Homework 4

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1. (a) i. $a + b + c$

$$\begin{aligned}
E &\Rightarrow E + T \\
&\Rightarrow E + F \\
&\Rightarrow E + T + F \\
&\Rightarrow T + T + F \\
&\Rightarrow T + F + F \\
&\Rightarrow F + F + F \\
&\Rightarrow a + F + F \\
&\Rightarrow a + b + F \\
&\Rightarrow a + b + c
\end{aligned}$$

ii. $a * b + c$

$$\begin{aligned}
E &\Rightarrow E + T \\
&\Rightarrow T + T \\
&\Rightarrow T * F + T \\
&\Rightarrow F * F + T \\
&\Rightarrow F * F + F \\
&\Rightarrow a * F + F \\
&\Rightarrow a * b + F \\
&\Rightarrow a * b + c
\end{aligned}$$

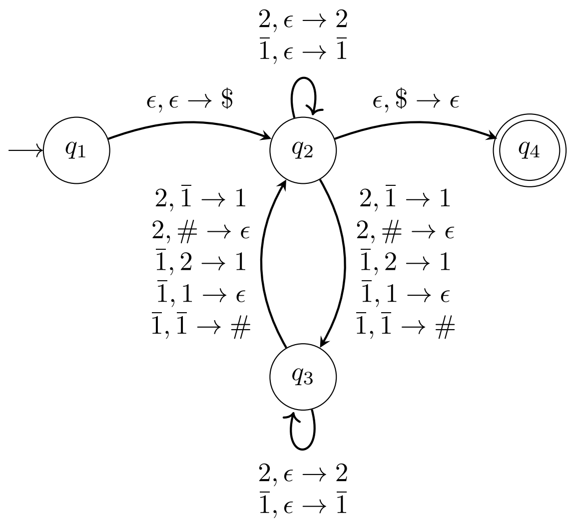
iii. $a * (b + c)$

$$\begin{aligned}
E &\Rightarrow T \\
&\Rightarrow T * F \\
&\Rightarrow T * (E) \\
&\Rightarrow F * (E) \\
&\Rightarrow F * (E + T) \\
&\Rightarrow F * (E + F) \\
&\Rightarrow F * (T + F) \\
&\Rightarrow F * (F + F) \\
&\Rightarrow a * (F + F) \\
&\Rightarrow a * (b + F) \\
&\Rightarrow a * (b + c)
\end{aligned}$$

(b)

$$\begin{aligned}
&E \to E + T \mid T \\
&T \to T * U \mid U \\
&U \to F \uparrow U \mid F \\
&F \to (E) \mid a \mid b \mid c
\end{aligned}$$

2. PDA for $L_2 = \{w \in \{2, \bar{1}\}^* | w \text{ has twice as many } \bar{1}\text{'s as 2's}\}$



This PDA works because we cannot pop elements from the stack and remain at states q_2 and q_3. The self loops ensure that we push the appropriate symbol to the stack, and that symbol is matched with another $\bar{1}, 2$ or another two $\bar{1}$'s. We can add as many $\bar{1}, 2$'s as we want on each self loop, but to get back to the bottom of the stack they must all be matched.

3. PDA and CFG for $L_3 = \overline{\{0^n 1^n \mid n \geq 0\}}$

We know that $L_3 = \overline{\{0^n 1^n \mid n \geq 0\}} = \{0^m 1^n \mid m \neq n\} \cup \overline{0^* 1^*}$

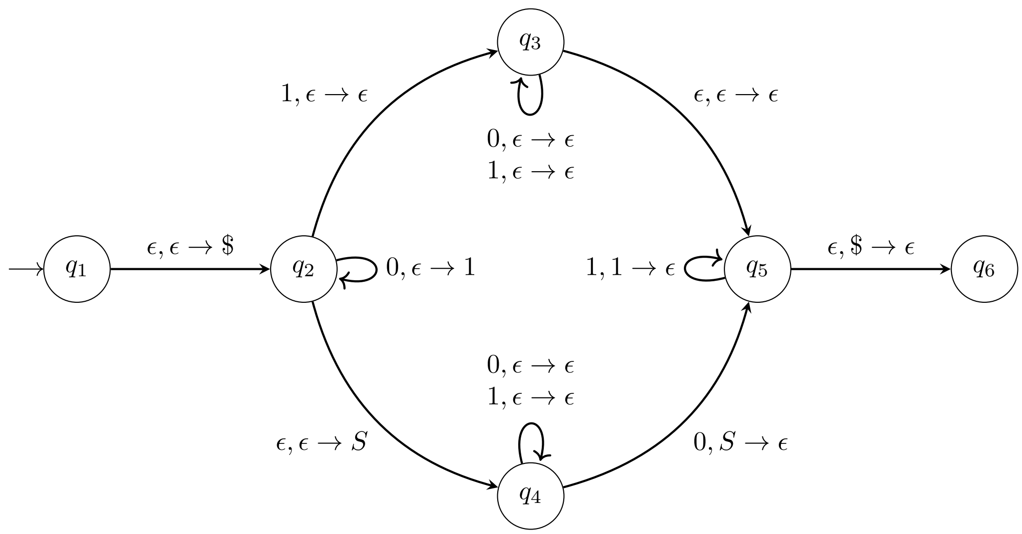
Proof. We can separate $L_3 = \overline{\{0^n 1^n \mid n \geq 0\}}$ into two subsets, one being the strings where the structure is 0^*1^* but with a different number of 0's and 1's. The other is all binary strings which do not match the structure of 0^*1^*. We can represent the first subset with $\{0^m 1^n \mid m \neq n\}$ and the second one with $\overline{0^*1^*}$. Combining them means that $L_3 = \{0^m 1^n \mid m \neq n\} \cup \overline{0^*1^*}$. □

CFG:

$$S \to 0S1 \mid 1T \mid T0$$
$$T \to 1T \mid 0T \mid \epsilon$$

This CFG works because any string that starts with a 1 or ends in a 0 should be accepted. T allows for any combination of 1's and 0's to be added onto a string and is the only terminal state. $0S1$ ensures that any string that starts with a 0 will not have the same number of 0's and 1's in a row.

PDA:



This PDA works because it represents the CFG from above. The path along q_2, q_3, q_5 accepts any string that starts with 1. The bottom path along q_2, q_4, q_5 accepts any string that ends with 0 by pushing and then popping a special symbol. The $0S1$ is represented with the two self loops on q_2 and q_5, which ensure that the CFG holds.