No.

Date 25 4 12

一、概统

卷积公式 孔

例1. X,Y独立 $\sim N(0,1)$ 求 $Z=X+Y$ 概率密度

$$f_Z(z)=\int_{-\infty}^{+\infty} f_X(z-y)f_Y(y)dy=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{(z-y)^2}{2}}\cdot\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{y^2}{2}}dy=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{-\frac{z^2}{2}+zy-\frac{y^2}{2}-\frac{y^2}{2}}dy$$

$$e^{-y^2+zy-\frac{z^2}{2}}=\cancel{e^{-(y-\frac{z}{2})^2-\frac{z^2}{4}}}\quad \cancel{e^{\frac{-[\sqrt2(y-\frac z2)]^2}{2}}}$$

$$\exp\left\{-\frac{[\sqrt2(y-\frac z2)]^2}{2}-\frac{z^2}{4}\right\}$$

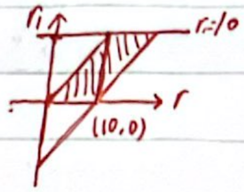
$$f_Z(z)=\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{z^2}{4}}\cdot\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}}\exp\left\{\frac{-[\sqrt2(y-\frac z2)]^2}{2}\right\}\frac{1}{\sqrt2}d[\sqrt2(y-\frac z2)]=\frac{1}{2\sqrt\pi}\cdot e^{-\frac{z^2}{4}}\Rightarrow Z\sim N(0,2)$$

其他做法: $f_Z(z)=\frac{1}{2\pi}e^{-\frac{z^2}{4}}\int_{-\infty}^{+\infty}e^{-(y-\frac z2)^2}dy=\frac{1}{2\pi}e^{-\frac{z^2}{4}}\int_{-\infty}^{+\infty}e^{-t^2}dt=\frac{1}{2\pi}e^{-\frac{z^2}{4}}\times\sqrt\pi=\frac{1}{2\sqrt\pi}e^{-\frac{z^2}{4}}$

例2. 电阻R_1,R_2串联 独立 $f(x)=\begin{cases}\frac{10-x}{50}, & 0\le x\le 10\\ 0, & 其他\end{cases}$ 求$R=R_1+R_2$…

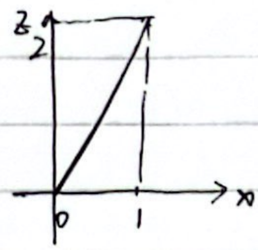
$$f_R(r)=\int_{-\infty}^{+\infty}f_{R_1}(r_1)f_{R_2}(r-r_1)dr_1\begin{cases}0\le r\le 10, & \int_0^r\frac{(10-r_1)}{50}\cdot\frac{(10-r+r_1)}{50}dr_1\\ 10<r\le 20, & \int_{r-10}^{10}\frac{10-r_1}{50}\cdot\frac{(10-r+r_1)}{50}dr_1\end{cases}$$ 其他$=0$

作图分析: $\begin{cases}0\le r_1\le 10\\ 0\le r-r_1\le 10\end{cases}\Rightarrow\begin{cases}0\le r_1\le 10\\ r-10\le r_1\le r\end{cases}$



线性组合 孔

例4. $f(x,y)=\begin{cases}1, & 0<x<1, 0<y<2x\\ 0, \end{cases}$ 求$Z=2X-Y$…



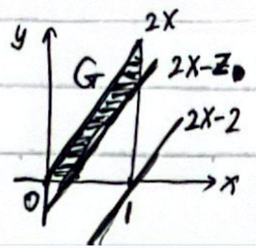
$f_Z(z)=\int_{-\infty}^{+\infty}f(x,2x-z)dx$

$0<x<1,\ 0<2x-z<2x\Rightarrow z>0,\ x>\frac z2$

当$0<z<2$时, $\cancel{f(x,y)}$ $f_Z(z)=\int_{\frac z2}^{1}1dx=1-\frac z2$

综上, $f_Z(z)=\begin{cases}1-\frac z2, & 0<z<2\\ 0, & 其他\end{cases}$

$\boxed{\text{用公式法注意y前面的系数}}$



$F_Z(z)=P\{Z\le z\}=P\{2X-Y\le z\}=P\{Y\ge 2X-z\}$

当$z\le 0$时, $P\{Y\ge 2X-z\}=0$; 当$z\ge 2$时, $P\{Y\ge 2X-z\}=1$

当$0<z<2$时, $P\{Y\ge 2X-z\}=\iint 1dxdy=\frac12-\frac12(1-\frac z2)^2$

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续概统

$$f_Z(z)=F_Z'(z)=\begin{cases}1-\frac{z}{2}, & 0<z<2\\ 0, & 其他\end{cases}$$

Z=XY 孔

~~[illegible]~~ 例1. $G=\{(x,y)\mid 0\le x\le 2,\ 0\le y\le 1\}$ 均匀分布 $S=XY$的矩形面积的分…

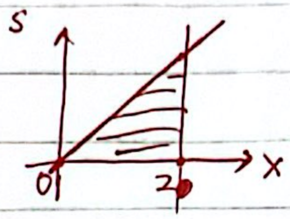
$$f_S(s)=\int_{-\infty}^{+\infty}\frac{1}{|x|}f\left(x,\frac{s}{x}\right)dx$$

$$f(x,y)=\begin{cases}\frac{1}{2}, & 0\le x\le 2, 0\le y\le 1\\ 0, & 其他\end{cases},\quad 0\le x\le 2,\ 0\le\frac{s}{x}\le 1\Rightarrow s\le x,\ s\ge 0$$

$$f_S(s)=\int_s^2\frac{1}{|x|}\cdot\frac{1}{2}dx=\frac{1}{2}\ln x\Big|_s^2=\frac{1}{2}\ln\frac{2}{s}$$

当$s>0$时,
$0<s\le x$

综上, $f_S(s)=\begin{cases}\frac{1}{2}\ln\frac{2}{s}, & s\ge 0\quad 0<s\le x\\ 0, & 其他\end{cases}$



画s-x图可以防止取值范围错误

例2. X、Y 分别服从λ_1,λ_2指数分布 独立 求$Z=\frac{X}{Y}$…

$X=ZY$, $f_Z(z)=\int_{-\infty}^{+\infty}f_X(zy)f_Y(y)dy$

$f_X(x)=\lambda_1e^{-\lambda_1x}$, $f_Y(y)=\lambda_2e^{-\lambda_2y}$

$$f_Z(z)=\lambda_1\lambda_2\int_0^{+\infty}ye^{-\lambda_1zy-\lambda_2y}dy\ \cancel{=\lambda_1\lambda_2\cdot\left(\frac{-1}{\lambda_1z+\lambda_2}\right)e^{-(\lambda_1z+\lambda_2)y}\Big|_0^{+\infty}=\frac{\lambda_1\lambda_2}{\lambda_1z+\lambda_2}}$$

$$\underset{分部积分}{=}-\frac{1}{\lambda_1z+\lambda_2}\left(y\cdot e^{-(\lambda_1z+\lambda_2)y}+\frac{1}{\lambda_1z+\lambda_2}e^{-(\lambda_1z+\lambda_2)y}\right)\Big|_0^{+\infty}=\frac{\lambda_1\lambda_2}{(\lambda_1z+\lambda_2)^2}$$

一维SB数学期望 孔

$\boxed{例3.}$ 车站8:00~9:00和9:00~10:00各有车到站 时间独立

8:10 9:10	8:30 9:30	8:50 9:50
$\frac{1}{6}$	$\frac{3}{6}$	$\frac{2}{6}$

8:20到站乘客等车时间期望?

X	10	30	50	70	90
P	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{2}{36}$

$E(X)=27.22$