Homework 2

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1. The set $\{\neg, \wedge\}$ forms a functionally complete set of connectives for zeroth-order logic because a form of $\{\neg, \wedge\}$ can replicate the truth values for $\vee$ as indicated by the truth table below.

p	q	$p \vee q$	$\neg(\neg p \wedge \neg q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

2. *Proof.* $(p \to q) \wedge (p \to r) \Leftrightarrow p \to (q \wedge r)$

$$\begin{aligned}
(p \to q) \wedge (p \to r) &\Leftrightarrow (\neg p \vee q) \wedge (p \to r) && \text{C.D.}\\
&\Leftrightarrow (\neg p \vee q) \wedge (\neg p \vee r) && \text{C.D.}\\
&\Leftrightarrow \neg p \vee (q \wedge r) && \text{Distributive}\\
(p \to q) \wedge (p \to r) &\Leftrightarrow p \to (q \wedge r) && \text{C.D.}
\end{aligned}$$

$\square$

3. (a) $\kappa(\text{“}\mathit{Macbeth}\text{”}) \wedge \vartheta(\text{“}\mathit{Banquo}\text{”})$
   (b) $\forall x \exists y(\kappa(x) \to \mu(y, x))$
   (c) $\exists x \forall y(\kappa(x) \to \mu(x, y))$
   (d) $\forall x \forall y(\nu(x) \to \neg\mu(x, y))$
   (e) $\forall x \forall y((\kappa(x) \wedge \vartheta(y)) \to \neg\mu(x, y))$
   (f) $\forall x(\omega(x) \to \neg\mu(x, \text{“}\mathit{Macbeth}\text{”}))$
   (g) $\forall x(\kappa(x) \to \vartheta(x))$
   (h) $\neg\kappa(\text{“}\mathit{Banquo}\text{”}) \wedge \exists x(\mu(x, \text{“}\mathit{Banquo}\text{”}))$

4. (a) Captain Richard Witterel shot all mates.
   (b) Not everyone was terribly ravaged by a sea monster.
   (c) There exists a sea monster that drowned all mates.

5. (a) $\neg\forall x \exists y(x \vee y)$
   $\exists x \forall y \neg(x \vee y)$
   $\exists x \forall y(\neg x \wedge \neg y)$

   (b) $\neg\forall x \forall y(\neg x \vee (x \leftrightarrow y))$
   $\exists x \exists y \neg(\neg x \vee (x \leftrightarrow y))$
   $\exists x \exists y(x \wedge \neg(x \leftrightarrow y))$

   (c) $\neg\exists x \forall y \forall z((x \to y) \to z)$
   $\forall x \exists y \exists z \neg((x \to y) \to z)$

   (d) $\neg\exists x(\varphi(x) \vee \forall y \exists z(\psi(y, z) \to \neg x))$
   $\forall x \neg(\varphi(x) \vee \forall y \exists z(\psi(y, z) \to \neg x))$
   $\forall x(\neg\varphi(x) \wedge \neg\forall y \exists z(\psi(y, z) \to \neg x))$
   $\forall x(\neg\varphi(x) \wedge \exists y \forall z \neg(\psi(y, z) \to \neg x))$