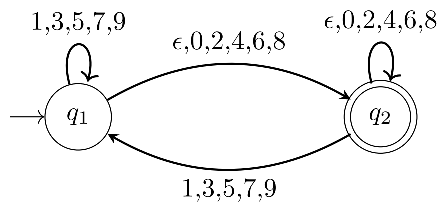
Homework 2

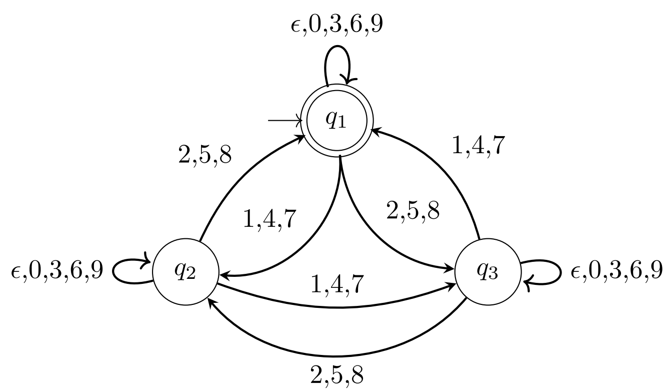
Walker Bagley

February 3, 2023

1. (a) DFA for D_2



(b) DFA for D_3



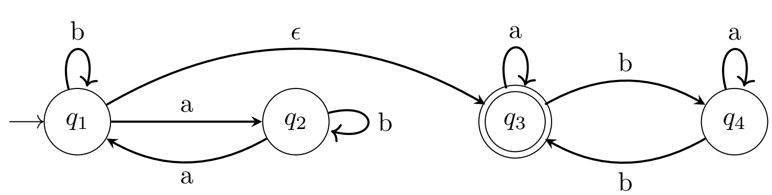
(c) Prove that D_6 is regular

Proof.

From the DFAs in parts (a) and (b), we know that D_2 and D_3 are regular
NTS that there is a DFA for D_6
We know that any multiple of 6 is a multiple of 2 and a multiple of 3
So, there exists a DFA that accepts for D_2 and D_3 (i.e. parts (a) and (b) multiplied)
This DFA will accept for D_6
The existence of this DFA by definition means that D_6 is regular

□

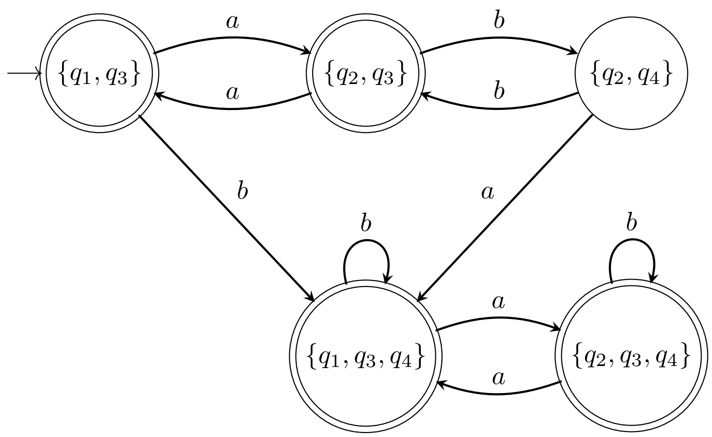
2. (a) NFA N_2 that recognizes L_2



(b) Show the path through N_2 for bab^n for $n = 1, 2, 3, 4$ and the boundary between u, v

$$n = 1 : bab \Rightarrow (q1) \xrightarrow{\epsilon} (q_3 \xrightarrow{b} q_4 \xrightarrow{a} q_4 \xrightarrow{b} q_3)$$
$$n = 2 : babb \Rightarrow (q_1 \xrightarrow{b} q_1) \xrightarrow{\epsilon} (q_3 \xrightarrow{a} q_3 \xrightarrow{b} q_4 \xrightarrow{b} q_3)$$
$$n = 3 : babbb \Rightarrow (q1) \xrightarrow{\epsilon} (q_3 \xrightarrow{b} q_4 \xrightarrow{a} q_4 \xrightarrow{b} q_3 \xrightarrow{b} q_4 \xrightarrow{b} q_3)$$
$$n = 4 : babbbb \Rightarrow (q_1 \xrightarrow{b} q_1) \xrightarrow{\epsilon} (q_3 \xrightarrow{a} q_3 \xrightarrow{b} q_4 \xrightarrow{b} q_3 \xrightarrow{b} q_4 \xrightarrow{b} q_3)$$

(c) Convert N_2 to DFA M_2 with subset construction



(d) Show the path through M_2 for bab^n for $n = 1, 2, 3, 4$ and the boundary between u, v if it is distinguishable

Consider the following states: $1 = \{q_1, q_3\}$, $2 = \{q_1, q_3, q_4\}$ and $3 = \{q_2, q_3, q_4\}$

$$n = 1 : bab \Rightarrow 1 \xrightarrow{b} 2 \xrightarrow{a} 3 \xrightarrow{b} 3$$
$$n = 2 : babb \Rightarrow 1 \xrightarrow{b} 2 \xrightarrow{a} 3 \xrightarrow{b} 3 \xrightarrow{b} 3$$
$$n = 3 : babbb \Rightarrow 1 \xrightarrow{b} 2 \xrightarrow{a} 3 \xrightarrow{b} 3 \xrightarrow{b} 3 \xrightarrow{b} 3$$
$$n = 4 : babbbb \Rightarrow 1 \xrightarrow{b} 2 \xrightarrow{a} 3 \xrightarrow{b} 3 \xrightarrow{b} 3 \xrightarrow{b} 3 \xrightarrow{b} 3$$

3. (a) *Proof.* Consider some regular language L and define an operation STRETCH by $\text{STRETCH}(w_1 w_2 \ldots w_n) = w_1 w_1 w_2 w_2 \ldots w_{n-1} w_{n-1} w_n w_n$. Since L is regular, there must be a DFA M that accepts it. The STRETCH operation simply converts every character of a word to that character twice. So, taking every transition in M and adding an intermediary node that accepts and outputs the same character, we get a new DFA that will accept STRETCH(L). The presence of this DFA means that STRETCH(L) must be regular. □

(b) *Proof.* Let the regular expression governing the DFA that accepts a regular language L be given by $\bigcup L$. This is equivalent to $w_1 \cup w_2 \ldots \cup w_n$ for all $w \in L$. For each $w \in L$, we can use the CHOP operation, leaving sue, where s is the first character and e is the last character of w. Appling this to every $w \in L$, we are left with the regular expression $u_1 \cup u_2 \ldots \cup u_n$, which must generate a DFA. Given a regular language L, this means that CHOP(L) is also regular. □