Intermediate Microeconomics Lecture 9

Cost Curves

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Overview of Last Class

- In the last chapter we described the long-run cost-minimizing behavior of a firm

$$\min_{x_1,x_2} wx_1 + w_2x_2$$
$$s.t.\ y = f(x_1,x_2)$$

- The solution can be written as $x_1(w_1,w_2,y)$ and $x_2(w_1,w_2,y)$
  - e.g. if $f(x_1,x_2) = x_1^a x_2^b$, $x_1 = [(\frac{b}{a}\frac{w_1}{w_2})^{-b} y]^{\frac{1}{a+b}}$
  - called the conditional factor demand functions (有条件的要素需求函数), or derived factor demands (派生的要素需求)
- The correspongding cost function is $c(w_1,w_2,y)$

Short-run Costs

- In producers' theory, the difference between short-run and long-run is whether all factors in production are free to vary
  - in the short-run, some factors are fixed
- Suppose x_2 is fixed in the short-run. Then the firm's cost minimization problem becomes

$$\min_{x_1} wx_1 + w_2\bar{x}_2$$
$$s.t.\ y = f(x_1, \bar{x}_2)$$

  - the firm now can only choose x_1
- We write the short-run cost function as $c_s(w_1, w_2, y; \bar{x}_2)$
  - the minimal costs needed to produce y units of the output holding the input of the second factor fixed at $\bar{x}_2$

Short-run Costs (cont.)

- Consider Cobb-Douglas technology $f(x_1, x_2) = x_1^a x_2^b$ again
- Suppose $x_2 > 0$ is fixed in the short-run.
- Then, to produce y units of the output, the firm needs at least

$$x_1 = \left(\frac{y}{\bar{x}_2^b}\right)^{\frac{1}{a}}$$

  units of the first factor.
- The firm's short-run cost function is

$$c_s(w_1, w_2, y; x_2) = w_1\left(\frac{y}{\bar{x}_2^b}\right)^{\frac{1}{a}} + w_2\bar{x}_2$$

Short-run Costs (cont.)

- In the short-run, even if the firm does not produce, it bears strictly positive costs.
  - $c_s(w_1, w_2, 0; x_2) = w_2\bar{x}_2 > 0$
  - comes from the fixed inputs which can not be changed in the short-run
  - called fixed costs (固定成本；不变成本)

Short-run Costs v.s. Long-run Costs

- Now let's consider the following minimization problem

$$\min_{x_2} c_s(w_1, w_2, y; x_2)$$

- Plugging in $c_s(w_1, w_2, y; x_2)$, we can write

$$\min_{x_2} w_1 \left(\frac{y}{x_2^b}\right)^{\frac{1}{a}} + w_2 x_2$$

- We can show that the minimized cost is

$$\frac{a+b}{a^{\frac{a}{a+b}} b^{\frac{b}{a+b}}} w_1^{\frac{a}{a+b}} w_2^{\frac{b}{a+b}} y^{\frac{1}{a+b}}$$

which is exactly the $c(w_1, w_2, y)$ we have calculated before

Short-run Costs v.s. Long-run Costs (cont.)

- In other words, for each $y > 0$, we have

$$c(w_1, w_{2,}y) = \min_{x_2} c_s(w_1, w_{2,}y; x_2)$$

  - the cost-minimizing amount of the variable factor in the long run is that amount that the firm would choose in the short run-if it happened to have the long-run cost-minimizing amount of the fixed factor
  - is true not only for this example, but for all production functions

Total Costs

- Next we will learn how to use cost curves to depict graphically the cost function of a firm
- In the rest of this chapter we will take the factor prices to be fixed so that we can write cost as a function of y alone, $c(y)$.
- We've learnt that $c(y)$ consists of two parts
  - fixed costs (不变成本)
    - costs that must be paid regardless of the level of production
    - $F \equiv c(0)$
  - variable costs (可变成本)
    - costs that change when output changes
    - $c_v(y) \equiv c(y) - c(0)$

Average Costs

- Average cost function $AC(y)$ (平均成本函数)
  - measures the costs per unit of output

$$AC(y) \equiv \frac{c(y)}{y}$$

- Average variable cost function $AVC(y)$(平均可变成本函数)
  - measures the variable costs per unit of output

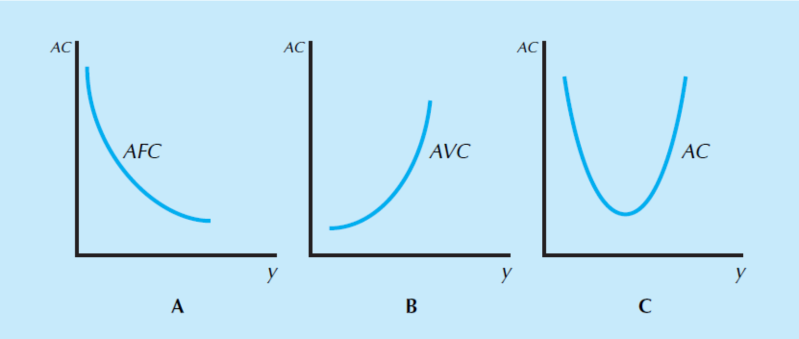
$$AVC(y) \equiv \frac{c_v(y)}{y}$$

- Average fixed cost function $AFC(y)$ (平均不变成本函数)
  - measures the fixed costs per unit of output

$$AVC(y) \equiv \frac{F}{y}$$

Average Costs (cont.)

- An example of average costs curves



Average Costs (cont.)

- The average fixed costs decrease as output is increased
- The average variable costs eventually increase as output is increased
  - if fixed factors are present, they will eventually constrain the production process
- The combination of these two effects produces a U-shaped average cost curve

Marginal Costs

- The marginal cost curve (边际成本曲线) measures the change in costs for a given change in output

$$MC(y) \equiv \lim_{\triangle y \to 0} \frac{c(y + \triangle y) - c(y)}{\triangle y} = c'(y)$$

  - additional costs if we produce one more unit of output
- It is easy to see

$$MC(y) = c_v'(y)$$

MC Curves v.s. AC Curves

- MC curve and AVC curve coincide at y = 0

$$MC(0) = \lim_{\triangle y \to 0} \frac{c(\triangle y) - c(0)}{\triangle y} = \lim_{\triangle y \to 0} \frac{c_v(\triangle y)}{\triangle y} = AVC(0)$$

- If $AVC(\cdot)$ decreases in the interval $(\underline{y}, \bar{y})$, then

$$MC(y) \leq AVC(y) \qquad \forall y \in (\underline{y}, \bar{y})$$

  - in a range of output where AVC curve is decreasing, MC curve must lie below AVC curve
  - intuition: If $AVC(n+1) \leq AVC(n)$, we must have $c_v(n+1) - c_v(n) \leq AVC(n)$

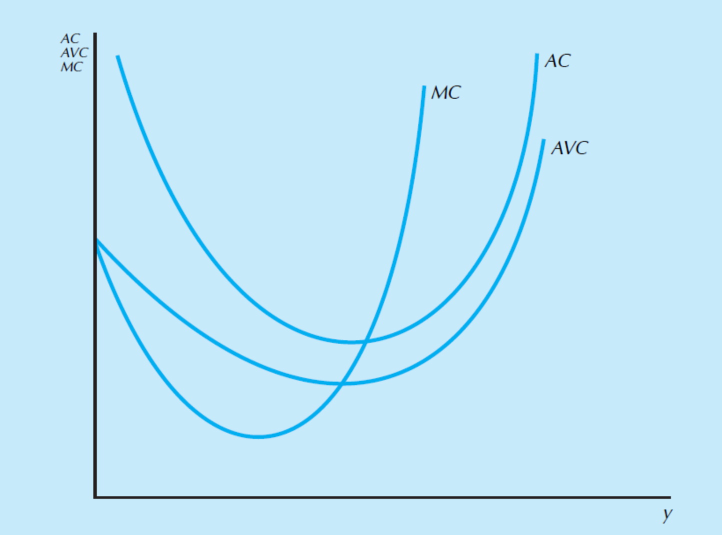
MC Curves v.s. AC Curves (cont.)

- Now consider a special case: "U-shaped" *AVC* curve.
- Assume that there exists $\tilde{y} > 0$ such that *AVC* decreases in $(0, \tilde{y})$ and increases in $(\tilde{y}, +\infty)$.
    - $\tilde{y}$ gives the lowest average variable costs
    - $MC(y) \leq AVC(y)$for $y < \tilde{y}$
    - $MC(y) \geq AVC(y)$for $y > \tilde{y}$
- Thus (by continuity), $MC(\tilde{y}) = AVC(\tilde{y})$
    - i.e. *MC* and *AVC* intersect at $\tilde{y}$

MC Curves v.s. AC Curves (cont.)

- The above analysis also applies to "U-shaped" AC curves.
- We also have:
  - In a range of output where AC is decreasing, MC must lie below AC
  - In a range of output where AC is increasing, MC must lie above AC

MC Curves v.s. AC Curves (cont.)

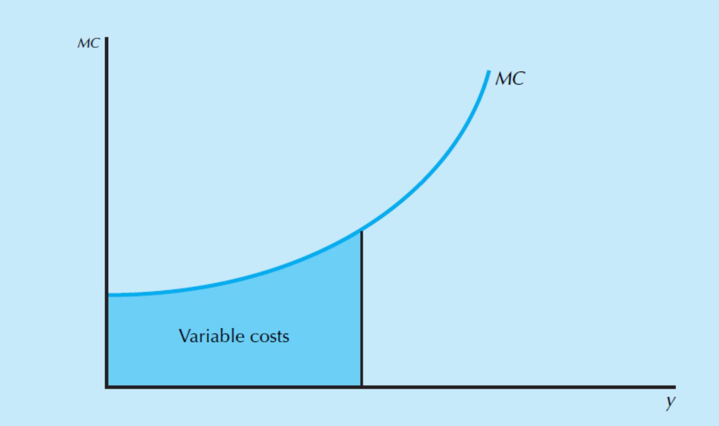


Marginal Cost v.s. Variable Costs

- There is also a relationship between marginal cost curve and variable costs.
- If the marginal cost is continuous on [0,+1), the area below a marginal cost curve is the associated variable costs
- Consider the case where the output good is produced in discrete amounts

$$\begin{aligned} c_v(y) &= [c_v(y)\text{-}c_v(y-1)] + [c_v(y-1) - c_v(y-2)] \\ &\quad + \ldots + [c_v(1) - c_v(0)] \\ &= MC(y-1) + MC(y-2) + \ldots + MC(0) \end{aligned}$$

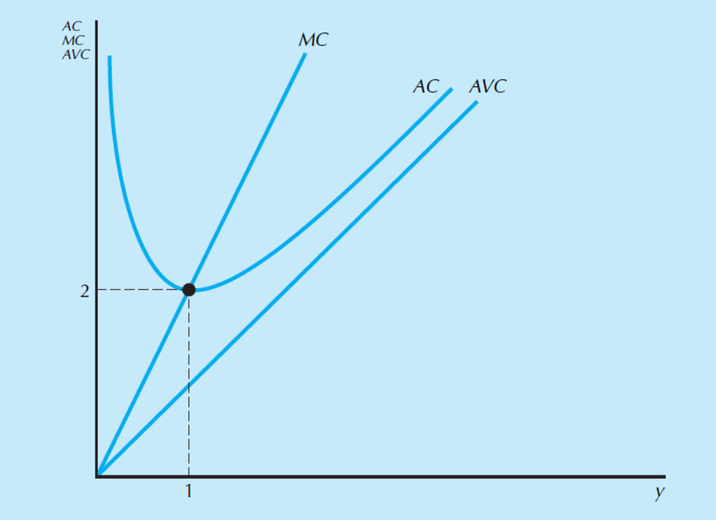
Marginal Cost v.s. Variable Costs (cont.)



An Example

- Let's consider the cost function $c(y) = y^2 + 1$
- We have the following derived cost curves:
  - variable costs:$c_v(y) = y^2$
  - fixed costs: $c_f(y) = 1$
  - average variable costs: $AVC(y) = y^2/y = y$
  - average fixed costs: $AFC(y) = 1/y$
  - average costs: $AC(y) = \frac{y^2+1}{y} = y + \frac{1}{y}$
  - marginal costs: $MC(y) = 2y$

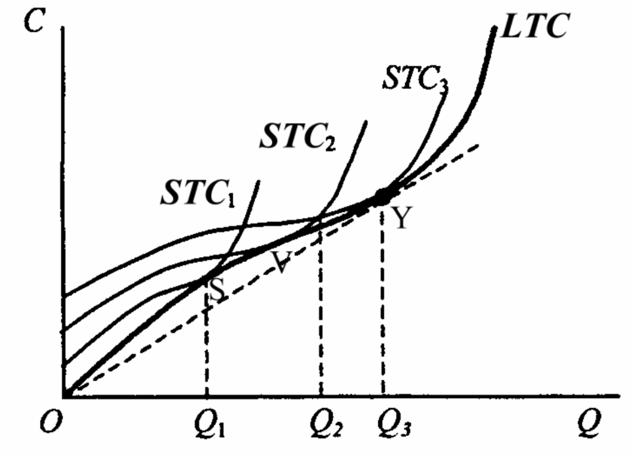
A Example (cont.)



Long-run v.s. Short-run Cost Curves

- Previous analysis applies to both long-run and short-run cost functions
- We only need to assume $F = 0$ when we think $c(\cdot)$ as a long-run cost function.
- What are the relationships between long-run and short-run cost curves?

Long-run v.s. Short-run Total Cost Curves



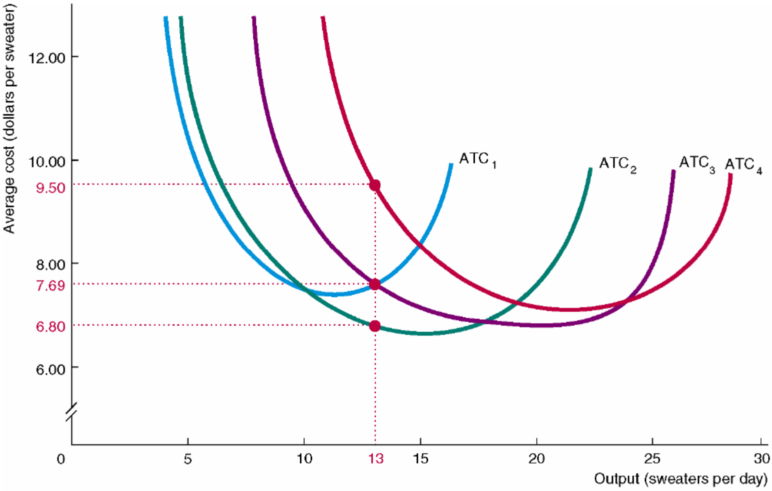
Long-run v.s. Short-run Total Cost Curves (cont.)

- For simplicity, assume that in the short-run capital k is the only fixed input
- Long-run cost function then can be expressed as

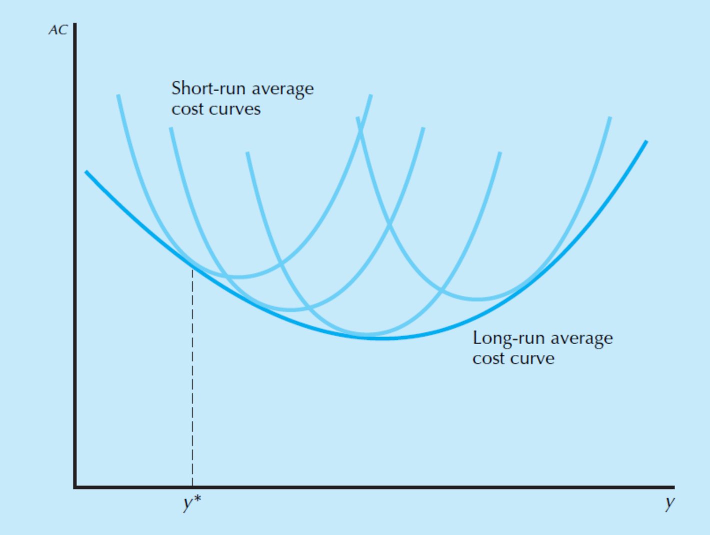
$$c(y) = \min_{k \geq 0} c_s(y; k)$$

- If for each y, $k(y)$ solves this minimization problem, we can also write $c(y) = c_s(y; k(y))$
  - long-run cost function is the lower envelop (下包络线) of all short-run cost functions

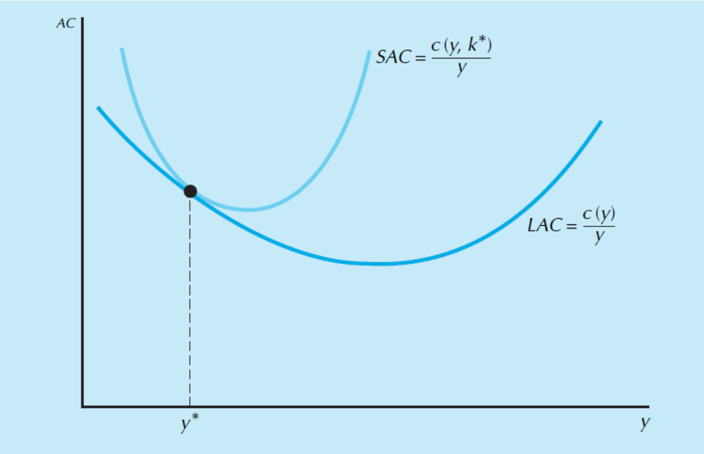
Long-run v.s. Short-run Average Cost Curves



Long-run v.s. Short-run Average Cost Curves (cont.)



Long-run v.s. Short-run Average Cost Curves (cont.)



Long-run v.s. Short-run Average Cost Curves (cont.)

- Moreover, we also have

$$LAC(y) \equiv \frac{c(y)}{y} = \frac{\min_k c_s(y;k)}{y} = \min_k \frac{c_s(y;k)}{y} = \min_k SAC(y;k)$$

where LAC:long-run average cost and SAC:short-run average cost

  - i.e. $LAC(y) = SAC(y; k(y))$
  - the long-run average cost function is also the lower envelop of all short-run average cost functions

Long-run v.s. Short-run Marginal Cost Curves (cont.)

- Because $c(y) = c_s(y; k(y))$, by chain rule, we have

$$MC(y) = \frac{\partial c_s(y; k(y))}{\partial y} + \frac{\partial c_s(y; k(y))}{\partial k} k'(y)$$

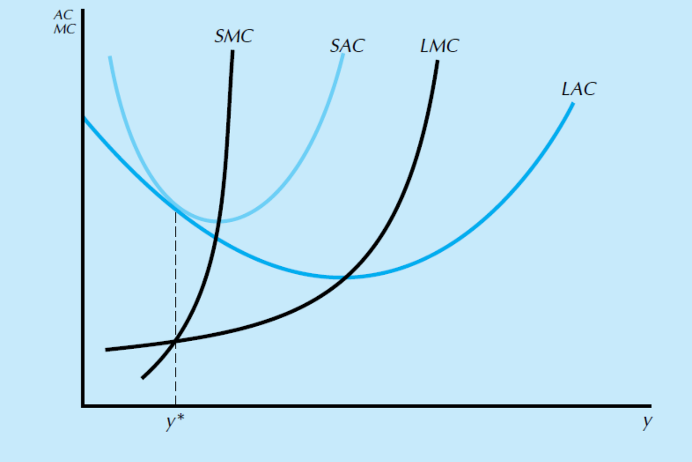
- Because $k(y)$ minimizes $c_s(y; k(y))$ for $k \geq 0$, first order condition yields

$$\frac{\partial c_s(y; k(y))}{\partial k} = 0$$

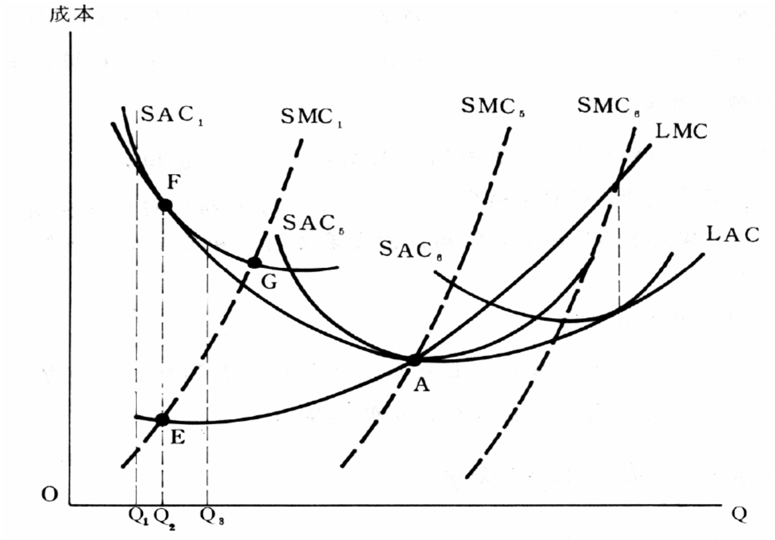
- Hence we have

$$LMC(y) = SMC(y; k(y))$$

Long-run v.s. Short-run Cost Curves (cont.)



Long-run v.s. Short-run Cost Curves (cont.)



Shape of LAC

- In the long-run,firms' long-run average cost is "U"shaped because they experience
  - economies of scale (规模经济):average cost falls as output expands
  - diseconomies of scale (规模不经济): average cost remains increases as output expands

Shape of LAC (cont.)

- Reasons for economies of scale
  - specialization of labour and capital
  - indivisibilities in plant size
  - marketing economies
  - transport and storage economies
  - bulk purchase of inputs
- Reasons for diseconomies of scale
  - plant size too big to manage
  - organisation too bureaucratic
  - input prices may rise due to scarcity
  - labour relations deteriorate