

## Lecture 01 - Static Fluids

0:00

So one of the first concepts that we need to talk about when we start thinking about fluids, and I should mention that we need to introduce fluid mechanics as a principle when we talk about unit operations, right?

0:11

Because in every unit operation, fluids are flowing, be them liquid or gas and these flowing fluids, we need to understand the principles of fluid mechanics and and the type, the effect of turbulence and how we can characterize that turbulence so that we can interpret what's happening in the unit operation itself.

0:33

However, before we can start talking about flowing fluids, we need to to think about what happens when a fluid's in equilibrium and static equilibrium and it's not moving.

0:41

So one of the first things we can look at is hydrostatic equilibrium, right?

0:46

And we can define that through this equation shown on top, where here we're saying little  $P$  is pressure multiplied by  $S$  is some surface area.

0:54

So that's a positive gain less than the pressure of this  $\Delta Z$  section,  $P$  plus or sorry,  $G \rho S \Delta Z$  on the right.

1:06

And then we have this  $P$  plus  $DP$  and that's the pressure differential of that section.

1:12

And that should all equilibrate out for the force balance.

1:15

And we can then of course recognize most importantly that when we're defining fluid head like this, we can identify our surface area terms.

1:25

And because these are all equilibrated, we can actually cancel surface area out of that.

1:30

So that yields an expression where we have the differential pressure being related to  $G$ , which is the gravitational constant 9 traditionally 9.81, but here we're going to actually look at a little more detail.

1:44

Rho is the density of the fluid over which it's applied, and DZ is the height of the column of interest.

1:50

So this allows us to relate the height of a hydrostatic column to the pressure that will be engendered by from that column.

1:58

Now if we define two heights relative to this diagram, ZA and ZB, then we can pop those into this expression and integrate it.

2:09

And so we end up with our pressure A on rho minus pressure B on rho is going to be equal to the gravity constant minor times, the difference between ZA and ZB.

2:22

And So what this does is also reveals to us a definition.

2:25

So when someone refers to the head of the pressure head or the fluid head, this is traditionally  $\rho G \Delta Z$ , which can equivalently as often written as just  $\rho GH$ , right, where H is being represented as the height of the column, right?

2:41

And this is a really important calculation to remember as an engineer because when we think of things, for instance, like the deepwater blowout in the Gulf of Mexico right at the Deepwater Horizon, you know, very quickly engineers have to get together and say, OK, we need to figure out what is the flow rate of oil that was spilling out into the Gulf of Mexico.

3:02

And we knew what the pressure in that pipeline was.

3:05

But we have to think, well, the flow rate is going to be defined by the difference in the pressure of that pipeline in the hydrostatic head of the ocean column, right?

3:13

And so how do we get there?

3:14

Well,  $\rho G$  times the height of the ocean column allows us to calculate that external pressure and from two points, then calculate how quickly the oil is going to flow out.

3:25

OK.

3:26

So the the main take home from this slide, right, is that this this pressure head term  $\rho G \Delta Z$  is going to come back a couple of times for us.

3:35

Now the a classical equation that I just wanted to highlight the barometric equation.

3:41

This comes from when we insert the ideal gas law into our hydrostatic equilibrium definition.

3:49

So here we can write ideal, the ideal gas law in a similar format, but  $RT$  equals  $P$  times the molecular weight on molar density and we can expand that.

3:59

If we put that definition of density into our our expression, we can expand this out now to have  $DP$  on  $P$  yeah,  $DPMP$  plus the gravity constant times molecular weight on  $RT$  times  $BZ$  and that will be equal to 0.

4:16

We can integrate this of course, at a constant temperature to yield an expression for the pressure difference as a function then of the difference in the sorry, the difference in the hydrostatic column height and parameters such as the molecular weight,  $RT$  and the gravity constant.

4:37

So this barometric equation is is quite things.

4:41

So I just wanted to to highlight one example.

4:43

Of course, if if you took Thermo, you'd remember that the ideal gas law makes a lot of assumptions, one of which is that we're operating at relatively low pressure.

4:51

So if we're thinking about things like water, this is probably not a very good derivation to use, but it's a historical point of context for you.

5:02

What we do use and a a very common application of hydrostatic equilibrium is manometers.

5:09

And so these are a very clever tool that exploit the working principle of hydrostatic equilibrium to measure pressure difference between two points in space.

5:20

If you are able to physically look measure the difference in the head of a fluid upstream and downstream at that point.

5:29

So we can take that same working equation that we have in the previous slide.

5:34

But if we look at that, we can simplify this to the difference in pressure.

5:37

So this might be something like my upstream pressure here and  $P_{\text{sub B}}$  here will be my downstream pressure, right?

5:47

And that will be equal to the gravity constant  $RM$ , which is the height difference between the manometer points two and four and row A minus row B, where in this case row A is the the manometer fluid density.

6:01

So, so that might be something like mercury that I'm going to fill this manometer with and fluid B, this will be the density in the fluid.

6:09

If I use red hatches here, this will be the density in the fluid above that, sorry, above that mercury.

6:19

And if we apply this right into the context of something like an orifice restriction, right, If we're engineers and we're going to introduce an orifice or if we have the ability to control the degree of choking of the orifice, right, how much restriction they're going to apply?

6:35

It's very useful if I can measure that pressure difference upstream and downstream, right?

6:40

And I can do that simply through applying this manometer into those conditions.

6:45

So in this case, that fluid B we can, sorry, fluid B I can imagine is actually the, the flowing field, right, all the way around this upper bit of the manometer.

6:59

And I'm only looking at then what happens in the mercury that makes up the lower part.

7:04

So to get a handle on that, let's do an example right and see where this comes into play.

7:09

So let's say we have a mercury manometer.

7:12

So the density of mercury will be 13,590kg per and it's used to measure the pressure drop of a flowing fluid.

7:23

In this case we'll think brine.

7:24

So maybe we're going to build a high, high pressure water pipeline that's going to go somewhere in Perth, right?

7:31

And so we're going to take ocean water, then take it for D cell.

7:33

So we need to measure the pressure drop of that brine using a manometer where the brine density is going to be 1260kg per cubic meter.

7:43

Now under no flow conditions, the level of mercury in the manometer is .9 meters below the orifice taps, right?

7:50

So that means that we have quite a large separation between where it contacts the orifice and where the mercury is.

7:56

And then when this pipeline is flowing at steady state, the gauge pressure upstream.

8:02

So that's this PA side is 0.14 bar and the pressure downstream of the tap we're going to say is 250mm of mercury below atmosphere.

8:21

So we want to know what is the reading of the manometer in millimeters, right?

8:24

This RM value, because that's what we're physically measuring in this case now, OK.

8:33

So one of the first points that we have to make is that it's very easy in this case, if we set the value of atmospheric pressure to be zero across the system.

8:44

So we're only going to be looking at changes from atmospheric, right?

8:47

And that will help us in characterizing the solution.

8:51

So we can say, OK, step one is we're going to call atmospheric pressure 0 the zero point, right?

9:04

And that will help us simplify so we can go back to our, our, our working equation.

9:09

Well, the first thing we need to know is  $P_A$  right now.

9:12

I was given that in bar, but I need that in Pascals and I'm going to use an SI unit set, which I, I always want to do for my calculations where possible.

9:20

So that's going to be 14,000 Pascals  $P_B$ .

9:27

All right.

9:27

So that's going to be the downstream pressure.

9:29

Now that pressure is dictated by the manometer reading.

9:33

So I can say that is rows of AG and then ZB.

9:40

So in this case, I have 13590 kilograms per cubic meter.

9:47

That's the density of my mercury, the gravitational constant.

9:51

Now I usually use 9.81 because again, McCabe was an author back, well, one of the original authors of chemical engineering, and his books date back to the 50s.

10:03

So McCabe uses 9.80665 meters per second squared, and then I have that ZB value, which is the difference in fluid height.

10:17

So this is going to be 250 millimetres of mercury below atmosphere.

10:23

So in this case I'm going to have -250 millimetres and I'm going to divide that by 1000 so I have the whole value in metres.

10:33

So this gives me a pressure B of negative 33318 pascals.

10:41

And that's negative because it's a pressure below atmosphere, right?

10:47

So this then means that I can pop those two together and I have  $P_A$  minus  $P_B$ .

10:52

But because  $P_B$  is a negative value, it in fact becomes 14,000 pascals plus 33,318 pascals, and that's going to be equal to  $R \cdot M$ . The height reading of the manometer times again 9.08665 meters per second squared times the density difference in my fluids.

11:20

So 13590 kilograms per cubic meter minus 1260 kilograms per cubic meter.

11:31

And if I solve for  $R \cdot M$ , this yields our value of 0.391 meters or 391 millimeters, right?

11:47

So it's important that this also tells me what the maximum differential pressure could be given the height of the manometer below the tap.

11:57

So I'm reaching just under 50% of the tap height through this measurement right now.

12:03

In this case, what we've done is to give you the pressure differential and ask you to solve for what is the height rating of that manometer using a principle of hydrostatics.

12:12

But equivalently, right.

12:13

And what would be more practical for an engineer is if you have a calibrated manometer, I'm going to give you a height rating and based on the known properties

of the fluid, ask you what is the downstream or the upstream pressure or simply the pressure drop right across some oracles restriction.

12:29

So do we have any questions on how to apply this kind of example or how to manipulate that hydrostatic equation for anomalous?

12:40

OK, so so shall it's a very good question.

12:49

Why is Pisa being negative?

12:51

This is negative.

12:52

Let me change colours so I can clarify.

12:56

Negative what?

13:00

It's negative because we're it's relative to atmospheric, right?

13:05

So it's 101325 minus 33318.

13:20

Does that make sense?

13:24

So it's a gauge pressure.

13:26

So obviously you can't have a negative pressure, right?

13:29

You can have a vacuum pressure, which is between zero and one absolute, but you only can have a negative value like this if you're in a gauge space.

13:38

So it's the atmospheric pressure minus 33318 pascals.

13:46

Yeah.

13:53

OK, let me know if that didn't clarify.



13:55

Yeah.

13:57

So another principle, right.

14:00

So that's a really common example of how we might use something like hydrostatic equilibrium in terms of an everyday measurement that's going to come up.

14:10

But another really common place to see hydrostatic equilibrium come up is actually in continuous gravity separation between 2 fluids.

14:18

And in this case, you might say to yourself, well, this is actually a constant flow scenario, right?

14:24

And so we have to make the assumption that we have a very slow liquid flow rate through the vessel relative to the size of the vessel as such that we can apply static fluid principles, right?

14:36

And within that assumption is inherent that we have no pressure grating within the vessel itself.

14:42

Now of course, we have a hydrostatic gradient, but we don't have a a momentum pressure driving force.

14:46

So there's not turbulent flow across that.

14:49

And of course, we have to assume there are no emulsions of water and oil mixing together or an aqueous and a hydrocarbon phase mixing.

14:57

And these are really comments on the top here.

14:59

This first diagram is a top view of the separator.

15:01

So you can see the feed coming in and feeding both sides of the separator where the the heavier fluid, the denser fluid will exit through the bottom.

15:11

And that's why it's a little hash circle.

15:13

Here we have a vent for a gas that might build up to keep it at a constant pressure.

15:18

And the light fluids will flow out horizontally through some point of the middle or upper bit of the separator.

15:25

And if we then break away and turn it and look at the side view here, we can see this would be our heavier fluid.

15:31

So this might be our aqueous system and this would be our hydrocarbon fluid, right, Where hydrocarbon's typically going to have a lower density than an aqueous system.

15:45

So if I very slowly feed these fluids in and I allow them to equilibrate in a separator for some amount of time, right, We're going to look at an example of what that separation time might be.

15:56

Then after some amount of time, I can apply this hydrostatic balance where I'm looking at the different heights.

16:01

So ZB being the height of that hydrocarbon or the the upper layer of liquid row B being it's density.

16:10

ZA one is the height of the aqueous or the bottom layer.

16:13

Row A is the the density of that layer and ZA 2 is the height difference between the inlet and the outlet streams for the heavy liquid, and row A is then the density of that denser.

16:28

So I can rearrange this hypostatic balance to solve explicitly for something like what is the height of that heavier liquid in the system?

16:39

And where this actually comes into play is if you are bringing together something like the offshore gas condensate systems on the Northwest Shelf.

16:48

In fact, you're typically producing condensate, which itself is a light hydrocarbon and you produce water at the same time.

16:55

And so in fact, this is the exact type of separator that a lot of these systems will have where they will bring these in from the wellhead through a fluid stream.

17:05

They run them into this gravity based separator.

17:08

These fluids then sit in the separator for some amount of time and they allow hydrostatic separation, right.

17:14

And so the same principles we're talking about here or, or what's governing that they would pull the condensate out through this light stream and then they would pull the oil out.

17:31

We're sorry, the water out through that heavier leaking stream.

17:36

So an example the and the textbook does go in a bit more detail about this.

17:41

But the example if we look at there's an empirical relationship for separation time for this type of separator presented by McCain and it's empirical, right?

17:51

So we have to be careful that we have to use the units exactly as they specify.

17:55

So we can't manipulate units in this.

17:57

But this empirical relationship is 100 times the value of  $\mu$  where  $\mu$  is going to be the viscosity of my continuous phase.

18:04

So in this case, whatever is the, the, the larger phase that's coming in requiring separation of a smaller phase out of it.

18:11

In this case, the continuous phase in our example is actually going to be the the condensate because that would be the the primary product of interest.

18:20

And then row A and row B is the density difference between MY2 immiscible liquids that I'm trying to separate.

18:26

So if I have a a condensate or a viscosity value 1.1 centipoise, right, And I say that my density of of fluid B.

18:35

So that's going to be the condensate is going to be 865 kilograms per cubic metre, right?

18:43

And a typical saline water density might be 1153 kilograms kilograms per cubic metre.

18:53

Then in fact, the the time that that outputs is about .38 hours, right?

19:01

So just under half an hour of separation time is going to be required if I have a what would be called a, a dead condensate and a sail line water in these conditions.

19:12

And of course because that you can see the separation time here is driven by the density difference between the fluids.

19:19

This continuous gravity separation doesn't always work when we have a very, very small density difference between the fluids.

19:27

So let's say I was trying to separate really heavy oil.

19:30

So if I'm working in Canada, for instance, some of the oil that comes out of Canada, it can actually have a density greater than one, right?

19:38

Because it has a lot of bitumen content in it.

19:42

And so if I'm trying to sit and then I have saline water that maybe the density is 1.2 and I'm trying to separate saline water from this heavy bitumen based oil, then I might need a more effective technique than just allowing that gravity based separation, right?

20:00

And this is where something like a centrifugal to cancer can come from.

20:04

And so we can introduce these centrifugal to cancers.

20:08

When we have a very, very small density difference between our two fluids And the idea is that my feedstock, right, would come into the centre of this thing and it's rotating.

20:17

So it's going to use a fluid motion to drive the heavier fluid toward the outside of the system, which I can then extract off.

20:25

And it's going to maintain the lighter fluid on the interior of the system based on the degree of rotation.

20:31

And McCabe in the textbook do not go into this in any great detail and there's no example that we're going to do on it.

20:36

But I just wanted to make you aware that even though the gravity decanters or gravity separators tend to be the most common operation that will do hydrostatic operation that we'll do for liquid, liquid separation.

20:48

When we get to that point of really small density differences, there are more elaborate methods we can go to such as a an eccentrifugal decanter.

20:57

Similarly, if I go back a few slides, that really simple manometer we looked at, you know this, this working equation is based on the density difference between the fluids.

21:08

Right now we'll choose my manometer fluid relative to the fluid that I'm going to measure in that space B.

21:15

But in fact, right, in cases where I have a very, very small density difference between the fluids, we can also look at something like an inclined manometer, right?

21:26

We're going to give it some.

21:29

A is going to be the degree of inclination of the system, right?

21:32

And then the difference between an equivalent fluid height level and the run that that takes through the pipe will be the characterization of  $R_{sub M}$ .

21:42

And that gives us some more sensitive measurement when we have a very, very small.

21:45

So we can take that monomer and basically instead of making it a perfect view, we can stretch that out and use that run of the fluid to get a more sensitive measurement.

21:55

So today was meant to be just a very, very simple, a quick introduction to section two of our textbook from McCabe on fluid mechanics.

22:04

And that's of course because some of us have not yet taken fluid mechanics and we really need to have the basics of fluids down before we can start interpreting unit operations in the laboratory.

22:15

The two kind of take home messages I wanted to provide were that once we look at the force balances on static fluids, it enables the definition of what engineers typically call head, right.

22:25

So this is either typically referenced as  $\rho GH$  or  $\rho G$  should be a little  $G$ ,  $\rho G$ ,  $\Delta Z$ .

22:34

And they're kind of used interchangeably in the field.

22:37

And we can in one lecture we can exploit these principles to look at something like a pressure drop measurement or looking at the separation of two fluids.

22:47

They're very remissible.

22:49

So with that.

Lecture 02 - Fluids in Motion

0:00  
OK.

0:01  
So if we first want to review Lecture 1 concepts, we learned early on that that if we're looking at static fluids, so these are things that are not in motion.

0:11  
Well, we can understand that the how to characterize these static fluids if we apply, apply a force balance and specifically that forest balance when we apply it to something like a, a column of fluid gives us the definition of head.

0:26  
And in particular, that's not just for the liquid head that you might think of as in the ocean, right?

0:31  
It's one of the most common examples that can also apply to things like the rock layers that sit underneath the ocean or if you have different fluids sitting in Ceres.

0:40  
And so it's very common that engineers when they talk will define head as  $\rho GH$ .

0:45  
But we recognize that that that H actually represents a  $\Delta Z$  or a delta height.

0:51  
And so that that together gives us a pressure that's being applied by that liquid.

0:55  
And one of the the really key principles to highlight here is this is independent of surface area.

1:04  
It is only a direct function of the height differential.

1:08  
So that's a really important clarification to kind of keep in your mind.

1:11  
So think about head is it's not a surface area phenomena.

1:15  
We then looked at a few kind of simple operations and measurements that can exploit this principle of static fluids, manometers being one of the most common.

1:25

These can either be simple manometers or a slightly more advanced inclined manometer shown to the right.

1:31

And we would use manometers to try to measure the differential pressure between two points.

1:36

And so that can be applied to something like a pipeline that can be applied to any system where we have a pressure differential.

1:43

And we like these because it's very easy to read out what that pressure difference looks like simply based on a difference in this  $R_{sub M}$ .

1:50

And so the pressure difference the manometer is going to read it's  $P_{sub A} - P_{sub B}$  is a function of that  $R_{sub N}$  that we measure and simply the known difference in the density of the two fluids.

2:03

We then talked a little bit about decanters.

2:06

And so gravity decanters are often used in oil and gas, and these will be used to separate hydrocarbon from an aqueous phase where we effectively allow gravity to do the work for us based again on that very simple density difference.

2:19

When the densities of these two fluids are very similar to each other, that's when we get into something more advanced like a central bugle decanter.

2:27

And so that's where we have these kind of spinning blades that are going to create an additional force pushing those fluids toward the outside.

2:34

So very simple introductory lecture, but getting us into thinking about these fluids in a static sense.

2:42

So now we get a little bit more complex, right?

2:44

And what happens when we start putting fluids into motion?

2:48

Well, initially, right, if we're operating in a laminar flow condition where we're not going to have a lot of turbulence.



2:54

So we're going to think about the most simple case where we're going to take a fluid column and very slowly try to create some motion through that column, right?

3:04

Not slashing it, not not creating a lot of turbulence, but just a little bit of motion through the column.

3:10

And one of the first things that we can recognize there is that if we imagine that for look at this plot on the left hand side that we have local fluid velocity on the X axis where we're going to go from a no velocity condition at the intercept and up into the bulk.

3:27

We can see this is asymptoting outward, right?

3:30

So it's coming into set an equilibrium velocity measurement out in the bulk away from a wall.

3:35

And that velocity will be defined as the net or the mean velocity of my fluid.

3:42

When I'm in this laminar flow condition where I'm not creating a lot of turbulence.

3:46

We have a very unique condition where as we approach the as we approach the wall itself, there is a no slip condition being applied from that fluid to the wall.

3:58

And where this comes from is we can consider that there's a molecular interface if I imagine.

4:04

Here's my wall of interest, right And they're at a molecular level.

4:10

I imagine all of these these molecules in my fluid, right where the very bottom layer is going to be absorbed to that static wall.

4:23

So because the wall itself, if I take that as the a static reference frame where the fluid is moving in the wall static, then that first layer of of molecules at the wall must have continuity with the wall itself.

4:37

So that means the fluid velocity in that first atomic layer of molecules or atomic layer of my species cannot be moving.

4:48

So that engenders then the definition that the fluid velocity must equal 0 when I'm at the wall.

4:56

And that's the so-called no slip condition.

4:59

And where this comes from is a consequence of interfacial tension or surface free energy.

5:04

And, and one of the, the points I wanted to make for those who took Thermo with me last semester, right, We'll remember that our definition DU so internal energy is going to be TDs minus PDV, all right.

5:18

And this could be for a, a single species.

5:21

So looking at something like water for instance, right now that's a volumetric definition.

5:28

But I wanted to point out that I can also write that if I don't have a three-dimensional species, but I only have a two-dimensional plane of existence, I can write it as DU and I can maybe say alpha is going to define an interfacial or A2 dimensional existence.

5:45

So the internal energy of that two-dimensional plane will be T times DS alpha.

5:54

But I don't have a volume, so my DV term goes away, right?

5:59

But what do I replace that with?

6:00

So what's the equivalent of PDV work or PDV energy in A2 dimensional plane of existence?

6:08

So imagine you're living in a cartoon, right?

6:10

When you don't get that third dimension in your life, you only have two to play with, so there's no  $V$  in that.

6:15

So we replace  $PDV$  with  $\gamma dA$ , right, And that  $dA$ , right?

6:23

So I can see  $dA$  is going to be my two-dimensional equivalent of my three-dimensional volume  $DV$ .

6:29

And  $\gamma$  is a 2 dimensional equivalent of pressure in three dimensions.

6:34

All right, So this is just a fun kind of a slide for you, but it points out that when we talk about interfacial tension, right, it's a term that comes up in engineering quite a bit.

6:43

And then I would assume you're going to hear it multiple times throughout your careers, right?

6:50

And interfacial tension itself is actually a product of the definition of internal energy when we're only looking at 2 dimensional plane existence.

7:00

And we also can call this surface free energy.

7:02

So as a consequence of having tension between my fluid and the wall, I, I then have this this reality that the fluid velocity itself must be zero at the point of the wall.

7:15

So if I know that that I have a zero condition at the wall and I have some equilibrium velocity as I come into the the bulk of the continuous phase, then it's going to to slowly come down toward a zero velocity as I hit the wall.

7:29

And what I can do is I can plot that gradient, right?

7:32

So DUDY on the right hand plot here, this is the local velocity gradient on the  $X$  axis as a function of the distance from the wall  $Y$ , right?

7:43

So what I can see is that gradient starts at an initially positive value, right?

7:49

And it's going to, yeah, an initially positive value, and it's going to come down as we go into the wall, meaning we can define this as a shear rate.

8:00

So what I can see is that my shear rate is going to traverse down in value until I come into the bulk of my species.

8:11

All right.

8:11

So what this should tell us is that we have the highest shear rate, right?

8:21

The highest shear rate can be found at the wall, right?

8:26

And that's a really important concept to understand as we start moving forward into to thinking about how to apply for looting mechanics.

8:33

So this shear rate then is you think about what is the the shear rate doing?

8:39

Well to understand the shear rate more, we need to think about shear stress and we can relate these two through a product we're going to call viscosity, right.

8:47

So if you've not heard or or seen the formal definition for viscosity, we'll go through it here pretty quickly.

8:54

So if I imagine that that the shear rates at a very high shear rate when I hit the wall and a relatively low shear rate approaching 0 is I'm in the very centre of a pipeline.

9:05

If I am looking at that wall condition, well, I imagine flowing fluid.

9:09

So if I draw a little, imagine we're looking at a cylindrical flow here.

9:14

Well, if I think about the amount of force, like the vector of the force as I go from the centre of a cylindrical flow channel and I look at different positions, right?

9:29

So different radii coming outward from that, the vector of that force being applied against the wall is going to be greater and greater as I go and finally hit the wall.

9:39

So it's at the wall at which the stress applied by my flowing fluid will be the greatest.

9:44

And then we can formally define this stress as the amount of force in Newtons or  $F$  sub  $S$  being applied across that area,  $A$  sub  $S$ .

9:53

And in fluid mechanics, we give that sheer stress the symbol  $\tau$ .

9:59

So in a cylindrical pipeline then because we know that wall is going to be the boundary of the highest stress we can apply, we give that  $\tau_w$ , meaning the wall shear stress, right?

10:10

And throughout your career, the wall shear stress is going to always be the point of reference, right?

10:15

Because that's that's the point at which Blues are going to either erode the pipeline wall or erode the unit operation wall.

10:22

So we can relate that shear stress either within the continuous phase or at the wall, which is shown here  $\tau$  to my shear rate  $\frac{du}{dy}$ , which we saw in the previous slide through the value  $\mu$ , which is viscosity.

10:41

So fundamentally, viscosity tells me for a given shear rate, how much stress that fluid is going to apply when it's flowing at any point in the system, right?

10:52

And if I define that the shear rate at the wall, then it fixes my shear stress at the wall similarly.

11:01

OK.

11:02

So I really want to make sure please feel free to type in questions into the meeting chat.

11:06

This is a really, really important concept that's going to come up.

11:11

Obviously, we're going to be using it in unidoms, but it's going to come up a lot, particularly in in the chemical engineering broadband units.

11:19

So it's really critical to understand that that relationship between sheer stress and sheer rate.

11:25

So please type questions in as we're going and I'll, I'll try to address them as we have them.

11:29

So if we then think about this viscosity, this, this particular relationship, as I showed here,  $\tau$  equals  $\mu$  times.

11:36

And we can also write the shear rate of say,  $\gamma$ , right?

11:39

But I'm not going to do that because we've already talked about  $\gamma$  in different contexts.

11:44

This particular relationship is a rather easy way to frame it.

11:48

And that's because we're looking at a Newtonian fluid.

11:51

So a Newtonian fluid means there's a linear relationship between the velocity gradient and the shear stress that's applied by that fluid.

11:59

But I can also have a few different types, right?

12:02

I can have a fluid that maybe the, the faster I shear it, the less stress it applies, right?

12:08

And that would be called a pseudo plastic.

12:10

And a good example of that is rubber latex.

12:13

My favorite example, if we have a fluid where the faster I shear it, the more stress it applies, that's a dilatant.

12:21

And the most famous example, of course, would be quicksand, right?

12:24

So if I really try to struggle to get out of quicksand, then I'm struggling.

12:28

I'm pulling a quick velocity gradient and the sheer stress applied by that quicksand to hold me in is going to go up.

12:35

A more kind of practical engineering example is some of the the high end car makers like Mercedes-Benz look at using the latent style fluids for brakes, right?

12:47

Such that if you compress that pad really quickly, it's going to apply a lot of stress and help slow the car down.

12:54

So you're not using a traditional brake pad, but a sheer thickening fluid.

12:59

So these three examples, AC and D on this plot here all assume that the shear stress applied rather the shear stress is 0 when the velocity gradient is 0, right?

13:13

But in some cases, I may need to apply some initial amount of shear stress before I can get any motion in the fluid, right?

13:24

And that's what's going to happen if I have what's called a Bingham plastic.

13:27

So if you have the most famous example of a Bingham plastic is going to be ketchup, right?

13:35

Or here we would say tomato sauce.

13:38

So if you've ever seen the the ketchup, the Heinz classical glass ketchup bottle and you've got to turn it upside down and bang it really hard to get the ketchup out right.

13:48

That's an example of a sheer stress that I have to apply.

13:52

So  $\tau$  not before it hits that activation energy to then create a velocity gradient and the ketchup can flow out of the bottle, right?

14:01

And incidentally, at normal conditions, the shear, we call this a yield stress.

14:07

So  $\tau$  not is going to be the yield stress, right?

14:14

And in normal conditions, so atmospheric pressure in say  $21^\circ\text{C}$ , the yield stress of ketchup tends to be  $\tau$  not around 20 Pascals, right.

14:28

So to, to put it in perspective, if we're forming something like a, a wax gel and a pipeline or we're going to form some kind of rock slurry and a mining operation, this is on the order of the shear stresses that concern our companies of that yield, right?

14:46

When we hit catch up level yield stresses, that's when people start to get worried because it's very difficult to flow across across a pipe network.

14:54

OK, now, so viscosity itself then this is a, a Thermo physical property of the fluid, right?

15:01

Whereas shear stress and the shear rate where the velocity gradient, right?

15:06

So I'll just write that in here are both properties of the flowing condition in which I'm studying the fluid.

15:15

So because viscosity is a Thermo physical property, it's sometimes convenient to also normalize it by the fluid density.

15:25

And when I do that, which is the  $\mu$  on  $\rho$ , I get  $\nu$ , which is we call the kinematic viscosity, right?

15:33

So kinematic viscosity tends not to be used all that often relative to dynamic viscosity or what we'll just refer to as normal viscosity  $\mu$ .

15:42

But it is something to be aware of.

15:44



OK, So in addition, this this table here shows us then two new fluids that I wanted to highlight where in in both cases there is a time dependence right to that flowing condition.

16:01

So a thixotropic fluid is one where if I'm flowing it right now, we have a  $\mu$  if I increase that shear rate where the velocity gradient, the fluid is going to thin out, right?

16:14

But a thixotropic fluid has the additional property that the longer I shear it, the thinner it gets.

16:21

So it is a time dependent phenomena on top of being a shear dependent phenomenon.

16:26

And a rheopectic fluid is the exact opposite.

16:29

So it's a shear thickening fluid where the as I increase the shear rate, it thickens up, but it is a time dependent phenomena.

16:38

So the longer I shear it and the higher I shear it, the thicker it gets, right.

16:43

So not, not things that we're going to be dealing with specifically in unit operations, but just something to be aware of that they're the viscosity of these systems to be a function of both the shear rate and in many cases, particularly for the oil industry, the time in which these species are flowing.

17:00

OK.

17:01

So when we have that plot of looking at if we go back one couple slides, this very simple gradient here of of shear rate as a function of distance away from the pipeline wall based on that local fluid velocity, right?

17:19

That's assuming laminar flow.

17:20

So I have a very simple well defined flow field.

17:24

But if I start increasing the amount of energy in the flow field, I can crossover from a laminar to a turbulent flow field, right?

17:32

And so turbulence originally came from Reynolds, right?

17:38

And Reynolds observed that if he was looking at a pipeline, right, it was filled with some fluid and he put a pressure drop, sorry, a velocity prompt, looking at a set velocity at relatively low flow rate, the pressure drop measured in that system through something like a manometer would increase directly with velocity.

17:58

But he noticed also that at high flow rates, that pressure drop measured would increase approximately with the velocity squared, right?

18:06

So this kind of starts thinking, oh, OK, so if something is happening that's different in the nature of the fluid as I turn up the flow rate and I put more and more energy into that flow stream.

18:17

So what we can see today, right, is we take that observation for Reynolds.

18:22

If we look on this left hand side here we're plotting.

18:25

So this is the turbulence kinetic energy plotted as a function of effectively the size of these different eddies that are observed.

18:34

And so if I'm putting energy right, energy into injecting energy into my flow stream, I will initially have these very large eddies and what those are going to do, we would normally think that an idiot.

18:47

It's going to be a rotating fluid motion, right?

18:50

We're not all these molecules are not pointed in the same direction, but rather they're rotating and they're moving in the X and the Y and the Z direction all at the same time.

18:58

Those rotations are not going to burn off heat straight away, right?

19:03

So if I have a very large rotation in that fluid, it is a very large Eddy.

19:08

What that's going to do is through a non heat dissipating mechanism, create a secondary and smaller Eddy, right?

19:15

So we, we transfer that energy down to a smaller length scale and that continues happening and continues happening until finally we convert the the smallest Eddy, we convert that energy to heat through what's called viscous dissipation, Right.

19:30

And this is not the actual experiment that Reynolds performed, but it's very similar in kind where you can inject dyes into fluids and watch these actual eddies come up.

19:40

And there are some really fun examples on YouTube if you want to Google turbulent eddies.

19:45

You'll find a lot of examples actually visualising what this behaviour looks like.

19:50

Another way to contextualise that in my kind of preferred way is this diagram on the bottom right.

19:57

So I can imagine if I'm putting energy into my system here, here's all my large eggs and, and I'm looking at this right across a huge space, I still have a lot of these large eggs.

20:06

Well, each of these large eggs is going to pass energy down and it's going to break into smaller equivalent eggs that are maybe half the size.

20:15

And that's going to continue downward right until I finally.

20:22

Get to the very, very smallest level here, right?

20:25

And this, this initial energy flow, we call this an energy cascade.

20:33

This initial energy flow, as we said is without heat loss until we get down to that very small scale of viscous dissipation.

20:39

And in 1949 was Kommelgrom that that led us to an understanding of what that length scale of viscous dissipation was.

20:47

So how small do my eddies have to be before they're going to burn off energy into heat forms versus just transferring energy down to smaller and smaller eddies?

20:57

So if we took Thermo with me last semester, you'll remember we started off in lecture #2 talking about the famous experiments of James Jewell, right?

21:05

And what James Jewell is doing is trying to introduce turbulence into a system with a crude calorimeter and measure the amount of heat that the fluid would take on and how warm it would become, right?

21:17

In so much the same way, what Kamalbrov recognizes, there's a certain length scale required at which below that we would be able to transition fluid velocity into this form of energy being heat.

21:30

And so this  $L_{sub D}$  being the Kolmogorov length scale at which we dissipate that energy into a viscous stress, is the velocity or the sorry, the kinematic viscosity to the power of 3 divided by the turbulence dissipation rate.

21:52

So the rate at which I'm dissipating this turbulence through the system which we give the Greek letter  $\eta$  and I raise that whole quantity to the power of  $1/4$ .

22:02

So what this allows engineers to do is if we know something about the turbulence dissipation rate and we know the kinematic viscosity, which itself is just a Thermo physical property of my fluid.

22:12

So my normal dynamic viscosity on the density, I can calculate how small of an Eddy am I going to have which I can transfer this which I get into that viscous dissipation regime, right.

22:25

And where this comes into play is if I'm designing a system, I can design it to operate in a turbulence region where I can stay above the point of passing fluid momentum energy down into a a form of heat.

22:39

OK.

22:39

So all of this is where we've come to these two diagrams.

22:43

This is where we've come to in kind of modern fluid mechanics and modern thinking about this.

22:48

But back in 1883, right now, I wanted to present this first because today most engineers use Reynolds number.

22:55

This the singular definition of Reynolds number we're going to go through to characterize turbulence, right.

23:02

But in fact, this definition of Reynolds number dates back to 1883 and has a host of assumptions with it and, and many times it's applied incorrectly, right.

23:12

So I wanted to show you first, this is how we actually think about turbulence and the nature of turbulence today as as thoughtful engineers.

23:22

And now we're going to take a step back by over a century to think about the actual definition of the Reynolds number.

23:29

So what that is, is the Reynolds number itself is a dimensionless quantity that balances the inertial forces in my system on the viscous forces.

23:38

And so the inertial forces we'll define as  $\rho V D$   $\rho$  being the fluid density,  $V$  being the velocity.

23:46

So the mean velocity of that fluid not accounting for that, that zero velocity or no slip condition at the wall, but simply looking at the mean velocity and  $D$  being the diameter of a cylindrical pipe, right.

24:00

And that's really critical.

24:01

That's one of the assumptions that has been lost over these years is that this this is productive flow, so classically cylindrical pipelines.

24:12

And we divide that by  $\mu$ , which is the viscosity of my system.

24:18

Now you'll notice because I have  $\mu$  and  $\rho$  in this, I can also replace that with the Greek letter  $\nu$ , which I apologise looks very similar to the, but that Greek letter  $\nu$  on the bottom, which corresponds to the kinematic viscosity.

24:35

So most engineers, we use this left hand side.

24:38

So row  $\text{Re}$  is what we typically memorize as the Reynolds number.

24:43

And if you haven't taken fluid mechanics yet, this is definitely something you want to memorize and you will use it frequently as you're talking about cylindrical pipe flow.

24:56

And under this definition of the Reynolds number, we say that we're in laminar flow when we that Reynolds number is below a value of 2100, right?

25:05

And what that means physically is that about that value 2100 of inertial to viscous forces.

25:11

The way I'm defining it here, I don't have any eddies in my system.

25:16

Or rather if an Eddy does crop up in the system, it's not going to be stable, right?

25:21

And the system itself is going to damp that Eddy back out and bring it back into that, that perfectly defined flow field.

25:28

Now, if my Reynolds number goes above 2100, then I get into a region for a little bit that I'm not sure sometimes, right?

25:35

So if an Eddy crops up, sometimes it might survive and it might propagate downstream and, and continue kicking on and dissipating its energy.

25:43

And sometimes it might be damped out, right?

25:46

So it's in this intermediary region where it might or might not be stable.

25:50

And that can depend on on the type of fluid that I have and how that Eddy was introduced.

25:56

And once the Reynolds number goes above 4000, right, then I know I'm in a turbulent flow regime.

26:01

So that means when Eddy's going to emerge in the fluid, it's going to persevere, right?

26:06

And it's going to kick on through this cascade and ultimately dissipate its energy into the heat form.

26:11

So a critical take away from this slide, right is this definition of Reynolds number that to memorize and the boundaries of below 2100 being laminar flow and above 4000 being turbulent in the flow.

26:23

But it's also, you know, for engineers in 2020, we're going to have a lot of work ahead of us in our careers, right?

26:30

So it's really important to remember that this definition right is almost 140 years old and it is very much an approximation of turbulence.

26:41

I'm not going to, I could go on about this literally for two hours.

26:45

But one of the the critical problems is that often times engineers will use quantities like the Reynolds number to scale phenomena they observe between different systems, right?

26:58

So maybe they're going to build something in the lab where they hire a company to build a pilot scale plant and they're going to try to match that Reynolds number and then scale it up to what they're going to do in the field.

27:07

It's some larger engineering operation.

27:10

You have to be really careful when you do that because again, if this Reynolds number is a very much an approximation and it is only applicable to ducted flow, OK, So let's do an example of this.

27:23

Something that's relevant to today of course is, is the Reynolds number in our lungs.

27:27

So if we take in error right being 79 just under 79% nitrogen, 21% oxygen, so at one atmosphere in 40° C, the viscosity of air is about point O 186 centipoise and the density is 1.14kg per cubic meter all right.

27:46

And the units of centipoise on viscosity, this will be dynamic viscosity.

27:57

So if my air velocity, so I've taken air here and we're going to see it traverse down the trachea and come through all of these bronchioli.

28:06

And if these traverse down and outward to be absorbed into the system, let's say we'll have a typical air velocity of .5 meters per second, right?

28:16

And I can see the the diameter in each of these cases, it's going to get smaller and smaller and smaller.

28:23

So I want to know then if I'm operating and I have an air velocity coming through at .5 meters per second, at what diameter will laminar flow be achieved?

28:33

So I can say, OK, my Reynolds number, if I assume that these are going to be cylindrical flow channels, my Reynolds number is going to be  $\rho V D$  unmutate, right?

28:45

And if I then know that the Reynolds number of interest is 2100, then I can say OK 2100 times  $\mu$  will be divided by  $\rho$  times.

29:03

The velocity will be equal to the diameter at which I have laminar flow.

29:08

So filling this out I have 2100 here now centipoise and this is a kind of a a trick to remember centipoise itself is not a SI unit of dynamic or dynamic viscosity.

29:25

SO1 centipoise is the same as one Milli Pascal 2nd and we like centipoise because it it tends to be a a pretty meaningful quantity.

29:37

The viscosity of water is 1 centipoise, right?



29:41

Typically the viscosity of something like olive oil is going to be 50 centipoise.

29:46

The viscosity of a really high, high viscosity oil like we might get out of Canada is going to be about 10,000 centipoise.

29:53

So that gives us really kind of a physical boundary, not for what those will be, but we have to remember that that set of poise is actually corresponding to a milli Pascal second.

30:03

So in fact, when I pull this into the definition of Reynolds number, I need to bring that into Pascal seconds.

30:09

So I'm going to multiply it by 0.000186 Pascal seconds and I can then divide that by 1.14kg per cubic meter and 0.5 meters per second of my defined flow velocity, which yields a diameter of 68.5mm.

30:36

So the point here is that as long as I'm working in a cylindrical flow channel, I can either ask you in a calculation.

30:44

Here are the properties of the fluid, something about the diameter and the velocity.

30:49

Can you calculate for me the Reynolds number and define whether or not that's going to be laminar transition or turbulent?

30:55

Or if I ask you about what that boundary will be, can you come back as to what the property or the flow conditions will be associated with that?

31:04

OK.

31:05

So we're doing a little bit early today, but kind of putting this all together.

31:10

As I start putting these fluids into motion, that motion itself is going to be characterized by fundamentally three things.

31:18

The viscosity of the fluid, which is itself a, which is itself a Thermo physical property,

the flow field in which that substance is being exposed, which and that's going to correspond to the shear rate, which I define as the gradient of the velocity  $DU/DY$  on  $DY$ .

31:50

And then the sheer stress that's going to be produced during flow, right?

31:53

And so that's if I'm, I'm flowing at a certain velocity for a known viscosity that will apply stress to throughout the fluid and ultimately up into the wall, right?

32:04

I'm going to give that the Greek letter Tau.

32:07

So if I'm in a laminar flow condition, then I do not have these eddies.

32:10

Rather, when the eddies do crop up, and that might be used through something like a differential pipe roughness, a little asperity might trigger an Eddy to start.

32:19

If I'm in a laminar flow condition, typically under Reynolds number of 2100 for a pipe, then those eddies are going to die out, right?

32:27

Because then you can imagine the damping factor of that fluid is such that it's going to absorb the Eddy back into it.

32:33

And a really good analogy here is that if I'm in laminar flow, I can kind of imagine that if I take a, a stack of playing cards and I spread them across the table laterally, that's kind of like the layers of fluid that are going to be interacting with each other, right?

32:49

So I'm not going to have a movement in the Y or Z directions.

32:53

I'm looking at flow in the X direction, but once I get into turbulent flow, then the creation of an Eddy either might propagate if it's in transition region or will absolutely propagate if it's in fully turbulent flow.

33:08

And it propagates through this energy and cascade, right?

33:15

So a lot of work goes in because this cascade itself is a function of both.

33:22

I'm going to write this as the dynamic viscosity, the density and the turbulence dissipation rate ADA.

33:30

A lot of work at present is in the field is going into better understanding and characterizing ADA for uncommon flow channels and uncommon flow conditions right outside of our ducted pipe flow.

33:45

Whereas traditionally we tend to know densities and dynamic viscosities reasonably well for most fluids of interest at this point.

33:53

But there's still quite a bit of work to understand what aid is going to look like in different geometries.

33:58

And so for that reason, we tend to use things like the Reynolds number to very simply characterize the balance of these inertial forces basically at what point I'm going to be accessing that viscous subregime and dissipating that momentum back into a heat itself.

34:18

And so the Reynolds number for my abducted flow will be  $\rho$  and here  $V$ .

34:23

This is the velocity times that pipe diameter divided by the dynamic viscosity.

34:30

OK, so we've done 10 minutes really today.

34:35

I was going to kick into the the statistics and the statistical distribution of it, but I thought I'd probably end on a this more slightly more conceptual note.

## Lecture 03 - Boundary Layers

0:00

As we're following along with the textbook, this is the last half of Chapter 3 in UK.

0:12

So I, I wanted to separate boundary layers.

0:15

If you've taken fluid mechanics before, you might not have talked at length about boundary layers, but I kind of wanted to separate in this into a, a different discussion unto itself.

0:24

Because boundary layers play a critical role when we go to analyse unit operations in the field.

0:31

And then when we start to compare how different simulation tools or things like ISIS are going to compare with our actual experimental results.

0:41

All right.

0:42

So if before we get into this, we'll pick up on a quick review of where we're at last time.

0:47

So at the end of chapter lecture 2:00, we, we talked about fluids in motion now being characterized by 1 Thermo physical property and that is viscosity and two flowing properties.

1:01

So one being the flow field that's produced or the flow field generated with the substance.

1:06

And so this is going to be characterized as the shear rate and that's typically given the Brig symbol  $\gamma \dot{\phantom{x}}$ , Here's again our viscosity and then the shear stress that's produced as the product of that viscosity and the shear rate when we're in flow.

1:23

And so if we think about something like a pipeline, in fact, I'm going to draw this a bit bigger because a lot of our unit operations tend to have cylindrical flow channels.

1:31

We can imagine that that shear stress, right, if we can draw little vectors here, right?

1:37

So the magnitude of the vector is going to indicate right, the extremity of the flowing shear stress.

1:45

And so this will be our  $\tau$  wall, our wall flowing shear stress.

1:49

And so this is really important because often times when we want to scale physical phenomena from the laboratory or even a pilot scale rig up to what's going to happen in the field and simply using something like a Reynolds or a Weber number is not going to be adequate, right?

2:05

It's not going to be quite enough.

2:06

We need to look towards these more fundamental characterizations of fluid mechanics to do that scaling and to understand that we're operating in regions of ship and similar shear stress.

2:17

So when you're operating now in a non turbulent region, which we call a laminar region, that region is characterized by the absence of turbulent eddies, right?

2:28

And so down here, these are our examples of what eddies might look like and they can exist at different length scales of the Eddy.

2:37

So different sizes of the Eddy and then different intensities as we're going to get into today.

2:43

So when we're in laminar flow, we, we talked about the fact that, you know, eddies might pop up, because if, if we're flowing, for instance, in this pipeline up here and we have a surface roughness on the edge of the pipe, will that flow past that those little asperities and the bumps and the roughness in the pipe, those can induce eddies, right?

2:59

But if we're in laminar flow, then it's characterized by the fact that the, the stress of the flow field itself is going to damp those eddies back out and bring it back into, into a, a region where the, the flow field will look like it's playing cards that are layering and being split out over one another.

3:18

Now, when we get into a transition region, so that'll be a Reynolds number somewhere between 2100 and 4000.

3:24

That's going to be where if we're generating some of those eddies, either naturally or because it's bumping into the pipeline wall, the eddies might propagate forward, but they might not.

3:33

And so there's this region that it kind of it's, it's in the middle of transitioning into what is ultimately a fully turbulent flow with the Reynolds number above 4000.

3:41

And that's when we're going to have those eddies being generated.

3:45

They will always propagate forward.

3:47

And what happens when they do propagate, there's these eddies contain energies we're going to talk about today.

3:53

And so how does, how does that energy flow through a system?

3:56

Well, it's going to start with a, a very large Eddy and the energy will flow from the large eddies into the intermediate and ultimately the smallest scale eddies.

4:05

And it's at that very smallest scale we call the Kolmogorov length scale.

4:10

That's the, the actual, how many microns or angstroms is the smallest Eddy in the system.

4:15

That's the point at which the energy is going to be lost to viscous dissipation, right?

4:27

So viscous dissipation, meaning we're burning that that momentum energy into heat.

4:33

So the Reynolds number then we can see as kind of this characterization of the inertia of the fluid.

4:38

So how much momentum that fluid is bringing to bear relative to the viscous forces in the system, right.

4:45

But we remember that this is for a cylindrical pipeline flow.

4:55

So if I can give you one warning, as future engineers, be very careful when you're applying Reynolds number to scale from 1 apparatus to another or from 1 field to another that you're following that that must be defined for the same dimensionality.

5:11

There are examples where we've seen researchers try to apply Reynolds numbers into contexts that they were never meant to be applied.

5:19

And that's where those scaling arguments start to break down.

5:22

OK, so at the very end of last lecture, we were starting to talk about this turbulence and the fact that if we were to, you know, take our pipeline and let's say here's our cross section of the line.

5:33

Imagine we could insert a sensor, right, and measure an instantaneous velocity at .1, and we can also measure an instantaneous velocity at .2, right?

5:41

These have different distances from the pipeline wall.

5:46

What we would notice is that in turbulent flow, right, if, if we're flowing in the, if  $U$  is in the positive  $X$  direction and if we're flowing in that positive  $X$  direction, if we're in laminar flow now, then the  $V$  and the  $W$  directions, which themselves might be corresponding to the  $Y$  direction or the  $Z$  direction would be 0, right?

6:11

We don't have any flow outside of the the primary because it's like those playing cards that are sliding over one another.

6:17

But when we get into turbulent flow right now we have flow in three dimensions, even if our net flow is in the positive  $X$  direction.

6:24

And so if we're measuring velocity instantaneously, say locations in the system, we notice a few things crop up.

6:34

Firstly, that the velocity itself is going to fluctuate, right, That we're actually measuring at that point.

6:42

And so this is an example that McKay publishes looking at measuring this velocity in the  $X$  direction as a function of time, right?

6:51

And he's showing in this case it was a measurement of  $\pm 5\%$  kind of bounded the region of that measurement.

6:57

But in fact, it's, it's indicative of a larger understanding here that what he's plotting is  $U$  prime.

7:05

So this is the deviation, right?

7:08

So the velocity deviation, right?

7:12

And this is the net velocity.

7:17

And so we, we kind of break this down to look at, OK, our net flow in the direction that X is going to have the value U.

7:24

But then we have these U prime, V prime and W prime values that are themselves the XY and Z quote deviating velocities.

7:33

And if we add our U&U prime together, they give us the instantaneous velocity that we're going to measure, right?

7:41

And that's going to be at .1, right?

7:44

That's when we stick our probing, we're actually going to be measuring that instantaneous velocity.

7:48

So because of this reality, then we have these fluctuations in the XYZ space or if we write our velocities, UV and W spaces, because this is a flow field, that also means we're going to have instantaneous pressure fluctuations.

8:04

So it's important to note that that this is a distributed property and we're going to kick on to talking more about that on the next slide.

8:12

Now these deviating velocities that we're talking about here have a unique property and that's that if I integrate the deviating velocities over only a few seconds, right?

8:22

So if I'm taking that measurement in time and I, I calculate what's this deviating velocity U prime, I'm going to integrate that numerically over a few seconds.

8:30

So if I draw a, a solid line through here, right?



8:33

What it's meaning is that each of these bits that I would fill in right over say 2 seconds are going to give me a net integration of the deviating velocity of 0, right.

8:47

So this is a nature deviating velocities that we have these these kind of pulsations of velocity in the system in the XY and Z directions of turbulent flow that that will integrate to zero only over just a few seconds of time.

9:02

So that gives you an idea of how, how instantaneously deviating these things are, right.

9:09

And of course, the more turbulent the flow becomes, the greater those deviating velocities are going to be.

9:15

So I wanted to take an aside now as we're talking about a distributed measurement because something that is going to start coming up both in the unit operations lab but also later on in your curriculum is the idea of a distributed versus a non distributed property.

9:30

So if we're measuring a non distributed property and that might be the height, let's say this is Bob, right?

9:36

We're going to measure Bob's height.

9:38

Well, you know Bob has a definitive height, right, for the point in time that we're measuring it.

9:43

So we can measure it 1000 times and it should always come back exactly the same.

9:48

So the measurement can be repeated with no variation in the result.

9:52

And so the uncertainty of our measurement in that context is not based on uncertainty of Bob's height, but rather the the method that we're using to measure that.

10:03

So here's an example of a very old school measuring type, right, where the uncertainty is going to be one inch.

10:11

Well, actually sorry, the uncertainty is .5 inches because we see that this is demarcated every half an inch, right.

10:21

And here's a more modern measurement tape where we're going to have it, 0.1 millimetre uncertainty, right.

10:28

So based on the type of apparatus that we're using to measure a property that we know to be not non distributed, that tells us the uncertainty or the accuracy of our measurement right now when we start to look at distributed properties.

10:42

And so the example here is if we have a pipeline containing flow and we're going to put a manometer from our previous class example, right?

10:49

So here's our manometer liquid of interest.

10:51

All right, this is filling up EU bend section well with that measurement because we know we are measuring a distributed property to begin with, right?

11:00

So every time we take that measurement, we don't expect to find the same result because itself it is a distribution within the system, right?

11:08

This is where we get if we plot not the deviating velocities, but in fact  $U$ , the instantaneous velocity being that net velocity plus the deviations with time.

11:20

If we were to measure this for say 20-30, forty seconds, we have all of these data points.

11:25

Well then we want to plot what's the probability of observation.

11:29

This is also called a probability density function, as in the X axis in this is the actual used by the instantaneous velocity that we're measuring, right?

11:43

And so this is where we then take twenty, 3040 seconds worth of measurement.

11:48

And here is the mean, but if it's a normally distributed property, here is the mean that we're going to get, right.

11:54

And we can characterize that normal distribution by a standard deviation, right, which we typically give here.

12:03

STDEV would be the action in Excel.

12:06

You can take on that data if you get this.

12:08

So because we're measuring something that we know to be fluctuating distributed through the system, this is where we tend to want to measure it for a longer period of time and to characterize the measurement based on the mean value that we're obtaining.

12:24

But we also tend to report the uncertainty bounds of the data point based on the precision, right?

12:29

Meaning is it always true, right.

12:35

So we can take that measurement and maybe we report a standard deviation error bound or if we know the measurement is going to be normally distributed, then we can go and use something like a Student's T test.

12:48

I'm going to ask a question using the forums here, just loading up for us.

13:11

OK, so I'm asking you just put this form up on the board.

13:18

Have you used a student's T test being a confidence interval in the past?

13:22

I'm just curious as to whether this is something anyone in the class has seen or done before.

13:27

So go ahead and if you open up the meeting chat and just tick the box yes or no and hit submit vote for me because it's something that when we get into the, the unit OPS lab, certainly we want to start as much as we can with that data.

13:43

I'm certainly making use of it because this is how, you know, as engineers, if we're reporting something like that instantaneous velocity, maybe it's a function of pump speed and RPM, right?

14:00

We would expect to see right some kind of probably exponential correlation or or in between the two, but we always want to report this with uncertainty bounds and so understanding.

14:17

Oh wow, 5050 split.

14:19

OK, cool.

14:22

And for those who said you have seen it before, if you take a few minutes, you can write out for me in the the comment section.

14:27

Where did you actually come across this?

14:29

What unit I would be interested to know.

14:34

So, so if we're trying to plot this then as engineers and figure out how is that that varying as a function of something like pump speed 3003, Good deal.

14:43

You know, we actually want to be able to assign this uncertainty bound to the system, right?

14:49

Because this is what's going to help us understand.

14:52

Oh, very nice, Shannon, you're 12.

14:54

Hold on.

14:55

And with those, uncertainty bounds will help us understand whether we have a correlation between the parameters we're looking at in the system.

15:03

OK.

15:03

So if we're trying to characterize turbulence, then there are fundamentally 2 properties that we'll look at.

15:08

One is the intensity of that turbulence, and the 2nd is the scale.

15:13

So the intensity itself depends on understanding the deviating velocity and the net velocity.

15:18

So we can of course we can get both of those by taking a velocity measurement with respect to time, right.

15:28

So if we put that velocity probe in there, we put our some kind of I can use an optical probe maybe, but to measure the velocity of that fluid as a function of time then over we would get both the net and the deviations around that net.

15:42

And the way we would calculate this is simply looking at what is the deviating velocity squared.

15:48

So the mean deviator square root of the mean deviating velocity squared divided by the net velocity.

15:53

And that's going to give us units of percent of the mean velocity.

15:57

So that's the intensity of the turbulence.

15:59

So you'll remember on the previous slide we were looking at deviations of  $\pm 5\%$ .

16:05

And that's a reasonably intense turbulence.

16:09

Well, the turbulence can go in a very highly turbulent system up to about 10%, right?

16:14

So that's kind of a, a nice intuitive upper limit.

16:17

We then also have the context of scale.

16:20

So the scale of turbulence, we'll look at how these deviating velocities are going to change in different locations across whatever volume I'm measuring it in.

16:29

So if you remember back to where am I putting these two probes?

16:34

And I might just for clarity, put a probe #2 at the wall, right?

16:39

So if I measure somewhere in the middle of the bulk and then somewhere at the wall, All right, so here's probe one and probe 2.

16:47

The scale of turbulence is going to tell me how broadly distributed that is across the volume of interest.

16:55

And the last thing I wanted to end on the point of this, talking about the isotropic turbulence.

17:01

So that will be defined if I have my XY and Z coordinates that the mean square flexible combinations in the XY and Z directions are all going to be equal, right?

17:11

Now where might you think that would happen?

17:19

Can anyone think of an example where you'd have isotropic turbulence in a a cylindrical flow channel?

17:35

OK, So what we would say is in the central line, so the absolute central line of flow we should in theory have isotropic turbulence, right?

17:43

And it was probably the the higher the velocity we go that the definition of central line's going to shrink.

17:49

But if we have one molecule, one set of molecules being the central line of the pipeline, if it's single phase flow, then we should have isotropic turbulence at that point.

18:00

So this then brings us to a discussion around boundary layers, which is the the main topic I wanted to to drive home today.

18:08

So a boundary layer, if we think we were going to begin with some uniform flow with a velocity  $U$  that's distributed Eden length and this flow is going to come across, this is going to be some substrate of interest, right?

18:21

So a wall, right?

18:24

And what's going to happen when that flow field interacts with the wall for the first time, we're going to see emerge what's called a boundary layer.

18:33

And so the boundary layer itself, what it is, is a region.

18:38

So if the uniform flow field is highly turbulent, then the boundary layer is basically a region where we go from a highly turbulent flow field and we decelerate into our no slip boundary condition at the wall itself.

18:52

So it's how we connect those principles of having a highly turbulent flow pattern and having a no slip boundary condition.

18:59

The boundary layer defines for us what is that field of deceleration.

19:04

All right, So up here above the boundary layer, we're going to have high turbulence.

19:11

And what this line OL is, is projecting here, we mark it up in red, is it's projecting for us where the extent of that boundary layer is.

19:21

Is that existing?

19:22

So we can see the second flow field is going to hit this wall, the boundary layer is going to start forming on the walls, and the flow field on top is being passed above the boundary layer.

19:32

And the boundary layer's going to grow as a function of distance in that flow field to the point that it will reach a steady state height.

19:43

Now in this context, right, we have, if we're plotting the magnitude of the velocity vectors, right, we can see that early on, right, it's going to relatively quickly, right, come into equilibrium that velocity field within the boundary layer whereby no slip condition at the wall, It's going to be slightly faster on top of that, slightly faster on top of that.

20:06

And it's going to very quickly come into equilibrium with that absolute velocity in the uniform flow field.

20:12

But if I go to Point C prime or C double prime further down in the boundary layer.

20:18

Then that boundary layer is going to be thicker, right?

20:21

And so it's going to take more time.

20:22

And it's it that's going to have more of a laminar region to it.

20:27

So near the wall itself, right?

20:29

This region will be essentially laminar.

20:31

And if I go toward the outside of that boundary layer, which we're going to define as 99% of the velocity of the uniform flow field itself, that's where I'm going to have my turbulence zone, OK.

20:44

So now if we think about the development of the boundary layer, all right, what we're plotting on the left hand side here is the thickness of that layer as a function of the distance from that leading edge, right?

20:56

So this right here would be the leading edge.

21:08

So if I think about the distance from that leading edge, how thick is the boundary layer going to be with time, right?



21:14

Well, ultimately the boundary layer is going to, if we're in a turbulent flow field, right?

21:18

The boundary layer is going to start and it's going to continue growing with time, right?

21:23

And we can characterize it in two regions.

21:26

Effectively, the first region will exist before I have any kind of turbulent eddies that are going to exist in the boundary layer.

21:34

Effectively, the boundary layer is going to grow so thick that it will begin having turbulent eddies of its own that can be sustained early on.

21:42

I'm going to only have laminar flow within that boundary layer.

21:45

Once I have the onset of turbulence, and I can see here are my eddies of interest, then the boundary layer is going to split into three different zones.

21:54

Here I have my turbulent layer, right, so that will be the largest.

21:59

I then have on the bottom this viscous sublayer.

22:02

All right, that's going to be laminar.

22:06

And then I have some kind of buffer layer or a transition layer that will exist with that.

22:12

It's a buffer of the two of them.

22:14

Now early on, the boundary layer of thickness will scale with distance from the leading edge to the power of .5. So the square root of distance into the the leading edge.

22:24

Once that boundary layer splits, I have a short region where it's going to grow with distance to the power of 1.5.

22:32

And then after I have that fully developed boundary layer, then it will scale in thickness with distance from the leading edge to the power of .8, right.

22:43

So as they come well into the truly into the pipeline, I think OK, it's going to grow basically to the the 4/5 power with the distance from that initiation to the flow field.

22:55

Now the, if we have a laminar boundary layer, these can grow quite large.

23:00

So an example might be two millimetres, right?

23:03

So a very thick laminar boundary layer before turbulence is going to onset.

23:07

But as soon as I have those turbulent eddies that are going to be sustained within the system, then it's going to be laminar portion of that where the bit that's in the viscous sub layer is going to be stripped down as you can see.

23:19

So this follows this pattern here, right?

23:22

And strip that into a very thin layer that's on this case about .2 millimetres.

23:26

And so if I want to calculate the flow across a flat plate, what is that approximate thickness of the boundary layer for either laminar or turbulent flow?

23:35

I can look at the approximations here for the boundary layer thickness, right?

23:39

So this delta will be the boundary layer thickness.

23:45

And in the laminar region, we'll go the equal to 5 divided by the square root of the Reynolds number defined at that cross section from the leading edge.

23:55

And the turbulence region will be .385 divided by Reynolds number to the power .2, right?

24:02

So these are approximations, but they'll start to tell you what is that thickness of the boundary layer going to be and how is it evolving with time.

24:10

I should note here that that if we're solving simply for the boundary layer thickness, right, I can write this also as  $\delta$  being equal to  $5 \cdot X$  divided by the square root of  $Re$  at the cross section  $X$ .

24:24

So as that evolves also with time.

24:28

So that means that if I'm going to put flow into a new pipeline, right, or a new flow channel of interest, the 2nd that that that flow field touches the new wall that it's going to come into equilibrium with.

24:42

That's that the distance of leading edge being 0.

24:45

But we can see that the nature of this relationship both on the left and the right means that  $\delta$  is going to grow directly with  $X$  right because Reynolds number is always going to be a positive quantity.

24:58

So in fact,  $\delta$  is going to continue to get larger and larger and larger with time both with the  $\Lambda$  and the turbulent solutions here.

25:06

So this leads to the idea then that if we start off, let's say this is at the, you know, our quote, leading edge position.

25:17

If I first put fluid into a pipeline, well as soon or any kind of flow channel, as soon as it starts forming, it's boundary layer that's going to continue growing and growing and growing with time until that boundary layer essentially cuts off and intersects itself from both sides of the pipe or the flow channel.

25:34

And that produces what we then call fully developed flow.

25:39

So this is a really, really important concept, right?

25:42

Because if I'm going to put materials, any kind of fluid material into a, a flow channel that as an engineer, I can only do calculations and wield that knowing that it is fully developed, right?

25:56

Because under fully developed flow, I now know what is my rate of mixing, degree of turbulence, rate of heat transfer, rate of mass transfer, all of these points that we'll be talking about throughout the rest of the unit.

26:08

So fully developed flow is critical.

26:10

And then you start to wonder then how long is it going to take to go from when I first dump fluids into a pipe to the point that they become fully developed?

26:22

And in this example, right, if we're in, we can also call this a transition length, right?

26:27

Or an entry length is also common to call it if I'm in a laminar bulk flow, right?

26:33

We can write this explicitly as the  $X$  so that transition length or entrance length divided by the diameter will be 0.5 times the Reynolds number.

26:45

And so in the example of a pipe ID of two inches with a Reynolds number of 1500, the entrance length is going to be about 12.3 feet or 3.75 meters, right?

26:58

So it's quite a long time that it takes for these boundary layers to grow and grow and grow and propagate, cut, you know, intersect each other to the point that we now have a fully developed flow pattern, right?

27:08

But more importantly is the heuristic.

27:11

It's a heuristic meaning, and it's a 1950s way of saying rule of thumb, but the heuristic that engineers will always use.

27:20

Because if I'm going to build a system on which I measure something, I must have fully developed flow to know that I can.

27:27

If I'm going to put a velocity probe, for instance, I need to know where in this to put that velocity probe to take a measurement.

27:34

So to get fully developed flow in laminar conditions is going to typically take around 100 pipe diameters if I'm in turbulent flow, right?

27:43

And it's scaled laminar flow will scale with this up here.

27:46

If I'm in turbulent flow, it's going to take 40 or rather 50 pipe diameters.

27:51

I, I tend to use book says 40 to 50.

27:53

I we always use 50 right?

27:56

And in highly turbulent flow, the entrance length required to get the fully developed flow is almost independent, not perfectly, but almost independent of that Reynolds number.

28:08

So if you're trying to design a piece of kit and you want to design the kit to then take a measurement down the stream, you know, we always use the heuristic of at least fifty  $L / D$  right?

28:19

Or or 50 length diameters if you're going to be very certain.

28:25

And in what I typically do, if we're designing a flow loop or a flowing apparatus, I double it, right?

28:30

Because we're going to build this once and do it properly.

28:32

So if I need fully developed flow, 50 is where I'm going to get it.

28:36

I'm going to add some to that, another 25 or 50 diameters to be absolutely certain that there is no way I have only partially developed flow in my system, right?

28:46

And it's important to note that rougher pipelines, so pipes with more asperities that might be to have a corrosion on them, these are also going to engender longer entrance lengths because they're going to disrupt that boundary layer as it's trying to form.

29:01

So here I wanted to play.

29:03

I found a really cool video from the University of Melbourne, right?

29:07

It's looking at boundary layer being tripped by one.

29:12

Are you able to see this in in real time or is it laggy for you guys?

29:23

Good.

29:23

OK.

29:24

So it's real really neat, right?

29:26

Because you can see here these turbulent eddies that are forming down below as they're turning higher and higher the Reynolds number.

29:33

And if we take a step up and we go to an even higher Reynolds number, now we'll see a very different nature starting to emerge and how that both the boundary layer is developing with time as we go further into the system, but also the thickness, right?

29:53

And then we can compare the two, which is really cool to see.

29:58

So I love watching those turbulent eddies on top.

30:01

You can actually see the swirls that correspond to those Eddy diameters, but it also gives you an idea about the nature, right, that that when we're in a highly turbulent system, we're going to see these boundary layers that are relatively thin, right?

30:16

And they're going to grow thinly with time.

30:18

But when we get into a highly or into a laminar system, I get much more of a

parabolic flow profile that would be consistent with with what we were looking at on the previous slide.

30:33

So I put the reference here.

30:35

If anyone's interested, you can actually go to the this this web link on the the web page itself and download this in high resolution.

30:44

It's made free as a part of the publication, but it's a really, really cool video and and certainly quite an uncommon 1 to see done it at this kind of resolution for sure.

31:08

OK, so finally, I want to end end with end with the point.

31:15

So so these boundary layer phenomena, so this is critical, right?

31:18

So boundary layers are going to be critical to designing pipe flow equipment of any kind, right?

31:38

So anytime I'm going to take a high pressure fluid stream or any kind of fluid stream and put this into a pipe network or a ducted network, right?

31:47

The 2nd that a flow channel or flow fluid is going to hit a flow channel, it's going to start developing these boundary layers.

31:55

But in fact, in the lavender flow sense, if I look at the cross section, right, I'm going to terrible draw, but I'm going to tend to see, right, Laminar flow will tend to give me a parabolic, parabolic flow flow profile.

32:17

And this will be for laminar when I get into turbulent flow, right?

32:22

What tends to happen is I'm going to have that little bump really on, but then it's going to be more or less just a straight head coming across, right?

32:34

So this will be my turbulent.

32:42

All right.

32:42

So we're going to start looking at this in Chapter 4 in, in the latter half of the the fluid mechanics work next week before we kick into unit operations.

32:51

But it is important to illustrate the nature of why that turbulent flow profile.

32:56

If I look at the cross section of velocities, that turbulent flow profile tends to have a little bit of a, a sharp knee early on and then a more of a blunt face across it.

33:06

Whereas laminar flow has that very nice parabolic flow profile there.

33:10

It's driven by the nature of these boundary layers as they come into development, right?

33:14

And if you play that video in slow motion and you really study it, you can actually see the causation in both cases.

33:21

You can see that boundary layer growing to actually meet the middle of that parabolic flow profile, where in a turbulence sense it's going to keep that boundary layer to relatively narrow bit of the pipe before it can then come into equilibrium.

33:38

So this is around what happens if if now I have pipe or fluid flowing into a pipe network, but what happens if my fluids come across an obstruction of some sort, right?

33:49

So if it's a parallel flow stream, I tend to get a downstream velocity variance here we can see this, just going to take into account for that.

33:56

And as soon as that parallel plates done right, some number of diameters downstream, usually about 10 to 20 diameters downstream, it will re establish and I'll come back into that uniform velocity.

34:08

But if I take that plate and I turn it perpendicular to the direction of flow, all of a sudden now I'm flowing and it's banging straight into the face of the plate.

34:17

Well, what happens is the flow of streams tend to come around this, right?



34:21

But what I get in the directly behind the plate are all of these turbulent eddies, right?

34:27

And so this is called a wake or the wake of the system.

34:31

And this is a really important phenomenon to be aware of as we start thinking about unit operations because it's often times things like sensors, right?

34:40

Anything I'm going to insert into a flow network to try to take a measurement, make a temperature, pressure, velocity, composition of some sort that directly behind that entry point, if I'm disturbing the flow pattern perpendicularly, directly behind that disturbance will be a wave zone.

35:01

And what will happen with that is the recirculation time, right?

35:09

The recirculation is going to take a long time, so fluids can be trapped in that wake area for quite some amount of time, right?

35:19

And where this can be a problem is if I have to worry about something like corrosion, right?

35:24

Where I'm worried about the fluids that might be interacting with my pipe materials of some sort, they can start to erode that over time.

35:33

Or in, for instance, if the fluids themselves can crystallize, maybe wax can form right from a hydrocarbon fluid or some kind of hydra can form from an aqueous fluid, or to scale right, the same kind of calcium carbon on your teeth.

35:46

If a lot of these fouling phenomena will depend on exposure time or residence time in the system and behind flow disturbances.

35:55

When I get a wake zone, that's an area where I can have a very long residence time relative to what the rest of the fluids look like, right?

36:04

So that's an important thing to kind of keep in the top of your head if you're designing materials or you're analyzing the performance of things that if I tend to have these

obstructions that can lead to a distribution of residents times in the system for this reason.

36:20

OK, so with that Friday afternoon, I'll let, I won't keep you all the way until 5:00, but to wrap up with some take home messages.

36:28

So the velocity itself and the turbulence, these exemplify what is a distributed property, meaning that every time we measure it, we do not expect it to be the same because the nature of the property we're measuring is one that will be distributed as a phenomenon.

36:43

So if I take multiple measurements, it's going to result in some mean and some distribution.

36:49

So we would typically use a Reynolds number to then estimate the turbulence in that system.

36:55

Now Reynolds number itself is simply going to be an estimate.

36:58

And I, I hope that's a point that you've taken home of probably beating it to the ground quite hard by now.

37:04

But the Reynolds number itself is very much an an estimate of that turbulence, where it's a simplification looking at the turbulent intensity and the scale of the turbulence.

37:16

But it does give us a very convenient way to connect what we do in the laboratory to what's done in pilot scale apparatus to what's done in the field scale.

37:25

And so it's a good scaling parameter between these.

37:28

And in talking about boundary layers, boundary layers start to then give us a, a sense of why we get different flow patterns, right?

37:37

So getting that parabolic flow profile if we're looking at something like laminar flow versus that kind of blunt head profile, if we're looking at a turbulent flow.

37:47

And these are a function of having this boundary layer interact with that wall.

37:52

So most importantly, they provide engineers with a basis on which they can expect when they can expect to encounter fully developed flow, right?

38:01

And the heuristics and these are things you should definitely memorize, 50 diameters for turbulent flow and 100 diameters for laminar flow.

38:10

Right now, I'd be very, very, very surprised if any engineers here ever work in laminar flow, with the exception of maybe looking at things like settling tanks, right?

38:18

We're actually trying to get into almost a hyperstatic condition.

38:22

But as you're going to see on the quiz this week, you know, virtually any flow energy is going to be enough to pop us into a turbulent region, right?

38:30

So laminar is really used as a, you know, honey flowing down wall is an example of laminar flow.

38:36

Outside of that, we tend to be in the transition of the turbulent.

## Lecture 04 - Incompressible Flow

0:00

So as we remember in the last lecture where we ended up was talking about velocity in turbulence and the turbulent quantities of velocity exemplifying what is a distributed property relative to something that might be called deterministic.

0:15

So the example that we used was height would be a good deterministic property.

0:20

But when we go to measure things like velocity, you know, if we take the same measurements 100 times, we're going to get a distribution.

0:27

So in that example, if we the probability density function, if we look, if we plot the actual instantaneous  $U$  of velocity that we're going to measure, it's going to look something like this.

0:40

And if it's a random variable or a random occurrence, then we should expect to see a normal distribution.

0:51

So normality in that context indicates we have a random event occurring.

0:57

So, you know, we talked that we can break turbulence itself into both the intensity and the scale as we start thinking about those eddies.

1:06

But typically, you know, this is a very difficult thing to do for engineers, right, because it's a complex calculation.

1:13

So things like the Reynolds number then tend to be really efficient scaling metrics to go from small or pilot scale systems up into the field scale.

1:25

And so this Reynolds number  $Re$ , which will typically write as  $\rho V D / \mu$ , where  $V$  is velocity in this case.

1:34

And we can write this Reynolds number.

1:36

And if we go through chapter 4 and chapter five of the textbook, you'll see a variation of ways of how to write this for different flow ducted or cylindrical flow channels.

1:46

But this allows us to then scale between these different slices of apparatus and it is well run the note of heuristics.

1:53

I thought it was helpful to note when you're thinking about as an engineer going from the lab to the pilot scale to the field, there is an engineering heuristic that you only scale by a factor of 10 in size.

2:15

So if you're going to work on a pipeline for say transporting water and you're going to have some kind of fouling in that system, you would only build a pilot scale apparatus to investigate that or trust a result if that is within the context of a factor of 10 in size.

2:34

And then finally, we started getting into the topic of boundary layers.

2:38

And so one of the, the points here is that if flow is going to end to reductive channel, be it cylindrical or a square or something else, the entry to that ductive channel would take some amount of time before the the flow is going to be what we call fully developed, right?

2:54

And that entrance dictates to us the creation or the formation of a boundary layer, right?

3:01

And so we have that that example of the boundary layer thickness, which we use little  $\delta$  as a function of entry distance  $X$  coming into the system.

3:09

And we saw that it would look something like this, right, where we'd have the onset of turbulence.

3:18

We have a laminar region down here on the very bottom and then a turbulent region up top.

3:23

If we were in a turbulent flow, so being able to anticipate how many pipe diameters do I require upstream before I have fully developed flow, the engineering heuristic would be 50 diameters in the turbulent system and 100 diameters in the laminar flow system, right.

3:40

So we're going to kind of pick up on this concept today and start running forward now into actual proper fluid mechanics calculations, but in the context of in, in the context of having these boundary layers fully established.

3:56

So when we talked about that, that established fully developed flow, then this is a really nice plot from the textbook.

4:05

And and so I should also mention, sorry, before I go too far, this lecture today we are very briefly covering Chapter 4.

4:16

So chapter four goes into the Navier Stokes equations, Reynolds average Navier Stokes.

4:21

And that's very much in the fluid mechanics realm that you're going to get in 3003 if you've not already taken it.

4:26

So I'm not going to beat that to death, but I'm just pulling forward the Bernoulli equation, which is really what matters when we start talking about incompressible fluid flow.

4:35

And so this covers briefly Chapter 4 and all of Chapter 5.

4:41

So if you've been following along in the textbook this week, we're going to knock out the second-half of the Section 2 on fluid mechanics.

4:49

OK?

4:49

So once we have these boundary layers starting to form and they're going to take some number of diameters until they reach that that equilibrium position, meaning we have fully developed flow and there's a couple of characters we're going to start looking at within this.

5:02

So laminar flow is shown by the dashed line here.

5:06

And what we're plotting on the ordinate over the Y axis is the relative distance from the centre line of the pipe, right?

5:12

And on the X axis is the fraction of maximum velocity.

5:16

So what this tells us is when we get to the wall, right?

5:19

In both cases, we see when we hit that wall either  $\pm 100\%$  from the distance of the center line of the pipe, we have no velocity because we have our no slip boundary condition enforced and our maximal velocity is in the center line of the pipe, right?

5:36

And that's true for both turbulent and laminar flow.

5:39

Now in the case of laminar flow, it has a unique character that no matter where we are in that laminar region, we're going to have a parabola, right?

5:48

And, and some of the mathematics we're going to look at today, it is actually a

mathematical parabola if we want to pick a point of velocity somewhere in the flow stream.

5:57

So if we say, well, what is the, what is the fraction of the maximum velocity going to be at say 70 percent, 75% right in on either side of the flow stream?

6:14

Well, I can trace across and in both cases I can say, OK, because it's a parabola, I can trace this across and say 70/75%, I'm going to be at about half of my maximum velocity.

6:27

Now when we get into a turbulent flow, right, we see more of a bullhead shape emerge where you'll remember that video from last time, the boundary layer that would form and come into equilibrium for a turbulent flow stream was actually much thinner, right?

6:41

So you can see here, right?

6:44

This is my highlighted up here.

6:47

This is my turbulent boundary layer that's emerging, right?

6:50

And quite quickly, right, it comes in to this Bullhead shape.

6:54

Now this is at a Reynolds number of 10,000.

7:00

If I were to turn that Reynolds number up to say 100,000, right, what would happen?

7:05

And I'll change colour here.

7:08

So at 100,000, we would expect that boundary layer to continue coming up, make a sharper turn, come across and come down.

7:18

So you can see the degree to which we have that bullhead front and the amount that's at that maximum velocity will depend on what that Reynolds number is when we're in the turbulent region.

7:32  
OK.

7:33  
Now at the same time, I also wanted to highlight that when we're talking about the distribution of velocity, this also begins a discussion on shear stress, right, which we've given the Greek symbol Tau.

7:45  
And in particular, if we plot this along the same central line, we can look at where is the the shear stress distribution as we move away from the central line of the pipe, right?

7:55  
So I can see that in the centre of the pipe, I have that condition that you referenced the last time of isotropic turbulence, right.

8:07  
And the condition for that was that my turbulent fluctuations in the XYZ direction, all three physical directions were equivalent or equal to each other.

8:19  
Now, as I move toward the wall, I'm going to have an increase from no shear stress in the central line to a maximal shear stress  $\tau_w$  wall at the edge.

8:31  
So keep that in mind because today that's going to start coming into some of the calculations.

8:36  
OK, So now if I want to start thinking about relating this, the average flowing velocity.

8:42  
So that would be something like the volumetric flow rate coming into the system divided by the constant surface area of flow.

8:51  
And I want to relate that to the absolute maximum of velocity in the system, right?

8:56  
This relation we would call  $\bar{V}$  is going to be the average velocity on the maximum, which will say  $U_{\text{sub Max}}$ .

9:05  
This is the notation from from the K.

9:09  
So I'm trying to to jump between notations here.



9:12

Now if we recall when we talked about shear stress, we said for a laminar nucleus turf for Newtonian system,  $\tau$  is going to be equal to the viscosity times,  $\dot{\gamma}$ , right, which we called this was the gamma dot or the shear rate.

9:35

Now when we talked about that a few lectures ago, we were defining that without the negative sign.

9:39

And I think somebody asked that in class that definition without the negative sign is because we were looking at the context of the wall, right?

9:46

And so everything moving away from the wall was going to have an increase in velocity.

9:51

Now in this context, you'll see the negative sign is going to show back up because now we're looking at the central line of the pipe and looking at how the shear or the velocity is going to fade away as we move toward the wall.

10:02

So just a, a heads up, that's where that negative sign comes from.

10:05

So I can take this definition here for the the shear stress and I can integrate this from the bounds of 0 to  $U$  for the velocity distribution and the radius of the wall versus the radius in any position they're in.

10:19

And I can generate an analytical expression, right?

10:22

And this is for  $\lambda$  flow only.

10:25

I can generate an analytical expression for the velocity  $U$  as a function of  $R$ , right?

10:31

Where  $R$  is going to be some distance away from the central line of the pipe, right?

10:37

So if I'm trying to evaluate maybe the sheer stress of an intermediate position, I can say this is my  $R$  that I'm evaluating, right?

10:46

And I'm trying to target the velocity at that point.

10:50

So here I can say  $U$  is a function of  $RI$ .

10:52

Need to know something about the wall shear stress.

10:54

$\tau$  of wall, the full pipe diameter, the sort of two  $RW$  is the pipe diameter, the fluid viscosity  $\mu$ .

11:03

And then I have this  $R$ -squared term show up.

11:07

Now if I define that at the centre line, right?

11:10

So at the centre line  $r = 0$ , so that second side of the term collapses and I can say, well the maximum velocity right in that centre line will be defined by theory as the wall shear stress, the radius of the pipe onto and divided by the fluid viscosity.

11:32

Now similarly I can take that same definition from above and evaluate it from in the context of my average flowing velocity right?

11:40

So I can get something in  $V$  bar, and if I go through that same process for  $V$  bar, I get the same  $\tau$ , so  $WRW$  but divided by four times the viscosity.

11:52

And so if I want to evaluate the quantity of the average velocity on the maximum velocity in the system for laminar flow, this is defined theoretically, right?

12:01

I can derive this condition and it's exactly .5, right?

12:06

So if I want to know how fast or what's the average velocity of a system or a flow moving through my pipeline relative to the centre line, velocity theory tells me for laminar flow, because it's perfectly defined, like those playing cards that are being smeared out across the table, it's exactly .5, right?

12:25

It's parabolic in nature.

12:27

Now when I get into turbulent flow, right?

12:29

And this is where a set of experiments from McCabe's set of addition comes in.

12:34

And here you'll notice this is actually Reynolds number on the axis or the X axis.

12:40

So I have 10 to the power of 3456.

12:43

So on a log scale.

12:44

And here I'm plotting that value of  $\bar{V}$  on the maximum velocity on the ordinate or the Y axis.

12:50

And I can see the intercept will be for laminar flow .5.

12:54

And the experiment show me that as I come into a a turbulent region, right?

12:59

I'm going to very quickly see this increase, right?

13:04

So the rest of the flow velocity is going to pick up as I become as my Reynolds number gets higher and higher and higher.

13:12

And so as that curve starts to approach one, it's meaning the entirety of the fluid is moving at that maximum velocity.

13:21

What that corresponds to is this bullhead, right, broadening out.

13:25

So majority of my fluid is moving at the higher velocity range, right, If that makes sense.

13:33

So that's the the relationship between this plot and what we were talking about with that bullhead kind of stretching out at higher Reynolds numbers.

13:42

OK.

13:43

So now what are we going to do with this natural flow, right?

13:46

What can we, when we're talking about incompressible systems?

13:50

Well, first let's start by thinking about a force balance that we're going to place on a volume of fluid, right?

13:56

So this would be a macroscopic momentum balance.

13:59

And so if I think about this, well, I can say, well, the sum of the forces acting right on a finite portion.

14:05

So we'll shade in here, what is our finite portion.

14:11

So the some of the forces acting on that finite portion include gravitational force, right?

14:16

If I have some pipe inclination, gravity is always going to be pulling it in One Direction of the pipeline.

14:22

I'm going to definitely have some kind of wall stress because I know that my wall shear stress is going to be interacting.

14:29

So I have an interaction term there.

14:31

And then we know that the flow of any kind through a pipe, be it incompressible or compressible, is driven fundamentally by a change in pressure, right?

14:41

So the pressure gradient across the system.

14:43

Now I'm kind of jumping around through the Chapter 5 and how I present this, but it seems when I read through it, easier to frame it in this term.

14:52

Once we look at this force balance, what this tells us is that we have the introduction of the momentum driving force that we call the pressure drop.

15:01

Now it's a bit tricky because the pressure drop itself we always define as the downstream pressure minus or less the upstream pressure, right?

15:12

And that's because fluids will flow from high pressure to low pressure.

15:15

So if we're engineers and we're talking about pressure drop all the time, we don't want to constantly be talking about a negative number, right?

15:22

So that pressure drop is defined as the downstream minus the upstream.

15:26

So we have a positive quantity for flowing from A to B.

15:31

And this but, this driving force of pressure drop is then balanced across a series of loss terms, right?

15:37

So I have my loss because I have an interaction at the wall.

15:40

I have a loss because I, I have gravity acting on the system potentially, right?

15:45

And what I put in here is to say and on and on and on until we hit the last lost term, right?

15:50

So what we're going to look at today is a summation for single phase incompressible flow in a ducted system, so say a water pipeline from here to Bunbury, right?

16:00

But these lost terms expand and they start to stack up as I get more and more elaborate systems.

16:06

So we'll get into that toward the very end.

16:09

Now, what chapter four goes through in quite a bit of detail is looking at all those derivations of fluid mechanics only to end at the point of saying the Bernoulli

equation where it is one that we can use to balance these kind of energetic energies, flowing energies in the system.

16:26

So the Bernoulli equation shown here from going from two points we'll say are upstream point A and downstream point B and the pressure  $P_a$  divided by.

16:37

Now the density of the fluid  $\rho$  is the gravitational field,  $Z_A$  is the upstream height,  $U_A$  is the upstream velocity squared.

16:48

And then we put in an efficiency  $\eta$  and a work of the pump just in case we might have a pump in this system.

16:54

And what we're going to see later on is, is we'll probably remove that for a few examples.

16:59

And then I look at balancing that with the downstream energy.

17:02

And this is every unit here has energy per unit  $\dot{M}$  is the the quantum.

17:08

So downstream I have the same pressure on density, whatever that downstream height  $Z_B$  is going to be.

17:14

Usually we're solving for velocity in most cases.

17:16

So the downstream velocity use of B and then I introduce  $H_F$ , right?

17:22

And  $H_F$  is what we're going to call a frictional loss term, right?

17:26

So in the same way that I can take a simple breeding of the equation and I if I have an upstream energy, I'm going to add the pump to that upstream.

17:35

I know that downstream I'm going to pay the penalty if any losses encountered in the system.

17:40

So I don't know if, if did anyone see it presented in this framework in 3003?

17:50

Let me know as we go, because I'm, I'm not quite sure how it's how it's structured in that curriculum.

17:54

But there are a few different ways to look at this.

17:57

The way we're going to be working through McCabe's nomenclature, everything is embodied within this  $H_{sub F}$  term, right?

18:04

We're going to look at expansions to this term as we go.

18:07

So it's worth noting.

18:09

Yeah, it's a bit.

18:09

OK.

18:11

So it's worth noting that everything within this equation, right, is defining an energetic condition at a point except the frictional loss term, right?

18:21

The frictional loss term is the accumulation of of energy lost between those two points.

18:28

So what do we do about this  $H_{sub F}$ , right?

18:30

The rest of the terms in this equation I can probably get at relatively easily, but what do I do about  $H_{sub F}$ ?

18:37

Well, let's let's first start thinking about just the wall shear stress.

18:42

So I've had a very simple pipeline, right, perfectly horizontal coming from here to Bunbury to carry freshwater.

18:48

That's it.

18:49

Now if I'm in single phase flow, has anyone seen shell balances in 3003?

18:56

Because that's usually more of a transport phenomena concept.

18:59

So I'd be surprised if anyone's seen a shell balance done properly yet.

19:04

But when you start getting into, into both advanced fluid mechanics and transport phenomena, we tend to, to think about analysing different points in a flow stream by these shell balances, right?

19:16

And all this is, is a geometric clone that's somehow smaller, right, than the representation of what the, the pipeline's taking on.

19:24

And we take an infinitesimal thickness and height, or in this case radius, so that each of our physical dimensions.

19:32

So if I'm working in simple and incompressible fluid flow, then the differential pressure across the system is going to be related to this frictional loss term, right?

19:42

And I can also go back to my differential equations and relate that pressure drop, right, which we can also write as DPDL.

19:50

If we're taking an instantaneous snapshot of a shell balance, I can relate that to the shear stress of the wall in a fundamental differential equation.

19:58

So if I combine these two definitions, I can then yield a very basic definition for the impact of wall shear stress on frictional loss.

20:15

OK, so I have  $4$  divided by the density of the fluid.

20:26

Now in here what we're saying is it's  $L / D$  but on the numerator I also have this  $\tau_w$  this wall shear stress, right?

20:36

And this might not be an easy thing to get at because you'll remember that's the product.



20:42

Now, the fluid viscosity, which I had better know, but also the shear rate, which I might not know, right?

20:48

Particularly if I'm in some kind of weirdly shaped channel or if I'm in a very highly turbulent region, I might not be able to easily get AT what is that shear rate right again?

21:00

Because if we go back to it in a turbulent region, the shear rate at the wall, right, is going to be the slope as I approach into the boundary layer, right?

21:09

So understanding where we are in that bullhead construct might not be a super easy thing to get at.

21:15

So as a consequence, we want to replace that with something that we can use pretty quickly as engineers to make it easier.

21:21

So in that context, we introduce the spanning friction factor and we give it so that's little F here.

21:27

And we give it the definition of two times the the wall shear stress divided by the density times the average velocity squared, right?

21:35

And I can put that up into the definition for my frictional loss term, which then generates  $8 \text{ sub } F$  of four times the fanning friction factor times  $L / D$  times that bulk or mean velocity squared divided by two, which itself is that energetic component from the Bernoulli equation.

21:57

So now if I need to then calculator figure out well what is this fanning friction factor going to be and how can I implement this for pipe flow?

22:07

There are a couple different ways that we can go about it.

22:09

So the 1st way is we'll look at the Moody diagram.

22:13

Now, if we're we're looking at the Moody diagram here, the ordinate or Y axis is the friction factor that I'm looking to find, right?

22:28

And the Reynolds number which I need to know is what I would have already calculated.

22:35

All right, You'll notice that the Moody diagram is designed with a few different regions in it.

22:42

So firstly, if I'm in a laminar flow, all right, that's going to follow this, this solid line, right, on a, a log linear scale.

22:52

But it's going to follow this line here, it's friction factor being equal to  $16$  divided by the Reynolds number, right?

22:59

And it's some critical Reynolds, it's some critical Reynolds number of transition, right?

23:05

You'll remember that we have that kind of transition regime where we sometimes have turbulent eddies that are going to persevere and sometimes they're going to be damped out.

23:13

And it's, it's kind of a, a distributed phenomenon once we get into the turbulent regime now, right?

23:20

This is where we start to see these different curves emerge.

23:23

Now, there are a couple different features I want to point out here.

23:26

This bottom curve, if we trace this bottom curve all the way back up, that is our baseline condition for a perfectly smooth pipe.

23:34

So if we looked at the example of something like a plastic pipe or a Teflon pipe where it's going to have minimal, if any roughness, that's the most efficient flow pipe that I can ever get.

23:46

Once I start to have more and more and more roughness, my friction factor is going to go up at a constant Reynolds number.

23:53

So let's look for an example here at Reynolds number of  $1 \times 10^6$ , right?

24:00

And I can find the points on that.

24:02

So I come down here right  $1 \times 10^6$  I find where I am on the smooth pipe and I can read across, right that's a friction factor of about 0.028 or 0.029 right now once I start to have roughness showing up in the pipes, this can occur from a variety of reasons.

24:21

So we're showing here on the right hand side a few examples.

24:25

I can have rough features that might look like teeth, right If I'm maybe that's the product of unanticipated corrosion of the pipeline material.

24:33

So if you use it at cheap steel and it's being goes to water and say  $\text{CO}_2$  and then over time it will corrode and it's not going to be a perfectly smooth pattern.

24:43

However, if I start to have something like mineral scale, So if I have hard water, meaning I have a lot of mineral or ion content in the water that can deposit like the calcium carbonate on your teeth on the pipeline wall over time.

24:57

And that's going to give me a much smoother profile.

25:00

But in both cases, what I'm looking to do is calculate this  $K$ , which is the we would call the average asperity size, right?

25:09

So that's basically how long is the average tooth if I measure all of the teeth in the system or OK, if it's a smooth phenomenon, is the difference between the peak and the trough, right?

25:21

Then I can trace this across and see we're defining the trough there and the peak here.

25:25

And so the way I'll frame this is then looking at the relative roughness of that.

25:31

So the height of the average is 30 size K divided by the pipe diameter.

25:36

All right, So what the Moody diagram then shows us is that on the right hand ordinate or Y axis, I have relative roughness  $K$  on  $D$ , right?

25:45

Where this would represent this highest condition would be 5% of the pipe diameter, right?

25:54

So my, my average asperity size is about 5% of the diameter of the pipe.

25:59

And then I go down from there, right?

26:01

So that once I know some characterization of the pipe, I can then pick where I am on the right hand axis.

26:08

And for instance, if I'm not using a plastic pipe, but I'm going to assume my pipe has been corroded for some number of years, some might have an average roughness of say .2%, right?

26:18

I can trace up on this and see that my friction factor now is instead of point OO29 is going to be .006, right?

26:28

So almost a a doubling, just over a doubling of my friction factor for the energy penalty I'm going to pay because I have a rough pipe surface, OK.

26:38

The other point that it's very important to make on this diagram is you can see I can draw a almost a vertical line at about 10 to the seven, right?

26:50

So once I have a Reynolds number in excess of about 10 to the seven, I tend to see little to no dependence on of the friction factor on Reynolds number, right?

27:01

Again.

27:02

So once I get above 100,000, it becomes minimally dependent on Reynolds number.

27:06

And once I'm well above a million, I no longer really have a dependence of that friction factor on how fast or turbulent the flow term is going to be.

27:15

Now traditionally, as you just saw, the approach is going to be that we're going to physically read the value off the Moody diagram.

27:23

But maybe I don't want to be having to reference this because I'm trying to build an algorithm to do calculations for me.

27:32

So in that context, I can look at a series of analytical relationships here.

27:37

And so these are presented through the top is analytical because it's actually derived and the textbook has a nice little derivation of it.

27:45

I just didn't have time to show it here.

27:47

And then there are three empirical relationships for turbulent flow, 2 for smooth pipes and the latter one obviously is covering a much larger area in terms of its Reynolds number space.

28:00

But then in turbulent flow with a steel pipe or a rough pipe, if I know something about that asperity size  $K$  on  $D$ , then I can also calculate the friction factor from that, right?

28:11

And so this would be a way if you're trying to then build that into a spreadsheet, you don't have to constantly reference against reference against the Moody diagram.

28:21

So that's the lost component that's going to come from the actual pipe wall interaction.

28:27

But what else can be included in this?

28:29

Well, if I have flow strings, for instance, that are going to go from a very narrow, right, if I'm going to suddenly expand a narrow flow stream into a very large pipeline, we remember back when we were looking at some of those examples, I'm going to have recirculation zones here, all right, that are going to be dead zones for quite some time.

28:54

So because of that, I'm going to lose energy because I'm burning that into turbulent eddies that are being created that otherwise would, you know, not be there if the flow stream was at steady state.

29:04

So if I have a friction loss and it's  $l$ , I hesitate to call it a friction loss.

29:09

It's just in the context of how it's presented here, it's an energetic loss.

29:13

But if I take a rapid expansion of flow stream, then I'm going to have this additional friction loss  $H_{sub F}$  with the subscript expansion.

29:23

And the way that we'll characterize that is the energy, the average energy of flow.

29:26

So  $V$  bar upstream.

29:28

So  $V$  bar a quantity squared divided by two times, case of  $E$ , right?

29:34

And case of  $E$  is the expansion loss coefficient.

29:37

And what that's going to be equal to is this one minus the surface area upstream on the surface area downstream quantity squared.

29:46

So I can calculate that directly if I know what my upstream and downstream surface areas are going to be similarly, right.

29:53

If I'm taking a large pipeline and I'm going to to instantaneously shovel that content into a small connecting pipeline, I'm going to engender a lot of friction around that.

30:03

And a lot of turbulent eddies will be created by the compression or, or the yeah, compression of that fluid.

30:10

So we can see here, this is my turbulent Eddy region and those flow streams are going to bend in their minimum point of curvature.

30:18

We call the Vena contractor and then it's going to come back out and restabilize.

30:23

So for instance, the point at which I, I hit my minimum plane, we would say here, right, this is the time to fully establish flow.

30:42

All right.

30:43

So I have to re establish the flow in both cases.

30:46

And I can see that really nicely in the contraction example.

30:49

So similarly, if I'm thinking about the friction loss in the contraction, I have the same flow stream or energetic content of the flow stream, only now I characterize it by the downstream velocity  $V$  bar squared and I'm multiplying it again by a contraction loss coefficient.

31:07

Now case of C, Now if I'm in the laminar flow, right, because laminar flow is just those cards being spread across the table, that case of  $C_{10}$  is less than .1, all right?

31:18

So 10 mean, meaning that the contribution of  $H_{sub F}$  contraction is negligible.

31:26

However, if I'm in turbulent flow, which most of the time I will be, then we characterize it similarly to the expansion loss coefficient, but it's simply  $.4 * 1$  minus the downstream surface area on the upstream surface area, right?

31:40

And this gets me KSM soon.

31:42

OK, so those are the first two editions of the pipe network.

31:45

I need to think about The 3rd and final is what happens if I have fittings, valves or bends in the pipeline, right?

31:51

So again, now our  $H_{sub F}$ , which is for the fittings part is going to be characterized by another case of  $F$  and again multiplied by that upstream kinetic energy of the fluid  $V$  bar squared on two.

32:08

And now I have to come into a table to look at what is the factor associated with each type of consideration.

32:18

So if I have something like an elbow in my, my flow stream, so let's say we're going to take a 90° bend in the system.

32:26

Well, I know that as I come around that bend, it's going to be some amount of time before I re establish flow, right?

32:32

So I'm no longer in a fully, fully developed flow pattern because I've turned a corner, which means I have additional frictional losses associated with that bend point, right?

32:42

So we come to this and say, well, how many bends do I have in my system?

32:46

What's the case of F value for each one of these bends?

32:50

And then I can put it into my equation and solve for the differential velocity upstream and downstream.

32:57

Similarly, if I'm flowing through something like ATA return bend, so a 180.

33:02

And then on the bottom here we have different types of valves, so a gate valve, angle valve and globe valve.

33:07

So you can see what will happen if I take my fluid through a globe valve, I'm going to have a very high friction loss right from that, from that entry.

33:18

So now what we've seen on this slide is that we can go way beyond just a simple backup a little bit a simple definition of the frictional loss encountered at the pipe wall.

33:30

I can take that a step further and say, well, the fluids also going to lose energy if it has to suddenly expand contract or it hit something it didn't anticipate like a bend OR gate or a valve, right?



33:43

And so I can take all three of these terms now and merge these together.

33:48

So if I recall the original Bernoulli equation on top where I had defined that wall friction here I can expand that outward and say, well, I'm going to pull my kinetic energy to the side for a minute because I know that that's a constant being multiplied through each of the each of the domains.

34:07

And so here I have my wall contribution, so the wall shear stress contribution.

34:12

And then I have my contraction coefficient, expansion coefficient and the fittings and bends coefficients right.

34:18

And I can simply sum these up, multiply them by the kinetic energy of the flow stream.

34:23

And that is what I can put back into my Bernoulli equation to account for the flow of those loads.

34:30

So if I want to then take this example on the right hand side.

34:35

So here we're going to go from an entrance header.

34:37

So I have a very large pipe that's going to be taken into a small pipe over some distance  $L$ , where I have a partial valve here that's going to be blocking it and then expand it to an exit header back into another large pipe, right?

34:49

How am I going to write this?

34:50

How do I balance the energy across this system?

34:53

Well, I can say, well, I have a momentum driving force  $\Delta P$ , I'm going to take the  $\rho$  and multiply it across.

35:00

So I have  $\rho G$ ,  $\Delta Z$ .

35:02

So if I, if it's all horizontal, that term for the potential energy change is going to drop out.

35:09

And then I have my resistance terms that are multiplying the kinetic energy term of the fluid.

35:16

So the, the final point that I wanted to make here was that, you know, as we go back and and talk about all these flowing fluids, one of the things that hopefully you've seen by now is that we have an important impact of the energy, right?

35:30

So we're talking about moving energy from a momentum standpoint to potential energy to kinetic energy.

35:36

So at some point, right, if, if we took 3006 with me last semester, we would have talked a lot about the relationship between heat and energy.

35:45

And in particular, if I look at the energy cascade, right, that we've been talking about for a few lectures now, we said that in in early bits of this cascade, going from large eddies down to intermediate to small eddies, I'm not losing energy.

36:02

I'm just transferring it from the large eddies to the small energies.

36:06

But once I or small eddies, right?

36:08

But once I hit that Kolmogorov link scale, now the small eddies are going to burn that energy off through viscous dissipation into heat.

36:16

So it might ask you how much heat can be generated.

36:21

Well, I can relate the amount of heat  $Q$  per unit volume  $V$  to the constant volume heat capacity of the system, the density of the fluid and the change in temperature, right?

36:36

And that itself will be related to the differential or the pressure drop across the system if I have an adiabatic incompressible fluid with a constant cross-sectional area.

36:48

So I can rewrite this now to say the amount of temperature change that will be engendered is the differential pressure applied across my system divided by the density times the constant volume and peak capacity.

37:02

So an example, right?

37:03

Then the constant volume and peak capacity is something I might get from other than NIST Chemical Web book, right?

37:11

Or write that here NIST Chemical Web book or HEISIS.

37:18

So an example of a hydrocarbon or a polymer flow, I might expect something like .4 to .6 Kelvin of temperature increase per mega Pascal of drop that I take across the system, right?

37:30

And that's due to the viscous, viscous dissipation at this lowest link scale.

37:42

OK.

37:43

So kind of bringing this back into take home messages, then when we talk about incompressible flow, I can deploy the Bernoulli equation, which itself is a simplification as some of the more complex fluid mechanics in momentum balance and models.

37:59

Now within this Bernoulli equation, I can see that I have my pressure terms right, potential energy terms in the middle and my kinetic energy terms on the right.

38:10

And then I'm going to add into this a constructive well.

38:13

I might have a little bit of upstream pump power or energy that's coming in from a pump, which itself will be the amount of work of the pump times its efficiency, ADA.

38:21

And on the downstream, I know that I'm paying an amount of energy for flow, right?

38:31

And so the loss from that frictional we characterize as  $H_{sub F}$ .

38:35

And I can take a traditional definition of that friction loss, which is itself going to incorporate the fanning friction factor  $F$ .

38:44

But I can also expand that out to account for contraction expansion of the pipe network where the inclusion fittings are bent, right?

38:51

So here we have the kind of sum of the equation, right?

38:55

And if I'm trying to calculate this  $F$ , there are two ways I can get there.

38:59

The Moody diagram is probably the most tried and true.

39:02

So that's definitely one you familiarize yourselves with, but certainly, you know, as it's becoming more commonplace now, I always want to be able to, to build analytical models and, and simulators and build calculation tools.

39:15

So the ability to have an analytical relationship for that is quite helpful.

39:19

So with the example, if I know something about the average disparity size  $K$  relative to the pipe diameter  $D$ , then here's a a model for that friction factor I can put in for a rough pipe network.

39:32

All right, so that's everything we have for today.

39:35

When we kick into lecture on Wednesday, we're going to start talking about compressible fluid flows.

39:40

So that's going to get into talking about gases.

39:46

Do you have any questions on this?

39:56

Can the surface area ratio be simplified to a diameter ratio?

40:01

Yes, as long as the pipe geometry is not changing.

40:08

All right.

40:08

So if you write out you're talking about the expansion and contraction loss coefficients show, yeah, So right.

40:22

If you look at one of those surface area, so say  $S$  of  $A$ , which is going to be my upstream area, right, will be  $\pi R A$  squared  $S$  of  $B$   $\pi B$  or  $R$  sub  $b$   $^2$ .

40:40

So it's not directly the ratio of the diameters, but it's the ratio of if I'm going to take a division between these two, right, it's going to be, I can knock out the pies, but it's going to be the the ratio of the square of the radius.

41:10

So  $SA$  is the cross-sectional area we've just highlighted here, not the surface area.

41:24

Do you have any other questions?

41:52

So  $KC$  for laminar flow.

41:54

So in in The Cave it talks about it  $KC$  for laminar flow has a maximum value of .1.

42:02

It doesn't give a model for that.

42:04

But if you plug that into the model and you look or this definition of the friction loss in, in a contracting environment for laminar flow, what the book talks about is the, the fact that the, the impact of that friction factor is so small that it becomes negligible, right?

42:24

So, so in that construct, if I think my average velocity is going to be something like .5, well in laminar flow, right?

42:31

And getting a meter a second is going to be a value of .5 as  $l / 2$ , but no way a meter a second is going to be laminar.

42:40

So realistically and down to say .1 or point O 5 meters a second at most, right?

42:46

That's going to be probably the limit of laminar flow for water in the pipeline.

42:50

So  $.1$  or point  $O 5^2$  itself divided by two, again multiplied by  $.1$ , right?

42:57

What they're saying is that this frictional loss from the contraction becomes so small of a term to add in that, that fundamentally I don't have to worry about it.

43:09

Whereas when I get into turbulent flow, right, this  $V$  bar squared becomes quite a high number.

43:15

And so that multiplied, you know, actually gives me a meaningful contribution to the friction loss.

43:29

So I should also mention, feel free to type in any other questions you have while we wrap up.

43:34

I should also mention there, there is another way if I want to account for things like bends and elbows, right?

43:41

And that's, that's to take an equivalent length.

43:45

So I, there's another approach that we can take we'll probably see a bit later in your curriculum if you haven't already.

43:51

But I can take an equivalent length of what how many diameters every bend OR elbow is going to introduce, and that will give me a longer total length of the system that I can then plug into my Bernoulli equation.

44:06

But the the structure we're following here is what McCabe uses.

44:10

I'm trying to stick with it for consistency.

Lecture 05 - Flow in Solid Beds

0:00

So here's our normal copyright notice, and this is lecture now #5 flow in solid pens.

0:07

So you'll recall the day in the last lecture we were talking about how we can take a very basic momentum balance.

0:14

And this is being applied for flow through incompressible flow through a pipe, right?

0:20

And so there's our back end of the pipe.

0:21

So we have some flow channel coming in here and we're looking primarily at the interaction of this wall shear stress as one domain of where we're going to lose energy because we pay a penalty for that fluid interacting with the wall.

0:36

All right.

0:37

Now we can write that general momentum balance on in terms of the Bernoulli equation for incompressible fluid flow.

0:44

And so with the Bernoulli equation is showing us here on either side, we have our our momentums or our pressure turns right.

0:52

Then we have our gravitational potential terms and our kinetic energy terms.

0:57

And so this third term  $U^2$  or velocity squared on two is often called the velocity head.

1:04

So you'll see that starting to emerge a bit in the text.

1:08

This is the velocity head relative to the pressure head being  $\frac{P}{\rho g}$  where the gravitational head being  $\Delta Z$ .

1:18

Now in this expanded Bernoulli equation we also looked at the inclusion of pump work.

1:24

At times its efficiency ADA and ended by talking about the frictional loss term, which we expanded beyond just that simple definition of wall shear stress.

1:35

But to look at both contraction and expansion as well as the inclusion of fittings where pipe bends, we're going to lose momentum energy so that HF.

1:45

This is the only term in the Bernoulli, Bernoulli equation that's accumulative.

1:51

So it accounts we look at the difference between 2 points and we accumulate all of these fluid losses.

1:57

Now within that for contractions, expansions and fittings or bends, we have these factors that can be calculated directly.

2:05

We see these are all multiplied by that fluid head because we're looking again at the the cumulative impact.

2:14

But for the wall stress, we introduced this friction factor, all right.

2:18

And that's really quite important because we're going to look today and what happens when we expand beyond that simple definition of friction factor for and we invert the flow scenarios.

2:28

So now we have an object that's sitting in the middle of our fluid as opposed to a wall interaction.

2:34

And so the wall shear stress and the friction factors, we talked at the very end about how these vary primarily with the the relative roughness where we define this as  $K/D$ , where  $K$  is the average asperity size and  $D$  of course is the diameter of our flow chart or sorry of our flow channel.

2:56

So this can be of course obtained from the Moody diagram.

3:00

But then we also looked at analytical relationships and these can be really helpful if we're going to, for instance, put together an Excel model where we want to start to, to vary some of the parameters.

3:10

And so here we have an analytical expression for turbulent flow in rough pipes or steel pipes.



3:17  
OK.

3:18  
So moving forward now, if we take a step back and instead of flowing a liquid down a pipeline and thinking about that wall shear stress, what happens if the pipe is significantly large and we want to focus in and we want to put an object in the middle of flow.

3:35  
So now I have this flow field here, right, the same kind of flow field.

3:39  
And maybe this is my pipe boundary at the very top, and that's going to be hitting some object.

3:46  
Now we can see that if we have a direct proportionality or a direct impact of the flow field on the front of the object, that's a relatively straightforward concept.

3:56  
But as we start to flow around the object, we're going to have some angle, right?

4:01  
So in this case, we're saying  $\alpha$  is the angle at which those flow streams are going to impact the side of the object.

4:10  
And so instead of having a wall shear stress, we introduce 2 new terminologies for immersed solids.

4:17  
And one of these will be the drag of the solid body.

4:21  
And that will equivalently because we're dealing with the fluid hitting a wall and of course, this has a no slip condition, we still have that shear stress term, but now we multiply it by the sine of  $\alpha$ .

4:33  
So that angle at which it's going to contact and  $DA$ , which is going to be the area of contact of a flow stream.

4:42  
So the way that we can start thinking about this, if I erase some of these lines for a minute, we're going to have flow or streamlines, right, that are going to be flowing around this fluid or sorry, this, this immersed object, right?

5:01

And so each of these streamlines, as we get further and further away from the immersed object is going to have less of an impact.

5:07

And we're really looking at what happens for the streamline that's impacting the edge of the object as we move.

5:13

So we have this drag that's being introduced, and we also have a pressure that's being applied into that boundary layer, the fluid again, the pressure times, then the cosine of  $\alpha$   $DA$ .

5:25

And this gives rise to two terms that we use for immersed objects.

5:29

Now, the wall drag  $T$  or  $\tau$  sub wall and the form drag that occurs from pressure.

5:35

So we're going to unpack these latter half of the equation.

5:40

So now if I'm thinking about an immersed object in flow.

5:44

So again, this, this little picture on the bottom here from the textbook, we can imagine that the wall, right, is going to be, if I flip around here, the wall is going to be some distance away from both sides of this.

6:02

So we're really narrowing in focusing only on one small object that's being submerged in a very large flow field.

6:10

And I can envision here that that the image, we were just looking at those individual streamlines impacting the surface of that.

6:17

Well, if I'm immersing a sphere and I want to look at how the fluid's going to flow around it, I can start to think about, right, how much energy lost or wall shear impact am I going to have because of that submerged object?

6:33

Now the way that we characterize this is first we use what's called a drag coefficient, right?

6:39

And the drag coefficient is fundamentally defined in blue here where of course that the quantity on the numerator.

6:47

So the the force of the drag divided by the area the  $A_{sub P}$ , which is the projected area I'll get to in just a minute.

6:55

That's an equivalent shear stress if we go back to the very fundamental definition of shear stress divided by the velocity head of the fluid coming in  $U^2$  on two times the fluid density.

7:08

So as we define this drag coefficient, So if we have a very high drag coefficient, it's going to mean we're burning a lot of shear stress, right?

7:16

Because this numerator is giving us a shear or a wall shear stress component and the denominator is giving us a velocity head component.

7:35

So if I have a very high drag coefficient, it's going to mean that I'm burning a lot of shear stress relative to the amount of velocity that I'm putting that I have in the system.

7:44

And I have a very low drag coefficient.

7:46

It means I'm not burning that much shear stress, right?

7:49

So I have an efficient flow as another way to think about it.

7:53

So most of the terms in this definition are relatively straightforward, but we have to think about this  $A_{sub P}$  the projected area a little bit more.

8:02

So the projected area is shown in this diagram on the bottom right.

8:06

This is if we take something like the easiest example is a sphere.

8:11

If this object here is a sphere, then if we project that into the two-dimensional plane that is perpendicular to the flow stream, we get a circle.

8:23

You can imagine taking a sphere and shining a torch on it.

8:28

And what's the image that's going to show up on the wall behind you, right?

8:31

That's the projected area that we want to manage.

8:35

Of course, now that we're not looking at some distance away because that torch is going to illuminate farther, but it does illustrate the point.

8:42

So we include this this projected area here as a way of thinking about, well, the fluid's primarily going to be impacting the front of the object.

8:49

So I have a two-dimensional surface in which it's going to be relevant.

8:53

Now I can further define this this drag coefficient by characterizing it relative to the Reynolds number of at the particle length scale, right?

9:06

So now instead of thinking of a Reynolds number for the pipeline or the duct or whatever large flow channel, I'm simply going to be focusing in on the particle itself.

9:16

Now  $\Phi$ , a little  $\Phi$  here is that proportionality function.

9:20

We're going to get into it in a little bit, but it illustrates that the drag coefficient is a direct function now of the fluid density coming in  $\rho$ , the velocity upstream of the fluid  $U$  and  $D_P$ , which is a characteristic length of the emerge, sorry, immersed object.

9:40

So in this case,  $D_P$  would be the diameter of the sphere, right?

9:44

But we'll look at a few variations to that.

9:47

And then the  $\mu$ , which is the the viscosity of the fluid coming in, right.

9:52

So we want to highlight this is our definition of the particle Reynolds number and it's going to come back a few times throughout the lecture.

10:03

So if I need to look up these or calculate the drag coefficients thing, you know, these can be found in a series of textbooks and look up tables.

10:13

A really good example here.

10:15

This comes from both the 7th edition and McCabe, but also references like Perry's chemical engineering handbook, right, which is just a big tome of relevant chemical engineering properties and coefficients that I need to characterize different systems.

10:31

These are the kind of resources that we look to, to identify different drag coefficients.

10:37

So the example that McKay presents in the textbook here is a similar style of plot that we've been looking at previously.

10:43

We're here on the ordinate or Y axis for looking at the drag coefficient and on the abscissa or the X axis, we have that particle Reynolds number, OK, where the only difference in that is we instead of using the diameter of the fluid stream, we use the diameter of the particle or the characteristic length of the particle.

11:04

So we have a look up tables then for the drag coefficient for each type of solid and each incarnation of that solid.

11:12

And we'll look at some examples in the last slide.

11:15

But here we can see as we look at this plot points that that we can identify.

11:23

So at very low Reynolds numbers below a value of about one, we see that there's a linear region, right?

11:32

And this corresponds to it.

11:34

We can actually derive from fundamental theory looking at Stokes law, we can derive

a relationship with that drag coefficient being 24 divided by the particle length scale Reynolds number, right?

11:45

So that gives us this linear type of relationship.

11:49

Now if we go into an intermediate relevance number, so this would be between 200, right and 300 here we see that we have a region where for with the exception of spheres, but for our disks and cylinders, and it's pronounced in the disks in particular, we have fluctuating vortices that will be induced when we flow fluids around these kinds of objects.

12:12

And the reason why, right?

12:14

If you imagine if I, if I have a flow stream here, if I drop a sphere right down, well, if that sphere is going to be rotating and and moving around the flow stream, the projected area is not going to change, right?

12:29

But if I have something like a cylinder right here, I'll, I'll draw out a hollow cylinder.

12:37

Well, if that's going to flow down this this flow channel or move down the flow channel, then I'm going to have a variation or fluctuation in that projected surface area.

12:48

So this means that the amount of turbulence or drag that's encountered in that scenario is going to be fluctuating.

12:57

So This is why as I have less and less spherical objects or my sphericity deviates from one which we're going to look at in the next slide, we start to see a higher drag coefficient pop up in this little intermediate region of the Reynolds number space.

13:13

Now once I get to a Reynolds number of about, well between 1100 thousand now, right, what I notice is that again now there's it's invariant, the drag coefficient tends to be relatively invariant to the Reynolds number and it will sit in that arrangement between .4 and .45 for most systems.

13:39

And if I go above 300,000 of this particle Reynolds number, I get a significant drop.

13:45

So in the case of spheres, now I'm down at the level of about .1.

13:49

So I should clarify both of these heuristics are for spheres.

13:56

So this, if we look at this chart, then it starts to tell us if we turn up the velocity of my flowing fluid or I increase the size of the immersed object in that flow, which is going to increase its DP or its characteristic length, I'm going to go to higher and higher Reynolds numbers.

14:14

And when I do that, I make flow more efficient, meaning I have less drag of the fluid around that immersed object.

14:22

So equivalently, the amount of sheer stress that I'm burning as I flow past that emerge object, immersed object decreases.

14:33

So what does this look like in practice?

14:35

Well, if I look at this intermediate range of Reynolds number, so the example of dropping a cylinder or a disk, and you'll watch it kind of fall through a column of liquid.

14:46

Equivalently, the disk isn't moving, but the disk is stationary and the liquid is flowing around it.

14:52

I can also look at what those flow streams will look like.

14:54

So this is a really cool image of of a dyed fluid or a ball that's being has a dye on it.

15:01

So as the fluids passing over it, it will pick up that dye and then I can look at the vortices that are induced downstream.

15:07

So here I have a a direction flow from left to right, very similar to to Reynolds early experiments, right?

15:15

And this behaviour downstream of that inverse object is known as the vortex St.

15:21

And so this is occurring in that intermediate range of Reynolds numbers.

15:25

And the reason why I see that kind of uptick and drag because I have those, if you think about the, the transition flow regime where those small eddies are going to sometimes propagate downstream and sometimes be damped out.

15:37

I'm in that kind of transition region flow.

15:40

But once I come up into this, this really high Reynolds number range.

15:45

Now if we look at the boundary of 100,000 of our submerged particle Reynolds number, I can see that if, if my flow is coming from left to right, well, it's going to contact the, the, in this case a sphere at point B.

16:01

But if I'm in a, a Reynolds number of 100,000, then I still have that relatively high drag coefficient, right?

16:08

And the flow will separate.

16:10

So I have an impact zone of where the eddies are going to be contacting the solid, an impact zone of about  $85^\circ$  in  $\alpha$ .

16:20

So beyond point C I no longer have direct contact between my incoming streamlines, right and the surface of the solid itself.

16:29

And that generates a relatively large wake right or a zona for circulation downstream of the object.

16:40

If I turn up the flow velocity from 100,000 to 300,000, now I can see that I now have an  $\alpha$  value that's going to go to  $140^\circ$  and it's not until the back edge, right?

16:53

So if here's the plane of the object, it's not until the back edge of the object that I get flow separation.

17:09

And that's a really important concept that we're going to continue talking about in a minute.

17:14



Once I move past the centre line of the object and my flow separates from the object on the backside of it, the size of the wake or the circulation zone decreases significantly.

17:27

And in fact, the higher I take the Reynolds number and the further that I push past that back edge of the object where the flow is going to separate from it, the more efficient the flow is going to become, right?

17:40

And if I extrapolate, This is why I get that sudden decrease right in the the drag coefficient as I pop above 130 or sorry above 300,000 Reynolds number, right?

17:54

Is that I've induced or I'm now separating on the back edge of the object.

17:59

Now this principle of where the flow is going to separate from the object, if I take that to an extreme, I can look at how do I minimise this drag coefficient.

18:10

So either I choose flow conditions because I don't want to burn all that momentum energy necessarily just in a wake.

18:16

So it's not recirculating.

18:19

I can either choose flow properties that get me into that region.

18:23

All right.

18:24

So that might be changing the local Reynolds number by varying the characteristic length of the particle or the flow velocity or the properties of the flow.

18:35

So the density and the viscosity, right?

18:39

So I can play with each parameter, the Reynolds number.

18:41

So what I'm selecting in the fluid, maybe instead of water, I want to use something like mono ethylene glycol or Meg.

18:46

And that's going to give me a significantly higher Reynolds number, sorry, a significantly higher viscosity.

18:52

So it will drop the Reynolds number if I want to play around in that region.

18:57

Or I can choose to position the object and choose its dimension of the object that itself minimizes this principle and minimizes this form drag.

19:08

So when I position the object correctly or I choose a geometry of the object that minimizes that drag.

19:16

What it looks like here is that I'll have if this is again the central line from the projected area, right?

19:22

I can see that I want an object that's going to have a very tapered tail, right?

19:27

So if I imagine that the pressure of that fluid is letting off ever so slightly as I go from the centre line pass toward the right hand side of the diagram, it's only the very, very tip of the object.

19:40

Now that I have a small recirculation zone.

19:43

And this process, whether I control this through the choice of my Reynolds number when I design the system or the selection of the immersed objects that I'm going to look at, this process is known as streamlining.

19:56

So streamlining is what happens when I have an object where I minimise that, that recirculation zone.

20:05

Now it's important that in all three of these examples from our last slide up through this current slide, this point B where my centre streamlining, right, that's going to pass through the very centre of the object, that centre streamline is going to impact the head of the object directly on.

20:23

And we call that the stagnation point.

20:25

At that point I have a zero velocity.

20:27

So this is point B on the diagram here I have zero velocity and it's at that point where I can fully convert my pressure head or  $\Delta P$  that to a velocity head or sorry, I fully convert my velocity head to a pressure head, right?

20:43

So whatever the upstream flow velocity is, use  $U$  naughty squared on two, I fully convert that to differential pressure from whatever the upstream pressure is to a higher pressure at the impact point B OK, this got messed up.

21:02

Shoot, hold on.

21:19

Sorry.

21:20

It's there we go.

21:24

It's a very busy slide and I don't want to.

21:27

And that's that, of course, here we go.

21:29

So now where does this matter?

21:30

Well, I'm not going to have one object immersed in my flow, right?

21:33

So I use these examples back here as kind of thinking theoretically, how do I go from all of my fluid mechanics of flow hitting the pipeline wall to thinking about a single immersed object?

21:45

But in reality, what I want to think about is a bed of these objects, right?

21:50

And this is a very common process in unit operations.

21:54

If I'm trying to control a reaction, maybe this bed of immersed objects is going to be a catalyst for that reaction.

22:01

So the surface area of that bed and the amount of material I can move through a bed of solids, you can imagine like a jar of marbles, right?

22:09

Is a really good metaphor for this, so that those marbles might be a catalyst for the reaction that I'm trying to control.

22:16

Or if I'm in a mass transfer operation, the surface area of those marbles is going to dictate something around the ability to transfer mass from a gas phase to a liquid phase.

22:26

So This is why we're spending so much time because this is going to inform most of the unit operations we're going to talk about over the coming month or two or these packed beds.

22:36

So when I'm thinking about the pack bed pen and and the metaphor here or an analogue is a jar of marbles.

22:51

If I'm flowing fluid, right?

22:54

So here's my jar, maybe I have a little outlet here.

22:58

And if I'm flowing fluid through this jar, I have to think about this impact of the form drag flow separation.

23:07

So where each for each of the individual marbles, where is that water or that liquid going to to separate from the marble as it contacts it, the downstream formation of wakes in each of these cases, all of these things together are going to give me a resistance to flow, which is the chemical engineers way to say pressure drop, right?

23:30

And I care about this because I'm going to use things like pumps or compressors to create an upstream pressure such that I can move a fluid of interest across this kind of packed bed.

23:41

So as I have this jar of marbles with a little hole in the bottom for an outlet, if I'm flowing fluid across this, then I think that, well, there's a certain volume fraction of those marbles that I can pack into the the jar itself.

23:56

And the rest of the volume of the jar is going to be available for me to fill with water and flow through.

24:01

But it's not going to be as simple as just water in a single streamline going from the top to the bottom of the jar.

24:07

Each molecule of water, if you think about it is going to have to take a very tortuous path because those solids are not going anywhere.

24:15

So it in fact means if my jar diameter right is something if we call the jar diameter  $D$  Well, actually the diameter available for flow, which we call an equivalent channel diameter  $D_{sub\ EQ}$ .

24:30

Is going to be less than that diameter of the jar  $D$ .

24:34

So how do I calculate this?

24:36

Well,  $D_{sub\ EQ}$  we can look toward just basic geometry is  $\frac{2}{3}$  times capital the sub  $S$ .

24:46

So this is this or this term is the sphericity that characteristic length of particle  $D_{sub\ P}$  and then I introduce epsilon which is the void fraction.

25:01

So the void fraction which is also called porosity.

25:04

If you look at something like flow through a rock, this is the amount of space available for me to fill with any kind of fluid of interest, right?

25:15

Any species that can be deformed and take the shape of its container where the value  $1 - \epsilon$ .

25:23

This is the particle or solids volume fraction in the system.

25:28

Now capital  $\Phi_{sub\ S}$  When I talk about sphericity, this is a very specific definition.

25:35

It's the surface to volume ratio for a sphere of of equivalent or characteristic length  $D$  sub P divided by the surface to volume ratio of the particle that I'm using.

25:48

And this is relevant because that particle may not be a sphere or anything sphere like.

25:55

It might be a ring structure, right?

26:01

That might look like a bit of helical DNA, but made out of metal, right?

26:04

And this is a common kind of packing here.

26:06

We'll look at some examples at the very end of the presentation of what these structures look like.

26:11

So I can go for materials that are, are actually spheres, like a, a jar of marbles all the way down to something with a very, very low sphericity because of the surface to volume ratio for that same equivalent diameter is going to be quite or depart quite significantly from that of a sphere.

26:29

So if, if the engineer's purpose then is to, to use pumps and compressors to, to control the pressure drop across this packed bed, then we get to, to choose what materials go in that.

26:41

And usually the, the choice of those materials will be dictated by whatever mass transfer process or reaction I'm trying to control.

26:49

But as the engineer, I have to think about am I providing enough energy for that fluid to get across that system, right?

26:57

And if I'm a chemical engineer that I'm managing and manipulating energy.

27:01

So I have to think about what's that energy requirement going to be?

27:05

And one good.

27:06

And that's what we're going to calculate on the next few slides.

27:09

But one good example as we think about the equivalent channel diameter here  $D_{sub EQ}$ .

27:15

Well if I have a void fraction of .4, so epsilon is .4, that means that my particle volume fraction is 60% or .6 which represents an upper limit for most systems.

27:28

If I'm dealing with semi spherical particles like sand, then my equivalent diameter  $D_{sub EQ}$  or the channel diameter will be .44 times the sphericity times the equivalent particle size.

27:42

So that tells me that the channel diameter available for flow is about half the particle's size, right?

27:50

So when we talk about these equivalent diameters, these are on the length scale of the particles themselves because the particles are filling up that that jar for my unit operation.

28:03

So if I need to then manage and manipulate flow and the pressure across the bed to make sure I have enough momentum energy to get from left to right, well, because I have this, if I go back to this jar of marbles now and I'm going to make very large marbles.

28:18

So we can look at an example here.

28:21

I think that my fluid here is going to have to take a very tortuous path.

28:26

So it's not going to be a direct shock from from top to bottom.

28:29

Each molecule is going to have to take a tortuous path to migrate through that flow channel.

28:35

So we introduce a correction factor Lambda to account for that virtuosity.

28:40

Now if you go through chapter 6 in the textbook, there are a number of equations presented on the basis of using  $\Lambda$ .

28:49

What I'm showing you here are just the take home.

28:52

Here's what we really need to know as engineers and not beating that to death because that correction factor itself is a topic of different interest.

29:00

Now what we see is that if we look at an empirical relationship, we can see that if we're in very low Reynolds number space, so back where the Stokes law was going to give us a solution for the particle Reynolds number, so up to value of Reynolds #1, then I can calculate that pressure drop.

29:21

So across the the whole packed bed  $DP$  divided by the length of the bed.

29:25

We typically like to report pressure drops in units of bars per meter or Pascals per meter, and that will be equal to 150 times the average upstream flow velocity  $V$  naughty.

29:37

We're just note here for conventions we're using  $V$  naughty instead of  $U$  naughty that the upstream flow velocity  $V$  naughty times the viscosity  $\mu$  divided by the sphericity squared and the particle diameter squared.

29:51

And I multiply this by the solids volume fraction  $1 - \epsilon$  to the power of 2 on the void fraction cubed.

30:01

And this relationship is known as the Cosini Carmen equation.

30:05

So this gives me, if I'm in a very low turbulence region, this allows me to relate the properties of my packed bed to the pressure drop engendered by the flow across it.

30:17

Now, where you'll see this crop up more is if you go into either oil and gas production and you're looking at moving hydrocarbons across a porous matrix like rock, or you go into carbon capture and sequestration as a field.

30:32

And you're going to take  $CO_2$  out of it as some production stream of interest, compress it to get it into the Super critical region and inject it into a porous matrix for storage.



30:43

Really the example of this, the Norwegians just announced a project to do this on an oil and gas length scale.

30:50

So using a huge depleted offshore field and they're going to counter inject CO<sub>2</sub> into that forest matrix for long term storage.

30:59

Well, then I have to this I, I will see this relationship emerge again for the closing Carmen equation as I'm thinking about how much energy I'm required to derive that CO<sub>2</sub> across that rock matrix.

31:12

And it depends on both the porosity, the fluid properties, as well as the characteristics of the fluids or the porous matrix itself.

31:23

And where that will come up for you is in Darcy's law.

31:26

So the Cozini Carmen equation shows us that the flow itself is proportional to delta, proportional to delta P.

31:35

So the flow velocity is proportional to delta P and inversely proportional to the fluid viscosity, right?

31:41

And that's the basis of Darcy's law.

31:43

So Cozini Carmen applies then if I have kind of a very low turbulence region, so I'm dealing a lot with viscous dissipation, but what happens if I turn up the energy to an extreme and I'm looking at very turbulent kinetic energy loss well above a Reynolds number of 1000?

32:01

The Burke plumber empirical equation here uses a similar basis.

32:06

So 1.75 now times the density times the velocity squared or the upstream velocity squared on now just the sphericity and particle diameter themselves times the solids volume fraction on the void fraction cubed.

32:24

So the Burke Plummer equation, this is for where the energy loss of flow is dominated by kinetic energy loss.

32:39

And the Cuisini Carmen relationship is where I have the dominance of viscous participation.

32:51

So do I have a relationship then that will govern both of these at once and can cross this whole spectrum?

32:58

Well, so Ergen is is a very famous name for packed bed flow.

33:03

And Ergen was looking at this problem of having a semi lavender kind of region and a very turbulent region and saying, OK, well, what happens if we can just combine these things together, right.

33:14

So the Ergen equation that's shown here, this is for that same pressure drop as a function of length.

33:21

We're going to see Ergen basically took directly the viscous loss contribution from Cozini carbon in blue and added to it the kinetic energy loss from the Burke plumber empirical relationship that looked at on the bottom.

33:38

And so when I look at this this set of relationships, then Ergen went through a series of validation trials for void fractions of .3 to .6, which is really in the upper range of what you would expect.

33:52

And when I look at what I'm calculating here, I don't actually need to know that much except the velocity and the fluid properties that are coming into the viscosity and density.

34:02

And then I need to know the three characteristic points about whatever solids are being used in the packed bed, and that's the sphericity of the equivalent particle length and the void fraction, which is usually something that I'm able to set overall.

34:16

So examples are shown here.

34:19

So if I have spheres, cubes or short cylinders, they will have a sphericity around 1.00, right?

34:25

So obviously posteriors.

34:27

But I've had very, very short cylinders that take on a spherical shape or a spherical length scale that in cubes give me a sphericity around 1:00.

34:37

But I have ratchet rings and two examples from the book are one where the inner diameter is 50% of the outer diameter.

34:44

So an example of ratchet rings are shown here.

34:47

And I have a sphericity of .58 and if my inner diameter is 75% of my outer diameter, then my sphericity is .33.

34:57

And barrel saddles, an example here, I have a sphericity about .3.

35:02

But you know, these are very, very simple shapes.

35:05

And now we're going to be looking at ratchet rings for one of our unit operation labs that will come up toward the end of semester.

35:12

But I wanted to highlight the on the left hand side, these are much more complex shapes that I can employ if I want to start controlling and really accentuating that surface area to the volume ratio of my packed bed.

35:28

And just to highlight, how do these, you know, synthetic materials that I can use in my pack to then relate to the sphericity I might expect for naturally occurring materials?

35:39

So Ottawa sand, this is a silicon sand that's quite commonly used as a laboratory standard across the world, has a sphericity about .95.

35:48

If I go to something like coal dust, I drop now crushed glass.

35:52

I'm sure we've all done that once or twice.

35:54

I have a sphericity of .65 and Micah, I don't know if anyone's ever seen Micah from a geology class, but Micah is a type of rock where I have layers of the Micah itself.

36:06

So as I go to break it, I get these plates almost and that will give me a sphericity of .228.

36:15

So let's put all this into to practice and look at an example of what we can do for calculating this pressure drop.

36:22

So in the first instance, if I have my ergot equation, so my pressure drop across the system is a function of length, it's going to be 150 times the upstream fluid velocity  $V$  naughty times the viscosity on cericity quantity squared and the characteristic length of the particle quantity squared times my solids volume fraction squared on the void fraction cubed right.

36:55

And then I add in that Plummer relationship for the kinetic energy contributions of 1.75 times the density of upstream velocity squared sphericity characteristic length or minus epsilon on epsilon cubed.

37:13

So if I say as a basic example, well let's look at a fluid velocity, so  $V$  not an average fluid velocity coming in of about .1 meters per second, right?

37:25

And because I'm using a use these typically for either a reaction or a mass transfer process, I tend not to have very high fluid velocities coming through.

37:36

If I'm going to use water, then I'll have a viscosity 1 centipoise or .001 Pascal seconds, right?

37:42

So I can remember that one centipoise is 1 millipascal seconds.

37:45

I'm going to be putting everything into base SI units here.

37:49

And if I'm using water, then again, my density I can assume will be 1000 kilograms per cubic meter.

37:56

Now the sphericity.

37:57

Let's start out and look at a sphericity for a value of 1.

38:03

And if I choose a 1mm particle, so then my characteristic length scale is .001 meters.

38:11

And for a void fraction, let's choose 40% as that was a very common example.

38:20

So now I'm going to do this calculation on the basis of how much pressure am I going to lose across a bed per meter, right?

38:27

So I'm dividing by L Well, I can write that out, so I'll change color here.

38:34

So I can write this out that the DP on L is going to be  $150 * 0.1$  meters per second times 0.001 Pascal seconds, 1 minus the quantity of .4.

38:51

My void fraction quantity cubed or squared, divided by the void fraction cubed.

38:59

And on the bottom here I have the sphericity.

39:01

So it's a value of  $1^2$  on the characteristic length point OO 1 meters squared.

39:09

Now I'm going to add to that and I'll change colors one more time.

39:13

I'm going to add to that the kinetic energy loss contribution.

39:18

So that will be 1.75 times the density 1000 kilograms per cubic meter, the average fluid velocity .1 meters per second quantity squared on the sphericity value of 1 \* .001 meters, the characteristic length.

39:38

And I multiply that now times the void fraction on or sorry the solids volume fraction on the void fraction cubed.

39:50

Now what I get out of this is the the top contribution right?

39:56

This is going to be equivalent to 84,375 pascals per meter per meter.

40:08

And if I look at the bottom contribution this will be 164,000 and 63 pascals per meter.

40:21

So if I look at the total of what these are going to be together, Delta P on L is going to be 248,438 Pascals per meter, right?

40:34

Or I would report that if I look at the significant figures that I put into it, say about 2.48 bar per meter.

40:46

Now if I take that as an example, so for a 1m column I'm going to lose two 2 1/2 bar of pressure for this set of common conditions that I might go for.

40:58

What happens to look at a variation to this?

41:02

What happens?

41:06

What if my sphericity instead is .33 right?

41:12

So if I go to the previous slide for a minute, what if I use a relatively thin wall ratchet ring instead of using a jar full of beads?

41:22

And how does that affect my pressure drop?

41:24

Well in that case my overall DP on L is going to be 12.72 bar.

41:35

And similarly if I go back to my base conditions and I think about what if my void fraction now.

41:44

So I'm going to use that same sphericity of one, but I want to go from a void fraction of .4 down to a void fraction of .25.

41:54

Then I get ADP on L value of 13.8 bar.

42:02

There we go.

42:03

So there's a consistency, no significant figures, There we go.

42:07

So the point that I wanted to illustrate here is that this urban equation is very, very sensitive to both the sphericity of the particles that I choose, but even more so to that void fraction, right?

42:20

So I have to think about if I have a, a jar full of marbles and those are mono dispersed, meaning I have a single marble size being used, I tend to have a maximum packing fraction or a solids fraction of of four sevens, meaning I have a maximum void fraction of three sevens or minimum void fraction of three sevens.

42:40

But as I start to manipulate that void fraction in this tight range, it's a significant control over how much pressure I'm going to lose in the flow stream, right?

42:50

So it's a very important parameter if I'm designing a process and have to know how much pressure I need to generate upstream of that for this.

43:00

OK.

43:01

So with that, that will bring us to the end of the packed bed sled.

43:05

So what we've seen now is if we take our initial example of looking at flow through a pipeline, losses are around fluid shear that's being applied to the wall or friction at that wall.

43:17

If I instead have objects that are immersed in a flow channel, then I can characterize those frictional losses through drag coefficient  $C_D$ , right?

43:26

And that's still going to to be balanced looking at the shear stress on the immersed object divided by the philosophy head and the drag that's in.

Lecture 06 - Fluidization

0:02

OK, welcome.

0:03

This is lecture number six.

0:04

I'm looking at the last lecture of our fluid mechanics review in preparation for getting into the United Operations.

0:12

This particular lecture is covering the second-half of Chapter 7, if you're following along in the textbook, and it will be the last lecture from Section 2, which is the review of fluid mechanics itself.

0:30

Now, we will be saving the discussion on transportation and metering fluids, so how we can measure and and move fluids through things like pumps and compressors.

0:40

We'll be covering some of that as well as vessel agitation, which is Chapter 9 in the latter part, after we've gone through some of the unit operations in the lab.

0:50

So with this, this lecture will correspond to quiz #2.

0:54

That will also be released today and we'll be doing in a week.

0:59

So getting into it in the last lecture now we talked about once we've gone through our our basis for fluid momentum in incompressible flow and we saw the derivation of the Bernoulli equation where we're looking at the gravitational head balanced against the pressure head balanced against the velocity head.

1:19

We also have included in that terms for frictional loss and in particular we were looking at what happens to interaction with the pipeline wall, maybe assign this wall shear stress and the wall which corresponds to that frictional loss.

1:37

Now if we instead moved from having a flow along the wall to thinking about an individual solid that was stationary within a flow stream, we then started to characterize the losses in the last lecture through the use of the drag coefficient.

1:52

So  $C_D$  here we can see is balanced on the top.

1:55

This is our shear stress by the very definition of shear stress in this case.



2:01

And on the bottom is our velocity.

2:08

So it's, it's representing, as we get a higher and higher drag coefficient is representing more of our velocity head that's being burned into shear stress by interaction between the fluid and that stationary object in the middle of flow.

2:21

And then we talked a bit about how we can either optimize the turbulence in the system or select a really clever particle shape that allows us to streamline that flow.

2:30

So we minimize those frictional losses from having a particle in fluid stream.

2:35

Now, moving forward, we then looked at, well, if we have a bed of these particles, so let's say we have a cylindrical column, right?

2:45

I think the metaphor that we were using was a jar filled with marbles.

2:50

So we have our jar of marbles and we're going to consider the pressure drop that's being applied across this DP on the length.

3:00

What is the pressure drop if we have some flow around all of these solid objects being packed together, Where the Greek symbol  $\epsilon$  shown in the equations here represents the void fraction or the porosity.

3:16

So how much room, how much volume in that cylinder is available for flow relative to the volume occupied by the particles?

3:24

And that's where we saw the derivation then of the Ergun equation where the left hand blue text represents a laminar or a low Reynolds number component.

3:33

So for a low velocity small particle space, it's that laminar component that will tend to dominate.

3:39

And when we go to a higher velocity space, we have a kinetic energy loss which is shown in red and that term tends to dominate.

3:46

So we like the Ergen equation here because then if we plug in some basic properties around the particle, notably the sphericity, which we give the Greek symbol capital Phi sub S, the characteristic length of the particles  $D_p$  So for a sphere that's obviously the diameter of the particle.

4:03

But if we get into a regular shapes, then we assign a characteristic length for that and we only need to know the void fraction  $\epsilon$ .

4:12

And this then lets us define that pressure drop as a function of the velocity fluids across the bed.

4:21

OK, So if we were looking at this, this picture previously, then this particle in our last lecture was stationary.

4:32

But what's going to happen then if the flow induces a motion in the particle or the solid itself?

4:38

Is that possible?

4:40

And, and what we can look at to answer that is a balance between the different forces that are going to be exerted on that particle.

4:47

So fundamentally we have a external force.

4:50

So this can be if if the particle you think it's single marble in a jar filled with water is going to to slowly float down to the bottom, that would be an example of a gravitational external force.

5:02

But if I take that jar and I start spinning it around really fast and I'll notice that that single marble is going to move through the outside of the jar and that will be an example of a centrifugal force being applied.

5:14

So we have that that external force.

5:15

Of course, this is then counteracted by the buoyancy force.

5:19

This comes from Archimedes principle looking at the volume of displacement of the fluid itself.

5:26

So  $F_B$  here being equal to the mass of the particle, the density  $\rho_P$  of the particle, the acceleration vector and divide it by the particle density itself.

5:38

And then finally counteracting these, we have the drag force.

5:42

So the drag force is then if we're going to create motion in that system, this is the force that will be exerted as shown in the picture here against that fluid motion.

5:54

So the conversion of the velocity head to the pressure head, that's going to slow down that particle motion.

5:59

And so if we then these three fundamental force contributions here into a force balance, so the mass of the particle times its velocity DUDT, oh, sorry, velocity gradient DUDT.

6:16

We can plug in those three definitions to fully define that equation and we can simplify it a bit.

6:22

So in the simplified version we have a DUDT.

6:25

So the acceleration of the particle being equal to  $a$ , which is the acceleration vector, the density difference  $\rho_P - \rho$  between the particle and its continuous fluid divided by the particle density itself.

6:46

And we subtract from that the contribution from our drag loss.

6:50

So  $C_D$  times the velocity head  $U^2$  on two times the continuous fluid density, the projected area of the particle divided by the mass of the particle.

7:03

Now when gravity is the external force acting on the particle, we can replace that  $a$  term so the acceleration vector with  $G$  9.81 meters per second squared.

7:15

And if centrifugal force is acting, we replace that  $a$  with  $R$  where  $R$  is going to

be at the radius shown here, the radius of motion or the radius of the orbit at times  $\Omega^2$ , right, where  $\Omega$  is going to be the centripetal velocity.

7:39

So in most cases, what we're going to be looking at and certainly through the rest of the lecture is going to be considering gravity as the acting force.

7:45

And when we look at that context to replace this here we see a really important principle emerge.

7:57

A really important principle emerge and that is that the the acceleration of the particle itself is effectively we see the gravitational component coming  $\rho$  times  $G$ , right?

8:10

And that's directly counteracted by the drag coefficient being a proportionality constant to that term.

8:18

So this actually gives us a really physical meaning for what that drag coefficient is.

8:24

And it's the proportionality of resistance to whatever external force is acting on the thing that's experiencing the drag itself.

8:34

Now, if we're thinking about a particle again, that's say, a single marble that's, that's falling through our jar filled with water, then we can also see an ability to calculate what is the terminal velocity of that particle.

8:49

So the terminal velocity is going to mean the maximum velocity that it can reach, right?

8:54

If I want to clock out something like time experienced as a function of the velocity of the particle, right?

9:01

I would expect some kind of curve of this nature where the point at which that curve plateaus out will be  $U$  not or the terminal or sorry  $U_T$  the terminal velocity in this case.

9:15

Now if gravity is the external force acting on the particle, then I define my terms of  $G$ , set that equation  $DUDT$  equal to 0 and I can solve directly for the velocity  $U$  in that case.

9:27

And that gives me the answer here where I have to take that square root, obviously, because  $U$  is squared on the right hand side.

9:33

And if some centrifugal separation is the the external force acting on the particle, then I simply replace that a sub  $U$  with  $R \Omega^2$  and again solve for for  $U$  sub  $T$ .

9:46

So what we're going to then is in this context, I can think about, well, let's apply this equation directly to a sphere.

9:54

So I have a single marble falling through that jar filled with water.

9:58

So in this case, geometrically, the mass of my particle, if I define it on the basis of the diameter will be  $\frac{1}{6} * \pi$  times the particle diameter cubed times the density of the particle and ace of  $B$  again, that's the the projected area, right?

10:19

So be projected into  $A^2$  dimensional space that's we used the metaphor last time at shining flashlight behind an object gives us that two-dimensional projection that would be  $\frac{1}{4} * \pi$  times the diameter of the particle square.

10:32

OK, So if gravity is the acting body, then let's look at a scenario where I have a rounds number 100,000, particle diameter 1mm, which we'll say is the density of the fluid, which in this case we just call  $\rho$ .

10:58

I will say it's 1000 grams cubic meter and that would correspond to it an assumed density for liquid water.

11:08

And let's say that the density of my solid rows of  $P$ , I'm going to assume is 2500 kilograms per cubic meter.

11:19

And of course, that's going to come into play if I have something like glass, right?

11:22

That's a typical sand or silica sand glass.

11:26

That's a pretty typical density for those kind of materials.

11:29

Now, because I have defined a Reynolds number that's reasonably high.

11:33

And if I recall back to lecture #5 when we were looking at the relationship between a drag coefficient and the velocity or the Reynolds number of the system of the particle.

11:45

And I can assume a drag coefficient to be equal to in this case because that's that region of constant constant drag coefficient.

12:00

So if I plug this in colors here.

12:03

So if I plug this in,  $U_{\text{sub T}}$  be equal  $481 \text{ meters per second squared} + 2500 \text{ kilograms per cubic meter} - 1000 \text{ kilograms per cubic meter} \times 0.001 \text{ meters}$  and I divide that by three times the  $C_{\text{sub D}}$  so the drag coefficient  $0.44$  times the density of the fluid.

12:39

What I get from this calculation is  $0.21 \text{ meters per second}$  as the terminal velocity of that particle.

12:49

Now as an example, if I were to increase the drag coefficient to a value of  $50$ , then in that case the terminal velocity would be  $0.02$  just per second.

13:12

So quite a sensitive calculation to that drag coefficient.

13:16

Obviously it's not linear because I have that square root relationship, but this is where we saw drag coefficients varying from around  $.1$  up to between a  $101,000$  for low relative number systems.

13:29

You can see where the proportionality response of that higher drag coefficient is quite significant when it's acting only on the difference of fluid density and particle density and the gravitational field.

13:42

So what this fundamentally means then is if I have an infinitely long jar filled with water and I drop a glass, glass, marble in at the very top, the maximum velocity that it's going to reach on the way down is  $.21 \text{ meters per second}$  for this set of conditions, All right.

14:01

If I measure that, that's going to be the maximum where it will take some time to reach that.

14:08

Now, that's for dense particles that would be falling through a fluid of interest.

14:14

It could be gas, could be liquid, but what happens if we invert that application when we think about bubbles or immiscible liquid droplets rising in another fluid?

14:24

So a really good example of this would be the deep water horizon blow out in the Gulf of Mexico back in 2010.

14:30

And if you we look at that case, what was happening is under about 1500 meters of the water column where I have a hydrostatic pressure.

14:38

So  $\rho$  GH hydro change color.

14:43

I have a hydrostatic pressure about 150 bar that's that's being exerted on that system.

14:50

But I had a blowout pressure here.

14:53

So just before the fluids were exiting that that failed blowout preventer stack, the fluids were about 230.

15:01

All right, So quite a significant pressure difference that was driving those fluids out with quite a bit of momentum.

15:07

Now in that case, you know, one of the the significant outstanding knowledge gaps that a lot of folks around the world are still trying to to work on is an understanding of what that particle size distribution.

15:18

So they both the oil size distribution and the gas bubble size distribution will be under these highly turbulent conditions.

15:26

And what McCabe in the textbook has a very brief section on this, but shows us is if

we look at something like the rise velocity, so a lot of estimates back in 2010 and in particular, we're estimating that the bubble size based on the rise time.

15:49

So if they made some modification of the wellhead through one of the remote operated vehicles and they were trying to, you know, put a blockade around it or something already published, he said, well, it would, you know, we were on the boats and it would take about 3 hours to see a surface expression change.

16:04

So from that rise time we can estimate what's the equilibrium diameter of both the bubbles and the oil droplets that are flowing upward.

16:13

Now what makes this much more complicated is that Reddy was using a linear relationship to estimate that.

16:19

And so he was saying, OK, well we can go back to those equations on the previous slide.

16:24

And from that we can estimate how quickly if we assume every increase in millimeter bubbler or droplet diameter is going to have a constant impact on the rise velocity.

16:36

And it turns out that assumption was not correct, right?

16:39

And we can go back to a publication from the early 70s that cited in decay to look at that rise velocity being bubbled in water.

16:47

So an air bubble with ambient pressure in liquid water as a function of the bubble diameter in millimeters, right?

16:53

And what we see is that in this early region, so within about a millimeter, the bubble itself will behave as though it were a rigid sphere.

17:04

So meaning the pressure from that bubble in the liquid phase, because it's starting to get quite small, the Laplace pressure inside the bubble will increase as the bubble gets smaller.

17:12

And so for that region, it obeys Stokes law and in the very limit of small bubble diameters.



17:18

Now as the bubble diameter increases, we see here these six data points are experimental measurements.

17:25

And in fact, there is a local maximum in the rise velocity as a function of the bubble diameter.

17:32

All right.

17:32

And the reason that this happens when we invert the this application and think about rising species, rising fluids in another immiscible fluid is that gases or liquids as they're rising can be deformed in flow.

17:46

So the larger that they get, right about the rigid sphere down here, by the time I get up to this point, I actually have something that looks a bit like our streamlined particle because that liquid or that gas can be deformed during the rising process.

18:03

And so it becomes more and more ellipsoidal the larger it gets.

18:07

So the liquid or the gas phase that's rising actually streamlines itself and we have this intermediate condition both maximum.

18:18

So I, I bring this up because it's actually still an outstanding point of research that a lot of folks are working on.

18:24

And something that McCabe quotes directly in the textbook is there is a tremendous uncertainty even today.

18:31

We have a lot of arguments about the impact of both interracial tension and not bulk viscosity as the textbook sites, but it's actually the viscous relationship between the particle and the continuous phase liquid that it's rising in.

18:45

So if I go back to my my jar of marbles for a minute, then one of the really common industrial applications for chemical engineers is to think about, well, if I have this jar and I fill it up with marbles, well, what happens if I start passing a fluid across that?

19:03

Well, if we go back to Ergen's equation for the pressure drop, so the pressure drop

as a function of the superficial velocity of air or or gas fluid that that's going to be passed across this.

19:15

Well, I begin of course with no fluid velocity, I have no pressure drop.

19:19

And Ergen's equation is going to show me that I'm going to start increasing.

19:23

If I increase the velocity, I'm going to get an increase, a linear increase in pressure drop because of course in the coming from zero, this is the low Reynolds number region.

19:33

So I have laminar, laminar flow early on, and that's going to correspond to that left hand blue term in the Ergen equation that we're using.

19:44

And what we see is that at some point, right, I'm going to have a point at which the bed will start to move, right?

19:53

So if I have those jars filled or that jar filled with marbles, I begin with a packed bed state shown here.

20:00

As I increase the gas velocity from left to right, I'm going to to initially.

20:05

Once I cross this this first transition point, all of a sudden those beads or those glass marbles are going to start rattling around.

20:13

Right.

20:14

And if I continue increasing the gas velocity, but I'm going to have a period where I have smooth fluidization, right?

20:21

And this is the target of interest for chemical engineers.

20:24

Oftentimes now I can continue past that gas velocity or that smooth fluidization point and I can start to get into turbulent flow where I'm going to actually be losing content all the way through, actually conveying particles through the system, at which point I start to have a plug flow style reaction.

20:44

Now, what's the consequence of this and where do we see it?

20:47

In early days, this type of fluidized bed approach or the process of fluidization was used in, in the petroleum field to increase the amount of gasoline or petrol that a single barrel of crude oil could produce.

21:02

So if you take a barrel of crude barrel, of course it can break into about 10,000 commercial products, but the most valuable of those is gasoline typically, right?

21:11

And so back in the the early 20th century, there's obviously need to increase the amount of gasoline that was coming out of that barrel.

21:19

And the lower portion of the barrel, the heavier molecular weight portion of the crude oil, you know, tends to not be very valuable or useful.

21:27

But what some of the chemical engineers figured out is they could take the the solids that were in that very heaviest portion of the crude oil and they could put them in these cat, what they called cat cracking or catalytic cracking units that were functionally fluidized beds.

21:43

And that would help induce a catalytic reaction to break down the heavy hydrocarbons and yield more octane or low molecular weight hydrocarbon compounds that can be used for more petrol.

21:56

So if you follow the the design and improvements to oil refinery efficiency over the course of about 80 years, you'll see the use of these fluidized cat cracking units increasing and it ultimately is designed to produce more and more gasoline from the same barrel of crude oil.

22:15

And now of course, you know that was one of the first applications.

22:17

But as we move toward more modern technology, we see a lot of use of these fluidized bed reactions or fluidized beds in processes requiring reactions.

22:28

So about 75% of polyolefins now again is that the hydrocarbon by product are produced and manipulated and reacted via fluidized beds.

22:38

We also see them as a a very useful tool when we're thinking about the absorption of gases, the combustion of coal and, and improving the combustion efficiency.

22:47

And more recently and and certainly as an eye on the future, the process of converting either utilizing biomass or converting biomass to biofuels, which itself is a very complex, complex catalytic reaction.

23:02

So in this state, I would expect you will certainly see a lot more fluidization applications as biofuels in the use of recycling material to produce biofuels becomes a more common practice.

23:14

Now the reason we call this a fluidization process is that when we hit this minimum fluidization point, the suspension of solids in whatever fluid I'm using, the suspension itself behaves as a dense fluid, right?

23:30

So that's where I, I come to the understanding that I have a fluidized bed because I have solids that are being suspended up in the air ever so slightly as though they act as a a uniform density.

23:43

Now you might think to yourself, OK, well, how do I then balance when I'm trying to design A fluidized bed?

23:47

How do I balance the velocity of gas that I put across this to make sure that I don't start losing particles, right?

23:54

Because I don't want that bed to go out of control.

23:55

I want a nice height on.

23:58

We can go back and think about the Ergun equation because that's going to tell us the amount of momentum, energy or pressure drop that we're going to lose across that bed.

24:05

And we can balance that against the gravitational force acting on the bed.

24:09

Here the density difference  $\rho_P$  minus  $\rho$  times  $G$  is the gravitational field.

24:16

And so we set these two to be equal to each other and that tells us  $V_{\text{naughty M}}$ .

24:22

So that's the average fluid velocity required for minimum fluidization and that's what the  $M$  subscript means.

24:32

Now, if you're following along in the textbook, then McCabe uses the nomenclature  $\epsilon_{\text{M}}$  and he replaces that through the Ergun equation just to correspond to the porosity or the void fraction at that point of minimum fluidization.

24:46

Simply because the void fraction of the bendable changes we're going to look at in a few slides as a function of that velocity.

24:54

So we're just tagging in what's that minimum fluidization point and the minimum or the corresponding porosity at that velocity.

25:02

Now, if I have a low Reynolds number condition so that the Reynolds number on the particle length scale is less than one, I can strip that back to the first blue term of the urban equation, right?

25:13

And if I solve them for  $V$ , not  $NI$ , get this relationship shown here, and we say it's approximately equal to, right?

25:21

Not a direct equality, but approximately equal to.

25:25

And if we have very large particles at high velocity, so Reynolds number greater than 10 to the three we have, again, we can only, well, we can neglect the laminar contribution and focus only on the kinetic energy loss contribution of the Ergon equation and again solve directly for  $V_{\text{naughty}}$ .

25:42

And we're here because  $V_{\text{naughty}}^2$ .

25:44

I'm going to have to take a square root of that as I rearrange and solve.

25:50

So watch in each of these cases then if I say, well, what's the, how much velocity can I put across this bed without losing material, right.

26:04

Well, what I can look at is comparing that minimum fluidization velocities, the point at which the bed first starts picking up with the terminal velocity of particles in that in that flow stream.

26:16

So if I focus on the worst case kind of low Reynolds number scenario, then I can ask the question, will the particles remain fluidized?

26:25

And so if I'm going to compare these two quantities, I can simply define  $U_{\text{sub T on}}$ .

26:30

So my definition for terminal velocity on the definition for the minimum fluidization velocity, which is expanded up to the right hand side here.

26:39

And if I simplify that down, I get this relationship on the body where I can see that balance between the terminal velocity and the minimum fluidization velocity is proportional to 1 minus the void fraction at the point of fluidization on the void fraction cubed.

26:57

And it's also proportional to one over the sphericity of the particle cubed.

27:02

So I can pop that out and what The Cave shows us in the textbook.

27:06

So these are density curves or density differences.

27:10

And he's using an old school unit for density of grams per cubic centimeter.

27:16

But I'm looking at different density units between the particles or between the particle and the continuous fluid.

27:23

And I can compare that to the terminal velocity for that same condition.

27:29

So I see as I go up in particle diameter where the characteristic length scale of the particle, I have a higher and higher both terminal velocity and minimum fluidization velocity in centimeters per second.

27:42

But the important thing to note, right, is that all four of these curves, right, these correspond to the minimum fluidization velocities.

27:54

And the singular curve for a density difference of one is going to correspond to the terminal velocity.

28:00

So at a given, right.

28:01

So that's this curve here for the minimum fluidization velocity.

28:07

So for a, a constant density difference, if I'm comparing the two quantities, what I can say is that at a given particle size, let's say 40 microns, 40 microns, I can see that my terminal velocity of the particles is going to be somewhere around 5 centimeters per second, whilst the minimum fluidization velocity is going to be on the order of about .1 meters per second per centimeters per second.

28:33

And this is quite healthy, right?

28:35

Because it means that I can control or have a range of how much fluid I can pass through this bed before I'm going to start losing material, right?

28:44

And the reason that I care about this is that because fluidized beds are often used for reactions and reaction control, then the amount of material I'm putting across in that fluid state is going to dictate the ultimate rate of reaction, right?

29:00

So it gives the engineer the ability to throttle the rate of reaction, looking at a comparison between the point at which the bed becomes functional or fluidized and the point at which it becomes ineffective.

29:10

If I'm OK now as I move to higher and higher velocities, one of the the points to make is that the bed itself will start expanding.

29:22

So this top curve here, this corresponds to the height of the bed  $L$  where I can see initially, right?

29:30

I have a region where the bed is at a constant height.

29:33

And as soon as I'll reach that minimum fluidization velocity here, right?

29:37

And these are experimental data compared with theory.

29:41

I'm plotting here the quantity on  $ETA$  cubed on one minus  $ETA$ .

29:46

So I can rearrange that directly from the laminar contribution of the Ergen equation and plot that as a function of velocity.

29:54

Where I here's my my bed velocity or fluid velocity in millimeters per second.

30:01

And this right hand turn here right effectively should correspond to the slope of this line.

30:09

So what I can see is that as I hit that minimum fluidization velocity, right, which in this case is going to be just under 2mm a second, as I increase the velocity beyond that, right?

30:22

If I'm, well, say a factor of 10 underneath my terminal velocity of the particles.

30:28

So I can certainly have some room to play in terms of how quickly I want to pass fluid over that bed, how much void fraction I want to engender in the bed.

30:36

As I turn that velocity up, the ratio of those void fractions increases, right?

30:42

And more importantly, the height of the bed that can be measured is also going to increase, right?

30:48

So in this case, I'm going to start out with a height of say .16 meters in total and it's going to increase to about .26 just over this range of two to 16mm a second.

31:02

And so just to show you, this is from a little diagram from a chemical one of the ASCE journals here.

31:09



We're looking at if we're going to design A fluidized bed, we would typically expect some grid plate to even out the gas with spargers to evenly distribute it into my bedding region here.

31:20

And then I have some amount of overhead where I can capture if the bed is going to move up and down, right.

31:29

So with that kind of summarizing the take home messages, we've gone from looking at the Bernoulli equation which showed us a balance of the different head contributions to incompressible fluid flow.

31:43

And if we then think about, well, what happens if I have not a single phase, but a two phase system where I have a particle or a solid in the middle of that flow.

31:52

I can determine whether that particle is going to come into motion based on a balance between the external force, either gravity or centrifugal against the drag force applied to the part.

32:05

And I can further to that then calculate if that particle does start moving, I can calculate the maximum velocity it's going to achieve, which we call the terminal velocity from this relationship here.

32:15

And this is defined because we see  $G$  in that we're obviously defining that specifically for gravitational, either gravitational settling or fluid movement.

32:27

Now in chemical engineering applications, one of the first operations we'll be talking about now is fluidized beds.

32:35

And in this context we call it a fluidized bed because once I pass enough fluid through that suspension or solid bed of particles, I can suspend them up in the air and the two phase system itself is going to behave as though it were a dense fluid.

32:50

And I can determine the conditions of that balancing porosity against velocity by setting the Ergen equation equal to the weight of the fluid or sort of the solid bed itself.

33:04

Now we need to maintain the fluid velocity well below that terminal velocity to prevent the loss of material, right?

33:11

If I'm thinking about having a bed that's in a constant state as opposed to something like a plug flow reactor.

33:17

But these two boundary points of the terminal velocity and the equivalence of the Ergen equation allow me to think about how much the bed is going to swell or how much more porous that bed section is going to become as I vary the fluid velocity from an engineering standpoint.

33:34

OK, so with that, that will bring us to the end of Chapter 7.

33:39

Now the next lecture will kick into heat transfer, I'm sorry, the end of lecture 6 and lecture 7 on Monday, we'll kick into section three of the textbook, which is going to correspond to heat transfer.

33:50

And specifically we're going to start looking at heat exchangers.

33:54

So if you have any time, it would be good to kind of brush up on your heat transfer background from the past few units.

34:00

And we'll also have quiz two, which corresponds to the second-half of the Fluid mechanics material, come online here for you to test your knowledge against from the past three lectures.

34:11

OK, wonderful.

34:12

We'll see you on Monday and give us a shout on Teams or the Teams channel or direct chat if you have any questions about the quiz or the lecture content you're reading.

34:21

Have a good weekend everyone.

Lecture 07 - Heat Transfer in Unit Operations

0:00

This text takes us into the next section of the textbook, which is going to be on heat transfer.

0:05

Obviously we've all had heat and mass transfer as a background to this unit.

0:09

We're now looking at heat transfer specifically in the context of unit operations.

0:14

So before we go too far, I wanted to do I can figure out how our options of doing polling had gone away, right?

0:35

OK, so can anyone tell me, have you looked at things like nestled number correlations to get at overall heat transfer coefficients?

0:43

Does that sound familiar so far?

0:50

Just so I can get a sense of where everyone's at.

0:59

OK, so today when we start talking about heat transfer, then if you've been following along in the textbook, this is getting into Section 3.

1:11

Specifically today we're going to be going through Chapters 10:00 and 11:00.

1:16

So chapters 10 and 11 are primarily around backgrounding some of the theory into heat transfer, where chapters 12, 13, 15 and 16 are more around the operations of heat transfer.

1:32

So I'm not spending a lot of time on the background because you would have had heat and mass transfer other than instead focusing more on the the applications.

1:41

So just as a summary point, in our last lecture to do with fluid mechanics, we looked at what would happen then if, if we start to generate motion within the silence.

1:52

And so this might be something like a, a fluidized bed as our primary application.

1:57

And in that context here we're looking at the balance between the drag force, which is going to resist flow in a bed filled with solids in the gravitational or potentially centrifugal forces that are going to try it act externally on that solid to keep it bound in a non moving bed.

2:14

And so as we apply that force balance, we can calculate the terminal velocity.

2:17

Here we see it's a function still with that projected area, which is what happens if I take the object of interest and shine a flashlight behind it.

2:27

Here the density difference between the solid of interest and my continuous phase.

2:32

And this is our drag coefficient.

2:34

So we can see in this construct as that drag coefficient goes higher, noting that in high realms, number of regions, it would be about .4 can go all the way up to between 10 and 100.

2:46

And the terminal velocity of how fast those solids can move will scale with  $1 / \sqrt{C_D}$  over that drag coefficient.

2:56

Now these fluidized beds, you know, this is where we ended up in fluid mechanics talking about our first primary chemical engineering operation.

3:04

I mean, what happens is when I take a whole bunch of solids and I start injecting fluid and that can be a gas or a liquid, but I'm injecting a fluid from beneath that solid bed.

3:14

Once I begin to lift that solid bed up and I can suspend those particles in the fluid of interest, the suspension itself, the two phase suspension will behave as though it were a singular dense fluid.

3:27

And so that's what we call it a fluidized bed.

3:30

And here we can see we can this is the the left hand side is our Ergun equation, which is the pressure drop across that bed.

3:37

And so the if I want to set that equal to the minimum or to solve the minimum fluidization, I can look at simply the gravity head of what the bed is itself to solve for the  $V_{mf}$  is that minimum fluidization velocity.

3:53

So the quiz that went up on Saturday and I set the due date of that to be the 17th of August, so a week from today by 5:00 PM.

4:03

And so that will cover some applications of calculating both terminal velocity as well as the fluidized bed.

4:10

Now it's important to note that when we do get into that point of fluidization, the bed itself will swell.

4:17

So that means the porosity or the void fraction, which we get the Greek, the Greek symbol  $\epsilon$  will increase with time or increase with fluid velocity.

4:29

So what that means is I turn the Reynolds number up beyond that minimum fluidization velocity.

4:36

I will basically space that out the solids as they're filling the bed fraction, OK.

4:45

So now from there, we're going to move into the next phase unit operations and this is going to look at the unit operations for heat transfer.

4:53

Now specifically, we'll be looking at heat exchangers that kicks into the first laboratory experience this Tuesday.

5:01

So tomorrow morning at 9:00 AM, we'll be having the first bit of the lab and that's based around looking at the refrigeration unit operation that that some of you would have seen in the dynamics last semester.

5:13

But we're taking a bit more complex approach to it in this lab.

5:16

So we're going to be looking at combining that with ISIS and then asking some more extrapolated questions beyond.

5:24

So just as a quick review for those who might not remember if it was a bit of a few, few semesters ago, within the context of heat transfer, we basically have three types of heat transfer we're concerned with.

5:36

So the first is conduction.

5:39

So Fourier's law tells us that the rate of heat transfer per area  $Q/QA$  is equal to a proportionality constant  $K$  and the thermal gradient  $DT/DX$  applied across some conductive surface.

5:56

And we'll be looking at that more in the next slide.

6:00

Convection is what happens if I start to have fluid motion.

6:02

And so there, that rate of heat transfer is going to be governed by an overall heat transfer coefficient  $U$  All right.

6:08

And this is the, the point in which we're really interested in solving  $T$  surface is the whatever the the surface that I'm transferring heat to and from and the bulk would be where we're losing energy at that point.

6:21

So radiation, and we're not going to talk a lot about radiation in this class, but the black body radiation here is going to go with the seven bolts in constant times temperature to the power, whatever the surface temperature is to the 4th power.

6:37

So really simple graphic that's going to illustrate these points as if I think about my household, right?

6:42

It's going to be heated by radiation from the sun on most days, right?

6:46

Potentially, I have a heater in here where I'm going to be putting some electrical work into that heater and then and generating heat within the room directly.

6:55

But most of the time, especially in Australia in the summer, right, the heat in the house is going to come directly from the radiation through those windows in the house.

7:03

I'll start to have convection because I'm going to have natural currents or natural convection just from thermal gradients within the house.

7:10

And then at night, right, I'm passing, if I don't have any radiation source anymore, the heat's going to have to transfer across that brick wall.

7:18

And that being example of conduction, OK, So if I'm thinking about conduction, right, and this is going to play a role when I start thinking about things like pipelines or especially I'm going to insulate my pipeline.

7:30

So conduction in the context of materials after unit operations, I can think here about what's going to happen if I have say, 3 solids that are all stacked together, where in this context we say  $B$  is going to be the thickness of the solid.

7:46

So I have BABB B&B sub  $C$  for the different solids together where I'm going to see a temperature gradient  $\Delta T$  sub  $A$ ,  $\Delta T$  sub  $B$ , and  $\Delta T$  sub  $C$  through each of these solids.

8:01

Now on the bottom left hand plot, I can look at the total amount of heat transfer  $\Delta T$ .

8:07

If I'm in a conductive series, it's basically going to be a summation of the different temperature gradients across these three different solids where I have continuity of temperature at the the connection point between the solids, right.

8:24

So what this is going to tell me is that, well, my solid  $T$  sub  $C$  or  $R$  sub  $C$  in this case is going to have a, a significant wall temperature on the outside and that's going to be drawing much of that thermal gradient.

8:40

And so if I can represent this mathematically through four years long, I can say that total  $\Delta T$  will simply be a sum of the heat that's being transferred across these three.

8:50

And if I expand the definition of  $\Delta T$  again, integrating Fourier's law here, right, I can go back and rewrite that as a function of the rate of heat transfer  $Q$  sub  $A$   $Q$  sub  $B$  &  $Q$  sub  $C$ , the thickness which we called  $X$  when it was written dimensionally in Fourier's law and  $A$  being the area across which that surface is going to be transferred.

9:14

Here again, we see that's all the same.

9:16

If we're in A1 dimensional construct and  $K_{AB}$  and  $C$ , we're calling  $K_{AB}$  the average thermal conductivity.

9:26

Now we use the the term average in this context just to mean that there's obviously a temperature gradient across these solids.

9:33

So thing properties, Thermo physical properties like thermal conductivity can certainly change as a function temperature.

9:41

So we want to typically represent these kind of properties in an average form and we'll be talking about that a little more later on.

9:48

So if I'm at steady state operations, then Fourier's law is going to tell me that the rate of heat transfer is going to be equilibrated.

9:56

So in fact,  $Q_{sub A} = Q_{sub B} = Q_{sub C} =$  a rate of heat transfer.

10:03

And I can also recognize that that surface area in A1 dimensional construct is equivalent.

10:07

So I can break that out to the left hand side.

10:10

$Q_{1A}$  being equal to.

10:11

Now the total temperature gradient across the system divided by what is the individual thickness of each conduction material  $B_{sub I}$  in this context divided by its thermal conductivity.

10:30

So this this quantum of  $B_I$  on the thermal conductivity or the thickness on conductivity is to what's representing the resistance to heat transfer, right?

10:41

Because I can write this equivalently in the form of some resistance term if I have times my driving force, right?

10:49

And that's a typical chemical engineering rate law structure.



10:53

So I can indicate from this that in a conductive series of materials, the resistance to heat transfer is proportional to the thermal conductivity normalized by the surface area.

11:04

If I invert that the bottom of that equation.

11:09

So now when I'm thinking about, you know, this was the solution at steady state, part of what the complexity that can come into play is what happens when I get to unsteady state or transient operations, right?

11:22

And all of a sudden in a transient context, this rate equation cannot for four days long cannot be taken to 0.

11:29

So in fact the temperature gradient with time, which I have to now represent as a partial derivative because the temperature of any space and time coordinate will both be changing is again proportion or equal to a set of combined rate constants Times Now the partial derivative of the 2nd partial derivative of temperature with respect to space.

11:54

So this this set of resistant terms, when I write this as a second order partial differential equation,  $K$  on  $\rho$ ,  $C_p$  can also call  $\alpha$ , which is classically called the thermal diffusivity.

12:09

Now, we're not going to be using this form of the equation today, and it's certainly not in this class.

12:15

If you get into it to graduate mathematics and really advanced chemical engineering, you will see this form of the equation.

12:22

And in fact, the solutions to forms like this crop up quite commonly, right?

12:27

And an example of that might be, you know, maybe I take a hot plate, right?

12:32

I'm done cooking with hot plate and I want it to cool down really quickly.

12:36

So I take it out of my kitchen and I'm just going to put it outside on a cold winter's day, unplugged.

12:41

It's initially sitting at  $130^{\circ}\text{C}$ .

12:44

Give me an equation, right?

12:46

If the hot plate is circular, give me an equation for the surface temperature of the hot plate as a function in both time and radial direction or space in the hot plate, right?

12:56

And that's the kind of problem that I'll want to solve through a second order pressure like this.

13:01

But for the purposes of this slide, what we're really looking at here and that the point to take home is what we've highlighted in blue, right?

13:08

And that's that when we're in a conduction scenario for a unit operation, we typically assume steady state when we're designing these things.

13:17

And we'll give this proportionality between the rate of heat transfer and both the thermal conductivity as well as the temperature gradient across the system.

13:29

OK.

13:30

Now often times where this comes into place pipe, we call cylindrical pipelines or really ducted systems, right?

13:38

And that's because I'm going to be passing material through these quite a bit.

13:42

So if I have a thick walled pipe, right?

13:44

And I'm going to specify a thick walled pipe by considering what is the internal radius  $R_i$  or the external radius  $R_o$  I'm sorry, the the outside radius are not on the internal radius  $R_i$ s of the body.

14:00

So if I have it an outside pipe that's say 1m right in diameter, so very, very large

pipeline and an internal radius that is, say .5 meters, then that the division between the two is going to give me a value of two.

14:15

All right, sorry, phone call coming.

14:23

So, OK, sorry.

14:26

So if you have a scenario where you have a thick walled pipe, right, so your, your division of the external pipe radius on the internal pipe radius is going to be significantly greater than one or unity, then I have to, to consider, consider that as a thick walled system, right.

14:45

So we we designed this for a thick walled system and then with the appreciation that this approach can be applied to most thin walled systems without much error coming in.

14:54

So if we have a thick walled pipe then we can write the Foyer's law here  $Q$  on  $A$  being equal to  $Q$ .

15:01

So the rate of heat transfer on  $2\pi RL$ .

15:04

So the surface area of that pipeline where  $R$  is going to be the average radius of the pipe.

15:09

So inside the thickness of the line, what is that average radius?

15:14

So you can see here from the diagram, I've been equal to the thermal conduct, one of the inverse of the thermal conductivity and the gradient of the temperature across it.

15:25

Now I can integrate that in cylindrical coordinates and what that gives me here I can see, I'm assuming that the thermal conductivity is not changing right with  $T$ .

15:35

So that's one of the assumptions you're going to see cropping up here.

15:39

Now what that gives me is an overall rate of heat transfer being equal to the thermal conductivity  $K$ .

15:47

And what I the weird, the way I'm writing this here is an  $A_{\text{bar sub } L}$ , So that's an average, average surface area.

15:57

And in fact, the way that we've expanded that out is defined as this  $A_{\text{bar sub } L}$  as  $2\pi L$  times the quantity of the outside radius minus the inside radius on the natural logarithm of the outside radius divided by the inside radius.

16:13

And what we call this, this blue quantum here is logarithmic mean radius, which can also be used for a logarithmic mean diameter of the system, right?

16:26

And the reason that we do this is that that log mean radius or log mean diameter will tend to give me a much more representative quantum for an average surface area when I'm in a thick walled pipe, particularly as the wall thickness increase increases.

16:42

And what McCabe is plotting for us here is that log mean radius divided by the average for the arithmetic radius of the pipelines.

16:50

That's just the thickness divided by two, where I'm plotting that as a function of the wall thickness effectively where an  $R_{\text{naughty}}$  or  $R_{\text{naughty}}$  divided by  $R_{\text{inside}}$ .

17:03

You know, if we're again near unity, I can see that the deviation between the two will be less than about 5%, right?

17:11

But as I go up to very, very thick walled systems, then in fact, just using an average thickness between the pipe inside the pipe outside is not going to give me a very representative solution.

17:23

So if we tend to to use this at the outset for most of our our cylindrical pipe calculations, then it's not too much of A concern, but just something, something to be aware of.

17:34

OK, so now that's if I'm conducting heat through system.

17:38

Now, what happens if I'm that fluid or, or the species that's that's conducting to and from the pipe wall is not static, but is in motion.

17:47

All right.

17:47

So now I have to think about a convective heat transfer.

17:50

Now, I, I wanted to start actually with a bit of Chapter 12 and then back up into it to what we're doing in Chapters 10 and 11.

17:57

But what happens if I'm going to have motion in that fluid?

18:00

Well, we've already talked about if, if my pipe wall here is going to conduct heat, right?

18:07

So here's a cue from a conduction that's pulling heat away from the surface of that wall.

18:11

Well, what happens if I and then thinking about the heat transfer between the wall itself and whatever fluid is inside the pipe?

18:18

Well, we've talked about the phenomena where when fluid is entering the pipeline, I'm going to have some kind of boundary layer effect, right?

18:27

And in the case of a highly turbulent system that will come into an equilibrium and then I'll have that kind of bullhead that will form the front of my fluid in the context of a laminar system, that that boundary layer will continue expanding outward until I generate that fully developed flow as a parabolic profile.

18:46

But in fact, because of this context, I also have to think about, well, there's going to be a thermal boundary layer, right, that will follow in the same approach.

18:56

The fluid of course, is going to be at some, you know, temperature that is not or that is unequal to the wall.

19:03

So if we say in this case that the wall is at  $T_{\text{sub wall}}$  or  $T_{\text{sub W}}$  is going to be greater in temperature than that of the fluid  $T$ , Well, as the fluid comes in contact with the wall, there is an infinite amount of heat transfer for the first molecule of fluid contacting that first molecule of wall.

19:23

And so as these flow together, that fluid temperature is going to continue heating up until it's coming into equilibrium with the wall.

19:30

All right, So because of that, I'll change colours here.

19:35

Because of that, I have what's called a thermal boundary layer, right that we're showing here.

19:41

And because I have a thermal boundary layer, I'm then going to specify that the boundary layers we've been talking about heretofore are in fact hydrodynamic boundary layers, right?

19:51

Where when they come into equilibrium, then the temperature will approach some temperature infinite, which would be the bulk being equilibrated against the wall, temperature and  $U$  here being driven by that bulk velocity,  $U$  not.

20:08

Now, it's important to note that in this context, I'm assuming that the heat generated from fluid friction is going to be negligible, right?

20:16

We talked about that in the end of the last lecture or a couple lectures ago where that that relationship that we showed is typically only valid in the context of having really a highly viscous fluid such as a heavy oil.

20:31

Now when we say heavy oil, that's going to be something like a fluid viscosity in excess of 1000 centipoise, right?

20:40

So 1000 times the viscosity of water or if we have a gas is the continuous phase.

20:47

Then if we need to have velocities approaching Mach one for that fluid friction to be meaningful and of course right, a good example, there is a spaceship or a space shuttle re entering order.

21:00

Now when that fluid is going to contact the wall we are going to create then this thermal gradient, this thermal boundary layer that's going to result in an initial thermal gradient in the liquid itself.

21:15

And it's a really, really important point to highlight in part of the the reason that that heat transfer gets so complex in these kind of systems is that because of that, the Thermo physical properties of whatever system I'm using.

21:30

So if water is my heat transfer medium in your laboratory tomorrow, we're going to be looking at R134A right as a common refrigerant being used for the heat transfer process.

21:42

The Thermo physical properties of that system are themselves a function of temperature, right?

21:48

So this makes them complex because if we apply that principle into what we're showing on top, well, in fact, if I take a little microscopic probe and I ask for what is the thermal conductivity of the fluid and the heat capacity of that fluid at this point, right?

22:04

That's going to be far different to what those properties are going to be at the outside of that thermal boundary layer.

22:11

All right.

22:11

So for this reason, we need to be very careful about how we're treating and selecting those Thermo physical properties.

22:20

Now, if I want to know something about how does my thermal boundary layer relate to my hydrodynamic boundary layer, that's when I, I am introducing now the Prandtl number.

22:31

So has anyone actually heard of the Prandtl number before?

22:34

Has this come up in any of your classes?

22:45

OK, excellent.

22:46

So the the Prandtl number is going to give us the relationship between the thermal boundary layer and the hydrodynamic boundary layer, which we're self defining here.

22:58

We can see thermal diffusivity,  $\alpha$  and kinematic viscosity  $\nu$  which we expand those definitions outward.

23:08

Then we'll see that's going to be ultimately the fluid's heat capacity times its viscosity on the thermal conductivity of the fluid itself.

23:18

So very simply, if the Prandtl number is greater than one for a given system or given fluid conditions, then that thermal boundary layer should be assumed to be thinner than the hydrodynamic boundary layer.

23:31

Now for water at 70° C, the Prandtl number is about 2.5.

23:36

Again, if I go to very viscous liquids where I have to think quite a bit about that dissipation, it can go as high as 600 and I typically see a decrease in the Prandtl number as the temperature increases.

23:47

So as I go to warmer and warmer systems, right, my thermal boundary layer should in theory be thinner and thinner relative to its hydrodynamic boundary layer in pipeline.

23:58

If the Prandtl number is going to be equivalent to one or round one, then I can typically assume both boundary layers are about the same thickness.

24:09

Now one point to highlight is that the profile here, the thermal boundary layer will not even in laminar flow, continue thickening it and then ultimately reach that same parabolic profile that we have with fully developed flow velocity.

24:25

But in fact, because I'm thinking about a heat transfer process, that boundary layer, even in laminar flow will continue to flatten as I move down the pipeline of interest, right?

24:36

Ultimately coming into equilibrium with the wall.

24:39

So because the purpose of the activity is transferring heat, you know, from the wall to the fluid in this case, then I need to appreciate that it's, it's ultimately if I go to an infinitely long pipeline, the temperature of the wall will be the temperature of the fluid.

24:55

OK.



24:56

Now something that that most of you should have certainly seen in heat mass transfer.

25:02

So can a can a couple of you let me know, please tell me this is not a new slide of, of content.

25:07

If we think about the the, the overall heat transfer coefficients.

25:12

Yeah.

25:17

So if we're thinking about a system where I have a, a cool fluid on the outside, a warm fluid on the inside, right.

25:23

And I use these in small quotes because it's obviously a yeah.

25:29

So if the Prandtl number is very, very small, then I can have a thermal boundary layer in theory that is significantly larger than my hydro dynamic boundary layer, correct.

25:38

Yeah.

25:44

OK.

25:44

So when I'm thinking about a complex system where I have two flow flow conditions and I also have conduction across a wall, and this is really where a lot of our heat transfer work is going to be, right?

25:57

And a lot of our unit operations follow this exact model.

26:00

I give them an outside and inside and little quotes because it doesn't really matter, right?

26:06

In fact, the only reason that I give this an inside and outside connotation is that I'm transferring heat from a warm system down to a cold system.

26:16

All right, so I hear what we're plotting.

26:19

We can see if we starting the warm fluid, there will be a thermal gradient and not dissimilar to the hydrodynamic gradient, right?

26:28

But if I'm still in a heat transfer process, then there's a thermal gradient as they go from the warm fluid and I approach that boundary layer of the wall.

26:38

Now as soon as I hit the boundary layer of the wall, right, I'm going to have a very steep descent into the wall temperature again because I'm going to have a no slip condition.

26:49

Now I have a a gradient across the wall that will be conductive in kind, where it's important to note the slope of that is proportional to thermal conductivity of the wall.

27:03

If I have another fluid on the outside that's going to be receiving that thermal energy, I first have to transit energy from that thermal gradient or through the thermal boundary layer, and then I pass through the hydrodynamic boundary layer and into the bulk temperature of the cooler fluid.

27:20

Now, in this case, if I'm thinking about, I know what that temperature difference overall is, and we've seen as a classic chemical engineering rate equation that the overall rate of transfer is going to be proportional to both the overall driving force, which is known, and the resistance to that driving force, which is usually what the chemical engineers design to to or ask to calculate.

27:42

Then I have to think about what are the individual resistances in this context and what is the sum of those.

27:48

So for what I've written here, well, I'm going to have a convective resistance that's coming from the cold fluid, all right.

27:57

So I have an initial convective resistance.

27:59

Then I'm going to have that conductive resistance across the wall and another convective resistance as I have to transit in or from the warm fluid.

28:07

So if I write it in terms of the total temperature difference  $\Delta T$  between the cold and warm fluids, well because it's steady state, that rate of heat transfer  $\dot{Q}$  again is going to be equilibrated.

28:19

I can see here in brackets I have the terms one on HDA or HIDI or DA sub I where that's going to be the internal or inside convective component  $K$  or  $X$  On  $kDa$ , these are defined for the wall conduction and one on HDA these are again for the outside now or cool fluid convection components.

28:44

So what we're showing is that the take home rule of thumb, the resistance for a convective system.

28:53

So the convective resistance is one on HA and the conductive resistance is going to be  $X$ , which is the thickness over which I'm conducting divided by  $K$ , the thermal conductivity  $A$ .

29:10

So I'm looking at the sum of these resistances.

29:12

And if I'm in a cylindrical system, I can simplify to get rid of those a terms, right?

29:20

And I, but I in order to represent them just as a differential diameter, then I'm going to have to pick either the inside or the outside diameter to be my base, right?

29:31

So I pick one of those and it's actually arbitrary what I choose, but I pick one of the diameters to be the base and then it will modify past that.

29:38

So what I end up what getting if I bring that this over to the left hand side, then my overall heat transfer coefficient  $U$  survive for this case will be 1 divided by each of the contributions themselves, which in this case I've chosen the inside surface area to be my base surface area.

29:59

And then I'm going to modify that as I go.

30:02

So  $1 / H$  sub I, that's just my internal convective heat transfer coefficient  $kW$  on or  $XW$  on.

30:11

$kW$  will be the thickness of the wall divided by the thermal conductivity of the wall.

30:17

Now  $D_{sub\ I}$  is my internal pipe diameter where  $D_{sub\ WL}$  with a bar on top.

30:23

It's a bit hard to see in the context of having a divisions line just to put it, but it's  $D_{bar}$  so  $WL$  that is my log mean diameter.

30:34

Now for the wall.

30:35

So I'm using a log mean approach assuming that that wall is thick and if it's not actually that thick in the context, then it's not going to make a big difference where finally I have one on  $H$  naughty or  $H$  outside.

30:48

So my external heat transfer coefficient times the quantity  $DI$ .

30:52

So the internal pipe diameter divided by external pipe diameter.

30:57

Now again, we would typically use that log mean temperature difference for the wall itself.

31:03

Now, if I'm considering each of these contributions, one of the really important points to think about and something that will play into your laboratory tomorrow is the effect of fouling.

31:15

All right, So anytime I have water or an aqueous system, I have to think about what the effective scale.

31:21

So things like calcium carbonate formation, where that water is going to flow and that's going to add an additional heat transfer resistance, the effects of dirt in my system or solids that might be deposited over time.

31:34

So each of these are going to show up as battling factors and then we will talk about these more as we go.

31:39

But these are added to the definition of use.

31:42

So I can add an additional 8 sub D to indicate filing and that filing can take place on both the inside and the outside of the line itself.

31:52

So all of this to say, we have to pay attention and we're starting to see large deviations between theory and our experiment.

32:00

Then the following factors are certainly one of the first places that we'll look to account for that and noting that the units for things like thermal conductivity are given watts per metre Kelvin, where the units for convection or my convective heat transfer coefficient  $H$  are given units of watts per metre squared Kelvin, right?

32:22

And these can vary from 1 something like air up to 100,000 if I have things like condensing steam and forced a forced fan.

32:31

OK, so again, this should be reviewed from heat and mass transfer, but I just wanted to get us out of the same page before we start kicking it into the unit operation now.

32:40

So when we start thinking then about, well, how do we actually apply this?

32:42

So has anyone in heat and mass transfer, it would be traditional that you would have seen Nusselt numbers applied for doing things like calculating the heat transfer coefficient, the convective heat transfer coefficient.

32:55

Has anyone on the call seen this kind of approach of using a Nusselt number before?

33:20

Yeah, no, OK.

33:31

It is just, I only ask because it will help me.

33:33

We're going to be doing a lot of this in the coming week and so I'm trying to get a feel for how comfortable.

33:38

I wish I could use the the Microsoft forum.

33:41

I don't know where it's where it's gone to.

33:44

I might try to bring it up in our go to the meeting chat real quick.

33:55

Oh, there we go.

33:56

I can put a form in here and a lot wouldn't let me before.

33:59

So I'm going to take a quick survey.

34:23

OK.

34:26

So while I'm, I'm lecturing on this slide, I just want to put up a survey, if you don't mind answering just so I can get a feel for the folks that are in the in the discussion with us, how comfortable we're going to be because this is going to dominate much of the next week or so as we start getting into how to design heat exchangers.

34:46

OK.

34:47

So when we're thinking about then conduction worker convection, we're going to be going through both laminar flow into transition flow into turbulent flow.

34:58

But because the nature of the thermal and hydrodynamic boundary layers is so different, right, then in fact, I'm going to need to take different courses to treat each one of these scenarios.

35:09

So today we're just going to touch on laminar flow because of course, that's the simplest thing.

35:14

And then in the next couple of lectures, we'll be getting into heat exchangers with transition and turbulent flow and what that means.

35:20

OK.

35:21

So in this context of laminar flow, this is really interesting because I talk about convection, but that's in the context of having fluid motion, right?

35:32

But when we say convection, right, in most cases, we're actually thinking about eddies that are transferring energy, right?

35:40

If I, if I'm in a or a highly turbulent system, then I'm going to have eddies that might be playing around at the boundary to the hydrodynamic boundary layer and certainly up in the fluid that are going to allow me to transfer vertically both mass and energy between the continuous bulk phase and that boundary layer itself.

36:01

Now, in the context of laminar flow, it's actually because I don't have eddies that are moving perpendicular to the wall.

36:10

I think again about my playing card analogy and imagine a whole deck of playing cards that's in motion right where I'm going to.

36:17

Each playing card is slightly cooler than the one below it, right?

36:22

And that's going to kind of govern how energy will be transferred.

36:25

So for laminar flow, it's, it's actually the, the case that for each of these layers of fluid, it's actually a conductive property that will transfer energy from a, a warmer layer of fluid to a cooler layer of fluid because these things lack eddies that are moving perpendicular to the flow.

36:43

Now in this case to in to handle this, we introduce nested numbers.

36:48

So Nusselt number, and this is a proper definition.

36:51

That's good to know.

36:53

This is the ratio of the heat of the convective to the conductive heat transfer resistance.

36:59

All right, So that's a really important thing to know.

37:02

If I look at a Nusselt number and if a Nusselt number is very high, then it's going to tell me if fluid is going to be very good at convecting energy.

37:10

I'm very poor at conducting it and the definition of the Nusselt number, so it's proper definition shown here on the bottom is the thermal or so the convective heat transfer coefficient  $H \cdot D$  So the Nusselt number for the system with a hydrodynamic diameter  $D$  and divided by its thermal conductivity  $K$  Now we'll get down to that, that latter bit in a minute.

37:35

But if I think about this scenario drawn on the right hand side, then we're going to introduce initially what is the Nusselt number defined for a position  $X$ , right?

37:48

That's along this plate.

37:50

So if the fluid is initially entering the plate at  $.0$ , I'm going to say, OK, this is my sum point  $X$  in the middle of I'm investigating and I want to define the Nusselt number at that point.

38:00

Now the reason we use Nusselt number correlations and the reason these are going to keep coming up is that this is a very convenient way.

38:09

If I know something about the position in the system or the dimensionality of the system and the thermophysical properties of the species that are transferring heat to or from, I can calculate the heat transfer coefficient for that system convectively.

38:25

So from theory, and we're not going to go into all of that detail, yeah, sure, you can say that.

38:35

That is a a simple way to say it.

38:36

Absolutely.

38:38

So from theory, we, we can see that there's a coefficient  $.332$  times the third root of the Prandtl number times the root of the Reynolds number.

38:47

And that's going to be defined at a position  $X$  into that line.



38:53

And here again, we can expand that out just to show the definitions of each of those.

38:58

And I wanted to highlight that a Nusselt number positioned at point X means I have a heat transfer coefficient, convective heat transfer coefficient defined at point X, which means my Reynolds number definition is at point X right now because that convective heat transfer coefficient  $h_x$  is going to vary with the square root of position X. I can integrate over the whole plate.

39:22

And it's going to give me the outcome that the overall heat transfer coefficient is going to be two times.

39:30

I'm sorry, the average heat transfer coefficient is 50% of the heat transfer coefficient along the entire plate, right?

39:38

If that distance is going to go all the way to X1.

39:41

So in fact, the Nusselt number correlation that we are going to use and the one that we need to know is shown here, right?

39:49

And this is the bit that we want to actually bring into a few calculations later on.

39:54

So this means, sorry, this should be 0.664.

40:01

So this is going to say now because I have that doubling effect, I can get rid of a positionally dependent Nusselt number and convective heat transfer coefficient, replace it with a Nusselt number defined for the length of the system.

40:14

We're going to call X of one.

40:17

We're now the convective heat transfer coefficient defined  $h$  that is simply going to be the coefficient for my system.

40:26

D is the hydrodynamic diameter for that system, K is the thermal conductivity of the fluid and the Nusselt number correlation in quotes is right here.

40:38

So I have .664 times the third root of the Prandtl number times the square root of the Reynolds number for that system.

40:46

And what I can do because these depend only on right.

40:50

So  $C_p$ ,  $\mu$ ,  $K$  and  $\rho$  these are all Thermo physical properties of my liquid  $U$  not and  $X$  are either hydrodynamic or geometric properties of my system.

41:06

So the velocity and the length of the tube or the length of the cylinder of interest, these are things that I should know.

41:13

In fact, I can calculate all of these to determine what is the Nusselt number, and if I do that, I can go back to the definition of vessel numbering because I clearly know  $K$  already and I know the geometry of the system.

41:27

I use that to get at the convective heat transfer coefficient for the system.

41:32

So we have this kind of correlation in this correlative approach.

41:36

And it's really important that we understand this and we'll do some examples.

41:39

We're not going to have time today, but we will do examples throughout the week on it.

41:43

But I can use this kind of approach because then I can select different refrigerants of interest, different fluids for heat transfer, and I can look at different pipe lengths in a heat exchanger and different pipe diameters to determine how that will ultimately affect my conductive heat transfer coefficient.

42:00

So what we're trying to say from above is that the in laminar flow, the average convective heat transfer coefficient is about twice that of the local heat transfer coefficient at the end of the plate.

42:13

And so that's where we go from that .332 from theory into .664 as the pre factor for this specific Nusselt correlation.

42:22

And as with every Nusselt number correlation, we have to define what is the range of conditions for which it's valid.

42:29

If I have a liquid as these spaces of interest, I have to have a Prandtl number greater than one, and if it's a gas, a Prandtl number greater than 7.

42:37

And generally, I want to use this at Nusselt numbers as a value of 10 or larger.

42:44

So what are some common applications and what are you going to see tomorrow morning with Angus and David as we get into the very first laboratory?

42:52

Well, we're going to be talking about refrigeration, but specifically heat exchangers and heat exchangers that can be applied to refrigeration.

42:59

So this is a simple application of a heat exchanger where we're looking at a shell and tube system.

43:05

Now we're going to go into a lot more detail next week about how these are designed, but we just wanted to layout, and this is coming from Chapter 11, layout some basic principles of operation as we get into discussing an application for these transcript coefficients.

43:22

So in this case, we would be looking at, we're going to insert some liquid that's of interest here on the left hand side.

43:33

And this is going to flow around a series of tubes, right?

43:36

So that's going to flow into this channel, right?

43:38

And it's going to be transiting, you can almost imagine through the background behind these tubes and what's running through these tubes.

43:46

We're going to have a vapour coming in that we're trying to condense.

43:49

And at the bottom, we'll come condensate from each of these two steps.

43:54

Now in this case, what we'll be looking at is if I have a temperature of the condensing vapour that's coming through those tubes side here, this is a constant temperature.

44:06

And here is the temperature of the inlet to my shell and tube system where I have  $\Delta T$  prescribed as that temperature.

44:14

So I initially have a very large temperature gradient and at the exit point I have a very small temperature gradient.

44:20

And I'm so I'm flowing here from left to right.

44:24

And what I can see is that as I'm moving along, right, I'm going to have a variable temperature gradient between those tubes and the shell at any given point in time.

44:35

So another just to generalize talking about heat exchangers a bit.

44:41

If I don't instead have one massive vessel containing a shell with a whole bunch of tubes running through it containing a secondary fluid that I'm trying to exchange heat to or from, I can also have what's called a double pipe heat exchanger, right?

44:55

And So what this is, is basically a jacketed pipe where I can see here, right?

44:59

If I take a cut away that there's a, an annulus around some internal pipe where I can flow one fluid, right?

45:10

And inside that pipe, I can have a second fluid.

45:17

Now in this context, I can choose if I have a traditional double pipe heat exchanger, I can either run the internal fluid Co current with which we also call parallel or counter current to what's flowing on the inside of the pipe.

45:33

So if I'm running in parallel flow, so the directions of flow that the fluids are travelling together, they're both going to enter right with the most extreme temperature and they're going to cool moving in the same direction.

45:46

They're going to cool with an attempt to equilibrate their temperatures toward the end.

45:51

And it actually turns out that it's quite rare that I'd be looking at parallel flow, but it's much more common and there are some applications where we can get into later.

46:00

But it's much more common that I'll be looking at counter current flow, right?

46:03

Where my, my warmer side of the fluid is going to enter at one point and my internal side of the pipe is going to be enter at the other side and they're going to flow across each other.

46:14

And what this gives me is a much more efficient heat transfer across that heat exchanger because that will tend to have a, a more constant temperature gradient maintained within the system, right?

46:27

If you think about that's our driving force heat transfer.

46:31

So again, this is just to set up and motivate a little bit of really thinking before we get into the more applications next week.

46:38

And the last kind of background in in theoretical point that we need to touch on is the log mean temperature difference.

46:44

So up until now we've been referencing that the thermal gradient between the pipeline inside and the outside of the pipeline be that the annulus or the ambient temperature that's going to be the driving force for heat transfer.

47:00

But in fact, I can represent that more accurately instead of just like we looked at our geometry instead of looking at an absolute difference in temperature by thinking about how those two positions right.

47:12

If this is my  $\Delta T$ ,  $\Delta T_1$ , and  $\Delta T_2$ , because I can have a dynamic amount of heat exchange over that whole exchanger, then for each of the sections that I might perform a calculation on to get a more accurate representation, I can combine both of these to give me an average temperature difference.

47:33

So if I know that I'm going to vary from  $\Delta T_2$  to  $\Delta T_1$ , I can represent that here on top.

47:40

And if I assume a constant heat transfer coefficient  $U$  and a constant specific heat in steady state operation, right?

47:47

So these are three major assumptions behind this that it's very, very good to know.

47:52

A lot of folks tend to forget what those assumptions behind LMTDR.

47:57

But if I have a constant heat transfer coefficient, constant specific heat and steady state operation, then I can redefine this equation here and I can integrate it, right?

48:09

And this integration step, what it gives me is a relationship between the natural logarithm for  $\Delta T_2$  on  $\Delta T_1$  to the absolute difference between  $\Delta T_2$  and  $\Delta T_1$ , right?

48:23

And if I rearrange that equation to solve for the rate of heat, the overall rate of heat exchange and bringing that back into my usual form for an overall heat convective heat transfer, then I can see that that we write this as a  $\Delta T_{\text{bar sub L}}$  where that itself we call the log mean temperature difference.

48:45

And that is the difference between the absolute difference between those, the minimum and maximum heat exchange in the system divided by the natural logarithm, the ratio between the two, right.

48:57

And again, that's coming from this, this point that either in a Sheldon tube system or some double pipe system, right, I'm going to have gradients of heat transfer within the system.

49:09

And this is the way I can take those gradients, right and wrap them into that same convective heat transfer relationship that I've used before.

49:18

Now again, there are three assumptions that we put up top that are required to inform this.

49:25

There are also two additional scenarios in which this is not an appropriate application.

49:32

One is that if I have a reacting flow, and so in reacting flow, I might have species inside that, that heat exchanger or pipeline, as they cool down, they might undergo either an exothermic or an endothermic reaction, right?

49:46

That's obviously going to mess with these thermal gradients in the system here.

49:51

So for that case, I can't use this approach because I can't guarantee that it's monotonic in its behaviour.

49:57

And it also doesn't necessarily work if I have multi pass exchange.

50:01

And as we'll learn about in the next lecture, it also fails to capture what happens if I have condensing right or latent heats being transferred through this as well.

50:13

OK, so the take home messages, this is a lot of kind of quick backgrounding to make sure we are on the same page.

50:19

Now we embark on the next kind of learning module within unit operations.

50:24

It will be about heat transfer and heat exchangers and heat exchanger design.

50:29

There are four primary take home messages here.

50:32

We will be including both conduction and convection when we start talking about heat exchanger design and that we will represent these by with an overall heat transfer coefficient that we're going to call  $U$ , right.

50:45

So this will be given that or not small  $U$ .

50:48

This will be given the capital  $U$  as a variable and we can calculate this by summing up each of the individual resistances, where convection is goes with one on each and conduction goes with  $KX$  on.

51:03

$K_i$  also can represent if I have thick walled tubes or I can just as a basic premise, and it tends to be accurate for thin walled tubes as well.

51:14

But I can use a long mean radius instead of the absolute or the average arithmetic mean radius of the pipe wall by looking at the difference between the outer and inner pipe radius divided by the logarithm of the ratio between the two.

51:30

Now here we've introduced 2 new reminded one and introduced the other of dimensionless quantities.

51:36

So the Prandtl number is the ratio of my momentum diffusivity on my thermal diffusivity.

51:42

So when I have values that are significantly greater than unity, it means that my thermal boundary layer will be sorry, that should read Hydrodynamic boundary layer is larger than my thermal boundary layer.

51:56

And if it's a value at unity or one, I will tend to have about the same thickness in the two boundary layers.

52:02

And we define the Prandtl number as the heat capacity.

52:05

The fluid times its viscosity on the thermal conductivity.

52:10

Now in the Nusselt number, this is the ratio of the convective heat transfer resistance to the conductive heat transfer resistance.

52:18

So this we have our convective heat transfer coefficient  $h$  and that's typically the point of why we want to do a Nusselt number correlation is to solve for  $h$  times the diameter of the system it's applied to divided again by the thermal conductivity  $k$ .

52:34

And what we've seen today is our first of what will be a handful of Nusselt number correlations.

52:40

Now it's, it's really important, you know, these are not things that you necessarily need to need to memorize.

52:47

If I'm asked you on an exam or you need to obviously do the different work, this is where you would go to the textbook or you'd look up what's the appropriate Nusselt correlation for the, the scenario that I'm trying to solve it for.



53:00

But these tend to be the most effective means chemical engineers can divine to look at calculating that convective heat transfer coefficient, which itself varies with things like velocity and the thermo physical properties of fluid of interest.

53:17

And when I bring these together, where these will play a role is then in thinking about the design of heat exchangers.

53:26

And I can calculate that total amount of heat transfer being my overall heat transfer coefficient  $U$ .

53:32

That's going to be defined from that Nusselt correlation, my total surface area in the system, and most importantly the log in temperature difference, where that's taking into account that I have a minimum and maximum temperature difference at the entry and exit points.

## Lecture 08 - Heat Transfer Rates

0:00

Chapter 12 in the textbook.

0:02

So this is Chapter 12, and specifically we'll be focusing on the latter portions talking about heat transfer in convective systems.

0:12

So at the end of this lecture, if you're reading along either at home or through the textbooks in the library, we should be at the end of Chapter 12, at the end of this lecture.

0:21

In the last lecture in seven, we introduced some concepts around heat transfer, specifically operations that require us to understand both conduction and convection.

0:34

And in that context, we were looking at the use of an overall transfer coefficient, which we will give the length of view as the designation to.

0:43

And we looked at the way that we calculate you.

0:45

And we're going to be revisiting this again in this lecture is the sum of the different heat transfer resistances.

0:51

So in the same way that a material is going to naturally resist the conduction of heat through it, and the same way I have a resistance if I'm trying to move heat through any convective process either to that wall or away from the wall.

1:08

And specifically, we looked at the resistances for convection as one on the convective heat transfer coefficient.

1:17

And for conduction is the thickness of the material divided by the thermal conductivity of that material.

1:26

Now, we also talked about the difference between having pipelines or ducts that have a thick wall versus a thin wall.

1:34

And specifically when that wall thickness, so the different ratio between the outer radius and the inner radius approaches 1, then we can use like an arithmetic mean diameter or an arithmetic mean radius for the pipe.

1:50

But when we have a very thick wall, so maybe we have a flowing space of two inches in diameter, but on top of that we also have a one inch thick wall.

2:00

So now that wall is going to be considered a thick wall.

2:04

And so when we consider the radius of that wall or the diameter of the wall or calculations, we need to use what's called a log mean radius and the definition of that.

2:13

That will be the difference between the wall thicknesses divided by the natural logarithm of the ratio of the outer wall radius or diameter on the inner wall radius or diameter.

2:23

And these together give me the log mean radius or the log mean diameter.

2:27

Now to start understanding a bit more about the calculations, we needed to introduce two dimensionless groups.

2:35

So the parameter number we talked about was the ratio of the momentum diffusivity on the thermal diffusivity.

2:43

And so when the Prandtl number is going to be greater than one, I can see that I have a  $\delta$ .

2:49

This should be reversed, sorry.

2:56

I have a hydrodynamic boundary layer that's larger than my thermal boundary layer when the value of the Prandtl number is above one and if the value is at 1.00, then those boundary layers together should be about the same thickness as I move from the leading edge of the plate inward.

3:12

We also introduced the Nusselt number and this is going to be the main focus of our topic today and that is the ratio of the convective on the conductive resistance to heat transfer.

3:23

So we can see here, right, those ratios  $k$  on  $D$  or  $k$  on  $X$  and  $h$  being a resistive heat transfer or the convective heat transfer resistance.

3:34

And most importantly, and a few people in the class said that they don't remember or might not have used a lot of muscle correlations.

3:42

So this is a really critical concept that we should need to be comfortable with.

3:46

And we're going to be looking at some examples today how to do this.

3:49

Muscle correlations are how we as engineers solve for the convective heat transfer coefficient in complex problems.

3:57

So it's a really critical way to understand what these are and how to use them.

4:00

Now the last point is that when we bring all of these together, we will estimate the total amount of heat transfer or the rate of heat transfer  $Q_A$  sub  $T$  being the overall heat transfer coefficient  $U$  the area  $a$  sub  $T$ , so the total heat transfer area.

4:17

And I can use something like the log mean area that's required.

4:21

But most importantly, when we have temperature differences across our heat exchangers, we now introduce a similar concept of the log mean temperature difference where if you'll recall, if we have something like right, so this might be the warm fluid secure platinum temperature as a function of distance.

4:48

So here's my warm fluid entry.

4:49

If I'm in a counter current jacketed heat exchanger, this is my warm fluid exit.

4:58

And here I have the cold fluid exit and the cold entry.

5:05

All right.

5:08

So the cold fluid is going to enter the lower temperature, it's going to be warming up as it transfers heat from the warm fluid.

5:15

And so because I have these these wide temperature gradients in the system, we call this difference at the very end.

5:23

This will be for instance, one of my approaches, we call them.

5:27

And this final temperature difference is my other approach.

5:31

And so I use these approaches as I define the log mean, temperature difference being the absolute difference between the approaches on the natural logarithm of those approaches.

5:42

And so this is 1 means that I can use then to more accurately represent the log mean or the the temperature driving force for overall heat transfer.

5:53

And there are a few caveats that we talked about in the last lecture that are good to know.

5:57

But in general, we'll be using this logging temperature difference as we move forward in today.

6:02

So before we go too far, all right, I just wanted to to kind of do one example since last lecture was primarily theory.

6:09

I wanted to do one example with everyone to look at this, this idea of counter current flow and what we'll call a single pass jacket heat exchanger.

6:18

Now, this is an example from the textbook on top of a jacketed heat exchanger.

6:23

In this case, we're not going to be having a, a multiple swinging passes like this, because in fact, that's one of the criteria that we can't use a lot of the temperature difference in.

6:35

So we're going to say this is just a, a, a single pass.

6:39

It's a straight pipe where I have some fluid in the middle and it has a jacket heat exchange fluid on the outside.

6:47

So the length of the system, let's say is 50 meters.

6:51

The internal diameter is going to be two meters, outer diameter going to be 4 meters, and the diameter of the jacket is 8.

7:05

So quite a large jacket in which we can assume there's not going to be any significant temperature gradient within that jacket that is well mixed.

7:14

I'm also going to say that the overall heat transfer coefficient is 475 watts per meter square.

7:27

Now if I look at this diagram down here, I need to prescribe what the temperatures are going to be in my inlet and exit fluids, right?

7:34

And often times these temperatures are dictated by the necessities of the unit operation I'm designing for.

7:40

So if I have my cold fluid enter here, I'm going to say that that cold fluid is going to enter at  $10^{\circ}\text{C}$  and it's going to exit at  $50^{\circ}\text{C}$ .

7:54

Now the warm fluid is going to enter here on the right hand side at  $80^{\circ}\text{C}$  That's going to exit.

8:06

It's going to exit at  $20^{\circ}\text{C}$  So before I go too far, let's first check do we have a, a, what is considered a thick wall in this case?

8:18

All right.

8:19

So if I look at my diameters, then I can calculate that log mean diameter.

8:27

We should give that an over bar to the outer diameter, the natural outer diameter diameter, right?

8:38

And in that case, I get a if I do that calculation with the number above 289 meters, where I can look at the arithmetic mean diameter, right, Which would just be .4 point 2 / 2 three.

9:06

That's that's just my sanity check.

9:08

I can see actually a quite a significant deviation if I go to use the log beam diameter versus the arithmetic beam diameter.

9:16

So in this case, because I've made a reasonably thick walled pipe of .2 meters or .1 meter in the radius, I actually need to use this log beam diameter.

9:29

OK, so now how am I going to calculate my surface area?

9:34

Total heat transfer area is then going to be  $2\pi L$  times my means.

9:41

I am right, which is just going to simplify times 50 meters times 22 eight and that's going to give me a value of five, three meters squared.

9:58

Now the approach is here.

10:01

This will approach one, you can see here.

10:04

Approach one is going to be  $10^{\circ}$  on the cold side.

10:07

On the warm side, approach two, if I just look at the absolute difference, it's going to be  $30^{\circ}$ .

10:13

Now again, I'll switch colors here, but if I want to look at the arithmetic mean temperature, get a subscript A to that, that's going to be  $30 + 10 / 2$  or degrees C.

10:34

Now if I want to calculate the temperature that is going to be  $30^{\circ}$  -,  $10^{\circ}$  C on the natural logarithm of  $30^{\circ}$  C,  $10^{\circ}$  C, and that gives me a value of 18.2.

10:58

And these are degrees here, degrees Kelvin because I'm talking about a differential temperature is equivalent.

11:03

And that's actually now becoming really important, right?

11:07

A minor variation of something like an overall surface area is one thing, but anytime I'm in chemical engineering that that delta right in my rate equation is my driving force.

11:18

And so here I can see what is significant difference in the driving force if I use an arithmetic versus my log mean temperature difference, right?

11:29

So now bringing these together, I can simply go back to here.

11:34

So I can say, well, my total heat transfer is going to be  $U$  over all times area total Times Now by log mean temperature difference.

11:46

So I have 475 watts per meter, square Kelvin times five, 3M square times 18 to Kelvin.

11:59

And remember I can interchange Kelvin and Celsius because I'm talking about a differential temperature.

12:04

And I'm going to then by that 1000 watts for one kilowatt just because I'm going to get a very large number out of this and that's going to give me in this case 392 kilowatts.

12:22

So for the system that's been prescribed with this geometry and these temperatures, this is the amount of heat transfer that that single pass jacket of heat exchanger or double pipe heat exchanger can deliver.

12:36

So then the next focus that we should take is to say, well, if I'm looking at this, I've been prescribed as overall heat transfer coefficient.

12:45

Well, that's not something I can just buy off the shelf, right?

12:47

That heat transfer coefficient exists for really if I know what the temperatures are, I have to calculate that.

12:53

So how are we going to do it?

12:55

Well, at the end of the last lecture, we have looked at 2 correlations for the Nusselt number, one that was prescribed for a flat plate and so much if I have a, a hot plate with liquid flowing on top of it.

13:10

What's a fundamental theoretical relationship for the Nusselt number that we can derive in the textbook goes through this in Chapter 11 and 12 to be some extent.

13:22

But I wanted to 1st show you that because this is the reason that we have this form of relating the Nusselt number to both the Grashof and the Reynolds number.

13:31

And in fact, this form is going to re emerge over and over as we look at different applications throughout this lecture.

13:39

So it's very uncommon that we're going to be looking at the laminar flow of fluid across a flat plate.

13:45

But what is more common is that we will have a pipe or a duct that we're trying to flow fluid through like a normal heat exchange.



13:53

And in that we have a very important correlation for laminar flow and at the cylindrical pipe or a duct.

13:59

And that relates our Nusselt number, which itself is the convective heat transfer coefficient  $H \cdot d$  the characteristic diameter of the pipe or the duct on the thermal conductivity  $K$ .

14:09

And the correlation itself is shown on the right hand side here.

14:13

In fact it includes this.

14:15

There we go.

14:17

So it's a coefficient of 1.85 times the product of the Reynolds number, Prandtl number, and the ratio of the diameter to the length all taken to the power of  $1/3$ .

14:30

And I then multiply this again by  $\Phi$  sub  $\Phi$ .

14:35

And we're going to talk about that in a minute.

14:37

But it's a correction for the viscosity of the flowing fluid.

14:41

So now if I look at this here, the Reynolds number, right, we have  $\rho V D$ .

14:45

The Reynolds number is going to be  $C_P \mu / K$ .

14:52

So in essence the entirety of this expression with the one, with the one exception of the velocity used to derive the Reynolds number or to calculate the Reynolds number, the entirety of the right hand expression relies on the Thermo physical properties of the fluid of interest.

15:13

And so this is the density, viscosity, thermal conductivity and heat capacity.

15:32

So if you remember in Thermo last semester how much time we spent talking about these different Thermo physical properties and the importance of calculating them.

15:40

And certainly what you will see in advanced Thermo 4404 is a much more rigorous deep dive into calculating these properties.

15:48

This is why, right, Because in very practical applications like fluid mechanics and unit operations, these are the properties that are informing how we govern and design a large scale operations.

16:01

So again, with the exception of the velocity and the diameter that come into that Reynolds number, the rest of these properties on the right hand side are of all Thermo physical in kind.

16:11

And this  $\Phi$  term which we can derive here is defined by the ratio of the fluid viscosity at the arithmetic mean temperature.

16:24

So the difference between the OR the difference between the inlet and outlet temperature, the average between those two on the viscosity of water at the wall temperature taken to the .14 power.

16:36

And so in theory, right, if we're heating up a fluid, all of its Thermo physical properties are going to change, right?

16:43

So everything we're putting in these four properties that we're putting into the muscle correlation will change as we change the temperature of the fluid.

16:50

The reason we only correct during viscosity is it's the one that's the most sensitive to temperature, right?

16:56

Heat capacity, conductivity and density really tend not to change all that much of temperature, so much so that we can actually neglect the calculation.

17:04

But viscosity, to give you an example, if we have something like a crude oil, the viscosity of a crude oil can change by a factor of 1000 as we take the temperature from 80° C to 100 degrees C, right?

17:18

So the viscosity fluids tends to be a very, very sensitive Thermo physical property temperature.

17:24

So for that reason, we include this this correction factor.

17:28

Now we only have to really worry about that correction factor when I'm dealing with things like crude oil or if I'm dealing with highly compressed gases.

17:37

The rest of the time, actually, if I'm dealing with water or, or most aqueous species, if I'm not at a really extreme temperature difference or a very high pressure, it tends not to be that impactful and in fact, I can neglect it.

17:49

So in the calculations you can do this week and and throughout the semester, if you're not told specifically here's the difference in viscosity or the viscosity modifier for this problem, then you can neglect that contribution.

18:02

OK, So if we know what the velocity diameter in the thermal physical properties are, right because they should begin in the definition of the Nusselt number.

18:11

In fact, what I can do is solve this right hand side of the expression and then by the definition of the Reynolds number solved directly for my target, which is the convective heat transfer coefficient.

18:22

In what Chapter 11 in the book goes through chapters 11 and 12 is that in most cases, if I take this down with the limit of very low Reynolds number, a Nusselt number of about 3.66 tends to be my theoretical limiting value for how low the Nusselt number can get, right?

18:40

So that's a good kind of benchmark to keep in mind when you're doing Nusselt calculations is that if you get a value below 3.66, that should send up a red flag because that's not a that's hitting below the theoretical value for laminar flow.

18:55

Now, obviously, in most applications, we want to move things quickly and we don't want to build big systems that take a long time.

19:02

So what happens when we're in turbulent flow, right?

19:05

So for a Reynolds number, in this case about 6000, there are two different

correlations that the text goes through that are quite famous and they're good to know.

19:14

I show them here in this form because in fact, they're quite the same thing, right?

19:19

So the did is bolted correlation on top.

19:21

This again is looking at that definition of Reynolds number.

19:24

And now the pre factor is point O 23 Reynolds number is taken to the .8 power and the Prandtl number in in the bit is bulk of correlation is just raised to the power of N where N is going to be .4 fluids being heated and .3 if it's being cooled.

19:40

Now, sometime after that, the sealer tape correlation came along and it follows along with it.

19:45

It's also called the Colbert equation.

19:48

The Colbert formulated it slightly differently.

19:50

And in fact, you can rearrange and get back to the same form.

19:54

So I'm only showing you one that I think is definitely worth knowing how to use.

19:59

But this latter formulation takes a very similar approach, but raises the Prandtl number to simply one third power.

20:05

All right.

20:06

So we're mimicking the Prandtl number dependency that was shown in the laminar flow case.

20:12

And it introduces that same viscosity multiplier, right?

20:15

You can see this folder were including two different variants of that Prandtl number power to account for the effect of heating and cooling.

20:24

But in fact, a more accurate representation from Colburn and and Sieder would be to use that theoretical Prandtl number power and then to follow that on with the viscosity multiplier, right.

20:36

So the take home messages from this, this we're dealing with cylindrical or ducted work.

20:41

This is a very important calculation for laminar flow on top and the Sieder in, in Colburn analogy equation on bottom would be what we use for turbulent flow.

20:50

What happens in transition flow?

20:52

Well, this becomes a little bit more nuanced and, and what we're showing here on the plot, this is where the, the name Colburn comes from is, is the Colburn J factor.

21:03

This was used to early on to try to understand what happens if I'm between around the number of 2100 and 6000.

21:11

Now in that case, so the Colburn defined his J factor as shown here is the product as a function of the Nusselt, Reynolds and Prandtl numbers.

21:24

And you can see here if I'm in a laminar flow region that we have a decrease in that Colburn J factor as I increase Reynolds number.

21:32

And as soon as I hit the onset of transition flow through the end of the period here, then I have this kind of parabolic behavior.

21:40

Now I only show this as an illustrative point because we can adapt Colburn's J factor equation, which shown on top to give us a Sieder like style of correlation for how we can do these calculations.

21:55

So here we don't quite have the Sieder number being defined on the left hand side, although I could very easily move this around and define it as a proper Sieder number if I wanted to, right?

22:05

But we have the heat transfer coefficient that we're solving for  $H$  divided by the heat capacity, density and velocity.

22:13

And that's being said equal to now pre factor of 1.86, the diameter on the length to the power of  $1/3$ .

22:20

And then we divide that by the product of the Reynolds number to the  $2/3$  power and the Prandtl number to the  $2/3$  power.

22:26

And we include that viscosity correction factor.

22:29

So in the same way, if I can do a calculation on the previous slide, looking at, you know, the Thermo physical properties and the geometry of the system, meaning the length and the diameter, I can pretty quickly get at the heat transfer coefficient  $H$  whether I am in a laminar transition or turbulent flow.

22:49

And in fact, we're going to do an example of that toward the end of the lecture today.

22:52

So these three approaches are all for what happens for internal pipe flow, right?

22:58

But what happens if I have an evaporator or a heat exchanger and I'm flowing things over the top of it, meaning air or subseas, using water as a cooling fluid or heating fluid in some cases.

23:12

So what if I have this external convection?

23:15

Well, as you may remember from heat transfer, we tend to break external convection into both forced and then natural because the characteristics change a little bit.

23:24

Now, if I'm in a forced external convection regime, this is going to be something, if I have wind or subsea, where I'm going to have that fan that's actually driving that.

23:33

So I have a more turbulent air flowing on top.

23:37

And what happens when I have a fan forcing that fluid atop?

23:40

Maybe this is my pipeline or my cylinder containing some temperature hot.

23:46

So a hot fluid that I'm trying to cool down.

23:49

Well, if I have wind passing in a form of process, as we saw in last week's Fluids review, we're going to start inducing wake shedding right around it.

24:00

So it means that the degree of turbulence that I can invoke in the heat transfer process is variable.

24:09

All right.

24:09

And then specifically, if I look at the effectiveness, so here we're plotting the Nusselt number.

24:15

And so we can see Nusselt numbers, right, very well above our limiting value of three.

24:19

But I'm plotting the Nusselt number as a function of the angular coordinates.

24:23

I look all the way around that pipeline where zero is going to represent the perpendicular contact between the wind or the the water current and the pipe of interest.

24:35

I can see here this variability where I get a huge spike right after I hit that that point.

24:43

And you'll recall we call the separation point, right?

24:46

So as as soon as I start increasing the angular coordinate, I get less and less contact until I get around that point of separation, right.

24:56

And you'll recall from the fluids lecture, we talked about these eddies that perform immediately after that separation point.

25:03

And This is why I tend to see a very pronounced heat transfer, so very high, not so number, very high heat transfer coefficient.

25:10

Immediately after that, that fluid separates from the object in which the fluid is or submerged in the fluid.

25:18

OK, now this is this is all experimentation that's just shown to illustrate the point.

25:23

But how do I calculate it for a variety of applications?

25:27

Well, here again, we're looking at that same right.

25:30

This is a Nusselt number definition.

25:33

So we have the now  $h_o$  is the OR  $h_o$  is the external heat transfer coefficient, the diameter of the external diameter and  $k_f$  when I have the subscript subscript F command for natural convection, I'm talking about the external transfer fluid.

25:57

So McCabe uses that subscript ectin the fluid and in this case the fluid is the external conduction fluid.

26:04

So that is all as we would expect equal to point some coefficient.

26:09

So in this case,  $.35 + .56$  times the Reynolds number of my external fluid raised to  $.5$  two power, all of the product of which is multiplied by the prandtl number of my external fluid.

26:23

PowerPoint 3 right in here on the bottom.

26:27

This is just an example of error that's flowing normal, so perpendicular to a single submerged tube.

26:34

And here we're looking at the nussle number on the the Y axis or ordinate as a function of Reynolds number.

26:42

So  $G$  being where they write this is a bit older, but  $G$  being the product of the density



of the mass flow rate, the mass flux times that outer diameter on the viscosity and that this is equal to the Reynolds number.

26:56

And so I can see I have to get very, very, very sub laminar region here before I can get a muscle number for the outside pipes that's below my theoretical value for normal internal flowing.

27:07

And in most cases, right, if I'm in a Reynolds number of anywhere from 100 right on up, I'm going to be above that limit.

27:14

And, and for virtually all wind applications and virtually all subsea applications, we will always be at a Reynolds number above 100.

27:22

And I can see here that for external forest convection, most of my Reynolds numbers of interest if I'm in a wind space will be on the order of 10 to 100, right?

27:32

Maybe as high as 500, but that's a pretty windy day or a lot of fan force that's going after that.

27:40

So again, the take home message here is I can use the same Nestle style correlation to perform a same calculation if I have a fan that's forcing a heat transfer fluid across the outside of the pipe.

27:54

The question is what happens if the convection is natural, right?

27:58

So the one caveat of natural convection for this external, right?

28:02

And this is, again, external.

28:05

One caveat for this external natural convection is that if we can imagine this example here of, of air that's flowing through a whole bunch of pipes in series, right?

28:15

So it's going to cool those down using a natural approach.

28:18

Well, if I just have natural convection, then there's no fan that's driving that fluid through, meaning that if I focus on an individual pipe fluid of interest, well, the temperature gradient, right, of the fluid as it passes across that pipe, it's going to

induce the temperature gradient in that fluid itself because it's just naturally floating up on top.

28:45

And so because I have a temperature gradient, that's the temperature is going to be, our heat's going to be flowing conductively through the air as it's passing or water as it's passing above my heat transfer pipe.

28:58

Because I have a temperature gradient, I now have to think about the effective buoyancy of that, that heat transfer fluid as it's changing its Thermo physical properties in proximity to the pipe itself.

29:14

And specifically the one that we'll use to characterize this.

29:17

And if you remember back from Thermo is the volume expansivity, which is given the Greek letter beta and it is that the partial derivative of and this is not velocity, but molar volume.

29:30

So apologies, it's a bit difficult when we jump from Thermo to fluids to heat transfer.

29:34

But this is the the partial derivative of the molar volume to temperature to constant pressure divided by the molar volume of the species.

29:42

So this is giving units you might recall.

29:49

And then so it tells me is a change the marginal degree in Kelvin.

29:53

How much is that fluid going to expand or shrink when it's a constant pressure?

29:59

And the ground shock number here is what we are going to use for natural convection, external convection to characterize this.

30:06

So this is the outer diameter cubed, the density of my external heat transfer fluid squared.

30:13

The volumetric expansivity beta times G is the gravitational acceleration constant.

30:21

And that's simply because we are assuming gravity is what's acting on these fluids.

30:27

We're not in a centrifugal capacity, but acting in a gravity field.

30:32

And delta T naughty here is the delta T naughty is the absolute temperature difference between the wall and what we call T Infinity.

30:41

So T Infinity in this case, right?

30:43

If I follow all the way out, staying a kilometer away, that's T Infinity, right?

30:50

So of my my natural convection field error, what is that T Infinity going to be?

30:56

So good example, if you were going to build a a natural heat exchanger to set out somewhere in Perth, a good proxy for T Infinity would be what the ABC weather report is for that day is the air temperature, right?

31:11

So once we have this scratch off numbers defined, again these are just thermal physical properties with one geometric definition for the system and some knowledge about the temperature difference.

31:26

We then pop that into this slightly different now Nusselt number approach.

31:31

So here B is going to vary.

31:35

So B is my pre factor.

31:37

B is going to vary based on the type of external heat transfer system I set up.

31:43

So if I have a vertical plates containing fluid or vertical cylinders then my B is going to be other .59 or .13 depending on the product range of the Grashoff and Prandtl numbers for the system.

31:58

So if I multiply them together I can look at what range they fit in and this tells me what BI should be using instead of vertical cylinders.

32:05

If I have horizontal cylinders, which is going to be more common than the criteria there is that we look at the base 10 log of the product of the Grashoff and Prandtl numbers and as long as that value is about four, then I can use these coefficients here.

32:20

So B in this case, I would take that right and pop that into B is  $.5$  three times the product of the Brankle and Grashof numbers raised to the power of  $N$  which if I have horizontal cylinders will be  $.25$  South in most cases.

32:38

We're going to do a calculation on the quiz this week that we'll, we'll look at the Grashof number in detail.

32:45

And when you actually take a reasonable fluid like air or water and you plug the properties in seawater or air and you plug the properties in for the definition, what you're going to see very quickly is the crash off number is a very, very big quantity.

32:58

So I'm looking at something like the base 10 log of the product of a Crandall and the Grashof number is usually going to be above 4.

33:08

That's a pretty reasonable assumption.

33:10

That's let's now look at an example of how they go back to the internal heat transfer coefficient now.

33:17

Now I want to do examples looking at the variation of velocity.

33:22

So if I have a single jacketed double pipe heat exchanger, terrible with the surface, there we go, a single jacket where the inner pipe diameter is going to be two meters, the length is going to be meters.

33:46

And I'm going to fill this with water and I assume that the conductivity of water is  $.598$  watts per meter.

33:56

The density of water is  $1000$  kilograms per cubic meter.

34:03

The viscosity of water is Pascal seconds and the heat capacity of water it's 4186 and eight joules per gram, right.

34:20

So as engineers, these are actually all good properties for you to know off the top of your head because we use water as a everyday basis for a lot of designs.

34:31

And in this case, these are all kind of the standard properties we'll be looking at.

34:37

So we want to know how does the convective?

34:45

So I'm just going to write  $H$  for that.

34:47

But how does the  $H$  value, so the convective  $H$  transfer coefficient change for laminar transition and turbulent flow?

35:09

OK, So in the lavender case, I'm going to say I have a velocity of, let's just pick one meters, something that we think will be reasonably loud, right?

35:24

So in this case, I can calculate the Reynolds number at that velocity  $\rho$  times the velocity at internal pipe diameter on  $\mu$ .

35:33

I know all of these petitions and I get a Reynolds number of exactly 2000.

35:39

The Prandtl number again is the heat capacity times the viscosity, thermal conductivity.

35:47

I've defined all of these already.

35:50

So I know that that's going to be 7.0 exactly.

35:56

And so I can then go into my Nusselt number calculation as my step, and I can say that's 1.85.

36:04

That turns the product of now 1000 times, 7.00 times the diameter, .2 meters of the length  $V$  meters, all to the power of  $1/3$ .

36:20

And that gives me a nestled number of 562, which itself is equal to 8 to five times the diameter on the thermal conductivity  $K$ .

36:31

So from that, all right, I can just multiply it by the thermal connectivity divided by the diameter, and I get an  $H$  value 16.8 watts per meter squared Kelvin.

36:44

OK, now what happens if I'm in transition?

36:53

Well, now I need to take that velocity up a little bit.

36:55

So let's assume the velocity is going to be one or 2 meters per second, right?

37:00

That's going to give me a relative number using the same definition of 4000.

37:06

Now it's interesting because if I look at the Prandtl number here again, nothing in the Prandtl number.

37:11

It's the heat capacity, viscosity and thermal conductivity.

37:15

Nothing in the definition of the Prandtl number varies with the velocity flowing through the system.

37:21

So I get the Prandtl number exactly 7.00 hasn't changed.

37:27

And if I for transition fluid, I have that perfect muscle correlation, but I can use the Colburn analogy.

37:35

So I can say that this  $H$  to the bottom on or the value of  $CP$  times row times the loss and it's going to be equal to 1.86 times.

37:49

So I have the diameter 2 on length 50 meters to the power of  $1/3$  divided by the Reynolds number.

37:59

So 4000 to the power of  $2/3$  times the prime #7 on to the power of  $2/3$ .

38:09

And again, because we're we're not talking about any viscosity changes here, I'm going to just assume that viscosity stays constant and so that the product of this expression for the Colburn analogy is going to be  $3.20 \times 4$ .

38:26

And so I can solve directly for  $H_{sub I}$  by multiplying this by the heat capacity, density and velocity.

38:33

And what I get out of that is an  $H_{sub I}$  bounding 26.8 watts per meter squared.

38:42

OK.

38:42

So as we've gone from a flowing pipe velocity point O5OO5 to point O2, we go from laminar to transition flow and our internal, our convective heat transfer coefficient increases from 16.8 watts per meter squared Kelvin to 26.8 watts per meter squared Kelvin.

39:01

So now let's look at.

39:07

So here I'm going to say the velocity is .2, right?

39:12

That's still quite a low velocity when you think about it, right?

39:15

And that's one of the points we made earlier with the review of fluid mechanics.

39:19

If an engineer says they have laminar flow, that's always a reason to raise an eyebrow unless they're talking about honey flowing down the wall, right?

39:27

That's our our model for laminar flow here.

39:29

We're talking about .2 meters a second.

39:31

And that's all we need to be well into the turbulent region.

39:35

All right.

39:36

So in that limit, same definition, the Reynolds number is now 30,000.

39:42

That's quite a high Reynolds number.

39:44

Again, the Prandtl number does not change.

39:46

It's not a function of velocity.

39:49

So I can use the Nelson correlation of point O2 three times the Reynolds #40,000 to the power of eight times the prandtl #7 to the power of 1/3.

40:03

And I'm not accounting for any viscosity variations, which gives me a Nelson value of 211, right?

40:10

Once again equal to H.

40:14

To H1 the diameter on the thermal conductivity.

40:19

So if I multiply that this 211 by the thermal conductivity and divide it by the diameter, I can get at my heat transfer coefficient, which in this case will be 602 watts per meter squared.

40:33

So now right I've gone up by another factor of 10 in velocity.

40:38

But now instead of having an increase of just under doubling my heat transfer coefficient, I've gone up by a factor of 20.

40:46

So in fact, what we can do is, is the reason we like muscle correlations is that we can put these into something like a spreadsheet, right?

40:54

And when we do that, we can calculate that's these are using three different correlations that we looked at earlier.



41:02

So this this bottom example in the Lambda region, this was using the vessel correlation 1.85.

41:12

And in fact the way I'll write this is that the heat transfer coefficient is  $K$  on  $d$  \* 1 a five times the Reynolds number and diameter on length to the power of 3.

41:33

So I can solve directly for that right times the viscosity, which I'm going to assume is equal to 1.

41:43

So I have no variations for viscosity, but I can solve directly for that if I go into the laminar region here.

41:54

This is now going to use the the cold room's modified analogy.

41:58

We're going to solve directly for  $H$  Sub I is now  $CP$  row velocity times 1.86, the diameter on the length to the power of  $1/3$  to the  $2/3$  and the number of  $2/3$  and and the opacity multiplier.

42:25

And then if I come up into the turbulent region here we use the the the final correlation that we looked at and I'll again resolve that.

42:36

So we have sub I, we now  $K$ , so it's multiplying that across to avoid that the Nelson calculation is a step point  $O$   $2/3$  through the power and viscosity factor that we're going to neglect for this.

42:55

So in each of these cases, what I've done is to look at for a variable water velocity in terms of millimeters a second, what is the convective heat transfer coefficient in watts per meter square Kelvin that I get out of the count these three calculations in series.

43:09

And if I bound them based on their realms number regions that they're appropriate for.

43:15

And So what I can see here, right if here is right below my lower limit, this bottom data point is 4.56.

43:23

So I'm, I'm just above what we would expect for that, that kind of limiting value 3.66, which is good to see.

43:32

And these correlations predict for me.

43:33

Then I have an increase and in fact these two that we're showing here from green and yellow are very tightly mapped to one another.

43:43

So as I transition from a Lambda to a transition flow regime, these agree quite well in what they predict.

43:49

And then all of a sudden when I hit a Reynolds number of 6000, I have a jump up to turbulent flow.

43:56

And that's going to be the effect of now having eddies that are persistent and that are distributing fluid perpendicular to the axis of flow.

44:07

OK.

44:08

So to summarize, we've looked now at 5 muscle correlations and these are really the five that are worth knowing.

44:14

Now I don't think you need to memorize them.

44:16

Certainly, if I ask you to use them on something like that, a closed book examination, I will give you the the muscle correlations, but I think it's really important to know how they work to be very familiar with them.

44:28

And on this week's quiz, you're going to get some practice with them.

44:31

So if we're talking about internal flow, then I can use these correlations here for laminar transition and turbulent flow.

44:44

And each of these ultimately gives me a relationship to the Nusselt number, which is itself the convective heat transfer coefficient, which is the object of my solution on times the diameter on the thermal conductivity.

44:56

If I'm an external flow, I either have natural convection or forced convection.

45:00

And in those cases, here again, I have slightly modified nested number of relationships and we still have that nature of being related to the Reynolds and Prandtl's number, Reynolds and Prandtl number.

45:11

But here we're slightly reformulating how we present them and we introduce one new concept and new dimensionless group is the Grashof number and that's for a natural convection on on pipes or external natural convection.

45:26

And that's simply because we start to have a buoyancy effects that are coming into play as that that air or water is going to hover around the pipe and have a thermal gradient within it.

45:36

Now when I'm done with all of this, what do I actually apply them to?

45:39

Well, if I have some, let's say I have a thick walled tube, so this is going to be a solid and I'm trying to think about then if I have heat that's going to come from the center of the tube, I need to convect it to the wall, conduct it across the wall and convect it away from that outside of the wall.

46:04

Well, to do that, I need to look at the overall heat transfer coefficient calculation.

46:10

And that was really what prompted this at the beginning of this lecture.

46:13

In the 1st place is how do we get to that U?

46:16

Well, the reason we've gone through this and so I'm going to focus on this right hand definition 1st and then we'll talk about the left.

46:22

But if I'm looking at calculating this U I need to be able to calculate the internal and external heat transfer coefficients, all right?

46:32

Now when it comes to thermal conduction through the wall, I need to use the typically for a thick wall too, which I'll just assume I'm going to use the log mean diameter.

46:44

And I also need to use the thermal conductivity of the wall, which we understand now is just a Thermo physical property of a species, right?

46:51

So that's actually quite a simple property to get at.

46:55

Now, in this case, I'm defining this  $U$  with respect to the internal diameter.

46:59

So I can take either an internal or an external diameter as a base and move from there.

47:05

But if I define all this with respect to the internal diameter, then I can see that my internal Fanning factor, my internal convective transfer coefficient, are just sitting on their own.

47:15

And it's only as I move away from that internal diameter that I start to modify and I have to put diameter or area modifications on that.

47:25

I can, conversely, on the left hand side, flip that around and define everything with respect to my outside diameter, in which case I just modify as I start looking at the inside of the pipe.

47:38

Right now, the only factors that we've not talked about how to calculate on their own are the fouling factors, right?

47:46

And in essence, we'll be getting into some of this a bit later in the unit.

47:50

But for now, if you are going to have a fouling factor, it will be something that you're given right to include in the calculations.

47:57

You don't need to worry about calculating those.

## Lecture 09 - Boiling and Condensing

0:00

On the theory of heat exchangers as we start getting into the actual design of heat exchangers next week.

0:05

So this lecture corresponds to chapter the end of chapter 12 and the rest of chapter 13 that we didn't touch on last time in the textbook.

0:15

So I'm going to grab my pen here.

0:18

So this is Chapter 13.

0:25

Now we are not going to discuss Chapter 14, which is on radiation heat transfer simply because we're not going to be working with a lot of unit operations that are radiative in kind.

0:35

But if you're following along, and I would really encourage you to take a few moments in the next week and make sure you've read up through the textbook.

0:41

In particular, the textbook goes down a number of pathways that we're not going to really have time to cover in just one semester's unit.

0:50

But what you'll see if you started reading this is if you're planning to go into the field of chemical engineering and practice and actually design things like boilers and heat exchangers and condensers.

1:01

The book has a lot of guidelines and heuristics and notes that come from very senior and experienced engineers.

1:08

It's a fantastic reference to have on hand.

1:11

OK, so with that, we'll review.

1:14

Now.

1:14

If you watched the lecture from Wednesday, and I'm sorry again that we weren't there live, but that lecture #8 focused on the rest of the Nusselt correlations that we need to know.

1:26

So in particular, we broke them into two halves.

1:30

These are for internal flow, so internal convective heat transfer where we have laminar, a laminar equation.

1:39

In general, we can typically look at 3.66 as the limiting value of a laminar Nusselt number for internal pipe flow, but if we want to calculate it to be slightly more accurate, then we can use this equation here.

1:54

When we get into transition flow, we use an analogy for Colburn.

1:59

Another way that this can be formulated, it goes through in the textbook, and what you might see in other textbooks is called the Colburn J factor.

2:06

I didn't go through the presentation of that because it's kind of an unnecessary complexity at this point when we can simply think of it as a Nusselt number correlation of sorts.

2:15

And of course, if I want to, I can rewrite this, this left hand side to represent a Nusselt number, right?

2:20

I just haven't heard for simplicity to keep it the way it's actually presented, but highlighting that it follows a very similar form of a dependency on both the Reynolds and the Prandtl number.

2:31

And of course, we had introduced this viscosity multiplier that typically doesn't matter if we're dealing with invisible fluids.

2:38

And then what you'll probably use most of the time for internal pipeline flow is the the seat or take correlation.

2:45

The only thing that really separates this from the Adidas bolt bolter correlation, this is the Prandtl number.

2:52

Here, instead of being defined as the Prandtl number here, instead of being defined as either .3 or .4 based on cooling or heating of the fluid respectively, it's taken to the power of  $1/3$  and then tape at this same viscosity multiplier.

3:09

So we can think about if the viscosity of the fluid is going to change significantly, maybe something like a heating oil, then we need to take that into account.

3:18

We also talked then about external convection, so external convection, so correlations for either forced or natural convection across pipes and introduce them as Grashof number where that dimensionless quantity takes into account buoyancy.

3:34

So we have a small pipe, right, that's that has some temperature hot inside and we're passing cold fluid on top of it.

3:42

We know that we're going to have a plot here.

3:45

We're going to have a temperature gradient as we go away from that pipe.

3:52

That's not a very high, high temperature at the pipe surface.

3:54

And as we go away, we'll come invert that.

3:58

But as we come away from the pipe into the bulk fluid, we will have a more equilibrium temperature so that the buoyancy that in a natural convection sense is quite relevant for that type of service.

4:11

And then the point is in every case using a Nestle style correlation, right?

4:15

And This is why I wanted to know early on, have you done Nestle numbers and is this a familiar thing?

4:21

Nestle correlation should be very familiar to every chemical engineer by the time they graduate.

4:26

And the whole point is that we use Nestle correlations to get a hold of  $H$  and we use  $H$  in defining the overall heat transfer coefficients both for the inside and the outside of both pipe systems and heat exchangers.

4:41

OK, so with that, let's kick forward.

4:45

So all of the models we've used to date assume that we're not going to have any phase change, right?

4:53

We have two fluids, one on the inside of the heat exchanger, so say the pipe side, one on the shell side and that we don't have phase change between them.

5:00

But often times right and, and where some of the roots of, of chemical engineering are traced back to in fact, is either boiler making or steam condensing to get fresh water, right.

5:13

So in, in thinking about heat exchanges where phases are changing, we can firstly think of the condensing species.

5:21

They can be either single or multi component.

5:23

And when they are a multi component, what happens is we just neglect the components that are condensing and only consider the partial pressure in the gas or the vapour phase of those components that are condensing.

5:34

So very common species that we'll think about condensing.

5:37

Obviously steam is the most common and we have an example on that we'll do today something like ethylene glycol glycerin, nitrobenzene as heptane and so on and so forth.

5:47

These are the the kind of five most common species that we'll put in to, to try to condense them out.

5:53

Now when we do go to to condense a vapor phase, one of two types of condensation can occur.

6:01

So drop wise is what happens if we have a otherwise dry.

6:06

So imagine if we have a warm vapour and a cold pipe or shelling tubes of cold series of pipes running through it.

6:13

Imagine that those pipes are dry, right?



6:16

So they have otherwise no vapour that have been present on the surface or no no liquid.

6:21

So drop wise condensation is what's going to happen when I have to have an initial nucleation of that liquid on the solid surface and that that initial nucleation site is then going to grow and cascade into a liquid film.

6:37

And a really good analogy to you to understand this is what happens if you take a cold pitcher of so like a pint or a pitcher of beer outside on a really warm summer day where that pitch was initially does not contain a liquid film, right?

6:52

So you'll initially see droplets forming.

6:54

The same analogy holds for a Coke can.

6:56

If you put a can of Coca-Cola outside, you'll see initial droplets of, of dew forming on it and that will eventually form a, a film of liquid.

7:05

Now, much more common than and what we'll typically see in heat exchanges of steady state is film type condensation.

7:11

And what that's going to mean is the wall is wetted.

7:18

So we have a wetted wall and now that film is going to run downward.

7:23

So it's very important that then when we have this film running downward and we're continuing to condense liquid in the system that that condensation process is going to affect the heat transfer profile and the rate.

7:34

So it introduces another limitation to heat transfer and ultimately we bind that up as modifying the heat transfer coefficient.

7:45

Now in calculating this, we'll assume that the heat transfer film is controlled, the action of the film is controlled by gravity and we typically assume that both the central vapour and the wall temperature are constant.

7:58

To be able to do this calculation.

8:00

We also make the assumption that the vapour momentum doesn't affect the film.

8:17

So in many cases this if you imagine a film flowing down a wall, it's going to be probably laminar.

8:23

We're very close to laminar in its flow profile and we assume that the momentum transfer of that vapour is minimal, so it doesn't affect the film's turbulence.

8:32

So one of the, the points we can see is that early on, the localised heat transfer coefficient is going to be inversely dependent on the film thickness, which we'll call little delta in this case.

8:43

And when we think about a tube, right, so we have a large vessel here, we're going to be, you know, condensing vapour on this wall.

8:51

So we start to have a liquid film that's running downward.

8:55

The thickness of that film, little delta will typically be between 101,000 times thinner than the diameter of the tube in which it's condensing.

9:06

So what does the profile look like for heat exchange?

9:09

Well, here and and apologies because the, the textbook gives these most in units of imperial.

9:17

So here we're looking at the film thickness in feet times  $10^{-4}$ .

9:23

So here we're plotting film thickness, sorry film thickness where we're going to start condensing at the top right and we're going to come down and by the very bottom of the profile we'll have about  $3 \times 10^{-4}$  feet.

9:38

So quite a thin film for this vessel.

9:41

And what we can see is that the heat transfer coefficient of that liquid film, right, initially begins at a very high value, right?

9:49

So this first axis or, or first abscissa X axis, is it 2000 BTU per hour Fahrenheit foot squared, but a very high heat transfer coefficient, right?

10:00

Because we don't have a film that's inhibiting that heat transfer.

10:03

Now as we begin the process of forming the film downward, we also see that the heat transfer coefficient is going to drop, right?

10:11

So thereby we can understand the thicker that film is going to be, the more it inhibits heat transfer.

10:17

So if we have laminar flow in a vertically orientated tube, then we can calculate the thickness of the film, little  $\delta$  and this is an average thickness and it's a function.

10:30

So in this case,  $\mu$  sub F, that's the viscosity of our our external or condensing fluid.

10:37

$\rho$  F is obviously the density of the fluid, and this capital  $\Gamma$  here is the mass rate of flow.

10:45

So something like kilograms per second per meter of surface or meter squared of surface area.

10:54

Now we can then bring that definition of thickness.

10:57

If we go back to the previous slide, we can see that the heat convective heat transfer coefficient is a function of that  $\delta$ .

11:03

So if we have a way to describe that average film thickness for the the fluid that we have, we can bring that into a definition of the heat transfer coefficient.

11:11

And here we see for internal or external convection, a generalized formula where

we're, we're depending on the thermal conductivity of the fluid case of  $F$  again, the density, viscosity and that mass rate of flow through the system.

11:31

Now if we're only considering external condensation of pure vapour, right?

11:35

And that's probably going to be a more common scenario is when we'll have something like cold tubes of fluid with a warm steam or warm vapour that's flowing atop.

11:47

Then we can simplify this a bit further and get rid of that mass fluid and and define it specifically on the thermal physical properties of the fluid, which we like, right?

11:56

So .943 times again that thermal conductivity of the fluid density.

12:02

Now this  $\Lambda$ , this is the latent heat of vaporization.

12:15

The textbook also calls it latent heat of compensation, right?

12:17

Which from Thermo we remember is the same thing.

12:21

So it's the latent heat of having to transition the phase itself.

12:24

And that's divided by this  $\Delta T$  naughty, which is the difference between the condensing fluid temperature and the wall temperature  $L$ , which is going to be the total length and surface area and  $\mu F$  again, the viscosity of my condensing phase.

12:40

Now it's important that that when we think about this heat transfer coefficient.

12:44

So here on the bottom plot, we're looking at the slightly modified version of the Nusselt number.

12:49

But again, I'm still directly proportional to the heat transfer coefficient as a function of the Reynolds number, right?

12:57

So effectively looking at how the rate of convective or the resistance to convective

heat transfer changes as a function of relevance over in turbulence when we have this liquid film condensing in the system.

13:09

And what we can see is that if we're in laminar or laminar waves or transition style flow, we have a relatively well, we have a decreasing depend heat transfer coefficient as the turbulence in the system is increased.

13:25

But once we cross that point of turbulent flow, we then see that it becomes much more effective to transfer heat through that liquid film, right.

13:35

And of course this upward trans here independent of the Prandtl number is going to be because we have eddies that are allowing us to cross cut flow and transfer heat through the profile of that system.

13:50

So let's then look at a more an actual example that we're going to do on the next slide of what happens instead of having a vertically oriented tube, it's probably going to be more common than I'll have horizontal tubes, right?

14:04

Because if I'm using a cooling liquid to condense vapour or something like steam, I don't want that cooling liquid in the pump behind it to have to fight gravity, right?

14:13

So I'd rather have horizontal tubes than I'm going to flow steam atop.

14:17

And this is the the design basis then for a shell and two heat exchanger.

14:21

So here I can expand it again on that same equation we looked at in the last slide, we can take one step forward and now define that with an additional term instead of the total length, we can look at the number of twos, right.

14:36

So we can define this directly in terms of Thermo physical properties of the fluid, which we'll put check marks by and three dimensions of the heat exchanger.

14:47

So the the external diameter of each tube, the temperature difference between the tube wall and the temperature of the condensing phase and the number of tubes in the system, right.

14:59

So as long as I know what fluid I'm condensing and the basic geometry of the the

exchanger itself, I can figure out what the resistance to heat transfer is going to look like, right?

15:10

And then I can put that outcome of  $H_{sub N}$  directly into this bottom equation here, where again that total rate of  $H$  transfer then is going to depend only on the convective transfer coefficient or if I'm looking at the internal side as well, overall, the surface area of transfer and the difference in temperature.

15:31

Now we note here that that is written as  $T_{sub SAT}$ , right?

15:35

So that's assuming we have a saturated vapour.

15:38

You might ask yourself what happens if we have a super heated vapor, right, because now we're going to be going beyond the Super heated or the saturation point.

15:46

And in that case we treat this, we actually break the operation in half and think of it as two portions.

15:52

One is AD super heater.

15:55

We're we're not changing phase.

15:57

So we're going to to treat that sensible heat.

16:06

So we in the D super heater portion of of the solution, we would simply say we're going to go from whatever dimension of superheater or degree of superheat we have down to the saturation point.

16:19

And then in the condensation process we can use this type of model to look at the latent heat and where that comes into play.

16:27

So we would then, yeah, with the D super heater, that would basically be a gas heat exchanger, right.

16:36

So using the models we looked at for heat transfer coefficients previously.

16:41

So with that in mind, let's look at an example now where we're going to be looking at turn this tab a little bit, looking at steam condensation.

17:00

And we have a system where we're going to say, well, the steam is coming in at 100°.

17:08

Now in this case, I'm going to say that we're going to use something like cold seawater, right?

17:14

Because we're going to draw up seawater, run it through these pipes and pump it back out to the ocean.

17:19

So it's it's a non destructive circuit and that cold seawater let's assume will come in at 20° C or sorry, 15° C and give me a wall temperature of 20° C.

17:33

And in this system, I'm going to have 10 tubes in the shell running through the shell together with an average length of five meters and an outside diameter of .15 meters right now, because I have saturated steam and I would assume that's 100° C saturated.

17:55

Edit saturation pressure.

17:58

What I can do is go to Appendix E of my thermodynamics textbook SVNAA 8th edition to look at the heat capacity being 1996 joules per kilogram Kelvin.

18:19

The density is going to be .6 kilograms per cubic meter.

18:25

Viscosity is going to be 0.3 centipoise.

18:31

The thermal conductivity of the fluid K will be .02457 watts per meter Kelvin.

18:41

And the latent heat of condensation, which of course Appendix E in the the Thermo textbook, we'll also refer to as the latent heat of vaporization will be 2256.9 joules per gram or kilojoules per kilogram is present.

19:06

OK, so with this in mind, and so all I need to know are these Thermo physical properties which I can get from my thermodynamic solution and some very basic design parameters of the heat exchanger that I'm thinking about.

19:18

Firstly, I don't know if I can change pen colour.

19:22

Oh, there we go.

19:23

So firstly, let's calculate what is the temperature difference.

19:27

Well, I know my delta T naughty is going to be  $80^{\circ}\text{C}$  in this case from the wall.

19:34

Now the area for this system, this is going to be  $\pi$  times the outer diameter, times the length, times the number of tubes.

19:46

So how much surface are I have and in that case I have 23.56 meters squared.

19:54

Now if I want to calculate  $H_7$ , so that's going to be the overall heat transfer coefficient for my system.

20:01

I can go back to that equation on the previous slide, so .729.

20:05

Now I'm going to put in the thermal conductivity.

20:08

So point O 2457 watts per meter Kelvin, and that quantity cubed times the density of the fluids of .6 kilograms per cubic meter.

20:25

And that that quantity is squared times 9.81 meters per second squared times Lambda.

20:32

And now it's important to look at the unit equivalency here in the dimensionality.

20:38

Lambda needs to be in joules per kilogram.

20:40



The way I've written these solutions, I'm using a kilogram basis, but I'm going to expand joules upwards.

20:45

So I need to present this as 225-6900 joules per kilogram, and then I divide all of that by the number of two, so 10 times the temperature difference, 80 Kelvin times the outer diameter, .15 meters times the viscosity of the fluid.

21:07

And again I've presented viscosity to the incentivoids.

21:11

But to be in a kilogram SLI basis, we need to have this as point 000, 3 Pascal seconds all raised to the one quarter power.

21:24

All right.

21:25

So when I do that, I get a valuation of 5.52 watts per meter squared Kelvin, which is a relatively low heat transfer coefficient, right, Relative to what we've seen previously for things like forced conduction, we're talking hundreds of of watts per meter squared Kelvin.

21:46

So this is a relatively low heat transfer.

21:48

And that kind of illustrates to you the impact of having that liquid film condensing on a surface.

21:53

And if I pop this into my Q equation, so what's the rate of heat transfer from my steam, right?

22:00

Again, that would be 8 sub N the surface area of condensation and the driving force that would be made a value of 10.4 kilowatts, right?

22:10

Four ten 104,000 watts.

22:15

So this is just a quick example.

22:18

We'll have another one of these come up on the quiz #3 that will be released at 5:00 today and do in a week.

22:26

And that quiz three is basically meant to give you a chance to practice all of these calculations for heat transfer efficiency.

22:33

You're really comfortable with sort of applying them into different heat exchanger designs.

22:38

So now everything up to to this point today has been about condensation of vapours onto a cold surface.

22:47

But what happens if I want to use a heat exchanger to boil a liquid and bring it into its vapour phase?

22:53

Right.

22:53

So this is obviously evaporation is the very old school name for this steam generation.

23:00

We've just talked about an example of using heat exchangers to drive steam or condensed steam.

23:05

But where this is really going to come up again is distillation, which we're going to cover on lab #3 in about 5 weeks time.

23:12

So the best metaphor that you can use to understand the process physically what happens when I start to boil the liquid is actually, and if you followed along with Thermo, you know, I love metaphors like this is to think about putting a, a big pot of water on the stove, like a metal pot, you know, fill it up with say a litre of water, turn on the gas and physically observe.

23:35

Just stare.

23:35

I know they say a washed pot never boils, but I promise it will.

23:38

But stare at the base of what's happening, the base of that pot as it starts to come up to the boiling.

23:46

And into that boiling process, and you will actually observe the four processes that I'm talking about now today.

23:54

So what happens here?

23:55

We're plotting on the ordinate or Y axis the overall rate of heat transfer, and we're presenting this as a function of the driving force  $\Delta T$ .

24:03

So this is the difference between the the wall temperature and the initial fluid temperature.

24:09

So if we think that fluid is going to begin very cold and we're going to expose it to a hot wall, right?

24:15

And This is why we like things like copper pans because they conduct heat very effectively.

24:20

So they spread it out and we have a very quickly warmed wall.

24:24

But this is effectively my driving force, right?

24:31

I'm at the wall.

24:35

So we're plotting on the the ordinate here, the rate of overall rate of heat transfer.

24:39

And on the same X axis on the right hand side, the ordinate is looking at the overall heat transfer coefficient at the film level.

24:46

So as we go from A to B, this is where the the fluid is quite cold, right?

24:53

I've not put a lot of heat into it.

24:55

And so I basically get natural convection.

25:01

It's the dominant force because I haven't put a lot of heat into that fluid.

25:05

So you'll see that that pot of water kind of move around ever so slightly, but it's not really changing that much.

25:13

Now at point B and from point B to Point C, which is the maximum flux for boiling.

25:19

This is where I'll see we call it nuclear boiling.

25:26

And what I tend to see is very small bubbles that will appear out of thin air, if you will.

25:32

They're very small bubbles that appear right on the base of the pan and then will over time release and come up to the surface.

25:38

And these are bubbles of of water vapour.

25:40

If I'm using water.

25:41

In this case right now, as I increase the driving force as the base of the pan and the base of the film gets warmer relative to the fluid, I will get more and more and more of this nuclear boiling.

25:51

And I can see on the right hand side through this process, I have a monotonic increase in the overall heat transfer coefficient because I still have liquid wetting the base of that pan, right?

26:02

And so as it heats up more and more, I get these massive bubbles of different sizes that will manifest or nuke at the surface and then release upward.

26:13

Now, I then have a phenomenon where, and you'll see this when you boil water as that, if you turn it, if you keep it at a very high temperature, at some point you start to see chaotic boiling, right?

26:24

And this is where you'll have those bubbles form and release.

26:28

And they're quite large relative to those tiny initial bubbles that were forming in the nucleic boiling phase.

26:34

I get these bubbles that release so large and so fast that they almost look like little jets, right?

26:41

So if I go back to that example from a couple of lectures ago showing that the map of the Deepwater Horizon and the physics of what was happening there, I can think of these jets that are almost coming out of the surface of the heating system.

26:54

And, and at that point, because I have a heating surface that's predominantly gas, right, I actually am going to decrease the amount of heat transfer, right?

27:06

And so it's a consequent, I have a decrease in the overall heat transfer coefficient and a decrease in the overall heat transfer itself because at this seated D transition, let me call this as you would transition boiling right here, I have a gas film now.

27:28

So in the same way we have that liquid film that would act as a barrier to heat transfer in a condensation process, we have this gas film as a barrier to our boiling process.

27:41

And the minimum, right, the minimum that we're going to achieve before we continue transferring more heat just because we have such a hot wall relative to the bulk fluid is called the laden frost point.

27:54

And so that's the D on this plot.

27:56

And of course, if we continue to turn that heat up, even though we have a very poor heat transfer coefficient, right, it will continue going down.

28:05

We call this this region here of an unacceptably poor film or poor heat transfer coefficient film boiling.

28:13

That's where we have a pretty much a pure gas film that we're trying to boil through.

28:16

And that gas film is going to act as a resistant to the heat transfer.

28:20

If I continue to turn up the driving force, I can overwhelm that the effect of a poor resistance and continue pushing work heat into the system.

28:30

So typically if you're designing a boiler right, this region here is somewhere between nuclear boiling and the maximum flux for nuclear boiling.

28:39

These are the, the range that we like to operate in as engineers because everything past that, as we can see on this graph, once we hit that maximum flux point, now we tend not to have a very efficient system and, and the physics starts to work against us because of that film process.

28:55

OK, so in general then how do we then apply this?

29:01

Well, if we're looking at the exact from from a few slides ago, we might take water vapour, steam maybe that's been boiled and we're going to try to condense this by using something like cooler seawater as a condensation mechanism.

29:17

And where that might come in handy is if we have only a supply of fresh sea water, right, And we're trying to make fresh water that we can drink.

29:24

This is one of the ways that we can do it.

29:26

We can put it into a boiling unit that we'll talk about on Monday.

29:29

When we boil this up, we can just select a heat or a range of temperature difference going to that boiler such that it's efficient heat transfer.

29:38

We take that pure water vapour since of course the salt is not going to be very significant in the water vapour phase and we can then put it through a shell into a heat exchanger to cool it back down.

29:50

So the general design of these then will follow this this team of approach where ASME is the American API standard for unfired pressure vessels.

30:02

And we'll go into more of these standards as we kick on through the unit.

30:05

Now we've covered this previous design of a heat exchanger in a couple lectures ago as we first started introducing heat exchange on Monday.

30:14

But it's important to note that the way that we drew this at the time, this is called a one to one heat exchanger, right?

30:21

Meaning I have a single passive of fluids coming through the shell side.

30:25

So I put a bulk fluid in, it runs the shell side and back out and I have a single passive of fluids running through my tube side.

30:35

Now what happens if we want to make this, oh, I should mention also Appendix 4 if you're looking along through the textbook, Appendix 4 is going to give a little more information around some of the standards in construction diameters for these kind of heat exchangers, right?

30:51

What happens if I want to make this process more efficient?

30:54

Well, in the first instance I can use I can start going into multi pass exchange so I can use a one to two heat exchanger, right?

31:01

And So what I can envision here is that initially my tube side fluids are going to come in.

31:09

I'm going to try to keep this within the boundaries terribly drawn, right?

31:13

So they come in and the fluids are going to trace through one set of tubes, right, that are about half the thickness of the heat exchanger.

31:20

They're then going to turn around, right and transit through a second passive tubes and come out the exit.

31:28

Meanwhile, if I change colors here, the warm fluid and it's going to come in, you imagine this, this warm fluid is passing around these cubes on the other side.

31:39

The warm fluid is going to come in my inlet and have to traverse all the way around these baffles that are used to break it up.

31:47

So we get good cross through in some heat exchange and then out the bottom.

31:52

Now one of the points that you would note here is that part of this is going to be in Co current and part of it or parallel and part of it is going to be in counter current flow just by the nature that I'm going to take my basis of heat exchange.

32:06

So if I'm coming in from the left to the right, well initially because my warm fluid is going from the left to the right hand side, I will be in parallel flow and then I'm going to turn it around and counter current flow.

32:18

So because of that, one to two heat exchangers tend to be an improvement to the overall efficiency, but they're not the most efficient design that I can use for the same size system and the same number of twos, right?

32:33

Because that parallel flow, as we talked about before, is very uncommon and it's it's not a very efficient means.

32:40

So what I can instead do is look at a two to four heat exchanger.

32:43

So in this case here I'm looking at this is my warm fluid tracing around the baffles here and it goes out the bottom.

32:51

And if I look at my cold fluid and blue coming in now, this is going to trace one of the pipes across, right?

32:59

It's going to turn around and come through a second pipe pass, a third pipe pass and a fourth pipe pass.

33:05

And now, so these two to two, two to four heat exchangers for shell and two designs tend to have a much higher overall heat transfer coefficient at the same flow rate than a one to two heat exchanger, right?

33:18

And I accomplish this by simply putting another horizontal baffle that's going to break up the background heating fluid flow.



33:29

So this here on top, we're going to, this is just a simplified diagram we can draw through.

33:35

But in particular, what I can do is plot the temperature at each of these points in and out of the system as a function of the tube length.

33:43

All right.

33:43

So if I have a one to two heat exchanger, my hot fluid is going to enter at some temperature up here, right?

33:49

It's simply going to cool down because it gets one pass through and that's where we get the one from.

33:55

But the cold fluid on the bottom is going to start at some initial temperature.

33:59

It's going to warm up a little bit and then warm up again.

34:02

All right, So I actually get this introduction of an intermediate temperature in that system.

34:09

Now when we go to A-24 heat exchanger and the reason we call it is that we have here this is my warm fluid entering on the top.

34:18

It initially gets its first pass in the shell and a second pass in the shell and that the bottom half of it before it exits.

34:26

Well, the cold fluid is going to get the benefit of that first pass in the shell in in countercurrent and the benefit of the second pass.

34:35

So I maximize that that approach from both ends in the heat exchanger.

34:40

And what this is going to require and where we're going to get to in the next lecture is this requires us to use a, a correction to the log mean temperature difference.

34:49

And those of you who are already into this phase of, of doing the calculations on the current lab project #1 we'll see.

34:57

This is where this is coming into play, right?

35:01

So to summarise for this lecture, when we go into systems where we're trying to change phase, right, and that's a common practice and a common purpose of why we design heat exchangers.

35:13

Obviously for the refrigeration lab last semester we would have seen that going, you know, beyond that standard carno refrigeration cycle and going into the subcooled and super heated space gives us a much greater efficiency for a similar amount of work going in.

35:28

If I have a phase change and let's assume it's coming from either a subcooled or a super heated position, I can effectively break the process into one of which I initially de super heat the fluid.

35:38

And so I can follow a very basic design of gas on a heat exchanger or a gas heat exchanger for that process followed by what we talked about today is the condensation, the condensation process across this.

35:51

So if I have vertical tubes, I can use this expression here as a direct equation to get the overall heat transfer coefficient.

35:58

And if I have a horizontal tube bundle right where I have N number of tubes in the bundle, then I can use this bottom equation here.

36:09

So to wrap that up then when I'm boiling liquids, it's important to take home that the transfer rates of that boiling process are not linear with the temperature difference.

36:20

But in fact, I have a local maximum right as they get into the realm of nucleic boiling.

36:25

And if you follow along with the lecture and just put a pan of cold water on the stove and watch it boil and physically observe what's happening with these micro bubbles that they start to form, you will see this entire process unfold in about 15 minutes.

## Lecture 10 - Shell and Tube Exchange

0:00

Changing phase.

0:01

So really common example is going to be using a super heated steam right now.

0:07

Might need to cool that super heated steam down or use that as an energy transfer mechanism.

0:11

And so the way that we'll we'll handle that is by breaking the problem in half and thinking about, you know, if we plot that in in super heat space, the amount of energy we're going to remove to take it from a super heated pressure and temperature condition to the saturation vapour.

0:27

And then how much we are going to condense from the saturated vapour side of the saturated liquid.

0:33

So we break that problem in half and we'll treat it as first heat transfer unit as AD super heater and 2nd then is a proper condenser.

0:42

And when we do have those condensers, we saw that the presence of a liquid film that exists on the heat transfer surface will act as an inhibitor to heat transfer.

0:52

So you can imagine that if in some cases that liquid film is flowing in a laminar sense, then we're actually going to have, if we recall back to, to the early bit of heat transfer in fluid mechanics, we're going to have a laminar flow, I'm sorry, a conductive flow of heat through that laminar film.

1:10

So in the context of vertical tubes or cylinders, if I'm condensing a vapor on these, I can use this correlation on top for the heat transfer, a convective heat transfer coefficient through that process, where in this case we're looking at  $\Lambda$ , right?

1:26

And this is the latent heat of condensation.

1:32

And then we saw that if we go to a horizontal tube bundle where we're no longer condensing on a single wall, but we're thinking about a rack of a rack of cooling

tubes where we're going to condense a saturated steam vapour on, we have that same  $\Lambda$  show up.

1:48

And then we simply include here  $N$  is the number of tubes in that bundle, right?

1:54

And the reason why is that if I'm flowing saturated steam vapour from the top of the tubes down to the bottom, I'm going to have a differential effectiveness of heat transfer for each layer of those tubes in the bundle.

2:05

So we use an approximation there together, get at the convective heat transfer coefficient.

2:11

Now we also talked about boiling.

2:13

Now as a quick survey, did anyone actually put a, a pot of cold water on the stove, turn it on and actually watch the boiling process and, and looking at what would happen as we, we plotted the long in the slide.

2:29

OK, So if you haven't already, OK James, what did you see?

2:33

Did it follow each of these mechanisms and and processes that we talked about?

2:48

Yeah.

2:48

Well, so if we if we look at that plug in from the the previous lecture, it was basically looking at if we look at the temperature difference between the surface and then the sorry the convective heat transfer coefficients.

3:04

So how effectively we were able to push that heat into the liquid phase, we saw that there was effectively a peak, right.

3:11

We might have an inflection point where it can turn at the bottom.

3:15

But we have this critical flux point in which as we have more and more discrepancy between the liquid temperature and the surface temperature of the pan where the boiler in this case, we start to get into this region of nuclear boiling, right?

3:28

And that's where we see very small bubbles emerging down here, kind of large bubbles that will start to nucleate on the surface and bubble up.

3:36

Midway point we hit our maximum flux on top.

3:39

And then as we cross over into this region down here, we get a retrograde, retrograde behaviour in the convective  $h$  transfer coefficient.

3:47

And this is film boiling.

3:51

And the major take away from that is that when we get to the point of having such a, a steep temperature difference between the heating surface and the fluid itself, we're actually able to vaporize that first.

4:04

If you imagine that first row of molecules that's sitting on that pans or the boiler surface.

4:10

And so it actually creates a vapour film that itself inhibits the heat transfer process in much the same way the liquid film does in the condensation.

4:19

And so the point to then take on is if we're responding boilers, which we're going to get to later in this week, we use nuclear boiling where we stay at, at or below that maximum flux point.

4:32

OK, I just got a message that there's bad network quality.

4:37

Is anyone having trouble hearing me type of message or something If it's starting to cut out or having trouble hearing.

4:48

So we also then kicked on to basic shell and two heat exchangers and talking about going from a single pass concept where you might have something like, you know, a jacketed right double pipe heat exchanger into a multi pass system.

5:07

So there we go.

5:14

OK, so an example of this.

5:19

OK, cool, give it a minute.

5:28

It's it's on 5G so maybe it's having a bad, bad day with storms and everything in town.

5:33

So let me know if it's getting worse or better as we go and I can back up and repeat it.

5:41

So when we're thinking about multi pass exchange then yeah.

5:48

So, so yeah, the the take home message from this last lecture is we were going through, it's something like this, although the ABC did have a very nice case on five GI think it was Four Corners a week or two ago.

6:03

That hopefully puts a little bit of that to them.

6:06

So the take home message shop in the last 30 seconds is that when we started to design boilers, which we're going to get into later in the week, that we typically design them to use this nucleic boiling point.

6:16

So we're looking at the some of the staying somewhere around the maximum flux.

6:22

We're below that, right.

6:23

But if we increase the surface temperature of the boiler too high, then we get retrograde behaviour and the effectiveness it drops down, we get this film boiling that we don't really want.

6:32

So when we start to think about actual shell into heat exchangers, then here we're looking at a design that's not a double pipe system, but actually a multi path system.

6:42

And if you'll remember, when we first introduced the lot of mean temperature differences, it means to characterize the driving force for heat exchangers.

6:50

We clarified that that multi path systems were in fact one of these areas in which the LMT approach as it was presented, it cannot be used straight away, right?

7:00

And so when we have a multi pass system, we can see here that I'm going to have my cold fluid entering here.

7:07

And this is a simplified right, I have one pass on each side, but the fluid is going to take this pathway through and then enter warm, right?

7:17

Now if I change colours, I can see what's happening in the shell.

7:21

So the shell side, now I'm going to have fluid coming from both of these and it's going to come around this battle, right?

7:37

And then out this warm slide.

7:38

So HD is the hot inlet, they're the hot fluid inlet and this is the hot fluid outlet.

7:51

So when we plot this up now we can see that that hot fluid, if we have a single or a double pass in the warm side is going to enter up here and right.

8:02

And then we have a secondary pass of the shell, so a double sided shell.

8:06

Whereas the OR the cold fluid that's to be heated, if we have 4 passes in the design that's been shown above, we'll have an initial pass and then a third pass and finally it's 4th pass.

8:18

And so the whole point of putting in multi pass exchanges is that the same initial approach, meaning the temperature difference between my inlet and outlet for the warm and cold sides, I can get much more efficient exchange and I can bring that the temperature of the cold fluid now as it exits above the temperature of the warm fluid as it leaves, right?

8:39

So I get more efficient exchange.

8:40

And this is running obviously and countercurrent.

8:43

Now because of that process being multi pass, as you said, LMTD won't work as it was first presented.

8:49

So we introduced this correction factor.

8:51

$F_{sub\ G}$  and  $F_{sub\ GS}$  is defined here, and it's a bit obnoxious, but it's defined as a numerical expression with respect to only two variables that go into it.

9:05

$Zed$  is defined as the temperature difference of the warm side, so inlet minus outlet, and the denominator is the temperature difference of the cold side or the cold fluid in the minus outlet.

9:18

So we're basically looking at the true temperature drop on the true temperature gain for the inlets and outlets of my heat exchanger.

9:25

So that's the definition of  $zed$ .

9:28

Now  $A$  to  $H$ , this is called heating effectiveness.

9:31

So it's the actual amount of heat exchange.

9:34

So the cold fluid outlet by this inlet on top or on the potential heat exchange.

9:41

So the warm fluid in the inlet minus the cold fluid inlet.

9:46

So if I know that the four temperatures that characterize my system, the inlet and outlet temperatures, I can write  $zed$  and  $AH$  and I can pop them into this expression here.

9:57

$F_{sub\ G}$  and this  $F_{sub\ G}$  then should be some value between zero and one, right?

10:06

And we'll go on to the next slide to look at an example of that.



10:08

So this basis was derived originally for 222 passes, but we can actually apply the principle to any number of two passes works out to be about the same.

10:22

So let's look at that in example.

10:26

We'll use red ink here.

10:28

Let's look at doing a calculation example.

10:31

If we have a one to two exchanger width.

10:38

Now the four temperatures TCA, that's my cold fluid inlet at 70° C TCB, that's my cold fluid outlet at 120° C THA.

10:52

So that's my warm fluid coming in at 240° C and THB at 120° C And what we want to do is figure out what's the correct driving force for temperature to be used in this scenario.

11:09

So what I would do is calculate ETA H.

11:13

So that's going to be  $120 - 70$  on  $240 - 70$ .

11:19

And that gives me a value of .294.

11:22

And I can also calculate Z which is going to be  $240 - 120$  on  $120 - 70$ .

11:31

And that gives me a value of 2.4.

11:34

Now as I put this into F sub GI can do a first preliminary calculation because I see in the definition of F, the correction factor F sub G that I have this term  $Z^2 + 1$  quantity to the 1/2.

11:48

So I'm going to go ahead and calculate that in advance just to make it a bit easier for me.

11:53

So  $Z^2 + 1$  to the power of  $1/2$  is going to be  $2.4^2 + 1^{1/2}$  which equals 2.60.

12:07

And then  $F_{\text{sub } G}$  is 2.6 times the natural log  $1 - 0.294$  on one minus then 2.4 times point 02 or sorry .294.

12:28

All of this is divided by 1.4 times the natural log of 2 -, .294, 3.4 - 2.6.

12:41

And very obnoxious isn't it?

12:43

2 - 0.294 and again 3.4, but now plus 2.6.

12:53

And this gives me a value, a correction factor  $F_{\text{sub } G}$  that's .807.

12:59

So what we can see is that the with these four temperatures, my normal.

13:05

So I'll switch colors here.

13:08

My normal log mean temperature, so  $\Delta T_{\text{sub } L \text{ bar}}$  is going to be 120 -, 50 on the natural log of 120 on 50.

13:23

And that gives me 80° C is my log mean temperature driving force.

13:27

And the way that I treat it is now the  $\Delta T$  corrected is going to be  $F_{\text{sub } G}$  times  $\Delta T$  of the log mean.

13:39

So .81 if I round up for point times 80° C carrying C things gives me 65° C is my actual driving force in the system.

13:54

OK, so instead of using a log mean temperature, what we do is we simply use the same expression where we'll have do one final colour just to break it apart and we'll have that hue total is going to be equal to.

14:10

So where's 50 coming from?

14:14

That's the the approaches, sorry.

14:17

So let me do this up here.

14:19

So 120 - , 70 this approach is 50° C and 240 - , 120 this approach is going to be 120° C.

14:37

So I can use then the same, the same definition that my total heat transfer is going to be that that well, you overall, however, we're going to define that the surface area.

14:50

And instead of just having a log mean temperature difference on its own, I can use that same definition, but then I multiply this times that correction factor  $F_{sub\ G}$ .

15:01

So Gus, yeah, it's a great question.

15:03

So this correction factor is applicable to two to four and other variations.

15:07

It was originally designed for as we're looking at it was originally designed for two tube passes, but in fact can be applicable to any other variation in terms of numbers of shell and two passes.

15:22

So the the take home message is that you obviously don't need to memorize, but be very comfortable using  $F_{sub\ G}$  and this is its definition.

15:31

And then we simply drop it into this expression here to correct or to to modify our chemical engineering rate equation, meaning we're modifying our driving force in this case.

15:45

OK, so when I look at this correction factor, then I can also plot this here.

15:50

We're plotting  $F_{sub\ G}$  for one to two, A12 exchanger on top and A2 for exchanger on bottom.

15:56

And we're plotting this as a function of that A to H, which if we back up a couple slides that A to H was going to be my heating effectiveness or the actual heating that

I can conduct on the potential heating or on the possible heating that I can put forward.

16:15

So if I plot the correction factor as a function of heating effectiveness, and here I'm looking at different curves of different  $Z$  values, I can see that I can also, you know, for these different heat exchangers use a table like this to read it.

16:29

Or I can just put that that expression into Excel.

16:34

And that way as I start varying my temperatures, I can use that as a design basis to get a correction.

16:40

Now one of the the important take home messages is that if you get a correction factor below .8, right?

16:47

So in our example, it was .81.

16:49

But it's common that heuristic or rule of thumb that if that correction factor is below .8, then the shell and tube system needs to be redesigned.

16:59

So it's not been done properly.

17:02

So one of the questions then is, you know, if I have this really chaotic flow, if we go back to our little diagram or back to our diagram here, if I have this this kind of chaotic flow of fluids coming in and out.

17:13

Well, we should remember from our introduction to fluid mechanics that I'm probably not going to have fully established flow and I in in most of the tubes and I'm certainly not going to have fully established flow in my slide sections here right in the shell where the flu, the tubes are going to turn around and move through.

17:34

So as a consequence of that, I don't have a purely analytical, a purely analytical way through which I can calculate that shell side heat transfer coefficient, right?

17:47

So in the tubes themselves that the fluid flowing through it, right?

17:51

I can assume that those are usually quite long.

17:53

So it might be something like 10 or 12 feet long for say 1/2 inch tube.

17:58

So I'm definitely going to get an  $L/D$  beyond fifty and have fully established flow profiling, meaning that I can use those kind of correlations of city state to get the internal heat transfer coefficient.

18:09

If I'm worried about the shell side, right?

18:11

Because I have fluid that's that's impacting these tubes, it's not going to be a fully established flow.

18:18

So what we can use is the Donahue equation, right, which you'll see is a again, another kind of modification or approach using a Nusselt number styled approach where here we can see our friend the Prandtl number showing up to the one third power.

18:35

This is obviously our viscosity correction factor we've seen before, and now we're just introducing these two terms on the bottom.

18:43

So instead of having a proper Reynolds number that's come into play, we're looking at the mass flow rate divided by each of the relevant areas in my system.

18:53

So  $S_{sub} B$  is the area in the baffle free window.

18:57

So if I'm using baffles to lock in each of these tubes and to define the flowing area, then my SFB term is defined or or defined by the area fraction occupied by the window.

19:12

So in in the case of something like a 25% baffle, an appendix 4 to the textbook goes through a a number of design cases around this.

19:21

But in a 25% baffle, the SFB term would be .1955, right?

19:28

So about 20, just 2020%.

19:31

So 25% of my area in a 20 in a 25% baffled shell.

19:39

I multiply that then obviously by the assuming a cylindrical shell to give me the the total area.

19:46

And then I subtract from that the number of tubes, right?

19:50

And so B times the outer diameter of each tube, right?

19:53

So then I'm just looking at that cross-sectional area.

19:56

So that's the amount that's going to be free for flow.

20:01

Now on the tube side does the same.

20:08

OK.

20:08

So it's so, so guys, let's actually get back to that question.

20:16

That's a good one.

20:17

But let's get back to that question toward the end of the lecture because we're going to get into to some of that later on.

20:26

So, so we then need to look at South of C, which is if we're looking at the area and South of B of the baffle free window, then what's the area of the tube side for a given pitch.

20:37

So here we're looking at a pitch in terms of P that, so that has a unit of meters on it, the diameter of the shell and then one minus the tube diameter divided by little P here, which is the center to center distance between the tubes.

20:51

So both the pitch and the little P, the center to center distance, these are geometric choices for the individual shell to the exchanger of choice.

21:00

And these then define my two relevant surface areas.

21:04

Those both come into my definition of the mass flow rate.

21:08

And you can see in the context of the typically having something like the, sorry, something like the Reynolds number in here, right?

21:20

I'm going to have a very similar row, row VD or row D on MU.

21:26

It's just modified a little bit in terms of how it's presented, but again to the .6 power.

21:31

And what I'm using this for is to get at the external heat transfer coefficient, OK.

21:39

So the approach that we've taken so far right is typically it's a very kind of old school approach of how we think about this.

21:49

Of course, there is another strategy that that's been looked at in terms of the number of heat transfer units.

21:56

And so this numbers of heat transfer units and how they're they're manipulated is a bit more of a modern approach to heat transfer or any heat transfer design, but it's one way that we can characterize the performance.

22:08

So the more heat transfer units that we have and give an operation, the more efficient or effective that's going to be at doing it's job.

22:15

And so we define the context of the heat transfer unit by the difference between the cold fluid.

22:22

So the fluid I'm attempting to warm up the difference between it's outlet and inlet temperatures on the corrected log mean temperature difference.

22:32

So if I'm using a single pass system, obviously  $F_{sub} G$  goes to one and I don't have to worry about defining it so that the denominator simplifies to just the log in temperature.

22:42

But in a multi pass exchanger or, or if I can't use that strict log mean temperature difference, then here I've written it up in in full.

22:50

So I can use the temperature difference of either string, but I just need to make sure the temperature change I'm looking at is a positive number right when I'm looking at heat transfer units.

23:00

So I can't have a negative number of heat transfer units, even if the if the fluid is decreasing, it's actually an absolute value difference I'm looking for there.

23:08

And we can see here in the plot we're looking at then the heat transfer effectiveness plotted against this number of heat transfer units.

23:14

And so most of my operations are going to be characterized by some, you know, number a positive quantity, say 234 heat transfer units in the system.

23:24

And I can see then if we follow the the curve for fluids of the same mass flow in heat capacity and that's this 1.0 here.

23:33

So  $R_{sub C}$  being these different curves here is the low stream heat capacity or the low capacity stream divided by the high capacity stream.

23:43

So it's a capacity ratio basically between the two fluids of interest.

23:47

So if those fluids are about the same, let's say I'm trying to transfer energy from warm water to cool water, then I should have about the same heat capacity and that will give me a value of one.

23:55

And what I can see is that as I increase the number of heat transfer units, quite the effectiveness of my heat transfer process goes up.

24:04

But what I can also pull out from this is that as I have a greater and greater difference in the capacity between MY2 fluids for the same number of transfer units, right?

24:16



So let's say we draw a vertical line here at, I draw a vertical line at 4 heat transfer units.

24:23

Well, the effectiveness of having a the same peak capacity mass fluid and fluids gives me an 80% effectiveness.

24:33

Whereas as I approach having a very significant difference in that ratio where the high capacity stream is say 100 times greater than the low capacity stream.

24:45

I've been through my manipulation of the mass flow rate or to the choice of fluid for the heat capacity, I can take that same number of heat transfer units and reach a much higher effectiveness, you know, approaching 1.

24:57

So this is one of then the reasons that is if you're an engineer designing this, you may not be able to afford to buy a really big heat exchanger.

25:07

But by selectively choosing whatever fluid you're using for the, the heat transfer process and by controlling its mass flow rate, you're able to kind of work in a bit of space of effectiveness within this.

25:20

OK, so that's a really good question chess.

25:22

So heat transfer unit, I didn't present this quite clearly enough.

25:27

Heat transfer unit is, is effectively it's, it's a different approach to characterising heat transfer.

25:38

And, and there are some textbooks that will go into a number of equations that look at deploying the number of heat transfer units.

25:49

But a unit of you can think of it this way, a unit of heat transfer is kind of like a, it's a quantum of how much heat I'm able to transfer.

26:00

So yes, yes, it is.

26:01

It is dimensionless by definition.

26:04

Another way that I can write this, this definition here based on the absolute value of my temperatures and the log mean difference, I can use that to say that the effectiveness is going to be equal to the number of heat transfer units divided by 1 plus the number of heat transfer units.

26:23

I know it's a, it's a really confusing nomenclature.

26:26

So heat transfer units are a much newer concept log mean temperature difference and the correction factors to it.

26:32

And the way we've been going through some of the early work, that's a much older style.

26:38

But it's important that you know both because certainly if you're going to buy heat exchangers off the shelf, most, most companies are going to characterize them for shell and tube systems through the file presented by McKay.

26:52

So the take home message from this slide, sorry if I've gotten confusing for you.

26:56

The take home message, right, is that if we think about something like the effectiveness of a heat exchange process, right?

27:03

Everything up until now, if we go to these correction factors and we look at our Nestle correlations here, everything up to this point has been looking at how we can characterize the process of heat transfer, but not necessarily how we can optimize it, right?

27:21

So going, we've seen one dimension of heat transfer optimization as we go from A1 to 1 to A12 to A2 for heat exchanger.

27:29

So increasing the number of passes, right, it's going to get me a more efficient heat transfer or effect yet efficient heat transfer process.

27:37

But I can also play with that engineering efficiency if I manipulate the mass flow rate in the system.

27:48

So how quickly my tube side is flowing or how quickly my shell side is flowing or I choose fluids that have an intentionally large heat capacity difference.

27:59

So I'm either maximizing CP difference or I'm choosing wildly different mass flow rates in an attempt, right to push up on these curves.

28:11

So heat transfer units in this context are just a way that I can characterize the system.

28:18

OK, now I'm getting to the earlier point.

28:20

So plate type heat exchangers.

28:23

So if I don't want to use a classic shell and tube, maybe I don't have enough room for that.

28:28

And I want something that's a bit more concise.

28:30

And I have very moderate pressures in temperatures, so I'm not working in extreme conditions.

28:34

What I can use here is a plate, sorry, a plate type heat exchange over the diagram on the bottom showing me I have my cold liquid coming in on on this top tube and out on the bottom.

28:46

And what will happen is I will be transferring that cold liquid on one side of these small, thin, these narrow gaps.

28:58

And if the hot liquid is then going to come in this side, I can see that it will traverse down, right?

29:05

And so I have these alternating spaces where I'm transferring heat between a highly ridged surface.

29:12

So this is an actual photo of what some of those plates will look like.

29:18

So they have a lot of surface very intentionally.

29:20

So I get really good heat transfer across them.

29:24

So I typically then will want to maintain a relatively low liquid or sorry, low Reynolds number in this case because I don't want a lot of turbulence in that system.

29:34

So that will tend to be because I have very, very narrow gaps, usually on the order of millimeters.

29:39

I'm going to have liquid velocities on the order of .2 to one meters per second that are flowing through these very thin gaps.

29:45

And that's going to get me around number below 2100.

29:48

And so in that context, I can then use a very classical Nusselt number correlation here.

29:53

The pre factor is .37.

29:56

Again, Reynolds to the  $2/3$  times  $\mu$  to the one third.

29:59

I'm seeing the same nature of these Nusselt style correlations coming up over and over and over.

30:06

Now  $H$  I'm going to base this, this  $H$  definition, we use the nominal area of the corrugated plate, right?

30:14

So that corrugated plate example is shown here and the  $D$  effective.

30:19

So the effective diameter that I'm going to use will either be four times the hydraulic radius or in the case of a plate, a plate type heat exchange will be twice, two times the plate spacing.

30:32

That's so nice.

30:33

Plates are say 3 millimetres apart, then that  $D$  effective will be 6 millimetres.

30:39

Now the reason that we really like these is that I can achieve right without having to optimize my process significantly.

30:45

I can achieve a performance that's almost perfectly counterflow.

30:50

So my correction factors need will approach Unity notice.

30:58

That's a good point.

30:58

So those those are that's absolutely right.

31:02

The corrugations if you zoom in on the slides when you download them, I don't think I can zoom in here now, but the corrugations that you can see in the photo here, these absolutely create turbulence.

31:13

But what we can remember is that when we're in a transition or nearly laminar transition flow regime, when we create turbulent eddies, they die out relatively quickly.

31:24

So what it means is that if I have a narrow gap between the two plates, the turbulence is going to be localized to the heat transfer surface as opposed to burning turbulence through the fluid, right?

31:35

So if I have a high Reynolds number, then that turbulence, every time the plate angle or corrugation is going to create a small Eddy, it will propagate forward into flow and it will remain stable.

31:46

Whereas if I'm approaching that transition flow, yeah, very localized turbulence, that's exactly what I guess.

31:52

So I'm going to transition flow regime as I create that turbulent Eddy.

31:56

It's going to die out after some time.

31:58

And so these allow that the localization of turbulence to the plate and where you'll find these very common links in the dairy industry and food processing industry, right.

32:07

So anyone planning to go to work in New Zealand definitely will come across this if you're working for the dairy industry in New Zealand for heat transfer.

32:16

OK, so, so one of the final points then we've seen 2 two points today of how we can start to optimize the effectiveness and the efficiency of our heat transfer process.

32:26

One in a shell and tube system is going to be increasing the number of tube passes and shell passes so I get a more and more effective heat transfer and counter flow.

32:36

But the other approach has been selectively or looking at the selection of my heat capacities and the manipulation of those mass flow rates, right?

32:47

So I get a higher capacity ratio, which of the same number of transfer units gets me in higher efficiency.

32:53

Now one additional mechanism that I can use to control and and propel my, my heat transfer forward is the use of extended surface equipment or fins.

33:01

Right now I'm sure everyone is is sort of fins in mass and heat transfer as a background.

33:07

So we're not going to go too far into the the basics for fins except to do a few brief definitions here and then we'll kick in in the next lecture to look into some examples with them.

33:18

So if I have very, very large deviations in my internal and external heat transfer coefficients, right, then I typically that's the scenario which I want to start using or thinking about fins.

33:32

And in general, I can look at two different types, right, either longitudinal fins, if I have flow that's right around that fin.

33:41

So the longitudinal fin means the direction of the fin is aligned with the axis of flow.

33:50

Whereas transverse fins, right, are ones that we will have flow coming right across this same point, but the flow is going to ripple as it comes across.

34:02

So if I recall looking at the, the, if I neglect the impact of conduction on this system, I look, I can look at a definition for the overall heat transfer coefficient here, which includes a contribution from the internal side.

34:18

And then of course the external side where I'm simply looking at the ratio of the two areas, not assuming it's a cylinder, right.

34:24

But I'm just going to look at the ratio of those two areas.

34:27

Now, if I want to add fins to the system to increase the rate of heat transfer, I can characterize the fin through basically three properties, right?

34:36

One is going to be  $T_{\text{sub } F}$ , right?

34:39

Where  $T_F$ , right?

34:41

As we're showing here,  $T_F$  is the temperature of the fin, right?

34:46

And what will happen is, let's say my, my fluid temperature is sitting at something like  $120^\circ\text{C}$  my wall temperature, right?

34:53

So the outside of the fin or the outside of the wall, and what's important, the very base of the fin is sitting at  $100^\circ\text{C}$ .

35:02

Then what I'm going to do is have a temperature gradient in this fin where it's going to warm up as it gets toward the outside of the fin, meaning the base of the fin starts at  $100^\circ\text{C}$  and it will increase in temperature as I go out in the fin.

35:18

So I have this temperature gradient I need to think about.

35:21

And the way that we'll characterize that is by looking at the difference between the average temperature distributed through the fin relative to the bulk fluid temperature.

35:32

So that's  $\Delta T_{\text{sub } F}$  and we have the over bar to indicate it's a spatial average divided by the temperature difference between the flowing fluid and the wall temperature, right?

35:42

$T_{\text{sub } W}$ .

35:44

So this gives me  $\eta_{\text{sub } F}$  which is my fin efficiency and then I can deploy that directly right in my revised definition for an overall heat transfer coefficient.

35:55

Right following from above, I have the same  $h$  in  $h_{\text{sub } N}$  on the bottom and then my outer convective heat transfer coefficient.

36:05

Instead of having a external I now think about  $A_{\text{sub } F}$  times the area of the fins of the surface area occupied by the fin plus the area of the pipe itself and this here is the area of the fin.

36:31

Now one of the oops give me to over.

36:36

So one of the points that I can look at is if I know that the height of the fin  $X_{\text{sub } F}$ , I can also define here a sub little a sub  $F$ .

36:46

And if I multiply these two together, I get a dimensionless quantity and I can plot something like the fin effectiveness against these two.

36:54

So I'm looking at as the fin gets longer or the properties of the fin change that I'm going to get a decreased fin effectiveness.

37:03

So I put this in here.

37:04



This comes from the book just to look at one example of how these properties work together.

37:09

But the idea is that if I have really short fins, right, the temperature difference between the fluid and the fin is going to be approaching one, right?

37:18

And that's all going to be about the wall temperature.

37:20

But as I start to get to very long fins or I have either a very high heat transfer coefficient.

37:28

In this case  $L$  is the thin perimeter,  $S$  is the cross-sectional area and  $K$  is the thermal conductivity of the thin material.

37:37

Then is is that value of little  $a$  sub  $S$  starts to increase.

37:41

Either that or the thin height will start to give me a, a very, very large temperature difference.

37:48

So the point being, we're not going to go through any examples yet today.

37:52

I just wanted to get everyone onto the same page of remembering this concept and kind of digging back into your, your thermal or your heat transfer knowledge as we start approaching this week.

38:03

But then effectively as we then wrap that up, there are three kind of processes that we've seen so far as to how we can manipulate our shell and tube heat transfer processes to become more effective, right?

38:18

So one of them at our first method for for modifying that will be increasing the number of passes.

38:32

The second method would be in choosing either  $\dot{M}$  and or the heat capacity.

38:46

So I maximize the ratio of the product of my mass flow rate and heat capacity between the two strings.

38:52

And the third method is extending the heat transfer service and then it can do something like use fins to accomplish that.

39:04

So in these cases, if I'm increasing the number of passes, then my my typical log mean temperature difference method fails.

39:11

And the equation that we looked at well, so it was drive for a 12 exchanger.

39:15

I can actually apply to any number of of modified exchanges and it will still give me a reasonable value.

39:22

So this this value simply then props in that I have  $Q$  total EU of raw a  $\Delta T$  log mean through a normal method now times the correction factor  $F$  sub  $G$  So it just modifies my temperature driving force in this.

39:39

When I do have a challenge in heat exchanger, then I'm going to use the Donahue equation.

39:43

This will give me a characterization of that outer heat transfer coefficient, bearing in mind of course that the variation or the inaccuracy in estimating this comes from not having fully established flow in that process.

39:57

And so I can see here I have a modified way of presenting a Reynolds number specific to that system, my Prandelt number to the one third, and my viscosity multiplier if I'm dealing with either something like a heavy oil or really dense gas.

40:11

And then when I start to use fins, these typically come into play only when I'm going to have a, a severe difference in the heat transfer coefficients.

40:19

But if I do, then I can look at, I can characterize the fin both on its surface area as well as its effectiveness so that that ratio of the temperatures across the fin, OK.

## Lecture 11 - Condensers and Evaporators

0:00

A combination of wrapping up Chapter 15 and an intro to Chapter 16.

0:10

So if you've followed along in the textbook, and I hope that you all have, Chapter 16 is the last chapter in the heat transfer section of McCain.

0:21

So after this, we're going to start kicking into mass transfer.

0:23

We're actually going to go a lot slower through the mass transfer text than we have through heat transfer just because obviously you've seen some of these operations to a limited extent in previous classes.

0:36

So if you haven't had a chance or haven't had time yet, I would strongly recommend in particular when we wrap up this week, you know, we're going to start mass transport on Monday and that's going to be a, a one month.

0:47

So four weeks of mass transport and different applications.

0:51

I would strongly recommend, you know, taking some time over the weekend and just making sure you're caught up with the reading so that we'd get kicked into Chapter 17 next week that we're on the same page with reading.

1:04

Because I, I can promise you, if you've not already appreciated this from Ritong's lectures, mass transport is a level up in terms of difficulty.

1:14

When we compare it with heat transport, there's a lot more nuance to the phenomenon because we have to account, of course, for that concentration gradient and that that differential Gibbs energy or molar Gibbs energy that's going to come into play.

1:27

So heat transport is a bit easier because we're only thinking about the flow of  $DT$ , right?

1:31

Which is that's, you know, one domain of Gibbs energy.

1:33

Mass transport's harder because now we're talking about two or it's sometimes 3 domains of Gibbs energy.

1:39

So a, a quick reminder.

1:42

So quiz #3 is due Friday at 5:00 PM.

1:52

So if just as a note, if you do submit your quizzes late, LMS will not automatically release the marks to you because it requires me to go in and look at both how you did in the quiz and then manually assign the deduction per university policy, which is 5% per day up to seven days and then a zero.

2:14

So if you submit your quiz a few minutes late, everyone else gets their marks back and yours is still showing up as nothing.

2:19

Don't panic that you might get a zero.

2:21

That's not the case necessarily, but just that it requires my intervention before the marks are released.

2:28

OK, so in lecture 10 we talked about the log mean temperature difference approach and how that will be used or how we can correct that when we go from a single pass to a multi pass heat exchanger.

2:41

And we introduced for that purpose this correction factor  $F_{sub\ G}$  which varies between zero and one.

2:48

And I think it bears worth repeating here when we talk about log mean diameters or log mean temperature differences, right?

2:58

So LMD and LMTD.

3:06

So log mean diameters and log mean temperature differences are relevant to thick walled pipes, right?

3:20

We looked at that one really critical plot a few lectures ago where we were comparing the difference between a log mean diameter and an arithmetic mean diameter to the ratio of the external to internal pipe radius, right?

3:35

And looking at what is the impact of that.

3:36

And what we see is that we need to have an external on internal radius in excess of about 1.5 before that that log mean approach starts to have a measurable impact, meaning more than 5% of the overall solution, right.

3:52

So just to note, you know, as you're carrying forward in your careers, log mean temperature and then the correction factor log mean temperature can get, you know, very cumbersome and certainly can turn out more accurate solutions.

4:03

But it's relevant to when we have a thick walled pipe right now.

4:07

When we apply this then to shell and tube heat exchangers, one of the points that we made is, you know because of our early backgrounding in fluid mechanics, we can understand there is a limited accuracy in understanding what the convective heat transfer coefficient of the shell side is going to be.

4:23

And that's because we're not going to have fully established flow because all of these pipes are going to be interrupting our flow patterns.

4:30

So the Donahue equation which is shown here, this is our our Reynolds components.

4:36

It's a modification of how we present Reynolds, but we're looking at a mass for areas so that the units work out.

4:41

So it's a modified Reynolds to the .6 or modified Reynolds definition to the .6, obviously our older than the Prandtl number to the .3.

4:49

And then our viscosity multiplier.

4:52

If we're dealing with a heavy oil or a highly a high pressure gas phase, this can then be used to estimate the overall heat transfer COVOD on the external side of the heat exchanger.

5:06

And one of the the concepts we introduced in that lecture was plate type heat exchangers.

5:10

And this is an alternative to classical shell into now, I'm not really going to go into a lot of depth in in plate type heat exchangers here simply because I think we Tom covers that in a good amount of detail in the heat and mass transfer.

5:24

But if you feel that that you want more examples and I can add some in online as extra work, let me know.

5:30

Not extra work, but extra material.

5:32

Let me know if you don't feel comfortable after the heat and mass unit.

5:37

So, so additionally, we talked about fins being used to increase heat transfer coefficient or the heat transfer coefficient.

5:44

Now I do want to well, we're in the class.

5:48

I do want to put up one quick here we go, one quick form.

6:16

OK, So I just added a form to the chat window because I'm always interested in how your experience in heat and mass transfer will inform your ability to do the unit operation calculation.

6:31

So I haven't put together specific examples of using fins.

6:35

So I'd like to know how comfortable you are on a a Likert scale.

6:40

Don't, don't use the shock face.

6:42

Just just more of the question of how comfortable you are.

6:45

So if I need to to add in some ancillary content over the weekend or next week, I can do that.

6:54

OK, so it looks like we're kind of flirting around the margin here, somewhat uncomfortable.

7:00

I'll wait to see how many responses we get together, but maybe it would be helpful for us to look at an example and I'll put that together and maybe do that offline.

7:09

So when we start talking about fins, then we basically consider fins have two, well, one modification and that is to modify our overall heat transfer coefficient.

7:21

That's the entire goal of adding a fin.

7:23

Now the way that we do that, if our normal expression for  $U_{sub I}$  is going to be one on  $H_{sub I}$  or one-on-one on  $H_{sub I}$  plus the internal area on the external area times the external heat transfer coefficient.

7:38

We modify this because the external area has now been modified by the fin right?

7:44

And in this case, a sub B is the pipe area and a sub F is the fin area where we modify that fin area because we assume that we know that fin is not going to be 100% efficient, right?

8:00

There's going to be a thermal gradient.

8:02

So the very tip of the fin which will be in equilibrium with the transfer fluid is not going to be as effective as a midpoint in the fin where we can both conduct and conduct.

8:12

And so we modify that area by this efficiency term, a sub F, which is just a function of the average fin temperature relative to my other two critical temperatures in the system, this being the fluid temperature and the wall temperature.

8:29

So these are the two characterization points for fins.

8:32

I'll put together an example later on so we can make sure we revise that.

8:35

And the other really in critical point that we talked about last time were the three basic methods.

8:41

If we're talking about using a heat exchanger, we're not necessarily changing phase.

8:48

We reviewed what were the three methods we've learned about to if we're engineers, how can we improve or manipulate the efficiency of that heat transfer process?

8:58

The first was in the use of multi pass exchangers.

9:01

So going from that single pass to multi pass, the second was the introduction of fins.

9:06

We're playing with surface area now.

9:08

And the 3rd and, and this is quite critical, the third is operational.

9:12

So it's the choice of the mass flow rate of my fluids on both sides and the heat capacity of those fluids.

9:21

And the reason that I'll highlight that Third Point as an operational point is that many times you as engineers might be walking into an industry or or a factory or an operation that's already the bend designed as an operation, right?

9:34

So you're not going to get to to choose whether the exchangers multi pass or single pass because that's already been decided as probably have whether their fins exist and what dimensionality they are.

9:44

So the one point of control that you have over an existing process is how it's treated operationally.

9:51

And that's where you can modify the mass flow rates to control those ratios or change out the fluids to look at a more advantageous heat capacity, right.

10:01

And again looking at maximizing the ratio of the mass flow and heat capacity for both the internal and external streams.

10:08



And we saw that the further that those are hard, the more efficient our heat transfer becomes.

10:14

So in in that context, that's where laboratory two starts delving into with Hisense, right?

10:21

Because you wouldn't necessarily, if you're a practicing engineer, sit down and do these calculations on the back of an envelope, right?

10:27

You'd probably map them out in Excel, but then definitely go to a simulation tool so you can get the most accurate result.

10:34

You can't possible.

10:35

And and that's where we want you to start looking at mass flow rates and heat capacities because that's the operational value of the engineer.

10:43

OK.

10:44

So let's let's kick forward from that.

10:46

And, and here we're going to talk today about, I want to touch on one final point about traditional heat exchange.

10:52

And then we're going to get kicked into evaporators and condensers.

10:56

So one of the the more common in really growing fields or growing domains of heat exchange is the use of air cooled heat exchangers, right?

11:06

And so because we don't necessarily want to use cooling water to the same extent that we have, you know, there's a good example, if you pull cooling water out of the river, you're then going to heat that cooling water back up and inject it back into the river.

11:20

So it's an open wind process.

11:22

We certainly don't want to consume the water or dump it into dry land.

11:27

And that's a very common thing to have been done with nuclear plants through Europe and the US, right?

11:32

Well, there's an environmental concern when you do that because the aquatic species that are that have evolved in that water or ecosystem are used to a certain temperature of water.

11:46

So if you start dialing up the temperature by dumping warm fluid into it constantly, it will affect the ecology and in some cases can be quite detrimental to the environment.

11:58

So for that reason, we've tried to start limiting the amount of cooling water we have.

12:03

And a really easy way to do that is to replace water by thinking about air, right?

12:08

We have quite a bit of air.

12:09

So it's a common, a common method that can be used.

12:13

Now when we talk about air cooled heat exchangers, we're going to expect horizontal bundles, right?

12:19

So we've talked about heat exchangers in either vertical or horizontal bundle design and they'll typically of course have thin tubes because air is not going to be as effective as a heat transfer medium as water will be.

12:31

So we need to do everything we can multi pass thin tubes, you name it, right, that will be carrying warm process fluids where air will be flowing atop them.

12:40

Now in the air heat exchanges we would typically expect a maximum tube diameter and again the tubes have the will be carrying the process fluid of interest of about one inch or 25 millimetres and they can be anywhere from 2:00 to 9:00 metres long for industrial process.

12:58

Obviously that you know when we look at it on our our refrigerator at home it's much smaller but at the industrial length scale it can get up to about 9 metres in length.

13:07

We would use transverse spins in this case.

13:10

So meaning if we have our pipe here and the fins, the fins are circling the outside, then the direction of flow is coming across them so that the direction of flow is orientated toward the surface area of the fin and then that that greatly increases our heat transfer.

13:27

So in a configuration like this of the one inch pipe, say 9 meters long, we can get upwards of 500 square meters of surface area.

13:36

And that allows us to cool process fluids in that configuration of up to about four 100° C.

13:41

So it can be quite effective.

13:42

They're large units, but they can be quite effective now because these are horizontal bundles and they're fan forced external convection right where air will be pushed between 3:00 and 6:00 meters a second.

13:56

We need to to introduce a slightly new well, we need to introduce a new Nestle correlation for this right?

14:02

But that that shouldn't be particularly concerning at this point.

14:06

So the external heat transfer coefficient  $h$  naughty right will be the  $Co$  group.

14:12

The pre factor here is .287.

14:14

I see my old friend, the Reynolds number showing up to the .61, the PRAM to the .33.

14:19

And it's worth noting that this definition of the Reynolds number, this is the air velocity, right?

14:29

And then we multiply this now by this factor  $F_{sub A}$  and  $F_{sub A}$  depends on how we're going to arrange that horizontal tube bundle, right?

14:38

So the table here that's from the textbook looks at the ratio of  $P$ , which is the spacing between my individual pipes on the diameter of the pipes.

14:48

So if I have a factor of two, it means that if I have a one inch pipe, there's space two inches apart, right?

14:56

And what I can see is for different Reynolds numbers, these are external Reynolds numbers.

15:04

For different external Reynolds numbers at the limiting case, right, I'm going to see a factor of about 1.

15:11

So it's not a huge modification to the system.

15:14

But if I start to have right the the other limiting boundary of very tightly packed tubes at a very almost well laminar air flow across them, then I'm going to throttle that heat efficiency a bit and it's going to take about 15% off the top.

15:27

But all of that to say, this  $F_A$  factor is not terribly critical, right?

15:31

It's it's going to play a relatively minor role in the overall solution.

15:35

So this nuzzle correlation as we apply it is both for air cooled heat exchangers and more generally for cross flow heat exchangers where I have if air swing in One Direction and the tube orientation is perpendicular to that.

15:51

OK, now this has all been up to this point about what's happening if I'm not going to have a phase change in my fluids.

15:59

But we've talked about a few lectures ago, what happens if I start to have phase

changes in either my, my heating fluid being a condenser or my, my, well, the fluid that's initially cold, which would be a vaporizer.

16:14

So if we think about condensing for a minute, right, we would have seen this left hand picture a few lectures ago.

16:21

And the idea of condensing is that we can use a number of processes.

16:25

In this case we'll look at a shelling tube system and if we're clever about it then we can use a coolant to remove the latent heat and allow that vapor to cross it's phase boundaries so we can liquefy it.

16:39

Now in the same way that we can design A non condensing or vaporizing shell and tube heat exchanger in a single or multi pass configuration, we can do the same thing when we apply our shell and tube style of heat exchange to condensing systems.

16:54

The one major difference here is that when we're condensing, we tend to have an incredibly large thermal gradient across the individual pipes, right.

17:04

And that's because they're going to go through that latent well, they might go through some super cooling or or super heating removal, which says de super heating then crossing their latent heat.

17:14

So in a period where we're transitioning in phase, we'll obviously be at at constant temperature.

17:19

We remember that from thermal and then we will continue potentially cooling down beyond that.

17:24

And so because I can have this incredible thermal gradient across the individual pipes, when I start getting to multi pass configurations, I will tend to design them with one of the heads.

17:35

So the reference or the wall to which the tubes are secured, I will allow that to float right, meaning it can move up and down.

17:45

And so this way I don't have to worry if an individual tube starts expanding or

contracting, we would call differential thermal expansion of the material that itself then I don't have to worry that that individual tube might break away from the wall or rupture, right?

18:00

So I allow one of these heads to float up and down and that gives me a little more flexibility.

18:05

So I typically won't use a single pass configuration when I'm condensing simply because it's, it's extremely uneconomical, right?

18:12

I have a very low heat transfer coefficient in a very large size.

18:16

And we'll remember from thermodynamics that the latent heat of phase transition, right, is usually in order of magnitude larger than is the sensible heat required for a typical industrial process temperature change.

18:32

So because I'm dealing with an incredibly high amount of heat flow that's required, I need to go to my most efficient design and that's going to be looking at a multi pass exchanger, you know, where I'm cleverly selecting the ratios of heat capacities and mass flow rates.

18:48

Now I can use that same cross flow Nusselt correlation, but I don't need that factor of  $F_{sub\ G}$  when I'm in a multi pass exchange that we looked at in the last slide or sorry, the  $F_{sub\ G}$  for the LMTD.

19:08

OK.

19:08

And that's simply because I'm not actually taking into account a very large overall temperature difference because most of my energy transfer is going to latent heats.

19:21

OK, so this this thing kind of begs the question, well, if I'm trying to condense something, let's say we're going to condense water vapor, then, you know, where would I have gotten that water vapor to begin with?

19:34

And so this is where we start to think about the combination of both reboilers or we call vaporizers and condensers working in tandem, right, in the same large industrial process.

19:47

So if I'm trying to generate steam that I can then put into that condenser, well, we're going to look at a few ways today that we can make vapors, right?

19:57

And the first will be a kettle type reboiler.

20:01

So this is the, the basis of operation here is that we will have a small, small tube bundle where I have two passes in the tube, right?

20:11

So I'll have my liquid feed coming in here.

20:16

So my, sorry, my steam inlet here will pass through the tubes, turn around and will condense and exit on the bottom.

20:25

We'll switch colours here.

20:28

I will then have so, so my steam is my heating fluid.

20:32

So I might have excess steam from a unit operation.

20:35

I might have generated it independently.

20:37

Now, if I'm trying to then transfer that to a different liquid, right?

20:42

So I can bring that liquid, say water to its boiling point, I will inject cold liquid from the, the bottom here.

20:51

And that cold liquid is going to form a cool that encapsulates the tube bundle, right?

21:00

And it's very critical.

21:01

I'll do one more color.

21:07

It's very critical to mark where this Weir is.

21:11

So this Weir gives me a physical block where the, the, the height of that liquid pool of water that I'm trying to heat up cannot pass that because if it does, it's going to spill over and through this hot liquid outlet on the bottom.

21:30

Now the the liquid that's occupying that pool.

21:33

So I have a whole bunch of these say copper tubes that are containing steam, right?

21:37

And that steam will be outputted as probably a partially saturated vapour.

21:43

So I won't be fully condensing the steam, but I'm going to be effectively submerging this liquid or submerging that tube bundle in the liquid, which means that it's steady state, it's going to bring the liquid up to the same temperature of the steam.

21:58

So I'm using the example of of water and steam, then it's going to bring the liquid up to the boiling point of water.

22:04

So this is commonly considered referred to then as a boiling cool, right?

22:15

And which is a bit counter intuitive because my boiling pool is actually my my fluid that I'm trying to heat up.

22:21

But as I'm adding more and more liquid to this, then I'm evaporating that and it will matriculate that to the vapor outlet.

22:29

And then I now have a generation of steam process.

22:31

So this is a typical kettle reboiler.

22:34

Now I wanted to also highlight we had talked earlier when we first started getting into how we make steam right?

22:41

Because steam is critical to our compensation process.

22:45

We looked at the heat flux for a single right for a single either tube or or even a a



single volume of interest as a function of the temperature difference applied at the surface.

23:00

And this is where that that suggestion of taking a look at boiling a a pot of water on the stove came from that you can see some of these different phenomena occurs.

23:08

So we looked at, you know, through this initial.

23:10

This will be our nucleate boiling obviously our critical heat flux on top and this of course was our laden frost point where after we start to get film boiling.

23:26

Now when we go into to thinking about a kettle reboiler it actually there there are two important differences here.

23:34

So as the data from The Cave is showing us, when I'm in the nuclear boiling region, right, So relatively low temperature differences between the surface of my steam tubes and the exterior of the fluid then at a constant temperature.

23:54

So let's say my temperature difference is, is going to be something like 20 Fahrenheit, right?

24:01

Well, if I have a 20 Fahrenheit temperature difference, then I'm going to have a much higher heat flux in a multi tube bundle for the kettle reboiler than I will for a single tube or a pot of water on the stove, right?

24:17

So for this reason, This is why kettle reboilers are designed with A2 pass tube bundle.

24:22

These baffles the spillover effect, right?

24:24

The design of this is based on how we can maximize for a very practical region of boiling space, how we can maximize the heat flux delivered to our liquid, thereby maximizing the efficiency.

24:37

But if I go to very high temperatures, so let's say I'm not using normal steam coming through these copper tubes, I'm going to use super heated steam, right?

24:46

When I get to very high temperatures, then my difference might be 200 Fahrenheit now.

24:53

And that's the difference between the copper surface again and the liquid in the boiling pool.

24:59

So that's going to mean I, because I get a shift in my overall heat flux spur to the left, we'll have that same condition.

25:06

Now I actually have a much lower overall heat flux for the tube bundle that I'm going to do for a single tube or for a pot of water on the stove, right.

25:17

So because I'm shifting the curve to the left, I enter the film boiling region for a kettle reboiler at a lower temperature difference.

25:26

So that's it's an important point that we when we talked about the theory behind nucleic boiling, the critical flux, the laden frost point and film boiling.

25:35

That's why I spent so much time around that.

25:36

Is that when, when you start getting into boiling processes, it will the, the configuration of these systems will shift this curve to the left or to the right depending on how we design the system.

25:48

And so it's very critical to inform for the engineer where can I operate with maximal efficiency without getting into a region where I'm unacceptable, unacceptably throttling my own process, right.

26:01

And we'll remember that that with film boiling the phenomenon of interest, right?

26:05

If this is my heating surface, then if I will have actually a thin film of gas followed by liquid on top of that.

26:17

So I'm, I'm vaporizing at that heating surface so quickly that I have a constant film of gas before it can transfer that heat into the liquid, which means that I start to have a, a series of heat transfer resistances to think about instead of just one.

26:34

OK, so that the kettle type reboiler is one way in which I can generate steam and as we saw on the slide before steam is critical, right?

26:44

If I'm going to use this as a condensation.

26:48

So the combination of a kettle reboiler and a multi pass A2 pass condenser is one way that I could take salt water and transform it into fresh water at large scale with a continuous flow rate, right, without having to do it as a batch distillation process.

27:07

I can put two of these unit operations together and all of a sudden I can produce 100 kilograms an hour of fresh drinkable pure water.

27:17

So that's one of the points I'll highlight here is that the combination of these two and we present them together is where we'll often see them.

27:24

Now it also occurs when thinking about the condensation process that I'm, if I apply it specifically to the context of water vapour and air or water in in some other fluid of interest like natural gas, then I'm going to have to think about a mixture, right?

27:43

So for the operating conditions of my condenser, I might have a mixture of both that a water vapour that I can easily condense and non condensable gases.

27:51

Now just to clarify, when we're saying non condensable, that means at, right?

27:58

So this is at the process PT conditions, right?

28:05

Because we do know that we can condense nitrogen, we get liquid nitrogen, we can generate liquid oxygen, we can do liquefied natural gas.

28:13

So we can condense most of the gases that we want to.

28:17

But at our process condition conditions of interest, it's only the water vapor that will be condensable.

28:25

So if I'm trying to strip water vapor out of air, right, And if anyone has you know, gone to Target or what not and purchased a dehumidifier, this is a good example of how this process works.

28:38

What I need to think about then is, is how I can selectively strip out that that water, condense it and then have the vapour flowing away.

28:47

So this is a typical dehumidifier and it's based around a vertically oriented tube bundle where my vapor inlet, right, will come through the top.

28:58

So this is my combination of water vapor, nitrogen, oxygen.

29:02

It will flow through these tubes and I will use a coolant.

29:09

So in this case, I'll typically use water, but the coolant will flow on the shell side, right?

29:15

So we'll come in through the bottom and it will flow out on top.

29:20

So I'm taking this cooling fluid and if I have vapour at say 6070°C that's containing water, then I'll use or water vapour that I'm trying to remove.

29:31

Then I'll use liquid water at say 15 or 20°C flowing up through the shell side.

29:36

And I can see here, if I trace through, I can see that the baffling structure here to give me multiple passes.

29:43

Then I'll use cool water to condense out that water vapour, right?

29:48

So this means that the when I come into this bottom region here, I'm going to have a combination of liquid water that is precipitated or condensed through these two passes alongside a little bit of water left in the vapour and the rest of the vapour itself.

30:07

So if we were using water vapour and air, it would be nitrogen and oxygen.

30:11

And then what happens is in this bottom portion, you can see here, we direct every all of those outlets through a channel which is basically a big separation vessel.

30:20

So my liquid right is going to pool on the bottom.

30:24

And if I change colors, then my vapor will come out of this nozzle and the only outlet for the vapor will flow above that liquid separation constants.

30:36

So it's a relatively simplistic concept when we get into it.

30:40

But the reason that we vertically orientate this instead of looking at something like a horizontal dehumidifier is that we can get pockets of these inert gases that might build up and create stagnation points and the two bundles or in my separation process as they come together.

30:59

And if I have these pockets of inert gas, it's going to function very similar to what happens with film oil, right.

31:05

So I'm going to if it's within the tube bundle, a pocket of inert gas.

31:09

I can imagine a horizontal tube if I start having a little bit of inert gas that's not condensing, it could sit up on the top of the pipeline because I have such a density difference between the the liquid water phase and the vapour phase.

31:22

So because of that, we turn it vertical, which means that I maintain a a vertical head up here so that any gases or inert gases that might have otherwise formed stagnation points will flow upward, right?

31:35

Or will be carried down.

31:36

So it doesn't allow them to build up.

31:38

And what I can look at here is that at a constant pressure condition, I can plot something like the equilibrium solubility of water in a vapour phase, in this case air.

31:53

And I can look at that as a function of temperature, where these different solid curves represent different relative humidity points.

32:00

So if I plotted something like this, that would be my 100% relative humidity, meaning the vapour phase is fully saturated with water.

32:09

And the point that I want to make is that in all cases, right, if I'm decreasing the temperature of my system by using cooling water, for instance, I'm able to decrease the amount of water content in the vapour phase.

32:24

All right?

32:25

So This is why we then get the context of a dehumidifier using cold water with a warm inlet.

32:31

And we put the two together and we will get a pool of liquid water on the bottom.

32:34

Now, I don't know if anyone has a dehumidifier.

32:36

Certainly if you own wood floors in your house or something like that, it's probably a good idea to have a dehumidifier.

32:43

But you'll if you do, then you know you need to empty for a batch operation.

32:48

You'll have to empty this little tank of water that will fill up, you know, every some number of hours or days depending on how much humidity is there.

32:57

So this will give you an idea of how much water we can actually pull out of a vapour phase if it's running continuously.

33:03

It's actually quite a large amount.

33:08

OK, So in talking about the condensation process, the last type of of condenser we'll think about is a well right now is a contact condenser.

33:20

So, so far we've looked at shell and tube configurations, either horizontal or if we're thinking about something like a dehumidifier, we're going to turn it vertically so that we don't have, we can utilize the density difference between water and vapour, but a contact condenser works on a completely different basis.

33:38

I mean, it's actually really cool.

33:40

I, I think that the premise of it is absolutely brilliant.

33:43

The idea here is that if I have some vapour phase of interest that I'm trying to condense, I will inject that from the top of the system.

33:53

And if I have a, let's say that vapor is warm, the idea, the principle of trying to dehumidify or condense is that I need to rapidly cool down the vapor phase, right?

34:04

Well, a really clever way that I can do that is by taking the water that I would otherwise use on a shell into heat exchanger.

34:11

It will be focusing on a, you know, water vapor in the inlet and water used for the cooling fluid.

34:18

I can inject that through a nozzle in a 360°, right?

34:26

In a 360° space, I can have these nozzles spraying water and atomizing.

34:32

And if we talk about atomization, right, then that's going to typically, if I atomize water droplets, I will typically see an atomized fluid stream between one and 20 microns, so micrometers in diameter.

34:49

So it gives me a very, very small droplet of water and it's going to create, you know, 10 to the 10 water droplets every minute or so sprayed into this vapour inlet.

35:00

Now each of those water droplets, if they're cold, is going to contact a warm vapour and it's going to exchange heat with the warm vapour, which means it's going to be a tremendous amount of surface area generated from the water itself instead of relying on the surface area of the heat exchanger right through something like fins.

35:19

So, so aerosolization is the general principle.

35:25

Atomization is what happens if I'm trying to create very small water droplets.

35:29

So the idea then is that if I have this atomization process on top, it will rapidly exchanges heat and draws heat out of the vapour phase, thereby condensing the water of interest.

35:41

I will have some potentially vapour flow out here in my my nitrogen oxygen and I'll have this liquid pool of water being driven downward.

35:50

So the nice part about this is that apart from, you know, I can imagine that both the water inlets is the jet and the spray.

35:59

So the the remainder of my water is injected to drive that discharge stream apart from the pumps that are required to pressurize the water inlet.

36:10

It's actually quite a passive process relative to something like a Shell and GP exchanger.

36:15

So the way I would describe it is something having much simpler construction by the contact condenser, right?

36:20

It doesn't.

36:21

It only works on a few physical principles, but the physics are quite complex by comparison, right?

36:28

In the complex physics come from understanding that atomization process and being able to optimize the size of the nozzle, the spray direction, right?

36:38

And that's an entire kind of engineering unit unto itself.

36:43

So contact condensers, right?

36:45

The take home message of that is that they're an alternative to using something like a shell and tube heat exchanger for condensing.

36:53

But as we can see, just comparing a simple 22 pass system in terms of construction



to what would basically be a singular vessel, a singular vessel spraying downward, I get a much simpler construction but utilizing a much more complex domain of physics being atomization.

37:14

So the last unit operation I want to talk to you today will get us into Chapter 16.

37:22

So Chapter 16 is now about evaporators and these are deployed if I want to partially vaporize a solution.

37:29

So the re boilers that we were talking about is what happens if I want to, you know, pretty much vaporize the vast majority of something like salt water, vaporize as much of it as I can, and then I'll dump the rest back out into the ocean.

37:42

So I can then use that steam, recondense it for drinkable need water.

37:47

Now, if I take that idea to a more general context, we're going to call it an evaporator.

37:54

And I don't always want to vaporize a huge majority of my liquid.

37:58

The point of an evaporator is to typically play with the concentration, right?

38:04

So when you think about evaporators, it's playing with the concentration of what's in my water phase and we call them evap.

38:10

When we reference the word evaporators, we're typically talking about manipulating an aqueous species, right.

38:16

So we tend not to talk about hydrocarbons or or liquid metals.

38:21

We're usually talking about a water based solution.

38:24

And that water based solution would contain either impurities that we're trying to get rid of.

38:29

So the example of of salt that we'll try to concentrate so we get clean water or more commonly for the chemical engineer valuable products that we're trying to purify.

38:39

So something like maybe our pharmaceuticals are reacted in the liquid phase and we then need to purify those by removing as much water as we can, right.

38:48

So the idea is that we'll, we'll approach this similar to how we have done for other the reboiler example earlier.

38:58

But we will be much more careful in how we're applying heat to this.

39:04

Because if we're thinking about a very valuable product like the pharmaceutical, but we don't want to increase the temperature too high and then destroy the chemistry or react the chemistry of the pharmaceutical in that process.

39:17

So when I'm thinking about it and evaporator again, typically using an aqueous basis, there are four kind of critical domains that we'll think about as engineers.

39:28

The 1st is that it works obviously on the basis of changing concentration.

39:32

And we need to think about the fact that when we change concentration, the Thermo physical properties of our fluid of our solution will change as a function of concentration, right.

39:43

So a good example, right, we'll remember from thermodynamics last semester, salt and water is an example of a colligative property.

39:51

So as I the boiling point of water increases with salt content, all right.

40:04

So if I start with say 3.5 weight percent salt water, which is a typical salt loading for oceanic sea water.

40:11

Well, if I start to turn up the heat on that, I'm going to evaporate some of that water up off.

40:17

I can actually concentrate salt in the liquid phase because I'm not going to be vaporizing my salt.

40:22

And the limit of that concentration is in fact usually around 26 weight percent salt in water.

40:31

So I can continue concentrating salt in the water phase up until the point that it hits about 26 weight percent, after which I'm going to start precipitating solids.

40:41

So solid salt will drop out.

40:45

So at the same time, right, that's an example of a very basic Thermo physical property and being the phase diagram changing.

40:53

But my densities, my viscosities, heat capacities and thermal conductivities will all change as a function of that concentration.

41:00

So when I'm designing it as an engineer, I need to think very carefully about what are the limits of that, the limits of my operating concentrations and what are the Thermo physical properties associated with each.

41:15

Right now I'm going to jump to the bottom real quick and at the same time scale formation.

41:19

So if I have, you know, ions in this water, I can certainly expect that these will at some point precipitate out and they're going to precipitate usually on the vessel wall, which means I'm going to have a calcium carbonate, the thing that the the dentist will scrape off your teeth or the hygienist that will form on the inside of the pipeline.

41:37

And that's where we get things like our internal fouling factors from a lot critically, because we are talking in this context usually about valuable products that the engineer is trying to preserve, I need to be very cognizant of the thermal sensitivity of those products.

41:54

So it's not just the absolute temperature that they experience, but even things like the heating time, right.

41:59

So the longer I heat them at, the higher the temperature will be at a greater degree of thermal degradation.

42:04

And the final consideration is if I have foaming, so I'm sure we've all had a chance of

being at the Oceanside and seeing that ocean foam that will will, you know, build up and sometimes can look really, really disgusting, right?

42:19

Well, that exists primarily because we have this diverse chemistry in the phase.

42:24

It's not just pure water in the ocean, right?

42:26

And so because I have these salt components, I have micro surfactants that can act, I can create a gas bubble.

42:33

So the idea or the definition of a foam, it's gas bubbles in metastable, in metastable equilibrium within water.

42:56

So they're being suspended in water and they're meta stable for some meaning meta meaning some amount of time.

43:02

So if I start to have a diverse set of chemistry and I'm thinking about it in the evaporator, well, this is a turbulent process, right?

43:08

I'm thinking about Reynolds number.

43:10

So I have flow going through this.

43:12

I the degree of foaming that I can get will depend heavily on the species chemistry and the amount of turbulence in place.

43:18

So for instance, if I start turning up the amount of salt in my system, I may be more prone to get a foam right?

43:26

So the gas bubbles will be more metastable suspended in water, meaning they will last for a longer amount of time at constant turbulence or they will require more turbulence to breakdown than otherwise.

43:40

So in this foaming phenomena that the issue that it can cause is that that foam is going to carry, right.

43:47

So if I think about a gas bubble, right, suspended liquid well on the surface of that gas bubble, it will carry a very thick layer of my, what we call that thick liquor.

44:04

It will carry a portion of that liquor with it as it goes up into the top of my evaporator.

44:11

So that means that the top side of my evaporator where I should be getting pure vapour could actually start to be contaminated by having a, a large amount of this liquor, heavy liquor that's carried over, right.

44:23

And the principle of heavy carryover is something that will come back into play when we start talking about mass transferring, in particular with distillation columns.

44:32

So the basic evaporator design, and in next lecture on Friday, we're going to be unpacking more examples and going into more detail on this With the basic evaporator design.

44:40

It's going to be based around like two different types.

44:43

What we're going to talk about first is a long tube vertical evaporator.

44:47

And we're then going to kick forward into talking about film evaporators, but in a vertical evaporator, right?

44:54

This is a relatively simple design and something that we've seen quite a bit.

44:59

Now we have a vertical tube bundle where I'm going to inject warm steam.

45:07

So my steam is going to be my method of transferring heat to evaporate something else.

45:14

That steam is going to then flow through the tube bundle vent out the top with any liquid condensate dropping out the bottom and then my liquid phase.

45:27

So I'm going to inject whatever liquid I'm trying to evaporate on the left side.

45:32

Hand side here will flow down through recirculation leg and that flows through the shell.

45:38

So here we go, sorry, sorry.

45:42

The steam as we inject it occupies the shell, not the tubes.

45:45

Occupies the shell of my vertical bundle where the vapour or the steam vapour will obviously exit on top and the condensate on the bottom where the liquid will then flow through the individual tubes from the bottom moving upward.

45:59

Now when I get to the very top, I notice that I'm going to dump that what should be now a purely vapour phase.

46:08

Well actually because I need to worry about potentially having some liquid carry over with it.

46:13

I'm going to dump that into what is effectively a separation vessel right where any liquids that are pulling down near the bottom again will be reinterring and taking down that liquid return line and any vapours will separate up and go out my product to vapour outlet stream.

46:30

So this is the shell side here and my feeds will run through the tube side.

46:40

So when I think about evaporators, this is a single effect evaporator meaning effect meaning I have one, one go through the tube or sorry, one set of heat transfer tubes.

46:53

It's so an effect is not quite the same as a pass, right?

46:57

I will begin looking at Friday on multi effect systems and that's where I'm going to be stacking these together, kind of like a complex turbine system from thermodynamics.

47:07

But this is an example of a single effect evaporator where and this is one of the most inefficient evaporation methods I can use.

47:18

And a good rule of thumb is I need about 1.3kg of steam to evaporate or vaporize 1 kilogram of water, right?

47:27

So I have to put way more work into the system than I do if I just wanted water vapor to begin with.

47:33

So the efficiency of this is very poor, but the reason that I might use it is if the materials.

47:39

So let's say we're dealing with a very valuable pharmaceutical drug that we cannot destroy.

47:45

So the amount of time it can be exposed to a high temperature environment might be limited to say 10 minutes and beyond that 10 minute exposure it's going to degrade.

47:55

Well in AI can use this then to limit the amount of thermal exposure of sensitive materials.

48:02

But if I'm not worried about that as much, then I'll start talking about the multi effective operators that we'll get into on Friday.

48:08

The very simple background to it is that the outlet vapor of the first whole unit operation, which is shown here can be an inlet to the second inlet to the third.

48:17

And so I get a counter flow effect, right?

48:19

And I can use that waste heat and waste energy to make the process more and more efficient.

48:26

OK, so it's the end of the hour, the take home messages for today.

48:29

We've looked at it a few different ways of generating vapor, either from reboilers is a very simple concept, or evaporators.

48:38

And I will typically use steam in these processes where in a kettle taper boiler, what that's defined by as a horizontal system where the steam is flowing through tubes.

48:47

And worse than that, you know, described boiling liquid where an evaporator the steam will flow through the shell side and the tube bundle is orientated vertically.

48:59

And the idea is that a kettle taper boiler is kind of a sledgehammer process.

49:04

So it's meant to to evaporate a large volume of fluid where an evaporator specifically is really designed around the concentration of what's in my liquid phase.

49:16

So it's a way of doing an evaporate a continuous process concentration control for some product of interest.

49:25

Now once I generate a vapor or a steam, right, it's common that I might want to condense that, right?

49:30

And the generation of fresh drinking water is obviously the most common example.

49:34

So we've looked today then at three ways that I can deploy that in a condensing fashion.

49:40

The first would be an air air cooled heat exchanger where I'm just using normal air.

49:44

And we looked at the only equation for today that you really need to to know and use on the quiz this week will be a Nestle correlation for cross flow heat exchangers.

49:57

We also then looked at deploying our our shell and tube technology either in a single or multi pass configuration for the purposes of condensation.

50:05

The only real difference in that is I need to one of the, the heads of the sheets that's binding all of my tubes together needs to float, right?

50:14

Because I'm going to have a, a much larger thermal gradient across these, the individual metal sections that is.

50:22

And then we looked at contact condensers, which is a method that's much simpler in the basis of construction, but a lot more complex than the physics that they're using.

50:30



Because we're gonna take our cooling fluid, aerosolize it and spray it into the warm inlet stream to condense it out.

50:38

OK, so mostly context and and concepts.

50:41

In today's lecture, I'm trying to to not put too many calculations because I appreciate that lab two or lab one is, is taking quite a bit of time.

50:50

So I'm trying to kind of keep this at a conceptual level, except for the one equation that we do need to know.

50:56

And that's the Nestle correlation for cross flow heat exchange, right?

50:59

And that's something that you can deploy quite rightly.

51:02

So with that, I will wrap up the formal lecture and, and let me know what questions you have from either today or Monday's lecture from last week.

51:12

This all makes sense.

51:14

Or is it starting to get a bit muddy or is it too easy?

51:16

Have you seen all of this before in in mass, mass and heat transfer?

51:42

Because I'm going to put up one final form.

51:44

I'll end the recording now, but I'll put up a final forms poll for everyone here.

## Lecture 12 - Advanced Evaporation

0:00

The critical operations for heat transfer and that's the use of equipment to generate vapour, right.

0:06

So we would break this into either a reboiler system or an evaporator.

0:10

And specifically we saw that that steam is a very common tool to use on the warm side of these systems.

0:18

We're a kettle type reboiler.

0:19

That's where we're going to have steam flowing through our horizontal tubes and it's going to be immersed in whatever liquid we're trying to heat.

0:26

Now the nature of bringing that liquid up to its boiling point means that I'm going to have this this boiling, boiling VAT of liquid with steam pipes running through it.

0:37

And that's a very brute force method of evaporating or generating a vapour where an evaporator itself works on a slightly different principle.

0:46

And what we talked about in the last lecture was a climbing film evaporator.

0:51

And the idea is that we would put some kind of feed into the system, right, That the initially liquid would come down here and it's running through the tubes, the individual tubes.

1:03

We'll draw little streamlines here for the flow of that liquid.

1:07

And that will transfer heat from the shell that contains steam into that liquid.

1:12

And in the process, what we can imagine is the vaporization of our liquid into its saturated vapour.

1:18

That vapour is going to climb up those tubes.

1:21

And so that's why we call it a climbing film evaporator.

1:24

Right now, at the top here or out of the top here, we'll have some combination of liquids and gas.

1:31

And any liquid that's going to cool up, right, is simply going to be taken in the vessel and it will be recirculated back down where we draw off the vapour of interest.

1:41

Now we're going to talk today a little bit more about different types of evaporators.

1:45

And these are all variations to this vertical, vertical, sorry, vertical condenser, sorry, vertical evaporator.

1:55

Now backing up a little bit in Section 3, we also learned about condensers and their ability to liquefy vapors of interest.

2:03

So if we generated some vapor, then we're probably going to want to do an operation with it and then re liquefy it, perhaps be purified something we want to then to liquefy.

2:13

So we looked at condensers where we can use air cooling and that's certainly becoming more common.

2:18

The correlation, we did introduce them in the last lecture and I'm not going to put it on the quiz this week just to give a bit of a break.

2:25

But it is one final Nusselt number correlation that would be good to to just take a look at and be familiar with because that's for cross flow.

2:33

So we designed the tubes of these air cooled heat exchangers to sit perpendicular to the axis that the wind is going to to flow or that the fan is going to across it.

2:44

Now we also saw the use of shell and tube heat exchangers applied to the the condensation process and that can go from a single to a multi pass system allowed to the tube or the shell side.

2:55

The one major difference is that we would need to include some kind of floating sheet that all of our pipes are anchored to.

3:02

And so that way the differential temperature across those pipes is not going to cause any kind of buckling or to rip them from the wall.

3:10

And then finally, we looked at that contact condenser and that's where we had an aerosol of droplets, say water, cool water being sprayed into our vapour to create a

tremendous amount of surface area and then rapidly cool the system down where all of the fluids are going to leave as that single stream.

3:28

Now when we think about this evaporator today, right?

3:32

So today we're going to go into advanced evaporators.

3:36

The evaporator that we showed last time, again we mentioned was a climbing film process.

3:41

We can also invert the design of that and think about a falling film so we can flow from the top to the bottom right.

3:48

And where that might come in handy, as we're going to see, is if we have materials that are very sensitive to heat, right?

3:54

So one of the concepts we introduced last lecture was that sometimes materials have a thermal history limit, so they can't be exposed to too high of a temperature for too long.

4:05

And if they do right, then we degrade the material or the valuable product.

4:09

And that that sidesteps the point of our unit operation.

4:13

So if we want to invert that into a falling film now where the liquid comes in on top and vapor primarily vapor will come out the bottom, then that's what we're going to see is an example today.

4:26

Now, what I have put together based on the feedback from our last class is a comprehensive example of designing a heat exchange system from front to back.

4:37

And so the majority of this lecture is going to go through that.

4:39

So I'm going to try to step quickly through a few of the context for look through some of the concept points and really get kicked into this example so we can get on the ground with an application of how to do all of these things put together.

4:51

So if we think about our evaporator for a minute that we were talking about last lecture, that relies on the process of natural convection, right?

4:59

So either I have a falling film or or a rising film, but I can also use what is called a forced circulation evaporator.

5:09

So I can insert a pump into this process, right?

5:12

Where I'm going to put my liquid feed here in the bottom, it's going to flow through a pump and in that process it's going to be pressurized.

5:20

Now once it it goes into a higher pressure condition, it then is going to flow through a shell into heat exchanger right where steam will enter on the shell side or steam will be surrounding on the shell side entering from the top.

5:35

But because I I use this pump instead of having a typical velocity of state between .3 and 1.2 meters a second, which would typically be associated with the natural convection of a falling film or a rising film.

5:48

Because I'm pumping it now I can control how much momentum energy I'm putting into the system.

5:53

So I get velocities on the order of two to 5 1/2 meters a second.

5:58

So it's much faster flow rate now because I have a a high pressure condition inside of my tubes for heat exchange as my fluids traverse the tube bundle.

6:07

Here is a tube house because I have a high pressure condition and it's a a fully liquid system.

6:14

As that X starts exiting the tube, the pressure profile is going to rapidly decay, right?

6:20

So I get this rapid pressure reduction toward the end of my tubes.

6:24

And then what happens is as I bring that into this flash vessel, well, I'm going to very quickly let off that pressure because I'm no longer in a liquid, 100% liquid condition.

6:34

I now have the majority vapour.

6:36

And so in that process of letting off the the pressure of the system, I effectively flash it into a vapour and spray small droplets, which is why we have a a deflector plate here to direct those droplets downward for the process to be recycled.

6:53

So the main take away is when I use a forced circulation evaporator, I get a much, I'm using a principle of superheat and flash to generate some of that vapor, but I get a much, much higher heat transfer coefficient.

7:08

So what the book is showing us here is an overall coefficient for a forced circulation evaporator of between 2 and 5000 watts per meter square Kelvin, where a natural circulation evaporator like we looked at last lecture here would operate usually between 11000 and 2500 watts per meter square Kelvin.

7:27

So keep those limits in mind because we're also going to be looking at today an example of doing these calculations.

7:34

So we'll ground it a bit more.

7:36

Now what we've drawn here is a horizontal 2 pass shell and tube.

7:40

I can also design this as a vertical tube bundle with a single pass design, right?

7:44

So I can follow that same design premise that I have on the falling film or rising film evaporator.

7:50

I can bring that into the force circulation.

7:53

What makes it forced is that I'm, I'm putting this pump onto it and I'm increasing the pressure of the system to create that super heat condition.

8:03

So the last type of evaporator of the three that we'll talk about is an agitated film evaporator.

8:09

So the chief advantage of this is that it, it's very, very good if I have viscous liquids that I'm trying to evaporate.

8:17

And so when I say viscous liquids, so viscosity is up to 1000 poise, sorry, past 1000, yeah, 1000 poise.

8:34

So if we're used to seeing water as one centipoise, we'll look at a few examples today.

8:39

I think one of the calculations from quiz three had steam or we're looking at a very low viscosity in the system when we talk about 1000 times the viscosity of water, right, If we're trying to evaporate that liquid, then we're going to have to put a little more work into it.

8:54

And the principle that this operates on is that I have a motor up here that's driving these blades.

9:01

So these internal blades, these are rotating with time.

9:06

And what happens is the liquid of interest, whatever my viscous feed is going to be enters from the top and it will, the vertical blades are going to slam it against the wall.

9:18

So the film that would naturally run down because there's a mechanical agitation going on.

9:24

I don't have that conductive heat transfer process going on in my liquid film on the wall, right?

9:30

And, and the liquid film on the wall is the issue because I'm heating this through a steam jacket.

9:35

So this internal bit where I have a mechanical almost like a blender that's extruded all the way up to the top.

9:41

And these blades break up that liquid film and prevent it from being a conductive heat transfer limitation.

9:49

So if I'm looking to estimate the internal heat transfer coefficient for these viscous liquids, I can actually go back to a previous example of a scrape surface system.

9:59

So this is a correlation for that.

10:02

Where I'm looking at  $K$  here is the thermal conductivity,  $\rho$  is the obviously the fluid density,  $CP$  is the heat capacity and represents the agitator speed.

10:13

So how quickly these blades, so how quickly these blades are turning around through here and, and  $B$  will be the number of blades per circumference.

10:26

So we're not going to do an example of this specifically, but I just wanted to make you aware.

10:30

Once we start getting into higher viscosity systems, right, we would start off with a falling film or a rising film if we have something like water, right?

10:39

We're a very inviscid species.

10:41

We would move to a forced circulation evaporator if we have a slightly higher viscosity, say 100 centipoise that we're trying to evaporate.

10:49

And if we get into a very high viscosity 1000 poise, right, then we'll move into something like an agitated film.

10:58

So the typical applications for this are things like gelatin, rubber, latex, antibiotics that might be put into a very viscous polymer solution and fruit juices that are being concentrated.

11:10

And the disadvantages of it, these are very high cost systems and they're high cost because of all of these moving parts required to drive agitation process.

11:20

So they tend to be a speciality application which is why we're not going to focus on them for our example.



11:26

That's coming up now.

11:28

OK.

11:28

So in the end of last lecture, we were kind of circling around the point that it's a bit difficult to look at each of these individual calculations and make sense of what what we need to do when we put all this together.

11:42

So I found a nice example that we can go through.

11:44

And that's what happens if we want to produce condensed milk, right?

11:49

So this is going to use a falling film evaporator and it's going to be made of stainless steel tubes.

12:02

So the diameter, the internal diameter of my tubes is going to be 32 millimeters and that will be, they will be two inches or sorry 2mm thick and they're going to be 6 meters long.

12:22

And the thermal conductivity of the stainless steel that I'm going to use in this example will be so  $K_{wall}$  is 16.3 and that watts per meter Kelvin.

12:34

OK.

12:35

Now, typically if you're an operating engineer, you would assume that any kind of steel is going to have a thermal connectivity around 50 watts per meter Kelvin.

12:44

That's a pretty common benchmark.

12:46

So 16 is a little bit low for a wall, but it might be a speciality steel or an alloy that's made specifically for the food industry.

12:55

OK, So that's the dimensionality of what I need to know for the system itself.

13:01

Now the feed, So the cool side is going to be milk, so we'll write this in blue.

13:08

So the feed is going to be coming in at 40 kilograms per hour and we usually report this per tube.

13:20

So you'll note that one of the well, I'm not giving you how many tubes are in this system and it actually doesn't matter because I'm going to design everything on the basis of a single tube and then it simply multiplies across for how large of a system we want to scale.

13:36

So it's 40 kilograms an hours to feed and that's done all on a per tube basis effectively.

13:43

And so that if I put that into, into base units, that's going to give me point O 111 kilograms per second of milk at 60° C.

13:55

Now the milk that I'm, I'm thinking about has a 13.5% fat content, right?

14:05

And the purpose of designing something like an evaporator for this milk is that I'm trying to increase the fat content of the milk, right?

14:12

Because that's the valuable product that I'm then trying to ship maybe overseas.

14:16

So it can then be diluted again and resold as normal milk.

14:21

So if I have 13.5% fat content and I'm trying to increase that fat content by boiling off the water or evaporating the water from my milk, then that's that's the design basis we're going to look at now.

14:34

This is going to have a feed viscosity of .94 centipoise, a density of 1010kg per cubic meter, a heat capacity 3930 joules per kilogram Kelvin, a thermal conductivity of .62 watts per meter Kelvin, and a heated vaporization, which in this textbook and unit we'll use is Lambda.

15:12

So heated vaporization of the milk is going to be 2357 Joules per gram and that's at 60° C.

15:22

Now I just oppose this with a warm fluid.

15:28

So my steam is going to be coming in on the shell side now.

15:32

So steam will enter at 70° C and at those conditions, right, I have a viscosity of .406 centipoise, a latent heated vaporization.

15:51

So Lambda S is going to be 2331 joules per gram and that's at 70° C because that's my my shell side temperature now.

16:01

Now the density because I'm going to be condensing this steam.

16:05

So the density of the water that it will condense into is 978 kilograms per cubic meter.

16:14

The heat capacity of my water sign is 4184 watts, as are joules per kilogram Kelvin and it's thermal conductivity K will be .662 watts per meter Kelvin.

16:32

OK, so all I've done right is to specify I need to evaporate some amount of water off of my milk that's coming in at 40 kilograms an hour.

16:42

That's really the only engineering throttle that I've said on this.

16:47

I've said that my tube side is going to be coming in at 60° C, my shell side is going to be at 70.

16:53

And that makes sense, right?

16:54

Because if any of you have ever been a barista, as soon as you take milk above about 80° C, you're going to start scalding it.

17:01

So this is a great example of one of those unit operations where you have to be careful about the thermal sensitivity of your product.

17:09

And all I've really done beyond that is to specify the Thermo physical properties of my fluids of interest.

17:16

So densities, heat capacities, viscosities, and then the latent heat of vaporization that we talked about in thermodynamics.

17:24

So to do this, right, one of the challenges here is that I don't know and I've not defined what is my mass flow rate of the steam, right?

17:35

And so from mass flow rate, I'll switch colors here from mass flow rate that's going to be related to the overall heat transfer coefficients, the area of exchange, the temperature driving force.

17:46

So in this case,  $70 - 60$  is  $10$  divided by the latent heat of vaporization  $\Lambda$ .

17:54

So if I write that out in equation form, the  $m_{\text{steam}}$  times the  $\Lambda$  of the steam will be equal to  $U_{\text{overall}} \Delta T$ .

18:04

Well, I don't know the mass flow rate of this team because that's not been specified.

18:10

And I don't know the overall heat transfer coefficient, right,  $U_{\text{overall}}$ .

18:15

And so that's a problem.

18:16

I have one equation and two unknowns.

18:17

So what am I going to do?

18:19

Well, we can go back one slide and we can say, well, we're going to be using a natural circulation.

18:27

And this is the value of having a textbook on hand.

18:30

We're going to be using a vertical long tube evaporator and it's going to be using natural circulation.

18:36

So I know that the overall coefficient should be somewhere between 1000 and 2500 watts per meter Kelvin.

18:42

So guess what, I'm going to take a guess.

18:45

I'm going to first guess that  $U$  overall is equal to 2000 and then I'm going to see how that gas unpacks and I'm going to refine it as I go.

19:00

So with that, I can then calculate directly the mass flow rate of the steam.

19:06

If that's the overall heat transfer coefficient would be 5.18g per second.

19:11

And if I then have a 5.18g per second, I can take a step forward and calculate the.

19:20

So  $\gamma$  is the liquid load that will be on the outside of the tube.

19:25

So  $\gamma$  for the steam.

19:28

And we defined that early on as  $\dot{M}$  on  $\pi$  times the diameter of the tube.

19:36

So it's the mass flow rate per time per surface perimeter of the tube.

19:44

And so that works out to be, in this case, 0.0515 kilograms per second meter.

19:54

Now, once I have a flow rate and I know the dimensionality of what I'm looking at, I can calculate the Reynolds number on the outside of the two, right.

20:02

We looked at the relationship for the Reynolds number.

20:05

This is back in about the first third of Chapter 13.

20:08

So the Reynolds number I can define in this case as four times that  $\gamma$  on  $\mu$ .

20:15

So if I don't know the OR if I'm not using a specific velocity, but I'm thinking about the mass flow rate per perimeter, I can also define Reynolds number this way.

20:24

And because I know the viscosity, that calculation works out to about 507 all right.

20:31

Now.

20:31

Similarly, my Prandtl number will be the heat capacity of my steam times the viscosity of it divided by the thermal conductivity.

20:41

And so that works out to 2.57.

20:44

OK.

20:44

Now what do we, you'll remember about 3 or 4 lectures ago, we were looking at what's going to happen on these external tubes and how do I find either a nestled number correlation or how am I going to find that heat transfer coefficient?

20:58

Well, the book here, so figure 13.2 is shown here from the textbook.

21:04

And I can see that we're in that laminar region where I'm going to have a not technically linear because the abscissa is logarithmic, but I have a, a steep decline of the heat transfer coefficient.

21:15

I'm using a modified Nusselt number.

21:17

So  $NU_{\text{prime}}$  and Nusselt prime is the way this is being written here.

21:24

And once I transition into that WAVY laminar region, right, I then have an increase again in the overall heat transfer or the heat transfer coefficient convectively.

21:35

Now one of the examples in the book that goes through something similar to this actually says, OK, I went to Figure 13.2.

21:43

I looked up my Reynolds number and we can see here is the the definition that we just employed on the bottom and they read out.

21:49

So if my Reynolds number is 500, they actually read out.

21:52

OK, right here is the approximate Nussle prime definition for my system.

22:00

Now personally, I tend not to like doing that unless I really have to.

22:04

So we do also have a correlation, so equation 13.13.

22:08

As long as we are to the right hand side of this WAVY laminar transition, I can use this correlation that's shown here in preference, which is only a function of the Reynold and Randall number that I just calculated.

22:21

So when I deploy that correlation, then what I end up with is a nussle, we'll call nussle prime of .260, right?

22:34

And that itself is equal to the H in this case, we'll call HO.

22:39

So that's the external heat transfer coefficient of the thermal conductivity of my steam, which I know the viscosity squared, the density squared, so the gravitational field 9.81 meters per second squared, all to the power of 1/3.

22:57

Well, because I know everything else except the external heat transfer coefficient, I can then solve directly for this and I get an H naughty or an H external of 6609 watts per meter squared Kelvin.

23:11

OK, so I've done the first domain of thinking about my external heat transfer coefficient and I'm using one of my Nestle correlations that's appropriate for a vertical flowing film.

23:24

So let's take a step forward down and think about well, what's going to happen at the wall and what's going to happen on the inside of the pipe.

23:30

Well, for the wall, right, the wall I can think of the the resistance contribution  $H_{sub} WI$  can also write as a thermal conductivity on the thickness of the wall.

23:44

All right, which in this case is going to be give me 8150 watts per meter squared Kelvin.

23:54

Now is the more complex point of starting to think about the milk and that's going to be flowing through the tube side.

24:00

Now, because we said this was a falling film evaporator, not a rising film like we drew in last lecture, we're looking at the opposite.

24:07

Now in a falling film evaporator, I'm going to begin with my colder, so in this case low fat melt content at the top and it's going to float downward through my pipes and pick up heat from the steam.

24:19

So I'm going to have a differential behaviour in the the flow properties of the Reynolds number, Prandtl number from the top of the tubes internally to the bottom.

24:32

So let's first consider what happens at the top of the tube.

24:35

So the liquid load on the milk side at the top.

24:40

I can write that again.

24:42

That will be defined as the mass flow rate divided by  $\pi$  times the internal diameter now.

24:47

And if I do that calculation, I get .1105 kilograms per second per meter.

24:57

And if I think about calculating the Reynolds number, right, which again 4 gamma on MU of milk at the top, assuming right, the milk properties I gave here, these are all defined as 60.

25:09

So before I've done my concentration process, so that four gamma on MU, that's going to give me a Reynolds number at the top of the pipe now of 470 and the Prandtl number of five .96.

25:25



So using the same, right, because I'm in a transition Reynolds number for this scenario, I can use that same Nusselt prime correlation approach to get a value of .274, right?

25:37

And what you'll see is if you go into the book and look at Figure 13.13, sorry, 13.2, in fact, you will get a very similar answer between a reading out from that figure as an experiment and doing this, this empirical correlation here.

25:55

So if my Nusselt prime value is .274, then again, I can solve that directly for  $h$  out inside the pipeline.

26:03

And that will give me a value at the top of 3807 watts per meter squared Kelvin.

26:12

All right, so now I need to think about, well, I don't know what happens, right?

26:20

If I think about and draw a little picture here.

26:23

All right, if this is my steam on the outside, I have milk coming in at 13.5% of fat.

26:33

It's going to pick up a little bit of heat from the steam and it with about a 10 Kelvin gradient across it.

26:39

And it's going to exit the tube with higher fat content.

26:47

And in this case, I don't know what's going to happen as a consequence thereof.

26:51

So what I'm going to initially do is calculate  $U$  overall on this basis and then think about how does that change as I go through the system and see if I'm getting consistency for my average guess, right?

27:04

And so the reason that I typically like to use correlations like calculations instead of reading it off the table is I can set all of this up in Excel and then iterate on value very quickly until I get convergence, right?

27:17

It turns out that 2000 watts per meter squared Kelvin is a very good initial guess, as I'm sure you would expect.

27:26

But but in this case, if I took a really bad initial guess, I could quickly rectify that and bring it into a convergence point.

27:33

So to figure out what happens from the top right down to the bottom of that individual pipe, the first thing I need to do is calculate  $1/U$  overall, which is going to be defined as one on the internal convection, one on the internal resistance, the wall resistance, the difference between the so the internal diameter and the wall, so the arithmetic mean.

27:59

Now in this case, an arithmetic mean is completely appropriate, right?

28:03

Because my wall is only 2mm thick and I'm talking about a 32 millimeter internal diameter pipe.

28:10

So that's  $D$  inside.

28:12

And then I have one on  $H$ , not that I just estimated previously on the internal diameter time of the internal diameter on now the external diameter, right?

28:22

And So what that's going to work out is that one on  $U$  is going to be or  $U$  over all point triple O 51263, meaning  $U$  over all will be estimated now at 1951 watts per meter squared Kelvin.

28:40

OK, so that tells me right away, you know, my initial guess of 2000 was actually pretty good right now from there now I can figure out well, if I know that overall rate of heat transfer and I'll change colours here for the last segment of the calculations.

28:58

If I know that overall rate of heat transfer, then the amount of mass I'm going to lose.

29:04

So how much yeah, how much mass are we going to lose in the evaporation process are so the rate of evaporation is going to be that  $U$  over all a  $\Delta T$  on now  $\Lambda$  of the thing that I'm trying to evaporate, right.

29:20

So if I go back to my problem statement, that's the heat of heat of vaporization for my milk.

29:26

So 2357 joules per gram.

29:29

So if I put in this, this estimated U over all try to have 1951 watts per meter squared Kelvin.

29:37

Now times the area that I'm looking at, this gets a little bit more complicated because I'm going to have to say it's  $\pi$  right Times Now the external diameter, I'm sorry, the internal diameter because that's what I'm evaporating in times 6, right?

29:59

Times Now, 10 Kelvin.

30:01

That's my driving force in temperature, all divided.

30:05

And what was my 2357 joules per gram?

30:12

All right, so my, my diameter is going to be 0032 meters.

30:21

We're sorry, 032 meters, OK.

30:27

And this gives me a rate of evaporation of now 4.992g per second.

30:34

So that's how much mass I'm losing out of my milk every second that it's flowing through this individual tube Now by simple mass balance, then the mass fluid of my milk at the end is going to be at my initial flow rate.

30:51

So 11 right.

30:52

If I look at this this value up here 11 point 0 5g per second.

30:59

In fact that will be that less 4.992 gives me an exit mass flow rate a product mass flow rate of 6.119 grams per second.

31:14

Now, once I know the exit mass flow rate, I'm assuming in this that I'm not going to be evaporating or vaporizing any of my fat content, right?

31:26

And that's quite reasonable because the milk solution itself is going to be aqueous and the fat content in milk is going to be a hydrocarbon chain that might be anywhere from C20 to C-40, right?

31:40

So I'm definitely at 70° seeing not going to have enough thermal energy to vaporize C36 wax or fat, right?

31:48

So I can assume that that map to solids content is going to stay completely in my product stream.

31:56

So then my percent solids is going to be, we're going to start off initially with.

32:04

So what is the initial content?

32:05

So 13.5%, right?

32:08

And then I'm going to multiply that by my initial 11.1.

32:15

So it's not to clarify, it's not from here, it's this mass flow rate.

32:19

So .111 on the previous slide, that's what we're drawing in for this mass flow rate in milk.

32:26

So point 11.1g per second, that's what it initially started out with.

32:31

So by basic mass balance divided by that by 4.992g per second, I get a yielded fat content now of 24.5% fat.

32:45

OK, so now what I've done is to effectively double the amount of fat in my milk without scalding it, right?

32:53

In a single pass configuration, right?

32:56

So this is quite good.

32:57

Now let's go to the the final step of the calculation.

33:02

And that's what we need to then think about at the end of the tube.

33:05

Now I don't have vapour right?

33:08

So I'm concentrating or it is the product is exiting.

33:12

I have 24 1/2% fat in liquid.

33:16

So the amount of liquid flow is going to be less, which means the liquid load gamma at the bottom of my tube will be defined as that exit mass flow rate M done right that we just said was 6.119 grams per second and divided again by  $\pi D$  So that gives me point O six O 9 kilograms per second per meter.

33:41

So at the bottom of the tube I can estimate right and I should clarify if I label those, it was the Reynolds number at the top and the Prandtl number at the top.

33:52

I can estimate the Reynolds number at the bottom as using the same formula now.

34:00

And well, sorry, jumped ahead of myself.

34:03

So because I have more fat in this, right?

34:08

If we look back, all of the Thermo physical properties that we were using for our milk assumed we had 12 1/2 percent fat.

34:17

However, these Thermo physical properties are likely to vary over the course of our heat exchange process, right?

34:23

And that's one of those things we've we've kind of beaten to death talking about all the time.

34:28

Now when I look at it for something like milk, if I'm going to increase the fat content, actually the density doesn't really change much.

34:36

Heat capacity doesn't change much, thermal conductivity doesn't change much.

34:40

But what does change is the viscosity.

34:43

The viscosity goes up a lot because I'm suspending these fats in in my aqueous system.

34:50

So I'm going to assume the only Thermo physical property that changes is the viscosity at 24.5% fat will be double the viscosity at 12.5% fat, right?

35:10

So what I'm saying is, if I'm doubling my fat content, I'm going to assume the viscosity is going to double as well.

35:16

Now in reality, it will probably be more than doubling, but I'm just going to throw up doubling because it's about right and I want to see what that does to the calculation.

35:27

So now the Reynolds number that we're going to think about is still that four now capital gamma at the bottom of the two, but now divided by the viscosity at the bottom, which itself is twice the viscosity at the inlet.

35:42

So that means my Reynolds number is going to decrease because my exit liquid load has decreased and my viscosity has increased.

35:51

So I get a Reynolds number at the exit point of the tubes of 130.

35:56

Similarly with the Prandtl number being a heat capacity times that viscosity on the thermal conductivity, I'm going to see the Prandtl number go up as well, right?

36:07

And that's, we'll estimate at 11.9.

36:11

So I'm just barely within this region, You know, I have WAVY, WAVY flow.

36:16

So I can use that same Nusselt style correlation.

36:19

So the Nusselt prime value for this will be .353 which gives me then an H internal at the exit, which we also call point B of 3097 watts per meter squared Kelvin.

36:38

OK, so at this bottom point, I have A convective transfer coefficient of about 3000 watts per meter squared Kelvin.

36:49

Whereas at the top point where I have cool milk and not a lot of fat, I have a convective transfer coefficient of almost 4000 watts per meter square Kelvin, right?

37:01

OK, so now we've calculated the gradient across this.

37:05

So to get my H internal as a final point, I'm simply going to take the average of the two, right?

37:12

So 3097 plus 3807 / 2 gives me my average internal coefficient of 3452 watts per meter squared per column.

37:28

Which means that one on U overall using that same premise is now point OOO 53964, which gives me an overall heat transfer coefficient of 1853 watts per meter squared Kelvin, right?

37:47

So my initial guess of 2000 was actually pretty good.

37:51

Now one of the benefits is when I pop all of this into a spreadsheet, right?

37:55

What I can do is vary.

37:57

I can vary any one of my input parameters or I can vary that initial guess until I have a consistency between the initial guess and my final condition.

38:06

So in conclusion then I have one extra blank slide just in case.

38:10

The last kind of topic I wanted to touch on is just to close out our discussion around evaporators is how we can increase the performance of our evaporators.

38:21

So this is all thinking about using a singular evaporator, but we characterize evaporators thinking about both the capacities, the amount or kilograms of water that they can vaporize in a given amount of time and their economy, meaning the amount of material I can vaporize per kilogram of steam being fed in fed in.

38:44

Now when we what we were just designing was around a single effect evaporator and the economy of that single effect is always almost less than one.

38:54

So as we saw last lecture, takes about 1.3kg of steam to to vaporize a kilogram of water in a single effect, which is why we go to multi effect.

39:04

So the example that we showed here is where we're going to think about a triple effect evaporator that's based around natural circulation, right?

39:13

And so the idea is that I'll have some feed coming in the bottom right.

39:18

I'm using my steam inlet here will initially come through, but it's only going to impact the first stage or the first effect of my multi effective operator.

39:33

Whereas my material is it's in this case flowing upward will come into my first separation vessel.

39:38

Liquids go down and then the warm vapour of that will go to a secondary system.

39:45

Now the liquids that come down, part of them will be split back into the first effect of the evaporation process.

39:52

The others will go across this valve, right?

39:55

And the purpose of the valve is to introduce a pressure drop.

39:57

So I maintain pressure consistency between each of these effects, and I'll talk about that at the very end.

40:03



So I now have liquid flowing up through this second effect where I have more condensate coming out on the bottom.

40:13

And this then comes into the second effect separator and the same process repeats until I get to that third effect where on the top here, the vapor that I'm pulling off this I'm going to actually introduce to the vacuum, right?

40:27

So I can hook up not just a pump in a vacuum pump.

40:30

So that my pressure in this third effect separator is going to be very low, right, in vacuum region somewhere.

40:37

And this, this all has the process.

40:38

And if what comes out the final side, this is my most concentrated solution.

40:43

And so the the benefit of using something like a multi effect evaporator or evaporation process is that I can use the waste heat from my first stage to get additional benefits downstream.

40:58

Now if I'm going to design something like this, right, the process that we just went through over three slides was thinking about what would a single effect evaporator be?

41:06

Well, if I start stacking these together, the outlet to one effect is going to be the input to the next.

41:12

And there are effectively four ways that I can introduce feed material to something like a multi effect system.

41:19

Now I'm not going to ask you to design any of the minutes.

41:22

It's not going to come up on a calculation.

41:23

It's just for your own knowledge.

41:25

And so we typically start thinking about a forward feed and that's what I just described.

41:31

And that would of course be the simplest idea.

41:33

Whereas a backward feed where we actually begin feeding the last stage that's at the lowest pressure and we go toward the stage at the highest pressure and we get out of that in the thick liquor.

41:45

A backward feed would be better for higher capacity systems that have a much more viscous feed stock.

41:52

But the downside is that I need more pumps to drive the liquid through each of these processes because I'm going against the pressure gradients and the separation doesn't.

42:04

Now, I can also use a mixed feed, meaning I can introduce a feedstock at multiple locations or in the middle of the system and look at getting a thick liquor out on one side that's allows me to maintain a high temperature and with few pumps.

42:20

And then probably most importantly that we'll talk about at the very end of the unit is if I want to do something like a parallel feed.

42:27

And that would be relevant if I have a crystallization process.

42:31

OK, So not doing too much on this, just to give you some basic working principles.

42:37

So the take home messages from this lecture that I'm sorry a bit late on Friday.

42:43

The evaporator performance, we define and characterize the operators based on their economy and their capacity.

42:49

So their capacity is how much, how much they can vaporize for time economy is how much they can vaporize per input.

42:55

In this case being a kilogram esteem.

42:58

If we have very invisible liquids, we would tend to use a feed forward design, right?

43:05

Any single or multi effect system that's shown here.

43:10

Once we get to the viscous liquids, then I'm going to have to think about it for circulation evaporators.

43:16

So even in the case of our milk examples today, we were not actually dealing with the viscous liquid, right?

43:21

Viscosity was still hovering just above that of water after we concentrated the fat into it.

43:27

And even then, it's not a highly viscous species.

43:31

If we go to something like really heavy oil, we're trying to evaporate off, you know, very significant components of 1000 points on up.

43:39

That's when we need to introduce mechanical turbulence to disrupt the liquid film that's going to form.

43:45

And then we call that an agitated film evaporator.

## Lecture 13 - Mass Transfer Operations

0:00

Transfer operations.

0:01

So if you've been following along in the book, this is starting now with chapter 17.

0:06

Now we're not going to get very far into chapter 17 today.

0:10

I'm just going to cover the kind of high level concepts because part of what I want to do is really close out what we've done with heat transfer and in particular the connection of heat transfer processes, methods to heat transfer operations.

0:23

So we can see the, the combination of the two.

0:25

Now of course you know the the material we've covered up till now, weeks one and two, we're all about a very quick introduction to fluid mechanics.

0:34

For those of us who haven't yet taken it weeks three and four, we're we're then covering heat, heat transfer operations with a very quick background to heat transfer methods that sit behind it.

0:48

So for that reason, if you've been following along in the book, I appreciate it's been quite a lot of reading up until now.

0:54

We've covered something like 12 chapters between the beginning of the year and now, you know, bearing in mind that that some of the background to this would have been covered already.

1:04

So that's where we we were accelerating so quickly through those.

1:07

Now from here on forward, we're going to be moving much more slowly through the book and we're going to talk today as why, right.

1:15

And that's because mass transfer is a concept is is much more complex and much more nuanced than something like heat transfer or certainly than fluid mechanics.

1:25

So to that end, I would strongly encourage, particularly now we're getting into the most difficult domain of the lecture material being mass transferred and it really take the time, you know, this is the the first quarter of Chapter 17 that we'll be looking at today and we're going to very slowly start unpacking mass transfer and we're not going to have to rush to the text.

1:52

So a quick review.

1:54

In the last lecture, we were tidying up the last concepts around heat transfer operations and specifically we're looking at heat transfer units that were designed to evaporate off a component that was undesirable.

2:08

So in the example that we did on Friday, that was looking at purifying milk.

2:14

So obviously the milk content is is priced based on how much fat it has.

2:19

That's what people are actually paying for it and the rest is water.

2:23

So, so in that context, we could look at how we could enrich the quality of milk, taking it to the point of of condensing it or creating condensed milk where we can then ship it at, at very high capacities.

2:33

So we're using an evaporator to do that.

2:35

And we saw that evaporators of course were based on economy and capacity.

2:40

We were using a single stage evaporator in that case with a falling film design.

2:46

So the milk started at the top with a very low concentration of solids, so 12 1/2 percent fat.

2:51

And by the time it made it to the bottom of the column, being a a falling film, so the film was going downward.

2:57

By the time it made it to the bottom, we had evaporated off quite a bit of our liquid resulting in a doubling of the fat concentration in the milk, right.

3:07

So that was an example of an in viscid system.

3:10

Even though our viscosity was doubling, it was still about 1 1/2 times that of water, so quite inviscid by nature.

3:17

And when we saw that, you know, in a typical example, we could improve the efficiency of this by looking at multi effect evaporation where we typically use a feed forward design for simplicity.

3:28

But there were other examples we went through around that.

3:30

Now if we were dealing with something much more viscous than milk, let's say heavy oils, right?

3:36

So maybe something like lubricating oil, then we would think about forced convection evaporators.

3:41

So that's where we'd add additional fluid momentum using the same horizontal or vertical tube bundle.

3:49

But this will allow us to increase the pressure upstream and entering that tube bundle, thereby accessing a region of super heat in the pressure temperature diagram where we would rely on flash drop more of that liquid.

4:03

And finally, if we get the highly viscous liquid, so not something like lubricating oil, but maybe a like a Canadian tar sands oil, right, where the viscosities are hitting up between 5,000 and 10,000 centipoise, that's where we want the agitated film evaporators where we're going to use a mechanical rotation to invoke turbulence and improve the quality of heat transfer.

4:24

So those were our three final concepts in talking about heat transfer.

4:28

Now, because we've hit the end of the fortnight, I wanted to take a minute before we get too deep into mass transfer and just kind of think critically for a minute about what are the methods or the processes that we've learned about in our heat transfer operations?

4:42

And then what are those actual operations that we've gone through to learn about?

4:47

So thinking about the processes we kind of led in by with our background of fluids, identifying some of the uncertainties, right?

4:55

And the reason that we don't have a perfect theoretical solution to heat transfer in most cases.

5:01

So the uncertainties in terms of having that thermal boundary layer, we saw early on that the Prandtl number shows us the ratio of something like if this is the boundary layer of thickness on the distance of entry we saw maybe here is our hydrodynamic boundary layer and here is our thermal boundary layer, right?

5:19

So I'll use a little  $H$  and a little  $T$  to denote them.

5:22

And so because we have those those boundary layer effects that are going to be non linear with entry into the heat exchanger because in the example of the shell and two right, the tubes or the shell side in particular is not going to have fully established slope.

5:37

Usually when we're doing multi passes in the shell side, we're churning this liquid around.

5:42

So we don't really have a theoretical solution that can be used to divine the heat transfer properties in that case.

5:50

So for that, those are the uncertainties from the fluids perspective that give rise to the highly empirical and correlative nature of designing heat transfer equipment.

6:01

Now when we go to design, we saw a couple of processes and methods fall out that we used in our example on on Friday, some of which, but these are the more common ones that we can apply to to multiple situations.

6:15

So first, the when thinking about heat transfer as an operation, you know, if we have a driving force and a thermal gradient, then the resistance to that driving force we can think about as a series that we can sum up and the resistances will come from conduction and convection put together, right.

6:36

And so this is where we get into that that model for you overall that we want to solve for being one on  $H$  internal plus the area internal on area external times the external heat transfer convective coefficient, right plus whatever conductive coefficients.

6:56

We might have a thin walled pipeline, we might have a thick walled pipeline or a multi layered pipeline, right.

7:03

So conventionally we would assign that conduction is going to be something like a, a

inside on a outside to get an arithmetic mean times the sorry thermal conductivity on the bottom.

7:23

So in that sense, right, we're looking at the ability to stack up.

7:28

If we have all of these going together, we can think of maybe we're going to have a pipeline where we have a copper sheets inside and a steel outer layer for security, right?

7:42

So we can have a higher pressure system.

7:45

So the copper on the inside would give us a very good heat transfer.

7:48

The steel is going to protect it mechanically.

7:50

So then we'll have this internal heat transfer coefficient and external heat transfer coefficient, right, And some conduction process going across that wall.

7:58

So to summarize, right, it's, it's important if you follow through the calculation on Friday, I would definitely take a few moments and sit down and actually work through that on your own, right?

8:09

It's all of the concepts that we brought into play, right, of talking about how do we get the internal, the external conductive coefficients right.

8:18

Thermal conductivity is obviously just a Thermo physical property of the fluids being used.

8:24

But but understanding how we get at those coefficients and the fact that they're always then deployed in this series approach is really critical.

8:32

Now what you'll notice here is that I've not written whether this is a arithmetic or a log mean diameter or a log mean driving force.

8:39

So we talked about when we will, we can use a simple arithmetic average.

8:45



So maybe the internal and plus the external diameter is divided by two, you know, versus when we need a log mean approach, right.

8:52

And the difference was when we compare the external radius on the internal radius and that would be for a value say significantly greater than one, right.

9:02

So if that value is approaching unity, we saw that that for instance, if the value of that ratio is 2, then we have a 5% deviation in the solution, right?

9:14

So you'll appreciate it because we're using correlations for everything here.

9:18

A 5% deviation in something like a heat transfer coefficient is probably within the uncertainty of the correlation we're using to begin with.

9:26

But it's really when we start to get to that thick walled pipeline where the outer radius on the inner radius is say two or greater, then it becomes quite significant that we're using a log beam diameter and a log mean driving force.

9:38

Now that leads into the Third Point of uncertainty is that we're using nested correlations and not for one thing for everything, right?

9:47

So the one take away message from this last fortnight is that, you know, you don't have to memorize those nested correlations.

9:53

Certainly that would be obnoxious, but the approach used in the design of heat transfer equipment is to deploy nested correlations.

10:02

And so it's very important that you're familiar with the breadth of what Nussel correlations are out there.

10:08

And by going through each of these, what we've in fact seen is they all share a common approach that the definition of the Nussel number itself, whether we're seeking an internal or an external convective coefficient.

10:18

And I won't say an = here, but has a proportionality.

10:22

So the Reynolds number raised to sum power A and the Prandtl number to sum power B.

10:27

And then we typically include that viscosity index to the power, say N, right?

10:33

And the viscosity index again is only going to come up in those scenarios where we're dealing with viscous systems.

10:38

And most of the time as engineers, that's not going to be the case.

10:41

So it's the, the take home message is the Nusselt number scales, usually it's a product of the Reynolds and and Prandtl numbers taken to some power right defined for our system.

10:52

And then we usually put a pre factor in front of it.

10:55

And that pre factor is going to account for a whole bunch of these uncertainties.

11:00

I'll draw a little dotted line, a whole bunch of these uncertainties that we were looking at up top from fluids.

11:06

So the the 3rd to the final point is that we talked a lot about limitations that will arise from films when we're dealing with processes that are changing phase.

11:19

So if we're in a simple, you know, God willing, a simple scenario of transferring energy between two fluids where they're both staying in the same phase and we're just heating one and cooling the other, that's a dream, right?

11:30

When we go to start changing phase, right, what's going to happen is we're either going to have a liquid film or a vapour film appearing at the point of heat transfer, right?

11:40

And that's the nature of what's happening.

11:41

And we saw that when those films emerge, because they're quite thin by definition, they invoke actually a conductive resistance, right?

11:49

So the way that we deal with that is by modifying our Nestle correlations.

11:53

You would think, right, that the way we deal with films is by adding in an additional conductive or conductive coefficient to our resistance term.

12:03

But in fact, the way the community has evolved is simply by modifying.

12:11

I'm not saying modifying that wouldn't be appropriate, say more more nestled number correlations.

12:20

OK.

12:21

Now in many of these cases and we'll we'll summarise these in a minute, but where we started looking at the shell and tube heat exchangers, these are obviously used in a number of heat transfer operations and by far they are the most common heat exchanger framework that we'll use.

12:38

So because of that, there are a couple concepts that we've identified if you're designing a heat exchanger.

12:44

So you have the luxury of being an engineer who's you know started out and you get to design a new process.

12:49

Then the two kind of critical points to bear in mind where you can optimize or improve the design is in the identification of multi paths flow either on the shell side, the tube side or both and the addition of thins to extend that heat transfer surface.

13:04

And I understand that many of you have gone through the use of thins already, so we didn't spend too much time covering that.

13:11

But one of the takeaways from this class should also have been that if you're walking into a operation where someone's already built the heat exchanger, right?

13:20

And like most things designed by humans, they've been done in a sub optimal fashion.

13:26

So if you're charged with further optimizing that existing equipment, one of the ways that you can do that is by maximizing the difference or the ratio between the product of the mass flow rate and the heat capacity.

13:39

So the thermal mass potential of both sides of the heat exchanger.

13:43

So by playing with that mass fluid, it gives you the ability to further optimize an existing design.

13:49

Now, what operations do we cover at the namesake of this class is that we should go through and understand what are the physical operations that engineers are going to do and how do they work?

13:58

So we spent quite some time at the beginning talking about double pass heat exchangers, right?

14:03

And that's where we're going to basically have an internal pipe carrying some fluid and an external pipe carrying another.

14:10

And we identified that these are commonly constructed in a counter flow fashion because that gives us the best log mean temperature difference.

14:19

It's a rare circumstance in which we're going to see a parallel double pipe heat exchanger.

14:24

But there are a few nuanced applications that the book goes through in which they would be required.

14:29

We looked at a variety of applications for our shell and two heat exchangers, right?

14:34

And these are going to be certainly the most common that we're going to come across that they can contain either horizontal or vertical tube bundles.

14:44

Certainly in thinking more about sustainability and and charging sustainability toward the engineering domain is, is the one to deliver the use of air cooled heat exchanges, right, as an infinite medium that that's available to us to control that.

14:57

We saw that that can handle process fluids up to 400°C, right?

15:03

So it's a rare situation that you're going to have a process fluid in excess of that temperature.

15:10

We also saw that the principle of a shell to the heat exchanger can be applied to condensers and reboilers, right?

15:15

Both of which are designed to change phases and utilize or deliver that latent heat vaporization.

15:24

Now we did talk about plate type heat exchangers a little bit about their characteristics, but we did not spend a lot of time around their design simply because Professor Ritong has touched on them before.

15:36

And finally, we finished up by talking about evaporation.

15:39

So the dehumidifying process of condensing water out of air contact condensers, you'll recall that's where we had a much simpler basis of physical operation, but we were actually employing much more complex physics because we had to deal with aerosolization or actually atomization of our water droplets to control the surface area of heat transfer, right.

16:06

And finally evaporators is where we did our our big example on this where again that natural film process is what we would use for most fluids unless we very high viscosity.

16:20

So you know, above maybe 100 to 1000 centipoise, I get to be up in that quite viscous nature for a forced film or further agitated evaporator.

16:30

So that's a kind of a summary I wanted to provide.

16:32

What are the key processes and the key operations that we've discussed and learned about over the last fortnight?

16:39

You know, you can go through the book and there are a number of examples from

these different types of heat exchangers that go through the process we did on Friday that follow these different processes and just apply them to different scenarios.

16:51

I think the one on Friday was probably one of the more complex that we can think of from a heat transfer characterization point, which is why I wanted to use that.

17:02

But if you're comfortable with what we did on Friday, the process that we followed, then most of the other heat exchanger principles and designs will be a bit simpler.

17:13

OK.

17:14

So that will end our discussion on heat transfer.

17:21

And so now we're going to take a step into the next six weeks of the unit that's going to focus on mass transfer, right.

17:28

And in fact, I appreciate thank you for those of you who have signed in and are with us today.

17:34

I do appreciate that that most people are working on the submission of their laboratory one report for tomorrow.

17:41

So I, I thank you for taking time out of your day to do this.

17:46

Now from tomorrow's laboratory moving forward will be a focus on mass transfer, right?

17:52

So we're going to go through what the layout of this is going to look like.

17:56

So before we begin looking at more detailed examples, I wanted to take a step back and kind of reinforce a core concept that hopefully by this point in your chemical engineering career would have come home and that's the use of a general rate equation.

18:13

So if it were to write a rate equation for something the chemical engineer is going to

describe, right, we describe that rate firstly as a a direct function of a driving force, right?

18:27

So the delta meaning driving and the F in this case indicating a generic force.

18:32

So there is something that is out of equilibrium in our system that we're, that the system itself is trying to, to reach equilibrium through.

18:42

Now the process by which it will reach equilibrium is acting across some interface.

18:47

So here I, I say surface area or a of action because it's very important when we're starting to, to generalize the rate equation and apply it across momentum, heat and mass transfer that we always think about what is the area of action, right.

19:02

There are many surface areas in most complex systems, but it's the one that my disequilibrium condition is acting across that matters.

19:12

That would be the controlling surface area.

19:14

And then we see for every driving force there is a resistance, right?

19:18

The yin and the Yang for life.

19:20

So we multiply out front this bar.

19:23

Now if we apply this to a momentum rate equation, then we can see that viscosity is the resistant term, right?

19:31

So viscosity by definition, is the resistance to a fluid deformation at a constant velocity field and the velocity gradient in a single direction is the driving force.

19:46

Now that gets a bit more complex and you're going to unpack that and transport phenomena because in fact, when we think about a momentum rate equation, that driving force and the rate, the rate equation when it equals on the left is actually a nine component tensor.

20:02

So if that's why transport comes after all of this, but fluids introduces you to the simple Bernoulli concept of having a linear velocity.

20:10

Gradient and the viscosity is that resistance to motion.

20:15

When we look at heat transfer, we have of course what we just talked about, our overall conductive convective coefficient.

20:21

So  $U$  overall, that's our resistance to the heat transfer and it's a thermal gradient that we're acting across.

20:29

Now that we're getting into the mass transfer, we're going to introduce the concept of diffusivity.

20:35

And I'm sure that that some of this will be hopefully is going to be a reminder or refresh since since you've taken heat and mass transfer.

20:44

But I wanted to kind of reframe this in a slightly different light than you probably would have seen it until now.

20:50

So the proportionality, the resistance term will be the diffusivity in mass transfer that with units of area per time.

21:00

And it's a concentration or an activity gradient that's giving rise to the disequilibrium condition the system's trying to resolve.

21:09

Now, if you took Thermo with me last semester, one of the the points that I hope I drove, I know I drove clean into the ground, was the idea of defining Gibbs energy as the matrix that's flowing through the world, right?

21:23

And I think it's important to point out, but Bernoulli showed you right that when we have a velocity gradient that invokes, right, a pressure gradient.

21:35

So having that velocity head, we can through the Bernoulli equation we can show is that there's an equivalency to the pressure head or a disequilibrium condition and pressure that gives rise to that velocity gradient.

21:48



So I wanted to, to point out then that as the chemical engineer, we think about these three domains of momentum rates, heat transfer rates and mass transfer rates.

21:56

And we can map this into Gibbs free energy, right?

22:01

So the momentum, this maps into our VDP term, right, where we have some disequilibrium on a local or a regional scale and space in the pressure.

22:12

Our heat transfer obviously maps into our entropy as a quality of energy.

22:18

I, I know, I hate that that phrasing, but but it does kind of demonstrate it here.

22:23

It's a about qualitative metric of the energy, but it is a thermal disequilibrium condition that the system is trying to resolve.

22:31

So that then leads.

22:32

Well, if we're talking about mass transfer, why is this the hardest concept to understand and why do we tend to put it last when we introduce all of these things?

22:40

And that's because in a mass transfer scenario, we're talking about this last term and that is so the chemical potential  $\mu_i$  times the  $dN_i$  is the differential in the number of moles of species  $i$  in my system, right?

22:56

And that's where we're going to get that concentration gradient from.

22:59

So for that reason, we can look at the reason the chemical engineer studies all three of these things together.

23:06

The, the pressure gradients giving rise to it to a momentum balance, the thermal gradients giving rise to heat transfer by equation and the chemical or the concentration gradients giving rise to a mass transfer equation.

23:19

These three constitute what is the Gibbs energy driving force in the gradient right now, as we talked about last semester, the field of thermodynamics has not really caught up to thinking about this quite yet, right.

23:33

So Thermo comes about really in its current form in the 50s and 60s, this whole fantastic ability to to design and and characterize systems.

23:44

But it's then transport phenomena that that take it a step further.

23:47

And that's why we're talking about heat transfer and the concept of correlative Nusselt numbers instead of talking about it from fundamental thermodynamics.

23:56

Now what I told you last semester is that non equilibrium Thermo is one of the newly emerging fields that chemical engineers are looking at.

24:10

And that's really coming from the fact of how do we look at the transfer of momentum of heat and of molecules.

24:19

So on a concentration basis, thinking about solving for a Gibbs energy grade, right?

24:25

And that is one that has literally come about in the past five years, 5 to 10 years arguably, and will certainly be the focal point of thermodynamics research for the coming decades.

24:37

So just kind of a point there to connect last semester to what we're doing now.

24:44

And I also want to point out as we're starting to think about these unit operations for mass transfer that this is where Gibbs phase rule is really going to start playing a rule or playing a role.

24:53

So the furthest we went down the path of proper Thermo is the evaporation problem.

24:59

We did right thinking about changing the amount of mass in our two different phases and we at that point said that we're going to neglect any of that fat in the milk that will be vaporized.

25:11

We don't think that's going to be a concern.

25:12

We're only thinking about the water.

25:15

Well, when we start talking about mass transfer now we're going to have at least a couple phases and usually more than a couple components, right.

25:24

If we're doing a two component problem, that would be a good day.

25:29

So, so certainly getting into remembering that the phase rule where the number of degrees of freedom we have as an engineer is 2 less the number of phases plus the number of components, right.

25:40

This is, is where that that end term is going to blow out.

25:44

And we're going to have a lot of degrees of freedom.

25:47

So that's why we slow down during mass transfer when we start to unpack this for the unit OPS and we're going to take it at a much more case by case basis.

25:56

So something to lead off with, and I think everyone should have seen this by this point, but Fick's law of diffusion and we're going to unpack this in greater detail in the next couple of slides, but shows us that the diffusional flux which is going to be represented in moles per meter squared per second will be equal to the diffusivity.

26:17

So the volumetric diffusivity and the concentration gradient.

26:21

Now we designed this and we introduced Fick's law.

26:25

It says 1D dimensional framework, right?

26:27

So we're in the spatial dimension of  $x$ .

26:31

And you'll also note that it's a steady state presentation, right?

26:34

So back when we first introduced heat transfer, we talked about and I briefly showed on one of the slides what a non steady state framework for heat transfer look like.

26:46

And that's where we have the partial derivatives in in time and that the 2nd order partial derivatives in space, right?

26:54

That's a very, very complex problem of if I put my cup of coffee out on let's say the deck on a cold winters morning, if I want to ascribe a formula for the temperature of that coffee as a function of space within the cylinder of coffee and time, that's where I get an unsteady state problem.

27:15

And that invokes second order partial differential equations.

27:18

So it's a very complex solution.

27:20

In the same vein, that's what happens with mass transfer.

27:24

So in many of the problems we're doing here, we are simplifying it to always assume steady state and present it as such, simply because the mathematics, to address what happens in unsteady state problems kind of right now hits up on humankind's ability to write and solve the mathematics.

27:45

You know, we're obviously turning toward an era of using empiricism and using computational tools to get at these answers.

27:52

And I don't think that's necessarily the wrong thing, but certainly the big breakthroughs in science have always come from the phenomenal logical or the phenomena based understanding of what's happening.

28:04

So the ability to solve those differential equations and something that we'll look at in a minute.

28:10

It is important to reinforce that we are considering fixed law in a system that is where our interface is perpendicular, sorry, our concentration gradients or our driving force is perpendicular to the interface.

28:24

So in this case, you know, the interface might be the table that you're sitting at right now and we would be looking at the concentration gradient from your eyes down to the table, right?

28:36

So it's perpendicular to that interface.

28:39

So what are the core unit operations that we're going to go through now we've embarked upon this and how are these going to be structured?

28:46

Well, we've designed these to follow up the layout and flow of the textbook effectively.

28:54

So we're going to start by kind of bringing up to speed on, on mass transfer and relating that to McCabe's thinking on mass transfer and how he's framed this problem by starting off with gas absorption.

29:08

Sorry, absorption.

29:10

I'm going to have to overly pronounce when we say differentiate between absorption and absorption moving forward because I tend to to bungle when I'm speaking.

29:19

So we're going to start off with gas absorption and that's going to be semester weeks, 5:00 and 6:00.

29:24

So this is a process where we might have some valuable product or some negative product in an otherwise in our gas phase, right?

29:33

The example that we're going to be looking at is CO<sub>2</sub> in here, right?

29:36

CO<sub>2</sub> is a negative that we do not want in our air outside.

29:39

So we're going to look at how do we get rid of it.

29:42

And so we're going to try to absorb that CO<sub>2</sub> from the air into a liquid phase.

29:48

Right now, the solubility, and This is why we kind of start with absorption from a mass transfer perspective, the solubility of that CO<sub>2</sub> in whatever liquid phase we choose is kind of one of the critical paths that controls the operation, right?

30:04

And one of the other examples beyond CO<sub>2</sub> that you're going to get at 9:00 AM

tomorrow morning will be the stripping of ammonia from the vapour phase using liquid water.

30:13

So if you're trying to purify ammonia from its vapour, then this would be a common application for it after absorption.

30:21

We're going to take a step forward and start talking about distillation.

30:24

Now.

30:25

I want to get a sense if I just pop it into a quick forms vote for everyone, I want to get a sense of how comfortable you all are.

30:35

Stream, there we go.

30:38

So I'm going to guess that that you've not done a lot of absorption before.

30:55

OK, So I'll pop this in the team or the meeting chat if you all can take a quick vote.

31:02

I'm really interested in how comfortable you all are with the concept of distillation, right?

31:08

So absorption is going to give us a chance to kind of go around a lot of the fundamentals of mass transfer and how we're going to bring mass transfer from a theory into the unit operation.

31:17

But distillation is when it gets really fun.

31:22

OK, so, so in the context of distillations, I'll wait for you all to vote here.

31:26

But in the context of distillation, the idea here is that we want to selectively vaporize one or more of our components in an otherwise complex liquid mixture.

31:38

OK, cool.

31:40

So we're all uncomfortable with it.

31:41

That's excellent.

31:42

So we are going to be very comfortable with distillation by the time we're done.

31:46

And I promise you, it's not actually that hard.

31:47

It's a wicked cool operation.

31:51

So the idea then is that because we are trying to take a complex, you know, and I say components, but it's in fact this last example that underscores the point, right?

32:01

I know that we don't do a lot with crude oil anymore.

32:03

And then this is certainly something that's being phased out across the world.

32:07

But distillation's most common application is both in creating alcohol, so ethanol and water splits.

32:14

But that's quite simple because it's only two components, crude oil, right?

32:17

If you pull a barrel of crude oil out of the ground, there are more than 10,000 chemical components in that crude oil.

32:24

And we use a series of complex distillation columns in sequence to split those 10,000 components into the 10% that can be put into gasoline or petrol for your car, the 10% that power Qantas jets, meaning kerosene or jet fuel, the 10% that are put into lubricants.

32:42

Fun fact, every wind turbine in the world is lubricated with Exxon Mobil based lubricants, which themselves are distilled from crude oil.

32:52

So how we create lubricants in the new energy economy is a very significant question.

32:57

We're starting to talk about hydrogen as a replacement for petrol and gasoline certainly, but how we get to these lubricants we don't yet know, right?

33:05

And so that's quite an open question.

33:07

And then bitumen so that the the black tar that binds right, a lot of rock together to make roadways.

33:15

This comes from the heaviest fraction of our crude oil.

33:18

So distillation, right?

33:19

When we talk about components, multiple components, the example that we've highlighted below, this is a 10,000 plus component distillation process, correct?

33:29

And that's where distillation can ultimately go, which is why it's worth taking some time on.

33:34

We won't be using crude oil for every example, so don't worry.

33:37

But it's one that underscores the necessity and the complexity of the operation.

33:42

Now, what controls distillation, right, is a bit more complex than gas absorption.

33:47

So in absorption, we're talking about solubility as one of the equilibrium or Thermo physical properties that we introduced.

33:54

When we get to distillation, right, solubility is certainly already inherent in the solution, but we now have to think about the vapour pressure.

34:04

So vapour pressure is our second physical component that we're exploiting as engineers to manipulate the distillation performance.



34:11

And we then have to think quite a bit about the design of the column and the operating conditions.

34:15

And what we'll see is the distillation columns can be operated from a variety, you know, number of choices in terms of pressures and temperatures where you put everything.

34:26

We're then going to kick forward to weeks 9 and 10 of semester where we get to gas absorption.

34:31

So that's where a fluid, and this can be a gas or a liquid is going to be interacting.

34:36

Let's say my fluid has some valuable product or some negative product, right?

34:41

It's the same framework as absorption, but we're trying to get rid of this.

34:45

And instead of trying to strip it from one fluid phase into another fluid phase, I'm going to introduce a fixed bed of particles in which this fluid is going to interact and that I'm going to select that bed and what particles are in it so that my target component, So now we're talking back down to a specific component, is going to interact and absorb to a site on that small or porous fixed bed.

35:12

So this is also why we spent so much time talking about a fixed bed from a fluid mechanic standpoint, because this is where it's going to come back into play.

35:20

Now, the interesting thing about adsorption, right, is that it does not operate on a steady state basis like distillation, right?

35:28

So in fact, when we start out, we can say our bed might have a million sites at which a molecule of my target component can adsorb and it will continue operating until all million sites are occupied.

35:41

Meaning I have a variable driving force and a variable capacity with time right after I've filled up that bed and all those sites are occupied, at that point, the bed becomes saturated and I can no longer use it to absorb.

35:54

So this is where we typically see absorption columns running in series where one will be absorbing and the other will be stripping, right?

36:01

Removing those components valuable or in negative.

36:04

However, we want to frame them from the surface of entrance.

36:08

So the engineer then gets to select, well, what's the bed material, the nominated pressure and temperature and the flow rate that will control my operation.

36:15

And then in week 11, we're going to do 1 brief segment on membrane separation.

36:20

I also appreciate that in 1318 you're, you're leading off and you probably either just finished or about to finish quite a large section on membranes.

36:28

So for that reason, you know, AR is certainly a membrane expert and I'm not we're not going to go heavily into membrane processes, but they are a very core unit operation of mass transfer.

36:40

So kind of bringing this toward the end, then as we get kicked off, we need to think a bit about what are the different quantities and frameworks that we're going to take to frame our discussion on mass transfer or the coming six, really seven weeks.

36:57

So first is velocity.

36:59

And velocity is kind of an interesting quantity to begin with because obviously this is length for time and we'll use the variable  $U$  to describe it.

37:08

But we're going to look in mass transfer at two different velocities, right?

37:12

There's not a singular velocity of interest as we get toward a mass transfer problem because mass transfer is often happening in a system where the fluid is already in motion, right?

37:23

Then the plane of my mass transfer process can also be in motion.

37:30

So we characterize the math of this by looking at the velocity relative to that plane, right.

37:37

So just to highlight, there's not not a singular definition, not a singular definition of velocity, but you're going to see this come up in a few different contexts.

37:52

Now, if I take this picture to the right hand side, then I can imagine that I have some region on the left where I have a high concentration of whatever component A.

38:01

So maybe my little particles here are component A and the white space is denoted by component B.

38:11

So this might be CO<sub>2</sub> particles in the water or CO<sub>2</sub> molecules in water.

38:16

And I'm going to have a concentration gradient from left to right across this.

38:19

Now what we're going to be characterizing and I'll switch colors to draw this, we're going to nominate a plane right where my mass transfer process and we, we looked at earlier, we, we introduced the requirement of being perpendicular to my plane of mass transfer.

38:44

So here is an engineer I've nominated a plane that sits perpendicular to the concentration gradient in my system.

38:53

So what we think about is then we characterize the flux across this plane N, right.

39:01

So this is our flux term and that will be in moles per area per time.

39:06

So the idea is that if the plane N remains stationary, well, if I, because this this mass transfer can be, it's taken as a steady state quantity, I can simply multiply the residence time of my fluid and the total surface area that I've designed into it to determine the total number of moles that I've pushed across that plane N.

39:28

Now, because the plane N can also be in motion, right?

39:31

And this is certainly going to start coming up in things like distillation columns because that plane can also be in motion.

39:38

I'm going to define two different types of flux.

39:41

So the first flux is if that plane N remains static and I'm only thinking about the total number of moles are going to cross it.

39:47

And I simply normalize it per area, per time, city state.

39:51

But if plane N is moving right or plane J, sorry.

39:55

So if I then want to think about a flux where plane J is going to move with my fluid, then I am going to have a nominally 0 differential velocity.

40:06

Now we obviously introduce concentration which will give the capital C and that is shown in this diagram.

40:12

And then most importantly, the concentration gradient, right?

40:15

And that's where Fick's law comes in.

40:17

So putting these in more mathematical quantities, my molar flux we would define.

40:23

Capital N sub A.

40:24

So in a 2 component system, so binary framework, if I'm thinking about this massive components A&B that are going to be transferring across a station plane, then my molar flux will be defined as the product of my concentration and my velocity of component A, where my molar flux of component B is its concentration, right moles per cubic metre times its velocity in metres per second.

40:50

Now my diffusional flux, this is where I think about the flux of A.

40:58

So the flux of component A for a stationary plane less the flux due to the total flow at a volume average velocity.

41:09

So if components A&B are going to be moving potentially in different velocities, then I, I can think about, well, what's that volume average velocity and how quickly is that moving?

41:20

So the difference between that concentration at the volume average velocity if the plane is in motion relative to if the plane were static, that's where we get the definition of what ultimately becomes fixed law of diffusion.

41:34

So  $J$  sub a being that diffusion flux.

41:37

That's what we're really interested in.

41:40

I can see here the concentration of A times its component velocity minus the concentration of A at the bulk velocity or the concentration of component A at the differential velocity of my component relative to A, a volumetric flow and the the E.

41:57

If I've confused this or if I've made it confusing here, the easier way to think about this is that if we give at some point the concentration of component A to be a static quantity, then if I am trying to move mass across my plane end that we drew up top.

42:16

Well, one way that I can do that is to put a pressure gradient across it and to create a velocity field in which all of those things are going to flow from left to right.

42:25

That's one way to do it.

42:26

Or I can leave everything static and I can allow Brownian motion to take over and those particles, those individual particles of component A, again called the molecules CO<sub>2</sub> in water, those individual particles of component A are going to buy Brownian motion, find their way from left to right.

42:46

The difference between those two frameworks of me putting a massive pressure gradient to just push everything from left to right including component B and the allowance of Brownian motion for these particles to wiggle their way from left to right.

42:59

That is the diffusional flux and that's what we're trying to get at as a quantity to predict.

43:06

That's our engineering framework.

43:09

So so is Brownie.

43:12

That's a good question guys.

43:13

Is Brownian motion considered when pressure is driven?

43:17

Yes, but a pressure driven solution will always well will be a much greater contribution, right.

43:25

So Brownian motion will become negligible when we start to have forced diffusion we would call it.

43:32

So in this framework of what we're doing in in first absorption laboratory and for the next couple of weeks, we're getting into the reference kind of thinking about this relative difference so that it's only the diffuse or the Brownian motion that's going to be controlling this gradient.

43:50

And we can then relate this diffusional flux, right?

43:52

This also is the definition of Fick's law or sorry, it's defined by Fick's first law of diffusion where we introduce then that concentration gradient in  $X$ .

44:03

So this would be the  $X$  direction on top that concentration gradient we can see as our driving force and we define the diffusivity relative to that.

44:13

Now it's an important clarification here, but this is the diffusivity is a property.

44:18

So it's a Thermo physical property, but it's probably one of the most complex and elusive of those Thermo physical properties that we can measure.

44:25

And certainly it's the one that we have the least information on in literature collected to date.

44:30

This is the diffusivity of component  $A$  when it is in a mixture with itself and component  $B$  only.

44:38

So if I start adding in multiple components to this problem, the diffusivity of component A is going to change, right.

44:45

And This is why doing practical like really large scale engineering solutions, right.

44:51

If we if we don't want to use crude oil, absolutely let's think about biomass reduction in biomass to biofuels generation, still going to get a complex distribution of hydrocarbons.

45:02

But I have to think about in that 8000 or 10,000 component solution of my biofuel that I've created.

45:09

I now have to think about what is the diffusivity of my one valuable component relative to the other 9999, right.

45:19

So to, to point out that we won't go any further than this today, but getting at this diffusivity and understanding how to predict this property, what the different tools at our disposal are.

45:30

This will be our focus of the next week as we start to get into the first unit operation of gas absorption.

45:38

OK.

45:39

So in summary, much like our heat transfer unit operations rely on thermal gradient, our mass transfer operations as you would have seen by now rely on a concentration gradient.

45:52

To Gus question, it's only in rare circumstance that we're going to have a thermal or a pressure gradient driving that mass transfer process.

45:59

And the theoretical framework we end adopt to describe this uses an understanding of the flux of our individual components and we think about that flux as a mole per area per time relative to a perpendicular interface in our system.

46:17

So the molar flux we can define as concentration and velocity of our component.

46:24

But what we're going to see is that we can reduce the simplicity of or, sorry, we can reduce the complexity of some of our more complex problems by thinking about the interface moving along with us.

46:34

So in that process, we're then thinking about only the differential between the Brownian motion and the volumetric flow.

46:41

So the diffusional flux we would define as the differential between molar flux with and without flow being present and diffusivity again is where we're going to end on.

46:51

And that's what we need to pick up in the next lecture to start unpacking how we calculate that because that's obviously the controlling quantity, the resistance term that's going to be dominating our overall flux calculation.

## Lecture 14 - Diffusivity

0:00

Actually, we talked about, we were going to get into, we had some questions come up about diffusivity is a concept.

0:06

And so now we're going to go into what diffusivity means and how we get there.

0:11

Now, if you're following along in the textbook, this is the middle pen tool.

0:17

This is the middle portion of Chapter 17.

0:25

So the idea is that we will finish up Chapter 17 this week talking about the application for diffusivity and how we actually calculate fluxes, and then next week work through all of the theory and backgrounding on gas absorption.

0:39

So we're kind of prepping it this week and then we'll deploy it next week all lined up then when you'll be working on your lab reports and the comparison of the highest equations.

0:49

So with that, so in Lecture 13, that was our our first, then yesterday getting into or sorry on Monday getting into mass transfer and what we identified in that I took a little bit of time just on the slide to talk about that.



1:04

Mass transfer is a principle or relies on a concentration gradient, whereas heat transfer in a much more simple context relies on a thermal or heat gradient.

1:14

And of course, momentum transfer relies on a pressure gradient.

1:17

And part of what we talked about in that lecture, what makes mass transfer such a complex field, right, is that we can look at our, our Gibbs free energy differential.

1:27

It's proportional to a momentum driving force, a thermal driving force, and what we will call  $\mu_{DN}$  so that that molecular gives free energy.

1:42

And this is in fact the term that's controlling our discussion around mass transfer.

1:47

So as opposed to the others, we're thinking about  $\Delta\mu$  or an entropy  $DT$ .

1:52

What makes mass transfer hard is that we're getting down to the level of molecular free energy.

1:57

And that's, that's kind of where diffusion starts to come in and what we're going to talk about today.

2:02

So when we frame up diffusion, we do that.

2:04

Then on the context of the flux of components, so how many moles or molecules of my component are moving across, say an interface or a surface, we call, I've lost my pen, there we go.

2:18

An interface that we'll call perpendicular to the plane of diffusion.

2:23

How many molecules are crossing that per metre squared per second, right?

2:28

And that's where we get this, the flux units of moles per meter squared per second.

2:34

And now we define the molar flux.

2:37

The total molar flux is a product of concentration and velocity.

2:41

And we're going to look at that again today.

2:42

So very briefly, we started thinking about diffusional flux, which we're going to give the term  $J_{\text{sub a}}$ .

2:53

And we saw that related to Fick's law, first law of diffusion, where the diffusional flux is a product of both the concentration gradient and this term diffusivities.

3:03

That's really the purpose of today's lecture is to start unpacking that term diffusivity, because we see that's the proportionality constant to our rate equation.

3:14

So if we think at a very high level, what is the diffusivity, right?

3:19

We'll start off thinking about an ideal gas, right?

3:21

And I do apologise because there's a lot of math in this particular lecture, but that just stopped me and asked a question if it gets a little too complex or it's getting frustrating.

3:31

So let's go to our very simplest case of thinking about an ideal gas where we have two components, right?

3:37

And of course, we have to be thinking about a binary system because that's when we're going to have a concentration gradient.

3:42

If it's a single component system, we don't have to worry about mass transfer.

3:47

So if we're thinking about diffusivity for an ideal gas, then we can write the molar density, right?

3:54

This row of  $N$  equal to the pressure on the product of the gas constant temperature  $RT$  and of course the molar density is just the inverse of molar volume.

4:05

So it's one on V7 and for an ideal gas, right, what you would have seen in Henry's law is that the concentrations of both components are additive, right?

4:15

Because we'll remember one of our assumptions in the ideal gas equation of state is that we have non interaction between the components.

4:21

So there are volumes, so thereby additive.

4:24

Now, if we have a constant pressure and temperature condition, right, you'll note that the right hand side of this term, both of those go to 0, all right?

4:34

Which means we have, if we differentiated that condition, then the differential concentration of component A and plus the differential concentration of component B or the change in the infinitesimally small change in is equal to the infinitesimally small change in the overall mole density, which itself is equal to 0, right?

4:57

So that implies that  $D_{CA}$  is equal to negative  $D_{CB}$ , right?

5:05

So whatever I lose in component B I gain in component A for a given location.

5:11

Now if I think about my original definition for the diffusional flux, right?

5:16

And that's the difference in concentration where I'm looking at the difference between the so  $U$  naughty is the bulk velocity flow in  $U_A$  is the flow rate by diffusion.

5:32

So the Brownian motion that's added to the diffusional equation or the diffusional behavior rather.

5:41

So my diffusional flux is then equal to that concentration of component A times the difference between the total velocity of component A going toward that plane minus the bulk velocity  $U$  not and fixed log first log diffusion relates that expression then to the diffusivity of component A&B times that concentration gradient.

6:04

So that's what we talked about last time.

6:06

Now if we take this reality here on the right hand side, we're going to plug that in to this equation shown below.

6:17

So when we choose our reference plane such that we have no flow.

6:22

So if I go back for a minute, right, imagine that this entire system that that cartoon that we've looked at is in motion.

6:29

Well, if we have a static reference plane, right?

6:32

So we're holding a, a, a say, a set of inoculars, right?

6:36

Looking at these molecules in motion, we are static and the molecules are in motion.

6:41

But if we choose our reference plane to move with those molecules, right, and it's moving with the plane of diffusion, then there is no difference in the velocities, right?

6:51

Meaning that I can set each of the fluxes.

6:55

The sum of the fluxes is going to be 0 if I have no flow in the system, right?

7:00

So that that would say  $U_A$  equals  $U_B$  in that case.

7:04

So if I then set the sum of both of my fluxes  $J_A$  plus  $J_B$ , well, I can invoke Fick's law of diffusion to expand both of those terms out.

7:17

So it's the diffusivity of A in B times the concentration gradient of A minus the diffusivity of component B in A times its concentration gradient, and all of that is equal to 0.

7:31

Now, if I take this reality here from my simple mass balance and I plug it in, then what I'm going to see is that in fact, I can manipulate that to knock out that differential concentration.

7:44

So in this limit, it means that my diffusivity of component A&B is equal to the diffusivity of component B and A.

7:54

Now, I'm only showing this to to illustrate the point that when we talked about last time, one of the questions that came up on Monday is if I'm thinking about the same methane diffusing in CO<sub>2</sub>, is it always going to be the same diffusivity as CO<sub>2</sub> diffusing in methane?

8:09

Generally they should be quite similar, right?

8:13

But we have to make some assumptions to get there.

8:15

And one of those assumptions is constant pressure, temperature and density.

8:19

So I can apply the same way of thinking to a liquid mixture, but that mixture must again have the assumption of constant density.

8:28

And I can again write both of my expressions.

8:31

This would be  $J_A$  plus  $J_B$ , right?

8:36

I can and expand that out.

8:38

I have, I can write the same type of mass balance for the differential concentration for both, plug that in and what I get out of that product is that again the diffusivity equivalent A&B is equal to that of B and A.

8:52

So when I start to then take this forward, I can think about, right?

9:01

So that was just a bit of background to, to explain that that question from Monday.

9:04

But it is an important concept to realise that they they should be equal, right?

9:08

But there are differences when we start looking at changes in pressure or

temperature or changes in density that are going to arise if I'm starting to, for instance, strip one of my components out of the system.

9:21

So if I want to think about how do I write this in a common form?

9:26

Well, the total flux in my system, right?

9:28

And the total flux, we gave that name  $N_{\text{sub A}}$  or the variable  $N_{\text{sub A}}$ , the total flux if I have a fixed plane.

9:36

So now I'm taking my binoculars again and zooming in to look at the flow of molecules.

9:41

The common form to write this is that the total flux is equal for a liquid system.

9:47

So the concentration of component A times the bulk velocity  $U$  not right?

9:52

And that's going to be what we call the convective term minus then fixed first law of diffusion.

9:58

So it's the volumetric diffusivity of of that component A in whatever it's surrounded by.

10:03

That could be  $D_{AB}$  or if I might have a ternary mixture  $D_{ABC}$  times the concentration gradient of  $C_A$  or sorry, the concentration gradient of component A with respect to  $X$ .

10:19

So this here, all right below it, This is my convective component and this is my diffusive component.

10:29

Now it's important to realise this is all framed for laminar flow.

10:35

So when I start to get into turbulent flow, and often times engineers want to use turbulent flow because we can move mass, move heat, move momentum more effectively, I can't use this same diffusivity.

10:49

So I have to modify by incorporating the Eddy diffusivity, which we're going to give the variable  $\epsilon_{\text{sub N}}$ .

10:59

Now the Eddy diffusivity itself, we don't actually go into to significant detail in this textbook about that.

11:07

And in fact, it's beyond the scope of this class, but I wanted to kind of bring it into the discussion today to realize when we're calculating diffusion coefficients from theory or or basic correlations that have been derived, these are for a laminar flowing system, right?

11:25

So this this framing of of mass flux or total flux might make a bit more sense where we're thinking that total flux coming through a system for liquid is going to be whatever conductively is happening because I have a bulk velocity behind it less the brownie motion component of diffusing along that concentration gradient.

11:45

Now I can apply a similar approach to the gas systems, But if we're thinking about common forms and, and how we traditionally write these, then for gas systems, we like to think about mole fractions, right?

11:56

And so instead of a concentration of a component, we want to write it as a mole fraction.

12:01

So I can rearrange that equation to plug in for  $Y_{sub a}$ .

12:05

And I can also replace the bulk velocity by thinking about the total flux on the molar density.

12:14

So I'll represent then the total flux of component A is going to be equal to the mole fraction of component A in the gas phase times the total gas flux  $N_{less}$ .

12:25

And that's again the convective component less the conductive component.

12:29

So the volumetric diffusivity of that component times it's molar density times the concentration gradient in that plane of X.

12:39

Now in the same way right whenever I'm going to have this volumetric diffusivity as I go into turbulent flow, I can expand that out to both have contribution to both the laminar volumetric diffusivity and the Eddy diffusivity.

12:54

So for this reason and the difficulty in calculating these Eddy diffusivities, we often times will rely on experimental data, either in the literature or using Thermo physical lookup systems like the NIST Chemical Web book to get at these diffusivities for complex systems.

13:17

OK, so let's let's look at a few exemplar cases of how we can actually make sense of this.

13:22

So one of the most basic will be equimolar diffusion.

13:26

So that's going to mean where I'm going to have a net volumetric and molar flow rates are going to be 0, right?

13:32

I'm just going to look at how these components cross over.

13:35

So I can take that equation from the previous slides and assuming I have a laminar system.

13:40

And what I can do is rearrange that.

13:43

Sorry, not for the liquid.

13:44

This will be for a gas phase, so I can take that equation and I can rearrange that to integrate both sides, where here I have my volumetric diffusivity, the molar density, and I'm simply looking at the concentration gradient.

13:58

So Dya and how that varies from the interface up to the bulk, right.

14:08

And similarly, and then the point of this is that we are diffusing from some point at the interface, which we're going to call a position 0 up through the thickness of a film that I'm diffusing through.

14:30

So in that context, I can integrate that quite easily and I give that Total Film thickness a value delta, a little delta.

14:39



And you'll see that that would correspond quite easily to what we're talking about with momentum and heat films as we're flowing.

14:47

Oh, that's good.

14:48

So what is the Eddy diffusivity actually mean?

14:52

So the Eddy diffusivity is going to come in turbulent flow.

14:55

If I think about, you know, go back to that what we draw or drew early on, right, this might be my hydrodynamic boundary layer and this would be my thermal boundary layer.

15:12

Well, those are going to be relatively well established if I'm in something like laminar flow.

15:19

But once I start getting into turbulent flow, that's where we talked about eddies that are going to cross down, right?

15:27

And they're going to transfer in the context of heat transfer, they're going to more easily move energy and thermal energy toward that interface of interest.

15:39

The same process will happen because the molecules actually transferring that heat are going to be diverse in the binary system or ternary system.

15:47

So those individual molecules that are functioning in the analogue of heat transfer to move heat more effectively to the surface in a turbulent system are also themselves the mass or the concentration of their components.

16:03

So they are going to move toward the surface themselves, thereby affecting my concentration gradient.

16:09

So the diffusivity is meaning how, how much more diffusion I'm going to get based on the size and intensity of those turbulent eddies, right.

16:19

If we go all the way back to the beginning of the unit, we're refreshing our

background, or in some cases by getting a quick background for the first time on fluid mechanics, we would have also seen that the eddies themselves are a function.

16:33

All right.

16:34

So the eddies themselves are a function of the dissipation rate of turbulence.

16:46

So the faster my turbulence is going to be distributed through a system where I go from the largest eddies transferring that energy down with very minor losses to the smallest eddies, at which point they break into and distribute their energy as a heat construct.

17:04

That transfer process governs the intensity of my eddies.

17:09

So what makes it complex and in mass transfer is that the eddies themselves are made of molecules and those molecules are the interest of the concentration gradient we're talking about now.

17:22

So with our equimolar diffusion case, now we can think about this, this integration, we're going to have some bulk concentration of component A or some bulk mole fraction and then an interfacial mole fraction  $Y_{Ai}$ , right?

17:39

And those will vary from the edge of the film all the way up to the beginning of the film.

17:44

So when I integrate this, right, I can see that because I have a net flow of 0, right?

17:51

Then my total mole or my total flux is equal to my diffusional flux, right?

17:57

That's because this first term here on the top, on the bottom, this first term  $N$  right, I have no contribution from that at that conductive component because my system's not in motion.

18:10

So because these two are equal, then they're equal to the product of that integration, which itself is then the diffusivity on the film thickness times my molar density and importantly times the differential concentration between the interface mole fraction and the volumetric or bulk mole fraction.

18:30

Now, because I can multiply the product, if I go back to my my previous, the product of my product of my molar density and the mole fraction gets me a concentration, I can also present this as the volumetric diffusivity on the film thickness, right?

18:52

So how much am I trying to diffuse through times the concentration driving force?

18:57

How many molecules are at my interface relative to how many molecules are at my bulk?

19:02

And an important take away is that if I have equimolar diffusion in gas phase, my concentration gradient is a linear function right of the.

19:13

So it's driven by A rather and has a linear driving force.

19:17

So the gradient that I get out of that in terms of mole fraction is itself linear.

19:23

Now if I plot this, I can look at those mole fractions and have a binary system.

19:28

Whereas if I'm moving away from the interface, I can see the concentration of component A in a bulk phase.

19:37

Let's say here is the the bulk or the interface to the bulk.

19:41

The concentration of component A in the bulk might be around, in this case .1.

19:46

But as I move toward the interface, I had a linear driving force for flux.

19:50

So it's going to be increasing linearly until I hit the interfacial concentration of Y sub I of component A, which in this case would be 90%.

19:59

Now the absolute value in mole fraction isn't irrelevant to your right because it's going to depend on what mass and what molecules I'm trying to move around.

20:08

But the point is it's a linear relationship between the two.

20:15

So Sha, can you clarify?

20:16

Sorry, can you draw what you mean by interfacing bulk in a diagram?

20:22

Yes, actually on the next slide might be helpful.

20:28

What we're looking at here is the interface.

20:34

So if I think about, right, let's say I'll draw it on this slide, just because I have a little more room, let's say that I have flowing water, right?

20:49

So the purpose of that water is going to be to pick up a component, let's say CO<sub>2</sub> from my gas phase, right?

20:57

Well, what this is representing then is that I'm not thinking about what's happening in the water right now.

21:03

I'm just thinking about my gas phase, where here this is the interface between my gas and the liquid.

21:13

And as I go out into the bulk, right, what I'm going to see is that I should have some relatively low concentration if I'm absorbing CO<sub>2</sub> into my liquid phase in the water.

21:25

I'm going to have, if we can weaken overlay concentration on this plot, I'm going to have a relatively low concentration of CO<sub>2</sub>, but because I'm absorbing it into the water phase.

21:39

And then if I go all the way out into the bulk, So we don't know how far that's going to be, but we'll just say here is hex bulk.

21:47

Now that might be a centimetre, could be 20 centimetres or could be A10 microns, right?

21:52

It's going to depend on the degree of turbulence and how fast the system's moving.

21:57

But as I get out into that bulk phase, well, now I'm going to have a much higher concentration between the two.

22:04

Oh, sorry, a much higher concentration of component A.

22:08

Now in the case of something like CO<sub>2</sub> and error, right, If I'm drawing error out of the, the environment, right, that would be its environmental composition.

22:18

And if I'm thinking about a cylindrical column, this is going to continue for say two or wow, give it a .5 to 1M until I do the same thing on the other side of that cylinder, right, Depending on how big it is.

22:33

And So what we're thinking about here then is if if nothing is moving right, and I have a completely static system, then I'm going to have a linear relationship between the two.

22:45

What we're going to look at in a couple slides is what happens when the system starts to become in motion and I have a single component in motion.

22:53

And there we're going to start to see curvature to the interface, right?

22:56

So I can also have something like that.

22:59

Does that answer your questions now?

23:03

But that, that's what we mean when we say the interface in the bulk.

23:06

Yeah, yeah, exactly.

23:12

So that's the the film thickness that we're thinking about here.

23:14

That's the dentin.

23:18

So the important thing is that in the exemplar case of equimolar diffusion of my gas,

now I have because I have no motion volumetric or molar flow, I'm only thinking about the diffusive component.

23:31

And because the driving force of my rate equation is linearly set up, I get a linear concentration gradient.

23:40

Sorry, yeah, I know it didn't mean to confuse you with the thickness of the plane.

23:43

Definitely.

23:46

So now let's think about one way diffusion, right?

23:48

I'm only going to have one component that's diffusing in is as opposed to having both components that are going to be diffused into some interface, right?

23:59

And that interface, as we were drawing a minute ago would be something like a water phase.

24:04

So in that context, now I need to think about both the convective component and the the diffusive component.

24:12

Now I can rewrite this so if I have only or I can take this right hand side sorry and and set this equal so I can  $1 - y$ .

24:24

So they let's me bring this over to the left hand side and in that I can break this apart and integrate it in the same fashion.

24:38

So on the left hand side, I can take all of my terms that are not part of the integral right.

24:44

And I can separate this out to think only about the DY component.

24:48

So I DX is going to directly integrate it to delta.

24:50

We've already established that, but it's the DY on Y sub I or  $1 - y$  sub a that that comes in when I have one way diffusions.

25:03

My other components is not diffusing now and what happens when I integrate that of  $AD_X$  on  $1 - X$  is I get a logarithm come out and I'm integrating again from the boundaries of the interface to the bulk concentration or mole fraction in the gas phase.

25:19

So my integrands dictate that the natural log that drops out has a numerator of  $1 - y_A$  and a denominator of  $1 - y_A$ .

25:30

Right?

25:31

Now, if I simplify that and rearrange it to write it for a product of the total molar flux of component A and  $y_A$  can see a similar kind of form emerged to what we just looked at.

25:43

So it's the volumetric diffusivity component A on the film thickness  $\delta$  times that molar density.

25:51

But now instead of having a linear driving force, I have one that's proportional to a logarithm.

25:57

And because I can take this logarithm and rewrite it to expand it, and just for fun, you'll notice a lot, I can rewrite this as a log mean, right?

26:08

So I can retain the linearity on the top if I want to.

26:12

Noting again, I can multiply this through to get concentration as a driving force, but I'm normalizing it relative to the log mean.

26:20

So that's a very important take home is that when I get into to one way diffusion, I'm starting to deal with a lot of mean driving force.

26:28

So in that context, I would have component A where it's going from some bulk concentration out here and it's diffusing through.

26:38

Sorry, the way this is drawn, this should actually be B and this should be A, but I can see that I'm I'm diffusing from some high concentration of B in the bulk phase out in

or down toward the interface where it's then being absorbed or absorbing of component A which will be aggregated.

26:57

The interface where component B is then increasing in concentration as it goes outward.

27:04

Sorry, I tried to flip that diagram around to make more sense, but I screwed up the label in that.

27:09

So those are just example cases to think about how I'm going to apply a diffusivity, but let's actually look at calculating a difference as an example.

27:20

So if I have a solute that's component A and it's being and that's in let's say component B and I'm looking at the diffusion of that solute through a layer of gas and it's being absorbed and it's being absorbed into liquid at the interface, right?

27:54

So that's the example we were just talking about of water.

27:57

Perhaps it's going to take on CO<sub>2</sub> at the interface.

28:00

Now in this construct, the bulk concentration or or mole fraction of components A will be .2 and the interfacial concentration of component A will be .1 and a mole fraction.

28:19

I'm going to say what's the difference in transfer rate for one way and equimol diffusion, right?

28:45

So relatively simple, we just want to compare the two constructs we've seen together.

28:50

And if I'm absorbing both components of the interface versus just one, how does it vary?

28:56

So we'll first write for equimol diffusion  $N_{\text{sub } a} = J_{\text{sub } a}$ , right?

29:05

Because again, I have no flow and that's going to be the volumetric diffusivity, the molar density on the film thickness Times Now 0.2 -, 0.1, right?



29:19

That's my driving force linearly.

29:22

And what we've done is, is to flip that around just so we have a gradient, right?

29:27

So the way that's presented, we rather because concentration or mass will flow from a point of high concentration to a point of low concentration in this context, even though we have a an inversion in that equation so that we see a driving force that's positive because that's the direction that mass will flow.

29:47

Now the one way approach, right?

29:51

The same total molar flux will be volumetric diffusivity, mass density, same film thickness.

29:59

But now I'm going to have the natural logarithm of .9 on .8, right?

30:04

If I use that first definition.

30:07

So I'm not going to do a log log mean approach that I have to, but it's the log of 1 minus the bulk and 1 minus the interface concentration, again inverted to reverse flux.

30:19

So we have a flux toward the interface because that's the direction that molecules are going to flow in this case.

30:26

And that's so even though I'm not going to calculate the film thickness, the diffusivity or the molar density in either of these cases, if I'm going to compare simply the transfer rates of the total mole or total mass flux in both modes of operation, I'm only looking at then the ratio of the two right hand sides.

30:49

So this driving force when it's linear in equimolar diffusion is a driving force of .1 where the natural log, if I have a one way diffusion, all right, and the 2nd component is stationary, not doing anything gives me a driving force of .1178.

31:07

So the ratio between the two, the ratio between these two of point 1178 on .1 is 1.18.

31:20

So it means that one way diffusion if I don't have, if that second component B is not interacting, is not absorbing is about 18% greater.

31:36

So I transfer mass 18% more effectively in my one way model than I do in my Equimodo model or scenario.

31:50

OK, so now the purpose of this is, is actually predicting what is this diffusivity.

31:56

So if I know something about a system, we'll go back a few slides.

32:00

If I know something about a system here and I say, well, I know my concentrations, right.

32:06

In my volumetric concentration, I'm going to measure perhaps what's the concentration of the mole fraction of component at the interface at the point of absorption.

32:15

I can calculate the film thickness, I can calculate the molar density from thermodynamics.

32:21

We do need to be able to calculate the diffusivity because that's the controlling point of our rate equations, the proportionality constant.

32:27

So how do we calculate this term?

32:29

We've actually used it yet, but we're going to break this into thinking about both the gas phase first and then the liquid phase second.

32:37

So if I'm thinking about diffusion in the gas phase and diffusivity, the best practice in this place to it applies to both gas and liquid as a, a construct.

32:54

In every case, you can start to see the the difficulty in doing this.

32:58

I always want to use experimental measurements wherever possible, right?

33:03

I do not want to rely on theory if I don't have to, But if I'm going to go to the gas phase, you want to look at the basic theory of how does this work?

33:12

Then the the volumetric diffusivity of a component in the gas phase is approximately, approximately equal.

33:21

It's a value of  $1/3$  times the average molecular velocity  $\bar{U}$  times the mean free path  $\Lambda$ .

33:30

And this relationship here that we're showing is typically independent of pressure up to about 10 atmospheres.

33:38

And that that is because we have a dependency of both pressure showing up in the diffusivity term.

33:48

So we, we tend to, to not see that actually play a role until I get very dense gases that are going to start moving as though and acting as though they were liquid like.

33:57

So if I then look at each of these terms, well, the mean free path varies directly with temperature to the first power and the molecular velocity tends to vary with temperature to the .5 power.

34:09

So put together, I see a temperature dependence of diffusivity for components in the gas phase, usually about temperature of the 1.5 power.

34:19

Now you don't need to apply this and this is not a construct I'm going to expect you to use, but I want to show it to you because it's relevant to what we're going to do on the next slide.

34:29

And at this point, I also wanted to highlight Appendix 18 of the textbook also gives basic volumetric diffusivities for some simple gases.

34:40

Right now the units on these are in feet squared per hour.

34:45

Typically we use centimetre squared per second at least in in most cases, but so that's using imperial units obviously.

34:55

But this starts to give you an example, right, of how diffusive certain types of gases are.

35:01

So if we look at something like a good example of the oxygen is .69 feet squared per hour, right?

35:08

Whereas if I go to a very polar, very a lot of hydrogen bonding potential in the molecule, something like ethyl ether, there you go as perfect as .3, right.

35:27

Whereas if I go to hydrogen itself, I think we have, it's hard to say hydrogen on here itself is 2.37.

35:35

So there we start.

35:36

If you take a look at this table and just think about the molecule that, that each component or each row of the table is talking about and you'll start to get an, A sense of, of how diffusive that molecule will be.

35:51

So Gus, it is, it is technically pressure dependent, but what this relationship is showing is an approximately equal to.

35:58

So this is a way to look at volumetric diffusivity on a molecular length scale.

36:04

It's highly dependent with temperature, but it's approximately equal to these properties, which themselves tend not to vary with pressure until we get to a quote high pressure environment where molecules will start interacting with each other more.

36:19

So it it starts to drive that mean free path down and there I start to get a change in diffusivity.

36:30

So where we can actually get into proper calculating this right.

36:33

So if we need to do a calculation on our own and we don't have a reference to look at, this is the Chapman and Encog equation.

36:42

And it's in fact the only one that we will be looking at for the the diffusion of in a binary gas.

36:52

And so Chapman Enskog uses the Leonard Jones 612 potential because of course, right.

36:57

And the purpose of showing this background, we need to think about how quickly are each of our molecules moving.

37:03

And they're most importantly, their mean free path is going to indicate when they're colliding with each other, right?

37:10

Because that will dictate how quickly they're able to move around.

37:13

So if I bring in the the Leonard Jones 612 potential, right?

37:18

And you'll recall, we then have to think about the energy, right?

37:22

So in this Leonard Jones construct, as the molecules move away or closer to each other, there's a different amount or a variable amount of energy in this.

37:32

So Appendix 19 to the textbook is going to give us our relevant properties here.

37:36

So if I want to calculate with this Chapman and Spock equation the diffusivity of component A in component B, now I have and this is in centimeter squared per second.

37:47

So this is back to using SI units.

37:50

I have a proportionality constant here, right?

37:53

One of the points from the last slide to carry over is we have temperature to the 1.5.

37:58

Now I have to think about weighting.

37:59

So taking the the root mean weight of the molecular weights of both components A&B which are showing here on the top and then I have 3 parameters on the bottom to show up.

38:09

So I have a pressure turn.

38:11

So there you go.

38:11

It is actually inversely proportional to pressure.

38:14

Absolutely.

38:16

And I have two molecular dependent terms that are going to rise.

38:20

So first Sigma is the effective collision diameter.

38:24

Now, because I don't have one molecule, but I have two, I need to think about the effective collision diameter of both.

38:31

Now I can look that up.

38:33

So in Appendix 19 on the right hand side, I can look up the effective collision diameter for these molecules and they're reported in angstroms and all I do is take the average of those two.

38:44

So for whatever two molecules I'm trying to employ.

38:47

Now if I take a step beyond that, I also need to calculate this, right?

38:52

So I calculate that effect collision diameter for AB together and square it.

38:57

And the only other term I need to think about is this, this last term, and that's the collision integral.

39:03

Now to get the collision integral right, that's going to say something around the propensity, right?

39:11

And how difficult it is for these two molecules to collide and how frequently that's going to happen.

39:17

Now what I do is I first right, I can look up the collision integral in Appendix 19, right where each collision integral right is a it's a non linear look up function.

39:32

So we treat this as a look up table right when we're doing the calculation.

39:36

So I need to 1st calculate the product of K times.

39:38

So the Bolton constant times the temperature divided by the epsilon AB, right, which there they report in this table as epsilon 1-2.

39:48

But it's it's the combined epsilon value for both of these molecules taken together.

39:54

So to get that term I take the root of the product of epsilon A and epsilon B and to get each of the epsilon A and epsilon B, that's the second component of my Leonard Jones look up table here for each of these compounds.

40:09

So the the calculation process is I nominate MY2 compounds, right?

40:14

And I go to this table and I find 2 characteristic properties.

40:18

One is their collision integral and one is the effective collision diameter.

40:23

And I can then take the collision diameter and take an average of the two.

40:27

For the collision integral I grab what is that epsilon value?

40:31

I take the root of the product of both epsilon values to get epsilon AB or epsilon 1-2.

40:36

And then I take the Boltzmann constant times the temperature divided by that combined collision integral and look up if that value is say .9.

40:46

I look up what the effect collision integral is going to be.

40:49

In this case, it would be 1.517, right?

40:52

And then I can pop that into this denominator here.

40:56

And all of that then returns to me the diffusivity of that component A in component B, again, thinking just about the gas phase within this absorbing film.

41:07

So the point to take away from this slide then is, is we might do one or two calculations in the coming weeks depending on how much time we have.

41:14

But this is one of the few fundamental equations that we can look at to get the diffusivity or predict the diffusivity of a component in the binary gas phase or in a binary gas mixture.

41:26

It goes with temperature to the 1.5.

41:28

And that's from theory.

41:30

So shall the symbol  $K$  is the Boltzmann constant.

41:34

And I think I have it listed on one of the next upcoming slides.

41:39

So it goes with temperature to the 1.5 power, it goes with the averaged weighted molecular weight to the .5 power.

41:49

And the diffusivity is inversely proportional to the pressure and the both the diameter and the integral of the collision between these molecules.

42:04

OK, So if I then so, so like we talked about, we should one kind of theoretical framing



around gas predicting gas to be so that he in in one equation to predict it for a binary gas system, right?

42:18

And that's the bulk right now volumetric.

42:21

Now, if I start to look at things like diffusing a gas through a very small pore, then it starts to change a little bit because now the ability for the gas to move around and what it's going to be interacting with are not varying with the collision between molecules necessarily.

42:38

But when that pore starts to get very, very small, we can see going from left to right here, all of a sudden that pore itself is going to restrict the collision between my individual molecules.

42:49

Right now where what I can do in this case is calculate what we call the the Knudsen diffusion  $D_{sub K}$ .

42:56

So that also has a units of centimeters squared per second.

43:00

It has a proportionality of 9700, right?

43:03

We'll use now  $R_{sub pore}$  here.

43:06

This is the pore radius in centimeters, and then it goes with the square root of the temperature in Kelvin and the molecular weight of whatever molecule I'm trying to diffuse.

43:19

So in this construct, the pore diameter is what's actually controlling.

43:24

Now where this starts to come into play, right?

43:26

Historically, this is one of those points that will govern the ability to recover natural gas from a deep, say an offshore or a land based porous medium where I'm going to have a very large distribution of porous sizes.

43:43

And if I go all the way down to the very smallest pore, then they will control the molecule's ability to diffuse outward.

43:51

But it also comes into play in a more modern context if I'm thinking about injecting CO<sub>2</sub> into either a depleted reservoir or into a porous media that's otherwise unoccupied.

44:03

And in that construct, if I start to look at very, very small pore sizes, then the ability to diffuse and Dr.

44:09

CO<sub>2</sub> through that system to sequester it is going to be limited.

44:14

So as you were mentioning, right, we don't have just the option of a volumetric or a tiny pore.

44:19

We might have a a small pore, right?

44:22

And in that construct, we think simply about calculating them in resistance.

44:26

So one on the diffusivity of an intermediate sized pore, we look at the sum of the one on the Knudsen diffusion and the one on the bulk diffusion.

44:39

OK, So then finally, what happens in the liquid phase?

44:41

If I want to calculate diffusivity in the liquid phase, I can look at the very simplest level, right?

44:47

This gets a bit easier right?

44:49

Now it's worth noting that that in general, we have a a slightly poorer development of theory, right, for understanding the process of diffusion in the liquid phase, whereas we have a bit more of a theoretical construct for what happens in the gas.

45:03

But in general, my liquid phase diffusivities will be 4:00 to 5:00 orders of magnitude smaller than those all find in the gas phase.

45:14

So to calculate these the simplest approach is to use Stokes Einstein correlation or equation and not correlation.

45:24

Where the volumetric diffusivity component we see here is this is the Boltzmann constant  $K$ .

45:30

So shout to your question.

45:31

This is the value of the Boltzmann constant  $1.38 \times 10^{-23}$  joules per Kelvin times temperature and I divide that by  $6 \pi R_M$  where  $R_M$  is the radius of the molecule of interest.

45:45

So Stokes Einstein assumes a spherical molecule at times in this case then the viscosity of the the diffusing component of interest on the bottom.

45:59

Now obviously this is a very simple approximation, right?

46:04

Because very few if any of our molecules are going to be spherical or that we can approximate spherically with some accuracy.

46:11

So the Wilk Chang correlation is actually what we use more often and that's where we can apply this if we have molecular weights below 400, right.

46:21

So that tends to apply to a lot of common species.

46:24

For chemical engineers, we have a proportionality constant sitting out front  $7.4 \times 10^{-8}$ .

46:32

Here I'm looking at the molecular weight of the component of interest.

46:36

Now size of  $B$  is the association parameter for the solvent.

46:42

So that's going to vary with what is the type of solvent that I'm using and where does that show up or what is its association parameter going to be.

46:52

It goes directly with temperature.

46:54

Now on the bottom MU is the again the viscosity of whatever component I'm trying to diffuse.

47:00

And  $V_{\text{sub A}}$  was not molar volume as we might often think, but it's the molar volume of the salt use.

47:08

So the component that's diffusing at its normal boiling point.

47:13

So if I take that solid unit, whatever conditions it's diffusing at pressure and temperature.

47:18

And I think that's the molecule.

47:20

What is the it's molar volume.

47:21

If I elevate that temperature to its boiling point, I then then have this this molar volume that comes in here.

47:30

Now this this correlation can be used for systems where I have both a mix of non electrolyte components and electrolyte components in the liquid phase.

47:42

But if I have a system where I have no electrolytes, right?

47:47

In fact, I can simplify, right?

47:48

Because what electrolytes do is they're going to start increasing the interaction between a lot of my molecules.

47:54

So that's where I need to think about the association of my solid.

47:58

So if I don't have any electrolytes, then I can simplify the Wilk chain correlation and look at here.

48:04

And I have this modified pre factor on top and it's going to go with that same molar volume of the solute that I'm diffusing at its boiling point.

48:16

But now to the .589 power right instead of .6 and  $\mu$  sub B is no longer the viscosity of my soul, my soul mute, but it's rather the viscosity of water come at those conditions incentive wise.

48:34

So the diffusivity that this reports, I'm going to here have these surveys again in cubic meters per gram mole and I'm going to again report diffusivity in a centimeter squared per second.

48:55

So often times in the liquid phase then because we don't have, we're not going to be dealing with systems where I'm have a complete set of electrolytes, right.

49:04

That's a whole, that's the neuron's equation.

49:06

And we're not going to touch on that here or I'm not going to have a go all the way back to using the Wolf Chang correlation when dealing with systems where I just don't have electrolytes present.

49:16

That's where I can simplify and use this approach down here.

49:21

Yeah, So that's an excellent question.

49:23

So what this represents is the volumetric diffusivity of a component.

49:28

And you are absolutely right, it does normally need to be either AB or BA.

49:35

But I'm not specifying here whether it's a binary mixture, right?

49:40

So if I have a ternary or quaternary mixture in the liquid phase, this is simply looking at calculating the binary diffusivity of a single component, right?

49:51

What you'll note is that when we looked at the Chapman and scoff equation, this is where I define  $D_{AB}$  because now I'm I'm locking the model in to only look at the diffusivity of 1 component in another and I'm specifying it must be binary.

50:08

But in the Wilk Chang correlation, I only have a volumetric diffusivity of a component of interest, component A.

50:18

And I'm going to put that in solvent.

50:19

But that solvent might have itself multiple components that are not diffusing or absorbing or absorbing.

50:28

So some of the take home messages I appreciate were were approaching the end of the hour.

50:35

So when we look at things like binary diffusivities, you know that we ended on in the last lecture.

50:40

In both the gas and liquid phases these are equal, but when and only when the molar density of my system remains constant throughout and a slightly easier way to think about I think then we ended on last time.

50:53

When we think about molar flux and the total molar flux and again specifying laminar flow.

50:59

I have both the convective contribution and a diffusive contribution for both the liquid and the gas side where what I need to calculate.

51:10

If I know something about the concentration gradient, I must be able to then calculate what is that diffusivity itself.

51:17

So looked at three ways to potentially do that for the gas phase and three ways to do that in the liquid phase.

51:22

That depending on the type of application we're looking at.

51:25

So if we're based only on an approximation from theory, then I can see there are two molecular parameters that we're going to require.

51:35

Now I can take a step beyond this and and if I ascribe something like a molecular potential to each of these molecules, so the Leonard Jones 612, then I can put in, I

end up with similar, right, similar uncertainties, but the Leonard Jones parameters themselves coming into play.

51:55

But those ultimately flow through into dictating the diffusivity.

52:00

And then we looked at a correlation for diffusion and porous media where that pore sizes on the same length scale as the molecular free path.

52:09

So it's going to start restricting flow.

52:11

And in that case in porous media, what I'll typically do is always look at the combination and that series contribution, both the bulk of the nuts and diffusion because in the context the nuts and diffusivity is going to be constraining, right?

52:23

That's what I'm ultimately going to walk away with.

52:26

Now in the liquid phase, it's, you know, less theoretical in construct because we we don't understand it quite as well, but we have a slightly more well developed series of, of correlative approaches.

52:37

Stokes Einstein assumes that we're dealing with the spherical molecule.

52:41

So in that context, I can do a calculation for the volumetric diffusivity.

52:46

And then I looked at two variations, both the original and the modified Wilkie Chang correlation.

52:52

Where the original deals with a combination of non and partially ionized systems, the simplified version is simply no electrolytes can be present.

53:03

And if right, if you ever need to go to a pure electrolyte system, that's the Nerdist equation.

53:19

We're not going to go into it in this class because it's simply beyond the scope of what we need to.

53:23

So one of the take homes, I apologise if this added more confusion than clarity on this topic, but one of the the kind of core take home messages here is that our interest when we talk about mass transfer is thinking about how we can characterize or predict the flux of a component from whatever bulk phase I'm flowing to an interface of interest, right?

53:48

And that flux is governed by both a volumetric of convective contribution and it's diffusion.

53:53

Calculating that diffusivity from theory or even correlative theory is extremely difficult, right?

54:02

That's why we end up with this final take home message that in every case, This is why we rely so heavily on experimental data as engineers because it's very, very difficult to calculate the individual diffusivities of these molecules in a in a solution.

54:19

OK, so that's it for today.

54:21

And if you're reading along in the text, we will finish up Chapter 17 and probably get kicked into the early part of Chapter 18 on gas absorption and the application of it on Friday.

54:32

Obviously, the application of all this is going to be much simpler and not rely so heavily on mathematics, but I wanted to make sure you had seen a proper and full background at least on this before we went into the application itself.

54:46

So one question that I have before we formally wrap up, I'll throw it as a team's pole for everyone.

55:31

OK, so I just threw a hole in the the Teams chat for you and I appreciate your question because hopefully the hopefully the laboratory too that you're starting to get into will be a bit easier.

55:47

That was the design of it certainly.

55:55

OK, so, so people have done some of these approaches before when you did these before.



56:02

I'm quite interested in what degree of detail you'd gone into and which ones you saw.

56:08

I'll stop the recording here, but hang around for a few minutes if if you're able to kind of expand on that.

## Lecture 15 - Mass Transfer Rates

0:00

So I'd just take a minute to kind of clarify this.

0:03

So we looked at 2 concepts.

0:05

And the way McCabe's book talks about equimolar counter diffusion is a bit, he uses equimolar diffusion.

0:14

So in that instance, we're looking at two components, each of which are diffusing at the same rate.

0:20

So there's no net velocity in the system.

0:23

And what we would look at and think about is that this top case could be an exemplar of having two kind of vessels maybe containing component A and component B, each of which are diffusing across a small pipe at a constant pressure and temperature.

0:43

So we have an equal and opposite diffusional flux.

0:45

And obviously any time we look at a mass transfer problem, the the flow and the concentration gradient of that component is going to dictate where that mass is flowing to.

0:56

So in this case, component A would be flowing from left to right and component B from right to left.

1:02

Now the bottom case that we looked at was one way diffusion.

1:06

And so in that we had a component B that was stagnant, right.

1:09

And we're simply looking at the diffusion of component A across this.

1:13

And we see that the top example of equimolar counter diffusion was driven by a linear gradient where the bottom example of one way diffusion used a log mean gradient.

1:28

So we had a different driving force that resulted in those curvatures.

1:32

Now the whole point of the last lecture and it got a little bit messy.

1:35

I do fully agree the point of last lecture was to drive home two key points of understanding.

1:42

First, when we talk about calculating things like total flux, right, which is what we're going to give the value or the variable  $N$ , there are two components to that equation.

1:53

The 1st right that we're seeing in this first column, the first component will be a convective contribution and the 2nd is a diffusive.

2:02

So in the context of mass transfer, we're we're thinking about how we calculate this diffusive element.

2:09

Now the formal, the formula that are provided here are for laminar flow.

2:14

And that's because when we get into turbulent flow, we can't define the diffusivity as a single component, but we have to think about the laminar diffusivity plus that any diffusivity that comes later.

2:24

All right, So we're going to kick on with some of that today and actually look at practical, how do we do this in real life?

2:30

Now the methods to calculate diffusivity.

2:33

The second take home message I wanted you to have from lecture 14 was that if we go to calculate something like diffusivity, it is really, really hard, right?

2:44

And there are 6 methods that we looked at.

2:46

There is theory for the gas phase, the application of a Leonard Jones potential then going into something like porous media with nuts and diffusion.

2:55

And in the liquid phase it's much more difficult.

2:58

We looked at a Stokes Einstein correlation for a spherical molecule and then the more common Wilk Chang correlation for either non or partially ionized systems.

3:09

And we referenced the Nernst equation if you have a fully ionized system.

3:15

So the point of this is that in all of these cases, we have a very limited capacity to calculate something like the volumetric diffusivity.

3:24

So that if we're going to look at an actual engineering case where we might have four or five different components, it's going to be intractable.

3:32

And so the the focus of today's lecture is what we can actually do in practice as engineers.

3:37

So before we move on, I do want to do one quick example with all of this using that that Wilk chain correlation.

3:45

And here we want to look at what is the diffusivity of benzene in toluene and similarly of toluene in benzene.

3:52

So if we first do the volumetric diffusivity of benzene and toluene.

3:59

So I'll give that the subscript BT.

4:02

So if we look at the actual correlation here on the right, so we have that pre factor  $7.4 \times 10^{-80}$ .

4:11

Now when we're dealing with a hydrocarbon or a carbon based system, we don't have to put in this association parameter.

4:19

The value of that goes to one if we're dealing with water, the association parameter.

4:25

So I'll write this on the left hand side.

4:27

So PSI of water is 2.6, PSI of methanol MEOH is 1.9, and the PSI of ethanol ETH will be 1.5.

4:43

And if we're dealing with something like non associating solvents, so hydrocarbons, that will be 1.0.

4:53

So we take that value as 1.0.

4:57

Now the molecular weight that Italian will plug into this is 92.13 and we raise that to the one half power.

5:05

We then multiply this by temperature, which in this case, because we're at  $110^{\circ}\text{C}$ , we put that in Kelvin.

5:11

So that's going to be 383 Kelvin.

5:17

And we divide all of this by the viscosity, so 0.26.

5:22

And we note that the viscosity when it's used here for this correlation is incentive wise, right?

5:27

So that that pre factor term is accounting for the units that we need to use.

5:34

And then we multiply that by the molar volume of our component at its boiling point.

5:41

So even though the boiling point of toluene is at 110, we're going to look at the example for benzene next.

5:49

We also take that at 110 not or sorry at 80.1 it's boiling point not at our operating condition of 110.

5:58

So that molar volume of this condition will be 96 .5 and that's cubic centimeters per mole.

6:10

We raise that to the .6 power and So what this yields is  $6.74 \times 10^{-5}$  cm<sup>2</sup>/s.

6:21

Now if I do the same calculation for volumetric diffusivity, but now for the toluene in the benzene, all right, I can plug those same numbers in across and I get  $5.95 \times 10^{-5}$  cm<sup>2</sup>/s.

6:37

And So what this tells me is that my benzene, right, has a lower volumetric diffusivity than my toluene, OK, or sorry, a higher volumetric diffusivity than my toluene, right?

6:57

And and if we think about that physically, the benzene has a lower boiling point.

7:02

So it means if I'm in a system at 110° C, it's going to be more mobile.

7:06

So more molecular energy is being driven in.

7:08

So I should see for that benzene a higher diffusivity show up.

7:15

OK.

7:15

Now from the last lecture that the complexity of doing these kind of calculations and certainly if we go to that the Leonard Jones 612 potential approach that becomes very, very difficult to do.

7:28

So what we do as engineers then is to look at mass transfer coefficients as an in preference to directly calculating the volume inter diffusivity.

7:37

And it's these mass transfer coefficients that we're going to look at correlations for today.

7:42

So the coefficients themselves can extend up through transient and turbulent flow.

7:50

So as opposed to having a diffusion or volumetric diffusivity, that's defined only, sorry, that's defined only for a laminar flow system.

8:05

Our mass transfer coefficients can run up into the range of turbulence and they can deal with transient operating conditions as well.

8:13

So I think one of the questions that came up during the discussion on Wednesday was what do we mean when we talk about this film thickness and what is that film?

8:22

We're going to unpack that a little bit more today and then apply it into an absorption column so we can start thinking about our calculation for laboratory #2 So what we're plotting here.

8:35

So first we'll define our mass transfer coefficient, and we give that the little  $K$  is the variable.

8:41

So we define our mass transfer coefficient.

8:43

We can think of it if we're first looking at the simplistic case of an equimolar counter diffusion basis, it's  $K_{C,A}$  and we give it a subscript of  $C$  to note that we're talking about concentration and not mole fraction is defined as the diffusional flux  $J_A$  on the concentration driving force.

9:02

So it's the concentration of the interface less the volumetric concentration.

9:07

Now this  $C_A$  that's drawn out here, it's important to note, right, that is not the maximum concentration in the central line.

9:16

So we can't draw a perfect analogue between momentum and mass transfer in this case.

9:22

But CA actually represents the volume averaged concentration and that's where we see here.

9:29

It's this dash line that sits underneath that peak concentration in this flow field.

9:34

So if I'm looking at a flow kind of diffusion around that at the periphery.

9:41

So if I start to to then unpack that and I substitute in my description for the diffusion flux  $J_{sub A}$  I can expand that out and I see my concentrations ultimately cancelled right as a definition.

9:55

So in the simplistic basis of economic counter diffusion, I define my mass transfer coefficient as the volumetric diffusivity on the film thickness  $\delta$ , right?

10:08

And so this film thickness here, this is what we're looking at is the film.

10:12

Now this tends to go with the momentum boundary layer, right?

10:18

And we're going to look at a further example of that later on.

10:23

But it represents the the film in which I'm actively diffusing, right?

10:26

And so at what point is that that gradient going to be linear coming into my interface?

10:32

Now as we were talking about earlier, it's more common that I want to represent this on the basis of a molar system instead of a concentration itself.

10:39

So we tend to refer to then the mass transfer coefficient of a gas phase is case of Y and the mass transfer coefficient in liquid is case of X.

10:49

And if we look at those same definitions and same expansions, we can see here a relationship between the two, where my molar basis for mass transfer coefficient is simply related by to the concentration basis by rows of  $M$  which is the molar volume in my system.

11:07

So how many moles per cubic metre am I holding right And I can expand that out.

11:12

This is for an ideal gas expansion, ideal gas equation and state expansion.

11:18

And here I just have to look at the Thermo physical calculations specifically for my system, right?

11:23

So the purpose of today's lecture is that we are going to get into how do we calculate case of X, case of Y and ultimately case of C through correlations.

11:34

Because these then allow me to get at my target variable, right, which is going to be the number of moles, right, that are going to be transferred either  $J_{sub A}$  or  $N_{sub A}$ .

11:48

But how many moles per second am I per interface am I putting across when I'm doing this transfer?

11:58

OK, now going from a simplistic system to a where where we don't have any mass transfer limitations on the other side of our interface, we're going to take a step up and think about, well, what happens if I have say, a very large gas phase in equilibrium with a large, say, water phase, right?

12:17

And this is where I'm going to have to think about diffusional limitations on both sides of that interface.

12:22

So historically we would call this two film theory and it comes from Whitman in 1923.

12:28

We operate under the assumption that we are at equilibrium at the interface.

12:34

And we can draw a lot of analogues to heat transfer here when we try to think about the cumulative mass transfer resistance on one side of the fluid interface and on the other side of the fluid interface.

12:46

So one of the unique aspects of mass transfer that does not correspond to momentum or heat transfer is that in mass transfer, I can have a concentration discontinuity at that interface, right?

13:00



So before we get into the maths on this slide, let's look down below of what we're actually talking about.

13:06

If I'm looking at a distillation system and remembering that that my mass is always going to flow from 1 concentration a high concentration to a low concentration, here I can see I'm losing mass of component A out of the liquid phase, but I'm gaining massive component A into the gas phase.

13:23

But at that point in the interface, the physical interface, I have a discontinuity in the interfacial concentrations.

13:31

Now I'll get to YA star in a minute.

13:36

If we're going to look at absorption like we're doing in laboratory #2 right there I have, I'm losing mass of component A out of my gas phase, and I'm picking up massive component A into my liquid phase.

13:50

So now I have these two offsets.

13:52

I can see two driving forces that are going to emerge here, right?

13:56

I'm going to have a case of X that's controlling my liquid phase diffusion, right?

14:01

And that driving force or that that resistance is related to the driving forces of X of A and XA at the interface.

14:08

And I'm going to have a case of Y that's related to the two driving forces of the gas phase concentration difference.

14:17

Now the way that we ultimately have to relate these is through the use of YA star, right?

14:24

So what YA star is defined by is the composition of vapor that would be in equilibrium with the liquid perhaps if I took the liquid phase composition XA.

14:39

So if I plot this in in Y versus X phase, I can see here this is my YA concentration.

14:45

So that's the volumetric concentration like gas phase  $X$  of  $A$ , this is the volumetric concentration of component  $A$  in my liquid phase.

14:56

And if I think about well on a single equilibrium line for that system, what is the vapour phase equilibrium concentration going to be, right.

15:06

And I can then see here is my interfacial concentration of of the on the liquid side of the film and similarly for the gas side of the film.

15:18

So going up to the maths for a minute, then the principle in equilibrium at these interfaces means that the rate of mass transfer is going to be equal across the not.

15:29

The absolute concentration is continuous, but the rate of mass transfer from one interface to the other side of the interface is equal.

15:38

So  $K$ , the liquid side case of  $X$  times its driving force is equal to the my mass transfer coefficient for gas case of  $Y$  times its dropping force.

15:48

But as an engineer, what I want to do is be able to represent these together and think about what the combined transport limitations going to be.

15:57

So there we introduce then an overall mass transport resistance, large case of  $Y$ , right?

16:03

So my overall mass transport resistance, that's why I bring in the definition of  $Y_A$ , star and  $Y_A$ .

16:11

So if I know two things, what is the volumetric concentration of component  $A$  in my gas?

16:16

Right.

16:17

And I'll write that here Component  $A$  in the gas phase and  $Y_A$  star is the equilibrium concentration of component  $A$ .

16:38

If if the gas were sorry for the liquid in equilibrium, sorry the vapor that would be in equilibrium with that liquid.

16:52

So I look at my liquid composition and figure out where is the equilibrium going to be there.

16:57

So if I want to then look at the resistances together, I can take these expressions and solve for case of Y large case of Y, right my overall mass transport resistance.

17:10

And when I do that, I get this intermediate expression here where I start to see some things cancelling out.

17:15

And in fact, what I get to is these these right hand terms cancel out.

17:21

So I have one on case of little case of Y and on the left hand side, what I ultimately reduce that to is this, this difference between the two  $Y_A$  star and  $y_A$  and  $X_A$ , these become the slope, this M the slope of my equilibrium curve down here, right, which I can see geometrically.

17:46

So it's the, it's the triangle that we cut through that point.

17:49

So ultimately then in thinking about the overall mass transport resistance, I calculated in reciprocity just like I did with my overall heat transfer coefficient, but it, it's modified by the slope of that equilibrium line.

18:08

So how do I then get at calculations of either an individual mass transport coefficient, mass transfer coefficient, or the overall mass transfer coefficient?

18:18

Well, much like complex heat transfer, right?

18:21

And we've just seen if the mass transfer coefficient is certainly related to diffusivity, right?

18:27

It's diffusivity on that film thickness, but it means it's going to vary with the components that I have in my system and the type of unit operation that I'm going to do.

18:37

So I can't simply predict this from theory when I start to get into complex non equilibrium systems or those where I have too many components to solve all at once.

18:48

So often times then we're going to go to experimental measurements to obtain this.

18:53

And what we'll look at in the latter half of today are the correlations that I can use to to get at it.

18:58

So the first approach experimentally will be to to choose a device with a known contact area.

19:05

So I don't have that separation effect in the boundary layer, right.

19:08

You'll recall that the boundary layer separation was when, right, we had that layer going up.

19:14

So the boundary layer is going to continue increasing and then it's some, it's some critical path.

19:21

This is the momentum boundary layer.

19:22

At some critical path, I'm going to have a breakdown in all of those turbulent eddies.

19:26

So I have that laminar boundary layer followed by a transition or buffer zone into the the turbulent region.

19:33

So I don't want that kind of boundary or boundary layer separation because I need a, a known contact area.

19:40

So a good example is this wedged wall column that we'll see here, right.

19:44

This uses gravity driven liquid flow going down the sides and it allows me to occasionally, I won't push this into a turbulent region where the uncertainty from that, and if I do the uncertainty is the turbulence will create a bit of ripples in the surface of the liquid.

20:03

But I have the benefit of knowing exactly what that contact area is.

20:08

And so if I'm measuring the composition of a component in the liquid phase at the end, I know the area, I know how much is there.

20:14

And by the principles that we've talked about heretofore, I can get at how much transferred over so I can calculate the mass transfer coefficient.

20:23

Now, when we start thinking about unit operations and what we're going to get into later today with the gas absorption ring, are things like packed towers, right?

20:33

And so often times, if I don't know the mass transfer coefficient or if I'm working on a principle or a process that's not been specifically designed in the past, a new process, what I will typically do is operate that unit over the maximal range of  $K \cdot A$ , right?

20:52

So whatever I can get in terms of the bounding conditions.

20:55

And then I'll look at correlating each the effective area and the effect of the overall or individual mass transfer coefficient.

21:03

And it's often times that I'll try to take the unit operation to a limiting condition where I either have a very small amount of liquid transfer or a small amount of vapour transfer to look at the minimal resistance that each phase can provide on its own.

21:20

Now in practice, most of the time when we design mass transfer operations as chemical engineers, we're doing that on the basis of things that have already been designed before.

21:29

We're just trying to modify or apply to a new scenario.

21:32

So the approach that we're going to take here, you're going to love, right?

21:36

Because if you didn't get enough of the Nestle style correlations in the past two weeks, we're going to have a whole lot more coming at you today.

21:43

But in fact, they're not all that different.

21:46

So I can use the Nusselt style correlations to try to solve directly for this mass transfer coefficient.

21:53

But I have to modify them in two ways.

21:56

The 1st is that coming from a heat transfer to a mass transfer, I need to introduce a new, a new dimensionless quantity that we'll call the Schmidt number.

22:06

You probably have seen mass transfer before and that is the momentum diffusivity on a molecular diffusivity, right?

22:12

So I can see here kinematic viscosity on the volumetric diffusivity on the bottom or related, if I expand out that kinematic viscosity, I get the dynamic viscosity, the density and the volumetric diffusivity.

22:30

So for the Schmidt number, this tells it's analogous to the Prandtl number, but for mass transfer operations.

22:36

So I will tend to see very low values if I have a high volumetric diffusivity.

22:42

So gases, the Schmidt number should be on the order of .5 to two.

22:45

But if I'm in a liquid where I have a very, very, very low diffusivity, then I'll expect Schmidt number in the liquid range to go from 10 to the two up to 10 to the five.

22:56

So the Schmidt number then replaces my Prandtl number for mass transfer operations.

23:05

But I also need something to replace my Nusselt number, right?

23:08

And that because we're no longer solving for a convective transfer coefficient, we want to solve for the mass transfer coefficient.

23:16

So the replacement there will be the Sherwood number, which is defined as the concentration definitions of case of C.

23:23

If I go back a few slides, I can see if I can get that case of Cl, can get it KY or KX, right?

23:32

So KC is my target variable.

23:34

It is defined as that mass transfer coefficient times the characteristic length or diameter of whatever unit operation I'm studying.

23:43

So if it's transferred across a cylindrical pipe, transferred across a, a spherical, say a sphere that's immersed in dissolving in liquid water, right?

23:55

That's my characteristic length and the volumetric diffusivity.

23:59

Now if I'm going to be looking at something like laminar flow across a flat plate, I think that was the first example we introduced in the heat transfer portion of the unit.

24:13

There I can relate the Sherwood number to the so .664 is the pre factor Schmidt number to the one third and the Reynolds number to the one half where BI calculate the Reynolds number for the length of that plate.

24:30

Now it's uncommon that I'm going to be doing calculations on a flat plate, but I want to think more now about pipe work and flow through pipe work as well as dissolving solids and flow around external objects, which is where we're getting to absorption.

24:44

So if I have laminar flow, I would note that the Sherwood number obeys a theoretical limit for internal ducted or piped flow 3.66.

24:53

And that should be a trigger in your mind, right?

24:55

But you saw that for Nussel correlations as well.

24:58

So here my Sherwood number is going to for laminar flow is going to be a 1.76 is the pre factor.

25:06

And then I take the product of  $\pi D L$  times the Schmidt number times the Reynolds number for that pipe flow all to the power of  $1/3$ , right?

25:14

And if I expand that out just for simplicity, I can see that a lot of these things, right?

25:20

My viscosity is going to cancel, my density is going to cancel.

25:26

And in fact, it it simplifies quite a bit.

25:30

Now one of the advantages that we've talked about that we need to do is, is getting into turbulent flow.

25:36

That's where engineers want usually to operate because we can be more effective.

25:42

So limiting value means that in fact we'll see it on a slide or two.

25:46

Limiting value means the lowest possible.

25:54

So you'll remember the Nusselt number if I take it in a laminar flow for a pipe, the lowest number of the Nusselt number is usually taken to be 3.66.

26:02

And the same kind of heuristic or rule of thumb applies for the Sherwood number if I'm an internal flow.

26:08

So now the the concept of the mass transfer coefficient and the use of both Schmidt and Sherwood correlations allows me to step in or we'll call Nusselt style correlations allows me to step into a very turbulent flow regime and still get reasonably accurate mass transfer coefficients.

26:25

So here we're looking at more Nestle correlations.

26:29

So the first I can actually take a direct adaptation of Seeler tape, which is of course my premier correlation for turbulent heat transfer.



26:40

But I replaced that Prandtl number with the Schmidt number of the  $1/3$ , right.

26:43

Otherwise the form of the correlation is the same.

26:46

Now across multiple studies of mass transfer, these exponents have been refined a little bit.

26:55

So in practice, although the Schmidt number is classically working around  $1/3$ , I can see that we can refine this if we're at Schmidt number is below say 400 and to have Reynolds number to the .81 power and Schmidt number to the .44 if I get into high Schmidt number regions.

27:11

And that's where I'm going to be thinking a lot about liquids, right, because it goes with the inverse of the volumetric diffusivity.

27:18

There I have a, a slightly different prefactor.

27:20

So point OO96, my Reynolds number goes up a little bit.

27:24

My, my Schmidt number stays around that one third mark.

27:29

Now for both scenarios where I have flow normal or external.

27:33

So now I'm going to step from the inside of the pipe to the outside of the pipe.

27:37

It's very similar approach what we looked at before.

27:39

I have a single cylinder.

27:41

I can define that reasonably well.

27:43

Prefactor is .61 rels to the half, Schmidt to the  $1/3$ .

27:48

If I start to think about a tube bundle that I'm transferring mass to, then it becomes a little bit more difficult, right, Because there's not a whole lot of data to inform that external flow across a tube bundle.

28:00

So here we say it's approximately 1.28, but again, Schmidt to the one third Reynolds.

28:05

Now the .4 and the analogue that we want to draw here is we can compare.

28:11

This is the Colger J factor and it's analogy, but it is effectively goes with the Sherwood or the Nestle numbers just to give us a kind of a benchmark and we can plot that against the Reynolds number.

28:25

And here I'm looking at the mass transfer solution shown in the solid black and the heat transfer solution shown in the dashed black curves.

28:34

So what this tells me is if I want to step outside of a a region, say here's my my 2100 value issue.

28:42

If I want to step outside of a region where I I have laminar flow right where I can calculate and have a definition for my volumetric diffusivity.

28:51

This, the style of using Sherwood correlations that themselves are Nestle correlations, allows me to step into that highly turbulent flow regime space whilst maintaining a a reasonably good degree of accuracy in obtaining those mass transfer coefficients.

29:09

Similarly, if I continue on from it thinking about external mass transfer coefficients, I can look at what happens with an isolated sphere, so just one sphere hanging out on its own.

29:19

And then maybe that's like something like a, a fish food, right, That we've dropped a pellet into water and I want to think about how quickly that's going to dissolve and release its mass into the continuous water phase.

29:31

So if I have a reasonably low Reynolds number, I can use this correlation here.

29:36

We we start off at two point O and then we add to it based on the Reynolds of the water and a Schmidt number again raised to the one third.

29:46

If I want to turn up the turbulence, I use a similar formulation.

29:49

I just have to take the entire square root of the entire thing and my pre factor goes to four.

29:56

But where this is going to become more impactful is when I think about a packed bed of either spheres or cylinders, right?

30:02

And that should start to sound really familiar if you're working on the laboratory right now.

30:07

So there my Sherwood number will go with the pre factor of 1.17 times Reynolds to the .585 and the Schmidt to the one third.

30:18

Now it's valid when I have between 40 and 45% packing or sorry 40 to 45% void fraction and if I get into a region where I have either cylinders, sorry rings or hollow particles.

30:32

Some of the other packings we'll look at in the final slide.

30:34

Then I need to look at different correlations.

30:36

But if I have cylinders or spheres, I can use the diameter of either of these to inform the definition of both the Reynolds and the Schmidt numbers, right?

30:46

And then finally, I can take a very similar approach if I think about rising bubbles or falling droplets, right?

30:51

So rising bubbles, obviously that would be something like the blowout in the Gulf of Mexico where we had to think about what are those, the gas bubbles rising and how quickly are they going to transfer mass and transfer that methane into the water phase?

31:04

And then what are the eco toxicity impacts of that?

31:07

That's where we start to.

31:08

We can use this same approach and use a Sherbet correlation to think about the mass transfer coefficient on that gap side.

31:17

OK, So what we're showing in the insight here, this comes from the textbook and so it's not something we need to go too far into.

31:26

But I just wanted to highlight that the particle diameters where these begin to, to really if we're looking at the top, this plotting the mass transfer coefficient as a function of the particle diameter, all right.

31:38

And where this starts to really deviate from theory is when I get into diameters that are around, right?

31:46

I can plot here that are around 3:00 to 4:00 microns, right?

31:50

So the point at which dust starts to feel its density and its buoyancy force, that's the point at which these particles and if I'm thinking about the mass transfer coefficients will diverge from their baseline.

32:05

So when we then let's do a quick example of what we can actually calculate together using some of this.

32:14

And let's think about the effective thickness of a gas film.

32:18

So we're going to consider a 2 inch diameter welded wall column with a Reynolds number of 10,000, a pressure of 11, atmosphere temperature of 40° C And we want to think about evaporating either water or ethanol into the air, right?

32:32

Where I can say the volumetric diffusivities at those for water or  $.288 \text{ cm}^2 \text{ per second}$  and for ethanol is  $.145 \text{ cm}^2 \text{ per second}$ .

32:43

So for error, the first calculation I need to do is to look at what's my Schmidt number.

32:50

So that Schmidt number will be and I need to put now the viscosity of water is going to come into play.

32:56

So point OOO, 186 Pascal seconds.

33:03

All right, And I'm going to look up, well, what is my density for the air water system point OO 129 grams per cubic centimeter multiplied by the volumetric diffusivity of the water, so  $.288\text{cm}^2$  per second.

33:25

And that gives me a Schmidt number for this air water system of .573.

33:31

So I can then calculate the Sherwood number as .023 times the Reynolds #10,000 to the power .81 times the Schmidt number .573 to the power .44.

33:49

So that gives me a Sherwood number now of 31.3.

33:53

So then go back to that.

33:55

That theory we had on the very beginning.

33:57

We can see that the concentration based mass transfer coefficient will be proportional to the volumetric diffusivity on the film thickness.

34:09

And the Sherwood number by definition is that mass transfer coefficient times the characteristic length or diameter of my system divided by the volumetric diffusivity.

34:22

So by commutative, I can think the Sherwood number is then equal to the characteristic length of my system divided by the film thickness.

34:31

And that's that controlling film that we looked at a backup for a minute.

34:38

This is the film thickness, little delta that we're trying to get at, OK.

34:45

So I can directly relate that Sherwood number to both the characteristic diameter and that film thickness.

34:51

And so for a Sherwood number now of rather I can rearrange this to say film thickness  $\delta$  is going to be two point O inches, and I can report the whole thing in inches to keep it consistent, divided by the Sherwood #31.3, which gives me a film thickness now at point O64 inches.

35:16

Now if I do the same thing but for the error ethanol system, similarly, my Schmidt number will be 1.14, my Sherwood number will be 42.3, and the film thickness is point O47 inches.

35:38

So what I can see is that if I'm in an air ethanol system, the inches and slightly the film thickness that's controlling the diffusion is going to be much thinner than for an air water system.

35:53

And that that does kind of make sense, right?

35:55

Because ethanol is going to be much more apt today for us and given these conditions.

36:02

So when we start to then apply this into Laboratory 2 and what we're going to kick off and spend the entirety of next week on is gas absorption theory, right?

36:11

And how we do this in practice.

36:13

So what I wanted to end on today was then thinking about this, we're going in to use the Sherwood correlations.

36:20

We can think about what's that the thickness of the film controlling that rate of mass transfer, right.

36:26

And using this approach, I can get at what is the mass transfer coefficient on a concentration basis and then relate that back to either the vapor or the liquid mass transfer coefficients as necessary.

36:40

So in the gas absorption unit operation right here, what we're showing is that a classical packed tower apparatus and the goal of this as you would have seen from the laboratory on Tuesday, is to typically strip a component from a vapor phase that's we don't want.

36:56

So a very good example is that if we have natural gas maybe flowing through our Bunbury to damp your pipeline or if we're taking raw natural gas out of the ground, we cannot have CO<sub>2</sub> or H<sub>2</sub>S in it because it will engender a corrosion list.

37:13

So to get rid of that, we can flow usually of water or or a similar liquid through a packed column and that will selectively absorb whatever antagonistic component we're trying to get rid of.

37:27

And then we can recover that solute, say, CO<sub>2</sub> or H<sub>2</sub>S in this case, by either distillation or running a reverse operation using the same kind of packed tower that we would call either stripping or desorption in the pack tower itself.

37:42

Here, what we're looking at are just 7 examples that are also shown in the laboratory materials, but what we can fill this tower with.

37:50

So the simplest of all would be the rash rings, right?

37:52

And that's where I can start to look at some of the mass transfer calculations today because I have a classical Sherwood number correlation for those that's using this packed bed with cylinders, right.

38:08

So with that, hopefully that's added a bit of clarity to what we talked about on Wednesday.

38:14

The main take home of Wednesday was that getting directly at volumetric diffusivity can be very difficult and in particular, it's not well defined as we get up into the turbulent flow regime.

38:25

So instead, what we want to do is replace that with something that scales well into the degree of turbulence.

38:32

And what we'll introduce to do this is a mass transfer coefficient, right?

38:36

We report these as other K<sub>X</sub> or K<sub>Y</sub>, and they relate directly to the volumetric diffusivity on the thickness of my diffusion layer that we just calculated doesn't examine.

38:49

So if you weren't tired of Nestle correlations yet, there's more to come and they're going to apply to the lab.

38:57

But we have our Schmidt number that's been introduced and both the Sherwood number one of the the take home messages is just as our Nestle correlations usually have a proportionality constant Reynolds to the power crammed to another power in mass transport.

39:11

We'll use the analogue or Sherwood number related to its coefficient Reynolds to a power and now Schmidt add to another power.

39:19

So the quiz for this week to to wrap up the quiz for this week is not hard.

39:24

I appreciate that lab one was a lot of work for everyone and you're already staring down Lab 2.

39:30

So I didn't want to make the quiz too difficult.

39:31

There are no calculations.

39:33

It's just theoretical questions to make sure we're all on the same page in terms of mass transfer.

39:38

And then we'll we'll actually use the Lab 2 exercise for a lot of the the calculations before we get kicked into this solution.

39:46

All right, so do you have any questions?

39:49

Hopefully that cleared things up a bit from Wednesday.

39:55

Well, sha, you weren't surprised on my Thermo exam.

39:58

I hope.



39:58

Do you know how I write exams?

40:08

Good to hear.

40:09

Yeah, I don't like surprising anyone on exams.

40:12

I don't think that's fair.

40:15

Yeah, I can't make it quite as easy, but.

40:19

But it'll be close.

40:20

I don't like surprises during the exams.

40:26

OK, well if any questions come up, I appreciate it's Friday afternoon.

40:29

Everyone's probably decided to get on to the weekend.

40:31

So if any questions come up over the weekend, feel free to throw it on the team's chat.

40:37

Otherwise, I will look forward to seeing you all or talking to you on Monday afternoon.

40:43

OK, bye.

40:43

Bye.

## Lecture 16 - Gas Absorption

0:00

18 in sequence.

0:01

Now, obviously absorption is the Laboratory 2 that you're working on right now.

0:05

So before we go too far, I wanted to throw a poll out to everyone, so I'm just going to send it into the meeting chat now.

0:13

I'm interested to see how you're finding Laboratory 2 so far with a particular comparison to Laboratory 1.

0:22

So I appreciate the incentive to vote for far too difficult.

0:27

But I'm I'm truly looking for your honest opinion on this, bearing in mind that we're going to go through some of these basics today and and unpack it a bit much.

0:40

Yeah, Portia, that's really good feedback.

0:41

Thanks for that.

0:43

Are you finding, we will make the questions more specific.

0:45

Are you finding enough support using the team's chat channel with David and Angus?

1:03

OK, well, definitely, you know, flag me into those questions if you're if you're finding that you're not getting enough specificity, part of the the exercise is in the exploration of it.

1:16

So when we go through the lecture content for this week, that should start to help unpack a little bit what we're talking about.

1:23

But if there are specific questions, we can raise them during or after lecture as they come up.

1:27

So please use the name chat and keep going.

1:30

So in Lecture 15, so on Friday for those of you who who were able to to join us, thank you for that.

1:38

The mass transfer coefficients that we started talking about, we used the little variable  $K$  or the yeah.

1:46

Do you have a question you guys, so we can use the, the OK, a little variable  $K$  for a, a component mass transfer or the big variable  $K$  as we'll look at in a little bit for an overall mass transfer resistance.

2:04

And we're looking at using mass transfer coefficients to effectively replace the use of volumetric diffusivity.

2:11

They can be a bit difficult to do so in the definition of these mass transfer coefficients we we can fundamentally think of them as that volumetric diffusivity on the thickness of the diffusion layer itself.

2:24

And these transfer coefficients are related to the diffusional flux divided by the difference between the interfacial concentration of component A in the vapour phase less the volumetrically averaged concentration of the component, so the driving force.

2:43

Now we also then looked at, we're going to see an application for absorption today, how we can bring both vapour and liquid side transport resistances together.

2:53

And we summed these in reciprocity much like we did with an overall heat transfer resistance.

2:58

And then of course we went through all of the Nestle correlations that can apply when we bring them into the context of mass transfer.

3:08

So we have can use the exact same muscle style correlations and apply them to mass transport problems, but we need to replace the Crandall number with the Schmidt number.

3:16

So that gives the same momentum diffusivity on a molecular diffusivity.

3:21

And we replaced the Nusselt number itself with a Sherwood number, which is the convective mass transfer on the diffusive mass transfer.

3:28

So the analogue in that is thinking about convective heat transfer on conduction, right?

3:33

So that kind of points out that if you hadn't taken it home from heat and mass transfer that the diffusion process is the mass transfer equivalent to conduction.

3:45

So generally these equations that we looked at will take the form of the Sherwood number being equal to some pre factor C times the Reynolds number to the nominal power typically around .8 and the Sherwood or the Schmidt number typically to around the power of about .3 or one third.

4:05

So on on Friday then we did start getting into what is an absorption column or a packed tower.

4:12

And so obviously this is what we're looking at within our Laboratory 2 this week.

4:16

So that the timing is we're going to set up the fundamentals and then get kicked into how we actually do the maths and the work behind this.

4:22

So in general, right, these towers, if you've not had a chance or spent much time with the lab yet, the idea is that we have gas flowing in from the bottom.

4:31

There is some nominal section of packing that can be a variable height, right, and a diameter that's a variable where we can choose the diameter as the engineers.

4:39

And we're going to be looking at Wednesday on what some of those design characteristics are.

4:45

The gas will flow upward and then leave through the top of the column where liquid will flow in the left hand side and be sprayed down into the packing material and leave through the bottom of the leg.

4:57

And the the concept behind this then is that we're able to remove or transfer some component of interest from the vapour phase.

5:06

So in the case of our lab, is CO<sub>2</sub> transfer a component from the vapour phase into a liquid phase, right.

5:12

In our case, we're using water.

5:15

So one of the questions in thinking about pack towers then is what type of packing material am I going to dump into this thing to be impactful?

5:23

And, and the core design principle behind pack towers is that the material should be cheap, right?

5:30

They should be cheap and they should be inert and readily plentiful.

5:33

So if you think back to the early 20th century, as engineers, we're just starting to do what we now call chemical engineering.

5:40

You know, they would use things like crushed stone or ceramic spheres, right?

5:46

So this is where the ideas came from is let's take a gas that contains something we don't like, get a whole bunch of inert crushed stone, dump it into a column to create a whole bunch of area.

5:55

And we'll we'll run liquid down that as we run gas up the column using the the buoyancy force to drive it.

6:02

And there we go.

6:02

We can transfer mass from one phase to another.

6:05

And what we're looking at today is simply the evolution of that thinking.

6:08

So when we started getting away from and ceramic stairs, that's where we started using ratchet rings and barrel ceramic and barrel saddles.

6:19

So these are not commonly used anymore.

6:21

I appreciate we use ratchet rings just because they're quite simple, so we can do a little more design in thinking about them than we could with something complex and it's flourishing.

6:30

But really where we've gone to that in the past, more more recent decades are looking at these right hand, right hand design.

6:41

Now this table here comes from chapter 18 in the book.

6:44

And we can see for the different types of packing material that I can dump in from the top of the column to fill it up.

6:50

These can be made from either ceramics, metals or plastics.

6:53

So in any case, we want something that's inert.

6:55

We have to make sure that that choice is appropriate.

6:58

So for instance, if we're thinking about a toluene and a xylene system, we would probably not use a plastic, right?

7:05

That would be dissolved in that we can choose the size of what we're looking at and most importantly, right each of these.

7:15

If we look at this right hand column from top to bottom, we can see that if we imagine ratchet rings are probably the first incarnation of proper chemical engineering trying to do this, as we go down that column toward more common and advanced materials, we get a much, much higher porosity.

7:31

So the idea is that we need something that generates a large porosity with a very large contact area.

7:39

Now, one of the constraints that's placed on this process is we don't want to pay a lot of money or use a lot of energy to power up this kind of system, right?

7:47

So the water pumps and the gas compressors that are going to be required, well, if I'm going to engender a huge pressure drop across this, this packing material that I need to buy a really big compressor, right?

7:59

And big compressors cost more money.

8:01

They take more electricity.

8:02

It's an inefficient process.

8:04

So the other design evolution for packed columns is that our  $F \text{ sub } P$  here.

8:10

So that's our our frictional, yeah, frictional pressure drop loss that we're going to see in the next couple of slides that I just sort of that goes down in general as we go from early evolutions of of pack material down into more common materials.

8:27

So for very simple say ceramic rashes rings, we started off with the porosity of 64% and a pressure drop packing factor of 580, right.

8:38

And if we look at a high pack metal material, roughly the same diameter, now we're talking about a crossing around .96 with a pressure drop factor of .4, sorry of 45 S.

8:53

An order of magnitude reduction in the amount of energy we're going to be losing across this material, which makes it much thin, much more inexpensive and much more efficient.

9:04

So we're going to come back to pressure drop on on the next slide after this.

9:09

But we wanted to to then take one aside and look at OK for one example of these interlocked saddles, which is shown here.

9:17

If we put a one inch interlocked saddle in a series of towers, then the McCabe reports a series of experiments looking at what is the type on the left hand side to the pressure drop.

9:30

And this was reported in inches of water per foot of packing, right?

9:35

So inch of H<sub>2</sub>O per foot.

9:40

So I apologise because in this to look at some of the examples, we're going to have to go into imperial units, but it should be a good kind of base case.

9:49

So when we look at these one inch interlocked saddles, we can look at the pressure drop in that column, that packed column as a function now of the mass velocity of the air.

9:59

So that's how many pounds per square foot per second?

10:04

So typically a mass velocity would be in pounds per, sorry, pounds per hour.

10:09

But we we normalize it to the square foot to effectively give it a superficial velocity in an unpacked column.

10:17

So if you just take the diameter of the column, we don't have any packing material in there.

10:21

What would be the mass velocity per surface area per time, right?

10:25

These are just conventional imperial units, but we could easily use kilograms or or grams per square meter per second.

10:32

That would be more common.

10:35

So one of the features to point out then is that this data includes what they call a dry line, right?

10:42

That's shown here.

10:43

And if we plot this on a log log axis like The Cave has done, we see that that dry line shows up as linear, right?



10:51

So this is for a 30 inch column with 10 feet of packing in it.

10:54

Now if I have no liquid flowing through the column, then my dry line forms kind of the basis now as I start increasing the amount of liquid flow.

11:06

So  $G$  sub  $X$ , right,  $G_X$  will also have units of pounds per foot squared per hour.

11:16

And so we're just looking at the comparison of varying the air and liquid velocities or mass, mass velocities.

11:22

Now when I start varying that liquid velocity, I can see here we go, I'm looking at an air velocity range from 100 to 5000 on this scale.

11:32

But for the liquid velocities, I'm going to go from 3000 to 36,000, right?

11:36

And that's going to be because my liquids will have a much higher density.

11:39

So I get a naturally much higher value.

11:41

And there are a couple of features to point out of this.

11:44

At very low liquid velocities and very low air mass velocities, I tend to get operating curves, pressure drop curves that are parallel to my dry line, right.

11:58

So if I don't have a lot of liquid in the system relative to the amount of air flowing through it and my air flow or my vapour flow is not highly turbulent.

12:06

So I'm working toward that laminar system.

12:09

It tends to behave as though the system were dry.

12:15

Once I start increasing the amount of liquid in the system, my pressure drop goes up now exponentially as I increase the air mass velocity, right?

12:25

So if I'm working at a constant, say 600 lbs of gas per foot squared per hour in this low region here, it's linear.

12:33

But as I go upward, I can see if I add more and more liquid to the system, I'm going to get an exponential curvature, which is ultimately going to give me much higher pressure drops.

12:43

So one to two inches of water per foot of packing.

12:46

Now we'll come back to that in a minute because it's, I know it's a very strange unit.

12:51

It's about 800, I think 817 pascals per metre is equivalent to one inch of water per foot.

12:58

It's just an imperial unit.

13:00

So when I start diverging from this point of linearity, right, which I would say if I'm at 3000, so very low liquid flow rate, my divergent is about here.

13:10

If I go up to a very high flow rate, I can see some parallelism here to my dry line.

13:17

So this is for £20,000 per foot square hour, but I'm getting a bit of divergent out at a slightly lower air mass velocity.

13:26

So the point of divergent is where we call the loading point.

13:29

So it's the the point of liquid flow at which the system's going to behave.

13:35

So it weren't dry.

13:37

Now similarly, if I continue increasing the liquid flow rate higher and higher and higher, at some point I'm going to get local pockets in my column where I'm going to have liquid as the continuous phase and gas bubbling through it, right, as opposed to a perfect contact between these two.

13:55

And I call that point at which I have a local liquid phase dominant or or dominating the column as the flooding point.

14:03

So for the same system of one inch interlock saddles in the same column conditions, we can look at the flooding velocity.

14:10

So this is the  $G$  sub  $Y$ .

14:14

The flooding velocity is a function of the liquid mass velocity and that's the variable that I would control as the engineer is my liquid flow rate.

14:21

And so I can see for a nominal, if I nominate say 1000 lbs per foot square per hour, right, that's the gas velocity of the design and that's the compressor I'm going to buy, right?

14:32

Remember that compressors are much more expensive than liquid phase pumps.

14:35

So once I make the investment in the compressor, I'm going to work around it.

14:40

If I have 1000 lbs per foot square hour flowing through the gas velocity, then I can look at what is the liquid mass velocity at which I get flooding for different packing sizes.

14:55

So in the case of 1/2 inch interlocked saddle, it's around 3000.

15:00

If I go to a 2 inch, it's a very large interlocked saddle, it's around 30,000, right?

15:07

So the size of the packing material that I use is critical to determine at what point how much liquid can I put through this before I start flooding the system.

15:21

Now when we want to calculate these pressure drops, we don't have a very well, we can't use a theoretical framework, right?

15:29

Because you imagine if you have something like two or three thousand of these little interlocked saddles all put together in a large scale column.

15:37

I can't calculate from first principles how the momentum is going to be transported through all of those things.

15:43

So I have to use a very empirical approach to this.

15:47

So Gus, that's a really good question.

15:49

Void fraction also changes.

15:51

If I look at the example here, what I can see if I go to let the Interlock saddles the void fraction at 1/2 inch or sorry, the porosity which will ultimately dictate my void fraction will go from .71 if I go up to a three inch to .79.

16:08

So it gives me more void in the gas space.

16:12

So in my packed columns, then they're OK.

16:15

So flooding, shout flooding is when, if you imagine, so I'm going to draw a column here.

16:24

And what I'm going to do is just draw these as spheres, right?

16:27

For simplicity, because I cannot draw interlocked saddles on demand right now.

16:33

The goal, if we started it, imagine our, our vapour velocity is set and we're loading a little bit of liquid on top.

16:40

Well, initially my goal is that I want just the surface of these spheres to be wetted by my liquid so I can maximise my contact area, right?

16:48

That's the point.

16:49

If I continue increasing that liquid flow, right, then I'm going to have more and more

coverage and the coverage is going to get thicker and then I have a thicker liquid layer on each of the spheres.

17:04

Flooding is the first point at which my liquid velocity is so high that I'm effectively flooding the liquid pores with water.

17:13

So we'll change colors one more time now.

17:16

So in this local region I have a localized flood, doesn't mean the entire column is now water filled and I'm running gas through it, but it means in this little localized region I've put so much liquid across it for the packing material that I've chosen, I now have a water continuous system in that little region.

17:45

So the the definition of flooding depends on the amount of pressure loss I'm going to put across the system, the choice of what my packing material is, it's diameter, and then my gas and liquid mass flow rates across it.

17:58

So the film thickness is a consequence of, well, the film thickness is a function of those in the same way that the flooding is a critical point of that.

18:08

So film thickness is going to increase until the point of flooding.

18:12

So on the next slide, I guess we're going to get to that question specifically, what is the pressure draw across that column as consistent with flooding as an onset, right.

18:23

So if I think about how much pressure am I going to lose, there are two correlations that we'll look at calculating this.

18:30

So the first is Eckert and that's shown on this slide.

18:33

And the, the correlation itself is not a, is not a, an equation.

18:40

It's actually the plot, right?

18:42

So for dumping packing materials into a column, the correlation works on two bases.

18:49

So I, I calculate the abscissa or the X axis here.

18:52

Now that's going to be the ratio of my gas on vapour mass flow rates.

18:57

Now, because it's normalized, I can use any units that I want as long as I'm consistent.

19:02

So I can do that pounds per square foot per hour or I can use kilograms per square meter per second, right.

19:07

And that's remember that square meter is of the unpacked column and that it's the ratio of that liquid on vapour mass flow rate times the square root now of the vapour phase density on the difference between the gap the liquid and the vapour phase density.

19:26

So again, because I'm, I'm looking at a ratio here, I can use either imperial or SI units to calculate this out of the system.

19:34

Now on the ordinate or the Y axis, I have a few different terms come in.

19:38

And in this ordinate I do have to, to work in imperial units when I calculate it.

19:44

So here I have the gas.

19:45

So  $G_{sub Y}$ .

19:47

In fact, I'll, I'll write this out of here so it's easier to read.

19:50

We have  $G_{sub y}^2 * F_{so P}$  So that's the pressure loss for defined for a given packing material.

20:02

Times  $\mu$  of the liquid phase, so the dynamic viscosity raised to the .1 power divided by.

20:09

So the gravitational acceleration constant  $J$  sub  $C$  which again I have to use imperial units because I can see I'm raising some of my properties to random powers.

20:19

So that tells us we're in an empirical territory and we have to stick with the units.

20:25

Again, the density difference between the phases times the density of the vapour phase.

20:31

So that's the, the Y axis or the ordinate that we're using here.

20:34

And the correlation then is if I know what my packing material is, the gas and liquid densities, the vapour and, and liquid flow rates.

20:44

And I can go back to this table to look up what is the, an estimate of the packing factor in the frictional pressure loss associated with that material, right.

20:54

So if I'm thinking about a Raschig ring, it's probably on the order of a couple 100.

21:00

I can drop that into this equation and I can calculate.

21:03

So if I say that my Y axis calculates at .06 and my X axis is going to calculate at .2, right?

21:11

So I calculate them independently.

21:13

I trace up on the graph here to see where do they intersect.

21:19

And what I can see is the intersection is at a value of about 1.5.

21:23

So that means I have 1.15.

21:26

I'll write it a little higher.

21:29

1.15 inches of H<sub>2</sub>O per foot of column and inches of water.

21:39

Meaning of of pressure measurement.

21:42

Sorry not 1.151.5.

21:44

And, and bearing in mind then that one inch of water per foot is equivalent to 817 Pascals per meter, right?

21:57

So that gives me a conversion over to my SI units.

21:59

Yeah, 1.5.

22:01

So this is one approach that I can use to figure out what is the pressure loss pressure drop across this column.

22:09

Now, the typical flooding value, right?

22:12

The point at which I'm going to hit flooding in in a system is when I get a pressure drop of about two inches of water per foot of column, right?

22:20

That's a heuristic or a rule of thumb.

22:22

We'll put that in here.

22:35

OK.

22:36

Now there's also an empirical relation that that McCabe goes through specifically for what is the limiting pressure drop I can put put across the column before I get flooding.

22:46

And so here we can see it's a pre factor of 1.15 or sorry .115 times that frictional pressure loss from the packing material raised to the .7 power.

22:56

And this returns units.



22:58

So the units of this are again inches of H<sub>2</sub>O per foot of packing.

23:04

And it's, it's this correlation itself is, is valid for packing factors from 10 to 60.

23:09

So some of those that I looked at earlier for ratchet rings, I can't, for instance, use these correlations on on that 200 to to 600 range for the the ceramic rash rings.

23:22

I would need to actually go to this graph and calculate it with a heuristic that once I hit 2, that's going to be the the flooding point.

23:29

Now the reason, right, that we're using pressure drop to indicate flooding is that my pressure loss is going to come from the friction of my liquid phase.

23:38

As that liquid film grows in thickness, it's going to burn more and more pressure out of the gas phase, right?

23:43

Bearing in mind that it's my gas phase expansion that drives momentum in the system.

23:50

Now there's an alternative that was proposed by Strugal and I personally think it's a little bit easier to think about right in the way that it's presented.

23:59

It still uses these imperial unit sets.

24:02

The the F system is similar, it's just we don't have that density difference.

24:06

It's the absolute liquid velocity or mass velocity on the absolute vapour velocity times the square root of the ratios of the vapour and liquid density.

24:18

And my my Y axis or or ordinate here introduces a capacity factor.

24:25

So U not is the superficial velocity of gas in the system.

24:31

So that's the velocity divided by the full cross-sectional area available times the ratios or the vapour phase density on the difference of the phase densities.

24:41

So that CS goes into my first term.

24:44

Now here again I'll write this out so we can read it more clearly.

24:48

It's CS times my packing factor to the .5 power times  $\nu$  which is kinematic viscosity to the point 05 power.

24:59

Remember that kinematic viscosity is my dynamic viscosity relative to the density of the material.

25:06

So one of the the take home messages where I do the same calculation.

25:10

Here I know what my vapour and liquid mass flow rates are and what their densities are and what materials driving the packing factor.

25:19

And again, I would annotate.

25:20

So if my value of the X axis is now .1, and let's say my ordinate calculates to be 1.2, then if I trace these up on the graph and intersect right now here I'm going to have to interpolate, right?

25:37

Because I'm not sitting directly out of lines.

25:38

My interpolation would be something like 0.48 inches of water per foot of packing.

25:50

So in both cases, the packing factor will increase the overall pressure drop in the propensity to flood the system, which is why the design of these systems has evolved to go from very high packing factors right to low packing factors.

26:10

What I want to point out is that in our, the Eckert correlation from the previous slide had F to the what is effectively the 1.0 power.

26:19

Striegel presents F to the .5 power and our correlation is to F to the .7 power, right?

26:29

So the dependency of my flooding behaviour and pressure drop on the packing factor is not a point of certainty, right?

26:38

It's, it's a range that we can talk about.

26:40

It's between .5 and one from studies that have been conducted.

26:45

But again, this is not an exact science now because we can't physically describe as a civilization the momentum processes and how gas actually flows through this at a detailed level.

26:56

So if I then bring this back into the context of absorption columns and oh, I'm very sorry, these equations got moved around a little bit.

27:07

They're not too obstructive though.

27:08

So it's it's OK.

27:10

So if I think about my absorption column or I told you I would make this mistake, absorption column because I do a lot of absorption and research.

27:19

So I write it too quickly and you don't think about it.

27:23

So the material balance for my absorption column now and the prompt lab to went through some of this early on.

27:29

If I think about the column as a whole, right?

27:32

And I, I'm not going to consider an internal part.

27:36

In fact, what, what this cross section shows here is that I can have a, a variable length.

27:44

But if I only think about the, the entirety of the column that I can do a terminal material balance.

27:48

So the terminal meaning what goes in and what comes out and nothing concerned with what goes inside the vapour phase leaving  $V$  survey liquid phase entering  $L$  sub A.

27:57

So in this context, we're going to use A as an entering point and B as a leaving point.

28:02

Similarly, the vapour entering  $V$  sub B or sorry, A is the top of the column, B is the bottom.

28:08

So vapour entering on the bottom is B and liquid leaving on the bottom is L.

28:15

Now, so those material balances will work together across all moles per second flowing through the column.

28:22

I can expand that to only consider a control surface where I cut the column in half, right and I'm only going to look at some what is either coming in the top, leaving the top and then the values at some intermediary point.

28:36

And that's going to become important when you get to the the second-half of the discussion.

28:41

And that gives me this total material balance within the column.

28:44

I can expand that out to talk about the component A.

28:49

So if my absorption component is component A material balance in that same column, I simply am multiplying  $X$  by the liquid side and  $Y$  by the vapour side right?

28:59

Where my entry and vapour exit points  $L$  sub A and  $B$  sub A have a defined composition.

29:07

And if I then apply that same thing to do material balance at a random point within the column, right, that gets me this bottom equation here.

29:15

So what this is saying is, I'm sorry, the the terminal component balance within the column.

29:21

So  $L_{\text{sub A}}$ , the liquid molar flow rate entering the tower times  $X_{\text{sub A}}$ , the composition of my absorbing component in the liquid as it enters.

29:31

Right now that can be balanced out by the liquid molar flow rate leaving and the composition of my absorbing component leaving.

29:37

So typically we would hope a column is going to absorb say 99% of that material  $V_{\text{sub B}}$ .

29:45

So that's going to be the vapour entering the bottom times the composition of my target component in the bottom as it enters.

29:51

And  $V_{\text{sub A}} \times y_{\text{sub A}}$  is how much of the moles per second of vapour leaving times that absorbing component mole fraction  $Y_{\text{sub a}}$  as it leaves.

30:05

Now I can take that terminal balance and I can rearrange that to solve for  $Y$  within the actual system.

30:12

And that gives me this operating line equation here.

30:17

Now the operating line equation I can represent as  $Y$  equals  $MX$  plus  $B$  effectively, right?

30:23

Because my entry and where my my top molar flow rates and compositions are defined and my liquid molar flow rate and vapour molar flow rate are also defined or they're set by the engineer.

30:37

So this right hand side right is my intercept.

30:41

Effectively that shouldn't be varying, but through the column my composition of component A in the liquid phase is going to vary as well as in the vapour phase.

30:56

So in this writing of it,  $L \cdot V$  is the slope of my operating line,  $X$  is the variable that's changing.

31:05

So I'm trying to move component A from the vapour phase into the liquid phase.

31:10

So I'm going to see an increase in the value of  $X$  as I go along and the right hand term is that intersect.

31:18

Now let's look for a minute then in in Apologies for the this overwriting here, but this enter this dashed line.

31:29

The ordinate intersection is  $Y_{sub B}$ , right?

31:32

That's what was meant to look on top.

31:33

When I upload these, I'll I'll realign those to the left that need delete these.

31:38

There we go.

31:40

So if I first think about let's plot A&B, right?

31:44

And the phase space that I'm plotting here is  $Y$  versus  $X$ .

31:49

So it's the vapour phase mole fraction of my, my absorbing sort of target component.

31:54

In your lab, it's going to be  $CO_2$  and  $X$  is going to be the liquid mole fraction of  $CO_2$  in the system.

32:05

Shabba, it's an excellent question.

32:06

The answer is yes, and I'm getting to it in about 3 sentences.

32:10

So a little bit ahead of the group, no pun intended.

32:14

So as we go from our initial conditions at the top of the column right  $X_{A1}$  to the final conditions at the bottom of the column  $Y_{B1}$ .

32:25

And I can plot this in, in, in my, my compositional space and noting right that my equilibrium curve, which we'll highlight here in blue.

32:36

So my equilibrium curve, this is a thermodynamic property or depth, it's a thermodynamic definition.

32:53

So this equilibrium curve is originating from basic thermodynamics and vapour liquid equilibrium, that's where I'm actually getting that curve.

33:02

So we need to go to something like the NIST chemical web book or use a tool like multi flash or again ISIS to be able to look at what is the equilibrium behaviour.

33:12

And this depends on the pressure, the temperature and the composition.

33:16

So the PTX conditions, we would commonly call them thermodynamics dynamics of my system.

33:21

So that's what defines my equilibrium here.

33:22

Now when I compare my operating line, we would think it should be straight given the linearity of the equation on the bottom.

33:31

However, we notice that it has a bit of curvature to it and the showers is beat me to the punch line.

33:37

The reason that that operating line is going to have curvature is that I'm losing moles right or or molecules from my vapour phase.

33:45

So  $V$  is decreasing ever so slightly as I go along and  $L$  is increasing ever so slightly as I go along because I'm absorbing molecules or moles from one phase to the other.

33:56

So for that reason, I get a bit of pitch or curvature into the operating line right now as an example that that's shown here.

34:05

Then if I as an engineer say OK, the B that's coming out right if if my defined conditions, I can easily look at YB star and you'll remember and we'll come back to you on the next couple of slides.

34:19

We talked about that overall driving force and overall resistance.

34:23

I need to know YB star.

34:24

That is, if I take a constant at bottoms composition for the liquid phase X of B, then YB star is going to be the vapour phase concentration that is in equilibrium with my liquid phase at that absorbent or solvent, sorry sorry solute concentration.

34:49

So the goal of my operation then is to increase the distance between Y or XA and XB as far as possible, right?

34:59

That's the the objective of what I'm trying to achieve as and that will by the nature of it then increase the distance between yb and YA.

35:09

Now when I plot it in this space, the difference between the final or the bottoms point of my column where my, my liquid is going to exit and the equilibrium curve, that's what constitutes my driving force.

35:24

So I can also then think about, well, if I'm the engineer and I'm trying to look at an operation that's already been built, you know, we're a small firm, We don't have enough money to buy a new absorption column.

35:35

We can't buy a new absorption gear.

35:38

We need to just use what we've got and make it more efficient.

35:41

One of the ways that I can do that is by throttling the liquid flow rate, right?

35:45

Again, noting that my compressor is usually the most expensive component.

35:49



So once that's set up and running, can't be changed, but I can change the liquid phase pumps usually within a reasonable range.

35:57

So if I start modifying the liquid moulder flow rate  $L$ , this gives me the ability to pitch shift the operating line, right?

36:06

And if I look at what happens if I, I'll switch colours into green here, what happens if I turn  $L$  down as far as it can go?

36:16

Well, that's going to give me an operating line where in theory at the furthest point it can go my exit composition  $y_x$  of  $B$  and the bottoms composition of the vapour entering  $Y_S$  of  $B$ .

36:29

So the maximum loading it can take in will directly intersect the equilibrium curve.

36:35

Right now when I hit that point of it, we don't want to actually plot it on the equilibrium curve, but put it ever so slightly to the left because we remember that the operating line must by definition be above the equilibrium line or equilibrium curve to have absorption taking place.

36:50

If that final point  $B$  prime intersects the equilibrium curve, I have need an infinitely large column to deal with that.

36:58

So that's where the the theory comes into play, right?

37:01

But this gives me then as an engineer, the ability to throttle the efficiency of the system because in turning down the liquid flow rate, what I'm doing, if I imagine that my intersection point is just to the left of that equilibrium curve, I now have a much, much higher liquid phase concentration,  $X_V$  star coming out of the bottoms.

37:23

So I've done a more effective job at capturing all of that mass for the same inlet gas composition from the bottom right.

37:33

OK, so I have a couple of questions here.

37:35

If  $L$  &  $V$  are not constant, would  $L$  &  $V$  have to be rearranged as a function of  $X$ , solve for  $Y$ ?

37:43

Yeah.

37:44

So, so this this relationship, that's a good question, Shaf.

37:50

If L&V are not constant, right, then this relationship is is you can't plot this in equilibrium space.

37:59

So do you mean, sorry, Shaf, can you clarify?

38:02

Do you mean not constant?

38:03

Isn't you're changing it during the operation or not constant?

38:07

Isn't They vary slightly through the height of the column because I don't want to go off on a tan.

38:16

The second one they very thin how to go so the column because it's operating in steady state.

38:21

I can take that to the equilibrium which allows me to then plot on YX diagram.

38:27

But because if if L&B were not changing at all, so my molar flow rates were practically constant, right, and I'm transferring only a few molecules from one phase to another, I would get an operating line that was perfectly straight, right?

38:41

The fact that they are changing means that over the height of the column, I have a variable flow rate.

38:47

And that's that's what gives that curvature to the operating line as I plot it out.

39:00

Sorry.

39:00

OK.

39:00

The terminal equation, of course, you do mean the terminal material, total material or terminal component A OK, yeah, that's that's using the mole fraction.

39:15

So  $x_A$  and or  $x_Y$  mean the liquid and vapour phase mole fraction of my target component, whereas  $L$  &  $V$  represent the liquid molar flow rates or sorry, the molar flow rates of the liquid and vapour phase.

39:30

So shout to your point of taking that into account.

39:33

That's why this is considering the bounding conditions of  $V_A$  and  $L_A$  when this equation is written for the operating line.

39:46

OK.

39:47

So are we comfortable with this or can I move on?

39:58

OK, so we'll kick forward now.

40:01

So if, if that's how I can plot up my.

40:03

OK, more about the equilibrium curve.

40:07

Yeah, so, so the equilibrium curve, this is what we started to talk about in 3006 thermodynamics, right?

40:15

So we're thinking about four, let's say two component system, right?

40:19

I'm going to have a, if I say I have a, a fixed pressure and temperature for argument's sake, so  $P$  &  $T$  are fixed.

40:29

If I'm operating in a pressure and temperature in which multiple phases are present, I'm going to have a variable amount of component A and component B in my binary system in each of those phases, we'll call them one and two.

40:43

And the amount of A&B that is in equilibrium between those two phases is dictated by the thermodynamics.

40:49

So this is where I would use, for instance, an equation of state.

40:53

So something like a cubic equation of state I would use and then apply that by thinking about individual components.

41:00

So I can calculate from those thermodynamic relationships what is the equilibrium amount the the the mole fraction of any given component in one phase versus another, right?

41:17

Does that answer?

41:18

So this equilibrium curve is assuming a constant pressure and temperature.

41:23

What is the what is the equilibrium relationship between the mole fraction of component A?

41:31

That's my target component in the vapour phase which is the Y axis and the liquid phase which is my X axis.

41:41

So what this shows me is that if I have, for instance, if I say that I have a two component system, so let's say that we're dealing with CO<sub>2</sub> in water, right, If I have a two component system, then this equilibrium curve is going to be for CO<sub>2</sub> in water, all right?

42:01

This whole plot would be for CO<sub>2</sub> in water.

42:04

So that means if I nominate what is my vapour phase?

42:09

And so I think about degrees of freedom.

42:10

If I nominate my vapour phase CO<sub>2</sub> concentration, I can call that Y CO<sub>2</sub>.

42:16

I can trace along to this equilibrium curve and figure out how much liquid is going to be in equilibrium with that.

42:24

So how much of my CO<sub>2</sub> is going to be in the liquid phase.

42:26

So this is a relationship then that's going to govern for A2 phase system the partitioning of each of these components.

42:33

Does that answer your question, Chao?

42:36

I'll erase that so it can confuse everyone.

42:39

OK, if you want to know more, I'm happy to go into it.

42:42

And we're going to definitely get a lot more into vapour liquid equilibrium when we kick into distillation.

42:48

But for now, I'll try to wrap this, wrap this up.

42:52

So when we think about, let's do it again, when you think about the absorption of rate, so the rates themselves can be written, the rates are all equal and they can be written as a function of any of my available driving forces as per the equilibrium diagram in the operating line just sketched out, right?

43:12

So I can define it with respect to my vapour phase, to the liquid phase, or if both my vapour and liquid are contributing resistance together, right?

43:22

Which is not always the case.

43:23

If I only have a little bit of liquid, then it's probably going to be my vapour contributing to resistance.

43:28

But if I have a reasonable amount of both and I'm increasing the amount of liquid in the system, then I'm probably going to have to think about an overall total resistance.

43:37

Then I can plot that.

43:39

So  $K$  or the rate as a function of those overall resistances either defined from the vapour side of the liquid side, right?

43:46

Which together we define over here as a function of the driving force in either the vapour or the liquid direction, right?

43:54

Because this is an equilibrium operation.

43:56

So what we're we're illustrating there is I can we already looked at a driving force.

44:06

If I think about the vertical separation between my operating line and the equilibrium line in your case for  $\text{CO}_2$  and water together, I can similarly represent that as a driving force for the water to take on  $\text{CO}_2$ , right.

44:19

So in the same way that that if you put a gas containing  $\text{CO}_2$  next to a a clean water phase that doesn't have any, if you think about the Gibbs energy flow between these two things, that's  $\text{CO}_2$ .

44:31

Well, the water wants some  $\text{CO}_2$  because it doesn't have any.

44:34

And the gas has way too much  $\text{CO}_2$  because it's looking at a water phase with nothing saying, hey, you need to take some of this.

44:40

So you can define the driving force to move a marginal  $\text{CO}_2$  molecule from either the vapours perspective or the liquids perspective and they're equal.

44:49

So if I define that with and I'll change colours to blue.

44:52

If I define it with respect to the liquid phase, I can also look at that horizontal difference between the equilibrium position or the equilibrium line and the operating line, right?

45:01

This also is a driving force.

45:04

And this for instance, would be  $X^*$  minus  $X$ , whereas my driving force on the bottom, if I look at the vertical intersect could be  $Y^*$  -  $Y$ .

45:19

So now something that you can use in the lab, you can also replace that  $Y$  driving force composition with a partial pressure difference, right?

45:28

Because those would be proportional to each other.

45:30

So that's where Henry's law can come into.

45:32

And my overall resistance is when I think about large  $K_X$  and large  $K_Y$  for my mass transport resistances, this is where I'm going to have an impact of, of mass transport.

45:44

And that's that that two film problem you were talking about last week where both my liquid film and my vapour film are going to have a resistance to transport port on either side.

45:55

They're going to apply resistance.

45:57

So I have to calculate both of them together.

45:59

So in general, this is a best practice not to use these top two equations, but rather these bottom 2 here.

46:05

It is a best practice to go ahead and just calculate what those vapour and liquid transport resistances are going to be, right?

46:13

And then we can, you'll see because we're adding these, we're summing them.

46:18

On summing these resistances, you will note at a time in which one of these resistances is not contributing anything meaningful, right?

46:27

In the same way that if you were to sum up conduction and convection for internal pipeline flow, then conduction through the wall, conduction through insulation, and convection out to the air, you would note that that internal convection is actually so negligible, say .001% of the overall solution that you can neglect at.

46:47

So let's see so, so then within the the individual diagram, right?

46:52

If we look down here, I can also relate that the slope of the curve that directly connects, right?

47:02

That that is between the operating line and equilibrium line, that slope is the ratio of my liquid phase transport resistance on the vapour phase transport resistance, right?

47:16

So the more the operating line pitch shifts and that's based on my variable liquid and vapour molar flow rates, that also changes the pitch of that intersected line between them, right?

47:29

And so the slope that we talked about here is this  $M$  term that shows up in my overall resistances, right?

47:35

So if I rearrange that to solve, I can see that  $M$  falls out.

47:40

Now it's rare.

47:42

We talked earlier on about the interfacial composition.

47:45

So if I have, you know, again that two film problem, so I'm transfer a resistance to mass transfer on the liquid side and resistance moving into the vapour side.

47:55

If I do want to know what those interfacial compositions are, that's where this that's also where this slope comes into play.

48:04

But it's, it's rare that I'm going to have a need to, to calculate that because we design these only on what goes in and what comes out.

48:10

So it's a volumetric basis.

48:12

I should note on this typically for a packed column  $K_X$ , so little sorry little  $K_X$  times  $A$  and we present them together because we don't always know what that interfacial area is going to be, is around 3 to 20 times  $K_Y$  times  $A$ .



48:38

So the vapour phase mass transfer coefficient times the interfacial area, Yeah.

48:44

So shout the A represents the interfacial area acting on on which the mass transfer is acting.

48:51

So even though my area of the column is calculable, I don't know exactly what the area of the liquid is going to be, right?

49:00

Because I don't know if I go back to the very beginning about flooding, I don't know exactly what condition of flooding I'm going to be in, exactly how much liquid film I'm going to have, right.

49:10

So we tend to to.

49:11

Then we solve these as a problem.

49:14

We bind K and a together as a variable when we're plotting out these equilibrium curves and operating lines.

49:27

OK, So right on the 50 minute mark, I'll hang around if you have more questions and we can chat through it.

49:32

But the take home messages from this right now that you're looking at Lab 2 and I think tomorrow morning, I think the plan is that David's going to be going through some more HYCIS and making sure because I know Hycis is a bit tricky to get working in this scenario, but but hopefully the instructions have been given should be helpful.

49:52

So shout out to your question.

49:54

Do the liquid film, do I mean the thickness of the water around the packing surface that when I say liquid film, yeah, that's what I mean.

50:01

So what's the thickness of the water?

50:03

But don't confuse that to mean that's not the interfacial thickness, right?

50:09

So if I go back to one of my earlier slides, if I zoom in on this, right?

50:15

So here I'm going to have that's this.

50:18

Here's my song.

50:20

Now I'm going to have within this and I'll draw in green.

50:23

Now let's say we have a reasonably thick right water film.

50:31

So that's my my water film.

50:32

And that might be something like 50 microns in thickness, right?

50:37

That would be reasonable.

50:39

However, what I'm really looking at, I'm going to change colours one more time.

50:45

So what I'm really looking at is A5 Micron thick layer right on either side of that interface where I have a gradient in the OK cool, a gradient in the vapour phase composition and a gradient in the liquid phase composition.

51:07

So part of what makes these columns tricky is then I don't always know what this height of that water thickness is going to be.

51:18

Yes.

51:19

And and so of course, absolutely the only reason we didn't kick into this lecture last time is that we had to get kind of the basics of mass transfer out of the way.

51:27

But we will be kicking into distillation as you were starting in fact, before you started the distillation lab.

51:33

So this kind of latter point won't occur again.

51:38

This distillation is using the same mass transfer principles as well gas adsorption.

51:43

So if I think about pressure drop then and pack columns to summarize and this is my my absorption application went back slide, sorry.

52:00

OK.

52:01

So the take home messages for my absorption application then are when I plot this in in YX space, in phase space, there are two points I need to think about.

52:13

One is the operating line, right?

52:15

And I get that by knowing what are my inlet and outlet molar flow rates and the composition of the streams at those points.

52:22

The other is the equilibrium curve and that's what we start talking about in 3006.

52:35

And where you'll get even a deeper into that is 4 four O 4.

52:39

So calculating those curves, you know, it's reasonably simple for a binary system.

52:43

It starts to get really, really complex if we go to say A5000 component system, right?

52:50

Or an intractable to do so.

52:52

These are the two take home messages of what I want to plot in YX space.

52:56

One is the operating line that I get through my unit operation.

52:59

The other is the equilibrium curve.

53:02

And then to to unpack that a bit, if I'm thinking about the rates of mass transfer, I can calculate that either based on the overall resistance, right, where I'm going to consider the individual contributions of my vapour or film thickness.

53:17

And then the liquid film thickness which I bind together is, is  $AK$  times  $A$  right again, because that variable thickness of the liquid film or the vapour film, I don't always know.

53:28

Now I can also use this to to think about as the pitch is shifting between these lines.

53:35

And I'll draw these two examples here, right?

53:38

I can see a variable pitch in terms of the intersect between the lines.

53:42

And that's going to to tell me what those interfacial compositions are going to be.

## Lecture 17 - Absorption Dependencies

0:00

So with that, let's first pick up on what we were talking about in Lecture 16.

0:06

So here we introduce now thinking about the material balances for typical absorption column, we were able to then divine what is the operating line equation, right?

0:16

So in that  $Y$  is going to be the mole fraction of my component that so mole fraction in the vapour phase  $L$  is the molar flow rate of the liquid,  $V$  is the molar flow rate of the gas.

0:32

$X$  is what we'll use for the mole fraction in the liquid phase.

0:39

And then on, on the numerator of my right hand side  $V$  sub  $A$  is the inlet, sorry, the outlet.

0:46

Sorry, inlet molar flow rate of the vapour phase  $L$  sub  $A$  is the outlet molar flow rate of

the liquid phase, and they're each multiplied by the compositions of my vapour of whatever component A I'm targeting for absorption in my vapour and liquid streams at that point divided by  $Z$ , which is the average molar flow rate of my vapour phase.

1:08

So what we take away from this operating line equation is that it has the form of  $MX$  plus  $B$  right?

1:17

Now, in general, that means that if I have some starting point A here, another point B on top, so the beginning the top of my column, I have A.

1:31

If I'm absorbing a component from the vapour phase into the liquid phase, I begin at point B, right?

1:37

And that may be a bit counter intuitive, but I begin at point B where I have a large amount of that component in the vapour phase.

1:44

So  $Y_{\text{sub B}}$  is high, right?

1:47

And I work down, I absorb that to a portion where I have a low amount to point A, a small amount of that component in the vapour phase  $Y_{\text{sub A}}$ .

2:00

So when I begin the absorption process, so here's the top of my column and here's the bottom of my column.

2:08

So when I begin that process, my driving force for absorption we look at is the difference between whatever the point at which we're injecting stuff into the column point B and the equilibrium curve, and that's the driving force for absorption.

2:25

So Gus is asking, is the different driving force always a pressure difference or are there other ways of forcing the absorption?

2:33

So it's not a pressure difference actually it's a concentration difference.

2:37

So YB star that we're showing here, this is the, if I look at the equilibrium curve, right, I can see I'm dashing down here to  $X_{\text{A/B}}$ .

2:49

So YB star is the equilibrium concentration of my component in the vapour phase that's in equilibrium with the liquid at the exit.

2:58

So it's a differential concentration driving force, right?

3:03

And this is where we started off last week talking about a differential molecular Gibbs free energy, right?

3:10

That's that third domain of the matrix equation that we talked about in the dynamics.

3:17

So as I proceed through the column now from the top toward the bottom, what I notice is that at the end of the column, I have a very small driving force, right?

3:26

And I can think about mapping out the driving force in different locations through this column.

3:31

So as I go from the top to the bottom, I'm going to have less driving force because I'm absorbing my component from the vapor phase into the liquid phase.

3:41

Now we also looked at an example in where if I want to maximize the amount of component that I'm driving from the vapor into the liquid, I can play with the either the vapor or the liquid mass or molar flow rates.

3:58

And so if I take that to an extreme, I can look at the operating line intersecting the equilibrium line for an absorption column, right?

4:07

And we're going to look today at an example of, of how we're, we can actually use this principle.

4:13

So I won't go any further about that.

4:16

But but often times and certainly what's coming up in your laboratory 2 reports, what we're showing here is that both the equilibrium lines and the operating lines have curvature, right?

4:25

And that's normal because we in the operating line, we are moving mass from the vapour phase into the liquid phase.

4:36

So over the process of flowing through that column, my instant, my cross-sectional liquid and vapour molar flow rates are very changing.

4:46

So that means the slope of that operating line is not going to be constant.

4:50

Now, in reality, I have a small amount of whatever species I'm targeting that's moving from one phase to another.

4:56

So I can actually do some calculations of what we're going to look at today, where I'm simply going to look at the average between these two.

5:04

This slope isn't going to change a whole lot, right?

5:08

But something that we haven't yet talked about is the equilibrium line, what McCabe calls here the equilibrium curve.

5:15

This can sometimes have curvature, but if I want to be very simple about it, then I can also draw that as a straight line, right?

5:21

And one of the ways that we can do that in the the textbook goes through in Chapter 17 is the use of Henry's Law.

5:26

And that's certainly now come up again in your Laboratory 2 project.

5:32

Sorry, Laboratory 2 prompt.

5:35

So if I have a Henry's Law coefficient right for CO<sub>2</sub> in water, the textbook gives an example that if that's at 20° C then the coefficient itself would be equal to 1430 atmospheres pressure per mole fraction.

5:59

So if I'm operating my absorption column now at one atmosphere, so  $H$ , then the  $HCO_2$  at 20° C and one ATM will be 1430.

6:14

And if I'm operating at 10 atmosphere, 20° C and 10 ATM, it's 143.

6:24

All right, so this allows us to look at a relationship.

6:27

If we go back to that equation that's in your Lab 2 prompt, this allows us to look at a relationship using Henry's law for the solubility of a component coming from the vector phase into the liquid phase where we can now, you know, we're going to see today we're going to use that linear relationship to do some calculations.

6:46

So one of the points that we need to think about then, if we're going to do a calculation to design an absorption column or a tower is, is the basis for how we rationalize the height and the ease with which we can move a component between the vapour and the liquid phase.

7:04

And the, the reason that we talked touched ever so briefly into the number of heat transfer units when we were talking about heat exchangers about 3 weeks ago, we first introduced the concept of a heat transfer unit.

7:17

And at the time I said, look, you know, don't pay too much attention to this.

7:20

It's just that it's a principle that's going to come back later when we talk about mass transfer.

7:24

Well, in this case, we use that that principle of heat mass transfer units or mass transfer stages and then that variable height between different mass transfer stages to design how big of a column we need and to determine if the column height is already set to determine how much mass can we move, right?

7:45

So this is the the working principle that we're going to design the absorption tower on.

7:49

And it's really important that we all are comfortable with this because the same principle now is going to come back into play when we start talking about distillation.

7:59

So just as one quick side note, I do appreciate that there's a bit of a learning curve as we're coming into really applying license on this Laboratory 2 prompt.



8:08

I would encourage you.

8:09

I, I know that although we are learning this on the fly as we go, these exact same principles are going to apply now when we start to do the distillation tower, which is itself an, a much, much more complex unit operation.

8:24

So we're doing a lot of the heavy lifting right now in this fortnight.

8:29

It will allow us to work through the operation with distillation column much more effectively in the coming Fortnite.

8:34

So that's part of the design.

8:35

I told you we're going to start the unit card and ease up as we move on and that will continue to play out.

8:41

So the next Fortnite unit operation shouldn't be quite such a struggle because we're going to have the same principles coming back at us.

8:49

So let's think about this.

8:51

If if we go back to to the equations that are governing both the operating line and the overall mass transfer resistance.

8:58

Now the example we're going to do, of course, when we have a capital K here, my pen's not working.

9:04

Is that showing up?

9:10

There we go.

9:11

When I have a capital K, that's an overall mass transport coefficient for the vapour phase.

9:17

And what we can see is that it's purported or equal to both the vapour film coefficient, the liquid film coefficient and  $M$  being the slope of that tangent line between the two.

9:29

So if I take this expression that of relating an overall mass transfer coefficient for the vapour phase, and I like to do that because that incorporates any liquid film resistance with any vapour film resistance.

9:41

So I don't need to worry about what side of the transport problem is, is governing the movement of the molecule.

9:49

It allows me to to incorporate both and what I can see what I can see here.

9:57

My pen is definitely not working.

10:00

Come on.

10:03

OK, that overall mass transport coefficient is being driven by the difference between the mole fraction of my component and a point in the column  $Y$  and the points  $Y^*$  in equilibrium.

10:23

So if I pick a random point here on my operating line, this is  $Y$ , and if I trace down to see what's the intersection of the equilibrium curve, that's what we'll call  $Y^*$ .

10:33

So in the same way that my driving force is varying from the top of the column to the bottom, I can identify there's a variable driving force that we showed here that's  $y - y^*$ , start at any point in the column, and I might multiply that driving force by its transport resistance to get a rate equation.

10:50

Right now what I've done is to split this up a little bit.

10:54

SDZ so cross-sectional area of my column will get  $S$  and if I focus only on an infinitesimally thin portion of the column, I call  $DZ$  and height because I'm trying to figure out the height of the tower and the number of transfer units involved.

11:10

So SDZ is now a differential volume of the column.

11:14

And here on the left hand side from the same basis we have  $V$  is the molar flow rate in the vapour phase and  $Y$  the composition of the component I'm targeting in the vapour phase.

11:26

So if I take this basis that we've just looked at and I rearrange that I'm integrating  $DZ$  from zero to the total height of the column  $Z_{\text{sub T}}$ .

11:36

And what I see is I'm integrating the right hand side now  $DY$  on  $y$  -,  $Y_{\text{star}}$  right, appreciating that I'm going from point A right where I have a very small driving force to point B on top where I have a very high driving force.

11:51

So I know this driving force is going to vary over the column.

11:55

Now I have a very simple solution for this.

11:58

So come back to the definition of the number of transfer units per minute.

12:01

But if I take this integral on the left bottom, left hand side, if my operating line and my equilibrium line are both linear and parallel, right, And that's a critical point.

12:14

If they're both linear and parallel, then this integral becomes quite simple because the driving force at the very beginning of the process is equal to the driving force at the very end of the process.

12:25

So this is an idealized operation, right?

12:28

And in that case, I can integrate this right hand side to be what I've shown in red brackets here.

12:34

And what we, what we demarcate is the number of, we call that the number of transfer units in the system.

12:44

So if I focus just for a minute, there's a principle here that's going to come back into play for distillation.

12:52

If I look at this ideal operation where my operating and equilibrium lines are both straight and parallel, I can integrate that right hand side quite easily.

13:01

And the number of transfer units here is related to the number of.

13:07

You can think of a transfer unit as the number of theoretical stages of transfer.

13:12

And the way that I'll think about a stage is I draw a stair step from the beginning of the process to the end of the process bouncing or ricocheting between my operating line and my equilibrium line.

13:30

Now, can you tell me, have you, has anyone seen this concept before in either heat and mass transfer?

13:37

Well, certainly not with energy material balances, but have you seen this before in heat and mass transfer?

13:46

OK.

13:47

So if you have let me know this, it's not around activity necessarily, but it's, it's around how we think about the number of theoretical stages.

13:56

So instead of having a continuum process right where every inch or every centimetre I'm going to have a, a changing driving force, I can simplify this and think about, OK, in this ideal operation, you see I've had five stair steps between my operating line and equilibrium line.

14:15

That means I have around 5 transfer units or transfer stages and then to get the total height of the column required to do this, I multiply that 5 transfer stages.

14:33

So  $N_{\text{sub OY}}$  in this case equals 5 by the height of each transfer stage.

14:40

Right Which we define here a transfer unit or a transfer stage?

14:43

It's height is  $V$  on South divided by  $y^* A$  where here is my resistance term so I can think of  $V$  the molar flow rate of my vapour phase normalized to the cross-sectional area  $S$  right.

14:59

So another way to write this is the is the quantity  $V$  on  $S$  divided by  $K$  sub  $y$  \*  $a$ .

15:16

So I can think of the height of the individual transfer stage or transfer unit as the molar fluoride on surface area normalized for surface area divided by the resistance to transfer, right.

15:29

So we we go through all of this because it's an easier way to define than trying to to think of a continuum process.

15:36

Now, in most cases, my operating line and my equilibrium line are not going to be parallel to each other, right?

15:43

They're probably going to be straight.

15:45

And we're going to invoke, obviously in Lab 2A, simplification of Hendry's law to make them straight or to make your equilibrium line straight.

15:54

The example we're going to do today is going to use a straight equilibrium line, straight operating line, but they're most likely not going to be parallel.

16:01

That means that if at the beginning of the process I have a very equal driving force to the end of the process.

16:08

Now to account for that, what am I going to do?

16:11

Same as we did with heat transfer thickness of the pipe.

16:14

If I have a thick pipe problem or if I have a a very sorry, a Co current heat exchanger, I need to think about a log mean temperature difference because I have a variable driving force between.

16:25

I'm going to invoke a log mean difference to account for the balance between my high driving force at the beginning of the unit operation and my small driving force at the end.

16:35

So in that way, when I integrate this right hand side and this is the the part that you need to be comfortable with and we're going to use quite a bit today, the number of transfer units for transfer stages is now equal to the amount of mass I'm going to transfer.

16:50

That's  $Y_B$  minus  $Y_{A^*}$  divided by the log mean driving force available.

16:58

So in this case,  $Y_B$  minus  $Y_{B^*}$  less  $Y_A$  minus  $Y_{A^*}$  on the natural logarithm of the ratio between the two.

17:10

All right, So we'll go through an example on the next.

17:14

We're going to look at one more slide to kind of cement this idea of the number of transfer units and the height.

17:19

And then we're going to go through a calculation example on it.

17:21

That'll be quite large.

17:24

OK.

17:24

So in the same way that I can write 4 different rate equations, right, I can define the number of transfer stages or units and the number of and the height of each one in four different ways with respect to the liquid film on top, with respect to the overall liquid.

17:44

And that overall is going to account for both liquid film resistance and gas film resistance.

17:49

So if it's a bit difficult to pull a molecule out of the vapour phase across that two phase boundary or across that phase boundary, then I need to go into the overall, right?

18:00

So we're going to tend to use these overall bases more often.

18:06

I can do with respect to the gas film if I know that my liquid film doesn't provide any resistance or the overall gas.

18:12

And in fact, it's this bottom that we're going to look at for our example today.

18:17

Now to visualize what I was trying to talk about in the last slide on the right hand here we can imagine what's not known by McCabe's diagram is  $Z_{sub} T$ , the overall of the column and what's involved in doing all of this.

18:30

So if I define my number of transfer stages with respect to the vapour phase, then I'm looking at these blue blocks, right?

18:40

So I have in this case 6 independent transfer stages, each that are relatively thin.

18:48

And if I want to define the problem with respect to the liquid Phase, I have 4 transfer stages, each of which are a bit thicker.

18:56

The height of each stage is higher.

19:00

Now in reality they're equal, right?

19:02

If I multiply the number of transfer stages of the vapour and liquid by the height of each stage, I should get about the same number come out or a balance between the two, right?

19:12

Because I'm ultimately moving molecules from one phase to another.

19:18

But they are not the absolute height and the absolute number are not equal to each other's together unless my operating lines are straight in parallel.

19:28

So before I create any more confusion, let's go into a proper example and look at how all of this is going to unpack.

19:35

So what I want to do, sorry, obviously this is a good question.

19:41

Is it  $Z_{sub} T$  meant to have an equal structure?

19:44

Yes.

19:45

So what we're targeting is to say the height of the column is defined as the same.

19:50

That's the product of the number of transfer stages and the height of each transfer stage.

19:56

But I can define that either on the liquid side or the vapour side.

20:01

So  $Z_{NT}$  is always going to be the same.

20:05

But if I define it with respect to my vapour side.

20:09

I have a different number of transfer stages and a different height of each transfer stage, right?

20:13

And that's this blue bit here.

20:15

If I define it with respect to my overall liquid side, I have a different number of transfer stages and a different height of each transfer stage.

20:23

So you can think of here this red box, this height is  $H_{OX}$  where  $N_{OX}$  equals 4.

20:35

Now on this, the height is  $H_{OY}$  where the  $N_{OY}$  in this case is equal to six.

20:49

Does that make a bit more sense?

20:51

Now the way I've drawn it on this right hand side is super easy, right?

20:55

Because it's a cartoon that I've made-up.

20:57



But what we're trying to do now is look at actual physical calculations of they're not going to be integer numbers.

21:05

So in the example that we're looking at, then what we want to do is think about a 1M, so 1M absorption column for ethanol, which I'm going to give ETOH.

21:26

So ethanol that I have in my vapour phase.

21:29

So let's say I'm making and reacting ethanol and I have some of this ethanol that's up in my vapour phase and I want to recover it, right?

21:35

Because this is a valuable product that I can sell if I can recover it and and purify it out.

21:43

So I want to absorb an absorption column for ethanol from the vapour phase via pure water.

21:53

So I'm going to take a pure water stream and try to use an absorption column with a whole bunch of cheap packing material, run these together and see if I can pull that ethanol out of the vapor phase.

22:04

So the column radius is going to be 10 centimeters, right?

22:13

Which means I haven't the surface area, which we'll just call  $S$  because we don't have any other  $S$  variables showing up here of point O3, 1416 square meters.

22:27

I'm operating the column at a temperature of 303 Kelvin, a pressure of 101.3 kilopascals, so atmospheric pressure.

22:38

And I'm going to give this an inlet.

22:43

So instead of using A&B to try to simplify, I'm going to use in and out, right?

22:47

Because we do the exam together, so it's a bit more intuitive.

22:50

So I have an inlet molar flow rate of my vapour phase.

22:54

That's 100 kilogram moles per hour, right?

23:01

And if I know the molar mass of my vapour phase, that's an easy conversion to go back and forth with an absolute mass and that inlet.

23:13

So my 100 kilogram moles of vapour per hour that contains 2.2 mole fraction ethanol or mole percent of ethanol.

23:26

So YETOH in is going to be 0.022.

23:37

Now I'm going to write here in blue.

23:41

I'm actually designing this problem with more uncertainty than you have in your laboratory #2 so the problem we're going to work through together today is harder, is less or is less constrained, so harder to solve than than laboratory 2, to try to help make sure we're all on the same page.

24:02

So I have a design spec right that I need 90% of my ethanol recovered and I want to operate at 150% of the minimum liquid velocity or liquid molar flow rate.

24:34

So if you recall back from oh, we'll give it a second to think.

24:43

It's not.

24:44

It's locked up for a minute.

24:46

You got to love Windows.

25:01

Oh, goodness.

25:05

OK, so.

25:06

Well, this is thinking about it.

25:10

I can still write.

25:11

What is up with that?

25:12

OK, I'm just going to keep writing and hopefully the thing will unfreeze and let me move slides in a minute.

25:18

So I can't change colour.

25:20

Well, it's it's frozen, but I can write.

25:24

So we've defined we want 90% of our ethanol recovered where we have only two point 2% by mold of my inlet vapor stream being ethanol in the 1st place.

25:34

But what I don't know is my liquid flow rate.

25:36

So I've given that as a constraint.

25:39

Now I am looking at the relationship for ethanol between air and water, and I can say that my equilibrium relationship is going to be  $y = 0.68 * X$ , right?

26:04

And if we've looked at Lab 2 so far, oh, there we go.

26:07

Now we can go back.

26:08

OK, If we've looked at Lab 2 so far, you can use Henry's law to give a a very similar constant equilibrium relationship.

26:16

So what this means is I'm going to have an equilibrium line, not a curve.

26:28

OK, So I'm going to switch back to red now.

26:32

So firstly, well, I know because I've specified that I'm using pure water to strip this ethanol out of the vapor phase.

26:40

I know that  $X$  of ETOA.

26:43

So the liquid mole fraction of ethanol coming in equals zero, right?

26:49

Starting off with pure water from the tap hopefully doesn't have any ethanol, right?

26:53

Otherwise per city water would be a lot more fun than it is.

26:59

So what I'm going to do on the 1st slide is think about my mass balances or material balances on a molar basis.

27:05

So first I need to calculate.

27:07

I'm going to free up some space here to do it.

27:11

I first need to calculate how much of my vapour that's coming in is going to be inert, meaning it's not going to participate in the absorption process.

27:21

Well the inert right is going to be equal to  $VN$ , so the total molar flow rate coming in times 1 minus the amount of ethanol coming in, which gives me 97.8 kilogram moles per hour.

27:45

Which means that I now have my inlet vapor is 2.2 kilogram moles of ethanol per hour.

28:02

So then if I think well how much of that ethanol is going to make it out the other side?

28:07

Well my design spec is that I want to recover 90%.

28:11

So if I employ that design spec now, the molar flow rate of ethanol out will be 0.1 times the molar flow rate of ethanol in total, which is 0.22 kilogram moles per hour.

28:37

So then I think, well, my total, my outlet vapor stream in terms of molar flow rate is going to be  $V_{\text{inert}}$  plus  $V_{\text{ethanol out}}$ , which gives me a value of 98.02 kilogram moles per hour.

29:00

Now the difference between the amount of ethanol in moles, kilogram moles per hour coming in and that leaving the difference between those two  $2.2 - 0.22$  gives me the amount of ethanol that's going to be in my liquid stream at the exit point.

29:16

So the liquid molar flow rate of ethanol out by material balance must be 1.98 kilogram moles per hour.

29:32

OK, now we're going to come back and because we need to next solve and figure out well how much of my liquid stream is going to be ethanol, right?

29:42

Because the difference between the two is going to give me my liquid molar flow rate.

29:49

And from my design spec, I can figure out my actual molar flow rate.

29:54

So the first thing I need to calculate is if I know the liquid number of moles of ethanol that are going out.

30:02

Well, I can look at it and say, OK, what is the molar composition, right.

30:08

So YETOH at the outlet.

30:14

All right.

30:16

And that is going to be defined by the number of moles.

30:22

So the molar flow rate of ethanol with the outlet on the total number of moles going out.

30:30

And that's going to give me .002244.

30:36

OK.

30:37

So I begin with a 2.2 mole percent ethanol stream.

30:43

I end with a .2244 mole percent ethanol stream.

30:50

Now this lets me get to right if I'm going to put this on an equilibrium YX spot.

30:55

Now I know that what my vapour phase compositions are going to be and I have a relationship for my equilibrium line.

31:03

Now I can figure out what is the amount of ethanol in the liquid phase that would be in equilibrium with that vapour.

31:10

So using this value, so the liquid composition of or molar composition of ethanol at the outlet, that's minimum, at the minimum conditions where I'm on the equilibrium line, all right, is going to be .022 divided.

31:46

Whoops, .2 two .022 divided now by .68 because that's the slope of my equilibrium line, right?

31:57

So I can convert between a liquid mole fraction and a vapour mole fraction using that that value.

32:04

This gives me now a value of point O3 2353 and that's a mole fraction.

32:17

OK, So what have we done on this slide so far?

32:21

What we've figured out?

32:23

I'm going to switch to a different colour here.

32:26

What we have figured out is that I begin right.

32:30

If I'm trying to put this on XY coordinates or in a YX plot, I need my two YX positions and the equilibrium line.

32:38

So my inlet ethanol concentration in the liquid is 0 and the outlet if I begin thinking about a minimum position is point O3 in my vapour Phase I begin with a point or 2.2 mole percent .022 and I end with .00224.

33:04

OK, so if I have these four different compositions, I can draw a line between the two.

33:12

I can also compare the line that I'm going to draw between the two with the relationship of  $Y$  equalling  $.68 X$ , right?

33:20

So these are the five parameters I need to think about.

33:22

And all of this is coming from a material balance only.

33:26

I'm not invoked a single principle of mass transfer yet.

33:32

OK.

33:33

So what I've done is I've just gone into Excel, mapped out that equilibrium line and mapped out my 2 operating points, connecting the two of them with a line, right?

33:45

So here is my outlet and here and here is my inlet.

33:50

Now, because I'm designing this right on the last slide, we said we wanted to know firstly, we don't know what the liquid molar flow rate is, and that's part of the problem that we have.

34:01

So what we've said is we want to pair that operating line at the maximum efficiency.

34:05

We want to take that all the way to intersect it with my equilibrium line.

34:11

So that means I see my intersection up here as would be expected, all right, and my inlet.

34:20

So here's my ethanol coming in in the liquid phase and my ethanol is leaving.

34:33

The liquid phase is leaving with about .032 mole fraction ethanol as we said, right?

34:41

And here are my differential gas compositions as we've already specified.

34:45

So take one more step now.

34:49

So if I know what that minimum liquid composition is going to be, then I can think, well, what is the if I know the amount of ethanol?

35:02

So remember that my total amount of ethanol that needs to be in the liquid phase is 1.98 kilogram moles per hour, all right?

35:13

And that came from my material balance.

35:15

Then the line or the molar fluid at the outlet at its minimum point because I'm intersecting with the equilibrium line is 1.9 divided by the composition at the minimum.

35:39

So it's divided now by 0.03, I'm sorry, 0.032353, which gives me a minimum liquid molar flow rate of 61.2 kilogram moles per hour.

35:59

Now what I've said on the previous design spec is I'm operating at 150% of the minimum because I know that if I am working at absolutely the minimum, I need an infinitely long column, right?

36:10

I can't afford to have an infinitely long column.

36:13

So that means my actual outlet molar flow rate is 150% of that or 91.8 kilogram moles per hour.

36:27

So from a mass balance of the outlet minus the ethanol.

36:33

So my inlet liquid molar fluoride is the out minus L of ETOH out right?



36:43

And that's assuming I'm not losing any water to the vapour phase.

36:48

That's going to give me 89.82 kilogram moles per hour, which means that because my outlet or I can figure out my actual outlet liquid composition now.

37:05

So liquid composition of ethanol at the outlet is going to be the total molar flow rate of ethanol going out on the actual flow rate of ethanol leaving which itself is 0.021569.

37:26

So 2.16 mole percent is the concentration of ethanol in the liquid phase as it leaves the column where the maximum I can get by intersecting with the equilibrium line is 3.235%.

37:44

OK, so now what I can see is that my actual liquid out so if it's 2.1 right, is actually going to be here.

37:57

So if I trace upward on the plot tear the drawing here, actually that's the the the other side of my YX diagram and my actual line is going to look something more like that, right?

38:13

So now do one final set of calculations to bring this back to transfer units.

38:21

Let's see actually what I'm going to do.

38:26

So on the next slide, I've actually mapped this out in Excel.

38:30

So ignore the stair stepping for a moment because we're going to come back to that, but in green.

38:36

Now this is the actual values plotted for this ethanol absorption system.

38:42

Now what I need to calculate here are a few critical points.

38:46

So if I want to do the numerical framework for the number of transfer units on the

vapor side and the number of transfer units on the liquid side, I need to 1st think about well, what are YB star and YA star so YB star switch back to red so YB star.

39:10

All right, this will be .68 times the actual composition of ethanol out, right?

39:20

So that's the vapor that's in equilibrium with my exit ethanol concentration, right?

39:26

And that gives me a value of point O 14667 YA star.

39:36

That is the amount of ethanol that's in equilibrium with the inlet.

39:43

Now in the inlet water.

39:46

So X inlet I have no ethanol.

39:50

So because my equilibrium plot goes through 00 here, the answer YA star is 0.

39:58

Now the I'm not going to break out the calculation for a delta T or sorry, delta.

40:05

It's a concentration log mean, but it's given here.

40:08

So if I know YA and YB.

40:10

And I've just defined YA star and yb star.

40:13

I can calculate the delta Y log.

40:20

OK.

40:21

So from that delta Y log is going to be equal to 0.004298.

40:34

Now if I go back to that definition of a number of transfer units, that's just the A -, b divided by that delta Y log mean.

40:44

So the number of transfer units in the vapor phase is 4.6.

40:52

Now from that, all right, I remember that the number of transfer units times the height of a transfer unit gives me the total height of a column.

40:59

But I know the height of a column, all right?

41:01

It's a 1m column.

41:03

So I can use the definition to get at what is capital  $K_{y,a}$ .

41:08

And I'll just go back a few slides and draw this out for a minute.

41:12

So we're using this bottom overall gas framework right now.

41:16

Now I know the number of transfer units.

41:18

I know the height of the column.

41:20

What I don't know is the overall mass transfer coefficient in the vapour phase capital  $K_Y$  times its area.

41:28

We always bind them together right?

41:30

Or we typically bind them together because it's not something we can easily calculate.

41:33

But I do know  $V$  which is the average molecular rate of the vapour phase and I know  $S$  which is that cross-sectional area.

41:39

So if I know everything else that I've circled here and  $H \cdot n$  equals  $ZI$  can solve this around directly for capital  $K_Y$  on  $A$ .

41:50

So from that relationship, what I get is capital  $K_y$  equals 14486 kilogram moles per meter cubed hour HR.

42:07

There we go.

42:08

OK, so now that's what I've done.

42:11

I have 4.6 transfer units in the vapor phase.

42:16

Now let me switch colors and we're going to talk about the liquid phase for a minute.

42:21

So  $X_A$  star, right?

42:24

The same principle, same framework apply.

42:26

So  $X_A$  star, I can take that definition as the ethanol concentration ETOH at the outlet divided by .68 because that's my equilibrium relationship to get the amount of liquid that would be in equilibrium with that vapor at that point.

42:47

And that gives me a value of point OO33O1  $X_B$  star is the ethanol concentration at the inlet again divided by .68.

43:01

And that gets me .032353.

43:05

We've seen that before, right?

43:08

And from that same framework of a log concentration difference, I'm just plugging in  $X$ 's instead of  $Y$ 's I can get a delta  $X$  log mean equal to -0.00632 right now.

43:26

That's OK because the number of transfer units is also  $X_B$  minus  $X_A$ , right?

43:32

If I go back to this definition here, it's  $X_B$  minus  $X_A$  on top of this number of transfer units for the liquid side.

43:43

So in fact, my two negatives are going to cancel out.

43:48

So this then means that the number of transmutants on the vapour side are going to be 3.41.

43:55

And if I use that same approach as I looked at from earlier, I can calculate capital KXA which comes out to 9864 kilogram moles per cubic metre per hour.

44:13

Now the point that I want you to take home from this is the average between the number of transfer units on my vapor side and the number of transfer units on the liquid side.

44:28

So the average of 4.6 and 3.41 is around 4.0, right?

44:36

So I have about four transfer units in total.

44:40

In this system I have exactly 4.6 units to transfer units or transfer stages if I'm coming from the vapor side and 3.4 transfer stages from the liquid side.

44:52

And where this comes into play is if I map out these two points, right?

44:59

I can stair step between this and the equilibrium line I have pretty much exactly, which is why I chose these parameters for the problem.

45:12

Exactly 4 stages that are mapped between these two equilibrium between the operating line and the equilibrium line.

45:23

So the purpose of this example is to illustrate to you a principle that's going to come back when we start talking about the design of distillation columns.

45:31

And that's that I can use both a numerical basis for design and most importantly the the reason that or the way chemical engineers have done this for decades, right before we had computers and graphics and all the fancy stuff, we would make a plot and do this, right?

45:52

So this visual basis of thinking about the number of stages inherent in my column, right?

45:57

What these four stages are representing or, or the 3.4 to 4.6 split is representing this difference here, right?

46:08

And the balance between the two.

46:11

But we can get there visually.

46:14

So I know we're approaching the end of the hour, but I hope that that example has helped everyone.

46:19

And I hope you can also see with the amount of data that you have in laboratory #2 it's basically a set of material balances.

46:28

And from there, we take one principle of an equilibrium line, operating line, and the rest of the questions will fall out.

46:35

And so there's just a few final points that I wanted to touch on before we wrap up today.

46:40

First is what happens at different pressures, right?

46:43

So the example that I did was atmosphere, ambient pressure.

46:47

But what happens if I increase the pressure?

46:49

Well, in general, right, a pressure increase is going to have an an increase in the partial pressure of a component in the vapour phase.

46:58

So what I can think is that it's going to take my operating equilibrium lines.

47:02

And if I'm at some low pressure  $P_1$ , right, that's the solid line shown here.

47:08

So I might have something like this, right?

47:11

And if I go into do it in redder, if I take it to a higher pressure  $P_2$ , it's going to smear or skew both of them to the right hand side.

47:22

All right, this means I'm going to have a different design basis.

47:29

The take home message from this is that because I'm working further to the right hand side of the higher pressure, it means I can achieve if we think about tilting.

47:41

So I'll delete this right hand step for a minute.

47:42

If we think about intersecting my operating line by playing with the LMV, so I'm going to play with the liquid molded flow rate, I can tilt that operating line to intersect with the equilibrium line.

47:53

That means that in a higher pressure, I have a higher potential amount that I can equilibrate between the vapour and liquid phase, right?

48:11

So I can drive a higher amount of my absorption product into the liquid if it's maximum operating condition relative to if I tilt this, this lower pressure operating line here, the intersect might be here, right?

48:25

So the goal of higher pressure is that I can get a richer product stream, right?

48:29

That's absolutely critical and I get that because my liquid film resistance is going to decrease, so I'm getting less resistance to mass transfer when I have a higher pressure, meaning the molecules can be pushed into the liquid phase more easily.

48:43

All right, But what we start here, it's a richer product stream that we're after when we talk about high pressure systems.

48:52

Finally, two last points.

48:54

I know we're just hitting up on the end of the hour here, but stripping columns, right?

48:59

So we've talked a lot about absorption and it's all around the design of an absorption column.

49:03

But certainly what we've looked at in the lab is a combination of absorption and stripping.

49:08

And these are often paired together, right?

49:09

And I'm going to use some gas that's containing a product which I'm trying to absorb out and put it into a liquid phase.

49:16

It only makes sense that I need some kind of unit operation to recover my product or my target chemical out of that liquid.

49:24

So fundamentally a stripping column is just an absorption column that's run in reverse to recover whatever the solute and sometimes to regenerate, right, that absorbing solution.

49:37

So I'm using a really valuable absorbing solution like an A mean.

49:41

I don't necessarily want to buy new a mean every time, but if I'm using water, then maybe it's possible I can dispose, use environmental water and dispose that back to the environment without having a consequence.

49:54

So that the rationale may vary.

49:56

And if I am trying to regenerate that absorption solution that was used in the 1st place, it's probable that I'm going to need multiple stages.

50:04

Because you saw in the last example that even operating for our ethanol, we could only get, you know, 90% of the way there.

50:11

So I probably need to put multiple stages together if I'm trying to create a pure inlet stream.

50:17

The only difference is that when I'm looking at a stripping column, the operating line sits beneath the equilibrium line or the equilibrium curve.



50:28

And if this is a typical operating line for a stripper, then I can think about the maximum operating line is the one in which it's not an end point condition.

50:44

So not a terminal point that's going to intersect the equilibrium line, but it's going to tangent.

50:49

It's going to intersect the tangent.

50:51

So at any point if my operating line touches the equilibrium line, right?

50:56

And this might be somewhere in between, right?

50:59

Typically not not an end member condition, but somewhere in between.

51:03

That's what I define as the minimum molecular rate for the vapour phase V submen.

51:09

And what we call this is a pinch point right now, I'm going to erase this for a second to switch colours just to draw it out.

51:17

But you can imagine 5070 years ago when early chemical engineers were trying to to describe the mathematics, right?

51:25

If we were using our graphical representation and doing our stair stepping to consider the number of stages and the height of each stage.

51:32

Well, as I tilt that operating line upward in the same way I can tilt the absorption operating line downward.

51:38

At some point, if I'm trying to do this right, the number of stair steps I'm going to take between my operating line and the equilibrium line is pinched right?

51:49

Like I, it pinches my, my stair or my stage steps in between this.

51:54

So this is the origin of the word pinch point.

51:58

And we're going to come back to this in quite some detail when we kick into distillation, but just to set up a context around it.

52:04

And then if I want to put the two together, here's an example for something like absorbing oxygen out of air right and into the water.

52:12

So if my equilibrium line on the bottom, this is for oxygen now in water, I can imagine my absorber on top where I'm going to have a slight pinch identified just because I have a very narrow gap between that operating line and the equilibrium line.

52:31

And here is the same operating line for the desorption process of pulling oxygen out of a water phase, right?

52:39

So typically it's common that if we're thinking of a combinatorial absorption stripping problem together we would plot both operating lines in the same YX space against the same equilibrium line.

52:50

So visually we can identify where is the pinch point, if there is one and how can I optimize both if I'm changing either the liquid molar flow rate or the vapor.

53:03

And then finally, one of the questions on your laboratories talks about structure packings, right?

53:08

So it's very, very simple.

53:10

Early days, engineers were taking things like rocks, dumping them into a column and using that to control surface area.

53:17

Then in the engineers thought, hey, well, if we stop using rocks and the evolution of rocks, which might be something like rash rings or how fancy can we get from there?

53:26

What happens if we just put a, a pack of material like gauze?

53:31

So wire gauze and we push that into the column and use that for contact area.

53:36

So that's what we define as a structured packing.

53:40

So where we've come since using gauze is most commonly perforated corrugated metal, right?

53:47

And this is a physical example that we can see here.

53:55

And what you're looking at is that we take this corrugated metal on 45° angles and we simply crisscross it and stack them together like a sandwich, right?

54:03

And there's punched little holes in this to allow the vapour to flow up and the liquid to wet that surface.

54:10

Each of these layers is typically one to two inches thick, right?

54:17

And when we go to use these, we tend to have a naturally high porosity, right?

54:23

This means that we are not going to have a very high pressure drop.

54:27

So we like structured packings because they tend to give us a very beneficial low pressure drop for the same amount of material that's going to be coming across.

54:37

And they tend to also align.

54:39

You imagine these porosities between 93 and 97% are on the same order as some of the best quote UN quote best packing materials that I can get for a randomly dumped column.

54:50

A few of the limitations around this is that the flooding velocity, I don't really have a, a way to estimate the flooding velocity easily, right?

55:00

So the graph here shows a handful of different packing or structured packings and what their relationship between the the flow rates and velocities might look like.

55:09

But we can estimate that that natural capacity, if we go back to the examples that we're doing on Monday for different randomly dumped packings, that maximum capacity is around 1.2 inches of water per foot.

55:23

So around the Pascal per meter, right, we do that conversion from Monday.

55:28

OK.

55:30

So the the purpose of mentioning structured packings is that these are very, very common in natural gas dehydration.

55:38

All right.

55:38

So if you if you intend to go work it with Woodside or Chevron in the natural gas production field, this is going to be the type of packing material that they'll typically use to pull the water out of a natural gas system.

55:50

But here for these examples, we're focusing more on the basis basics of absorb.

55:57

I told you I do this multiple times.

55:59

Absorption, desorption in randomly packed columns.

## Lecture 18 - Mass Transfer in Staged Operations

0:00

So the last time that we were meeting on a Wednesday, we were looking at not just the basis of absorption or desorption columns, but how we design them.

0:08

And in particular, we introduced this concept around the number of transfer units, which I also call the number of transfer stages, right?

0:15

And you'll see it written both ways.

0:19

So at it's core, a transfer unit, you can think of a transfer unit as responsible for transferring a certain amount of mass from one phase to another.

0:30

So if we're looking at the overall gas phase number of transfer units, that's capital N sub OY, right?

0:35

And this does not have a dimension with it, it's just a number.

0:38

So that might be 4.5 units, for instance.

0:41

And then we multiply that by the height of each unit to get the total height of the column right.

0:46

And one of the things we articulated was that if our operating and our equilibrium lines are both straight and parallel, that we can simplify that number of transfer units definition.

0:56

But in most cases and in everything that we've seen, they're they might be straight, but they're not parallel.

1:01

So we have to use this log mean driving force.

1:04

And we can visualize this on the right hand side, where perhaps our gas phase will have 6 transfer units of a rather thin height.

1:11

And our liquid phase, if we look at the overall, we'll have 4 transfer units of a slightly larger height.

1:17

And so whether we design the column from the liquid side or the gas side, we actually get to the same answer, right?

1:23

And so it's most common that in gas absorption applications, engineers will use this overall gas phase, number of transfer units and height.

1:32

That's the convention.

1:34

And then we also talked a little bit about what a pinch point looks like.

1:37

And we're going to look at another example today, but that's where we have some kind of contact between other operating or sorry, our operating and equilibrium lines.

1:46

So if we're thinking about an absorption column where we're moving mass from the gas phase into the liquid phase, a pinch point's typically identified by a contact, a terminal contact.

2:00

So my inlet, for instance, of the vapour phase by inlet concentration of component A is going to be on the equilibrium line.

2:09

Now if we flip that around and think about a stripping column, then I don't necessarily have to have a terminal point touching, but rather the line just has to any point in the middle of my my operating line has to graze the equilibrium line or equilibrium curve.

2:26

We also then talked about the effect of pressure in these systems and specifically saw that high pressure tends to skew my results and my operating and equilibrium lines to give me a richer product.

2:39

So I, I move more mass of component A from the vapour phase into the liquid phase at at otherwise the same flow rate.

2:47

And then we touched very briefly on the difference between dumped packings and structured packings.

2:52

And of course that's playing into some of the content in your lab 2 reports the most, the one big application here.

2:59

And if you take CHPR 5522, which is gas processing, two, you'll go through LNG plant design, natural gas dehydration.

3:12

And that's where the idea of a structured packing Polym is going to come into play.

3:18

OK, so before we go too far, I wanted to do one more kind of main example on absorption of a component from the vapour phase into the liquid phase.

3:30

And this time we're going to talk about acetone at one atmosphere and 20° C.

3:35

Now the reason I like this example is because we had to go to get some equilibrium data from the back of the textbook.

3:44

And so if you have the 7th edition of the textbook, you can go to Appendix 3.

3:48

And this shows us the number of equilibrium relationships between or four different components and these are between the liquid and the vapour phase.

3:57

So picking up on some of the discussion from undergraduate thermodynamics 3006, when we think about a vapour liquid equilibrium relationship, it's dictating for a given pressure and a given temperature, how what's the partitioning of that component in the vapour phase versus the liquid phase.

4:18

So the back of the book then gives us this table that we showed here on the right hand side where what's been happening here?

4:23

And I like this because it came from Sherwood in 1937 was the original publication.

4:29

And so that's Sherwood as in like the Sherwood number.

4:33

So what Sherwood then measured as he went into the lab and had mixtures of acetone and water and error and for different operating conditions was measuring what's the for different amounts of acetone in the system, how much acetone is in the liquid phase and how much is in the vapour phase.

4:51

And so these are the actual experimental measurements that Sherwood was taking on the left and right side.

4:57

Now it's typically reported in terms of the partial pressure.

5:01

And so the table itself shows us the partial pressure of acetone in the vapour phase with units of millimetres of mercury.

5:07

And so of course we can divide that by 760.

5:11

So P of acetone on 700 and 760mm of mercury because that's our our operating pressures, one ATM that gets us the vapor phase concentration.

5:29

So what I've done below here is to plot out in YX space each of these five points that Sherwood reported in 1937.

5:38

And one of the side problems then is to figure out, OK, well, we talked a lot about linearity.

5:44

So if we have a linear relationship between our equilibrium versus a curved relationship, how do I go about actually dealing with this?

5:53

Well, I've, I've fit a linear regression.

5:57

Now you'll notice I did not fit the OR or set the linear regression to intersect at zero, and I probably should have done that, but we can see here.

6:05

The point of this exercise is a line does not really fit this data all that well, right?

6:09

It's not that linear, but I can fit it reasonably well with a a second order parabolic and that gives me a good R-squared as long as I don't use this parabolic to extrapolate, right?

6:20

So as long as I stay within my boundaries of .17 to 0 in terms of the liquid mole fraction, then this this parabolic relationship will give me a reasonably good connection between the vapour and liquid equilibrium.

6:36

So the problem set up that we're going to have here, we have a a column that has a surface area of .186 meters squared.

6:49

My acetone at the inlet mole fraction is going to be .026.

6:56

My acetone at the outlet is going to be .005 S Now I'm specifying what those those compositional boundaries are and I might do something like this if for instance, I plan to discharge this water to the environment and I have a regulation governing what's the acceptable concentration of acetone in that before I can discharge to the environment.

7:22



And of course, acetone being nail Polish remover, this is not something that's going to be inherently highly toxic, but there will be a limit to it.

7:30

Now I'm going to use in this absorption column, I'm going to use air that's going to be coming in.

7:36

So the inlet of the air that's containing the acetone, that will be thirteen .65 kilogram moles per hour.

7:50

And I'm going to be so that air is now starting off with 2.6 mole percent acetone.

7:55

At the end it's .5 mole percent acetone.

7:58

And to to pull that out of the air, I'm going to hit it with fresh liquid water flowing at 45.36 kilogram moles per hour.

8:12

And the one thing I have to specify here to solve this straight away is that the overall vapour phase mass transfer coefficient capital  $k_y$  is going to be equal to .0218183 kilogram moles per meter squared per second.

8:41

OK, now, so because I'm looking at the boundaries of point OO5 and point O26 in my vapour phase, I actually don't need all of the equilibrium data that Sherwood plotted here, right?

8:55

I don't need to have a second order polynomial that's going all the way up to .17.

9:00

I can actually get away with only looking at the relationship between the OO point and that very first data point of point O 333.

9:09

So if I write that out, I can say, OK, I'm going to change colours here to go into the solution.

9:14

I can say that within the region of interest point O395.

9:20

So that's the mole fraction in the vapour phase.

9:23

I've just rounded it to the the same number of sig figs is going to be equal to my slope of that line times point O333.

9:34

This gives me now a slope of 1.186.

9:39

So I can say  $y = 1.186 * X$  right as long as I'm working within that region of a maximum vapour phase concentration of just under point O 4.

9:53

So when I then do the calculations on this, it gives me that the vapour phase inlet if I account for that acetone.

10:03

So I should clarify the vapour phase inlet I wrote up here.

10:06

This is at the inert component.

10:11

So if I add in, if I know how much error is the inert component is flowing and I know the mole fraction inside of that, I can get the vapour phase of the inlet.

10:20

In total that will be 14 point OO kilogram moles per hour and the vapour phase of the outlet.

10:35

Actually, before I do that, I know that I'm going to be losing.

10:40

Where do we have here?

10:42

Oh, sorry, I've already specified it up above.

10:44

So the vapor phase of the outlet or sorry, the molar flow rate of the outlet of the vapor phase is going to be 13.72 kilogram moles per hour, which gives me an average vapor phase mole flow rate of 13.86 kilogram moles per hour.

11:05

So now the height of an individual transfer units in the gas phase  $H_{OG}$  right?

11:13

I can define from that same relationship  $V$  on  $S$  times.

11:19

So right out here  $V$  the quantity  $V$  on  $S$  divided by  $k_y a$  which gives me .948 meters.

11:31

So this means that for every one of my vapour phase transfer units, each one is going to be just under a meter in height.

11:39

So the final bit of the solution is round compositions.

11:43

Now I know that the composition of acetone at the liquid inlet is 0 and what I can do through that mass balance is to then think about what is the composition of the acetone at the outlet.

12:01

So the acetone at the outlet is going to be point OO644 and this is in mole fractions.

12:11

This means that because I have an acetone inlet composition of 0  $Y_A$  star or  $Y_A$  inlet star is going to be equal to 0 and at this outlet composition  $Y_A$  out star.

12:31

So the amount that's in equilibrium with that is point OO764, right?

12:37

And where am I getting that?

12:38

I'm getting that by applying this relationship that I derived up here.

12:43

So if I know the  $X_I$  can solve for the  $Y$  in both cases.

12:48

So now I can calculate the log mean driving force, which I'll, I'll spare you the unpacking of, but we've looked at them on a previous example.

12:58

So  $\Delta Y_{\text{bar } L}$  what I calculate here is point O1O27 which means the number of transfer units  $N_{\text{sub } OY}$  is going to be equal to  $Y_{\text{out}} \text{ versus sorry } Y_{\text{in}} \text{ minus } Y_{\text{out}}$  on  $\Delta Y_{\text{log mean}}$ .

13:22

And what I calculate is 2 point O 4-5 transfer units and I know that the total column height has to be the number of any given transfer unit times the height of that corresponding transfer unit, which gives me a total column height of 1.94 meters.

13:43

So this is a really simple example then, where as an engineer I simply specified I need to pull this much material out of my vapour phase.

13:52

Here's the vapour flow rate, the liquid flow rate and the one critical point that we'll talk about in the next slide.

13:59

The one thing that I would need to have looked up or found somewhere else is the vapour phase mass transfer coefficient.

14:07

So, OK, so when we then think about absorption and stripping problems and principles, there are basically two knowledge gaps.

14:23

Every time you approach a problem like this, there are two knowledge gaps that the engineer's going to have to fill.

14:29

The 1st is in the mass transfer coefficients.

14:32

Now that we can report that as a little K or, or big K, right, depending on whether we're simplifying it to the film coefficient, we're thinking about the overall coefficient.

14:40

But the take home message is these are system specific, all right?

14:44

So that means I, I can, they're going to vary with what type of operation I'm running, whether it's a packed column or a, a well as we're going to look at later, a plate column or distillation column and with the components that I'm using.

15:02

So the example of CO<sub>2</sub> and water is going to be different to acetone and water and air for instance.

15:09

So the second knowledge gap that I need to satisfy is around equilibrium data.

15:17

So what I just looked at on the last slide.

15:19

So some of the examples that we've seen before would be predicting relationships

from something like multi flash, or they might use a really fundamental basis like UNIFAC or UNIFAC.

15:30

But in reality, the single best practice that an engineer can take is that you find equilibrium data from sources that have been measured, right, not using equation state predictions like we would talk about in thermodynamics.

15:46

You know, the hope is that in some number of years thermodynamics will evolve enough that we can get meaningful predictions.

15:54

But right now the engineer should always divert to using equilibrium data wherever possible.

16:00

So what resources do you have available to you for as an engineer?

16:05

Well, first that I think David highlighted in the last lab session would have been the NIST chemical web book.

16:10

And this is really good for pure component information.

16:13

Certainly one of the gold standards if you're a practicing engineer is to be very, very familiar with the peer reviewed literature, right?

16:20

So from 1930 through about 1970, the vast majority of chemists and chemical engineers worked.

16:29

The publications that came out of the field were all around equilibrium and measuring equilibrium and reporting that in peer reviewed paper, right?

16:37

And that's where a lot of our forefathers in the field come from or those kind of publications.

16:43

So I can go to the literature as long as I have a suitably matched example to find both mass transfer coefficients and equilibrium lines slash data, right?

16:54

And the following source is, of course, you have your textbooks.

16:56

And don't underestimate the value of these textbooks.

17:00

You know, keep them with you if you go to be a practicing engineer.

17:03

McCabe has quite a few data sets in this book that make it worthwhile, but in particular the gold standard for any chemical engineer is Perry's Chemical Engineering Handbook.

17:15

This, I think one of the editions I had some number of years ago was about 900 pages and it is all data.

17:23

It is basically a tone of reference data.

17:27

So if you need to look something up and you're trying to plot one of these relationships out, any chemical engineer over the age of 40 will tell you, well, have you checked Perry's?

17:35

And if it's not there, search the literature.

17:37

And if it's not there, then we'll go to something like an equation state conviction.

17:42

So I just wanted to to kind of wrap that up talking about what are the resources because I appreciate filling these knowledge gaps can be a bit of a challenge sometimes.

17:50

So what happens now if we start going from our simplistic system, thinking around we have one component that we're trying to move from phase A to phase B, what happens if we have multiple components, right?

18:02

And a really common example of this is, again, in natural gas treatment, it's not uncommon to see both CO<sub>2</sub> and H<sub>2</sub>S show up in natural gas reservoirs.

18:14

And so before we can process or do anything with this gas, we need to remove that CO<sub>2</sub> and H<sub>2</sub>S content.

18:21

But of course we don't like CO<sub>2</sub> and H<sub>2</sub>S because they are corrosive.

18:25

But in particular H<sub>2</sub>S is lethal at about 100 parts per million.

18:30

So if you inhale H<sub>2</sub>S, if you get one solid breath of it, it's lights out.

18:35

It's a very, very fatal chemical.

18:37

So for this reason it occurs naturally in reservoirs or or in sub sea porous medium through a microbial biodegradation.

18:45

And so we need to pull that H<sub>2</sub>S content out of the natural gas.

18:49

So if we are thinking about using a singular system to absorb more than one component, in this case, H<sub>2</sub>S and CO<sub>2</sub> go together because they're both highly soluble in water.

18:59

So it would make sense to put them both through the same absorption column.

19:03

What we see I'm just showing here is on the left hand side that single component absorption.

19:07

So you have kind of a benchmark.

19:09

But on the right hand side, now we're looking at here is the operating line for component B, and this is the operating line for component A, all right, each with respect to their individual equilibrium lines.

19:21

So we can plot the whole thing together, right?

19:23

We just have to plot two sets of operator A set of two operating lines and two equilibrium lines now, and there are a few points to take away.

19:31

So in the process that you've shown here, can anyone look at it really quick and identify if there is a pinch point?

19:41

Have a think for a second.

19:43

Do you see a pinch point showing up anywhere?

19:52

So the answer is absolutely, the pinch point is showing up right here.

19:56

So on this first bolded line, this is the equilibrium line for B right naturally and on the right hand side, this is the equilibrium line for A.

20:07

So we see that we keep some amount of distance through the entire absorption of a process from the equilibrium line and B.

20:17

Similarly, stays away from its equilibrium line except for this one pinch point.

20:24

So when we then look at this example, we can see beautifully illustrated.

20:27

This process was designed to remove most of A because we go from the top of the column to the bottom of the column, right, and we maintain a pretty decent driving force for component A to absorb.

20:42

But for B, we give it a pinch point toward the top of the column, right, which is the uppermost data point.

20:48

So that means that the absorption of B is going to be very inefficient, right?

20:53

That's a pinch point B very inefficient for component B at the top and it will become more efficient through the bottom of the column.

21:01

Now it's also important to note that both of these operating lines are parallel to each other and that's because we have the same liquid and vapour molar flow rates coming through this system.

21:11

So that's a characteristic by absorbing multiple components.



21:15

The  $L/V$  ratio controls both operating lines or not controls it is a product of both operating lines.

21:25

OK.

21:25

So we're not going to do much on multi movement absorption because in fact we're going to move straight into a more advanced topic, which leads us to distillation shortly.

21:35

So instead of having a absorption column where I'm going to fill it, remember absorption originally came around, engineers were taking a whole heap of rocks and they were were crushed down and they were filling up a big column bubbling gas through running water down and trying to see if they can move mass from one face to another.

21:53

Well, so, so the modern day absorption column is really just an evolution of that thinking.

22:01

Whereas in the middle of the 1940s, late 1930s to the mid 1940s, we started going from thinking around having one operation where we're going from right a, an initial equilibrium points better on this left hand graph, right our initial equilibrium point here to a subsequent outlet equilibrium point on the bottom.

22:29

We started thinking, OK, well, this is a singular I'm, I'm extorting a singular equilibrium difference to go from one to the other.

22:38

So then what happens if I start to think about a column not that's packed with ratchet rings or, or any kind of material that's going to be in a random fashion, but I start thinking about segregating my fluids and gas per plate.

22:53

And so instead of having a one absorption column, I'm going to have all of these little absorption columns stacked one on top of another where the outlet of vapour from one of those, if we look at, for instance, this internal plate here, well, the, the vapour that's flowing out of that and the liquid that's flowing into it is an equilibrium stage.

23:18

All right.

23:20  
OK.

23:21  
So this then gives me a basis for thinking around packed columns.

23:26  
Oh, sorry, from moving from packed to plate columns.

23:29  
So what we're going to do initially is apply our thinking of of that single component absorption into the context of a plate column.

23:39  
And what we're going to see is the mass stays pretty much the same, but because I have some N number of plates right in this number of plates, I might have 5/10/15 different plates.

23:52  
Because I have an N number of plates, it means I have that same N number of equilibrium stages that I'm walking through.

23:59  
So it gives me more control as an engineer.

24:02  
Now, the one difference when I move from a pack to a plate column is I now need to think about the efficiency of an individual plate.

24:12  
And the nomenclature we're actually going to use is a stage, right?

24:15  
This is why I started harping on transfer units and going down that path, because that sets us up to think about the stages of plate column, which are ultimately related to the stages of a distillation column.

24:27  
So I can apply the same steady state balances.

24:29  
Now the way that this is being drawn, this is for the dashed line shown to the right hand side.

24:37  
I can draw the same overhead material balance governing the liquid and vapour flow through this dash portion of the column.

24:46

I can also break that out on component A.

24:48

Now if I have a two component system, I don't get anything new by doing component B.

24:53

Of course, because they're going to equal out.

24:58

The one advantage is that we tend to neglect mechanical potential energy and kinetic energy.

25:05

So after all that diatribe, if you took Thermo with me last last semester, after all that diatribe of wrapping these things together, actually we're going to neglect both of those contributions.

25:15

And what we can do is balance the enthalpies of the two streams across every one of these stages.

25:20

So when we start mocking up the distillation column in ISIS over the next couple weeks, this is one of the principles that ISIS is going to work on is balancing those enthalpies between our different equilibrium stages, right?

25:32

And in the same way that I have my operating lines I define for a single absorption column, if I'm just stacking together N number of absorption columns, I can define an operating line for each one of my plates or stages.

25:47

So instead of going through it from a theoretical context, let's put together an example, right?

25:53

So what we want to do is it absorb so sulfur dioxide.

26:07

So we're going to absorb the sulfur dioxide at a temperature of 293 Kelvin pressure of one O 1.3 kilopascals.

26:18

Now the vapour molar flow rate will be 150 kilograms per meter squared per hour.

26:30

And this is the air that's coming in containing my sulfur dioxide I need to get rid of and running against this in a counterflow fashion, I'm going to wash that with 6000 kilograms.

26:40

So my liquid inlet to the plate column is going to be 6000 kilograms per meter squared per hour.

26:47

And we're just normalizing that per surface area because we're designing this independent of the cross-sectional area, right?

26:54

So defining my flow rates that way is an easy way to knock out consideration of the total size of the column and let me scale.

27:03

So I'm going to wash that with 6000 kilograms per meter squared per hour of fresh water that contains no sulphur dioxide.

27:12

Now the inlet right, my SO<sub>2</sub> concentration in the vapor phase is going to be 20 mole percent and so quite, quite some amount.

27:26

And what I need to get that down to for discharge is point O<sub>2</sub> mole mole fractions.

27:34

Sorry.

27:35

OK.

27:36

And actually that's everything I need to to define the problem except for the efficiency of every tray.

27:43

So what I'm looking for here, how many trays are in this column?

27:54

Well, to say that what I need to to ascribe is the efficiency of a given tray, which I'll give the characteristic a.

28:01

So I'm going to say that each tray is 25% efficient right at at creating this separation.

28:09

So then what I can do, All right, same process that we just went into.

28:14

We know that our inlet composition of SO<sub>2</sub> in the liquid phase is naturally zero because we're using fresh water.

28:21

So I can then put this into.

28:27

So the amount of SO<sub>2</sub> that's going to be absorbed, so transferred from the vapour phase into the liquid Phase, I can directly calculate based on the known concentration or mole fraction difference as .93 kilogram mole per hour per meter squared.

28:52

Which gives me a vapor phase outlet of 4.24 per meter squared per hour and a liquid phase outlet or sorry, that's meter squared per second and a liquid phase outlet of 334 kilograms kilogram.

29:35

This tablet's frozen in kilogram moles per meter squared per second.

29:47

So sorry, sorry, sorry.

29:51

Sorry.

29:51

No, no, no, this is per hour.

29:54

My bad.

29:55

Where that was coming from is I've taken the kilograms and divided it by the molar, or the molecular weight of air at 29 and water at 18 to go from kilograms to kilogram moles.

30:06

So these are still kilogram moles per hour per meter squared.

30:12

OK, sorry for anyone that's got confusing.

30:16

So this means that if I know how much SO<sub>2</sub> I'm going to absorb into my liquid phase and I know the molar flow rate of the liquid phase leaving, I can calculate what is the final concentrate or mole fraction of SO<sub>2</sub> leaving in the liquid.

30:32

And that's going to be point OO28.

30:36

So if I go to appendix three in the 7th edition of McKay, there is the same equilibrium data for now sulphur dioxide in an air water system and the equilibrium data goes from zero to to actually .9 mole fraction SO<sub>2</sub> in the vapour phase.

31:06

So on the next slide, what I've done is to plot all that out.

31:09

But before we get there, right, we're not going to use a number of transfer units to think about this plate column because that concept no longer applies, right?

31:18

The transfer units were a way that I can formalize my thinking about it an A packed absorption column where it's a operating with a gradient through the entire thing.

31:28

Now that I'm working on the basis of having physically separated stages that are in equilibrium, I don't actually need to go to all that calculation trouble to think about the number of transfusions.

31:40

I can do it graphically.

31:42

So what I've done on the next slide is to take that equilibrium data from The Cave and plot it.

31:47

And what I'm going to plot it against is an operating line, right, that comes from the four points I'm circling here.

31:58

OK, so I have my, my 2X, and MY<sub>2</sub>Y concentrations.

32:02

So now what this gives me I can see here's the bottom of the column and here's the top and what I'm looking at here on the right hand side.

32:12

These are all of the equilibrium data from McKay.

32:16

Now, I've fit a line to that, and you can see that in this case, the equilibrium is actually relatively linear.

32:22

It's not perfect, but what I'm going to be doing is instead of thinking about calculating the height of each transfer unit and the number of transfer units, I'm going to solve this problem visually, right?

32:34

So the approach that was done in the 1950s and the one that's still done today is that we actually can either do this on the computer or we print this out and we go from the left hand side, we're going to follow across and hit the equilibrium line and then we're going to go vertical and hit our operating line.

32:56

That is our first plate or first stage.

33:03

Now, once I hit that operating line again, so I'll just label these real quick.

33:11

Once I hit that operating line again, I'm going to trace to the right hand side until I hit the equilibrium line and I'm going to trace vertically upward.

33:21

Now my my hand drawing is not clearly straight, but mapping it on the computer or pulling out a ruler is what we used to do until I hit that operating line again.

33:30

That is my second stage.

33:33

And then what I can see is if I start to trace to the right hand side here, well, I'm only separated between the second stage intercept and the final point in my column by a very short distance, right?

33:47

Whereas if I start tracing to the right, I'm going to go far too, too.

33:51

It's going to have way too much room to run.

33:54

So in fact, I go a little bit over and then I go vertical again, right until I hit that final point.

34:01

And what I look at is that the length of that, that short horizontal bit relative to its potential to intersect the equilibrium line is about 10%.

34:17

So you think, well, how many plates or stages do I need?

34:25

Well, theoretically, you guessed it, is 2.1, right?

34:33

So I've seen both stages and then I have .110% of that final stage is what's going to be required.

34:40

Now, because I had an efficiency, a stage efficiency of 25%, my actual stages are going to be  $2.1 / .25$ , which gives me an outcome of 8.4.

34:58

So if I'm designing this column all right, and I'm going to have equilibrium stages where SO<sub>2</sub> is going to contact water, they're going to come into equilibrium, the SO<sub>2</sub> and the vapour continues rising, the water falls down through the column.

35:12

If I were designing this in a perfect world, I only need just over two stages.

35:17

So as an engineer I'd have to design three to meet my specifications.

35:21

But because the stages and plates themselves are not perfectly efficient, I then go to 8.4 stages when I account for that.

35:29

So it means as an engineer I'm going to have to build in nine stages.

35:33

Now if someone gives you a question on quiz or, or an exam, you don't want to round one way or another.

35:39

You want to report here's the actual number of stages.

35:41

Because of course we can affect this quantity by playing with things like the L / V ratio, right?

35:47

So the number of stages we'd actually need here is 8.4.

35:51

So if I take this concept of absorbing a single component, but instead of doing it



through this continuous column where there's a concentration gradient from left or from top to bottom, I break this into little mini columns where I have each one of these plates operating in equilibrium.

36:08

If I do this for not one component, but I do it for say 1000 components that are going to be fed in the inlet, this is where distillation comes from.

36:17

So the application is distillation, and the reason why we like it is that we can deploy very complex feed systems and separate them out based on a particular key component.

36:30

So we're not going to do any distillation calculations today, but I wanted to set up the context of it.

36:35

So Monday we can actually get kicked into doing some problems.

36:38

So by Tuesday when the lab rolls around, this, none of this is a foreign concept.

36:44

So the first thing I'll call your attention to is this cartoon on the bottom right, right.

36:48

And so this comes from the textbook.

36:50

So that the context of the distillation column is that plate style column that you just looked at, right?

36:56

And we did a calculation on where the single, you know, a single component being absorbed or moved from one phase to another.

37:04

This is the plate column we were talking about, right?

37:09

Where I have my vapour flowing upward and my reflux flowing downward.

37:14

Now traditionally what will happen is I will also have a feed point somewhere in the height of this column, right?

37:22

And we're going to get to that in a little bit.

37:24

Now the difference between a plate style absorption column right?

37:28

In that system, I'm feeding it as a once through.

37:31

So it's going to go through eight or nine equilibrium stages, but once it exits, its exits and that's it.

37:37

What makes a distillation column a little bit different is that I've added in two of my heat transfer operations, one on top and one on bottom.

37:48

On the bottom we introduce a reboiler, right?

37:51

And this might be something like a kettle type reboiler, right?

37:54

So that's why we spent a little bit of time refreshing that concept as well, where I'm going to pump heat through a coil, boil some of those fluids up, for instance, through a shell and tube style kettle boiler.

38:04

And now in this example of McCabe, he's putting the the full feed through the reboiler.

38:08

That's quite an uncommon practice to do right.

38:12

So I'm going to cross that out and say we're going to feed the columns somewhere in the middle usually.

38:16

So that re boiler now is going to start vaporizing everything in the liquid phase and that's going to create vapour that's flowing upward.

38:24

Now my feed that's coming in can be a liquid or a vapour or some combination of the two.

38:29

It can be multi phase.

38:30

Traditionally we use a liquid when we feed distillation columns.

38:34

Now once the vapour carries all the way up through the top of my plate column instead of exiting as it would do in a typical plate absorption column, we just looked at what makes a distillation column unique is that I put this now through a condenser.

38:49

So I'm going to remove heat from that vapour product.

38:52

And this means that I have a, I'm going to condense out a portion of that liquid and I'm going to recycle it, right?

39:01

So this is where reflux comes in.

39:04

So a fraction, a fraction of what's taken as the vapour and condensed back into the liquid phase is taken as a product and a portion of it is fed through as reflux, meaning it's recycled as liquid back into the column.

39:17

And the purpose of doing this is that I can create significantly greater efficiencies than using just a plate absorption column.

39:26

So that's why we don't really go through a lot of detailed unit operations on plate style absorption.

39:31

We deal with what's the simple, cheap and easy way to do an absorption, a gas absorption column.

39:37

But if we're going to start thinking about plates and multi phase, then we're going to go straight into a distillation column because that's typically the efficiency we want to get out of it.

39:47

Now we will over the coming two weeks, we will plot and map the performance of what happens in this column through a very similar kind of YX relationship, right, that we looked at.

39:58

And in fact, we're going to look at both numerical ways and graphical ways called

them a cave Tilley diagram for how we can plot the number of stages to figure out how many stages do I need or how many trays in this column.

40:10

And a good rule of thumb is if you're a chemical engineer and you go to build a distillation column.

40:15

Every stage that you put in it costs about equal to a Mercedes-Benz.

40:23

That's a a heuristic AUS heuristic from like 1950 and it's remained relatively true to this day.

40:29

So on the left hand side then what we're looking at, this is a typical industrial distillation column right now.

40:36

These things are used for a variety of applications that are used to obviously make vodka, which is one of the the that's going to be the basis for lab #3 as the purification of ethanol.

40:47

They're also used to take crude oil and split it into multiple byproducts.

40:51

They're used for biomass to biofuels, refining and subsequent splitting.

40:56

But one of the take homes, if you look at this image, is these columns are really, really high.

41:01

And each one of these columns contains a significant number.

41:04

It wouldn't be uncommon to say 20 or 30 different stages.

41:08

So you can imagine for the chemical engineer, this is a lot of money that you're responsible for.

41:13

So if we take that column on the left and we break it into what I like is this much simpler cartoon.

41:18

Now we're looking at just the column itself where we're feeding a liquid on the left hand side.

41:27

And what happens is that liquid comes in, I can see in it what we've done is to blow up this middle section here.

41:33

So here's my feet coming in on the left.

41:35

Well, what I have are little bubble caps.

41:38

So once the pressure in this vapour down here starts to build up enough, it will start flowing up into this bubble cap.

41:46

And if the pressure rises just enough, the cap will pop up right, allowing that vapour to bubble through the liquid.

41:54

Now if there's not enough pressure, the cap stays sealed until it builds up.

41:57

So I can guarantee the pressure at that condition, right?

42:08

Because of the use of these these caps now that means the liquid is then spilling over and flowing through.

42:15

So if I switch colours to blue for a minute means the liquid is going to be flowing around each of these caps until it finally falls over this Weir and will fall down into the the next stage, right?

42:37

Meanwhile the vapour has now exchanged a lot of mass with the liquid at that known pressure and temperature condition and it will continue flowing upward.

42:46

So this process will happen.

42:48

You know the diagram here shows 7 stages.

42:50

In reality it's usually 20 or 30, each one costing a Mercedes-Benz.

42:56

And the final point right is that I have pure vapour leaving the top of my plate column.

43:03

Now that vapour is what goes to the condenser.

43:05

The condenser transforms all of it to a liquid phase.

43:08

So it's that usually might be something like a shell and tube heat exchanger, might be an air cooled heat exchanger.

43:14

The lab you're going to start on next week is a water cooled shell and tube heat exchanger.

43:19

So this should all be quite familiar.

43:21

And then what I can decide is how much of that liquid is am I going to take this product could be 100% or if I'm trying to get a more efficient system, I could recycle maybe 50% of that liquid through the reflux stream back into the column.

43:37

So the two sources of liquid into my distillation column are the feet of the reflux now on the bottom right.

43:44

Once this liquid has now lost all of its lighter components, all all the light ends are in the vapour phase.

43:50

The liquid down on bottom flows to its reboiler and this should be the the liquid content at the lowest vapour pressure now.

43:57

So then I put that into a similar style shell into heat exchanger, pump a lot of energy into it.

44:03

I'm going to bring it to its boiling point and I can either take a portion of that as a product, or it's not uncommon to see a vast majority of that liquid content be recycled and put back in as a vapour flowing upward, right?

44:18

So this whole process comes to equilibrium.

44:21

And for a typical industrial distillation column, it might be something like three to 10 days that it's going to take for these columns to come into equilibrium.

44:30

It's a very, very delicate unit operation and the levers that the engineer has to play with are usually the amount of energy going into the re boiler, right.

44:40

So the energy content here, the amount of energy pulled out through the condenser and what we call this reflux ratio.

44:51

So how much of that that product stream am I, am I going to be recycling in right.

44:56

And for instance, when we create reactions to make ethanol, usually water and methanol are a by product.

45:02

This is one of the ways that we separate out the ethanol, which is a valuable component versus the methanol, right that can kill people if they drink.

45:10

So with that, I won't go any further into it, but I wanted to kind of set the stage as we go from a, a single absorption column now into a plate style system where each one of these, right?

45:24

So this would be equilibrium condition one, equilibrium condition 23456 and seven.

45:34

I have seven different equilibrium stages that are in this column, right?

45:39

And that would correspond to something like the number of stair steps that I'm going to have to take on the right hand side.

45:45

So we won't touch any more on distillation.

45:47

There's nothing on the quiz this week about it.

45:49

I just wanted to kind of set the stage for going from one topic to the next.

45:54

So the take home messages now are that as we wrap up our discussion on gas absorption and I and I was quite happy to see the team's content.

46:02

Some of the questions for David and Angus look like everyone's kind of at the final point of the lab.

46:06

So thank you all for hanging in there.

46:09

I hope you've learned a lot through the process.

46:11

Gas absorption is one of those things that that chemical engineers are uniquely qualified to do because we're the only engineering discipline that's trained in mass transfer principles.

46:21

We like gas absorption because it's cheap, it's very cheap and it's relatively easy as long as we have a knowledge of the data and the literature.

## Lecture 19 - Distillation

0:00

In the last lecture 18 we talked about the kind of wrap up to absorption columns, right?

0:07

And the take home message I wanted to highlight here was that we frame and we think about designing the height of the column and the amount of material that we need to put into it based on this concept of mass transfer units.

0:20

So if going from the gas to the liquid phase, how many transfer units do I have on either one and what is the height of that transfer unit?

0:28

And the product of those two gives me the height of the overall column.

0:33

Now when we go from an absorption column to one where there are physical plates and this is our introduction to distillation, that's where we go from having this mass transfer units to actual number of stages that we're going to think about.

0:49

We take a very brief look on on on Friday as to what a distillation column looks like, right?



0:56

So the purpose of the distillation column is that if I want to separate two chemical materials, right, that are somehow mixed together, I can do that through an absorption column.

1:07

And that's a very cheap and easy way because remember, early absorption columns used crushed rocks and crushed tone to get to the surface area.

1:15

The reason I go to a distillation column is that I can get a really high efficiency and an incredibly high purity in my product streams, both from the liquid side and the vapor side.

1:26

But it's a much more complex operation that now introduces what we learned in heat exchange for both the RE boiler and a water or an air condenser.

1:36

It introduces these as components to the column.

1:40

So this is the focus now of, of the laboratory #3 coming up.

1:44

So before we go into the, the distillation column as a concept, it's, I think it's, it's easier to begin by thinking about what happens in a single stage flash distillation.

1:54

Now for the, the handful on the call.

1:56

Have you ever gone through single stage flash calculations in any of your, your previous coursework today?

2:08

Do me a favour and type in the the meeting chat if you have, because it'd be helpful to to know whether this is coming into heat and mass transfer or or material and energy balances like it should.

2:18

So the basis for flash distillation, and this is one of our simplest principles.

2:24

If if we go away from that, that cheap and easy absorption, how long we've taken a step higher?

2:29

What the flash, just a single stage flash will do is that I have basically an inlet here on the bottom left and I'm going to take the inlet to distillation.

2:39

Columns tends to be liquid and I have two components in that liquid or more, and I'm trying to separate them.

2:46

So when I'm thinking about a flash vessel, the principle that it's going to exploit is pressure.

2:53

So I'll take this inlet feed and I'm going to put it through a shell and tube heat exchanger, right?

2:58

So this might be steam coming in on the tube side and that steam will flow across.

3:03

Obviously I just have a single pass in the steam and a handful of passes in the in the tube side.

3:11

So the feed that I'm trying to heat up goes from say  $T_{7L}$  to some  $T_{warm}$  or  $T_{7H}$ , right?

3:20

And in the process of heating it up, right, I also increase it's pressure, right?

3:25

So I go from a lower pressure to a higher pressure condition.

3:30

And then what's going to happen is I put that that liquid, as I increase its pressure by heating it up, I put it across the valve to rapidly expand to some low pressure again.

3:48

And in the process of expanding that, my liquid, my heavier component of the two, right, will drop out and my vapour will be enriched in the lighter components of the two.

4:01

So the working principle here that's going to separate them is vapour pressure.

4:08

Now.

4:12

So taking that a step further, right, if I'm looking at this diagram here, when I think

about doing a single stage flash on a, say a binary mixture, right, I'm going to have two components.

4:24

And as a convention for the next two weeks, when we talk about distillation, we will always refer to the composition of the lighter component, which we'll call  $y$  that tends to be the basis that we work on in most text textbooks.

4:38

So if we think about the single stage flash of a binary mixture now with components A&B that are together in this inlet stream, well, firstly I have two unknowns, right?

4:51

So if I have my feed mole fraction, I know that because that's what I've either trying to clean up or I've paid money to get and I'm feeding that into the system.

4:59

I do not know the quality, I shouldn't say quality, it isn't steam.

5:04

I do not know the outlet concentration of my component in either the vapour stream or the liquid stream.

5:12

So in talking about distillation, we always talk about our vapour product with a subscript D So  $y_D$  is the vapour concentration of my light component.

5:23

Now in the again the convention we're going to use for the next two weeks when we refer to the liquid side so that the slide that's being enriched in my non, my non volatile component, in this case component B, we call the bottoms product and we get an  $x_B$ .

5:43

So in this case,  $x_B$  is the mole fraction of component A in liquid.

5:55

And I write that down just because it can get a little bit confusing when we're saying A&B.

6:00

But in this case, when we say B, we're always referring to the bottoms unless we somehow explicitly clarify.

6:07

So if I think about this, I have two unknowns.

6:09

I don't know the composition that will be coming out either the top or the bottom of the flash separate flash stage.

6:17

So there are basically three ways that I can get out of those.

6:21

The best thing we can do is find actual data measured for equilibrium for our systems.

6:29

It's often the case, right, that if you're doing something like a simple, if you're trying to make your own vodka, there is a lot of equilibrium data dating back to the 1800s that's in literature that you can look up as a best practice to guide yourself with.

6:43

When you're using simple systems, again, like ethanol and water, because that's a relatively well studied system and there's a lot of data, then you can also use equations to take predictions like multflash.

6:54

So if we want to generate PXY or TXY diagrams like we're going to look at today, multflash will do a reasonably good job as long as our system is simple, right?

7:04

We're not using 50 components that are very uncommon to find in engineering.

7:12

But if we don't have either of these, if we don't trust our equation and state predictions because we have too many esoteric components and we don't trust or we can't find any equilibrium data.

7:22

The other approach that I can take is relative volatility.

7:24

All right?

7:25

And this kind of underscores the working principle of distillation that we're looking at.

7:30

So we define relative volatility through the Greek letter alpha.

7:34

And because we're talking about a binary mixture, we give it alpha AB.

7:39

So this is the relative volatility of component A, or rather the volatility of component A relative to component B.

7:47

And it's fundamentally defined right as the vapor on liquid equilibrium of component A on the favorite liquid equilibrium ratio of component B.

8:13

Now obviously, if I have equilibrium data right, that's something I can calculate directly.

8:19

But if I have data, then I don't necessarily need to go after relative volatility in the 1st place.

8:25

But here's where I can also understand that this Y on X ratio for component A and Y on X for component B.

8:32

These can also be defined from Ralph's law, right?

8:35

And I can either use Rouse law for kids if I could.

8:38

Everyone on the call today is from thermal estimates.

8:40

But so you remember my Rouse law for kids example, Well, if I divide both sides of that by X of A and by P, then I have Y on X = P stat on P.

8:54

And because everything is operating at the same pressure in the flash vessel, then the relative volatility of component A to B in the context of Rat's law for kids simply becomes the ratio of their vapour pressures at the same temperature.

9:17

So now if I look at this this equation, obviously I'm omitting the fugacity coefficient and the activity coefficient, right?

9:25

So our correction factors for non ideality of both the gas and liquid phases respectively.

9:31

So if I did write Rat's law for chemical engineers, so YACAP equals  $X_A \gamma_A$  gamma APA sat.

9:44

I can also rearrange that to say  $Y_A$  on  $X_A$  is going to be equal to now  $P_A^{\text{sat}}$  times  $\gamma_A / P$  times the fugacity coefficient of the vapour phase  $\Phi_A$ .

9:59

And if I take the same ratios of the two, then  $\alpha_{AB}$  as defined by as defined by Ralph's law for chemical engineers with all non ideality included, the  $P$  is still cancelled out.

10:15

But now I'm looking at.

10:16

In fact, I should probably write this down here so it's a bit easier.

10:27

$\alpha_{AB}$  is going to be equal to  $P_A^{\text{sat}} \gamma_A$  divided by  $P_B^{\text{sat}} \gamma_B$ .

10:44

And then because I have that fugacity coefficient  $\Phi_A$  on the denominator, that's going to flip around for both sides.

10:51

So on top I have  $\Phi_B$  and on bottom I have  $\Phi_A$ .

10:55

All right, So this is a really easy way.

11:00

Then if I start to have multiple components in my mixture or very unusual components that I might not be able to find equilibrium data for.

11:08

Well, if I know there are vapour pressures which I can get off NIST chemical web book if I know or I want to account for liquid phase activity, which is usually the most common, right?

11:18

We're not high enough pressures for the fugacity coefficient to be all that meaningful.

11:22

Should be that one.

11:24

But liquid activity is reasonable.

11:26

So we've gone through liquid activity models last semester like Margulis that we can deploy to estimate what is this relative volatility for new components.

11:36

So once I have that, then I can do a material balance on the this process, the single stage flash process from my feed to both my tops and bottoms product.

11:49

And when I do that, right, I can relate  $X$  sub  $V$ .

11:52

And what we're looking at here this  $F$  and  $1 - F$  in this case,  $F$  is the molar fraction that's vaporized.

12:00

So it's out of 100%, right?

12:03

So the zero is less than or equal to  $F$ , less than or equal to 100%.

12:07

Those are the bounding conditions.

12:08

And  $F$  has units of percent.

12:11

So if I'm vaporizing in my flash vessel 100% of my content, that means that  $1 - F$ , 100% is 0.

12:18

So I have no component A coming out of my bottom stream, and 100% of my feed of component A is going into the vapour side.

12:29

Now if I don't know exactly what that ratio is going to be, I can rewrite it in the context of relative volatility to say a relationship between  $Y$  &  $X$  for this operation can be related solely through that relative volatility principle.

12:44

And then I finally in the same way that I can balance my material on a component level through this process, I can balance enthalpy.

12:51

And of course we talked last time we very briefly mentioned we tend to neglect molecular kinetic and potential energy when we are balancing energy on a, on a per molecule basis.

13:09

So we only use the enthalpy of our inlet and exit streams.

13:14

So this is a, a very quick introduction to what a, a flash process looks like.

13:19

Now you can imagine that if you're going to go through all of the expense of building one of these things, right?

13:24

And on the last slide here, this is an example, you can see what a typical flash vessel might look like, right?

13:31

It's quite large, has a lot of mechanical engineering around it, a lot of integrity to consider, right?

13:38

The next stage is to say what happens if that outlet, so the quality of component A in the vapours tree Y is D.

13:48

Maybe I can only get that in a single stage.

13:51

If I have a feed that's 50/50, maybe that's going to split and my vapour product is going to be 80% component A.

13:58

So it's going to enrich the vapour product.

14:02

Now the next step that the engineer is going to want to ask then is what happens if I put another flash vessel in line, right?

14:12

And maybe that takes component A in the vapour side to 90%.

14:16

What happens if I put a third, It's going to go to 92%.

14:19

If I take a fourth, it's going to go to 95%, right?

14:22

So the idea of stacking together multiple flash vessels, right, is where we're going to



start getting a lot of efficiency and we can get to a much higher purity in component A, right?

14:34

So this is now the context that we come into when we talk about multi stage flash and a continuous or a distillation column, right?

14:44

These are designed to run in continuous mode.

14:46

So this is the actual name of our unit operation continuous distillation.

14:51

But this is effectively a multi stage flash process, right?

14:55

So every one of these sections we call plates in here and we looked on Friday in an image of what some of those plates might be.

15:04

Each one of these sections is itself operating like an independent flash vessel, right?

15:11

The only difference is that the pressure drop across these columns is, is measurable, right?

15:17

That tends to not be that significant.

15:20

And certainly the pressure drop, we are not exploiting the the principle of pressure to drive this kind of system like we are with a a single stage flash vessel, right?

15:30

Where it's in fact that that pressure drop across the valve right here, it's that differential pressure that's going to drive my actual separation process.

15:40

And an example might be if, if this is a binary TPXY diagram, if I've heated my fluids, right, let's say they're up here.

15:51

Well, if I flash downward in pressure and go from 80 to 50 kilopascals, then the fluids are going to split right?

15:59

And my do curve, this will be my for a single stage flash, my liquid composition and on the left hand where it intersects the bubble curve, this is my vapour composition.

16:12

So in this particular orientation of the TXY diagram, this would be enriching component A in the liquid phase, right?

16:19

And so I'm looking at equilibrium day.

16:21

So the idea is that a single stage flash is designed around exploiting the principle of pressure as it's working.

16:27

Whereas when I go to a distillation column now I don't have pressure that's being exploited.

16:32

It's a relatively constant pressure, but it's temperature that's actually the exploited of separation.

16:39

So when I'm unpacking this column for a minute, which we're going to kind of go through the basic operations, there are kind of four core elements to take a look at.

16:48

There's what we call the stripping section.

16:50

So if I'm feeding the column, you can see here's a feed plate.

16:54

That means that I have a mass inlet, right?

16:57

That's coming in right here to this middle of the column, right?

17:02

And usually as an engineer, I get to decide where I want to feed the column.

17:06

And if I'm going to build or buy a column, I'm probably going to choose one where I can change the feed as I'm optimising the process.

17:15

And so it's not uncommon that you might change feed locations over time.

17:19

So if the column is being fed in the middle here and everything from that feed plate below, we refer to as a stripping section.

17:27

So that that functions kind of like a, a stripping process of an absorber or or the inverse of an absorption, absorption column where we're trying to remove our volatile component A or our valuable component from the liquid phase.

17:42

Everything above that feed plate we refer to as the rectifying section.

17:47

And that's where we are purifying component A in the vapour phase.

17:51

So I have these two sections that I break my working column into, right?

17:55

So I take my absorber and my stripper.

17:58

I'm going to consider multiple of those in sequences, right?

18:01

So in this case, this is going to function on top like 12345 absorption columns running in in series.

18:14

And on the bottom I have the same 1234.

18:23

Oh, hold on, Siri.

18:30

You got a lot of Siri.

18:31

She's always listening in.

18:33

So on the bottom now I have these four, what are actually stripping columns that are being run in parallel, right?

18:42

So what I'm doing is taking my principles of absorption and stripping, putting the two together and then adding to this, the two unit operations.

18:50

We started out with a heat transfer.

18:52

So I have a condensing system, right, which is typically an air cooled or a water cooled heat exchanger shell and two.

18:59

And I have a, a reboiler on the bottom, right.

19:02

That's going to be the same kind of reboiling process we've talked about before.

19:06

So I have 4 unit operations effectively that I'm bringing together in one large scale unit operation that we call a distillation column.

19:15

Now what makes this particular column, most of these will have reflux and you're going to see that in the lab tomorrow.

19:21

So there are when we talk about reflux, this means that when we are condensing our product stream and this is going to be very rich, right, in component A remember that's our volatile component that we're trying to to purify.

19:40

What makes this very efficient is that I take that that column or sorry that that product as it comes through the condenser, it's going to be liquefied, right?

19:49

And the way this is being drawn here, it's using a water cooled, a dual stage, a water cooled exchanger.

19:55

But this can be any configuration, right?

19:57

I can use an air cooled heat exchanger to be more environmentally friendly.

20:01

I can use a refrigeration system if I want to, but I take this outlet and here now it's 100% liquid.

20:12

Where the inlet to this was 100% vapour once that all liquefies I can choose at the outlet of that how much of that liquid is going to be recycled?

20:23

And now this is liquid that's that's very high purity of component A be recycled through a pump back into the top stage of the column.

20:34

And how much is going to be taken out as a product.

20:37

All right.

20:37

And remember we used that subscript D to denote the top product of my column.

20:44

So that's the distillate product.

20:46

Also in older nomenclature, we call this the overhead product.

20:51

So this ratio then of the engineer will get to usually choose pump power, right?

21:02

That means I get to choose from zero to 100% how much of that liquid stream do I want to recycle into the top of the column and how much do I want to take out as a product.

21:14

And so we called it the ratio of that  $L&L$  is  $a$ , a molar flow rate as  $D$  is a molar flow rate.

21:26

We call the ratio of these molar flow rates the reflux ratio because it's dictating how much reflux am I putting back in or how much liquid am I refluxing into the column at the top.

21:39

Now it's when we go to this week, we are going to focus on how we design a column.

21:45

So that will be primarily a consideration of, sorry, not the number of columns, the number of plates or, or stages, right?

21:58

So what I've drawn here is a system that has 9 stages and I'm feeding it in the middle.

22:03

Now if I feed this at the very bottom stage, then I have 8 absorption columns running in series and one stripping column on the bottom right.

22:12

So that's where we might build a system that's going to have multiple feed locations that I can change over time.

22:18

So the engineer then needs to think about in a design construct, where am I going to feed this and how many plates do I need to get the degree of separation that I'm looking for?

22:30

Now, you can imagine that because I'm pumping a lot of heat into this reboiler, right?

22:35

So this is a very high temperature at the bottom of the column.

22:41

And because I'm using cold water at the top to condense everything, you can imagine that there is now a very clear temperature gradient from the top to the bottom of the column, right?

22:51

So instead of exploiting the principle of pressure to separate two or more components, distillation columns exploit the principle of temperature.

23:01

So they have an inbuilt temperature gradient and that's what I'm using to create this separation.

23:08

So once the engineer is designed, okay, I'm going to need in this case A9 stages or plates for my operation, I'm going to feed it in the middle.

23:18

Then these latter 3 considerations, these are what the engineer gets to play with in operation, right?

23:24

So if you walked into a job and more likely than not, because remember each one of these stages in an industrial process usually costs in a Mercedes-Benz.

23:34

So you can imagine that a column building a column like this with its condenser and reboiler, it's going to be a multi \$1,000,000 operation, right?

23:42

So it's more likely than not, if you were an engineer, you would walk in to a column system that's already been built.

23:48

So these latter 3 points, right?

23:50

If you, if you can't change feed location, this is where the engineer can then optimize the process further or make it more efficient.

23:57

And so we get to choose, well, what's that duty?

24:00

So the amount of energy we're putting into the re boiler, which ultimately dictates it's working temperature, what's the condenser duty, right?

24:08

How much energy am I going to need to fully liquefy the product, right?

24:12

So this is the condenser duty tends to to be a consequence of the re boiling duty and the amount of energy I'm pumping in.

24:20

But then the number one thing the engineer gets to usually control is the reflux ratio.

24:28

And that reflux ratio is controlled alongside the feed rate, right?

24:35

So one of the the poor parts that we're really excited about is that historically we've had to run batch distillation in the unit OPS lab, right?

24:43

But getting a continuous distillation column online can be very tricky and take quite some time.

24:48

So Angus and David have been hard at work in the lab actually getting this thing running.

24:52

In continuous operations, you can get a very practical, meaningful, real experience in how a distillation column is going to work.

25:01

It is rare if ever to see a multi \$1,000,000 rig like this would be an industry deployed in a batch context right.

25:10

That is rarely you've ever done because of the expense of these kind of things.

25:14

So in when you go to industry, because these will stand right, the height of the new E zone building, that would be a, a short distillation column and set up, you can imagine that it takes because we're thinking about individual absorption and stripping columns that are all kind of stitched together, right and massive re condensers and re boilers that are adding and subtracting heat from the system.

25:41

Well, these things will take from days to weeks to come into equilibrium operation.

25:46

So if you're the engineer, right, one of the points to think about is how you throttle the feed rate and the reflux ratio.

25:54

So if you're trying to start up a distillation column from scratch, a very typical operation would be that you start feeding it nothing, right?

26:02

We fill up our re boiler on the bottom, we fill that up with our initial feed.

26:07

So, and then we start that heating process to generate some vapour and we initially we will tend to start with a reflux ratio of Infinity.

26:17

All right.

26:17

And you can imagine this thing goes to Infinity because we're dividing by zero D, the overhead or the distillate product is 0.

26:25

We're not taking anything out to begin with.

26:27

We're just starting the, the engine up and letting this vapour start flowing through the column, condensing and recycling back.

26:36

So if we are at 100% reflux, right means that 100% of my liquid product stream is being reflexed into the column.

26:45

That's the same as having a reflux ratio of Infinity.



26:49

So I will typically start up a column with no feed and a reflux ratio of Infinity.

26:55

And as I start to see that distillate product emerge, right, I will then start turning on the feed rate and very slowly backing down the reflux ratio at the same time, right?

27:09

Because you think if you, if you decrease the reflux ratio from Infinity, that means I'm starting to take a distillate product.

27:17

Well, because this is a series of equilibrium operations, right?

27:21

Anytime I make a change to one domain in the system, it's going to take for a small column like hours for teaching, it's going to take 30 minutes to propagate all the way through the system.

27:32

For a large column like you'll find in an industry, it's going to take six hours, right?

27:37

So the principle is I slowly turn on my, my inlet feed, it may be a 1% full capacity and I back off the reflux ratio, I'm going to back it down.

27:47

So I go from 100% reflux to 99% reflux, right?

27:52

And, and the reflux ratio in there might be somewhere in the the highest or low hundreds, right.

27:58

And then I'm going to back it to 98% reflux and 2% feed.

28:03

So I, I try to maintain a constant mass in the column as I'm bringing it into the goal, which is 100% of my feed rate.

28:11

So just kind of a slide note, when you go to start these columns up and operate them, they, they tend to be very, very finicky to get into to full operation.

28:20

And this is part of the process.

28:22

So there are those two design considerations.

28:25

How many plates do I need?

28:26

How many equilibrium stages to achieve my desired separation?

28:32

And then I have these kind of 3/4 variables that I can play with.

28:35

If we combine both re boiler and condenser duty, it's kind of I have one thermal variable, 1 reflux ratio variable, and the feed rate that kind of complement complements the reflux ratio.

28:46

These are the three things that I'm changing on the fly if I'm trying to manage a distillation column.

28:55

OK, so this picture to the right hand side, if if you have gone to any breweries or distilleries, you would notice this is actually a distillation column shown here, right?

29:05

And what this is obviously built for show, but what they're including is windows.

29:10

So you can actually see each of the individual stages, right?

29:13

And I can see the the kind of boiling and the the bubbling that's happening in each stage.

29:18

And this is a fun example from a distillery.

29:21

You can see here, this is one of the outlet tanks, right?

29:25

That's probably going down to a re boiler.

29:27

So if I'm thinking about each one of these individual stages that would be clipped, these colour other than red.

29:35

But if I think about each one of these individual stages that would be clipped right across the system, well, I can theoretically then however I want to apply my theory, I can break these into  $N$  number of stages, right?

29:49

And I say end number because we don't know from the outset how many plates we're going to need.

29:55

If every plate in an industrial process is going to cost us a Mercedes-Benz, we don't want any more than we need to.

30:02

But we have to begin the design process without a clear understanding of what, how many will be necessary.

30:08

So if I think about just the end plate, all right, so I'm going to circle here.

30:13

Here's my end plate.

30:15

And this plate is going to have, as we saw earlier, a whole bunch of caps on it so that the gas from the plate beneath can bubble up and the liquid is going to spill down the sides and overflow down to the plate beneath.

30:28

So on this end plate I have vapour that's entering and the vapour entering will have the composition  $Y_{sub N}$  I have liquid that's leaving that plate and that will have the composition or sorry, vapor leaving the plate has the composition  $Y_{sub N}$  at a molar flow rate of  $V_{sub N}$ .

30:48

The liquid leaving the plate has a composition of  $X_{sub N}$  and this is a liquid spilling over the side of this equilibrium plate and it's spilling over the molar flow rate of  $L_{sub N}$ .

31:00

Now I have two inlets to my plate.

31:02

That's my upwelling vapor, right?

31:05

So that's why  $n + 1$ , if I'm numbering my plates from the top to the bottom, which is an arbitrary standard or convention that we use.

31:15

And the liquid that's going to spill down to the plate from the, the plate above will have a, a liquid composition of  $X$  and  $y_{n-1}$ , right?

31:26

So I have these on any given plate, I have these four compositions that I need to think about and I'm framing this around my most volatile component, A.

31:36

So all of this is about a binary distillation right now.

31:39

And next week we'll get into what happens when you have a multi component.

31:42

So in each of these equilibrium stages, my vapour is going to be enriched in its concentration of A as it flows from the bottom to the top of the column.

31:53

So that means I can write with certainty  $Y$ .

31:56

If I look at that diagram,  $Y_{N-1}$  will be greater than  $Y_N$  greater than  $Y_{n+1}$ , greater than  $Y_{n+2}$ , so on and so forth from the top to the bottom.

32:09

And the liquid is going to be depleted in that rich component A as it flows from both now the top of the column, right in my reflux and wherever I'm feeding it, either the middle, top or bottom of the column, the liquid is going to be depleted in A as it flows from the top toward the bottom of the column.

32:30

Which means I can guarantee  $X_{n+1}$  will be greater than  $X_N$ , which is greater than  $X_{N-1}$ , so on and so forth.

32:42

So you imagine then each of these, if I take that that framework of my absorption column, right, there was a very simple mass transfer operation in each case, right?

32:52

I have two liquid and two vapour compositions, 2 that are coming in and two that are going out, right?

32:58

And the purpose of that plate is to try to achieve an equilibrium between the inlet vapour and liquid strings.

33:08

So how do we do that?

33:11

Well, because we are exploiting the principle of temperature, when we do distillation columns, then we're going to plot this up on TXY space, right?

33:21

So switch back to red here, right?

33:24

This is a traditional TXY diagram, and in fact, this one comes from the book.

33:29

But if you were in Thermo with me last semester, we would have seen a handful of examples, right, of what these TXY diagrams look like.

33:38

So because my column has a gradient temperature across it with the reboiler at the bottom, condenser at the top, I know the direction of, of thermal, the thermal gradient.

33:47

If I modify that duty.

33:49

So the amount of energy I put into either the reboiler or the condenser or pull out of the condenser, I'm going to modify where I'm at in this TXY space.

34:01

So to to frame this around a TXY diagram, let's say that plate N that we're thinking about on the previous slide where we had our two liquid and two vapour compositions that were meaningful, that plate N as a temperature  $T_{\text{sub } N}$ , right.

34:20

So in TXY space, if this is a binary mixture of only components A&B.

34:28

So on the left I have 0% A and on the right I have 100% component A.

34:34

Well, if I demarcate a horizontal line corresponding to  $T_{\text{sub } N}$ , which is the temperature of the plate N, in the same way, I could look at a PXY diagram to unpack what's the phase split going to be when I flash that fluid and I drop its pressure rapidly.

34:56

I can use a TXY diagram to identify well the intersection of the bubble curve, which

on a TXY is that lower curve and the intersection of my DO curves respectively, give me the liquid concentration of component A at, call it plate N.

35:15

So  $X_{N-1}$  and the vapour composition of component A at plate N or  $Y_N$ .

35:24

Now I can take it a step further and I can say at plate  $N-1$ , I know the temperature is going to be cooler, right, because plate  $N-1$  is above plate N. So if I then say, OK, this is  $T_{N-1}$ , well, now I can use that same txy.

35:50

And I'm not trying to agree with the dash line here, actually just it's an arbitrary right.

35:56

I can see now my composition split at the plate above plate N I'm going to have a richer composition of both the liquid and the vapor phase.

36:11

So on the left this would be  $X_{N-1}$  and on the right this would be  $Y_{N-1}$ .

36:21

Now I can do this again and say, OK, plate  $N-2$ .

36:30

So the one that's going to sit above this, all right, that's going to be say,  $T_{N-2}$  a colder temperature.

36:39

And here again in txy space, I can map out exactly what those liquid and vapor compositions are going to be for component A.

36:53

And because it's a binary system,  $1 - x_B$ , that is the concentration of component B, all right?

37:00

And I can do the same thing going downward into the column.

37:04

So if I grab a different color, I can say, well, here is maybe N the  $T_N$  plus one, so that the next stage lower, the second stage lower, third stage lower, so on and so forth,  $N+2$  and  $T_{N+3}$ .

37:22

All right.

37:25

So this is one of the most fantastic examples of directly exploiting thermodynamic data for a very important unit operation, right?

37:33

And a connection between the two.

37:34

And, and in fact, one of the things that you're going to do, I think it's question two or three, is that we're going to look at that temperature distribution in the column for making our vodka.

37:45

And we are going to plot that out directly on ATXY diagram for ethanol and water, right?

37:53

OK, so kind of wrapping up the discussion for today, the once we have this working principle in play, where you should be at is to say, OK, I know that a flash, a single stage flash vessel exploits the working principle of pressure, but a distillation column has a temperature gradient.

38:16

And the way around, the way I can relate the temperature of the stages to the composition on each stage is in thinking through a TXY diagram for whatever system of interest I have, right?

38:29

And I can generate this from multi flash.

38:31

I can go to Google Scholar and look up equilibrium data to plot ATXY for ethanol and water.

38:37

I think David and I just going to give you that data so you don't have to go hunting for it.

38:42

But this is very simply how I can lay out to say, OK, if I need a vapour product that is 98% ethanol, for instance, for 98% component, AI need to drive the system right down to this very bottom point that that vapor stream should be so cured.

39:02

Maybe my requirement is only 80% ethanol.

39:05

That means I can knock off a few stages and save a few Mercedes-Benz.

39:08

Yeah.

39:11

So where we're going to wrap up is the last content slide for today.

39:15

And where we're going to get kicked into on Wednesday is how we use our material balances to to draw both the operating and the stripping lines.

39:25

Because you remember that we have this rectifying section and then switch colours.

39:31

The rectifying section is where I'm, this is my purification or purifying my vapor where the stripping section which is everything at the feed plate and below that's going to be where I'm removing component A from the liquid phase as much as possible.

39:53

So when I start to to map out these material balances for the column, I have to do so with the context of my reflux product or sorry, reflux stream and the overhead product.

40:10

So what we've shown here, this diagram comes from the textbook and there's a few conventions I want to highlight.

40:16

We tend to to think about distillation columns because they're being driven by temperature as the exploitation principle.

40:23

We tend to think of them as basically the rectifying section that includes the condenser and the stripping section that includes the reboiler.

40:32

So we think about all of them at once.

40:35

Now on the entire unit, right, I can see I have two things going out a bottoms product with a quality or a composition and a overhead product or a distillate product with a molar flow rate D.

40:52

So these this would be something like moles per hour and a composition of my of X sub D, right?

41:01



Because remember, I've liquefied all of that in the condenser and if I have AC flowing in at some number of moles per hour, then the mass or material balance isn't going to do.

41:12

And we put this into a design basis.

41:15

This assumes steady state operation, right?

41:18

So the, the critical bit of having a continuous distillation lab is that this is the only way we can actually apply the, the design principles to look at how our steady state material balances play in, right.

41:30

Otherwise we're just going from point to point in thermodynamics.

41:34

So if I'm thinking about the more volatile component and that's AI can do an overall mass balance on the system or I can do a component mass balance on a, right?

41:44

And of course, because it's binary in this case,  $1 - A$  equals the composition of component B.

41:51

Now I can rearrange this, this component material balance to think about two kind of critical ratios that are not going to be used today, but they're going to come into how we start designing the and deciding on how many stages we have, right?

42:06

And that's the KT only diagram that we're going to go through on Wednesday.

42:09

And in fact, we've already done an early version of that with gas absorption and our number of transfer operator transfer units.

42:17

So I can take the ratio of my overhead product molar flow rate on the feed molar flow rate and that is going to be equal to the difference between the feed and bottoms composition and the distillate and bottoms composition.

42:33

Similarly, if I rearranged it to get the bottoms molar flow rate, so that's the amount leaving on the feed flow rate that will be equal to the distillate composition minus the feed composition on the distillate composition minus the bottoms composition.

42:49

So by rearranging my material balance, I'm going to create these two identities that I'm bringing to create my operating lines.

42:58

So I can apply this same principle then to balance both the rectifying and the pure stripping section.

43:05

So I do an overall balance on the rectifying section is our first equation here.

43:10

And if I make it a component balance, right, for component A, we have our second equation.

43:16

So none of this should be new because we've actually seen we've done this already for an absorption column.

43:22

One of the things that will make it a bit different here is that when we start talking about constant mole overflow, right, that's going to mean that I'm not remembering with the absorption discussion, our operating line would start to take on a bit of curvature, right, because we are moving moles from one phase to another.

43:48

Now we made the assumption that we were going to straighten out that operating line because it made our calculations a bit easier.

43:55

One of the principles we apply to designing distillation columns is effectively that process.

44:01

So constant mole overflow means I straighten my operating lines.

44:14

So when I think about the context of saying, well, this  $V_{N+1}$ , right?

44:20

That's going to be where I cut the control surface.

44:23

I can account account for how many stages are within this.

44:26

I know that that molar flow rate is not going to be quite the same molar flow rate as  $V_{\text{sub A}}$ , my overall molar flow rate at the top, right?

44:37

There's going to be a bit of variation because I'm moving mass from one phase to another, right?

44:42

When I go to the bottoms, it's convention instead of using N or I'll use M as the stage number just because we don't quite know when these are all going to meet up yet until we design it well.

44:55

I know that my vapour and liquid molar flow rates on the M or M or M + 1 stage are not necessarily going to be equal to that molar flow rate is coming out L right?

45:09

So in the same process of the convention, what it does is when I make the assumption of constant mole overflow, I can get rid of using this right hand side of both equations and simply use this right.

45:22

This left hand side is my component mass balance or component material balance.

45:27

So we're not going to do anything with these today.

45:30

I just wanted to go through, here's the basic material balance, understanding that these four equations on the left hand side, I'll get rid of these crosshounds.

45:39

But these are our four material balances that are actually going to come into play that we're going to manipulate around to get the operating lines that we've made straight for both the rectifying and the stripping sections.

45:56

OK, so with that take home messages, flash distillation, this is a single stage unit operation.

46:03

Where I tend to see flash distillation used is if I have a separation where there's a large deviation in the boiling points of the two materials, right?

46:14

So if I'm heating it up, one of those materials is going to vaporize much more easily or readily than the other.

46:21

That means when I flash them and drop the pressure, I get greater separation.

46:26

And I like flash distillation because it tends to be cheap, right?

46:30

As long as I can afford the energy to power that that boiling unit or I have leftover steam that I can use to make the process more efficient.

46:38

It's a very cheap way to separate phases, but it can only take one.

46:43

If I go to that Pxy diagram, it can only do 1 split in pressure or one split in composition.

46:51

So if I think here was my right pxy alright and I had something looked bit like that right?

47:05

So the maximum width of that Pxy bubble between my dew curve and my bubble curve, the maximum width of that thing represents the, the, this maximum space that I can run, right?

47:19

And it's actually on a horizontal basis.

47:22

But the fatter this thing is, the more I can separate in a single stage, right?

47:28

But that's the, the fundamental constraint of my, my single stage flash distillation.

47:33

So as we've talked about, when we go to continuous distillation now, now I split temperature instead of pressure.

47:38

So I have my Txy on the curves, but the benefit here, right, is that I'm employing multiple stages so I can make use of the split from the low concentration range to a very high concentration range.

47:56

And if you are an engineer that's thinking about designing or or mapping out the design of one of these columns, like we're going to start tomorrow morning and we're going to spend the rest of the week on one of the things that we start with.

48:08

Number one question, how many trays?

48:10

Each tray is a Mercedes-Benz.

48:12

It's very expensive to build.

48:13

All right.

48:15

The engineer then gets to decide, OK, if I know how many trays I need because it's a temperature exploitation.

48:23

The duty that I nominate for the reboiler, which is usually a shell and tube and the condenser, which is maybe water cooled, that duty dictates how high or how low I can go in the TXY region.

48:36

All right.

48:36

So these are the two really core design bases of distillation columns.

48:42

And then the engineer gets to both design and typically uses as a point of manipulation or optimization both the reflux ratio and the feed flow rate to counterbalance the total mass in the column and ones.

## Lecture 20 - Distillation Column Design

0:00

We talked about in lecture 19, we started off by thinking about a flash distillation.

0:05

So that's a, a unit operation that takes place in a single stage, right?

0:10

And, and the reason we like flash distillation is that it's very easy to build, but to, to employ it, we're trying to take a feed mixture and separate it into its two or more components.

0:22

We need to have components that are have a variation in their boiling point.

0:27

Now the way that we exploit Flash installation is if we think about APXY diagram, right?

0:34

And that was going to look something like this.

0:36

Here's our phase envelope.

0:38

Well, if we initially pressurize our inlet stream up to some high temperature, sorry, warm it up to some high temperature, then the pressure of that stream is going to go up with it.

0:49

And if we then rapidly expand the stream across a valve, we're going to drop into the two phase region, right?

0:56

And what will then happen is our lines will split.

1:02

And so we have our vapor line, our vapor composition, this will be of component A that we're trying to separate our vapor line will demonstrate a higher content of component A in the vapor side and there will be less component A in the liquid side compared to, I'm just going to erase this one compared to the original feed concentration, right.

1:31

So it's a single stage process where we're exploiting a principle of pressure.

1:35

And when we look at this on a PXY diagram, we can see whatever our feed is, it's going to split across that phase envelope so that we can enrich one of our phases, the Baker phase in that component.

1:49

Now as we talked about, the limitation of that is that the width that we can see, I'll highlight in green the width of this envelope at a unknown pressure that we're dropping down to.

2:00

That's the maximum separation that we're ever going to get out of a single flash distillation because it is a single stage unit operation.

2:09

And so This is why chemical engineers then deploy continuous distillation because this allows us to put multiple flash distillation units together in a column and each of these will achieve a some degree of phase separation.

2:23

So the column of course is going to be warm at the bottom and that's because it's being heated by a reboiling unit, maybe that's a kettle type boiler like we looked at previously.

2:31

And it will be cool at the top, right.

2:34

And we can imagine the top is going to be driven in the case of Lab 3 by something like a water fed condenser, right?

2:43

And so this means we have a temperature gradient across the column.

2:46

And in each case we have equilibrium stages that are distributed throughout the column.

2:51

And part of the question that we're addressing today is how many stages and how do we design it?

2:56

And the difference between a single stage flash and a continuous distillation is that here we are exploiting curves in Txy space.

3:04

All right.

3:05

So if I can look at now a Txy envelope, what we saw is that if we draw lines across each of these points of temperature going from our warm stages to our cool stages, we can see what that split of our X&Y compositions is going to be.

3:33

So what you'll know is that if we think about the prompt for laboratory #3 of course, the first question is simply take the data from the lab, plot it out as a function of time, and that's it, right?

3:46

Figure out when that reaches steady state and report that with the plot.

3:50

So that's easy question #2 is doing what we just did here, right?

3:55

So this TXY diagram drawing those horizontal lines, if you know that what are the temperatures at each stage when they're at steady state, that's all we're doing.

4:03

And you have already, you have been given that TXY diagram for ethanol and water at this mixture ratio and we know what the temperatures of each stage are.

4:14

So actually from Monday's lecture, we have question one, question two done.

4:18

All right.

4:19

So now what we're going to think about in this lecture is how do we carry on and go through questions 3-4 and in particular five.

4:27

Question five at the lab is all about the McCabe Taylor diagram.

4:30

And so that's what we're going to go through today.

4:33

Now, when we design one of these distillation columns, there are four kind of parameters that chemical engineers will use to figure out what works best.

4:44

Now, two of these parameters are typically set at the point the column is designed.

4:50

So we must, we must determine the total number of stages that we need.

4:56

Each stage represents one of these trays and with the number of stages that are present, this is a fixed quantity and we remember a heuristic from the old days, meaning the rule of thumb is that each stage is going to cost something on the order of a Mercedes-Benz to buy.

5:16

So getting correct the number of stages is quite important.

5:21

Now we choose the number of stages based on how pure right we need our final vapor stream, our product stream and our bottoms are a liquid stream to be.

5:33

Now the duty, this is the amount of energy that our reboiler and our condenser are going to take.

5:38

Now the condenser is usually a consequence of the reboiler.

5:43

So we tend to think about it as sizing up the duty in terms of what the reboiler is



going to take because that's what's generating the actual temperature distribution in the column.

5:52

So we can choose a reboiler and in typically reboiling units, we will have a range of potential duty that we can employ, but we do need to pick a reboiling unit that has an appropriate range, right?

6:07

So these are the two parameters that we need to design up front and then the two operating parameters, right?

6:12

And it will look at these in a design sense.

6:14

But once you're working on a column, the two things chemical engineers will have at your disposal to modify are both the reflux ratio and the feed flow rate.

6:24

So how quickly moles or moles per hour are coming into the column and what is the amount that is recycled back to the column being  $L$  and that's in moles per hour as well divided by how much is being taken out of the top of the column as a distillate product also in moles per hour.

6:44

So the ratio of these two gives us our reflux ratio.

6:49

Now as we were talking about, you would have seen in the lab, it's very common if we're starting up a continuous distillation rig, we would start it with a reflux ratio of Infinity and a feed flow rate of 0, right?

7:01

We're going to allow the column to warm up.

7:03

So each of our stages is working near its set point temperature.

7:09

And then as we turn that reflux ratio down, meaning we're biasing toward more distillate and we're taking some out of our recycle stream, we need to increase slowly the feed flow rate so that our total, right, because the feed flow rate is also in moles per hour.

7:27

So this means that our total mass in that column stays roughly the same, right?

7:32

It doesn't need to be perfect, but it needs to be roughly the same.

7:36

So understanding this nuance of the operation, This is why an industrial distillation column might be used to make hand sanitizer, could be used to make vodka, could be used to make biofuels.

7:50

Bringing one of these online for industry can take on the order of days to weeks to reach steady state because these operations must be tuned very, very slowly.

8:01

So today we're going to move forward and talk about the next stage in this process, which is how we design each of the stages that are going into this column and how we figure out what the individual molar flow rates are going to be.

8:20

OK?

8:21

So the number of stages is the critical path for design like we talked about as a reminder on the previous slide.

8:28

Now where you've seen this come up before is when we introduce gas absorption, right?

8:33

And the reason that we emphasize the use of mass transfer units in the gas absorption design process is this gives us kind of conceptual way to think about a distillation column that's actually quite simple.

8:48

All right.

8:49

So you remember from this example up here, we had here in green on top, this is our operating line, right?

8:55

And in black, this was our equilibrium line, right?

8:58

We were thinking about stair stepping down between the two to visually identify how many transfer units there would be.

9:07

Now the difference between a gas absorption column and a distillation column is that our distillation column has both a rectifying section, which is analogous.

9:23

It's analogous to our absorption section, but it also has a stripping section, which is analogous to a stripping column.

9:32

So in the same way that a gas absorption rig, right, is going to have one absorbing column and perhaps one stripping column to purify whatever fluid is being used as an absorb for absorption fluid, a distillation column puts all of these things together.

9:49

So now we don't have one operating line, we have two, right?

9:53

And that's one of the primary differences.

9:55

So we have an operating line for our rectifying section.

9:58

And throughout this lecture, I'm going to use blue to indicate rectifying sections.

10:02

So you can think about a high purity of my product in the vapor phase.

10:07

So I'm trying to purify that high vapor pressure component.

10:11

So in the case of lab three will be ethanol and our operating line, this corresponds to the stripping section.

10:18

So that's where we're going to see a greater purity of water and less ethanol being present.

10:25

Now what we were talking about in this diagram on the right hand side, this shows we have an uncertain number of stages, right, connecting the middle.

10:35

And So what we're going to do is draw a box or a control surface around the top of the column that includes our condenser and the outlet of our overhead product, right.

10:45

So that's going to flow at a molar flow rate of  $D$  moles per hour and a composition or

a mass, a mole fraction of ethanol that is  $X_D$  Now leaving, right, we have liquid flowing out of that control surface.

11:03

So the molar flow rate of our liquid leaving is  $L_N$  and that's shown on the left here.

11:10

And the mole fraction of ethanol in that liquid is  $X_N$ , OK.

11:17

So our operating line for the rectifying section and this can, we can define this on any given stage, right, Which is why we have this arbitrary box, but it's  $Y$  equals.

11:28

And we look at this right hand side.

11:30

So I'm showing the left just because when we look at the origin of this, it's actually based in the same mass that we were using for gas absorption.

11:39

But because we don't necessarily know how many stages there are going to be when we're designing it up front, we can replace some of these terms by thinking about an overall material balance on the column itself.

11:51

So that's where we're going to use this blue colored definition for the operating line.

11:56

So I can relate the mole fraction of ethanol in my vapor  $Y$  to now the molar flow rate of liquid leaving  $L_N$ , the molar flow rate of distillate product  $D$  and the concentration or mole fraction of ethanol in my liquid flowing down  $X_N$  and the distillate product  $X_D$ , right?

12:18

So four terms are required from that material balance to do.

12:22

Now similarly, the operating line for the stripping section right has the same basis.

12:28

So the mole fraction of ethanol in the leaving that stripping column or the stripping portion of the column, that's  $Y$ , that's related to both the incoming liquid molar flow rate, right,  $L_M$ , the product molar flow rate.

12:45

So we call it the bottoms product molar flow rate,  $B$  again in moles per hour and the two relevant mole fractions of ethanol in the liquid phase.

12:55

So  $X$  of  $N$  is the mole fraction of ethanol entering the bottom of my column and  $X$  of  $B$  is the mole fraction of ethanol leaving in the bottoms product.

13:05

So one of the points that you've been given in the lab is the compositions or we'll figure out the compositions by knowing that the masses of the product streams if it's ethanol in water, All right.

13:15

So once we know the compositions, we can get AT what are the molar flow rates through the method that we're going to go through next, which is OK.

13:26

So the design basis right there are just as we said, that we don't have a single operating line.

13:34

What makes distillation columns tricky is that we have 3 operating lines.

13:39

So let's unpack.

13:41

This is a theoretical McCabe Teal diagram, but in fact, it's one that you can directly apply to your Laboratory 3.

13:50

And in fact, this will take you up through the full answer to sector to question 5C by the end of this lecture.

13:57

So everything that's outside of Hisis will be sorted through what we're going to do in this lecture.

14:03

And then on Friday, I'm going to take you through a fully worked example of McCabe, Thiel, so you can see it put into practice.

14:10

So really the only uncertainty in Lab 3 after this will be configuring that distillation column in crisis.

14:18

OK.

14:19

So firstly, when we look at this, we're now using that same YX diagram that we were employing for absorption and stripping columns.

14:28

But we can see now we have these three operating lines and we're going to go through each of these in a little bit of detail.

14:33

Now, first, when I construct my YX diagram, and what I would do is actually construct this in Excel, right?

14:42

So that you can physically then eventually print off a copy and by hand go through to map out each of these stages with a straight edge and a pencil or a pen, right?

14:56

Probably a pencil because we might need to erase them throughout a few times.

15:00

So when I make my YX diagram, I make it a square, all right?

15:06

And I need to make it a square because when I construct it, I'm also going to add in this  $45^\circ$  line.

15:13

All I'm doing is manually connecting the bottom left and the top right portions of my square.

15:21

And this is very helpful because the Y or the  $45^\circ$  line is where  $X = y$ .

15:27

So here I have  $X = 0$  and  $y = 0$ , and this on the right is  $X = 1$ .

15:39

That means I have 100% ethanol in the liquid and on the top is  $y = 1$ .

15:45

OK, so I use this  $45^\circ$  line and we're going to look at Y in a little bit.

15:51

So I draw my square, I draw my  $45^\circ$  line and then I, before I begin my design process, I need equilibrium data.

15:59

So I need to know what is the equilibrium relationship for what's in my column.

16:03

In your case, that's going to be ethanol and walk right now, I think David in the lab is walking through how we're, how we generated those equilibrium curves from multi flash.

16:18

And that's just as a fun example for you.

16:20

So you know, if you need to do this on the job somewhere, a resource that you can go to, as we've talked about previously, it's always a best practice if you can get this from peer reviewed literature, Perry's chemical engineering handbook, right?

16:34

Data that's actually been measured.

16:36

But for a simple system like ethanol and water, where we know that the species has studied this ad nauseam and we know that our phase and property simulators are going to capture this reasonably well, it's perfectly acceptable if you want to use an equation of state approach in multflash to grab this curve.

16:56

So the first thing I'll do, this is actually step zero, is I'm going to plot that equilibrium curve out on my FYX diagram.

17:06

So now what I've got is this 45° black line, my green line or curve, and now all I need to do is add my 3 operating lines.

17:17

Now we talked about having a stripping section and a rectifying section.

17:21

There's one more choice that the engineer gets to make, and that's where I put my feed and what kind of feed I have.

17:29

So the feed line is our third operating line.

17:33

Now, the McCabe Thiel method, right?

17:34

And it should, for what it's worth, the reason we're using McCabe's textbook here and not anything else, all right, is this was one of the folks responsible for the design of what is the most elaborate unit operation in all of chemical engineering.

17:49

All right?

17:49

So that's why we like this textbook and it's a good resource to have on hand.

17:54

So the first thing we're going to do, you can see we have our stripping line in red, rectifying line in blue, and our feed line in purple.

18:00

The first thing we do in the McCabe TL method is we locate that feed line and the feed line itself crosses over.

18:14

So the position of the feed line in the X spades that is equal to the composition of the feed of whatever lighter component we're using.

18:27

So in this case ethanol, the composition or the mole fraction of ethanol in the feed.

18:31

So your feed is at at 25 weight percent.

18:33

You need to convert that to mole percent and that will tell you where from left to right this feed line is going to start.

18:42

Now, once we know the X position of the feed line, so here what I've drawn is a feed of 50 mole percent, right, just for theoretical.

18:52

But once we know the mole fraction, and in fact I'll draw dashed lines, you can see the connection there.

18:58

Once we know the mole fraction in our feed, we can say the start of the feed line.

19:03

We then need to figure out what's the pitch of the feed line, right?

19:08

So where is it going?

19:09

Is it going to be pitched vertical?

19:11

Is it going to be horizontal or downward?



19:14

And the the pitch of that is dictated solely by the quality of the feed and we're going to go through that in a few slides.

19:21

Quality in this case meaning the thermodynamic quality.

19:25

So is it pure liquid, saturated liquid, saturated vapor, right sub cooled liquid or super heated vapor.

19:32

So the quality is a number typically between zero and one and we would have gone through that in 3006.

19:39

So the quality is the only variable we need to dictate the slope.

19:43

So now we know the starting point and the slope and from there we can move on.

19:49

So once we have that feed line, the next thing we calculate is the rectifying operating line.

19:54

That's the top part of our column where we're purifying the we're increasing the amount of ethanol in our vapor phase.

20:02

Now, if we know or if we have said so, if we measured, we're trying to model out a column like your lab or if we're designing from scratch.

20:12

We need to either measure or assume what we want is this, and I'm going to switch to blue here as this distillate purity, OK?

20:23

So in this case, maybe we wanted distillate purity of something like 80% ethanol in our vapor phase, right?

20:30

So the distillate that comes out once it's condensed goes from say 30 mole percent ethanol to 80 mole percent ethanol.

20:38

Now the way that we do this is by first the OR the rectifying line can be done visually.

20:46

It has a beginning point and that is the mole fraction of, in this case, ethanol that we've measured in that distillate.

20:57

And we've plot that on the YX line, the  $45^\circ$  line.

21:02

And because it's on the  $45^\circ$  line, right, the values are the same for both the Y and the X domain.

21:09

So once we know the origin point, we then have to figure out, well where is that going to intercept the Y axis, right?

21:16

Where does that blue line terminate to the left?

21:19

And the terminal point has a value of the distillate mole fraction  $X_D$  divided by our reflux ratio plus one.

21:32

So we know the distillate mole fraction anyway because we've just used it.

21:37

We need to know the reflux ratio.

21:40

All right?

21:40

So that's something that's a part of the lab that you would have gotten in the data already, which means we can plunk the intercept.

21:48

And what we would, what I would do and certainly what we did back in the day, is pull out a straight edge and draw that blue line between these two blue points.

22:00

So now we have our rectifying line drawn.

22:03

We know where our feed line starts.

22:05

And I'll show you in a few slides how we calculate the pitch.

22:07

But in this case, this would be a having a pitch that's perpendicular to the  $45^\circ$  tells me I have somewhere between zero and one of vapor quality, right?

22:17

So I have some saturated liquid, sorry, some liquid and some vapor in my incoming stream.

22:23

We're almost done.

22:24

The last thing we need to do is calculate our stripping line.

22:27

Now the stripping operating line begins, right?

22:31

If we know the concentration or the mole fraction of ethanol in our bottoms stream, right?

22:37

So that's the amount of ethanol in the water intensive side coming out of the reboiler.

22:42

We know that, we've measured that.

22:45

Then we plot that as the same intercept on the  $45^\circ$  line.

22:49

And that's the origin story of our stripping line And the trick of McCabe teal, and the reason we like doing it visually is that I don't need to worry about where the stripping line intercepts the X axis up here, right?

23:05

Don't worry about that.

23:07

Instead, I connected that all three of my operating lines must have the same intersect.

23:16

So that's points 1-2 and three of the McCabe Thiel method.

23:23

Now what I can do is just like in gas absorption, the slopes of both my rectifying and

my stripping lines are physically related to the ratios of my molar flow rates in the liquid vapor phase.

23:40

So if I have manually drawn these, then I can manually estimate what's the slope of that line.

23:47

And I can relate that value to a material balance.

23:53

So that's something I'm going to go through on the next few slides is what that slope means.

23:57

But when I say take the slope, I mean literally calculate physically on a using a straight edge.

24:04

You can do it through Excel if you want, but using a straight edge, calculate physically what is the slope of this line rise over run and that's going to be equal to now our material balance or ratios of the molar flow rate.

24:17

So the final bit of McCabe Teal is this diagram not only allows me to very easily and quickly connect all three operating lines, it also allows me to figure out how many ideal stages do I need.

24:34

So when we say ideal, that means we have a stage efficiency.

24:39

We don't have any inefficiency in the stage.

24:41

So the ideal stages will always be the minimal number of stages required.

24:46

And the only way I need to worry about this is I simply step from the top right to the bottom left of my graph, starting at the blue point, going to the red point.

24:54

And I do so bouncing between my operating lines and the equilibrium line.

24:59

So now assume that my drawing here is straight.

25:02

Y'all know I'm terrible at drawing, right?

25:06

There's our first stage, second stage, third stage, fourth stage, fifth stage, right?

25:15

I'm going to pause for a minute.

25:16

So here's 1234.

25:20

Once I hit this fifth stage, it's very important.

25:24

What I'm seeing here is I was using that that blue line, all right, So as my guardrail bouncing between the equilibrium curve and the rectifying operating line, once I cross over the feed line, now I'm no longer going to use the operating line as a guardrail, but I'm going to use the stripping line as a guardrail, OK, SO567.

25:54

So I need about  $7 \frac{1}{2}$ .

25:59

So this, this would be my eighth stage down here.

26:02

But I can see here I, I hit that, that XV concentration at about  $7 \frac{1}{2}$ .

26:07

Now, as we were talking about, you can't buy  $\frac{1}{3}$  of a stage, right?

26:11

These are physical stages, as you saw in the lab example.

26:16

So at the end of the day, this means you're going to need to buy 8 stages, right?

26:20

But that's OK because these are ideal stages.

26:22

It's highly likely we're going to add a stage to that and need a little bit more.

26:27

Now there's one final point, right, that I want to make here, and that's the stage that we always see the feed on, right?

26:35

This should make a triangle.

26:40

But that feed encapsulates.

26:43

We can see we need to be feeding on the fifth stage here, right?

26:47

So the fifth stage, from the way that I've drawn the K and TL, the fifth stage is our feed stage.

26:52

Now, if we put the feed up at stage 3, we're going to have a very inefficient column and we're going to be wasting money and energy.

27:00

If we put the feed down at stage six or seven, we're going to have an inefficient column, wasting money and wasting energy.

27:08

So McCabe, Thiel, the reason this is such a powerful method is that right here we have designed the column, figured out the number of stages and done all of this only knowing what are the boundaries of our target separation and what is our reflux ratio and the quality of what's coming into the process.

27:28

So the feed quality and the composition, all right?

27:34

So now when we think about the rectifying operating line, right, and this is where we start on that top right bit, the rectifying operating line itself and what we're showing here.

27:45

We can also relate this.

27:47

Instead of writing it, writing it directly as a function of the molar flow rates, we can also write it as a function of the reflux ratios.

27:58

Let's switch to blue here.

28:02

So our Y is related to X plus an intercept.

28:06

This is where the intercept definition comes from.

28:08

We can invoke that definition of the reflux ratio to simplify how we present the operating line.

28:15

But the take home message here is that the slope of the operating line.

28:19

So the intercept we've already shown on the McKay teal diagram is  $x_D$  on  $R_d$ .

28:25

plus one.

28:26

So the distillate mole fraction on the reflux ratio plus one.

28:30

But the slope of the reflux ratio or the operating line is  $L/V$  for the rectifying column or the rectifying side of the column.

28:40

So when I'm thinking about my material balance for the rectifying side of the column, one of the unknowns that I need to solve for is figuring out.

28:51

I can write.

28:51

I know my feed, I know my concentration or mole fraction coming in, I know the mole fraction leaving and I know the amount of material leaving, but I don't know how much material is flowing from the upper, the rectifying portion as liquid back down the column.

29:12

Well, this right here, looking at that slope, this is how I have one more data point to plug in for the unknown.

29:21

All right, So if I've mapped that McCabe teal, I take a manual measurement and I can do it analytically, but it's more fun with a straightedge.

29:29

What is the slope of that line?

29:32

And that equals  $L/V$ .

29:34

So the liquid molar flow rate in the operating section divided by the vapor molar flow rate.

29:42

That means I can write my material balance for that section in terms of only one variable and solve for it.

29:49

Now we are using a total condenser in this process, and the reason that's a simpler solution, as you can see here, what McCabe shows us, a total condenser transforms all of our vapor into only liquid, and our reflux ratio dictates how much of that's product and how much is recycled.

30:05

If we have a partial condenser, it means that we now have a little bit of liquid that's leaving and a little bit of vapor that's leaving.

30:13

So it's another equilibrium stage that's introduced, right?

30:17

That adds another layer of complexity.

30:20

We're not going to be using a partial condenser, so it's not something we need to worry about right now.

30:28

So the final point that we want to make on this slide is that if our liquid is cooled below the bubble point, all right, so we don't have a saturated liquid, but actually we have a subcooled liquid.

30:42

Now our reflux ratio needs to be modified a little bit.

30:47

And that's meaning we our condenser is going to be driving the liquid cooler than just taking out the latent heat of condensation.

30:56

So if it is colder than the saturated liquid should be at that mole fraction of ethanol in water, we simply use a modified reflux ratio right where we're adding in  $\Delta L$  and  $\Delta L$  is defined here on the left hand side.

31:14

It's the difference between so  $T_1$ , meaning this is the temperature at the top tray.

31:21

We can also take that to be the bubble point temperature if we want.



31:26

It's a common simplification, but it's the molar flow rate, heat capacity and  $\Delta T$  right between the condenser and what's coming into it divided by the latent heat, the  $\lambda$ , the latent heat of vaporization.

31:41

So we simply add in this  $\Delta L$  to get a modified reflux ratio if and only if our condenser is cooling beyond just creating pure liquid.

31:51

All right.

31:53

Now, in our case, the only thing you need to take away from this slide is that we are using a total condenser, right?

32:00

And so once we plot that out, we know how much is covered, the quality or the mole fraction of ethanol that's coming out of the system.

32:08

So we can get at  $L$  on  $V$  inside the column based on the slope of that operating line.

32:15

Similarly with stripping the stripping operating line, now we're in both cases we're assuming constant overflow.

32:26

And as we talked about on Monday, all that means, all right, is it allows us to straighten our operating lines, right without the assumption of constant mole overflow, then our operating lines are going to have to have a beta curvature and the calculations well and subtractable get a little bit more complex, OK.

32:51

But we're assuming constant mole overflow, which gives us these straight operating lines, OK.

32:57

Now again, the stripping operating line terminates as we saw whatever the the bottoms outlet mole fraction of ethanol is going to be as we've measured and the slope of that line is again  $L$  on  $V$ .

33:11

Now we use  $\bar{L}$  and  $\bar{V}$  in this context simply because we're relating that to the bottom, to the stripping side of the columns.

33:20

I'm using a bar here, not to indicate a mean, but just to indicate these are the liquid

and vapor mole flow rates on average that relate to the stripping side or the bottom side of the column, right?

33:31

And then if we need to close the balances a bit further, these are the two take home critical bits from Monday is material balance, where we can relate the bottoms molar flow rate, which is  $B$  to the feed.

33:44

Oh, and I've copied and pasted the same equation twice.

33:47

OK, I will update that when I upload these marked up lectures for us.

33:51

But I can relate  $D$  on  $F$ , so the distillate molar flow rate to the feed or  $B$  on  $F$ , the bottoms molar flow rate to the feed simply based on the three mole fractions of ethanol incoming being  $X$  of  $F$ , outgoing on the top  $X$  of  $D$  or the outgoing on the bottom  $X$  of  $B$ .

34:10

Now when we're looking at the bottom of this column and we have a reboiler, right, one of the assumptions that we do invoke is that we have a singular bottoms product, right?

34:19

So we're not pulling out multiple liquid strings from the bottom.

34:23

We only have one.

34:26

But as long as that, that assumption is valid, we can use any type of reboiler.

34:30

Now the, the diagram that the KB uses here is a steam, right?

34:34

So it's a kettle type steam reboiler.

34:39

And we, this tells us then that the reboiler is returning vapor.

34:44

So the return line is going to be vapor that's in equilibrium with the liquid coming in, right?

34:51

So all we're doing is vaporizing it and we pull a little bit of the liquid out as a product.

34:56

So it's important that this we understand this reboiler functions as a single ideal plate or a stage, right?

35:06

So if we have 8 stages in the column, well, the fact we have a reboiler down bottom means this is a ninth stage.

35:14

So if we go back to that McCabe Teal diagram, it's important to remember when we do have a reboiler, and we call this a partial reboiler, right?

35:22

Because it's only partially reboiling and the rest we're taking is liquid.

35:27

But the use of a partial reboiler, it inherently means this is an equilibrium stage that we need to worry about or that we need to include in our calculations.

35:36

If we are using a total condenser as in the lab, that's not an equilibrium stage.

35:42

We're just transforming a full phase transformation of our vapor product to liquid.

35:47

But if we start to invoke that partial condenser, that acts as an equilibrium stage, not something we need to worry about in the lab.

35:55

So finally, the feed line.

35:57

Now, as we talked about, the feed line very simply crosses right at the feed mole fraction of ethanol  $X_{\text{sub F}}$ .

36:07

And fundamentally, if we go back to thermodynamics last semester, that feed can be in one of five states, a cold feed meaning pure liquid, what Smith, Dennis and Abbott would call it, subcooled feed in that condition, right?

36:22

If our feed is coming in here, the left, all of our liquid is going to flow directly down immediately, right?

36:28

And it's going to cool the remaining liquid as it contacts in that stage.

36:35

And in that consequence, the quality which we're going to give  $Q$ , that's quality, the quality is going to be greater than one.

36:44

So I can calculate the quality directly by saying  $Q = 1$  plus heat capacity.

36:50

The bubble point temperature of this mixture at a known pressure this sorry, this is  $T$  of the feed.

37:06

So the temperature of the feed coming in.

37:07

So what is the difference between the bubble point temperature and my feed temperature?

37:12

And I divide  $CP \Delta T$  by  $\Lambda$  which is the latent heat of vaporization.

37:18

So if I think of it this way, if my feed is at the bubble point temperature, I have a saturated liquid and by this definition  $\Delta T$  is 0 which means  $Q = 1$ .

37:30

Now, the most common condition is that I use a saturated liquid, so I have no vapor.

37:37

And in that my quality is one.

37:40

If I Preheat my feed by quite some margin, now I have a quality between zero and one.

37:48

And we'll remember this from thermodynamics, right?

37:50

So how do I calculate exactly what that quality is going to be?

37:54

Well, now, right, I can't use a  $CP \Delta T$  relationship because that's either subcooling or super heating.

38:02

Instead, I need to think about the enthalpy of my feed at known pressure, known

temperature, the enthalpy of the saturated liquid and the enthalpy of the saturated vapor.

38:15

And I use that inverse lever rule from undergraduate thermodynamics to calculate the quality.

38:22

So as an example, if a very simple example grant you, if my enthalpy, let's see, my enthalpy of the saturated liquid would be 2000 joules per mol and the enthalpy of my saturated vapor is 4000 joules per mole.

38:40

And I calculate well my incoming enthalpy because I have a say 30 mole percent ethanol at say 50°C and one bar.

38:53

Well, if I know the pressure, temperature and composition, I can calculate the enthalpy of that string.

38:57

And in fact I can use something like multi flash to calculate it.

39:01

It's not a plug in composition, pressure, temperature.

39:04

Select your model, do a PT flash and it will tell you the enthalpy.

39:08

So if I know the enthalpy of my stream, let's assume it's 3000 joules per mole, then I'm halfway between find if I I'm just going to draw a very, very small drawing here.

39:21

If I Draw Something like a an enthalpy, right?

39:26

I look at that pressure, pressure enthalpy diagram right now, remember we had that kind of characteristic hump.

39:37

Well, if I'm at constant pressure, right on the left is my saturated liquid, my right is the saturated vapor.

39:43

So if I'm at a liquid saturated liquid enthalpy of 2000, saturated vapor on the right of 4000, and my feed is sitting at 3000 directly between them, then I have a 5050 mixture of saturated vapor and saturated liquid or a quality of .5.

40:03

All right, if I'm instead, if my, if I'm at 3500, I would have a quality of 25.

40:13

So I'm almost I'm 75% of the way I've vaporized.

40:17

So my quality is 0.25.

40:20

OK.

40:21

Now if I'm at a saturated vapor, it means I have no liquid left, but I'm not super heated yet.

40:25

So I'm sitting at the saturation temperature.

40:27

And in that case quality equals 0.

40:33

Now finally, if I have a super heated vapor, then the only question is how far above the dew point am I?

40:39

And in this case the quality is going to go below 0.

40:43

So that means I have a negative constant volume heat capacity  $C_V$  the actual temperature of the feed.

40:51

What's the dew point temperature?

40:53

So  $C_V \Delta T$  divided by  $\Lambda$ , which is the latent heat of vaporization.

40:58

Now you shouldn't have a super heated vapor and you in fact, you won't have a vapor feed.

41:02

You can see from the the lab that we're dealing with the liquid.

41:05

So the only question is to think about how do we calculate that quality?

41:10

And if you know the composition of what's coming in, which is listed in the lab, right?

41:13

We have 25 weight percent.

41:15

So we convert that to mole percent.

41:17

If we know the temperature and pressure coming in as a feed, then we simply go to a something like multflash and say, OK, how much is that enthalpy greater than or equal to the saturated liquid enthalpy at that condition.

41:37

Now once I know the quality, so let's assume we're using a quality of 1.

41:42

So we have a saturated liquid coming in and this is the most common application for distillation because it means we've preheated.

41:48

We're probably going to use the waste heat from the reboiler to Preheat our feed stage.

41:54

So if I have a quality of one, then what it means is my bottom's vapor flow rate  $V_{\text{bar}}$  is equal to  $V$  and my bottom's liquid molar flow rate  $L_{\text{bar}}$  is equal to my liquid molar flow rate on the top of the column  $L$  plus the molar flow rate of the feed  $F$ . If I have a quality greater than one, maybe we haven't put enough energy in it.

42:19

We have a cool, a cold liquid feed.

42:23

Then I can calculate and relate.

42:26

Rather I can relate the liquid molecular rates in the stripping section, in the rectifying section.

42:33

Remember stripping section uses the  $L_{\text{bar}}$ ,  $V_{\text{bar}}$  and the rectifying section uses  $L$  and  $V$  I can relate that directly to the  $Q$ , the fraction of the feed that's going to be sorry, that's going to be vaporized or were liquefied.

42:51

Now what we're drawing here in this diagram, this shows us for each of these cases, ABCD and E, what are the different pitches of that operating line.

43:04

So remember, we could figure out where do we begin that operating line as a point, but we don't know the pitch yet.

43:10

Well, the pitch or the slope as we can see here, right is related error is negative  $Q$  on  $1 - Q$ .

43:24

So if I have a quality of 1, so I have a saturated liquid coming in again, the most common case I see I'm dividing one dot divided by zero.

43:34

I have a slope of Infinity.

43:37

That gives me a perfectly vertical operating line.

43:40

I'm sorry, a perfectly vertical feed line.

43:42

So if I know where the point begins, I draw a vertical line straight up.

43:47

If I have a quality somewhere, or rather a cold feed, so it's not at that saturated temperature yet, then it pitches to the right ever so slightly, right?

44:01

And all I need to know is the latent heat of vaporization, the bubble point temperature, and the feed temperature and the heat capacity.

44:07

So just thermodynamic properties in my feed.

44:11

And I can very quickly calculate that  $Q$  and that  $Q$  negative  $Q / 1 - Q$  is the slope of that line.

44:21

OK.

44:21

Now if I have a partially vaporized feed, this is what's going to now give me a slope to the left.



44:30

So I'm no longer vertical, but I'm tilting it downward or toward the left side rather.

44:36

And by the time I get to a super heated vapor, you can see now we've shifted even farther downward.

44:44

So determining our feed conditions, which is just the composition and then by the temperature and pressure, we can figure out the quality.

44:53

This tells me where my operating line slope is going to be.

44:56

All right.

44:57

And we should be somewhere either between a cold feed or a saturated liquid.

45:03

OK, and that's it.

45:05

So the take home message is here.

45:07

This is again that theoretical kind of mock up for the McCabe Thiele method.

45:13

The beauty of this is that it's visual in nature, right?

45:17

So questions a six through nine of the lab.

45:22

These are using Hisis, right?

45:25

But you imagine the amount of money and time and effort that goes into using Hisis to design A distillation column that's going to turn out an answer pretty darn near what McCabe Thiele will.

45:36

There is an elegance in this method that it can be done with a straight edge and a little bit of equilibrium data.

Lecture 21 - Distillation Worked Example

0:00

So in our last lecture on Wednesday, we introduced the McCabe Thiele concept for how we size and characterize distillation columns.

0:09

And this is competitive to what folks would use either through calculating directly the number of transfer units is another method.

0:17

Also using something like Hisis directly is another method.

0:20

I like this, the McCabe Thiele method because it's visual, right?

0:24

And so most importantly, it requires the least amount of information to get started.

0:29

So what we looked at as opposed to our either absorption column or stripping column here when we use a distillation column, because we have our absorption or rectifying section as well as a stripping section, we have two operating lines.

0:44

And the feed line is where our our product or our our feed is going to come into the column.

0:50

And that bisects both the operating and the stripping lines.

0:54

So the power of this method is that we don't need to know the internal flow rates, either the liquid or vapour molar flow rates in either the rectifying or stripping sections of the column.

1:04

But rather if we know our product compositions that we're desiring and what our feedstock looks like, that's really all we need to get started.

1:15

So with that, we'll get kicked into the example for today.

1:20

So this will be a long form worked example for us.

1:24

So the example that we're looking at, we want to use a distillation column where we have a combination of feed is benzene and toluene.

1:38

So our benzene in the feed will be 40 weight percent and toluene is going to be 60 weight percent.

1:51

Right now, it's not uncommon that we would use something like a distillation column to separate benzene and toluene.

1:58

If we remember back, something like a flash vessel would be really important if we were trying to separate products that have a very different boiling point.

2:07

But because benzene and toluene are so similar, right, they're not going to be easily separated in a flash vessel.

2:14

So that means we need to use something more elaborate like a distillation column to pull the two apart.

2:18

And it would, it would be common that we find them together because they're typical by products if we look at a petrochemical feedstock.

2:26

So that might be something like a, a crude oil that's been distilled down and now we end up with our aromatic cut.

2:32

We're trying to separate out benzene and toluene.

2:35

One of the reasons we would want to in this case is these are both very good solvents.

2:39

So if we're trying to clean materials or clean up the laboratory, benzene and toluene are both things that can dissolve soils.

2:48

They can dissolve dirt very effectively, but notably they have very different ecotoxic properties.

2:56

So in this example we might want to say we're going to purify benzene away from toluene.

3:00

In that way.

3:01

We have two individual chemistries that we can use to clean up different types of contaminants.

3:10

So we've defined our feed as containing benzene and toluene and we're going to say that that feed the rate that we need to clean it up in a continuous distillation column.

3:20

So the feed flow rate is at 30,000 kilograms per hour, right?

3:36

And this is also this flow rate underscores why we would use a distillation column as opposed to a simple batch process or something like a flash vessel, right?

3:46

When we start talking about 10s of thousands of kilograms of something coming through every hour, that's the level that we're operating at when we start to invoke a proper distillation column.

3:57

Now, because we would be separating these and selling them to market cleaning agents or something like this, we need a very pure distillate stream in a very pure product stream.

4:10

So let me just move a few things around here.

4:13

So to do that, we're going to set that we want.

4:16

So benzene in this example, right, That's our lighter component of the two.

4:20

So we call this component A as is convention and our Toluene which will have slightly heavier will be component B.

4:31

Now our distillate target.

4:33

So the engineer is then going to decide, well, I need a purity of my distillate stream to be X percentage, right.

4:40

So we're going to decide that the mass fraction of distillate will be 97% benzene and that's by mass or by weight.

4:59

And we're also going to then set that we want a bottoms product of 98% toluene.

5:10

And that's also by mass, right?

5:14

So this is if you go to a typical chemical vendor like Sigma Aldrich is the largest worldwide, but there are a handful.

5:20

When you go to buy a, a product like I'm going to buy a litre of benzene or a litre of tell you need to clean up the laboratory, you will typically specify, do I want 95% pure, 97% pure?

5:33

And that pure you get, the more money you pay, right?

5:36

So a typical example, buying a litre of benzene for cleaning the lab might cost 80 Australian dollars.

5:42

If I buy the 95% purity, if I go to 98%, it's going to be probably 200.

5:49

And if I ask for the 99.9%, I'm going to pay about \$500.00 for that same leader in today's currency.

5:56

So that kind of gives you an idea of when we say we're going to take in a feedstock that's roughly the same and split it into this degree of purity, that's where that degree of cost comes from.

6:05

So this would be a relatively pure product stream.

6:08

And it would also tell you when you go to buy a chemical for this kind of purpose, the typical contaminants were what makes up that other percentage, 5%, two percent, it's going to be whatever wasn't able to be distilled around the same boiling point, right?

6:26

So in this case, if I bought a 97% pure benzene, I'm going to expect that I'm probably going to have one or 2% toluene in that mix because that's likely the chemistry that's going to be of a similar molecular weight boiling point.

6:41

OK, so now I've set my targets and this is by mass percent, so we're going to have to convert it over to a mole percent, right?

6:47

I like this example because it maps really well into your laboratory.

6:51

3 Now the final thing the engineer is going to set is the reflux ratio.

6:58

So the reflux ratio we're going to set is 3.5 and that's going to be moles of reflux per mole of product.

7:11

All right, now I want to look in this at three feed options.

7:19

It's rather 33 feed conditions, right?

7:24

So feed condition one will be liquid at the boiling point, feed condition 2 is going to be liquid at 20° C, and feed condition three will be 2/3 vapor and one third liquid, right?

8:00

So what we want to do is size up this distillation column and figure out how it's going to operate with three different potential feed conditions, right?

8:09

And you can imagine the simplest of these is going to be liquid at 20° C, meaning I'm not going to have any preheating to it.

8:16

I'm just going to pull it in at room temperature, inject it into the distillation column and see what happens.

8:22

If I use a preheater, and that might be shown on the bottom here, I can take my feed, put it through right through this, this bottoms counter heat exchanger, right?

8:33

So here, if here's my reboiler, then my product stream coming out can run through a secondary heat exchanger to cool that product stream down and rather pass that heat into a preheater for my feed.

8:47

Now if I do that somewhat effectively, I'm going to bring the liquid to its boiling point.

8:52

If I do that really effectively using big heat exchanger that's really efficient, maybe I've put extended surfaces or fins on that bottom heat exchanger, then I'm going to start to pre vaporize some of that feed.

9:04

And so that's where I end up with this condition three of having 1/3 liquid and now 2/3 in my vapour phase.

9:10

OK, so these are our initial conditions.

9:13

So now the second thing we're going to have to do is identify what's the, what equilibrium data are going to correspond to this.

9:22

So step #2 is identify.

9:30

And I'm not going to call it equilibrium data, but rather my thermodynamic data, right?

9:41

So in the first instance, I'm going to say, OK, for the enthalpy of of vaporization for benzene and I'll call that component A, that's going to be 7360 calories per mole, right?

10:02

And remember that a calorie is just a, it's related to a Joule, I think what is it 4.186 joules per calorie for memory.

10:12

The heat of vaporization for toluene in component B is 7960 calories per mole.

10:21

Now the boiling point of my feed at one atmosphere, because I need to know the boiling point to figure out well if I have 20° versus at the boiling point versus in the two phase region, you know, I need the boiling point to figure out where that 20° is going to sit.

10:40

So the boiling point for this feed will be 95° C.

10:46

Also need to know for the liquid if it's at 20° C what's my specific heat?

10:52

So the specific heat capacity at 20° C for the feed will be 0.44 calories per gram degrees centigrade.

11:07

And the final two points, the molecular weight of component A, my benzene will be 78g per mole and the molecular weight of my timing component B will be 92g per mole.

11:28

Now the final thing is I've pulled some equilibrium data, right?

11:32

Because I needed to know what is the YX relationship at this working pressure of one ATM.

11:38

So I have X of A and Y of A.

11:41

So they're going to go from 1:00 to 1:00, 0.780.90.5810.7770.411.632.258.456, the last point, 0.13 and 0.261.

12:16

And then of course, I have my 00, right.

12:19

So this equilibrium data, this comes from a different textbook, but I could just as easily get this from Perry's Handbook of Chemical Engineering or another option.

12:29

So now what?

12:31

I have change back.

12:35

OK.

12:36

So on the first step we've identified step one, we're going to set up the problem, right, Step 2, because I've now mapped this to my or identified my relevant thermodynamic data for this system.



12:58

And the things on this diagram on the right hand side that we don't know.

13:02

We don't know what the overhead product flow rate is.

13:06

We don't know what the bottom product flow rate is.

13:09

We also don't know what the number of stages is going to be or where the feed should be located.

13:23

All right.

13:25

So those are our degrees of uncertainty that we're trying to solve for now.

13:28

And what we're going to do is use the McCabe Thiele method to look at these three feed conditions.

13:36

So the first will be liquid at the boiling point.

13:41

Now obviously we'll come back to the graph on the right hand side where we set the problem up here.

13:47

So firstly, on the fractions that I've been given are mass fractions, right, or weight fractions.

13:54

And that makes sense because that's probably what we're going to measure in the laboratory or, or the engineering unit.

14:01

We're not going to go out and specifically measure in A, in a mole fraction.

14:06

So the first thing I need to do is figure out in my feedstock what is the mole fraction of component A in my feedstock.

14:13

So for component A, and I'm just going to call this  $X_{\text{sub feed}}$  that will be related to now 40 on 78 / 40 on 78.

14:29

Actually I'll write this linearly, SO40 on 78 / 40 on 78 + 60 on 92, right?

14:45

So those are the 78 and 92, these are the molecular weights.

14:49

40 and 60 are the mass percentages coming in.

14:52

So I'm just taking 100 gram basis and I'm using a gram mole.

14:57

So that will convert me into a mole fraction.

14:59

So that means my feed mole fraction of component A.

15:02

And remember, once I'm I'm talking about designing the column, everything's in the lighter components of component A.

15:09

That gives me a feed mole fraction of 0.4402.

15:14

And of course you'll see here then that if I look at the .44, I can see everything line up vertically with that.

15:25

Next, what is the mole fraction of component A in the distillate string?

15:32

Well that is going to be 97 on 78 divided by 97 on 78 + 3 on 92, which gives me a distillate mole fraction of .9744.

15:56

In here we can see on the right hand side I can mark  $X_{\text{sub DX sub feed}}$  and I can see that top blue point line up on my  $45^\circ$  line with the  $X_{\text{sub D}}$ .

16:11

That's the distillate string and then finally X of the bottom string.

16:16

Now I said it was 98 weight percent of the toluene, which means I have two weight percent of my benzene, my lighter component.

16:24

So now it's two on 78 divided by two on 78 + 98 on 92, which is going to give me point O235 mole fraction of benzene, my lighter component in my my bodice product, right.

16:54

And here is X of B.

16:56

And so by the nature of The Cave theory, before we go any further, what we can also say is that the left hand point on my  $45^\circ$  line, so my bottoms point marked in red the the abscissa or X axis value of .0235 is equivalent to the Y value .0235.

17:19

Similarly for my distillate product .97 and the X will also be the equivalent for the Y.

17:28

So I can mark both the feed point in the purple so that the bottom purple point as well as both of the blue points right away.

17:38

OK, so the next question is OK my feed I have is given to me in a mass so I need to convert that into a molar flow rate so I can do a material balance in moles across the column.

17:50

So the molecular weight of my feed averaged right is going to be 100 if I take 100 gram bases divided by 40 on 78 + 60 on 92.

18:08

And that's going to give me the molecular weight now of 85.84g per mole.

18:17

OK, so all I've done is convert everything over into a molar basis so far.

18:21

Now because I realize I have a blended feedstock, I also need to calculate what is the average latent heated vaporization for my feed given that feedstock.

18:32

So that'll be .44 O2 because remember when we looked up those latent heats of vaporization, they were per mole, right?

18:42

So I'm now using molar basis to average those times 7360 plus 0.56 times 7960 and that's going to give me 7695.9 calories per mole.

19:04

Then finally I need to convert my feed flow rate into a molar flow rate.

19:11

So then we're just going to call that capital F and that will be 30,000 kilograms per hour divided by the molecular weight 85.8, which gives me 349.5 kilogram moles per hour.

19:31

So when we think about closing the mass balance now there are two critical equations and I've highlighted them on Monday's lecture and said we were going to use these multiple back multiple times.

19:45

So to get the distillate molar flow rate, what I can do is say, remember the distillate divided by the feed in in molar flow rates were related to the differences.

19:58

So if I write up here just a few equations, distillate on the feed is going to be equal to  $X_D$  minus  $X_B$  on  $X$ .

20:13

I've quoted that incorrectly.

20:17

Whoops.

20:17

OK XF.

20:23

Sorry, XF.

20:27

Minus  $X_B$  on  $X_D$  minus  $X_B$ , Yes, that's correct.

20:35

And the other material balance I can invoke is the bottoms on the feed flow rate is going to be equal to now  $X_D$  minus  $X_F$  divided by  $X_D$  minus  $X_B$ .

20:52

So if I know the feed flow rate and the molar compositions of both the feed bottoms and in distillate conditions, I can directly calculate from these equations what are the distillate and bottoms molar molar flow rate.

21:06

So first I'll calculate the distillate molar flow rate.

21:09

So that is going to be the feed molar flow rate.

21:12

So I have 349.5 Times Now by XF, so .44 O 2 minus the bottoms point O235 divided by the distillate which is .9744 minus the bottoms point O235.

21:44

And that gives me a distillate molar flow rate of 153.14 kilogram moles per hour.

21:53

Now I can calculate the bottoms flow rate through the same principle right or alternative and it works out the same.

21:59

Alternatively, I can say the bottoms is going to be equal to the molar flow rate of the feed minus the distillate right?

22:05

Because if I draw a control surface around the entire distillation column, then the material balance must hold the steady state.

22:14

So if I can do either operation, I get a bottoms flow rate of 196.35 kilogram moles per hour.

22:24

So this is really quite good, right?

22:26

Because I have a feed molar flow rate and it's being pretty much split in half at a reasonably pure condition for both the tops and the bottoms where I can sell both of these at a reasonably good price.

22:38

Does anyone else's stream frozen?

22:40

I just saw Gus's message here.

22:43

So I have it on a separate computer just to check it and it it still shows up live to me.

22:54

OK, So what I've done then is to then say now if I and I'll switch to a different colour now because I know in my first condition the feed is at the boiling point, right?

23:12

Well, we say what is the quality of the boiling point, right.

23:15

So for feed condition one, the quality is saying what does it equal at the boiling point?

23:26

Well, the quality of the boiling point equals one, right?

23:31

Now if I go to my definition, what is the slope of the feed line, right?

23:39

That's going to be defined as negative  $Q$  on  $Q - 1$ .

23:44

Well, if the quality is one, then I have  $1 / 0$ .

23:48

That means the slope of my feed line is Infinity.

23:52

And a slope of Infinity means we have we're perfectly vertical.

23:57

So what we've done is to say, OK, we know our feed is going to intersect.

24:04

We know the feed molar condition and that will be placed as a dot right on the  $45^\circ$  line.

24:11

We've defined our left hand point, the bottoms point, and the distillate point.

24:16

We can draw a vertical line.

24:18

So what I just said is I want the same feed condition and I'm going to set that now all the way up to 1.

24:23

So I draw a vertical line up here to get the switch.

24:30

Here we go.

24:32

I then need to think about where does my operating line crossover?

24:36

Well, to get the operating line, I'm going to invoke or rather the rectifying operating line.

24:44

There I'm going to invoke the definition of the intercept.

24:47

So for that intercept now then the definition is  $X_D$  divided by  $R_d$ .

24:55

plus one.

24:57

Well,  $X_D$  .9744  $R_d$ .

25:03

is 3.5.

25:04

That's our reflux ratio.

25:06

So if I calculate  $.9744 / R_d + 1$  or 4.5 in total, I get a value here of .21654.

25:25

OK, so the first step I did was to draw my my operating line or my feed line because in this case it's quite simple.

25:31

My second step is I draw my rectifying operating line right.

25:35

I know the right hand point, I know the left hand intercepts.

25:38

So I connect the two and my final stage, I know the bottoms point and what I don't know, which I'm not going to calculate directly because I can do it visually with McKay Tiely.

25:50

It must be the ternary intersection between all three of these operating lines, right?

25:57

And this green curve on top, this comes from the equilibrium data.

26:00

All I've done is to plot it in the same YX space.

26:05

So this is everything I need to set up.

26:06

Example one.

26:10

Now when I do that here is the same thing, but I've cleaned all my comments off or all the annotations off it.

26:18

And now what we're looking at is the McCabe Thiele step method coming through this right.

26:24

So if I do all of this stair stepping, what I find is for condition one, I have 11 stages and that includes the reboiler, right?

26:48

So remember we're using because we're using a total condenser right here.

26:54

We have all of our product is going to be condensed that does not act as a partial equilibrium stage, but the reboiler will always act as an equilibrium stage.

27:13

So what this means is I have 11 stages including the reboiler, which means I have 10 stages in the column.

27:23

And I can identify that this this feed line comes in on the sixth stage.

27:38

OK, So now we might say, OK, what happens if we want to look at condition two, right?

27:45

That's my feed, which is now going to be a liquid at 20° C Well, if we're at 20° C, we know we're below the boiling point of 95° C So one of the first things I need to do is the convert my.

28:11

Sorry, 2 seconds.

28:12

I need to convert my heat of vaporization of the feed from a molar basis.

28:19



We go back to our thermodynamic portion, convert it from a molar basis to a mass basis.

28:30

Here we go.

28:31

So this quantity I've calculated to set up the problem.

28:34

This is calories per mole, but I need to convert that over to a mass basis now.

28:43

OK, So what I'm going to say is the Lambda of the feed is going to be 7695.9 calories per mole divided by the molecular weight, the average molecular weight which was 85.84g per mole, right?

29:10

That equals 89.7 calories per gram.

29:16

So now if I know that the Lambda of that feed I can invoke the definition of Q is going to be equal to 1 plus now X to the feed.

29:38

So the mole fraction of component A in the feed times the temperature difference between the boiling point and my current temperature.

29:45

So  $95^{\circ}\text{C}$  minus my operating or inlet temperature  $20^{\circ}\text{C}$  divided by the Lambda of that feed and this gives me AQ of 1.368.

30:07

Now because I have a feed quality below the boiling point, I would expect AQ above 1, which means my slope of my operating or my feed line again negative Q on Q minus or plus one, I'm sorry,  $1 - Q$ , the slope is going to be equal to 3.72.

30:39

So if I look at my McCabe Thiele diagram, then, well, the rectifying line doesn't change.

30:48

My initial point doesn't change.

30:54

However, the slope of this feed line is going to tilt to the right now because it's not going to be a vertical line, it's going to have a positive slope.

31:04

So what I've done is to just estimate well, where does that that equal 1 on my graph such that the slope of this is going to be equal to 3.72.

31:17

And then finally this point on the bottom doesn't change, but I do have to manually find where does it intersect this ternary point of my 3 operating lines right now personally, I did this visually right?

31:41

You could, you could certainly calculate it directly or you could just play around with it in Excel until you find an agreement between the two.

31:50

So I found this intersection visually and when I do a stair step method on this from McCabe Thiele, what do I get?

31:58

Well, now McCabe Thiele says I have 12 stages, again including the reboiler.

32:11

So that means I have 11 in the column with the feed now still on the 6th.

32:24

Now finally, we'll look at an example.

32:27

What happens if my my preheating feed stage is really, really efficient.

32:32

All right, I do a good job preheating that feed.

32:36

Well, now I have feed condition three is one in which I have  $2/3$  vapor and one third liquid.

32:47

Now by definition, right?

32:50

Because I'm in the two phase region for my feed, by definition the quality of that feed equals  $1/3$  what fraction is liquid.

32:59

So the feed slope again as negative  $Q$  on one minus  $Q$  - ,  $Q$ , the slope of that line will equal negative .5 South.

33:19

Again this blue line doesn't move.

33:26

My initial point is the same, but what I've done is to simply rise over run, do a calculation, rise over run where I know the slope is equal to negative to negative .5 until I can find the intercept on the X axis to plot the other point, right?

33:44

So this slope is equal to -.5.

33:50

This point stays the same and now all I do is visually identify what are my new conditions for this intercept, right?

33:59

And when I visually identify it, I see it at .35 and .49.

34:07

So I'll go back real quick and right on the other slide, where did I visually identify that that for me was .48 and .59, right?

34:21

And ideally, I did this within Excel where the lines are a bit thick and it's not all perfect.

34:25

So ideally I would print this off on like a an A4 paper and high clarity and actually use a straight edge to connect very thin one point lines so I have a more accurate estimate.

34:38

Actually doing this with thick lines on the computer is really not best practice.

34:41

The method's usually done by hand for this reason.

34:45

But when I do all of this now, right, I've identified this, this new intersect point between the three.

34:50

I do that same stair stepping method between them and my McCabe thieling.

34:54

Turns out now 12 stages with the re boiler, which means I have 11 in the column and now the feed is on the 7th stage right.

35:14

So I can see here this is the end of the sixth stage right, Which means my feet is just now barely touched into the seventh stage.

35:23

So because my I've done more effective work in preheating this.

35:29

Oh sorry, been missing the the chat.

35:31

Yeah, Q represents the quality.

35:33

So Q is the quality of my feet.

35:36

So it's liquid quality.

35:37

Thank you touch out for for correct that.

35:40

So because I've done a really effective job in preheating my feed.

35:43

Now I am going to inject my feed on a slightly lower stage.

35:48

So that should tell you that the more work you do on the feed, the more energetic the content of the feed is coming into the system, the lower in the system you're going to inject it right.

35:58

And if you were injecting that in a pure vapour condition, you would typically pull that to the very bottom of the column, right?

36:05

Whereas if you're in a very cold condition, if you put all right.

36:12

So with that, that's the end of our example for today.

36:15

So the take home message is, I hope it's been useful to take one example and look at a variety of inlet conditions following the same basis as you were given.

36:23

So you have mass fractions coming into your lab 3 convert them over to mole fractions.

36:28

We need the heats of vaporization which we can get from multi flash or or any of our look up tables.

36:35

The initial conditions that the engineer needs to specify for the feedstock, how much, what composition and what condition, and that dictates the quality and what is the objective, what do we want to achieve?

36:48

We set the distillate and the bottoms purity, right?

36:51

Because these are the things that we're going to take to market and sell or we're going to put them into another process.

36:56

So that's what we need to determine as the engineer.

36:59

And then we get to decide what the reflux ratio that we want to use.

37:03

And that's effectively what's going to balance the distillate flow rate over the ultimate bottoms flow rate.

37:10

So that, that reflux ratio of 3.5 was a really good choice because that let us get a relatively equal kilogram moles per hour in both product strings, right.

37:21

And the method that we used, well, the initial condition one, once we determine what feedstock we have, that's all we need that quality for the feed line.

37:31

The operating line itself is set by conditions two and three.

37:34

So the objective we're trying to achieve and the reflux ratio we choose and the stripping line is determined by the bottom security that we choose and the intersection of three operating lines, OK?

37:46

So I hope that that makes everyone very comfortable with the Katy.

37:50

It's a fantastic method.

37:52

When you run this up in ISIS either this week or next, you know, ISIS is certainly shouldn't be that hard to set up for a distillation column.

38:01

But what I hope you walk away with is that this very simple method back from the 1960s, there's a reason that this is still the standard that's used for sizing distillation columns.

38:12

OK.

38:13

And, and if you get a chance, feel free to like, let me know if you take a shot at this at home.

38:19

Certainly we can get a lot more clear than the the stair stepping you can see I've done here, which is a pretty rough estimate using the computer itself versus doing it by hand.

38:30

OK.

38:30

For the few folks that have hung in there with us on a Friday afternoon, do we have any?

38:35

OK.

38:35

Did you say the higher if you Preheat the feed, the higher you put the feed?

38:39

I know.

38:40

So shadow the, the higher we Preheat the feed, the lower we would put the feed because we're going to have more vapour, right.

38:46

So the more vapour we have, the more time we want that vapour to be in contact with liquid flowing upward.

38:51

So we're going to put a hot, more vaporized feed lower in the column.

38:55

And that's where you can see going from our middle example to the end.

39:01

We go from a feed on the 6th stage when we have a a cold feed or a feed at the boiling point.

39:08

Once we create vapor, it drops down to the seventh stage.

## Lecture 22 - Practical Distillation

0:00

In the whole take home message, of course this is our McKay Deely diagram.

0:08

So on Wednesday we talked a lot about the theory of how we make this, and on Friday it was all about an example of how we can actually use it.

0:16

And what we saw were the process to create this firstly requires us to know something about our bottoms composition, right?

0:25

What is our feed composition and our distillate composition?

0:30

So these are three absolutely critical points that we want to set as the engineer and we get to decide effectively what those are.

0:39

And from there, what McCabe Teeley allows us to do is identify what's our reflux ratio and what's the quality of our incoming feed queue.

0:48

And if we determine those two points and then we as engineers can control what those are, right?

0:53

Because this, this feed quality, as we would have seen from Friday's lecture, will largely be determined by the effectiveness of our heat transfer operation upstream.

1:02

So if we decide to not Preheat our feed, we're going to be coming in cold and we'll see that feed line, the pitch of it shift to the right, right.

1:11

So we have a less efficient system.

1:13

If we do a lot of preheating, we'll see that shift to the left and typically see a more effective system.

1:19

But this is where the engineer can then make a choice as to how we want to Preheat and the degree of the level of quality we want in that incoming stream.

1:30

And once we decide on the reflux ratio, that dictates where our our rectifying operating line in blue is going to intercept.

1:38

And then from there, this is the region of the McCabe TL diagram in which we're going to do our stair step.

1:45

So what I wanted to to highlight out of that then is that what, what is it that you as the engineer will get to the sign and that's the incoming composition.

1:55

What do we want to achieve and what's the reflux ratio that's best suited for that?

2:00

And then the McCabe tailing method allows us to basically step around this and and figure out that in size of the column.

2:07

A key take home message from this is that these are the number of ideal stages.

2:17

So coming back to one of the earlier questions then when we talk about ideal stages, an equilibrium stage includes that partial reboiler and would also include a partial condenser if we were drawing off some of that toothpaste product before we were fully liquefying.

2:37

So the number of trays in the column is is a contribution to this.

2:42

But that partial reboiler is also included as an equilibrium stage.

2:47

Now it's finally, you know everything that we're doing here is in the compositional space and we're using material balances to close a lot of these calculations.

2:58

So a few of the examples from last we could look at for instance, the ratios of of the distillate flow rate on the feed flow rate  $D$  on  $F$  and with that will be at steady state equal to the difference in compositions.



3:10

Similarly, the bottoms molar flow rate on the feed molar flow rate  $D$  on  $F$  is is can be defined on a compositional basis.

3:17

It's also worth noting that we can draw in the use of enthalpy balances.

3:22

If we are dealing in a binary system, we can think about the enthalpy balance of the composition and the flow rate.

3:30

And to help us close some of that, I've not gone into a lot of detail around that because largely that would be something we'd invoke if we didn't have a domain information.

3:40

For instance, in a McCabe Thiele diagram like this, we said we don't know what is the best distillate flow rate we can get to.

3:47

We might use something like an enthalpy balance to help us close that relationship if and only if we needed to.

3:54

So I'm taking a step forward then if we think about the number of plates in our column, right?

4:00

So to put it clearly, stages equals the number of plates in the column plus the partial re boiler plus the partial condenser.

4:19

And that's if and only if we have a partial condenser.

4:21

Most distillation columns will operate with a complete condenser, right?

4:25

So we're transforming 100% of that vapour to 100% of the liquid.

4:29

So then would be the the question of, well, we know our partial reboiler from Friday's example.

4:33

Every time we did one of those in The Cave Tilley examples, we saw that we

calculated the number of stages and we would subtract 1 in every example from Friday to determine the number of plates in the column.

4:45

Well, how do you then identify what's the minimum number of plates that I can get away with if I'm an engineer?

4:52

So the minimum number of plates is dictated by the condition of total reflux, right?

4:58

And that's one where our reflux ratio is defined as Infinity, right?

5:04

Because the reflux ratio by definition is the liquid return rate, which I think we called  $L$  in our previous example, divided by the distillate molar flow rate.

5:17

So if my distillate molar flow rate is 0, then I'm defining some number divided by zero, so the reflux ratio will go to Infinity.

5:27

So if I have an infinite reflux ratio, meaning I have a total reflux condition that represents one of the bounding conditions of a distillation column.

5:39

And in that case the rectifying operating line slope.

5:43

If  $R_{sub D}$  on  $R_{sub d + 1}$ , if  $R_{sub D}$  goes to Infinity, then I have Infinity divided by Infinity.

5:51

So that means that my rectifying operating line from this example will fall on the 45, right?

6:02

And in a total reflux condition, I also have a corollary that I have no inlet or outlet flow.

6:11

So anytime you're going to operate at a operate a distillation column with total reflux, you do not draw bottoms or tops products and you do not have a feed.

6:22

So you start the column up, the reboilers is filled with some liquid of nominal composition and you're firing up that reboiler and the condensing system and simply allowing that column to start coming into equilibrium.

6:35

Now at that condition, I'll switch colours here at that condition, right here are my operating lines.

6:43

They sit on the 45.

6:45

So that means if I have my equilibrium line, which we'll plot here in green.

6:55

So this is my equilibrium line for the two.

6:59

Then as I'm going through to determine the number of steps in the total reflux case, right?

7:08

I might have 5:00-ish stages in here with the feed here's 123.

7:15

My feed should be placed around the third stage, right?

7:19

Four.

7:20

And then I would say here's my 4.5 stage.

7:25

So in my total reflux condition, I'm not pulling liquid from either the outlet, either outlet to the distillate or the bottoms outlet.

7:33

And I do not allow any inlet into the column.

7:37

So we use this as the start up condition to the distillation process.

7:42

Now what we're going to talk about towards the end of the lecture is we can also use total reflux as a way to get at batch distillation.

7:51

But that is a very, very niche case.

7:54

And I'm sure if you're looking at examples of what's been done in years, in the past in this unit, it's been using batch distillation.

8:02

This is a reason why we replaced it this year, because that is not how chemical engineers actually operate.

8:08

Chemical engineers will always use a continuous reflux or continuous operation for distillation columns.

8:15

So when we're in a total reflux condition, it means we're trying to start the column up in that condition.

8:21

Our operating lines collapse to the 45 and we can very simply figure out, OK, where do I need to feed the column to start it up.

8:27

And that might be different than where I'm going to ultimately feed the column during steady state mode.

8:33

So it's not uncommon that if you have an industrial distillation column, you're going to be able to control what feed out of let's say 12 or 13 industrial stages.

8:42

The engineer will have a little switch saying how much I'm going to feed on each stage, right?

8:48

And if you have no feed coming in during a start up, as you very first put the first bit of say milli moles per hour into that feed section to pull it out of total reflux.

8:59

This total reflux condition will tell you what stage that feed needs to initially come in on, right?

9:05

As we move from the bounding condition of total reflux into our steady state condition that we've been looking at previously.

9:15

So now if we have ideal mixtures, right?

9:18

And these are ones where we can define  $\alpha_{AB}$ , right?

9:22

This is going to be that PA SAT and PB SAT.

9:28

So we're invoking Ralph's Law for kids.

9:30

We have no activity in the liquid phase, no fugacity in the vapour phase, but we have ideal mixtures where we are.

9:38

Our alpha AB is going to be constant, right?

9:43

Or through the entire column.

9:45

Then we can also use an analytical expression to determine the minimum number of stages.

9:49

But again, I prefer to use The Cave Tilly method just because it makes a bit more sense intuitively.

9:56

So if the well, if the minimum number of plates in the column itself, appreciating that that partial reboiler is always going to act as a stage.

10:06

If the minimum number of plates in the column is dictated by a total reflux condition, then the maximal number of plates is going to be dictated by the pinch point, right?

10:16

So if by controlling my reflux ratio, I'm going to be shifting the pitch of my rectifying operating line, right?

10:26

And here are 4 examples, right?

10:29

So this bottom one would be consistent with the total reflux condition.

10:36

And as I can see, as I turn the reflux ratio down, so I'll draw this in a different color.

10:48

So this is decreasing R, so D.

10:50

So as I, if you imagine an engineer sitting in a control panel, I have a little dial to control the reflux ratio.

10:58

So as I turn that reflux ratio down, I'm going to be shifting the pitch of that rectifying operating line toward the the upper part of the diagram.

11:09

Well, at some point, right, that operating line is going to bisect the equilibrium curve.

11:16

That's if I go back and just highlight this in green for a minute.

11:20

Here is my equilibrium curve.

11:22

So I can see that bi section is that red circle.

11:25

And if we think back to our absorption examples, that's the same as a pinch point, right?

11:31

The second my operating lines touch or graze an equilibrium curve in in composition space, I have a pinch point.

11:39

Well, if we imagine trying to do a stair step right all the way down into this space that I'm going to have an infinite number of little stairs I'm trying to cram in there as my operating line bisects the equilibrium line.

11:55

So this pinch point then core dictates when I have my minimum reflux ratio.

12:02

So that's the lowest I can drop the reflux ratio while still trying to get a separation.

12:09

And in reality that's a minimum in theory, right?

12:12

I have to be have a reflux ratio of slightly above that to still have some amount of of separation going on.

12:21

Now when I have a minimum reflux ratio, the number of stages required for the separation go to Infinity, right?

12:30

So I'm going from on the left or previously a total reflux condition where I have as few stages as possible.

12:37

So in this example of four and a half, like we said on Friday, this would be 3 1/2 in the column plus the partial reboiler.

12:50

And if I go from that reflux ratio of Infinity to, let's say a reflux ratio of two, for example, if that's our minimum reflux ratio, now the number of stages required goes to Infinity.

13:02

So these are the two bounding points I can think about in my system.

13:05

And at this minimum reflux condition, now I can determine this based on the definition of where that slope is going to bisect the equilibrium curve.

13:19

So  $X_D$  being the distillate product and  $Y'$  prime being the as I can see here, the vapour composition at the bisection of the operating line and the equilibrium line and  $X'$  prime being the bisection, the liquid composition at that bisection.

13:34

I can determine directly what is that minimum reflux ratio for any column, right?

13:40

As I start thinking about that bisection point.

13:47

So I, I don't think we go into great detail on the lab about this, but as you're unpacking your Laboratory 3 exercise, I would absolutely, since you're going to plot this McCabe teal out anyway, right?

13:59

I would absolutely recommend that everyone actually plot this out and say, well, if I start changing that reflux ratio, where is the minimum, right?

14:09

And can I determine this visually and then based on these compositions?

14:14

So the, the point that I want to take home from these two slides then is that the

actual columns we're going to be operating will be bounded between our minimum reflux condition and our total reflux condition, right.

14:27

And the trade off, and this is what the engineer is always asked to do, the trade off is that as we turn the reflux ratio down from Infinity, we are trading off having more product for more stages, right.

14:42

If you go back to that principle that every stage costs the Mercedes-Benz.

14:46

So if, if in Australia that might be \$100,000, if I'm going to build a 10 tray column, that's going to be a \$1,000,000 column, All right.

14:57

So I'm turning down or turning up the CapEx.

15:00

I need more stages, but allowing more product because I have a lower reflux ratio.

15:07

So it's a question of CapEx versus OpEx, right?

15:09

It's fundamentally where this comes into play.

15:14

So from a theoretical perspective, we can think about that cost.

15:19

So here are fixed cost.

15:20

This is more modernly what we would call CapEx, right?

15:24

What do we have to pay for that system and the heating and cooling costs?

15:29

We think, well, the more flow we have we have through this system, the more heating we're going to have to apply.

15:36

So lower reflux ration means we're pulling more material at the top.

15:40

Well, if we pull more material at the top, that column needs to have a pretty much



constant number of moles in it at any given time, which means we need a larger feedstock.

15:49

A larger feedstock means we need a larger reboiler and a larger condenser.

15:54

So as we pull more product out, right, the column is producing more for the number of stages that it has, but I also need to have much higher heating and cooling requirements, right.

16:05

And because heating and cooling, when we go back to heat exchange, well, the the cost of building those kind of column, say a shell into heat exchanger or an air cooled condenser, Yeah, the cost of building those things isn't really that extreme, right.

16:19

We can see here our our minimum cost for building kind of intersected or plateau out, but it's actually the energy requirements of of driving heat exchange processes that are expensive.

16:31

So the heating and cooling cost comes into play as an off X consideration.

16:35

So what I can think about here now is the optimal reflux ratio for a practicing column is going to be the balance between the CapEx to buy it and the off X to run it.

16:47

So if I take these two curves, here's my, my CapEx curve, we'll draw in red.

16:53

And if I have my off X curve, we draw on blue, then in green.

17:05

What I would do is add these two curves together because I'm going to have to pay all that money at the end of the day.

17:10

And this would dictate to me what is the minimum reflux ratio or the optimum reflux ratio for which I'm getting spending the minimal amount of money to make this process work.

17:23

So the most efficient reflux ratio, so that this comes from the 5th edition of McCabe's book on the left hand side.

17:31

The more modern 7th edition actually gives an example where it breaks this down into a total cost of dollars.

17:38

And this is obviously a very small column that we're looking at for less than \$100,000.

17:44

And we can see here my minimum reflux ratio has been identified.

17:48

The fixed charges on the equipment plant to wrap pretty quickly.

17:51

That's the bottom curve here.

17:53

We're seeing a relatively linear dependence if we're using steam to heat and cooling water to condense.

17:59

So adding those two together, I can quickly identify a minimum total cost.

18:06

So the take home message right is that I can balance my minimum reflux and my total reflux conditions based on thermodynamics and and theory.

18:15

And I would ultimately select that that optimal reflux ratio based on the cost of the equipment both buying it and running it.

18:26

OK.

18:27

So something else important to highlight is that everything we've talked about with the cake deal, I've tried to highlight every time that this is around an ideal number of stages.

18:38

So we need to think about then well, how does my actual stage vary when I compare that to my ideal stage.

18:47

So before we go into the upper bit, thinking about overall and mercury efficiency on the bottom here, this is an example of a valve tray.

18:56

So inside the little distillation column that we're using for our laboratory and what we

might see on a more industrial scale, this would be a part of a plate tray where it's being, you know, bound into the side of the wall here.

19:12

And we can see here a whole bunch of these little what look like caps that are sitting down on the top of the tray.

19:18

And because this is just taken out and taken photograph of, they're all flat and they're all sitting down.

19:23

And what happens is that if I have a lot of pressure build up, so a lot of pressure coming from underneath.

19:31

So the vapour space is going to pop that cap up and that's going to allow, if I switch to red, that's going to allow my vapour to escape and interact with the liquid on the tray above it.

19:45

All right?

19:45

So I can imagine if I have 102 hundred of these little caps that are sitting there, as soon as that pressure from the tray beneath starts to build, the cap will pop up, release that vapour and then it will bubble through the liquid.

19:59

So a diagrammatic way to show this, we can see here is a partial cap, that liquid will bubble through that and on each of these trays, right?

20:07

This is where I'm getting my mixing zone.

20:12

Well, hopefully one of the points that you've taken home by now is we start talking about, you know, the example of gas absorption being very simple by comparison to distillation column.

20:22

One of those take home messages is that for say an absorption column.

20:27

Well, depending on the amount of liquid in the system and depending on how quickly I decide to move this material through, I get differences in efficiency.

20:36

All right.

20:36

So you remember in gas absorption, if we have too much content flowing through that column, I'm going to flood out the absorption column.

20:44

So hopefully that that nomenclature of flooding is, is reminiscent for some people.

20:51

So if I'm flooding out the column, that means I have a variable efficiency based on how much material either in the vapour phase or the liquid phase I'm pulling through this column at once.

21:01

And distillation columns are knowing they're not exempt from this process either.

21:06

So each of these trays, when we talk about McKay and Teeley, we talk about the ideal number of stages.

21:12

But in reality, I will have a, an efficiency of each of the individual plates in my column, right?

21:19

So the plates being the physical stages of the column, I will have a variable efficiency and this will give rise to having an overall efficiency of the entire column.

21:29

So we categorize this as engineers in either one of two ways.

21:33

I've been talking about the overall efficiency of the unit that we're planning to buy.

21:37

And that might be something that the vendor is going to give you.

21:40

If you're buying a distillation column with a reboiler with a condenser, you buy the whole package for say 1.5, they're going to give you here's the overall efficiency of this column for the operation that you're buying it right through which you're buying it.

21:53

So an example of an overall efficiency if we needed six ideal plates inside the column, and then we go to the vendor and say, OK, this is what I've calculated.

22:05

They say, well, we have this one column for sale.

22:07

It has an overall efficiency of 60%, right?

22:10

Then in this calculation, it's  $6 / .6$ , meaning we have 10 actual plates.

22:18

So if you've done your McKay Tilley diagram and you've said I think we need 6 plates, if it's ideal, then if you're buying a column of 60% overall efficiency, you need to buy a column of 10 plates.

22:29

And so it's quite an important difference, right?

22:31

Because if each of these plates is going to cost us some Mercedes-Benz, that's 4 Mercedes-Benz is different or 400,000 in this case.

22:39

So it's quite a bit of money when we start thinking about it.

22:42

Now, instead of thinking about an overall efficiency, where we can start to get a more theoretical construct to use is the Murphy efficiency of every stage itself.

22:53

And the definition here is quite simple, right?

22:56

So  $Y_{sub N}$ .

22:58

So the Murphy efficiency  $ETA_M$  that we're defining on the left here is equal to.

23:04

So  $Y_{sub N}$ , This is the actual composition.

23:14

So the actual vapour composition of my lighter component minus  $Y_N$  plus one.

23:22

So that's the the composition of the tray coming from beneath it coming in and  $Y_N$  star this is the composition of the vapour that's in equilibrium with the liquid on that stage.

23:36

So an easy way to think about this, right?

23:38

In an ideal world, our vapour should be leaving that stage at YN star.

23:46

All right.

23:46

We've always indicated that if here we look at on the right hand side, that YN star corresponds to that vapour phase that's in equilibrium with the liquid on that stage.

24:03

So in reality, what will happen is that my vapour leaving won't quite make it to equilibrium.

24:09

And this could be that I'm trying to push mass through this column too quickly.

24:13

Maybe the rate at which the gas is bubbling through these little caps is going too quickly and so I'm not getting enough mass transfer on each of these individual stages.

24:25

And so the way this manifests in practice is that if I'm thinking about stair stepping through McCabe tiling, so here's my operating line, right, here's my equilibrium line.

24:36

Well, if I'm going to stair step through this right on any given stage, I'm not going to quite make it to intersect with the equilibrium line, but rather all right.

24:48

In this example that I've drawn, I think, OK, here are my my three positions.

24:56

This might be 90% of the distance, right?

25:04

So, and I'm going to go 90% of the way there before I start going down again.

25:09

And so this will give me that a 10% inefficiency on the tray.

25:13

So I have a 90% Murphy efficiency for that tray.

25:19

So all it means is if I think about my equilibrium line, that's my ideal system.

25:24

And if my tray efficiency, my Murphy tray efficiency is not going to be 100%, then I'm going to take that equilibrium line.

25:31

And in practice I'm going to pull it down into the right by the percent to which I'm being inefficient.

25:40

So on the bottom right, if I draw right, here's my actual equilibrium line dictated by data.

25:52

And if I'm going to draw then what is a Murphy efficiency of say 90% in this case that we've looked at, I'm simply going to pull that equilibrium line down a little bit.

26:09

So as I stair step through this, I have a slightly lower bounding condition and that's going to account for an inefficiency of each of my trays.

26:16

Or perhaps I've, I've picked a poorly designed a really cheap set of valves for each of these trays and that's why I'm getting that inefficiency.

26:25

So not something that we need to think about for this laboratory.

26:29

This is one of the reasons that the number of theoretical stages might vary from the number of actual stages in practice, but it's something that if you're going to go out and actually work on distillation columns, you do need to be aware of.

26:46

OK, so, so as I, I kind of mentioned very briefly early on, if you've, I'm sure everyone has seen the, or been passed down the lecture notes from previous years in this unit.

26:57

And one of the things you would have seen in that is that historically unit operations has always used batch distillation or I think it was actually done with some of the unit OPS or gas processing too.

27:10

But UWA chemical engineering has always used batch distillation when we teach this.

27:16

And in practice, if you're going to go out and be chemical engineers, nobody uses batch distillation, right, unless we're in a very, very special scenario.

27:26

And and where we apply that batch distillation is when we have small plants where we're trying to only distill a specific set of compounds that have very different relative volatilities, right.

27:44

So applicable to small plants where  $\alpha_{AB}$ .

27:49

So the relative volatility of my two components that I'm trying to separate is quite odd, right?

27:58

So a really good example of this is ethanol and water where you would definitely expect to see batch distillation as if you're making it still, right?

28:07

But if you're going to go be a chemical engineer, maybe doing bio fuels production, you are definitely not going to be using batch distillation if you're spending one to \$2,000,000 per distillation column that you're trying to build.

28:19

So one of the challenges with batch distillation, if we think about being the batch still here that's filled with some initial liquid of interest and this might be something like a a kettle type or boiler or we're using steam to heat this up.

28:34

We're going to vaporize our components.

28:36

Now these might be, we might have a column in here looking at a set number of stages.

28:42

The way we've drawn it here is that we simply then recondense everything out with cooling water and we received the product right.

28:48

So this, this drawing of it neglects the use of multiple stages to create purity, but instead thinks about one, simply one product.

28:58

What we could easily have right or a set of independent stages in here before our vapor is finally going to leave.

29:07



And when we have this product receiver, we might also, we could have an incarnation where we're going to refeed and reflux part of this.

29:15

So batch can be a batch distillation can work on either a single pass principle or a multi pass principle with reflux.

29:24

Either of these things is possible.

29:27

We can use this equation here, this Rayleigh equation.

29:32

The challenge with batch distillation is that if we think about well, we need to conceptualise it in terms of the marginal, say, set of moles that are being vaporized at a given time.

29:45

The nature of being batched means that we don't have a steady state process, but rather the composition of the what's in that batch still is going to change over time.

29:55

So if we are refluxing this right before we take our distillate products in, whether we have no reflux or partial reflux going back, either the product quality is going to be or the product composition is going to be changing with time or we're going to have to be playing with that reflux ratio constantly if we want to maintain the same product quality.

30:19

So This is why if you're a chemical engineer, you do not want to be changing these reflux ratios.

30:25

What you should have seen in question one by now is this little column that we have, tiny little column in the laboratory is going to take quite some amount of time to come into city state.

30:34

Every time we make a change, we'll take that column diameter and height and multiply it by 200.

30:41

Right now we're talking about industrial process.

30:44

So this is not something that you want to be tweaking the the reflex ratio on because the ability, right, with the exception of perhaps an AI that's been highly trained, the

ability to tweak that reflux ratio just right to get that and maintain the product composition that you're looking for is very, very hard.

31:09

So for this reason, we tended to look at batch distillation only in, not in terms of an internal column, but simply in terms of vaporizing, condensing and whatever comes out, which means we have a variable composition coming into the product procedure with time.

31:26

We can also use McCabe Till for this or the McCabe Tilly solution for this.

31:33

But what we can see here is that you know as that reflux ratio or as our bottoms composition changes with time.

31:41

So if here's our initial bottoms composition of what's in the batch still as we're heating it up and here might be our final right, the number of equilibrium stages or the number the ability to separate this is going to be dynamic.

31:54

So we're not going to do much on batch distillation simply because it's not very common of a principle and it's easier to think about it as A2 component flash where you're simply going to move those bottoms boundaries with time.

32:08

OK.

32:08

Now where we're going to go this week and where this lecture really should start to hit home is a multi component distillation.

32:16

So up until now, everything we've talked about has been a binary distillation.

32:22

Either we're trying to take two components and absorb them out in a gas absorption column or use a stripping column to separate them.

32:31

But in reality, it is a very good day for an engineer.

32:36

If you've been given pure ethanol and pure water in someone's sense, please separate the two.

32:41

That's definitely not what's going to happen.

32:44

What you're going to be handed is a suite of components sitting in a jug and someone's going to say, OK, I need one of those components pulled out because that's the component of value and that's what I'm trying to sell or that's what we need to use for some product.

32:59

And this is where we get into multi component distillation.

33:01

So this is now going to be the topic for this group this week, Monday, Wednesday and Friday's election.

33:08

So in many ways this is a focus now on on real distillation processes.

33:13

Now historically the example that we would always use for multi component distillation would be crude oil, right That we take some crude oil and we're trying to fractionate that out for the the variety of different fuel sources that that is used worldwide.

33:28

And while that is still very true, I think for engineers of your generation, it's also much more important to contextualise this against biofuels.

33:37

All right, In particular because one of the the points to appreciate is it's not just things like replacing petrol or replacing kerosene, which of course is the what we use for jet fuel right now.

33:52

There's been a large argument around electric cars and hydrogen based engines, either fuel cell or otherwise in airplanes that will allow us to run those processes in the absence of carbon based fuels.

34:04

But one of the challenges we have is all of the petrochemical derivatives, so something like 10,000 products that are used worldwide.

34:13

Everything from the dyeing or clothing that virtually everyone has in their closet right now to every single type of plastic that's used, including your Tupperware.

34:24

These are all somehow manipulated from petrochemical feedstocks, meaning they have carbon chain links that are being selected and brought into those fuel sources.

34:35

So one of the great challenges for the chemical engineer over the next 30 to 40 years is going to be how can we create a renewable biofuel source that can be used to replace all of these potential petrochemical uses today.

34:53

And so if we imagine we can take biofuels from a variety of sources, this might be something like food by products that are being biodegraded.

35:01

Algae has been a very popular topic for the use in creation of biofuels.

35:05

But at the end of the day, our biofuel is going to come into how we imagine we're going to have a carbon chain length distribution, right?

35:14

So we go all the way down.

35:16

This might be something like methane and ethane, right?

35:22

And here is C50 we typically refer to.

35:26

To give you an example, C35 to C36 is what we typically refer to as candalilla wax.

35:37

So if you have a candle lying around the house, usually the carbon chain length that makes up the wax of that candle is between C30 and C36.

35:46

And there are IUPAC names, so tricontaine to Hexatri contain or if you use an old school imperial name as candelilla.

35:55

So all of these carbon chain links if we're at warm temperatures will be solubelized together, right?

36:01

And so hereby what we have, right?

36:03

And this is going to go, if we look at this same range, right, your vegetable oils will be say C10 to C-15, Bio gas is going to be that C, methane, ethane, propane, butane, biodiesel is going to be C7 to C-12, so on and so forth.

36:22

You're going to have this massive distribution of hydrocarbons.

36:25

And what a chemical engineer will do is then say, OK, if I've generated this from mashing up algae and say a trash or compact trash that we're going to put into a kiln and not burn, but liquefying under pressure to create a biofuel.

36:41

And then emerge all of this together to get a biofuels feedstock.

36:45

We need to then take that complex distribution of hydrocarbons and separate it out into each of these different product streams, right?

36:52

Because I can't be putting vegetable oil into my biodiesel engine, right?

36:57

And if I'm going to use bio gas, I can't be putting bio diesel into my bio gas engine.

37:03

And so I need to the ability to separate these to extremely high purity.

37:08

And that's where distillation columns running in continuous mode are going to come into play.

37:12

And it's worth noting that when we start applying this problem, we'll typically see between 10 and 20 trays per column and that's multiplied usually between 5:00 and 10:00 columns.

37:28

So if you look at a typical oil refinery, right, the same number of columns and trays set up will be consistent.

37:35

So if we say 20 trays per column and This is why those, those distillation columns get so tall and that's two.

37:43

So 20 trays at \$100,000 a tray, right is going to be \$2,000,000 per column times 10 columns is 20 million just in CapEx, independent of the amount of money that we're going to need to spend to operate this for both cooling and heating requirements and biofuels production has the same constraint placed faucet.

38:04

So we as chemical engineers are virtually the only, no, not virtually, we are the only discipline that knows how to do this right.

38:13

Nobody in civil, mechanical, electrical, no other engineer knows how to actually design and manipulate this.

38:20

The chemists don't know, the physicists don't know.

38:23

So it's going to be increasingly important as we come into a world of biofuels that we're all very comfortable with what's going on here because there's a lot of money sunk in the CapEx of these systems and an equal amount, if we go back to that previous example, an equal amount of money as we can see here, that's going to be put into that heating and cooling requirement relative to the CapEx of the equipment itself, right?

38:51

So the way that we're going to be tackling tackling this and we'll get into a very detailed example on Wednesday is we start thinking about, well, if I have not two components now, but let's say I have 50 components, Every one of my distillation columns is going to be tasked with taking let's say my 50 components.

39:14

Like imagine a, you'd play 50 card pick up, right?

39:16

So take a deck of playing cards.

39:18

Every one of my distillation columns is going to be like going to, to play blackjack at the casino where I'm going to take one, one of those colour cards where I want the dealer to split the deck.

39:30

So take that red card and I'm going to insert it somewhere in the deck and that's going to take my playing card deck and split it in half.

39:38

So I have these two halves of the deck, right?

39:40

Each half is going to come out one side of the distillation column.

39:46

And then that's I take that that half partial deck again.

39:50

I insert that red card to split the deck a second time and take it into a second column and a third column and a fourth column.

39:57

And this is how I can take a complex feedstock and split that into eight to 10 different products.

40:05

So identifying what are these key, I call them my key components.

40:09

If I have 50 components, if I'm going to create the separation based on C20 and C21, those become my keys for separation.

40:19

And the way that we're going to think about this is using what's called AK factor.

40:23

Now K factors are also called distribution coefficients, but here I'm going to call them K factors because distribution coefficients can also start to get into to thermodynamics and I don't want to get too confusing with nomenclature, but it is fundamentally distribution coefficients.

40:38

So our K factor,  $K_{sub\ I}$  is defined as the vapour phase composition of component I in equilibrium with the liquid phase composition of component IX sub I.

40:49

And you can imagine in the same definition of an ideal system, it's the ratio of my vapour pressure on the operating pressure.

40:57

So we're actually going to use a bio fuels and petrochemical example for this because that's where if I'm using all hydrocarbons, I can assume I have a relatively ideal system because my hydrocarbons are not going to interact antagonistically with one another.

41:14

And this is where we'll go to something like the NIST chemical web book, look up what are the vapour pressures as a function of temperature for each one.

41:22

And then I can start to calculate in real time what is the outline and distribution of these K factors.

41:28

And historically they were also called Watson K factors.

41:31

But a more modern nomenclature to say K factors and the nature of what these K factors are allows me to draw two really critical relationships at the bubble point, right?

41:43

Right.

41:43

The sum of my  $Y_{sub I}$  components and I have from 1:00 to north sub C so I have N components in my system.

41:51

The sum of my  $Y_S$  has to equal 1.

41:54

And by the definition of an ideal system up here, then the same sum of  $X_{sub I}$  times the Watson K factor  $K_{sub I}$  for that component must also equal 1.

42:08

And the same I can apply for a two point relationship on the right hand side, except I'm dividing my vapor phase  $Y_{sub IS}$  by that Watson K factor  $K_{sub I}$  and it's on a per component basis.

42:19

So if I have 50 components in my system, I have 50  $Y_{sub IS}$  50  $X_{sub IS}$  and 50 Watson K factors.

42:26

And each of these are these K factors are temperature dependent.

42:30

So it's critical then if I'm starting to sum these up and I say, OK, well, my K factors, when I apply them, having these summations are less than or greater than one, I'm not working at an appropriate temperature.

42:42

And that's where I start to tweak the temperature.

42:44

All right.

42:45

So you can see why I want to put these up into a spreadsheet because I can then change one value in that spreadsheet and it will allow me to separate back out and figure out what is the optimal temperature for separating what I want to here.

42:58

OK, so we're going to get much deeper into this on Wednesday, but I just wanted to get a quick kind of scene setting in play.



43:07

So the take home messages.

43:08

If we think about distillation, we can fundamentally break this into three bits.

43:12

Batch distillation is a niche application as rarely used.

43:17

The most common example will be doing something like a still to make a vodka right where I'm going to start off with a very high purity product and I'll get an average purity from that on.

43:29

Most commonly for chemical engineers will be using continuous fed distillation.

43:34

And there one of the the real critical paths is to figure out the optimal number of trains.

43:39

So my minimum boundary is set when I have an infinite reflux and then I can use the  $45^\circ$  line for the KO2 diagram.

43:46

So in fact, I don't need to even construct my operating for the rectifying or stripping section lines.

43:53

The maximum boundary is when I have a pinch point.

43:55

So that's when my rectifying line will intersect or bisect the equilibrium curve.

44:02

And so these are my two boundaries for how I get to choose my reflux ratio.

44:07

And the practice is a trade off between the CapEx for the column and the productivity of the system.

44:13

So when I go to implement this too, it's also important to think about none of my trays are probably going to be 100% efficient.

44:20

So in practice, the engineer should always be thinking about what is the tray

efficiency, meaning that that when we use McKay and Keeley, that's always going to give us a minimum number of trays required.

44:32

I'm probably going to have anywhere from 10 to 30% more trays than that to account for these inefficiencies.

44:40

And when I apply this all to what we're really going to be working on out in the field as engineers, we'll be dealing with multi component feedstocks.

44:48

And so that's where we introduce these Watson K factors that are effectively thermodynamic distribution coefficients.

44:53

So I can partition now between each of my components and select what where in this 50 card deck do I want to chop to to separate the lighter side from the heavier side and go in from there?

## Lecture 23 - Multicomponent Distillation

0:00

Lecture 22, the kind of summary that we got to, and I'll go through this summary in a minute, but I wanted to highlight on the bottom left this diagram around biofuels again, because I think one of the, the key messages I hope you can take home from this distillation Fortnite discussion is that once we figure out how to make a biofuel, right?

0:24

And that's, that's something that we as a, a civilization are still working quite hard on.

0:30

So there's a lot of research going into what is the right upstream feedstock that we need to take.

0:36

You know, obviously the interest in taking trash, taking recyclable materials, putting these on in kiln and high pressure, not igniting them necessarily.

0:46

They're trying to melt them down to recreate that bio well to to recreate the the fuel generation process of of compacting material in the earth at high temperature and high pressure for a long time to generate that in biofuel from a renewable feedstock.

1:04

I would highlight that if if anyone's watched Michael Moore's new movie Planet of the Humans, one of the really big problems going on in North America and Germany right now is the sources of biofuels that are being used or wood chips.

1:17

So if you want to take trash and try to burn it as a more sustainable economy, that's a great thing.

1:24

But trash in a wrong form does not ignite very easily.

1:27

And So what they're doing is chopping down forests because we know trees burn and that's being supplemented in the biofuels process.

1:35

And so that's a great concern, I think for a lot of people.

1:39

However, certainly within your lifetimes as an engineer, we will have a much clearer focus on how to create renewable biofuels and sustainable fuels.

1:50

And So what the CEO of Dow Chemical was going on about was then it's not just the use of of a of a hydrocarbon distribution for something like jet fuel being kerosene, diesel or, or petrol.

2:04

It's actually the diversity of all of those petrochemical derivatives.

2:10

Again, 2/3 of everything we touch, including the clothing, every plastic material, much of our glassware and the coatings on that.

2:18

Certainly half of the components that go into every computer that's being used to listen to this right now are derivatives of the petrochemical feedstock.

2:26

And chemical engineers are the only engineering discipline that is taught how to take this complex feedstock of materials and distribute that out into its necessary components for processing.

2:40

So the problem in many ways is cut in half.

2:43

How do we move from using a fossil fuel based, so either a crude oil or a natural gas that has recombination, but a fossil fuel based feedstock to then distribute into our petrochemical derivatives.

2:54

How do we move from that to a sustainable feedstock that can be used to replace many of these components?

3:01

But then the processes that will be invoked in doing that to use them will be the same.

3:07

So we will still need our distillation columns and our learning.

3:10

In fact, we're going to need to amp it up to be able to accomplish this.

3:14

So if we look at this bottom diagram for women, if the current generation of what we consider biofuel would be containing right in its natural multi component feedstock, this variety of variety of products.

3:30

So everything from light alcohols, right all the way through diesel, which is usually C8.

3:35

So octane to dodecane, bio gas is obviously methane to say butane, but then solid fuels will be something like C18 to C24.

3:45

So we've got a large set of hydrocarbon distributions.

3:48

And what will happen is this whole feedstock will be put into the first distillation column and that column pressure and temperature are set to split to effectively split say if we have six key components coming into this and, and bearing in mind each of these components listed here is actually made-up of hundreds of different isomers and, and, and a hetero atom molecules of the same carbon chain line.

4:14

But if we take it into the first installation column, we would say, OK, we're going to cut that in half.

4:19

And the purpose of that column is to split these components in half.

4:23

So if we do that, then we're going to take something, maybe a product out of our upper tray and that will contain our lighter components and we're going to put that as an input to a secondary distillation column, right?

4:35

And that is also going to split those components in half again.

4:39

All right.

4:40

So I'll make a little you imagine now we're talking about  $1/4$  will be the output and that will go into a third distillation column, which itself will be split in half again and then a fourth to target that final say eighth of the original feedstock that we're going to need to make, for instance, a renewable set of Tupperware or renewable bags for sandwiches for lunch, right.

5:09

So this is the the class of problem that chemical engineers are faced with and it's very much connected intimately to the biofuels discussion and biofuels utilization.

5:20

So that's part of why I wanted to, to make sure we brought in proper, just continuous distillation into this, because this will be something that if you're doing the proper chemical engineering for your career, it's, it's going to be more important now than it ever has been, is the understanding of these principles and how to manipulate them.

5:41

So batch distillation, which we've taught previously at this university, this is a niche operation and it is rarely if ever used in the industry, largely because every time we build one of these columns, again, the rule of thumb is every tray costs a Mercedes-Benz.

5:57

So if we're talking about 11:50 trays, that's \$1,000,000 easily.

6:02

Now optimising those trays and how many we need, you know, over the diversity of potential feedstocks, appreciating that that more in a biofuels world than a crude oil world.

6:13

The diversity of what's coming in as our feedstock is likely to be much broader.

6:18

So our operation needs to account for that.

6:21

The minimum number of trays that we'll have is when we have an infinite reflux condition, right?

6:26

And that's when the the column will be usually starting up.

6:29

So we have no input infinite reflux ratio.

6:33

And then as we start to draw distant product off that we turn the reflux ratio down, turn the feed flow rate up and we can bring the two into balance very, very slowly.

6:44

The maximum boundary, so the infinite number of trays corresponds to when we have a pinch point or we start to approach a pinch point.

6:51

So the trade off between the two is if we have an infinite number of trays, we will tend to get a higher or we have an access to get a higher quality, I'm sorry, lower quality.

7:03

But as we have an infinite reflux ratio, we tend to get a higher concentration distillate product.

7:08

So it's a trade off then between the CapEx that we're willing to spend to make those trades in the 1st place and the productivity of the column and how much we can pull out in terms of kilomoles per hour.

7:20

So once we are talking this week now about multi component distillation, this is the real problem that practical chemical engineering is faced with, right.

7:29

It's an ideal world to have water and ethanol.

7:32

But in reality we do have these multi component feedstocks and and we're asked to carve out 1/8 of that chemical diversity in the feedstock and this is the the chemical that we're after.

7:45

So in the last lecture, one of the focal points that we took was to introduce what we call K factors.

7:53

And so K factors by definition are at equilibrium the vapor mole fraction of component I that's in equilibrium with the liquid mole fraction of component I.

8:04

Now you can see here that we can also rewrite this in terms of Ralph's law.

8:08

And what we've highlighted in green, if we go back to thermodynamics last semester, is the definition of Ralph's law for kids.

8:15

So it's the vapour pressure on the system pressure gives us that K factor.

8:21

Now I can also see that if I want to take a more accurate approach I can write and I'll draw this down some ways in green, but I can also write a more accurate version in terms of that K factor being  $\gamma P_i^{\text{sat}}$ .

8:44

So the vapour pressure times its activity coefficient divided by the fugacity coefficient  $\Phi_i$  times the pressure.

8:52

So accounting for the non ideality in both the vapour and liquid phases will give me a more accurate K factor representation.

9:01

Now the challenge here and if we go back to thermodynamics last semester, one of the challenges you might remember is that it's very difficult for us to define activity coefficients when we get away from binary only systems.

9:16

So that that tends to mean that that we're restricted.

9:20

So in the case of ethanol and water, we can absolutely write K factors and use this approach invoking an activity coefficient to account for that.

9:28

And in fact, in something like ethanol and water, the activity is not negligible, right.

9:34

These are pretty strongly hydrogen bonding compounds.

9:37

And so there is quite a bit of chemical activity going on.

9:39

That might be one of the explanations that you can use when you're thinking about the experimental data versus theory in your lab.

9:49

But if we then take these K factors and we want to say we don't want two components, right, we're going to talk about 20 components.

9:55

Well, at that point, getting activity coefficients and fugacities tends to be or fugacity coefficients.

10:00

These of  $\gamma$  tends to be pretty difficult.

10:03

So we will tend to write these K factors simply in terms of their idealized systems with that vapor pressure on the system pressure.

10:11

Now when we do that we can define just through Raoult's law a relationship to the bubble points on the left hand side blue.

10:21

So our bubble point relationship says that for an N number of component system I can sum up all of the Y fractions  $Y_{sub\ i}$ .

10:30

By definition those must equal 1.

10:32

And because of the definition of the K factor, if I'm using Raoult's law for kids, then the summation from component 1 to N of that K factor for component on times  $X_{sub\ i}$  will also equal 1, right?

10:45

And I can do the same thing on the dew point side of the relationship.

10:48

So what this means is that I'm searching for a bubble point temperature or pressure, right?

10:53

Once that condition, meaning this summation on the right or the left hand side once that reaches unity.

11:02

And when we say unity, we mean one once that equals one.

11:06

I can then use this equation on the bottom here to calculate for each of these K factors because remember the K factor itself is a function now of temperature, right?



11:17

So I say  $K_i$  is a function of  $P_i^{\text{sat}}$ , which is a function of temperature, right?

11:29

And so it's a function of both temperature and pressure, right?

11:34

So because I have a function of temperature and pressure coming in here, once I identify the temperature and pressure conditions I'm searching for, I can then use this equator, the definition of the K factor on the right hand side to solve directly for the vapor phase compositions at that condition, at that condition.

11:52

Similarly for the dew point relationship, if I go to red, right now my K factor is being with the  $Y_{\text{sub}}$ .

11:59

I divided by that K factor, but again they must be with unity.

12:04

And so as that relationship approaches unity, I can now calculate my liquid phase compositions through that same approach.

12:10

So instead of going on about this just in theory, what I thought might be helpful is to put together a quick example that we can do together.

12:20

So this is for a multi component do and bubble point right?

12:36

And the reason that of course I need to know this is that if I'm designing a unit operation, I need to be within a two phase region to split maps between these two phases.

12:46

So before I go any further, I'm just curious for the folks on the call right now, have you heard the phrase Ratchford Rice ever thrown around in your curriculum up until now except for by me, I think I might have mentioned it last semester episode briefly.

13:01

OK, cool.

13:03

So we're going to learn something really cool today.

13:05

And in fact, Ratchford Rice is one of the most powerful tools that chemical engineers have at their disposal.

13:12

So the multi component distillation that I want to do this will involve pentane, hexane, heptane and octane.

13:21

So pentane C5, hexane C6, heptane C7 and octane C8.

13:28

So what you might think about is that we're trying to consider the bubble and dew points of say a bio fuel, right.

13:34

So if our cars and our combustion engines typically run on an octane or a high octane fuel, then we're going to try to make up a mixture that has from a bio fuel speed stock that's going to have this diversity from pentane through octane.

13:48

And we want to figure out what are the operating conditions of that and is that suitable to be driving in my car at high temperature and some reasonable pressure.

13:57

So one of the first things I need to do that if I'm going to employ this K factor structure, I need to know my vapor pressures.

14:05

Where do we think we're going to go for that?

14:08

Well, we're going to go to the NIST chemical web book.

14:16

And what I've done is to look at, I've just pulled up the web book written down here is all the ABC correlation or ABC values for base 10 correlation for vapor pressure that we'll see here.

14:29

And so I can calculate then that the so P the vapor pressure pentane, right.

14:38

And I should clarify, firstly what I'm going to do is pull these values and this is going to be at 54° C.

14:48

So I've set my temperature and the reason we came up with this is to say, well, what's what's likely the hottest my car is going to get sitting out in the sun on a cruise day 50 ish right 50 sitting in the sun observing all that heat.

15:02

This is probably the upper limit of how warm that fuel tank is going to get.

15:07

So the vapour pressure might have C5 component.

15:09

Then if I define it at 54° C, I'm going to use, there's only one correlation in the web book.

15:17

So this gives me a vapour pressure and I have all right, just one of these out for anyone who might want to refresh it from last semester.

15:26

3.9892 -.

15:29

1070.617 / 327 and that's temperature in Kelvin now -40.454.

15:44

So C in this case is negative and that gives me a pentane vapor pressure of 1.79 O3 bar.

15:53

Now my heptane vapor pressure will be .6191 bar by my heptane vapor pressure is .22 O 6 bar and my octane, my octane vapor pressure point O 8 OO bar.

16:25

OK.

16:26

So what this allows me to say then is I can if I'm operating at 327 Kelvin, I can very clearly see the vapour pressure is going to increase with or as the molecular weight of my hydrocarbon gets lower and lower.

16:41

So one of the first questions is going to arise is to say, OK, if I'm I'm just going to switch colors here.

16:48

So if I'm thinking about a traditional PXY diagram, right, you can see I have my liquid region here, my vapor region here.

17:04

And what I want to know is for this multi component feedstock, if I'm coming from a high pressure condition, let's say I'm keeping my fuel tank at a reasonably high pressure, at what point, All right, What's the bubble point?

17:28

And if I continue decreasing in pressure, what's the dew point?

17:43

So how low can I go in this region and still have a two phase operation allowing me to split my components?

17:51

So the first thing that we'll do is think about calculating this bubble point.

17:56

So what I'm going to do is write out a table here.

18:12

So if I have pentane, hexane, heptane and octane.

18:20

So for if I have a feedstock coming in of point O, 50 mole percent, 15 mole percent hexane, 10 mole percent heptane and 25 mole percent octane, that's probably pretty good because I need a lot of octane for a traditional combustion engine if I'm going to convert it over.

18:43

Well, I know my vapor pressures now and these are going to be in bar.

18:50

So 1.79, 0.619, 0.220, 0.106 and point O 8.

19:06

This allows me to now calculate my K factor.

19:15

So for each one of these, I have 6.8742 point, 3770.847 and 0.307.

19:28

And finally I can invoke that relationship from Raoult's Law to say that my  $P_i$  which will be  $K_i X_i$  multiplied together  $P_i$  will be zero point.

19:42

Well, sorry, before I go there, this pressure that I've set here, what I've done in excel is to goal seek this.

19:52

So I gave an initial estimate of .5 bar, right?

19:55

I'm looking for some kind of split in the middle of my vapor pressures that should be an operating pressure.

19:59

So if I set that pressure to be equal to .26 O 4 bar, then I get this distribution of K factors, which then gives me a resultant Y sub I.

20:11

If I multiply the K factor times the XI component, I get Y sub i's of 3437.

20:19

Point 3566.0847 and .2151 right and the sum to 1.000.

20:36

Now if I also then go back into this this same system and I guess maybe incorrectly as I did early on, well, I'm going to guess a much higher pressure.

20:47

So if I do a second set of calculations at a pressure of .5 bar, then I get a new set of K factors.

20:57

I have the same vapor pressure because I'm not changing my temperature, but my denominator pressure has changed.

21:02

So I get a new set of K factors cropping up.

21:05

And so these are three-point 5811.238.441 and .160 and subsequently my Y sub YS, right?

21:22

So Kixi, these change as well.

21:26

Point 1790.1857, point O 441 and .1120 and the sum of these is .5 two O 9, right?

21:44

So that's a sad face which is not letting me draw that low.

21:49

But that's a sad face because that's not summing up to one which is the requirement of the equation, the working equation that I'm using.

21:58

So if I'm trying to to guess and check my bubble point pressure, my initial guess of .5 bar is not correct.

22:13

But as I start reducing this and I can either do it manually or use a goal seek or

optimization function in Excel, I do find that the actual answer for this if I'm looking for the bubble point pressure of a multi component feedstock at 54 Celsius or 327 K is .26 bar.

22:34

So on this particular feedstock, I'm going to have to pull a reasonable vacuum in order to get that to start bubbling.

22:41

Now if I take that a step through and I say, OK, well what's the dew point pressure?

22:57

So I'll very quickly draw the same table again.

23:10

Now if I initially guess that I have a bubble point pressure, I'm sorry, a dew point pressure of .10, not .1.

23:27

If I say .5 again, then my K factors that I'm going to plot out, these are the same ones that I had before.

23:54

Which means if I now have all my components are in the vapor phase  $Y_i$  .05, point one 5.1 and .7.

24:07

Which means  $X_i$  of I is going to be  $Y_i$  on  $K_i$  .014 point 1211.2267 and 4.3731.

24:27

Which means the sum of these is 4.7349.

24:33

Right?

24:34

Which we give a sad face right?

24:36

So I'm not summing.

24:37

When I sum these up to for the solution, it does not equal 1.

24:41

So that means if I'm looking for the dew point pressure.

24:44

Now my initial guess of .5 bar was too high.

24:50

So again through a goal seek function in excel, what I can do is guess to find and solve for this summation equal 1.

25:00

If the pressure is .1056 bar then my K factors are going to be 16 point 9535.8632, point O 8-9 and 0.758 which gives me.

25:20

Now I have the same Y sub I's, but it means my X sub I, my liquid compositions, that same Y sub I on my new K factors are going to be point OO29, point 0256.0479 and .9236, which sums to one exactly.

25:47

So the point of this approach, right, is it lets me instead of thinking about a dew, a dew point or a vapor, sorry, dew point or a bubble point and either a single phase or a binary system where I'm going to use experimental data.

26:01

What I can instead do is invoke Ralph's law, either for kids or chemical engineers, depending on the complexity.

26:06

In most cases, particularly for hypercarbon systems where this will come up often in your career, we we assume not for ideality in these phases because hydrocarbons tend not to be very non ideal until we get to to highly complex feedstocks.

26:23

Which then allows me to use Graf's law for kids to very simply invoke this definition, the K factor and identify now a phase envelope for the bubble point and the dew point as a function of composition in feedstock.

26:36

OK, so it's a way instead of representing this on a PXY or a TXY diagram, because I now have more than a binary system, I I have to use AK factor, which gives me a table for my solutions.

26:51

All right, so then where do we actually bring this into play?

26:54

Well, the, so we're going to wrap up on Friday talking about multi component, multi stage distillation.

27:03

But as you would expect, right, the simplest distillation we can perform as engineers will be a single stage distillation.

27:09

So something like a flash distillation.

27:12

And this is where K factors really come into play.

27:15

So if I have this, these K factors that allow me to incorporate Raoult's law over multiple components, I can write out for that in a single stage flash system we can define.

27:29

So ydi.

27:31

So that's the vapour composition of component I in the distillate stream will be related to the feedstock right of component I.

27:41

The bottoms composition of component I and F that we're using here, F is again that Burdwell fraction and that tends to have a molar basis of the feed that's vaporized and that's on it.

28:05

This single stage distillation.

28:07

What you will also see some more modern textbooks, we'll also call this beta.

28:15

So you'll see the same form of the equation emerge, but some texts will use italics F and some will use beta, but they're meaning the same thing.

28:24

How much of my feed is going to be vaporized on a molar basis?

28:27

So because we're relating  $y$  to  $x$  here, we can also bring that  $x$  down to the left hand side.

28:32

And once we have  $y$  on  $x$ , we can say, well, hey, that's equal to  $K$  factor.

28:38

And so this means that we can write our K factors in terms of the amount of product that's vaporized and the composition relationship above the feed and what comes out the bottom.



28:52

So one of the the two products closing your mass balance.

28:56

So in a single stage equilibrium operation, then what Ranchford, Wright and Rice put together was in a model equation using the same principle that we just looked at to say, hey, often times what I don't know as a chemical engineer is this F or beta term.

29:14

So I know my feed composition because I just went and bought it, right?

29:17

I know my K factors because I can use the NIST web book where back in the day they would use Paris Chemical Engineer and Handbook.

29:23

And I've set the operating pressure and temperature so I can get K pretty easily.

29:27

But what makes this a multi component flash operation is that I don't know how much of the components are going to go up and how much are going to go down, right?

29:36

And that's what I'm really solving for.

29:38

And this, this is the approach that we would take before tools like ISIS existed.

29:43

So if you're stranded maybe out in the middle of of the desert and you don't have access to the Internet or VPN and you cannot use ISIS, this is the approach that we have always taken as chemical engineers to solve it.

29:56

So the Ratchford Rice equation, right, is shown here.

29:59

And this is actually a solution methodology for multi component systems.

30:04

So I think the easiest way again to illustrate this is to look at an example.

30:10

So this will be a single stage flash at 305 Kelvin, so 32° C reasonable work in temperature.

30:26

And what we're going to have is pentane, hexane, and now nonning, so C9 and decking C10.

30:33

So what I can calculate, and I'm just going to write this out as a quick table, I have C5C6C9 and C10.

30:59

Now I've used the exact same NIST web book equation, so they're base 10 logs for vapor pressure.

31:08

So I can get  $P_i$  sat and that's in bar for each of these components.

31:18

This gives me at 32 centigrade.

31:20

I have .8754 bar, .2693 bar, point 00679 and point 00172 bar, OK.

31:37

So the next thing I need to decide is, OK, what's my operating pressure going to be and what's my feedstock ratio?

31:45

And that will get me to, that will get me to so I can invoke this Rashford resolution.

31:53

So let's look at 11 operating pressure.

31:56

So if I have a feedstock composition coming in of let's say 40 more percent pentane, 10 more, 10 more percent hexane, 30 more percent nonning and 20 more percent decane, then if my pressure that I'm going to set.

32:18

So if  $P$  is equal to .25 bar, then I can calculate  $K_{sub\ i}$  for each of these components, right?

32:28

And that's just my  $P_i$  sat on  $P$ .

32:30

So that will give me a  $K_i$  of 3.5 O2 one point O 7/7.

32:39

Oh OK, computer just froze but that's cool.

32:52

There we go, we're back.

32:53

A  $K_i$  of nonning of point O2 seven and a  $K_i$  of decking point OO7.

33:03

So then the the last part that I invoke is my Rashford rice, my Rashford Rice summation here, right?

33:20

So in each of these cases, what I'm going to tabulate is the it's taking a very long time to load each individual slide here.

33:38

Oh wait, it's showing up for you, but it's not changing for me.

33:41

Cool.

33:42

So is the ratchet right estimation?

33:44

So this right hand side of the equation,  $x_i$  on one plus my  $F$ , the amount that's vaporized times  $K_i - 1$ .

33:55

So I have  $X$  OK, so  $X$  feed I on one plus the amount that's vaporized  $K_i - 1$ .

34:14

So when I tabulate that for each of these components, what I call my Ratchford Rice contribution, OK, what I'm going to get is point O2 or sorry .2266, point O977, .4272 and then .2873, right?

34:47

And if I take the sum of these, which is the ratchet res condition to equal 1, I have 1.0388.

34:58

So what this means is I have a reasonably good solution because I have searched for right When I built my spreadsheet, I didn't know what this  $F$  equaled.

35:07

So I guessed initially that  $F$  might be .550% vaporized, but that turned out to be too high, so in fact I'm solving iteratively.

35:21

The F in this case is 30.6% of my feedstock will be vaporized all right.

35:30

And once I know the amount vaporized, then I can go back and split that in to figure out what's the individual composition.

35:36

So if I take this a step further now what I can see on the right hand side here, this is the then fraction of feed that's vaporized if I have that same operating temperature of 305 Kelvin as a function of the operating pressure.

35:54

So what I can see showing up on the right hand side and for some reason I can't draw here, but the amount that once I hit an operating pressure of say .42 point 43 I no longer have any of my feed that's going to be vaporized.

36:09

So to hit that bubble point I can see showing up from the previous ratchet rights style solution is identifying the bubble point.

36:17

And the more that I, I pull vacuum on this system and the lower the operating pressure gets, the more and more of this speed is going to be vaporized.

36:26

And the composition of the left hand diamond and what's in the vapour phase is going to be very different to the composition of what's in the right hand diamond, right?

36:35

So this is an introduction to what is the ratchet for the rights method in advance of a dynamics 4 four O 4 is going to go through this in greater detail, but I think this is a really good kind of introduction to have under your belt.

36:48

So what are some of the take home messages?

36:52

Well, as we started off with distillation, right, which is fundamentally our our core separation process.

36:59

If you think back, what are the ways that we've separated things so far?

37:04

Well, it's by running a liquid over a gas or running a gas through a liquid or a lot of surface area, right?

37:10

That's our simplest approach.

37:12

Then we decided, OK, instead of having one continuous column, what if I had create a whole bunch of surface area on individual plates and that's where we get our distillation columns from.

37:22

So really we're not that many incarnations of technology into thinking about how to move mass and move components between phases.

37:30

We invoke then Ralph's Law from good old thermodynamics to define the equilibrium content between  $Y_i$  and  $X_i$ , and that's where we get these K factors from.

37:40

So up until now we've been talking about, you know,  $y = .61 * X$ .

37:46

This is where that K factor is going to come from.

37:48

And usually Y and something simple like gas absorption, our operating line will be linear and not curve like we would.

37:56

So the key take home is that then we relate this vapor liquid equilibrium on a per component basis to what is the equilibrium distribution on each of our stages in the system.

38:10

So the more equilibrium stages we add to a column or to a system, right?

38:15

Because we can put these single stage flash vessels, one in contact with another in contact the 3rd and so on and so forth.

38:22

But we are taking each of our equilibrium stages and splitting that equilibrium equilibrium for each component.

38:29

So we can then also use this approach, this Rashford or S approach with K factors to either identify our bubble points or dew points in polyphases or multi component systems or to figure out the extent of vaporization, which is typically what chemical engineers will be targeting because it's the thing that that we need something like ISIS to solve for.

38:50

But as a take on when ISIS is running all of this, these calculations on a distillation call, it's actually solving ratchet rice in real time, right?

38:59

And it's just incorporating more accurate, more accurate.

## Lecture 24 - Introduction to Adsorption

0:00

The the next step going from a binary distillation column to multi component distillation and specifically where this plays a role for chemical engineers.

0:09

So as opposed to the other engineering disciplines, chemical engineering is known for its ability to separate multi component feedstocks.

0:18

And where this really comes into play is when we think about either a natural fossil fuel source or a naturally occurring fossil fuel source or a biofuel that we've synthetically manufactured from something like a recycled feedstock.

0:31

And however we have these, what we end up is usually with a distribution of hydrocarbons that we're looking to then separate into various products that are required for either additional feedstocks downstream in the chemical engineer or the engineering manufacturing realm or directly for sale.

0:51

And what separates the chemical engineer from any other discipline in engineering is that this process of multi component separation on a chemical level, we are the only ones who are uniquely qualified to do that.

1:04

So we talked in the last lecture and over the past couple weeks about the use of McCabe Healy diagram and certainly through laboratory #3 you would have gotten experience in how to apply that directly to the ethanol water feedstock.

1:19

And of course what you're learning in 3018 is then the the integration of that with when you start having multi component systems requires us to now use K factors and key components.

1:31

So heavy keys and light keys and so on and so forth.

1:34

The final bit that I really wanted to highlight and what will come up when you take advanced thermodynamics.

1:40

So CHPR.

1:41

4 four O 4, is that the Ratchford Rice equation, right.

1:45

This is now the solution to which a chemical engineer can use to marry up all of those K factors for the different components.

1:53

If we know the composition of the feed and determine at a set pressure and temperature, what the amount or the extent of flash vaporization will occur based on that feed composition.

2:05

So that's a really critical equation to be comfortable with.

2:08

It's not something I'm going to go very deep into in this unit, but it's one to be comfortable with.

2:12

So when it does come up in four O 4 O four, you're prepared for it.

2:17

OK, so let's go ahead and get started.

2:20

So I wanted to, before we go too far, I wanted to take a moment and just think back a little bit about some of the mass transfer unit operations that we've learned and in fact, the order that we've done them in and why we've done them in that order.

2:34

So we began with gas absorption with AB in weeks five and six of the semester, right?

2:39

And that of course had our packed column or we might have something like our ratchet rings, right?

2:47

Or even, right, our old school approach.

2:54

Let's put crushed stone in here, right?

2:58

So this is one of the original chemical engineering operations.

3:01

And so we could take a column, crush up a whole bunch of stone, have liquid flowing down from the top, gas coming up from the inner meter from the bottom.

3:09

And in the intermediary section, these two phases would transfer a component, right?

3:16

So we're using A2 phase system to transfer component and where chemical engineers have really made an impact over the past couple decades as in identifying and developing new packing materials.

3:30

But in many ways, because this column is operated with very minimal pressure drop, we've obviously learned the equations for that, you know, relatively little temperature change.

3:41

This is quite a simple unit operation.

3:44

Now in weeks seven and eight, we've gone one step further and said, OK, we're not going to be transferring mass from one phase to another through a column.

3:51

But what happens if we try to bring these two phases into equilibrium, right, if they're coming from a different equilibrium position?

4:00

And so the simplest approach that we looked at was a flash distillation column, right?

4:05

So that is one equilibrium stage.

4:11

Now in that process, this got a little bit more complex because now right, we have our shell and tube heat exchanger and This is why weeks four or three and four were so critical beforehand because we had to get some backgrounding around what these heat exchange processes might look like.

4:29



Now a flash distillation column that uses is drawn here using a shell and two heat exchanger.

4:34

But in fact what we learned in weeks three and four is that we could use a diversity in heat exchangers based on the material that we have available.

4:42

So if we're trying to recycle an energy stream, a shell and two might not be the most effective, but we can design this up in a number of ways and look at what would be most appropriate.

4:52

The critical point to take home is that a flash distillation column is looking at a massive pressure drop across a valve.

5:00

So our fluid comes in cold T low.

5:04

It heats up through the shell and tube system and by the time it exits that we have the temperature is increased.

5:12

And because we know temperature and pressure are directly related, the pressure of our process feed is going to increase as well.

5:19

And the principle of flash distillation when it's single stage is that we then take a massive pressure drop across that valve C.

5:28

So we are exploiting the component vapor liquid equilibrium in PXY space.

5:33

So if we go back to basic Thermo and we draw a PXY diagram here, what we're looking at is how what the width of that PXY phase envelope is going to look like and the the width at any given pressure and that we drop across it.

5:55

So if we're coming from from green, right if here is here is P initial and we drop down to a lower pressure, then that's going to determine the vapor liquid equilibrium split of that component.

6:13

Now that is of course the simplest method of doing distillation.

6:17

Once we go to multi stage distillation, now we have some N number of stages inside the column that are physical plates.

6:27

We have one stage that is our partial reboiler and we have either zero or one stage stages that are added that are added on top with a condenser.

6:43

So if we have a partial condenser and that's going to mean we bring that vapor back or the sorry the liquid back in, then that partial condenser will act as an equilibrium stage much like our partial reboiler acts as an equilibrium stage.

6:58

But if it's a complete condenser where we take the vapor stream liquefy 100%, then either pull off our reflux or take some distillate product that does not, that complete condenser does not act as an equilibrium stage.

7:12

And of course then this is the most complex unit operation we've looked at so far.

7:16

And it's one of the most complex in chemical engineering because we're employing both, both of our size of the heat transfer equation, but the condenser and our shell and tube say kettle type or boilers in a multi stage equilibrium column.

7:30

So each one of these stages is going to act right like this whole unit operation does with with a single stage flash.

7:42

The only difference here is that now you're exploiting TXY space.

7:47

And what you would have seen in your lab #3 reports is the ability to plot for each of the temperatures in this column, going from the warmest where the reboiler comes back in to the coldest at the top where we go out for condensation.

8:03

Each of these temperatures, we have a different VLE split.

8:06

All right, So if I draw here now my TXY diagram, and if I switch to green, I can see for each of these stages that comes in, I'm going to have a different split across that TXY space.

8:31

So for these three mass transfer operations, the reason that we have taught these to you first is that all three of these we operate or we try to operate on the basis of steady state.

8:41

And that's because when we as engineers sit down to solve the problem, of course the, the real solutions come from partial differential equations, usually first or second order.

8:53

Now those become very difficult to solve.

8:56

And so if we can make the assumption of steady state, it means that anything varying with time.

9:02

So the DDT terms go to 0 and of course then the solution becomes quite simple.

9:08

Now the reason that we have waited for gas absorption to the end of our mass transfer set is that it, although it is not conceptually more difficult, it does not work on the principle of steady state as we're going to see today.

9:24

So when we start wanting to transfer mass in the context of gas absorption, right, and we differentiate with the  $D$ , So adsorption means that on a molecular level, we have a, a molecule that's absorbing to some surface of interest.

9:41

So now in the column that we're going to talk about, this is another packed bed column.

9:46

And it, it has this similar kind of constraint to fluidization, right, as you might expect that we talked about back in the the absorption point.

9:56

But here, now we take our column and we have a fluid of interest, then it can be a gas or a liquid feed, and we're going to contact that with a porous solid that's placed in the column.

10:07

So when we think about absorption, which we've highlighted in purple, just to differentiate for a minute, when we think about absorption, the purpose of the solids in our column is actually to act as an enhancement for surface area.

10:20

So our liquid and vapor phases have more contact with each other, hoping to reach greater, well, hoping to come to equilibrium right within the column itself.

10:32

When we talk about gas adsorption, the second phase in the adsorption column is the solid itself.

10:40

So in fact, our adsorption column, if we have the same cylinder has only two phases, whatever fluid we're thinking about and our porous solids and that's it.

10:52

And these solids themselves will absorb the components from our fluid that we're targeting on a molecular level.

10:59

And so the process looks like this if this is our on the bottom left, our solid right.

11:05

So maybe this is the zeolite, maybe it's some kind of simple aluminum derivative.

11:11

If we have a component that we're trying to absorb from the vapor phase, say something like CO<sub>2</sub> here, what will happen is when we expose that solid, if we've selected correctly to CO<sub>2</sub>, maybe we have 20% CO<sub>2</sub> by mole in, in our vapor phase.

11:29

We want to get rid of that, right?

11:30

Because we don't want to be venting the CO<sub>2</sub> to the environment.

11:33

Well, now that solid is able to absorb the CO<sub>2</sub> to its surface and bind it, right?

11:39

So this is the process, the physical process.

11:53

Sorry, this is the physical process of absorption.

11:59

Now, when we think about this, the column itself is going to contain a fixed amount of solid.

12:06

Now that means that we have a fixed surface area and a fixed number of absorption sites.

12:13

Now, when we think about the number of absorption sites, we need to be very careful because we're talking about a molecular level here, right?

12:21

So even though the column might be something like 1m high, because these are very porous, very small solids, usually we're getting surface areas that are going to extend anywhere up to 500 to 1000 meters squared per gram of solid, right?

12:40

So very, very high surface area and you can imagine that each solid is going to have anywhere from say 5 to 50 potential absorption sites on it.

12:51

So the number of absorption sites in a column filled with material can be extraordinarily high.

12:57

But the point to take home is that those number of sites are fixed.

13:02

There are a finite quantity.

13:05

So once we start the physical process of absorption, the sites become filled up, right?

13:10

Because the solid if we've again, if we've selected it correctly, is not going to release that CO<sub>2</sub> unless we force it to.

13:19

So what happens is then this bed becomes saturated over time.

13:23

And the time dependence of this right means that we cannot apply a steady state assumption for balances in general, right?

13:34

And for the for chemical engineers, that goes, oh, oh, oh, right now we have to start thinking about transient processes.

13:43

And of course, chemical engineering is the only domain of engineering where we do ultimately learn how to solve partial differential equations with time.

13:54

And so the math behind gas absorption actually gets very tricky and is one of the reasons that ISIS itself cannot directly solve absorption problems.

14:06

But there is a specialty software that Aspen offers called Aspen Absorption specifically for this purpose.

14:15

So what happens is then over time, if we plot the concentration relative to the concentration of whatever component we're targeting in the from the vapor phase that's or say vapor phase in this example that's being injected into the column.

14:30

So we're normalizing it to the feedstock composition.

14:33

What will happen is if we're measuring the content of what comes out of our column as a function of time.

14:40

Well, initially from say T1 to about T3 in this this example diagram, initially we're not going to measure any effluent.

14:50

So if we're targeting CO<sub>2</sub>, we're not going to measure any effluent CO<sub>2</sub> coming out because the material that we've put into that column is absorbing every molecule of CO<sub>2</sub> it's touching.

15:01

However, at some amount of time, the absorption sites start to become filled up, meaning we have a lower and lower capacity for absorption.

15:12

And as a consequence, we are starting to release CO<sub>2</sub> into the effluent stream, right?

15:18

And this is going to pick up with time, right?

15:22

And this is what we call an S curve now.

15:29

It's an S curve, right, Because it's not following a linear trend.

15:33

And we're going to get into some of the examples of the types of absorption that we can follow here, but it's following an S curve because of that partial differential relation, right?

15:43

And that's what's governing that type of behaviour.

15:46

So once this happens, what the engineer will do is switch the feed fluid to a second bed where the adsorption sites are brand new, right.

15:54

So we have a second bed of new material.

15:56

Once the first bed becomes saturated and once we switch that feedstock to the second bed, then we begin the process of either replacing physically or preferentially we want to regenerate the adsorption sites in our first bed.

16:12

So this is what this process looks like.

16:15

So this is an industrial, industrial example shown here.

16:25

And here we're looking at on the bottom, this is a simplified diagram where we're assuming vapor phase absorbance, we're using a gas.

16:32

Again, assume that we're talking about methane containing CO<sub>2</sub> or air containing CO<sub>2</sub>.

16:37

So it's a gas type absorption setup that we're trying to strip the CO<sub>2</sub> out of the air instead of venting it to the environment.

16:44

So in these types of columns, it's typical pack particles again up to about 1m deep.

16:49

And what we would typically do for is have a plate here, right, a plate with perforations to support that packing material.

17:03

Now if we're using a liquid, we would certainly have to downflow this.

17:09

But if we're using a gas, we have the option of either using upflow of the gas or downflow because again, that's the only fluid phase present.

17:17

We tend to prefer downflow in most cases simply because if you can imagine if the gas or certainly if it's a liquid is flowing vertically upward, well that's going to increase

the risk that we're going to fluidize that bed and potentially start to lose fines material or lose some of our absorption, absorption sites.

17:38

So we then measure, right, if we have our feedstock coming in the top here, it's going to come through and all the color you can imagine, this is now filled with a whole bunch of say, zeolite material that's going to be absorbing that CO<sub>2</sub>.

17:54

And then we have the rest of our air flowing outward with no CO<sub>2</sub> in it.

17:57

And what we do is we take a measurement and if CO<sub>2</sub> is our example, we'll take a measurement of CO<sub>2</sub> at that outlook point and we want to measure that with time.

18:07

So once we start measuring that, it reaches saturation.

18:11

Now imagine that this is all being done at a relatively constant temperature.

18:14

Assume it's room lab temperature.

18:18

Well, once we start detecting CO<sub>2</sub> in that effluent stream, we might run it for a few additional minutes to finish packing out that absorption or those absorption sites.

18:29

But we're going to shut it off pretty quickly and we're instead going to direct the feed, which I'll draw on here.

18:35

We're going to direct the feed to the second absorption column, all right, which is itself containing now an additional amount of Co light.

18:42

That's that's brand new.

18:44

Now this absorption process is going to start again, right?

18:48

And we're going to have some amount of time where we don't detect any CO<sub>2</sub> in the effluent stream.

18:53

But we have this problem.



18:53

This first column is now packed, right?

18:56

All of those sites are absorbed of CO<sub>2</sub> and we need to regenerate that.

19:01

Well, typically what we're going to look at is, is temperature swing absorption, right?

19:06

And that's the process that we're focusing on here.

19:09

So this means that the adsorbent material, so the zeolite that we've chosen, the binding potential of those molecules to the surface is a temperature dependent physical phenomenon.

19:22

So if we use something like steam that's and we insert counter injecting steam into this column, they want to take the column temperature from say 21°C, so lab temperature up to 100°C.

19:36

If it's a temperature dependent absorption process, that means that that in that the CO<sub>2</sub> will desorb from the surface, right?

19:46

And that is a temperature, temperature swing absorption example.

19:52

So then what we have is a coming out the top, we have our steam that's containing a lot of CO<sub>2</sub>, right?

19:59

It's released all of the CO<sub>2</sub> from this absorbing site.

20:05

Now the example of steam in CO<sub>2</sub> is actually quite poor because CO<sub>2</sub> is going to be soluble in water as we go to then recondense.

20:12

All right.

20:13

So we're going to recondense this usually using like a cold water condenser, and we're going to have our fresh water coming out the bottom, our vent vapor and whatever solvent we might have present if we were using CO<sub>2</sub> as an example.

20:26

In fact, we don't want to use steam because we have a lot of that.

20:28

CO<sub>2</sub> is soluble in the effluent water stream that we have produced.

20:31

But instead we want to use a hot gas, right, Some inert gas, maybe it's really hot nitrogen that can warm that zeolite up.

20:40

But as we condense it down, we'll have be able to capture the ZO<sub>2</sub>.

20:47

OK.

20:47

So some of the practical examples of this, if we want to remove toxic or unwanted components from our process stream, this might be organic.

20:56

So hydrocarbon based solvents that would be present in paints, inks, solutions, fabrics.

21:02

As I mentioned a couple days ago, you know 2/3 of the materials that we use every day from fabric dyes all the way through Tupperware, through naming is a hydrocarbon derivative.

21:14

So we have a lot of hydrocarbon solvents that are present throughout our life cycle streams.

21:20

So it's not uncommon that we want to remove these from a process stream, and absorption is a very common way to do that.

21:28

It's also very effective at pollutants like H<sub>2</sub>S and CO<sub>2</sub> that might be present in vapor phase systems.

21:35

And we can also use it to purify drinking water if there are specific contaminants in that.

21:41

I think where we're going to see absorption, you know, since you're beginning your

engineering careers now, where you're going to see absorption taken up to quite a lot is around the ability to eliminate waste and reduce emissions.

21:54

So making processes more sustainable and more efficient.

21:58

And a really good example of this is a UW, A process that was developed about five years ago and is now patented.

22:06

And there are some companies working on it both in China and Australia around the use of absorbent material to reconcentrate very low purity natural gas.

22:18

So methane containing CO<sub>2</sub> that has been in China normally vented to the atmosphere, right?

22:25

And of course, methane has 100 times greater greenhouse warming potential than CO<sub>2</sub>.

22:29

So venting methane is a very, very bad thing to do from a greenhouse emissions standpoint.

22:35

Well, this absorbent material developed at EWA allows these companies to take what might be 30 mole percent methane, right?

22:44

That's not high enough methane content to combust.

22:48

And that's what they've been venting.

22:49

It allows them to put it through an absorption column, capture the impurities and produce up to 80 and 80 to 90% of pure methane, which they can actually use to provide power as opposed to venting the entire thing to the atmosphere.

23:05

So this is a very much an emerging technology that there's a lot of work going on and it's all around what are the solids that we can use to capture this material and then of course, dehydration, right?

23:19

So you'll commonly hear older engineers talk about mole sieves or molecular sieves.

23:25

And this is where we use zeolites to remove water from a vapour phase.

23:30

So a mole sieve dehydration is a very common practice, certainly in the natural gas industry to capture that water content.

23:39

But we can also use the same zeolite based solids to separate oxygen and nitrogen.

23:45

So certainly once we get through the basics of absorption in the next couple lectures, then in the following week we'll kick into more discussion around absorption materials and what those development processes look.

23:58

So to think about how we're going to characterize absorption, right?

24:03

We can't really use our steady state mass and heat under heat and mass balances because they're not steady state, right?

24:10

This process is not steady state anymore.

24:12

It's transient and this presents a problem for us.

24:16

So at the first instance, the way we're going to characterize this is through the use of an isotherm, right?

24:21

What an isotherm means, right?

24:23

They mean constant temperature.

24:25

So the column is operating at some constant low temperature where we can plot here if we think about a bed length from zero to a total length  $L$  that at time one when we first begin injecting our gas again, let's say it's air containing  $\text{CO}_2$ , we inject that into the column.

24:46

Well, if we measure that concentration profile, so this is concentration of  $\text{CO}_2$  relative to the concentration in the feedstock.

24:55

So say if we have 20 mole percent in the feedstock, then a value of 1 on this ordinate means 20% mole or 20 mole percent, whereas .5 will mean 10 mole percent.

25:08

So initially at time one, we have very clean zeolite in this this packing material and we're going to see that concentration profile fall off, meaning all of that material is captured right in the first block of the column, maybe the first third of its length and the last 2/3 is doing no work, right?

25:28

But very quickly, right.

25:30

If we look at here by time 2, right, which might be a couple minutes later.

25:36

In fact, that concentration doesn't begin to fall off until about 10% of the way into the column.

25:44

So that means the 1st 10% of the bed is doing no work because all of those absorption sites are taken up already.

25:51

So at time 2, now that fall off curve only begins to happen after a few minutes.

25:57

And what we can do is instead of looking at the concentration of the CO<sub>2</sub> or whatever our target component is in the solid itself, what we instead plot is what is the concentration in equilibrium between our liquid or vapor, fluid phase liquid or vapor and the solid.

26:16

And in fact, when we start plotting that, and we'll get into the next lecture, this is the driving force that we're now thinking, all right?

26:24

So this is our driving force for adsorption, all right?

26:27

And this process goes on and on until we notice at time 4.

26:32

Now this is where at the end of the column, uh oh, right, We don't have a zero percent CO<sub>2</sub> leaving.

26:39

We have some measurable amount of CO<sub>2</sub> that's being ejected from the column.

26:44

Now if instead of plotting this as a function of length with at different times and just for reference, this is what we would call a profile plot, right?

26:54

Because it's a profile of the column where we take different time snapshots on the right hand side.

27:00

This is a trend plot.

27:04

We're looking at a temporal distribution at one point physically in the column or multiple points physically if you want.

27:11

Now what we're representing here is a trend plot of the effluent stream.

27:16

So it's at the very end of the column.

27:19

So this is a trend plot at  $L$  sub  $T$  where we can see again up through that beginning of breakthrough, which we call  $T$  sub  $B$  at the beginning of breakthrough this and this is a value of about 5% at the beginning of breakthroughs where we start to detect CO<sub>2</sub> rising, right?

27:40

Here's that  $T_4$  that lines up.

27:42

And then at some time we as engineers are going to decide, OK, now the process is not efficient enough and we're going to flip the stream over and feed it to the second column.

27:52

So what we're going to get into in the next lecture is actually the maths around how we characterize it and calculate this.

27:58

But it's important to then understand this mass transfer zone, right, which is going to be a concentration range of between 5 and 95% of this  $C$  on  $C$ , not value.

28:12

It moves, the mass transfer zone moves from the column inlet to the column outlet.

28:17

All right.

28:18

And this is where this S shaped curve in our trend plot comes from.

28:23

OK, so with that, we'll wrap up our introductory electron gas absorption.

28:27

So the take home message here, we're no longer at steady state, right?

28:31

We've left Oz, we're not Nas on it anymore.

28:34

And and now we need to think about transient processes.

28:37

And This is why we save it to the very end, because the maths behind actually doing this can get quite difficult, right?

28:43

So we're going to only look at the, the simplistic framework of how to design it, but this is certainly one of the most elaborate math solutions for a mass transfer process.

28:55

And it's, it's really the, the selection and the technology of the poorest song that we use.

29:01

That is the, the trajectory of the field for gas absorption.

29:04

That's where all the work is going on at present.

29:06

And there's quite a lot worthwhile.

29:09

OK.

29:10

So the when we want to, I think that the last point to highlight here is when we want to regenerate these beds, right?

29:16

We're talking about temperature swing absorption, meaning we're going to go from a

low temperature, we're going to heat that bed up and in the process of heating it, we're going to desorb, all right?

29:26

So we can go the other way if we increase the temperature, right?

29:34

Whereas this absorption process I'll call low temperature, right.

29:43

So in a temperature swing absorption, we're exploiting the the fact that the physical process of absorption depends on temperature to be effective.

29:52

Now we can also look at pressure swing absorption where that that material is chosen to be more sensitive to the working pressure of the column than it is it's absolute temperature.

30:05

And in some ways, that can be a more effective method, but it also assumes that we have access to compression equipment.

30:13

So both of these methods are used.

30:15

The one that we're going to focus on here is temperature swing absorption.

30:18

But again, we'll start getting into that and unpacking that over the coming couple lectures.

30:23

OK.

30:23

So if you haven't had a chance yet, certainly watch the laboratory number.

30:29

Well, the 4th unit operations lab, but I think it's workshop number 7.

30:35

And yeah, let us know if you have any or, sorry, workshop #8 let us know if you have any questions.

30:41

Now, we will not be having any lectures next week, so I'll send cancellation notices shortly because of course, it's study break and we're not going to have any work during study break.



30:50

So we're going to push that, that 4th lab on, on gas absorption due date out to the 13th of October at 9:00 AM.

30:56

Let me just triple check that sort of this quote.

31:00

Yep, 13th of October at 9:00 AM, 2-3 weeks from now before it's due.

31:05

And as David and Angus went through this morning, it should be relatively straightforward and there will not be a ISIS component for this because ISIS cannot capture adsorption.

31:15

So wonderful.

31:16

Have a good Tuesday everyone, and we will talk to you tomorrow.

## Lecture 25 - Adsorption Methods

0:00

Pen tool, there we go.

0:00

In Lecture 24, we talked about the introduction of absorption and the reason why we bring this as the last unit operation that we're going to go through when we talk about mass transfer, or rather I should say the last of the three experimental opportunities.

0:16

And the reason why is that the process of absorption is actually quite simple relative to something like a distillation column.

0:24

But the mathematics of it is quite difficult, right?

0:27

And that's because we're operating now with a transient system.

0:30

So we're no longer able to invoke that assumption of steady state, which allows us to both take our time differentials, so the DDT turns to 0 and subsequently simplify the residual or the other differentials in the system, things like concentration differences.

0:48

So we're going to look at later in the lecture today where these come into play and

what the complexity of some of those solutions look like and what the more advanced versions of chemical engineering get into.

1:01

So the principle in the process of absorption itself is actually, as I said, quite simple.

1:07

Instead of having a column where we have two fluid phases that are interchanging material to come to the vapor liquid equilibrium, Now the two phases involved are simply one fluid and that can be a liquid or a vapor and one solid.

1:19

And that's our absorbent.

1:23

So the selection of picking the right kind of absorb, it's solid is critical to informing how much of our target component we can remove.

1:33

And we cannot, as I said earlier, represent this process using a steady state balance.

1:39

So let's think about what happens and kind of review the process of absorption.

1:43

What initially will be down on the bottom here.

1:46

This is our our adsorbent material.

1:51

So that might be something like a silica bed.

1:54

It could be an activated carbon.

1:56

And we have some molecule here.

1:58

This might be CO<sub>2</sub> right up in the vapor phase or the liquid phase that we're trying to absorb from our fluid to that adsorbent solid.

2:06

And so the process of adsorption is one of having those solids go from the fluid to be bound right.

2:15

And this is usually a phys absorption or a chem absorption process, but they will be bound to that absorbent solid.

2:22

And so over time then these beds that will initially be filled with a vacant solid are going to all of those little porous sites are going to pack up and be filled with whatever target component we're trying to absorb.

2:35

So that means if we plot something like here, we're looking at the concentration of the effluent stream relative to the concentration at the feed inlets or feed inlet would be  $C_{in}$  and our measured effluent would be  $C$ .

2:49

So the concentration of the outlook that we measure relative to what we know is coming in, we can plot this over time and initially see from  $T_1$  to  $T_3$ , say we have a period where all of that material is being absorbed to our solid.

3:04

And then at some point we have a breakthrough and we're going to talk more about that today.

3:08

And we start detecting this target material in our effluent stream, which means we now need to switch beds, right?

3:15

So we go from a bed that's becoming saturated to a second bed that's clean, right, filled with the same solid but has no absorbent material on it or no target material on the absorbent.

3:29

And that means we then need to regenerate our initial bed.

3:31

And we'll go through a little on the next slide what that regeneration process looks like.

3:37

So when we're thinking about how we regenerate, there are two basic methods that we can use when we talk about absorption.

3:45

One is temperature swing absorption or TSA, and one is pressure swing absorption or PSA.

3:52

So temperature swing absorption is what you're using.

3:54

And this will be in laboratory #4 All right, So that's what's being used in the laboratory.

4:02

And the idea is that absorption is favored by low temperatures and or high pressures.

4:24

So if I have a column that's initially at, say, ambient pressure and room temperature, right, that will be one in which I'm going to be favorable to absorb to the Saphora solid that I've nominated.

4:38

But once that solid packs up and the bed becomes filled, to regenerate it, if I don't have control over pressure, I'm going to have to increase the temperature of the system.

4:48

So that's where we're going to use either steam or an inert hot gas in a temperature swing absorption system to push that warm fluid through our saturated bed.

4:59

And because we are increasing the temperature, we're no longer favoring absorption, we're favoring desorption.

5:06

So you imagine if I had done that surface, right, here's my absorption surface.

5:11

Well, if I have some, I'm just going to draw a spherical molecule here.

5:15

But if that is normally bound to the surface, you imagine it's it's bound because of the kinetic energy of that molecule is quite low or because the pressure, the energy applied volumetrically above it is quite high.

5:32

So there are two ways that I can get that molecule off the surface to regenerate my solid.

5:37

And that's either I'm going to increase the temperature of the system.

5:40

So I'm giving I'm adding kinetic energy to the molecule so it pops off the surface.

5:53

Or if I'm using pressure in the system to hold that molecule at the surface, then as I release the pressure of the system, there's less of a driving force in the to keep it at the surface in The molecule will then desorb back into the fluid of interest.

6:08

So these are the two basic mechanisms that I have.

6:11

If I'm using a temperature swing absorption, that's typically what I'm going to employ for targeting low concentrations of materials.

6:19

So that might be, for instance, if I have say 5 to 10 mole percent of some contaminant or some ecotoxic species I'm trying to get rid of, I can use a temperature swing absorption in that context.

6:32

And these beds tend to be relatively small, so maybe 0.3 to 1 meter in height and a typical cycle time, right?

6:40

So how long are we going to run this before I hit saturation and I need to switch beds and start regenerating my first bed?

6:46

We'll be anywhere from 2 to 24 hours.

6:49

So we're on the time scale of within a day.

6:52

Now pressure swing absorption is designed to work at much, much higher concentrations.

6:58

So that's where we maybe have 30 to 50 mole percent of either a contaminant or a product that we're trying to use.

7:06

And that's we consider then a bulk separation now is the example we talked about last time of one of the UWA technologies that's now up in China, is that some of it, the Chinese coal seam gas tends to have very, very low methane content, say 5 to 10 mole percent, right?

7:26

So what has typically been happening is all that methane is being vented to the atmosphere.

7:31

Well, part of what UWA chemical engineering researchers have done is to put together and design a new absorbent material in a pressure swing absorption system that allows it builds quite large vessels and allows these companies to take that five to 10 mole percent of CO<sub>2</sub> and absorb it to this material.

7:53

So that when we desorb we have a stream that say 80 to 90% and that can then be combusted for electricity using a conventional gas fired turbine.

8:04

So what it means is that we're able to use a very large scale process to take something that was being vented right as a very significant greenhouse gas and turn that into a product that can be used for electricity generation.

8:20

So when we start thinking that about how we characterize adsorption and we referenced, we introduced in the last lecture the idea of adsorption isotherms.

8:30

So in general, we can look on the right hand side the way we characterize adsorption and we talk about an isotherm because typically if we're thinking temperature swing absorption, then this is all being done at a relatively constant temperature.

8:44

So what we want to do is relate the concentration and as I should say is concentration of our target species in the fluid versus the solid.

8:59

So if I'm thinking about a gas, right, then we typically report that in concentration.

9:03

And this can also be done as a partial pressure where if I'm in a liquid, I'll typically use parts per million, which is equivalent to milligrams or Megs per liter and or parts per billion, which is equivalent to micrograms or Mics per litre.

9:23

And so in when we're talking about liquids, we typically think about PPM and PPB concentrations because we're trying to pull out a highly ecotoxic or or you know, poor environmental species.

9:37

So it could be high equitoxicity, high bioaccumulation or low biodegradability.

9:43

But for some reason we have a very minute stream of this and we need to pull that out before we can release materials to the environment.

9:50

Now in this context, we're going to be plotting now on the right hand side, this is the West.

9:56

So the characteristic loading, and we think about loading in terms of the grams of material absorbed per gram of solid, right?

10:07

So if I'm absorbing, let's say I'm trying to pull CO<sub>2</sub> out of my gas phase, if I'm absorbing 1 gram of CO<sub>2</sub> per gram of adsorbent solid or activated carbon in the system, right, then that W value equals one.

10:23

And when we think about adsorption isotherms, then we characterize this as a function of the concentration of my target material.

10:32

So that might be CO<sub>2</sub> if I'm trying to absorb out of a gas phase, If I'm thinking about something like a maybe a industry surfactant, So highly toxic soap species that's used in a lot of industrial processes, those will typically show up in parts per million to parts per billion, but I cannot release them to the environment because they're quite damaging.

10:54

So in that context, I need to then think about, well, how much can I pull out relative to the size and the amount of material in my column?

11:03

So in general, there are four classes of absorption isotherms we can think about, and I'm going to highlight these.

11:12

First, the irreversible absorption, right?

11:15

This is shown on the top, and it's a horizontal line.

11:19

So let's think about what does that mean?

11:22

Well, that means that if I have a very low concentration of target material and I'll just keep using CO<sub>2</sub> for the example.

11:30

But it means if I have a very low concentration of material, I can absorb the same amount on my solid, the same 15 grams per 15 grams, right?

11:43

Or 15 grams of CO<sub>2</sub> per 1g of CO<sub>2</sub>.

11:45

If I pick a better solid, I can absorb the same amount at a low concentration of CO<sub>2</sub> in the system as I can at a high concentration of CO<sub>2</sub> in the system, right?

11:55

So it's independent of the amount of material I'm putting at it.

11:59

It's saying that this absorb material I've selected is going to grab these molecules at pretty much any concentration.

12:06

And so we talk about irreversible adsorption because it's an ideal framework, right?

12:12

It's a model framework for us to use in our minds.

12:15

Now the other model framework we can think about is linear adsorption, right?

12:20

And this makes sense.

12:21

This is again a benchmark model that we'll think about.

12:25

As I have more and more CO<sub>2</sub> present in the system, I'm able to adsorb more and more CO<sub>2</sub> to my solid.

12:34

Now on either side of this, I can see two.

12:38

If I look at that linear adsorption, I can either have a solid that's being selected that's worse than that or that's better than that.

12:47

So if I'm worse that I'm going to be in an unfavorable absorption region.

12:53

And So what this means if I think about two points here, I'm able to absorb less material at a low concentration than I am at a high concentration.

13:10



So if I have 5 percent CO<sub>2</sub> versus 20 percent CO<sub>2</sub> in my inlet stream, the higher the concentration of CO<sub>2</sub>, the more effective my absorbent material will be added at absorbing it, right?

13:23

But at the end of the day, that's an unfavorable kind of region because it means the the solid, the porous solid itself doesn't want that CO<sub>2</sub> or doesn't want my target material.

13:35

So it's only accepting it because I'm giving it a higher and higher concentration that eventually it can't ignore.

13:41

All right.

13:42

So the way I would think about this when I say unfavorable is that we're ultimately talking about this region down here, right?

13:49

When I introduce a little bit of CO<sub>2</sub> to the system, the absorbent solids like, Nah, I don't want any of that, right?

13:57

And I have to really drive that concentration up before it's able to bring it into the solid.

14:04

Now, conversely, if I'm sitting above that linear region, I can have favorable or strongly favorable.

14:11

This means if I think again about that initial pitch, well, the second I introduce a little bit of CO<sub>2</sub> to that absorbent solid, it's going to freak out and grab everything it can because it really wants to have the CO<sub>2</sub> absorbed.

14:26

So that's a favorable absorption region because the solid wants what I'm offering it.

14:33

So these are the four characteristic regions, irreversible and linear represent kind of our idealized models.

14:40

And then we can be either favorable or unfavorable in a practical engineering context.

14:45

Now obviously engineers wouldn't use an unfavorable material.

14:49

There's not really an argument for that.

14:50

So we tend to to always work in our favorable region, right?

14:54

So we need to pick a porous solid that aligns well with the type of material I'm trying to absorb.

15:03

So we think, well, when you talk about these isotherms, then how do I actually plot this out and characterize it if I'm an engineer designing a system like this?

15:11

Well, if I have a favorable adsorption, not strongly favorable, but just favorable, so I'm above a linear region, my solid wants the material I'm offering it.

15:21

The characteristic isotherm, and this is one to certainly know is the Langmuir isotherm, right?

15:27

And you're going to see this start to show up in a reaction kinetics.

15:30

You're going to see the context of the linear isotherm propping up in multiple units from here on out.

15:37

And what this tells us is that the  $W$  at any point, so the amount of material that I can absorb relative to  $W_{\text{Max}}$ , which is the maximum loading that I can achieve, right?

15:51

So if I take every site on that porous solid and I pack it, what is the maximum packing I can achieve?

15:59

And this  $W_{\text{Max}}$ ?

16:04

This is defined as concentration goes to Infinity so that the amount of material relative to the maximum material is equal to.

16:15

And what makes this the line your isotherm is I have  $K$  times the concentration divided by  $1 + K$  times the concentration, right?

16:23

And you're going to learn in kinetics in the next year the nature of what type of reaction this is going to be or the type of kinetic relationship.

16:34

Now the Langmuir isotherm is by far the simplest.

16:37

And the reason that we like it is that it only employs 1 empirical constant.

16:41

We call the Langmuir adsorption constant  $K$  and that constant is system dependent.

16:47

So it varies with the porous solid, the material I'm absorbing, the operating pressure and temperature.

16:53

It is a typically something we'll determine empirically and then we can extend that linear isotherm to think about what happens if I don't have one component but I have two 3-4.

17:05

Well, we like this because we had simply the constant for component one times the concentration of component 1, right?

17:14

And then I normalize that on one plus  $K_1C_1$  plus the constant component 2 times its concentration.

17:21

And I can extend that out for an  $N$  component system.

17:26

So the Langmuir isotherm is something we're going to use for favorable absorption systems.

17:31

If I go to a strongly favorable region, right?

17:34

You can see here then I need to really account for that initial curvature because the 2nd, I have a few molecules flowing by that solid, they're going to be picked up.

17:43

And so in this context, we can use the Fremlich equation, which says the West, the amount that will be absorbed is a function of concentration.

17:52

But I now have two specific absorption constants coming into play, one that's multiplying out front and one that takes that concentration to a power, right?

18:03

So B&M in this context are just the two absorption constants.

18:08

Now, you don't need this characterization necessarily for your laboratory exercise, but it's one that I wanted you to be familiar with because in particular, the nature of looking at in the Langmuir system,  $K$  on  $1 + K$ , right, as a function of concentration versus  $B * C$  to the power of  $M$ , These are two different types of kinetics that we can look at, right?

18:34

And so this is going to come back into play when you start doing more advanced kinetic work.

18:40

So how does this play out in in real life, right?

18:43

Well, let's think about trying to desiccate air.

18:46

So if if we have a a lot of water vapor in our air and we're going to characterize this as engineers thinking about relative humidity.

18:53

So this is a plot that comes from The Cave where we're going to look at relative humidity as an ambient pressure from 100% to down to 0%.

19:01

And this is over a temperature range of 20 to 50°C.

19:05

So, right, because the relative humidity, so the saturation of water in air is inversely proportional to temperature.

19:15

I can also then realize that I have a 50° condition here and 0% relative humidity and a 20° C condition at 100% relative humidity, right?

19:28

So as I get warmer, it's difficult, it's more and more difficult to keep water vapor in air.

19:36

So what McCabe is showing us here then is that same  $W$  value and that's pounds of water per 100 lbs of dry solid that's being used.

19:48

And we use just say dry solid because we want to compare three different absorbent materials.

19:53

So the first is aluminum, the 2nd is a molecular sieve.

19:58

So this might be something like activated carbon.

20:01

And what we're seeing here is a four angstrom pore size and that's going to come back in a few minutes.

20:07

And on the top, the silica gel.

20:10

So there are a couple regions we want to point out here.

20:13

Let's first look at the silica gel amount.

20:16

So the loading of the silica gel  $W$  as a function of the relative humidity, right?

20:22

So this is meaning the amount of water vapor in air.

20:26

Well, what I see is the silica gel is actually linear or more or less linear up through about 50% humidity, right?

20:36

And it's only very high humidities that I'm going to see that silica gel start to tail off.

20:43

So I'm not able to get a higher and higher capacity.

20:45

It's not linear through the entire region, but rather I can use silica gel if I'm at low

relative humidities perhaps like we typically stand for to pull that water vapor out of the air.

20:57

Now if I go to somewhere like Brisbane, I cannot use silica gel, right, because we're going to be talking relative humidities of 80 to 100% on most days and the temperature is certainly up to 40 or 50° C.

21:10

Can't use silica gel anymore because it's not going to become an effective material.

21:15

Now one point to take home is that if I want to think about the capacity of my material, so that's the W Max, right?

21:26

I can see silica gel tails off at about 40 lbs of water per 100 LB of dry solid where my moles is right and the aluminum are both tailing off between say 19 and 2223 lbs.

21:42

So the silica gel ultimately has a much higher capacity or W Max than the other materials when I look at this mole.

21:52

So I'm not going to go through alumina in detail because it's basically a comparison point for silica gel as a poorer vestigant, right?

22:00

So looking at this isotherm, I can see if I have to pick between silica gel and alumina, I'm always going to have a better bang for my buck thinking or buying silica gel in this context.

22:11

Now, the molecular sieve, this is an interesting case, right?

22:15

Because now what I see here, if I highlight this in green, I have very, very little change in the amount of water that I can drive into that solid as a function of concentration.

22:27

So this is an exemplar of my irreversible absorption case.

22:34

It's not perfect, right, because I have a bit of a tail off of low relative humidity, but the behavior is generally constant.

22:41

And we think about now if I have molecular sieve that has a pore size of four

angstroms, part of the reason why it's relatively constant is that those little pore spaces are going to hold water very, very tightly, right?

22:56

And by type, I mean, if I think that, right, So here I'll draw some solid, right?

23:03

And here it's going to be a little pore in there.

23:06

It has a four angstrom diameter, right?

23:12

There we go A4 angstrom diameter.

23:14

Now when I start to think about the effect of what's called Laplace pressure, once I start to create a small volume.

23:23

So this is going to be water that's driving, water that's driving into that little porous space.

23:33

Once that porous space goes below about one Micron, right, once we go down certainly to the nanometer and absolutely to the angstrom length scale, the effective pressure inside of that pore increases geometrically.

23:47

So if I'm at a pore size of say 20 microns, then in reality the pressure of the fluid inside of that pore is going to be pretty much equal to the pressure of the bulk system.

24:00

But as I decrease the pore size, let's say my bulk system is operating at 2 bar by the 20 Micron pore, then that's going to be at about two bar the fluid inside of that.

24:11

But as I decrease that pore size, once I go below 1 Micron, the pressure in the pore is greater than the pressure outside.

24:19

So if my pressure outside is 2 bar, once I decrease below 1 Micron, it's going to be perhaps 2.1 bar inside the pore.

24:27

Once I get down to the angstrom length scale, that's where I start to see pressures of hundreds of bar inside the pore.

24:34

So the ability then to pull that material back out, I require a lot of energy to do this as a process.

24:43

And this also underscores then the point that as I go to more of an idealized irreversible absorption system, I'm going to have to pump in a lot of energy to reverse that process, right?

24:58

Because this material is absolutely going to want to hold on to that material, right?

25:04

So this discussion around Laplace pressure, this is the explanation as to why in a molecular sieve of something like activated carbon we would expect that kind of behavior.

25:13

But where do we see this crop up?

25:16

Well, mole sieves are and we call them mole sieves for short are very common for high temperature industrial processes, right?

25:23

And one of the most common examples is if we're trying to to flow natural gas.

25:29

So in the Danbury or Bunbury to to damp your pipeline, there's a requirement for how much water.

25:39

So the relative humidity in that natural gas that's moving between these two locations for two reasons.

25:45

One is that if I have too much water content, then there will always be a little bit of CO<sub>2</sub> in the gas phase.

25:52

That water can react with the CO<sub>2</sub> form of carbonic acid and start eroding or corroding my pipeline.

26:00

But the second is that if I get into very cold regions, I can form a gas hydrate or a plaster of hydrate.



26:05

You'll start to learn about this in Advanced Dynamics.

26:08

And certainly if you take CHPR 5521, we go quite a bit into corrosion and hydrate processes.

26:16

So for that reason, it's very important when I'm moving natural gas in through any location, right?

26:22

And this is certainly relevant to the discussion going on right now about building a gas pipeline from West to east.

26:28

Mold sieves will absolutely be the preferred technology to remove water content from that gas because it is a very delicate operation.

26:37

And I need to make sure that if I'm operating my system at say 50-60 to 50° C would be not uncommon.

26:45

Certainly if it's going to go West to east, it's going to get a lot hotter than that as it goes through the middle, right.

26:52

It's critical that whatever technology I'm using, it can heat up to 50-60 degrees C and still be highly effective at capturing any molecule of water that runs through it.

27:03

So for that reason, This is why a lot of natural gas companies employ molse as a preferred method.

27:10

OK.

27:11

So in the context of our adsorption column, now there are a couple characteristic points that we can draw and of course the lab prompt that David and Angus put together.

27:23

You know what you're noticing, I hope by now is that the absorption lab is not going to have a HISIS component with it.

27:30

And that's because HYSIS can't solve this type of unit operation.

27:36

It is too complex for HYSIS because HYSIS is a steady state engine.

27:41

There is a tool called Aspen absorption and we're going to get into to what are the mathematics driving that on the next couple slides.

27:49

But the laboratory for then what you're going to see is that it's relatively straightforward because we're basically taking experimental data and there's only so many operations that we can do with that to try to characterize the system because we simply don't have the kind of mathematics to predict from fundamental theory what an absorption column should do and then to compare that.

28:16

So there are two characteristic points in an absorption column that I need to think about.

28:22

One is the mass transfer zone, the other is the point of breakthrough.

28:27

So the mass transfer zone or MTZ, right?

28:30

This is the region in which I'm picking up a majority of that material, right?

28:35

So you'll remember in the last lecture showing those different curves in time where the mass transfer zone was moving down the column, right?

28:43

And we typically would identify if we plot concentration of our effluent versus the concentration of the inlet  $C_{naughty}$ .

28:50

The mass transfer zone is wherever I'm picking up material from 5 to 95%, right?

28:58

So if I plot, I'll plot down here on the bottom  $C$  on  $C_{naughty}$ , right as a function of bed length, right?

29:10

And I can see that fall off.

29:16

These are my 3 temporal curves.

29:17

So I have T1T2 and T3 and I'm characterizing that mass transfer zone from 95% to 5%.

29:31

So if I think about then integrating this, so I'll do this on that.

29:36

The middle one, I can numerically integrate to figure out how much mass of absorbing or of my target component, maybe CO<sub>2</sub>, how much mass is being absorbed in my mass transfer zone.

29:52

All right, So if I plot this in time space, then and then we'll go back up on top.

29:59

Now, if I'm plotting this in a time space, one of the characteristic points that we need to look at as the engineer is where we're going to breakthrough.

30:08

So where does my effluent string leaving the column start to contain too much of my target material and I now need to switch to my secondary column.

30:18

And as a heuristic or a rule of thumb, this would be between 5 and 10% of that C on C, not value.

30:26

So let's be conservative engineers and take 5% is the threshold we use.

30:31

So on this plot, we can identify the breakthrough point then, right is where you're going to be meeting this target of 5% and that's T sub V.

30:43

Now we're going to come back to in a minute, but it's also once I hit that right, that's when I moved to my second bed.

30:51

Now if I think about when is this breakthrough going to happen as a function of the width or the height of the mass transfer zone.

30:59

So if I have a very large mass transfer zone or a wide mass transfer zone relative to the column height, right?

31:08

So if I think about the ratio, the mass transfer zone might be, oh, .8 meters.

31:14

And if that's on a 1m column, I have a really wide mass transfer zone because it means pretty early on in the process I'm going to hit that breakthrough point, right?

31:24

And I'm going to need to switch to the second column, whereas if I have a very narrow mass transfer zone, maybe that's .1 meters on a 1m column, well then I can operate it for some time with absolutely no of my target material, again using CO<sub>2</sub> as the exemplar, no target material leaving that column.

31:43

And it's only after some time that we hit our breakthrough criteria.

31:47

So as engineers, we want to select materials and design the column such that we have as narrow of a mass transfer zone as possible.

31:56

That's one of our optimization criteria.

31:59

Now when we think then about how much material can be absorbed in the column in total, right?

32:05

And this is to connect C on C naughty, right?

32:09

If I numerically integrate, so I have a value of one on top if I numerically integrate.

32:16

So that's this hashed area shown here that tells me the total amount of material right now C on C notes are ratios.

32:25

So if I do that numerical integration over time and I multiply that by C not, I get back to the total amount of material that's been absorbed into that column throughout the time period T.

32:37

Now one of the characteristic points here, and in that I should say before I finish, that is equivalent then W Max.

32:45

So the maximum amount that I can drive into that material W Now one of the things you'll notice is we talked about, well, at  $T_{sub V}$ , that's my breakthrough point.

32:55

That's when the engineer needs to switch the column over and go to the second clean column.

33:00

But what do we do, right?

33:03

We need to integrate this entire area as my breakthrough curve goes from the 0% to the 100% mark.

33:11

Well, if I'm not going to operate the column past the breakthrough time  $T_{sub V}$ , how on earth can I do this integration?

33:18

And that's where this  $T_{star}$  criterion comes from.

33:21

So in an ideal world,  $T_{star}$  is going to intersect my breakthrough curve at 50% of the inlet concentration.

33:35

So how do I calculate  $T_{star}$ ?

33:39

So this is the length of the column  $L_{sub B}$ .

33:43

This is the bulk density of the material  $U_0$ .

33:48

So length bulk density  $U_0$  is the inlet velocity.

34:00

$C_0$  is obviously the inlet concentration.

34:03

Now  $W_0$ , this is my initial loading, and that's probably going to be 0 if I have a clean material, right?

34:12

So we typically make the assumption that's zero and  $W_{sat}$ .

34:16

Well this is what I'm trying to understand.

34:20

What is the saturation limit of my material?

34:22

So once I know the amount that can be absorbed, I can calculate what does this T star value.

34:30

And the reason that we like to do this is that because T star bisects my S curve, my breakthrough curve at 50%, then the rectangle of T star - 0 times AC not or C on C not value of 1 -, 0, that rectangle area is equivalent to the integrated area here on the bottom.

35:06

So if I catch this region, we calculate T star because this integral area, these integrated areas are equal, right?

35:24

So one of the things that the only thing we need to do this integration is we either need to run the column for a full breakthrough once to measure what is the amount of saturated material that I can drive into this system.

35:40

Or if I already know that value, then I can calculate T star and figure out once I hit my breakthrough point, what is the total amount of material that I can drive into this?

35:51

OK, now don't freak out.

35:55

We don't need to actually, we're not going to be using these mathematics, but I wanted to make a point.

36:00

So when we talked about heat transfer and when we first got into mass transfer, I mentioned very briefly that engineers will have partial differentials showing up as we go to advanced chemical engineering.

36:13

All right?

36:14

And that's typically late 4th year or fifth year type of chemical engineering material.

36:19

Now what we've done in many cases is to invoke an assumption of steady state so that we can kill those differentials and simplify the system.

36:29

But because absorption is inherently a transient process, we cannot invoke that assumption.

36:34

So I wanted to take just a few minutes and illustrate to you the approach that engineers will take when we have a problem like this.

36:42

Now, if we think about our column, right?

36:45

So you hear this little diagram going from left to right is the column.

36:49

So this is our inlet and this is going to be our outlet.

36:54

And let's think about a small slice of this DL, right?

36:57

And we're going to simply evaluate what happens at DL, assuming that our velocity  $U$  not is not going to change, right?

37:05

So we have the same velocity over the column, but we start it's midway through the column with some concentration  $C$  of material, not  $C$ , not anymore because we're in the middle of our column.

37:16

So the concentration on the other side of the our cross section DL is  $C$  plus  $DC$ , where  $DC$  is the amount of material absorbed or removed.

37:26

So it will be a negative value, right?

37:29

So I can write a differential mass balance or material balance across this which we showed here.

37:35

Now  $\epsilon$ , this is the overall bed porosity, or I should say that porosity, this is the external void fraction because porosity in this context is, is a characteristic of the absorbent material itself.

37:58

So the particle fraction is  $1 - \epsilon$ .

38:07

So all I'm saying here is that I'm breaking my bed.

38:11

So the time derivative of my bed into two components, the left, the first term here is what's flowing and how that concentration DC changes with time.

38:21

And it's not DC rather, but partial C, partial time.

38:24

So concentration is changing with both time and space.

38:28

That's what makes this really, really difficult, right, is now then I have this second bit, the partial W partial time.

38:37

This is what's governing the pickup of my material in the absorbent phase.

38:40

And that's related to then simply the difference of the upstream and downstream concentration conditions.

38:48

So I can then rewrite this and this is the take home point here.

38:52

I can rewrite this to say, well, concentration right of my target material, maybe that CO<sub>2</sub> in my gas phase is changing now as a function of both time and space.

39:05

So it's no longer DCDT or DCDL, it's partial C partial T and partial C partial L. Because if I want to write an equation for concentration, it must be a function of both time and space.

39:22

And as I differentiate it, return this equation here.

39:26

So if we the solution, if we go all the way up through our understanding of chemical engineering, right, which fits your curriculum plus there are only about 20 mathematical solutions that exist to problems of this or greater complexity, right?

39:46

So we as engineers in the past 50 years of having advanced mathematics at our fingertips and no pun intended, have really only been able to define 20 solutions to common problems that fit this kind of form.



40:03

And one of the ways that we'll then we're going to look at the next slide, the solution to this.

40:07

Now, if we don't want to solve this directly, right, which we're going to look at on the next slide, the direct solution.

40:13

But if we want to approximate this, then one way that we can do this is thinking about the pickup of our adsorbent material partial  $W$  partial  $T$ .

40:23

And we're going to set that equal to an overall driving force  $C - C^*$  and an overall resistance case of  $C$ , right?

40:30

And we would get our overall resistance using that same kind of series resistance summation.

40:38

But what I can do if I think I have an internal and an external mass transfer convective resistance, my external convection resistance is going to come from a Sherwood correlation that we looked at back with Chapter 17 about a month ago.

40:55

And my internal convective resistance for a porous solid, I can relate.

41:02

So  $DP$  is the particle size and I have to use this approximately equal to because I'm approximating the internal resistance.

41:13

But  $D_{sub E}$ , this is now going to be the effective the effect of diffusion coefficient.

41:27

So that's going to change with the particle characteristics.

41:29

So if I'm trying to think about the amount of material I can drive in in this partial differential framework, I can use this approximation that we see here, right?

41:40

Using both the Sherwood correlation and knowledge of the effect of diffusion coefficient for my solid to drive that overall mass transfer coefficient.

41:49

But what happens if I want to solve this top equation directly?

41:55

All right, I need a solution for a that is a partial function of both space and time.

42:02

And this is what it looks like.

42:03

All right.

42:04

So remember that time is the element that's flowing through both our solid and our liquid phase here.

42:11

So one of the methods that chemical engineers will use, and I should mention right as we talk about the role of chemical engineering, we are the only discipline and the entire faculty of engineering that gets into the this level of mathematics for solutions to physical problems.

42:29

So it's quite a cool domain to go into if you like mathematics, but it's one of those that I do find quite difficult.

42:36

So when we think about the partial differential equation that we're solving here, the chemical engineers will often invoke dimensionless quantities.

42:44

We will try to remove the dimensions from our X&Y axis for solutions so that we can take a mathematical solution and apply it to a diversity of problems.

42:56

So what we're introducing here is a dimensionless time that we call Tau and that will be the absolute time on P star, which was that 50% breakthrough criteria.

43:10

So this right hand side, this is the solution to that problem that we looked at on the previous slide, right?

43:17

So it is a function of both the bed or the three primary domains, the bed characteristics, so the length, the void fraction and the bulk density rows of beam, the flow characteristics, my inlet velocity and inlet concentration which we expect and the absorbent characteristics, absorbent characteristics that I choose.

43:40

So my initial loading  $W$  naughty and the saturation loading that it can hold four of the material I've selected  $W_{SAT}$ .

43:47

So I use this dimensionless time and this would go from zero to one as it's defined because at  $T^*$  that's the point at which the bed is fully saturated.

43:58

So I liked  $\tau$  because it goes from zero to one and the solution then invokes a second definition of  $N$ , which is the number of transfer units.

44:09

So in this, I do have that overall transfer coefficient resistance, right?

44:19

And I can get that through a combination of my Sherwood number and the effective diffusivity of the particle or the porous solid itself, the area of the bed, length of the bed and the inlet velocity, right?

44:33

So I can use these two parameters to then solve that problem that we looked at previously.

44:40

OK.

44:41

So you don't need to worry and fret about doing the partial differential solution.

44:47

I just wanted to show you where the field of chemical engineering goes as you start to get up into the 4th, 5th plus right more advanced material.

44:58

That's the direction that our discipline will take on a practical construct.

45:05

What we need to, to understand, to characterize lab four and to solve it is an understanding that our mass transfer zone will run from 5 to 95% of  $C$ . So if I think about plotting that same  $C$  on  $C$  now is a function of bed length.

45:25

Well, at any given time snapshot, I'm looking at the mass transfer zone.

45:34

If this is a value of 1, be just below that just off here, this is the mass transfer zone that I'm concerned.

45:46

And then in temporal space once my effluent, so what's coming out of that first column reaches about 5% of that threshold value.

45:56

That's the point at which I as an engineer, I'm going to switch the feed to jump to the second bed, right.

46:02

So when we wanted to determine then the extent of absorption, we do need to have some knowledge of the bed, the flow rate, the particle characteristics that we're trying to calculate it without any experimental data.

46:14

But if we do have data, then we can take that that effluent concentration and look at this breakthrough profile and we can integrate that, right.

46:24

And we can either integrate that directly if we've measured all the way through breakthrough where my effluent out of the column returns to the initial inlet concentration.

46:35

So I've characterized the entire breakthrough process experimentally or if I already know what the loading potential of my solid is, is  $W_{sat}$ , I can calculate this  $T^*$  criteria and simply integrate this rectangle here.

46:52

All right.

46:54

So the final point to take away that the characteristic type of absorption we're thinking about here is going to be favorable absorption.

47:02

And so the most important isotherm that you remember as you take it forward into reaction kinetics is the Langmuir isotherm, right?

47:11

So I can equivalently write write this as  $W$  on  $W_{max}$ .

47:18

So that's the amount on my solid absorbed my solid at any given time divided by the maximum potential of the solid, right.

47:26

You can see again, engineers, we chemical engineers, we like to take things from zero to one.

47:31

We like those boundaries will be equal to  $K$  times the concentration  $1 + K$  times the concentration where  $K$  is my characteristic, my characteristic fitting factor for whatever solid and fluid I have and  $C$  is the concentration I'm thinking about.

47:49

OK, so again, we don't have a lot of theory that we're applying into this lab because now that we push into transient systems, this is starting to push our basic premise of mathematics.

48:01

So we're going to leave it there for now.

48:04

Next week, of course, is study breaks.

48:06

So there will be no quiz that will be released then.

48:09

I'll post a note on that as well.

48:11

You do not need to work on the laboratory over that study break.

48:14

I think the intention is to take a deep breath in what has been a very gnarly 2020 and just take a deep breath next week because the laboratory for will not be due until the 13th of October, right.

48:27

So of course, I think David and Angus and I, with the acceptance of Monday for the, the public holiday will be working all of next week.

48:33

So if you do have questions or anything that you want to clarify, of course, you're always welcome to pop us a note.

48:39

Otherwise, we'll plan to, to see you again for the next lecture, which will kick into membrane processes and that will be on Monday, the 5th of October, right.

48:50

And, and we're not going to hit membranes all that hard because I do appreciate you've spent quite a bit of time with them in 3018.

48:57

And as you're kind of reflecting over the study break, I do also hope that you appreciate that this unit started off as very front loaded in the amount of work.

49:06

And I do appreciate it's one of the hardest units in any good chemical engineering curriculum worldwide.

49:12

But I hope that as we approach the end of semester, you've been able to take a deep breath and things are starting to slow down in the demand of this unit relatively get to your others.

49:20

So it will hopefully create some sense of balance.

49:22

OK.

49:24

If anything else comes up, feel free to pop me an e-mail or a Teams chat.

49:27

Otherwise, I will look forward to seeing you all on Monday the 5th of October, OK?

## Lecture 26 - Adsorption Rates

0:00

And characterize this initial breakthrough point, we call it  $T$  breakthrough as being about 5% of the initial concentration.

0:11

So whatever  $C_{not}$  is coming into.

0:13

So  $C_{not}$  being the initial concentration of our target material that we're trying to pull out of a fluid.

0:20

When we hit about 5% of that in our effluent stream leaving the bed, that's the point at which we'll say, OK, the bed is becoming saturated and we need to move towards our new fresh bed and sort of desorbing material off the current bed.

0:34

Now if we take, if we have one of these curves, there are a couple different ways that we can figure out how much material we can pack on of these adsorption sites and how much we can remove then as a consequence from our inlet streams.

0:48

And so if we don't know anything about the material and we don't know what it's, so the adsorbent material and we don't know what its saturation limit is, we can determine this experimentally by actually measuring right from the point at which we

first see that the first molecule of target material move through into our effluent stream all the way up through seeing an effluent stream that's equivalent to what we had at the inlet.

1:13

And so that would be an experimental trial we'd run to then numerically integrate from zero to one what that curve looks like.

1:21

Now when we see this T star definition, this would typically cut the breakthrough curve at a point of 50%.

1:31

So when our effluent stream is about 50% of it of C on C naughty.

1:35

And if we have a symmetric breakthrough curve, the reason we like T star is that allows us to simply multiply right zero to T star and 0 to one on the C to C naughty curve.

1:47

So simply multiplying by C naughty and that will give us then the total integrated area if that breakthrough curve is symmetric.

1:56

But in order to calculate T star, if we're able to do that from the outset, we must first know what is the saturation amount of how much material or target material I can pack into this adsorbent.

2:11

And we assume that W naughty we would assume that to be 0.

2:14

So we're beginning with a clean adsorb material.

2:17

Now we'll typically consider we we introduced in the very last lecture this Langmuir isotherm and you're going to see isotherms like this show up in a number of your follow on units.

2:29

So we won't go into great detail here, but we can characterize the performance in the ability for a target molecule to adsorb by a singular experimental constant.

2:41

So in this case, K is the Langmuir absorption constant for a given molecules of concentration C and we like the Langmuir isotherm because it can also be extended to consider multiple components.

2:57

So in fact, I'll wrap this up here on a different colour, but if we have  $W_{\text{Max}}$ , let's say we have two components, we can say, well, there's the first Langmuir absorption constant,  $K_1$  times concentration of component one, and then one plus  $K_1C_1$  plus  $K_2C_2$ , so on and so forth.

3:22

So the Langmuir absorption constant is very convenient, which is why we like to use it in this context.

3:30

Now one of the things we really haven't talked about too much up until now are the types of absorption.

3:35

So we just assume if we have some kind of absorbent material of interest and we flood it with a fluid containing a target compound, that compound is going to absorb to our sites.

3:46

But in fact, there are different Ways and Means that that can happen.

3:50

Now, one of the most common is called physisorption.

3:53

So for those of us in the room, can I get a quick, quick poll?

3:57

Has anyone heard the term physisorption before?

4:01

Physisorption versus chemisorption?

4:03

Is that familiar to anyone?

4:11

Cool.

4:11

OK, so you're going to hear about this quite a bit when you get into reaction engineering.

4:16

But this is chemical engineers tend to characterize the interaction of some material in their fluid.



4:25

So let's say we have some kind of environmental contaminant flowing in wastewater.

4:29

We characterize the interaction between that target molecule and a substrate.

4:34

So our absorbent material or maybe steel if it's absorbing the steel in one of two ways, either and the easiest way to think about this, either there is a chemical bond between the target material and the solid or not.

4:50

If there's a target bond, we call it chem absorption, and typically the bonds will form a covalent.

4:56

So that means we're going to react the material, our target material, with the solid, and it's going to form this chemical bond together.

5:03

So it's a very, very strong interaction, meaning we need a tremendous amount of energy to desorb it.

5:09

This is a very simple plot from Wikimedia Commons.

5:13

So what we're showing here, this  $E_d$ , this is the Gibbs free energy as a function of distance of absorption.

5:20

So if I imagine on the right hand side, we're at an infinite distance.

5:25

And as I bring that molecule closer to the surface, if it's a chemisorption molecule, and I'll change to blue here, right then that the minimum energy, the well of that minimum energy as a function of distance is very, very deep, meaning that there's a huge driving force that molecule is going to want to go to that surface and form a chemical bond.

5:47

Now again, this is quite rare because it we then require a tremendous amount of energy to desorb to reverse the chemical reaction.

5:54

So much more common is what we'd call phys absorption.

5:57

And that's that's what we're talking about here with your laboratory for.

6:01

So that means we have typically Van der Waals interactions could be a bit of hydrogen bonding that's going to hold that target material at the substrate.

6:10

So it's a very weak bond or a weak interaction relative to a chemical bond.

6:15

But here we can see we begin at the same infinite distance.

6:18

The target well is going to be much, much shallower.

6:22

So there's a small amount of energy.

6:24

There's still a minimum.

6:25

And the depth of that well dictates both the propensity for the molecule to absorb and the amount of energy that we're going to have to pump into the system either thermally or in changing pressure to desorb it.

6:38

But this is a much more common type now the isotherms that we talked about and again on the last slide we referenced Langner as a very common one.

6:47

When we go to phys absorption, it's not uncommon that if we have our substrate of interest up here, let's say we'll have a molecule, let's say surfactant molecule that's going to have three hydrocarbon tails coming off it.

7:03

This is, you know, it's not uncommon that these kind of surfactants are going to be highly eco toxic.

7:07

All right.

7:14

Now it's not uncommon when we're talking about Phys's origin that what I've drawn here so far is a first monolayer of molecules that will be absorbed.

7:25

It is occasional that we can sometimes have a second layer, right?

7:31

And typically what will happen with things, if they have a hydrocarbon tail, they'll turn their tails around so they create a a bit of a bilayer.

7:41

But what we can do as engineers is we can model this behaviour through what we call multi layer absorption.

7:46

So that can be monolayer, bilayer, Tri layer, so on and so forth.

7:51

And we're not going to go into detail because this is where reaction engineering is going to take you.

7:56

But in that case, when we have more than one layer of molecules at the interface, we can't use a Langmuir isotherm, but in fact, we use a BET isotherm as the characteristic.

8:08

Now I wanted to to then also frame this in terms of what's happening at a thermodynamic level.

8:15

So you can imagine in both of these cases, right, if this is Gibbs energy, they were applying on the Y axis or ordinate here.

8:26

Well, the Gibbs energy is negative in the absorption process, right?

8:30

So we can say our definition of Gibbs free energy is shown here.

8:33

Enthalpy plus temperature times change in entropy, and that whole value has to be less than 0.

8:40

So the fact that it's zero tells you critically that it is a spontaneous process.

8:54

And by spontaneous, what we mean is that if we simply dangle a molecule anywhere near this surface, once we get within a few nanometres where that potential energy well is felt by the molecule, because this is again the Gibbs energy in a molecule.

9:10

Once that potential energy well is filled by the molecule, it will spontaneously absorb down to the surface, be it a chem absorption or phys absorption process.

9:18

So when we have a negative Gibbs free energy of a process, that tells us that process is going to be spontaneous.

9:26

Now one, one thing we recall of course from thermodynamics is that any process will any real process will engender a positive change in entropy.

9:39

So if I know that my Gibbs energy change is going to be less than 0 and my entropy change is going to be greater than 0, then by definition, right, I know that my enthalpy change must also be less than 0, less than 0.

10:02

So that enthalpy is going to make up for whatever entropy is being caused in this case.

10:09

So this means that because I have a  $\Delta H$  of the molecule that's going to go down, that energy, that or that enthalpy must be made-up elsewhere, meaning the process is going to be exothermic.

10:21

So it's going to release energy from the molecule and the molecule's enthalpy is going to go down.

10:29

So when we think about absorption, then we're again, we're not going to go very deep into the kinetic rates and behaviour.

10:35

But I wanted to give you kind of an introduction to where this dovetails with the next step in your curriculum.

10:41

So one of the simplest cases you'll recall in the last lecture, we're looking at really complex definitions of partial differential equations and the solutions to them.

10:54

Now one of the ways that we can simplify that to solve directly is if we have the case of irreversible absorption.

11:02

So that means we have a constant mass transfer coefficient.

11:05

And in the case of a constant mass transfer coefficient, we can only have that if we have only external resistance to mass transfer in the film.

11:15

So we don't have any internal and poor resistance in the absorption process, but it's only that liquid film that's going to control.

11:23

And in that case, if we have a constant transfer coefficient, then the rate of mass transfer is directly proportional to the fluid concentration.

11:30

So the mathematical solution that we originally had was a partial differential of concentration with both space and time.

11:38

So partial C partial L and partial C partial T, we can get rid of that partial C partial T component, right?

11:45

Because it's only proportional to the fluid concentration, which simplifies our mathematical set up here.

11:53

And when we directly integrate to get the profile, the active profile concentration in the bed, I get a natural log of C on C naughty and it's proportional right to KC store constant mass transfer coefficient, the area cross sectionally the bed, the inlet velocity U naughty and L being the position in the bed, right?

12:14

So it's only a function of position at this point.

12:17

Now you'll recall that this right hand side was also our dimensionless number of transfer units N that we looked at in the last lecture, which means that we can define the concentration at the end of the bed as C being equal to C naughty times the exponential of negative N right?

12:41

Now the reason that we show this and we'll go into the next slide, but this is a classical example of a zero order reaction kinetic.

12:51

And so this will be the first of the type of reactions that you look at next in the coming year.

12:58

So what we can then do is plot the concentration of our target component on the initial concentration as a function of what is effectively time.

13:07

And the way we're representing this here, this is  $N$ , So that is defined our  $k_{cal}$  on  $MU$  naughty.

13:14

And this  $\tau$  -1  $\tau$  is again that dimensionless time or  $TNT$  star -1.

13:20

So what we can see as we go from left to right across this graph, we are increasing in time, right?

13:26

And what this solid curve is showing us is if we have irreversible absorption, so we have only external film resistance, then the amount of material as a function of time at the end of the bed is going to go up, right?

13:39

And it's going to be a continuous increase.

13:42

Whereas if I have diffusional control within the pore, right, then I get more, I get more of a logarithmic behaviour just shown here.

13:56

And then we'll draw in blue.

13:59

What happens if I have a combination of the two?

14:01

So my my assumption of irreversible absorption might not be quite right, but it's a balance instead.

14:07

Then I have my large hash curve here, which will look more like an S curve.

14:13

So again, this this is an example of our zeroth order kinetics.

14:18

And the reason we know it's zero order, we call it zero order, is that the solution itself is not time dependent, right?

14:26

So when we think about the order and the rate, it's going to depend on where we are as a function of both space and time.

14:33

But in this case, we don't have any temporal dependence, right?

14:36

So if I write out the definition of  $N$  on the top, I can see here that I'm only a function of physical parameters like the velocity area and the length of the bed, and then the material parameter being the mass transfer coefficient.

14:50

Now I can compare this with both a first order rate equation, which we showed here.

14:55

So this is typical of something like trying to pull contaminants from an environmental engineering solution, trying to absorb contaminants of something like wastewater to clean it up.

15:07

And here what we're looking at is we now have the rate of mass transfer such as  $Q$ .

15:11

In general, the rate of mass transfer with time will be proportional or equal to a first order rate constant.

15:18

And this will be different to our our mass transfer coefficient capital  $K$ .

15:23

But a first order rate constant that has units of one on time right times some driving force.

15:35

So  $Q_{\infty}$  is the equilibrium capacity of the absorbent material and  $Q$  is the time dependent capacity.

15:44

So how much capacity have I taken up right?

15:48

How much material have I absorbed relative to that equilibrium quantity?

15:53

Now I can also get a bit more complex if I start thinking about multi layer absorption or I'm trying to pull a select component out of a complex hydrocarbon system.

16:01

So it's something like a biofuel.

16:03

And in that case we'll write.

16:05

If we frame this around a complex liquid solution, we'll write the concentration as a function of time.

16:11

DCDT now is a function of the 2nd order rate constant, and this will typically have units of volume per mass per time.

16:28

So an example might be liters per gram second.

16:33

All right.

16:34

So it's a more complex 2nd order kinetic rate constant Times Now the instantaneous concentration of my material squared, right.

16:46

And that that square is what makes it a second order rate equation.

16:50

So again, I'm not going to go into too much detail around this, but just to highlight what we're talking about in adsorption is very, quite, quite simple for irreversible adsorption being 0 TH order kinetic.

17:01

And once you get into the next year's curriculum, you're going to start unpacking first and 2nd order rate equations.

17:08

Now one kind of final note that I wanted to talk about in terms of how we design adsorbers is in your lab 4 reports, you would have been working on a batch process, right?

17:20

And if there's one thing I hope you've learned so far needed operations, it's that we tend not to like batch processes because they're quite inefficient and wasteful.

17:29

So as you can imagine, the engineer would sit back and say, how can we make this a continuous operation?

17:35



And, and this starts to get very interesting if we look at a continuous process here I can see I have my feed containing some Y compositions.

17:45

So here we have a vapour feed where Y said F is the initial concentration of my target component in that vapour Y the extract.

17:55

That's what's leaving out of the Desorber.

17:59

The raffinate and the desorbent are shown here, but what we're actually doing in this case is that I don't have a liquid that's balancing against it, but I'm using a solid to pick this up.

18:10

That means I have a physical process.

18:13

So it could be something like a conveyor belt, right?

18:17

Or a bucket system that's going to physically move material that's been fully packed.

18:26

So, so absorb material that needs to go through a desorption process.

18:29

It is going to physically move from the bottom of my absorption column when it, when it's fully packed back up and drop it in the top of the desorber right where I'm using a counter current flow because now my feed's coming in.

18:42

So the feed will contact solid material that's near it's it's capacity.

18:49

And when I design something that's a continuous process like this, I can then go back to a McKay Teeley diagram and I can use that with both absorption lines.

18:57

If I have a raffinate desorbent characterization, I can I use these to determine the ideal number of stages, right.

19:05

So we're not going to go through this in any great detail, but just to make you aware if you are working with an absorption system, one of the ways that we can make this both continuous mode and to make it more efficient is to think about it using this physical migration of path material so that we can apply our McCabe to healing lessons yet again.

19:27

OK, so that's actually it for today because I wanted to spare a few minutes toward the end to go through the exam setup.

19:35

So the kind of take home messages from this with the absorption rate as we get into rates this the way we model this will vary with the type of absorption that we're talking about.

19:47

And most critical around that is that we have two potential types.

19:54

The 1st is phys absorption where we have a weak chemical bond or sorry a weak interaction, something like a van der Waals interaction between our target material and the surface of interest.

20:09

And the 2nd is chem absorption where we have chemical bonding and to the actual surface itself.

20:15

And this is typically.

20:20

Covalent and it's it's quite uncommon to see chem absorption because of the amount of energy required to go through this.

20:30

Now the kinetic rates in our mass transfer zone, if we're using a batch process, we looked at quick examples of 0 first and 2nd order kinetic rates.

20:39

And these kind of set you up for a little bit of thinking about where reaction engineering, so CHPR 4406 and Lanced Reaction engineering 55 O1 are going to come into play.

20:51

And I think that the really critical connection I wanted to make there, right, is that if we've gone from ENSE 3006, so undergraduate thermodynamics, one of the, the lessons we would have learned in that is that we are looking at a Gibbs energy equilibrium, right?

21:08

So we're always trying to find that Gibbs energy equilibrium.

21:11

Now what starts to change a little bit is if we have a situation in which we're out of

equilibrium, right, the system itself will try to relax back toward whatever our Gibbs energy minimum or that equilibrium point is.

21:28

Now where this gets quite interesting is that when we're using a steady state solution, right?

21:38

In that case we can knock out a lot of the complexity, right?

21:44

But when we start coming into transient processes, we cannot get rid of the time dependence.

21:51

So this is when we start talking about reaction and advanced reaction engineering.

21:55

This is where we go from the field of equilibrium thermodynamics to non equilibrium Thermo and what we call non equilibrium Thermo, right?

22:10

That's where we have tried to characterize this as based on the the mathematical nature of how these systems behave.

22:19

So that leads us to a more empirical field that we call kinetics, right?

22:26

But as you're approaching kinetics and reaction kinetics, the way to think about this is in fact we're just in a non equilibrium position in thermodynamic space.

22:35

So it gives energy space and the system itself is trying to relax back toward that equilibrium position.

22:41

And so the, the, the farther we're out and the nature of which we're out of equilibrium will dictate the pathway the system's going to take to go back to equilibrium.

22:50

And so we, we have studied the commonalities in that pathway and the types of mathematical relationships that that are similar between them.

23:00

And this then forms the field of reaction kinetics.

Lecture 27 - Chromatography

0:00

Started getting into absorption kinetics a little bit.

0:02

And so looking at what is the, the rate of absorption in systems.

0:06

And one of the, the critical lessons there was looking at the type of physical absorption process that's happening.

0:12

And so we talked about two fundamental types and that's phys absorption where there's a Van der Waals type interaction between our target molecule and our solid interest and chem absorption where there's that actual chemical bond form.

0:25

So chem absorption might be more comparable with something like irreversible absorption.

0:30

And then as we started unpacking the rates at which these different things can happen, we saw that the different types if we if we plot, I think this, this UWA laptop is definitely dying.

0:46

So if we plot something like the Gibbs free energy as a function of separation distance between our target molecule and our wall, right.

0:55

So here would be that might be the core wall.

0:59

We saw that that there's a free energy minimum.

1:04

And so chemisorption is going to have quite a deep, we call it a free energy well.

1:09

And so it's in fact, in fact, I should draw this such that it comes back up to zero.

1:14

There we go.

1:15

And so the difference, the difference between that, that zero line and the minimum of the free energy well, right, this is  $\Delta G$  or that's the free energy associated with the process of absorption.

1:31

And so that, that deep well would be something like chemisorption right?

1:35

Now it's, it's important to note that this goes toward Infinity.

1:39

So as we bring that separation in distance, closer and closer, you can imagine the molecule itself is going to occupy a finite volume.

1:47

And if we bring that volume, or rather we we bring the diameter of that, assume it's a spherical molecule within.

1:55

So if that  $D$  goes less than the diameter of the molecule, we're going to start compacting the molecule.

2:00

And so then the free energy difference will go up toward Infinity because the molecule's not going to want to be crushed on a on a subatomic level.

2:11

Now, we can also look at phys absorption.

2:14

And that was an example more shown here, where we have a much shallower energy well.

2:21

So there's a lower driving force for absorption in these systems.

2:26

I'll plot  $D$  up there, a bit of separation.

2:30

So the the free energy difference associated with having a molecule out in the continuous phase and absorb to the wall.

2:38

This free energy difference is what drives the rate of absorption that we then subsequently measure.

2:45

And we characterize these as engineers as a kinetic grade and they're going to vary with the materials that we use, so the type of molecule and the type of wall.

2:53

And we gave very, very brief examples of things called zero first and 2nd order kinetics.

2:59

And this is what you're going to start unpacking in reaction engineering or advanced reaction engineering in greater detail.

3:06

But just to make you aware to kind of connect from undergraduate thermodynamics all the way through advanced kinetics, this is the pathway we're taking us ultimately looking at these free energy differentials.

3:17

And that's what's driving this kind of behaviour.

3:20

And then we very briefly just demonstrated that of course we can, if we have a continuous absorption operation, we're going to have to physically move our fully saturated absorbent material to the desorbing side of the process.

3:32

And so that's going to invoke something like a conveyor belt or a set of buckets.

3:36

But if we do have a continuous process, we can go back to our Tri and trusted McCabe Thiele diagram that's now shown up in three different mass transfer processes.

3:50

OK, it's jumping around.

3:53

There we go.

3:54

So today I wanted to go in to spend our last lecture on absorption, talking about one of the growing applications for engineers.

4:02

And and this is quite specific actually for chemical engineers in particular, both chemical engineers and chemists tend to use chromatography quite a bit.

4:10

So it's a quick show of hands that a handful of us on the call.

4:15

Has anyone heard of chromatography or used it in the laboratory before?

4:21

Yeah, one.

4:24

Cool.

4:24

OK.

4:26

Now where did you use it out of curiosity?

4:29

Would have been undergraduate chemistry ISM.

4:39

Cool.

4:40

So I did they go into the background of how a chromatograph works.

4:48

Cool.

4:48

OK, so we have a, a good a good corollary here then.

4:53

So.

4:53

So chromatography then this comes into play when we have multi component gas or liquid mixtures.

5:02

So fundamentally if you would have used this, then it sounds like an undergraduate chemistry, we're going to have a column, so a chromatography column that's going to be used where we're going to take a carrier gas or a carrier liquid and use that carrier to move our feed through the bed.

5:17

Now I'm going to focus most of the time here on gas chromatography and we'll go into some examples for the end of lecture.

5:25

But the idea is that if we are using a carrier fluid that's inert and relatively low molecular weight to move a single batch of material through a long narrow pipeline.

5:39

So in this case we use like a coiled column, then it's going to take this multi component mixture and spread it out based on the partitioning or sorry, distribution coefficients of each component in that mixture.

5:49

So we'll look at in a few slides with the mass of that is now once that column where that coil gets long enough, the components are going to emerge at the end of that.

6:01

So the effluent stream of our absorption column in this case is going to contain or present pulses of material.

6:10

So the goal here is not to slowly saturate a column and have, you know, stop the the process once we hit a breakthrough curve.

6:17

But what happens if we want to take a small batch of material and and basically spread it out inside of the column?

6:25

And So what comes out the other end?

6:27

The highest vapour pressure and lowest activity components will come out first and the lowest vapour pressure components will come out last.

6:35

So it allows us to smear out this multi component mixture and what we get out of this then is a signal that corresponds to the retention time of some of these components within the column.

6:46

And so the example from the textbook here then looks at 5 different.

6:50

So this is a 5 component mixture.

6:54

Now when we talk about a chromatograph, then this is a combination of basically that column, the analyzer that's going to to be processing that effluent stream and whatever relevant flow equipment or measurement equipment we're going to use associated with this.

7:10

So for instance, we can be measuring something like hydrocarbon composition for signal.



7:15

We can also measure density, right, for each of these compounds.

7:18

So there's a number of different types of flow equipment arrangements we can employ.

7:23

So what we can then plot, we can see here if we have these five components in the mixture, well, the 5th component is going to have a much be much heavier than the first component, right?

7:35

So the longer the retention time of these components in the column, as you would imagine, the higher their molecular weights.

7:42

And so if we're going to, you know, it's not one thing to do this on A5 component mixture, but where engineers really come in is if we're dealing with an N component mixture.

7:51

So maybe we have something like 10,000 components that are going to show up in the biofuels process, right?

7:56

And that's a, a meaningful #10,000 components is exactly what we'd expect to see.

8:01

So, so this then helps to start to characterize the distribution of those components and what type of biofuel have we manufactured in this process.

8:11

Now one of the the key characteristics to highlight here is that we can see as we go from the left to the right hand side of this plot, the peaks will naturally broaden and smear out.

8:21

And that's a a consequence of molecular diffusion inside of the column itself.

8:26

As you can imagine, the longer that that pocket of material is going to be sitting in the column and moving through it, it will have natural diffusion both in the primary direction flow, but also laterally and backward, right.

8:38

So even if my net velocity of the molecules is all moving from the front to the the back of the column, I'm still going to have a spreading material because a few

molecules as I move forward are going to be pointed and and moving against the general flow.

8:54

So that's where I get that smearing effect.

8:59

So the types of chromatography sweep, I will get into that.

9:05

That's a very good question.

9:06

How do we get appropriate resolution to determine between heavy molecules?

9:09

I will get into that as an example in a few slides.

9:11

That's a very good question.

9:13

So the types of chromatography we'll touch on briefly.

9:16

We can either use gas or liquid chromatography.

9:20

So gas chromatography is one of the most common bases and this is for chemical engineers working in the energy industry.

9:27

So I should clarify that energy engineers so, and that's because we're typically working with some kind of liquid fuel.

9:39

And so the idea is that we can either have a solid adsorbent or a liquid that's going to be held in a porous solid.

9:45

We need a carrier gas.

9:47

So helium will be the most common, about 90% of cases where nitrogen and hydrogen can also be used.

9:54

The idea is that we're going to take a small batch of our feed liquid, we inject it into this column.

9:59

So on the example shown here, here's our carrier gas and our injector.

10:04

So a small amount of liquid batch is going to be injected into this column, which is really a coiled tube in a temperature controlled environment that will smear out.

10:13

And as it comes out now that the column itself, the wall will be coated in our adsorbent material.

10:29

So if we imagine a cross section of it, here's our adsorbent material all the way around that coiled column and that X marks the flow space as we're going into the word.

10:42

So the the material will flow through this column, the heavier components will interact with that material for a longer time, and then we bring the outlet or the effluent into this detector and we then read that out.

10:57

So I mentioned there were a few different ways that we could characterize this.

11:00

So we can measure thermal conductivity, the density of each of these components coming out, or what's more common as we deal with energy systems and things like biofuels or flame ionization detectors or Fids.

11:12

And we typically use very small liquid pulses simply because if we start to get a lot of liquid in the column or liquid building up the activity contributions, we're going to see activity coefficients coming back from undergraduate thermodynamics.

11:26

The amount of activity in the liquid phase will start to become a a bit of a problem.

11:29

And now we can also use liquid chromatography similarly, either with a solid adsorbent or a more heavier, so higher molecular weight liquid that's held in the porous space.

11:41

And this is relevant when we start having or if we have systems where we're going to have a large diffusion resistance.

11:48

So the the one limitation if we go to liquid chromatography is that of course, this whole space is now packed with liquid.

11:55

So we can tend to get a quite large pressure drop on the order of several bar over the column.

12:02

So if we think about the masks that are driving this sorry, tab was just frozen for a minute, there we go.

12:11

So if we think about the the masks that are driving this for a minute, well, we can initially say that the we can define of course the equilibrium mole fraction of a components.

12:20

So we'll call component A in our mixture.

12:23

We can write here we all remember what law this is the routes law for for chemies, right?

12:35

But of course this is routes law for chemies assuming ideal ideal vapour behaviour.

12:50

So we want to be operating at high temperatures and low pressures, which is quite appropriate for AGC system.

12:57

Now we can write this also for the concentration of component A and the liquid and the vapour phase or liquid and gas phase.

13:04

We'll get L meaning liquid and G meaning gas.

13:08

We can define this as the mass fraction of the component X of A times the density of the liquid rows of L divided by the molecular weight of the liquid itself and taken on an average times the molecular weight component A.

13:25

And we can simplify that through the introduction.

13:28

So  $\rho M_{\text{sub L}}$  in this case is molar volume and that will have units of moles per meter cubed.

13:41

So this is just a Thermo physical property of our individual component.

13:45

So it's the mass fraction, the molar volume and the molecular weight of that component A.

13:50

And then we can write the same parallel relationship in the vapor phase.

13:54

So it can amount or the concentration of component A in the gas phase.

13:58

And then most importantly, we can use Route's law to relate the two.

14:03

So we can relate the amount of component in the liquid phase to the amount of component in vapour phase.

14:08

And we have this equilibrium relationship show up here where we're looking at the difference in the average molecular weights between our two phases and then our routes law contribution of activity and vapour pressure.

14:21

So, OK, now when we're passing our material through the column, there's one additional thing we need to consider and that's the amount of solvent that will be in the unit volume itself.

14:35

So the mass of the solvent will give the variable  $W$  will be the density of the bed rows of B and S will be the surface or the solvent loading.

14:58

So this just allows us to correct for how much solvent is there per unit volume.

15:04

Now we can write this if we're doing a a mass balance or a material balance on component A.

15:09

And we can define that at equilibrium by thinking about firstly the concentration of component A in the gas phase times.

15:18

Now the epsilon in this case is the core, sorry the the void fraction of the bed.

15:26

So this is the volume available to the vapor phase to flow.

15:32

Now, if we go back to our example of these coiled tubings, right, this is where Epsilon's going to come in.

15:44

So the fraction that's available for the concentration of the component and the gas phase to move through.

15:50

And then I'm needing to think about, well, how much of my component is going to be tied up in that solvent.

15:56

And so This is why I'm going to need to, if my, my solvent, for instance, is a liquid held in the porous phase where it's partitioning directly to the adsorbent material to, to physisorption, then I mean, and certainly these are all physisorption reactions, not chemisorption in the renewable column.

16:15

But then I need to think about, well, OK, what's the mass of solvent per volume?

16:19

And then I can multiply that by my Raoult's law relationship to figure out how much of that component of that same CA of the gas, which how can I relate that to the concentration of component A that will be tied up in that solid or or porous phase.

16:36

And so we give this bracketed term that's highlighted in red, we call, we can truncate this and call it the partition coefficient case of A.

16:46

So effectively this represents the balance of partitioning of a molecule between our vapour phase and whatever pore of solid or liquid in the porous space we're using in the absorption column now the carrier gas.

17:01

So this is going to move through that highlighted space here, right?

17:05

That carrier gas moves through with the superficial velocity  $U$  not.

17:10

And we divide that by the flow area where the the void fraction of the bed epsilon.

17:16

So this means that the time at which an individual molecule entering the column is going to leave the column will be defined as the length of the column times that were divided by that  $U$  on epsilon relationships.

17:32

So it gives me a residence time for the marginal molecule.

17:36

Now we've defined it this way because we can then take a final step and figure out what is the breakthrough time, right?

17:43

The midpoints of the the we're thinking go back a few slides.

17:51

When we say peak or the midpoint, we're meaning what's the middle of each of these, the middle of each of these individual breakthrough relationships.

18:05

So the midpoint of that breakthrough pulse, we can define that with the retention or the retention time of that based on the length of the column, the superficial velocity of how quickly the carrier gas is moving through.

18:18

And we multiply that by the sum of our void fraction and this partitioning coefficient.

18:26

Now, when we use liquid chromatography, it's not uncommon that our partitioning coefficient or partitioning coefficient will be significantly greater than the void fraction.

18:34

So this means that I get a nearly linear relationship between the breakthrough time and the partition coefficient.

18:42

It's not exactly linear, but it tends to be very, very close.

18:46

And so when that void fraction is low enough that we can neglect it in the calculation, we can start relating the breakthrough times of multiple components.

18:55

If let's say we have A and B, so we're using a binary mixture and trying to study it, we can relate the ratio of those two breakthrough times to the ratio of the vapour pressures and the activity coefficients of the components.

19:09

And it's in fact the product that we're measuring.

19:11

So this is one way.

19:12

If we have a multi component system, this is 1 analytical chemistry tool that we can use to start getting at things like activity coefficients experimentally if we know, for instance, what the vapour pressures look like.

19:26

Hence the dramatic importance of using Antoine relationship so much last semester.

19:36

OK, so where do you see this show up in practising engineering?

19:39

So one of the most common applications and certainly if you go into the energy industry here in Perth, one of the very common applications you're going to see is high temperature gas chromatography or HTGC.

19:51

So what we do here is instead of working at an ambient temperature or a near room temperature, we're going to equip this column with the ability to change its temperature.

20:01

So we can ramp the temperature from say 50° C up to in this case we're using an example of 320° C, but it's not uncommon to see HTG CS go up to 450° C And so the example that we're showing here, this is looking at AGC with a flame ionization detector.

20:23

So we're looking at picking up hydrocarbon based molecules and then we'll transform that if when we put an FID on this, it transforms the area under each of these peaks to a weight percent.

20:33

So it helps us read out this weight percentage for our different hydrocarbons in the mixture.

20:39

Now on the left hand side, and this is examples provided by Brandon Graham, if you've not met them, you will soon.

20:47

But on the left hand side, we're looking at a relationship now between boiling point.

20:51



So boiling point shown in red and the carbon number of a given hydrocarbon distribution shown in blue as a function of retention time in AGC column.

21:02

And so one of the points that we can make is that the thermophysical characteristics of the molecules are closely related to their retention time in these columns.

21:11

And in fact, we can use that reality to take a very, very complex hydrocarbon mixture.

21:17

Maybe we're going to have 10,000 components if we're synthesizing a biofuel, and we can then characterize that in AGC column.

21:24

And that's where HDGCS come into play.

21:27

So this is an actual real example.

21:30

Now, this is a heavy oil, right?

21:35

So this might be a biofuel or a natural, well, natural bit of fossil fuel oil.

21:40

If you're looking at the fraction that's going to be used for something like bitumen, for example, that's a very heavy oil that can create this kind of GC signature.

21:50

And what we're looking at is the on the, the Y axis we call relative content.

21:55

But this is effectively the signal intensity of the GC as a function of its retention time.

22:01

Now, the point that we want to make here is this is a really, really messy plot, right?

22:05

So if we zoom in on one portion of the plot here, in fact, we're not able to even see an individual resolution in these peaks, right?

22:14

So we see this start to smear and get very difficult to read.

22:18

So we have a few options as engineers for trying to use this to interpret something on the left hand side.

22:25

This is an HTGC equipped with high resolution mass spectrometer.

22:31

So a GCMS is a common pairing of instrument because it allows me to take my GC and what comes out the other end I can break it apart.

22:38

But because we can see this is coming from a massive distribution, this would be something like a lighter bio fuel or a, a condensate material.

22:48

So usually the types of biofuels or fuel sources you'd expect to create jet fuel or gasoline.

22:56

We, if we equip this with a high resolution mass spectrometer, we can then start breaking and looking at those individual molecules.

23:02

Now the limitation with this is that we can't see isomers, right?

23:06

So it merges all of the isomers together because they have the same molecular weight.

23:10

But it does allow us to start differentiating if we're trying to pick up one particular contaminant that we can't allow into our fuel or into our hydrocarbon or biofuel source, this is a clever way to pick that up on the right hand side.

23:25

This starts to then look at a lower resolution GC.

23:30

So the type that most companies are going to have in their labs unless they're super, super rich.

23:35

This is the type of GC that we're going to tend to find in most labs where we see a huge scatter between low resolution and a lot of scatter between these individual compounds.

23:46

Now the way that you as engineers can go about using this, if you take a full for this distribution, then go behind it and run benchmarks.

23:56

So pure, for instance, in a hydrocarbon for biofuel, run pure decking, pure C20, pure C30 and pure C-40, right?

24:06

And that allows you to identify, well, this right here is the C-40 peak.

24:11

Now, if we blow that up right for a minute, so if we blow that peak up and we can see the baselines moving around a bit here, but I'll try to replicate the drawing.

24:26

So if that's that C-40 peak, one of the the ways that we go about determining relative content and I'll switch colours for this is that we numerically integrate everything in that peak, right?

24:43

And if the next peak is going to start there, we numerically integrate everything up through the start of the next peak.

25:00

And so this thing gives us a relative proportionality of what we might call our C39 or C-40 pseudo fraction, right?

25:08

Because we're not going to be able to break out individual molecules once we get to this scale if we have 10,000 components in a relatively cheap, do you see?

25:16

But it turns out for most thermodynamics calculations, we don't actually need to know all 10,000 of those components.

25:22

We can simply group them into their average molecular weight.

25:26

And so this is one of the ways that practicing engineers will go about using an HDGC when they're trying to characterize a fuel source.

25:33

So, Gus, then back to your original question, how do you get appropriate resolution to determine between the heavy molecules?

25:39

So this is where it's a bit of a nuanced answer.

25:42

How do you pay for a really expensive measurement in a high temperature GC with high resolution mass spec?

25:50

Or you group the components together and run benchmarks for C40C50C60 and it groups all of them together.

25:58

And then we do our our thermodynamic calculations on that basis.

26:02

But it's not uncommon that you're going to see both of these crop up in your careers at different times.

26:08

And so kind of bringing that to, to a close.

26:10

And the reason why we tend to as as engineers, we've been working for the past, well, for the 20th century, primarily around using pressure and temperature measurements, right?

26:23

That's what we had readily available to us.

26:25

And so if, if a chemical engineer was hired to design A process or optimize it or try to figure out what's wrong with it, the data that they had to go from is typically pressure and temperature or the real time data we can get.

26:39

One of the really big movements in, in chemical engineering in particular in the past 20, say 25 years has been the uptake of compositional measurements.

26:51

And so if you think about you go back to Gibbs free energy, right?

26:56

We have DGVDP minus SDT come on here, plus some of the components I of MU IDN.

27:08

All right.

27:09

So if we're measuring pressure and we're measuring temperature, right, we've developed a number of Ways and Means to do this.

27:17

Well, measuring composition is really the final frontier for a lot of chemical

engineering applications that start to allow us to get into direct estimates of, of changes in Gibbs free energy, right?

27:30

The, the limitation is we need to know the chemical potential, but This is why we're seeing such an uptick in online systems, meaning they're plugged into the process and they're measuring this in real time of online compositional measurements.

27:43

And GCS are a really, really good way to do that because they're cheap.

27:47

So the the type of GC we were looking at on the previous slide, the cost of this might be something on the order of 30 to 40,000 Australian, right.

27:56

And that's going to say be cut in half as we move forward and they start to become more common.

28:01

So by the time you're practicing engineers, you might expect 20 to 30,000 is going to be reasonable to buy GC.

28:07

And if you're managing, you're trying to optimize a multi \$1,000,000 or \$100 million process, then at 30 to 40,000 to bring a compositional measurement, which is fundamentally that the 3rd and final domain of Gibbs energy is a complementary technique is really a powerful thing to do.

28:27

So one of the, the points we've started to see is then you can use these in online systems to get a very high resolution identification contaminants.

28:36

If you're trying to, to put in place, for instance, an AI machine learning algorithm that's plugged into your online management for process control or process optimization, right, as many domains of data as you can give that machine learning algorithm better.

28:49

And then finally, if you're working in what will certainly be a massive industry in the coming 50 years of biofuels and in biofuel manufacturing, you're going to have to be very OK with compositional and characterisation of both the raw feedstocks and the process fuel stocks.

29:07

So the the take away from GC methods then is that when we feed these in and we use the carrier pads for it, the lighter components are going to breakthrough first.

29:15

So that retention time is related to the partitioning coefficient.

29:19

So the more activity there is and the higher or the lower the vapour pressure, the more it's going to drag that molecule down and keep it toward the end of the breakthrough.

29:29

So we see that inverse relationship to both component activity and vapour pressure.

29:33

All right.

29:34

So this is our final application of absorption and I'm glad that people have used GCS before.

29:38

And if you didn't know, this was one of our hallmark applications of absorption for chemical engineers.

## Lecture 29 - Reverse Osmosis

0:00

OK.

0:00

So the last lecture we looked at liquid membrane applications of dialysis as well as or talked about rather liquid membrane applications of both dialysis and desalination.

0:12

And so here what we're doing is focusing pretty exclusively on desalination to support your kind of final miniature design project.

0:20

So we want to, in particular in this lecture, then go into reverse osmosis membranes.

0:24

And we talked in the last lecture that in a typical system, the osmotic gradient is one that means that we're going to flow at a solvent from a region of low concentration of solute to a region of high concentration of solute.

0:37

So that's the process of osmosis.

0:40

But if we think about that, so that rather flows along a chemical potential gradient.

0:46

But if we think about that and it gives energy framework, then we can start applying pressure to reverse to maintain the same Gibbs energy minimum and reverse that chemical potential gradient effectively, which means we start applying pressure to the side that has more salt in it.

1:02

So a high concentration of solute, we can actually pull or push water out of that and into a, a secondary phase.

1:14

So in this case, what we're looking at is then the application of that to a membrane where we're going to have a little bit of solute and quite a bit of solvent flowing across into a highly concentrated water phase.

1:24

So one of the critical bits from the last lecture was then how we calculate the osmotic pressures.

1:28

So there was a table with some common examples of seawater and sodium chloride water, so kind of synthetic pure salt water.

1:37

But we also looked at a, a way to calculate that with an, with an equation.

1:43

So if you wanted to put this into a spreadsheet and look at different examples, we went through that.

1:49

So today in this lecture, what we're going to do is focus on a very kind of brief introduction to membrane design.

1:57

But then because you've gotten this previously in 3018, I'm going to focus most of our time today on examples.

2:03

So on the top here, this is a spiral.

2:07

So this is a more common type used in reverse osmosis systems where it's laid out much like a roll a sandwich.

2:15

So we have a a base plastic screen within membrane and covering.

2:22

So we have our our outer screen here and then we have an inner mesh covering that holds our membrane in place.

2:29

And so that's our target material.

2:32

And what happens is we'll feed across this and then we can add different layers of this membrane and then we roll it up like a sandwich around a pipe that is has holes in it to take in our permeate.

2:44

And so this then allows us on both sides, if both ends are open to we push the feed solution in around the style and so it's flowing across, it's shown here.

2:59

And then in that center pipe is where we're going to collect the output of the membrane.

3:03

And an alternative approach is to look at a hollow fiber membrane where the feed is going to move across this membrane section here.

3:10

So this is our hollow fiber, the feed is going to flow through the outside and then we can go through the two and then you can see the permeate is going to flow from the outside here as it moves across and through that hollow fiber membrane.

3:24

So in this the hollow fiber setup we can see that then the direction of flow is going to be normal to the membrane itself that's going to go through the membrane whereas in the spiral wands separated the flow is going to be across the length of the pipeline.

3:48

So in terms of the typical setup, and this is where we'll see today, you know, one of the points that your groups are going to have to look at in this miniature design project is to determine how much salt you need to pull out.

4:00

Oh, sorry, the sound quality is dead.

4:02

Is it?



4:03

Has it come back yet?

4:12

OK, give me two seconds.

4:47

OK, how's that?

5:03

Just joined from my phone.

5:31

OK.

5:31

Is that any better?

5:44

Shoot.

5:44

OK, let me think.

6:07

OK, is that getting any better?

6:15

Ah, OK, good.

6:18

It was the, it was the fans on the computer.

6:24

OK, cool.

6:25

They were getting too loud.

6:28

OK.

6:29

So when we look at then a typical reverse osmosis system, one of the things we're going to be calculating today are examples of the purity and the rejection rate that we can get.

6:40

So what one of the the points that we'll make here is that we would organise these potentially in two different series.

6:46

So we'll have a first stage of membranes and then these will feed into a second stage or both of these output pure water.

6:52

Now one of the points we were talking about early on is the miniature design project has a bit of a variation around what is the energy balance that's going to be required.

7:04

And and certainly here we can see in this case the actual energy balance.

7:09

We can think about what is the feedstock and the energy or the enthalpy of that feedstock compared to the enthalpy of the residue in pure water.

7:19

But we also, all right, want to think about the energy input to the pump, right?

7:25

Because this is going to be a critical, this is where we're consuming all of our energy for the most part, maybe 90 some percent of it.

7:33

So it's really, in this case, the pump that we need to think about what pressure do we need to achieve.

7:36

And we'll look at an example, 2 examples, in fact, about today.

7:40

And that will start to dictate the amount of energy that we're planning to use on an annual basis.

7:45

So we can, as we're talking to, we can either pass feed radially across or parallel to the fibers depending on the type of membrane that we're going to use.

7:56

And we can collect water at one or both ends.

7:58

If we have those ends open, we tend to get a bit of a lower pressure drop.

8:03

And the pressure drop across these fibers is the thing we're most concerned about.

8:07

And that's because it's the physical pressure drop that balances against that osmotic gradient.

8:12

So we need to be very careful in terms of how much pressure drop we're engendering.

8:16

Now, the Hagen Poiseuille equation that gives us our skin friction factors that can be modified.

8:26

So we can take a term out of that here to look at that skin pressure drop.

8:31

So this is the DPDL for laminar flow.

8:34

And if we have a constant flux of material through this, we can rewrite this in terms of that flux  $J_S$  and  $W$ .

8:41

So that's one of our target variables that we'll solve for here.

8:44

And then we can directly integrate this thing to get an equation for the pressure drop across one of these membranes at under a constant flux condition.

8:55

So instead of going on about the too much about these theory, let's actually go through a few examples and then look at the calculations.

9:05

So in example one, we have a hollow fiber Meg rating.

9:16

So this has an outer diameter of 300 microns, an inner diameter of 200 microns and it produces.

9:31

So our flux moving through this is going to be 10 gallons per foot squared per day.

9:43

And the feed concentration, so  $C_1$  will be 0.1 molar sodium chloride in water at 20° C And then we're going to give this membrane a rejection percent of 97%.

10:07

So 97% salt will be rejected.

10:09

And then we need to say something about the average velocity to be able to calculate some of these pressure drops.

10:21

So we'll give this an average velocity or superficial velocity of  $0.5 \text{ cm}^2$  per second.

10:32

Now, does this look familiar to anyone?

10:37

So this is a similar set up to one of the examples you did in 3018.

10:43

We're going to take the same example basis and go a little bit farther to think about the pressure drop across this and the mass transfer coefficients around it.

10:51

So in 3018, you saw that the polarization was around 6.8%.

11:13

So that's meaning we have a difference in concentration that's 6.8% higher at the membrane surface than in the bulk solution.

11:21

So one of the the points that we identify from this is that we need to maintain good flow distribution to minimize this, so to minimize the concentration polarization so that the calculations here, excuse me, the calculations here, what is the mass transfer coefficient and pressure drop if the membrane is 3 meters long?

12:33

OK, so let's let's look at some of these calculations now.

12:37

So J of our water.

12:40

So this is going to be the flux of the water.

12:44

We said it was 10 gallons per foot squared per day, right?

12:50

And if I take a a 3M long and I have my based on the inner area, then I can say that's equal to  $4.72 \times 10^{-4}$  centimeters per second in flow.

13:04

And that's, that's based on the inner area.

13:15

Now of course the Reynolds number would be  $3 \times 10^{-2}$  centimeters times .5 centimetres per second times 1g per centimetre cubed divided by the point the viscosity .01g per centimetre second and that will be a Reynolds number of 1.5.

13:46

We will we will assume a diffusivity of  $1.6 \times 10^{-5}$  cm<sup>2</sup> per second.

13:58

And so to calculate first our mass transfer coefficient, we need to go after our Schmidt and Sherwin numbers.

14:06

So first we'll do the Schmidt number.

14:08

So that's defined as viscosity on the density times the times the diffusivity.

14:20

So .01 / 1 times and then  $1.6 \times 10^{-5}$ .

14:29

So I get a Schmidt number of 625.

14:31

And then for a Sherwood number, what I can do is use that correlation that we looked at earlier.

14:39

So it's 1.28 times Reynolds number to the .4 times Schmidt number to the .33.

14:47

So we typically see that and that gives me a Sherwood number of 12.6 which itself is equal to the mass transfer coefficient times the unit length divided by the diameter.

15:02

So.

15:05

So I can then solve this directly if I know the length and the diameter to look directly for the mass transfer coefficient.

15:13

So 12.6 times  $1.6 \times 10^{-5}$  to the negative 5th.

15:23

Sorry this is not a diameter, this is diffusivity divided by the length at 0.03 which gives me a mass transfer coefficient of  $6.72 \times 10^{-3}$  centimeters per second.

15:41

OK.

15:42

So this is our first calculation now with we also then want to take a step further and think about what is the pressure drop.

15:52

So that's a good question.

15:54

Where, where did the 1.28?

15:56

So this is one of the Sherwood correlations just for a hollow fiber membrane.

16:01

So the 1.28, if we remember back to the Nelson correlations and Sherwood correlations, there are always three different types.

16:08

Yeah, yeah.

16:09

There are always three different types of fitting factors and that's the pre factor and then the powers associated with both Reynolds and Schmidt.

16:17

So the second calculation we want to do is around pressure drop.

16:21

So if my flux is initially was based on the internal area, So if you want to convert that to look at the external area for the pressure drop  $4.72 \times 10^{-4} \times 300 \times 200$ , which gives me 7.08 times  $10^{-6}$ .

16:43

And I'll put that in meters per second now instead of centimeters per second.

16:46

And of course the viscosity water will be always we consumed as one centipoise, even if it's salt water, which is the same as one point or  $1 \times 10^{-3}$  Pascal seconds.

17:02

And then the tube diameter is going to be  $200 \times 10^{-6}$  meters.

17:12

I'm just putting that taking it out of microns.

17:16

So for pressure drop, then what I can directly apply this equation.

17:21

So one half  $32 \mu L$  on  $D$  times times  $4 J_{SWL}$  on  $D$  and this gives me an output.

17:42

Now in fact, I'll I'll plug the numbers in here, but we can simplify this quite a bit,  $.425$  factor of  $3210$  to the  $-3$  by three shows up and then I have my diameter on the bottom, so  $2 \times 10^{-4}$  quantity squared times  $1/2$ .

18:05

So this gives me now a pressure drop of  $5.1 \times 10^5$  Pascals, which is equal to  $5.03$  atmospheres, right?

18:20

So across this membrane, if it's three meters long, I'm going to take a  $5$  atmosphere drop.

18:24

So if I'm pressurizing up my initial salt water side, whatever my permeate's going to come out at will be  $5$  around  $5$  atmospheres lower right.

18:33

And this is obviously not highly precise because it's a Sherwood correlation that got me halfway here, but it but it gives me a good estimate.

18:42

So if I my feed is initially.

18:45

Just to think about the practicality of this, if the feed was initially at  $50$  bar, then my osmotic gradient.

19:09

So the difference rather in the osmotic gradient between the raffinate and the permeate will initially be  $5$  atmospheres and it will go to  $10$  atmospheres at the exit because of this this pressure drop.

19:42

OK, Now on this example, I'll do one more kind of quick modification to this.

19:50

If we for instance, half the length and the velocity, then our pressure drop is going to go to 1.26 atmospheres, all right?

20:17

So we can repeat the calculation with half the length, half the velocity and we get a significantly lower pressure drop.

20:22

So one of the take home messages here is that when we look back at the nature.

20:27

And this equation is certainly not linear, but rather with the square of length.

20:32

So the longer I make these twos, the more pressure drop I'm going to take across them.

20:38

Now, for that reason, this is where we tend to see relatively short membranes and a whole bunch of them running in parallel, right?

20:46

And why we go from right some integer  $N$  1st stage units and we take that and drop it into a second stage unit because in fact we don't need a whole lot of performance out of that.

21:00

We only need to grab the last few percent salt that that we might be able to extract.

21:12

Sorry, that's a good question.

21:14

The pressure doesn't increase across the membrane.

21:17

It's the differentials of the osmotic gradient.

21:21

So this delta, now I'll switch back to blue.

21:26

This delta  $\Pi$  term is the osmotic gradient.

21:30

So the difference in the osmotic gradient between the what's on each side of the membrane.



21:48

So, so that, that is because we are changing the concentration of salt in the system.

22:00

So if you imagine at the beginning of the membrane when the the feedstock first comes across, it is you would say it was rich in water and it's going to start losing water into this using a a spiral Wan, it's going to start losing water into the product water stream, which means the concentration of salt is going to increase in the feedstock as it's coming through the membrane.

22:27

So the osmotic pressure of that feedstock is going to increase at the end of the membrane relative to the beginning of the membrane where we're going to maintain relatively pure water in that centre pipe.

22:41

So if I think about if the feedstock is going to come in on, on this side, then we're looking at the difference, the difference in.

23:19

So the osmotic gradient is defined at the feed point relative to the osmotic gradient at the end of the membrane.

23:36

Does that make sense guys?

23:44

OK.

23:44

So one of the kind of key questions if you are going to use an RO membrane for this miniature design project is that you want to make sure the pressure drop that you take across the system because of course it's  $\Delta P$  minus  $\Delta \Pi$  is our driving force.

23:59

So we we need to maintain a relatively consistent driving force in both or throughout the membrane process.

24:10

So if that's why we want to do some of these calculations, you can actually look at that and say we're going to take an average driving force as we go into the design on this membrane.

24:18

OK, so next switch back to red here.

24:24

Let's, that was an example where we already knew what the the flow conditions were going to look like and we wanted to calculate some of the the properties, or rather we knew what the concentrations were in the membrane properties and we wanted to target the flow conditions.

24:39

So let's flip that around and say that we know something about the membrane, about the sorry, we know something around the flow conditions and we want to target what are the concentrations in the product streams that we can get out of this.

24:54

So in this case, we're going to look at an RO membrane to purify water with 2.5 grams of sodium chloride per litre of water at 25° C.

25:24

Now this initial condition, right?

25:28

This will give me a density, we'll call that density one.

25:33

So my initial density will be 999 kilograms per cubic meter, and this also gives me a concentration 1 of 2.5.

25:48

I can write kilograms of system chloride per cubic meter of water.

25:58

So that also means that feed contains 596.5 kilograms of H<sub>2</sub>O.

26:08

Your cubic meter solution.

26:14

Now from the table or the analytical method we looked at last time, we can calculate the osmotic pressure at .1 will be 1.97 atmospheres.

26:25

So this is taken directly off that table and we know that the membrane has a permeability constant or has a permeability constant of.

26:52

So for the water slide will be  $4.81 \times 10^{-4}$  kilograms per second per meter squared atmosphere and for salt solution 4.42 times the  $-7$  meters per second.

27:13

And our calculations, we want to 1st calculate so water and salt flux across the membrane, so the solvent and the solute flux if the pressure drop is set at 27.2 atmospheres.

27:42

Secondly is the solute rejection.

27:48

So if we know what the membrane permeability looks like and we've already set what the pressure's going to be, then once we know the feedstock, we can calculate the solute rejection And then finally from that knowledge, the product concentration.

28:11

OK.

28:12

So to start what we'll do, we would take an iterative design process here because we're not in, we need to assume the properties of the product to 1st or to then calculate the driving force to get our flux to then calculate the properties of the product.

28:28

So to start, we're going to make what is ultimately going to be a reasonably good assumption.

28:37

We know that  $C_2$  has to be less than  $C_1$  significantly, so we'll assume that  $C_2 = .1$  kg of sodium chloride per cubic meter, which yields a density of 997 and a osmotic pressure of point O 8.

29:09

All right, so this means that my change in osmotic pressure is 1.89 ATM.

29:20

So I can relate the water flux to that solubility or sorry permeability constant times the driving force for flow or for mass transfer.

30:12

OK, So this is my water flux across that enbrane on a per square meter basis.

30:19

Now I can also if I want to start unpacking this, I can to get to the rejection fraction.

30:29

And so this is pulling a method from Gene Coppelis.

30:31

It's not gone into in detail in The Cave, but we can define the rejection fraction in terms of B.

30:41

So this parameter B that we'll look at in a minute.

30:43

$\Delta P \text{ minus } \Delta P_i \text{ on } 1 + b \text{ of } \Delta P \text{ minus } \Delta P_i$ .

30:52

And that's also equal to the difference in concentrations divided by the inner concentration.

31:09

And so in that definition, we've defined B as  $PW$  on the diffusivity as transfer coefficient and concentration of water at the outlet.

31:25

And that's also equal to the relative permeability on the water relative to the soda salt.

31:33

So then relative to the solute times the outlet concentration.

31:37

So this is just a way to rearrange some of our working equations.

31:41

So because I've defined  $C_{2AW}$  and ASI can define B here as one point O92 inverse atmospheres.

31:53

OK.

31:54

So we'll go to the next slide is a blank.

31:59

OK.

32:00

So now because I have that definition of RI can get at R directly if I have knowledge of B.

32:07

So R is going to be one point O nine 2 \* 27.20 - 1.89 on one plus one point O nine 27.20 - 1.89 which equals 96.5%.

32:27

So we have a rejection of 98 six .5%.

32:31

Now I I can also up top I've re related that rejection fraction to  $C_1 \text{ minus } C_2 / C_1$ .

32:40

So now if I go back and I say well I don't know what  $C_2$  is so I want to solve it here.

32:48

I can solve for  $C_2$  directly is point O 875 and that will have units of kilograms of sodium chloride per meter cubed.

33:00

So this is close enough to our assumed value of  $C_2$ , which is 0.1, that the osmotic pressure will not change significantly.

33:24

So in this case we can accept this assumption or we can repeat the calculation to converge it a little more finely.

33:32

Now if I wanted to calculate the salt flux across this similarly, it would be the relative permeability of the salt times that same concentration gradient.

33:44

So I can write this as  $4.42 \times 10^{-7} \times 0.50 - 0.0875$ , which gives me a salt flux across the membrane at one point  $0.6 \times 10^{-6}$  kilograms of sodium chloride per meter squared.

34:07

Second.

34:07

OK, so this will bring us to the end of the the lecture for today.

34:15

So what we've seen now is that if we're going to go through a design process for an RO membrane, we need to either know or assume the permeability constants for in this case, our water and our salt.

34:28

We need to know or assume the inlet water composition.

34:31

So that's not too hard.

34:33

We need to make an engineering decision as to what our pressure upstream is going to be and then figure out what that pressure drop is going to look like.

34:41

So our ultimate engineering decision is the pressure upstream and then we make an

engineering decision on the type, thickness and length of the of the membrane and we can see examples.

34:52

Then we need to either know the mass transfer coefficients and pressure drop.

34:56

I'm sorry, we actually know the flow conditions and we can go out to the mass transfer coefficients and pressure drop, or we need to know the properties of the membrane and how to go after the performance of it, meaning the flow conditions.

35:08

So I think this latter example that we've done is, is quite directly applicable, I hope to what you'd be looking at in your design, miniature design projects.

35:18

So it should be relatively easy to apply this in and we're simply looking to get at a slightly more purified solution, but then additionally with a much, much higher throughput.

35:31

So you're going to need a number of these membranes working together.

## Lecture 28 - Membrane Separation

0:00

We're wrapping up kind of our, our last point around adsorption in particular focusing on a, a unit operation deploying adsorption as chromatography and then so we can either use liquid or gas as the basis for this.

0:15

And we like chromatography because it allows us to, you know, certainly it's useful for batch characterization.

0:22

So if if you were working with fluid and you want what would be called a chemical assay of that to figure out what's in that fluid and what's the distribution, that's where you might hire someone to do a batch characterization for you.

0:34

But also if you're an engineer designing a process and you want to put that make that an online sampling system, chromatography is one of the kind of two core methods that we as engineers have available to us.

0:47

So one of the the points that we talked about there was that you imagine if an engineer is trying to design A process and in particular starting to use things like machine learning to refine that process.

0:59

Then fundamentally all of those kind of machine learning algorithms, even if they're based on a phenomenology.

1:06

So some kind of set of fundamental first principles equations that are predicting what should happen in the process and then refining, for instance, a valve opening or flow rate to optimize the process for real time.

1:20

The ability for those machine learning algorithms to deploy their optimizations is limited by both the number of data points you have or whether you're collecting on the process, but also the depth domains of the data points you can collect.

1:36

And so if we go back to our fundamental Thermo, we have DGSVDP minus SDT plus the sum of MU DN.

1:50

So we can see that there is a, a pressure domain, a temperature domain, or rather a mechanical, thermal and chemical potential.

1:58

These are the three domains.

1:59

And so part of the reason that chromatography is quite critical and it's not taught in in a lot of the earlier versions of books from say the 90s or even early 2000s.

2:10

But historically, engineers have always relied on pressure and temperature measurements to optimize their process or to figure out how well the process is performing.

2:19

But when you start getting into online chromatography and the other domain method that can be used to capture this chemical potential.

2:33

So we can use either chromatography, I shouldn't say, or spectroscopy.

2:41

So what we start to see is the more and more online processes, and this is certainly an emerging field, but more and more online processes are using other chromatography or spectroscopy to characterize the chemical content of the flow stream.

2:57

And in particular because most flowing systems that we as chemical engineers deal with are going to usually be in the cross section of the flow stream at mechanical equilibrium.

3:08

So meaning if we're using a square ducted pipe, then the top of the duct is the same pressure as the bottom of the duct.

3:15

In general temperature, we tend not to have a lot of cross-sectional temperature distribution.

3:20

So wherever we measure, it's going to be the same.

3:24

But you imagine each of those, if we have the cross section, then pressure and temperature tend to usually be equilibrated in the cross section of the line, meaning that every time we take a measurement, we only get one measurement as a function of length down a pipe, we're down a duct.

3:41

Now chemical potential or measuring the chemical composition right is quite useful because in particular of something like high resolution spectroscopy or chromatography as basis will give you the breakdown of how many chemicals are present and what their distribution is.

3:58

So the fidelity of that data.

4:00

And we think about chromatography, if we have a 5 component mixture.

4:08

So in a 5 component mixture we get five data points right?

4:19

So in the world of machine learning and and so-called big data, the fidelity of the data that we get by measuring compositions is becoming more and more important.

4:29

So that's something that that you as kind of budding engineers should think about because there's a lot of impact to be made in current generation process design and optimization when we start collecting data on chemical composition.

4:44

And ultimately these are all informing that overall flow of Gibbs energy, be it mechanical, thermal or chemical.



4:52

So when we do take chromatography measurement, these are designed for multi component feedstocks and the components with the highest pressure, meaning lowest molecular weight are going to breakthrough that column first, so they have the lowest retention time.

5:05

And it's related to how the components are going to partition between the feed stream and whatever solid adsorbent material we're using.

5:12

So the partition coefficient is going to be inversely proportional to the chemical activity of the component and its vapour pressure.

5:20

And in fact, today we're going to see another application of component activity as we go into membranes.

5:28

So you know, I appreciate that, that you all had quite a few lectures in 3018.

5:34

I think lectures 7 to 10 were all around membranes, both their theory and their operation on the gas side of the liquid side and that there were some exemplar discussions of reverse osmosis for water desalination.

5:47

So obviously, as I, I appreciate everyone can see at this point, we have crammed in quite a bit to this unit to really flesh it out the way it should be done.

5:57

So because we have previous knowledge in 3018, I'm not going to go through and repeat the basics on membrane operation.

6:04

You would have seen applications of mass flux and examples of using diffusivity up to this point.

6:11

So instead what we're going to focus on is the application of membranes specifically for desalination, but we're going to target into the actual engineering.

6:22

So how do we design a membrane system to deliver a desal product?

6:26

And obviously, this is one of the critical tools that we have at our disposal when either, you know, addressing or combating climate change, right?

6:35

Because the need for fresh and drink a pot of water is certainly going to increase in many regions on the planet, many of which had never had to think about access to freshwater, right?

6:48

They've either had inland aquifers.

6:49

For instance, the the middle of the United States has one of the largest freshwater aquifers in the world called the Ogallala aquifer.

6:57

And unfortunately, about 60 years ago, it was recognized that the farmers were pumping freshwater out of that aquifer significantly faster than its replenishing rate.

7:08

And even though this was brought up by the scientific community, the farmers never stopped.

7:13

So the Oglala aquifer has been depleting at a, at a rate of a few percent per year for about 50 years.

7:21

Obviously it's, it's a, a graded scale and so if you do the calculus on that, you can pretty quickly identify that actually in about 10 to 20 years, then that aquifer will run dry and the evidence for its depletion.

7:36

If you go to the states Nebraska, Kansas, Iowa and and Oklahoma, it's kind of that middle of the US where it's a farm built.

7:46

You can actually measure the entire four states in the middle of the US are sinking at a rate of a few centimeters per year because of the depletion of that aquifer.

7:56

And there's really no plan as to how all of those farm communities are going to survive outside of what's probably going to be desalination methods, right?

8:06

And putting these along the coastline, pulling in that seawater and then figuring out how do we get the salt, what's the best vehicle as we can continue farming and agriculture without sending these communities into to bankruptcy.

8:22

So this is going to be a really critical tool as we start trying to engineer our way forward with climate change and figure out how to move with it.

8:32

Now, this also constitutes a really core application when we start thinking about membranes.

8:43

Oh, sorry, is that can you hear me better now?

8:50

OK.

8:53

I'm using a 5G Internet, so it might dip in and out every now and then.

8:58

So this is a really cool, but please let me know if it dies and I can repeat what I just said.

9:02

So this is a really core unit or core application of membrane separation and it's relevant.

9:09

When we kick off tomorrow morning at 9:00 AM, I'll actually be walking you through this final laboratory report and what we're going to do here.

9:17

So I appreciate in 3018 the project was thinking about the CapEx and optics of a membrane based desalination system and then the pipe network to get that down to birth.

9:28

What we're going to be looking at in lab report 5 is the engineering of desalination.

9:34

So the project is to deliver desalinated water, so potable water to Derby using a method of your choice, your groups, I should say choice.

10:03

So I'll go through the group layout tomorrow and it's going to be a relatively quick project.

10:08

There's only a handful of questions I'm going to put up in it, but the idea is that I'm going to ask you to think about, you know, we've talked in this class now and also in thermodynamics about a number of methods that are available to the engineer to what will all effectively be purifying seawater.

10:27

And these membrane based reverse osmosis, these cells is one of the, you know, more common applications, but it's certainly not the only one.

10:36

So we can think about something like flash distillation, so a single stage flash tip to pull the water out and then condense it.

10:43

We can think about a kettle type boiler.

10:47

We can think about using maybe an absorption process, although I don't think that would be very efficient.

10:54

And certainly we can think about reverse osmosis membranes.

10:57

So the idea is basically pick a process, either heat transfer or either the common mass transfer processes.

11:04

And because we've already worked these up in with the exception of absorption, we've worked these up in Isis.

11:09

Then it's relatively straightforward with a few lectures we're going to have this week on RL membranes and how we design them and what their flow rates are going to be.

11:17

That you can basically say here's a, a nominal engineering design for a plant and here's the scale of the energy requirements that will be required to deliver the nominated freshwater rate for Derby.

11:32

So it's a relatively simple project.

11:34

We're not covering any new material, but I want to to give you an open-ended design project that's going to let you kind of flex your muscles as engineers a little bit and make some choices.

11:44

It's one of those projects where there's no wrong answer.

11:47

The only answer that's wrong is the answer that's not justified.

11:50

So as long as you make a call and justify, I'm happy with the call you made and we'll go through that in more detail tomorrow.

12:01

OK, so with that, let's get into a little bit about a little bit about membrane separation.

12:08

So I'm not covering the gas side.

12:10

We're instead looking at membrane separations for solutions, meaning we have some kind of contaminant in our liquid phase that we want to remove.

12:23

Some really classical examples here are dialysis and desalination.

12:27

So an example of a dialysis membrane shown below.

12:30

Now this is designed on the basis of having three species in our feedstock.

12:35

So species A, this is a low molecular weight solute, Species B is an intermediate molecular weight solute, and species C is a colloid.

12:45

So a very large molecule that's actually an amalgamation of other smaller molecules.

12:50

So that colloid, because there's a large molecular diameter, isn't going to be able to fit through this membrane.

12:56

And what we can see is if we go from the feed side, we can see that there's a boundary layer.

13:02

So this is boundary layer, boundary layer one and the nomenclature, this is boundary layer two.

13:12

And then we have some kind of transport resistance from the membrane.

13:16

So if we start with our low molecular weight solute one or A, we have our initial concentration.

13:22

So  $C_A$  one is the concentration of species A in the feedstock.

13:27

We can see there's going to be a slight dip in the concentration as we go from the middle of the continuous phase into that boundary layer one and that film where the boundary layer will engender a film mass transport resistance of  $K_1$ , little  $K_{1A}$ .

13:48

We're then going to take a torturous diffusion path across that membrane.

13:53

So we have little  $K_{MA}$  and we have a a smaller concentration gradient across the 2nd boundary layer on the product side of the membrane.

14:04

So that's a little  $K_{2A}$ .

14:07

And then we go from there out into the bulk concentration and the product stream of component A, so which we call  $C_A$  two.

14:15

So if we were to write the mass flux, right, we can see that they're here basically 4 concentrations of interest.

14:24

If we were to write the the mass flux of component A, then we write that or the general flux equation shown here is an overall mass transfer resistance multiplied by the driving force, which is the concentration difference between the feedstock and the product stream of component A.

14:45

Now this overall mass transport resistance, we can break in just like we did with our with any of our unit operations for mass transfer to date by thinking about if we have a convective contribution, a conductive contribution and a secondary convective contribution.

15:07

So we have  $K_{1A}$ ,  $K_{MA}$  and  $K_{2A}$  and these are added in, in, in a series resistance stream.

15:15

So we take the reciprocal.

15:18

And what this means is that effectively the in, in most cases, one of these will dominate, all right.

15:27

And so whatever the dominant value is tends to, to typically be.

15:31

So in this case, it's the smallest value of transport resistance tends to be what will dictate the overall mass transport resistance.

15:39

Now if we focus in just for a minute on this KMA, so that's the membrane diffusion.

15:48

This is equal to the diffusivity across the membrane divided by its thickness  $Z$ .

15:58

And this is the effective diffusivity.

16:09

And this is commonly what controls diffusion across the membrane itself.

16:25

So little KMA tends to be the controlling constant.

16:29

And so we can typically characterize the mass flux based on or the general flux based on this effect of diffusivity.

16:35

And if we know the thickness of the membrane, that tends to give us a reasonable estimate for our transport terms.

16:41

Now this illustration of dialysis, of course, one of the most common examples of this is in medical dialysis.

16:49

And so they're using this process to replace the the original functioning of the kidney to strip out contaminants.

16:56

So we're not going to go through an example of that, but instead we're going to focus on Desalination.

17:01

And this works on really interesting principles.

17:03

So I went through some of the 3018 lecture slides and I can see how osmosis, osmotic pressure and reverse osmosis had been framed in the past for you.

17:12

And I wanted to, to kind of take a moment because I think I don't know if I can get a quick survey up.

17:22

It's 2 seconds and I'll I'll pull up the lecture chat and give you a quick Teams poll to vote in.

17:45

There we go.

17:51

OK, so I just put a quick poll up for you.

17:53

If you can, I'll take a quick vote.

17:55

How comfortable are you with reverse osmosis?

18:00

Just because I'm interested to see.

18:02

I think if we've gone through Thermo in the past semester, we can also frame this in a slightly different way.

18:09

So go ahead, if you can pull up the Teams chat and just take a a quick vote when you get a second.

18:14

So when we think about the process first of osmosis itself, we'll get into osmotic pressure in a minute.

18:24

But the idea behind osmosis is that if I have some kind of membrane that is permeable to my solvent.

18:31

So of course we're going to focus on the application here of seawater.

18:35

So if we take our example of seawater or I'll say it's salt water.



18:40

So we'll talk about that in, in a minute.

18:53

So in the example of salt water, my solvent is water and my solute will be sodium chloride.

19:00

So the ions that I'm trying to get rid of.

19:02

OK.

19:02

So thanks for the vote.

19:04

Let's see if the end of this whether this framing helps a little bit for the concepts to reverse osmosis.

19:10

So if I have this salt water, I have a solvent that's water and a solute that's the salt.

19:15

And I'm trying to to to purify.

19:17

So I want to concentrate sodium chloride to to one side of my bin and I want to purify, get purified water on the other side of my bin, right?

19:27

So I'm basically going against the gradient of entropy here, right?

19:30

Because I'm taking what is a very mixed up system and I'm trying to separate it into two, one that's more pure and salt and one that's significantly less pure and salt.

19:39

Now the if I'm thinking about osmosis, then the membrane that I have of course permeable to my solvent, but nearly so.

19:48

It's not perfect but nearly impermeable to my salt used so water can flow through it but salt can't.

19:56

Now what makes this then osmosis is that my Sol bent, meaning my water.

20:02

If I have these two bins, my water is going to want to naturally diffuse from if my bin of low concentration.

20:12

So on the bottom here I'll have low sodium fluoride concentration.

20:16

And in the right hand bin, I'll have high sodium chloro concentration.

20:22

If that's the case, then naturally the water that's right here on this barrier, right?

20:28

So I can see if I zoom into those molecules that are sitting on the barrier, right, they're going to notice and especially the, the molecules that are inside the, the, the membrane in the 1st place.

20:40

So I focus in there.

20:42

Those molecules that see a lot of salt in that water are going to look up and say, hey, well, I have a low concentration of salt on one side of me and a very high concentration of salt on the other.

20:55

But for the the way that the water molecules look at it, they see a lot of themselves on one side and not a lot of themselves on the other.

21:04

So they're going to want to spread out, right, so that they distribute themselves evenly everywhere, right?

21:11

So they're going to naturally flow by water's going to flow from a region of low concentration of solute, meaning there's a lot of water and it's all it's seeing is it's own cell to a region of high concentration of salt, meaning there's not there's not a lot of water, right?

21:26

And so the the water's going to naturally want to flow back into the salt water stream.

21:32

Now a way to think about this is that activity is lower in the the more concentrated.

21:46

So before I write this, if I think about this region here I have low salt content on one side, high salt content on the other.

21:54

Another way to look at this, I have high activity of water on the left, right and I have low activity of water on the right.

22:13

And I can think about that, that let's say when I say high concentration.

22:17

So I'll get rid of these qualitative ways of looking at it.

22:21

Let's say I have 20% sodium chloride on the right and 1% so I didn't chloride on the left.

22:31

Well you'd remember from from undergraduate thermodynamics that activity, so a metric of non ideality in solution activity is always going to be the highest as I approach the infinite dilution coefficient or the infinite dilution state.

22:47

So as I go to lower and lower concentration of my solute, the activity will go up and up and up right until I hit that infinite dilution boundary.

22:56

So what brings this home is that activity is proportional to chemical potential, right when I get to dilute systems.

23:08

So if I'm at a reasonably low say a few percent, kind of a few mole percent of my cell mute, then in fundamentally the activity of the solution is directly proportional to the chemical potential.

23:24

So this means, right, this gives rise to the the reality that it is a chemical potential driving force, right?

23:33

So my water flows, the water from my solvent flows from high chemical potential to low chemical potential in this system, right?

23:54

So if I think about if I go back to that relationship for Gibbs energy, all right, it's this right hand term, the chemical potential and the number of molecules.

24:06

So I multiply these two together.

24:08

So it's a molecular Gibbs free energy that's driving the migration of water from the water side to the high salt content side.

24:16

All right, So this is kind of a thermodynamic framework to understand the process of osmosis.

24:23

Now where that starts to get really interesting is if I apply.

24:28

So I'll switch colors here.

24:30

Do not get too confusing if I start applying pressure right to the sorry, sorry.

24:42

If I start applying pressure or if I modify the temperature of the system or if I modify the temperature of one of the containers, I can use that as a way to control the chemical potential.

24:56

So if I go back to this relationship for a minute,  $GDP$  minus  $SDT$  plus the summation over  $I$  of  $MU_{IDN I}$ .

25:08

Well, if it's fundamentally the product of this term that's driving my process of osmosis, I can play with either the mechanical potential or the thermal potential across the system to counteract that, right?

25:25

So in this case, osmotic pressure is meaning if I start increasing or dialling up the pressure in the system and we could do temperature, but what you're going to see shortly is the amount of pressure required to revert or to control or reverse the process of osmosis that is extraordinarily high.

25:45

So you can only imagine the amount of temperature that we're talking about and that's not quite tractable from a materials integrity standpoint.

25:53

So if I start dialing up that pressure, I can throttle the ability for chemical potential to control that grading.

26:02

And if I start dialing up the pressure enough, then at some point, right, that is going to hit the same driving force, right.

26:13

So if I, I dial up pressure on the salt water side, then at some point that's going to hit the exact same Gibbs energy driving force that my natural osmosis process is using in chemical potential, right.

26:36

So that chemical potential in my driving force for osmosis and I can dial up the pressure turn to reverse osmosis.

26:52

So if I keep continue increasing that pressure above, so, so when they're equal and I have no net Gibbs energy driving force, we call that the osmotic pressure.

27:04

And we're going to give that the variable  $\pi$  So that's the exact pressure required to stop the osmosis process.

27:10

But if I dial up my pressure even more than I can reverse it and I can drive fresh water from the high salt side to the low salt side right against the chemical potential gradient.

27:27

So that's fundamentally where we get the term reverse osmosis from.

27:33

But realistically we can think about this as a, a Gibbs energy distribution across the system, right?

27:39

And, and fundamentally we'd only ever have three levers to pull and that's our mechanical potential, our thermal potential and our chemical potential.

27:47

So hopefully that adds a bit of clarity to to the process of reverse osmosis that we're going to be using.

27:55

So when we think about reverse osmosis, then the as at least the the writing of the McCabe textbook and and from my understanding thereafter, the exact mechanism behind this is not really that well understood.

28:12

So we we have proposed a mechanism called solution diffusion to explain what we observe in reverse osmosis.

28:20

And this mechanism invokes that the concentration of water, water inside my polymer will be proportional to the water activity in solution.

28:33

So when I'm on the low pressure side, what we say is that the activity is going to be unity and that means a value of one point O if I have pure water at one atmosphere.

28:45

All right.

28:46

So the the low pressure sign for reverse osmosis, this is my product water product or potable water.

29:02

And on the high pressure side which is my saltwater inland this will typically have a pressure of 20 to 50 atmospheres.

29:09

So if we go back to thermodynamics, we can see that the amount of, if we were to use the thermal potential to, to drive against chemical potential, the amount of energy that we have to drive in from a thermal standpoint is extraordinary.

29:23

And if we're talking a high, high pressure side of 50 atmospheres, right?

29:27

So that's a lot of pressure.

29:30

So on that high pressure side, if I have something like 5% sodium chloride, then my activity is going to be less than one point O if I'm at the same pressure as the as the product side be equal to 1.0 when I'm at the osmotic pressure and greater than 1.0 ever so slightly if I'm above that osmotic pressure.

29:52

So I'm using the the absolute pressure of the system that throttles activity and as a consequence chemical potential.

29:59

All right.

30:00

And So what we can see here is we go from a feedstock to our permeate or our product water.

30:10

I can see that my activity of the OR the concentration of water is going to dip on the feed side while the concentration of my salt lines is going to increase.

30:24

And at the end of the day, I have this little skin effect in the night.

30:28

I ultimately get a concentration here of activity that or is equal to my activity of water.

30:35

Now I can calculate the flux of water right through this this porous membrane if I can think about.

30:45

So  $D$  is the water or sorry, the diffusivity of the membrane  $C_{\text{sub } W}$  is the average water concentration row  $MW$ .

31:17

I'll draw it over here.

31:18

Row  $MW$ .

31:20

This is the partial molar volume of water.

31:33

And then it's this is multiplied by a netted driving force.

31:38

So  $\Delta P$ ,  $\Delta P$  is going to be the pressure difference applied and that's less the difference in osmotic pressure between the feed and product.

32:14

So the amount of pressure I applied, this is one of my engineering decisions that I'm going to choose, right?

32:19

And that's going to ultimately dictate the amount of water that I get through.

32:24

So you can immediately see that, you know, one of the constraints in a process like this is I need to apply a significant amount of pressure to dial all this up because my membrane diffusivity has been selected.

32:36

The water average water concentrations will be dictated as an outcome and flux.

32:42

The partial molar volume of water and  $RT$ , these are all from a physical constants or Thermo physical conditions or properties.

32:51

But it's really the osmotic pressure difference is a product of the amount of pressure that I apply.

32:56

So that's, that's really my key engineering decision that I get to make.

33:01

And that's where the the primary expense is going to come from the design of this kind of system.

33:06

Meanwhile then the flux of the solute, so my salt in this case the flux of that moving through my membrane and into my product stream will be equal to the diffusivity.

33:26

Now this S term, this is the we can call selectivity or distribution coefficient.

33:41

So that's our second application of that distribution coefficient that we were looking at.

33:45

And this  $\Delta C$  here, this is the difference in the solution concentrations before and after, right?

33:51

So that  $\Delta C$  is going to come from the  $C_{sub F}$  or  $C_{sub SF}$  and the  $C_{sub SP}$  or permeate as I, I exit the system and then  $Z$  in both cases is the thickness of my membrane.

34:06

OK, So one of the, the key observations here is that our our top equation for the flux of fresh water through or pure water through the membrane that this the water flux increases directly with the pressure difference once I exceed the osmotic pressure of the system.

34:26

But I do not really have a dependence of salt or or solute flow on the pressure difference.

34:34

So the next question then is how do we calculate this osmotic pressure specifically for sodium chloride solutions and sea salt solutions?

34:46

No, Gus, I don't think they do.



34:49

I'm just illustrating that from a fundamental standpoint, I think.

34:51

I don't know if they did the exploit temperature ever, but but to my knowledge, every system I've seen is driven by pressure.

35:01

So if we think about reverse osmosis for desalination, then it's not uncommon that earlier systems can get 10 to 20 gallons.

35:09

And I apologize for the imperial units here, but 10 to 20 gallons per square foot.

35:14

So that's per square foot of surface area and the membrane or square foot of surface area of flow into the membrane, not membrane surface area per day with a higher salt projection.

35:27

So 95 plus percent and that was the employing asymmetric membranes as polypers and DuPont Holo fiber membranes.

35:36

These can support up to one to three gallons per square foot of pipe area per day.

35:42

And the current generation of what we're targeting now that we're going to get into this week and that which would be available to you if your group does want to select it would be a 99% salt rejection membrane.

35:53

So to get there, what we'll do is first think about sodium chloride solutions, which is the left hand side of this table.

36:03

And so if I characterize this as so gram bowls of sodium chloride per kilogram of water, right.

36:13

So I can see here that the density is going to increase as I add more and more salt to the system.

36:19

Interestingly, this is actually a really obviously an example of a colligative property and something that a lot of engineers tend to neglect.

36:30

So if you're putting in salt, sodium chloride or any other into an aqueous system, a lot of folks tend to not think about, well, is this going to affect my density?

36:39

And in fact, as you can see, just a couple percent of salt in this system will have a very significant impact on the density.

36:50

And then I also can see that the osmotic pressure is going to increase directly with the amount of sodium chloride in my system.

36:57

Now sea salt is a bit interesting because this tends to be or is an amalgam of a huge diversity of different ions.

37:06

So it's not uncommon if you're working in a laboratory environment that you might use sodium chloride to initially make up it just an exemplar solution.

37:17

It's going to be replicated of sea salt, but in fact you can see if we look at and convert those the the moles per kilogram H<sub>2</sub>O over to a weight percentage basis.

37:29

In fact they're not quite 1 to one when they get to an osmotic pressure standpoint.

37:34

Now, it's worth noting that nominal seawater from anywhere on the planet has a salt content of about 3.45%.

37:43

So this is a a really important number that's going to be useful to remember that whenever you're dealing with seawater, you're just under 3 1/2 weight percent of salt ions, and that leads to an osmotic pressure of 25 atmospheres.

37:55

So as we're thinking about this equation, right, if we need to bring this down to a salt content of near zero, then we can see the osmotic pressure difference there.

38:04

And that tells us how much differential pressure we're going to need to apply.

38:10

And So what I can, I can also plot this if I think about the both the flux of water through the system as well as the salt rejection.

38:19

And I can see here that there's virtually no impact on the salt flux and it's very, very low.

38:25

But I can see that I need to 1st exceed the osmotic pressure before I get any breakthrough of flux of water flux in the system.

38:34

And then I have the linear dependence.

38:36

So the higher the pressure, the more water I can get out of that system.

38:39

All right.

38:40

So this then gives us a trade off between the amount of pressure that we want to pay for in terms of our our pump water phase pump and the surface area of the membrane, which is going to be dictated by the OR dictate the cost of the membrane.

38:55

And I also do get an improved rejection here.

38:59

So the higher the pressure I go, the more salt I will concentrate and the more fresh water I will I will push out of the system as a, as a product of the material balance.

39:11

So a couple common values then to think about.

39:18

South DW is typically for these kind of systems, 10 to the -6 centimeters squared per second.

39:27

So we can use this value.

39:29

The fusion tests in a polymer film, and these were were shown in the text, show us that our selectivity or partition coefficient for salt in these systems tends to be around 10 to the -9 centimeters squared per second.

39:53

Oh sorry, Nope, read the wrong line.

39:55

There tends to be around point O35 and the diffusivity of salt is around 10 to the -9 centimeters squared per second.

40:11

So if you know what your initial concentration of salt from your sea water is going to be right, you can assume if you want 3 1/2 weight percent salt water or 3.45 weight percent so.

40:23

If we know what that initial concentration is going to be, we can use a tool like multi flash to quickly run that up.

40:31

And I'll, I'll run through demo tomorrow.

40:33

But to quickly run that up into get what the molar volume of that's going to be.

40:37

And then pretty much we know what our operating temperature and our membrane think this is going to be.

40:41

We can use that to calculate what's the flux of water across this thing as well as the flux of salt.

40:49

OK.

40:50

So finally, in terms of the osmotic pressure and, and calculating this outside of having, for instance, a, a look up table to use, we might want to be able to calculate this directly.

41:01

And so we get van Hoff demonstrated there.

41:03

This follows a similar relationship to that of an ideal gas.

41:07

So our osmotic pressure that we're targeting, and that's the, the Greek symbol  $\pi$  this is what will be equal to  $nRT$ , right?

41:16

Where  $N$  is the the number of kilogram moles of solute and I multiply this by the number of ions.

41:39

So in the case of sodium chloride, when I dissolve this into water I generate 2 ions, so I multiply that value by two.

41:48

$V_{\text{sub M}}$  is the volume of the pure solvent.

42:04

So it's salt and water in this case in cubic meters that are associated with that kilograms or kilogram moles of solute or this is going to be my gas constant.

42:20

So that will be  $82.1 \times 82.057 \times 10^{-3}$  meters cubed atmosphere per kilogram mole per Kelvin.

42:40

All right.

42:41

And  $T$  is obviously my temperature in Kelvin at that point.

42:44

So if we look at a quick example of doing this kind of calculation to think about calculating the osmotic pressure for .1g moles of sodium chloride in 1000 grams of water, this gives a density of point or sorry, 997 kilograms per meter cubed at  $25^{\circ}\text{C}$  just to get our firm of physical properties first.

43:26

So in this case, I can write  $\pi$  is going to be equal now to two times.

43:33

So I have my  $N$  values going to be first, but I'm going to multiply this by two because sodium chloride is going to dissociate into two ions, the sodium and the chloride.

43:42

So two Times Now, one times  $10^{-4}$  because I need to put this in kilogram moles and I've given you the prompt and gram moles just to demonstrate divided by the volume of pure solvent.

44:05

So this is going to be one point O Keep seeing sig things divided by the density 997 shouldn't say point O be consistent.

44:18

And I multiply this now by my gas constant  $82.057 \times 10^{-3} \times 298.15$  and this gives me 4.88 app series.

44:37

So if I go back, because I'm dealing with, if I go back to to this table here, I can see that .1 my osmotic pressure was around 4.56 atmospheres and what I've generated from this equation is 4.88 atmospheres.

44:54

So we're reasonably close.

44:56

And this then gives me the ability if I want to plug and shut this in a spreadsheet and figure out well, how does this system vary?

45:03

How does the performance vary with the amount of over pressure that I apply beyond the osmotic pressure?

45:07

This will be a more convenient means to do that instead of interpolating between those different table points.

45:15

OK, So I know we're right on the 50 minute marks.

45:18

So to wrap up for today, so the the membrane applications, we're focusing here on liquid membranes.

45:25

And so obviously dialysis is probably one of the most common from a medical standpoint, but water desalination but critical in terms of what we're talking about as a unit operation.

45:35

And then if we base then this on reverse osmosis, typically osmotic systems will have a solvent flowing in the from the direction of low concentration to solute to high concentration solutes.

45:48

It will naturally follow the chemical potential gradient.

45:51

But if I start applying pressure, I can equilibrate the Gibbs energy in the system.

45:55

So bring a Gibbs energy differential to zero.

45:57

And if I dial up the pressure even more, I can reverse that chemical and overwhelm the chemical potential gradient with my pressure gradient, so I can effectively flow solvent from my concentrated system to my fresh system.

Lecture 30 - Liquid Pumping and Metering

0:00

The the lecture 29 concept.

0:01

So in the last lecture we worked through membranes specifically for reverse osmosis.

0:09

So one of the the key kind of points of that is that in working through two specific examples, one where we were trying to determine the mass transfer coefficient within the membrane and the pressure across it, if we knew something about flow conditions leading into it.

0:26

And the 2nd, determining what those flow conditions are, look like if we know what the membrane properties are going to be.

0:32

And in fact, that's quite relevant because those membrane properties are ones that you can use to inform your RO membrane design if you want to.

0:39

So that means the only thing you really need to determine is the flow condition.

0:42

And if you're going to to determine those, then the missing link is how do we pump them?

0:55

And then what are the energy requirements behind that?

1:05

So what I've put together for today, this kind of steps back into Chapter 8 where it's now more appropriate to talk specifically about pumps and how we characterize them and, and how we use them.

1:17

But of course, any time we're going to talk about a pump, it's because we're connecting a pump to pipe work.

1:22

So it's a quick aside before we get into it.

1:25

I don't know if any of the other units have gone through this in great detail, but appendix through your textbook actually lays out, and this is the 7th edition lays out some typical pipe specifications and pipe schedules.

1:41

So if we turn to that appendix in the little clip is shown here, when we talk about nominal size for most of our pipe work worldwide, all of the outer diameters will be standardized and that's so that we can use have confidence in using bonding fittings for these different pipes.

1:56

But when an engineer will reference something like a nominal diameter, that will typically correspond to the internal diameter if we're thinking about tubing, a very small pipe work, or corresponds to the outer diameter if we're thinking about large pipe work.

2:11

So one of the the characteristic properties that are sitting inside the nominal diameter is the pipe schedule.

2:20

And so that schedule is an indication of the thickness.

2:24

So these are the 10 schedule numbers that are typically used.

2:28

Once we get down to really small pipe work, we tend to have to have fewer schedules available to us.

2:35

So an example here what what's shown in Appendix 3.

2:38

This is for we can see they're showing two different pipe schedules 40 and 80 corresponding to every nominal pipe size, right?

2:47

So you can imagine this is where we get a very large matrix of potential pipe work that we can buy.

2:52

So I'm not going to go into this in in any great detail, but if you find yourself needing to put or order pipe work or have this put together, we would typically select and and we'll talk.

3:03

We'd go into this in some additional detail when we in gas processing one, but we select pipe work and schedules based on what is the optimal velocity.

3:15



Now the textbook itself in chapter, it actually lays out a few exemplar equations for water flow.

3:21

But one of the points I wanted to make is that that optimal velocity and it's relation to the minimum pressure drop across the pipe, because that's really the constraint here.

3:29

Pressure drop cost and material compatibility is more of an environmental check.

3:35

That minimum pressure drop across the pipe is not a linear relationship or even a directed relationship as was presented in chapter in the textbook.

3:45

But in fact what we'll find is that something like the pressure drop across the pipe as a function of its diameter.

3:52

If all of the things remain constant, for instance, the mass flux of of the fluids, meaning gas or multiple liquids that are going to be upstream of it.

4:03

If if that mass flux is constant, we actually tend to see a non linear relationship, something that's going to look like this.

4:10

So this is going to indicate to us, right, Different, right, that ABC, these are different flow machines.

4:25

That's family.

4:29

So has anyone actually learned or seen anything about flow regimes yet in your curriculum in fluid accounts?

4:44

Good.

4:45

Yeah.

4:45

So this is where you start to get more and more the the influence of flow regimes is identifying and balancing that minimum.

4:52

So I didn't want to bring in the example from the textbook because I actually think it's a bit too simplistic and it doesn't really do it justice.

5:00

So then when we get into pumps then, and this gets to to the question that was online from your portion, it's certainly something that I've got a few emails about as well.

5:08

So the fundamental purpose of our pump, if we think about it from a thermodynamics perspective, is to translate our electrical potential into a mechanical potential.

5:17

But then specifically the pump takes that mechanical energy and transfers it as into the kinetic energy of the liquid.

5:25

And we'll often refer to this thinking about the different heads.

5:28

So remember we had our pressure head, that's pressure divided by density, our gravitational heads, so the gravitational field constant or acceleration constant multiplied by the height and then the velocity head, which is the up or downstream velocity divided by two.

5:46

And So what we can do is write the rewrite the Bernoulli equation directly to solve in terms of pump work and pump efficiency that we're looking at here.

5:55

And these bracketed terms we can see what we're looking at is final minus initial or downstream minus upstream.

6:08

So we're looking at the effect of the pump.

6:11

These are often referred to as a total head if we Add all three of these together.

6:16

So in in the context of your projects, if you're thinking about using a liquid phase pump to drive the membrane, certainly we wouldn't be worrying a whole lot about the differences in height at this point because it's it's going to be within a meter or two.

6:32

And we're talking about the generation of 50 to 100 atmospheres of pressure.

6:36

So realistically, realistically, it's going to be that that pressure head that we're concerned with.

6:43

Now if we write the head in terms of the in terms of this total head that we're going to apply, we can also identify the pump work directly as a function or being equal to the difference in these heads.

6:56

So  $\Delta H$  divided by the pump efficiency.

7:00

So that can be as low as say 25% or as high as 9095%.

7:05

And of course, an ideal pump, we would expect an efficiency of one.

7:10

Now getting to the the more detailed question, the power that's going to be supplied.

7:15

And so this is the, the power that we're plugging in and and drawing into the unit for an energy balance.

7:20

This will be the mass flow rate of material moving through the pump times the work applied by the pump, right, Where that same efficiency factor is going to come in the power delivered to the system is ultimately the efficiency as that power is supplied.

7:36

And we can see the efficiency, efficiency showing up here as well.

7:41

OK.

7:42

So when we think about the energy balance, then really what we're talking about is the primary energy in a membrane system is going to be translating that electrical energy into a mechanical energy so that we can increase the pressure of the fluid.

7:58

OK.

7:58

So does that answer your question portion over yes, so so but be careful that this  $\Delta H$  this is the change in total head.

8:19

So it's not a change in enthalpy of the fluid.

8:30

Yeah, so the you're correct.

8:32

So the power delivered is  $M \dot{}$ .

8:40

Yeah.

8:41

So the efficiencies will cancel.

8:44

So one of the one of the concerns that we have about pumping and this won't be directly relevant to pumping something like seawater.

8:53

But certainly if you ever work with organic species or you work in a chemical plant, one of the really critical things to think about is a big progression.

9:01

So if you are drawing on liquid to your upstream suction side, it's going particularly when it starts going into a of vacuum conditions, so we're below atmospheric pressure, then it's relatively easy to generate a suction pressure that's within the same magnitude or slightly above of the vapour pressure of the species that we're drawn into the pump.

9:25

And if that happens, then we can have a kind of periodic or minor flashes, meaning a small amount of that vapour or liquid is going to be vaporized as it's coming into the pump.

9:36

So this means now our liquid phase pump is dealing with A2 phase environment and and that's particularly concerning if we've not purchased a multi phase pump.

9:46

So certainly there are pumps that you can buy that are designed to either deal with a solid liquid slurry or a gas liquid mixture or a liquid liquid mixture without a problem.

9:57

But if we purchased a pump that's intended to only work with one fluid, say seawater, and we start adding gas coming into that stream, then that secondary phase meaning that the dispersed gas bubbles will act to erode the material of the pump itself.

10:15

So these are just a few kind of example images, but we can identify the cavitation right, of the species that we're highlighting here.

10:22

So the way that McKay presents this is to add in an engineering safety margin called the net positive section header NPSH.

10:32

That effectively is the amount of pressure we need to be above that.

10:36

That pump inlet needs to sit at above the vapour pressure of the species.

10:40

So we get that by looking at the one on  $G$  is the gravitational constant.

10:46

This piece of a prime, this is the absolute pressure at the inlet piece of  $V$ , this is the vapour pressure of the species and this  $HFS$ , this is the friction.

11:21

So that's the friction in the suction line it connecting that and actually not the inlet, but the reservoir.

11:29

So in the application of your design, your kind of miniature design project, if you were drawing from seawater and you have to run that pipe say 30 meters offshore to get it away from near shore ecosystems, then your pressure at the suction point.

11:45

So the absolute pressure of the reservoir is going to be 1 bar if you have that surface of the seawater pipe just barely touching the ocean, right.

11:53

So that would be atmosphere pressure, but then you're going to have to connect say a 3040 meter line between that surface of the ocean pump and or sorry, the surface of the ocean pipe and the connection point to your pump.

12:05

So you have a 3040 meter run of pipeline that's going to engender friction.

12:10

So that  $H_{FS}$  is the friction engendered in that suction line.

12:14

So we're accounting for it when you think about the pump inlet because we don't want to be at a lower pressure at the point we get to the inlet than we thought we would.

12:21

And then see.

12:22

So there's obviously the the difference in height it will be.

12:25

So if you know, this is a good thing to think about because you are going to have to locate your pubs somewhere on shore and you're going to have to be accessing seawater and it's probably not a good idea to be taking it directly from the beach, right?

12:40

But we're going to have to go out a little bit.

12:42

So it's important to kind of think about that as a small consideration that we're guarding against the ability to have vapour in our pump either from an air entrainment standpoint of where we get the water from or if the pressure at that is below the vapour pressure of water or rather is different by this NPSH factor.

13:04

So if we look at the types of pumps that we can have, then other two general classes we'll talk about, one is a reciprocating.

13:11

So we're talking 1st about positive displacement pumps and these come in two flavours.

13:16

The first is a reciprocating positive displacement pump and, and we can look at this in a two different styles.

13:24

Piston pumps are designed to have relatively lower output pressures.

13:31

So you can see what's happening here is our fluids are coming into the suction lines.

13:35

We withdraw the piston all right.

13:37

And this is just working on a a mechanical motor turning this crankshaft fluids are drawn in and this is a, a gate valve.

13:45

So you can see as it's drawn up, it allows the fluids to move around the side of this.

13:50

But once the pressure starts to increase and the pneumatic pressure, that thing shuts.

13:54

And so the only thing available is to go through our exit valve, right?

13:59

And so in this way, we can have a relatively constant motion.

14:03

And it's not uncommon to see two of these connected.

14:05

So one of these on each side if that Cranch that so we get dual effect or dual stroke for the same motor driving it.

14:13

And so these will typically give us discharge pressures of up to 15 or sorry 50 atmospheres on a commercial scale.

14:20

Now if we go to something with higher pressure, this is where we get to plunger plunge.

14:24

So we can either have a reciprocating plunger or we can use something called a diaphragm plunger.

14:31

So this is going to replace a steel plunger with a rubber or a more flexible material.

14:38

So our heavy walled reciprocating plungers, these can get up to 15108 atmospheres.

14:44

And so you can see the difference in this reconnect figure between a piston pump and a plunger pump where that that plunging effect is basically going to it.

14:55

Well, it minimizes the strain placed on the the shaft of the piston.

15:00

And so in that way we can deliver a lot more momentum into it and as a consequence generating much higher pressure.

15:08

So both of these pump styles have mechanical efficiencies.

15:13

So serving at ADA value somewhere around the 50% mark if we're in lower volumes or can be up to 70 or 90% if they're large.

15:21

So the higher the rather the larger the pump reciprocating or positive displacement pump, the higher the efficiency of it.

15:31

Now this is contrasted to something called Rotary positive displacement pump.

15:37

And so there are a few styles in this and they can go by a variety of names.

15:40

And so on the left here is a gear pump.

15:44

In the middle this is a vein pump and on the right is a screw pump.

15:49

And these are common styles and I quite like this right hand when you can see how old this thinking actually is, right?

15:55

The idea is that you're actually just turning a set of screws and if it's 100% liquid, this will simply raise the fluid level and dub it out on the top here.

16:04

Where it gets a bit more interesting is when you look at the the gear and the vein pumps.

16:09

So it's not uncommon.

16:11

The tolerances on our gear pumps, both of these edges and in the middle here are incredibly tight, as are the tolerances associated with where those bangs are going to come through.

16:21

You can catch them before.

16:24

So what will happen is this thing is turning, it's going to draw in liquids, right?

16:31



And as these liquids come around, they're going to to be increased in their velocity and then throughput through the discharge line.

16:38

Now one of the the applications of this that you will also see when you come into like biochemical applications or pharmaceutical manufacturing will be the use of peristaltic pumps.

16:50

And so these will work by having some kind of flexible tube where we're going to have a number of rollers placed across it, right.

16:59

And so much like those kind of chiropractic tables, you get with the, the rollers on them to, to try to relax muscles before a chiropractic session.

17:07

These will basically massage liquid out of these flexible tubes.

17:12

So they're squeezed by the succession of rollers.

17:14

The value of periscald pumps is they give us a, a leak proof ish if they're connected correctly.

17:21

We have a few in the lab and they don't work particularly well, but they give us a leak proof design at a nearly constant flow rate.

17:28

So where this gets really useful is if you're looking at really precious materials and that's why they come into pharmaceuticals quite often.

17:36

An alternative to these it's something called a high pressure syringe.

17:47

And so these are starting, one of the the most common manufacturers is Isco, but syringe pumps are starting to come into fashion because we're we're able to get control motors that are much, much more precise.

18:00

So we can control up to say one or two millilitre level per minute what the injection rate looks like.

18:08

And so this tends to give us quite a nice setup if we're trying to inject a constant flow stream over some amount of time.

18:15

And most of the high pressure systems can go anywhere from they can go up to 250 bar and you can get them in really durable materials like Hastaway.

18:26

So if you want to work with exotic or or very precious materials, you can buy these now at a manufacturing scale that are quite durable.

18:37

So sitting apart from our positive displacement concept, the other approach is to look at a centrifugal pump.

18:46

And so these are in fact some of the most common, in fact common in plant machinery.

18:50

So they're an alternative design context where the liquid is initially drawn in from the right hand side here.

18:57

And so you can see we're looking at two different orientations, right.

19:01

So the initial point of contact is one this will be as it's the liquid is first coming toward the centre axis, it's rotating of this centrifugal fan and the liquid is thrown through centrifugal motion toward the outside.

19:18

It's thrown toward the outside of these impellers and they have grooves in them to to allow liquid to flow through.

19:23

And so it's at the outside.

19:25

And so the second point in our Bernoulli consideration and what makes these effective is that they these impellers can drive really, really fast.

19:34

So these are moving somewhere the realm of a car engine in terms of speed.

19:39

Now the limitation is that they can typically only get to about 60 meters of head being created.

19:44

So for water that's around by the atmospheres, right.

19:51

So really not getting us the same kind of pressure considerations.

19:57

So if we're thinking in the context of AD cell plant, if you're going to need 50 to 80 bars, say after your membranes, then you're going to have to look at those positive displacement pumps.

20:08

And this is part of where the consideration of the expensive D cell comes from and is that we usually need a relatively large offset.

20:17

Because of the amount of energy dropping in, now we can get more energy out of these centrifugal designs if we put them in multiple stages or we put them in series and parallel together.

20:27

Working in a matrix, we can see high energy one designs up to 200 meters ahead.

20:33

So again, for water that's sitting in about 18 atmospheres, but still is a relatively high throughput, low energy design, whereas a positive displacement pump is a slower throughput, high energy, sorry, a high pressure design.

20:49

So if you're ever going to go to buy a pump, particularly in your engineering practice, it's absolutely critical that you ask the vendor to to share with you an actual pump curve, right, that they've measured physically for a system that's appropriate or similar to yours.

21:12

And so this particular example, this is a 5 inch centrifugal impellar design.

21:21

Now the way that we can look at these pump curves and these are collected from experimental data.

21:26

These relate the head that we're going to generate on on the pump to the flow rate as a function of the speed that we're we're running at in Palloran.

21:35

So we can see here is the 1750 RPM line 2880 and 3450.

21:39

So they were measuring these at three different, three different RPMS, but that would allow the engineer to interpolate.

21:49

Now if we look at an example, so at the 3450 RPM, if we said, OK, our design spec is going to be 200 gallons per minute, then I can trace this thing down right down this, this curve black line.

22:02

And I can trace up, up from 200 and say, OK, there is my nominal operating point that I'm that I'm going to think about now that we can overlay with these liquid heads to say, OK, at that point that will generate a liquid head of about 88 feet of liquid.

22:20

And, and apologies, because I know McCabe's units are all in, in Imperial.

22:25

So it's a bit of experience to go back and forth.

22:28

But that will generate 88 feet in terms of the head of the liquid.

22:32

But I can also read out that my efficiency with that point will be sitting around 80%, and if I'm in this case extrapolating right from my 5 horsepower line, then I can see at that point I need around 5.5 horsepower.

22:50

Now interestingly, right, this is a great condition to look at for this particular example because I'm sitting at the highest efficiency that's been characterized if I go to either 150 or say 240 gallons per minute flow rate.

23:15

So if I increase or decrease the flow rate away from this 200 gallons per minute set point, my efficiency is going to go down in both cases.

23:26

So this is the reason that we always want equipment vendors.

23:30

It's, it's really their engineering kind of a moral obligation to provide pump curves to you specifically for the application you're talking about or for an appropriate system of similar like an APS system or something.

23:44

So that you can read it and say, what exactly is this efficiency of the system going to be?

23:50

And, and what is my flexibility in terms of the RPM, which is going to dictate my energy requirements and so on and so forth.

23:57

So I can also look at kind of theoretical point curves instead of just considering the head.

24:02

I can also plot the pressure.

24:04

So I'm sorry, the power where I can look the fluid powers that deliver to the fluid versus the total power in the system.

24:11

And that's what I need to actually buy, right?

24:15

So that's what I'm paying to get from the grid or what I get from renewables, right?

24:23

So either I'm going to pull it in from the grid or I'm going to renewable system and in which case I'm probably going to need a battery pack if we're talking about an isolated, an isolated diesel plant.

24:34

And then I can also look at the efficiency where we're generally going to see, we just observed the example, right, of a fall off between our peak efficiency and what happens on the other side.

24:44

So then what happens if you're working with a pump Bender that says, OK, we don't actually have any pump curves and we don't have any plans to measure.

24:51

But we did measure a pump curve for something that's about, say half the diameter of the system, but for exactly the materials you're looking to use, right, if you're trying to ballpark it.

25:02

And and I don't like that that the field calls these affinity laws because it's not law, but it's based on the ideal relationship for pump behaviour.

25:11

If you are looking to upscale from a smaller pump design and pump curve to what might happen in a larger pump, then we can either look at this through a constant diameter lens or a constant impeller speed, right?

25:24

If we're a constant diameter, and I think it's quite convenient, the capacity scales directly with the impeller speed, changes in the impeller speed.

25:34

So if our scaling, if our little pump is say the same, well, if pump A that they've measured the curve for is the same diameter as what you have and you're looking to scale to a a impeller speed that they didn't test, then the capacity of the pump scales directly with impeller speed, head scales with impeller speed squared or the the change in head and the power scales with impeller speed cubed.

25:59

Whereas if they've measured something at the same impeller speed but not maybe not the right diameter, and you're looking for a slightly different diameter pump, then the capacity is again directly with diameter, the change in head with diameter squared, and the power with diameter cubed.

26:14

Right now these relationships come from ideal pump behavior.

26:26

So it's important to note that this is not a perfect relationship.

26:30

So if you have to employ these so-called affinity laws, they are going to give you a bit of uncertainty that you need to account for.

26:37

But the other big take home message on this is that the power in both cases is scaling with both the impeller speed and the diameter of the system cubed right?

26:48

So whilst large pumps tend to be more efficient, the amount of power that these require, because we fundamentally are using a mechanical motor to drive these kind of fans, the amount of power that these require is extraordinary, right.

27:06

So these tend to be some of the highest electrical requirements in large systems are pumps and compression systems.

27:17

So if we are moving fluids through the use of a pump, then I want it to take a few minutes and and talk about how do we actually measure this and how do we know that what we're doing is the right thing.

27:27

So if we go back to what we're talking about in Thermo for a minute, well, fundamentally what we're trying to measure as engineers is changes in the Gibbs free energy, right?

27:35

And a lot of our field hasn't really caught up to this way of thinking yet.

27:38

It's starting to trickle out from bits here and there, but fundamentally that's then our measurements in the mechanical potential.

27:46

So changes in pressure, measurements of the thermal potential, changes to temperature or measurements in the number of species present related then to the chemical potential.

27:59

So if we're trying to assess the mechanical potential, we can either directly measure pressure, right?

28:05

And this might be through the use of something like a strange gauge pressure measurement or, or a diaphragm or even a we used to have a, a sapphire crystal pressure measurement.

28:15

So it would compress a crystal and put in very high pressures in a differential state.

28:20

But when we're measuring pressure and flowing systems, this allows the engineer to deconvolute that flowing state.

28:26

And this is the problem invoked by complex flow regimes that we talked about earlier in the lecture or one of the a few things we'll talk about today.

28:45

We can characterize either directly the velocity or the turbulence in the system if we know what the materials are and from that velocity or turbulence measurement related to the pressure conditions or we can do both, right.

28:57

So one of the common applications that we're trying to assess the type of flow regime that's going on is to actually do both of these things together as a way to get at those unpacking those complex fluid flow machines.

29:11

So well, I'll use thermocouples if we're being cheap with thermometers, if we need precision to measure temperature potential.

29:17

And then as we talked about last week, we can use gasolipid chromatography as an exemplar application of adsorb machine and we want to start measuring the number of molecules and really the concentration of different species in the system.

29:32

Or we can get into more advanced technique that is the use of spectroscopic methods for inline measurements.

29:38

So things like inline Roman spectroscopy start to become more and more popular and then really goes a lot of power as the engineers start running these online systems.

29:49

So I think you're going to see, you know, in the next 10 years as AI machine learning or coming in to vogue, these are going to get very powerful in the context of being able to work with complex data sets.

30:07

So measuring flow rate, if any, if any of you stop by the lab, we have upstairs examples of rotimeters.

30:14

And so these are really simple design.

30:16

They work best usually for Baker systems that I'm trying to meter something like compressed air.

30:23

And what will happen is that if I have a float in here with a known drag force and a known weight, then as I put flow across this, it will suspend this float somewhere in the rotator where I'll have gradations on here right?

30:39

Indicating something like the ACF Ms., the actual cubic feet per minute inflow.

30:48

Whereas if we look at vortex shedding meters, these are a bit more interesting because what we're then placing is some kind of sort of obstacle or a blunt body, in this case an anoxin, it's a trap is open.

31:03

We're going to put this directly in the centre of of flow line.

31:06

And this could be with air or or preferentially with liquids.

31:10

And So what will happen is we get this flow induced vortex shedding, right.



31:14

So here's our little vortices coming behind this.

31:16

Now if we look at the side view, what will then happen is that this is the the piezoelectric elements that are placed here.

31:26

So the piezoelectrics allow us to relate fluctuations and pressure to fluctuations in current or vice versa, right.

31:33

And so if I have these fluctuations and pressure because of my vortices that are contacting this piezoelectric element, then the extent and the severity of those fluctuations relate to the degree of turbulence.

31:46

And if I know what the fluid composition is, then that turbulence gets me back to a velocity and from velocity I get back to pressure.

31:54

So these are particularly useful if I have high turbulence or high temperature flow systems.

32:00

And then finally, just to we'll talk about Coriolis flowing.

32:04

So these are very, very useful.

32:07

They tend to only show up in rather extreme and expensive applications, but they give us a measure of total mass flow rate, right?

32:15

And So what happens here is if we're doing something that these can work at very high pressure, but if we're flowing through a high pressure, some complex and they're really useful for complex flow regimes.

32:27

And so we have multiple phases present then we'll we'll redirect flow through both of these U-shaped tubes here on the right hand side.

32:36

These are just V shaped tubes.

32:37

And I can see there's a bit of an exciter that's going to move these up and down.

32:40

Right now what will happen is the force that's exerted by the the fluids running around this U shape that forces proportional to the mass, the forward velocity and the angular velocity.

32:54

So as a function of that mass flow rate variation, I will get secondary random deviations and the excited motion of the tubes.

33:03

And it's it's random because we can plot those deviations as something like the the probability of observation.

33:15

So we'll look at a probability distribution.

33:17

But if I look at the median of that probability distribution, the magnitude of the deviation, so where it sits on the absolute scale corresponds to that total mass flow rate.

33:34

So if I combine something like a Coriolis flow here, that's going to give me how many kilograms of material are coming through the system.

33:41

Now, if I'm working in a complex system that might have a bit of oil, a bit of water, a bit of gas, could be 3-4 phases present, could be sand, right?

33:49

I don't know what the distribution of those phases is, but if I combine the Coriolis flow meter with something like a pressure measurement, maybe multiple pressure measurements, so I'm getting a series of inline pressure drops, then the engineer can start to deconvolute what's actually present in the system.

34:10

OK.

34:11

So to wrap up hopefully that that added some clarity around what we can do with pumps.

34:17

And in looking at the pump powers, basically the application we want to go after her balances.

34:23

So the liquid phase pumps where we've talked about primarily, we would categorize

these as either positive displacement or being reciprocating Rotary or centrifugal pumps being an array of designs and names that can be used.

34:35

But the idea is that centrifugal pumps tend to have lower pressure at the outlet and it's high throughput, whereas positive displacement pumps tend to have lower throughput that can reach very high pressures.

34:48

So if we're going as an engineer, then to purchase these pumps, we must always look for pump curves, right?

34:53

And these allow us to relate head efficiency and power to the flow rate that we've nominated and make sure that we're operating in the sweet spot for that pump.

35:01

And if we don't have a correctly sized curve rather in terms of the intensity of the pumping activity or the size, then I can use these so-called affinity laws to help interpolate and scale in fact, especially if we're going outside of the measured region.

35:19

Now for your applications in this design project, certainly I think a Coriolis flooding would be overkill.

35:26

These tend to be very expensive, so \$100,000 for openers.

35:31

But you know, something like a, a simple rectimeter for a water or in fact you can even use APO tube.

35:37

If you remember back to I think the early part of the first couple of lectures in 3019, we talked about the use of PO tubes or, or exemplar manometers to, to get inline pressure measurements or inline flow rate measurements.

35:51

So it's useful then to have some kind of measurement on or thinking about, you know, we do want to measure these characteristics at a few points that are critical to our system.

36:02

Because for instance, if we are drawing in seawater, but all of a sudden something happens and we start drawing in sludge for maybe down in the dirt, the the pipe cracks and it, it starts to draw sand and dirt instead, then we want some kind of early measurements being taken in that system.

36:18

So the control system, the machine learning algorithm that's sitting behind it can kill that before those are going to hit our membranes, right and foul up the entire system.

## Lecture 31 - Dispersed Systems

0:00

Pumping liquids either through positive displacement pumps and we looked at a reciprocating pump and that was that.

0:06

One example was where we'd be using like a piston to displace or or create a vacuum or a Rotary pump.

0:15

We also looked at the use of centrifugal pumps as an idea where we would have a much lower pressure head that could be delivered, but a higher volumetric throughput.

0:24

And one of the points that we learned in this is that because we're playing often with pressure and using vacuum to maybe a bit of a vacuum process to draw fluids in and then a high pressure process to reject fluids that cavitation.

0:42

And so this was engendered by having gas pockets.

0:53

So this cavitation process was critical to avoid.

0:58

So typically if we're pulling in a liquid, if we create a vacuum and that has the potential to create a small bubble of gas, but that running gas through these kind of high pressure liquid pumps would often erode them.

1:12

And in pump parlance, we call that cavitation.

1:16

So we then talked about if we're going to purchase pumps as an engineer, it's really critical that we look at the pump we're purchasing relative to the application.

1:25

So the vendor of that pump should already have background information being pump curves to show us how that pump's going to perform for a specific application of interest.

1:34

And so these pumpers allow us to relate the injection head or pressure to the efficiency and the power all is a function, the flow rate of what's moving through that pump so we can operate in in in our optimal design process or our optimal space.

1:51

And then if we don't have specific pumpers that have been generated for our size and flow rate or RPM, that we can often times use affinity laws as kind of a basic scaling parameter.

2:05

Either to be at the same RPM we intend to operate and scale in terms of pump diameter or looking to be at the same diameter and scale in terms of RPM.

2:15

And then finally, if we're starting to move fluids around, it's critical that we know how much.

2:22

And so we can use metering applications to understand this.

2:25

Things like vortex shedding meters, rotimeters, Coriolis flow meters, these are all useful.

2:30

Particularly when we combine them together with pressure and temperature measurements, we can start to get at what might be happening in complex flow patterns.

2:38

And we're going to start to look at a little bit of that today as an introduction to the next part of your curriculum.

2:44

So when we talk about dispersed systems and I, I love to tell this very small story, but dispersed systems, we got into an argument, a friendly academic argument with a professor from MIT probably 8 years ago.

3:00

Now, 7-8 years ago, we were at a conference.

3:04

If we were arguing about nomenclature, right, as an academic conference, we tend to have the argument we were having was the correct labeling of what we should be calling the oil that had been dispersed during the deepwater rise and blow out in the Gulf of Mexico.

3:20

And in fact, we said, well, that's an emulsion.

3:24

You know, we're chemical engineers.

3:25

Trust us on this.

3:25

That's called an emulsion.

3:26

And we can go into emulsion theory to help us inform what some of those dispersion processes in the deepwater gulf look like.

3:34

So we'd better know where and when to start cleaning up oil and where we're looking for it.

3:38

And this MIT professor is just adamant it should be called only a suspension, but we should not be calling it an emulsion because that's the kind of foamy material that's generated on the surface of water through things like wave action.

3:54

And so in fact, this this table is very common in chemical engineering.

3:58

It's funny, we were arguing with the mechanical engineering professor, but this table comes from chemical engineering.

4:03

It shows us that in fact, when we have our dispersed phase, all right, So these are our three columns.

4:08

You can have either a gas, a gas, liquid or solid being dispersed in a continuous phase of being a gas liquid or solid.

4:17

And when we have one liquid being that oil dispersed in a secondary liquid being the ocean water, we would call that an emulsion.

4:27

And if we had a bit of gas coming out with liquid oil, we would call that a foamy emulsion.

4:32

But emulsion would be the the correct term for characterizing it well droppings.

4:36

And so we can read across this table and see a number of common examples in parentheses.

4:40

So things like liquid aerosols, this might be a fog.

4:43

We have those little small droplets of dew.

4:47

Bush virus smoke is a very common example of a a solid aerosol with things like whipped cream.

4:54

If anyone's ever made homemade whipped cream, certainly that's an example of a foam.

4:59

And glass is actually an example of solid solve or a solid in solid suspension.

5:05

So in general, dispersions as an engineering practice can emerge either from something that's intentional in our unit operation design if we intend to react fluid phases of some sort to form a product.

5:19

And so that can be from vapour or liquid.

5:22

And a really common example, as I mentioned is the crystallization for pharmaceutical manufacturing.

5:26

And we'll look at one of those unit operations toward the end of the lecture today.

5:31

They can also be unintentional.

5:33

And this might be happening if we have like mineral scaling or mineral scale formation that's a drop out of say calcium carbonate during the flow of water of hard water across some interface.

5:49

So an application to our miniature design project to be thinking about mineral scaling during the membrane separation process.

5:55

And I think as Inar talked about toward the end of his lecture, something that he looks at the laboratory is membrane fouling, right and how we characterize this for our large scale RO systems.

6:07

And also quite common example is what happens if we start dropping out hydrocarbon solids from fuel oil transfer.

6:14

So we would call those waxes if they drop out.

6:17

So these unintentional consequences engineers talk about as being fouling processes and we use a really foul word to describe that because these cost us a lot of money.

6:30

They typically introduce severe health hazards to clean out and they can also introduce severe HSE hazards for the operations team during steady state flow as the following is taking place or during the removal process of the following system.

6:46

So in many ways from the environment to operator safety to cost filing is quite a critical issue for many engineering domains.

6:56

So if we so, well, I guess that was an so who was right?

7:03

Well, of course we were right.

7:05

Not because we were from UWA, but because we were chemical engineers.

7:09

And in fact, we just last year published 2 textbooks on the topic, I think was 25 chapters each.

7:16

It was a very fun experience.

7:17

We all worked together for about 8 years, but very much we were right.

7:21



But of course, we've continued to to fight with papers back and forth across the literature.

7:28

Yeah, yeah, we definitely kind of overshot that.

7:31

But we made the point and I think we published a comment 2018 and 2019 in one of the Nature imprints talking about manipulation of terms and how that can have a knock on consequence for the field.

7:48

So if we think about dispersion in where this can arise naturally, one of the most common locations you're going to see is multi phase flow.

7:57

All right.

7:59

And if we're thinking about multi phase flow generally here we're talking about liquid is a common carrier meeting, but we can have a dispersion of other liquid and liquid emulsion and gas in liquid dispersion.

8:11

So maybe we're transporting a bit of waters or pipe loading is not 100% liquid or as I mentioned the other day, if you put the inlet to your, if you're doing an RO system, you put the inlet to the seawater too hot in the surface, maybe the waves or the tide are going to entrain some air in that.

8:28

So that would be an application of gas and liquid flow.

8:31

Or as we're going to talk about a bit later, if you have solids forming and reacting in that system, that would be solid and liquid flow.

8:38

So however you cut it, these multi phase flow systems are then called them are critically driven by a balance of two energies.

8:47

And so one is a term called certs tension.

8:51

So the simplest definition here, I know that many of us that have been working together can have two semesters between dynamics and unit operations is the simplest way to look at it is the free energy at the interface, right?

9:06

So the volume, the bulk volume has its free energy.

9:10

The interface itself has a different free energy because we're going from a three-dimensional system to a 2 dimensional system.

9:16

So I'll show that and then how we get there in just a minute.

9:21

So that free energy at the interface between our 2 phases that is resisted or counter balanced by the shear or so the fluid shear meaning it corresponds to the velocity of flow of our ducted flow pattern.

9:37

And this is that fluid shear.

9:40

That's one of the ways that our turbulence is going to be distributed or dissipated and that will be generating surface areas.

9:48

So if we think about something like gas and liquid, it's a very common flow pattern.

9:53

If we increase the shear on the system, right, If we begin where maybe we have a 50% filled liquid pool, if we start turning up the shear of the system, one of the ways that that shear energy or turbulence is going to be dissipated is by trapping small particles of gas in the liquid phase.

10:10

But because we have a higher free energy system and having all of those liquid or gas bubbles in there, each little interface itself is going to want to combine with other interfaces to reduce the total free energy.

10:27

So what do I mean by that?

10:29

Well, if we think about the relation for Gibbs free energy of a volumetric system, OK, so we have our same mechanical, thermal and chemical potentials in this term.

10:49

What we can also do right, if we're thinking about an interface, and I'll, I'll use a superscript  $\sigma$  here, but the Gibbs free energy of the interface can also be defined.

11:16

So of course, an interface, what makes it different is it, it does not occupy a volume, but by definition it is A2 dimensional quantum.

11:24

So it has no volume.

11:25

And as a consequence, it cannot have a pressure.

11:28

So the the origin of physics of pressure requires A3 dimensional species or three-dimensional application, which is why we multiply our pressure differentials by the volume in which we're talking about.

11:41

Now when we go to A2 dimensional system at the interface, we can see here our volume is replaced by an area term, right?

11:50

Because now we have 2 dimensions, that's the two-dimensional collapse, and our pressure driving force is replaced by surface tension.

11:58

So if we look at it from a thermodynamic construct, in fact, the easiest way to rationalize this concept of surface tension is as A2 dimensional equivalent to pressure in 3D space, right?

12:10

In fact, we can collapse that to one dimension, right?

12:13

And this starts to get into to thinking about total free energy considerations between the different dimensions, 3:00 to 2:00 to 1:00.

12:25

Now, of course, entropy is always defined.

12:28

It exists whether independent of the dimension that it's inhabiting, as does temperature, as do the number of molecules.

12:36

Now if we get into this, and I think this, you'll see a little bit more on this if you take gas processing one, so CHPR 5521.

12:45

But the way we treat that chemical potential gets a little bit different because the interface is comprised of an integer number of molecules sitting across.

12:54

So it's not quite the same, but how we treat that is a later topic.

12:59

All right.

12:59

And so we can see here on the top left, this is a plot of the surface tension of water.

13:04

We report this with units of Dimes per centimeter, which is equal to a millinewton per meter.

13:12

And that is going to typically decrease with with temperature, it will also tend tend to decrease the pressure.

13:18

And in fact, if we have a a single phase system, so let's say a hydrocarbon system where it's a hydrocarbon gas in equilibrium of its liquid, the definition of surface tension being zero is when we hit the critical point of that species.

13:34

Now if this kind of thermodynamic framework is a bit uncomfortable, another way to think about it is at the interface.

13:41

And I don't like this definition, but it is, it is easier, I think for some folks to understand on the left hand side of the interface.

13:49

If you think about our one marginal, and I'll make this red, our one marginal atom or molecule, it will have a smaller number of nearest neighbours than that same molecule when it's sitting in the middle of the bulk bags.

14:08

And so for this reason, it has fewer collisions available to it with which it can dissipate energy, meaning it has a higher apparent energy when we look at that interfacial layer, right?

14:21

That's a more physical way to look at it.

14:24

But these are both accurate characterizations.

14:28

And so if we take for instance, a system more or less at rest and then we're going to look at 2:00.

14:35

So we have the bottom two quadrants are a low interfacial tension or low energy, meaning there's a, a not a significant penalty for creating new interface.

14:53

And at the top, this is a intermediate.

15:01

It's an intermediate penalty for creating interface.

15:05

Notice that that as we start putting out a force across these simulation boxes, the low energy system tends to generate smaller, smaller and more dispersed species where here the feature on the top where we have a higher energy penalty larger, larger and more elongated bubbles is or disperse species as they start to come into flow.

15:49

So again, this this topic in the way we handle this is CHPR 5521 put in a few call codes through today so you can see where this tracks into the rest of the curriculum.

16:02

And so the consequence if we start wanting to transport these kind of systems, and I'll focus here, this is for gas liquid.

16:18

And so I understand that Jeremy started getting into a little bit of multi phase flow.

16:22

Some of the penalties that we have to pay when we start having these polyphasic or multi phase systems to available to us is that we burn energy because we are transporting.

16:33

Now it's not just the creation of all that interface.

16:36

If we want to move it, we pay a penalty for every square meter of interface in the system, an additional penalty, right?

16:44

So as we go from our simplest case of say stratified flow between 2 phases and we start then going to stratified WAVY flow and into sled flow, that's the process that we're showing here, right?

16:57

And this is a largely simulation by trans AT now it's not very common that folks will use computational fluid dynamics to characterize multi phase flow.

17:08

It's typically to help interpret what's happening because all of our relationships in more advanced chemical engineering topics will be based around very, very elaborate and expensive pilot plant studies.

17:20

And that's simply that we lack the mathematics and the physics to understand from a first principles perspective what's happening in these systems.

17:30

So this particular slug flow simulation, this is actually showing the generation of slugs.

17:35

If we look at this repeating video, we initially have that that very even stratifying flow.

17:42

And then at this first stage, we're going to see small waves crop up as the video repeats right there.

17:48

The waves that directed crop up as soon as that liquid peaks and hits the top of the pipe.

17:53

Now we have that.

17:54

We have a gas pocket that builds up and pressurizes behind it and that drives it 100%.

18:00

It's a pure liquid.

18:01

And that's why we call it a slug, because it's a slug of liquid.

18:05

Now, I bring this up because you as engineers, it's really critical to be aware if you're ever, ever, ever transporting a system where you have gas and liquid flowing at the same time, it is absolutely critical to remember this phenomenon of slug flow.

18:21

This is one of the most dangerous things that you can worry about if you are transporting gas and liquid together and you're putting pressure behind it for this reason.

18:31

And what happens is think about this slug that's generated here.

18:35

What the engineer really needs to be worried about is what happens when that reaches the end of the pipe or the end of the duct or whatever unit operation we're thinking about.

18:44

So it's not uncommon that we might include something like slug catchers, we call them, which are basically gigantic flash vessels to allow the receipt of these large liquid volumes.

18:56

Now, if we don't, as soon as that thing hits, say if we want to take it across a valve, if that slug of liquid hits a valve, it will burst the end of the pipe because it will rapidly pressurize, right?

19:08

And so this is a very, very important process consideration.

19:12

And again, one of those things that I think Jeremy's probably introduced and we'll go into more advanced applications in in gas processing.

19:19

One, if you take it now on a slightly separate topic that was more around gas and liquid transport.

19:27

But then we wonder, well, what's going to happen if we're going to look at solids generation or or often will be called crystallization during the transport of our fluids.

19:38

And so here we can generally break this into thinking about two different types.

19:43

The first will be what happens again, if it's a fouling process, so my my solids formation is unintentional.

19:51

And the the 2nd is what happens if it's a desirable process.

19:54

And so on the next slide, we'll go through an example of of a unit operation designed to create crystals.

20:02

So if we're crystallizing in a fouling type of environment, well rather no matter how we're crystallizing but but what's often thought about in a fouling environment, this can be either ice formation.

20:13

If we have water flowing, it can also be classic hydrates with a slightly different structure and phase on molecular level of the water.

20:21

Again, the example of solid mineral scaling if we have a lot of AQS ions present or wax, solid wax that can drop out from hydrocarbon liquid flow.

20:31

Now what makes crystallization so interesting is that if you want to to form crystals.

20:36

So if you take a one of those trays of water, right, that you put it in your freezer to make ice.

20:43

Let's imagine that you could put a, a thermometer in there and measure, right.

20:48

The temperatures of your freezer are usually sitting at -16 negative 617 Celsius.

20:53

Well, if you put a little thermometer ideally in each one of those little cubes out of the water as you put them in the tray, right, you measure what's the actual temperature is I cross below 0° C what's the actual temperature that those form ice at?

21:09

What you would find is that in fact it does not happen ever at 0° C So if we apply that the probability density function of what we would call then that subcooling, right?

21:23

So how far below 0° CMI, what you would find is that it should typically be a log normal distribution.

21:34

All right.

21:35

So just to type a statistical distribution, and we would typically expect in a freezer something on the order of say .5 to 1.5 Kelvin, maybe 2 Kelvin or Celsius because we're talking about a temperature difference here being the point at which the first crystal of solid water is going to form in each of those little buckets.

21:54

Now ice is one of the easier things to negate.



21:57

But when you start talking about things like cloud 300, wax, mineral scaling, the more elaborate solids, these can have subcooling requirements or induction requirements that go significantly, significantly beyond that, that .5 or one Kelvin.

22:14

And in some cases we would be talking 10 to 20 Kelvin below that temperature, which the solid phase is stable.

22:21

So we identify this as a nucleation induction, meaning we have to induce nucleus formation, crystal nucleus formation.

22:32

And again, one of the ways that we can think about this is going back to our, our Gibbs free energy framework.

22:39

All right.

22:39

And so you'll often see in, in any kind of textbook or literature, folks, we use this Delta T or sub cooling to characterize that crystal induction.

22:53

But in fact, sub cooling is a really it's a classical way to think about it.

23:00

So it's much of our historical literature that does that.

23:03

But but where the field is largely turning toward, I'm just thinking a bit more globally about it.

23:09

So if in fact, if we write our again Gibbs energy definition for volume now in fact when I'll write this instead of DG, I'm going to write delta.

23:17

So state to state and so state one.

23:31

So state one will be our equilibrium melting conditions.

23:42

So if we were to form ice and pull back out, we know that that first the first molecule of water that's going to melt from the ice phase will occur exactly at 0° C, never before.

23:58

So often times engineers will use melting as a way to characterize the actual thermodynamic phase equilibrium where the induction requirements are considered as our our nucleation.

24:12

So stage one you would take is the the melting conditions over stage two would be the nucleation conditions.

24:29

So if I instead of looking at  $\Delta G_{PDT}$  I'm going to look at  $\Delta$  as the state state difference not stage two state 2.

24:39

So  $\Delta G$  or  $\Delta G$  will be  $V$  times  $\Delta P$  minus entropy times  $\Delta$  temperature plus  $\Delta N$ , right.

25:00

And it's, it's likely because our chemical potentials are entropies and our volumes are going to change.

25:06

In fact, these are going to be showing up as as multipliers in each case.

25:11

So we often times use right some cooling, which is this kind of middle way to characterize what is actually a free energy difference between our two states simply because it's easier.

25:24

And this came into vogue in the the mid to late 1980s.

25:28

Prior to that, it was often times we would use what we call or or some truncated to over pressure for systems.

25:37

And now the more modern way to think about it is we try to characterize it between chemical potential driving force between state one to state 2.

25:46

But in fact, one of the yet more kind of futuristic ways to look at this is the balance of summing up all three driving forces and thinking what is that net driving force across the system.

25:59

So when we do then induce that first crystal or crystal of nucleus, it very interesting

depending on where we are in this, this overall driving force here it's presented in The Cave as a function of sub cooling.

26:14

If we are at a relatively low sub cooling, which again is we've drawn as probably less than 4° Calvin or centigrade, right, we tend to see a relatively normal, normal crystal.

26:31

Now, if we were to nucleate at exactly 0.001 Celsius, we would see what would be typically called a perfect crystal, meaning we don't have any crystalline defects because the crystal is not rushing to grow.

26:46

If we start getting into higher and higher and higher subcoolings, which for engineers typically means we're rushing the process, and that's when we start to see, we have to remember that growth is an exothermic process.

26:59

So the driving force at which we induce nucleation or at which nucleation occurs spontaneously dictates how rapidly might support a very high driving force.

27:11

That means our Gibbs energy difference between where the the system wants to be, where it's currently at is very large.

27:18

And the bigger the difference, the more it's going to rush to get there.

27:22

So because growth is exothermic and the system is rushing, it's going to release a lot of heat at once.

27:28

Well, as it starts to release that heat, it needs to dissipate it because if it keeps all that heat trapped locally at the crystal surface shown here, it's going to start building up, right, Which is going to to warm that local region above.

27:45

It's going to warm that local region above the equilibrium conditions and thereby slow the process down again.

27:50

So what will typically happen the higher and higher the driving force at the point of nucleation, right, which is shown here is subcooling.

27:58

Once we get the high subcooling systems, we start to see spike growth and there are a number of different terms that crystal growth and researchers have coined around this, but kind of spike wise growth is one of them.

28:13

I will see a very, very rigid and spike surface.

28:17

In physics.

28:17

They will often talk about crystalline defects that arise as a function of this, which is why different types of crystals will have different opacities, right?

28:31

But if we can look perfectly through a diamond versus maybe if we drop a diamond and it cracks a little bit and induced a small crystal defect, that then changes how light's going to penetrate through.

28:45

So that's an example of then crystallization that we talked about.

28:49

If that's happening in a duct or happening in a membrane, it's obviously fouling process.

28:54

But what happens if we want to generate crystals because maybe we're producing a pharmaceutical or we're producing a salt, right?

29:04

And so this is an example of vacuum crystallization, right, where that our body of the unit operation is maintained typically a vacuum pressure condition, so less than that of the atmospheric pressure.

29:16

And it's, it's uses a one or more vacuum pumps to maintain this alongside a condenser on top.

29:25

So our feed solution then here is maintained at a warm temperature where the IT is above the boiling point at the pressure of our unit operation, right?

29:38

So we're going to feed it across the valve and it's going to take a pressure drop, but it's already at a temperature where it's well above the boiling point, which means it's going to have to cool to that equilibrium temperature.

29:50

The enthalpy of the vaporization process is balanced by the enthalpy of both cooling the process stream and the enthalpy of crystallization.

30:00

So it basically allows that enthalpy balance to go from condensation or vaporization and counter balances that through the endothermic.

30:12

I'm sorry, the endothermic nature of vaporization or condensation is balanced by the energy requirement to cool that process stream.

30:20

That was it.

30:20

It flows upwardly alongside our crystallization.

30:26

So as we start forming crystals, then they will be taken off, right?

30:31

We can see here, this is the valve in which they're drawn off.

30:35

So I we could take it through a slurry pump, all right?

30:38

And this is then put into a centrifuge.

30:40

Because when we have crystals that are going to be dispersed in a liquid phase, now we need to mechanically separate these where we then siphon off.

30:49

So Centrifuge is going to spin off the liquids and then we have a product stream here where our liquids would then be returned down into our process stream and recycle through the body of that operation.

31:03

So if this type of process and thinking about the rates of crystallization here, these will be more appropriate for CHPR 4, four O 6 and five five O 1.

31:22

So reaction engineering and advanced reaction engineering, OK.

31:29

Now one of the the issues to think about is if we are looking at what I will touch on here is not that great a crystallization and how we handle it, but in particular the slurry pump.

31:41

So if we're going to start moving a solid and liquid suspension around, right, the energy requirements placed on this pump here we'll annotate with the star are going to be significantly higher than that which we talked about in Friday's lecture, right.

31:57

So when, when it gets into the pumping or transport of suspended solids in liquid, I should say there is a heuristic that would be very good to keep in mind and that is anytime, I'll start on the left here.

32:13

Anytime you maintain a solid liquid suspension, that liquid velocity should be above 1M a second if it drops below about 1m per second.

32:23

So here is because it is an engineering rule of thumb that's accurate.

32:26

Usually 80 plus percent of the time, usually when liquid velocity is independent of their duct size drop below 1m a second, it will allow the solids to settle.

32:37

So if the density of the solid is greater than that of the liquid.

32:40

So an example would be beach sand and water right?

32:43

Beach sand will have a density of around 2500 kilograms per DV meter.

32:48

The bottom is obviously around 1010 fifty.

32:51

So if we get below a meter a second, that beach sand is going to start settling downward in our flow stream and it's going to collect near the bottom and, and maybe it's going to maintain what we'd call a moving bed.

33:03

So that will mean the sand is kind of in a dense layer at the bottom, but it is moving along.

33:07

If it gets too far below that, that heuristic of 1 meter a second, it's going to form what we call a stationary bed, meaning that part of that sand is now solid on the bottom and we're going to, it's not going to be flowing with time, right?

33:20

And you can also have a hybrid of the two of a moving and a stationary bed together.

33:24

So we have a density stratification of basically no sand at the top, all sand at the bottom.

33:29

Now if we're forming an ice in water, for instance, and an example of this might be a desalination process, right, of wanting to freeze out ice in a salt water stream.

33:42

Because of course the formation of ice instead of relatively low, so cool will exclude ions from its formation and concentrate ions in the remaining residual aqueous stream.

33:56

But now ice has a density about 10% below that water.

33:59

So if it does form, it's going to float near the top.

34:02

And if you maintain a velocity below, or if your velocity dips below that 1m a second, then that ice is going to start collecting near the top of your pipe or duct and it's going to start freezing together over time.

34:15

So this 1m a second rule is what we use to prevent those solids from upward or downward settling.

34:21

But as we mentioned, if we get to that point of a stationary bed, then the viscosity or the amount of stress that we need to apply to the system to restart motion will be significantly higher than that which we were using to maintain it.

34:38

So if we draw this out for a minute, so if we think about, we would typically say the shear rate.

34:45

So that's the shear rate has units one per second, but it's well not equivalent.

34:56

It scales with velocity.

35:02

So shear rate depends heavily on the geometry that we're flowing in, but it's basically the dissipation or the rate of change of velocity from the centre line of our flow system to the edge.

35:14

So it's it's a scalar with velocity.

35:17

So if we look at that shear rate and then we look at the shear stress, right?

35:24

And that typically will have units of Pascal, right?

35:28

If we have a linear relationship this would be called a Newtonian fluid where the slope is the definition of viscosity right?

35:45

So a common or an important rule that is shear stress is equal to viscosity which has the units or the the Greek symbol  $\mu$  is viscosity times shear rate for a Newtonian system.

36:00

Now if we commonly have a solid and liquid suspension, this will typically take on either a shear thinning or shear thickening behaviour.

36:21

But importantly, if we allow those solids to settle out, then the stress that we need to apply  $\tau$  to restart flow in the system right of a solid bed we would call it will have a minimum value of stress that we need to apply before the first bit of shear rate is realized.

36:47

So if we have a Newtonian system, a shear thinning system or shear thickening system, the 2nd we have solids that have settled out where we need to think about a yield stress to restart flow in the system, all right.

36:59

And so this yield stress in some cases can exceed, if we've not thought about it ahead of time, can exceed the stress available from my pump or the amount of energy that it can impart.

37:11

So this can be quite a significant penalty to pay.

37:14

Now when it comes to viscosity, whether it's Newtonian thickening or thickening or thinning, this is very much dependent on the surface or interfacial tension between the solid and liquid, whether the solid is reacting with time, the nature, nature of its surface area, that sort of thing.

37:35



And it typically is going to require a lot of pilot scale testing experimentation to understand exactly what type of unit operation you're applying it to.

37:44

But one of the simplest viscosity models for the apparent viscosity of the slurry was actually reported by Einstein and he showed us that the volume fraction of salts and so this would be from zero to one in our system times the the base viscosity of liquid.

38:17

So let's say we're thinking about sand and water.

38:20

Well, we can reasonably assume water viscosity of you one centipoise.

38:24

So that means  $\mu_{sub}$  or  $\mu_{naughty}$  is 1 centipoise.

38:28

And so we can estimate the slurry viscosity or  $\mu_{sub S}$  as the base liquid viscosity times the product of  $1 + 2.5$  times the volume fraction of solids.

38:40

For zero means no, no solids and one would be 100%.

38:44

So we can never get into one, right?

38:45

Because they'll, they'll eventually jam together.

38:48

I mean usually between 4 sevens and five sevens.

38:52

So this model is only accurate to about two volume percent of solids in the system.

38:59

Just to give you an idea of how complex this type of calculation the research can be.

39:04

So This is why then translating this kind of dispersion of solid flow in these plants can actually be quite a significant domain.

39:14

And it's it's an area where you could spend multiple pH DS and still have a lot of ground to cover.

39:21

So finally, when we think about this in the context of a unit operation, it will typically require engineers to then invoke assumptions, right?

39:29

Because we're obviously not going to know everything we need to upfront.

39:34

So we would typically, you know, densities are relatively easy property because we would average it out in the system on a volumetric basis.

39:41

So if I have 25% solids at a density of say 25175% liquid in a density of 1000, I would simply say  $.75 * 1000 + .25 * 2500$  will give me a volumetric average density for that system.

40:00

Viscosity, as we just talked about, is a much, much harder problem.

40:04

So the the typical way that engineers will handle is simply because we don't have yet for the fundamental physics to describe more than two phase transport.

40:16

That's that's kind of an area of current research.

40:20

We would typically take a three phase system like gas plus liquid plus solid and reduce this using something called a pseudo phase approximation into the transport of gas plus a liquid suspension.

40:32

So we treat the secondary phase as all liquid and then we modify it's properly as either density or viscop and or as you know density and viscosity based on the amount of solid present.

40:45

OK, So in summary, dispersion is a very, very advanced kind of direction in chemical engineering, certainly an area very active for research.

40:56

But we can think about dispersion if we're we're in a system that's separable.

41:01

So gas, you know, air and water is the best example to think about.

41:06

The dispersion between two phases that can mix it is dictated by the force we put into it.

41:12

So that's the dissipation of turbulent energy, which acts to break up and create surface area.

41:18

And the interfacial tension, which is a kind of A2 dimensional well, is A2 dimensional, equivalent to pressure.

41:26

So that acts to restore phases.

41:29

And you'll also see this report in the literature, interfacial tension, surface tension and it's also called surface free energy in more advanced tests.

41:44

And so when we start transporting or moving around, so mixing or transporting a dispersion whether it's a gas liquid, liquid liquid or solid liquid, this can evoke with very high energy penalty.

41:59

And so although we have significantly higher viscosities during steady state operation and then the base liquid or fluid right, which that would have the viscosity need not we have it yet even higher yield stress requirement to re entrain any kind of solid material in particular if we're restarting.

42:22

So then this this is critical on the field of crystallization, which is a more shouldn't say as crystallization and advanced chemical engineering.

42:35

So one of the critical points to take home is that crystallization phenomena will always, always, always associate with it in induction requirement in both time and energy.

42:48

So the higher the energy driving force so that total DG or  $\Delta G$ , the lower the induction time.

42:56

But if I'm sitting at a very low driving force of  $\Delta G$ , which I can characterize others  $\Delta P$ ,  $\Delta T$ , or  $\Delta T_0$ , then  $\Delta \mu \times \Delta N$ , then I will also have a much longer induction time requirement.

43:20

This can apply to the desirable outcome of crystallization if I have for instance purification via like a vacuum crystallizer, but it can also apply to an undesirable phenomena being filing in some unit operations.