# Intermediate Macroeconomics: Problem Set 3

Due Tuesday, April 1

## 1. Solow Model with "Learning by Doing" (Midterm 2022)

a.

$$C_t + I_t = Y_t$$

b.

$$\begin{aligned} K_{t+1} &= K_t(1-\delta) + I_t \\ &= K_t(1-\delta) + sK_t^{\alpha}(A_t N_t)^{1-\alpha} \end{aligned}$$

c.

不是 CRS, 解释。

d.

$$K^* = \left(\frac{B^{1-\alpha}S}{\delta}\right)^{\frac{1}{(1-\phi)(1-\alpha)}}$$

e.

Slope:

$$1-\delta+[\alpha+\phi(1-\alpha)]SBK_t^{\alpha+\phi(1-\alpha)-1}$$

## 2. Industrial Policy in a Discrete-Time Ramsey Model

### (a) Resource Constraint and Externality.

The government collects $\tau\, y_t = \tau\, A\, k_t^{\alpha}$ from household output and uses it for targeted investments. Hence, the household has:

$$c_t + i_t = (1-\tau)\, A\, k_t^{\alpha}.$$

Equivalently, substituting $i_t = k_{t+1} - (1-\delta)k_t$,

$$c_t + k_{t+1} - (1-\delta)k_t = (1-\tau)\,A\,k_t^{\alpha}.$$

Economically, the factor $(1-\tau)$ reduces the household's disposable output for consumption and capital formation in period $t$. However, these government-funded activities raise next period's total factor productivity (TFP) from $A$ to $A+\phi(\tau)$. Since we assume

$$\phi(\tau) = \gamma\,\tau^{\eta}, \quad \gamma > 0,\ 0 < \eta < 1,$$

it captures a positive externality or spillover effect: the more is spent on strategic industries, the larger the productivity boost in the subsequent period (albeit with diminishing returns).

**(b) Household Optimization and Euler Equation.**

*Without industrial policy ($\tau = 0$):*

- The household's problem is

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t), \quad \text{subject to} \quad c_t + k_{t+1} - (1-\delta)k_t = A\,k_t^{\alpha}.$$

- Using a Lagrangian approach or standard dynamic optimization methods, one obtains the Euler equation:

$$U'(c_t) \;=\; \beta\,U'(c_{t+1})\left[(1-\delta) + \alpha\,A\,(k_{t+1})^{\alpha-1}\right].$$

*With industrial policy ($\tau > 0$) and boosted TFP:*

- The *modified* resource constraint is $c_t + k_{t+1} - (1-\delta)k_t = (1-\tau)\,A\,k_t^{\alpha}$.
- In period $t+1$, the production function is

$$f(k_{t+1}) \;=\; \big[A + \phi(\tau)\big]\,(k_{t+1})^{\alpha}.$$

- Re-deriving the first-order condition (or adapting the above Euler equation) gives:

$$U'(c_t) \;=\; \beta\,U'(c_{t+1})\Big[(1-\delta) \;+\; \alpha\,\big(A+\phi(\tau)\big)\,(k_{t+1})^{\alpha-1}\Big].$$

- Hence, $\phi(\tau)$ raises the next-period marginal product of capital, affecting the household's saving/consumption choice.

**(c) Steady-State Analysis.**

Let $k^*$ and $c^*$ be the steady-state capital and consumption. At the steady state:

$$k_{t+1} = k_t = k^*, \quad c_{t+1} = c_t = c^*.$$

1. From the resource constraint (accounting for depreciation),

$$c^* = (1-\tau)\,A\,(k^*)^{\alpha} \; - \; \delta\,k^*.$$

2. From the Euler equation, in steady state, we have

$$1 \; = \; \beta\Big[(1-\delta) \; + \; \alpha(1-\delta)\,(A+\phi(\tau))\,(k^*)^{\alpha-1}\Big].$$

3. *Comparative statics in $\tau$.* - As $\tau$ increases, $(1-\tau)$ shrinks current disposable output, potentially reducing $c^*$ unless the subsequent productivity gain $\phi(\tau)$ is substantial. - Since $\phi(\tau) = \gamma\tau^{\eta}$ with $\eta < 1$, there are diminishing returns to raising $\tau$. If $\tau$ is too large, the deadweight loss to current consumption could outweigh the future productivity gains, possibly decreasing overall welfare.

**(d) Transition and Optimal Policy.**

1. *Transition Dynamics.* Starting from $k_0$, the path $\{k_t\}$ and $\{c_t\}$ depends on $\tau$. A moderate $\tau$ yields a modest rise in future productivity, whereas a larger $\tau$ can provide a bigger TFP boost but imposes a larger immediate consumption loss. The interplay of these factors determines how quickly the economy converges to the new steady state.

2. *Optimal $\tau^*$.* In principle, a social planner (or a government) might choose $\tau$ to maximize

$$\sum_{t=0}^{\infty} \beta^t U(c_t)$$

subject to the resource and accumulation constraints. One would compare welfare across different feasible $\tau$ values. Closed-form solutions are rare in such models, so typically a *numerical approach* is used: for each candidate $\tau$, solve for the dynamic path and compute the discounted utility, then pick the $\tau$ that yields the highest total welfare.

3. *Further Considerations.* In real settings, $\phi(\tau)$ might differ across sectors, labor mobility could be imperfect, or other distortions (e.g., credit constraints, technological spillovers that vary by industry) come into play. Each of these factors can shift the trade-off and may justify a different tax/subsidy structure or additional instruments besides $\tau$ to maximize overall welfare.