Exam 1 - Part A

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(2)

$$\begin{aligned}
E(FV) &= \frac{1}{2}\cdot 400 + \frac{1}{2}\cdot 100 = 250\\
E(PV) &= \frac{250}{1.04} = 240.38\\
z &= \begin{cases} z_1 = \frac{400}{1.04} - 240.38 = 144.23\\ z_2 = \frac{100}{1.04} - 240.38 = -144.23\end{cases}\\
E(z) &= 0\\
\ln(900 + 0 - \rho) &= \frac{1}{2}\ln(900 + 144.23) + \frac{1}{2}\ln(900 - 144.23)\\
&= 3.4755 + 3.3139 = 6.7894\\
\rho &= 900 - e^{6.7894} = \$11.63
\end{aligned}$$

$$\begin{aligned}
S_0 &= E(PV) - \rho = 240.38 - 11.63 = 228.75\\
\text{return} &= \frac{E(FV) - S_0}{S_0} = \frac{250 - 228.75}{228.75} = 0.0923\\
\text{equity premium} &= 0.0923 - 0.04 = 0.0523
\end{aligned}$$

(3)

$$\begin{cases} S_0 = 179.66\\ S_1 = 175.10\\ r = 0.0422\\ \sigma = 0.1777\\ K = 180\\ T = 90 \text{ days}\end{cases}$$

Calculate V_0:

$$\begin{aligned}
d_1 &= \frac{\ln\frac{179.66}{180} + (0.0422 + \frac{1}{2}0.1777^2)\cdot\frac{90}{365}}{0.1777\sqrt{\frac{90}{365}}}\\
&= \frac{-0.00189 + (0.5799)\cdot 0.2466}{0.0882} = 0.1406\\
d_2 &= d_1 - 0.1777\sqrt{\frac{90}{365}} = 0.0524\\
V_0 &= 179.66N(0.1406) - 180e^{-0.0422\cdot\frac{90}{365}}N(0.0524)\\
&= 179.66(0.5559) - 178.1367(0.5209) = \$7.08
\end{aligned}$$

Calculate the "Greeks":

$$
\begin{aligned}
\Delta &= N(d_1) = 0.5559 \\
\Gamma &= \frac{1}{S_0\sigma\sqrt{2\pi T}} e^{-\frac{1}{2}d_1^2} = \frac{e^{-\frac{1}{2}0.1406^2}}{179.66 \cdot 0.1777\sqrt{2\pi \cdot \frac{90}{365}}} \\
&= \frac{e^{-0.00988}}{31.9255\sqrt{1.5493}} = 0.02492 \\
\theta &= -\frac{\sigma S_0}{2\sqrt{2\pi T}} e^{-\frac{1}{2}d_1^2} - rKe^{-rT}N(d_2) \\
&= -\frac{0.1777 \cdot 179.66}{2\sqrt{2\pi\frac{90}{365}}} e^{-\frac{1}{2}0.1406^2} - 0.0422 \cdot 180 e^{-0.0422 \cdot \frac{90}{365}} N(0.0524) \\
&= -\frac{31.9255}{2.4894} e^{-0.00988} - 7.596 e^{-0.0104}(0.5209) \\
&= -12.6985 - 3.9158 = -16.6143
\end{aligned}
$$

Calculate V_1:

$$
\begin{aligned}
d_1 &= \frac{\ln\frac{175.10}{180} + (0.0422 + \frac{1}{2}0.1777^2) \cdot \frac{89}{365}}{0.1777\sqrt{\frac{89}{365}}} \\
&= \frac{-0.0276 + (0.05799) \cdot 0.2438}{0.0877} = -0.1534 \\
d_2 &= d_1 - 0.1777\sqrt{\frac{89}{365}} = -0.2411 \\
V_1 &= 175.10N(-.1534) - 180e^{-0.0422 \cdot \frac{89}{365}} N(-0.2411) \\
&= 175.10(0.4390) - 178.1573(0.4047) = \$4.77
\end{aligned}
$$

Exact Profit:

$$
\begin{aligned}
\text{exact profit} &= 100[-(V_1 - V_0) + \Delta(S_1 - S_0) - (e^{\frac{r}{365}} - 1)(\Delta S_0 - V_0)] \\
&= 100[-(4.77 - 7.08) + 0.5559(175.10 - 179.66) - (e^{\frac{0.0422}{365}} - 1)(0.5559 \cdot 179.66 - 7.08)] \\
&= 100[2.31 + 0.5559(-4.56) - (0.000115)(92.7930)] \\
&= 100[2.31 - 2.5349 - 0.0107] \\
&= 100(-0.2327) = -\$23.27
\end{aligned}
$$

Approximate Profit:

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\begin{aligned}
\text{approximate profit} &= 100[-\theta\frac{1}{365} - \frac{1}{2}\Gamma(S_1 - S_0)^2 - \frac{r}{365}(\Delta S_0 - V_0)] \\
&= 100[\frac{16.6143}{365} - \frac{0.02492}{2}(175.10 - 179.66)^2 - \frac{0.0422}{365}(0.5559 \cdot 179.66 - 7.08)] \\
&= 100[0.0455 - (0.01246)(20.7936) - (.0001156)(92.7930)] \\
&= 100(-0.2243) = -\$22.43
\end{aligned}
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(4a)

$$
\begin{aligned}
d_1 &= \frac{\ln \frac{S_0}{K} + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \\
\lim_{\sigma \to 0} d_2 &= \lim_{\sigma \to 0} d_1 - \sigma\sqrt{T} = \lim_{\sigma \to 0} d_1 - 0 = \lim_{\sigma \to 0} d_1 \\
\text{Case 1: } \ln \frac{S_0}{K} + rT > 0 & \\
\lim_{\sigma \to 0^+} d_1 &= \frac{\ln \frac{S_0}{K} + rT}{0^+\sqrt{T}} = \infty \\
\lim_{\sigma \to 0} V_0^{ECO} &= S_0 N(\infty) - e^{-rT} K N(\infty) \\
V_0^{ECO}(0) &= S_0 - e^{-rT} K \\
\text{Case 2: } \ln \frac{S_0}{K} + rT = 0 & \\
\lim_{\sigma \to 0} d_1 &= \frac{0}{0} \\
\lim_{\sigma \to 0} d_1 &= \frac{0}{\sqrt{T}} = 0 \\
\lim_{\sigma \to 0} V_0^{ECO} &= S_0 N(0) - e^{-rT} K N(0) \\
V_0^{ECO}(0) &= \frac{1}{2}[S_0 - e^{-rT} K] \\
\text{Case 3: } \ln \frac{S_0}{K} + rT < 0 & \\
\lim_{\sigma \to 0^+} d_1 &= \frac{\ln \frac{S_0}{K} + rT}{0^+\sqrt{T}} = -\infty \\
\lim_{\sigma \to 0} V_0^{ECO} &= S_0 N(-\infty) - e^{-rT} K N(-\infty) \\
V_0^{ECO}(0) &= 0
\end{aligned}
$$

(4b)

$$
\begin{aligned}
d_1 &= \frac{\ln \frac{125}{100} + (0.03 + \frac{1}{2}\sigma^2)}{\sigma} = \frac{0.5063 + \sigma^2}{2\sigma} \\
d_2 &= \frac{0.5063 - \sigma^2}{2\sigma}
\end{aligned}
$$

(i)

$$
\begin{aligned}
V_0 = 28.59 &= 125N(d_1) - 100e^{-0.03}N(d_2) \\
&= 125N(d_1) - 97.0446N(d_2) \\
V_0(0.1) &= \$27.98 \\
V_0(0.2) &= \$29.04 \\
V_0(0.15) &= \$28.27 \\
V_0(0.175) &= \$28.59 \\
\text{implied volatility } \sigma &= 0.175
\end{aligned}
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(ii)

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V_0(0) = 125 - 97.0446 = \$27.96
$$

Since the lowest volatility ($\sigma = 0$) gives us a minimum market price of \$27.96, there cannot possibly exist a value of σ that would yield $V_0 = 24$.