Kinematics

Position

	Cartesian	Cylindrical	Spherical
r	$\chi_{p_c} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	$\chi_{p_z} = \begin{bmatrix} \rho \\ \theta \\ z \end{bmatrix} \quad {}_{\mathcal{A}}r = \begin{bmatrix} \rho\ \cos\theta \\ \rho\ \sin\theta \\ z \end{bmatrix}$	$\chi_{p_s} = \begin{bmatrix} r \\ \theta \\ \phi \end{bmatrix} \quad {}_{\mathcal{A}}r = \begin{bmatrix} r\ \cos\theta\ \sin\phi \\ r\ \sin\theta\ \sin\phi \\ r\ \cos\phi \end{bmatrix}$
$\dot{r} = \mathbf{E}_p(\chi_P)\dot{\chi}_P$	$\mathbf{E}_{p_c} = \mathbb{I}$	$\dot{r}(\chi_{p_z}) = \begin{bmatrix} \dot{\rho}\ \cos\theta - \rho\dot{\theta}\ \sin\theta \\ \dot{\rho}\ \sin\theta + \rho\dot{\theta}\ \cos\theta \\ \dot{z} \end{bmatrix}$ $\mathbf{E}_{p_z}(\chi_{p_z}) = \begin{bmatrix} \cos\theta & -\rho\ \sin\theta & 0 \\ \sin\theta & \rho\ \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\mathbf{E}_{p_z}^{-1}(\chi_{p_z}) \begin{bmatrix} \cos\theta\sin\theta & 0 & \\ -\frac{\sin\theta}{\rho} & \frac{\cos\theta}{\rho} & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\mathbf{E}_{p_s} = \begin{bmatrix} \cos\theta\sin\phi & -r\sin\phi\sin\theta & r\cos\phi\cos\theta \\ \sin\phi\sin\theta & r\cos\theta\sin\phi & r\cos\phi\sin\theta \\ \cos\phi & 0 & -r\sin\phi \end{bmatrix}$ $\mathbf{E}_{P_s}^{-1} = \begin{bmatrix} \cos\theta\sin\phi & \sin\phi\sin\theta & \cos\phi \\ -\frac{\sin\theta}{r\sin\phi} & \frac{\cos\theta}{r\sin\phi} & 0 \\ \frac{\cos\phi\cos\theta}{r} & \frac{\cos\phi\sin\theta}{r} & -\frac{\sin\phi}{r} \end{bmatrix}$

Rotation

Quaternion

$\xi = [\xi_0, \check{\xi}] \in \mathbb{R}^4, \check{\xi} = [\xi_1, \xi_2, \xi_3] \in \mathbb{R}^3$

$$[x]_\times = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}$$

- Rotation matrix : $R = (2\xi_0 - 1)I + 2\xi_0 \left[\check{\xi}\right]_\times + 2\check{\xi}\check{\xi}^\top$
- multiplication : $q \otimes p = \begin{bmatrix} q_0 & -\check{q}^\top \\ \check{q} & q_0 I + [\check{q}]_\times \end{bmatrix} p = \begin{bmatrix} p_0 & -\check{p}^\top \\ \check{p} & p_0 I - [\check{p}]_\times \end{bmatrix} q$