Appendix Tables

Table A.1 Cumulative Binomial Probabilities

$$B(x; n, p) = \sum_{y=0}^{x} b(y; n, p)$$

a. $n = 5$

							p								
x	**0.01**	**0.05**	**0.10**	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.75**	**0.80**	**0.90**	**0.95**	**0.99**
0	.951	.774	.590	.328	.237	.168	.078	.031	.010	.002	.001	.000	.000	.000	.000
1	.999	.977	.919	.737	.633	.528	.337	.188	.087	.031	.016	.007	.000	.000	.000
2	1.000	.999	.991	.942	.896	.837	.683	.500	.317	.163	.104	.058	.009	.001	.000
3	1.000	1.000	1.000	.993	.984	.969	.913	.812	.663	.472	.367	.263	.081	.023	.001
4	1.000	1.000	1.000	1.000	.999	.998	.990	.969	.922	.832	.763	.672	.410	.226	.049

b. $n = 10$

							p								
x	**0.01**	**0.05**	**0.10**	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.75**	**0.80**	**0.90**	**0.95**	**0.99**
0	.904	.599	.349	.107	.056	.028	.006	.001	.000	.000	.000	.000	.000	.000	.000
1	.996	.914	.736	.376	.244	.149	.046	.011	.002	.000	.000	.000	.000	.000	.000
2	1.000	.988	.930	.678	.526	.383	.167	.055	.012	.002	.000	.000	.000	.000	.000
3	1.000	.999	.987	.879	.776	.650	.382	.172	.055	.011	.004	.001	.000	.000	.000
4	1.000	1.000	.998	.967	.922	.850	.633	.377	.166	.047	.020	.006	.000	.000	.000
5	1.000	1.000	1.000	.994	.980	.953	.834	.623	.367	.150	.078	.033	.002	.000	.000
6	1.000	1.000	1.000	.999	.996	.989	.945	.828	.618	.350	.224	.121	.013	.001	.000
7	1.000	1.000	1.000	1.000	1.000	.998	.988	.945	.833	.617	.474	.322	.070	.012	.000
8	1.000	1.000	1.000	1.000	1.000	1.000	.998	.989	.954	.851	.756	.624	.264	.086	.004
9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.994	.972	.944	.893	.651	.401	.096

c. $n = 15$

							p								
x	**0.01**	**0.05**	**0.10**	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.75**	**0.80**	**0.90**	**0.95**	**0.99**
0	.860	.463	.206	.035	.013	.005	.000	.000	.000	.000	.000	.000	.000	.000	.000
1	.990	.829	.549	.167	.080	.035	.005	.000	.000	.000	.000	.000	.000	.000	.000
2	1.000	.964	.816	.398	.236	.127	.027	.004	.000	.000	.000	.000	.000	.000	.000
3	1.000	.995	.944	.648	.461	.297	.091	.018	.002	.000	.000	.000	.000	.000	.000
4	1.000	.999	.987	.836	.686	.515	.217	.059	.009	.001	.000	.000	.000	.000	.000
5	1.000	1.000	.998	.939	.852	.722	.402	.151	.034	.004	.001	.000	.000	.000	.000
6	1.000	1.000	1.000	.982	.943	.869	.610	.304	.095	.015	.004	.001	.000	.000	.000
7	1.000	1.000	1.000	.996	.983	.950	.787	.500	.213	.050	.017	.004	.000	.000	.000
8	1.000	1.000	1.000	.999	.996	.985	.905	.696	.390	.131	.057	.018	.000	.000	.000
9	1.000	1.000	1.000	1.000	.999	.996	.966	.849	.597	.278	.148	.061	.002	.000	.000
10	1.000	1.000	1.000	1.000	1.000	.999	.991	.941	.783	.485	.314	.164	.013	.001	.000
11	1.000	1.000	1.000	1.000	1.000	1.000	.998	.982	.909	.703	.539	.352	.056	.005	.000
12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.973	.873	.764	.602	.184	.036	.000
13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.920	.833	.451	.171	.010
14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.987	.965	.794	.537	.140

(continued)

Table A.1 Cumulative Binomial Probabilities *(cont.)*

$$B(x; n, p) = \sum_{y=0}^{x} b(y; n, p)$$

d. $n = 20$

	p														
x	**0.01**	**0.05**	**0.10**	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.75**	**0.80**	**0.90**	**0.95**	**0.99**
0	.818	.358	.122	.012	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
1	.983	.736	.392	.069	.024	.008	.001	.000	.000	.000	.000	.000	.000	.000	.000
2	.999	.925	.677	.206	.091	.035	.004	.000	.000	.000	.000	.000	.000	.000	.000
3	1.000	.984	.867	.411	.225	.107	.016	.001	.000	.000	.000	.000	.000	.000	.000
4	1.000	.997	.957	.630	.415	.238	.051	.006	.000	.000	.000	.000	.000	.000	.000
5	1.000	1.000	.989	.804	.617	.416	.126	.021	.002	.000	.000	.000	.000	.000	.000
6	1.000	1.000	.998	.913	.786	.608	.250	.058	.006	.000	.000	.000	.000	.000	.000
7	1.000	1.000	1.000	.968	.898	.772	.416	.132	.021	.001	.000	.000	.000	.000	.000
8	1.000	1.000	1.000	.990	.959	.887	.596	.252	.057	.005	.001	.000	.000	.000	.000
9	1.000	1.000	1.000	.997	.986	.952	.755	.412	.128	.017	.004	.001	.000	.000	.000
10	1.000	1.000	1.000	.999	.996	.983	.872	.588	.245	.048	.014	.003	.000	.000	.000
11	1.000	1.000	1.000	1.000	.999	.995	.943	.748	.404	.113	.041	.010	.000	.000	.000
12	1.000	1.000	1.000	1.000	1.000	.999	.979	.868	.584	.228	.102	.032	.000	.000	.000
13	1.000	1.000	1.000	1.000	1.000	1.000	.994	.942	.750	.392	.214	.087	.002	.000	.000
14	1.000	1.000	1.000	1.000	1.000	1.000	.998	.979	.874	.584	.383	.196	.011	.000	.000
15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.994	.949	.762	.585	.370	.043	.003	.000
16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.984	.893	.775	.589	.133	.016	.000
17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.965	.909	.794	.323	.075	.001
18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.992	.976	.931	.608	.264	.017
19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.997	.988	.878	.642	.182

(continued)

Table A.1 Cumulative Binomial Probabilities *(cont.)*

$$B(x; n, p) = \sum_{y=0}^{x} b(y; n, p)$$

e. $n = 25$

	p														
x	0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	.778	.277	.072	.004	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
1	.974	.642	.271	.027	.007	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.998	.873	.537	.098	.032	.009	.000	.000	.000	.000	.000	.000	.000	.000	.000
3	1.000	.966	.764	.234	.096	.033	.002	.000	.000	.000	.000	.000	.000	.000	.000
4	1.000	.993	.902	.421	.214	.090	.009	.000	.000	.000	.000	.000	.000	.000	.000
5	1.000	.999	.967	.617	.378	.193	.029	.002	.000	.000	.000	.000	.000	.000	.000
6	1.000	1.000	.991	.780	.561	.341	.074	.007	.000	.000	.000	.000	.000	.000	.000
7	1.000	1.000	.998	.891	.727	.512	.154	.022	.001	.000	.000	.000	.000	.000	.000
8	1.000	1.000	1.000	.953	.851	.677	.274	.054	.004	.000	.000	.000	.000	.000	.000
9	1.000	1.000	1.000	.983	.929	.811	.425	.115	.013	.000	.000	.000	.000	.000	.000
10	1.000	1.000	1.000	.994	.970	.902	.586	.212	.034	.002	.000	.000	.000	.000	.000
11	1.000	1.000	1.000	.998	.980	.956	.732	.345	.078	.006	.001	.000	.000	.000	.000
12	1.000	1.000	1.000	1.000	.997	.983	.846	.500	.154	.017	.003	.000	.000	.000	.000
13	1.000	1.000	1.000	1.000	.999	.994	.922	.655	.268	.044	.020	.002	.000	.000	.000
14	1.000	1.000	1.000	1.000	1.000	.998	.966	.788	.414	.098	.030	.006	.000	.000	.000
15	1.000	1.000	1.000	1.000	1.000	1.000	.987	.885	.575	.189	.071	.017	.000	.000	.000
16	1.000	1.000	1.000	1.000	1.000	1.000	.996	.946	.726	.323	.149	.047	.000	.000	.000
17	1.000	1.000	1.000	1.000	1.000	1.000	.999	.978	.846	.488	.273	.109	.002	.000	.000
18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.993	.926	.659	.439	.220	.009	.000	.000
19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.971	.807	.622	.383	.033	.001	.000
20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.910	.786	.579	.098	.007	.000
21	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.967	.904	.766	.236	.034	.000
22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.968	.902	.463	.127	.002
23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.993	.973	.729	.358	.026
24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.996	.928	.723	.222

Table A.2 Cumulative Poisson Probabilities

$$F(x; \lambda) = \sum_{y=0}^{x} \frac{e^{-\lambda}\lambda^y}{y!}$$

	λ									
x	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
0	.905	.819	.741	.670	.607	.549	.497	.449	.407	.368
1	.995	.982	.963	.938	.910	.878	.844	.809	.772	.736
2	1.000	.999	.996	.992	.986	.977	.966	.953	.937	.920
3		1.000	1.000	.999	.998	.997	.994	.991	.987	.981
4				1.000	1.000	1.000	.999	.999	.998	.996
5							1.000	1.000	1.000	.999
6										1.000

(continued)

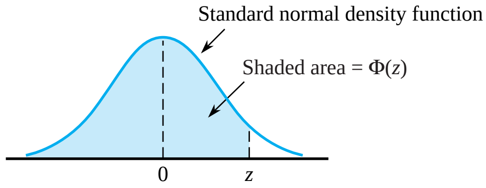
Table A.2 Cumulative Poisson Probabilities *(cont.)*

$$F(x;\lambda)=\sum_{y=0}^{x}\frac{e^{-\lambda}\lambda^{y}}{y!}$$

		λ										
		2.0	**3.0**	**4.0**	**5.0**	**6.0**	**7.0**	**8.0**	**9.0**	**10.0**	**15.0**	**20.0**
	0	.135	.050	.018	.007	.002	.001	.000	.000	.000	.000	.000
	1	.406	.199	.092	.040	.017	.007	.003	.001	.000	.000	.000
	2	.677	.423	.238	.125	.062	.030	.014	.006	.003	.000	.000
	3	.857	.647	.433	.265	.151	.082	.042	.021	.010	.000	.000
	4	.947	.815	.629	.440	.285	.173	.100	.055	.029	.001	.000
	5	.983	.916	.785	.616	.446	.301	.191	.116	.067	.003	.000
	6	.995	.966	.889	.762	.606	.450	.313	.207	.130	.008	.000
	7	.999	.988	.949	.867	.744	.599	.453	.324	.220	.018	.001
	8	1.000	.996	.979	.932	.847	.729	.593	.456	.333	.037	.002
	9		.999	.992	.968	.916	.830	.717	.587	.458	.070	.005
	10		1.000	.997	.986	.957	.901	.816	.706	.583	.118	.011
	11			.999	.995	.980	.947	.888	.803	.697	.185	.021
	12			1.000	.998	.991	.973	.936	.876	.792	.268	.039
	13				.999	.996	.987	.966	.926	.864	.363	.066
	14				1.000	.999	.994	.983	.959	.917	.466	.105
	15					.999	.998	.992	.978	.951	.568	.157
	16					1.000	.999	.996	.989	.973	.664	.221
	17						1.000	.998	.995	.986	.749	.297
	18							.999	.998	.993	.819	.381
x	19							1.000	.999	.997	.875	.470
	20								1.000	.998	.917	.559
	21									.999	.947	.644
	22									1.000	.967	.721
	23										.981	.787
	24										.989	.843
	25										.994	.888
	26										.997	.922
	27										.998	.948
	28										.999	.966
	29										1.000	.978
	30											.987
	31											.992
	32											.995
	33											.997
	34											.999
	35											.999
	36											1.000

Table A.3 Standard Normal Curve Areas

$\Phi(z) = P(Z \leq z)$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

(continued)

Table A.3 Standard Normal Curve Areas *(cont.)*

$\Phi(z) = P(Z \leq z)$

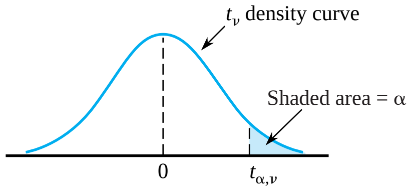
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table A.4 The Incomplete Gamma Function

$$F(x;\alpha)=\int_0^x \frac{1}{\Gamma(\alpha)}y^{\alpha-1}e^{-y}\,dy$$

x \ α	1	2	3	4	5	6	7	8	9	10
1	.632	.264	.080	.019	.004	.001	.000	.000	.000	.000
2	.865	.594	.323	.143	.053	.017	.005	.001	.000	.000
3	.950	.801	.577	.353	.185	.084	.034	.012	.004	.001
4	.982	.908	.762	.567	.371	.215	.111	.051	.021	.008
5	.993	.960	.875	.735	.560	.384	.238	.133	.068	.032
6	.998	.983	.938	.849	.715	.554	.394	.256	.153	.084
7	.999	.993	.970	.918	.827	.699	.550	.401	.271	.170
8	1.000	.997	.986	.958	.900	.809	.687	.547	.407	.283
9		.999	.994	.979	.945	.884	.793	.676	.544	.413
10		1.000	.997	.990	.971	.933	.870	.780	.667	.542
11			.999	.995	.985	.962	.921	.857	.768	.659
12			1.000	.998	.992	.980	.954	.911	.845	.758
13				.999	.996	.989	.974	.946	.900	.834
14				1.000	.998	.994	.986	.968	.938	.891
15					.999	.997	.992	.982	.963	.930

Table A.5 Critical Values for *t* Distributions

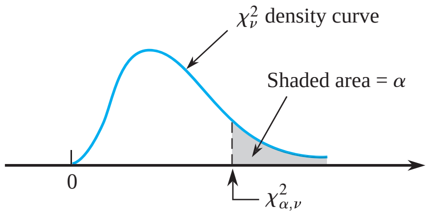


	α						
ν	.10	.05	.025	.01	.005	.001	.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	1.299	1.676	2.009	2.403	2.678	3.262	3.496
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Table A.6 Tolerance Critical Values for Normal Population Distributions

		Two-sided Intervals						One-sided Intervals					
Confidence Level		95%			99%			95%			99%		
% of Population Captured		≥ 90%	≥ 95%	≥ 99%	≥ 90%	≥ 95%	≥ 99%	≥ 90%	≥ 95%	≥ 99%	≥ 90%	≥ 95%	≥ 99%
	2	32.019	37.674	48.430	160.193	188.491	242.300	20.581	26.260	37.094	103.029	131.426	185.617
	3	8.380	9.916	12.861	18.930	22.401	29.055	6.156	7.656	10.553	13.995	17.370	23.896
	4	5.369	6.370	8.299	9.398	11.150	14.527	4.162	5.144	7.042	7.380	9.083	12.387
	5	4.275	5.079	6.634	6.612	7.855	10.260	3.407	4.203	5.741	5.362	6.578	8.939
	6	3.712	4.414	5.775	5.337	6.345	8.301	3.006	3.708	5.062	4.411	5.406	7.335
	7	3.369	4.007	5.248	4.613	5.488	7.187	2.756	3.400	4.642	3.859	4.728	6.412
	8	3.136	3.732	4.891	4.147	4.936	6.468	2.582	3.187	4.354	3.497	4.285	5.812
	9	2.967	3.532	4.631	3.822	4.550	5.966	2.454	3.031	4.143	3.241	3.972	5.389
	10	2.839	3.379	4.433	3.582	4.265	5.594	2.355	2.911	3.981	3.048	3.738	5.074
	11	2.737	3.259	4.277	3.397	4.045	5.308	2.275	2.815	3.852	2.898	3.556	4.829
	12	2.655	3.162	4.150	3.250	3.870	5.079	2.210	2.736	3.747	2.777	3.410	4.633
	13	2.587	3.081	4.044	3.130	3.727	4.893	2.155	2.671	3.659	2.677	3.290	4.472
	14	2.529	3.012	3.955	3.029	3.608	4.737	2.109	2.615	3.585	2.593	3.189	4.337
	15	2.480	2.954	3.878	2.945	3.507	4.605	2.068	2.566	3.520	2.522	3.102	4.222
	16	2.437	2.903	3.812	2.872	3.421	4.492	2.033	2.524	3.464	2.460	3.028	4.123
Sample Size *n*	**17**	2.400	2.858	3.754	2.808	3.345	4.393	2.002	2.486	3.414	2.405	2.963	4.037
	18	2.366	2.819	3.702	2.753	3.279	4.307	1.974	2.453	3.370	2.357	2.905	3.960
	19	2.337	2.784	3.656	2.703	3.221	4.230	1.949	2.423	3.331	2.314	2.854	3.892
	20	2.310	2.752	3.615	2.659	3.168	4.161	1.926	2.396	3.295	2.276	2.808	3.832
	25	2.208	2.631	3.457	2.494	2.972	3.904	1.838	2.292	3.158	2.129	2.633	3.601
	30	2.140	2.549	3.350	2.385	2.841	3.733	1.777	2.220	3.064	2.030	2.516	3.447
	35	2.090	2.490	3.272	2.306	2.748	3.611	1.732	2.167	2.995	1.957	2.430	3.334
	40	2.052	2.445	3.213	2.247	2.677	3.518	1.697	2.126	2.941	1.902	2.364	3.249
	45	2.021	2.408	3.165	2.200	2.621	3.444	1.669	2.092	2.898	1.857	2.312	3.180
	50	1.996	2.379	3.126	2.162	2.576	3.385	1.646	2.065	2.863	1.821	2.269	3.125
	60	1.958	2.333	3.066	2.103	2.506	3.293	1.609	2.022	2.807	1.764	2.202	3.038
	70	1.929	2.299	3.021	2.060	2.454	3.225	1.581	1.990	2.765	1.722	2.153	2.974
	80	1.907	2.272	2.986	2.026	2.414	3.173	1.559	1.965	2.733	1.688	2.114	2.924
	90	1.889	2.251	2.958	1.999	2.382	3.130	1.542	1.944	2.706	1.661	2.082	2.883
	100	1.874	2.233	2.934	1.977	2.355	3.096	1.527	1.927	2.684	1.639	2.056	2.850
	150	1.825	2.175	2.859	1.905	2.270	2.983	1.478	1.870	2.611	1.566	1.971	2.741
	200	1.798	2.143	2.816	1.865	2.222	2.921	1.450	1.837	2.570	1.524	1.923	2.679
	250	1.780	2.121	2.788	1.839	2.191	2.880	1.431	1.815	2.542	1.496	1.891	2.638
	300	1.767	2.106	2.767	1.820	2.169	2.850	1.417	1.800	2.522	1.476	1.868	2.608
	∞	1.645	1.960	2.576	1.645	1.960	2.576	1.282	1.645	2.326	1.282	1.645	2.326

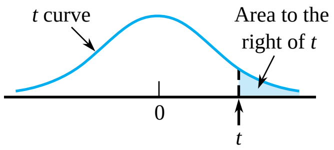
Table A.7 Critical Values for Chi-Squared Distributions



	α									
ν	.995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	0.000	0.000	0.001	0.004	0.016	2.706	3.843	5.025	6.637	7.882
2	0.010	0.020	0.051	0.103	0.211	4.605	5.992	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.344	12.837
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.085	16.748
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.440	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.012	18.474	20.276
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.534	20.090	21.954
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.022	21.665	23.587
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.724	26.755
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.041	19.812	22.362	24.735	27.687	29.817
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.600	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.577	32.799
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.407	7.564	8.682	10.085	24.769	27.587	30.190	33.408	35.716
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.843	7.632	8.906	10.117	11.651	27.203	30.143	32.852	36.190	38.580
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.033	8.897	10.283	11.591	13.240	29.615	32.670	35.478	38.930	41.399
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.195	11.688	13.090	14.848	32.007	35.172	38.075	41.637	44.179
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	10.519	11.523	13.120	14.611	16.473	34.381	37.652	40.646	44.313	46.925
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.807	12.878	14.573	16.151	18.114	36.741	40.113	43.194	46.962	49.642
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.120	14.256	16.147	17.708	19.768	39.087	42.557	45.772	49.586	52.333
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
31	14.457	15.655	17.538	19.280	21.433	41.422	44.985	48.231	52.190	55.000
32	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328
33	15.814	17.073	19.046	20.866	23.110	43.745	47.400	50.724	54.774	57.646
34	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.966	56.061	58.964
35	17.191	18.508	20.569	22.465	24.796	46.059	49.802	53.203	57.340	60.272
36	17.887	19.233	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581
37	18.584	19.960	22.105	24.075	26.492	48.363	52.192	55.667	59.891	62.880
38	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.162	64.181
39	19.994	21.425	23.654	25.695	28.196	50.660	54.572	58.119	62.426	65.473
40	20.706	22.164	24.433	26.509	29.050	51.805	55.758	59.342	63.691	66.766

For $\nu > 40$, $\chi^2_{\alpha,\nu} \approx \nu\left(1 - \frac{2}{9\nu} + z_\alpha\sqrt{\frac{2}{9\nu}}\right)^3$

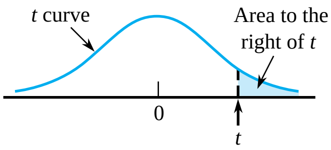
Table A.8 *t* Curve Tail Areas



t \ ν	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0.0	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500
0.1	.468	.465	.463	.463	.462.	.462	.462	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461
0.2	.437	.430	.427	.426	.425	.424	.424	.423	.423	.423	.423	.422	.422	.422	.422	.422	.422	.422
0.3	.407	.396	.392	.390	.388	.387	.386	.386	.386	.385	.385	.385	.384	.384	.384	.384	.384	.384
0.4	.379	.364	.358	.355	.353	.352	.351	.350	.349	.349	.348	.348	.348	.347	.347	.347	.347	.347
0.5	.352	.333	.326	.322	.319	.317	.316	.315	.315	.314	.313	.313	.313	.312	.312	.312	.312	.312
0.6	.328	.305	.295	.290	.287	.285	.284	.283	.282	.281	.280	.280	.279	.279	.279	.278	.278	.278
0.7	.306	.278	.267	.261	.258	.255	.253	.252	.251	.250	.249	.249	.248	.247	.247	.247	.247	.246
0.8	.285	.254	.241	.234	.230	.227	.225	.223	.222	.221	.220	.220	.219	.218	.218	.218	.217	.217
0.9	.267	.232	.217	.210	.205	.201	.199	.197	.196	.195	.194	.193	.192	.191	.191	.191	.190	.190
1.0	.250	.211	.196	.187	.182	.178	.175	.173	.172	.170	.169	.169	.168	.167	.167	.166	.166	.165
1.1	.235	.193	.176	.167	.162	.157	.154	.152	.150	.149	.147	.146	.146	.144	.144	.144	.143	.143
1.2	.221	.177	.158	.148	.142	.138	.135	.132	.130	.129	.128	.127	.126	.124	.124	.124	.123	.123
1.3	.209	.162	.142	.132	.125	.121	.117	.115	.113	.111	.110	.109	.108	.107	.107	.106	.105	.105
1.4	.197	.148	.128	.117	.110	.106	.102	.100	.098	.096	.095	.093	.092	.091	.091	.090	.090	.089
1.5	.187	.136	.115	.104	.097	.092	.089	.086	.084	.082	.081	.080	.079	.077	.077	.077	.076	.075
1.6	.178	.125	.104	.092	.085	.080	.077	.074	.072	.070	.069	.068	.067	.065	.065	.065	.064	.064
1.7	.169	.116	.094	.082	.075	.070	.065	.064	.062	.060	.059	.057	.056	.055	.055	.054	.054	.053
1.8	.161	.107	.085	.073	.066	.061	.057	.055	.053	.051	.050	.049	.048	.046	.046	.045	.045	.044
1.9	.154	.099	.077	.065	.058	.053	.050	.047	.045	.043	.042	.041	.040	.038	.038	.038	.037	.037
2.0	.148	.092	.070	.058	.051	.046	.043	.040	.038	.037	.035	.034	.033	.032	.032	.031	.031	.030
2.1	.141	.085	.063	.052	.045	.040	.037	.034	.033	.031	.030	.029	.028	.027	.027	.026	.025	.025
2.2	.136	.079	.058	.046	.040	.035	.032	.029	.028	.026	.025	.024	.023	.022	.022	.021	.021	.021
2.3	.131	.074	.052	.041	.035	.031	.027	.025	.023	.022	.021	.020	.019	.018	.018	.018	.017	.017
2.4	.126	.069	.048	.037	.031	.027	.024	.022	.020	.019	.018	.017	.016	.015	.015	.014	.014	.014
2.5	.121	.065	.044	.033	.027	.023	.020	.018	.017	.016	.015	.014	.013	.012	.012	.012	.011	.011
2.6	.117	.061	.040	.030	.024	.020	.018	.016	.014	.013	.012	.012	.011	.010	.010	.010	.009	.009
2.7	.113	.057	.037	.027	.021	.018	.015	.014	.012	.011	.010	.010	.009	.008	.008	.008	.008	.007
2.8	.109	.054	.034	.024	.019	.016	.013	.012	.010	.009	.009	.008	.008	.007	.007	.006	.006	.006
2.9	.106	.051	.031	.022	.017	.014	.011	.010	.009	.008	.007	.007	.006	.005	.005	.005	.005	.005
3.0	.102	.048	.029	.020	.015	.012	.010	.009	.007	.007	.006	.006	.005	.004	.004	.004	.004	.004
3.1	.099	.045	.027	.018	.013	.011	.009	.007	.006	.006	.005	.005	.004	.004	.004	.003	.003	.003
3.2	.096	.043	.025	.016	.012	.009	.008	.006	.005	.005	.004	.004	.003	.003	.003	.003	.003	.002
3.3	.094	.040	.023	.015	.011	.008	.007	.005	.005	.004	.004	.003	.003	.002	.002	.002	.002	.002
3.4	.091	.038	.021	.014	.010	.007	.006	.005	.004	.003	.003	.003	.002	.002	.002	.002	.002	.002
3.5	.089	.036	.020	.012	.009	.006	.005	.004	.003	.003	.002	.002	.002	.002	.002	.001	.001	.001
3.6	.086	.035	.018	.011	.008	.006	.004	.004	.003	.002	.002	.002	.002	.001	.001	.001	.001	.001
3.7	.084	.033	.017	.010	.007	.005	.004	.003	.002	.002	.002	.002	.001	.001	.001	.001	.001	.001
3.8	.082	.031	.016	.010	.006	.004	.003	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001
3.9	.080	.030	.015	.009	.006	.004	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001
4.0	.078	.029	.014	.008	.005	.004	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.000	.000

(continued)

Table A.8 *t* Curve Tail Areas *(cont.)*



t \ ν	19	20	21	22	23	24	25	26	27	28	29	30	35	40	60	120	$\infty(= z)$
0.0	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500
0.1	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.460	.460	.460	.460	.460
0.2	.422	.422	.422	.422	.422	.422	.422	.422	.421	.421	.421	.421	.421	.421	.421	.421	.421
0.3	.384	.384	.384	.383	.383	.383	.383	.383	.383	.383	.383	.383	.383	.383	.383	.382	.382
0.4	.347	.347	.347	.347	.346	.346	.346	.346	.346	.346	.346	.346	.346	.346	.345	.345	.345
0.5	.311	.311	.311	.311	.311	.311	.311	.311	.311	.310	.310	.310	.310	.310	.309	.309	.309
0.6	.278	.278	.278	.277	.277	.277	.277	.277	.277	.277	.277	.277	.276	.276	.275	.275	.274
0.7	.246	.246	.246	.246	.245	.245	.245	.245	.245	.245	.245	.245	.244	.244	.243	.243	.242
0.8	.217	.217	.216	.216	.216	.216	.216	.215	.215	.215	.215	.215	.215	.214	.213	.213	.212
0.9	.190	.189	.189	.189	.189	.189	.188	.188	.188	.188	.188	.188	.187	.187	.186	.185	.184
1.0	.165	.165	.164	.164	.164	.164	.163	.163	.163	.163	.163	.163	.162	.162	.161	.160	.159
1.1	.143	.142	.142	.142	.141	.141	.141	.141	.141	.140	.140	.140	.139	.139	.138	.137	.136
1.2	.122	.122	.122	.121	.121	.121	.121	.120	.120	.120	.120	.120	.119	.119	.117	.116	.115
1.3	.105	.104	.104	.104	.103	.103	.103	.103	.102	.102	.102	.102	.101	.101	.099	.098	.097
1.4	.089	.089	.088	.088	.087	.087	.087	.087	.086	.086	.086	.086	.085	.085	.083	.082	.081
1.5	.075	.075	.074	.074	.074	.073	.073	.073	.073	.072	.072	.072	.071	.071	.069	.068	.067
1.6	.063	.063	.062	.062	.062	.061	.061	.061	.061	.060	.060	.060	.059	.059	.057	.056	.055
1.7	.053	.052	.052	.052	.051	.051	.051	.051	.050	.050	.050	.050	.049	.048	.047	.046	.045
1.8	.044	.043	.043	.043	.042	.042	.042	.042	.042	.041	.041	.041	.040	.040	.038	.037	.036
1.9	.036	.036	.036	.035	.035	.035	.035	.034	.034	.034	.034	.034	.033	.032	.031	.030	.029
2.0	.030	.030	.029	.029	.029	.028	.028	.028	.028	.028	.027	.027	.027	.026	.025	.024	.023
2.1	.025	.024	.024	.024	.023	.023	.023	.023	.023	.022	.022	.022	.022	.021	.020	.019	.018
2.2	.020	.020	.020	.019	.019	.019	.019	.018	.018	.018	.018	.018	.017	.017	.016	.015	.014
2.3	.016	.016	.016	.016	.015	.015	.015	.015	.015	.015	.014	.014	.014	.013	.012	.012	.011
2.4	.013	.013	.013	.013	.012	.012	.012	.012	.012	.012	.012	.011	.011	.011	.010	.009	.008
2.5	.011	.011	.010	.010	.010	.010	.010	.010	.009	.009	.009	.009	.009	.008	.008	.007	.006
2.6	.009	.009	.008	.008	.008	.008	.008	.008	.007	.007	.007	.007	.007	.007	.006	.005	.005
2.7	.007	.007	.007	.007	.006	.006	.006	.006	.006	.006	.006	.006	.005	.005	.004	.004	.003
2.8	.006	.006	.005	.005	.005	.005	.005	.005	.005	.005	.005	.004	.004	.004	.003	.003	.003
2.9	.005	.004	.004	.004	.004	.004	.004	.004	.004	.004	.004	.003	.003	.003	.003	.002	.002
3.0	.004	.004	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.002	.002	.002	.001
3.1	.003	.003	.003	.003	.003	.002	.002	.002	.002	.002	.002	.002	.002	.002	.001	.001	.001
3.2	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.001	.001	.001	.001	.001
3.3	.002	.002	.002	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.000
3.4	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.000	.000
3.5	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.000	.000	.000
3.6	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.000	.000	.000	.000	.000
3.7	.001	.001	.001	.001	.001	.001	.001	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
3.8	.001	.001	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
3.9	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
4.0	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

Table A.9 Critical Values for F Distributions

ν_2 = denominator df	α	ν_1 = numerator df 1	2	3	4	5	6	7	8	9
1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
	.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
	.010	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
	.001	405284	500000	540379	562500	576405	585937	592873	598144	602284
2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
	.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39
3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
	.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86
4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47
5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
	.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24
6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69
7	.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33
8	.100	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	.050	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
	.010	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
	.001	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77
9	.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
	.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
	.001	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11
10	.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
	.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
	.001	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96
11	.100	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
	.050	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
	.010	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
	.001	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12
12	.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
	.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
	.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48

(continued)

Table A.9 Critical Values for *F* Distributions *(cont.)*

ν_1 = numerator df										
10	**12**	**15**	**20**	**25**	**30**	**40**	**50**	**60**	**120**	**1000**
60.19	60.71	61.22	61.74	62.05	62.26	62.53	62.69	62.79	63.06	63.30
241.88	243.91	245.95	248.01	249.26	250.10	251.14	251.77	252.20	253.25	254.19
6055.8	6106.3	6157.3	6208.7	6239.8	6260.6	6286.8	6302.5	6313.0	6339.4	6362.7
605621	610668	615764	620908	624017	626099	628712	630285	631337	633972	636301
9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.49
19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.48	19.49	19.49
99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.48	99.49	99.50
999.40	999.42	999.43	999.45	999.46	999.47	999.47	999.48	999.48	999.49	999.50
5.23	5.22	5.20	5.18	5.17	5.17	5.16	5.15	5.15	5.14	5.13
8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.58	8.57	8.55	8.53
27.23	27.05	26.87	26.69	26.58	26.50	26.41	26.35	26.32	26.22	26.14
129.25	128.32	127.37	126.42	125.84	125.45	124.96	124.66	124.47	123.97	123.53
3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.80	3.79	3.78	3.76
5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.63
14.55	14.37	14.20	14.02	13.91	13.84	13.75	13.69	13.65	13.56	13.47
48.05	47.41	46.76	46.10	45.70	45.43	45.09	44.88	44.75	44.40	44.09
3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.12	3.11
4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.44	4.43	4.40	4.37
10.05	9.89	9.72	9.55	9.45	9.38	9.29	9.24	9.20	9.11	9.03
26.92	26.42	25.91	25.39	25.08	24.87	24.60	24.44	24.33	24.06	23.82
2.94	2.90	2.87	2.84	2.81	2.80	2.78	2.77	2.76	2.74	2.72
4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.75	3.74	3.70	3.67
7.87	7.72	7.56	7.40	7.30	7.23	7.14	7.09	7.06	6.97	6.89
18.41	17.99	17.56	17.12	16.85	16.67	16.44	16.31	16.21	15.98	15.77
2.70	2.67	2.63	2.59	2.57	2.56	2.54	2.52	2.51	2.49	2.47
3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.32	3.30	3.27	3.23
6.62	6.47	6.31	6.16	6.06	5.99	5.91	5.86	5.82	5.74	5.66
14.08	13.71	13.32	12.93	12.69	12.53	12.33	12.20	12.12	11.91	11.72
2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.30
3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.02	3.01	2.97	2.93
5.81	5.67	5.52	5.36	5.26	5.20	5.12	5.07	5.03	4.95	4.87
11.54	11.19	10.84	10.48	10.26	10.11	9.92	9.80	9.73	9.53	9.36
2.42	2.38	2.34	2.30	2.27	2.25	2.23	2.22	2.21	2.18	2.16
3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.80	2.79	2.75	2.71
5.26	5.11	4.96	4.81	4.71	4.65	4.57	4.52	4.48	4.40	4.32
9.89	9.57	9.24	8.90	8.69	8.55	8.37	8.26	8.19	8.00	7.84
2.32	2.28	2.24	2.20	2.17	2.16	2.13	2.12	2.11	2.08	2.06
2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.64	2.62	2.58	2.54
4.85	4.71	4.56	4.41	4.31	4.25	4.17	4.12	4.08	4.00	3.92
8.75	8.45	8.13	7.80	7.60	7.47	7.30	7.19	7.12	6.94	6.78
2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	1.98
2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.51	2.49	2.45	2.41
4.54	4.40	4.25	4.10	4.01	3.94	3.86	3.81	3.78	3.69	3.61
7.92	7.63	7.32	7.01	6.81	6.68	6.52	6.42	6.35	6.18	6.02
2.19	2.15	2.10	2.06	2.03	2.01	1.99	1.97	1.96	1.93	1.91
2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.40	2.38	2.34	2.30
4.30	4.16	4.01	3.86	3.76	3.70	3.62	3.57	3.54	3.45	3.37
7.29	7.00	6.71	6.40	6.22	6.09	5.93	5.83	5.76	5.59	5.44

(*continued*)

Table A.9 Critical Values for *F* Distributions *(cont.)*

ν_2 = denominator df	α	ν_1 = numerator df 1	2	3	4	5	6	7	8	9
13	.100	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16
	.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
	.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
	.001	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98
14	.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12
	.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
	.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
	.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58
15	.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
	.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26
16	.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
	.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
	.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
	.001	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98
17	.100	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
	.050	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
	.010	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68
	.001	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75
18	.100	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00
	.050	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
	.010	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
	.001	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56
19	.100	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98
	.050	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
	.010	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
	.001	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39
20	.100	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
	.050	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
	.010	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
	.001	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24
21	.100	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95
	.050	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
	.010	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
	.001	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11
22	.100	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93
	.050	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
	.010	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
	.001	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99
23	.100	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92
	.050	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
	.010	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30
	.001	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89
24	.100	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
	.050	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
	.010	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
	.001	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80

(continued)

Table A.9 Critical Values for F Distributions *(cont.)*

ν_1 = numerator df

10	12	15	20	25	30	40	50	60	120	1000
2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.85
2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.31	2.30	2.25	2.21
4.10	3.96	3.82	3.66	3.57	3.51	3.43	3.38	3.34	3.25	3.18
6.80	6.52	6.23	5.93	5.75	5.63	5.47	5.37	5.30	5.14	4.99
2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.87	1.86	1.83	1.80
2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.24	2.22	2.18	2.14
3.94	3.80	3.66	3.51	3.41	3.35	3.27	3.22	3.18	3.09	3.02
6.40	6.13	5.85	5.56	5.38	5.25	5.10	5.00	4.94	4.77	4.62
2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.83	1.82	1.79	1.76
2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.18	2.16	2.11	2.07
3.80	3.67	3.52	3.37	3.28	3.21	3.13	3.08	3.05	2.96	2.88
6.08	5.81	5.54	5.25	5.07	4.95	4.80	4.70	4.64	4.47	4.33
2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.79	1.78	1.75	1.72
2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.12	2.11	2.06	2.02
3.69	3.55	3.41	3.26	3.16	3.10	3.02	2.97	2.93	2.84	2.76
5.81	5.55	5.27	4.99	4.82	4.70	4.54	4.45	4.39	4.23	4.08
2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.76	1.75	1.72	1.69
2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.08	2.06	2.01	1.97
3.59	3.46	3.31	3.16	3.07	3.00	2.92	2.87	2.83	2.75	2.66
5.58	5.32	5.05	4.78	4.60	4.48	4.33	4.24	4.18	4.02	3.87
1.98	1.93	1.89	1.84	1.80	1.78	1.75	1.74	1.72	1.69	1.66
2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.04	2.02	1.97	1.92
3.51	3.37	3.23	3.08	2.98	2.92	2.84	2.78	2.75	2.66	2.58
5.39	5.13	4.87	4.59	4.42	4.30	4.15	4.06	4.00	3.84	3.69
1.96	1.91	1.86	1.81	1.78	1.76	1.73	1.71	1.70	1.67	1.64
2.38	2.31	2.23	2.16	2.11	2.07	2.03	2.00	1.98	1.93	1.88
3.43	3.30	3.15	3.00	2.91	2.84	2.76	2.71	2.67	2.58	2.50
5.22	4.97	4.70	4.43	4.26	4.14	3.99	3.90	3.84	3.68	3.53
1.94	1.89	1.84	1.79	1.76	1.74	1.71	1.69	1.68	1.64	1.61
2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.97	1.95	1.90	1.85
3.37	3.23	3.09	2.94	2.84	2.78	2.69	2.64	2.61	2.52	2.43
5.08	4.82	4.56	4.29	4.12	4.00	3.86	3.77	3.70	3.54	3.40
1.92	1.87	1.83	1.78	1.74	1.72	1.69	1.67	1.66	1.62	1.59
2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.94	1.92	1.87	1.82
3.31	3.17	3.03	2.88	2.79	2.72	2.64	2.58	2.55	2.46	2.37
4.95	4.70	4.44	4.17	4.00	3.88	3.74	3.64	3.58	3.42	3.28
1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.60	1.57
2.30	2.23	2.15	2.07	2.02	1.98	1.94	1.91	1.89	1.84	1.79
3.26	3.12	2.98	2.83	2.73	2.67	2.58	2.53	2.50	2.40	2.32
4.83	4.58	4.33	4.06	3.89	3.78	3.63	3.54	3.48	3.32	3.17
1.89	1.84	1.80	1.74	1.71	1.69	1.66	1.64	1.62	1.59	1.55
2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.88	1.86	1.81	1.76
3.21	3.07	2.93	2.78	2.69	2.62	2.54	2.48	2.45	2.35	2.27
4.73	4.48	4.23	3.96	3.79	3.68	3.53	3.44	3.38	3.22	3.08
1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.57	1.54
2.25	2.18	2.11	2.03	1.97	1.94	1.89	1.86	1.84	1.79	1.74
3.17	3.03	2.89	2.74	2.64	2.58	2.49	2.44	2.40	2.31	2.22
4.64	4.39	4.14	3.87	3.71	3.59	3.45	3.36	3.29	3.14	2.99

(continued)

Table A.9 Critical Values for *F* Distributions *(cont.)*

ν_2 = denominator df	α	ν_1 = numerator df: 1	2	3	4	5	6	7	8	9
25	.100	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89
	.050	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
	.010	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22
	.001	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71
26	.100	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88
	.050	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
	.010	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18
	.001	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64
27	.100	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87
	.050	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
	.010	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15
	.001	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57
28	.100	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87
	.050	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
	.010	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
	.001	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50
29	.100	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86
	.050	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
	.010	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
	.001	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45
30	.100	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
	.050	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
	.010	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
	.001	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
40	.100	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79
	.050	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
	.010	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
	.001	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02
50	.100	2.81	2.41	2.20	2.06	1.97	1.90	1.84	1.80	1.76
	.050	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07
	.010	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78
	.001	12.22	7.96	6.34	5.46	4.90	4.51	4.22	4.00	3.82
60	.100	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
	.050	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
	.010	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
	.001	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
100	.100	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69
	.050	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97
	.010	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59
	.001	11.50	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44
200	.100	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66
	.050	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93
	.010	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50
	.001	11.15	7.15	5.63	4.81	4.29	3.92	3.65	3.43	3.26
1000	.100	2.71	2.31	2.09	1.95	1.85	1.78	1.72	1.68	1.64
	.050	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89
	.010	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43
	.001	10.89	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13

(*continued*)

Table A.9 Critical Values for F Distributions *(cont.)*

ν_1 = numerator df										
10	**12**	**15**	**20**	**25**	**30**	**40**	**50**	**60**	**120**	**1000**
1.87	1.82	1.77	1.72	1.68	1.66	1.63	1.61	1.59	1.56	1.52
2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.84	1.82	1.77	1.72
3.13	2.99	2.85	2.70	2.60	2.54	2.45	2.40	2.36	2.27	2.18
4.56	4.31	4.06	3.79	3.63	3.52	3.37	3.28	3.22	3.06	2.91
1.86	1.81	1.76	1.71	1.67	1.65	1.61	1.59	1.58	1.54	1.51
2.22	2.15	2.07	1.99	1.94	1.90	1.85	1.82	1.80	1.75	1.70
3.09	2.96	2.81	2.66	2.57	2.50	2.42	2.36	2.33	2.23	2.14
4.48	4.24	3.99	3.72	3.56	3.44	3.30	3.21	3.15	2.99	2.84
1.85	1.80	1.75	1.70	1.66	1.64	1.60	1.58	1.57	1.53	1.50
2.20	2.13	2.06	1.97	1.92	1.88	1.84	1.81	1.79	1.73	1.68
3.06	2.93	2.78	2.63	2.54	2.47	2.38	2.33	2.29	2.20	2.11
4.41	4.17	3.92	3.66	3.49	3.38	3.23	3.14	3.08	2.92	2.78
1.84	1.79	1.74	1.69	1.65	1.63	1.59	1.57	1.56	1.52	1.48
2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.79	1.77	1.71	1.66
3.03	2.90	2.75	2.60	2.51	2.44	2.35	2.30	2.26	2.17	2.08
4.35	4.11	3.86	3.60	3.43	3.32	3.18	3.09	3.02	2.86	2.72
1.83	1.78	1.73	1.68	1.64	1.62	1.58	1.56	1.55	1.51	1.47
2.18	2.10	2.03	1.94	1.89	1.85	1.81	1.77	1.75	1.70	1.65
3.00	2.87	2.73	2.57	2.48	2.41	2.33	2.27	2.23	2.14	2.05
4.29	4.05	3.80	3.54	3.38	3.27	3.12	3.03	2.97	2.81	2.66
1.82	1.77	1.72	1.67	1.63	1.61	1.57	1.55	1.54	1.50	1.46
2.16	2.09	2.01	1.93	1.88	1.84	1.79	1.76	1.74	1.68	1.63
2.98	2.84	2.70	2.55	2.45	2.39	2.30	2.25	2.21	2.11	2.02
4.24	4.00	3.75	3.49	3.33	3.22	3.07	2.98	2.92	2.76	2.61
1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.48	1.47	1.42	1.38
2.08	2.00	1.92	1.84	1.78	1.74	1.69	1.66	1.64	1.58	1.52
2.80	2.66	2.52	2.37	2.27	2.20	2.11	2.06	2.02	1.92	1.82
3.87	3.64	3.40	3.14	2.98	2.87	2.73	2.64	2.57	2.41	2.25
1.73	1.68	1.63	1.57	1.53	1.50	1.46	1.44	1.42	1.38	1.33
2.03	1.95	1.87	1.78	1.73	1.69	1.63	1.60	1.58	1.51	1.45
2.70	2.56	2.42	2.27	2.17	2.10	2.01	1.95	1.91	1.80	1.70
3.67	3.44	3.20	2.95	2.79	2.68	2.53	2.44	2.38	2.21	2.05
1.71	1.66	1.60	1.54	1.50	1.48	1.44	1.41	1.40	1.35	1.30
1.99	1.92	1.84	1.75	1.69	1.65	1.59	1.56	1.53	1.47	1.40
2.63	2.50	2.35	2.20	2.10	2.03	1.94	1.88	1.84	1.73	1.62
3.54	3.32	3.08	2.83	2.67	2.55	2.41	2.32	2.25	2.08	1.92
1.66	1.61	1.56	1.49	1.45	1.42	1.38	1.35	1.34	1.28	1.22
1.93	1.85	1.77	1.68	1.62	1.57	1.52	1.48	1.45	1.38	1.30
2.50	2.37	2.22	2.07	1.97	1.89	1.80	1.74	1.69	1.57	1.45
3.30	3.07	2.84	2.59	2.43	2.32	2.17	2.08	2.01	1.83	1.64
1.63	1.58	1.52	1.46	1.41	1.38	1.34	1.31	1.29	1.23	1.16
1.88	1.80	1.72	1.62	1.56	1.52	1.46	1.41	1.39	1.30	1.21
2.41	2.27	2.13	1.97	1.87	1.79	1.69	1.63	1.58	1.45	1.30
3.12	2.90	2.67	2.42	2.26	2.15	2.00	1.90	1.83	1.64	1.43
1.61	1.55	1.49	1.43	1.38	1.35	1.30	1.27	1.25	1.18	1.08
1.84	1.76	1.68	1.58	1.52	1.47	1.41	1.36	1.33	1.24	1.11
2.34	2.20	2.06	1.90	1.79	1.72	1.61	1.54	1.50	1.35	1.16
2.99	2.77	2.54	2.30	2.14	2.02	1.87	1.77	1.69	1.49	1.22

Table A.10 Critical Values for Studentized Range Distributions

		m										
ν	α	2	3	4	5	6	7	8	9	10	11	12
5	.05	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.17	7.32
	.01	5.70	6.98	7.80	8.42	8.91	9.32	9.67	9.97	10.24	10.48	10.70
6	.05	3.46	4.34	4.90	5.30	5.63	5.90	6.12	6.32	6.49	6.65	6.79
	.01	5.24	6.33	7.03	7.56	7.97	8.32	8.61	8.87	9.10	9.30	9.48
7	.05	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16	6.30	6.43
	.01	4.95	5.92	6.54	7.01	7.37	7.68	7.94	8.17	8.37	8.55	8.71
8	.05	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05	6.18
	.01	4.75	5.64	6.20	6.62	6.96	7.24	7.47	7.68	7.86	8.03	8.18
9	.05	3.20	3.95	4.41	4.76	5.02	5.24	5.43	5.59	5.74	5.87	5.98
	.01	4.60	5.43	5.96	6.35	6.66	6.91	7.13	7.33	7.49	7.65	7.78
10	.05	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60	5.72	5.83
	.01	4.48	5.27	5.77	6.14	6.43	6.67	6.87	7.05	7.21	7.36	7.49
11	.05	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49	5.61	5.71
	.01	4.39	5.15	5.62	5.97	6.25	6.48	6.67	6.84	6.99	7.13	7.25
12	.05	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.39	5.51	5.61
	.01	4.32	5.05	5.50	5.84	6.10	6.32	6.51	6.67	6.81	6.94	7.06
13	.05	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32	5.43	5.53
	.01	4.26	4.96	5.40	5.73	5.98	6.19	6.37	6.53	6.67	6.79	6.90
14	.05	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36	5.46
	.01	4.21	4.89	5.32	5.63	5.88	6.08	6.26	6.41	6.54	6.66	6.77
15	.05	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20	5.31	5.40
	.01	4.17	4.84	5.25	5.56	5.80	5.99	6.16	6.31	6.44	6.55	6.66
16	.05	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15	5.26	5.35
	.01	4.13	4.79	5.19	5.49	5.72	5.92	6.08	6.22	6.35	6.46	6.56
17	.05	2.98	3.63	4.02	4.30	4.52	4.70	4.86	4.99	5.11	5.21	5.31
	.01	4.10	4.74	5.14	5.43	5.66	5.85	6.01	6.15	6.27	6.38	6.48
18	.05	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07	5.17	5.27
	.01	4.07	4.70	5.09	5.38	5.60	5.79	5.94	6.08	6.20	6.31	6.41
19	.05	2.96	3.59	3.98	4.25	4.47	4.65	4.79	4.92	5.04	5.14	5.23
	.01	4.05	4.67	5.05	5.33	5.55	5.73	5.89	6.02	6.14	6.25	6.34
20	.05	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01	5.11	5.20
	.01	4.02	4.64	5.02	5.29	5.51	5.69	5.84	5.97	6.09	6.19	6.28
24	.05	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92	5.01	5.10
	.01	3.96	4.55	4.91	5.17	5.37	5.54	5.69	5.81	5.92	6.02	6.11
30	.05	2.89	3.49	3.85	4.10	4.30	4.46	4.60	4.72	4.82	4.92	5.00
	.01	3.89	4.45	4.80	5.05	5.24	5.40	5.54	5.65	5.76	5.85	5.93
40	.05	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.73	4.82	4.90
	.01	3.82	4.37	4.70	4.93	5.11	5.26	5.39	5.50	5.60	5.69	5.76
60	.05	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65	4.73	4.81
	.01	3.76	4.28	4.59	4.82	4.99	5.13	5.25	5.36	5.45	5.53	5.60
120	.05	2.80	3.36	3.68	3.92	4.10	4.24	4.36	4.47	4.56	4.64	4.71
	.01	3.70	4.20	4.50	4.71	4.87	5.01	5.12	5.21	5.30	5.37	5.44
∞	.05	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.47	4.55	4.62
	.01	3.64	4.12	4.40	4.60	4.76	4.88	4.99	5.08	5.16	5.23	5.29

Table A.11 Chi-Squared Curve Tail Areas

Upper-tail Area	$\nu = 1$	$\nu = 2$	$\nu = 3$	$\nu = 4$	$\nu = 5$
> .100	< 2.70	< 4.60	< 6.25	< 7.77	< 9.23
.100	2.70	4.60	6.25	7.77	9.23
.095	2.78	4.70	6.36	7.90	9.37
.090	2.87	4.81	6.49	8.04	9.52
.085	2.96	4.93	6.62	8.18	9.67
.080	3.06	5.05	6.75	8.33	9.83
.075	3.17	5.18	6.90	8.49	10.00
.070	3.28	5.31	7.06	8.66	10.19
.065	3.40	5.46	7.22	8.84	10.38
.060	3.53	5.62	7.40	9.04	10.59
.055	3.68	5.80	7.60	9.25	10.82
.050	3.84	5.99	7.81	9.48	11.07
.045	4.01	6.20	8.04	9.74	11.34
.040	4.21	6.43	8.31	10.02	11.64
.035	4.44	6.70	8.60	10.34	11.98
.030	4.70	7.01	8.94	10.71	12.37
.025	5.02	7.37	9.34	11.14	12.83
.020	5.41	7.82	9.83	11.66	13.38
.015	5.91	8.39	10.46	12.33	14.09
.010	6.63	9.21	11.34	13.27	15.08
.005	7.87	10.59	12.83	14.86	16.74
.001	10.82	13.81	16.26	18.46	20.51
< .001	> 10.82	> 13.81	> 16.26	> 18.46	> 20.51

Upper-tail Area	$\nu = 6$	$\nu = 7$	$\nu = 8$	$\nu = 9$	$\nu = 10$
> .100	< 10.64	< 12.01	< 13.36	< 14.68	< 15.98
.100	10.64	12.01	13.36	14.68	15.98
.095	10.79	12.17	13.52	14.85	16.16
.090	10.94	12.33	13.69	15.03	16.35
.085	11.11	12.50	13.87	15.22	16.54
.080	11.28	12.69	14.06	15.42	16.75
.075	11.46	12.88	14.26	15.63	16.97
.070	11.65	13.08	14.48	15.85	17.20
.065	11.86	13.30	14.71	16.09	17.44
.060	12.08	13.53	14.95	16.34	17.71
.055	12.33	13.79	15.22	16.62	17.99
.050	12.59	14.06	15.50	16.91	18.30
.045	12.87	14.36	15.82	17.24	18.64
.040	13.19	14.70	16.17	17.60	19.02
.035	13.55	15.07	16.56	18.01	19.44
.030	13.96	15.50	17.01	18.47	19.92
.025	14.44	16.01	17.53	19.02	20.48
.020	15.03	16.62	18.16	19.67	21.16
.015	15.77	17.39	18.97	20.51	22.02
.010	16.81	18.47	20.09	21.66	23.20
.005	18.54	20.27	21.95	23.58	25.18
.001	22.45	24.32	26.12	27.87	29.58
< .001	> 22.45	> 24.32	> 26.12	> 27.87	> 29.58

(*continued*)

Table A.11 Chi-Squared Curve Tail Areas *(cont.)*

Upper-tail Area	$\nu = 11$	$\nu = 12$	$\nu = 13$	$\nu = 14$	$\nu = 15$
> .100	< 17.27	< 18.54	< 19.81	< 21.06	< 22.30
.100	17.27	18.54	19.81	21.06	22.30
.095	17.45	18.74	20.00	21.26	22.51
.090	17.65	18.93	20.21	21.47	22.73
.085	17.85	19.14	20.42	21.69	22.95
.080	18.06	19.36	20.65	21.93	23.19
.075	18.29	19.60	20.89	22.17	23.45
.070	18.53	19.84	21.15	22.44	23.72
.065	18.78	20.11	21.42	22.71	24.00
.060	19.06	20.39	21.71	23.01	24.31
.055	19.35	20.69	22.02	23.33	24.63
.050	19.67	21.02	22.36	23.68	24.99
.045	20.02	21.38	22.73	24.06	25.38
.040	20.41	21.78	23.14	24.48	25.81
.035	20.84	22.23	23.60	24.95	26.29
.030	21.34	22.74	24.12	25.49	26.84
.025	21.92	23.33	24.73	26.11	27.48
.020	22.61	24.05	25.47	26.87	28.25
.015	23.50	24.96	26.40	27.82	29.23
.010	24.72	26.21	27.68	29.14	30.57
.005	26.75	28.29	29.81	31.31	32.80
.001	31.26	32.90	34.52	36.12	37.69
< .001	> 31.26	> 32.90	> 34.52	> 36.12	> 37.69
Upper-tail Area	$\nu = 16$	$\nu = 17$	$\nu = 18$	$\nu = 19$	$\nu = 20$
> .100	< 23.54	< 24.77	< 25.98	< 27.20	< 28.41
.100	23.54	24.76	25.98	27.20	28.41
.095	23.75	24.98	26.21	27.43	28.64
.090	23.97	25.21	26.44	27.66	28.88
.085	24.21	25.45	26.68	27.91	29.14
.080	24.45	25.70	26.94	28.18	29.40
.075	24.71	25.97	27.21	28.45	29.69
.070	24.99	26.25	27.50	28.75	29.99
.065	25.28	26.55	27.81	29.06	30.30
.060	25.59	26.87	28.13	29.39	30.64
.055	25.93	27.21	28.48	29.75	31.01
.050	26.29	27.58	28.86	30.14	31.41
.045	26.69	27.99	29.28	30.56	31.84
.040	27.13	28.44	29.74	31.03	32.32
.035	27.62	28.94	30.25	31.56	32.85
.030	28.19	29.52	30.84	32.15	33.46
.025	28.84	30.19	31.52	32.85	34.16
.020	29.63	30.99	32.34	33.68	35.01
.015	30.62	32.01	33.38	34.74	36.09
.010	32.00	33.40	34.80	36.19	37.56
.005	34.26	35.71	37.15	38.58	39.99
.001	39.25	40.78	42.31	43.81	45.31
< .001	> 39.25	> 40.78	> 42.31	> 43.81	> 45.31

Table A.12 Critical Values for the Ryan–Joiner Test of Normality

		α		
		.10	**.05**	**.01**
n	**5**	.9033	.8804	.8320
	10	.9347	.9180	.8804
	15	.9506	.9383	.9110
	20	.9600	.9503	.9290
	25	.9662	.9582	.9408
	30	.9707	.9639	.9490
	40	.9767	.9715	.9597
	50	.9807	.9764	.9664
	60	.9835	.9799	.9710
	75	.9865	.9835	.9757

Table A.13 Critical Values for the Wilcoxon Signed-Rank Test

$P_0(S_+ \geq c_1) = P(S_+ \geq c_1$ when H_0 is true)

n	c_1	$P_0(S_+ \geq c_1)$	n	c_1	$P_0(S_+ \geq c_1)$
3	6	.125		78	.011
4	9	.125		79	.009
	10	.062		81	.005
5	13	.094	14	73	.108
	14	.062		74	.097
	15	.031		79	.052
6	17	.109		84	.025
	19	.047		89	.010
	20	.031		92	.005
	21	.016	15	83	.104
7	22	.109		84	.094
	24	.055		89	.053
	26	.023		90	.047
	28	.008		95	.024
8	28	.098		100	.011
	30	.055		101	.009
	32	.027		104	.005
	34	.012	16	93	.106
	35	.008		94	.096
	36	.004		100	.052
9	34	.102		106	.025
	37	.049		112	.011
	39	.027		113	.009
	42	.010		116	.005
	44	.004	17	104	.103
10	41	.097		105	.095
	44	.053		112	.049
	47	.024		118	.025
	50	.010		125	.010
	52	.005		129	.005
11	48	.103	18	116	.098
	52	.051		124	.049
	55	.027		131	.024
	59	.009		138	.010
	61	.005		143	.005
12	56	.102	19	128	.098
	60	.055		136	.052
	61	.046		137	.048
	64	.026		144	.025
	68	.010		152	.010
	71	.005		157	.005
13	64	.108	20	140	.101
	65	.095		150	.049
	69	.055		158	.024
	70	.047		167	.010
	74	.024		172	.005

Table A.14 Critical Values for the Wilcoxon Rank-Sum Test

$P_0(W \geq c) = P(W \geq c$ when H_0 is true)

m	*n*	*c*	$P_0(W \geq c)$	*m*	*n*	*c*	$P_0(W \geq c)$
3	3	15	.05			40	.004
	4	17	.057		6	40	.041
		18	.029			41	.026
	5	20	.036			43	.009
		21	.018			44	.004
	6	22	.048		7	43	.053
		23	.024			45	.024
		24	.012			47	.009
	7	24	.058			48	.005
		26	.017		8	47	.047
		27	.008			49	.023
	8	27	.042			51	.009
		28	.024			52	.005
		29	.012	6	6	50	.047
		30	.006			52	.021
4	4	24	.057			54	.008
		25	.029			55	.004
		26	.014		7	54	.051
	5	27	.056			56	.026
		28	.032			58	.011
		29	.016			60	.004
		30	.008		8	58	.054
	6	30	.057			61	.021
		32	.019			63	.01
		33	.010			65	.004
		34	.005	7	7	66	.049
	7	33	.055			68	.027
		35	.021			71	.009
		36	.012			72	.006
		37	.006		8	71	.047
	8	36	.055			73	.027
		38	.024			76	.01
		40	.008			78	.005
		41	.004	8	8	84	.052
5	5	36	.048			87	.025
		37	.028			90	.01
		39	.008			92	.005

Table A.15 Critical Values for the Wilcoxon Signed-Rank Interval

$(\overline{x}_{(n(n+1)/2-c+1)}, \overline{x}_{(c)})$

n	Confidence Level (%)	*c*	*n*	Confidence Level (%)	*c*	*n*	Confidence Level (%)	*c*
5	93.8	15	13	99.0	81	20	99.1	173
	87.5	14		95.2	74		95.2	158
6	96.9	21		90.6	70		90.3	150
	93.7	20	14	99.1	93	21	99.0	188
	90.6	19		95.1	84		95.0	172
7	98.4	28		89.6	79		89.7	163
	95.3	26	15	99.0	104	22	99.0	204
	89.1	24		95.2	95		95.0	187
8	99.2	36		90.5	90		90.2	178
	94.5	32	16	99.1	117	23	99.0	221
	89.1	30		94.9	106		95.2	203
9	99.2	44		89.5	100		90.2	193
	94.5	39	17	99.1	130	24	99.0	239
	90.2	37		94.9	118		95.1	219
10	99.0	52		90.2	112		89.9	208
	95.1	47	18	99.0	143	25	99.0	257
	89.5	44		95.2	131		95.2	236
11	99.0	61		90.1	124		89.9	224
	94.6	55	19	99.1	158			
	89.8	52		95.1	144			
12	99.1	71		90.4	137			
	94.8	64						
	90.8	61						

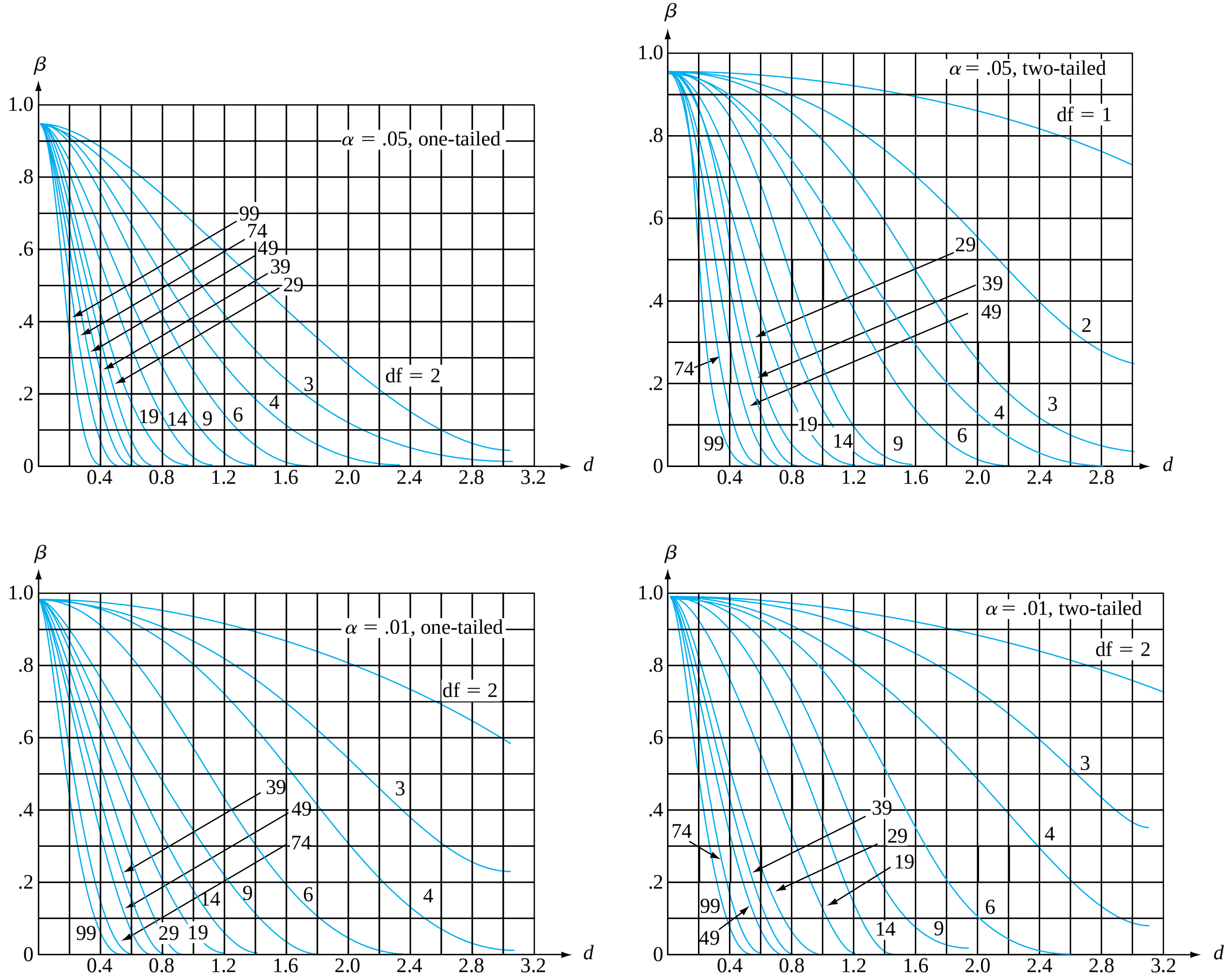
Table A.16 Critical Values for the Wilcoxon Rank-Sum Interval

$(d_{ij(mn-c+1)}, d_{ij(c)})$

Larger Sample Size	Smaller Sample Size 5: Confidence Level (%)	5: c	6: Confidence Level (%)	6: c	7: Confidence Level (%)	7: c	8: Confidence Level (%)	8: c
5	99.2	25						
	94.4	22						
	90.5	21						
6	99.1	29	99.1	34				
	94.8	26	95.9	31				
	91.8	25	90.7	29				
7	99.0	33	99.2	39	98.9	44		
	95.2	30	94.9	35	94.7	40		
	89.4	28	89.9	33	90.3	38		
8	98.9	37	99.2	44	99.1	50	99.0	56
	95.5	34	95.7	40	94.6	45	95.0	51
	90.7	32	89.2	37	90.6	43	89.5	48
9	98.8	41	99.2	49	99.2	56	98.9	62
	95.8	38	95.0	44	94.5	50	95.4	57
	88.8	35	91.2	42	90.9	48	90.7	54
10	99.2	46	98.9	53	99.0	61	99.1	69
	94.5	41	94.4	48	94.5	55	94.5	62
	90.1	39	90.7	46	89.1	52	89.9	59
11	99.1	50	99.0	58	98.9	66	99.1	75
	94.8	45	95.2	53	95.6	61	94.9	68
	91.0	43	90.2	50	89.6	57	90.9	65
12	99.1	54	99.0	63	99.0	72	99.0	81
	95.2	49	94.7	57	95.5	66	95.3	74
	89.6	46	89.8	54	90.0	62	90.2	70

Larger Sample Size	Smaller Sample Size 9: Confidence Level (%)	9: c	10: Confidence Level (%)	10: c	11: Confidence Level (%)	11: c	12: Confidence Level (%)	12: c
9	98.9	69						
	95.0	63						
	90.6	60						
10	99.0	76	99.1	84				
	94.7	69	94.8	76				
	90.5	66	89.5	72				
11	99.0	83	99.0	91	98.9	99		
	95.4	76	94.9	83	95.3	91		
	90.5	72	90.1	79	89.9	86		
12	99.1	90	99.1	99	99.1	108	99.0	116
	95.1	82	95.0	90	94.9	98	94.8	106
	90.5	78	90.7	86	89.6	93	89.9	101

Table A.17 β Curves for t Tests



9.3 Summary and examples

Summary of hypothesis tests of two independent samples

Population	**Sample**	σ^2	**Equal** σ^2	**Test statistic**	**CI**
-	Large	Known	-	$\frac{(\bar{X}_1-\bar{X}_2)-c}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} \sim N(0,1)$	$\left[(\bar{x}_1-\bar{x}_2) \pm Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\right]$
Normal	-				
-	Large	Unknown	-	$\frac{(\bar{X}_1-\bar{X}_2)-c}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} \sim N(0,1)$	$\left[(\bar{x}_1-\bar{x}_2) \pm Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}\right]$
Normal	Small	Unknown	Equal	$s_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$ $\frac{(\bar{X}_1-\bar{X}_2)-c}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim t(n_1+n_2-2)$	$\left[(\bar{x}_1-\bar{x}_2) \pm t_{\alpha/2}^{(n_1+n_2-2)} s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\right]$
			Not Equal	$v = \left\lfloor \frac{(s_1^2/n_1+s_2^2/n_2)^2}{\frac{(s_1^2/n_1)^2}{n_1-1}+\frac{(s_2^2/n_2)^2}{n_2-1}} \right\rfloor$ $\frac{(\bar{X}_1-\bar{X}_2)-c}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} \sim t(v)$	$\left[(\bar{x}_1-\bar{x}_2) \pm t_{\alpha/2}^{(v)}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}\right]$

Chapter 1 Stuff

Construct a **stem-and leaf plots**:

1. Split each observation into a

• Stem: one or more of the leading, or left-hand, digits; and a

• Leaf: the trailing, or remaining, digit(s) to the right.

2. Write the stems in a column, from the smallest to the largest. Include all stems between the smallest and largest, even if there are no corresponding leaves.

3. List all the digits of each leaf next to its corresponding stem. It is not necessary to put the leaves in increasing order, but make sure the leaves line up vertically.

4. Indicate the unites for the stems and leaves.

Example: Number of weekly client of one store are recorded. Construct the stem-and-leaf plot, and describe the distribution. $n=16$

420 395 208 581 443 353 496 528 544 389 399 498 627 618 808 469

1. split the data

42 | 0 too many stems

4 | 20

stem, leaf

2. 208 - 808

3.

Stem	Leaf
2	08
3	95 53 89 99
4	20 43 96 98 69
5	81 28 44
6	27 18
7	
8	08

← Peak

Stem: hundreds of digits
Leaf: tens and ones digits

Distribution
1. Representative value: 300-400
2. Spread: not too far from the center
3. Gap: One minor gap at 700
4. Extent of symmetry: not symmetric
5. Peak: one peak at 400
6. Outlier: example value would be 1600

Histogram

→ Equal Class Width

\# of rows/classes: $\sqrt{n}$

Width of a class: max - min

→ Unequal Class Width:

\# of rows/classes: anything

Y-axis: density: $\frac{\text{rel freq}}{\text{width}}$

Mean ($\bar{x}$)

Median ($\tilde{x}$)

Trimmed Mean ($\bar{x}_{tr}(p)$)

↳ $n \cdot p$ = amount of numbers to trim

Sample Proportion of Success ($\hat{p}$)

$\hat{p} = \frac{n(S)}{n}$

Sample Range

↳ $R = x_{max} - x_{min}$

Lower Forth (f_1)

Upper Forth (f_3)

Fourth Spread (f_s)

↳ $f_s = f_3 - f_1$

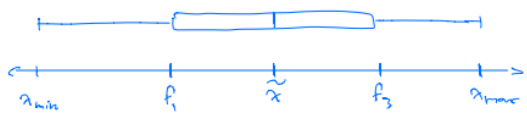
Sample Variance (s^2)

↳ $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$

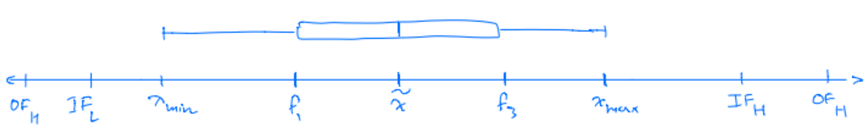
Sample Standard Deviation (s)

$s = \sqrt{s^2}$

Simple Box Plot



Modified Box Plot



Inner Fence $= [f_1 - 1.5f_s \; , \; f_3 + 1.5f_s]$

Outer Fence $= [f_1 - 3f_s \; , \; f_3 + 3f_s]$

Example 1.21 Sodium content values in food product. Construct simple box-plot, boxplot with outliers using the sample.

$n=12$

211 408 171 178 359 249 205 203 201 223 234 256.

mild

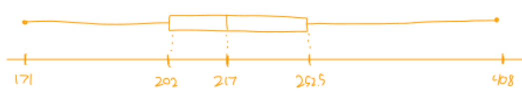
Extreme

171 178 201 | 203 205 211 | 223 234 249 | 256 359 408

$\tilde{x} = \frac{211+223}{2} = 217$ $\quad f_1 = \frac{201+203}{2} = 202$ $\quad f_3 = \frac{249+256}{2} = 252.5$

$x_{min} = 171$ $\quad x_{max} = 408$

simple box plot



modified box plot

$f_s = f_3 - f_1 = 252.5 - 202 = 50.5$

$IF = [f_1 - 1.5f_s \quad f_3 + 1.5f_s] = [202 - 1.5 \times 50.5 \quad 252.5 + 1.5 \times 50.5]$

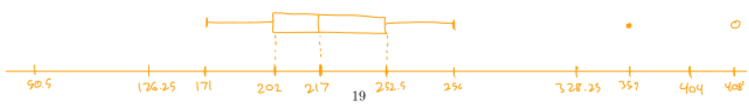
$= [126.25, \ 328.25]$

$OF = [f_1 - 3f_s \quad f_3 + 3f_s] = [202 - 3 \times 50.5 \quad 252.5 + 3 \times 50.5]$

$= [50.5 \quad 404]$

mild outlier: 359

extreme outlier: 408



Chapter 2 Stuff

Conditional probability of A given that the event B has occurred:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) > 0.$$

Multiplication rule:

$$P(A \cap B) = P(A|B) * P(B).$$

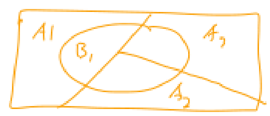
The law of total probability: Let $A_1, A_2, ..., A_k$ be mutually exclusive and exhaustive events, then for any other event B.

$$P(B) = \sum_{i=1}^{k} P(B|A_i)*P(A_i) = P(B|A_1)*P(A_1)+P(B|A_2)*P(A_2)+...+P(B|A_k)*P(A_k).$$

Because $A_1, A_2, \dots A_k$ are disjoint and exhaustive

$B = (A_1 \cap B) \cup (A_2 \cap B) \cup \dots \cup (A_k \cap B)$

$P(B) = \sum_{i=1}^{k} P(A_i \cap B) \xrightarrow[\text{rule}]{\text{multiplication}} \sum_{i=1}^{k} P(B/A_i) \cdot P(A_i)$



Baye's Theorem Let $A_1, A_2, ..., A_k$ be a collection of k mutually exclusive and exhaustive events, with prior probability $P(A_i), i = 1, 2, ..., k$. Then for any other event B for which $P(B) > 0$, the posterior probability of A_j given that B has occurred is

multiplication rule

$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B|A_j) * P(A_j)}{\sum_{i=1}^{k} P(B|A_i) * P(A_i)}.$$

→ the law of total probability

Chapter 3 Stuff

3.3 Expected value of X

3.3.1 Expected value

Example: Bernoulli $D = \{0, 1\}$
$P(0) = 0.5$ $P(1) = 0.5$
$E(x) = 0 \times 0.5 + 1 \times 0.5 = 0.5$

Let X be a discrete rv with set of possible values D and pmf $p(x)$. the **expected value** or **mean value** of X, denoted by $E(X)$ or μ_X:

$$E(X) = \mu_X = \sum_{x \in D} \overset{\text{value}}{x} * \overset{P(X=x)}{p(x)}.$$

The **expected value** of any function $h(X)$, denoted by $E[h(X)]$ or $\mu_{h(X)}$:

$$E[h(X)] = \sum_{x \in D} h(x) * p(x).$$

Question: what is the different between $\bar{x}$, μ and μ_X?

$\bar{x}$: sample mean; average of a given sample
μ: population mean; average of the entire population
μ_x: mean value of

3.3.2 Variance of X

σ^2: variance of data

Let X be a discrete rv with pmf $p(x)$, and expected value μ. Then the **variance** of X, denoted by $V(X)$ or σ_X^2, or σ^2 is

$$V(X) = \sum_{x \in D} (x - \mu)^2 * p(x) = E[(X - \mu)^2], \text{ or}$$

$$V(X) = \sum_{x \in D} x^2 * p(x) - \mu^2 = E(X^2) - [E(X)]^2. \longrightarrow \text{very useful}$$

The **standard deviation (SD)** of X is $\sigma_X = \sqrt{\sigma_X^2}$.

The variance of function $h(X)$:

$$V[h(X)] = \sigma_{h(X)}^2 = \sum_{x \in D} (h(x) - E[h(X)])^2 * p(x) = E[(h(X) - E(h(X)))^2].$$

Binomial random variable, X, is defined as the number of success in n trials.

And the probability of success is denoted by p, the pmf of X:

$$X \sim b(x, n, p).$$

$$b(x, n, p) = p(x) = p(X = x) = \binom{n}{x} p^x (1-p)^{n-x} = \frac{n!}{x!\,(n-x)!} \cdot p^x (1-p)^{n-x}$$

$x = 0, 1, 2 \ldots n-1, n$

Mean of X: expected value of x: $n \cdot p$

Variance of X: $n \cdot p \cdot (1-p)$

Standard deviation of X: $\sqrt{n \cdot p(1-p)}$

Cumulative probability for a binomial random variable, X, is defined as:

3.5 The Poisson probability distribution

The **poisson distribution** is often used to count rare events.

discrete distribution

$P(x \leq 4) = P(0) + P(1) + \underbrace{P(2) + P(3) + P(4)}_{P(2 \leq x \leq 4)}$

$P(x \leq 1) = P(0) + P(1)$

Poisson experiment:

1. The probability of a single event occurs in a given interval (of time, length, volumn...) is the same for all interval. = 3 / hour

2. The number of events that occur in any interval is independent of others.

Poisson random variable, X, is a count of the number of times the specific event occurs during a given interval.

e: euler # $e \approx 2.71828$

$x: 0, 1, 2, 3, 4 \ldots$

The pmf: $P(X = x) = \frac{e^{-\mu} \cdot \mu^x}{x!}$

The mean: $E(x) = \mu$

The variance: $V(x) = \mu$

The cdf: Appendix A2

The Poisson distribution as a limit of Binomial

Suppose that in the binomial pmf $b(x; n, p)$, we let $n \longrightarrow \infty$ and $p \longrightarrow 0$. In such a way that np approaches a value $\mu > 0$, then

$$b(x; n, p) \xrightarrow{\text{approximate}} p(x; \mu)$$

This approximation can safely be applied if $n > 50$ $n \cdot p < 5$ → p to be small

Poisson process

Let α be the average event occurring rate in a unit time period.

Let rv X be the number of events occurs during any time interval of length t.

$X \sim p(x; \alpha t)$.

$\alpha t = 3 \times 24 = 72 / \text{day}$

$x \sim p(x, 72)$

Chapter 4 Stuff:

4.1 Probability density functions

A **continuous probability distribution** completely describes the random variable and is used to compute probabilities associated with random variable.

Probability density function (pdf), $f(x)$:

1. is a function defined for all real numbers. i.e. $x \in (-\infty, +\infty)$.

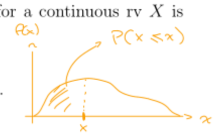
2. is a smooth curve describes the **probability distribution** for a continuous random variable X through area under the curve. Let $a \le b$, the probability

$$P(a \le X \le b) = \int_a^b f(x)dx.$$

Is $f(a) = p(x=a)$?

The **cumulative distribution function (cdf)** $F(x)$ for a continuous rv X is defined for every number x by

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(y)dy.$$

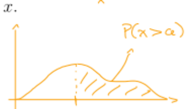


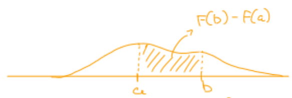
$F(x)$ is the area under the density curve to the left of x.

Note: For any numbers $a, b, a \le b$.

1. $P(X > a) =$ $1 - P(x \le a) = 1 - F(a)$

2. $P(a \le X \le b) =$ $F(b) - F(a)$



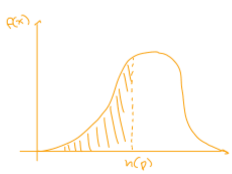


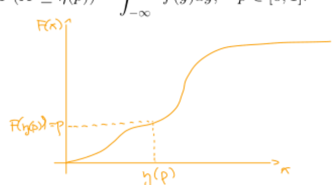
3. $f(x) = F'(x)$. $\Longleftrightarrow$ $F(x) = \int_{-\infty}^{x} f(x)dx$

pdf

4. The $(100p)$th percentile, $\eta(p)$, is defined by

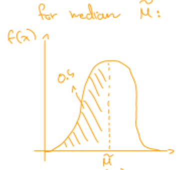
$$p = F(\eta(p)) = P(X \le \eta(p)) = \int_{-\infty}^{\eta(p)} f(y)dy, \quad p \in [0, 1].$$

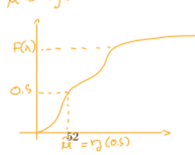




5. The **median** of a continuous distribution, denoted by $\tilde{\mu}$, is the 50th percentile. so $F(\tilde{\mu}) = 0.5$. Half the area under the pdf curve is to the left of μ.

for median $\tilde{\mu}$: $p = 0.5$ $\tilde{\mu} = \eta(0.5)$ $F(\tilde{\mu}) = F(\eta(0.5)) = 0.5$





6. The **Expected value** and variance of a continuous rv X:

$\mu_X = E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

$\mu_{h(X)} = E[h(X)] = \int_{-\infty}^{\infty} h(x) \cdot f(x)\, dx$

$\sigma_X^2 = V(X) = \int_{-\infty}^{\infty} (x - E(x))^2 f(x)\, dx$

$\sigma_X = \sqrt{\sigma_x^2}$

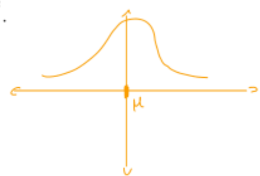
4.2 The normal distribution

Family of distributions

Normal distribution has two parameters: μ, σ (or μ, σ^2), $-\infty < \mu < +\infty$ and $\sigma > 0$. We write the random variable $X \sim N(\mu, \sigma^2)$. The pdf is:

$p(x<1) = \int_{-\infty} f(x)\, dx \rightarrow$ uneasy

$$f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}.$$

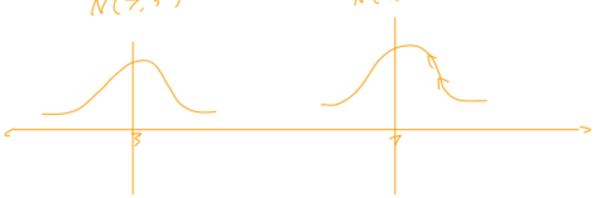


1. Bell shaped, unimodal
2. Symmetric about $x = \mu$
3. σ (σ^2) represent the spread $V(X) = \sigma^2$

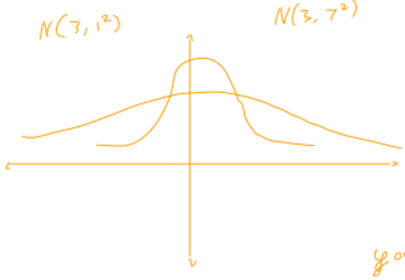
Side question: How many normal distributions are there?
Infinitely many

Example:

$N(7, 5^2)$ $N(3, 5^2)$



$N(3, 1^2)$ $N(3, 7^2)$



you may have missed something
Mar 5

specific normal distribution

rv: $Z \to N(0,1)$

Standard normal distribution $N(0,1)$: That is $\mu = 0, \sigma = 1$. The pdf is

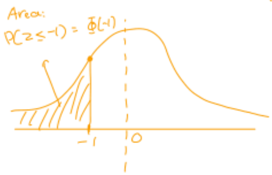
$$f(z;0,1) = \frac{1}{2\pi}e^{-z^2/2}.$$

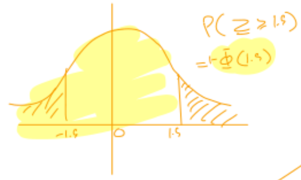
The cdf: Appendix A.3

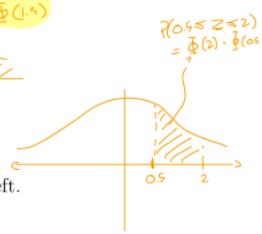
cdf: $F(x)$

$$\Phi(z) = P(Z \le \underset{\text{value}}{z}) = \int_{-\infty}^{z} f(y)dy = \int_{-\infty}^{z} \frac{1}{2\pi}e^{-z^2/2}dy$$

$P(X \le x)$



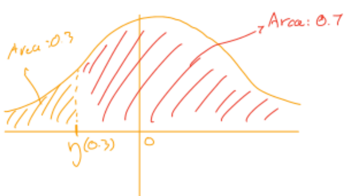


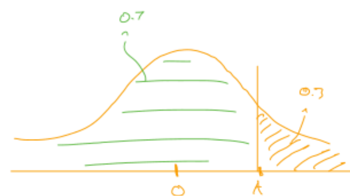


(100p)th percentile vs critical value z_α

- **(100p)th percentile** $\eta(p)$: p refers to the area on the left.
- **Critical value** z_α: α refers to the area on the right.

Example: $P = 0.3$ $\quad \eta(0.3)$





$\underset{\text{30th percentile}}{\eta(0.3)} = \underbrace{Z_{0.7}}$ Z-critical value with $\alpha = 0.7$

A: $\eta(0.7)$

$Z_{0.3}$ $\quad \alpha = 0.3$

Standardization a normal rv X: $X \sim N(\mu, \sigma^2)$

$$X \sim N(\mu, \sigma^2) \xrightarrow{\text{Standardization}} Z \sim N(0,1)$$

$$Z = \frac{x - \mu}{\sigma}$$

$$P(x \le a) = P\Big(\underbrace{\frac{x-\mu}{\sigma}}_{\substack{\text{new rv.} \\ \sim N(0,1)}} \le \frac{a-\mu}{\sigma}\Big) = P\Big(Z \le \frac{a-\mu}{\sigma}\Big)$$

$$= \Phi\Big(\frac{a-\mu}{\sigma}\Big)$$

* $X \le a$ and $\frac{x-\mu}{\sigma} \le \frac{a-\mu}{\sigma}$ are equivalent

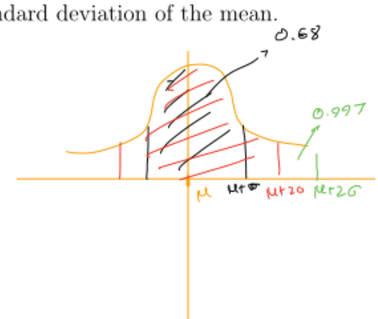
The empirical rule: If the population distribution of a variable is (approximately) normal, then

1. Roughly 68% of the values are within 1 standard deviation of the mean.
2. Roughly 95% of the values are within 2 standard deviation of the mean.
3. Roughly 99.7% of the values are within 3 standard deviation of the mean.

$P(\mu-\sigma \le x \le \mu+\sigma) \approx 0.68$

$P(\mu-2\sigma \le x \le \mu+2\sigma) \approx 0.95$

$P(\mu-3\sigma \le x \le \mu+3\sigma) \approx 0.997$



Approximating the binomial distribution

$= 0.1717$

Let X be a binomial rv based on n trials with success probability p, so $X \sim b(n,p)$.

Let Y be a normal rv, $\mu = n*p$, $\sigma = \sqrt{np(1-p)}$, $Y \sim N(\mu, \sigma^2)$.

When $np \geq 10$ and $n(1-p) \geq 10$,

$$P(X \leq x) = b(x; n, p) \approx P(Y \leq x + 0.5).$$

5.2 The distribution of the sample mean

Proposition 1 The rv's $X_1, X_2, ..., X_n$ be a random sample from a distribution with mean value μ and standard deviation σ. Let $T_0 = X_1 + X_2 + ... + X_n$, the sample total. Then

1. $E(\bar{X}) = \mu$. The mean of sample mean is the same as population mean
2. $V(\bar{X}) = \sigma^2/n$, $\sigma_{\bar{X}=\sigma/\sqrt{n}}$. The variance of sample mean is smaller than population variance
3. $E(T_0) = n\mu$, $V(T_0) = n\sigma^2$, $\sigma_{T_0} = \sqrt{n}\sigma$.

Proposition 2 The rv's $X_1, X_2, ..., X_n$ be a random sample from a **normal** distribution with mean value μ and standard deviation σ, then for any n

1. $E(\bar{X}) = \mu$, $V(\bar{X}) = \sigma^2/n$. same as above
2. $\bar{X} \sim N(\mu, \sigma^2/n)$. normal distribution for $\bar{x}$
3. $T_0 \sim N(n\mu, n\sigma^2)$.

Central limit theorem (CLT)

The rv's $X_1, X_2, ..., X_n$ be a random sample from a distribution with mean value μ and standard deviation σ. If n is sufficiently large $(n > 30)$,then

1. $E(\bar{X}) = \mu$, $V(\bar{X}) = \sigma^2/n$.
2. $\bar{X} \sim N(\mu, \sigma^2/n)$
3. $T_0 \sim N(n\mu, n\sigma^2)$

Example (exercise 47 P237) $X \sim N(70, 1.6^2)$, $n = 16$, find $P(69 \leq \bar{X} \leq 71)$.

$$\left.\begin{matrix} X \sim N(70, 1.6^2) \\ n=16 \end{matrix}\right\} \Rightarrow \bar{x} \sim N\left(70, \frac{1.6^2}{16}\right)$$

$$P(69 \leq \bar{x} \leq 71) = P\left(\frac{69-70}{1.6/4} \leq Z \leq \frac{71-70}{\frac{1.6}{4}}\right)$$

$$= P(-2.5 \leq Z \leq 2.5) = 0.9938 - 0.0062 = 0.9876$$

$$\sigma^2 = \frac{1.6^2}{16} \qquad \sigma = \sqrt{\frac{1.6^2}{16}} = \frac{1.6}{4}$$

Chapter 6 Stuff

To estimate the population mean μ, we an choose the following point estimators.

μ: population parameter, denote as Θ,

want to find point estimator $\hat{\Theta}$ in this case is $\hat{\mu}$ —> sample measurement

① $\hat{\mu} = \bar{x}$ sample mean —> point estimation

$\bar{x} = \frac{1}{20} \sum_{i=1}^{20} x_i = 27.793$ —> point estimate

② $\hat{\mu} = \tilde{x}$ sample median —> point estimator

$\tilde{x} = \frac{27.94 + 27.98}{2} = 27.96$ —> point estimate

③ $\hat{\mu} = \frac{x_{max} - x_{min}}{2}$ —> point estimator $\quad \frac{24.46 + 30.88}{2} = 27.67$ } Point estimate

P: population proportion

point estimator: $\hat{p} = \frac{x}{n}$ —> sample proportion

point estimate: $\hat{p} = \frac{x}{n} = \frac{15}{25} = 0.6$ —> calculated by the sample

A point estimator $\hat{\theta}$ is said to be

1. **unbiased estimator** of θ, if $E(\hat{\Theta}) = \Theta$
1. **biased estimator** of θ, if $E(\hat{\Theta}) \neq \Theta$

Which is better? unbiased provides a better estimate

Some unbiased estimator:

parameter	unbiased estimator	Comments
$p : X \sim b(n,p)$	$\hat{p} = \frac{x}{n}$	$E(\frac{x}{n}) = \frac{1}{n} E(x) = \frac{1}{n}(n \cdot p) = p$
$\mu = \sum_{i=1}^{N} X_i/N$	$\hat{\mu} = \bar{x}$	$E(\bar{x}) = E(\frac{\sum x_i}{n}) = \frac{1}{n} E(\sum x_i) = \frac{1}{n} \sum E x_i = \frac{1}{n} \sum \mu$
$\mu = \sum_{i=1}^{N} X_i/N$	$\hat{\mu} = \tilde{x}$	if the distribution is continuous and symmetric
$\mu = \sum_{i=1}^{N} X_i/N$	$\hat{\mu} = x_{tr(p)}$	
$\sigma^2 = \sum_{i=1}^{N} (X_i - \mu)^2/N$	$S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$	$E(S^2) = \sigma^2$

Minimum variance unbiased estimation (MVUE): Among all estimators that are unbiased, the one that has minimum variance is called **MVUE**.

Example

$\bar{X}$ is MVUE of μ

$V(\bar{x}) \leq V(\tilde{x})$

$V(\bar{x}) \leq V(\bar{x}_{tr(p)})$

is MVUE good?

MVUE is better smaller variance for an estimation is good

Standard error of an estimator $\hat{\theta}$: describes the magnitude of a typical or representative deviation between an estimate and the true value of θ

$\sigma_{\hat{\theta}} = \sqrt{V(\hat{\theta})} = \sqrt{\sigma^2_{\hat{\theta}}}$ → Standard deviation of $\hat{\theta}$

Estimated standard error of an estimator $\hat{\theta}$: If the standard error of the estimator itself involves unkown parameters, whose value can be estimated.

denoted by $\hat{\sigma}_{\theta}$ or $S_{\hat{\theta}}$ means the standard error involves variables

Example 6.9 P259 A normal distribution $N(\mu, \sigma)$.

S^2 = sample variance

$S^2 = \frac{\Sigma (x_i - \bar{x})^2}{n-1}$

$\bar{x}$ is estimator of μ (MVUE) let $n=20$

$\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$ by proposition 2

$\sigma_{\bar{x}} = \sqrt{\frac{\sigma^2}{n}} = \frac{\sigma}{\sqrt{n}}$ → Standard error of estimation

① if σ is Known $\sigma_{\bar{x}} = \frac{1.5}{\sqrt{20}} = 0.335$ → standard error

② if σ is unknown $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{20}}$ involves a variable

use sample sd s to estimate σ, let's say $s = 1.462$

$\hat{\sigma}_{\bar{x}} = \frac{s}{\sqrt{20}} = \frac{1.462}{\sqrt{20}} = 0.32$ → estimated standard error

6.2.1 The method of moments

Let $X_1, X_2, ..., X_n$ be a random sample from a pmf or pdf $f(x)$. For $k = 1, 2, 3, ...$ the k**th population moment**(k**th moment of the distribution** $f(x)$): $E(X^k)$.

the k**th sample moment**: $\frac{1}{n}\sum_{i=1}^{n} X_i^k$.

	1st	2nd	3rd	
Population moment	$E(x)$	$E(x^2)$	$E(x^3)$	
Sample moment	$\frac{1}{n}\sum_{i=1}^{n} x_i = \bar{x}$	$\frac{1}{n}\sum_{i=1}^{n} x_i^2$	$\frac{1}{n}\sum_{i=1}^{n} x_i^3$	....

Example:

Let $X_1, X_2, ..., X_n$ be a random sample from a pmf or pdf $f(x; \theta_1, ..., \theta_m)$, where $\theta_1, ..., \theta_m$ are parameters whose values are unknown.

the **Moment estimators** $\hat{\theta}_1, ..., \hat{\theta}_m$ are obtained by equating the first m sample moments to the corresponding first m population moments and solving for $\theta_1, ..., \theta_m$.

Example 6.12 P265 Let $X_1, X_2, ..., X_n$ be a random sample from exponential distribution. Find the moment estimator of parameter λ. $X \sim Exp(\lambda)$

1st P.M $E(x) = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{E(x)}$

1st S.M $\bar{x}$ hence $\hat{\lambda} = \frac{1}{\bar{x}}$ by equating $E(x)$, $\bar{x}$

6.2.2 Maximum likelihood estimation

Let $X_1, X_2, ..., X_n$ be a random sample from a pmf or pdf $f(x; \theta_1, ..., \theta_m)$, $x_1, x_2, ..., x_n$ are the observed sample values. $x_1, x_2, x_3 \dots x_m$ are variables

$\theta_1, \theta_2, \theta_3, \dots \theta_m$ are given

The **joint pmf/pdf**:

Product

$$f(x_1, x_2, ..., x_n; \theta_1, ..., \theta_m) = \prod_{i=1}^{n} f(x_i; \theta_1, ..., \theta_m)$$

$$= f(x_1; \underbrace{\theta_1, ..., \theta_m}_{\text{variables}}) f(\underbrace{x_2}_{\text{given}}; \theta_1, ..., \theta_m) ... f(x_n; \theta_1, ..., \theta_m)$$

The **likelihood function**:

$$f(x_1, x_2, ..., x_n; \theta_1, ..., \theta_m) = \prod_{i=1}^{n} f(x_i; \theta_1, ..., \theta_m)$$

$$= f(x_1; \theta_1, ..., \theta_m) f(x_2; \theta_1, ..., \theta_m) ... f(x_n; \theta_1, ..., \theta_m)$$

The **natural logarithm of likelihood function**:

$$\ln f(x_1, x_2, ..., x_n; \theta_1, ..., \theta_m) = \sum_{i=1}^{n} \ln f(x_i; \theta_1, ..., \theta_m)$$

$$= \ln f(x_1; \theta_1, ..., \theta_m) + \ln f(x_2; \theta_1, ..., \theta_m) + ... + \ln f(x_n; \theta_1, ..., \theta_m)$$

The **maximum likelihood estimators (mle)** $\hat{\theta}_1, ..., \hat{\theta}_m$: Those values of θ_i's that maximize the likelihood function.

Choose MLE $\hat{\theta}_1, \hat{\theta}_2 \dots \hat{\theta}_m$ to make observed $x_1, x_2, \dots x_m$ most likely

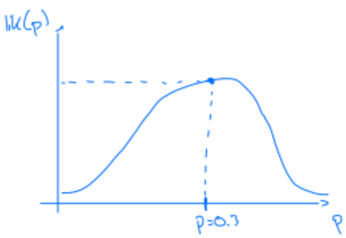
Hence $P(X = 1) = p, P(X = 0) = 1 - p$. Suppose 10 individuals are randomly selected. $x_1 = x_3 = x_{10} = 1$, other seven x_i's are all zero. Find the mle of p.

$Lik(p) = P(x_1=1) \cdot P(x_2=0) \cdot P(x_3=1) \cdot \dots \cdot P(x_{10}=1) = p(1-p) \cdot p \cdots p$

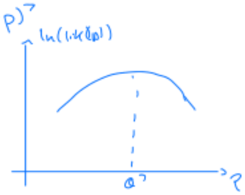
$$lik(p) = \begin{cases} p^3(1-p)^7 = (0.25^3)(0.75^7) = 0.002086 \\ p^3(1-p)^7 = (0.5^3)(0.5^7) = 0.000977 \end{cases}$$

$p=0.25$ make the sample more likely

$p=0.5$ $\quad p \in [0,1]$



when $p=0.3$ $lik(p) = p^3(1-p)^7$ has the largest value



$\ln(lik(p)) = \ln[p^3(1-p)^7]$

$= \ln(p^3) + \ln(1-p)^7 = 3\ln p + 7\ln(1-p)$

$$\frac{\partial \ln(lik(p))}{\partial p} = \frac{3}{p} + \frac{7}{1-p} \times (-1)$$

$$= \frac{3}{p} - \frac{7}{1-p} = 0$$

$p = 0.3$ maximizes $\ln(lik(p))$ also maximizes $lik(p)$

$\hat{p} = 0.3$ is mle

Chapter 7 Stuff

7.1.4 The width of CI and sample size

Given the formula of $100(1-\alpha)\%$ confidence interval $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$. The width is

$$W = 2z_{\alpha/2}\frac{\sigma}{\sqrt{n}}.$$

We can then solve for n, which will derive the desired CI width.

$$n = \left(2z_{\alpha/2}\frac{\sigma}{W}\right)^2$$

Note: The following factors may effect the width of a CI.

1. $\alpha \downarrow \rightarrow CL \uparrow \rightarrow z_{\frac{\alpha}{2}} \uparrow \rightarrow CI \uparrow$

2. $\sigma \uparrow \rightarrow CI \uparrow$

3. $n \uparrow \rightarrow CI \downarrow$

7.2.4 One-sided CI (confidence bounds)

Large sample one-sided CI gives upper bound for μ:

→ Large Sample $n > 40$

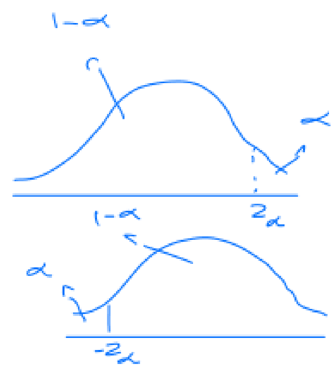
→ σ unknown

→ Population's distribution is unknown

$$(-\infty, \bar{x} + z_\alpha\frac{s}{\sqrt{n}}).$$

Large sample one-sided CI gives lower bound for μ:

$$(\bar{x} - z_\alpha\frac{s}{\sqrt{n}}, \infty).$$



7.3.3 Two-sided CI under different assumptions

	population	sample size	variance σ	distribution	CI
1	Normal	-	known	$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$	$[\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}]$
2	-	Large ($n \geq 40$)	known	$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$	$[\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}]$
3	-	Large ($n \geq 40$)	unknown	$\frac{\bar{X}-\mu}{s/\sqrt{n}} \sim N(0,1)$	$[\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}]$
4	Normal	Small ($n \leq 40$)	unknown	$\frac{\bar{X}-\mu}{s/\sqrt{n}} \sim t(n-1)$	$[\bar{x} \pm t^{n-1}_{\alpha/2}\frac{s}{\sqrt{n}}]$

7.3.4 A prediction interval for a single future value

A random sample $X_1, X_2, ..., X_n$ from a normal distribution, we would like to predict the value to be observed at some future time, X_{n+1}.

A point predictor of X_{n+1} is $\bar{X}$. Hence the predict error is: $\bar{x} - x_{n+1}$

$\bar{x}$ ↓ prediction x_{n+1} ↘ true value

1. The expected value of predict error:

$E(\bar{x} - x_{n+1}) = E(\bar{x}) - E(x_{n+1}) = \mu - \mu = 0$

2. The variance of predict error:

$V(\bar{x} - x_{n+1}) = V(\bar{x}) + V(x_{n+1}) = \frac{\sigma^2}{n} + \sigma^2 = \sigma^2(1 + \frac{1}{n})$

$\bar{x}, x_{n+1}$ are independent $V(x \pm y) = V(x) + V(y)$ when x, y are independent

3. The distribution of the predict error:

$\bar{x}, x_{n+1}$ are both normal, and independent

$\bar{x} - x_{n+1}$ is also normal

$\bar{x} - x_{n+1} \sim N(0, \sigma^2(1 + \frac{1}{n}))$

$$\frac{(\bar{x} - x_{n+1}) - 0}{\sigma\sqrt{1 + \frac{1}{n}}} \sim N(0,1)$$

A prediction interval (PI) for a single observation to be selected from a normal population is

if σ is unknown, we will use s to replace σ

$$= \frac{(\bar{x} - x_{n+1}) - 0}{\frac{s}{\sqrt{n}}} \sim t(n-1)$$

2 sided: the $100(1-\alpha)\%$ P.I $\bar{x} \pm t_{\frac{\alpha}{2}}(n-1)\, s \cdot \sqrt{1 + \frac{1}{n}}$

1 sided: P.I with upper bound $(-\infty, \bar{x} + t_\alpha(n-1)(s)\sqrt{1 + \frac{1}{n}})$

lower bound $(\bar{x} - t_\alpha(n-1)\, s\sqrt{1 + \frac{1}{n}}, +\infty)$

Chapter 8 Stuff

We now show how to solve it using P-**value**:

1. Write down the hypotheses:

   $H_0 : \mu = 76$ $\qquad\qquad$ $H_a : \mu > 76$

2. Figure out the distribution of $\bar{X}$, calculate the P-value $P(\bar{X} \geq \bar{x} | H_0$ is true)

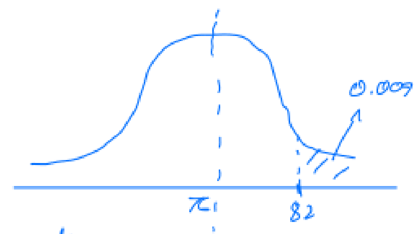
   X : weight of canadian males (30-44) $\quad X \sim N(\mu, 25^2)$

   $\bar{x}$: sample mean $\quad \bar{X} \sim N(\mu, \frac{25^2}{n}) = N(\mu, (\frac{25}{10})^2)$

Assume H_0 is true, calculate P-Value

$P(\bar{X} \geq \bar{x} | H_0 \text{ is true}) = P(\bar{X} \geq 82 / \mu = 76)$

$= P(\frac{\bar{X} - 76}{\frac{25}{10}} \geq \frac{82-76}{\frac{25}{10}}) = P(Z \geq 2.4) = 0.009$

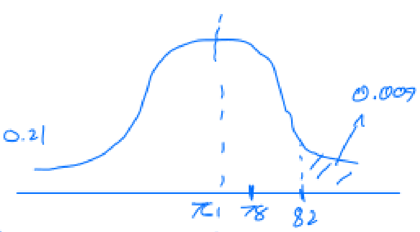


If H_0 is true, it is very unlikely ($p = 0.009$) to get a sample mean of 82 or above. But now, a rare event ($\bar{x} = 82$) occurs hence there is evidence that H_0 is wrong

3. Choose a threshold α, compare with P-value.

What if $\bar{x} = 78$, calculate p-value

$P(\bar{X} \geq 78 \mid H_0 \text{ is true}) = P(\frac{\bar{X} - 76}{\frac{25}{10}} \geq \frac{78-76}{\frac{25}{10}}) = P(Z \geq 0.8) = 0.21$



Is $\bar{x} = 78$ a rare event or a reasonable event?

Hence, we will set a threshold α (significance level)

4. Make conclusion.

   Choose small values for α $\qquad$ Example: $\alpha = 0.01, 0.05, 0.1$

   Compare p-value with α

   P-value $\leq \alpha$ $\quad$ rare event (unlikely to occur), reject H_0 in favor of H_a

   P-value $> \alpha$ $\quad$ reasonable event (likely to occur) accept H_0

We then show how to solve it using **reject point, reject region (RR)**:

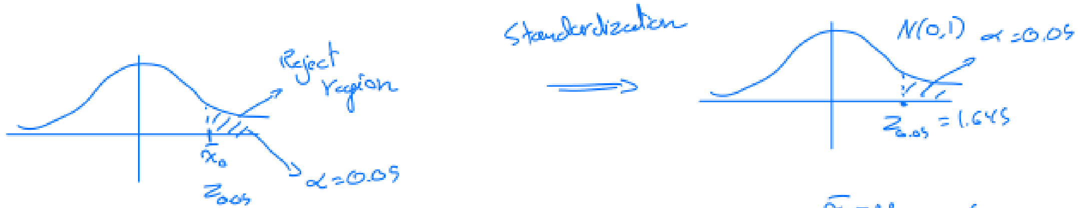
1. Write down the hypotheses:

$H_0: \mu = 76$ $\quad\quad$ $H_a: \mu > 76$

2. Figure out the distribution of $\bar{X}$, determine/calculate the test statistic (TS).

$X \sim N(\mu, \sigma^2) \implies \bar{x} \sim N(\mu, \frac{\sigma^2}{n})$

Start to assume H_0 is true, $\mu_0 = \mu = 76$ $\quad$ $\bar{x} \sim N(76, (\frac{25}{10})^2)$



Define test statistic (TS): $z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$ (z-score of $\bar{x}$)

3. Choose a threshold α, calculate reject point and reject region (RR).

ussually α small value 0.05, 0.01, 0.1

In this case we choose $\alpha = 0.05$, we want to calculate the cut off (reject point) $\bar{x}_0$

such that if H_0 is true, then 95% sample mean will be less than $\bar{x}_0$

That is $\frac{\bar{x}_0 - \mu_0}{\frac{\sigma}{\sqrt{n}}} = Z_\alpha = Z_{0.05} = 1.645$, then we can solve $\bar{x}_0$, then compare

$\bar{x} = 82$ with $\bar{x}_0$

Note: compare $\bar{x}$ with $\bar{x}_0$ is equivalent to compare TS $z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$ with Z_α

Define: Reject Point as Z_α

Reject Region as $(Z_\alpha, +\infty)$

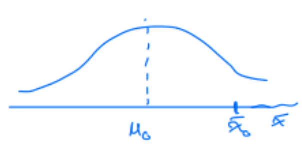
4. Make conclusion

TS

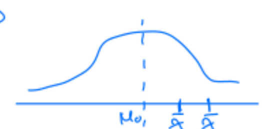
① $z \geq Z_\alpha$ (ie: $z \in (Z_\alpha, +\infty)$): rare event $\to$ reject H_0

② $z < Z_\alpha$ (ie: $z \notin (Z_\alpha, +\infty)$) reasonable $\to$ accept H_0

①



②

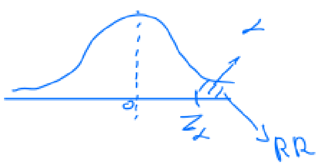


8.1.2 Three types alternative hypothesis

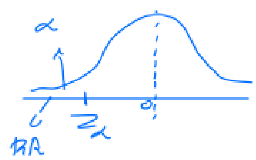
Upper one-tailed, lower one-tailed, two-tailed

$H_0: \mu = \mu_0$

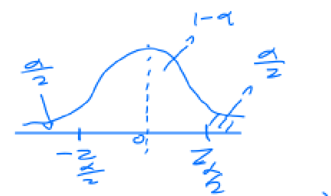
$H_a: \mu > \mu_0$ | $H_a: \mu < \mu_0$ | $H_a: \mu \neq \mu_0$ ($\mu < \mu_0$ or $\mu > \mu_0$)



Upper



lower



RR: $(-\infty, -Z_{\frac{\alpha}{2}}) \cup (Z_{\frac{\alpha}{2}}, +\infty)$

Check both tails

8.1.5 Hypothesis test - normal distribution, unknown σ, and small sample

Recall: when

1. Underlying distribution is normal, and
2. Unknown σ^2. We have

$$T = \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t(n-1).$$

Hence, When the underlying population is normal, but the population variance σ^2 is unknown. To implement the hypothesis test, we define:

Test statistic (TS): $T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$ $\quad t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$

Z_α $\quad -Z_\alpha$ $\quad \pm Z_{\frac{\alpha}{2}}$

	$H_a : \mu > \mu_0$	$H_a : \mu < \mu_0$	$H_a : \mu \neq \mu_0$
Reject Point	$t_\alpha(n-1)$	$-t_\alpha(n-1)$	$\pm t_{\frac{\alpha}{2}}(n-1)$
RR	$(t_\alpha(n-1), +\infty)$	$(-\infty, -t_\alpha(n-1))$	$(-\infty, -t_{\frac{\alpha}{2}}(n-1)) \cup (t_{\frac{\alpha}{2}}(n-1), +\infty)$
P-value	$P(T \geq t)$	$P(T \leq t)$	$2P(T \geq t)$

We reject H_0, when:

1. P-Value $\leq \alpha$

or

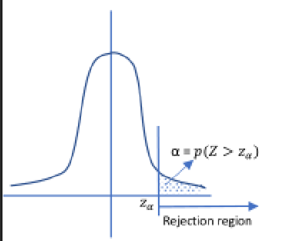
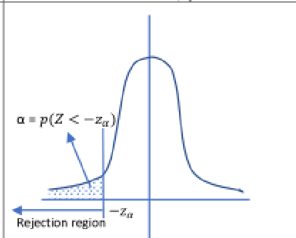
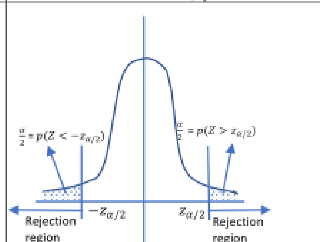
2. TS: $t \in$ RR

Determine test statistic for different assumptions

	population	sample size	variance σ	statistic	distribution
1	Normal	-	known	$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$	$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$
2	Normal	Small ($n \leq 40$)	unknown	$t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$	$\frac{\bar{X}-\mu}{s/\sqrt{n}} \sim t(n-1)$
3	-	Large ($n \geq 40$)	known	$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$	$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$
4	-	Large ($n \geq 40$)	unknown	$z = \frac{\bar{x}-\mu}{s/\sqrt{n}}$	$\frac{\bar{X}-\mu}{s/\sqrt{n}} \sim N(0,1)$

CLT

Summary of solving hypothesis tests:

$H_0: \mu = \mu_0$		
Upper-tailed test	Lower-tailed test	Two-tailed test
Ha: $\mu > \mu_0$	Ha: $\mu < \mu_0$	Ha: $\mu \neq \mu_0$
		
Solve using Reject region (RR): reject H0, when test statistic falls in the RR.		
Test statistic: $Z \sim N(0,1)$		
Reject point: z_α	Reject point: $-z_\alpha$	Reject point: $\pm z_{\alpha/2}$
RR: (z_α, ∞)	RR: $(-\infty, -z_\alpha)$	RR: $(-\infty, -z_{\alpha/2}) \cup (z_{\alpha/2}, \infty)$
Test statistic: $T \sim t(n-1)$		
Reject point: $t_\alpha(n-1)$	Reject point: $-t_\alpha(n-1)$	Reject point: $\pm t_{\alpha/2}(n-1)$
RR: $(t_\alpha(n-1), \infty)$	RR: $(-\infty, -t_\alpha(n-1))$	RR: $(-\infty, -t_{\alpha/2}(n-1)) \cup (t_{\alpha/2}(n-1), \infty)$
Solve using P-value: reject H0, when P-value $\leq \alpha$.		
Test statistic: $Z \sim N(0,1)$		
P-value = $P(\bar{X} \geq \bar{x} \mid$ H0 is true$)$ = $P(Z \geq z)$	P-value= $P(\bar{X} \leq \bar{x} \mid$ H0 is true$)$ = $P(Z \leq z)$	P-value=2 $* P(\bar{X} \geq \lvert\bar{x}\rvert \mid$ H0 is true$)$ = 2*$P(Z \geq \lvert z \rvert)$
Test statistic: $T \sim t(n-1)$		
P-value = $P(\bar{X} \geq \bar{x} \mid$ H0 is true$)$ = $P(T \geq t)$	P-value= $P(\bar{X} \leq \bar{x} \mid$ H0 is true$)$ = $P(T \leq t)$	P-value=2 $* P(\bar{X} \geq \lvert\bar{x}\rvert \mid$ H0 is true$)$ = 2*$P(T \geq \lvert t \rvert)$
Solve using confidence interval: reject H0, when $\mu_0 \notin$ CI		
Test statistic: $Z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$		
$(\bar{x} - z_\alpha \frac{\sigma}{\sqrt{n}}, \ +\infty)$	$(-\infty, \ \bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}})$	$(- z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \ \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}})$
Test statistic: $Z = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} \sim N(0,1)$		
$(\bar{x} - z_\alpha \frac{s}{\sqrt{n}}, \ +\infty)$	$(-\infty, \ \bar{x} + z_\alpha \frac{s}{\sqrt{n}})$	$(\bar{x} - z_{\alpha/2} \frac{s}{\sqrt{n}}, \ \bar{x} + z_{\alpha/2} \frac{s}{\sqrt{n}})$
Test statistic: $T \sim t(n-1)$		
$(\bar{x} - t_\alpha^{(n-1)} \frac{s}{\sqrt{n}}, \ +\infty)$	$(-\infty, \ \bar{x} + t_\alpha^{(n-1)} \frac{s}{\sqrt{n}})$	$(\bar{x} - t_{\alpha/2}^{(n-1)} \frac{s}{\sqrt{n}}, \ \bar{x} + t_{\alpha/2}^{(n-1)} \frac{s}{\sqrt{n}})$

Example 8.5 P319 The drying time of a type of paint under specified test conditions is known to be normally distributed with mean value 75 mins and standard deviation 9 mins. A new additive is designed to decrease average drying time. It is believed that drying time with this additive will remain normally distributed with $\sigma = 9$. We want to test if this additive will decrease the drying time. $\alpha = 0.01$, $n = 25$. Compute the probability of type II error $\beta(\mu_a)$.

$H_0: \mu = 75$ $\quad H_a: \mu < 75$

normal, σ unknown $\Rightarrow$ TS: $Z = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{\bar{x} - 75}{\frac{9}{\sqrt{25}}} = \frac{\bar{x} - 75}{1.8}$$

To reject H_0 we need p-value $< \alpha$, or R.P to be $-Z_\alpha = -Z_{0.01} = -2.325$

That is equivalent to $\dfrac{\bar{x}_0 - \mu_0}{\frac{\sigma}{\sqrt{n}}} = -2.325 \Rightarrow \bar{x}_0 = 70.8$

Type II error: Accept H_0 given H_a is true

assume the true mean $\mu_a = 72 < 75$

① $\beta(72) = P(\text{Accept } H_0 \mid \mu_a = 72)$

$= P(\bar{x} > 70.8 \mid \bar{x} \sim N(72, 1.8^2))$

$= P\left(Z > \dfrac{70.8 - 72}{1.8}\right) = P(Z > -0.67) = 0.7486$

μ_a is further from μ_0 then $\beta(\mu_a) \downarrow$

② $\beta(70) = P(\bar{x} > 70.8 \mid \bar{x} \sim N(70, 1.8^2))$

$= P\left(Z > \dfrac{70.8 - 70}{1.8}\right) = P(Z > 0.44) = 0.33$

③ $\beta(67) = P(\bar{x} > 70.8 \mid \bar{x} \sim N(67, 1.8^2))$

$= P(Z > 2.11) = 0.174$

Alternative Hypothesis	Type II Error Probability $\beta(\mu')$ for a Level α Test
H_a: $\mu > \mu_0$	$\Phi\left(z_\alpha + \dfrac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right)$
H_a: $\mu < \mu_0$	$1 - \Phi\left(-z_\alpha + \dfrac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right)$
H_a: $\mu \neq \mu_0$	$\Phi\left(z_{\alpha/2} + \dfrac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right) - \Phi\left(-z_{\alpha/2} + \dfrac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right)$

where $\Phi(z) =$ the standard normal cdf.

The sample size n for which a level α test also has $\beta(\mu') = \beta$ at the alternative value μ' is

$$n = \begin{cases} \left[\dfrac{\sigma(z_\alpha + z_\beta)}{\mu_0 - \mu'}\right]^2 & \text{for a one-tailed (upper or lower) test} \\ \left[\dfrac{\sigma(z_{\alpha/2} + z_\beta)}{\mu_0 - \mu'}\right]^2 & \text{for a two-tailed test (an approximate solution)} \end{cases}$$

Chapter 9 Stuff

Comparing $\mu_1 - \mu_2$

The properties of $\bar{X}_1 - \bar{X}_2$: difference of sample

1. $E(\bar{x}_1 - \bar{x}_2) = E(\bar{x}_1) - E(\bar{x}_2) = \mu_1 - \mu_2$ unbiased

2. $V(\bar{x}_1 - \bar{x}_2)$: $\bar{x}_1$ and $\bar{x}_2$ are independant $V(\bar{x}_1) + V(\bar{x}_2) = \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{n}$

   $\bar{x}_1$ and $\bar{x}_2$ independant because 2 samples are independant

   $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

3. $\bar{x}_1$, $\bar{x}_2$ both normal $\longrightarrow$ $\bar{x}_1 - \bar{x}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$

   $$\frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

4. $$P\left(-Z_{\frac{\alpha}{2}} \leq \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \leq Z_{\frac{\alpha}{2}}\right) = 1 - \alpha$$

   $$P\left(\bar{x}_1 - \bar{x}_2 - Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \underbrace{\mu_1 - \mu_2}_{\text{(population) mean difference}} \leq (\bar{x}_1 - \bar{x}_2) + Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right) = 1 - \alpha$$

The hypothesis:

$H_0 : \mu_1 - \mu_2 = c.$

Population	Sample	σ^2	Equal σ^2	Test statistic	CI
-	Large	Known	-	$\frac{(\bar{X}_1 - \bar{X}_2) - c}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$	$\left[(\bar{x}_1 - \bar{x}_2) \pm Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right]$
Normal	-				
-	Large	Unknown	-	$\frac{(\bar{X}_1 - \bar{X}_2) - c}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim N(0,1)$	$\left[(\bar{x}_1 - \bar{x}_2) \pm Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right]$
Normal	Small	Unknown	Equal	$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$ $\frac{(\bar{X}_1 - \bar{X}_2) - c}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1 + n_2 - 2)$	$\left[(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}^{(n_1+n_2-2)} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right]$
			Not Equal	$v = \left[\frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}}\right]$ $\frac{(\bar{X}_1 - \bar{X}_2) - c}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim t(v)$	$\left[(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}^{(v)}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right]$

The hypothesis:

(population)
mean
difference

$H_0 : \mu_1 - \mu_2 = c.$

$H_a : \mu_1 - \mu_2 > c$ **or** $H_a : \mu_1 - \mu_2 < c$ **or** $H_a : \mu_1 - \mu_2 \neq c$

TS: $z = \frac{\bar{X}_1 - \bar{X}_2 - c}{\sqrt{\sigma^2/n_1 + \sigma^2/n_2}}$

RR:

	$H_a : \mu_1 - \mu_2 > c$	$H_a : \mu_1 - \mu_2 < c$	$H_a : \mu_1 - \mu_2 \neq c$
RR	(z_α, ∞)	$(-\infty, -z_\alpha)$	$(-\infty, -z_{\frac{\alpha}{2}}) \cup (z_{\frac{\alpha}{2}}, +\infty)$

We reject H_0 when TS z lies in the RR, or P-value is less than or equal to α.

$\underbrace{\lfloor \quad n_1-1 \quad , \quad n_2-1 \quad \rfloor}$ Floor / round down

The hypothesis:

$H_0 : \mu_1 - \mu_2 = c.$

$H_a : \mu_1 - \mu_2 > c$ **or** $H_a : \mu_1 - \mu_2 < c$ **or** $H_a : \mu_1 - \mu_2 \neq c$

TS:

$$T = \frac{\bar{X}_1 - \bar{X}_2 - c}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} \sim t(v).$$

RR:

	$H_a : \mu_1 - \mu_2 > c$	$H_a : \mu_1 - \mu_2 < c$	$H_a : \mu_1 - \mu_2 \neq c$
RR	$(t_\alpha(v), +\infty)$	$(-\infty, -t_\alpha(v))$	$(-\infty, -t_{\frac{\alpha}{2}}(v)) \cup (t_{\frac{\alpha}{2}}(v), +\infty)$