1.

100

a. $C_t+I_t\le Y_t=K_t^{\alpha}(BK_t^{\phi}N_t)^{1-\alpha}$

b. $K_{t+1}=K_t(1-\delta)+I=K_t(1-\delta)+sK_t^{\alpha}(BK_t^{\phi}N_t)^{1-\alpha}$

c. $f(\lambda K_t,\lambda N_t)=(\lambda K_t)^{\alpha}(B\lambda^{\phi}K_t^{\phi}\lambda N_t)^{1-\alpha}$

$=\lambda^{1+\phi-\phi\alpha}f(K_t,N_t)>\lambda f(K_t,N_t)$

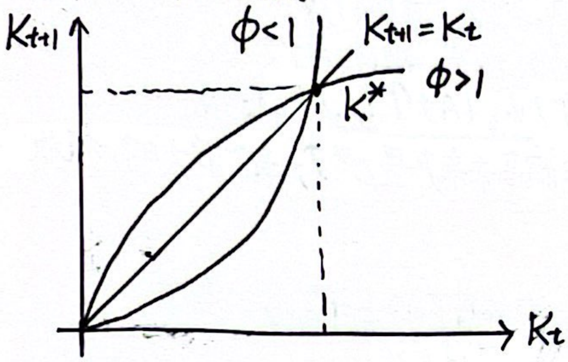
d. $N_t=1,\ K_{t+1}=K_t(1-\delta)+sB^{1-\alpha}K_t^{\alpha+\phi-\phi\alpha}$

$(K_{t+1})''=sB^{1-\alpha}\cdot(\alpha+\phi-\phi\alpha)(\alpha+\phi-\phi\alpha-1)K_t^{\alpha+\phi-\phi\alpha-2}$

$\alpha+\phi-\phi\alpha-1=(1-\phi)(\alpha-1)$

当$\phi>1$时，$(K_{t+1})''<0$，

当$\phi<1$时，$(K_{t+1})''>0$，



$K^*=K^*(1-\delta)+sB^{1-\alpha}K^{*\,\alpha+\phi-\phi\alpha}$

$\Rightarrow K^*=\left(\frac{\delta}{sB^{1-\alpha}}\right)^{\frac{1}{(1-\phi)(\alpha-1)}}$

e. $\left.\frac{\partial K_{t+1}}{\partial K_t}\right|_{K_t=K^*}=(1-\delta)+sB^{1-\alpha}(\alpha+\phi-\phi\alpha)(K^*)^{(1-\phi)(\alpha-1)}=\delta(\alpha+\phi-\phi\alpha)+1-\delta$

$=1+\delta(\alpha-1)(1-\phi)$

当$\phi>1$时，$\frac{\partial K_{t+1}}{\partial K_t}<1$；当$\phi<1$时，$\frac{\partial K_{t+1}}{\partial K_t}>1$

$\phi<1$： $K_t<K^*$，$K_{t+1}>K_t$，回到稳态

$K_t>K^*$，$K_{t+1}<K_t$，回到稳态

f. $K^*=\left(\frac{sB^{1-\alpha}}{\delta}\right)^{\frac{1}{(1-\phi)(1-\alpha)}}$

$s\uparrow,\ K^*\uparrow$
$B\uparrow,\ K^*\uparrow$
$\delta\uparrow,\ K^*\downarrow$

2.

(a) $C_t+k_{t+1}-(1-\delta)k_t=(1-\tau)Ak_t^{\alpha}$

$(1-\tau)Ak_t^{\alpha}<Ak_t^{\alpha}$，可支配收入减少

$t+1$期的产出$=A[1+\phi(\tau)]>A$

所以未来的产出可能增加。

(b) 令 $\mathcal{L}=\sum_{t=0}^{\infty}\beta^t u(C_t)+\sum_{t=0}^{\infty}\lambda_t[Ak_t^{\alpha}-C_t-k_{t+1}+(1-\delta)k_t]$

$\tau=0$,

$[C_t]$: $\beta^t u'(C_t)-\lambda_t=0$

$[k_{t+1}]$: $-\lambda_t+\lambda_{t+1}(1-\delta+\alpha A k_t^{\alpha-1})=0$

欧拉方程: $u'(C_t)=\beta u'(C_{t+1})(1+\alpha A k_{t+1}^{\alpha-1}-\delta)$

一次性产业政策, $[C_t]$: $\beta^t u'(C_t)-\lambda_t=0$

$[k_{t+1}]$: $-\lambda_t+\lambda_{t+1}[1+\alpha(A+\phi(T))k_{t+1}^{\alpha-1}-\delta]=0$

欧拉方程: $u'(C_t)=\beta u'(C_{t+1})[1+\alpha(A+\phi(T))k_{t+1}^{\alpha-1}-\delta]$

(c) 无限期产业政策, 欧拉方程: $u'(C_t)=\beta u'(C_{t+1})[1+\alpha(1-T)(A+\phi(T))k_{t+1}^{\alpha-1}-\delta]$

$C_t=C_{t+1}$, $1=\beta[1+\alpha(1-T)(A+\phi(T))(k^*)^{\alpha-1}-\delta] \Rightarrow k^*=\left[\frac{\alpha\beta(1-T)(A+\phi(T))}{1-\beta(1-\delta)}\right]^{\frac{1}{1-\alpha}}$

$C^*=(1-T)(A+\phi(T))(k^*)^{\alpha}-\delta k^*$

1阶导: $\eta\gamma T^{\eta-1}-A-\gamma(\eta+1)T^{\eta}$ $T\to 0, \to +\infty$; $T\to+\infty, \to -\infty$

T对k^*的影响取决于$(1-T)(A+\gamma T^{\eta})$。

2阶导: $\eta(\eta-1)T^{\eta-2}-\gamma\eta(\eta+1)T^{\eta-1}<0$

所以k^*随着T的上升先增后减

$C^*=\left[\left(\frac{\alpha\beta}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}}-\delta\left(\frac{\alpha\beta}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}}\right]\cdot[(1-T)(A+\gamma T^{\eta})]^{\frac{1}{1-\alpha}}$

C^*有意义, $\left[\left(\frac{\alpha\beta}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}}-\delta\left(\frac{\alpha\beta}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}}\right]>0$

T对C^*的影响取决于$(1-T)(A+\gamma T^{\eta})$, 所以C^*随着T的上升先增后减

long-run welfare $=\sum_{t=0}^{\infty}\beta^t u(C^*)=\frac{u(C^*)}{1-\beta}=\frac{(C^*)^{1-\sigma}}{(1-\sigma)(1-\beta)}$, $0<\sigma<1$

设$T=T^*$时, C^*取得最大值, 当$T>T^*$时, long-run welfare 减少。

(满足 $\eta\gamma T^{*\eta-1}-A-\gamma(\eta+1)T^{*\eta}=0$)

(d) $T^*=0.1119$, $k^*=5.7136$, $C^*=1.6494$

γ增大, T^*也增大

η增大, T^*先增后减

α增大, T^*不变

我使用了Kimi, 我让它帮我调整缩进, 解释代码。好用。