

$F$  dikatakan sebagai **transformasi linear** jika:

1.  $F(\vec{u} + \vec{v}) = F(\vec{u}) + F(\vec{v})$  untuk semua vector  $\vec{u}$  dan  $\vec{v}$  pada  $V$
2.  $F(k\vec{u}) = kF(\vec{u})$  untuk semua scalar  $k$  dan vector  $\vec{u}$  pada  $V$

1. Apakah  $F: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  dengan  $F(\vec{x}) = (2x_1, x_2 + x_2)^T$ ; merupakan transformasi linear?
2. Apakah  $F: \mathbb{R}^3 \rightarrow \mathbb{R}^2$  dengan  $F((x, y, z)^T) = \begin{bmatrix} x+z \\ x-2y \end{bmatrix}$ ; merupakan transformasi linear?
3. Misalkan Himpunan  $S = \{\vec{v}, \vec{w}\}$  adalah basis dari  $\mathbb{R}^2$  dengan  $\vec{v} = (1, 0)$  dan  $\vec{w} = (-2, 1)$ . Misalkan sebuah transformasi linear  $F: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  memetakan:

$$F(\vec{v}) = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}; F(\vec{w}) = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}.$$

Temukan fungsi transformasi tersebut?

①  $F: \mathbb{R}^2 \rightarrow \mathbb{R}; F([x_1, x_2]) = (2x_1, x_2 * x_2)^T$

a) bukti  $F(k\vec{u}) = kF(\vec{u})$ :

$$F\left(k \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = F\left(\begin{bmatrix} kx_1 \\ kx_2 \end{bmatrix}\right) = \begin{pmatrix} 2kx_1 \\ kx_2 \cdot kx_2 \end{pmatrix} = \begin{pmatrix} 2kx_1 \\ k^2x_2^2 \end{pmatrix}$$

$$kF\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = k \begin{pmatrix} 2x_1 \\ x_2 \cdot x_2 \end{pmatrix} = \begin{pmatrix} 2kx_1 \\ kx_2^2 \end{pmatrix}$$

$$F\left(k \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) \neq kF\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) \rightarrow F \text{ bukan transformasi linear}$$

②  $F: \mathbb{R}^3 \rightarrow \mathbb{R}^2; F([x, y, z]^T) = \begin{bmatrix} x+z \\ x-2y \end{bmatrix}$

a) bukti  $F(k\vec{u}) = kF(\vec{u})$ :

$$\rightarrow F\left(k \begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = F\left(\begin{bmatrix} kx \\ ky \\ kz \end{bmatrix}\right) = \begin{pmatrix} kx + kz \\ kx - k2y \end{pmatrix} = k \begin{pmatrix} x+z \\ x-2y \end{pmatrix}$$

$$= kF\left(k \begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$$

$$\hookrightarrow F(k\vec{u}) = kF(\vec{u}) \checkmark$$

b) bukti  $F(\vec{u} + \vec{v}) = F(\vec{u}) + F(\vec{v})$

$$\rightarrow \text{misalkan: } \vec{a} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \text{ \& } \vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\vec{a} + \vec{x} = \begin{bmatrix} a+x \\ b+y \\ c+z \end{bmatrix}$$

$$\vec{a} + \vec{x} = \begin{bmatrix} a+x \\ b+y \\ c+z \end{bmatrix}$$

$$\begin{aligned} \rightarrow F(\vec{a} + \vec{x}) &= F\left(\begin{bmatrix} a+x \\ b+y \\ c+z \end{bmatrix}\right) = \begin{bmatrix} a+x+(c+z) \\ a+x-2(b+y) \end{bmatrix} \\ &= \begin{bmatrix} a+x+c+z \\ a+x-2b-2y \end{bmatrix} \\ &= \begin{bmatrix} a+c+x+z \\ a-2b+x-2y \end{bmatrix} \\ &= \begin{bmatrix} a+c \\ a-2b \end{bmatrix} + \begin{bmatrix} x+z \\ x-2y \end{bmatrix} \\ &= F(\vec{a}) + F(\vec{x}) \\ &\rightarrow F(\vec{u} + \vec{v}) = F(\vec{u}) + F(\vec{v}) \checkmark \end{aligned}$$

$$\left. \begin{aligned} c) F(\vec{u} + \vec{v}) &= F(\vec{u}) + F(\vec{v}) \checkmark \\ F(k\vec{u}) &= k F(\vec{u}) \checkmark \end{aligned} \right\} F \text{ adalah transformasi linier}$$

$$(3) [\vec{v} \ \vec{w}] = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} ; F([\vec{v} \ \vec{w}]) = \begin{bmatrix} -1 & 1 \\ 1 & -2 \\ 2 & -3 \end{bmatrix}$$

$$\begin{aligned} \rightarrow F^{m \times n} \cdot [\vec{v} \ \vec{w}] &= F([\vec{v} \ \vec{w}]) \\ \underline{F^{m \times n}} &= F([\vec{v} \ \vec{w}]) \cdot [\vec{v} \ \vec{w}]^{-1} \end{aligned}$$

$\rightarrow$  mencari  $[\vec{v} \ \vec{w}]^{-1}$ :

$$\left( \begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array} \right) \xrightarrow{E_{12}(2)} \left( \begin{array}{cc|cc} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{array} \right)$$

$$[\vec{v} \ \vec{w}]^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$\rightarrow$  mencari matriks  $F^{m \times n}$ :

$$F^{m \times n} = \begin{bmatrix} -1 & 1 \\ 1 & -2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$$

## Sistem Persamaan Linear:

Selesaikan SPL berikut dengan metode matriks balikan

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 5 \\ 2x_1 - 4x_2 - x_3 &= 7 \\ 3x_1 &+ 2x_3 = 8 \end{aligned}$$

1. Dengan Eliminasi Gaus-Jordan
2. Dengan Faktorisasi LU

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 5 \\ 2x_1 - 4x_2 - x_3 &= 7 \\ 3x_1 &+ 2x_3 = 8 \end{aligned} \rightarrow \begin{pmatrix} 1 & -1 & 2 \\ 2 & -4 & -1 \\ 3 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 8 \end{pmatrix}$$

① Metode Gauss-Jordan :

$$\left( \begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 2 & -4 & -1 & 7 \\ 3 & 0 & 2 & 8 \end{array} \right) \xrightarrow{\substack{E_{21}(-2) \\ E_{31}(-3)}} \left( \begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 0 & -2 & -5 & -3 \\ 0 & 3 & -4 & -7 \end{array} \right) \xrightarrow{\substack{E_{12}(-0,5) \\ E_{32}(1,5)}} \left( \begin{array}{ccc|c} 1 & 0 & 4,5 & 6,5 \\ 0 & -2 & -5 & -3 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$\xrightarrow{E_3\left(\frac{1}{11,5}\right)} \left( \begin{array}{ccc|c} 1 & 0 & 4,5 & 6,5 \\ 0 & -2 & -5 & -3 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{\substack{E_{13}(-4,5) \\ E_{23}(5)}} \left( \begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$\xrightarrow{E_2\left(\frac{-1}{2}\right)} \left( \begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

② Faktorisasi LU

$$\begin{aligned}
 &\text{matriks } A = L \cdot U \\
 &Ax = B \\
 &\underline{L \cdot U} x = B \\
 &\quad \rightarrow \text{misalkan ini } Y \\
 &L \cdot Y = B \\
 &U \cdot x = Y
 \end{aligned}$$

→ Cari nilai  $U$  dan  $Y$

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & -9 & -1 \\ 3 & 0 & 2 \end{pmatrix} \rightarrow \text{ubah ke segitiga}$$

$$\begin{aligned}
 &\xrightarrow{-L_{21}} \begin{pmatrix} 1 & -1 & 2 \\ 0 & -2 & -5 \\ 0 & 3 & -9 \end{pmatrix} \xrightarrow{-L_{31}} \begin{pmatrix} 1 & -1 & 2 \\ 0 & -2 & -5 \\ 0 & 3 & -9 \end{pmatrix} \xrightarrow{-L_{32}} \begin{pmatrix} 1 & -1 & 2 \\ 0 & -2 & -5 \\ 0 & 0 & -11,5 \end{pmatrix}
 \end{aligned}$$

$$\Rightarrow L = \begin{pmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1,5 & 1 \end{pmatrix}$$

→ Cari nilai  $Y$ :

$$LY = B$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1,5 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 8 \end{pmatrix}$$

$$\Rightarrow y_1 = 5$$

$$\Rightarrow 2y_1 + y_2 = 7$$

$$10 + y_2 = 7$$

$$y_2 = -3$$

$$\Rightarrow 3y_1 - 1,5y_2 + y_3 = 8$$

$$15 + 4,5 + y_3 = 8$$

$$y_3 = -11,5$$

$$\Rightarrow \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ -11,5 \end{pmatrix}$$

→ Cari nilai  $X$ :

$$U \cdot X = Y$$

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & -2 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

$$U \cdot X = Y$$

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & -2 & -5 \\ 0 & 0 & -11,5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ -11,5 \end{pmatrix}$$

$$\Rightarrow -11,5 x_3 = -11,5$$

$$x_3 = 1$$

$$\Rightarrow -2x_2 - 5x_3 = -3$$

$$-2x_2 - 5 = -3$$

$$-2x_2 = 2$$

$$x_2 = -1$$

$$\Rightarrow x_1 - x_2 + 2x_3 = 5$$

$$x_1 + 1 + 2 = 5$$

$$x_1 + 3 = 5$$

$$x_1 = 2$$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$