

$$z_3 = \frac{z_1 + z_2}{|z_1 + z_2|}$$

where $z_1, z_2, z_3 \in \mathbb{C} \wedge |z_1| = |z_2| = 1$

**mathematical formulation
from the software requirements**



```
IComplex midarc(IComplex z1, IComplex z2) {
    if(abs(z1)!=1 || abs(z2)!=1)
        throw PreConditionException;
    REAL r = realpart(z1) + realpart(z2);
    REAL i = imaginary(z1) + imaginary(z2);
    IComplex sum(r,i);
    IComplex z3 = sum / abs(sum);
    return z3;
}
```

direct numerical program
(written in infinite-precision arithmetic,
easy to maintain,
however, may be unstable in finite-precision arithmetic)



software developers
(may not be familiar with numerical analysis,
just program directly following the
mathematical formulation)



**Global Numerical
Optimization
Framework**



optimization rules



**numerical
experts**



```
FComplex midarc(FComplex z1, FComplex z2) {
    if((abs(z1)-1)>epsi || (abs(z2)-1)>epsi)
        throw PreConditionException;
    double r = realpart(z1) + realpart(z2);
    double i = imaginary(z1) + imaginary(z2);
    FComplex sum(r,i); FComplex z3;
    if(abs(sum)<epsi){
        double theta1 =
            atan2(imaginary(z1), realpart(z1));
        double theta2 =
            atan2(imaginary(z2), realpart(z2));
        double theta3 = (theta1+theta2)/2;
        z3 = FComplex(cos(theta3), sin(theta3));
    }else{
        z3 = sum / abs(sum);
    }
    return z3;
}
```

optimized numerical program
(in fixed-precision floating-point arithmetic
with numerically stable algorithms)