Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Robotic Systems Lab

Msc. - Written Exam | January 31th, 2023

# Robot Dynamics - Exam HS 2022

Marco Hutter, Roland Siegwart

| Question | Points | Score |
|---|---|---|
| Multiple Choice | 20 | |
| Kinematics | 19 | |
| Dynamics | 10 | |
| Floating Base System | 15 | |
| Rotary Wing | 9 | |
| Fixed-wing | 8 | |
| Total: | 81 | |

| | |
|---|---|
| Duration: | 120min |
| Number of pages: | 21 |
| Allowed aids: | Calculator |
| | Two A4 sheets of personal notes, written on both sides |
| | Dictionary for foreign students |

Write your name on every page in the box in the footer.

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Cooperation is strictly forbidden.

Please draw your answer in the respective figure if required to do so in the respective questions.

Name: ______________________________

Student number: ______________________________

Signature: ______________________________

**A. Multiple Choice** 20 pts

Decide whether the following statements are true or false. Cross the checkbox on the corresponding answer. You will be credited 1 point for a correct answer, while 1 pt will be subtracted from the total, if your answer is wrong.

| | | | |
|---|---|---|---|
| (1) The quaternion $\mathbf{q} = \left(\frac{\sqrt{2}}{2}, 0, 0, \frac{\sqrt{2}}{2}\right)^T$ describes a 90° rotation around the $z$ axis. The quaternion $\tilde{\mathbf{q}} = \left(\frac{-\sqrt{2}}{2}, 0, 0, \frac{\sqrt{2}}{2}\right)^T$ therefore describes a 90° rotation around the negative $z$ axis. | ◯ True | ◯ False | [1 pt] |
| (2) Given a desired end-effector position and orientation, the inverse kinematics problem applied to a 6 degrees-of-freedom serial linkage manipulator has a unique solution. | ◯ True | ◯ False | [1 pt] |
| (3) Given a point on a body, the velocity vector and the time derivative of the position vector are equal when they are expressed in the same frame. | ◯ True | ◯ False | [1 pt] |
| (4) Geometric Jacobians can be added and subtracted when represented in the same reference frame. | ◯ True | ◯ False | [1 pt] |
| (5) For very slow motions (i.e., $\dot{\mathbf{q}} \approx \ddot{\mathbf{q}} \approx \mathbf{0}$), an inverse-dynamics control law on joint-level is equivalent to a joint-level PD-controller with gravity compensation for the same PD gains. | ◯ True | ◯ False | [1 pt] |
| (6) For a 7 degrees of freedom robotic arm, multiple end-effector accelerations can be achieved from a unique torque vector. | ◯ True | ◯ False | [1 pt] |
| (7) When choosing an angle-axis representation as part of the generalized coordinates of a free-floating rigid body in 3D space, the gravitational torques vector must have dimensions $7 \times 1$. | ◯ True | ◯ False | [1 pt] |
| (8) Series elastic actuators can achieve higher torque control bandwidth than pseudo direct drive systems. | ◯ True | ◯ False | [1 pt] |
| (9) Consider the task of a 3 degrees-of-freedom robotic arm pushing a box in a desired direction. One could design a control law that attains proper tracking of a position trajectory along with an arbitrary force reference, both assigned in the same direction. | ◯ True | ◯ False | [1 pt] |
| (10) In the hierarchical quadratic programming approach the user is allowed to specify more tasks than there are degrees of freedom in the robot. | ◯ True | ◯ False | [1 pt] |
| (11) For a bipedal system with two point feet on the ground, every torque command results in a unique acceleration. | ◯ True | ◯ False | [1 pt] |
| (12) A spider (8-legged) robot in non-singular configuration with 3DOF legs and point feet that are all in contact, has a 18-dimensional manifold to modify the joint torque without causing the system to move. | ◯ True | ◯ False | [1 pt] |
| (13) Due to the under-actuated nature of classic multi-rotors, position and attitude dynamics are coupled. | ◯ True | ◯ False | [1 pt] |
| (14) Feed forward terms in trajectory tracking controller for multi-rotor systems are necessary for the system stability. | ◯ True | ◯ False | [1 pt] |
| (15) Two counter-rotating identical propellers in coaxial configuration rotating at the same speed will ideally generate zero drag torque. | ◯ True | ◯ False | [1 pt] |
| (16) Classic linear attitude controllers (such as PD) can stabilize a multi-rotor from any configuration. | ◯ True | ◯ False | [1 pt] |
| (17) Control surface deflections on fixed-wing aircraft need to be scaled proportionally to the inverse of the squared groundspeed to maintain constant effectiveness (force, torque). | ◯ True | ◯ False | [1 pt] |
| (18) Guidance logic (position control) for fixed-wing aircraft often feeds back the airspeed, as opposed to the groundspeed, to keep some level of robustness to wind. | ◯ True | ◯ False | [1 pt] |
| (19) In steady, horizontal flight at a fixed angle of attack, the aerodynamic drag of a fixed-wing aircraft depends on its mass. | ◯ True | ◯ False | [1 pt] |
| (20) In a coordinated turn, no aerodynamic forces are acting along the body-y axis of a fixed-wing aircraft. | ◯ True | ◯ False | [1 pt] |

Please fill in your name: ______________________________

**B. Kinematics** 19 pts

The robot depicted in Fig. 1 is Thiago from PAL Robotics, a Spanish company. For reference, you can find the full specifications about the system. We will analyze in this exam the kinematics and dynamics of this system. For simplicity, we ignore the hand/gripper as well as the head. The system features a 7DOF arm, a vertical extendable main body, and a mobile differential drive base that can drive on flat ground. (Picture sources: wikipedia, PAL webpage, github)

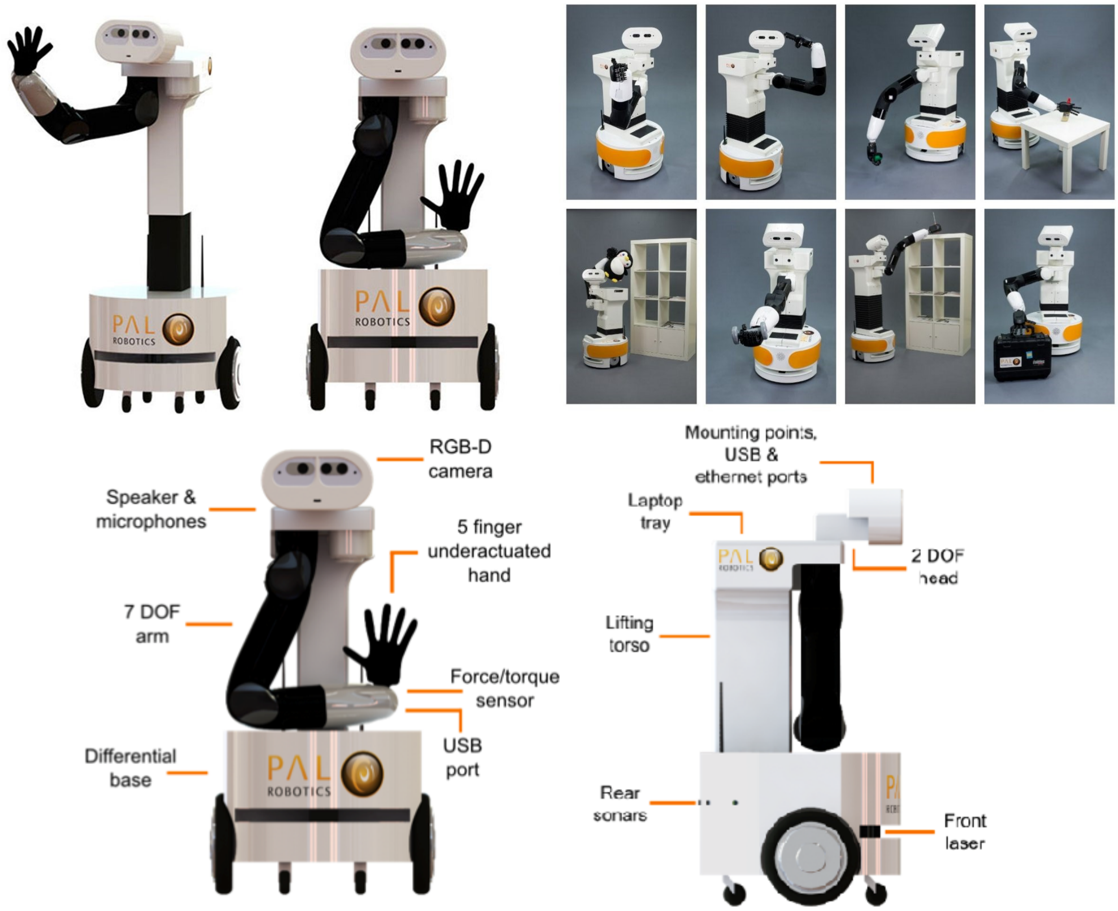



Figure 1: Thiago, a service robot from PAL.

In the first part, we analyze the full 3D system depicted in Fig. 2, whos state is defined by

$$\mathbf{q}_n = \begin{pmatrix} \mathbf{q}_b \\ \mathbf{q}_a \end{pmatrix} \in \mathbb{R}^n, \quad with \quad \mathbf{q}_b = \begin{pmatrix} x_b \\ y_b \\ \gamma_b \end{pmatrix} \tag{1}$$

The base pose can be described by $n_b = 3$ generalized coordinates, namely the position in the plane ($x_b$ and $y_b$) and the yaw rotation $\gamma_b$ defined in the positive direction around the vertical axis. $\mathbf{q}_a$ includes all the remaining actuated degrees of freedom such such as the lifting column or arm rotation joints.

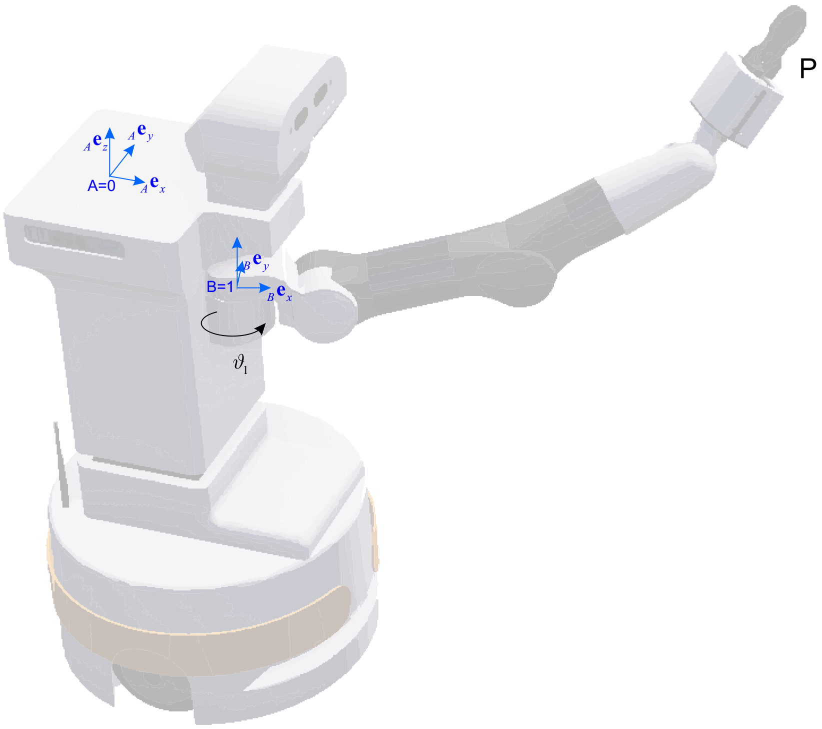



Figure 2: 3D model of Thiago with two coordinate frames.

(1) Define a set actuated generalize coordinates $\mathbf{q}_a$. What is the total number of $n$ generalized coordinates required to describe the full pose of the system? As mentioned before, ignore the gripper/hand and only consider an end-effector link. [1 pt]

$\mathbf{q}_a =$ $n = n_b + n_a =$

(2) Write down one possible parameterization for position and orientation of the end-effector [1 pt]

$\boldsymbol{\chi}_P =$

$\boldsymbol{\chi}_R =$

(3) What is the dimension of the analytical Jacobian for an end-effector parameterization $\boldsymbol{\chi}_E = [\boldsymbol{\chi}_P; \boldsymbol{\chi}_R]$ when choosing an Angle Axis representation for orientation and spherical coordinates for position? [1 pt]

$\mathbf{J}_{EA} \in$

(4) What is the dimension of the geometric Jacobian for the end-effector parameterization $\boldsymbol{\chi}_E = [\boldsymbol{\chi}_P; \boldsymbol{\chi}_R]$ when choosing Quaternion and Cartesian coordinates for rotation and position parameterization correspondingly? [1 pt]

$\mathbf{J}_{E0} \in$

As depicted in Fig. 2, the first arm joint corresponds to a rotation around the z-Axis with the generalized angle $\vartheta_1$. Frame $\mathcal{A}$ corresponds to an upper-body fixed frame. Frame $\mathcal{B}$ corresponds to a fixed frame on the first moving link of the arm. B, the center of frame $\mathcal{B}$, is positioned on the first rotation axis with offset ${}_{\mathcal{A}}\mathbf{r}_{AB} = const$ from A, the center of Frame $\mathcal{A}$.

(5) What is the geometric Rotation Jacobian of frame $\mathcal{B}$ with respect to frame $\mathcal{A}$ expressed in world-fixed frame $\mathcal{W}$? What is the dimension of the matrix? [1 pt]

$${}_{\mathcal{W}}\mathbf{J}_{\mathcal{AB}_R} = \begin{bmatrix} \quad \end{bmatrix}$$

(6) What is the geometric Position Jacobian of frame $\mathcal{B}$ with respect to frame $\mathcal{A}$ expressed in frame $\mathcal{A}$? [1 pt]

$${}_{\mathcal{A}}\mathbf{J}_{\mathcal{AB}_P} = \begin{bmatrix} \quad \end{bmatrix}$$

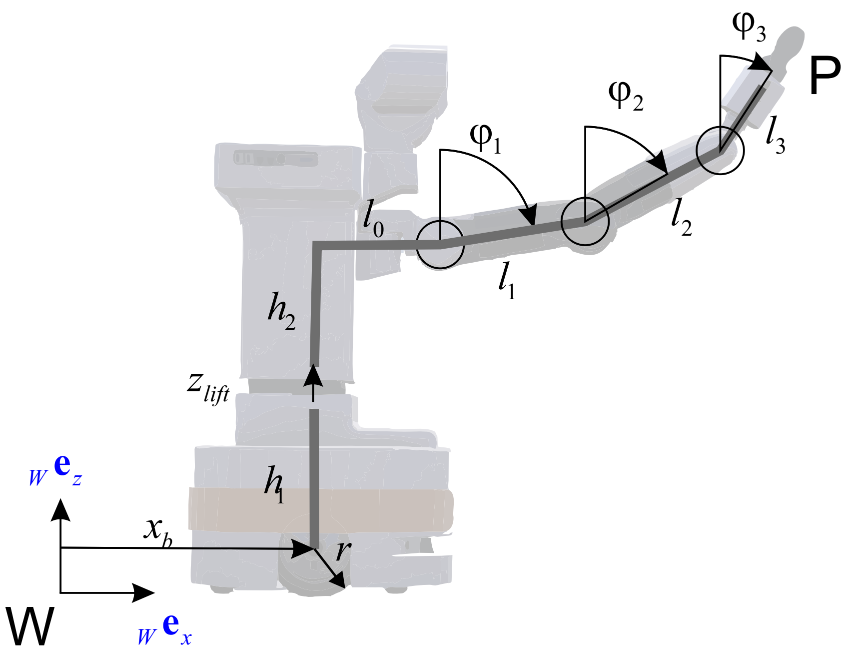



Figure 3: 2D model of Thiago.

In the second part of the kinematics section, we will analyze the planar problem of the mobile manipulator as displayed in Fig. 3. The system consists of a base that can move in the horizontal direction, a body that can vertically extend by $z_{lift}$, and three consecutive rotary joints. The geometric lengths $r$, $h_1$, $h_2$, $l_0$, $l_1$, $l_2$, $l_3$ are known and indicated in the drawing. In our illustration, we have selected a set of generalized coordinates that is consistent with all active constraints

$$\mathbf{q}_{2D} = \begin{pmatrix} x_b \\ z_{lift} \\ \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}. \tag{2}$$

(7) Is this set of generalized coordinates unique? If yes, argue why. If no, draw an alternative set in the image. [1 pt]

(8) What is the end-effector position ${}_W\mathbf{r}_P \in \mathbb{R}^2$ expressed in world-fixed frame $W$ as a function of $\mathbf{q}_{2D}$ defined in equation (2)? [2 pts]

${}_W\mathbf{r}_P =$

(9) What is the end-effector position Jacobian $\mathbf{J}_{P,t} = {}_W\mathbf{J}_P\left(\mathbf{q}_{2D,t}\right)$ for the configuration $\mathbf{q}_{2D,t} = (1, 1, 90°, 45°, 45°)^T$? [2 pts]

$\mathbf{J}_{P,t} =$

(10) Provide a singular configuration $\mathbf{q}_{2D,sing}$. Explain your answer, i.e. tell us which directions are not controllable. [1 pt]

$\mathbf{q}_{2D,sing} =$

Assume a non-singular configuration $\mathbf{q}_t$ as displayed in Fig. 3. We would like to move the end-effector $P$ in horizontal direction with velocity $\mathbf{v}_P^* = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \mathrm{m\,s^{-1}}$.

(11) What is the least square minimal desired generalized velocity $\dot{\mathbf{q}}_{2D}^*$ that ensures tracking the end-effector velocity $\mathbf{v}_P^*$? You can use the previously introduced variables. [1 pt]

$\dot{\mathbf{q}}_{2D}^* =$

(12) There exist multiple $\dot{\mathbf{q}}_{2D}^*$ that result in an end-effector velocity of $\mathbf{v}_P^*$. Write down the entire solution space and explain how many additional tasks you can possibly fulfill together with the end-effector velocity tracking. [2 pts]

(13) Beside tracking the end-effector velocity $\mathbf{v}_P^*$ (Task 1), we would like to make sure that the first and second rotary joints of the arm ($\varphi_1$ and $\varphi_2$) do not move at all (Task 2). Task 2 two can be formulated as a linear constraint $\mathbf{A}_2\dot{\mathbf{q}}_{2D} = \mathbf{b}_2$. Determine the corresponding matrix $\mathbf{A}_2$ and vector $\mathbf{b}_2$. [1 pt]

$\mathbf{A}_2 =$

$\mathbf{b}_2 =$

(14) How do you determine the desired joint space velocity $\dot{\mathbf{q}}_{2D}^*$ if you give Task 1 and Task 2 the same priority? [1 pt]

$\dot{\mathbf{q}}_{2D}^* =$

(15) Is the solution unique? Explain why or why not. [1 pt]

(16) How do you determine the desired joint space velocity $\dot{\mathbf{q}}^*_{2D}$ if you give Task 2 higher priority than Task 1? [1 pt]

$\dot{\mathbf{q}}^*_{2D} =$

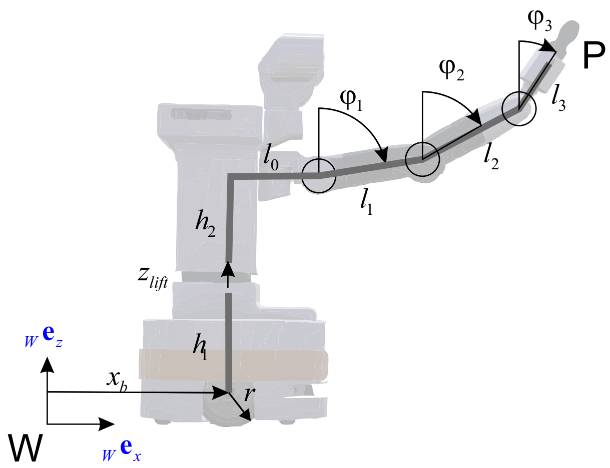



Figure 4: 2D model of Thiago. (same picture as Fig. 3)

**C. Dynamics** 10 pts

In this section, we consider the same system again as before (Fig. 4) with the generalized coordinates $\mathbf{q}_{2D}$. The equations of motion of the complete system can be described by

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{b} + \mathbf{g} = \boldsymbol{\tau} \tag{3}$$

For the following questions you can consider all the formulas for the end-effector Jacobian $\mathbf{J}_P = {}_0\mathbf{J}_{AE_P}(\mathbf{q})$, $\mathbf{J}_R = {}_0\mathbf{J}_{AE_R}(\mathbf{q})$ as given.

(1) Explain why Equation (3) can be written without a selection matrix before $\boldsymbol{\tau}$? [1 pt]

(2) Provide a simple impedance-based torque controller without gravity compensation that tries to bring the system to the target location $\mathbf{q}^*$ . [1 pt]

$\boldsymbol{\tau} =$

Please fill in your name: ____________________

(3) What is the steady state configuration of the above introduced controller? [1 pt]

$\mathbf{q}_{t=\infty} =$

(4) Given the current state defined by $\dot{\mathbf{q}}_t$ and $\mathbf{q}_t$, what is the torque required to move point $P$ with acceleration $\ddot{\mathbf{r}}_P^*$ while producing zero acceleration at the base in x-direction? Is the torque unique? Use joint space inverse dynamics control. [2 pts]

$\boldsymbol{\tau} =$

(5) How does the equation of motion Equation (3) change in case the hand mounted at the end-effector is $0.1\,\mathrm{kg}$ heavier? Write down the new equation of motion using previously introduced terms. You can model the additional mass as a point-mass at point P. [2 pts]

In the following, we consider the arm in contact with a horizontal surface, e.g. a table. The arm has no additional end-effector, i.e. has the original mass.

(6) Formulate a joint space inverse dynamics controller that produces the desired end-effector acceleration $\ddot{\mathbf{r}}_P^* = (\ddot{x}_P^*; 0)$ and a vertical pressure force of $10\,\mathrm{N}$ on to the table. The table surface is considered friction less. [3 pts]

$\boldsymbol{\tau}^* =$

Please fill in your name: ____________________

**D. Floating Base System** 15 pts

In this exercise we consider the free floating version of the planar Thiago (Fig. 5). Use the variables indicated in the figure.

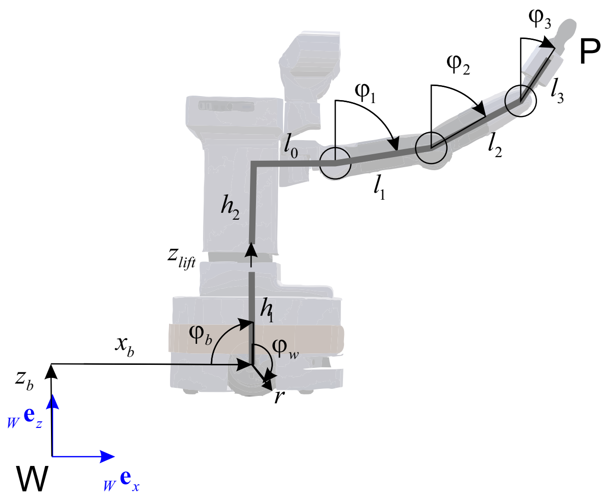



Figure 5: Free floating model in 2D of Thiago

(1) Define the generalized coordinates and generalized velocities for the floating base 2D system (Fig. 5). What is the number of actuated ($n_a$) and un-actuated ($n_b$) degrees of freedom (DOFs)? [1 pt]

$\mathbf{q}_{ff} =$ $\mathbf{u} =$

$n_a =$
$n_b =$

(2) Now we assume the robot to be in contact with the floor. What is the contact Jacobian $\mathbf{J}_{cb} = {}_W\mathbf{J}_{cb}$ between the robot and the ground for the 2D model of Thiago (Fig. 5)? The ground is assumed to have infinite friction such that the wheel will never slip. [2 pts]

$\mathbf{J}_{cb} =$

Please fill in your name: ______________________

(3) The support consistent dynamics of the Thiago can be expressed as [2 pts]

$$\mathbf{N}_{cb}^T \mathbf{M}\dot{\mathbf{u}} + \mathbf{N}_{cb}^T \mathbf{b} + \mathbf{N}_{cb}^T \mathbf{g} = \mathbf{N}_{cb}^T \mathbf{S}^T \boldsymbol{\tau} \tag{4}$$

Provide the dimension or rank for the following variables:

| |
|---|
| Dimension of $\mathbf{M}$: |
| Dimension of $\boldsymbol{\tau}$: |
| Dimension of $\mathbf{N}_{cb}$: |
| Rank of $\mathbf{N}_{cb}$: |

In the following exercise we assume that Thiago is cleaning a window. We will formulate a hierarchical optimization-based controller to identify the necessary torque to drive the motion and interaction force.

(4) Draw all the reaction forces from window and ground acting on Thiago in steady state in Fig. 5 if the robot must apply a specific horizontal force at point $P$. [2 pts]

| |
|---|
| Draw directly in Fig. 5 |

(5) Given the end-effector Jacobian $\mathbf{J}_P = {}_W\mathbf{J}_P\left(\mathbf{q}_{ff}\right)$, what is the additional constraint on generalized acceleration $\dot{\mathbf{u}}$ imposed by the fact that the wall is a vertical surface preventing horizontal motion? Clearly indicate the dimensions of the used matrices and vectors. [1 pt]

(6) While interacting with the environment at point P, the robot can produce an interaction force $F_{Px}$. How do you have to modify or extend Equation 4 on the left hand side to include this additional interaction force? Write down the complete equation of motion and indicate the dimensions of the used variables [1 pt]

Please fill in your name: ______________________________

(7) To clean the window, we would like to formulate a multi-task inverse dynamics controller using hierarchical least square optimization. As optimization variable we use [3 pts]

$$\mathbf{x} = \begin{pmatrix} \dot{\mathbf{u}} \\ F_{Px} \\ \boldsymbol{\tau} \end{pmatrix} \tag{5}$$

Please provide $\mathbf{A}_i$, $\mathbf{b}_i$ for each task $i$ and the decision variable $\mathbf{x}$ that allow formulating the following objectives in the form $\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{b}\|^2$:

1. Fulfil the support consistent equation of motion
2. Fulfil the motion constraint imposed by the ground contact
3. Ensure that the hand (point P) has zero horizontal acceleration
4. Ensure a desired normal contact force of $10\,\mathrm{N}$ to the window at point P in horizontal direction
5. Provide a tracking controller to follow a desired vertical motion of point P along a desired motion $z_P^*(t), \dot{z}_P^*(t)$
6. $\boldsymbol{\tau}$ has minimal two-norm

Clearly indicate the dimensions of all matrices, in particular also if you introduce identity or block-zero matrices.

(8) Is the solution to the previous question unique or not? Provide an explanation/argumentation. [1 pt]

(9) Is it possible to fulfill the window cleaning task as described in exercise (7) while ensuring that the base does not move, the body lift degree of freedom remains in the middle, and the hand (last link after rotation joint 3 of the arm) remains perpendicular to the wall? Justify why it is not possible or how it can be realized [2 pts]

**E. Rotary Wing** 9 pts

Figure 6 shows the schematic of a hexarotor in free flight. The body frame $\mathcal{B}$ is fixed to its center and the body $z$-axis $\boldsymbol{e}_z^{\mathcal{B}}$ points upwards. For each rotor, its rotor speed and spinning direction are indicated by $\omega_{1,\ldots,6}$ and the arrow, respectively. The spinning directions can not be inverted and every rotor produces thrust along the positive body $z$-axis.

The inertial frame $\mathcal{I}$ is fixed to the ground and the gravity vector $\boldsymbol{g}$ points along its negative z-axis.

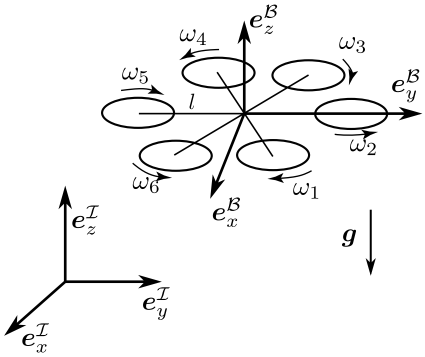



Figure 6: Schematic of the hexarotor.

| Property | Value |
|---|---|
| Mass | $m = 1.4\,\mathrm{kg}$ |
| Thrust constant | $b = 8.6 \times 10^{-6}\,\mathrm{N\,rad^{-2}\,s^2}$ |
| Drag constant | $d = 1.4 \times 10^{-7}\,\mathrm{N\,m\,rad^{-2}\,s^2}$ |
| Maximum rotor speed | $\omega_{max} = 700\,\mathrm{rad\,s^{-1}}$ |
| Gravity vector | ${}_{\mathcal{I}}\boldsymbol{g} = \begin{bmatrix} 0 & 0 & -9.81 \end{bmatrix}^\top \mathrm{m\,s^{-2}}$ |
| Inertia tensor | ${}_{\mathcal{B}}\boldsymbol{I} = \begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0.02 & 0 \\ 0 & 0 & 0.04 \end{bmatrix} \mathrm{kg\,m^2}$ |
| Arm length | $l = 0.2\,\mathrm{m}$ |

Table 1: Properties of the hexarotor.

(1) Assuming all propellers spin at the same rate, what is their speed (in $\mathrm{rad\,s^{-1}}$) required for static hover? We now want to accelerate horizontally at $2\,\mathrm{m\,s^{-2}}$ without loosing altitude: what is the required pitching angle $\theta$? Consider a whole body air drag coefficient of $c_D = 0.6$, air density $\rho = 1.255\ \mathrm{kg\,m^{-3}}$ and aerodynamic surface $S = 0.8\,\mathrm{m}^2$: what is the achieved lateral velocity of the hexarotor when maintaining the previous pitching angle and thrust? [3 pts]

(2) What is the allocation matrix to transform rotors speeds into the thrust force and torques applied on the body? When designing a flight controller, the derived relation needs to be inverted: how many possible solutions exist and why? Write the analytical form of the most efficient one (i.e. the solution that minimizes the rotor speeds to save battery)? Suppose that motors 1 and 4 suddenly stop working ($\omega_1 = \omega_4 = 0$): how many allocation solutions do we have in this case if we want to keep flying? [6 pts]

**F. Fixed-wing** 8 pts

Use the parameters from the table in Figure 7 with the additional environmental and platform specific information, below, to answer the following questions. **Do not interpolate** when using the table, simply take the closest value. Assume the aircraft's *lift* and *drag* coefficients are only a function of angle of attack, $c_L = f(\alpha)$, $c_D = f(\alpha)$. **Please show all work used for your calculations!**

Environment: density $\rho = 1.225 kg/m^3$, gravity $g = 9.81 m/s^2$ (assume sea-level values)

Aircraft specs: wing area $S = 0.39 m^2$, mass $m = 2.65 kg$

| $\alpha$ [deg] | $c_L$ | $c_D$ | $c_L/c_D$ |
|---|---|---|---|
| -5 | -0.227 | 0.0268 | -8.470 |
| -4 | -0.0544 | 0.0313 | -1.738 |
| -3 | 0.106 | 0.0367 | 2.888 |
| -2 | 0.256 | 0.0429 | 5.967 |
| -1 | 0.394 | 0.0499 | 7.896 |
| 0 | 0.519 | 0.0578 | 8.979 |
| 1 | 0.633 | 0.0664 | 9.533 |
| 2 | 0.735 | 0.0759 | 9.684 |
| 3 | 0.826 | 0.0862 | 9.582 |
| 4 | 0.904 | 0.0973 | 9.291 |
| 5 | 0.971 | 0.109 | 8.908 |
| 6 | 1.025 | 0.121 | 8.471 |
| 7 | 1.068 | 0.135 | 7.911 |
| 8 | 1.099 | 0.149 | 7.376 |
| 9 | 1.118 | 0.165 | 6.776 |
| 10 | 1.125 | 0.181 | 6.215 |
| 11 | 1.121 | 0.197 | 5.690 |
| 12 | 1.105 | 0.215 | 5.140 |
| 13 | 1.0767 | 0.234 | 4.601 |
| 14 | 1.036 | 0.253 | 4.095 |
| 15 | 0.984 | 0.273 | 3.604 |

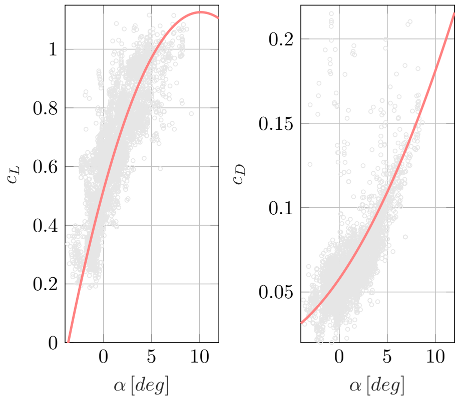



Figure 7: Aerodynamic data.

(1) The UAV returns from a mission and flies horizontally in steady-state with an airspeed of $14.5\,\mathrm{m\,s^{-1}}$. [2 pts]
The home position is still $800\,\mathrm{m}$ away (horizontal distance) and $100\,\mathrm{m}$ below the UAV (vertical distance) when, all for sudden, the propulsion system fails (thrust $T = 0\,\mathrm{N}$). Assume no wind and no obstacles (i.e. flat ground).

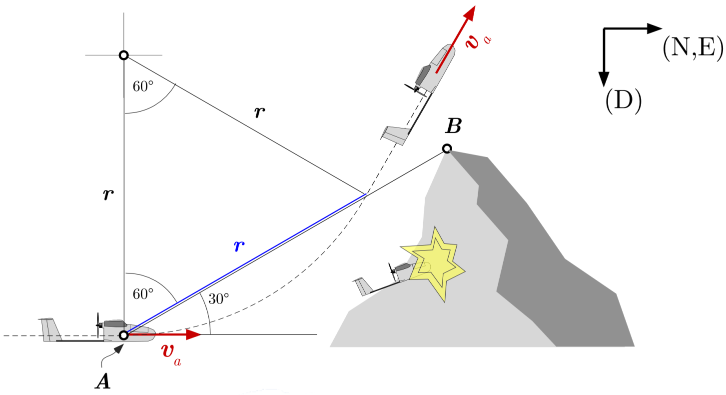
a) At what angle of attack is the UAV flying just prior to the propulsion system failure, assuming the thrust acts only in flight direction? What thrust force is required to maintain this state?

b) Regulating to an angle-of-attack of $5°$, is the UAV able to glide back home? Is there an angle of attack that would yield a longer gliding range? Justify your answers. *Hint: Consider the flight-path angle.*

(2) The UAV flies level when its perception system detects a mountain lying straight ahead. The mountain top is located at a distance $d_{AB}$ and an elevation of $30°$ (simplifies geometry, note $r_{safe} \leq d_{AB}$) from the current position and path, respectively. Assume that thrust is not a limiting factor and that there is no wind. [4 pts]

a) Assume the UAV maintains a constant airspeed of $||v_a|| = 11\,\mathrm{m\,s^{-1}}$ and pitches up so that its path describes a circular arc (cf. figure). Calculate the minimum range $d_{AB}$ of an obstacle detection system so that the UAV can evade the terrain (i.e. fly the dashed arc) without stalling the wing. *Hint: the required lift is largest in point A*

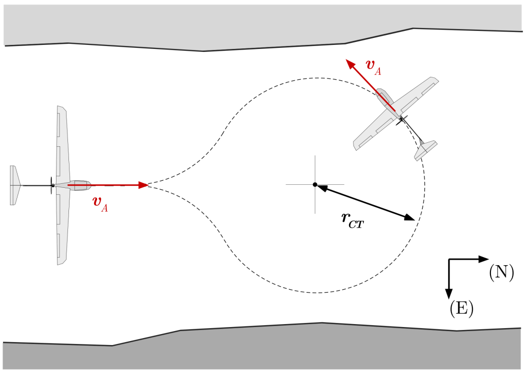
b) In the presented scenario, if the range of the obstacle detection system reduces, should the UAV fly slower to still be safe? Justify your answer.



(3) In another mission taking place in the mountains, the UAV is tasked to explore a deep, narrow valley. The UAV operator has to evaluate, whether it is safe to enter the valley, i.e. if the UAV is able to turn back around by flying a level coordinated turn within the given space. Assume zero wind. [2 pts]

a) Express the lift coefficient $c_L$ required to fly a level, coordinated turn as a function of the turn radius $r_{CT}$ and the airspeed $v_a$. *Hint: Start with calculating the required lift force.*

b) Is it safe to fly down a valley with a minimum width of $50\,\mathrm{m}$ at an airspeed of $v_a = 12\,\mathrm{m\,s^{-1}}$? Neglect the collision volume of the aircraft. *Hint: What limits the minimum (aerodynamically) feasible turn radius if the airspeed is given?*




Please fill in your name: ____________________