

Polynomial Regression

Simple
Linear
Regression

$$y = b_0 + b_1 x_1$$

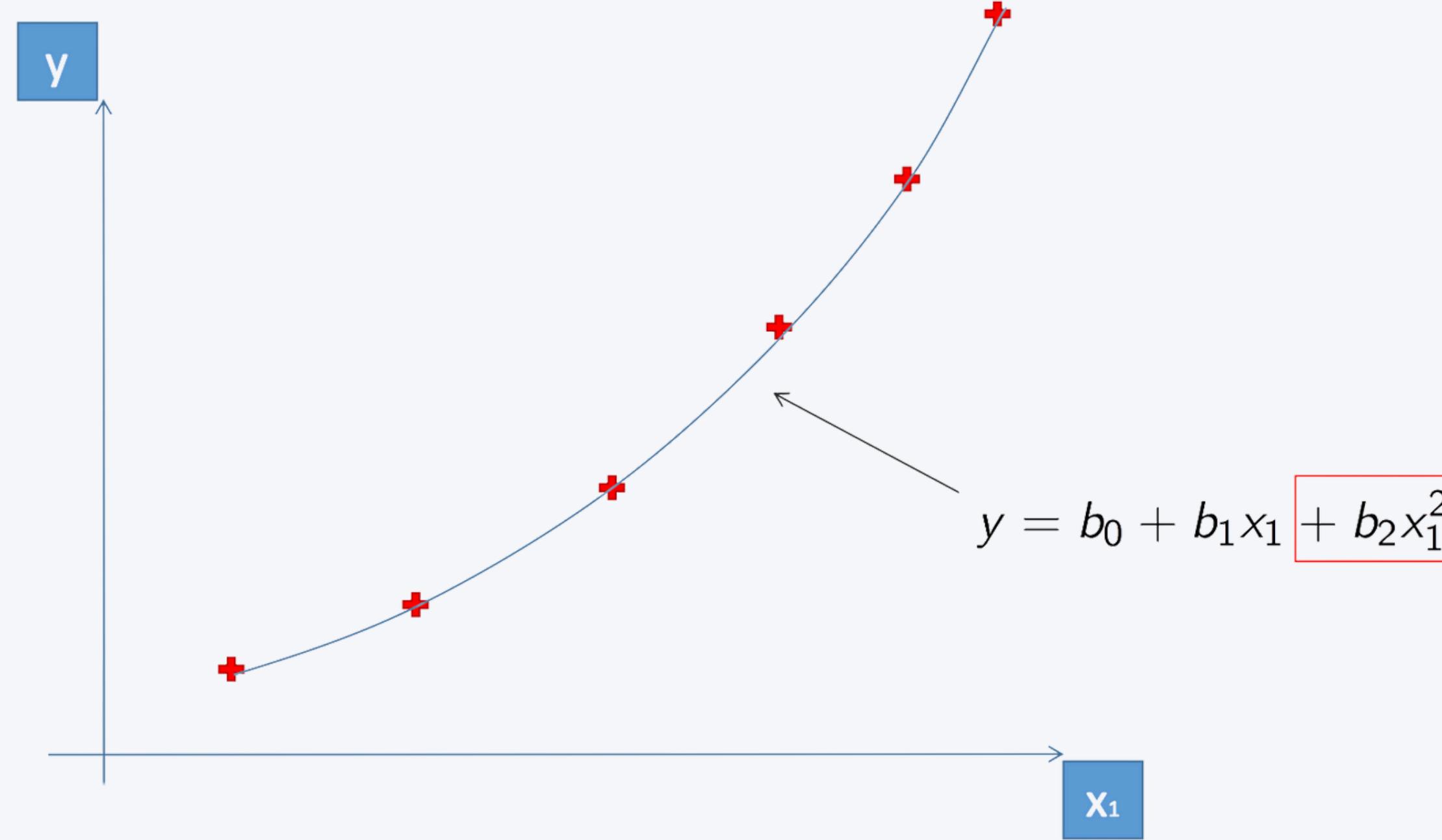
Multiple
Linear
Regression

$$y = b_0 + b_1 x_1 + b_2 x_2 + \dots + b_n x_n$$

Polynomial
Linear
Regression

$$y = b_0 + b_1 x_1 + b_2 x_1^2 + \dots + b_n x_1^n$$

Polynomial Regression



Polynomial Linear Regression is an extension of linear regression that **models non-linear relationships by transforming the input variables into polynomial terms**. Even though the relationship between the variables may **not** be linear, **the regression itself is still considered "linear" because the model is linear in terms of the coefficients**.

Polynomial Regression

Key Features of Polynomial Regression

Transformation of Variables:

- In regular linear regression, the model is: $y=b_0+b_1x$
- In polynomial regression, higher-order terms are added: $y=b_0+b_1x+b_2x^2+b_3x^3+\dots$

These polynomial terms allow the model to **capture curves or more complex relationships.**

Fitting Curves:

- Useful when the data shows a **non-linear relationship**, such as U-shaped, S-shaped, or exponential patterns.

Still a "Linear Model":

- Even though the relationship is non-linear, it is called "linear" because the equation remains linear in terms of the coefficients (b_0, b_1, b_2).
- This is why it's called **Polynomial Linear Regression**.

When to Use Polynomial Regression?

- When the data shows **clear non-linear patterns** that cannot be captured by regular linear regression.
- When adding **flexibility** is needed for better accuracy but without overfitting (important to choose the right degree of the polynomial).

Polynomial
Linear
Regression

$$y = b_0 + b_1x_1 + b_2x_1^2 + \dots + b_nx_1^n$$

Polynomial Regression

Why Do We Need Polynomial Regression?

- Linear regression works only **for straight-line relationships.**
- Many real-world relationships are non-linear, such as:
 - Population growth.
 - Disease progression.
 - Economic data (e.g., sales trends).

Example: Stock price data that linear regression cannot capture.

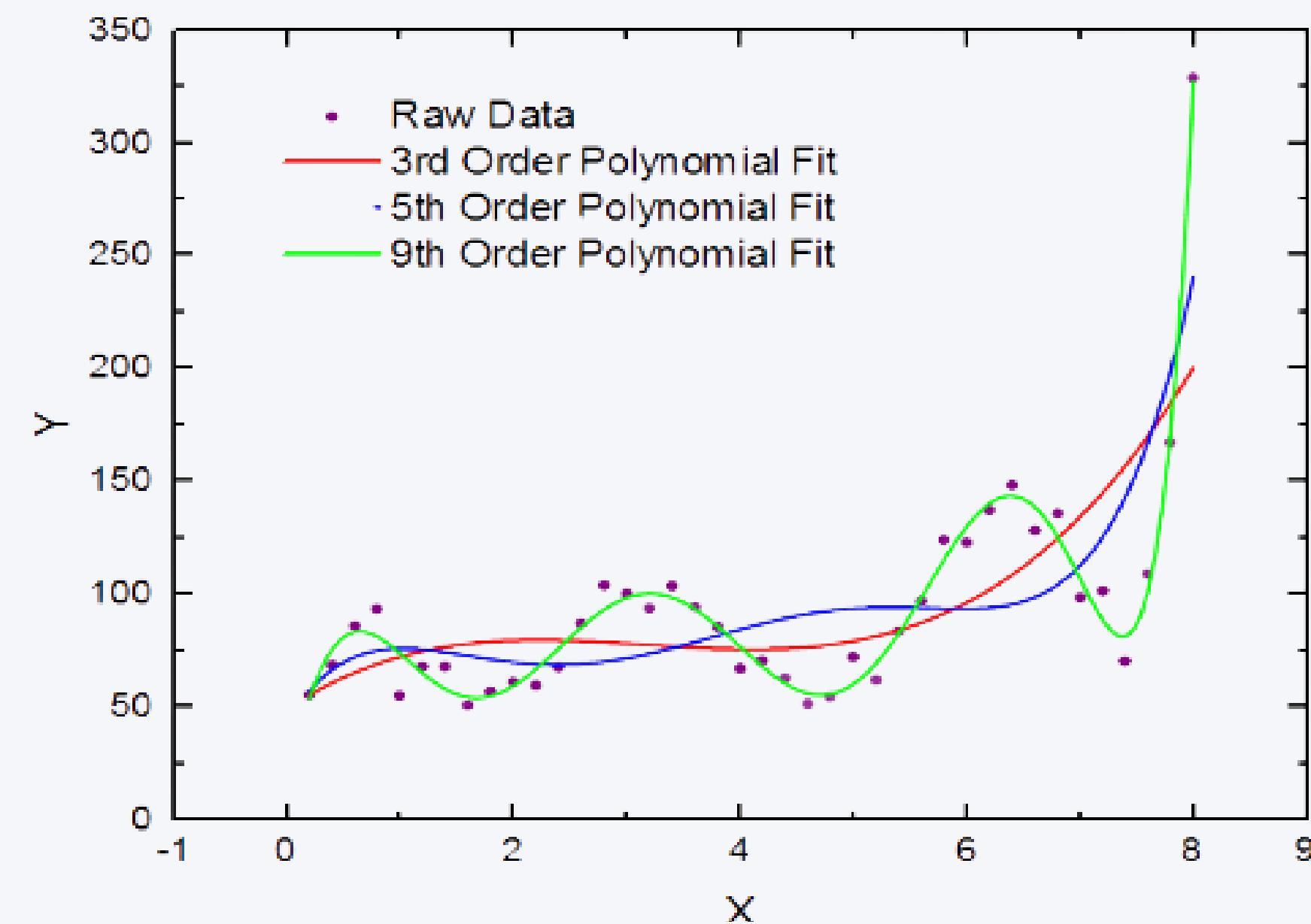


https://www.investopedia.com/terms/p/polynomial_trending.asp

Polynomial Regression

How Polynomial Regression Works

- Convert the input variable (x) into polynomial terms (x^2, x^3).
- Apply linear regression to estimate coefficients (β_0, β_1).
- Higher-order terms allow the model to capture curves.



Polynomial Regression

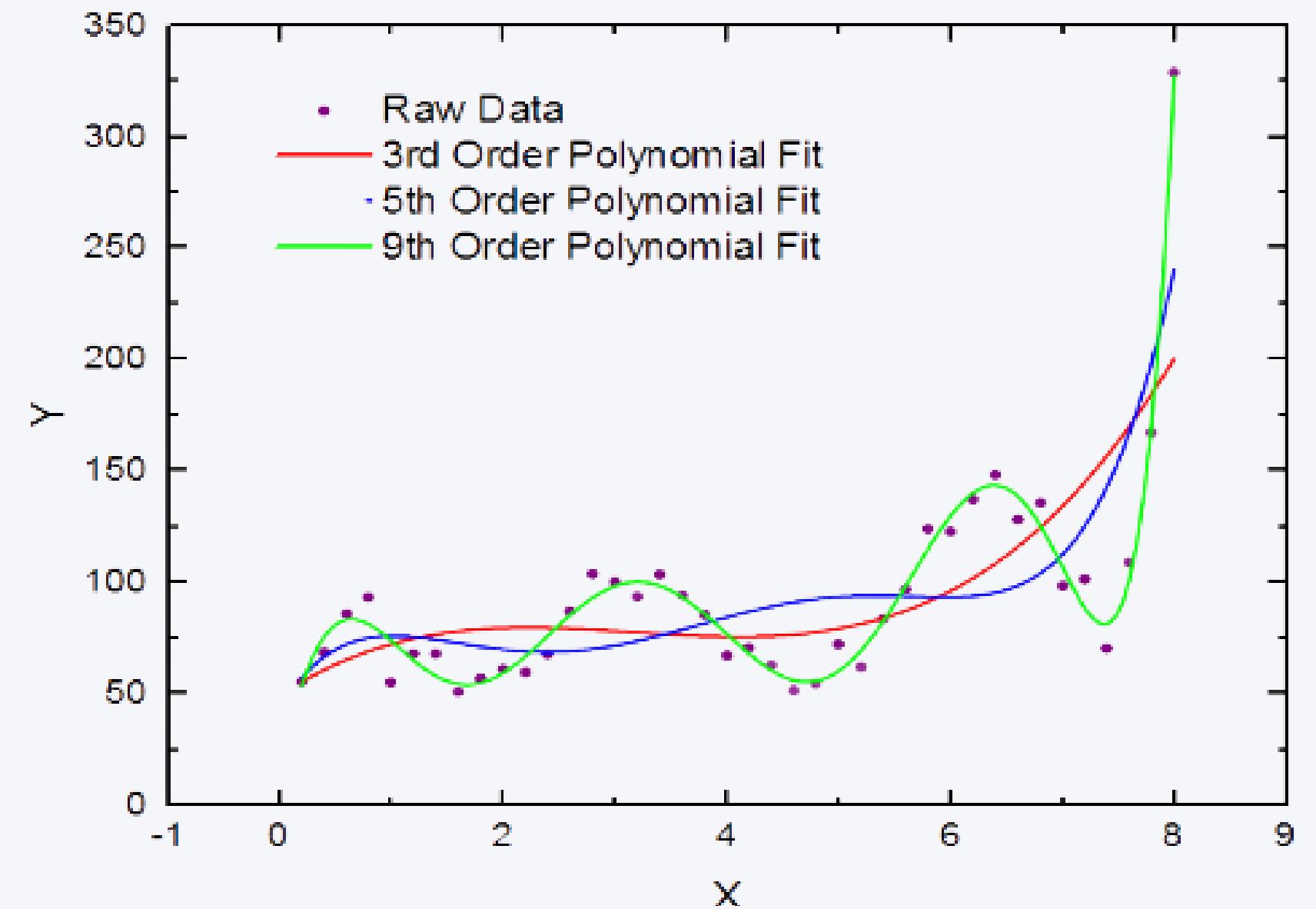
Choosing the Polynomial Degree

The degree of the polynomial affects the model's complexity:

- **Low degree:** Underfitting (model too simple).
- **High degree:** Overfitting (model too complex, fits noise).

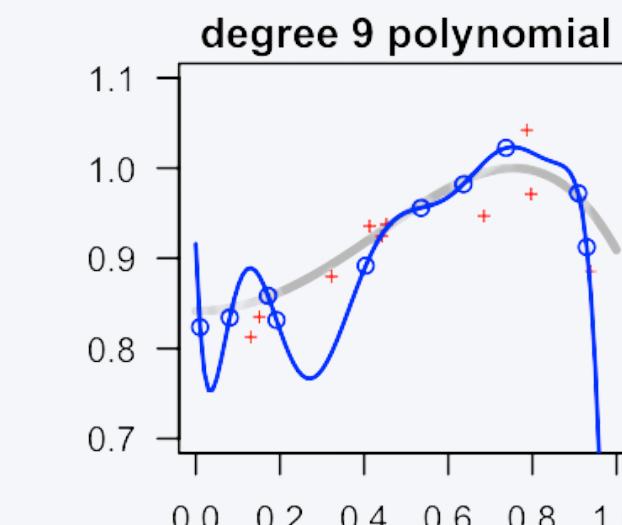
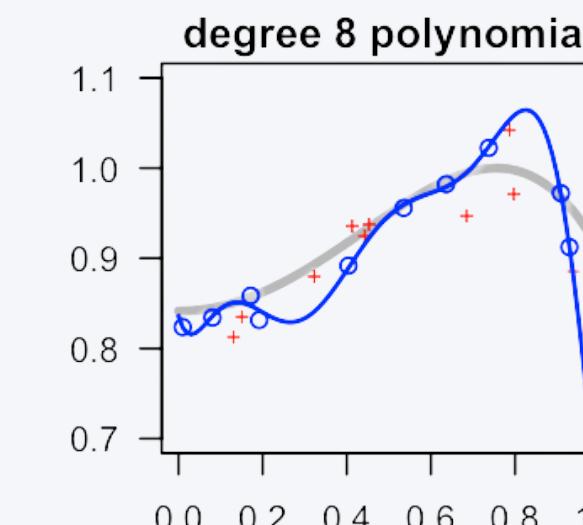
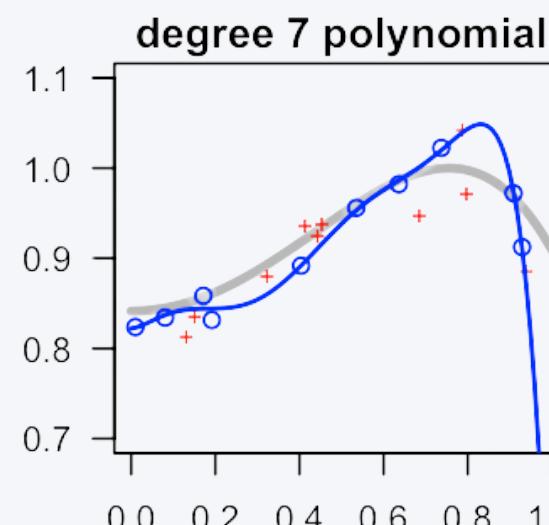
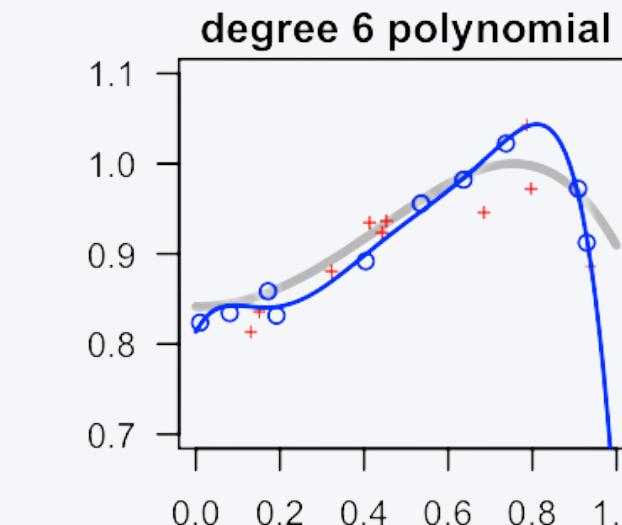
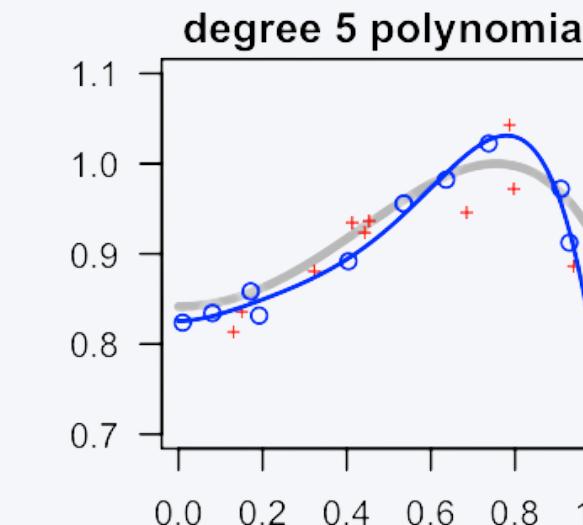
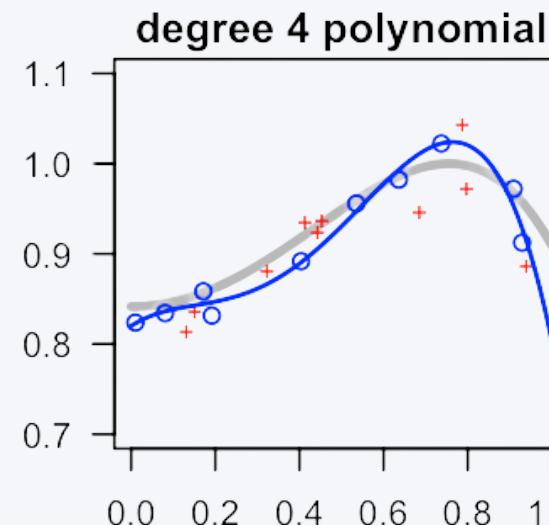
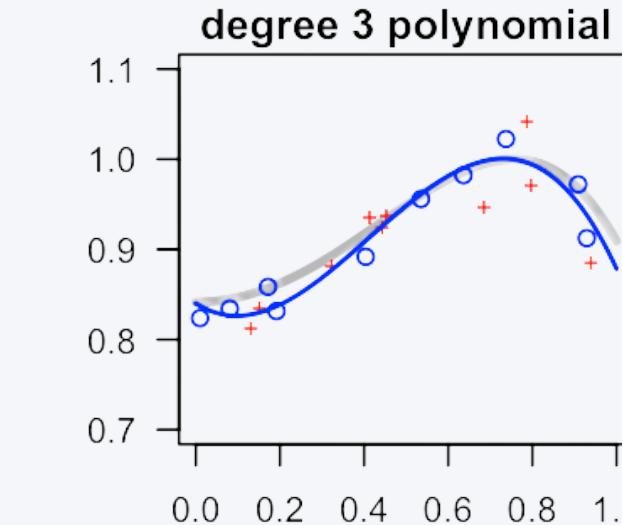
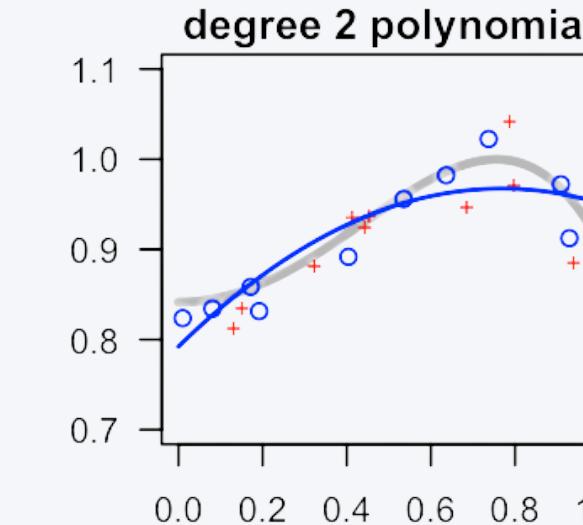
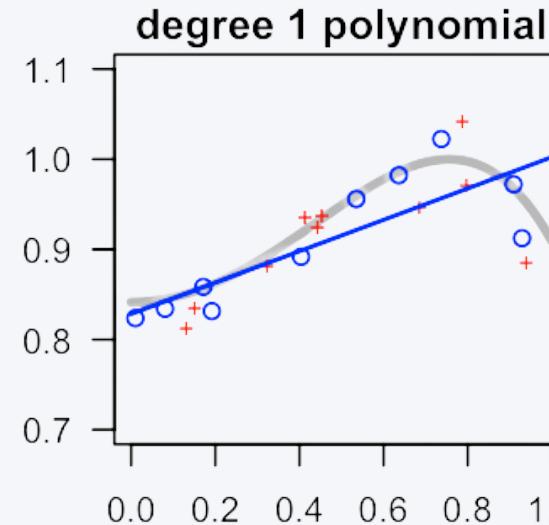
Use tools like:

- Cross-validation.
- Adjusted R².



Polynomial Regression

Choosing the Polynomial Degree



Polynomial Regression

Practical Applications

Use cases for polynomial regression:

- Forecasting trends in sales or demand.
 - Modeling natural phenomena (e.g., rainfall vs. crop yield).
 - Predicting disease progression.
 - Engineering data (e.g., stress-strain curves).
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Limitations of Polynomial Regression

- Overfitting when the degree is too high.
- Sensitive to outliers.
- May not generalize well to unseen data.
- Not suitable for truly complex, non-linear relationships (consider other models like decision trees or neural networks).

Hands-On Code

Polynomial Linear Regression

