# Lecture 11 Valuation and Capital Budgeting

In this section, we combine capital budgeting decision and capital structure decision together, because two decisions are actually related.

The goal of this chapter is to value a project, or the firm itself, when leverage is employed.



### 1 Adjusted-Present-Value Approach

(调整净现值法)

The adjusted present value method is best described by the following formula:

$$APV = NPV + NPVF$$

The value of a project to a levered firm (APV) is equal to the value of the project to an unlevered firm (NPV) plus the net present value of the financing side effects (NPVF).

In general we will think about the following side effects:

- (1) The Tax Subsidy to Debt.
- (2) The Costs of Issuing New Securities.
- (3) The Costs of Financial Distress.
- (4) Some Other Subsidies to Debt Financing.



In general, we have this formula:

$$\begin{aligned} APV &= NPV + NPVF \\ &= -Initial \ investment + \sum_{t=1}^{\infty} \frac{UCF_t}{(1+r_0)^t} + Additional \ effects \ of \ debt \end{aligned}$$

 $UCF_t$  is the project's cash flow at date t to the equityholders of an unlevered firm

 $r_0$  is the cost of capital for project in an unlevered firm.

#### Example 1:

Consider a project of the P.B. Singer Co. with the following characteristics:

Cash inflows: \$500,000 per year for the indefinite future.

Cash costs: 72% of sales

Initial investment: \$475,000.

$$T_C = 34\%$$

$$r_0 = 20\%$$

First we calculate the NPV of the project, that is, the value of the project to an all equity firm.

$$NPV = -Initial \ investment + \sum_{t=1}^{\infty} \frac{UCF_t}{(1+r_0)^t}$$

$$= -475,000 + \frac{(500,000 - 72\% \times 500,000) \times (1-34\%)}{0.2}$$

$$= -475,000 + \frac{92,400}{0.2}$$

$$= -475,000 + 462,000$$

$$= -13,000$$

Since the NPV is negative, the project would be rejected by an all equity firm.

Now imagine that the firm finances the project with \$126,229.5 in debt, so that the remaining investment of \$348,770.50 (475,000-126,229.5) is financed with equity.

Then the net present value of the project under leverage, which we call the adjusted present value of the APV, is

$$APV = NPV + T_CB$$
  
= -13,000 + 34% × 126,229.5  
= \$29,918



Now let's think about why we choose \$126,229.5 in debt.

Actually, we chose it so that the ratio of the debt to value or debt to the present value of the project under leverage is 0.25.

So debt is a fixed proportion of the present value of the project, not a fixed proportion of the initial investment of \$475,000.



## 2 Flow-to-Equity Approach (权益现金流量法)

It means that we discount the cash flow from the project to the equityholders of the levered firm at the cost of equity capital  $r_s$ .

$$\sum_{t=1}^{\infty} \frac{LCF_t}{(1+r_S)^t} - (Initial\ investment - Amount\ borrowed)$$

 $LCF_t$  is the project's cash flow at date t to the equityholders of a levered firm.

 $r_s$  is the cost of equity capital with leverage.

#### We assume the interest rate is 10 percent. Then

Cash inflows	500,000,000
Cash costs	360,000,000
Interest	12,622.95
Income after interest	127,377.05
Corporate tax	43,308.20
Levered cash flow (LCF)	\$84,068.83

We can also calculate levered cash flow directly from unlevered cash flow.

The difference between the cash flow that equityholders receive in an unlevered firm and the cash flow that equityholders receive in a levered firm is the **after tax interest payment**.

$$UCF - LCF = (1 - T_C)r_BB$$

Then in our example:

$$LCF = UCF - (1 - T_C)r_BB$$
  
= 92,400 - (1 - 34%) × 0.1 × 126,229.5  
= \$84,068.85

Now we calculate  $r_S$  by MM proposition II. Our debt to value ratio is 1/4.

$$r_S = r_0 + \frac{B}{S} \times (1 - T_C) \times (r_0 - r_B)$$

$$= 0.2 + \frac{1}{3} \times (1 - 34\%) \times (0.2 - 0.1)$$

$$= 0.222$$

Then the present value of the project's LCF is

$$\frac{LCF}{r_S} = \frac{84,068.85}{0.222} = \$378,688.5$$

The net present value of the project is simply the difference between the present value of the project's LCF and the investment not borrowed.

Thus we have



$$NPV = \$378,688.5 - (\$475,000 - \$126,229.5) = \$29,918$$

This is identical with the result found with the APV approach.

## 3 Weighted Average Cost of Capital Method (加权平均资本成本法)

In this method, we discount the unlevered cash flow of the project at the weighted average cost of capital  $r_{WACC}$ .

The net present value of the project is

$$\sum_{t=1}^{\infty} \frac{UCF_t}{(1+r_{WACC})^t} - Initial investment$$

*rwacc* is the weighted average cost of capital.



So in our example:

$$r_{WACC} = \frac{S}{B+S}r_S + \frac{B}{B+S}r_B(1-T_C)$$

$$= \frac{3}{4} \times 0.222 + \frac{1}{4} \times 0.1 \times (1-34\%)$$

$$= 0.183$$



We know UCF is \$92,400. Then the present value of the project is

$$\frac{92,400}{0.183} = \$504,918$$

The net present value of the project is

$$$504,918 - $475,000 = $29,918$$

In this example, all three approaches yield the same value.

#### 4 An APV Example

#### Example 2:

Bicksler Enterprises is considering a \$10 million project that will last five years, implying straight line depreciation per year of \$2 million.

The cash revenues less cash expenses per year are \$3,500,000. The corporate tax bracket is 34 percent. The risk free rate is 10 percent, and the cost of unlevered equity is 20 percent.

#### **(1) NPV**

Assuming the project is financed with all equity, the value of the project is

$$-10,000,000 + 0.34 \times 2,000,000 \times A_{0.2}^5 + 3,500,000 \times (1 - 34\%) \times A_{0.2}^5 = -1,058,106$$

So an all equity firm would clearly reject this project, because the NPV is negative.

However, debt financing may add enough value to the project to justify acceptance.

#### **(2) NPVF**

Bicksler Enterprises can obtain a five year, **nonamortizing** loan for \$7,500,000 after flotation costs at the risk free rate of 10 percent.

(Flotation costs are fees paid when stock or debt is issued. These fees may go to printer, lawyers, and investment banker, among others.).

Bicksler enterprises is informed that flotation costs will be 1 percent of the gross proceeds of its loan.

#### **Flotation Costs**

Given that flotation costs are 1 percent of the gross proceeds, we have

 $$7,500,000 = (1 - 0.01) \times Gross \ Proceeds = 0.99 \times Gross \ Proceeds$ 

Then the gross proceeds are \$7,575,758. This implies flotation costs is \$75,758. (1%\*\$7,575,758).

The flotation costs are paid immediately but are deducted from taxes by amortizing on a straight line basis over the life of the loan.

The cash flows from flotation costs are:

When discounting at 10 percent, the net cost of flotation is

$$-75,758 + 0.34 \times \frac{75,758}{5} \times A_{0.1}^{5} = -56,228$$

The net present value of the project after the flotation costs of debt but before the benefits of debt is

$$-1.058,106 - 56,228 = -1,114,334$$

#### **Tax Subsidy**

Interest must be paid on the gross proceeds of the loan, even though intermediaries receive the flotation costs.

Since the gross proceeds of the loan are \$7,575,758, annual interest is \$757,576 (\$7,575,758\*0.1). The interest cost after taxes is \$500,000(757,576\*(1-0.34)). Because the loan is nonamortizing, the entire debt of \$7,575,758 is repaid at date 5.

The cash flows are as follows:

#### NPV(loan)

=  $Amount\ Borrowed - PV(after\ tax\ interest\ payment)$ - $PV(loan\ repayment)$ 

$$NPV(loan) = 7,575,758 - 500,000 \times A_{0.1}^5 - \frac{7,575,578}{1.1^5}$$
  
= 976,415

NPV is positive, reflecting the interest tax shield.

The adjusted present value of the project with this financing is

#### APV

- $= NPV(all\ equity\ value) Flotation\ cost\ of\ debt + NPV(loan)$
- = -1.058.106 56,228 + 976,415
- =-137,919

A firm would reject the project even though a \$7,500,000 loan could be obtained.





#### **Non-Market Rate Financing**

Suppose that the project of Bicksler Enterprises is deemed socially beneficial and the state of New Jersey grants the firm a \$7,500,000 loan at 8 percent interest. In addition, all flotation costs are absorbed by the state.

Clearly, the company will choose this loan over the one we previously calculated.

The cash flows from the loan are:

$$NPV(loan)$$
= 7,500,000 - 396,000 ×  $A_{0.1}^5$  -  $\frac{7,500,000}{1.1^5}$ 
= 1,341,939

The NPV of the project with subsidized debt financing is:

# APV $= NPV(all\ equity\ value) - Flotation\ cost\ of\ debt + NPV(loan)$ = -1.058,106 - 0 + 1,341,939 = 283,833



#### Example 3:

Triad Corporation has established a joint venture with Tobacco Road Construction, Inc., to build a toll road in North Carolina.

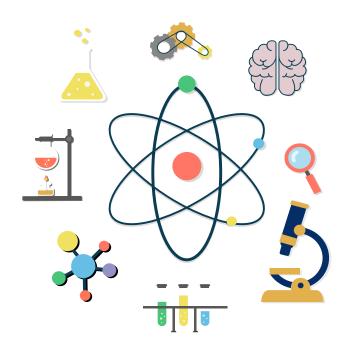
The initial investment in paving equipment is \$20 million. The equipment will be fully depreciated using the straight-line method over its economic life of five years.

Earnings before interest, taxes, and depreciation collected from the toll road are projected to be \$3 million per annum for 20 years starting from the end of the first year. The corporate tax rate is 25 percent. The required rate of return for the project under all-equity financing is 12 percent. The pre-tax cost of debt for the joint partnership is 9 percent per annum.

In order to encourage investment in the country's infrastructure, the U.S. government will subsidize the project with a \$10 million, 15-year loan at an interest rate of 5 percent per year. All principal will be repaid in one balloon payment at the end of year 15.

What is the adjusted present value (APV) of this project?

## Next



In this chapter, we provide three approaches to capital budgeting for the levered firm.

The adjusted present value (APV) approach first values the project on an all equity basis. That is, the project's after tax cash flows under all equity financing (UCF) are placed in the numerator of the capital budgeting equation. The discount rate, assuming all equity financing, appears in the denominator.

We then add the net present value of the debt.

The flow to equity (FTE) approach discounts the after tax cash flow from a project going to the equityholders of a levered firm (LCF).

LCF, which stands for levered cash flow, is the residual to equityholders after interest has been deducted. The discount rate is  $r_s$ , the cost of capital to the equityholders of a levered firm.



The WACC technique calculates the project's after tax cash flows assuming all equity financing (UCF).

The UCF is placed in the numerator of the capital budgeting equation. The denominator  $r_{WACC}$  is a weighted average of the cost of equity capital and the cost of debt capital.

The tax advantage of debt is reflected in the denominator because the cost of debt capital is determined net of corporate tax. The numerator does not reflect debt at all.

#### 5 The Features of the Three Approaches



#### **5.1 APV versus WACC**

Both approaches adjust the basic NPV formula for unlevered firms in order to reflect the tax benefit of leverage.

The APV approach makes this adjustment directly. It simply adds in the present value of the tax shield as a separate term. The WACC approach makes the adjustment in a more subtle way. The discount rate is lowered below  $r_0$ . These two adjustments always have the same quantitative effect.

#### **5.2 Entity Being Valued**

For both the APV and WACC approaches, the initial investment is subtracted out in the final step.

However, for the FTE approach, only the firm's contribution to the initial investment (Initial investment-amount borrowed) is subtracted out. This occurs because under the FTE approach, only the future cash flows to the levered equityholders (LCF) are valued. By contrast, future cash flows to the unlevered equityholders (UCF) are valued in both the APV and WACC approaches.

Thus since LCFs are net of interest payments, whereas UCFs are not, the initial investment under the FTE approach is correspondingly reduced by debt financing.

In this way, the FTE approach produces the same answer that the other two approaches do.



#### **5.3** A Suggested Guideline

- Use WACC or FTE if the firm's target debt to value ratio applies to the project over its life.
- Use APV if the project's level of debt is known over the life of the project.
- If the risk of a project stays constant throughout its life, it is plausible to assume that  $r_0$  remains constant throughout the project's life. This assumption of constant risk appears to be reasonable for most real world project's life.
- In addition, if the debt to value ratio remains constant over the life of the project, both  $r_S$  and  $r_{WACC}$  will remain constant as well. Either the FTE and WACC approach is easy to apply.

Both the FTE and WACC present difficulties when the debt to value ratio changes over time.

Financial managers generally think in terms of target debt value ratios. The WACC and FTE approaches are more appropriate than is the APV approach when a firm focuses on a target debt value ratio.

While the APV approach is based on the level of debt in each future period. Consequently, when the debt level can be specified precisely for future periods, the APV approach is quite easy to use. However, when the debt level is uncertain, the APV approach becomes more problematic.

# 6 Capital Budgeting When the Discount Rate Must Be Estimated



#### Example 4:

World-Wide Enterprises (WWE) is a large conglomerate thinking of entering the widget business, where it plans to finance projects with a debt to value ratio of 25 percent. There is currently one firm in the widget industry, American Widgets (AW). This firm is financed with 40 percent debt and 60 percent equity. The beta of AW's equity is 1.5. AW has a borrowing rate of 12 percent, and WWE expects to borrow for its widget venture at 10 percent.

The corporate tax rate for both firms is 0.4, the market risk premium is 8.5 percent, and the riskless interest rate is 8 percent.

What is the appropriate discount rate for WWE to use for its widget venture?

(1) Determining AW's Cost of Equity Capital.

$$r_S = r_f + \beta \times (\bar{R}_M - r_f)$$
  
20.75% = 8% + 1.5 × 8.5%

(2) Determining AW's Hypothetical All Equity Cost of Capital.

$$r_S = r_0 + \frac{B}{S} \times (1 - T_C) \times (r_0 - r_B)$$

$$20.75\% = r_0 + \frac{0.4}{0.6} \times 0.6 \times (r_0 - 12\%)$$

We find that  $r_0 = 0.1825$ 

(3) Determining  $r_S$  for WWE's Widget Venture.

$$r_S = r_0 + \frac{B}{S} \times (1 - T_C) \times (r_0 - r_B)$$



$$19.9\% = 18.25\% + \frac{1}{3} \times 0.6 \times (18.25\% - 10\%)$$

(4) Determining  $r_{WACC}$  for WWE's Widget Venture.

$$r_{wacc} = \frac{B}{B+S}r_B(1-T_C) + \frac{S}{B+S}r_S$$

$$16.425\% = \frac{1}{4} \times 10\% \times 0.6 + \frac{3}{4} \times 19.9\%$$

# 7 Beta and Leverage

The relationship between the beta of the unlevered firm and the beta of the levered equity is

$$\beta_{equity} = (1 + \frac{(1 - T_C)Debt}{Equity})\beta_{Unlevered\ firm}$$

If we rearrange the formula we have

$$\beta_{Unlevered\ firm} = \beta_{equity} \times \frac{Equity}{Equity + (1 - T_C)Debt}$$

Because 
$$(1 + \frac{(1 - T_C)Debt}{Equity})$$
 must be more than 1 for a levered firm, we have  $\beta_{Unlevered\ firm} < \beta_{equity}$ 

## Example 5:

C.F.Lee Incorporated is considering a **scale-enhancing** project. The market value of the firm's debt is \$100 million, and the market value of the firm's equity is \$200 million.

The debt is considered risk less. The corporate tax rate is 34 percent. Regression analysis indicates that the beta of the firm's equity is 2. The risk free rate is 10 percent, and the expected market premium is 8.5 percent.

What would the project's discount rate be in the hypothetical case that C.F.Lee, Inc., is all equity?

#### Example 6:

The J.Lowes Corporation, which currently manufactures staples, is considering a \$1 million investment in a project in the aircraft adhesives industry. The corporation estimates unlevered after tax cash flows of \$300,000 per year into perpetuity from the project. The firm will finance the project with a debt to value ratio of 0.5. Assume that the beta of debt is zero.

The three competitors in this new industry are currently unlevered, with betas of 1.2, 1.3, and 1.4. Assuming a risk free rate of 5 percent, a market risk premium of 9 percent, and a corporate tax rate of 34 percent, what is the net present value of the project?

### Example 7:

Blue Angel, Inc., a new firm in the holiday gift industry, is considering a project. To better assess the risk of the project, the firm obtained the following information on 10 other firms in the industry.

	Industry Average	Blue Angel
Target D/E	30%	35%
$oldsymbol{eta}_{equity}$	1.5	?
$r_{\scriptscriptstyle B}$	10%	10%

The expected return on the market portfolio is 17 percent, and the risk free rate is 9 percent. Blue Angel is subject to a corporate tax rate of 40 percent.

The project requires an initial outlay of 325,000 and is expected to result in a 55,000 cash inflow (after tax) at the end of the first year. The project will be financed at Blue Angel's target debt equity ratio. Annual cash flow from the project will grow at a constant rate of 5 percent until the end of the fifth year and remain constant forever thereafter.

Should blue Angel invest in the project?

## Example 8:

ABC, Inc., is an unlevered firm with expected annual earnings before taxes of \$30 million in perpetuity. The required return on the firm's unlevered equity is 18 percent, and the firm distributes all of its earnings as dividends at the end of each year. ABC has 1 million shares of common stock outstanding and is subject to a corporate tax rate of 34 percent.

The firm is planning a recapitalization under which it will issue \$50 million of perpetual 10 percent debt and use the proceeds to buy back shares.

A Calculate the value of ABC before the recapitalization plan is announced. What is the value of ABC's equity before the announcement? What is the price per share?

B Use the APV method to calculate the value of ABC after the recapitalization plan is announced. What is the value of ABC's equity after the announcement? What is the price per share?

C How many shares will be repurchased? What is ABC's equity value after the repurchase has been completed? What is the price per share?

D Use the flow to equity method to calculate the value of ABC's equity after the recapitalization.



# Please check the webpage for the fourth homework!

Due on Dec, 23rd in lecture!

Less than one week late: one grade penalty

More than one week late: no accepted

( no excuse, no make-up)

You have to finish the homework individually!

