Intermediate Macroeconomics: Midterm Exam
Suggested Solution

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DO NOT TURN OVER THIS PAGE UNTIL THE PROCTOR SAYS YOU MAY.

Instructions

- This exam is out of 100 points.
- You have **120** minutes to complete the exam.
- **Write down your answers on the provided answer sheets, in either English, Chinese, or a combination of both**. Don't forget to write down your name.
- No calculators, phones, notes or books of any kind are permitted.

Good Luck!

1. True/False/Uncertain (20 points, or 5 points each)

Assess whether the following statements are true, false or uncertain and justify your answers. Points are given for explanations only.

a. Assuming countries have the same rates of technological progress and population growth, the Solow model predicts that along their balanced growth paths, the countries with a lower saving rates will grow more slowly than those with higher saving rates.

b. In a economy with positive inflation rates, investing in nominal bonds are always better than investing in real capital, since the nominal interest rate from the bond would always be higher than the real interest rate from the capital production.

c. Holding the rate of technological growth equal, a lower population growth rate would raise per capita income and capital in the long run.

d. If the asset market is complete, it is possible to find a risky asset that pays more than the safe bond in all states of the world, but still cost less than the safe bond.

Solution:

a. False. On the balanced growth path, the long term growth rates are $g_A + g_N$, which is independent of the saving rates.

b. False. Even though Fisher equation states that $i = \pi + r$, and when $\pi > 0$ it implies $i > r$, it does not mean the nominal interest rates give higher return.

c. True. In both Solow and Ramsey model with tech and population growth, a higher g_n corresponds to a lower steady state k^* and therefore a lower steady state per capita capital.

d. False/Uncertain. As long as the state-contingent claims are priced positively, a risky asset that pays more in every state should always be more expensive.

2. OLG Model with Money in Utility (30 points)

Consider an overlapping generations model with money in the utility function. In each period, a cohort of constant size 1 is born. Each individual lives for 2 periods. There is an initial old cohort, endowed with capital K_0 and money M. The preferences for the initial old are defined as

$$U = u(c_0^o)$$

Agents born after $t = 0$ are endowed with one unit of labor when young, and do not supply labor when old. They have utility function defined as

$$U(c_t^y, c_{t+1}^o, m_t) = u(c_t^y) + v(m_t) + \beta u(c_{t+1}^o)$$

Where $m_t = \frac{M_t}{P_t}$ is real money holding for young agents at time t, and $u(\cdot), v(\cdot)$ satisfy $u' > 0, v' > 0, u'' < 0, v'' < 0$. Output is produced using a contant return to scale production function

$Y_t = F(K_t, N_t)$, with capital depreciation rate $\delta = 1$. There is no storage technology other than money and investment. Also, there is no government, so no new money is ever issued.

a. (6 pts) Find the budget constraints for the initial old cohort, as well as cohorts born after $t = 0$. Use w_t and R_t to represent (real) wage and rental rates at t.

b. (6 pts) Define a competitive equilibrium for this economy.

c. (6 pts) Solve the firm's problem for its first order conditions.

d. (6 pts) Find the first order conditions for agent born after $t = 0$, and write down the Euler Equation.

e. (6 pts) Using the first order condition for money and the fact that $v' > 0$, show that in a competitive equilibrium the real return on capital R_{t+1} is always greater than real return on money, $\frac{P_t}{P_{t+1}}$. Explain this result.

Solution:

a.

$$
\begin{aligned}
c_0^o &= \frac{M}{P_0} + r_0 K_0 \\
c_t^y + \frac{M_t}{P_t} + K_{t+1} &= w_t N_t \\
c_{t+1}^o &= \frac{M_t}{P_{t+1}} + R_{t+1} K_{t+1}
\end{aligned}
$$

b. The competitive equilibrium is defined as the set of variables $\{K_{t+1}, c_t^y, c_{t+1}^o, c_0^o\}$ and $\{w_t, R_t, P_t\}$ such that

(a) Initial old household choose c_0^o to maximize utility subject to its budget constraint, taking r_0, P_0 as given.

(b) All households born after $t = 0$ choose $c_t^y, c_{t+1}^o and K_{t+1}^s$ to maximize utility subject to their budget constraints, taken w_t, R_t, P_t as given;

(c) Given w_t, R_t, firm choose K_t^d, N_t^d to maximize profit.

(d) All markets clear. $c_t^o + c_t^y = Y_t$, $M_t = M$,$K_t^d = K_t^s$, $N_t^d = 1$.

c.

$$
\begin{aligned}
F_K(K_t, N_t) &= R_t \\
F_N(K_t, N_t) &= w_t
\end{aligned}
$$

d.

$$
\mathcal{L} = u(c_t^y) + v(m_t) + \beta u(c_{t+1}^o) + \lambda_1(w_t - c_t^y - m_t - K_{t+1}) + \lambda_2(R_{t+1}K_{t+1} + \frac{M_t}{P_{t+1}} - c_{t+1}^o)
$$

$$[c_t^y]: \; u'(c_t^y) = \lambda_1$$
$$[c_{t+1}^o]: \; \beta u'(c_{t+1}^o) - \lambda_2 = 0$$
$$[m_t]: \; v(m_t) - \lambda_1 + \lambda_2 \frac{P_t}{P_{t+1}} = 0$$
$$[K_{t+1}]: \; -\lambda_1 + \lambda_2 R_{t+1} = 0$$

Euler Equation: $u'(c_t^y) = \beta R_{t+1} u'(C_{t+1}^o)$

e.

$$v'(m_t) = \beta u'(c_{t+1}^o)[R_{t+1} - \frac{P_t}{P_{t+1}}] > 0$$

So $R_{t+1} > \frac{P_t}{P_{t+1}}$

3. Trade and Tariff in a Two-Period Model (35 Points)

"Tariffs don't protect, they backfire." - Ronald Reagan

Consider the following two-period model with a simple production function. Suppose the social planner of a country (country A) is solving the following problem:

$$\max_{c_0, l_0, c_1, l_1} \; \ln(c_0) + \ln(1 - l_0) + \beta[\ln(c_1) + \ln(1 - l_1)]$$
$$s.t. \; c_0 + b_0 = A_0 l_0,$$
$$c_1 = A_1 l_1 + b_0(1 + r)$$

where c_t, l_t, A_t stands for consumption, labor, and technology of country A in period t.

Questions: For questions (a) to (c), assume that country A cannot borrow or trade, so $b_0 = 0$.

a. (6 pts) Write down the Lagrangian for the social planner, and derive the first order conditions.

b. (6 pts) Solve the optimal levels of c_0, c_1, l_0, l_1 as functions of A_0, A_1, and β.

c. (6 pts) Solve the equilibrium interest rate r_1^* as an expression of A_0, A_1 and β.

For questions (d) onwards, assume there are another similar country (country C) that can trade with country A. Its population, utility function are both identical to country A's, but the technology levels are different. In particular, country C's social planner solves the following problem:

$$\max_{\tilde{c}_0, \tilde{l}_0, \tilde{c}_1, \tilde{l}_1} \; \ln(\tilde{c}_0) + \ln(1 - \tilde{l}_0) + \beta[\ln(\tilde{c}_1) + \ln(1 - \tilde{l}_1)]$$
$$s.t. \; \tilde{c}_0 + \tilde{b}_0 = \tilde{A}_0 \tilde{l}_0,$$
$$\tilde{c}_1 = \tilde{A}_1 \tilde{l}_1 + \tilde{b}_0(1 + r)$$

where we use $\tilde{c}_t, \tilde{l}_t, \tilde{A}_t$ to represent the consumption, labor, and technology of country C in period t. Importantly, we no longer impose the restriction that b_0 or $\tilde{b}_0$ =0; instead, we allow countries to borrow/lend to each other, as long as the total borrowing or lending between the countries add up to 0, or $b_0 + \tilde{b}_0 = 0$.

For (d), suppose $A_0 = \tilde{A}_0$ and $A_1 > \tilde{A}_1$.

d. (7 pts) Using the global product market clearing conditions (when two countries are trading with each other):

$$c_0 + \tilde{c}_0 = A_0 l_0 + A_0 \tilde{l}_0$$
$$c_1 + \tilde{c}_1 = A_1 l_1 + \tilde{A}_1 \tilde{l}_0$$

Solve the equilibrium interest rate r_2^* as functions of $A_0, A_1, \tilde{A}_1$ and β. Is this interest rate higher or lower than the equilibrium interest rate r_1^* that you solve in part (c)?

For (e) and (f), assume $A_0 = \tilde{A}_0 = \tilde{A}_1 = 1$, $A_1 = 3$, and $\beta = 1$.

e. (5 pts, **Difficult**) First solve the equilibrium levels of c_0, c_1, l_0, l_1 of country A. We can define country A's net export in period t as

$$NX_t^A = A_t l_t - c_t$$

If $NX_t^A > 0$, we call country A a *net exporter* in period t, and it has *positive trade balance* with country C. Otherwise, we call it a *net importer*. Using the results above, show whether country A is a net exporter or importer in period 0 and period 1.

f. (5 pts, **Very difficult**) Suppose country A imposes a tariff of $\tau = 100\%$ on its imports from country C, and the revenue from the tariff are invested into wasteful government expenditure (therefore not entering the total income of its citizens); the budget constraint of country A becomes

$$c_0 + \frac{b_0}{1+\tau} = A_0 l_0 \tag{1}$$
$$c_1 = A_1 l_1 + b_0(1+r) \tag{2}$$

Discuss the impact of this tariff τ on interest rates, trade balances and the total utility of citizens in country A.

Solution:

a.

$$\mathcal{L} = \ln(c_0) + \ln(1-l_0) + \beta[\ln(c_1) + \ln(1-l_1)] + \lambda\left[A_0 l_0 + \frac{A_1 l_1}{1+r} - c_0 - \frac{c_1}{1+r}\right]$$

The first order conditions are

$$
\begin{aligned}
[c_0] \quad & \frac{1}{c_0} = \lambda \\
[l_0] \quad & \frac{1}{1-l_0} = \lambda A_0 \\
[c_1] \quad & \frac{\beta}{c_1} = \frac{\lambda}{1+r} \\
[l_1] \quad & \frac{\beta}{1-l_1} = \frac{\lambda A_1}{1+r}
\end{aligned}
$$

b. From the first order conditions, we have

$$
\begin{aligned}
\frac{c_1}{c_0} &= \beta(1+r) \quad \text{(Euler Equation)} \\
\frac{c_0}{1-l_0} &= A_0 \quad \Rightarrow c_0 = A_0(1-l_0) \\
\frac{c_1}{1-l_1} &= A_1 \quad \Rightarrow c_1 = A_1(1-l_1)
\end{aligned}
$$

From the assumption that $b_0 = 0$ and $c_0 = A_0 l_0$, $c_1 = A_1 l_1$, we can solve that $l_0 = l_1 = 0.5$, and $c_0 = 0.5A_0, c_1 = 0.5A_1$.

c. From the Euler equation,

$$
1 + r = \frac{c_1}{\beta c_0} = \frac{A_1}{\beta A_0}
$$

d. Now there is another country with very similar problem. From the first order conditions, we can get that

$$
\begin{aligned}
\tilde{c}_0 &= A_0(1-\tilde{l}_0) \\
\tilde{c}_1 &= \tilde{A}_1(1-\tilde{l}_1)
\end{aligned}
$$

Now using the goods market clearing condition, we know that

$$
\begin{aligned}
c_0 + \tilde{c}_0 &= A_0(1 - l_0 + 1 - \tilde{l}_0) = A_0 l_0 + A_0 \tilde{l}_0 \\
c_1 + \tilde{c}_1 &= A_1(1-l_1) + \tilde{A}_1(1-\tilde{l}_1) = A_1 l_1 + \tilde{A}_1 \tilde{l}_1
\end{aligned}
$$

From the first equation, we get

$$
c_0 + \tilde{c}_0 = A_0 l_0 + A_0 \tilde{l}_0 = A_0
$$

From the second equation, we get

$$
c_1 + \tilde{c}_1 = A_1 l_1 + \tilde{A}_1 \tilde{l}_1 = \frac{A_1 + \tilde{A}_1}{2}
$$

Since both countries have identical Euler equations, (i.e. $c_1 = \beta(1+r)c_0$, $\tilde{c}_1 = \beta(1+r)\tilde{c}_0$), we have that

$$
1 + r = \frac{\frac{A_1 + \tilde{A}_1}{2}}{\beta A_0}
$$

Because $\tilde{A}_1 < A_1$, the interest rate here is lower than the previous interest rate.

e. This problem is harder than it looks. There are 4 equations and 4 unknowns to solve simultaneously for each country. We can first solve for $1+r$, using the numbers given:

$$1 + r = 2/(1 * 1) = 2$$

Then we go back to the original problem:

$$\begin{aligned} c_1 &= 2c_0 \\ c_0 &= 1 - l_0 \\ c_1 &= 3(1 - l_1) \\ c_0 + \frac{c_1}{2} &= l_0 + \frac{3l_1}{2} \end{aligned}$$

We can solve that $l_0 = \frac{3}{8}$, $l_1 = \frac{7}{12}$, $c_0 = \frac{5}{8}$, $c_1 = \frac{5}{4}$. We find that country A is a net importer in period 0 (since $c_0 > A_0 l_0$) and a net exporter in period 1 (since $c_1 < A_1 l_1$)

f. The most difficult part of this problem is to understand what the new market clearing conditions are. As there are only two countries and net "borrowing" equals net "lending", we should have that $b_0 + \tilde{b}_0 = 0$ and $b_1 + \tilde{b}_1 = 0$. Let's analyze the change closely. First, the optimization problem of country A is changed, and the new Lagrangian is

$$\mathcal{L} = \ln(c_0) + \ln(1 - l_0) + \beta[\ln(c_1) + \ln(1 - l_1)] + \lambda \left[A_0 l_0 + \frac{A_1 l_1}{(1+r)(1+\tau)} - c_0 - \frac{c_1}{(1+r)(1+\tau)} \right]$$

So the new Euler equation for country A is

$$\frac{c_1}{c_0} = \beta(1 + r)(1 + \tau)$$

The Euler equation for country C is still $\tilde{c}_1 = \tilde{c}_0 \beta(1+r)$. Also, the tradeoff between consumption and labor does not change in each period, meaning that $c_0 = A_0(1 - l_0), c_1 = A_1(1 - l_1)$ still hold true. The other difference is that the new global market clearing conditions are

$$\begin{aligned} c_0(1 + \tau) + \tilde{c}_0 &= A_0 l_0 (1 + \tau) + A_0 \tilde{l}_0 \\ c_1 + \tilde{c}_1 &= A_1 l_1 + \tilde{A}_1 \tilde{l}_0 \end{aligned}$$

We can still use the condition that $c_t = A_t(1 - l_t)$, $\tilde{c}_t = \tilde{A}_t(1 - \tilde{l}_t)$ to solve for the interest rate. Interestingly it's easier to solve than expected, and the result is

$$1 + r = \frac{A_1 + \tilde{A}_1}{\beta A_0 (2 + \tau)}$$

As τ increases, the interest rate becomes smaller than the case in (d), meaning that country A is likely to increase its borrowing in period 0. We can calculate the numbers when $\tau = 1$, and what I get is $c_0 = 17/32, l_0 = 15/32, c_1 = 17/12, l_1 = 19/36$. The import in period 0 decreases from $\frac{1}{4}$ to $\frac{1}{16}$, and export in period 1 decreases from $\frac{1}{2}$ to $\frac{1}{6}$. The total utility changes from $\ln(5/8) + \ln(5/8) + \ln(5/4) + \ln(5/12) \approx -1.592$ to $\ln(17/32) + \ln(17/32) + \ln(17/12) + \ln(17/36) \approx -1.667$, reflecting a lower level of welfare.

4. Short Answer (15 points)

Using what you have learned from this class so far, discuss whether you agree with the statement below, as carefully and thoroughly as possible.

Statement: sometimes it can be beneficial to let people borrow from their future income and accumulate some debt, as long as they can repay it.

Solution: Could answer from many persepctives. Using the insights learned from two period models (with or without uncertainty), infinite period models, OLG, Ramsey models, all are fine. Basically, allowing people to borrow when they want to can usually bring equal or higher utility (the effect of relaxing borrowing constraint) but may also expose them to higher risks.