

10. interpolasi

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Polinom langrange

derajat 1

$$P_1(x) = y_0 \frac{(x-x_1)}{(x_0-x_1)} + y_1 \frac{(x-x_0)}{(x_1-x_0)}$$

derajat 2

$$P_2(x) = y_0 \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_1-x_2)} + y_1 \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + y_2 \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

derajat 3

$$\begin{aligned} P_3(x) &= y_0 \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_1-x_2)(x_0-x_3)} + y_1 \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \\ &+ y_2 \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + y_3 \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \end{aligned}$$

Polinom Newton

- Rekursif

$$P_1(x) = a_0 + a_1(x-x_0)$$

$$P_2(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1)$$

$$P_3(x) = P_2(x) + a_3(x-x_0)(x-x_1)(x-x_2)$$

$$P_N(x) = P_{N-1}(x) + a_N(x-x_0)(x-x_1)\dots(x-x_{N-1})$$

$$a_0 = f(x_0)$$

$$a_1 = f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$a_2 = f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$$a_N = f[x_0, \dots, x_N] = \frac{f[x_1, \dots, x_N] - f[x_0, \dots, x_{N-1}]}{x_N - x_0}$$

Tabel terbagi newton

x_k	$f[x_k]$	Ordo 1 $f[,]$	Ordo 2 $f[, ,]$	Ordo 3 $f[, , ,]$	Ordo 4 $f[, , , ,]$
x_0	$f[x_0]$				
x_1	$f[x_1]$	$f[x_0, x_1]$			
x_2	$f[x_2]$	$f[x_1, x_2]$	$f[x_0, x_1, x_2]$		
x_3	$f[x_3]$	$f[x_2, x_3]$	$f[x_1, x_2, x_3]$	$f[x_0, x_1, x_2, x_3]$	
x_4	$f[x_4]$	$f[x_3, x_4]$	$f[x_2, x_3, x_4]$	$f[x_1, x_2, x_3, x_4]$	$f[x_0, x_1, x_2, x_3, x_4]$

untuk memudahkan mengerjakan soal Selesaikan segitiganya

Buatlah tabel beda terbagi Newton untuk berdasarkan pada 5 titik ($k, \cos(x)$), $k = 0, 1, 2, 3, 4$

Gunakan tabel tersebut untuk menemukan koefisien-koefisien a_k dan polinom interpolasi $P_1(x)$, $P_2(x)$, $P_3(x)$ dan $P_4(x)$.

x_k	$f(X_k)$	$f[x_0, x_1]$	$f[x_0, x_1, x_2]$	$f[x_0, x_1, x_2, x_3]$	$f[x_0, x_1, x_2, x_3, x_4]$
0	0.5403023				
1	-0.9161968	-0.9564991			
2	-0.9899925	-0.538457	0.1913017		
3	-0.6536336	0.3163489	0.4550973	0.0879	
4	0.2836622	0.9373058	0.3009789	-0.0515	0.0390

$$P_0(x) = 0.5403023$$

$$P_1(x) = 0.54 + (-0.95)(x-x_0)$$

$$P_2(x) = 0.54 + (-0.95)(x-x_0) + 0.19(x-x_0)(x-x_1)$$

$$P_3(x) = P_2(x) + 0.088(x-x_0)(x-x_1)(x-x_2)$$

$$P_4(x) = P_3(x) + 0.0398(x-x_0)(x-x_1)(x-x_2)(x-x_3)$$