Intermediate Microeconomic

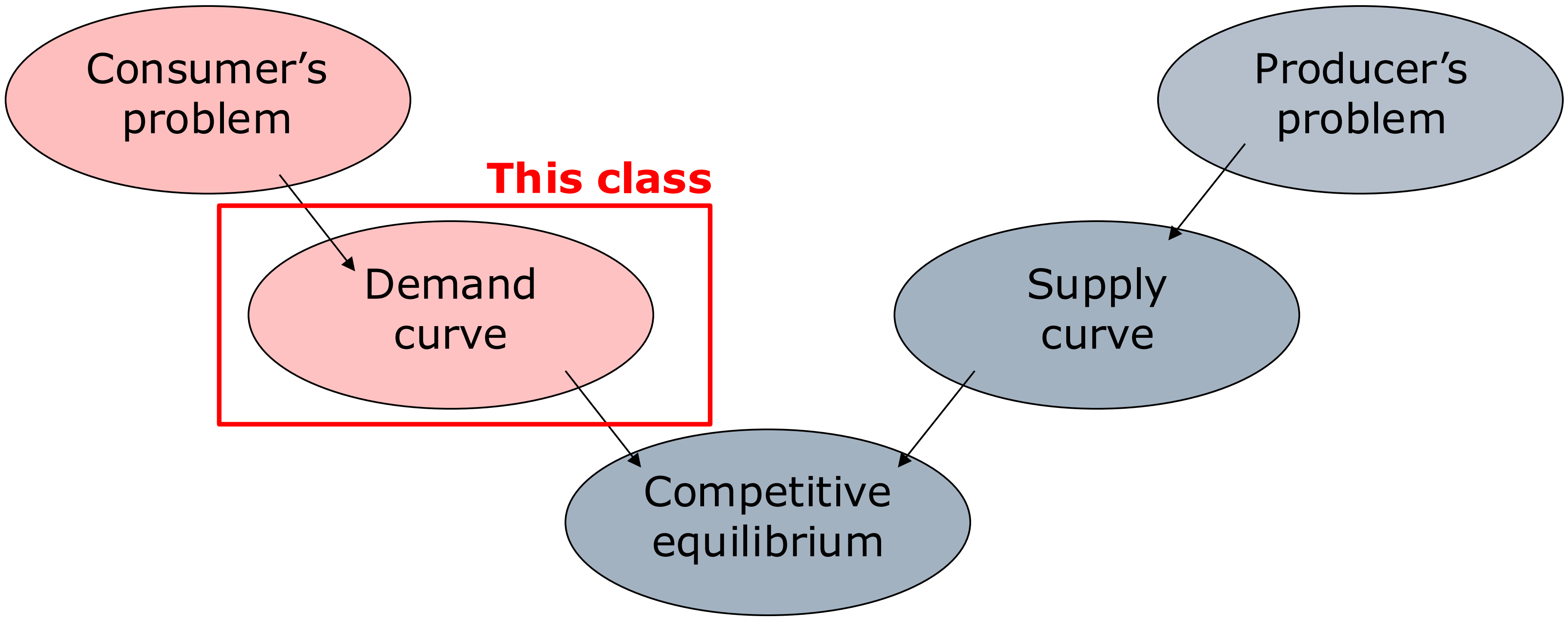
Spring 2025

Part two: Choice and Demand

Week 3a: Demand

Yuanning Liang

Big Picture



Consumer's problem
This class
Demand curve
Producer's problem
Supply curve
Competitive equilibrium

Demand Functions/Curves

- When we solved for x^* and y^*, the solutions were functions of $\text{income}\, m$, price of x p_x, and price of y p_y.

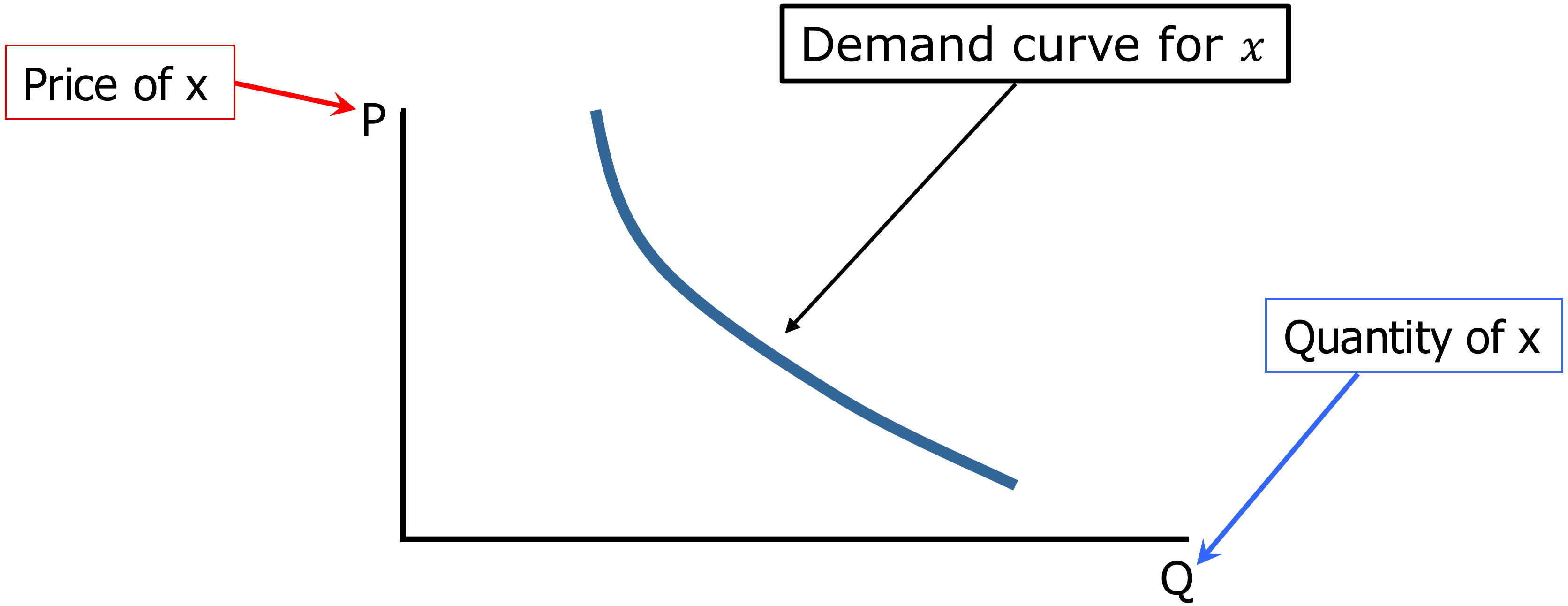
$$x^* = \frac{\alpha m}{(\alpha+\beta)p_x} \quad \text{and} \quad y^* = \frac{\beta m}{(\alpha+\beta)p_y}$$

- When viewed as functions of m and the p's, we call the solution the ***demand function.***

- Sometimes, like when we graph, we think only of the relationship between x or y and one or both prices. When we want to emphasize this, we call it the ***demand curve.***

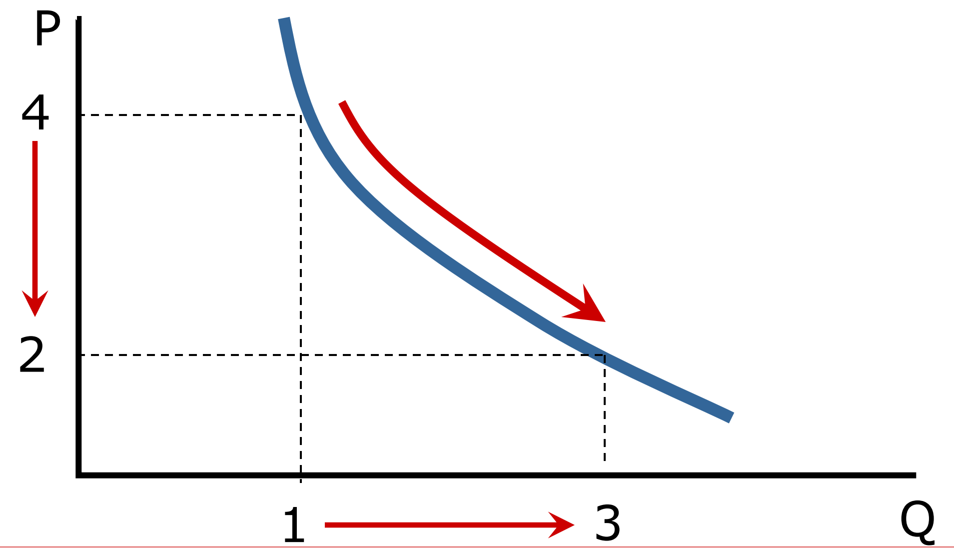
Individual’s Demand Curve

- Graphical representation of demand as function of price of good.
- Note that the axes are backward!



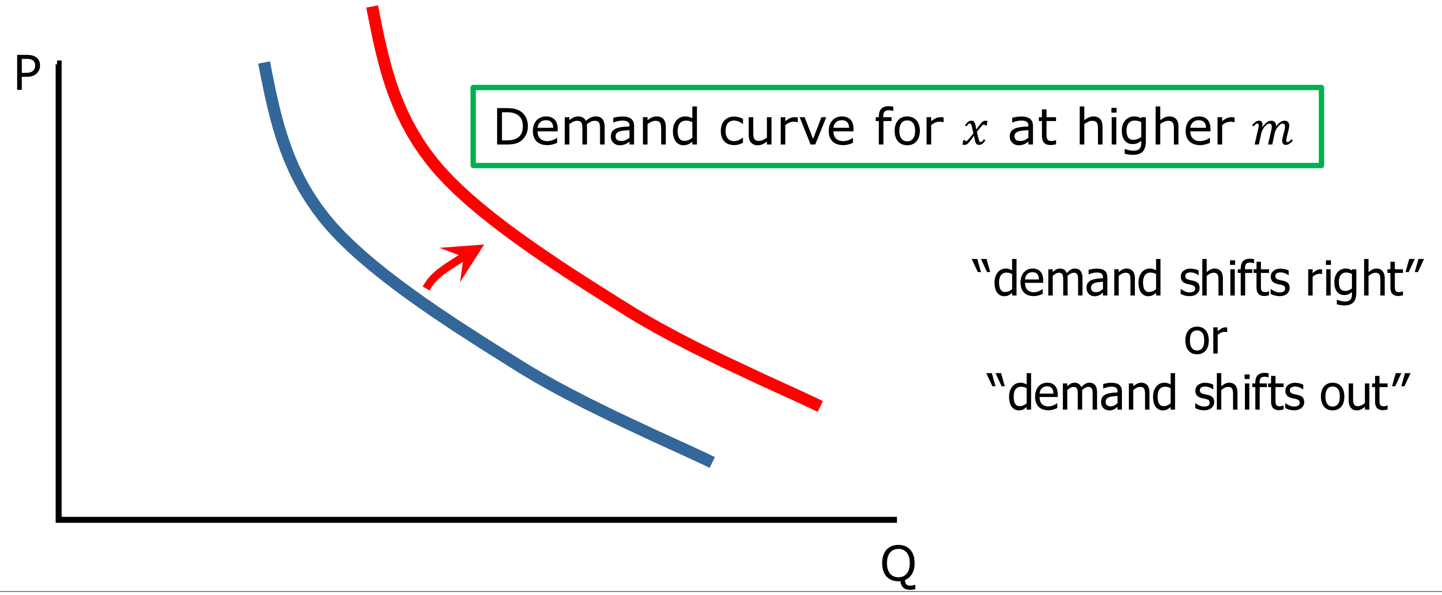
Change in Demand

- As price of good x changes (holding everything else equal, such as price of y and income), we "move along" the demand curve:



Change in Demand

- As <u>other</u> factors change, we "shift" the demand curve
- Other factors change the relationship between price and quantity demanded ***at any price***.



Homogeneous functions

- Definition: A function $f(x_1, x_2, \ldots x_n)$ is **homogeneous of degree *k*** if:

$$f(tx_1, tx_2, \ldots tx_n) = t^k f(x_1, x_2, \ldots x_n)$$

  - When k = 1, a **doubling of all** of its arguments **doubles** the value of the function itself.
  - When k = 0, a **doubling of all** of its arguments leaves the value of the function **unchanged**.

Homogeneity

- If we were to double all prices and income, the optimal quantities demanded will not change
  - the budget constraint is unchanged

$$x_i^* = d_i(p_1,p_2,\ldots,p_n,I) = d_i(tp_1,tp_2,\ldots,tp_n,tI)$$

- Individual demand functions are <u>homogeneous of degree</u> ____?____ in all prices and income

Homogeneity, examples

- With a Cobb-Douglas utility function

$$\text{utility} = U(x,y) = x^{0.3}y^{0.7}$$

the demand functions are

$$x^* = \frac{0.3I}{p_x} \qquad y^* = \frac{0.7I}{p_y}$$

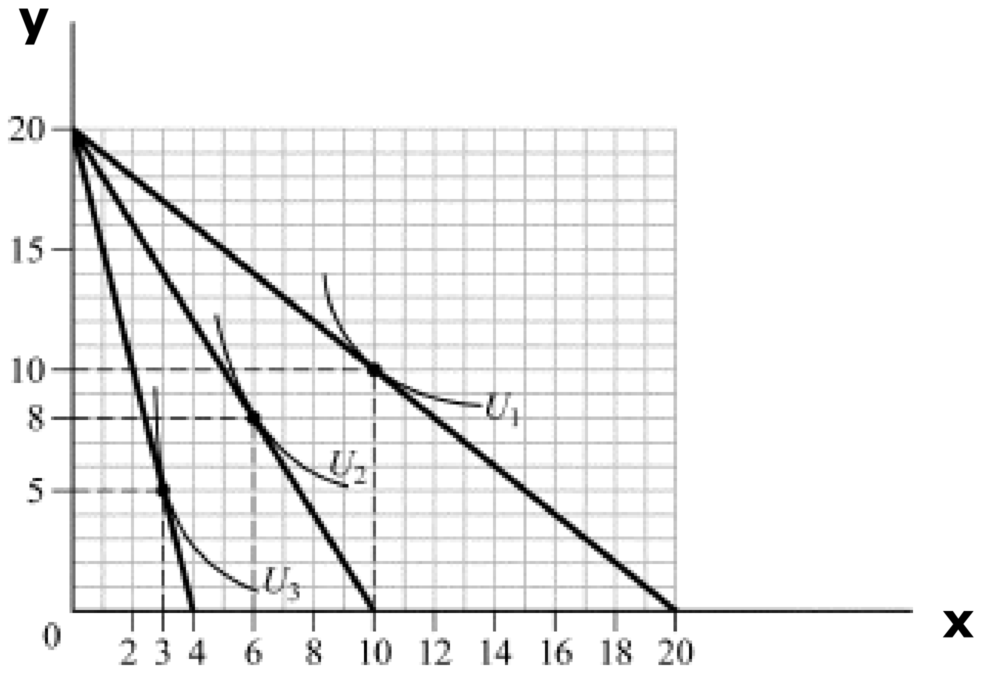
- Note that a doubling of both prices and income would leave x^* and y^* unaffected

Key Takeaway Points

- Most rational preferences can be represented by a **utility function**.
- **Indifference curves** tell us how individuals trade off consumption of different goods, holding utility constant.
- A **budget constraint** helps us pin down the solution to the consumer's utility maximization problem.
- **Demand functions** tell us how much of a particular good a consumer would buy as a function of prices and wealth.
- **Demand curves** tell us how much a particular good a consumer would buy as a function of *that* good's price, holding constant prices for other goods and wealth

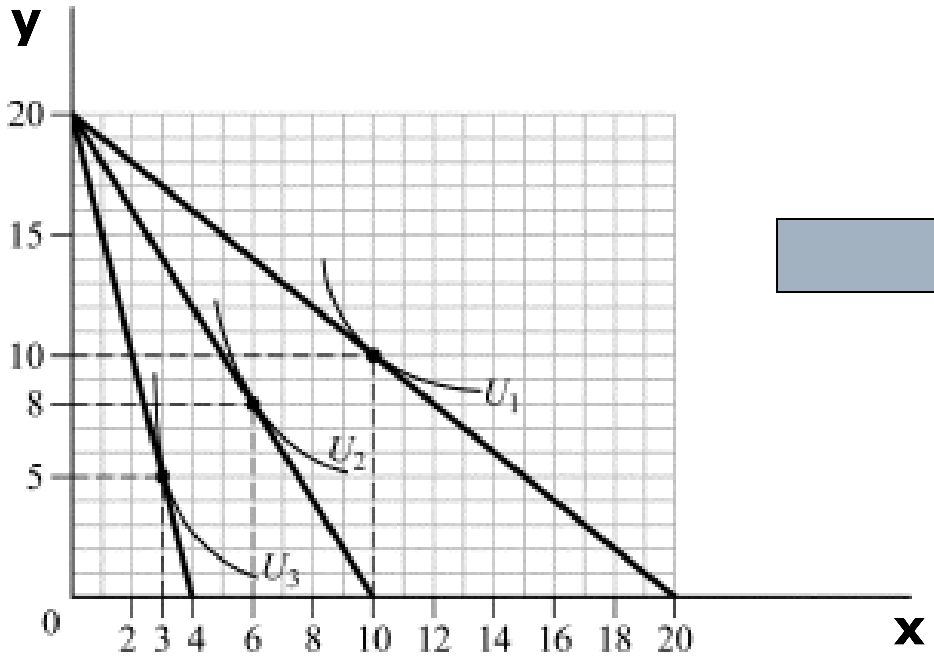
Practice Example

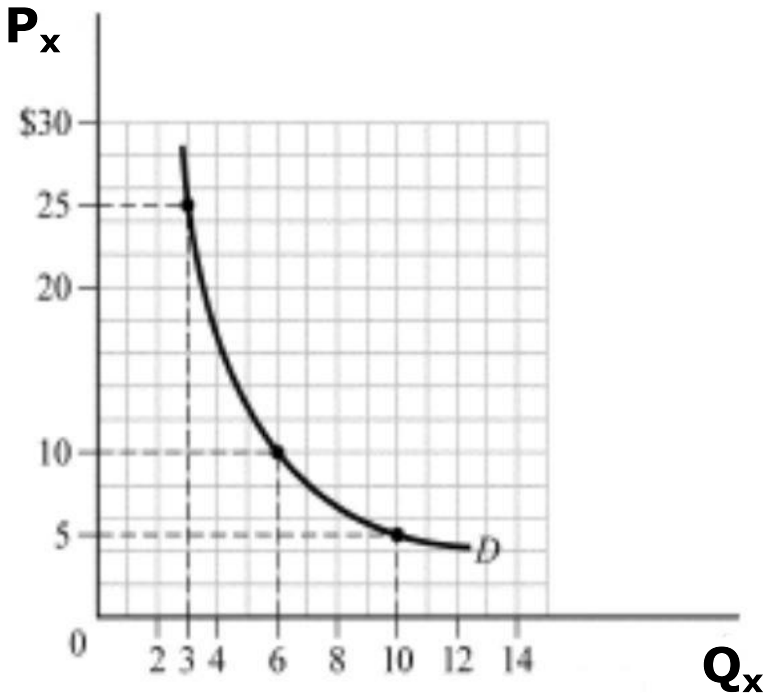
- Consider a consumer with the following indifference curves and budget constraints. Suppose the consumer has a wealth of $100. Plot the consumer's demand curve for good x.



Demand and Utility

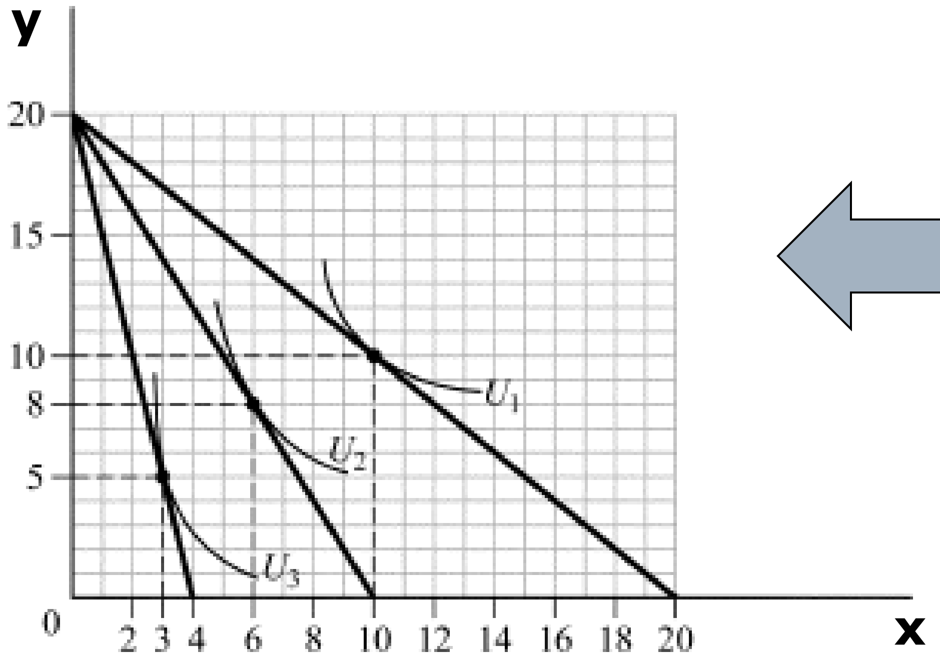
- So far: given a utility function, one can find the associated demand function

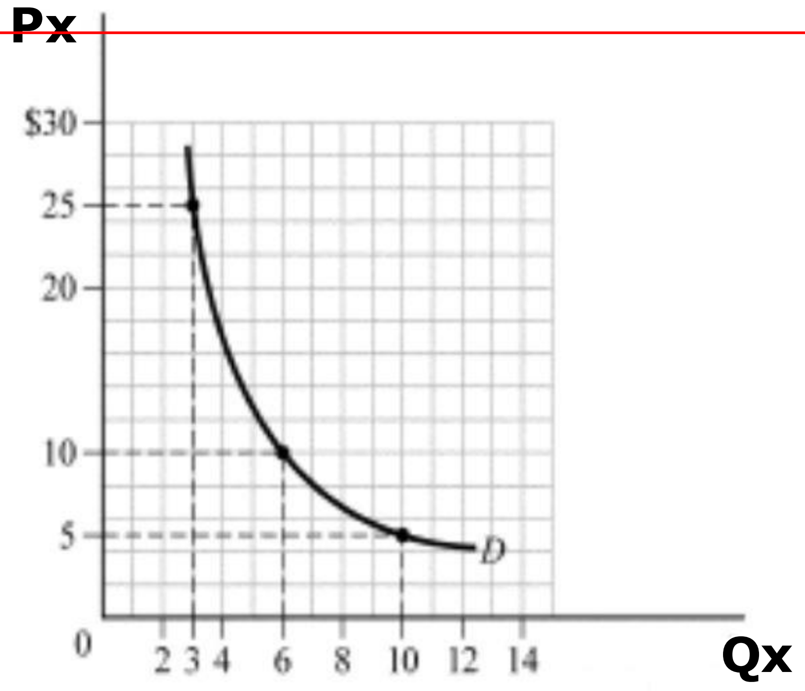




Demand and Utility

- **(Integrability Theorem)** Turns out the reverse is (often) also true: given a demand function, one can construct a utility function that would have generated the demand





Demand Analysis

- Suppose demand for good is $x(p_x, p_y, m)$.
- Demand analysis: what happens to demand for x if we change p_x (or p_y, or m)?
  - **Algebraic analysis: elasticity**
  - Graphical analysis: income and substitution effects

Elasticities: a reminder

- An elasticity answers the following question: *By what percent does the demand for x change if I change some other variable (e.g., price or wealth) by 1%*?

- Formally, the elasticity of demand for x with respect to a change in some factor, z, is:

$$\frac{\%\Delta x}{\%\Delta z} = \frac{\Delta x / x}{\Delta z / z} = \frac{\Delta x}{\Delta z} * \frac{z}{x}$$

- In calculus terms, $\frac{\Delta x}{\Delta z} \approx \frac{\partial x}{\partial z}$, and the elasticity formula becomes

$$\varepsilon = \frac{\partial x}{\partial z} * \frac{z}{x}$$

(Own) Price Elasticity

- By what % does quantity of x change in response to a 1% increase in its own price?

$$\varepsilon_{xp_x} = \frac{\partial x}{\partial p_x} * \frac{p_x}{x}$$

- Generally negative in sign
- Some terminology:
  - Demand is **elastic** if $|\varepsilon| > 1$
  - Demand is **inelastic** if $|\varepsilon| < 1$
  - Demand is **unit-elastic** if $|\varepsilon| = 1$

e < 1, curve steeper;
e > 1, curve flatter

Special Case 1: Perfectly Elastic Demand

P_x

If demand is perfectly elastic, it means that:

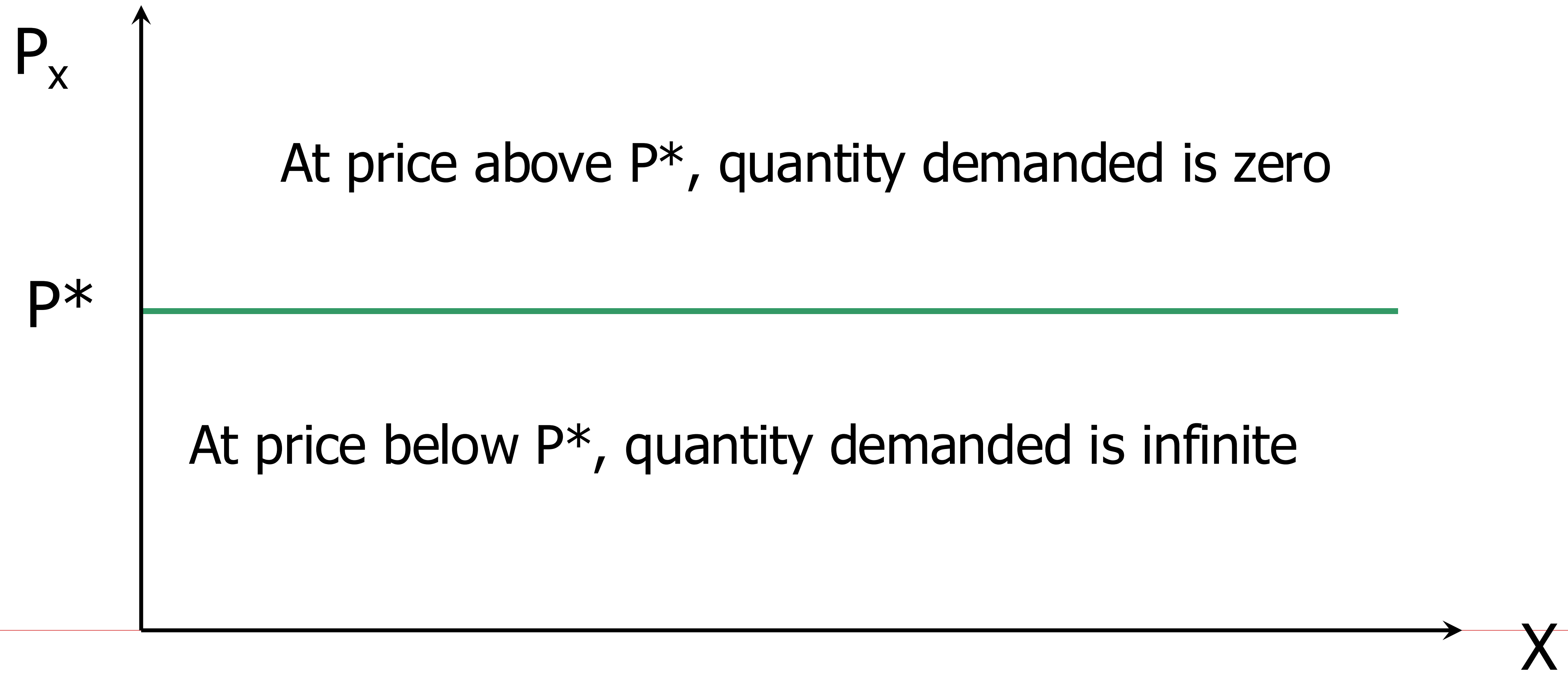
$$\varepsilon = \frac{\partial X(P_x, P_y, M)}{\partial P_x} \cdot \frac{P_x}{X} = -\infty$$

What would this look like?

Hint: note that the axes are "backwards", so -∂x/∂p is actually 1/slope.

X

Special Case 1: Perfectly Elastic Demand



Special Case 2: Perfectly Inelastic Demand

P_x

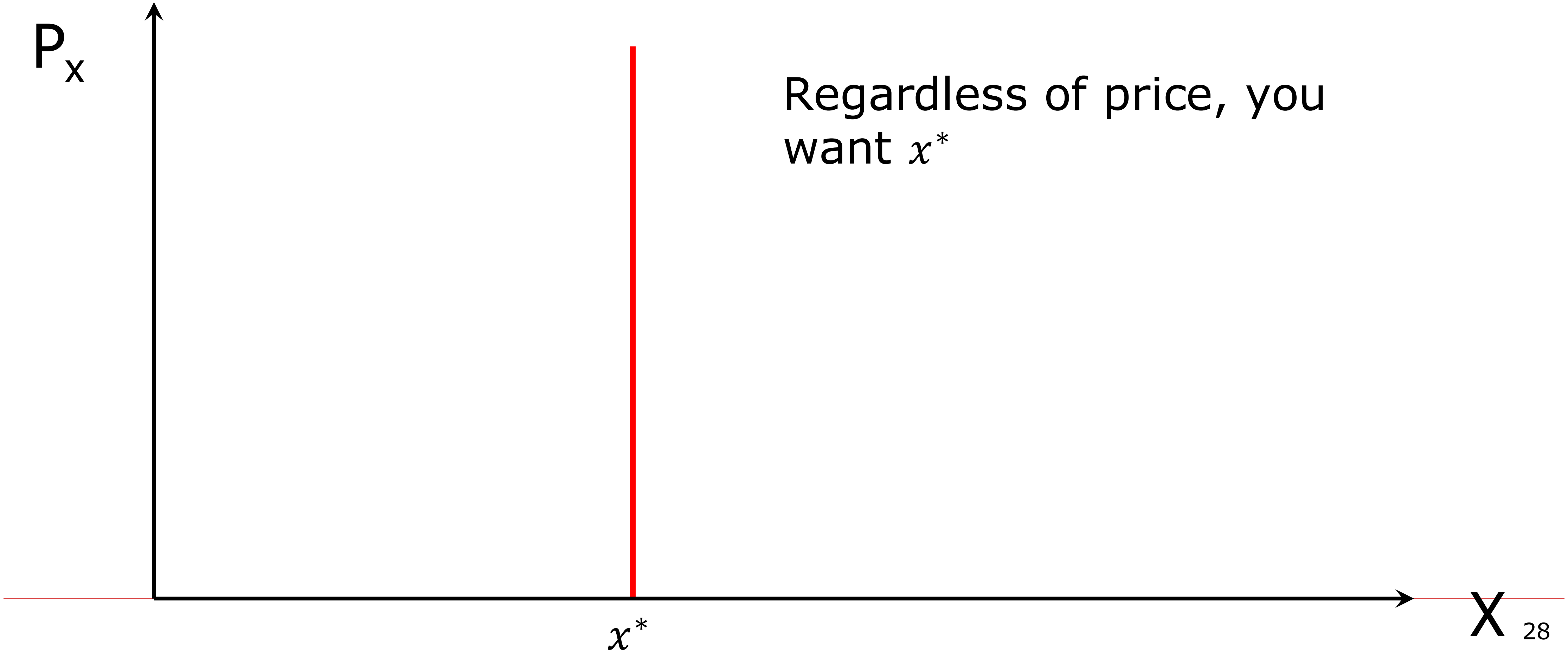
If demand is perfectly inelastic, it means that:

$$\varepsilon = \frac{\partial X(P_x, P_y, M)}{\partial P_x} \cdot \frac{P_x}{X} = 0$$

What would this look like?

X

Special Case 2: Perfectly Inelastic Demand



Regardless of price, you want x^*

Income Elasticity

- By what % does quantity of x change if income increases by 1%?

$$\varepsilon_{xm} = \frac{\partial x}{\partial m} * \frac{m}{x}$$

- Unlike own-price elasticity, income elasticity isn't necessarily positive
  - A good is **normal** good if $\varepsilon > 0$
  - A good is **inferior** good if $\varepsilon < 0$

- Some other terminology:
  - A good is luxury good if $\varepsilon > 1$ (e.g., 10% increase in m increases consumption by more than 10%)
  - A good is necessity good if $\varepsilon < 1$

Cross-Price Elasticity

- By what % does quantity of x change if price of y increases by 1%?

$$\varepsilon_{xp_y} = \frac{\partial x}{\partial p_y} * \frac{p_y}{x}$$

- Some terminology:
  - Goods x and y are **substitutes** if $\varepsilon_{xp_y} > 0$
  - Goods x and y are **complements** if $\varepsilon_{xp_y} < 0$
  - Neither complements nor substitutes if $\varepsilon_{xp_y} = 0$

Understanding Check

- **Consider demand function**

$$x(p_x, p_y, m) = 100 - 2p_x + 3p_y + 10m$$

and suppose current price and income is $p_x = 6, p_y = 4, m = 14$

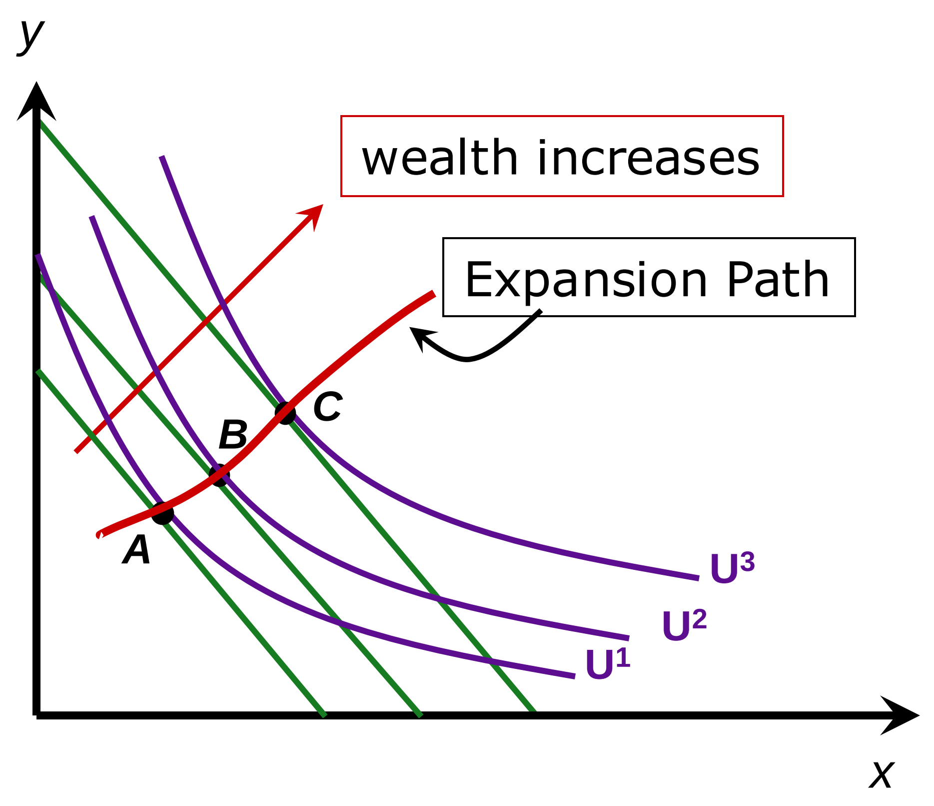
  - **Compute income elasticity of x**
  - **Compute cross-price elasticity of x**
  - **Is x a normal good?**
  - **Are x and y substitutes?**

Demand Analysis

- Suppose demand for good is $x(p_x, p_y, m)$.
- Demand analysis: what happens to demand for x if we change p_x (or p_y, or m)?
  - Algebraic analysis: elasticity
  - **Graphical analysis: income and substitution effects**

Effect of a change in wealth

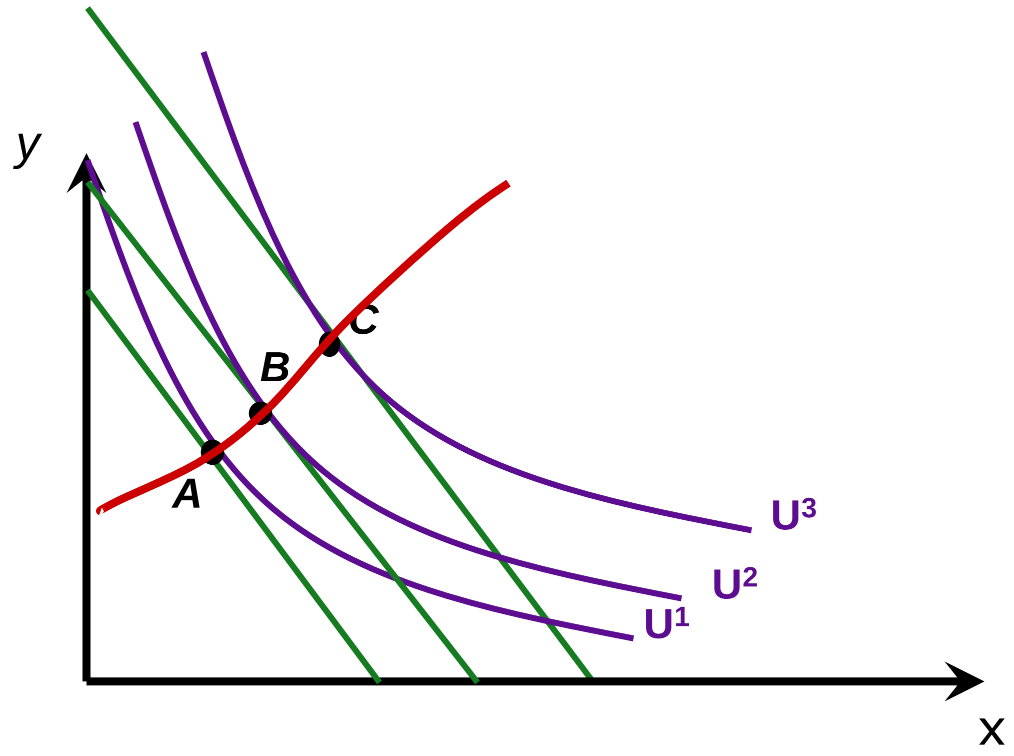
- Recall that an increase in m shifts the budget line out.
- The consumer maximizes utility for each budget.
- Done many times, this traces out a curve called an "Expansion path."
- **Expansion Path:** How does demand change as wealth increases.



Normal Goods

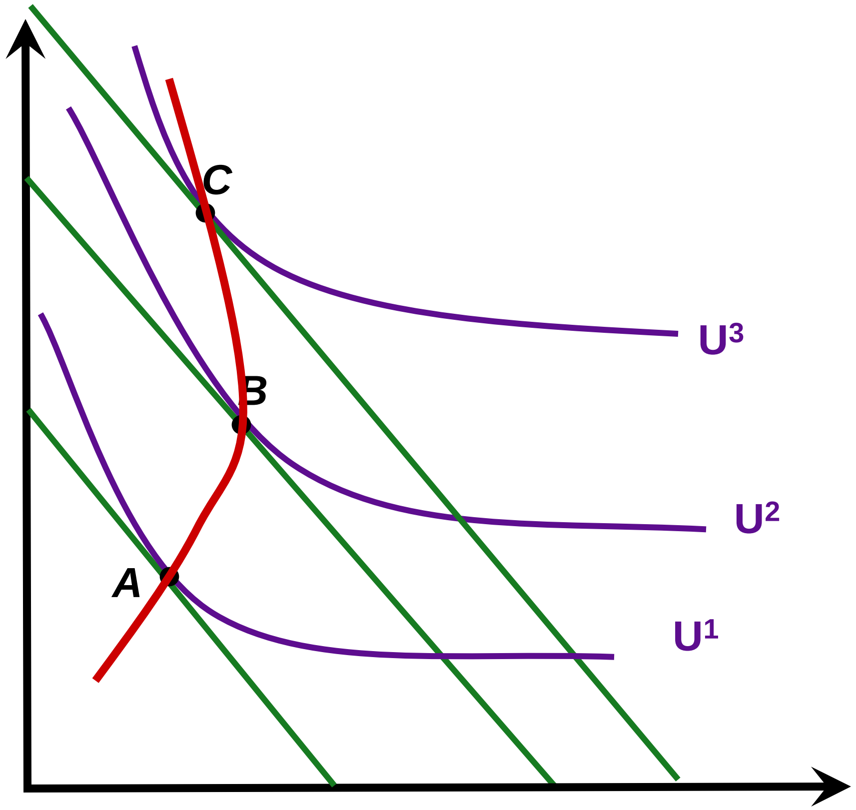
- **Normal Good:** if an increase in m increases the quantity demanded of a good, we call it a normal good.

- In this diagram, both goods are normal goods.



Inferior Goods

- But, not all goods are normal.
- Here, the expansion path bends backward from B to C.
- Consumption of x decreases over this range.
- **Inferior Good:** an increase in wealth reduces quantity demanded.
- Examples?
  - Low quality options.



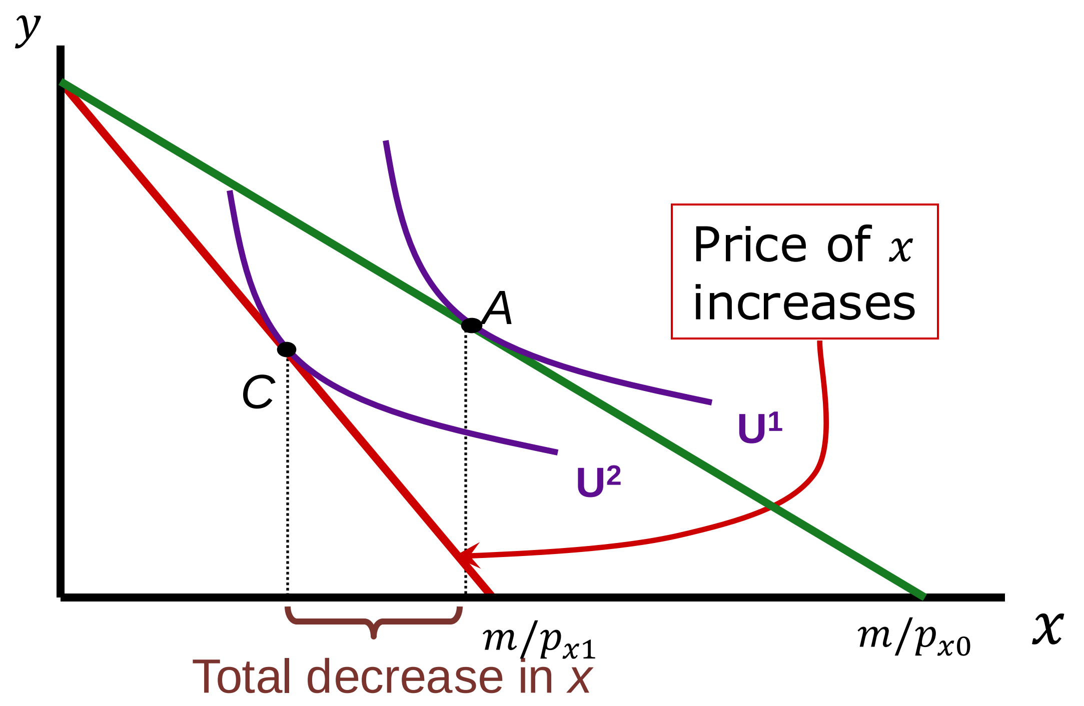
Normal and Inferior Goods: Clarification

- To some extent, normal (inferior) “good” is a misnomer
  - The same good can be normal for consumer A, but inferior to consumer B
  - Even for the same consumer, a good can be normal for some (low) income levels, and inferior at others

- So, normal (inferior) is less of a property of the good itself, but more of a property of the consumer’s preference

Effect of a change in price

- How does a consumer respond to the change in the price of a good?
- If price of good x goes up, budget line gets steeper (i.e., p_x/p_y goes up).
- This means that MRS must increase.
- ***Usually,*** this involves consuming less x.
- Demand moves from A to C (yes, I have a reason for calling it C!)

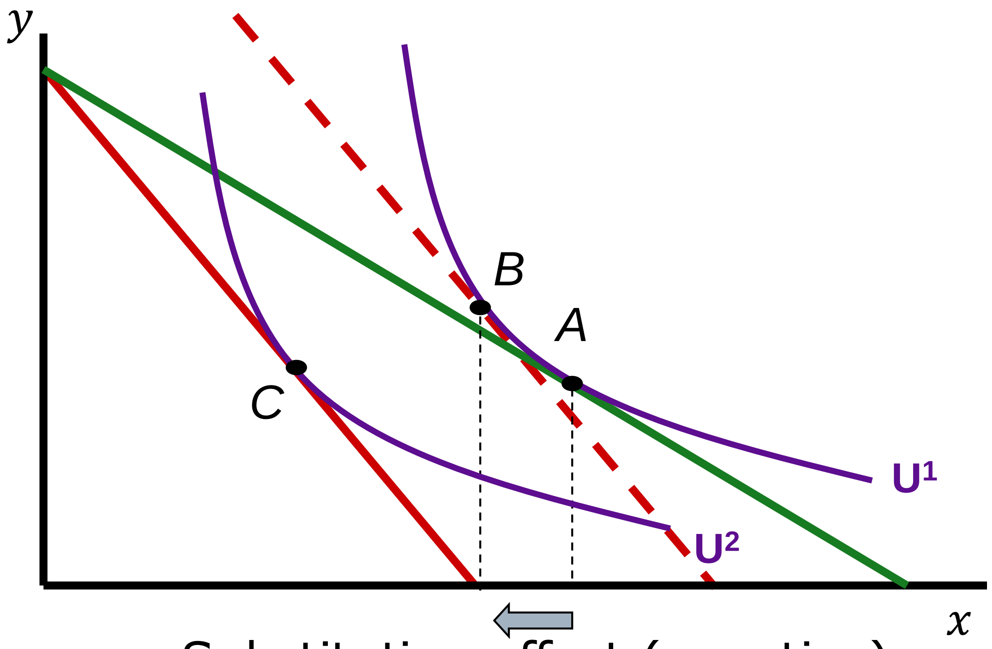


Effect of a price change

- In the case of a normal good, quantity demanded always decreases.
- Why?
- There are actually two reasons:
  - When the relative price of x goes up, substitute toward y since it is relatively cheaper now.
  - When the price of x goes up, you feel poorer, and poorer people reduce consumption of normal goods.
- The first is called the ***substitution effect***. The second is called the ***income effect.***

Income and Substitution Effects: Normal Good

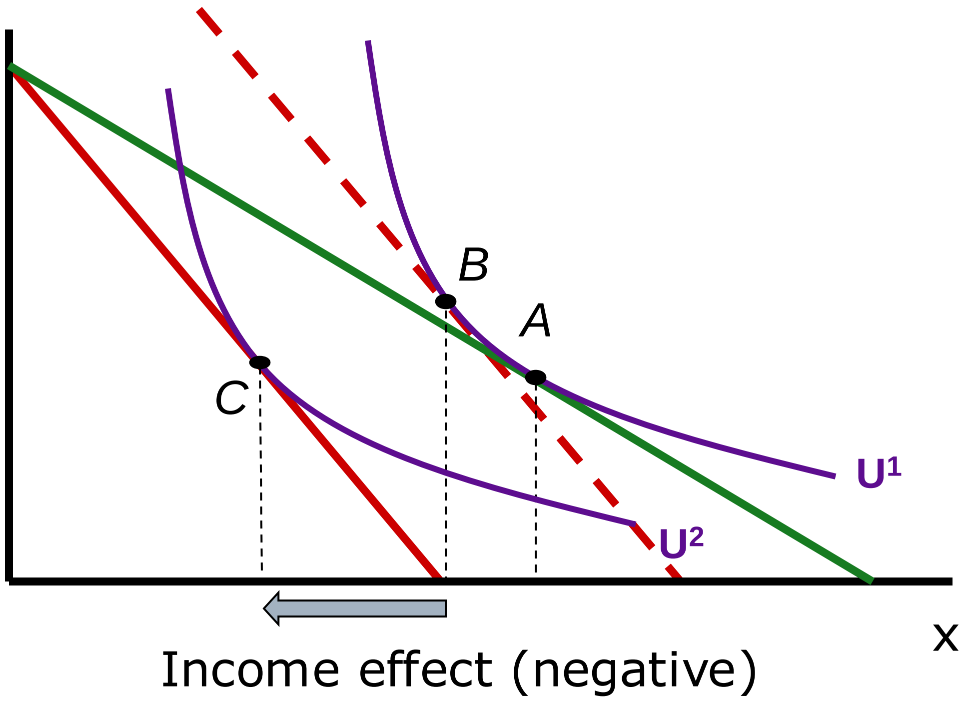
- Let's isolate the two effects.
- The **substitution effect** captures how you respond to the relative price change, even if there were no income effect.
- Substitution effect assumes you can still reach **original utility level**, but face the new prices.
- So, look for the point along U^1 where MRS = new price ratio.
- Look for the point along U^1 where MRS = new price ratio.
- Here, it is from A to B.
- The price increase leads to more y and less x being chosen.



Substitution effect (negative)

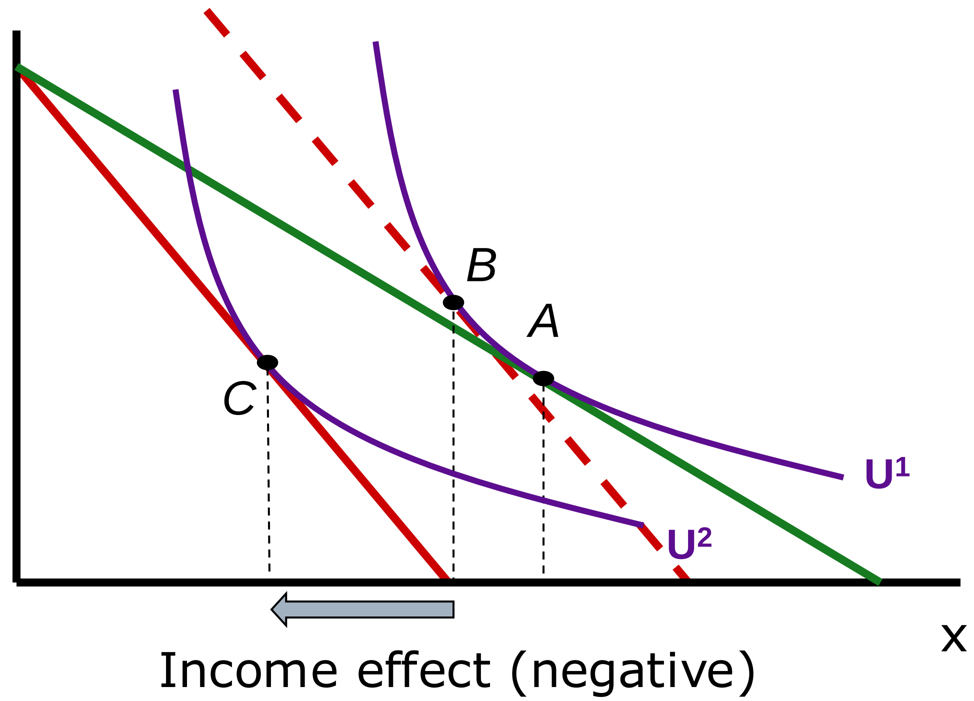
Income and Substitution Effects: Normal Good

- Next, the **income effect**.
- Recall that changes in income lead to parallel shifts in the budget line.
- The price increase makes the consumer poorer.
- Income effect is captured by the parallel shift in the budget line from the orange dashed line to the orange solid budget line.



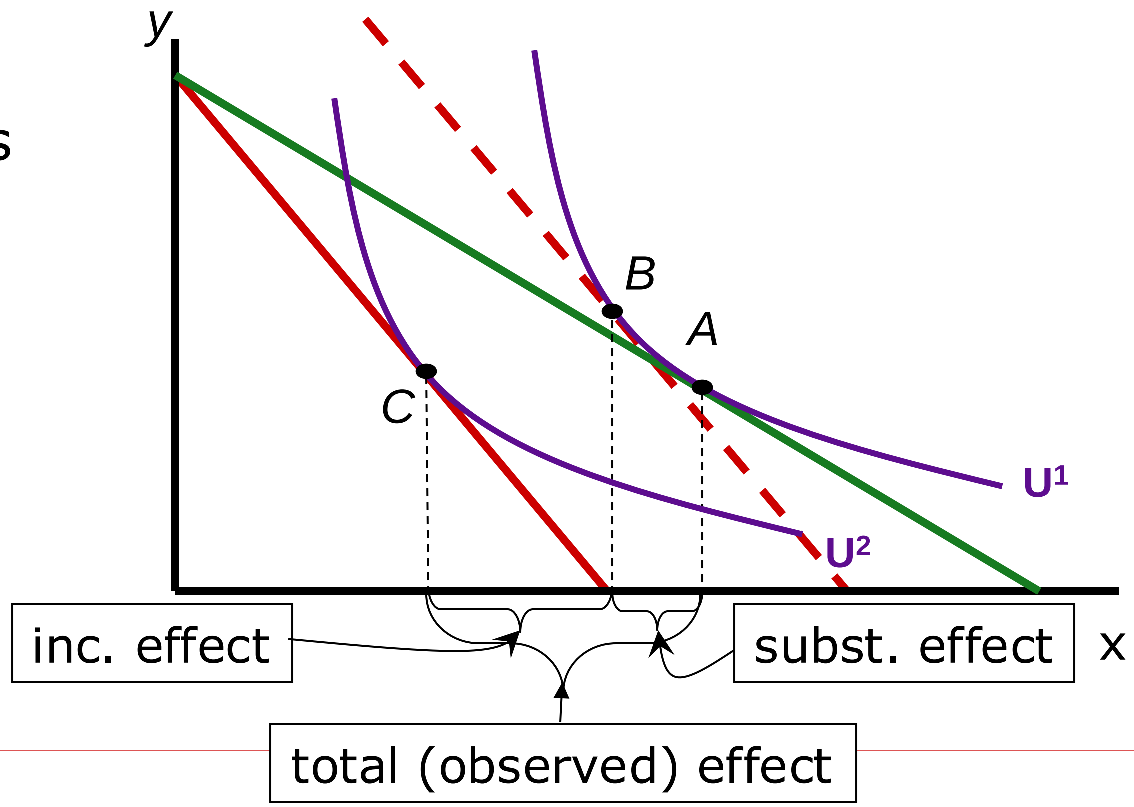
Income and Substitution Effects: Normal Good

- The income effect is the change from point B to point C.
- The income effect is also negative → x must be a normal good.
- The income effect is negative for a normal good because a price increase makes the consumer poorer.



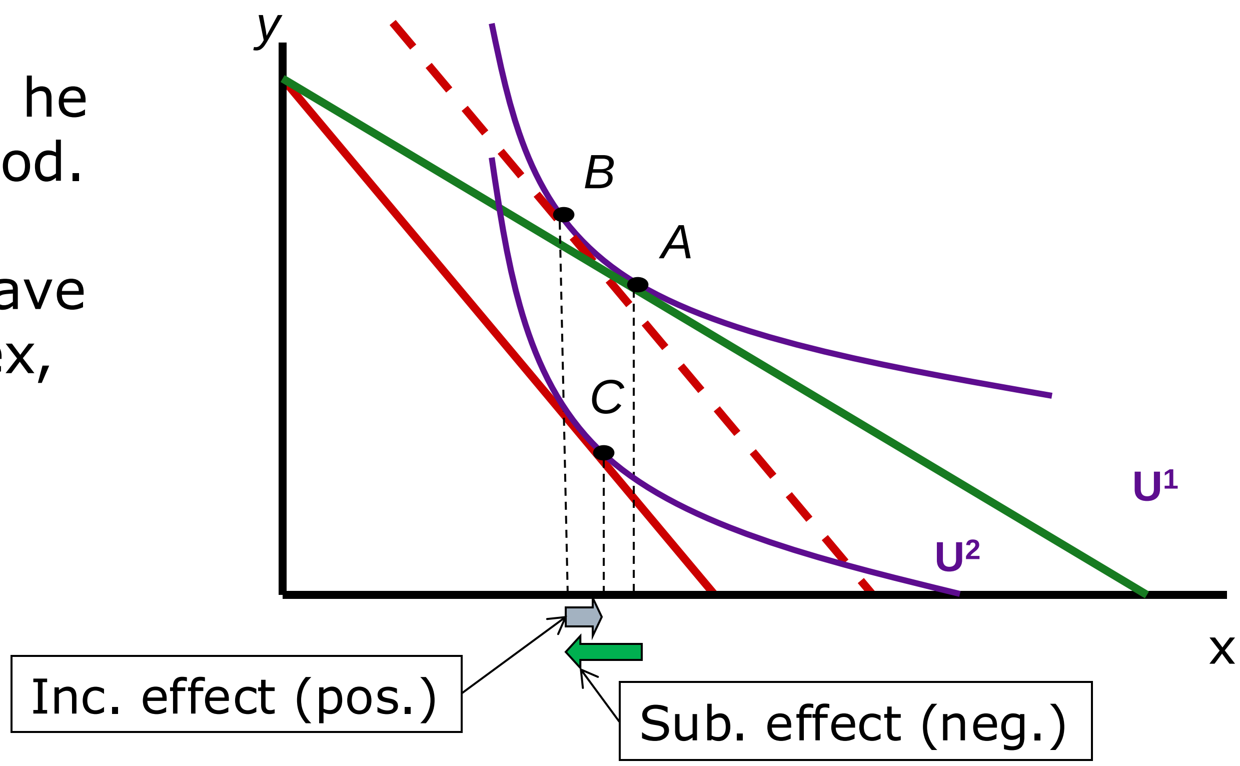
Income and Substitution Effects: Normal Good

- The total effect is the sum of the income and substitution effects (i.e., A ➔ C).
- In the case of a normal good, both the income and substitution effects are negative.
- So, the overall effect must be negative.



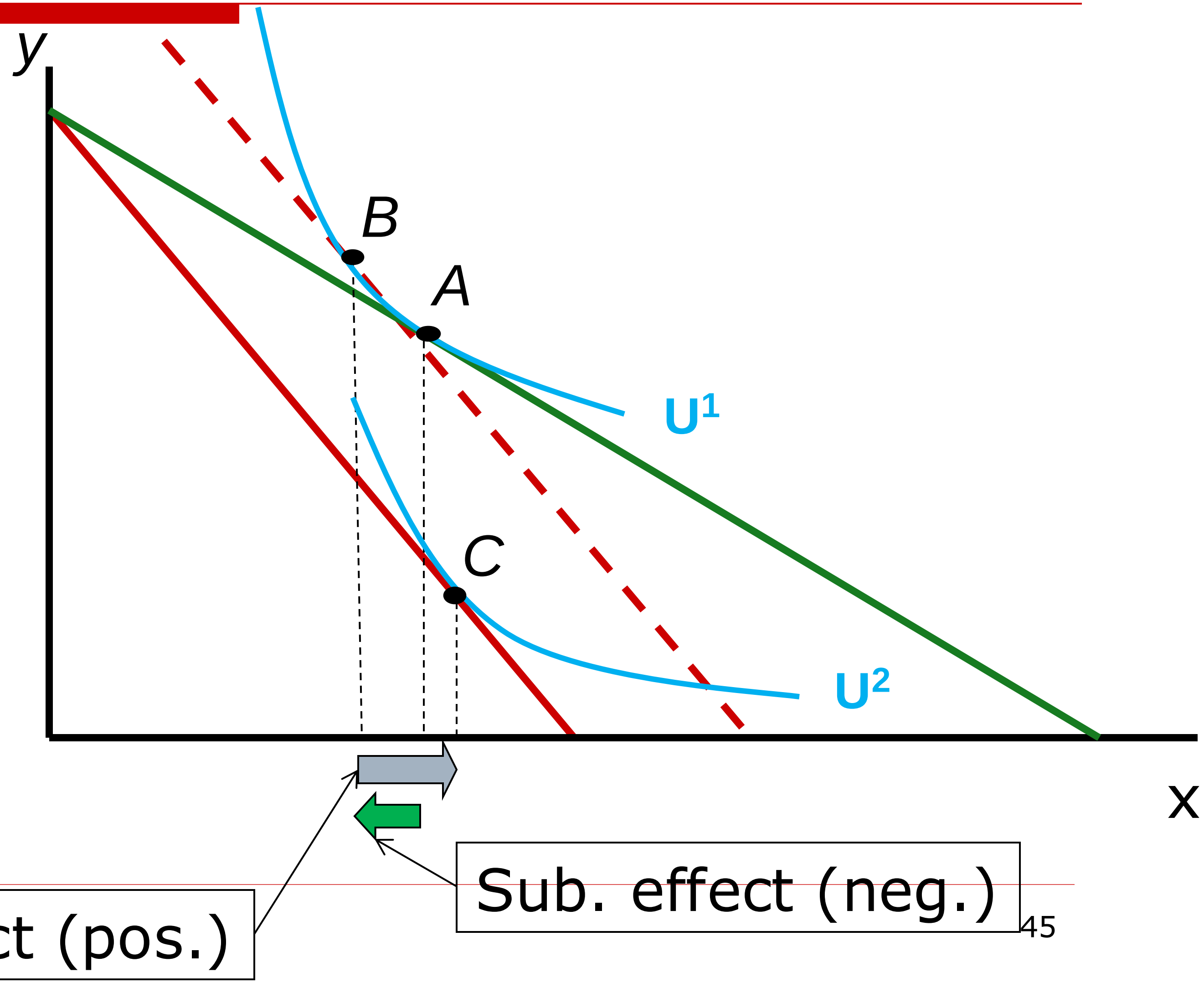
Income and Substitution Effects: Inferior Good

- For an inferior good, the income effect is positive.
- When individual becomes poorer, he consumes more of the inferior good.
- Requires indifference curves to have a particular shape, but still convex, more is better.
- Here, although total effect is negative, income effect reduces substitution effect.



Income and Substitution Effects: Giffen Good

- For an inferior good, income effect opposes substitution effect.
- So, if substitution effect is small, income effect is large, the price increase can actually increase demand.
- Goods with this property are known as Giffen goods.
- Substitution effect: neg.
  Income effect: pos.
  Total effect: pos.!



Giffen Good

- Named after economist Sir Robert Giffen (1837-1910) who came up with the idea

- Example: consider a VERY poor consumer who eats mostly staple (e.g., rice) to maintain basic nutritional need, and can barely afford some meat
  - When the price of staple increases, the consumer has no choice but to *increase* consumption of staple to survive.
  - In this case, the staple food is a Giffen good

- There is real-world evidence that Giffen-style behavior exits, but in quite extreme cases
  - For those of you interested, read this paper in the *American Economic Review*: https://www.aeaweb.org/articles?id=10.1257/aer.98.4.1553

Income and Substitution Effects: Summary

- When the price of a good increases, how does demand for that good change?

- It depends.

- If normal good:
  - Substitution effect is negative
  - Income effect is negative
  - Demand for that good decreases

- If inferior good:
  - Substitution effect is negative
  - Income effect is positive
  - Demand for that good decreases, unless income effect is VERY positive ("Giffen good")

Understanding Check

- **Consider a world with two goods: rice and meat**
  - **Suppose when rice price increases, demand for meat decreases, then meat is normal/inferior/either?**
  - **Suppose when meat price increases, demand for rice increases, then rice is normal/inferior/either?**

The Law of Demand

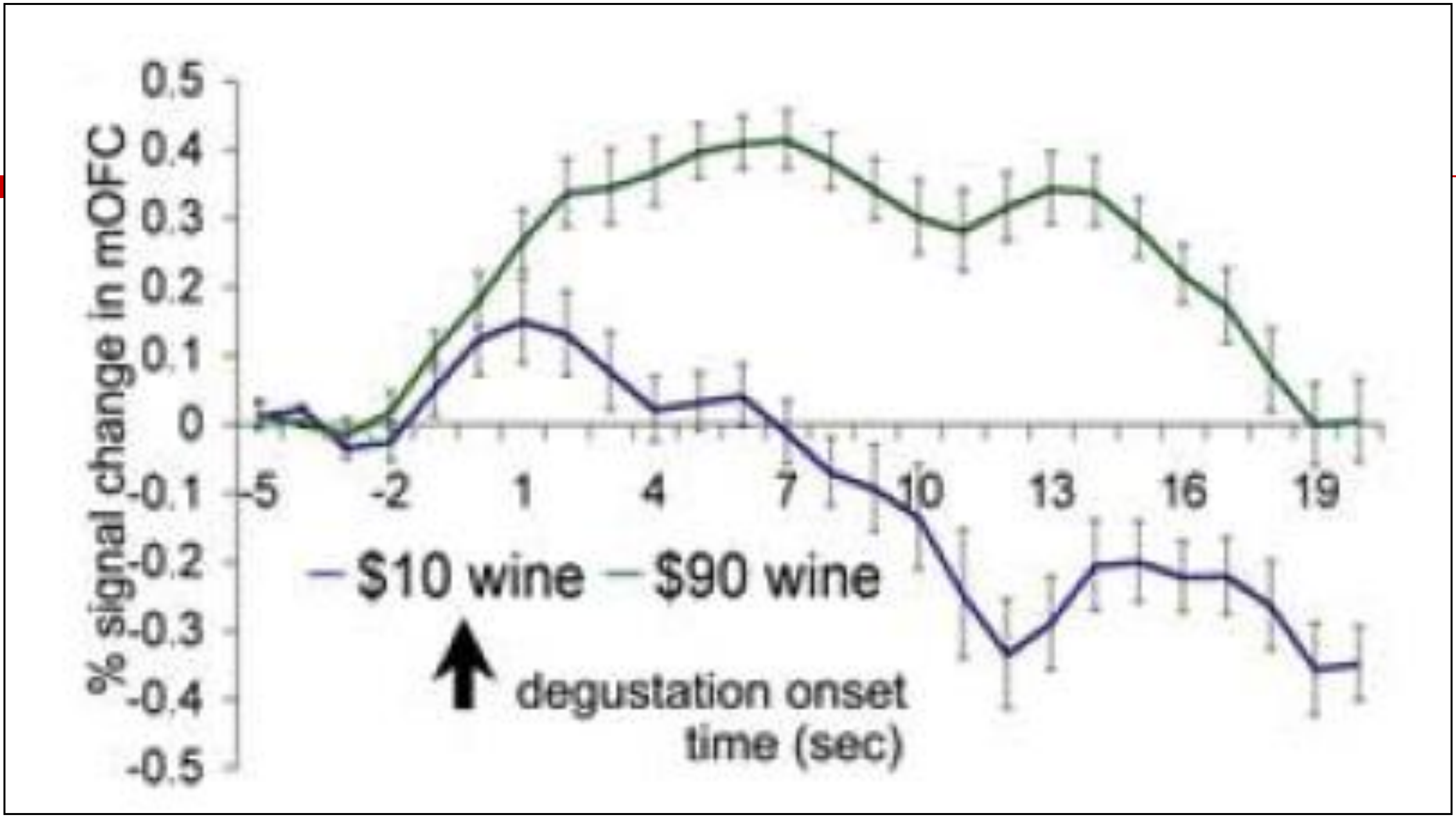
- When price of a good increases, demand for that good generally decreases
  - So, individual demand curve usually slopes downward
- In very rare cases (Giffen good), demand curve can slope upward
- But, even if some consumers have upward sloping demand, it is unlikely that enough would have upward sloping demand that the market demand curve could slope upward.
- **Law of Demand:** Market demand curves slope downward.

Some Related Phenomena

- We’ve focused on situations where prices only matter through their effects on budget sets.
- But, sometimes people care about prices directly.
  - Status (Veblen) goods: people want to convey status, high-priced goods do that, so consume more when price goes up.
  - Snob goods: people like to consume goods that others do not consume. High-priced goods are often that way.
  - Price as a signal of quality: people interpret high-priced goods as high-quality goods (e.g., wine).

Article: “Study: $90 wine tastes better than the same wine at $10.” (cnet news, January 14, 2008).

- People say that the same wine tastes better when it has a price tag of $90 than when it had a price tag of $10.
- This has long been attributed to snob effects.
- Using f-MRI brain scans, this study shows that people actually enjoy the wine more when it has a higher price tag.
- "changes in the price of a product can influence neural computations associated with experienced pleasantness,“
- Point to studies saying marketing doesn’t just trick people intellectually. They actually enjoy the products more.



This graph shows the activity in the brain's pleasure center; there's more activity with wine subjects think costs $90 a bottle (top line) than the same wine priced at $10. The arrow shows the moment when the subjects started tasting the wine.

Demand Curve in the Real World

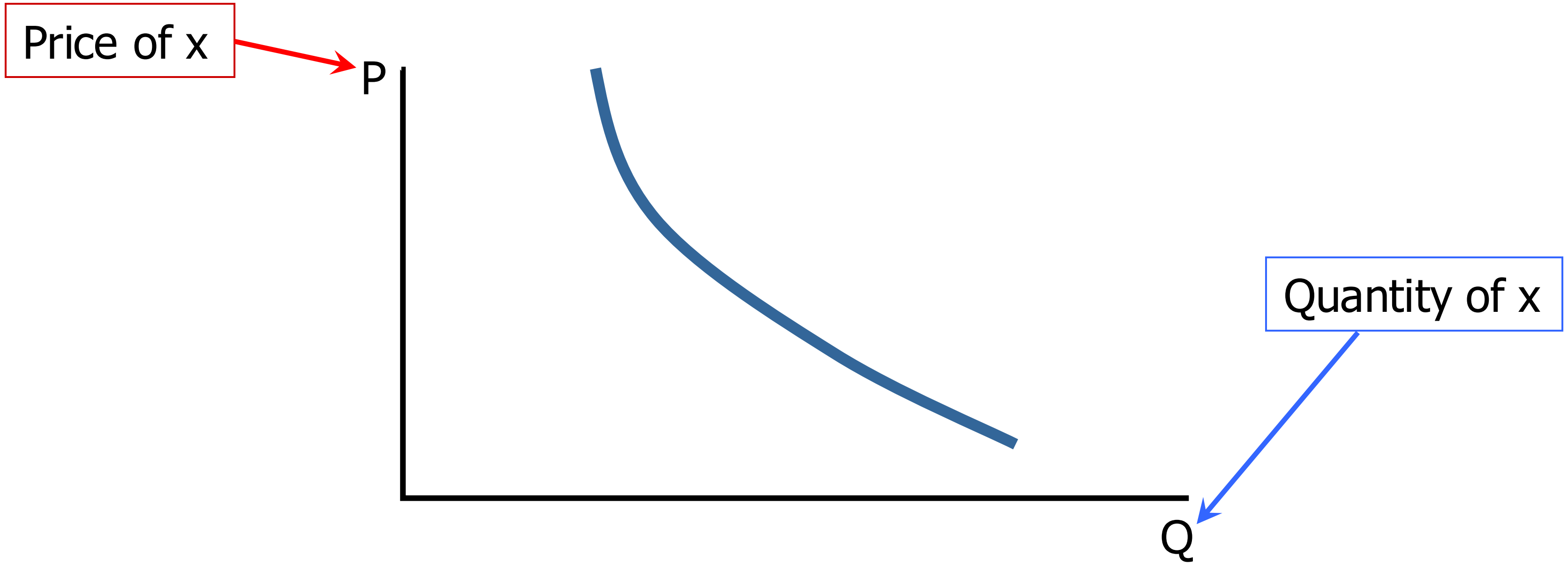
Empirical Microeconomics

- In this class, we spend a lot of time talking about **microeconomic theory**
  - Begin with some assumptions (e.g. rationality of preference, utility-maximizing behavior)
  - Show under these assumptions, something is true (e.g. market demand curve slopes downward)

- Theory is only a part of microeconomics. Another major branch of microeconomics is empirical
  - e.g., can we actually see “market demand curve slopes downward” in real-world data?

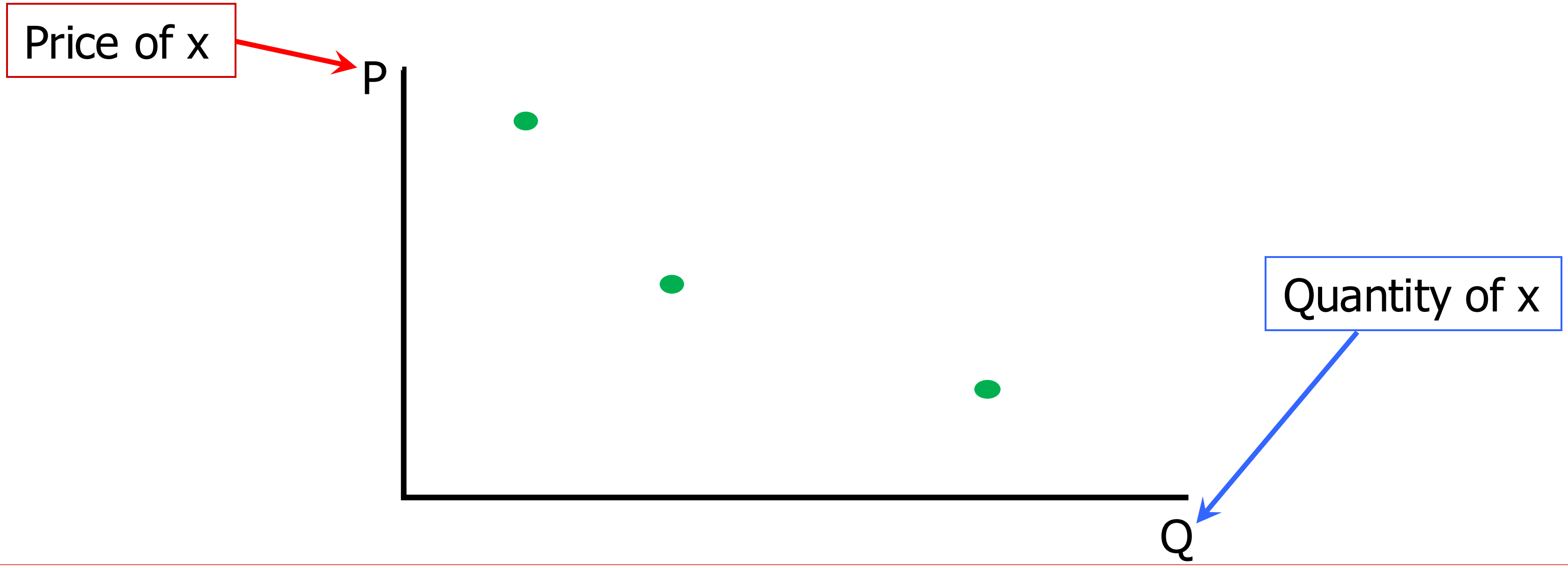
Estimating Demand

- Imagine there is a market demand curve for good x
- Theoretical analysis (this class): what assumptions do we need to generate a demand curve? (Ex: rationality of preference, utility function, UMP..)
- **Empirical analysis: how do we estimate demand curve using data?**



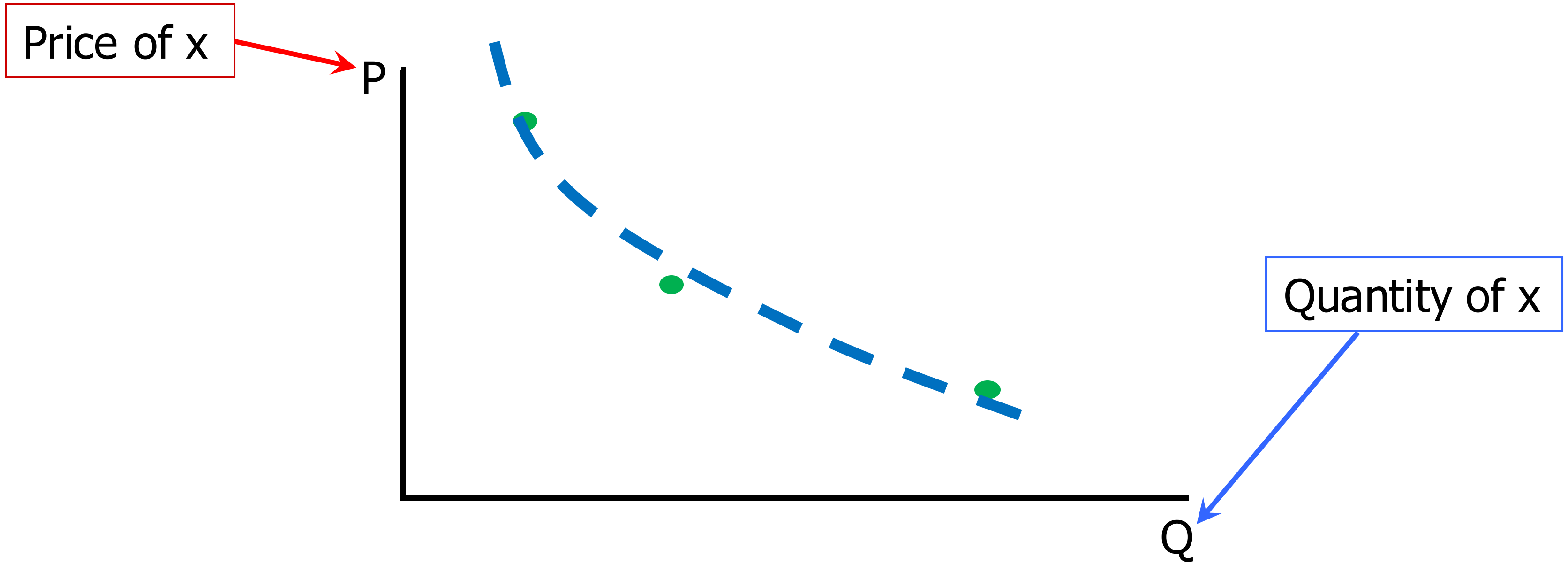
Estimating Demand

- Suppose you have three data points on price and quantity
  - When price for x is 15, quantity demanded is 1
  - When price for x is 7, quantity demanded is 2
  - When price for x is 5, quantity demanded is 8



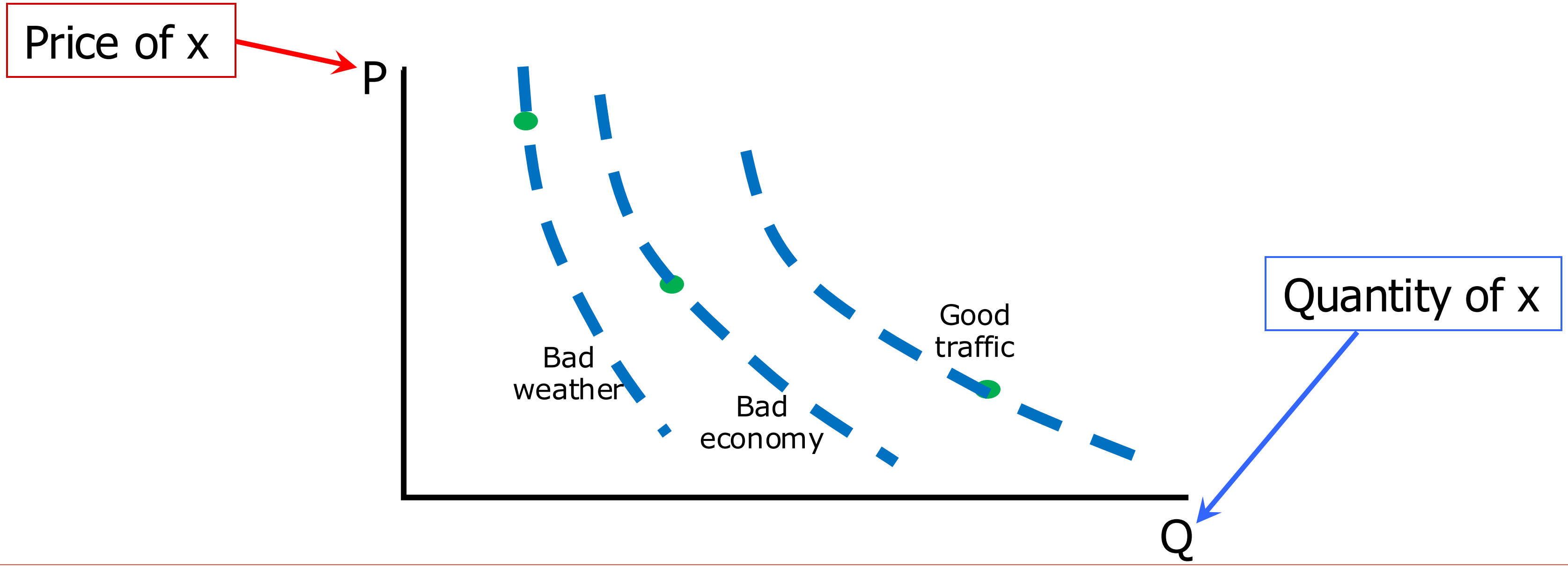
Estimating Demand

- Intuitively, you might want to “fit” a curve across these data points, and the best-fitting curve is the demand curve
- But this can be problematic. Why?



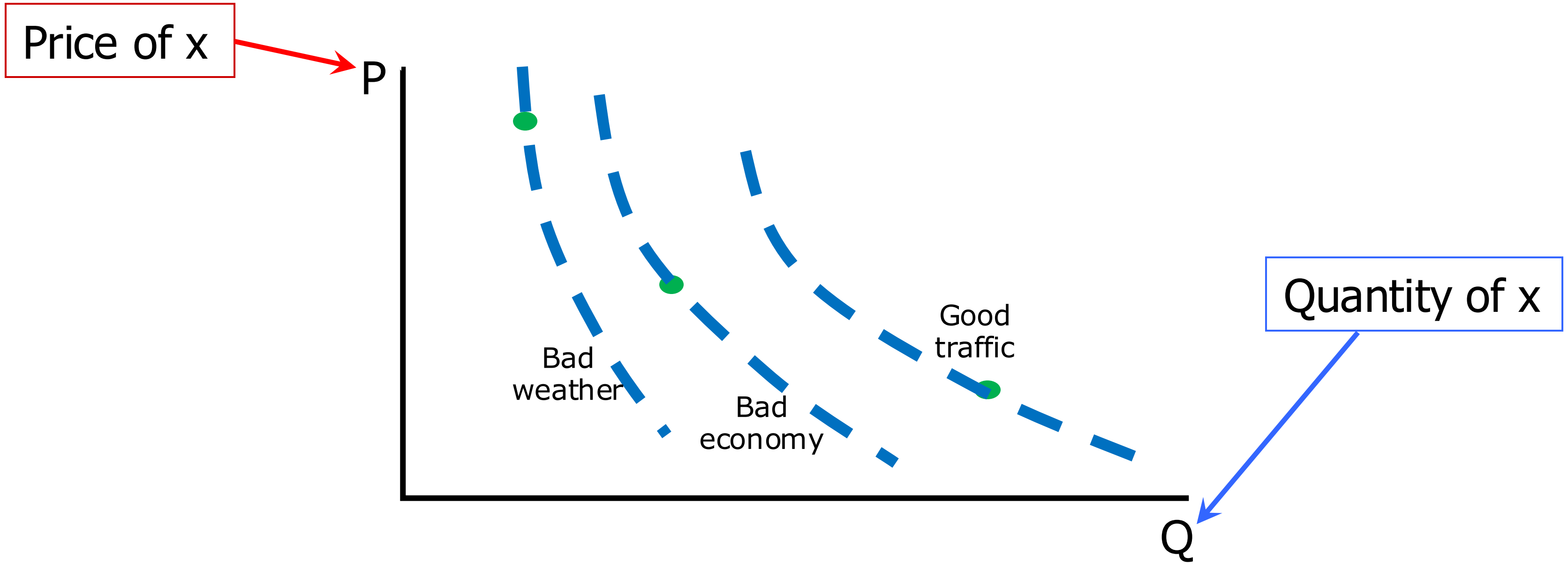
Estimating Demand

- There can be so many other things that change at the same time with price
  - When price for x is 15 **and weather is bad**, quantity demanded is 1
  - When price for x is 7 **and economy is in recession**, quantity demanded is 2
  - When price for x is 5 **and no traffic jams**, quantity demanded is 8



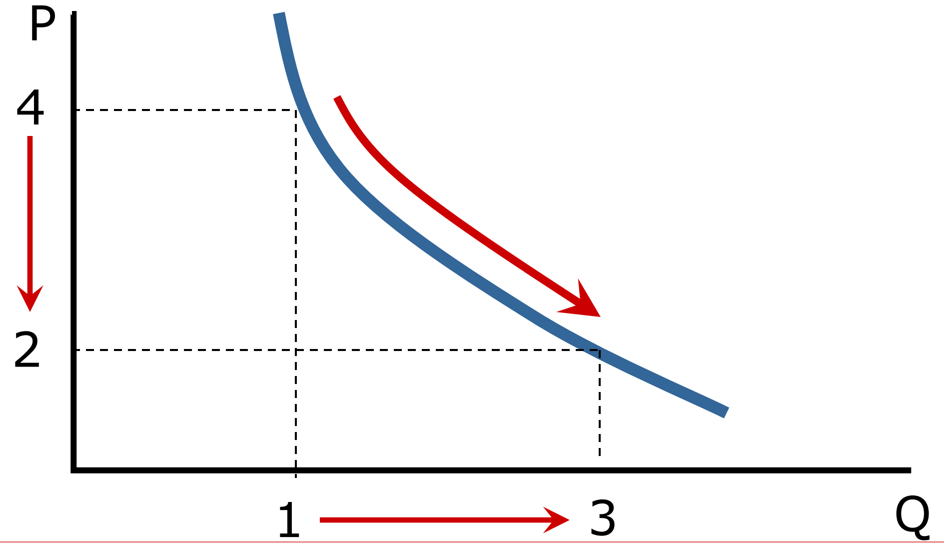
Estimating Demand

- Simply observe different quantities at different price levels isn't enough to estimate a demand curve b/c the change in quantities may not be caused by price changes alone
- Have to find variations in your data that "hold everything else equal" EXCEPT for price changes



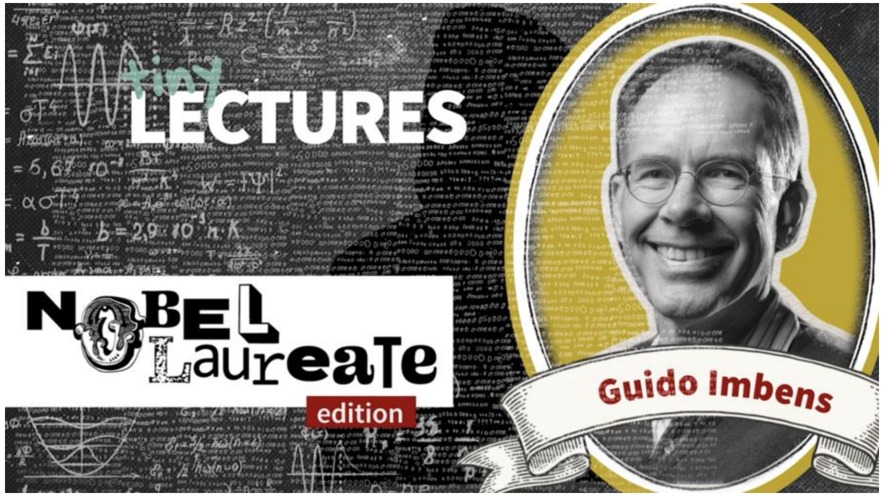
Estimating Demand

- In the language of this class: we want to find (P,Q) pairs in the data that are likely "moving along the demand curve".
- This exercise is also known as finding the "causal relationship" between P and Q



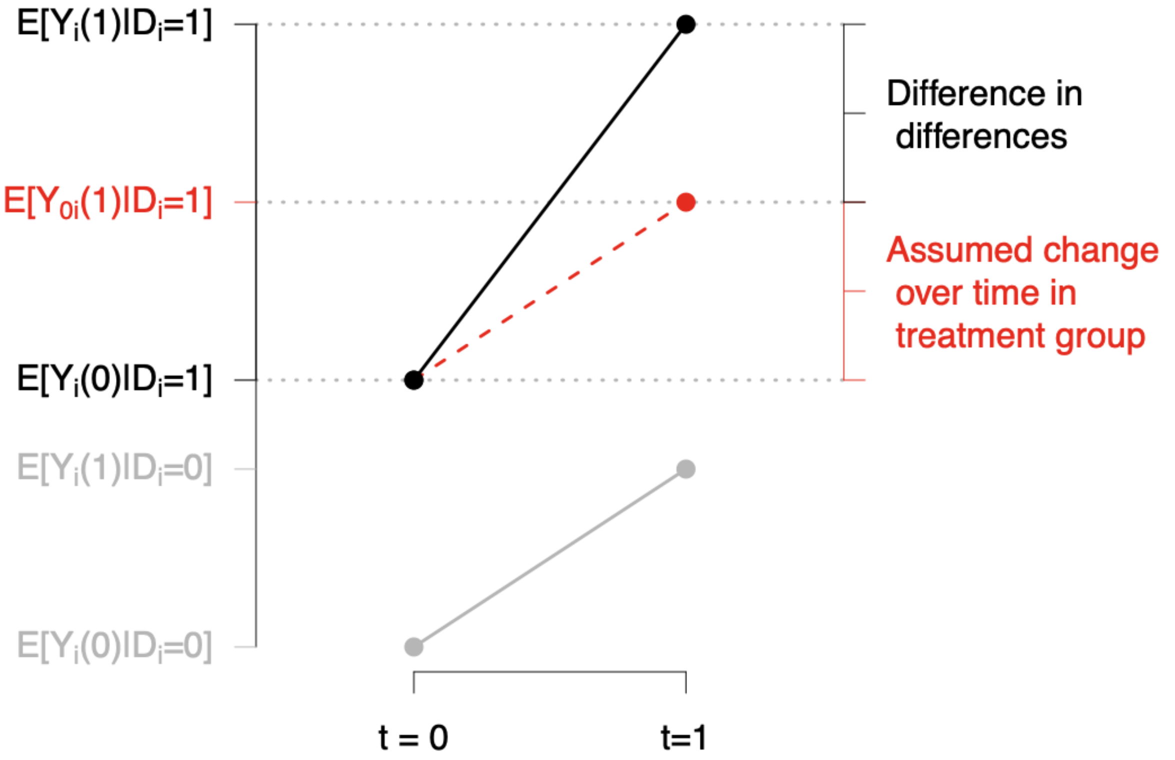
A little digression: causal inference

- Guido Imbens, Joshua Angrist
  - local average treatment effect (LATE), introduced in a 1994 *Econometrica* paper, called "Identification and Estimation of Local Average Treatment Effects," that showed researchers how to draw causal inferences from observational data.
  - Tiny lecture: Guido Imbens



A little digression: causal inference

- David card & Alan Krueger
  - [minimum wage]() (Difference-in-Difference framework)



A little digression: causal inference

- A more recent paper by Cohen et al (2016)
  - Regression discontinuity

nber.org

NBER | NATIONAL BUREAU of ECONOMIC RESEARCH

Subscribe | Media | Open Calls

Welcome, Yuanning!

Research Programs & Projects Conferences Affiliated Scholars NBER News Career Resources About

Search

Home › Research › Working Papers › Using Big Data to Estimate Consumer...

Using Big Data to Estimate Consumer Surplus: The Case of Uber

Peter Cohen, Robert Hahn, Jonathan Hall, Steven Levitt & Robert Metcalfe

WORKING PAPER 22627 **DOI** 10.3386/w22627 **ISSUE DATE** September 2016

Estimating consumer surplus is challenging because it requires identification of the entire demand curve. We rely on Uber's "surge" pricing algorithm and the richness of its individual level data to first estimate demand elasticities at several points along the demand curve. We then use these elasticity estimates to estimate consumer surplus. Using almost 50 million individual-level observations and a regression discontinuity design, we estimate that in 2015 the UberX service generated about $2.9 billion in consumer surplus in the four U.S. cities included in our analysis. For each dollar spent by consumers, about $1.60 of consumer surplus is generated. Back-of-the-envelope calculations suggest that the overall consumer surplus generated by the UberX service in the United States in 2015 was $6.8 billion.

Download a PDF

Cohen et al.(2016) paper

- Uses administrative data on about 50 million Uber trips to estimate demand curve for Uber

- Uber’s “surge pricing”
  - Uber has a baseline pricing for a given trip. This price reflects baseline operation costs (e.g., driver wages, gasoline)
  - Depending on peak hours, bad road traffic etc., Uber charges a higher price by calculating a “surge factor” (e.g., surge price = 1.5 x baseline price during a peak hour)

Cohen et al.(2016) paper

- There is an interesting "glitch" in the surge factor calculation
  - Internally, surge factor is calculated as a continuous number
  - But, when applied to actual price that consumer faces, surge factor has only 1 decimal point
  - So, when surge factor is 1.249, the consumer faces 1.2 x base price; when surge factor is 1.251, the consumer faces 1.3 x base price

- This generates a situation of "everything else equal EXCEPT for price"
  - On average, market conditions (weather, economy, traffic, etc.) are nearly identical when the algorithm suggests a surge of 1.249x and when it suggests 1.251x
  - … EXCEPT that, the consumer faces a surge factor of 1.2x in the former, and 1.3x in the latter

- So, changes in Q when surge factor goes from 1.249 to 1.251 (i.e., P goes from 1.2 to 1.3) likely reflects **the causal effect of P on Q**

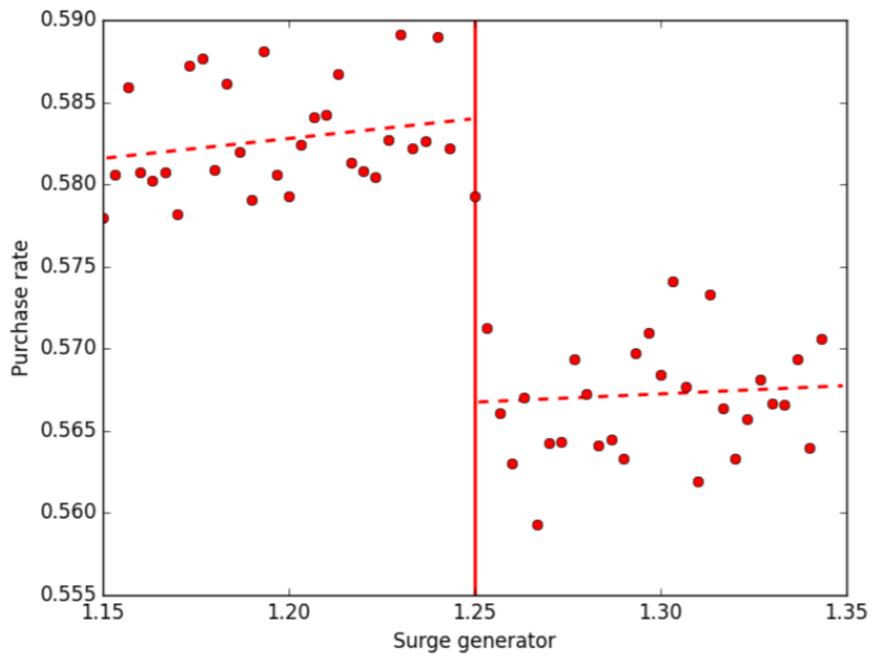
Cohen et al.(2016) paper

Figure 4: Example of purchase rate changes at price discontinuity

Note: This figure illustrates how purchase rates vary as a function of the surge generator over the range 1.15x to 1.35x. The vertical line when the surge generator equals 1.25 identifies the point at which the surge price changes from 1.2x to 1.3x.

Cohen et al.(2016) paper

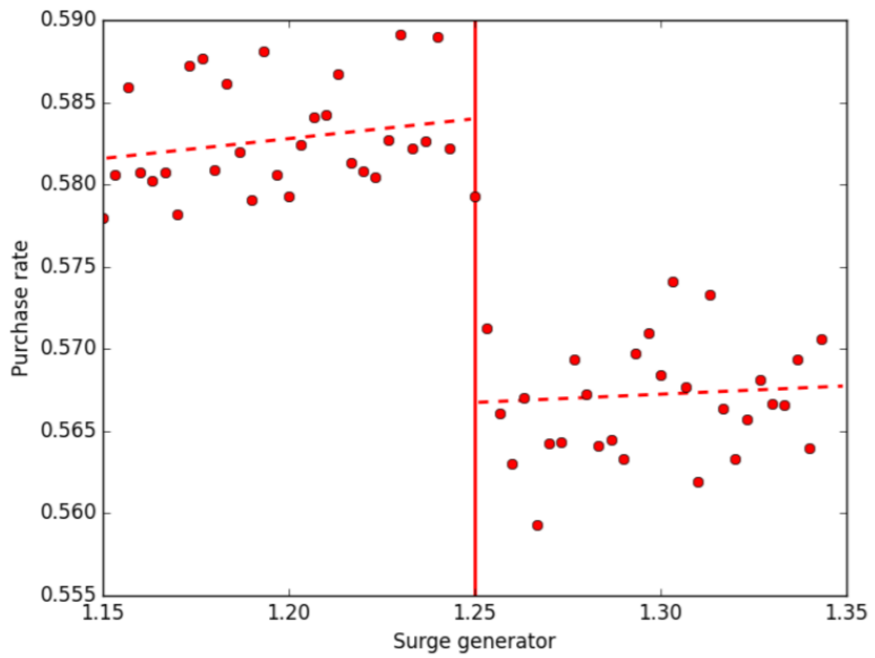
Figure 4: Example of purchase rate changes at price discontinuity



Note: This figure illustrates how purchase rates vary as a function of the surge generator over the range 1.15x to 1.35x. The vertical line when the surge generator equals 1.25 identifies the point at which the surge price changes from 1.2x to 1.3x.

Cohen et al.(2016) paper

Figure 4: Example of purchase rate changes at price discontinuity

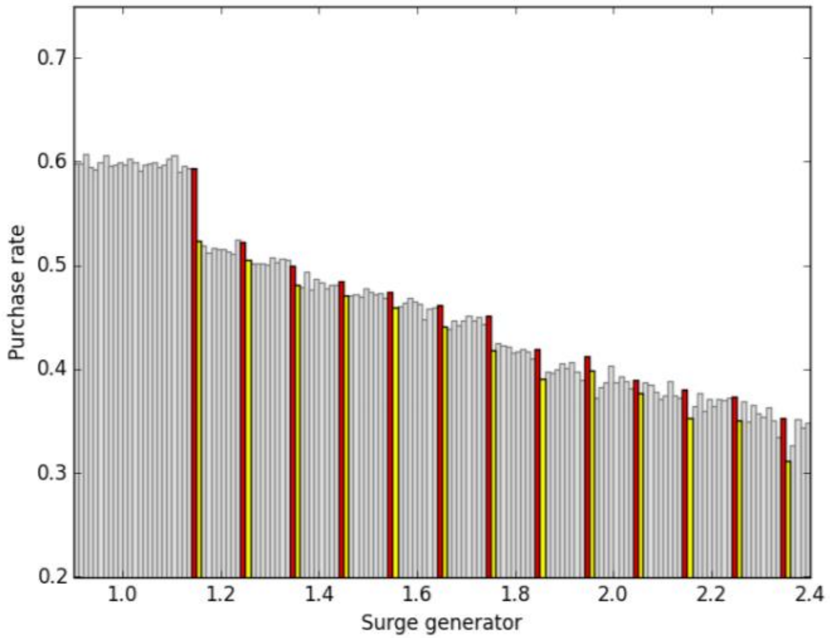


Note: This figure illustrates how purchase rates vary as a function of the surge generator over the range 1.15x to 1.35x. The vertical line when the surge generator equals 1.25 identifies the point at which the surge price changes from 1.2x to 1.3x.

Method formally known as "Regression Discontinuity" design

Cohen et al.(2016) paper

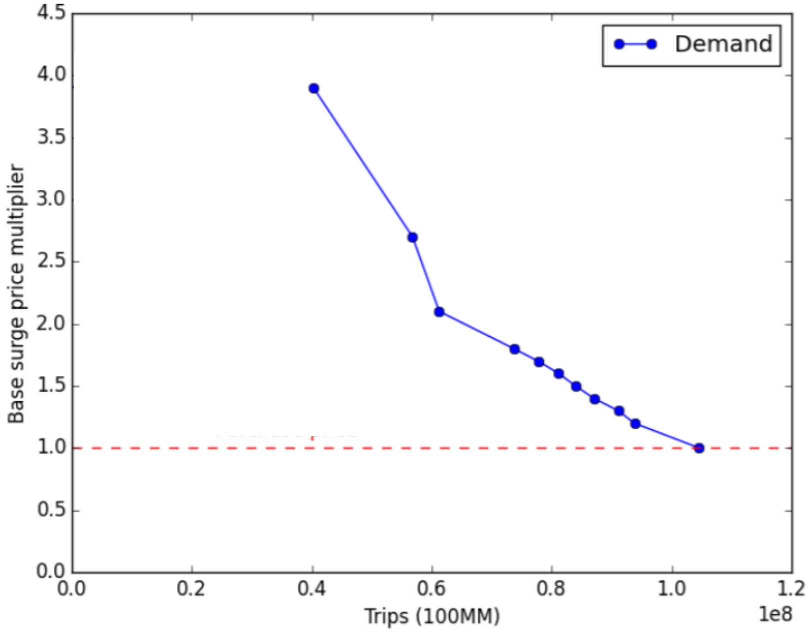
Figure 5: Request rate drops at pricing discontinuities



Note: This figure illustrates how purchase rates vary as a function of the surge generator when the surge generator is less than 2.4x. Red bars identify all observations within .01 units to the left of a price discontinuity. Yellow bars identify all observations within .01 units to the right of a price discontinuity. All observations not within these windows are depicted in gray.

Cohen et al.(2016) paper

Figure 6: Visual representation of demand curve for transactions at 1.0x



Note: This figure presents a piecewise linear demand curve with jumps at each price discontinuity. The curve is generated from the underlying elasticities estimated for each price discontinuity and for consumers facing transactions at 1.0x.

Theory & Empirics

- Theory can provide guidance to the empirical analysis
- In this case, nice to see that empirical evidence provides support to theory
- Empirical microeconomics is one of the fastest growing branches of economics in the past 20 years
  - Very different questions: Does policy X really matter? How much is price elasticity for good Y?
  - Very different approach: big data, causal inference, machine learning, etc.
- Take upper level economics class to know more about empirical research in microeconomics