# How Does Warren Buffett Make So Much Money?


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# Contents

# 1 Introduction

In this paper, we seek to explain how Warren Buffett has increased his wealth through insurance. We start with a thorough explanation of term life insurance. This includes how policies work, who purchases these policies, and the use of term life insurance. Although we outline many varieties of term life insurance, we hone in on basic level-premium term life policies without any extra riders. Using probabilities of death of individuals based on various characteristics such as age, sex, and race, we can determine the probability that a policyholder dies within the given year using empirical data. Using these probabilities, we are able to develop a binomial model to obtain the total present value of potential payments of a given policy. We provide an example calculation of the monthly premium cost of a 25 year old Black female.

From there we shift to a discussion of insurance premiums. We use a simplified model to discuss how activities such as smoking and alcohol consumption can have a large impact on the premiums paid. We then use the previously developed policy present values to determine a payment schedule for policies. Having provided a thorough introduction to insurance premiums and how they are calculated, we then enter a detailed discussion on a specific type of insurance known as catastrophe insurance, and how Warren Buffett engages with it. There, we introduce the concept of reinsurance and provide an illustrative example of how these companies work, and discuss the intricacies of catastrophe bonds as both an alternative to reinsurance and an important portfolio diversification tool.

We then take a dive into the basics of terrorism insurance. Terrorist catastrophe laws, exemplified by initiatives like the Terrorism Risk Insurance Act in the United States, aim to mitigate the economic fallout of terrorism by providing insurance coverage and fostering collaboration between government and private sectors. These laws not only shift terrorism-related risks to the insurance market, relieving pressure on government resources, but also promote resilience, deterrence, and societal cohesion in the face of terrorist threats, thereby safeguarding national security and economic stability. Finally, we explore general stochastic models in life insurance. Particularly, we will go through basic properties of Markov Chains and provide its applications in life insurance.

# 2 Life Insurance and Probability of Death

## 2.1 Term Life Insurance Basics

Term life is the simplest form of life insurance in which policyholders pay a premium throughout the duration of their policy in exchange for a lump sum payment that is to be distributed to the named beneficiaries if the protected life dies during the specified term of the contract. These policies typically range from 10-30 years. According to the textbook *Actuarial Mathematics for Life Contingent Risks*, these policies carry premiums that are relatively small compared to the sum insured.[5] It can be noted that although these contracts usually have a fixed duration, policyholders can choose to "lapse" at any time—this means that policyholders stop paying premiums and thus the contract terminates. Thus, although the insured is locked in at a given rate for a given number of years, they can end their contract at any time without penalty. Furthermore, there is no surrender value if you decide to end our policy early which distinguishes term life from more complicated forms of life insurance like universal life and whole life.

The textbook *Actuarial Mathematics for Life Contingent Risks* contends that the primary purpose of term life insurance is family protection, but it can also be used to protect businesses against loss associated with the death of key employees—this subset of term life insurance is known as Company Owned Life Insurance.[5] Notice that most of the time, individuals use term life as protection for the spouse and children that would arise if a household's primary earner were to die. In fact, Guardian Life suggests a rule of thumb is to aim for coverage long enough to house and see your children through college (How Term Life Insurance Works). Thus it seems reasonable to assume that most people purchasing term life insurance are middle aged. It would not make much sense for a 20 year old to purchase life term life insurance since they likely don't have to support anyone and don't have much disposable income to pay for the insurance anyways. Likewise, people past the age of retirement (65) are likely not going to need term life insurance coverage since their death would not cause much financial hardship due to minimal to no loss of income.

Another interesting variant of term life insurance is "decreasing term life insurance" which has a decreasing death benefit over the time the policy is in force. *Actuarial Mathematics for Life Contingent Risks* notes that such term policies are "used in conjunction with home loans" such that the death benefit is set to much the outstanding loan balance over the course of the loan under a prespecified payment plan.[5] Such an insurance policy is used to ensure that if the policyholder dies, the family of the policyholder will not struggle to repay their mortgage. Additionally flexibility can be provided by renewable and convertible term life insurance. Renewable term life insurance is term life insurance with a rider that specifies that the insured is able to renew their contract at the end of the term whereas convertible term life insurance allows the option to convert to whole life insurance at the end of a term. Note that neither of these require reevaluation of health status, but rates can be adjusted based on the age, and there can be a maximum age for which the renewability and convertibility of a policy is no longer permitted. These such features can be included as additional riders to policies in which policy owners can pay higher premiums in order to include such options.

Although rates are in large part determined by age, prospective policyholders must undergo the underwriting process which includes an evaluation of health, lifestyle, and occupation risk. Guardian Life specifically notes that hobbies like scuba diving as well as dangerous occupations like working at an oil rig can raise your rates (How Term Life Insurance Works). Other common characteristics included in the underwriting process are smokers vs. non-smokers, family medical history, personal medical history, and a medical examination. Note however that Guardian Life (and other insurance companies as well) do offer Guaranteed issue policies which do not require medical examination and rather just require a few health questions. Such policies carry higher premiums due to the assumption that people seeking those policies have poor health, and they may even pay only a partial death benefit for the first few years of the coverage (How Term Life Insurance Works). Notice that these inflated costs as well as the decreased death benefit in the first few years are used to protect insurance companies against adverse selection—this occurs when somebody who knows they will require an insurance payout soon based on unseen characteristics decides to purchase a large sum of insurance. Life insurance companies seek to eliminate this risk as much as possible using the underwriting process, but when there is minimal underwriting, they can simply employ the strategies above to deter uninsurable individuals from trying to beat the system. With the proper safeguards in place, life insurance companies are able to rest assured that in the long run, on average, they will turn a profit while still being able to protect individuals against risk.

## 2.2 Statistics for the Probability of Death in Individuals

Many actuaries at life insurance companies specialize in ratemaking and loss reserving. Ratemaking is the process of setting policy rates based on the expected losses associated with a policy over the course of several years while reserving involves the process of estimating the amount of excess funds that should be set aside in order to ensure that the insurance company is able to pay off its liabilities. For life insurance companies, these liabilities are the expected payouts associated with the sum of all the policies they have in force. For term life insurance specifically, both of these processes rely on accurate estimation of the probability of death of individuals.

Although many insurance companies have their own proprietary data that employ the use of various metrics obtained throughout the underwriting process—from personal health history to the medical examination—to extract a more informed probability of death, we will employ a simpler model for predicting mortality based on a 2013 National Center for Health Statistics report on mortality statistics from 1999-2001. In this simplified report, the CDC breaks the population out between males and females as well as Black vs. White. We extract the probability of death from these files using python and generate a table of values where each column indicates a population group and rows are indexed by a given attained age; the values are probabilities of dying in the given year if a person lives to that age. Although our dataset includes ages 1-108, Table 1 depicts the extracted probabilities for ages by race and sex. As can clearly be seen, death probabilities increase with age, and they are higher for males and Black individuals.

| Age | White Female | White Male | Black Male | Black Female |
|---|---|---|---|---|
| 21 | 0.00043 | 0.00125 | 0.00235 | 0.00068 |
| 22 | 0.00043 | 0.00129 | 0.00253 | 0.00074 |
| 23 | 0.00043 | 0.00128 | 0.00259 | 0.00078 |
| 24 | 0.00044 | 0.00124 | 0.00255 | 0.00082 |
| 25 | 0.00044 | 0.00119 | 0.00247 | 0.00085 |
| 26 | 0.00045 | 0.00115 | 0.00242 | 0.00090 |
| 27 | 0.00047 | 0.00113 | 0.00239 | 0.00095 |
| 28 | 0.00049 | 0.00114 | 0.00241 | 0.00102 |
| 29 | 0.00052 | 0.00118 | 0.00247 | 0.00110 |
| 30 | 0.00055 | 0.00123 | 0.00255 | 0.00118 |
| 31 | 0.00059 | 0.00128 | 0.00262 | 0.00128 |
| 32 | 0.00064 | 0.00134 | 0.00273 | 0.00139 |
| 33 | 0.00070 | 0.00143 | 0.00286 | 0.00152 |
| 34 | 0.00077 | 0.00153 | 0.00301 | 0.00167 |
| 35 | 0.00084 | 0.00164 | 0.00317 | 0.00181 |
| 36 | 0.00092 | 0.00175 | 0.00336 | 0.00196 |
| 37 | 0.00099 | 0.00188 | 0.00359 | 0.00214 |
| 38 | 0.00108 | 0.00203 | 0.00388 | 0.00235 |
| 39 | 0.00118 | 0.00219 | 0.00423 | 0.00259 |
| 40 | 0.00129 | 0.00237 | 0.00460 | 0.00285 |

Table 1: Probability of Death by Gender and Race

## 2.3 Multi-Period Binomial Model For Expected Insurance Payout

The python code attached in Appendix A.1 includes the full process for calculating the expected cost an insurance company will incur in term life claims over the course of 30 years given a certain set of assumptions in our toy model. We first assume that everybody is starting their policy today, and all policies are simple term life policies which last 30 years each having a $500,000 term payout upon death. We also assume that we insure 1,000,000 individuals identified as either black or white and either male or female. We further assume that they live in the US, and their ages range from 25-55 at the time of purchase with 40 being the most common age whereas ages further from 40 are less common for purchasing life insurance. Finally, we assume a 5% risk free interest rate for evaluating the present value, and we assume that our death benefit occurs at the end of the year in which the death takes place for present value calculations.

The code attached in Appendix A.1 contains concise descriptions of the process of generating expected costs the insurance company will incur due to the term life claims; however, a brief description of how our binomial model works is included below.

We can define a new random variable $X_t$ indexed by time where $X_t = 0$ if the policyholder survives year $t$, but $X_t = 1$ if the policyholder dies in that year. We start with a given individual who is either Black or White as well as either male or female—they also have a given starting age. Let us observe for example a 25 year old black woman who has just purchased term life insurance of 30 years with a \$500,000 payout upon death. Although normally underwriters are able to obtain more details, we will use a simplistic model based on age, race, and sex. Starting from the final year of the policy, given that our individual survives 29 years to the age of 54, there is 0.805% of deaths in that final year whereas the survival probability is the complement which is 99.195%. Thus we have a 0.805% chance of a \$500,000 payout and a 99.195% chance of a \$0 payout. Thus the present value of the policy heading into year 29 is

$$V_{29} = \frac{1}{1.05}[0.00805 \cdot 500,000 + 0.99195 \cdot 0] = \$3833.33$$

We can work backwards from here defining value iteratively as

$$V_t = \frac{1}{1+r}[p_t \cdot 500,000 + (1 - p_t)V_{t+1}] \tag{2.1}$$

where $p_t$ is the probability of death at a given attained age. We can do the calculation at time 28 (age 53 in our case) as such:

$$V_{28} = \frac{1}{1.05}[0.00747 \cdot 500,000 + 0.99253 \cdot 3833.33] = \$7,180.67$$

Continuing this process until we get to the present time, we are able to obtain an expected present value of \$18,558 when evaluating present value at a 5% interest rate. The full table of calculations is provided in Appendix A.1. Furthermore, we are able to complete this evaluation for the 1,000,000 person sample we created in our code. The information for the demographic breakdown is also included in Appendix A.1, and completing the same process for 1,000,000 individuals with their own given death probabilities in each year, we find the the total expected present value of all of their policies combined is approximately \$48.56 billion. The exact number is included in the code.

# 3 Insurance Premiums

## 3.1 Overview

Insurance premiums are the terms of payment between an insurance company and the entity being insured. They are the cost of acquiring a particular insurance policy over the course of a given period.[9] The structure of the payment schedule and how the premiums are calculated can vary greatly across different liabilities and policies. Premiums may be paid at prearranged intervals or entirely up front. For example, car insurance is required by law in the United States and thus if you own a car, you must have a car insurance policy to protect that asset. As a result, car insurance is usually charged month by month as long until the policyholder cancels the policy.

Premiums are also subject to a range of policies or benefits offered by the insurer. Some policies may have a much wider array of benefits or more enhanced coverage in specific areas in exchange for an increased premium. Back to the car insurance example, vehicle theft coverage may be an additional benefit that only applies to specific policies. Even more interesting is that these particular policies or benefits can be exclusive based on the risk of the policy holder both in terms of specific coverage risk as well as financial risk, or the inability to afford the premium.

Generally, insurers make money when they sell policies that holders do not file (many) claims for. When a policy is sold, the insurer is accepting the risk of paying out whatever the policy covers in the event of realized risk by the policy holder. An insurance company is paid for their policies regularly, but only spends when claims are made, which is rather dependent on outside factors like bad storms causing more car accidents during certain parts of the year. So, what do insurance companies do with the premiums they are paid? They invest! As long as the insurance company maintains enough capital to pay off expenses and policy claims, they can invest in financial instruments that can maximize their profits. In 2011 alone, the average U.S. life insurance company held less than 3% of their general account in liquid assets like cash and short term investments[11] that would be able to pay off claims. The rest was invested in longer term assets that would become liquid at a future date.

## 3.2 Calculating Insurance Premiums

Calculating premiums is a complex task that is left to internal actuaries at any major insurance company, though this is increasingly being done with the aid of computational algorithms and artificial intelligence. When an insurance company calculates a premium, they are assessing the risk of offering a specific policy, i.e. the expected expense of selling that policy. The expected expense is influenced by a multitude of factors as mentioned above. For example, smoking habits, prior health history or even job industry are factors that could influence how much an individual might cost a health or life insurance company. These trends can shift over time as people become healthier or unhealthier, which insurance companies also take into account. This means that insurance companies often adjust premiums when a policy holder files fewer or greater claims than expected or an outside factor important to their premium calculation changes.

The wide range of factors that go into calculating a premium means that it is more complex than a binomial model. In fact, far more complex. In any insurance industry but especially health and life insurance, the metrics that impact an individual's premium are incredibly intertwined and thus not independent variables that can be calculated easily. For example, 50 year old women in the United States have a 0.69% chance of dying by age 60 if they smoke as opposed to 0.37% if they do not, marking an 85.5% increase in risk for smokers in this demographic.[16] However, this is not independent from alcohol related deaths, as smokers are usually prone to addictive substances and thus may have a higher rate of alcohol related deaths than a non-smoker.

For the sake of simplicity, suppose we can calculate the overlap between these two risk factors, which in actuality is modeled and statistically inferred by actuaries. Suppose we continue working in the demographic of 50 year old American women looking for a 10 year term life insurance policy. The chance of death for women from ages 50-60 as a whole is 0.402% according to Appendix A.2. Let's say that 64% of adults aged 50-64 drink regularly[15] and 10.1% of women in this age group smoke regularly.[3] Assuming these are purely independent factors, which we know not to be the case but going with it for simplicity, then we define the following probabilities given $a$ = alcohol consumer and $s$ = smoker:

$$P = \begin{cases} P(a \wedge s) = 0.64 \cdot 0.101 = 0.06464 \\ P(a \wedge \bar{s}) = 0.64 \cdot 0.899 = 0.57536 \\ P(\bar{a} \wedge s) = 0.36 \cdot 0.101 = 0.03636 \\ P(\bar{a} \wedge \bar{s}) = 0.36 \cdot 0.899 = 0.32364 \end{cases}$$

Let's define risk with $R$, and understand that there are 15699 alcohol related deaths per year for women aged 50-64[1] times 10 years for a total estimate of 156,990 deaths over this period, divided by $10.37 + 10.6 + 10.82 = 31.79$ million people in this demographic[12] for an expected alcohol death rate of $156990/31790000 = 0.004938 = R(a)$. Given our average risk of 0.00402, we can calculate the risk for people who don't consume alcohol $R(\bar{a})$:

$$
\begin{aligned}
0.00402 &= R(a) \cdot p(a) + R(\bar{a}) \cdot p(\bar{a}) \\
R(\bar{a}) &= \frac{0.00402 - R(a) \cdot p(a)}{p(\bar{a})} = \frac{0.00402 - 0.004938 \cdot 0.64}{0.32} \\
&= \frac{0.00402 - 0.00316032}{0.32} = \frac{0.00085968}{0.32} \\
&= 0.0026865
\end{aligned}
$$

Recall that we already have $R(s) = 0.0069$ and $R(\bar{s}) = 0.0037$ from above, so we can create a system of four equations with four unknowns to solve for the four types of risk we have:

$$
R = \begin{cases}
R(s) = R(a \wedge s)p(a \wedge s) + R(\bar{a} \wedge s)p(\bar{a} \wedge s) \\
R(\bar{s}) = R(a \wedge \bar{s})p(a \wedge \bar{s}) + R(\bar{a} \wedge \bar{s})p(\bar{a} \wedge \bar{s}) \\
R(a) = R(a \wedge s)p(a \wedge s) + R(a \wedge \bar{s})p(a \wedge \bar{s}) \\
R(\bar{a}) = R(\bar{a} \wedge s)p(\bar{a} \wedge s) + R(\bar{a} \wedge \bar{s})p(\bar{a} \wedge \bar{s})
\end{cases}
$$

This happens to be an unsolvable system of equations, though we can use some linear algebra to calculate the least squares solution to give us a decent approximation of a solution. It is important to remember that this is real world data and often times we must model a solution that may not be as exact as desired. So let's convert this into a matrix system of equations:

$$
A = \begin{bmatrix} 0.06464 & 0 & 0.03636 & 0 \\ 0 & 0.57536 & 0 & 0.32364 \\ 0.06464 & 0.57536 & 0 & 0 \\ 0 & 0 & 0.03636 & 0.32364 \end{bmatrix} \qquad b = \begin{bmatrix} 0.0069 \\ 0.0037 \\ 0.004938 \\ 0.0026865 \end{bmatrix} \tag{3.1}
$$

Using the code from Figure 3.1, we can generate the following approximate solutions to our system of equations:

$$
R = \begin{cases}
R(a \wedge s) = 0.07210712 \\
R(a \wedge \bar{s}) = 0.00177432 \\
R(\bar{a} \wedge s) = 0.04111993 \\
R(\bar{a} \wedge \bar{s}) = 0.00597965
\end{cases} \tag{3.2}
$$

These risks then serve as our probabilities of death given the two factors we used to calculate

```python
import numpy as np
from scipy import linalg

A = np.array([[0.06464, 0, 0.03636, 0],
              [0, 0.57536, 0, 0.32364],
              [0.06464, 0.57536, 0, 0],
              [0, 0, 0.03636, 0.32364]])
b = np.array([0.0069, 0.0037, 0.004938, 0.0026865]).reshape((4, 1))

p, *_ = linalg.lstsq(A, b)
print(*p)
```

Figure 3.1: Code to calculate the least squares solution of equation 3.1

them, so we can generate what's called a premium table. A premium table outlines the given rates for specific factors, plans and demographics so that companies can easily lookup a premium for an individual looking to purchase a policy. This saves the company lots of time in determining premiums on a case by case basis, though they update these tables as the data impacting them changes. For this example, let's consider a ten year term life insurance policy with an payout of \$500,000 upon the expiration of the policy if the insured individual dies and a risk-free interest rate of 5%. Note that we have calculated the expected risk for the entire duration of the policy, though much more calculation would need to be done to account for the risk-free interest rate over the course of the policy depending on when the payout may occur. As such the risks would shift over the duration of the policy and therefore a more comprehensive integration of risks and interest rates would be necessary to calculate that. As we are only interested in how premiums work, that is beyond the scope of this particular example. Using the risks from equation 3.2, we calculate the expected value of this policy as follows:

$$E = \begin{cases} E(a \wedge s) & = \frac{1}{1.05^{10}}[0.07210712 \cdot 500000] = \$22133.76 \\ E(a \wedge \bar{s}) & = \frac{1}{1.05^{10}}[0.00177432 \cdot 500000] = \$544.64 \\ E(\bar{a} \wedge s) & = \frac{1}{1.05^{10}}[0.04111993 \cdot 500000] = \$12622.04 \\ E(\bar{a} \wedge \bar{s}) & = \frac{1}{1.05^{10}}[0.00597965 \cdot 500000] = \$1835.49 \end{cases}$$

Now that we've calculated the expected premiums, we can construct the following premium table for future use:

## 3.3 Premium Schedules

While Section 3.2 works with a simple example that pays the entire value of the policy at the end of the term and requires an upfront premium of the entire policy value, this is largely not the

| | Smokes? | | | |
|---|---|---|---|---|
| | Yes | | No | |
| | Alcohol? | | Alcohol? | |
| | Yes | No | Yes | No |
| Example Policy | \$22,133.76 | \$12,622.04 | \$544.64 | \$1,835.49 |

Table 2: Premium Table for Smoking and Alcohol Example

case for life insurance policies. Generally, policies require month by month premiums in order for the insurers to maintain enough cash to payout other claims as well as payout claims in the event the insured person dies.

Using the example from Section 2.3, let's examine what a month by month premium schedule may look like. First, we calculate the value of the policy for the entire term, which would be a summation of the year by year value adjusted for the risk-free interest rate $r$. For a policy $P$ with a term of $t$ years, payout of $m$ and yearly probability of death $p(t)$, we have the following equation defining the present value of the policy:

$$PV(P) = \sum_{\tau=1}^{t} \frac{1}{(1+r)^{\tau}} \left[ \left( \prod_{s=0}^{\tau} 1 - p(s) \right) \cdot p(\tau) \cdot m \right] \tag{3.3}$$

Notice that the probability of death in the $n$th year is the product of the probabilities of dying in none of the prior years times the probability of dying in that year. The issue is that this assumes a year by year payout and premium schedule as the data we have is in yearly increments, though with enough data (as insurance companies have) this equation can be modified to produce the present value of a monthly or even daily updated policy. Working with this equation for present value, we can find the monthly premiums using the recurring payment formula from *Mathematical Methods in Finance and Economics*[2]

$$PV = \frac{C}{r} \left[ 1 - \left( \frac{1}{(1+r)^n} \right) \right] \tag{3.4}$$

where $C$ is the payment per term, $r$ is the risk-free interest rate per term and $n$ is the number of terms. In our case, we divide $r$ by 12 and multiply $n$ by 12 since they are currently per year instead of per month. So, setting Equation 3.3 and Equation 3.4 equal to each other yields

$$\frac{12C}{r}\left[1-\left(\frac{1}{(1+\frac{r}{12})^{12t}}\right)\right]=\sum_{\tau=1}^{t}\frac{1}{(1+r)^{\tau}}\left[\left(\prod_{s=0}^{\tau}1-p(s)\right)\cdot p(\tau)\cdot m\right]$$

which, when solved for our monthly payment $C$, gives us

$$C=\frac{\sum_{\tau=1}^{t}\frac{1}{(1+r)^{\tau}}\left[\left(\prod_{s=0}^{\tau}1-p(s)\right)\cdot p(\tau)\cdot m\right]}{\frac{12}{r}\left[1-\left(\frac{1}{(1+\frac{r}{12})^{12t}}\right)\right]} \tag{3.5}$$

Using the code in Appendix A.3, we find that the monthly premium for a policy on a 25 year old white male lasting 30 years with an annual risk-free interest rate of 5% and $500,000 of coverage is $79.96, with a present policy value of $14,895.35. Notice that the level premium structure does not in fact lineup with the expected annual loss amounts by year. In fact the expected payment each year is actually increasing as you age so later years in the policy are actually more expensive. Still, the customer typically pays one flat rate rather than an increasing rate to match the expected present value of loss each year. This allows the insurance company to have larger profit margins on policies early on which offsets the cost of underwriting and sale as well as provides additional income they can invest in long term assets. Furthermore, notice that the premium schedule calculated above provide the payments required to break even *on average* if the only cost to the insurer is the potential insurance payout. Of course, policy premiums may need to be adjusted to account for fees, commissions and other operational costs. An important cost adjustment for insurance companies is the risk premium, where consumers pay an additional fee for the security of a guaranteed outcome—insurance companies may include this risk premium as part of the added value they provide through their product.

# 4 Catastrophe Insurance

## 4.1 Introduction to Warren Buffett and Catastrophe Insurance

Warren Buffett is one of the most successful investors of all time, with a net worth over $100 billion. He relies on the principles of value investing pioneered by Benjamin Graham, which involves the detailed fundamental analysis of securities to identify those that are underpriced, and then purchasing them in the hopes that their prices will increase to their “intrinsic values”. Intrinsic value is an objective measure of an asset’s value, which is different from an asset’s price in the market. By understanding and purchasing assets that are priced below their intrinsic value, profit is generated as the asset’s price then increases towards its intrinsic value. Buffett purchases these underpriced securities through his holding company, Berkshire Hathaway Inc. The purpose of a holding company is to "hold" the stock of other companies. In the case of Berkshire Hathaway, it purchases and holds the stock of companies which Buffett, and his Vice Chairman Charlie Munger, believe to be great companies that are undervalued by the broader market. Buffett and Munger were so successful at this that Berkshire has grown to a market cap of almost $900 billion, but it was not always like this.

Berkshire began as a textile manufacturer which Buffett started to buy shares of in 1962. By 1964, Buffett had a controlling interest in the company and started to expand into other areas of business. This was the beginning of Berkshire's transformation into a conglomerate holding company. The most notable industry of expansion was insurance, which offered Berkshire the foundation of its success, and still remains a significant portion of the business today. Berkshire entered into the insurance industry originally through its purchase of National Indemnity Company, and later GEICO (which is the insurance core today). However, the type of insurance of focus for this section falls under the reinsurance category. Reinsurance is, essentially, insurance for insurance companies. Insurance companies purchase insurance from a reinsurance company as an attempt to protect themselves from major claim events. These events are often things like hurricanes, earthquakes, and other natural disasters which is why this insurance is sometimes known as "catastrophe insurance". Buffett and Berkshire entered the reinsurance business in 1998 through their purchase of the General Reinsurance Corporation ("Gen Re"), an American multinational reinsurance company specializing in property/casualty and life/health reinsurance. Since then, reinsurance has been a large part of Berkshire's business. In 2023, the property and casualty reinsurance arm alone produced $3.5 billion in earnings.[6]

Catastrophe insurance and reinsurance work slightly differently than the typical insurance one might be used to. In the case of homeownership, catastrophes (natural disasters) are often excluded from standard home insurance policies because they are much lower in probability and much higher in cost than other insurance events. Additionally, "a catastrophic event often results in an extremely large number of claims being filed at the same time."[8] These characteristics of catastrophe insurance make it very difficult for catastrophe insurers to effectively manage risk and estimate total potential exposure. To remedy this, reinsurance comes into play. Reinsurance allows insurers (the "ceding company") to pass off some of their risk to the reinsurance company for a premium similar to that paid by an insurance policy holder. In the specific case of catastrophe reinsurance, this is typically done through an "excess of loss" reinsurance contract. This is when "the reinsurer indemnifies–or compensates–the ceding company for losses that exceed a specified limit."[8] An illustrative example of how one one these contracts might be formulated will be carried out in the next section.

## 4.2 Estimate of Insurance Payout Each Period

Suppose we want to start a catastrophe insurance company, and we want to provide insurance to one million people (so we have one million policies) for earthquake damage. We let each of these people have a square root utility function, that is $U(C) = \sqrt{C}$. Let the average price of the property we are insuring be \$150,000. Since most people have roughly 75% of their total wealth in their home, we can assume that on average $C = 150000 \cdot 1.25 = 200000$. Let us further suppose that earthquakes happen with a 5% chance every year in the parts of the world we are providing insurance in, and that an earthquake would completely destroy any given home. Then our policyholders are faced with a gamble $z$ such that

$$E(z) = .05(-150000) + (1 - .05) \cdot 0 = -7500$$

Using the rest of the information above, we can find $\rho$, the risk premium for our average policyholder through the following:

$$\begin{aligned} \sqrt{200000 - 7500 - \rho} &= 0.95\sqrt{200000 + 0} + 0.05\sqrt{200000 - 150000} \\ \rho &= 2375 \end{aligned}$$

As the insurance company, we would be receiving this premium in addition to the expected loss from policyholders in a given year times the amount of policyholders we have, or \$9,875,000,000, every year. Now, with insurance covering different regions of the world, an earthquake is likely to occur somewhere at some point during the year. Suppose that an earthquake strikes a particular area that represents 1% of our policyholders, or 10,000 homes, and destroys them completely. We would then owe \$1,500,000,000 in damage coverage, making our profits for the year decrease by just over 15% to \$8,375,000,000. This is a very large risk, which is why reinsurance companies exist.

As mentioned before, by entering into a contract with a reinsurance company, we can mitigate our risk exposure by spreading some of the risk to them. Continuing with the example above, let us enter into a proposed "excess of loss" contract with a reinsurance company to reduce our risk. For this type of contract, we agree to a specified loss amount that the reinsurance company will reimburse us for if we incur losses above that threshold. Let us set that threshold at \$750,000,000, 50% of our loss in the earthquake event above. Using similar logic, we are faced with a gamble $w$ such that

$$E(w) = .05(-750000000) + (1 - .05) \cdot 0 = -37500000$$

Therefore, our risk premium would be:

$$\sqrt{9875000000 - 37500000 - \rho} = 0.95\sqrt{9875000000 + 0} + 0.05\sqrt{9875000000 - 750000000}$$
$$\rho = 703400$$

Thus, we would pay $37500000 + 703400 = 38203400$, or \$38,203,400 to a reinsurance company for a reinsurance contract that protects us from up to \$750,000,000 in losses.

To summarize the above, by reinsuring ourselves we have reduced our risk by limiting the variance in our expected yearly profits. Table 3 illustrates this difference.

| In (\$) | With Reinsurance | Without Reinsurance |
|---|---|---|
| Maximum Profit | 9,836,796,600 | 9,875,000,000 |
| Expected Profit | 9,837,500,000 | 9,800,000,000 |
| Minimum Profit | 9,125,000,000 | 8,375,000,000 |
| Maximum - Minimum | 711,796,600 | 1,500,000,000 |

Table 3: Example Reinsurance Comparison

## 4.3 Catastrophe Bonds as Part of the Reinsurance Mix

What was described in the previous section is known as the "traditional" form of reinsurance. In the past two decades, alternative risk transfer has become a growing part of the reinsurance mix due to its access to the capital markets, which are much larger than the insurance market. The largest type of alternative risk transfer are insurance-linked securities (ILS), which are funded by investors and supported through the premium payments by the underlying insurance company. The amount of ILS instruments issued in 2021 was \$14 billion, with 96% of these being catastrophe bonds (cat bonds).[7] This is done through a Special Purpose Vehicle (SPV) created by the insurance company to act as an intermediary between themselves and the capital markets. This SPV collects "premium" in the form of insurance payments from the insurer, and issues bonds into the capital markets for investors. When investors buy the bonds, the SPV uses the proceeds along with the premium collection to produce an investment income through the purchase of T-bills. This income is then used to cover interest payments to the investors along with repayment of principal. If a "triggering event" occurs, which is detailed in the SPV bond contracts (e.g. an earthquake causes a specified amount in damages to houses insured by the insurance company), the income is instead used to pay off the insurance claims. For investors, this results in bond default. An illustration of a typical cat bond can be found in Figure 4.1.

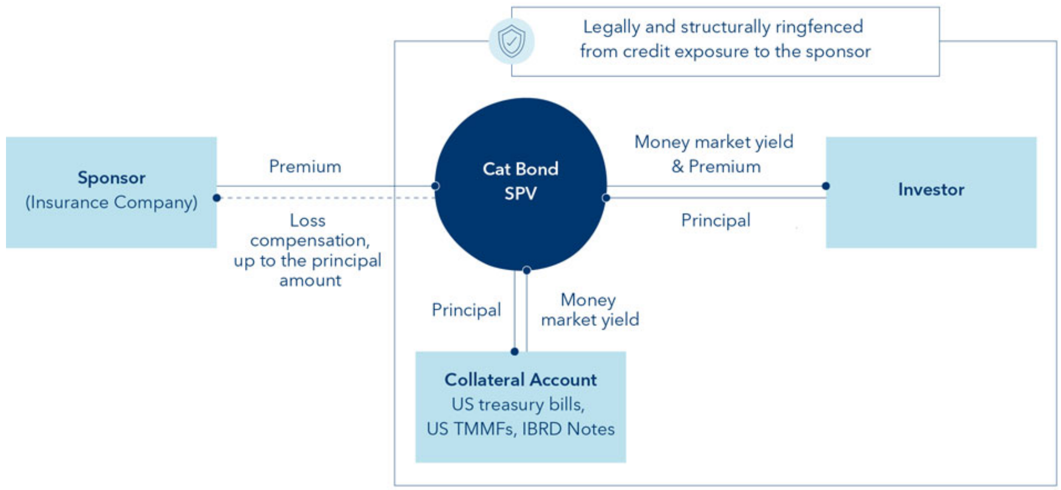


Figure 4.1: Typical Cat Bond Structure[13]

The benefit of cat bonds to insurers is obvious in that it provides them access to a larger capital base to help offset their risk, but there are also many benefits to investors. For one, it provides investors with access to the insurance industry and risk premiums from natural disasters as another source of cash flow. Cat bonds also exhibit low credit risk since T-bills are their form of collateral, and are mostly structured on a floating rate basis which also eliminates interest rate risk. But perhaps the most important benefit to investors offered by cat bonds is diversification. Cat bonds have very low correlation to other financial instruments and asset classes by definition. They are dependant solely on natural disasters, not specific companies or industries. This makes cat bonds great additions to an already diversified investor portfolio, generating exposure to a new industry with little correlation to existing financial products.

Many investment professionals have realized the importance and opportunity associated with cat bonds in recent years, including Warren Buffett. In 2023, "the cat bond market was approximately $40 billion,"[13] and had peak new issuance levels of over $15 billion. Over the past 20 years, the Swiss Re Cat Bond Index returned 7.13% with a volatility of only 5.1% and a Sharpe Ratio of 1.01. This was over a period with natural disasters like Hurricane Katrina and the Tohoku earthquake in Japan, highlighting the resiliency of cat bonds and the stability of returns. Overall, cat bonds are becoming an increasingly more important part of the reinsurance mix, and should be utilized by both insurance companies and investors to diversify and spread risk systematically throughout the broader market.

# 5 Terrorist Catastrophe Laws

## 5.1 Background

Terrorism, with its destructive potential and capacity to sow fear and chaos, has become an unfortunate reality of the modern world. The events of September 11, 2001, serve as a stark reminder of the devastating impact that terrorist attacks can have not only on human lives but also on economies and societies at large. In the aftermath of 9/11, the American economy experienced significant turmoil, with the stock market plummeting, businesses facing unprecedented disruptions, and the airline industry grappling with massive losses. According to some estimates, the total economic cost of the attacks exceeded $1.4 trillion, making it one of the most costly events in U.S. history.[4]

The shockwaves of 9/11 reverberated far beyond the immediate physical and emotional devastation, highlighting the interconnectedness of global economies and the vulnerability of key industries to terrorist threats. Businesses across various sectors, from finance to tourism, felt the impact as consumer confidence plummeted, supply chains were disrupted, and insurance costs soared. The attacks exposed the inadequacies of traditional insurance models in addressing terrorism-related risks, prompting governments to reassess their approaches to managing such threats and safeguarding their economies.

In response to the economic challenges posed by terrorism, governments worldwide have grappled with the need to develop effective strategies for mitigating the financial fallout of such catastrophic events. One such strategy has been the development and implementation of terrorist catastrophe laws, which aim to provide a framework for managing the economic consequences of terrorism and promoting resilience in the face of persistent threats.

## 5.2 Provisions of the Law

Terrorist catastrophe laws encompass a wide range of provisions aimed at facilitating the provision of insurance coverage and managing risks associated with terrorist activities. For example, the Terrorism Risk Insurance Act (TRIA) in the United States, enacted in response to the events of 9/11, established a federal backstop for insurance claims related to acts of terrorism. Under TRIA, the government provides a financial backstop to insurers in the event of a major terrorist attack, helping to ensure the availability of terrorism insurance coverage in the private market.[14]

In addition to federal backstops, terrorist catastrophe laws may include provisions for the establishment of terrorism insurance pools or reinsurance mechanisms to help spread the risk of terrorism-related losses among insurers and reinsurers. These programs may also mandate or incentivize the purchase of terrorism insurance for certain types of properties or businesses deemed to be at elevated risk, such as major infrastructure facilities, commercial centers, and critical government buildings.

Furthermore, these laws may stipulate the creation of public-private partnerships aimed at enhancing collaboration between government agencies, insurers, and other stakeholders in the areas of risk assessment, preparedness, and response. By fostering greater coordination and information-sharing among key actors, these partnerships can improve the effectiveness of efforts to mitigate terrorism-related risks and enhance the resilience of communities and critical infrastructure.

## 5.3 Benefits of Catastrophe Insurance

Catastrophe insurance under terrorist catastrophe laws offers a range of benefits to both individuals and society at large. For businesses and property owners, such insurance provides financial protection against the potentially ruinous effects of terrorist attacks, allowing them to recover and rebuild more quickly. By spreading the risk across a broad pool of insured parties, catastrophe insurance helps to stabilize premiums and ensure the availability of coverage in high-risk areas, reducing the likelihood of market failure in the event of a major terrorist event.

Moreover, catastrophe insurance promotes social resilience by promoting solidarity and mutual support within communities affected by terrorism. By providing a financial safety net for victims of terrorism and their families, these programs help to mitigate the psychological and emotional impact of such events and facilitate the process of recovery and healing. From a broader economic perspective, catastrophe insurance can mitigate systemic risks and prevent cascading financial crises, thereby safeguarding jobs and preserving economic stability in the aftermath of terrorist attacks.

## 5.4 Reasons for the Government to Pass Such a Law

Governments have compelling reasons to enact terrorist catastrophe laws as part of their national security and economic resilience strategies. By providing a mechanism for transferring terrorism-related risks to the insurance market, these laws reduce the burden on taxpayers and government resources in the event of a catastrophic attack. Moreover, the existence of terrorism insurance programs can serve as a deterrent to would-be attackers by increasing the perceived costs and consequences of terrorism, thereby helping to prevent future attacks.

Additionally, by promoting private-sector involvement in risk management and recovery efforts, terrorist catastrophe laws foster innovation and efficiency in addressing complex security challenges. The involvement of insurers and reinsurers in assessing and pricing terrorism risks can help to improve the understanding of these risks and develop more effective strategies for managing them. Furthermore, the enactment of terrorist catastrophe laws demonstrates a government's commitment to protecting its citizens and critical infrastructure from the impacts of terrorism, bolstering public confidence and social cohesion in the face of adversity.

Overall, terrorist catastrophe laws represent a proactive and pragmatic approach to managing the evolving threat landscape posed by terrorism in the modern world. By providing a framework for addressing the economic consequences of terrorist attacks and promoting resilience and solidarity within affected communities, these laws play a crucial role in safeguarding national security and preserving the prosperity and well-being of citizens in an uncertain world.

# 6 Stochastic Models in Life Insurance

## 6.1 Markov Chains

Let $(\Omega, \mathscr{F}, \mathbb{P})$ be a fixed probability space and we fix a measurable space $\mathcal{S}$. Denote by $\mathscr{B}_{\mathcal{S}}$ the sigma-algebra of Borel subsets of $\mathcal{S}$.

**Definition 6.1.** *A* stochastic process *with state space in $\mathcal{S}$ parametrized by a set $T$ is a family of measurable maps*

$$X(t) : (\Omega, \mathscr{F}, \mathbb{P}) \to (\mathcal{S}, \mathscr{B}_{\mathcal{S}}), \ \ \omega \mapsto X_\omega(t), \ \ t \in T,$$

Denote by $\mathcal{S}^T$ the set of functions $T \to \mathcal{S}$. The set $\mathcal{S}^T$ is equipped with natural projections

$$\mathrm{Ev}_t : \mathcal{S}^T \to \mathcal{S}, \ \ \mathcal{S}^T \ni X \mapsto X(t) \in \mathcal{S}, \ \ t \in T.$$

These define a canonical sigma-algebra $\mathscr{B}_{\mathcal{S}}^T$ on $\mathcal{S}^T$. More precisely, $\mathscr{B}_{\mathcal{S}}^T$ is the smallest sigma-algebra such that all the maps $\mathrm{Ev}_t$ are measurable. Equivalently, it is the sigma-algebra generated by the sets

$$C_{B,t} := \left\{ X \in \mathcal{S}^T; \ \ X(t) \in B \right\} = \mathrm{Ev}_t^{-1}(B),$$

where $t \in T$, and $B$ is a Borel subset of $\mathscr{B}_{\mathcal{S}}$

**Definition 6.2.** *A stochastic process defines a measurable map*

$$X : (\Omega, \mathscr{F}, \mathbb{P}) \to (\mathcal{S}^T, \ \ \mathscr{B}_{\mathcal{S}}^T), \ \ \Omega \ni \omega \mapsto X_\omega(-) \in \mathcal{S}^T.$$

*The push-forward of the probability measure $\mathbb{P}$ by $X$ is a probability measure $\mathbb{P}_X$ on $(\mathcal{S}^T, \mathscr{B}_{\mathcal{S}}^T)$ called the* distribution *of the stochastic process $X$.*

In the following, we assume the state space $\mathcal{S}$ is countable.

**Definition 6.3.** *Let $(X_t)_{t\in T}$ be a stochastic process on probability space $(\Omega, \mathscr{F}, \mathbb{P})$ with state space in $\mathcal{S}$ parametrized by a set $T$. The process is called* Markov Chain *if for all $n \geq 1$,*

$$t_1 < t_2 < ... < t_n < t_{n+1} \in T, \quad i_1, i_2, ..., i_n, i_{n+1} \in \mathcal{S}$$

*satisfying the following* Markov Property*:*

$$\mathbb{P}\big[\, X_{t_{n+1}} = i_{n+1} \,|\, X_{t_1} = i_1, X_{t_2} = i_2, ..., X_{t_n} = i_n \,\big] = \mathbb{P}\big[\, X_{t_{n+1}} = i_{n+1} \,|\, X_{t_n} = i_n \,\big]$$

The Markov Property states that the probability of a future event is conditionally independent of the past given the present information. We list an important example of Markov Chain:

**Example 6.1.** Let $(X_k)_{k\in\{1,2,3,...\}}$ be a sequence of independent random variables and let $(S_n)_{n\in\{1,2,3,...\}}$ be a random walk, i.e.

$$S_n := \sum_{k=1}^{n} X_k,$$

then the random walk is a Markov Chain.

## 6.2 Some Properties of Markov Chains

**Definition 6.4.** *Let $(X_t)_{t\in T}$ be a stochastic process on probability space $(\Omega, \mathscr{F}, \mathbb{P})$. Then $p_{ij}(s,t) := \mathbb{P}\big[\, X_t = j \,|\, X_s = i \,\big]$, where $s < t$ and $i, j \in \mathcal{S}$, is called transitional probability.*

**Theorem 6.2.** *Chapman–Kolmogorov equation*
*Let $(X_t)_{t\in T}$ be a Markov Chain. For $s \leq t \leq u \in T$ and $i, k \in \mathcal{S}$ such that $\mathbb{P}\big[\, X_s = i \,\big] > 0$, then the following holds:*

$$p_{ik}(s,u) = \sum_{j\in\mathcal{S}} p_{ij}(s,t) p_{jk}(t,u)$$

*In another word, we have following transition probability matrix multplication,*

$$P(s,u) = P(s,t)P(t,u)$$

*Proof.*

$$p_{ik}(s,u) = \mathbb{P}\big[\, X_u = k \,|\, X_s = i \,\big] = \sum_{j\in\mathcal{S}} \mathbb{P}\big[\, X_u = k, X_t = j \,|\, X_s = i \,\big]$$

$$= \sum_{j\in\mathcal{S}} \mathbb{P}\big[\, X_t = j \,|\, X_s = i \,\big] \cdot \mathbb{P}\big[\, X_u = k \,|\, X_s = i, X_t = j \,\big]$$

Now, by simple Markov property, the above probability becomes

$$= \sum_{j\in\mathcal{S}} \mathbb{P}\big[\, X_t = j \,|\, X_s = i \,\big] \cdot \mathbb{P}\big[\, X_u = k \,|\, X_t = j \,\big] = \sum_{j\in\mathcal{S}} p_{ij}(s,t)p_{jk}(t,u),$$

as desired. □

**Remark 6.1.** *The matrix in Chapman-Kolmogorov equation is called transition matrix.*

**Definition 6.5.** *Let $(X_t)_{t\in T}$ be a Markov chain in continuous time with finite state space $\mathcal{S}$. $(X_t)_{t\in T}$ is called regular if*

$$\mu_i(t) = \lim_{\Delta t\to 0} \frac{1 - p_{ii}(t, t+\Delta t)}{\Delta t}$$

*and*

$$\mu_{ij}(t) = \lim_{\Delta t\to 0} \frac{p_{ij}(t, t+\Delta t)}{\Delta t}$$

*are well-defined and continuous with respect to $t$. Moreover, $\mu_i(t)$ and $\mu_{ij}(t)$ are called transition rates of Markov Chains and we define $\mu_{ii}(t) = -\mu_i(t)$*

**Remark 6.2.** *Note that we can understand the transition rates as derivatives of the transition probabilities for the following reason:*

$$\mu_{ij}(t) = \lim_{\Delta t\to 0} \frac{p_{ij}(t, t+\Delta t)}{\Delta t} = \lim_{\Delta t\to 0} \frac{p_{ij}(t, t+\Delta t) - p_{ij}(t,t)}{\Delta t}$$

$$= \frac{d}{ds} p_{ij}(t,s)\big|_{s=t}$$

**Definition 6.6.** *Let $(X_t)_{t\in T}$ be a regular Markov chain on a finite state space $\mathcal{S}$ Then we denote the conditional probability to stay during the time interval $[s,t]$ in $j$ by*

$$\bar{p}_{jj}(s,t) := \mathbb{P}\big[ \bigcap_{\xi\in[s,t]} \{X_\xi = j \,|\, X_s = j\} \,\big],$$

*where $s, t \in \mathbb{R}, s \leq t$ and $j \in \mathcal{S}$*

In the setting of a life insurance, this probability can be used to calculate the probability that the insured survives in 5 years. The following theorem provides a tool to compute this probability based on the transition rate.

**Theorem 6.3.** *Let $(X_t)_{t\in T}$ be a regular Markov chain. Then*

$$\bar{p}_{jj}(s,t) = \exp\big( -\sum_{j\neq k} \int_s^t \mu_{ik}(\tau)\, d\tau \,\big)$$

*Proof.* For the detailed proof of this theorem, we refer to theorem 2.3.7 in [10] □

## 6.3 Applications in Insurance

In this section, we give applications to the theory developed above.

**Example 6.4.** We begin with an example in the field of life insurance, which provides a sum of money to the heirs in case of the death of the insured. The state space of a permanent life insurance consists of the states "dead" and "alive". Hence, $\mathcal{S} = \{*, \dagger\}$, where $*$ indicates "alive" and $\dagger$ indicates "dead". Now, we will use the following death rate explained in section 4.3 of Chapter 4. [10]

$$\mu_{*\dagger}(x) = \exp\big( -9.13275 + 8.09438 \cdot 10^{-2}x - 1.10180 \cdot 10^{-5}x^2 \big)$$

Consequently, by theorem 6.3 above, we derive the survival rate of a 35 year old man is

$$\bar{p_{**}}(35, x) = \exp\Big( -\int_{35}^{x} \mu_{*\dagger}(\tau)\, d\tau \Big)$$

Our second example relates to disability pension.

**Example 6.5.** We consider a model of disability pension with three states: "Active", "Disabled", and "Dead". Hence, our state space is $\mathcal{S} = \{*, \diamond, \dagger\}$, where $*, \diamond, \dagger$ represents being active, disabled, and dead, respectively.
Now, the transition rates are defined by

$$\sigma(x) := 0.0004 + 10^{0.060x - 5.46}$$

$$\mu(x) := 0.0005 + 10^{0.038x - 4.12}$$

$$\mu_{*\diamond}(x) := \sigma(x), \quad \mu_{*\dagger}(x) := \mu(x), \quad \mu_{\diamond\dagger}(x) := \mu(x)$$

The transition rate $\sigma$ is the infinitesimal probability of being disabled and $\mu$ is the the infinitesimal probability of being dying. Next, we aim to compute transition probabilities by solving following Kolmogorov's differential equations with the boundary condtions $p_{ij}(s, s) = \delta_{ij}$, where if $i = j$, $\delta_{ij} = 1$ and if $i \neq j$, $\delta_{ij} = 0$.

$$\frac{d}{dt} p_{**}(s, t) = -p_{**}(s, t) \cdot (\mu(t) + \sigma(t))$$

$$\frac{d}{dt} p_{*\diamond}(s, t) = -p_{*\diamond}(s, t)\mu(t) + p_{**}(s, t)\sigma(t)$$

$$\frac{d}{dt}\,p_{*\dagger}(s,t) = \big(\,p_{**}(s,t) + p_{*\diamond}(s,t)\,\big) \cdot \mu(t)$$

$$\frac{d}{dt}\,p_{\diamond *}(s,t) = 0$$

$$\frac{d}{dt}\,p_{\diamond\diamond}(s,t) = -p_{\diamond\diamond}(s,t)\mu(t)$$

$$\frac{d}{dt}\,p_{\diamond\dagger}(s,t) = p_{\diamond\diamond}(s,t)\mu(t)$$

$$\frac{d}{dt}p_{\dagger\dagger}(s,t) = 0$$

Solving these differential equation yields

$$p_{**}(x,y) = \exp\big(\,-\int_x^y \big[\,\mu(\tau) + \sigma(\tau)\,\big]\,d\tau\,\big)$$

$$p_{*\diamond}(x,y) = \exp\big(\,-\int_x^y \mu(\tau)\,d\tau\,\big) \times \big(\,1 - \exp\big(\,-\int_x^y \sigma(\tau)\,d\tau\,\big)\,\big)$$

$$p_{\diamond\diamond}(x,y) = \exp\big(\,-\int_x^y \mu(\tau)\,d\tau\,\big)$$

We also list several numerical solutions of the equation above.

| **Age** | $p_{**}(30,x)$ | $p_{*\diamond}(30,x)$ | $p_{*\dagger}(30,x)$ | $p_{\diamond\diamond}(30,x)$ | $p_{\diamond\dagger}(30,x)$ |
|---|---|---|---|---|---|
| 30.00 | 1.00000 | 0.00000 | 0.00000 | 1.00000 | 0.00000 |
| 35.00 | 0.98743 | 0.00354 | 0.00903 | 0.99097 | 0.00903 |
| 40.00 | 0.96998 | 0.00850 | 0.02152 | 0.97849 | 0.02152 |
| 45.00 | 0.94457 | 0.01620 | 0.03923 | 0.96077 | 0.03923 |
| 50.00 | 0.90624 | 0.02903 | 0.06474 | 0.93526 | 0.06474 |
| 55.00 | 0.84725 | 0.05106 | 0.10169 | 0.89831 | 0.10169 |
| 60.00 | 0.75677 | 0.08832 | 0.15491 | 0.84509 | 0.15491 |
| 65.00 | 0.62287 | 0.14700 | 0.23013 | 0.76987 | 0.23013 |

Table 4: Transition probabilities for the disability insurance

Notice that we set up our initial age to be 30 in the table above.

# A Appendices

## A.1 Mortality Notebook

```
import pandas as pd
import os
import numpy as np
import warnings
warnings.filterwarnings('ignore')

# First we read in the probabilities of death in a given year given a current age for 4 different categories of people. We have data for ages 0-108, but we display below ages 21-80 as these are likely the most commonly considered ages for term life.

dfs = []
folder_path = "Probability of Death Data"
for filename in os.listdir(folder_path):
    if filename.endswith(".xls"):
        df = pd.read_excel(os.path.join(folder_path, filename), skiprows=24, names=['Index', f'Prob{filename}'])
        dfs.append(df)
merged_df = dfs[0]
for df in dfs[1:]:
    merged_df = merged_df.merge(df, on='Index', how='inner')
merged_df.set_index("Index",inplace=True)
merged_df.columns= ["White Female","White Male","Black Male","Black Female"]
probs=merged_df
probs.iloc[20:80,:]

# We relatively arbitrarily assign black and white men/women percentages of the population so that our sample of people is reasonable. The percents are based off of US population percentages found online if we exclude people that are not either Black or White.

b_perc = .076 #percentage of our population that is black women (equals percentage that is black men)
w_perc =.424 #percentage of our population that is white women (equals percentage that is white men)
wF, wM, bM, bF = w_perc, w_perc,b_perc, b_perc #get number of each population group

# Generate a random sample of individuals based on the percentages defined above. We observe 1 million individuals of various ages where we place 40
```

```
as the mode and select ages from ages 25-55 in a triangular sort of distribution. Although this is not a perfect representation of who a term insurance company may insure, it is a reasonable assumption for our toy model. We create the people dataframe for easy calculation where we group by race and sex showing counts of people in our dataset.

import random
percentages = {
    'Black Male': bM,
    'Black Female': bF,
    'White Male': wM,
    'White Female': wF
}

# Generate 1 million random observations for race/sex based on percentages
n = 1000000
random.seed(1)
race_sex = np.random.choice(list(percentages.keys()), n, p=list(percentages.values()))

# Generate random ages for our population with the heaviest ages on 40 and lower weights the further from 40 you are
#this simiple model assumes people between 25 and 55 are purchasing term life insurance from our company
ages = np.random.triangular(left=25, mode=40, right=55, size=n)

# Create a DataFrame
people = pd.DataFrame({'Race/Sex': race_sex, 'Age': ages})
people['Age']=people['Age'].astype(int)

people = people.groupby(['Race/Sex','Age']).size().reset_index().rename(columns={0:"Count"})
#PeopleView is used later for plotting
peopleView = people.copy()
peopleView[['Race','Sex']]= peapleView['Race/Sex'].str.split(' ', 1, expand=True)
people #one column for race, sex, one for age, and another for count of people  in that demographic

# Below we plot our people subsets letting us visual the random distribution of age. Note that although random, due to the high number of samples, we approach a normal distribution.

import matplotlib.pyplot as plt
```

```
grouped = peopleView.groupby(['Race', 'Sex'])

# Create a figure and axes for subplots
fig, axs = plt.subplots(len(grouped), figsize=(10, 8))

# Loop through the groups and plot
for (key, ax), group in zip(enumerate(axs), grouped):
    (race, sex), data = group
    data.plot(kind="bar", x='Age', y='Count', ax=ax, title=f"{race} - {sex}")
    ax.set_ylabel("Count")
    ax.set_xlabel("Age")
    ax.grid(True)

# Adjust layout and display
plt.tight_layout()
plt.show()

# Below, our expected payouts function is used to calculate overall death probability at each age using a binomial model. The input is data contained in a single row of the people dataframe, and we have included some fixed variables for calculation puproses that can be adjusted if needed. Note that the expected payouts are based on present value of expected payouts using a risk free interest rate of 5%.

def binomial_model(race_sex,age,count=1,n=30,payout=500000,probs=probs,r=.05):
    '''
    input: race_sex: is string of race and sex space seperated,
        age: is the age at the start of the policy,
        count: is the number of individuals carrying these characteristics
        n: length of the period of term life
        payout: term life payout
        probs: mortality data table defined above for different probabilities of death at various ages
        r: assumed risk free interest rate to calculate present value
    '''
    probs2 = probs.iloc[age-1:age-1+n,] #take all data of ages within n years
    death_probs = probs2[[race_sex]]
    death_probs.columns = ["DeathProb"]
    death_probs["Expected Present Value of Payout If Alive"] = 0
    death_probs["Expected Present Value of Payout If Already Dead"] = 0
    for i in range(n):
        current_prob = death_probs.iloc[n-1-i,0]
        if i == 0:
            death_probs.iloc[n-1-i,1]= (1/(1+r))*payout*current_prob
```

```
        else:
            death_probs.iloc[n-1-i,1]= (1/(1+r))*(death_probs.iloc[n-i,1]*(1-current_prob)+payout*current_prob)
    death_probs["Expected Present Value of Payout If Alive"] = count*death_probs["Expected Present Value of Payout If Alive"]
    death_probs.reset_index(inplace=True)
    return death_probs[['Expected Present Value of Payout If Alive']].transpose().round(2)
def extract_pv(race_sex,age,count=1,n=30,payout=500000,probs=probs,r=.07):
    df=binomial_model(race_sex=race_sex,age=age,count=count,n=n,payout=payout,probs=probs,r=r)
    return '{:,.0f}'.format(df["Expected Present Value of Payout If Alive"].iloc[0,].round(2))

# Example results for a Black Female of Age 25. Below you can see the present value of expected death benefit for each year of the policy given she is still alive as calculated by the binomial model.

bf25 = binomial_model("Black Female",25).transpose()
bf25

# Below is the total present value of the policy (or the present value at year 0)

pv = '{:,.0f}'.format(bf25["Expected Present Value of Payout If Alive"].iloc[0,].round(2))
print(f"Total Present Value of Expected Costs = ${pv}")

# Now we apply the above function to all groups of people to get our total present value of expected death benefits which is nearly 55 Billion dollars in present value over the course of the 30 years.

observations = [binomial_model(row["Race/Sex"],row["Age"],row["Count"]) for index,row in people.iterrows()]
costs = pd.concat(observations, ignore_index=True)
total=format(costs.iloc[:,0].sum(),",")
print(f"Total Present Value of Expected Costs = ${total}")
people.merge(costs,left_index=True,right_index=True)
```

## A.2 Chance of Death for 50 Year Old Women Within 10 Years

```
import pandas as pd
import os
import numpy as np
```

```

ww = pd.read_excel('t2027.xls')
bw = pd.read_excel('t2029.xls')

ww.columns = ['Age', 'Prob']
bw.columns = ['Age', 'Prob']

ww = ww.iloc[23:]
bw = bw.iloc[23:]

ww_remaining = 1
prob50_60ww = 0
bw_remaining = 1
prob50_60bw = 0

for i in range(50,61):
    p = float(ww.iloc[23+i]['Prob'])
    prob50_60ww += p*ww_remaining
    ww_remaining *= 1 - p
    p = float(bw.iloc[23+i]['Prob'])
    prob50_60bw += p*bw_remaining
    bw_remaining *= 1 - p

print(prob50_60ww, prob50_60bw)

b_perc = .076
w_perc =.424

perc = b_perc + w_perc

avg_prob = w_perc * prob50_60ww / perc + b_perc * prob50_60bw / perc
print(avg_prob)
```

## A.3 Calculating Periodic Premiums

```
import pandas as pd

class Data():
    def __init__(self):
        self.wm = pd.read_excel('t2026.xls')
        self.ww = pd.read_excel('t2027.xls')
        self.bm = pd.read_excel('t2028.xls')
        self.bw = pd.read_excel('t2029.xls')
        self.map = {'wm': self.wm, 'ww': self.ww, 'bm': self.bm, 'bw': self.bw
```

```
            }

        for demo in [self.wm, self.ww, self.bm, self.bw]:
            demo.columns = ['Age', 'Prob']
            demo = demo.iloc[23:]
            demo.reset_index(drop=True, inplace=True)
            demo = demo.apply(pd.to_numeric)

    def presentValue(self, s: float, t: float, m: float, r: float, demo='wm'):
        '''
        s: starting age of policy
        t: length of policy in years
        m: policy payout
        r: risk-free interest rate
        demo: demographic
        '''

        prob = self.map[demo]
        pv = 0

        for i in range(1, t+1):
            age = s + i
            interest = 1 / ((1+r)**i)

            chanceOfSurvival = 1
            for j in range(1, i):
                chanceOfSurvival *= (1 - float(prob.iloc[age + j]['Prob']))

            chanceOfDeath = chanceOfSurvival * float(prob.iloc[age + i]['Prob'])
            ev = interest * chanceOfDeath * m
            pv += ev

        return pv

    def periodCost(self, s: float, t: float, n: float, m: float, r: float, demo='wm'):
        '''
        s: starting age of policy
        t: length of policy in years
        n: number of periods per year
        m: policy payout
        r: risk-free interest rate
        demo: demographic
```

```
        '''
        pv = self.presentValue(s, t, m, r, demo)
        interest = 1 - (1 / ((1 + r / n) ** (n * t)))
        adjustment = n * interest / r
        return pv, pv / adjustment

data = Data()
pv, c = data.periodCost(s=25, t=30, n=12, m=500000, r=0.05, demo='wm')

print(f'Present value of policy: ${pv:8.2f}')
print(f' Monthly policy premium: ${c:8.2f}')
```

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