

Q1

Consider the scalar system

$$\dot{x} = -x + u + w$$

w is zero-mean process noise with a variance of Q . The control has a mean value of u_0 , an uncertainty of 2 (one standard deviation), and is uncorrelated with w . Rewrite the system equations to obtain an equivalent system with a normalized control that is perfectly known. What is the variance of the new process noise term in the transformed system equation?

Solution: The variance of the new process noise, w_u is $\Sigma_{w_u} = Q + \sigma_u^2 = Q + 4$.

$$\dot{x} = -x + u_0 + \underbrace{w + \Delta u}_{w_u}, \quad w_u \sim (0, Q + \sigma_u^2).$$

Q2

Consider the system

$$x_{k+1} = \phi x_k + w_k,$$

$$y_k = x_k,$$

where $w_k \sim (0, 1)$, and $\phi = 0.9$ is an unknown constant. Design an extended Kalman filter to estimate ϕ . Simulate the filter for 100 time steps with $x_0 = 1, P_0 = I, \hat{x}_0 = 0$, and $\hat{\phi}_0 = 0$. Hand in your source code and a plot showing $\hat{\phi}$ as a function of time.

Solution: Perform the measurement update of the state estimate and estimation error covariance as follows

$$\begin{aligned} K_k &= P_k^- H_k^\top (H_k P_k^- H_k^\top + R_k)^{-1} = P_k^- H_k^\top (H_k P_k^- H_k^\top)^{-1}, \quad \text{Since } R_k = 0, \\ \hat{x}_k^+ &= \hat{x}_k^- + K_k (y_k - h_k(\hat{x}_k^-, 0)) \\ &= \hat{x}_k^- + K_k (y_k - \hat{x}_k^-), \quad \text{Since } \hat{\phi}_k^- = 0, \\ P_k^+ &= (I - K_k H_k) P_k^- \end{aligned}$$

Q5

For our problem we have $L = 1500$ Du, Travel costs of 3 Mu/Du and cargo transportation needs per time unit as shown below:

$$\delta(d) = \begin{cases} 5, & \text{if } 0 \leq d \leq 500 \\ 1, & \text{if } 500 < d < 1000 \\ 5, & \text{if } 1000 \leq d \leq 1500 \end{cases}$$

Solution:

$$CumulativeDemand(d) = CD(d) = \begin{cases} 5d, & \text{if } 0 \leq d \leq 500 \\ 2500 + (d - 500), & \text{if } 500 < d < 1000 \\ 3000 + 5(d - 1000), & \text{if } 1000 \leq d \leq 1500 \end{cases}$$

A. The approach that I will use for this part is solve use the integrals in the formulation to get the answer.

1. For this case, the optimal location is given by:

$$d^* = \frac{\int_a^b CD(x)dx + CD(a) \cdot a - CD(b) \cdot b}{CD(a) - CD(b)}$$

which is obtained by setting the areas on both sides of the d^* equal to each other. For our particular problem:

$$d^* = \frac{\int_0^{1500} CD(x)dx + CD(0) \cdot 0 - CD(1500) \cdot 1500}{CD(0) - CD(1500)}$$

$$d^* = \frac{4,125,000 + 0 - 8,250,000}{0 - 5500} = 750$$

$$\boxed{d^* = 750}$$

The best location for the terminal ($n = 1$) is at 750 Du.

2.