PS1 Answer Sheet (For Reference Only)

Question 1 (Utility functions)

Do the following utility functions represent the same preferences?

Please provide your reasoning.

$(a) u(x; y) = xy, v(x; y) = 3(xy)^2 + 6$

$(b) u(x; y) = xy, v(x; y) = -3(xy)^2 + 6$

$(c) u(x; y) = xy, v(x; y) = lnx + lny$

$(d) u(x; y) = xy, v(x; y) = x + y$

Note that in this question we take it for granted that both x and y are no less than 0.

(a) Use the affine tranformation as follows.

$U^*(x; y) = 3 * [u(x; y)]^2 + 6 = 3(xy)^2 + 6 = v(x; y).$

Further, $u(x; y)$ and $v(x; y)$ shares the same monotonicity for $xy > 0$,

then we can conclude that u and v represent the same preference.

(b) Watch out for the negative sign here!

Under this circunstance, u and v have excatly the opposite monotonicity.

That is to say, for two terms $x_1 * y_1 < x_2 * y_2$ and the underlying consumption bundles Z_1 and Z_2,

consumers with a particular preference featuring a utility function $u(x; y)$ would prefer Z_1 to Z_2, and vice versa.

Consequently, u and v represent different preferences.

(c) Similar to (a), but the transformation is no longer afffine in this case. (We only require that the tranformation is order-preserving)

$U^*(x; y) = ln[u(x; y)] = ln(xy) = lnx + lny = v(x; y)$.

u and v represent the same preference.

(d) With the help of certain counter-examples, we can show that the transformation from $u(x; y)$ to $v(x; y)$ is not order-preserving.

e.g. for two bundles $Z_1 = (1, 5)$ and $Z_2 = (2, 3)$,

$u(1; 5) = 5 < u(2; 3) = 6, v(1; 5) = 6 > v(2; 3) = 5,$

hence Z_1 is preferred to $Z_2 (Z_1 \succ Z_2)$ given utility function v, and Z_2 is preferred to $Z_1 (Z_2 \succ Z_1)$ given utility function u.

Consequently, u and v represent different preferences.

Tips: we need only ONE counter-example to show that preferences are different; however EVERY possible example shall be checked for the conclusion to hold.

Consequently, we cannot say that u and v represent the same preference by simply ckecking only one pair of bundles (4.4) and (9,1))

Question 2 (Indifference curves)

Indifference curves (monotonic and convex) are downward sloping. True or false?

Please prove your answer.

See the applendix.

Question 3 (Budget Sets)

This question concerns a consumer who is choosing how many of two goods to buy:

Footballs (the round ones, that you kick with your foot) and cricket balls (like baseballs, but better).

The consumer has an income of $20, and the cost of a football is $4 and a cricket ball is $2.

1.Write down the equation for the consumer's budget constraint and graph it in the commodity space.

2.The government decides that football is evil and needs to be taxed. They introduce a 50% tax on each football sold.

Rewrite and re-graph the budget constraint.

3.A new government is elected that hates all sports. They now tax both footballs and cricket balls at 50%.

What does the budget constraint look like now?

4.Due to a threat of revolt amongst sports fans, the government hands out a subsidy of $10 to the consumer.

What does their new budget constraint look like?

How would you expect consumer behavior to differ between this situation and the no-tax, no-subsidy situation described in part 1?

5.Revolution comes, and all taxes and subsidies are abolished. Even better, the consumer finds a new shop that offers bulk discounts.

In this shop, footballs cost $4 each if you buy 3 or fewer. However, the cost of any additional football after 3 is $2.

What does the budget set look like now? graph it in the commodity space.

1. Let x,y denote the amounts of footballs and cricket balls respectively.

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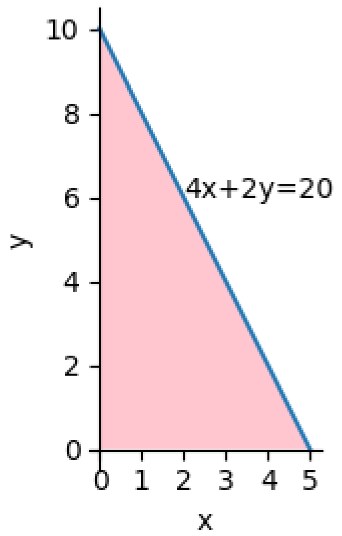
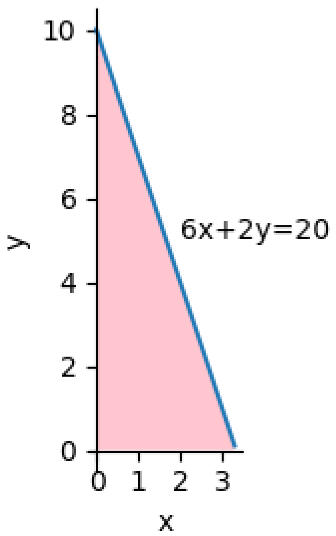
Equation for the consumer's budget constraint: $4x + 2y = 20$

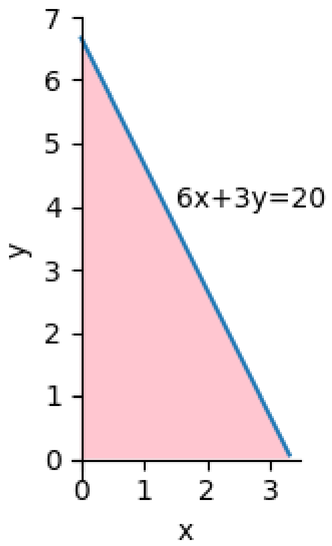


2. Equation for the consumer's budget constraint: $4 * (1 + 50\%) * x + 2 * y = 20 \ \ i.e. \ \ 6x + 2y = 20$



3. Equation for the consumer's budget constraint:

$4 * (1 + 50\%) * x + 2 * (1 + 50\%) * y = 20 \ \ i.e. \ \ 6x + 3y = 20$



4. (Note that this question is set on the basis of all the previous parts)

With the subsidy in place, equation for the consumer's budget constraint:

$4 * (1 + 50\%) * x + 2 * (1 + 50\%) * y = 20 + 10 \quad i.e. \quad 6x + 3y = 30.$

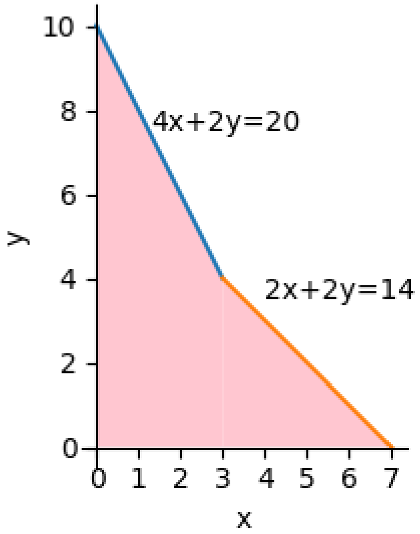
Compared to the equation in the first part $4x + 2y = 20$, they are exactly the same, total effects of all taxes and subsidy cancelled out.

Therefore consumer behavior shall be identical to the no-tax, no-subsidy situation described in part 1.

5. for $0 \leq x \leq 3$, footballs cost $4 each, and the budget constraint is the same as in the first part: $4x + 2y \leq 20$

for $3 < x \leq 7$, footballs cost $2 as additional ones after a necessary purchase of 3 balls at $4 each.

Therefore, the budget constraint is $[4 * 3 + 2 * (x - 3)] + 2y \leq 20, i.e. 2x + 2y \leq 14$.



Question 4 (UMP)

Edmund consumes two commodities, sewage and punk rock video cassettes.

He doesn't drink sewage of course, but he gets paid for taking it away at \$$2$ per ton.

Edmund can accept as much sewage as he wishes at that price. He has no other source of income. Video cassettes cost him \$6 each.

He has a utility function $u(s; v)$ on sewage (s) and videos (v) which is decreasing in s and increasing in v.

1. If Edmund's accepts 0 tons of sewage, how many video cassettes can he buy?

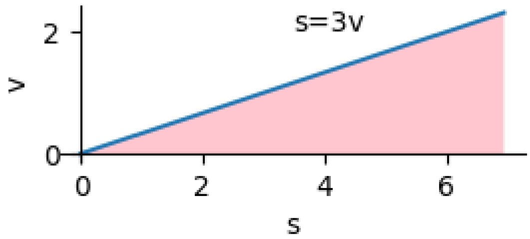
2. Write down Edmund's constrained optimization problem.

3. Draw Edmund's budget line and shade his budget set.

1. If Edmund's accepts 0 tons of sewage, he then has an income of \$$0$, and can thus afford no video cassettes at all!
2. Edmund's constrained optimization problem:

$\max_{s,v} u(s, v)$, constraint to $2s - 6v \geqslant 0$ (budget constraint) and $s, v \geqslant 0$ (ofc Edmund cannot consume a negative amount of cassettes or pour sewage back!).

3. The graph is as follows.



Question 5 (UMP)

A consumer has utility function $u(x, y) = x^{\frac{1}{2}} y^{\frac{1}{2}}$.

Let the prices of goods x and y be given by p_x and p_y respectively, and let m denote the consumer's wealth.

Assume that p_x, p_y and m are all strictly positive.

1. Write down the consumer's utility maximization problem.

2. Derive the consumer's optimal choice of x and y.

3. Are there any prices and wealth (p_x, p_y and m) at which good x is a *Giffen good*? Briefly explain why.

1. $\max_{x,y} u(x, y) = x^{\frac{1}{2}} y^{\frac{1}{2}}$, constraint to $p_x * x + p_y * y \leq m$ (and $x, y \geqslant 0$).

(Note that in some cases x and y may be smaller than 0, where there does not exist any short-sale constraints(in Chinese aka "卖空限制"), as in the American stock markets.

However in our course of Microeconomics, we generally assume that x and y is at least 0 but for particular declaration!)

2. To solve **UMP** problems, we usually follow **certain steps**.

Step 1, we write down the consumer's **utility maximization problem**, which has already been done in the first part.

Step 2, we set up ***Lagrange* function**:

$$L = x^{\frac{1}{2}} y^{\frac{1}{2}} + \lambda * (m - p_x * x - p_y * y)$$

(we organize the term after λ in such ways $s.t.$ it is always larger than or equal to 0 when the constraint is satisfied)

Step 3, from the *Lagrange* function above, we derive the **first order consitions (F.O.C)**:

$$\begin{cases} \frac{\partial L}{\partial x} = \frac{1}{2} x^{-\frac{1}{2}} y^{\frac{1}{2}} - \lambda * p_x \\ \frac{\partial L}{\partial y} = \frac{1}{2} x^{\frac{1}{2}} y^{-\frac{1}{2}} - \lambda * p_y \\ \frac{\partial L}{\partial m} = m - p_x * x - p_y * y \end{cases}$$

Step 4, **solving F.O.C**, we have the solution to UMP as follows

$$\begin{cases} x = \frac{m}{2p_x} \\ y = \frac{m}{2p_y} \end{cases}$$

Step 5, **check** whether the solution is really available under the assumptions!

Question: What should we do under the case of corner solutions?

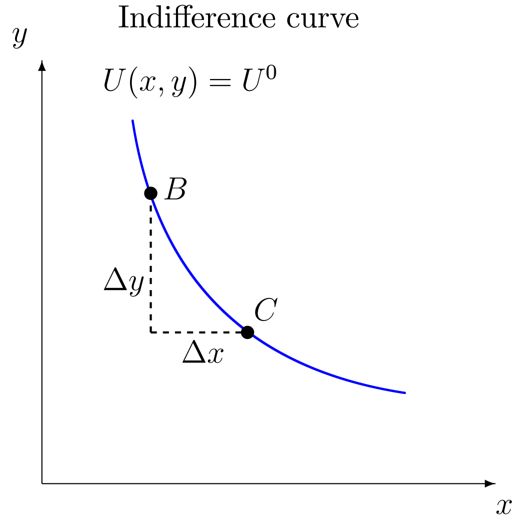
3. From what has been derived, $x = \frac{m}{2p_x}$.

That is to say, if we hold ***all*** other variables(e.g. m, p_y) constant, the demand for x would fall, if p_x increases.

According to the definition (increases in price leading to increases in demand), good x is ***never*** a *Giffen good*.

Appendix

II. Proof of Property 2:



1. Intuition

Diagrammatic proof: (i) proof by contradiction, or (ii) dominated and dominating regions

2. Mathematical Proof

The total differential of $U(x,y)$ is given by

$$dU(x,y) = \frac{\partial U(x,y)}{\partial x}dx + \frac{\partial U(x,y)}{\partial y}dy. \tag{1}$$

Since $U(x,y) = U^0$, taking total differential on both sides, we obtain $dU(x,y) = dU^0 = 0$. If follows from (1) that

$$dU(x,y) = \frac{\partial U(x,y)}{\partial x}dx + \frac{\partial U(x,y)}{\partial y}dy = 0. \tag{2}$$

Rearranging (2),

$$\left.\frac{dy}{dx}\right|_{U(x,y)=U^0} = -\frac{\frac{\partial U(x,y)}{\partial x}}{\frac{\partial U(x,y)}{\partial y}} \equiv -\frac{\text{marginal utility of } x}{\text{marginal utility of } y} = -\frac{U_x}{U_y}. \tag{3}$$

The marginal rate of substitution (MRS) of x for y is defined as

$$MRS = -\left.\frac{dy}{dx}\right|_{U(x,y)=U^0} = \frac{U_x}{U_y}. \tag{4}$$

By (3), the indifference curve is downward-sloping (i.e., $\frac{dy}{dx} < 0$) if $U_x > 0$ and $U_y > 0$. In other words, if "more is preferred to less" (preferences are monotonic), then the indifference curve will be downward-sloping.