Macro-Financial Preferred Habitat Models

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November 2024
USC Macro-Finance Reading Group

Motivation

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Bernanke: "QE works in practice but not in theory"

- *"...in practice..."*
  - Effects on asset prices: Treasuries, MBS, ...?
  - State-dependence? Mean reversion? Stock vs. flow?
  - Risk premia vs. wealth effects vs. demand for reserves vs. ...?
  - Transmission of non-Fed-initiated purchases?
- *"QE works..."*
  - Effects on real outcomes?
  - Unintended consequences of QE?
  - Long-run size of balance sheet?
  - Optimal design of QE (along with conventional policy and FG)?
- *"...but not in theory"*
  - Many theories of QE transmission; how to validate/falsify?
  - Frontiers of "preferred habitat" models of market segmentation

Motivation

- Today: focus on an overview of macro-finance preferred habitat models
- Question: how are bond prices determined?
  - Macro-finance 101: by consumption smoothing of representative household
  - Naive preferred habitat: demand and supply of investor clienteles for bonds of specific maturities
- Modern preferred habitat answer: interaction of clientele investors and arbitrageurs with limited risk-bearing capacity
  - "Asset demand systems" and "intermediary asset pricing"
- Transmission mechanisms of QE:
  - ...across assets (localization)
  - ...to the real economy
  - ...through intermediary risk-bearing capacity
  - ...and others

Preferred Habitat Model Set-Up

Vayanos-Vila in 2 slides

- Continuum of zero coupon bonds with maturity $0 \leq \tau \leq T \leq \infty$, price $P_t^{(\tau)}$
- Arbitrageurs: mean-variance optimization

$$\max \mathbb{E}_t \,\mathrm{d}\omega_t - \frac{a}{2} \mathbb{V}\mathrm{ar}_t \,\mathrm{d}\omega_t \,, \quad \text{s.t. } \mathrm{d}\omega_t = \omega_t i_t \,\mathrm{d}t + \int_0^T X_t^{(\tau)} \left(\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} - i_t \,\mathrm{d}t \right) \mathrm{d}\tau$$

- Habitat bond demand for maturity τ:

$$Z_t^{(\tau)} = -\alpha(\tau) \log P_t^{(\tau)} - \beta_t^{(\tau)}$$

- Exogenous nominal short rate $i_t = \lim_{\tau \to 0} y_t^{(\tau)}$. Endogenous bond prices:

$$\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} \equiv \mu_t^{(\tau)} \,\mathrm{d}t + \boldsymbol{\sigma}_t^{(\tau)} \,\mathrm{d}\mathbf{B}_t$$

  - $\mathbf{B}_t$ collects innovations to risk factors affecting short rate i_t, demand $\beta_t^{(\tau)}$
- Arbitrageur optimality conditions and market clearing:

$$\mu_t^{(\tau)} - i_t = \boldsymbol{\sigma}_t^{(\tau)} \boldsymbol{\Lambda}_t, \quad \boldsymbol{\Lambda}_t^\top = a \int_0^T X_t^{(\tau)} \boldsymbol{\sigma}_t^{(\tau)} \,\mathrm{d}\tau = -a \int_0^T Z_t^{(\tau)} \boldsymbol{\sigma}_t^{(\tau)} \,\mathrm{d}\tau$$

Vayanos-Vila in 2 slides

- Key insight: risk premia move if arbitrageur risk exposure changes
  - Market price of risk depends on risk aversion, volatility, and aggregate positions
  - Quantity shocks change prices through portfolio rebalancing
  - Other shocks (eg, short rate) affect prices and thus also lead to rebalancing
- Key insight: state-dependence
  - If risk-bearing capacity is high: QE shocks have global effects
  - If risk-bearing capacity is low: QE shocks have localized effects
- What's missing?
  - Other assets/risks?
    - Corporate bonds [Ray, Droste, Gorodnichenko 2024]; international bonds [Greenwood, Hanson, Stein, Sunderam 2023; Gourinchas, Ray, Vayanos 2024]; sovereign default risk [Costain, Nuno, Thomas 2024]
  - Other "limits to arbitrage"?
    - Arbitrageur wealth [Kekre, Lenel, Mainardi 2024]; repo markets [Pelizzon, Ruggero, Subrahmanyam 2024; Wu 2024]; holding costs [Ray, Vayanos 2024]
  - Regime selection and multiple equilibria [Droste, Gorodnichenko, Ray 2024]
  - General equilibrium and welfare [Kamdar, Ray 2024]

Vayanos-Vila Extensions

- Other assets? $\implies$ spillovers across maturities *and asset classes*

$$\boldsymbol{\Lambda}_t^\top = a \int_0^T \left(X_t^{(\tau)} \boldsymbol{\sigma}_t^{(\tau)} + \tilde{X}_t^{(\tau)} \boldsymbol{\sigma}_t^{(\tau)} \right) \mathrm{d}\tau$$

- Other risks? Poisson default risk $\implies$ changes to duration risk *and default risk*

$$\mathrm{d}\omega_t = \omega_t i_t \,\mathrm{d}t + \int_0^T X_t^{(\tau)} \left(\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} - \delta \,\mathrm{d}N_t - i_t \,\mathrm{d}t \right) \mathrm{d}\tau$$

- Other limits to arbitrage? CRRA vs CARA $\implies$ arbitrageur wealth is a *state variable*

$$v(\omega_0) = E_0 \int_0^\infty e^{-\rho t} \frac{\omega_t^{1-\varsigma}}{1-\varsigma} \,\mathrm{d}t$$

- Other limits to arbitrage? Holding costs $\implies$ higher degree of localization, apparent deviations from risk-neutral arbitrage

$$\mathrm{d}\omega_t = \omega_t i_t \,\mathrm{d}t + \int_0^T X_t^{(\tau)} \left(\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} - i_t \,\mathrm{d}t \right) \mathrm{d}\tau - \int_0^T \phi(\tau) \left[X_t^{(\tau)} \right]^2 \mathrm{d}\tau$$

Solution Details

Overview: Solving the Model (Gourinchas, Ray, Vayanos 2024)

1. Collect state variables $\mathbf{q}_t \equiv \begin{bmatrix} i_{Ht} & i_{Ft} & \beta_{Ht} & \beta_{Ft} & \gamma_t \end{bmatrix}^\top$. Vector OU (exogenous):

$$\mathrm{d}\mathbf{q}_t = -\boldsymbol{\Gamma}\left(\mathbf{q}_t - \overline{\mathbf{q}}\right)\mathrm{d}t + \boldsymbol{\sigma}\,\mathrm{d}\mathbf{B}_t$$

2. Conjecture affine (log) prices:

$$-\log P_{jt}^{(\tau)} = \mathbf{A}_j(\tau)^\top \mathbf{q}_t + C_j(\tau)$$
$$-\log e_t = \mathbf{A}_e^\top \mathbf{q}_t + C_e$$

3. Ito's Lemma + arbitrageur optimality conditions pins down excess returns as a function of arbitrageur holdings
4. Market clearing + habitat demand characterizes the solution to the unknown coefficients $\mathbf{A}_j(\tau), \mathbf{A}_e, \ldots$
5. Solve! (confirm affine conjecture holds)

Details: Solution Characterization

- Substitute market clearing into arbitrageur optimality conditions, collect $\mathbf{q}_t$ terms:

$$\mathsf{A}_j'(\tau) + \mathsf{M}\mathsf{A}_j(\tau) - \mathbf{e}_j = \mathbf{0}, \quad \mathsf{M}\mathsf{A}_e - (\mathbf{e}_H - \mathbf{e}_F) = \mathbf{0} \quad (\text{where } \mathbf{e}_j^\top \mathbf{q}_t = i_{jt})$$

- The matrix M is defined as

$$\begin{aligned}\mathsf{M} = \mathbf{\Gamma}^\top - a\Big\{ &\int_0^T \left[-\alpha_H(\tau)\mathsf{A}_H(\tau) + \mathbf{\Theta}_H(\tau)\right] \mathsf{A}_H(\tau)^\top \,\mathrm{d}\tau \\ &+ \int_0^T \left[-\alpha_F(\tau)\mathsf{A}_F(\tau) + \mathbf{\Theta}_F(\tau)\right] \mathsf{A}_F(\tau)^\top \,\mathrm{d}\tau \\ &+ \left[-\alpha_e \mathsf{A}_e + \mathbf{\Theta}_e\right] \mathsf{A}_e^\top \Big\} \mathbf{\Sigma}\end{aligned} \tag{1}$$

- Initial conditions $\mathsf{A}_j(0) = \mathbf{0}$. Hence

$$\mathsf{A}_j(\tau) = \left[\mathbf{I} - e^{-\mathsf{M}\tau}\right] \mathsf{M}^{-1}\mathbf{e}_j \tag{2}$$

$$\mathsf{A}_e = \mathsf{M}^{-1}(\mathbf{e}_H - \mathbf{e}_F) \tag{3}$$

Details: Existence and Uniqueness

- Note: $\mathbf{M}$ appears on both sides of equation (1), through the solution of the affine coefficients (2), (3)
    - Interpretation: risk-adjusted dynamics of the risk factors
- In general: system of J^2 nonlinear equations in J^2 unknowns, where $J = \dim \mathbf{q}_t$
- Under risk neutrality ($a = 0$), the solution is simple: $\mathbf{M} = \mathbf{\Gamma}^\top$
- When $a > 0$, the solution may not exist, or there may be multiple equilibria
- Can show (using IFT) that in a neighborhood of $a = 0$, the solution exists and is (locally) unique. Beyond that, very difficult to prove anything analytically in the fully general version of the model
    - See (soon!) Droste, Gorodnichenko, Ray (2024): policy as *equilibrium selection*
    - Related to DeLong, Shleifer, Summers Waldmann (1989)

Numerical Solution: Algorithm

- How to solve for $\mathbf{M}$ in the general model?
- Continuation algorithm:
  1. For $\hat{a} = \hat{a}^{(0)} = 0$, the known solution is $\mathbf{M}^{(0)} = \mathbf{\Gamma}^\top$
  2. Given a solution $\mathbf{M}^{(n)}$ for $\hat{a} = \hat{a}^{(n)}$, use this as the initial value for $\hat{a}^{(n+1)} = \hat{a}^{(n)} + \epsilon$
  3. Solution $\mathbf{M}^{(N)} = \mathbf{M}$ for $\hat{a}^{(N)} = a$
- Notes:
  - Many many ways to make this more sophisticated (see "homotopy continuation")
  - In step 2: any fixed point/root finding algorithm can be used (which can exploit the structure of the problem; again see homotopy continuation)
- For our purposes, we use a fine grid (small fixed step size ϵ) for two reasons:
  1. The code is fast (enough)
  2. The algorithm doubles as an equilibrium selection criteria: we trace out the solution which uniquely converges to the risk-neutral benchmark when $a \to 0$

Numerical Solution: Laplace Transformations

- In order to solve the model numerically, we need to parameterize the habitat functions $\alpha_j(\tau), \theta_j(\tau)$. Our approach:

$$\alpha_j(\tau) = \alpha_{j0} e^{-\alpha_{j1}\tau}$$
$$\theta_j(\tau) = \theta_{j0} \tau e^{-\theta_{j1}\tau}$$

    - Implies price elasticities are declining in τ, yield elasticities are single peaked
    - Demand functions are single-peaked
- If we take maximum maturity $T \to \infty$, then we can use properties of Laplace transforms to simplify the fixed point problem characterizing M
- Turns diff-eqs into algebraic (gets rid of matrix exponentials): $\mathcal{A}(s) \equiv \mathcal{L}\left\{\mathsf{A}(\tau)\right\}(s)$ given by:

$$s\mathcal{A}(s) + \mathsf{M}\mathcal{A}(s) - \frac{1}{s}\mathbf{e}_i = \mathbf{0} \implies \mathcal{A}(s) = \left[s\mathbf{I} + \mathsf{M}\right]^{-1}\left[\frac{1}{s}\mathbf{e}_i\right]$$

- Can get rid of all the integral terms in the fixed-point problem for M (but still require the solution algorithm described above)

Towards General Equilibrium

PE vs. GE Solution

- Given some microfoundations...we eventually end up with (linearized) dynamics

$$\begin{bmatrix} \mathrm{d}\mathbf{y}_t \\ E_t \, \mathrm{d}\mathbf{x}_t \end{bmatrix} = -\boldsymbol{\Upsilon} \begin{bmatrix} \mathbf{y}_t - \bar{\mathbf{y}} \\ \mathbf{x}_t - \bar{\mathbf{x}} \end{bmatrix} \mathrm{d}t + \boldsymbol{\sigma} \, \mathrm{d}\mathbf{B}_t$$

- $\mathbf{y}_t$: state (predetermined) variables. $\mathbf{x}_t$: jump (non-predetermined) variables
- Assume REE determinacy conditions are met (number of eigenvalues of $\boldsymbol{\Upsilon}$ with positive real part is equal to $\dim \mathbf{y}_t$). Eigendecomposition:

$$\boldsymbol{\Upsilon} = \mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^{-1}, \quad \boldsymbol{\Lambda} = \begin{bmatrix} \boldsymbol{\Lambda}_1 & 0 \\ 0 & \boldsymbol{\Lambda}_2 \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} \end{bmatrix},$$

- Then REE dynamics (see Buiter 1984, cts time version of Blanchard-Kahn 1980)

$$\mathrm{d}\mathbf{y}_t = -\boldsymbol{\Gamma}\left(\mathbf{y}_t - \bar{\mathbf{y}}\right) \mathrm{d}t + \boldsymbol{\sigma} \, \mathrm{d}\mathbf{B}_t, \qquad \mathbf{x}_t - \bar{\mathbf{x}} = \boldsymbol{\Omega}\left(\mathbf{y}_t - \bar{\mathbf{y}}\right)$$
$$\boldsymbol{\Gamma} = \mathbf{Q}_{11}\boldsymbol{\Lambda}_1\mathbf{Q}_{11}^{-1}, \qquad \boldsymbol{\Omega} = \mathbf{Q}_{21}\mathbf{Q}_{11}^{-1}$$

  - Note: dynamics matrix $\boldsymbol{\Upsilon}$ may be a function of long-term bonds, exchange rates
  - Solution method is similar (but now $\boldsymbol{\Gamma}$ is endogenous)

Optimal Policy

Optimal Macro-Financial Stabilization in a New Keynesian Preferred Habitat Model

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Our Model

- This paper: develops a tractable general equilibrium model with market segmentation, nominal frictions, and household heterogeneity
- Arbitrageurs with imperfect risk-bearing capacity absorb supply and demand shocks in bond markets
- Clientele investors introduce a degree of market segmentation
    - Bonds of different maturities traded by specialized investors (eg pension funds, MMMF)
    - Arbitrageurs (eg hedge funds, broker-dealers) partly overcome segmentation
- Households have differentiated access to asset markets
    - Households borrow with bonds of different maturities (eg mortgages)
    - Introduces imperfect risk-sharing, consumption and labor dispersion across households
- Formally: embed a Vayanos-Vila model in a New Keynesian model, where households are heterogeneous in access to financial markets

Findings: Policy Transmission

- Key mechanisms of conventional monetary policy:
  - Changes in the short rate affect required rates of return of all assets
  - Interaction of arbitrageurs and investor clienteles leads to portfolio rebalancing
  - Implies variation in risk premia, imperfect transmission to households
- Key mechanisms of balance sheet policy:
  - Imperfect arbitrage breaks QE neutrality
  - Central bank asset purchases induce portfolio rebalancing and hence reduce risk premia
  - Vice versa for central bank asset sales (QT)
- Thus, short rate and balance sheet policies are substitutes when targeting aggregate borrowing rates
  - A fall in aggregate borrowing rates is stimulative for the usual NK reasons

Findings: Welfare Consequences

- If the policymaker only cares about macroeconomic stabilization, conventional and unconventional policies are essentially equivalent
  - Nominal rigidities $\implies$ welfare losses due to inflation volatility
  - Policy stabilizes inflation by keeping aggregate borrowing rates at some “natural” rate
  - Triumphalist view: even with short rate constraints, QE is equally effective
- However, both policies imply variation in risk premia
  - Excess fluctuations in risk premia lead to dispersion in borrowing rates
- Social welfare depends not only on macroeconomic fluctuations:
  - Imperfect risk sharing $\implies$ welfare losses from consumption dispersion
  - Labor market inefficiencies $\implies$ welfare losses from labor dispersion

Findings: Optimal Policy

- Hence, when policy is unconstrained we derive an **optimal separation result**:
  - Conventional policy targets macroeconomic stability
  - Unconventional policy targets financial stability
- However, when policy constraints bind, policy must balance trade-offs:
  - Balance sheet constraints: short rate must be less reactive in order to minimize financial disruptions (at the cost of macroeconomic stability)
  - Short rate constraints: QE must be used to offset macroeconomic shocks (at the cost of financial stability)
- With full commitment, forward guidance is welfare-improving (short rate and QE)
  - Policymaker uses the entire expected path of borrowing rates to minimize macroeconomic volatility
  - But reduces short-run fluctuations to keep risk premia volatility low
  - However, dynamics are complicated and suffer from time-inconsistency
- General message: implementation matters for welfare

Related Literature

- Preferred habitat models
  - Vayanos & Vila (2009, 2021), Ray, Droste, & Gorodnichenko (2024), Greenwood & Vayanos (2014), Greenwood et al (2016), King (2019, 2021) , Kekre, Lenel, & Mainardi (2024), ...
- Empirical evidence: QE and preferred habitat
  - Krishnamurthy & Vissing-Jorgensen (2012), D'Amico & King (2013), Li & Wei (2013), Cahill et al. (2013), King (2019), Fieldhouse et al (2018), Di Maggio et al (2020), Gali, Debortoli, & Gambetti (2020) , ...
- Macroeconomic QE models
  - Andres, Lopez-Salido, & Nelson (2004), Gertler & Karadi (2011), Curdia & Woodford (2011), Chen et al (2012), Gertler & Karadi (2013), Sims & Wu (2020), Karadi & Nakov (2020), Iovino and Sergeyev (2023), Carlstrom et al (2017), Ippolito et al (2018) , ...
- Market segmentation, macro-prudential monetary policy
  - Cui & Sterk (2021), Auclert (2016), Collard et al (2017), Kaplan et al (2017), Debortoli & Gali (2017) , ...
- International
  - Itskhoki & Mukhin (2022), Greenwood et al (2023), Gourinchas, Ray, & Vayanos (2022) , ...

Set-Up

Model Set-Up

- Continuous time New Keynesian model with embedded Vayanos-Vila bond markets
- Agents:
  - Firms: monopolistic competitors produce using labor, face nominal pricing frictions
  - Households: supply differentiated labor, consume, save via bond markets
  - Arbitrageurs: imperfect risk-bearing capacity, conduct bond carry trades
  - Habitat funds: buys and sell bonds of a specific maturity
- Policymakers:
  - Central bank: conducts short rate and balance sheet (QE) policy
  - Government: optimal subsidies, otherwise passive
- Bond markets:
  - Continuum of zero coupon bonds with maturity $0 \leq \tau \leq T \leq \infty$
  - Bond price $P_t^{(\tau)}$ with yield to maturity $y_t^{(\tau)} = -\log P_t^{(\tau)} / \tau$
  - Nominal short rate: in equilibrium, $i_t = \lim_{\tau \to 0} y_t^{(\tau)}$

Firms

- Continuum of intermediate goods $j \in [0,1]$ (and CES final good with elasticity ϵ)
- Linear production in differentiated labor $Y_t(j) = e^{z_t} L_t(j)$:

$$\mathrm{d}z_t = -\kappa_z z_t \,\mathrm{d}t + \sigma_z \,\mathrm{d}B_{t,z}\,, \quad L_t(j) = \left[\int_{h\in\mathcal{H}} L_t(j,h)^{\frac{\epsilon_w - 1}{\epsilon_w}} \,\mathrm{d}h\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

- Face costs $\Theta(\pi_t(j)) = \frac{\theta}{2}\pi_t(j)^2 P_t Y_t$ when setting prices $\frac{\mathrm{d}P_t(j)}{P_t(j)} = \pi_t(j)\,\mathrm{d}t$. Maximizes:

$$U_0 \equiv \max \mathbb{E}_0 \int_0^\infty e^{-\rho t} Q_t^{\mathcal{H}} \frac{\mathcal{F}_t}{P_t} \,\mathrm{d}t$$

$$\text{s.t. } \mathcal{F}_t = (1+\tau^y) P_t(j) Y_t(j) - \mathcal{W}_t L_t(j) - \Theta(\pi_t(j)) - \mathcal{T}_t$$

  - Take as given CES demand, wage index, price index, τ^y subsidy, taxes $\mathcal{T}_t$
  - Profits are discounted by HH sector real SDF $Q_t^{\mathcal{H}}$

Key takeaway: inefficiencies due to pricing frictions, differentiated labor

Households

- Continuum of HH members $h \in \mathcal{H}$, differentiated by access to bond markets τ
- Mass $\eta(\tau)$ of each $h = (i, \tau)$ HH where $\int_0^T \eta(\tau)\,\mathrm{d}\tau = 1$ (otherwise identical)
    - Intuition: HHs sluggishly rebalance (our model is limiting case)
- A τ-type HH chooses consumption and labor $C_t(\tau), N_t(\tau)$ in order to solve

$$V_0(\tau) \equiv \max \mathbb{E}_0 \int_0^{\infty} e^{-\rho t} \left(\frac{C_t(\tau)^{1-\varsigma} - 1}{1-\varsigma} - \frac{N_t(\tau)^{1+\varphi}}{1+\varphi} \right) \mathrm{d}t$$

$$\text{s.t. } \mathrm{d}A_t(\tau) = \left[(1+\tau^W)\mathcal{W}_t(\tau)N_t(\tau) - P_t C_t(\tau)\right]\mathrm{d}t + A_t(\tau)\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} + \mathrm{d}F_t(\tau)$$

- $A_t(\tau)$ nominal savings earn $\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}}$; $\mathcal{W}_t(\tau)$ is nominal (differentiated) wage
- Take as given CES labor demand, τ^W labor subsidy, transfers $\mathrm{d}F_t(\tau)$

Key takeaway: consumption/labor choices differ when bond returns not equalized

Arbitrageurs

- Mean-variance optimization

$$\max \mathbb{E}_t \, \mathrm{d}\omega_t - \frac{a}{2} \mathbb{V}\mathrm{ar}_t \, \mathrm{d}\omega_t$$

$$\text{s.t.} \;\; \mathrm{d}\omega_t = \omega_t i_t \, \mathrm{d}t + \int_0^T X_t^{(\tau)} \left(\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} - i_t \, \mathrm{d}t \right) \mathrm{d}\tau$$

- Arbitrageurs invest $X_t^{(\tau)}$ in bond carry trade of maturity τ
- Remainder of wealth ω_t invested at the short rate
- Risk-return trade-off governed by a
    - Formally: risk aversion coefficient
    - More generally: proxies for any limits to risk-bearing capacity or intermediation frictions
    - Arbitrageurs transfer gains/losses to HHs, so a represents any frictions which hinder ability to trade on behalf of HHs

Key takeaway: risk averse arbitrageurs' holdings increase with expected return

Preferred Habitat Funds

- Habitat bond demand (exogenous) for maturity τ:

$$Z_t^{(\tau)} = -\alpha(\tau) \log P_t^{(\tau)} - \beta_t^{(\tau)}$$

- $\alpha(\tau)$: demand elasticity for τ fund
- $\beta_t^{(\tau)}$: additional time-varying ("noise") demand factor
  - Noise demand $\beta_t^{(\tau)} = \theta(\tau)\beta_t$ follows a factor structure across habitat funds, eg

$$\mathrm{d}\beta_t = -\kappa_\beta \left(\beta_t - \bar{\beta}\right) \mathrm{d}t + \sigma_\beta \, \mathrm{d}B_{\beta,t}$$

  - $\theta(\tau)$: mapping from demand factor β_t to τ-habitat demand
  - $Z_t^{(\tau)}$ financed at the short rate (zero-cost position)
  - Habitat funds also transfer gains/losses to HHs

Key takeaway: habitat funds introduce noise; price movements require portfolio rebalancing

Government

- Central bank chooses policy rate i_t and bond holdings $S_t^{(\tau)}$
- Potentially subject to costs:

$$Y_t \Psi_t^S \equiv Y_t \int_0^T \frac{\psi^{(\tau)}}{2} \left(S_t^{(\tau)}\right)^2 \mathrm{d}\tau \,, \quad Y_t \Psi_t^i \equiv Y_t \frac{\psi^i}{2} \left(i_t - \bar{i}_t\right)^2$$

- In the background: fiscal authority chooses production/labor subsidies τ^y, τ^w, balances the budget period by period
- Optimal policy: maximize social welfare

$$\max \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left(\int_0^T \eta(\tau) u\left(C_t(\tau), N_t(\tau)\right) \mathrm{d}\tau \right) \mathrm{d}t$$

Key takeaway: policy attempts to undo frictions:

1. Nominal pricing frictions $\implies$ deadweight loss
2. Differentiated labor $\implies$ production inefficiencies
3. Market segmentation $\implies$ consumption dispersion, imperfect risk-sharing

Equilibrium

Equilibrium Overview

- Equilibrium bond price dynamics and arbitrageur optimality conditions:

$$\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} \equiv \mu_t^{(\tau)}\,\mathrm{d}t + \boldsymbol{\sigma}_t^{(\tau)}\,\mathrm{d}\mathbf{B}_t\,, \quad \mu_t^{(\tau)} - i_t = \boldsymbol{\sigma}_t^{(\tau)}\boldsymbol{\Lambda}_t, \quad \boldsymbol{\Lambda}_t^\top = a\int_0^T X_t^{(\tau)}\boldsymbol{\sigma}_t^{(\tau)}\,\mathrm{d}\tau$$

  - Term premia depend on risk aversion a and equilibrium holdings $X_t^{(\tau)}$
- Approximation around “small risk, low risk bearing capacity” (nonzero premia)
- The first-best allocation obtained when $\theta = 0$ and $a = 0$. Output gap $X_t \equiv \frac{Y_t}{Y_t^n}$ and inflation evolve according to (linearized)

$$\mathrm{d}x_t = \varsigma^{-1}\left(\tilde{\mu}_t - \pi_t - r_t^n\right)\mathrm{d}t$$
$$\mathrm{d}\pi_t = \left(\rho\pi_t - \delta x_t\right)\mathrm{d}t$$

- $r_t^n \equiv -\kappa_z z_t$ is the usual natural rate and $\tilde{\mu}_t$ is the effective borrowing rate:

$$\tilde{\mu}_t = \int_0^T \eta(\tau)\mu_t^{(\tau)}\,\mathrm{d}\tau$$

  - Up to first-order, our model is the same as Ray, Droste, & Gorodnichenko (2024) details

Social Welfare

- Per-period social welfare loss (second-order expansion relative to first-best):

$$\begin{aligned}\mathcal{L}_t \equiv\ & (\varsigma + \varphi)x_t^2 + \theta\pi_t^2 \\ & + \frac{\varsigma}{\varphi}\left(\varphi + \varsigma\left[\frac{\varphi\epsilon_w}{1+\varphi\epsilon_w}\right]^2\right)\mathbb{V}\mathrm{ar}_\tau\, c_t(\tau) + \epsilon_w\,\mathbb{V}\mathrm{ar}_\tau\, w_t(\tau) \\ & + \int_0^T \psi^{(\tau)}\left(s_t^{(\tau)}\right)^2 \mathrm{d}\tau + \psi^i\left(i_t - \bar{i}_t\right)^2\end{aligned}$$

- First line: losses from nominal rigidities (same as in textbook RANK)
- Next line: losses also depends on consumption and wage dispersion *across* HHs
- Final line: losses from policy frictions (when $\psi^i > 0, \psi^{(\tau)} > 0$)

Aggregate and Welfare Consequences: Simple Policy Rules

- In order to better understand the model, simplify to a version of the model which only includes natural rate shocks r_t^n

$$\mathrm{d}r_t^n = -\kappa_z r_t^n \,\mathrm{d}t + \sigma_r \,\mathrm{d}B_{z,t}$$

- Consider policy rules which implement

$$i_t = \chi_i r_t^n$$
$$S_t^{(\tau)} = \chi_S^{(\tau)} r_t^n$$

- Simple policy rules: function of natural state variables only
    - Time-consistent: policymaker seeks to minimize unconditional social welfare loss
- We will examine the outcome of these policies in different versions of the model

Risk Neutral Arbitrageur

Benchmark: Risk Neutral Arbitrageur (“Standard Model”)

- Consider the benchmark case of a risk neutral arbitrageur: $a = 0$
- The expectations hypothesis holds: $\mu_t^{(\tau)} = i_t \implies$ model collapses to RANK

$$\mathbb{V}\text{ar}_\tau\, c_t(\tau) = 0, \quad \mathbb{V}\text{ar}_\tau\, w_t(\tau) = 0$$

- Recover the standard QE neutrality result: balance sheet policies do not affect bond prices (and therefore have no aggregate effects)
- Divine coincidence holds: conventional policy can achieve first-best

$$\chi_i = 1 \implies \mu_t^{(\tau)} = r_t^n \implies x_t = \pi_t = 0$$

- ‘Woodford-ian’ equivalence: the role of policy on aggregate dynamics and welfare is fully summarized by policy rate i_t

Imperfect Arbitrage

Imperfect Arbitrage

- Now assume $a > 0$ and the central bank continues to implement $i_t = r_t^n$

Proposition (Imperfect Arbitrage and Ad-hoc Policy)

Assume risk aversion $a > 0$ and price elasticities $\alpha(\tau) > 0$

Bond markets: bond carry trade return $\mu_t^{(\tau)} - i_t$

- Decreases with the short rate i_t
- Decreases with QE shocks $S_t^{(QE)}$

Aggregate dynamics: output gaps x_t and inflation π_t

- Not identically zero: $\mathbb{V}\mathrm{ar}\, x_t \neq 0$ and inflation $\mathbb{V}\mathrm{ar}\, \pi_t \neq 0$;
- QE increases the output gap and inflation

Dispersion: consumption and wage dispersion $\mathbb{V}\mathrm{ar}_\tau\, c_t(\tau) \neq 0, \mathbb{V}\mathrm{ar}_\tau\, w_t(\tau) \neq 0$

Imperfect Arbitrage Intuition: Policy Pass-Through

- Consider a fall in the natural rate inducing a cut in the policy rate:
  - When $\downarrow i_t$, bond arbitrageurs want to invest more in the BCT
  - $\implies$ bond prices increase $\uparrow P_t^{(\tau)}$
  - As $\uparrow P_t^{(\tau)}$, price-elastic habitat bond investors ($\alpha(\tau) > 0$) reduce their holdings: $\downarrow Z_t^{(\tau)}$
  - Bond arbitrageurs increase their holdings $\uparrow X_t^{(\tau)}$, which requires a larger BCT return
- Now consider a QE shock
  - QE purchases: $\uparrow S_t^{(\tau)}$
  - Bond arbitrageurs reduce holdings $\downarrow X_t^{(\tau)}$, reducing risk exposure and pushing down yields

Imperfect Arbitrage Intuition: Aggregate Effects

- Therefore, a shock to the natural rate does not fully pass through to the effective borrowing rate $\tilde{\mu}_t \neq i_t$
  - Thus aggregate borrowing demand changes, and hence $x_t \neq 0$
  - Through the NKPC, $\pi_t \neq 0$

- On the other hand, a QE shock stimulates the economy
  - QE reduces borrowing rates $\downarrow \tilde{\mu}_t$ and therefore stimulates aggregate consumption $\uparrow x_t$
  - Through the NKPC, inflation $\uparrow \pi_t$

- Additionally, in general $\mu_t^{(\tau)} \neq \mu_t^{(\tau')}$
  - Hence individual Euler equations differ
  - $\implies c_t(\tau) \neq c_t(\tau'), n_t^{(\tau)} \neq n_t(\tau')$ and therefore $\mathbb{V}\mathrm{ar}_\tau\, c_t(\tau) \neq 0, \mathbb{V}\mathrm{ar}_\tau\, w_t(\tau) \neq 0$

Optimal Policy

Imperfect Arbitrage and Macroeconomic Stabilization

- Can conventional policy alone close the output gap?
- Yes but the short rate must react more than one-for-one with the natural rate:

$$\exists\, \chi_i^n > 1: \; i_t = \chi_i^n r_t^n \implies \tilde{\mu}_t = r_t^n$$

- However, this does not achieve first-best since $\mathbb{V}\mathrm{ar}_\tau\, c_t(\tau) \neq 0, \mathbb{V}\mathrm{ar}_\tau\, w_t(\tau) \neq 0$
- In fact, relative to the policy $i_t = r_t^n$, in general we have $\uparrow \mathbb{V}\mathrm{ar}_\tau\, c_t(\tau), \uparrow \mathbb{V}\mathrm{ar}_\tau\, w_t(\tau)$
  - Short rate is more volatile, hence $\uparrow$ term premia volatility
  - This implies higher dispersion across borrowing rates $\mu_t^{(\tau)}$ and therefore an increase in consumption/labor dispersion
- Optimal short rate policy: if $\psi^{(\tau)} \to \infty$, then optimal policy implements

$$i_t = \chi_i^* r_t^n, \;\; \chi_i^* < \chi_i^n \implies \frac{\partial \tilde{\mu}_t}{\partial r_t^n} < 1$$

- With access to frictionless balance sheet policies, we obtain the following

Proposition (Optimal Policy Separation Principle)

Assume risk aversion $a > 0$ and price elasticities $\alpha(\tau) > 0$, and policy costs $\psi^i = \psi^{(\tau)} = 0$. Suppose the central bank implements short rate and balance sheet policy according to

$$i_t = r_t^n$$

$$S_t^{(\tau)} = \alpha(\tau) \log P_t^{(\tau)}$$

Then first-best is achieved:

- Macroeconomic stabilization: $x_t = \pi_t = 0 \ \forall t$
- Financial stabilization: $\mu_t^{(\tau)} = \tilde{\mu}_t \ \forall \tau$
- Consumption and wage equalization: $\mathbb{V}\mathrm{ar}_\tau \, c_t(\tau) = 0, \mathbb{V}\mathrm{ar}_\tau \, w_t(\tau) = 0 \ \forall t$

Optimal Policy Separation Principle: Intuition

- The results follow naturally from our findings regarding ad-hoc policy
- QE policy stabilizes shocks to bond markets by offsetting all habitat portfolio rebalancing shocks:

$$S_t^{(\tau)} = -Z_t^{(\tau)} \implies \boldsymbol{\sigma}_t^{(\tau)} \boldsymbol{\Lambda}_t = 0$$

- This equalizes borrowing rates across HHs: $\mu_t^{(\tau)} = \tilde{\mu}_t$
- Hence the model collapses to a standard RANK model, in which case divine coincidence implies $i_t = r_t^n$ is optimal

Separation principle for optimal policy:

- Optimal balance sheet policy stabilizes financial markets
- Optimal short rate policy stabilizes macroeconomic aggregates

Separation Principle with Balance Sheet Constraints

- When the central bank faces balance sheet constraints ($\Psi^{(\tau)} > 0$), policy can no longer achieve first-best
- However, as long as $\Psi^{(\tau)} < \infty$, optimal policy implies the central bank still uses balance sheet tools
- Let $\Psi^{(\tau)} = a \cdot \boldsymbol{\sigma}^{(\tau)} \left[\boldsymbol{\sigma}^{(\tau)}\right]^\top$
    - $\implies$ same friction a as arbitrageurs, except policymaker cannot net out positions
- Even with "large" balance sheet costs, the central bank still uses QE to (partially) stabilize term premia
- Intuition:
    - The central bank faces holding costs which imply it is worse than private arbitrageurs at financial intermediation
    - But the central bank internalizes the social benefits of minimizing fluctuations in term premia
    - Nevertheless, non-negligible balance sheet costs imply that optimal policy is less reactive

Financial Stabilization Policy with Short Rate Constraints

- Suppose that short rate policy is constrained, and implements

$$i_t = \tilde{\chi}_i r_t^n, \quad 0 < \tilde{\chi}_i < 1$$

  - Formally: assume costs $\psi^i \left(i_t - \tilde{\chi}_i r_t^n\right)$ and take $\psi^i \to \infty$
- If the central bank continues to implement the balance sheet policy derived above, then borrowing rates are still equalized $\mu_t^{(\tau)} = \tilde{\mu}_t$
- However, $\tilde{\mu}_t \neq r_t^n$ and hence this policy does not achieve macroeconomic stabilization

$$x_t \neq 0, \pi_t \neq 0$$

Macroeconomic Stabilization with Short Rate Constraints

- Can balance sheet policy alone close the output gap?
- Yes but must sacrifice equalizing borrowing rates:

$$\mu_t^{(\tau)} = i_t + \sigma_t^{(\tau)} \lambda_t$$

$$\lambda_t \equiv a \int_0^T \left[\alpha(\tau) \log P_t^{(\tau)} - S_t^{(\tau)} \right] \sigma_t^{(\tau)} \, \mathrm{d}\tau$$

- Hence, can always choose $\left\{ S_t^{(\tau)} \right\}$ such that

$$\lambda_t^* = \frac{r_t^n - i_t}{\int_0^T \eta(\tau) \sigma_t^{(\tau)} \, \mathrm{d}\tau} \implies \tilde{\mu}_t = r_t^n$$

- However, because $\sigma_t^{(\tau)} \neq \sigma_t^{(\tau')}$ this necessitates

$$\mu_t^{(\tau)} = i_t + \sigma_t^{(\tau)} \left(\frac{r_t^n - i_t}{\int_0^T \eta(\tau') \sigma_t^{(\tau')} \, \mathrm{d}\tau'} \right) \neq r_t^n \qquad (\text{unless } i_t = r_t^n)$$

Stabilization with Short Rate Constraints: Intuition

- Balance sheet policy works by affecting term premia through changes in the market price of risk
- Although arbitrage is imperfect in this model, arbitrageurs still enforce tight restrictions between between market price of risk and term premia across maturities
- Hence, while in principle the central bank has a continuum of policy tools $\left\{S_t^{(\tau)}\right\}$, in practice it can only manipulate λ_t
- Related to localization results in Vayanos & Vila (2021) and Ray, Droste, & Gorodnichenko (2024)
  - In the one-factor model considered here, the effects of QE are fully global
  - Even with more complicated risk structure, localization is not strong enough to allow balance sheet policy alone to achieve first-best

Extensions Overview

- "Noise" Demand Shocks details
  - Optimal separation principle still holds with stochastic habitat demand $\beta_t^{(\tau)}$, but QE must be more reactive
  - Additional result: if noise demand dynamics are such that $\uparrow\uparrow \beta_t^{(\tau)}$ in response to $\uparrow r_t^n$, then it is optimal to expand the balance sheet $\uparrow S_t^{(\tau)}$ while hiking rates $\uparrow i_t$
- Cost-Push Shocks details
  - Adding shocks to NKPC (eg, wage rigidity in labor markets) breaks divine coincidence but unfortunately, our separation principle still holds
  - Despite multiple policy instruments, (un)conventional policy only affects aggregate outcomes through changes in effective borrowing rate $\tilde{\mu}_t$
- Nonzero First-Best Term Premia details
  - When first-best BCT returns are $\nu^{(\tau)} \neq 0$
  - Our results hold when $\nu^{(\tau)}$ is achievable but optimal short rate policy is a function of $\nu^{(\tau)}$

History-Dependent Policy

Monetary Policy with Commitment

- When policy frictions bind, simple policy rules fail to achieve first-best
- Instead suppose the policymaker chooses policy tools $\mathbf{u}_t$ as a function of entire history of predetermined and nonpredetermined variables $\mathbf{Y}_t \equiv \begin{bmatrix} \mathbf{y}_t^\top & \mathbf{x}_t^\top \end{bmatrix}^\top$
- Minimizes conditional social loss

$$\begin{aligned} \mathcal{W}_0 &= \frac{1}{2}\mathbb{E}_0 \int_0^\infty e^{-\rho t}\mathcal{L}_t \, \mathrm{d}t \\ &= \frac{1}{2}\mathbb{E}_0 \int_0^\infty e^{-\rho t} \left(\mathbf{Y}_t^\top \mathbf{R}\mathbf{Y}_t + \mathbf{u}_t^\top \mathbf{Q}\mathbf{u}_t\right) \mathrm{d}t \,, \quad \mathbf{y}_0 \text{ given} \end{aligned}$$

- By setting policy in a history-dependent manner, full commitment policies can improve on simple policy rules

Characterizing Optimal Policy with Commitment

Theorem (Optimal Policy with Commitment)

Given $\mathbf{y}_0$, the policymaker minimizes $\mathcal{W}_0$ by choosing $\mathbf{u}_t = \mathbf{F}\mathbf{Y}_t$, which induce equilibrium dynamics $\mathrm{d}\mathsf{Y}_t = -\boldsymbol{\Upsilon}(\mathsf{F})\mathsf{Y}_t\,\mathrm{d}t + \mathsf{S}(\mathsf{F})\,\mathrm{d}\mathsf{B}_t$. Necessary conditions are given by

$$\mathbf{y}_0^\top\left(\partial_i \mathsf{P}_{11} - \partial_i \mathsf{P}_{12}\mathsf{P}_{22}^{-1}\mathsf{P}_{21} - \mathsf{P}_{12}\mathsf{P}_{22}^{-1}\partial_i \mathsf{P}_{21} + \mathsf{P}_{12}\left(\mathsf{P}_{22}^{-1}\partial_i \mathsf{P}_{22}\mathsf{P}_{22}^{-1}\right)\mathsf{P}_{21}\right)\mathbf{y}_0 = 0$$

where $\rho\mathsf{P} = \mathsf{R} + \mathsf{F}^\top\mathsf{Q}\mathsf{F} - \mathsf{P}\boldsymbol{\Upsilon} - \boldsymbol{\Upsilon}^\top\mathsf{P}$. Dynamics are given by $\mathbf{q}_0 = \begin{bmatrix}\mathbf{y}_0 & \mathbf{0}\end{bmatrix}^\top$ and

$$\mathrm{d}\mathbf{q}_t = -\begin{bmatrix}\mathsf{I} & \mathbf{0}\\ \mathsf{P}_{21} & \mathsf{P}_{22}\end{bmatrix}\boldsymbol{\Upsilon}\begin{bmatrix}\mathsf{I} & \mathbf{0}\\ -\mathsf{P}_{22}^{-1}\mathsf{P}_{21} & \mathsf{P}_{22}^{-1}\end{bmatrix}\mathbf{q}_t\,\mathrm{d}t + \begin{bmatrix}\mathsf{I} & \mathbf{0}\\ \mathsf{P}_{21} & \mathsf{P}_{22}\end{bmatrix}\mathsf{S}\,\mathrm{d}\mathsf{B}_t \equiv -\boldsymbol{\Gamma}\mathbf{q}_t\,\mathrm{d}t + \boldsymbol{\sigma}\,\mathrm{d}\mathsf{B}_t$$

Bond prices are affine in $\mathsf{A}(\tau)^\top\mathbf{q}_t$ with $\mathsf{A}(\tau) = \left[\mathbf{I} - e^{-\mathsf{M}\tau}\right]\mathsf{M}^{-1}\mathbf{e}_i$ and

$$\mathbf{e}_i^\top\mathbf{q}_t = i_t,\quad \mathsf{M} = \boldsymbol{\Gamma}^\top - \int_0^T\left[-\alpha(\tau)\mathsf{A}(\tau) + \boldsymbol{\Theta}(\tau)\right]\mathsf{A}(\tau)^\top\,\mathrm{d}\tau\,\tilde{\boldsymbol{\Sigma}}$$

Monetary Policy with Commitment: Intuition

- Policymaker chooses tools i_t, $\left\{S_t^{(\tau)}\right\}$ which:
  - Directly affect optimality conditions of arbitrageurs
  - Indirectly affect HHs through changes in equilibrium borrowing rates
  - Indirectly affect firms through changes in marginal costs
- Trade-off: more aggressive policy reactions to shocks:
  - Greater pass-through to HHs
  - Larger and more volatile term premia
- Commitment partially relaxes this link:
  - HH decisions depend on entire expected path of borrowing rates $\int_0^\infty \mu_t^{(\tau)}\,\mathrm{d}\tau$
  - Arbitrageur risk compensation depends on volatility of short-run fluctuations $\mathrm{d}i_t$, $\mathrm{d}S_t^{(\tau)}$
- Characterizing dynamics of optimal policy with commitment is difficult
  - Ongoing work studies optimal policy numerically
  - Suffers from time inconsistency; simple rules may be more practical

Concluding Remarks

- This paper develops a tractable general equilibrium model with market segmentation, nominal frictions, and household heterogeneity
- We find that optimal short rate and balance sheet policy is characterized by a sharp **optimal separation result**:
  - Conventional policy targets macroeconomic stability
  - Unconventional policy targets financial stability
- Optimal policy removes excess volatility of risk premia and hence improves risk-sharing across households, while reducing excess macroeconomic volatility
  - Policy constraints on either the short rate or balance sheets imply trade-offs between these two policy objectives
- When considering social welfare, cannot abstract from the policy tools used to conduct monetary policy

Thank You!

Equilibrium Details

Aggregation

- Firms, arbitrageurs, and funds transfer profits equally to HHs
- Symmetric firm equilibrium $Y_t(j) = Y_t, P_t(j) = P_t, \pi_t(j) = \pi_t, \frac{\mathrm{d}P_t}{P_t} = \pi_t \,\mathrm{d}t$
- Clearing in production and goods markets:

$$Y_t = Z_t L_t \equiv Z_t \left[\int_0^T \eta(\tau) N_t(\tau)^{\frac{\epsilon_W - 1}{\epsilon_W}} \,\mathrm{d}\tau \right]^{\frac{\epsilon_W}{\epsilon_W - 1}}$$

$$C_t \equiv \int_0^T \eta(\tau) C_t(\tau) \,\mathrm{d}\tau = Y_t \left(1 - \frac{\theta}{2} \pi_t^2 - \Psi_t^S - \Psi_t^i \right)$$

- Bond market clearing implies

$$X_t^{(\tau)} + Z_t^{(\tau)} + \eta(\tau) A_t(\tau) + S_t^{(\tau)} = 0$$

back

Optimality Conditions

- Equilibrium bond price dynamics:

$$\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} \equiv \mu_t^{(\tau)}\,\mathrm{d}t + \boldsymbol{\sigma}_t^{(\tau)}\,\mathrm{d}\mathbf{B}_t$$

  - $\mathbf{B}_t$ collects innovations to risk factors (technology, noise demand, ...)
- Arbitrageur optimality conditions:

$$\mu_t^{(\tau)} - i_t = \boldsymbol{\sigma}_t^{(\tau)}\boldsymbol{\Lambda}_t, \quad \boldsymbol{\Lambda}_t^\top = a\int_0^T X_t^{(\tau)}\boldsymbol{\sigma}_t^{(\tau)}\,\mathrm{d}\tau$$

  - Term premia depend on risk aversion a and equilibrium holdings $X_t^{(\tau)}$
- HH optimality conditions (log-linearized) :

$$w_t = \varsigma c_t(\tau) + \phi n_t(\tau) + \frac{1}{\epsilon_w}\left(n_t(\tau) - \ell_t\right), \quad \mathbb{E}_t\,\mathrm{d}c_t(\tau) = \varsigma^{-1}\left(\mu_t^{(\tau)} - \pi_t - \rho\right)\mathrm{d}t$$

- Firm optimality conditions (log-linearized):

$$\mathbb{E}_t\,\mathrm{d}\pi_t = \left(\rho\pi_t - \delta_w w_t\right)\mathrm{d}t$$

back

Simplifying Assumptions

- Tractability assumption: a "head of HH" sets transfers such that in equilibrium, wealth is equalized: across τ HH groups, $A_t(\tau) \equiv A_t$
    - Pros: clear focus on the role market segmentation plays on consumption dispersion
    - Cons: ignores the impact of market segmentation on wealth inequality
- Approximation: around a limiting case: risk $\hat{\boldsymbol{\sigma}}_t^{(\tau)} \equiv \hat{h}^{\frac{1}{2}} \cdot \boldsymbol{\sigma}_t^{(\tau)} \to \mathbf{0}$ but arbitrageur risk aversion $\hat{a} \equiv a/\hat{h} \to \infty$ such that $\hat{a}^{\frac{1}{2}} \cdot \hat{\boldsymbol{\sigma}}_t^{(\tau)} \equiv a^{\frac{1}{2}} \cdot \boldsymbol{\sigma}_t^{(\tau)}$ remains non-zero and bounded
    - Pros: clear focus on the idea of "imperfect arbitrage"
    - Cons: less realistic risk premia (particularly in first-best)
- Allows us to derive simple first-order approximations of dynamics and second-order approximations for welfare

back

Aggregate Dynamics

- The first-best (natural) allocation obtained when $\theta = 0$ and $a = 0$. Output gap:

$$X_t \equiv \frac{Y_t}{Y_t^n}$$

- Output gap evolves according to modified aggregate Euler equation:

$$\mathrm{d}x_t = \varsigma^{-1}\left(\tilde{\mu}_t - \pi_t - r_t^n\right)\mathrm{d}t$$

- $r_t^n \equiv -\kappa_z z_t$ is the usual natural rate and $\tilde{\mu}_t$ is the effective borrowing rate:

$$\tilde{\mu}_t = \int_0^T \eta(\tau)\mu_t^{(\tau)}\,\mathrm{d}\tau$$

- We recover a standard NKPC:

$$\mathrm{d}\pi_t = \left(\rho\pi_t - \delta x_t\right)\mathrm{d}t$$

back

Extensions

Extensions: "Noise" Demand Shocks

- We obtain identical results when allowing for shocks to habitat demand $\beta_t^{(\tau)}$
- Optimal separation principle still holds with $\psi^{(\tau)} = 0$, but QE must be more reactive:

$$S_t^{(\tau)} = \alpha(\tau) \log P_t^{(\tau)} + \beta_t^{(\tau)}$$

- Optimal short rate policy still implements $i_t = r_t^n$
- Additional result: if noise demand dynamics are such that $\uparrow\uparrow \beta_t^{(\tau)}$ in response to $\uparrow r_t^n$, then it is optimal to expand the balance sheet $\uparrow S_t^{(\tau)}$ while hiking rates $\uparrow i_t$
- Intuition:
  - Suppose during a hiking cycle and in the absence of QE we have an increase in term premia
  - Then the optimal balance sheet policy is to conduct additional QE purchases in order to offset spike in term premia
  - $\implies$ conventional and unconventional policy seem to be at odds with one another
  - Otherwise, short rate policy and balance sheet policy tend to be reinforcing back

Extensions: Cost-Push Shocks

- What if divine coincidence does not hold? Eg, wage rigidity in labor markets
- More generally, introduce exogenous cost-push shocks u_t in NKPC:

$$\mathrm{d}\pi_t = (\rho\pi_t - \delta x_t - u_t)\,\mathrm{d}t$$

- Unfortunately, our separation principle still holds:
  - Optimal QE stabilizes term premia
  - Short rate policy must contend with the output gap/inflation trade-offs
- Intuition: despite multiple policy instruments, (un)conventional policy only affects aggregate outcomes through changes in effective borrowing rate $\tilde{\mu}_t$
  - Take any feasible path $\{x_t, \pi_t, \tilde{\mu}_t\}_t$ from an implementation implying policies $\left\{\hat{i}_t, \hat{S}_t^{(\tau)}\right\}_t$
  - Can also be achieved with $i_t = \tilde{\mu}_t, S_t^{(\tau)} = \alpha(\tau)\log P_t^{(\tau)} + \beta_t^{(\tau)}$
  - This guarantees $\mathbb{V}\mathbf{ar}_\tau\, c_t(\tau) = \mathbb{V}\mathbf{ar}_\tau\, w_t(\tau) = 0$ and hence strictly dominates
- However, room for policy improvement with endogenous cost-push shocks (eg, working capital channel) back

Extensions: Non-Zero First-Best Carry Trade Returns

- Our approximation approach implies that in the first-best, expected carry trade returns are zero
- This simplifies our analytical results but of course is an extreme assumption
- Suppose instead that first-best BCT returns are $\nu^{(\tau)} \neq 0$
- Our separation principle still holds when $\nu^{(\tau)}$ is achievable but optimal short rate policy is a function of $\nu^{(\tau)}$
- Intuition: combination of previous results
  - Aggregate outcomes through changes in effective borrowing rate $\tilde{\mu}_t$ (as before)
  - Optimal QE policy guarantees $\mu_t^{(\tau)} - i_t \equiv \nu^{(\tau)}$ and hence $\tilde{\mu}_t = i_t + \int_0^T \eta(\tau)\nu^{(\tau)}\,\mathrm{d}\tau \equiv i_t + \tilde{\nu}$
  - Thus, optimal short rate policy implements $i_t = r_t^n - \tilde{\nu}$ back