Intermediate Microeconomic
Spring 2025

Part two: Choice and Demand

Week 3b: Demand

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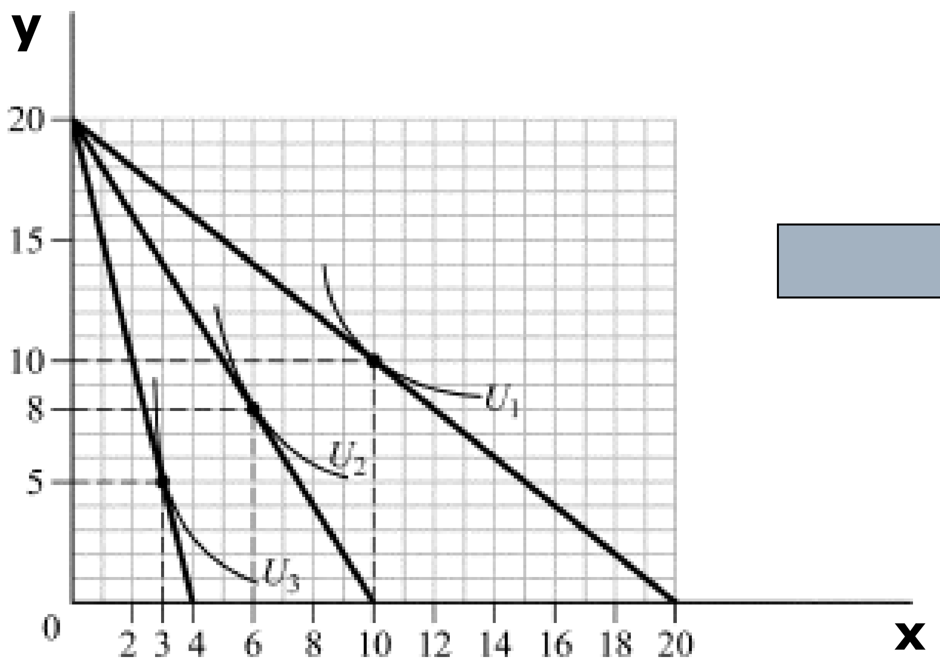
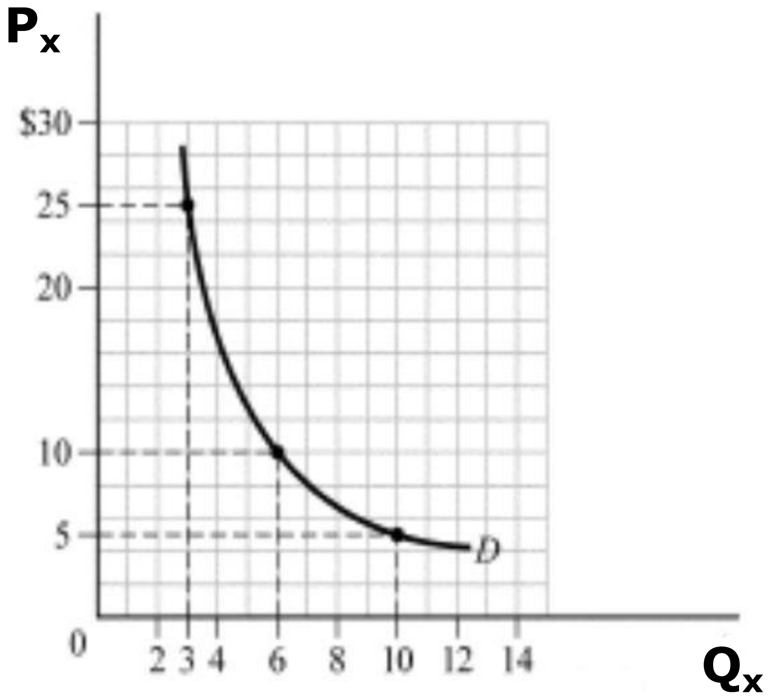
Compensated Demand Curves

- The actual level of utility varies along the Marshallian demand curve



y
20
15
10
8
5
0
2 3 4 6 8 10 12 14 16 18 20
x
U_1
U_2
U_3



P_x
$30
25
20
10
5
0
2 3 4 6 8 10 12 14
Q_x
D

Compensated Demand Curves

- The actual level of utility varies along the Marshallian demand curve

- As the price of x falls, the individual moves to higher indifference curves
  - it is assumed that nominal income is held constant as the demand curve is derived
  - this means that “real” income rises as the price of x falls

- An alternative approach holds “real” income (or utility) constant while examining reactions to changes in p_x
  - the effects of the price change are “compensated” so as to constrain the individual to remain on the same indifference curve
  - reactions to price changes include only substitution effects

Compensated Demand Curves

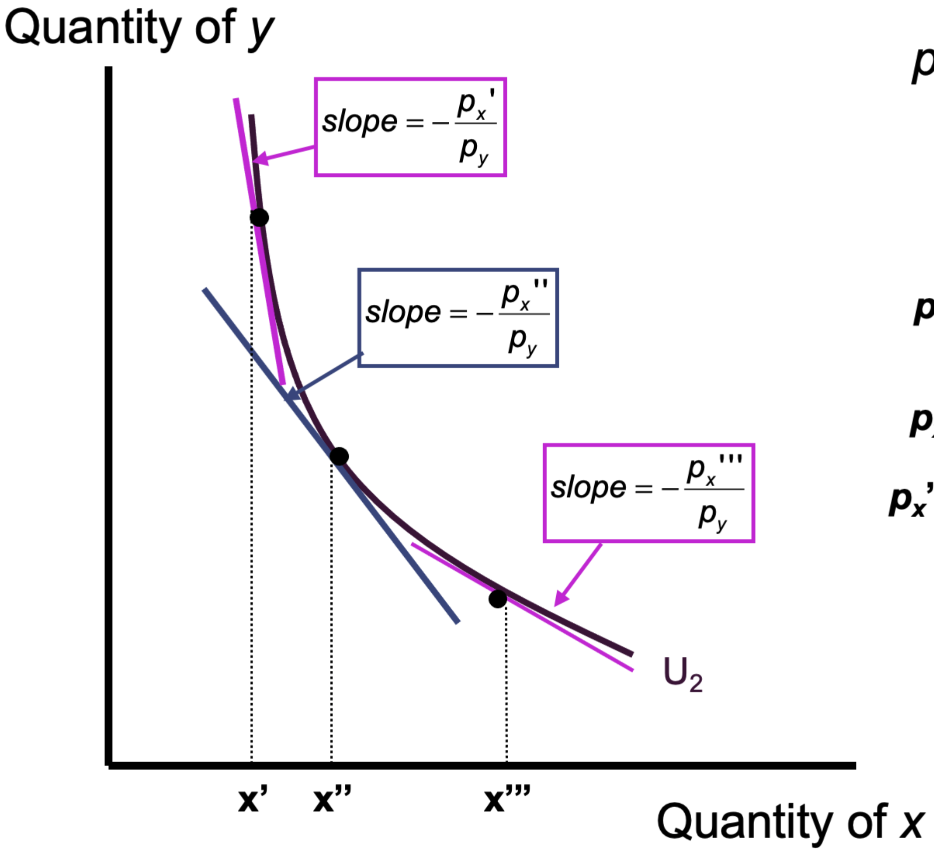
- A compensated (Hicksian) demand curve shows the relationship between the price of a good and the quantity purchased assuming that other prices and utility are held constant.

- The compensated demand curve is a two-dimensional representation of the compensated demand function

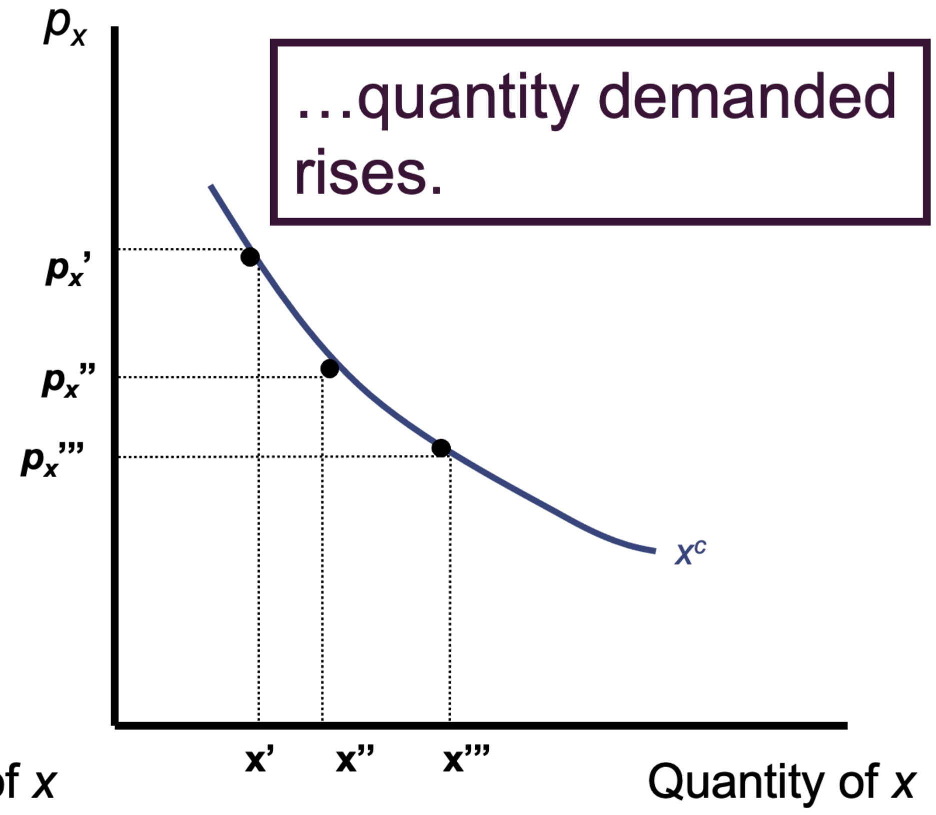
$$x^* = x^c(p_x,p_y,U)$$

Compensated Demand Curves

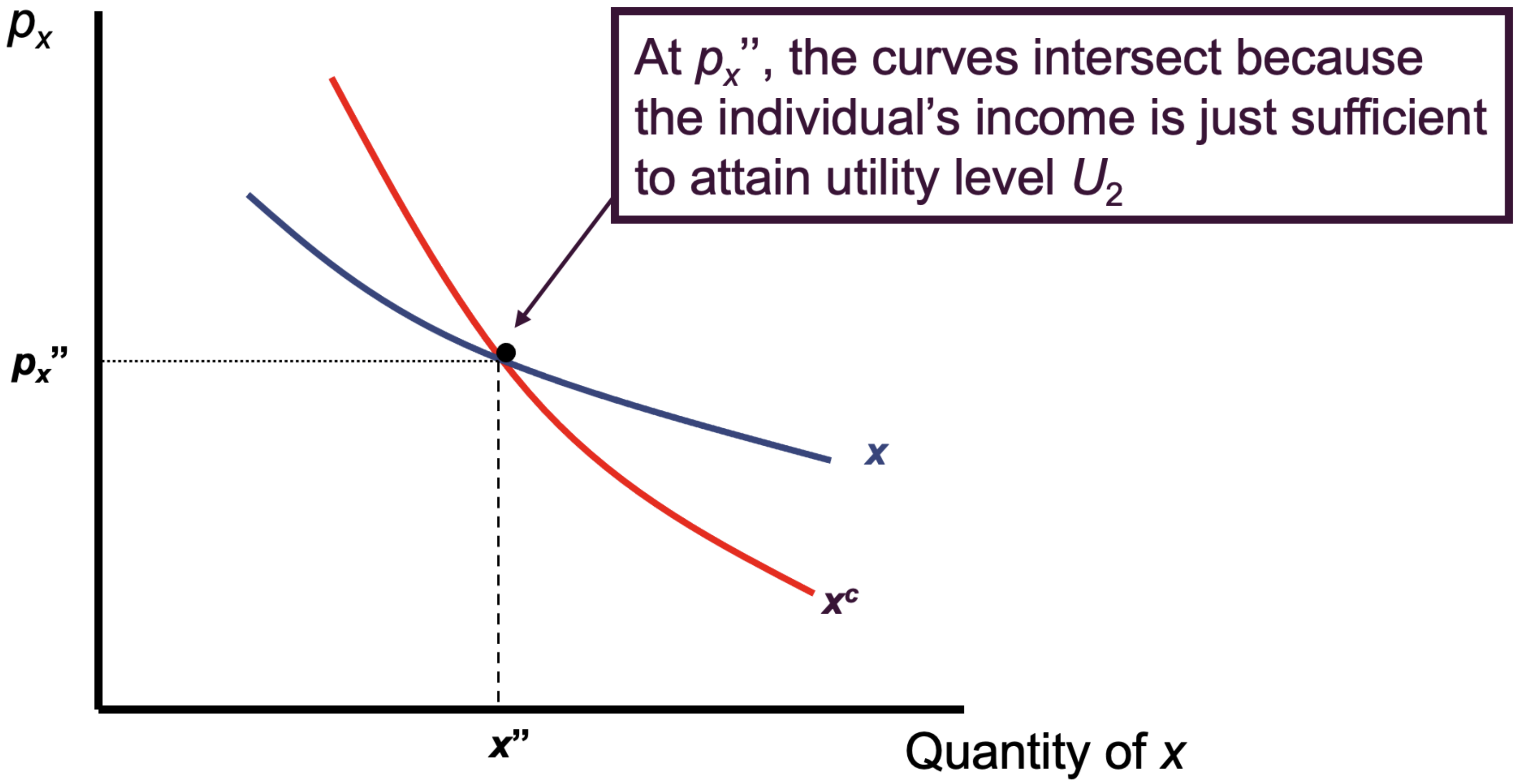
Holding utility constant, as price falls...



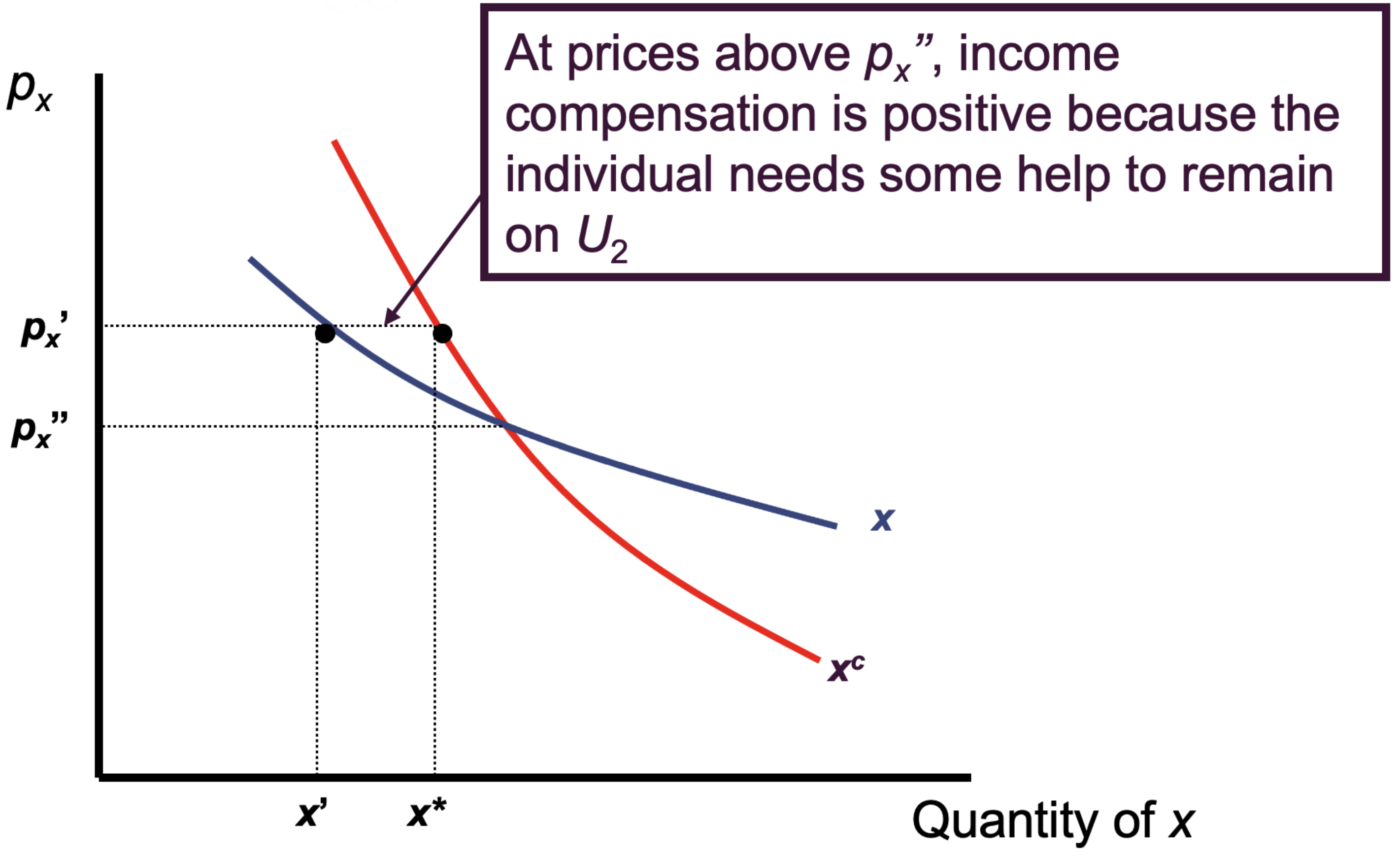
…quantity demanded rises.



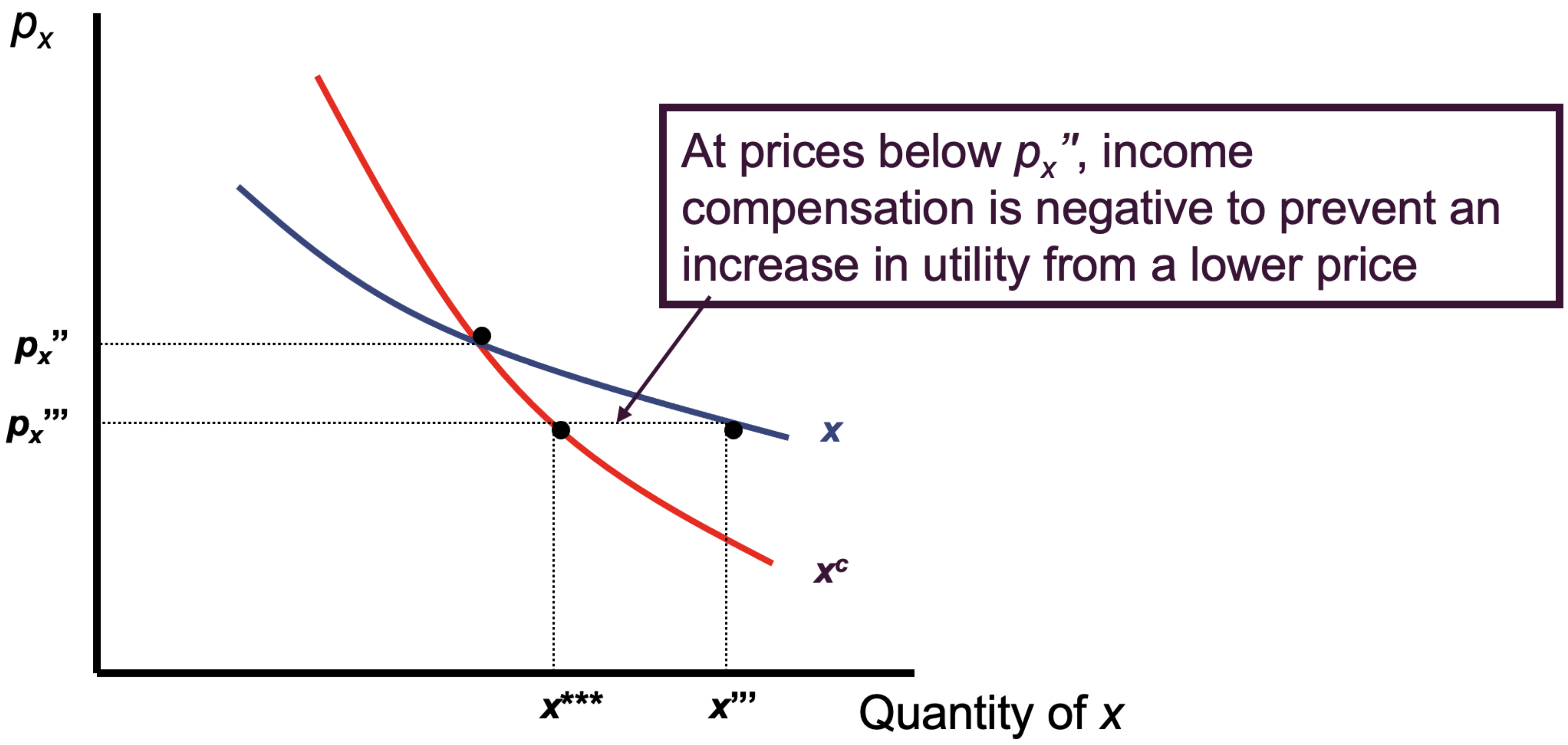
Compensated & Uncompensated Demand



Compensated & Uncompensated Demand



Compensated & Uncompensated Demand



Compensated & Uncompensated Demand

- For a normal good, the compensated demand curve is less responsive to price changes than is the uncompensated demand curve
  - the uncompensated demand curve reflects both income and substitution effects
  - the compensated demand curve reflects only substitution effects (because income is compensated)

Compensated Demand Functions

- Suppose that utility is given by

$$\text{utility} = U(x,y) = x^{0.5}y^{0.5}$$

- The uncompensated (Marshallian) demand functions are

$$x = I/2p_x \qquad\qquad y = I/2p_y$$

- The indirect utility function is

$$\text{utility} = V(I, p_x, p_y) = \frac{I}{2p_x^{0.5}p_y^{0.5}}$$

Compensated Demand Functions

- To obtain the compensated demand functions, we can solve the indirect utility function for I and then substitute into the Marshallian demand functions

$$x^c = \frac{Vp_y^{0.5}}{p_x^{0.5}} \qquad y^c = \frac{Vp_x^{0.5}}{p_y^{0.5}}$$

- Demand now depends on utility (V) rather than income
- Increases in p_x reduce the amount of x demanded
  - only a substitution effect

Duality

- Remember the expenditure function

$$\text{minimum expenditure} = E(p_x,p_y,U)$$

- Then, by definition

$$x^c\,(p_x,p_y,U) = x\,[p_x,p_y,E(p_x,p_y,U)]$$

- The quantity demanded is equal for both Marshallian and Hicksian demand functions when income is exactly what is needed to attain the required utility level.

Deriving the Slutsky equation

- Slutsky discovered that changes to demand from a price change are always the sum of a pure substitution effect and an income effect.

- We can differentiate the compensated demand function

$$x^c(p_x, p_y, U) = x[p_x, p_y, E(p_x, p_y, U)]$$

- and get

$$\frac{\partial x^c}{\partial p_x} = \frac{\partial x}{\partial p_x} + \frac{\partial x}{\partial E} \cdot \frac{\partial E}{\partial p_x}$$

Deriving the Slutsky equation

$$\frac{\partial x}{\partial p_x} = \frac{\partial x^c}{\partial p_x} - \frac{\partial x}{\partial E} \cdot \frac{\partial E}{\partial p_x}$$

- The first term is the slope of the compensated demand curve
  - the substitution effect
- The second term measures the way in which changes in p_x affect the demand for x through changes in purchasing power
  - the income effect

A mathematical representation of income and substitution effects

The substitution effect can be written as

$$\text{substitution effect} = \frac{\partial x^c}{\partial p_x} = \left.\frac{\partial x}{\partial p_x}\right|_{U=\text{constant}}$$

The income effect can be written as

$$\text{income effect} = -\frac{\partial x}{\partial E} \cdot \frac{\partial E}{\partial p_x} = -\frac{\partial x}{\partial I} \cdot \frac{\partial E}{\partial p_x}$$

The Slutsky Equation

- Note that $\partial E/\partial p_x = x$ (as E = $p_x x + p_y y$)
  - a $1 increase in p_x raises necessary expenditures by x dollars

- The utility-maximization hypothesis shows that the substitution and income effects arising from a price change can be represented by

$$\frac{\partial x}{\partial p_x} = \text{substitution effect} + \text{income effect}$$

$$\frac{\partial x}{\partial p_x} = \left.\frac{\partial x}{\partial p_x}\right|_{U=\text{constant}} - x\frac{\partial x}{\partial I}$$

The Slutsky Equation

$$\frac{\partial x}{\partial p_x} = \boxed{\left.\frac{\partial x}{\partial p_x}\right|_{U=\text{constant}}} \boxed{- x \frac{\partial x}{\partial I}}$$

- The first term is the substitution effect
  - always negative as long as *MRS* is diminishing
  - the slope of the compensated demand curve must be negative
- The second term is the income effect
  - if x is a normal good, then $\partial x/\partial I > 0$
    - the entire income effect is negative
  - if x is an inferior good, then $\partial x/\partial I < 0$
    - the entire income effect is positive

A Slutsky Decomposition

- We can demonstrate the decomposition of a price effect using the Cobb-Douglas example studied earlier
- The Marshallian demand function for good *x* is

$$x(p_x,p_y,I) = \frac{0.5I}{p_x}$$

- The Hicksian (compensated) demand function for good *x* is

$$x^c(p_x,p_y,V) = \frac{Vp_y^{0.5}}{p_x^{0.5}}$$

A Slutsky Decomposition

- The overall effect of a price change on the demand for x is

$$\frac{\partial x}{\partial p_x} = \frac{-0.5I}{p_x^2}$$

- This total effect is the sum of the two effects that Slutsky identified

A Slutsky Decomposition

- The substitution effect is found by differentiating the compensated demand function

$$\text{substitution effect} = \frac{\partial x^c}{\partial p_x} = \frac{-0.5Vp_y^{0.5}}{p_x^{1.5}}$$

- We can substitute in for the indirect utility function ($V(p_x,p_y,I)$)

$$\text{substitution effect} = \frac{-0.5(0.5Ip_x^{-0.5}p_y^{-0.5})p_y^{0.5}}{p_x^{1.5}} = \frac{-0.25I}{p_x^2}$$

A Slutsky Decomposition

- Calculation of the income effect is easier

$$\text{income effect} = -x\frac{\partial x}{\partial I} = -\left[\frac{0.5I}{p_x}\right]\cdot\frac{0.5}{p_x} = -\frac{0.25I}{p_x^2}$$

Practice Example: Demand for nonmarket goods

- The theory of **revealed preference** was proposed by Paul Samuelson in the late 1940s
- The theory defines a principle of rationality based on observed behavior and then uses it to approximate an individual's utility function
- For example, it helps us to estimate demand for nonmarket goods.

Revealed Preference

- Consider two bundles of goods: ***A*** and ***B***
- If the individual can afford to purchase either bundle but chooses ***A***, we say that ***A*** had been revealed preferred to ***B***
- Under any other price-income arrangement, ***B*** can never be revealed preferred to ***A***

Applications of revealed preference

- There is no clear way to directly value changes in quantities of environmental/recreational goods.

- Why?

Applications of revealed preference

- There is no clear way to directly value changes in quantities of environmental/recreational goods.
- Why?
- There aren't any markets for them!
- Is there a way we can reveal the value of these goods?

Applications of revealed preference

- There is no market for orcas
- Suppose there's a massive decline in orcas off the Washington coast, what happens?
- We will likely see demand for sightseeing tours go down.
- This drops the price of tours.
- A non-market good had an effect on a market price.

Applications of revealed preference

- One way to circumvent this problem is to look at private goods that interact with the environmental good

- If there are changes in the environmental good, holding everything else fixed, that should be reflected in *some way* in changes in the price of the related private good

- This change in price can tell us something about how people value the change in the environmental good

- 1. Hedonics method
- 2. Travel cost model

2. Travel cost model

- Recreational areas **have value**
- Their quality also has value
- Not placing a value on recreation is essentially giving it a value of **zero**
- This is likely inappropriate
- If someone dumped toxic waste in Yellow River does that have zero cost?

2. Travel cost model

- The travel cost method uses observable data on recreation visitation to infer the recreational value of environmental amenities
- The central idea is that the time and travel cost expenses that people incur to visit a site represent the **price** of access to the site
- This means that people's willingness-to-pay to visit can be estimated based on the number of visits they make to sites of different prices
- This gives us a **demand curve** for sites/amenities, so we can value changes in these environmental amenities

Hotelling

- After WWII, the U.S. national park service solicited advice from economists on methods for quantifying the value of specific park properties
- Would total entrance fee that people pay measure the value?

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- After WWII, the U.S. national park service solicited advice from economists on methods for quantifying the value of specific park properties
- Would total entrance fee that people pay measure the value?
- **No!**
- Harold Hotelling proposed the first indirect method for measuring the demand of a non-market good in 1947

Theoretical foundation

- Here's the theory for how we can use observed data to tell us something about willingness to pay

- Consider a single consumer and a single recreation site

- The consumer has:
  - Total number of recreation trips: x, to site of quality: q
  - Total budget of time: T
  - Working time: H
  - Non-recreation, non-work time: l
  - Time to recreation site: t
  - Hourly wage: w
  - Money cost of reaching the site: c
  - consumption: z

Theoretical foundation

- This lets us write down the consumer's utility maximization problem:

$$\max_{x,z,l} U(x,z,l,q) \quad \text{subject to:} \quad \underbrace{wH = cx + z}_{\text{money budget}}, \ \underbrace{T = H + L + tx}_{\text{time budget}}$$

- Combine the two constraints to get:

$$\max_{x,z,l} U(x,z,l,q) \quad \text{subject to:} \quad \underbrace{wT = z + (c + wt)x + wl}_{\text{combined money/time budget}}$$

Theoretical foundation

- Let $Y = wT$ be the consumer's *full income*, their money value of total time budget
- Let $p = c + wt$ be the consumer's *full price*, their total cost to reach the site

- Then we can write the problem as:

$$\max_{x,z,l} U(x,z,l,q) \quad \text{subject to:} \quad \underbrace{Y = z + px + wl}_{\text{combined budget}}$$

- Solve the constraint for z and substitute into the utility function...

Theoretical foundation

$$\max_{x,l} U\left(x, Y - px - wl, l, q\right)$$

This has first-order conditions:

$$[x] \;\; U_x - pU_z = 0 \rightarrow \frac{U_x}{U_z} = p$$

and

$$[l] \;\; -wU_z + U_l = 0 \rightarrow \frac{U_l}{U_z} = w$$

Theoretical foundation

- $\frac{U_x}{U_z} = p$ tells us the consumer equates the marginal rate of substitution between recreational trips and consumption to be the full price of the recreational trip

- What does this mean?

- The value of the recreational trip to the consumer, in dollar terms, is revealed by the full price p

Theoretical foundation

$$U_x - pU_z = 0 \qquad -wU_z + U_l = 0$$

- The above FOCs are two equations, the consumer had two choices (x,l) so we had two unknowns

- We can thus solve for x (and l) as a function of the parameters (p,Y,q):

$$x = f(p, Y, q)$$

- This is simply the consumer's **demand curves** for recreation as a function of the full price p, full budget Y, and quality q

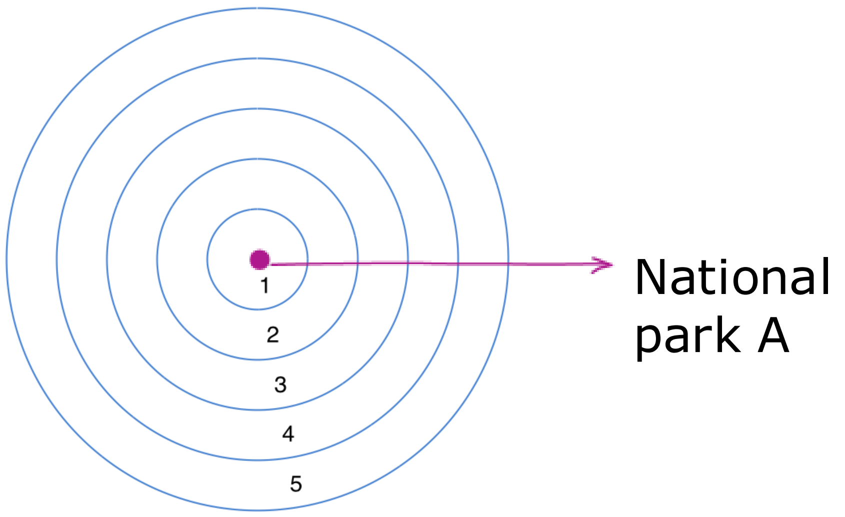
Theoretical foundation

$$x = f(p, Y, q)$$

- If we observe consumers going to sites of different full prices $p_1, p_2, \ldots, p_n$, we are moving up and down their recreation demand curve
- This lets us trace out the demand curve
- Changing Y or q shifts the demand curve in or out: these are income and quasi-price effects

Zonal (single-site) model

- Here's the most basic travel cost model to start:
- A simple example of zones 1-5



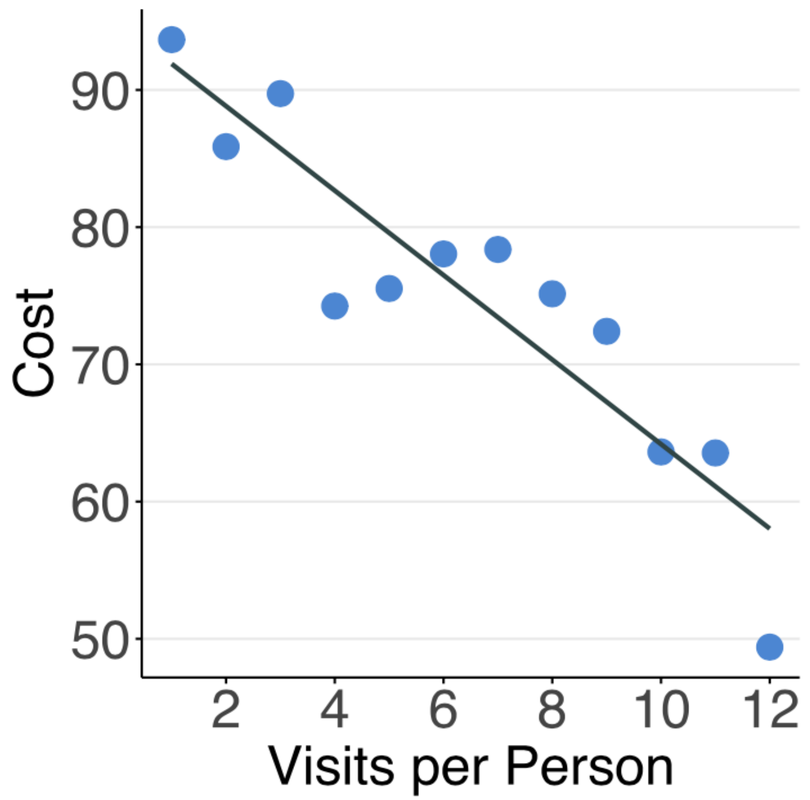
Zonal (single-site) model

Demand curve is simply from estimating

Visits per person = f[travel cost per visit, entrance fee, (and travel costs to other sites), relevant socioeconomic data (income and education, etc)] + ϵ

Or even more simply,

$$\text{Visits per person} = \beta_0 + \beta_1 \text{travel cost} + \epsilon$$



Issues and concerns with the single-site model?

- It ignores non-use value (automatically zero for non-users)
- What are the right zones to choose?
- What is the right functional form for demand?
- How do we measure the opportunity cost of time?
- How do we treat multi-purpose trips?
- How do we value particular site attributes? Can't disentangle them at a single site