Intermediate Macroeconomics: Final Exam

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DO NOT TURN OVER THIS PAGE UNTIL THE PROCTOR SAYS YOU MAY.

Instructions

- This exam is out of 100 points.
- You have **120** minutes to complete the exam.
- **Write down your answers on the provided answer sheets, in either English, Chinese, or a combination of both**. Don't forget to write down your name.
- No calculators, phones, notes or books of any kind are permitted.

Good Luck!

1. True/False/Uncertain (15 points, or 5 points each)

Assess whether the following statements are true, false or uncertain and justify your answers. Points are given for explanations only.

a. (5 points) In the Ramsey model, a permanent positive shock in technology from A to $A' > A$ will increase capital and consumption levels in the long run, but may initially decrease consumption in the short run.

b. (5 points) According to the permanent income hypothesis, in a two-period economy with borrowing and saving, an agent with an income of \$10 in period 1 and \$0 in period 2 will have the exact same consumption in both periods as an agent with an income of \$5 in both periods.

c. (5 points) According to the Solow model, a country's GDP can grow with a positive annual growth rate only when this country's population is also growing at the same constant rate.

2. Ramsey Model with Habit Persistence (30 Points)

Consider a Ramsey model with constant technology A and population $N = 1$. The representative household's utility at period t is given as

$$u(C_t, C_{t-1}) = \log(C_t) - \gamma \log(C_{t-1})$$

Which means the representative household penalizes itself for changing current consumption C_t too far from consumption in the previous period, C_{t-1}. The parameter $\gamma > 0$ measures persistence of habit. For example, when $\gamma = 0$, there is no habit persistence, and the utility is simply the logarithm function.

The household's problem is:

$$\max_{\{C_t, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(C_t, C_{t-1})$$
$$\text{s.t. } C_t + K_{t+1} - (1-\delta)K_t = AK_t^\alpha$$

For completeness, assume we know the value of $C_{-1} > 0$.

Questions:

a. (3 points) Argue that C_{t-1} is NOT a choice variable at time t.

b. (6 points) Write down the Lagrangian for the household problem, and solve for the first-order conditions.

c. (5 points) Find the Euler Equation, and explain its intuition.

d. (4 points) How do the steady-state values of K_t and C_t change when the habit persistence parameter γ changes?

For part e-f, assume the economy is initially at the steady state.

e. (6 points) Suppose a natural disaster destroys half of the capital stock. Draw the phase diagram on the $K_t - C_t$ space to illustrate the change, with directional arrows and saddle paths.

f. (6 points) Following part e, draw the transition paths of K_t, C_t and $I_t = K_{t+1} - (1-\delta)K_t$ after the disaster. Explain the change in investment. Is there a jump and/or convergence? Why?

3. Overlapping Generations with Labor Supply (20 points)

Consider a variation of the overlapping generations model. In each period, the economy is occupied by two cohorts of two generations of households – the young and the old – living for two periods. Instead of exogenous income, assume that the output Y_t^y and Y_{t+1}^o for cohort born in t is produced linearly by using

$$Y_t^y = A_t l_t^y \tag{1}$$
$$Y_{t+1}^o = A_{t+1} l_{t+1}^o \tag{2}$$

where A_t is the productivity parameter at time period t. Household maximizes its lifetime utility subject to the budget constraints:

$$\max_{c_t^y, c_{t+1}^o, l_t^y, l_{t+1}^o} \quad u(c_t^y) + v(l_t^y) + \beta[u(c_{t+1}^o) + v(l_{t+1}^o)] \tag{3}$$
$$s.t. \quad c_t^y + s_t^y = Y_t^y \tag{4}$$
$$c_{t+1}^o = Y_{t+1}^o + (1 + r_{t+1}) s_t^y \tag{5}$$

where s_{t+1}^y is the saving by household born at time t when it is young. The interest rate at time $t+1$ is r_{t+1}.

Let the utility functions be:

$$u(c) = \log(c) \tag{6}$$
$$v(l) = -0.5 l^2 \tag{7}$$

Finally, there is no storage technology, social security or any form of money in this economy.

Questions:

a. (6 points) What is the saving s_t^y by households born at t?

b. (5 points) Solve for the optimal consumption and labor supply choices of household born in t, when it is young and old.

c. (5 points) Solve the equilibrium real interest rate r_{t+1}.

d. (4 points) How does an increase in A_{t+1} change the labor supply of households in both periods? Why?

4. Two-Period Model with Labor Income Tax and Government (20 points)

Consider an economy that lasts for 2 periods, $t = 0, 1$. There is one representative household in this economy with 1 unit of time to allocate between labor n_t and leisure l_t. The utility function for the household is given by:

$$U(c_0, n_0, c_1, n_1) = \log(c_0) + \theta \log(1 - n_0) + \beta[\log(c_1) + \theta \log(1 - n_1)]$$

The household can choose to save/borrow s between time period 0 and 1 by purchasing a "treasury bond", whose interest rate r is taken as given by the household. The wage for household's labor services is (w_0, w_1), but in both periods the household faces labor income taxes with rate (τ_0, τ_1) .

The household's $t = 0$ budget constraint is:

$$c_0 + s = w_0 n_0 (1 - \tau_0)$$

There is a representative firm with constant return to scale production function that only uses labor as input, i.e.

$$y_t = A n_t$$

where A is fixed. There is a government with **fixed** expenditures (g_0, g_1) in each period. The goverment expenditures are financed by labor income taxes (τ_0, τ_1) and "treasury bonds" b. The government's budget constraints are given by:

$$g_0 = w_0 n_0 \tau_0 + b$$
$$g_1 + b(1 + r) = w_1 n_1 \tau_1$$

You can think of the "treasury bond" as government borrowing from households in $t = 0$ and repaying the debt in $t = 1$ with some interest rate r. The bond market clearing condition is:

$$b = s$$

a. (5 points) Write the household's period $t = 1$ budget constraint, as well as its intertemporal budget constraint.

b. (5 points) Solve the household's problem and derive the first-order conditions for labor (n_0, n_1) and consumption (c_0, c_1) in both periods. Show that the Euler Equation for consumption does not depend on tax rates.

c. (5 points) Show that the firm's problem and market clearing conditions imply that $w_0 = w_1 = A$. Combine this result and the first order conditions to show how equilibrium consumption-leisure ratio $\frac{c_0^*}{1-n_0^*}$ in period 0 depends on tax rate τ_0.

d. (5 points) **(Hard)** Suppose the labor tax rates are $(\bar{\tau}_0, \bar{\tau}_1)$ before, with $\bar{\tau}_0 > 0$ and $\bar{\tau}_1 > 0$. A new administration wants to cut taxes in period 0 by setting $\tau_0 = 0$. Discuss the impact of this policy on equilibrium consumption (c_0^*, c_1^*) and labor supply (n_0^*, n_1^*).

5. Short Answer (15 points)

In models of intertemporal consumption choice, we typically assume that a representative household can freely save or borrow any amount from a bank at a given interest rate, choosing its consumption and saving to maximize utility.

However, in reality, various **borrowing constraints** limit households' ability to borrow freely. These constraints might include the inability to obtain any loan from the bank, a cap on the total borrowing amount, or differences between borrowing and savings interest rates.

Using the models and techniques discussed in class, analyze how borrowing constraints may affect households' optimal consumption-saving choices. Discuss whether relaxing these constraints would necessarily lead to better outcomes.