

Pembahasan kuis 4

Monday, 02 December 2024 23:13

1. Diketahui peubah acak X dengan fkp

$$f_X(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{lainnya} \end{cases}$$

- a. Tentukan FKP bagi $Y = \frac{1}{2}X^3$! (skor 20)
- b. Tentukan FKP bagi $W = X^5 + 4$ (skor 20)
- c. Tentukan FKP bagi $T = |1 - X|$ (skor 20)

$$(1) f_X(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{else} \end{cases}$$

$$\begin{aligned} a) \Rightarrow F_Y(Y) &= p(Y \leq y) \Rightarrow f_Y(Y) = \frac{d}{dy} (2y)^{2/3} \\ &= p\left(\frac{1}{2}x^3 \leq y\right) = \frac{2}{3}(2y)^{-1/3}(2) \\ &= p(x \leq \sqrt[3]{2y}) \\ &= \int_0^{\sqrt[3]{2y}} 2x \, dx = \frac{4}{3\sqrt[3]{2y}} \\ &= x^2 \Big|_0^{\sqrt[3]{2y}} \Rightarrow \begin{aligned} 0 < x < 1 \\ 0 < \frac{1}{2}x^3 < \frac{1}{2} \\ 0 < y < \frac{1}{2} \end{aligned} \\ &= (2y)^{2/3} \end{aligned}$$

$$F_Y(Y) = \begin{cases} 0, & y \leq 0 \\ (2y)^{2/3}, & 0 < y < \frac{1}{2} \\ 1, & y \geq \frac{1}{2} \end{cases} \quad f_Y(Y) = \begin{cases} \frac{4}{3\sqrt[3]{2y}}, & 0 < y < \frac{1}{2} \\ 0, & \text{else} \end{cases}$$

$$\begin{aligned} b) \Rightarrow F_W(W) &= p(W \leq w) \Rightarrow f_W(W) = \frac{d}{dw} (w-4)^{2/5} \\ &= p(x^5 + 4 \leq w) = \frac{2}{5}(w-4)^{-3/5} \\ &= p(x \leq \sqrt[5]{w-4}) \\ &= \int_0^{\sqrt[5]{w-4}} 2x \, dx \Rightarrow \begin{aligned} 0 < x < 1 \\ 4 < x^5 + 4 < 5 \\ 4 < w < 5 \end{aligned} \\ &= x^2 \Big|_0^{\sqrt[5]{w-4}} \\ &= (w-4)^{2/5} \end{aligned}$$

$$F_W(W) = \begin{cases} 0, & w \leq 4 \\ (w-4)^{2/5}, & 4 < w < 5 \\ 1, & w \geq 5 \end{cases} \quad f_W(W) = \begin{cases} \frac{2}{5}(w-4)^{-3/5}, & 4 < w < 5 \\ 0, & \text{else} \end{cases}$$

$$\begin{aligned}
 C) \Rightarrow F_t(T) &= P(T \leq t) \\
 &= P(1-x \leq t) \\
 &\text{Karena } 0 < x < 1, \\
 &\quad 1-x \text{ akan selalu positif} \\
 &= P(1-x \leq t) \\
 &= P(-x \leq t-1) \\
 &= P(x \geq 1-t) \\
 &\Rightarrow 0 < x < 1 \\
 &\quad -1 < -x < 0 \\
 &\quad 0 < 1-x < 1 \\
 &\quad 0 < |1-x| < 1 \\
 &\quad 0 < T < 1 \\
 &= \int_{1-t}^1 2x \, dx \\
 &= x^2 \Big|_{1-t}^1 \\
 &= 1 - (t^2 - 2t + 1) \\
 &= 2t - t^2
 \end{aligned}$$

$$\Rightarrow F_t(T) = \begin{cases} 0, & T \leq 0 \\ 2t - t^2, & 0 < T < 1 \\ 1, & T \geq 1 \end{cases} \quad f_t(T) = \begin{cases} 2-2t, & 0 < T < 1 \\ 0, & \text{else} \end{cases}$$

2. Diketahui peubah acak $X \sim \exp(4)$. Tentukan fungsi kepekatan peluang bagi $Y = \frac{1}{2}X$ (gunakan metode momen) (skor 20)

$$\begin{aligned}
 X &\sim \exp(4) \\
 \text{mgf buat } X \Rightarrow M_X(t) &= E(e^{tx}) = (1-4t)^{-1}
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow M_Y(t) &= E(e^{ty}) \\
 &= E(e^{t\frac{1}{2}x}) \\
 &= E(e^{(\frac{1}{2}t)x})
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow E(e^{tx}) &= (1-4t)^{-1} \\
 E(e^{\frac{1}{2}tx}) &= (1-2t)^{-1} \\
 E(e^{ty}) &= (1-2t)^{-1}
 \end{aligned}$$

$$Y \sim \exp(2)$$

$$\text{fkpnya : } f_y(y) = \begin{cases} \frac{1}{2} e^{-\frac{y}{2}}, & y \geq 0 \\ 0, & \text{else} \end{cases}$$

3. Diketahui peubah acak $X \sim \text{Gamma}(2, \beta)$. Tentukan fungsi kepekatan peluang bagi $W = \frac{X}{4\beta}$ (gunakan metode momen) (skor 20)

$$X \sim \text{Gamma}(2, \beta)$$

$$\text{mgf } X : M_X(t) = E(e^{tx}) = (1 - \beta t)^{-\alpha}$$

$$\hookrightarrow \alpha = 2 \rightarrow M_X(t) = E(e^{tx}) = (1 - \beta t)^{-2}$$

$$\begin{aligned} M_W(t) &= E(e^{tw}) \\ &= E(e^{t \frac{x}{4\beta}}) \\ &= E(e^{\frac{t}{4\beta}x}) \end{aligned}$$

$$\hookrightarrow E(e^{tx}) = (1 - \beta t)^{-2}$$

$$E(e^{\frac{t}{4\beta}x}) = (1 - \beta \frac{t}{4\beta})^{-2}$$

$$= (1 - \gamma_1 t)^{-2}$$

$$E(e^{tw}) = (1 - \gamma_1 t)^{-2}$$

$$\Rightarrow Y \sim \text{Gamma}(2, \gamma_1)$$

$$\Rightarrow \frac{1}{\delta(\alpha) \beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}} = \frac{x \cdot e^{-\frac{x}{2}}}{\delta(2) \frac{1}{\gamma_1^2}}$$

$$\Rightarrow f_w(w) = \begin{cases} \frac{w \cdot e^{-\frac{w}{2}}}{\delta(2) \frac{1}{\gamma_1^2}}, & w \geq 0 \\ 0, & \text{else} \end{cases}$$