Intermediate Microeconomic

Spring 2025

Part three: Production and supply

Week 4(b): Production Function

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Definition of Economic Cost

- Economic cost of any input
  - The payment required to keep that input in its present employment
  - The remuneration the input would receive in its best alternative employment

Definitions of Costs

- It is important to differentiate between accounting cost and economic cost
  - Accountants: out-of-pocket expenses, historical costs, depreciation, and other bookkeeping entries
  - Economists focus more on opportunity cost

Economic Costs

- For decision making, the critical question is whether a cost is avoidable or not.
- If a cost is not avoidable, it should not affect your decisions.
  - E.g., if you have an unbreakable lease on your factory, then your rent should not factor into your decision making.
- Unavoidable costs are also called "sunk costs."
- Over a long-enough time horizon, all costs are avoidable. This is what we mean when we talk about "long-run" costs.

Opportunity cost

- Opportunity cost:
  - the **opportunity cost** of an asset is its value in its **next-best use**.
  - It is what you must give up to use the asset in the way you're considering.

- "Opportunity cost" is the right concept of cost for decision-making, since decision-making is about deciding whether to use an asset (including yourself!) in a particular project or *in its next-best use*.

- If you understand opportunity cost, you will be in a better place to understand whether a company is doing the right thing and whether you're doing the right thing!

Should Google be making self-driving cars? (note: spun off into Waymo)

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- The answer of course depends on (a) how profitable Google's self-driving cars are likely to be.

- But it's not just about whether the revenue from making self-driving cars is greater than the cost of doing so. It also depends on (b) what the best alternative use of Google's money is.

- If Google can, for example, come up with something patentable that will replace smart phones, it should *not* be using those resources to pursue self-driving cars. The right question is NOT whether (a)>0, but whether (a)>(b)!

- (Google can also give that money back to shareholders! And then the question is: what's the profit from the best use of the money available to shareholders?)

Economic Cost and Profit

- The **total economic cost** of a project is what you must earn in order to justify not employing the assets in their best alternative uses.

- If revenue ≥ **economic** cost, then the enterprise is worthwhile.

- Economic profit of 0 ➔ fair return on the assets used in the project.
  - If you are a star and earn $$$$ in an alternative job, zero economic profit means that you could earn just as much in the new project.
  - Economic profit = 0 doesn’t mean “just getting by.”

Economic Cost and Profit

- Economic profit > 0 ➔ assets are getting more than their “fair” return.
  - Generally arises only if some asset used in the enterprise that is in short supply.
  - This asset earns an extraordinary return, which we call an “economic rent.”

- “Economic rents” vary with the context. I’m (hopefully!) getting an economic rent from having a PhD in my current job, but I wouldn’t get an economic rent if I became a baker.

The cost of a startup

- You quit your $120,000/year job as a financial analyst to start a hedge fund. You buy $10,000 in computer equipment, raise $12,000 from friends and family, and start working away. What are your monthly costs?

- **Accountant**: $1,000/month (assuming even spending of friends & family money) + depreciation of computer equipment.

- **Economist**: $10,000/month from not getting a salary, $1,000/month in friends & family money, and $$$ you would get if you rented out the computer equipment.

Producer's Problem

- Producer's **Profit Maximization Problem (PMP)**: Given the production technology $Q=F(K,L)$, find the optimal choice of output (Q^*), capital (K^*), labor(L^*) that maximizes firm's profit $\pi = pQ - vK - WL$

- If you think about it, PMP has two parts
  - How many output Q should I produce?
  - Given I want to produce Q, what's the optimal combination of K and L that I should choose

The "Cost Minimization Problem" (CMP)

Cost Minimization Problem (CMP)

- Let v be the price of capital, w be the wage.
- The cost of pair $(L, K) = w\,L + v\,K$.
- Suppose the firm wants to produce Q units of output.
- Then it solves the **Cost Minimization Problem (CMP)**:

$$\min_{L,K} w\,L + v\,K$$

subject to: $\mathrm{Q} \leq F(K, L)$

- Read the constraint this way: “with K and L inputs, the producer can produce at most F(K,L)”. Usually, assume Q=F(K,L), i.e. no reason to throw away output.

Isocost Curves

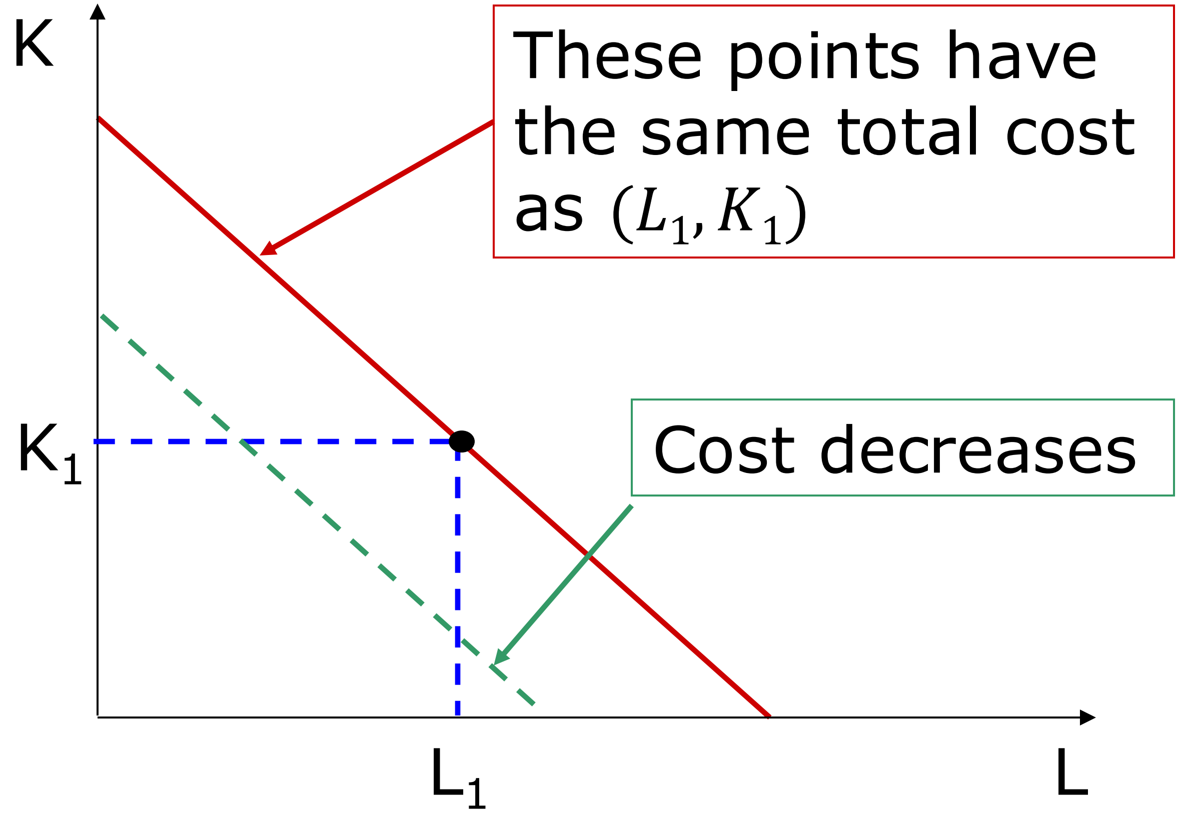
- The set of (K, L) that has cost C is given by:

$$vK + wL = C, \text{ or}$$

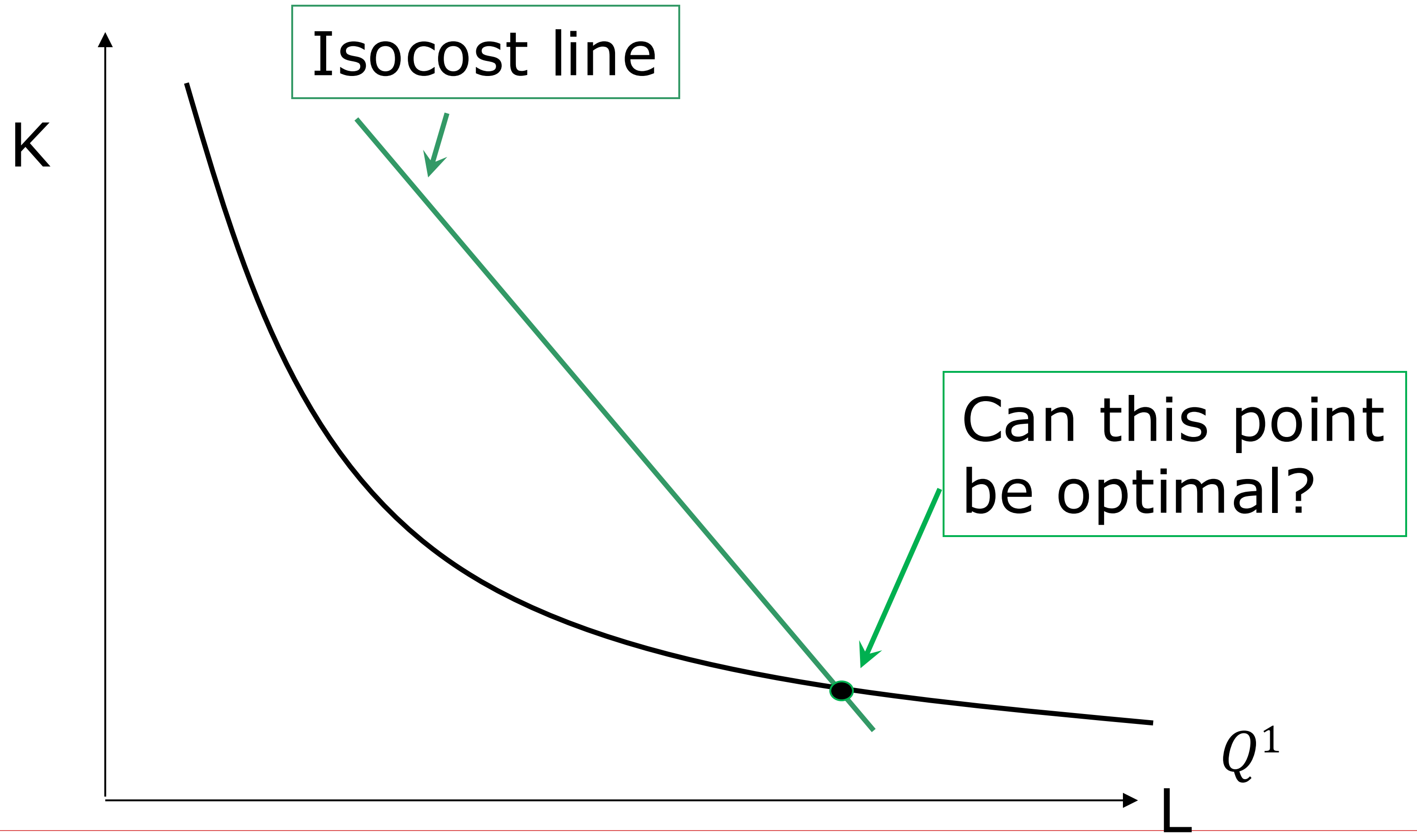
$$K = -w/v * L + C/v$$

- "Isocost curves" are lines with slope $-w/v$.

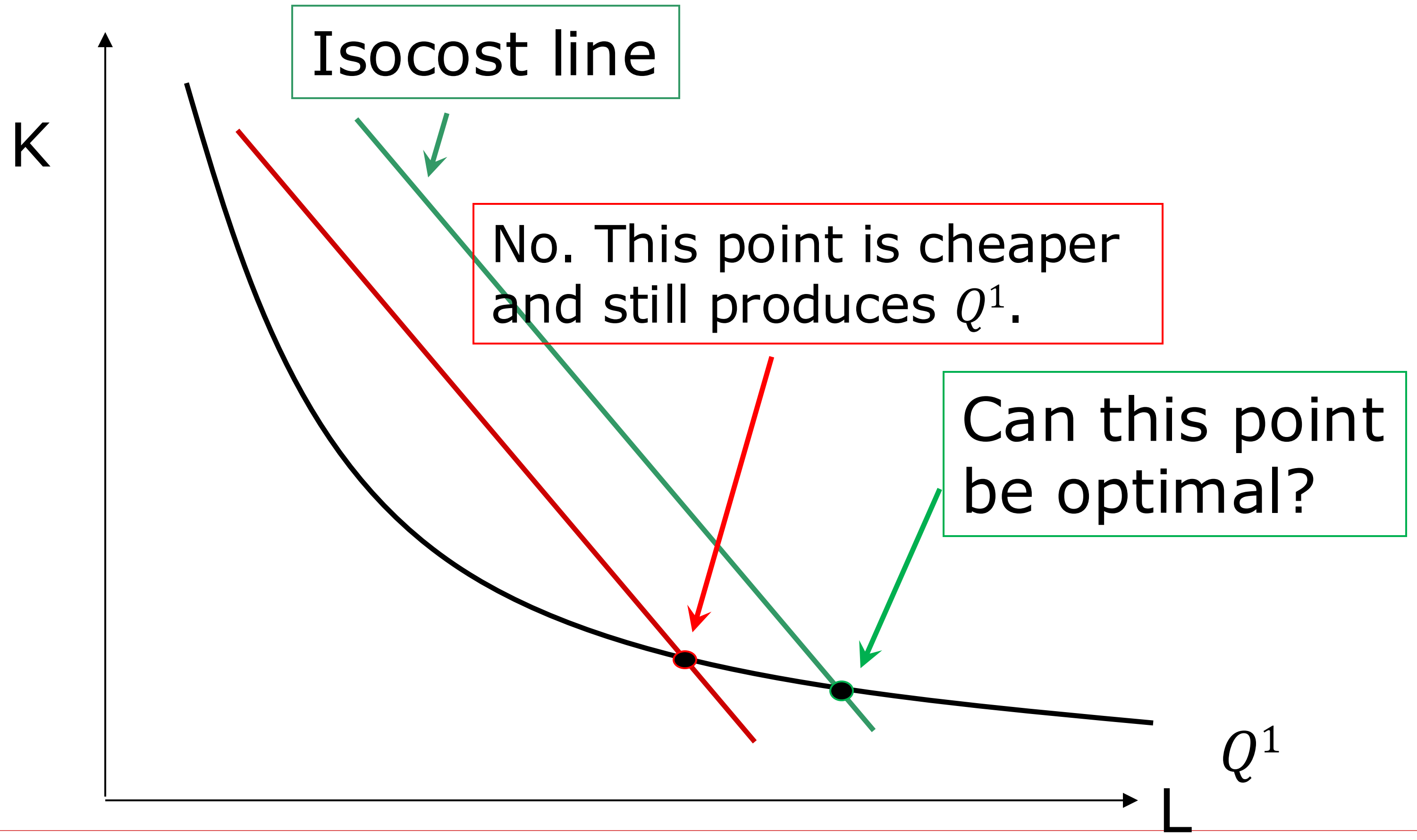
- Lower costs are closer to the origin.



Cost Minimization: Graphically



Cost Minimization: Graphically



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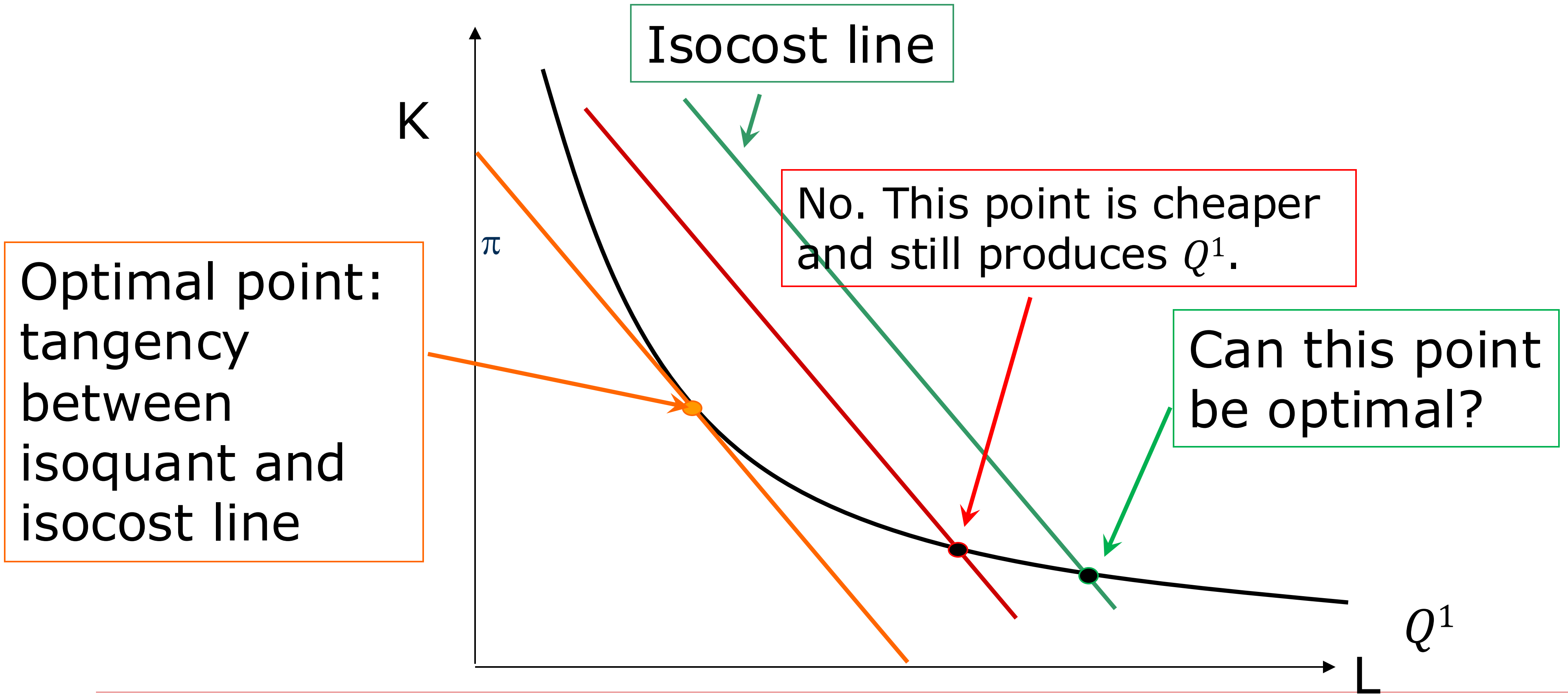


Cost-Minimizing Input Choices

- Minimize total costs given $q = f(k,l) = q_0$
- Setting up the Lagrangian:

$$\mathscr{L} = wl + vk + \lambda[q_0 - f(k,l)]$$

- First-order conditions:

$$\partial\mathscr{L}/\partial l = w - \lambda(\partial f/\partial l) = 0$$

$$\partial\mathscr{L}/\partial k = v - \lambda(\partial f/\partial k) = 0$$

$$\partial\mathscr{L}/\partial\lambda = q_0 - f(k,l) = 0$$

Cost-Minimizing Input Choices

- Dividing the first two conditions we get

$$\frac{w}{v} = \frac{f_l}{f_k} = \text{MRTS}$$

  - The cost-minimizing firm should equate the MRTS for the two inputs to the ratio of their prices

Cost-Minimizing Input Choices

- Useful reciprocal relationship:

$$\frac{w}{f_l} = \frac{v}{f_k} = \lambda$$

  - The Lagrangian multiplier shows how the extra costs that would be incurred by increasing the output constraint slightly

Interpreting the Tangency Condition

- The tangency condition can be rewritten as:

$$\frac{MP_L}{w} = \frac{MP_K}{v}$$

- $\frac{MP_L}{w}$ and $\frac{MP_K}{v}$ are the additional production the producer will get by spending another dollar in labor and capital, respectively.

- Q: What if $\frac{MP_L}{w} > \frac{MP_K}{v}$?

Conditional Factor Demands and the Cost Function

- The solution to the CMP depends on w, v, and Q.
- We can write $L^* = L(w, v, Q)$ and $K^* = K(w, v, Q)$.
- We call these functions the **conditional factor demand functions**.
  - *Conditional* on producing output Q.
- The minimum cost of producing Q when input prices are w and v is therefore:

$$C(w, v, Q) = w\,L(w, v, Q) + v\,K(w, v, Q)$$

- This function is the firm's **cost function**.

Example

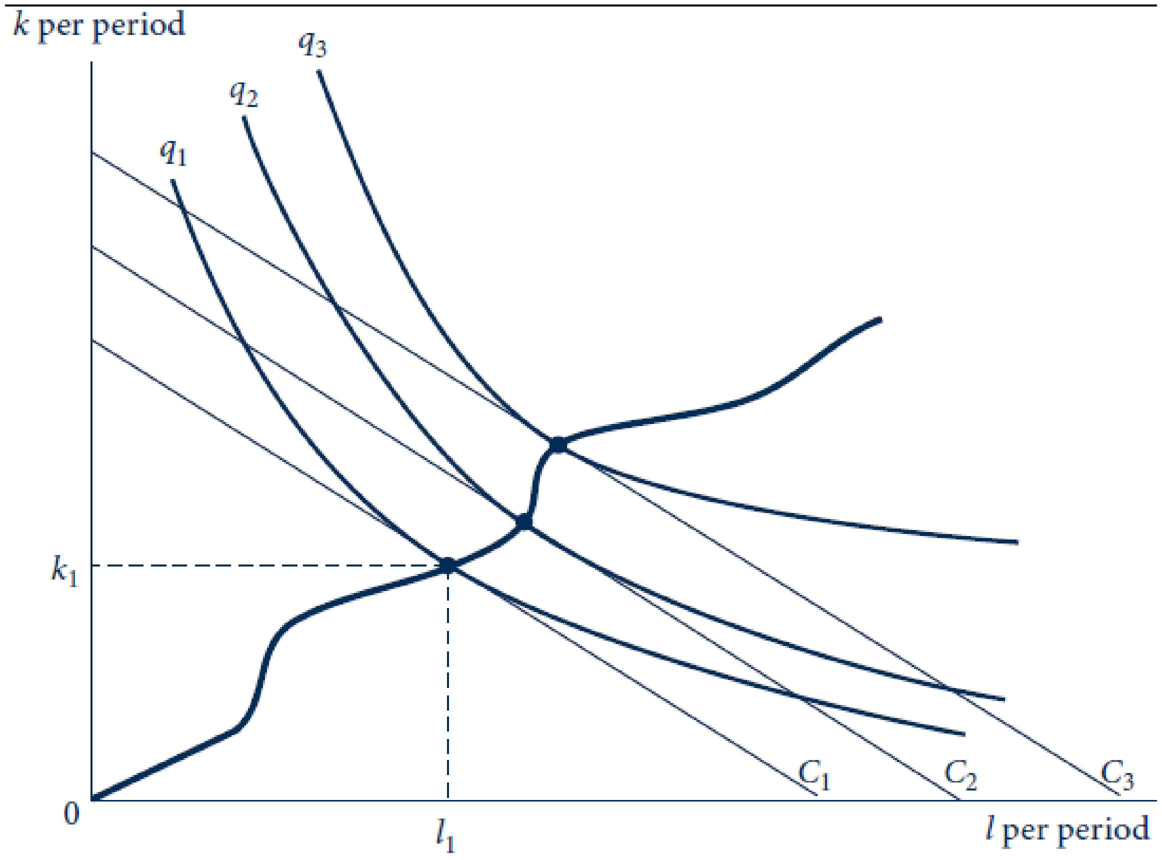
- Suppose a firm has production function $F(K,L) = 2K^{0.5} + 2L^{0.5}$
- a) Write down firm's cost minimization problem
- b) Solve for firm's conditional factor demand functions
- c) Solve for firm's cost function
- d) If $v = 1$ and $w = 2$, what K and L minimize the cost of producing 24 units of output?
- e) Solve for firm's cost when v=1 and w=2

Firm’s Expansion Path

- The firm can determine
  - The cost-minimizing combinations of k and l for every level of output

- If input costs remain constant for all amounts of k and l
  - We can trace the locus of cost-minimizing choices
    - Called the firm’s expansion path

Firm's Expansion Path



The firm's expansion path is the locus of cost-minimizing tangencies.

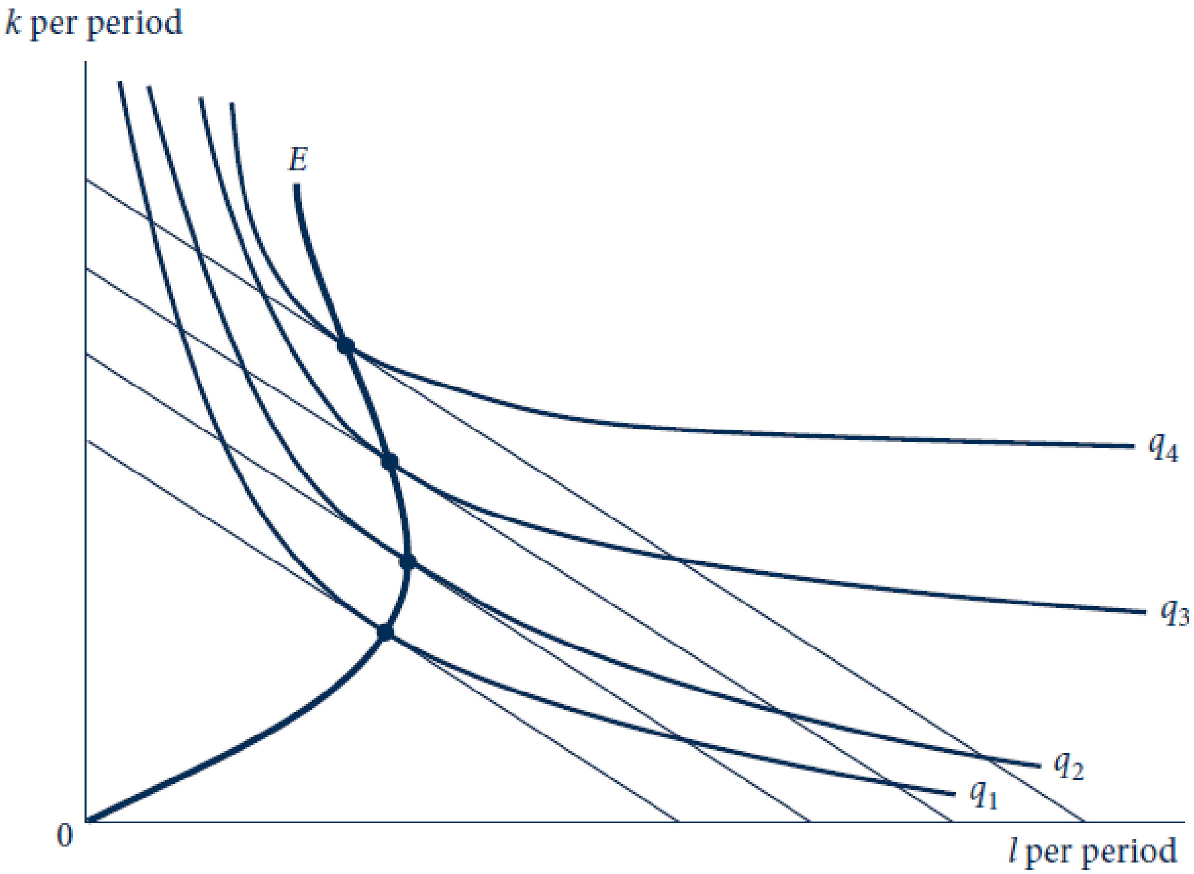
Assuming fixed input prices, the curve shows how inputs increase as output increases.

The Firm’s Expansion Path

- The expansion path does not have to be a straight line
  - The use of some inputs may increase faster than others as output expands
    - Depends on the shape of the isoquants

- The expansion path does not have to be upward sloping
  - If the use of an input falls as output expands, that input is an inferior input

Input Inferiority



With this particular set of isoquants, labor is an inferior input

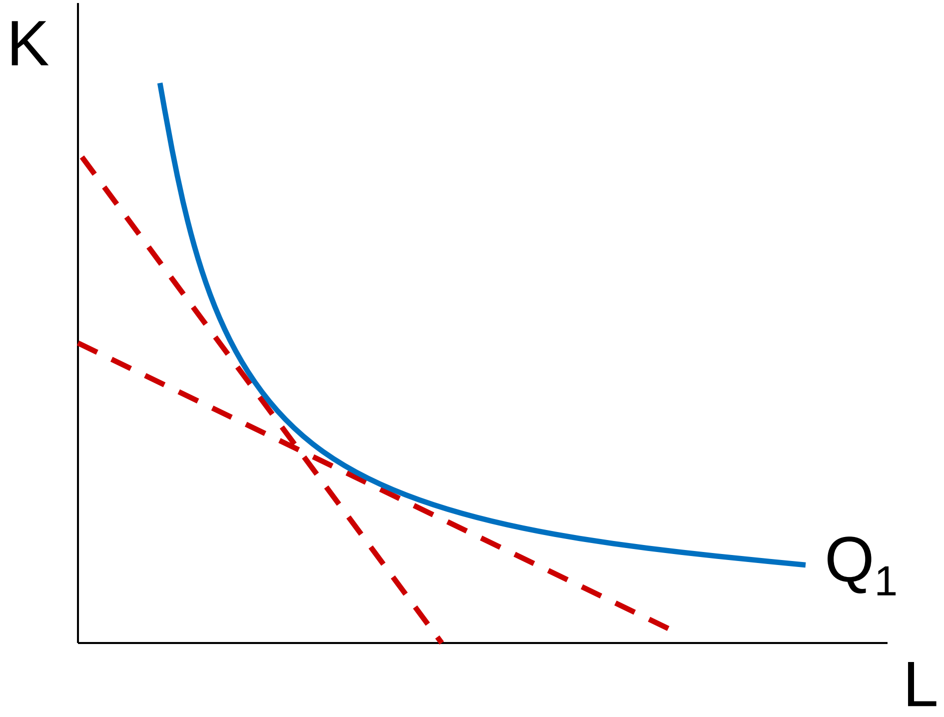
because less *l* is chosen as output expands beyond q_2.

Cobb-Douglas production function

$$F(K, L) = AK^{\alpha}L^{\beta}$$

$$1 > \alpha > 0; 1 > \beta > 0; A > 0$$

- $1 > \alpha, \beta$ ensures decreasing marginal product of capital, labor.
- Very flexible yet mathematically tractable, captures tradeoff between capital and labor.
- Choice of capital and labor depends on their price ratio.



Cobb-Douglas production function: $q = k^{\alpha} l^{\beta}$

- The Lagrangian expression for cost minimization of producing q_0 is

$$\mathcal{L} = vk + wl + \lambda(q_0 - k^{\alpha} l^{\beta})$$

- First-order conditions for a minimum

$$\partial\mathcal{L}/\partial k = v - \lambda\alpha k^{\alpha-1} l^{\beta} = 0$$

$$\partial\mathcal{L}/\partial l = w - \lambda\beta k^{\alpha} l^{\beta-1} = 0$$

$$\partial\mathcal{L}/\partial\lambda = q_0 - k^{\alpha} l^{\beta} = 0$$

Cobb-Douglas production function

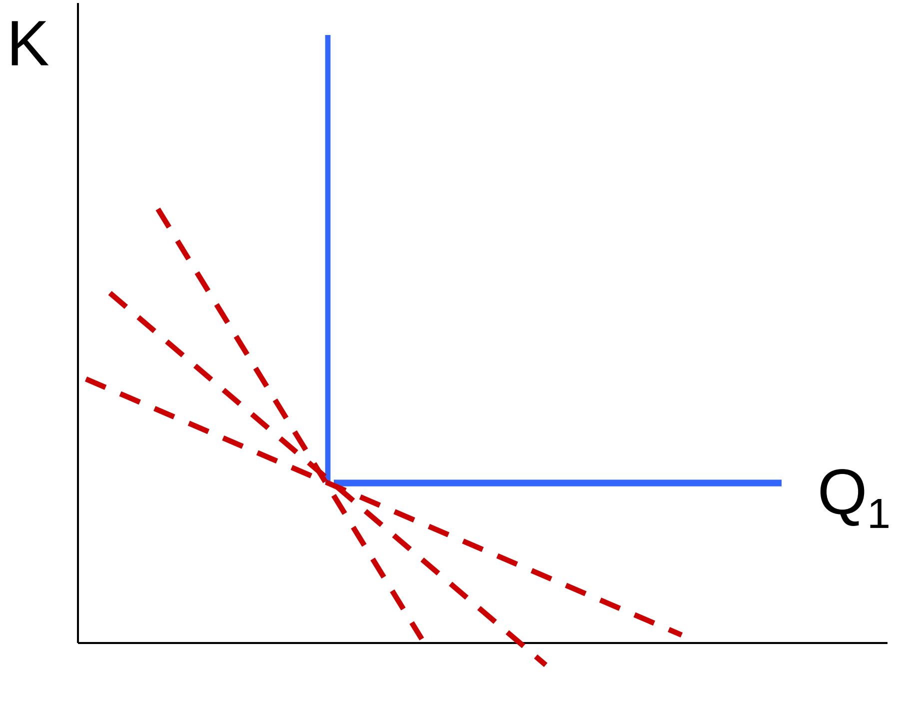
- Dividing the first equation by the second gives us

$$\frac{w}{v} = \frac{\beta k^{\alpha} l^{\beta-1}}{\alpha k^{\alpha-1} l^{\beta}} = \frac{\beta}{\alpha} \cdot \frac{k}{l} = \text{MRTS}$$

  – The MRTS depends only on the ratio of the two inputs
  – The expansion path is a straight line

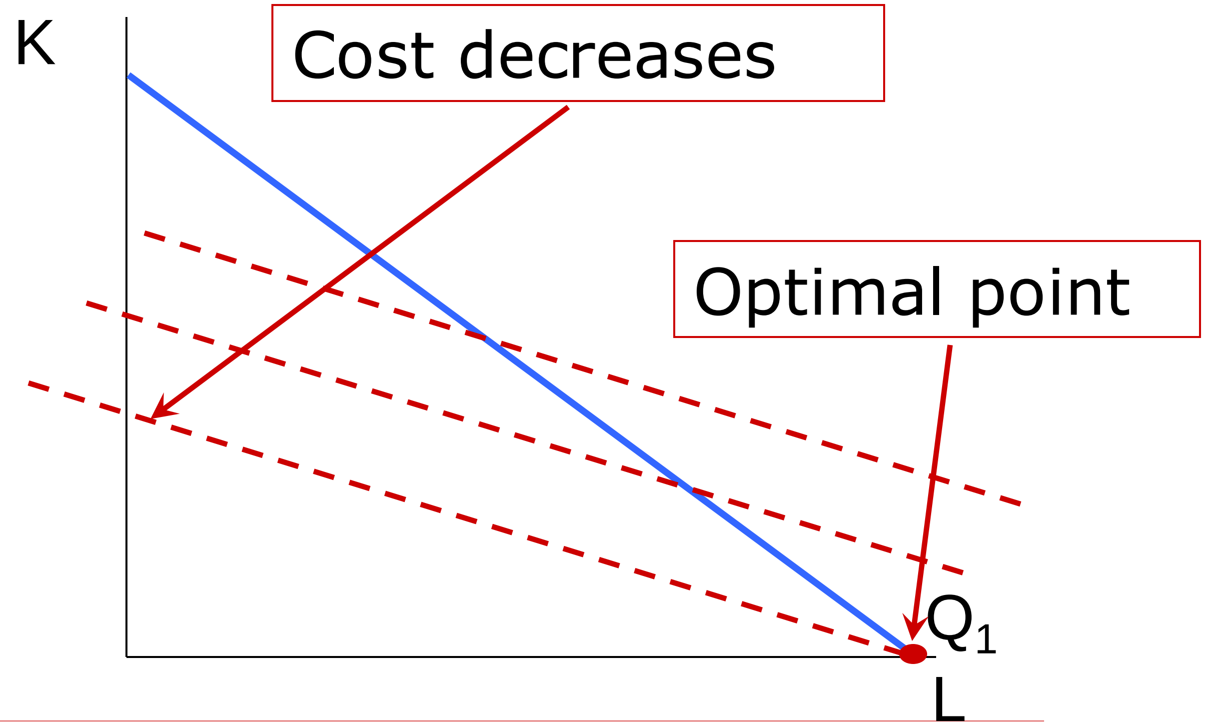
Perfect complements production function

- Perfect complements: inputs are used in fixed proportions.
- E.g., analysts and computers they work on; farmers and tractors.
- For any positive prices, the cost-minimizing point is **at the kink**.



Perfect substitutes production function

- E.g., Dell and HP computers.
- MP_K and MP_L are constants.
- MP_K/v and MP_L/w are constants.
- Use only the input for which (MP/factor price) is larger.



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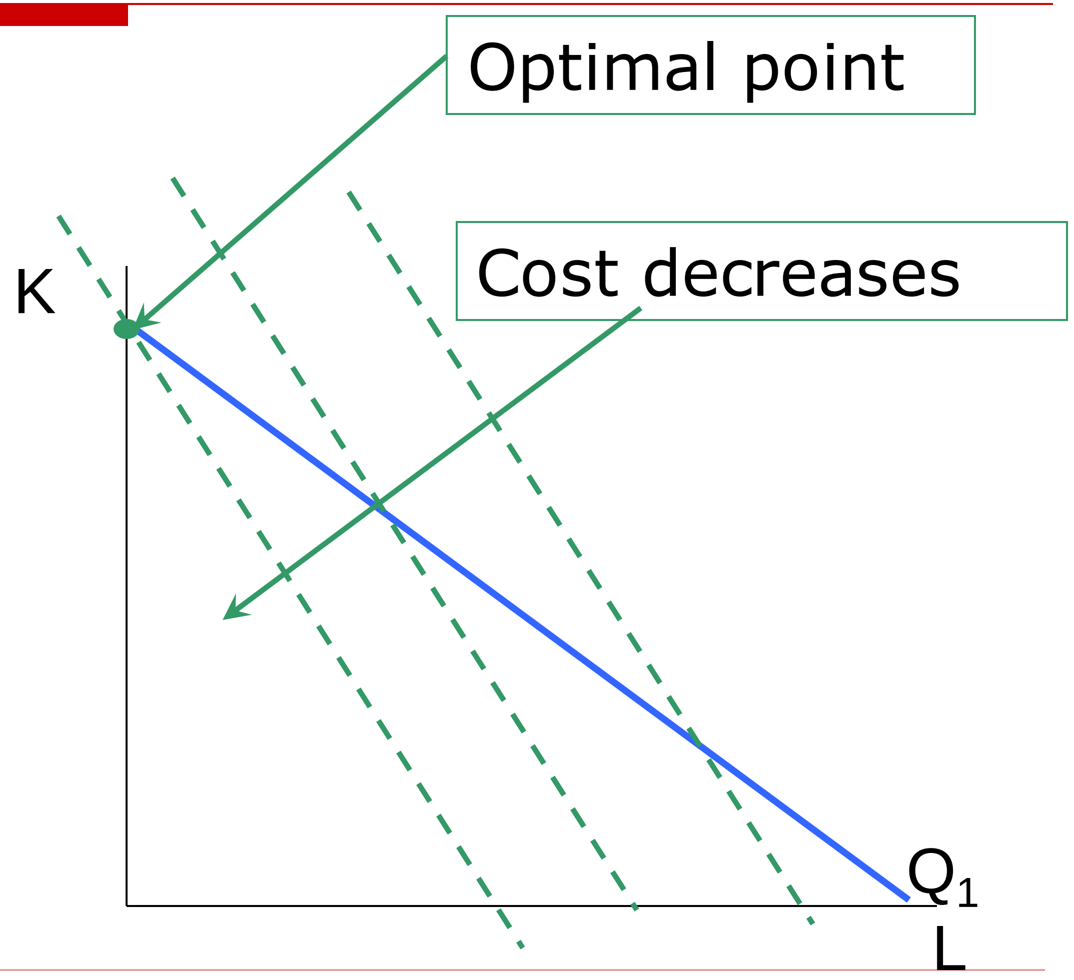


Summary

- Cost Minimization Problem (CMP): Given a required level of output Q, and input prices v and w, how to choose cost-minimizing input pairs (K*, L*)
  - Optimal (K*, L*) satisfies the condition MRTS= the ratio of the inputs' prices.
  - Optimal (K*, L*) satisfies the technology constraint Q=F(K*,L*)

- Solving CMP gives us **conditional factor demand functions**: K*(v,w,Q) and L*(v,w,Q)

- The **cost function** is C(v,w,Q)=vK*(v,w,Q) + wL*(v,w,Q)

- A **cost curve** C(Q) is cost function at particular levels of v and w.

Total Cost Function

- Total cost function
  - Shows that for any set of input costs and for any output level
  - The minimum cost incurred by the firm is

$$C = C(v,w,q)$$

  - As output (q) increases, total costs increase

A Taxonomy of Costs

- Fixed vs. Variable.
  - Variable costs are costs that increase with Q.
  - Fixed costs do not.

Cost Function: an example

- Consider an example of the cost function:

$$C(Q) = 100 + 5Q + Q^2$$

- Fixed cost?
  - 100 (the part that doesn't have any Q's in it).
- Variable cost?
  - $5Q + Q^2$

A Taxonomy of Costs

- Average vs. marginal.
  - Let $C(Q) = FC + VC(Q)$ denote the firm's total cost function. Note: we have suppressed w and v, since we're interested in how cost varies with Q.

- Average cost: $AC(Q) = \frac{C(Q)}{Q} = \frac{FC+VC(Q)}{Q}$.
- Average variable cost: $AVC(Q) = \frac{VC(Q)}{Q}$.

  Notice AVC ≤ AC at all Q.

- Marginal cost: $MC(Q) = C'(Q)$. "the change in $C(Q)$ when Q changes by a small amount"

Cost Function: an example

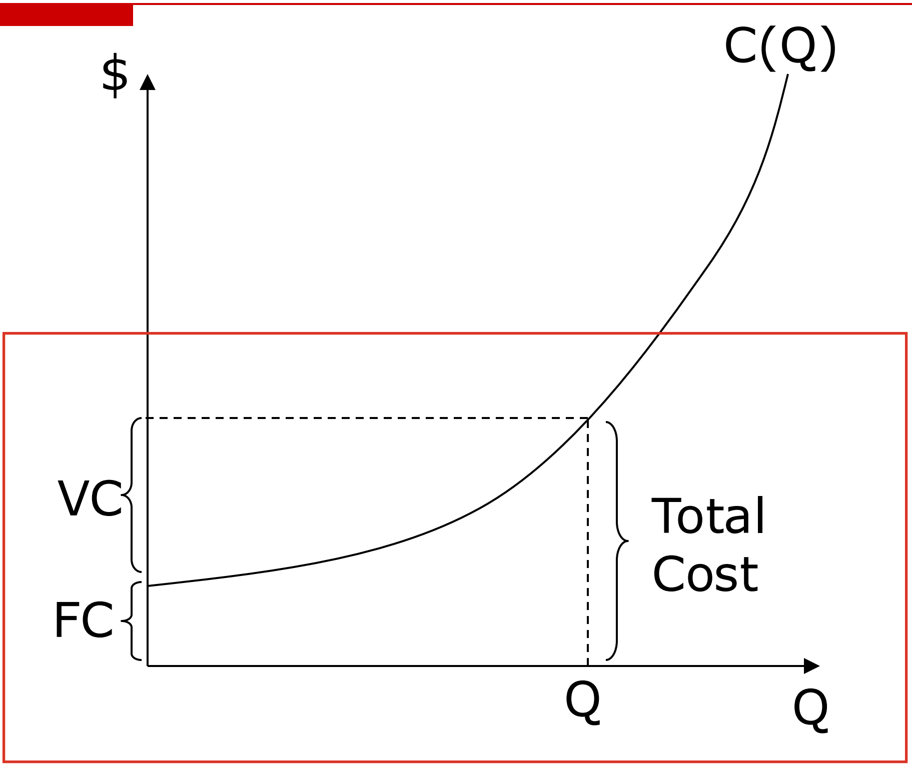
- **Consider the cost function**

$$C(Q) = 100 + 5Q + Q^2$$

- **Find**
  - **a) AC(Q)**
  - **b) AVC(Q)**
  - **c) MC(Q)**

Graphical relationship between cost curves.

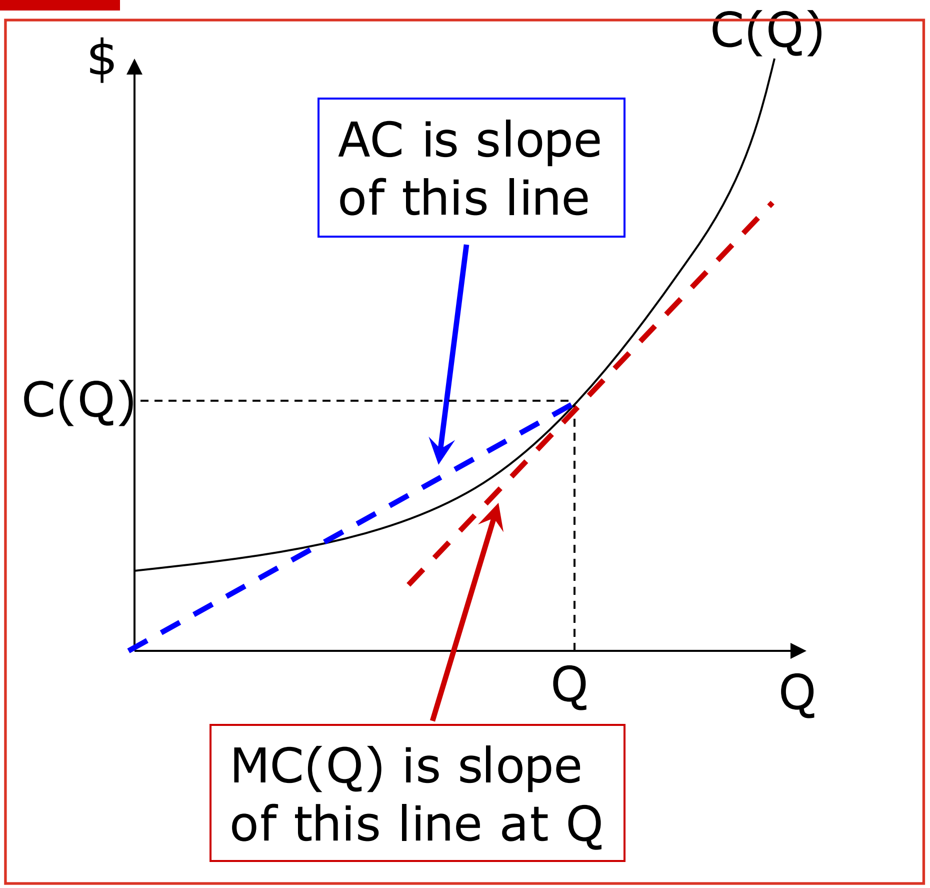
- Consider this cost curve.
- Fixed Cost: C(0) ➔ intercept.
- Total Cost: C(Q) ➔ height of curve.
- Variable Cost: C(Q) – C(0)
- Total Cost = Fixed + Variable Cost.



Graphical relationship between cost curves.

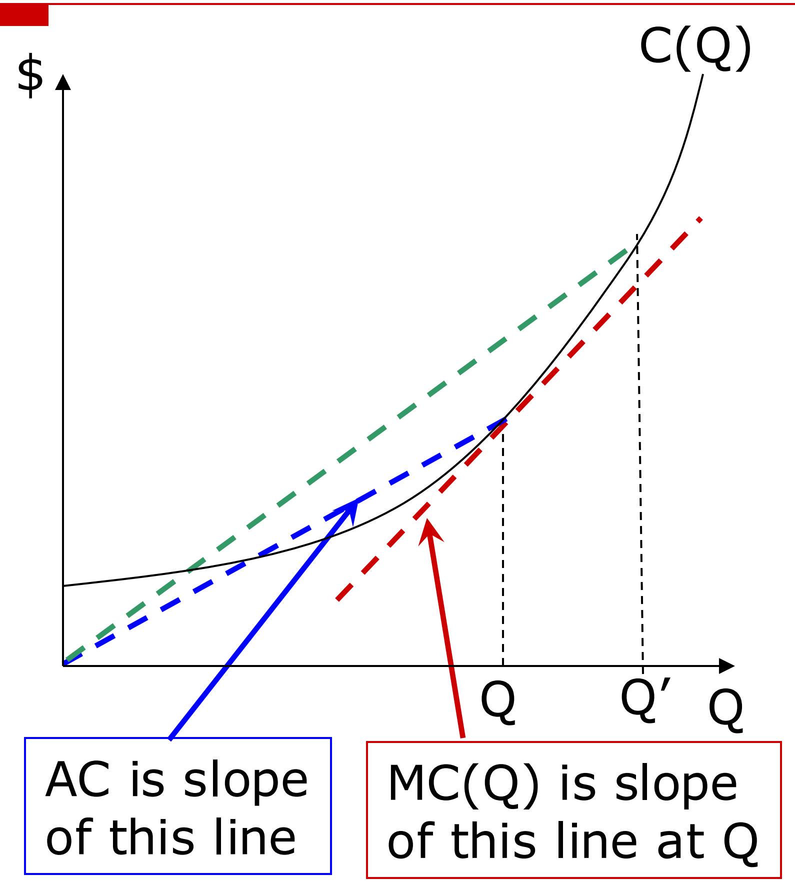
- Marginal Cost: MC(Q) = C′(Q) ➔ slope of curve.

- Average Cost: AC(Q) = C(Q)/Q.
  - C(Q) is height of curve at Q
  - Q is distance to right from 0
  - So, AC(Q) is slope of a line from the origin to (Q,C(Q)).



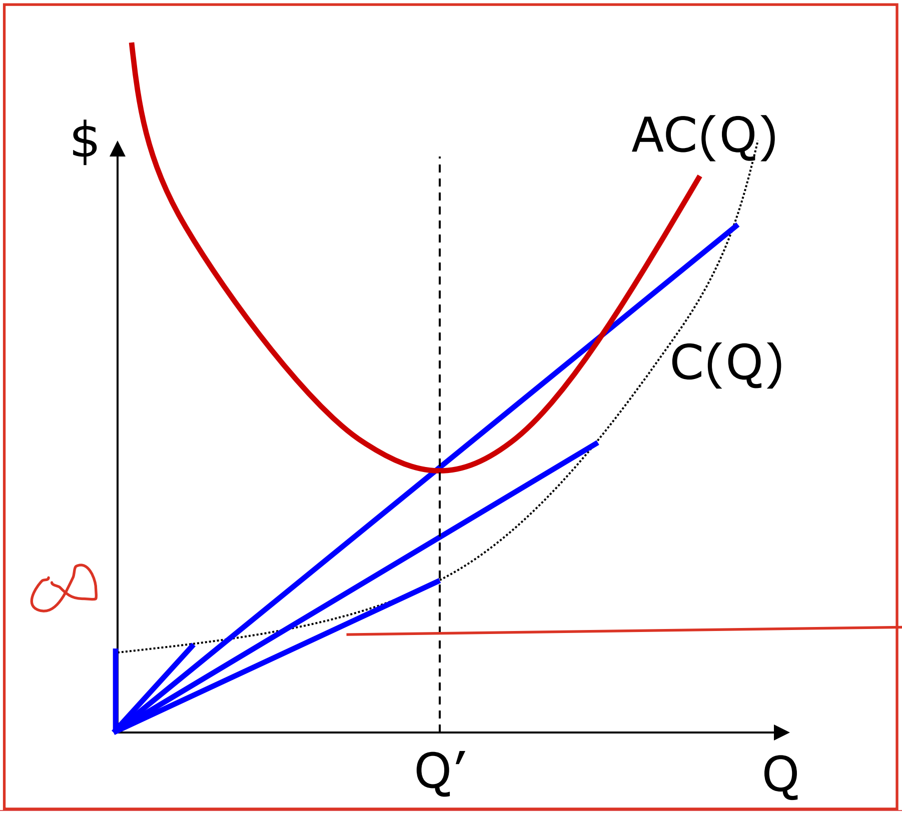
Graphical relationship between cost curves.

- What happens to MC, AC as Q increases?
- Think about your test average. If your last (marginal) test score is higher than your average, average goes up.
- If MC > AC, AC rises; if MC < AC, AC falls.
- Here, MC > AC at Q. If you increase Q, AC increases.
- Green AC line at Q' steeper than blue AC line at Q.



Graphical relationship between cost curves.

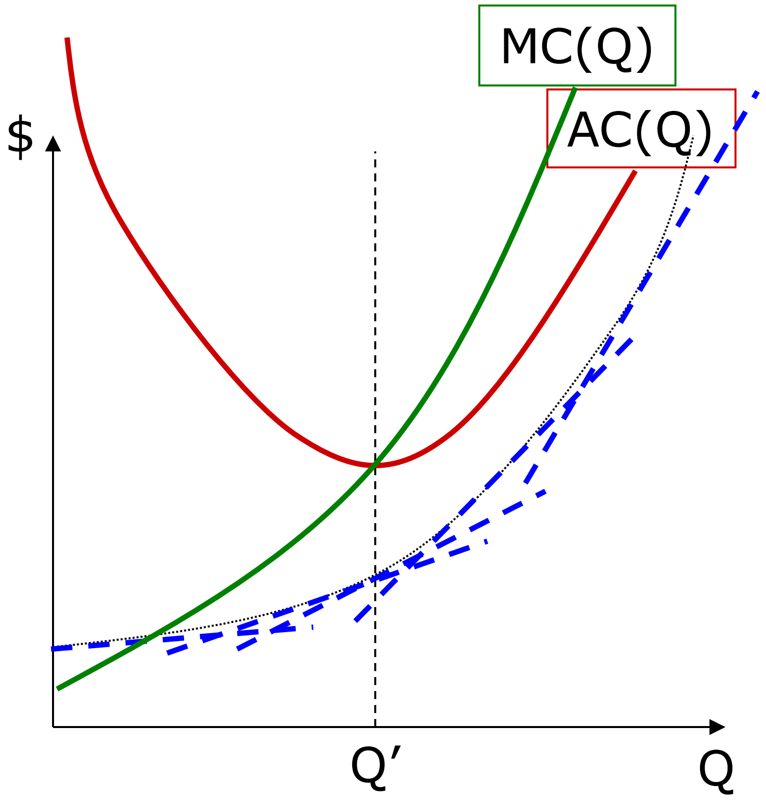
- Using this idea, you can generate the entire AC curves and MC curves.
- AC starts infinite, then declines until Q′, then increases again.
- The AC curve is U-shaped, with a minimum at Q′.



happen to be the tangent line of the cost curve AC(Q) = MC(Q)

Graphical relationship between cost curves.

- The MC curve starts flat and gets steeper.
- And, MC is below AC at low Q, and above AC at high Q.
- Where do they cross?
  - If MC < AC, AC falls; if MC > AC, AC rises.
  - **At the point of min AC.**
- So, the MC curve looks like this:



Graphical Analysis of Total Costs

- To produce one unit of output we need
  - k_1 units of capital
  - And l_1 units of labor input

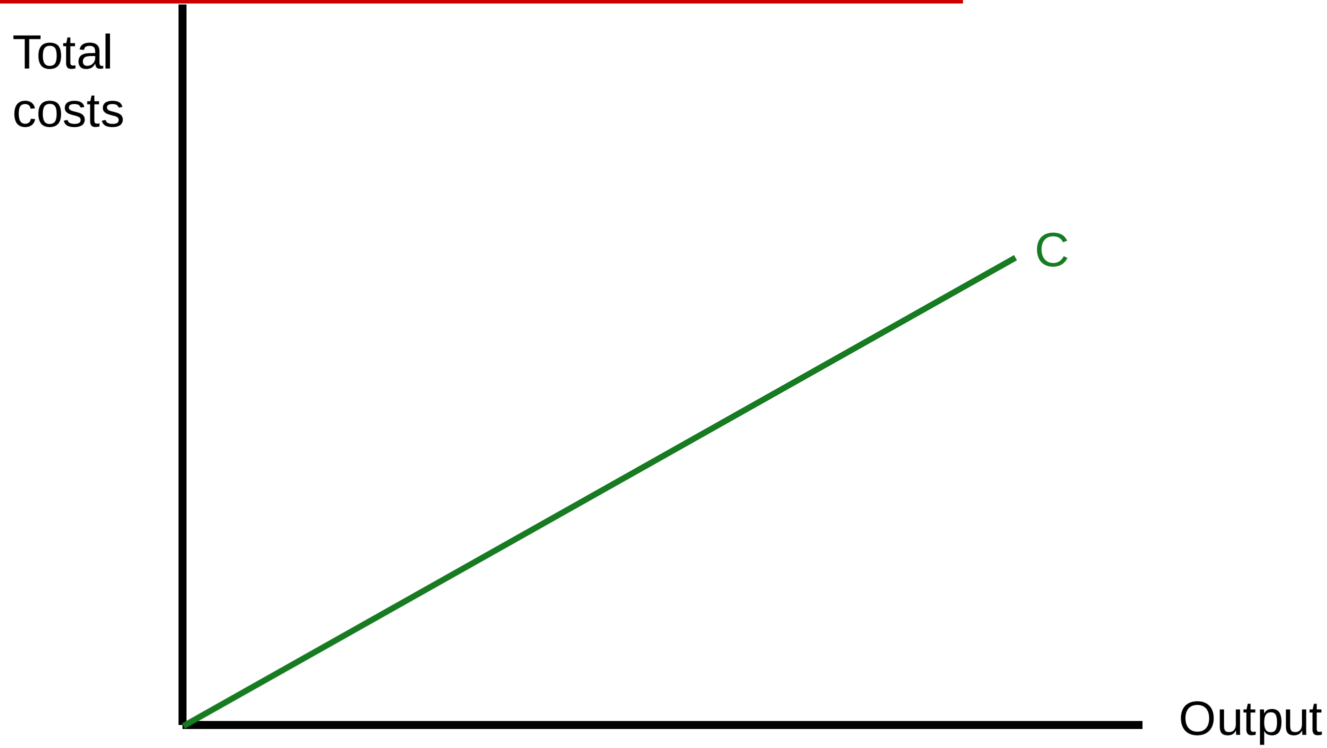
$$C(v,w,1) = vk_1 + wl_1$$

- To produce m units of output
  - Assuming constant returns to scale

$$C(v,w,m) = vmk_1 + wml_1 = m(vk_1 + wl_1)$$

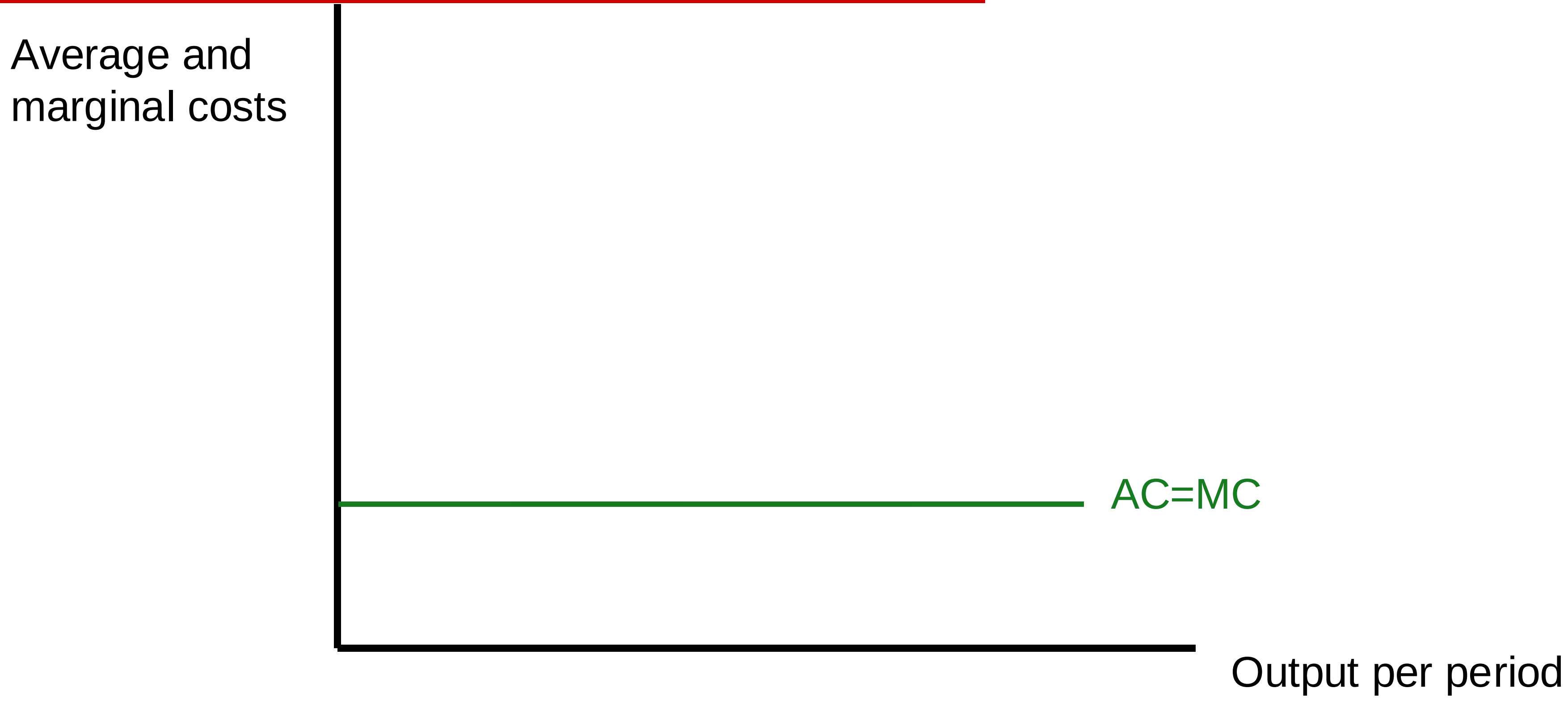
$$C(v,w,m) = m \cdot C(v,w,1)$$

Cost Curves in the Constant Returns-to-Scale Case



Total costs are proportional to output level.

Cost Curves in the Constant Returns-to-Scale Case

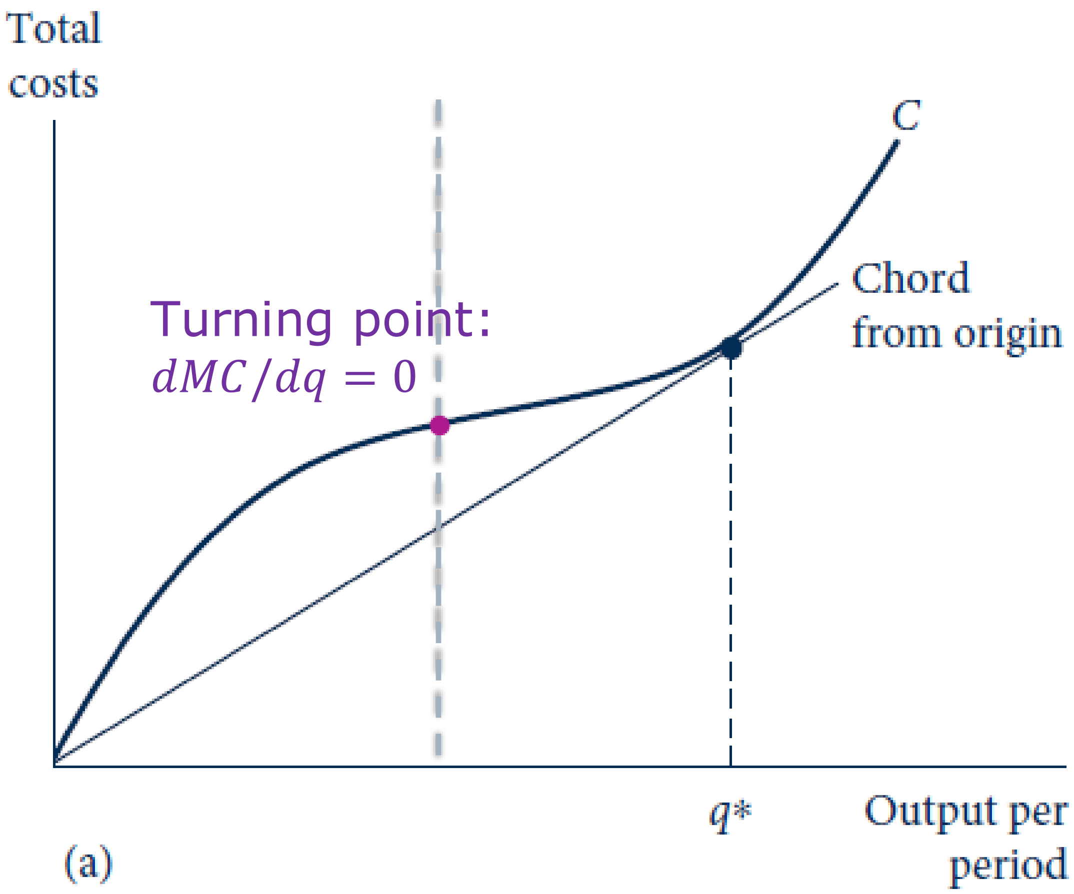


Average and marginal costs are equal and constant for all output levels.

Graphical Analysis of Total Costs

- Suppose that total costs start out as concave and then becomes convex as output increases
  - One possible explanation for this is that there is a third factor of production that is fixed as capital and labor usage expands
  - Total costs begin rising rapidly after diminishing returns set in

Total, Average, and Marginal Cost Curves for the Cubic Total Cost Curve Case

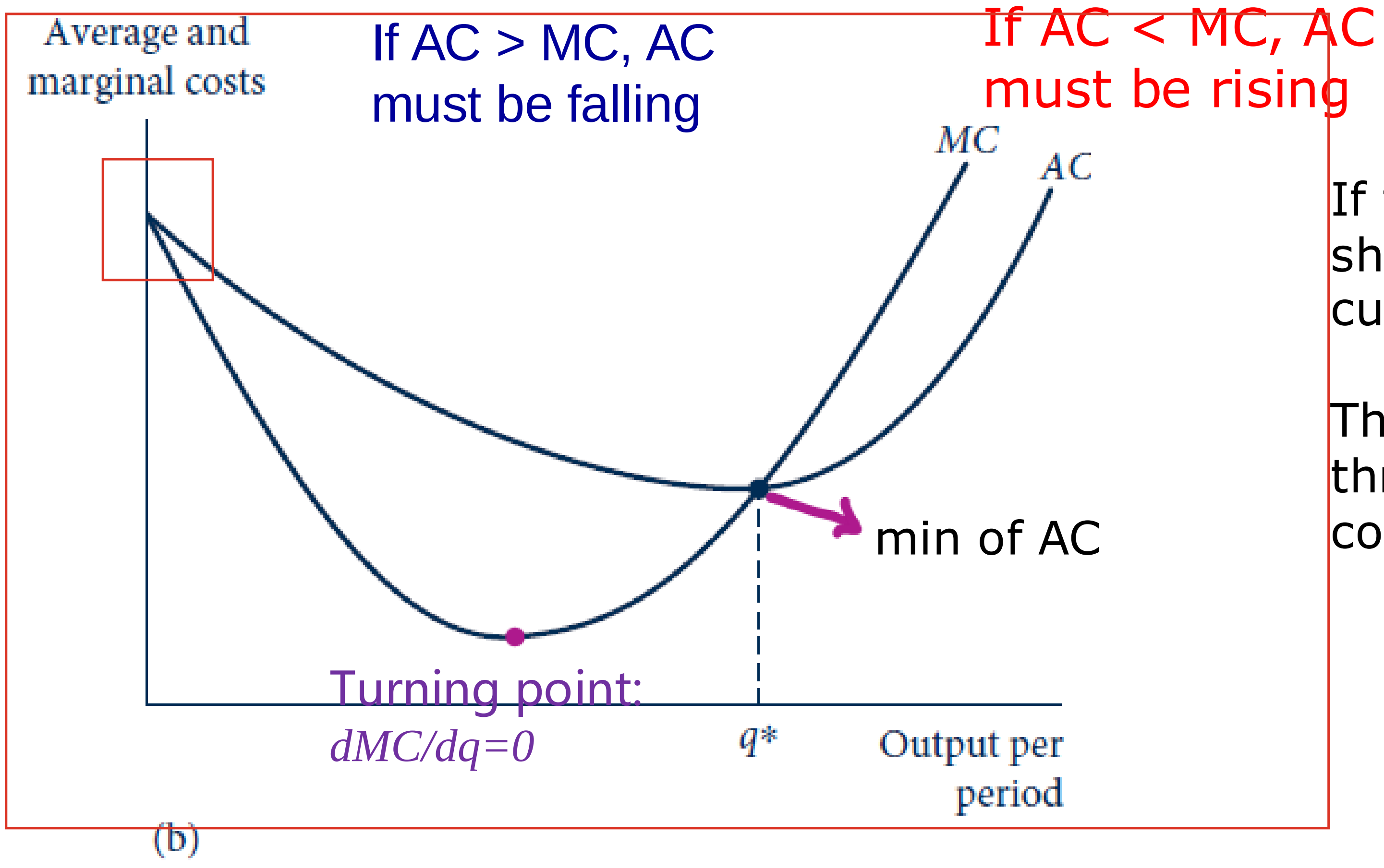


If the total cost curve has the cubic shape,

average and marginal cost curves will be U-shaped.

Slope of the total cost curve is the marginal cost curve.

Total, Average, and Marginal Cost Curves for the Cubic Total Cost Curve Case



If the total cost curve has the cubic shape, average and marginal cost curves will be U-shaped.

The marginal cost curve passes through the low point of the average cost curve at output level q*.

Shifts in Cost Curves

- Cost curves
  - Are drawn under the assumption that input prices and the level of technology are held constant
  - Any change in these factors will cause the cost curves to shift

Properties of Cost Functions

- Homogeneity
  - Cost functions are all homogeneous of degree one in the input prices
  - A doubling of all input prices will not change the levels of inputs purchased
  - Inflation will shift the cost curves up

Properties of Cost Functions

- Nondecreasing in q, v, and w
  - Cost functions are derived from a cost-minimization process
  - Any decline in costs from an increase in one of the function's arguments would lead to a contradiction

Properties of Cost Functions

- Some of these properties carry over to average and marginal costs
  - Homogeneity

$$\text{Because } C(tv, tw, q) = tC(v, w, q)$$

$$AC(tv, tw, q) = \frac{C(tv, tw, q)}{q} = \frac{tC(v, w, q)}{q} = tAC(v, w, q)$$

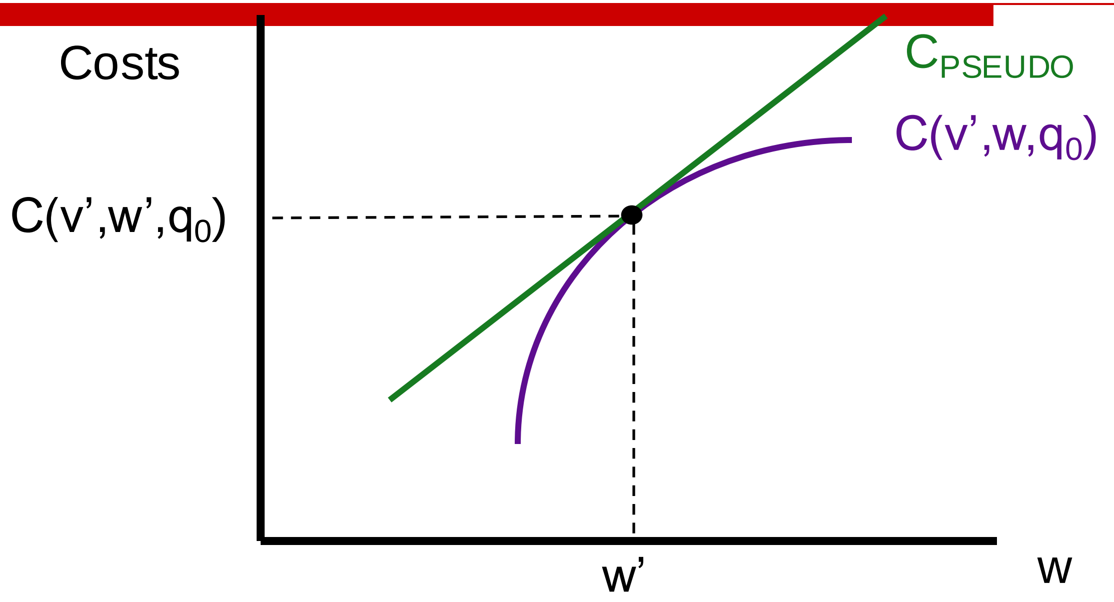
$$MC(tv, tw, q) = \frac{\partial C(tv, tw, q)}{\partial q} = \frac{t\partial C(v, w, q)}{\partial q} = tMC(v, w, q)$$

  - But effects of *v*, *w*, and *q* are ambiguous

Properties of Cost Functions

- Concave in input prices
  - Costs will be lower when a firm faces input prices that fluctuate around a given level than when they remain constant at that level
  - The firm can adapt its input mix to take advantage of such fluctuations

Cost Functions Are Concave in Input Prices



- With input prices w' and v' , total costs of producing q_0 are C (v', w', q_0). If the firm does not change its input mix, costs of producing q_0 would follow the straight line C_{PSEUDO}.
- With **input substitution**, actual costs C (v', w, q_0) will fall below this line, and hence the cost function is concave in w.

Input Substitution

- A change in the price of an input
  - Will cause the firm to alter its input mix
- Recall the formula for elasticity of substitution

$$\sigma = \frac{d(k/l)}{dMRTS} * \frac{MRTS}{k/l} = \frac{d\ln(k/l)}{d\ln MRTS}$$

Input Substitution

- Cost-minimization principle:
  - *MRTS(of l for k)* = *w/v* at an optimum
  - Substituting, we get input elasticity of substitution:

$$s = \frac{d(k/l)}{d(w/v)} \cdot \frac{w/v}{k/l} = \frac{d\ln(k/l)}{d\ln(w/v)}$$

  - In the two-input case, s must be nonnegative
  - Large values of s indicate that firms change their input mix significantly if input prices change

Quantitative Size of Shifts in Costs Curves

- The increase in costs will be largely influenced by
  - The relative significance of the input in the production process
  - The ability of firms to substitute another input for the one that has risen in price

Some Illustrative Cost Functions

- Fixed proportions

$$q = f(k,l) = \min(\alpha k, \beta l)$$

  - Production will occur at the vertex of the L-shaped isoquants ($q = \alpha k = \beta l$)

$$C(w,v,q) = vk + wl = v(q/\alpha) + w(q/\beta)$$

  - Constant returns-to-scale:

$$C(w,v,q) = qC(w,v,1)$$

  - What is the input elasticity of substitution?

Some Illustrative Cost Functions

- Cobb-Douglas, $q = f(k,l) = k^{\alpha} l^{\beta}$
  - Cost minimization requires that:

$$\frac{w}{v} = \frac{\beta}{\alpha} \cdot \frac{k}{l} \text{ , so } \quad k = \frac{\alpha}{\beta} \cdot \frac{w}{v} \cdot l$$

  - Solve for l(v,w,q) and k(v,w,q).
  - Derive the total cost function C(v,w,q).
  - What is the input elasticity of substitution?

Some Illustrative Cost Functions

- Cobb-Douglas, $q = f(k,l) = k^{\alpha} l^{\beta}$
  - Cost minimization requires that:

$$\frac{w}{v} = \frac{\beta}{\alpha} \cdot \frac{k}{l}, \text{ so } \quad k = \frac{\alpha}{\beta} \cdot \frac{w}{v} \cdot l$$

  - Substitute into the production function and solve for l, then for k

$$l = q^{1/\alpha+\beta} \left(\frac{\beta}{\alpha}\right)^{\alpha/(\alpha+\beta)} w^{-\alpha/(\alpha+\beta)} v^{\alpha/(\alpha+\beta)}$$

$$k = q^{1/\alpha+\beta} \left(\frac{\alpha}{\beta}\right)^{\beta/(\alpha+\beta)} w^{\beta/(\alpha+\beta)} v^{-\beta/(\alpha+\beta)}$$

Some Illustrative Cost Functions

- Cobb-Douglas
  - Now we can derive total costs as

$$C(v,w,q) = vk + wl = q^{1/(\alpha+\beta)} B v^{\alpha/(\alpha+\beta)} w^{\beta/(\alpha+\beta)}$$

  - Where B is a constant that involves only the parameters α and β

$$B = (\alpha+\beta)\alpha^{-\alpha/(\alpha+\beta)} \beta^{-\beta/(\alpha+\beta)}$$

  - Input elasticity S=1