Exam 1 Cheatsheet

Walker Bagley

March 6, 2024

2. Fixed Interest Rates

$$r = \frac{FV - PV}{PV}$$

$$FV = PV\left(1 + \frac{r}{n}\right)^{nt} \qquad n \text{ is number of periods}$$

$$\text{Effective annual interest rate } r = \left(1 + \frac{r}{n}\right)^{n} - 1$$

$$\text{Treasury bills: } r_{dask} = \frac{360}{d}\frac{1 - P_{ask}}{1} \qquad d \text{ is time to maturity}$$

$$\text{Mortgage payments: } PV = \frac{C}{r}\left[1 - \left(\frac{1}{1+r}\right)^{n}\right] \qquad C \text{ is recurring payment}$$

$$\textbf{Continuous compounding: } FV = PVe^{rt}$$

$$\text{Effective annual rate (continuous) } r_e = e^{r} - 1$$

$$FV = PVe^{\int_0^t r(s)ds} \qquad r(s) \text{ is variable interest rate}$$

$$FV = PVe^{rt} + \int_0^t S(\tau)e^{r(t-\tau)}d\tau \qquad S \text{ gives rate of deposits per year}$$

$$S_0 = \frac{\text{Market value}}{\#\text{ shares}} \qquad S_0 \text{ is stock price}$$

3. Assessing Risk

$$\text{equity premium} = \text{stock return} - r$$

$$U(C + E(z) - \rho) = pU(C + z_1) + (1-p)U(C + z_2) \qquad \text{utility } U\text{, consumption } C$$

$$E(z) = pz_1 + (1-p)z_2 \qquad \text{risk } z\text{, risk premium } \rho$$

$$\textbf{Arrow's theorem: } \rho \approx -\frac{1}{2}Var(z)\frac{U''(C)}{U'(C)} > 0 \qquad z \text{ is actuarially fair}$$

4. Binomial Asset Pricing Model

$$\text{Binomial model: } \begin{cases} u = \frac{S_1(H)}{S_0} \\ d = \frac{S_1(T)}{S_0} \end{cases}$$

$$\text{No Arbitrage Axiom (NA): } d < 1 + r < u$$

$$\text{1 period ECO price: } V_0 = \frac{1}{1+r}[\tilde{p}V_1(H) + \tilde{q}V_1(T)] \qquad \begin{cases} \tilde{p} = \frac{(1-r)-d}{u-d} \\ \tilde{q} = \frac{u-(1-r)}{u-d} \end{cases}$$

$$\text{Delta hedging: } \Delta_0 = \frac{V_1(H) - V_1(T)}{S_1(H) - S_1(T)}$$

Put-Call Parity: $S_0 + V_0^{EPO} = \dfrac{K}{1+r} + V_0^{ECO}$

5. Continuous Time BSM

Log normal model: $S(t) = S_0 e^{(r-\frac{1}{2}\sigma^2)t+\sigma\sqrt{t}z_t}$

Black-Scholes-Merton (BSM): $V_0^{ECO} = S_0 N(d_1) - Ke^{-rt}N(d_1 - \sigma\sqrt{t})$ $\qquad d_1 = \dfrac{\ln\frac{S_0}{K} + (r+\frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$

$V_0^{EPO} = -S_0 N(-d_1) + Ke^{-rt}N(-d_1 + \sigma\sqrt{t})$

Put-Call Parity: $S_0 + V_0^{EPO} = e^{-rT}K + V_0^{ECO}$

The Greeks: $\begin{cases} \Delta = \frac{\partial V_0}{\partial S_0} = N(d_1) \\ \Gamma = \frac{\partial^2 V_0}{\partial S_0^2} = \frac{1}{\sigma S_0\sqrt{T}}\frac{dN}{dd_1} \\ \nu = \frac{\partial V_0}{\partial \sigma} = S_0\sqrt{T}\frac{dN}{dd_1} \\ \theta = -\frac{\partial V_0}{\partial T} = -\frac{\sigma S_0}{2\sqrt{T}}\frac{dN}{dd_1} - Kre^{-rT}N(d_2) \\ \rho = \frac{\partial V_0}{\partial r} = KTe^{-rT}N(2) \end{cases}$ $\qquad \dfrac{dN}{dd_1} = \dfrac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1^2}$

6. Applications of BSM

Log normal stock w/ dividends: $V_0^{\delta} = e^{-\delta T}S_0 N(d_+) - e^{-rT}KN(d_-)$ $\qquad d_{\pm} = \dfrac{\ln\frac{S_0}{K} + (r-\delta\pm\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$

$\delta = \ln\left(\dfrac{S_0}{S_0 - De^{-r}}\right)$ $\qquad D$ is dividends per year

Call on forward contract: $V_0^c = e^{-rT}F_{0,T}N(d_1) - e^{-rT}KN(d_1 - \sigma\sqrt{t})$ $\qquad d_1 = \dfrac{\ln\frac{F_{0,T}}{K} + \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}$

Forward price: $F_{0,T} = e^{rT}S_0$ $\qquad$ forward exp T, call exp t

Implied volatility: $V_0^{mkt} = S_0 N(d_1) - Ke^{-rT}N(d_1 - \sigma\sqrt{T})$ $\qquad$ solve for σ

Probability of default: $P(V_A < D) = N(-d_-)$

Distance to default: $d(V_0^A, \sigma_A) = d_- = \dfrac{\ln\frac{V_0^A}{D} + (r-\frac{1}{2}\sigma_A^2)T}{\sigma_A\sqrt{T}}$ $\qquad D$ is value of debt