

Optimal Unconventional Policy in a New Keynesian Preferred Habitat Model

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October 2025

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 - Obvious: reduce long-term yields
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Bernanke: “**QE works (??)** in practice but not in theory”

Our Model

- **This paper**: develops a tractable general equilibrium model with market segmentation, nominal frictions, financial frictions, and household heterogeneity
- **Preferred habitat** tradition: assets traded by specialized investors
 - Pension funds hold long-maturity bonds
 - Money market funds hold short-maturity bonds

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- **Preferred habitat** tradition: assets traded by specialized investors
 - Pension funds hold long-maturity bonds
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- Our model: **households and firms** have differentiated access to asset markets
 - Households borrow with assets of different maturities (eg pension funds, mortgages)
 - Firms face working capital constraint
 - Introduces imperfect risk-sharing, **consumption and saving dispersion** across households
- **Arbitrageurs** (eg hedge funds, broker-dealers) with imperfect risk-bearing capacity intermediate bond markets

Findings: Policy Transmission

- Key mechanisms of conventional monetary policy:
 - Changes in the short rate affect required rates of return of all assets, but imperfect transmission to household borrowing rates
- Key mechanisms of balance sheet policy:
 - Imperfect arbitrage breaks QE neutrality: induces portfolio rebalancing and hence reduces term premia

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- Key mechanisms of balance sheet policy:
 - Imperfect arbitrage breaks QE neutrality: induces portfolio rebalancing and hence reduces term premia
- Thus, short rate and balance sheet policies are substitutes when targeting aggregate borrowing rates
 - A fall in aggregate borrowing rates is stimulative for the usual NK reasons

Findings: Welfare Consequences

- If the policymaker only cares about **macroeconomic stabilization**, conventional and unconventional policies are essentially equivalent
 - **Nominal rigidities** \implies welfare losses due to inflation and output gap volatility
 - **Triumphalist view**: even with short rate constraints, QE is equally effective

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- However, **imperfect risk sharing** \implies welfare losses from excess fluctuations in risk premia
- **Triple mandate**: social welfare depends on volatility of output, inflation, and long-term rates

“Promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates.”

Findings: Optimal Policy

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 - **Balance sheet constraints**: short rate less reactive to minimize bond disruptions
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- General message: **implementation matters** for welfare

Related Literature

- Preferred habitat models
 - Vayanos & Vila (2009, 2021), Ray, Droste, & Gorodnichenko (2024), Greenwood & Vayanos (2014), Hamilton & Wu (2012), Greenwood et al (2016), King (2019, 2021) , Kekre, Lenel, & Mainardi (2024), ...
- Empirical evidence: QE and preferred habitat
 - Krishnamurthy & Vissing-Jorgensen (2012), Hamilton and Wu (2012), D'Amico & King (2013), Li & Wei (2013), Cahill et al. (2013), King (2019), Fieldhouse et al (2018), Di Maggio et al (2020), Gali, Debortoli, & Gambetti (2020) , ...
- Macroeconomic QE models
 - Andres, Lopez-Salido, & Nelson (2004), Gertler & Karadi (2011), Curdia & Woodford (2011), Chen et al (2012), Gertler & Karadi (2013), Sims & Wu (2020), Karadi & Nakov (2020), Iovino and Sergeyev (2023), Carlstrom et al (2017), Ippolito et al (2018), Dordal & Lee (2024) , ...
- Market segmentation, macro-prudential monetary policy
 - Cui & Sterk (2021), Auclert (2016), Collard et al (2017), Kaplan et al (2017), Debortoli & Gali (2017) , Campbell & Nemtyrev (2025) ...
- International
 - Itskhoki & Mukhin (2023), Greenwood et al (2023), Gourinchas, Ray, & Vayanos (2025) , ...

Model Setup


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- Continuous time New Keynesian model with embedded Vayanos-Vila **bond markets:**
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 -  Imperfect risk-bearing capacity

Aggregate Risk Factors and Risk-Bearing Capacity

- Aggregate technology shock to firm production

$$Z_t = Ze^{z_t}, \quad dz_t = -\kappa_z z_t dt + \sigma dB_t$$

- More generally (in paper): $N_z \times 1$ vector z_t exogenous risk factors where $\text{Var}_t dz_t = \text{Var}_t [\sigma dB_t] = \sigma \sigma^\top dt$ (cost-push, portfolio rebalancing, firm financing, ...)
- Arbitrageur optimally chooses portfolio $\{X_t(\tau)\}$ given risk aversion a :

$$E_t \frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - i_t dt = a \cdot \int_0^T X_t(\tau') \text{Cov}_t \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}, \frac{dP_t^{(\tau')}}{P_t^{(\tau')}} \right) d\tau'$$

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-  Limits to arbitrage: parameterize $\sigma(\xi), a(\xi)$ such that

$$\lim_{\xi \rightarrow 0} \sigma(\xi) \rightarrow 0, \quad \lim_{\xi \rightarrow 0} a(\xi) \sigma(\xi) \rightarrow \hat{a} \hat{\sigma}$$

First-Best Allocation

Proposition (First-best allocation)

Consider the limiting riskless case ($\xi \rightarrow 0$).

- With perfect arbitrage ($\hat{a} = 0$), the model admits a representative agent representation.
- Given an optimal production subsidy, the first-best allocation is obtained under flexible prices.

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- *Corollary:* steady state is efficient with zero inflation and zero arbitrageur positions
- Perturbation around “low risk, low risk-bearing capacity” point [details](#)

Equilibrium Aggregate Dynamics I

- **Bond returns:** $dP_t^{(\tau)} / P_t^{(\tau)} = \mu_t(\tau) dt + \sigma(\tau) dB_t$
- **Firm** production and marginal costs (symmetric equilibrium):

$$y_t = z_t + n_t, \quad m_t = (1 + \beta\varrho)(w_t - z_t) + \hat{\mu}_t, \quad \beta = \int_0^T \beta(\tau) d\tau$$

- **Household i** optimality conditions:

$$\begin{aligned} E_t dc_t(i) &= \varsigma^{-1} [\tilde{\mu}_t(i) - \pi_t] dt, \quad w_t = \varsigma c_t(i) + \varphi n_t(i) \\ \implies E_t dc_t &= \varsigma^{-1} [\tilde{\mu}_t - \pi_t] dt, \quad w_t = \varsigma c_t + \varphi n_t \end{aligned}$$

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-  “effective” rates:

$$\tilde{\mu}_t(i) \equiv \int_0^T \eta_i(\tau) \mu_t(\tau) d\tau, \quad \tilde{\mu}_t = \int_0^1 \tilde{\mu}_t(i) di, \quad \hat{\mu}_t = \int_0^T \beta(\tau) \mu_t(\tau) d\tau$$

Equilibrium Aggregate Dynamics II

- Output gap $x_t \equiv y_t - y_t^n = y_t - \frac{1+\varphi}{\varsigma+\varphi} z_t$ and natural rate $v_t \equiv -\varsigma \kappa_z \frac{1+\varphi}{\varsigma+\varphi} z_t$
- \implies modified NK equations [cf. Ray, Droste, & Gorodnichenko 2024]:

$$E_t dx_t = \varsigma^{-1} [\tilde{\mu}_t - \pi_t - v_t] dt$$


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-  In general, $\mu_t(\tau) = i_t + \lambda_t(\tau)$ where $\lambda_t(\tau) \neq 0$. Arbitrageur optimality conditions:


$$\lambda_t(\tau) = \hat{a} \sigma(\tau) \Lambda_t, \quad \Lambda_t = \int_0^T x_t(\tau) \sigma(\tau) d\tau$$

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- Arbitrageur positions $x_t(\tau)$ pinned down by market clearing conditions:
 - Firm borrowing: function of wage w_t , labor supply n_t , portfolio weights $\beta(\tau)$
 - Household bond holdings: function of wealth $b_t(i)$, portfolio weights $\eta_i(\tau)$
 - Central bank holdings $qe_t(\tau)$

Equilibrium Risk Prices I

- Assume **steady state household wealth** $B = 0$
 - More generally (in paper): $B \neq 0$ allows for more complicated wealth effects
 - \implies arbitrageur holding $x_t(\tau)$ feature own- and cross-price elasticities wrt $p_t(\tau)$ and $\int_0^T \int_0^1 \eta_i(\tau) p_t^{(\tau')} di d\tau'$

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- **Household i wealth** dynamics:

$$db_t(i) = \left[\underbrace{\varrho b_t(i) + B \tilde{\mu}_t(i)}_{\text{relative portfolio returns}} + \underbrace{B \varrho \check{p}_t(i)}_{\text{relative wealth effects}} - \underbrace{(1 + \varsigma/\varphi) \check{c}_t(i)}_{\text{relative consumption/income}} \right] dt$$

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- \implies **Arbitrageur market clearing:**

$$\underbrace{x_t(\tau) + qe_t(\tau)}_{\text{holdings net of QE}} = \underbrace{\beta(\tau) \left((1 + \varsigma + \varphi)x_t - \frac{1}{\varsigma \kappa_z} v_t \right)}_{\text{firm borrowing}} - \underbrace{\int_0^1 \eta_i(\tau) \left(b_t(i) + B \left[\tilde{p}_t(i) - p_t^{(\tau)} \right] \right) di}_{\text{HH savings}}$$

Equilibrium Risk Prices II

- Risk prices Λ_t depend on risk-weighted objects: endogenous volatility $\sigma(\tau)$ of bonds

$$\Lambda_t = \int_0^T x_t(\tau) \sigma(\tau) d\tau \equiv x_t^\sigma = -qe_t^\sigma + \beta^\sigma \left((1 + \varsigma + \varphi)x_t - \frac{1}{\varsigma \kappa_z} v_t \right) - b_t^\sigma$$

- Risk-weighted aggregate wealth $b_t^\sigma \equiv \int_0^1 \int_0^T \sigma(\tau) \eta_i(\tau) b_t(i) d\tau di, \dots$
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- **Heterogeneity matters** for first-order dynamics: **risk-weighted market clearing**

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- Final line: losses from policy frictions (when $\psi^i > 0, \psi(\tau) > 0$)

- Per-period social welfare loss (second-order expansion relative to first-best):

$$\begin{aligned}\mathcal{L}_t \equiv & (\varsigma + \varphi)x_t^2 + \vartheta\pi_t^2 \\ & + \frac{\varsigma}{\varphi}(\varphi + \varsigma)\text{Var}_i \check{c}_t(i) \\ & + \int_0^T \psi(\tau)qe_t(\tau)^2 d\tau + \psi^i \left(i_t - \bar{i}_t\right)^2\end{aligned}$$

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Key takeaway: policy attempts to undo frictions:

1. Nominal rigidities \implies **pricing inefficiencies**
2. Firm financing friction \implies **production inefficiencies**
3. Household market segmentation \implies **imperfect risk-sharing**

Benchmark I: Risk Neutral Arbitrageur

- Consider the benchmark case of a risk neutral arbitrageur: $\hat{a} = 0$
- The [expectations hypothesis](#) holds: $\mu_t(\tau) = i_t$

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- Recover the standard **QE neutrality result**: balance sheet policies do not affect bond prices (and therefore have no aggregate effects)
- **‘Woodford-ian’ equivalence**: the role of policy on aggregate dynamics and welfare is fully summarized by policy rate i_t


Benchmark II: Representative Agent Representation

- Even with imperfect arbitrage ($\hat{a} > 0$), consider **special case**:
 1. Zero wealth steady state ($B = 0$) and initially equal wealth distribution ($b_t(i) = 0$)
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 - **No wealth dynamics** ($b_{t+k}(i) = 0$)
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- \implies EH holds; RANK representation; conventional policy achieves first best
-  **QE non-neutrality**: balance sheet policies affect arbitrageur positions
 - \implies QE affects bond prices and aggregates
 - \implies **induces heterogeneity across households**
- \approx 'Woodford-ian' equivalence but QE \neq short rate policy

Dynamics: Analytical Results

Simplified Aggregate Dynamics: Rigid Prices

- **Simplifications:** rigid prices
- Along with the dynamics of natural rate shocks, we have

$$db_t^\sigma = [\varrho b_t^\sigma - (1 + \varsigma/\varphi) \check{c}_t^\sigma] dt$$

$$E_t dx_t = \varsigma^{-1} [i_t + \eta^\sigma \Lambda_t - v_t] dt$$

$$E_t d\check{c}_t^\sigma = \hat{a}\varsigma^{-1} \check{\Sigma} \Lambda_t dt$$

$$\Lambda_t = -qe_t^\sigma + \beta^\sigma \left((1 + \varsigma + \varphi)x_t - \frac{1}{\varsigma\kappa_Z} v_t \right) - b_t^\sigma$$

- Ad-hoc **Taylor policy rules** close the model

$$i_t = \phi_x x_t + \epsilon_{i,t}, \quad qe_t(\tau) = \phi_x(\tau) x_t + \epsilon_{q,t}(\tau)$$

- Paper: existence and uniqueness, solution algorithm
- Simple linear REE model, except endogenous coefficients $\eta^\sigma, \beta^\sigma, \check{\Sigma}$ due to endogenous volatility $\sigma(\tau)$ when $\hat{a} > 0$

Simplified Aggregate Dynamics: Rigid Prices

Proposition (Rigid price dynamics, general case)

Assume $\hat{a} > 0$, $\beta(\tau) > 0$, and $0 < \phi_x < \bar{\phi}_x$ for some upper bound $\bar{\phi}_x$.

- Following a natural rate shock:

$$\frac{\partial x_t}{\partial v_t} > 0, \quad \frac{\partial \Lambda_t}{\partial v_t} > 0, \quad \text{Cov}(i_t, \Lambda_t) > 0, \quad \exists k > 0 : \frac{\partial x_{t+k}}{\partial v_t} > 0, \quad \frac{\partial \Lambda_{t+k}}{\partial v_t} < 0$$

- The reaction of risk prices Λ_t is stronger if $\phi_x(\tau) > 0$
- Following a conventional monetary policy shock: $\frac{\partial x_t}{\partial v_t} < 0$, $\frac{\partial \Lambda_t}{\partial \epsilon_{i,t}} < 0$

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- **Intuition:**

- Recession $\implies \downarrow i_t, \downarrow$ firm borrowing on impact, \searrow HH saving over time
- Arbitrageur rebalancing $\implies \downarrow$ term premia on impact, \nearrow over time
- Contraction policy shock $\implies \uparrow i_t, \downarrow$ firm rebalancing

Empirical Evidence

Stylized Model Predictions:

1. **Unconditionally**, increases in short rates associated with contemporaneous increases in term premia
2. Larger unconditional reactions **during QE periods**
3. Over **longer horizons**, unconditional reaction of term premia to short rates weakens or becomes negative
4. **Conditional** reaction of term premia to monetary policy *shocks* are small or negative

Model Predictions and Evidence

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Empirical Specification:

- Utilize movements in **long forward rates** (Gurkaynak et al 2005, Hanson et al 2021)

$$f_{t+h}^{(\tau)} - f_{t-1}^{(\tau)} = \alpha(\tau) + \beta(\tau)D_t + \epsilon_t(\tau)$$

- **Unconditional vs conditional** regressions: D_t are daily change in short-term yields (Gurkayank et al 2007) vs high-frequency MP shocks (Nakamura and Steinsson 2018)

Empirical Results: Unconditional, Varying Maturities

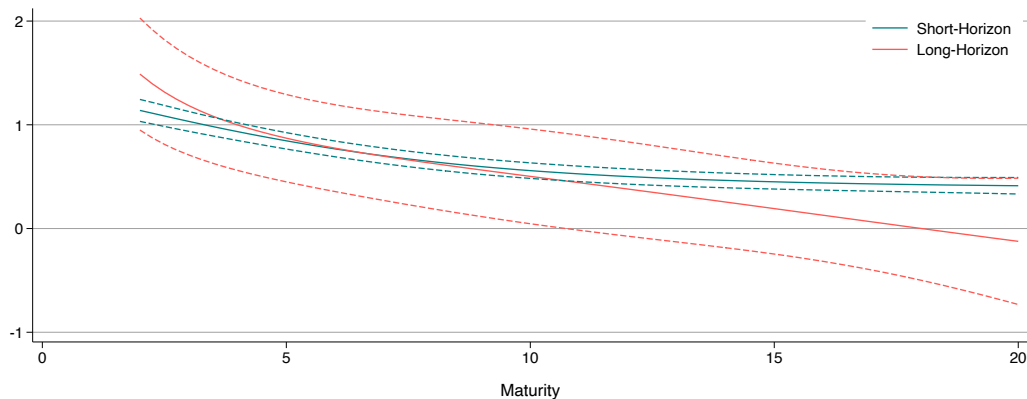


Figure 1: Forward Rates (Unconditional, Varying Maturities)

Full sample (1982-2020), $h = 0$ and $h = 90$, $\tau = 2, \dots, 20$

Empirical Results: Unconditional, Varying Horizon

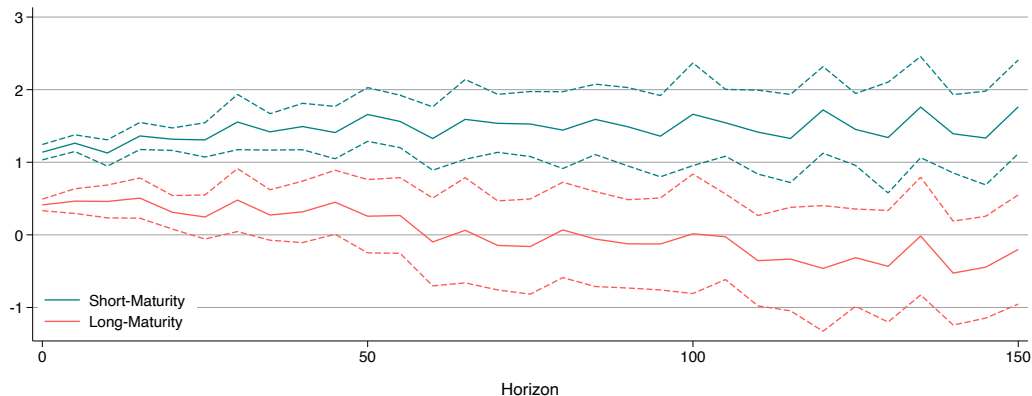


Figure 2: Forward Rates (Unconditional, Varying Horizon)

Full sample (1982-2020), $h = 0 \dots 150$, $\tau = 2$ and $\tau = 20$

Empirical Results: Unconditional, Rolling Short Horizon

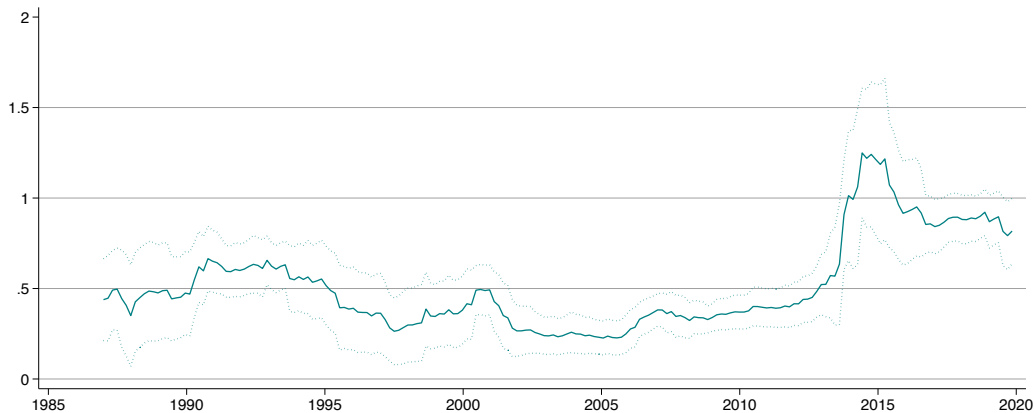


Figure 3: Forward Rates (Unconditional, Rolling Short Horizon)

Rolling window (5 year), $h = 0$, $\tau = 20$

Empirical Results: Unconditional, Rolling Long Horizon

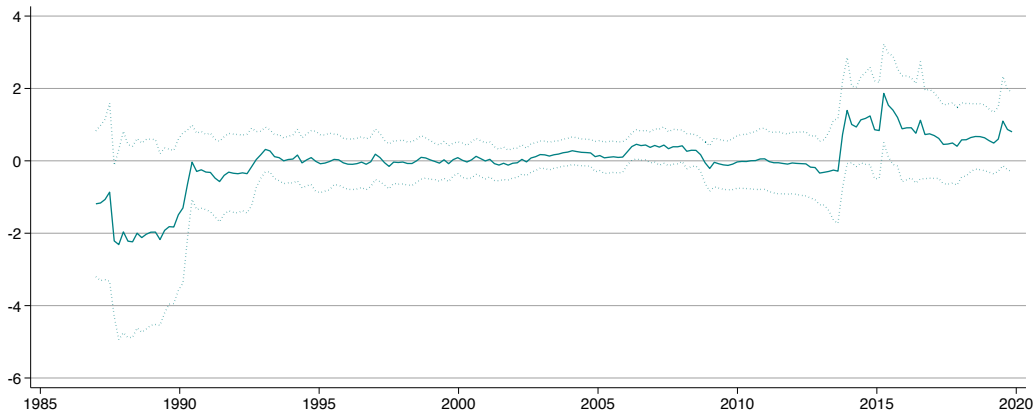


Figure 4: Forward Rates (Unconditional, Rolling Long Horizon)

Rolling window (5 year), $h = 90$, $\tau = 20$

Empirical Results: Conditional, Varying Maturity

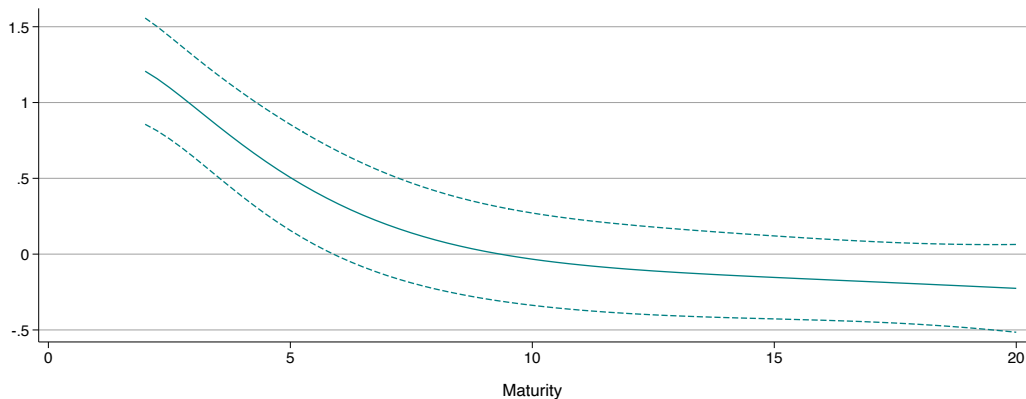


Figure 5: Forward Rates (Shocks, Varying Maturity)

Full sample (FOMC meetings 1995-2020), $h = 0$, $\tau = 2, \dots, 20$

Empirical Results: Conditional, Rolling

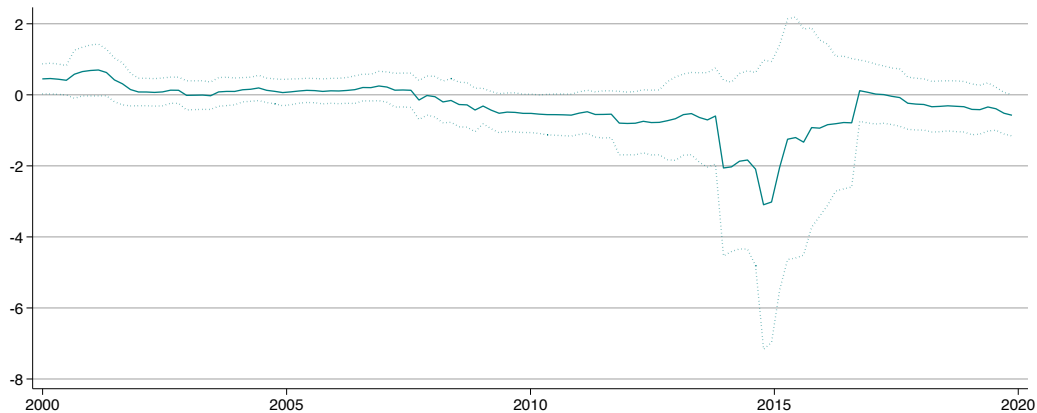


Figure 6: Forward Rates (Shocks, Rolling)

Rolling window (5 year), $h = 0$, $\tau = 20$

Welfare

Welfare Consequences: Simple Policy Rules

- For simplicity, continue assuming rigid prices
- Consider **policy rules** which implement

$$i_t = \chi_{i,v} v_t + \chi_{i,b} b_t^\sigma$$
$$qe_t(\tau) = \chi_{q,v}(\tau) v_t + \chi_{q,b}(\tau) b_t^\sigma$$

- **Simple policy rules**: function of natural state variables only
 - Time-consistent: policymaker seeks to minimize **unconditional** social welfare loss
- We will examine the outcome of these policies in different versions of the model
- **Risk-neutral benchmark**: perfect arbitrage ($\hat{a} = 0$) implies $\chi_{i,v} = 1$ is optimal

Optimal Policy: Short Rate Only

- First consider short rate tools only (formally, balance sheet frictions $\psi^{(\tau)} \rightarrow \infty$)

Proposition (Optimal short rate policy rule)

Assume risk aversion $\hat{a} > 0$ and $\beta(\tau) > 0$. If bond dispersion across households $\check{\Sigma} = 0$:

- $\exists \chi_{i,v}^n \leq 1$ along with $\chi_{i,b} = 0$ which guarantees $x_t = 0 \forall t$.
- Sign of $\chi_{i,v}^n - 1$ is determined by the endogenous reaction of firm borrowing to v_t .

With $\check{\Sigma} > 0$:

- Optimal short rate policy $i_t = \chi_{i,v}^* v_t + \chi_{b,i} b_t^\sigma$ where $\chi_{b,i} \neq 0$ and $\chi_{i,v}^* < \chi_{i,v}^n$.
- Implications
 1. Bond carry trade returns $\mu_t(\tau) - i_t$ move in the same direction as i_t iff firm borrowing declines in response to natural rate shocks.
 2. Output gaps x_t are not identically zero.
 3. Consumption dispersion is non-zero: $\text{Var}_i \check{c}_t(i) \neq 0$.

Optimal Short Rate Intuition

- Follows from intuition derived studying ad-hoc rules
- Consider recessionary shock $\downarrow v_t \implies \downarrow i_t$
 - If \downarrow firm borrowing, then arbitrageur rebalancing $\implies \downarrow \Lambda_t$
 - Vice-versa if \uparrow firm borrowing
 - In order to keep $\tilde{\mu}_t = v_t$, policy must be react less/more strongly than RANK benchmark (depending on firm borrowing reaction)

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- If policy is such that $\tilde{\mu}_t = v_t$, fluctuations in risk prices Λ_t imply $\exists \mu_t(\tau) \neq v_t$
- Fluctuations in borrowing rates across the term structure imply $\text{Var}_i \check{c}_t(i) > 0$

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- Fluctuations in borrowing rates across the term structure imply $\text{Var}_i \check{c}_t(i) > 0$
- All else equal:
 - **Reducing policy rate volatility** \implies term premia volatility \downarrow
 - **Reducing policy rate response** to shocks \implies macro volatility \uparrow
- Optimal policy balances these objectives

Optimal Policy: Unconstrained Case

- With access to frictionless [balance sheet policies](#), we obtain the following

Proposition (Optimal policy separation principle)

Assume risk aversion $\hat{a} > 0$ and $\beta(\tau) > 0$. Suppose the central bank implements short rate and balance sheet policy according to

$$i_t = v_t, \quad \int_0^T \sigma(\tau) q e_t(\tau) d\tau = -\frac{\beta^\sigma}{\varsigma \kappa_z} v_t.$$

If short rate policy is frictionless ($\psi^i = 0$) and the central bank does not face holding costs ($\psi^{(\tau)} = 0$), then first-best is achieved:

1. Macroeconomic stabilization: $x_t = 0 \ \forall t$.
2. Term premia stabilization: $\mu_t(\tau) = \tilde{\mu}_t \ \forall \tau$.
3. Consumption equalization: $c_t(i) = c_t(i') \ \forall i, i'$.

Optimal Policy Separation Principle: Intuition

- The results follow naturally from our findings regarding ad-hoc policy
- QE policy **stabilizes shocks to bond markets** by offsetting all firm borrowing movements
- Implies net zero arbitrageur positions so

$$\int_0^T \sigma(\tau) x_t(\tau) d\tau = 0 \implies \Lambda_t = 0$$

- This **equalizes borrowing rates** across HHs: $\mu_t(\tau) = \tilde{\mu}_t$
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Separation principle for optimal policy:

- Optimal balance sheet policy **stabilizes bond markets**
- Optimal short rate policy **stabilizes macroeconomic aggregates**

Optimal Policy with Constraints

- Even with “large” [balance sheet constraints](#) the central bank still uses QE to (partially) stabilize term premia [details](#)
 - Even if “worse arbitrageur” central bank internalizes welfare costs of risk price fluctuations

Optimal Policy with Constraints

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- With **short rate constraints**, balance sheet tools are **capable** of stabilizing output or term premia, but not both details
 - QE works by affecting **term premia** through changes in the **market price of risk**
 - Although arbitrage is imperfect in this model, arbitrageurs still enforce **tight restrictions** between between market price of risk and term premia across maturities
 - Hence, while in principle the central bank has a **continuum of policy tools** $\{qe_t(\tau)\}_{\tau=0}^T$, can **only manipulate risk price** Λ_t
 - Related to **localization results** (Vayanos & Vila 2021, Ray, Droste, & Gorodnichenko 2024)

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- Other extensions (sticky prices, cost-push shocks, noise demand, nonzero first-best term premia): [details](#)

History-Dependent Policy

Monetary Policy with Commitment

- When policy frictions bind, simple policy rules fail to achieve first-best
- Instead suppose the policymaker chooses policy tools \mathbf{u}_t as a function of **entire history** of predetermined and nonpredetermined variables $\mathbf{Y}_t \equiv \begin{bmatrix} \mathbf{y}_t^\top & \mathbf{x}_t^\top \end{bmatrix}^\top$
- Minimizes conditional social loss

$$\begin{aligned}\mathcal{W}_0 &= \frac{1}{2} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \mathcal{L}_t dt \\ &= \frac{1}{2} \mathbb{E}_0 \int_0^\infty e^{-\rho t} (\mathbf{Y}_t^\top \mathbf{R} \mathbf{Y}_t + \mathbf{u}_t^\top \mathbf{Q} \mathbf{u}_t) dt, \quad \mathbf{y}_0 \text{ given}\end{aligned}$$

- By setting policy in a history-dependent manner, full commitment policies can improve on simple policy rules

Characterizing Optimal Policy with Commitment (*work in progress!*)

Theorem (Optimal Policy with Commitment)

Given \mathbf{y}_0 , the policymaker minimizes \mathcal{W}_0 by choosing $\mathbf{u}_t = \mathbf{F}\mathbf{Y}_t$, which induce equilibrium dynamics $d\mathbf{Y}_t = -\mathbf{\Upsilon}(\mathbf{F})\mathbf{Y}_t dt + \mathbf{S}(\mathbf{F})d\mathbf{B}_t$. Necessary conditions are given by

$$\mathbf{y}_0^\top \left(\partial_i \mathbf{P}_{11} - \partial_i \mathbf{P}_{12} \mathbf{P}_{22}^{-1} \mathbf{P}_{21} - \mathbf{P}_{12} \mathbf{P}_{22}^{-1} \partial_i \mathbf{P}_{21} + \mathbf{P}_{12} \left(\mathbf{P}_{22}^{-1} \partial_i \mathbf{P}_{22} \mathbf{P}_{22}^{-1} \right) \mathbf{P}_{21} \right) \mathbf{y}_0 = 0$$

where $\rho \mathbf{P} = \mathbf{R} + \mathbf{F}^\top \mathbf{Q} \mathbf{F} - \mathbf{P} \mathbf{\Upsilon} - \mathbf{\Upsilon}^\top \mathbf{P}$. Dynamics are given by $\mathbf{q}_0 = \begin{bmatrix} \mathbf{y}_0 & \mathbf{0} \end{bmatrix}^\top$ and

$$d\mathbf{q}_t = - \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \mathbf{\Upsilon} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{P}_{22}^{-1} \mathbf{P}_{21} & \mathbf{P}_{22}^{-1} \end{bmatrix} \mathbf{q}_t dt + \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \mathbf{S} d\mathbf{B}_t \equiv -\mathbf{\Gamma} \mathbf{q}_t dt + \mathbf{\sigma} d\mathbf{B}_t$$

Bond prices are affine in $\mathbf{A}(\tau)^\top \mathbf{q}_t$ with $\mathbf{A}(\tau) = [\mathbf{I} - e^{-\mathbf{M}\tau}] \mathbf{M}^{-1} \mathbf{e}_i$ and

$$\mathbf{e}_i^\top \mathbf{q}_t = i_t, \quad \mathbf{M} = \mathbf{\Gamma}^\top - \int_0^T \mathbf{\Theta}(\tau) \mathbf{A}(\tau)^\top d\tau \tilde{\mathbf{\Sigma}}$$

Monetary Policy with Commitment: Intuition

- Policymaker chooses tools $i_t, \{qe_t(\tau)\}_{\tau=0}^T$ which:
 - Directly affect optimality conditions of arbitrageurs
 - Indirectly affect HHs through changes in equilibrium borrowing rates
 - Indirectly affect firms through changes in marginal costs

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 - Greater pass-through to HHs
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 - Arbitrageur risk compensation depends on volatility of short-run fluctuations $di_t, dqe_t(\tau)$

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- Characterizing dynamics of optimal policy with commitment is difficult
 - Ongoing work studies optimal policy numerically
 - Suffers from time inconsistency; simple rules may be more practical

Concluding Remarks

- This paper develops a tractable general equilibrium model with market segmentation, nominal frictions, financial frictions, and household heterogeneity
- We find that optimal short rate and balance sheet policy is characterized by a sharp **optimal separation result**:
 - Conventional policy targets **macroeconomic stability**
 - Unconventional policy targets **bond market stability**
- Optimal policy removes excess volatility of bond returns and hence improves risk-sharing across households, while reducing excess macroeconomic volatility
 - Policy constraints on either the short rate or balance sheets imply trade-offs between these policy objectives
- When considering social welfare, **cannot abstract from the policy tools** used to conduct monetary policy

Thank You!

Households

- Continuum of HH members $i \in [0, 1]$, differentiated by access to bond markets
 - Captures the observed differentiated HH portfolios (eg, due to demographics, market access via investment funds, mortgage market structure, etc)
 - Formalization: HHs [sluggishly rebalance](#) (our model is limiting case)
- HH i chooses consumption and labor $C_t(i), N_t(i)$ in order to solve

$$V_0(i) \equiv \max E_0 \int_0^\infty e^{-\rho t} \left(\frac{C_t(i)^{1-\varsigma}}{1-\varsigma} - \frac{N_t(i)^{1+\varphi}}{1+\varphi} \right) dt$$

$$\text{s.t. } d\mathcal{B}_t(i) = [\mathcal{W}_t N_t(i) - \mathcal{P}_t C_t(i)] dt + \mathcal{B}_t(i) d\tilde{R}_t(i) + d\mathcal{F}_t$$

- $\mathcal{B}_t(i)$ nominal savings earn [d \$\tilde{R}_t\(i\)\$](#)
- Taken as given (as well as nominal wage \mathcal{W}_t , price index \mathcal{P}_t , transfers $d\mathcal{F}_t$)

[Key takeaway](#): consumption/savings choices differ when bond returns not equalized [back](#)

Firms

- Continuum of intermediate goods $j \in [0, 1]$ (and CES final good with elasticity ϵ)
- Produce using labor $Y_t(j) = Z_t L_t(j)$
- Revenue and costs of production:

$$d\Pi_t(j) = [(1 + \tau^y) \mathcal{P}_t(j) Y_t(j) - \mathcal{W}_t L_t(j) - \mathcal{T}_t^y] dt - d\Theta_t(j)$$

$$d\Theta_t(j) = \frac{\vartheta}{2} (\pi_t(j) - \varpi_t)^2 \mathcal{P}_t Y_t dt + \mathcal{W}_t L_t(j) d\hat{R}_t$$

- **Rotemberg costs** when setting prices $d\mathcal{P}_t(j) = \mathcal{P}_t(j) \pi_t(j) dt$ (away from target ϖ_t)
- **Working capital** friction: finance a fraction $\int_0^T \beta_t(\tau) d\tau$ of wage bill
- Taking as given CES demand, τ^y subsidy, taxes \mathcal{T}_t^y , SDF $Q_t^{\mathcal{H}}$, firm j solves:

$$U_0(j) \equiv \max E_0 \int_0^\infty e^{-\rho t} Q_t^{\mathcal{H}} d\Pi_t(j)$$

Key takeaway: inefficiencies due to pricing frictions, financing friction [back](#)

Arbitrageurs

- Mean-variance optimization

$$\begin{aligned} & \max E_t d\mathcal{X}_t - \frac{a_t}{2} \text{Var}_t d\mathcal{X}_t \\ \text{s.t. } & d\mathcal{X}_t = \mathcal{X}_t i_t dt + \int_0^T \mathcal{X}_t(\tau) \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - i_t dt \right) d\tau \end{aligned}$$

- Arbitrageurs invest $\mathcal{X}_t(\tau)$ in bond carry trade of maturity τ
- Risk-return trade-off governed by a_t
 - Formally: risk aversion coefficient
 - More generally: proxies for any limits to risk-bearing capacity or intermediation frictions
 - Arbitrageurs transfer gains/losses to HHS, so a_t represents any frictions which hinder ability to trade on behalf of HHS

Key takeaway: risk averse arbitrageurs' holdings increase with expected return

[back](#)

Central Bank and Fiscal Authority

- Central bank sets policy rate i_t and buys/sells bonds $\mathcal{QE}_t(\tau)$
- Both policy actions potentially subject to costs:

$$Y_t \Psi_t^S \equiv Y_t \int_0^T \frac{\psi(\tau)}{2} (\mathcal{QE}_t(\tau))^2 d\tau$$

$$Y_t \Psi_t^i \equiv Y_t \frac{\psi^i}{2} (i_t - \bar{i}_t)^2$$

- In the background: fiscal authority chooses subsidies τ^y
- Fiscal authority also supplies bonds: $\mathcal{S}_t^{(\tau)}$ total supply net of QE holdings
- Financed lump-sum via households
- Optimal policy: maximize social welfare

$$\max E_0 \int_0^\infty e^{-\rho t} \left(\int_0^T \eta(\tau) u(C_t(\tau), N_t(\tau)) d\tau \right) dt$$

- $\eta(\tau)$: fraction of HHs with access to τ bonds (so $\int_0^T \eta(\tau) d\tau = 1$)

Aggregation and Market Clearing

- Firms, arbitrageurs, and funds transfer profits equally to HHs
- **Symmetric firm equilibrium** $Y_t(j) = Y_t, P_t(j) = P_t, \pi_t(j) = \pi_t, \frac{dP_t}{P_t} = \pi_t dt$
- Clearing in production and goods markets:

$$Y_t = Z_t N_t, \quad C_t \equiv \int_0^1 \eta(i) C_t(i) di = Y_t \left(1 - \frac{\vartheta}{2} \pi_t^2 - \Psi_t^S - \Psi_t^i \right)$$

- **Bond market clearing** implies

$$\mathcal{X}_t(\tau) - \bar{\beta} \theta(\tau) \mathcal{W}_t N_t + \int_0^1 \eta_i(\tau) \mathcal{B}_t(i) di + \mathcal{S}_t(\tau) = 0$$

Aggregate Risk Factors and Risk Pricing

- Aggregate technology $Z_t = \bar{Z}e^{z_t}$

$$dz_t = -\kappa_z z_t dt + \sigma_z dB_{z,t}$$

- Generic set of N_z exogenous risk factors \mathbf{z}_t with associated Brownian motions \mathbf{B}_t (where $z_t \in \mathbf{z}_t, B_{z,t} \in \mathbf{B}_t$) with volatility

$$\text{Var}_t d\mathbf{z}_t = \text{Var}_t \boldsymbol{\sigma} d\mathbf{B}_t = \boldsymbol{\sigma} \boldsymbol{\sigma}^\top dt$$

- Allow for exogenous cost-push shocks, firm financing shocks, discount factor shocks...
- Thus, instantaneous return of τ bond is

$$\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} = \mu_t(\tau) dt + \boldsymbol{\sigma}_t(\tau) d\mathbf{B}_t$$

- Arbitrageur optimality conditions imply

$$\mu_t(\tau) - i_t = a_t \boldsymbol{\sigma}_t(\tau) \boldsymbol{\Lambda}_t, \text{ where } \boldsymbol{\Lambda}_t^\top = \int_0^T \mathcal{X}_t(\tau) \boldsymbol{\sigma}_t(\tau) d\tau$$

Simple Optimal Short Rate: PE Illustration I

- Partial equilibrium illustration with ad-hoc loss function, simple policy rules
- Dynamics and loss function

$$dz_t = -\kappa_z z_t dt + \sigma_z dB_t, \quad x_t(\tau) = -\theta(\tau) z_t$$

$$\mathcal{L}_t \equiv (z_t - i_t)^2 + \left(\int_0^T \theta(\tau) (\mu_t(\tau) - i_t) d\tau \right)^2, \quad \min E \mathcal{L}_t$$

- Risk prices $\Lambda_t = \int_0^T -\sigma(\tau) \theta(\tau) d\tau z_t \equiv -\tilde{\sigma} z_t$

$$\mu_t(\tau) - i_t = \hat{\alpha} \sigma(\tau) \Lambda_t \implies \left(\int_0^T \theta(\tau) (\mu_t(\tau) - i_t) d\tau \right)^2 = \hat{\alpha}^2 \tilde{\sigma}^2 z_t^2$$

- Simple policy rule: choose χ such that $i_t = \chi z_t$

Simple Optimal Short Rate: PE Illustration II

- Unconditionally, $E(z_t - i_t)^2$ is decreasing in χ for $\chi < 1$
- Is $\chi = 1$ optimal? Not if $\hat{a} > 0$, since $\tilde{\sigma}$ is endogenous
- Solving for $\tilde{\sigma}$: conjecture affine term structure

$$-\log P_t^{(\tau)} = A_z(\tau)z_t + C(\tau)$$

- Ito's Lemma and market clearing:

$$A'_z(\tau) + MA_z(\tau) = \chi \implies A_z(\tau) = \chi \frac{1 - e^{-M\tau}}{M}, \text{ where } M = \kappa_z + a\sigma_z^2 \int_0^T \theta(\tau)A_z(\tau) d\tau$$

$$\implies \tilde{\sigma}^2 = \sigma_z^2 \left(\int_0^T \theta(\tau)A_z(\tau) d\tau \right)^2$$

- Hence, unconditionally $E \left(\int_0^T \theta(\tau)(\mu_t(\tau) - i_t) d\tau \right)^2$ is increasing in χ
- Optimal $0 < \chi^* < 1$

Separation Principle with Balance Sheet Constraints

- When the central bank faces **balance sheet constraints** ($\psi^{(\tau)} > 0$), policy can no longer achieve first-best
- However, as long as $\psi^{(\tau)} < \infty$, optimal policy implies the central bank still uses balance sheet tools
- Let $\psi^{(\tau)} = a \cdot \sigma(\tau)\sigma(\tau)^\top$
 - \implies same friction a as arbitrageurs, except policymaker **cannot net out** positions
- Even with “large” balance sheet costs, the central bank still uses QE to (partially) stabilize term premia
- **Intuition:**
 - The central bank faces holding costs which imply it is **worse than private arbitrageurs** at financial intermediation
 - But **internalizes the social benefits** of minimizing fluctuations in term premia
 - Nevertheless, non-negligible balance sheet costs imply that optimal policy is less reactive

Optimal Policy: Short Rate Constraints

- Suppose that **short rate policy is constrained**, and implements

$$i_t = \tilde{\chi}_i v_t, \quad 0 < \tilde{\chi}_i \ll 1$$

- Formally: assume costs $\psi^i (i_t - \tilde{\chi}_i v_t)$ and take $\psi^i \rightarrow \infty$

Proposition (Optimal balance sheet rule)

Assume risk aversion $\hat{a} > 0$, $\beta(\tau) > 0$, and constrained short rates.

- *Bond market stabilization*: $qe_t^\sigma = \beta^\sigma \left((1 + \varsigma + \varphi)x_t - \frac{1}{\varsigma\kappa_z} v_t \right)$ implies
 1. Borrowing rates are stabilized, consumption and wealth dispersion are zero.
 2. Output gaps x_t are no longer identically zero.
- *Macroeconomic stabilization*: there exist parameters $\chi_{q,v} \neq 0, \chi_{q,b} \neq 0$ such that
 1. Output gaps are zero.
 2. Borrowing rate, consumption, and wealth dispersion are non-zero.

Extensions Overview

- Sticky prices, cost-push shocks
 - If firm borrowing is a small part of marginal costs, then all results go through
 - Exogenous cost-push shocks breaks divine coincidence but unfortunately, our separation principle still holds
 - Despite multiple policy instruments, (un)conventional policy only affects aggregate outcomes through changes in effective borrowing rate $\tilde{\mu}_t$
 - If firm borrowing is large, then policymaker can in principle manipulate HH and firm effective borrowing rates $\tilde{\mu}_t, \hat{\mu}_t$ (though this is suboptimal due to risk-sharing motives)
- “Noise” demand shocks
 - Optimal separation principle still holds with firm financing shocks β_t
 - QE policy must be more reactive than the benchmark
 - The optimal rule may imply conventional and unconventional policies seemingly acting against one another
- Nonzero first-best term premia
 - When first-best BCT returns are $\nu(\tau) \neq 0$
 - Results hold when $\nu(\tau)$ is achievable but optimal short rate policy is a function of $\nu(\tau)$

Full Commitment Optimal Short Rate: PE Illustration I

- Partial equilibrium illustration with ad-hoc loss function, full commitment
- Dynamics and loss function

$$dz_t = -\kappa_z z_t dt + \sigma_z dB_t, \quad x_t(\tau) = -\theta(\tau) z_t$$

$$\mathcal{L}_t \equiv (z_t - i_t)^2 + \left(\int_0^T \theta(\tau) (\mu_t(\tau) - i_t) d\tau \right)^2, \quad \min E_0 \int_0^\infty e^{-\rho t} \mathcal{L}_t dt$$

- Risk prices $\Lambda_t = \int_0^T -\sigma(\tau) \theta(\tau) d\tau z_t \equiv -\tilde{\sigma} z_t$

$$\mu_t(\tau) - i_t = \hat{\alpha} \sigma(\tau) \Lambda_t \implies \left(\int_0^T \theta(\tau) (\mu_t(\tau) - i_t) d\tau \right)^2 = \hat{\alpha}^2 \tilde{\sigma}^2 z_t^2$$

- Policy rule with commitment: choose χ, κ_i, i_0 such that

$$di_t = -\kappa_i (i_t - \chi z_t) dt$$

Full Commitment Optimal Short Rate: PE Illustration II

- Dynamics

$$\mathbf{x}_t = e^{-\mathbf{\Gamma}t} \mathbf{x}_0 + \int_0^t e^{-\mathbf{\Gamma}(t-u)} \boldsymbol{\sigma}_x dB_u, \quad \mathbf{\Gamma} = \begin{bmatrix} \kappa_z & 0 \\ -\kappa_i \chi & \kappa_i \end{bmatrix}, \quad \boldsymbol{\sigma}_x = \begin{bmatrix} \sigma_z \\ 0 \end{bmatrix}$$

- Affine term structure

$$\begin{aligned} -\log P_t^{(\tau)} &= A_z(\tau) Z_t + A_i(\tau) i_t + C(\tau) \equiv \mathbf{A}(\tau)^\top \mathbf{x}_t + C(\tau) \\ \implies \mathbf{A}(\tau) &= \mathbf{M}^{-1} [\mathbf{I} - e^{-\mathbf{M}\tau}] \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{M} \equiv \mathbf{\Gamma}^\top + \begin{bmatrix} \hat{\alpha} \sigma_z^2 \int_0^\tau \theta(\tau) A_z(\tau) d\tau & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

- If $\hat{\alpha} = 0$, then $i_0 = z_0, \chi = 1, \kappa_i \rightarrow \infty$
- As with simple policy rules, $\chi \rightarrow 0 \implies A_z(\tau) \rightarrow 0$
- But policymaker still utilizes choices of i_0 and $\kappa_i < \infty$ (smoothing)