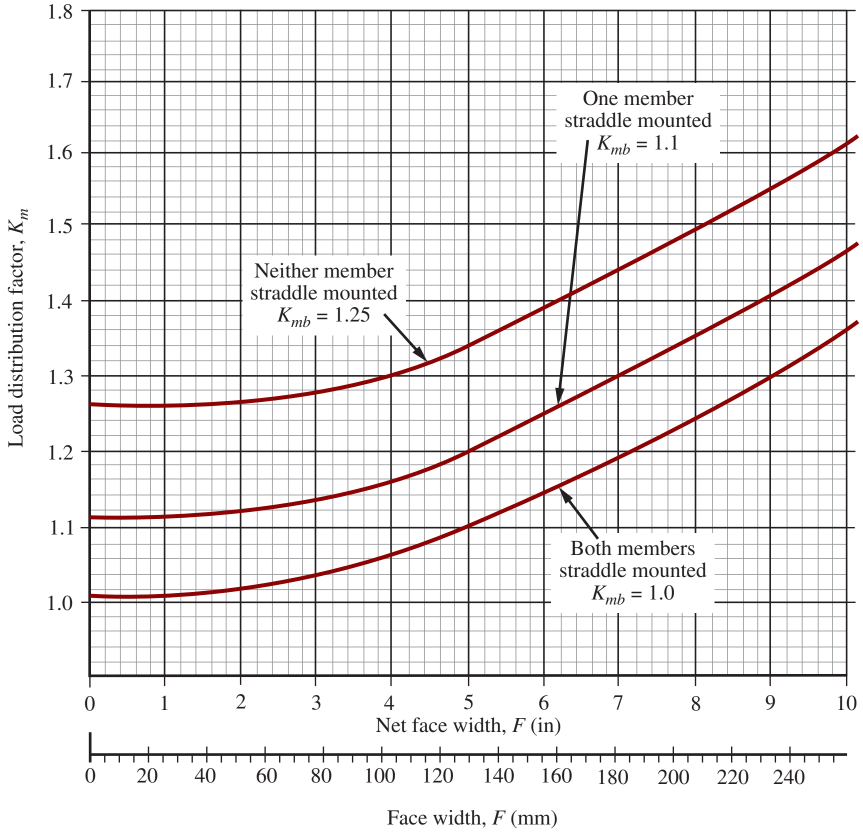


FIGURE 10–14 Load distribution factor, K_m, for crowned bevel gear teeth (Adapted from AGMA 2003-C10, *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, Zerol Bevel and Spiral Bevel Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th Floor, Alexandria, VA.)

Example Problem 10–6

Compute the bending stress in the teeth of the bevel pinion shown in Figure 10–9. The data from Example Problem 10–4 apply: $N_P = 16$; $N_G = 48$; $n_P = 600$ rpm; $P = 2.50$ hp; $P_d = 8$; $d = 2.000$ in; and $F = 1.00$ in. Assume that the pinion is driven by an electric motor and that the load provides moderate shock. The quality number, A_v, is to be 11.

Solution

$$W_t = \frac{T}{r} = \frac{63\,000(P)}{n_p}\frac{1}{d/2} = \frac{63\,000(2.50)}{600}\frac{1}{2.000/2} = 263 \text{ lb}$$

$$v_t = \pi d n_P/12 = \pi(2.000)(600)/12 = 314 \text{ ft/min}$$

$$K_o = 1.50 \text{ (from Table 9–1)}$$

$$K_s = 0.4867 + 0.2133/P_d = 0.4867 + 0.2133/8 = 0.513$$

$$K_m = 1.004 \text{ (both gears straddle mounted, general commercial quality)}$$

$$J_P = 0.230 \text{ (from Figure 10–5)}$$

$$K_v = 1.24 \text{ (Use } A_v = 11 \text{ and } v_t = 314 \text{ ft/min}$$

(Read from Figure 9–16 or computed from equations in Table 9–6)

Then, from Equation (10–14),

$$s_t = \frac{W_t P_d K_o K_s K_m K_v}{FJ} = \frac{(263)(8)(1.50)(0.5131)(1.004)(1.24)}{(1.00)(0.230)} = 8764 \text{ psi}$$

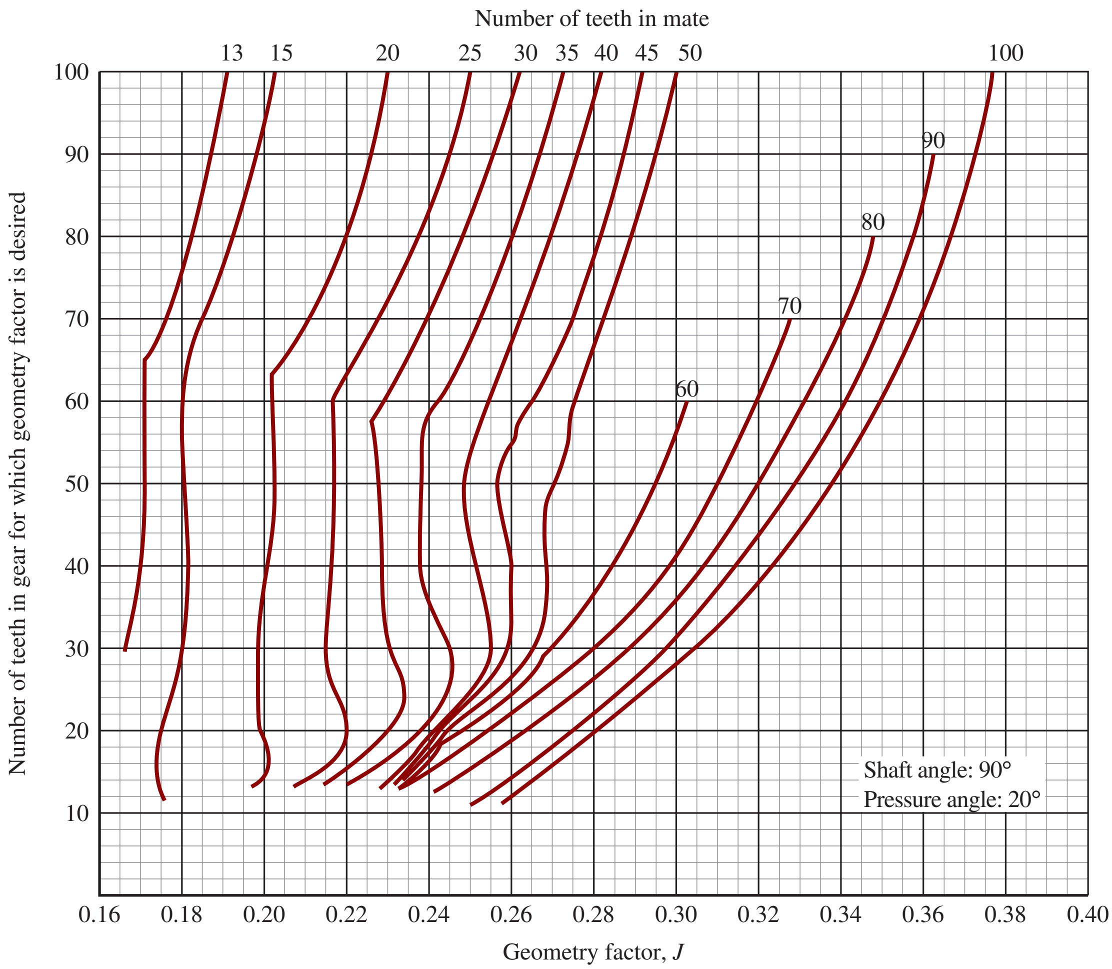


FIGURE 10–15 Geometry factor, J, for straight bevel gears with 20° pressure angle and 90° shaft angle (Adapted from AGMA 2003-C10, *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, Zerol Bevel and Spiral Bevel Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th Floor, Alexandria, VA.)

Allowable Bending Strength Number: The process of finding the required allowable bending strength, s_{at}, for bevel gears is similar to that used for spur and helical gears. The fundamental equation is

$$s_{wt} = \frac{s_{at}K_L}{(SF)(K_R)} \tag{10–17}$$

where

s_{wt} = permissible bending stress number considering design life and reliability

K_L = Stress cycle factor for bending
For carburized case-hardened steel bevel gears, use Figure 10–16. Most of the test data for these curves were developed for carburized case-hardened gears so use for through-hardened steel is only approximate. In Chapter 9, a similar factor was called Y_N and that is the term used in ISO standards as well. Note that the curve for life factors above about 3×10^6 is used for most commercial gear drives. The standard permits a lower value for some critical applications.

K_R = Reliability factor. Use Table 10–3. Note that for bending, the reliability factor for bevel gears is the same as for spur and helical gears in Table 9–11.

SF = Safety factor. Typically taken to be 1.00, but values up to about 1.50 can be used for greater uncertainty or for critical systems.

For design where the goal is to specify a suitable gear material, Equation (10–17) can be combined with the equation for s_t from Equation (10–14) and we can solve for the required value of the allowable bending strength, s_{at}. That is,

$$\text{Let } s_t = s_{wt} = \frac{s_{at}K_L}{(SF)(K_R)}$$

Then, the required value of s_{at} is

$$s_{at} = \frac{s_t(SF)(K_R)}{K_L} \tag{10–18}$$

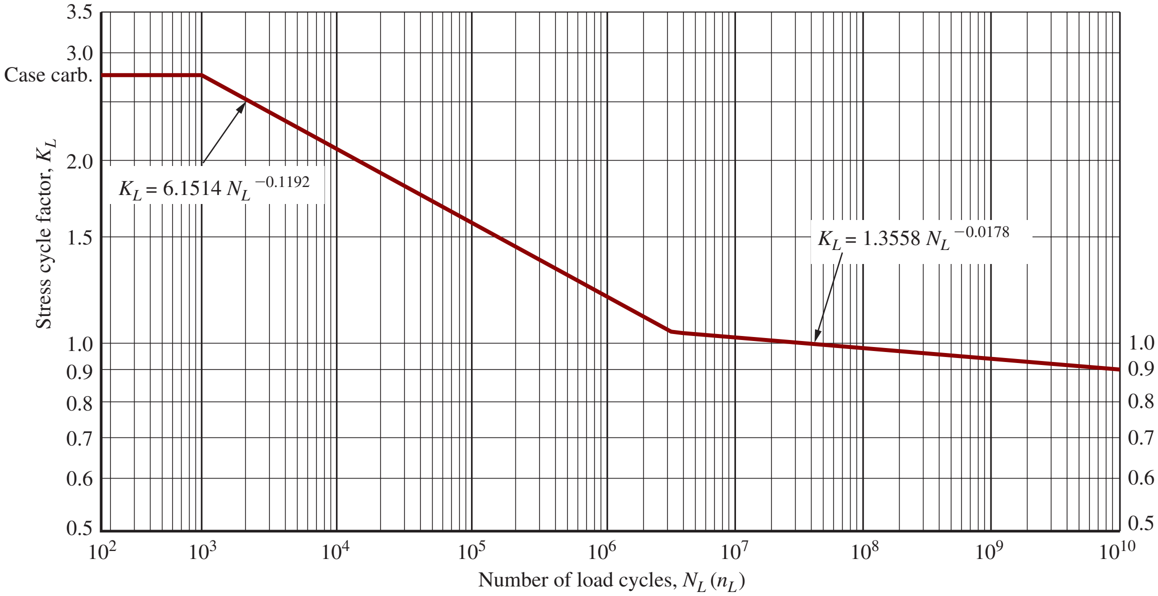


FIGURE 10–16 Stress cycle factor for bending strength, K_L (carburized case-hardened steel bevel gears) (Adapted from AGMA 2003-C10, *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, Zerol Bevel and Spiral Bevel Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th Floor, Alexandria, VA.)

TABLE 10–3 Reliability Factors for Allowable Bending and Contact Stresses

Reliability R	Interpretation	Reliability factors: Bending K_R	Reliability factors: Contact C_R
0.9	Fewer than one failure in 10	0.85	0.92
0.99	Fewer than one failure in 100	1.00	1.00
0.999	Fewer than one failure in 1000	1.25	1.12
0.9999	Fewer than one failure in 10 000	1.50	1.22

Source: Adapted from AGMA 2003-C10, *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, Zerol Bevel and Spiral Bevel Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th Floor, Alexandria, VA.

Use Figure 10–17 to determine the required hardness for through-hardened steel, recognizing that HB 400 is the maximum recommended value. Above that, flame, induction, or carburized case-hardened steel should be used up to the limits shown in Table 10–4.

Next, we develop the similar approach for determining the required allowable contact stress number, s_{ac}. Then we will show another example problem in which we identify the critical value on which material selection is based.

Contact Stress Number, s_c: The maximum contact stress number occurs on the face of the teeth as it does in spur and helical gears; the equation is the same for both U.S. and SI units with due attention to units for W_t, F, D_P, *and* C_p:

U.S. or SI Units:

$$s_c = C_p\sqrt{\frac{W_t K_o K_m K_v C_s C_{xc}}{F D_p I}} \text{ psi or MPa} \qquad \textbf{(10–19)}$$

TABLE 10–4 Allowable Stress Numbers for Bevel Gears

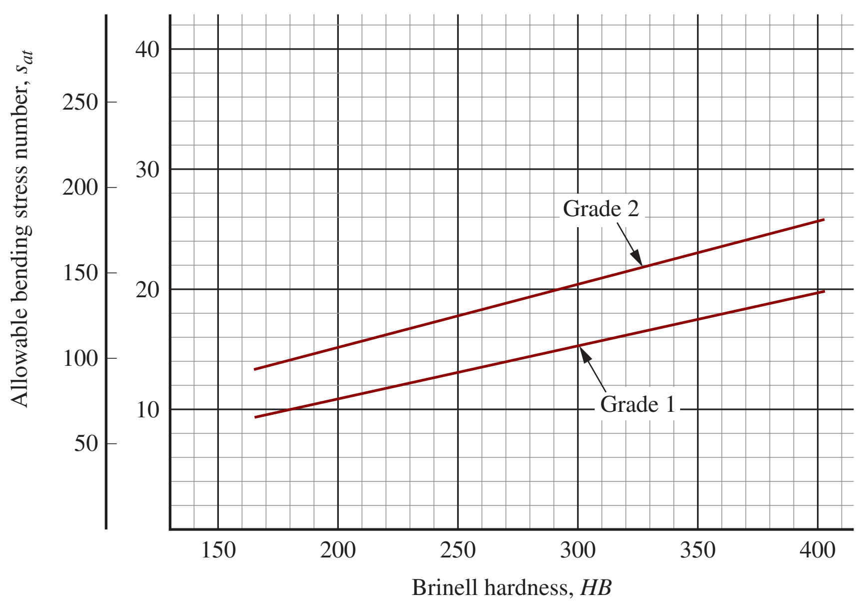
Hardness at surface	Allowable bending stress number, s_{at} (ksi)	Allowable bending stress number, s_{at} (Mpa)	Allowable contact stress number, s_{ac} (ksi)	Allowable contact stress number, s_{ac} (Mpa)
Case hardened Grade 1 steel materials				
Flame or induction hardened				
50 HRC—Unhardened roots	12.5	86	175	1207
50 HRC—hardened roots	22.5	155	175	1207
Carburized and case hardened				
55–64 HRC	30	207	200	1379

Source: Adapted from AGMA 2003-C10, *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, ZEROL Bevel and Spiral Bevel Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th Floor, Alexandria, VA.

Grade 2 steel:

U.S.: $s_{at} = 0.048\,HB + 5.980$ ksi   SI: $s_{at} = 0.33\,HB + 41.24$ MPa

Required $HB = (s_{at} - 5.980)/0.048$ Required $HB = (s_{at} - 41.24)/0.33$



Grade 1 steel:

U.S.: $s_{at} = 0.044\,HB + 2.10$ ksi   SI: $s_{at} = 0.30\,HB + 14.48$ MPa

Required $HB = (s_{at} - 2.10)/0.044$ Required $HB = (s_{at} - 14.48)/0.30$

FIGURE 10–17 Allowable bending stress number, s_{at}, for through-hardened steel for bevel gears (Adapted from AGMA 2003-C10, *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, Zerol Bevel and Spiral Bevel Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th Floor, Alexandria, VA.)

Note that this equation is used for both the pinion and the gear because the basic contact stress is equal on each. *Do not use the pitch diameter of the gear in this equation.*

All variables in this equation have been discussed in regard to bending stress number except for I, C_p, C_s, *and* C_{xc}, discussed next.

C_p: The elastic coefficient depends on the modulus of elasticity and Poisson's ratio for the materials of the pinion and the gear. For two steel gears, $C_p = 2300\ \text{psi}^{0.5}(191\ \text{MPa}^{0.5})$. Use the values from Chapter 9 in Table 9–7 for other materials.

C_s: The *size factor for contact stress* is different from the value of K_s used for bending stress. See Figure 10–18. The factor is based on the face width, F. For $F \leq 0.50$ in (12.5 mm), use $C_s = 0.50$. For $F \geq 3.14$ in (80 mm), use $C_s = 0.83$. Between those limits, use the figure or compute the value from the equations given here:

U.S. Units: $$C_s = 0.125F + 0.4375 \qquad \textbf{(10–20)}$$

SI Metric Units: $$C_s = 0.00492F + 0.4375 \qquad \textbf{(10–20M)}$$

C_{xc}: The *crowning factor for pitting* accounts for the contact pattern between mating teeth. Typical bevel gear production processes create crowning both along the tooth flank and across the full width of the face. This is the preferred approach. However, some bevel gears are not crowned. AGMA Standard 2003 (Reference 10) recommends the following factors:

$$C_{xc} = 1.5 \text{ for properly crowned teeth}$$

$$C_{xc} = 2.0 \text{ or larger for non-crowned teeth}$$

I: The *geometry factor for pitting resistance* incorporates the radii of curvature of the pinion and gear teeth and the degree of load sharing between teeth. It is a function of the number of teeth in both the pinion and the gear and, therefore, to the gear ratio. Use Figure 10–19 for values for problem solving in this book. AGMA Standard 2003 (Reference 10) provides a formula for I and a detailed procedure for acquiring the necessary data and performing the calculations.

We now revisit the example design problem used throughout this chapter and compute the contact stress.

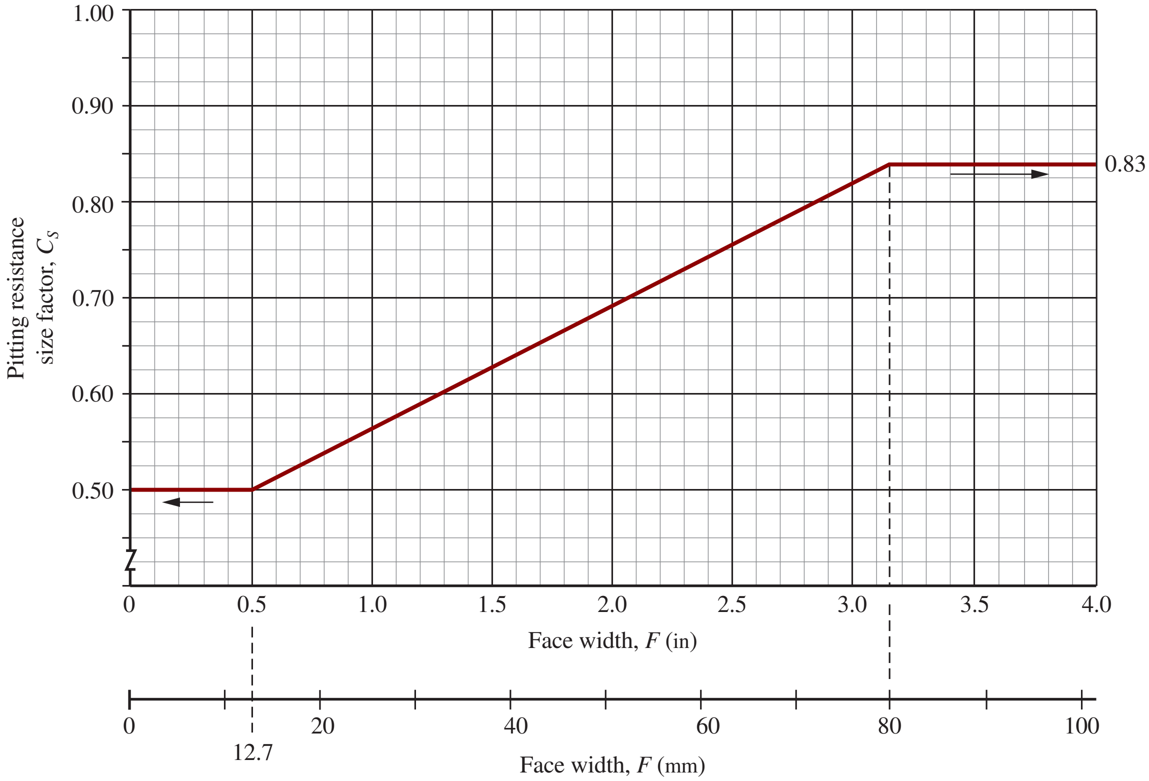


FIGURE 10–18 Pitting resistance size factor, C_s (Adapted from AGMA 2003-C10, *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, Zerol Bevel and Spiral Bevel Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th Floor, Alexandria, VA.)

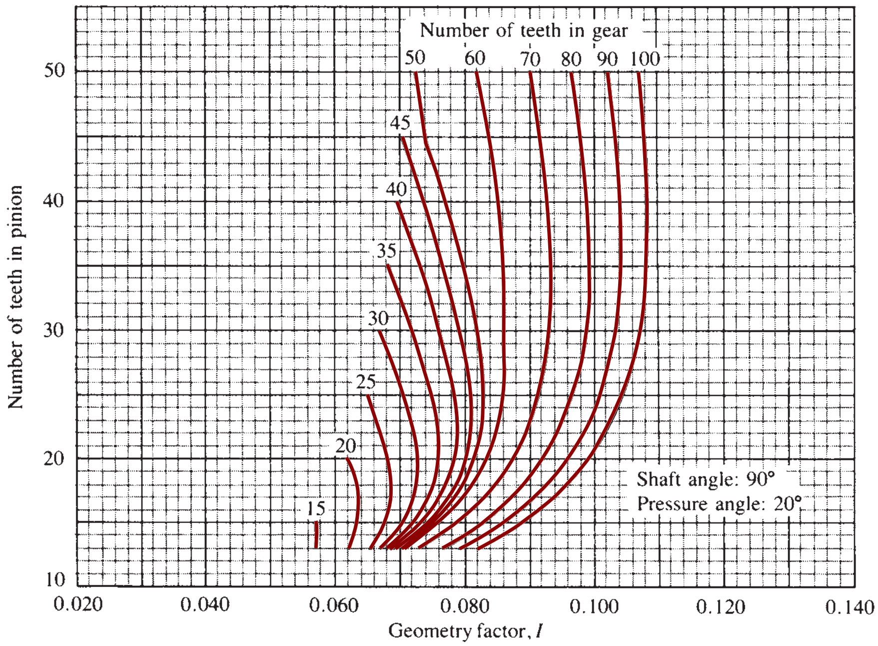


FIGURE 10–19 Geometry factors for straight and ZEROL® bevel gears (Extracted from AGMA 2003 (Reference 10), *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, ZEROL® Bevel and Spiral Bevel Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314.)

Example Problem 10–7 Compute the contact stress for the gear pair in Figure 10–9 for the conditions used in Example Problem 10–5: $N_P = 16$; $N_G = 48$; $n_P = 600$ rpm; $P_d = 8$; $F = 1.00$ in; and $D_P = 2.000$ in. Both gears are to be steel.

Solution We use Equation (10–19) with U.S. units.

$$s_c = C_p\sqrt{\frac{W_t K_o K_m K_v C_s C_{xc}}{F D_p I}}\ \text{psi}$$

From Example Problem 10–5: $W_t = 263$ lb; $K_o = 1.50$; $K_m = 1.004$; and $K_v = 1.24$. Other factors are as follows:

Elastic coefficient $C_p = 2300\ \text{psi}^{0.5}$ for two steel gears

Size factor $C_s = 0.56$ (Figure 10–18)

Crowning factor $C_{xc} = 1.5$ (Specify properly crowned teeth)

Geometry factor for pitting resistance $I = 0.077$ (Figure 10–19)

Then

$$s_c = 2300\sqrt{\frac{(263)(1.50)(1.004)(1.24)(0.56)(1.5)}{(1.00)(2.000)(0.077)}}\ \text{psi} = 119\,044\ \text{psi}$$

Allowable Contact Stress Number: The process of finding the required allowable contact stress, s_{ac}, for bevel gears is similar to that used for spur and helical gears. The fundamental equation is

$$s_{wc} = \frac{s_{ac} C_L}{(SF)(C_R)} \tag{10–21}$$

where

s_{wc} = permissible contact stress number considering design life and reliability.

C_L = Stress cycle factor for contact stress. Use Figure 10–20, developed for carburized case-hardened steel bevel gears. Application to through-hardened gears is approximate.

C_R = Reliability factor. Use Table 10–3. Note that the reliability factor for pitting resistance is equal to the square root of that for bending. The AGMA standard adjusts the allowable bending strengths accordingly.

SF = Safety factor. Typically taken to be 1.00, but values up to about 1.50 can be used for greater uncertainty or for critical systems.

For design where the goal is to specify a suitable gear material, Equation (10–21) can be combined with the equation for s_c from Equation (10–19) and we can solve for the required value of the allowable bending strength, s_{ac}. That is,

$$\text{Let } s_c = s_{wc} = \frac{s_{ac} C_L}{(SF)(C_R)}$$

Then, the required value of s_{ac} is

$$s_{ac} = \frac{s_c (SF)(C_R)}{C_L} \tag{10–22}$$

Use Figure 10–21 to determine the required hardness for through-hardened steel, recognizing that HB 400 is the maximum recommended value. Above that, flame, induction, or carburized case-hardened steel should be used up to the limits shown in Table 10–4.

Now we look again at the design analysis example considered before and evaluate both the required value of s_{at} and s_{ac}. Finally, we identify the critical value and specify a suitable material for the two gears.

Example Problem 10–8 Specify suitable materials for the bevel pinion and gear for the data of Example Problems 10–5 to 10–7. Design for a life of 15 000 hours.

Solution From Example Problems 10–6 and 10–7, we find the following data:

Rotational speed of pinion $= n_P = 600$ rpm

Bending stress number, $s_t = 8764$ psi

Contact stress number, $s_c = 119\,044$ psi

We apply Equations (10–18) and (10–22).

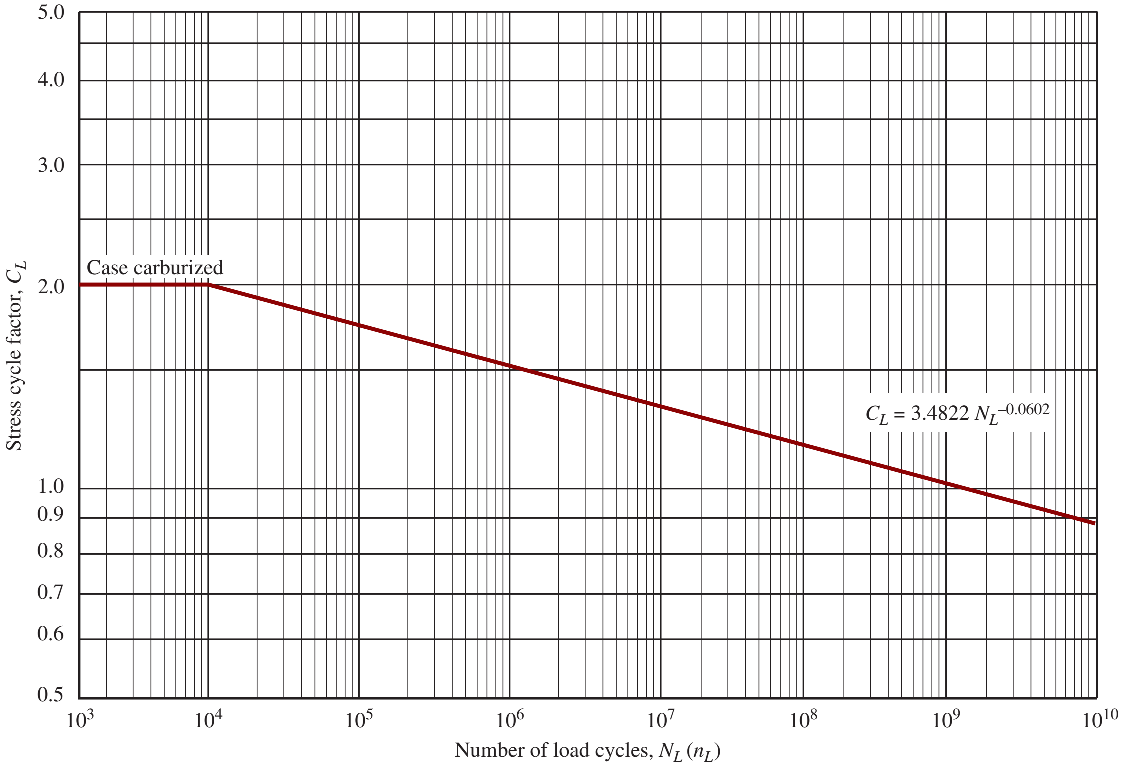


FIGURE 10–20 Stress cycle factor for pitting resistance, C_L (carburized case-hardened steel bevel gears) (Adapted from AGMA 2003-C10, *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, Zerol Bevel and Spiral Bevel Gear Teeth,* with permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th Floor, Alexandria, VA.)

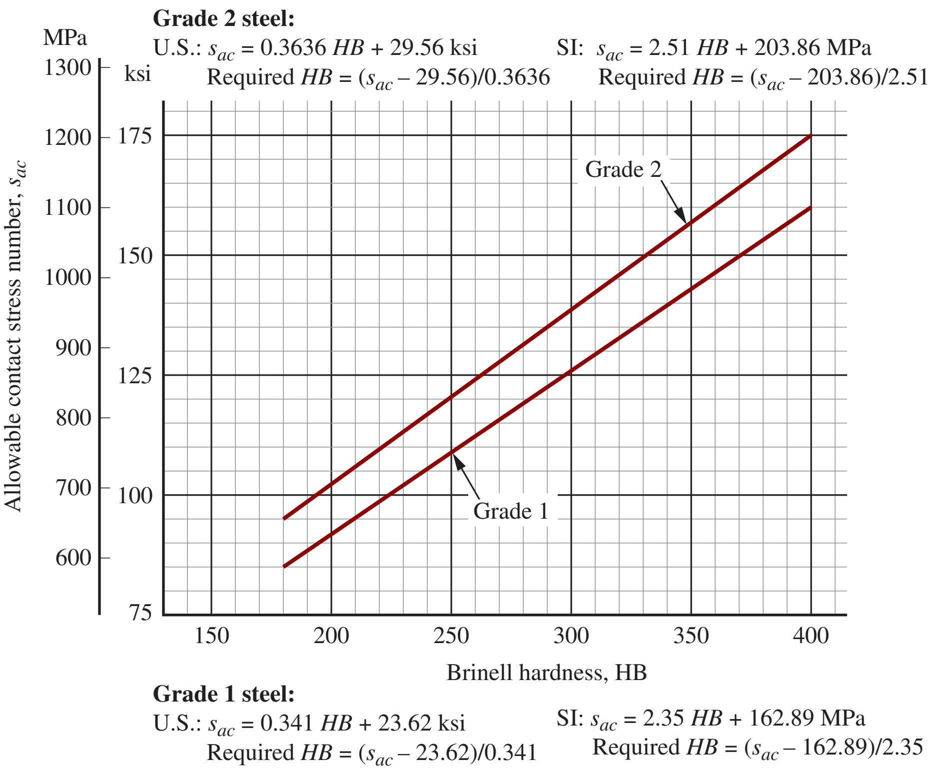


FIGURE 10–21 Allowable contact stress number, S_{ac}, for through-hardened steel for bevel gears (Adapted from AGMA 2003 (Reference 10), *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, Zerol Bevel and Spiral Bevel Gear Teeth,* with permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th Floor, Alexandria, VA.)

$$s_{at} = \frac{s_t(SF)(K_R)}{K_L} \qquad \textbf{(10–18)}$$

$$s_{ac} = \frac{s_c(SF)(C_R)}{C_L} \qquad \textbf{(10–22)}$$

Two of the factors in each equation are design decisions.

Safety factor *SF*:

No additional uncertainties or special requirements are known. Then $SF = 1.00$.

Reliability factors K_R and C_R: Assume a reliability of 0.99, one failure in 100.

Then, $K_R = 1.00$ and $C_R = 1.00$.

Life factors K_L and C_L: We use Figures 10–16 and 10–20 that require the expected number of load cycles, N_c. The equation shown below was used in Chapter 9 with q indicating the number of stress cycles per revolution of the gear, typically 1.0. For planetary or split power systems, q can be 2.0 or more.

$$N_c = 60(L)(n_P)(q) = (60)(15\,000)(600)(1) = 5.4 \times 10^8 \text{ load cycles}$$

For bending: $K_L = 0.948$ (computed from equation in Figure 10–16)

For pitting resistance: $C_L = 1.038$ (computed from equation in Figure 10–20)

We can then complete the calculations:

$$s_{at} = \frac{s_t(SF)(K_R)}{K_L} = \frac{(8764)(1.0)(1.0)}{0.948} = 9245 \text{ psi} = 9.245 \text{ ksi}$$

$$s_{ac} = \frac{s_c(SF)(C_R)}{C_L} = \frac{(119\,044)(1.0)(1.0)}{1.038} = 114\,686 \text{ psi} = 114.7 \text{ ksi}$$

Required hardness of steels and material specification:

Use Figure 10–17 for bending: Required $HB = 162$; Low; Use almost any steel.

Use Figure 10–21 for contact stress: Required $HB = 267$.

This value is quite suitable for through-hardened steel.

Use Figure A4–1: Specify SAE 1040 WQT 1000; $HB = 269$

22% elongation; $s_y = 88$ ksi; $s_u = 114$ ksi

Summary and comments: A bevel gear drive has been designed with straight teeth for transmission of power to shafts oriented 90° to each other. Key data were given and used in Chapter 8 and this chapter in Example Problems 8–3, and 10–4 to 10–8. Key data were given and used in this chapter and in Example Problems 10–4 to 10–8. Results are summarized as follows:

1. ***Example Problem 8–3***: Values for geometrical features were calculated for the following input data: $P_d = 8$; 20° pressure angle; $N_P = 16$; $N_G = 48$; 90° shaft angle.
   a. Output data used in subsequent problems include: Gear Ratio = 3.00; $D_P = 2.000$ in; $D_G = 6.000$ in; Pitch cone angle for the pinion = $\gamma = 18.43°$; Pitch cone angle for the gear = $\Gamma = 71.57°$; Face width = $F = 1.00$ in (design decision); Pinion outside diameter = $D_{oP} = 2.368$ in; Pinion outside diameter = $D_{oP} = 2.368$ in; Gear outside diameter = $D_{oG} = 6.041$ in. Several other detailed gear tooth features were also calculated.
2. ***Example Problem 10–4:*** Forces and torques on the pinion and gear were calculated for a given power transmitted = $P = 2.50$ hp at a pinion speed of 600 rpm.
   a. Results included: Torque on the pinion = $T_P = 263$ lb·in; Torque on the gear = $T_G = 788$ lb·in; Tangential transmitted force at the mean radii = $W_t = 313$ lb; Radial force = $W_r = 108$ lb; Axial force = $W_x = 36$ lb; Mean radius of the pinion teeth = $R_{mP} = 0.84$ in; Mean radius of the gear teeth = $R_{mG} = 2.53$ in;
   b. The forces at the mean radii, torques, and mean radii for the pinion and the gear were used in the analysis of forces on the shafts and bearings in Example Problem 10–5. Note that tangential forces are recalculated in Example Problem 10–6 to conform to conventions for stress analysis of bevel gear teeth.
3. ***Example Problem 10–5:*** See Figure 10–10 for a graphical display of the free-body diagrams of both shafts showing all forces and torques acting on the two shafts carrying the pinion and the gear along with bearing reactions. This required analysis of forces, shearing forces, and

bending moments on both shafts in two planes. All four bearings experience radial reaction forces in two planes. Resultants were calculated for each bearing. By a design decision, the thrust forces created by the bevel gears were taken at Bearings *B* and *D*. Results are as follows:

a. Bearing *A*: Radial load = 224 lb
b. Bearing *B*: Radial load = 108 lb; Thrust load = 36 lb
c. Bearing *C*: Radial load = 148 lb
d. Bearing *D*: Radial load = 188 lb; Thrust load = 108 lb

4. ***Section 10–8:*** The equilibrium analysis to determine forces in Example Problem 10–5 was extended to complete the shearing forces and bending moments in two planes for both shafts. See Figure 10–11 for the pinion shaft and Figure 10–12 for the gear shaft. Resultant bending moments were also calculated that would be needed for completing the design for the shafts, as will be discussed in Chapter 12.

5. ***Example Problem 10–6:*** The bending stress number for the teeth of the pinion was calculated using data from previous problems and additional given data:
   a. The pinion is driven by an electric motor and the load provides moderate shock. The quality number is $A_v = 11$, achievable by general commercial processing.
   b. Note that the transmitted forces on the pinion and the gear are recalculated based on the torque transmitted and the outer pitch radii for the pinion and the gear. The differences between these forces and those used in the shaft force analysis are accounted for in the geometry factors *J* and *I*. Here we use $W_t = 263$ lb.
   c. Bending stress number on pinion teeth = $s_t = 8764$ psi. The stress on the gear teeth will be lower because of the relative value of the geometry factor.

6. ***Example Problem 10–7:*** The contact stress number for the teeth of the pinion was calculated using data from previous problems. A design decision chose to use steel for both the pinion and the gear.
   a. Contact stress number, $s_c = 119\,044$ psi

7. ***Example Problem 10–8:*** The minimum required allowable bending stress number and the minimum allowable contact stress numbers were calculated based on results from Example Problems 10–6 and 10–7 along with the following design decisions:
   a. Reliability = $R = 0.99$ ($<$ 1 failure in 100); $K_R = C_R = 1.0$
   b. Safety factor = $SF = 1.0$, assuming no unusual uncertainties beyond other *K*-factors
   c. Design for a life of 15 000 hours. This is reasonable for general industrial applications, fully utilized.
   d. Results: Bending: $s_{at} = 9245$ psi = 9.245 ksi; $s_{ac} = 114\,686$ psi = 114.7 ksi

8. ***Required hardness of steels and material specification:***

   Bending: Required $HB = 162$; Low; Use almost any steel.

   Contact stress–Pitting resistance: Required $HB = 267$.

   This value controls the material specification and is quite suitable for through-hardened steel.

   Using Figure A4–1 Specify SAE 1040 WQT 1000; $HB = 269$; 22% elongation;
   $s_y = 88$ ksi; and $s_u = 114$ksi

9. ***Comments on the analysis and design:*** Taken together, the procedures and results summarized here demonstrate a reasonable approach to designing bevel gear pairs with straight teeth. The design is satisfactory as shown. However, alternate designs are practical and other iterations are recommended to explore what improvements can be made. Possible choices for different design decisions are as follows:
   a. A smaller overall drive may be practical by permitting the use of case hardening by flame or induction hardening or carburizing. These would be capable of operating at higher bending and contact stresses produced by smaller gears, but with corresponding higher processing costs.
   b. A more accurate gear quality could be chosen instead of $A_v = 11$ to decrease stresses and improve smoothness of operation and noise, but this also has higher associated costs.
   c. Consider using shot peening or other means of improving the life of the gears.
   d. Spiral bevel gears may permit a more compact design and reference should be made to AGMA Standard 2003 (Reference 10) for analysis methodology and additional data required.

Practical Considerations for Bevel Gearing

Factors similar to those discussed for spur and helical gears should be considered in the design of systems using bevel gears. The accuracy of alignment and the accommodation of thrust loads discussed in the example problems are critical.

Figure 10–22 shows the exterior view of a heavy-duty gear reducer for an industrial right angle drive. The input shaft is to the left and the output shaft is the larger one at the right extending through the side of the housing. Figure 10–23 shows a reducer similar to that in Figure 10–22 but with the upper part of the housing removed so that the entire three-stage reduction can be seen. The first stage is a spiral bevel gear pair and stages two and three are helical gear pairs. The large speed reduction results in a correspondingly large increase in the torque on the output shaft, requiring its diameter to be larger as shown. Observe, also, the careful placement of the bearings that support all shafts in the housing and how the housing allows for lubrication of the gears and assembly of all components.

10–10 FORCES, FRICTION, AND EFFICIENCY IN WORMGEAR SETS

See Chapter 8 for the geometry of wormgear sets. Also see References 2, 14, 16–18, and 21.

The force system acting on the worm/wormgear set is usually considered to be made of three perpendicular components as was done for helical and bevel gears. There are a tangential force, a radial force, and an axial force acting on the worm and the wormgear. We will use the same notation here as in the bevel gear system.

Figure 10–24 shows two orthogonal views (front and side) of a worm/wormgear pair, showing only the pitch diameters of the gears. The figure shows the separate worm and wormgear with the forces acting on each. Note that because of the 90° orientation of the two shafts,

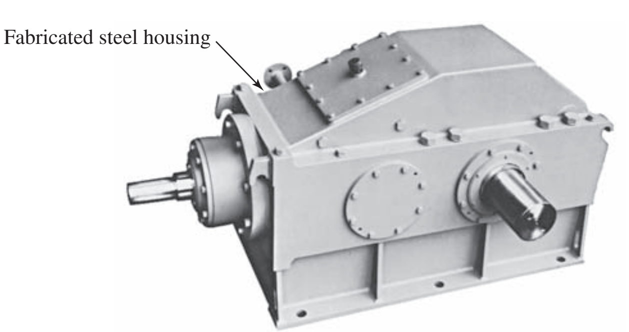


FIGURE 10–22 Heavy-duty right angle gear reducer (Sumitomo Machinery Corporation of America, Teterboro, NJ)

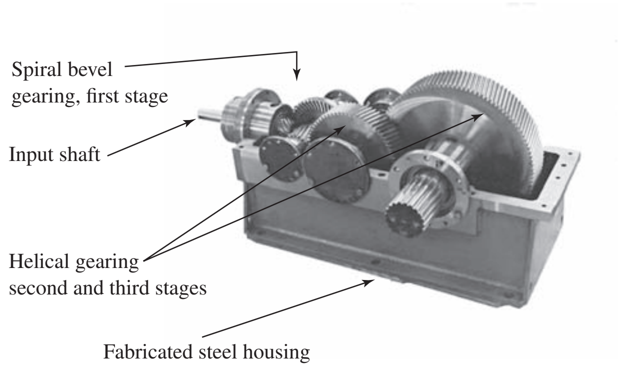


FIGURE 10–23 Three-stage industrial gear reducer employing spiral bevel and helical gears (Sumitomo Machinery Corporation of America, Teterboro, NJ)

⇨ **Forces on Worms and Wormgears**

$$\left.\begin{aligned} W_{tG} &= W_{xW} \\ W_{xG} &= W_{tW} \\ W_{rG} &= W_{rW} \end{aligned}\right\} \qquad (10\text{–}23)$$

Of course, the directions of the paired forces are opposite because of the action/reaction principle.

The tangential force on the wormgear is computed first and is based on the required operating conditions of torque, power, and speed at the output shaft.

Pitch Line Speed, v_t

As stated in Chapter 8, the pitch line velocity is the linear velocity of a point on the pitch line for the worm or the wormgear. For the worm having a pitch diameter D_W in, rotating at n_W rpm,

⇨ **Pitch Line Velocity for Worm**

$$v_{tW} = \frac{\pi D_W n_W}{12} \text{ ft/min} \quad \text{or} \quad v_{tW} = \frac{\pi D_W n_W}{60\,000} \text{ m/s}$$

For the wormgear having a pitch diameter D_G in, rotating at n_G rpm,

⇨ **Pitch Line Velocity for Gear**

$$v_{tG} = \frac{\pi D_G n_G}{12} \text{ ft/min} \quad \text{or} \quad v_{tG} = \frac{\pi D_G n_G}{60\,000} \text{ m/s}$$

Note that these two values for pitch line velocity are *not* equal.

Velocity Ratio, *VR*

It is most convenient to calculate the velocity ratio of a worm and wormgear set from the ratio of the input rotational speed to the output rotational speed:

⇨ **Velocity Ratio for Worm/Sot**

$$VR = \frac{\text{speed of worm}}{\text{speed of gear}} = \frac{n_W}{n_G} = \frac{N_G}{N_W}$$

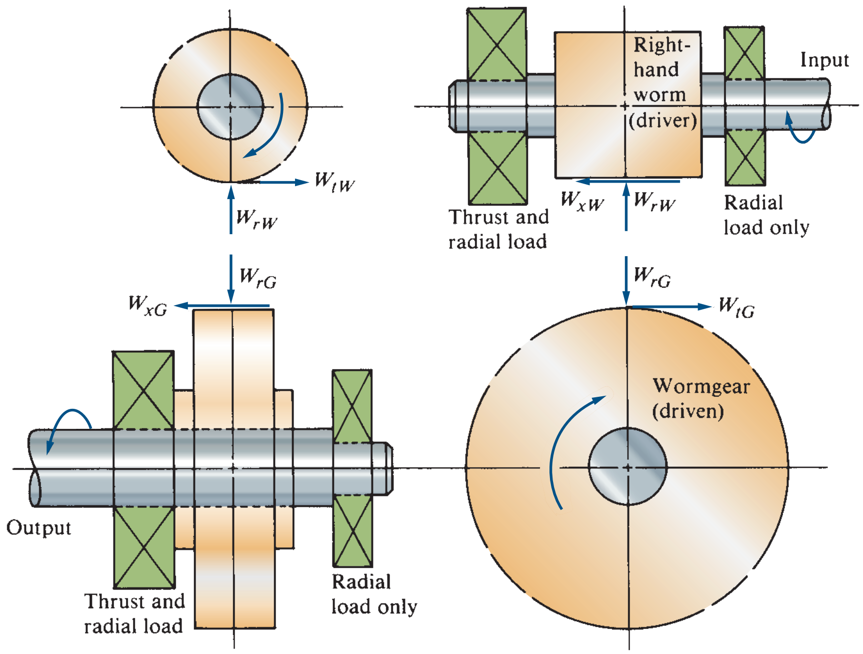


FIGURE 10–24 Forces on a worm and a wormgear

Coefficient of Friction, μ

Friction plays a major part in the operation of a wormgear set because there is inherently sliding contact between the worm threads and the wormgear teeth. The coefficient of friction is dependent on the materials used, the lubricant, and the sliding velocity. Based on the pitch line speed of the gear, the sliding velocity is

➩ Sliding Velocity for Gear

$$v_s = v_{tG}/\sin \lambda \qquad (10\text{–}24)$$

Based on the pitch line speed of the worm,

➩ Sliding Velocity for Worm

$$v_s = v_{tW}/\cos \lambda \qquad (10\text{–}25)$$

The term λ is the lead angle for the worm thread as defined in Equation (8–24).

The AGMA (see Reference 14) recommends the following formulas to estimate the coefficient of friction for a hardened steel worm (58 HRC minimum), smoothly ground, or polished, or rolled, or with an equivalent finish, operating on a bronze wormgear. The choice of formula depends on the sliding velocity. *Note:* v_s must be in ft/min in the formulas; 1.0 ft/min = 0.0051 m/s.

Static Condition: $v_s = 0$

$$\mu = 0.150$$

Low Speed: $v_s < 10$ **ft/min (0.051 m/s)**

$$\mu = 0.124e^{(-0.074 v_s^{0.645})} \qquad (10\text{–}26)$$

Higher Speed: $v_s > 10$ **ft/min**

$$\mu = 0.103e^{(-0.110 v_s^{0.450})} + 0.012 \qquad (10\text{–}27)$$

Figure 10–25 is a plot of the coefficient μ versus the sliding velocity v_s.

Output Torque from Wormgear Drive, T_o

In most design problems for wormgear drives, the output torque and the rotating speed of the output shaft will be known from the requirements of the driven machine. Torque and speed are related to the output power by

➩ Output Torque from Wormgear

$$T_o = \frac{63\,000(P_o)}{n_G} \qquad (10\text{–}28)$$

By referring to the end view of the wormgear in Figure 10–24, you can see that the output torque is

$$T_o = W_{tG} \cdot r_G = W_{tG}(D_G/2)$$

Then the following procedure can be used to compute the forces acting in a worm/wormgear drive system.

Procedure for Calculating the Forces on a Worm/Wormgear Set

Given:

Output torque, T_o, in lb·in
Output speed, n_G, in rpm
Pitch diameter of the wormgear, D_G, in inches
Lead angle, λ
Normal pressure angle, ϕ_n

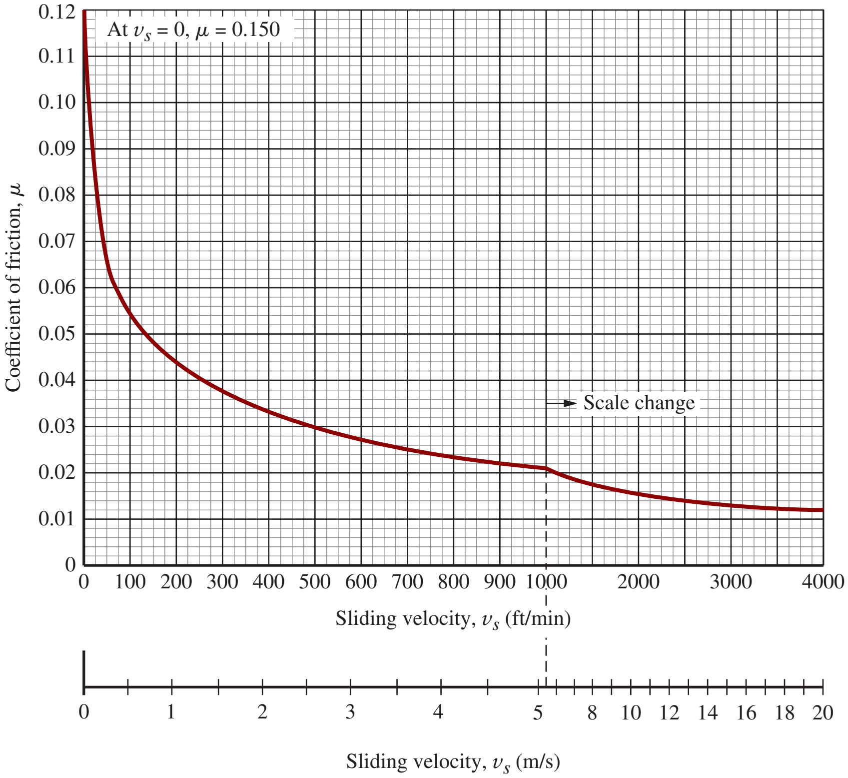


FIGURE 10–25 Coefficient of friction versus sliding velocity for steel worm and bronze wormgear

Compute:

$$W_{tG} = 2\,T_o/D_G \tag{10–29}$$

$$W_{xG} = W_{tG}\frac{\cos\phi_n \sin\lambda + \mu\cos\lambda}{[\cos\phi_n\cos\lambda - \mu\sin\lambda]} \tag{10–30}$$

$$W_{rG} = \frac{W_{tG}\sin\phi_n}{\cos\phi_n\cos\lambda - \mu\sin\lambda} \tag{10–31}$$

The forces on the worm can be obtained by observation, using Equation (10–23). Equations (10–30) and (10–31) were derived using the components of both the tangential driving force on the wormgear and the friction force at the location of the meshing worm threads and wormgear teeth. The complete development of the equations is shown in Reference 15.

Friction Force, W_f

The friction force, W_f, acts parallel to the face of the worm threads and the gear teeth and depends on the tangential force on the gear, the coefficient of friction, and the geometry of the teeth:

$$W_f = \frac{\mu W_{tG}}{(\cos\lambda)(\cos\phi_n) - \mu\sin\lambda} \tag{10–32}$$

Power Loss Due to Friction, P_L

Power loss is the product of the friction force and the sliding velocity at the mesh. That is,

$$P_L = \frac{v_s W_f}{33\,000} \tag{10–33}$$

In this equation, the power loss is in hp, v_s is in ft/min, and W_f is in lb.

Input Power, P_i

The input power is the sum of the output power and the power loss due to friction:

$$P_i = P_o + P_L \tag{10–34}$$

Efficiency, η

Efficiency is defined as the ratio of the output power to the input power:

$$\eta = P_o/P_i \tag{10–35}$$

Efficiency for a wormgear drive with the usual case of the input coming through the worm can also be computed directly from the following equation.

$$\eta = \frac{\cos\phi_n - \mu\tan\lambda}{\cos\phi_n + \mu/\tan\lambda} \tag{10–36}$$

Factors Affecting Efficiency

As can be seen in Equation (10–32), the lead angle, the normal pressure angle, and the coefficient of friction all affect the efficiency. The one that has the largest effect, and the one over which the designer has the most control, is the lead angle, λ. The larger the lead angle, the higher the efficiency, up to approximately $\lambda = 45°$. (See Figure 10–26.)

Now, looking back to the definition of the lead angle, note that the number of threads in the worm has a major effect on the lead angle. Therefore, to obtain a high efficiency, use multiple-threaded worms. But there is a disadvantage to this conclusion. More worm threads require more gear teeth to achieve the same ratio, resulting in a larger system overall. The designer is often forced to compromise.

Example Problem: Forces and Efficiency in Wormgearing

Review now the results of Example Problem 8–4, in which the geometry factors for a particular worm and wormgear set were computed. The following example problem extends the analysis to include the forces acting on the system for a given output torque.

Example Problem 10–9 The wormgear drive described in Example Problem 8–4 is transmitting an output torque of 4168 lb · in. The transverse pressure angle is 20°. The worm is made from hardened and ground steel, and the wormgear is bronze. Compute the forces on the worm and the wormgear, the output power, the input power, and the efficiency.

Solution Recall from Example Problem 8–4 that

$$\lambda = 14.04° \qquad D_G = 8.667 \text{ in} \qquad n_G = 101 \text{ rpm}$$

$$n_W = 1750 \text{ rpm} \qquad D_W = 2.000 \text{ in}$$

The normal pressure angle is required. From Equation (8–26),

$$\phi_n = \tan^{-1}(\tan\phi_t \cos\lambda) = \tan^{-1}(\tan 20° \cos 14.04°) = 19.45°$$

Because they recur in several formulas, let's compute the following:

$$\sin\phi_n = \sin 19.45° = 0.333$$

$$\cos\phi_n = \cos 19.45° = 0.943$$

$$\cos\lambda = \cos 14.04° = 0.970$$

$$\sin\lambda = \sin 14.04° = 0.243$$

$$\tan\lambda = \tan 14.04° = 0.250$$

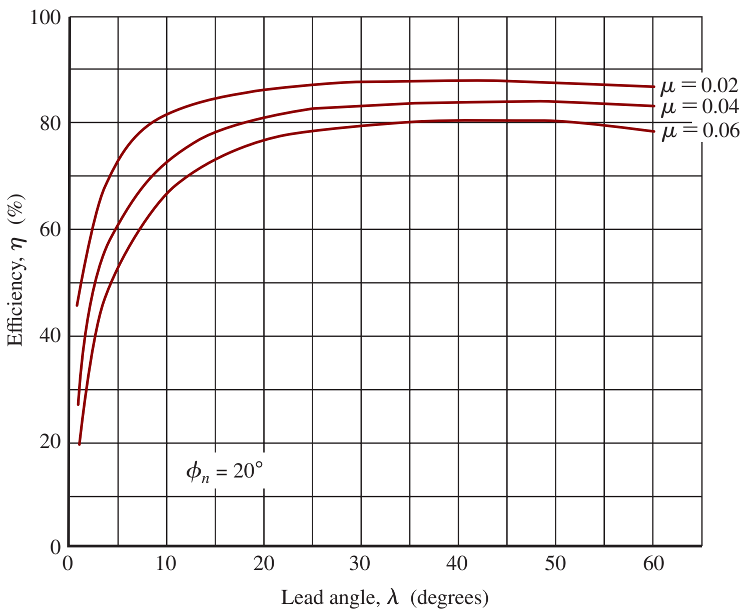


FIGURE 10–26 Efficiency of wormgear drive versus lead angle

We can now compute the tangential force on the wormgear using Equation (10–29)

$$W_{tG} = \frac{2T_o}{D_G} = \frac{(2)(4168\ \text{lb}\cdot\text{in})}{8.667\ \text{in}} = 962\ \text{lb}$$

The calculations of the axial and radial forces require a value for the coefficient of friction that, in turn, depends on the pitch line speed and the sliding velocity.

Pitch Line Speed of the Gear:

$$v_{tG} = \pi D_G n_G/12 = \pi(8.667)(101)/12 = 229\ \text{ft/min}$$

Sliding Velocity: [Equation (10–24)]

$$v_s = v_{tG}/\sin\lambda = 229/\sin 14.04° = 944\ \text{ft/min}$$

Coefficient of Friction: From Figure 10–25, at a sliding velocity of 944 ft/min, we can read $\mu = 0.022$.

Now the axial and radial forces on the wormgear can be computed.

Axial Force on the Wormgear: [Equation (10–30)]

$$W_{xG} = 962\ \text{lb}\left[\frac{(0.943)(0.243) + (0.022)(0.970)}{(0.943)(0.970) - (0.022)(0.243)}\right] = 265\ \text{lb}$$

Radial Force on the Wormgear: [Equation (10–31)]

$$W_{rG} = \left[\frac{(962)(0.333)}{(0.943)(0.970) - (0.022)(0.243)}\right] = 352\ \text{lb}$$

Now the output power, input power, and efficiency can be computed.

Output Power: [Equation (10–28)]

$$P_o = \frac{T_o n_G}{63\,000} = \frac{(4168\ \text{lb}\cdot\text{in})(101\ \text{rpm})}{63\,000} = 6.68\ \text{hp}$$

The input power depends on the friction force and the consequent power loss due to friction.

Friction Force: [Equation (10–32)]

$$W_f = \frac{\mu W_{tG}}{(\cos\lambda)(\cos\phi_n) - \mu\sin\lambda} = \frac{(0.022)(962\ \text{lb})}{(0.970)(0.943) - (0.022)(0.243)} = 23.3\ \text{lb}$$

Power Loss Due to Friction: [Equation (10–33)]

$$P_L = \frac{v_s W_f}{33\,000} = \frac{(944\ \text{ft/min})(23.3\ \text{lb})}{33\,000} = 0.666\ \text{hp}$$

Input Power: [Equation (10–34)]

$$P_i = P_o + P_L = 6.68 + 0.66 = 7.35\ \text{hp}$$

Efficiency: [Equation (10–35)]

$$\eta = \frac{P_o}{P_i}(100\%) = \frac{6.68\ \text{hp}}{7.35\ \text{hp}}(100\%) = 90.9\%$$

Equation (10–36) could also be used to compute efficiency directly without computing friction power loss.

Self-Locking Wormgear Sets

Self-locking is the condition in which the worm drives the wormgear, but, if torque is applied to the gear shaft, the worm does not turn. It is locked! The locking action is produced by the friction force between the worm threads and the wormgear teeth, and this is highly dependent on the lead angle. It is recommended that a lead angle no higher than about 5.0° be used in order to ensure that self-locking will occur. This lead angle usually requires the use of a single-threaded worm; the low lead angle results in a low efficiency, possibly as low as 60% or 70%.

10–11 STRESS IN WORMGEAR TEETH

We present here an approximate method of computing the bending stress in the teeth of the wormgear. Because the geometry of the teeth is not uniform across the face width, it is not possible to generate an exact solution. However, the method given here should predict the bending stress with sufficient accuracy to check a design because most worm/wormgear systems are limited by pitting, wear, or thermal considerations rather than strength.

The AGMA, in its Standard 6034-B92 (Reference 14), does not include a method of analyzing wormgears for strength. The method shown here was adapted from Reference 15. Only the wormgear teeth are analyzed because the worm threads are inherently stronger and are typically made from a stronger material.

The stress in the gear teeth can be computed from

$$\sigma = \frac{W_d}{yFp_n} \tag{10–37}$$

where W_d = dynamic load on the gear teeth

y = Lewis form factor (see Table 10–5)

F = face width of the gear

p_n = normal circular pitch = $p \cos \lambda = \pi \cos \lambda / P_d$ **(10–38)**

The dynamic load can be estimated from

$$W_d = W_{tG}/K_v \tag{10–39}$$

and

$$K_v = 1200/(1200 + v_{tG}) \tag{10–40}$$

$v_{tG} = \pi D_G n_G/12$ = pitch line speed of the gear **(10–41)**

Only one value is given for the Lewis form factor for a given pressure angle because the actual value is very difficult to calculate precisely and does not vary much with the number of teeth. The actual face width should be used, up to the limit of two-thirds of the pitch diameter of the worm.

The computed value of tooth bending stress from Equation (10–37) can be compared with the fatigue strength of the material of the gear. For manganese gear bronze, use a fatigue strength of 17 000 psi; for phosphor gear bronze, use 24 000 psi. For cast iron, use approximately 0.35 times the ultimate strength, unless specific data are available for fatigue strength.

TABLE 10–5 Approximate Lewis Form Factor for Wormgear Teeth

ϕ_n	y
$14\frac{1}{2}°$	0.100
20°	0.125
25°	0.150
30°	0.175

10–12 SURFACE DURABILITY OF WORMGEAR DRIVES

AGMA Standard 6034-B92 (Reference 14) gives a method for rating the surface durability of hardened steel worms operating with bronze gears. The ratings are based on the ability of the gears to operate without significant damage from pitting or wear. All equations from the standard are in the U.S. system. For designs in the SI system, values of parameters should be converted to U.S. units for which the equations were written to determine the corresponding factors. The units are as follows:

W_{tR}—Rated tangential load: lb

Diameters and face width: in

v_s—Sliding velocity: ft/min

Equivalent values in SI units are shown only for reference.

The procedure calls for the calculation of a *rated tangential load,* W_{tR}, from

Rated Tangential Load on Wormgears

$$W_{tR} = C_s D_G^{0.8} F_e C_m C_v \tag{10–42}$$

where C_s = materials factor (from Figure 10–27)

D_G = pitch diameter of the wormgear, in inches

F_e = effective face width, in inches. Use the actual face width of the wormgear up to a maximum of 0.67 D_W

C_m = ratio correction factor (from Figure 10–28)

C_v = velocity factor (from Figure 10–29)

Conditions on the Use of Equation (10–42)

1. The analysis is valid only for a hardened steel worm (58 HRC minimum) operating with gear bronzes specified in AGMA Standard 6034-B92. The classes of bronzes typically used are tin bronze, phosphor bronze, manganese bronze, and aluminum bronze. The materials factor, C_s, is dependent on the method of casting the bronze, as indicated in Figure 10–27. The values for C_s can be computed from the following formulas.

Sand-Cast Bronzes:

For $D_G > 2.5$ in (64 mm),

$$C_s = 1189.636 - 476.545 \log_{10}(D_G) \tag{10–43}$$

For $D_G < 2.5$ in (64 mm),

$$C_s = 1000$$

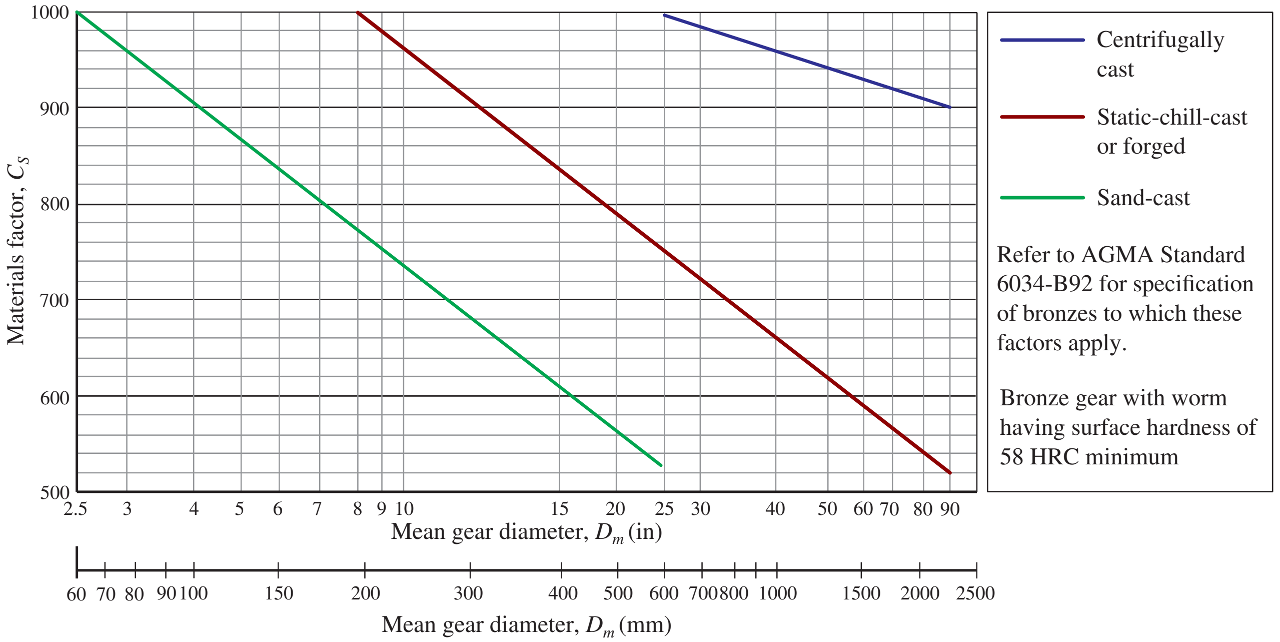


FIGURE 10–27 Materials factor, C_s, for hardened steel worms operating with bronze gears for center distance $C > 3.0$ in (76 mm) (Extracted from AGMA Standard 6034–B92, *Practice for Enclosed Cylindrical Wormgear Speed Reducers and Gearmotors,* with permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th Floor, Alexandria, VA 22314.)

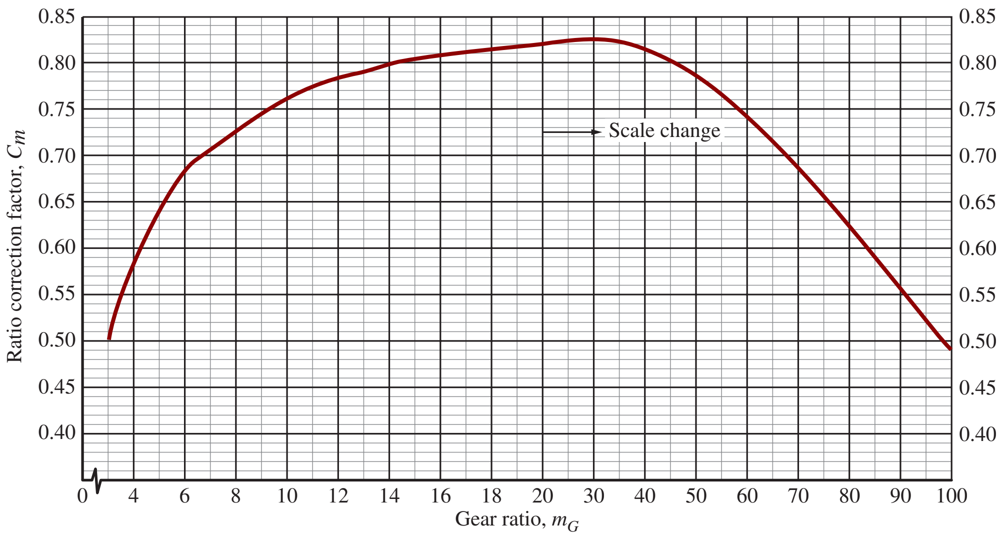


FIGURE 10–28 Ratio correction factor, C_m, versus gear ratio, m_G

Static-Chill-Cast or Forged Bronzes:

For $D_G > 8.0$ in (203 mm),

$$C_s = 1411.651 - 455.825 \log_{10}(D_G) \qquad \textbf{(10–44)}$$

For $D_G < 8.0$ in (203 mm)

$$C_s = 1000$$

Centrifugally Cast Bronzes:

For $D_G > 25$ in (635 mm),

$$C_s = 1251.291 - 179.750 \log_{10}(D_G) \qquad \textbf{(10–45)}$$

For $D_G < 25$ in (635 mm),

$$C_s = 1000$$

2. The wormgear diameter is the second factor in determining C_s. The *mean diameter* at the midpoint of the working depth of the gear teeth should be used. If standard addendum gears are used, the mean diameter is equal to the pitch diameter.
3. Use the actual face width, F, of the wormgear as F_e if $F < 0.667(D_w)$. For larger face widths, use $F_e = 0.67(D_w)$, because the excess width is not effective.

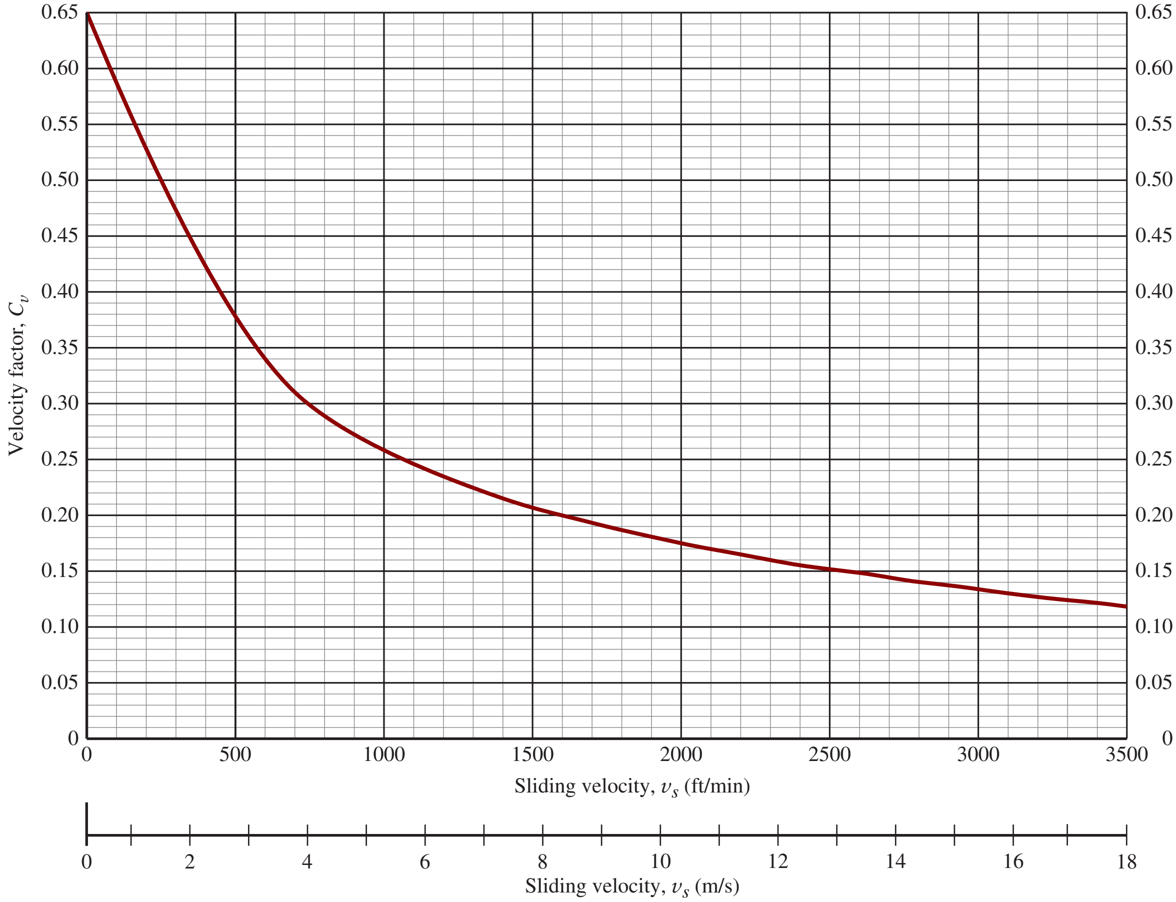


FIGURE 10–29 Velocity factor, C_v, versus sliding velocity

4. The ratio correction factor, C_m, can be computed from the following formulas.

For Gear Ratios, m_G, from 6 to 20

$$C_m = 0.0200(-m_G^2 + 40m_G - 76)^{0.5} + 0.46 \quad \textbf{(10–46)}$$

For Gear Ratios, m_G, from 20 to 76

$$C_m = 0.0107(-m_G^2 + 56m_G + 5145)^{0.5} \quad \textbf{(10–47)}$$

For $m_G > 76$

$$C_m = 1.1483 - 0.00658m_G \quad \textbf{(10–48)}$$

5. The velocity factor depends on the sliding velocity, v_s, computed from Equation (10–24) or (10–25). Values for C_v can be computed from the following formulas.

For v_s, from 0 to 700 ft/min (0–3.56 m/s)

$$C_v = 0.659e^{(-0.0011v_s)} \quad \textbf{(10–49)}$$

For v_s, from 700 to 3000 ft/min (3.56 to 15.24 m/s)

$$C_v = 13.31v_s^{(-0.571)} \quad \textbf{(10–50)}$$

For $v_s > 3000$ ft/min (>15.24 m/s)

$$C_v = 65.52v_s^{(-0.774)} \quad \textbf{(10–51)}$$

6. The proportions of the worm and the wormgear must conform to the following limits defining the maximum and minimum pitch diameters of the worm in relation to the center distance, C, for the gear set. All dimensions are in inches.

$$\text{Maximum } D_W = C^{0.875}/1.6 \quad \textbf{(10–52)}$$

$$\text{Minimum } D_W = C^{0.875}/3.0 \quad \textbf{(10–53)}$$

7. The shaft carrying the worm must be sufficiently rigid to limit the deflection of the worm at the pitch point to the maximum value of $0.005\sqrt{P_x}$, where P_x is the axial pitch of the worm, which is numerically equal to the circular pitch, p, of the wormgear.
8. When you are analyzing a given wormgear set, the value of the rated tangential load, W_{tR}, must be greater than the actual tangential load, W_t, for satisfactory life.
9. Ratings given in this section are valid only for smooth systems such as fans or centrifugal pumps driven by an electric or hydraulic motor operating under 10 hours per day. More severe conditions, such as shock loading, internal combustion engine drives, or longer hours of operation, require the application of a service factor. Reference 14 lists several such factors based on field experience with specific types of equipment. For problems in this book, factors from Table 9–1 can be used.

Example Problem 10–10 Is the wormgear set described in Example Problem 8–4 satisfactory with regard to strength and wear when operating under the conditions of Example Problem 10–9? The wormgear has a face width of 1.25 in.

Solution From previous problems and solutions,

$$W_{tG} = 962 \text{ lb} \qquad VR = m_G = 17.33$$
$$v_{tG} = 229 \text{ ft/min} \qquad v_s = 944 \text{ ft/min}$$
$$D_G = 8.667 \text{ in} \qquad D_W = 2.000 \text{ in}$$

Assume 58 HRC minimum for the steel worm. Assume that the bronze gear is sand cast.

Stress:

$$K_v = 1200/(1200 + v_{tG}) = 1200/(1200 + 229) = 0.84$$
$$W_d = W_{tG}/K_v = 962/0.84 = 1145 \text{ lb}$$
$$F = 1.25 \text{ in}$$
$$y = 0.125 \text{ (from Table 10–5)}$$
$$p_n = p\cos\lambda = (0.5236)\cos 14.04° = 0.508 \text{ in}$$

Then,

$$\sigma = \frac{W_d}{yFp_n} = \frac{1145}{(0.125)(1.25)(0.508)} = 14\,430 \text{ psi}$$

The guidelines in Section 10–11 indicate that this stress level would be adequate for either manganese or phosphor gear bronze.

Surface Durability: Use Equation (10–42):

$$W_{tR} = C_s D_G^{0.8} F_e C_m C_v \tag{10–42}$$

C Factors: The values for the *C* factors can be found from Figures 10–27 to 10–29. We find

$$C_s = 740 \text{ for sand-cast bronze} \quad \text{and} \quad D_G = 8.667 \text{ in}$$
$$C_m = 0.814 \quad \text{for } m_G = 17.33$$
$$C_v = 0.265 \quad \text{for } v_s = 944 \text{ ft/min}$$

We can use $F_e = F = 1.25$ in if this value is not greater than 0.67 times the worm diameter. For $D_W = 2.000$ in,

$$0.67D_W = (0.67)(2.00 \text{ in}) = 1.333 \text{ in}$$

Therefore, use $F_e = 1.25$ in. Then the rated tangential load is

$$W_{tR} = (740)(8.667)^{0.8}(1.25)(0.814)(0.265) = 1123 \text{ lb}$$

Because this value is greater than the actual tangential load of 962 lb, the design should be satisfactory, provided that the conditions defined for the application of Equation (10–42) are met.

10–13 EMERGING TECHNOLOGY AND SOFTWARE FOR GEAR DESIGN

Comprehending the fundamentals of gear design presented in Chapters 8–10 in this book should provide a foundation on which further growth in capabilities to perform reasonable design decisions for gear-type power transmissions can be built. The gear design field is continually adding new technologies and improving upon the methods presented here. Literature in the field often promotes the advantages of gaining additional experience to enable more refined designs and to explore emerging technologies. This section discusses a few emerging technologies and topics beyond the scope of this book. While some technologies mentioned are not truly new and emerging, they are becoming more readily adopted by gear drive design practitioners. Also presented is a brief overview of software available to assist in the gear design process.

Emerging Gearing Technologies

It is strongly recommended that gear design professionals remain aware of the continual upgrading and updating of gear design technology as represented in the extensive suite of standards published by the American Gear Manufacturers Association (AGMA), the International Organization for Standardization (ISO), and other such organizations around the world. With experience, design professionals should also become involved in the standards development process as most of the standards are produced by volunteers representing prominent companies who produce gear drive products and the machinery to make them, or who employ such drives in their products and systems. Industry publications such as *Power Transmission Engineering* and *Gear Technology* provide timely, ongoing commentary and reporting on the state of the art in gear drives and their manufacture. (See Internet sites 10 and 14 in Chapter 9.)

Non-standard Gearing and Gear Tooth Forms. To provide a foundation for further study, this book has focused on standard gear tooth forms of the full-depth involute type with standard fillet geometry, addenda, dedenda, center distances, and other features, using methods that were discovered in the late eighteenth century. Major equipment manufacturers provide general-purpose machines to produce standard designs and suppliers of gears and gear drives benefit from standardization in terms of replacement components and are able to specify products from a variety of vendors.

Experience has shown that modifications to some standard features can be beneficial in special applications and in fields where extensive testing can be performed. Examples are automotive, ship propulsion, industrial machinery, construction equipment, and aerospace propulsion. Crowning of teeth and tip relief to promote smooth engagement, geometry modifications that accommodate deformations of teeth under heavy loads, and modified fillets to optimize bending stresses are a few areas of constant exploration. Materials technologies are also areas where fruitful developments are made for steels, other metallic materials, and the widely diverse array of plastic materials. Others pursue radical changes in gear tooth form to enhance strength, stiffness, noise reduction, or pitting resistance.

Internet site 1 describes proprietary designs called Megagear® and Unimegagear® for which increased power density, efficiency, and durability are obtained. The gear tooth profiles are optimized resulting in greater contact area between gear teeth that reduces compressive stress, increases fatigue life, and makes load distribution more uniform.

Internet site 2 describes Direct Gear Design®, an alternative method of analysis and design of involute gears, which separates gear geometry definition from tool selection to achieve the best possible performance for a particular product and application. The result is gear geometry with asymmetric teeth and optimized fillets at the root area. Two involutes of two different base circles are defined that simultaneously increase the contact ratio and operating pressure angle beyond conventional gear limits. Specially designed gear cutting tools are used to produce the gears by hobbing or shaping. Plastic gear molding, die casting, gear forging, and powder metal processes can also be used. The application of Direct Gear Design must be justified by a significant improvement in gear performance. See also Reference 23.

Internet site 3 describes the patented MGT Frictionless Drive System® that uses the repulsive forces of magnets to provide a link between a drive shaft and the driven one while allowing both shafts to rotate completely independently of each other. Consequently there is no friction generated between them and the efficiency and life of the equipment are improved. The system can be applied either as gearing or couplings. Safety is improved because if the system is overloaded by a certain amount, the driver slips with respect to the driven member. Coupler faces do not touch and the gap between them is typically a few millimeters.

Internet site 4 and Reference 24 describe the field of non-circular gears and their manufacture. Forms can be elliptical, square, or designed to a wide variety of special functions. Sometimes used as cam substitutes in applications where torque or radial loads must be handled. Applications include speed matching on assembly lines, variable cutoff cycles for cutting products from continuous webs, linear motion with quick return, positive and negative rotation mechanisms, flowmeters, bicycle sprockets, variable-speed windshield wipers, and stop and dwell motion devices.

Peening. We have emphasized that successful gear design must demonstrate that the bending stress and contact stress in gears must be controlled to limits of the fatigue strength and pitting resistance of the material from which the gear is made. It stands to reason, therefore, that enhancing the fatigue strength and pitting resistance of the material can lead to safer, lighter, and longer lasting gears. One method of accomplishing that is called *peening*, a process of bombarding the high-stress surfaces of gears with hard materials, called *shot*, at high velocity in a controlled manner. Each impact of a piece of shot permanently dents the target material producing the final condition of compression. Shot media come in a variety of shapes such as follows:

- Cast spheres (steel, stainless steel, cast iron, aluminum oxide, zinc, titanium, and others)
- Cut wire (steel, aluminum, zinc, copper, and others)
- Beads (ceramic, glass, aluminosilicate, and plastic)
- Crushed glass, coal slag, hard rock, and garnet

The process leaves the surface with a desirable residual compressive stress and a high hardness that extends the load-carrying ability and life of the treated component. One major objective for using shot peening is to selectively treat areas of a part that have inherently high tensile stress due to operating conditions or those for which tensile stresses at the surface are created by previous processing steps such as grinding, welding, and aggressive machining. Peening counteracts the residual tensile stresses resulting in lower net final stresses under load. Applications are found in numerous types of products from aerospace (landing gear; turbine components), automotive (gears, engine components, and structural elements), engines (crankshafts, camshafts, connecting rods, shafts, leaf springs, and coil springs), construction equipment (high-wear surfaces, actuator arms, and shafts), medical devices (replacement knees), energy production and transmission, and recreation equipment.

Internet sites 5 and 6 describe the types of equipment used for shot peening and the materials and forms of shot available. Two common types are as follows:

- Wheel blasting—Shot is introduced at the center of a rapidly spinning wheel and centrifugally accelerated as it passes to the periphery and is then flung at high velocity toward the target area.
- Air blasting—Pressurized air passes through a nozzle into which the shot is injected and then blown at high velocity toward the target.

Related products commonly called sand blasters are used to clean surfaces of scale and to produce a textured surface finish without necessarily producing the residual compressive stress in the surface. Vibratory finishing employs hard media of many materials and shapes into which parts are discharged to remove scale and deburr machined edges and surfaces, by repetitive rubbing of the media against the parts.

Software for Gear Design

As may be obvious from working through the topics and design methodologies covered in this chapter and Chapter 9, the subject of gear design is computationally demanding. This book includes a few focused calculation aids in the form of spreadsheets that give a hint to the advantages of using such computer-aided engineering approaches. Developers of several commercially available software packages with far greater capabilities for gear design covering a wide range of gear types are listed as Internet sites 7–13. Some of the developers also offer consulting services to aid in planning and implementing complex gear drives.

REFERENCES

1. American Gear Manufacturers Association. Standard 2008-D11. *Assembling Bevel Gears*. Alexandria, VA: American Gear Manufacturers Association, 2011.
2. American Gear Manufacturers Association. Standard 6022-C93 (R2014). *Design Manual for Cylindrical Wormgearing*. Alexandria, VA: American Gear Manufacturers Association, 2014.
3. American Gear Manufacturers Association. Standard AGMA 917-B97. *Design Manual for Parallel Shaft Fine-Pitch Gearing*. Alexandria, VA: American Gear Manufacturers Association, 1997.
4. American Gear Manufacturers Association. Standard 6001-E08 (R2014). *Design and Selection of Components for Enclosed Gear Drives*. Alexandria, VA: American Gear Manufacturers Association, 2014.
5. American Gear Manufacturers Association. Standard 2015/915-1-A02. *Accuracy Classification System—Tangential Measurement Tolerance Tables for Cylindrical Gears*. Alexandria, VA: American Gear Manufacturers Association, 2002.
6. American Gear Manufacturers Association. Standard AGMA 908-B89 (R2015). *Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical, and Herringbone Gear Teeth*. Alexandria, VA: American Gear Manufacturers Association, 2015.
7. American Gear Manufacturers Association. Standard AGMA 1012-G05 (R2011). *Gear Nomenclature, Definitions of Terms with Symbols*. Alexandria, VA: American Gear Manufacturers Association, 2011.
8. American Gear Manufacturers Association. Standard AGMA 2000-A88. *Gear Classification and Inspection Handbook—Tolerances and Measuring Methods for Unassembled Spur and Helical Gears (Including Metric Equivalents)*. Alexandria, VA: American Gear Manufacturers Association, 1988. Partial replacement of AGMA 390.03.
9. American Gear Manufacturers Association. Standards 2001-D04 (R2010) (based on P_d) and 2101-D04 (R2010) (based on Metric Module, m). *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*. Alexandria, VA: American Gear Manufacturers Association, 2010.
10. American Gear Manufacturers Association. Standard AGMA 2003-C10 (R2015). *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, ZEROL® Bevel, and Spiral Bevel Gear Teeth*. Alexandria, VA: American Gear Manufacturers Association, 2015.
11. American Gear Manufacturers Association. Standard AGMA 2004-C08 (R2014). *Gear Materials, Heat Treatment and Processing Manual*. Alexandria, VA: American Gear Manufacturers Association, 2014.
12. American Gear Manufacturers Association. Standard AGMA 22849-A12. *Design Recommendations for Bevel Gears*. Alexandria, VA: American Gear Manufacturers Association, 2012.
13. American Gear Manufacturers Association. Standard AGMA 6013-A06 (R2011). *Standard for Industrial Enclosed Gear Drives*. Alexandria, VA: American Gear Manufacturers Association, 2011.
14. American Gear Manufacturers Association. Standard AGMA 6034-B92 (R2010). *Practice for Enclosed Cylindrical Wormgear Speed Reducers and Gearmotors*. Alexandria, VA: American Gear Manufacturers Association, 2010.

15. Budynas, R. G., and K. J. Nisbett. *Shigley's Mechanical Engineering Design*. 10th ed. New York: McGraw-Hill, 2015.
16. Crosher, W. P. *Design and Application of the Worm Gear*. New York: American Society of Mechanical Engineers, 2002.
17. Drago, Raymond J. *Fundamentals of Gear Design*. New York: Elsevier Science & Technology Books, 1988.
18. Radzevich, Stephen. *Dudley's Handbook of Practical Gear Design and Manufacture*. 3rd ed. Boca Raton, FL: CRC Press, 2016.
19. International Standards Organization. Standard ISO 54:1996. *Cylindrical Gears for General Engineering and for Heavy Engineering—Modules*. Geneva, Switzerland: International Standards Organization, 1996.
20. Kimotho, J., and Kihiu, J. *Design Optimization of Multistage Gear Trains*. Saarbruken, Germany: VDM Verlag Dr. Muller GMBH & Co., 2010.
21. Lynwander, Peter. *Gear Drive Systems: Design and Application*. Boca Raton, FL: CRC Press, 1983.
22. Oberg, Erik, Franklin D. Jones, Holbrook L. Horton, and Henry H. Ryffel. *Machinery's Handbook*. 30th ed. New York: Industrial Press, 2015.
23. Kapelevich, Alexander. *Direct Gear Design*. Boca Raton, FL: CRC Press, 2013.
24. Litvin, F. L., A. Fuentes, I. Gonzalez-Perez, and K. Hayasaka. *Noncircular Gears: Design and Generation*. Cambridge, UK: Cambridge University Press, 2014.

INTERNET SITES RELATED TO HELICAL GEARS, BEVEL GEARS, AND WORMGEARING

Refer to the list of Internet sites at the end of Chapter 9, Spur Gears. Virtually all sites listed there are also relevant to the design of helical gears, bevel gears, and wormgearing.

1. **Power Engineering and Manufacturing, Ltd.** Manufacturer of custom speed reducers and creator of the Megagear® gear tooth form.
2. **AKGears, LLC.** Creator of the Direct Gear Design® gear tooth form and developer of its application to a variety of products and mechanical systems.
3. **Magnetic Gearing and Turbine Corporation.** Developer and producer of the MGT Frictionless Drive System® that uses the repulsive forces of magnets to provide torque and power transmission without physical contact between the mating parts.
4. **Cunningham Industries, Inc.** Manufacturer of elliptical and other non-circular gears.
5. **The Shot Peener.** A site dedicated to providers of shot peening services or machinery to perform shot peening. The site contains valuable information about the technology and its applications.
6. **Wheelabrator, Inc.** Manufacturer of equipment and media for shot peening including wheelblast and airblast processes along with vibratory finishing equipment.
7. **TEDATA.** Developer of the MDESIGN calculation and analysis software for numerous mechanical components including gears.
8. **Smart Manufacturing Technology, Ltd.** Developer of the MASTA gear design and analysis software and provider of design services for gear-type transmissions.
9. **Universal Technical Systems, Inc.** Developer of the versatile UTS technical analysis software that includes the Integrated Gear Software (IGS) for gear design.
10. **Drive System Technology, Inc.** Mechanical power transmission consultants and developer of the PowerGear gear design software. Company founded by Raymond Drago, Chief Engineer of Drive Systems Technology, Inc. (see Reference 17).
11. **Gleason Plastic Gears.** Engineering consultants for the design and production of plastic gears, utilizing their own custom plastic gear design software. The website's *Engineering and Design* page includes an animated, dynamic, full-color FEA image of gear teeth in contact that displays bending and contact stresses during power transmission.
12. **KISSsoft, U.S.A., LLC.** Developer of the KISSsoft gear design software for several common gear styles along with related machine components and complete power transmissions.
13. **AGMA Software Products.** Developer of a set of software tied directly to the AGMA standards. Included are Bevel Gears Rating Suite, Gear Rating Suite (for spur and helical gears), AGMA 2015-1-A01 Calculator (Accuracy Classification System–Tangential Measurements for Cylindrical Gears). From the home page, select *Software Products*.

PROBLEMS

Helical Gearing

1. A helical gear has a transverse diametral pitch of 8, a transverse pressure angle of $14\frac{1}{2}°$, 45 teeth, a face width of 2.00 in, and a helix angle of 30°.
   (a) If the gear transmits 5.0 hp at a speed of 1250 rpm, compute the tangential force, the axial force, and the radial force.
   (b) If the gear operates with a pinion having 15 teeth, compute the bending stress in the pinion teeth. The power comes from an electric motor, and the drive is to a reciprocating pump. Specify a quality number for the teeth.
   (c) Specify a suitable material for the pinion and the gear considering both strength and pitting resistance.
2. A helical gear has a normal diametral pitch of 12, a normal pressure angle of 20°, 48 teeth, a face width of 1.50 in, and a helix angle of 45°.
   (a) If the gear transmits 2.50 hp at a speed of 1750 rpm, compute the tangential force, the axial force, and the radial force.
   (b) If the gear operates with a pinion having 16 teeth, compute the bending stress in the pinion teeth. The power comes from an electric motor, and the drive

is to a centrifugal blower. Specify a quality number for the teeth.

(c) Specify a suitable material for the pinion and the gear considering both strength and pitting resistance.

3. A helical gear has a transverse diametral pitch of 6, a transverse pressure angle of $14\frac{1}{2}°$, 36 teeth, a face width of 1.00 in, and a helix angle of 45°.
   (a) If the gear transmits 15.0 hp at a speed of 2200 rpm, compute the tangential force, the axial force, and the radial force.
   (b) If the gear operates with a pinion having 12 teeth, compute the bending stress in the pinion teeth. The power comes from a six-cylinder gasoline engine, and the drive is to a concrete mixer. Specify a quality number for the teeth.
   (c) Specify a suitable material for the pinion and the gear considering both strength and pitting resistance.
4. A helical gear has a normal diametral pitch of 24, a normal pressure angle of $14\frac{1}{2}°$, 72 teeth, a face width of 0.25 in, and a helix angle of 45°.
   (a) If the gear transmits 0.50 hp at a speed of 3450 rpm, compute the tangential force, the axial force, and the radial force.
   (b) If the gear operates with a pinion having 16 teeth, compute the bending stress in the pinion teeth. The power comes from an electric motor, and the drive is to a winch that will experience moderate shock. Specify a quality number for the teeth.
   (c) Specify a suitable material for the pinion and the gear considering both strength and pitting resistance.

For Problem 5–11, complete the design of a pair of helical gears to operate under the stated conditions. Specify the geometry of the gears and the material and its heat treatment. Assume that the drive is from an electric motor unless otherwise specified. Consider both strength and pitting resistance.

5. A pair of helical gears is to be designed to transmit 5.0 hp while the pinion rotates at 1200 rpm. The gear drives a reciprocating compressor and must rotate between 385 and 390 rpm.
6. A helical gear pair is to be a part of the drive for a milling machine requiring 20.0 hp with the pinion speed at 550 rpm and the gear speed to be between 180 and 190 rpm.
7. A helical gear drive for a punch press requires 50.0 hp with the pinion rotating at 900 rpm and the gear speed at 225 to 230 rpm.
8. A single-cylinder gasoline engine has the pinion of a helical gear pair on its output shaft. The gear is attached to the shaft of a small cement mixer. The mixer requires 2.5 hp while rotating at approximately 75 rpm. The engine is governed to run at approximately 900 rpm.
9. A four-cylinder industrial engine runs at 2200 rpm and delivers 75 hp to the input gear of a helical gear drive for a large wood chipper used to prepare pulpwood chips for paper making. The output gear must run between 4500 and 4600 rpm.
10. A small commercial tractor is being designed for chores such as lawn mowing and snow removal. The wheel drive system is to be through a helical gear pair in which the pinion runs at 450 rpm while the gear, mounted on the hub of the wheel, runs at 75 to 80 rpm. The wheel has an 18-in diameter. The two-cylinder gasoline engine delivers 20.0 hp to the wheels.
11. A water turbine transmits 15.0 hp to a pair of helical gears at 4500 rpm. The output of the gear pair must drive an electric power generator at 3600 rpm. The center distance for the gear pair must not exceed 4.00 in.
12. Determine the power-transmitting capacity of a pair of helical gears having a normal pressure angle of 20°, a helix angle of 15°, a normal diametral pitch of 10, 20 teeth in the pinion, 75 teeth in the gear, and a face width of 2.50 in, if they are made from SAE 4140 OQT 1000 steel. They are of typical commercial quality. The pinion will rotate at 1725 rpm on the shaft of an electric motor. The gear will drive a centrifugal pump.
13. Repeat Problem 12 with the gears made from SAE 4620 DOQT 300 carburized, case-hardened steel. Then compute the axial and radial forces on the gears.

Bevel Gears

14. A straight bevel gear pair has the following data: $N_P = 15$; $N_G = 45$; $P_d = 6$; and 20° pressure angle. If the gear pair is transmitting 3.0 hp, compute the forces on both the pinion and the gear. The pinion speed is 300 rpm. The face width is 1.25 in. Compute the bending stress and the contact stress for the teeth, and specify a suitable material and heat treatment. The gears are driven by a gasoline engine, and the load is a concrete mixer providing moderate shock. Assume that neither gear is straddle mounted.
15. A straight bevel gear pair has the following data: $N_P = 25$; $N_G = 50$; $P_d = 10$; and 20° pressure angle. If the gear pair is transmitting 3.5 hp, compute the forces on both the pinion and the gear. The pinion speed is 1250 rpm. The face width is 0.70 in. Compute the bending stress and the contact stress for the teeth, and specify a suitable material and heat treatment. The gears are driven by a gasoline engine, and the load is a conveyor providing moderate shock. Assume that neither gear is straddle mounted.
16. Design a pair of straight bevel gears to transmit 5.0 hp at a pinion speed of 850 rpm. The gear speed should be approximately 300 rpm. Consider both strength and pitting resistance. The driver is a gasoline engine, and the driven machine is a heavy-duty conveyor.
17. Design a pair of straight bevel gears to transmit 0.75 hp at a pinion speed of 1800 rpm. The gear speed should be approximately 475 rpm. Consider both strength and pitting resistance. The driver is an electric motor, and the driven machine is a reciprocating saw.

Wormgearing

18. A wormgear set has a single-thread worm with a pitch diameter of 1.250 in, a diametral pitch of 10, and a normal pressure angle of 14.5°. If the worm meshes with a wormgear having 40 teeth and a face width of 0.625 in, compute the gear pitch diameter, the center distance, and the velocity ratio. If the wormgear set is transmitting 924 lb·in of torque at its output shaft, which is rotating

at 30 rpm, compute forces on the gear, efficiency, input speed, input power, and stress on the gear teeth. If the worm is hardened steel and the gear is chilled bronze, evaluate the rated load, and determine whether the design is satisfactory for pitting resistance.

19. Three designs are being considered for a wormgear set to produce a velocity ratio of 20 when the worm gear rotates at 90 rpm. All three have a diametral pitch of 12, a worm pitch diameter of 1.000 in, a gear face width of 0.500 in, and a normal pressure angle of 14.5°. One has a single-thread worm and 20 wormgear teeth; the second has a double-thread worm and 40 wormgear teeth; the third has a four-thread worm and 80 wormgear teeth. For each design, compute the rated output torque, considering both strength and pitting resistance. The worms are hardened steel, and the wormgears are chilled bronze.

20. For each of the three designs proposed in Problem 19, compute the efficiency.

The data for Problems 21–23 are given in Table 10–6. Design a wormgear set to produce the desired velocity ratio when transmitting the given torque at the output shaft for the given output rotational speed.

24. Compare the two designs described in Table 10–7 when each is transmitting 1200 lb·in of torque at its output shaft, which rotates at 20 rpm. Compute the forces on the worm and the wormgear, the efficiency, and the input power required.

TABLE 10–6 Data for Problems 21–23

Problem	*VR*	Torque (lb·in)	Output speed (rpm)
21.	7.5	984	80
22.	3	52.5	600
23.	40	4200	45

TABLE 10–7 Designs for Problem 24

Design	P_d	N_t	N_G	D_W	F_G	Pressure angle
A	6	1	30	2.000	1.000	14.5°
B	10	2	60	1.250	0.625	14.5°

CHAPTER ELEVEN

KEYS, COUPLINGS, AND SEALS

THE BIG PICTURE — Keys, Couplings, and Seals

Discussion Map

- Keys and couplings connect functional parts of mechanisms and machines, allowing moving parts to transmit power or to locate parts relative to each other.
- Retaining rings hold assemblies together or hold parts on shafts, such as keeping a sprocket in position or holding a wheel on an axle.
- Seals protect critical components by excluding contaminants or by retaining fluids inside the housing of a machine.

Discover

Look around you and identify several examples of the use of keys, couplings, retaining rings, and seals in automobiles, trucks, home appliances, shop tools, gardening equipment, or bicycles.

This chapter will help you understand the functions and design requirements of such devices. In addition, you will learn to recognize commercially available designs and apply them properly.

Think of how two or more parts of a machine can be connected for the purpose of locating one part with respect to another. Now think about how that connection must be designed if the parts are moving and if power must be transmitted between them.

This chapter presents information on commercially available products that accomplish these functions. The generic categories of keys, couplings, and seals actually encompass numerous different designs.

A *key* is used to connect a drive member such as a belt pulley, chain sprocket, or gear to the shaft that carries it. (See Figure 11–1.) Torque and power are transmitted across the key to or from the shaft. But how does the power get into or out of the shaft? One way might be that the output from the shaft of a motor or engine is connected to the input shaft of a transmission through a *flexible coupling* that reliably transmits power but allows for some misalignment between the shafts during operation because of the flexing of frame members or through progressive misalignment due to wear.

Seals may be difficult to see because they are typically encased in a housing or are covered in some way. Their function is to protect critical elements of a machine from contamination due to dust, dirt, or water or other fluids while allowing rotating or translating machine elements to move to accomplish their desired functions. Seals exclude undesirable materials from the inside of a mechanism, or they hold critical lubrication or cooling fluids inside the housing of the mechanism.

Look at machines that you interact with each day, and identify parts that fit the descriptions given here. Look in the engine compartment of a car or a truck. How are drive pulleys, linkages, latches, the hinges for the hood, the fan, the windshield wipers, and any other moving part connected to something else—the frame of the car, a rotating shaft, or some other moving part? If you are familiar with the inner workings of the engine and transmission, describe how those parts are connected. Look at the steering and suspension systems, the water pump, the fuel pump, the brake fluid reservoir, and the suspension struts or shock absorbers. Try to see where seals are used. Can you see the universal joints, sometimes called the *constant-velocity* (CV) joints, in the drive train? They should be connecting the output shaft from the transmission to the final parts of the drive train as the power is delivered to the wheels.

Find a small lawn or garden tractor at home or at a local home store. Typically their mechanisms are accessible, although protected from casual contact for safety reasons. Trace how power is transmitted from the engine, through a transmission, through a drive chain or belt, and all the way to the wheels or to the blade of a mower. How are functional parts connected together?

Look at home appliances, power tools in a home shop, and gardening equipment. Can you see parts that are held in place with *retaining rings?* These are typically thin, flat rings that are pressed onto shafts or inserted into grooves to hold a wheel on a shaft, to hold a gear or a pulley in position along the length of a shaft, or to simply hold some part of the device in place.

How are the keys, couplings, seals, retaining rings, and other connecting devices made? What materials are used? How are they installed? Can they be removed? What kinds of forces must they resist? How does their special geometry accomplish the desired function? How could they fail?

This chapter will help you become familiar with such mechanical components, with some of the manufacturers that offer commercial versions, and with correct application methods.

YOU ARE THE DESIGNER

In the first part of Chapter 8, you were the designer of a gear-type speed reducer whose conceptual design is shown in Figure 1–12. It has four gears, three shafts, and six bearings, all contained within a housing. How are the gears attached to the shafts? One way is to use keys at the interface between the hub of the gears and the shaft. You should be able to design the keys. How is the input shaft connected to the motor or engine that delivers the power? How is the output shaft connected to the driven machine? One way is to use flexible couplings. You should be able to specify commercially available couplings and apply them properly, considering the amount of torque they must transmit and how much misalignment they should permit.

How are the gears located axially along the shafts? Part of this function may be provided by shoulders machined on the shaft. But that works only on one side. On the other side, one type of locating means is a retaining ring that is installed into a groove in the shaft after the gear is in place. Rings or spacers may be used at the left of gears *A* and *B* and at the right of gears *C* and *D*. Notice that the input and output shafts extend outside the housing. How can you keep contaminants from the outside from getting inside? How can you keep lubricating oil inside? Shaft seals can accomplish that function. Seals may also be provided on the bearings to retain lubricant inside and in full contact with the rotating balls or rollers of the bearing.

You should be familiar with the kinds of materials used for seals and with their special geometries. These concepts are discussed in this chapter.

11–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will be able to:

1. Describe several kinds of *keys*.
2. Specify a suitable size key for a given size shaft.
3. Specify suitable materials for keys.
4. Complete the design of keys and the corresponding keyways and keyseats, giving their complete geometries.
5. Describe *splines* and determine their torque capacity.
6. Describe several alternate methods of fastening machine elements to shafts.
7. Describe *rigid couplings* and *flexible couplings*.
8. Describe several types of flexible couplings.
9. Describe *universal joints*.
10. Describe *retaining rings* and other means of locating elements on shafts.
11. Specify suitable seals for shafts and other types of machine elements.

11–2 KEYS

A *key* is a machinery component placed at the interface between a shaft and the hub of a power-transmitting element for the purpose of transmitting torque [see Figure 11–1(a)]. The key is demountable to facilitate assembly and disassembly of the shaft system. It is installed in an axial groove machined into the shaft, called a *keyseat*.

A similar groove in the hub of the power-transmitting element is usually called a *keyway*, but it is more properly also a keyseat. The key is typically installed into the shaft keyseat first; then the hub keyseat is aligned with the key, and the hub is slid into position.

Square and Rectangular Parallel Keys

The most common type of key for shafts up to $6\frac{1}{2}$ in in diameter is the square key, as illustrated in Figure 11–1(b). The rectangular key, Figure 11–1(c), is recommended for larger shafts and is used for smaller shafts where the shorter height can be tolerated. Both the square and the rectangular keys are referred to as *parallel keys* because the top and bottom and the sides of the key are parallel. (See Internet sites 1 and 20.)

Table 11–1 gives the preferred dimensions for parallel keys as a function of shaft diameter for both U.S. sizes and SI Metric sizes. The width is approximately one-quarter of the diameter of the shaft. See References 7 and 9 for more detailed dimensions and tolerances.

The keyseats in the shaft and the hub are designed so that exactly one-half of the height of the key is bearing on the side of the shaft keyseat and the other half on the side of the hub keyseat. Figure 11–2 shows the resulting geometry. The distance Y is the radial distance from the theoretical top of the shaft, before the keyseat is machined, to the top edge of the finished keyseat to produce a keyseat depth of exactly $H/2$. To assist in machining and inspecting the shaft or the hub, the dimensions S and T can be computed and shown on the part drawings. The equations are given in Figure 11–2. Tabulated values of Y, S, and T are available in References 7 and 9.

As discussed later in Chapter 12, keyseats in shafts are usually machined with either an end mill or a circular milling cutter, producing the profile or sled runner keyseat, respectively (refer Figure 12–7). In general practice, the keyseats and keys are left with essentially square ends and edges. But radiused keyseats and chamfered keys can be used to reduce the stress concentrations. Table 11–2 shows suggested values from ANSI Standard B17.1.

As alternates to the use of parallel keys, taper keys, gib head keys, pin keys, and Woodruff keys can be used to provide special features of installation or operation. Figure 11–3 shows the general geometry of these types of keys.

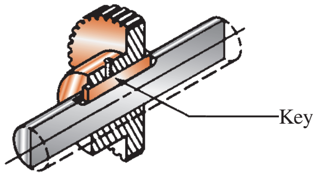
Taper Keys and Gib Head Keys

Taper keys [Figure 11–3(a) and (b)] are designed to be inserted from the end of the shaft after the hub is in position rather than installing the key first and then sliding the hub over the key as with parallel keys. The taper extends over at least the length of the hub, and the height, H, measured at the end of the hub, is the same as for the parallel key. The taper is typically 1/8 in per foot. Note that this design gives a smaller bearing area on the sides of the key, and the bearing stress must be checked.

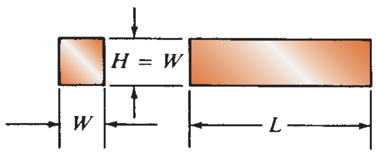
The *gib head key* [Figure 11–3(c)] has a tapered geometry inside the hub that is the same as that of the plain taper key. But the extended head provides the means of extracting the key from the same end at which it was installed. This is very desirable if the opposite end is not accessible to drive the key out.

Pin Keys

The *pin key*, shown in Figure 11–3(d), is a cylindrical pin placed in a cylindrical groove in the shaft and hub. Lower stress concentration factors result from this design as compared with parallel or taper keys. A close

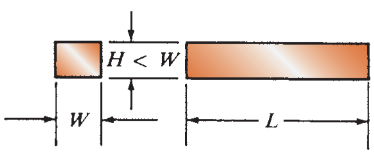


(*a*) Key and keyseat applied to a gear and shaft



(*b*) Square key

(*d*) Commercially available keys



(*c*) Rectangular key

FIGURE 11–1 Parallel keys

TABLE 11–1 Key Size vs. Shaft Diameter

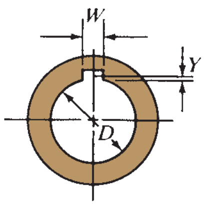
U.S. inch sizes				SI metric sizes			
Nominal shaft diameter		Key dimensions		Nominal shaft diameter		Key dimensions	
Over (in)	to-including (in)	Width, *W* (in)	Height, *H* (in)	Over (mm)	to-including (mm)	Width, *W* (mm)	Height, *H* (mm)
0.3125	0.4375	0.09375	0.09375	6	8	2	2
0.4375	0.5625	0.1250	0.1250	8	10	3	3
0.5625	0.875	0.1875	0.1875	10	12	4	4
0.875	1.250	0.2500	0.2500	12	17	5	5
1.250	1.375	0.3125	0.3125	17	22	6	6
1.375	1.75	0.375	0.375	22	30	8	7
1.75	2.25	0.500	0.500	30	38	10	8
2.25	2.75	0.625	0.625	38	44	12	8
2.75	3.25	0.750	0.750	44	50	14	9
3.25	3.75	0.875	0.875	50	58	16	10
3.75	4.50	1.00	1.00	58	65	18	11
4.50	5.50	1.25	1.25	65	75	20	12
5.50	6.50	1.50	1.50	75	85	22	14
6.50	7.50	1.75	1.50	85	95	25	14
7.50	9.00	2.00	1.50	95	110	28	16
9.00	11.00	2.50	1.75	110	130	32	18
11.00	13.00	3.00	2.00	130	150	36	20
13.00	15.00	3.50	2.50	150	170	40	22
15.00	18.00	4.00	3.00	170	200	45	25
18.00	22.00	5.00	3.50	200	230	50	28
22.00	26.00	6.00	4.00	230	260	56	32
26.00	30.00	7.00	5.00	260	290	63	32
				290	330	70	36
				330	380	80	40
				380	440	90	45
				440	500	100	50

Note: Key sizes above the horizontal line are square; others are rectangular.

fit between the pin and the groove is required to ensure that the pin does not move and that the bearing is uniform along the length of the pin.

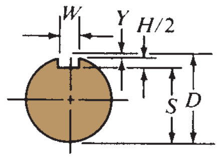
Woodruff Keys

Where light loading and relatively easy assembly and disassembly are desired, the *Woodruff key* should be considered. Figure 11–3(e) shows the standard configuration. The circular groove in the shaft holds the key in position while the mating part is slid over the key. The stress analysis for this type of key proceeds in the manner discussed for the parallel key, taking into consideration the particular geometry of the Woodruff key. ANSI Standard B17.2–1967 lists the dimensions for a large number of standard Woodruff keys and their mating keyseats. (See Reference 8.) Table 11–3 provides a sampling. Note that the *key number* indicates the nominal key dimensions. The last two digits give the nominal diameter, *B*, in eighths of an inch, and the digits preceding the last two give the nominal width, *W*, in thirty-seconds of an inch. For example, key number 1210 has a diameter of 10/8 in ($1\frac{1}{4}$ in), and a width of 12/32 in (3/8 in). The actual size of the key is slightly smaller than half of the full circle, as shown in dimensions *C* and *F* in Table 11–3.



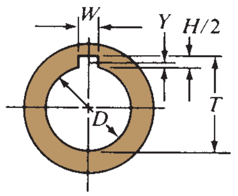
(*a*) Chordal height

$$Y = \frac{D - \sqrt{D^2 - W^2}}{2}$$



(*b*) Depth of shaft keyseat

$$S = D - Y - \frac{H}{2} = \frac{D - H + \sqrt{D^2 - W^2}}{2}$$



(*c*) Depth of hub keyseat

$$T = D - Y + \frac{H}{2} + C = \frac{D + H + \sqrt{D^2 - W^2}}{2} + C$$

Symbols
- C = Allowance
  - $+0.005$-in clearance for parallel keys
  - -0.020-in interference for taper keys
- D = Nominal shaft or bore dia., in
- H = Nominal key height, in
- W = Nominal key width, in
- Y = Chordal height, in

FIGURE 11–2 Dimensions for parallel keyseats

TABLE 11–2 Suggested Fillet Radii and Key Chamfers

H/2, keyseat depth			
Over	**To (incl.)**	**Fillet radius**	**45° chamfer**
1/8	1/4	1/32	3/64
1/4	1/2	1/16	5/64
1/2	7/8	1/8	5/32
7/8	$1\frac{1}{4}$	3/16	7/32
$1\frac{1}{4}$	$1\frac{3}{4}$	1/4	9/32
$1\frac{3}{4}$	$2\frac{1}{2}$	3/8	13/32

Note: All dimensions are given in inches.

Selection and Installation of Keys and Keyseats

The key and the keyseat for a particular application are usually designed after the shaft diameter is specified by the methods of Chapter 12. Then, with the shaft diameter as a guide, the size of the key is selected from Table 11–1. The only remaining variables are the length of the key and its material. One of these can be specified, and the requirements for the other can then be computed.

Typically the length of a key is specified to be a substantial portion of the hub length of the element in which it is installed to provide for good alignment and stable operation. But if the keyseat in the shaft is to be in the vicinity of other geometric changes, such as shoulder fillets and ring grooves, it is important to provide some axial clearance between them so that the effects of the stress concentrations are not compounded.

The key can be cut off square at its ends or provided with a radius at each end when installed in a profile

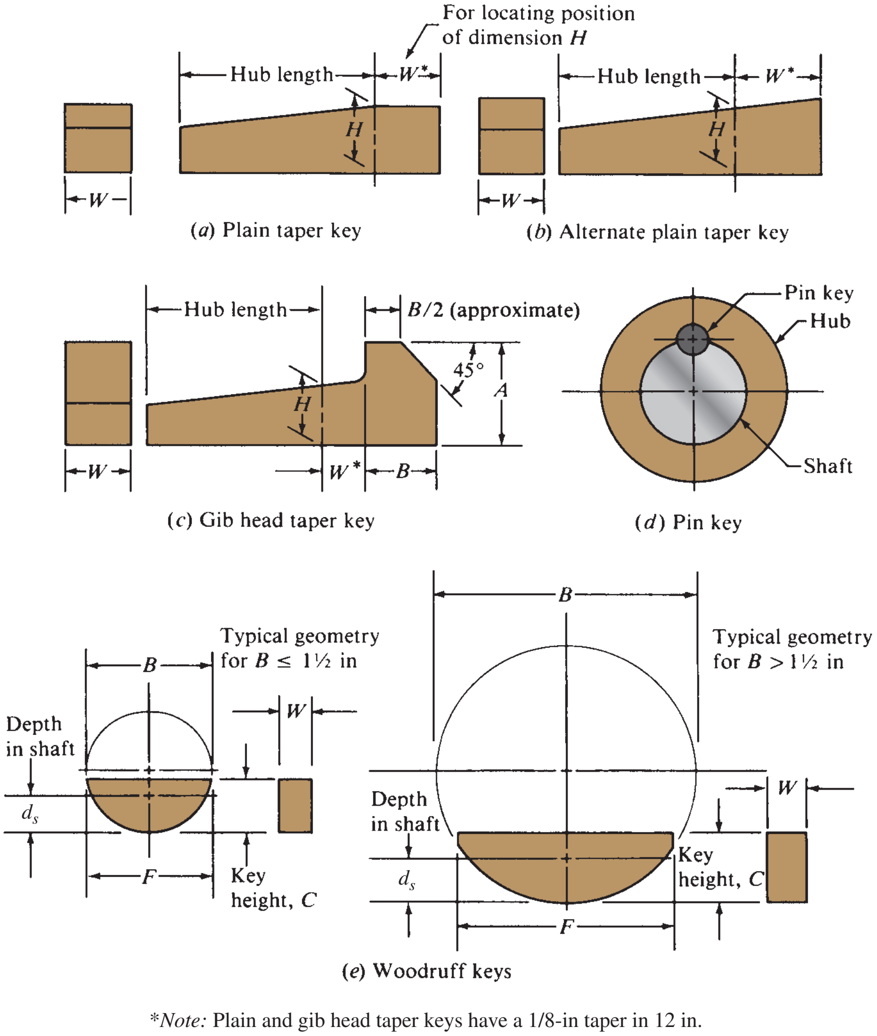


FIGURE 11–3 Key types

TABLE 11–3 Woodruff Key Dimensions

Key number	Nominal key size, $W \times B$	Actual length, F	Height of key, C	Shaft keyseat depth	Hub keyseat depth
202	$1/16 \times 1/4$	0.248	0.104	0.0728	0.0372
204	$1/16 \times 1/2$	0.491	0.200	0.1668	0.0372
406	$1/8 \times 3/4$	0.740	0.310	0.2455	0.0685
608	$3/16 \times 1$	0.992	0.435	0.3393	0.0997
810	$1/4 \times 1\frac{1}{4}$	1.240	0.544	0.4170	0.1310
1210	$3/8 \times 1\frac{1}{4}$	1.240	0.544	0.3545	0.1935
1628	$1/2 \times 3\frac{1}{2}$	2.880	0.935	0.6830	0.2560
2428	$3/4 \times 3\frac{1}{2}$	2.880	0.935	0.5580	0.3810

Note: All dimensions are given in inches.

keyseat to improve location. Square-cut keys are usually used with the sled-runner-type keyseat.

The key is sometimes held in position with a set screw in the hub over the key. However, the reliability of this approach is questionable because of the possibility of the set screw's backing out with vibration of the assembly. Axial location of the assembly should be provided by more positive means, such as shoulders, retaining rings, or spacers.

11–3 MATERIALS FOR KEYS

Keys are made from plain carbon steel, alloy steels, stainless steels, and some nonferrous metals. Even plastics are used for small devices under low loads. This book will focus primarily on steels for general industrial applications. Table 11–4 lists a variety of steels and one aluminum alloy to illustrate the types of materials available. (See Internet site 20.)

For problem solutions in this book, it is recommended that the low-carbon steel SAE 1018 be considered for most applications. It is a low-cost, readily available material and its strength is generally adequate.

If higher strength is necessary to produce a design with a reasonable length, medium-carbon SAE 1035 or 1045, or alloy steels SAE 4140 or 8630 are recommended. The high-carbon steel SAE 1095 may be used but it may have low ductility.

Where corrosion resistance is necessary, the stainless steels listed in Table 11–4 may be considered. Aluminum 6061 is less frequently used for keys but may be desirable for material compatibility reasons.

11–4 STRESS ANALYSIS TO DETERMINE KEY LENGTH

There are two basic modes of potential failure for keys transmitting power: (1) shear across the shaft/hub interface and (2) compression failure due to the bearing action between the sides of the key and the shaft or hub material. The analysis for either failure mode requires an understanding of the forces that act on the key. Figure 11–4 shows the idealized case in which the torque on the shaft creates a force on the left side of the key. The key in turn exerts a force on the right side of the hub keyseat. The reaction force of the hub back on the key then produces a set of opposing forces that place the key in direct shear over its cross section, $W \times L$. The magnitude of the shearing force can be found from

$$F = T/(D/2)$$

The shearing stress is then

$$\tau = \frac{F}{A_s} = \frac{T}{(D/2)(WL)} = \frac{2T}{DWL} \qquad \textbf{(11–1)}$$

In design, we can set the shearing stress equal to a design stress in shear for the maximum shear stress theory of failure:

$$\tau_d = 0.5s_y/N$$

Then the required length of the key is

➯ Minimum Required Key Length for Shear

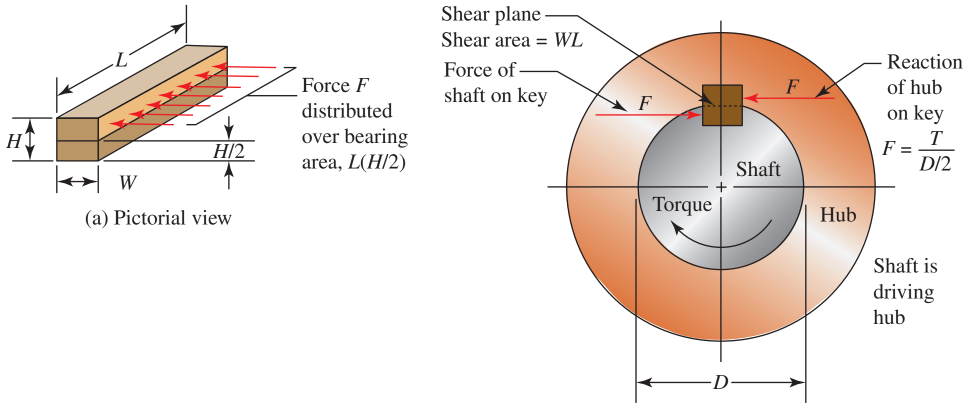
$$L_{\min} = \frac{2T}{\tau_d DW} \qquad \textbf{(11–2)}$$

TABLE 11–4 Examples of Materials Used for Keys

Material designation	Tensile strength s_u (ksi)	Tensile strength s_u (MPa)	Yield strength s_y (ksi)	Yield strength s_y (MPa)
Carbon steels (SAE)				
1018	64	441	54	372
1035	72	496	39.5	272
1045	91	627	77	531
1095	140	965	83	572
Alloy steels (SAE)				
4140	102	703	90	621
8630	100	690	95	655
Stainless steels (SAE)				
303	90	621	35	241
304	85	586	35	241
316	85	586	35	241
416	75	517	40	276
Aluminum				
6061	18	124	12	83

Source: Adapted from Internet site 20.

Note: Strength properties typical, not guaranteed.



(a) Pictorial view

(b) Side view

FIGURE 11–4 Forces on a key

The failure in bearing is related to the compressive stress on the side of the key, the side of the shaft keyseat, or the side of the hub keyseat. The area in compression is the same for either of these zones, $L \times (H/2)$. Thus, the failure occurs on the surface with the lowest compressive yield strength. Let's define a *design stress for compression* as

$$\sigma_d = s_y/N$$

Then the compressive stress is

$$\sigma = \frac{F}{A_c} = \frac{T}{(D/2)(L)(H/2)} = \frac{4T}{DLH} \qquad \textbf{(11–3)}$$

Letting this stress equal the design compressive stress allows the computation of the required length of the key for this mode of failure:

➩ Minimum Required Key Length for Compression

$$L_{\min} = \frac{4T}{\sigma_d DH} \qquad \textbf{(11–4)}$$

For the design of a square key in which the strength of the key material is lower than that of the shaft or the hub, Equations (11–2) and (11–4) produce the same result. Substituting the design stress into either equation would give

➩ Minimum Required Key Length if Key Material Is Weakest

$$L_{\min} = \frac{4TN}{DWs_y} \qquad \textbf{(11–5)}$$

But be sure to evaluate the length from Equation (11–4) if either the shaft or the hub has a lower yield strength than the key.

DESIGN PROCEDURE FOR PARALLEL KEYS

1. Complete the design of the shaft into which the key will be installed, and specify the actual diameter at the location of the keyseat.
2. Select the size of the key from Table 11–1.
3. Specify a suitable design factor, N. In typical industrial applications, $N = 3$ is adequate to accommodate accidental overloads and shock.
4. Specify the material for the key, usually SAE 1018 steel. A higher-strength material can be used.
5. Determine the yield strength of the materials for the key, the shaft, and the hub.
6. If a square key is used and the key material has the lowest strength, use Equation (11–5) to compute the minimum required length of the key. This length will be satisfactory for both shear and bearing stress.
7. If a rectangular key is used, or if either the shaft or the hub has a lower strength than the key, use Equation (11–4) to compute the minimum required length of the key based on bearing stress. Also, use Equation (11–2) or (11–5) to compute the minimum required length based on shear of the key. The larger of the two computed lengths governs the design. Check to be sure that the computed length is shorter than the hub length. If not, a higher-strength material must be selected and the design process repeated. Alternatively, two keys or a spline can be used instead of a single key.
8. Specify the actual length of the key to be equal to or longer than the computed minimum length. A convenient standard size should be specified using the preferred basic sizes shown in Appendix A2–1. ***The key should extend over all or a substantial part of the length of the hub. But the keyseat should not run into other stress raisers such as shoulders or grooves.***
9. Complete the design of the keyseat in the shaft and the keyway in the hub using the equations in Figure 11–2. ANSI Standard B17.1 should be consulted for standard tolerances on dimensions for the key and the keyseats.
10. See also Chapter 15 for additional details concerning tolerancing and chamfering.

Example Problem 11–1 A portion of a shaft where a gear is to be mounted has a diameter of 2.00 in. The gear transmits 2965 lb · in of torque. The shaft is to be made of SAE 1040 cold-drawn steel. The gear is made from SAE 8650 OQT 1000 steel. The width of the hub of the gear mounted at this location is 1.75 in. Design the key.

Solution From Table 11–1, the standard key dimension for a 2.00-in-diameter shaft would be 0.500-in square. See Figure 11–5 for the proposed design.

Material selection is a design decision. Let's choose SAE 1018 steel with $s_y = 54\,000$ psi as listed in Table 11–4.

A check of the yield strengths of the three materials in the key, the shaft, and the hub indicates that the key is the weakest material. Then Equation (11–5) can be used to compute the minimum required length of the key:

$$L = \frac{4TN}{DWs_y} = \frac{4(2965\text{ lb}\cdot\text{in})(3)}{(2.00\text{ in})(0.500\text{ in})(54\,000\text{ lb/in}^2)} = 0.659\text{ in}$$

This length is well below the width of the hub of the gear. Notice that the design of the shaft includes retaining rings on both sides of the gear. It is desirable to keep the keyseat well clear of the ring grooves. Therefore, let's specify the length of the key to be 1.50 in.

Summary In summary, the key has the following characteristics:

Material: SAE 1018 steel keystock
Width: 0.500 in
Height: 0.500 in
Length: 1.50 in

Figure 11–5 shows some details of the completed design. A profile keyseat in the shaft is shown.

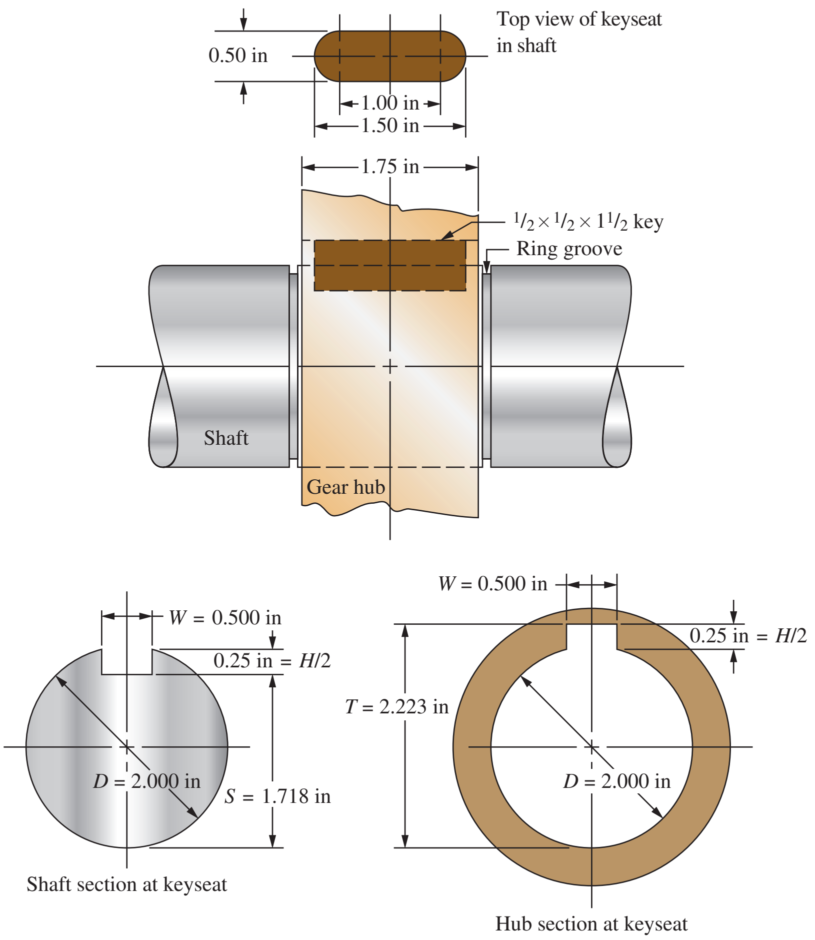


FIGURE 11–5 Details for proposed design of key and keyseats

Shear and Bearing Areas for Woodruff Keys

The geometry of Woodruff keys makes it more difficult to determine the shear area and the bearing area for use in stress analyses. Figure 11–3(e) shows that the bearing area on the side of the key in the keyseat is a segment of a circle. The shear area is the product of the chord of that segment times the thickness of the key. The following equations describe the geometry:

Given

B = nominal diameter of the cylinder of which the key is a part

W = width (thickness) of the key

C = full height of the key

d_s = depth of the keyseat in the shaft

Results

$$\text{Shear area} = A_s = 2W\sqrt{d_s(B - d_s)} \quad \textbf{(11–6)}$$

To define the equations for the bearing areas on the side of the key in the shaft and in the hub, we first define three geometric variables, G, L, and J, as follows:

$$G = (\pi/180)B\cos^{-1}\{2[(B/2) - d_s]/B\}$$

$$L = 2\sqrt{d_s(B - d_s)}$$

$$J = (\pi/180)B\cos^{-1}\{2[(B/2) - C]/B\}$$

Bearing Area in Shaft

$$A_{c\ \text{shaft}} = 0.5\{G(B/2) - L[(B/2) - d_s]\} \quad \textbf{(11–7)}$$

$$A_{c\ \text{hub}} = 0.5\{J(B/2) - F[(B/2) - C]\} - A_{c\ \text{shaft}} \quad \textbf{(11–8)}$$

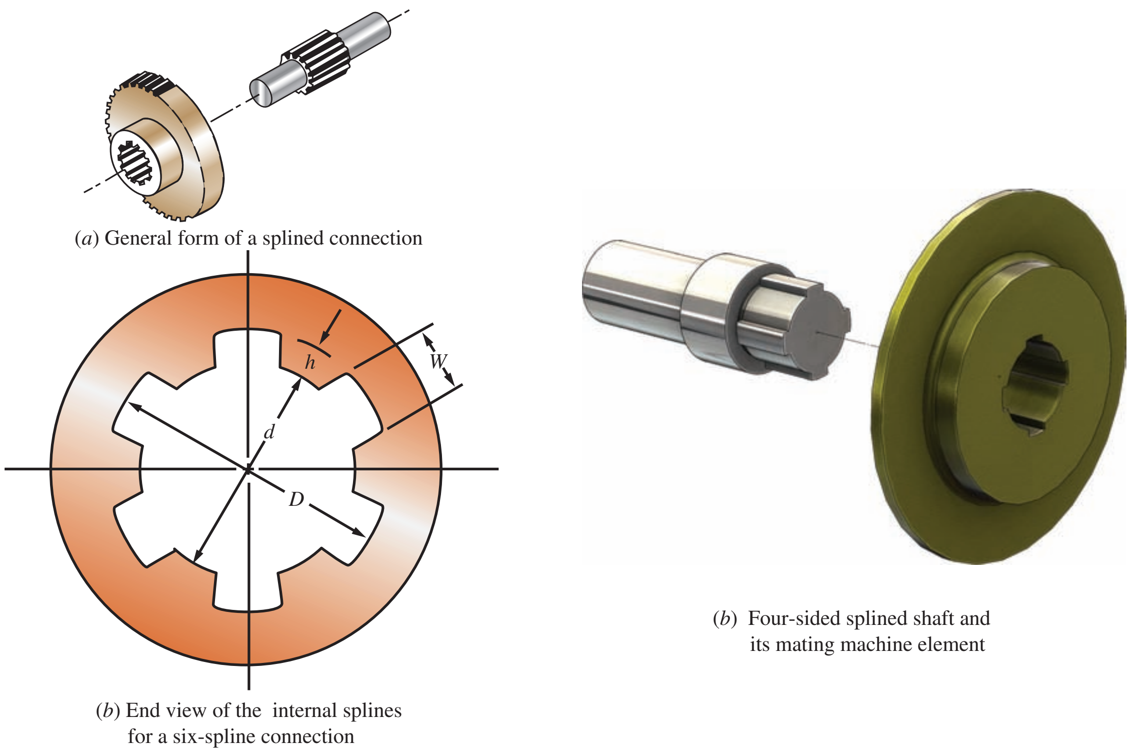
11–5 SPLINES

A *spline* can be described as a series of axial keys machined into a shaft, with corresponding grooves machined into the bore of the mating part (gear, sheave, sprocket, and so on; see Figure 11–6). The splines perform the same function as a key in transmitting torque from the shaft to the mating element. The advantages of splines over keys are many. Because usually four or more splines are used, as compared with one or two keys, a more uniform transfer of the torque and a lower loading on a given part of the shaft/hub interface result. The splines are integral with the shaft, so no relative motion can occur as between a key and the shaft. Splines are accurately machined to provide a controlled fit between the mating internal and external splines. The surface of the spline is often hardened to resist wear and to facilitate its use in applications in which axial motion of the mating element is desired. Sliding motion between a standard parallel key and the mating element should not be permitted. Because of the multiple splines on the shaft, the mating element can be indexed to various positions.

Splines can be either straight-sided or involute. The involute form is preferred because it provides for self-centering of the mating element and because it can be machined with standard hobs used to cut gear teeth.

Straight-Sided Splines

Straight splines are made according to the specifications of the Society of Automotive Engineers (SAE) and usually contain 4, 6, 10, or 16 splines. Figure 11–6 shows three styles. The six-spline version shows the basic design



(*a*) General form of a splined connection

(*b*) End view of the internal splines for a six-spline connection

(*b*) Four-sided splined shaft and its mating machine element

FIGURE 11–6 Straight-sided splines with different numbers of splines

TABLE 11–5 Formulas for SAE Straight Splines

Number of splines	*W*, for all fits	A: Permanent fit		B: To slide without load		C: To slide under load	
		h	*d*	*h*	*d*	*h*	*d*
Four	0.241*D*	0.075*D*	0.850*D*	0.125*D*	0.750*D*		
Six	0.250*D*	0.050*D*	0.900*D*	0.075*D*	0.850*D*	0.100*D*	0.800*D*
Ten	0.156*D*	0.045*D*	0.910*D*	0.070*D*	0.860*D*	0.095*D*	0.810*D*
Sixteen	0.098*D*	0.045*D*	0.910*D*	0.070*D*	0.860*D*	0.095*D*	0.810*D*

Note: These formulas give the maximum dimensions for *W, h,* and *d.*

parameters of D (major diameter), d (minor diameter), W (spline width), and h (spline depth). The dimensions for d, W, and h are related to the nominal major diameter D by the formulas given in Table 11–5. Note that the values of h and d differ according to the use of the spline. The permanent fit, A, is used when the mating part is not to be moved after installation. The B fit is used if the mating part will be moved along the shaft without a torque load. When the mating part must be moved under load, the C fit is used.

The torque capacity for SAE splines is based on the limit of 1000-psi bearing stress on the sides of the splines, from which the following formula is derived:

➩ Torque Capacity for a Spline

$$T = 1000NRh \tag{11–9}$$

where N = number of splines
R = mean radius of the splines
h = depth of the splines (from Table 11–5)

The torque capacity is per inch of length of the spline. But note that

$$R = \frac{1}{2}\left[\frac{D}{2} + \frac{d}{2}\right] = \frac{D + d}{4}$$

$$h = \frac{1}{2}(D - d)$$

Then

$$T = 1000N\frac{(D + d)}{4}\frac{(D - d)}{2} = 1000N\frac{(D^2 - d^2)}{8} \tag{11–10}$$

This equation can be further refined for each of the types of splines in Table 11–5 by substitution of the appropriate relationships for N and d. For example, for the six-spline version and the B fit, $N = 6$, $d = 0.850D$, and $d^2 = 0.7225D^2$.
Then

$$T = 1000(6)\frac{[D^2 - 0.7225D^2]}{8} = 208D^2$$

Thus, the required diameter to transmit a given torque would be

➩ Required Spline Diameter for a given Torque

$$D = \sqrt{T/208}$$

In these formulas, dimensions are in inches and the torque is in pound-inches. We use this same approach to find the torque capacities and required diameters for the other versions of straight splines (Table 11–6).

The graphs in Figure 11–7 enable you to choose an acceptable diameter for a straight-sided spline to carry a given torque, depending on the desired fit, A, B, or C. The data were taken from Table 11–6.

Involute Splines

Involute splines are typically made with pressure angles of 30°, 37.5°, or 45°. The 30° form is illustrated in Figure 11–8, showing the two types of fit that can be

TABLE 11–6 Torque Capacity for Straight Splines per Inch of Spline Length

Number of splines	Fit	Torque capacity	Required diameter
4	*A*	$139D^2$	$\sqrt{T/139}$
4	*B*	$219D^2$	$\sqrt{T/219}$
6	*A*	$143D^2$	$\sqrt{T/143}$
6	*B*	$208D^2$	$\sqrt{T/208}$
6	*C*	$270D^2$	$\sqrt{T/270}$
10	*A*	$215D^2$	$\sqrt{T/215}$
10	*B*	$326D^2$	$\sqrt{T/326}$
10	*C*	$430D^2$	$\sqrt{T/430}$
16	*A*	$344D^2$	$\sqrt{T/344}$
16	*B*	$521D^2$	$\sqrt{T/521}$
16	*C*	$688D^2$	$\sqrt{T/688}$

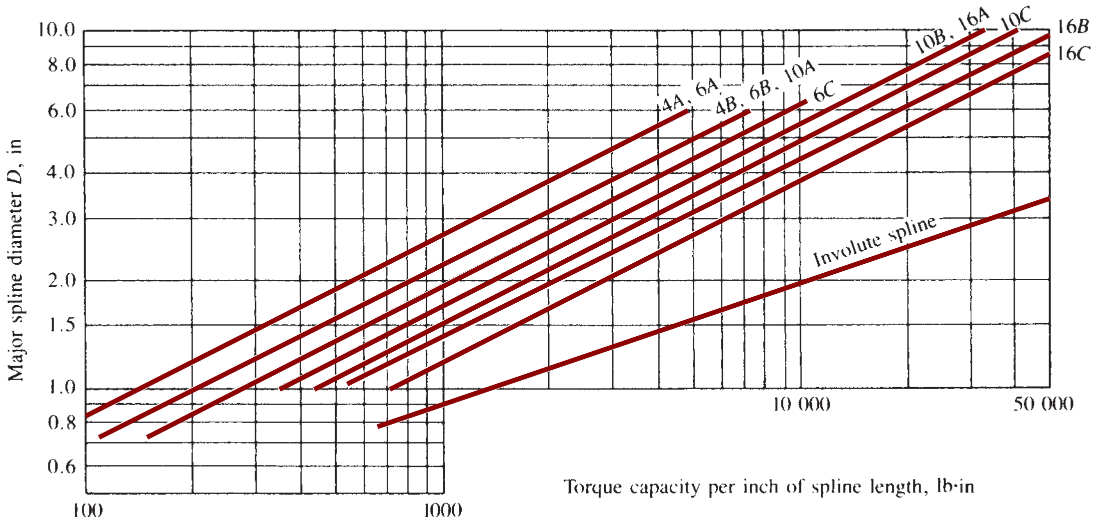
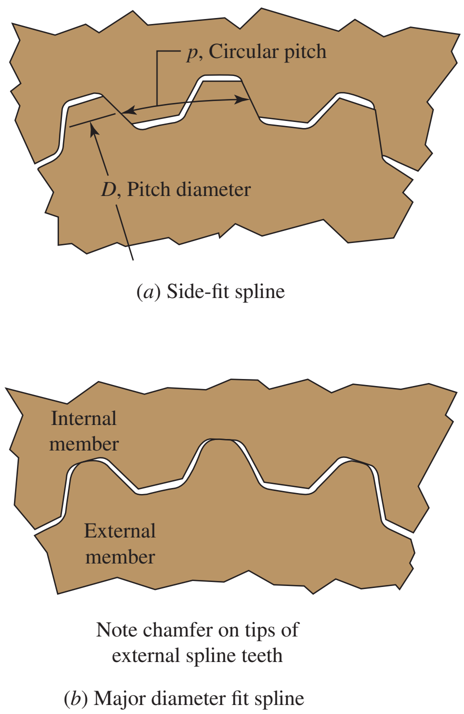


FIGURE 11–7 Torque capacity per inch of spline length, lb · in for straight-sided splines



N = Number of spline teeth
P = Diametral pitch
$D = N/P$ = Pitch diameter
$p = \pi/p$ = Circular pitch

Minor diameter:

Internal: $\dfrac{N-1}{P}$

External: $\dfrac{N-1.35}{P}$

Major dia.:

Internal: $\dfrac{N+1.35}{P}$ side fit

$\dfrac{N+1}{P}$ major diameter fit

External: $\dfrac{N+1}{P}$

FIGURE 11–8 30° involute spline

specified. The *major diameter fit* produces accurate concentricity between the shaft and the mating element. In the *side fit,* contact occurs only on the sides of the teeth, but the involute form tends to center the shaft in the mating splined hub.

Figure 11–8 also gives some of the basic formulas for key features of involute splines in the U.S. Customary Unit System with dimensions in inches. (See Reference 5.) The terms are similar to those for involute spur gears, which are discussed more completely in Chapter 8. The basic spline size is governed by its *diametral pitch, P:*

$$P = N/D \tag{11–11}$$

where N = number of spline teeth
D = pitch diameter

The diametral pitch, then, is *the number of teeth per inch of pitch diameter.* Only even numbers of teeth from 6 to 60 are typically used. Up to 100 teeth are used on some 45° splines. Note that the pitch diameter lies *within* the tooth and is related to the major and minor diameters by the relationships shown in Figure 11–8.

The *circular pitch, p,* is the distance from one point on a tooth to the corresponding point on the next adjacent tooth, measured along the pitch circle. To find the nominal value of p, divide the circumference of the pitch circle by the number of spline teeth. That is,

$$p = \pi D/N \tag{11–12}$$

But because $P = N/D$, we can also say

$$p = \pi/P \tag{11–13}$$

The *tooth thickness*, t, is the thickness of the tooth measured along the pitch circle. Then the theoretical value is

$$t = p/2 = \pi/2P$$

The nominal value of the width of the tooth space is equal to t.

Standard Diametral Pitches. The following are the 17 standard diametral pitches in common use:

2.5	3	4	5	6	8	10	12	16
20	24	32	40	48	64	80	128	

The common designation for an involute spline is given as a fraction, P/P_s, where P_s is called the *stub pitch* and is always equal to $2P$. Thus, if a spline had a diametral pitch of 4, it would be called a 4/8 pitch spline. For convenience, we will use only the diametral pitch.

Length of Splines. Common designs use spline lengths from $0.75D$ to $1.25D$, where D is the pitch diameter of the spline. If these standards are used, the shear strength of the splines will exceed that of the shaft on which they are machined.

Metric Module Splines. The dimensions of splines made to metric standards are related to the *module, m,* where

$$m = D/N \tag{11–14}$$

and both D and m are in millimeters. (See Reference 6.) Note that the symbol Z is used in place of N for the number of teeth in standards describing metric splines. Other features of metric splines can be found from the following formulas:

$$\text{Pitch diameter} = D = mN \tag{11–15}$$

$$\text{Circular pitch} = p = \pi m \tag{11–16}$$

$$\text{Basic tooth thickness} = t = \pi m/2 \tag{11–17}$$

Standard Modules. There are 15 standard modules:

0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
2.50	3	4	5	6	8	10	

Design Aids

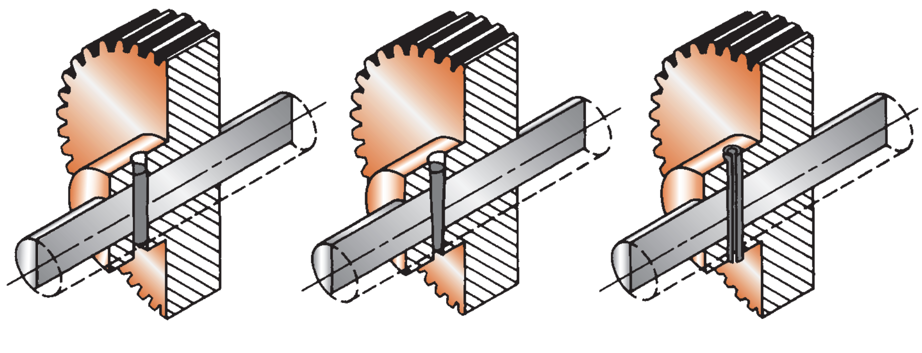
Refer References 9–11 for additional design guidelines. Reference 9 includes extensive tables of data for splines and offers application and design information. Reference 10 gives information on the application, operation, dimensioning, and manufacture of involute splines as applied to automotive applications. Data are included on allowable shear stress, allowable compressive stress, wear life factor, spline overload factor, and fatigue life factor. Reference 11 focuses on metric module splines.

11–6 OTHER METHODS OF FASTENING ELEMENTS TO SHAFTS

The following discussion will acquaint you with some of the ways in which power-transmitting elements can be attached to shafts without keys or splines. In most cases the designs have not been standardized, and analysis of individual cases, considering the forces exerted on the elements and the manner of loading of the fastening means, is necessary. In several of the designs, the analysis of shear and bearing will follow a procedure similar to that shown for keys. If a satisfactory analysis is not possible, testing of the assembly is recommended.

Pinning

With the element in position on the shaft, a hole can be drilled through both the hub and the shaft, and a pin can be inserted in the hole. Figure 11–9 shows three examples of this approach. The straight, solid, cylindrical pin is subjected to shear over two cross sections. If there is a



(*a*) Cylindrical pin (*b*) Taper pin (*c*) Spring pin

FIGURE 11–9 Pinning

force, F, on each end of the pin at the shaft/hub interface, and if the shaft diameter is D, then

$$T = 2F(D/2) = FD$$

or $F = T/D$. With the symbol d representing the pin diameter, the shear stress in the pin is

$$\tau = \frac{F}{A_s} = \frac{T}{D(\pi d^2/4)} = \frac{4T}{D(\pi d^2)} \qquad \textbf{(11–18)}$$

Letting the shear stress equal the design stress in shear as before, solving for d gives the required pin diameter:

➩ Required Diameter for a Pin

$$d = \sqrt{\frac{4T}{D(\pi)(\tau_d)}} \qquad \textbf{(11–19)}$$

Sometimes the diameter of the pin is purposely made small to ensure that the pin will break if a moderate overload is encountered, in order to protect critical parts of a mechanism. Such a pin is called a *shear pin*.

One problem with a cylindrical pin is that fitting it adequately to provide precise location of the hub and to prevent the pin from falling out is difficult. The *taper pin* overcomes some of these problems, as does the *split spring pin* shown in Figure 11–9(c). For the split spring pin, the hole is made slightly smaller than the pin diameter so that a light force is required to assemble the pin in the hole. The spring force retains the pin in the hole and holds the assembly in position. But, of course, the presence of any of the pin-type connections produces stress concentrations in the shaft. (See Internet site 1.)

Keyless Hub to Shaft Connections

Using a steel ring compressed tightly around a smooth shaft allows torque to be transmitted between the hub of a power-transmitting element and a shaft without having a key between the two elements. Figure 11–10 shows a commercially available product called a *Locking Assembly*™ from Ringfeder® Corporation that employs this principle.

The Locking Assembly™ employs steel rings with opposing mating tapers held together with a series of fasteners. With the Locking Assembly™ placed completely within a counter bore of the hub of virtually any kind of power-transmitting element such as a gear, sprocket, fan wheel, cam, coupling, or turbine rotor, the fasteners can then be tightened. Initially there is a small clearance between the inside diameter of the locking device and the shaft as well as the hub bore. This clearance facilitates easy assembly and positioning of the hub. After the hub is positioned in the desired location on the shaft, the fasteners are tightened to a specified torque in a specific sequence. As the bolts are tightened, they draw the opposing tapered rings together, generating a radial movement of the inner ring toward the shaft and a simultaneous outward movement of the outer ring toward the ID of the hub. Once the initial clearances are eliminated, further tightening of the bolts results in a high pressure against the shaft and the hub. When the bolts are properly torqued using a torque wrench, the final contact pressure combined with friction allows for the transmission of a predetermined and predictable amount of torque between the hub and the shaft.

(*a*) A variety of styles

(*b*) Locking assembly applied to a gear

FIGURE 11–10 Ringfeder® Locking Assemblies (Used by Ringfeder Power Transmission USA Corp.)

The connection can transmit axial forces in the form of thrust loads as well as torque as with helical gears, for example. Table 11–7 lists examples of torque and axial force capacities for selected sizes of one model of the Ringfeder Locking Assemblies®. All the metric data are for catalog metric sizes with capacities given for torque in kN·m and axial force in kN. Data in the upper nonshaded part for the inch-size models are from catalog data with capacities given for torque in lb·ft and axial force in lb. The shaft sizes listed are fairly close to the metric sizes in the left part of the table to illustrate comparisons. The lower shaded part shows data in U.S. units converted from the metric data for the larger sizes because the maximum standard inch-size model is for a shaft size of 7.875 in.

Unlike a thermal or pressure fit connection, the locking device can be easily removed since it is a *mechanical shrink fit*. The pressures generated within the locking device itself allow the stresses to remain within the elastic limits of the materials. Removal is simply done by carefully loosening the screws, thus allowing the rings to slide apart and return the Locking Assembly™ to its original relaxed condition. The element can then be repositioned or removed at any time. Some locking devices have self-releasing taper angles, which allow them to self-release when the fasteners are loosened. Other locking devices have self-locking tapers that require the gentle pressing

TABLE 11–7 Capacities of Selected Sizes of Ringfeder Locking Assemblies

Metric sizes			Inch sizes		
Shaft size (mm)	Transmissible torque (kN · m)	Transmissible axial force (kN)	Shaft size (in)	Transmissible torque (lb · ft)	Transmissible axial force (lb)
25	0.460	30.0	1.000	337	8088
50	2.07	80.0	2.000	1808	21 696
75	5.81	160	3.000	4332	34 656
100	11.1	220	4.000	8489	50 934
150	28.0	380	6.000	22 762	91 048
200	63.5	640	7.875	46 707	142 345
300	183	1220	11.8	134 987	274 281
400	384	1920	15.7	283 252	431 655
600	896	2989	23.6	660 921	671 987
800	1550	3876	31.5	1 143 334	871 402
1000	2375	4749	39.4	1 751 145	1 067 670
			Converted from metric data		

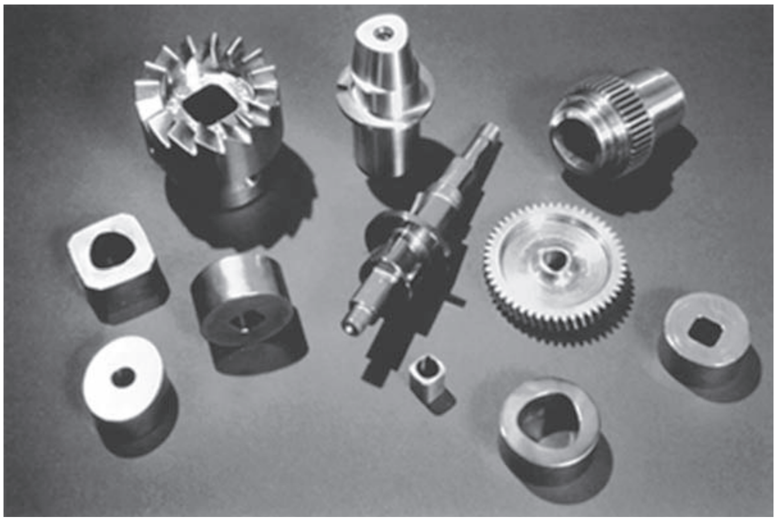
Note: All unshaded data are from catalog listings.

apart of the locking device parts. Different applications dictate which type of device is better suited.

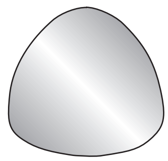
Advantages of the keyless connection are the elimination of keys, keyways, or splines, and the cost of machining them; tight fit of the driving element around the shaft; the ability to transmit reversing or dynamically changing loads; and easy assembly, disassembly, and adjustment of the elements. General engineering information is provided on Internet site 2 for Ringfeder® Corporation for installation dimensions, tolerances, lubrication, and surface finishes required. The rated torque values are for use on solid shafts; hollow shafts require additional analysis. Special considerations are necessary for the hub design to ensure that stresses remain below the yield strength of the hub material.

Polygon Hub to Shaft Connection

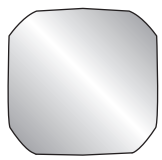
Figure 11–11 shows a shaft to hub connection that employs special mating polygon shapes to transmit torque without keys or splines. See Internet site 18 for available sizes and application information. German standards DIN 32711 and 32712 describe the forms. They can be produced on shaft sizes from 0.188 in



Polygon profile produced in a variety of products



Three-sided P3 external profile



Four-sided PC4 external profile

FIGURE 11–11 Polygon hub to shaft connections (Used by General Polygon Systems, Inc.)

(4.76 mm) to 8.00 in (203 mm). The three-sided configuration is called the P3 profile, and the four-sided design is called the PC4 profile. CNC turning and grinding can be used to produce the external form while broaching typically produces the internal form. Torque is transmitted by distributing the load on each side of the polygon, eliminating the shearing action inherent with keys or splines. Dimensions can be controlled with tight or press fits for precision, backlash-free location or with a sliding fit for ease of assembly.

Split Taper Bushing

A *split taper bushing* (see Figure 11–12) uses a key to transmit torque. Axial location on the shaft is provided by the clamping action of a split bushing having a small taper on its outer surface. When the bushing is pulled into a mating hub with a set of capscrews, the bushing is brought into tight contact with the shaft to hold the assembly in the proper axial position. The small taper locks the assembly together. Removal of the bushing is accomplished by removing the capscrews and using them in push-off holes to force the hub off the taper. The assembly can then be easily disassembled.

Set Screws

A *set screw* is a threaded fastener driven radially through a hub to bear on the outer surface of a shaft (see Figure 11–13). The point of the set screw is flat, oval, cone-shaped, cupped, or any of several proprietary forms. The point bears on the shaft or digs slightly into its surface. Thus, the set screw transmits torque by the friction between the point and the shaft or by the resistance of the material in shear. The capacity for torque transmission is somewhat variable, depending on the hardness of the shaft material and the clamping force created when the screw is installed. Furthermore, the screw may loosen during operation because of vibration. For these reasons, set screws should be used with care. Some manufacturers provide set screws with plastic inserts in the side among the threads. When the set screw is screwed into a tapped hole, the plastic is deformed by the threads and holds the screw securely, resisting vibration. Using a liquid adhesive also helps resist loosening. (See Internet site 21.)

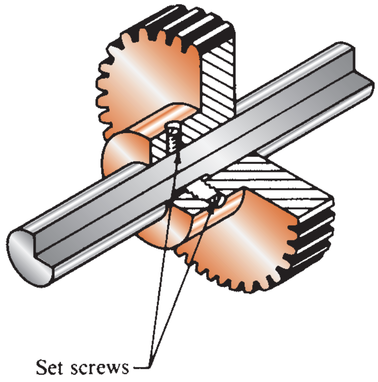


FIGURE 11–13 Set screws

Another problem with using set screws is that the shaft surface is damaged by the point; this damage may make disassembly difficult. Machining a flat on the surface of the shaft may help reduce the problem and also produce a more consistent assembly.

When set screws are properly assembled on typical industrial shafting, their force capability is approximately as follows (see Reference 9):

Screw Diameter (in)	*Holding Force (lb)*
1/4	100
3/8	250
1/2	500
3/4	1300
1	2500

Taper and Screw

The power-transmitting element (gear, sheave, sprocket, or other) that is to be mounted at the end of a shaft can be secured with a screw and a washer in the manner shown in Figure 11–14(a). The taper provides good concentricity and moderate torque transmission capacity. Because of the machining required, the connection is fairly costly. A modified form uses the tapered shaft with

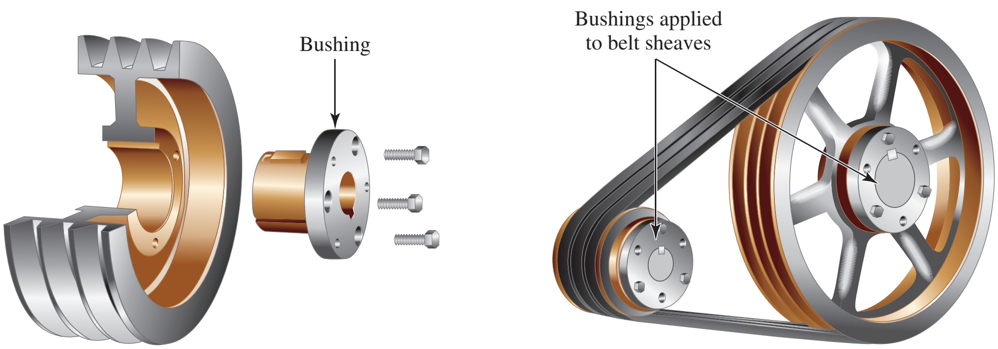


FIGURE 11–12 V-belt sheaves having three grooves with a split taper bushing for shaft mounting

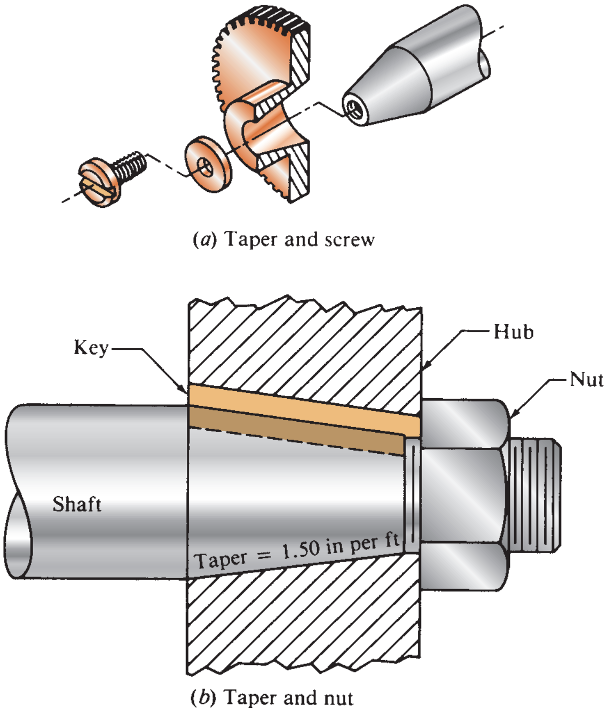


FIGURE 11–14 Tapered shaft for fastening machine elements to shafts

a threaded end for the application of a nut, as shown in Figure 11–14(b). The inclusion of a key lying in a keyseat machined parallel to the taper increases the torque-transmitting capacity greatly and ensures positive alignment.

Press Fit

Making the diameter of the shaft greater than the bore diameter of the mating element results in an interference fit. The resulting pressure between the shaft and the hub permits the transmission of torque at fairly high levels, depending on the degree of interference. This is discussed in more detail in Chapter 13. Sometimes the press fit is combined with a key, with the key providing the positive drive and the press fit ensuring concentricity and holding the part in position axially.

Molding

Plastic and die cast gears can be molded directly to their shafts. Often the gear is applied to a location that is knurled to improve the ability to transmit torque. A modification of this procedure is to take a separate gear blank with a prepared hub, locate it over the proper position on a shaft, and then cast zinc into the space between the shaft and the hub to lock them together.

11–7 COUPLINGS

The term *coupling* refers to a device used to connect two shafts together at their ends for the purpose of transmitting power. There are two general types of couplings: rigid and flexible.

Rigid Couplings

Rigid couplings are designed to draw two shafts together tightly so that no relative motion can occur between them. This design is desirable for certain kinds of equipment in which precise alignment of two shafts is required and can be provided. In such cases, the coupling must be designed to be capable of transmitting the torque in the shafts.

Figure 11–15 shows three styles of rigid couplings. (See Internet site 3.) For the style in part (a), the two flanged parts are installed on the shafts to be coupled and the bolts in the flange are drawn together by a series of bolts. The load path is then from the driving shaft to its flange, through the bolts, into the mating flange, and out to the driven shaft. The torque places the bolts in shear. The total shear force on the bolts depends on the radius of the bolt circle, $D_{bc}/2$, and the torque, T. That is,

$$F = T/(D_{bc}/2) = 2T/D_{bc}$$

Letting n be the number of bolts, the shear stress in each bolt is

$$\tau = \frac{F}{A_s} = \frac{F}{n(\pi d^2/4)} = \frac{2T}{D_{bc}n(\pi d^2/4)} \tag{11–20}$$

Letting the stress equal the design stress in shear and solving for the bolt diameter, we have

➪ Required Bolt Diameter for Rigid Coupling

$$d = \sqrt{\frac{8T}{D_{bc}n\pi\tau_d}} \tag{11–21}$$

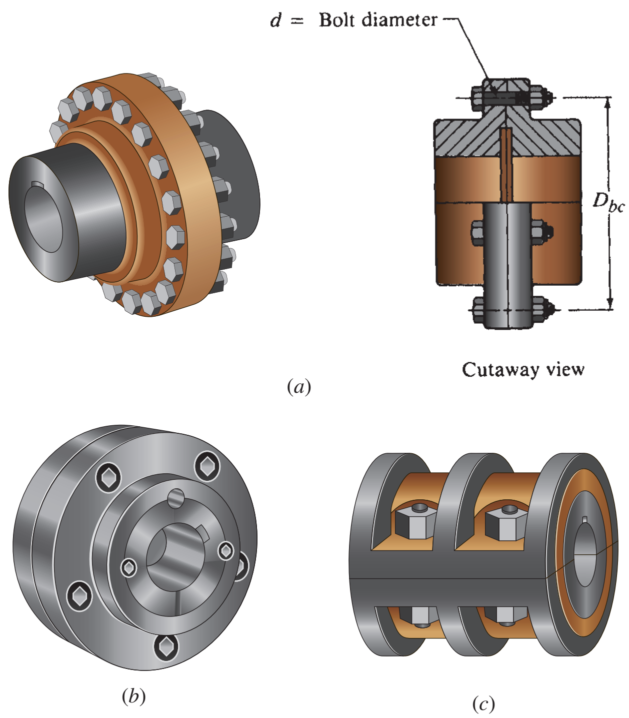


FIGURE 11–15 Rigid coupling

Notice that this analysis is similar to that for pinned connections in Section 11–6. The analysis assumes that the bolts are the weakest part of the coupling.

The style in Figure 11–15(b) is called a *taper-lock rigid coupling,* available in eight sizes capable of transmitting torques from 5050 lb·in to 254 500 lb·in (655 N·m to 33 000 N·m) and accommodating shaft sizes from 0.50 in to 6.00 in (12.7 mm to 152 mm). Figure 11–15(c) is called a *ribbed rigid coupling* available in bore sizes up to 7.00 in (178 mm) and can transmit up to 254 400 lb·in (33 000 N·m) of torque. (See Internet site 3.)

Rigid couplings should be used only when the alignment of the two shafts can be maintained very accurately, not only at the time of installation but also during operation of the machines. If significant angular, radial, or axial misalignment occurs, stresses that are difficult to predict and that may lead to early failure due to fatigue will be induced in the shafts. These difficulties can be overcome by the use of flexible couplings.

Flexible Couplings

Flexible couplings are designed to transmit torque smoothly while permitting some axial, radial, and angular misalignment. The flexibility is such that when misalignment does occur, parts of the coupling move with little or no resistance. Thus, no significant axial or bending stresses are developed in the shaft.

Many types of flexible couplings are available commercially, as shown in Figures 11–16 through 11–25. Each is designed to transmit a given limiting torque. The manufacturer's catalog lists the design data from which you can choose a suitable coupling. Remember that torque equals power divided by rotational speed. So for a given size of coupling, as the speed of rotation increases, the amount of power that the coupling can transmit also increases, although not always in direct proportion. Of course, centrifugal effects determine the upper limit of speed.

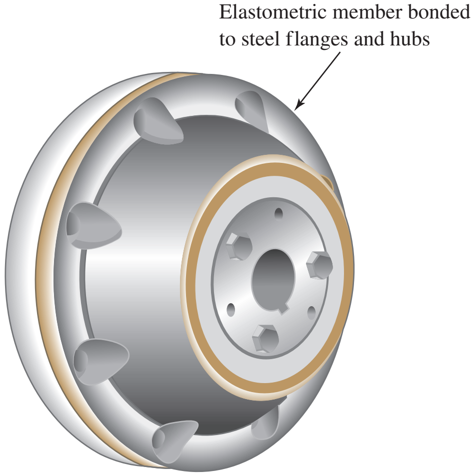


FIGURE 11–17 Elastomeric coupling. The features of this coupling are that it (1) generally minimizes torsional vibration; (2) cushions shock loads; (3) compensates for parallel misalignment up to 1/32 in; (4) accommodates angular misalignment of ±3°; and (5) provides adequate end float, ±1/32 in

The degree of misalignment that can be accommodated by a given coupling should be obtained from the manufacturer's catalog data, with values varying with the size and design of the coupling. Small couplings may be limited to parallel misalignment of 0.005 in, although larger couplings may allow 0.030 in or more. Typical allowable angular misalignment is ±3°. Axial movement allowed, sometimes called *end float,* is up

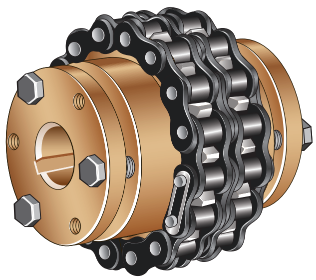


FIGURE 11–16 Chain coupling. Torque is transmitted through a double roller chain. Clearances between the chain and the sprocket teeth on the two coupling halves accommodate misalignment

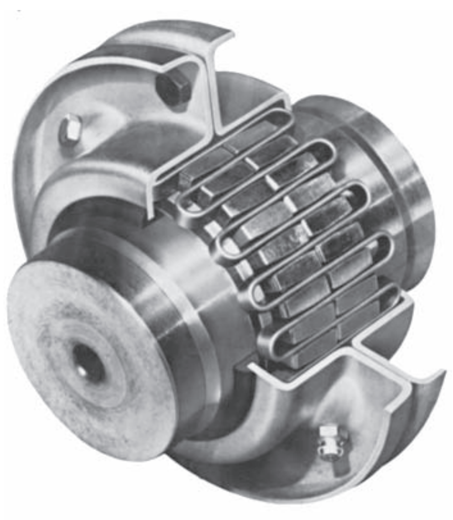


FIGURE 11–18 Flexible grid coupling. Torque is transmitted through a flexible spring steel grid. Flexing of the grid permits misalignment and makes it torsionally resilient to resist shock loads

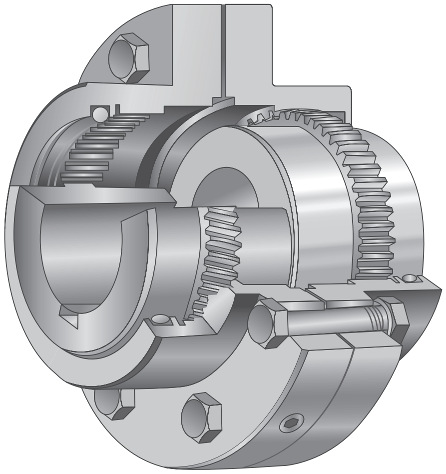


FIGURE 11–19 Gear coupling. Torque is transmitted between crown-hobbed teeth from the coupling half to the sleeve. The crown shape on the gear teeth permits misalignment

FIGURE 11–20 Bellows coupling. The inherent flexibility of the bellows accommodates the misalignment (Stock Drive Products, New Hyde Park, NY)

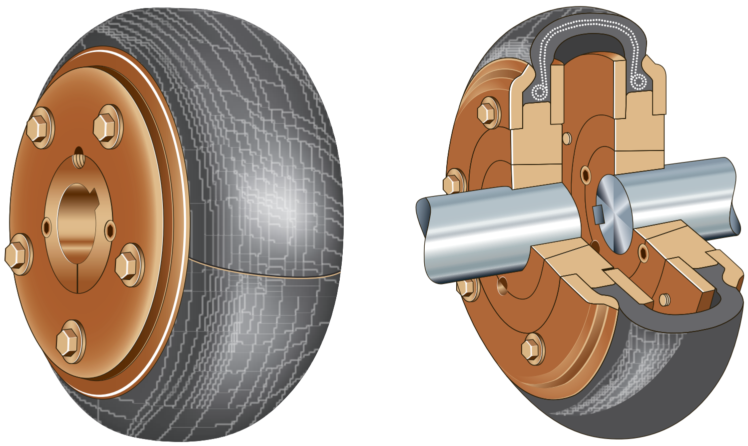


FIGURE 11–21 PARA-FLEX® coupling. Using an elastomeric element permits misalignment and cushions shocks

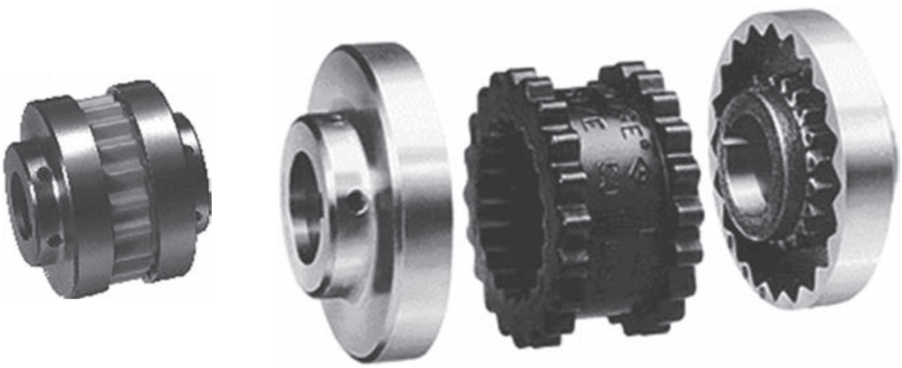


FIGURE 11–22 D-Flex coupling used mainly for connecting pumps and motors (Baldor/Dodge, Greenville, SC)

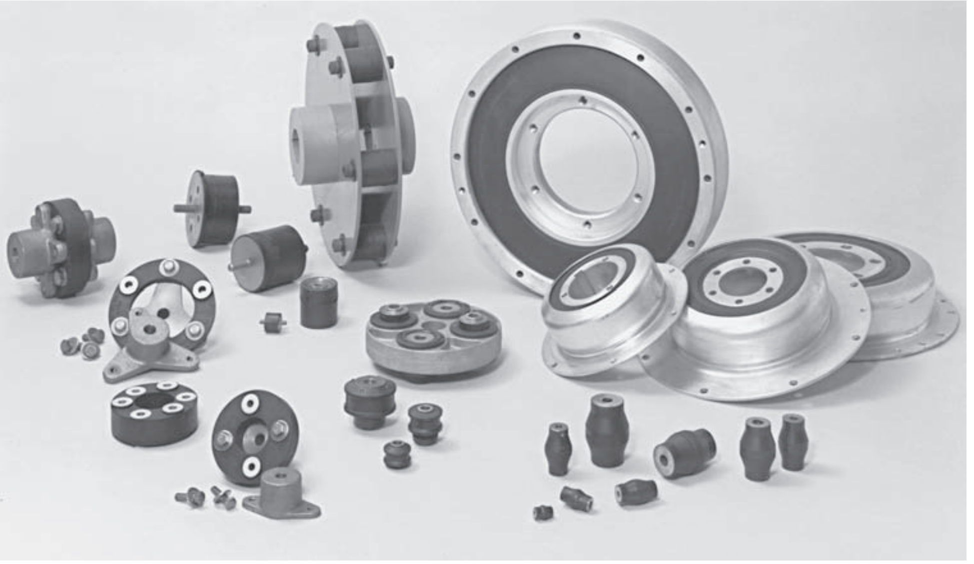


FIGURE 11–23 Dynaflex® coupling. Torque is transmitted through elastomeric material that flexes to permit misalignment and to attenuate shock loads (Courtesy of LORD Corporation)

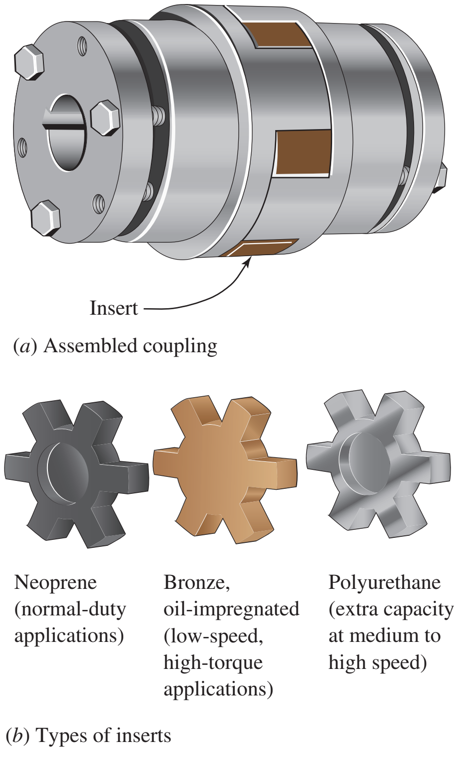


FIGURE 11–24 Jaw-type coupling

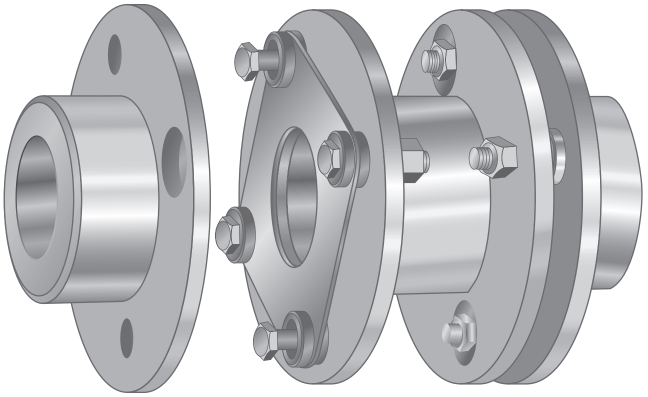


FIGURE 11–25 Flexible disk-type coupling. Torque is transmitted from hubs through laminated flexible elements to the spacer (T. B. Wood's Incorporated, Chambersberg, PA)

to 0.030 in for many types of couplings. (See References 1–4 and Internet sites 3–8, 17, 19, 22, and 24 for the manufacturers.)

The following is an example of the procedure used to select flexible couplings. We are focusing on the flexible disc type as illustrated in Figures 11–26 and 11–27.

Figure 11–26 shows a spacer-type flexible coupling, specifically used in pump drive applications. The spacer allows the coupling to be removed without moving the pump or motor, providing access to change the pump seal.

Figure 11–27 shows a floating shaft–type flexible coupling used for long-span applications. Each side of the flexible coupling needs to be rigidly supported by the connecting shafts.

The data given in Tables 11–8 and 11–9 can be used for problems in this book and they are only examples of the types of data available; different suppliers may show their data in different formats. See the Internet sites listed earlier.

Flexible Disc Coupling Selection Procedure

The procedure uses a pump drive as an example and assumes that the following data are known: (Using U.S. units)

- Power being transmitted by the coupling in horsepower, P
- The normal rotational speed of the coupling in rpm, n

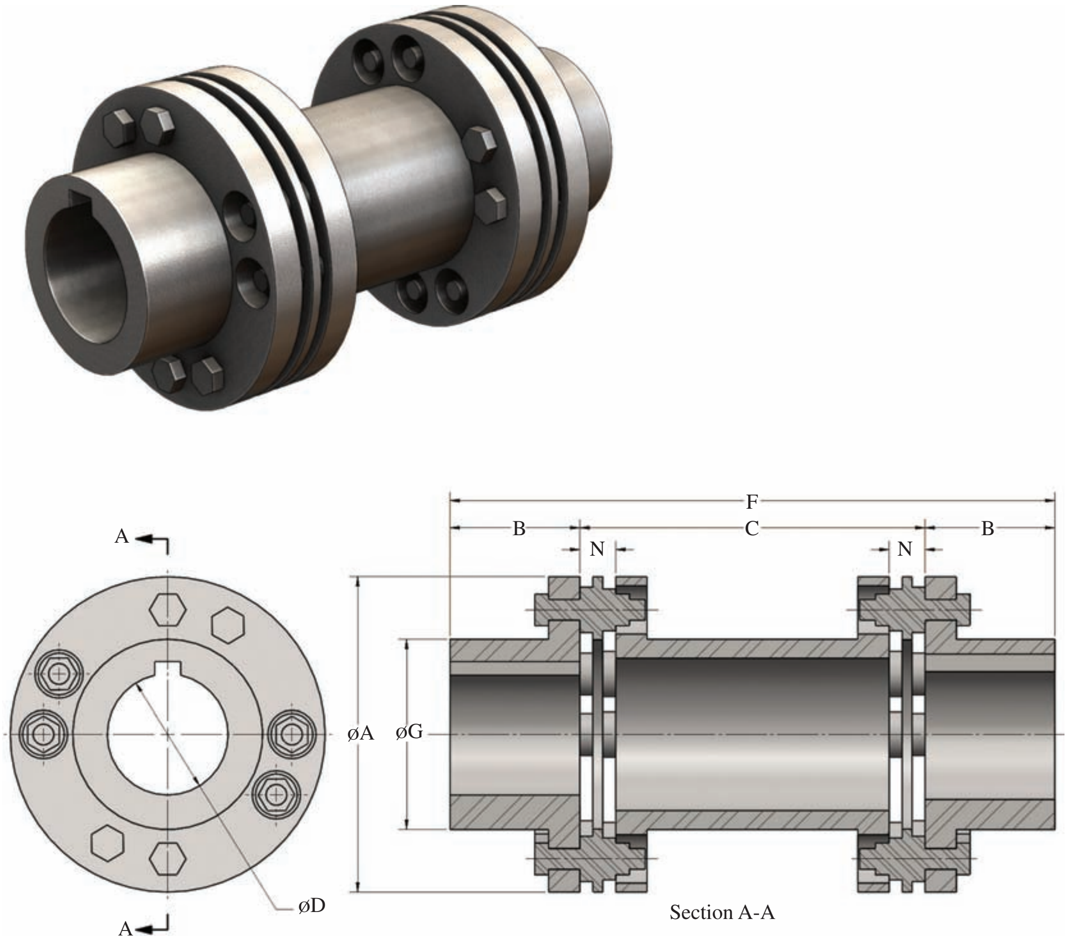


FIGURE 11–26 Flexible disc coupling—Spacer type

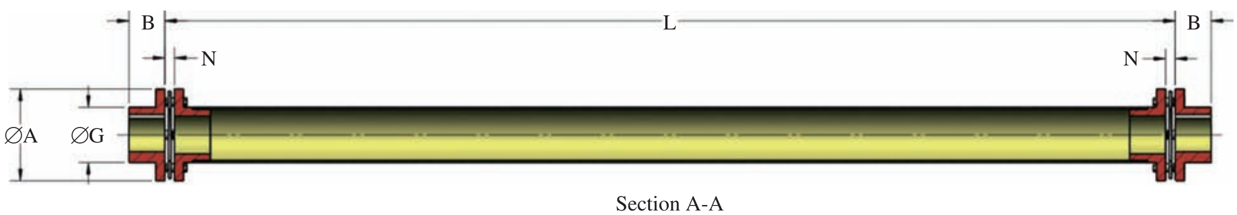


FIGURE 11–27 Flexible disc coupling—Floating shaft type

TABLE 11–8 Service Factor of Motor- and Turbine-Driven Equipment

Application	Service factor	Application	Service factor
Blowers		**Centrifugal pump**	
Centrifugal	1.0	General Duty	1.0
Lobe	1.5	Boiler Feed	1.0
Vane	1.5	Slurry	1.5
Compressors		Dredge	2.0
Centrifugal	1.0	**Reciprocating pump**	
Lobe	1.5	Double Acting	2.0
Vane	1.5	Single Acting 1 or 2 cylinders	2.5
Screw	1.5	Single Acting 3 or more cylinders	2.0
Fans		**Pump**	
Centrifugal	1.5	Rotary	1.5
Axial	1.5	Lobe	1.5
Light-Duty Blower	1.0	Vane	1.5
Conveyors		**Printing presses**	
Uniformly loaded	1.5	General Equipment	1.5

TABLE 11–9 Spacer-Type Flexible Coupling Engineering Data

								Max. horsepower per 100 rpm			Max			
								Service factor			rpm		Max. continuous torque (lb · in)	Peak overload torque (lb · in)
Coupling size	Max bore	A	B	C	F	G	N	1	1.5	2	Not balanced	balanced		
A	7/8	2 11/16	1 1/8	3 1/2	5 3/4	1 7/16	21/64	0.65	0.43	0.33	6000	8200	410	820
B	1 5/16	3 7/32	1 3/8	3 1/2	6 1/4	1 11/16	7/16	1.30	0.87	0.65	6000	7500	820	1640
C	1 3/8	3 27/32	1 5/8	3 1/2	6 3/4	2 1/16	1/2	2.20	1.50	1.10	5500	6800	1400	2800
D	1 7/8	4 9/16	1 7/8	3 1/2	7 1/4	2 3/4	1/2	3.10	2.10	1.60	5000	6300	1950	3900
E	2 1/4	5 11/32	2 1/8	3 1/2	7 3/4	3 9/32	19/32	5.60	3.70	2.80	4600	5700	3530	7060
F	2 5/8	6 1/16	2 5/8	5	10 1/4	3 25/32	31/32	10.00	6.70	5.00	4100	5500	6300	12 600
G	3	7	3	5	11	4 7/16	1 1/32	15.70	10.50	7.90	3700	5000	9900	19 800

- The maximum rotational speed that the coupling will see during any operating condition.
- The desired distance between the pump shaft and the drive motor shaft
- The diameters of the drive motor shaft and the pump shaft
- The nature of the application, as listed in Table 11–8, used to determine a service factor

Selection Procedure

1. Determine the power capacity required per 100 rpm:

$$\frac{\text{HP}}{100\ \text{rpm}} = \frac{\text{HP} \cdot 100}{n_{\text{coupling}}}$$

2. Determine the service factor, *SF*, for the given application from Table 11–8.
3. Select one or more candidate couplings from Table 11–9 using the computed power rating capacity per 100 rpm at the given service factor.
4. Verify that the maximum bore size of the coupling will fit the given shaft diameters. Bore diameters smaller than the maximum can be ordered.
5. Check that the maximum speed of the coupling does not exceed the maximum running speed of the equipment.
6. Verify that the system running torque and the maximum torque due to start up or shock loading does

not exceed the coupling's peak continuous torque and overload torque. Use the equation

$$\text{Torque} = \text{Power/rotational speed} = P/n$$

7. Summarize the pertinent data for sizes and allowable misalignment.

Example Problem 11–2

Figure 11–28 shows a 30-hp electric motor driving a centrifugal pump with a flexible coupling connecting the two shafts. The motor shaft diameter is 1.875 in and the pump shaft diameter is 1.75 in. The normal rotational speed of the drive motor is 1760 rpm. The desired spacing between the motor shaft and the pump shaft is 4.0 in minimum. Specify a suitable coupling using the sample data in Table 11–9.

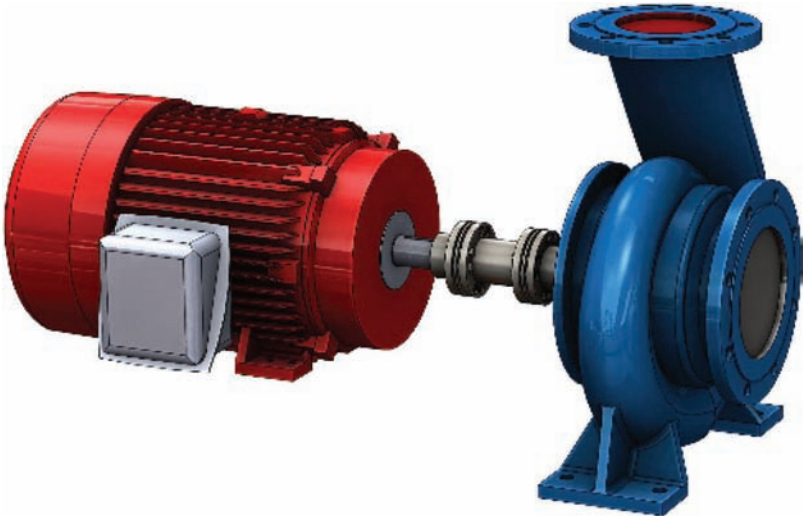


FIGURE 11–28 Electric motor driving a centrifugal pump through a flexible coupling

Solution

1. Determine the power capacity required per 100 rpm:

$$30\text{ hp }(100)/n = 30\text{ hp}(100)/1760\text{ rpm} = 1.70\text{ hp/100 rpm}$$

2. Determine the service factor for the given application from Table 11–8:
   For a centrifugal pump drive, $SF = 1.0$.
3. Select one or more candidate couplings from Table 11–9 using the 1.70 hp/100 rpm at 1.0 service factor.
   We choose coupling sizes C and D as candidates:
   1. Coupling C is rated at 2.2 hp/100 rpm.
   2. Coupling D is rated at 3.3 hp/100 rpm.
4. Check the maximum allowable bore sizes for the two candidate couplings:
   1. Coupling C has a maximum bore size of 1 3/8 in (1.375 in)—Too small for either shaft.
   2. Coupling D has a maximum bore size of 1 7/8 in (1.875 in)—Fits both shaft sizes.

   We continue the selection process using Coupling D.
5. Check that the maximum speed of the selected Coupling D:
   Maximum speed is 5000 rpm > 1760 rpm of the drive motor—Satisfactory
6. Check the running torque for Coupling D:

$$\text{Actual running torque, } T = \frac{P}{n} = \frac{30\text{ hp}}{1760\text{ rev/min}} \cdot \frac{33\,000\frac{\text{lb}\cdot\text{in}}{\text{min}}}{1.0\text{ hp}} \cdot \frac{1\text{ rev}}{2\pi\text{ rad}} \cdot \frac{12\text{ in}}{\text{ft}} = 1074\text{ lb}\cdot\text{in}$$

The listed maximum continuous torque for Coupling D is 1950 lb · in—Satisfactory

The allowable peak overload torque is 3900 lb · in and it is unlikely that this level will be reached.

7. Pertinent data for Coupling D:

   a. The spacer length is 3.50 in.
   b. The length between the flanges and the spacer ends is 0.50 in on each end.
   c. Assuming that the motor and pump shafts are inserted to the end of the two parts of the coupling, the total spacing is

$$\text{Spacing length} = 3.50 \text{ in} + 0.50 \text{ in} + 0.50 \text{ in} = 4.50 \text{ in}$$

   This is greater than the specified minimum length of 4.00 in—Satisfactory.

   d. Specify the bores of the two ends of Coupling D:

   Left (motor) side: Bore = 1.875 in

   Right (pump) side: Bore = 1.75 in

   e. Total length of assembled Coupling D (see Table 11–9):

   Length $= C + 2(B) = 3.50 \text{ in} + 2(1.875 \text{ in}) = 7.25 \text{ in}$ (Dimension F in the table)

   f. Diameters:

   End pieces and middle tube: 2.75 in

   OD of flanges: 4.5625 in

   g. A key is required for both end pieces, specified by the motor and pump supplier.
   h. The manufacturer's data must be consulted to determine the allowable misalignment. Typical values for this type and size of coupling are as follows:

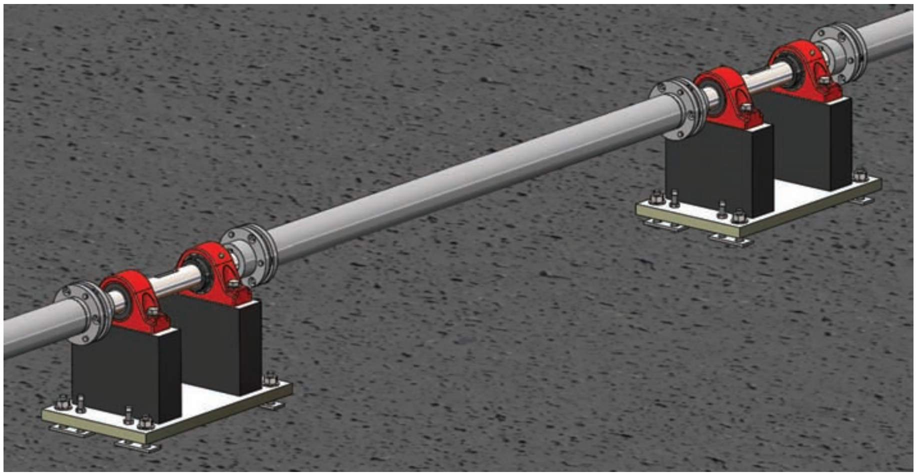
$$\text{Maximum angular misalignment} = 0.50°$$
$$\text{Allowable axial end float} = \pm 0.057 \text{ in}$$

The procedure for selecting a floating shaft–type flexible coupling is virtually the same as that just demonstrated for the spacer-type coupling in Example Problem 11–2 with only small differences. Refer to Table 11–10 for the floating shaft–type coupling and compare the data with that in Table 11–9 for the spacer-type coupling.

Whereas the spacer-type coupling has a specified length value indicated by the C dimension in Table 11–9,

TABLE 11–10 Floating Shaft–Type Flexible Coupling Engineering Data

						Max. Horsepower per 100 rpm			Max. Span L [in] For various angular speeds				
						Service Factor							
Coupling size	Max Bore	A	B	G	N	1	1.5	2	500 rpm	1000 rpm	1500 rpm	Max. Continuous torque (lb · in)	Peak overload torque (lb · in)
A2	7/8	2 11/16	1 1/8	1 7/16	21/64	0.65	0.43	0.33	94	94	88	410	820
B2	1 5/32	3 7/32	1 3/8	1 11/16	7/16	1.30	0.87	0.65	102	102	93	820	1640
C2	1 3/8	3 27/32	1 5/8	2 1/16	1/2	2.20	1.50	1.10	114	114	105	1400	2800
D2	1 7/8	4 11/32	1 7/8	2 3/4	17/32	3.90	2.60	2.00	133	133	122	2460	4920
E2	2 1/4	5 7/16	2 1/8	3 9/32	9/16	6.20	4.10	3.10	161	147	135	3900	7800
F2	2 5/8	5 13/16	2 5/8	3 25/32	19/32	11.20	7.50	5.60	172	157	144	7100	14 200
G2	3 1/8	6 5/8	3	4 1/2	15/32	19.40	12.90	9.70	191	173	160	12 200	24 400



(*a*) A floating shaft–type coupling mounted between two rigid supports

(*b*) Side view of the coupling in (a)

FIGURE 11–29 Example of an installation for a floating shaft–type coupling

the floating shaft–type coupling does not and that column has been deleted in Table 11–10. However, Table 11–10 shows new data for "Max Span L" toward the right side of the table. The maximum values listed are dependent on the rotational speeds because of the dynamics of long rotating cylinders. Therefore, an additional Step 8 can be added to the procedure given earlier:

8. Specify the length L for the coupling, ensuring that it is smaller than the Max Span L value.

Figure 11–29 shows the floating shaft–type coupling installed in a typical machinery arrangement.

11–8 UNIVERSAL JOINTS

When an application calls for accommodating misalignment between mating shafts that is greater than 3° typically provided by flexible couplings, a *universal joint* is often employed. Figures 11–30 to 11–33 show some of the styles that are available. See Internet sites 5, 6, 9, 10, and 22 for commercially available universal joints. Angular misalignments of up to 45° are possible at low rotational speeds with single universal joints like that shown in Figure 11–30(a), consisting of two yokes, a center bearing block, and two pins that pass through the block at right angles. Approximately 20° to 30° is more reasonable for speeds above 10 rpm. Single universal joints have the disadvantage that the rotational speed of the output shaft is nonuniform in relation to the input shaft.

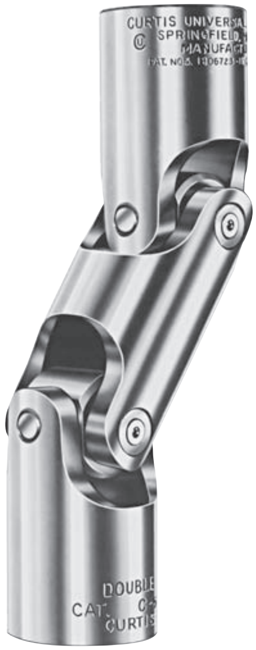
A double universal joint, shown in Figure 11–30(b), allows the connected shafts to be parallel and offset by large amounts. Furthermore, the second joint cancels the nonuniform oscillation of the first joint so the input and output shafts rotate at the same uniform speed.

Figure 11–31 shows a vehicular universal joint connecting an engine or transmission to the drive wheels used in some rear-wheel-drive cars, light and heavy duty trucks, agricultural equipment, and construction vehicles. The spider assembly contains needle-bearing rollers on each arm. The right end shows a ball stud yoke, a flange yoke, and a center coupling yoke that make up a *double Cardan universal joint*. Another style, called a *constant velocity joint*, or simply a *CV joint*, is often used as a key component of front-wheel-drive and all-wheel-drive vehicle drivelines.

Figure 11–32 shows a heavy-duty industrial type double universal joint. Some of this type will have a two-part connecting tube that is splined to allow for sizable changes in axial position as well as accommodating the angular or parallel misalignment.

Figure 11–33 shows a novel design called the Cornay™ universal joint that produces true constant velocity of the output shaft through all drive angles up to 90°. Compared with standard universal joint designs, the Cornay™ joint can operate at higher speeds, carry

(*a*) Single universal joint components



(*b*) Double universal joint

FIGURE 11–30 Single and double universal joints (Curtis Universal Joint Co., Inc., Springfield, MA)

higher torque levels, and produce less vibration. See Internet site 10.

Manufacturers' literature should be consulted for specifying a suitable size of universal joint for a given application. The primary variables involved are the torque to be transmitted, the rotational speed, and the angle at which the joint will be operating. A *speed/angle factor* is computed as the product of the rotational speed and the operating angle. From this value, an *operating use factor* is determined that is applied to the basic torque load to compute the required rated torque for the joint. All manufacturers provide such data in their catalogs.

Some drive shafts incorporating universal joints with relatively long, tubular sections between the end joints are often called *torque tubes*. This terminology is used in the following section giving an example of the selection of such a torque tube for an industrial machine.

Procedure to select an industrial universal joint or torque tube

As stated above, the selection of a universal joint or torque tube should be guided by the catalog data of the specific brand and model to be specified for the joint. The lists of Internet sites at the end of this chapter include information about several such manufacturers.

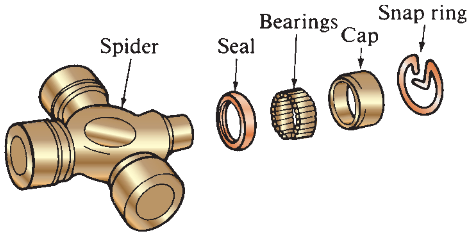
Here we show an example of a typical selection procedure for a torque tube of the type shown in Figure 11–34. We use data from Tables 11–11 to 11–13 that are representative of industry data. The uses of the tables are described within the following procedure.

TABLE 11–11 Service Factor for Industrial Universal Joint Applications

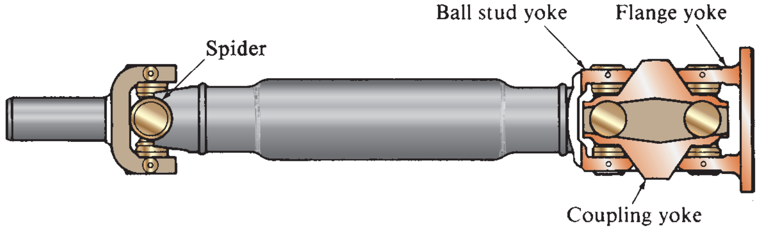
Application	Service factor
Blower	1.5
Conveyor	1.5
Fans	2.5
Food Industry	1.5
Centrifugal Pump	1
Reciprocating Pump	2
Paper Mills Press Roll Drives	2.5
Steel Mills: Cold Rolling Mills	5

TABLE 11–12 Industrial Universal Joint

Series	Peak torque rating (lb · in)	Bearing life factor	Min L (in)	Length compensation ΔL (in)	A (in)	B (in)	C (in)	D (in)	E (in)	F (in)	α (deg)	Maximum operating speed (rpm)	Speed to angle factor
A	800	230	13.66	3.12	1.38	2.375	3.88	2.50	3/8	3.125	20	5000	16 250
B	1240	366	14.96	3.62	1.56	2.750	4.56	3.00	7/16	3.750	20	5000	16 250
C	1500	443	15.81	3.47	1.69	2.750	4.56	3.50	7/16	3.750	22	5000	16 250
D	2000	591	15.5	2.5	2.00	3.750	5.88	3.50	1/2	4.750	22	5000	16 250
E	2400	756	15.75	2.5	2.00	3.750	5.88	3.50	1/2	4.750	22	5000	16 250

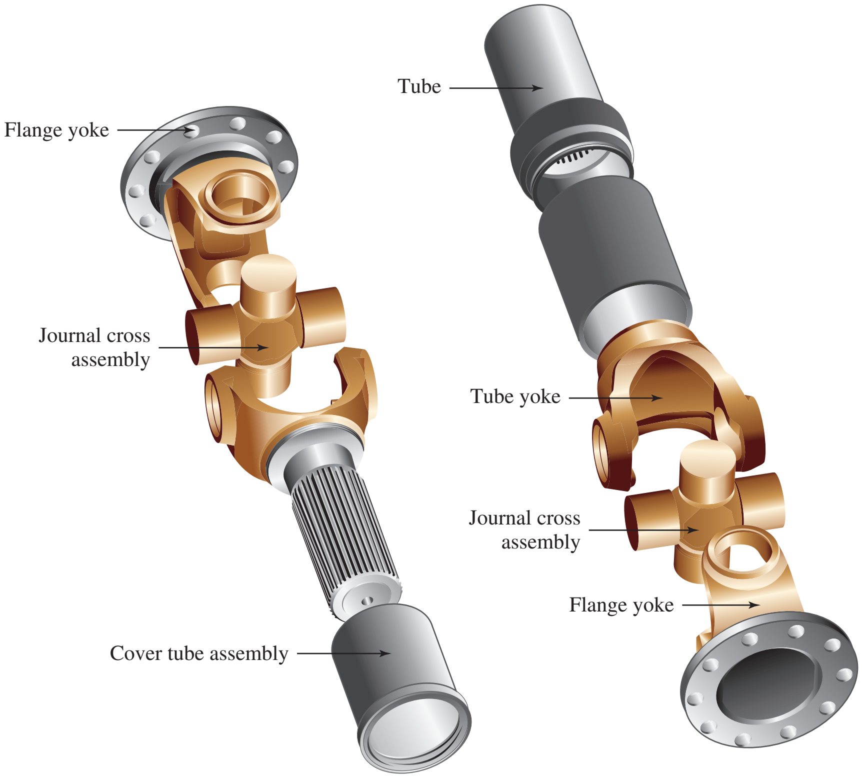


(*a*) Automotive universal joint components

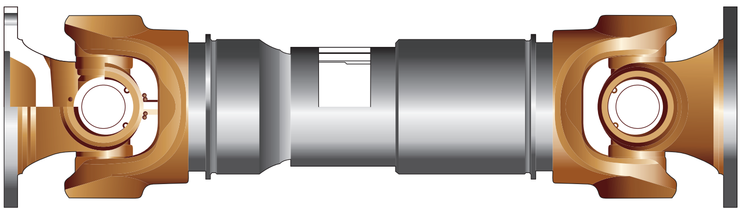


(*b*) Automotive propeller shaft with double Cardan universal joint

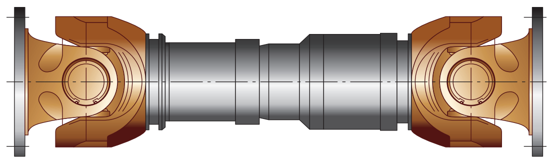
FIGURE 11–31 Automotive universal joints



(*a*) Exploded view of driveshaft components



(*b*) Side view of assembly with partial cutaway sections



(*c*) Drawing of a drive shaft assembly

FIGURE 11–32 Industrial drive shaft incorporating a pair of universal joints (GWB - A Dana Brand)

FIGURE 11–33 Cornay™ universal joint (Drive Technologies, Inc., Longmont, CO)

TABLE 11–13 Adjusted Normal Torque

Prime mover	Service factor
Electric Motor	1
Gas Engine	1.75
Diesel Engine	2

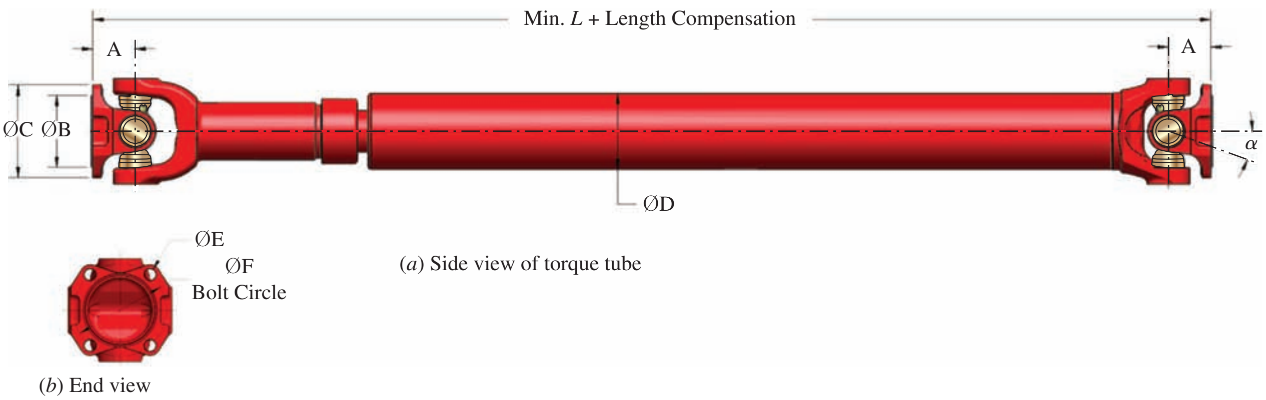


FIGURE 11–34 Type of torque tube used in the selection procedure

Typical data required to select a torque tube include the following:

1. Power to be transmitted by the torque tube.
2. Rotational speed of the torque tube, n.
3. The approximate length available between the two ends of the torque tube.
4. The expected angle of inclination of the torque tube.
5. The required life of the torque tube assembly.

With these data available, the following procedure can be used to specify a suitable torque tube.

1. Calculate the *equivalent torque* in the unit of lb·ft, using the following equations and a service factor from Table 11–11 for the particular application.

$$\text{Calculated Torque} = \frac{\text{Power}}{n}$$

$$\text{Equivalent Torque} = \text{Calculated Torque} \cdot \text{Service Factor}$$

2. Select a universal joint from Table 11–12, ensuring that the Equivalent Torque is less than the *Peak Torque Rating* of the joint.
3. Calculate the expected L_{10} life of the bearings using the following equation and the data for the selected joint.

$$L_{10} = \left(\frac{1.5 \cdot 10^7}{n \cdot \beta}\right) \cdot \left(\frac{\text{Bearing Life Factor}}{\text{Adjusted Normal Torque}}\right)^{10/3}$$

where

n: rotational speed of the universal joint (rpm)
β: Universal joint operating angle (degrees)
Bearing Life Factor: from Table 11–12
Adjusted normal torque in lb·ft from the following equation, using data from Table 11–13

$$\text{Adjusted Normal Torque} = \text{Equivalent Torque} \cdot \text{Prime Mover Service Factor}$$

4. Ensure that the rotational speed of the universal joint does not exceed the *maximum safe operating speed* (MSOS) from Table 11–12.
5. Verify that the universal joint operating speed times the operating angle is less than the *speed to angle factor* from Table 11–12.

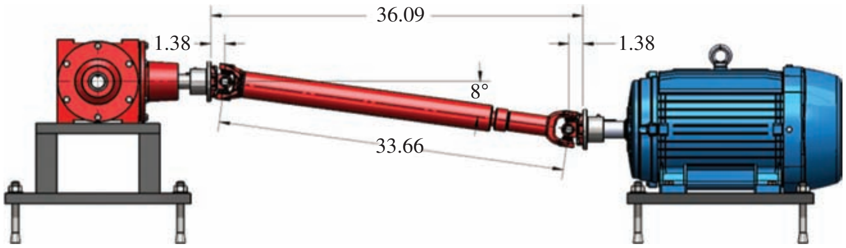
$$n \cdot \beta < \textit{Speed to angle factor}$$

6. Specify pertinent dimensions and other data for the specified joint from Table 11–12.

Example Problem 11–3

Figure 11–35 shows part of a piece of machinery for the food industry in which an electric motor is driving a right-angle gearbox through a torque tube. The motor supplies 20 hp at a rotational speed of 1000 rpm. The nominal length between the motor and the gearbox is 30 in and the angle of inclination is 8°. Specify a suitable torque tube.

(*a*) Pictorial view of the drive system



(*b*) Side view

FIGURE 11–35 Electric motor driving a gearbox through a torque tube

Solution

Given: Power = 20 hp; Rotational speed = 1000 rpm; Drive shaft joint angle = β = 8°
Use the **Procedure to select an industrial universal joint or torque tube**.

Results:

1. Calculate the *equivalent torque* in the unit of lb · ft, using the following equations and a service factor from Table 11–11 for the particular application.

   From Table 11–11, the application service factor = 1.5

   $$\text{Calculated Torque} = \frac{\text{Power}}{n} = \frac{20\text{ hp}}{1000\text{ rpm}} \cdot \frac{33\,000\text{ lb}\cdot\text{ft/min}}{1\text{ hp}} \cdot \frac{1\text{ rev}}{2\pi\text{ rad}} = 105\text{ ft}\cdot\text{lb}$$

   $$\text{Equivalent Torque} = \text{Calculated Torque} \cdot \text{Service Factor} = (105\text{ lb}\cdot\text{ft})(1.5) = 157.6\text{ lb}\cdot\text{ft}$$

2. Select a universal joint from Table 11–12. The Series A joint has a peak torque rating of 800 lb · ft. Use this as a preliminary selection.
3. Calculate the expected L_{10} life of the bearings. The bearing life factor for the Series A joint is 230. The prime mover service factor from Table 11–13 for an electric motor is 1.0. Therefore, the Adjusted Normal Torque is the same as the equivalent torque, 157.6 lb · ft.

   $$L_{10} = \left(\frac{1.5 \cdot 10^7}{n \cdot \beta}\right) \cdot \left(\frac{\text{Bearing Life Factor}}{\text{Adjusted Normal Torque}}\right)^{10/3}$$

   $$L_{10} = \left(\frac{1.5 \cdot 10^7}{1000\text{ rpm} \cdot 8°}\right) \cdot \left(\frac{230}{157.6\text{ lb}\cdot\text{ft}}\right)^{10/3} = 25\,594\text{ hours}$$

   This is deemed to be acceptable for the application.
4. Ensure that the rotational speed of the universal joint does not exceed the *maximum safe operating speed* from Table 11–12. For a Series A joint, MSOS = 5000 rpm which is far greater than the 1000 rpm speed of this joint.
5. Verify that the universal joint operating speed times the operating angle is less than the *speed to angle factor* from Table 11–12.

$$n \cdot \beta < \textit{Speed to angle factor}$$

The Speed to angle factor = 16 250 from Table 11–12.

Then $n \cdot \beta = (1000 \text{ rpm}) \cdot (8°) = 8000 < 16\,250$, which is satisfactory.

6. Pertinent data: The OD of the torque tube is 2.50 in. Other data are available from manufacturers' catalogs.

Summary: The Series A joint from Table 11–12 has been specified for this application to transmit 20 hp at a rotational speed of 1000 rpm while operating at an angle of 8°.

11–9 OTHER MEANS OF AXIAL LOCATION

The preceding sections of this chapter have focused on means of connecting machine elements to shafts for the purpose of transmitting power. Therefore, they emphasized the ability of the elements to withstand a given torque at a given speed of rotation. It must be recognized that the axial location of the machine elements must also be ensured by the designer.

The choice of the means for axial location depends heavily on whether or not axial thrust is transmitted by the element. Spur gears, V-belt sheaves, and chain sprockets produce no significant thrust loads. Therefore, the need for axial location affects only incidental forces due to vibration, handling, and shipping. Although not severe, these forces should not be taken lightly. Movement of an element in relation to its mating element in an axial direction can cause noise, excessive wear, vibration, or complete disconnection of the drive. Any bicycle rider who has experienced the loss of a chain can appreciate the consequences of misalignment. Recall that for spur gears, the strength of the gear teeth and the wear resistance are both directly proportional to the face width of the gear. Axial misalignment decreases the effective face width.

Some of the methods discussed in Section 11–6 for fastening elements to shafts for the purpose of transmitting power also provide some degree of axial location. Refer to the discussions of pinning, keyless hub to shaft connections, split taper bushings, set screws, the taper and screw, the taper and nut, the press fit, molding, and the several methods of mechanically locking the elements to the shaft.

Among the wide variety of other means available for axial location, we will discuss the following:

- Retaining rings
- Collars
- Shoulders
- Spacers
- Locknuts

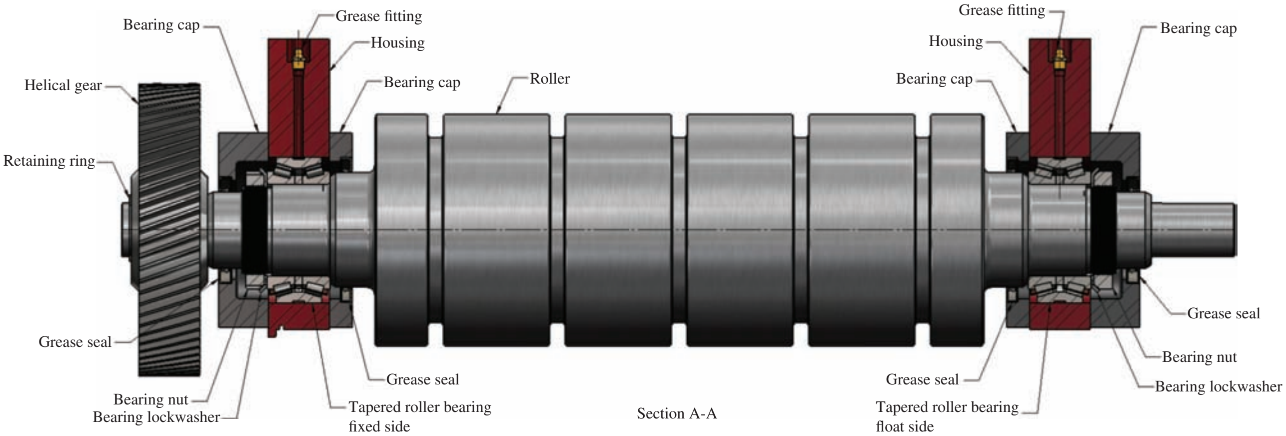
Some of these items are shown in Figure 11–36. A machine shaft is supported on two tapered roller bearings with a gear on the left end and the middle portion is a roller used as part of a paper handling system. The outer races of the bearings are supported by bearing caps and the housing of the machine. Because of the different widths of the inner and outer races, spacers are used to ensure axial positioning. The inner races of the bearings are press-fitted on the shaft, as discussed in Chapter 14. In addition, the outside faces of the inner race are held by locknuts.

The gear on the left end of the shaft is seated against a shoulder on its right side. Then, after the gear is in place, a retaining ring is inserted into a groove on the shaft to ensure that the gear does not move axially out of position or fall off the shaft. An alternative means of maintaining the axial position of the helical gear is to use a *retaining cap* that fastens to the end of the shaft and clamps the gear against the shaft shoulder.

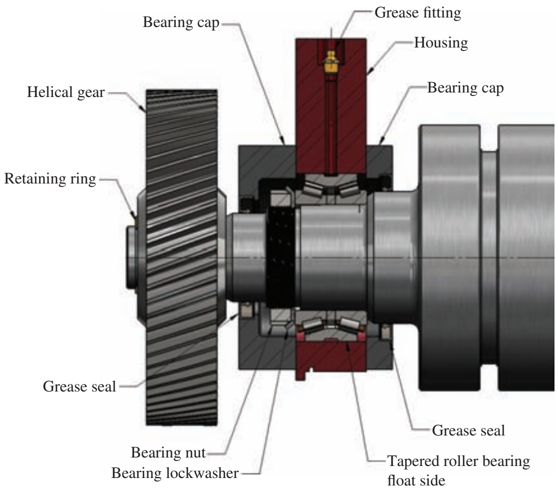
More information on these other means of location follows.

Retaining Rings

Retaining rings are placed on a shaft, in grooves cut into the shaft, or in internal recesses to prevent the axial movement of a machine element or to hold internal components in place. Figure 11–37 shows sketches of common generic forms of retaining rings. The basic external design [Figure 11–37(a)] is most often used for retaining gears and other power transmission elements in axial position on shafts. This is discussed further in Chapter 12 on shaft design. There are numerous styles available from several vendors, including those listed in Internet sites 7, 11, and 22, from which catalog data may be found along with design guidance for preparing a shaft or housing to accept the rings. In general, retaining rings are classified as either *external* or *internal*. Some install in carefully sized grooves, while others hold their position with gripping force between the ring and the mating element. The amount of axial thrust capacity and the shoulder height provided by the different ring styles vary and catalog data should be consulted. Rings are made to either U.S. Customary dimensions or SI metric sizes. See the left end of the shaft in Figure 11–36 for an example of the use of retaining rings.

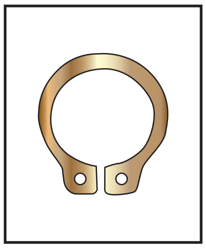


(*a*) Shaft assembly

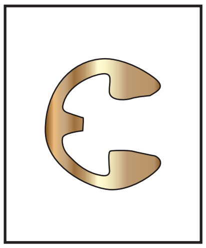


(*b*) Detail for left end of shaft

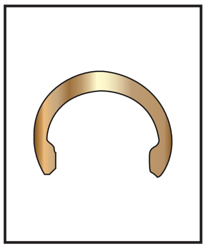
FIGURE 11–36 Shaft assembly showing mounting details



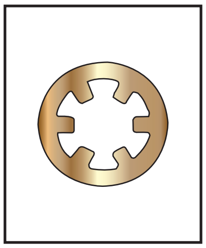
(*a*) Basic external ring for shaft applications



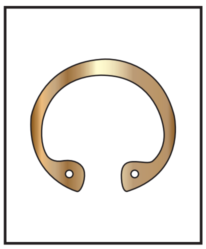
(*b*) External E-ring style



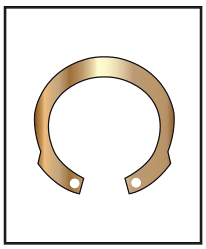
(*c*) Radially installed external ring



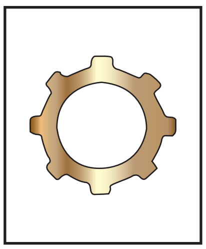
(*d*) Pressed-on external ring



(*e*) Basic internal ring with inward projections



(*f*) Internal ring with outward projections



(*g*) Pressed-in internal ring

FIGURE 11–37 Examples of retaining ring styles

Collars

A *collar* is a ring slid over the shaft and positioned adjacent to a machine element for the purpose of axial location. It is held in position, typically, by set screws. Its advantage is that axial location can be set virtually anywhere along the shaft to allow adjustment of the position at the time of assembly. The disadvantages are chiefly those related to the use of set screws themselves (Section 11–6).

Shoulders

A *shoulder* is the vertical surface produced when a diameter change occurs on a shaft. Such a design is an excellent method for providing for the axial location of a machine element, at least on one side. Several of the shafts illustrated in Chapter 12 incorporate shoulders. The main design considerations are providing (1) a sufficiently large shoulder to locate the element effectively and (2) a fillet at the base of the shoulder that produces an acceptable stress concentration factor and that is compatible with the geometry of the bore of the mating element (see Figures 12–2 and 12–7). Note the use of shoulders in the shaft shown in Figure 11–36 to locate one side of the gear and the two bearings.

Spacers

A *spacer* is similar to a collar in that it is slid over the shaft against the machine element that is to be located. The primary difference is that set screws and the like are not necessary because the spacer is positioned *between* two elements and thus controls only the relative position between them. Typically, one of the elements is positively located by some other means, such as a shoulder or a retaining ring.

Locknuts

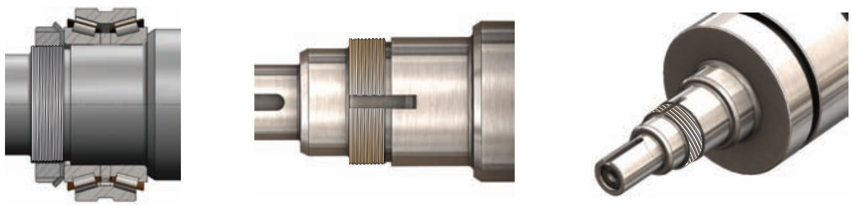
When an element is located at the end of a shaft, a *locknut* can be used to retain it on one side. Figure 11–38 shows a bearing retainer type of locknut. These are available as stock items from bearing suppliers.

The shaft is threaded to match the threads on the locknut which can be seen in part (c) of this figure. Note that the lockwasher with the peripheral teeth has a tang on the ID that is positioned in a groove machined into the shaft. After inserting the lockwasher on the shaft against the element being held, the nut is screwed on so that it seats against the lockwasher, thus securing the bearing in place. Furthermore, four of the external tangs on the lockwasher are bent into the four grooves in the outer periphery of the nut to ensure that the nut is not rotated off the shaft.

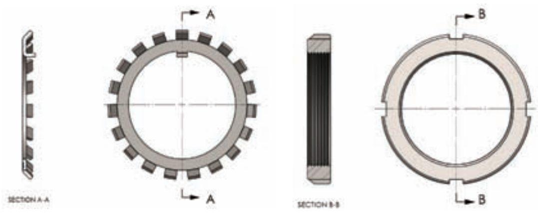
Caution against Overconstraint

One practical consideration in the matter of axial location of machine elements is exercising care that elements are not *overconstrained*. Under certain conditions of differential thermal expansion or with an unfavorable tolerance stack, the elements may be forced together so tightly as to cause dangerous axial stresses. At times it may be desirable to locate only one bearing positively on a shaft and to permit the other to float slightly in the axial direction. The floating element may be held lightly with an axial spring force accommodating the thermal expansion without creating dangerous forces.

(*a*) Pictorial view of bearing locknut and lockwasher



(*b*) Shaft assembly for use of lockwasher and locknut



Lockwasher Locknut

(*c*) Lockwasher and locknut

FIGURE 11–38 Locknut and lockwasher for retaining a bearing on a shaft.

In Figure 11–36, the left bearing is fixed into its housing, while the right bearing is permitted to slide (float) axially a small amount to accommodate thermal expansion and varying tolerances on shaft dimensions.

11–10 TYPES OF SEALS

Seals are important parts of machine design in situations where the following conditions apply:

1. Contaminants must be excluded from critical areas of a machine.
2. Lubricants must be contained within a space.
3. Pressurized fluids must be contained within a component such as a valve or a hydraulic cylinder.

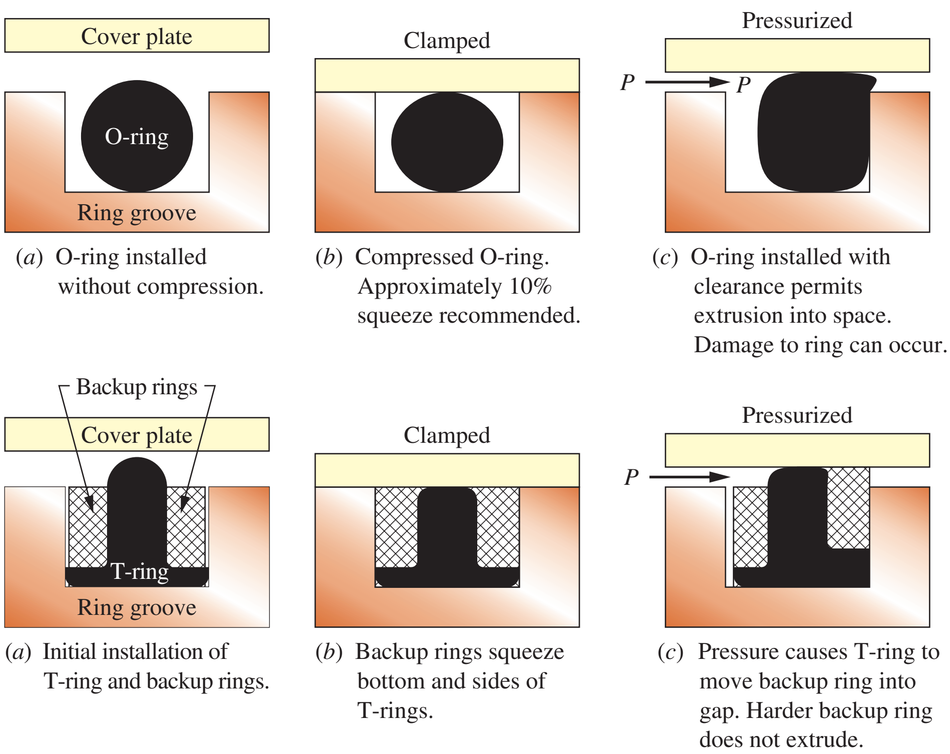
Some of the parameters affecting the choice of the type of sealing system, the materials used, and the details of its design are as follows:

1. The nature of the fluids to be contained or excluded
2. Pressures on both sides of the seal
3. The nature of any relative motion between the seal and the mating components
4. Temperatures on all parts of the sealing system
5. The degree of sealing required: Is some small amount of leakage permissible?
6. The life expectancy of the system
7. The nature of the solid materials against which the seal must act: corrosion potential, smoothness, hardness, wear resistance
8. Ease of service for replacement of worn sealing elements

The number of designs for sealing systems is virtually limitless, and only a brief overview will be presented here. More comprehensive coverage can be found in References 12–15. Often, designers rely on technical information provided by manufacturers of complete sealing systems or specific sealing elements. Also, in critical or unusual situations, testing of a proposed design is advised. See Internet sites 12–15, 22, and 23.

The choice of a type of sealing system depends on the function that it must perform. Common conditions in which seals must operate are listed here, along with some of the types of seals used.

1. Static conditions such as sealing a closure on a pressurized container: elastomeric O-rings; T-rings; hollow metal O-rings; and sealants such as epoxies, silicones, and butyl caulking (Figure 11–39).
2. Sealing a closed container while allowing relative movement of some part, such as diaphragms, bellows, and boots (Figure 11–40).
3. Sealing around a continuously reciprocating rod or piston, such as in a hydraulic cylinder or a spool valve in a hydraulic system: lip seal; U-cup seal;



(*a*) O-ring installed without compression.
(*b*) Compressed O-ring. Approximately 10% squeeze recommended.
(*c*) O-ring installed with clearance permits extrusion into space. Damage to ring can occur.

(*a*) Initial installation of T-ring and backup rings.
(*b*) Backup rings squeeze bottom and sides of T-rings.
(*c*) Pressure causes T-ring to move backup ring into gap. Harder backup ring does not extrude.

FIGURE 11–39 O-rings and T-rings used as static seals

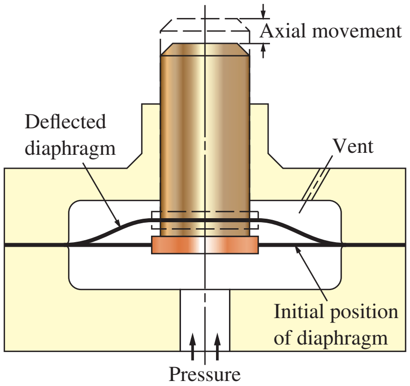


FIGURE 11–40 Application of a diaphragm seal

V-packing; and split ring seals, sometimes called *piston rings* (Figure 11–41).

4. Sealing around a rotating shaft such as the input or output shafts of a speed reducer, transmission, or engine: lip seal, wipers and scrapers, and face seals (Figure 11–42).
5. Protection of rolling-element bearings supporting shafts to keep contaminants from the balls and rollers (Figure 11–43).
6. Sealing the active elements of a pump or compressor to retain the pumped fluid: face seals and V-packing.
7. Sealing infrequently moved elements such as a valve stem of a fluid-flow control valve: compression packings and V-packings.
8. Sealing between hard, rigid surfaces such as between a cylinder head and the block of an engine: resilient gaskets. See Internet site 15.
9. Circumferential seals, such as at the tips of turbine blades, and on large, high-speed rotating elements: labyrinth seals, abradable seals, and hydrostatic seals.

11–11 SEAL MATERIALS

Most seal materials are resilient to permit the sealing points to follow minor variations in the geometry of mating surfaces. Flexing of parts of the seal cross section also occurs in some designs, calling for resiliency in the materials. Alternatively, as in the case of hollow metal O-rings, the shape of the seal allows the flexing of hard materials to occur. Face seals require rigid, hard materials that can withstand constant sliding motion and that can be produced with fine accuracy, flatness, and smoothness.

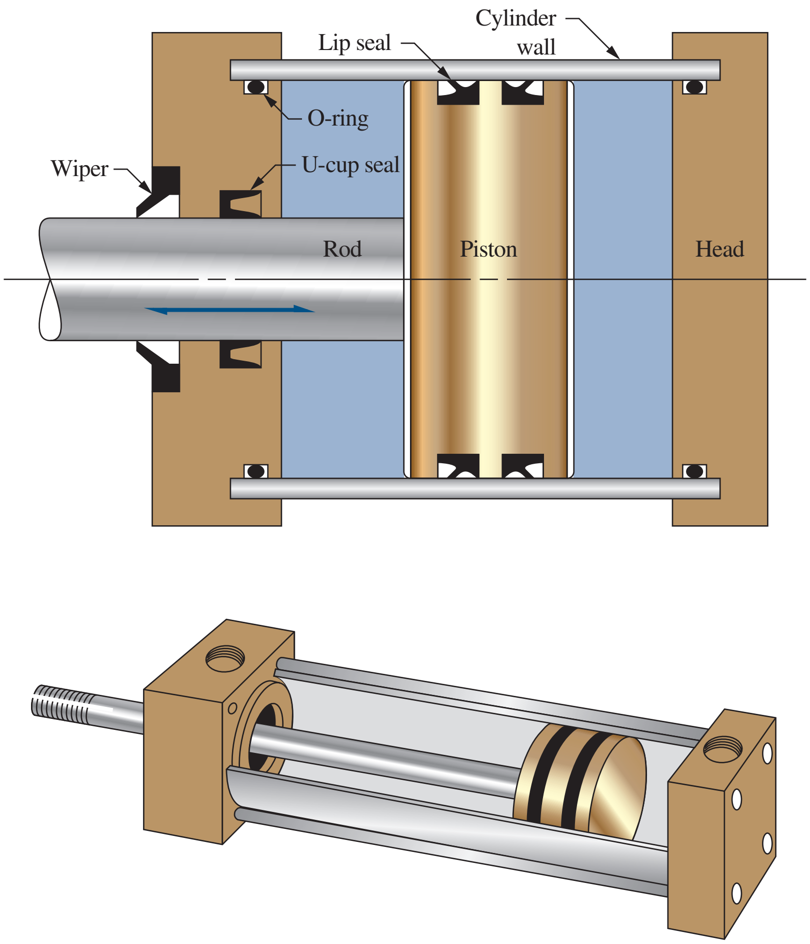
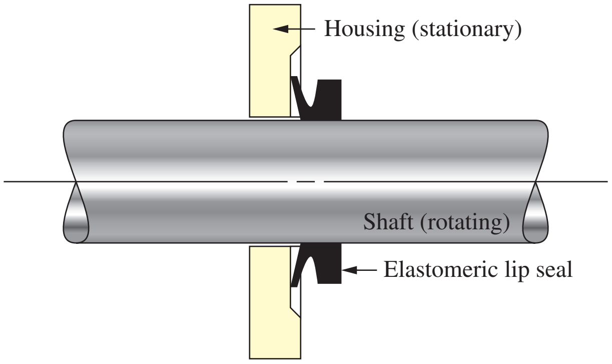
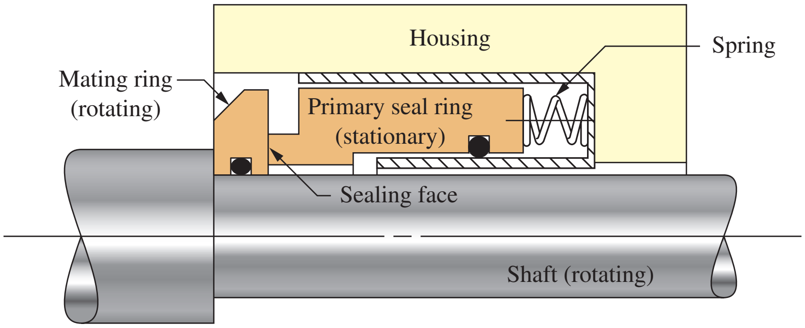


FIGURE 11–41 Lip seals, U-cup seal, wiper, and O-rings applied to a hydraulic actuator



(*a*) Lip-type face seal



(*b*) Mechanical face seal

FIGURE 11–42 Face seals

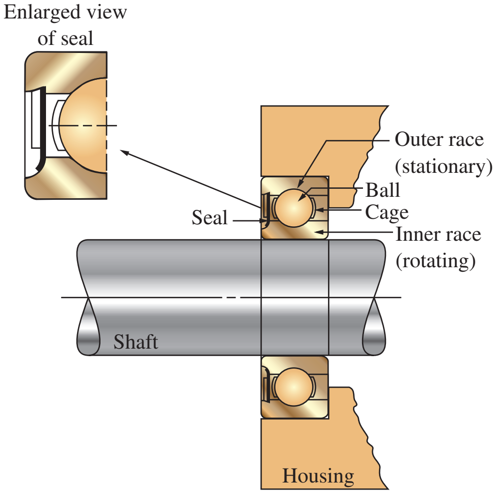


FIGURE 11–43 Seal for ball bearing

Elastomers

Resilient seals such as O-rings, T-rings, and lip seals are often made from synthetic elastomers such as the following:

Neoprene	Butyl	Nitrile (Buna-*N*)
Fluorocarbon	Silicone	Fluorosilicone
Butadiene	Polyester	Ethylene propylene
Polysulfide	Polyurethane	Epichlorohydrin
Polyacrylate	PNF (Phosphonitrilic fluoroelastomer)	

Many proprietary formulations within these general classifications are available under trade names from seal producers and plastics manufacturers.

Properties needed in a given installation will limit the selection of candidate materials. The following list gives some of the more prevalent requirements for seals and some of the materials that meet those requirements:

Weather resistance: Silicone, fluorosilicone, fluorocarbon, ethylene propylene, polyurethane, polysulfide, polyester, neoprene, epichlorohydrin, and PNF.

Petroleum fluid resistance: Polyacrylate, polyester, PNF, nitrile, polysulfide, polyurethane, fluorocarbon, and epichlorohydrin.

Acid resistance: Fluorocarbon.

High-temperature operation: Ethylene propylene, fluorocarbon, polyacrylate, silicone, and PNF.

Cold-temperature operation: Silicone, fluorosilicone, ethylene propylene, and PNF.

Tensile strength: Butadiene, polyester, and polyurethane.

Abrasion resistance: Butadiene, polyester, and polyurethane.

Impermeability: Butyl, polyacrylate, polysulfide, and polyurethane.

Rigid Materials

Face seals, and the parts of other types of sealing systems against which elastomers seal, require rigid materials that can withstand the sliding action and that are compatible with the environment around the seal. Some typical rigid materials used in sealing systems are described in the following list:

Metals: Carbon steel, stainless steel, cast iron, nickel alloys, bronze, and tool steels.

Plastics: Nylon, filled polytetrafluoroethylene (PTFE), and polyimide.

Carbon, ceramics, and tungsten-carbide.

Plating: Chromium, cadmium, tin, nickel, and silver.

Flame-sprayed compounds.

Packings

Packings for sealing shafts, rods, valve stems, and similar applications are made from a variety of materials, including leather, cotton, flax, several types of plastics, braided or twisted wire made from copper or aluminum, laminated cloth and elastomeric materials, and flexible graphite.

Gaskets

Common gasket materials are cork, cork and rubber compounds, filled rubber, paper, resilient plastics, and foams. See Internet site 15.

Shafts

When radial lip seals are required around shafts, the shafts are typically steel. They should be hardened to HRC 30 to resist scoring of the surface. Tolerance on the diameter of the shaft on which the seal bears should conform to the following recommendations to ensure that the seal lip can follow the variations:

Shaft Diameter (in)	*Tolerance (in)*
$D \leq 4.000$	± 0.003
$4.000 < D \leq 6.000$	± 0.004
$D > 6.000$	± 0.005

The surface of the shaft and any areas over which the seal must pass during installation should be free of burrs to protect against tearing the seal. A surface finish of 10 to 20 μin is recommended, with adequate lubrication to ensure full contact and to reduce friction between the seal and the shaft surface.

REFERENCES

1. American Gear Manufacturers Association. AGMA 9009-D02 (R2014). *Nomenclature for Flexible Couplings.* Alexandria, VA: American Gear Manufacturers Association, 2014.
2. American Gear Manufacturers Association. ANSI/AGMA 9002-C14. *Bores and Keyways for Flexible Couplings (Inch Series)*. Alexandria, VA: American Gear Manufacturers Association, 2014.
3. American Gear Manufacturers Association. AGME 9112-B15. *Bores and Keyways for Flexible Couplings (Metric Series)*. Alexandria, VA: American Gear Manufacturers Association, 2015.
4. American Gear Manufacturers Association. ANSI/AGMA 9001-B97 (R2014). *Lubrication of Flexible Couplings*. Alexandria, VA: American Gear Manufacturers Association, 2014.
5. American National Standards Institute. ANSI B92.1–1986. *Involute Splines*. New York: American National Standards Institute, 1996.
6. American National Standards Institute. ANSI B92.2M–1989 (R1989). *Metric Module Involute Splines*. New York: American National Standards Institute, 1989.
7. American Society of Mechanical Engineers. ANSI B17. 1–67.(R2013). *Keys and Keyseats*. New York: American Society of Mechanical Engineers, 2013.
8. American Society of Mechanical Engineers. ANSI B17.2–67.R98. *Woodruff Keys and Keyseats*. New York: American Society of Mechanical Engineers, 1998.
9. Oberg, Erik et al. *Machinery's Handbook*. 30th ed. New York: Industrial Press, 2015.
10. Society of Automotive Engineers. *Standard AS-84D,* (R2014-02-06) *Splines, Involute (Full Fillet)*. Warrendale, PA: Society of Automotive Engineers, 2014.
11. International Standards Organization. *ISO Standard 4156–1:2005, Straight cylindrical involute splines–Metric Module*. Geneva, Switzerland: International Standards Organization, 2005.
12. Flitney, R. K. *Seals and Sealing Handbook*. 6th ed. New York: Elsevier Science, 2014.
13. Summers-Smith, J. D. (Editor). *Mechanical Seal Practice for Improved Performance*. New York: John Wiley & Sons, 2005.
14. Bickford, John. *Gaskets and Gasketed Joints*. Boca Raton, FL: CRC Press, 1998.
15. Czernik, Daniel. *Gaskets: Design, Selection, and Testing*. New York: McGraw-Hill, 1996.

INTERNET SITES FOR KEYS, COUPLINGS, AND SEALS

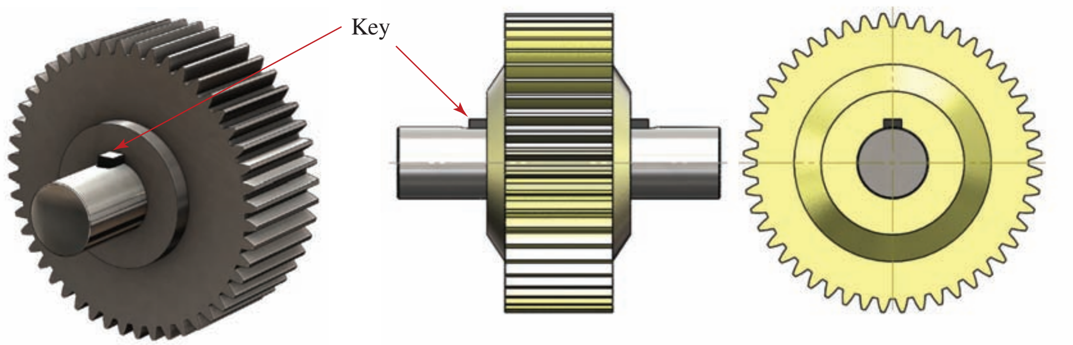
1. **Fastenal Company.** Provider of a wide variety of mechanical products including keystock in low-, medium-, and high-carbon steel, 18-8 and 316 stainless steel. Both U.S. and metric sizes. Square, rectangular, and woodruff types.
2. **Ringfeder Power Transmission GMBH.** Manufacturer of Ringfeder and ECOLOC keyless hub to shaft locking devices, along with Tschan and Gerwah couplings.
3. **Baldor/Dodge.** Manufacturer of a wide variety of power transmission components including flexible couplings, gearing, belt drives, clutches and brakes, and bearings.
4. **Regal Beloit Americas, Inc.** Manufacturer of a wide variety of power transmission equipment including flexible couplings and universal joints under the Kop-Flex, Browning, and Morse brands. Other brands include McGill, Rollway, and Sealmaster for gearing, belt drives, chain drives, bearings, conveyor components, drive shafts, and clutches.
5. **Dana Limited—Spicer Drivetrain Components.** Manufacturer of universal joints, and drive shafts for vehicular and industrial applications using the Dana and Spicer brand names.

6. **GWB—A Dana Brand.** Manufacturer of heavy-duty drive shafts for industrial equipment, locomotives, and similar applications.
7. **Stock Drive Products/Sterling Instrument.** Manufacturer and distributor of precision machine components and assemblies, including flexible couplings, gearing, clutches and brakes, and fasteners.
8. **T. B. Wood's Sons Company.** Manufacturer of mechanical, industrial power transmission products including flexible couplings, synchronous belt drives, V-belt drives, gearmotors, and gearing.
9. **Curtis Universal Joint Company.** Manufacturer of universal joints for the industrial and aerospace markets in U.S. and metric sizes.
10. **Cooper Tools/Apex Universal Joints.** Manufacturer of universal joints for the military, aerospace, performance racing, and industrial power transmission markets. Product line includes the Cornay™ universal joints.
11. **Rotor Clip Company, Inc.** Manufacturer of a wide variety of retaining rings and other products in U.S. and metric sizes for industrial, commercial, military, and consumer products.
12. **Federal Mogul Powertrain.** Manufacturer of seals for engines, transmissions, wheels, differentials, and industrial applications. Select Products, then Engine or Sealing.
13. **American Seal & Packing Company.** Manufacturer of O-rings, gaskets, mechanical seals, and related products.
14. **Mechanical Seals.net.** Part of the site for American Seal and Packing Company providing technical information about the design of various types of seals, how they work, how they are maintained, and the materials used.
15. **IGS Industries.** Manufacturer of gaskets, shims, and custom fabricated seals, and industrial sealants.
16. **American High Performance Seals, Inc.** Manufacturer of a wide variety of wipers and seals for rods, pistons, and rotary applications. Site includes numerous graphic representations of seal cross sections and tables of materials and their properties.
17. **Lord Corporation—Vibration & Motion Control Products.** Manufacturer of flexible couplings, vibration mounts, and shock isolation mounts using elastomeric materials bonded to metals. Select Products & Solutions, then Vibration & Motion Control.
18. **General Polygon Systems, Inc.** Providers of mechanical shaft to hub connections using the General Polygon System.
19. **Belden Universal.** Manufacturer of universal joints, drive shaft assemblies, and couplings for industrial, nuclear, aerospace, and other applications.
20. **G. L. Huyett Co.** Supplier of numerous machine components, including square and rectangular keys and keystock, bars, shafts, shim and shimstock in both U.S. and SI sizes and in several grades of material. From the home page, select *Power Transmission.*
21. **LOCTITE® Industrial Adhesives & Sealants/Henkel Corporation.** Manufacturer of numerous types of adhesives and sealants, including LOCTITE® Threadlockers® that inhibit loosening of screws and other threaded products.
22. **Grainger.** Distributor of a huge array of products for machinery components, hardware, motors, power transmission drive products, bearings, flexible couplings, universal joints, fasteners, retaining rings, keys and key stock, raw materials, welding products, adhesives, seals and sealants, threadlocking materials, linear motion devices, and many more.
23. **Federal Mogul Corporation/Powertrain.** Manufacturer of automotive components and products for industrial uses, energy, and transport, including seals crankshafts, camshafts, and oil control rings on pistons.
24. **Rexnord – Thomas Flexible Disc Couplings.** Manufacturer of a broad line of flexible disc couplings for pump drives, fans, compressors, and general industrial machinery.

PROBLEMS

For Problems 1–4 and 7, determine the required key geometry: length, width, and height. Use SAE 1018 steel for the keys if a satisfactory design can be achieved. If not, use a higher-strength material from Table 11–4. Unless otherwise stated, assume that the key material is weakest when compared with the shaft material or the mating elements.

1. Specify a key for a gear to be mounted on a shaft with a 2.00 in diameter. The gear transmits 21 000 lb·in of torque and has a hub length of 4.00 in. See Figure P11–1.
2. Specify a key for a gear carrying 21 000 lb·in of torque if it is mounted on a 3.60-in-diameter shaft. The hub length of the gear is 4.00 in.



(*a*) Assembly (*b*) Side view (*c*) End view

FIGURE P11–1 Gear, shaft, and key for Problem 11–1

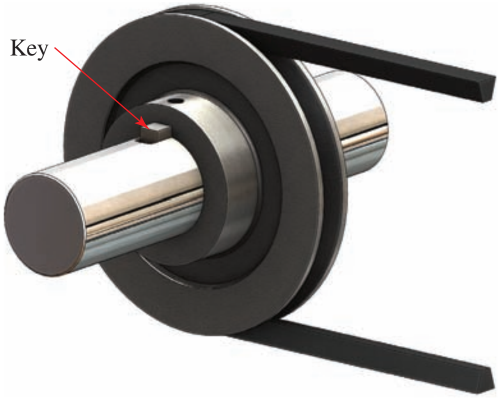


FIGURE P11–3 Belt sheave, shaft, and key for Problem 11–3

3. A V-belt sheave transmits 1112 lb·in of torque to a 1.75-in-diameter shaft. The sheave is made from ASTM class 20 cast iron and has a hub length of 1.75 in. See Figure P11–3.
4. A chain sprocket delivers 110 hp to a shaft at a rotational speed of 1700 rpm. The sprocket has a bore diameter of 2.50 in. The hub length is 3.25 in.
5. Specify a suitable spline having a *B* fit for each of the applications in Problems 1–4.
6. Design a cylindrical pin to transmit the power, as in Problem 4. But design it so that it will fail in shear if the power exceeds 220 hp.
7. Specify a key for both the sprocket and the wormgear from Example Problem 12–3. Note the specifications for the final shaft diameters at the end of the problem.
8. Describe a Woodruff key no. 204.
9. Describe a Woodruff key no. 1628.
10. Make a detailed drawing of a Woodruff key connection between a shaft and the hub of a gear. The shaft has a diameter of 1.500 in. Use a no. 1210 Woodruff key. Dimension the keyseat in the shaft and the hub.
11. Repeat Problem 10, using a no. 406 Woodruff key in a shaft having a 0.500 in diameter.
12. Repeat Problem 10, using a no. 2428 Woodruff key in a shaft having a 3.250 in diameter.
13. Compute the torque that could be transmitted by the key of Problem 10 on the basis of shear and bearing if the key is made from SAE 1018 cold-drawn steel with a design factor of $N = 3$.
14. Repeat Problem 13 for the key of Problem 11.
15. Repeat Problem 13 for the key of Problem 12.
16. Make a drawing of a four-spline connection having a major diameter of 1.500 in and an *A* fit. Show critical dimensions. See Figure P11–16 for the general layout.
17. Make a drawing of a 10-spline connection having a major diameter of 3.500 in and a *B* fit. Show critical dimensions.
18. Make a drawing of a 16-spline connection having a major diameter of 2.500 in and a *C* fit. Show critical dimensions.
19. Determine the torque capacity of the splines in Problems 16–18.
20. Describe the manner in which a set screw transmits torque if it is used in place of a key. Discuss the disadvantages of such an arrangement.
21. Describe a press fit as it would be used to secure a power transmission element to a shaft.

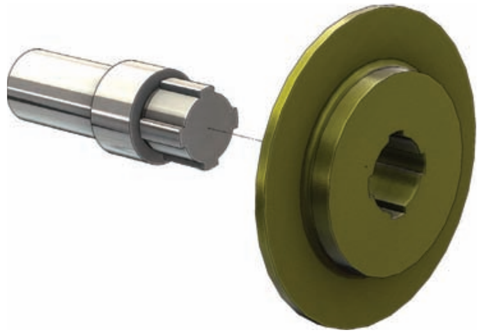
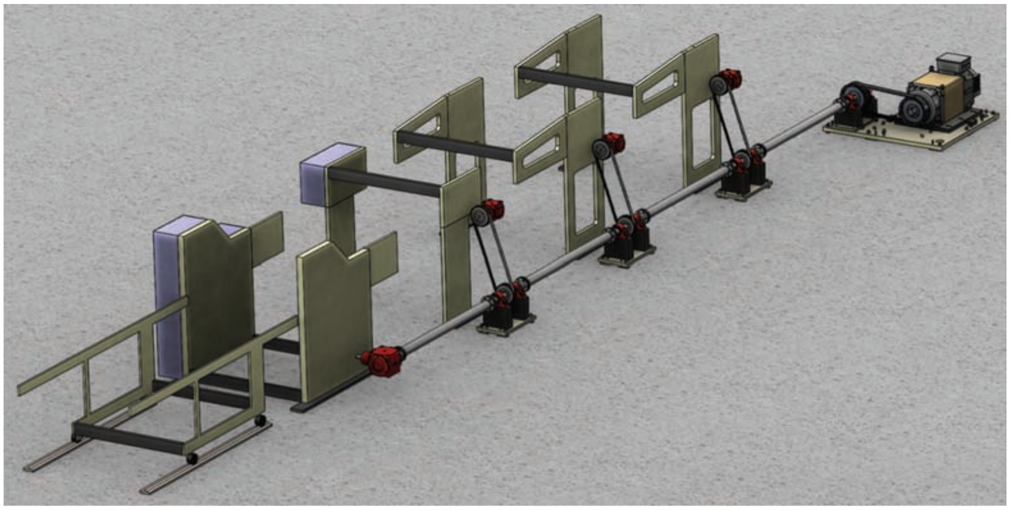


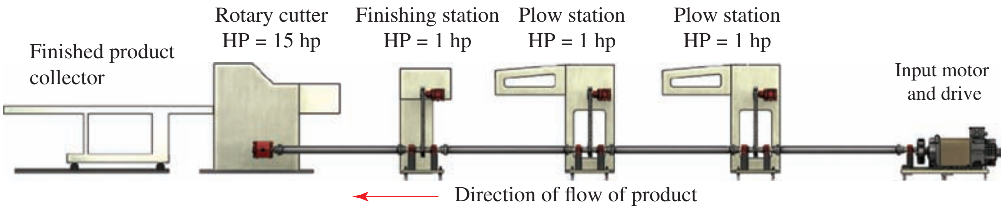
FIGURE P11–16 Pictorial view of splined shaft connection

22. Describe the main differences between rigid and flexible couplings as they affect the stresses in the shafts that they connect.
23. Discuss a major disadvantage of using a single universal joint to connect two shafts with angular misalignment.
24. Describe five ways to locate power transmission elements positively on a shaft axially.
25. Describe three situations in which seals are applied in machine design.
26. List eight parameters that should be considered when selecting a type of seal and specifying a particular design.
27. Name three means of sealing a pressurized container under static conditions.
28. Name three methods of sealing a closed container while allowing relative movement of some part.
29. Name four types of seals used on reciprocating rods or pistons.
30. Name three types of seals applied to rotating shafts.
31. Describe the method of sealing a ball bearing from contaminants.
32. Describe an O-ring seal, and sketch its installation.
33. Describe a T-ring seal, and sketch its installation.
34. Describe some advantages of T-ring seals over O-rings.
35. Describe a diaphragm seal and the type of situation in which it is used.
36. Describe suitable methods of sealing the sides of a piston against the inner walls of the cylinder of a hydraulic actuator.
37. Describe the function of a scraper or wiper on a cylinder rod.
38. Describe the essential elements of a mechanical face seal.
39. Name at least six kinds of elastomers commonly used for seals.
40. Name at least three kinds of elastomers that are recommended for use when exposed to weather.
41. Name at least three kinds of elastomers that are recommended for use when exposed to petroleum-based fluids.
42. Name at least three kinds of elastomers that are recommended for use when exposed to cold-temperature operation.
43. Name at least three kinds of elastomers that are recommended for use when exposed to high-temperature operation.
44. A sealing application involves the following conditions: exposure to high-temperature petroleum fluids and impermeability. Specify a suitable elastomer for the seal.
45. A sealing application involves the following conditions: exposure to high temperature and weather, impermeability, and high strength and abrasion resistance. Specify a suitable elastomer for the seal.
46. Describe suitable shaft design details where elastomeric seals contact the shaft.

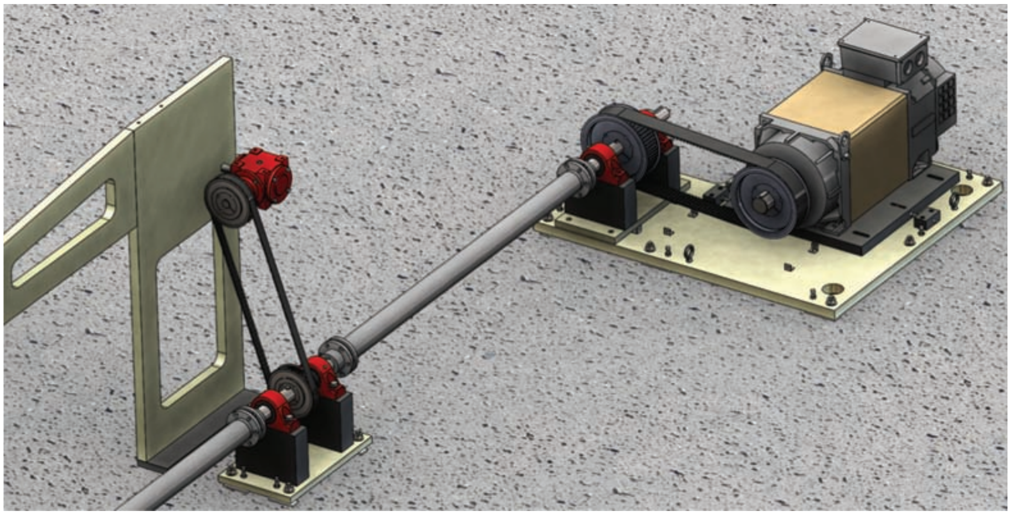
47. A driveline is to be designed to drive a series of four pieces of equipment for the printing industry as shown in Figure P11–47(a). An electric motor at the right drives the driveline at a rotational speed of 500 rpm and the power demand for each piece of equipment is shown in Figure P11–47(b). Figure P11–47(c) of the figure shows an enlarged view of the drive motor and the first segment of the driveline. All four segments are to be identical. Figure P11–47(d) of the figure shows the general design of one driveline segment. The distance between the ends of each drive shaft segment is to be 63.00 in. Select a floating shaft–type coupling to connect each piece of equipment to a 2.00-in-diameter rigid jack shaft assembly. Refer Section 11–7 for design data.



(*a*) General layout of the machinery for a press line for the printing industry.



(*b*) Power demand for the four pieces of equipment



(*c*) Detail for the drive motor, synchronous belt reducer from the motor to the drive shaft, the floating shaft–type coupling and the belt drive to the first plow station

(*d*) General design of the coupling for each segment of the driveline

FIGURE P11–47

CHAPTER TWELVE

SHAFT DESIGN

THE BIG PICTURE Shaft Design

Discussion Map

- A shaft is a rotating machine component that transmits power.
- Designers must create a practical shape for the shaft and specify suitable diameters that ensure that the shaft is safe under expected loads.

Discover

Identify examples of mechanical systems that incorporate power-transmitting shafts. Describe their geometry and the forces and torques that are exerted on them.

What kinds of stresses are produced in the shaft?

How are other elements mounted on the shaft? How does the shaft geometry accommodate them? How is the shaft supported? What kinds of bearings are used?

This chapter provides approaches that you can use to design shafts that are safe for their intended use. But you have the final responsibility for the design.

A *shaft* is the component of a mechanical device that transmits rotational motion and power. It is integral to any mechanical system in which power is transmitted from a prime mover, such as an electric motor or an engine, to other rotating parts of the system. Can you identify some kinds of mechanical systems incorporating rotating elements that transmit power?

Here are some examples: gear-type speed reducers, belt or chain drives, conveyors, pumps, fans, agitators, and many types of automation equipment. What others can you think of? Consider household appliances, lawn maintenance equipment, parts of a car, power tools, and machines around an office or in your workplace. Describe them and discuss how shafts are used. From what source is power delivered into the shaft? What kind of power-transmitting element, if any, is on the shaft itself? Or is the shaft simply transmitting the rotational motion and torque to some other element? If so, how is the shaft connected to that element?

Visualize the forces, torques, and bending moments that are created in the shaft during operation. In the process of transmitting power at a given rotational speed, the shaft is inherently subjected to a torsional moment, or *torque*. Thus, torsional shear stress is developed in the shaft. Also, a shaft usually carries power-transmitting components, such as gears, belt sheaves, or chain sprockets, which exert forces on the shaft in the transverse direction (perpendicular to its axis). These transverse forces cause bending moments to be developed in the shaft, requiring analysis of the stress due to bending. In fact, most shafts must be analyzed for combined stress. Often, the forces applied to shafts act in virtually any direction, not just in a single plane.

Describe the specific geometry of shafts from some types of equipment that you can examine. Make a sketch of any variations in geometry that may occur, such as changes in diameter, to produce shoulders, grooves, keyseats, or holes. How are any power-transmitting elements held in position along the length of the shaft? How are the shafts supported? Typically, bearings are used to support the shaft while permitting rotation relative to the housing of the machine. What kinds of bearings are used? Do they have rolling elements such as ball bearings? Or, are they smooth-surfaced bearings? What materials are used?

It is likely that you will find much diversity in the design of the shafts in different kinds of equipment. You should see that the functions of a shaft have a large influence on its design. Shaft geometry is greatly affected by the mating elements such as bearings, couplings, gears, chain sprockets, or other kinds of power-transmitting elements.

This chapter provides approaches that you can use to design shafts that are safe for their intended use. But the final responsibility for the design is yours because it is impractical to predict in a book all conditions to which a given shaft will be subjected.

YOU ARE THE DESIGNER

Consider the gear-type speed reducer shown in Figure 1–12. Note that there are three shafts that must be designed. The input shaft carries the first gear in the gear train and rotates at the speed of the prime mover, typically an electric motor or an engine. The middle shaft carries two gears and rotates more slowly than the input shaft because of the first stage of speed reduction. The final gear in the train is carried by the third shaft that rotates slower than shaft 2 and which also transmits the power to the driven machine. From what material should each shaft be made? What torque is being transmitted by each shaft, and over what part of the shaft is it acting? How are the gears to be located on the shafts? How is the power to be transmitted from the gears to the shafts, or vice versa? What forces are exerted on the shaft by the gears, and what bending moments result? What forces must the bearings that support each shaft resist? What are the minimum acceptable diameters for the shafts at all sections to ensure safe operation? What should be the final dimensional specifications for the many features of the shafts, and what should be the tolerances on those dimensions? The material in this chapter will help you make these and other shaft design decisions.

12–1 OBJECTIVES OF THIS CHAPTER

After completing this chapter, you will be able to:

1. Propose reasonable geometries for shafts to carry a variety of types of power-transmitting elements, providing for the secure location of each element and the reliable transmission of power.
2. Compute the forces exerted on shafts by gears, belt sheaves, and chain sprockets.
3. Determine the torque distribution on shafts.
4. Prepare shearing force and bending moment diagrams for shafts in two planes.
5. Account for stress concentration factors commonly encountered in shaft design.
6. Specify appropriate design stresses for shafts.
7. Apply the shaft design procedure shown in this chapter to determine the required diameter of shafts at any section to resist the combination of torsional shear stress and bending stress.
8. Specify reasonable final dimensions for shafts that satisfy strength requirements and installation considerations and that are compatible with the elements mounted on the shafts.
9. Consider the influence of shaft rigidity on its dynamic performance.

12–2 SHAFT DESIGN PROCEDURE

Because of the simultaneous occurrence of torsional shear stresses and normal stresses due to bending, the stress analysis of a shaft virtually always involves the use of a combined stress approach. The recommended approach for shaft design and analysis is the *distortion energy theory of failure*. This theory was introduced in Chapter 5 and will be discussed more fully in Section 12–5. Vertical shear stresses and direct normal stresses due to axial loads may also occur. On very short shafts or on portions of shafts where no bending or torsion occurs, such stresses may be dominant. The discussions in Chapters 3–5 explain the appropriate analysis.

The specific tasks to be performed in the design and analysis of a shaft depend on the shaft's proposed design in addition to the manner of loading and support. With this in mind, the following is a recommended general approach for the design of a shaft.

This process will be demonstrated after the concepts of force and stress analysis are presented.

PROCEDURE FOR DESIGN OF A SHAFT

1. Determine the rotational speed of the shaft.
2. Determine the power or the torque to be transmitted by the shaft.
3. Determine the design of the power-transmitting components or other devices that will be mounted on the shaft, and specify the required location of each device.
4. Specify the location of bearings to support the shaft. Normally two and only two bearings are used to support a shaft. The reactions on bearings supporting radial loads are assumed to act at the midpoint of the bearings. For example, if a single-row ball bearing is used, the load path is assumed to pass directly through the balls. If thrust (axial) loads exist in the shaft, you must specify which bearing is to be designed to react against the thrust load. Then the bearing that does not resist the thrust should be permitted to move slightly in the axial direction to ensure that no unexpected and undesirable thrust load is exerted on that bearing.
   Bearings should be placed on either side of the power-transmitting elements if possible to provide stable support for the shaft and to produce reasonably well-balanced loading of the bearings. The bearings should be placed close to the power-transmitting elements to minimize bending moments. Also, the overall length of the shaft should be kept small to keep deflections at reasonable levels.
5. Propose the general form of the geometry for the shaft, considering how each element on the shaft will be held in position axially and how power transmission from each element to the shaft is to take place. For example, consider the shaft in Figure 12–1, which is to carry two gears as the intermediate shaft in a double-reduction, spur gear-type speed reducer. Gear *A* accepts power from gear *P* by way of the input shaft. The power is transmitted from gear *A* to the shaft through the key at the interface between the gear hub and the shaft. The power is then transmitted down the shaft to point *C*, where it passes through another key into gear *C*. Gear *C* then transmits the power to gear *Q* and thus to the output shaft. The locations of the gears and bearings are dictated by the overall configuration of the reducer.
   It is now decided that the bearings will be placed at points *B* and *D* on the shaft to be designed. But how will the bearings and the gears be located so as to ensure that they stay in position during operation, handling, shipping, and so forth? Of course, there are many ways to do this. One way is proposed in Figure 12–2. Shoulders are to be machined in the shaft to provide surfaces against which to seat the bearings and the gears on one side of each element. The gears are restrained on the other side by retaining rings snapped into grooves in the shaft. The bearings will be held in position by the housing acting on the outer races of the bearings. Keyseats will be machined in the shaft at the location of each gear. This proposed geometry provides for positive location of each element.
6. Determine the magnitude of torque that the shaft sees at all points. It is recommended that a torque diagram be prepared, as will be shown later.
7. Determine the forces that are exerted on the shaft, both radially and axially.
8. Resolve the radial forces into components in perpendicular directions, usually vertically and horizontally.
9. Solve for the reactions on all support bearings in each plane.

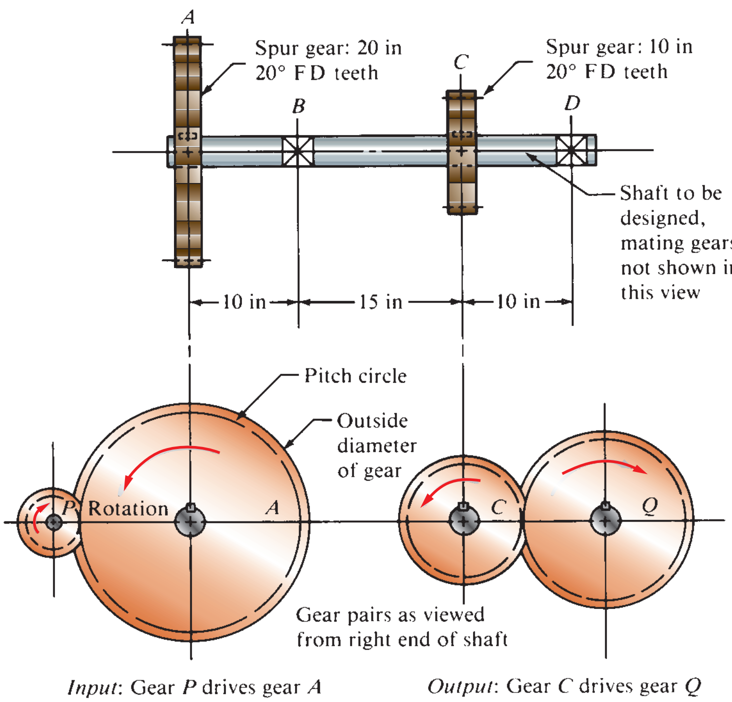


FIGURE 12–1 Intermediate shaft for a double-reduction, spur gear-type speed reducer