Intermediate Macroeconomics: Problem Set 2

Due Tuesday, March 18 in class

1. Can I Borrow from You? (Midterm 2023)

Consider the following infinite-period model with two types of agents. Both types have the same utility function over their lifetime consumption:

$$U^A = \sum_{t=0}^{\infty} \beta^t u\left(c_t^A\right), \quad U^B = \sum_{t=0}^{\infty} \beta^t u\left(c_t^B\right)$$

where U^i, c_t^i represents the type i agents' utility and consumption, respectively. The difference between the two types is that:

- Type A agents earn real income y in odd periods (i.e. $t = 1, 3, 5, \dots$), but no income in even periods (i.e. $t = 0, 2, 4, \dots$).
- Type B agents are just the opposite, receiving income y in even periods and no income in odd periods.

Agents are allowed to save/borrow with real interest rate r, which they take as given. **Assume there are equal numbers of type A and type B agents**, and their utility functions $u(\cdot)$ satisfies $u' > 0, u'' < 0$.

Note that the budget constraint for type A agents in period $t = 0$ is:

$$c_0^A + b_0^A = 0$$

where b_0^A represents type A agents' savings in period 0. Similarly, the budget constraint for type B agents in period $t = 0$ is:

$$c_0^B + b_0^B = y$$

Questions:

a. (4 points) Based on the information above, write down type A and type B agents' budget constraints in $t = 1, 2$.

b. (4 points) Write down the intertemporal budget constraints for type A and type B agents, which should take the following form:

$$\sum_{t=0}^{\infty} \frac{c_t^i}{(1+r)^t} = PI_i$$

where PI_i represents type i agents' *permanent income*, and should be expressed as a function of y and r. Show that when $r > 0$, type B agents have higher permanent income than type A agents.

(Hint: you may find the following formula helpful.)

$$1 + \alpha + \alpha^2 + \alpha^3 + \cdots = \frac{1}{1-\alpha} \quad \text{, for } 0 < \alpha < 1$$

c. (4 points) Define type A agent's optimization problem, and write down the corresponding Lagrangian function and first order conditions.

d. (3 points) Using the first order condition with respect to $[c_t^A]$, derive the Euler Equation:

$$\frac{u'(c_t^A)}{\beta u'(c_{t+1}^A)} = ?$$

From part e) onwards, assume the utility function takes the logarithmic form: $u(c_t^i) = \ln c_t^i$

e. (3 points) Using your answers in b) and d), solve c_0^A as a function of y, r and β.

f. (3 points) Write down the market clearing conditions in this economy.

g. (4 points, **Difficult**) Calculate the interest rate level r^* at the competitive equilibrium, when both types of agents optimize their utility functions subject to budget constraints, and all markets clear.

h. (5 points) Show that in a steady state with $c_{t+1}^A = c_t^A = c^A$ and $c_{t+1}^B = c_t^B = c^B$, type A agents would occasionally borrow from type B agents, but type B agents never borrow from type A agents. Calculate the amount that type A agents borrow in both odd and even periods.

2. Cash-In-Advance Model with Two Goods (Midterm 2022)

Consider the following cash-in-advance model. There are two consumption goods, C_t and F_t, in the economy. The former good C_t can be paid by any means, but the latter good F_t can only be paid in cash. Both goods have identical prices P_t in each period. The good F_t is subject to the following CIA constraint:

$$P_t F_t \leq M_{t-1}$$

There are no other saving technology except money in this economy. The representative agent is endowed with real income Y_t each period, and maximizes her utility function by choosing C_t, F_t and the real money balance m_t:

$$\max_{\{C_t, F_t, m_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t [u(C_t) + u(F_t)]$$

$$\text{s.t. } C_t + F_t + m_t = Y_t + m_{t-1}\frac{P_{t-1}}{P_t} + \frac{\tau_t}{P_t}$$

$$[\text{+ CIA Constraint}]$$

The monetary authority sets the following policy:

$$M_t = M_{t-1} + \tau_t = (1+\mu)M_{t-1}$$

Where μ is the money growth rate, and $\tau_t = \mu M_{t-1}$ is the additional cash printed each period. Finally, define the inflation rate as

$$\pi_t = \frac{P_{t+1}}{P_t} - 1$$

Questions:

a. Write the CIA constraint in real terms (by dividing P_t on both sides), and form the representative agent's Lagrangian function. (For Lagrange multipliers, please use $\lambda_{1,t}$ for the budget constraint and $\lambda_{2,t}$ for the CIA constraint.)

b. Find the first order conditions with respect to $[C_t]$, $[F_t]$, and $[m_t]$.

c. Combine the three first-order conditions into one equation, which does not contain the Lagrange multipliers.

d. When CIA constraint is binding, compare the marginal utilities $u'(F_t)$ and $u'(C_t)$. Explain why they are (or are not) equal.

e. Suppose the real income is $Y_t = Y$ in each period, and the economy is at a steady state:

$$C_{t+1} = C_t = C^*, F_{t+1} = F_t = F^*$$

Using the answers from parts c and d, what do you think should be the optimal money growth rate μ? Explain.