# Intermediate Macroeconomics:
# Job Search, Employment and Unemployment

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## Instructions

In this project, you will construct and solve a McCall job search model and a Lake model, and study their implications on labor market aggregates, taxation and welfare. Hopefully, this project will help you understand how macroeconomic research is conducted, and inspire you to start your own research topic in the future!

## Part I. The McCall Job Search Model

### 0. Setting up the Model

Since first introduced in 1970, the McCall search model reshaped the way economists think about labor markets. Using a simple framework, this model provides important implications, such as workers may voluntarily stay unemployed in order to search for acceptable jobs in the future. In this exercise, we will consider a version of McCall model with **stochastic job offers** and **possibility of job termination/job destruction**.

The worker is infinitely lived, and could be in one of the two states: employed or unemployed. He wants to maximize the total discounted utility of his income, given by

$$\sum_{t=0}^{\infty} \beta^t u(y_t)$$

Where $u(\cdot)$ is a utility function that satisfies the Inada conditions. In particular, $u'(x) > 0$ and $u''(x) < 0$ for all $x > 0$. The depreciation rate $\beta$ lies in $(0, 1)$. The worker's income $y_t$ is equal to

- Wage $w_t$ when he is employed
- Unemployment benefit $b$ when he is unemployed.

At the beginning of each period, the worker faces the following decisions:

*This project is inspired by the lecture series on quantitative economics modelling by Thomas J. Sargent and John Stachurski. All errors are mine. Please contact me at lunl@pku.edu.cn for any queries, corrections or comments.

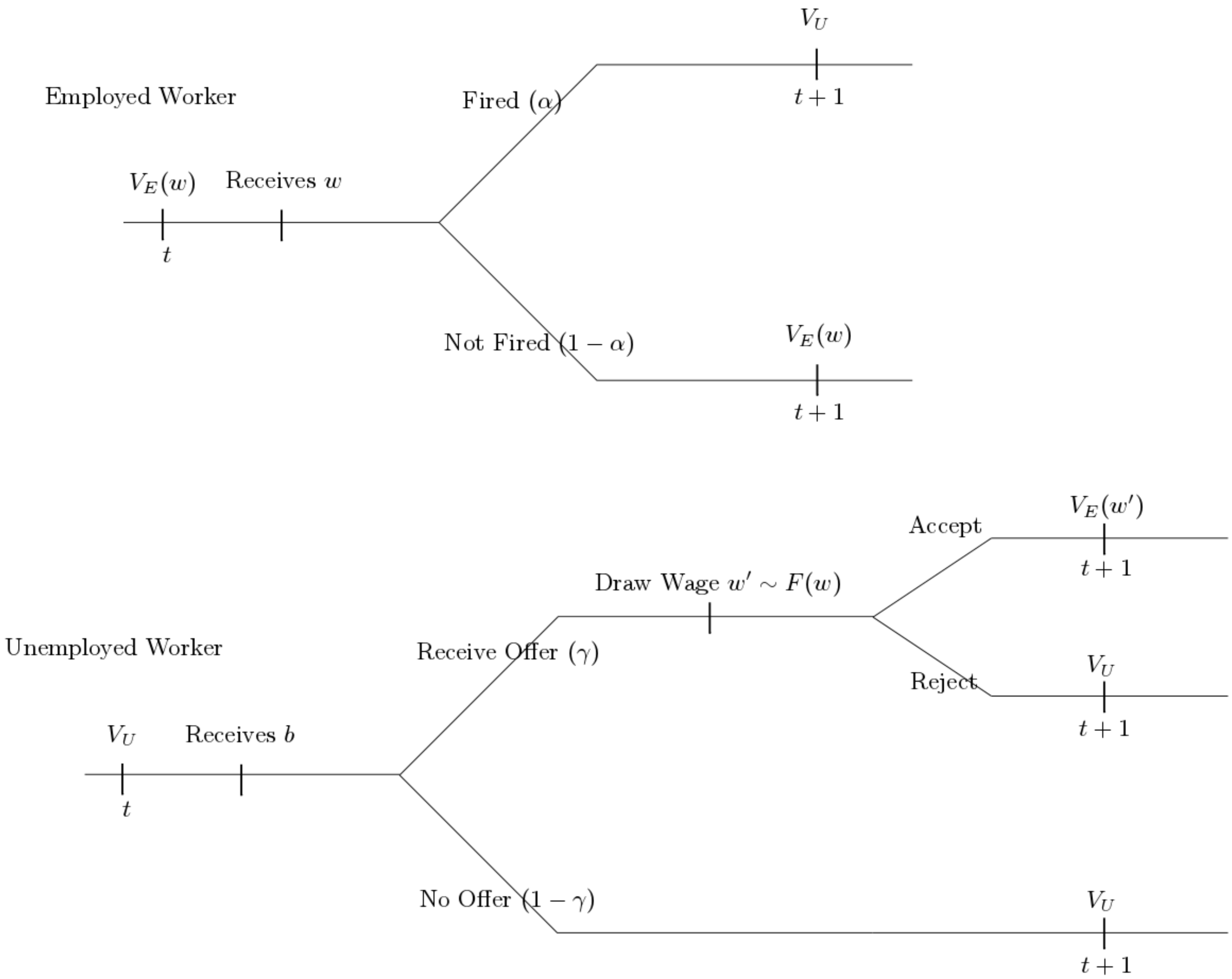


**Figure 1:** Timeline for the McCall Model

- If he is employed, he receives wage $w$ and derive utility $u(w)$. There is a constant probability $\alpha$ of being fired at the end of period.
- If he is unemployed, he receives the unemployment benefit $b$. In addition, he receives an job offer next period with probability $\gamma$. If he does not receive any offers, he will enter the next period remaining unemployed. However, if he receives an offer, the wage $w'$ is drawn from a known distribution $F(w)$. He then chooses one of the following:
  - Accept: he enters the next period employed with wage $w'$.
  - Reject: he enters next period unemployed

We do not allow recall: a worker cannot accept a previously rejected offer. Also, there is no job search while employed.

The timeline of this model is illustrated by Figure 1.

### 1. Write the Bellman Equations

Let $V_E$ and $V_U$ denote the value functions of employed and unemployed worker, respectively. Use the timeline in Figure 1 to write the **Bellman Equations** for $V_E$ and $V_U$.

**Hint:** $V_U$ must be defined before wage offer $w'$ is realized, because it's possible that unemployed worker gets no offers in the current period. In other words, the unemployed value function $V_U$ should **not** be a function of $w'$, and should take the following form:

$$V_U = \cdots + \cdots + \beta\gamma\mathbb{E}_{w'}[\max\{V_U, V_E(w')\}]$$

### 2. Solve the Bellman Equations

The Bellman Equations are apparently non-linear, and it is difficult to solve them analytically. Follow the steps outlined below, and write Matlab scripts to approximate the solution using value function iterations.

#### Step 1: Discretize the State Space

A major challenge when solving Bellman Equations is to compute the **expectations**. To get around that challenge, we will first transform the state space so that wage becomes a "discrete variable".

Assume wage offers $w$ satisfies $w \in [w_{min}, w_{max}]$. Take $n$ evenly spaced points in the interval, and define

$$w_0 = w_{min}, w_1 = w_{min} + \frac{w_{max} - w_{min}}{n}, w_2 = w_{min} + 2 * \frac{w_{max} - w_{min}}{n} \ldots, w_n = w_{max}$$

We will assume wage could only take values in this finite set $W = \{w_0, w_1, \ldots, w_n\}$

The next step is to assign some probability distribution $\{p_1, p_2, \ldots, p_n\}$ over the space $W$. For simplicity, you can assume the distribution is uniform:

$$p_i = Pr(w = w_i) = \frac{1}{n+1} \;\; \forall i$$

Alternatively, you could also pick a probability distribution of your choice. For example, you can make the agent more likely to draw close-to-average offers, than extremely high or low offers. (Illustrated by Figure 2).

#### Step 2: Define the Utility Function

For tractability, we will assume utility takes the CRRA form, with parameter $\sigma$:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

In your scripts, define a Matlab function "utility" such that for any given value of consumption $c$ and parameter $\sigma$, it generates the utility $u(c, \sigma)$ according to the formula given above.

#### Step 3: Set Parameter Values

We will pick the following parameter values:

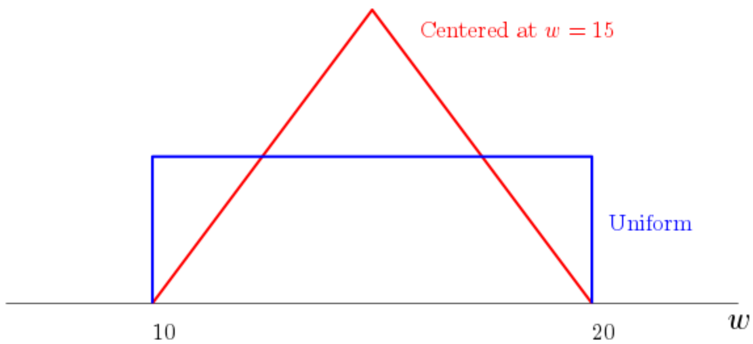


**Figure 2:** Example of Alternative Probability Distribution

- Job separation rate $\alpha = 0.2$
- Discount rate $\beta = 0.98$.
- Job offer rate $\gamma = 0.7$
- Utility parameter $\sigma = 2$
- Unemployment benefit $b = 6$
- Minimum wage $w_{min} = 10$
- Maximum wage $w_{max} = 40$
- Number of discrete points $n = 1000$

**Step 4: Value Function Iterations**

Make guesses for $V_0^E(w)$ and $V_0^U$ (say, constant functions at 0). Repeat the following procedures until the last step, convergence criterion, is satisfied.

1. For all $w \in W$ and given $V_j^E$ and $V_j^U$, compute from Bellman Equations the new values $V_{j+1}^E(w)$ and $V_{j+1}^U$:

$$\begin{aligned} V_{j+1}^E(w) &= g(V_j^E(w), V_j^U) \\ V_{j+1}^U &= k(V_j^E(w), V_j^U) \end{aligned}$$

   Where $g()$ and $k()$ are relationships according to the Bellman Equations solved in question 1.

2. Use some measure to check if both $V_{j+1}^E(w)$ and $V_{j+1}^U$ are sufficiently close to $V_j^E(w)$ and $V_j^U$. For example, you can use

$$\rho_j = \sum_{w' \in W} (V_{j+1}^E(w') - V_j^E(w'))^2 + (V_{j+1}^U - V_j^U)^2$$

3. If $\rho > \tau$, where $\tau$ is a tolerance level of your choice (say $10^{-9}$), run (1) and (2) again. If $\rho < \tau$, then stop the iteration, and report $V_E = V_{j+1}^E$ and $V_U = V_{j+1}^U$ as the final result.

### Step 5: Plot a Figure, and Find the Reservation Wage

Plot $V_E$ and $V_U$ for all values of $w$. Your result should look like Figure 3 on the following page. As we learned from class, the intersection between $V_E$ and $V_U$ determines the reservation wage $w_R$. From your output, find the smallest level of $w$ that makes $V_E(w) > V_U$ – that will be your approximation for $w_R$!

**Congratulations! You have successfully completed your first value function iteration algorithm.**

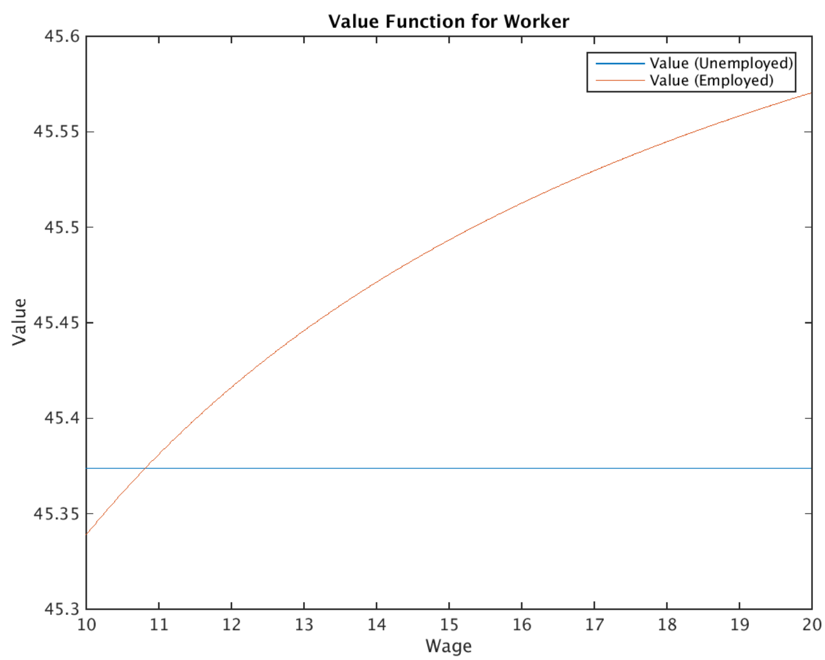


**Figure 3:** Sample Output For Question 2, Part I

## 3. Comparative Statics of Reservation Wage

Reservation wage $w_R$ is defined such that the agent chooses to work if and only if his offer satisfies $w > w_R$. When the job itself becomes "less desirable" in some dimensions, the reservation wage will change in the equilibrium. Alternatively, if the unemployment benefit becomes greater, the worker will also ask for a different reservation wage.

1. Repeat last question with different values of job destruction rate $\alpha$ (say from 0.05 to 0.5, with increments of 0.01). What happens to reservation wage when losing a job becomes easier ($\alpha$ higher)? Show a graph and explain your result in words.

2. Repeat last question with different values of unemployment benefit $b$ (say from $[2, 15]$, with increments of 0.1). What happens to reservation wage when unemployment benefit is higher? Show a graph and explain your result in words.

# Part II: A Lake Model of Employment and Unemployment

The McCall model focuses on the decision rule of a single worker, but it could not explain well the dynamics of aggregate variables like employent, unemployment, and labor force participation. In this section, we will solve a Lake model of unemployment, where "lakes" in the model are pools of employed and unemployed workers. We can then study the flows between "lakes", caused by labor market churning (firing and hiring, entry and exit from labor force), and decide the optimal level of unemployment benefit in this economy.

## 0. Setting up the Model

The economy is inhabited by a large number of identical workers. They live forever, and spend their lives moving between unemployment and employment. The rate of transition between employment and unemployment are governed by the following parameters.

- Job finding rate for currently unemployed workers $\lambda$.
- Job separation rate for currently employed workers $\alpha$.
- Entry rate into the labor force $r$.
- Exit rate from the labor force $d$.

We also define the following aggregate variables:

- $E_t$: the total number of employed worker at date $t$
- $U_t$: the toal number of unemployed worker at date $t$
- $N_t$: the total number of workers in the labor force at $t$.

By definition, the employment and unemployment rate are given by:

- Employment rate:
$$e_t = \frac{E_t}{N_t}$$
- Unemployment rate:
$$u_t = \frac{U_t}{N_t}$$

The law of motion for employed workers will be:

$$E_{t+1} = (1-d)(1-\alpha)E_t + (1-d)\lambda U_t$$

And for unemployed worker:

$$U_{t+1} = (1-d)\alpha E_t + (1-d)(1-\lambda)U_t + r(E_t + U_t)$$

We can write the two equations into a linear system, by defining

$$X_t = \begin{bmatrix} E_t \\ U_t \end{bmatrix} \tag{1}$$

And

$$X_{t+1} = AX_t$$

Where

$$A := \begin{pmatrix} (1-d)(1-\alpha) & (1-d)\lambda \\ (1-d)\alpha + r & (1-d)(1-\lambda) + r \end{pmatrix} \tag{2}$$

This tells us how employment and unemployment evolves over time. Also note that the total stock of workers $N_t = E_t + U_t$ evolves as

$$N_{t+1} = (1 + r - d)N_t$$

## 1. Transition Paths of Employment and Unemployment

For our empirical part, assume $\lambda = 0.3$ and $\alpha = 0.2$. We can also match the entry and exit rate $r$ and $d$ with monthly birth and death rates in the U.S., respectively. This gives us $r = 0.0124$ and $d = 0.00822$. Set initial values for aggregate employment $E_0$ and unemployment $U_0$ (say, $E_0 = 138, U_0 = 12$), then use Matlab to plot the transition paths of $E_t$, $U_t$ and $N_t$ over time. Your answer should look like the graph below:

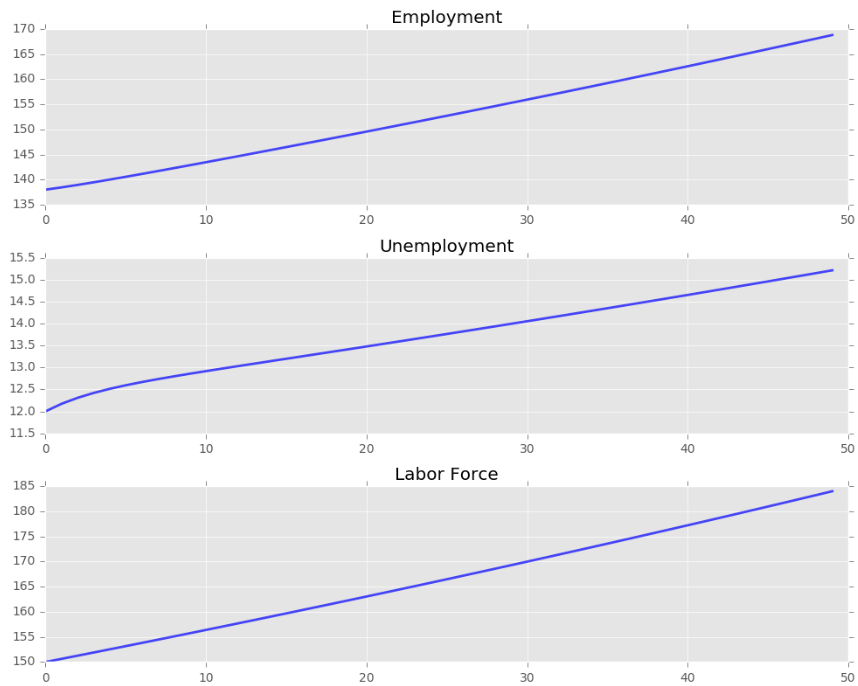


**Figure 4:** Sample Output for Question 1, Part II

It looks like aggregate employment and unemployment have constant growth rates. What about **employment and unemployment rates?**

## 2. Transition Paths of Employment and Unemployment Rates

To find the dynamics of employment and unemployment **rates**, normalize both sides of $X_{t+1} = AX_t$ by labor force $N_{t+1}$:

$$x_{t+1} = \frac{1}{1+r-d} A x_t$$

Where $x_t = \begin{bmatrix} e_t \\ u_t \end{bmatrix}$ is the employment and unemployment rate of time $t$. Use the same initial aggregate employment and unemployment vector $X_0$, generate a Matlab graph to show that as $t \to \infty$, $e_t$ and $u_t$ converge to some steady state level $\bar{e}$ and $\bar{u}$. Your answer should look like the Figure 5 below.

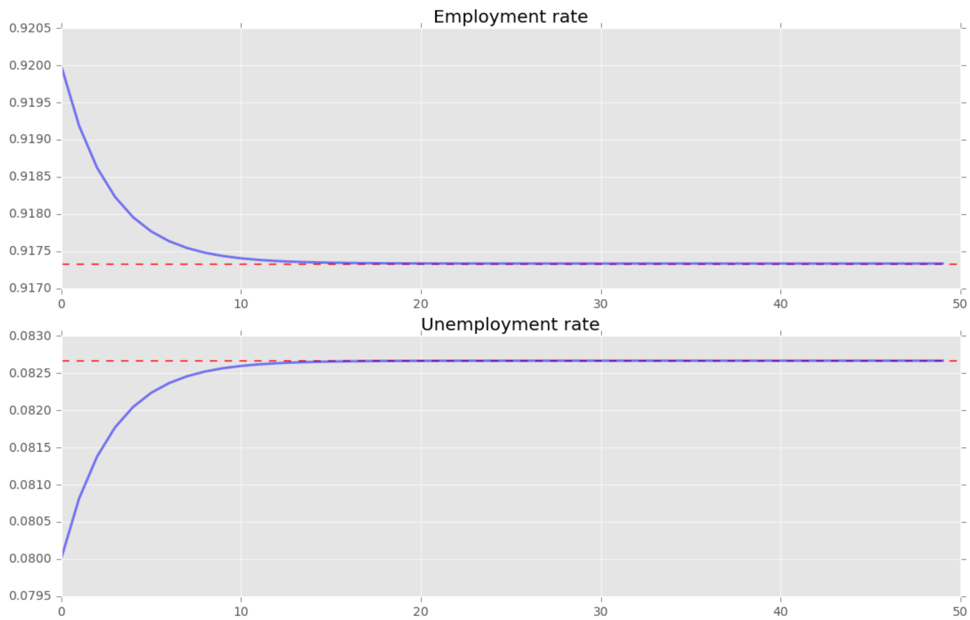


**Figure 5:** Sample Output for Question 2, Part II

## 3. Compute the Steady State Employment Rates

Write a Matlab function to compute the steaday state employment and unemployment rates $\bar{e}$ and $\bar{u}$. You may use the following algorithm to approximate the steady state:

1. Use $x_0$ from last part as the initial value. Also set a tolerance level, say $tol = 1 \times 10^{-9}$.
2. For each $t \geq 0$, compute

$$x_{t+1} = \frac{1}{1+r-d} A x_t$$

And measure the total squared difference between $x_{t+1}$ and $x_t$ by

$$\rho = |x_{t+1} - x_t|^2 = (e_{t+1} - e_t)^2 + (u_{t+1} - u_t)^2$$

3. Repeat step 2 until the convergence condition $\rho < tol$ is satisfied. Report $x_{t+1}$ as the steady state employment and unemployment rates.

**Congratulations! You have numerically solved the steady state employment and unemployment rates, from a first order difference equation in two variables.**

### 4. Impulse Response of a Baby Boom

Suppose the employment and unemployment rates $x_t$ stays at the steady state level $\bar{x}$. However, an exogenous event makes the birth rate $r$ increase from 0.0124 to 0.0248 for 10 periods, and return to 0.0124 afterwards. Assuming all other parameters stay the same during this event. Use Matlab to plot the transition paths of employment and unemployment rates after the shock.

Is it realistic to assume that all other parameters will stay the same during a "baby boom"? In your opinion, which parameter would change as well?

## Part III: Linking McCall Search Model to Lake Model

### Setting up

Suppose that all workers inside a Lake model behave according to the McCall search model, where he takes a job offer if and only if the wage is higher than reservation wage. We can then connect the parameters between two models in a realistic way, and study the welfare implications of a particular fiscal policy.

Recall in Lake model, the probability of leaving unemployment is $\lambda$. Under the assumption that all agents behave according to the McCall model, $\lambda$ satisfies

$$\lambda = \gamma \mathbb{P}(w_t \geq \bar{w}) = \gamma \sum_{w' \geq w_R} p(w')$$

Where $\gamma \mathbb{P}(w_t \geq \bar{w})$ is the probability of getting an offer better or equal to the reservation wage. In other words, we endogenized the job finding rate $\lambda$, and no longer assume $\lambda = 0.3$ as in the previous section.

We also assume there is a government that charges a lumpsum tax $\tau$ to each agent in the labor force, including unemployed workers. The tax revenue is used to finance unemployment benefit. To attain a balanced fovernment budget, the following relationship needs to hold:

$$\tau N = Ub$$
$$\tau = ub$$

The post-tax income for an employed worker with wage $w$ becomes $w - \tau$, and for an unemployed worker it becomes $b - \tau$. Define welfare of this economy by the average value, i.e.

$$W := e * \mathbb{E}[V_E(w) | w - \tau > w_R] + u * V_U$$

We want to find the level of unemployment benefit that maximizes the welfare in this economy.

### Solving the Model

Refer to the following algorithm to find a range for optimal unemployment benefit $b$.

1. Define a sequence of unemployment benefit $(b_1, b_2, b_3, \ldots, b_n) = (2, 3, 4, \ldots, 20)$. Take $b = b_1$, the first value in the sequence. Take some initial guess of employment and unemployment rates $x_0 = (e_0, u_0)'$.

2. For each $i \geq 0$, compute the lumpsum tax
$$\tau_i = u_i b$$

3. Compute the new reservation wage $w_R$ using lumpsum tax $\tau = \tau_i$. You may use the McCall model in Part I for this question, but remember to replace wage by $w - \tau$ and unemployment benefit by $b - \tau$, while keeping other inputs the same.

4. Using the reservation wage $w_R$, compute job finding rate
$$\lambda_i = \gamma \sum_{w' \geq w_R + \tau} p(w')$$
Use $\lambda_i$ to compute the steady state employment rates $\bar{x}$, using the Lake model you solved in Part II.

5. Take $x_{i+1} = \bar{x}$. Repeat steps 1 to 4 until $x_i$ and $x_{i+1}$ is sufficiently close, and record $x_{i+1} = (e_{i+1}, u_{i+1})$ as the solution for employment rate $e$ and unemployment rate $u$. Also compute the welfare
$$W = u * V_U + e \sum_{w' \geq w_R + \tau} V_E(w') p(w')$$

6. Repeat steps 2 to 5 for different values of unemployment benefit $b_2, b_3, b_4, \ldots, b_n$. Generate Matlab graphs to show that
   - Employment rate dereases with unemployment benefit;
   - Unemployment rate increases with unemployment benefit;
   - Welfare first increases then decreases with unemployment benefit, so there is a level of $b$ that maximizes welfare. **Find a range of the optimal unemployment benefit.**

The resulting graphs should look like Figure 6 and 7 below.

## Conclusion

In this project, we studied behaviors of labor market aggregates, including reservation wage, employment/ unemployment rates, and aggregate welfare. First, we set up and solved a McCall search model with job destruction and stochastic offers, and examined the influence of a higher unemployment benefit and job destruction rate on reservation wage. Next, we simulated the transition paths of employment(unemployment) rates, computed their steady states values, and studied the counterfactual implications of a baby boom. Finally, we introduced government taxation into the model, and show that there is an optimal unemployment benefit level that maximizes total social welfare.

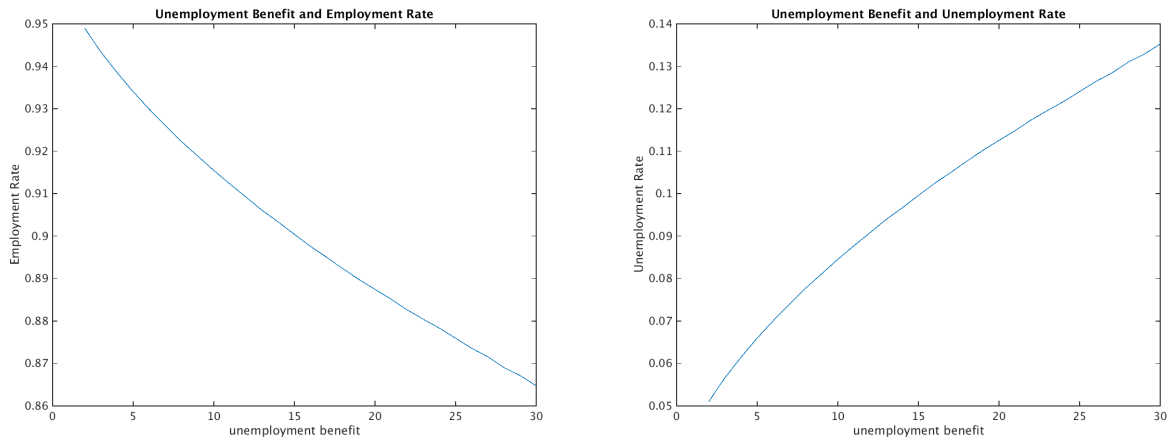


**Figure 6:** Sample Graphs for Question 1, Part III

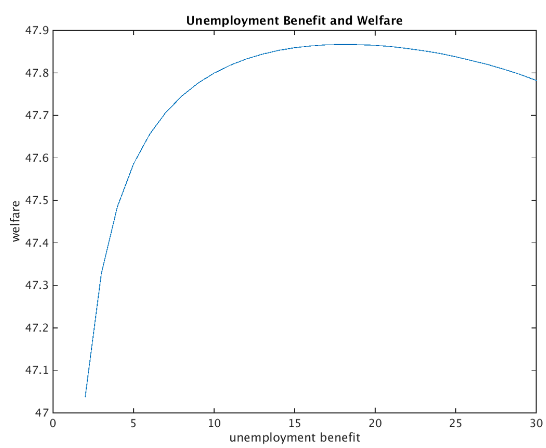


**Figure 7:** Sample Graphs for Question 1, Part III