Optimal Unconventional Policy in a New Keynesian Preferred Habitat Model

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October 2025
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    - Obvious: reduce long-term yields
    - Less obvious: stimulate the economy
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Bernanke: "QE **works (??)** in practice but not in theory"

Our Model

- This paper: develops a tractable general equilibrium model with market segmentation, nominal frictions, financial frictions, and household heterogeneity
- **Preferred habitat** tradition: assets traded by specialized investors
  - Pension funds hold long-maturity bonds
  - Money market funds hold short-maturity bonds

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- **Preferred habitat** tradition: assets traded by specialized investors
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- Our model: households and firms have differentiated access to asset markets
  - Households borrow with assets of different maturities (eg pension funds, mortgages)
  - Firms face working capital constraint
  - Introduces imperfect risk-sharing, consumption and saving dispersion across households
- Arbitrageurs (eg hedge funds, broker-dealers) with imperfect risk-bearing capacity intermediate bond markets

Findings: Policy Transmission

- Key mechanisms of conventional monetary policy:
  - Changes in the short rate affect required rates of return of all assets, but imperfect transmission to household borrowing rates

- Key mechanisms of balance sheet policy:
  - Imperfect arbitrage breaks QE neutrality: induces portfolio rebalancing and hence reduces term premia

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- Key mechanisms of balance sheet policy:
  - Imperfect arbitrage breaks QE neutrality: induces portfolio rebalancing and hence reduces term premia
- Thus, short rate and balance sheet policies are substitutes when targeting aggregate borrowing rates
  - A fall in aggregate borrowing rates is stimulative for the usual NK reasons

Findings: Welfare Consequences

- If the policymaker only cares about macroeconomic stabilization, conventional and unconventional policies are essentially equivalent
  - Nominal rigidities $\implies$ welfare losses due to inflation and output gap volatility
  - Triumphalist view: even with short rate constraints, QE is equally effective

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- **Triple mandate**: social welfare depends on volatility of output, inflation, and long-term rates

  "Promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates."

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  - Balance sheet constraints: short rate less reactive to minimize bond disruptions
  - Short rate constraints: QE used to offset macroeconomic shocks
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- General message: implementation matters for welfare

Related Literature

- Preferred habitat models
  - Vayanos & Vila (2009, 2021), Ray, Droste, & Gorodnichenko (2024), Greenwood & Vayanos (2014), Hamilton & Wu (2012), Greenwood et al (2016), King (2019, 2021) , Kekre, Lenel, & Mainardi (2024), ...
- Empirical evidence: QE and preferred habitat
  - Krishnamurthy & Vissing-Jorgensen (2012), Hamilton and Wu (2012), D'Amico & King (2013), Li & Wei (2013), Cahill et al. (2013), King (2019), Fieldhouse et al (2018), Di Maggio et al (2020), Gali, Debortoli, & Gambetti (2020) , ...
- Macroeconomic QE models
  - Andres, Lopez-Salido, & Nelson (2004), Gertler & Karadi (2011), Curdia & Woodford (2011), Chen et al (2012), Gertler & Karadi (2013), Sims & Wu (2020), Karadi & Nakov (2020), Iovino and Sergeyev (2023), Carlstrom et al (2017), Ippolito et al (2018), Dordal & Lee (2024) , ...
- Market segmentation, macro-prudential monetary policy
  - Cui & Sterk (2021), Auclert (2016), Collard et al (2017), Kaplan et al (2017), Debortoli & Gali (2017) , Campbell & Nemtyrev (2025) ...
- International
  - Itskhoki & Mukhin (2023), Greenwood et al (2023), Gourinchas, Ray, & Vayanos (2025) , ...

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- **Firms:** monopolistic competitors face nominal frictions firms
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  - ⚠ Imperfect risk-bearing capacity

Aggregate Risk Factors and Risk-Bearing Capacity

- Aggregate technology shock to firm production

$$Z_t = Ze^{z_t}, \quad \mathrm{d}z_t = -\kappa_z z_t \,\mathrm{d}t + \sigma \,\mathrm{d}B_t$$

  - More generally (in paper): $N_z \times 1$ vector $\mathbf{z}_t$ exogenous risk factors where $\mathsf{Var}_t \,\mathrm{d}\mathbf{z}_t = \mathsf{Var}_t [\boldsymbol{\sigma} \,\mathrm{d}\mathbf{B}_t] = \boldsymbol{\sigma}\boldsymbol{\sigma}^\top \mathrm{d}t$ (cost-push, portfolio rebalancing, firm financing, ...)

- Arbitrageur optimally chooses portfolio $\{X_t(\tau)\}$ given risk aversion a:

$$\mathsf{E}_t \frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} - i_t \,\mathrm{d}t = a \cdot \int_0^T X_t(\tau') \,\mathsf{Cov}_t\left(\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}}, \frac{\mathrm{d}P_t^{(\tau')}}{P_t^{(\tau')}}\right) \mathrm{d}\tau'$$

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- ⚠ Limits to arbitrage: parameterize $\sigma(\xi), a(\xi)$ such that

$$\lim_{\xi \to 0} \sigma(\xi) \to 0, \quad \lim_{\xi \to 0} a(\xi)\sigma(\xi) \to \hat{a}\hat{\sigma}$$

First-Best Allocation

Proposition (First-best allocation)

Consider the limiting riskless case ($\xi \to 0$).

- With perfect arbitrage ($\hat{a} = 0$), the model admits a representative agent representation.
- Given an optimal production subsidy, the first-best allocation is obtained under flexible prices.

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- *Corollary*: steady state is efficient with zero inflation and zero arbitrageur positions
- Perturbation around "low risk, low risk-bearing capacity" point details

Equilibrium Aggregate Dynamics I

- Bond returns: $\mathrm{d}P_t^{(\tau)} / P_t^{(\tau)} = \mu_t(\tau)\,\mathrm{d}t + \sigma(\tau)\,\mathrm{d}B_t$
- Firm production and marginal costs (symmetric equilibrium):

$$y_t = z_t + n_t, \quad m_t = (1 + \beta\varrho)(w_t - z_t) + \hat{\mu}_t, \quad \beta = \int_0^T \beta(\tau)\,\mathrm{d}\tau$$

- Household i optimality conditions:

$$\mathrm{E}_t\,\mathrm{d}c_t(i) = \varsigma^{-1}\left[\tilde{\mu}_t(i) - \pi_t\right]\mathrm{d}t\,, \quad w_t = \varsigma c_t(i) + \varphi n_t(i)$$
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- ⚠ "effective" rates:

$$\tilde{\mu}_t(i) \equiv \int_0^T \eta_i(\tau)\mu_t(\tau)\,\mathrm{d}\tau\,, \quad \tilde{\mu}_t = \int_0^1 \tilde{\mu}_t(i)\,\mathrm{d}i\,, \quad \hat{\mu}_t = \int_0^T \beta(\tau)\mu_t(\tau)\,\mathrm{d}\tau$$

Equilibrium Aggregate Dynamics II

- Output gap $x_t \equiv y_t - y_t^n = y_t - \frac{1+\varphi}{\varsigma+\varphi} z_t$ and natural rate $v_t \equiv -\varsigma \kappa_z \frac{1+\varphi}{\varsigma+\varphi} z_t$
- $\implies$ modified NK equations [cf. Ray, Droste, & Gorodnichenko 2024]:

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- ⚠ In general, $\mu_t(\tau) = i_t + \lambda_t(\tau)$ where $\lambda_t(\tau) \neq 0$. Arbitrageur optimality conditions:

$$\lambda_t(\tau) = \hat{a} \sigma(\tau) \Lambda_t, \quad \Lambda_t = \int_0^T x_t(\tau) \sigma(\tau) \, \mathrm{d}\tau$$

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- Arbitrageur positions $x_t(\tau)$ pinned down by market clearing conditions:
  - Firm borrowing: function of wage w_t, labor supply n_t, portfolio weights $\beta(\tau)$
  - Household bond holdings: function of wealth $b_t(i)$, portfolio weights $\eta_i(\tau)$
  - Central bank holdings $qe_t(\tau)$

Equilibrium Risk Prices I

- Assume steady state household wealth $B = 0$
  - More generally (in paper): $B \neq 0$ allows for more complicated wealth effects
  - $\implies$ arbitrageur holding $x_t(\tau)$ feature own- and cross-price elasticities wrt $p_t(\tau)$ and $\int_0^T \int_0^1 \eta_i(\tau) p_t^{(\tau')} \, \mathrm{d}i \, \mathrm{d}\tau'$

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$$\mathrm{d}b_t(i) = \Big[\underbrace{\varrho b_t(i) + B\tilde{\mu}_t(i)}_{\text{relative portfolio returns}} + \underbrace{B\varrho\check{p}_t(i)}_{\text{relative wealth effects}} - \underbrace{(1 + \varsigma/\varphi)\,\check{c}_t(i)}_{\text{relative consumption/income}} \Big] \mathrm{d}t$$

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- $\implies$ Arbitrageur market clearing:

$$\underbrace{x_t(\tau) + qe_t(\tau)}_{\text{holdings net of QE}} = \underbrace{\beta(\tau)\left((1+\varsigma+\varphi)x_t - \frac{1}{\varsigma\kappa_z}v_t\right)}_{\text{firm borrowing}} - \underbrace{\int_0^1 \eta_i(\tau)\left(b_t(i) + B\left[\tilde{p}_t(i) - p_t^{(\tau)}\right]\right) \mathrm{d}i}_{\text{HH savings}}$$

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- Risk prices Λ_t depend on risk-weighted objects: endogenous volatility $\sigma(\tau)$ of bonds

$$\Lambda_t = \int_0^T x_t(\tau)\sigma(\tau)\,\mathrm{d}\tau \equiv x_t^\sigma = -qe_t^\sigma + \beta^\sigma \left((1+\varsigma+\varphi)x_t - \frac{1}{\varsigma\kappa_z}v_t \right) - b_t^\sigma$$

  - Risk-weighted aggregate wealth $b_t^\sigma \equiv \int_0^1 \int_0^T \sigma(\tau)\eta_i(\tau)b_t(i)\,\mathrm{d}\tau\,\mathrm{d}i, \ldots$
  - More generally (in paper): $N_z \times 1$ vectors with multiple risk factors

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- $\check{\Sigma}$ measures risk-weighted portfolio dispersion

$$\check{\Sigma} \equiv \int_0^1 \eta_i^\sigma\,[\eta_i^\sigma - \eta^\sigma]\,\mathrm{d}i\,, \quad \eta_i^\sigma \equiv \int_0^T \eta_i(\tau)\sigma(\tau)\,\mathrm{d}\tau\,, \quad \eta^\sigma \equiv \int_0^1 \eta_i^\sigma\,\mathrm{d}i$$

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$$\check{\Sigma} \equiv \int_0^1 \eta_i^\sigma\,[\eta_i^\sigma - \eta^\sigma]\,\mathrm{d}i\,, \quad \eta_i^\sigma \equiv \int_0^T \eta_i(\tau)\sigma(\tau)\,\mathrm{d}\tau\,, \quad \eta^\sigma \equiv \int_0^1 \eta_i^\sigma\,\mathrm{d}i$$

- Heterogeneity matters for first-order dynamics: risk-weighted market clearing

Social Welfare

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Key takeaway: policy attempts to undo frictions:

1. Nominal rigidities $\implies$ pricing inefficiencies
2. Firm financing friction $\implies$ production inefficiencies
3. Household market segmentation $\implies$ imperfect risk-sharing

Benchmark I: Risk Neutral Arbitrageur

- Consider the benchmark case of a risk neutral arbitrageur: $\hat{a} = 0$
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- Recover the standard QE neutrality result: balance sheet policies do not affect bond prices (and therefore have no aggregate effects)
- 'Woodford-ian' equivalence: the role of policy on aggregate dynamics and welfare is fully summarized by policy rate i_t

Benchmark II: Representative Agent Representation

- Even with imperfect arbitrage ($\hat{a} > 0$), consider special case:
  1. Zero wealth steady state ($B = 0$) and initially equal wealth distribution ($b_t(i) = 0$)
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- $\implies$ EH holds; RANK representation; conventional policy achieves first best
- ⚠ QE non-neutrality: balance sheet policies affect arbitrageur positions
    - $\implies$ QE affects bond prices and aggregates
    - $\implies$ induces heterogeneity across households
- $\approx$ 'Woodford-ian' equivalence but QE $\neq$ short rate policy

Dynamics: Analytical Results

Simplified Aggregate Dynamics: Rigid Prices

- Simplifications: rigid prices
- Along with the dynamics of natural rate shocks, we have

$$
\begin{aligned}
\mathrm{d}b_t^\sigma &= \left[\varrho b_t^\sigma - (1+\varsigma/\varphi)\,\check{c}_t^\sigma\right]\mathrm{d}t \\
\mathrm{E}_t\,\mathrm{d}x_t &= \varsigma^{-1}\left[i_t + \eta^\sigma\Lambda_t - v_t\right]\mathrm{d}t \\
\mathrm{E}_t\,\mathrm{d}\check{c}_t^\sigma &= \hat{a}\varsigma^{-1}\check{\Sigma}\Lambda_t\,\mathrm{d}t \\
\Lambda_t &= -qe_t^\sigma + \beta^\sigma\left((1+\varsigma+\varphi)x_t - \frac{1}{\varsigma\kappa_z}v_t\right) - b_t^\sigma
\end{aligned}
$$

- Ad-hoc Taylor policy rules close the model

$$
i_t = \phi_x x_t + \epsilon_{i,t}, \quad qe_t(\tau) = \phi_x(\tau)x_t + \epsilon_{q,t}(\tau)
$$

  - Paper: existence and uniqueness, solution algorithm
  - Simple linear REE model, except endogenous coefficients $\eta^\sigma, \beta^\sigma, \check{\Sigma}$ due to endogenous volatility $\sigma(\tau)$ when $\hat{a} > 0$

Simplified Aggregate Dynamics: Rigid Prices

Proposition (Rigid price dynamics, general case)

Assume $\hat{a} > 0$, $\beta(\tau) > 0$, and $0 < \phi_x < \bar{\phi}_x$ for some upper bound $\bar{\phi}_x$.

- Following a natural rate shock:

$$\frac{\partial x_t}{\partial v_t} > 0, \quad \frac{\partial \Lambda_t}{\partial v_t} > 0, \quad \mathrm{Cov}(i_t, \Lambda_t) > 0, \qquad \exists k > 0 : \frac{\partial x_{t+k}}{\partial v_t} > 0, \quad \frac{\partial \Lambda_{t+k}}{\partial v_t} < 0$$

  - The reaction of risk prices Λ_t is stronger if $\phi_x(\tau) > 0$
- Following a conventional monetary policy shock: $\frac{\partial x_t}{\partial v_t} < 0, \quad \frac{\partial \Lambda_t}{\partial \epsilon_{i,t}} < 0$

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- Intuition:
  - Recession $\implies \downarrow i_t, \downarrow$ firm borrowing on impact, $\searrow$ HH saving over time
  - Arbitrageur rebalancing $\implies \downarrow$ term premia on impact, $\nearrow$ over time
  - Contraction policy *shock* $\implies \uparrow i_t, \downarrow$ firm rebalancing

Empirical Evidence

Model Predictions and Evidence

Stylized Model Predictions:

1. Unconditionally, increases in short rates associated with contemporaneous increases in term premia
2. Larger unconditional reactions during QE periods
3. Over longer horizons, unconditional reaction of term premia to short rates weakens or becomes negative
4. Conditional reaction of term premia to monetary policy *shocks* are small or negative

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4. Conditional reaction of term premia to monetary policy *shocks* are small or negative

Empirical Specification:

- Utilize movements in long forward rates (Gurkaynak et al 2005, Hanson et al 2021)

$$f_{t+h}^{(\tau)} - f_{t-1}^{(\tau)} = \alpha(\tau) + \beta(\tau) D_t + \epsilon_t(\tau)$$

- Unconditional vs conditional regressions: D_t are daily change in short-term yields (Gurkayank et al 2007) vs high-frequency MP shocks (Nakamura and Steinsson 2018)

Empirical Results: Unconditional, Varying Maturities

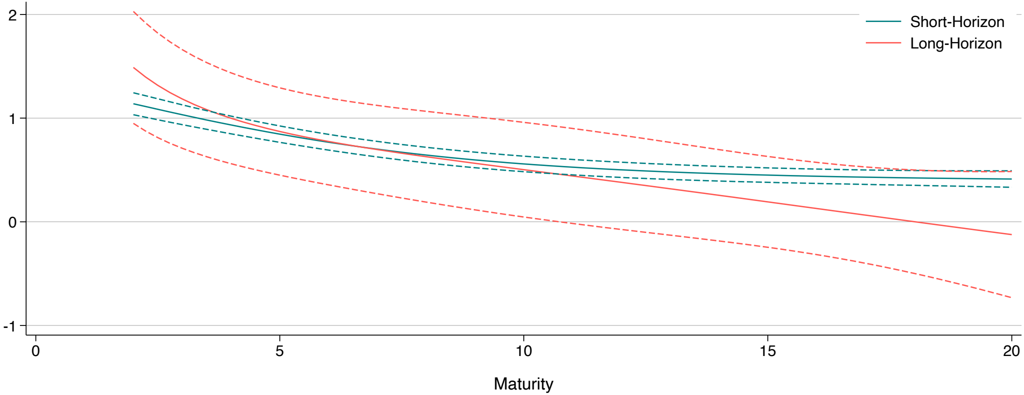

Figure 1: Forward Rates (Unconditional, Varying Maturities)

Full sample (1982-2020), $h = 0$ and $h = 90$, $\tau = 2, \ldots 20$

Empirical Results: Unconditional, Varying Horizon

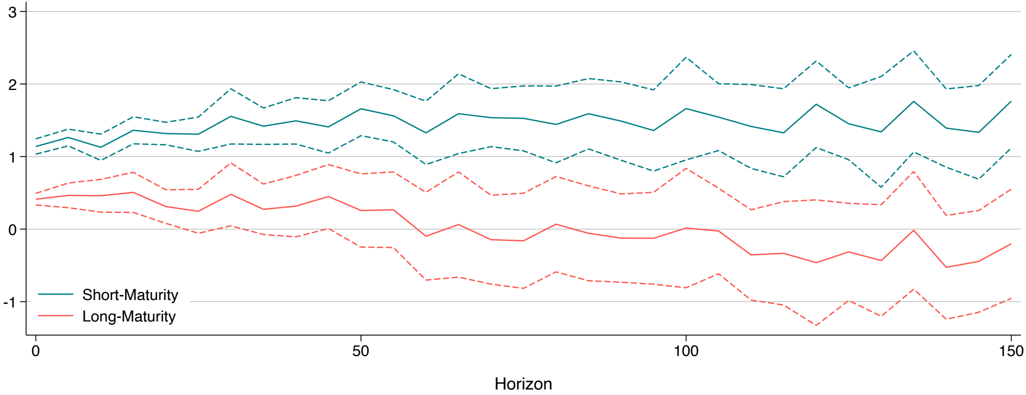

Figure 2: Forward Rates (Unconditional, Varying Horizon)

Full sample (1982-2020), $h = 0 \ldots 150$, $\tau = 2$ and $\tau = 20$

Empirical Results: Unconditional, Rolling Short Horizon

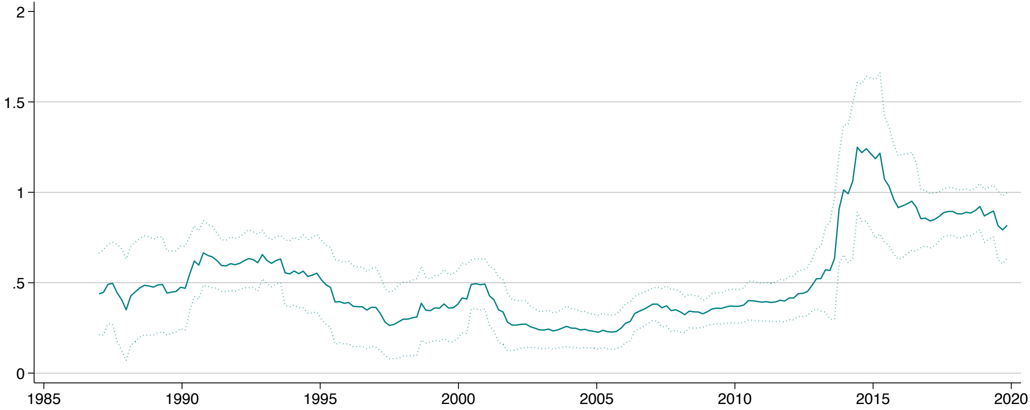

Figure 3: Forward Rates (Unconditional, Rolling Short Horizon)

Rolling window (5 year), $h = 0$, $\tau = 20$

Empirical Results: Unconditional, Rolling Long Horizon

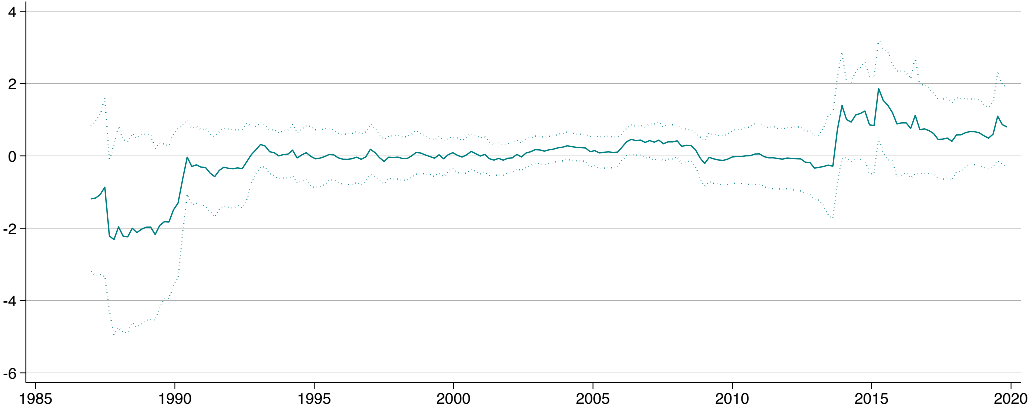

Figure 4: Forward Rates (Unconditional, Rolling Long Horizon)

Rolling window (5 year), $h = 90$, $\tau = 20$

Empirical Results: Conditional, Varying Maturity

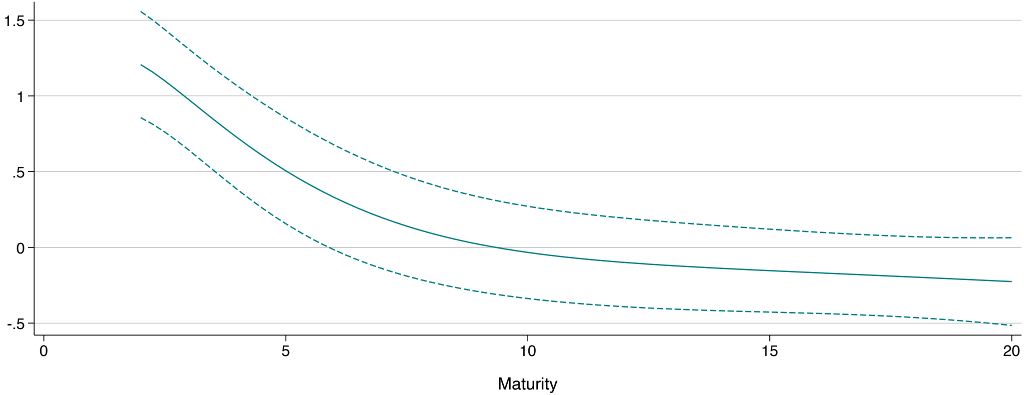

Figure 5: Forward Rates (Shocks, Varying Maturity)

Full sample (FOMC meetings 1995-2020), $h = 0$, $\tau = 2, \ldots, 20$

Empirical Results: Conditional, Rolling

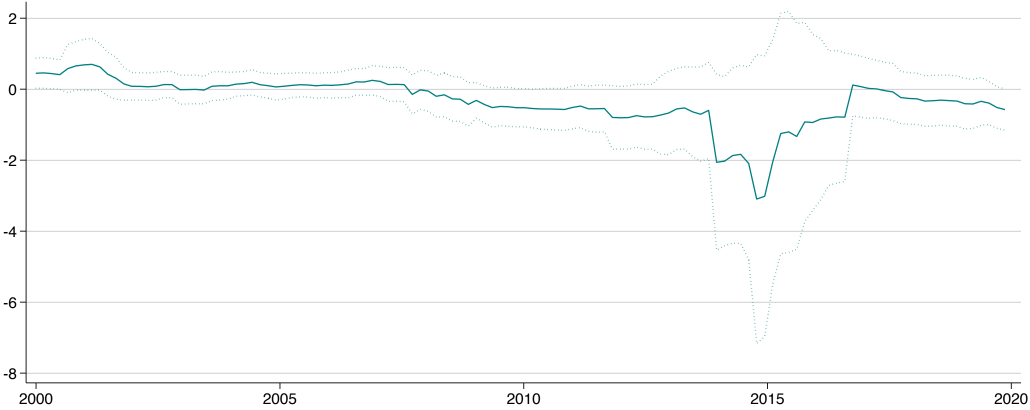

Figure 6: Forward Rates (Shocks, Rolling)

Rolling window (5 year), $h = 0$, $\tau = 20$

Welfare

Welfare Consequences: Simple Policy Rules

- For simplicity, continue assuming rigid prices
- Consider policy rules which implement

$$i_t = \chi_{i,v} v_t + \chi_{i,b} b_t^{\sigma}$$
$$qe_t(\tau) = \chi_{q,v}(\tau) v_t + \chi_{q,b}(\tau) b_t^{\sigma}$$

- Simple policy rules: function of natural state variables only
    - Time-consistent: policymaker seeks to minimize unconditional social welfare loss
- We will examine the outcome of these policies in different versions of the model
- Risk-neutral benchmark: perfect arbitrage ($\hat{a} = 0$) implies $\chi_{i,v} = 1$ is optimal

Optimal Policy: Short Rate Only

- First consider short rate tools only (formally, balance sheet frictions $\psi^{(\tau)} \to \infty$)

Proposition (Optimal short rate policy rule)

Assume risk aversion $\hat{a} > 0$ and $\beta(\tau) > 0$. If bond dispersion across households $\check{\Sigma} = 0$:

- $\exists \chi^n_{i,v} \lesseqgtr 1$ along with $\chi_{i,b} = 0$ which guarantees $x_t = 0 \ \forall t$.
- Sign of $\chi^n_{i,v} - 1$ is determined by the endogenous reaction of firm borrowing to v_t.

With $\check{\Sigma} > 0$:

- Optimal short rate policy $i_t = \chi^*_{i,v} v_t + \chi_{b,i} b^\sigma_t$ where $\chi_{b,i} \neq 0$ and $\chi^*_{i,v} < \chi^n_{i,v}$.
- Implications
    1. Bond carry trade returns $\mu_t(\tau) - i_t$ move in the same direction as i_t iff firm borrowing declines in response to natural rate shocks.
    2. Output gaps x_t are not identically zero.
    3. Consumption dispersion is non-zero: $\mathrm{Var}_i\, \check{c}_t(i) \neq 0$.

Optimal Short Rate Intuition

- Follows from intuition derived studying ad-hoc rules
- Consider recessionary shock $\downarrow v_t \implies \downarrow i_t$
  - If $\downarrow$ firm borrowing, then arbitrageur rebalancing $\implies \downarrow \Lambda_t$
  - Vice-versa if $\uparrow$ firm borrowing
  - In order to keep $\tilde{\mu}_t = v_t$, policy must be react less/more strongly than RANK benchmark (depending on firm borrowing reaction)

PE Illustration

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- If policy is such that $\tilde{\mu}_t = v_t$, fluctuations in risk prices Λ_t imply $\exists \mu_t(\tau) \neq v_t$
- Fluctuations in borrowing rates across the term structure imply $\mathrm{Var}_i\, \check{c}_t(i) > 0$

PE Illustration

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- Fluctuations in borrowing rates across the term structure imply $\text{Var}_i\, \check{c}_t(i) > 0$
- All else equal:
  - Reducing policy rate volatility $\implies$ term premia volatility $\downarrow$
  - Reducing policy rate response to shocks $\implies$ macro volatility $\uparrow$
- Optimal policy balances these objectives

PE Illustration

Optimal Policy: Unconstrained Case

- With access to frictionless balance sheet policies, we obtain the following

Proposition (Optimal policy separation principle)

Assume risk aversion $\hat{a} > 0$ and $\beta(\tau) > 0$. Suppose the central bank implements short rate and balance sheet policy according to

$$i_t = v_t, \quad \int_0^T \sigma(\tau) q e_t(\tau) \, \mathrm{d}\tau = -\frac{\beta^\sigma}{\varsigma \kappa_z} v_t.$$

If short rate policy is frictionless ($\psi^i = 0$) and the central bank does not face holding costs ($\psi^{(\tau)} = 0$), then first-best is achieved:

1. Macroeconomic stabilization: $x_t = 0 \; \forall t$.
2. Term premia stabilization: $\mu_t(\tau) = \tilde{\mu}_t \; \forall \tau$.
3. Consumption equalization: $c_t(i) = c_t(i') \; \forall i, i'$.

Optimal Policy Separation Principle: Intuition

- The results follow naturally from our findings regarding ad-hoc policy
- QE policy stabilizes shocks to bond markets by offsetting all firm borrowing movements
- Implies net zero arbitrageur positions so

$$\int_0^T \sigma(\tau) x_t(\tau)\, \mathrm{d}\tau = 0 \implies \Lambda_t = 0$$

- This equalizes borrowing rates across HHs: $\mu_t(\tau) = \tilde{\mu}_t$
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Separation principle for optimal policy:

- Optimal balance sheet policy stabilizes bond markets
- Optimal short rate policy stabilizes macroeconomic aggregates

Optimal Policy with Constraints

- Even with “large” balance sheet constraints the central bank still uses QE to (partially) stabilize term premia details
  - Even if “worse arbitrageur” central bank internalizes welfare costs of risk price fluctuations

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- With short rate constraints, balance sheet tools are capable of stabilizing output or term premia, but not both details
  - QE works by affecting term premia through changes in the market price of risk
  - Although arbitrage is imperfect in this model, arbitrageurs still enforce tight restrictions between between market price of risk and term premia across maturities
  - Hence, while in principle the central bank has a continuum of policy tools $\{qe_t(\tau)\}_{\tau=0}^{T}$, can only manipulate risk price Λ_t
  - Related to localization results (Vayanos & Vila 2021, Ray, Droste, & Gorodnichenko 2024)

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  - Related to localization results (Vayanos & Vila 2021, Ray, Droste, & Gorodnichenko 2024)
- Other extensions (sticky prices, cost-push shocks, noise demand, nonzero first-best term premia): details

History-Dependent Policy

Monetary Policy with Commitment

- When policy frictions bind, simple policy rules fail to achieve first-best
- Instead suppose the policymaker chooses policy tools $\mathbf{u}_t$ as a function of entire history of predetermined and nonpredetermined variables $\mathbf{Y}_t \equiv \begin{bmatrix} \mathbf{y}_t^\top & \mathbf{x}_t^\top \end{bmatrix}^\top$
- Minimizes conditional social loss

$$\begin{aligned} \mathcal{W}_0 &= \frac{1}{2}\mathbb{E}_0 \int_0^\infty e^{-\rho t}\mathcal{L}_t \, \mathrm{d}t \\ &= \frac{1}{2}\mathbb{E}_0 \int_0^\infty e^{-\rho t}\left(\mathbf{Y}_t^\top \mathbf{R}\mathbf{Y}_t + \mathbf{u}_t^\top \mathbf{Q}\mathbf{u}_t\right) \mathrm{d}t\,, \quad \mathbf{y}_0 \text{ given} \end{aligned}$$

- By setting policy in a history-dependent manner, full commitment policies can improve on simple policy rules

Characterizing Optimal Policy with Commitment (*work in progress!*)

Theorem (Optimal Policy with Commitment)

Given $\mathbf{y}_0$, the policymaker minimizes $\mathcal{W}_0$ by choosing $\mathbf{u}_t = \mathbf{F}\mathbf{Y}_t$, which induce equilibrium dynamics $\mathrm{d}\mathbf{Y}_t = -\boldsymbol{\Upsilon}(\mathbf{F})\mathbf{Y}_t\,\mathrm{d}t + \mathbf{S}(\mathbf{F})\,\mathrm{d}\mathbf{B}_t$. Necessary conditions are given by

$$\mathbf{y}_0^\top \left(\partial_i \mathbf{P}_{11} - \partial_i \mathbf{P}_{12}\mathbf{P}_{22}^{-1}\mathbf{P}_{21} - \mathbf{P}_{12}\mathbf{P}_{22}^{-1}\partial_i \mathbf{P}_{21} + \mathbf{P}_{12}\left(\mathbf{P}_{22}^{-1}\partial_i \mathbf{P}_{22}\mathbf{P}_{22}^{-1}\right)\mathbf{P}_{21}\right)\mathbf{y}_0 = 0$$

where $\rho\mathbf{P} = \mathbf{R} + \mathbf{F}^\top\mathbf{Q}\mathbf{F} - \mathbf{P}\boldsymbol{\Upsilon} - \boldsymbol{\Upsilon}^\top\mathbf{P}$. Dynamics are given by $\mathbf{q}_0 = \begin{bmatrix}\mathbf{y}_0 & \mathbf{0}\end{bmatrix}^\top$ and

$$\mathrm{d}\mathbf{q}_t = -\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{P}_{21} & \mathbf{P}_{22}\end{bmatrix}\boldsymbol{\Upsilon}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ -\mathbf{P}_{22}^{-1}\mathbf{P}_{21} & \mathbf{P}_{22}^{-1}\end{bmatrix}\mathbf{q}_t\,\mathrm{d}t + \begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{P}_{21} & \mathbf{P}_{22}\end{bmatrix}\mathbf{S}\,\mathrm{d}\mathbf{B}_t \equiv -\boldsymbol{\Gamma}\mathbf{q}_t\,\mathrm{d}t + \boldsymbol{\sigma}\,\mathrm{d}\mathbf{B}_t$$

Bond prices are affine in $\mathbf{A}(\tau)^\top\mathbf{q}_t$ with $\mathbf{A}(\tau) = \left[\mathbf{I} - e^{-\mathbf{M}\tau}\right]\mathbf{M}^{-1}\mathbf{e}_i$ and

$$\mathbf{e}_i^\top\mathbf{q}_t = i_t, \quad \mathbf{M} = \boldsymbol{\Gamma}^\top - \int_0^T \boldsymbol{\Theta}(\tau)\mathbf{A}(\tau)^\top\,\mathrm{d}\tau\,\tilde{\boldsymbol{\Sigma}}$$

Monetary Policy with Commitment: Intuition

- Policymaker chooses tools $i_t, \{qe_t(\tau)\}_{\tau=0}^{T}$ which:
  - Directly affect optimality conditions of arbitrageurs
  - Indirectly affect HHs through changes in equilibrium borrowing rates
  - Indirectly affect firms through changes in marginal costs

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- Characterizing dynamics of optimal policy with commitment is difficult
  - Ongoing work studies optimal policy numerically
  - Suffers from time inconsistency; simple rules may be more practical

PE Illustration

Concluding Remarks

- This paper develops a tractable general equilibrium model with market segmentation, nominal frictions, financial frictions, and household heterogeneity
- We find that optimal short rate and balance sheet policy is characterized by a sharp **optimal separation result**:
  - Conventional policy targets macroeconomic stability
  - Unconventional policy targets bond market stability
- Optimal policy removes excess volatility of bond returns and hence improves risk-sharing across households, while reducing excess macroeconomic volatility
  - Policy constraints on either the short rate or balance sheets imply trade-offs between these policy objectives
- When considering social welfare, cannot abstract from the policy tools used to conduct monetary policy

Thank You!

Households

- Continuum of HH members $i \in [0,1]$, differentiated by access to bond markets
  - Captures the observed differentiated HH portfolios (eg, due to demographics, market access via investment funds, mortgage market structure, etc)
  - Formalization: HHs sluggishly rebalance (our model is limiting case)
- HH i chooses consumption and labor $C_t(i), N_t(i)$ in order to solve

$$V_0(i) \equiv \max \mathrm{E}_0 \int_0^\infty e^{-\rho t} \left(\frac{C_t(i)^{1-\varsigma} - 1}{1-\varsigma} - \frac{N_t(i)^{1+\varphi}}{1+\varphi} \right) \mathrm{d}t$$

$$\text{s.t.} \quad \mathrm{d}\mathcal{B}_t(i) = [\mathcal{W}_t N_t(i) - \mathcal{P}_t C_t(i)]\,\mathrm{d}t + \mathcal{B}_t(i)\,\mathrm{d}\tilde{R}_t(i) + \mathrm{d}\mathcal{F}_t$$

- $\mathcal{B}_t(i)$ nominal savings earn $\mathrm{d}\tilde{R}_t(i)$
- Taken as given (as well as nominal wage $\mathcal{W}_t$, price index $\mathcal{P}_t$, transfers $\mathrm{d}\mathcal{F}_t$)

Key takeaway: consumption/savings choices differ when bond returns not equalized back

Firms

- Continuum of intermediate goods $j \in [0,1]$ (and CES final good with elasticity ϵ)
- Produce using labor $Y_t(j) = Z_t L_t(j)$
- Revenue and costs of production:

$$\mathrm{d}\Pi_t(j) = \left[(1+\tau^y)\mathcal{P}_t(j)Y_t(j) - \mathcal{W}_t L_t(j) - \mathcal{T}_t^y\right]\mathrm{d}t - \mathrm{d}\Theta_t(j)$$

$$\mathrm{d}\Theta_t(j) = \frac{\vartheta}{2}\left(\pi_t(j) - \varpi_t\right)^2 \mathcal{P}_t Y_t\,\mathrm{d}t + \mathcal{W}_t L_t(j)\,\mathrm{d}\hat{R}_t$$

- Rotemberg costs when setting prices $\mathrm{d}\mathcal{P}_t(j) = \mathcal{P}_t(j)\pi_t(j)\,\mathrm{d}t$ (away from target ϖ_t)
- Working capital friction: finance a fraction $\int_0^T \beta_t(\tau)\,\mathrm{d}\tau$ of wage bill
- Taking as given CES demand, τ^y subsidy, taxes $\mathcal{T}_t^y$, SDF $Q_t^{\mathcal{H}}$, firm j solves:

$$U_0(j) \equiv \max \mathrm{E}_0 \int_0^\infty e^{-\rho t} Q_t^{\mathcal{H}}\,\mathrm{d}\Pi_t(j)$$

Key takeaway: inefficiencies due to pricing frictions, financing friction back

Arbitrageurs

- Mean-variance optimization

$$\max \mathsf{E}_t \, \mathrm{d}\mathcal{X}_t - \frac{a_t}{2} \mathsf{Var}_t \, \mathrm{d}\mathcal{X}_t$$
$$\text{s.t.} \;\; \mathrm{d}\mathcal{X}_t = \mathcal{X}_t i_t \, \mathrm{d}t + \int_0^T \mathcal{X}_t(\tau) \left(\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} - i_t \, \mathrm{d}t \right) \mathrm{d}\tau$$

- Arbitrageurs invest $\mathcal{X}_t(\tau)$ in bond carry trade of maturity τ
- Risk-return trade-off governed by a_t
  - Formally: risk aversion coefficient
  - More generally: proxies for any limits to risk-bearing capacity or intermediation frictions
  - Arbitrageurs transfer gains/losses to HHs, so a_t represents any frictions which hinder ability to trade on behalf of HHs

Key takeaway: risk averse arbitrageurs' holdings increase with expected return back

Central Bank and Fiscal Authority

- Central bank sets policy rate i_t and buys/sells bonds $\mathcal{QE}_t(\tau)$
- Both policy actions potentially subject to costs:

$$Y_t \Psi_t^S \equiv Y_t \int_0^T \frac{\psi(\tau)}{2} \left(\mathcal{QE}_t(\tau)\right)^2 \mathrm{d}\tau$$

$$Y_t \Psi_t^i \equiv Y_t \frac{\psi^i}{2} \left(i_t - \bar{i}_t\right)^2$$

- In the background: fiscal authority chooses subsidies τ^y
- Fiscal authority also supplies bonds: $\mathcal{S}_t^{(\tau)}$ total supply net of QE holdings
- Financed lump-sum via households
- Optimal policy: maximize social welfare

$$\max \mathrm{E}_0 \int_0^\infty e^{-\rho t} \left(\int_0^T \eta(\tau) u\left(C_t(\tau), N_t(\tau)\right) \mathrm{d}\tau \right) \mathrm{d}t$$

- $\eta(\tau)$: fraction of HHs with access to τ bonds (so $\int_0^T \eta(\tau)\,\mathrm{d}\tau = 1$)

back

Aggregation and Market Clearing

- Firms, arbitrageurs, and funds transfer profits equally to HHs
- Symmetric firm equilibrium $Y_t(j) = Y_t, P_t(j) = P_t, \pi_t(j) = \pi_t, \frac{\mathrm{d}P_t}{P_t} = \pi_t \,\mathrm{d}t$
- Clearing in production and goods markets:

$$Y_t = Z_t N_t, \quad C_t \equiv \int_0^1 \eta(i) C_t(i) \,\mathrm{d}i = Y_t \left(1 - \frac{\vartheta}{2}\pi_t^2 - \Psi_t^S - \Psi_t^i\right)$$

- Bond market clearing implies

$$\mathcal{X}_t(\tau) - \bar{\beta}\theta(\tau)\mathcal{W}_t N_t + \int_0^1 \eta_i(\tau)\mathcal{B}_t(i) \,\mathrm{d}i + \mathcal{S}_t(\tau) = 0$$

back

Aggregate Risk Factors and Risk Pricing

- Aggregate technology $Z_t = \bar{Z}e^{z_t}$

$$\mathrm{d}z_t = -\kappa_z z_t \,\mathrm{d}t + \sigma_z \,\mathrm{d}B_{z,t}$$

- Generic set of N_z exogenous risk factors $\mathbf{z}_t$ with associated Brownian motions $\mathbf{B}_t$ (where $z_t \in \mathbf{z}_t, B_{z,t} \in \mathbf{B}_t$) with volatility

$$\mathrm{Var}_t \,\mathrm{d}\mathbf{z}_t = \mathrm{Var}_t \,\boldsymbol{\sigma} \,\mathrm{d}\mathbf{B}_t = \boldsymbol{\sigma}\boldsymbol{\sigma}^\top \,\mathrm{d}t$$

  - Allow for exogenous cost-push shocks, firm financing shocks, discount factor shocks...
- Thus, instantaneous return of τ bond is

$$\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} = \mu_t(\tau)\,\mathrm{d}t + \boldsymbol{\sigma}_t(\tau)\,\mathrm{d}\mathbf{B}_t$$

- Arbitrageur optimality conditions imply

$$\mu_t(\tau) - i_t = a_t \boldsymbol{\sigma}_t(\tau)\boldsymbol{\Lambda}_t, \quad \text{where } \boldsymbol{\Lambda}_t^\top = \int_0^T \mathcal{X}_t(\tau)\boldsymbol{\sigma}_t(\tau)\,\mathrm{d}\tau$$

back

Simple Optimal Short Rate: PE Illustration I

- Partial equilibrium illustration with ad-hoc loss function, simple policy rules
- Dynamics and loss function

$$\mathrm{d}z_t = -\kappa_z z_t \,\mathrm{d}t + \sigma_z \,\mathrm{d}B_t \,, \quad x_t(\tau) = -\theta(\tau) z_t$$

$$\mathcal{L}_t \equiv (z_t - i_t)^2 + \left(\int_0^T \theta(\tau)(\mu_t(\tau) - i_t) \,\mathrm{d}\tau \right)^2 \,, \quad \min \mathrm{E}\, \mathcal{L}_t$$

- Risk prices $\Lambda_t = \int_0^T -\sigma(\tau)\theta(\tau)\,\mathrm{d}\tau\, z_t \equiv -\tilde{\sigma} z_t$

$$\mu_t(\tau) - i_t = \hat{a}\sigma(\tau)\Lambda_t \implies \left(\int_0^T \theta(\tau)(\mu_t(\tau) - i_t) \,\mathrm{d}\tau \right)^2 = \hat{a}^2 \tilde{\sigma}^2 z_t^2$$

- Simple policy rule: choose χ such that $i_t = \chi z_t$

back

Simple Optimal Short Rate: PE Illustration II

- Unconditionally, $\mathrm{E}(z_t - i_t)^2$ is decreasing in χ for $\chi < 1$
- Is $\chi = 1$ optimal? Not if $\hat{a} > 0$, since $\tilde{\sigma}$ is endogenous
- Solving for $\tilde{\sigma}$: conjecture affine term structure

$$-\log P_t^{(\tau)} = A_z(\tau) z_t + C(\tau)$$

- Ito's Lemma and market clearing:

$$A_z'(\tau) + M A_z(\tau) = \chi \implies A_z(\tau) = \chi \frac{1 - e^{-M\tau}}{M}, \quad \text{where } M = \kappa_z + a\sigma_z^2 \int_0^T \theta(\tau) A_z(\tau)\,\mathrm{d}\tau$$

$$\implies \tilde{\sigma}^2 = \sigma_z^2 \left(\int_0^T \theta(\tau) A_z(\tau)\,\mathrm{d}\tau \right)^2$$

- Hence, unconditionally $\mathrm{E}\left(\int_0^T \theta(\tau)(\mu_t(\tau) - i_t)\,\mathrm{d}\tau \right)^2$ is increasing in χ
- Optimal $0 < \chi^* < 1$

back

Separation Principle with Balance Sheet Constraints

- When the central bank faces balance sheet constraints ($\psi^{(\tau)} > 0$), policy can no longer achieve first-best
- However, as long as $\psi^{(\tau)} < \infty$, optimal policy implies the central bank still uses balance sheet tools
- Let $\psi^{(\tau)} = a \cdot \boldsymbol{\sigma}(\tau)\boldsymbol{\sigma}(\tau)^\top$
  - $\implies$ same friction a as arbitrageurs, except policymaker cannot net out positions
- Even with "large" balance sheet costs, the central bank still uses QE to (partially) stabilize term premia
- Intuition:
  - The central bank faces holding costs which imply it is worse than private arbitrageurs at financial intermediation
  - But internalizes the social benefits of minimizing fluctuations in term premia
  - Nevertheless, non-negligible balance sheet costs imply that optimal policy is less reactive

back

Optimal Policy: Short Rate Constraints

- Suppose that short rate policy is constrained, and implements

$$i_t = \tilde{\chi}_i v_t, \quad 0 < \tilde{\chi}_i \ll 1$$

  - Formally: assume costs $\psi^i\,(i_t - \tilde{\chi}_i v_t)$ and take $\psi^i \to \infty$

Proposition (Optimal balance sheet rule)

Assume risk aversion $\hat{a} > 0$, $\beta(\tau) > 0$, and constrained short rates.

- *Bond market stabilization*: $qe_t^\sigma = \beta^\sigma \left((1 + \varsigma + \varphi)x_t - \frac{1}{\varsigma\kappa_z} v_t\right)$ implies
  1. Borrowing rates are stabilized, consumption and wealth dispersion are zero.
  2. Output gaps x_t are no longer identically zero.
- *Macroeconomic stabilization*: there exist parameters $\chi_{q,v} \neq 0, \chi_{q,b} \neq 0$ such that
  1. Output gaps are zero.
  2. Borrowing rate, consumption, and wealth dispersion are non-zero.

back

Extensions Overview

- Sticky prices, cost-push shocks
  - If firm borrowing is a small part of marginal costs, then all results go through
  - Exogenous cost-push shocks breaks divine coincidence but unfortunately, our separation principle still holds
  - Despite multiple policy instruments, (un)conventional policy only affects aggregate outcomes through changes in effective borrowing rate $\tilde{\mu}_t$
  - If firm borrowing is large, then policymaker can in principle manipulate HH and firm effective borrowing rates $\tilde{\mu}_t$, $\hat{\mu}_t$ (though this is suboptimal due to risk-sharing motives)
- "Noise" demand shocks
  - Optimal separation principle still holds with firm financing shocks β_t
  - QE policy must be more reactive than the benchmark
  - The optimal rule may imply conventional and unconventional policies seemingly acting against one another
- Nonzero first-best term premia
  - When first-best BCT returns are $\nu(\tau) \neq 0$
  - Results hold when $\nu(\tau)$ is achievable but optimal short rate policy is a function of $\nu(\tau)$

back

Full Commitment Optimal Short Rate: PE Illustration I

- Partial equilibrium illustration with ad-hoc loss function, full commitment
- Dynamics and loss function

$$\mathrm{d}z_t = -\kappa_z z_t \,\mathrm{d}t + \sigma_z \,\mathrm{d}B_t \,, \quad x_t(\tau) = -\theta(\tau) z_t$$

$$\mathcal{L}_t \equiv (z_t - i_t)^2 + \left(\int_0^T \theta(\tau)(\mu_t(\tau) - i_t) \,\mathrm{d}\tau \right)^2 , \quad \min \mathsf{E}_0 \int_0^\infty e^{-\varrho t} \mathcal{L}_t \,\mathrm{d}t$$

- Risk prices $\Lambda_t = \int_0^T -\sigma(\tau)\theta(\tau)\,\mathrm{d}\tau \, z_t \equiv -\tilde{\sigma} z_t$

$$\mu_t(\tau) - i_t = \hat{a}\sigma(\tau)\Lambda_t \implies \left(\int_0^T \theta(\tau)(\mu_t(\tau) - i_t) \,\mathrm{d}\tau \right)^2 = \hat{a}^2 \tilde{\sigma}^2 z_t^2$$

- Policy rule with commitment: choose χ, κ_i, i_0 such that

$$\mathrm{d}i_t = -\kappa_i (i_t - \chi z_t) \,\mathrm{d}t$$

back

Full Commitment Optimal Short Rate: PE Illustration II

- Dynamics

$$\mathbf{x}_t = e^{-\boldsymbol{\Gamma} t}\mathbf{x}_0 + \int_0^t e^{-\boldsymbol{\Gamma}(t-u)}\boldsymbol{\sigma}_x \,\mathrm{d}B_u\,, \quad \boldsymbol{\Gamma} = \begin{bmatrix} \kappa_z & 0 \\ -\kappa_i\chi & \kappa_i \end{bmatrix}, \quad \boldsymbol{\sigma}_x = \begin{bmatrix} \sigma_z \\ 0 \end{bmatrix}$$

- Affine term structure

$$-\log P_t^{(\tau)} = A_z(\tau)z_t + A_i(\tau)i_t + C(\tau) \equiv \mathbf{A}(\tau)^\top \mathbf{x}_t + C(\tau)$$

$$\implies \mathbf{A}(\tau) = \mathbf{M}^{-1}\left[\mathbf{I} - e^{-\mathbf{M}\tau}\right]\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{M} \equiv \boldsymbol{\Gamma}^\top + \begin{bmatrix} \hat{a}\sigma_z^2 \int_0^T \theta(\tau)A_z(\tau)\,\mathrm{d}\tau & 0 \\ 0 & 0 \end{bmatrix}$$

- If $\hat{a} = 0$, then $i_0 = z_0, \chi = 1, \kappa_i \to \infty$
- As with simple policy rules, $\chi \to 0 \implies A_z(\tau) \to 0$
- But policymaker still utilizes choices of i_0 and $\kappa_i < \infty$ (smoothing)

back