# Homework 6

Walker Bagley

April 9, 2024

## 1 Simple Cycles

A graph $C_n$ with $n$ nodes would have $2n$ automorphisms. This is because there exist automorphisms for the $n$ possible rotations as well as for mappings created by a reflection about an axis followed by $n$ possible rotations. Thus there are $n + n = 2n$ automorphisms of $C_n$.

## 2 Complete Graphs

A complete graph $K_n$ means that every node is connected to every other node. Thus, the number of automorphisms is equivalent to the number of combinations we can create of the $n$ nodes. For the first node, there are $n$ options, second has $n-1$ options and so on until there is 1 node left. So, the number of ways to arrange the $n$ nodes and therefore number of automorphisms is $n \cdot (n-1) \ldots 2 \cdot 1 = n!$.

## 3 Automorphism Orbits

In a graph $P_n$, there can be an even or odd number of nodes stretched along a line. Let's number the nodes on the line from left to right counting from 1 to $n$, so node 1 is the leftmost and $n$ the rightmost. Then, we count orbits working from both ends inwards. Counting nodes 1 and $n$ as an orbit, nodes 2 and $n-1$ and so on. If $n$ is even, then the middle two nodes will form one orbit and thus we are done, otherwise if $n$ is odd, then the very middle node will not have a match and thus be its own orbit. So we have $\frac{n}{2}$ orbits in the even case and $\frac{n+1}{2}$ orbits in the odd case, which we can rewrite as $\lceil \frac{n}{2} \rceil$ to generalize both cases.

## 4 GDV

| Orbit | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| GDV | 5 | 2 | 9 | 1 | 0 | 6 | 0 | 7 | 1 | 0 | 0 | 3 | 0 | 0 | 0 |