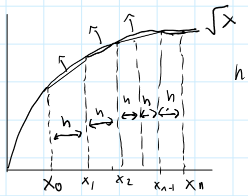
11. Rangkuman integral numerik

Sunday, 08 December 2024 10:18

BAB 11 : INTEGRAL NUMERIK SATU VARIABEL



$h = \dfrac{x_n - x_0}{n}$

-> Metode trapesium :

$$\int_{x_0}^{x_n} f(x)\,dx \approx T_n = \frac{h}{2}\left(f(x_0) + 2\left[\sum_{i=1}^{n-1} f(x_i)\right] + f(x_n)\right)$$

$$h = \frac{x_n - x_0}{n}$$

n -> banyak titik

h -> besar perbedaan antara titik acuan

->> Rumus galat metode trapesium

$$E_n^T(f) = \frac{-h^2(b-a)}{12} f''(c) ; \quad x_0 \le c_n \le x_n$$

> Metode Simpson -> garis aproximasinya melengkung dibandingkan metode trapesium yang garisnya lurus

$$\int_{x_0}^{x_n} f(x)\,dx \approx S_n = \frac{h}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right)$$

$$h = \frac{x_n - x_0}{n}$$

n -> harus genap

->> rumus error simpson

$$E_n^S = \frac{-h^4(b-a)}{180}$$

Tentukan integral tentu fungsi $f(x) = \frac{1}{1+x}$ dari 0 ke 1. $I = ln(2) = 0.693147$

$$h = \frac{x_n - x_0}{n} = \frac{1-0}{3} = 1/3$$

$$\int_{x_0}^{x_n} f(x)\,dx \approx T_n = \frac{h}{2}\left(f(x_0) + 2 \cdot \sum_{i=1}^{n-1} f(x_i) + f(x_n)\right)$$

$x_0 = 0$

$x_1 = x_0 + h = 0 + 1/3 = 1/3$

$x_2 = x_1 + h = 1/3 + 1/3 = 2/3$

$x_3 = x_2 + h = 2/3 + 1/3 = 1$

$$T_n = \frac{1/3}{2}\left(f_0 + 2(f_1 + f_2) + f_3\right)$$

$$= \frac{1}{6}\left(\frac{1}{1+0} + 2\left(\frac{1}{1+1/3} + \frac{1}{1+2/3}\right) + \frac{1}{1+1}\right)$$

$$= \frac{1}{6}\left(1 + 2\left(\frac{3}{4} + \frac{3}{5}\right) + \frac{1}{2}\right)$$

nilai aproximasi = 0,7

nilai asli = 0,693147

Tentukan integral tentu fungsi $f(x) = \frac{1}{1+x}$ dari 0 ke 1. $I = ln(2) = 0.693147$

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 4f(x_{n-1}) + f(x_n)\right)$$

Pada integral simmpson, untuk setiap xi:
Jika i ganjil (x1, x3, ...) -> konstanta = 4
Jika i genap (x2, x4, ...) -> konstanta = 2

$$h = \frac{x_n - x_0}{n} = \frac{1-0}{4} = \frac{1}{4}$$

$x_0 = 0$

$x_1 = 1/4$

$x_2 = 1/2$

$x_3 = 3/4$

$x_4 = 1$

$$S_N = \frac{1/4}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right)$$

$$= \frac{1}{12}\left(1 + 4 * \frac{1}{1+1/4} + 2 * \frac{1}{1+1/2} + 4 * \frac{1}{1+3/4} + \frac{1}{1+1}\right)$$

$$= \frac{1}{12}\left(1 + 4 \cdot \frac{4}{5} + 2 \cdot \frac{2}{3} + 4 \cdot \frac{4}{7} + \frac{1}{2}\right)$$

nilai aproximasi = 0,693254

nilai asli = 0,693147

Ubahlah fungsi integrasi berikut agar dapat dilakukan integrasi numerik trapesium (menjadi non-singular)

$$I = \int_0^1 \frac{\cos(x)}{\sqrt{x}} dx$$

$$\int_0^1 \frac{\cos(x)}{\sqrt{x}} dx \quad \rightarrow \quad \int_{\sqrt{0}}^{\sqrt{1}} \frac{\cos(u^2)\, 2u}{u} du$$

-> $u = \sqrt{x}$

$$2\int^1 \cos(u^2)\, du$$

Ubahlah fungsi integrasi berikut agar dapat dilakukan integrasi numerik trapesium (menjadi non-singular)

1. $I = \int_{0.3}^{1} \frac{dx}{\sqrt{(1-x^3)}}$

$$\rightarrow \int_{0.3}^{1} \frac{1}{\sqrt{1-x} \cdot \sqrt{x^2 + x + 1}} dx \quad \rightarrow \int_{u(0.3)}^{u(1)} \frac{-2u\, du}{u\sqrt{(1-u^2)^2 + 2 - u^2 + 1}}$$

->> $u = \sqrt{1-x}$

$$\frac{d}{}u = \frac{-1}{}$$

$$= \int^{\sqrt{0}} \frac{-2}{\sqrt{(u^4 - 2u^2 + 1) + 2 - u^2}} du$$

$$\int_{0} \quad \sqrt{x}$$

$\Rightarrow u = \sqrt{x}$

$\frac{d}{dx} u = \frac{1}{2\sqrt{x}}$

$2\sqrt{x}\, du = dx$

$2u\, du = dx$

$$\int_{\sqrt{0}} \frac{\quad}{u}$$

$$2 \int_0^1 \cos(u^2)\, du$$

$\Rightarrow u = \sqrt{1-x}$

$\frac{d}{dx} u = \frac{-1}{2\sqrt{1-x}}$

$-2\sqrt{1-x}\, du = dx$

$-2u\, du = dx$

$\Rightarrow u^2 = 1-x$

$x = 1-u^2$

$$= \int_{\sqrt{0.7}}^{\sqrt{0}} \frac{-2}{\sqrt{(u^4 - 2u^2 + 1) + 2 - u^2}}\, du$$

$$= \int_0^{\sqrt{0.7}} \frac{2}{\sqrt{u^4 - 3u^2 + 3}}\, du$$