

# Optimal Macro-Financial Stabilization in a New Keynesian Preferred Habitat Model

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# Motivation

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Bernanke: “QE works in practice but not in theory”

- By now the gap between practice and theory is small
- But what do we mean by *QE works*?
  - Obvious: reduce long-term yields
  - Less obvious: stimulate the economy
  - Even less obvious: improve social welfare
  - Reis: “QE’s original sin”
- Especially relevant today now that central banks are implementing QT while increasing short rates
- **Question**: what is the optimal QE policy, and how does this interact with short rate policy?

# Our Model

- [This paper](#): develops a tractable general equilibrium model with market segmentation, nominal frictions, and household heterogeneity
- [Arbitrageurs](#) must absorb supply and demand shocks in bond markets
- [Clientele investors](#) introduce a degree of [market segmentation](#)
  - Bond markets populated by different investor clienteles (pension funds, mutual funds)
  - Arbitrageurs (hedge funds, broker-dealers) partly overcome segmentation
- [Households](#) have differentiated access to bond markets
  - Introduces imperfect risk-sharing and [consumption dispersion](#) across households
- Formally: embed a Vayanos-Vila model in a New Keynesian model, where households are heterogeneous in their savings vehicles

## Findings: Policy Transmission

- Key mechanisms of conventional monetary policy:
  - Policy rate changes are transmitted to households via segmented bond markets
  - Interaction of arbitrageurs and investor clienteles implies portfolio rebalancing
  - Hence, short rate changes lead to variation in risk premia
- Key mechanisms of balance sheet policy:
  - Central bank asset purchases induce portfolio rebalancing and hence reduce risk premia
  - Vice versa for central bank asset sales (QT)
- Thus, short rate and balance sheet policies are substitutes when targeting aggregate borrowing rates
  - A fall in aggregate borrowing rates is stimulative for the usual NK reasons
- However, both policies imply variation in risk premia
  - Excess fluctuations in risk premia implies dispersion in borrowing rates and therefore consumption across households

## Findings: Optimal Policy

- Hence, when policy is unconstrained we derive an **optimal separation result**:
  - Conventional policy targets **macroeconomic stability**
  - Unconventional policy targets **financial stability**
- However, when **policy constraints bind**, policy must balance trade-offs:
  - **Balance sheet constraints**: short rate must be less reactive in order to minimize financial disruptions (at the cost of macroeconomic stability)
  - **Short rate constraints**: QE must be used to offset macroeconomic shocks (at the cost of financial stability)
- General message: **implementation matters** for welfare

## Related Literature

- Preferred habitat models
  - Vayanos & Vila (2021), Ray, Droste, & Gorodnichenko (2023), Greenwood & Vayanos (2014), Greenwood et al (2016), King (2019, 2021) , ...
- Empirical evidence: QE and preferred habitat
  - Krishnamurthy & Vissing-Jorgensen (2012), D'Amico & King (2013), Li & Wei (2013), Cahill et al. (2013), King (2019), Fieldhouse et al (2018), Di Maggio et al (2020), Gali, Debortoli, & Gambetti (2020) , ...
- Macroeconomic QE models
  - Andres, Lopez-Salido, & Nelson (2004), Gertler & Karadi (2011), Curdia & Woodford (2011), Chen et al (2012), Gertler & Karadi (2013), Sims & Wu (2020), Karadi & Nakov (2020), Iovino and Sergeyev (2023), Carlstrom et al (2017), Ippolito et al (2018) , ...
- Market segmentation, macro-prudential monetary policy
  - Cui & Sterk (2021), Auclert (2016), Collard et al (2017), Kaplan et al (2017), Debortoli & Gali (2017) , ...
- International
  - Itskhoki & Mukhin (2022), Greenwood et al (2023), Gourinchas, Ray, & Vayanos (2022) , ...

## Set-Up

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# Model Set-Up

- Continuous time New Keynesian model with embedded Vayanos-Vila bond markets
- **Agents:**
  - **Households:** supply labor, consume, save via differentiated habitat bond funds
  - **Firms:** monopolistic competitors produce using labor, face nominal pricing frictions
  - **Habitat funds:** buys and sell bonds of a specific maturity
  - **Arbitrageurs:** imperfect risk-bearing capacity, conduct bond carry trades
  - **Central bank:** conducts short rate and balance sheet (QE) policy
  - **Government:** optimal production subsidy, otherwise passive
- **Bond markets:**
  - Continuum of **zero coupon bonds** with maturity  $0 \leq \tau \leq T \leq \infty$
  - Bond price  $P_t^{(\tau)}$  with yield to maturity  $y_t^{(\tau)} = -\log P_t^{(\tau)} / \tau$
  - Nominal short rate  $i_t = \lim_{\tau \rightarrow 0} y_t^{(\tau)}$

# Households

- Continuum of HHs, differentiated by access to bond markets  $\tau$
- There is a mass  $\eta(\tau)$  of each  $\tau$  HH where  $\int_0^T \eta(\tau) d\tau = 1$  (but otherwise identical)
- A  $\tau$ -HH chooses consumption and labor  $C_t^{(\tau)}, N_t^{(\tau)}$  in order to solve

$$V_0^{(\tau)} \equiv \max E_0 \int_0^\infty e^{-\rho t} \left( \frac{[C_t^{(\tau)}]^{1-\varsigma}}{1-\varsigma} - \frac{[N_t^{(\tau)}]^{1+\phi}}{1+\phi} \right) dt$$

$$\text{s.t. } dA_t^{(\tau)} = [\mathcal{W}_t N_t^{(\tau)} - P_t C_t^{(\tau)}] dt + A_t^{(\tau)} \frac{dP_t^{(\tau)}}{P_t^{(\tau)}} + dF_t^{(\tau)}$$

- $A^{(\tau)}$  is nominal wealth earning  $\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}$  and  $dF_t^{(\tau)}$  are (flow) nominal transfers
- $\mathcal{W}_t$  is the nominal wage and  $P_t$  is the price index (same for all HHs)

**Key takeaway:** differentiated consumption and labor choices when bond returns not equalized

# Firms

- Continuum of intermediate goods  $j \in [0, 1]$  (and CES final good)
- Linear production in labor  $Y_{t,j} = Z_t N_{t,j}$  where  $Z_t = \bar{Z} e^{z_t}$  is aggregate technology:

$$dz_t = -\kappa_z Z_t dt + \sigma_z dB_{t,z}$$

- Face Rotemberg costs  $\Theta(\pi_{t,j}) = \frac{\theta}{2} \pi_{t,j}^2 P_t Y_t$  when setting prices  $\frac{dP_{t,j}}{P_{t,j}} = \pi_{t,j} dt$
- Nominal profits are given by

$$\mathcal{F}_t(P_{t,j}, Y_{t,j}, \pi_{t,j}) = (1 + \tau^*) P_{t,j} Y_{t,j} - W_t N_{t,j} - \Theta(\pi_{t,j}) - \mathcal{T}_t$$

- $\tau^*$  is the (optimal) production subsidy funded by lump-sum taxes  $\mathcal{T}_t$
- Firms choose  $\pi_{t,j}$  in order to solve

$$U_0 \equiv \max E_0 \int_0^\infty e^{-\rho t} Q_t \frac{\mathcal{F}_t}{P_t} dt$$

- Since HHs own firms, profits are discounted by weighted real SDF  $Q_t \equiv \int_0^T \eta(\tau) Q_t^{(\tau)} d\tau$

**Key takeaway:** pricing frictions create deadweight loss

- Mean-variance optimization

$$\begin{aligned} & \max E_t dW_t - \frac{\gamma}{2} \text{Var}_t dW_t \\ \text{s.t. } & dW_t = W_t i_t dt + \int_0^T X_t^{(\tau)} \left( \frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - i_t dt \right) d\tau \end{aligned}$$

- Arbitrageurs invest  $X_t^{(\tau)}$  in bond carry trade of maturity  $\tau$
- Remainder of wealth  $W_t$  invested at the short rate
- Risk-return tradeoff governed by  $\gamma$

**Key takeaway:** risk averse arbitrageurs' holdings increase with expected return

# Preferred Habitat Funds

- Habitat bond demand for maturity  $\tau$ :

$$Z_t^{(\tau)} = -\alpha(\tau) \log P_t^{(\tau)} - \theta(\tau) \beta_t$$

- $\alpha(\tau)$ : demand elasticity for  $\tau$  fund
- $\beta_t$ : additional time-varying (“noise”) demand factor

$$d\beta_t = -\kappa_\beta (\beta_t - \bar{\beta}) dt + \sigma_\beta dB_{\beta,t}$$

- $\theta(\tau)$ : mapping from demand factor to  $\tau$ -habitat demand

**Key takeaway:** price movements require portfolio rebalancing

- Central bank chooses the policy rate  $i_t$  and bond holdings  $S_t^{(\tau)}$
- Optimal policy: maximize social welfare

$$\max E_0 \int_0^\infty e^{-\rho t} \left( \int_0^T \eta(\tau) u \left( C_t^{(\tau)}, N_t^{(\tau)} \right) \right) dt$$

- In the background: fiscal authority chooses production subsidy  $\tau^*$

Key takeaway: policy attempts to undo frictions:

1. Monopolistic competition  $\implies$  inefficient production
2. Nominal pricing frictions  $\implies$  deadweight loss
3. Market segmentation  $\implies$  consumption dispersion

# Equilibrium

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# Simplifying Assumptions

- **Tractability assumption:** a “head of HH” equalizes wealth: across  $\tau$  HH groups,  
 $A_t^{(\tau)} \equiv A_t$ 
  - Pros: clear focus on the role market segmentation plays on consumption dispersion
  - Cons: ignores the impact of market segmentation on wealth inequality
- **Approximation:** around a limiting case: risk  $\sigma_z, \sigma_\beta \rightarrow 0$  but arbitrageur risk aversion  $\gamma \rightarrow \infty$ 
  - Pros: clear focus on the idea of “imperfect arbitrage”
  - Cons: quantitatively less realistic risk premia
- Allows us to derive simple first-order approximations of dynamics and second-order approximations for welfare and focus on **analytical results**



# Bond Market Equilibrium

- Bond price dynamics:

$$\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} \equiv \mu_t^{(\tau)} dt + \sigma_t^{(\tau)} dB_t$$

- $B_t$  collects innovations to risk factors (technology, noise demand, ...)
- Arbitrageur optimality conditions:

$$\begin{aligned}\mu_t^{(\tau)} - i_t &= \sigma_t^{(\tau)} \Lambda_t \\ \Lambda_t &= \gamma \int_0^T \chi_t^{(\tau)} [\sigma_t^{(\tau)}]^\top d\tau\end{aligned}$$

- Term premia depend on risk aversion  $\gamma$  and equilibrium holdings  $\chi_t^{(\tau)}$ 
  - In our limiting case,  $\sigma_t^{(\tau)} \Lambda_t \neq 0$

# Aggregation

- Symmetric equilibrium:  $Y_{t,j} = Y_t, P_{t,j} = P_t, \pi_{t,j} = \pi_t, \frac{dP_t}{P_t} = \pi_t dt$  and we have

$$Y_t = Z_t N_t \equiv Z_t \int_0^T \eta(\tau) N_t^{(\tau)} d\tau$$

$$C_t \equiv \int_0^T \eta(\tau) C_t^{(\tau)} d\tau = Y_t \left( 1 - \frac{\theta}{2} \pi_t^2 \right)$$

- Firms, arbitrageurs, and funds transfer profits to HHs. **Bond market clearing** implies

$$X_t^{(\tau)} + Z_t^{(\tau)} + S_t^{(\tau)} = 0$$

- **Natural benchmark:**  $\theta \rightarrow 0$  and  $\gamma \rightarrow 0$  along with optimal  $\tau^*$  implies first-best

$$Y_t^n = C_t^n = Z_t^{\frac{1+\phi}{\varsigma+\phi}}, \quad N_t^n = Z_t^{\frac{1-\varsigma}{\varsigma+\phi}}, \quad \frac{W_t^n}{P_t^n} = Z_t$$

- Output gap  $X_t \equiv \frac{Y_t}{Y_t^n}$

details

# Household and Firm Optimality Conditions

- Bond price dynamics and household (log-linearized) optimality conditions give:

$$dc_t^{(\tau)} = \varsigma^{-1} \left( \mu_t^{(\tau)} - \pi_t - \rho \right) dt$$

- Also gives us a **modified dynamic IS curve**:

$$dx_t = \varsigma^{-1} (\tilde{\mu}_t - \pi_t - r_t^*) dt$$

- $r_t^* \equiv -\kappa_z z_t$  is the usual natural rate and  $\tilde{\mu}_t$  is the **effective borrowing rate**:

$$\tilde{\mu}_t = \int_0^T \eta(\tau) \mu_t^{(\tau)} d\tau$$

- Firm (log-linearized) optimality conditions give a **standard NKPC**:

$$d\pi_t = (\rho\pi_t - \delta x_t) dt$$

- $\implies$  to a **first-order**, our model is essentially the same as Ray, Droste, & Gorodnichenko (2023)

- A second-order expansion of social welfare relative to the first best gives

$$L_0 \equiv -\frac{1}{2} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left( (\varsigma + \phi) x_t^2 + \theta \pi_t^2 + \frac{\varsigma}{\phi} (\varsigma + \phi) \text{Var}_\tau c_t^{(\tau)} \right) dt$$

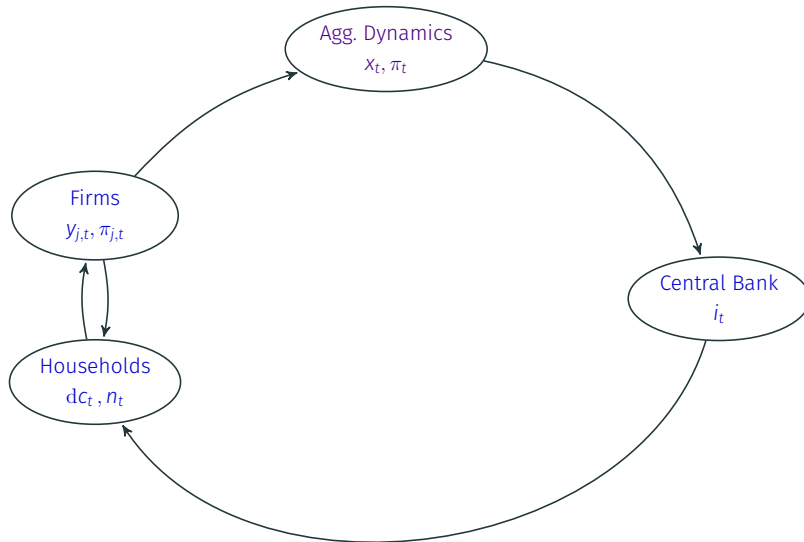
- Compared to a standard RANK model, there is the addition of the term  $\text{Var}_\tau c_t^{(\tau)}$

$$\text{Var}_\tau c_t^{(\tau)} \equiv \int \eta(\tau) \left( c_t^{(\tau)} \right)^2 d\tau - \left[ \int \eta(\tau) c_t^{(\tau)} d\tau \right]^2$$

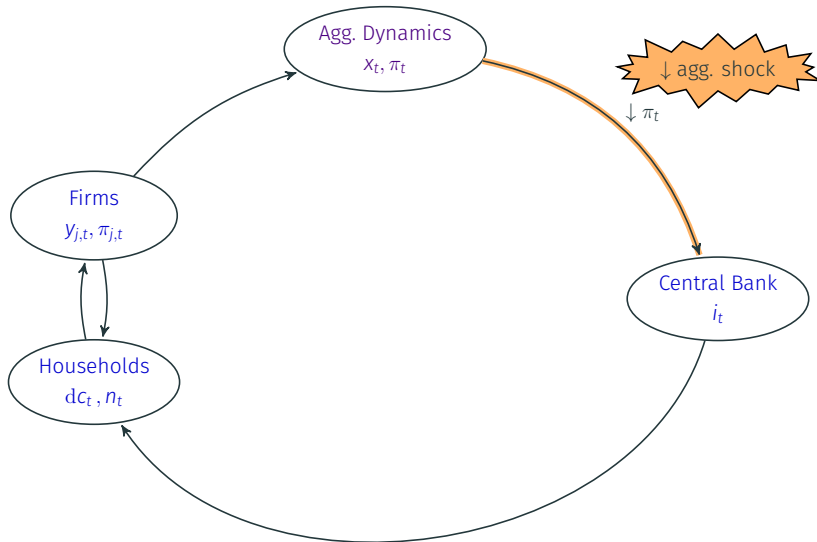
- Increased **consumption dispersion** across HHs implies welfare losses

details

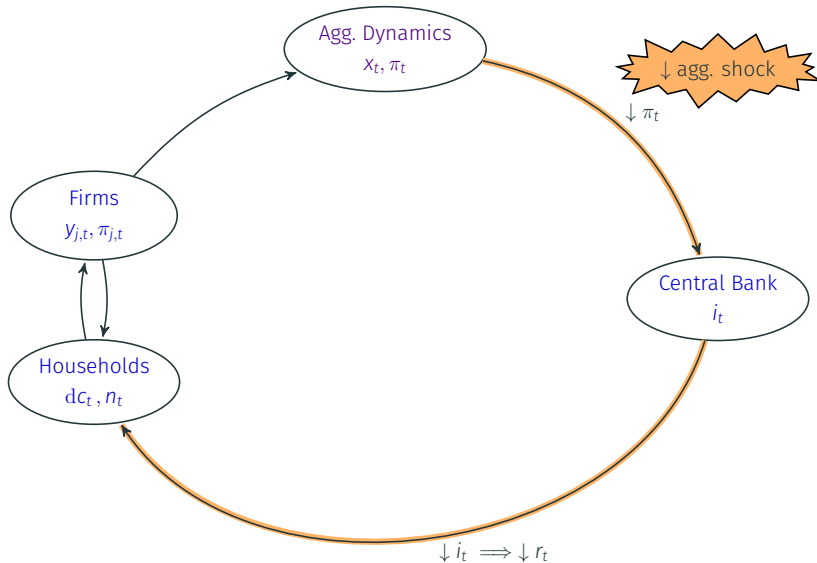
# Equilibrium and Welfare Illustration: Standard Model



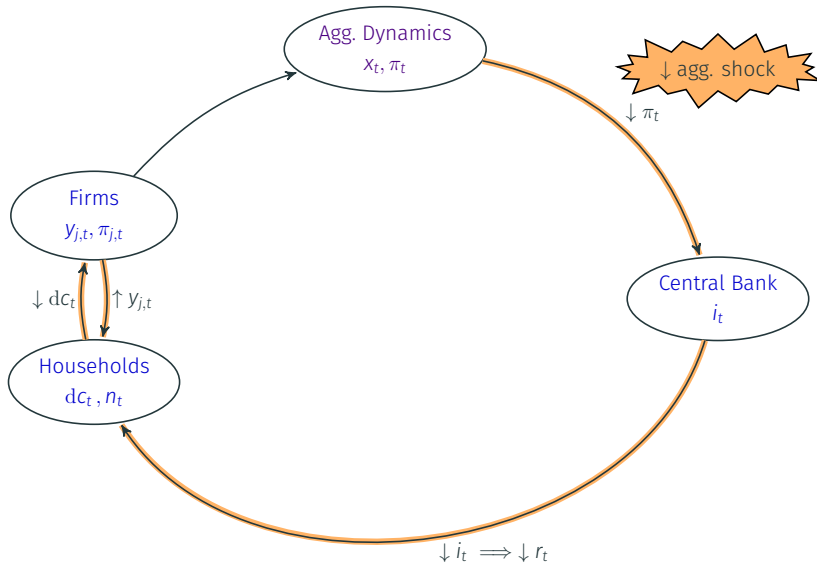
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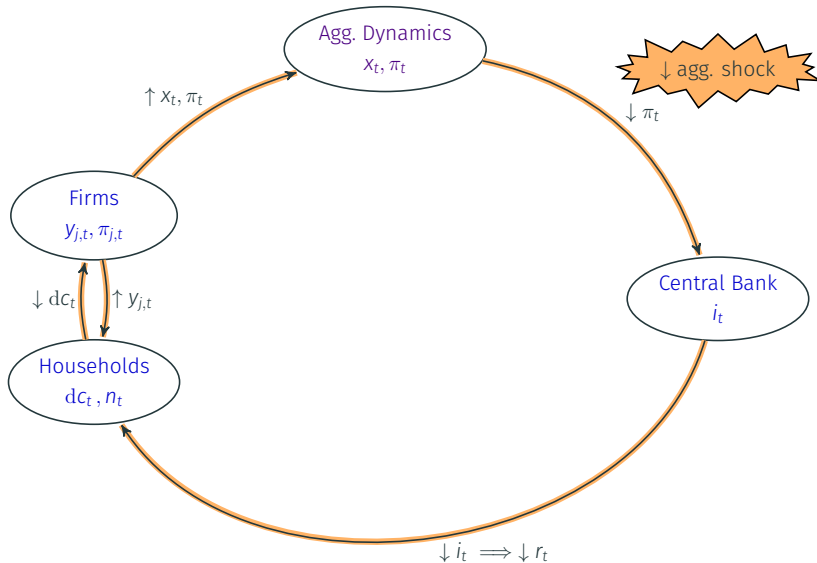


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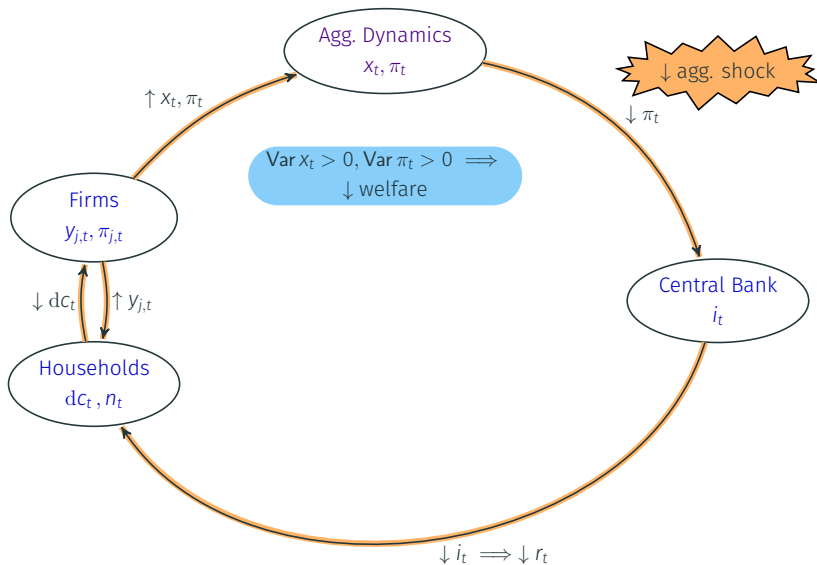




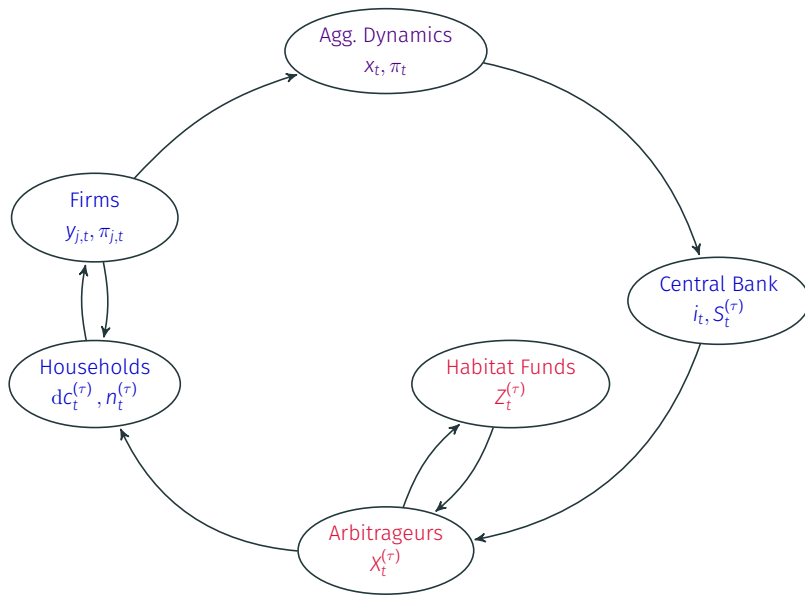
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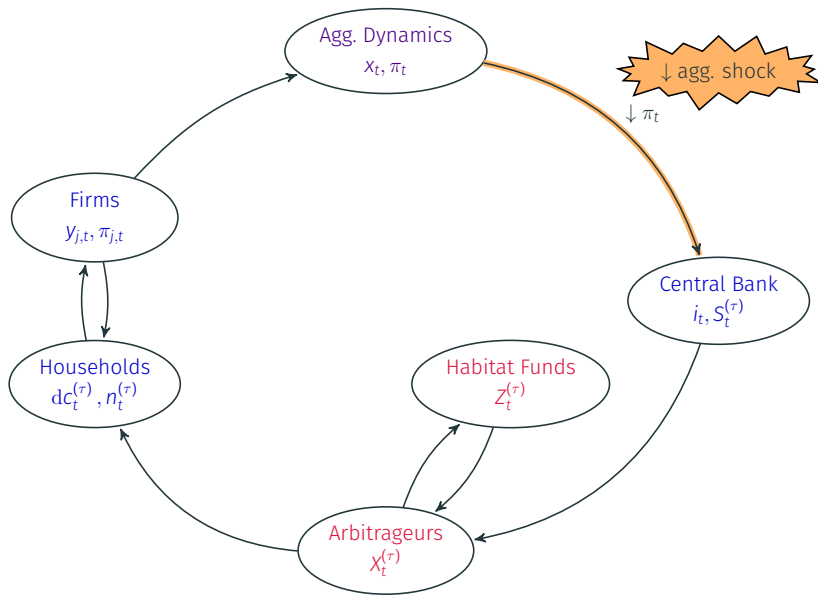
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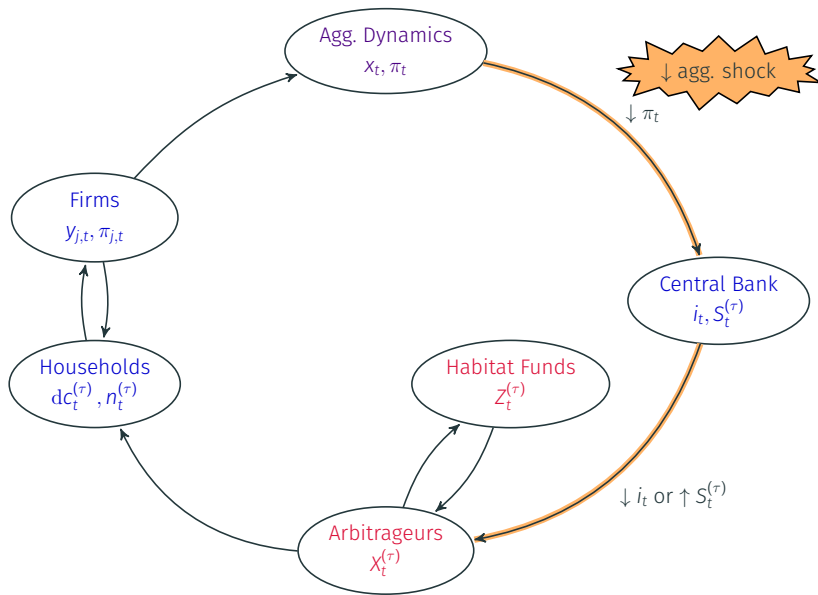
# Equilibrium and Welfare Illustration: Imperfect Arbitrage



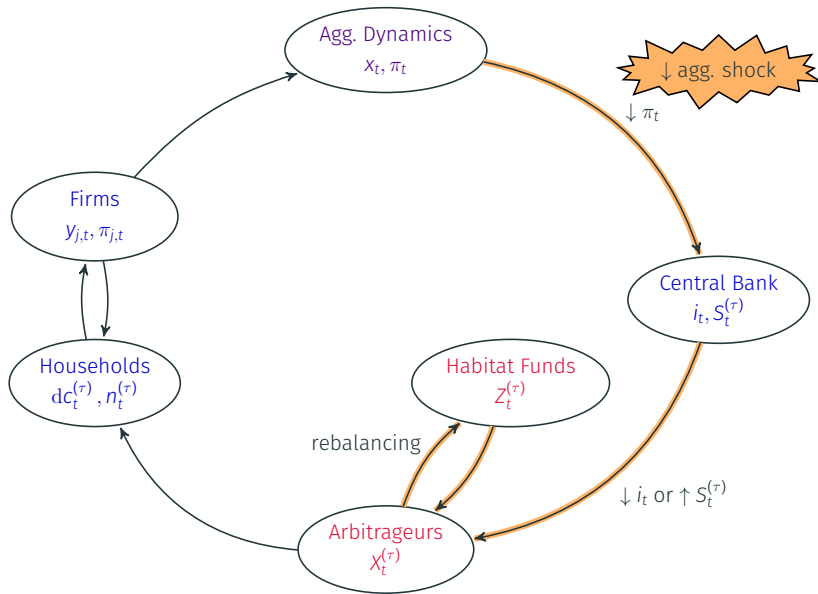
# Equilibrium and Welfare Illustration: Imperfect Arbitrage



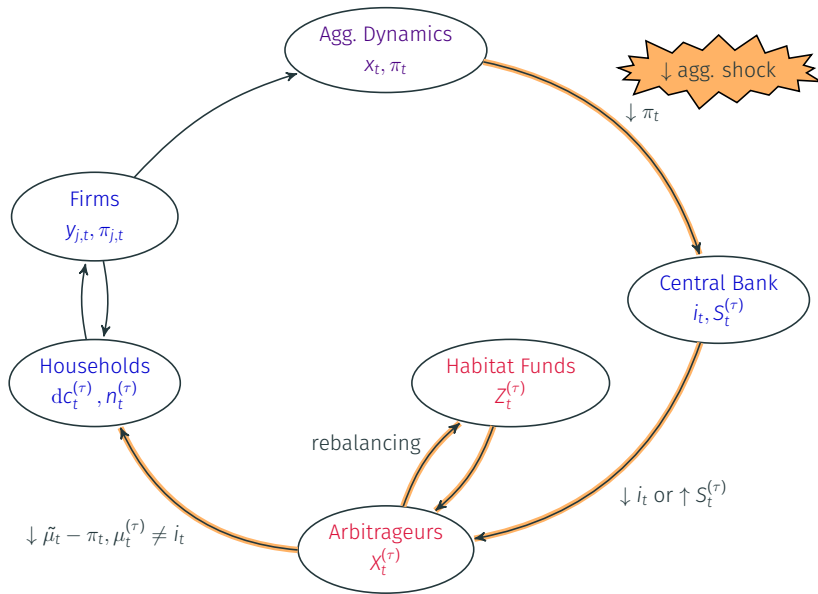
# Equilibrium and Welfare Illustration: Imperfect Arbitrage



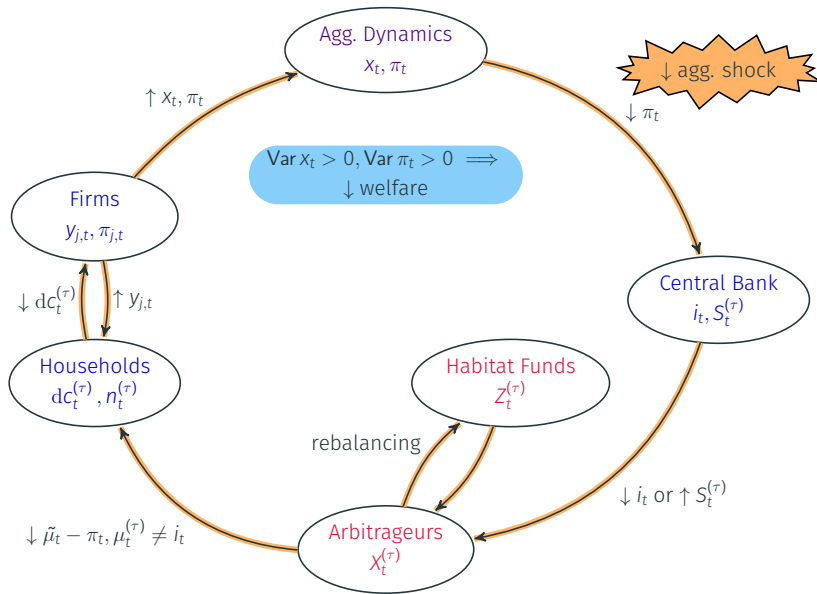
# Equilibrium and Welfare Illustration: Imperfect Arbitrage



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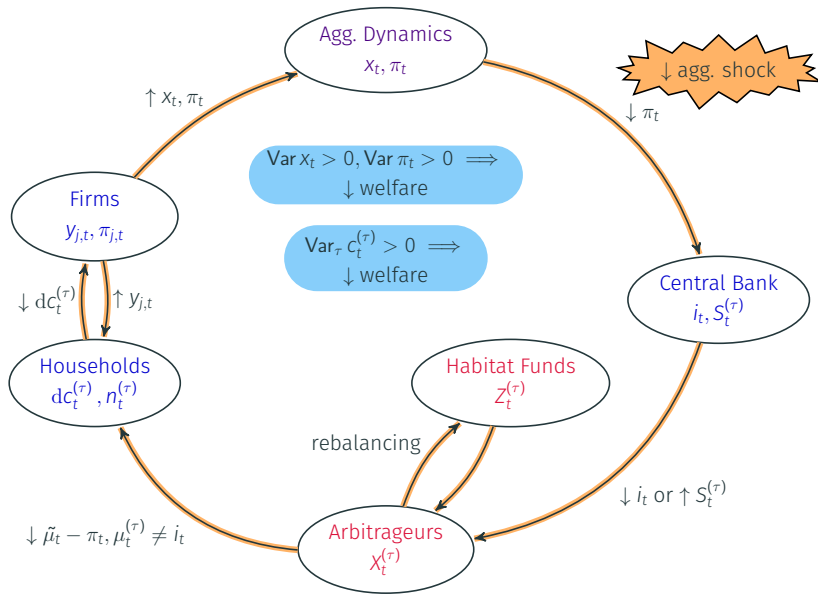


## Equilibrium and Welfare Illustration: Imperfect Arbitrage





# Equilibrium and Welfare Illustration: Imperfect Arbitrage



# Ad-hoc Policy Rule

- In order to better understand the model, simplify to a version of the model which only includes **natural rate shocks**  $r_t^*$
- Consider a **policy rule** which implements

$$i_t = r_t^*$$

- Also consider an ad-hoc **QE shock**:

$$S_t^{(\tau)} \equiv \zeta(\tau) \beta_t^{(QE)}$$
$$d\beta_t^{(QE)} = -\kappa_{QE} \beta_t^{(QE)} dt$$

- We will examine the outcome of these policies in different versions of the model

## Risk Neutral Arbitrageur

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## Benchmark: Risk Neutral Arbitrageur (“Standard Model”)

- Consider the benchmark case of a risk neutral arbitrageur:  $\gamma = 0$
- The **expectations hypothesis** holds:

$$\mu_t^{(\tau)} = i_t = r_t^*$$

- $\implies$  model collapses to a **standard RANK model** and so

$$\text{Var}_\tau c_t^{(\tau)} = 0$$

- Recover the standard **QE neutrality result**: balance sheet policies do not affect bond prices (and therefore have no aggregate effects)
- **Divine coincidence holds**: conventional policy can achieve first-best  $x_t = \pi_t = 0$ 
  - With the addition of cost-push shocks, instead face an output-inflation trade-off
- **‘Woodford-ian’ equivalence**: the role of policy on aggregate dynamics and welfare is fully summarized by policy rate  $i_t$

# Imperfect Arbitrage

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# Imperfect Arbitrage

- Now assume  $\gamma > 0$  and the central bank continues to implement  $i_t = r_t^*$

## Proposition (Imperfect Arbitrage and Ad-hoc Policy)

Assume risk aversion  $\gamma > 0$  and price elasticities  $\alpha(\tau) > 0$

Bond markets: bond carry trade return  $\mu_t^{(\tau)} - i_t$

- Decreases with the short rate  $i_t$
- Decreases with QE shocks  $\beta_t^{(QE)}$

Aggregate dynamics: output gaps  $x_t$  and inflation  $\pi_t$

- Not identically zero:  $\text{Var } x_t \neq 0$  and inflation  $\text{Var } \pi_t \neq 0$ ;
- QE increases the output gap and inflation

Dispersion: consumption dispersion  $\text{Var}_\tau c_t^{(\tau)} \neq 0$

# Imperfect Arbitrage Intuition: Policy Pass-Through

- Consider a fall in the natural rate inducing a cut in the policy rate:
  - When  $\downarrow i_t$ , bond arbitrageurs want to invest more in the BCT
  - $\implies$  bond prices increase  $\uparrow P_t^{(\tau)}$
  - As  $\uparrow P_t^{(\tau)}$ , price-elastic habitat bond investors ( $\alpha(\tau) > 0$ ) reduce their holdings:  $\downarrow Z_t^{(\tau)}$
  - Bond arbitrageurs increase their holdings  $\uparrow X_t^{(\tau)}$ , which requires a larger BCT return
- Now consider a QE shock
  - QE purchases:  $\uparrow S_t^{(\tau)}$
  - Bond arbitrageurs reduce holdings  $\downarrow X_t^{(\tau)}$ , reducing risk exposure and pushing down yields

# Imperfect Arbitrage Intuition: Aggregate Effects

- Therefore, a shock to the natural rate does not fully pass through to the effective borrowing rate  $\tilde{\mu}_t \neq i_t$ 
  - Thus aggregate borrowing demand changes, and hence  $x_t \neq 0$
  - Through the NKPC,  $\pi_t \neq 0$
- On the other hand, a QE shock stimulates the economy
  - QE reduces borrowing rates  $\downarrow \tilde{\mu}_t$  and therefore stimulates aggregate consumption  $\uparrow x_t$
  - Through the NKPC, inflation  $\uparrow \pi_t$
- Additionally, in general  $\mu_t^{(\tau)} \neq \mu_t^{(\tau')}$ 
  - Hence individual Euler equations differ
  - $\implies c_t^{(\tau)} \neq c_t^{(\tau')}$  and therefore  $\text{Var}_\tau c_t^{(\tau)} \neq 0$



## Optimal Policy

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# Imperfect Arbitrage and Macroeconomic Stabilization

- Can conventional policy alone close the output gap?
- Yes but the short rate must react **more than one-for-one** with the natural rate:

$$i_t = \chi_i r_t^*, \quad \chi_i > 1$$

- The parameter  $\chi_i$  can be chosen so that

$$\tilde{\mu}_t = r_t^*$$

- However, this does not achieve first-best since  $\text{Var}_\tau c_t^{(\tau)} \neq 0$
- In fact, relative to the policy  $i_t = r_t^*$ , in general we have  $\uparrow \text{Var}_\tau c_t^{(\tau)}$ 
  - Short rate is **more volatile**, hence  $\uparrow$  **term premia volatility**
  - This implies **higher dispersion across borrowing rates**  $\mu_t^{(\tau)}$  and therefore an increase in consumption dispersion

# Imperfect Arbitrage and Macro-Financial Stabilization

- If the central bank also utilizes [balance sheet policies](#), we obtain the following

## Proposition (Optimal Policy Separation Principle)

Assume risk aversion  $\gamma > 0$  and price elasticities  $\alpha(\tau) > 0$

Suppose the central bank implements short rate and balance sheet policy according to

$$\begin{aligned} i_t &= r_t^* \\ S_t^{(\tau)} &= \alpha(\tau) \log P_t^{(\tau)} + \theta(\tau) \bar{\beta} \end{aligned}$$

Then first-best is achieved:

- **Macroeconomic stabilization:**  $x_t = \pi_t = 0 \ \forall t$
- **Financial stabilization:**  $\mu_t^{(\tau)} = \tilde{\mu}_t \ \forall \tau$
- **Consumption equalization:**  $c_t^{(\tau)} = c_t^{(\tau')} \ \forall \tau, \tau'$  and hence  $\text{Var}_\tau c_t^{(\tau)} = 0 \ \forall t$

# Optimal Policy Separation Principle: Intuition

- The results follow naturally from our findings regarding ad-hoc policy
- QE policy **stabilizes shocks to bond markets** by offsetting all habitat portfolio rebalancing shocks:

$$S_t^{(\tau)} = -Z_t^{(\tau)} \implies \sigma_t^{(\tau)} \Lambda_t = 0$$

- This **equalizes borrowing rates** across HHs:  $\mu_t^{(\tau)} = \tilde{\mu}_t$
- Hence the model collapses to a standard RANK model, in which case divine coincidence implies  $i_t = r_t^*$  is optimal

**Separation principle** for optimal policy:

- Optimal balance sheet policy **stabilizes financial markets**
- Optimal short rate policy **stabilizes macroeconomic aggregates**

# Constrained Optimal Policy

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# Financial Stabilization Policy with Short Rate Constraints

- Suppose that **short rate policy is constrained** and so cannot implement the policy derived above
  - Note: we do not model an explicit ZLB as the non-linearities make solving for equilibrium in bond markets much more tedious
  - Instead, assume that the short rate in equilibrium evolves according to

$$i_t = \chi_i r_t^*, \quad 0 < \chi_i < 1$$

- If the central bank continues to implement the balance sheet policy derived above, then **borrowing rates are still equalized**  $\mu_t^{(\tau)} = \tilde{\mu}_t$
- However,  $\tilde{\mu}_t \neq r_t^*$  and hence this policy does not achieve **macroeconomic stabilization**

$$x_t \neq 0, \pi_t \neq 0$$

# Macroeconomic Stabilization with Short Rate Constraints

- Can balance sheet policy alone close the output gap?
- Yes but must sacrifice equalizing borrowing rates:

$$\mu_t^{(\tau)} = i_t + \sigma_t^{(\tau)} \lambda_t$$

$$\lambda_t \equiv \gamma \int_0^T \left[ \alpha(\tau) \log P_t^{(\tau)} + \theta(\tau) \bar{\beta} - S_t^{(\tau)} \right] \sigma_t^{(\tau)} d\tau$$

- Hence, can always choose  $\{S_t^{(\tau)}\}$  such that

$$\lambda_t^* = \frac{r_t^* - i_t}{\int_0^T \eta(\tau) \sigma_t^{(\tau)} d\tau} \implies \tilde{\mu}_t = r_t^*$$

- However, this necessitates

$$\mu_t^{(\tau)} = i_t + \sigma_t^{(\tau)} \left( \frac{r_t^* - i_t}{\int_0^T \eta(\tau) \sigma_t^{(\tau)} d\tau} \right) \neq r_t^* \quad (\text{unless } i_t \neq r_t^*)$$

# Stabilization with Short Rate Constraints: Intuition

- Balance sheet policy works by affecting **term premia** through changes in the **market price of risk**
- Although arbitrage is imperfect in this model, arbitrageurs still enforce **tight restrictions** between between market price of risk and term premia across maturities
- Hence, while in principle the central bank has a **continuum of policy tools**  $\{s_t^{(\tau)}\}$ , in practice it can **only manipulate**  $\lambda_t$
- Related to **localization results** in Vayanos & Vila (2021) and Ray, Droste, & Gorodnichenko (2023)
  - In the one-factor model considered here, the effects of QE are **fully global**
  - Even with more complicated risk structure, localization is not strong enough to allow balance sheet policy alone to achieve first-best



## Extensions

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## Extensions: “Noise” Demand Shocks

- We obtain identical results when allowing for shocks to habitat demand  $\beta_t^{(\tau)}$
- Optimal separation principle still holds, but optimal QE must be more reactive:

$$S_t^{(\tau)} = \alpha(\tau) \log P_t^{(\tau)} + \beta_t^{(\tau)}$$

- Optimal short rate policy still implements  $i_t = r_t^*$
- **Additional result:** if noise demand dynamics are such that  $\uparrow\uparrow \beta_t^{(\tau)}$  in response to  $\uparrow r_t^*$ , then it is optimal to **expand** the balance sheet  $\uparrow S_t^{(\tau)}$  while hiking rates  $\uparrow i_t$
- **Intuition:**
  - Suppose during a **hiking cycle** and in the absence of QE we have an **increase in term premia**
  - Then the optimal balance sheet policy is to conduct **additional QE purchases** in order to offset spike in term premia
  - $\implies$  conventional and unconventional policy **seem to be at odds** with one another
  - Otherwise, short rate policy and balance sheet policy tend to be reinforcing

## Extensions: Cost-Push Shocks

- What if divine coincidence does not hold? **Cost-push shocks**:

$$d\pi_t = (\rho\pi_t - \delta x_t - v_t) dt$$

- Unfortunately, our **separation principle still holds**:
  - Optimal QE stabilizes term premia
  - Short rate policy must contend with the output gap/inflation trade-offs
- **Intuition**: despite multiple policy instruments, (un)conventional policy only affects aggregate outcomes through changes in **effective borrowing rate**  $\tilde{\mu}_t$ 
  - Take any feasible path  $\{x_t, \pi_t, \tilde{\mu}_t\}_t$  from an implementation implying policies  $\{\tilde{i}_t, \tilde{S}_t^{(\tau)}\}_t$
  - Can also be achieved with  $i_t = \tilde{\mu}_t, S_t^{(\tau)} = \alpha(\tau) \log P_t^{(\tau)} + \beta_t^{(\tau)}$
  - This guarantees  $\text{Var}_\tau c_t^{(\tau)} = 0$  and hence strictly dominates

## Extensions: Non-Zero First-Best Term Premia

- Our approximation approach implies that in the first-best, term premia are zero
- This simplifies our analytical results but of course is an extreme assumption
- Suppose instead that **first-best** term premia are  $\nu^{(\tau)} \neq 0$
- Our **separation principle still holds** when  $\nu^{(\tau)}$  is achievable but optimal short rate policy is a function of first-best term premia
- **Intuition:** combination of previous results
  - Aggregate outcomes through changes in **effective borrowing rate**  $\tilde{\mu}_t$  (as before)
  - Optimal QE policy guarantees  $\mu_t^{(\tau)} - i_t \equiv \nu^{(\tau)}$  and hence  $\tilde{\mu}_t = i_t + \int_0^T \eta(\tau) \nu^{(\tau)} d\tau \equiv i_t + \tilde{\nu}$
  - Thus, optimal short rate policy implements  $i_t = r_t^* - \tilde{\nu}$
  - Note: if first-best term premia are not achievable, optimal policy is more complicated

# Measuring Balance Sheet Objectives: Return Predictability

- Fama-Bliss regression:

$$\frac{1}{\Delta\tau} \log\left(\frac{P_{t+\Delta\tau}^{(\tau-\Delta\tau)}}{P_t^{(\tau)}}\right) - y_t^{(\Delta\tau)} = a_{FB}^{(\tau)} + b_{FB}^{(\tau)} \left(f_t^{(\tau-\Delta\tau, \tau)} - y_t^{(\Delta\tau)}\right) + \varepsilon_{t+\Delta\tau}$$

- Measures how the slope of the term structure predicts excess returns
- In our model, when the central bank does not use balance sheet policies:

$$\bar{b}_{FB}^{(\tau)} > 0$$

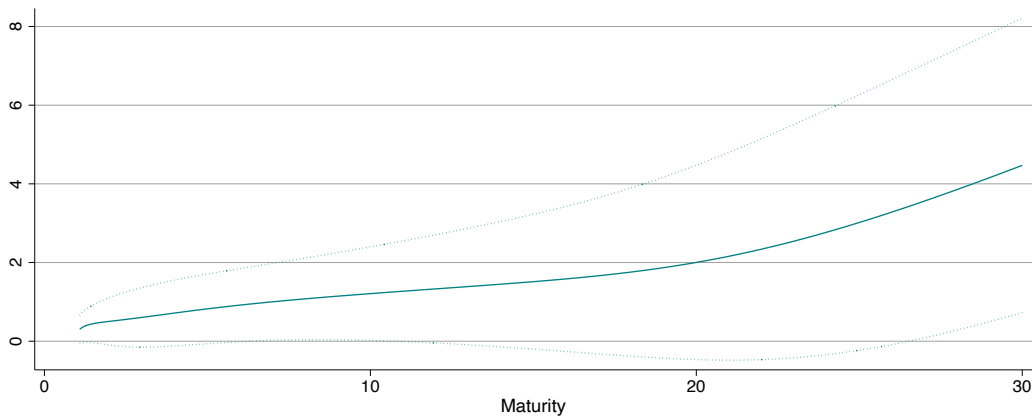
- If balance sheet policy is pursuing **financial stabilization**:

$$\bar{b}_{FB}^{(\tau)} > b_{FB}^{(\tau)} \rightarrow 0$$

- Instead, if balance sheet policy is pursuing **macroeconomic stabilization**:

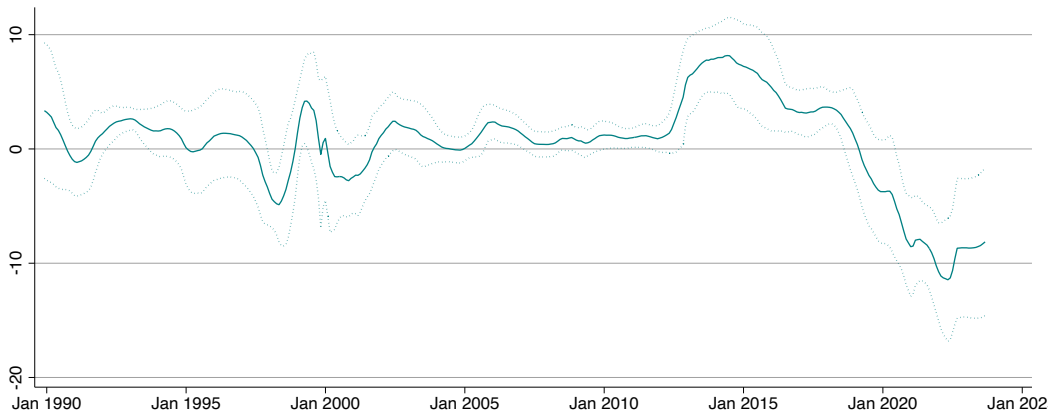
$$b_{FB}^{(\tau)} > \bar{b}_{FB}^{(\tau)}$$

## Fama-Bliss Coefficients: Treasuries, Full Sample



FB coefficients are non-zero (and increasing across maturities)

## Fama-Bliss Coefficients: 10-year Treasuries, Rolling Sample



FB coefficients **increase during initial QE regime**, but have fallen and even become **negative** in recent years

## Concluding Remarks

- This paper develops a tractable general equilibrium model with market segmentation, nominal frictions, and household heterogeneity
- We find that optimal short rate and balance sheet policy is characterized by a sharp **optimal separation result**:
  - Conventional policy targets **macroeconomic stability**
  - Unconventional policy targets **financial stability**
- Optimal policy removes excess volatility of risk premia and hence improves risk-sharing across households, while reducing excess macroeconomic volatility
  - Policy constraints on either the short rate or balance sheets imply trade-offs between these two policy objectives
- When considering social welfare, **cannot abstract from the policy tools** used to conduct monetary policy



Thank You!

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# Aggregation Details I

- Aggregating across HH members:

$$C = \int \eta(\tau) C^{(\tau)} d\tau, \quad N = \int \eta(\tau) N^{(\tau)} d\tau, \quad A = \int \eta(\tau) A^{(\tau)} d\tau, \quad a = \int \eta(\tau) a^{(\tau)} d\tau$$

- Hence, aggregate HH real wealth evolves:

$$da = [wN - C] dt + a \left( \int \eta(\tau) \frac{dP^{(\tau)}}{P^{(\tau)}} d\tau - \pi dt \right) + \frac{1}{P} dF$$

- Formally,  $\tau$  HHs borrow through the relevant  $\tau$ -habitat fund  $\implies$  budget constraint

$$dW^{(\tau)} = Z^{(\tau)} \frac{dP^{(\tau)}}{P^{(\tau)}} + \left[ W^{(\tau)} - Z^{(\tau)} + \eta(\tau) A^{(\tau)} \right] i dt - \eta(\tau) A^{(\tau)} \frac{dP^{(\tau)}}{P^{(\tau)}}$$

- Flow budget constraint for the central bank:

$$dW^{(CB)} = W^{CB} i dt + \int S^{(\tau)} \left( \frac{dP^{(\tau)}}{P^{(\tau)}} - i dt \right) d\tau$$

## Aggregation Details II

- Total transfers from arbitrageurs, central bank, and habitat funds to HHs:

$$dW + \int dW^{(\tau)} d\tau + dW^{(CB)} = \left[ W + W^{(CB)} + \int W^{(\tau)} d\tau + A \right] i dt - A \int \eta(\tau) \frac{dP(\tau)}{P(\tau)} d\tau$$

- Follows from market clearing  $\int X^{(\tau)} + Z^{(\tau)} + S^{(\tau)} d\tau = 0$
- Term in brackets is aggregate demand for short-term bonds (reserves):  $B = 0$  in equilibrium
- Output and goods market clearing gives nominal firm profits transferred to HHs:

$$\int_0^1 \mathcal{F}_j dj = PY \left( 1 - \frac{W}{Z} - \frac{\theta}{2} \pi^2 \right) = PC - \mathcal{W} \frac{Y}{Z} = PC - \mathcal{W} N$$

- Hence, aggregate nominal transfers to the HH sector are given by

$$dF = [PC - WN] dt - A \int \eta(\tau) \frac{dP(\tau)}{P(\tau)} d\tau$$
$$\implies dA = 0, \quad da = -a\pi dt = 0 \quad (\text{if } A = 0)$$

## Aggregation Details III

- Finally, the “head of HH” ensures that each member has identical wealth  $A^{(\tau)} \equiv A$
- With  $A^{(\tau)} = A = 0$ , we have that aggregate HH transfers are given by

$$dF = [PC - \mathcal{W}N] dt$$

- Wealth of a  $\tau$  member in equilibrium is given by

$$dA^{(\tau)} = [\mathcal{W}N^{(\tau)} - PC^{(\tau)}] + dF^{(\tau)}$$

- Hence, the head of HH redistributes wealth according to

$$\begin{aligned} dF^{(\tau)} &= [PC^{(\tau)} - \mathcal{W}N^{(\tau)}] dt \\ \implies dF &= \int \eta(\tau) dF^{(\tau)} d\tau \end{aligned}$$

- Note: recall that there is a mass  $\eta(\tau)$  of each  $\tau$ -HH type; while transfers depend on  $\tau$ , each  $\tau$  member takes these as given

# Equilibrium General Characterization I

- Collect all state variables  $\mathbf{y}_t$  and jump variables  $\mathbf{x}_t$  into a vector  $\mathbf{Y}_t$
- Assume the central bank implements policy which in equilibrium satisfies

$$i_t = \boldsymbol{\chi}_i^\top \mathbf{y}_t$$
$$S_t^{(\tau)} = \boldsymbol{\zeta}(\tau)^\top \mathbf{y}_t$$

- Then (assuming determinacy conditions hold), the first-order approximation described above implies the unique REE

$$d\mathbf{Y}_t = -\boldsymbol{\Upsilon} (\mathbf{Y}_t - \bar{\mathbf{Y}}) dt + \mathbf{S} d\mathbf{B}_t$$
$$\implies d\mathbf{y}_t = -\boldsymbol{\Gamma} (\mathbf{y}_t - \bar{\mathbf{y}}) dt + \boldsymbol{\sigma} d\mathbf{B}_t$$
$$\mathbf{x}_t - \bar{\mathbf{x}} = \boldsymbol{\Omega} (\mathbf{y}_t - \bar{\mathbf{y}})$$

- $\boldsymbol{\Gamma}, \boldsymbol{\Omega}$  are functions of the eigen-decomposition of  $\boldsymbol{\Upsilon}$ , which depends endogenously on sensitivity of bond prices to state

## Equilibrium General Characterization II

- Bond prices are (log) affine functions of the state

$$-\log P_t^{(\tau)} = \mathbf{A}(\tau)^\top (\mathbf{y}_t - \bar{\mathbf{y}}) + C(\tau)$$

- Affine coefficients solve the following fixed point

$$\mathbf{A}(\tau) = \int_0^\tau e^{-\mathbf{M}s} \mathrm{d}s \boldsymbol{\chi}_i$$

$$\mathbf{M} = \boldsymbol{\Gamma}^\top - \int_0^T [-\alpha(\tau)\mathbf{A}(\tau) + \boldsymbol{\Theta}(\tau) - \zeta(\tau)] \mathbf{A}(\tau)^\top \mathrm{d}\tau \overline{\gamma \boldsymbol{\Sigma}}$$

- Note:  $\overline{\gamma \boldsymbol{\Sigma}} \neq \mathbf{0}$  in the limiting case described above
- Bond returns are given by

$$\mu_t^{(\tau)} = \hat{\mathbf{A}}(\tau)^\top (\mathbf{y}_t - \bar{\mathbf{y}}) + C'(\tau)$$

$$\begin{aligned} \hat{\mathbf{A}}(\tau) &= \mathbf{A}'(\tau) + \boldsymbol{\Gamma}^\top \mathbf{A}(\tau) \\ &= \boldsymbol{\chi}_i + (\boldsymbol{\Gamma}^\top - \mathbf{M})\mathbf{A}(\tau) \end{aligned}$$

## Equilibrium General Characterization III

- In general, welfare loss can be written

$$\begin{aligned} L_0 &\equiv -\frac{1}{2} \mathbb{E}_0 \int_0^T \eta(\tau) \mathbf{B}(\tau)^\top \left[ \int_0^\infty e^{-\rho t} (\mathbf{y}_t - \bar{\mathbf{y}}) (\mathbf{y}_t - \bar{\mathbf{y}})^\top dt \right] \mathbf{B}(\tau) d\tau \\ &= -\frac{1}{2} \int_0^T \eta(\tau) \mathbf{B}(\tau)^\top \tilde{\boldsymbol{\Sigma}}^\infty \mathbf{B}(\tau) d\tau \end{aligned}$$

- Both the vector functions  $\mathbf{B}(\tau)$  and the long-run discounted variance  $\tilde{\boldsymbol{\Sigma}}^\infty$  terms may depend on policy choices

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