Optimal Unconventional Policy in a New Keynesian Preferred Habitat Model

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Motivation

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Bernanke: "QE works in practice but not in theory"

- By now the gap between practice and theory is small
- But what do we mean by *QE works*?
  - Obvious: reduce long-term yields
  - Less obvious: stimulate the economy
  - Even less obvious: improve social welfare
  - Reis: "QE's original sin"
- Especially relevant today now that central banks are implementing QT while increasing short rates
- **Research Question**: what is the optimal QE policy, and how does this interact with short rate policy?

Our Model

- This paper: develops a tractable general equilibrium model with market segmentation, nominal frictions, financial frictions, and household heterogeneity
- Households have differentiated access to asset markets
  - Households borrow with assets of different maturities (eg pension funds, mortgages)
  - Introduces imperfect risk-sharing, consumption and saving dispersion across households
- Firms face nominal pricing frictions and financial frictions
- Arbitrageurs with imperfect risk-bearing capacity intermediate bond markets
- **Preferred habitat** tradition:
  - Bonds of different maturities traded by specialized investors (eg pension funds, MMMF)
  - Arbitrageurs (eg hedge funds, broker-dealers) partly overcome segmentation
  - Formally: embed Vayanos-Vila in a New Keynesian model, where households and firms have imperfect access to financial markets which induce preferred habitat segmentation

Findings: Policy Transmission

- Key mechanisms of conventional monetary policy:
  - Changes in the short rate affect required rates of return of all assets
  - Interaction of arbitrageurs, firms, and households leads to portfolio rebalancing
  - Implies variation in term premia, imperfect transmission to household borrowing rates
- Key mechanisms of balance sheet policy:
  - Imperfect arbitrage breaks QE neutrality
  - Central bank asset purchases induce portfolio rebalancing and hence reduce term premia
  - Vice versa for central bank asset sales (QT)
- Thus, short rate and balance sheet policies are substitutes when targeting aggregate borrowing rates
  - A fall in aggregate borrowing rates is stimulative for the usual NK reasons

Findings: Welfare Consequences

- If the policymaker only cares about macroeconomic stabilization, conventional and unconventional policies are essentially equivalent
  - Nominal rigidities $\implies$ welfare losses due to inflation and output gap volatility
  - Policy stabilizes inflation by keeping aggregate borrowing rates at some "natural" rate
  - Triumphalist view: even with short rate constraints, QE is equally effective
- However, both policies imply variation in term premia
  - Excess fluctuations in term premia lead to dispersion in borrowing rates
- Social welfare depends not only on macroeconomic fluctuations. Imperfect risk sharing $\implies$ welfare losses from consumption dispersion
- **Triple mandate**: social welfare depends on volatility of output, inflation, and long-term rates

Findings: Optimal Policy

- Hence, when policy is unconstrained we derive an **optimal separation result**:
  - Conventional policy targets macroeconomic stability
  - Unconventional policy targets bond market stability
- However, when policy constraints bind, policy must balance trade-offs:
  - Balance sheet constraints: short rate must be less reactive in order to minimize bond market disruptions (at the cost of macroeconomic stability)
  - Short rate constraints: QE must be used to offset macroeconomic shocks (at the cost of bond market stability)
- With full commitment, forward guidance is welfare-improving (short rate and QE)
  - Policymaker uses the entire expected path of borrowing rates to minimize macroeconomic volatility
  - But reduces short-run fluctuations to keep term premia volatility low
  - However, dynamics are complicated and suffer from time-inconsistency
- General message: implementation matters for welfare

Related Literature

- Preferred habitat models
  - Vayanos & Vila (2009, 2021), Ray, Droste, & Gorodnichenko (2024), Greenwood & Vayanos (2014), Hamilton & Wu (2012), Greenwood et al (2016), King (2019, 2021) , Kekre, Lenel, & Mainardi (2024), ...
- Empirical evidence: QE and preferred habitat
  - Krishnamurthy & Vissing-Jorgensen (2012), Hamilton and Wu (2012), D'Amico & King (2013), Li & Wei (2013), Cahill et al. (2013), King (2019), Fieldhouse et al (2018), Di Maggio et al (2020), Gali, Debortoli, & Gambetti (2020) , ...
- Macroeconomic QE models
  - Andres, Lopez-Salido, & Nelson (2004), Gertler & Karadi (2011), Curdia & Woodford (2011), Chen et al (2012), Gertler & Karadi (2013), Sims & Wu (2020), Karadi & Nakov (2020), Iovino and Sergeyev (2023), Carlstrom et al (2017), Ippolito et al (2018), Dordal & Lee (2024) , ...
- Market segmentation, macro-prudential monetary policy
  - Cui & Sterk (2021), Auclert (2016), Collard et al (2017), Kaplan et al (2017), Debortoli & Gali (2017) , ...
- International
  - Itskhoki & Mukhin (2022), Greenwood et al (2023), Gourinchas, Ray, & Vayanos (2024) , ...

Model Setup

Model Setup

- Continuous time New Keynesian model with embedded Vayanos-Vila bond markets
- **Agents**:
  - Households: supply labor, consume, save via bond markets
  - Firms: monopolistic competitors face nominal frictions, finance labor with borrowing
  - Arbitrageurs: imperfect risk-bearing capacity, conduct bond carry trades
- **Policymakers**:
  - Central bank: conducts short rate and balance sheet (QE) policy
  - Government: optimal subsidies, otherwise passive
- **Bond markets**:
  - Continuum of zero coupon bonds with maturity $0 \leq \tau \leq T \leq \infty$
  - Bond price $P_t^{(\tau)}$ with yield to maturity $y_t^{(\tau)} = -\log P_t^{(\tau)}/\tau$
  - Nominal short rate: in equilibrium, $i_t = \lim_{\tau \to 0} y_t^{(\tau)}$

Households

- Continuum of HH members $i \in \mathcal{H}$, differentiated by access to bond markets τ
  - Captures the observed differentiated HH portfolios (eg, due to demographics, market access via investment funds, mortgage market structure, etc)
  - Formalization: HHs sluggishly rebalance (our model is limiting case)
- A τ-type HH chooses consumption and labor $C_t(\tau), N_t(\tau)$ in order to solve

$$V_0(\tau) \equiv \max \mathrm{E}_0 \int_0^\infty e^{-\rho t} \left(\frac{C_t(\tau)^{1-\varsigma} - 1}{1-\varsigma} - \frac{N_t(\tau)^{1+\varphi}}{1+\varphi} \right) \mathrm{d}t$$

$$\text{s.t.} \;\; \mathrm{d}\mathcal{B}_t(\tau) = \left[\mathcal{W}_t N_t(\tau) - \mathcal{P}_t C_t(\tau)\right] \mathrm{d}t + \mathcal{B}_t(\tau) \frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} + \mathrm{d}\mathcal{F}_t$$

- $\mathcal{B}_t(\tau)$ nominal savings earn $\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}}$
- Take as given nominal wage $\mathcal{W}_t$, price index $\mathcal{P}_t$, transfers $\mathrm{d}\mathcal{F}_t$

Key takeaway: consumption/savings choices differ when bond returns not equalized

Firms

- Continuum of intermediate goods $j \in [0,1]$ (and CES final good with elasticity ϵ)
- Linear production in differentiated labor $Y_t(j) = Z_t L_t(j)$
- Revenue and costs of production:

$$\mathrm{d}\Pi_t(j) = \left[(1+\tau^y)\mathcal{P}_t(j)Y_t(j) - \mathcal{W}_t L_t(j) - \mathcal{T}_t^y\right]\mathrm{d}t - \mathrm{d}\Theta_t(j)$$

$$\mathrm{d}\Theta_t(j) = \frac{\vartheta}{2}\pi_t(j)^2\mathcal{P}_t Y_t\,\mathrm{d}t + \bar{\beta}\mathcal{W}_t L_t(j)\left(\int_0^T \theta(\tau)\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}}\,\mathrm{d}\tau\right)$$

- Rotemberg costs when setting prices $\frac{\mathrm{d}P_t(j)}{P_t(j)} = \pi_t(j)\,\mathrm{d}t$
- Working capital friction: finance a fraction $\bar{\beta}$ of wage bill with portfolio $\theta(\tau)$
- Taking as given CES demand, τ^y subsidy, taxes $\mathcal{T}_t^y$, SDF $Q_t^{\mathcal{H}}$, firm j solves:

$$U_0(j) \equiv \max \mathrm{E}_0 \int_0^\infty e^{-\rho t} Q_t^{\mathcal{H}}\,\mathrm{d}\Pi_t(j)$$

Key takeaway: inefficiencies due to pricing frictions, financing friction

Arbitrageurs

- Mean-variance optimization

$$\max \mathsf{E}_t \, \mathrm{d}\mathcal{X}_t - \frac{a_t}{2} \mathsf{Var}_t \, \mathrm{d}\mathcal{X}_t$$

$$\text{s.t. } \mathrm{d}\mathcal{X}_t = \mathcal{X}_t i_t \, \mathrm{d}t + \int_0^T \mathcal{X}_t(\tau) \left(\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} - i_t \, \mathrm{d}t \right) \mathrm{d}\tau$$

- Arbitrageurs invest $\mathcal{X}_t(\tau)$ in bond carry trade of maturity τ
- Risk-return trade-off governed by a_t
  - Formally: risk aversion coefficient
  - More generally: proxies for any limits to risk-bearing capacity or intermediation frictions
  - Arbitrageurs transfer gains/losses to HHs, so a_t represents any frictions which hinder ability to trade on behalf of HHs

Key takeaway: risk averse arbitrageurs' holdings increase with expected return

Government

- Central bank sets policy rate i_t and holdings $\mathcal{S}_t(\tau)$, potentially subject to costs:

$$Y_t \Psi_t^S \equiv Y_t \int_0^T \frac{\psi(\tau)}{2} \left(\mathcal{S}_t(\tau)\right)^2 \mathrm{d}\tau \,, \quad Y_t \Psi_t^i \equiv Y_t \frac{\psi^i}{2} \left(i_t - \bar{i}_t\right)^2$$

- In the background: fiscal authority chooses subsidies τ^y, balances the budget
- Optimal policy: maximize social welfare

$$\max \mathrm{E}_0 \int_0^\infty e^{-\rho t} \left(\int_0^T \eta(\tau) u\left(C_t(\tau), N_t(\tau)\right) \mathrm{d}\tau \right) \mathrm{d}t$$

- $\eta(\tau)$: fraction of HHs with access to τ bonds (so $\int_0^T \eta(\tau)\,\mathrm{d}\tau = 1$)

Key takeaway: policy attempts to undo frictions:

1. Nominal rigidities $\implies$ pricing inefficiencies
2. Firm financing friction $\implies$ production inefficiencies
3. Household market segmentation $\implies$ imperfect risk-sharing

Equilibrium

Aggregation and Market Clearing

- Firms, arbitrageurs, and funds transfer profits equally to HHs
- Symmetric firm equilibrium $Y_t(j) = Y_t, P_t(j) = P_t, \pi_t(j) = \pi_t, \frac{\mathrm{d}P_t}{P_t} = \pi_t \,\mathrm{d}t$
- Clearing in production and goods markets:

$$Y_t = Z_t N_t, \quad C_t \equiv \int_0^T \eta(\tau) C_t(\tau)\,\mathrm{d}\tau = Y_t \left(1 - \frac{\vartheta}{2}\pi_t^2 - \Psi_t^S - \Psi_t^i\right)$$

- Bond market clearing implies

$$\mathcal{X}_t(\tau) - \theta(\tau)\mathcal{W}_t N_t + \eta(\tau)\mathcal{B}_t(\tau) + \mathcal{S}_t(\tau) = 0$$

Aggregate Risk Factors and Risk Pricing

- Aggregate technology $Z_t = \bar{Z}e^{z_t}$

$$\mathrm{d}z_t = -\kappa_z z_t \,\mathrm{d}t + \sigma_z \,\mathrm{d}B_{z,t}$$

- Generic set of N_z exogenous risk factors $\mathbf{z}_t$ with associated Brownian motions $\mathbf{B}_t$ (where $z_t \in \mathbf{z}_t, B_{z,t} \in \mathbf{B}_t$) with volatility

$$\mathrm{Var}_t \,\mathrm{d}\mathbf{z}_t = \boldsymbol{\sigma}\boldsymbol{\sigma}^\top \,\mathrm{d}t$$

  - Allow for exogenous cost-push shocks, firm financing shocks, discount factor shocks...
  - Generally richer dynamics for risk factors
- Thus, instantaneous return of τ bond is

$$\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} = \mu_t(\tau)\,\mathrm{d}t + \boldsymbol{\sigma}_t(\tau)\,\mathrm{d}\mathbf{B}_t$$

- Arbitrageur optimality conditions imply

$$\mu_t(\tau) - i_t = a_t\boldsymbol{\sigma}_t(\tau)\boldsymbol{\Lambda}_t, \;\; \text{where } \boldsymbol{\Lambda}_t^\top = \int_0^T \mathcal{X}_t(\tau)\boldsymbol{\sigma}_t(\tau)\,\mathrm{d}\tau$$

First-Best Approximation

- Approximation method: parameterize risk and arbitrageur risk aversion

$$\lim_{\xi \to 0} \boldsymbol{\sigma}(\xi) = \mathbf{0}, \quad \lim_{\xi \to 0} a(\xi)\boldsymbol{\sigma}(\xi) = \boldsymbol{\sigma} \text{ where } 0 < \|\boldsymbol{\sigma}\| < \infty$$

- Allows for tractable first- and second-order approximations with meaningful (first-order) variation in risk prices
- Approximate around the first-best:

Proposition (First-best allocation)

Consider the riskless model ($\xi \to 0$) with initially equalized wealth ($\mathcal{B}_0(i) \equiv \mathcal{B}_0$).

- With perfect arbitrage ($a = 0$), the model is equivalent to a representative agent model.
- With the optimal production subsidy τ^y, first-best is obtained with flexible prices ($\vartheta = 0$).

- Define the output gap relative to first-best path of output Y_t^n:

$$X_t \equiv \frac{Y_t}{Y_t^n}$$

Linearized Aggregate Dynamics

- Output gap evolves according to modified aggregate Euler equation:

$$\mathsf{E}_t \, \mathrm{d}x_t = \varsigma^{-1} \left[\tilde{\mu}_t - \pi_t - v_t\right] \mathrm{d}t$$

- $v_t \equiv -\varsigma \kappa_z \frac{1+\varphi}{\varsigma+\varphi} z_t$ is the usual natural rate and $\tilde{\mu}_t$ is the HH effective borrowing rate:

$$\tilde{\mu}_t = \int_0^T \eta(\tau) \left(\mu_t(\tau) - \varrho\right) \mathrm{d}\tau$$

- Inflation evolves according to a modified NKPC:

$$\mathsf{E}_t \, \mathrm{d}\pi_t = \left[\rho \pi_t - \kappa \left(x_t + \bar{\beta} \hat{\mu}_t\right)\right] \mathrm{d}t$$

- κ measures aggregate price rigidity and $\hat{\mu}_t$ is the firm effective borrowing rate:

$$\hat{\mu}_t = \int_0^T \theta(\tau)(\mu_t(\tau) - \varrho) \, \mathrm{d}\tau$$

Linearized Household Dynamics

- Let $\check{\cdot}$ denotes differences from HH average, eg

$$\check{c}_t(\tau) \equiv c_t(\tau) - \int_0^T \eta(\tau')c_t(\tau')\,\mathrm{d}\tau' \equiv c_t(\tau) - \tilde{c}_t$$

- Dynamics of relative consumption of τ-type household:

$$\mathsf{E}_t\,\mathrm{d}\check{c}_t(\tau) = \varsigma^{-1}\check{\mu}_t(\tau)\,\mathrm{d}t$$

- Given transfers, the dynamics of relative wealth of τ-type household:

$$\mathrm{d}\check{b}_t(\tau) = \left[-\left(1 + \frac{\varsigma}{\varphi}\frac{\bar{W}}{\bar{Z}}\right)\check{c}_t(\tau) + \bar{b}(\check{\mu}_t(\tau) - \pi_t) + \varrho\check{b}_t(\tau)\right]\mathrm{d}t + \bar{b}\check{\boldsymbol{\sigma}}(\tau)\,\mathrm{d}\mathsf{B}_t$$

- Steady-state wealth $\bar{b}$ (identical across HHs)

Social Welfare

- Per-period social welfare loss (second-order expansion relative to first-best):

$$\begin{aligned}\mathcal{L}_t \equiv (\varsigma + \varphi)x_t^2 + \vartheta\pi_t^2 \\ + \int_0^T \psi(\tau)\left(s_t(\tau)\right)^2 \mathrm{d}\tau + \psi^i\left(i_t - \bar{i}_t\right)^2 \\ + \frac{\varsigma}{\varphi}(\varsigma + \varphi)\,\mathrm{Var}_\tau\, \check{c}_t(\tau)\end{aligned}$$

- First line: losses from nominal rigidities (same as in textbook RANK)
- Next line: losses from policy frictions (when $\psi^i > 0, \psi(\tau) > 0$)
- Final line: losses also depends on consumption dispersion *across* HHs

Equilibrium

- Difficulty: how to characterize $\{\mu_t(\tau)\}_{\tau=0}^{T}$? From arbitrageur optimality conditions and market clearing, seemingly need the entire distribution of household wealth
- We show only need an additional N_z set of moments of wealth, consumption, and balance sheet tools:

$$\tilde{\mathbf{b}}_t \equiv \int_0^T \boldsymbol{\sigma}(\tau)^\top \eta(\tau) \check{b}_t(\tau)\,\mathrm{d}\tau\,, \quad \tilde{\mathbf{c}}_t \equiv \int_0^T \boldsymbol{\sigma}(\tau)^\top \eta(\tau) \check{c}_t(\tau)\,\mathrm{d}\tau\,, \quad \tilde{\mathbf{s}}_t \equiv \int_0^T \boldsymbol{\sigma}(\tau)^\top s_t(\tau)\,\mathrm{d}\tau$$

- Linearized dynamics:

$$\mathrm{d}\tilde{\mathbf{b}}_t = \left[-\left(1 + \frac{\varsigma}{\varphi}\frac{\bar{W}}{\bar{\bar{Z}}}\right)\tilde{\mathbf{c}}_t + \bar{b}\tilde{\boldsymbol{\Sigma}}\boldsymbol{\Lambda}_t + \varrho\tilde{\mathbf{b}}_t\right]\mathrm{d}t + \bar{b}\tilde{\boldsymbol{\Sigma}}\,\mathrm{d}\mathbf{B}_t\,, \quad \mathrm{E}_t\,\mathrm{d}\tilde{\mathbf{c}}_t = \varsigma^{-1}\tilde{\boldsymbol{\Sigma}}\boldsymbol{\Lambda}_t\,\mathrm{d}t$$

$$\boldsymbol{\Lambda}_t = \int_0^T \boldsymbol{\sigma}(\tau)^\top x_t(\tau)\,\mathrm{d}\tau = -\tilde{\mathbf{s}}_t + \hat{\boldsymbol{\sigma}}^\top \bar{\beta}\frac{\bar{W}}{\bar{\bar{Z}}}(w_t + n_t) - \tilde{\mathbf{b}}_t - \tilde{\boldsymbol{\sigma}}^\top(\tilde{b}_t - \bar{b})$$

- Endogenous risk objects

$$\tilde{\boldsymbol{\Sigma}} \equiv \int_0^T \eta(\tau)\boldsymbol{\sigma}(\tau)^\top \check{\boldsymbol{\sigma}}(\tau)\,\mathrm{d}\tau\,, \quad \tilde{\boldsymbol{\sigma}} \equiv \int_0^T \eta(\tau)\boldsymbol{\sigma}(\tau)\,\mathrm{d}\tau\,, \quad \hat{\boldsymbol{\sigma}} \equiv \int_0^T \theta(\tau)\boldsymbol{\sigma}(\tau)\,\mathrm{d}\tau$$

Benchmark: Risk Neutral Arbitrageur (“Standard Model”)

- Consider the benchmark case of a risk neutral arbitrageur: $a = 0$
- The expectations hypothesis holds: $\mu_t(\tau) = i_t \implies$ model collapses to RANK

$$\mathrm{Var}_\tau\, \check{c}_t(\tau) = 0$$

- Recover the standard QE neutrality result: balance sheet policies do not affect bond prices (and therefore have no aggregate effects)
- When divine coincidence holds ($\bar{\beta} = 0$) and no policy constraints ($\psi^i = 0$): conventional policy can achieve first-best

$$i_t = v_t \implies \mu_t(\tau) = v_t \implies x_t = \pi_t = 0$$

- ‘Woodford-ian’ equivalence: the role of policy on aggregate dynamics and welfare is fully summarized by policy rate i_t

Dynamics: Analytical Results

Simplified Aggregate Dynamics

- Simplifications: rigid prices ($\vartheta \to \infty$), natural rate shocks only ($N_z = 1$), zero wealth steady state ($\bar{b} = 0$)
- Along with the dynamics of natural rate shocks, we have

$$
\begin{aligned}
\mathrm{d}\tilde{b}_t &= \left[-\left(1+\frac{\varsigma}{\varphi}\right)\tilde{c}_t - \varrho\tilde{b}_t\right]\mathrm{d}t \\
\mathrm{E}_t\,\mathrm{d}x_t &= -\varsigma^{-1}\left[i_t + \tilde{\sigma}\lambda_t - v_t\right]\mathrm{d}t \\
\mathrm{E}_t\,\mathrm{d}\tilde{c}_t &= -\varsigma^{-1}\tilde{\Sigma}\tilde{\sigma}\lambda_t\,\mathrm{d}t \\
\lambda_t &= -\tilde{s}_t + \bar{\beta}\hat{\sigma}\left((1+\varsigma+\varphi)x_t - \frac{1}{\varsigma\kappa_z}v_t\right) - \tilde{b}_t
\end{aligned}
$$

- Ad-hoc Taylor policy rules close the model

$$
i_t = \phi_x x_t + \epsilon_{i,t}, \quad s_t(\tau) = \phi_x(\tau)x_t + \epsilon_{s,t}(\tau) \implies \tilde{s}_t = \tilde{\phi}_x x_t + \tilde{\epsilon}_{s,t}
$$

Simplified Aggregate Dynamics: No Firm Borrowing

- Simplest case: no firm borrowing

Proposition (Rigid price dynamics, no firm borrowing)

Suppose that $\bar{\beta} = 0$, $\phi_x(\tau) = 0$, and household wealth is initially equalized.

- With short rates only, dynamics are equivalent to RA. Risk prices $\lambda_t = 0$ and

$$\frac{\partial x_t}{\partial v_t} > 0, \quad \frac{\partial \lambda_t}{\partial v_t} = 0, \quad \frac{\partial x_t}{\partial \epsilon_{i,t}} < 0, \quad \frac{\partial \lambda_t}{\partial \epsilon_{i,t}} = 0$$

- However, following an unconventional monetary shock

$$\frac{\partial x_t}{\partial \tilde{\epsilon}_{s,t}} > 0, \quad \frac{\partial \lambda_t}{\partial \tilde{\epsilon}_{s,t}} < 0, \tilde{b}_{t+k} \neq 0$$

  and dynamics are not equivalent to a representative agent case

- Intuition: without firm borrowing
  - Conventional policy $\not\Longrightarrow$ HH rebalancing $\Longrightarrow$ constant term premia
  - QE $\Longrightarrow$ arbitrageur rebalancing $\Longrightarrow$ $\downarrow$ term premia $\Longrightarrow$ consumption dispersion

Simplified Aggregate Dynamics: Firm Borrowing

- With firm borrowing, more complicated dynamics

Proposition (Rigid price dynamics, general case)

Suppose that $\bar{\beta} > 0$ and $0 < \phi_x < \bar{\phi}_x$ for some upper bound $\bar{\phi}_x$.

- Following a natural rate shock:

$$\frac{\partial x_t}{\partial v_t} > 0, \quad \frac{\partial \lambda_t}{\partial v_t} > 0, \quad \text{Cov}(i_t, \lambda_t) > 0, \qquad \exists k > 0 : \frac{\partial x_{t+k}}{\partial v_t} > 0, \quad \frac{\partial \lambda_{t+k}}{\partial v_t} < 0$$

- The reaction of risk prices λ_t is stronger if $\phi_x(\tau) > 0$
- Following a conventional monetary policy shock

$$\frac{\partial x_t}{\partial v_t} < 0, \quad \frac{\partial \lambda_t}{\partial \epsilon_{i,t}} < 0$$

- Intuition: with firm borrowing
  - Recession $\implies \downarrow i_t, \downarrow$ firm borrowing $\implies$ arbitrageur rebalancing $\implies \downarrow$ term premia
  - Over longer horizons, $\downarrow$ HH saving $\implies$ arbitrageur rebalancing $\implies \uparrow$ term premia
  - Contraction policy *shock* $\implies \uparrow i_t, \downarrow$ firm rebalancing

Stylized Model Predictions

1. Unconditionally, increases in short rates associated with contemporaneous increases in term premia
2. Larger unconditional reactions during QE periods
3. Over longer horizons, unconditional reaction of term premia to short rates weakens or becomes negative
4. Conditional reaction of term premia to monetary policy *shocks* are small or negative

Empirical Evidence

Empirical Specification

- Utilize movements in long-dated forward rates (similar to Hanson et al 2021)

$$f_{t+h}^{(\tau)} - f_{t-1}^{(\tau)} = \alpha(\tau) + \beta(\tau) D_t + \epsilon_t(\tau)$$

- Unconditional regressions: D_t are daily change in short-term yields

$$D_t = y_t^{(1)} - y_{t-1}^{(1)}$$

- Conditional regressions: D_t are high-frequency MP shocks
- Data:
    - Daily nominal yield curve data from Gurkayank et al (2007)
    - High frequency shocks from Nakamura and Steinsson (2018)

Empirical Results: Unconditional, Varying Maturities

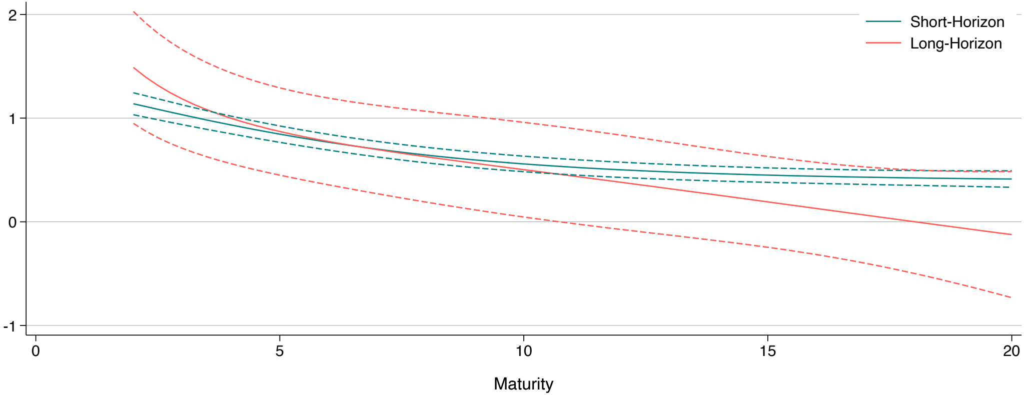


Figure 1: Forward Rates (Unconditional, Varying Maturities)

Full sample (1982-2020), $h = 0$ and $h = 90$, $\tau = 2, ... 20$

Empirical Results: Unconditional, Varying Horizon

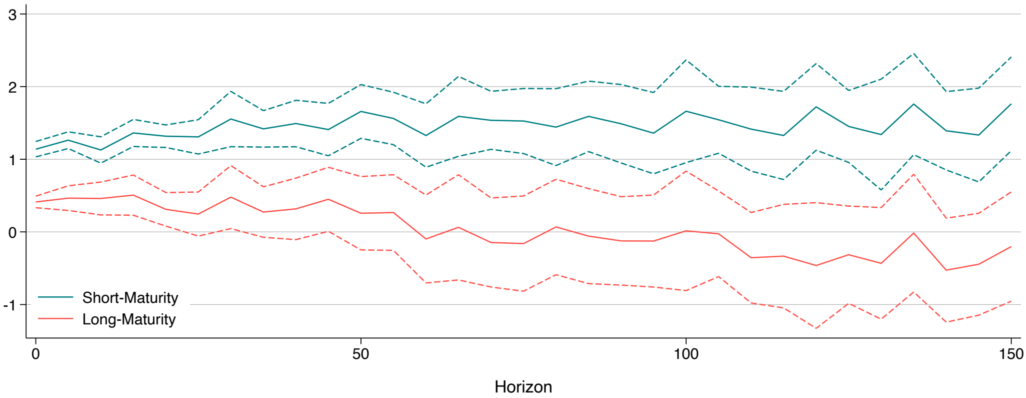


Figure 2: Forward Rates (Unconditional, Varying Horizon)

Full sample (1982-2020), $h = 0 \ldots 150$, $\tau = 2$ and $\tau = 20$

Empirical Results: Unconditional, Rolling Short Horizon

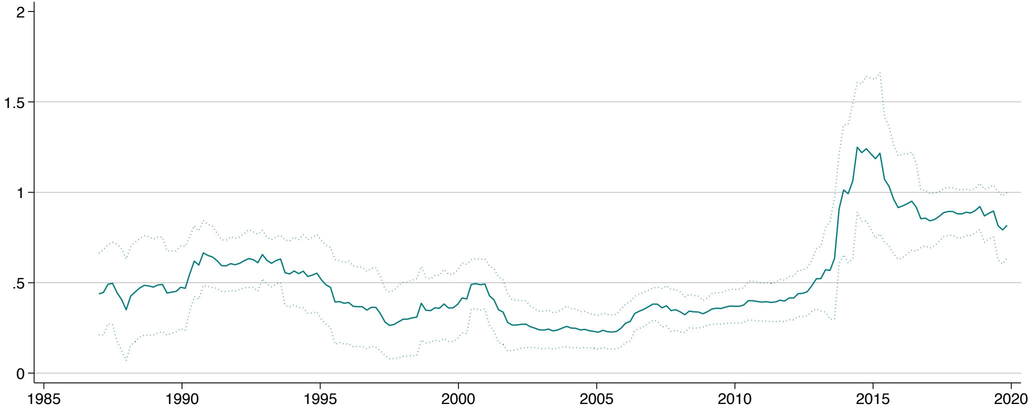


Figure 3: Forward Rates (Unconditional, Rolling Short Horizon)

Rolling window (5 year), $h = 0$, $\tau = 20$

Empirical Results: Unconditional, Rolling Long Horizon

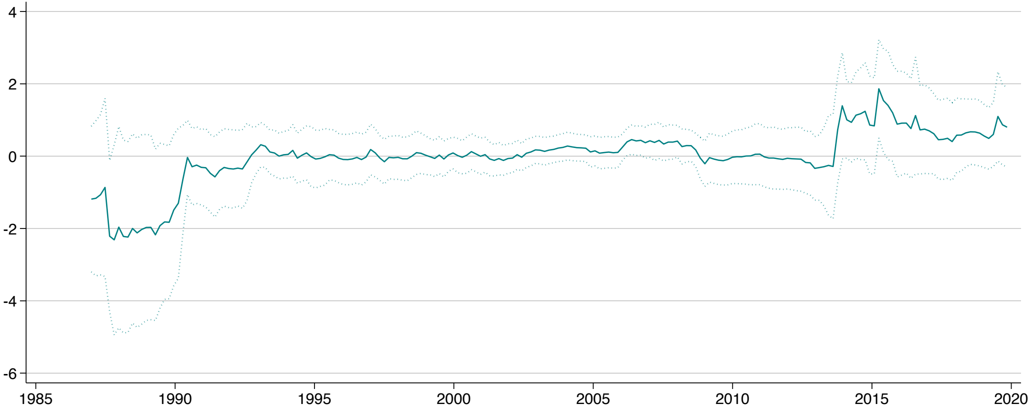


Figure 4: Forward Rates (Unconditional, Rolling Long Horizon)

Rolling window (5 year), $h = 90$, $\tau = 20$

Empirical Results: Conditional, Varying Maturity

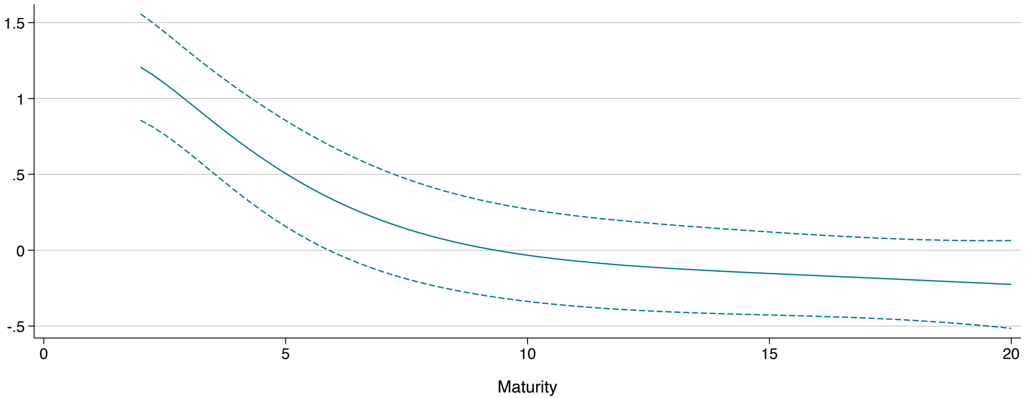


Figure 5: Forward Rates (Shocks, Varying Maturity)

Full sample (FOMC meetings 1995-2020), $h = 0$, $\tau = 2, \ldots, 20$

Empirical Results: Conditional, Rolling

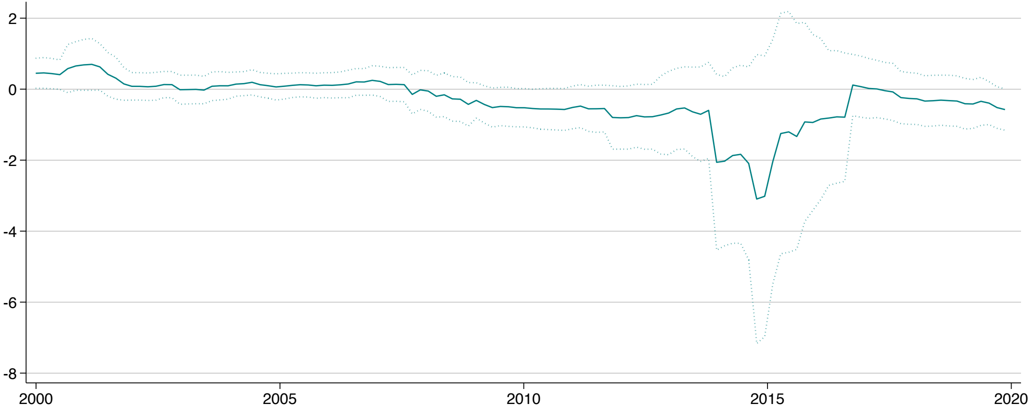


Figure 6: Forward Rates (Shocks, Rolling)

Rolling window (5 year), $h = 0$, $\tau = 20$

Welfare

Welfare Consequences: Simple Policy Rules

- For simplicity, continue assuming rigid prices, natural rate shocks only
- Consider policy rules which implement

$$i_t = \chi_{i,v} v_t + \chi_{i,b} \tilde{b}_t$$
$$s_t(\tau) = \chi_{s,v}(\tau) v_t + \chi_{s,b}(\tau) \tilde{b}_t \implies \tilde{s}_t = \tilde{\chi}_{s,v} v_t + \tilde{\chi}_{s,b} \tilde{b}_t$$

- Simple policy rules: function of natural state variables only
    - Time-consistent: policymaker seeks to minimize unconditional social welfare loss
- We will examine the outcome of these policies in different versions of the model
- Risk-neutral benchmark: perfect arbitrage ($a = 0$) implies $\chi_{i,v} = 1$ is optimal

Optimal Policy: Short Rate Only

- First consider short rate tools only (formally, balance sheet frictions $\psi^{(\tau)} \to \infty$)

Proposition (Optimal short rate policy rule)

Assume risk aversion $a > 0$ and $\bar{\beta} > 0$. If bond risk dispersion across households $\tilde{\Sigma} = 0$:

- $\exists \chi^n_{i,v} \lessgtr 1$ along with $\chi_{i,b} = 0$ which guarantees $x_t = 0 \ \forall t$.
- Sign of $\chi^n_{i,v} - 1$ is determined by the endogenous reaction of firm borrowing to v_t.

With $\tilde{\Sigma} > 0$:

- Optimal short rate policy $i_t = \chi^*_{i,v} i_t + \chi_{b,i} \tilde{b}_t$ where $\chi_{b,i} \neq 0$ and $\chi^*_{i,v} < \chi^n_{i,v}$.
- Implications
  1. Bond carry trade returns $\mu_t(\tau) - i_t$ move in the same direction as i_t iff firm borrowing declines in response to natural rate shocks.
  2. Output gaps x_t are not identically zero.
  3. Consumption dispersion is non-zero: $\mathrm{Var}_\tau \, \check{c}_t(\tau) \neq 0$.

Optimal Short Rate Intuition

- Follows from intuition derived studying ad-hoc rules
- Consider recessionary shock $\downarrow v_t \implies \downarrow i_t$
  - If $\downarrow$ firm borrowing, then arbitrageur rebalancing $\implies \downarrow \lambda_t$
  - Vice-versa if $\uparrow$ firm borrowing
  - In order to keep $\tilde{\mu}_t = v_t$, policy must be react less/more strongly than RANK benchmark (depending on firm borrowing reaction)
- If policy is such that $\tilde{\mu}_t = v_t$, fluctuations in risk prices λ_t imply $\exists \mu_t(\tau) \neq v_t$
- Fluctuations in borrowing rates across the term structure imply $\mathrm{Var}_\tau\, \check{c}_t(\tau) > 0$
- All else equal, reducing policy rate volatility $\implies$ term premia volatility declines
- All else equal, reducing policy rate response to natural rate shocks $\implies$ macroeconomic volatility increases
- Optimal policy balances these objectives

PE Illustration

Optimal Policy: Unconstrained Case

- With access to frictionless balance sheet policies, we obtain the following

Proposition (Optimal policy separation principle)

Assume risk aversion $a > 0$, $\bar{\beta} > 0$, and household wealth is initially equalized. Suppose the central bank implements short rate and balance sheet policy according to

$$i_t = v_t, \quad \tilde{s}_t = -\frac{\bar{\beta}\hat{\sigma}}{\varsigma \kappa_z} v_t.$$

If short rate policy is frictionless ($\psi^i = 0$) and the central bank does not face holding costs ($\psi^{(\tau)} = 0$), then first-best is achieved:

1. Macroeconomic stabilization: $x_t = 0 \ \forall t$.
2. Term premia stabilization: $\mu_t(\tau) = \tilde{\mu}_t \ \forall \tau$.
3. Consumption equalization: $c_t(\tau) = c_t(\tau') \ \forall \tau, \tau'$.

Optimal Policy Separation Principle: Intuition

- The results follow naturally from our findings regarding ad-hoc policy
- QE policy stabilizes shocks to bond markets by offsetting all firm borrowing movements
- Implies net zero arbitrageur positions so

$$\int_0^T \boldsymbol{\sigma}(\tau)^\top x_t(\tau)\,\mathrm{d}\tau = \mathbf{0} \implies \boldsymbol{\Lambda}_t = 0$$

- This equalizes borrowing rates across HHs: $\mu_t(\tau) = \tilde{\mu}_t$
- Hence the model collapses to a standard RANK model, where $i_t = v$ is optimal

Separation principle for optimal policy:

- Optimal balance sheet policy stabilizes bond markets
- Optimal short rate policy stabilizes macroeconomic aggregates

Separation Principle with Balance Sheet Constraints

- When the central bank faces balance sheet constraints ($\psi^{(\tau)} > 0$), policy can no longer achieve first-best
- However, as long as $\psi^{(\tau)} < \infty$, optimal policy implies the central bank still uses balance sheet tools
- Let $\psi^{(\tau)} = a \cdot \boldsymbol{\sigma}(\tau)\boldsymbol{\sigma}(\tau)^\top$
  - $\implies$ same friction a as arbitrageurs, except policymaker cannot net out positions
- Even with "large" balance sheet costs, the central bank still uses QE to (partially) stabilize term premia
- Intuition:
  - The central bank faces holding costs which imply it is worse than private arbitrageurs at financial intermediation
  - But internalizes the social benefits of minimizing fluctuations in term premia
  - Nevertheless, non-negligible balance sheet costs imply that optimal policy is less reactive

Optimal Policy: Short Rate Constraints

- Suppose that short rate policy is constrained, and implements

$$i_t = \tilde{\chi}_i v_t, \quad 0 < \tilde{\chi}_i \ll 1$$

  - Formally: assume costs $\psi^i (i_t - \tilde{\chi}_i v_t)$ and take $\psi^i \to \infty$

Proposition (Optimal balance sheet rule)

Assume risk aversion $a > 0$, $\bar{\beta} > 0$, and constrained short rates.

- *Bond market stabilization*: $\tilde{s}_t = \bar{\beta}\hat{\sigma}\left((1 + \varsigma + \varphi)x_t - \frac{1}{\varsigma\kappa_z} v_t\right)$ implies
    1. Borrowing rates are stabilized, consumption and wealth dispersion are zero.
    2. Output gaps x_t are no longer identically zero.
- *Macroeconomic stabilization*: there exist parameters $\tilde{\chi}_{s,v} \neq 0, \tilde{\chi}_{s,b} \neq 0$ such that
    1. Output gaps are zero.
    2. Borrowing rate, consumption, and wealth dispersion are non-zero.

Stabilization with Short Rate Constraints: Intuition

- Balance sheet policy works by affecting term premia through changes in the market price of risk
- Although arbitrage is imperfect in this model, arbitrageurs still enforce tight restrictions between between market price of risk and term premia across maturities
- Hence, while in principle the central bank has a continuum of policy tools $\{s_t(\tau)\}_{\tau=0}^{T}$, can only manipulate risk price λ_t
- Policymaker has to decide between macroeconomic and bond market stabilization:

$$\tilde{s}_t^{(macro)} \implies \tilde{\mu}_t = v_t \implies x_t = 0, \mathrm{Var}_\tau\, \check{c}_t(\tau) \gg 0$$

$$\tilde{s}_t^{(bond)} \implies \mu_t(\tau) = i_t \neq v_t \implies x_t \neq 0, \mathrm{Var}_\tau\, \check{c}_t(\tau) = 0$$

- Related to localization results in Vayanos & Vila (2021) and Ray, Droste, & Gorodnichenko (2024)
    - In the one-factor model considered here, the effects of QE are fully global
    - Even with more complicated risk structure, localization is not strong enough to allow balance sheet policy alone to achieve first-best

Extensions Overview

- Sticky prices, cost-push shocks
  - If firm borrowing is a small part of marginal costs, then all results go through
  - Exogenous cost-push shocks breaks divine coincidence but unfortunately, our separation principle still holds
  - Despite multiple policy instruments, (un)conventional policy only affects aggregate outcomes through changes in effective borrowing rate $\tilde{\mu}_t$
  - If firm borrowing is large, then policymaker can in principle manipulate HH and firm effective borrowing rates $\tilde{\mu}_t$, $\hat{\mu}_t$ (though this is suboptimal due to risk-sharing motives)
- "Noise" demand shocks
  - Optimal separation principle still holds with firm financing shocks β_t
  - QE policy must be more reactive than the benchmark
  - The optimal rule may imply conventional and unconventional policies seemingly acting against one another
- Nonzero first-best term premia
  - When first-best BCT returns are $\nu(\tau) \neq 0$
  - Results hold when $\nu(\tau)$ is achievable but optimal short rate policy is a function of $\nu(\tau)$

History-Dependent Policy

Monetary Policy with Commitment

- When policy frictions bind, simple policy rules fail to achieve first-best
- Instead suppose the policymaker chooses policy tools $\mathbf{u}_t$ as a function of entire history of predetermined and nonpredetermined variables $\mathbf{Y}_t \equiv \begin{bmatrix} \mathbf{y}_t^\top & \mathbf{x}_t^\top \end{bmatrix}^\top$
- Minimizes conditional social loss

$$\begin{aligned} \mathcal{W}_0 &= \frac{1}{2}\mathbb{E}_0 \int_0^\infty e^{-\rho t}\mathcal{L}_t \,\mathrm{d}t \\ &= \frac{1}{2}\mathbb{E}_0 \int_0^\infty e^{-\rho t}\left(\mathbf{Y}_t^\top \mathbf{R}\mathbf{Y}_t + \mathbf{u}_t^\top \mathbf{Q}\mathbf{u}_t\right)\mathrm{d}t\,, \quad \mathbf{y}_0 \text{ given} \end{aligned}$$

- By setting policy in a history-dependent manner, full commitment policies can improve on simple policy rules

Characterizing Optimal Policy with Commitment (*work in progress!*)

Theorem (Optimal Policy with Commitment)

Given $\mathbf{y}_0$, the policymaker minimizes $\mathcal{W}_0$ by choosing $\mathbf{u}_t = \mathbf{F}\mathbf{Y}_t$, which induce equilibrium dynamics $\mathrm{d}\mathbf{Y}_t = -\boldsymbol{\Upsilon}(\mathbf{F})\mathbf{Y}_t\,\mathrm{d}t + \mathbf{S}(\mathbf{F})\,\mathrm{d}\mathbf{B}_t$. Necessary conditions are given by

$$\mathbf{y}_0^\top\left(\partial_i \mathbf{P}_{11} - \partial_i \mathbf{P}_{12}\mathbf{P}_{22}^{-1}\mathbf{P}_{21} - \mathbf{P}_{12}\mathbf{P}_{22}^{-1}\partial_i \mathbf{P}_{21} + \mathbf{P}_{12}\left(\mathbf{P}_{22}^{-1}\partial_i \mathbf{P}_{22}\mathbf{P}_{22}^{-1}\right)\mathbf{P}_{21}\right)\mathbf{y}_0 = 0$$

where $\rho\mathbf{P} = \mathbf{R} + \mathbf{F}^\top\mathbf{Q}\mathbf{F} - \mathbf{P}\boldsymbol{\Upsilon} - \boldsymbol{\Upsilon}^\top\mathbf{P}$. Dynamics are given by $\mathbf{q}_0 = \begin{bmatrix}\mathbf{y}_0 & \mathbf{0}\end{bmatrix}^\top$ and

$$\mathrm{d}\mathbf{q}_t = -\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{P}_{21} & \mathbf{P}_{22}\end{bmatrix}\boldsymbol{\Upsilon}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ -\mathbf{P}_{22}^{-1}\mathbf{P}_{21} & \mathbf{P}_{22}^{-1}\end{bmatrix}\mathbf{q}_t\,\mathrm{d}t + \begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{P}_{21} & \mathbf{P}_{22}\end{bmatrix}\mathbf{S}\,\mathrm{d}\mathbf{B}_t \equiv -\boldsymbol{\Gamma}\mathbf{q}_t\,\mathrm{d}t + \boldsymbol{\sigma}\,\mathrm{d}\mathbf{B}_t$$

Bond prices are affine in $\mathbf{A}(\tau)^\top\mathbf{q}_t$ with $\mathbf{A}(\tau) = \left[\mathbf{I} - e^{-\mathbf{M}\tau}\right]\mathbf{M}^{-1}\mathbf{e}_i$ and

$$\mathbf{e}_i^\top\mathbf{q}_t = i_t, \quad \mathbf{M} = \boldsymbol{\Gamma}^\top - \int_0^T \boldsymbol{\Theta}(\tau)\mathbf{A}(\tau)^\top\,\mathrm{d}\tau\,\tilde{\boldsymbol{\Sigma}}$$

Monetary Policy with Commitment: Intuition

- Policymaker chooses tools $i_t, \{s_t(\tau)\}_{\tau=0}^{T}$ which:
    - Directly affect optimality conditions of arbitrageurs
    - Indirectly affect HHs through changes in equilibrium borrowing rates
    - Indirectly affect firms through changes in marginal costs
- Trade-off: more aggressive policy reactions to shocks:
    - Greater pass-through to HHs
    - Larger and more volatile term premia
- Commitment partially relaxes this link:
    - HH decisions depend on entire expected path of borrowing rates $\int_0^\infty \mu_t(\tau)\,\mathrm{d}\tau$
    - Arbitrageur risk compensation depends on volatility of short-run fluctuations $\mathrm{d}i_t$, $\mathrm{d}s_t(\tau)$
- Characterizing dynamics of optimal policy with commitment is difficult
    - Ongoing work studies optimal policy numerically
    - Suffers from time inconsistency; simple rules may be more practical

PE Illustration

Measures of Policy Optimality

Measuring Balance Sheet Objectives: Return Predictability

- Fama-Bliss regression:

$$\frac{1}{\Delta\tau}\log\left(\frac{P_{t+\Delta\tau}^{(\tau-\Delta\tau)}}{P_t^{(\tau)}}\right) - y_t^{(\Delta\tau)} = a_{FB}^{(\tau)} + b_{FB}^{(\tau)}\left(f_t^{(\tau-\Delta\tau,\tau)} - y_t^{(\Delta\tau)}\right) + \varepsilon_{t+\Delta\tau}$$

- Measures how the slope of the term structure predicts excess returns
- In our model, when the central bank does not use balance sheet policies:

$$\bar{b}_{FB}^{(\tau)} > 0$$

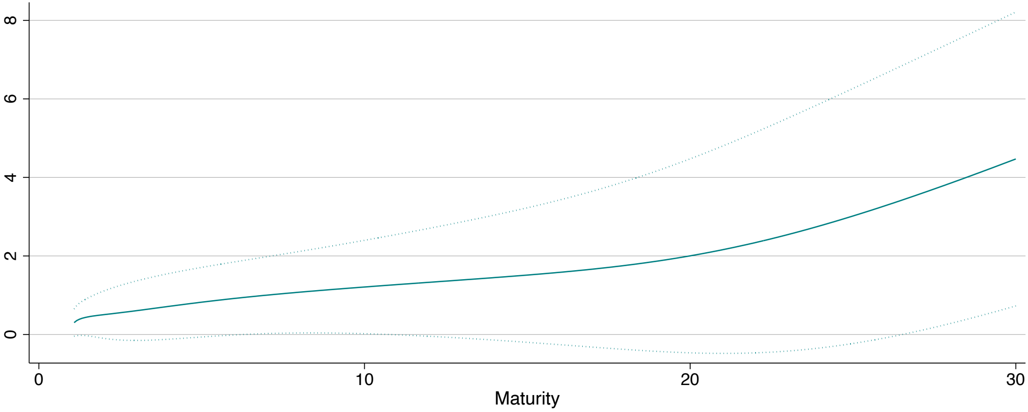
- If balance sheet policy is pursuing bond market stabilization:

$$\bar{b}_{FB}^{(\tau)} > b_{FB}^{(\tau)} \to 0$$

- Instead, if balance sheet policy is pursuing macroeconomic stabilization:

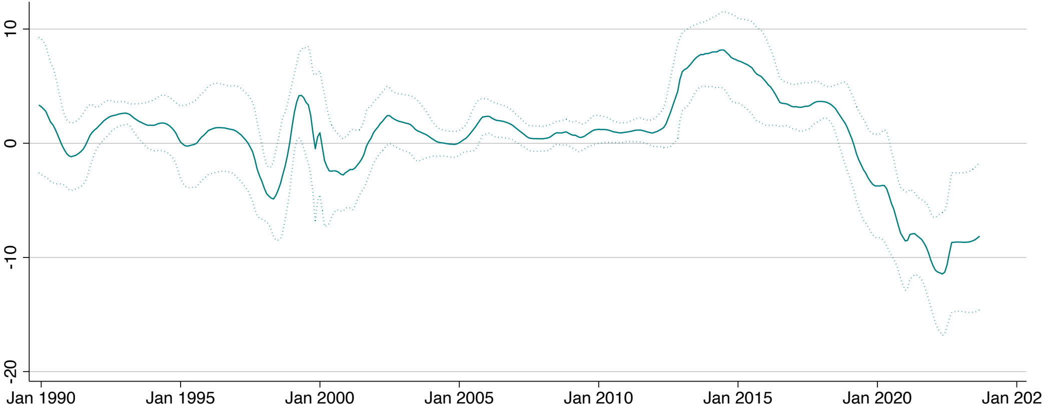
$$b_{FB}^{(\tau)} > \bar{b}_{FB}^{(\tau)}$$

Fama-Bliss Coefficients: Treasuries, Full Sample



FB coefficients are non-zero (and increasing across maturities)

Fama-Bliss Coefficients: 10-year Treasuries, Rolling Sample



FB coefficients increase during initial QE regime, but have fallen and even become negative in recent years

Concluding Remarks

- This paper develops a tractable general equilibrium model with market segmentation, nominal frictions, financial frictions, and household heterogeneity
- We find that optimal short rate and balance sheet policy is characterized by a sharp **optimal separation result**:
    - Conventional policy targets macroeconomic stability
    - Unconventional policy targets bond market stability
- Optimal policy removes excess volatility of bond returns and hence improves risk-sharing across households, while reducing excess macroeconomic volatility
    - Policy constraints on either the short rate or balance sheets imply trade-offs between these policy objectives
- When considering social welfare, cannot abstract from the policy tools used to conduct monetary policy

Thank You!

Simple Optimal Short Rate: PE Illustration I

- Partial equilibrium illustration with ad-hoc loss function, simple policy rules
- Dynamics and loss function

$$\mathrm{d}z_t = -\kappa_z z_t \,\mathrm{d}t + \sigma_z \,\mathrm{d}B_t\,, \quad x_t(\tau) = -\theta(\tau) z_t$$

$$\mathcal{L}_t \equiv (z_t - i_t)^2 + \left(\int_0^T \theta(\tau)(\mu_t(\tau) - i_t) \,\mathrm{d}\tau \right)^2 , \quad \min \mathsf{E}\, \mathcal{L}_t$$

- Risk prices $\lambda_t = \int_0^T -\sigma(\tau)\theta(\tau)\,\mathrm{d}\tau\, z_t \equiv -\tilde{\sigma} z_t$

$$\mu_t(\tau) - i_t = a\sigma(\tau)\lambda_t \implies \left(\int_0^T \theta(\tau)(\mu_t(\tau) - i_t) \,\mathrm{d}\tau \right)^2 = a^2 \tilde{\sigma}^2 z_t^2$$

- Simple policy rule: choose χ such that $i_t = \chi z_t$

back

Simple Optimal Short Rate: PE Illustration II

- Unconditionally, $\mathsf{E}(z_t - i_t)^2$ is decreasing in χ for $\chi < 1$
- Is $\chi = 1$ optimal? Not if $a > 0$, since $\tilde{\sigma}$ is endogenous
- Solving for $\tilde{\sigma}$: conjecture affine term structure

$$-\log P_t^{(\tau)} = A_z(\tau) z_t + C(\tau)$$

- Ito's Lemma and market clearing:

$$A_z'(\tau) + M A_z(\tau) = \chi \implies A_z(\tau) = \chi \frac{1 - e^{-M\tau}}{M}, \quad \text{where } M = \kappa_z + a\sigma_z^2 \int_0^T \theta(\tau) A_z(\tau)\, \mathrm{d}\tau$$

$$\implies \tilde{\sigma}^2 = \sigma_z^2 \left(\int_0^T \theta(\tau) A_z(\tau)\, \mathrm{d}\tau \right)^2$$

- Hence, unconditionally $\mathsf{E}\left(\int_0^T \theta(\tau)(\mu_t(\tau) - i_t)\, \mathrm{d}\tau \right)^2$ is increasing in χ
- Optimal $0 < \chi^* < 1$

back

Full Commitment Optimal Short Rate: PE Illustration I

- Partial equilibrium illustration with ad-hoc loss function, full commitment
- Dynamics and loss function

$$\mathrm{d}z_t = -\kappa_z z_t \,\mathrm{d}t + \sigma_z \,\mathrm{d}B_t\,, \quad x_t(\tau) = -\theta(\tau) z_t$$

$$\mathcal{L}_t \equiv (z_t - i_t)^2 + \left(\int_0^T \theta(\tau)(\mu_t(\tau) - i_t)\,\mathrm{d}\tau \right)^2 , \quad \min \mathsf{E}_0 \int_0^\infty e^{-\varrho t} \mathcal{L}_t \,\mathrm{d}t$$

- Risk prices $\lambda_t = \int_0^T -\sigma(\tau)\theta(\tau)\,\mathrm{d}\tau\, z_t \equiv -\tilde{\sigma} z_t$

$$\mu_t(\tau) - i_t = a\sigma(\tau)\lambda_t \implies \left(\int_0^T \theta(\tau)(\mu_t(\tau) - i_t)\,\mathrm{d}\tau \right)^2 = a^2 \tilde{\sigma}^2 z_t^2$$

- Policy rule with commitment: choose χ, κ_i, i_0 such that

$$\mathrm{d}i_t = -\kappa_i (i_t - \chi z_t)\,\mathrm{d}t$$

back

Full Commitment Optimal Short Rate: PE Illustration II

- Dynamics

$$\mathbf{x}_t = e^{-\boldsymbol{\Gamma} t}\mathbf{x}_0 + \int_0^t e^{-\boldsymbol{\Gamma}(t-u)}\boldsymbol{\sigma}_x \, \mathrm{d}B_u \,, \quad \boldsymbol{\Gamma} = \begin{bmatrix} \kappa_z & 0 \\ -\kappa_i\chi & \kappa_i \end{bmatrix}, \quad \boldsymbol{\sigma}_x = \begin{bmatrix} \sigma_z \\ 0 \end{bmatrix}$$

- Affine term structure

$$-\log P_t^{(\tau)} = A_z(\tau)z_t + A_i(\tau)i_t + C(\tau) \equiv \mathbf{A}(\tau)^\top \mathbf{x}_t + C(\tau)$$

$$\implies \mathbf{A}(\tau) = \mathbf{M}^{-1}\left[\mathbf{I} - e^{-\mathbf{M}\tau}\right]\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{M} \equiv \boldsymbol{\Gamma}^\top + \begin{bmatrix} a\sigma_z^2 \int_0^T \theta(\tau)A_z(\tau)\,\mathrm{d}\tau & 0 \\ 0 & 0 \end{bmatrix}$$

- If $a = 0$, then $i_0 = z_0, \chi = 1, \kappa_i \to \infty$
- As with simple policy rules, $\chi \to 0 \implies A_z(\tau) \to 0$
- But policymaker still utilizes choices of i_0 and $\kappa_i < \infty$ (smoothing)

back