

F dikatakan sebagai **transformasi linear jika:**

1. $F(\vec{u} + \vec{v}) = F(\vec{u}) + F(\vec{v})$ untuk semua vector \vec{u} dan \vec{v} pada V

2. $F(k\vec{u}) = kF(\vec{u})$ untuk semua scalar k dan vector \vec{u} pada V

1. Apakah $F: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ dengan $F(\vec{x}) = (2x_1, x_2 * x_2)^T$; merupakan transformasi linear?

2. Apakah $F: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ dengan $F((x, y, z)^T) = \begin{bmatrix} x+z \\ x-2y \end{bmatrix}$; merupakan transformasi linear?

3. Misalkan Himpunan $S = \{\vec{v}, \vec{w}\}$ adalah basis dari \mathbb{R}^2 dengan $\vec{v} = (1, 0)$ dan $\vec{w} = (-2, 1)$. Misalkan sebuah trasnformasi linear $F: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ memetakan :

$$F(\vec{v}) = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}; F(\vec{w}) = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}.$$

Temukan fungsi transformasi tersebut?

$$\textcircled{1} \quad F: \mathbb{R}^2 \rightarrow \mathbb{R}; \quad F([x_1, x_2]) = (2x_1, x_2 * x_2)^T$$

a) bukti $F(k\vec{u}) = kF(\vec{u})$:

$$F(k \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}) = F \left(\begin{bmatrix} kx_1 \\ kx_2 \end{bmatrix} \right) = \begin{pmatrix} 2k \cdot x_1 \\ kx_2 \cdot kx_2 \end{pmatrix} = \begin{pmatrix} 2kx_1 \\ k^2 x_2^2 \end{pmatrix}$$

$$kF \left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = k \begin{pmatrix} 2x_1 \\ x_2 \cdot x_2 \end{pmatrix} = \begin{pmatrix} 2kx_1 \\ kx_2^2 \end{pmatrix}$$

$$F(k \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}) \neq F \left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) \rightarrow F \text{ bukan transformasi linear}$$

$$\textcircled{2} \quad F: \mathbb{R}^3 \rightarrow \mathbb{R}^2; \quad F([x, y, z]^T) = \begin{bmatrix} x+2 \\ x-2y \end{bmatrix}$$

a) bukti $F(k\vec{u}) = kF(\vec{u})$:

$$\begin{aligned} F \left(k \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) &= F \left(\begin{bmatrix} kx \\ ky \\ kz \end{bmatrix} \right) = \begin{pmatrix} kx + kz \\ kx - kz \end{pmatrix} \\ &= k \begin{pmatrix} x+2 \\ x-2y \end{pmatrix} \end{aligned}$$

$$= kF \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right)$$

$$\hookrightarrow F(k\vec{u}) = kF(\vec{u}) \checkmark$$

b) bukti $F(\vec{u} + \vec{v}) = F(\vec{u}) + F(\vec{v})$

$$\rightarrow \text{misalkan: } \bar{a} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \& \quad \bar{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\bar{a} + \bar{x} = \begin{bmatrix} a+x \\ b+y \\ c+z \end{bmatrix}$$

v<1

v>2

$$\widehat{a+x} = \begin{bmatrix} a+x \\ b+y \\ c+z \end{bmatrix}$$

$$\begin{aligned}
\Rightarrow F(\widehat{a+x}) &= F\left(\begin{bmatrix} a+x \\ b+y \\ c+z \end{bmatrix}\right) = \begin{bmatrix} a+x+(c+z) \\ a+x-2(b+y) \end{bmatrix} \\
&= \begin{bmatrix} a+x+c+z \\ a+x-2b-2y \end{bmatrix} \\
&= \begin{bmatrix} a+c+x+z \\ a-2b+x-2y \end{bmatrix} \\
&= F(\bar{a}) + F(\bar{x}) \\
\textcolor{red}{\hookrightarrow} F(\bar{u}+\bar{v}) &= F(\bar{u}) + F(\bar{v}) \quad \checkmark
\end{aligned}$$

c) $F(\bar{u}+\bar{v}) = F(\bar{u}) + F(\bar{v}) \quad \checkmark$ } F adalah transformasi linier
 $F(k\bar{u}) = kF(\bar{u}) \quad \checkmark$

(3) $[\bar{v} \bar{w}] = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}; \quad F([\bar{v} \bar{w}]) = \begin{bmatrix} -1 & 1 \\ 1 & -2 \\ 2 & -3 \end{bmatrix}$

$$\begin{aligned}
\Rightarrow F^{M \times n} \cdot [\bar{v} \bar{w}] &= F([\bar{v} \bar{w}]) \\
\textcolor{red}{F^{M \times n}} &= F([\bar{v} \bar{w}]) \cdot [\bar{v} \bar{w}]^{-1}
\end{aligned}$$

\Rightarrow mencari $[\bar{v} \bar{w}]^{-1}$:

$$\left(\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array} \right) \xrightarrow{E_{12}(v)} \left(\begin{array}{cc|cc} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{array} \right)$$

$$[\bar{v} \bar{w}]^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

\Rightarrow mencari matriks $F^{M \times n}$:

$$F^{M \times n} = \begin{bmatrix} -1 & 1 \\ 1 & -2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$$

Sistem Persamaan Linear:

Selesaikan SPL berikut dengan metode matriks balikan

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 5 \\ 2x_1 - 4x_2 - x_3 &= 7 \\ 3x_1 + 2x_3 &= 8 \end{aligned}$$

1. Dengan Eliminasi Gaus-Yourdan
2. Dengan Faktorisasi LU

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 5 \\ 2x_1 - 4x_2 - x_3 &= 7 \\ 3x_1 + 2x_3 &= 8 \end{aligned} \rightarrow \begin{pmatrix} 1 & -1 & 2 \\ 2 & -4 & -1 \\ 3 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 8 \end{pmatrix}$$

(1) Metode Gauss-Jordan :

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 2 & -4 & -1 & 7 \\ 3 & 0 & 2 & 8 \end{array} \right) \xrightarrow{E_{21}(-2)} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 0 & -2 & -5 & -3 \\ 3 & 0 & 2 & 8 \end{array} \right) \xrightarrow{E_{12}(-0,5)} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 0 & -2 & -5 & -3 \\ 0 & 3 & -4 & -7 \end{array} \right) \xrightarrow{E_{32}(1,5)} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 0 & -2 & -5 & -3 \\ 0 & 0 & 1 & 15 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 4,5 & 6,5 \\ 0 & -2 & -5 & -3 \\ 0 & 0 & -11,5 & -11,5 \end{array} \right) \xrightarrow{E_3\left(\frac{1}{11,5}\right)} \left(\begin{array}{ccc|c} 1 & 0 & 4,5 & 6,5 \\ 0 & -2 & -5 & -3 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{E_{13}(-4,5)} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{E_{23}(5)} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{E_2\left(\frac{1}{2}\right)} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$\therefore \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

(2) Faktorisasi LU

matriks $A = L \cdot U$

$$A \cdot x = B$$

$$L \cdot U \cdot x = B$$

↪ misalkan ini Y

$$L \cdot Y = B$$

$$U \cdot x = Y$$

⇒ Cari nilai U dan Y

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & -9 & -1 \\ 3 & 0 & 2 \end{pmatrix} \rightarrow \text{ubah ke segitiga}$$

$-L_{32} R$

$$\begin{matrix} E_{21}(-2) \\ \sim \\ E_{31}(-3) \end{matrix} \begin{pmatrix} 1 & -1 & 2 \\ 0 & -2 & -5 \\ 0 & 3 & -9 \end{pmatrix} \sim \begin{matrix} E_{32}(1/5) \\ \sim \\ E_{31}(-3) \end{matrix} \begin{pmatrix} 1 & -1 & 2 \\ 0 & -2 & -5 \\ 0 & 0 & -11.5 \end{pmatrix}$$

$$\Rightarrow L = \begin{pmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1.5 & 1 \end{pmatrix}$$

⇒ Cari nilai Y :

$$LY = B$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1.5 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 8 \end{pmatrix}$$

$$\Rightarrow y_1 = 5$$

$$\Rightarrow 2y_1 + y_2 = 7$$

$$10 + y_2 = 7$$

$$y_2 = -3$$

$$\Rightarrow 3y_1 - 1.5y_2 + y_3 = 8$$

$$15 + 4.5 + y_3 = 8$$

$$y_3 = -11.5$$

$$\Rightarrow \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ -11.5 \end{pmatrix}$$

⇒ Cari nilai X :

$$U \cdot x = Y$$

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & -2 & -5 \\ 0 & 0 & -11.5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ -11.5 \end{pmatrix}$$

$$U \cdot x = Y$$

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & -2 & -5 \\ 0 & 0 & -11.5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ -11.5 \end{pmatrix}$$

$$\Rightarrow -11.5x_3 = -11.5$$

$$x_3 = 1$$

$$\Rightarrow -2x_2 - 5x_3 = -3$$

$$-2x_2 - 5 = -3$$

$$-2x_2 = 2$$

$$x_2 = -1$$

$$\Rightarrow x_1 - x_2 + 2x_3 = 5$$

$$x_1 + 1 + 2 = 5$$

$$x_1 + 3 = 5$$

$$x_1 = 2$$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$