Homework 5

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E. Textbook Exercises

Exercise 8

$$\begin{aligned}\text{Dividends } &= S_0e^r(1-e^{-\delta})\\ 3.69 &= 34.10e^{0.0009}(1-e^{-\delta})\\ e^{-\delta} &= 0.89189\\ \delta &= 0.1144\end{aligned}$$

$$\begin{aligned}d_\pm &= \frac{\ln\frac{34.10}{30}+(0.0009-0.1144\pm\frac{1}{2}0.2^2)\cdot 0.2795}{0.2\sqrt{0.2795}}\\ &= \frac{0.1281+(-0.1135\pm 0.02)\cdot 0.2795}{0.1057}\\ d_+ &= 0.9647\\ d_- &= 0.8589\\ V_0 &= 34.10e^{-0.1144\cdot 0.2795}N(0.9647)-30e^{-0.0009\cdot 0.2795}N(0.8589)\\ &= 34.10e^{-0.032}(0.8327)-30e^{-0.00025}(0.8048)\\ &= 33.026(0.8327)-29.993(0.8048)\\ &= \$3.36\end{aligned}$$

The value of the option is less than the one without dividends as the option holder does not receive them, so they decrease the value of the underlying asset to the options holder.

Exercise 11

$$\begin{aligned}d_1 &= \frac{\ln\frac{200}{250}+(0.05+\frac{1}{2}0.4^2)}{0.4} = \frac{-0.2231+0.13}{0.4} = -0.2328\\ d_2 &= -0.2328-0.4 = -0.6328\end{aligned}$$

$$\begin{aligned}\Delta &= \frac{\partial C}{\partial S_0}(S_0,T) = \frac{\partial V_0}{\partial S_0}(S_0,T) = N(d_1) = N(-0.2328) = 0.4079\\ \Gamma &= \frac{\partial^2 C}{\partial S_0^2}(S_0,T) = \frac{1}{S_0\sigma\sqrt{2\pi T}}e^{-\frac{1}{2}d_1^2} = \frac{1}{200\cdot 0.4\sqrt{2\pi}}e^{-\frac{1}{2}0.2328^2}\\ &= 0.00499e^{-0.0271} = 0.00486\\ \theta &= -\frac{\partial C}{\partial T}(S_0,T) = -\frac{\sigma S_0}{2\sqrt{2\pi T}}e^{-\frac{1}{2}d_1^2}-rKe^{-rT}N(d_2)\\ &= -\frac{0.4\cdot 200}{2\sqrt{2\pi}}e^{-0.0271}-0.05\cdot 250e^{-0.05}N(-0.6328)\end{aligned}$$

$$= -15.9577e^{-0.0271} - 12.5e^{-0.05}(0.2634)$$
$$= -15.5311 - 3.1319 = -18.663$$

Exercise 13

$$\begin{aligned}
2.38 &= 100e^{0.02}(1 - e^{-\delta}) \\
e^{-\delta} &= 0.97667 \\
\delta &= 0.0236 \\
d_{\pm} &= \frac{\ln \frac{100}{125} + (0.02 - 0.0236 \pm \frac{1}{2}0.4^2) \cdot 0.25}{0.4\sqrt{0.25}} \\
&= \frac{-0.2231 + (-0.0009 \pm 0.02)}{0.2} \\
d_+ &= -1.02 \\
d_- &= -1.22 \\
V_0 &= e^{-0.0236 \cdot 0.25} 100N(-1.02) - e^{-0.02 \cdot 0.25} 125N(-1.22) \\
&= 100e^{-0.0059}(0.1539) - 125e^{-0.005}(0.1112) \\
&= 15.39e^{-0.0059} - 13.9e^{-0.005} \\
&= \$1.47
\end{aligned}$$

F. Homework Exercises

(1) Corn Forward Option

$$\begin{aligned}
d_1 &= \frac{\ln \frac{F_{0,T}}{K} + \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}} = \frac{\ln \frac{4.4}{4.2} + \frac{1}{2}0.25^2 \cdot 0.25}{0.25\sqrt{0.25}} \\
&= \frac{0.04652001563 + 0.0078125}{0.125} = 0.4347 \\
V_0^c &= e^{-rT} F_{0,T} N(d_1) - e^{-rT} K N(d_1 - \sigma\sqrt{t}) \\
&= e^{-0.015} 4.4 N(0.4347) - e^{-0.015} 4.2 N(0.4347 - 0.125) \\
&= e^{-0.015}[4.4N(0.4347) - 4.2N(0.3097)] \\
&= e^{-0.015}[4.4(0.6681) - 4.2(0.6216)] \\
&= e^{-0.015}[2.9396 - 2.6107] \\
&= e^{-0.015} \cdot 0.3289 \\
&= \$0.32
\end{aligned}$$

(2) Implied Volatility

$$\begin{aligned}
d_1 &= \frac{\ln \frac{100}{100e^{0.005}} + (0.02 + \frac{1}{2}\sigma^2) \cdot 0.25}{\sigma\sqrt{0.25}} = \frac{-0.005 + 0.005 + \frac{1}{8}\sigma^2}{\frac{1}{2}\sigma} = \frac{1}{4}\sigma \\
d_1 - \sigma\sqrt{0.25} &= \frac{1}{4}\sigma - \frac{1}{2}\sigma = -\frac{1}{4}\sigma = -d_1 \\
V_0 &= 100N(d_1) - 100e^{0.005} N(-d_1) \\
&= 100N(d_1) - 100e^{0.005}(1 - N(d_1)) \\
&\approx 200N(d_1) - 100 \\
4.50 &= 200N(d_1) - 100 \\
N(d_1) &= 0.5225
\end{aligned}$$

$$d_1 \approx 0.05643$$
$$\sigma \approx 0.2257$$

(4) Applications

1. Forward contract on stock: the buyer of the contract has agreed to pay some price for the stock at a future time T, so that price must be equal to the expected future value of the current stock price
2. Forward contracts as a combination of European call and put options: replicates the forward contract with the purchase of a call option and sale of a put option with the same strike price
3. Forward contract on stock with dividends: if we receive dividends at some rate, we need to purchase less of the stock at the initial time, otherwise this works the same as a normal forward contract
4. Pricing a call option on an expiring forward contract: gives the buyer the right but not the obligation to purchase the forward contract at expiration for the specified strike price, good for hedging strategies
5. Pricing a call option on a forward contract on a stock paying dividends: similar to the normal call option, just accounting for the rate at which dividends are paid out when determining the present value of the option
6. Options on future contracts: only different from options of forward contracts due to marking to market, so the value of the option can vary depending on when it is exercised

Example of a forward contract on stock: Assume ground beef (in dollars per pound) is moving log-normally with standard deviation $\sigma = 0.15$. Also assume the annual risk free interest rate is $r = 0.04$. Find the price of a call option on a 3 month forward contract with $F_{0,1/4} = 5.2$ if the expiration is $t = 1$ month and the strike price at $t = 3$ months is $K = \$4.9$.