Intermediate Microeconomics – Spring 2025

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Problems Set 2

**Due Monday March 24th**

**Instructions:**

1. You can either write down your answers by hand or type out your solutions. Please hand in a hard copy of your answers and make sure your work is stapled. Alternatively, you can submit a digital copy to the TA via email. Most importantly, please remember to indicate your name and student number on your homework.

2. The problem sets should be submitted to the TA before the beginning of the lecture on the due day. Late submissions will NOT be accepted. If you have any emergency that prevents you from coming to the lecture, please email an electronic version of your problem set to the TA before the class.

3. You are encouraged to discuss with your classmates, but please write down your answers individually. Directly copying others' answers may result in zero points.

**Question 1: (Consumer's Problem)**

A utility-maximizing consumer has an income m > 0, which he/she allocates between two goods (1 and 2). For each good, the consumer faces a constant price, $p_x > 0$ and $p_y > 0$, respectively. For each of the following utility function, derive the consumer's optimal demand $x^*(p_x, p_y, m)$ and $y^*(p_x, p_y, m)$.

(1) $u(x, y) = \sqrt{xy}$

(2) $u(x, y) = min\{x^2, y\}$

(3) $u(x, y) = 2\sqrt{x} + y$

(4) $u(x, y) = \sqrt{x} + \sqrt{y}$

(5) $u(x, y) = \begin{cases} x + y, & \text{if } 0 \leq x \leq 20 \\ 20 + y, & \text{if } x > 20 \end{cases}$

**Question 2: (Choice Problem)**

Dave is deciding how much to work in the coming year. He derives utility from consumption, C, but he also really likes taking leisure time L. He must divide his available hours between work and leisure - for every hour of leisure he takes he must work one fewer hours. The function that describes his preferences is given by

$$U(C, L) = \sqrt{CL}$$

He can earn a wage of *w*, and suppose the price of consumption is given by $p = 1$.

a. What is Dave's full income if he can work 2000 hours in a year? What is his measured income if he takes *L* hours for leisure? Write down his budget constraint.

b. Solve Dave's utility maximization problem and write down his optimal consumption of C and L. What would Dave's choice of leisure and consumption be if w=20?

c. A payroll tax is proposed that would tax Dave's labor income by 25%. Write Dave's new budget constraint and graph it together with the budget constraint where there is not tax. How much does Dave effectively earn per hour?

d. What is Dave's new choice for leisure and consumption?

e. Graphically depict the income and substitution effects on leisure hours associated with the tax.

f. Will the income and substitution effects have the same sign? What is the intuition for this?

**Question 3 (Consumer Price Index (CPI) and Cost-of-living Adjustments (COLA). )**

Motivation: Prices change over time (inflation or deflation)

Consumer Price Index (CPI) : an index to measure the cost of a standard bundle of goods over time.

We have a reference year or base year (with the base year changing every few years).

Consider two years: year 0 and year 1

$$I_0 = p_x^0 x_0 + p_y^0 y_0$$

$$I_1 = p_x^1 x_0 + p_y^1 y_0$$

$p_x^0 =$ price of $x$ in year 0, $p_y^0 =$ price of $y$ in year 0, $(x_0, y_0) =$ market basket of goods.

$p_x^1 =$ price of $x$ in year 1, $p_y^1 =$ price of $y$ in year 1,

$I_0 =$ expenditure in year 0, $I_1 =$ expenditure in year 1.

Dividing $I_1$ by $I_0$, then

$$CPI = \frac{I_1}{I_0} = \frac{p_x^1 x_0 + p_y^1 y_0}{p_x^0 x_0 + p_y^0 y_0}$$

(a) Show that CPI is a weighted average of the price increase for each good, $\frac{p_x^1}{p_x^0}$ and $\frac{p_y^1}{p_y^0}$, where the weights are each good's budget share in the base year, $s_x^0 = \frac{p_x^0 x_0}{I_0}$ and $s_y^0 = \frac{p_y^0 y_0}{I_0}$.

(b) Problems with CPI: Substitution bias.

By confining to a fixed basket of goods, the index is a biased measure of the cost of living because it does not consider the possibility that consumers make substitutions in the basket of goods when their prices change. The result is that economists estimate that CPI in the U.S. overstates the cost of living by approximately 1.1 percentage points annually.

(b.1) Fill in the blanks in the table:

Cost-of-Living Adjustments (COLA):

$$U(x, y) = 20\sqrt{xy}$$

| | $p_x$ | $p_y$ | $I$ | $x^*$ | $y^*$ | $U$ |
|---|---|---|---|---|---|---|
| Base year | \$1 | \$4 | \$400 | 200 | 50 | 2,000 |
| First year | \$2 | \$5 | | | | |
| No adjustment | | | \$400 | 100 | 40 | 1,265 |
| CPI adjustment | | | \$650 | | | |
| True COLA | | | | | | 2,000 |

Where $x^*$ and $y^*$ are optimal level of consumption choices of the two goods x and y given the utility maximization problem.

Let $I^* =$ income that would keep utility at the base year level (true COLA income), i.e.,

$U(p_x^0, p_y^0, I_0) = U(p_x^1, p_y^1, I^*)$

Calculate:

(b.2) $CPI = \frac{I_1}{I_0}$;

(b.3) increase in the true cost of living = $\frac{I^*}{I_0}$;

(b.4) substitution bias in the CPI = $\frac{I_1}{I^*}$ .

**Question 4 (Cost Function)**

Consider the following production functions:

$$F(K, L) = K^2 L^2$$

$$F(K, L) = K^{\frac{1}{3}} L^{\frac{2}{3}}$$

a. What is the returns to scale for each function (use formal argument with $\lambda$)?

Let $w_L = w_K = 1$.

b. Find the cost functions for each of the production functions.

c. Plot the cost functions on the same graph with y on the horizontal axis and cost on the vertical one.

d. Find and plot the average and marginal cost functions with y on the horizontal axis and average cost on the vertical one.