Lecture 8 Risk and Return

- In real world, most capital budgeting projects involve risky cash flows, so a different discount rate must be used.

Now we will determine the discount rate of the risky projects.

- Risk and return: the relationship between expected return and risk for portfolios and individual assets.

When capital markets are in equilibrium, they determine a trade off between expected return and risk.

1 Two Statistics

1.1 Return Statistics

- The average return is the first and most natural number describing the past annual returns on the stock market. It tells us the best estimate of the return that an investor could have realized over a particular time period.

- The formula for the average return is:

$$Mean = \overline{R} = \frac{(R_1 + R_2 + ... + R_T)}{T}$$

1.2 Risk Statistics

- The second number that we use to characterize the distribution of returns is a measure of risk in returns.
- The variance and its square root, the standard deviation, are the most common measures of variability or dispersion. It measures how spread out the frequency distribution is.

- We use *Var* and σ^2 to denote the variance, and *SD* and σ to denote the standard deviation.

$$Var = \frac{1}{T-1} \times [(R_1 - \overline{R})^2 + (R_2 - \overline{R})^2 + ... + (R_T - \overline{R})^2]$$

For a portfolio of n stocks:

$$E(R_P) = w_1 r_1 + w_2 r_2 + \cdots + w_n r_n$$
$$= \sum_{i=1}^{n} w_i r_i$$

$$Var(R_P) = \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i \, w_j \sigma_{ij}$$

Note w_i may be negative. Portfolio weights sum to 1.

2 The Historical Record

The study was conducted by *Roger Ibbotson and Rex Sinquefield.* They present year-by-year historical rates of return for the following five important types of financial instruments:

A Large-Company Common Stocks.

B Small-Company Common Stocks.

C Long-Term Corporate Bonds.

D Long-Term U.S. Government Bonds.

E U.S. Treasury Bills.

- Risk-free rate is the rate you can earn by leaving money in risk-free assets such as T-bills, or the bank. In the U.S. we usually use T-bills or Treasury bills as the risk free rate.

- The following Table shows the average stock return, bond return, T-bill return and inflation rate of the period from 1926 through 2014 in the U.S.

Total Annual Returns, U.S. Securities Markets, 1926-2014

Series	Arithmetic Mean (%)	Risk Premium (relative to U.S. Treasury bills) (%)	Standard Deviation (%)
Small-company stocks	16.7	13.2	32.1
Large-company stocks	12.1	8.6	20.1
Long-term corporate bonds	6.4	2.9	8.4
Long-term government bonds	6.1	2.6	10.0
Intermediate-term government bonds	5.4	1.9	5.6
U.S. Treasury bills	3.5		3.1
Inflation	3.0		4.1

- The difference between risky returns and risk free returns are called **the excess return on the risky asset.** It is called excess because it is the additional return resulting from the riskiness of common stocks and is interpreted as an **equity risk premium**.

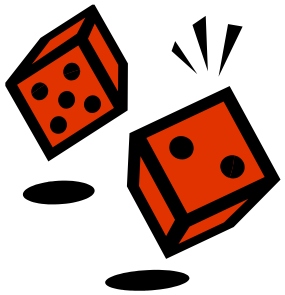
- Long run excess of the stock return over the risk-free return. An investor for this period was rewarded for investment in the stock market with an extra or excess return over that would have been achieved by simply investing in T-bills.

- Why was there such a reward?

- Risk compensation is the amount of deterministic consumption one would have to charge an agent in exchange for exposing him of a risk.

- In finance applications, it is always referred to as the **risk premium**.

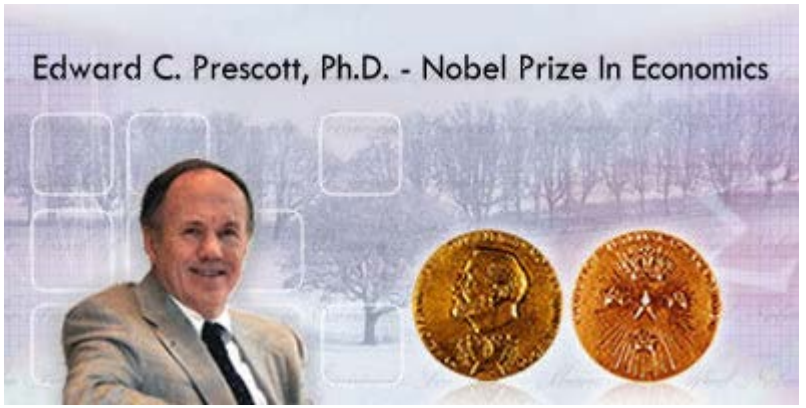
- Risk premium refers to the expected return on a security less the risk free return.



The degree to which investors are willing to commit funds to stocks depends on **risk aversion**.

Financial analysts generally assume investors are risk averse in the sense that, if the risk premium were zero, people would not be willing to invest any money in stocks.

In theory, then, there must always be a positive risk premium on stocks in order to induce risk averse investors to hold the existing supply of stocks instead of placing all their money in risk-free assets.



Mehra, Rajnish* and *Prescott, Edward, 1985

" The Equity Premium: A Puzzle"

Suppose that you are the only income earner in your family, and you have a good job guaranteed to give you your current (family) income every year for life.

You are given the opportunity to take a new and equally good job, with an even chance it will double your (lifetime family) income and an even chance it will cut your (lifetime family) income.

Indicate exactly what the percentage cut X would be that would leave you indifferent between keeping your current job or taking the new job and facing a 50-50 chance of doubling your income or cutting it by X percent.

3 The Capital Asset Pricing Model (CAPM)

The model gives us a precise prediction of the relationship that we should observe between the risk of an asset and its expected return.

3.1 The Underlying Assumptions

(1) There are many investors, each with an endowment (wealth) that is small compared to the total endowment of all investors.

Investors are price-takers, in that they act as though security prices are unaffected by their own trades.

(2) Investments are limited to a universe of publicly traded financial assets, such as stocks and bonds, and to risk-free borrowing or lending arrangements.

(3) Investors pay no taxes on returns and no transaction costs on trades in securities.

(4) All investors are rational mean-variance optimizers.

(5) All investors analyze securities in the same way and share the same economic view of the world. It is also called **homogeneous expectations** (共同期望假设). It states that all investors have the same beliefs concerning returns, variances, and covariances.

3.2 Definition of Risk when Investors Hold the Market Portfolio

- The market portfolio is derived from the sum or the aggregation of the portfolios of all individual investors. Any investor will choose to hold a portfolio of risky assets in proportions that duplicate representation of the assets in the market portfolio.

- In practice, financial economists use a broad-based index such as the Standard & Poor's (S & P) 500 as a proxy for the market portfolio.

- The best measure of the risk of a security in a large portfolio is **the beta of the security** β.

- **Beta measures the responsiveness of a security to movements in the market portfolio**.

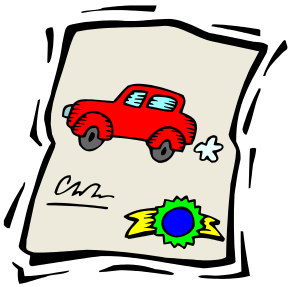
Example 1:

State	Type of Economy	Return on Market (percent)	Return on Security A (percent)
1	Bull	15	25
2	Bull	15	15
3	Bear	-5	-5
4	Bear	-5	-15

Type of Economy	Return on Market (percent)	Expected Return on Security A (percent)
Bull	15	$20\% = 25\% \times \frac{1}{2} + 15\% \times \frac{1}{2}$
Bear	-5	$-10\% = -5\% \times \frac{1}{2} + (-15\%) \times \frac{1}{2}$

The relationship appears in the following figure: **characteristic line of the security**（证券特征线）.

- The figure tells us that the return of security A is magnified 1.5 times over those of the market. When the market does well, security A's stock is expected to do even better. When the market does poorly, security A's stock is expected to do even worse.



- Then how about securities with negative betas?

These securities are acting as either hedges or insurance policies. The security is expected to do well when the market does poorly and vice versa.

The actual definition of beta is:

$$\beta_i = \frac{Cov(R_i, R_M)}{\sigma^2(R_M)}$$

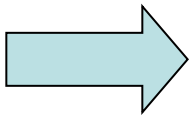
Where $Cov(R_i, R_M)$ is the covariance between the return on asset i and the return on the market portfolio and $\sigma^2(R_M)$ is the variance of the market.

Back to our example:

$$\sigma^2(R_M) = \frac{1}{4}(15-5)^2 + \frac{1}{4}(15-5)^2 + \frac{1}{4}(-5-5)^2 + \frac{1}{4}(-5-5)^2 = 100$$

$$Cov(R_A, R_M) = \frac{1}{4}(15-5)(25-5) + \frac{1}{4}(15-5)(15-5)$$

$$+\frac{1}{4}(-5-5)(-5-5) + \frac{1}{4}(-5-5)(-15-5) = 150$$



$$\beta_A = \frac{Cov(R_A, R_M)}{\sigma^2(R_M)} = \frac{150}{100} = 1.5$$

Another property is that the average beta across all securities, when weighted by the proportion of each security's market value to that of the market portfolio, is 1. That is

$$\sum_{i=1}^{N} X_i \beta_i = 1$$

Where X_i is the proportion of security i 's market value to that of the entire market and N is the number of securities in the market.

Which portfolio has the beta of 1?

Now think about the following two questions:

(1) What sort of investor rationally views the variance or standard deviation of an individual security's return as the security's proper measure of risk?

(2) What sort of investor rationally views the beta of a security as the security's proper measure of risk?

- A rational, risk-averse investor views the variance or standard deviation of her portfolio's return as the proper measure of the risk of her portfolio.

If the investor can hold only one security, the variance of that security's return becomes the variance of the portfolio's return. Hence, the variance of the security's return is the security's proper measure of risk.

- If an individual holds a diversified portfolio, she still views the variance or standard deviation of her portfolio's return as the proper measure of the risk of her portfolio.

  However, she is no longer interested in the variance of each individual security's return. Rather, she is interested in the contribution of an individual security to the variance of the portfolio.

Under the assumption of homogeneous expectations, all individuals hold the market portfolio.

Thus, we measure risk as the contribution of an individual security to the variance of the market portfolio. This contribution, when standardized properly, is the beta of the security.

3.3 The Capital Asset Pricing Model

First, we look at the expected return on market.

$$E(R_M) = R_F + Risk\ \text{Pr}emium$$

Note that the equation refers to the expected return on the market, not the actual return in a particular month or year.

- The study conducted by *Roger Ibbotson and Rex Sinquefield.*

- The average return on large company common stocks was 12.1 percent. The average risk free rate over the same time interval was 3.5 percent. Thus the average difference between the two was 8.6 percent. This is a useful estimate for the market risk premium in the future.

- *For example, if the risk free rate, right now estimated by the current yield on a one year T-bill, is 1 percent, the expected return on the market is _____.*

Second, we talk about the expected return on individual security.

The Capital Asset Pricing model (CAPM) articulates a relationship between any asset's expected return and its risk.

$$E(R_i) = R_F + \frac{\sigma_{i,m}}{\sigma_m^2} \times (E(R_M) - R_F)$$

The relationship between expected return and beta can be represented by the following equation.

$$E(R_i) = R_F + \beta_i \times (E(R_M) - R_F)$$

Expected return on a security = Risk free rate +

Beta of the security × Difference between expected return on market and risk free rate

- **Total risk**, which is $Var,$ is the risk that one bears by holding onto one security only.

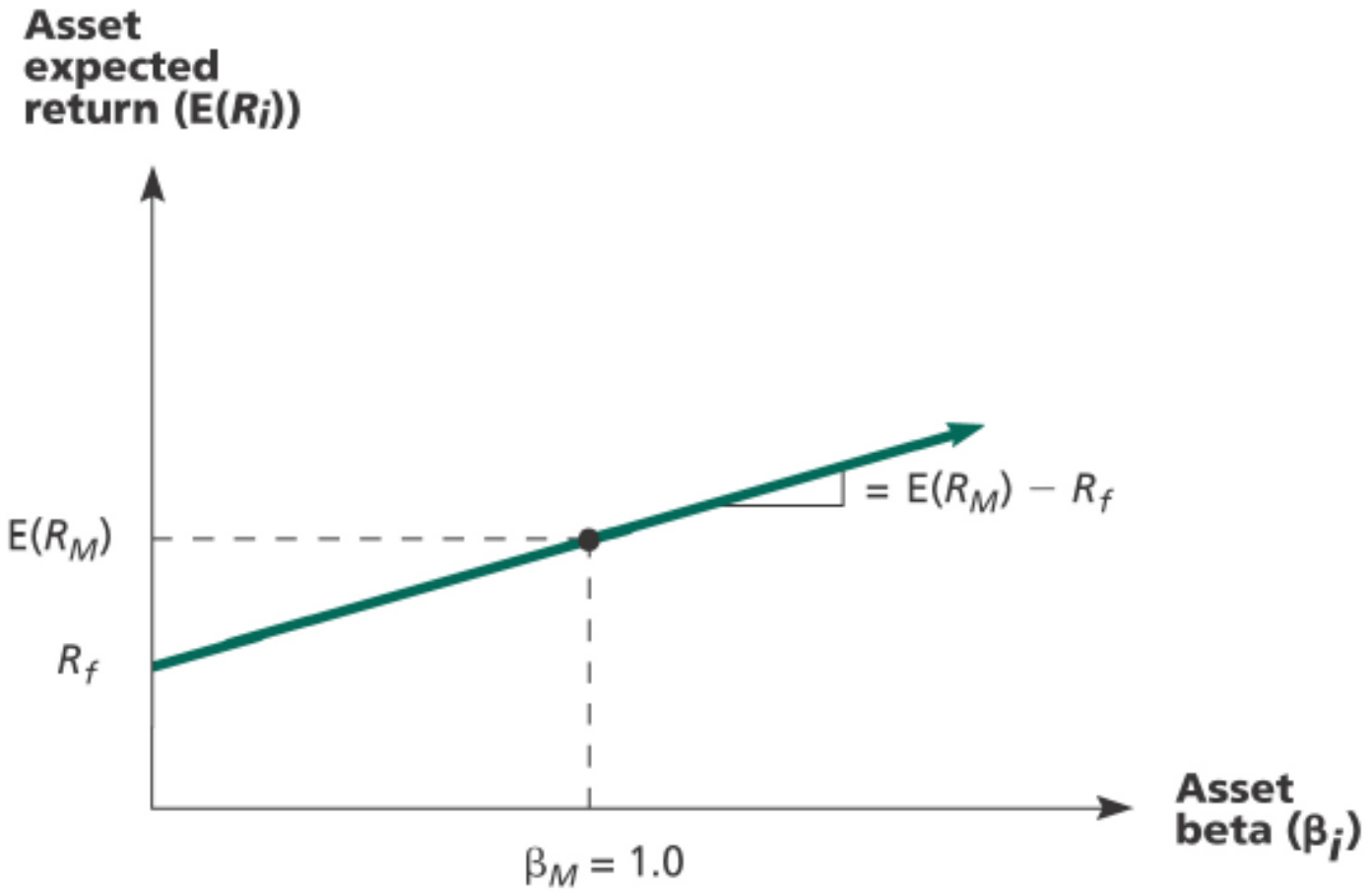
$$Variance = Total\ risk = Systematic + Unsystematic\ risk$$

- **Systematic risk** is the risk that one still bears after achieving full diversification, which cannot be eliminated by combining assets in a large portfolio. It is often called **portfolio risk, market risk or nondiversifiable risk .**

- **Unsystematic risk, diversifiable, or unique risk** is the risk that can be diversified away in a large portfolio. It is also known as firm specific risk.

- CAPM is derived from a well diversified portfolio with no unsystematic risk in equilibrium. β measures the systematic risk of a asset.

- To an individual who selects a diversified portfolio, the total risk of an individual security is not important.
- When considering adding a security to a diversified portfolio, the individual cares about only that portion of the risk of a security that cannot be diversified away, which is the beta of the security.



The slope of the security market line is equal to the market risk premium; i.e., the reward for bearing an average amount of systematic risk. The equation describing the SML can be written:

$$E(R_i) = R_f + \beta_i \times [E(R_M) - R_f]$$

which is the capital asset pricing model (CAPM).

- CAPM can be represented graphically by the upward-sloping line in the figure. The line is frequently called the **security market line (SML，证券市场线).**

- The line will be upward-sloping as long as the expected return on the market is greater than the risk-free rate. Because the market portfolio is a risky asset, theory suggests that its expected return is above the risk free rate.

(1) Linearity

- Because beta is the appropriate measure of risk, high beta securities should have an expected return ____(above/below) that of low beta securities. The relationship between expected return and beta corresponds to a straight line.

Given assumptions we made at the beginning, all securities must lie on the SML in market equilibrium.

- In other words, if a stock is perceived to be a good buy, or underpriced, it will provide an expected return in excess of the fair return stipulated by SML.

Underpriced stocks therefore plot above the SML: Given their betas, their expected returns are greater than dictated by the CAPM. Overpriced stocks, plot below the SML.

The difference between the fair and actually expected rates of return on a stock is called the stock's alpha, denoted α .

For example, if the market return is expected to be 14%, a stock has a beta of 1.2, and the T-bill rate is 6%, the SML would predict an expected return on the stock of _____.

If one believed the stock would provide an expected return of 17%, the implied alpha would be _____. See the figure.

(2) Portfolios as well as securities.

Our discussion of the CAPM considered individual securities. The relationship also holds for portfolios as well.

The beta of a portfolio is a weighted average of the betas of the individual securities.

$$\beta_p = \sum_{i=1}^{n} w_i \beta_i$$

We consider the portfolio formed by investing equally in our two securities, A and B. The expected return on the portfolio is

$$E(R_p) = x_A E(R_A) + x_B E(R_B) = 0.5 \times 15\% + 0.5 \times 8.6\% = 11.8\%$$

The beta of the portfolio is simply a weighted average of the betas of the two securities. Thus

$$\beta_p = x_A\beta_A + x_B\beta_B = 0.5\times 1.5 + 0.5\times 0.7 = 1.1$$

Under CAPM, the expected return on the portfolio is

$$E(R_p) = R_F + \beta_p \times (E(R_M) - R_F) = 3\% + 1.1\times 8\% = 11.8\%$$

Since two equations have the same answer, the CAPM holds for portfolios as well as for individual securities.

Summary:

- Stock prices are affected by firm specific and market wide risks. Investors care only about risk that is non-diversifiable.
- A stock's non-diversifiable risk is measured by beta, the slope when the stock is regressed on the market. In practice, betas are calculated by regressing historical returns against the return of the market portfolio.

$$E(R_{i,t}) = \alpha_i + \beta_i (R_{m,t}) + \tilde{\varepsilon}_{i,t}$$

- The slope of the regression is beta. β_i captures the sensitivity of the individual asset to the broad market movement. $\tilde{\varepsilon}_{i,t}$ reflects firm specific return, that is, diversifiable, idiosyncratic, or unsystematic risk.

Example 2:

The risk free rate is 8% and the expected return on the market portfolio is 16%. A firm considers a project that is expected to have a beta of 1.3.

A What is the required rate of return on the project?

B If the expected IRR of the project is 19%, should it be accepted?

Example 3:

Stock XYZ has an expected return of 12% and risk of $\beta = 1$. Stock ABC has expected return of 13% and $\beta = 1.5$. The market's expected return is 11%, and $R_f = 5\%$.

A According to the CAPM, which stock is a better buy?

B What is the alpha of each stock? Plot the SML and each stock's risk return point on one graph. Show the alphas graphically.

Example 4:

There are two stocks in the market, stock A and stock B. The price of stock A today is $50, the price of stock A next year will be $40 if the economy is in a recession, $55 if the economy is normal, and $60 if the economy is expanding. The probabilities of recession, normal times and expansion are 0.1, 0.8, and 0.1, respectively. Stock A pays no dividends and has a correlation of 0.8 with the market portfolio.

Stock B has an expected return of 9 percent, a standard deviation of 12 percent, a correlation with the market portfolio of 0.2, and a correlation with stock A of 0.6.

The market portfolio has a standard deviation of 10 percent. Assume the CAPM holds.

A If you are a typical, risk averse investor with a well diversified portfolio, which stock would you prefer? Why?

B What are the expected return and standard deviation of a portfolio consisting of 70 percent of stock A and 30 percent of stock B?

C What is the beta of the portfolio in part B?