Monetary Policy and the Limits to Arbitrage: Insights from a New Keynesian Preferred Habitat Model

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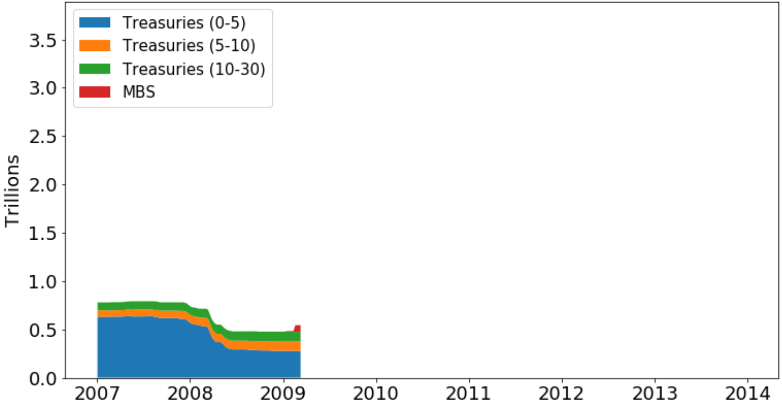
June 10, 2019

University of Surrey

Policy Response to Great Recession



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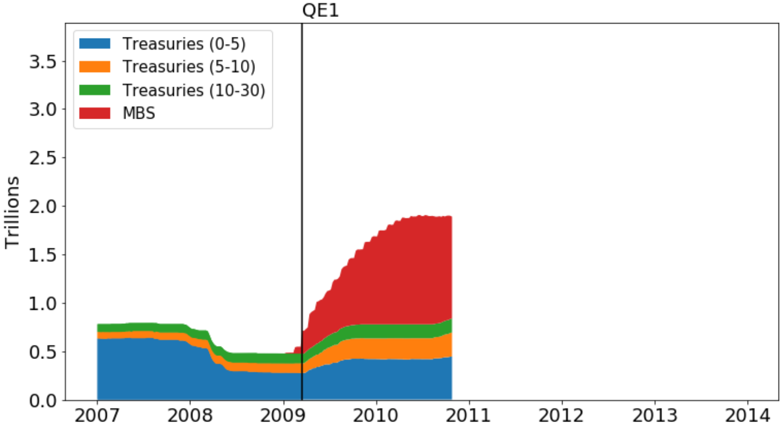

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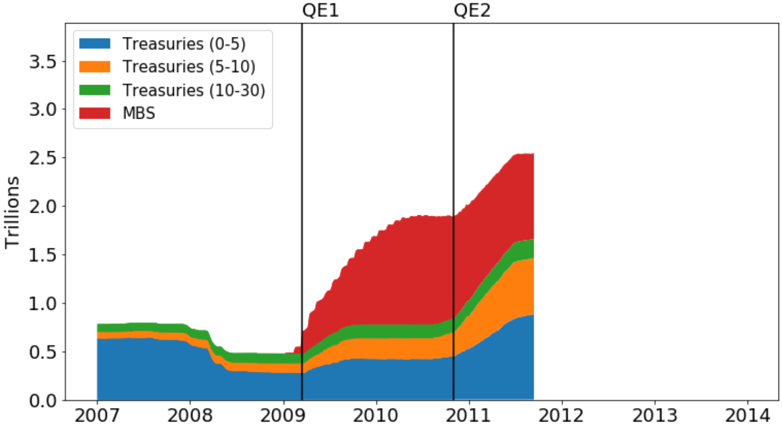

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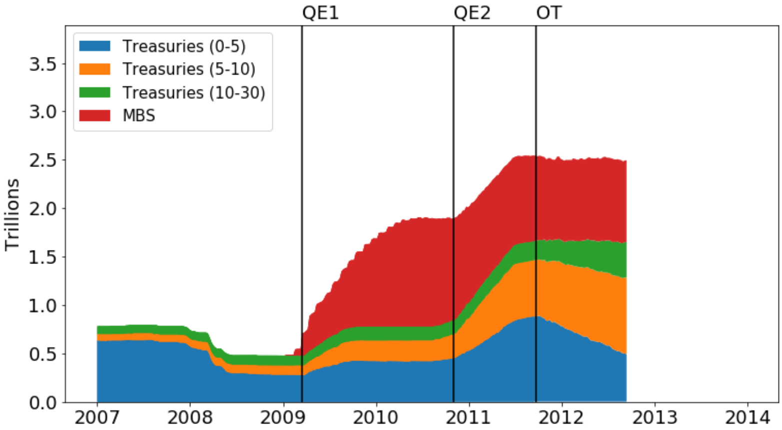

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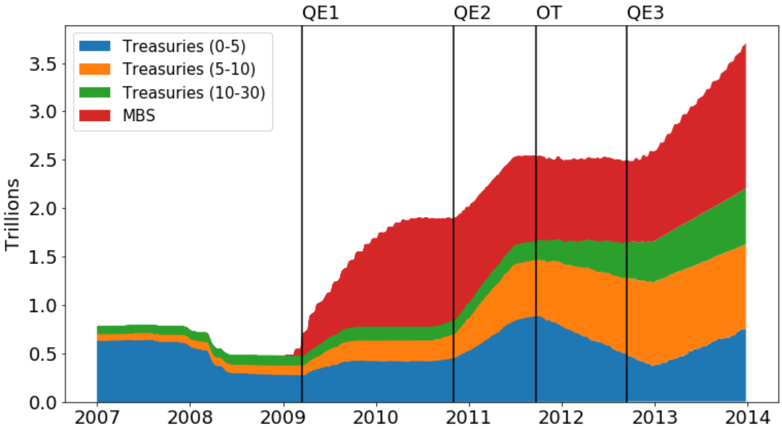

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- Bond market imperfections play a role in the transmission of **conventional** monetary policy
- Crucial for designing monetary policy going forward

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- Monetary policy works through both channels

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  - QE rule can be stabilizing

Literature Contributions

- "Preferred habitat" as a key channel for understanding bond markets
  - D'Amico and King (2013), Hamilton and Wu (2012), Greenwood and Vayanos (2014), Gorodnichenko and Ray (2017), Greenwood and Vissing-Jorgensen (2018)
- Few formal models
  - Vayanos and Vila (2009)
- QE in general equilibrium: Market segmentation vs. forward guidance
  - Gertler and Karadi (2013), Chen et al (2012), Carlstrom et al (2017), Christensen and Rudebusch (2012), Bauer and Rudebusch (2014), Bhattarai et al (2015)
- Frictions and expected future policy
  - McKay et al (2016), Farhi and Werning (2017), Gabaix (2016), Angeletos and Lian (2018)

New Keynesian Preferred Habitat Framework

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  - Infinitely-lived **households** work and consume
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- Government:
  - **Central bank** sets the short nominal rate (and conducts QE)
  - Lump-sum taxes/transfers from investors to HHs

Aggregate Dynamics

- Modification of a benchmark NK model ($\pi_t \equiv 0$):

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- Closing the model: equilibrium term structure determination

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Zero-coupon bond yields and prices $R_{t,\tau} = -\frac{\log P_{t,\tau}}{\tau}$ determined by interactions of two types of investors [Vayanos and Vila 2009]:

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- Market clearing: $b_{t,\tau} = -\tilde{b}_{t,\tau}$

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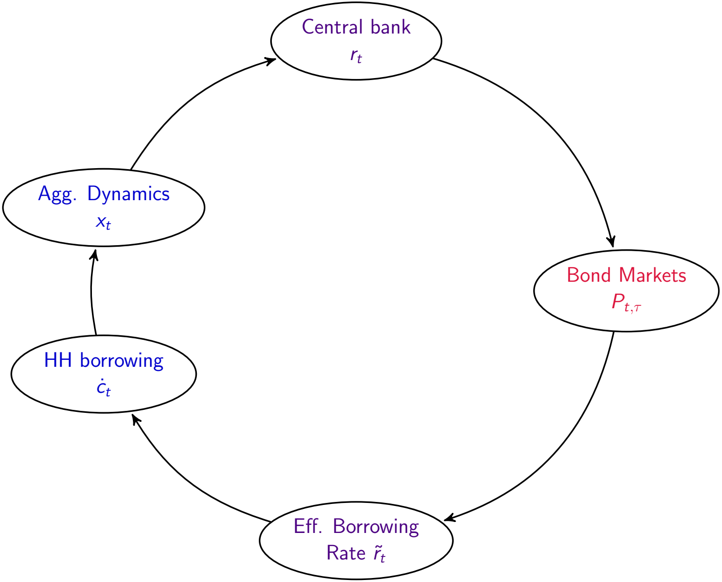

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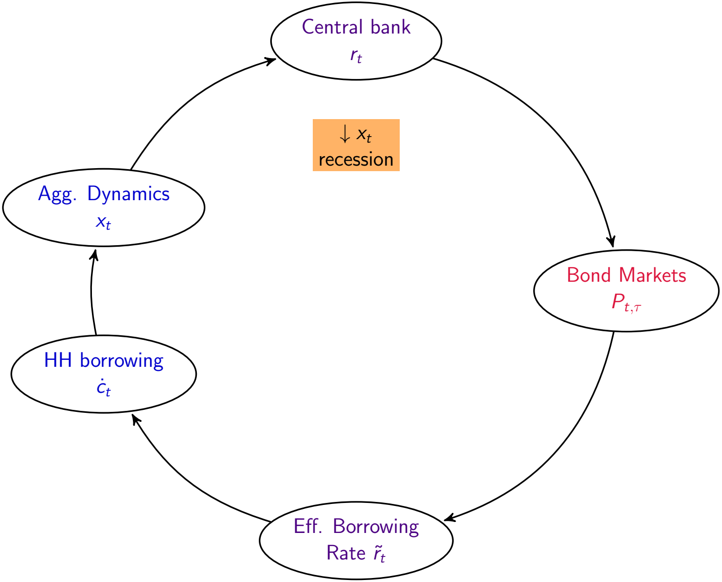

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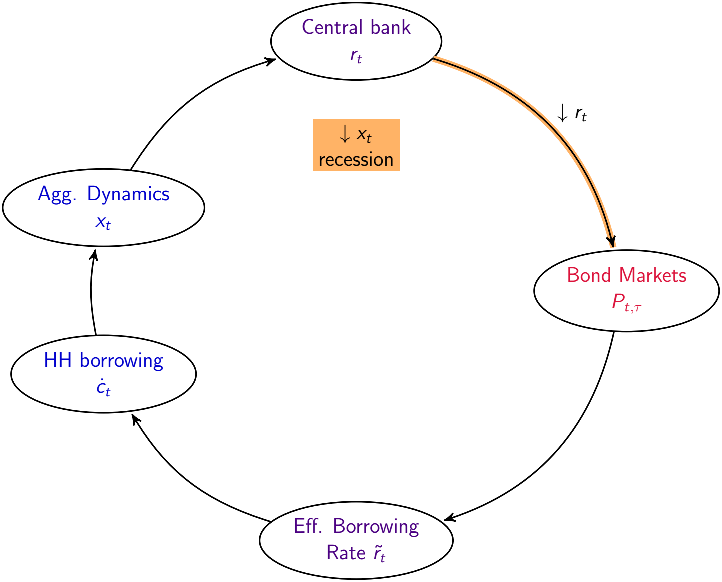

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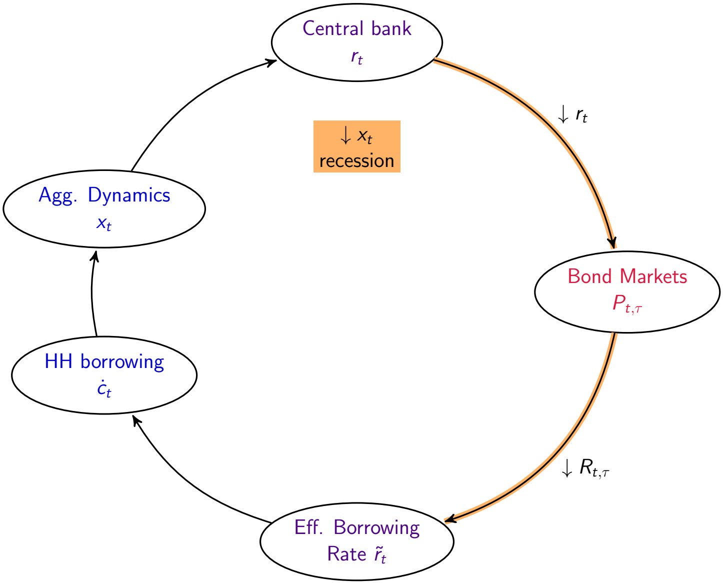

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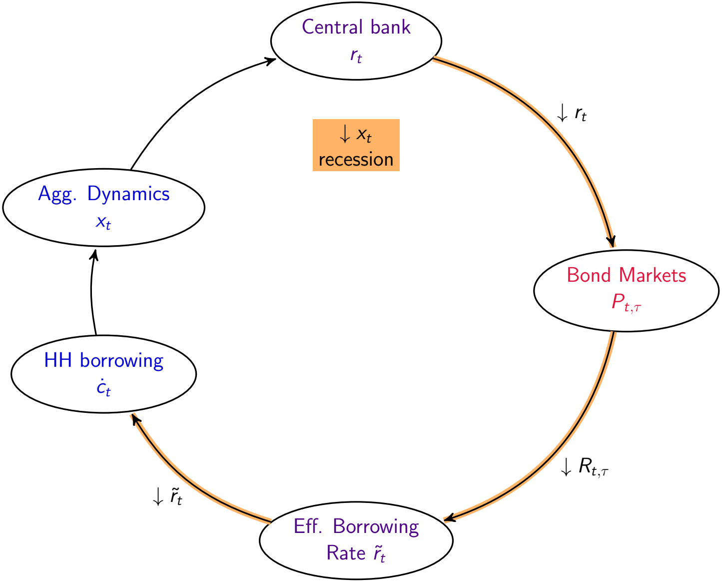

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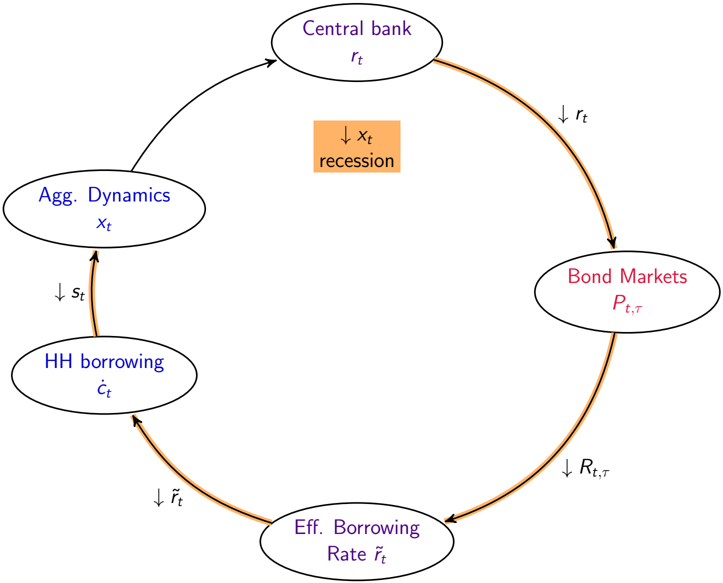

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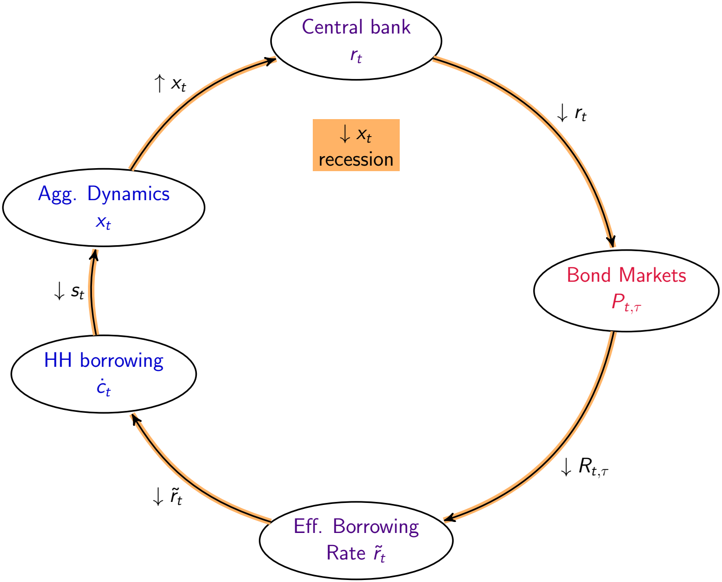

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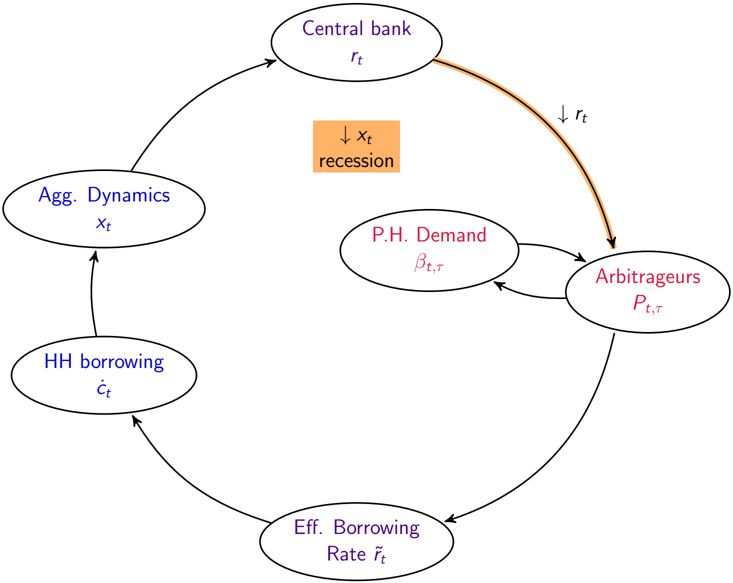

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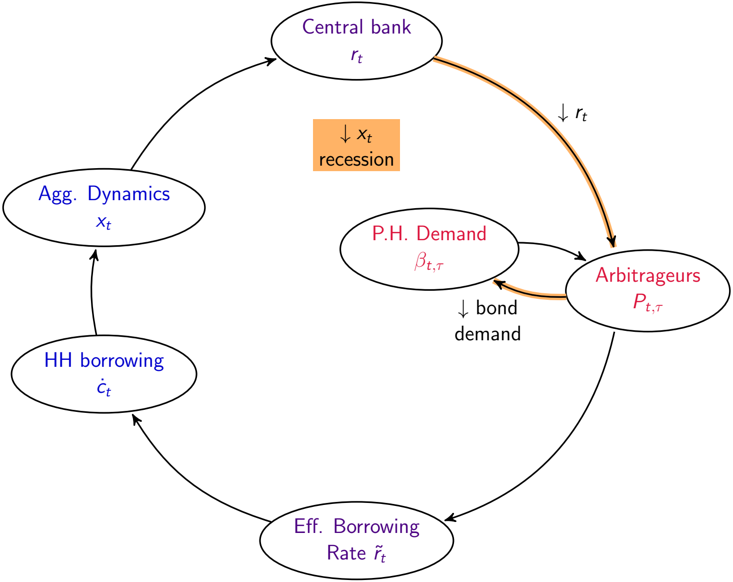

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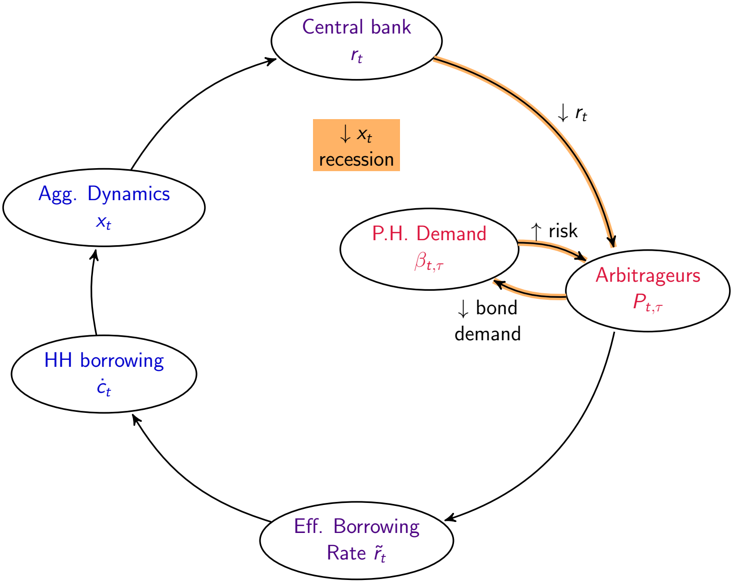

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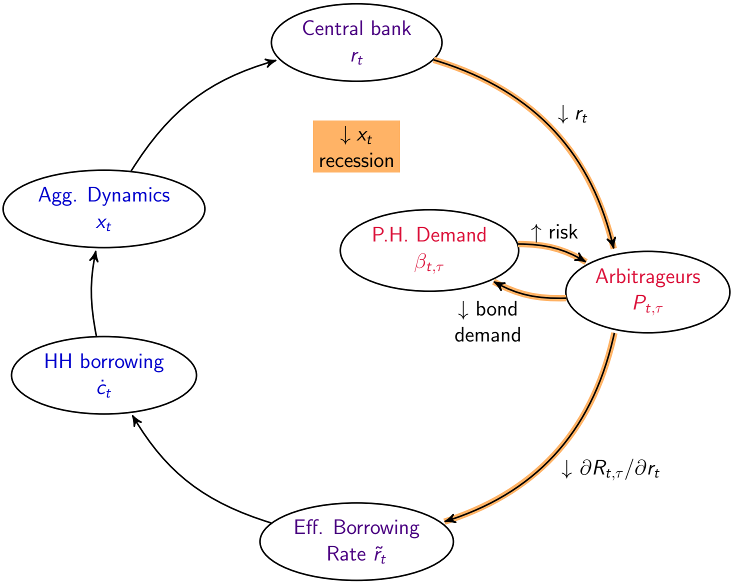

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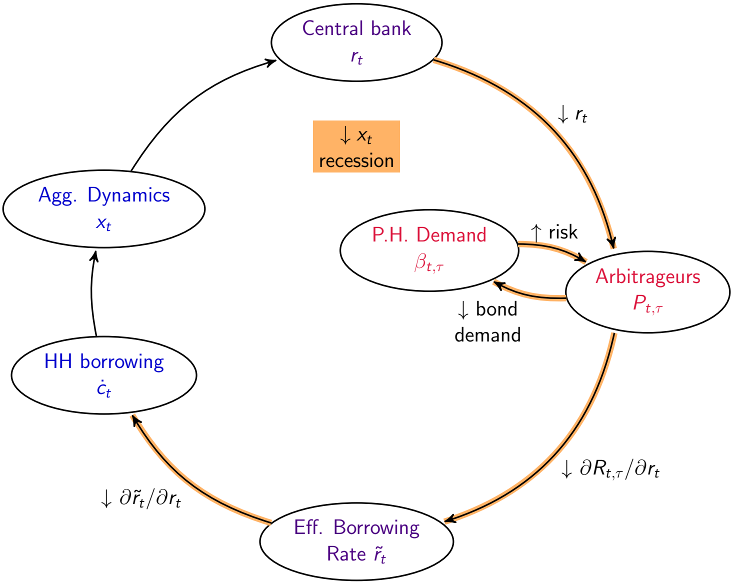

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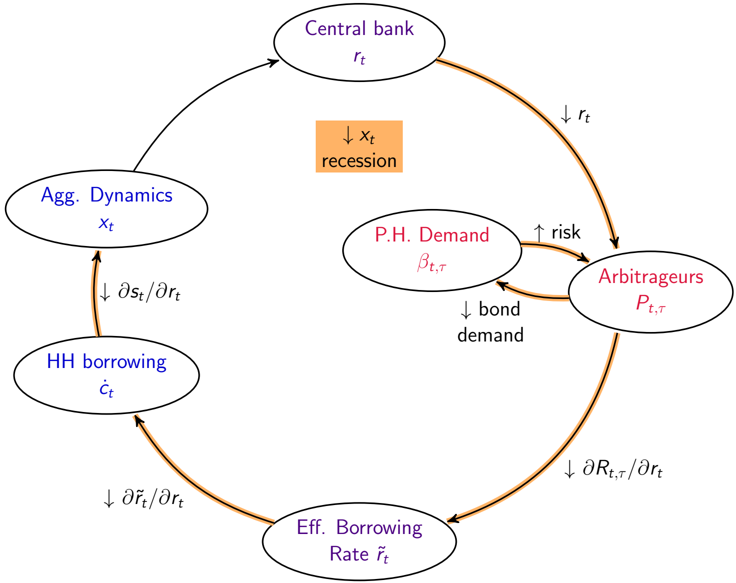

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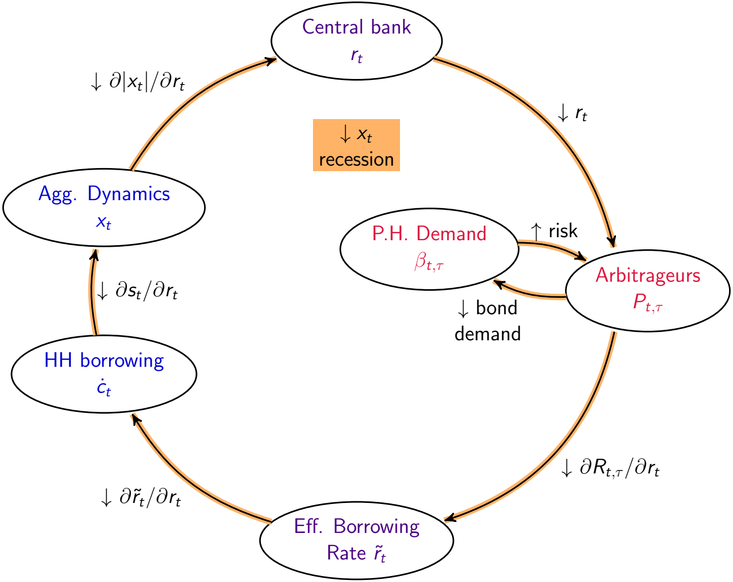

Solving the Model

- Equilibrium affine term structure

$$-\log P_{t,\tau} = A_r(\tau) r_t + C(\tau)$$

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- Linear stochastic differential equation:

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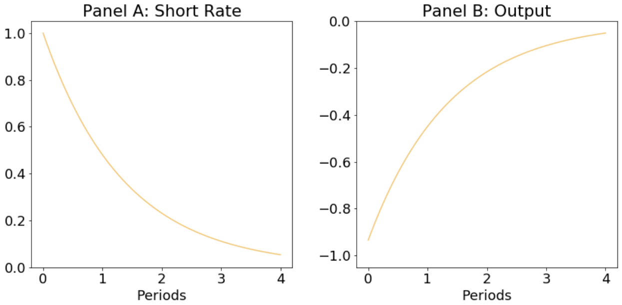

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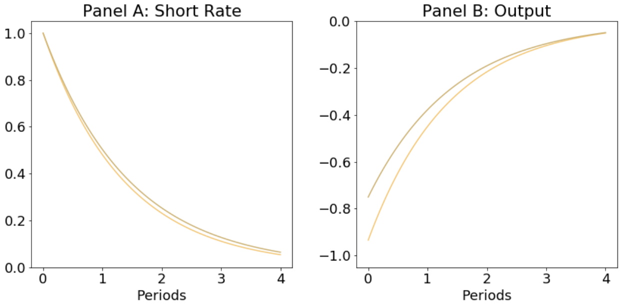

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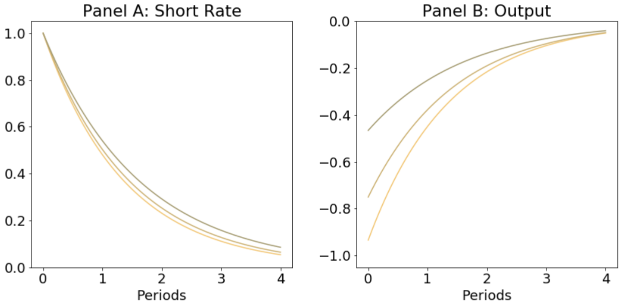

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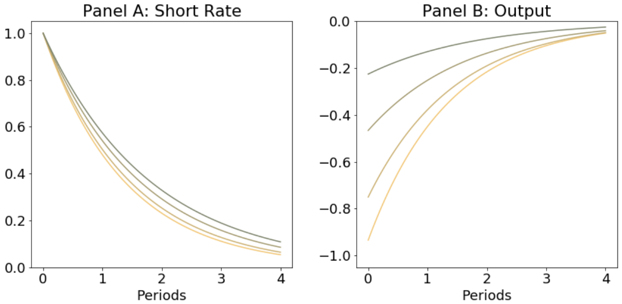

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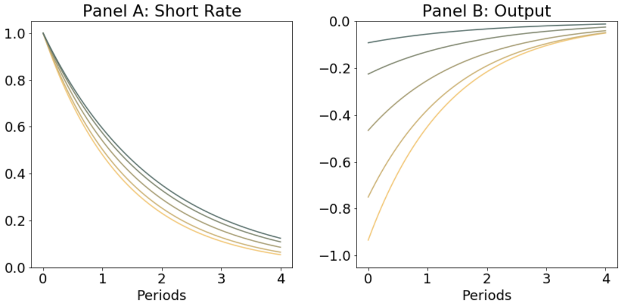

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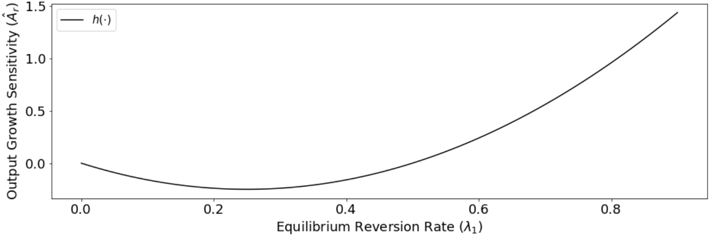

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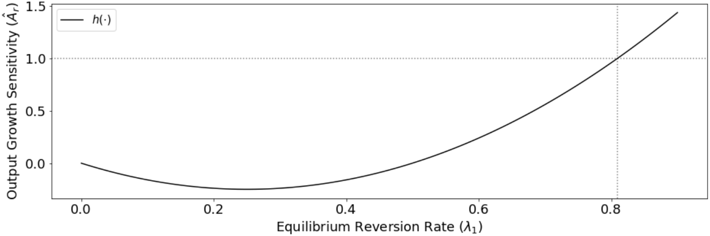

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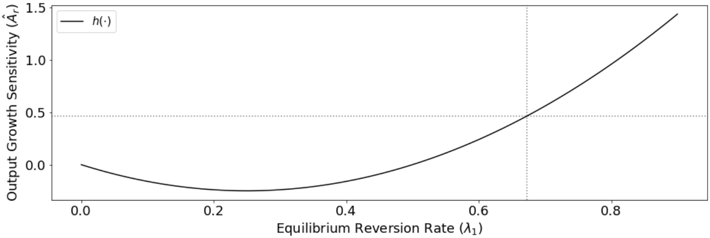

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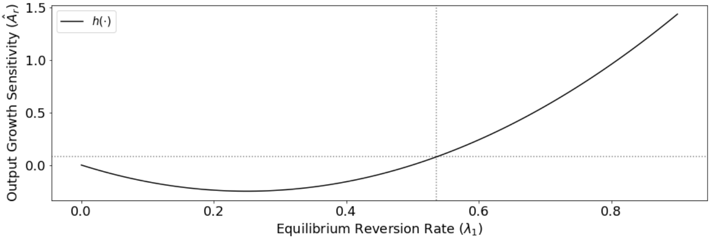

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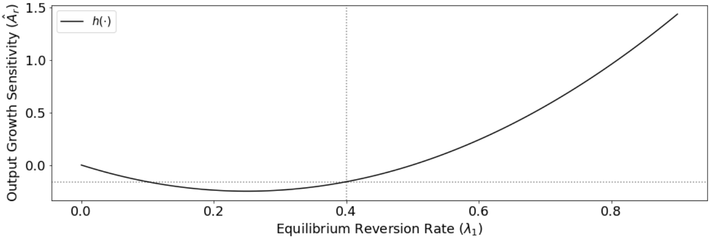

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$$\frac{\mu_{t,\tau} - r_t}{A_r(\tau)} = a\sigma_r^2 \int_0^T b_{t,\tau} A_r(\tau)\, \mathrm{d}\tau$$

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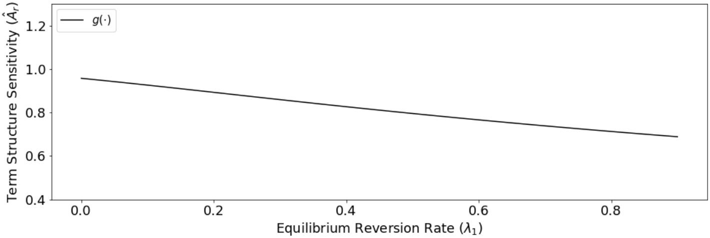
- Prices adjust to balance demand and optimality conditions
- $\implies$ differential equation which solves affine coefficients

$$\hat{A}_r \equiv \int_0^T \frac{\eta(\tau)}{\tau} A_r(\tau)\,\mathrm{d}\tau$$

Term Structure Equilibrium



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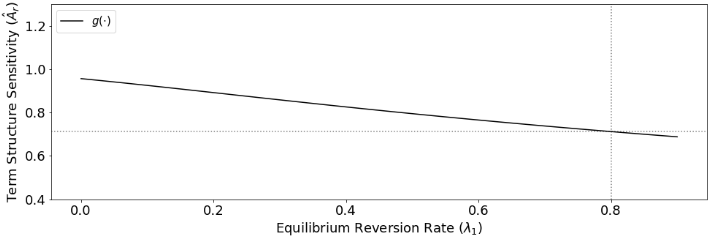

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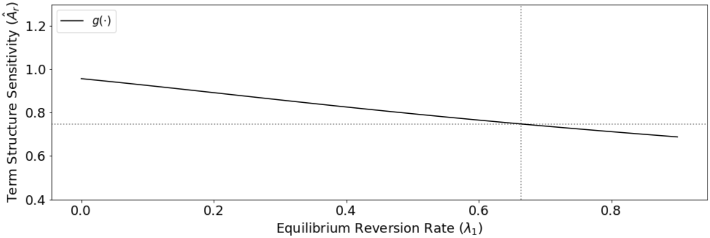

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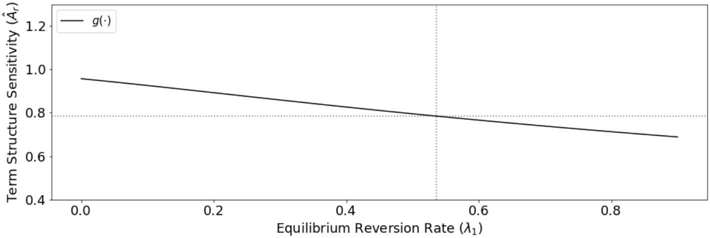

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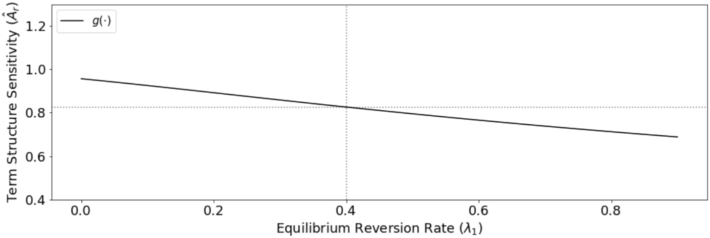

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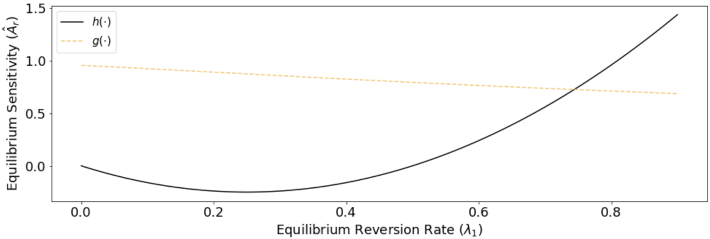
- EH: two responses should be identical (only when $a=0$)

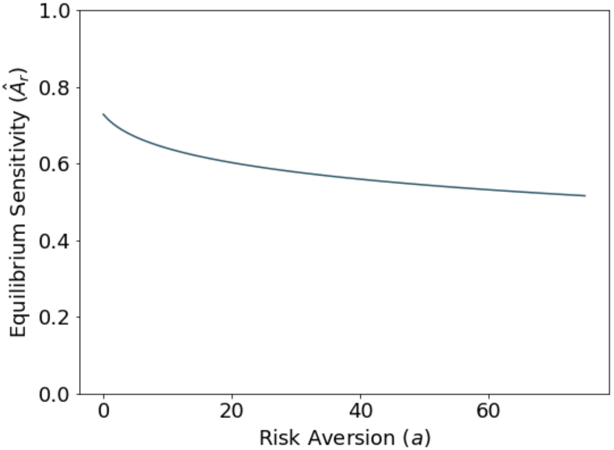
General Equilibrium



Existence and Uniqueness

There exists a unique positive eigenvalue of Υ $\lambda_1 > 0$ for which $g(\lambda_1) = h(\lambda_1)$, which fully characterizes the model equilibrium. Further, this implies $0 < \hat{A}_r < 1$.

Conventional Policy and Financial Disruptions



Notes: equilibrium changes in sensitivity to the short rate $\hat{A}_r$ as risk aversion a increases.

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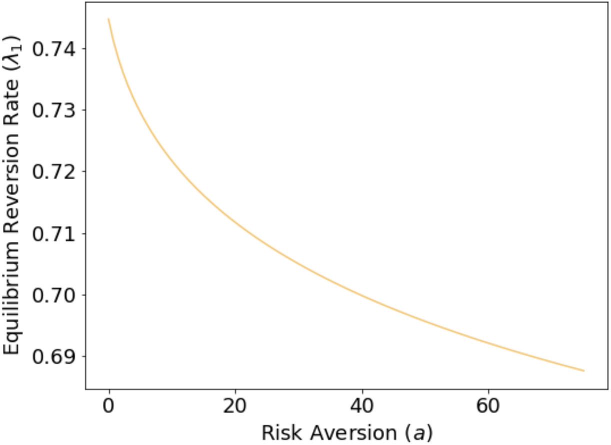

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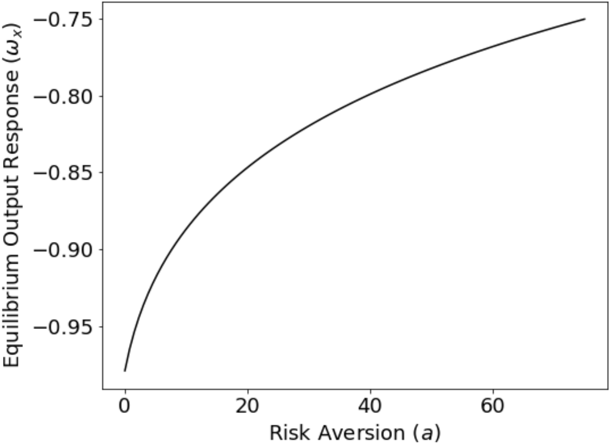

Notes: equilibrium changes in output response ω_x to monetary shocks as risk aversion a increases.

Policy Implications

- More aggressive response to output ϕ_x results
- Higher inertia κ_r results
- Shifts in effective rate weights $\eta(\tau)$ results
- Forward guidance less effective as risk aversion increases details

Modeling LSAPs

- Suppose the central bank directly purchases bonds through open market operations
- Change to the demand shifter in PH demand

$$\tilde{b}_{t,\tau} = \alpha(\tau)\tau(R_{t,\tau} - \beta_{t,\tau})$$

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- Change to the demand shifter in PH demand

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\begin{aligned}
\tilde{b}_{t,\tau} &= \alpha(\tau)\tau(R_{t,\tau} - \beta_{t,\tau}) \\
\beta_{t,\tau} &= \bar{\beta}(\tau) + \theta(\tau)\beta_t \\
\mathrm{d}\beta_t &= -\kappa_\beta \beta_t \, \mathrm{d}t
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- Affine functional form of bond prices

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-\log P_{t,\tau} = A_r(\tau) r_t + A_\beta(\tau) \beta_t + C(\tau)
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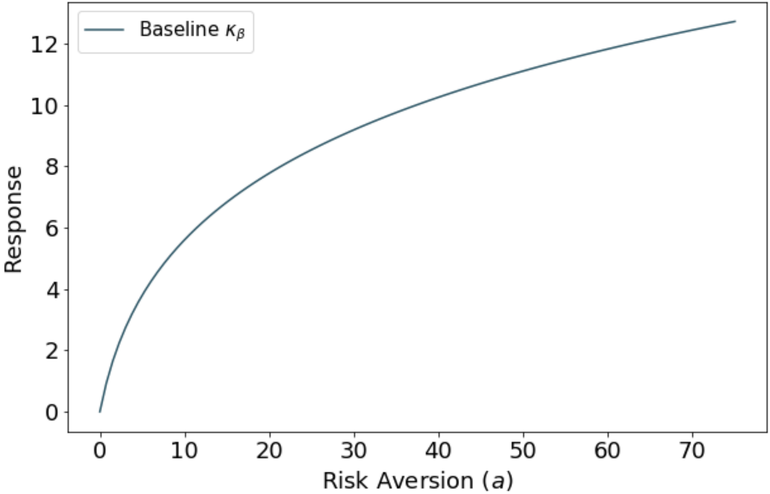
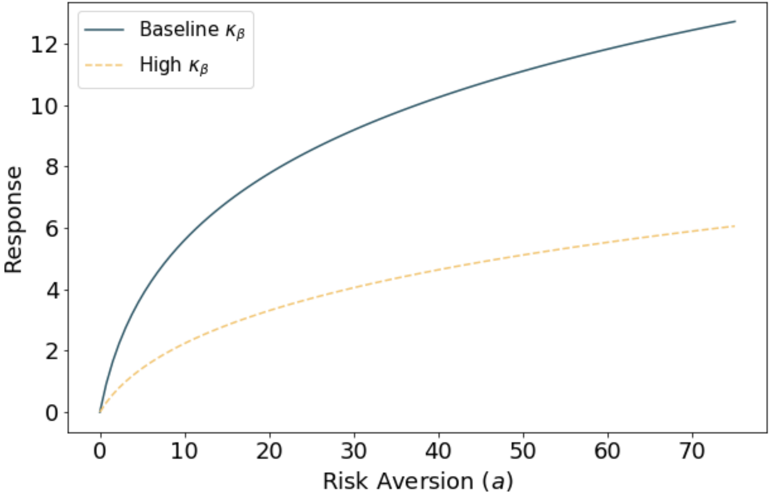
Output Response to QE



Notes: plots of output gap response to a QE shock as risk aversion increases.

Output Response to QE



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Sticky Prices

- What about when prices are not fixed?

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$$\hat{A}_r > \frac{\delta}{\delta\phi_\pi + \rho\phi_x}$$

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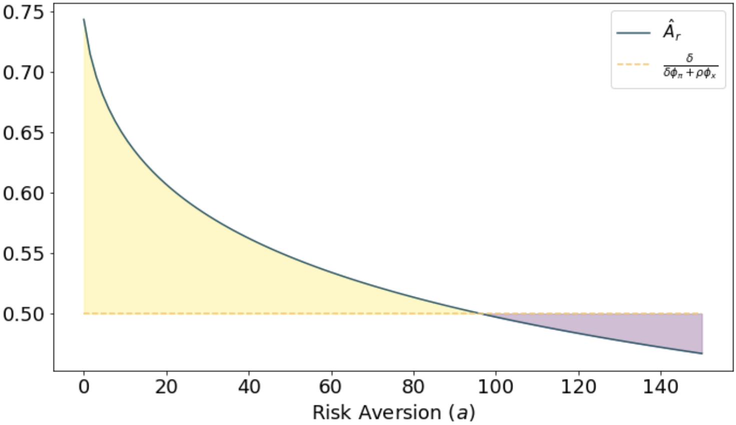
- If $\hat{A}_r = 1$ and $\phi_x = 0$, reduces to $\phi_\pi > 1$

Implications – Determinacy



Notes: determinacy condition as risk aversion a increases.
The model is determinate if the solid dark line lies above the dotted light line (light shaded region) and is indeterminate otherwise (dark shaded region).

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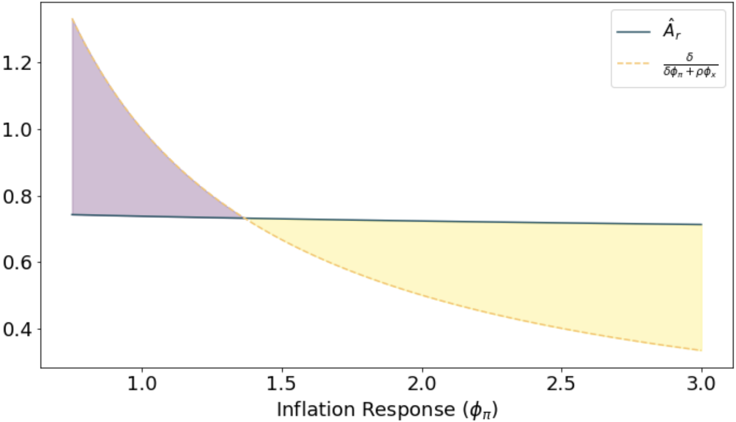

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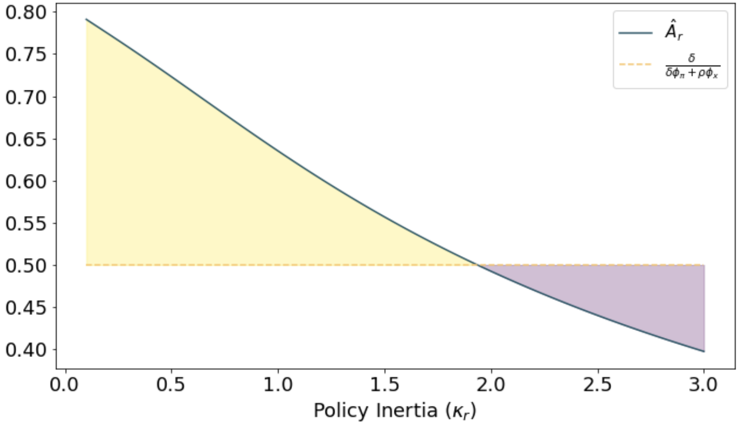

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Quantitative (Generalized) Model

- Sticky price model with shocks

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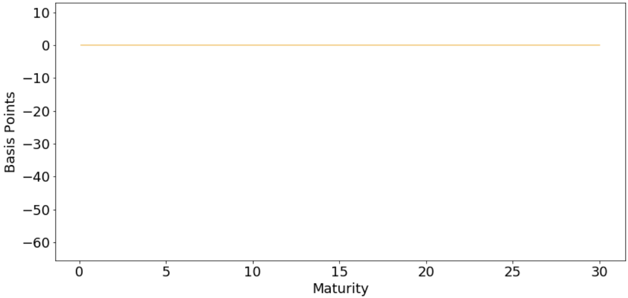
- Requires numerical solution methods

Yield Curve (QE, long end)



Notes: yield curve response to a "long" QE shock, for different levels of risk aversion *a*. Darker lines correspond to higher levels of risk aversion.

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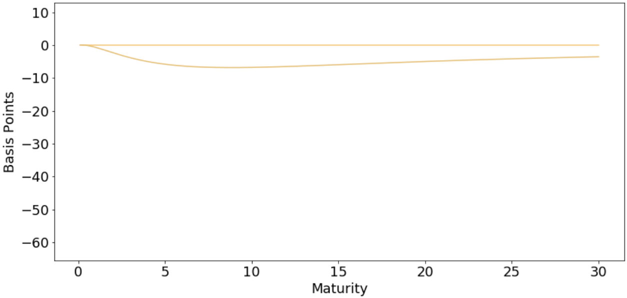

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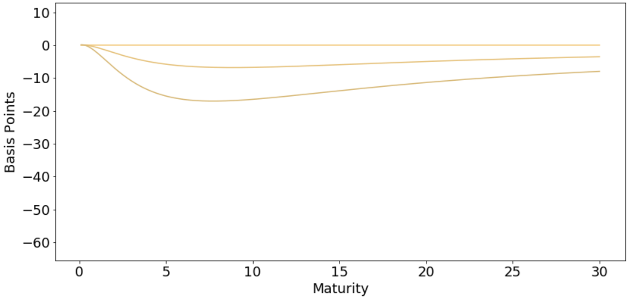

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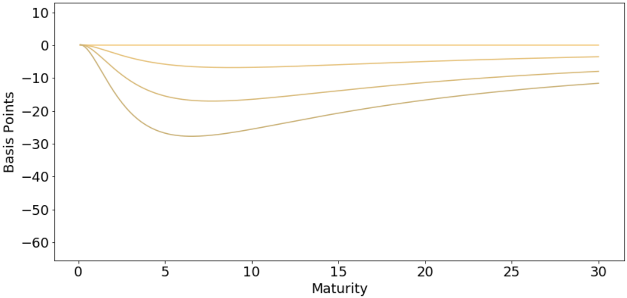

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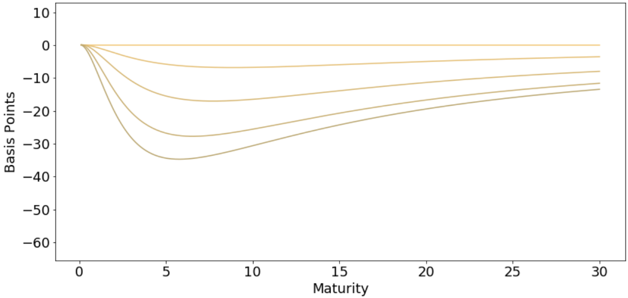

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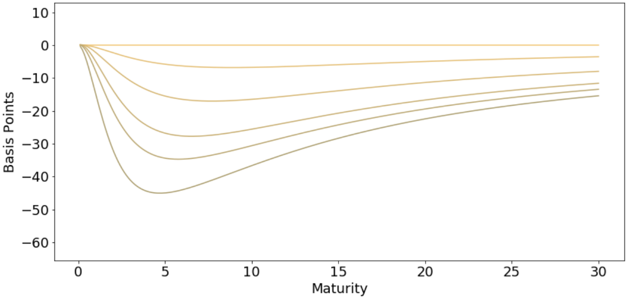

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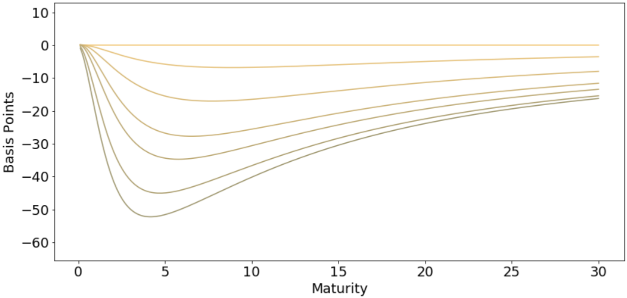

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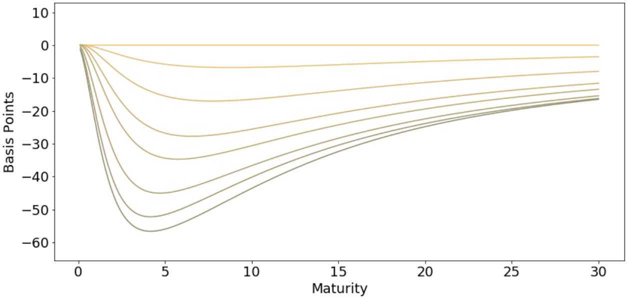

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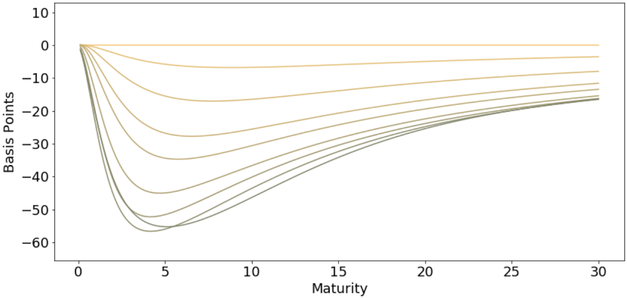

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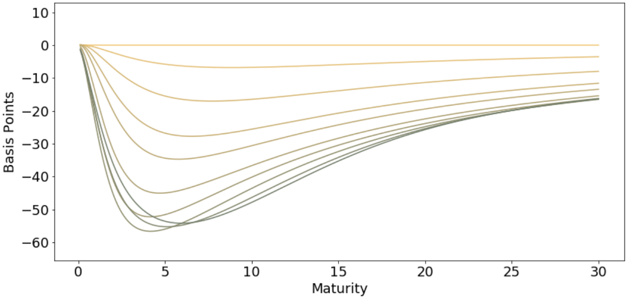

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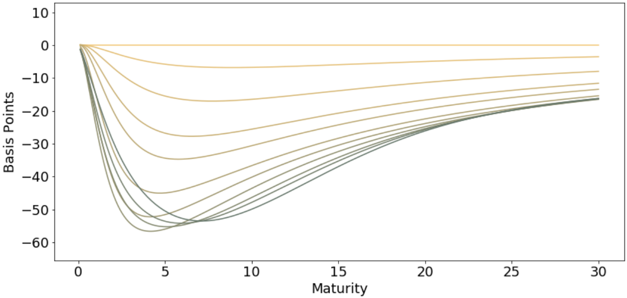

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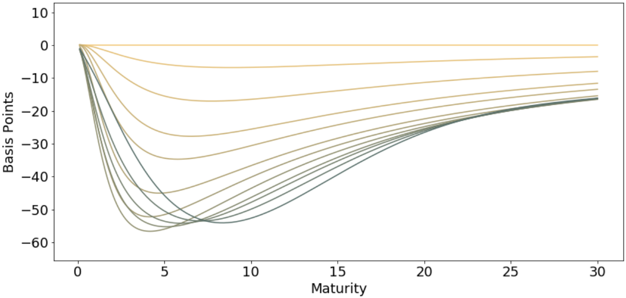

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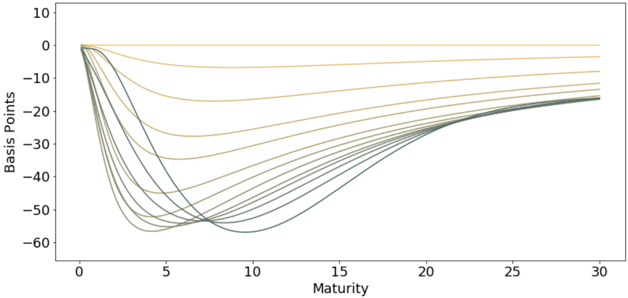

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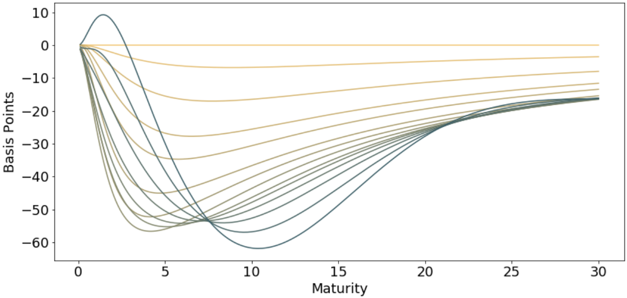

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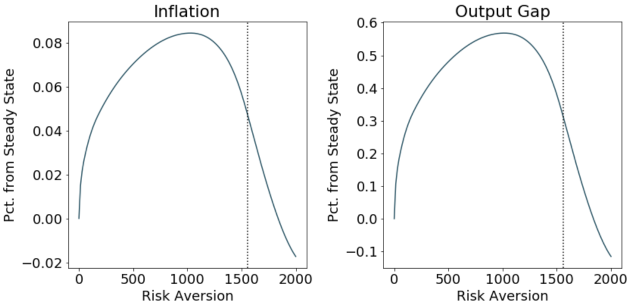
Aggregate Response (QE, long end)



Notes: inflation and output response to “long” QE shock on impact, for different levels of risk aversion a.

Stabilizing LSAPs

- Can LSAPs be used to ensure determinacy?
- Endogenous QE purchases:

$$\mathrm{d}\beta_t = -\kappa_\beta \left(\beta_t - \phi_\pi^\beta \pi_t \right) \mathrm{d}t$$

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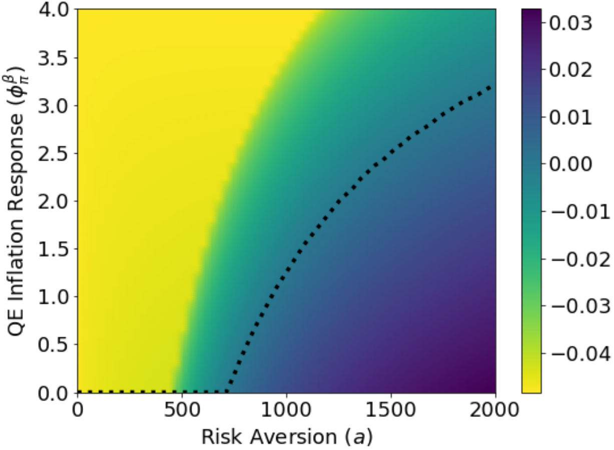
QE and Determinacy



Notes: determinacy conditions as a function of risk aversion (x-axis) and endogenous response of QE to inflation (y-axis). Darker colors correspond to larger values of the unstable eigenvalue. The dotted black line demarcates the region of determinacy.

Concluding Remarks

- Develops a unified, parsimonious framework to study conventional and unconventional monetary policies
- Transmission depends crucially on the risk-bearing capacity of financial markets

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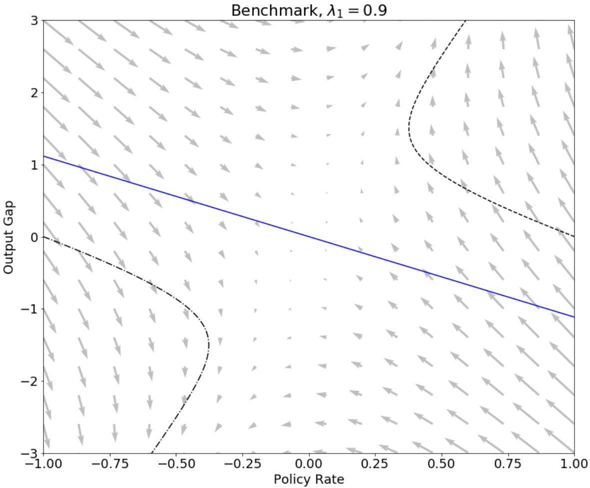
- Transmission depends crucially on the risk-bearing capacity of financial markets
- Future work:
  - Macroprudential policies, default risk
  - Monetary policy in open economies
  - Debt management

APPENDIX

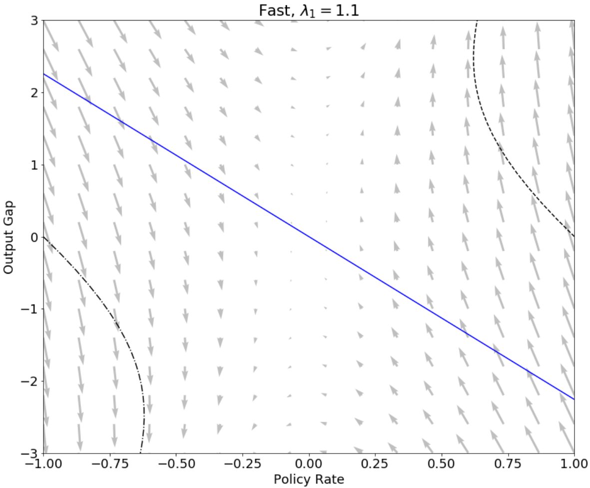
Phase Diagrams



Notes: phase diagrams of the dynamics of output and the policy rate as the equilibrium mean reversion rate of shocks varies.

back

Phase Diagrams



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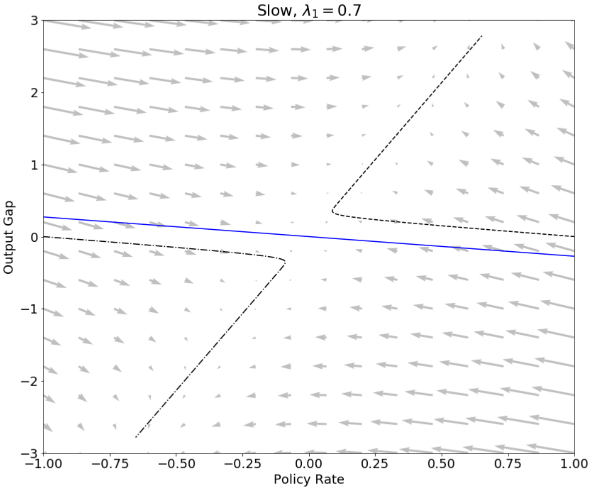

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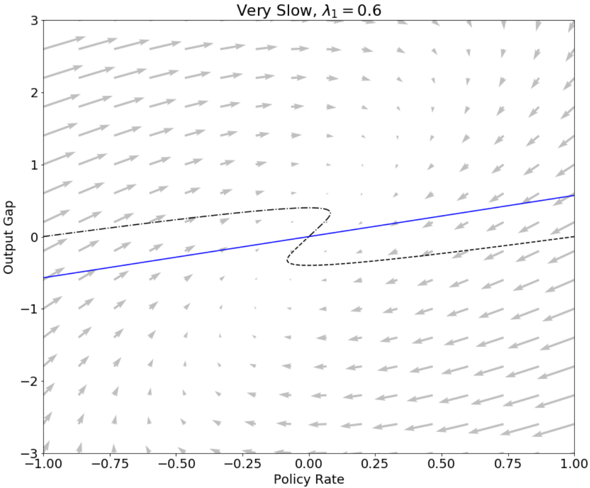

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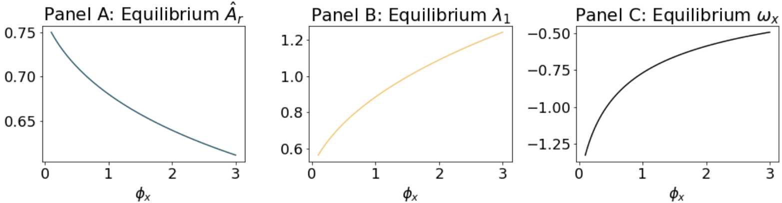
back

Implications – Conventional Policy



Notes: equilibrium changes in sensitivity to the short rate $\hat{A}_r$ and monetary shock reversion λ_1 as central bank response to output ϕ_x increases.

back

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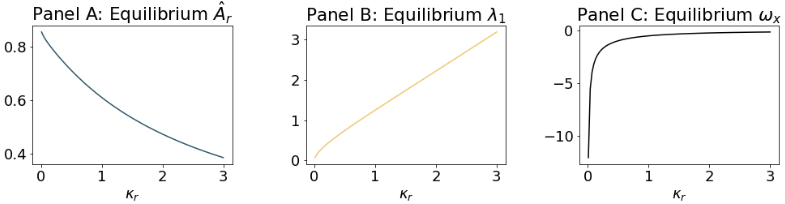
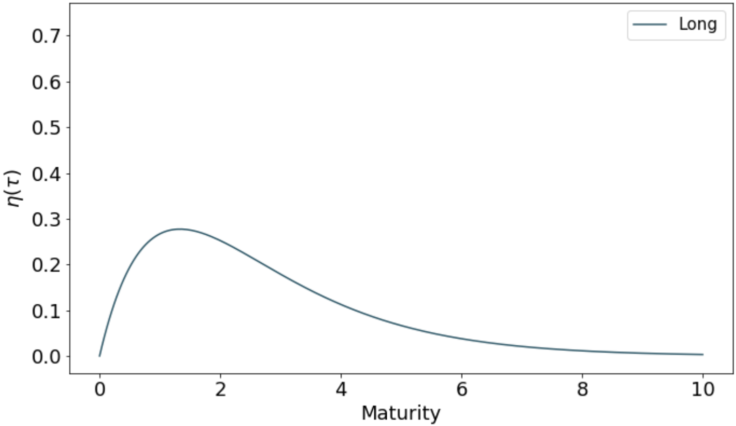


Notes: equilibrium changes in sensitivity to the short rate $\hat{A}_r$ and monetary shock reversion λ_1 as central bank inertia κ_r increases.

back

Sensitivity to Long Rates



Notes: different weighting function $\eta(\tau)$ in the determination of the effective borrowing rate $\tilde{r}_t$.

back

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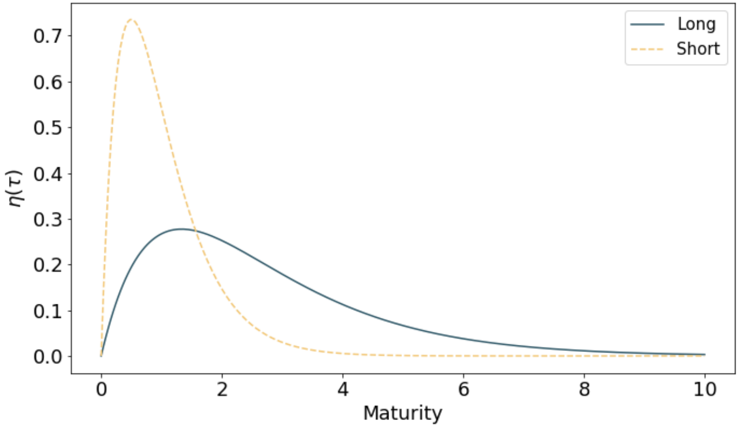

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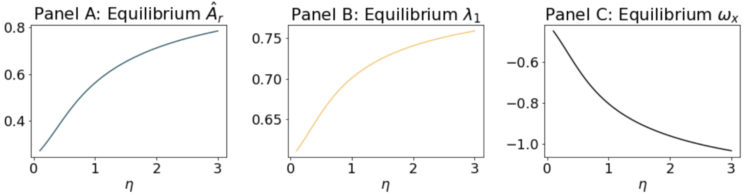


Notes: equilibrium changes in sensitivity to the short rate $\hat{A}_r$ and monetary shock reversion λ_1 as the weighting function $\eta(\tau)$ shifts towards short-term bonds.

back

Forward Guidance

- Central bank announces a peg: $r_0 = r^\diamond$ and

$$\mathrm{d}r_t = \begin{cases} -\kappa_r^\diamond (r_t - r^\diamond)\,\mathrm{d}t + \sigma_r^\diamond\,\mathrm{d}B_{r,t} & \text{if } 0 < t < t^\diamond \\ -\kappa_r (r_t - \phi_x x_t - r^*)\,\mathrm{d}t + \sigma_r\,\mathrm{d}B_{r,t} & \text{if } t \geq t^\diamond \end{cases}$$

- Affine coefficient functions during peg:

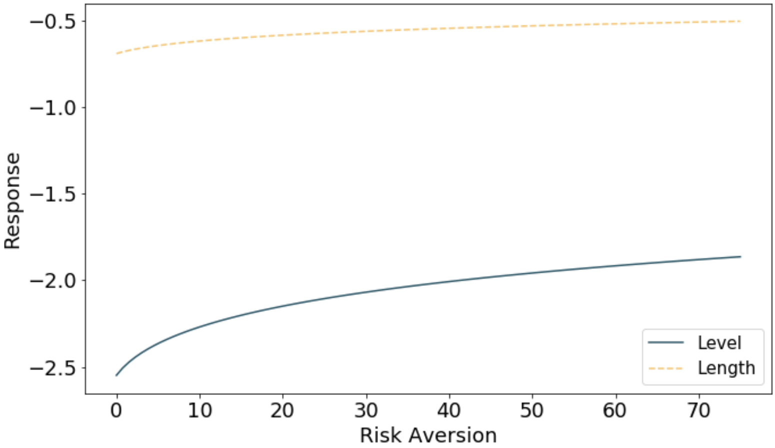
$$-\log P_{t,\tau} = A_r^\diamond(\tau) r_t + C^\diamond(\tau)$$
$$\implies \tilde{r}_t = \hat{A}_r^\diamond r_t + \hat{C}^\diamond$$

- Rational expectations dynamics for output:

$$\frac{\partial x_0}{\partial r^\diamond} = \omega_x - t^\diamond \varsigma^{-1} \hat{A}_r^\diamond\,, \quad \frac{\partial^2 x_0}{\partial r^\diamond \partial t^\diamond} = -\varsigma^{-1} \hat{A}_r^\diamond$$

back

Response to Forward Guidance



Notes: plots of $\frac{\partial x_0}{\partial r^\diamond}$ ("level") and $\frac{\partial^2 x_0}{\partial r^\diamond \partial t^\diamond}$ ("length") as risk aversion increases.

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Long-Run Variance

- State-space representation

$$\mathrm{d}\mathbf{y}_t = -\Gamma\left(\mathbf{y}_t - \mathbf{y}^{SS}\right)\mathrm{d}t + \mathbf{S}\,\mathrm{d}\mathbf{B}_t\,,\quad \mathbf{x}_t = \Omega\left(\mathbf{y}_t - \mathbf{y}^{SS}\right)$$

- Conditional distribution $\mathbf{y}_t|\mathbf{y}_0 \sim N\left(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t\right)$ where

$$\boldsymbol{\mu}_t = \mathbf{y}^{SS} + e^{-\Gamma t}(\mathbf{y}_0 - \mathbf{y}^{SS}),\quad \boldsymbol{\Sigma}_t = \int_0^t e^{\Gamma(u-t)}\boldsymbol{\Sigma} e^{\Gamma^T(u-t)}\,\mathrm{d}u$$

- Present-discounted value

$$\widetilde{\boldsymbol{\Sigma}}_\infty \equiv \int_0^\infty e^{-\rho t}\boldsymbol{\Sigma}_t\,\mathrm{d}t$$

$$\Longrightarrow\ \operatorname{vec}\widetilde{\boldsymbol{\Sigma}}_\infty = (\Gamma \oplus \Gamma)^{-1}(\rho\mathbf{I} + \Gamma \oplus \Gamma)^{-1}\operatorname{vec}\boldsymbol{\Sigma}$$

- Jump variables

$$\widetilde{\boldsymbol{\Sigma}}^{\mathbf{x}}_\infty = \Omega\widetilde{\boldsymbol{\Sigma}}_\infty\Omega^T$$

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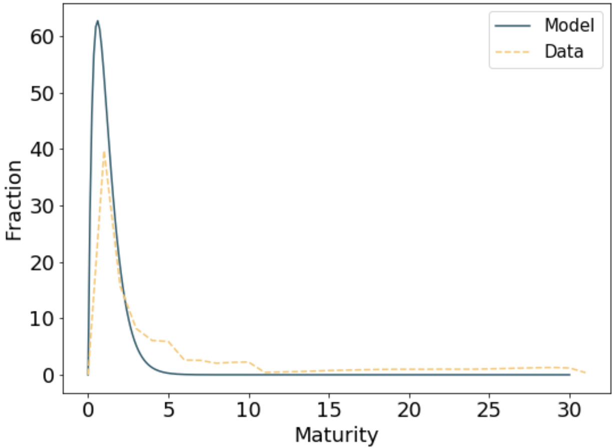
Effective Borrowing Rate Weights



Notes: average maturity distribution of outstanding Treasury debt (light dotted line). The dark line corresponds to the effective borrowing rate weights in the model. Source: FRED.

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