Intermediate Microeconomics
Spring 2025

Week 10: Monopoly

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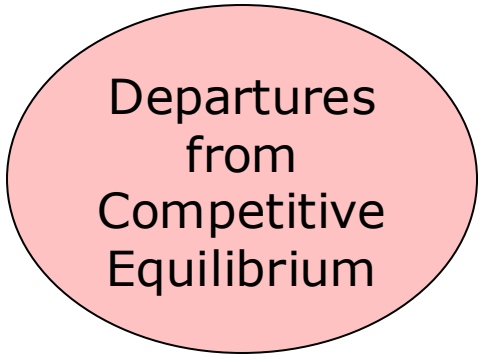
Big Picture



1. Violation of the "private good" assumption
2. Violation of the "price-taking" assumption **This class**
3. Violation of the "complete market" assumption

Road Map

1. Set up a simple model of a "monopolist"

- A firm that can single-handedly change market price for a good (by choosing to produce difference quantities)
- Study welfare consequences (quantity distortion, DWL) of such market power

2. Study pricing strategies monopolists can use to improve profits, when the market is populated with different types of consumers

- 1st degree price discrimination
- 2nd degree price discrimination
- 3rd degree price discrimination

Competitive Assumptions and Market Failures

- In the last lecture(s), we considered the impact of relaxing the competitive assumption on "private goods"
  - Market allocations in the presence of externalities are not efficient.

Competitive Assumptions and Market Failures

- In the last lecture(s), we considered the impact of relaxing the competitive assumption on "private goods"
  - Market allocations in the presence of externalities are not efficient.
- Today, we consider situations where the price taking assumption does not hold.
  - Price taking: firms believe that they can sell as much as desired at the posted price.
  - When price taking does not hold, firms believe that if they make more available for sale, the price will go down.
- We will focus on a particular case, Monopoly.
  - Monopoly: there is a single firm that sells the good.

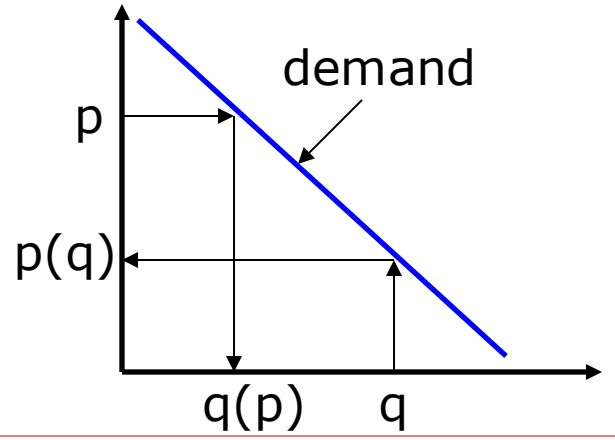
"Monopolist": Clarification

- We will use the term "monopolist" without any sentiment
  - It simply refers to a market structure where a firm can influence price of good
  - It does not mean the market structure is necessarily "bad" (we will have some discussion on why)
- Our goal is to study *what happens* when a firm has market power
- Whether a firm *should* be allowed to have market power is a completely different (and more complicated) issue
  - Depends on what type of commodity we are talking about, legal institutions, cultural context, etc.

Monopoly Pricing

- We typically think of the monopolist as choosing price and letting quantity be determined by the market.

- However, the demand curve gives a one-to-one relationship between prices and quantities.
  - We could also think of the monopolist as choosing quantity and letting the price be determined by the market.
  - This turns out to be a bit more convenient.

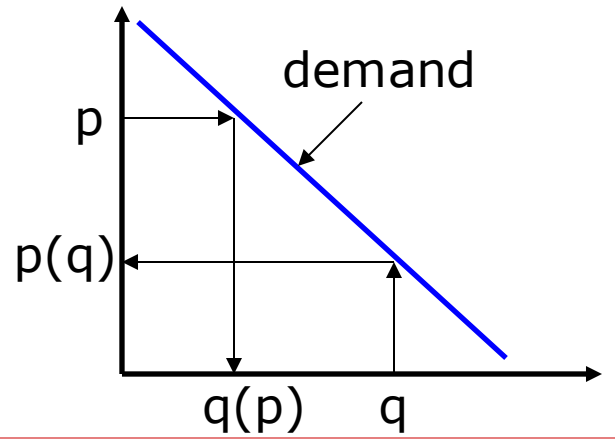


Monopoly Pricing

- Let p(q) denote the monopolist's (inverse) demand curve.
- The monopolist's problem can also be written:

$$\max_q \; p(q)\, q - c(q).$$

- We will generally work with this formulation.

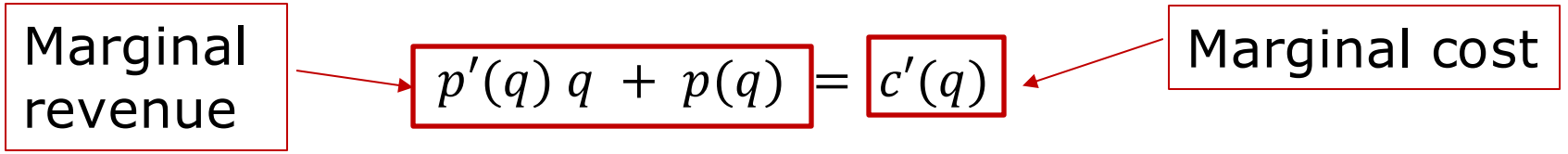


Profit Maximization

- The monopolist chooses q to maximize profit $p(q)\,q - c(q)$.
- Take derivative with respect to q, set equal to zero:

$$p'(q)\,q + p(q) - c'(q) = 0$$

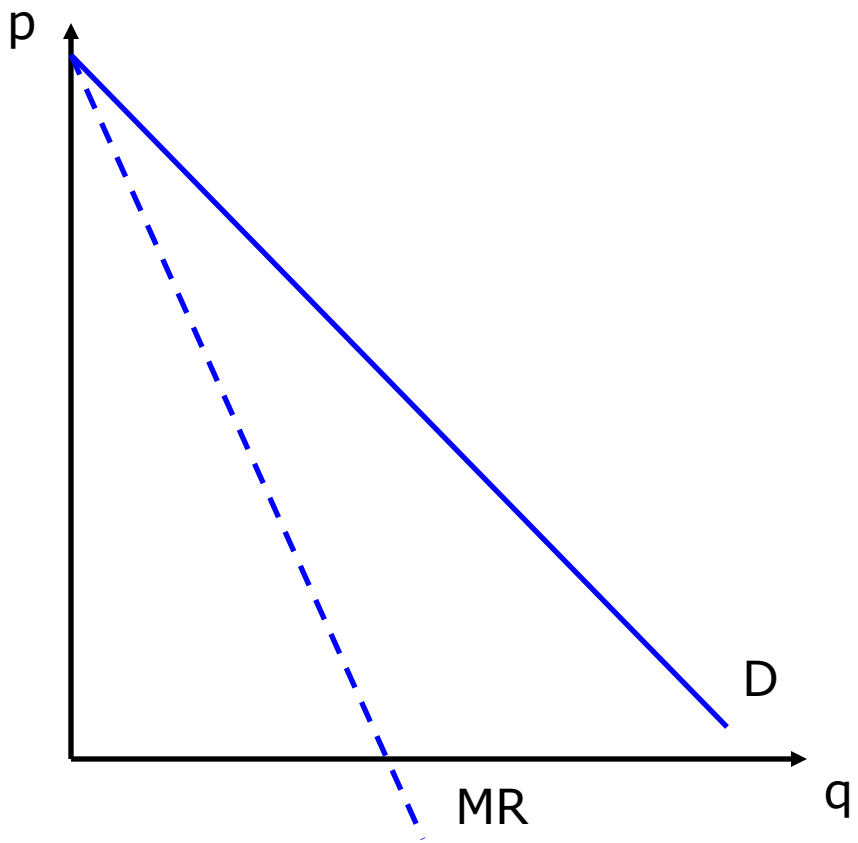
- Rearrange:



- **Marginal Revenue:** the rate at which revenue changes when you increase q by a small amount.
- Also need to consider whether it's better to produce $q^* = 0$

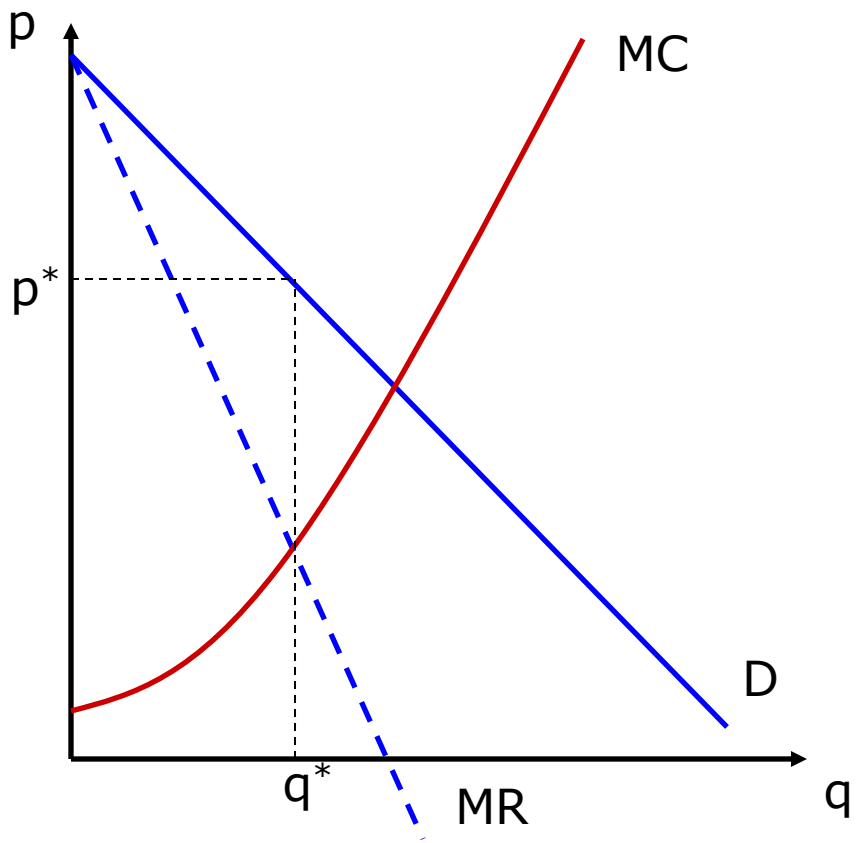
Optimal choice of q^*

- How do MR and p(q) compare?
- Demand slopes down: $p'(q)<0$
- So, $p'(q)\, q + p(q) < p(q)$
- Marginal revenue lies below D.



Optimal choice of q^*

- Monopolist chooses q^* where MR = MC.
- Optimal price p^* is found by evaluating p(q) at q^*.
- Frequent mistake: don't plug q^* into MR by mistake!



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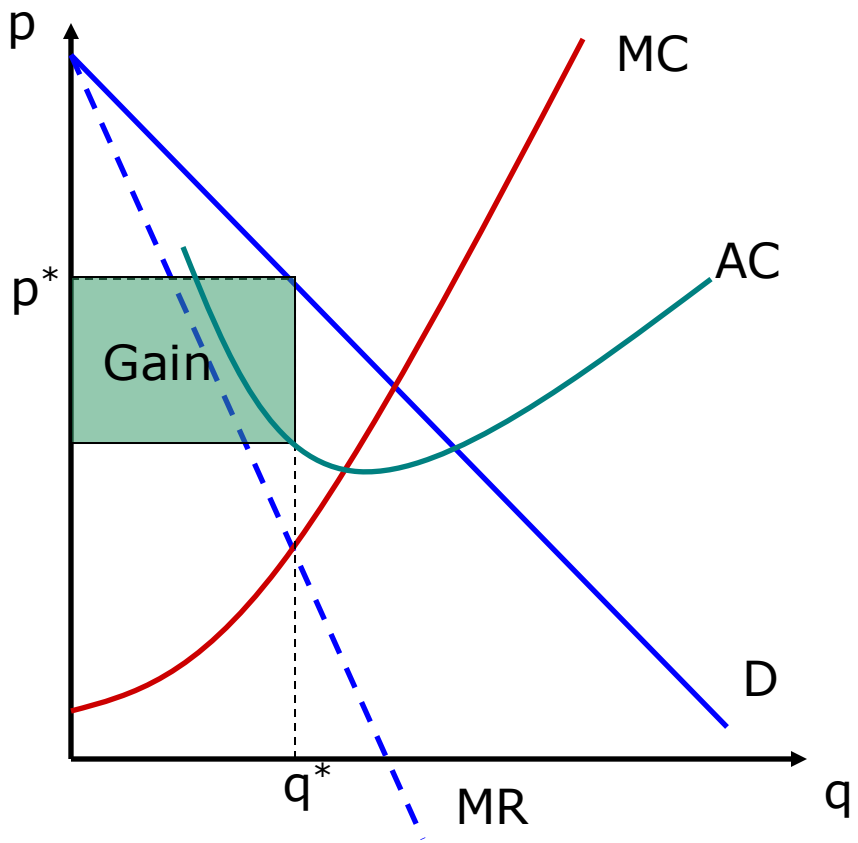
Optimal choice of q^*

- After finding q^*, compute profit: $p(q^*)\, q^* - c(q^*)$.
- Compare to profit if produce 0.
- If profit for q^* > profit for 0, produce.
- Otherwise, shut down/exit.



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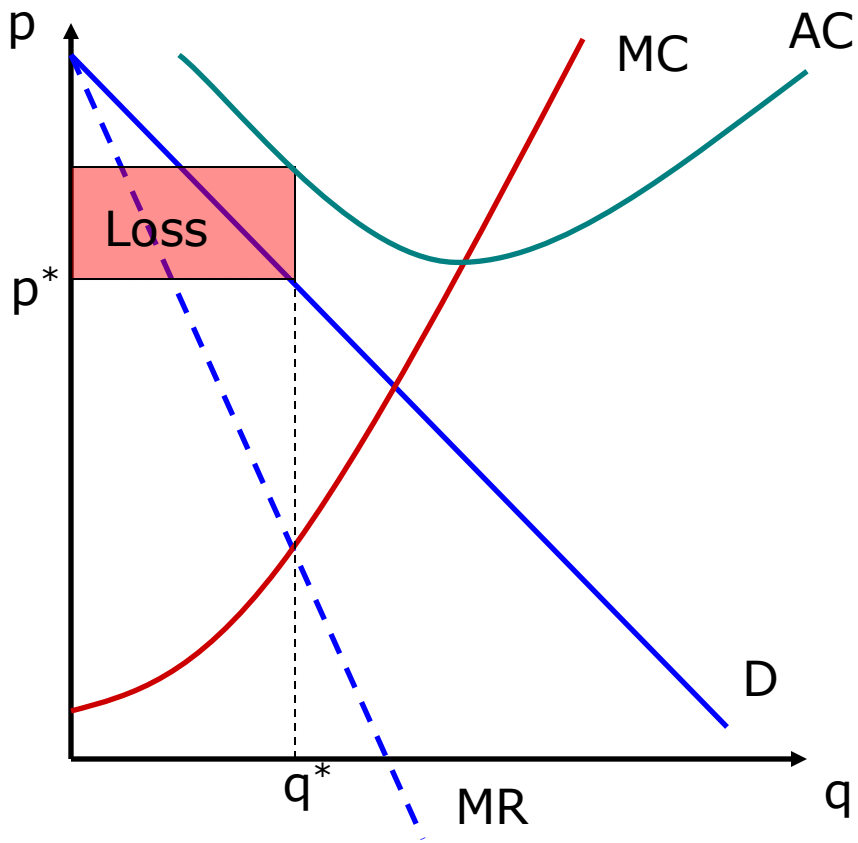

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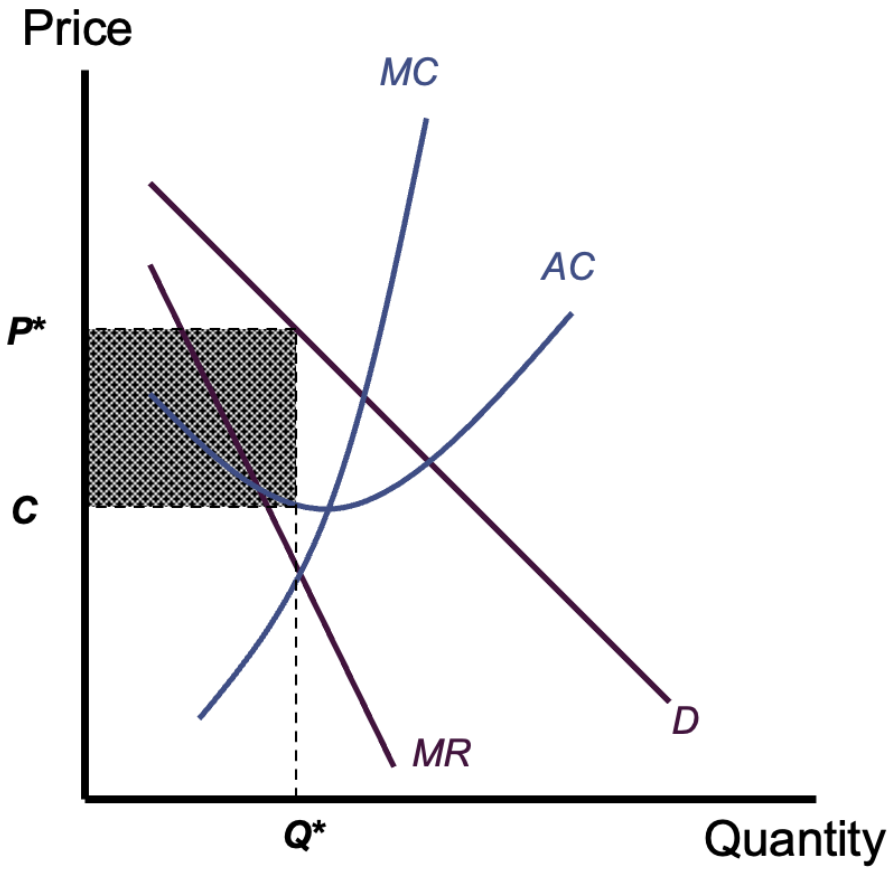
Monopoly Profits

- Monopoly profits will be positive as long as $P > AC$
- Monopoly profits can continue into the long run because entry is not possible
  - some economists refer to the profits that a monopoly earns in the long run as monopoly rents
    - the return to the factor that forms the basis of the monopoly

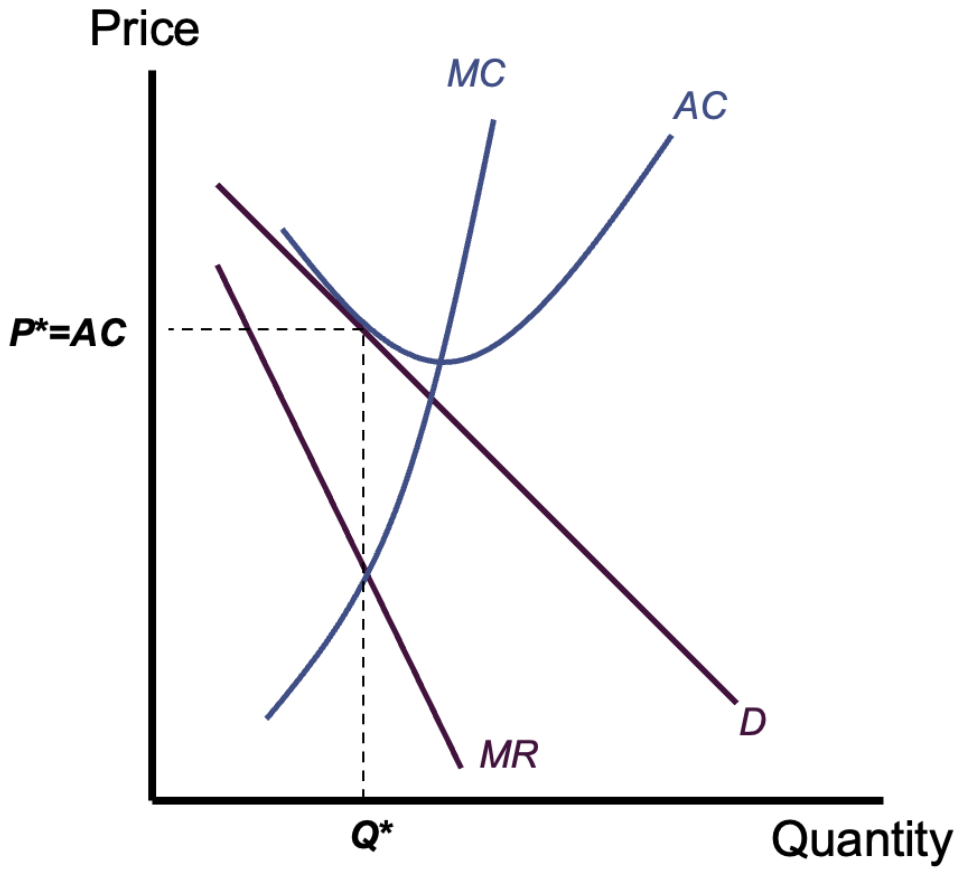
Monopoly Profits

- The size of monopoly profits in the long run will depend on the relationship between average costs and market demand for the product

Monopoly Profits



Positive profits



Zero profit

No Monopoly Supply Curve

- With a fixed market demand curve, the supply "curve" for a monopolist will only be one point
  - the price-output combination where $MR = MC$

- If the demand curve shifts, the marginal revenue curve shifts and a new profit-maximizing output will be chosen

Practice Example:
Monopoly with Linear Demand

- Suppose that the market for frisbees has a linear demand curve of the form

$$Q = 2{,}000 - 20P$$

- The total costs of the frisbee producer are given by

$$C(Q) = 0.05Q^2 + 10{,}000$$

- What is the monopolist's optimal price and quantity?

Monopoly with Linear Demand

- To maximize profits, the monopolist chooses the output for which $MR = MC$
- We need to find total revenue

$$TR = P \cdot Q = 100Q - Q^2/20$$

- Therefore, marginal revenue is

$$MR = 100 - Q/10$$

while marginal cost is

$$MC = 0.01Q$$

Monopoly with Linear Demand

- Thus, $MR = MC$ where

$$100 - Q/10 = 0.01Q$$

$$Q^* = 500 \qquad P^* = 75$$

- At the profit-maximizing output,

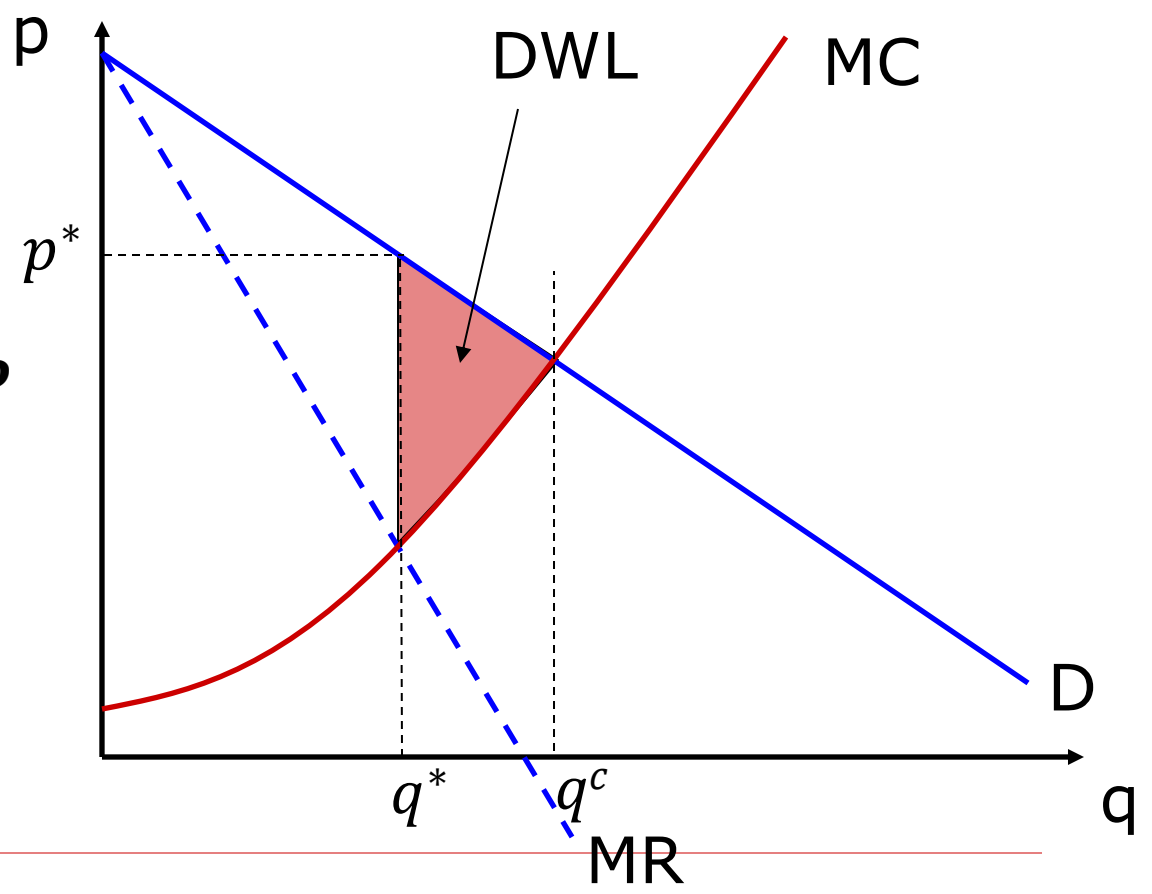
$$C(Q) = 0.05(500)^2 + 10{,}000 = 22{,}500$$

$$AC = 22{,}500/500 = 45$$

$$\pi = (P^* - AC)Q = (75 - 45)\cdot 500 = 15{,}000$$

DWL of Monopoly

- Because $MR < D$, $q^c > q^*$ → there is a deadweight loss from monopoly.
  - Units between q^* have a positive surplus (MU > MC) but are not produced.
  - Left to their own devices, ***monopolists always produce less than firms with no market power***.
- Questions to ponder:
  - Is there always a DWL?



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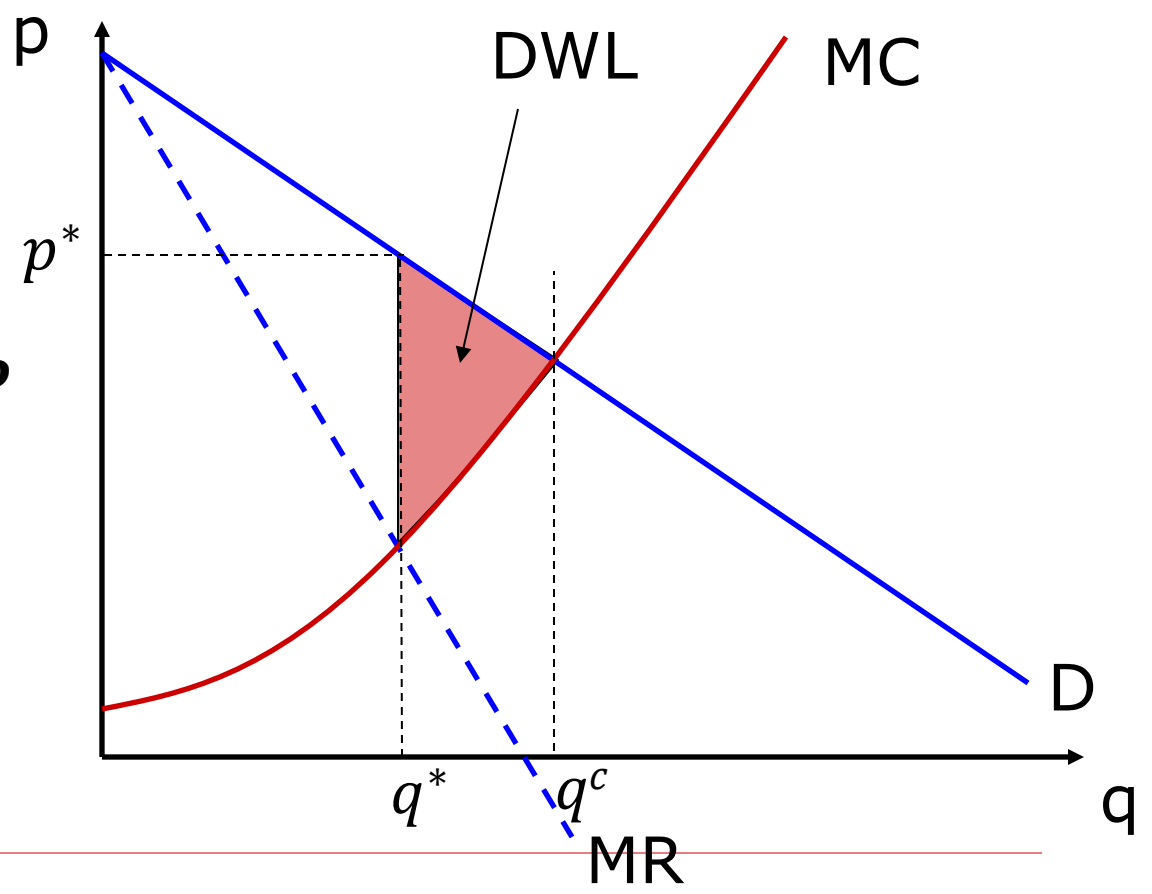


- Questions to ponder:
  - Is there always a DWL?
  - It depends. If there is a negative externality AND a monopolist, a monopolist could produce too much OR too little relative to the socially optimal quantity.

Barriers to Entry

- The reason a monopoly exists is that other firms find it unprofitable or impossible to enter the market

- Barriers to entry are the source of all monopoly power

- there are two general types of barriers to entry
  - technical barriers
  - legal barriers

Technical Barriers to Entry

- The production of a good may exhibit decreasing marginal and average costs over a wide range of output levels

- in this situation, relatively large-scale firms are low-cost producers
  - this situation is known as natural monopoly
  - once the monopoly is established, entry of new firms will be difficult

- Another technical basis of monopoly is special knowledge of a low-cost productive technique
  - it may be difficult to keep this knowledge out of the hands of other firms

- Ownership of unique resources may also be a lasting basis for maintaining a monopoly

Legal Barriers to Entry

- Many monopolies are created as a matter of law
  - with a patent, the basic technology for a product is assigned to one firm
  - the government may also award a firm an exclusive franchise to serve a market

Creation of Barriers to Entry

- Some barriers to entry result from actions taken by the firm
  - research and development of new products or technologies
  - purchase of unique resources
  - lobbying efforts to gain monopoly power

- The attempt by a monopolist to erect barriers to entry may involve real resource costs

Monopoly and Product Quality

- The market power enjoyed by a monopoly may be exercised along dimensions other than the market price of its product
  - type, quality, or diversity of goods

- Whether a monopoly will produce a higher-quality or lower-quality good than would be produced under competition depends on demand and the firm’s costs

Monopoly and Product Quality

- Suppose that consumers' willingness to pay for quality (X) is given by the inverse demand function $P(Q,X)$ where

$$\partial P/\partial Q < 0 \text{ and } \partial P/\partial X > 0$$

- If costs are given by $C(Q,X)$, the monopoly will choose Q and X to maximize

$$\pi = P(Q,X)Q - C(Q,X)$$

Monopoly and Product Quality

- First-order conditions for a maximum are

(1) $$\frac{\partial \pi}{\partial Q} = P(Q, X) + Q\frac{\partial P}{\partial Q} - C_Q = 0$$

  - $MR = MC$ for output decisions

(2) $$\frac{\partial \pi}{\partial X} = Q\frac{\partial P}{\partial X} - C_X = 0$$

  - Marginal revenue from increasing quality by one unit is equal to the marginal cost of making such an increase

Monopoly and Product Quality

- The level of product quality that will be opted for under competitive conditions is the one that maximizes net social welfare

$$SW = \int_0^{Q^*} P(Q, X)dQ - C(Q, X)$$

- Maximizing with respect to X yields

$$\frac{\partial SW}{\partial X} = \int_0^{Q^*} P_X(Q, X)dQ - C_X = 0$$

Monopoly and Product Quality

- The difference between the quality choice of a competitive industry and the monopolist is:
  - the *monopolist* looks at the marginal valuation of one more unit of quality assuming that Q is at its profit-maximizing level

$$\frac{\partial \pi}{\partial X} = Q^M \frac{\partial P}{\partial X} - C_X = 0$$

  - the *competitive* industry looks at the marginal value of quality averaged across all output levels

$$\frac{\partial SW}{\partial X} = \int_0^{Q^*} P_X(Q, X) dQ - C_X = 0$$

Monopoly and Product Quality

- Even if a monopoly and a perfectly competitive industry chose the same output level, they might opt for different quality levels
  - each is concerned with a different margin in its decision making

Practice example: product quality

- Suppose a monopolist produces alkaline batteries that may have various useful lifetimes (X). Suppose also that consumers' (inverse) demand depends on batteries' lifetimes and quantity (Q) purchased according to the function

$$\mathrm{P}(\mathrm{Q}, X) = g(X * Q),$$

  where $g' < 0$.
- That is, consumers care only about the product of quantity times lifetime (X∗Q): They are willing to pay equally for many short−lived batteries or few long−lived ones. (*Hint:* Treat XQ as a composite commodity.)
- Assume also that battery costs are given by

$$C(Q, X) = C(X) * Q$$

  where C'(X)>0.
- Show that, in this case, the monopoly will opt for the same level of X as does a competitive industry even though levels of output and prices may differ.

Road Map

1. Set up a simple model of a "monopolist"
   - A firm that can single-handedly change market price for a good (by choosing to produce difference quantities)
   - Study welfare consequences (quantity distortion, DWL) of such market power

2. **Study pricing strategies monopolists can use to improve profits, when the market is populated with different types of consumers**
   - 1st degree price discrimination
   - 2nd degree price discrimination
   - 3rd degree price discrimination

Price Discrimination

- We will now look at pricing techniques the monopolist can use to improve profit when it faces *heterogeneous* consumers.

- **Price discrimination**: charging different prices to different consumers, or different groups of consumers. Categorized into one of three **degrees**:

  - **First degree:** charging each consumer his willingness to pay.
  - **Second degree:** offering a menu of options from which consumers can choose (self-selection mechanisms).
  - **Third degree:** charging different prices to different types of consumers based on an observable characteristic.

- Note: I've never been clear on why these are ordinal. They're really just three different types of techniques.

First Degree Price Discrimination

- Each consumer is charged an individual price.
  - Also known as perfect price discrimination.

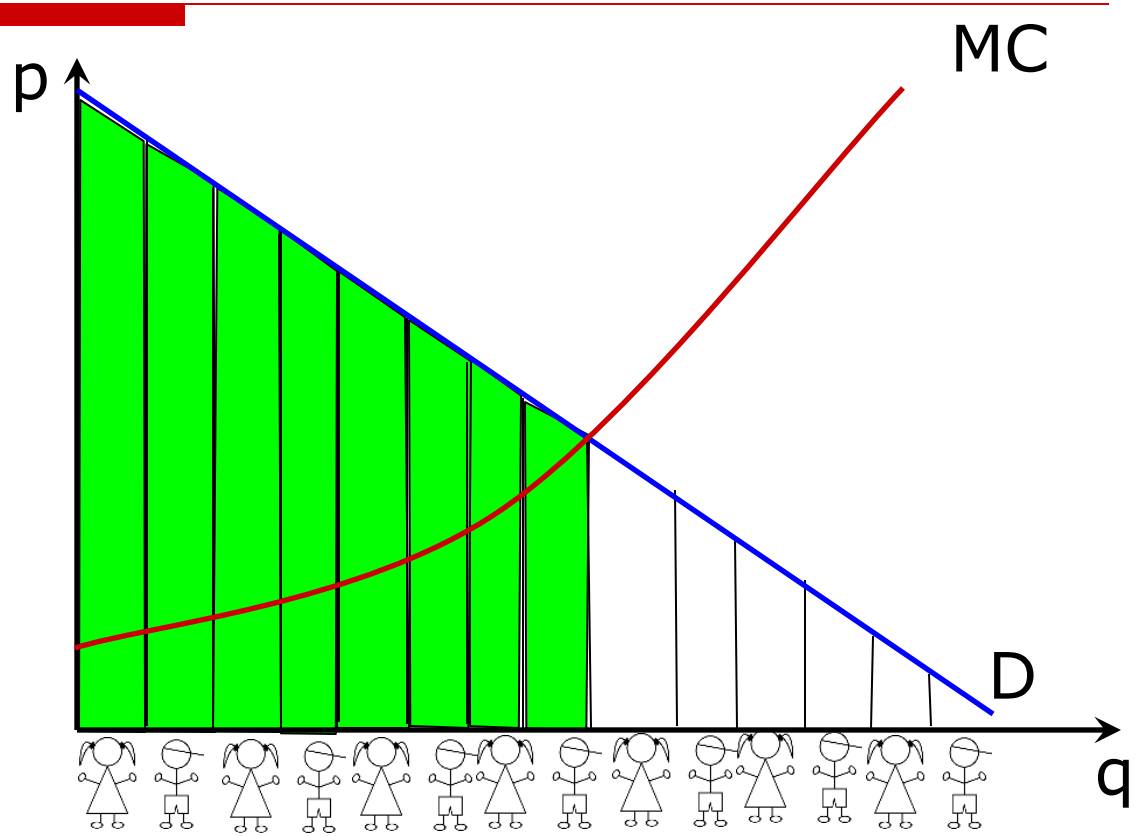
- Assumption: monopolist can observe each individual's **willingness to pay** for the product.

- Because the monopolist can observe willingness to pay and charge an individually tailored price, the monopolist will charge each individual his/her full willingness to pay.

- Just like our perfectly competitive market assumptions, it's unrealistic to expect this ever to hold perfectly. But it serves as a useful benchmark (and something for the firm with market power to aspire to!).

First Degree P.D.

- Think of the demand curve as consisting of a lot of people, each of whom wants to buy one unit.
- Monopolist can observe each person's wtp ➔ for every unit sold, charge wtp.
- Profit can be made if $p > MC$ ➔ Sell to where D and MC cross.
- Profit is equal to maximum total surplus.
- Note: can also think of this as a single consumer wanting multiple units of a good and the monopolist charging different price for each unit.



Practice example: First Degree P.D.

- Recall the example of the frisbee manufacturer
- $Q = 2{,}000 - 20P$
- $C(Q) = 0.05Q^2 + 10{,}000$

- If this monopolist practices perfect price discrimination, calculate the quantity produced and the profit.

DWL of Monopoly and Uniform Pricing

- Notice there is NO deadweight loss in 1st degree P.D.!
  - Monopolist quantity = competitive quantity

- The DWL of monopoly arises partly because the monopolist charges the same price for all units (**uniform pricing or linear pricing**).

- But, if we allow the monopolist to use a more complicated pricing scheme, then monopolist can increase profit and reduce DWL.
  - Underscores the idea that deadweight loss arises because of *quantity* distortion, not because of *price* distortion.
  - Getting the price "wrong" doesn't matter for deadweight loss if you get quantity right (but obviously results in a very different distribution of surplus).

Is first degree price discrimination possible?

- To be able to first-degree price discriminate effectively, the monopolist would need to know your income, savings, debts, history of purchases, etc.
- Most firms are unlikely to have access to such information.
- One reason people worry about online privacy is that the more information firms have about you, the easier it is for them to price discriminate.

Online shopping

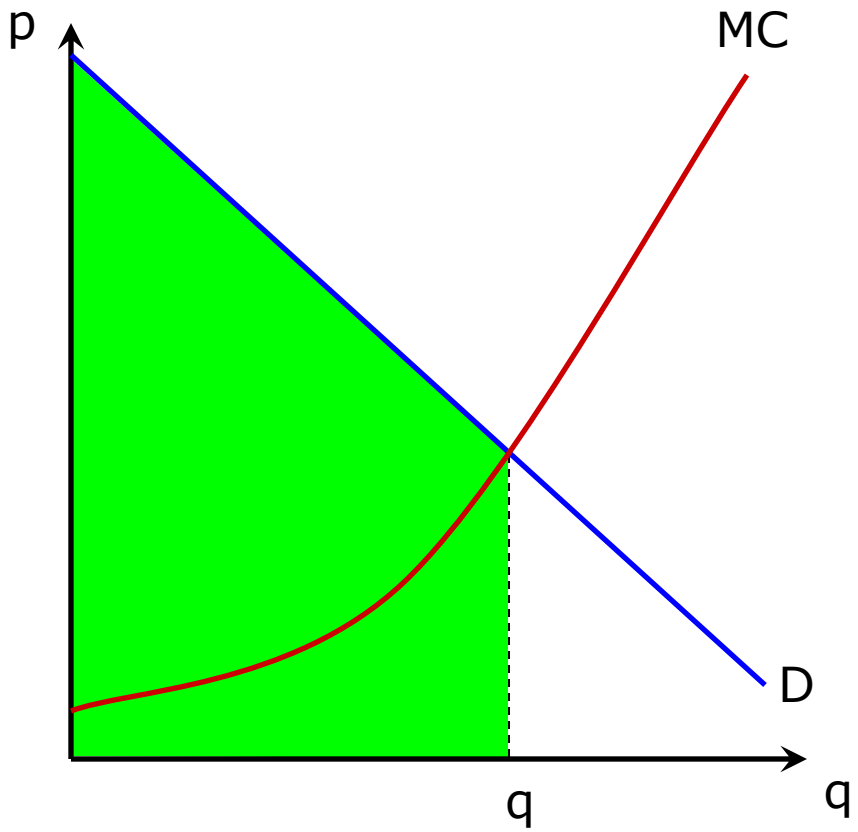
Two part tariffs

- Suppose there is uniform price regulation (i.e., monopolist is only allowed to charge a same price for different units)

- Monopolist can still achieve the 1st Degree P.D. revenue by running a "two part tariff" strategy:
  - Produce competitive quantity Q*, charge competitive price P* for each unit
  - Charge a "fixed fee" = 1st Degree P.D profit – competitive profit

Two part tariffs

- Suppose M charges a "two-part tariff" consisting of:
  - A fixed fee, F, and
  - A per-unit price, p.
- Consumer's total payment for q is F +p*q.
- The right two-part tariff also extracts max social surplus.



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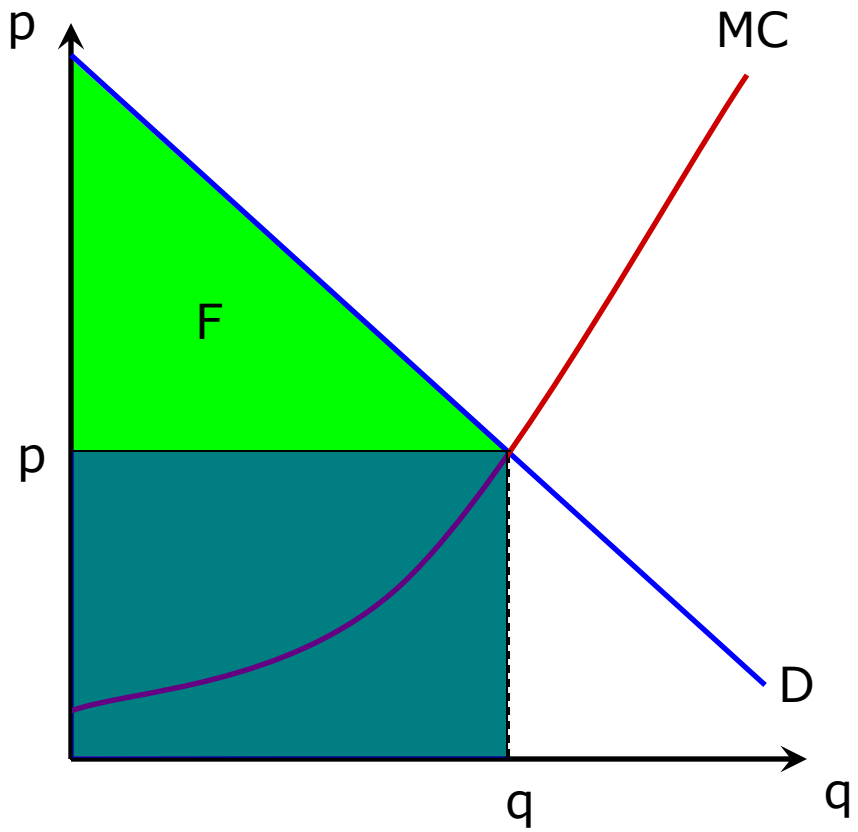
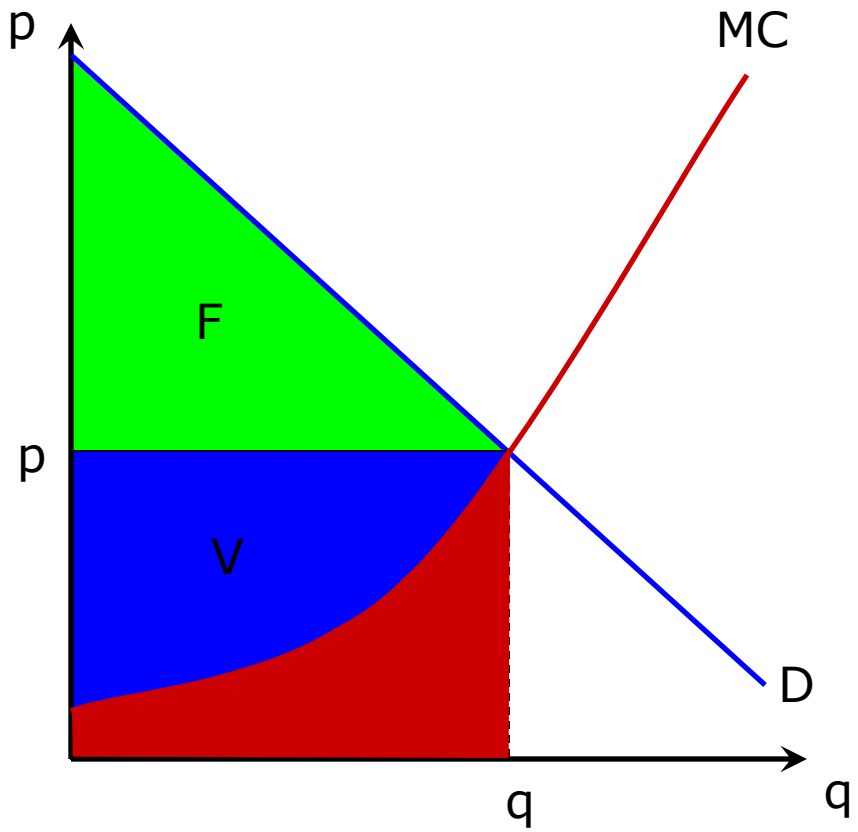
Two part tariffs

- To extract full surplus, consumer must consume q.
- So, p = p(q).
- Consumer pays p*q for q units.
- What should F be?
  - Remaining CS is area above p, below D.
  - Set F = this area.



Two part tariffs

- Revenue from per-unit price (blue+red).
- Revenue from fixed fee (green).
- Variable cost (red).
- Profit (except for fixed cost) is area F + V.
- F+V is maximum social surplus, so efficient.



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Practice example: Two-Part Tariffs

- Suppose there are two different buyers with the demand functions

$$q_1 = 24 - p_1$$

$$q_2 = 24 - 2p_2$$

and $MC = 6$.

- Suppose the monopolist wants to use the two-part tariffs pricing strategy, find out the per-unit price and the fixed fee.

Third Degree Price Discrimination

- 3rd degree P.D.: Charging different prices to different groups of consumers based on an observable characteristic.

- Suppose there are two groups of consumers that differ in their willingness to pay and on an observable characteristic.
  - Examples
    - Age (senior citizen discount at the movies)
    - Gender (ladies' night promotions, car insurance).
    - Student/non-student (shows, concerts).

- Be careful: it can be illegal to charge higher prices based on certain characteristics.

3rd Degree Price Discrimination

- Suppose there are two groups, 1 and 2, that differ on an observable characteristic.
- Demand curve for group i is given by $p_i(q_i)$.
- Marginal cost depends on total production $c(q_1 + q_2)$.
- How should monopolist set q_1 and q_2 (alternately, p_1 and p_2) to maximize profit?

3rd Degree P.D.: solving.

- Step 1: write down profit:

$$p_1(q_1)\, q_1 + p_2(q_2)\, q_2 - c(q_1 + q_2)$$

- To maximize, take derivative wrt q_1, q_2, set = 0.

$$p'_1(q_1)\, q_1 + p_1(q_1) = c'(q_1 + q_2)$$

$$p'_2(q_2)\, q_2 + p_2(q_2) = c'(q_1 + q_2)$$

- So, monopolist sets $MR_1 = MR_2 = MC$.
- Note that this is not the same as setting *price* equal for the two groups.
- Why set MR equal? If $MR_1 > MR_2$, what should monopolist do?

Practice Example: Football tickets

- Suppose there are two groups, alumni and students.
- Marginal cost of a seat is 0.

- Demand:
  - Alumni: $p_a(q_a) = 100 - q_a$.
  - Students: $p_s(q_s) = 20 - 0.1q_s$.
  - Students have lower wtp, but there are more of them.

- Compute the price set for alumni and students, and total profit.

Example: Football tickets

- Thought experiment: suppose you have to sell one additional ticket because University wants a “sell out.” BUT all tickets for a group have to sell at the same price. Which group should get the ticket?

卖给学生。因为q**增加1，**学生组的价格下降更小。

Market Separation

- All the monopolist needs to know in this case is the price elasticities of demand for each market
  - set price according to the inverse elasticity rule

$$\frac{P - MC}{P} = -\frac{1}{e_{Q,P}}$$

- If the marginal cost is the same in all markets,

$$P_i(1+\frac{1}{e_i}) = P_j(1+\frac{1}{e_j})$$

Market Separation

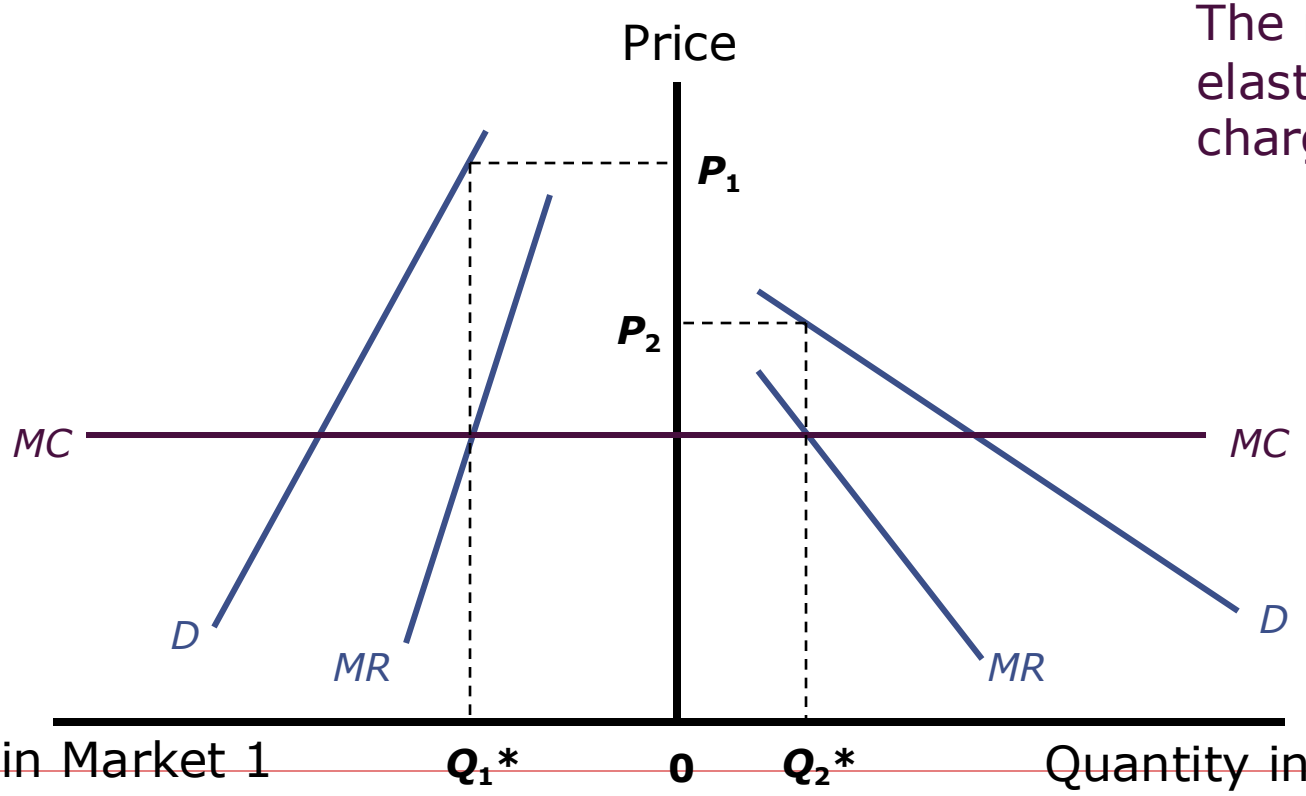
- This implies that

$$\frac{P_i}{P_j} = \frac{(1+\frac{1}{e_j})}{(1+\frac{1}{e_i})}$$

- The profit-maximizing price will be higher in markets where demand is less elastic.

Market Separation

If two markets are separate, maximum profits occur by setting different prices in the two markets



The market with the less elastic demand will be charged the higher price

Regulation of Monopoly

- Sometimes, even though monopolies tend to result in DWL, it is still the best way to organize a market.
- For example, consider an industry with large fixed costs and low marginal costs, like a power plant.
- Here, there are economies of scale (i.e., AC is decreasing).
- So, the average cost of production is minimized by having only one producer.
- Industries such as this are known as "natural monopolies."
  - Examples: electric, phone, water utilities.

Google, Facebook, Amazon And The Future Of Antitrust Laws

Natural Monopoly: Definition

- Formally, an industry is a natural monopoly if its cost function is **sub-additive.**

- That is, for any Q and Q'<Q, C(Q) < C(Q-Q') + C(Q').
  - For any Q, it is cheaper to produce Q at one plant than to divide it among two identical plants.

- Decreasing AC ➔ sub-additivity, proof?

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步骤 2：将分割后的成本 $C(Q')$ 和 $C(Q-Q')$ 分别表示为：

$$C(Q') = Q' \cdot AC(Q'), \quad C(Q-Q') = (Q-Q') \cdot AC(Q-Q').$$

步骤 3：利用平均成本递减的性质：

- 因为 $Q' < Q$，有 $AC(Q) < AC(Q')$；
- 同理，$Q - Q' < Q$，故 $AC(Q) < AC(Q-Q')$。

步骤 4：比较总成本与分割后的成本之和：

$$C(Q') + C(Q-Q') = Q' \cdot AC(Q') + (Q-Q') \cdot AC(Q-Q').$$

由于 $AC(Q') > AC(Q)$ 且 $AC(Q-Q') > AC(Q)$，可得：

$$Q' \cdot AC(Q') + (Q-Q') \cdot AC(Q-Q') > Q' \cdot AC(Q) + (Q-Q') \cdot AC(Q).$$

右侧化简为：

$$Q' \cdot AC(Q) + (Q-Q') \cdot AC(Q) = Q \cdot AC(Q) = C(Q).$$

因此：

$$C(Q') + C(Q-Q') > C(Q),$$

即：

$$C(Q) < C(Q') + C(Q-Q').$$

Natural Monopoly and Regulation

- If an industry is a natural monopoly, it may be efficient to have a single firm produce.

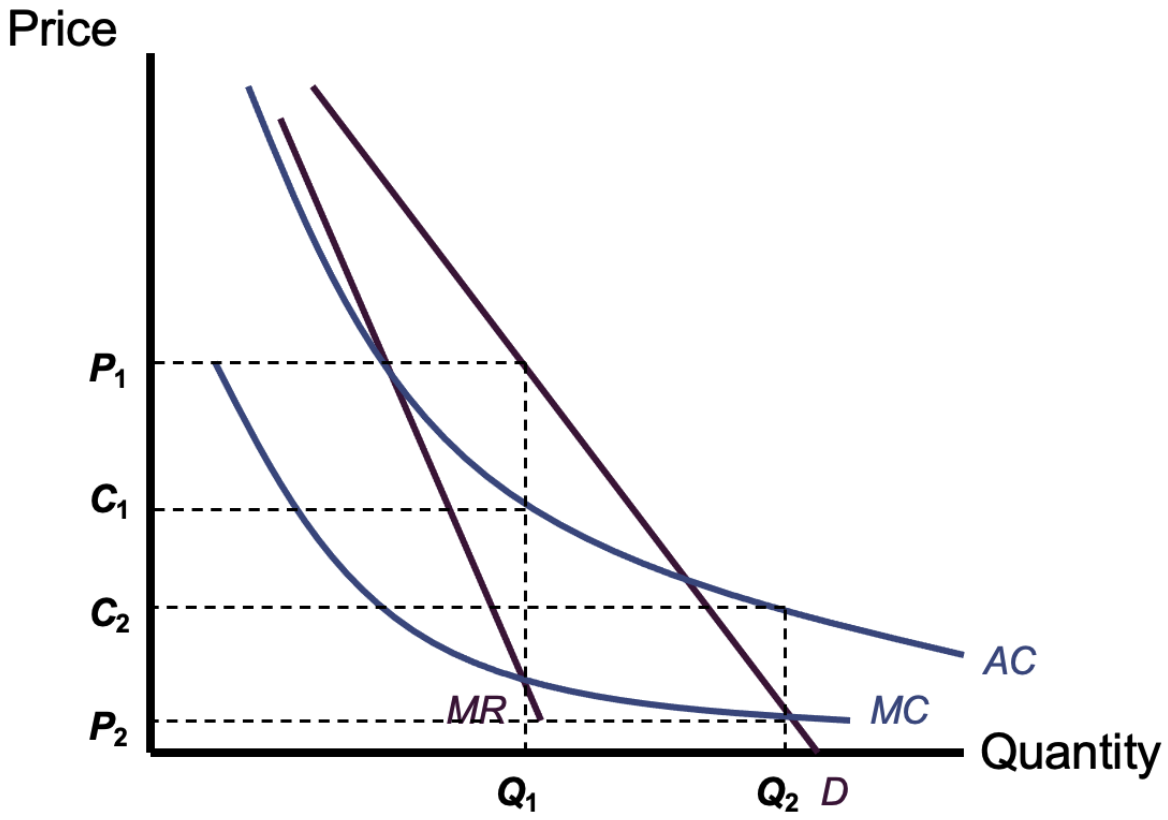
- But, if the producer is a monopoly, it may exercise its monopoly power.
  - Leads to DWL.
  - In the case of utilities, we think that *access* is important
    - electricity or gas for heat in winter.
    - phone to call 110.

- In many cases governments solve this problem by designating a monopolist to operate but then regulating the prices that the monopolist can charge.

Regulation of Monopoly

- Natural monopolies such as the utility, communications, and transportation industries are highly regulated in many countries
- Many economists believe that it is important for the prices of regulated monopolies to reflect marginal costs of production accurately
- An enforced policy of marginal cost pricing will cause a natural monopoly to operate at a loss
  - natural monopolies exhibit declining average costs over a wide range of output

Regulation of Monopoly

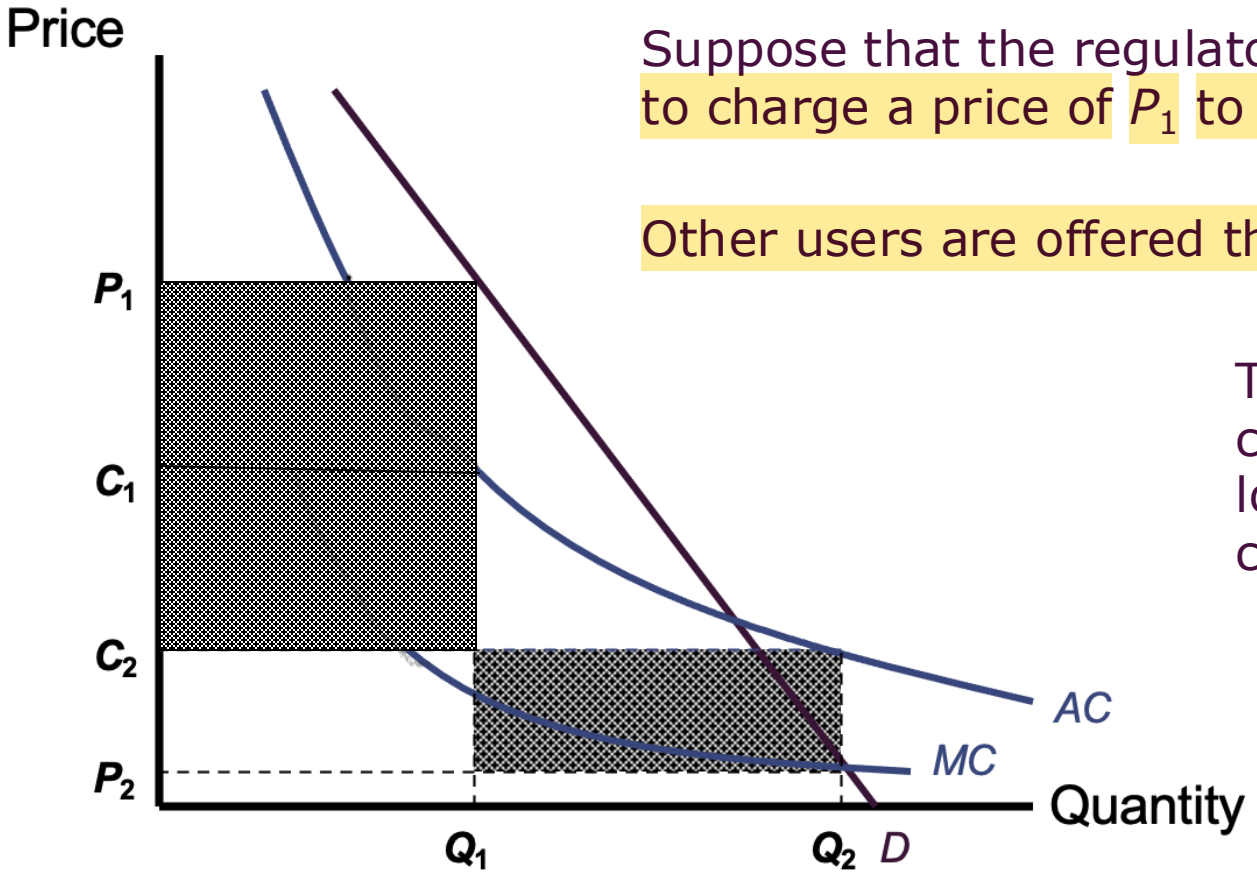


Because natural monopolies exhibit decreasing costs, *MC* falls below *AC*

An unregulated monopoly will maximize profit at Q_1 and P_1

An enforced policy of marginal cost pricing: If regulators force the monopoly to charge a price of P_2, the firm will suffer a loss because $P_2 < C_2$

Regulation of Monopoly



Suppose that the regulatory commission allows the monopoly to charge a price of P_1 to some users

Other users are offered the lower price of P_2

The profits on the sales to high-price customers are enough to cover the losses on the sales to low-price customers

Rate of return regulation

- Another approach followed in many regulatory situations is to allow the monopoly to charge a price above marginal cost that is sufficient to earn a “fair” rate of return on capital investment

- If this rate of return is greater than that which would occur in a competitive market, there is an incentive to use relatively more capital than would truly minimize costs

Regulation of Monopoly

- Suppose that a regulated utility has a production function of the form

$$q = f(k,l)$$

- The firm's actual rate of return on capital is defined as

$$s = \frac{pf(k,l) - wl}{k}$$

Regulation of Monopoly

- Suppose that s is constrained by regulation to be equal to s_0, then the firm's problem is to maximize profits

$$\pi = pf(k,l) - wl - vk$$

subject to this constraint $s=s_0$

- The Lagrangian for this problem is

$$\mathbf{L} = pf(k,l) - wl - vk + \lambda[wl + s_0k - pf(k,l)]$$

Regulation of Monopoly

- If $\lambda=0$, regulation is ineffective and the monopoly behaves like any profit-maximizing firm

- If $\lambda=1$, the Lagrangian reduces to

$$\mathbf{L} = (s_0 - v)k$$

which (assuming $s_0 > v$), will mean that the monopoly will hire infinite amounts of capital – an implausible result

Regulation of Monopoly

- Therefore, $0<\lambda<1$ and the first-order conditions for a maximum are:
  - (1) $\dfrac{\partial \mathbf{L}}{\partial l} = pf_l - w + \lambda(w - pf_l) = 0$
  - (2) $\dfrac{\partial \mathbf{L}}{\partial k} = pf_k - v + \lambda(s_0 - pf_k) = 0$
  - (3) $\dfrac{\partial \mathbf{L}}{\partial \lambda} = wl + s_0 - pf(k,l) = 0$

Regulation of Monopoly

- Because $s_0 > v$ and $\lambda < 1$, this means that
  - (2) $\frac{\partial \mathbf{L}}{\partial k} = pf_k - v + \lambda(s_0 - pf_k) = 0$

$$(1-\lambda)pf_k = v - \lambda s_0 < v - \lambda v = (1-\lambda)v$$

$$pf_k < v$$

- The firm will use more capital than it would under unregulated conditions ($pf_k = v$),
- it will also achieve a lower marginal productivity of capital

Practice example: subsidy

- Suppose the government wishes to combat the undesirable allocational effects of a monopoly through the use of a subsidy.

- What is the desirable allocation?

- Why would a lump-sum subsidy not achieve the government's goal?

Practice example: subsidy

- Use a graphical proof to show how a per-unit-of-output subsidy might achieve the government's goal.

Practice example: subsidy

- Suppose the government wants its subsidy to maximize the difference between the total value of the good to consumers and the good's total cost.

- Show that, to achieve this goal, the government should set

$$\frac{t}{P} = -\frac{1}{e_{D,P}}$$

- where *t* is the per-unit subsidy and *P* is the competitive price. Explain your result intuitively.

Dynamic Views of Monopoly

- Some economists have stressed the beneficial role that monopoly profits can play in the process of economic development
- these profits provide funds that can be invested in research and development
- the possibility of attaining or maintaining a monopoly position provides an incentive to keep one step ahead of potential competitors