Homework 1

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D. Leverage

Leverage is the idea of using borrowed resources to make investments. Of course, borrowed resources will always need to be paid back, but a successful use of leverage means that the investment should generate more profit or value than the borrowed sum itself. Leveraging amplifies the gains and losses from investments because it basically multiplies the amount invested. Used correctly, one can profit greatly off of good investments. Used incorrectly and one might find themselves unable to pay off the debt they acquired and losing substantially.

For a simple example, consider an individual wanting to put \$1000 into an index fund. They get a \$4000 loan to increase their capital to \$5000 total, investing all of it in this index fund. Suppose the index fund increases 50% over the course of the year (extreme, I know), so the individual now has \$7500 worth of said index fund which they withdraw entirely. Paying off their \$4000 loan plus \$500 of interest for the time, they are left with \$3000 after all is said and done. They were able to triple their original capital when their investment only increased by half. Conversely, if the fund had decreased 50%, their investment would be worth \$2500 which they withdraw, leaving them out \$2000 after they pay off their loan. This is one of the downsides to leverage, as the individual now lost their entire initial investment and then some. In some cases this can be incredibly damaging to the individual's finances as their loss was amplified. Obviously this is a simple example, but highlights the amplification of leveraging capital.

There are other advantages and disadvantages to leverage. One advantage is the ability to afford more expensive investments such as factories, real estate, acquisitions and more. Often, these investments will net the company or individual more value than the cost of the investment itself, so leveraging can be very beneficial in this context. Because of this, leverage reduces the barrier to investing in more expensive opportunities for individuals and small businesses alike. On the other hand, leverage can induce more brokerage fees, interest, and risk and thus is a much more complex way to trade. So individuals and businesses must have a good understanding of their finances in order to leverage well. Another element of complexity is also introduced, time. Some investments take longer than others to benefit, and most financial instruments for raising capital (loans, issuing bonds, etc) stay on a fixed timeline. The time discrepancy between the investment and debt can induce more risk and potentially more benefit if the investment is yielding faster than the debt is owed.

One example of an important use of leverage is Blackstone's buyout of the Hilton Hotel chain in 2007 for about \$26 billion. Nearly 80% of this sum was leveraged from a variety of financial institutions. During the housing crisis, Blackstone lost about 70% of their investment, but was able to turn the Hilton brand around, taking them to an IPO in 2013, where Blackstone profited roughly \$9 billion. They sold the rest of their stake in Hilton in 2018 to end up netting about \$14 billion on the whole deal. This deal implies the risk of such large deals and leverage when economic events like a recession influence the value of investments. Blackstone had to lose a lot to gain even more and the time it takes for some investments to pay off can be very long. It also highlights the benefits of leverage as this deal is one of the best and most well known leveraged buyouts of all time.

Another example of leverage is governments during economic recessions and depressions. Governments will issue bonds among other methods of raising capital in an effort to stimulate the economy. These are called expansionary fiscal policies. The government will make payouts to citizens and companies in an attempt to maintain the economy and incentivize people to produce and consume. In exchange, the government will continue to collect taxes and generate income from sustained economic activity and hopefully pay off the capital they raised. This is a much more complex economic topic, but is still a great example of what leverage can do for the general public. It is also less conventional than the above buyout and what people generally consider when they think of leverage and has broad implications in our society today. In fact, there is an entire market on government issued bonds and lots of research in this area.

E. Chapter Exercises

Exercise 1

$$\begin{aligned} PV &= \left(1 + \frac{0.05}{12}\right)^{-12\cdot 10} \cdot 100000 \\ &= (1.00417)^{-120} \cdot 100000 \\ &= 0.60716 \cdot 100000 \\ &= \$60,716.10 \end{aligned}$$

Exercise 2

$$\begin{aligned} \text{min/yr} &= 60 \cdot 24 \cdot 365 = 525,600 \\ r_a &= \left(1 + \frac{0.04}{525600}\right)^{525600} - 1 \\ &= 1.04081 - 1 = 0.04081 \\ &= 4.081\% \end{aligned}$$

Exercise 3

$$\begin{aligned} 0.0008 &= \frac{360}{34}\frac{1 - P_{bid}}{1} \\ P_{bid} &= 1 - 0.0008 \cdot \frac{34}{360} \\ &= 1 - 0.0000755555 = 0.9999244444 \\ &= \$999,924.44 \\ \text{Profit} &= \$999,943.33 - \$999,924.44 = \$18.89 \end{aligned}$$

Exercise 5

$$\begin{aligned} 0.0002 &= \frac{360}{40}\frac{1 - P_{ask}}{1} \\ P_{ask} &= 1 - 0.0002 \cdot \frac{40}{360} \\ &= 1 - 0.0000222222 = 0.9999777777 \\ &= \$999,977.78 \end{aligned}$$

$$\begin{aligned} 0.0003 &= \frac{360}{40}\frac{1 - P_{bid}}{1} \\ P_{bid} &= 1 - 0.0003 \cdot \frac{40}{360} \\ &= 1 - 0.00003333333 = 0.9999666666 \\ &= \$999,966.67 \end{aligned}$$

Exercise 6

$$
\begin{aligned}
0.0009 &= \frac{360}{355}\frac{1-P_{ask}}{1} \\
P_{ask} &= 1 - 0.0009 \cdot \frac{355}{360} \\
&= 1 - 0.0008875 = 0.9991125 \\
&= \$999,112.50
\end{aligned}
$$

$$
\begin{aligned}
0.0011 &= \frac{360}{355}\frac{1-P_{bid}}{1} \\
P_{bid} &= 1 - 0.0011 \cdot \frac{355}{360} \\
&= 1 - 0.0010847222 = 0.9989152778 \\
&= \$998,915.28
\end{aligned}
$$

Exercise 7

$$
\begin{aligned}
c &= \frac{r_m PV}{1-(\frac{1}{1+r_m})^n} \\
PV &= \frac{c \cdot (1-(\frac{1}{1+r_m})^n)}{r_m} \\
r_m &= \frac{0.0703}{12} = 0.0058583333 \\
PV &= \frac{1500 \cdot (1-(\frac{1}{1+0.0058583333})^{360})}{0.0058583333} = \frac{1500 \cdot (1-(\frac{1}{1.0058583333})^{360})}{0.0058583333} \\
&= \frac{1500 \cdot (1-0.122108)}{0.0058583333} = \frac{1500 \cdot 0.877892}{0.0058583333} \\
&= \frac{1316.84}{0.0058583333} = \$224,780.66
\end{aligned}
$$

Exercise 11

$$
\begin{aligned}
10000 &= PV \cdot e^{4 \cdot 0.05} \\
PV &= 10000e^{-0.2} \\
&= \$8,187.31
\end{aligned}
$$

F. Homework Exercises

(1) Annuity

$$\begin{aligned}
c &= \frac{r_m PV}{1-(\frac{1}{1+r_m})^n} \\
PV &= \frac{c\cdot(1-(\frac{1}{1+r_m})^n)}{r_m} \\
r_m &= \frac{0.06}{12} = 0.005 \\
PV &= \frac{1800\cdot(1-(\frac{1}{1+0.005})^{180})}{0.005} = \frac{1800\cdot(1-(\frac{1}{1.005})^{180})}{0.005} \\
&= \frac{1800\cdot(1-0.40748)}{0.005} = \frac{1800\cdot 0.59252}{0.005} \\
&= \frac{1066.53}{0.005} = \$213,306.33
\end{aligned}$$

(2) Perpetuity

$$PV = \frac{\$15}{0.03} = \$500$$

(4) Perpetual Stream

$$\begin{aligned}
r_a &= e^r - 1 \\
r &= \ln(r_a+1) = \ln(0.05+1) = \ln(1.05) = 0.04879 \\
PV &= \frac{\$10000}{0.04879} = \$204,959.34
\end{aligned}$$

(5) Arithmetic and Geometric Returns

1. $r_1 = r_2 = r_3 = 0.4$, $r_4 = 0.3$ and $r_5 = -1$

$$\begin{aligned}
r_a &= \frac{0.4+0.4+0.4+0.3-1}{5} = \frac{0.5}{5} = 0.1 \\
r_g &= [(1+0.4)(1+0.4)(1+0.4)(1+0.3)(1-1)]^{\frac{1}{5}} - 1 \\
&= [(1.4)(1.4)(1.4)(1.3)(0)]^{\frac{1}{5}} - 1 = 0 - 1 = -1
\end{aligned}$$

2. Show that $r_a = r_g$ if $r_1 = r_2 = \ldots = r_n$

$$\begin{aligned}
\text{Let } r &= r_1 = r_2 = \ldots = r_n \\
\text{Then } r_a &= \frac{r+r+\ldots+r}{n} = \frac{n\cdot r}{n} = r \\
\text{Also, } r_g &= [(1+r)(1+r)\ldots(1+r)]^{\frac{1}{n}} - 1 \\
&= [(1+r)^n]^{\frac{1}{n}} - 1 = (1+r) - 1 = r \\
\text{So, } r_a &= r_g
\end{aligned}$$