Life Tables and Survival Analysis

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Definition

- In actuarial science and demography, a life table (also called a mortality table or actuarial table) is a table which shows, for each age, what the probability is that a person of that age will die before their next birthday ("probability of death").
- In other words, it represents the survivorship of people from a certain population.
- This is a broad number but a pretty indicator of a populations longevity. It should go up or down slowly unless something dramatic happens… COVID, toxic drug overdoses.

https://en.wikipedia.org/wiki/Life_table

Key Points

- To predict if a population will grow or shrink, ecologists need to know birth and death rates for organisms at different ages as well as the current age and sex makeup of the population.
- **Life tables** summarize birth and death rates for organisms at different stages of their lives.
- **Survivorship curves** are graphs that show what fraction of a population survives from one age to the next.
- An **age-sex pyramid** is a "snapshot" of a population in time showing how its members are distributed among age and sex categories.

https://www.khanacademy.org/science/biology/ecology/population-ecology/a/life-tables-survivorship-age-sex-structure

Life Tables

- Scary because lots of notation.
- Basic life tables are relatively simple… but everything gets complicated

Table 3.1.2: Abridged Life Table for the Total United States Population, 1997

(1) Age Interval $(x, x+n)$	(2) ${}_nq_x$	(3) l_x	(4) ${}_nd_x$	(5) ${}_nL_x$	(6) T_x	(7) e_x^0
< 1	0.00723	100000	723	99371	7650789	76.5
1-4	.00144	99277	143	396774	7551418	76.1
5-9	.00092	99135	91	495432	7154644	72.2
10-14	.00116	99043	115	494997	6659212	67.2
15-19	.00374	98929	370	493801	6164215	62.3
20-24	.00492	98558	485	491596	5670414	57.5
25-29	.00509	98073	499	489137	5178818	52.8
30-34	.00630	97574	615	486397	4689680	48.1

https://www.measureevaluation.org/resources/training/online-courses-and-resources/non-certificate-courses-and-mini-tutorials/multiple-decrement-life-tables/lesson-3.html

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Life table notation and definitions

Notation	Definition
$(x, x+n)$	Age interval or period of life between two exact ages stated in years
l_x	Of the starting number of newborns in the life table (called the radix of the life table, usually set at 100,000) the number living at the beginning of the age interval (or the number surviving to the beginning of the age interval)
${}_nd_x$	The number of persons in the cohort who die in the age interval $(x, x+n)$
${}_nq_x$	Proportion of persons alive at the beginning of the age interval who die during the age interval

https://www.measureevaluation.org/resources/training/online-courses-and-resources/non-certificate-courses-and-mini-tutorials/multiple-decrement-life-tables/lesson-3.html

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Ordinary life table

Age interval in years x, x+ n	Number surviving at beginning of age interval out of 100,000 born l_x	The number of persons in the cohort who die in the age interval ${}_nd_x$	Proportion of persons alive at the beginning of the age interval who due during the age interval ${}_nq_x$
0-1	100000	5400	0.05 (100000/5400)
1-5	94600 (100000 – 5400)	1450	0.02 (94600/1450)
5-10	93150 (94600 – 1450)	1200	0.01
10-20	91950 …	1300	0.01
20-30	90650	1200	0.01

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Key Points

Notation	Definition
$(x, x+n)$	We set this too. Depends on how much data we have for difference age ranges.
l_x	We just set this to a value. Usually 100,000
${}_nd_x$	This is the key thing you need to get right. If your death data is wrong, the entire table is not very useful.
${}_nq_x$	This is just math based on l_x *and* ${}_nq_x$

https://www.measureevaluation.org/resources/training/online-courses-and-resources/non-certificate-courses-and-mini-tutorials/multiple-decrement-life-tables/lesson-3.html

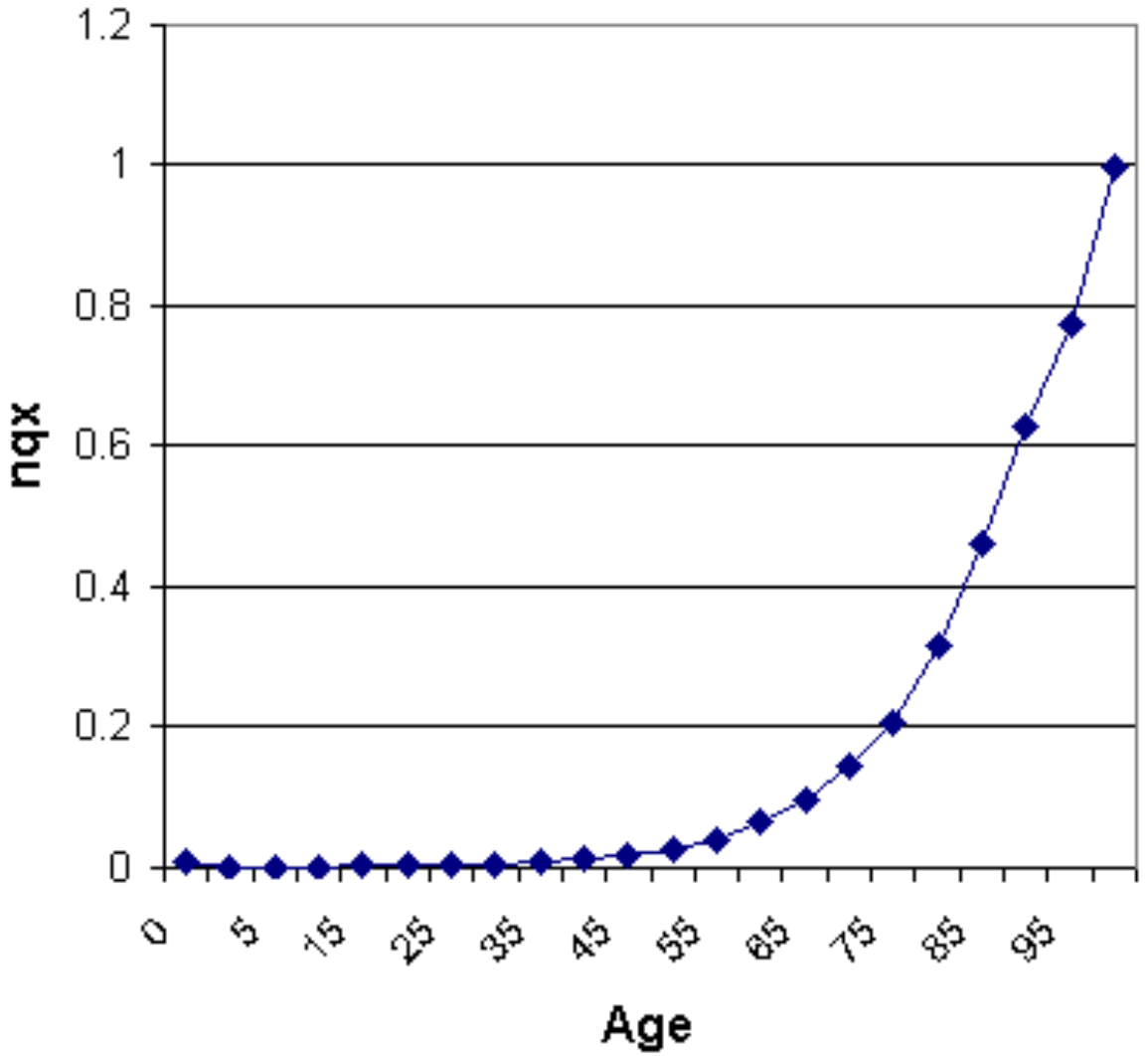
From the Ordinary life table

- We can derive useful figures from the ordinary life table based on
  - ${}_nq_x$ (ie. proportion of persons alive at the beginning of the age interval who die during the age interval)
  - l_x (ie. the number surviving to the beginning of the age interval)

https://www.measureevaluation.org/resources/training/online-courses-and-resources/non-certificate-courses-and-mini-tutorials/multiple-decrement-life-tables/lesson-3.html

${}_nq_x$ by age

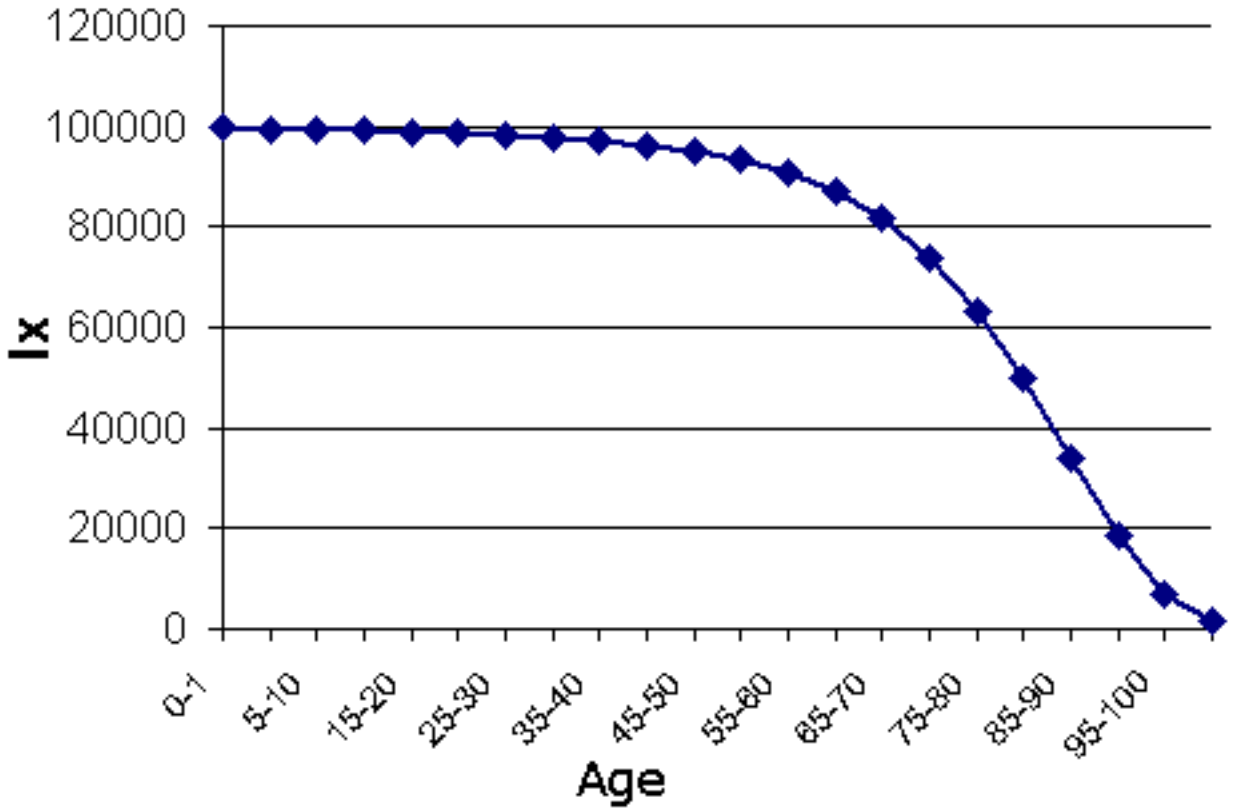
- Probability that a person of age x will die in the age interval



https://www.measureevaluation.org/resources/training/online-courses-and-resources/non-certificate-courses-and-mini-tutorials/multiple-decrement-life-tables/lesson-3.html

l_x by age

- Number of newborns surviving to a given age.
- Could also give this a proportion.



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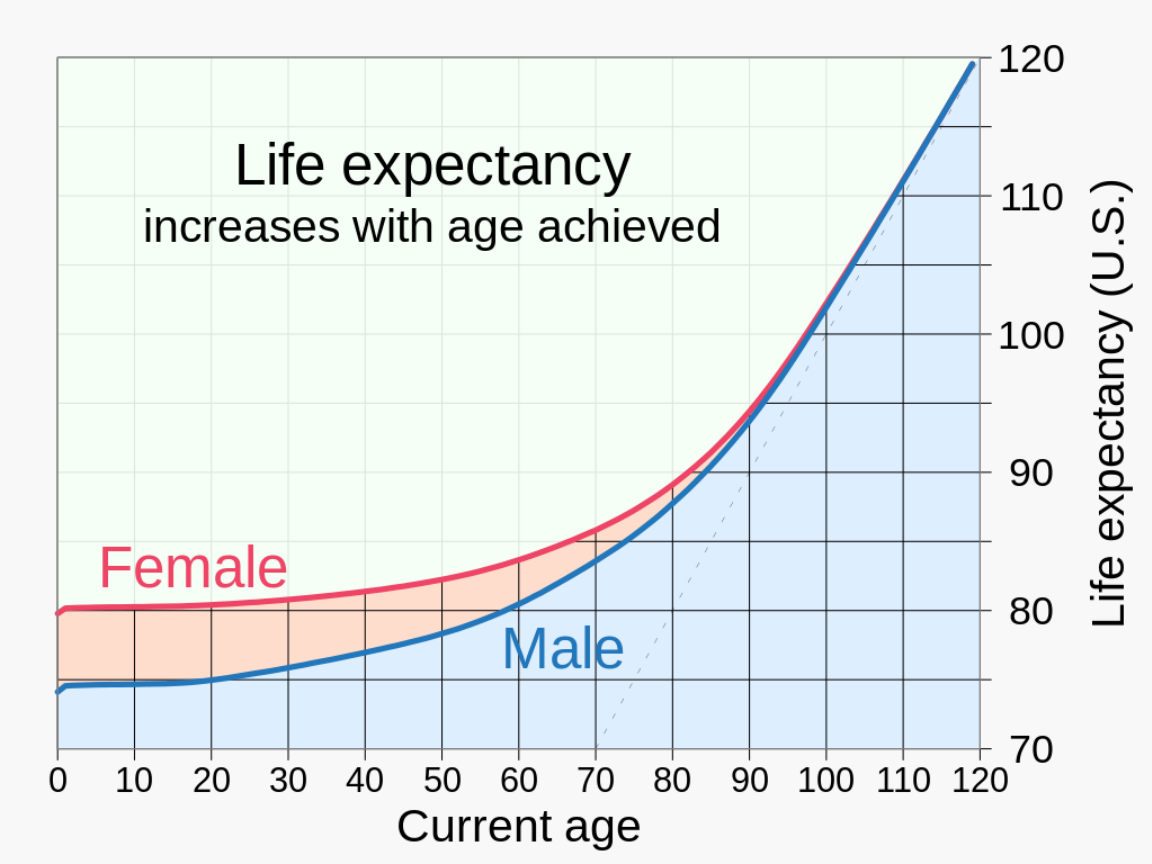
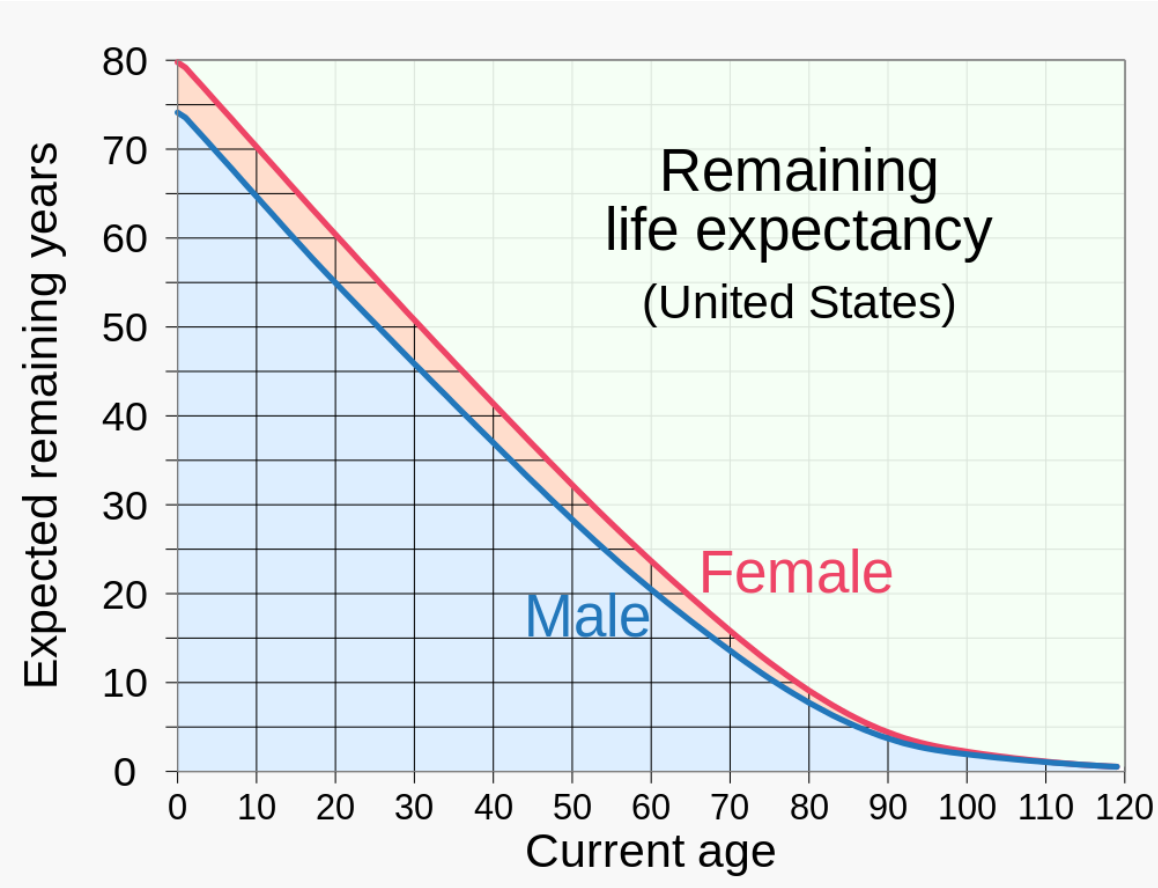
${}_nd_x$	The number of persons in the cohort who die in the age interval $(x, x+n)$
$\boldsymbol{{}_nL_x}$	**Number of years of life lived by the cohort within the indicated age interval $(x, x+n)$ (or person-years of life in the age interval)**
$\boldsymbol{T_x}$	**Total person-years of life contributed by the cohort after attaining age x**
$\boldsymbol{e_x^0}$	**Average number of years of life remaining for a person alive at the beginning of age interval x**

https://www.measureevaluation.org/resources/training/online-courses-and-resources/non-certificate-courses-and-mini-tutorials/multiple-decrement-life-tables/lesson-3.html





https://en.wikipedia.org/wiki/Life_table

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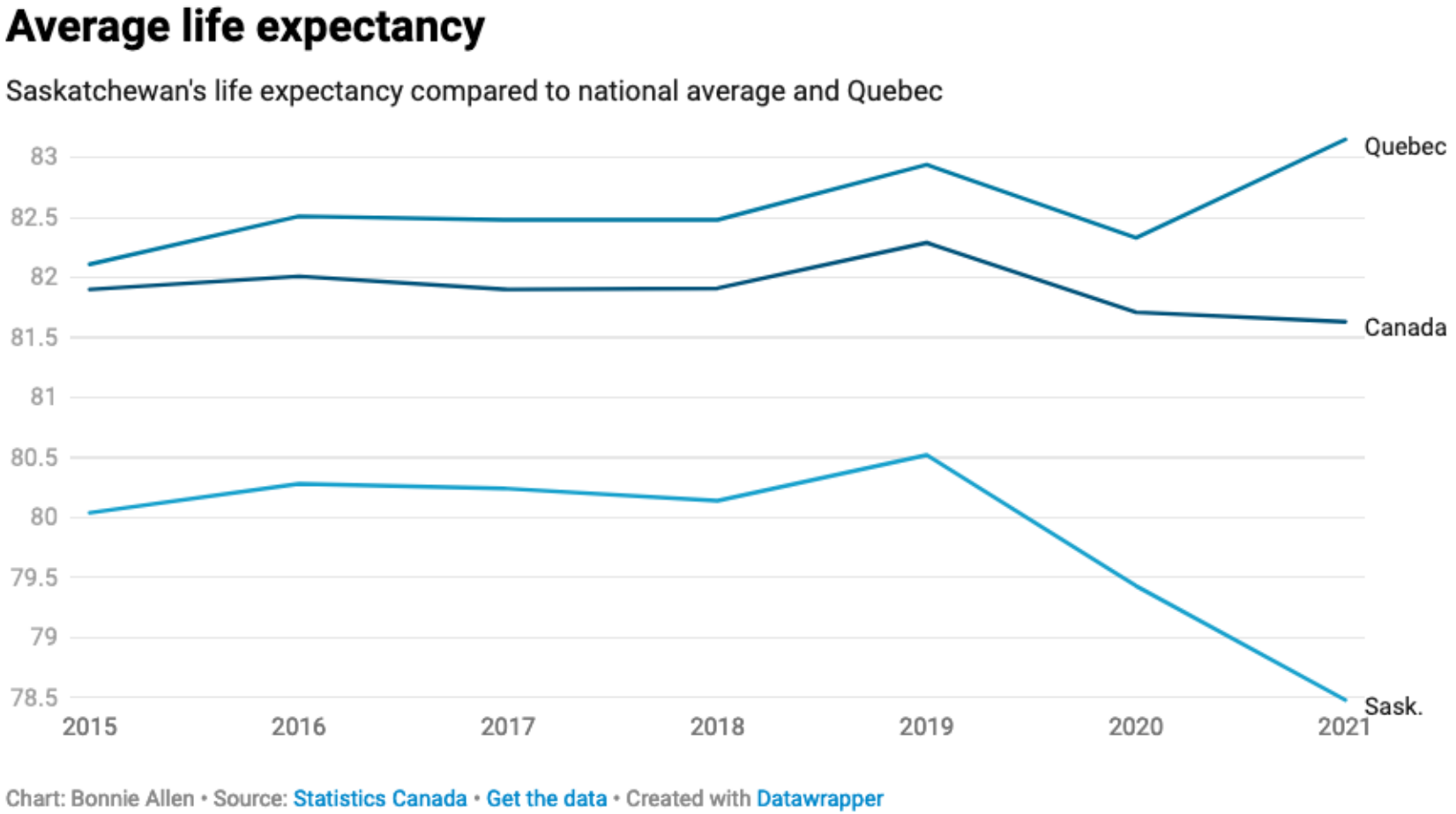
# Indicates larger trends

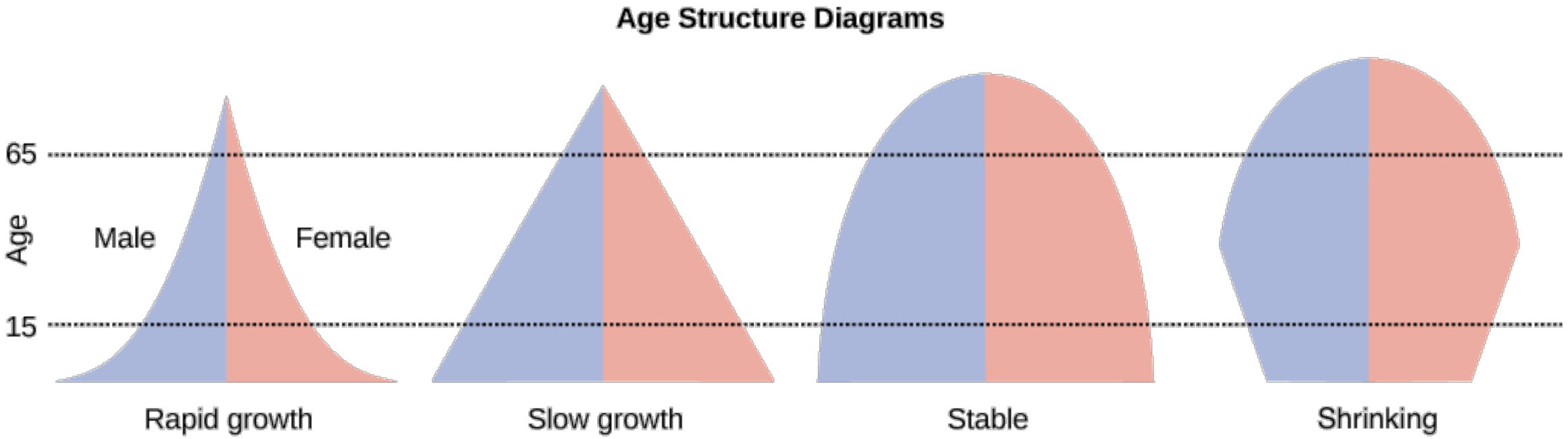


https://www.cbc.ca/news/canada/saskatchewan/drug-overdoses-covid-19-drive-sask-life-expectancy-to-lowest-level-in-22-years-1.6959685

Population pyramids

- A population pyramid (age structure diagram) or "age-sex pyramid" is a graphical illustration of the distribution of a population by age groups and sex
  a) typically takes the shape of a pyramid when the population is growing.
  b) Males are usually shown on the left and females on the right, and they may be measured in absolute numbers or as a percentage of the total population.
  c) The pyramid can be used to visualize the age of a particular population.
  d) It is also used in ecology to determine the overall age distribution of a population; an indication of the reproductive capabilities and likelihood of the continuation of a species.

https://en.wikipedia.org/wiki/Population_pyramid



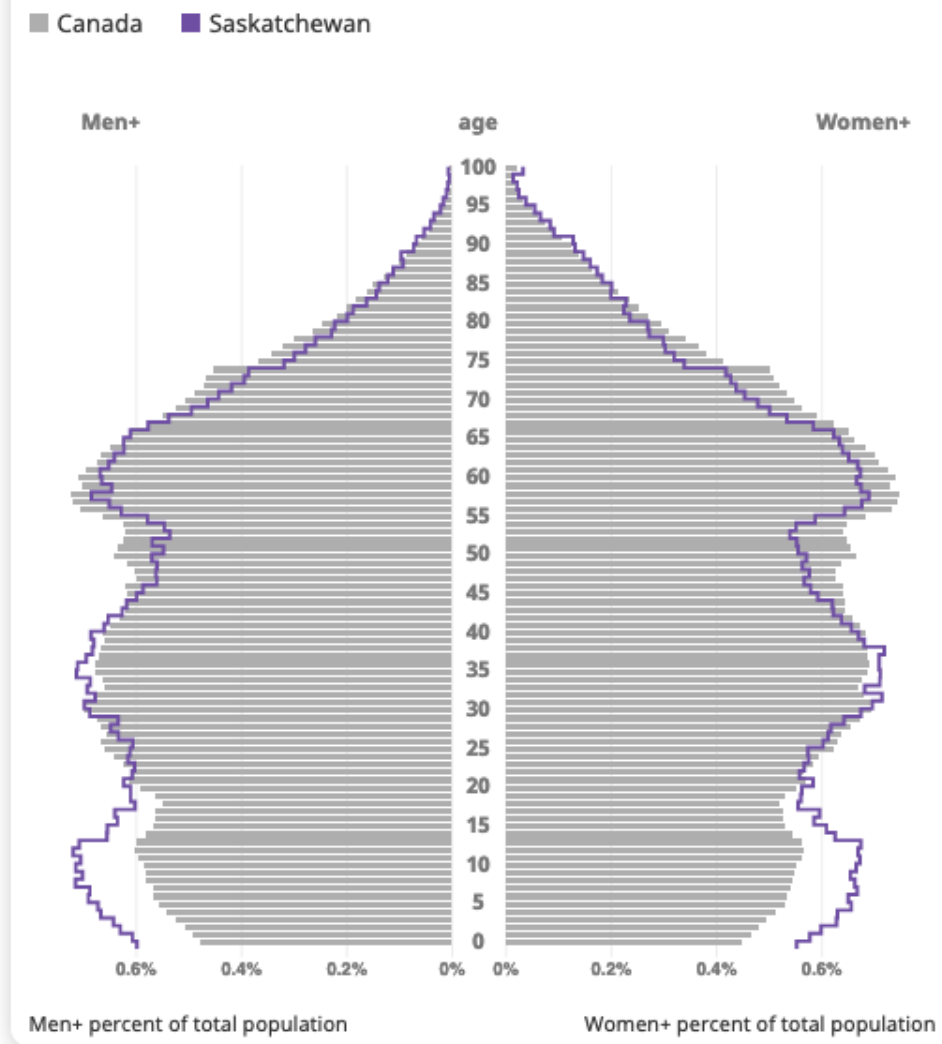
https://www.khanacademy.org/science/biology/ecology/population-ecology/a/life-tables-survivorship-age-sex-structure

Geography 1 Canada

Geography 2 Saskatchewan

Change visualization Comparison | Historical

Canada vs Saskatchewan—2021



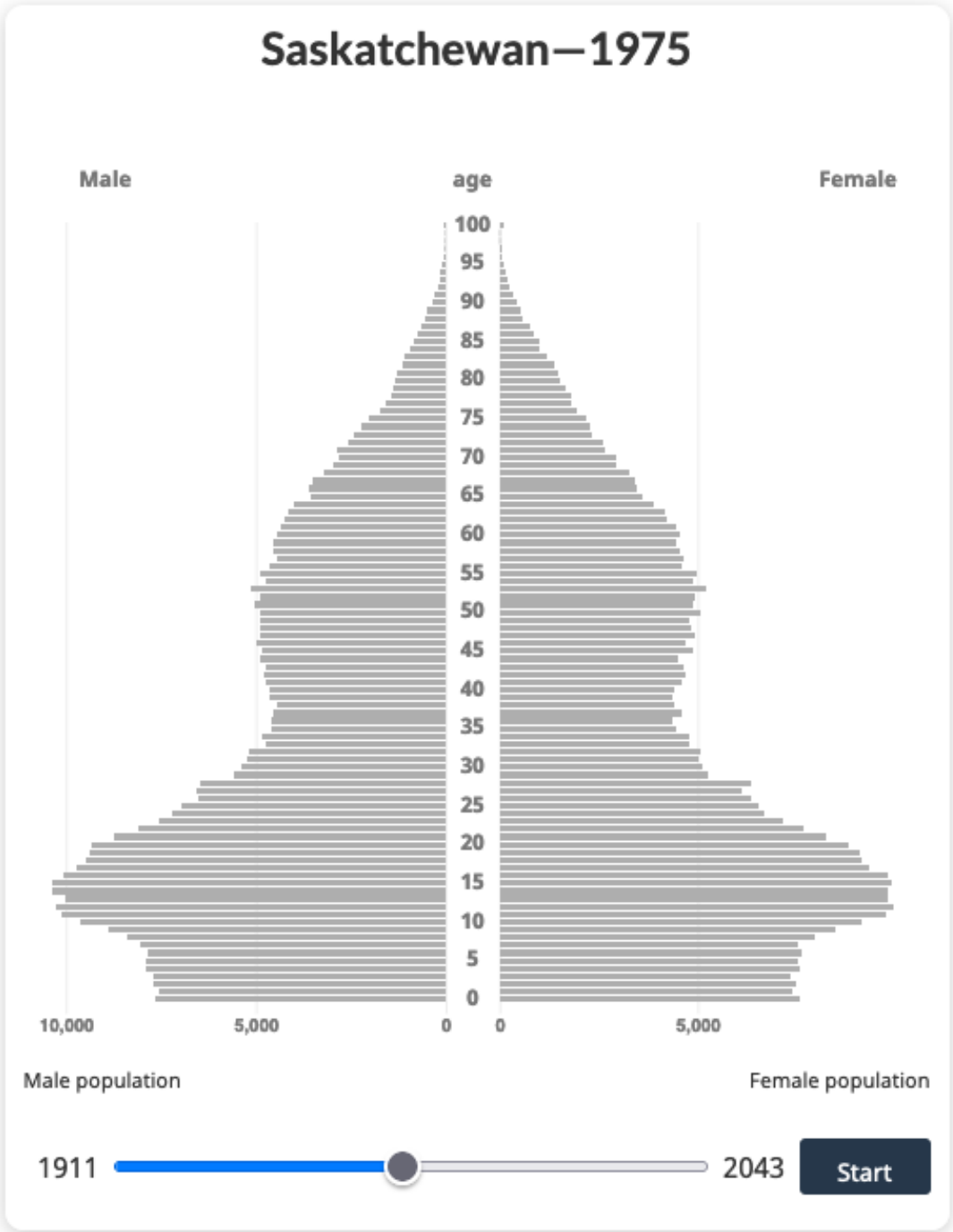
Number (proportion)—Age Total

	Canada	Saskatchewan
Men+	18,226,240 (49.3%)	563,120 (49.7%)
Women+	18,765,745 (50.7%)	569,385 (50.3%)
Total	36,991,985	1,132,505

Median age (Average age)

	Canada	Saskatchewan
Men+	40.4 (41.0)	38.0 (38.9)
Women+	42.8 (42.8)	39.6 (40.7)
Total	41.6 (41.9)	38.8 (39.8)

https://www12.statcan.gc.ca/census-recensement/2021/dp-pd/dv-vd/pyramid/index-en.htm


Saskatchewan—1975
Male
age
Female
100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5
0
10,000
5,000
0
0
5,000
Male population
Female population
1911
2043
Start


Saskatchewan—2023
Male
age
Female
100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5
0
8,000
6,000
4,000
2,000
0
0
2,000
4,000
6,000
8,000
Male population
Female population
1911
2043
Start


Saskatchewan—2023
Male
age
Female
100
95
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85
80
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25
20
15
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8,000
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2,000
4,000
6,000
8,000
Male population
Female population
1911
2043
Start


Saskatchewan—2043
Male
age
Female
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25
20
15
10
5
0
10,000
5,000
0
0
5,000
10,000
Male population
Female population
1911
2043
Start

Regression model life tables

- If life tables are not available for a particular country, use model life tables to obtain survival rates.
- Model life tables are derived from life tables and mortality experiences of a number of countries. They are primarily used to assist countries that do not have vital statistics systems to record deaths.

Survival Analysis

- Survival analysis is a branch of statistics for analyzing the expected duration of time until one event occurs, such as death in biological organisms and failure in mechanical systems.
- In survival analysis, we use information on event status and follow up time to estimate a survival function.
- Survival analysis attempts to answer certain questions
  a) What is the proportion of a population which will survive past a certain time?
  b) Of those that survive, at what rate will they die or fail?
  c) Can multiple causes of death or failure be taken into account?
  d) How do particular circumstances or characteristics increase or decrease the probability of survival?

Survival Analysis

- There are unique features of time to event variables.
  a) Time to event are always positive and their distributions are often skewed.
  b) Participants are enrolled over a period of time and the study ends on a specific calendar date. Participants who enroll later are followed for a shorter period than participants who enroll early.
  c) Some participants may drop out of the study before the end of the follow-up period (e.g., move away, become disinterested) and others may die during the follow-up period (assuming the outcome of interest is not death).
  d) True survival time (sometimes called failure time) is not known because the study ends or because a participant drops out of the study before experiencing the event.

Censoring

- There are several different types of censoring.
- Right censoring
  a) We don't know when or if the event happens but, if it does, we know that it is after a specific time point.
- Left censoring
  a) We know only that an event happened before a specific time point.
  b) For example, in a study of the time between HIV infection and onset of AIDS, we do not know the time of infection—only that it occurred before the positive HIV test.

https://www.nature.com/articles/s41592-022-01563-7

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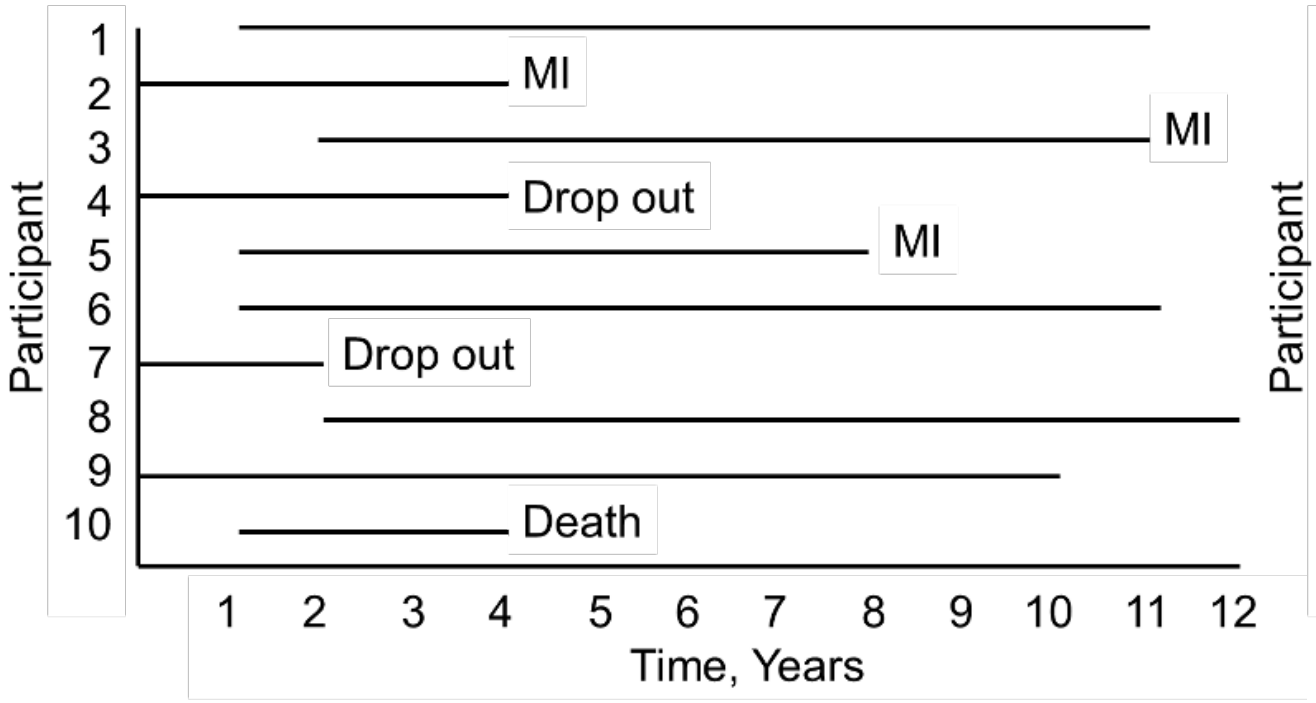
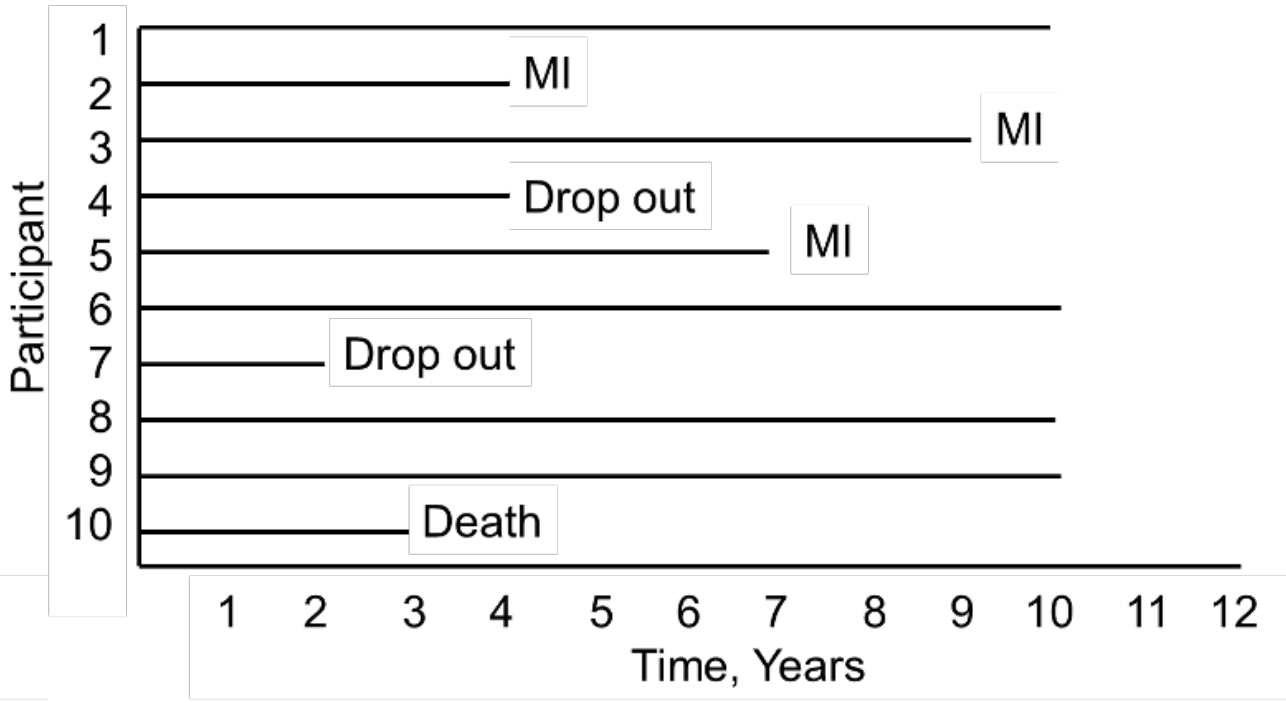
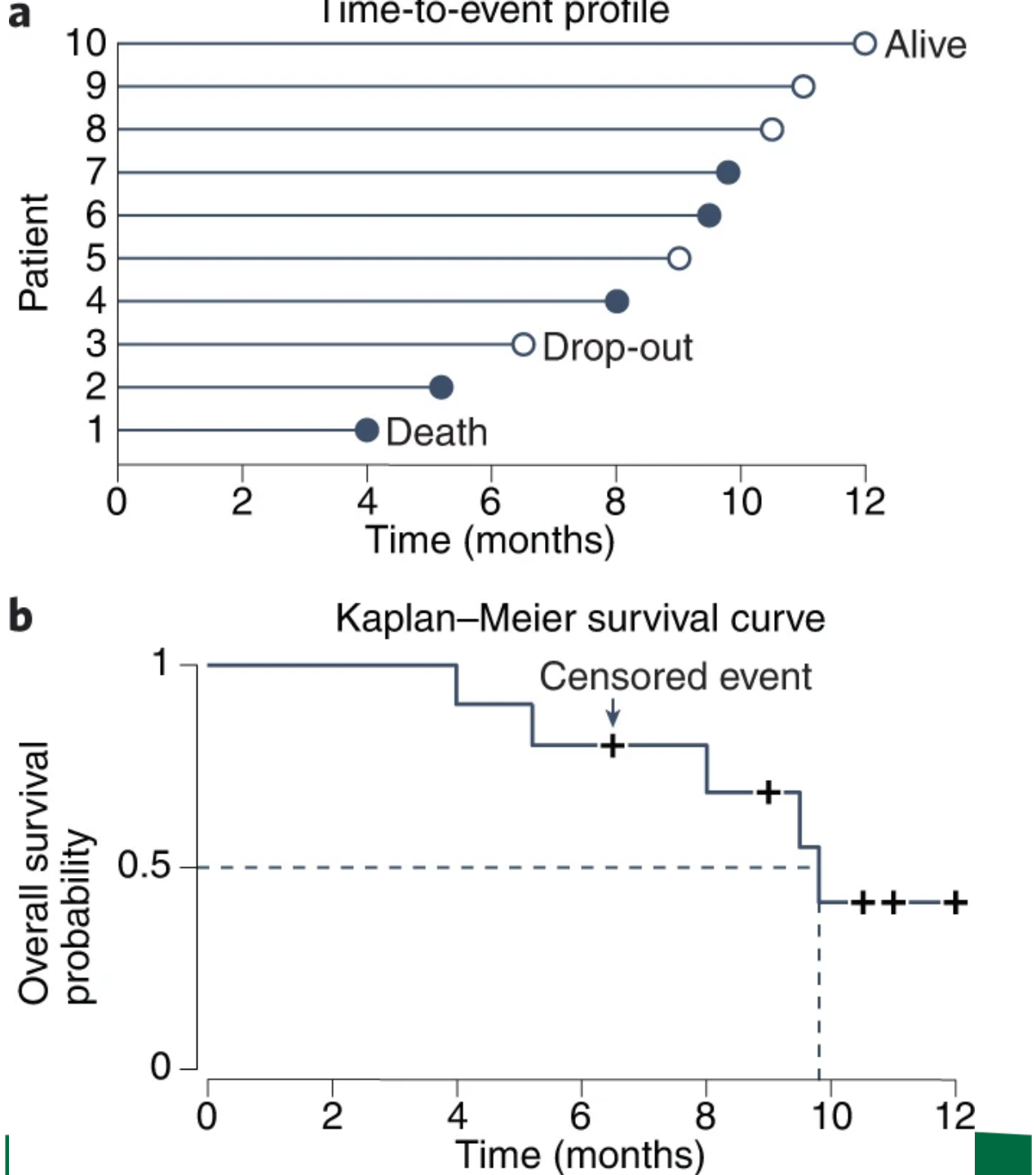
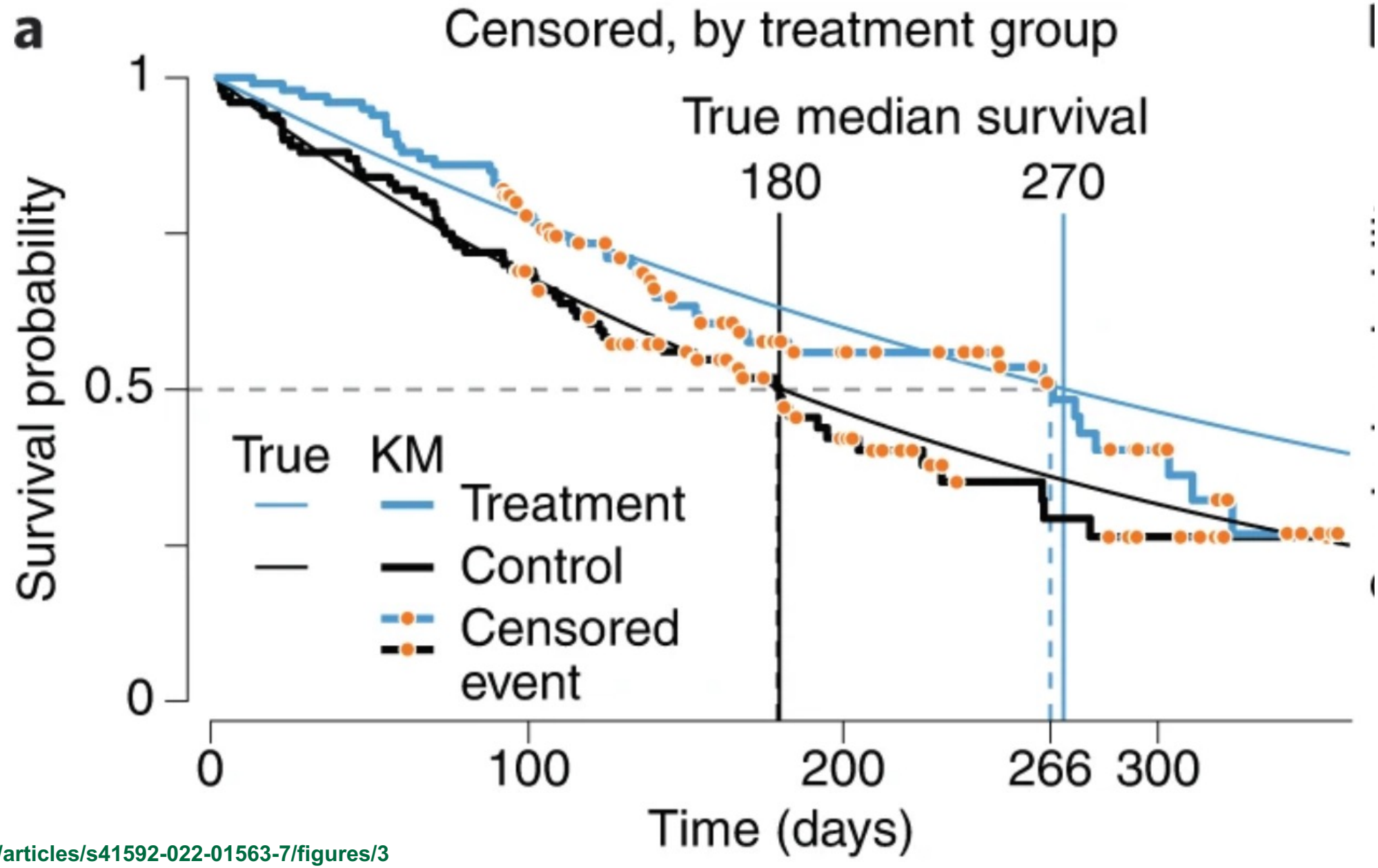
Censoring






a
Time-to-event profile
Patient
10
9
8
7
6
5
4
3
2
1
Alive
Drop-out
Death
0
2
4
6
8
10
12
Time (months)
b
Kaplan–Meier survival curve
Censored event
Overall survival probability
1
0.5
0
0
2
4
6
8
10
12
Time (months)

From: Survival analysis—time-to-event data and censoring



https://www.nature.com/articles/s41592-022-01563-7/figures/3

Creating survival functions

- In survival analysis, we use information on event status and follow up time to estimate a survival function.
  1. Life Table
  2. Kaplan-Meier (Product Limit) Approach
  3. Log rank test (compare 2 curves)
  4. Regression based approaches (compare 2+ curves)
     - Cox Proportional Hazards Regression Analysis
     - Parametric survival models
     - Survival trees
     - Survival random forests

University of Saskatchewan

Canadian Data

- Life Tables
  a) Tables (https://www150.statcan.gc.ca/n1/pub/84-537-x/84-537-x2023002-eng.htm)
  b) Methods (https://www150.statcan.gc.ca/n1/pub/84-538-x/84-538-x2023001-eng.htm)
- Population Pyramids
  a) https://www12.statcan.gc.ca/census-recensement/2021/dp-pd/dv-vd/pyramid/index-en.htm

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Ressources

- https://www.measureevaluation.org/resources/training/online-courses-and-resources/non-certificate-courses-and-mini-tutorials/multiple-decrement-life-tables/lesson-3.html
- https://www.measureevaluation.org/resources/training/online-courses-and-resources/non-certificate-courses-and-mini-tutorials/population-analysis-for-planners/lesson-7/lesson-7-overview-of-life-tables-and-survival-rates
- https://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_Survival/BS704_Survival_print.html
- https://www.khanacademy.org/science/biology/ecology/population-ecology/a/life-tables-survivorship-age-sex-structure
- https://academic.oup.com/book/27487/chapter/197435699
- https://en.wikipedia.org/wiki/Survival_analysis