Intermediate Microeconomics-Practice Problems 5

Due: Saturday, December 28 in class

Question 1. Dollar Split Game

Player 1 and 2 are bargaining over how to split one dollar. Both players simultaneously name shares they would like to have, s_1 and s_2 , where $0 \leq s_1, s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the players receive the shares they named; if $s_1 + s_2 > 1$, then both players receive zero. What are the pure-strategy Nash equilibria of this game?

Question 2. Advertisement and Firm's Entry[1]

Consider the following game. An incumbent makes a decision to advertise at a cost K or not. This action is observed by a challenger who has the option of entering the market at cost F or staying out. If the challenger stays out of the market, the incumbent firm is a monopolist. If the challenger enters the market, the two parties compete as Cournot competitors. Advertisement increases demand at any given price. The inverse demand curves when incumbent advertises and when not are(correspondingly):

$$\begin{aligned} P_{Ad}(q_1, q_2) &= 60 - Q \\ P_{NoAd}(q_1, q_2) &= 48 - Q \end{aligned}$$

where $Q = q_1 + q_2$. Assume that all parties has zero marginal costs.

1. Draw the extensive form of this game.
2. Suppose that $F = 350$. Should the incumbent advertise?
3. Suppose that $F = 100$. For what levels of K should the incumbent advertise?

Question 3. [2]

Consider an industry with 3 firms, each having marginal costs equal to 0. The inverse demand curve facing this industry is

$$P(q_1, q_2, q_3) = 60 - (q_1 + q_2 + q_3)$$

1. If each firm behaves as a Cournot competitor, what is the industry equilibrium? What are the associated payoffs?
2. Firm 2 and 3 decide to merge and form a single firm (MC is still 0). Calculate the new industry equilibrium. Is firm 1 better or worse off as a results? Are the combined profits from firm 2 and 3 greater or less than before? Would it be a profitable idea for all three firms to organize into a cartel?
3. Suppose firm 1 can commit to a certain level of output in advance. If the choice of firm 1 is q_1, what would be the optimal choices of firm 2 and 3?

[1] Modified from Izmalkov (2006, MIT)
[2] Modified from Izmalkov (2006, MIT)

Question 4. Pure Exchange Economy

Consider the following pure exchange economy with two consumers and two goods, x and y. Consumer 1 has the utility function $u_1(x, y) = x^2y$ and his endowment is $(2, 0)$. Consumer 2 has the utility function $u_1(x, y) = xy^2$ and her endowment is $(0, 2)$. Compute the competitive equilibrium price and allocation.