Intermediate Microeconomics Lecture 14

General Equilibrium I: Pure Exchange Economies

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Partial v.s. General Equilibrium

- Up until now we have been ignoring the effect of these other prices on the market equilibrium.
- When we discussed the equilibrium conditions in a particular market, we only looked at how demand and supply were affected by the price of the particular good we were examining.
- This is called partial equilibrium analysis (局部均衡分析)
- In this chapter we will begin our study of general equilibrium analysis (一般均衡分析):
  - how demand and supply conditions interact in several markets to determine the prices of many goods

Pure Exchange Economies

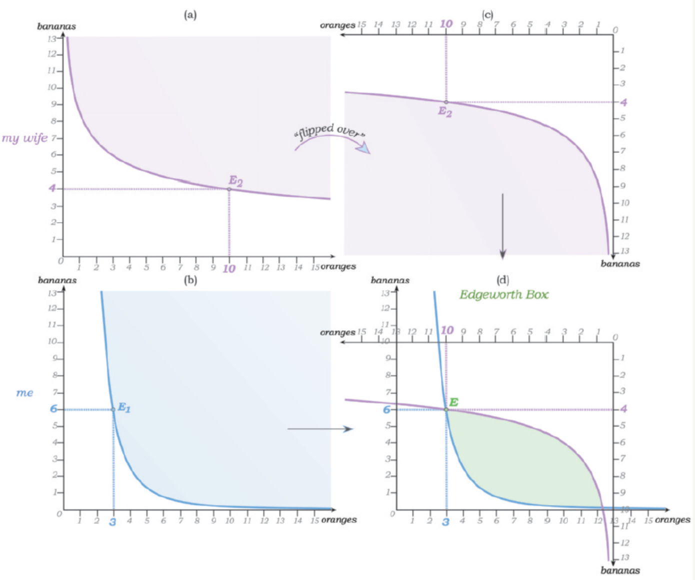
- To keep things simple, we start with pure exchange economy (纯交换经济)
- A pure exchange economy is an economy without production:
  - people have fixed endowments of goods
  - they can trade these goods among themselves
- To further simplify, we will focus on an economy consisting of:
  - 2 goods, denoted by 1 and 2
  - 2 agents, denoted by A and B

Pure Exchange Economies (cont.)

- Agent $i = A, B$ has $\omega_i = (\omega_i^1, \omega_i^2)$ initial endowments (初始禀赋) and utility function $u_i(x_i^1, x_i^2)$
- The total endowments in this economy are thus $\omega_A + \omega_B$
- The agent will exchange some of these goods with each other to end up at a final allocation (最终配置)
  - an allocation is a pair of vectors $x_A = (x_A^1, x_A^2)$ and $x_B = (x_B^1, x_B^2)$
  - a feasible allocation (可行的配置) is an allocation such that

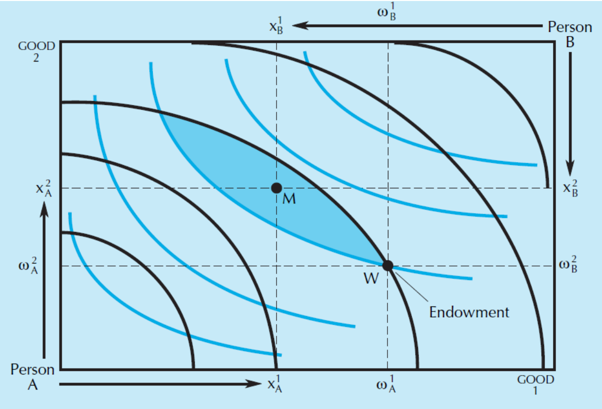
$$x_A + x_B = \omega_A + \omega_B$$

Graphical Depiction of a Pure Exchange Economy



Mutually Beneficial Trades in the Edgeworth Box

- Edgeworth box (埃奇沃斯盒) can be used to analyze the exchange of two goods between two people



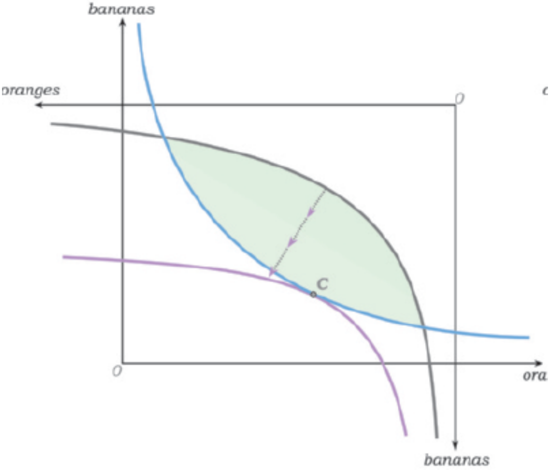
Pareto Efficiency and Contract Curve

- Formally, in our 2 agents economy, we say a feasible allocation (x_A, x_B) Pareto dominates another feasible allocation $(\tilde{x}_A, \tilde{x}_B)$ if

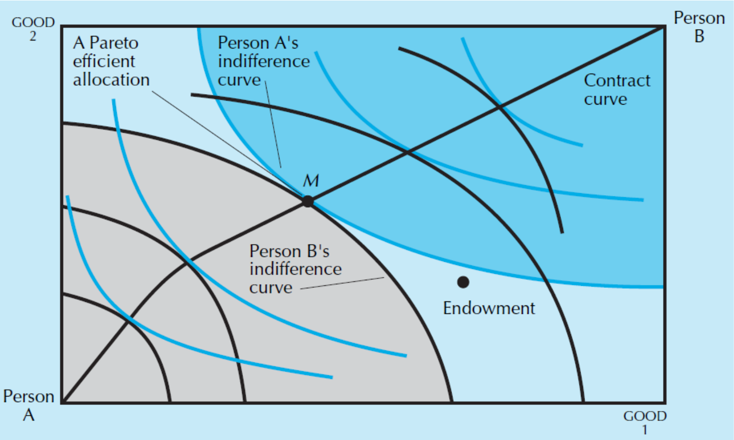
$$u_i(x_i) > u_i(\tilde{x}_i) \text{ and } u_j(x_j) \geq u_j(\tilde{x}_j)$$

  where $(i, j) = (A, B)$ or $(i, j) = (B, A)$
- A feasible allocation (x_A, x_B) is Pareto efficient if it is not Pareto dominated by any other feasible allocation.
- In words, a feasible allocation is Pareto efficient if no one can be made strictly better off without hurting others.

Pareto Efficiency and Contract Curve (cont.)



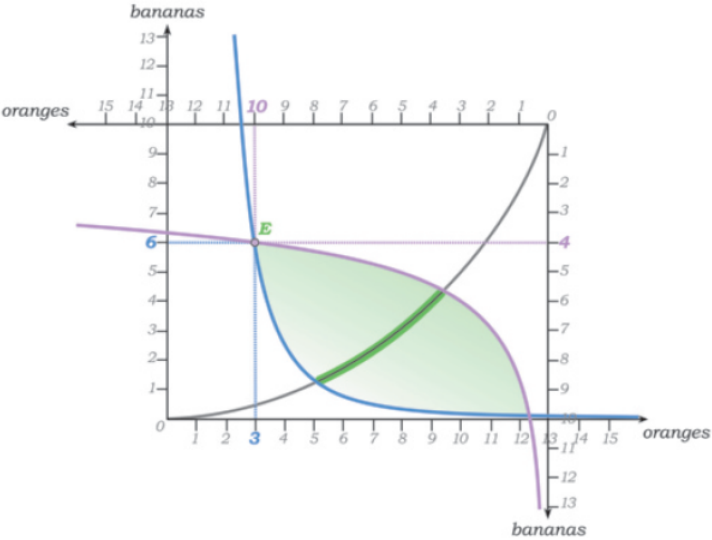
Pareto Efficiency and Contract Curve (cont.)



Pareto Efficiency and Contract Curve (cont.)

- At a Pareto efficient allocation such as M, each person is on his highest possible indifference curve, given the indifference curve of the other person.
- The indifference curves of the two agents must be tangent at any Pareto efficient allocation in the interior of the box.
- The set of all Pareto efficient points in the Edgeworth box is known as the Pareto set (帕累托集), or the contract curve (契约曲线)
  - all "final contracts" for trade must lie on the Pareto set

Mutually Beneficial Efficient Trades and the “Core”



Competitive Equilibrium:Utility Maximization

- Agents are assumed to take the market price as given and then to choose a consumption bundle out of their budget set to maximize their utilities.
- Recall if the agent i's income is w_i , then his utility maximization problem is:

$$\begin{aligned} \max_{(x_i^1, x_i^2)} u_i(x_i^1, x_i^2) \\ \text{s.t. } p^1 x_i^1 + p^2 x_i^2 &\leq w_i \end{aligned}$$

- But in this economy, i's income comes from "selling" his endowments at the market price p:

$$\begin{aligned} \max_{(x_i^1, x_i^2)} u_i(x_i^1, x_i^2) \\ \text{s.t. } p^1 x_i^1 + p^2 x_i^2 &\leq p^1 \omega_i^1 + p^2 \omega_i^2 \end{aligned}$$

Demand and Excess Demand

- Let $d_i(p) \equiv (d_i^1(p), d_i^2(p))$ denote a solution to the above utility maximization problem.
    - $d_i(p)$ is agent i's (gross) demand given the market price p.
- The excess demand (超额需求)(or net demand 净需求) is then

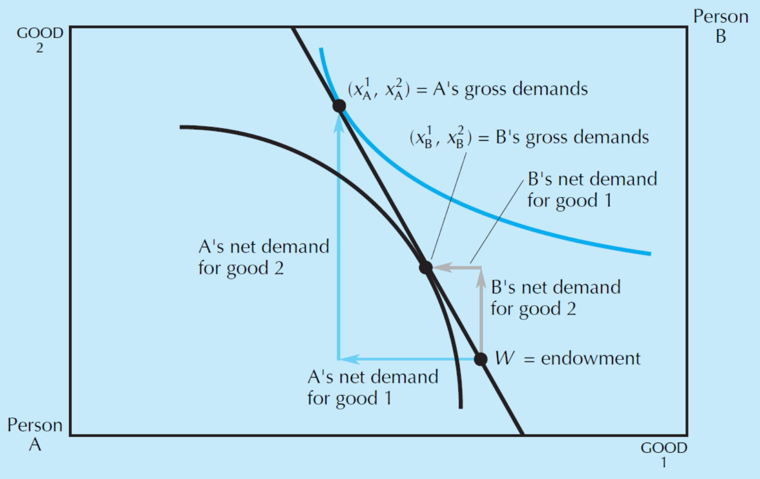
$$e_i(p) \equiv d_i(p) - \omega_i$$

- The sum of excess demand across agents

$$z(p) \equiv e_A(p) + e_B(p)$$

is called the aggregate excess demand (总超额需求)

Demand and Excess Demand (cont.)



Competitive Equilibrium

- As we can see from previous figure, for arbitrary market price p, it might be the case that

$$d_A(p) + d_B(p) \neq \omega_A + \omega_B$$

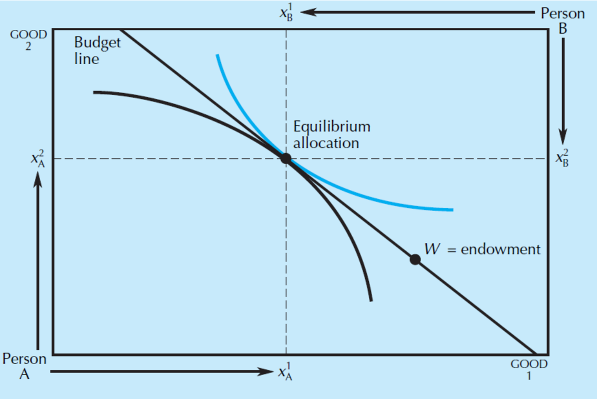
- Or equivalently, $z(p) \neq 0$
- We are intersted in those p's under which the above inequalities becomes equalities.

Competitive Equilibrium (cont.)

- A competitive (Walrasian) equilibrium of this economy consists of a market price p^* and an allocation (x_A^*, x_B^*) such that:
  - given price p^* , x_i^* maximizes i's utlity, i.e. $d_i(p^*) = x_i^*$;
  - market clears: $x_A^* + x_B^* = \omega_A + \omega_B$
- Condition 1 states that agents behave optimally given the price.
- Condition 2 states that the price equates demand and supply.
- Condition 1 and 2 together is equivalent to $z(p^*) = 0$

Competitive Equilibrium (cont.)

- Equilibrium in the Edgeworth box



Competitive Equilibrium (cont.)

- The model is silent about how equilibrium price is formed.
- We only know that this equilibrium price mysteriously clear the markets.
- One way to think about the price formation is to consider a hypothetical auctioneer who adjusts the price so that markets clear.
- Price-taking is hardly the case for trade between two agents.
- One way to reconcile is to think about the two agents as two types of agents and assume each type has many many identical agents

A Numerical Example

- Assume the two consumers have utilities:

$$u_A(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$$

$$u_B(x_1, x_2) = x_1^{\beta} x_2^{1-\beta}$$

- Endowments: ω_A and ω_B
- We try to find its competitive equilibrium.

A Numerical Example (cont.)

- Assume the market price is p
- We can calculate the demand for both agents:

$$d_A^1(p) = \alpha \frac{p_1\omega_A^1 + p_2\omega_A^2}{p_1}$$

$$d_A^2(p) = (1-\alpha)\frac{p_1\omega_A^1 + p_2\omega_A^2}{p_2}$$

$$d_B^1(p) = \beta \frac{p_1\omega_B^1 + p_2\omega_B^2}{p_1}$$

$$d_B^2(p) = (1-\beta)\frac{p_1\omega_B^1 + p_2\omega_B^2}{p_2}$$

A Numerical Example (cont.)

- Market clearing conditions states:

$$d_A^1(p) + d_B^1(p) = \omega_A^1 + \omega_B^1$$

$$d_A^2(p) + d_B^2(p) = \omega_A^2 + \omega_B^2$$

- Solving these two equations yields:

$$\frac{p_1^*}{p_2^*} = \frac{\alpha\omega_A^2 + \beta\omega_B^2}{(1-\alpha)\omega_A^1 + (1-\beta)\omega_B^1}$$

- Plugging this relative price back into the demand functions will pin down the equilibrium allocations.

A Numerical Example (cont.)

- Two observations from previous example:
  - If $(p^*, x_A^*, x_B^*\ )$ is a competitive equilibrium, so is $(\lambda p^*, x_A^*, x_B^*\ )$ where λ is a positive scalar.
  - If $p \gg 0$, i.e. prices are positive, then the fact that one market clears implies that the other clears as well.
- These two results hold for any arbitrary pure exchange economy.

Competitive Equilibrium (cont.)

- The first observation comes from the fact that individual agent's demands will be the same under the prices p and λp.
- Therefore, when solving for equilibrium prices, we usually normalize the price of one good to be 1.
  - this good is called a numeraire
  - the choice of the numeraire can be arbitrary

Competitive Equilibrium (cont.)

- The second observation is a simple implication of Walras' Law (瓦尔拉斯法则):

$$p_1 z_1(p) + p_2 z_2(p) \equiv 0$$

  - where $z_1(p)$ and $z_2(p)$ are the aggregate excess demand for good 1 and 2 respectively.
- Walras' Law is nothing but an accounting identity
  - the sum of all agents' budget constraints

$$p_1 x_i^1(p) + p_2 x_i^2(p) \equiv p_1 \omega_i^1 + p_2 \omega_i^2$$

- If there are n markets, $n - 1$ market clear at a positive price, then the nth market must clear as well.

Competitive Equilibrium (cont.)

- One of the central questions in general equilibrium analysis is the existence of competitive equilibrium.
- In 1954, Kenneth Arrow (1972 Nobel Prize) and Gérard Debreu (1983 Nobel Prize) proved that under fairly general conditions on the preferences, a competitive equilibrium always exists.
- Roughly speaking, the conditions are continuity, convexity and monotonicity.

First Welfare Theorem

- Are competitive equilibria outcomes Pareto efficient?
- First welfare theorem (福利经济学第一定理)
  - if agents' preferences are monotonic (utility functions are increasing), then any competitive equilibrium allocation is Pareto efficient.
- As a result, this theorem is widely considered as a formal statement of "invisible hand".

First Welfare Theorem (cont.)

- Assume (p^*, x_A^*, x_B^*) is a competitive equilibrium.
- A feasible allocation (x_A, x_B) Pareto dominates (x_A^*, x_B^*)
- Assume w.o.l.g $u_A(x_A) > u_A(x_A^*)$ and $u_B(x_B) \geq u_B(x_B^*)$
- Then we know

$$p_1^* x_A^1 + p_2^* x_A^2 > p_1^* x_A^{1*} + p_2^* x_A^{2*}$$

$$p_1^* x_B^1 + p_2^* x_B^2 \geq p_1^* x_B^{1*} + p_2^* x_B^{2*}$$

- But

$$p_1^*(\omega_A^1 + \omega_B^1) + p_2^*(\omega_A^2 + \omega_B^2) > p_1^*(\omega_A^1 + \omega_B^1) + p_2^*(\omega_A^2 + \omega_B^2)$$

a contradiction.

Second Welfare Theorem

- The FWT asks whether competitive equilibrium allocation is Pareto efficient.
- The second welfare theorem (福利经济学第二定理) asks the converse:
  - can every Pareto efficient allocation be a competitive equilibrium if we adjust the initial endowment (e.g. by redistributing wealth)?
- Second welfare theorem:
  - if agents' preferences are continuous, monotonic and convex, then for any Pareto efficient allocation there exists some initial endowment redistribution such that the efficient allocation is a competitive equilibrium allocation after redistribution.

Second Welfare Theorem (cont.)