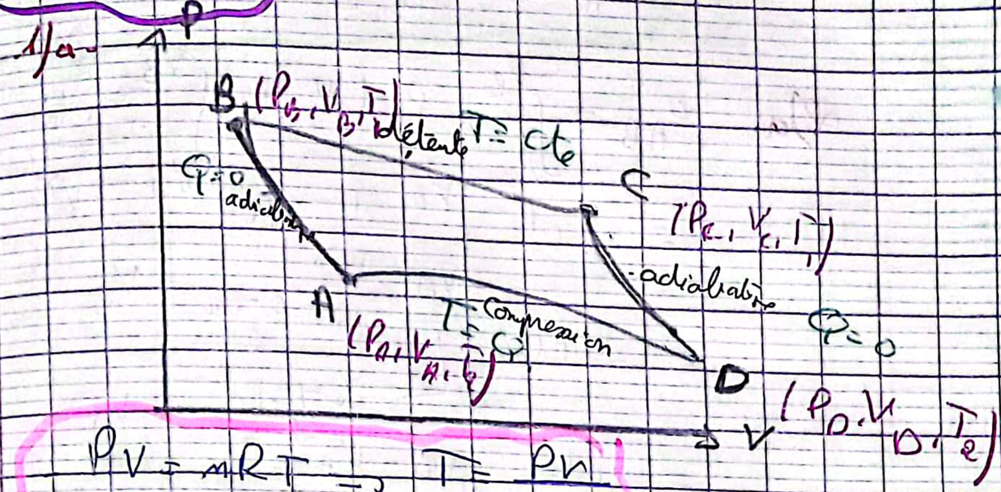
$\gamma = \frac{C_p}{C_V} = \frac{5}{3}$

1sal tégro d'abord
mécanique

Série N°2 :

Exercice 2 :

1/ a-



$PV = nRT \Rightarrow T = \frac{PV}{nR}$

détente $V \nearrow$ $P \searrow$ $T = cte$

compression $V \searrow$ $P \nearrow$

b/ on a pour adiabatique

$TV^{\gamma-1} = cte$

$A \longrightarrow B$: $T_2 V_A^{\gamma-1} = T_1 V_B^{\gamma-1} \Rightarrow \frac{T_1}{T_2} = \left(\frac{V_A}{V_B}\right)^{\gamma-1}$

$C \longrightarrow D$: $T_1 V_C^{\gamma-1} = T_2 V_D^{\gamma-1}$

$\Longrightarrow \frac{T_1}{T_2} = \left(\frac{V_D}{V_C}\right)^{\gamma-1}$

$\left(\frac{V_A}{V_B}\right)^{\gamma-1} = \left(\frac{V_D}{V_C}\right)^{\gamma-1}$

$\frac{V_A}{V_B} = \frac{V_D}{V_C} \Longrightarrow V_A V_C = V_B V_D$

2 ème : les adiabatiques AB et CD

$PV^\gamma = cte$

$P_A V_A^\gamma = P_B V_B^\gamma$

$P_C V_C^\gamma = P_D V_D^\gamma$

$P_A P_C (V_A V_C)^\gamma = P_B P_D (V_B V_D)^\gamma$ (1)

2 ème isotherme : BC et DA

$PV = cte$

$\left.\begin{array}{l} P_B V_B = P_C V_C \\ P_D V_D = P_A V_A \end{array}\right] \Longrightarrow P_A P_C (V_A V_C) = P_B P_D (V_B V_D)$ (2)

À partir de ① et ②, on déduit que :

$$P_A P_C = P_B P_D \quad \text{et} \quad V_A V_C = V_B V_D$$

2) a. $W_{AB} = \int_{V_A}^{V_B} -P\,dV = \int_{V_A}^{V_B} -\frac{cte}{V^\gamma}\,dV$

$= -cte \int_{V_A}^{V_B} \frac{1}{V^\gamma}\,dV$

$= -cte \left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_A}^{V_B}$

$= cte \cdot \frac{V_B^{1-\gamma} - V_A^{1-\gamma}}{\gamma - 1}$

$= \frac{P_B V_B - P_A V_A}{\gamma - 1} = \boxed{\frac{3}{2} nR (T_1 - T_2)}$

Autre méthode

$W_{AB} = \Delta U_{AB} = n C_V \Delta T = n C_V (T_B - T_A)$

$= n \frac{R}{\gamma - 1}(T_1 - T_2) = \frac{m}{M}\frac{R}{\gamma - 1}(T_1 - T_2)$

$= \frac{3}{2} nR (T_1 - T_2)$

$W_{C \to D} = -\frac{3}{2} nR (T_2 - T_1)$.

$W_{B \to C} = \int -P\,dV = \int -\frac{nrT}{V}\,dV = -nrT \int \frac{1}{V}dV$

$= -nrT [\ln V]$

$= nrT (\ln V_B - \ln V_C)$

$= nrT_1 \ln\left(\frac{V_B}{V_C}\right)$

< 0

$W_{D \to A} = nrT_2 \ln\left(\frac{V_D}{V_A}\right) > 0$

b) On a : $W_{AB} = \frac{3}{2} nR (T_1 - T_2)$

$= -\frac{3}{2} nR (T_2 - T_1)$

$W_{AB} = -W_{CD}$

(*) on a $\Delta U_{cycle} = 0 = W_{cycle} + Q_{cycle}$

Donc $\Delta U = W_{AB} + W_{BC} + W_{CD} + W_{DA} + \cancel{Q_{AB}} + Q_{BC} + \cancel{Q_{CD}} + Q_{DA} = 0$

or B → C est isotherme

Donc $\Delta U = 0 = W_{BC} + Q_{BC} \Rightarrow Q_{BC} = -W_{BC}$

de même D → A isotherme : $Q_{DA} = -W_{DA}$

Donc $W_{AB} + W_{CD} = 0$

Alors $\boxed{W_{AB} = -W_{CD}}$

3) a) AB et CD sont adiabatique

alors $Q_{AB} = Q_{CD} = 0\,J$

BC est isotherme

$Q_{BC} = -W_{BC} = -nRT_1 \ln\left(\frac{V_B}{V_C}\right) > 0$

DA est isotherme

$Q_{DA} = -W_{DA} = -nrT_2 \ln\left(\frac{V_D}{V_A}\right) < 0$

b) $V_A V_C = V_B V_D$

$$\frac{V_A V_C}{V_B V_B} = 1$$

$$\ln\left(\frac{V_A V_C}{V_B V_B}\right) = \ln(1) = 0 \Rightarrow nR\ln\left(\frac{V_C}{V_B}\right) + nR\ln\left(\frac{V_A}{V_D}\right) = 0$$

$$nR\frac{T_1}{T_1}\ln\left(\frac{V_C}{V_B}\right) + nR\frac{T_2}{T_2}\ln\left(\frac{V_A}{V_B}\right) = 0$$

$$= \frac{Q_{BC}}{T_1} + \frac{Q_{DA}}{T_2} = 0$$

4) $W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$

on a $W_{CD} = -W_{AB}$

Donc $W = \cancel{W_{AB}} + W_{BC} - \cancel{W_{AB}} + W_{DA}$

$$\boxed{W = W_{BC} + W_{DA}}$$

$$W = W_{BC} + W_{DA}$$
$$= nRT_1 \ln\left(\frac{V_B}{V_C}\right) + nRT_2 \ln\left(\frac{V_D}{V_A}\right)$$
$$= nRT_1 \ln\left(\frac{V_B}{V_C}\right) - nRT_2 \ln\left(\frac{V_A}{V_D}\right)$$

On sait que :

$$V_A V_C = V_B V_D \Rightarrow \frac{V_A}{V_D} = \frac{V_B}{V_C}$$

Donc : $W = nRT_1 \ln\left(\frac{V_B}{V_C}\right) - nRT_2 \ln\left(\frac{V_B}{V_C}\right)$

$$= nR \underbrace{\ln\left(\frac{V_B}{V_C}\right)}_{<0} \underbrace{\left[T_1 - T_2\right]}_{>0}$$

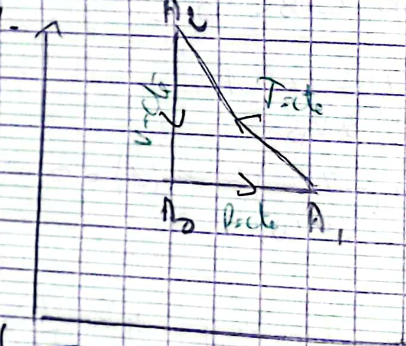
On a $V_B < V_C \Rightarrow \frac{V_B}{V_C} < 1 \Rightarrow \ln\left(\frac{V_B}{V_C}\right) < 0$

Le signe dépend du diagramme

et $T_1 > T_2 \Rightarrow T_1 - T_2 > 0 \Rightarrow$ Donc $W < 0$

Exercice 1.

1.



- détente $P = cte$; $V_1 = 2V_0$
- isotherme $T = cte$; $V_2 = V_0$

$V_2 < V_1$; $P_2 < P_1$

2) Transformation 1 : $P = cte$

On a $W_{A_0 \to A_1} = \int_{V_0}^{V_1} -P\,dV$

$$= -P \int_{V_0}^{V_1} dV$$
$$= -P(V_1 - V_0)$$
$$= -P(2V_0 - V_0) = -P_0V_0$$

Transformation 2 : $T = cte$

On a : $\left(P + \frac{a}{V^2}\right)(V - b) = RT \Rightarrow P = \frac{RT}{V-b} - \frac{a}{V^2}$

et : $W_{A_1 \to A_2} = \int_{V_1}^{V_2} -P\,dV = \int_{V_1}^{V_2} -\left(\frac{RT}{V-b} - \frac{a}{V^2}\right) dV$

$W_{A_1 \to A_2} = a \int_{V_1}^{V_2} \frac{1}{V^2} dV - RT \int_{V_1}^{V_2} \frac{1}{V-b} dV$

$W_{A_1 \to A_2} = a \left[-\frac{1}{V}\right]_{V_1}^{V_2} - RT \left[\ln(V-b)\right]_{V_1}^{V_2}$

$W_{A_1 \to A_2} = a\left(-\frac{1}{V_2} + \frac{1}{V_1}\right) - RT\left(\ln(V_2 - b) - \ln(V_1 - b)\right)$

$W_{A_1 \to A_2} = a\left(\frac{1}{2V_0} - \frac{1}{V_0}\right) - RT \ln\left(\frac{V_0 - b}{2V_0 - b}\right)$

et comme $R.T = \left(P_2 \frac{a}{V^2}\right)(V - b) = RT$

donc on obtient :

$$W_{A_1 \to A_2} = a\left(\frac{1}{2V_0} - \frac{1}{V_0}\right) - \left(P_0 + \frac{a}{(2V_0)^2}\right)(2V_0 - b) \times \ln\left(\frac{V_0 - b}{2V_0 - b}\right)$$

$= \frac{-a}{2V_0} - //$

3ème transformation $V = cte$ $(dV = 0)$

$W_{A_2 \to A_0} = \int_{V_2}^{V_0} -P\,dV = 0$ car $dV = 0$.

3) $W_{cycle} = W_{0 \to 1} + W_{1 \to 2} + W_{2 \to 0}$

$= -P_0 V_0 - \frac{a}{2V_0} - \left(P_0 + \frac{a}{(2V_0)^2}\right)(2V_0 - b) - \ln\left(\frac{V_0 - b}{2V_0 + b}\right) + 0$

$\Delta_{cycle} = W_{cycle} + Q_{cycle} = 0$

$W_{cycle} = -Q_{cycle}$