Homework 1

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D. Chapter Exercises 1-5

Exercise 4

$$\begin{aligned} E(z) = 0 &= 0.9 \cdot 5000 + 0.1 \cdot z_2 \\ &= 4500 + 0.1 \cdot z_2 \\ -4500 &= 0.1 \cdot z_2 \\ z_2 &= -\$45,000 \end{aligned}$$

Exercise 5

$$\begin{aligned} C &= 50000 \\ U(C) &= 2\sqrt{C} \\ z &= \begin{cases} z_1 = 5000 & p = 0.5 \\ z_2 = -5000 & p = 0.5 \end{cases} \\ E(z) &= 0 \\ 2\sqrt{50000 - \rho} &= 0.5 \cdot 2\sqrt{55000} + 0.5 \cdot 2\sqrt{45000} \\ &= \sqrt{55000} + \sqrt{45000} \\ \rho &= 50000 - \left(\frac{\sqrt{55000} + \sqrt{45000}}{2} \right)^2 \\ &= 50000 - \left(\frac{446.652822}{2} \right)^2 = 50000 - (223.3264)^2 \\ &= 50000 - 49874.69 = \$125.31 \end{aligned}$$

E. Chapter Exercises 6-22

Exercise 16

Proof of utility:

$$\begin{aligned} U(C) &= C - 0.005C^2 \\ U'(C) &= 1 - 0.01C \\ U'(C) &\geq 0 \text{ when } 0 \leq C \leq 100 \\ U''(C) &= -0.01 < 0 \text{ for all } C \end{aligned}$$

Explicit Risk Premium:

$$\begin{aligned}
C-\rho-0.005(C-\rho)^2 &= p(C+z_1-0.005(C+z_1)^2)+(1-p)(C+z_2-0.005(C+z_2)^2)\\
C-\rho-0.005(C^2-2C\rho+\rho^2) &=\\
C-\rho-0.005C^2+0.01C\rho-0.005\rho^2 &=\\
-0.005\rho^2+0.01C\rho-\rho &= p(C+z_1-0.005(C+z_1)^2)+(1-p)(C+z_2-0.005(C+z_2)^2)-C+0.005C^2\\
\rho^2+(2C-200)\rho &= p((C+z_1)^2-200C-200z_1)+(1-p)((C+z_2)^2-200C-200z_2)+200C-C^2\\
&= p((C+z_1)^2-200z_1)+(1-p)((C+z_2)^2-200z_2)-C^2\\
&= p(C+z_1)^2-200pz_1+(1-p)(C+z_2)^2-200(1-p)z_2-C^2\\
&= p(C+z_1)^2+(1-p)(C+z_2)^2-C^2\\
\rho^2+(2C-200)\rho+(C-100)^2 &= p(C+z_1)^2+(1-p)(C+z_2)^2-C^2+(C-100)^2\\
&= p(C^2+2Cz_1+z_1^2)+(1-p)(C^2+2Cz_2+z_2^2)-C^2+(C-100)^2\\
&= C^2+2C(pz_1+(1-p)z_2)+pz_1^2+(1-p)z_2^2-C^2+(C-100)^2\\
&= pz_1^2+(1-p)z_2^2+(C-100)^2\\
(\rho+C-100)^2 &= pz_1^2+(1-p)z_2^2+(C-100)^2\\
\rho+C-100 &= \sqrt{pz_1^2+(1-p)z_2^2+(C-100)^2}\\
\rho &= \sqrt{pz_1^2+(1-p)z_2^2+(C-100)^2}-C+100
\end{aligned}$$

Approximate Risk Premium:

$$\begin{aligned}
Var(z) &= p(z_1-0)^2+(1-p)(z_2-0)^2\\
&= pz_1^2+(1-p)z_2^2\\
\rho &\approx -\frac{1}{2}Var(z)\frac{U''(C+E(z))}{U'(C+E(z))}\\
&\approx -\frac{1}{2}Var(z)\frac{-0.01}{1-0.01(C+0)}\\
&\approx \frac{Var(z)}{200-2C}\\
&\approx \frac{pz_1^2+(1-p)z_2^2}{200-2C}
\end{aligned}$$

Exercise 18

Investor 1:

$$\begin{aligned}
\sqrt{1.05S_0} &= 0.8\cdot\sqrt{100}+0.2\cdot\sqrt{40}\\
&= 8+1.2649=9.2649\\
1.05S_0 &= 85.8386\\
S_0 &= \$81.75
\end{aligned}$$

Investor 2:

$$\begin{aligned}
\ln(1.05S_0) &= 0.8\cdot\ln(100)+0.2\cdot\ln(40)\\
&= 3.6841+0.7378=4.4219\\
1.05S_0 &= 83.2553\\
S_0 &= \$79.29
\end{aligned}$$

Exercise 19

$$
\begin{aligned}
-e^{-0.0001 \cdot 1.05^5 \cdot S_0} &= -0.6e^{-0.0001 \cdot 20000} - 0.4e^{-0.0001 \cdot 15000} \\
e^{-0.0001 \cdot 1.05^5 \cdot S_0} &= 0.6e^{-2} + 0.4e^{-1.5} \\
&= 0.0812 + 0.08925 = 0.17045 \\
0.0001 \cdot 1.05_0^5 &= -1.769 \\
1.05^5 \cdot S_0 &= 17692.943 \\
S_0 &= \$13862.88
\end{aligned}
$$

F. Homework Exercises

(2) Stock and Bond Equity Premium

$$
\begin{aligned}
E(FV) &= 0.5 \cdot 200 + 0.5 \cdot 50 = 100 + 25 = 125 \\
E(PV) &= \frac{125}{1.05} = 119.05 \\
C &= 350 + E(PV) = 350 + 119.05 = 469.05 \\
z &= \begin{cases} z_1 = \frac{200}{1.05} - 119.05 = 190.48 - 119.05 = 71.43 \\ z_2 = \frac{50}{1.05} - 119.05 = 47.62 - 119.05 = -71.43 \end{cases} \\
E(z) &= 0.5 \cdot 71.43 - 0.5 \cdot 71.43 = 0 \\
\ln(469.05 + 0 - \rho) &= 0.5 \ln(469.05 + 71.43) + 0.5 \ln(469.05 - 71.43) \\
&= 0.5(\ln(540.48) + \ln(397.62)) = 0.5 \cdot 12.27795 = 6.138977 \\
469.05 - \rho &= e^{6.138977} = 463.58 \\
\rho &= 469.05 - 463.58 = 5.47 \\
\text{expected stock return} &= \frac{125 - (119.05 - 5.47)}{(119.05 - 5.47)} = \frac{11.42}{113.58} = 0.1005 \\
\text{equity premium} &= 0.1005 - 0.05 = 0.0505
\end{aligned}
$$

(3) Risk Premium

$$
\begin{aligned}
C &= 90000 \\
z &= \begin{cases} z_1 = 10000 & p = 0.99 \\ z_2 = 0 & p = 0.01 \end{cases} \\
E(z) &= 0.99 \cdot 10000 + 0.01 \cdot 0 = 9900 + 0 = 9900 \\
\sqrt{90000 + 9900 - \rho} &= 0.99 \cdot \sqrt{90000 + 10000} + 0.01 \cdot \sqrt{90000 + 0} \\
&= 0.99 \cdot \sqrt{100000} + 0.01 \cdot \sqrt{90000} \\
&= 0.99 \cdot 316.22776 + 0.01 \cdot 300 \\
&= 313.065 + 3 = 316.065 \\
99900 - \rho &= 316.065^2 = 99897.39 \\
\rho &= \$2.61
\end{aligned}
$$

(4) Insurance Premium Comparison

1. My Insurance Premium

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\begin{aligned}
C &= 100000 \\
z &= \begin{cases} z_1 = 100000 & p = 0.99 \\ z_2 = 0 & p = 0.01 \end{cases} \\
E(z) &= 0.99 \cdot 100000 + 0.01 \cdot 0 = 99000 + 0 = 99000 \\
\text{expected premium} &= \$1000 \\
\sqrt{100000 + 99000 - \rho} &= 0.99 \cdot \sqrt{100000 + 100000} + 0.01 \cdot \sqrt{100000 + 0} \\
&= 0.99 \cdot \sqrt{200000} + 0.01 \cdot \sqrt{100000} \\
&= 0.99 \cdot 447.2135955 + 0.01 \cdot 316.227766 \\
&= 442.7415 + 3.1623 = 445.90 \\
199000 - \rho &= 445.90^2 = 198830.14 \\
\rho &= \$169.86 \\
\text{premium} &= 1000 + 169.86 = \$1169.86
\end{aligned}
$$

2. Cousin's Insurance Premium

$$
\begin{aligned}
C &= 999,900,000 \\
E(z) &= 99000 \\
\text{expected premium} &= \$1000 \\
\sqrt{999,900,000 + 99000 - \rho} &= 0.99 \cdot \sqrt{999,900,000 + 100000} + 0.01 \cdot \sqrt{999,900,000 + 0} \\
&= 0.99 \cdot \sqrt{1,000,000,000} + 0.01 \cdot \sqrt{999,900,000} \\
&= 0.99 \cdot 31622.7766 + 0.01 \cdot 31621.19542 \\
&= 31306.548835 + 316.2119542 = 31622.7608 \\
999,999,000 - \rho &= 31622.7608^2 = 999,998,999.98 \\
\rho &= \$0.02 \\
\text{premium} &= 1000 + 0.02 = \$1000.02
\end{aligned}
$$

(5) Life Insurance Example

Suppose you and your wife's current wealth is \$1,000,000 and your wife wants to take out a life insurance policy that would pay \$100,000 in the chance that you die, which has a likelihood of 0.5% during a year given you are a fairly healthy 50 year old. Assuming you and your wife share the same utility function ($U(C) = \ln C$), what is the expected insurance premium your wife must pay in this situation?