

1. Diketahui peubah acak  $X$  dengan fkp

$$f_X(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & x \text{ lainnya} \end{cases}$$

- Tentukan FKP bagi  $Y = \frac{1}{2}X^3$  (skor 20)
- Tentukan FKP bagi  $W = X^5 + 4$  (skor 20)
- Tentukan FKP bagi  $T = |1 - X|$  (skor 20)

①  $f_X(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{else} \end{cases}$

a)  $\Rightarrow F_Y(Y) = P(Y \leq y)$   $\Rightarrow f_Y(Y) = \frac{d}{dy} (2y)^{2/3}$   
 $= P\left(\frac{1}{2}x^3 \leq y\right)$   $= \frac{2}{3} (2y)^{-1/3} (2)$   
 $= P(x \leq \sqrt[3]{2y})$   $= \frac{4}{3 \sqrt[3]{2y}}$   
 $= \int_0^{\sqrt[3]{2y}} 2x \, dx$   
 $= x^2 \Big|_0^{\sqrt[3]{2y}}$   $\Rightarrow 0 < x < 1$   
 $= (2y)^{2/3}$   $0 < \frac{1}{2}x^3 < \frac{1}{2}$   
 $0 < y < \frac{1}{2}$

$$F_Y(Y) = \begin{cases} 0, & y \leq 0 \\ (2y)^{2/3}, & 0 < y < 1/2 \\ 1, & y \geq 1/2 \end{cases} \quad f_Y(Y) = \begin{cases} \frac{4}{3 \sqrt[3]{2y}}, & 0 < y < 1/2 \\ 0, & \text{else} \end{cases}$$

b)  $\Rightarrow F_W(W) = P(W \leq w)$   $\Rightarrow f_W(W) = \frac{d}{dw} (w-4)^{2/5}$   
 $= P(X^5 + 4 \leq w)$   $= \frac{2}{5} (w-4)^{-3/5}$   
 $= P(X \leq \sqrt[5]{w-4})$   $\Rightarrow 0 < x < 1$   
 $= \int_0^{\sqrt[5]{w-4}} 2x \, dx$   $4 < x^5 + 4 < 5$   
 $= x^2 \Big|_0^{\sqrt[5]{w-4}}$   $4 < w < 5$   
 $= (w-4)^{2/5}$

$$F_W(W) = \begin{cases} 0, & w \leq 4 \\ (w-4)^{2/5}, & 4 < w < 5 \\ 1, & w \geq 5 \end{cases} \quad f_W(W) = \begin{cases} \frac{2}{5} (w-4)^{-3/5}, & 4 < w < 5 \\ 0, & \text{else} \end{cases}$$

$$c) \rightarrow F_t(T) = P(T \leq t)$$

$$= P(1-x \leq t)$$

karena  $0 < x < 1$ ,  
 $1-x$  akan selalu positif

$$= P(1-x \leq t)$$

$$= P(-x \leq t-1)$$

$$= P(x \geq 1-t)$$

$$= \int_{1-t}^1 2x \, dx$$

$$= x^2 \Big|_{1-t}^1$$

$$= 1 - (t^2 - 2t + 1)$$

$$= 2t - t^2$$

$$\rightarrow f_t(T) = \frac{d}{dt} (2t - t^2)$$

$$= 2 - 2t$$

$$\rightarrow 0 < x < 1$$

$$-1 < -x < 0$$

$$0 < 1-x < 1$$

$$0 < |1-x| < 1$$

$$0 < T < 1$$

$$\rightarrow F_t(T) = \begin{cases} 0, & T \leq 0 \\ 2t - t^2, & 0 < T < 1 \\ 1, & T \geq 1 \end{cases}$$

$$f_t(T) = \begin{cases} 2-2t, & 0 < T < 1 \\ 0, & \text{else} \end{cases}$$

2. Diketahui peubah acak  $X \sim \exp(4)$ . Tentukan fungsi kepekatan peluang bagi  $Y = \frac{1}{2}X$  (gunakan metode momen) (skor 20)

$$X \sim \exp(4)$$

$$\text{mgf buat } x \Rightarrow M_x(t) = E(e^{tx}) = (1-4t)^{-1}$$

$$\rightarrow M_y(t) = E(e^{ty})$$

$$= E(e^{t \cdot \frac{1}{2}x})$$

$$= E(e^{(\frac{1}{2}t)x})$$

$$\rightarrow E(e^{tx}) = (1-4t)^{-1}$$

$$E(e^{\frac{1}{2}tx}) = (1-2t)^{-1}$$

$$E(e^{ty}) = (1-2t)^{-1}$$

$$\rightarrow Y \sim \exp(2)$$

$$\text{fknya: } f_y(y) = \begin{cases} \frac{1}{2} e^{-y/2}, & y \geq 0 \\ 0, & \text{else} \end{cases}$$

3. Diketahui peubah acak  $X \sim \text{Gamma}(2, \beta)$ . Tentukan fungsi kepadatan peluang bagi  $W = \frac{X}{4\beta}$  (gunakan metode momen) (skor 20)

$$X \sim \text{gamma}(2, \beta)$$

$$\text{mgf } X : M_X(t) = E(e^{tx}) = (1 - \beta t)^{-\alpha}$$

$$\hookrightarrow \alpha = 2 \rightarrow M_X(t) = E(e^{tx}) = (1 - \beta t)^{-2}$$

$$\begin{aligned} M_W(t) &= E(e^{tw}) \\ &= E\left(e^{t \frac{x}{4\beta}}\right) \\ &= E\left(e^{\frac{t}{4\beta} x}\right) \end{aligned}$$

$$\begin{aligned} \rightarrow E(e^{tx}) &= (1 - \beta t)^{-2} \\ E\left(e^{\frac{t}{4\beta} x}\right) &= \left(1 - \beta \frac{t}{4\beta}\right)^{-2} \\ &= \left(1 - \frac{1}{4} t\right)^{-2} \\ E(e^{tw}) &= \left(1 - \frac{1}{4} t\right)^{-2} \end{aligned}$$

$$\rightarrow Y \sim \text{gamma}(2, \frac{1}{4})$$

$$\rightarrow \frac{1}{\Gamma(\alpha) \beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}} = \frac{x \cdot e^{-\frac{x}{2}}}{\Gamma(2) \frac{1}{4^2}}$$

$$\rightarrow f_W(w) = \begin{cases} \frac{w \cdot e^{-\frac{w}{2}}}{\Gamma(2) \frac{1}{4^2}} & , w \geq 0 \\ 0, & \text{else} \end{cases}$$