

Optimal Macro-Financial Stabilization in a New Keynesian Preferred Habitat Model

Rupal Kamdar
Indiana University

Walker Ray
LSE & CEPR

July 2024
CEPR AP Meeting

Motivation

Bernanke: “QE works in practice but not in theory”

- By now the gap between practice and theory is small
- But what do we mean by *QE works*?
 - Obvious: reduce long-term yields
 - Less obvious: stimulate the economy
 - Even less obvious: improve social welfare
 - Reis: “QE’s original sin”
- Especially relevant today now that central banks are implementing QT while increasing short rates
- **Question**: what is the optimal QE policy, and how does this interact with short rate policy?

Our Model

- [This paper](#): develops a tractable general equilibrium model with market segmentation, nominal frictions, and household heterogeneity
- [Arbitrageurs](#) with imperfect risk-bearing capacity absorb supply and demand shocks in bond markets
- [Clientele investors](#) introduce a degree of [market segmentation](#)
 - Bonds of different maturities traded by specialized investors (pension funds, MMMF)
 - Arbitrageurs (hedge funds, broker-dealers) partly overcome segmentation
- [Households](#) have differentiated access to bond markets
 - Introduces imperfect risk-sharing, [consumption and labor dispersion](#) across households
- Formally: embed a Vayanos-Vila model in a New Keynesian model, where households are heterogeneous in access to financial markets

Findings: Policy Transmission

- **Key mechanisms** of conventional monetary policy:
 - Changes in the short rate affect required rates of return of all assets
 - Interaction of arbitrageurs and investor clientele leads to **portfolio rebalancing**
 - Implies **variation in risk premia**, imperfect transmission to households
- **Key mechanisms** of balance sheet policy:
 - Imperfect arbitrage breaks QE neutrality
 - Central bank asset purchases induce portfolio rebalancing and hence **reduce risk premia**
 - Vice versa for central bank asset sales (QT)
- Thus, short rate and balance sheet policies are **substitutes** when targeting aggregate borrowing rates
 - A fall in aggregate borrowing rates is stimulative for the usual NK reasons

Findings: Welfare Consequences

- If the policymaker only cares about **macroeconomic stabilization**, conventional and unconventional policies are essentially equivalent
 - **Nominal rigidities** \implies welfare losses due to inflation volatility
 - Policy stabilizes inflation by keeping aggregate borrowing rates at some “natural” rate
 - **Triumphalist view**: even with short rate constraints, QE is equally effective
- However, both policies imply variation in **risk premia**
 - Excess fluctuations in risk premia lead to dispersion in borrowing rates
- **Social welfare** depends not only on macroeconomic fluctuations:
 - **Imperfect risk sharing** \implies welfare losses from consumption dispersion
 - **Labor market inefficiencies** \implies welfare losses from labor dispersion

Findings: Optimal Policy

- Hence, when policy is unconstrained we derive an **optimal separation result**:
 - Conventional policy targets **macroeconomic stability**
 - Unconventional policy targets **financial stability**
- However, when **policy constraints bind**, policy must balance trade-offs:
 - **Balance sheet constraints**: short rate must be less reactive in order to minimize financial disruptions (at the cost of macroeconomic stability)
 - **Short rate constraints**: QE must be used to offset macroeconomic shocks (at the cost of financial stability)
- With full commitment, **forward guidance** is welfare-improving (short rate and QE)
 - Policymaker uses the entire expected path of borrowing rates to minimize macroeconomic volatility
 - But reduces short-run fluctuations to keep risk premia volatility low
 - However, dynamics are complicated and suffer from time-inconsistency
- General message: **implementation matters** for welfare

Related Literature

- Preferred habitat models
 - Vayanos & Vila (2009, 2021), Ray, Droste, & Gorodnichenko (2023), Greenwood & Vayanos (2014), Greenwood et al (2016), King (2019, 2021) , Kekre, Lenel, & Mainardi (2024), ...
- Empirical evidence: QE and preferred habitat
 - Krishnamurthy & Vissing-Jorgensen (2012), D'Amico & King (2013), Li & Wei (2013), Cahill et al. (2013), King (2019), Fieldhouse et al (2018), Di Maggio et al (2020), Gali, Debortoli, & Gambetti (2020) , ...
- Macroeconomic QE models
 - Andres, Lopez-Salido, & Nelson (2004), Gertler & Karadi (2011), Curdia & Woodford (2011), Chen et al (2012), Gertler & Karadi (2013), Sims & Wu (2020), Karadi & Nakov (2020), Iovino and Sergeyev (2023), Carlstrom et al (2017), Ippolito et al (2018) , ...
- Market segmentation, macro-prudential monetary policy
 - Cui & Sterk (2021), Auclert (2016), Collard et al (2017), Kaplan et al (2017), Debortoli & Gali (2017) , ...
- International
 - Itskhoki & Mukhin (2022), Greenwood et al (2023), Gourinchas, Ray, & Vayanos (2022) , ...

Set-Up

Model Set-Up

- Continuous time New Keynesian model with embedded Vayanos-Vila bond markets
- **Agents:**
 - **Firms:** monopolistic competitors produce using labor, face nominal pricing frictions
 - **Households:** supply differentiated labor, consume, save via bond markets
 - **Arbitrageurs:** imperfect risk-bearing capacity, conduct bond carry trades
 - **Habitat funds:** buys and sell bonds of a specific maturity
- **Policymakers:**
 - **Central bank:** conducts short rate and balance sheet (QE) policy
 - **Government:** optimal subsidies, otherwise passive
- **Bond markets:**
 - Continuum of **zero coupon bonds** with maturity $0 \leq \tau \leq T \leq \infty$
 - Bond price $P_t^{(\tau)}$ with yield to maturity $y_t^{(\tau)} = -\log P_t^{(\tau)} / \tau$
 - Nominal short rate: in equilibrium, $i_t = \lim_{\tau \rightarrow 0} y_t^{(\tau)}$

Firms

- Continuum of intermediate goods $j \in [0, 1]$ (and CES final good with elasticity ϵ)
- Linear production in differentiated labor $Y_t(j) = e^{z_t} L_t(j)$:

$$dZ_t = -\kappa_Z Z_t dt + \sigma_Z dB_{t,Z}, \quad L_t(j) = \left[\int_{h \in \mathcal{H}} L_t(j, h)^{\frac{\epsilon_W - 1}{\epsilon_W}} dh \right]^{\frac{\epsilon_W}{\epsilon_W - 1}}$$

- Face costs $\Theta(\pi_t(j)) = \frac{\theta}{2} \pi_t(j)^2 P_t Y_t$ when setting prices $\frac{dP_t(j)}{P_t(j)} = \pi_t(j) dt$. Maximizes:

$$U_0 \equiv \max \mathbb{E}_0 \int_0^\infty e^{-\rho t} Q_t^{\mathcal{H}} \frac{\mathcal{F}_t}{P_t} dt$$

s.t. $\mathcal{F}_t = (1 + \tau^y) P_t(j) Y_t(j) - \mathcal{W}_t L_t(j) - \Theta(\pi_t(j)) - \mathcal{T}_t$

- Take as given CES demand, wage index, price index, τ^y subsidy, taxes \mathcal{T}_t
- Profits are discounted by HH sector real SDF $Q_t^{\mathcal{H}}$

Key takeaway: inefficiencies due to pricing frictions, differentiated labor

Households

- Continuum of HH members $h \in \mathcal{H}$, differentiated by access to bond markets τ
- Mass $\eta(\tau)$ of each $h = (i, \tau)$ HH where $\int_0^T \eta(\tau) d\tau = 1$ (otherwise identical)
- A τ -type HH chooses consumption and labor $C_t(\tau), N_t(\tau)$ in order to solve

$$V_0(\tau) \equiv \max \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left(\frac{C_t(\tau)^{1-\varsigma} - 1}{1-\varsigma} - \frac{N_t(\tau)^{1+\varphi}}{1+\varphi} \right) dt$$

$$\text{s.t. } dA_t(\tau) = [(1 + \tau^w) \mathcal{W}_t(\tau) N_t(\tau) - P_t C_t(\tau)] dt + A_t(\tau) \frac{dP_t^{(\tau)}}{P_t^{(\tau)}} + dF_t(\tau)$$

- $A_t(\tau)$ nominal savings earn $\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}$
- $\mathcal{W}_t(\tau)$ is nominal (differentiated) wage (baseline: set frictionlessly)
- Take as given CES labor demand, τ^w labor subsidy, transfers $dF_t(\tau)$

Key takeaway: consumption/labor choices differ when bond returns not equalized

Arbitrageurs

- Mean-variance optimization

$$\begin{aligned} & \max \mathbb{E}_t d\omega_t - \frac{a}{2} \text{Var}_t d\omega_t \\ \text{s.t. } & d\omega_t = \omega_t i_t dt + \int_0^T \chi_t^{(\tau)} \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - i_t dt \right) d\tau \end{aligned}$$

- Arbitrageurs invest $\chi_t^{(\tau)}$ in bond carry trade of maturity τ
- Remainder of wealth ω_t invested at the short rate
- Risk-return trade-off governed by a
 - Formally: risk aversion coefficient
 - More generally: proxies for any limits to risk-bearing capacity or intermediation frictions
 - Arbitrageurs transfer gains/losses to HHS, so a represents any frictions which hinder ability to trade on behalf of HHS

Key takeaway: risk averse arbitrageurs' holdings increase with expected return

Preferred Habitat Funds

- Habitat bond demand (exogenous) for maturity τ :

$$Z_t^{(\tau)} = -\alpha(\tau) \log P_t^{(\tau)} - \beta_t^{(\tau)}$$

- $\alpha(\tau)$: demand elasticity for τ fund
- $\beta_t^{(\tau)}$: additional time-varying (“noise”) demand factor
 - Noise demand $\beta_t^{(\tau)} = \theta(\tau)\beta_t$ follows a factor structure across habitat funds, eg

$$d\beta_t = -\kappa_\beta (\beta_t - \bar{\beta}) dt + \sigma_\beta dB_{\beta,t}$$

- $\theta(\tau)$: mapping from demand factor β_t to τ -habitat demand
- $Z_t^{(\tau)}$ financed at the short rate (zero-cost position)
- Habitat funds also transfer gains/losses to HHs

Key takeaway: habitat funds introduce noise; price movements require portfolio rebalancing

Government

- Central bank chooses policy rate i_t and bond holdings $S_t^{(\tau)}$
- Potentially subject to costs:

$$Y_t \Psi_t^S \equiv Y_t \int_0^T \frac{\psi^{(\tau)}}{2} \left(S_t^{(\tau)} \right)^2 d\tau, \quad Y_t \Psi_t^i \equiv Y_t \frac{\psi^i}{2} \left(i_t - \bar{i}_t \right)^2$$

- In the background: fiscal authority chooses production/labor subsidies τ^y, τ^w , balances the budget period by period
- Optimal policy: maximize social welfare

$$\max \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left(\int_0^T \eta(\tau) u(C_t(\tau), N_t(\tau)) d\tau \right) dt$$

Key takeaway: policy attempts to undo frictions:

1. Nominal pricing frictions \implies deadweight loss
2. Differentiated labor \implies production inefficiencies
3. Market segmentation \implies consumption dispersion, imperfect risk-sharing

Equilibrium

Equilibrium Overview

- Equilibrium bond price dynamics and arbitrageur optimality conditions:

$$\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} \equiv \mu_t^{(\tau)} dt + \sigma_t^{(\tau)} dB_t, \quad \mu_t^{(\tau)} - i_t = \sigma_t^{(\tau)} \boldsymbol{\Lambda}_t, \quad \boldsymbol{\Lambda}_t^\top = a \int_0^T \chi_t^{(\tau)} \sigma_t^{(\tau)} d\tau$$

- Term premia** depend on risk aversion a and equilibrium holdings $\chi_t^{(\tau)}$
- Approximation** around “small risk, low risk bearing capacity” (nonzero premia)
- The **first-best** allocation obtained when $\theta = 0$ and $a = 0$. Output gap $X_t \equiv \frac{Y_t}{Y_t^n}$ and inflation evolve according to (linearized)

$$dX_t = \varsigma^{-1} (\tilde{\mu}_t - \pi_t - r_t^n) dt$$

$$d\pi_t = (\rho\pi_t - \delta X_t) dt$$

- $r_t^n \equiv -\kappa_Z Z_t$ is the usual natural rate and $\tilde{\mu}_t$ is the **effective borrowing rate**:

$$\tilde{\mu}_t = \int_0^T \eta(\tau) \mu_t^{(\tau)} d\tau$$

- Up to **first-order**, our model is the same as Ray, Droste, & Gorodnichenko (2023) [details](#)

- Per-period social welfare loss (second-order expansion relative to first-best):

$$\begin{aligned}\mathcal{L}_t \equiv & (\varsigma + \varphi)x_t^2 + \theta\pi_t^2 \\ & + \frac{\varsigma}{\varphi} \left(\varphi + \varsigma \left[\frac{\varphi\epsilon_w}{1 + \varphi\epsilon_w} \right]^2 \right) \mathbb{V}\text{ar}_\tau c_t(\tau) + \epsilon_w \mathbb{V}\text{ar}_\tau w_t(\tau) \\ & + \int_0^T \psi^{(\tau)} \left(S_t^{(\tau)} \right)^2 d\tau + \psi^j \left(i_t - \bar{i}_t \right)^2\end{aligned}$$

- First line: losses from **nominal rigidities** (same as in textbook RANK)
- Next line: losses also depends on **consumption and wage dispersion** across HHs
- Final line: losses from policy frictions (when $\psi^j > 0, \psi^{(\tau)} > 0$)

Aggregate and Welfare Consequences: Simple Policy Rules

- In order to better understand the model, simplify to a version of the model which only includes **natural rate shocks** r_t^n

$$dr_t^n = -\kappa_z r_t^n dt + \sigma_r dB_{z,t}$$

- Consider **policy rules** which implement

$$i_t = \chi_i r_t^n$$
$$S_t^{(\tau)} = \chi_S^{(\tau)} r_t^n$$

- **Simple policy rules**: function of natural state variables only
 - Time-consistent: policymaker seeks to minimize **unconditional** social welfare loss
- We will examine the outcome of these policies in different versions of the model

Risk Neutral Arbitrageur

Benchmark: Risk Neutral Arbitrageur (“Standard Model”)

- Consider the benchmark case of a risk neutral arbitrageur: $a = 0$
- The **expectations hypothesis** holds: $\mu_t^{(\tau)} = i_t \implies$ model collapses to **RANK**

$$\text{Var}_{\tau} c_t(\tau) = 0, \quad \text{Var}_{\tau} w_t(\tau) = 0$$

- Recover the standard **QE neutrality result**: balance sheet policies do not affect bond prices (and therefore have no aggregate effects)
- **Divine coincidence holds**: conventional policy can achieve first-best

$$\chi_i = 1 \implies \mu_t^{(\tau)} = r_t^n \implies x_t = \pi_t = 0$$

- ‘**Woodford-ian**’ **equivalence**: the role of policy on aggregate dynamics and welfare is fully summarized by policy rate i_t

Imperfect Arbitrage

Imperfect Arbitrage

- Now assume $a > 0$ and the central bank continues to implement $i_t = r_t^n$

Proposition (Imperfect Arbitrage and Ad-hoc Policy)

Assume risk aversion $a > 0$ and price elasticities $\alpha(\tau) > 0$

Bond markets: bond carry trade return $\mu_t^{(\tau)} - i_t$

- Decreases with the short rate i_t
- Decreases with QE shocks $S_t^{(QE)}$

Aggregate dynamics: output gaps x_t and inflation π_t

- Not identically zero: $\text{Var } x_t \neq 0$ and inflation $\text{Var } \pi_t \neq 0$;
- QE increases the output gap and inflation

Dispersion: consumption and wage dispersion $\text{Var}_\tau c_t(\tau) \neq 0, \text{Var}_\tau w_t(\tau) \neq 0$

Imperfect Arbitrage Intuition: Policy Pass-Through

- Consider a fall in the natural rate inducing a cut in the policy rate:
 - When $\downarrow i_t$, bond arbitrageurs want to invest more in the BCT
 - \implies bond prices increase $\uparrow P_t^{(\tau)}$
 - As $\uparrow P_t^{(\tau)}$, price-elastic habitat bond investors ($\alpha(\tau) > 0$) reduce their holdings: $\downarrow Z_t^{(\tau)}$
 - Bond arbitrageurs increase their holdings $\uparrow X_t^{(\tau)}$, which requires a larger BCT return
- Now consider a QE shock
 - QE purchases: $\uparrow S_t^{(\tau)}$
 - Bond arbitrageurs reduce holdings $\downarrow X_t^{(\tau)}$, reducing risk exposure and pushing down yields

Imperfect Arbitrage Intuition: Aggregate Effects

- Therefore, a shock to the natural rate does not fully pass through to the effective borrowing rate $\tilde{\mu}_t \neq i_t$
 - Thus aggregate borrowing demand changes, and hence $x_t \neq 0$
 - Through the NKPC, $\pi_t \neq 0$
- On the other hand, a QE shock stimulates the economy
 - QE reduces borrowing rates $\downarrow \tilde{\mu}_t$ and therefore stimulates aggregate consumption $\uparrow x_t$
 - Through the NKPC, inflation $\uparrow \pi_t$
- Additionally, in general $\mu_t^{(\tau)} \neq \mu_t^{(\tau')}$
 - Hence individual Euler equations differ
 - $\implies c_t(\tau) \neq c_t(\tau'), n_t^{(\tau)} \neq n_t(\tau')$ and therefore $\text{Var}_\tau c_t(\tau) \neq 0, \text{Var}_\tau w_t(\tau) \neq 0$

Optimal Policy

Imperfect Arbitrage and Macroeconomic Stabilization

- Can conventional policy alone close the output gap?
- Yes but the short rate must react **more than one-for-one** with the natural rate:

$$\exists \chi_i^n > 1 : i_t = \chi_i^n r_t^n \implies \tilde{\mu}_t = r_t^n$$

- However, this does not achieve first-best since $\text{Var}_\tau c_t(\tau) \neq 0, \text{Var}_\tau w_t(\tau) \neq 0$
- In fact, relative to the policy $i_t = r_t^n$, in general we have $\uparrow \text{Var}_\tau c_t(\tau), \uparrow \text{Var}_\tau w_t(\tau)$
 - Short rate is **more volatile**, hence \uparrow term premia volatility
 - This implies **higher dispersion across borrowing rates** $\mu_t^{(\tau)}$ and therefore an increase in consumption/labor dispersion
- **Optimal short rate policy**: if $\psi^{(\tau)} \rightarrow \infty$, then optimal policy implements

$$i_t = \chi_i^* r_t^n, \quad \chi_i^* < \chi_i^n \implies \frac{\partial \tilde{\mu}_t}{\partial r_t^n} < 1$$

Imperfect Arbitrage and Macro-Financial Stabilization

- With access to frictionless **balance sheet policies**, we obtain the following

Proposition (Optimal Policy Separation Principle)

Assume risk aversion $a > 0$ and price elasticities $\alpha(\tau) > 0$, and policy costs $\psi^i = \psi^{(\tau)} = 0$. Suppose the central bank implements short rate and balance sheet policy according to

$$\begin{aligned} i_t &= r_t^n \\ S_t^{(\tau)} &= \alpha(\tau) \log P_t^{(\tau)} \end{aligned}$$

Then first-best is achieved:

- **Macroeconomic stabilization:** $x_t = \pi_t = 0 \ \forall t$
- **Financial stabilization:** $\mu_t^{(\tau)} = \tilde{\mu}_t \ \forall \tau$
- **Consumption and wage equalization:** $\text{Var}_\tau c_t(\tau) = 0, \text{Var}_\tau w_t(\tau) = 0 \ \forall t$

Optimal Policy Separation Principle: Intuition

- The results follow naturally from our findings regarding ad-hoc policy
- QE policy **stabilizes shocks to bond markets** by offsetting all habitat portfolio rebalancing shocks:

$$S_t^{(\tau)} = -Z_t^{(\tau)} \implies \sigma_t^{(\tau)} \Lambda_t = 0$$

- This **equalizes borrowing rates** across HHs: $\mu_t^{(\tau)} = \tilde{\mu}_t$
- Hence the model collapses to a standard RANK model, in which case divine coincidence implies $i_t = r_t^n$ is optimal

Separation principle for optimal policy:

- Optimal balance sheet policy **stabilizes financial markets**
- Optimal short rate policy **stabilizes macroeconomic aggregates**

Separation Principle with Balance Sheet Constraints

- When the central bank faces **balance sheet constraints** ($\Psi^{(\tau)} > 0$), policy can no longer achieve first-best
- However, as long as $\Psi^{(\tau)} < \infty$, optimal policy implies the central bank still uses balance sheet tools
- Let $\Psi^{(\tau)} = a \cdot \sigma^{(\tau)} [\sigma^{(\tau)}]^\top$
 - \implies same friction a as arbitrageurs, except policymaker **cannot net out** positions
- Even with “large” balance sheet costs, the central bank still uses QE to (partially) stabilize term premia
- **Intuition:**
 - The central bank faces holding costs which imply it is **worse than private arbitrageurs** at financial intermediation
 - But the central bank **internalizes the social benefits** of minimizing fluctuations in term premia
 - Nevertheless, non-negligible balance sheet costs imply that optimal policy is less reactive

Financial Stabilization Policy with Short Rate Constraints

- Suppose that **short rate policy is constrained**, and implements

$$i_t = \tilde{\chi}_i r_t^n, \quad 0 < \tilde{\chi}_i < 1$$

- Formally: assume costs $\psi^i (i_t - \tilde{\chi}_i r_t^n)$ and take $\psi^i \rightarrow \infty$
- If the central bank continues to implement the balance sheet policy derived above, then **borrowing rates are still equalized** $\mu_t^{(\tau)} = \tilde{\mu}_t$
- However, $\tilde{\mu}_t \neq r_t^n$ and hence this policy does not achieve **macroeconomic stabilization**

$$x_t \neq 0, \pi_t \neq 0$$

Macroeconomic Stabilization with Short Rate Constraints

- Can balance sheet policy alone close the output gap?
- Yes but must sacrifice equalizing borrowing rates:

$$\mu_t^{(\tau)} = i_t + \sigma_t^{(\tau)} \lambda_t$$

$$\lambda_t \equiv a \int_0^T \left[\alpha(\tau) \log P_t^{(\tau)} - S_t^{(\tau)} \right] \sigma_t^{(\tau)} d\tau$$

- Hence, can always choose $\{S_t^{(\tau)}\}$ such that

$$\lambda_t^* = \frac{r_t^n - i_t}{\int_0^T \eta(\tau) \sigma_t^{(\tau)} d\tau} \implies \tilde{\mu}_t = r_t^n$$

- However, because $\sigma_t^{(\tau)} \neq \sigma_t^{(\tau')}$ this necessitates

$$\mu_t^{(\tau)} = i_t + \sigma_t^{(\tau)} \left(\frac{r_t^n - i_t}{\int_0^T \eta(\tau') \sigma_t^{(\tau')} d\tau'} \right) \neq r_t^n \quad (\text{unless } i_t = r_t^n)$$

Stabilization with Short Rate Constraints: Intuition

- Balance sheet policy works by affecting **term premia** through changes in the **market price of risk**
- Although arbitrage is imperfect in this model, arbitrageurs still enforce **tight restrictions** between between market price of risk and term premia across maturities
- Hence, while in principle the central bank has a **continuum of policy tools** $\{s_t^{(\tau)}\}$, in practice it can **only manipulate** λ_t
- Related to **localization results** in Vayanos & Vila (2021) and Ray, Droste, & Gorodnichenko (2023)
 - In the one-factor model considered here, the effects of QE are **fully global**
 - Even with more complicated risk structure, localization is not strong enough to allow balance sheet policy alone to achieve first-best

Extensions Overview

- “Noise” Demand Shocks details
 - Optimal separation principle still holds with stochastic habitat demand $\beta_t^{(\tau)}$, but QE must be more reactive
 - **Additional result**: if noise demand dynamics are such that $\uparrow\uparrow \beta_t^{(\tau)}$ in response to $\uparrow r_t^n$, then it is optimal to **expand** the balance sheet $\uparrow S_t^{(\tau)}$ while hiking rates $\uparrow i_t$
- Cost-Push Shocks details
 - Adding shocks to NKPC (eg, wage rigidity in labor markets) breaks divine coincidence but unfortunately, our separation principle still holds
 - Despite multiple policy instruments, (un)conventional policy only affects aggregate outcomes through changes in **effective borrowing rate** $\tilde{\mu}_t$
- Nonzero First-Best Term Premia details
 - When **first-best** BCT returns are $\nu^{(\tau)} \neq 0$
 - Our results hold when $\nu^{(\tau)}$ is achievable but optimal short rate policy is a function of $\nu^{(\tau)}$

History-Dependent Policy

Monetary Policy with Commitment

- When policy frictions bind, simple policy rules fail to achieve first-best
- Instead suppose the policymaker chooses policy tools \mathbf{u}_t as a function of **entire history** of predetermined and nonpredetermined variables $\mathbf{Y}_t \equiv \begin{bmatrix} \mathbf{y}_t^\top & \mathbf{x}_t^\top \end{bmatrix}^\top$
- Minimizes conditional social loss

$$\begin{aligned}\mathcal{W}_0 &= \frac{1}{2} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \mathcal{L}_t dt \\ &= \frac{1}{2} \mathbb{E}_0 \int_0^\infty e^{-\rho t} (\mathbf{Y}_t^\top \mathbf{R} \mathbf{Y}_t + \mathbf{u}_t^\top \mathbf{Q} \mathbf{u}_t) dt, \quad \mathbf{y}_0 \text{ given}\end{aligned}$$

- By setting policy in a history-dependent manner, full commitment policies can improve on simple policy rules

Characterizing Optimal Policy with Commitment

Theorem (Optimal Policy with Commitment)

Given \mathbf{y}_0 , the policymaker minimizes \mathcal{W}_0 by choosing $\mathbf{u}_t = \mathbf{F}\mathbf{Y}_t$, which induce equilibrium dynamics $d\mathbf{Y}_t = -\mathbf{\Upsilon}(\mathbf{F})\mathbf{Y}_t dt + \mathbf{S}(\mathbf{F}) d\mathbf{B}_t$. Necessary conditions are given by

$$\mathbf{y}_0^\top \left(\partial_i \mathbf{P}_{11} - \partial_i \mathbf{P}_{12} \mathbf{P}_{22}^{-1} \mathbf{P}_{21} - \mathbf{P}_{12} \mathbf{P}_{22}^{-1} \partial_i \mathbf{P}_{21} + \mathbf{P}_{12} \left(\mathbf{P}_{22}^{-1} \partial_i \mathbf{P}_{22} \mathbf{P}_{22}^{-1} \right) \mathbf{P}_{21} \right) \mathbf{y}_0 = 0$$

where $\rho \mathbf{P} = \mathbf{R} + \mathbf{F}^\top \mathbf{Q} \mathbf{F} - \mathbf{P} \mathbf{\Upsilon} - \mathbf{\Upsilon}^\top \mathbf{P}$. Dynamics are given by $\mathbf{q}_0 = \begin{bmatrix} \mathbf{y}_0 & \mathbf{0} \end{bmatrix}^\top$ and

$$d\mathbf{q}_t = - \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \mathbf{\Upsilon} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{P}_{22}^{-1} \mathbf{P}_{21} & \mathbf{P}_{22}^{-1} \end{bmatrix} \mathbf{q}_t dt + \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \mathbf{S} d\mathbf{B}_t \equiv -\mathbf{\Gamma} \mathbf{q}_t dt + \mathbf{\sigma} d\mathbf{B}_t$$

Bond prices are affine in $\mathbf{A}(\tau)^\top \mathbf{q}_t$ with $\mathbf{A}(\tau) = [\mathbf{I} - e^{-\mathbf{M}\tau}] \mathbf{M}^{-1} \mathbf{e}_i$ and

$$\mathbf{e}_i^\top \mathbf{q}_t = i_t, \quad \mathbf{M} = \mathbf{\Gamma}^\top - \int_0^T [-\alpha(\tau) \mathbf{A}(\tau) + \mathbf{\Theta}(\tau)] \mathbf{A}(\tau)^\top d\tau \tilde{\mathbf{\Sigma}}$$

Monetary Policy with Commitment: Intuition

- Policymaker chooses tools $i_t, \{S_t^{(\tau)}\}$ which:
 - Directly affect optimality conditions of arbitrageurs
 - Indirectly affect HHs through changes in equilibrium borrowing rates
 - Indirectly affect firms through changes in marginal costs
- **Trade-off**: more aggressive policy reactions to shocks:
 - Greater pass-through to HHs
 - Larger and more volatile term premia
- Commitment partially relaxes this link:
 - HH decisions depend on entire expected path of borrowing rates $\int_0^\infty \mu_t^{(\tau)} d\tau$
 - Arbitrageur risk compensation depends on volatility of short-run fluctuations $di_t, dS_t^{(\tau)}$
- Characterizing dynamics of optimal policy with commitment is difficult
 - Ongoing work studies optimal policy numerically
 - Suffers from time inconsistency; simple rules may be more practical

Concluding Remarks

- This paper develops a tractable general equilibrium model with market segmentation, nominal frictions, and household heterogeneity
- We find that optimal short rate and balance sheet policy is characterized by a sharp **optimal separation result**:
 - Conventional policy targets **macroeconomic stability**
 - Unconventional policy targets **financial stability**
- Optimal policy removes excess volatility of risk premia and hence improves risk-sharing across households, while reducing excess macroeconomic volatility
 - Policy constraints on either the short rate or balance sheets imply trade-offs between these two policy objectives
- When considering social welfare, **cannot abstract from the policy tools** used to conduct monetary policy

Thank You!

Equilibrium Details

Aggregation

- Firms, arbitrageurs, and funds transfer profits equally to HHs
- **Symmetric firm equilibrium** $Y_t(j) = Y_t, P_t(j) = P_t, \pi_t(j) = \pi_t, \frac{dP_t}{P_t} = \pi_t dt$
- Clearing in production and goods markets:

$$Y_t = Z_t L_t \equiv Z_t \left[\int_0^T \eta(\tau) N_t(\tau)^{\frac{\epsilon_W - 1}{\epsilon_W}} d\tau \right]^{\frac{\epsilon_W}{\epsilon_W - 1}}$$
$$C_t \equiv \int_0^T \eta(\tau) C_t(\tau) d\tau = Y_t \left(1 - \frac{\theta}{2} \pi_t^2 - \psi_t^S - \psi_t^i \right)$$

- **Bond market clearing** implies

$$\chi_t^{(\tau)} + Z_t^{(\tau)} + \eta(\tau) A_t(\tau) + S_t^{(\tau)} = 0$$

Optimality Conditions

- Equilibrium bond price dynamics:

$$\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} \equiv \mu_t^{(\tau)} dt + \sigma_t^{(\tau)} dB_t$$

- B_t collects innovations to risk factors (technology, noise demand, ...)
- Arbitrageur optimality conditions:

$$\mu_t^{(\tau)} - i_t = \sigma_t^{(\tau)} \Lambda_t, \quad \Lambda_t^\top = a \int_0^T \chi_t^{(\tau)} \sigma_t^{(\tau)} d\tau$$

- Term premia depend on risk aversion a and equilibrium holdings $\chi_t^{(\tau)}$
- HH optimality conditions (log-linearized) :

$$w_t = \varsigma c_t(\tau) + \phi n_t(\tau) + \frac{1}{\epsilon_w} (n_t(\tau) - \ell_t), \quad \mathbb{E}_t dc_t(\tau) = \varsigma^{-1} \left(\mu_t^{(\tau)} - \pi_t - \rho \right) dt$$

- Firm optimality conditions (log-linearized):

$$\mathbb{E}_t d\pi_t = (\rho \pi_t - \delta_w w_t) dt$$

Simplifying Assumptions

- **Tractability assumption:** a “head of HH” sets transfers such that in equilibrium, wealth is equalized: across τ HH groups, $A_t(\tau) \equiv A_t$
 - Pros: clear focus on the role market segmentation plays on consumption dispersion
 - Cons: ignores the impact of market segmentation on wealth inequality
- **Approximation:** around a limiting case: risk $\hat{\sigma}_t^{(\tau)} \equiv \hat{h}^{\frac{1}{2}} \cdot \sigma_t^{(\tau)} \rightarrow \mathbf{0}$ but arbitrageur risk aversion $\hat{a} \equiv a/\hat{h} \rightarrow \infty$ such that $\hat{a}^{\frac{1}{2}} \cdot \hat{\sigma}_t^{(\tau)} \equiv a^{\frac{1}{2}} \cdot \sigma_t^{(\tau)}$ remains non-zero and bounded
 - Pros: clear focus on the idea of “imperfect arbitrage”
 - Cons: less realistic risk premia (particularly in first-best)
- Allows us to derive simple first-order approximations of dynamics and second-order approximations for welfare

Aggregate Dynamics

- The **first-best** (natural) allocation obtained when $\theta = 0$ and $a = 0$. Output gap:

$$X_t \equiv \frac{Y_t}{Y_t^n}$$

- Output gap evolves according to **modified aggregate Euler equation**:

$$dX_t = \varsigma^{-1} (\tilde{\mu}_t - \pi_t - r_t^n) dt$$

- $r_t^n \equiv -\kappa_z Z_t$ is the usual natural rate and $\tilde{\mu}_t$ is the **effective borrowing rate**:

$$\tilde{\mu}_t = \int_0^T \eta(\tau) \mu_t^{(\tau)} d\tau$$

- We recover a **standard NKPC**:

$$d\pi_t = (\rho\pi_t - \delta X_t) dt$$

Extensions

Extensions: “Noise” Demand Shocks

- We obtain identical results when allowing for shocks to habitat demand $\beta_t^{(\tau)}$
- Optimal separation principle still holds with $\psi^{(\tau)} = 0$, but QE must be more reactive:

$$S_t^{(\tau)} = \alpha(\tau) \log P_t^{(\tau)} + \beta_t^{(\tau)}$$

- Optimal short rate policy still implements $i_t = r_t^n$
- **Additional result:** if noise demand dynamics are such that $\uparrow\uparrow \beta_t^{(\tau)}$ in response to $\uparrow r_t^n$, then it is optimal to **expand** the balance sheet $\uparrow S_t^{(\tau)}$ while hiking rates $\uparrow i_t$
- **Intuition:**
 - Suppose during a **hiking cycle** and in the absence of QE we have an **increase in term premia**
 - Then the optimal balance sheet policy is to conduct **additional QE purchases** in order to offset spike in term premia
 - \implies conventional and unconventional policy **seem to be at odds** with one another
 - Otherwise, short rate policy and balance sheet policy tend to be reinforcing

Extensions: Cost-Push Shocks

- What if divine coincidence does not hold? Eg, wage rigidity in labor markets
- More generally, introduce exogenous **cost-push shocks** u_t in NKPC:

$$d\pi_t = (\rho\pi_t - \delta x_t - u_t) dt$$

- Unfortunately, our **separation principle** still holds:
 - Optimal QE stabilizes term premia
 - Short rate policy must contend with the output gap/inflation trade-offs
- **Intuition**: despite multiple policy instruments, (un)conventional policy only affects aggregate outcomes through changes in **effective borrowing rate** $\tilde{\mu}_t$
 - Take any feasible path $\{x_t, \pi_t, \tilde{\mu}_t\}_t$ from an implementation implying policies $\{\hat{i}_t, \hat{S}_t^{(\tau)}\}_t$
 - Can also be achieved with $i_t = \tilde{\mu}_t, S_t^{(\tau)} = \alpha(\tau) \log P_t^{(\tau)} + \beta_t^{(\tau)}$
 - This guarantees $\mathbb{V}\text{ar}_\tau c_t(\tau) = \mathbb{V}\text{ar}_\tau w_t(\tau) = 0$ and hence strictly dominates

Extensions: Non-Zero First-Best Carry Trade Returns

- Our approximation approach implies that in the first-best, expected carry trade returns are zero
- This simplifies our analytical results but of course is an extreme assumption
- Suppose instead that **first-best** BCT returns are $\nu^{(\tau)} \neq 0$
- Our **separation principle still holds** when $\nu^{(\tau)}$ is achievable but optimal short rate policy is a function of $\nu^{(\tau)}$
- **Intuition:** combination of previous results
 - Aggregate outcomes through changes in **effective borrowing rate** $\tilde{\mu}_t$ (as before)
 - Optimal QE policy guarantees $\mu_t^{(\tau)} - i_t \equiv \nu^{(\tau)}$ and hence $\tilde{\mu}_t = i_t + \int_0^T \eta(\tau) \nu^{(\tau)} d\tau \equiv i_t + \tilde{\nu}$
 - Thus, optimal short rate policy implements $i_t = r_t^n - \tilde{\nu}$