PS4 Answer Sheet (For Reference Only)

Question 1 (Externality)

Consider a plant that manufactures dynamite d and a nearby farm producing tomatoes t. The dynamite's production cost is:

$$TC_d(d, x) = (\frac{1}{2})d^2 + (x - 2)^2$$

where d is the amount of dynamite produced and x is the intensity of use of nitrogen in the production process. The side product associated with use of nitrogen is NH_3- a fertilizer that is released to the air. Such fertilizer promotes growth of tomatoes, making the production cheaper. In particular, the higher the intensity x, the lower is the farmer's cost:

$$TC_t(t, x) = (\frac{1}{2})t^2 + 2t - xt$$

The prices of tomatoes and dynamite are: $P_d = P_t = 1$

a. Is the market interaction associated with a positive or negative externality?

b. Find the level of production of dynamite d and intensity x that maximizes the profit of the dynamite manufacturer. What is the maximum level of profit?

c. What is the marginal benefit from using x in optimum? Give one number and show it on the graph with x is on the horizontal axis.

d. Given the intensity x from part b) , find the optimal level of production of tomatoes t, and the profit of the farmer.

e. Find the joint profit of the dynamite manufacturer and the farmer.

f. Find the pareto efficient level of production of d, t and use of nitrogen x. Compare these values to the ones obtained in parts b) and d).

g. Is the marginal benefit from using x in f) positive, zero, or negative? Why?

a. With side product as a cost-cutting fertilizer, the market interaction is associated with a positive externality.

b. As for the dynamite manufacturer, we have:

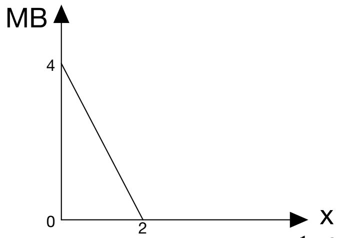
$$\begin{aligned}\max_{d,x} \Pi^D(d, x) &= P_d d - (\frac{1}{2}d^2 + (x - 2)^2) \\ &= -\frac{1}{2}(d - P_d)^2 + \frac{1}{2}P_d^2 + (x - 2)^2 \\ &= -\frac{1}{2}(d - 1)^2 + (x - 2)^2 + \frac{1}{2}\end{aligned}$$

Then the level of profit for the dynamite manufacturer is maximized when $d = P_d = 1,\ x = 2$

And $\Pi^D_{max} = \frac{1}{2}$

c. Marginal benefit is $\frac{\partial \Pi^D}{\partial x} = -2(x - 2) = -2x + 4$

When $x = 2$, the marginal benefit is then $-2x + 4\ |_{x=2} = 0$



d. Given $x = 2$, we now maximize $\Pi^T(t, x = 2) = -\frac{1}{2}t^2 + t = -\frac{1}{2}(t-1)^2 + \frac{1}{2}$

The optimal level of production of tomatoes is 1 and the profit is $\Pi^T_{max} = \frac{1}{2}$

e.

$$\begin{aligned}\Pi^S(d, t, x) &= \Pi^D(d, x) + \Pi^T(t, x) \\ &= [d - (\frac{1}{2}d^2 + (x-2)^2)] + [t - (\frac{1}{2}t^2 + 2t - xt)]\end{aligned}$$

f. Calculating the first order conditions, we have:

$$\begin{cases} \frac{\partial \Pi}{\partial d} = -d + 1 = 0 \\ \frac{\partial \Pi}{\partial t} = -t - 1 + x = 0 \\ \frac{\partial \Pi}{\partial x} = t - 2(x-2) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} d' & = & 1 \\ t' & = & 2 \\ x' & = & 3 \end{cases}$$

However, when we optimize the profits separately as in b) and d), we have:

$$\begin{cases} d & = & 1 \\ t & = & 1 \\ x & = & 2 \end{cases}$$

And $d = d',\ t < t',\ x < x'$

g. Marginal benefit is $\frac{\partial \Pi^S}{\partial x} = t - 2(x-2)$

Hence marginal benefit is still 0 when $t = 2$ and $x = 3$

Question 2: Taxing Pollution

(a)

The marginal benefit (MB) is : $MB = \frac{d\pi}{dQ} = 10 - 2Q$

The marginal damage (MD) is : $MD = \frac{dD}{dQ} = 2Q + 2$

MB Curve ($MB = 10 - 2Q$):

- Y-intercept (when Q=0): $MB = 10$
- X-intercept (when MB=0): $10 - 2Q = 0 \Rightarrow 2Q = 10 \Rightarrow Q = 5$
- Slope: -2

MD Curve ($MD = 2Q + 2$):

- Y-intercept (when Q=0): $MD = 2$
- Slope: 2

(b)

If Buchanan Industries does not have to pay for the damage, it will maximize its own profit by producing where its marginal benefit from polluting is zero.

- **How much would it produce?** Set $MB = 0$: $10 - 2Q = 0$ $2Q = 10$ $Q = 5$ tons
- **profit** Substitute $Q = 5$ into the profit formula: $\pi = 10Q - Q^2 = 10(5) - (5)^2 = 50 - 25 = \$25$
- **total damages** Substitute $Q = 5$ into the total damage formula: $D = Q^2 + 2Q = (5)_2 + 2(5) = 25 + 10 = \$35$
- **net benefits** Net Benefits = Profit - Damages $= 25 - 35 = -\$10$

(c)

The socially efficient level of Q is where the marginal benefit of pollution equals the marginal damage of pollution ($MB = MD$).

- **Socially efficient level of Q:** $10 - 2Q = 2Q + 2$ $8 = 4Q$ $Q = 2$ tons
- **profit** Substitute $Q = 2$ into the profit formula: $\pi = 10Q - Q^2 = 10(2) - (2)^2 = 20 - 4 = \$16$
- **total damages** Substitute $Q = 2$ into the total damage formula: $D = Q^2 + 2Q = (2)^2 + 2(2) = 4 + 4 = \$8$
- **net benefits** Net Benefits = Profit - Damages $= 16 - 8 = \$8$

(d)

Deadweight loss (DWL) occurs because the firm produces at $Q = 5$ instead of the socially efficient $Q = 2$. The DWL is the area between the MB and MD curves from the socially efficient quantity to the quantity produced by the firm.

On the graph, the deadweight loss is the triangular area bounded by the MB curve, the MD curve, and the vertical lines at Q = 2 and Q = 5.

The DWL is the area of the triangle formed by the points $(2, 6)$, $(5, 0)$, and $(5, 12)$. DWL $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 3 \times 12 = \$18$.

(e)

If production is restricted to $Q = 1$, this is below the socially efficient level of $Q = 2$.

On the graph, this deadweight loss is the triangular area bounded by the MB curve, the MD curve, and the vertical lines at $Q = 1$ and $Q = 2$. This triangle has vertices at $(1, 4)$, $(1, 8)$, and $(2, 6)$.

DWL $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 1 = \$2$.

(f)

- **Who benefits** Reducing Q from $Q = 5$ to $Q = 2$ means less pollution. The **neighbors/society** benefit from reduced damages. Total damages decrease from $D(5) = \$35$ to $D(2) = \$8$, a reduction of $35 - 8 = \$27$.

- **Who bears the costs** Buchanan Industries bears the cost as its profit decreases. Profit decreases from $\pi(5) = \$25$ to $\pi(2) = \$16$, a reduction of $25 - 16 = \$9$.

- **Pareto improving or potential Pareto improvement** This change is **not Pareto improving** because Buchanan Industries is worse off.

  However, it is a **potential Pareto improvement** if the gainers (neighbors/society) could compensate the losers (Buchanan Industries). For example, if society pays Buchanan Industries \$9 to reduce pollution, society is still $\$27 - \$9 = \$18$ better off.

Question 3 (Monopoly)

PineApple Company decides how many packets of the new operating system it is going to sell on the market. The research (fixed) costs associated with the development of the new system amounts to $F = \$1000$. The variable costs of the packet is negligible, variable cost = 0. PineApple's inverse demand for the new operating system is given by

$$p(y) = 100 - y$$

a. What would consumer and producer surplus be if PineApple was a price taker (a competitive firm)? Give exact numbers and show corresponding areas on a graph.

b. Assume that PineApple is a monopoly, and it cannot discriminate among its customers. Find geometrically and analytically the level of sales that maximizes profit, the market price, and the maximum profit.

c. Is outcome in b) *pareto efficient*? If not, find the deadweight loss (DWL) geometrically and analytically.

d. Find consumer's and producer's surpluses (CS and PS) geometrically and analytically, with a monopolistic firm.

e. Find the elasticity of the demand at the optimal level of production. Is the monopolistic firm operating on elastic or inelastic part of the demand?

f. Suppose that PineApple can perfectly price discriminate among the consumers. What is PineApple's profit if in that case? Is the allocation *pareto efficient*? What is consumer surplus?

Let's get more realistic and assume that PineApple can charge different prices on two segments of the market: individual buyers and firms. The demands on two segments are:

$$y^I(p^I) = 50 - \frac{4}{5}p^I$$

$$y^F(p^F) = 50 - \frac{1}{5}p^F$$

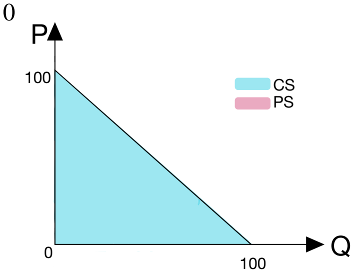
g. Find the level of sales, price, profit and elasticity of demand for each segment of the market.

a. As a price taker, PineApple chooses to produce at $p = MC = 0$

Substitute $p = 0$ into the inverse demand function $p(y) = 100 - y$, we have:

$$y = 100 - p = 100$$

$$CS = \int_0^{100} (p(q) - p)dq = 5000, \ PS = 0$$



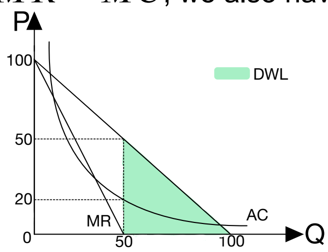
b. As a monopoly, PineApple chooses to:

$$\max_y \Pi = p(y)y - F$$

$$F.O.C.\ \frac{\partial \Pi}{\partial y} = p(y) + yp'(y) = 100 - 2y = 0$$

$$\Rightarrow y^* = 50,\ p^*(y) = 50,\ \Pi^* = 1500$$

or else, $MR = 100 - 2y$, $MC = 0$, let $MR = MC$, we also have $y^* = 50$



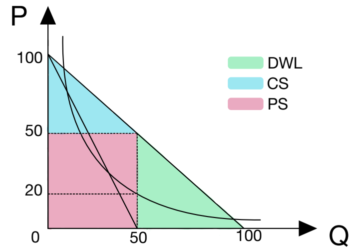
c. No, monopoly **alone** produces no *pareto efficiency*

$$DWL = \int_{q^*}^{100} (p(q) - p)dq = 1250$$

d.

$$CS = \int_0^{q^*} (p(q) - p^*)dq = 1250$$

$$PS = \int_0^{q^*} (p^* - p)dq = 2500$$



e.

$$e_{Q,P} = \frac{\partial Q/Q}{\partial P/P} = -1$$

Then the monopolistic firm operates on the **unit elastic** part of the demand. Nevertheless, we generally refer to this case as neither "elastic" nor "inelastic".

f. Under this perfect price discrimination situation,

$$profit = \int_0^{100} (p(q) - p)dq - F = 4000$$

This allocation is *pareto efficient*, with $CS = 0$

g. In the individual segment of the market:

$$y^I(p^I) = 50 - \frac{4}{5}p^I$$

$$\Rightarrow p^I = \frac{125}{2} - \frac{5}{4}y^I$$

$$\Rightarrow MR^I = \frac{125}{2} - \frac{5}{2}y^I$$

To maximize monopoly profit in this segment, let:

$$MR^I = MC = 0$$

$$\Rightarrow y^I = 25,\ p^I = \frac{125}{4}$$

$$e^I = \frac{\partial Q/Q}{\partial P/P} = -\frac{4}{5} * \frac{\frac{125}{4}}{25} = -1$$

Similarly on the other segment we have:

$$y^F = 25,\ p^F = 125$$

$$e^F = \frac{\partial Q/Q}{\partial P/P} = -\frac{1}{5} * \frac{125}{25} = -1$$

Then for the whole market:

$$profit = y^I * (p * I) + y^F * (p * F) - F = 25 * \frac{125}{4} + 25 * 125 - 1000 = 2906.25$$