Intermediate Macroeconomics: Final Exam

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Instructions

- This exam is out of 120 points.
- You have 120 minutes to complete the exam.
- **Write your answers on 5-10 pieces of A4 paper, in either English, Chinese, or a combination of both**. Don't forget to write down your name.
- No calculators, phones, notes or books of any kind are permitted.

Good Luck!

1. True/False/Uncertain (15 points, or 5 points each)

Assess whether the following statements are true, false or uncertain and justify your answers. Points are given for explanations only.

a. In the neoclassical growth model, a permanent positive shock in technology from A to $A' > A$ will increase capital and consumption levels in the long run, but may decrease consumption in the short run.

b. People will go to the ATM more often if the nominal interest rate becomes greater.

c. (Credit: Xiang A.) An unemployed worker will accept any job offer with wage $w > 0$ if the government doesn't provide any unemployment benefit.

2. OLG Model with Money in Utility (40 points)

Consider an overlapping generations model with money in the utility function. In each period, a cohort of constant size 1 is born. Each individual lives for 2 periods. There is an initial old cohort, endowed with capital K_0 and money M. The preferences for the initial old are defined as

$$U = u(c_0^o)$$

Agents born after $t = 0$ are endowed with one unit of labor when young, and do not supply labor when old. They have utility function defined as

$$U(c_t^y, c_{t+1}^o, m_t) = u(c_t^y) + v(m_t) + \beta u(c_{t+1}^o)$$

Where $m_t = \frac{M_t}{P_t}$ is real money holding for young agents at time t, and $u(\cdot), v(\cdot)$ satisfy $u' > 0, v' > 0, u'' < 0, v'' < 0$. Output is produced using a contant return to scale production function $Y_t = F(K_t, N_t)$, with capital depreciation rate $\delta = 1$. There is no storage technology other than money and investment. Also, there is no government, so no new money is ever issued.

a. (5 pts) Find the budget constraints for the initial old cohort, as well as cohorts born after $t = 0$. Use w_t and R_t to represent (real) wage and rental rates at t.

b. (5 pts) Define a competitive equilibrium for this economy.

c. (5 pts) Solve the firm's problem for its first order conditions.

d. (5 pts) Find the first order conditions for agent born after $t = 0$, and write down the Euler Equation.

e. (5 pts) Using the first order condition for money and the fact that $v' > 0$, show that in a competitive equilibrium the real return on capital R_{t+1} is always greater than real return on money, $\frac{P_t}{P_{t+1}}$. Explain this result.

From part (f) to (g), assume the following functional form:

$$\begin{aligned} Y_t &= K_t^\alpha N_t^{1-\alpha} \\ u(c) &= \log c \\ v(m) &= \log m \end{aligned}$$

f. (5 pts) Find an expression of $\frac{c_t^y}{c_{t+1}^o}$ as a function of R_{t+1} and β.

g. (5 pts) Find the consumption of young agents c_t^y as a function of w_t, R_{t+1}, π_t, m_t and β only, where $\pi_t = \frac{P_{t+1} - P_t}{P_t}$ is the inflation rate between t and $t + 1$. Show that the young agent's savings rate, defined as

$$s_t = 1 - \frac{c_t^y}{w_t}$$

Satisfies $s_t > \frac{\beta}{1+\beta}$.

h. (5 pts) Find the young agent's money demand function $m_t(w_t, R_{t+1}, \pi_t)$ at the competitive equilibrium.

3. Asset Pricing with Lucas Tree (45 Points)

Consider a 2-period economy with a "Lucas tree", which grows consumption goods (fruit) each period for the representative agent. In period 0, the tree bears 1 units of fruit with certainty. In period 1, the tree bears $1 + x$ units of fruit in the good state (with probability $\pi > 0$) and $1 - x$ units in the bad state(with probability $1 - \pi$). The agent's utility function is:

$$U(c_0, c_g, c_b) = u(c_0) + \beta \left[\pi u(c_g) + (1 - \pi)u(c_b)\right]$$

where $\beta < 1$ and $u(c)$ satisfies $u'(c) > 0, u''(c) < 0$. c_0, c_g, c_b denote consumption in period 0, consumption in the good state, and consumption in the bad state respectively.

The agent may save from period 0 to period 1 by purchasing state contingent claims. In particular, she has access to a bond that pays one unit of good in the good state, and another bond that pays one unit of good in the bad state. We use q_g and q_b to denote the price of good and bad state bonds, and a_g and a_b to denote the quantity of the good and bad state bonds.

a. (5 pts) Write down the utility maximization problem for the agent as well as a separate budget constraint for each period and state. Also specify which variables are the agent choosing.

By combining the state-by-state budget constraints in your answer above, we would arrive at a single lifetime budget constraint:

$$c_0 + q_g c_g + q_b c_b = 1 + q_g(1 + x) + q_b(1 - x) \tag{1}$$

b. (5 pts) Using the above budget constraint, write the Lagrangian of the agent's problem, and solve for its first order conditions.

c. (5 pts) Solve for the prices of the good and bad state bond q_g and q_b in terms of x, β, π, and derivatives of the utility function u.

d. (5 pts) Now suppose that there are two additional assets available in period 0. The first asset is a stock which pays the same as the tree: i.e. the stock pays $1 + x$ in the good state and $1 - x$ in the bad state. The second asset is a safe bond which pays 1 in both circumstances. What is the price of each of these assets in terms of q_g, q_b, and other parameters of the model (i.e. $x, u(\cdot), \beta$)? Call the price of the stock P_S and the price of the bond P_B.

e. (5 pts) Compute the expected return of the stock and the safe bond, denoting them R_S and R_B, respectively.

For the rest of this problem, assume $\pi = \frac{1}{2}$, so that each state is equally likely.

f. (5 pts) Compare prices between the stock and the safe bond. Is $P_S > P_B$? Is $R_S > R_B$? What is the intuition for this result?

g. (5 pts) From part (g) to (i), assume that consumers have constant relative risk aversion (CRRA) utility, given by

$$u(c) = \frac{c^{1-\alpha}}{1 - \alpha}$$

where a higher α indicates consumers are more risk averse. Use this utility function to explicitly solve for P_S, P_B, R_S and R_B in terms of x, α, and β.

h. (5 pts) Suppose that x increases to $x_h > x$. Does the excess return on stocks, $R_s - R_B$ increase or decrease? What is the intuition behind this result?

i. (5 pts) Suppose that risk aversion α increases to $\alpha_h > \alpha$. Does the excess return on stocks increase or decrease? What is the intuition behind this result?

4. Short Answer (20 points)

Last week, the Federal Reserve raised its benchmark interest rate by 0.75 percentage points, the biggest hike since 1994. Some economists worry that this policy would bring the US into a new recession, while other economists disagree.

As carefully and completely as possible, discuss what you think would be the effect of this new monetary policy on the US economy. You may rely on any frameworks you learned from this class, write an illustrative model of your own, or not use any models at all.