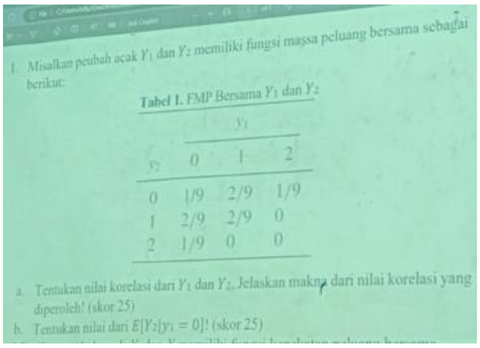
Pembahasan kuis 3

Monday, 02 December 2024 16:35



1. Misalkan peubah acak Y_1 dan Y_2 memiliki fungsi massa peluang bersama sebagai berikut:

Tabel 1. FMP Bersama Y_1 dan Y_2

y_2 \ y_1	0	1	2
0	1/9	2/9	1/9
1	2/9	2/9	0
2	1/9	0	0

a. Tentukan nilai korelasi dari Y_1 dan Y_2. Jelaskan makna dari nilai korelasi yang diperoleh! (skor 25)

b. Tentukan nilai dari $E[Y_2|y_1 = 0]$! (skor 25)

$Var(x) = E(x^2) - \mu^2$

$=$

a) korelasi $= \rho(x,y) = \dfrac{cov(y_1, y_2)}{\sqrt{Var(y_1) \cdot Var(y_2)}}$

$\cdot > cov(y_1, y_2) = E(Y_1 Y_2) - E[Y_1] \cdot E[Y_2]$

$\cdot >> E(Y_1 Y_2) = \sum_{Y_1}\sum_{Y_2} (Y_1 \cdot Y_2) \cdot f(Y_1, Y_2)$

$= 0\,[0 \cdot f(0,0) + 1\, f(0,1) + 2\, f(0,2)]$

$+ 1\,[0 \cdot f(1,0) + 1 \cdot f(1,1) + 2 \cdot f(1,2)]$

$+ 2\,[0 \cdot f(2,0) + 1 \cdot f(2,1) + 2 \cdot f(2,2)]$

$= 0 + 1\left[\frac{2}{9} + 2(0)\right] + 2\,[0]$

$= \frac{2}{9}$

$\cdot >> E[Y_1] = \sum_{Y_1}\sum_{Y_2} Y_1 \cdot f(Y_1, Y_2)$

$= 0\,(f(0,0) + f(0,1) + f(0,2))$

$+ 1\,(f(1,0) + f(1,1) + f(1,2))$

$+ 2\,(f(2,0) + f(2,1) + f(2,2))$

$= 0 + 1\left(\frac{2}{9} + \frac{2}{9} + 0\right) + 2\left(\frac{1}{9} + 0 + 0\right)$

$= 0 + \frac{4}{9} + \frac{2}{9} = \frac{6}{9} = \frac{2}{3}$

$\cdot >> E(Y_2) = \sum_{Y_1}\sum_{Y_2} Y_2 \cdot f(Y_1, Y_2)$

$= 0\,[f(0,0) + f(1,0) + f(2,0)]$

$+ 1\,[f(0,1) + f(1,1) + f(2,1)]$

$+ 2\,[f(0,2) + f(1,2) + f(2,2)]$

$= 0 + 1\left(\frac{2}{9} + \frac{2}{9} + 0\right) + 2\left(\frac{1}{9} + 0 + 0\right)$

$= \frac{4}{9} + \frac{2}{9} = \frac{6}{9} = \frac{2}{3}$

$\cdot >> cov(y_1, y_2) = \frac{2}{9} - \frac{2}{3} \cdot \frac{2}{3} = \frac{-2}{9}$

$\cdot > Var(Y_1) = E(Y_1^2) - [E(Y_1)]^2$

$\cdot >> E(Y_1) = \frac{2}{3} \rightarrow [E(Y_1)]^2 = \frac{4}{9}$

$E(Y_1^2) = \sum\sum Y_1^2\, f(Y_1, Y_2)$

$= \frac{8}{9}$

$$E(Y_1^2) = \sum\sum Y_1^2 f(Y_1, Y_2)$$
$$= \frac{8}{9}$$

$$\Rightarrow Var(Y_1) = \frac{8}{9} - \frac{4}{9} = \frac{4}{9}$$

$$\Rightarrow Var(Y_2) = E(Y_2^2) - [E(Y_2)]^2$$
$$= \frac{8}{9} - \frac{4}{9} = \frac{4}{9}$$

$$\Rightarrow \rho(x, y) = \frac{cov(y_1, y_2)}{\sqrt{Var(y_1) \cdot Var(y_2)}} = \frac{-2/9}{\sqrt{4/9 \cdot \frac{4}{9}}} = \frac{-2/9}{4/9}$$
$$= -2/4$$
$$= -1/2$$

Korelasi selalu bernilai diantara -1 dan 1. Semakin nilainya mendekati 0, kekuatan hubungan antara peubahnya semakin lemah, sedangkan semakin nilainya mendekati 1 atau -1, kekuatan hubungan antara peubahnya semakin kuat.

Karena nilai korelasinya bernilai -1/2, kekuatan antara hubungannya adalah sedang. Karena nilainya negatif, maka ketika salah satu peubahnya meningkat, peubah lainnya menurun

b) $E(Y_2 \mid Y_1 = 0) = \sum\sum Y_2 f(y_2 \mid y_1 = 0)$

$$\Rightarrow f(y_2 \mid y_1 = 0) = \frac{f(y_1 = 0, y_2)}{g(y_1 = 0)}$$

$$\Rightarrow g(y_1 = 0) = f(0,0) + f(0,1) + f(0,2)$$
$$= \frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \frac{4}{9}$$

$$\Rightarrow E(Y_2 \mid 0) = 0 \cdot \frac{f(0,0)}{g(0)} + 1 \cdot \frac{f(0,1)}{g(0)} + 2 \cdot \frac{f(0,2)}{g(0)}$$

$$= 0 + \frac{2/9}{4/9} + 2\left(\frac{1/9}{4/9}\right)$$

$$= \frac{2}{4} + \frac{2}{4} = \frac{4}{4} = \underline{\underline{1}}$$

2. Misalkan peubah acak X dan Y memiliki fungsi kepekatan peluang bersama

$$f(x,y) = \begin{cases} 3x, & 0 \le y \le x \le 1 \\ 0, & \text{lainnya,} \end{cases}$$

a. Tentukan fungsi marginal bagi X! (skor 10)
b. Tentukan nilai dari $P(0.4 \le X < 0.6)$! (skor 10)
c. Tentukan nilai dari $E[XY]$! (skor 15)
d. Tentukan fungsi kepekatan bersyarat $f(x|y)$! (skor 10)
e. Tentukan nilai dari $P(X < 0.5|y = 0.1)$ (skor 5)

$$f(x,y) = \begin{cases} 3x, & 0 \le y \le x \le 1 \\ 0, & \text{else} \end{cases}$$

a) $g(x) = \int_0^x 3x \, dy$

$= 3x \, (y)_0^x$

$= 3x^2, \quad 0 \le x \le 1$

b) $p(0{,}4 \le x < 0{,}6) = \int_{0,4}^{0,6} g(x) \, dx$

$= \int_{0,4}^{0,6} 3x^2 \, dx$

$= (x^3)_{0,4}^{0,6}$

$= 0{,}6^3 - 0{,}4^3$

$= \frac{19}{125}$

c) $E[XY] = \int_0^1 \int_0^x xy \, (3x) \, dy \, dx$

$= \int_0^1 \int_0^x 3x^2 y \, dy \, dx$

$= \int_0^1 3x^2 \left(\frac{1}{2} y^2\right)_0^x dx$

$= \int_0^1 3x^2 \cdot \frac{1}{2} x^2 \, dx$

$= \int_0^1 \frac{3}{2} x^4 \, dx$

$= \frac{3}{10} (x^5)_0^1$

$= \frac{3}{10}$

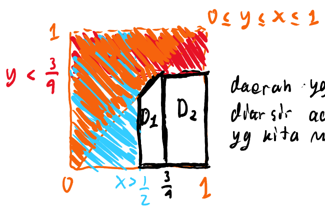
d) $f(x|y) = \dfrac{f(x,y)}{h(y)}$

$\to$ marginal y:

$h(y) = \int_y^1 3x \, dx$

$= \left(\frac{3}{2} x^2\right)_y^1 = \frac{3}{2}(1-y^2)$

$\to \dfrac{f(x,y)}{h(y)} = \dfrac{3x}{\frac{3}{2}(1-y^2)} = \dfrac{2x}{1-y^2}$



daerah yg tidak diarsir adalah yg kita mau

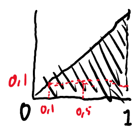
Total D = D_1 + D_2

$p(x > \frac{1}{2}, y < \frac{3}{4}) = \int_{1/2}^{3/4} \int_0^x 8x \, dy \, dx + \int_{3/4}^1 \int_0^{3/4} 8x \, dy \, dx$

e) marginal y:

$$h(y) = \int_y^1 3x \, dx$$

$$= \left(\frac{3}{2}x^2\right)_y^1 = \frac{3}{2}(1-y^2)$$



$0 \leq y \leq x \leq 1$

$$p(x < 0,5 \mid y = 0,1) = \int_{0,1}^{0,5} \frac{3x}{\frac{3}{2}(1-0,1^2)} \, dx$$

$$= \frac{2}{0,99} \cdot \frac{1}{2}\left(x^2\right)_{0,1}^{0,5}$$

$$= \frac{1}{0,99} \cdot 0,25 - 0,01$$

$$= \frac{0,29}{0,99} = \frac{29}{99} = \frac{8}{33}$$