

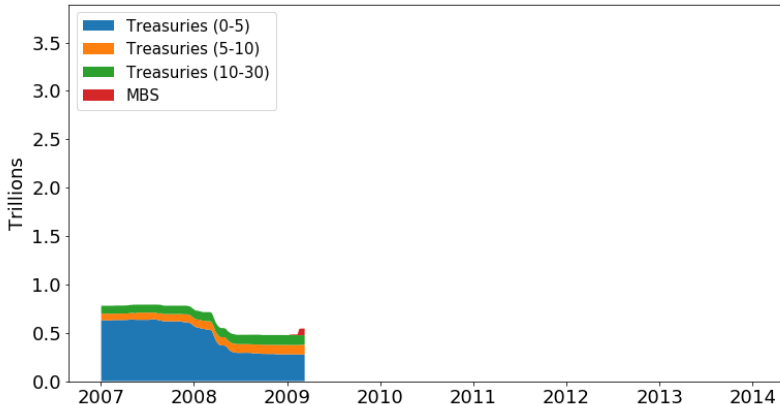
Monetary Policy and the Limits to Arbitrage: Insights from a New Keynesian Preferred Habitat Model

Walker Ray
UC Berkeley

January 28, 2019

LSE Job Market Seminar

Policy Response to Great Recession



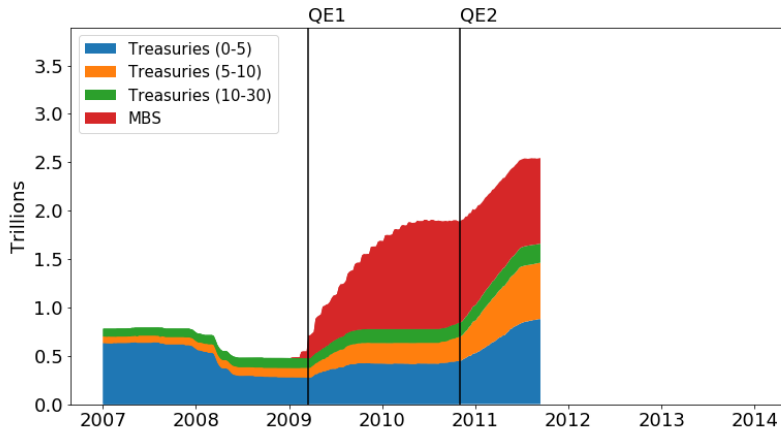
Notes: Federal Reserve holdings of Treasuries (by maturity) and Mortgage-Backed Securities. Vertical lines indicate the start of LSAP programs. Source: FRED.

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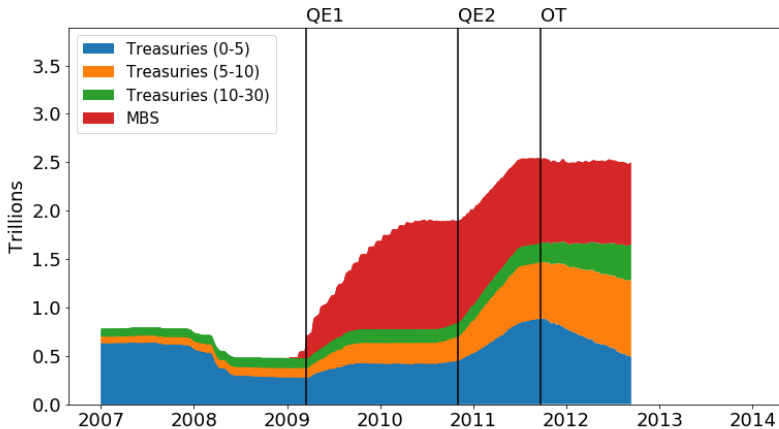
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- Bond market imperfections play a role in the transmission of **conventional** monetary policy
- Crucial for designing monetary policy going forward

Model Overview

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- Monetary policy works through both channels

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- Designing policy going forward:
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 - ▶ QE rule can be stabilizing

Literature Contributions

- “Preferred habitat” as a key channel for understanding bond markets
 - ▶ D’Amico and King (2013), Hamilton and Wu (2012), Greenwood and Vayanos (2014), Gorodnichenko and Ray (2017), Greenwood and Vissing-Jorgensen (2018)
- Few formal models
 - ▶ Vayanos and Vila (2009)
- QE in general equilibrium: Market segmentation vs. forward guidance
 - ▶ Gertler and Karadi (2013), Chen et al (2012), Carlstrom et al (2017), Christensen and Rudebusch (2012), Bauer and Rudebusch (2014), Bhattarai et al (2015)
- Frictions and expected future policy
 - ▶ McKay et al (2016), Farhi and Werning (2017), Gabaix (2016), Angeletos and Lian (2018)

New Keynesian Preferred Habitat Framework

- Time $t \in [0, \infty)$ is continuous. Consumption and production:
 - ▶ Infinitely-lived **households** work and consume
 - ▶ **Firms** produce using labor, face price frictions

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- Government:
 - ▶ **Central bank** sets the short nominal rate (and conducts QE)
 - ▶ Lump-sum taxes/transfers from investors to HHs

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- Modification of a benchmark NK model ($\pi_t \equiv 0$):

$$dx_t = \varsigma^{-1} (r_t - \bar{r}) dt \quad (\text{IS})$$

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- Closing the model: equilibrium term structure determination

Term Structure and Preferred Habitat

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$$\begin{aligned}\tilde{b}_{t,\tau} &= -\alpha(\tau) \log P_{t,\tau} + \varepsilon_{t,\tau} \\ &= \alpha(\tau)\tau(R_{t,\tau} - \beta_{t,\tau})\end{aligned}\tag{PH}$$

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$$\begin{aligned}\max_{\tilde{b}_{t,\tau}} E_t dW_t - \frac{a}{2} \text{Var}_t dW_t \\ \text{s.t. } dW_t = \left(W_t - \int_0^T \tilde{b}_{t,\tau} d\tau \right) r_t dt \\ + \int_0^T \tilde{b}_{t,\tau} \frac{dP_{t,\tau}}{P_{t,\tau}} d\tau\end{aligned}\tag{BC}$$

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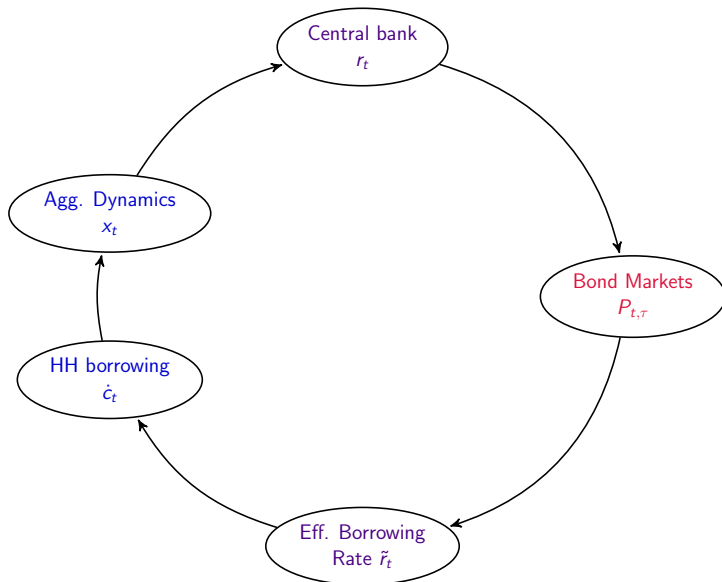
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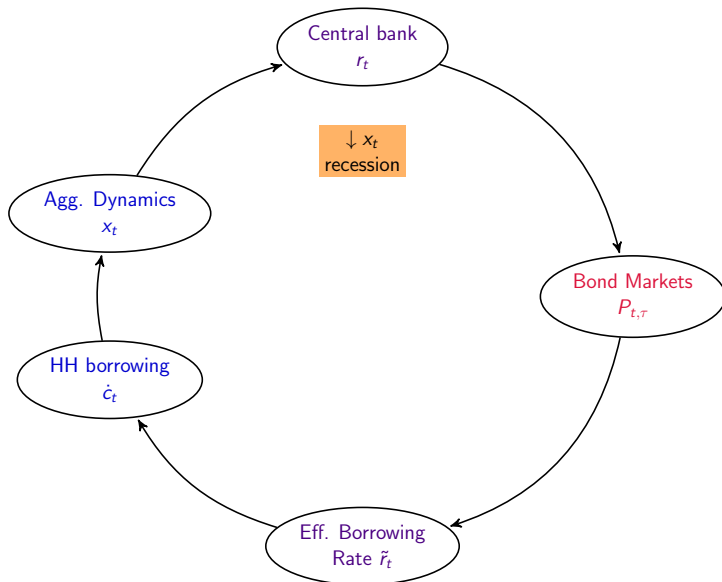
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- Market clearing: $b_{t,\tau} = -\tilde{b}_{t,\tau}$

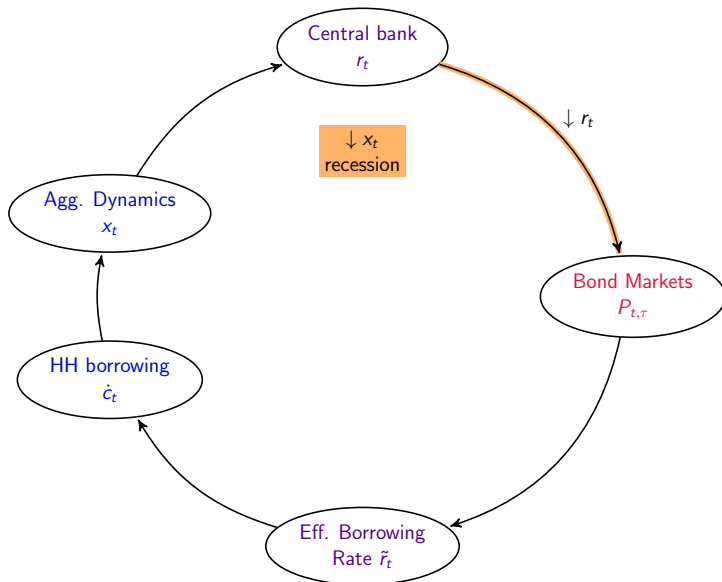
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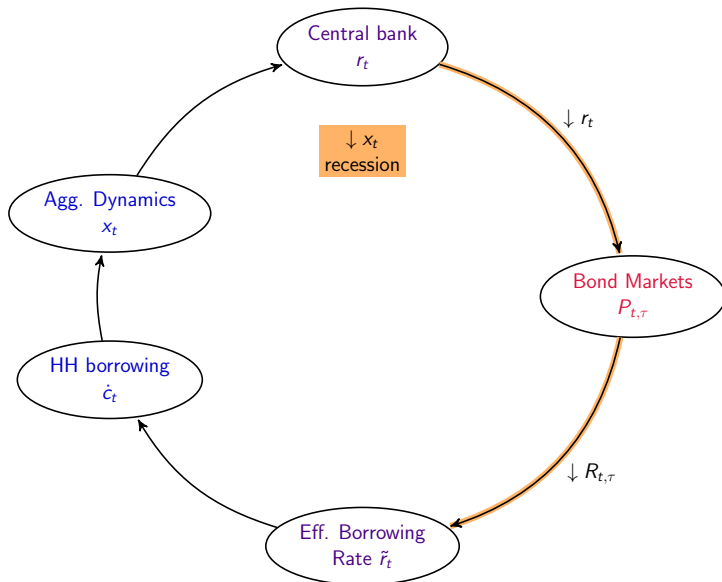
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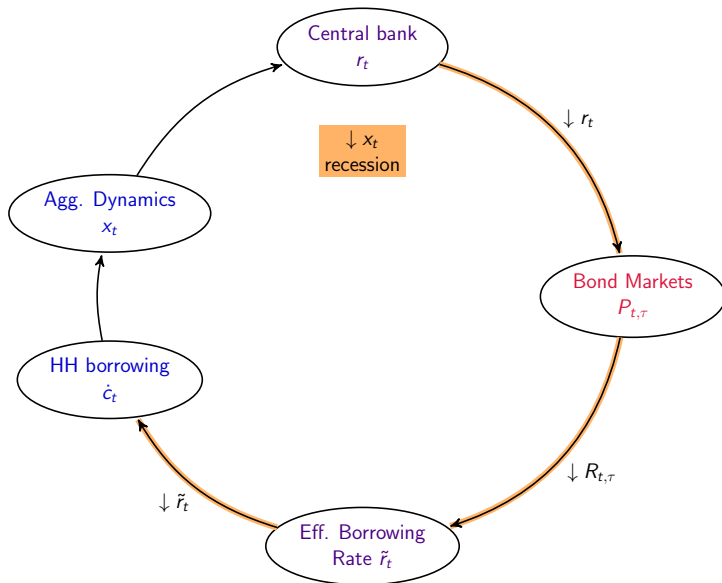
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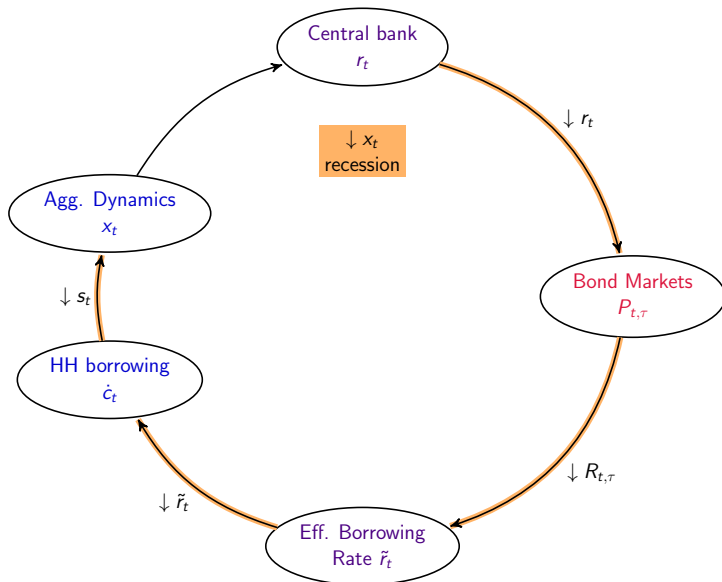
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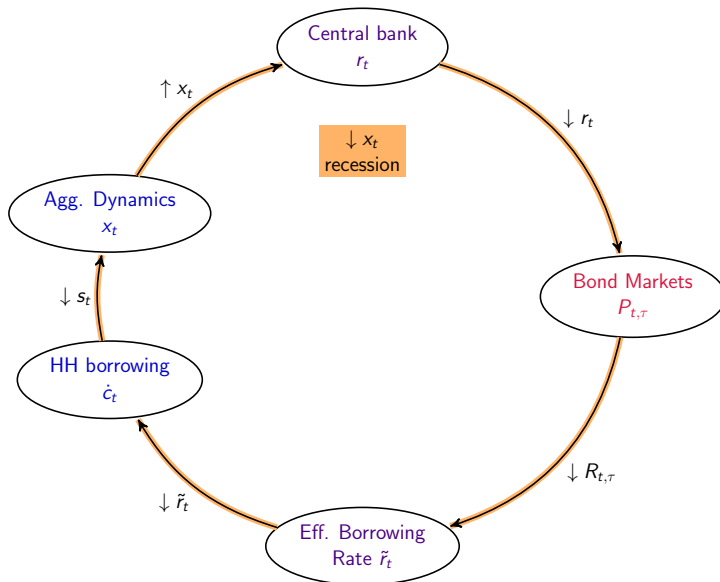
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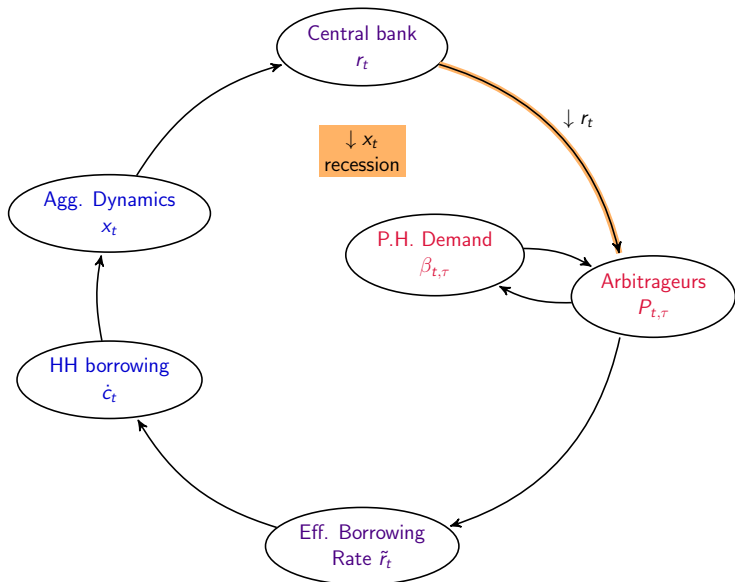
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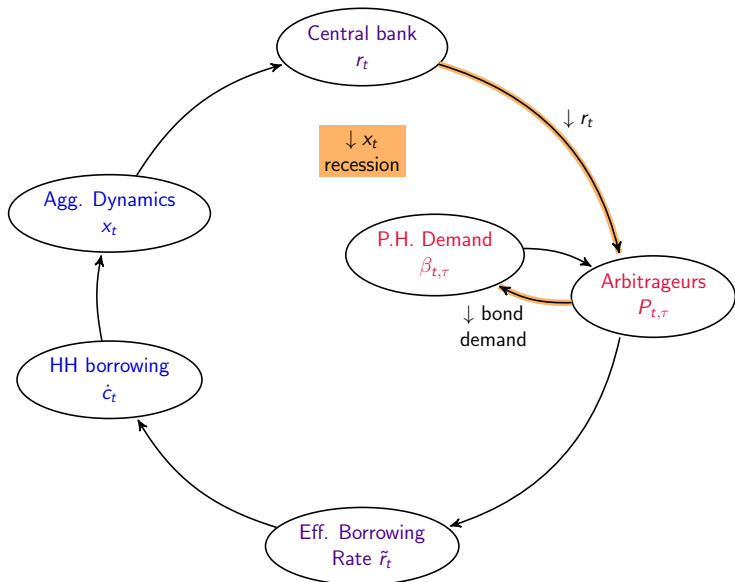
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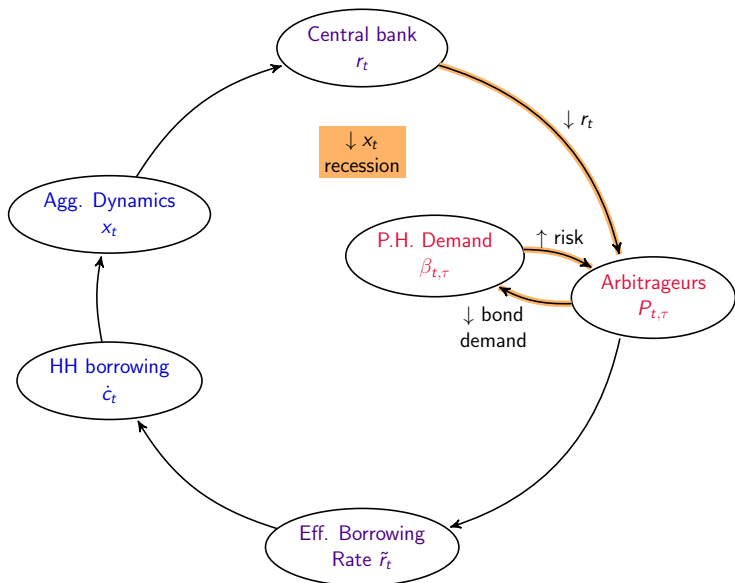
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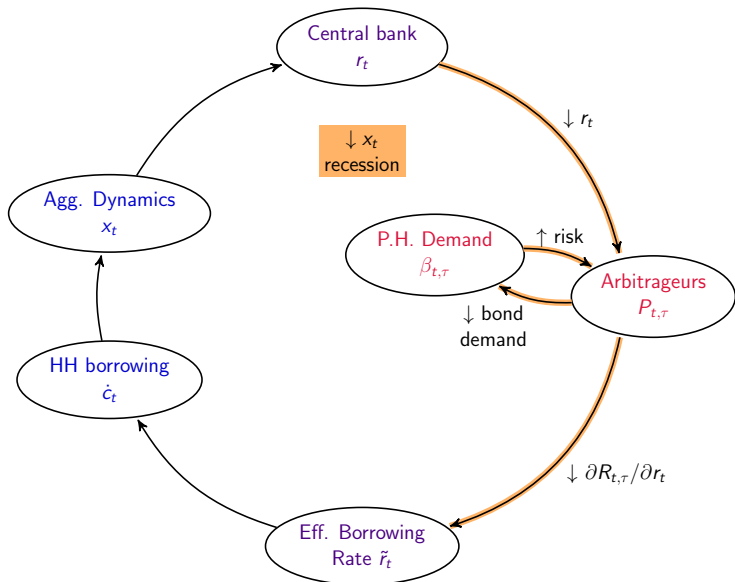
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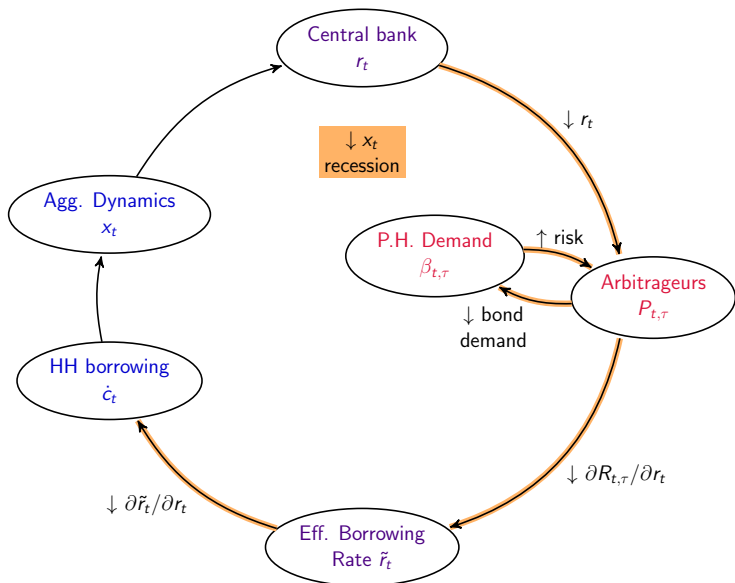
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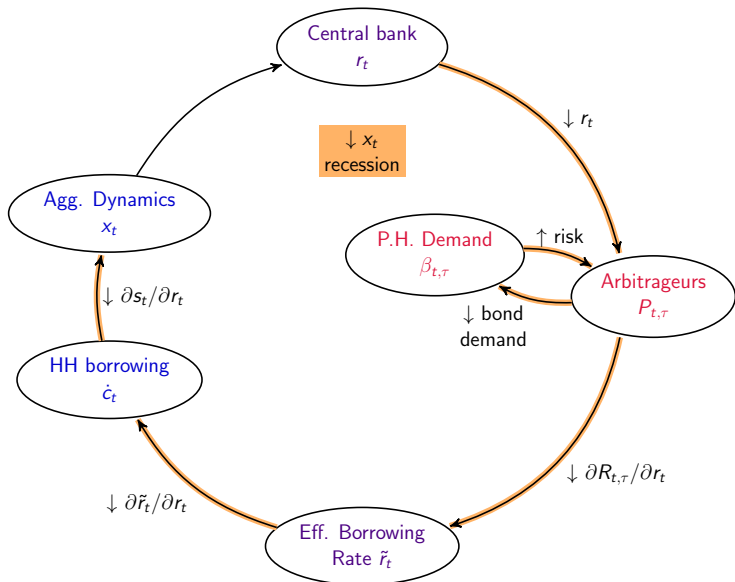
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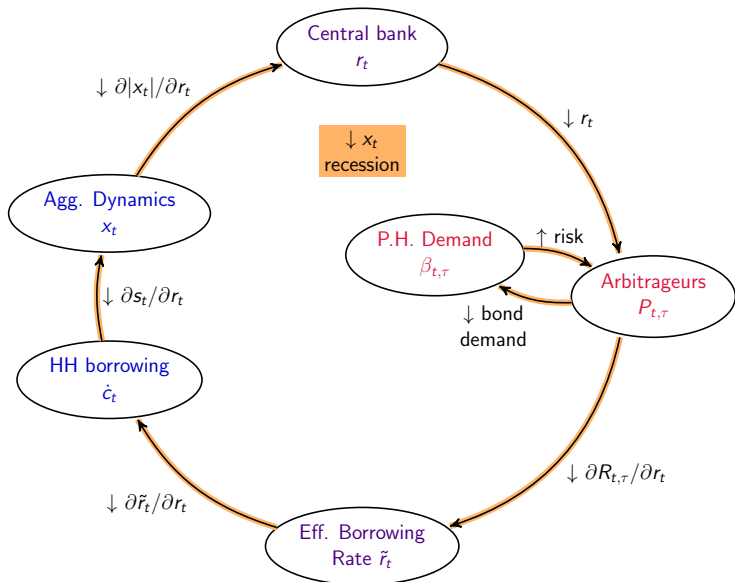
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- Linear stochastic differential equation:

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Rational Expectations Dynamics

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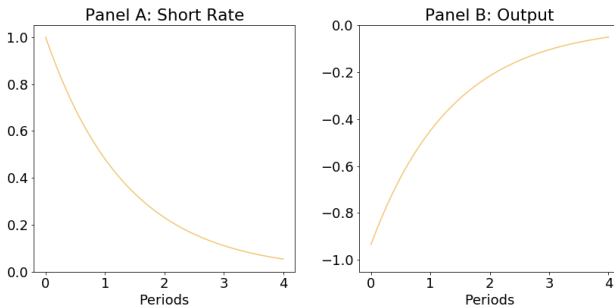
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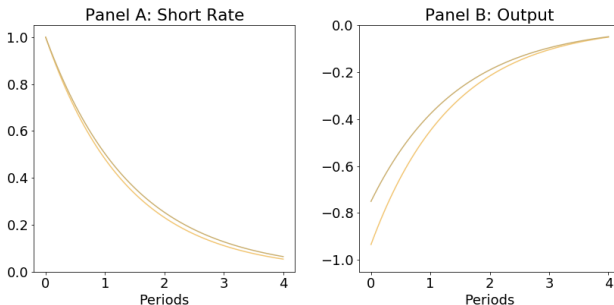
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Dynamics and Output Growth Sensitivity



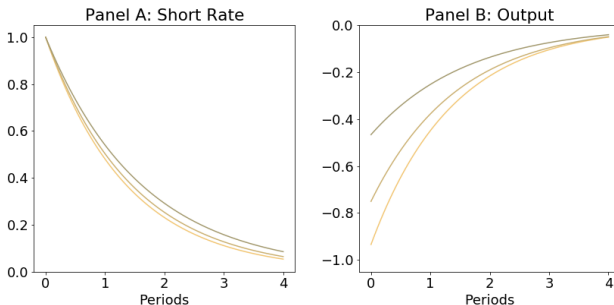
Notes: impulse response functions of the policy rate and output in response to a unit monetary shock, as equilibrium output growth sensitivity to the policy rate falls. Darker lines correspond to lower output growth sensitivity \hat{A}_r . [phase diagrams](#)

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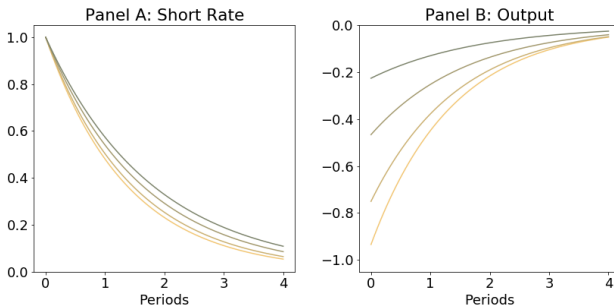
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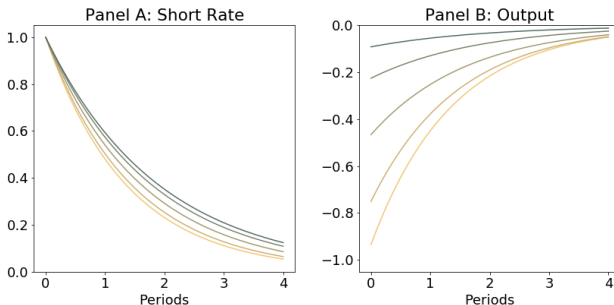
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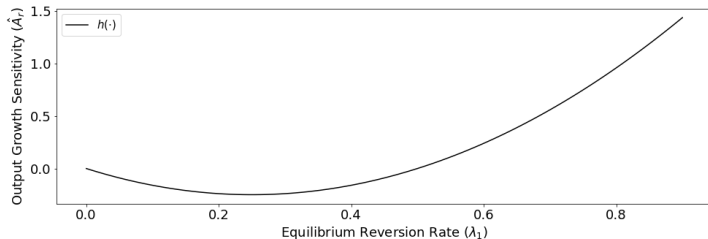
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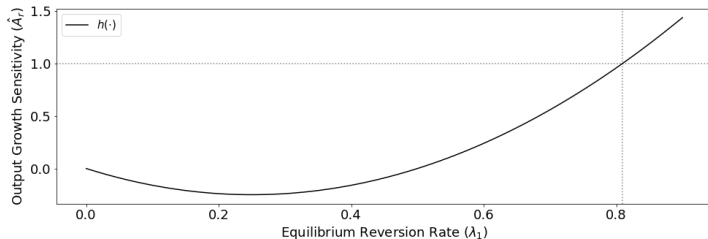


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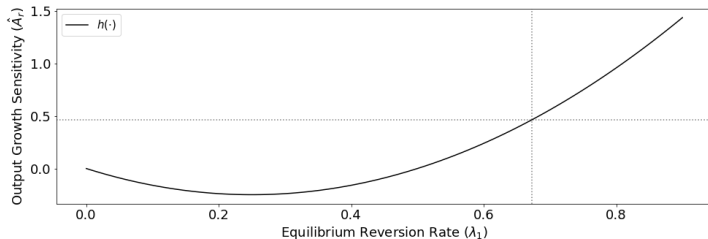


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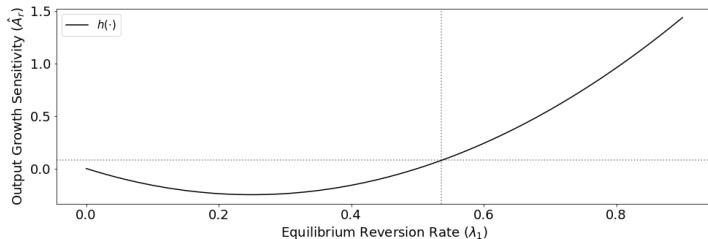


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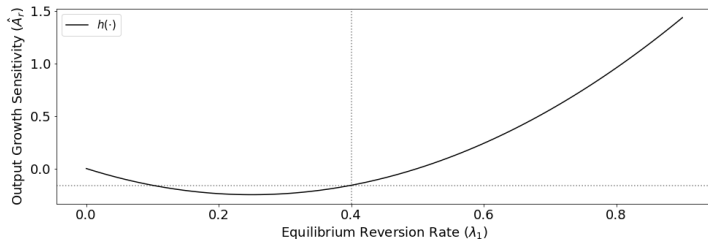


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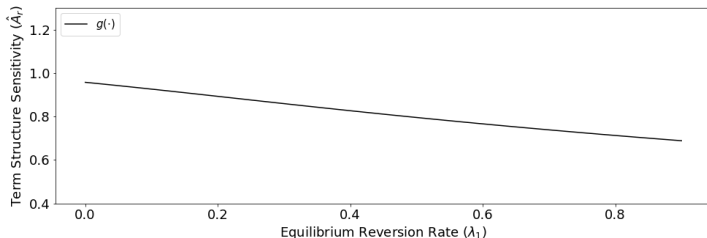
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- Prices adjust to balance demand and optimality conditions
- \implies differential equation which solves affine coefficients

$$\hat{A}_r \equiv \int_0^T \frac{\eta(\tau)}{\tau} A_r(\tau) d\tau$$

Term Structure Equilibrium



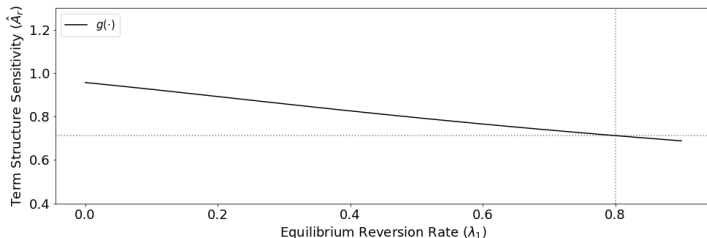
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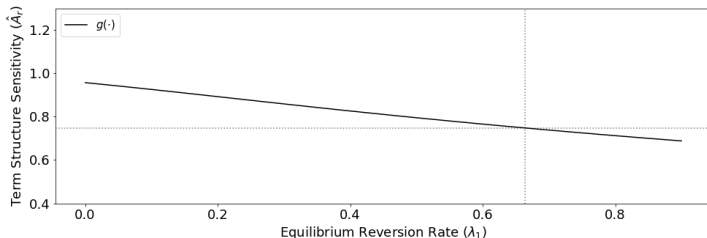
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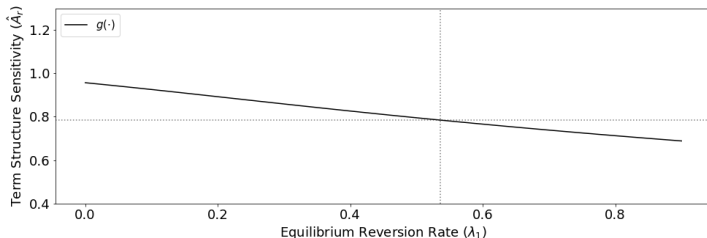
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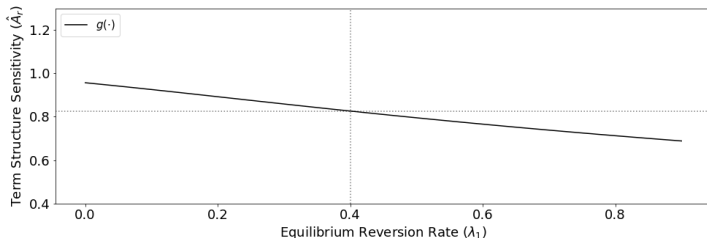
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$$\nu(\lambda) = \lambda + a\sigma_r^2 \int_0^T \alpha(\tau)\tau^2 f(\nu(\lambda)_\tau)^2 d\tau$$

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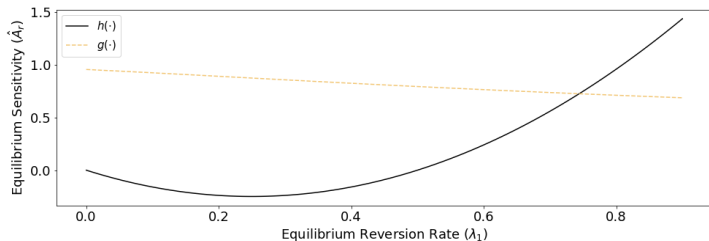
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- EH: two responses should be identical (only when $a = 0$)

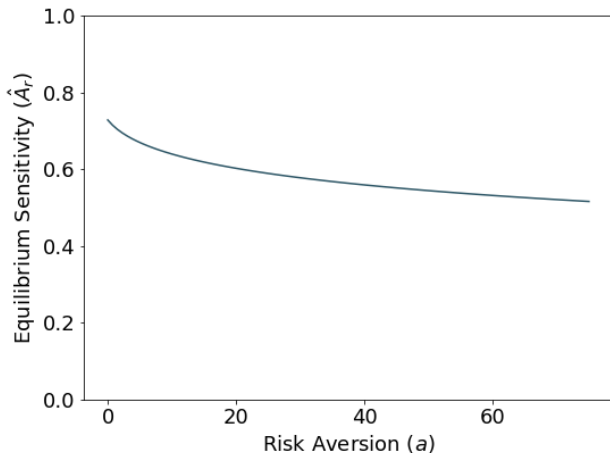
General Equilibrium



Existence and Uniqueness

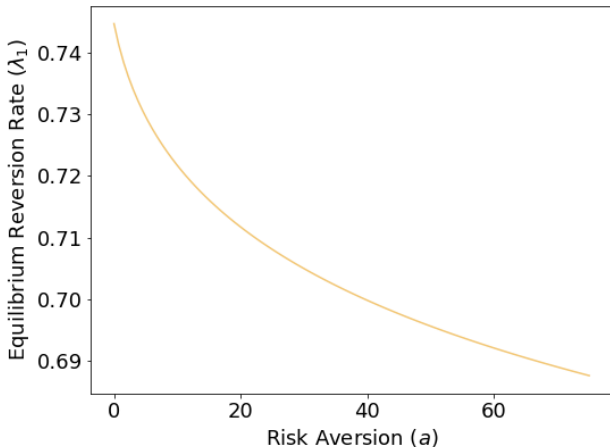
There exists a unique positive eigenvalue of Υ $\lambda_1 > 0$ for which $g(\lambda_1) = h(\lambda_1)$, which fully characterizes the model equilibrium. Further, this implies $0 < \hat{A}_r < 1$.

Conventional Policy and Financial Disruptions



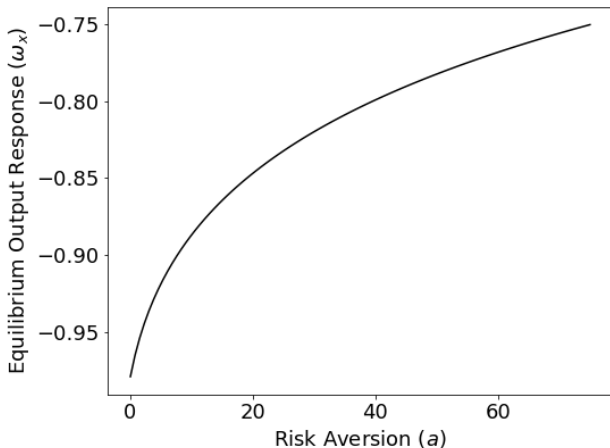
Notes: equilibrium changes in sensitivity to the short rate \hat{A}_r as risk aversion a increases.

Conventional Policy and Financial Disruptions



Notes: equilibrium changes in monetary shock reversion λ_1 as risk aversion a increases.

Conventional Policy and Financial Disruptions



Notes: equilibrium changes in output response ω_x to monetary shocks as risk aversion a increases.

Policy Implications

- More aggressive response to output [\(\$\phi_x\$ results\)](#)
- Higher inertia [\(\$\kappa_r\$ results\)](#)
- Shifts in effective rate weights [\(\$\eta\(\tau\)\$ results\)](#)
- Forward guidance less effective as risk aversion increases [\(details\)](#)

Modeling LSAPs

- Suppose the central bank directly purchases bonds through open market operations
- Change to the demand shifter in PH demand

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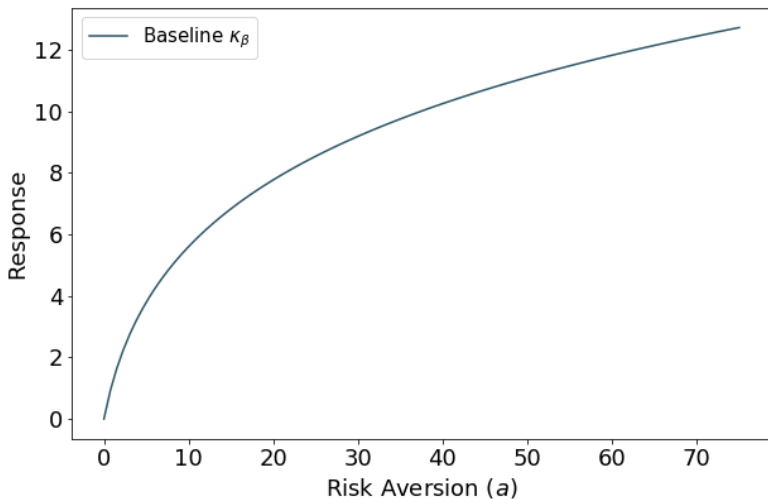
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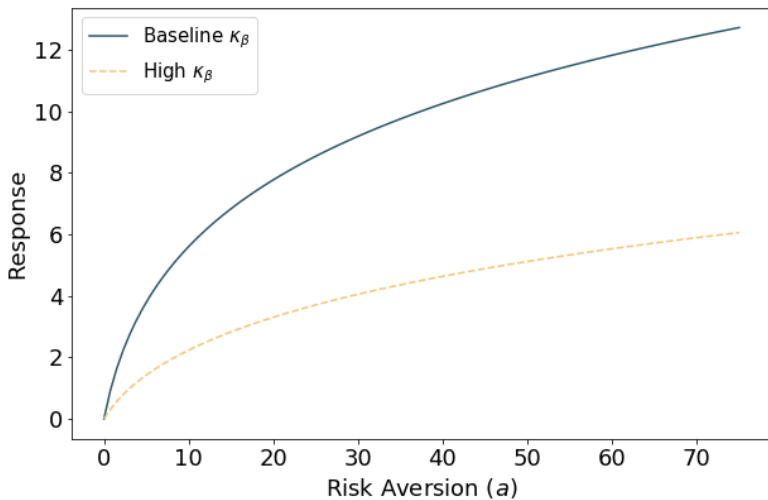
$$\implies \tilde{r}_t = \hat{A}_r r_t + \hat{A}_\beta \beta_t + \hat{C}$$

Output Response to QE



Notes: plots of output gap response to a QE shock as risk aversion increases.

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Sticky Prices

- What about when prices are not fixed?

$$dx_t = \varsigma^{-1}(\tilde{r}_t - \pi_t - \bar{r}) dt$$

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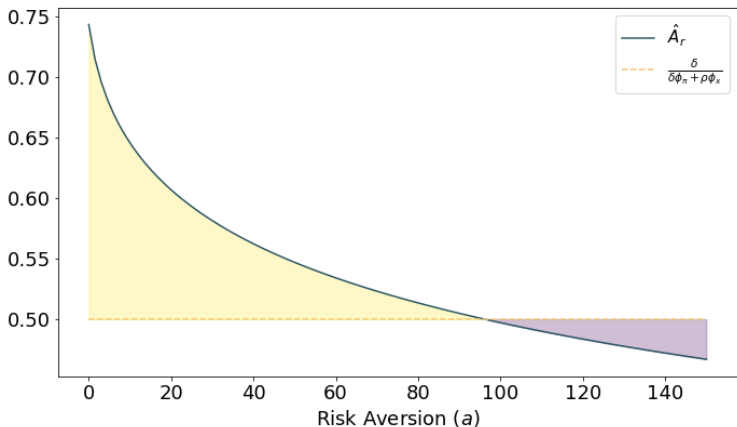
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- If $\hat{A}_r = 1$ and $\phi_x = 0$, reduces to $\phi_\pi > 1$

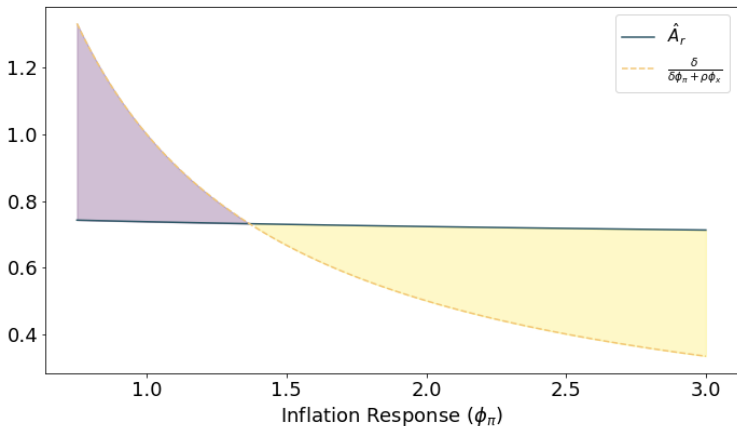
Implications – Determinacy



Notes: determinacy condition as risk aversion a increases.

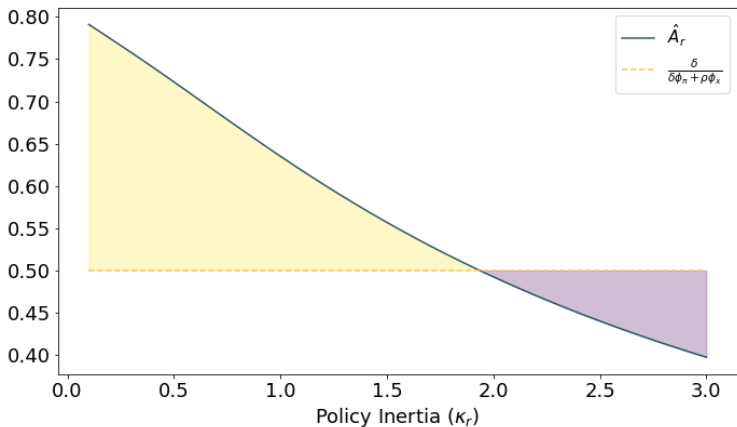
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Notes: determinacy condition as central bank response to inflation ϕ_π increases. The model is determinate if the solid dark line lies above the dotted light line (light shaded region) and is indeterminate otherwise (dark shaded region).

Implications – Determinacy



Notes: determinacy condition as central bank inertia κ_r increases.

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Quantitative (Generalized) Model

- Sticky price model with shocks

$$dx_t = \varsigma^{-1} (\tilde{r}_t - \pi_t - \bar{r} - z_{x,t}) dt$$

$$d\pi_t = (\rho\pi_t - \delta x_t - z_{\pi,t}) dt$$

$$dr_t = -\kappa_r(r_t - \phi_\pi\pi_t - \phi_x x_t - r^*) dt + \sigma_r dB_{r,t}$$

Quantitative (Generalized) Model

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$$dx_t = \varsigma^{-1} (\tilde{r}_t - \pi_t - \bar{r} - z_{x,t}) dt$$

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- Shocks

$$dz_{i,t} = -\kappa_{z_i} z_{i,t} dt + \sigma_{z_i} dB_{z_i,t}$$

Quantitative (Generalized) Model

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- Shocks

$$dz_{i,t} = -\kappa_{z_i} z_{i,t} dt + \sigma_{z_i} dB_{z_i,t}$$

- Demand factors

$$\beta_{t,\tau} = \bar{\beta}(\tau) + \sum_k \beta_{k,t} \theta_k(\tau)$$

$$d\beta_{k,t} = -\kappa_{\beta_k} \beta_{k,t} dt + \sigma_{\beta_k} dB_{\beta_k,t}$$

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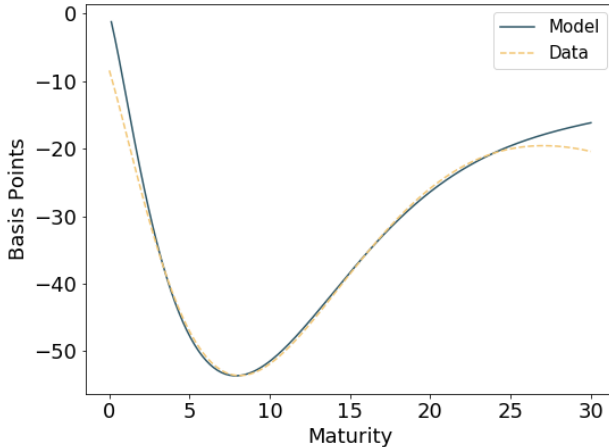
- Requires numerical solution methods

Calibration

Table: Numerical Exercise Calibration

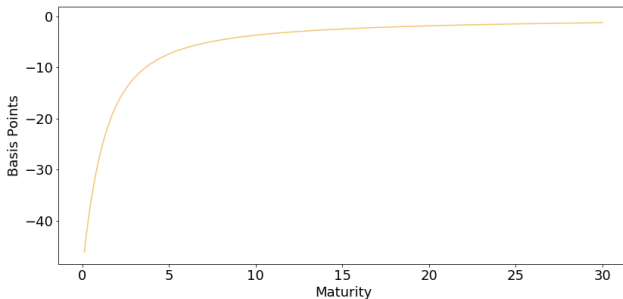
| Parameter | Value | Description | Target |
|---------------------------------|---------------------|--------------------------|---|
| <i>Effective Borrowing Rate</i> | | | |
| η_1 | 1.7069 | Weight Scaling Factor | Treasury Maturity Distribution |
| <i>Macroeconomic Dynamics</i> | | | |
| ρ | 0.0400 | Discount Factor | Long-Run Interest Rate |
| ς^{-1} | 1.0000 | Intertemporal Elasticity | Balanced Growth |
| κ_r | 0.9473 | Monetary Policy Inertia | $\text{Cov}[r_t, r_{t-1}] = 3.5013$ |
| $\kappa_{z\pi}$ | 0.5863 | Cost-Push Shock Inertia | $\text{Cov}[\pi_t, \pi_{t-1}] = 0.9141$ |
| κ_{zx} | 0.2554 | Demand Shock Inertia | $\text{Cov}[x_t, x_{t-1}] = 2.2908$ |
| ϕ_π | 2.0420 | Inflation Taylor Coeff. | $\text{Cov}[r_t, \pi_t] = 1.0006$ |
| ϕ_x | 0.9709 | Output Taylor Coeff. | $\text{Cov}[r_t, x_t] = 0.7722$ |
| δ | 0.0459 | Nominal Rigidity | $\text{Cov}[\pi_t, x_t] = -0.3015$ |
| σ_r | 0.0116 | Monetary Shock Vol. | $\text{Var}[r_t] = 2.7066$ |
| $\sigma_{z\pi}$ | 0.0068 | Cost-Push Shock Vol. | $\text{Var}[\pi_t] = 0.5097$ |
| σ_{zx} | 0.0126 | Demand Shock Vol. | $\text{Var}[x_t] = 1.5192$ |
| <i>Term Structure</i> | | | |
| $\theta_s(\tau)$ | $\delta(\tau - 2)$ | Short Factor Location | LSAP Targets |
| $\theta_\ell(\tau)$ | $\delta(\tau - 10)$ | Long Factor Location | LSAP Targets |
| $\alpha(\tau)$ | 1.0000 | Habitat Elasticity | Normalized |
| κ_β | 0.1710 | Habitat Factor Inertia | QE1 Yield Curve Response |
| $\sigma_{z\beta}$ | 0.0142 | Habitat Factor Vol. | QE1 Yield Curve Response |
| a | 1559.7 | Risk Aversion | QE1 Yield Curve Response |

QE: Model vs. Data



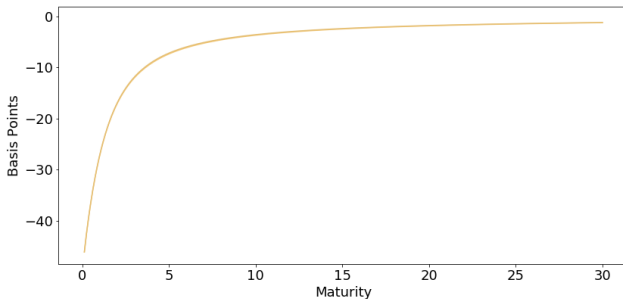
Notes: Yield curve response to the announcement of the initial round of QE on March 18, 2009 (light dotted line). The dark line corresponds to the yield curve response to a QE shock in the model. Source: Gurkaynak, Sack, and Wright (2007). $\eta(\tau)$

Yield Curve (Monetary Policy)



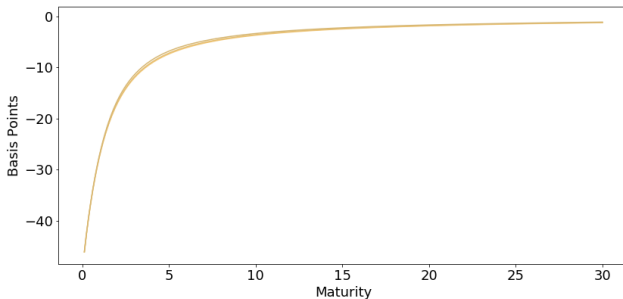
Notes: yield curve response to a 50 b.p. monetary shock on impact, for different levels of risk aversion a . Darker lines correspond to higher levels of risk aversion.

Yield Curve (Monetary Policy)



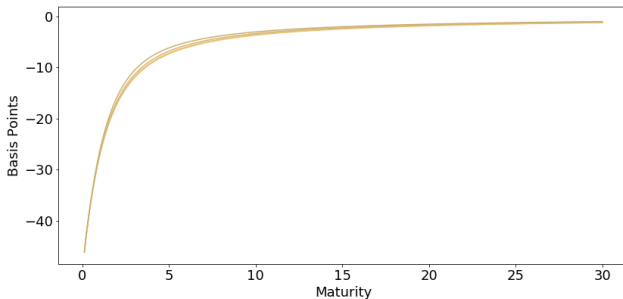
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Yield Curve (Monetary Policy)



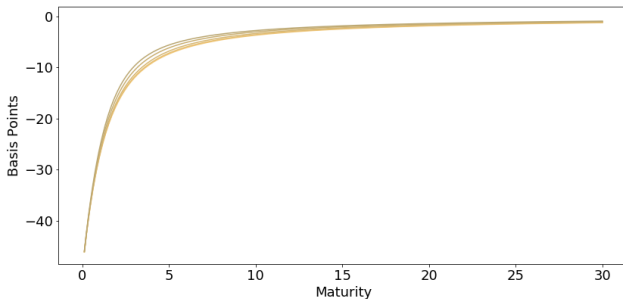
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Yield Curve (Monetary Policy)



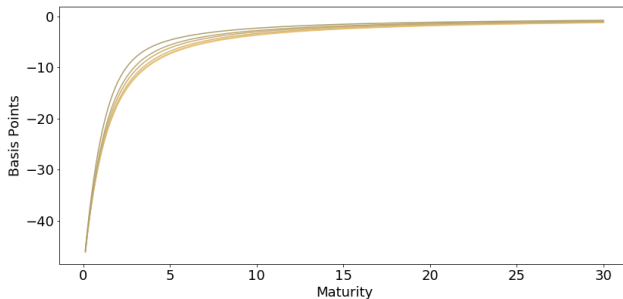
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Yield Curve (Monetary Policy)



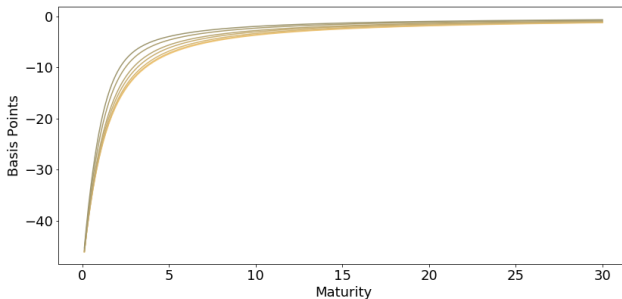
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Yield Curve (Monetary Policy)



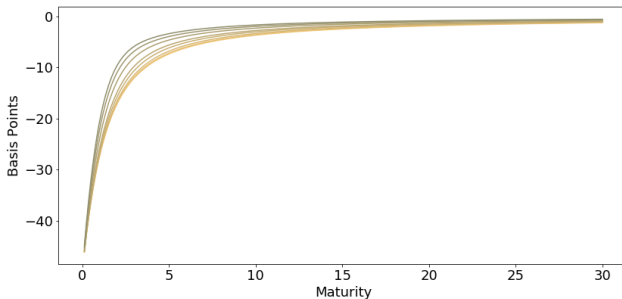
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Yield Curve (Monetary Policy)



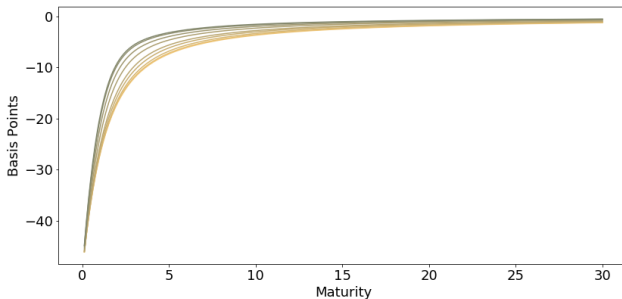
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Yield Curve (Monetary Policy)



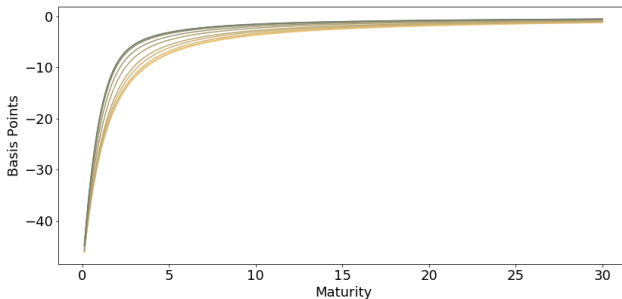
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Yield Curve (Monetary Policy)



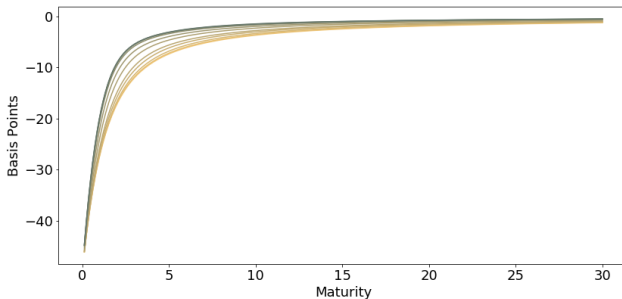
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Yield Curve (Monetary Policy)



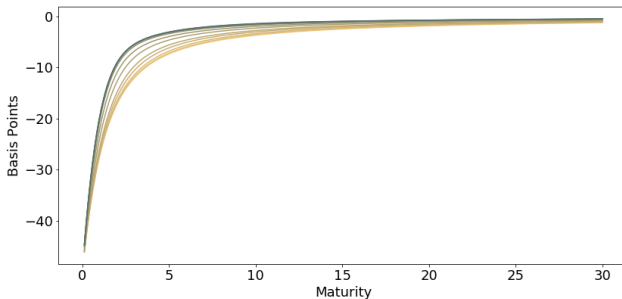
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Yield Curve (Monetary Policy)



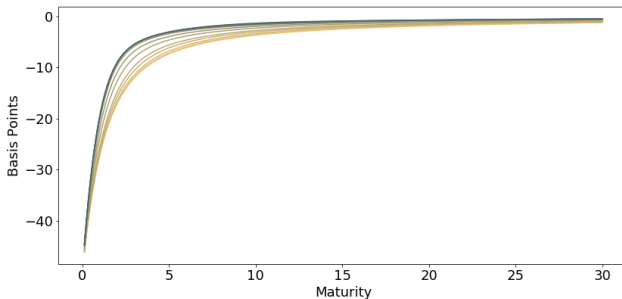
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Yield Curve (Monetary Policy)



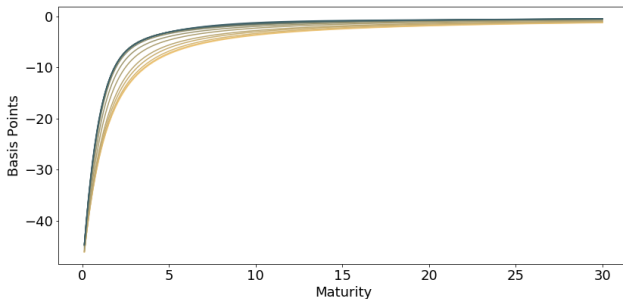
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Yield Curve (Monetary Policy)



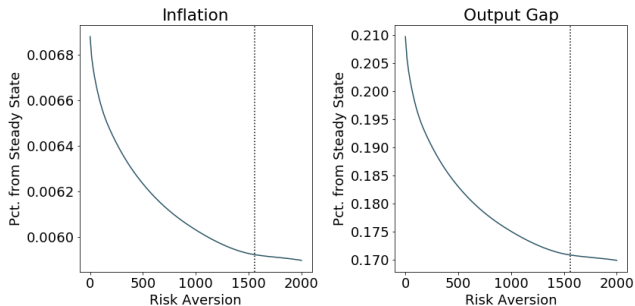
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Yield Curve (Monetary Policy)



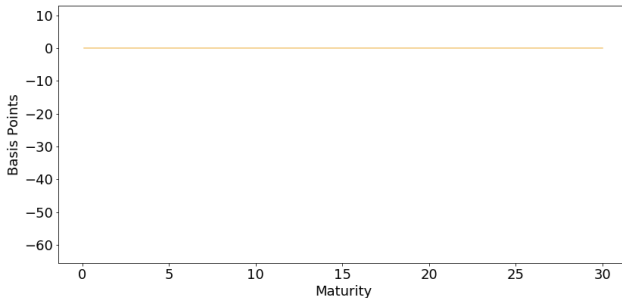
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Aggregate Response (Monetary Policy)



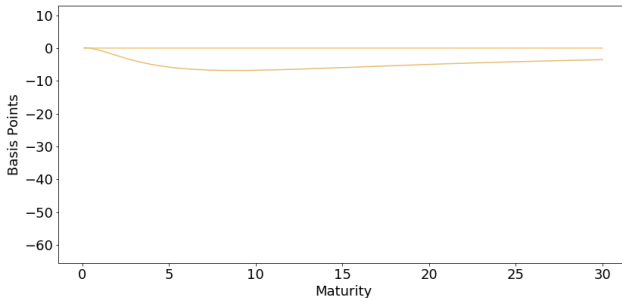
Notes: inflation and output response a 50 b.p. monetary shock, for different levels of risk aversion a .

Yield Curve (QE, long end)



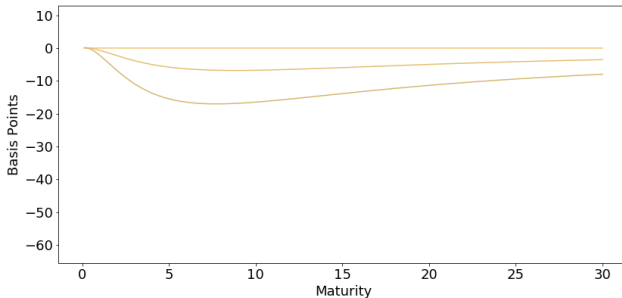
Notes: yield curve response to a “long” QE shock, for different levels of risk aversion α . Darker lines correspond to higher levels of risk aversion.

Yield Curve (QE, long end)



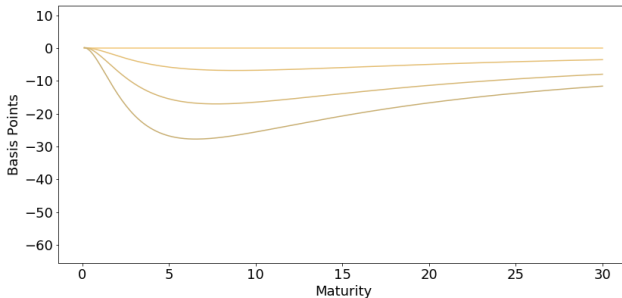
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Yield Curve (QE, long end)



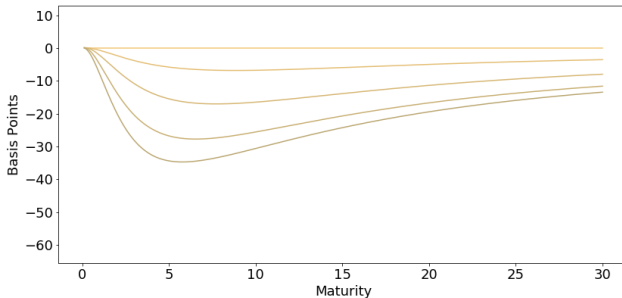
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Yield Curve (QE, long end)



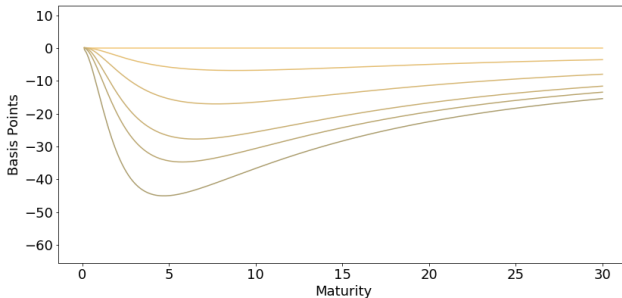
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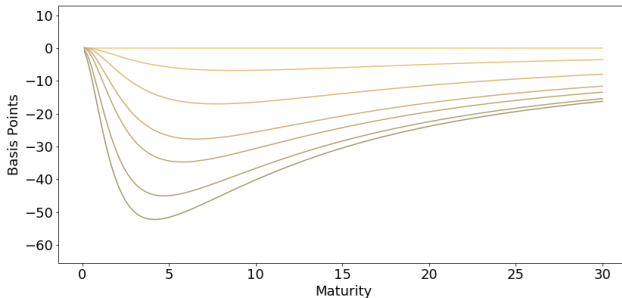
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Yield Curve (QE, long end)



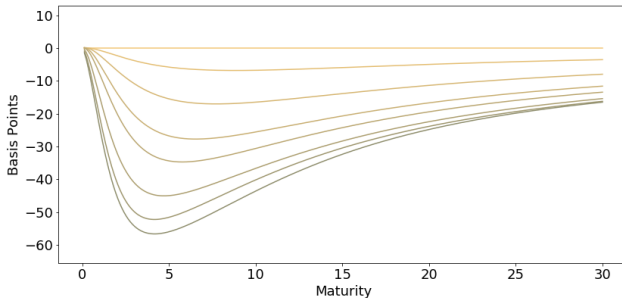
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Yield Curve (QE, long end)



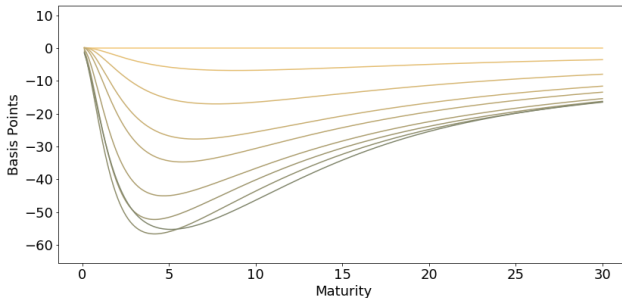
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Yield Curve (QE, long end)



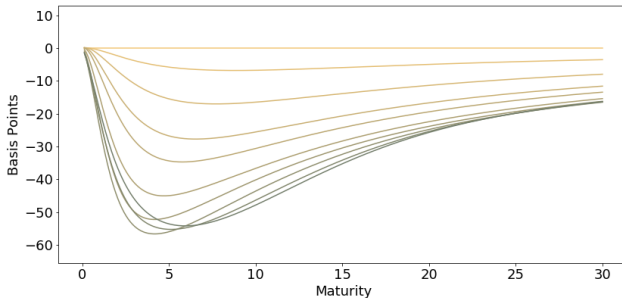
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Yield Curve (QE, long end)



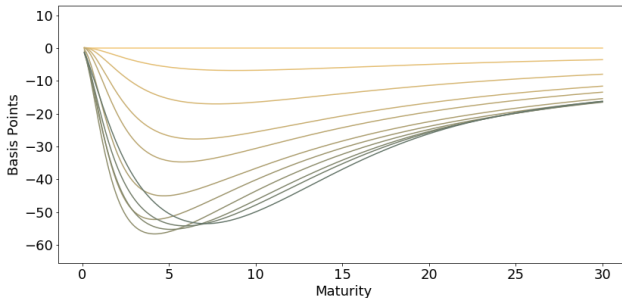
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Yield Curve (QE, long end)



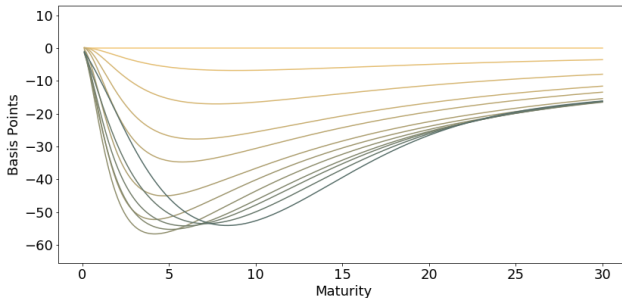
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Yield Curve (QE, long end)



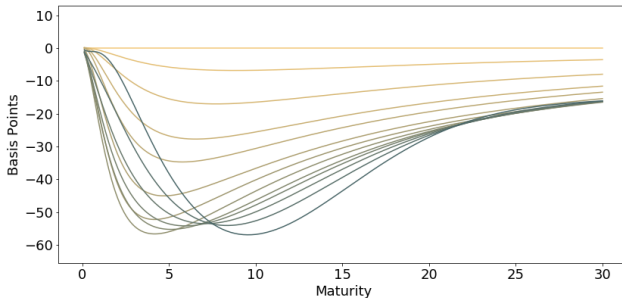
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Yield Curve (QE, long end)



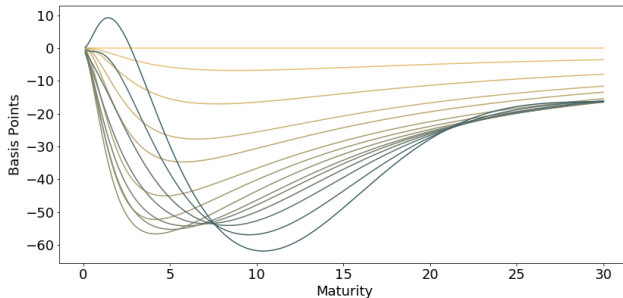
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Yield Curve (QE, long end)



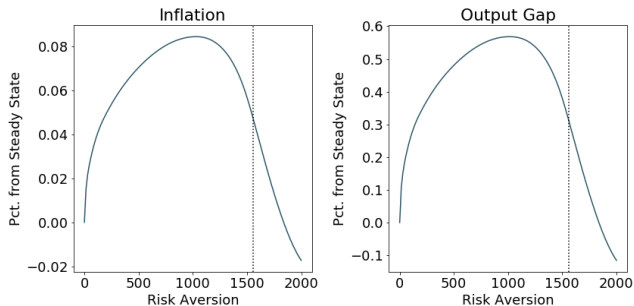
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Yield Curve (QE, long end)



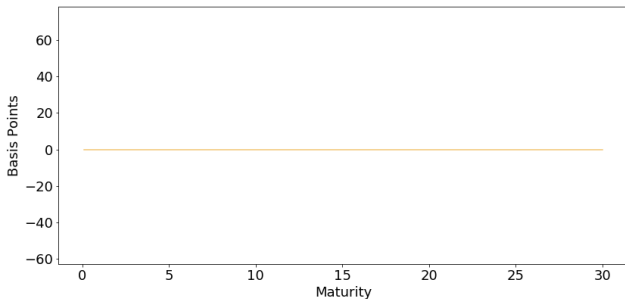
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Aggregate Response (QE, long end)



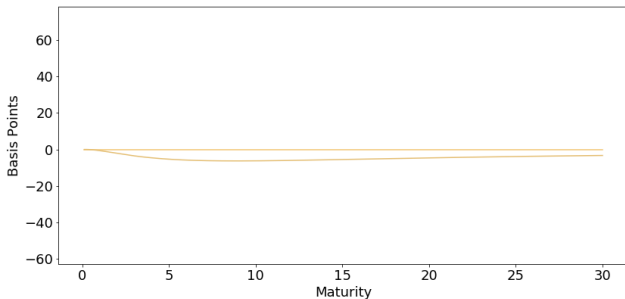
Notes: inflation and output response to “long” QE shock on impact, for different levels of risk aversion a .

Yield Curve (Operation Twist)



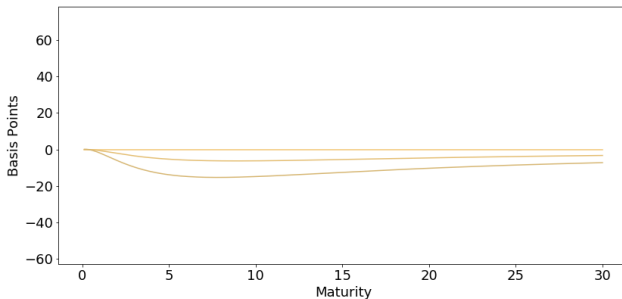
Notes: yield curve response to an “Operation Twist” shock on impact, for different levels of risk aversion α . Darker lines correspond to higher levels of risk aversion.

Yield Curve (Operation Twist)



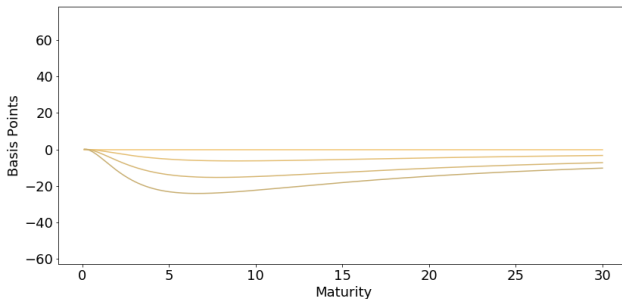
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Yield Curve (Operation Twist)



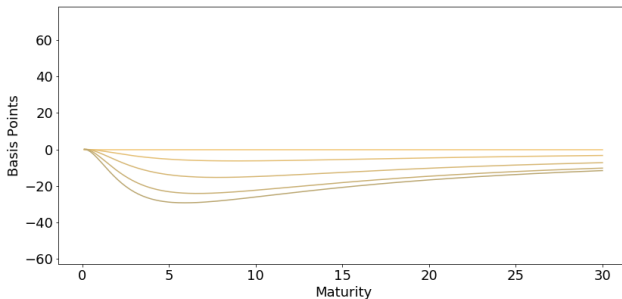
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Yield Curve (Operation Twist)



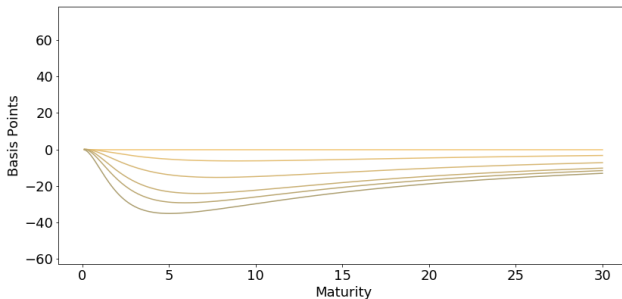
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Yield Curve (Operation Twist)



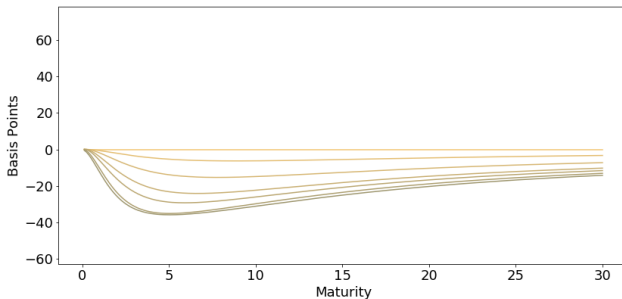
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Yield Curve (Operation Twist)



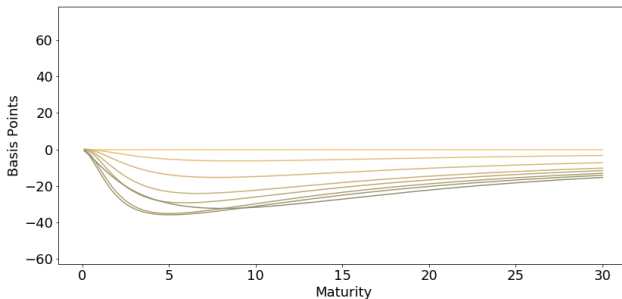
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Yield Curve (Operation Twist)



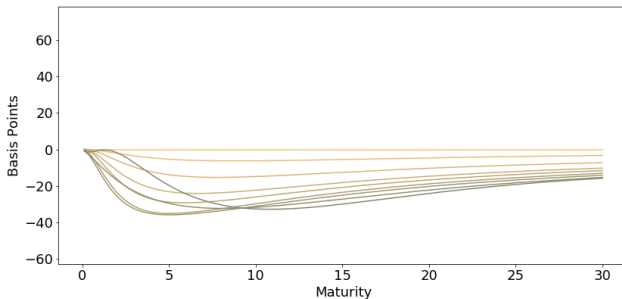
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Yield Curve (Operation Twist)



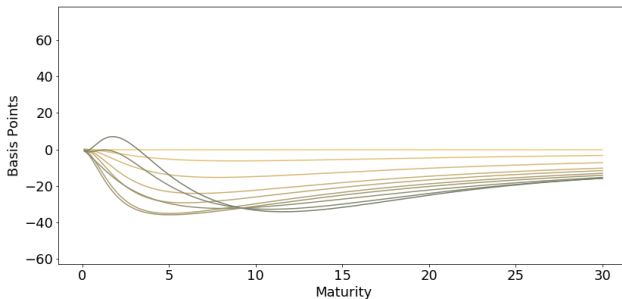
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Yield Curve (Operation Twist)



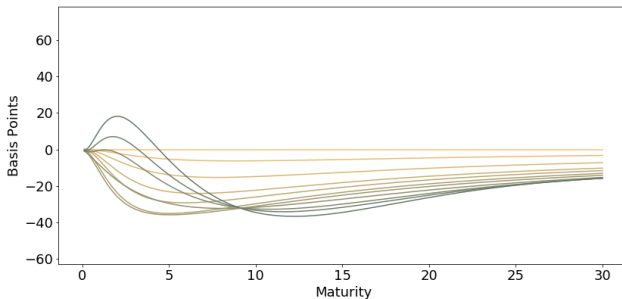
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Yield Curve (Operation Twist)



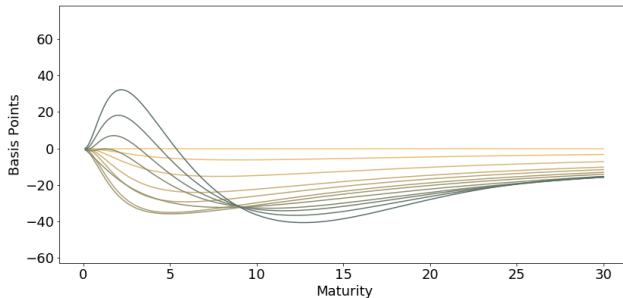
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Yield Curve (Operation Twist)



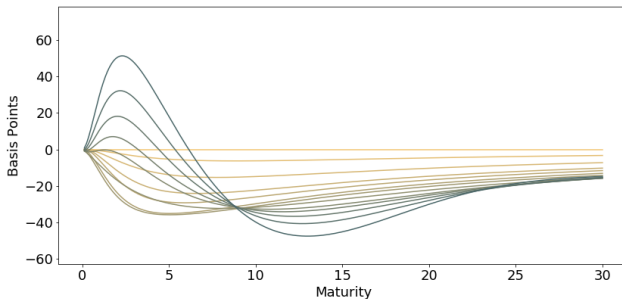
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Yield Curve (Operation Twist)



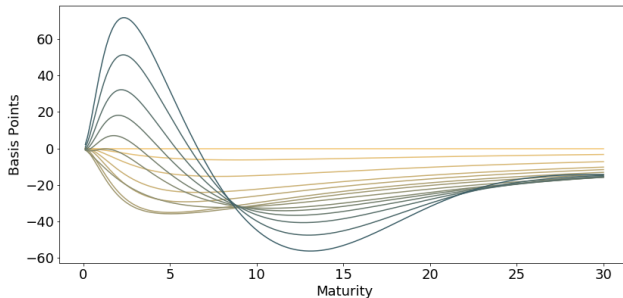
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Yield Curve (Operation Twist)



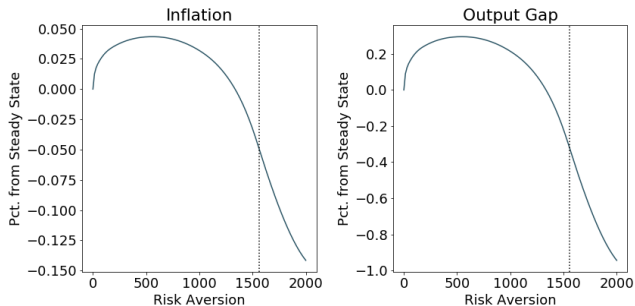
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Yield Curve (Operation Twist)



Notes: yield curve response to an "Operation Twist" shock on impact, for different levels of risk aversion α . Darker lines correspond to higher levels of risk aversion.

Aggregate Response (Operation Twist)



Notes: inflation and output response an “Operation Twist” shock, for different levels of risk aversion a .

Optimal Conventional Policy

- Can the planner improve outcomes?
- Loss function

$$E_0 \int_0^{\infty} e^{-\rho t} (w_{\pi} \pi_t^2 + w_x x_t^2) dt$$

Optimal Conventional Policy

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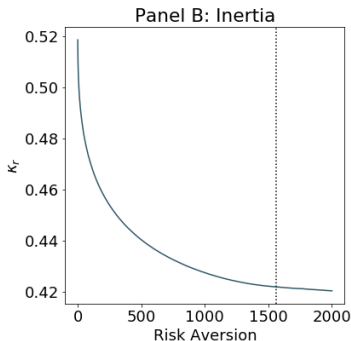
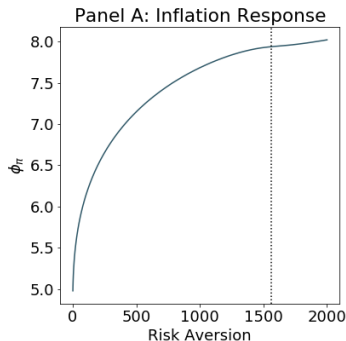
Optimal Conventional Policy

- Can the planner improve outcomes?
- Loss function

$$\min_{\phi_{\pi}, \kappa_r} E_0 \int_0^{\infty} e^{-\rho t} (w_{\pi} \pi_t^2 + w_x x_t^2) dt$$

- Optimal inflation response and inertia as financial disruptions increase conditional distribution

Optimal Response: More Aggressive in Crises



Notes: optimal policy coefficients on inflation (Panel A) and inertia (Panel B) as risk aversion increases. Planner weights: $w_\pi = 1$, $w_x = 0.1$.

Stabilizing LSAPs

- Can LSAPs be used to ensure determinacy?
- Endogenous QE purchases:

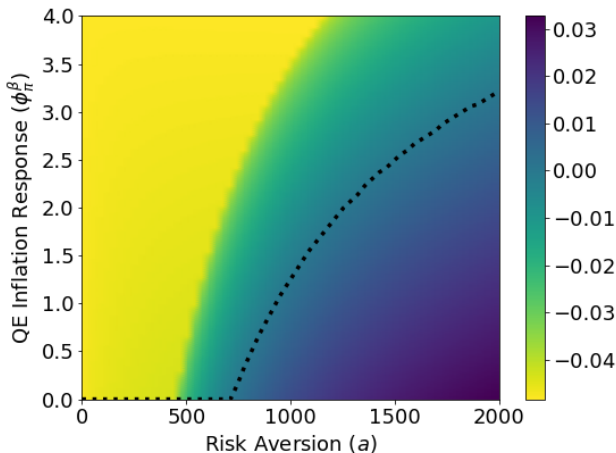
$$d\beta_t = -\kappa_\beta \left(\beta_t - \phi_\pi^\beta \pi_t \right) dt$$

Stabilizing LSAPs

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QE and Determinacy



Notes: determinacy conditions as a function of risk aversion (x-axis) and endogenous response of QE to inflation (y-axis). Darker colors correspond to larger values of the unstable eigenvalue. The dotted black line demarcates the region of determinacy.

Concluding Remarks

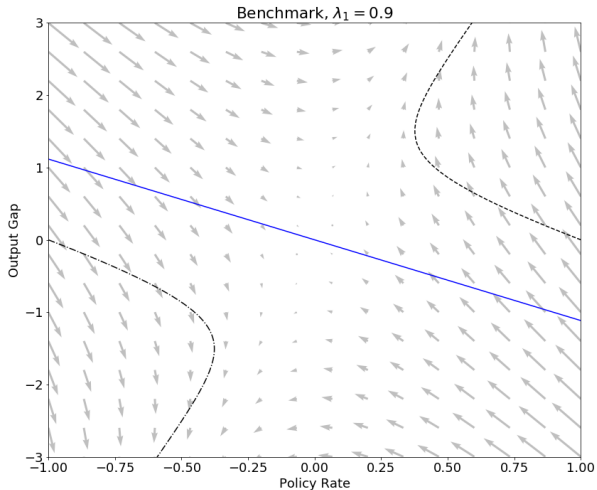
- Develops a unified, parsimonious framework to study conventional and unconventional monetary policies
- Transmission depends crucially on the risk-bearing capacity of financial markets

Concluding Remarks

- Develops a unified, parsimonious framework to study conventional and unconventional monetary policies
- Transmission depends crucially on the risk-bearing capacity of financial markets
- Future work:
 - ▶ Macroprudential policies, default risk
 - ▶ Monetary policy in open economies
 - ▶ Debt management

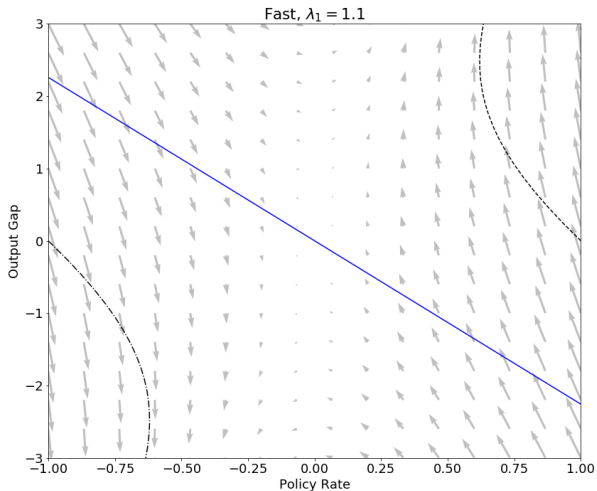
APPENDIX

Phase Diagrams



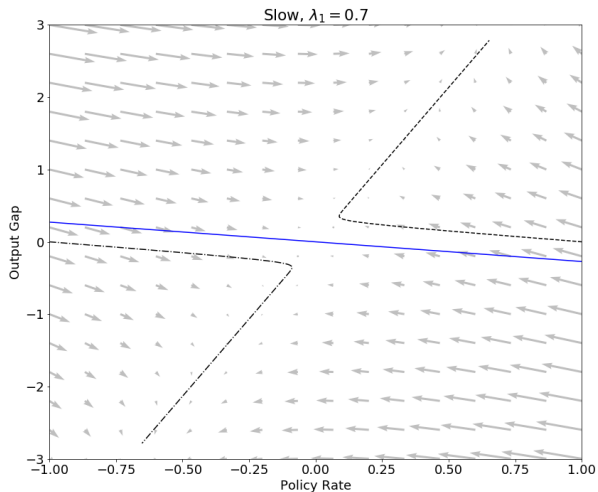
Notes: phase diagrams of the dynamics of output and the policy rate as the equilibrium mean reversion rate of shocks varies.

Phase Diagrams



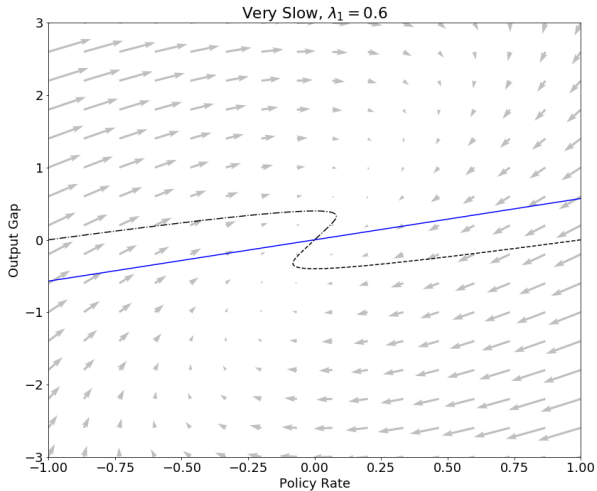
Notes: phase diagrams of the dynamics of output and the policy rate as the equilibrium mean reversion rate of shocks varies.

Phase Diagrams



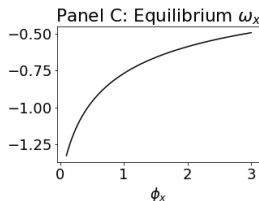
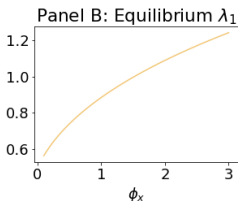
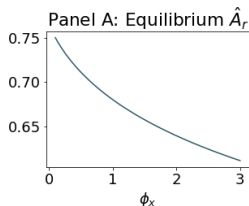
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Phase Diagrams



Notes: phase diagrams of the dynamics of output and the policy rate as the equilibrium mean reversion rate of shocks varies.

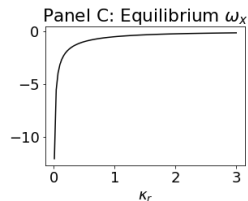
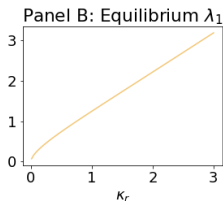
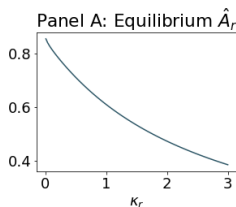
Implications – Conventional Policy



Notes: equilibrium changes in sensitivity to the short rate \hat{A}_r and monetary shock reversion λ_1 as central bank response to output ϕ_x increases.

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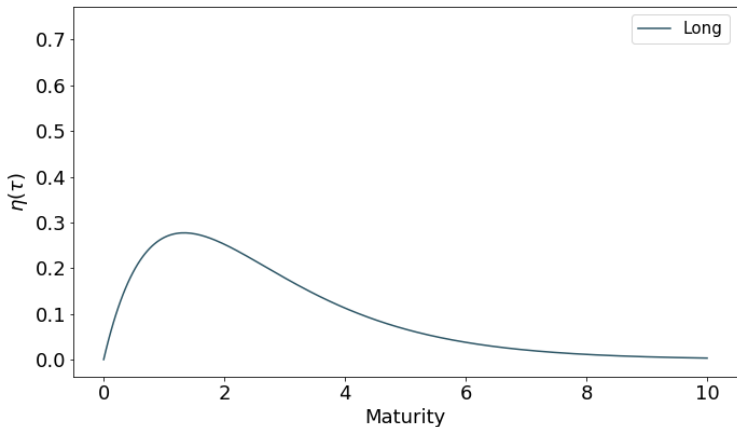
Implications – Conventional Policy



Notes: equilibrium changes in sensitivity to the short rate \hat{A}_r and monetary shock reversion λ_1 as central bank inertia κ_r increases.

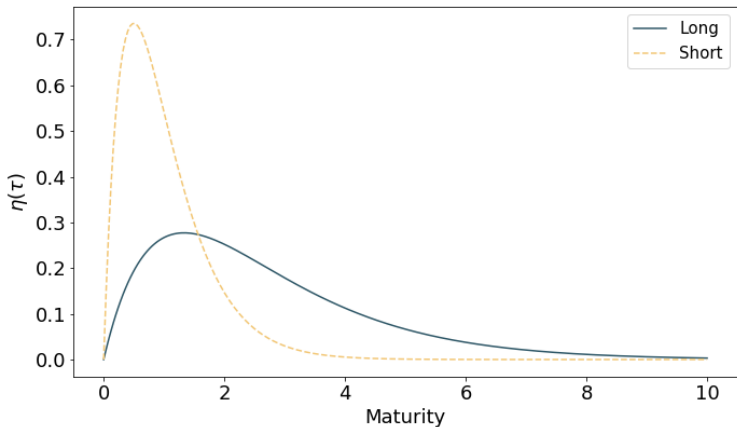
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Sensitivity to Long Rates



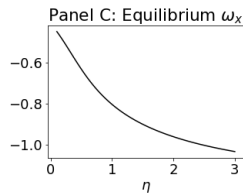
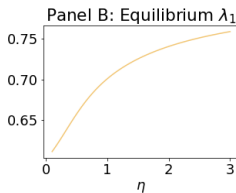
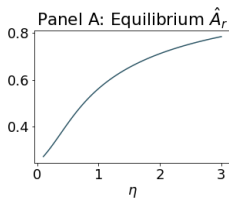
Notes: different weighting function $\eta(\tau)$ in the determination of the effective borrowing rate \tilde{r}_t .

Sensitivity to Long Rates



Notes: different weighting function $\eta(\tau)$ in the determination of the effective borrowing rate \tilde{r}_t .

Implications – Sensitivity to Long Rates



Notes: equilibrium changes in sensitivity to the short rate \hat{A}_r and monetary shock reversion λ_1 as the weighting function $\eta(\tau)$ shifts towards short-term bonds.

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Forward Guidance

- Central bank announces a peg: $r_0 = r^\diamond$ and

$$dr_t = \begin{cases} -\kappa_r^\diamond(r_t - r^\diamond)dt + \sigma_r^\diamond dB_{r,t} & \text{if } 0 < t < t^\diamond \\ -\kappa_r(r_t - \phi_x x_t - r^*)dt + \sigma_r dB_{r,t} & \text{if } t \geq t^\diamond \end{cases}$$

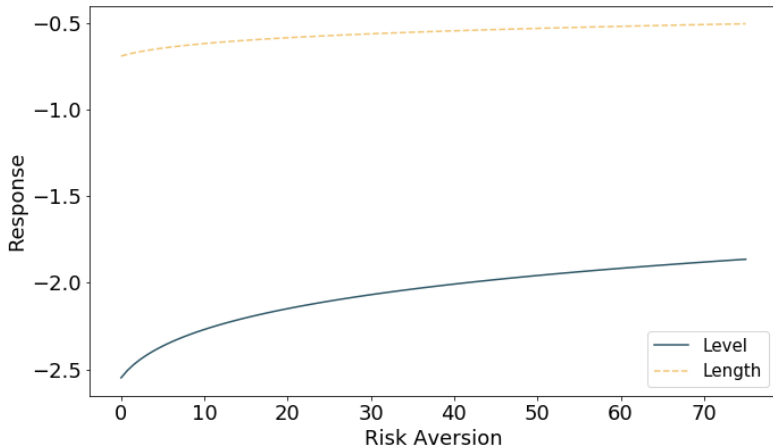
- Affine coefficient functions during peg:

$$\begin{aligned} -\log P_{t,\tau} &= A_r^\diamond(\tau)r_t + C^\diamond(\tau) \\ \implies \tilde{r}_t &= \hat{A}_r^\diamond r_t + \hat{C}^\diamond \end{aligned}$$

- Rational expectations dynamics for output:

$$\frac{\partial x_0}{\partial r^\diamond} = \omega_x - t^\diamond \varsigma^{-1} \hat{A}_r^\diamond, \quad \frac{\partial^2 x_0}{\partial r^\diamond \partial t^\diamond} = -\varsigma^{-1} \hat{A}_r^\diamond$$

Response to Forward Guidance



Notes: plots of $\frac{\partial x_0}{\partial r^\diamond}$ ("level") and $\frac{\partial^2 x_0}{\partial r^\diamond \partial t^\diamond}$ ("length") as risk aversion increases.

Long-Run Variance

- State-space representation

$$d\mathbf{y}_t = -\Gamma (\mathbf{y}_t - \mathbf{y}^{SS}) dt + \mathbf{S} d\mathbf{B}_t, \quad \mathbf{x}_t = \Omega (\mathbf{y}_t - \mathbf{y}^{SS})$$

- Conditional distribution $\mathbf{y}_t | \mathbf{y}_0 \sim N(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$ where

$$\boldsymbol{\mu}_t = \mathbf{y}^{SS} + e^{-\Gamma t}(\mathbf{y}_0 - \mathbf{y}^{SS}), \quad \boldsymbol{\Sigma}_t = \int_0^t e^{\Gamma(u-t)} \boldsymbol{\Sigma} e^{\Gamma^T(u-t)} du$$

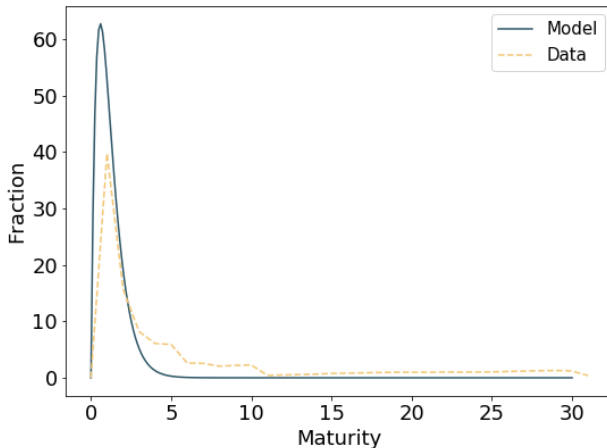
- Present-discounted value

$$\begin{aligned} \tilde{\boldsymbol{\Sigma}}_{\infty} &\equiv \int_0^{\infty} e^{-\rho t} \boldsymbol{\Sigma}_t dt \\ \implies \text{vec } \tilde{\boldsymbol{\Sigma}}_{\infty} &= (\Gamma \oplus \Gamma)^{-1} (\rho \mathbf{I} + \Gamma \oplus \Gamma)^{-1} \text{vec } \boldsymbol{\Sigma} \end{aligned}$$

- Jump variables

$$\tilde{\boldsymbol{\Sigma}}_{\infty}^{\mathbf{x}} = \Omega \tilde{\boldsymbol{\Sigma}}_{\infty} \Omega^T$$

Effective Borrowing Rate Weights



Notes: average maturity distribution of outstanding Treasury debt (light dotted line). The dark line corresponds to the effective borrowing rate weights in the model. Source: FRED.