A Preferred-Habitat Model of Term Premia, Exchange Rates, and Monetary Policy Spillovers

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Outline

1. ~~Motivation~~
2. Model Setup
3. Equilibrium Conditions
4. Special Cases
5. General Solution Numerical Algorithm
6. Towards General Equilibrium

Model Setup

Setup: Two-Country Vayanos & Vila (2021)

Goal: a model of segmented markets which can help us understand the joint behavior of term premia and currency risk premia (including in reaction to QE/QT)

- Continuous time $t \in (0, \infty)$, 2 countries $j = H, F$
- Nominal exchange rate e_t: H price of F (increase $\equiv$ depreciation of H's currency)
- In each country j, continuum of zero coupon bonds in zero net supply with maturity $0 \leq \tau \leq T$, and $T \leq \infty$
- Bond price (in local currency) $P_{jt}^{(\tau)}$, with yield to maturity $y_{jt}^{(\tau)} = -\log P_{jt}^{(\tau)} / \tau$
- Nominal short rate ("monetary policy") $i_{jt} = \lim_{\tau \to 0} y_{jt}^{(\tau)}$ (exogenous, stochastic)

Overview: Arbitrageurs and Preferred-Habitat Investors

- Home and foreign preferred-habitat bond investors
  (hold bonds in a specific currency and maturity: $Z_{jt}(\tau)$)
  - Eg, pension funds, money market mutual funds
  - Time-varying demand β_{jt}, downward sloping in terms of bond price (elasticity $\alpha_j(\tau)$)
- Preferred-habitat currency traders
  (hold foreign currency: Z_{et})
  - Eg, importers/exporters
  - Time-varying demand γ_t, downward sloping in terms of exchange rate (elasticity α_e)
- Global rate arbitrageurs
  (can trade in both currencies, in domestic and foreign bonds: $W_{Ft}, X_{jt}(\tau)$)
  - Eg, global hedge funds
  - Mean-variance preferences (risk aversion a)
  - Engage in currency carry trade, domestic and foreign bond carry trade

Overview: Solving the Model

1. Collect state variables $\mathbf{q}_t \equiv \begin{bmatrix} i_{Ht} & i_{Ft} & \beta_{Ht} & \beta_{Ft} & \gamma_t \end{bmatrix}^\top$. Vector OU (exogenous):

$$\mathrm{d}\mathbf{q}_t = -\boldsymbol{\Gamma}\left(\mathbf{q}_t - \overline{\mathbf{q}}\right)\mathrm{d}t + \boldsymbol{\sigma}\,\mathrm{d}\mathbf{B}_t$$

2. Conjecture affine (log) prices:

$$-\log P_{jt}^{(\tau)} = \mathbf{A}_j(\tau)^\top \mathbf{q}_t + C_j(\tau)$$
$$-\log e_t = \mathbf{A}_e^\top \mathbf{q}_t + C_e$$

3. Ito's Lemma + arbitrageur optimality conditions pins down excess returns as a function of arbitrageur holdings
4. Market clearing + habitat demand characterizes the solution to the unknown coefficients $\mathbf{A}_j(\tau), \mathbf{A}_e, \ldots$
5. Solve! (confirm affine conjecture holds)

Equilibrium Conditions

Details: Global Rate Arbitrageur

- Mean-variance preferences

$$\begin{aligned}
&\max \mathbb{E}_t(\mathrm{d}W_t) - \frac{a}{2}\mathbb{V}\mathrm{ar}_t(\mathrm{d}W_t) \\
\text{s.t.}\ \ \mathrm{d}W_t =& W_t i_{Ht}\,\mathrm{d}t + W_{Ft}\left(\frac{\mathrm{d}e_t}{e_t} + (i_{Ft} - i_{Ht})\,\mathrm{d}t\right) \\
&+ \int_0^T X_{Ht}^{(\tau)}\left(\frac{\mathrm{d}P_{Ht}^{(\tau)}}{P_{Ht}^{(\tau)}} - i_{Ht}\,\mathrm{d}t\right)\mathrm{d}\tau + \int_0^T X_{Ft}^{(\tau)}\left(\frac{\mathrm{d}(P_{Ft}^{(\tau)}e_t)}{P_{Ft}^{(\tau)}e_t} - \frac{\mathrm{d}e_t}{e_t} - i_{Ft}\,\mathrm{d}t\right)\mathrm{d}\tau
\end{aligned}$$

- Wealth W_t:
  - W_{Ft} invested in country F short rate (CCT)
  - $X_{jt}^{(\tau)}$ invested in bond of country j and maturity τ (BCT_j)
  - Remainder in country H short rate

Details: Arbitrageur Optimality Conditions

- Ito's Lemma:

$$\frac{\mathrm{d}P_{jt}^{(\tau)}}{P_{jt}^{(\tau)}} = \mu_{jt}^{(\tau)}\,\mathrm{d}t + \boldsymbol{\sigma}_j^{(\tau)}\,\mathrm{d}\mathsf{B}_t$$

$$\frac{\mathrm{d}e_t}{e_t} = \mu_{et}\,\mathrm{d}t + \boldsymbol{\sigma}_e\,\mathrm{d}\mathsf{B}_t$$

where

$$\mu_{jt}^{(\tau)} = \mathsf{q}_t^\top \mathsf{A}_j'(\tau) + C_j'(\tau) + [\boldsymbol{\Gamma}(\mathsf{q}_t - \overline{\mathsf{q}})]^\top \mathsf{A}_j(\tau) + \frac{1}{2}\,\mathsf{Tr}\left[\boldsymbol{\sigma}\mathsf{A}_j(\tau)\mathsf{A}_j(\tau)^\top\boldsymbol{\sigma}\right]$$

$$\mu_e = [\boldsymbol{\Gamma}(\mathsf{q}_t - \overline{\mathsf{q}})]^\top \mathsf{A}_e + \frac{1}{2}\,\mathsf{Tr}\left[\boldsymbol{\sigma}\mathsf{A}_e\mathsf{A}_e^\top\boldsymbol{\sigma}\right]$$

$$\boldsymbol{\sigma}_j^{(\tau)} = -\mathsf{A}_j(\tau)^\top\boldsymbol{\sigma}$$

$$\boldsymbol{\sigma}_e = -\mathsf{A}_e^\top\boldsymbol{\sigma}$$

Details: Arbitrageur Optimality Conditions

- Arbitrageurs' optimality conditions imply expected excess returns are given by:

$$\mu_{jt}^{(\tau)} - i_{jt} = \mathbf{A}_j(\tau)^\top \boldsymbol{\Lambda}_t$$
$$\mu_{et} + i_{Ft} - i_{Ht} = \mathbf{A}_e{}^\top \boldsymbol{\Lambda}_t$$

- Endogenous coefficients $\mathbf{A}_j(\tau), \mathbf{A}_e$ govern sensitivity to market price of risk $\boldsymbol{\Lambda}_t$

$$\boldsymbol{\Lambda}_t = a\boldsymbol{\Sigma}\left(W_{Ft}\mathbf{A}_e + \sum_{j=H,F}\int_0^T X_{jt}^{(\tau)}\mathbf{A}_j(\tau)\,\mathrm{d}\tau\right)$$

where $\boldsymbol{\Sigma} \equiv \boldsymbol{\sigma}\boldsymbol{\sigma}^\top$

Details: Preferred-Habitat Bond and FX Investors

- Demand for bonds in currency j, of maturity τ:

$$Z_{jt}^{(\tau)} = -\alpha_j(\tau) \log P_{jt}^{(\tau)} - \theta_j(\tau)\beta_{jt}$$

  - $\alpha_j(\tau)$: demand elasticity for τ investor in country j
  - $\theta_j(\tau)$: how variations in factor β_{jt} affect demand for τ investor in country j
- Demand for foreign currency (spot):

$$Z_{et} = -\alpha_e \log e_t - \theta_e \gamma_t$$

  - Can accommodate forward demand. Under CIP, equivalent to spot + H and F bond trades
- Market clearing and zero net supply: $X_{jt}^{(\tau)} = -Z_{jt}^{(\tau)}$ and $W_{Ft} = -Z_{et}$
  - WLOG: can rewrite intercept terms to include positive supply
- Rewrite using affine functional form:

$$X_{jt}^{(\tau)} = -\alpha_j(\tau)\left[\mathbf{A}_j(\tau)^\top \mathbf{q}_t + C_j(\tau)\right] + \boldsymbol{\Theta}_j(\tau)^\top \mathbf{q}_t + \zeta_j(\tau)$$
$$W_{Ft} = -\alpha_e\left[\mathbf{A}_e^\top \mathbf{q}_t + C_e\right] + \boldsymbol{\Theta}_e^\top \mathbf{q}_t + \zeta_e$$

Details: Solution Characterization

- Substitute market clearing into arbitrageur optimality conditions, collect $\mathbf{q}_t$ terms:

$$\mathsf{A}_j'(\tau) + \mathsf{M}\mathsf{A}_j(\tau) - \mathbf{e}_j = \mathbf{0}, \quad \mathsf{M}\mathsf{A}_e - (\mathbf{e}_H - \mathbf{e}_F) = \mathbf{0} \quad (\text{where } \mathbf{e}_j^\top \mathbf{q}_t = i_{jt})$$

- The matrix M is defined as

$$\begin{aligned}\mathsf{M} = \mathbf{\Gamma}^\top - a\Big\{ &\int_0^T \left[-\alpha_H(\tau)\mathsf{A}_H(\tau) + \mathbf{\Theta}_H(\tau)\right] \mathsf{A}_H(\tau)^\top \,\mathrm{d}\tau \\ &+ \int_0^T \left[-\alpha_F(\tau)\mathsf{A}_F(\tau) + \mathbf{\Theta}_F(\tau)\right] \mathsf{A}_F(\tau)^\top \,\mathrm{d}\tau \\ &+ \left[-\alpha_e\mathsf{A}_e + \mathbf{\Theta}_e\right] \mathsf{A}_e^\top \Big\}\mathbf{\Sigma}\end{aligned} \tag{1}$$

- Initial conditions $\mathsf{A}_j(0) = \mathbf{0}$. Hence

$$\mathsf{A}_j(\tau) = \left[\mathbf{I} - e^{-\mathsf{M}\tau}\right] \mathsf{M}^{-1}\mathbf{e}_j \tag{2}$$

$$\mathsf{A}_e = \mathsf{M}^{-1}(\mathbf{e}_H - \mathbf{e}_F) \tag{3}$$

Details: Existence and Uniqueness

- Note: $\mathbf{M}$ appears on both sides of equation (1), through the solution of the affine coefficients (2), (3)
  - Interpretation: risk-adjusted dynamics of the risk factors
- In general: system of J^2 nonlinear equations in J^2 unknowns, where $J = \dim \mathbf{q}_t$
- Under risk neutrality ($a = 0$), the solution is simple: $\mathbf{M} = \mathbf{\Gamma}^\top$
- When $a > 0$, the solution may not exist, or there may be multiple equilibria
- Can show (using IFT) that in a neighborhood of $a = 0$, the solution exists and is (locally) unique. Beyond that, very difficult to prove anything analytically in the fully general version of the model

Special Cases: Independence, No Demand Shocks

1. Benchmark: Risk Neutral Global Rate Arbitrageur ("Standard Model")

Consider the benchmark case of a risk neutral global rate arbitrageur: $a = 0$

- Expectation Hypothesis holds:

$$\mathbb{E}_t \, \mathrm{d}P_{Ht}^{(\tau)} / P_{Ht}^{(\tau)} = i_{Ht}, \quad \mathbb{E}_t \, \mathrm{d}P_{Ft}^{(\tau)} / P_{Ft}^{(\tau)} = i_{Ft}$$

  - No effect of QE on yield curve, at Home or Foreign
  - Yield curve independent from foreign short rate shocks

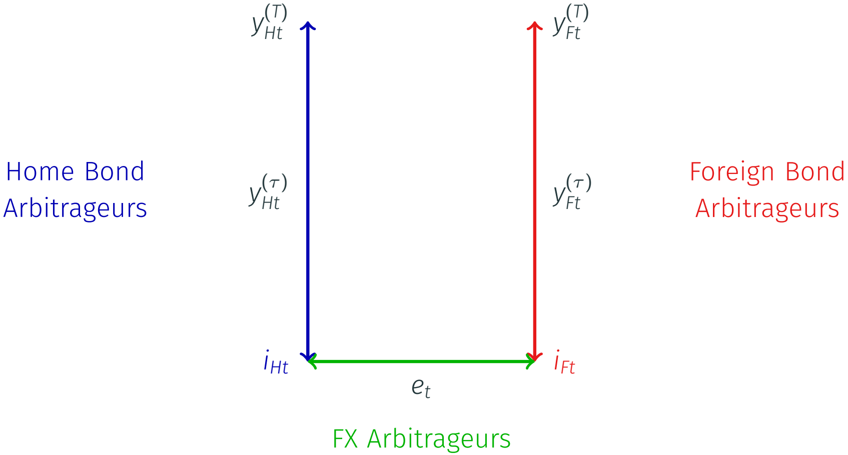
- Uncovered Interest Parity holds:

$$\mathbb{E}_t \, \mathrm{d}e_t / e_t = i_{Ht} - i_{Ft}$$

  - 'Mundellian' insulation: shock to short rates 'absorbed' into the exchange rate
  - Classical Trilemma: capital flows and floating exchange rates deliver monetary autonomy

2. Intermediate Step: Segmented Arbitrage and No Demand Shocks

Assume foreign currency and bonds traded by three disjoint sets of arbitrageurs



2. Intermediate Step: Segmented Arbitrage and No Demand Shocks

Proposition (Segmented Arbitrage, Currency Carry Trade CCT and UIP Deviations)

When arbitrage is segmented, risk aversion $a > 0$ and FX price elasticity $\alpha_e > 0$

- Attenuation relative to risk-neutral case
- CCT expected return $\mathbb{E}_t \, de_t / e_t + i_{Ft} - i_{Ht}$ decreases in i_{Ht} and increases in i_{Ft} (UIP deviation)

Intuition: Similar to Kouri (1982), Gabaix and Maggiori (2015)

- When $i_{Ht} \downarrow$ or $i_{Ft} \uparrow$, FX arbitrageurs want to invest more in the CCT
- Foreign currency appreciates ($e_t \uparrow$)
- As $e_t \uparrow$, price elastic FX traders ($\alpha_e > 0$) reduce holdings: $Z_{et} \downarrow$
- FX arbitrageurs increase their holdings $W_{Ft} \uparrow$, which requires a higher CCT return

2. Intermediate Step: Segmented Arbitrage and No Demand Shocks

Proposition (Segmented Arbitrage and Bond Carry Trade BCT)

When arbitrage is segmented, $a > 0$ and $\alpha(\tau) > 0$ in a positive-measure subset of $(0, T)$:

- Attenuation relative to risk-neutral case
- Bond prices in country j only respond to country j short rates (no spillover)
- BCT_j expected return $\mathbb{E}_t \, \mathrm{d}P_{jt}^{(\tau)} / P_{jt}^{(\tau)} - i_{jt}$ decreases in i_{jt}

Intuition: Similar to Vayanos & Vila (2021)

- When $i_{jt} \downarrow$, bond arbitrageurs want to invest more in the BCT
- Bond prices increase ($P_{jt}^{(\tau)} \uparrow$)
- As $P_{jt}^{(\tau)} \uparrow$, price-elastic habitat bond investors ($\alpha_j(\tau) > 0$) reduce their holdings: $Z_{jt}^{(\tau)} \downarrow$
- Bond arbitrageurs increase their holdings $X_{jt}^{(\tau)} \uparrow$, which requires a larger BCT return

2. Macro Implications of the Segmented Model

Assume $a > 0$, $\theta_j(\tau) > 0$ and $\theta_e > 0$:

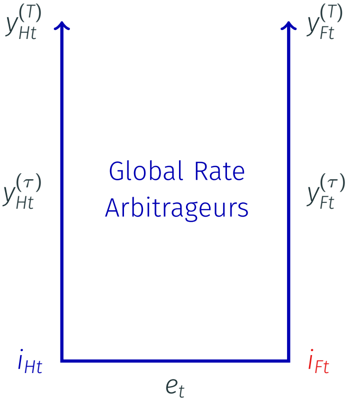
- Unexpected increase in bond demand in country j (QE_j) reduces yields in country j
- No effect on bond yields in the other country or on the exchange rate
    - QE purchases: $Z_{jt}^{(\tau)} \uparrow$
    - Bond arbitrageurs reduce holdings $X_{jt}^{(\tau)} \downarrow$, reducing risk exposure and pushing down yields
    - Arbitrageurs in other markets are unaffected

Open Economy Macro Implications:

- Changes in Home monetary conditions (conventional or QE) have no effect on the Foreign yield curve. Full insulation
- Insulation is even stronger in the case of QE: exchange rate is unchanged
- Insulation is due to arbitrageur segmentation (eg, limited capital flows), not because of floating exchange rates

3. Global Rate Arbitrageur and No Demand Shocks

Assume now global rate arbitrageur can invest in bonds (H and F) and FX



3. Global Rate Arbitrageur and No Demand Shocks

Proposition (Global Arbitrage and Carry Trades CCT, BCT)

When arbitrage is global, risk aversion $a > 0$ and price elasticities $\alpha_e, \alpha_j(\tau) > 0$:

- The results of the previous propositions obtain: both CCT and BCT_H return decrease with i_{Ht}, and attenuation is stronger than with segmented markets
- ⚠ In addition, BCT_F increases with i_{Ht}
- The effect of i_{jt} on bond yields is smaller in the other country: $A_{jj'}(\tau) < A_{jj}(\tau)$

Intuition: Bond and FX Premia Cross-Linkages

- When $i_{Ht} \downarrow$ global arbitrageurs want to invest more in CCT and BCT_H
- e_t and $W_{Ft} \uparrow$: increased FX exposure (risk of $i_{Ft} \downarrow$)
- Hedge by investing more in BCT_F since price of foreign bonds increases when i_{Ft} drops: foreign yields decline and BCT_F decreases

3. Macro Implications of Global Rate Arbitrageur Model

Assume $a > 0$ and $\alpha_e, \alpha_j(\tau) > 0$:

- Unexpected QE_H reduces yields in country H
- ⚠ Also reduces yields in country F, and depreciates the Home currency
  - Arbitrageurs decrease H bond exposure (less exposed to risk of $i_{Ht} \uparrow$)
  - More willing to hold assets exposed to this risk: increase holdings of F bonds and currency, pushing down F yields and depreciating the H currency

Open Economy Macro Implications:

- Changes in Home monetary conditions (conventional or QE) affect both yield curves and the exchange rate: potential spillovers from monetary policy
- Imperfect insulation even with floating rates
- QE (or FX interventions) in one country affect monetary conditions in both countries, and depreciate the currency

General Solution Numerical Algorithm

Numerical Solution: Algorithm

- How to solve for $\mathbf{M}$ in the general model?
- Continuation algorithm:
  1. For $\hat{a} = \hat{a}^{(0)} = 0$, the known solution is $\mathbf{M}^{(0)} = \mathbf{\Gamma}^\top$
  2. Given a solution $\mathbf{M}^{(n)}$ for $\hat{a} = \hat{a}^{(n)}$, use this as the initial value for $\hat{a}^{(n+1)} = \hat{a}^{(n)} + \epsilon$
  3. Solution $\mathbf{M}^{(N)} = \mathbf{M}$ for $\hat{a}^{(N)} = a$
- Notes:
  - Many many ways to make this more sophisticated (see "homotopy continuation")
  - In step 2: any fixed point/root finding algorithm can be used (which can exploit the structure of the problem; again see homotopy continuation)
- For our purposes, we use a fine grid (small fixed step size ϵ) for two reasons:
  1. The code is fast (enough)
  2. The algorithm doubles as an equilibrium selection criteria: we trace out the solution which uniquely converges to the risk-neutral benchmark when $a \to 0$

Numerical Solution: Laplace Transformations

- In order to solve the model numerically, we need to parameterize the habitat functions $\alpha_j(\tau), \theta_j(\tau)$. Our approach:

$$\alpha_j(\tau) = \alpha_{j0} e^{-\alpha_{j1}\tau}$$
$$\theta_j(\tau) = \theta_{j0} \tau e^{-\theta_{j1}\tau}$$

  - Implies price elasticities are declining in τ, yield elasticities are single peaked
  - Demand functions are single-peaked
- If we take maximum maturity $T \to \infty$, then we can use properties of Laplace transforms to simplify the fixed point problem characterizing M
- Turns diff-eqs into algebraic (gets rid of matrix exponentials): $\mathcal{A}(s) \equiv \mathcal{L}\left\{\mathsf{A}(\tau)\right\}(s)$ given by:

$$s\mathcal{A}(s) + \mathsf{M}\mathcal{A}(s) - \frac{1}{s}\mathbf{e}_i = \mathbf{0} \implies \mathcal{A}(s) = [s\mathbf{I} + \mathsf{M}]^{-1}\left[\frac{1}{s}\mathbf{e}_i\right]$$

- Can get rid of all the integral terms in the fixed-point problem for M (but still require the solution algorithm described above)

Model Calibration

- Current approach: choose unknown parameters $\boldsymbol{\rho}$ to minimize the weighted sum of squares:

$$L(\boldsymbol{\rho}) = \sum_{n=1}^{N} w_n(\hat{m}_n - m_n(\boldsymbol{\rho}))^2$$

  - $\{\hat{m}_n\}_{n=1}^N$ are empirical moments and $\{m_n(\boldsymbol{\rho})\}_{n=1}^N$ are model-implied counterparts
  - $\{w_n\}_{n=1}^N$ are weights placed on each target moment
- The art of moment selection: moments are chosen which intuitively relate to unknown coefficients
  - Short-term rates $y_t^{(1)}$: short rate parameters in $\boldsymbol{\Gamma}, \boldsymbol{\sigma}$
  - Long-term rates $y_t^{(\tau)}$: bond demand factor parameters in $\boldsymbol{\Gamma}, \boldsymbol{\sigma}$
  - Exchange rates e_t: currency demand factor parameters in $\boldsymbol{\Gamma}, \boldsymbol{\sigma}$
  - Cross-correlations and deviations from EH/UIP: risk-adjusted parameters $\mathbf{M}$, which indirectly pin down elasticity functions $\alpha(\tau), \alpha_e$

Model Calibration

- Exploring a new approach. Finite-differenced structural model (where $\mathbf{q} : J \times 1$ is not observed):

$$\mathsf{q}_t = [\mathbf{I} - \boldsymbol{\Gamma}]\, \mathsf{q}_{t-1} + \boldsymbol{\sigma}\boldsymbol{\varepsilon}_t$$

- Observation model: $\mathbf{p} : K \times 1$, with $K \geq J$:

$$\mathsf{p} = \mathsf{A}\mathsf{q} \implies \mathsf{A}^{+}\mathsf{p} = \mathsf{q}$$
$$\implies \mathsf{p}_t = \mathsf{A}\, [\mathbf{I} - \boldsymbol{\Gamma}]\, \mathsf{A}^{+}\mathsf{p}_{t-1} + \mathsf{A}\boldsymbol{\sigma}\boldsymbol{\varepsilon}_t$$

- Hence

$$\mathsf{p}_t | \mathsf{p}_{t-1} \sim N\left(\mathsf{B}\mathsf{p}_{t-1}, \mathsf{S}\right)$$
$$\mathsf{B} \equiv \mathsf{A}\, [\mathbf{I} - \boldsymbol{\Gamma}]\, \mathsf{A}^{+}, \quad \mathsf{S} \equiv \mathsf{A}\boldsymbol{\sigma}\boldsymbol{\sigma}^{\top}\mathsf{A}^{\top}$$

- Log-likelihood is given by

$$\mathcal{L} \equiv -\frac{1}{2}\sum_t \left(\mathsf{p}_t - \mathsf{B}\mathsf{p}_{t-1}\right)^{\top} \mathsf{S}^{+} \left(\mathsf{p}_t - \mathsf{B}\mathsf{p}_{t-1}\right) - \frac{T}{2} \log\det \mathsf{S}^{+}$$

Towards General Equilibrium

PE vs. GE Solution

- Given some microfoundations...we eventually end up with (linearized) dynamics

$$\begin{bmatrix} \mathrm{d}\mathsf{y}_t \\ \mathrm{d}\mathsf{x}_t \end{bmatrix} = -\boldsymbol{\Upsilon} \begin{bmatrix} \mathsf{y}_t - \bar{\mathsf{y}} \\ \mathsf{x}_t - \bar{\mathsf{x}} \end{bmatrix} \mathrm{d}t + \boldsymbol{\sigma}\, \mathrm{d}\mathsf{B}_t$$

- y_t: state (predetermined) variables. x_t: jump (non-predetermined) variables
- Assume REE determinacy conditions are met (number of eigenvalues of $\boldsymbol{\Upsilon}$ with positive real part is equal to $\dim \mathsf{y}_t$). Eigendecomposition:

$$\boldsymbol{\Upsilon} = \mathsf{Q}\boldsymbol{\Lambda}\mathsf{Q}^{-1}, \quad \boldsymbol{\Lambda} = \begin{bmatrix} \boldsymbol{\Lambda}_1 & 0 \\ 0 & \boldsymbol{\Lambda}_2 \end{bmatrix}, \quad \mathsf{Q} = \begin{bmatrix} \mathsf{Q}_{11} & \mathsf{Q}_{12} \\ \mathsf{Q}_{21} & \mathsf{Q}_{22} \end{bmatrix},$$

- Then REE dynamics (see Buiter 1984, cts time version of Blanchard-Kahn 1980)

$$\begin{aligned} \mathrm{d}\mathsf{y}_t &= -\boldsymbol{\Gamma}\,(\mathsf{y}_t - \bar{\mathsf{y}})\,\mathrm{d}t + \boldsymbol{\sigma}\,\mathrm{d}\mathsf{B}_t, \qquad & \mathsf{x}_t - \bar{\mathsf{x}} &= \boldsymbol{\Omega}\,(\mathsf{y}_t - \bar{\mathsf{y}}) \\ \boldsymbol{\Gamma} &= \mathsf{Q}_{11}\boldsymbol{\Lambda}_1\mathsf{Q}_{11}^{-1}, \qquad & \boldsymbol{\Omega} &= \mathsf{Q}_{21}\mathsf{Q}_{11}^{-1} \end{aligned}$$

  - Note: dynamics matrix $\boldsymbol{\Upsilon}$ may be a function of long-term bonds, exchange rates
  - Solution method is similar (but now $\boldsymbol{\Gamma}$ is endogenous)

Small Selection of Preferred Habitat Papers

- Seminar papers: Vayanos & Vila (2009, 2021), Greenwood & Vayanos (2014)
    - Closed economy, PE
- FX markets: in addition to our paper, Greenwood et. al. (2022)
    - Two-country, PE
    - Simplified discrete time, two-bond model (but also explore CIP deviations)
- Ray (2019), Droste, Gorodnichenko, Ray (2022)
    - Closed economy, GE
    - Also allows for risky assets (uncertain payoffs)
- Costain, Nuno, Thomas (2022)
    - Currency union, PE
    - Fixed exchange rates, but allows for default (Poisson shocks)
- Early stages of a GE version of this model...
- For discrete time people: eg, Hamilton & Wu (2012)