Intermediate Microeconomic
Spring 2025

Part three: Production and supply

Week 5(a): Cost functions

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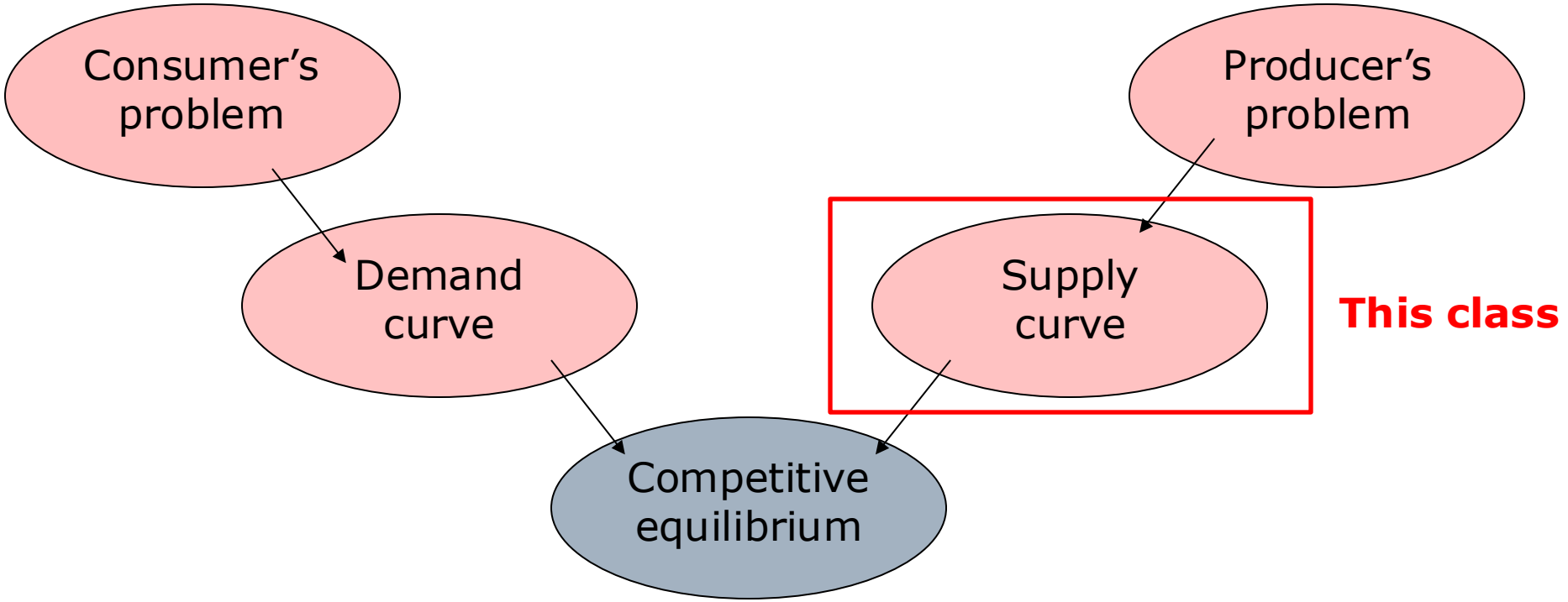
Big Picture


Consumer's problem
Producer's problem
Demand curve
Supply curve
This class
Competitive equilibrium

Producer's Problem

- Producer's **Profit Maximization Problem (PMP)**: Given the production technology Q=F(K,L), find the optimal choice of output (Q*), capital (K*), labor(L*) that maximizes firm's profit: pQ – vK – wL

- If you think about it, PMP has two parts
  - How many output Q should I produce?
  - Given I want to produce Q, what's the optimal combination of K and L that I should choose

"Cost Minimization Problem" (CMP)

Producer's Problem

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Now we look at the first part of the question ("Profit Maximization")

Profit Maximization

- For simplicity, from this point on we will assume input prices v and w are fixed. So, a firm faces a simple cost function C(Q)

- Firm's problem:
  - "My cost-minimizing manager told me that, if I decide to produce Q unit of output, the associated cost will be C(Q)"
  - "I also know the market price for output is p"
  - "The Question is: how much Q* should I choose to produce to maximize firm's profit"

- It turns out that to solve this problem, one simply needs to take a closer look at the cost function

Profit-Maximizing Decisions

- To think about **HOW MUCH** output should a firm produce, focus on the "margin."
  - Does producing 1 more unit increase my profit?

- To think about **WHETHER** a firm should produce, focus on the "average"
  - Does producing give me positive profit at all?

- Average cost can tell you only whether the firm is making money or not. It cannot tell you whether it could make ***more*** money by changing output.

"How much" to produce?

$$C(Q) = 100 + 5Q + Q^2$$

- Suppose the firm is currently producing $Q = 20$ units of output. The price of output is $p = 40$. ***Should the firm increase output to $Q = 21$?***

- How should the decision be made?
  - $C(20) = 600$
  - Average cost: $C(20)/20 = 30$
  - Marginal cost of 21st unit: $C(21) - C(20) = 46$

"How much" to produce?

- Wrong answer: Since 40 = p > AC = 30, the firm is making money on each unit. So, it "looks like" it should produce another unit. But:
  - Profit (20) = 40*20 – 600 = 200
  - Profit (21) = 40*21 – 646 = 194

- Right answer:
  - Marginal cost of 21st unit: C(21) – C(20) = 46
  - Producing another unit adds 46 to total cost and only brings in revenue of 40. So, it should not produce another unit. The cost of the 21st unit is not its average cost! Its cost is its marginal cost!

"How much" to produce?

- The key to thinking about the firm's decision of whether to increase output is to focus on the "margin."

- In asking whether to increase output, firm asks whether the additional ("marginal") benefit of doing so is greater than its marginal cost.

- Should I produce another unit?
  - Marginal benefit: sell it for p (assuming for now that the firm takes price as given).
  - Marginal cost: marginal cost at current Q, $MC(Q)$.
  - Produce another unit if $p > MC(Q)$.

"How much" to produce?

- The firm's profit is Revenue – Cost:
  - Revenue: $R(Q) = P * Q$.
  - Profit: $P * Q - C(Q)$.

- To find profit-maximizing quantity, take the derivative with respect to Q, set it equal to zero:
  - $\frac{d(P*Q - C(Q))}{dQ} = 0$ ➔ $P - C'(Q^*) = 0$

  - If the firm produces a positive quantity, it produces Q^* where **price = marginal cost.**

“Whether” to produce

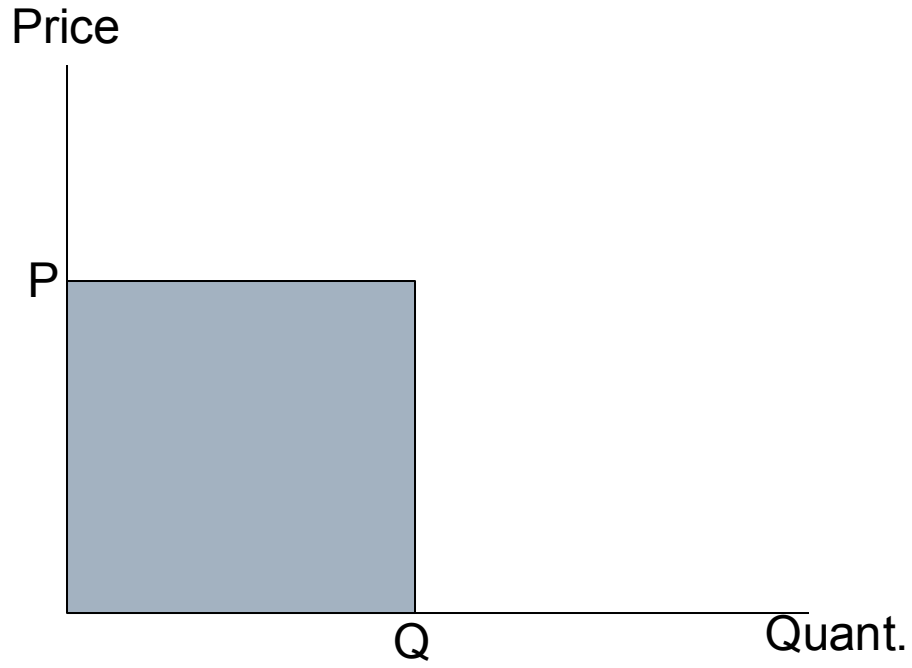
- We have shown that, **if a firm decides to produce**, it should produce Q such that P=MC(Q)

- Then a firm needs to think about this: whether it wants to produce a positive amount in the first place
  - .. because P=MC(Q) alone does not ensure that firm will earn a positive profit at Q
  - .. in which case it’s in the firm’s best interest to NOT produce (Q*=0)

- Easiest to see this on graphs

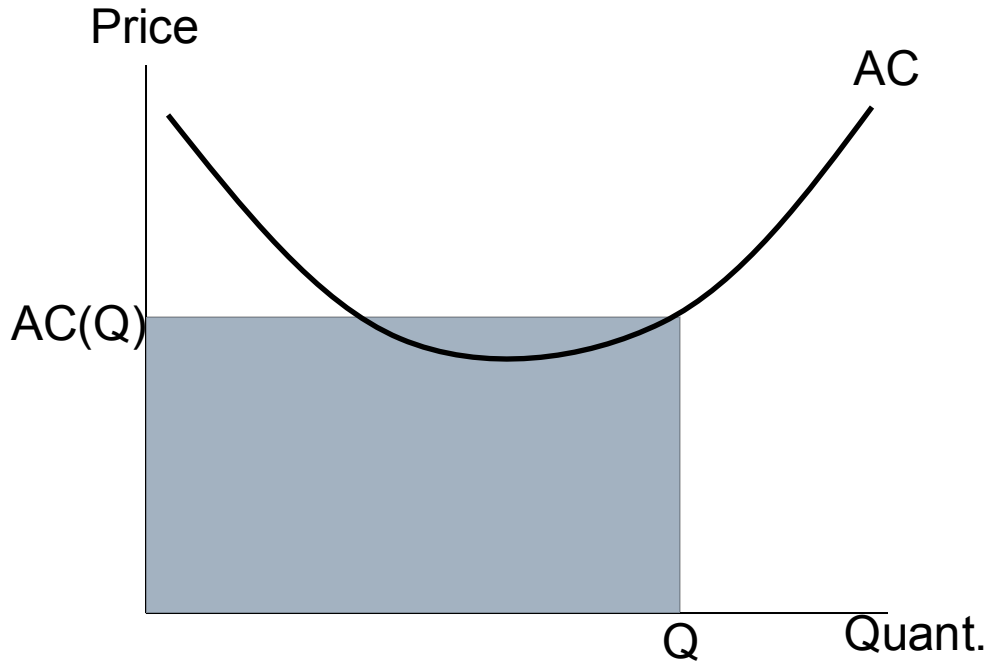
“Whether” to produce

- On a graph like this, areas represent dollars.
  - Price = $/unit
  - Quantity = units
  - P*Q = $
- If the firm sells Q units at price P, the revenue is given by the rectangle.



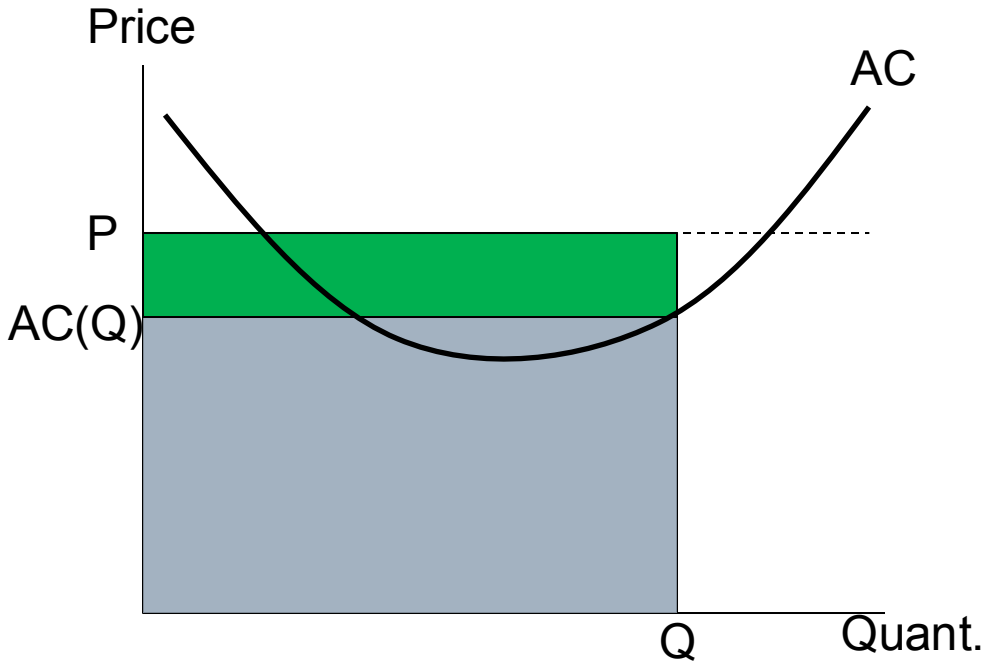
“Whether” to produce

- If the firm sells Q units, its cost is given by the rectangle bounded by Q, AC(Q) and the axes.



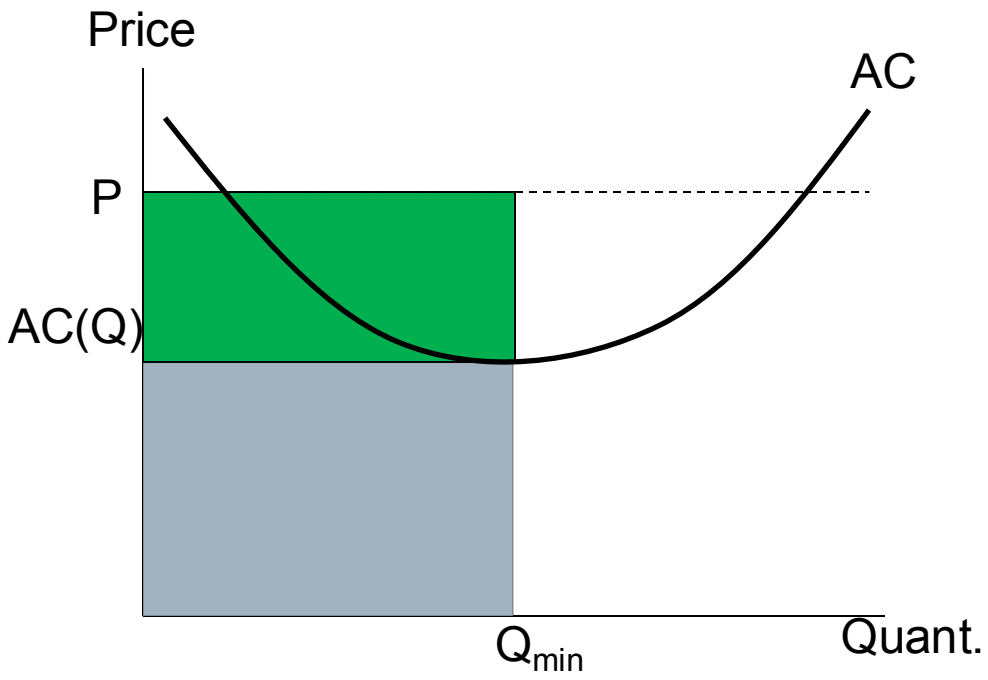
“Whether” to produce

- If the firm sells Q units, its cost is given by the rectangle bounded by Q, AC(Q) and the axes.
- If the firm sells these Q units at a price of P, the revenue is given by the rectangle bounded by P, Q and the axes.
- Profit is the excess of revenue over cost.
- Profit = Green rectangle.



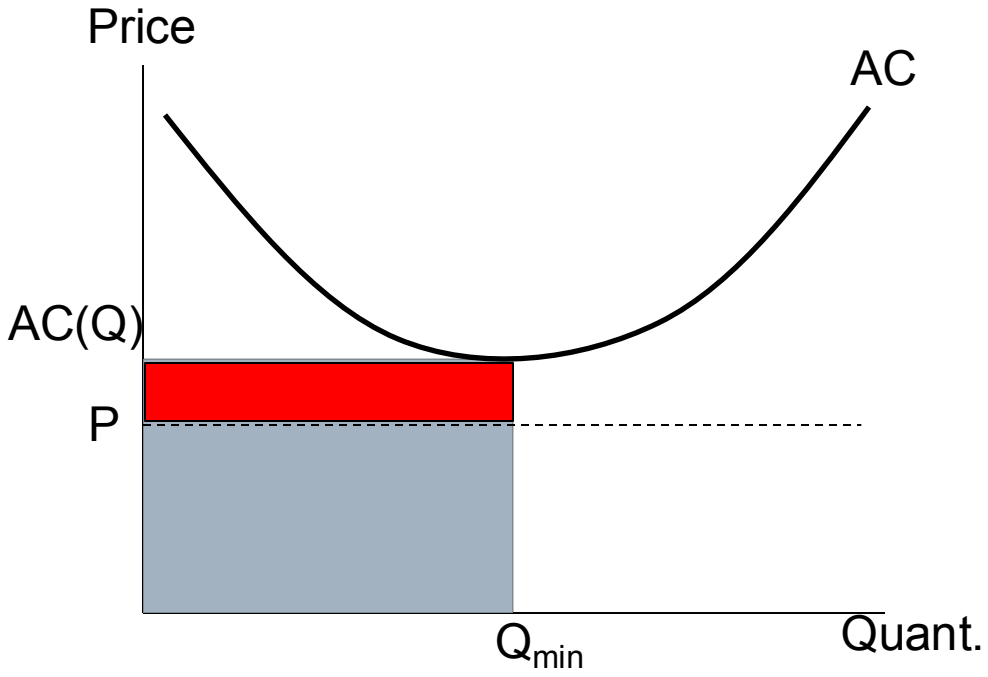
“Whether” to produce

- Whenever P ≥ min AC, it is possible for the firm to choose a quantity that makes a profit.
- In particular, the Q that minimizes AC will make a profit.



“Whether” to produce

- Whenever P ≥ min AC, it is possible for the firm to choose a quantity that makes a profit.
- In particular, the Q that minimizes AC will make a profit.
- If P < min AC, there is no Q the firm can choose the offers a positive profit.

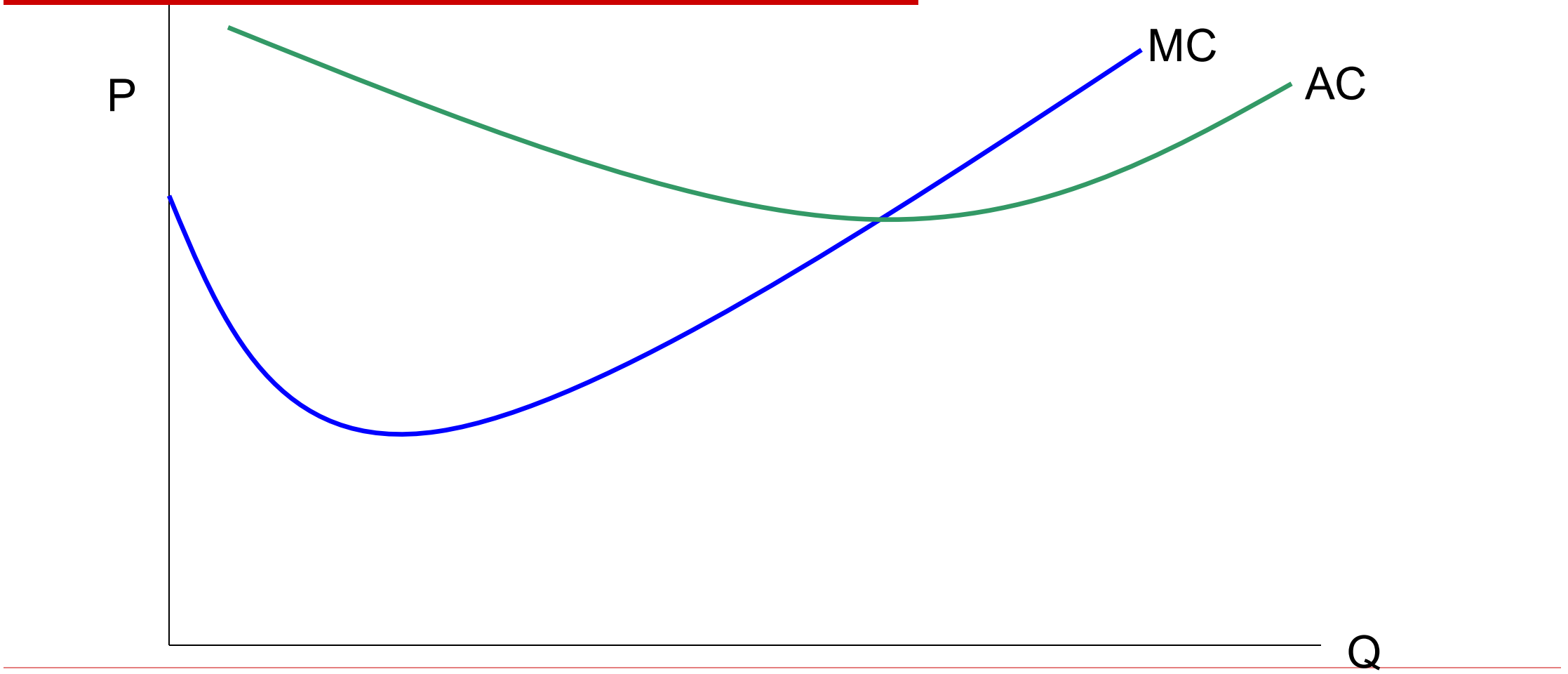


“Whether” to produce

- So, if market price of output P < min AC, there is no way for the firm to make profit. The firm should choose Q=0

- If P ≥ min AC, the firm should produce Q>0.
  - How much to produce? Choose Q such that P=MC(Q)

Putting Things Together



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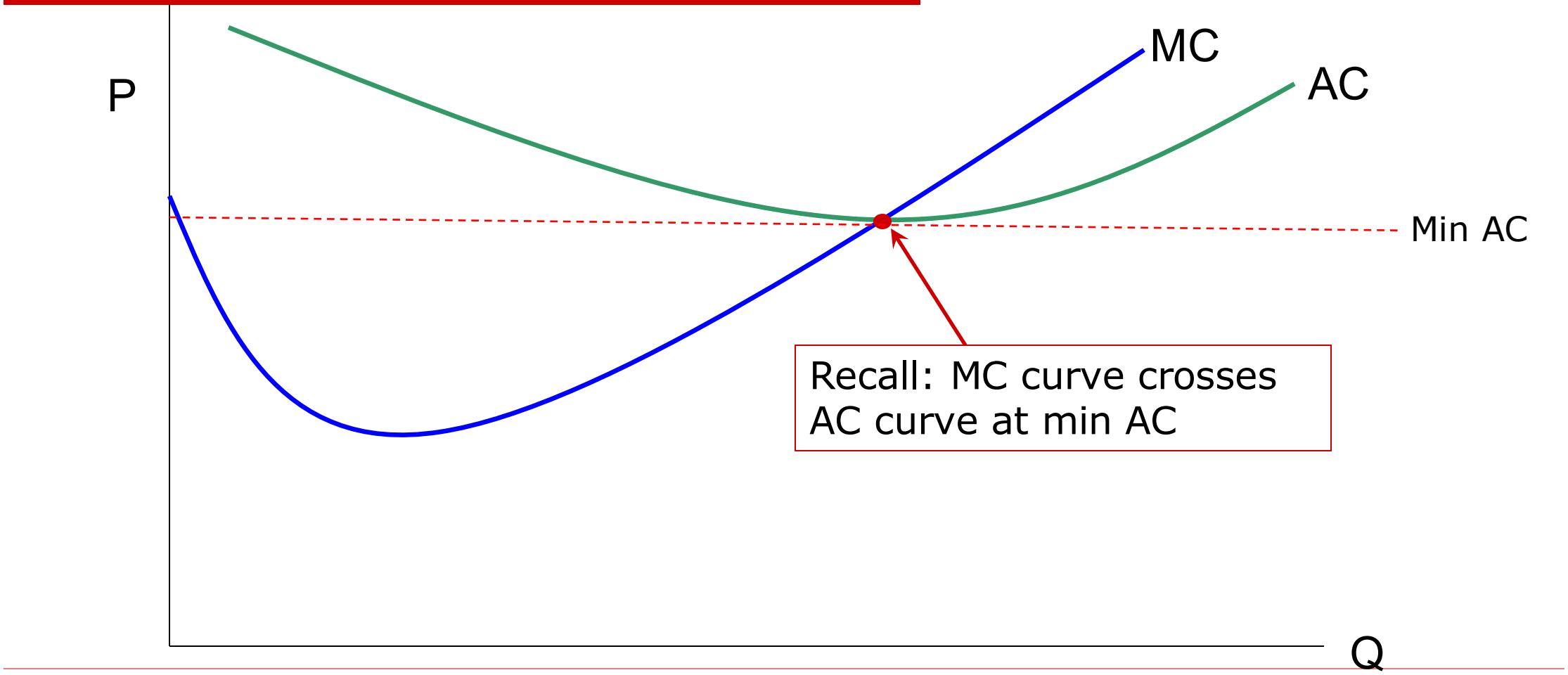

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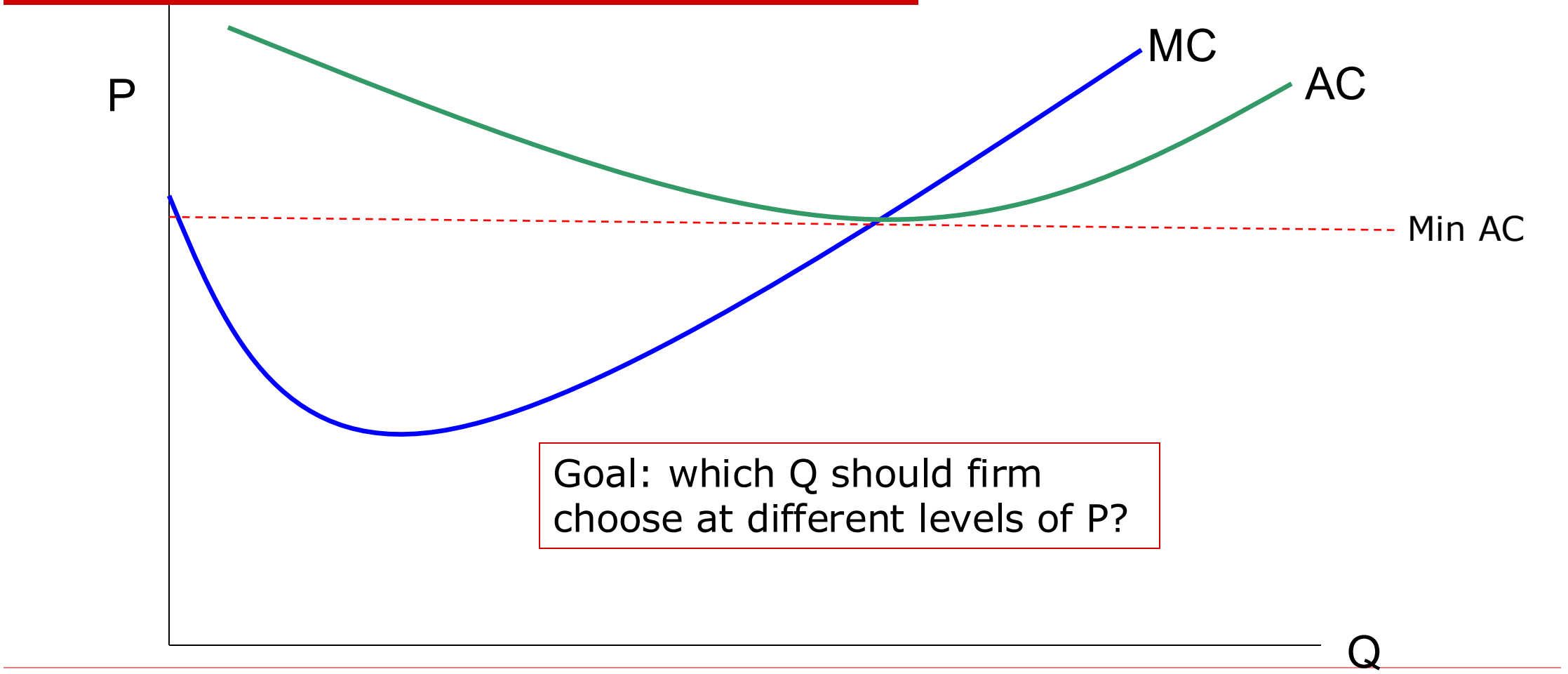

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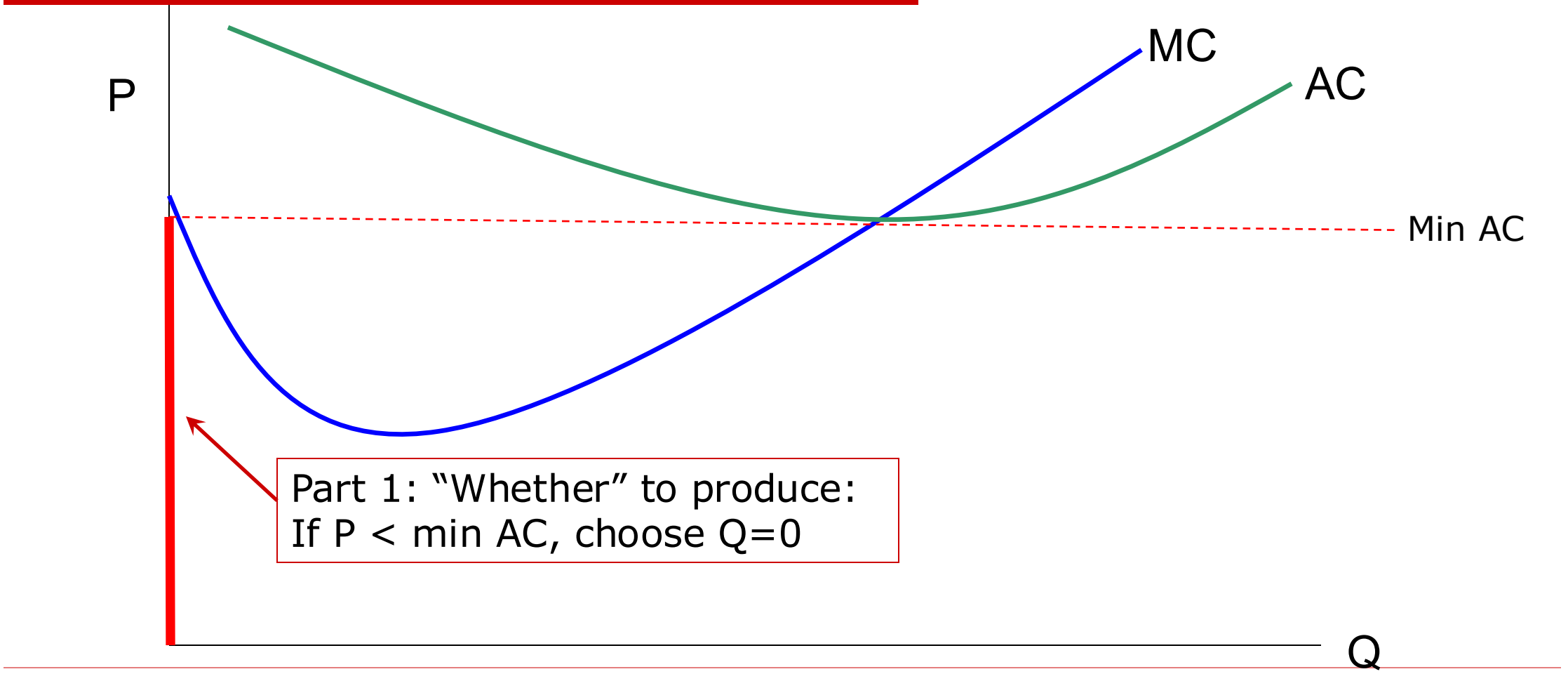

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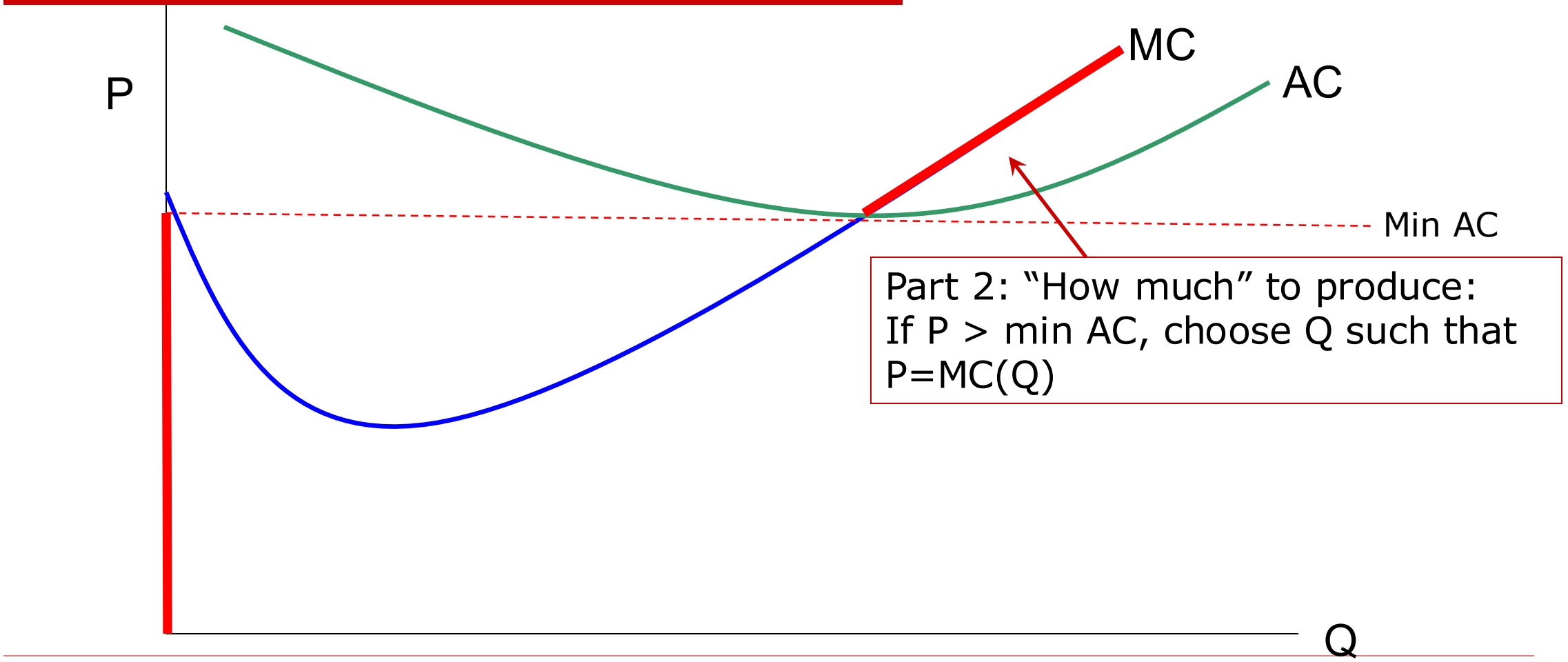

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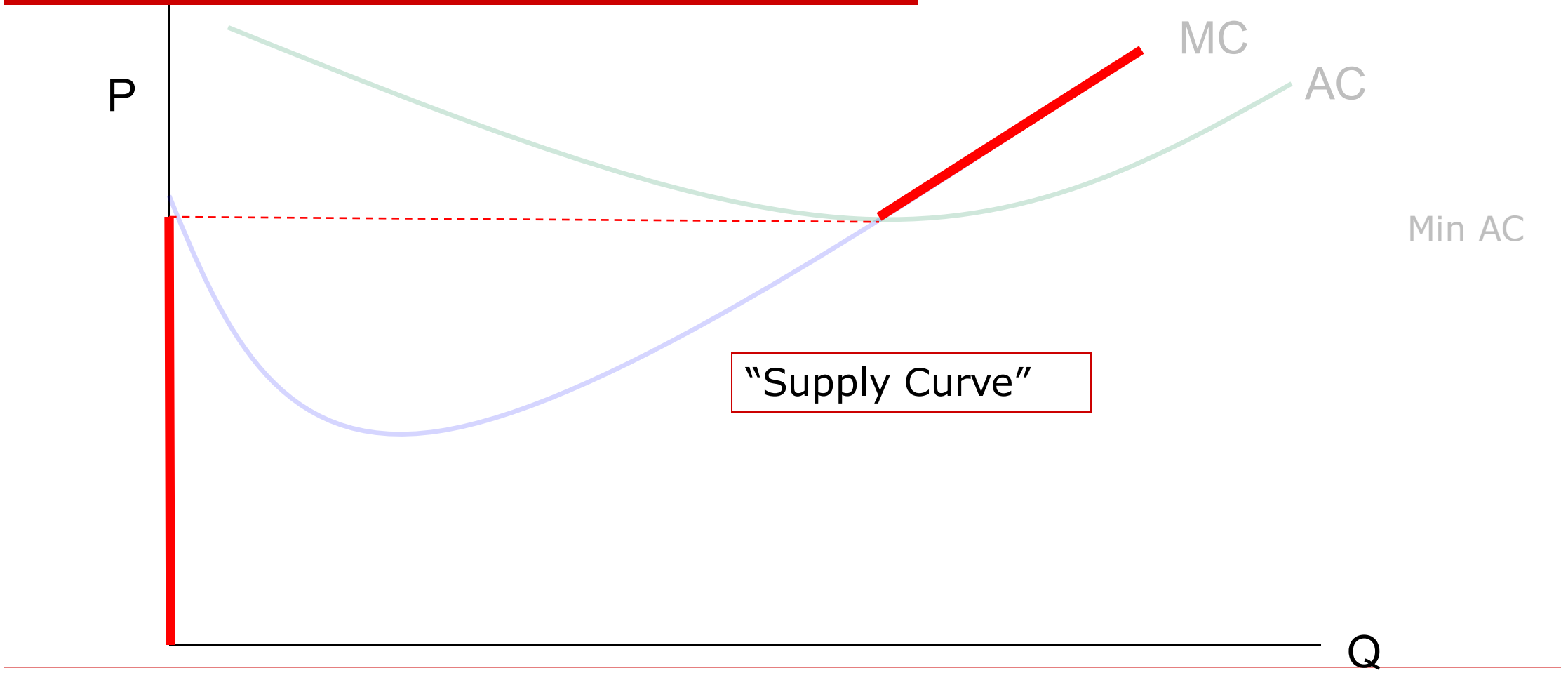


Example

- **Suppose $C(Q) = 32 + 5Q + Q^2$. Price of output P = 10. Find how much Q would a profit maximizer choose to produce**

But here is a puzzle..

- In the previous example, suppose Q = 0
  - $C(0) = 32 + 5Q + Q^2 = 32$
  - Revenue = 0
  - Profit = 0 – 32 = -32 (i.e. firm lose 32 by choosing Q=0)
- But if firm chooses Q = 1
  - C(1) = 32 + 5 + 1 = 38
  - Revenue = 10
  - Profit = 10 – 38 = -28 (i.e. firm lose 28 by choosing Q=1)
- So, this firm is actually better off by producing Q = 1!
- ???

Short-Run and Long-Run Costs

- The trick here is the difference between short-run cost and long-run cost

- **In the short-run, $C(0) = 32 + 5Q + Q^2 = 32$**
  - Think of "32" as what you already paid for renting the store
  - Even if you produce Q=0, you still have to pay 32

- **In the long-run, C(0) = 0**
  - Technically, in the long run, you choose to "exit" the market
  - If you choose to exit, you don't even need to rent the store

- In other words, in the short-run fixed cost is not avoidable, but in the long run fixed cost is avoidable

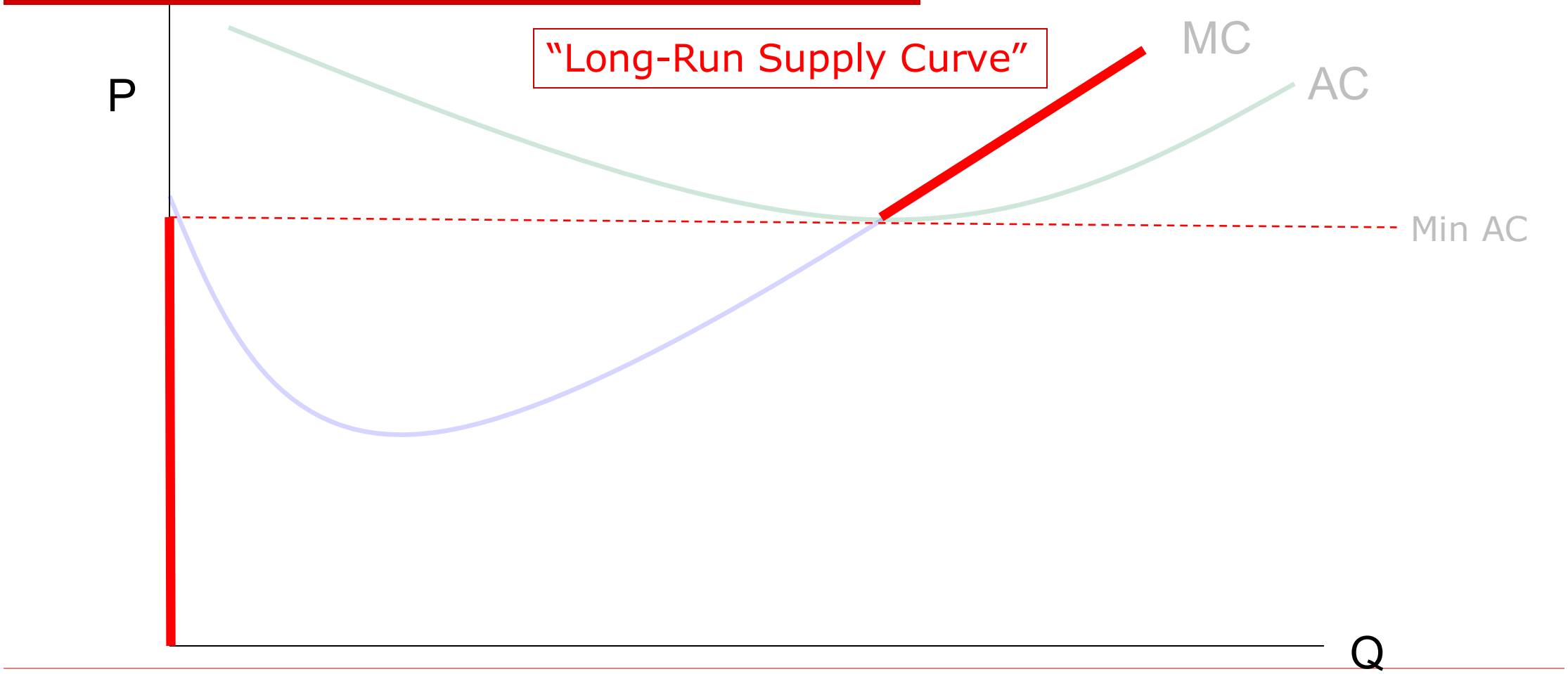
Short-Run and Long-Run Costs

- An equivalent way to think about this is to imagine firms in two situations:
  - Short run: Firm has already entered the market and paid fixed cost
  - Long run: Firm is deciding whether to enter the market; if not, doesn't have to pay fixed cost

- In short-run decision, you should consider only variable cost
  - "I've already paid the rent and can do nothing about it. What's gone is gone. Given I am already in this business, I only need to worry about whether choosing to produce can at least cover my operation (variable) costs"
  - Produce Q such that P=MC(Q) if P > min AVC; choose Q=0 otherwise

- In long-run decision, you should consider total cost
  - "I can decide whether to start the business. If I start, I pay rent and operation costs. But if starting the business cannot cover costs, I won't enter this market."
  - Produce Q such that P=MC(Q) if P > min AC; do not enter (or "exit") the market otherwise

Short-Run and Long-Run Supply Curves



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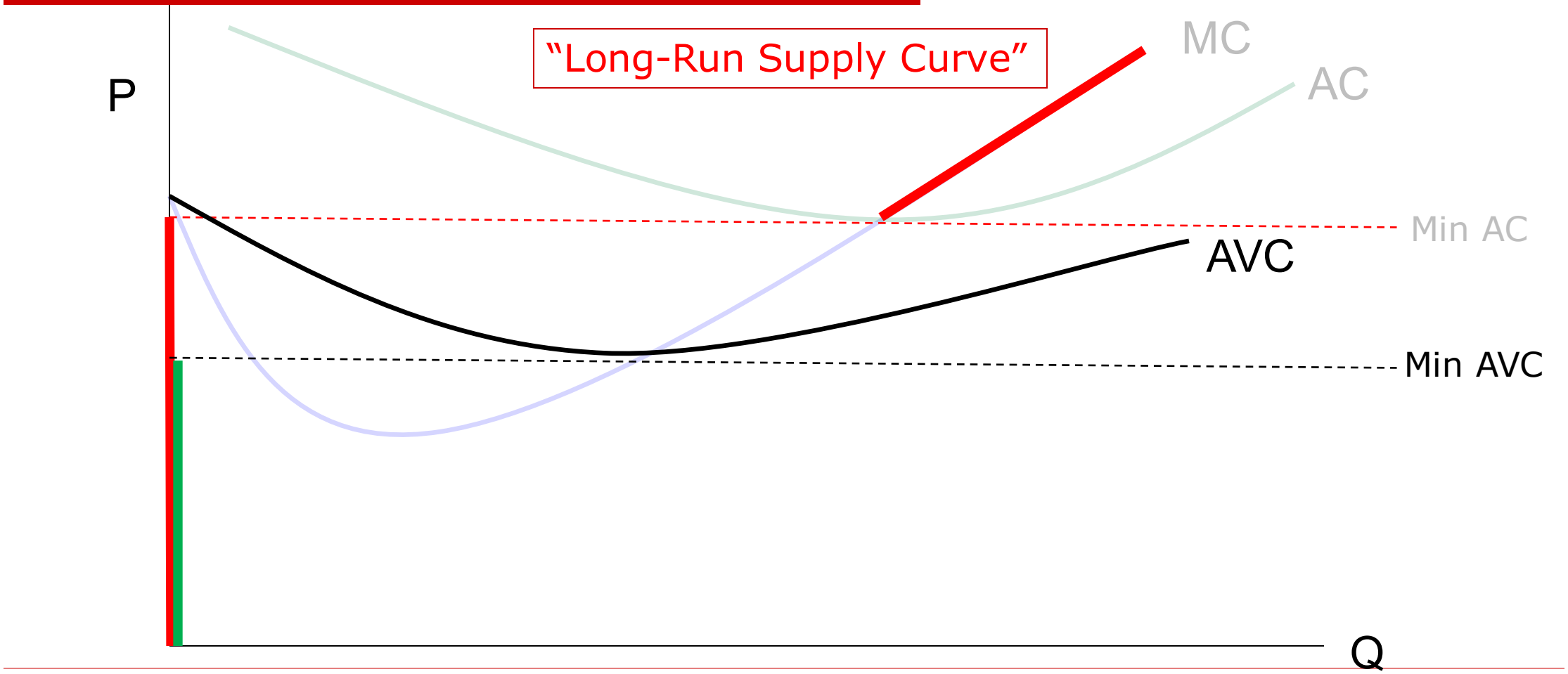
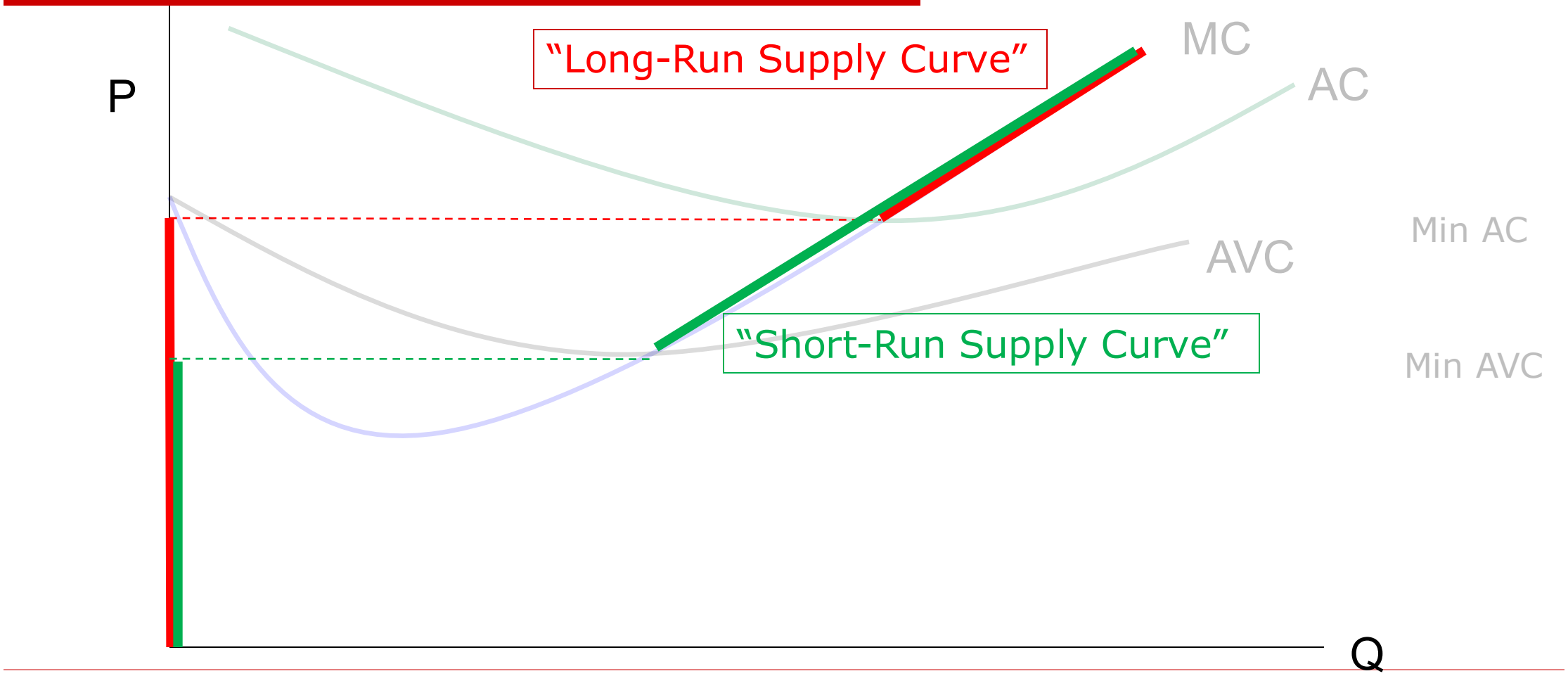


Short-Run and Long-Run Supply Curves



Short-Run/Long-Run Supply

- In the short run, a firm's supply curve is given by its marginal cost curve above the point of minimum average variable cost.

Short Run Supply	
If	Then
P ≥min AVC	Produce Q where P = MC(Q)
P < min AVC	Produce 0.

- In the long run, fixed costs are avoidable. If p < min AC, exit.

Long Run Supply	
If	Then
P ≥min AC	Produce Q where P = MC(Q)
P < min AC	Exit.

Supply: an example

- **A firm's total cost function is:**

$$C(Q) = 100 + 2Q + 0.1\,Q^2$$

- **What is the firm's short run and long run supply curve?**

- To answer this question, you need to know:
  1. The firm's MC curve
  2. Min AVC
  3. Min AC