Intermediate Microeconomics Lecture 12

General Equilibrium II: Production Economies

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One-consumer, One-producer Economy

- We extend our pure exchange economy to incorporating production.
- To start with, we first consider the simplest possible setting.
- There is one price-taking consumer and one price-taking firm.
- There are two goods:
  - labor (or leisure) of the consumer
  - a consumption good produced by the firm
- The consumer has a utility function $u(c, l)$ where c stands for consumption good and l for leisure.
- He is endowed with $\bar{L}$ units of leisure initially

One-consumer, One-producer Economy (cont.)

- The firm can hire the consumer and produce the consumption good.
  - the production function is $f(L)$
- Firms in any economy must be owned by someone.
- Thus in our economy, the single consumer is the sole owner of the firm and receives all the profits earned by the firm.
- This economy is sometimes referred to as Robinson Crusoe economy (鲁滨逊.克鲁索经济).

Profit Maximization

- Given price p for the consumption good and wage w for labor, the firm seeks to maximize its profits by choosing how much labor to hire:

$$\max_{L \geq 0} pf(L) - wL$$

- Let $\pi(p, w)$ be the corresponding profits. We have

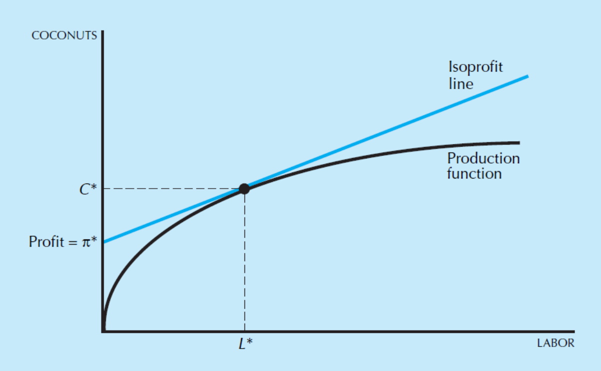
$$\pi(p, w) = pf(L) - wL$$

or

$$f(L) = \frac{\pi}{p} + \frac{wL}{p}$$

Profit Maximization (cont.)

- The firm's problem can be illustrated as:



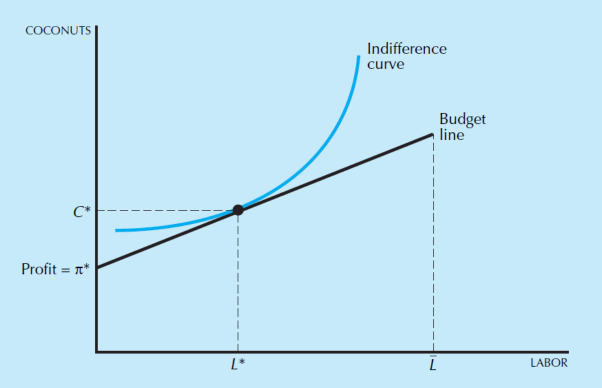
Utility Maximization

- The consumer decides how much leisure to enjoy and how much consumption good to buy to maximize his utility.
- If the consumer enjoys l units of leisure, then he supplies $\bar{L} - l$ units of labor and earns a total wage $w(\bar{L} - l)$
- Because the consumer also owns the profits made by the firm, his total income is $w(\bar{L} - l) + \pi(p, w)$
- Thus the consumer's problem is:

$$\max_{(c,l)} u(c, l)$$
$$\text{s.t } pc \leq w(\bar{L} - l) + \pi(p, w)$$

Utility Maximization (cont.)

- The consumer's problem can be illustrated as:



Competitive Equilibrium

- A competitive equilibrium of this economy consists of prices (p^*, w^*) and allocations (c^*, l^*, L^*) such that
  - Given (p^*, w^*), L^* solves firm's profit maximization:

$$\max_{L \geq 0} p^* f(L) - w^* L$$

  - Given (p^*, w^*), (c^*, l^*) solves consumer's utility maximization:

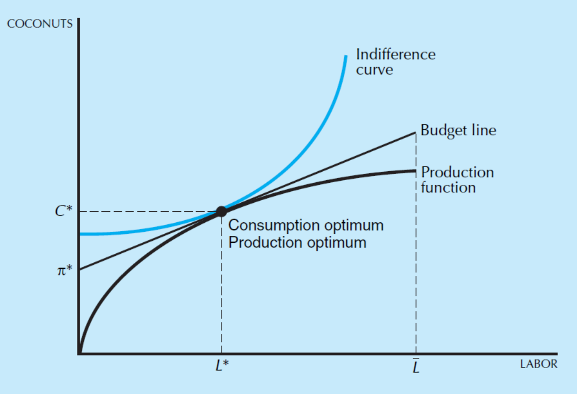
$$\max_{(c,l)} u(c, l)$$
$$\text{s.t } pc \leq w(\bar{L} - l) + \pi(p, w)$$

  - Market clears:

$$\begin{aligned} c^* &= f(L^*) \\ l^* &= \bar{L} - L^* \end{aligned}$$

Competitive Equilibrium (cont.)

- The equilibrium can be illustrated as:



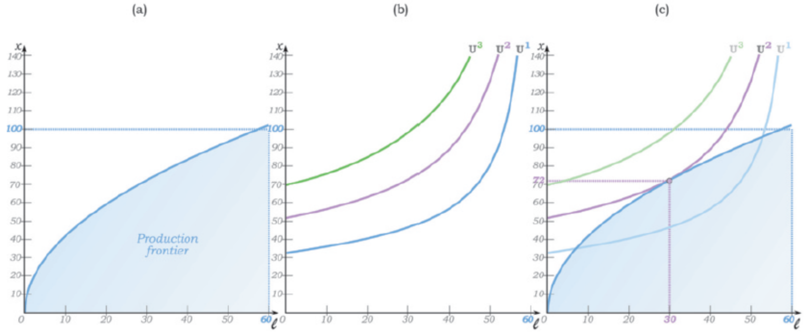
Competitive Equilibrium (cont.)

- In this particular graph, we have assumed nice properties of the preference and production function.
  - preference: continuous, monotone and convex
  - production function: increasing and concave
  - if preferences are continuous, monotone and convex and if the production set if convex, then a competitive equilibrium always exists.

Competitive Equilibrium (cont.)

- Moreover, a particular consumption-leisure bundle can be a competitive equilibrium if and only if it maximizes the consumer's utility subject to the economy's technological and endowment constraints
  - the first welfare theorems
  - the second welfare theorems

Competitive Equilibrium (cont.)



Production with Several Inputs and One Output

- We want to extend our baseline model to more general settings where firms use several inputs to produce several outputs.
- This extension will allow us to learn how firms optimally choose their productions among the different combinations of inputs and outputs.
- To facilitate the understanding, we take a detour and analyze firm's behavior alone first.

Production with Several Inputs and One Output (cont.)

- Let us start with a firm who uses inputs $x_1, \cdots, x_n$ to produce a single output y_1
- The set
$$\{(y_1, x_1, \cdots, x_n) | y_1 \leq f_1(x_1, \cdots, x_n)\}$$
is called the production (possibilities) set, where $f_1(x_1, \cdots, x_n)$ is the production function

Production with Several Inputs and One Output (cont.)

- Let us define a function

$$F(y_1, x_1, \cdots, x_n) \equiv y_1 - f_1(x_1, \cdots, x_n)$$

- Then the production possibilities set can be characterized as

$$\{(y_1, x_1, \cdots, x_n) | F(y_1, x_1, \cdots, x_n) \leq 0\}$$

- The function $F(y_1, x_1, \cdots, x_n) = 0$ describes the production possibilities frontier (生产可能性边界)
- The function F is called a production transformation function.

Production with Several Inputs and Outputs

- Now suppose the firm can use the same set of inputs to produce another output, y_2
- Now the production transformation function $F : \mathbb{R}^2 \times \mathbb{R}^n \to \mathbb{R}$
- Production possibilities set:

$$\{(y_1, y_2, x_1, \cdots, x_n) | F(y_1, y_2, x_1, \cdots, x_n) \leq 0\}$$

- The boundary of this set

$$\{(y_1, y_2, x_1, \cdots, x_n) | F(y_1, y_2, x_1, \cdots, x_n) = 0\}$$

  is the production transformation frontier (or production possibilities frontier)

Production with Several Inputs and Outputs (cont.)

- The slope of PPF measures the number of $y_2$that one unit of y_1 can be "transformed" into , holding inputs fixed
  - if we lower the output of y_1 by one unit, how many additional units of y_2 can be produced?
  - if we raise the output of y_1 by one unit, how many units of y_2 must be sacrificed?
  - this is the marginal rate of transformation (MRT 边际转换率) between y_1 and y_2

Production with Several Inputs and Outputs (cont.)

- Consider two inputs x_1 and x_2
- Holding x_1 and x_2 fixed, $F(y_1, y_2, x_1, x_2) = 0$ implies

$$\frac{\partial F}{\partial y_1} dy_1 + \frac{\partial F}{\partial y_2} dy_2 = 0$$

or equivalently,

$$\frac{dy_2}{dy_1} = -\frac{\partial F}{\partial y_1} \Big/ \frac{\partial F}{\partial y_2}$$

- So if the inputs are fixed: one unit of y_1 can be transformed into $\frac{\partial F}{\partial y_1} / \frac{\partial F}{\partial y_2}$ units of y_2, and vice versa

Profit Maximization

- Several inputs and outputs with $F(y_1, y_2, x_1, x_2)$ and prices (p_1, p_2, w_1, w_2)
- Profit maximization of a competitive firm:

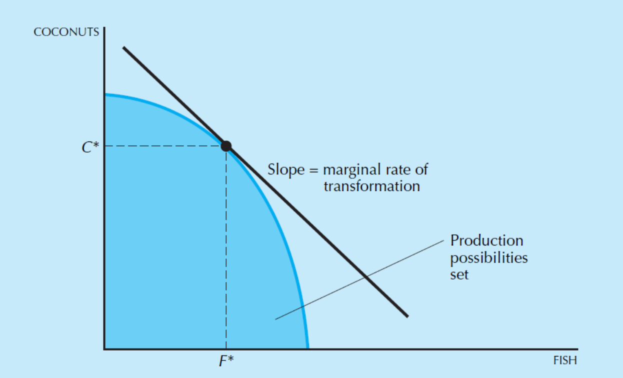
$$\max_{y_1,y_2,x_1,x_2 \geq 0} p_1 y_1 + p_2 y_2 - w_1 x_1 - w_2 x_2$$
$$\text{s.t } F(y_1, y_2, x_1, x_2) = 0$$

- Optimality:

$$-\frac{\partial F}{\partial x_i} \Big/ \frac{\partial F}{\partial y_j} = \frac{w_i}{p_j} \qquad \text{for } i, j \in \{1, 2\}$$
$$\frac{\partial F}{\partial y_1} \Big/ \frac{\partial F}{\partial y_2} = \frac{p_1}{p_2} \qquad \frac{\partial F}{\partial x_1} \Big/ \frac{\partial F}{\partial x_2} = \frac{w_1}{w_2}$$

- *MRT* equals price ratio

Profit Maximization (cont.)



Two-input, Two-output Economy

- Now we are ready to extend our model to an economy with several inputs and outputs.
- There are two consumers, A and B.
- Consumer $i \in \{A, B\}$ is endowed with $\bar{L}_i$ units of labor.
- There are two consumption goods, x and y.
- Consumer i's utility function is $u_i(x_i, y_i)$

Two-input, Two-output Economy (cont.)

- There is only one firm as before.
- The firm can hire A or B or both to produce consumption goods x and y
- Let F be the production transformation function.
- This firm must be owned by consumers, again. Let δ_A and δ_B denote A and B's share of this firm respectively. Of course $\delta_A + \delta_B = 1$.

Two-input, Two-output Economy (cont.)

- In the market, we have four prices: $p \equiv (p_x, p_y, w_A, w_B)$
- The firm maximizes its profits given the prices:

$$\begin{gathered} \max_{x,y,l_A,l_B \geq 0} p_x x + p_y y - w_A l_A - w_B l_B \\ \text{s.t } F(x, y, l_A, l_B) = 0 \end{gathered}$$

- Consumers i $(i = A, B)$ maximize their utilities given the prices:

$$\begin{aligned} &\max_{(x_i, y_i)} u_i(x_i, y_i) \\ &\text{s.t } p_x x_i + p_y y_i \leq \quad w_i \bar{L}_i + \delta_i \pi(p) \end{aligned}$$

Two-input, Two-output Economy (cont.)

- A competitive equilibrium of this economy consists of $((p_x^*, p_y^*, w_A^*, w_B^*), (x_A^*, y_A^*), (x_B^*, y_B^*), (x^*, y^*, l_A^*, l_B^*))$ such that
  - consumers maximize their utilities
  - the firm maximize its profits
  - four markets clear

Two-input, Two-output Economy (cont.)

- Because consumers maximize their utilities, at the equilibrium allocation, we know

$$MRS_A = MRS_B = \frac{p_x^*}{p_y^*}$$

- From previous analysis, we know the firm's marginal rate of transformation between x and y must equal the price ratio too.

$$MRT = \frac{\partial F}{\partial x} \Big/ \frac{\partial F}{\partial y} = \frac{p_x^*}{p_y^*}$$

- In sum, we have

$$MRT = MRS_A = MRS_B = \frac{p_x^*}{p_y^*}$$

Two-input, Two-output Economy (cont.)