Latihan UAS (untuk dishare)

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Singular Value Decomposition (SVD):

Tentukan SVD dari $A = \begin{bmatrix} 0 & 2 & 2 \\ 2 & 1 & 0 \end{bmatrix}$

Jika baris > kolom : $A^TA \rightarrow$ kolom V

Jika kolom > baris : $AA^T \rightarrow$ kolom U

$u_1 = \frac{1}{\sigma_1} A v_1$

$u_2 = \frac{1}{\sigma_2} A v_2$

> Kolom V

>> $A^TA = \begin{pmatrix} 0 & 2 \\ 2 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 2 & 2 \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 5 & 4 \\ 0 & 4 & 4 \end{pmatrix}$

>> eigenvalues : $\begin{pmatrix} 4-\lambda & 2 & 0 \\ 2 & 5-\lambda & 4 \\ 0 & 4 & 4-\lambda \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$

$\det \begin{pmatrix} 4-\lambda & 2 & 0 \\ 2 & 5-\lambda & 4 \\ 0 & 4 & 4-\lambda \end{pmatrix} = 0$

$= [(4-\lambda)(5-\lambda)(4-\lambda) + 2\cdot4\cdot0 + 0\cdot2\cdot4]$

$- [0\cdot(5-\lambda)\cdot0 + 4\cdot4(4-\lambda) + (4-\lambda)\cdot2\cdot2] = 0$

$= -\lambda^3 + 13\lambda^2 - 36\lambda = 0$

$= -\lambda\ (\lambda^2 - 13\lambda - 36\lambda) = 0$

$\lambda = 0;\ \lambda = 4;\ \lambda = 9 \rightarrow \sigma_1 = 3;\ \sigma_2 = 2$

>> eigenvectors :

$\lambda = 9$

$\hookrightarrow \begin{pmatrix} 4-9 & 2 & 0 \\ 2 & 5-9 & 4 \\ 0 & 4 & 4-9 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} -5 & 2 & 0 \\ 2 & -4 & 4 \\ 0 & 4 & -5 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\sim \begin{pmatrix} 0 & -8 & -10 \\ 2 & -4 & 4 \\ 0 & -4 & -5 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 \\ 2 & -4 & 4 \\ 0 & -4 & -5 \end{pmatrix}$

$\begin{pmatrix} 0 & 0 & 0 \\ 2 & -4 & 4 \\ 0 & -4 & -5 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

>>> $-4b - 5c = 0$

$b = \frac{5}{4} c$

>>> $2a - 4b + 4c = 0$

$2a - 5c + 4c = 0$

$a = \frac{1}{2} c$

>>> $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \frac{1}{2}c \\ \frac{5}{4}c \\ c \end{pmatrix} = \begin{pmatrix} 2c \\ 5c \\ 4c \end{pmatrix} = c \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix}$

>>> $\vec{v}_1 = \frac{1}{\sqrt{2^2 + 5^2 + 4^2}} \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix}$

$\lambda = 4$

$\hookrightarrow \begin{pmatrix} 4-4 & 2 & 0 \\ 2 & 5-4 & 4 \\ 0 & 4 & 4-4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 2 & 0 \\ 2 & 1 & 4 \\ 0 & 4 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\sim \begin{pmatrix} 0 & 2 & 0 \\ 2 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

>>> $2y = 0$

$y = 0$

$2x + y + 4z = 0$

$2x = -4z$

$x = -2z$

>>> $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2z \\ 0 \\ z \end{pmatrix} = z \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$

>>> $\vec{v}_2 = \frac{1}{\sqrt{(-2)^2 + (1)^2 + (0)^2}} \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -2/\sqrt{5} \\ 0 \\ 1/\sqrt{5} \end{pmatrix}$

$\lambda = 0$

$\hookrightarrow \begin{pmatrix} 4 & 2 & 0 \\ 2 & 5 & 4 \\ 0 & 4 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\sim \left(\begin{array}{ccc|c} 4 & 2 & 0 & 0 \\ 0 & 4 & 4 & 0 \\ 0 & 4 & 4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 4 & 2 & 0 & 0 \\ 0 & 4 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\begin{pmatrix} 4 & 2 & 0 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

>>> $4x + 2y = 0$

$x = -\frac{1}{2} y$

>>> $4y + 4z = 0$

$z = -y$

>>> $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}y \\ y \\ -y \end{pmatrix} = \begin{pmatrix} -y \\ 2y \\ -2y \end{pmatrix} = y \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$

>>> $\vec{v}_3 = \frac{1}{\sqrt{(-1)^2 + 2^2 + (-2)^2}} \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} -1/3 \\ 2/3 \\ -2/3 \end{pmatrix}$

$$= \begin{pmatrix} 2/3\sqrt{5} \\ 5/3\sqrt{5} \\ 4/3\sqrt{5} \end{pmatrix}$$

→ Kolom U

$$\bar{u}_1 = \frac{1}{\sigma_1} A \bar{v}_1 = \frac{1}{3} \begin{pmatrix} 0 & 2 & 2 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2/3\sqrt{5} \\ 5/3\sqrt{5} \\ 4/3\sqrt{5} \end{pmatrix} = \begin{pmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{pmatrix}$$

$$\bar{u}_2 = \frac{1}{\sigma_2} A v_2 = \frac{1}{2} \begin{pmatrix} 0 & 2 & 2 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} -2/\sqrt{5} \\ 0 \\ 1/\sqrt{5} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{5} \\ -2/\sqrt{5} \end{pmatrix}$$

→ $A = U \cdot \Sigma \cdot V^T$

$$= [u_1 \ u_2] \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \end{pmatrix} \begin{bmatrix} v_1^T \\ v_2^T \\ v_3^T \end{bmatrix}$$

$$= \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} \frac{2}{3\sqrt{5}} & \frac{5}{3\sqrt{5}} & \frac{4}{3\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}} \\ \frac{-1}{3} & \frac{2}{3} & \frac{-2}{3} \end{pmatrix}$$

→ SVD

Tentukan persamaan Ciri (polynomial characteristic), Eigen Value dan Eigen Vector dari matriks berikut :

$$A = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A\bar{x} = \lambda \bar{x}$$
$$A\bar{x} - \lambda \bar{x} = 0$$
$$(A - \lambda)\bar{x} = 0$$

$$\begin{pmatrix} 1-\lambda & -2 & -2 \\ -1 & 1-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

→ $\det \begin{pmatrix} 1-\lambda & -2 & -2 \\ -1 & 1-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{pmatrix} = 0$

→ metode Sarrus:

$$[(1-\lambda)^3 + 0 + 2] - [0 + (1-\lambda) + 2(1-\lambda)]$$
$$= (1-\lambda)^3 - 3(1-\lambda) + 2 = 0$$
$$= -\lambda^3 + 3\lambda^2 - 3\lambda + 1 - 3 + 3\lambda + 2 = 0$$
$$= 3\lambda^2 - \lambda^3 = 0$$
$$\lambda = 0, \ \lambda = 3$$

→ eigenvalue

→ $\lambda = 0$

↳ $\begin{pmatrix} 1 & -2 & -2 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$

$$\sim \begin{pmatrix} 0 & -1 & -1 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

→ $\lambda = 3$

↳ $\begin{pmatrix} 1-3 & -2 & -2 \\ -1 & 1-3 & 1 \\ 0 & 1 & 1-3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} -2 & -2 & -2 \\ -1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 0 & 2 & -4 \\ -1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 \\ -1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix}$$

$$\sim \begin{pmatrix} \cdots \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} \cdots \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

» $y + z = 0$

$z = -y$

» $-x + y + z = 0$

$-x + y - z = 0$

$x = 0$

» $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ y \\ -y \end{pmatrix} = y \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

eigenvector

$$\sim \begin{pmatrix} 0 & 2 & -9 \\ -1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 \\ -1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ -1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

» $y - 2z = 0$

$y = 2z$

» $-x - 2y + z = 0$

$-x - 4z + z = 0$

$-x - 3z = 0$

$x = -3z$

» $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3z \\ 2z \\ z \end{pmatrix} = z \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$

eigen vector

Tentukan nilai a dan b sedemikian sehingga persamaan $xy = a + bx^2$ mendekati data berikut

X	1	2	4	6	8
y	5.43	6.28	10.32	14.86	19.51

nilai asli : y_k

nilai pendekatan : $xy = a + bx^2$

$y = a\left(\frac{1}{x}\right) + bx$

$\rightarrow a\left(\frac{1}{x_k}\right) + bx_k$

nilai error : $E_k = a\left(\frac{1}{x_k}\right) + bx_k - y_k$

$E_k^2 = \left[a\left(\frac{1}{x_k}\right) + bx_k - y_k\right]^2$

total error fungsi : $E(f) = \sum_{k=1}^{n} E_k^2$

$= \sum_{k=1}^{n} \left[a\left(\frac{1}{x_k}\right) + bx_k - y_k\right]^2$

Minimasi total error fungsi : $\frac{\partial}{\partial a} E(f) = 0$ & $\frac{\partial}{\partial b} E(f) = 0$

› $\frac{\partial}{\partial a} E(f) = 0$

» $2\sum_{k=1}^{n} \left[a\left(\frac{1}{x_k}\right) + bx_k - y_k\right]\left(\frac{1}{x_k}\right) = 0$

› $\frac{\partial}{\partial b} E(f) = 0$

» $2\sum_{k=1}^{n} \left[a\left(\frac{1}{x_k}\right) + bx_k - y_k\right] x_k = 0$

$$\Rightarrow 2\sum_{k=1}^{n}\left[a\left(\frac{1}{x_k}\right)+bx_k-y_k\right]\left(\frac{1}{x_k}\right)=0$$

↳ 2 hilangin karena hasilnya sama sama 0

$$\sum_{k=1}^{n}\left[a\left(\frac{1}{x_k^2}\right)+b-\frac{y_k}{x_k}\right]=0$$

$$a\sum_{k=1}^{n}\left(\frac{1}{x_k^2}\right)+bn-\sum_{k=1}^{n}\frac{y_k}{x_k}=0$$

$$\Rightarrow a\sum_{k=1}^{n}\left(\frac{1}{x_k^2}\right)+bn=\sum_{k=1}^{n}\frac{y_k}{x_k}$$

$$\Rightarrow 2\sum_{k=1}^{n}\left[a\left(\frac{1}{x_k}\right)+bx_k-y_k\right]x_k=0$$

↳ 2 hilangin karena hasilnya sama sama 0

$$\sum_{k=1}^{n}\left[a+bx_k^2-y_kx_k\right]=0$$

$$an+b\sum_{k=1}^{n}x_k^2-\sum_{k=1}^{n}y_kx_k=0$$

$$\Rightarrow an+b\sum_{k=1}^{n}x_k^2=\sum_{k=1}^{n}y_kx_k$$

Tabel nilai

k	x_k	y_k	x_k^2	$\frac{1}{x_k^2}$	y_k/x_k	y_kx_k
1	1	5,43	1	1	5.43	5.43
2	2	6.28	4	1/4	3.14	12.56
3	4	10.32	16	1/16	2.58	41.28
4	6	14.86	36	1/36	2.476667	89.16
5	8	19.51	64	1/64	2.43857	156.08
SUM	21	56,4	121	1.355093	16.06542	304.51

$$\Rightarrow a\sum_{k=1}^{n}\left(\frac{1}{x_k^2}\right)+bn=\sum_{k=1}^{n}\frac{y_k}{x_k}\rightarrow 1.355093a+5b=16.06542$$

$$\Rightarrow an+b\sum_{k=1}^{n}x_k^2=\sum_{k=1}^{n}y_kx_k\rightarrow 5a+121b=304.51$$

Cari Nilai konstanta:
$$a=\frac{\begin{vmatrix}16.06542 & 5\\ 304.51 & 121\end{vmatrix}}{\begin{vmatrix}1.355093 & 5\\ 5 & 121\end{vmatrix}}=3.032145$$

$$b=\frac{\begin{vmatrix}1.355093 & 16.06542\\ 5 & 304.51\end{vmatrix}}{\begin{vmatrix}1.355093 & 5\\ 5 & 121\end{vmatrix}}=2.391316$$

nilai fungsi pendekatan: $y=a\frac{1}{x}+bx$

$$y=3.032145\left(\frac{1}{x}\right)+2.391316x$$

fungsi pendekatan