

Homework 1

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January 21, 2022

1. (a) Proposition - Santa either lives in the South Pole or he does not
 (b) Proposition - Sets either contain themselves or they do not, and by the definition of a set, sets cannot contain themselves, making this proposition false
 (c) Not a Proposition - is a contradiction because if we assume that it is true, the set contradicts its rule of being a set of sets that do not contain themselves by nature of containing itself. Conversely, if we assume the statement is false, the set should not contain itself and yet it does because it follows the rule of being a set that does not contain itself.
 (d) Not a Proposition - is a question
 (e) Proposition - Madrid is the capital of Spain, or it is not
2. (a) $(p \wedge q) \rightarrow (r \wedge \neg s)$
 (b) $\neg p \rightarrow \neg r$
 (c) $\neg q \leftrightarrow s$
 (d) $s \wedge r$
 (e) $s \rightarrow (\neg p \vee \neg q)$
 (f) $(\neg s \rightarrow p) \wedge (q \rightarrow \neg s)$
 (g) $p \rightarrow r$
 (h) $\neg s \rightarrow q$
 (i) $\neg(p \wedge q)$
 (j) $(p \wedge q) \rightarrow (r \wedge \neg s)$
3. (a) If it does not snow tonight, then I will not stay home.
 (b) If I stay home, then it will snow tonight.
 (c) If I do not stay home, then it will not snow tonight.
4. (a) $p \leftrightarrow q \Leftrightarrow (p \wedge q) \vee (\neg p \wedge \neg q)$

$$\begin{aligned}
 (p \leftrightarrow q) &\Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p) && \text{B.C.D.} \\
 &\Leftrightarrow (\neg p \vee q) \wedge (q \rightarrow p) && \text{C.D.} \\
 &\Leftrightarrow (\neg p \vee q) \wedge (\neg q \vee p) && \text{C.D.} \\
 &\Leftrightarrow ((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p) && \text{Distributive} \\
 &\Leftrightarrow ((\neg q \wedge \neg p) \vee (\neg q \wedge q)) \vee ((\neg p \vee q) \wedge p) && \text{Distributive} \\
 &\Leftrightarrow ((\neg q \wedge \neg p) \vee F) \vee ((\neg p \vee q) \wedge p) && \text{Negation} \\
 &\Leftrightarrow (\neg q \wedge \neg p) \vee ((\neg p \vee q) \wedge p) && \text{Domination} \\
 &\Leftrightarrow (\neg q \wedge \neg p) \vee ((p \wedge \neg p) \vee (p \wedge q)) && \text{Distributive} \\
 &\Leftrightarrow (\neg q \wedge \neg p) \vee (F \vee (p \wedge q)) && \text{Negation} \\
 &\Leftrightarrow (\neg q \wedge \neg p) \vee (p \wedge q) && \text{Domination} \\
 (p \leftrightarrow q) &\Leftrightarrow (p \wedge q) \vee (\neg q \wedge \neg p) && \text{Commutative} \\
 &&& \text{Q.E.D.}
 \end{aligned}$$

$$(b) \quad (p \rightarrow r) \vee (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$$

$$\begin{aligned}
 (p \rightarrow r) \vee (q \rightarrow r) &\Leftrightarrow (\neg p \vee r) \vee (\neg q \vee r) && \text{C.D.} \\
 &\Leftrightarrow (\neg p \vee r) \vee \neg q \vee r && \text{Associative} \\
 &\Leftrightarrow (\neg p \vee \neg q) \vee r \vee r && \text{Associative} \\
 &\Leftrightarrow (\neg p \vee \neg q) \vee (r \vee r) && \text{Associative} \\
 &\Leftrightarrow (\neg p \vee \neg q) \vee r && \text{Idempotent} \\
 &\Leftrightarrow \neg(p \wedge q) \vee r && \text{DeMorgan} \\
 (p \rightarrow r) \vee (q \rightarrow r) &\Leftrightarrow (p \wedge q) \rightarrow r && \text{C.D.} \\
 &&& \text{Q.E.D.}
 \end{aligned}$$

$$(c) \quad (p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r) \Leftrightarrow T$$

$$\begin{aligned}
(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r) &\Leftrightarrow \neg((p \vee q) \wedge (\neg p \vee r)) \vee (q \vee r) && \text{C.D.} \\
&\Leftrightarrow (\neg(p \vee q) \vee \neg(\neg p \vee r)) \vee (q \vee r) && \text{DeMorgan} \\
&\Leftrightarrow ((\neg p \wedge \neg q) \vee (p \wedge \neg r)) \vee (q \vee r) && \text{DeMorgan} \\
&\Leftrightarrow (\neg p \wedge \neg q) \vee ((p \wedge \neg r) \vee (q \vee r)) && \text{Associative} \\
&\Leftrightarrow (\neg p \wedge \neg q) \vee (((p \wedge \neg r) \vee r) \vee q) && \text{Associative} \\
&\Leftrightarrow (\neg p \wedge \neg q) \vee (((p \vee r) \wedge (\neg r \vee r)) \vee q) && \text{Distributive} \\
&\Leftrightarrow (\neg p \wedge \neg q) \vee (((p \vee r) \wedge T) \vee q) && \text{Negation} \\
&\Leftrightarrow (\neg p \wedge \neg q) \vee ((p \vee r) \vee q) && \text{Domination} \\
&\Leftrightarrow ((\neg p \wedge \neg q) \vee q) \vee (p \vee r) && \text{Associative} \\
&\Leftrightarrow ((\neg p \vee q) \wedge (\neg q \vee q)) \vee (p \vee r) && \text{Distributive} \\
&\Leftrightarrow ((\neg p \vee q) \wedge T) \vee (p \vee r) && \text{Negation} \\
&\Leftrightarrow (\neg p \vee q) \vee (p \vee r) && \text{Domination} \\
&\Leftrightarrow ((\neg p \vee q) \vee p) \vee r && \text{Associative} \\
&\Leftrightarrow ((\neg p \vee p) \vee q) \vee r && \text{Associative} \\
&\Leftrightarrow (\neg p \vee p) \vee (q \vee r) && \text{Associative} \\
&\Leftrightarrow T \vee (q \vee r) && \text{Negation} \\
(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r) &\Leftrightarrow T && \text{Domination} \\
&&& \text{Q.E.D.}
\end{aligned}$$

$$(d) \quad (p \rightarrow q) \rightarrow r \not\Leftrightarrow p \rightarrow (q \rightarrow r)$$

$$\begin{aligned}
p &:= F \\
q &:= T \\
r &:= F \\
(p \rightarrow q) \rightarrow r &\Leftrightarrow (F \rightarrow T) \rightarrow F && \text{Left side} \\
&\Leftrightarrow T \rightarrow F && \text{Left side} \\
&\Leftrightarrow F && \text{Left side} \\
p \rightarrow (q \rightarrow r) &\Leftrightarrow F \rightarrow (T \rightarrow F) && \text{Right side} \\
&\Leftrightarrow F \rightarrow F && \text{Right side} \\
&\Leftrightarrow T && \text{Right side} \\
F &\not\Leftrightarrow T && \\
&&& \text{Q.E.D.}
\end{aligned}$$

$$(e) \quad (p \wedge q) \rightarrow r \not\Leftrightarrow (p \rightarrow r) \wedge (q \rightarrow r)$$

$$\begin{aligned}
(p \rightarrow r) \wedge (q \rightarrow r) &\Leftrightarrow (\neg p \vee r) \wedge (q \rightarrow r) && \text{C.D.} \\
&\Leftrightarrow (\neg p \vee r) \wedge (\neg q \vee r) && \text{C.D.} \\
&\Leftrightarrow (\neg p \wedge \neg q) \vee r && \text{Distributive} \\
&\Leftrightarrow \neg(p \vee q) \vee r && \text{DeMorgan} \\
&\Leftrightarrow (p \vee q) \rightarrow r && \text{C.D.} \\
(p \wedge q) \rightarrow r &\not\Leftrightarrow (p \vee q) \rightarrow r \\
p &:= T \\
q &:= F \\
r &:= F \\
(p \wedge q) \rightarrow r &\Leftrightarrow (T \wedge F) \rightarrow F && \text{Left side} \\
&\Leftrightarrow F \rightarrow F && \text{Left side} \\
&\Leftrightarrow T && \text{Left side} \\
(p \vee q) \rightarrow r &\Leftrightarrow (T \vee F) \rightarrow F && \text{Right side} \\
&\Leftrightarrow T \rightarrow F && \text{Right side} \\
&\Leftrightarrow F && \text{Right side} \\
T &\not\Leftrightarrow F && \\
&&& \text{Q.E.D.}
\end{aligned}$$