Homework 1

Walker Bagley

September 10, 2024

(3)

Show that the number $20888\ldots821$ is always composite.

$$
\begin{aligned}
20821 &= 47 \cdot 443 \\
208821 &= 47 \cdot 3 \cdot 1481 \\
2088821 &= 47 \cdot 7^2 \cdot 907 \\
20888821 &= 47 \cdot 444443
\end{aligned}
$$

It seems like 47 is a prime factor of all numbers of this form, so let's try long division.

$$
\begin{array}{l}
444\ldots443 \\
47\overline{)20888\ldots821} \\
\underline{188} \\
208 \\
\underline{188} \\
208 \\
\underline{188} \\
208 \\
\underline{188} \\
208 \\
\ldots \\
208 \\
\underline{188} \\
202 \\
\underline{188} \\
141 \\
\underline{141} \\
0
\end{array}
$$

Clearly, 47 will always divide this number without a remainder, so it must be composite.

(5)

How many digits does $2^{2^{2^{2^2}}}$ have when written in base 10?

$$
\begin{gathered}
2^{2^{2^{2^2}}} = 2^{2^{2^4}} = 2^{2^{16}} = 2^{65536} \\
\log\left(2^{65536}\right) = 65536 \log 2 = 65536 \cdot 0.30103 = 19728.302 \Rightarrow 19729
\end{gathered}
$$

(7)

What is more precise: knowing 7 decimals of π in base 10 or knowing 8 decimals in base 7?
Well, 7 decimals in base 10 gives us a precision of $\frac{1}{10^7} = \frac{1}{10{,}000{,}000}$. 8 decimals in base 7 gives a precision of $\frac{1}{7^8} = \frac{1}{5{,}764{,}801}$. We clearly get more precise with 7 decimals in base 10, in fact, nearly twice as precise.

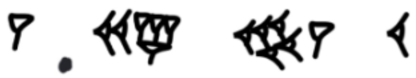
(8)

Write 3.06015625 in base 20.

$$\begin{aligned} 3.06015625 &= 3_{(20)} + \frac{1.203125}{20} \\ &= 3.1_{(20)} + \frac{4.0625}{20^2} \\ &= 3.14_{(20)} + \frac{1.25}{20^3} \\ &= 3.141_{(20)} + \frac{5}{20^4} \\ &= 3.1415_{(20)} \end{aligned}$$

(9)

What is the following cuneiform (base 60) number approximating?



$$\begin{aligned} 1 \cdot 60^0 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} &= 1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} \\ &= 1 + 0.4 + \frac{51}{60^2} + \frac{10}{60^3} \\ &= 1 + 0.4 + 0.0141\overline{6} + \frac{10}{60^3} \\ &= 1.4141\overline{6} + 0.0000462963 \\ &= 1.4142129629\overline{6} \end{aligned}$$

It becomes apparent after converting to base ten that this number is approximating $\sqrt{2}$.