

1) **Algorithm** initResult(G)

Input: graph G

Output: create new empty sequence

$S \leftarrow$ create new sequence

Algorithm preComponentVisit(G, v)

Input: graph G and vertex v

Output: add vertex v to a global sequence

s.insertLast(v)

Algorithm result(G)

Input: graph G

Output: sequence S containing a vertex from each connected component

Return s

2) a)

Algorithm BFS(G)

Input graph G

Output labeling of the edges and partition of the vertices of G

initResult(G)

for all $u \leftarrow G.vertices()$

 setLabel(u, UNEXPLORED)

for all $e \leftarrow G.edges()$

 setLabel(e, UNEXPLORED)

for all $v \leftarrow G.vertices()$

 if getLabel(v) \leftarrow UNEXPLORED

preComponentVisit(G, v)

 BFS(G, v)

postComponentVisit(G, v)

result(G)

Algorithm BFS(G, s)

$L_0 \leftarrow$ new empty sequence

$L_0.insertLast(s)$

setLabel(s, VISITED)

$i \leftarrow 0$

while not $L_i.isEmpty()$

$L_{i+1} \leftarrow$ new empty sequence

 for all $v \in L_i.elements()$

preVertexVisit(G, v)

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    for all  $e \in G.\text{incidentEdges}(v)$ 
        preEdgeVisit( $G, v, e$ )
        if  $\text{getLabel}(e) = \text{UNEXPLORED}$ 
             $w \leftarrow \text{opposite}(v, e)$ 
            if  $\text{getLabel}(w) = \text{UNEXPLORED}$ 
                preDiscoveryEdgeVisit ( $G, v, e, w$ )
                 $\text{setLabel}(e, \text{DISCOVERY})$ 
                 $\text{setLabel}(w, \text{VISITED})$ 
                 $L_{i+1}.\text{insertLast}(w)$ 
                postDiscoveryEdgeVisit ( $G, v, e, w$ )
            else
                 $\text{setLabel}(e, \text{CROSS})$ 
                crossEdgeVisit( $G, v, e, w$ )
        postEdgeVisit( $G, v, e$ )
    postVertexVisit( $G, v$ )
     $i \leftarrow i+1$ 

```

b)

Algorithm $\text{initResult}(G)$

$S \leftarrow \text{create new stack}$

Algorithm $\text{preDiscoveryEdgeVisit}(G, v, e, w)$

$w.\text{setParent}(v)$

$w.\text{setEdge}(e)$

if $w = z$ then

$\text{tracePath}(w)$

Algorithm $\text{tracePath}(v)$

$s.\text{push}(v)$

if $v.\text{parent}() \neq \text{null}$ then

$s.\text{push}(v.\text{edge}())$

$\text{tracePath}(v.\text{getParent}())$

Algorithm $\text{result}(G)$

return $s.\text{elements}()$

c)

Algorithm initResult(G)

S \leftarrow create new stack

Algorithm preDiscoveryEdgeVisit(G, v, e, w)

If v.getLevel() = null then

 currentLevel \leftarrow 0

else

 currentLevel \leftarrow v.getLevel()

w.setLevel (currentLevel)

w.setParent(v)

w.setEdge(e)

Algorithm crossEdgeVisit(G, v, e, w)

j \leftarrow v.getLevel()

k \leftarrow w.getLevel()

If j > k then

 While j>k do

 s.insertLast(v)

 s.insertLast(v.getEdge())

 v \leftarrow v.getParent()

 j \leftarrow j-1

else if k > j then

 while k>j do

 S.insertLast(w)

 S.insertLast(w.getEdge())

 w \leftarrow w.getParent()

 k \leftarrow k-1

t \leftarrow create new stack

while v.getParent() \neq w.getParent() do

 S.insertLast(v)

 S.insertLast(v.getEdge())

 v \leftarrow v.getParent()

 t.push(w)

 t.push(w.getEdge)

while \neg t.isEmpty() do

 S.insertLast(t.pop())

Algorithm result(G)

return S.elements()

d) No, because DFS goes to the deepest level before it backtracks. As a result, It won't necessarily find the shortest path.

3)

Algorithm DijkstraDistances(G, s)

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Q  $\leftarrow$  new heap-based priority queue
initResult( $G, s$ )
for all  $v$  in  $G.vertices()$ 
    if  $v = s$  then
        setDistance( $v, 0$ )
    else
        setDistance( $v, \text{inf}$ )
     $l \leftarrow Q.insert(\text{getDistance}(v), v)$ 
    setLocator( $v, l$ )
while not  $Q.isEmpty()$ 
     $u \leftarrow Q.removeMin()$ 
    for all  $e$  in  $G.incidentEdges(u)$ 
        { relax edge  $e$  }
         $z \leftarrow G.opposite(u, e)$ 
        preEdgeRelax( $G, u, z, e$ )
         $r \leftarrow \text{getDistance}(u) + \text{weight}(e)$ 
        if  $r < \text{getDistance}(z)$ 
            preDistanceUpdate( $G, u, z, e, r$ )
            setDistance( $z, r$ )
             $Q.replaceKey(\text{getLocator}(z), r)$ 
            postDistanceUpdate( $G, u, z, e, r$ )
        postEdgeRelax( $G, u, z, e$ )

Result( $G, s$ )
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Algorithm shortestPath(G, u, v)

Input: graph G , vertex u and vertex v
Output: shortest path between u and v
 $target \leftarrow v$
DijkstraDistances (G, u)

Algorithm preDistanceUpdate(G, u, z, e, r)

$z.setEdge(e)$

Algorithm Result(G, s)

```
T  $\leftarrow$  create new sequence
While  $target.getEdge() \neq \text{null}$  do
    T.insertLast(target)
    T.insertLast(target.getEdge())
     $target \leftarrow G.getOpposite(target, target.getEdge())$ 

result  $\leftarrow T.getElements()$ 
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4)

Algorithm initResult(G)

 componentIndex \leftarrow 0

Algorithm preComponentVisit(G,v)

 componentIndex \leftarrow componentIndex +1

Algorithm startVertexVisit(v)

 v.setLabel(componentIndex)