Intermediate Microeconomics

Spring 2025

Week 11a: Imperfect Competition

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Oligopoly

- A market with relatively few firms but more than one
- Possibility of strategic interaction among firms
- Difficult to predict exactly the possible outcomes for price and output

Pricing Under Homogeneous Oligopoly

- We will assume that the market is *perfectly competitive on the demand side*
  - there are many buyers, each of whom is a price taker
- We will assume that the good obeys the law of *one price*
  - this assumption will be relaxed when product differentiation is discussed

Pricing Under Homogeneous Oligopoly

- We will assume that there is a *relatively small number of identical firms (n)*
  - we will initially start with n fixed, but later allow n to vary through entry and exit in response to firms' profitability

- The output of each firm is q_i (i=1,…,n)
  - symmetry in costs across firms will usually require that these outputs are equal

Pricing Under Homogeneous Oligopoly

- The inverse demand function for the good shows the price that buyers are willing to pay for any particular level of industry output

$$P = f(Q) = f(q_1+q_2+\ldots+q_n)$$

- Each firm's goal is to maximize profits

$$\pi_i = f(Q)q_i - C_i(q_i)$$
$$\pi_i = f(q_1+q_2+\ldots q_n)q_i - C_i$$

Oligopoly Pricing Models

- The quasi-competitive model assumes price-taking behavior by all firms
  - *P* is treated as fixed
- The cartel model assumes that firms can collude perfectly in choosing industry output and *P*

Oligopoly Pricing Models

- The <u>Cournot model</u> assumes that firm *i* treats firm *j*'s output as fixed in its decisions
  - $\partial q_j / \partial q_i = 0$

- The <u>conjectural variations model</u> assumes that firm *j*'s output will respond to variations in firm *i*'s output
  - $\partial q_j / \partial q_i \neq 0$

Quasi-Competitive Model

- Each firm is assumed to be a price taker
- The first-order condition for profit-maximization is

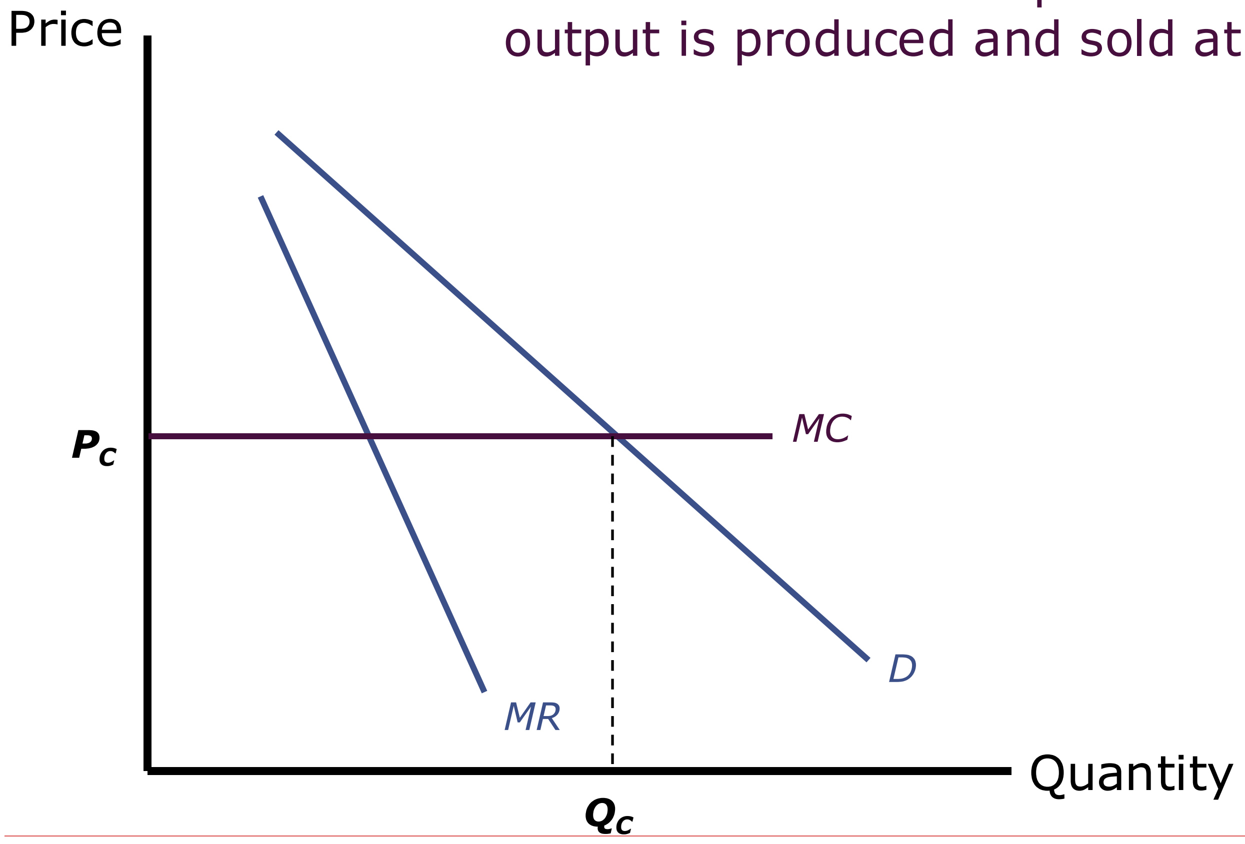
$$\partial\pi_i / \partial q_i = P - (\partial C_i / \partial q_i) = 0$$

$$P = MC_i(q_i) \quad (i=1,\ldots,n)$$

- Along with market demand, these *n* supply equations will ensure that this market ends up at the short-run competitive solution

Quasi-Competitive Model

If each firm acts as a price taker, $P = MC_i$ so Q_C output is produced and sold at a price of P_C



Bertrand Model

- Two identical firms
  - Producing ***identical*** products at a constant $MC = c$
  - Choose prices p_1 and p_2 ***simultaneously***
    - Single period of competition
  - How Sales get split
    - All sales go to the firm with the lowest price
    - Sales are ***split evenly*** if $p_1 = p_2$

Bertrand Model: The Only Pure-strategy Nash equilibrium

- The Only Pure-strategy Nash equilibrium: $p_1^*=p_2^*=c$
  - ***Both*** firms are playing a ***best response*** to each other
  - ***Neither*** firm has an incentive to ***deviate*** to some other strategy

- A formal proof should verify that all other cases are not Nash equilibrium
  - Let's focus on cases where $p_1 \leq p_2$
  - Three cases: $p_1^*<c$, $p_1^*>c$, $p_1^*=c$

Bertrand Model: The Only Pure-strategy Nash equilibrium

- If $p_1 < c$ (and $p_1 \leq p_2$)
  - Profit would be negative, should deviate to $p_1 = c$
- If $p_1 > c$ (*and* $p_1 \leq p_2$)
  - Firm 2 could gain by ***undercutting*** the price of firm 1 and captures all the market
- If $p_1 = c$ (*and* $p_1 \leq p_2$)
  - If $p_1 < p_2$, then firm 1 can raise price ***slightly over*** c but still lower than P_2, and earn higher profit (because it still gets the whole market)
- The Only Pure-strategy Nash equilibrium: $p_1^* = p_2^* = c$

Bertrand Model

- For any number of firms $n \geq 2$
  - The same result
  - Nash equilibrium of the n-firm Bertrand game is $p_1^* = p_2^* = \ldots = p_n^* = c$
- The <u>Bertrand paradox</u>
  - The Nash equilibrium of the Bertrand model is the same as the perfectly competitive outcome ***even though there are only two firms***
    - Price is set to marginal cost
    - Firms earn zero profit

Bertrand Model

- The Bertrand paradox
  - General : holds for ***any c*** and ***any downward-sloping*** demand curve
  - ***Not*** general: ***can be undone*** by changing assumptions:
    - Choosing quantity rather than price
    - Facing capacity constraint
    - Products slightly differentiated (not perfect substitute)
    - Repeated interaction

Cartel Model

- The assumption of price-taking behavior may be inappropriate in oligopolistic industries
  - each firm can recognize that its output decision will affect price
- An alternative assumption would be that firms act as a group and coordinate their decisions so as to achieve monopoly profits

Cartel Model

- In this case, the cartel acts as a multiplant monopoly and chooses q_i for each firm so as to maximize total industry profits

$$\pi = PQ - [C_1(q_1) + C_2(q_2) + \ldots + C_n(q_n)]$$

- If write everything in terms of q_i

$$\pi = f(q_1 + q_2 + \ldots + q_n)[q_1 + q_2 + \ldots + q_n] - \sum_{i=1}^{n} C_i(q_i)$$

Cartel Model

- The first-order conditions for a maximum are that

$$\frac{\partial}{\partial q_i}\left(\sum_{j=1}^{n}\pi_j\right) = P(Q) + P'(Q)\sum_{j=1}^{n} q_j + C'_i(q_i) = 0 \quad \text{for } i = 1,\ldots,n$$

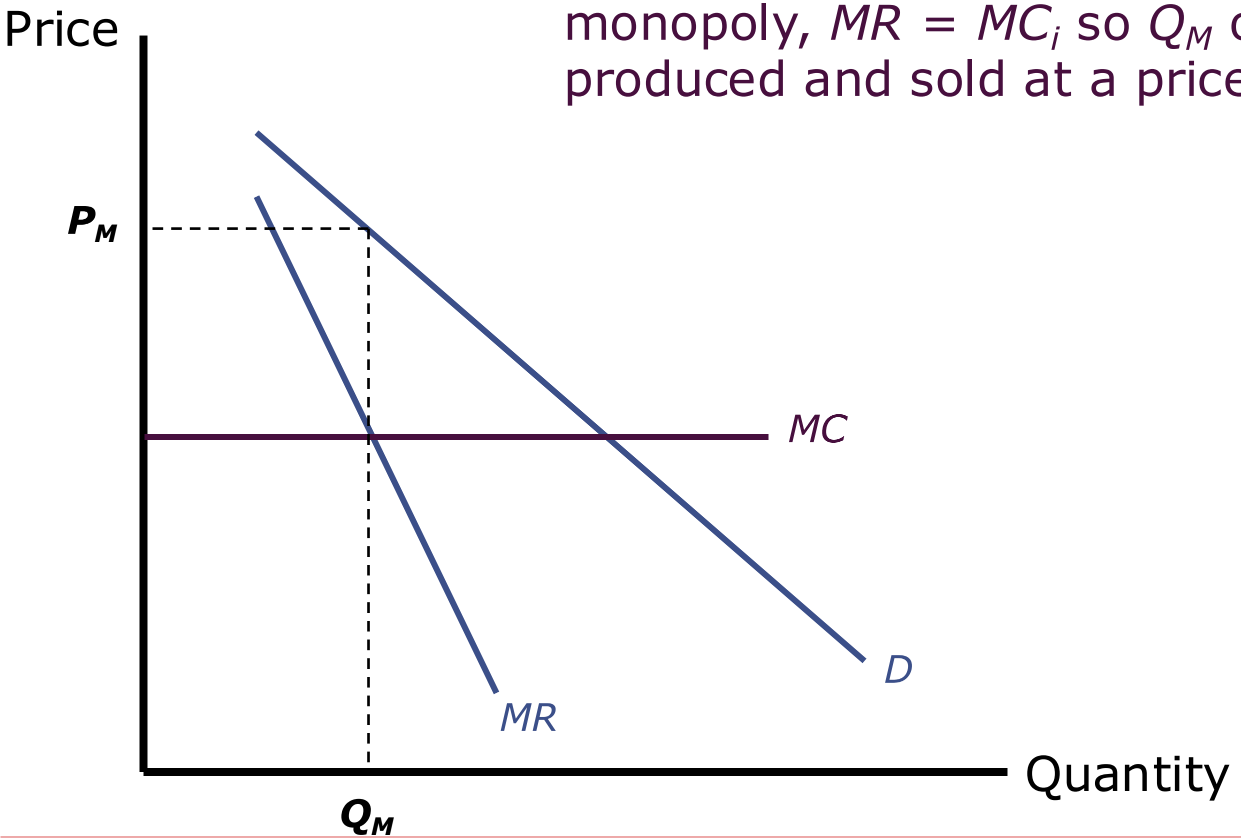
- This implies that

$$MR(Q) = MC_i(q_i)$$

- At the profit-maximizing point, marginal revenue will be equal to each firm's marginal cost

Cartel Model

If the firms form a group and act as a monopoly, $MR = MC_i$ so Q_M output is produced and sold at a price of P_M



Cartel Model

- There are three problems with the cartel solution
  - these monopolistic decisions may be illegal
  - it requires that the directors of the cartel know the market demand function and each firm’s marginal cost function
  - the solution may be unstable
    - each firm has an incentive to expand output because $P > MC_i$

Cournot Model

- Each firm recognizes that its own decisions about q_i affect price
  - $\partial P/\partial q_i \neq 0$
- However, each firm believes that its decisions do not affect those of any other firm
  - $\partial q_j /\partial q_i = 0$ for all $j \neq i$

Cournot Model

- Firm i's profit = total revenue – total cost

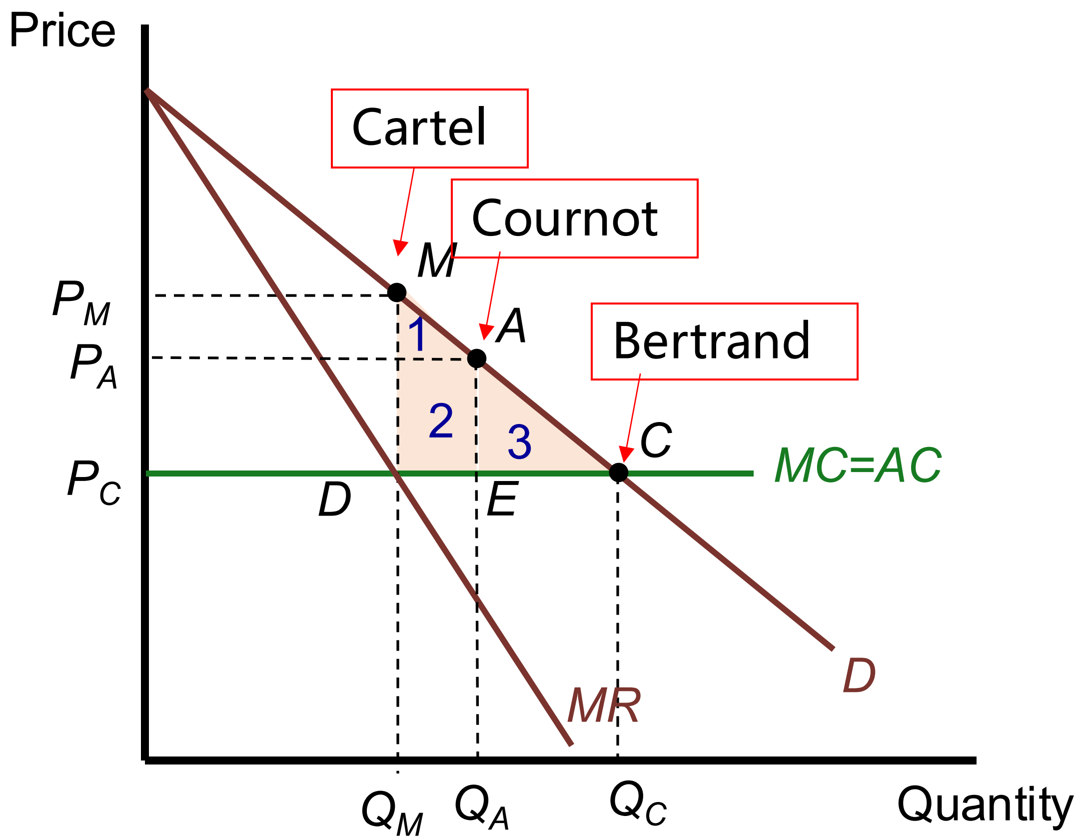
$$\pi_i = P(Q)q_i - C_i(q_i)$$

- First-order conditions for profit maximization:

$$\frac{\partial \pi_i}{\partial q_i} = \underbrace{P(Q) + P'(Q)q_i}_{\text{MR}} - \underbrace{C'_i(q_i)}_{\text{MC}} = 0$$

  - Maximize profit where $MR_i = Mc_i$
    - the firm assumes that changes in q_i affect its total revenue only through their direct effect on market price

Bertrand vs. Cournot vs. Cartel



- In Cournot game, industry profits
  - Lower than in the cartel model ($P_A AEP_C < P_M MDP_C$)
- DWL
  - Smaller in the Cournot model (3) than in the cartel situation (1+2+3)

Varying the Number of Cournot Firms

- The Cournot model
  - Can represent the whole range of outcomes by varying the number of firms
  - $n = \infty \Rightarrow$ perfect competition
  - $n = 1 \Rightarrow$ perfect cartel / monopoly
- n identical firms
  - Same cost function $C(q_i)$
  - In equilibrium, each produces $q_i = Q/n$

Varying the Number of Cournot Firms

- Difference between price and marginal cost: $P'(Q)Q/n$
  - The wedge term disappears as n grows large; firms become infinitesimally small – price takers
    - Price approaches marginal cost
    - Market outcome approaches the perfectly competitive one
  - As n decreases to 1: the Cournot outcome approaches that of a perfect cartel

Conjectural Variations Model

- In markets with only a few firms, we can expect there to be strategic interaction among firms
- One way to build strategic concerns into our model is to consider the assumptions that might be made by one firm about the other firm's behavior

Conjectural Variations Model

- For each firm *i*, we are concerned with the assumed value of $\partial q_j / \partial q_i$ for $i \neq j$

- because the value will be speculative, models based on various assumptions about its value are termed <u>conjectural variations models</u>
  - they are concerned with firm *i*'s conjectures about firm *j*'s output variations

Conjectural Variations Model

- The first-order condition for profit maximization becomes

$$\frac{\partial \pi_i}{\partial q_i} = P + q_i \left[\frac{\partial P}{\partial q_i} + \sum_{j \neq i} \frac{\partial P}{\partial q_j} \cdot \frac{\partial q_j}{\partial q_i} \right] - MC_i(q_i) = 0$$

The firm must consider how its output decisions will affect price in two ways

- directly
- indirectly through its effect on the output decisions of other firms

Practice example: Natural Springs Duopoly

- Assume that there are two owners of natural springs
  - each firm has no production costs
  - each firm has to decide how much water to supply to the market

- The demand for spring water is given by the linear demand function

$$Q = q_1 + q_2 = 120 - P$$

Natural Springs Duopoly

- In a Bertrand model, what are the market price and the quantity supplied?

Natural Springs Duopoly

- In a Cartel model, what are the market price and the quantity supplied?

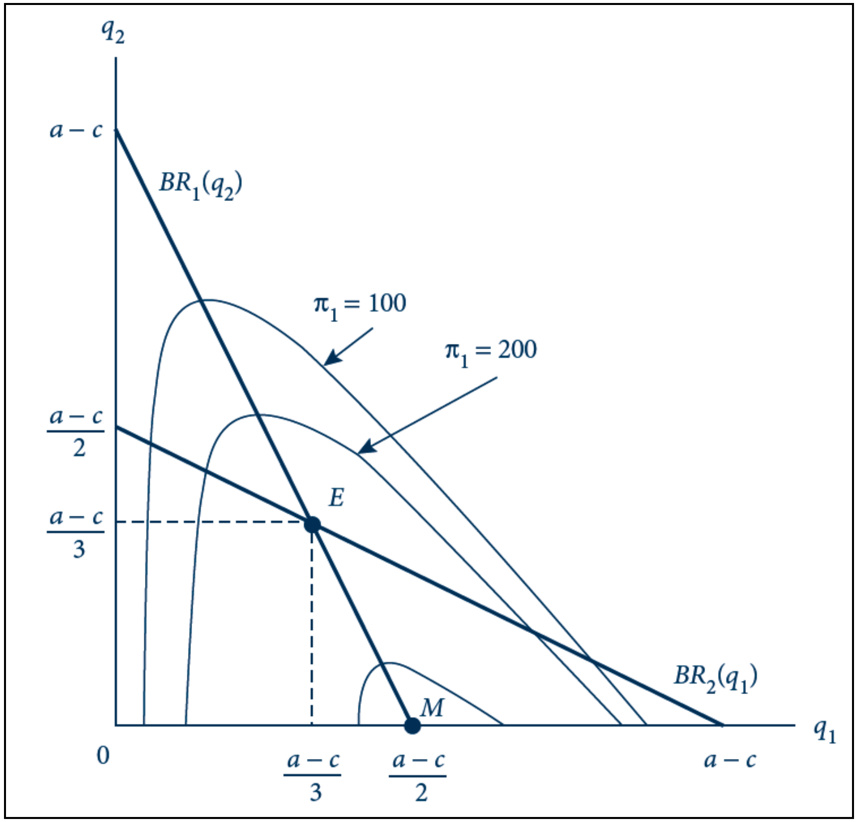
Cournot's Natural Springs Duopoly

- In a Cournot model, what are the market price and the quantity supplied?

EXAMPLE 15.2 Cournot Best-Response Diagrams

- Solve for the Nash equilibrium using graphical methods
  - Graph the intercepts of the best-response functions
  - Intersection between the best responses is the Nash equilibrium
- An isoprofit curve for firm 1
  - Is the locus of quantity pairs providing it with the same profit level

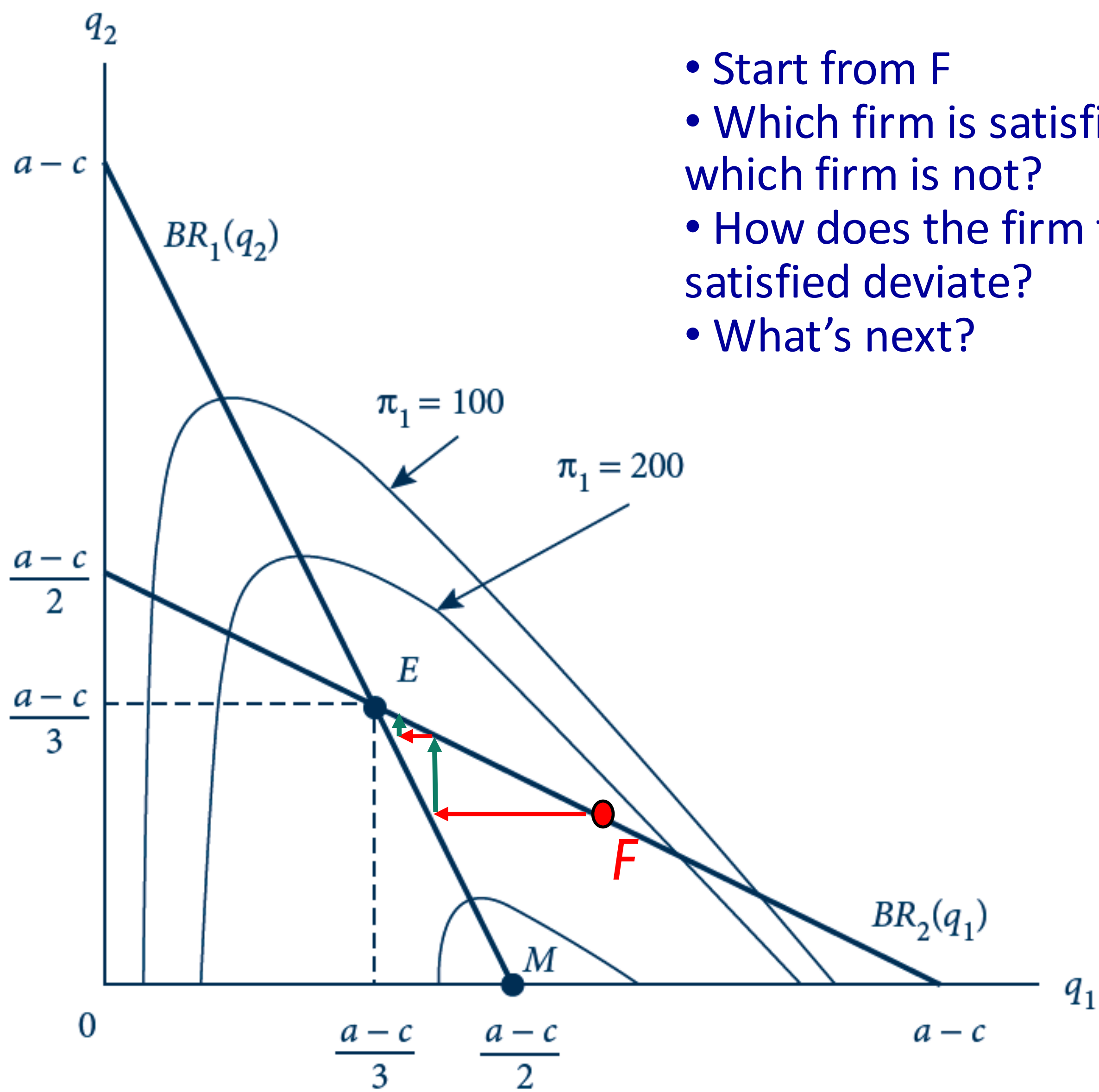
Best-Response Diagram for Cournot Duopoly



Demand: $P(Q) = a\text{-}Q$
Cost: $C_i(q_i)=cq_i$

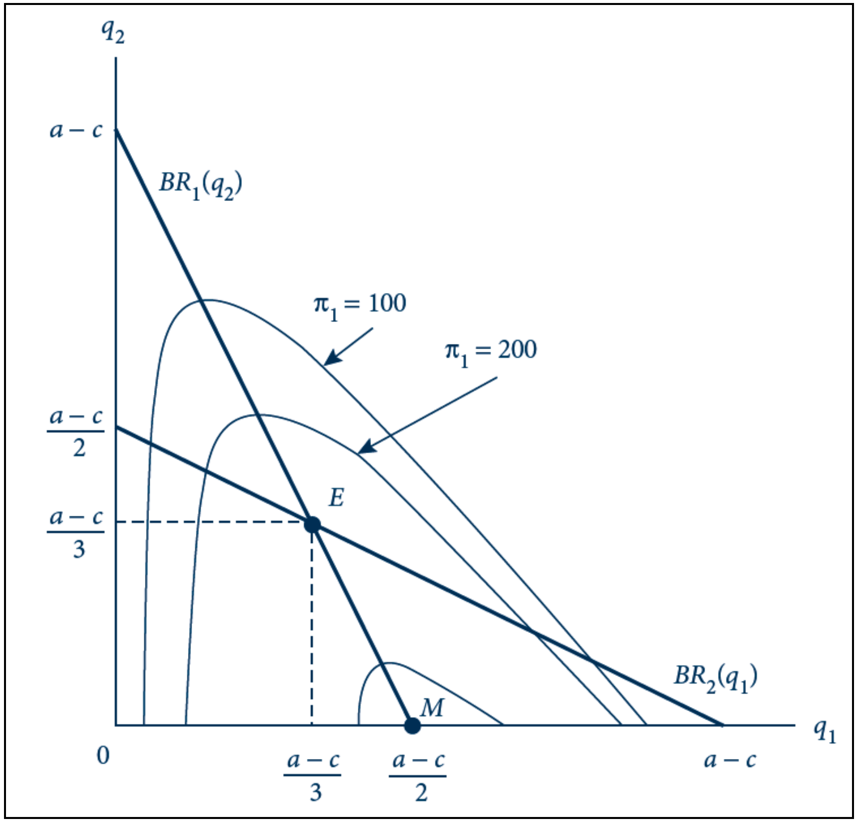
- Solve for the Cournot firms' best response functions.

Best-Response Diagram for Cournot Duopoly



- Start from F
- Which firm is satisfied at F, which firm is not?
- How does the firm that's not satisfied deviate?
- What's next?

Best-Response Diagram for Cournot Duopoly



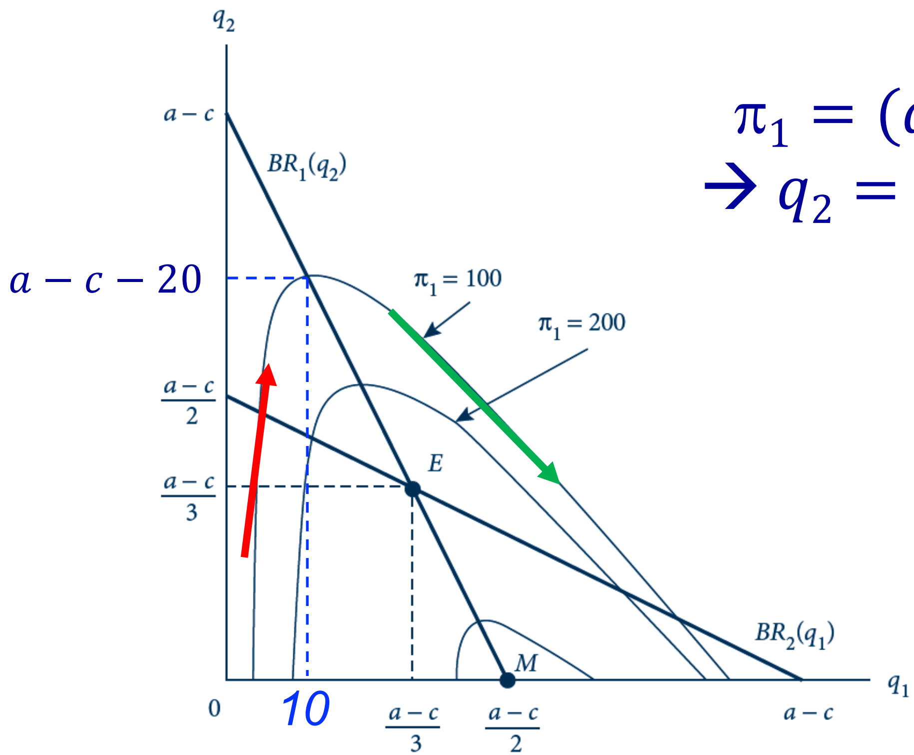
Demand: $P(Q) = a\text{-}Q$
Cost: $C_i(q_i)=cq_i$

- Firms' best responses are drawn as thick lines;
  - Their intersection (E) is the Nash equilibrium of the Cournot game.

$$q_1 = \frac{a-q_2-c}{2} \qquad q_2 = \frac{a-q_1-c}{2}$$

- An iso-profit curve for firm 1
  - Is the ***locus*** of quantity pairs providing it with the same profit level

Iso-profit curve: inverse U-shape

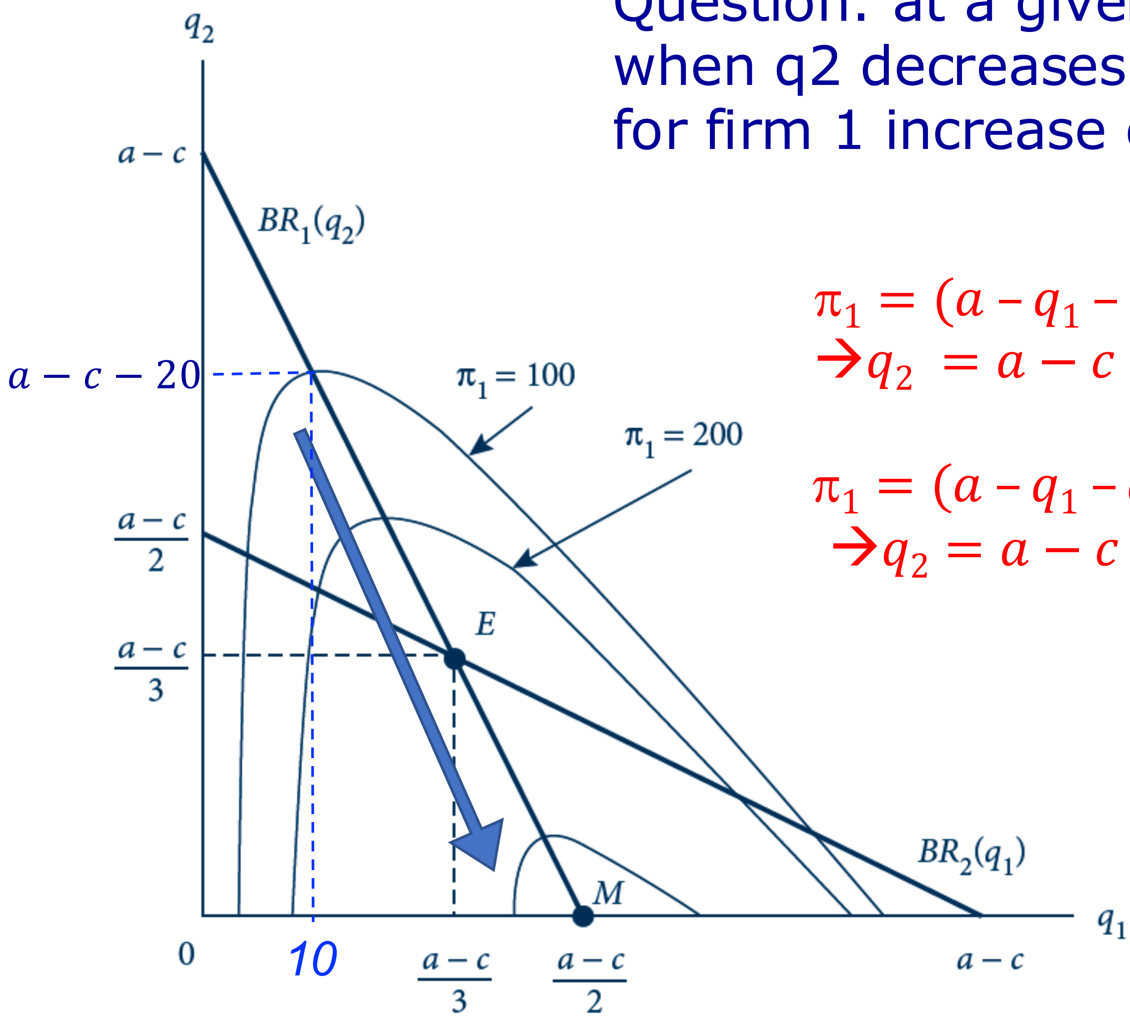


$$\pi_1 = (a - q_1 - q_2 - c)q_1 = 100$$
$$\rightarrow q_2 = a - c - (q_1 + 100/q_1)$$

- As q_1 was close to 0 and q_1 increases, $100/q_1$ dominates, and q_1+100/q_1 decreases if $q_1<10$
  - So if $q_1<10$, q_2 must be increasing to keep profit constant at 100

- As q_1 increases further (>10), q_1 will begin to dominate, and q_1+100/q_1 increases
  - So q_2 must be decreasing to keep profit constant at 100

Iso-profit curve



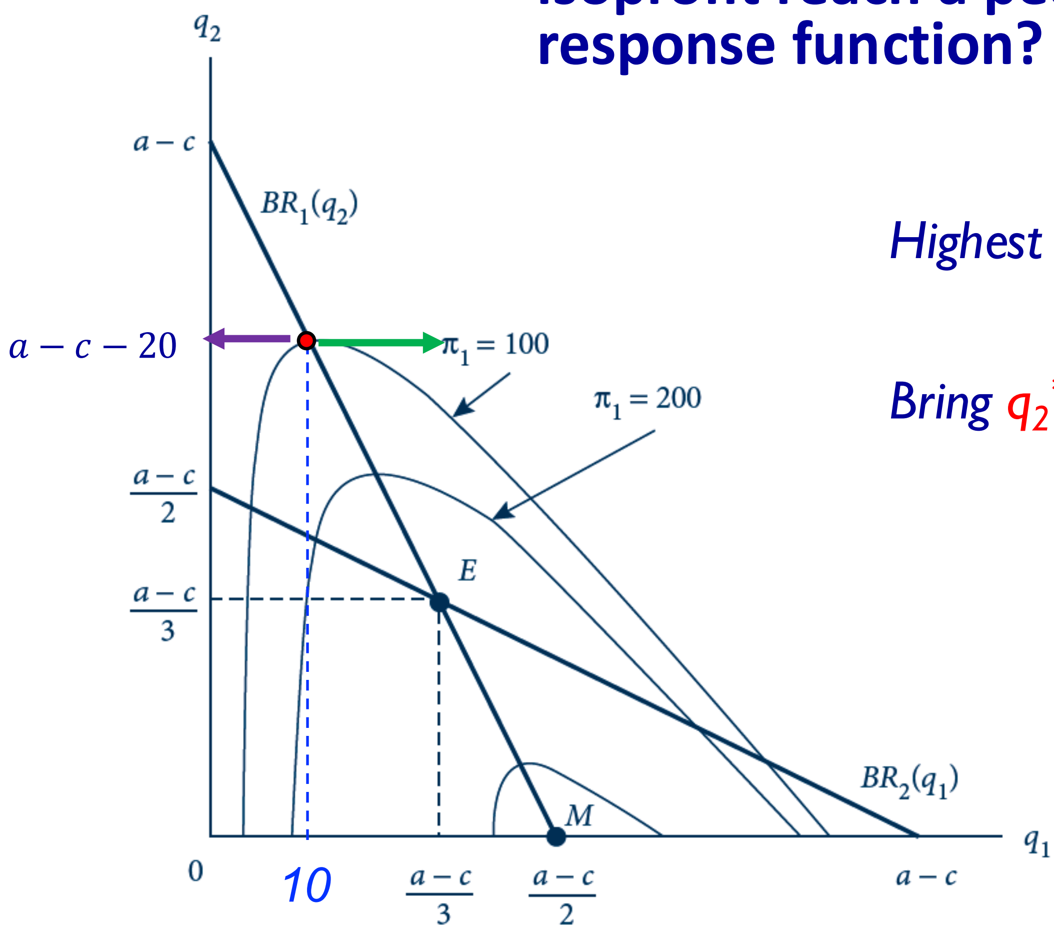
Question: at a given level of q1, when q2 decreases, does profit for firm 1 increase or decrease?

$\pi_1 = (a - q_1 - q_2 - c)q_1 = 100$
$\rightarrow q_2 = a - c - q_1 - 100/q_1$

$\pi_1 = (a - q_1 - q_2 - c)q_1 = 200$
$\rightarrow q_2 = a - c - q_1 - 200/q_1$

- As profit increases from 100 to 200 to yet higher levels, the associated isoprofits shrink down to the monopoly point, which is the highest isoprofit on the diagram.

Question: Why does firm 1's individual isoprofit reach a peak on its best-response function?



Highest q_2 On this curve:

$q_1^*=10, q_2^*=a\text{-}c\text{-}20$

Bring q_2^* to the *best-response curve:*

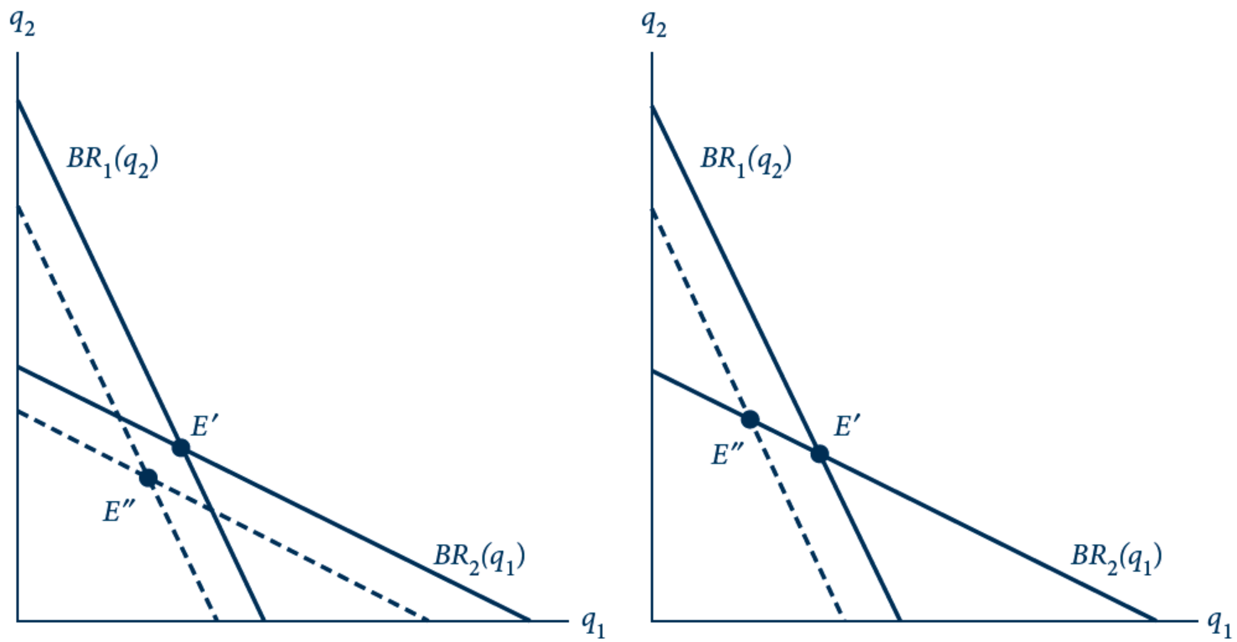
$$q_1 = \frac{a-q_2-c}{2}$$
$$= \frac{a-c-(a-c-20)}{2}$$
$$= 10$$

Intuition: On firm 1's best-response function, for a given level of q2

- If firm 1 increases its output q1, profit will decrease.
- If firm 1 decreases its output q1, profit will also decrease.

Hence, the point on the best-response function is at the peak of the isoprofit curve.

Best-response diagrams $q_1 = \frac{a - q_2 - c}{2}$ $q_2 = \frac{a - q_1 - c}{2}$



(a) Increase in both firms' marginal costs (b) Increase in firm 1's marginal cost

- Panel (a) depicts an increase in both firms' marginal costs, c, shifting their best responses ***inward***.
- If marginal costs are different as in Panel (b), output q_1 is lower, q_2 is higher.
- What about an increase in the preference parameter, a ?

Practice example:

- Let c_i be the constant marginal and average cost for firm i (so that firms may have different marginal costs). Suppose demand is given by $P=1-Q$.

- 1. Calculate the Nash equilibrium quantities assuming there are two firms in a Cournot market. Also compute market output, market price, firm profits, industry profits, consumer surplus, and total welfare.
- 2. Represent the Nash equilibrium on a best-response function diagram. Show how a reduction in firm 1’s cost would change the equilibrium. Draw a representative isoprofit for firm 1.

Prices or Quantities?

- Bertrand model - price competition
  - Discontinuous jump from monopoly to perfect competition if just two firms enter
  - Additional entry beyond two has no additional effect on the market outcome
- Cournot model - quantity competition
  - Industry grows more competitive as the number n of firms entering the market increases

Capacity Constraints

- For the Bertrand model to generate the Bertrand paradox
  - Firms must have unlimited capacity
  - More realistically, firms may not have an unlimited ability to meet all demand

- Starting from equal prices, if a firm lowers its price the slightest amount, then its demand essentially doubles. The firm can satisfy this increased demand because it has no capacity constraints.
- If the undercutting firm could not serve all the demand because of capacity constraints, that would leave some residual demand for the higher-priced firm and would decrease the incentive to undercut.

Capacity Constraints

- Two-stage game
  - Firms build capacity in the first stage
  - Firms choose prices p_1 and p_2 in the second stage
  - Sales of firms cannot exceed the capacity chosen in the first stage
  - If the cost of building capacity is sufficiently high
    - Equilibrium - the same as the Nash equilibrium of the Cournot model
    - As if firms choose quantities rather than price.

假设市场上有两家企业A和B，生产同质化产品，边际成本均为c。在没有产能约束的情况下，按照Bertrand模型，两家企业会不断降价，最终价格都降至c，利润为零。但如果引入产能约束，假设每家企业的最大产能为q，市场总需求为D(p)。在选择价格时，企业A和B都需要考虑自己的产能限制。如果企业A设定价格p_1，企业B设定价格p_2，消费者会购买价格较低的产品，但如果价格相同，则按比例分配。由于产能有限，即使某家企业价格略低，它也无法满足所有需求，因此企业不会无限制地降价，而是会在产能约束下选择一个最优价格，使得自己的产量和价格组合能够最大化利润，这类似于Cournot模型中企业选择产量的过程，最终的市场结果也会更接近Cournot模型的均衡结果

Product Differentiation

- To avoid the Bertrand paradox
  - Assume that firms produce differentiated products
- Market
  - A group of closely related products
    - That are more substitutable among each other
      - As measured by cross-price elasticities
    - Than with goods outside the group

Bertrand competition with differentiated products

- There are n firms competing in a particular market
  - Each product has its own attributes, a_i
- The product's attributes affect its demand

$$q_i(p_i, P_{-i}, a_i, A_{-i})$$

  - Where P_{-i} is a list of all other firms' prices
  - And A_{-i} is a list of the attributes of other firms' products

Bertrand competition with differentiated products

- Firm *i*'s
  - Total cost: $C_i(q_i, a_i)$
  - Profit: $\pi_i = p_i q_i - C_i(q_i, a_i)$
- First-order conditions for a maximum:

$$\frac{\partial \pi_i}{\partial p_i} = q_i + p_i \frac{\partial q_i}{\partial p_i} - \frac{\partial C_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial p_i} = 0$$

$$\boxed{\frac{\partial \pi_i}{\partial a_i} = p_i \frac{\partial q_i}{\partial a_i} - \frac{\partial C_i}{\partial a_i} - \frac{\partial C_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial a_i} = 0}$$

Bertrand competition with differentiated products

- First-order conditions for a maximum:

$$\frac{\partial \pi_i}{\partial p_i} = \boxed{q_i + \boxed{p_i \frac{\partial q_i}{\partial p_i}}} - \boxed{\frac{\partial C_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial p_i}} = 0$$

Blue box: marginal revenue from an increase in price.
Red box: cost savings associated with the reduced sales that accompany an increased price.

Regarding the choice of a_i is more complex, let's take a look at the next two examples.

EXAMPLE 15.4 Toothpaste as a Differentiated Product

- Two firms produce toothpaste
  - One a green gel and the other a white paste
  - Suppose that production is costless
- Demand for product *i*, $q_i = a_i - p_i + p_j/2$
  - The goods are gross substitutes (positive coefficient on p_j)
  - Suppose that attribute a_i is an endowment rather than a choice variable for the firm.

EXAMPLE 15.4 Toothpaste as a Differentiated Product

- Two firms produce toothpaste
  - One a green gel and the other a white paste
  - Suppose that production is costless
- Demand for product *i*, $q_i = a_i - p_i + p_j/2$
- Firm *i*'s profit: $\pi_i = p_i q_i - C_i(q_i) = p_i(a_i - p_i + p_j/2)$
  - Where $C_i(q_i) = 0$ for simplicity
  - First-order condition for profit maximization

$$\frac{\partial \pi_i}{\partial p_i} = a_i - 2p_i + p_j/2 = 0$$

EXAMPLE 15.4 Toothpaste as a Differentiated Product

- Best-response functions

$$p_1 = \frac{1}{2}\left(a_1 + \frac{p_2}{2}\right), \quad p_2 = \frac{1}{2}\left(a_2 + \frac{p_1}{2}\right)$$

- Nash equilibrium prices

$$p_i^* = \frac{8}{15}a_i + \frac{2}{15}a_j$$

- Firm i 's equilibrium price is not only increasing in its own attribute, a_i, but also in the other product's attribute, a_j.
- An increase in a_j causes firm j to increase its price, which increases firm *i* 's demand and thus the price firm *i* charges.

EXAMPLE 15.4 Toothpaste as a Differentiated Product

- Best-response functions

$$p_1 = \frac{1}{2}\left(a_1 + \frac{p_2}{2}\right), \quad p_2 = \frac{1}{2}\left(a_2 + \frac{p_1}{2}\right)$$

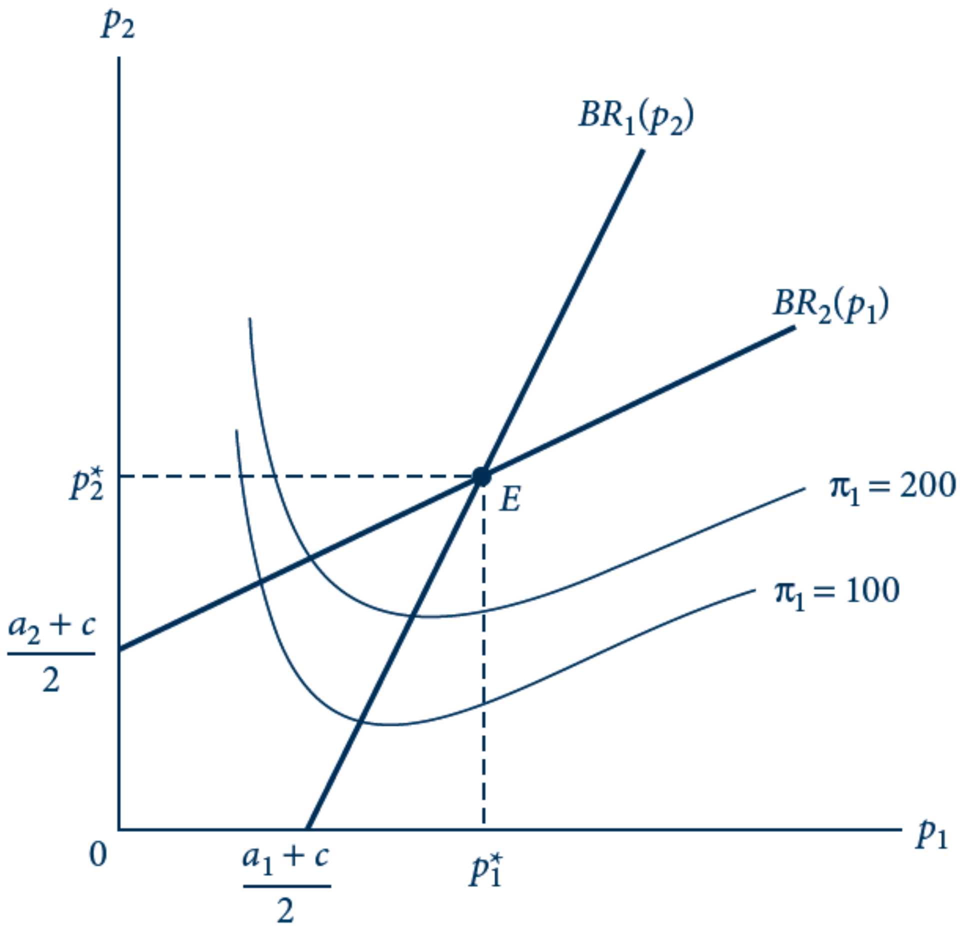
- Nash equilibrium prices

$$p_i^* = \frac{8}{15}a_i + \frac{2}{15}a_j$$

- Profits

$$\pi_i^* = \left(\frac{8}{15}a_i + \frac{2}{15}a_j\right)^2$$

FIGURE 15.4 Best Responses for Bertrand Model with Differentiated Products

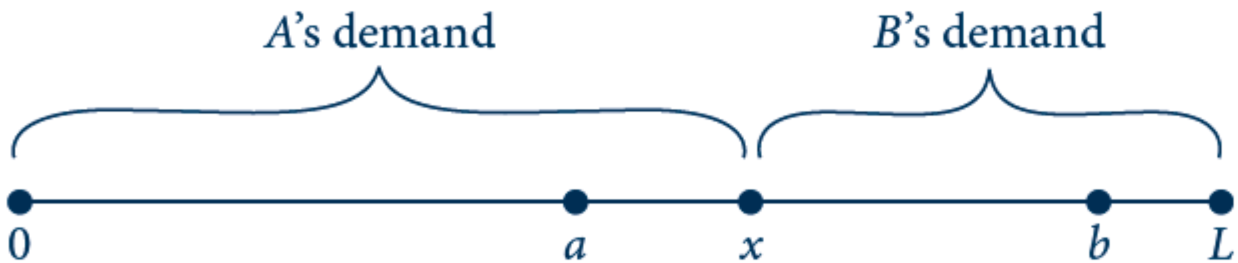


Firm' best responses are drawn as thick lines; their intersection (E) is the Nash equilibrium.

EXAMPLE 15.5 Hotelling's Beach

- Ice cream stands located on a beach
  - We will take the locations of the ice cream stands as given.
  - Demanders are located uniformly along the beach
    - One at each unit of beach
  - Ice cream cones are costless to produce
  - But carrying them back to one's place on the beach results in a cost of td^2
    - t = temperature
    - d = distance

FIGURE 15.5 Hotelling's Beach



Ice cream stands A and B are located at points a and b along a beach of length L. The consumer who is indifferent between buying from the two stands is located at x. Consumers to the left of x buy from A and to the right buy from B.

EXAMPLE 15.5 Hotelling's Beach

- A person located at point x will be indifferent between stands A and B if

$$p_A + t(x-a)^2 = p_B + t(b-x)^2$$

  – Where p_A and p_B are the prices charged by each stand, and $t(x\text{-}a)^2$ is the transportation cost.

- Solving for x we get

$$x = \frac{b+a}{2} + \frac{p_B - p_A}{2t(b-a)}$$

  – If the two stands charge an equal price, the indifferent consumer is located midway between a and b

EXAMPLE 15.5 Hotelling's Beach

- Consumers 0 to x buy from A;

$$q_A(p_B, p_A, a, b) = x = \frac{b+a}{2} + \frac{p_B - p_A}{2t(b-a)}$$

- the remaining L-x consumers buy from B.

$$q_B(p_B, p_A, b, a) = L - x = L - \frac{b+a}{2} + \frac{p_A - p_B}{2t(b-a)}$$

- Solve for Nash Equilibrium price and profits for the two firms.

接下来表示出A和B的利润，求导，得到best response function，然后联立求纳什均衡。

EXAMPLE 15.5 Hotelling's Beach

- The Nash equilibrium prices:

$$p_A^* = \frac{t}{3}(b-a)(2L+a+b)$$

$$p_B^* = \frac{t}{3}(b-a)(4L-a-b)$$

- Profits for the two firms:

$$\pi_A^* = \frac{t}{18}(b-a)(2L+a+b)^2$$

$$\pi_B^* = \frac{t}{18}(b-a)(4L-a+b)^2$$