Intermediate Microeconomic

Spring 2025

Part four: Competitive markets

Week 7: Markets and efficiency (III)

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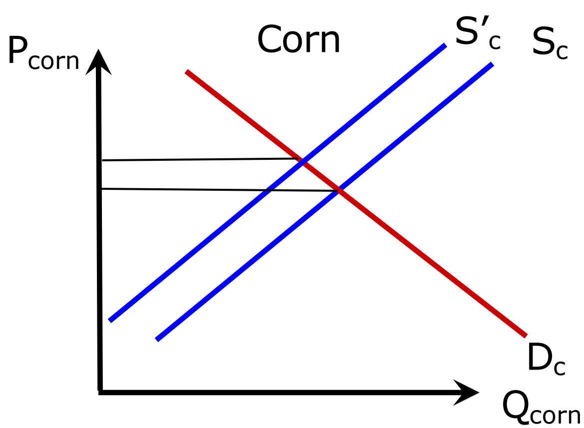
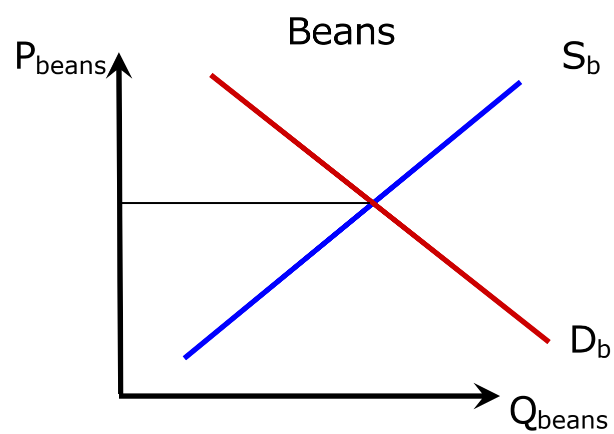
General Equilibrium

- Up until now, we have focused on a single market at a time. But economy is made up of many markets, and they often interact with each other
  - When the price of gasoline rises, you have less money to spent on EVERYTHING else. So, what happens on the gasoline market has an impact on all other markets
- This topic – markets cannot and should not be treated separately – is called "General Equilibrium" (GE) in advanced microeconomics
  - What we've learned so far is called "Partial Equilibrium" (PE), and you can consider PE as a "beginner" version of GE.
  - In fact, most insights of PE originate from GE. In other words, most important conclusions we learn in PE also apply in GE model
  - For that reason, this class focuses on PE
- The goal of the lecture today is to give you a sense of GE.

Example of General Equilibrium

- Suppose that people consume either corn or beans.
- Bad weather reduces the supply of corn.
- This shifts supply in, increases the price of corn.





Example of General Equilibrium

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- The higher corn price shifts demand for beans out.
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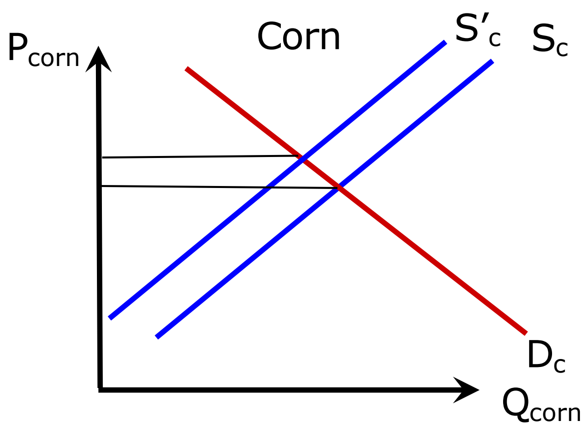

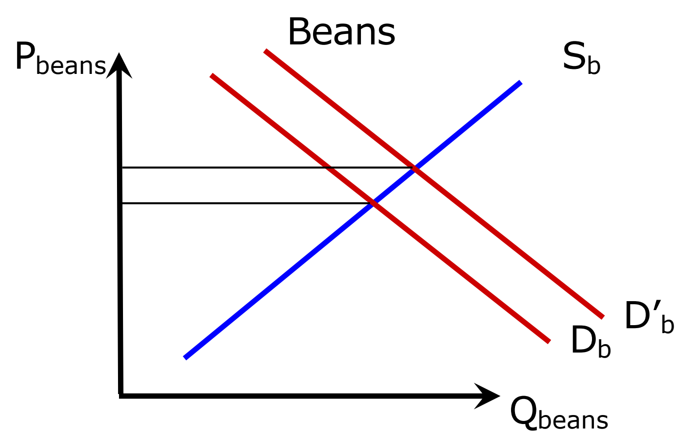

Example of General Equilibrium

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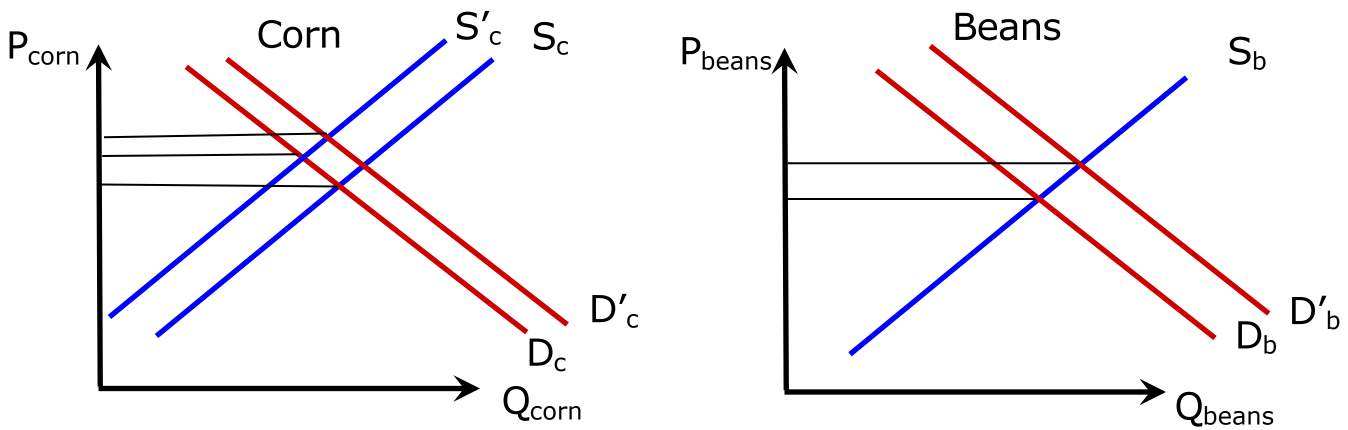
- Bad weather reduces the supply of corn.
- This shifts supply in, increases the price of corn.
- The higher corn price shifts demand for beans out.
- The price of beans rises.
- This, in turn, shifts demand for corn out, price of corn rises.
- Feedback effects tend to grow smaller until new equilibrium is reached.



General Equilibrium

Roadmap: GE & Markets

- The aim is to introduce a general model of behavior of consumers and firms that captures the consequences of spillovers across markets.
- Since markets are interdependent, extending the theory of PE to GE is a natural step.

- The analysis proceeds as follows:
  - Exchange Economies
  - Definition of a Walrasian Equilibrium
  - Efficiency & Pareto Efficiency
  - Welfare Theorems
  - GE with production

Exchange Economies

Exchange Economies

- First we abstract from production and study how goods are traded by consumers endowed of a fixed amounts.
- Consider an **exchange economy**:
  - with n commodities and k consumers;
  - in which each consumer j is described by:
    - a utility function $U^j(x^j) = U^j(x_1^j, ..., x_n^j)$;
    - an initial endowment $\omega^j = (\omega_1^j, ..., \omega_n^j)$.
- A **consumption bundle** for consumer j is $x^j = (x_1^j, ..., x_n^j)$.
- An **allocation** $x = (x^1, ..., x^k)$ assigns a bundle to each consumer.
- An allocation x is **feasible** if $\sum_{j=1}^k x^j \leq \sum_{j=1}^k \omega^j$.
- A feasible allocation obtains by redistributing or destroying endowments.

Walrasian Equilibrium

Definition

A **Walrasian equilibrium** $(\bar{x}, \bar{p})$ consists of an allocation $\bar{x} = (\bar{x}^1, ..., \bar{x}^k)$ and of a price vector $\bar{p} = (\bar{p}_1, .., \bar{p}_n)$ such that:

1. $\bar{x}$ is feasible;
2. $\bar{x}^j$ maximizes $U^j(x^j)$ subject to $\bar{p}x^j \leq \bar{p}\omega^j$ for all $j = 1, ..., k$.

Walrasian equilibria are also referred to as competitive or general equilibria.
A competitive equilibrium consists of an allocation and of a price vector st:

- consumers choices are optimal when taking as given such prices;
- the implied demands are feasible (i.e. all markets clear).

Introductory example: market clearing

Consider a simple economy with:

- $k = 2$ and $n = 2$;
- $p = (p_1, p_2)$;
- $\omega^1 = (3, 1)$ and $U^1(x^1) = min\{x_1^1, x_2^1\}$;
- $\omega^2 = (1, 3)$ and $U^2(x^2) = x_1^2 x_2^2$.

Find the equilibrium allocation $x_1^1, x_2^1, x_1^2, x_2^2$.

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- $\omega^2 = (1, 3)$ and $U^2(x^2) = x_1^2 x_2^2$.

$m_1 = 3P_1 + P_2$

$m_2 = P_1 + 3P_2$

Solving the consumers' problems we obtain that

$$x_1^1(p, p\omega^1) = \frac{m_1}{2P_1}$$

$$x_2^1(p, p\omega^1) = \frac{m_1}{2P_2}$$

$$x_1^2(p, p\omega^2) = \frac{m_2}{2P_1}$$

$$x_2^2(p, p\omega^2) = \frac{m_2}{2P_2}$$

Equilibrium conditions require the two markets to clear:

$$Good1 : x_1^1 + x_1^2 = 4 = 2 + 2\frac{p_2}{p_1}$$

$$Good2 : x_2^1 + x_2^2 = 4 = 2 + 2\frac{p_2}{p_1} \tag{1}$$

equilibrium in market 1 implies equilibrium in market 2

Introductory example: Edgeworth Box

A useful exercise is to plot the example in an Edgeworth Box.
Proceed by:

1. plotting the Edgeworth box and the feasible allocations;
2. finding the endowment, and drawing budget constraints
3. plotting the equilibrium itself;
4. showing that $p_1/p_2 > 1$ cannot be an equilibrium, since consumer 1 demands of good 2 more than 2 wants to give.

Show more generally that an equilibrium is a price ratio for which consumers' demands are optimal and markets clear.

At an interior solution consumers equalize MRS. But boundary solutions, multiplicity of equilibria and non-existence are other possible phenomena.

Normalizing prices

Recall that:

- $x_i^j(p, p\omega^j)$ denotes j's demand for good i;
- $x^j(p, p\omega^j)$ denotes vector of j's demands;
- $x_i^j(p, p\omega^j)$ is homogeneous of degree 0 in p.

Fact

Because demands are homogeneous of degree 0 if $\bar{p}$ is an equilibrium price vecotr, $t\bar{p}$, $t > 0$, is also an equilibrium price vector.

Hence, we can normalize one price.
Without loss, set $\bar{p}_i = 1$ for one arbitrary good i.
Call good i the **numeraire good**.

Excess Demand and LNS

Assume LNS (Local Non-Satiation) holds, then

$$px^j(p, p\omega^j) = p\omega^j, \text{for any } j = 1, ..., k.$$

Define the vector of j's **excess demand** as

$$z^j(p, p\omega^j) = x^j(p, p\omega^j) - \omega^j.$$

Then, by LNS, it must be that

$$pz^j(p, p\omega^j) = 0.$$

Define the vector of **aggregate excess demands** as

$$z(p) = \sum_{j=1}^{k} z^j(p, p\omega^j),$$

where $z_i(p)$ denotes the aggregate excess demand for good i.

Walras Law

Lemma

The value of aggregate excess demand is equal to zero, $pz(p) = 0$.

Walras Law can be restated in terms of goods as follows

$$p_1 z_1(p) + ... + p_n z_n(p) = 0.$$

The following two conclusions immediately hold:

- If $p >> 0$ and $z_i(p) = 0$ for any market $i \neq n$, then $z_n(p) = 0$. The intuition follows along the lines of the previous example.
- If $\bar{p}$ is an equilibrium price vector, $z_i(\bar{p}) < 0$ implies $\bar{p}_i = 0$. Intuitively, if there are 4 units of good i, but in aggregate we only demand 2 units, then what happens is that either the price should fall so that we clear the market in equilibrium, or $\bar{p}_i$ is already zero so that we cannot drop the price anymore.

Efficiency and welfare

Pareto Efficiency

A common justification for markets is that markets attain efficiency.
The next definition is crucial for the welfare properties of equilibria.

Definition

An allocation $\bar{x} = (\bar{x}^1, ..., \bar{x}^k)$ is **Pareto efficient** if and only if:

1. it is feasible
2. there is no other feasible allocation $\hat{x} = (\hat{x}^1, ..., \hat{x}^k)$ such that:
   a. $U^j(\hat{x}^j) \geq U^j(\bar{x}^j)$ for any $j \in \{1, ..., k\}$;
   b. $U^j(\hat{x}^j) > U^j(\bar{x}^j)$ for some $j \in \{1, ..., k\}$.

Comments:

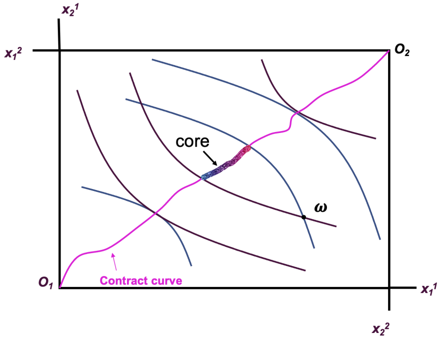
- there are no prices involved in the definition of Pareto efficiency.

The Pareto problem & the core

A Pareto efficient allocation $(\bar{x}^1, \bar{x}^2)$ maximizes $U^1(x^1)$ subject to:

$$U^2(x^2) = U^2(\bar{x}^2) \text{ and } x^1 + x^2 = \omega^1 + \omega^2.$$

The **contract curve** identifies all Pareto efficient allocations, while the **core** identifies those Pareto efficient allocations in which all players are better off than at their endowment.



Optimality in the Pareto problem

If $(\bar{x}^1, \bar{x}^2)$ is interior, FOC for the Pareto problem imply

$$MRS^1 = MRS^2 \ \& \ \bar{x}^1 + \bar{x}^2 = \omega^1 + \omega^2.$$

Example: Consider an economy with:

- $k = 2$ and $n = 2$;
- $\omega^1 = (3, 1)$ and $U^1(x^1) = x_1^1 x_2^1$;
- $\omega^2 = (1, 3)$ and $U^2(x^2) = x_1^2 x_2^2$.

Find the contract curve (the relationship between x_2^1 and x_1^1).

The First Welfare Theorem

Theorem (The First Welfare Theorem)

If locally non-satiation holds, any competitive equilibrium allocation is Pareto efficient.

Key assumptions:

- Price taking behavior (agents take prices as given)
- No externalities (utility only depends on own consumption)
- Complete markets (any commodity can be traded)

Example: externalities

Consider an economy with:

- $k = 2$ and $n = 2$;
- $\omega^1 = (3, 1)$ and $U^1(x^1) = min\{x_1^1, x_2^1\} - 2x_1^2$;
- $\omega^2 = (1, 3)$ and $U^2(x^2) = x_1^2 x_2^2 - 2x_2^1$.

- What is the equilibrium allocation?
- What are the initial utilities and the equilibrium utilities U^1 and U^2?

The Second Welfare Theorem (Easy version)

Theorem

Consider a Pareto efficient allocation $\bar{x}$. Suppose that a competitive equilibrium $(\hat{p}, \hat{x})$ exists when $\omega = \bar{x}$. If so, $(\hat{p}, \bar{x})$ is a competitive equilibrium.

Intuitively, Second Welfare Theorem says that any Pareto efficient allocation can be achieved as a competitive equilibrium outcome, provided initial endowments are adjusted properly.

In other words, suppose you have a particular allocation in mind you like, and it is Pareto efficient, the Second Welfare Theorem says that, you can "make that allocation happen" by doing the following:

1. redistribute by reallocating resources initially (frequently wealth, but sometimes land, labor, etc.)
2. Let the market run

Comments on the Second Welfare Theorem

The second welfare states that any efficient allocation can be decentralized into a competitive equilibrium if transfers are feasible.

To decentralize an efficient allocation planner would need:

- information about preferences and endowments;
- power to enact transfers.
- to make sure that all prices are known.

This exercise will fail however if there are: externalities, market power, public goods, or incomplete information.

Existence of a competitive equilibrium is proven via fixed point theorems, if preferences are continuous, convex, and locally non-satiated.

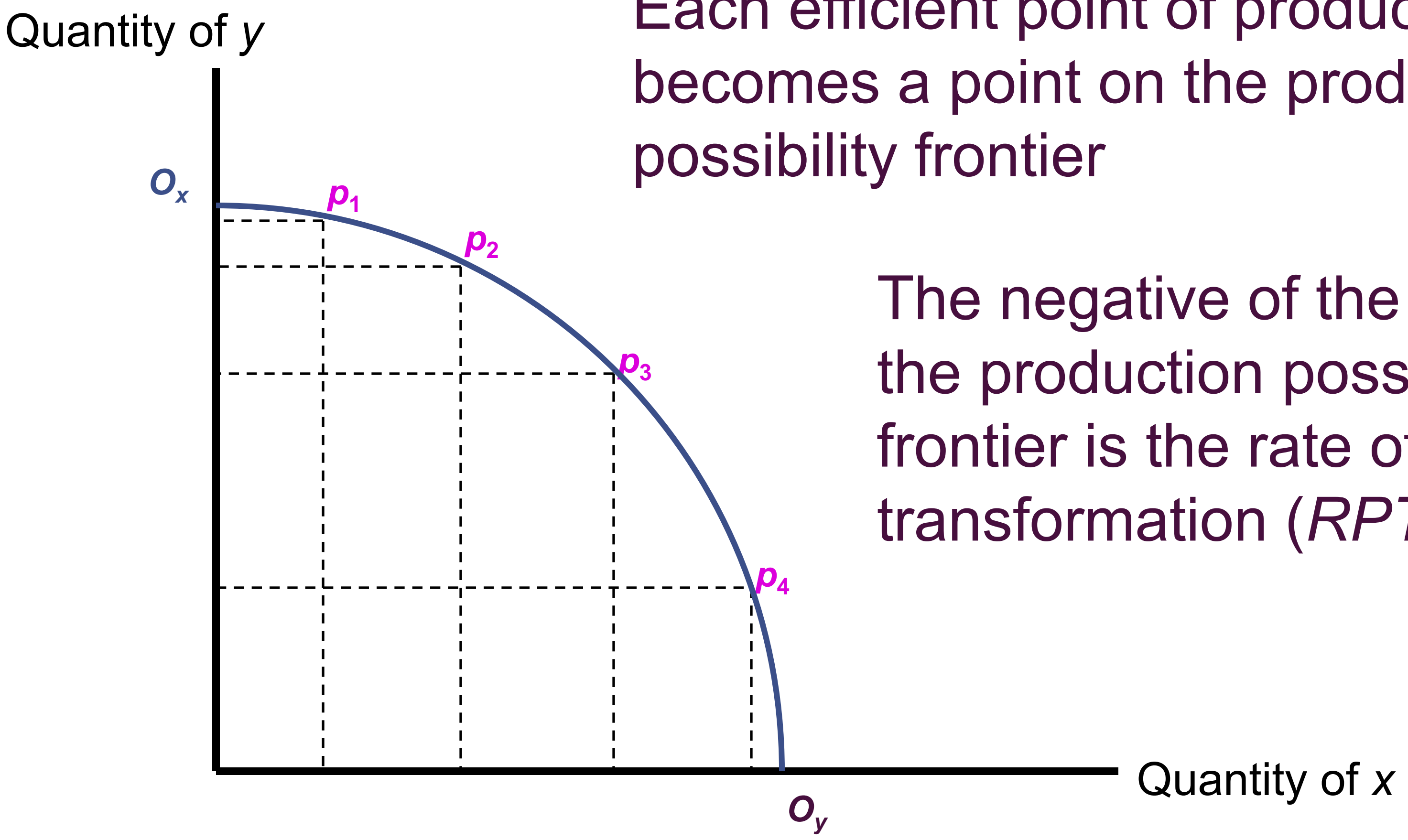
GE is a closed interrelated system, as opposed to partial equilibrium. It is suitable to address problems which relate to the whole economy. Its beauty lies in the ambitious results obtained with few free parameters.

Production Economies

Production Possibility Frontier

- The production possibility frontier shows the alternative outputs of x and y that can be produced with the fixed amounts of capital and labor inputs that are employed efficiently

Production Possibility Frontier



Each efficient point of production becomes a point on the production possibility frontier

The negative of the slope of the production possibility frontier is the rate of product transformation (*RPT*)

Rate of Product Transformation

- The rate of product transformation (*RPT*) between two outputs is the negative of the slope of the production possibility frontier

$$RPT \text{ (of } x \text{ for } y) = -\text{slope of production possibility frontier}$$

$$RPT \text{ (of } x \text{ for } y) = -\frac{dy}{dx} \text{ (along } O_xO_y)$$

Rate of Product Transformation

- The rate of product transformation shows how *x* can be technically traded for *y* while continuing to keep the available productive inputs efficiently employed

Determination of Equilibrium Prices

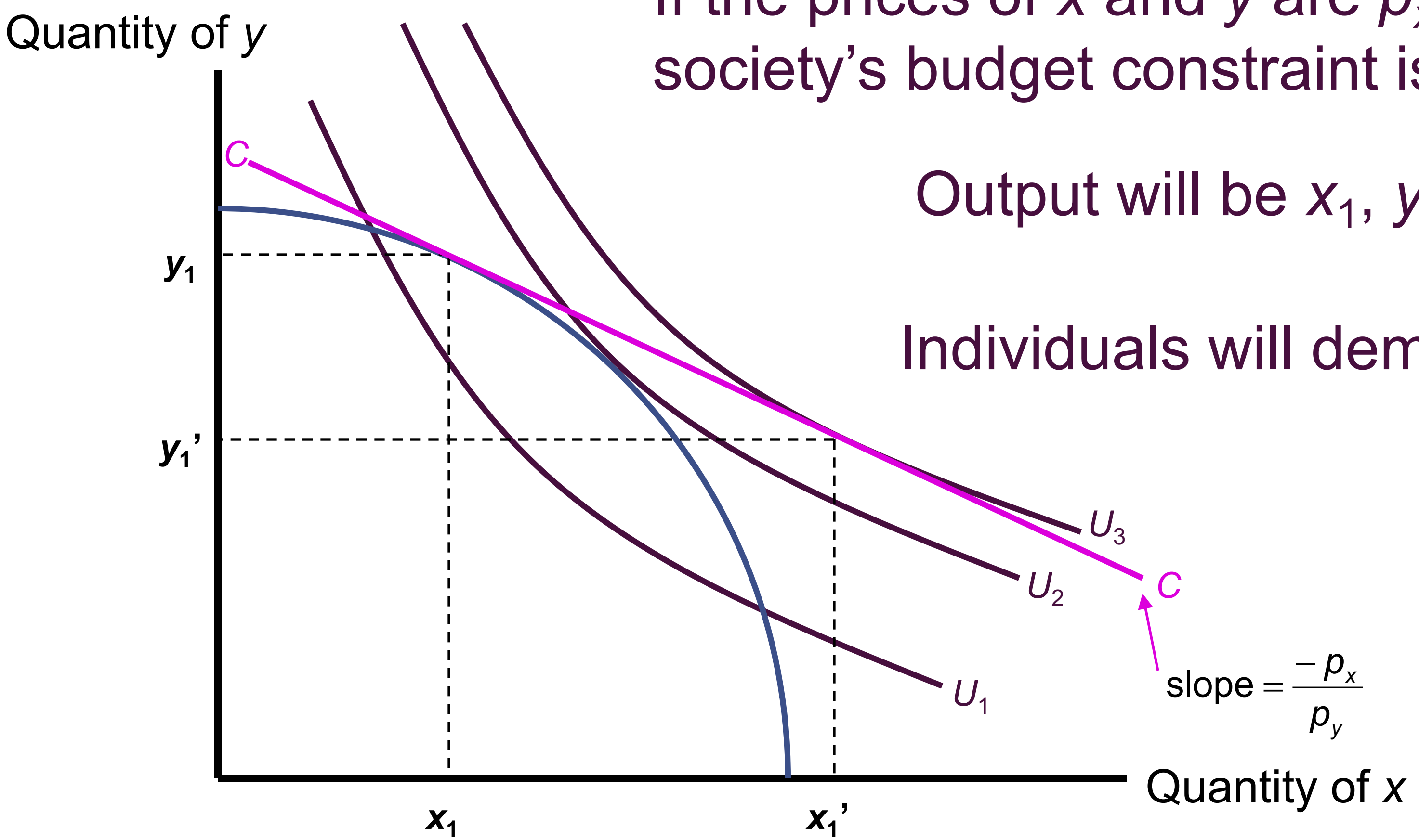
- We can use the production possibility frontier along with a set of indifference curves to show how equilibrium prices are determined
  - the indifference curves represent individuals' preferences for the two goods

Determination of Equilibrium Prices

If the prices of x and y are p_x and p_y, society's budget constraint is C

Output will be x_1, y_1

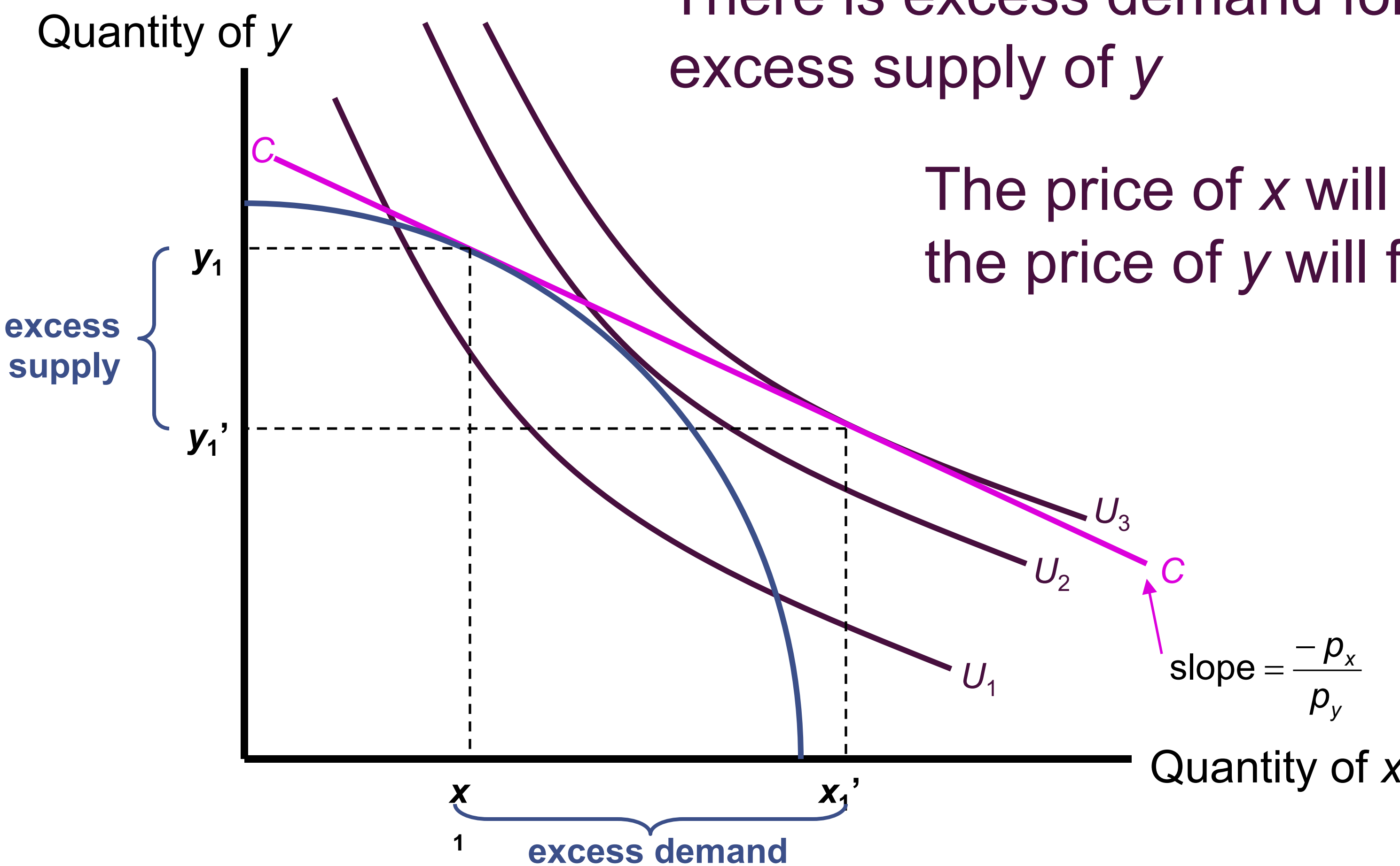
Individuals will demand x_1', y_1'



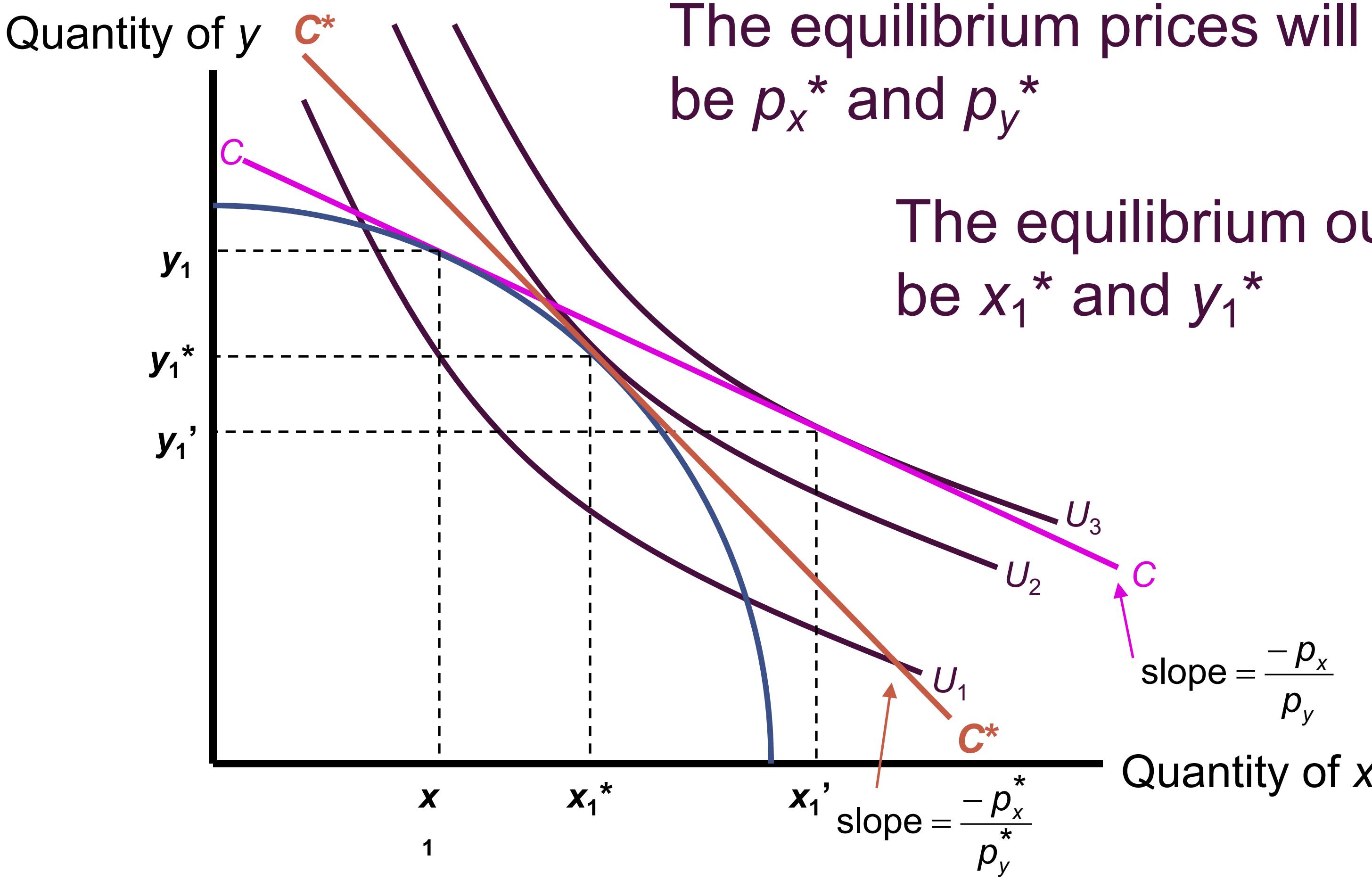
Determination of Equilibrium Prices

There is excess demand for *x* and excess supply of *y*

The price of *x* will rise and the price of *y* will fall



Determination of Equilibrium Prices



The equilibrium prices will be p_x^* and p_y^*

The equilibrium output will be x_1^* and y_1^*

Comparative Statics Analysis

- The equilibrium price ratio will tend to persist until either preferences or production technologies change
- If preferences were to shift toward good x, p_x/p_y would rise and more x and less y would be produced
  – we would move in a clockwise direction along the production possibility frontier

Comparative Statics Analysis

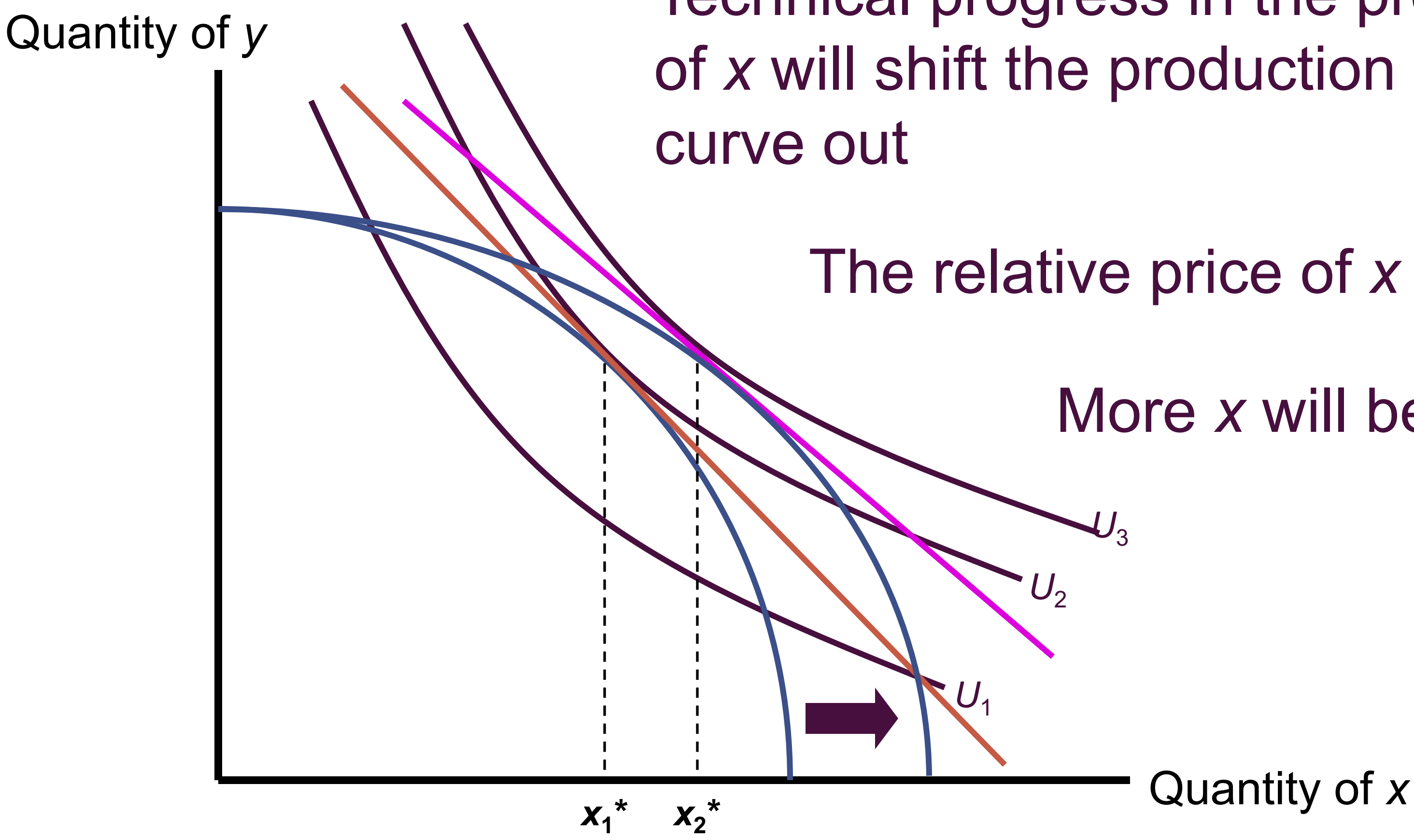
- Technical progress in the production of good x will shift the production possibility curve outward
  - this will lower the relative price of x
  - more x will be consumed
    - if x is a normal good
  - the effect on y is ambiguous

Technical Progress in the Production of x

Technical progress in the production of *x* will shift the production possibility curve out

The relative price of *x* will fall

More *x* will be consumed



GE with production

Consider an economy with:

- n commodities;
- k consumers;
- m firms.

Suppose that each firm maximizes profits.
Let $\pi_i(p)$ be the profits of firm i.
Let α_i^j be consumer j's share of the profits of firm i.
Consumer j's budget constraint becomes

$$px^j \leq p\omega^j + \sum_{i=1}^{m} \alpha_i^j \pi_i(p)$$

All the main results derived for exchange economies extend to production economies with minor modifications.

Example: production I

Consider the following economy:

1. two firms, 1 and 2, produce goods x and y using the same input k.
   Production functions satisfy
   $$x = \sqrt{k_x} \text{ and } y = \sqrt{k_y},$$
   where k_i is the amount of k used for producing i.
2. Two consumers, A and B, have utility functions
   $$U^A = x^A y^A \text{ and } U^B = x^B y^B.$$
   Consumers derive no utility from input k.
3. The total endowment of x and y is zero.
   Consumer A owns firm 1 and 5 units of k.
   Consumer B owns firm 2 and 3 units of k.

Let p_x and p_y denote the price of output, let r denote the price of input k. Normalize $p_y = 1$.
Find the competitive equilibrium prices (p_x, p_y, r), and equilibrium quantities of x^A, y^A, x^B, y^B.

Pareto efficiency I

Extend the above example to general production functions

$$x = f_x(k_x) \text{ and } y = f_y(k_y).$$

Suppose that total input endowment is $k_x + k_y = \gamma$.
By allocating the input k between the two production processes, obtain the production possibility frontier

$$y = T(x).$$

Formally, if $f_x^{-1}(x)$ denotes the inverse of $f_x(k_x)$, we have that

$$T(x) = f_y(\gamma - f_x^{-1}(x)).$$

Pareto efficiency II

The rate of product transformation, RPT, is

$$\frac{dT}{dx} = -\frac{\partial f_y/\partial k}{\partial f_x/\partial k}.$$

Pareto efficient allocations can now be found by choosing (x^A, y^A, x^B, y^B) so to maximize

$$U^A(x^A, y^A) \text{ subject to}$$
$$U^B(x^B, y^B) = \bar{u} \text{ and } y^A + y^B = T(x^A + x^B).$$

FOC immediately yield that

$$MRS^A = MRS^B = RPT$$

Concluding comments

GE is a parsimonious and flexible theory, which relies on price taking, individual maximization, and market clearing to derive implications about market outcomes.

Conclusions on welfare further require the absence of externalities and private information, and the completeness of markets.