Monetary Policy and the Limits to Arbitrage: Insights from a New Keynesian Preferred Habitat Model

Walker Ray
SF Fed & LSE

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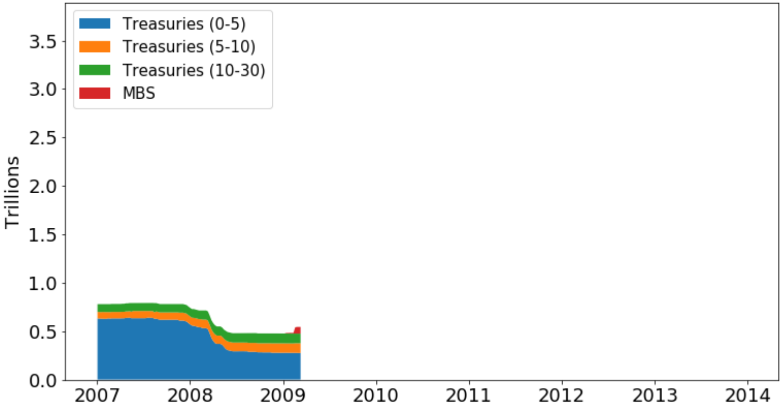
St. Louis Fed

Policy Response to Great Recession



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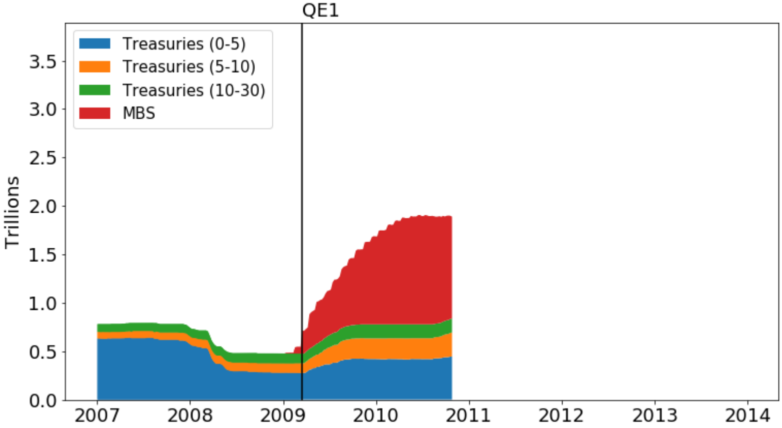

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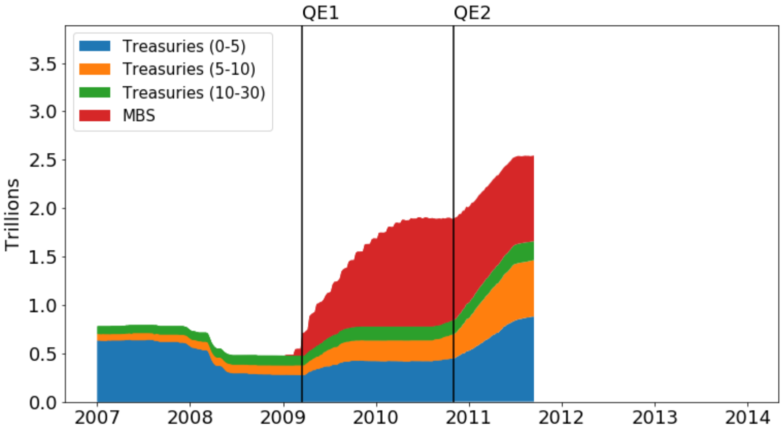

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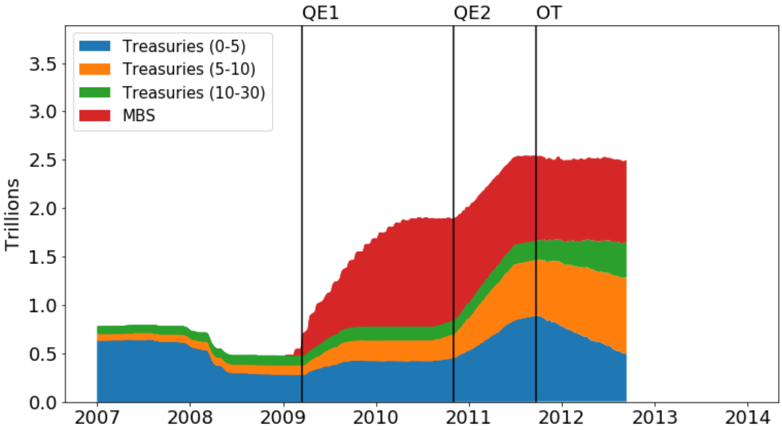

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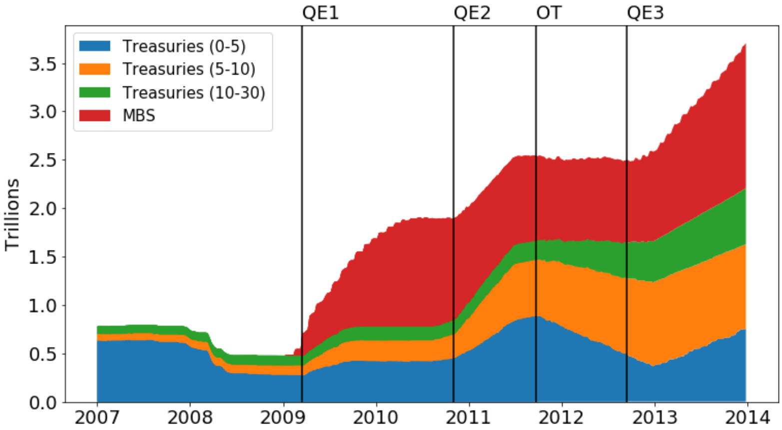

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- Develop a GE model which takes seriously limits to arbitrage
  - Derive **theoretical conditions** under which QE works
  - **Quantify** the aggregate effects of QE
- Bond market imperfections play a role in the transmission of **conventional** monetary policy
- Crucial for designing monetary policy going forward

Model Overview

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  - Households do not have full access to bond markets
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- Dual equilibrating role of the yield curve:
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- By affecting equilibrium bond **prices** and **allocations**, policy works through both channels

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  - Conventional policy: more aggressive in financial crises
  - QE rule can be stabilizing

Literature Contributions

- "Preferred habitat" as a key channel for understanding bond markets
  - D'Amico and King (2013), Hamilton and Wu (2012), Greenwood and Vayanos (2014), Gorodnichenko and Ray (2017), Greenwood and Vissing-Jorgensen (2018)
- Few formal models
  - Vayanos and Vila (2009)
- QE in general equilibrium: Market segmentation vs. forward guidance
  - Gertler and Karadi (2013), Chen et al (2012), Carlstrom et al (2017), Christensen and Rudebusch (2012), Bauer and Rudebusch (2014), Bhattarai et al (2015)
- Frictions and expected future policy
  - McKay et al (2016), Farhi and Werning (2017), Gabaix (2016), Angeletos and Lian (2018)

New Keynesian Preferred Habitat Framework

- Time $t \in [0, \infty)$ is continuous. Consumption and production:
  - Infinitely-lived **households** work and consume
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- Government:
  - **Central bank** sets the short nominal rate (and conducts QE)
  - Lump-sum taxes/transfers from investors to HHs

Aggregate Dynamics

- Modification of a benchmark NK model ($\pi_t \equiv 0$):

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- Closing the model: equilibrium term structure determination

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Zero-coupon bond yields and prices $R_{t,\tau} = -\frac{\log P_{t,\tau}}{\tau}$ determined by interactions of two types of investors [Vayanos and Vila 2009]:

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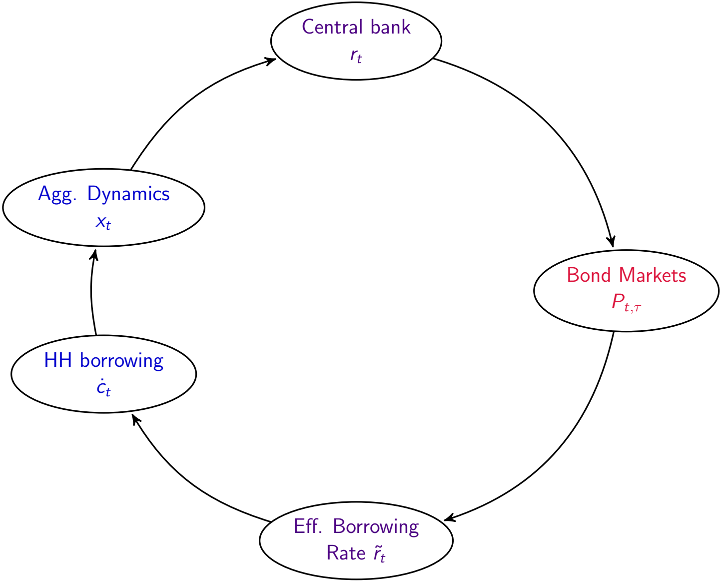
- Market clearing: $b_{t,\tau} = -\tilde{b}_{t,\tau}$

Equilibrium Under Perfect Arbitrage



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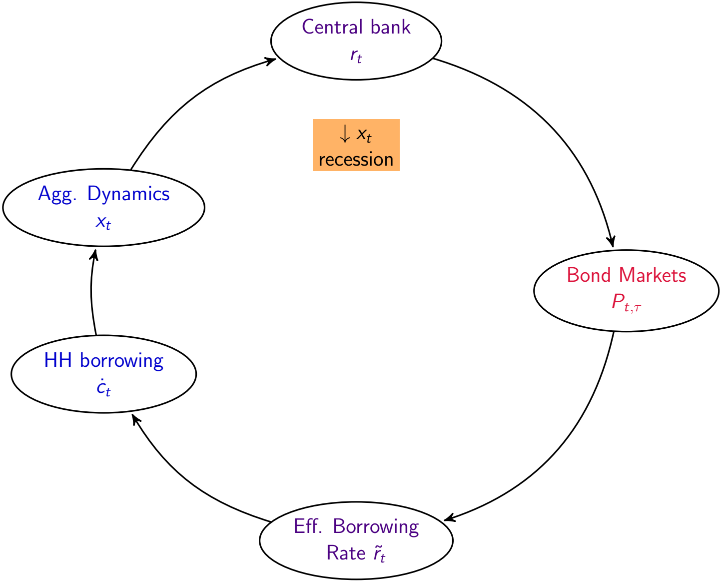

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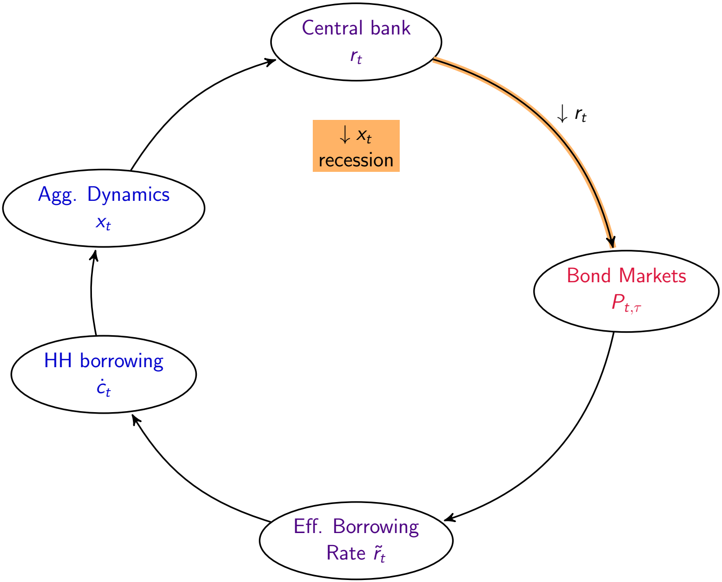

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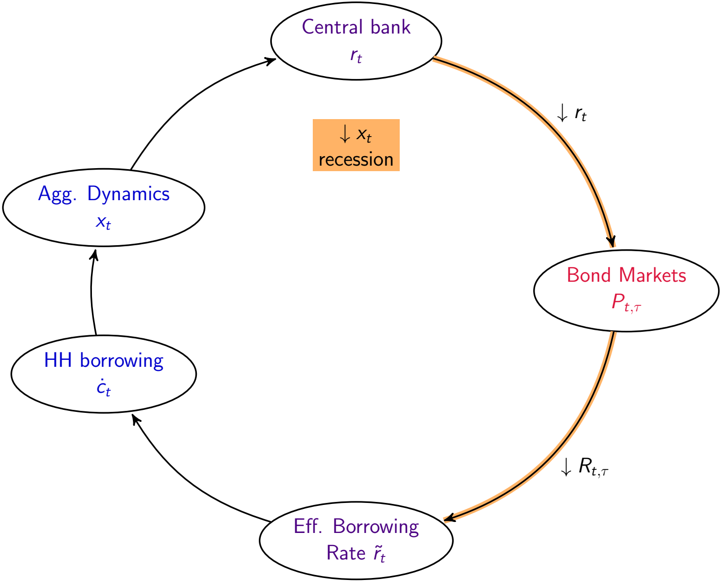

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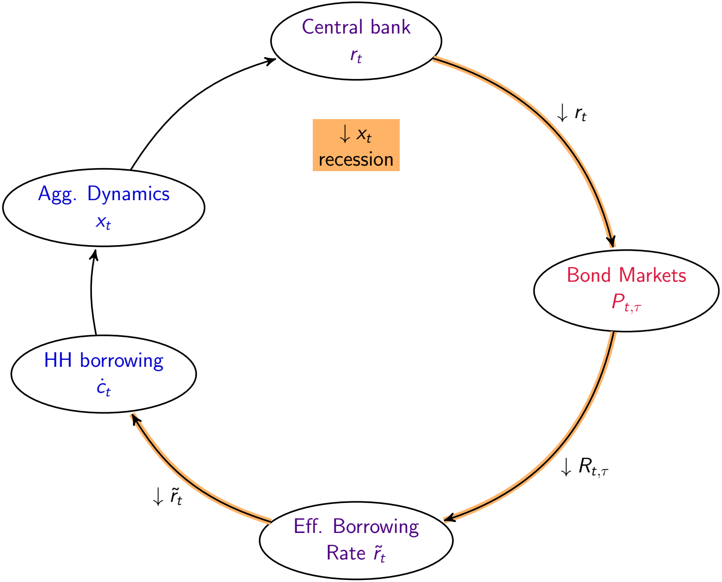

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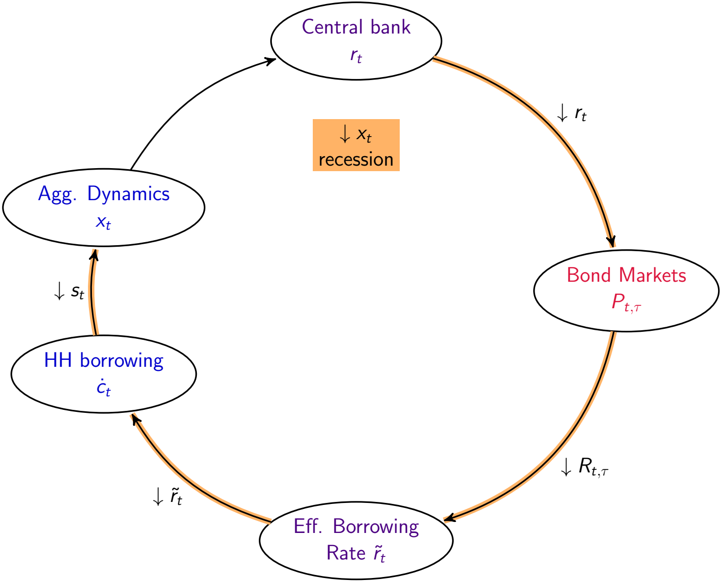

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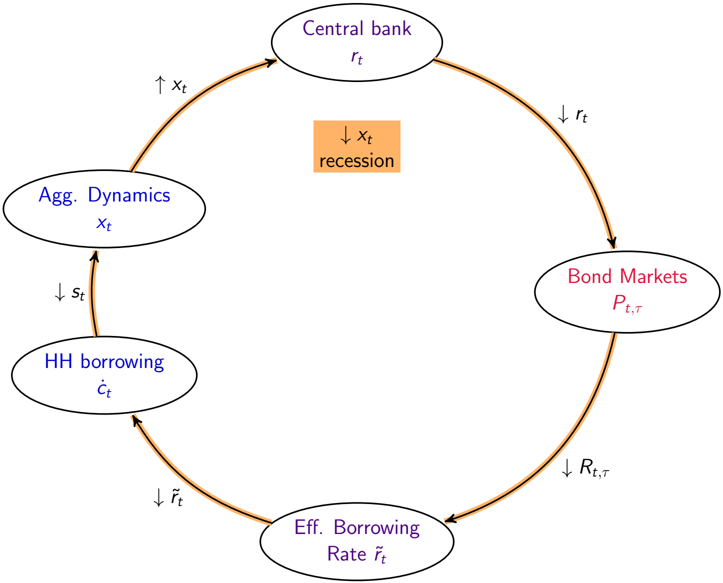

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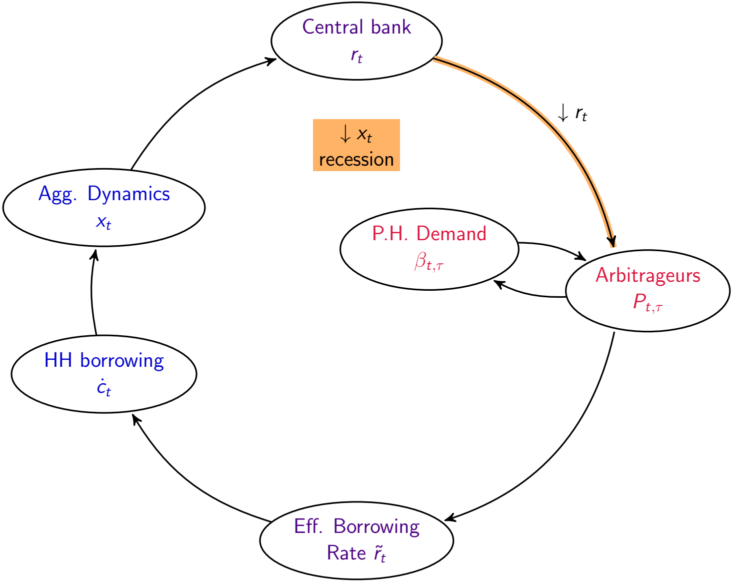

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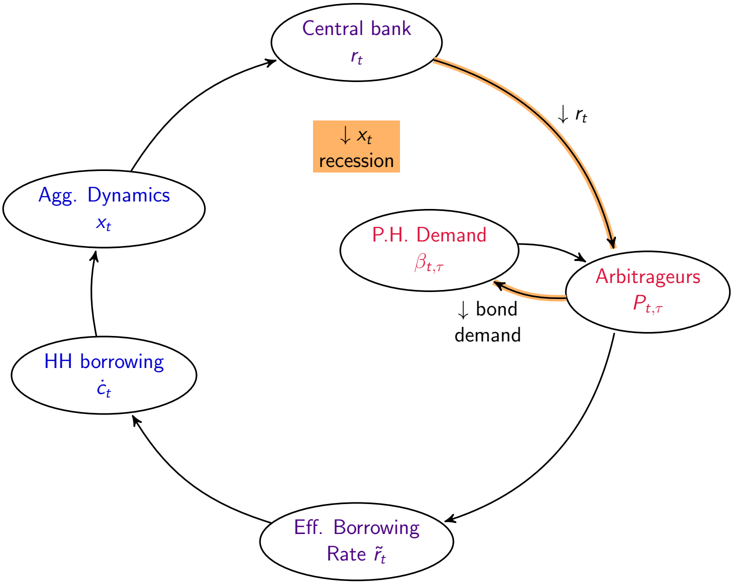

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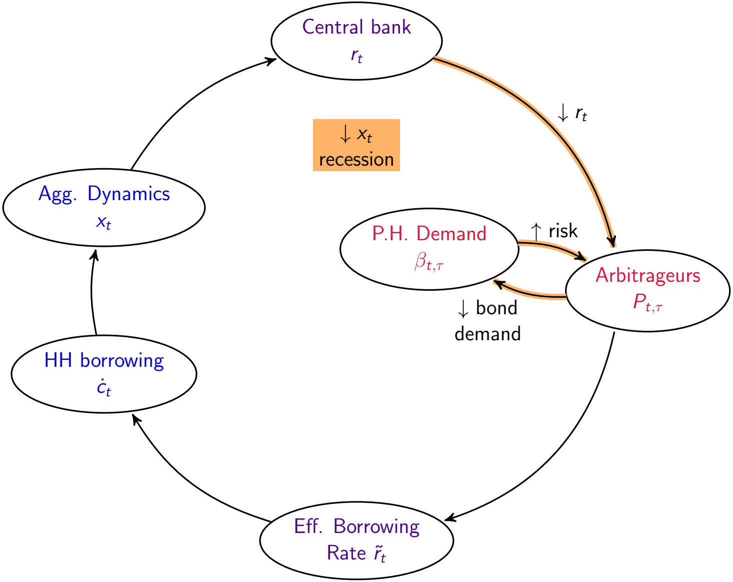

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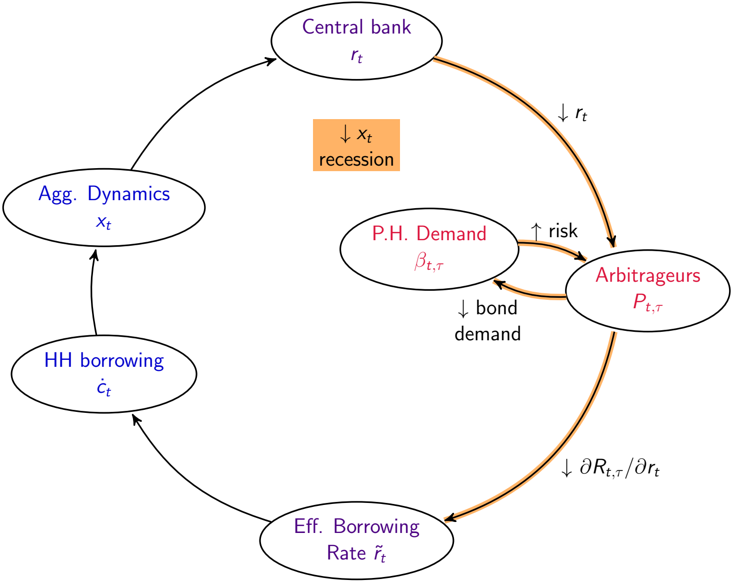

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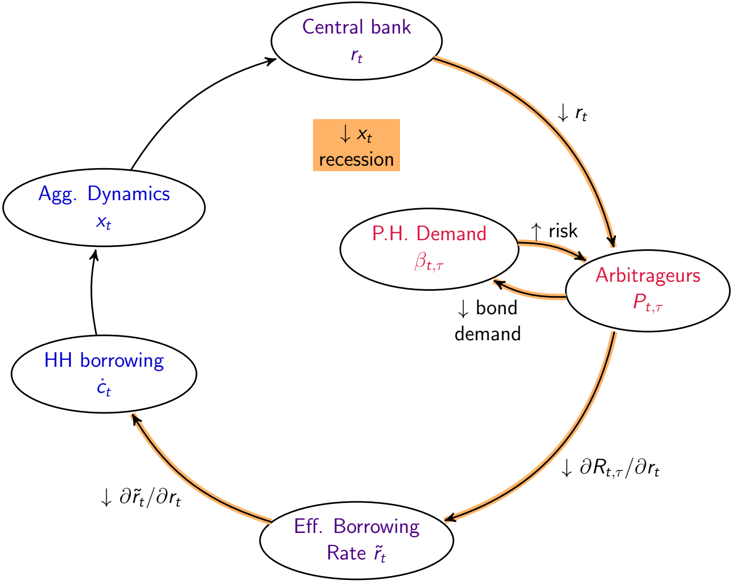

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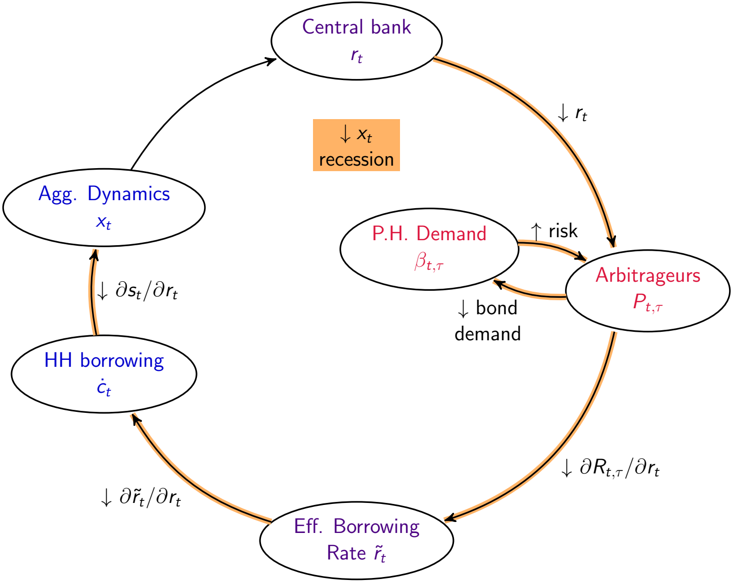

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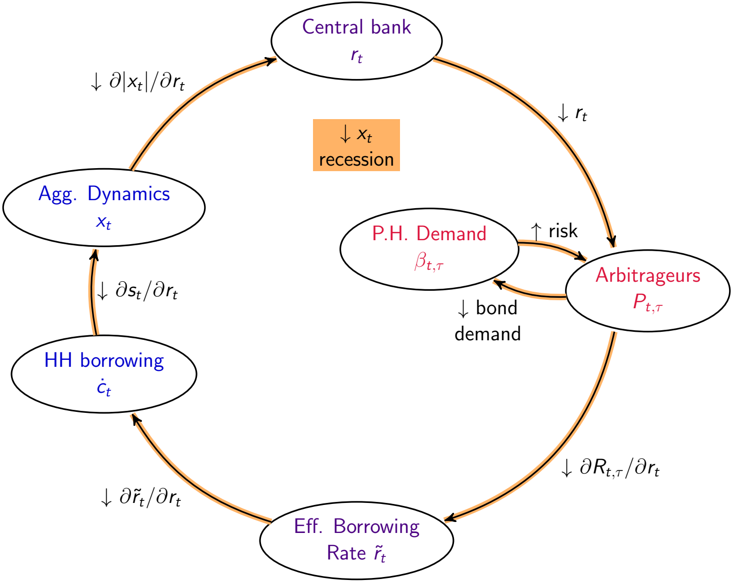

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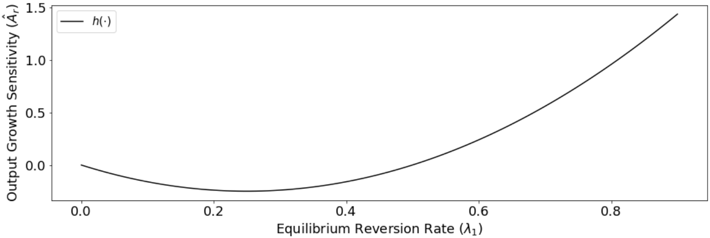
Rational Expectations Equilibrium



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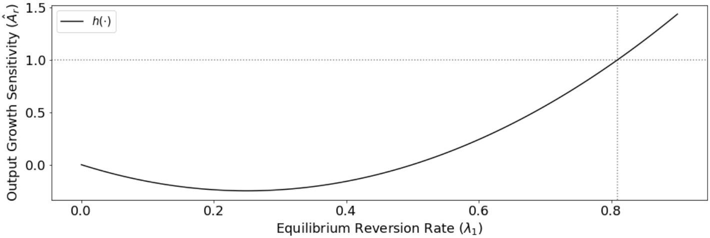

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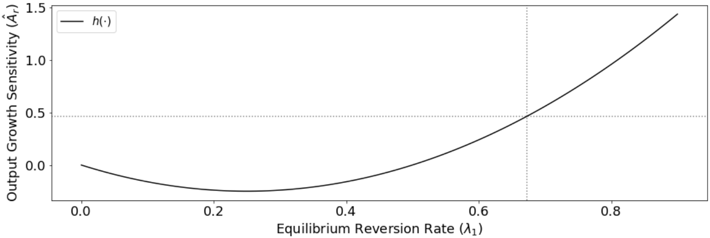

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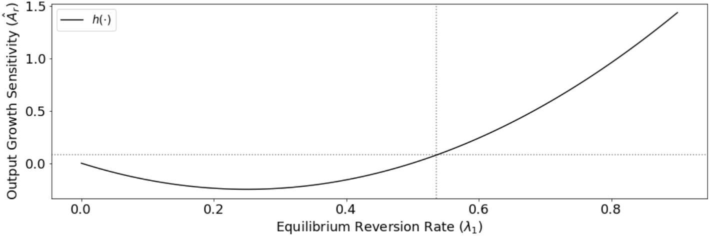

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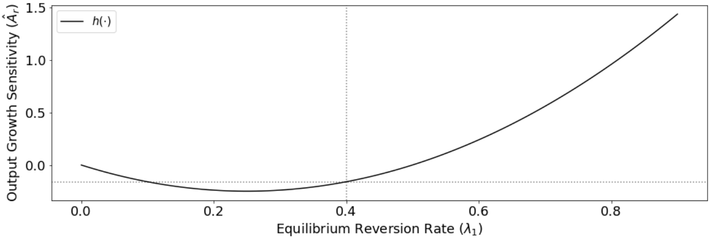

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Arbitrageur Portfolio Choice

- Take as given equilibrium dynamics of the short rate

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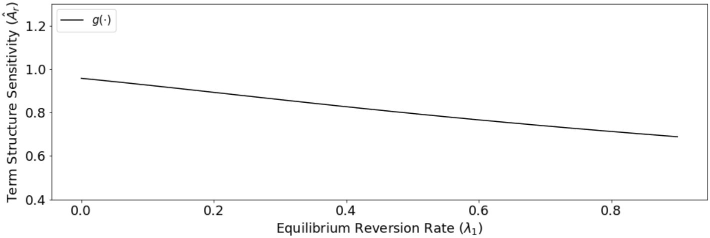

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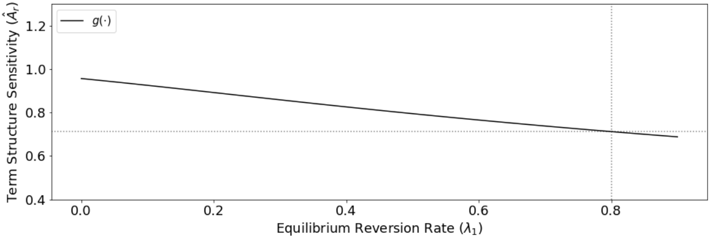

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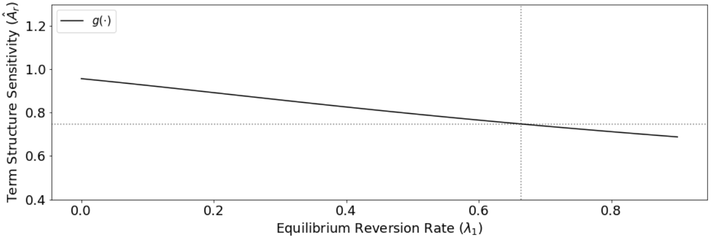

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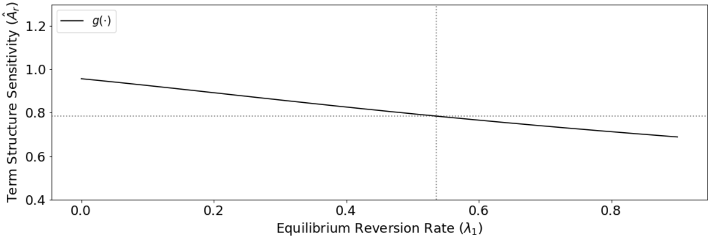

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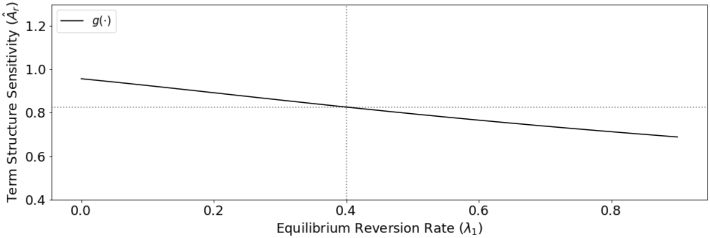

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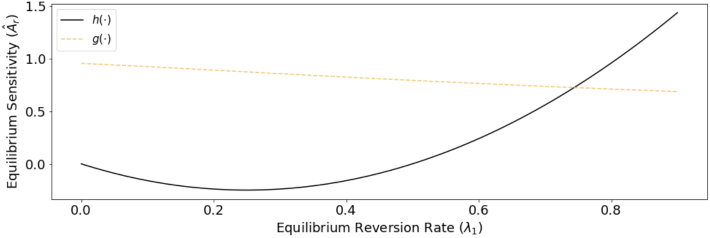
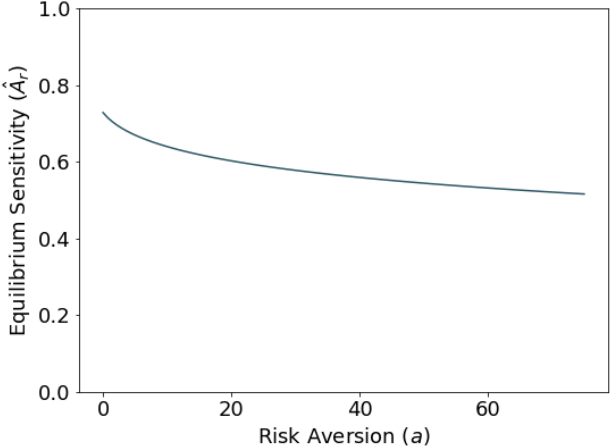


Existence and Uniqueness

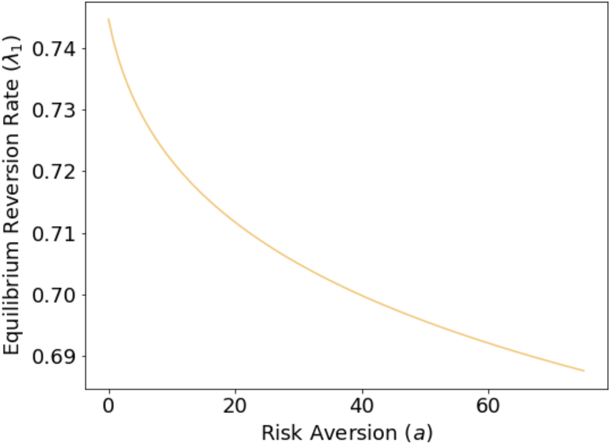
There exists a unique positive eigenvalue of Υ $\lambda_1 > 0$ for which $g(\lambda_1) = h(\lambda_1)$, which fully characterizes the model equilibrium. Further, this implies $0 < \hat{A}_r < 1$.

Conventional Policy and Financial Disruptions



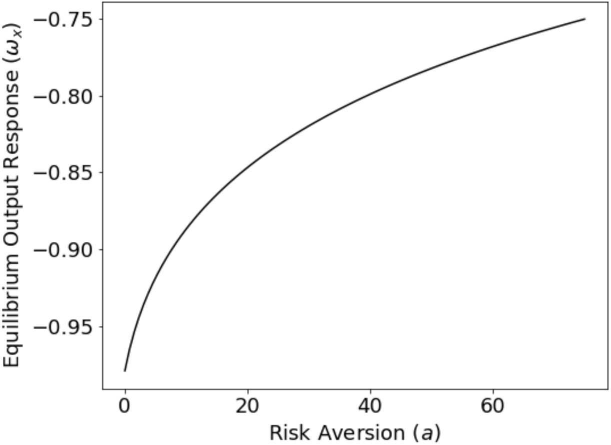
Notes: equilibrium changes in sensitivity to the short rate $\hat{A}_r$ as risk aversion a increases.

Conventional Policy and Financial Disruptions



Notes: equilibrium changes in monetary shock reversion λ_1 as risk aversion a increases.

Conventional Policy and Financial Disruptions



Notes: equilibrium changes in output response ω_x to monetary shocks as risk aversion a increases.

Policy Implications

- More aggressive response to output ϕ_x results
- Higher inertia κ_r results
- Shifts in effective rate weights $\eta(\tau)$ results
- Forward guidance less effective as risk aversion increases details

Modeling LSAPs

- Suppose the central bank directly purchases bonds through open market operations
- Change to the demand shifter in PH demand

$$\tilde{b}_{t,\tau} = \alpha(\tau)\tau(R_{t,\tau} - \beta_{t,\tau})$$

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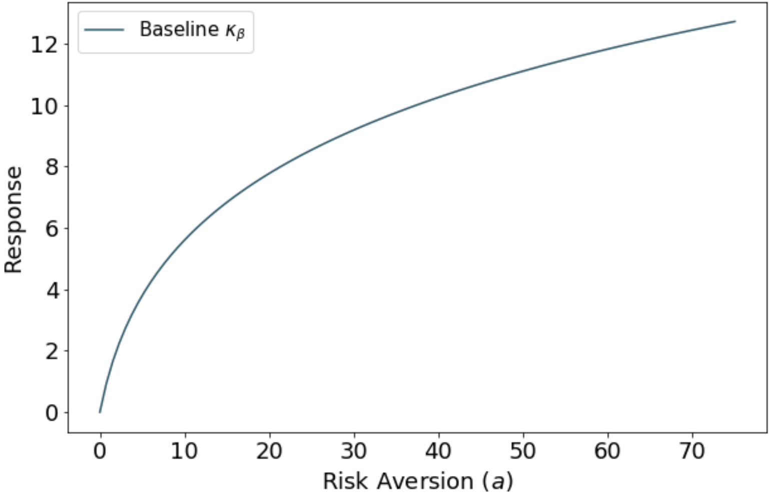
Output Response to QE



Notes: plots of output gap response to a QE shock as risk aversion increases.

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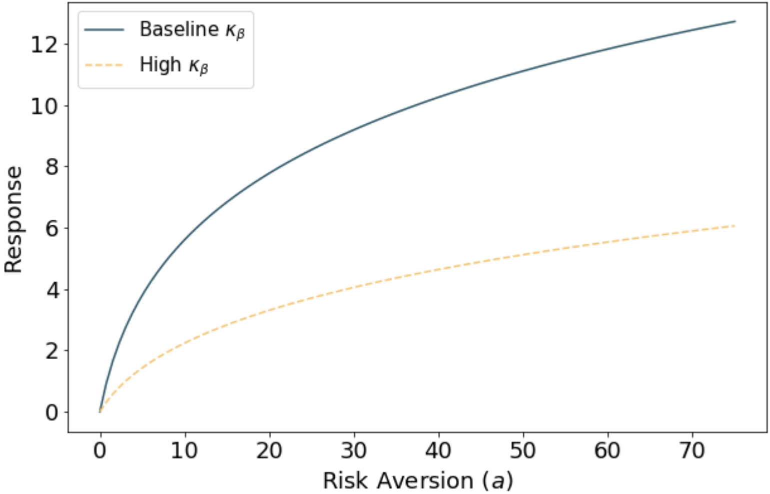

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Sticky Prices

- What about when prices are not fixed?

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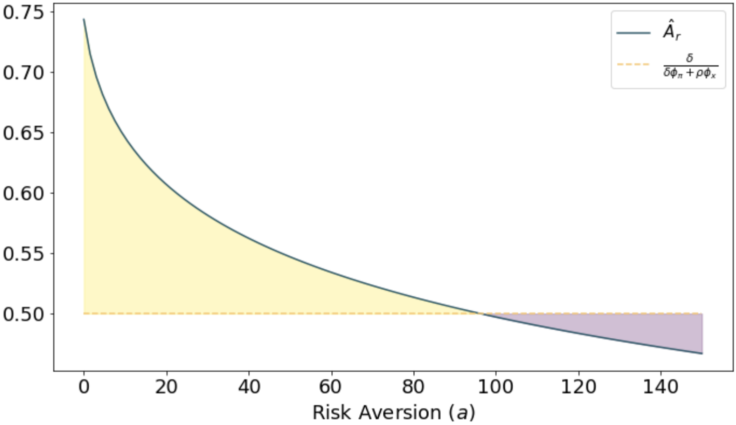
- If $\hat{A}_r = 1$ and $\phi_x = 0$, reduces to $\phi_\pi > 1$

Implications – Determinacy



Notes: determinacy condition as risk aversion a increases.
The model is determinate if the solid dark line lies above the dotted light line (light shaded region) and is indeterminate otherwise (dark shaded region).

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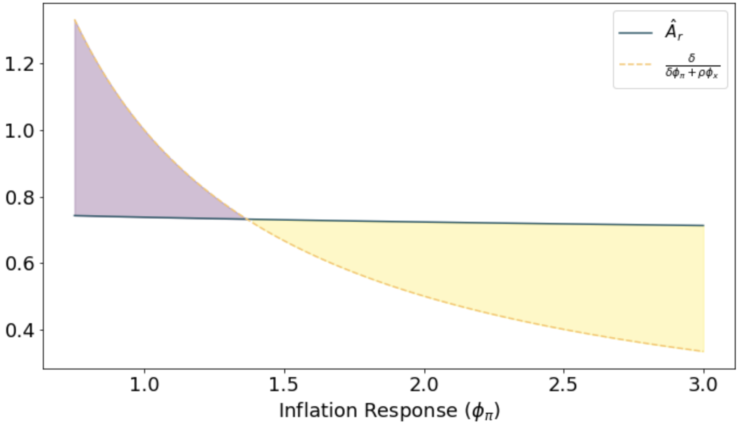

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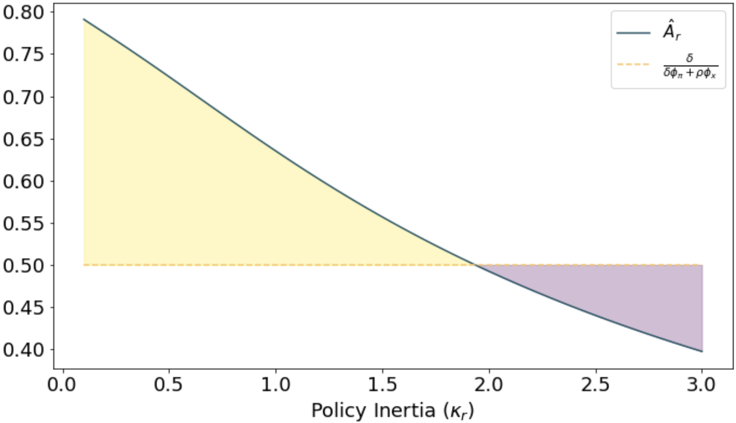

Notes: determinacy condition as central bank inertia κ_r increases.
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Generalized Model

- Sticky price model with shocks

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- Shocks

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Generalized Model

- Sticky price model with shocks

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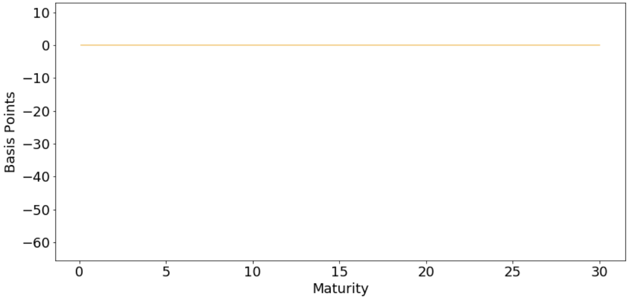
Yield Curve (QE, long end)



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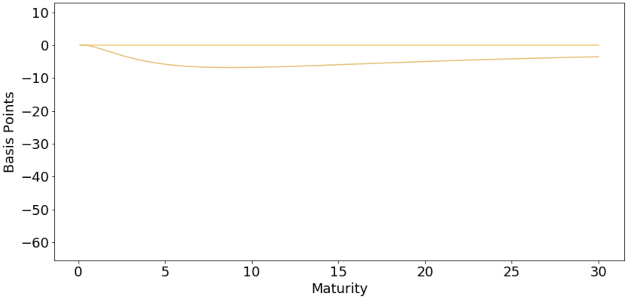

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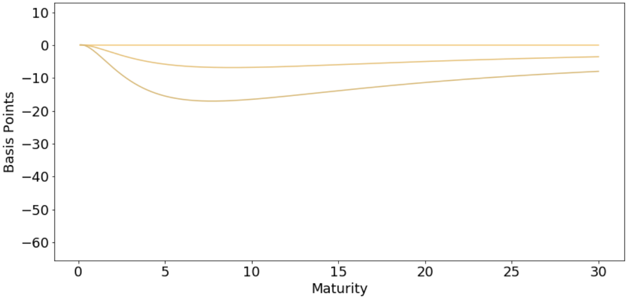

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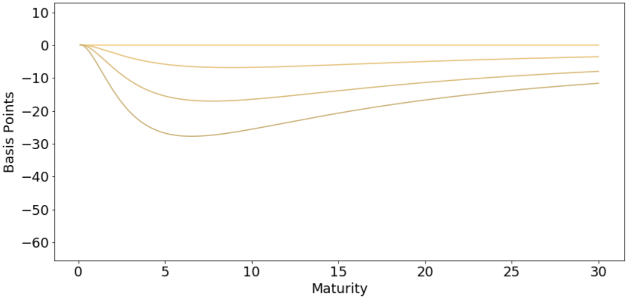

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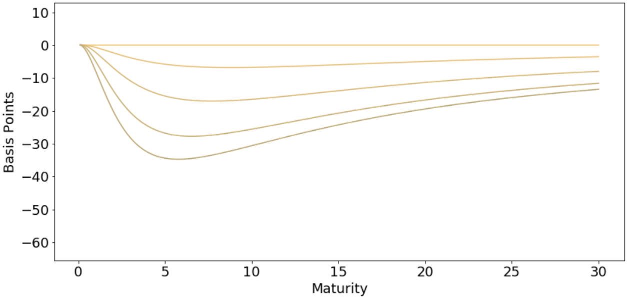

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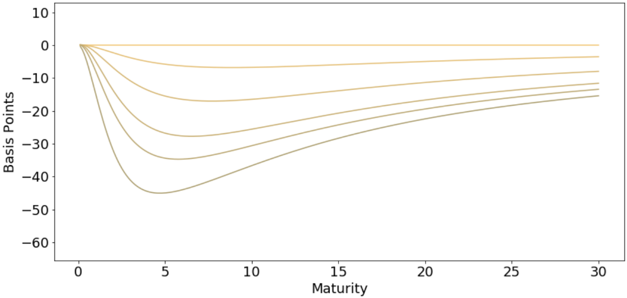

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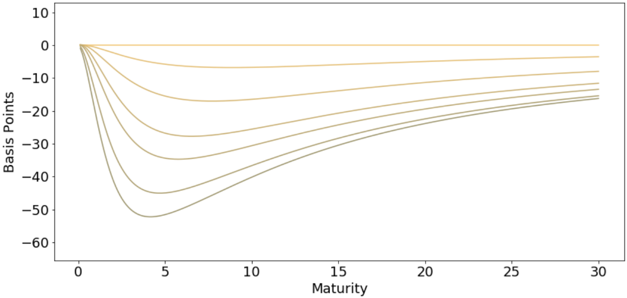

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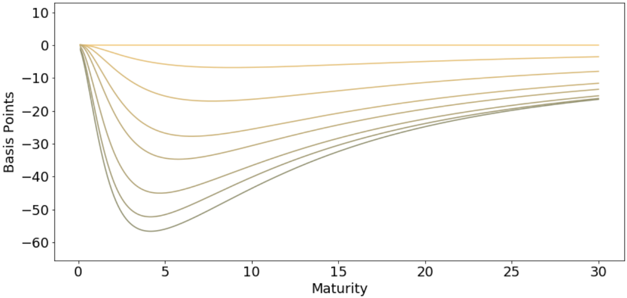

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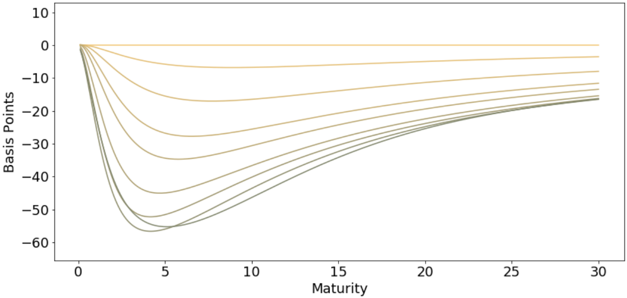

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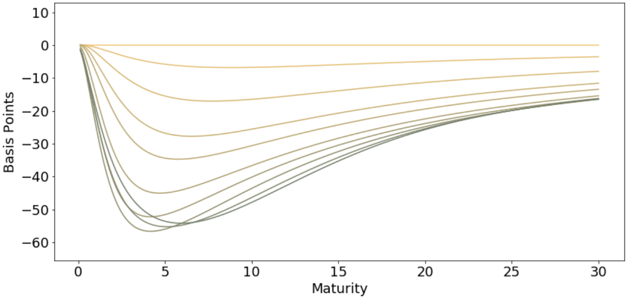

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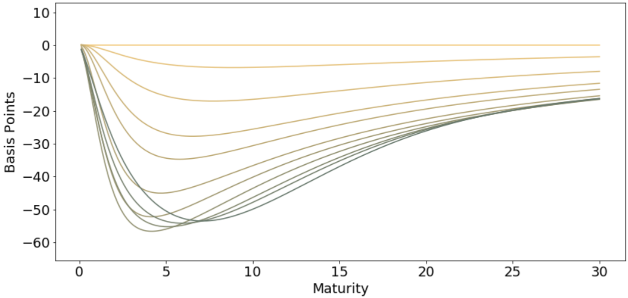

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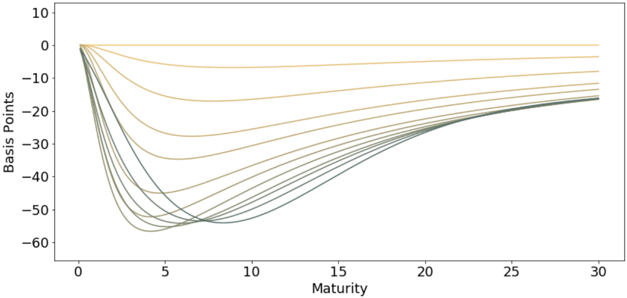

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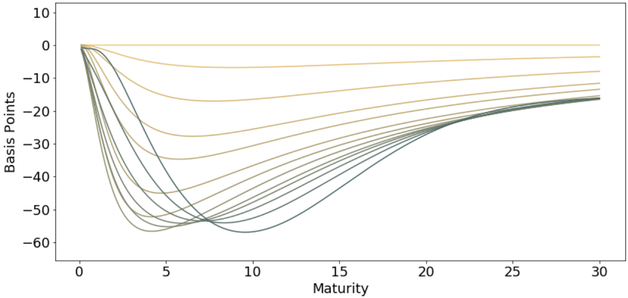

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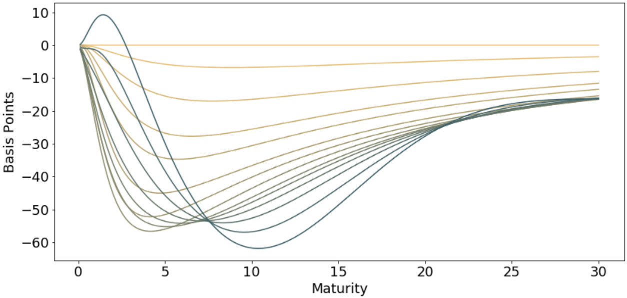

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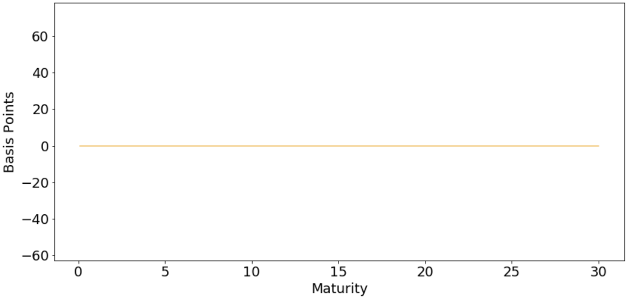
Yield Curve (Operation Twist)



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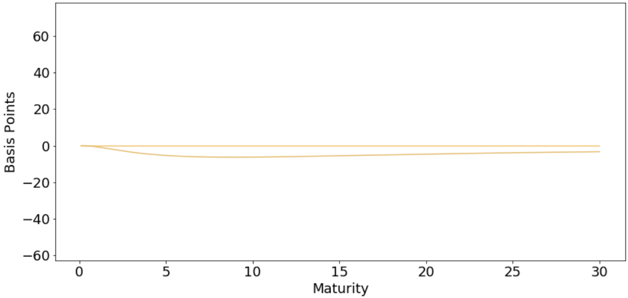

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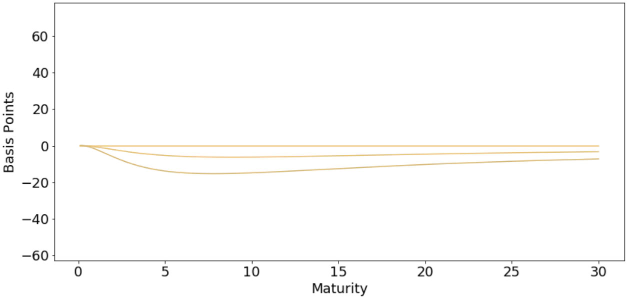

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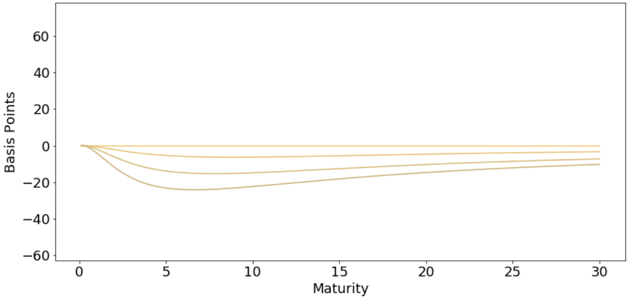

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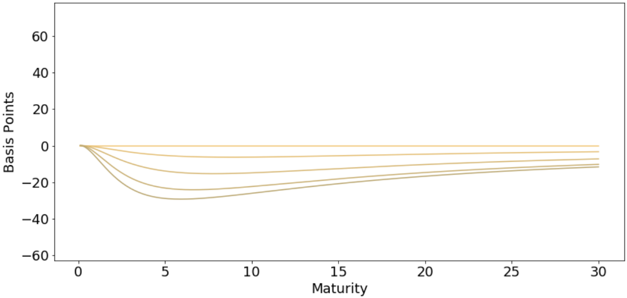

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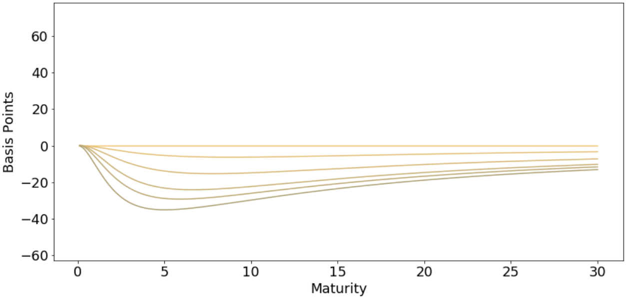

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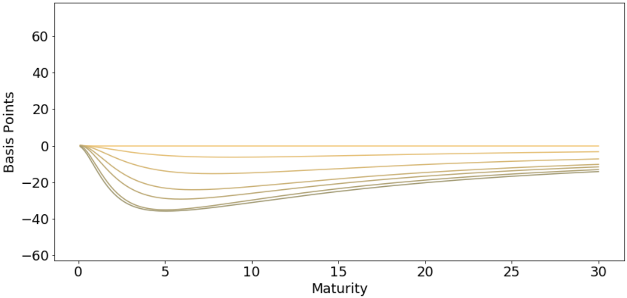

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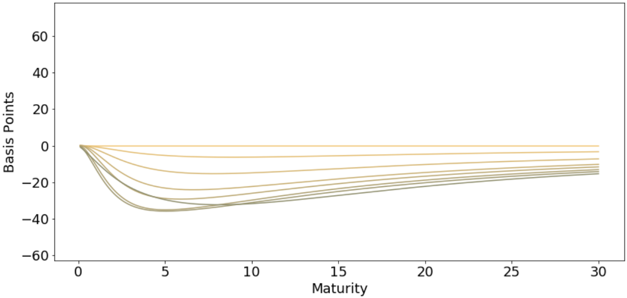

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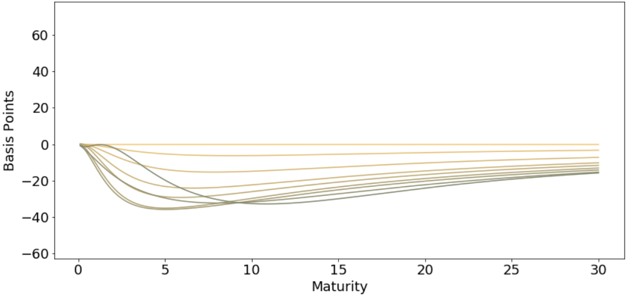

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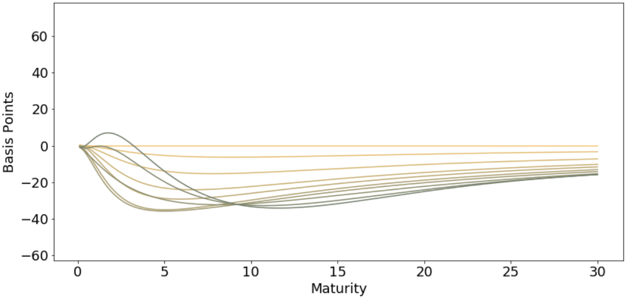

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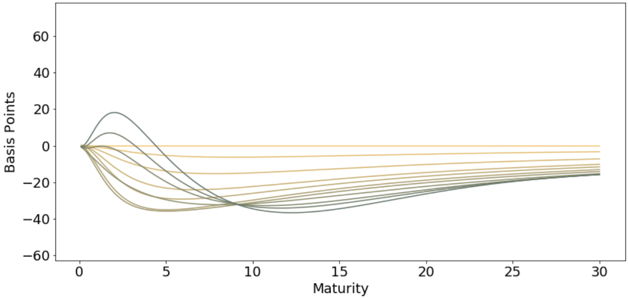

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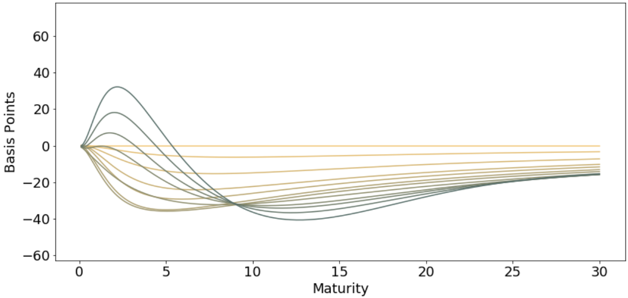

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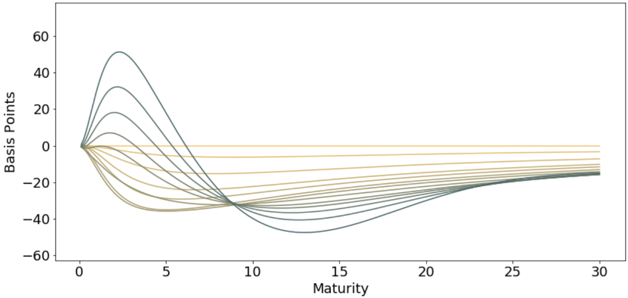

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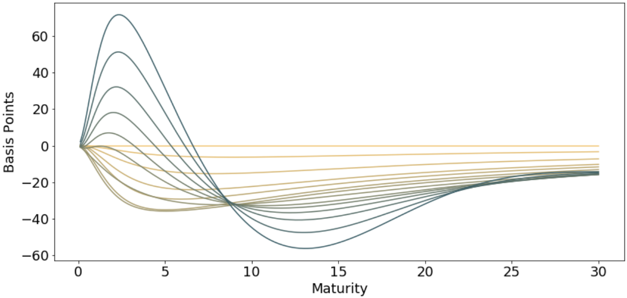

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Stabilizing LSAPs

- Can LSAPs be used to ensure determinacy?
- Endogenous QE purchases:

$$\mathrm{d}\beta_t = -\kappa_\beta \left(\beta_t - \phi_\pi^\beta \pi_t\right) \mathrm{d}t$$

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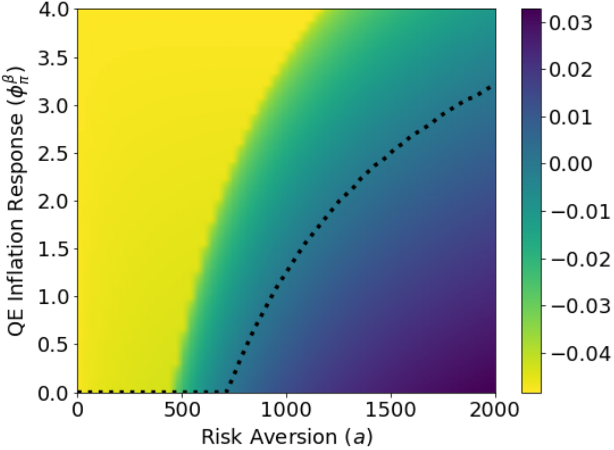
QE and Determinacy



Notes: determinacy conditions as a function of risk aversion (x-axis) and endogenous response of QE to inflation (y-axis). Darker colors correspond to larger values of the unstable eigenvalue. The dotted black line demarcates the region of determinacy.

Concluding Remarks

- Develops a unified, parsimonious framework to study conventional and unconventional monetary policies
- Transmission depends crucially on the risk-bearing capacity of financial markets

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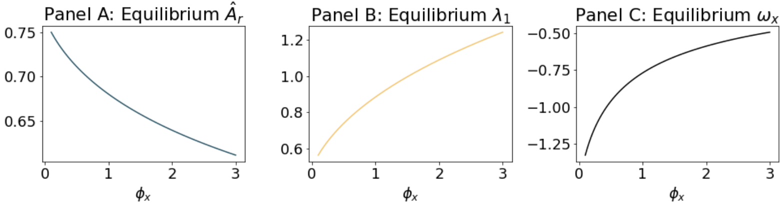
- Transmission depends crucially on the risk-bearing capacity of financial markets
- Future work:
  - Monetary policy in open economies [Gourinchas, Ray, Vayanos (2019)]
  - Macroprudential policies
  - Debt management

APPENDIX

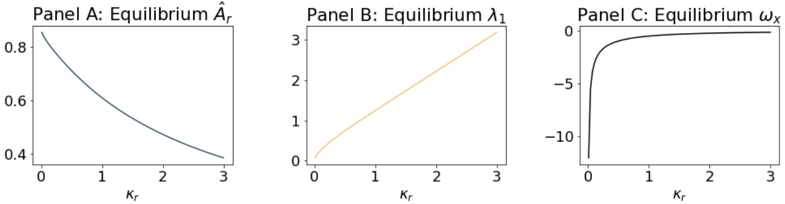
Implications – Conventional Policy



Notes: equilibrium changes in sensitivity to the short rate $\hat{A}_r$ and monetary shock reversion λ_1 as central bank response to output ϕ_x increases.

back

Implications – Conventional Policy



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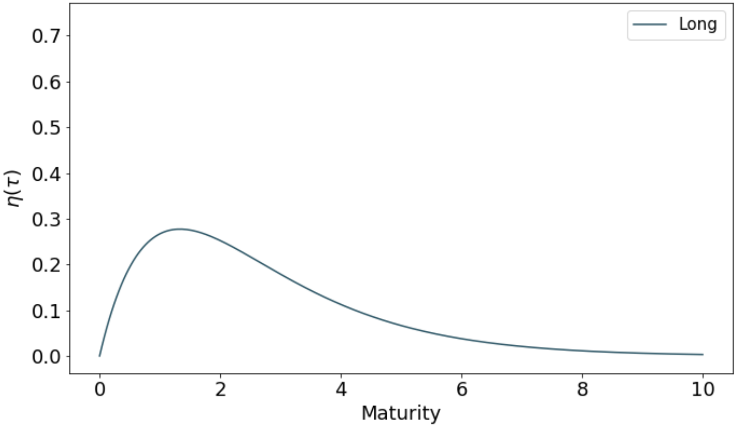
back

Sensitivity to Long Rates



Notes: different weighting function $\eta(\tau)$ in the determination of the effective borrowing rate $\tilde{r}_t$.

back

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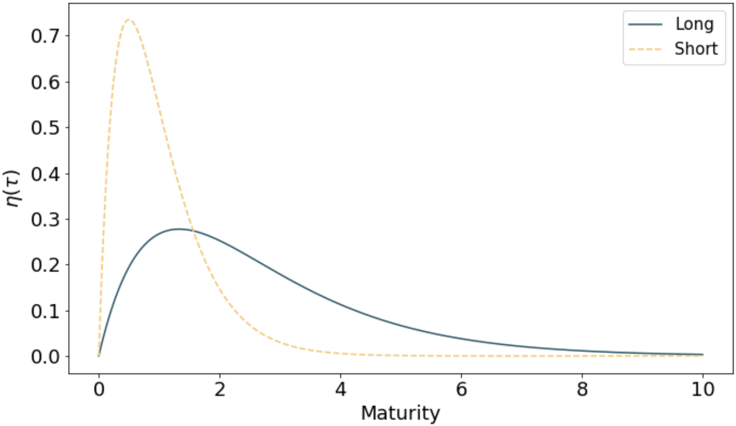

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back

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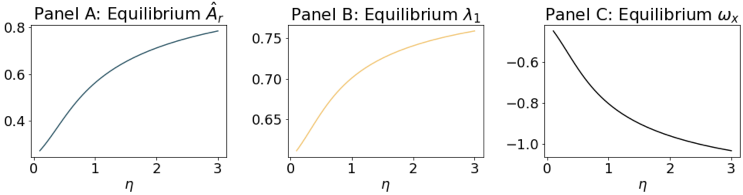


Notes: equilibrium changes in sensitivity to the short rate $\hat{A}_r$ and monetary shock reversion λ_1 as the weighting function $\eta(\tau)$ shifts towards short-term bonds.

back

Forward Guidance

- Central bank announces a peg: $r_0 = r^\diamond$ and

$$\mathrm{d}r_t = \begin{cases} -\kappa_r^\diamond(r_t - r^\diamond)\,\mathrm{d}t + \sigma_r^\diamond\,\mathrm{d}B_{r,t} & \text{if } 0 < t < t^\diamond \\ -\kappa_r(r_t - \phi_x x_t - r^*)\,\mathrm{d}t + \sigma_r\,\mathrm{d}B_{r,t} & \text{if } t \geq t^\diamond \end{cases}$$

- Affine coefficient functions during peg:

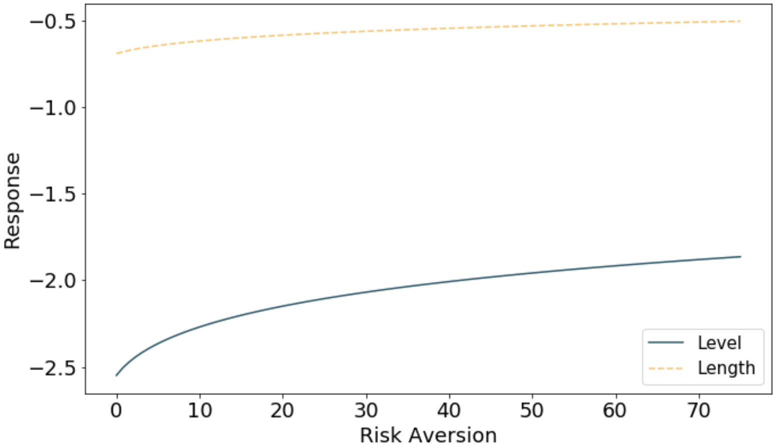
$$-\log P_{t,\tau} = A_r^\diamond(\tau) r_t + C^\diamond(\tau)$$
$$\implies \tilde{r}_t = \hat{A}_r^\diamond r_t + \hat{C}^\diamond$$

- Rational expectations dynamics for output:

$$\frac{\partial x_0}{\partial r^\diamond} = \omega_x - t^\diamond \varsigma^{-1} \hat{A}_r^\diamond\,, \quad \frac{\partial^2 x_0}{\partial r^\diamond \partial t^\diamond} = -\varsigma^{-1} \hat{A}_r^\diamond$$

back

Response to Forward Guidance



Notes: plots of $\frac{\partial x_0}{\partial r^\diamond}$ ("level") and $\frac{\partial^2 x_0}{\partial r^\diamond \partial t^\diamond}$ ("length") as risk aversion increases.

back