Intermediate Macroeconomics: Problem Set 1

Due Tuesday, March 4 in class

1. Nominal and Real GDP (20 points)

There are only two goods in the economy – apples and iPads. In Year 1, 110 iPads are sold at $420 each and 9,500 apples are sold at $1 each. In Year 2, 160 iPads are sold at $520 each and 8,000 apples are sold at $1.25 each.

Questions:

a. Compute the nominal GDP for both years.

b. Compute the real GDP for Year 2 using Year 1 as the base year. What is the growth rate of real GDP?

c. Suppose in Year 2 iPads are upgraded, being twice as productive (faster, more features, etc.) as the Year 1 version. How would this change your answers to parts (a) and (b), and why?

2. National Accounting (30 points)

Consider an economy with a coal producer, a steel producer, and some consumers who want to buy steel (as final goods). In a given year:

- The coal producer produces 18 tons of coal and sells it for $6 per ton. The coal producer pays $60 in wages.
- The steel producer uses 28 tons of coal as input for steel production, purchased at the market price. Of this, 18 tons come from the domestic producer and 10 tons are imported.
- The steel producer produces 12 tons of steel and sells it for $22 per ton. Domestic consumers buy 9 tons of steel, and 3 tons are exported. The steel producer pays $50 in wages.
- All profits are distributed to domestic consumers.

Questions:

a. Calculate GDP using all three approaches discussed in class, and show that the results are the same.

b. Gross National Product (GNP) is defined as the total value of all final products and services produced in a given period by the means of production owned by a country's entities. Calculate the GNP of this economy.

c. Suppose that the coal producer is owned by a foreign nation, and the coal producer's profits are distributed abroad. Recompute GNP and GDP in this case. Briefly explain the changes compared to part (b).

3. Calculating Growth Rates (20 points)

For this exercise, visit the National Bureau of Statistics and FRED Economics Data's websites to answer the following questions.

Questions:

a. Calculate the average annual growth rate of (nominal) GDP per capita between 1980 and 2020 for China and U.S.

b. Assume that U.S. GDP per capita grows at 2% over the next 50 years. What annual growth rate will bring China's GDP per capita equal to the United States by 2075?

4. A Two-Period Fertility Model with Transfers (30 Points)

(Inspired by Ward and Butz, 1980) Assume a household lives for two periods, $t = 1, 2$. In each period t, it consumes $c_t \geq 0$ out of a fixed income y_t. It also decides how many children $n_t \geq 0$ to have, paying a per-child cost $p_t > 0$. In addition, parents receive a transfer $w \geq 0$ in period 2 for each child born in period 1. The utility from consumption and having children is represented by $u(\cdot)$ and $v(\cdot)$. We let $\beta \in (0, 1)$ denote the household's discount factor.

The household's objective is to maximize

$$U = u(c_1) + \beta\, u(c_2) + v\big(n_1 + n_2\big),$$

subject to the budget constraints

$$c_1 + p_1 n_1 = y_1 \quad \text{and} \quad c_2 + p_2 n_2 = y_2 + w\, n_1.$$

Questions:

a. Write down the Lagrangian for the household's problem.

b. Write down the first-order conditions for the Lagrangian and show that, in equilibrium, we have

$$u'(c_1)\, p_1 \;=\; \beta\, u'(c_2)\,\big(w + p_2\big).$$

Explain the intuition behind this equation.

From part (c) onward, assume $u(c) = \ln c$ and $v(n) = n$.

c. Solve for the optimal consumption and number of children $\{c_1^*,\, c_2^*,\, n_1^*,\, n_2^*\}$ as functions of the model parameters $(y_1,\, y_2,\, p_1,\, p_2,\, w,\, \beta)$.

d. Analyze how total fertility $n_1 + n_2$ changes if current income y_1, future income y_2, or the per-child transfer w increases.

Extra Credit: Growth Rate Approximations (5 points)

A function $F(x)$ can be approximated by the Taylor series expansion around any values a to the n-th order. That is,

$$F(x) = \sum_{n=0}^{\infty} \frac{\partial^n F(a)/\partial x^n}{n!} = F(a) + F'(a)(x-a) + \frac{F''(a)}{2}(x-a)^2 + \dots$$

When $a = 0$, the first-order Taylor approximation of $F(x)$ is

$$F(x) \approx F(0) + xF'(0)$$

a. Using first-order Taylor approximations, show that the growth rate of a variable X_t, defined as

$$g = \frac{X_{t+1} - X_t}{X_t}$$

can be approximated by $g \approx \ln X_{t+1} - \ln X_t$.