Homework 3

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1. (a) $\alpha(x)$:= "x is a man"
$\beta(x)$:= "x is featherless"
$\gamma(x)$:= "x is a biped"

$\forall x(\beta(x) \wedge \gamma(x) \rightarrow \alpha(x))$
$\beta(\text{“}Diogenes\text{”})$
$\gamma(\text{“}Diogenes\text{”})$

$\therefore \alpha(\text{“}Diogenes\text{”})$

(b) Yes, this argument is valid because if Diogenes is featherless and a biped, then the condition for the material implication is true, implying that the right side is true. By this logic, and assuming that Diogenes is within the universal discourse Ω, then Diogenes must be a man.

2. (a) Ω := "sentient humanoid beings (humans and androids) in the year 2029 in Japan"
$\alpha(x)$:= "x is a person"
$\beta(x)$:= "x is an android"
$\gamma(x)$:= "x is a ghost in a shell"

$\forall x(\beta(x) \rightarrow \gamma(x))$
$\beta(\text{“}Major\ Kusanagi\text{”})$
$\exists x(\alpha(x) \wedge \beta(x))$
$\alpha(\text{“}Chief\ Nakamura\text{”})$

(b) i. $\alpha(\text{“}Major\ Kusanagi\text{”})$ - Doesn't Follow
While Major Kusanagi is an android, and some androids are people, we don't necessarily know whether or not Major Kusanagi is a person. He could be a case that satisfies premise 3, or he could not; there is no way to tell.

ii. $\gamma(\text{“}Major\ Kusanagi\text{”})$ - Follows

Proof.

$\beta(\text{“}Major\ Kusanagi\text{”})$	premise	(1)
$\forall x(\beta(x) \rightarrow \gamma(x))$	premise	(2)
$\beta(\text{“}Major\ Kusanagi\text{”}) \rightarrow \gamma(\text{“}MajorKusanagi\text{”})$	U.I. on (2)	(3)
$\gamma(\text{“}Major\ Kusanagi\text{”})$	Modus Ponens on (1)&(3)	(4)

□

iii. $\gamma(\text{“}Chief\ Nakamura\text{”})$ - Doesn't Follow
We know that Chief Nakamura is a person from premise 4, but according to premise 3, he may or may not be an android. If he is an android, then this conclusion follows, but since we cannot assume he is an android, then we cannot guarantee that he is a ghost in a shell.

iv. $\neg\gamma(\text{“}Chief\ Nakamura\text{”})$ - Doesn't Follow
Once again, we cannot say with certainty whether Chief Nakamura is or is not an android. By this reasoning in conjunction with premise 3, we also cannot say that he is not a ghost in a shell.

v. $\forall x(\beta(x) \rightarrow \alpha(x))$ - Doesn't Follow
If some androids are people, then some androids must not be people. By this logic, not every android can be a person.

3. *Proof.* Valid Argument

$\forall x(\varphi(x) \to \psi(x))$	premise	(5)
$\forall x(\psi(x) \to \xi(x))$	premise	(6)
$\exists x(\varphi(x) \wedge \chi(x))$	premise	(7)
$\varphi(c) \to \psi(c)$	U.I. on (5)	(8)
$\psi(c) \to \xi(c)$	U.I. on (6)	(9)
$\varphi(c) \to \xi(c)$	Hypothetical on (8)&(9)	(10)
$\varphi(c) \wedge \chi(c)$	E.I. on (7)	(11)
$\xi(c) \wedge \chi(c)$	Modus Ponens on (10)&(11)	(12)
$\exists x(\xi(x) \wedge \chi(x))$	E.G. on (12)	(13)

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