Intermediate Microeconomics Lecture 13

Oligopoly

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Oligopoly

- A number of firms compete in a certain market.
- Each of them has non-negligible effects on the market.
- Decisions of one firm infuence and are infuenced by other firms.
- In other words, firm strategically interact in the market.
- Examples: automobiles, airlines, aircraft, phone...
- Game theory provides a tool to study firms' behavior in these markets.
- Unless otherwise indicated, we shall consider only pure strategies in our analysis.

Traditional Models

- Quantity competition
  - Simultaneous quantity setting: the Cournot model
  - Quantity leadership: the Stackelberg model
- Price competition
  - Simultaneous price setting: the Bertrand model
- Co-operative behavior: collusion

Games with Continuum of Actions

- The games we will deal within this chapter are different from what we have learned in that players choose actions from infnitely many possible actions
- Although we can no long express a normal form game by a matrix, there is nothing conceptually new.
- Consider a two-player game.
- Each player i can choose an action from A_i. The set A_i might contain infnitely many actions,e.g. quantity, price.
- Player i has a payoff function $u_i(a_i, a_{-i})$ (if $i = 1$, then $-i$ means 2 and if $i = 2$ then $-i$ means 1)

Games with Continuum of Actions (cont.)

- Given a_{-i}, let $BR(a_{-i})$ be i's optimal choice(s).
- The correspondence (function when it is single-valued) BR_i is i's best response correspondence (function).
- A strategy profle$(a_1^* \ , a_2^* \ )$ is a Nash equilibrium of this game if it is a pair of mutual bestresponses:
  - $a_1^* \in BR_1(a_2^*)$
  - $a_2^* \in BR_2(a_1^*)$
- The same idea of backward induction also applies when we talk about sequential move games.

Cournot Model

- Two firms $i = 1, 2$ compete in quantities in a market with (inverse) demand curve:

$$p(y) = a - by$$

- Two firms simultaneously choose their own outputs.
- If firm i chooses output y_i, then the total output of this market is $y_1 + y_2$ and the market price is $a - b(y_1 + y_2)$ where $a > 0$ and $b > 0$
- For simplicity, assume there is no production cost for both firms.

Cournot Model (cont.)

- If firm 2 produces y_2, then firm 1 faces the following profit-maximizing problem:

$$\max_{y_1 \geq 0} [a - b(y_1 + y_2)]y_1$$

- Firm 1's optimal output choice is (assume a is large enough so that we do not need to worry about the constraint $y \geq 0$)

$$y_1 = \frac{a - by_2}{2b}$$

- Similarly, if firm 1 produces y_1, we know the optimal output choice for firm 2 is

$$y_2 = \frac{a - by_1}{2b}$$

- These two functions are firm 1 and 2's best response functions (最优反应函数) respectively.

Cournot Model (cont.)

- Therefore, a pair of outputs (y_1^*, y_1^*) is a Nash equilibrium of this game if and only if it solves

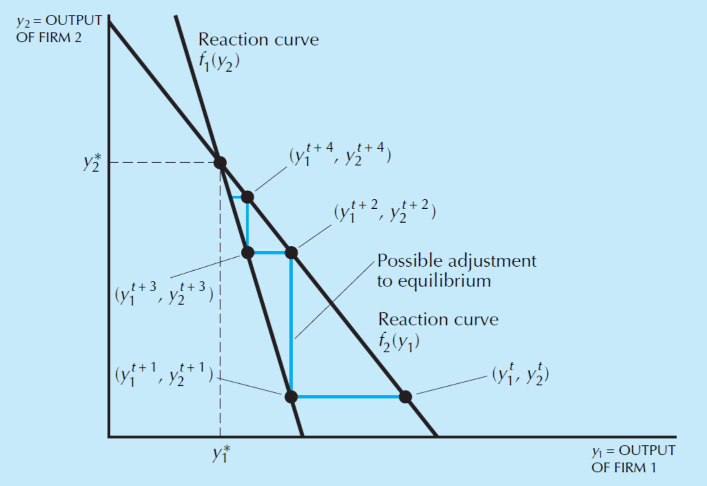
$$y_1^* = \frac{a - by_2^*}{2b}$$

$$y_2^* = \frac{a\text{-}by_1^*}{2b}$$

- Solving these two equations yields $y_1^* = y_2^* = \frac{a}{3b}$
- This is the unique Nash equilibrium, also known as Cournot equilibrium.
- The profits for these two firms are

$$\pi_1^* = \pi_2^* = \frac{a^2}{9b}$$

Cournot Model (cont.)



Cournot Model (cont.)

- The Cournot model can be naturally extended to multiple firms.
- Assume there are $n > 1$ firms.
- The demand function is as above.
- So if firm i produces y_i for $i = 1, \cdots, n$, then the price is

$$a - b \sum y_i$$

- Assume again for simplicity, production cost is 0 for all firms.

Cournot Model (cont.)

- Adopting previous analysis, we can show firm i's best response function is

$$y_i = \frac{a - b\sum_{j \neq i} y_j}{2b}$$

- Rearranging yields

$$y_i + \sum_{j=1}^{n} y_j = \frac{a}{b}$$

- A strategy profile $(y_1^*, \cdots, y_n^*)$ is a Nash equilibrium if and only if it solves

$$y_i^* + \sum_{j=1}^{n} y_j^* = \frac{a}{b} \; \forall i$$

Cournot Model (cont.)

- We get

$$y_i^* = \frac{a}{(n+1)b}$$

- As the number of firms n becomes large, $n \to \infty$
  - the total output $\frac{na}{(n+1)b} \to \frac{a}{b}$
  - the price $p(\frac{na}{(n+1)b}) \to 0$
- That is the market becomes pure competition

Cournot Model (cont.)

- More generally, suppose that there are again two firms, 1 and 2, compete in quantities in a market with the inverse demand function being $p(.)$
- The constant marginal cost of each firm is $c \geq 0$
- The profits of firms i are

$$\pi_i = y_i p(y_i + y_j) - cy_i$$

- Each firm chooses its own output to maximize its own profit, taking the other firms output as given
- The first order condition is

$$p(y_i + y_{-i}) + y_i p'(y_i + y_{-i}) - c \leq 0, \text{ with equality if } y_i > 0$$

Cournot Model (cont.)

- Suppose that $(y_1^*, y_1^*) \gg 0$ is a Nash equilibrium of the model, then

$$p(y_1^* + y_2^*) + y_1^* p'(y_1^* + y_2^*) - c = 0$$

$$p(y_1^* + y_2^*) + y_2^* p'(y_1^* + y_2^*) - c = 0$$

- Therefore,

$$p(y_1^* + y_2^*) + \frac{(y_1^* + y_2^*)}{2} p'(y_1^* + y_2^*) = c$$

- The market price is greater than c $(p'(.) < 0)$

Cournot Model (cont.)

- We can show that the market price is smaller than the monopoly price.
- To show $p(y_1^* + y_2^*) < p^m$; we need to show that $y_1^* + y_2^* > y^m$
- If $y_1^* + y_2^* < y^m$:
  - by increasing y_1 to $y^m - y_2^*$ the total profit of the two firms will be higher, while the market price will be lower
  - this means that firm 2's profit will be lower, while firm 1's profit must therefore be higher
  - contradicts the assumption that y_1^* is an equilibrium price

Cournot Model (cont.)

- If $y_1^* + y_2^* = y^m$:
  - we would have

$$p(y^m) + \frac{(y^m)}{2} p'(y^m) = c$$

  - contradicts the definition of y^m as the solution to

$$p(y^m) + y^m p'(y^m) = c$$

- Hence, $y_1^* + y_2^* > y^m$

Cournot Model (cont.)

- With linear demand $p = a - by$:
  - $y^m = \frac{(a-c)}{2b}$
  - $y^{Cournot} = \frac{2(a-c)}{3b}$
  - $y^c = \frac{a-c}{b}$
- In addition, the joint profit in the Cournot duopoly is lower than the monopoly profit.
- This is because when a firm increases its output, it reduces the other firms profit, which is not taken into account when a firm decides its own optimal output.

Stackelberg Model

- We modify the Cournot game so that firm 1 chooses first and firm 2 chooses after observing firm 1's output.
- Firm 1 is the leader firm (领导者厂商) and firm 2 is the follower (跟随着厂商)
- Assume the market demand is the same as the Cournot game:

$$p(y) = a - by$$

- We again assume that the production cost is 0 for both firms.

Stackelberg Model (cont.)

- This is a sequential move game.
- We use backward induction.
- Assume firm 1 chooses y_1
- After observing y_1, firm 2 faces its profit-maximization problem:
$$\max_{y_2 \geq 0} [a - b(y_1 + y_2)] y_2$$
- We know the solution:
$$y_2 = \frac{a - b y_1}{2b}$$

Stackelberg Model (cont.)

- Firm 1 knows that firm 2 will choose quantity after observing its quantity choice.
- Backward induction also states that 2 will choose $y_2 = \frac{a-by_1}{2b}$ if firm 1 choosesy_1.
- Therefore, firm 1 faces the profit-maximization problem

$$\max_{y_1 \geq 0} [a - b(y_1 + y_2)]y_1$$

$$\text{s.t. } y_2 = \frac{a\text{-}by_1}{2b}$$

- Substituting y_2 by the constraint, firm 1's problem can be rewritten as

$$\max_{y_1 \geq 0} [a - b(y_1 + \frac{a - by_1}{2b})]y_1$$

Stackelberg Model (cont.)

- This implies

$$y_1^* = \frac{a}{2b}$$

and

$$\pi_1^* = \frac{a^2}{8b}$$

- Therefore, in equilibrium, firm 2 will choose

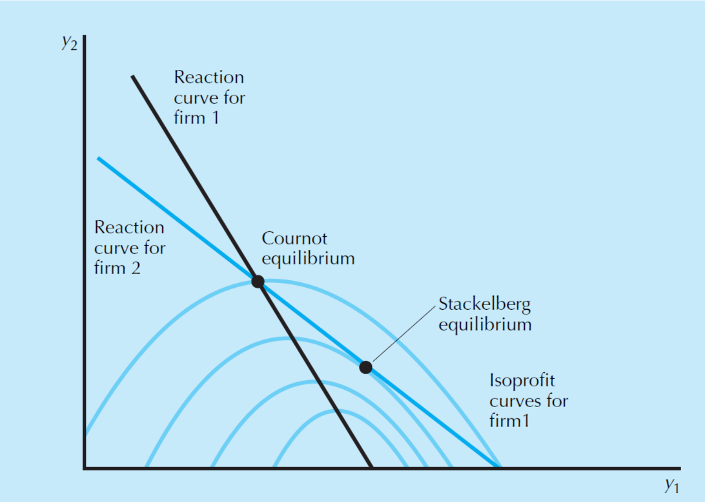
$$y_2^* = \frac{a}{4b}$$

and

$$\pi_2^* = \frac{a^2}{16b}$$

Cournot v.s Stackelberg Model

- Firm 1, the leader, chooses the point on firm 2's reaction curve that touches firm 1's lowest possible isoprofit line



Cournot v.s Stackelberg Model (cont.)

- $\pi_{1S}^* > \pi_{1C}^*$
  - this is obvious because in the Stackelberg model firm 1 always has the opportunity to choose the quantity that it would choose in the Cournot model.
  - this is usually called the first mover advantage (先动优势)
- $y_{1S}^* > y_{1C}^*$
  - Firm 1 always would like firm 2 to choose a lower quantity.
  - Intuitively, by committing to a quantity higher than it would choose in the Cournot model, firm 1 forces firm 2 to choose a lower quantity.

Cournot v.s Stackelberg Model (cont.)

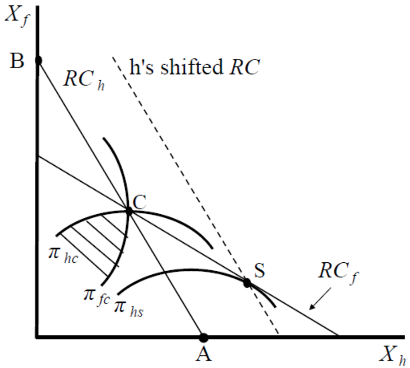
- $y^*_{2S} < y^*_{2C}$
  - this is because of $y^*_{1S} > y^*_{1C}$ and the fact that firm 2's best response decreases in y_1
- $y^*_{1S} + y^*_{2S} > y^*_{1C} + y^*_{2C}$
  - because of $y^*_{1S} > y^*_{1C}$ and the fact that the slope of firm 2's best response function is larger than -1
- $\pi^*_{2S} < \pi^*_{2C}$

An Example: Strategic Government Interactions

- Market Structure
  - Two firms
    - one in country A, the other in B
    - sell in market C
    - no domestic sales
  - They engage in Cournot competition
- Objective
  - maximize the profits of the domestic firm
  - would like foreign firm to produce less
- If home firm cannot do this on is own, home government can give it the advantage using strategic trade policy
  - before firms choose their quantities, the government of country A can set an output tax (subsidy)

Strategic Government Interactions (cont.)

- Cournot equilibrium and profit levels



Strategic Government Interactions (cont.)

- The Cournot equilibrium for the Home and Foreign is point C
  - each firm is choosing its optimal output given the output of the other firm's output
- Now consider a government in the Home country
  - RC_f can be considered as a constraint
  - the best possible point for Home is S
  - foreign firm is worse off

Strategic Government Interactions (cont.)

Strategic trade policy for the Home:

- induce a shift in Home firm's best-response function so that Home firm makes the highest possible profits subject to being on the Foreign firm's best-response function
- we want Home firm to produce more output at each level of Foreign firm's output
- this can be done via a production subsidy
- profits in the world market are shifted from Foreign to Home
  - "profit-shifting" argument

Bertrand Model

- Price competition as opposed to quantity competition.
- There are two firms$i = 1, 2$ selling homogeneous products.
- There is one consumer with a unit demand.
- The two firms announces their prices simultaneously.
- Seeing the prices, the consumer simply buys from the firm which charges a lower price.
- When there is a tie in prices, the consumer randomizes between the two firms with equal probability.
- Constant marginal cost is $c > 0$ for both firms.

Bertrand Model (cont.)

- The only consumer in this setting can also be (is usually) interpreted as a continuum of identical consumers with total unit mass.
- The randomization when there is a tie in prices is then interpreted as that half of the consumers buy from one firm and the other half buy from the other firm.

Bertrand Model (cont.)

- Assume firm 2 announces a price p_2, then firm 1's profit function can be written as

$$\pi_1(p_1, p_2) = \begin{cases} p_1 - c & \text{if } p_1 < p_2 \\ \frac{1}{2}(p_1 - c) & \text{if } p_1 = p_2 \\ 0 & \text{if } p_1 > p \end{cases}$$

- We can similarly write out firm 2's profit function

Bertrand Model (cont.)

- Suppose (p_1^*, p_2^*) is a Nash equilibrium
- Is it possible that $p_1^* > p_2^*$, i.e., firm 2 gets all the demand and firm 1 gets nothing?
  - If $p_2^* > c$, firm 1 can lower its price to p_1 such that $p_2^* > p_1 > c$. In this way, firm 1 can get positive profit.
  - If $p_2^* = c$, then firm 2 can raise its price to p_2 such that $p_1^* > p_2 > p_2^*$. In this way, firm 2 can get positive profit.
  - If $c > p_2^*$, then firm 2 gets negative profits. It can get zero by announcing any price $p_2 > p_1^*$.
- So this is can not be a Nash equilibrium.
- Similarly, any pair (p_1^*, p_2^*) with $p_1^* < p_2^*$ can not be a Nash equilibrium.

Bertrand Model (cont.)

- What about $p_1^* = p_2^* = p^*$
  - If $p^* < c$, then both firm get negative profits. Again one firm can announces a higher price to get zero profit.
  - If $p^* > c$, then each of them gets $(p^* - c)/2 > 0$. But one firm can lower its price by an innitely small amount $\varepsilon > 0$ and obtain the whole demand with profits $(p^* - \varepsilon - c)$.
    - as long as
      $$0 < \varepsilon < \frac{p^* - c}{2}$$
      this firm can get higher profits.

Bertrand Model (cont.)

- Now we are left with $p_1^* = p_2^* = c$.
- This is indeed a Nash equilibrium:
  - given $p_2^* = c$, firm 1 can not gain more by raising its price. Firm 1 will get negative profits if it lowers its price. Hence $p_1^* = c$ is a best response to p_2^*
  - similarly, given $p_1^* = c$, $p_2^* = c$ is a best response for firm 2.

Bertrand Model (cont.)

- This equilibrium is a little counter intuitive.
- This market with only two firms behaves as if it were a competitive market
  - market price is equal to marginal cost.
- The Bertrand model serves as a benchmark to think of sharp small-number price competition.

Bertrand Model (cont.)

- The striking result that with only two firms the competitive price is obtained is troubling to observers of most markets
  - the Bertrand Paradox
- There are several ways to resolve this paradox
  - firms compete in quantities (the Cournot model)
  - capacity constraints
  - differentiated products
  - dynamic models with repeated consumer purchases
  - repeated interactions between firms

Co-operative Behavior: Collusion

- Collusion is illegal in US, but not for international cartels
  - OPEC
- Goal of cartel: Joint profit maximization
  - can achieve (joint) monopoly profits
  - must divide output, profits among cartel members
- Instability:
  - successful cartel has $p > MC$
  - a firm gets huge profits if lowers own price while others hold price constant (cheat on agreement)

Co-operative Behavior: Collusion