Intermediate Microeconomics

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Week 14a: Asymmetric Information (I)

Yuanning Liang

Big Picture

Departures from Competitive Equilibrium

1. Violation of the “private good” assumption

2. Violation of the “price-taking” assumption

3. Violation of the “complete market” assumption

Missing Market

- First Welfare Theorem (i.e., competitive equilibrium is efficient) relies on the assumption of “complete market”: there is a market for every good, and trade is free

- We’ve already seen an example of violation: externalities
  - The reason we have inefficiency is because the market for “pollution” is missing
  - If there is a market for pollution (say you have to pay $50 to emit a ton of CO2), then there wouldn’t be an externality problem, b/c polluters will factor that cost in making decision

- Here, we study another famous example in economics: missing market for information

Asymmetric Information

- Transactions can involve a considerable amount of uncertainty
  - Can lead to inefficiency when one side has better information
- Asymmetric information
  - The side with better information
    - Private information

The Value of Contracts

- Contractual provisions
  - Can be added in order to circumvent some of the inefficiencies associated with asymmetric information
  - Rarely do they eliminate them

Principal-Agent Model

- Principal
  - The party who proposes the contract
- Agent
  - The party who decides whether or not to accept the contract
  - And then performs under the terms of the contract
  - Typically the party with asymmetric information

Two Leading Models

- Moral hazard model
  - The agent's actions affect the principal, but the principal does not observe the actions directly
  - Hidden-action model
- Adverse selection model
  - The agent has private information before signing the contract (his type)
  - Hidden-type model

TABLE 18.1 Applications of the Principal-Agent Model

		Agent's Private Information	
Principal	**Agent**	**Hidden Type**	**Hidden Action**
Shareholders	Manager	Managerial skill	Effort, executive decisions
Manager	Employee	Job skill	Effort
Homeowner	Appliance repairer	Skill, severity of appliance malfunction	Effort, unnecessary repairs
Student	Tutor	Subject knowledge	Preparation, patience
Monopoly	Customer	Value for good	Care to avoid breakage
Health insurer	Insurance purchaser	Preexisting condition	Risky activity
Parent	Child	Moral fiber	Delinquency

First, Second, and Third Best

- First-best contract
  - Full-information environment
  - The principal could propose a contract that maximizes joint surplus
    - Could capture all of the surplus for himself
    - Leaving the agent just enough to make him indifferent between agreeing to the contract or not

First, Second, and Third Best

- Second-best contract
  - The contract that maximizes the principal's surplus
  - Subject to the constraint that he is less well informed than the agent
- Adding further constraints
  - for example, restricting contracts to some simple form such as constant per-unit prices
  - leads to the third best, the fourth best, and so on, depending on how many constraints are added.

Hidden Actions

- The principal
  - Would like the agent to take an action that maximizes their joint surplus
- The agent's actions
  - May be unobservable to the principal
  - The agent will prefer to shirk
- Contracts
  - Can mitigate shirking by tying compensation to observable outcomes

Hidden Actions

- The principal
  - More concerned with outcomes than actions
    - May as well condition the contract on outcomes
- The problem
  - Outcome may depend on random factors
  - Tying the agent’s compensation to outcomes exposes the agent to risk
  - If the agent is risk averse: payment of a risk premium before he will accept the contract

Owner-Manager Relationship

- A firm: one owner and one manager
  - The owner (principal) offers a contract to the manager
  - The manager (agent) decides whether to accept the contract and what action $e \geq 0$ to take
    - An increase in e increases the firm's gross profit but is personally costly to the manager

Owner-Manager Relationship

- The firm's gross profit: $\pi_g = e + \varepsilon$
  - Where ε represents demand, cost, and other economic factors outside of the agent's control
    - Assume $\varepsilon \sim N(0,\sigma^2)$
  - $c(e)$ is the manager's personal disutility from effort; assume $c'(e) > 0$ and $c''(e) > 0$
- Firm's net profit: $\pi_n = \pi_g - s$
  - Where s is the manager's salary

Owner-Manager Relationship

- Risk-neutral owner
  - Owner represents individual shareholders who each own a small share of the firm as part of a diversified portfolio, we will assume that she is risk neutral.
  - Maximize the expected value of profit

$$E(\pi_n) = E(e + \varepsilon - s) = e - E(s)$$

Owner-Manager Relationship

- Risk adverse manager
  - Constant absolute risk aversion parameter, $A > 0$
  - Manager's expected utility: $U(W) = -e^{-AW}$

$$E(U) = \boxed{E(s) - \frac{A}{2}\mathrm{Var}(s)} - c(e)$$

*Details on derivation see Example 7.3 in Nicholson & Snyder

First Best (Full-Information Case)

- Optimal salary contract
  - The owner can pay the manager
    - A fixed salary s^* if he exerts a first-best level of effort e^*
    - And nothing otherwise
  - For the manager to accept the contract (participation constraint)

$$E(U) = s^* - c(e^*) \geq 0$$

First Best (Full-Information Case)

- The owner
  - Will pay the lowest salary possible [$s^* = c(e^*)$]
  - Net profit: $E(\pi_n) = e^* - E(s^*) = e^* - c(e^*)$
  - Maximize profit for e^* satisfying the first-order condition
  - At the optimum, the marginal cost of effort equals the marginal benefit, $c'(e^*) = 1$

Second Best (Hidden-Action Case)

- If the owner cannot observe effort
  - The contract cannot be conditioned on *e*
  - The owner may still induce effort if some of the manager's salary depends on gross profit, π_g
  - The owner offers a salary such as

$$s(\pi_g) = a + b\pi_g$$

    - *a* is the fixed salary
    - *b* is the power of the incentive scheme

Second Best (Hidden-Action Case)

- This relationship can be viewed as a three-stage game
  - Owner sets the salary (choosing *a* and *b*)
  - The manager decides whether or not to accept the contract
  - The manager decides how much effort to put forth (conditional on accepting the contract)

Second Best (Hidden-Action Case)

- We will solve for the subgame-perfect equilibrium of this game by using backward induction,
- starting with the manager's choice of *e in the last stage* and taking as given that the manager was offered salary scheme $a + b\pi_g$ and accepted it.

Second Best (Hidden-Action Case)

- Manager
  - Accepted the salary scheme: $a + b\pi_g$
  - Expected utility from the linear salary:

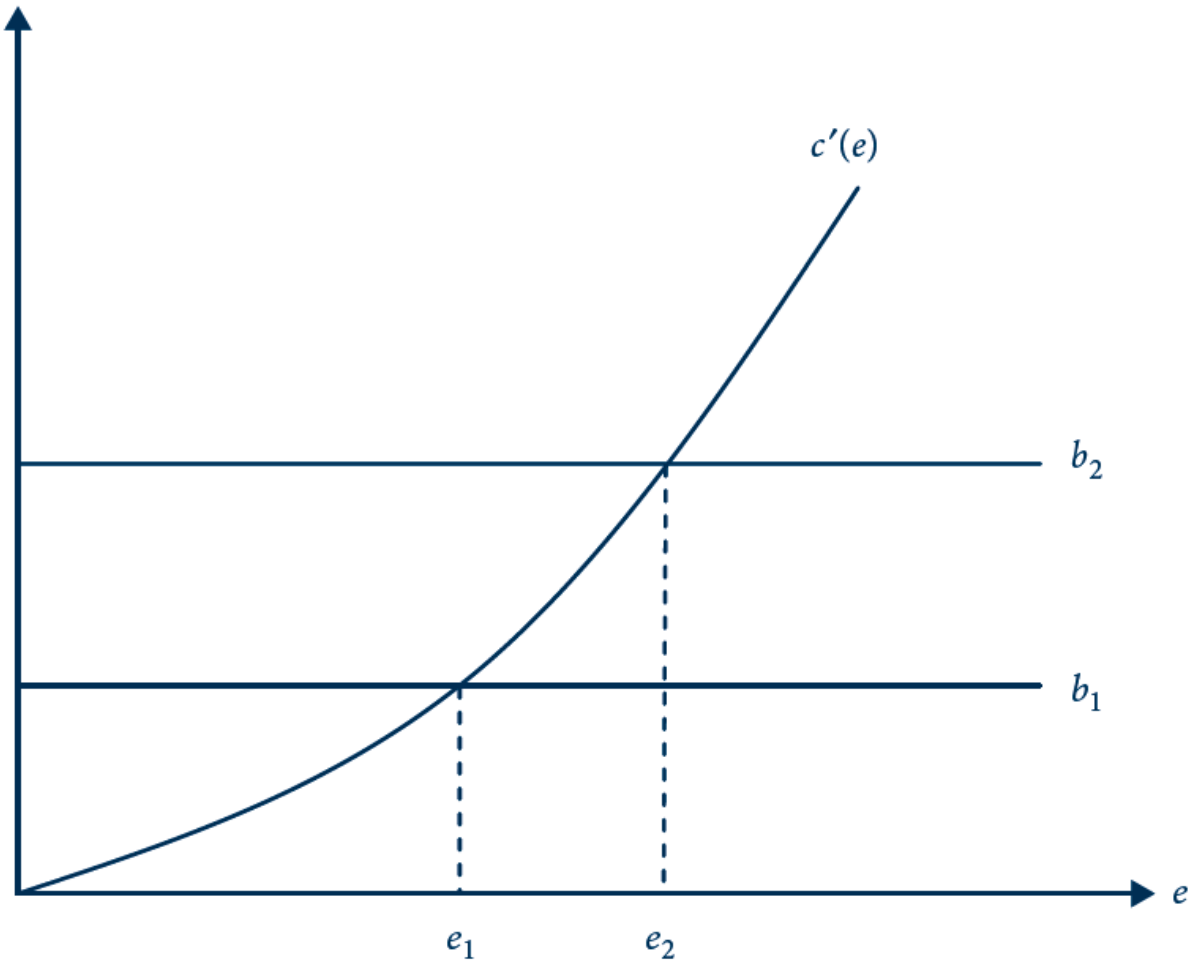
$$E(a + b\pi_g) - (A/2)\,Var(a + b\pi_g) - c(e)$$

  - Note: $E(a + b\pi_g) = E(a + be + b\epsilon) = a + be + bE(\epsilon) = a + be$
  - Variance: $Var(a + b\pi_g) = Var(a + be + b\epsilon) = b^2 Var(\epsilon) = b^2\sigma^2$
  - Manager's Expected utility $= a + be - (A\,b^2\sigma^2/2) - c(e)$

Second Best (Hidden-Action Case)

- First-order condition for the e
  - Maximizing the manager's expected utility
  - $c'(e) = b$
    - $c(e)$ is convex; $c'(e)$ is increasing in e
    - The higher is the power b of the incentive scheme, the more effort e the manager exerts
- The manager's effort
    - Depends only on the slope, b, and not on the fixed part, a, of his incentive scheme

FIGURE 18.2 Manager's Effort Responds to Increased Incentives



Because the manager's marginal cost of effort, c'(e), slopes upward, an increase in the power of the incentive scheme from b_1 to b_2 induces the manager to increase his effort from e_1 to e_2.

Second Best (Hidden-Action Case)

- Back to manager's *second-stage choice*:
- The manager accepts the contract
  - If his expected utility is non-negative

$$a \geq c(e) + (A\, b^2\sigma^2 /2) - be$$

Second Best (Hidden-Action Case)

- Back to owner's *first-stage choice* of parameters a and b of the salary scheme
- The owner's objective
  - Maximize expected surplus = $e(1 - b) - a$
  - Subject to two constraints
    - Manager must accept contract in second stage – *participation constraint*
    - Manager will choose *e* to suit himself rather than the owner, who cannot observe *e* - *incentive compatibility constraint*

Second Best (Hidden-Action Case)

- Owner's surplus as a function of manager's effort
- Substituting the constraint ($a \geq c(e) + (Ab^2\sigma^2/2) - be$) into the objective function

$$\text{Owner's surplus} = e - c(e) - \frac{A\sigma^2[c'(e)]^2}{2}$$

- The second-best effort e** satisfies the first-order condition

$$c'(e^{**}) = \frac{1}{1 + A\sigma^2 c''(e^{**})} = b^{**}$$

- $C'(e^{**})=b^{**}$ from maximizing the manager's expected utility

Second Best (Hidden-Action Case)

- Because $c'(e^{**}) < 1 = c'(e^{*})$,
- The convexity of $c(e)$ implies $e^{**} < e^{*}$
  - The presence of asymmetric information leads to lower equilibrium effort
  - The fundamental trade-off in the owner-manager relationship is between incentives and insurance

Second Best (Hidden-Action Case)

- If the owner cannot specify *e* in a contract, then she can induce effort only by tying the manager's pay to firm profit;
- however, doing so introduces variation into his pay for which the risk-averse manager must be paid a risk premium.
- This risk premium ($\frac{A\sigma^2[c'(e)]^2}{2}$) adds to the owner's cost of inducing effort.
- The more risk averse is the manager, the more important is insurance relative to incentives

EXAMPLE 18.1 Owner-Manager Relationship

- Assume
  - Firm's gross profit = $e + \varepsilon$
  - Manager's cost of effort: $c(e) = e^2/2$
  - $\sigma^2 = 1$
- First best
  - What is the optimal effort e^*? 1
  - Manager's fixed salary = ? 1/2
  - Owner's net profit = ? 1/2

EXAMPLE 18.1 Owner-Manager Relationship

- Second best, assume risk aversion A = 1
  - e^{**} = ? 1/2
  - b^{**} = ? 1/2
  - a^{**} = ? 0
  - Owner's expected net profit = ? 1/4

EXAMPLE 18.1 Owner-Manager Relationship

- Still second best, what if risk aversion A = 2?
  - $e^{**} =$ 1/3
  - $b^{**} =$ 1/3
  - $a^{**} =$ 1/18
  - Owner's expected net profit = 1/6

A increase，more risk averse, a increase.

Moral Hazard in Insurance

- If a person is fully insured
  - He will have a reduced incentive to undertake precautions
  - May increase the likelihood of a loss occurring
- Moral hazard
  - The effect of insurance coverage on an individual's precautions
  - Which may change the likelihood or size of losses

Mathematical Model

- Risk-averse individual
  - Faces the possibility of a loss (l)
  - That will reduce his initial wealth (W_0)
  - The probability of loss is π
  - An individual can reduce π by spending more on preventive measures (e)

Mathematical Model

- An insurance company (principal)
  - Offers a contract involving a payment of x to the individual if a loss occurs
  - The premium is p
- If the individual takes the coverage
  - Expected utility: $(1-\pi)U(W_1) + (\pi)U(W_2)$
    - Wealth in state 1 (no loss): $W_1 = W_0 - e - p$
    - Wealth in state 2 (loss): $W_2 = W_0 - e - p - l + x$
- The risk-neutral insurance company's objective is to maximize expected profit = $p - \pi x$

First Best Insurance Contract

- Insurance company - perfectly monitor *e*
  - Set the terms (e, p, x) to maximize its expected profit - subject to the participation constraint that individual will take the insurance contract

$$(1-\pi)U(W_1) + \pi U(W_2) \geq \bar{U}$$

Where $\bar{U}$ is the highest utility the individual can attain in the absence of insurance.

First Best Insurance Contract

- Insurance company - perfectly monitor *e*
  - Set the terms (e, p, x) to maximize its expected profit - subject to the participation constraint that individual will take the insurance contract

    $(1-\pi)U(W_1) + \pi U(W_2) \geq \bar{U}$
  - Will result in full insurance with $x = l$
  - The individual will choose the socially efficient level of precaution

First Best Insurance Contract

Lagrangian: $L = p - \pi x + \lambda[(1-\pi)U(W_1)+\pi U(W_2) - \bar{U}]$

First-order conditions:

$$0 = \frac{\partial \mathcal{L}}{\partial p} = 1 - \lambda[(1-\pi)U'(W_0 - e - p) + \pi U'(W_0 - e - p - l + x)],$$

$$0 = \frac{\partial \mathcal{L}}{\partial x} = -\pi + \lambda \pi U'(W_0 - e - p - l + x),$$

$$0 = \frac{\partial \mathcal{L}}{\partial e} = -\frac{\partial \pi}{\partial e}x - \lambda\{(1-\pi)U'(W_0 - e - p) + \pi U'(W_0 - e - p - l + x) + \frac{\partial \pi}{\partial e}[U(W_0 - e - p) - U(W_0 - e - p - l + x)]\}.$$

First Best Insurance Contract

- (1) and (2) imply

$$\frac{1}{\lambda} = (1-\pi)U'(W_0 - e - p) + \pi U'(W_0 - e - p - l + x)$$
$$= U'(W_0 - e - p - l + x),$$

- Will result in full insurance with $x = l$

First Best Insurance Contract

- (3) implies $-\frac{\partial \pi}{\partial e} l = 1$.
- At an optimum, the marginal social benefit of precaution (the reduction in the probability of a loss multiplied by the amount of the loss) equals the marginal social cost of precaution (which here is just 1).
- The individual will choose the socially efficient level of precaution

First Best Insurance Contract

- At full insurance, $x=l$ and $W_1 = W_2$.
- The insured party's expected utility is
$$U(W_1) = U(W_0 - e - p)$$
- This is maximized by choosing the lowest level of precaution possible, $e = 0$

Second-Best Insurance Contract

- Insurance company cannot monitor *e*
  - Add incentive compatibility constraint:
    - Agent is free to choose the level of precaution that suits him and maximizes his expected utility $(1-\pi)U(W_1)+\pi(W_2)$
- The second-best contract will typically not involve full insurance
  - Exposing the individual to some risk induces him to take some precaution