Homework 7

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April 15, 2024

B. Chapter Exercises

Exercise 9

Using the following average returns which were calculated from closing price 5 years ago to today:

1. Apple: 28.21%
2. Ford: 5.72%
3. GE: 27.11%
4. Coca Cola: 22.01%
5. ExxonMobil: 8.21%

And the following standard deviations for each company's returns:

1. Apple: 31.5%
2. Ford: 42.8%
3. GE: 40.9%
4. Coca Cola: 20.9%
5. ExxonMobil: 34.3%

We have the following correlation matrix:

$$\begin{bmatrix} 1 & 0.08 & 0.31 & 0.16 & 0.23 \\ 0.08 & 1 & 0.08 & 0.05 & 0.06 \\ 0.31 & 0.08 & 1 & 0.18 & 0.41 \\ 0.16 & 0.05 & 0.18 & 1 & 0.14 \\ 0.23 & 0.06 & 0.41 & 0.14 & 1 \end{bmatrix}$$

Which we can generate the covariance matrix from by multiplying by the standard deviations for each company:

$$\begin{bmatrix} 0.0922 & 0.0108 & 0.0399 & 0.0105 & 0.0335 \\ 0.0108 & 0.1832 & 0.014 & 0.0045 & 0.0088 \\ 0.0399 & 0.014 & 0.1673 & 0.0154 & 0.0575 \\ 0.0105 & 0.0045 & 0.0154 & 0.0437 & 0.01 \\ 0.0335 & 0.0088 & 0.0575 & 0.01 & 0.1176 \end{bmatrix}$$

Then the notebook gives us the following portfolio breakdown:

1. Apple: -2.29%
2. Ford: 25.08%
3. GE: -9.43%
4. Coca Cola: 48.22%
5. ExxonMobil: 38.42%

And a standard deviation of 20.02%.

C. Homework Exercises

(1) The Magic of Diversification

$$\begin{aligned}
\mu_P &= \sum_{i=1}^{10000} \frac{1}{10000}\mu_i = 10000 \cdot \left(\frac{1}{10000}\mu\right) \\
&= 0.2 \\
\sigma_P^2 &= \sum_{i=1}^{10000}\sum_{j=1}^{10000} k_i k_j \sigma_{ij} = 10000\left(\frac{1}{10000^2}\sigma^2\right) \\
&= \frac{1}{10000} \cdot 0.4^2 = 0.000016 \\
\sigma_P &= \sqrt{0.000016} = 0.004
\end{aligned}$$

In this case we would prefer the stock portfolio over the risk free portfolio with a return of 15%. This is because the standard deviation of the stock portfolio is incredibly low due to the high diversification and independence amongst the stocks it contains. We are all but guaranteed a 20% return with the stock portfolio so we take that option.

(2) The One Fund Economy

$$\begin{aligned}
\mu_P &= k_m\mu_m + (1-k_m)\mu_r \\
k_m &= \frac{\mu_P - \mu_r}{\mu_m - \mu_r} \\
\text{Sharpe Ratio } &= \frac{0.2-0.05}{0.4} = \frac{0.15}{0.4} = 0.375 \\
\text{Conservative Friend: } 0.375 &= \frac{0.1-0.05}{\sigma} \\
\sigma &= \frac{0.05}{0.375} = 0.1333 \\
k_m &= \frac{0.1-0.05}{0.2-0.05} = \frac{0.05}{0.15} = \frac{1}{3} \\
\text{Risky Friend: } 0.375 &= \frac{0.7-0.05}{\sigma} \\
\sigma &= \frac{0.65}{0.375} = 1.7333 \\
k_m &= \frac{0.7-0.05}{0.2-0.05} = \frac{0.65}{0.15} = \frac{13}{3}
\end{aligned}$$

(4) Sharpe Ratio

(a) Ratio Comparison

I would choose the manager with a higher Sharpe Ratio. This is because the Sharpe Ratio measures both return and volatility. So, a manager with a higher Sharpe Ratio may offer higher returns, lower volatility or both. I am more likely to get a consistently higher return with a manager who has a higher Sharpe Ratio than their competitors. The Sharpe Ratio is better for measuring performance because it accounts for both returns and volatility, thus rewarding consistent portfolio managers who also yield high returns. Remember we want to maximize returns with respect to volatility, so for these reasons the Sharpe Ratio is better at measuring performance than simply returns.

(b) Portfolio Comparison

When comparing the market portfolio to the efficient frontier, we find that the slope, or Sharpe Ratio, of any portfolio on the market line is higher than that of the equivalent return on the efficient frontier. This is because there is more standard deviation on the efficient frontier.

(5) Beta Factor

(a) Excess Return

$$\text{excess return } = 1.5 \cdot 0.1 = 0.15$$

(b) Companies

$\beta > 1$:

1. Wayfair: 3.2
2. Carnival Corporation: 2.46
3. Royal Caribbean Cruises: 2.45
4. Caesars Entertainment: 2.9
5. Tesla: 2.29

$\beta < 1$:

1. General Mills: 0.15
2. Vita Coco: 0.16
3. Campbell Soup: 0.24
4. Hershey: 0.34
5. Clorox: 0.43

(c) Determining Beta Factor

To calculate beta of a stock using historical data, we must calculate the returns of the stock as well as the returns of its index (like the NYSE or NASDAQ). Then we calculate the covariance of these returns as well as the variance of the index price over the historical data and divide them so that $\beta = \frac{COV(\mu_{stock}, \mu_m)}{\sigma_m^2}$.