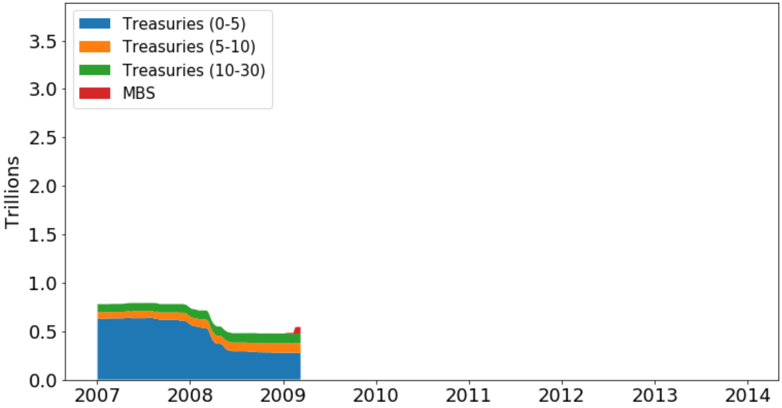
Monetary Policy and the Limits to Arbitrage: Insights from a New Keynesian Preferred Habitat Model

Walker Ray
UC Berkeley

January 15, 2019

Wharton Job Market Seminar

Policy Response to Great Recession



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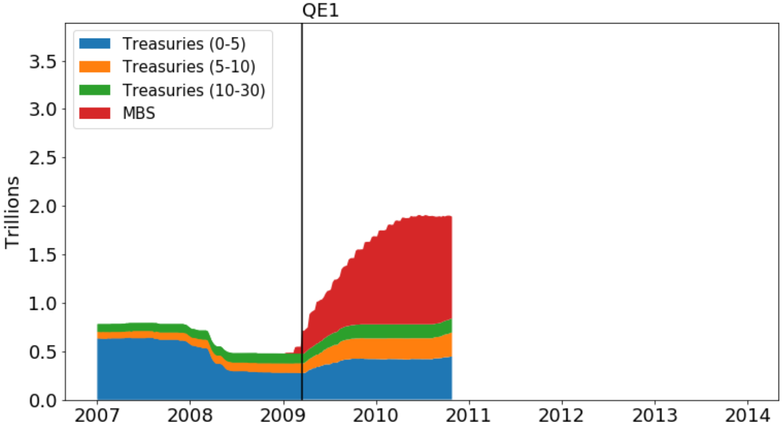

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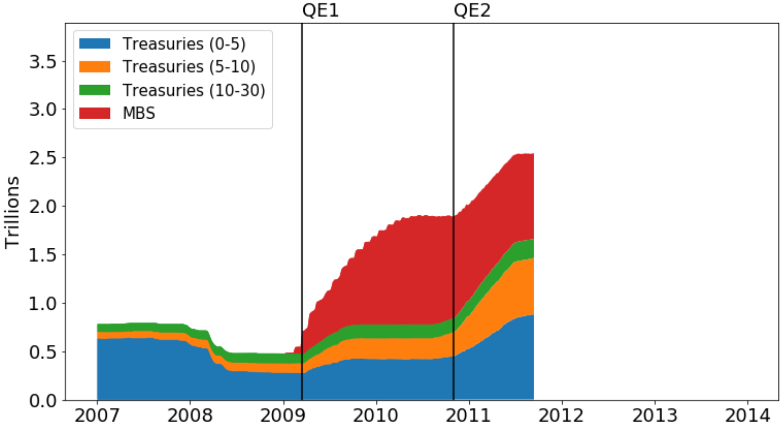

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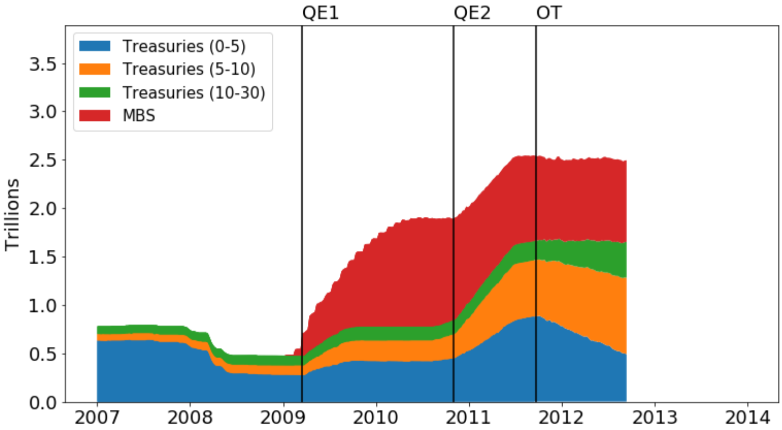

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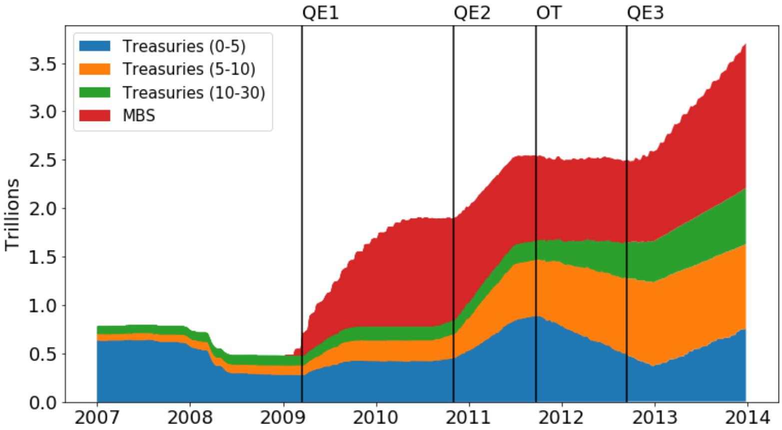

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- Bond market frictions play a role in the transmission of **conventional** monetary policy
- Crucial for designing monetary policy going forward

Model Overview

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- Monetary policy works through both channels

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- Designing policy going forward:
  - Conventional policy: more aggressive in financial crises
  - QE rule can be stabilizing

Literature Contributions

- "Preferred habitat" as a key channel for understanding bond markets
  - D'Amico and King (2013), Hamilton and Wu (2012), Greenwood and Vayanos (2014), Gorodnichenko and Ray (2017), Greenwood and Vissing-Jorgensen (2018)
- Few formal models
  - Vayanos and Vila (2009)
- QE in general equilibrium: Market segmentation vs. forward guidance
  - Gertler and Karadi (2013), Chen et al (2012), Carlstrom et al (2017)
  - Bauer and Rudebusch (2014), Bhattarai et al (2015)
- Frictions and expected future policy
  - McKay et al (2016), Farhi and Werning (2017), Gabaix (2016), Angeletos and Lian (2018)

New Keynesian Preferred Habitat Framework

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- Closing the model: equilibrium term structure determination

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- Arbitrageurs with mean-variance trade-off in wealth:

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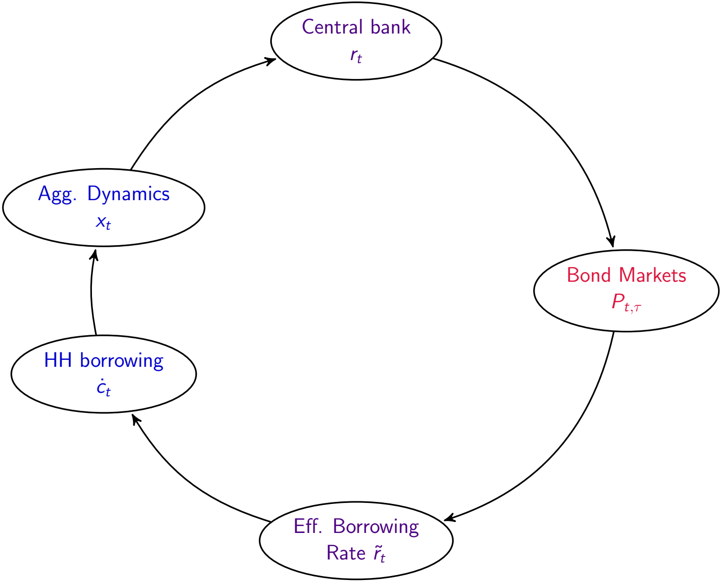
- Market clearing: $b_{t,\tau} = -\tilde{b}_{t,\tau}$

Equilibrium Under Perfect Arbitrage



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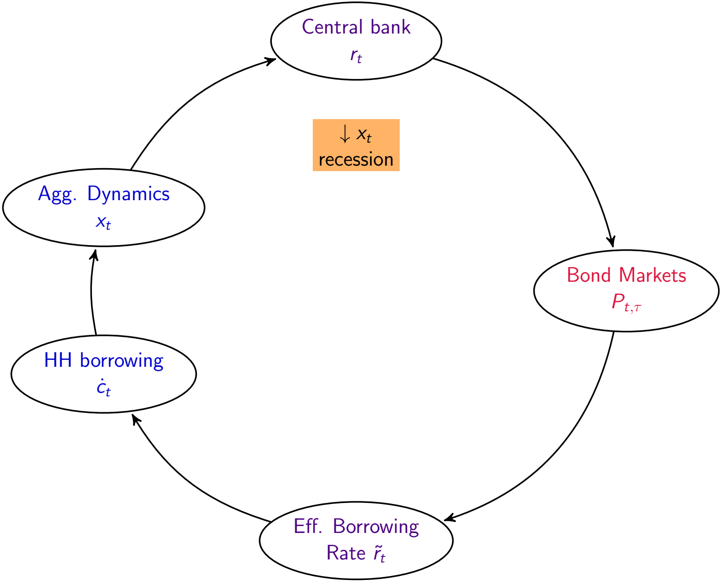

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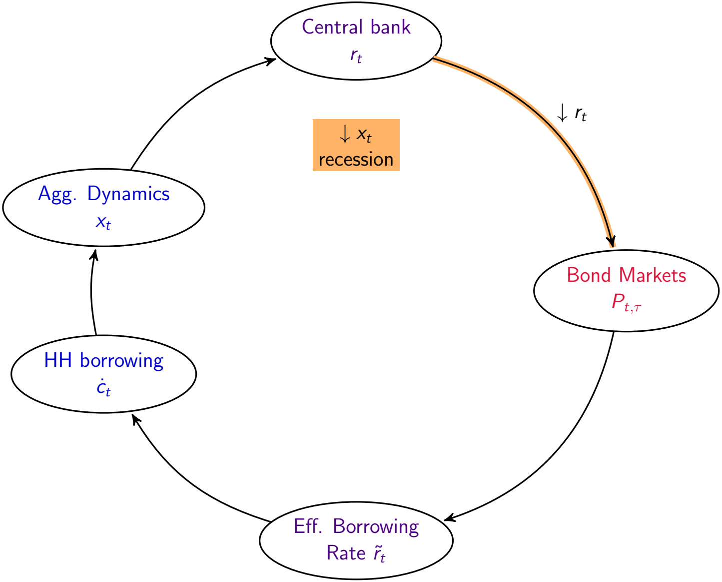

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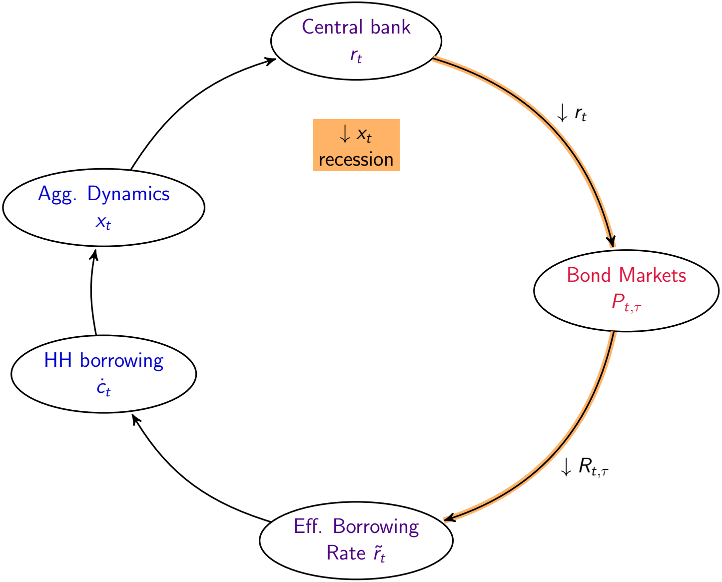

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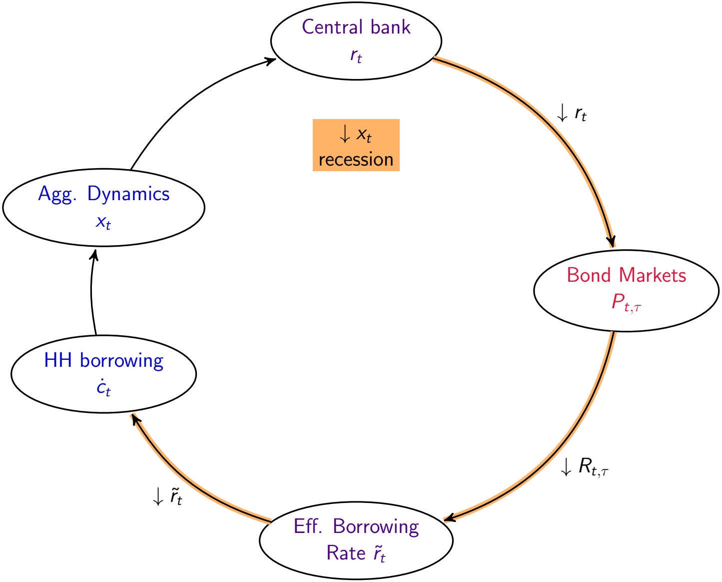

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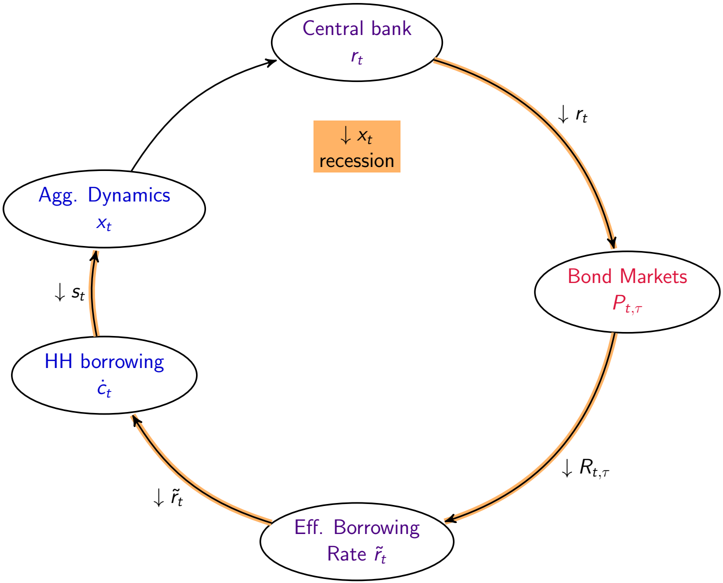

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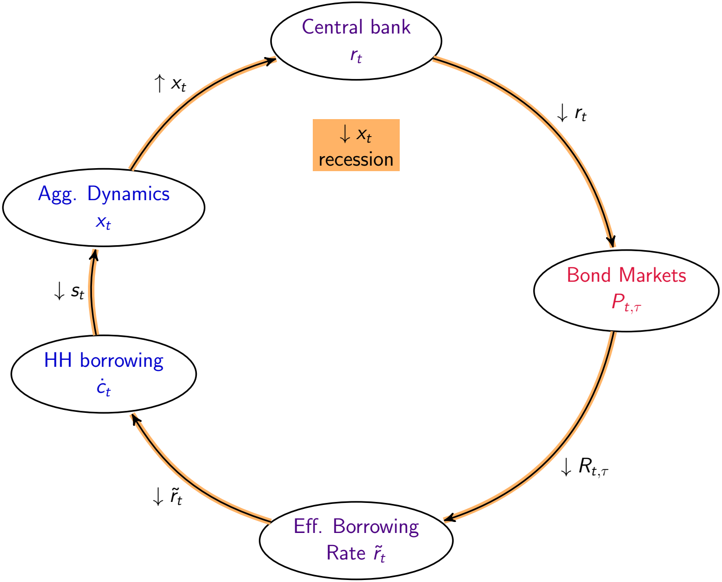

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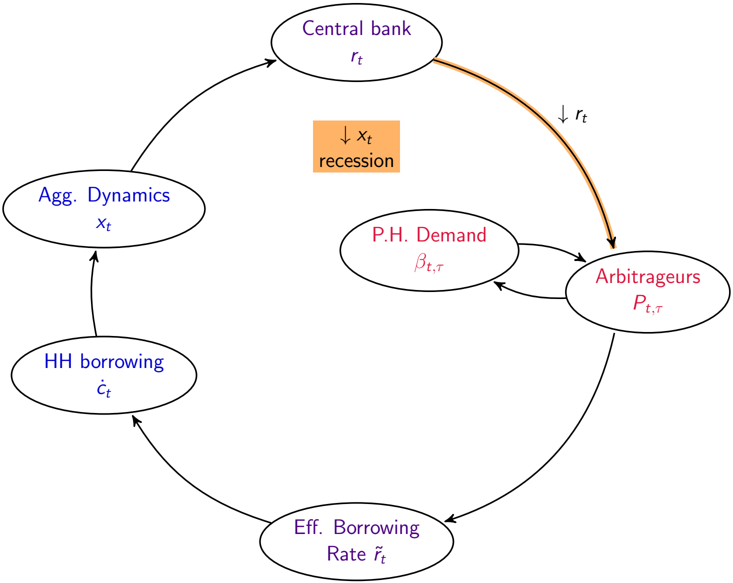

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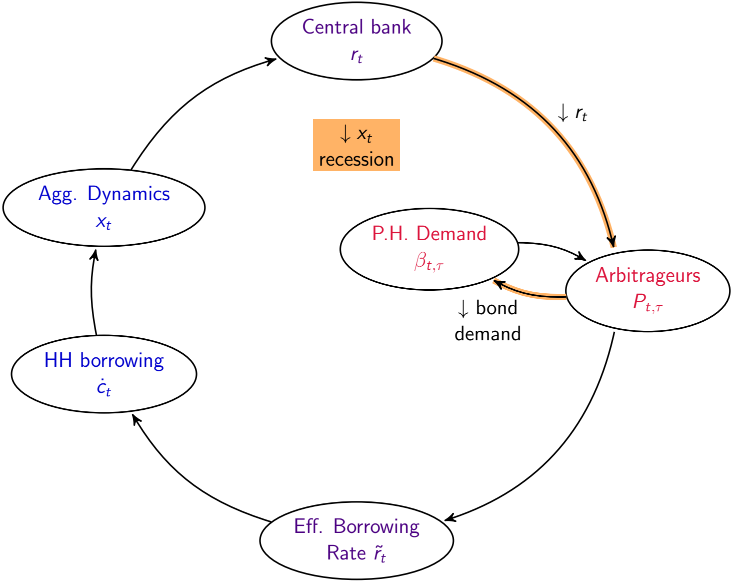

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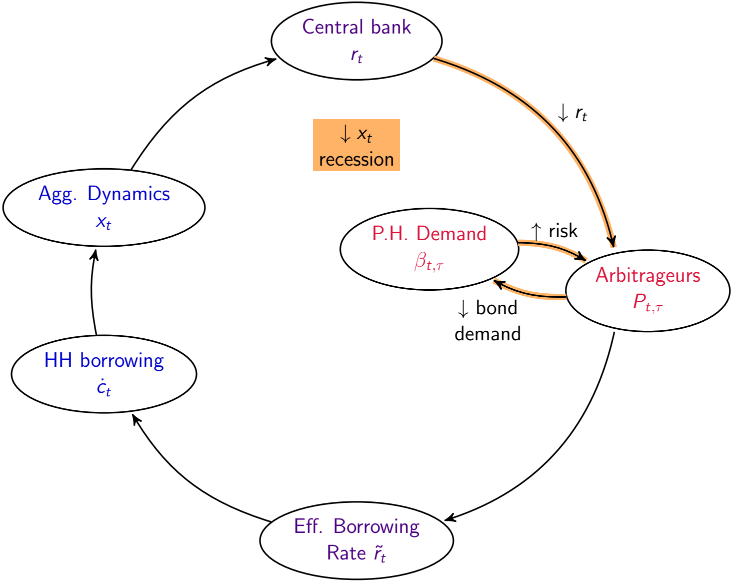

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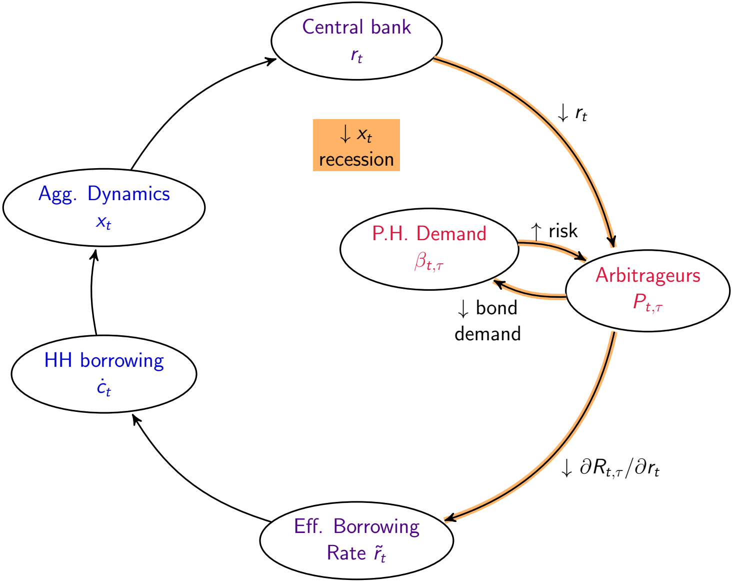

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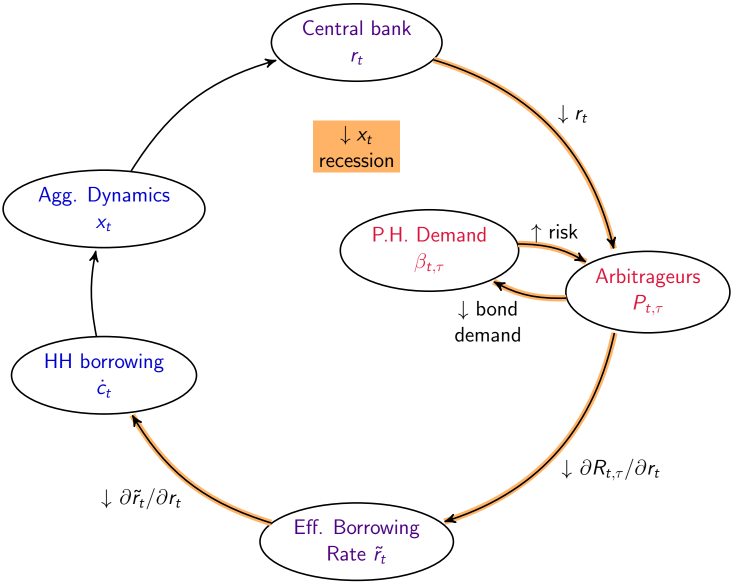

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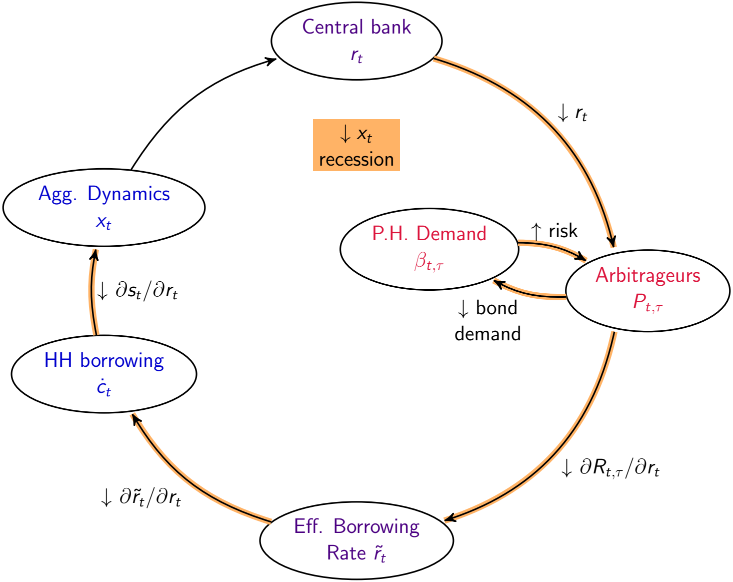

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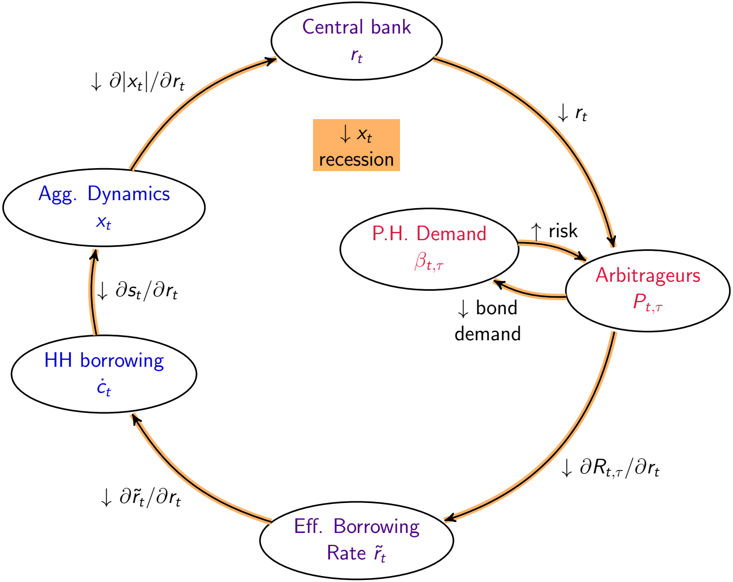

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- Conjecture: affine bond prices

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Rational Expectations Dynamics

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- If determinacy conditions are met:

$$\mathrm{d}r_t = -\lambda_1 (r_t - r^{SS})\, \mathrm{d}t + \sigma_r\, \mathrm{d}B_{r,t}$$
$$x_t = \omega_x (r_t - r^{SS})$$

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$$\mathrm{d}\mathbf{Y}_t = -\Upsilon\left(\mathbf{Y}_t - \mathbf{Y}^{SS}\right)\mathrm{d}t + \mathbf{S}\,\mathrm{d}\mathbf{B}_t$$

$$\Upsilon = \begin{bmatrix} \kappa_r & -\kappa_r\phi_x \\ -\varsigma^{-1}\hat{A}_r & 0 \end{bmatrix}$$

- If determinacy conditions are met:

$$\mathrm{d}r_t = -\lambda_1(r_t - r^{SS})\,\mathrm{d}t + \sigma_r\,\mathrm{d}B_{r,t}$$
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Rational Expectations Equilibrium

Characterizing $\hat{A}_r$

1. Υ has exactly one eigenvalue with positive real part if and only if $\hat{A}_r > 0$. Further, this stable root is real: $\lambda_1 > 0$.
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$$h(\lambda) = \frac{\lambda(\lambda - \kappa_r)}{\varsigma^{-1}\kappa_r\phi_x}$$

3. The output gap dynamics are given by

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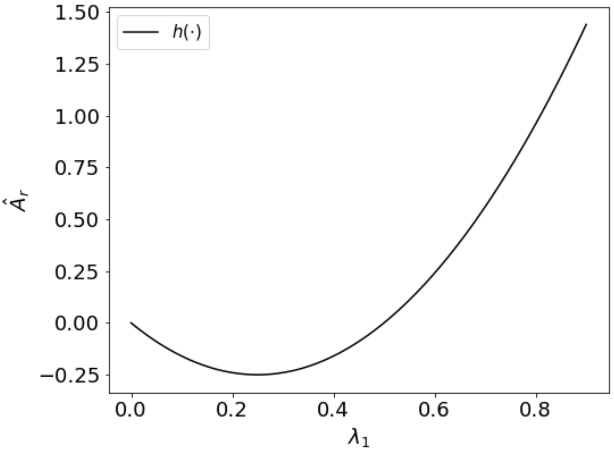
$h(\cdot)$: sensitivity of output growth to the policy rate

Varying Output Growth Sensitivity $h(\cdot)$



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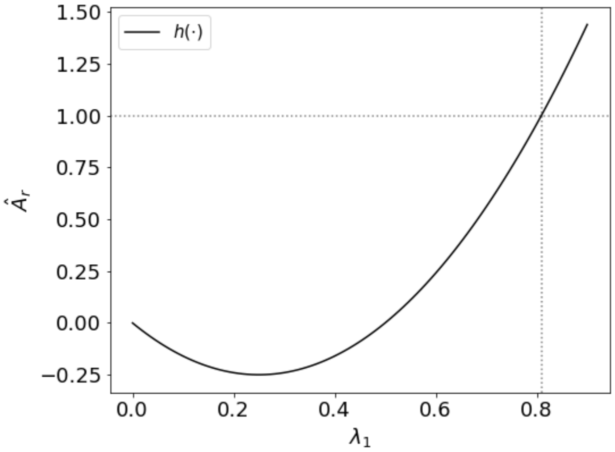

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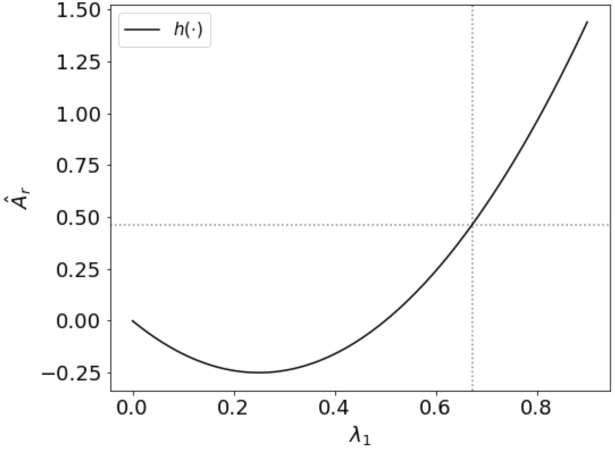

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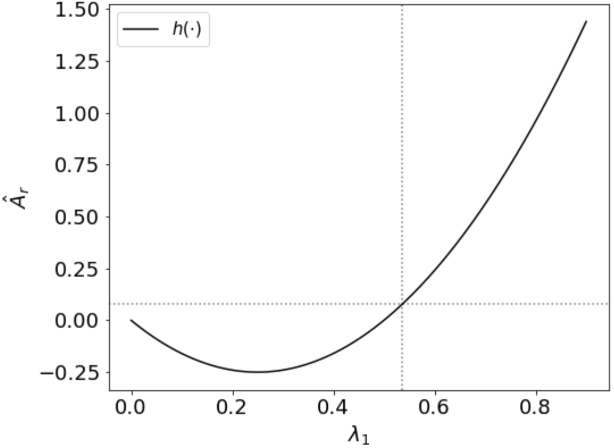

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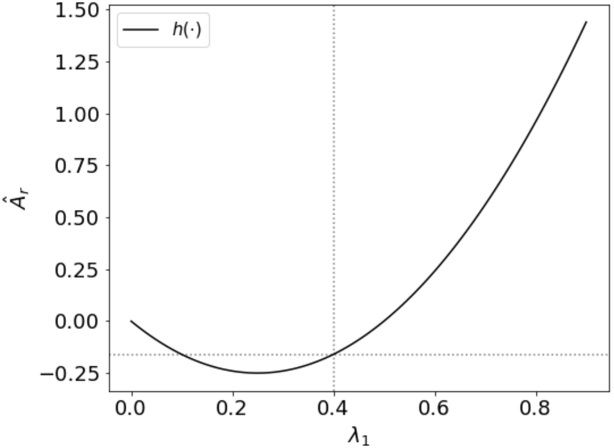

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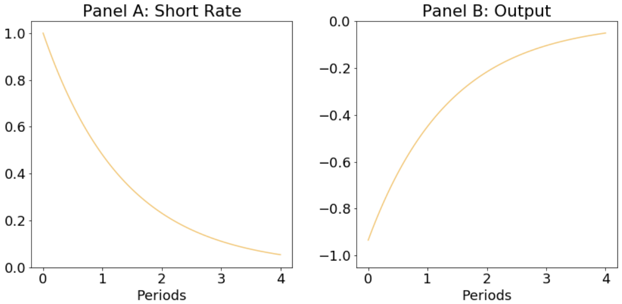
Dynamics and Output Growth Sensitivity



Notes: impulse response functions of the policy rate and output in response to a unit monetary shock, as equilibrium output growth sensitivity to the policy rate falls. Darker lines correspond to lower output growth sensitivity $\hat{A}_r$. phase diagrams

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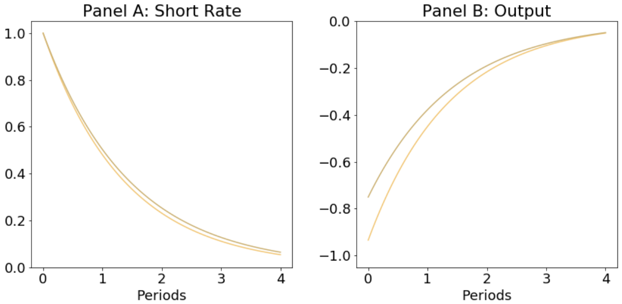

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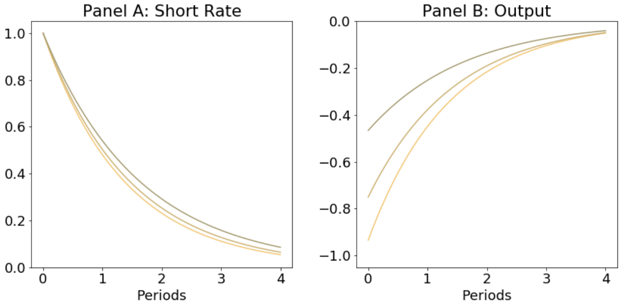

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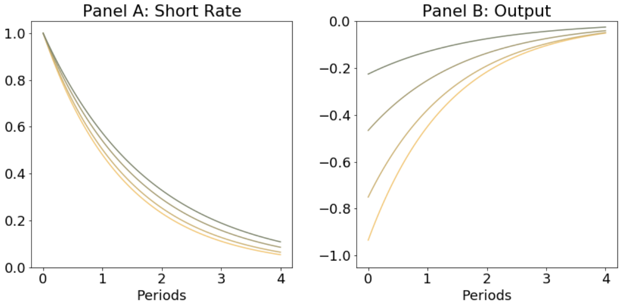

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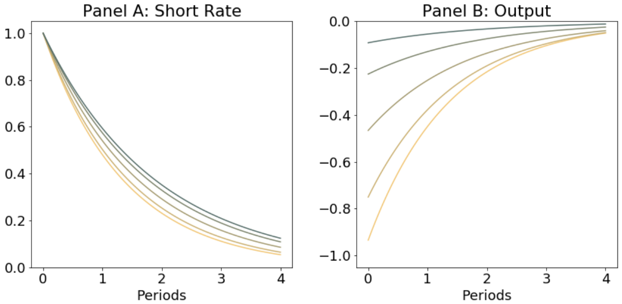

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Absorbing Demand Shocks

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- Prices adjust to balance demand and optimality conditions
- Solution for affine coefficients and risk sensitivity

$$\hat{A}_r \equiv \int_0^T \frac{\eta(\tau)}{\tau} A_r(\tau)\,\mathrm{d}\tau$$

Term Structure Equilibrium

Characterizing $\hat{A}_r$

$\hat{A}_r = g(\lambda_1)$ where $g : \mathbb{R}_+ \to \mathbb{R}$:

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where $f(x) = \frac{1-e^{-x}}{x}$ and

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$g(\cdot)$: maturity-weighted sensitivity of bonds to short rate

ν: risk-adjusted reversion rate

Under-Reaction of Yields

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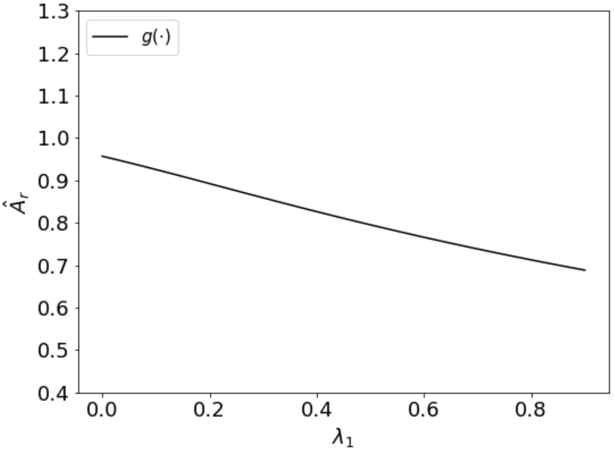
which is the immediate response of τ-bond yields

- EH: two responses should be identical (only when $a = 0$)

Varying Short-Rate Sensitivity $g(\cdot)$



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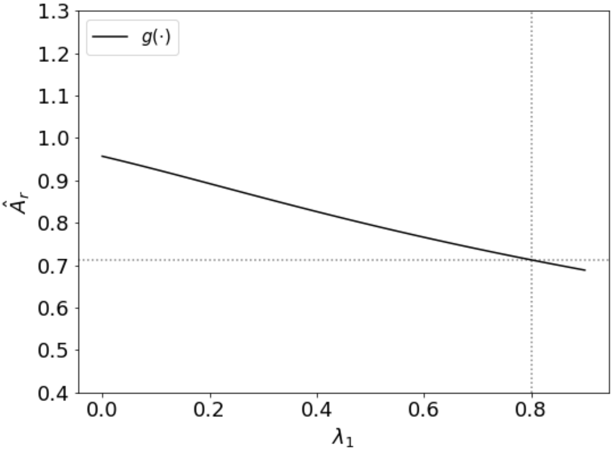

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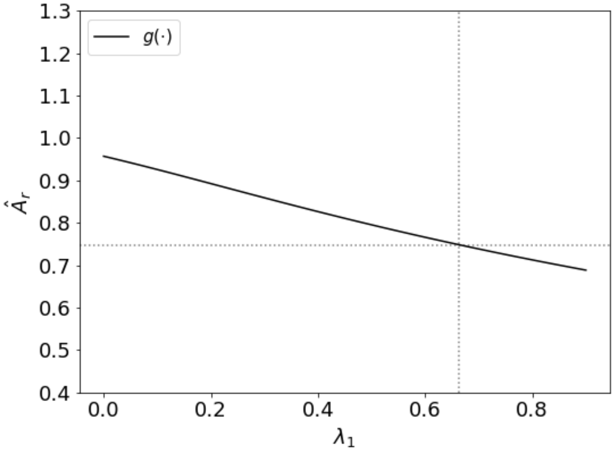

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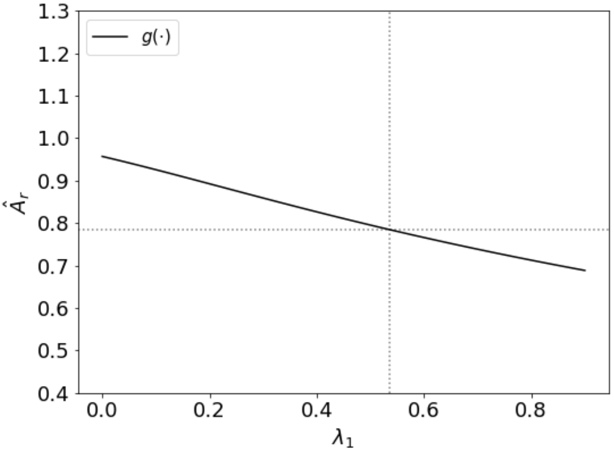

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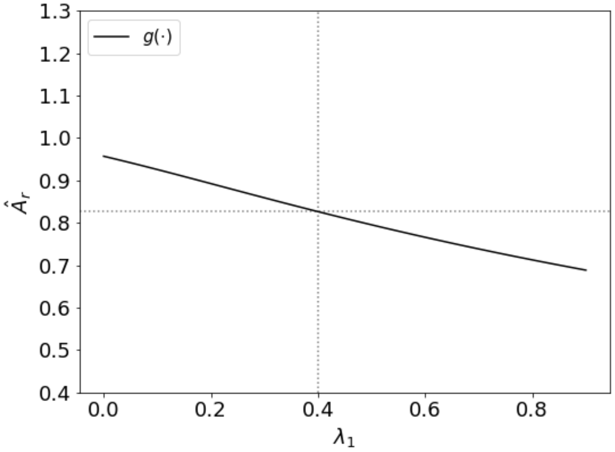

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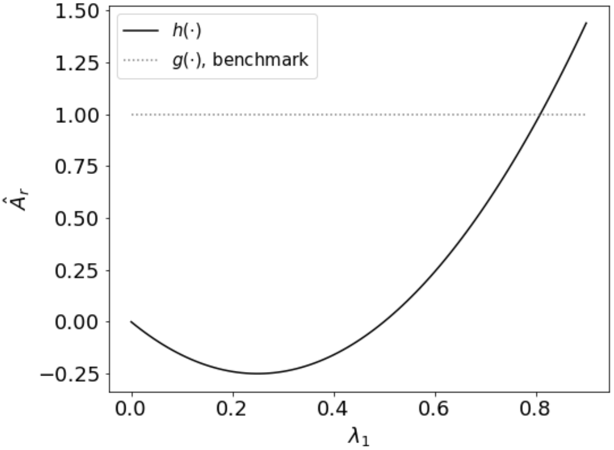
General Equilibrium

Existence and Uniqueness

There exists a unique positive eigenvalue of Υ $\lambda_1 > 0$ for which $g(\lambda_1) = h(\lambda_1)$, which fully characterizes the model equilibrium. Further, this implies $0 < \hat{A}_r < 1$.

Balancing $\hat{A}_r$



Notes: functions $g(\lambda)$ and $h(\lambda)$. The solid black line is $h(\cdot)$, which governs output growth sensitivity to the policy rate. The dashed lines are $g(\cdot)$, which determines the maturity-weighted sensitivity to the short rate (for different risk aversion and weightings).

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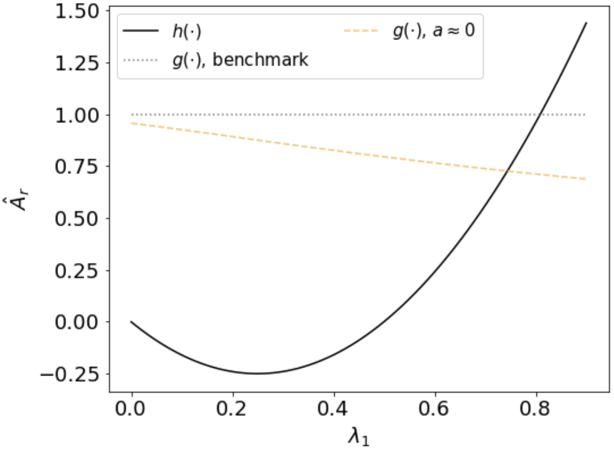

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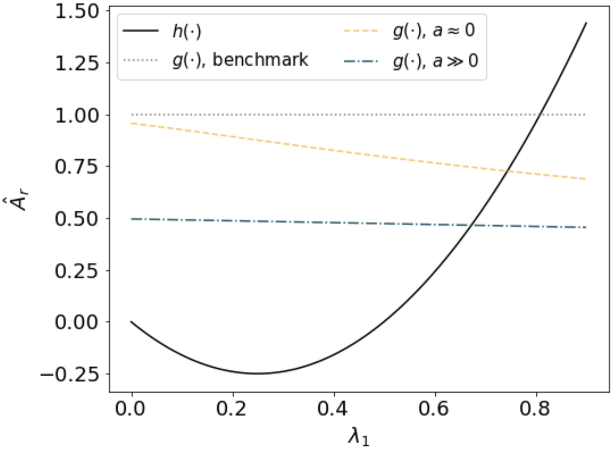

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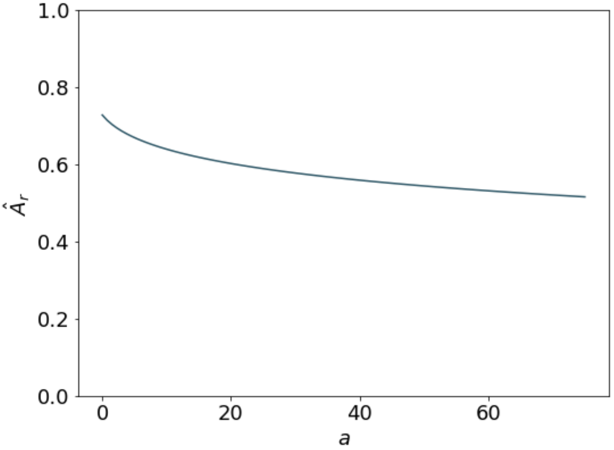
Conventional Policy and Financial Disruptions



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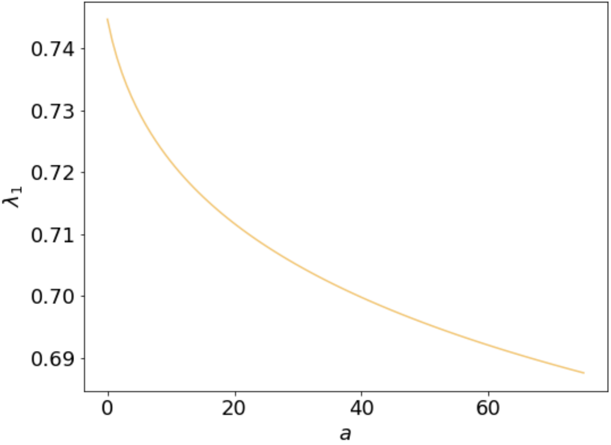

Notes: equilibrium changes in monetary shock reversion λ_1 as risk aversion a increases.

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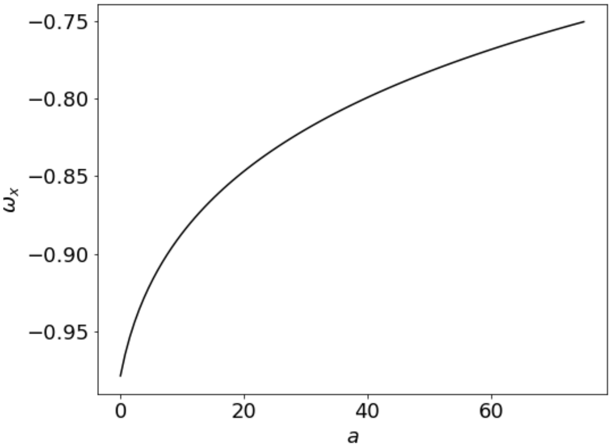

Notes: equilibrium changes in output response ω_x to monetary shocks as risk aversion a increases.

Policy Implications

- More aggressive response to output ϕ_x results
- Higher inertia κ_r results
- Shifts in effective rate weights $\eta(\tau)$ results
- Forward guidance less effective as risk aversion increases details

Modeling LSAPs

- Suppose the central bank directly purchases bonds through open market operations
- Change to the demand shifter in PH demand

$$\tilde{b}_{t,\tau} = \alpha(\tau)\tau(R_{t,\tau} - \beta_{t,\tau})$$

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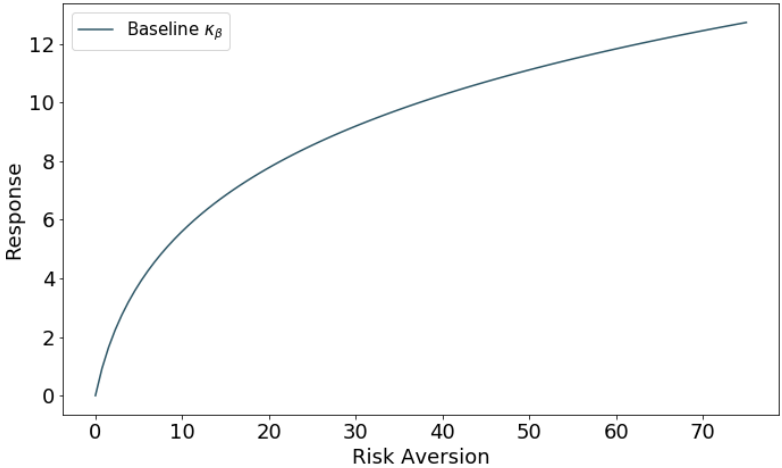
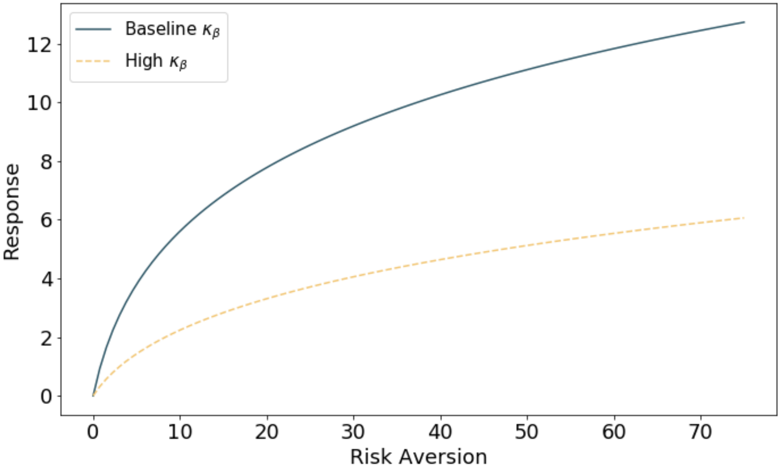
\end{aligned}$$

Output Response to QE



Notes: plots of output gap response to a QE shock as risk aversion increases.

Output Response to QE



Notes: plots of output gap response to a QE shock as risk aversion increases.

Sticky Prices

- What about when prices are not fixed?

$$
\begin{aligned}
\mathrm{d}x_t &= \varsigma^{-1}(\tilde{r}_t - \pi_t - \bar{r})\,\mathrm{d}t \\
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- Results go through if determinacy condition is met:

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\hat{A}_r > \frac{\delta}{\delta\phi_\pi + \rho\phi_x}
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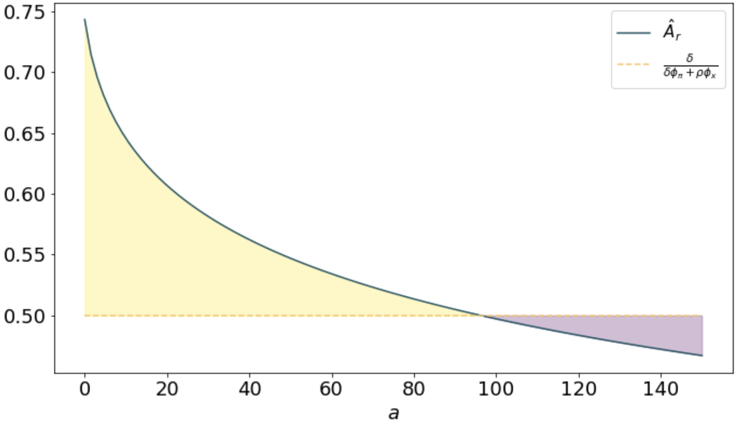
- Results go through if determinacy condition is met:

$$\hat{A}_r > \frac{\delta}{\delta\phi_\pi + \rho\phi_x}$$

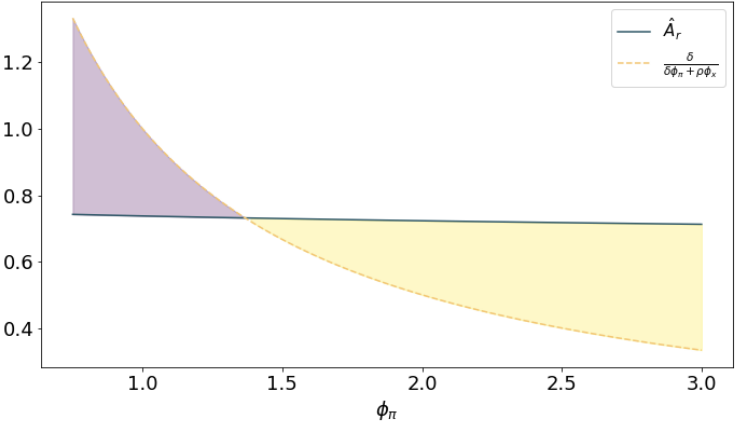
- If $\hat{A}_r = 1$ and $\phi_x = 0$, reduces to $\phi_\pi > 1$

Implications – Determinacy



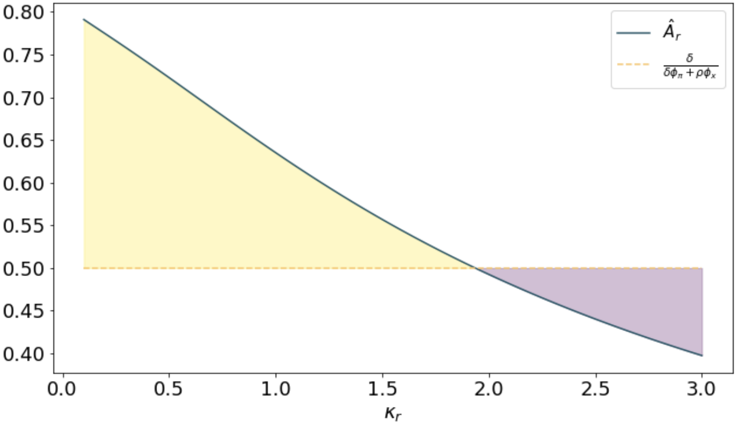
Notes: determinacy condition as risk aversion a increases.
The model is determinate if the solid dark line lies above the dotted light line (light shaded region) and is indeterminate otherwise (dark shaded region).

Implications – Determinacy



Notes: determinacy condition as central bank response to inflation ϕ_π increases. The model is determinate if the solid dark line lies above the dotted light line (light shaded region) and is indeterminate otherwise (dark shaded region).

Implications – Determinacy



Notes: determinacy condition as central bank inertia κ_r increases.
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Quantitative (Generalized) Model

- Sticky price model with shocks

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dz_{i,t} = -\kappa_{z_i} z_{i,t}\,\mathrm{d}t + \sigma_{z_i}\,\mathrm{d}B_{z_i,t}
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- Demand factors

$$\beta_{t,\tau} = \bar{\beta}(\tau) + \sum_k \beta_{k,t}\theta_k(\tau)$$
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$$

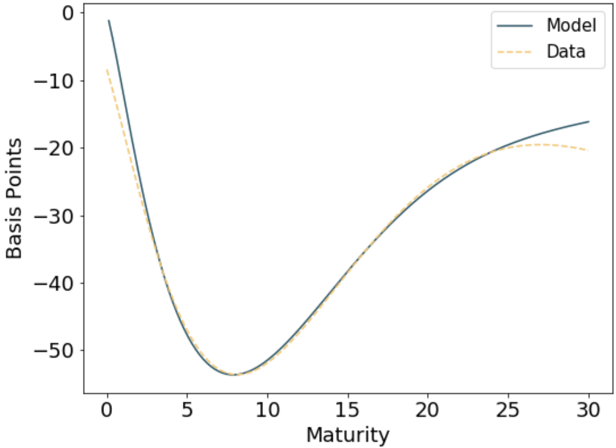
- Requires numerical solution methods

Calibration

Table: Numerical Exercise Calibration

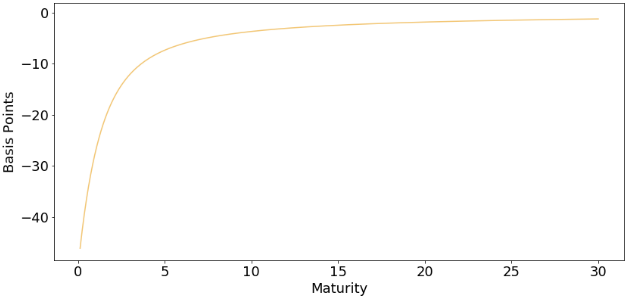
Parameter	Value	Description	Target
Effective Borrowing Rate			
η_1	1.7069	Weight Scaling Factor	Treasury Maturity Distribution
Macroeconomic Dynamics			
ρ	0.0400	Discount Factor	Long-Run Interest Rate
ς^{-1}	1.0000	Intertemporal Elasticity	Balanced Growth
κ_r	0.9473	Monetary Policy Inertia	$\text{Cov}[r_t, r_{t-1}] = 3.5013$
κ_{z_π}	0.5863	Cost-Push Shock Inertia	$\text{Cov}[\pi_t, \pi_{t-1}] = 0.9141$
κ_{z_x}	0.2554	Demand Shock Inertia	$\text{Cov}[x_t, x_{t-1}] = 2.2908$
ϕ_π	2.0420	Inflation Taylor Coeff.	$\text{Cov}[r_t, \pi_t] = 1.0006$
ϕ_x	0.9709	Output Taylor Coeff.	$\text{Cov}[r_t, x_t] = 0.7722$
δ	0.0459	Nominal Rigidity	$\text{Cov}[\pi_t, x_t] = -0.3015$
σ_r	0.0116	Monetary Shock Vol.	$\text{Var}[r_t] = 2.7066$
σ_{z_π}	0.0068	Cost-Push Shock Vol.	$\text{Var}[\pi_t] = 0.5097$
σ_{z_x}	0.0126	Demand Shock Vol.	$\text{Var}[x_t] = 1.5192$
Term Structure			
$\theta_s(\tau)$	$\delta(\tau - 2)$	Short Factor Location	LSAP Targets
$\theta_\ell(\tau)$	$\delta(\tau - 10)$	Long Factor Location	LSAP Targets
$\alpha(\tau)$	1.0000	Habitat Elasticity	Normalized
κ_β	0.1710	Habitat Factor Inertia	QE1 Yield Curve Response
σ_{z_β}	0.0142	Habitat Factor Vol.	QE1 Yield Curve Response
a	1559.7	Risk Aversion	QE1 Yield Curve Response

QE: Model vs. Data



Notes: Yield curve response to the announcement of the initial round of QE on March 18, 2009 (light dotted line). The dark line corresponds to the yield curve response to a QE shock in the model. Source: Gurkaynak, Sack, and Wright (2007). $\eta(\tau)$

Yield Curve (Monetary Policy)



Notes: yield curve response to a 50 b.p. monetary shock on impact, for different levels of risk aversion a. Darker lines correspond to higher levels of risk aversion.

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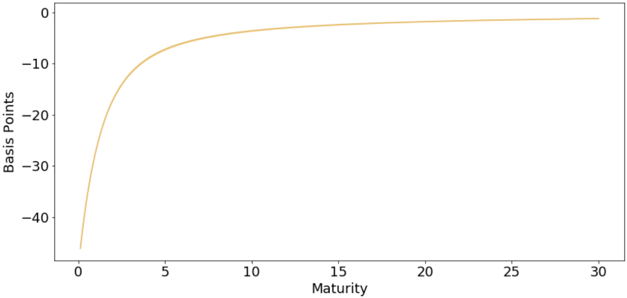

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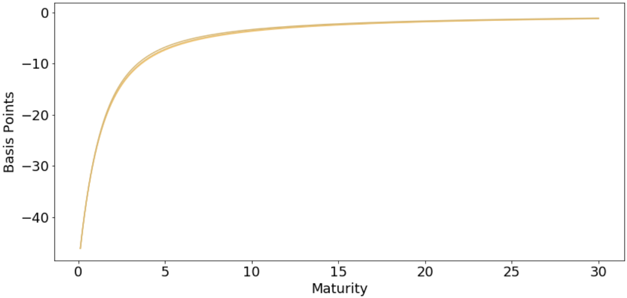

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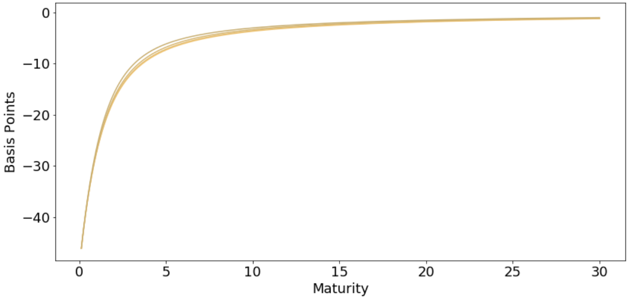

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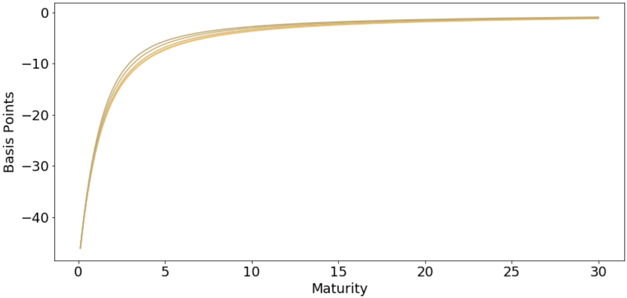

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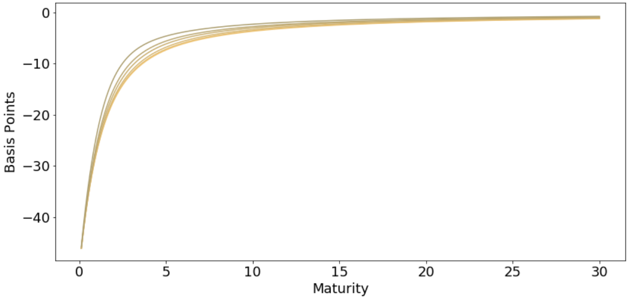

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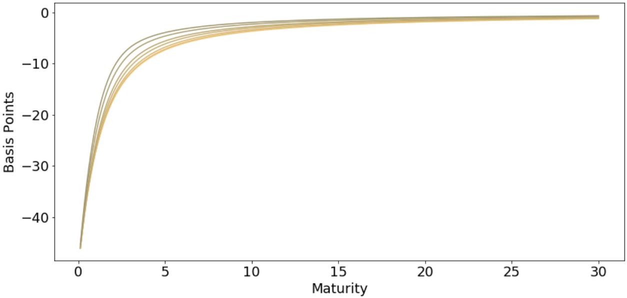

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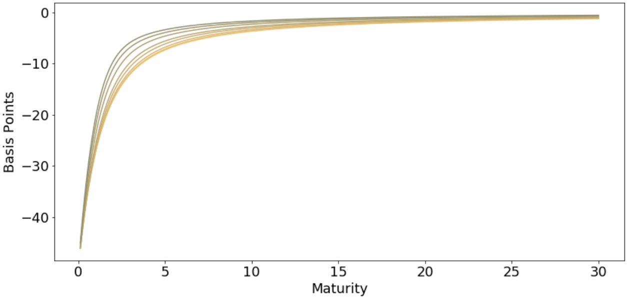

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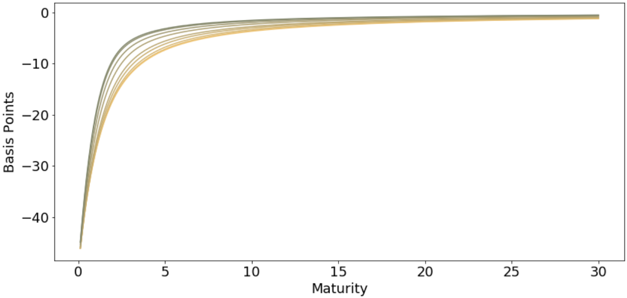

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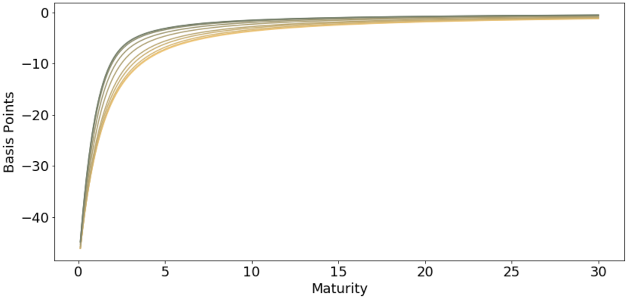

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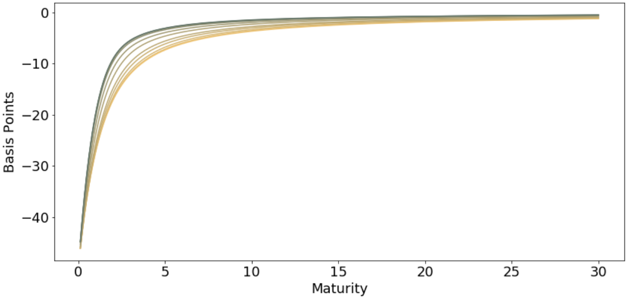

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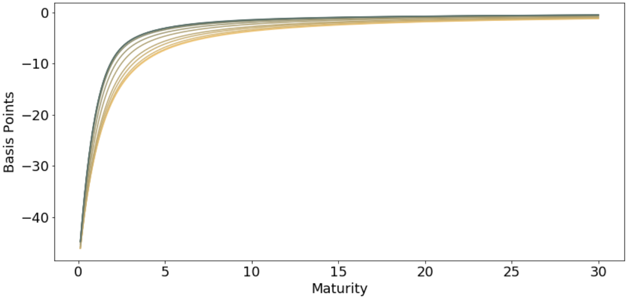

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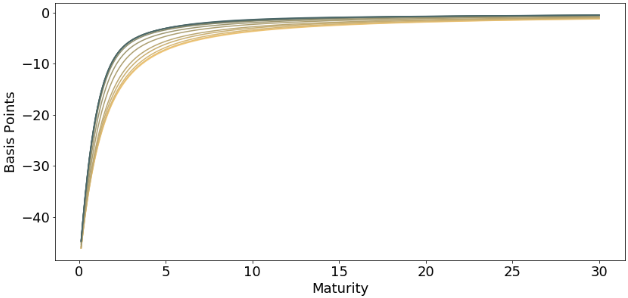

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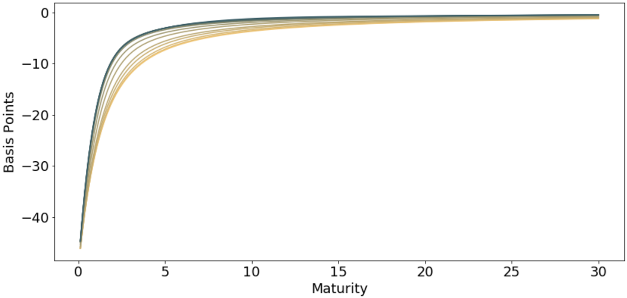

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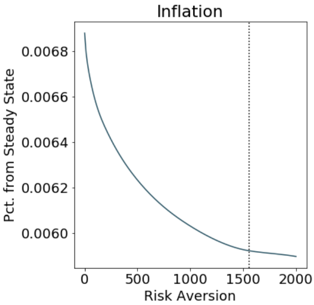
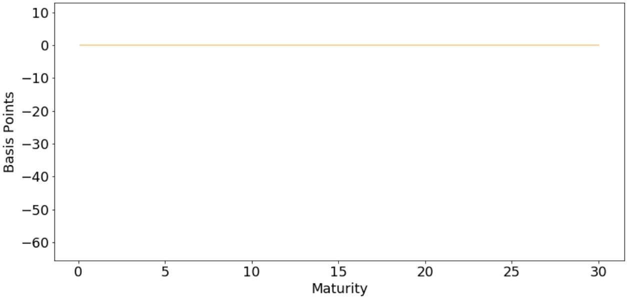
Aggregate Response (Monetary Policy)



Notes: inflation and output response a 50 b.p. monetary shock, for different levels of risk aversion *a*.

Yield Curve (QE, long end)



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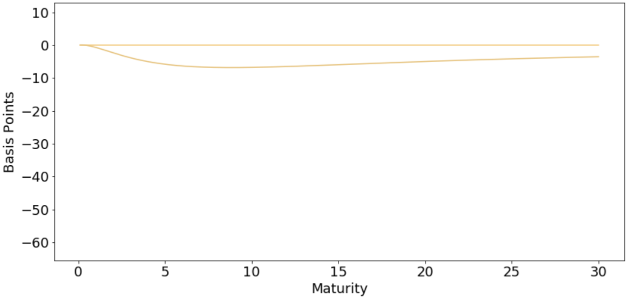

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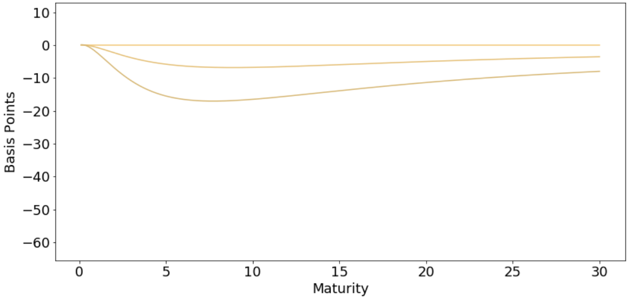

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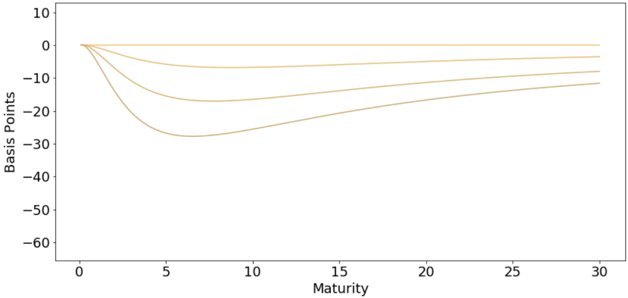

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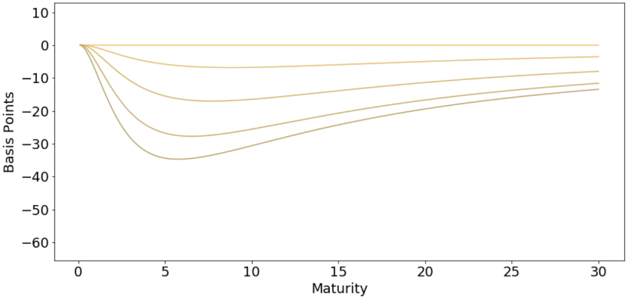

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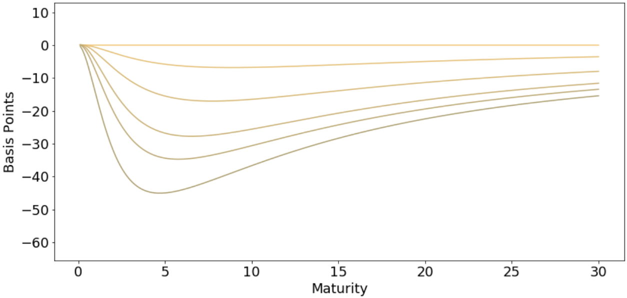

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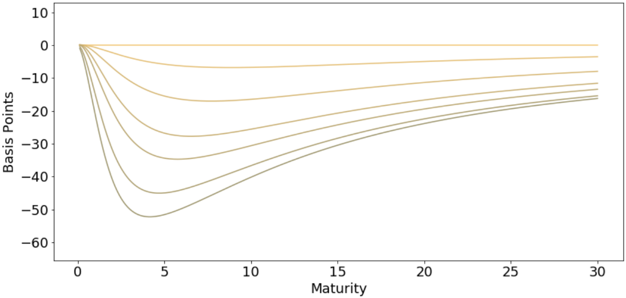

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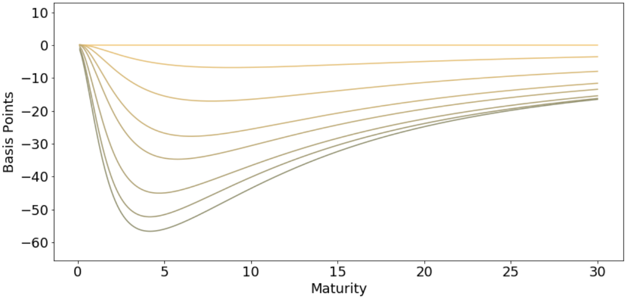

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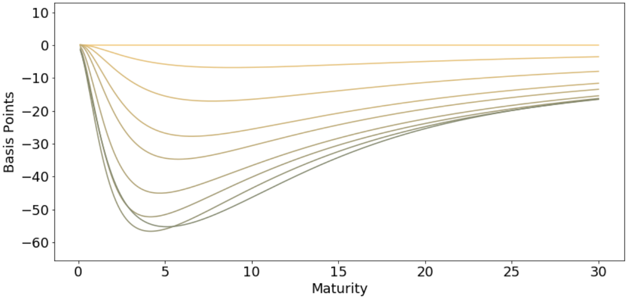

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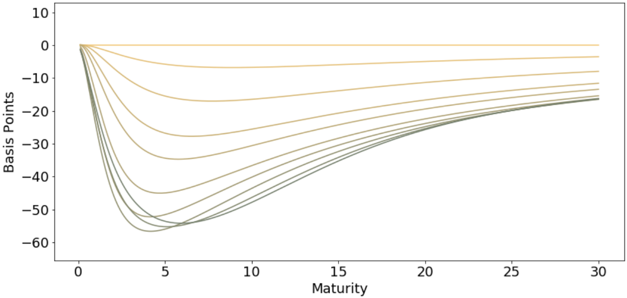

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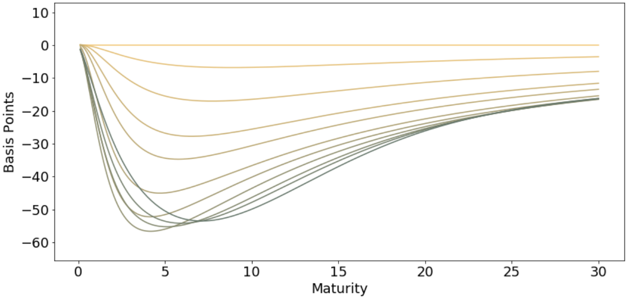

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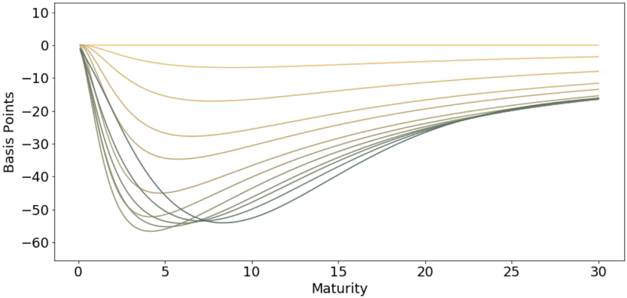

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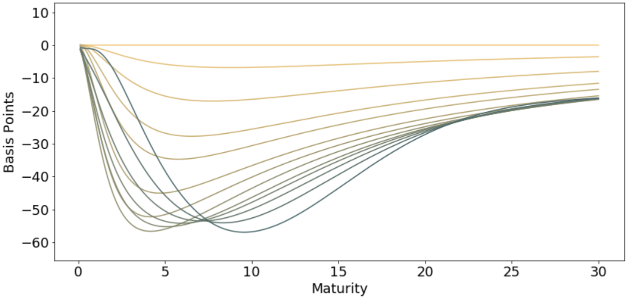

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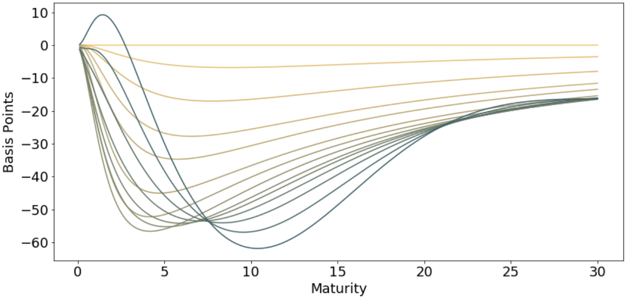
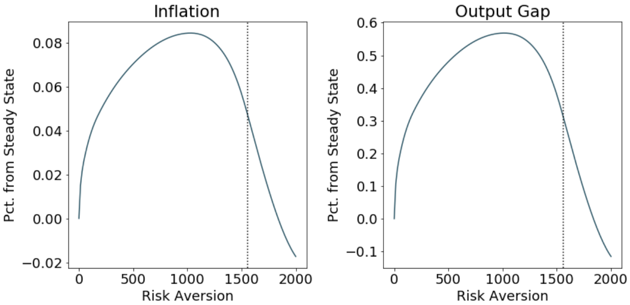
Yield Curve (QE, long end)



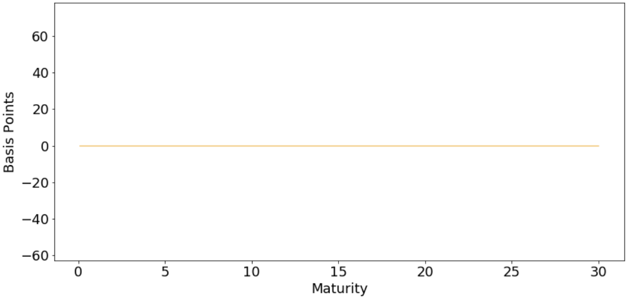
Notes: yield curve response to a "long" QE shock, for different levels of risk aversion a. Darker lines correspond to higher levels of risk aversion.

Aggregate Response (QE, long end)



Notes: inflation and output response to "long" QE shock on impact, for different levels of risk aversion *a*.

Yield Curve (Operation Twist)



Notes: yield curve response to an "Operation Twist" shock on impact, for different levels of risk aversion *a*. Darker lines correspond to higher levels of risk aversion.

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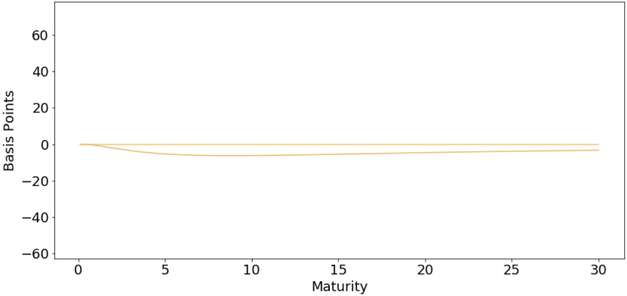

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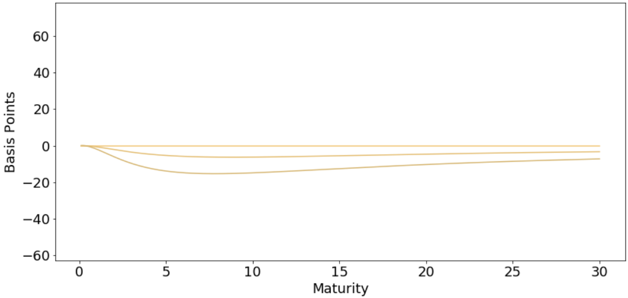

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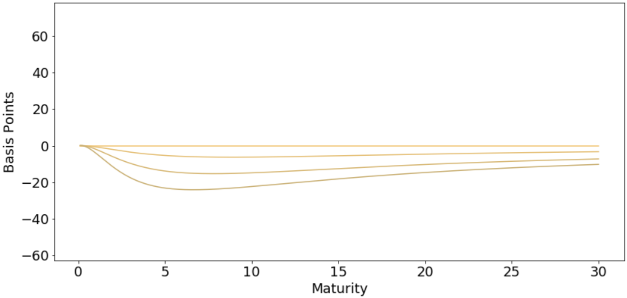

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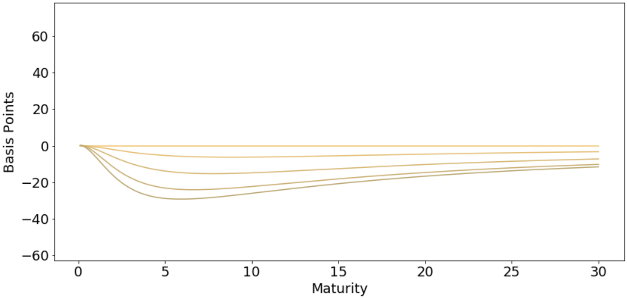

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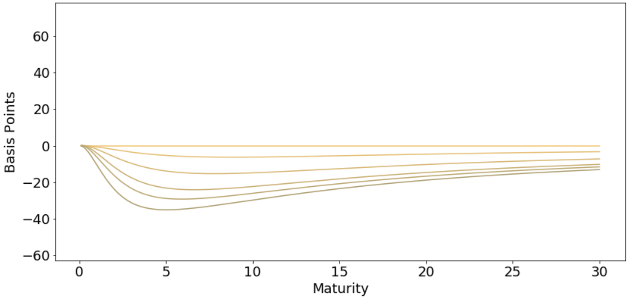

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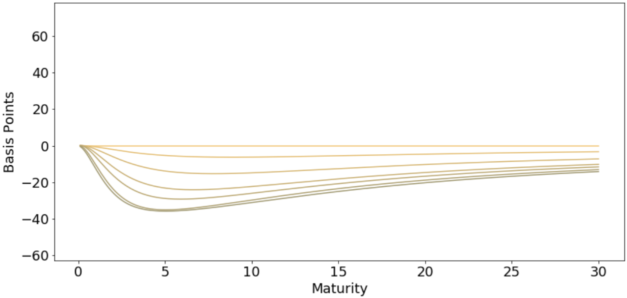

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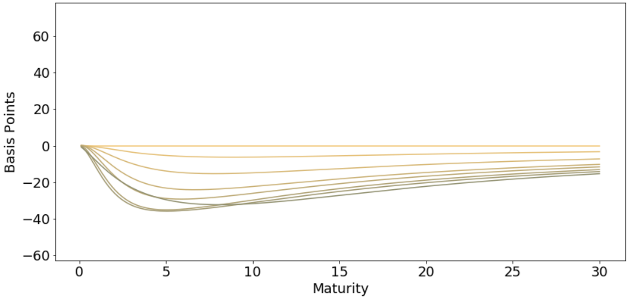

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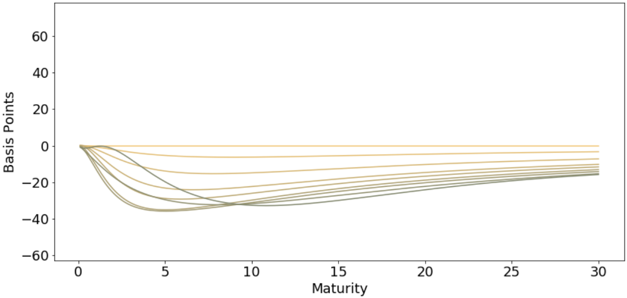

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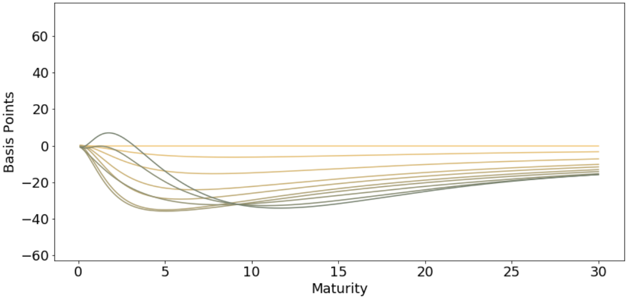

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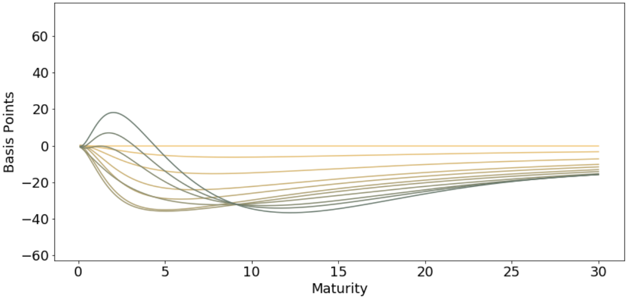

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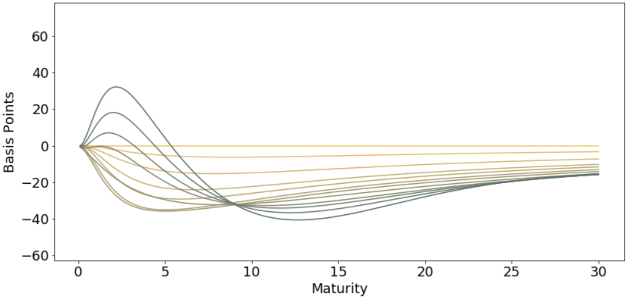

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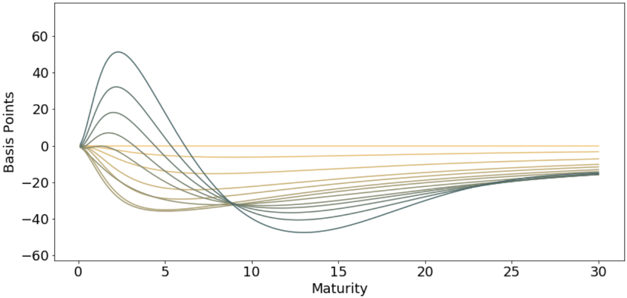

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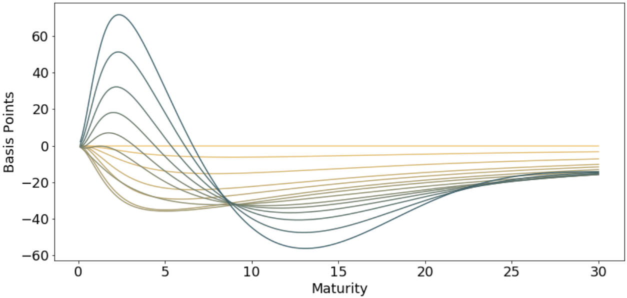

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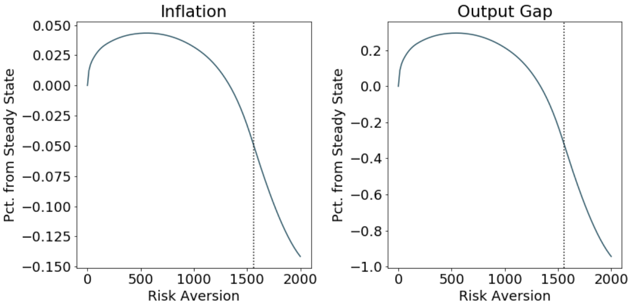
Aggregate Response (Operation Twist)



Notes: inflation and output response an "Operation Twist" shock, for different levels of risk aversion a.

Optimal Conventional Policy

- Can the planner improve outcomes?
- Loss function

$$E_0 \int_0^\infty e^{-\rho t} \left(w_\pi \pi_t^2 + w_x x_t^2 \right) \mathrm{d}t$$

Optimal Conventional Policy

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Optimal Conventional Policy

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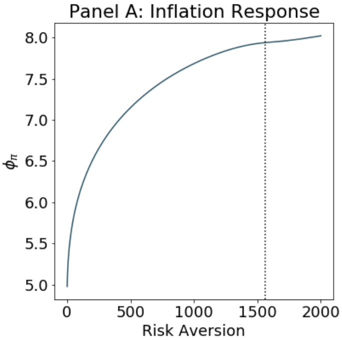
- Loss function

$$\min_{\phi_\pi, \kappa_r} E_0 \int_0^\infty e^{-\rho t} \left(w_\pi \pi_t^2 + w_x x_t^2 \right) \mathrm{d}t$$

- Optimal inflation response and inertia as financial disruptions increase conditional distribution

Optimal Response: More Aggressive in Crises



Notes: optimal policy coefficients on inflation (Panel A) and inertia (Panel B) as risk aversion increases. Planner weights: $w_\pi = 1$, $w_x = 0.1$.

Stabilizing LSAPs

- Can LSAPs be used to ensure determinacy?
- Endogenous QE purchases:

$$\mathrm{d}\beta_t = -\kappa_\beta \left(\beta_t - \phi_\pi^\beta \pi_t \right) \mathrm{d}t$$

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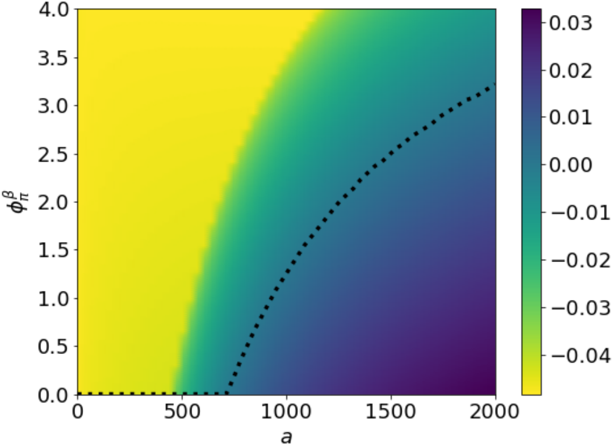
QE and Determinacy



Notes: determinacy conditions as a function of risk aversion (x-axis) and endogenous response of QE to inflation (y-axis). Darker colors correspond to larger values of the unstable eigenvalue. The dotted black line demarcates the region of determinacy.

Concluding Remarks

- Develops a unified, parsimonious framework to study conventional and unconventional monetary policies
- Transmission depends crucially on the health of financial markets

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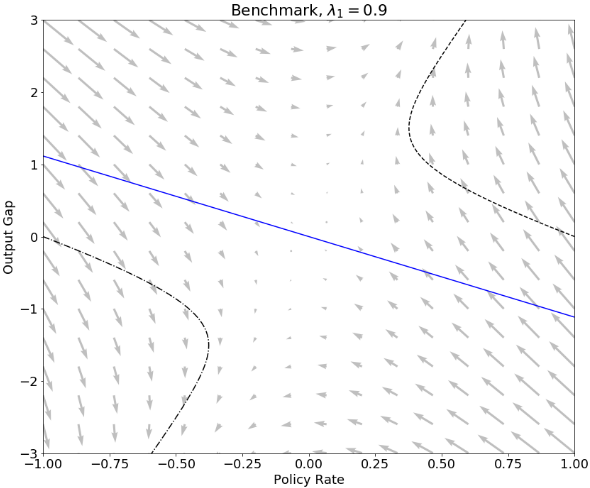
- Transmission depends crucially on the health of financial markets
- Future work:
  - Macroprudential policies
  - Monetary policy in open economies

APPENDIX

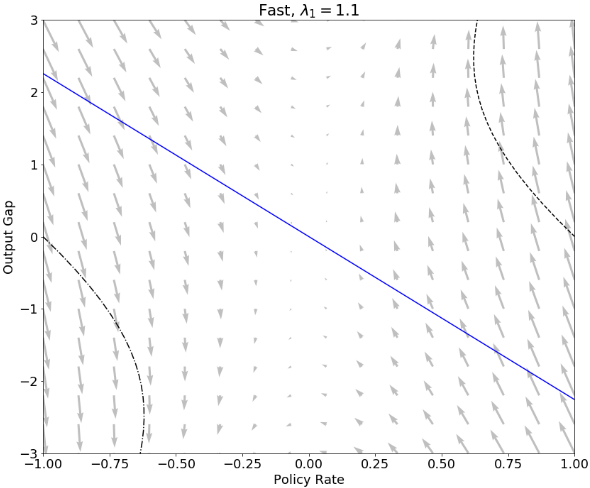
Phase Diagrams



Notes: phase diagrams of the dynamics of output and the policy rate as the equilibrium mean reversion rate of shocks varies.

back

Phase Diagrams



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back

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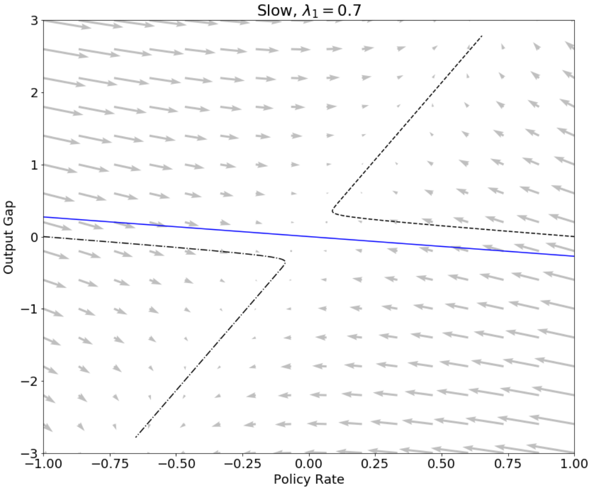

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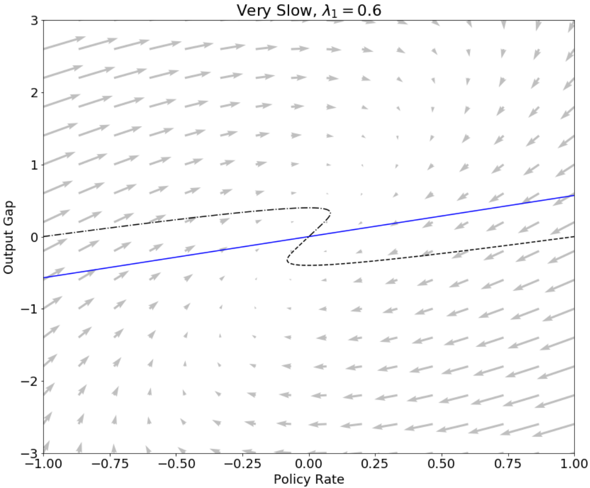
back

Phase Diagrams



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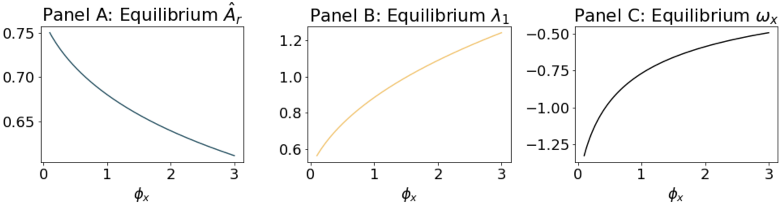
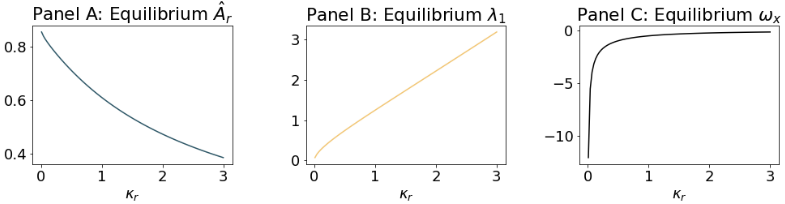
Implications – Conventional Policy



Notes: equilibrium changes in sensitivity to the short rate $\hat{A}_r$ and monetary shock reversion λ_1 as central bank response to output ϕ_x increases.

back

Implications – Conventional Policy



Notes: equilibrium changes in sensitivity to the short rate $\hat{A}_r$ and monetary shock reversion λ_1 as central bank inertia κ_r increases.

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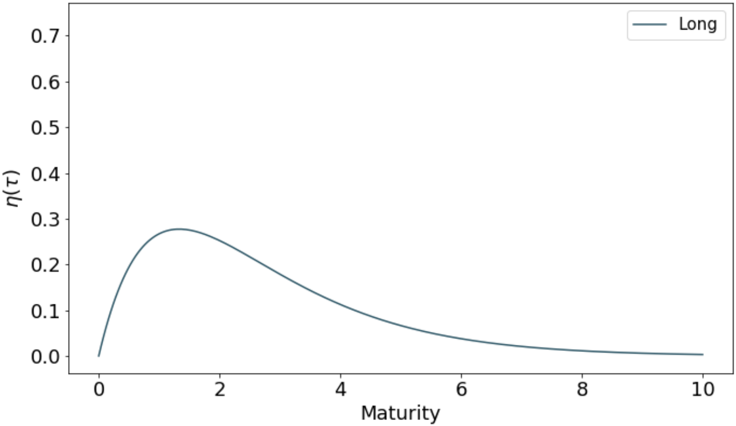
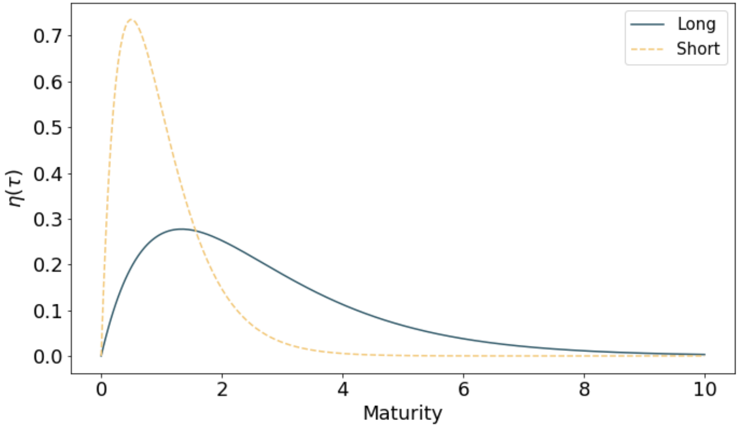
Sensitivity to Long Rates



Notes: different weighting function $\eta(\tau)$ in the determination of the effective borrowing rate $\tilde{r}_t$.

back

Sensitivity to Long Rates



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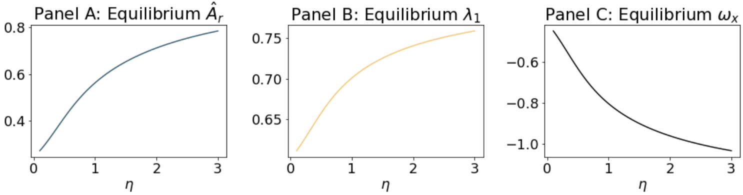
back

Implications – Sensitivity to Long Rates



Notes: equilibrium changes in sensitivity to the short rate $\hat{A}_r$ and monetary shock reversion λ_1 as the weighting function $\eta(\tau)$ shifts towards short-term bonds.

back

Forward Guidance

- Central bank announces a peg: $r_0 = r^\diamond$ and

$$\mathrm{d}r_t = \begin{cases} -\kappa_r^\diamond (r_t - r^\diamond)\,\mathrm{d}t + \sigma_r^\diamond\,\mathrm{d}B_{r,t} & \text{if } 0 < t < t^\diamond \\ -\kappa_r (r_t - \phi_x x_t - r^*)\,\mathrm{d}t + \sigma_r\,\mathrm{d}B_{r,t} & \text{if } t \geq t^\diamond \end{cases}$$

- Affine coefficient functions during peg:

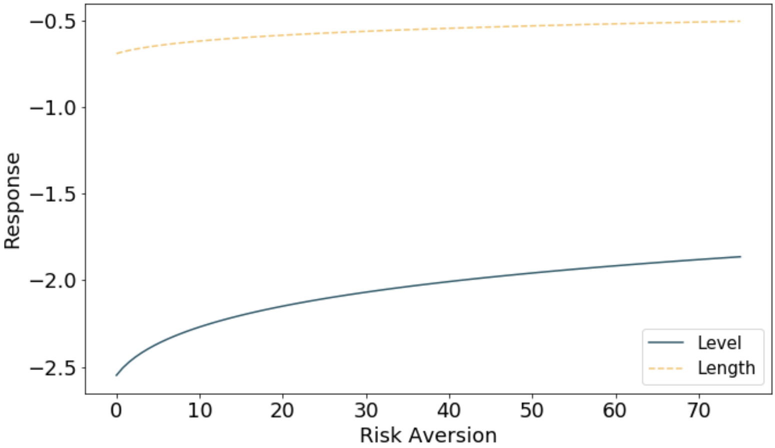
$$-\log P_{t,\tau} = A_r^\diamond(\tau) r_t + C^\diamond(\tau)$$
$$\implies \tilde{r}_t = \hat{A}_r^\diamond r_t + \hat{C}^\diamond$$

- Rational expectations dynamics for output:

$$\frac{\partial x_0}{\partial r^\diamond} = \omega_x - t^\diamond \varsigma^{-1} \hat{A}_r^\diamond \,, \quad \frac{\partial^2 x_0}{\partial r^\diamond \partial t^\diamond} = -\varsigma^{-1} \hat{A}_r^\diamond$$

back

Response to Forward Guidance



Notes: plots of $\frac{\partial x_0}{\partial r^\diamond}$ ("level") and $\frac{\partial^2 x_0}{\partial r^\diamond \partial t^\diamond}$ ("length") as risk aversion increases.

back

Long-Run Variance

- State-space representation

$$\mathrm{d}\mathbf{y}_t = -\Gamma\left(\mathbf{y}_t - \mathbf{y}^{SS}\right)\mathrm{d}t + \mathbf{S}\,\mathrm{d}\mathbf{B}_t\,, \quad \mathbf{x}_t = \Omega\left(\mathbf{y}_t - \mathbf{y}^{SS}\right)$$

- Conditional distribution $\mathbf{y}_t|\mathbf{y}_0 \sim N\left(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t\right)$ where

$$\boldsymbol{\mu}_t = \mathbf{y}^{SS} + e^{-\Gamma t}(\mathbf{y}_0 - \mathbf{y}^{SS}), \quad \boldsymbol{\Sigma}_t = \int_0^t e^{\Gamma(u-t)}\boldsymbol{\Sigma} e^{\Gamma^T(u-t)}\,\mathrm{d}u$$

- Present-discounted value

$$\widetilde{\boldsymbol{\Sigma}}_\infty \equiv \int_0^\infty e^{-\rho t}\boldsymbol{\Sigma}_t\,\mathrm{d}t$$

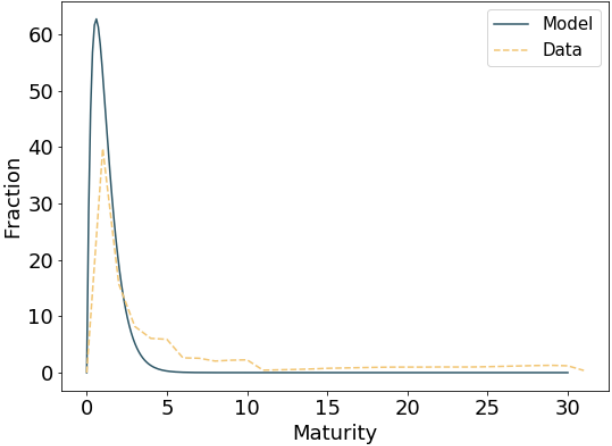
$$\implies \operatorname{vec}\widetilde{\boldsymbol{\Sigma}}_\infty = (\Gamma \oplus \Gamma)^{-1}(\rho\mathbf{I} + \Gamma \oplus \Gamma)^{-1}\operatorname{vec}\boldsymbol{\Sigma}$$

- Jump variables

$$\widetilde{\boldsymbol{\Sigma}}^{\mathbf{x}}_\infty = \Omega\widetilde{\boldsymbol{\Sigma}}_\infty\Omega^T$$

back

Effective Borrowing Rate Weights



Notes: average maturity distribution of outstanding Treasury debt (light dotted line). The dark line corresponds to the effective borrowing rate weights in the model. Source: FRED.

back