Intermediate Macroeconomics: Problem Set 4

Due Thursday, May 8

1. RBC Model with Consumption Habits (60 points)

In the standard RBC model studied in class, utility in period t depends only on consumption in that period and is not affected by past consumption. Here we consider an RBC model with consumption habits. Assume the representative agent's utility function is

$$u(C_t, C_{t-1}, L_t) = \gamma \log\bigl(C_t - \phi\, C_{t-1}\bigr) + (1-\gamma)\log\bigl(1 - L_t\bigr),$$

where C_t is consumption in period t, C_{t-1} is consumption in period $t-1$, L_t is labor supply in period t, and $\gamma \in (0,1)$ is the consumption–leisure weight. The agent's budget constraint is

$$C_t + I_t = W_t\, L_t + R_t^k\, K_t,$$

where I_t is total investment (savings), W_t is the real wage, R_t^k is the real rental rate of capital, and K_t is the capital stock. **In parts (a)–(g), assume no stochastic technology shocks exist in the economy.**

a. Assume capital depreciates at rate δ. Write down the law of motion for capital.

b. Substitute the investment I_t from the capital accumulation equation into the agent's budget constraint to obtain a new budget constraint.

c. State the agent's dynamic optimization problem and construct the Lagrangian.

d. Take derivatives of the Lagrangian and write down the first-order conditions with respect to C_t, K_{t+1}, and L_t.

e. Using the first-order conditions for C_t and K_{t+1}, derive the intertemporal equilibrium (Euler) equation,

$$\frac{?}{?} = \beta\bigl[R_{t+1}^k + 1 - \delta\bigr].$$

Then show the special case when $\phi = 0$. How should we interpret this result?

f. Suppose firms have a Cobb–Douglas production function

$$Y_t = A_t \, K_t^{\alpha} \, L_t^{1-\alpha}.$$

Write down the firm's profit-maximization problem and derive expressions for the factor prices W_t and R_t^k from the first-order conditions.

g. List the seven equations that characterize the dynamics of the seven endogenous variables

$$\{Y_t,\ C_t,\ I_t,\ K_t,\ L_t,\ R_t^k,\ W_t\}.$$

(Hint: just list the equations; do not log-linearize or solve them.)

h. Now let the stochastic technology shock $\tilde{A}_t$ follow an AR(1) process, with $g = 0$:

$$\begin{aligned} \log A_t &= \log \bar{A} + \tilde{A}_t, \\ \tilde{A}_t &= \rho_A \, \tilde{A}_{t-1} + \epsilon_t. \end{aligned}$$

Compare this habit-formation model to the baseline RBC model: for a given technology shock that occurs once, will the instantaneous response of consumption C_t be larger or smaller? How about investment I_t? (Hint: it's not necessary to solve the problem; using words or intuition is sufficient.)

2. Two-Period Model with Labor Income Tax and Government (Final 2024, 40 points)

Consider an economy that lasts for 2 periods, $t = 0, 1$. There is one representative household in this economy with 1 unit of time to allocate between labor n_t and leisure l_t. The utility function for the household is given by:

$$U(c_0, n_0, c_1, n_1) = \log(c_0) + \theta \log(1 - n_0) + \beta[\log(c_1) + \theta \log(1 - n_1)]$$

The household can choose to save/borrow s between time period 0 and 1 by purchasing a "treasury bond", whose interest rate r is taken as given by the household. The wage for household's labor services is (w_0, w_1), but in both periods the household faces labor income taxes with rate (τ_0, τ_1) .

The household's $t = 0$ budget constraint is:

$$c_0 + s = w_0 n_0 (1 - \tau_0)$$

There is a representative firm with constant return to scale production function that only uses labor as input, i.e.

$$y_t = An_t$$

where A is fixed. There is a government with **fixed** expenditures (g_0, g_1) in each period. The goverment expenditures are financed by labor income taxes (τ_0, τ_1) and "treasury bonds" b. The government's budget constraints are given by:

$$g_0 = w_0 n_0 \tau_0 + b$$
$$g_1 + b(1+r) = w_1 n_1 \tau_1$$

You can think of the "treasury bond" as government borrowing from households in $t = 0$ and repaying the debt in $t = 1$ with some interest rate r. The bond market clearing condition is:

$$b = s$$

a. Write the household's period $t = 1$ budget constraint, as well as its intertemporal budget constraint.

b. Solve the household's problem and derive the first-order conditions for labor (n_0, n_1) and consumption (c_0, c_1) in both periods. Show that the Euler Equation for consumption does not depend on tax rates.

c. Show that the firm's problem and market clearing conditions imply that $w_0 = w_1 = A$. Combine this result and the first order conditions to show how equilibrium consumption-leisure ratio $\frac{c_0^*}{1-n_0^*}$ in period 0 depends on tax rate τ_0.

d. Suppose the labor tax rates are $(\bar{\tau}_0, \bar{\tau}_1)$ before, with $\bar{\tau}_0 > 0$ and $\bar{\tau}_1 > 0$. A new administration wants to cut taxes in period 0 by setting $\tau_0 = 0$. Discuss the impact of this policy on equilibrium consumption (c_0^*, c_1^*) and labor supply (n_0^*, n_1^*).