Intermediate Microeconomic

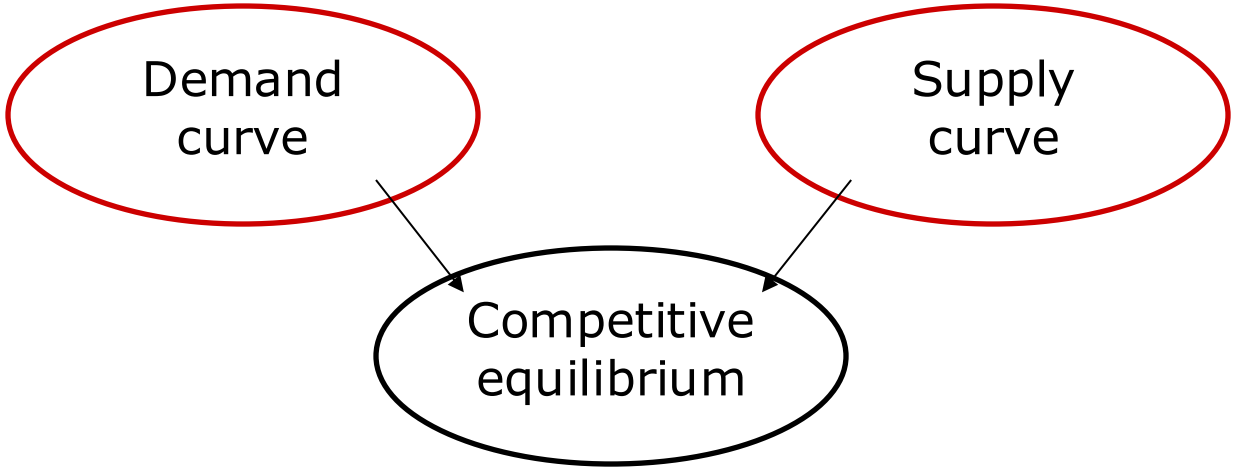
Spring 2025

Part two: Choice and Demand

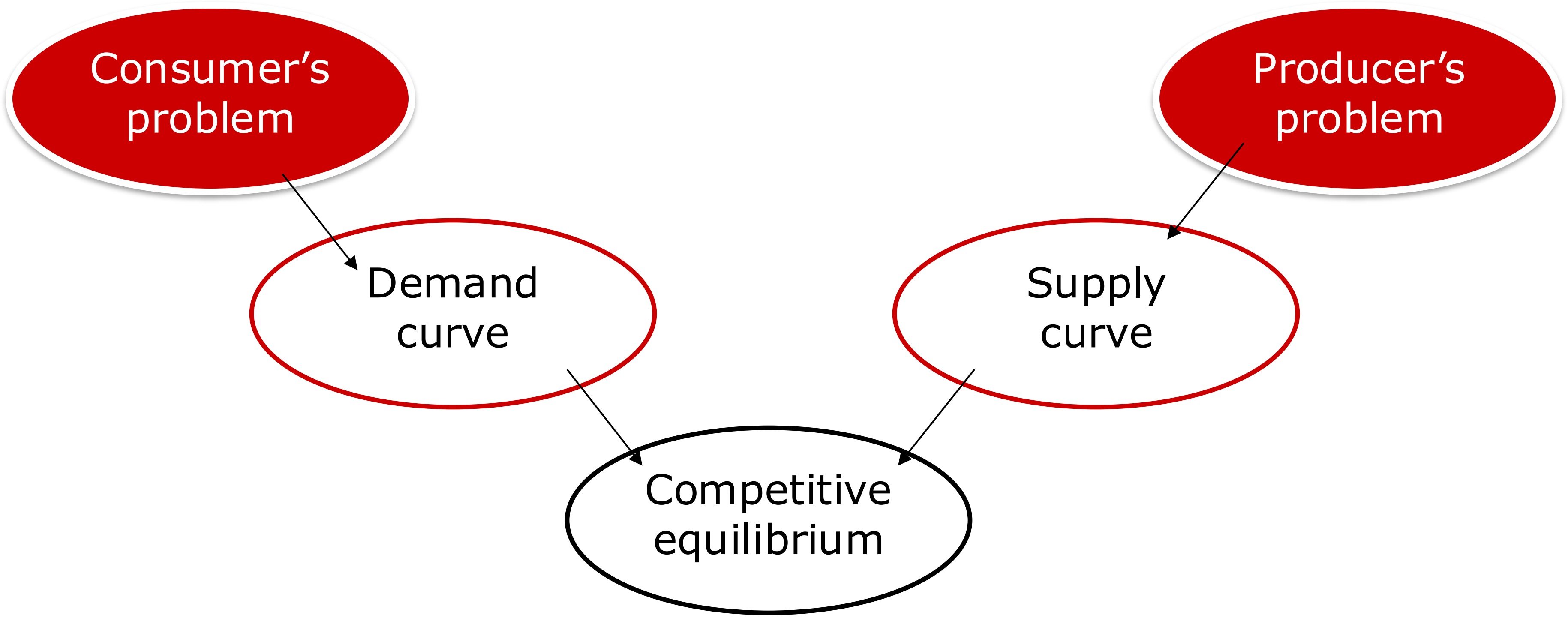
Week 2(a): Preference and Utility

Yuanning Liang

Big Picture

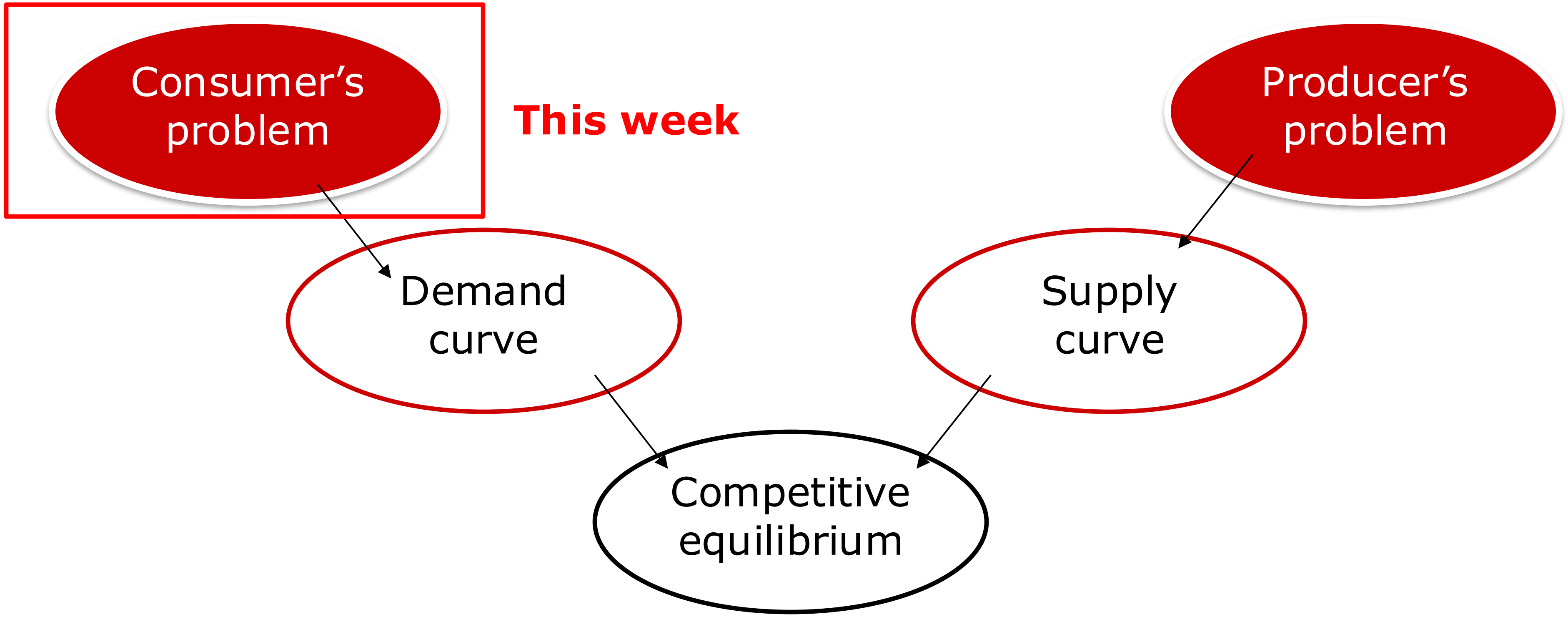


Big Picture


Consumer's problem
Producer's problem
Demand curve
Supply curve
Competitive equilibrium

Big Picture

This week



Consumer Theory

- A consumer wakes up in the morning. The world has two goods (commodities) for consumption: x and y. He looks outside the window and observe prices P_x and P_y. He reaches into his pocket and find M dollars there; he knows his preferences over goods x and y.

- Consumer's problem: find the <u>best</u> <u>affordable</u> combination of goods x and y for him to eat

- Two building blocks of the consumer theory:
  - **Preference**
  - **Budget constraint**

Consumer Theory

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Preferences

- People have preferences over “commodity bundles” that characterize how they feel about various choices, e.g.

  Bundle A: (4,4)
  Bundle B: (9,1)

  1 unit of good y

  9 units of good x

- Definition: X = consumption set (e.g., $X = R^n_+$, the set of nonnegative real numbers), x = consumption bundle, $x \in X$.
- Definition: $x \succsim y$ means that x is at least as good as y.
- Definition: $x \succ y$ means that x is strictly preferred to y.
- Definition: $x \sim y$ means that x is indifferent to y, if and only if $x \succsim y$ and $y \succsim x$.

Axioms of **rational** choice:

1. **Completeness**: if A and B are any two situations, an individual can always specify exactly one of these possibilities:
   - A is preferred to B
   - B is preferred to A
   - A and B are equally attractive (A is indifferent to B)

2. **Transitivity**: if A is preferred to B and B is preferred to C, then A is preferred to C.

3. **Continuity:** if A is preferred to B, then situations suitably "close to" A must also be preferred to B
   - used to analyze individuals' responses to relatively small changes in income and prices

Preferences

- Definition of economic rationality differs from how a layperson would define it.
  - It says (1) people can compare any two alternative, and (2) preferences are internally consistent
- Nothing about people only care about money, or people only care about himself, or people never make mistakes
- Rationality (**completeness** and **transitivity**) is in fact a very weak assumption on preference. It is essentially impossible to find a real-world choice that <u>definitely</u> violates completeness and/or transitivity.

Preferences: Example 1

- Suppose I give you $100. You can give me back some amount, or you can take the entire $100.

- How much would you keep?

- Often when exercises like this are studied in laboratory environments, people keep only about $70.

- Is this irrational?
  - Probably not. Nothing about this violates completeness or transitivity or continuity.
  - Nobody said you have to always choose more money over less in order to be rational.

Preferences: Example 2

- At a restaurant you are given the choice between steak and chicken, and you choose chicken. Then, the waiter tells you that fish is also available. You change your mind and choose steak.

- Is this irrational?
  - Maybe. Your first choice may show that you prefer chicken to steak, but your second choice then shows that you prefer steak to chicken.

- But, maybe not. Why?

Preferences: Example 2

- At a restaurant you are given the choice between steak and chicken, and you choose chicken. Then, the waiter tells you that fish is also available. You change your mind and choose steak.

- Is this irrational?
  - Maybe. Your first choice may show that you prefer chicken to steak, but your second choice then shows that you prefer steak to chicken.

- But, maybe not. Why?
  - Maybe you think that you need to have a great chef to serve a great steak. And, you think that a restaurant that serves fish is more likely to have a great chef. So, this new information leads you to change your behavior.

Preferences

- The point here is that many choices can be “rationalizable”. It is essentially impossible to find a real-world choice that <u>definitely</u> violates completeness and/or transitivity.

What are the criteria for decision-making?

- Need a measure (yardstick) to make choices. The yardstick measures the desirability (value) of the choices.

- Mathematically, we need to find a function $f(x)$ to measure the value of x; where x is the quantity of goods purchased (or income, wealth, etc.).

- In economics, the criterion function $f(x)$ is known as the **utility function.**

Utility Function

- **Theorem** (Debreu (1954)): If individual's preference is complete, transitive, and continuous, then there exists a numerical function U() such that U(A)≥U(B) if and only if bundle A is preferred to bundle B.
  - That is, utility functions assign higher numbers to better bundles.
  - We call this **U()** function the **utility function**

- The theorem says that whenever a decision maker has rational preferences, those preferences can be represented by a utility function.

- In other words, the utility function fully characterizes the consumer's preference, so that we don't need to worry about preference itself which is very abstract and hard to work with.

Utility Function

- Consider our previous example. A consumer is facing the following bundles

  Bundle A: (4,4)
  Bundle B: (9,1)

- Suppose the consumer's utility function is $\mathrm{U}(\mathrm{x}, y) = \sqrt{x} + 2\sqrt{y}$

- **Question: Does the consumer prefer Bundle A or Bundle B?**

Utility Function

- Consider our previous example. A consumer is facing the following bundles

  Bundle A: (4,4)
  Bundle B: (9,1)

- Suppose the consumer’s utility function is $U(x,y)=x*y$

- **Question: Does the consumer prefer Bundle A or Bundle B?**

Utility Function: Clarification

- It turns out that knowing one's utility function is also equivalent to knowing one's **demand function**.
  - That is, under some assumptions, each utility function is uniquely associated with a demand function
  - So, preference, utility, demand are essential the same characterization of a consumer

- Note that demand (how much you consumer at different prices and income levels) is directly observable. So, the equivalence between utility function and demand function implies that **you can interpret people's behavior "as if" they have utility functions in mind**

- We'll talk about this interpretation later

Utility Function

- For the rest of the class, you can "loosely" interpret utility function U(9,1) as "how much utility you get by consuming 9 units of x and 1 unit of y"

- But, you have to remember this: utility function is **ordinal** (only the order matters, not magnitudes).
  - If U(Bundle A) = 3, U(Bundle B) = 6, we can only say that B is better than A, not that it is twice as good.
  - This is because the same rational preference can be represented by many utility functions
  - More generally, if $g(\cdot)$ is an increasing function, then $g(U(\cdot))$ represents the same preferences as $U(\cdot)$.

- For the same reason, utility function represent a **particular** consumer's preferences. Cannot be used to compare two consumers' preferences.

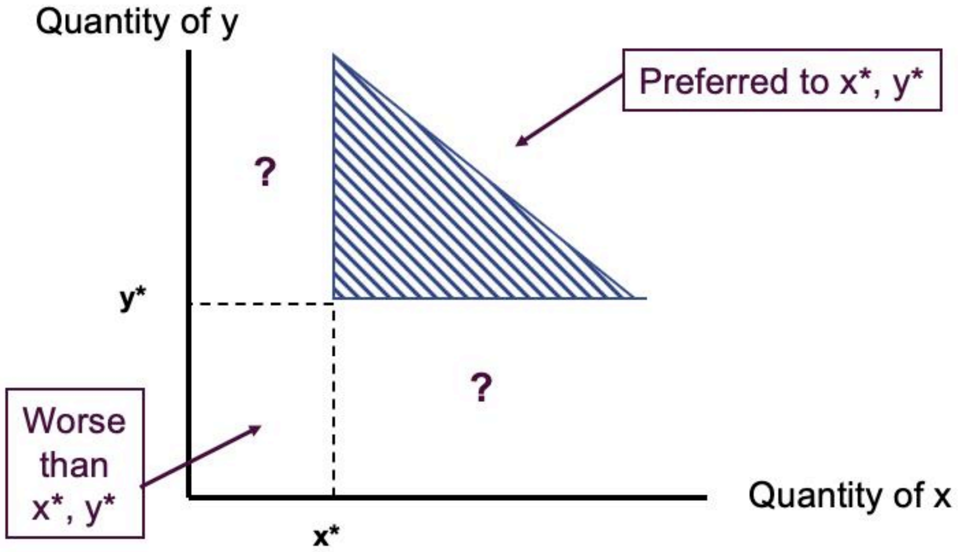
Utility Function

- For most of this class, we'll focus on a utility function with two goods, x and y
  - U(x,y)
  - Apples and oranges
  - Apples and "everything else"
  - Apples and money unspent

- Virtually all conclusions from a two-good world generalizes to an N-good world

Economic goods

- In the utility function, the x's are assumed to be "goods"
  - more is preferred to less

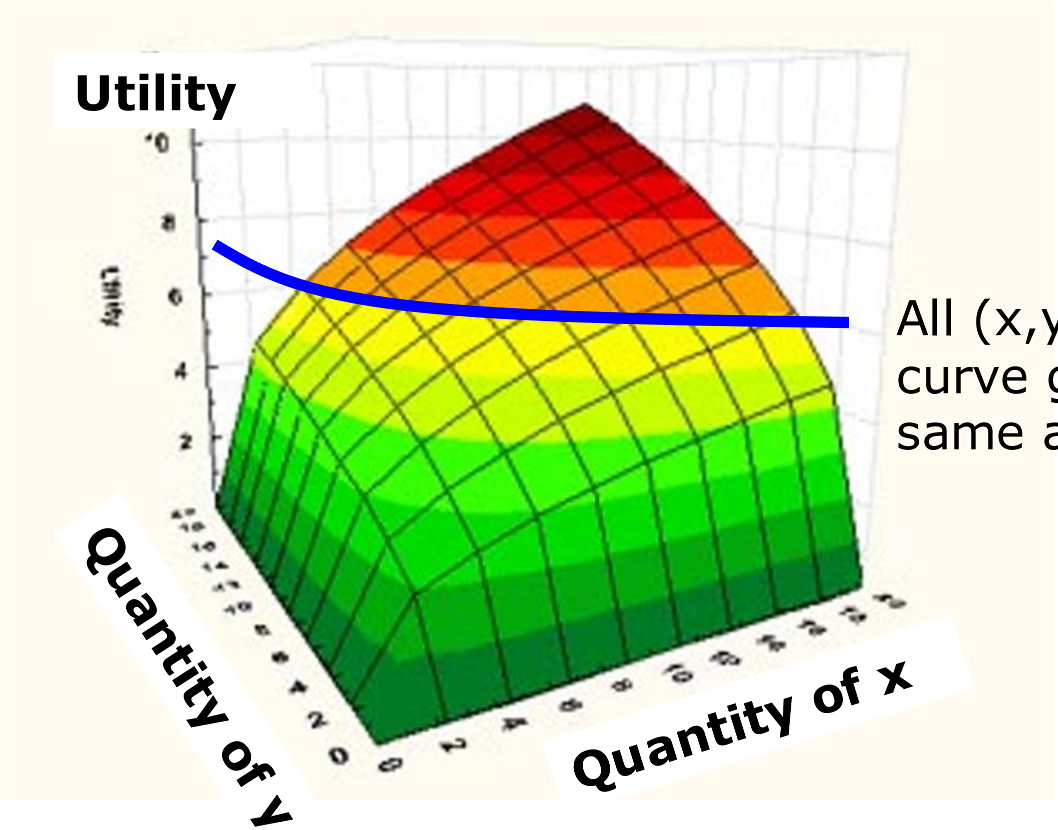


Indifference Curves

- The function U(x,y) is a three-dimensional entity.
- In order to obtain a more tractable diagrammatic analysis, we employ two-dimensional indifference curves (level sets).
- An indifference curve shows a set of consumption bundles over which the decision maker is indifferent.

What does U(x,y) look like?

Indifference Curves

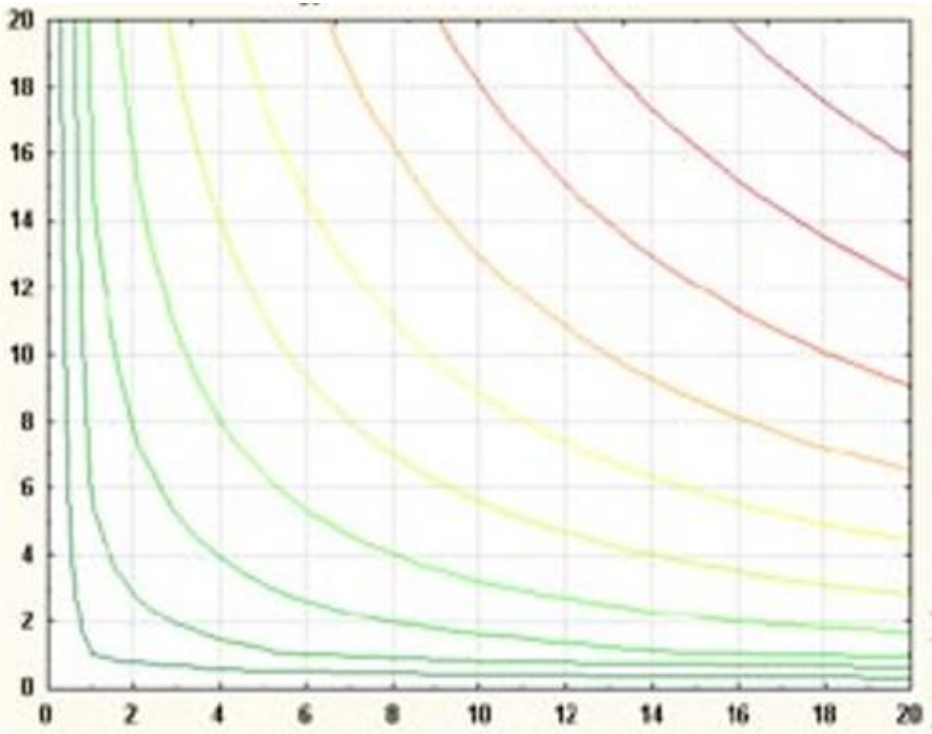


All (x,y) bundles along this curve gives the consumer the same amount of utility

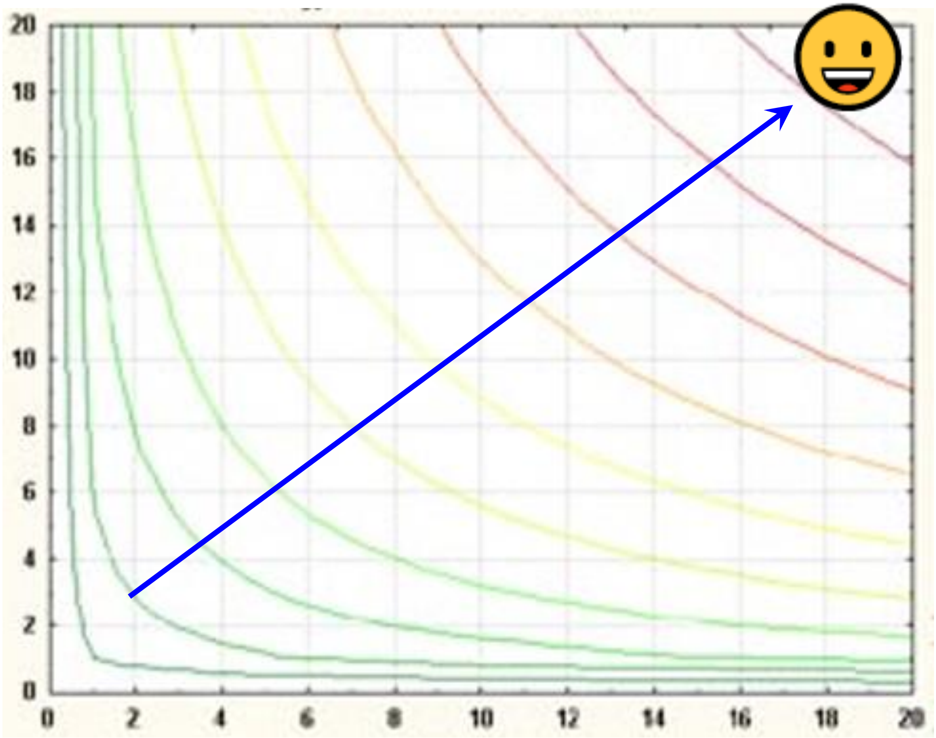
Indifference Curves

Bundles along this curve contains more x and more y, so they give more utility

Top-Down View of U(x,y)



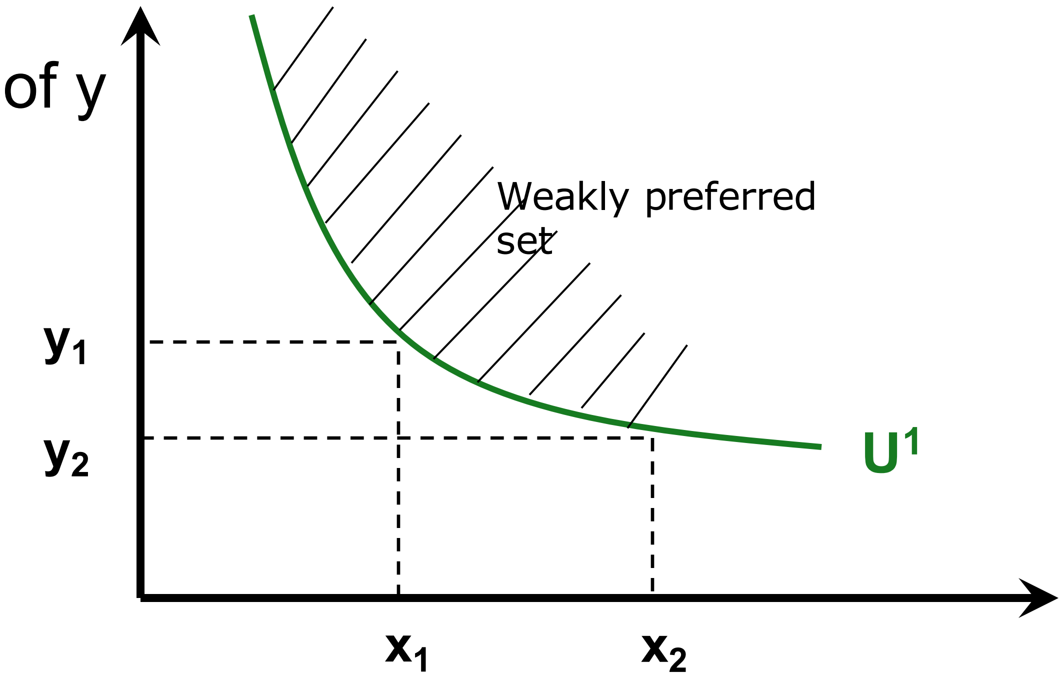
Top-Down View of U(x,y)



Utility increases up and to the right (usually).

Indifference Curves

- An indifference curve shows a set of consumption bundles over which the decision maker is indifferent.
- The decision maker is indifferent between all bundles on curve U^1.



Combinations (x_1, y_1) and (x_2, y_2) provide the same level of utility $U(x_1, y_1) = U(x_2, y_2) = U^1$.

Quantity of x

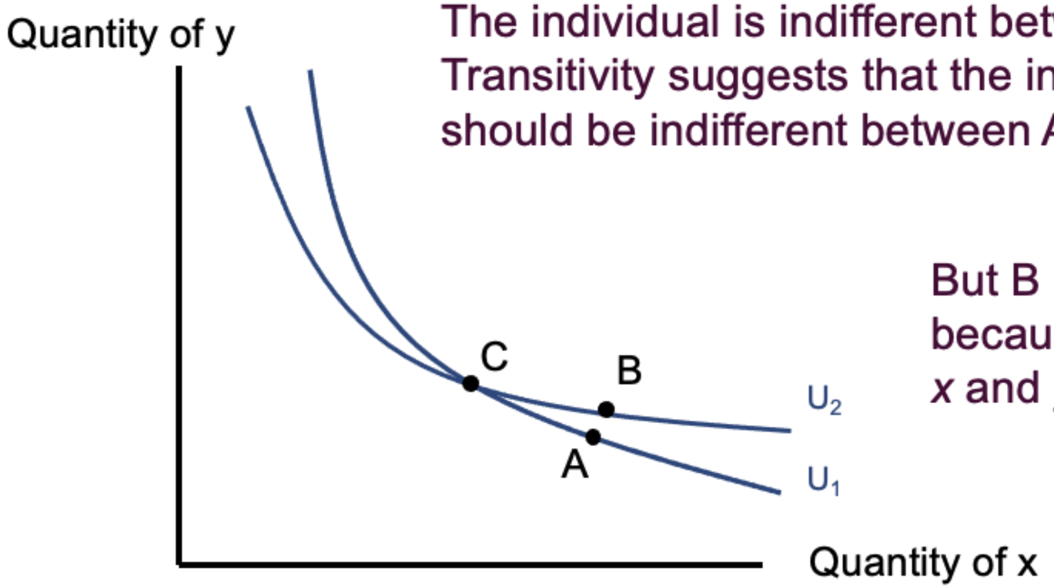
Transitivity

- **Question: Can a rational consumer's indifference curves intersect?**

Transitivity

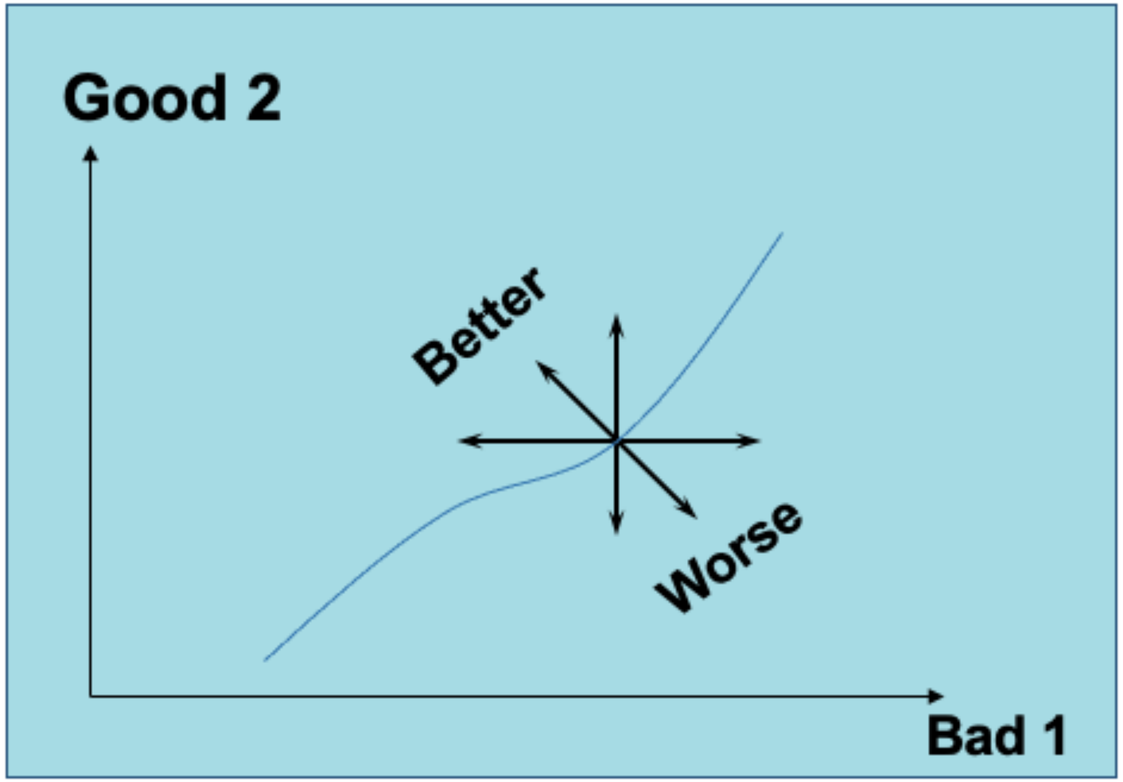
- **Question: Can a rational consumer’s indifference curves intersect?**

The individual is indifferent between A and C.
The individual is indifferent between B and C.
Transitivity suggests that the individual should be indifferent between A and B

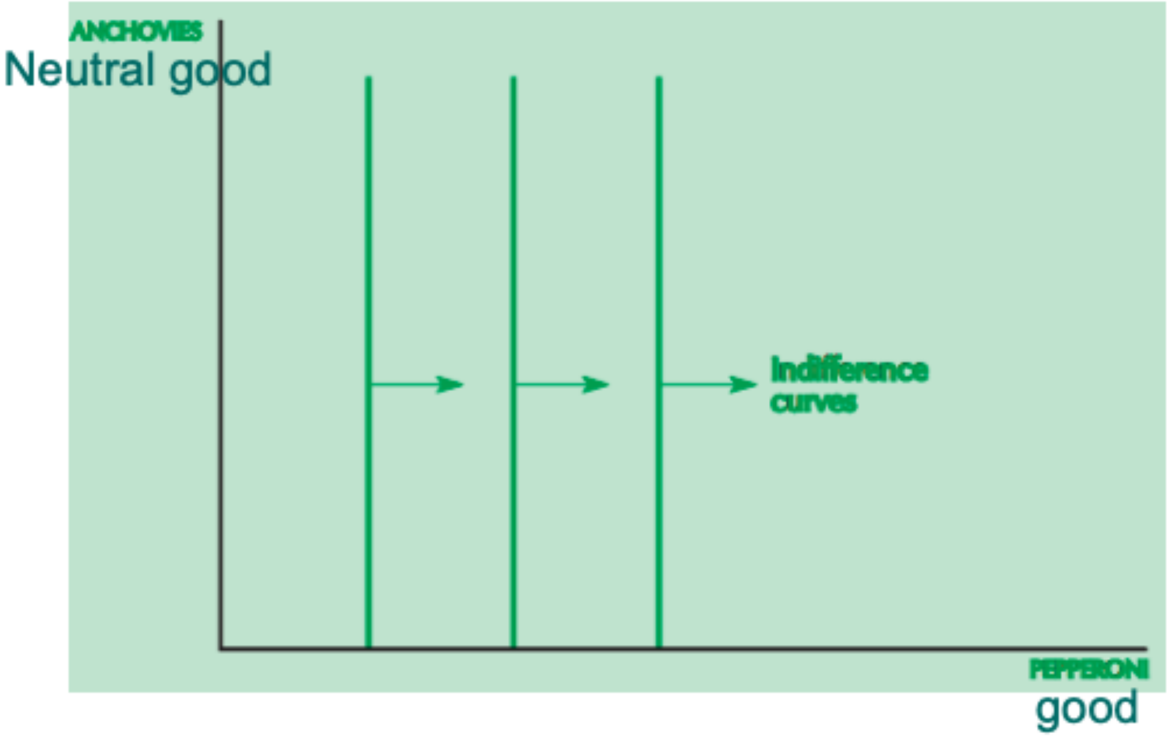


But B is preferred to A because B contains more *x* and *y* than A

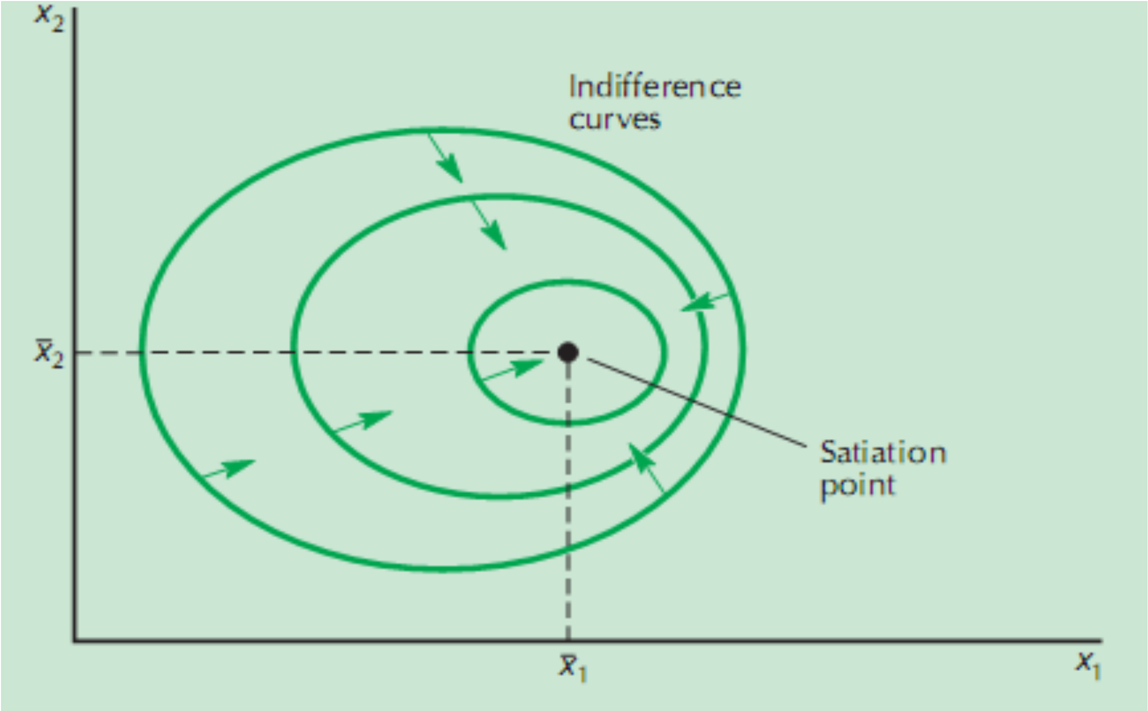
Various kinds of preferences



Various kinds of preferences



Various kinds of preferences



Well-behaved preferences

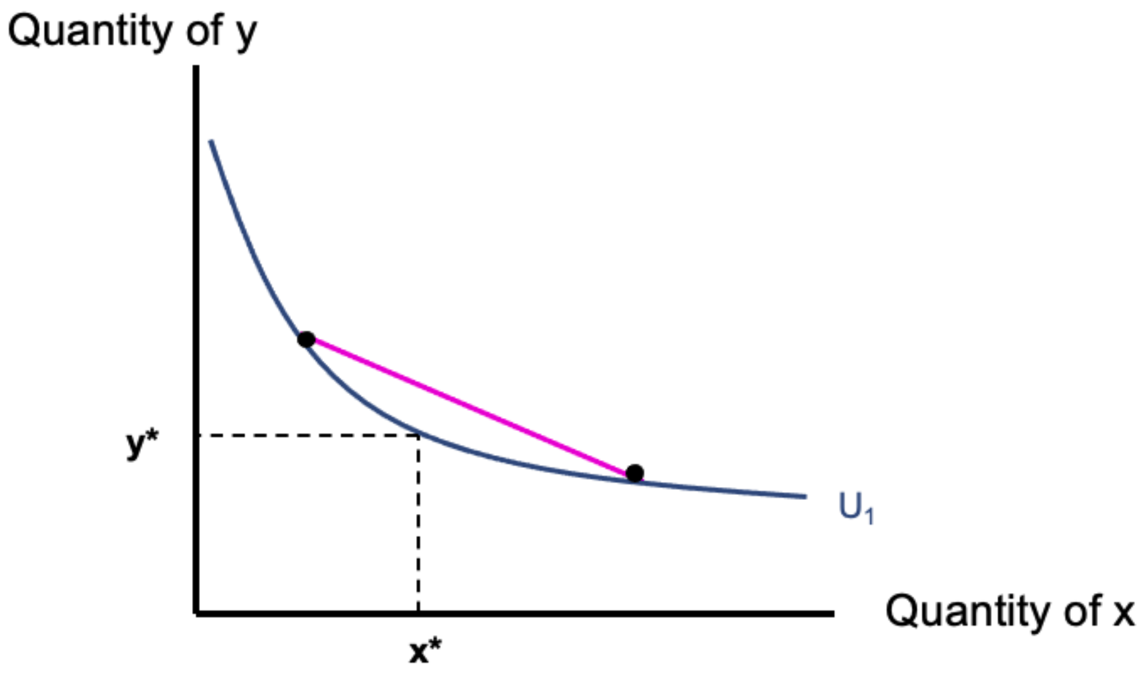
- A preference relation is "well-behaved" if its indifference curve is
  - monotonic and convex.

- Monotonicity: More of any commodity is always preferred (*i.e.* no satiation and every commodity is a good).

- Convexity: Mixtures of bundles are (at least weakly) preferred to the bundles themselves.

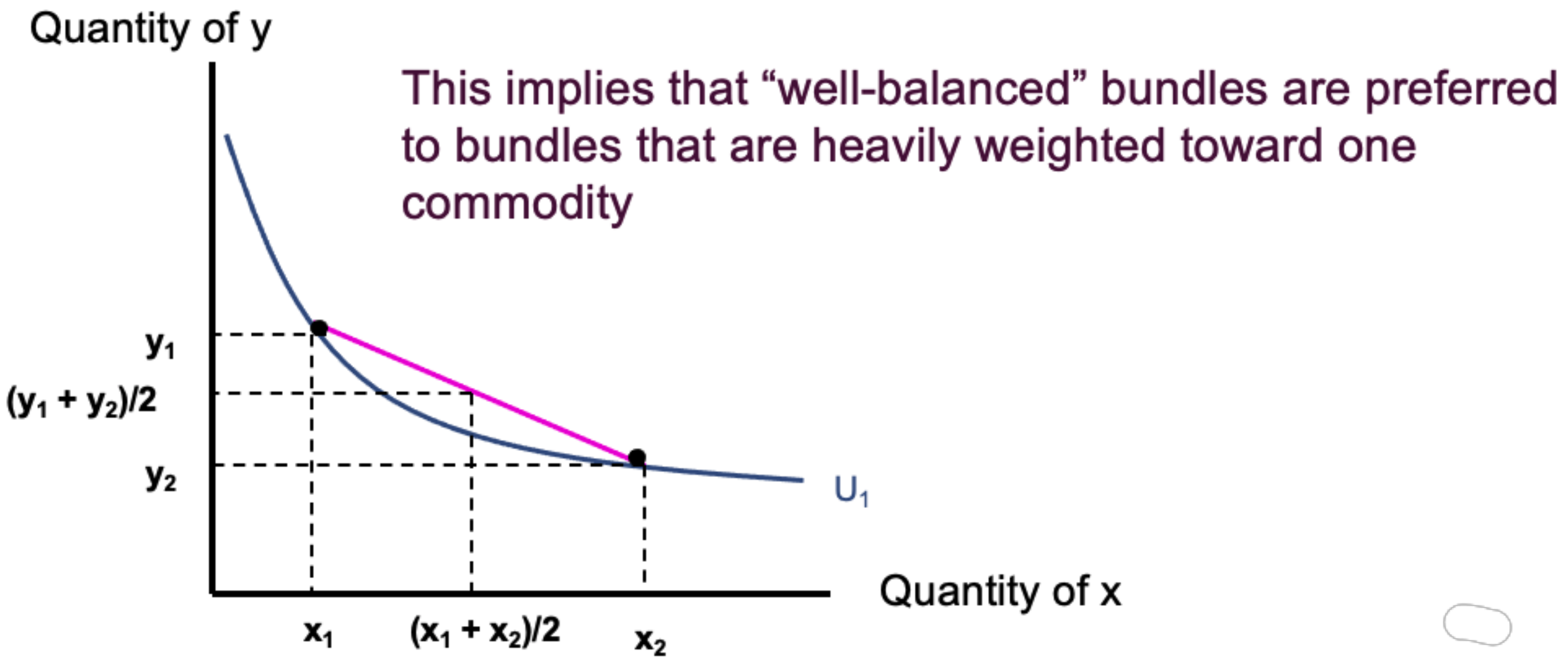
Convexity of indifference curves

- A set of points is <u>convex</u> if any two points can be joined by a straight line that is contained completely within the set.



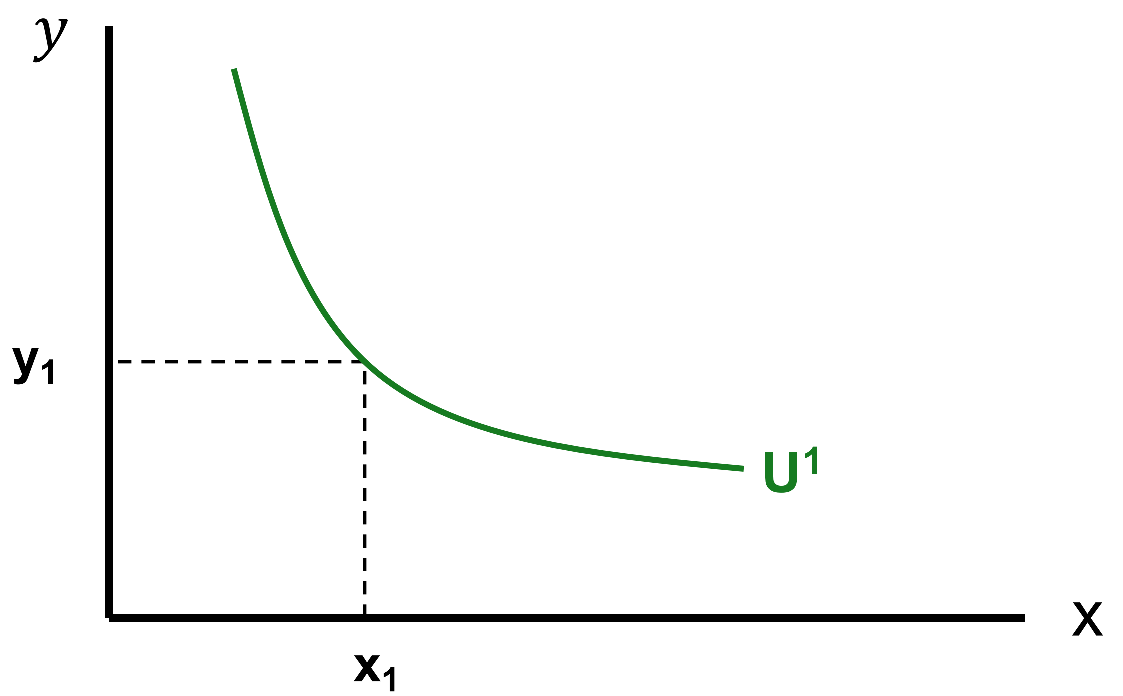
Convexity of indifference curves

- If the indifference curve is convex, then the combination $(x_1 + x_2)/2$, $(y_1 + y_2)/2$ will be preferred to either (x_1,y_1) or (x_2,y_2)



Slope of Indifference Curve

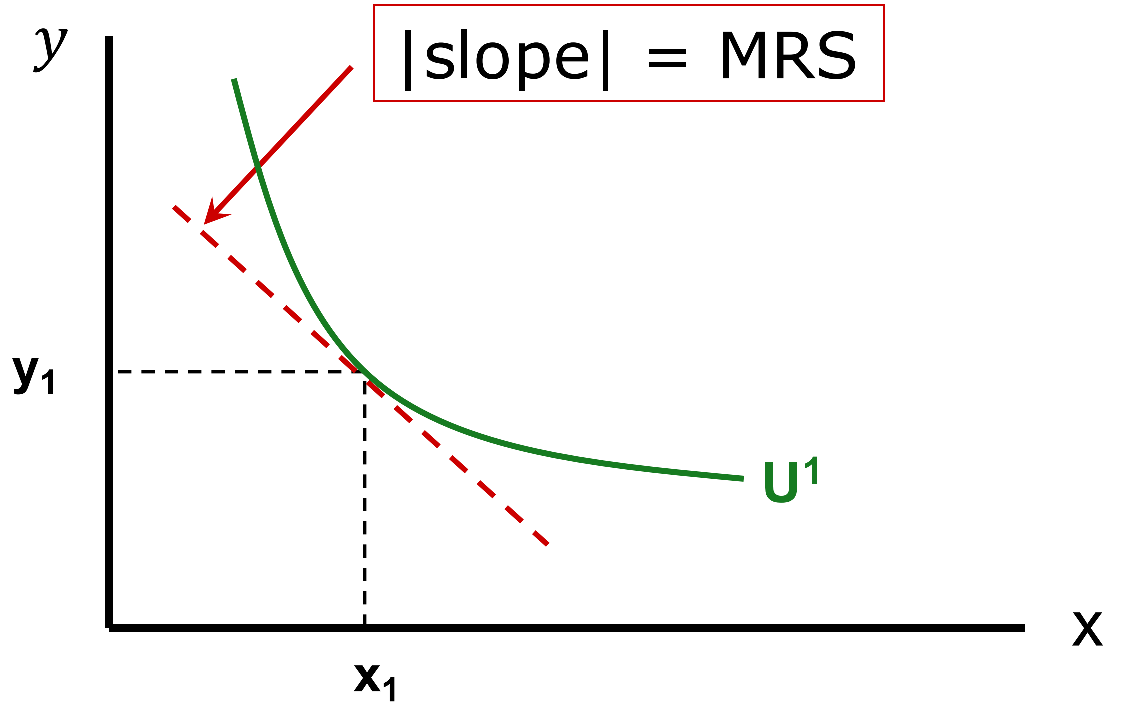
- The slope of the indifference curve captures how the individual trades off x against y, holding utility constant.
- How much y is the individual willing to give up to get a little bit more x?



Slope of Indifference Curve

- The slope of the indifference curve captures how the individual trades off x against y, holding utility constant.
- How much y is the individual willing to give up to get a little bit more x?

- For a very small change in x, we have to use calculus to define the slope of the indifference curve
- We call this ratio the **Marginal Rate of Substitution (MRS)**.
- By convention, MRS is positive even though slope is negative.



Marginal Rate of Substitution

- The MRS is the (absolute value of the) slope of an indifference curve at a point (x,y).

$$\boxed{MRS = -\left.\frac{dy}{dx}\right|_{U=constant} = \left|\frac{dy}{dx}\right| = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}}}$$

- To find the MRS, simply
  - (a) differentiate $U(x, y)$ with respect to x,
  - (b) differentiate $U(x, y)$ with respect to y,
  - and divide the results in (a) by the result in (b).

An Aside on Math

- Where does the MRS formula come from?
- Consider the total differential of $U(x, y)$

Amount of increase in x

$$dU = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy$$

- Since utility is constant along the indifference curve, $dU = 0$, and we can solve for dy/dx.

$$0 = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy$$

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An Aside on Math

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"Marginal utility": rate of utility change when x increases

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An Aside on Math

- Where does the MRS formula come from?
- Consider the total differential of $U(x, y)$

Utility change when x changes by dx amount

$$dU = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy$$

- Since utility is constant along the indifference curve, $dU = 0$, and we can solve for dx/dy.

$$0 = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy$$

$$MRS = -\left.\frac{dy}{dx}\right|_{U=constant} = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}}$$

An Aside on Math

- Where does the MRS formula come from?
- Consider the total differential of $U(x, y)$

Utility change when x changes by dx and y changes by dy

$$dU = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy$$

- Since utility is constant along the indifference curve, $dU = 0$, and we can solve for dx/dy.

$$0 = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy$$

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$$\boxed{MRS = -\left.\frac{dy}{dx}\right|_{U=constant} = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}}}$$

Utility Functions

- There is no unique utility function representation of a preference relation.
- Suppose $U(x_1,x_2) = x_1x_2$ represents a preference relation.
- Again consider the bundles (4,1), (2,3) and (2,2).

Utility Functions

- $U(x_1,x_2) = x_1x_2$, so

  $U(2,3) = 6 > U(4,1) = U(2,2) = 4$;

  that is, $(2,3) \succ (4,1) \sim (2,2)$.

Utility Functions

- $U(x_1,x_2) = x_1x_2$ $\Rightarrow$ $(2,3) \succ (4,1) \sim (2,2)$.
- Define $V = U^2$.
- Then $V(x_1,x_2) = x_1^2x_2^2$ and
  $V(2,3) = 36 > V(4,1) = V(2,2) = 16$
  so again
  $(2,3) \succ (4,1) \sim (2,2)$.
- V preserves the same order as U and so represents the same preferences.

Utility Functions

- $U(x_1,x_2) = x_1x_2 \Rightarrow (2,3) \succ (4,1) \sim (2,2).$
- Define $W = 2U + 10.$
- Then $W(x_1,x_2) = 2x_1x_2+10$ so $W(2,3) = 22 > W(4,1) = W(2,2) = 18.$ Again, $(2,3) \succ (4,1) \sim (2,2).$
- W preserves the same order as U and V and so represents the same preferences.

Utility Functions

- If
  - U is a utility function that represents a preference relation $\succsim$ and
  - f is a strictly increasing function,
- then V = f(U) is also a utility function representing $\succsim$.

A few examples

1. Suppose that the utility function is

$$\text{utility} = \sqrt{x \cdot y}$$

Calculate the MRS.

A few examples

2. Suppose that the utility function is

$$U(x,y) = x + xy + y$$

Calculate the MRS.

A few examples

3. Suppose that the utility function is

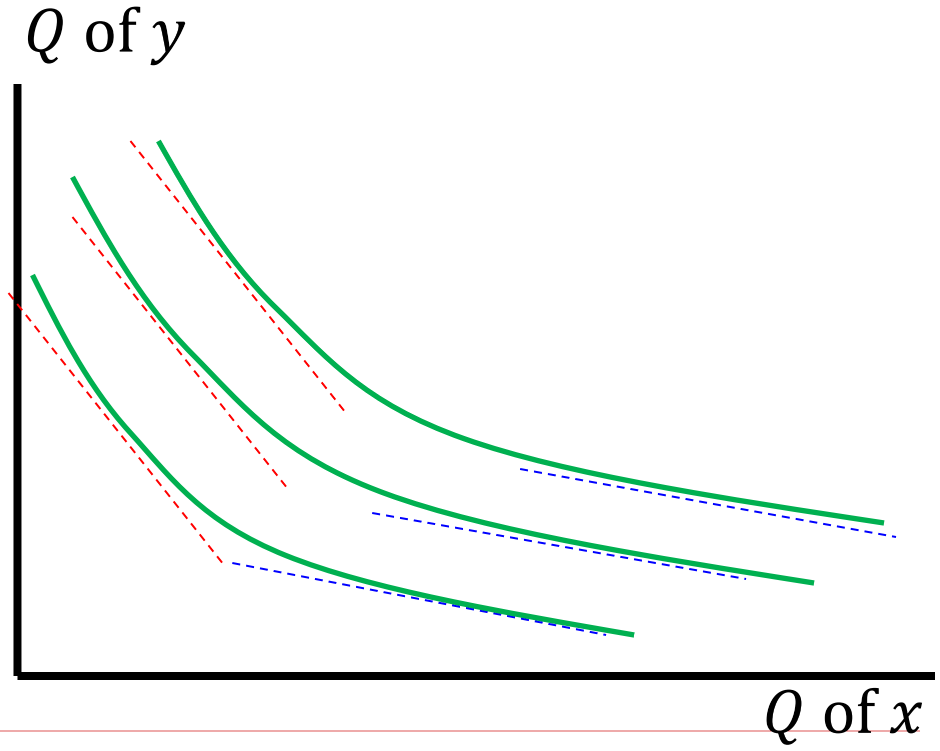
$$\text{utility} = \sqrt{x^2 + y^2}$$

Calculate the MRS.

Some popular forms of utility functions

Cobb-Douglas Utility: $U(x, y) = x^{\alpha}y^{\beta}$, α and β are positive constants.

- Relative sizes of α and β indicate the relative importance of the goods.
- MRS decreases smoothly along indifference curves.



Cobb-Douglas Utility

- **Question: Consider a Cobb-Douglas utility function $U(x, y) = x^{\alpha}y^{\beta}$.**
- **Show that MRS = $\frac{\alpha}{\beta} \cdot \frac{y}{x}$**

Cobb-Douglas Utility

- The relative sizes of α and β indicate the **<u>relative importance</u>** of the goods.

- We can also write it in a normalized form:

$$U(x, y) = x^{\delta} y^{1-\delta}$$

where $\delta = \alpha/(\alpha + \beta)$, $1 - \delta = \beta/(\alpha + \beta)$.

- We can also log transform it:

$$U(x,y) = \delta \ln x + (1-\delta) \ln y$$

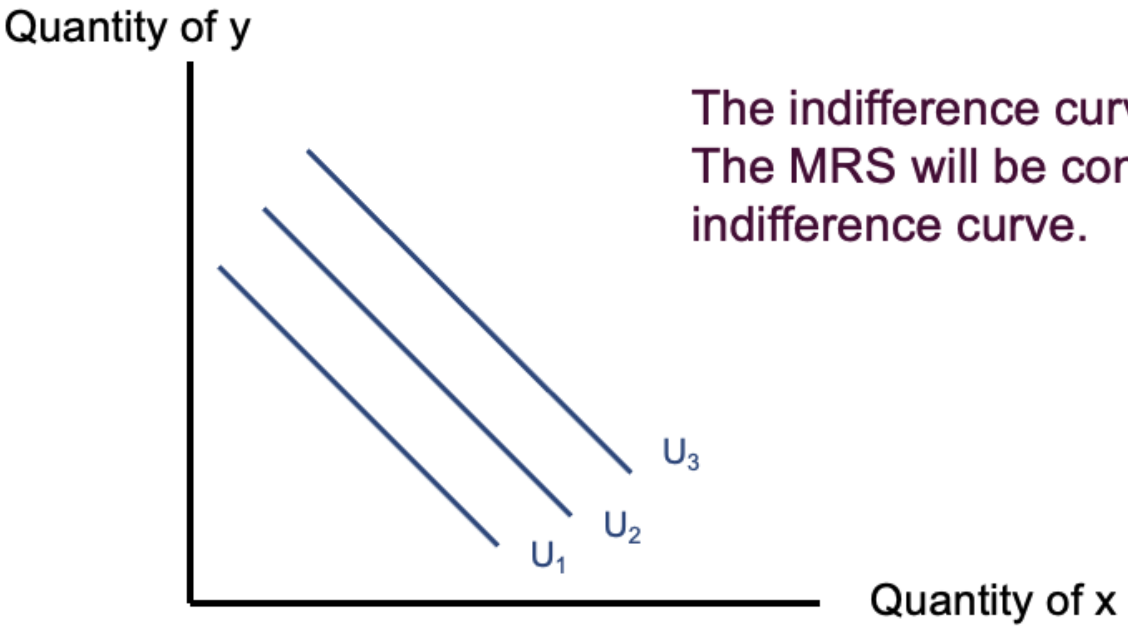
Perfect Substitutes

- U(x,y) = $\alpha x + \beta y$

→ MRS is constant

Perfect Substitutes

- U(x,y) = $\alpha x + \beta y$



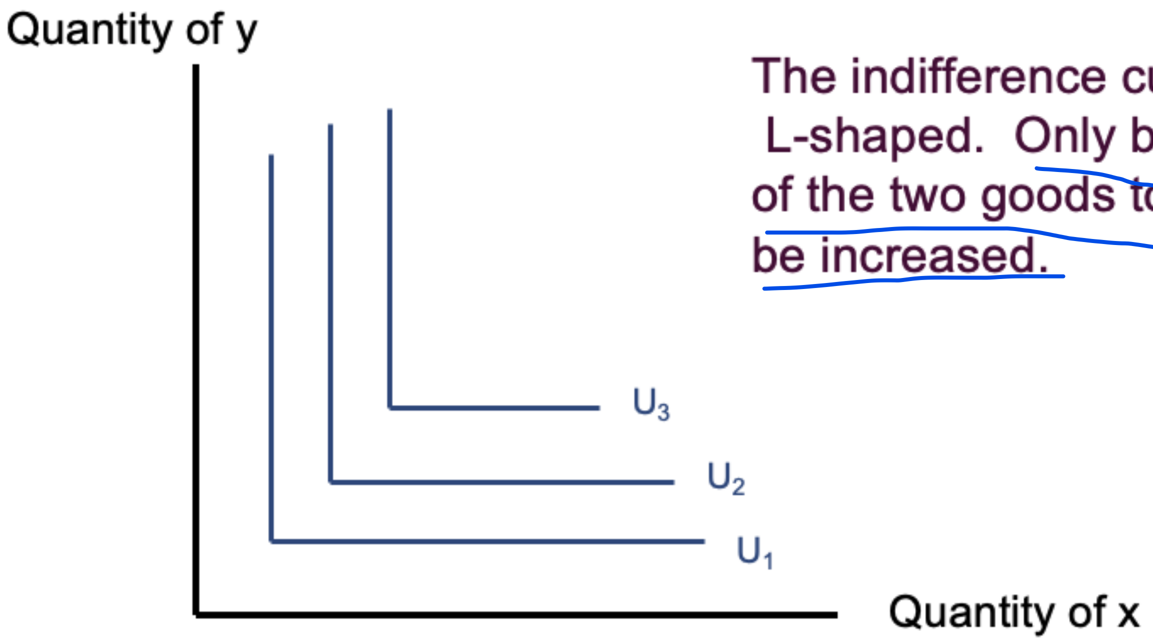
The indifference curves will be linear. The MRS will be constant along the indifference curve.

Perfect complements

- $U(x,y) = \min(\alpha x, \beta y)$

Perfect complements

- U(x,y) = min (αx, βy)



The indifference curves will be L-shaped. Only by choosing more of the two goods together can utility be increased.

CES utility (constant elasticity of substitution)

- $U(x,y) = a\frac{x^{\delta}}{\delta} + b\frac{y^{\delta}}{\delta}$

- Definition:

$$\sigma = \text{ elasticity of substitution} = \frac{\text{percentage change in } \frac{y}{x}}{\text{percentage change in } MRS}$$

$$= \frac{d(\frac{y}{x})}{dMRS} \times \frac{MRS}{\frac{y}{x}} = \frac{d\ \ln\frac{y}{x}}{d\ \ln MRS} = \frac{d\ \ln\frac{y}{x}}{d\ \ln\frac{U_x}{U_y}}$$

- Show that the elasticity of substitution (σ) is equal to $1/(1 - \delta)$.

Quasi-linear utility

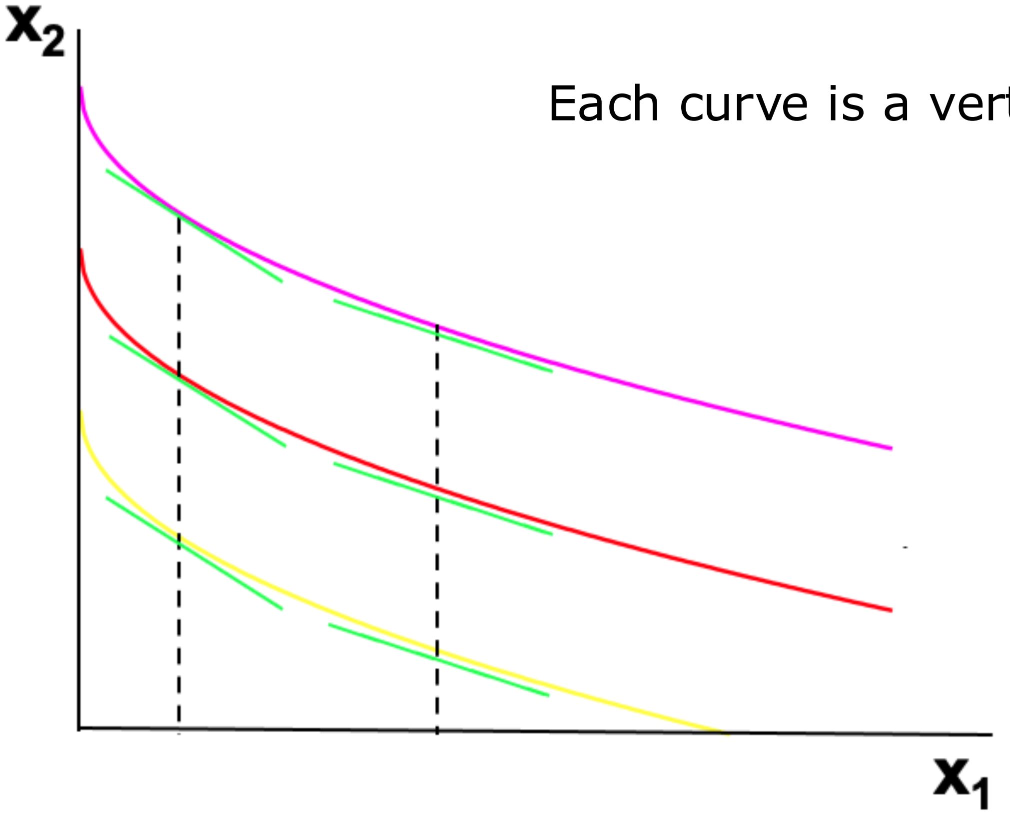
- A utility function of the form

$$U(x_1,x_2) = f(x_1) + x_2$$

  is linear in just x_2 and is called **quasi-linear.**

- *E.g.* $U(x_1,x_2) = 2x_1^{1/2} + x_2$.

Quasi-linear utility



Each curve is a vertically shifted copy of the others.