

Solution to Problem Set 2

Shijian Liu, April 2022

Updated by

Shilong Zhuang, March 2025

1 Question 1

$$(1) x^* = \frac{m}{2p_x} \quad y^* = \frac{m}{2p_y}$$

$$(2) y^* = x^{*2} = \frac{p_x^2 - p_x \sqrt{p_x^2 + 4mp_y} + 2mp_y}{2p_y^2}$$

$$(3) \begin{cases} x^* = \left(\frac{p_y}{p_x}\right)^2 y^* = \frac{mp_x - p_y^2}{p_x p_y^2} & \text{if } p_x > \frac{1}{m} p_y^2 \\ x^* = \frac{m}{p_x} \quad y^* = 0 & \text{if } p_x \leq \frac{1}{m} p_y^2 \end{cases}$$

$$(4) x^* = \frac{mp_y}{p_x^2 + p_x p_y} \quad y^* = \frac{mp_x}{p_y^2 + p_x p_y}$$

$$(5) \begin{cases} x^* = 0 \quad y^* = \frac{m}{p_y} & \text{if } p_x > p_y \\ x^* \in [0, 20] \quad y^* = \frac{m - p_x x^*}{p_y} & \text{if } p_x = p_y \text{ and } 20p_x < m \\ x^* = \alpha \frac{m}{p_x} \quad y^* = (1 - \alpha) \frac{m}{p_y} & \text{if } p_x = p_y \text{ and } 20p_x \geq m \\ x^* = 20 \quad y^* = \frac{m - 20p_x}{p_y} & \text{if } p_x < p_y \text{ and } 20p_x < m \\ x^* = \frac{m}{p_x} \quad y^* = 0 & \text{if } p_x < p_y \text{ and } 20p_x \geq m \end{cases}$$

2 Question 2

a. Full income: $2000w$

budget constraint: $p \cdot c \leq (2000 - L) \cdot w$

Measured income: $(2000 - L)w$

$$b. c^* = \frac{1}{2} TW, L^* = \frac{1}{2} T$$

c. New budget constraint: $c \leq (T - L) \cdot 0.75w$ effective hourly wage: $0.75w$

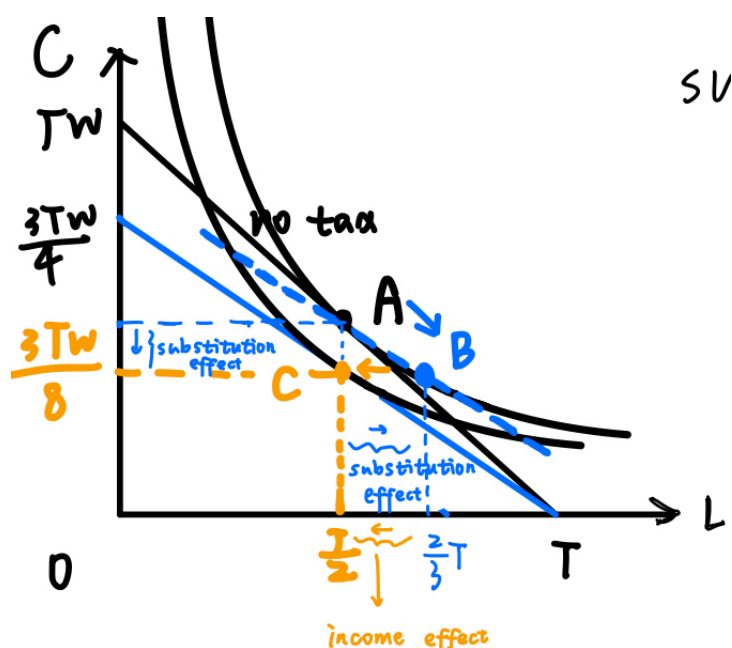
Figure 1: Answer from Boran Zhang



$$d. C^* = \frac{3}{8} TW, L^* = \frac{1}{2} T$$

e. See the picture below. **Notice** that since $L^* = \frac{1}{2}T$ in both sub-question b and d, the substitution effect and income effect of tax on L should completely offset each other in your graph.

Figure 2: Answer from Ruizhu Mei



f. Same/Different/It depends.

- If your answer is "Yes, they have the same sign." Your answer should regard to the sub/income effect of the tax on **consumption**. (which is not exactly the same as the income effect of tax on consumption is 0 in this case.)
Intuition (Any reasonable answer is OK.): When there is tax, the unit return of labour decreases. Therefore, leisure is "cheaper" than before, and one would prefer more leisure (substitution effect). However, the total income decreases as tax increases. Therefore one would have to work more and reduce her leisure (income effect).
- If your answer is "No, they have different sign." Your answer should regard to the sub/income effect of the tax on **leisure hours**.
- If your answer is "It depends", you should mention both situations above.

3 Question 3

a.

$$S_x^0 \cdot \frac{P_x^1}{P_x^0} + S_y^0 \cdot \frac{P_y^1}{P_y^0} = \frac{P_x^0 \cdot x_0}{I_0} \cdot \frac{P_x^1}{P_x^0} + \frac{P_y^0 \cdot y_0}{I_0} \cdot \frac{P_y^1}{P_y^0} \quad (1)$$

$$= \frac{P_x^1 \cdot x_0}{I_0} + \frac{y_0 \cdot P_y^1}{I_0} = \frac{P_x^1 \cdot x_0 + y_0 \cdot P_y^1}{I_0} = \frac{I_1}{I_0} \quad (2)$$

b.1 Approximate value is allowed.

	p_x	p_y	I	x^*	y^*	U
Base year	\$1	\$4	\$400	200	50	2,000
First year	\$2	\$5				
No adjustment			\$400	100	40	1,265
CPI adjustment			\$650	$325/2$	65	$650\sqrt{10}$
True COLA			$\$100\sqrt{10}$	$50\sqrt{10}$	$20\sqrt{10}$	2,000

b.2 $\frac{13}{8}$ b.3 $\frac{200\sqrt{10}}{400} \approx 1.6$ b.4 $\frac{650}{200\sqrt{10}} \approx 1.03$

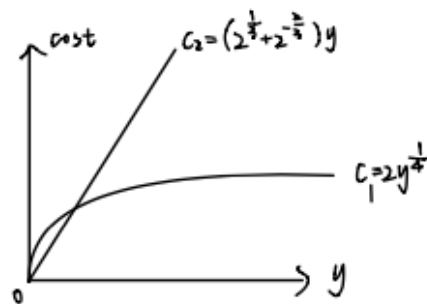
4 Question 4

a. IRTS CRTS

b. $C_1^* = 2q_0^{\frac{1}{4}}$ $C_2^* = (2^{\frac{1}{3}} + 2^{-\frac{2}{3}})q_0$

c.

Figure 3: Answer from Ruizhu Mei



d.

$$AC_1 = 2q_0^{-\frac{3}{4}}$$

$$MC_1 = \frac{1}{2}q_0^{-\frac{3}{4}}$$

$$AC_2 = 2^{\frac{1}{3}} + 2^{-\frac{2}{3}}$$

$$MC_2 = 2^{\frac{1}{3}} + 2^{-\frac{2}{3}}$$

