

# Monetary Policy and the Limits to Arbitrage: Insights from a New Keynesian Preferred Habitat Model

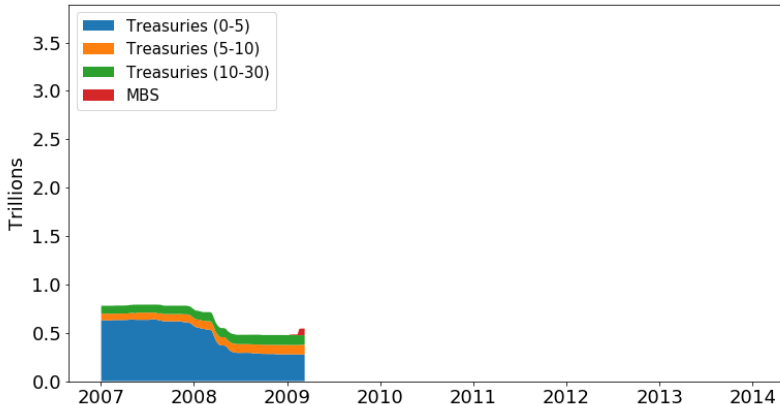
Walker Ray  
UC Berkeley

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INSEAD Job Market Seminar

# Policy Response to Great Recession

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Notes: Federal Reserve holdings of Treasuries (by maturity) and Mortgage-Backed Securities. Vertical lines indicate the start of LSAP programs. Source: FRED.

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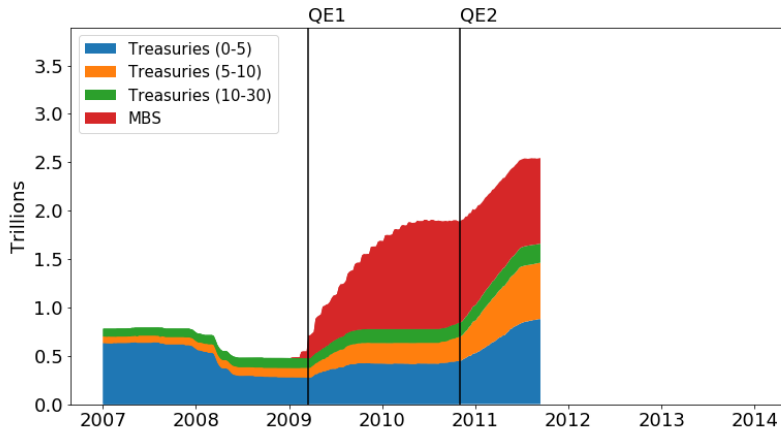
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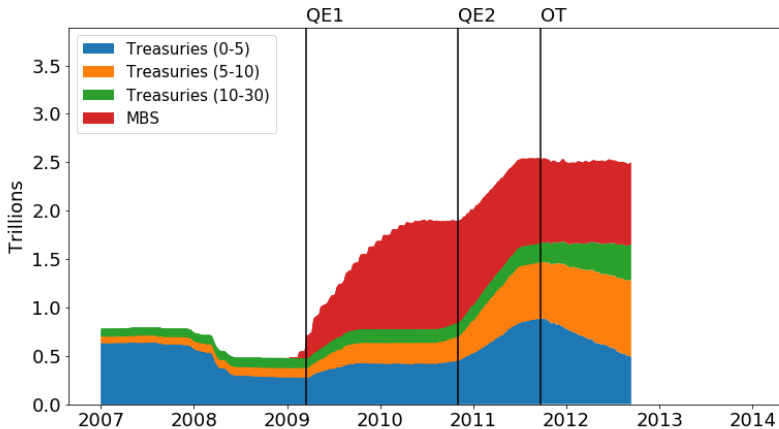
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- Bond market imperfections play a role in the transmission of **conventional** monetary policy
- Crucial for designing monetary policy going forward

# Model Overview

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- Monetary policy works through both channels

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- Designing policy going forward:
  - ▶ Conventional policy: more aggressive in financial crises
  - ▶ QE rule can be stabilizing

# Literature Contributions

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- “Preferred habitat” as a key channel for understanding bond markets
  - ▶ D’Amico and King (2013), Hamilton and Wu (2012), Greenwood and Vayanos (2014), Gorodnichenko and Ray (2017), Greenwood and Vissing-Jorgensen (2018)
- Few formal models
  - ▶ Vayanos and Vila (2009)
- QE in general equilibrium: Market segmentation vs. forward guidance
  - ▶ Gertler and Karadi (2013), Chen et al (2012), Carlstrom et al (2017), Christensen and Rudebusch (2012), Bauer and Rudebusch (2014), Bhattarai et al (2015)
- Frictions and expected future policy
  - ▶ McKay et al (2016), Farhi and Werning (2017), Gabaix (2016), Angeletos and Lian (2018)

# New Keynesian Preferred Habitat Framework

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  - ▶ Infinitely-lived **households** work and consume
  - ▶ **Firms** produce using labor, face price frictions

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  - ▶ HHs save and borrow through a passive **index fund**
  - ▶ **Habitat investors** with maturity-specific demand
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- Government:
  - ▶ **Central bank** sets the short nominal rate (and conducts QE)
  - ▶ Lump-sum taxes/transfers from investors to HHs



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- Closing the model: equilibrium term structure determination

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$$\begin{aligned}\tilde{b}_{t,\tau} &= -\alpha(\tau) \log P_{t,\tau} + \varepsilon_{t,\tau} \\ &= \alpha(\tau)\tau(R_{t,\tau} - \beta_{t,\tau})\end{aligned}\tag{PH}$$



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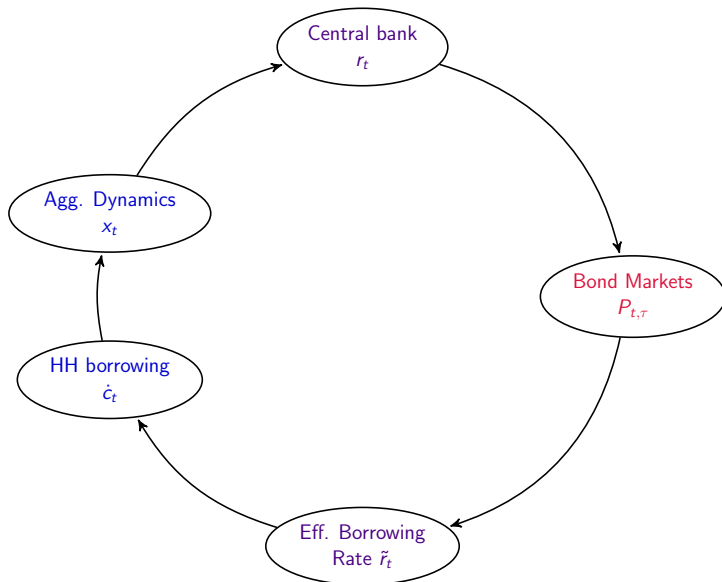
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- Market clearing:  $b_{t,\tau} = -\tilde{b}_{t,\tau}$

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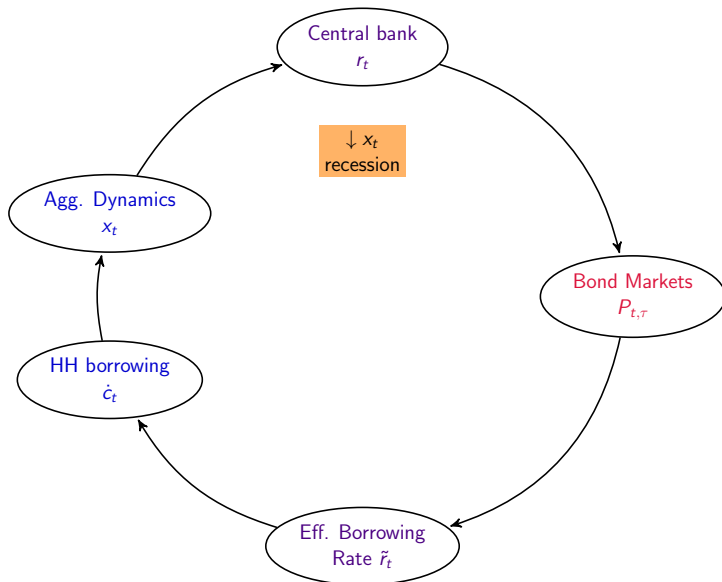
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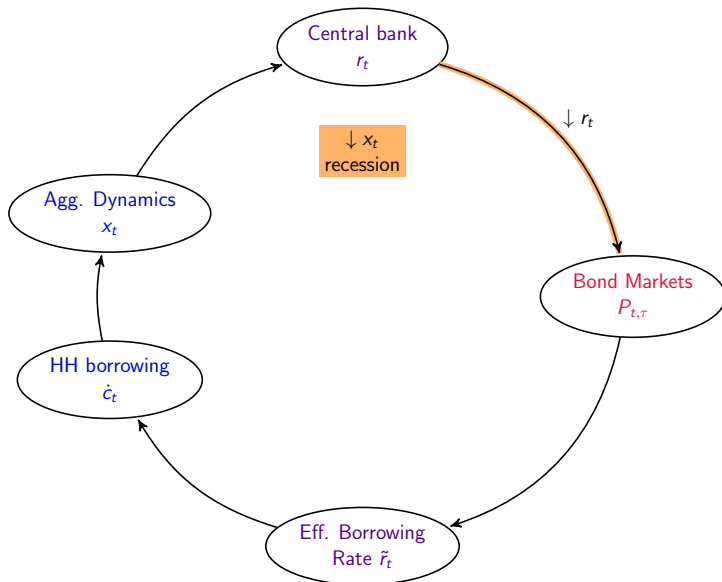
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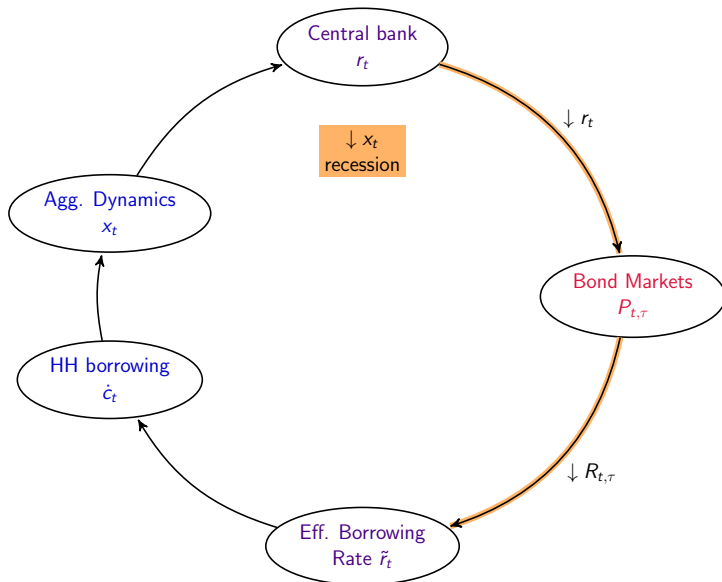
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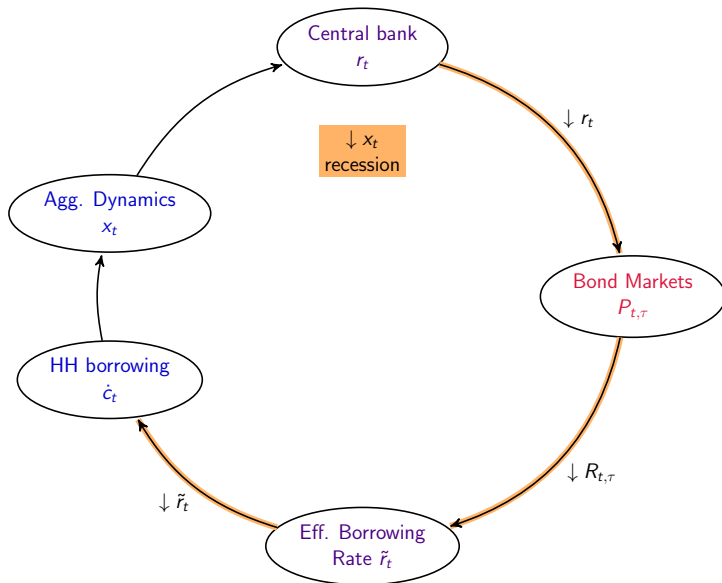
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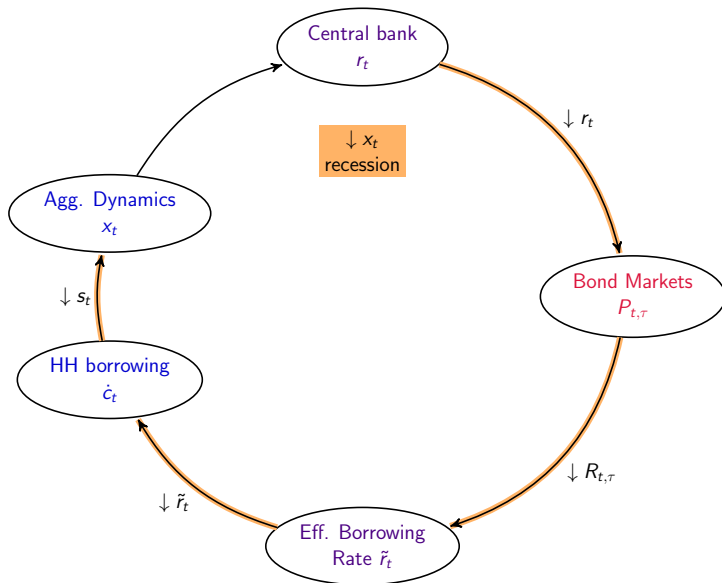


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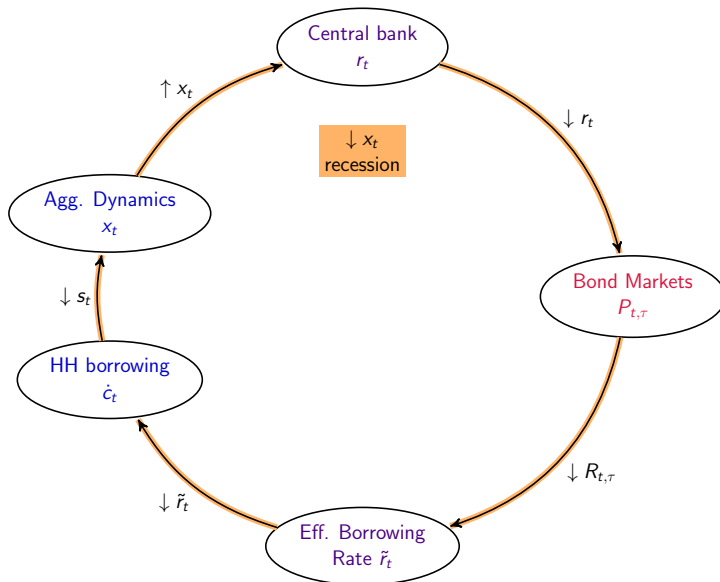
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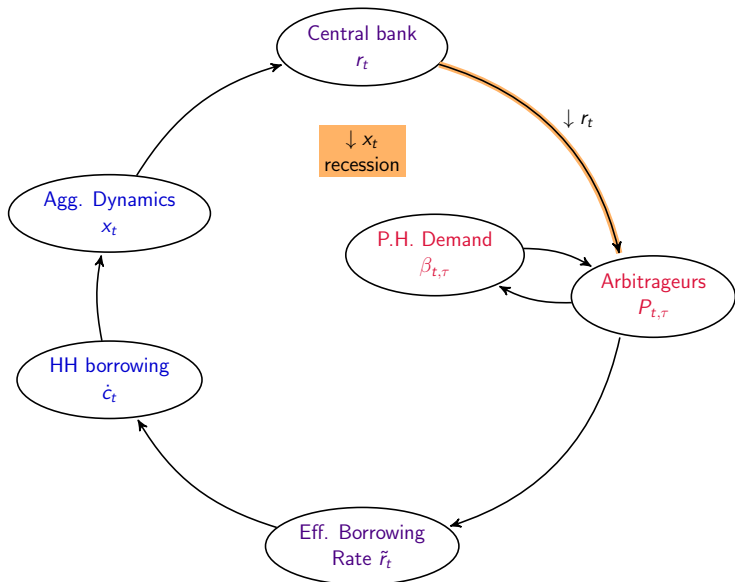
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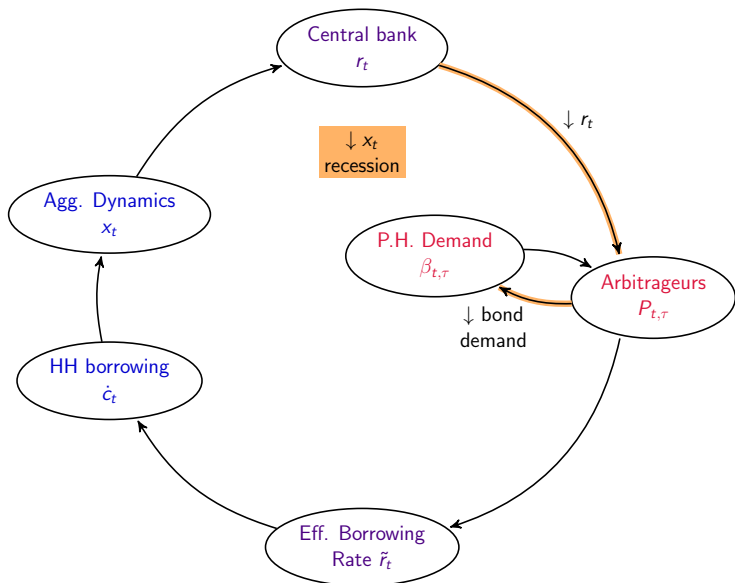
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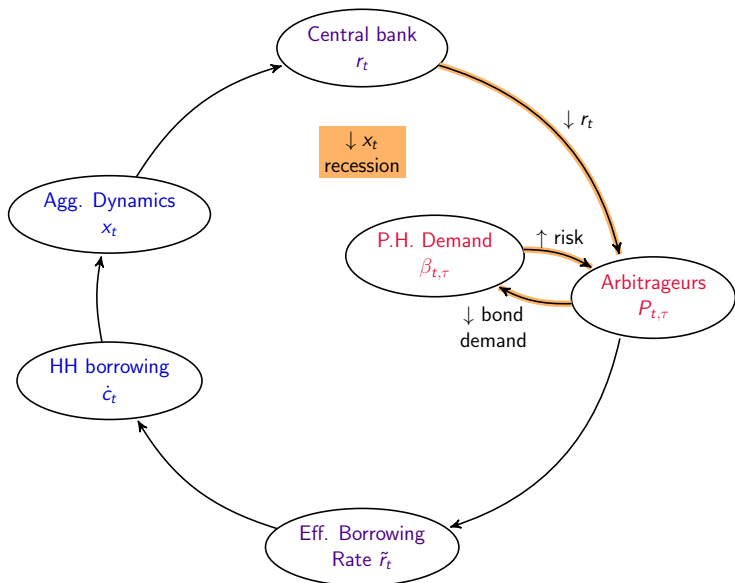


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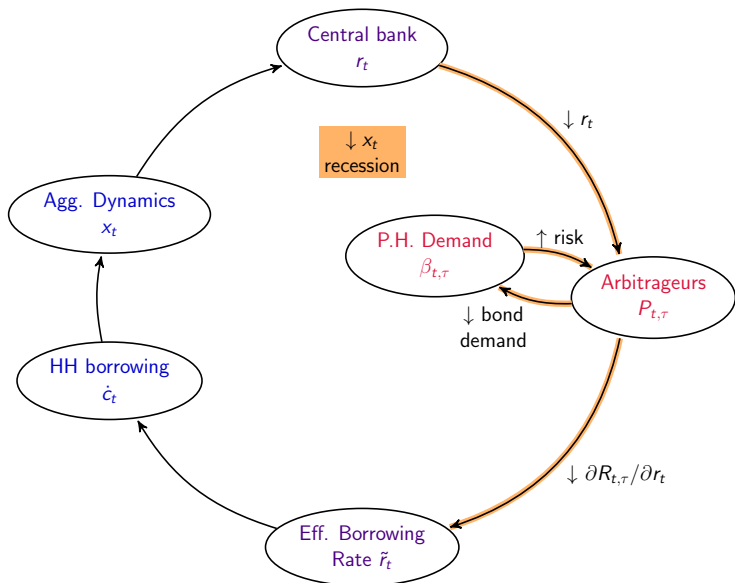




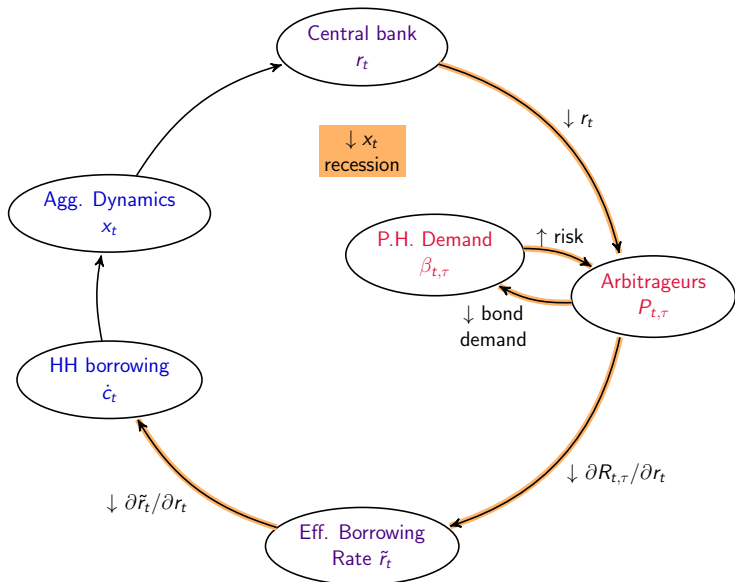
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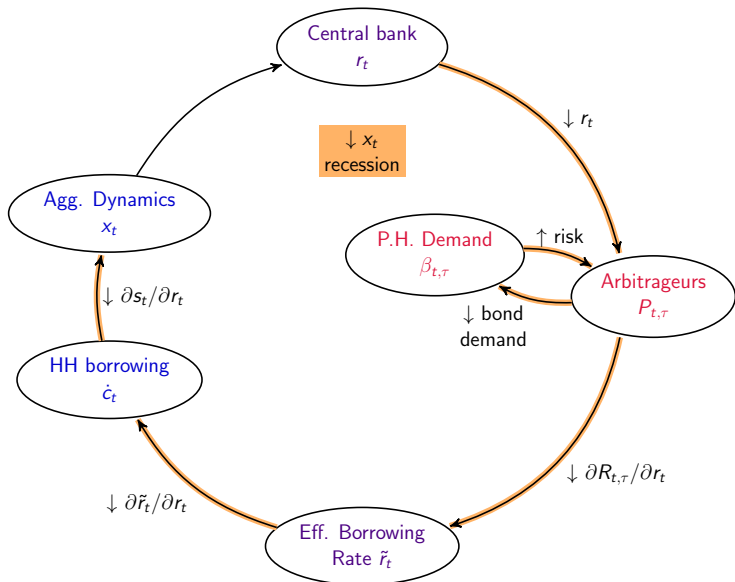
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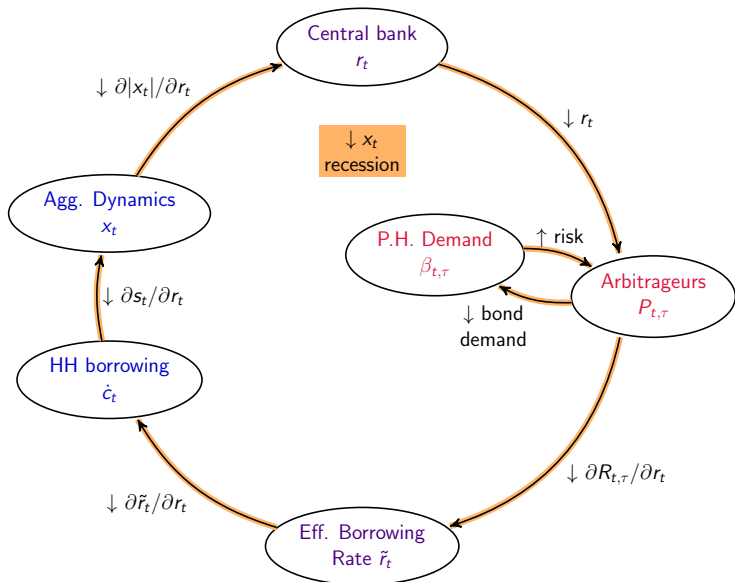
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# Rational Expectations Dynamics

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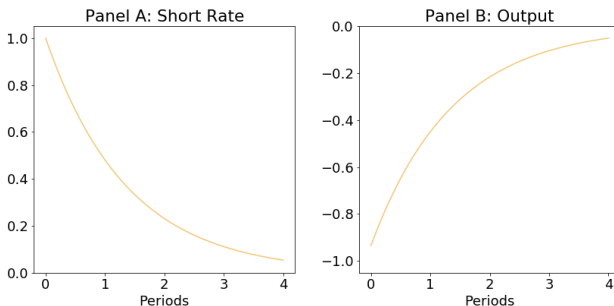
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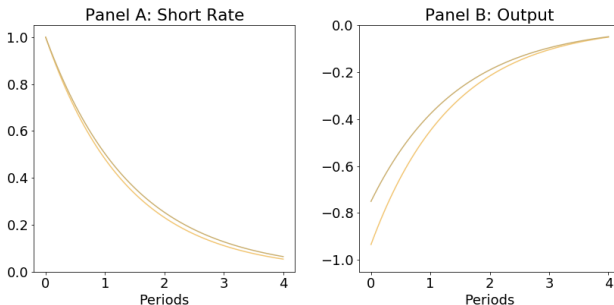
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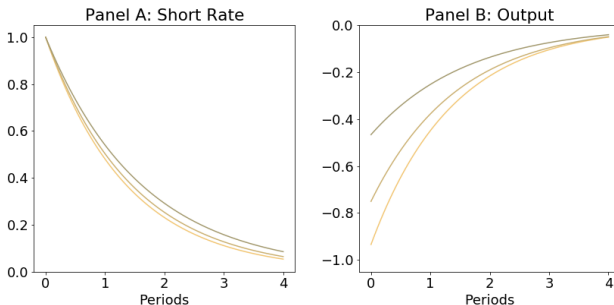
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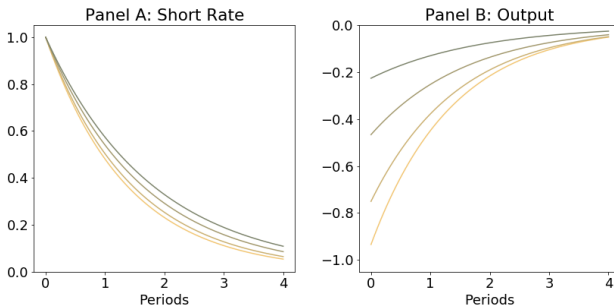
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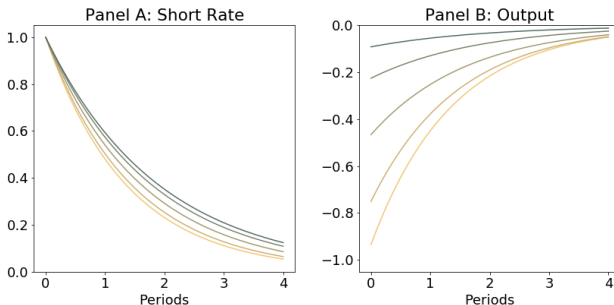
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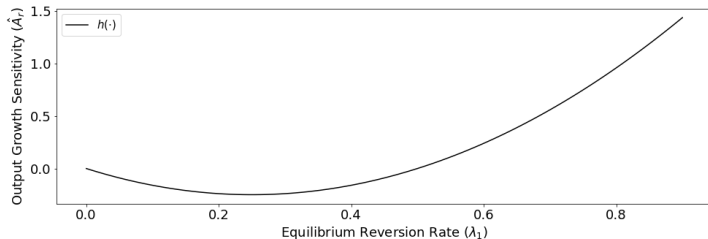


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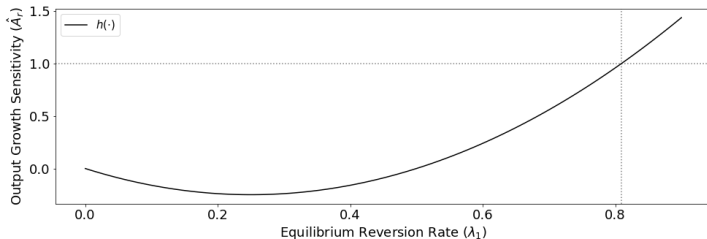


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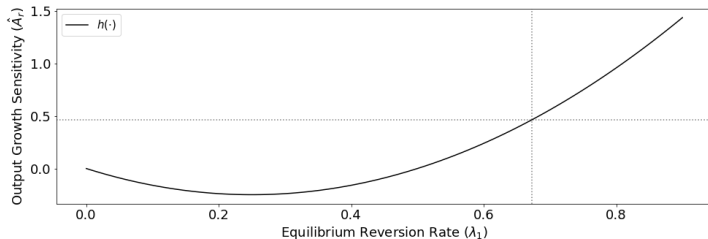


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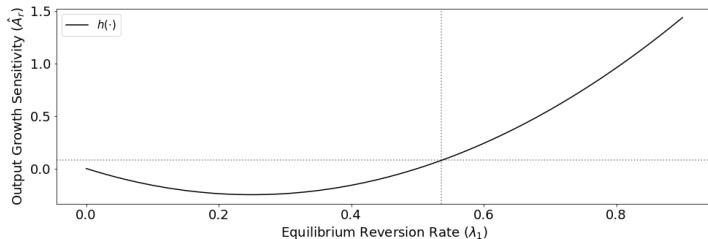


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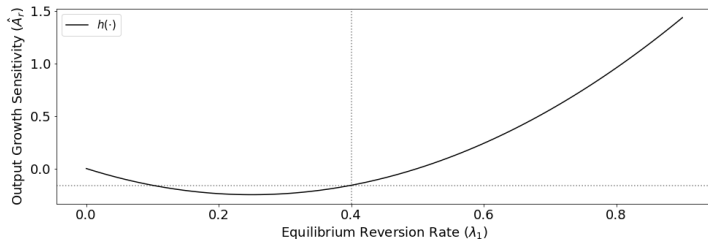


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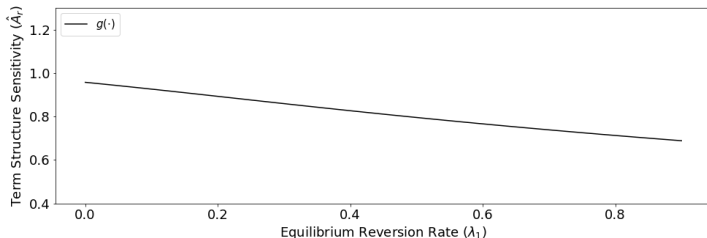
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- Prices adjust to balance demand and optimality conditions
- $\implies$  differential equation which solves affine coefficients

$$\hat{A}_r \equiv \int_0^T \frac{\eta(\tau)}{\tau} A_r(\tau) d\tau$$



# Term Structure Equilibrium



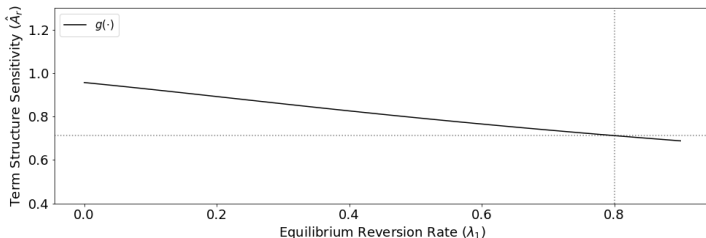
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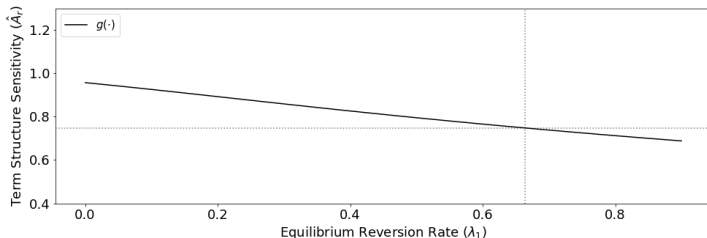
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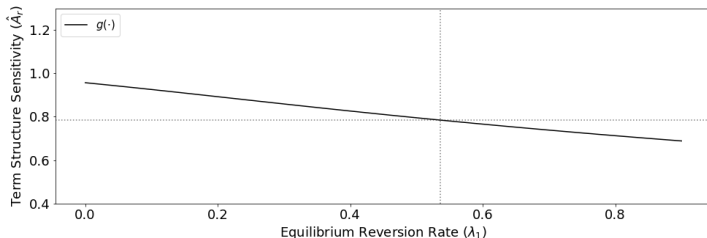
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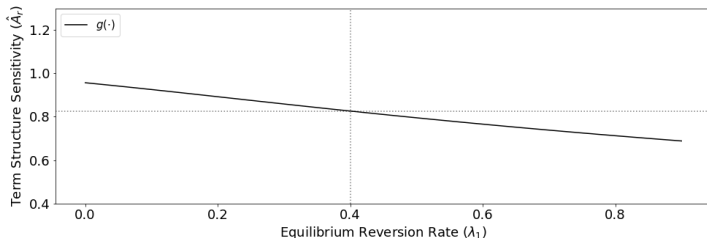
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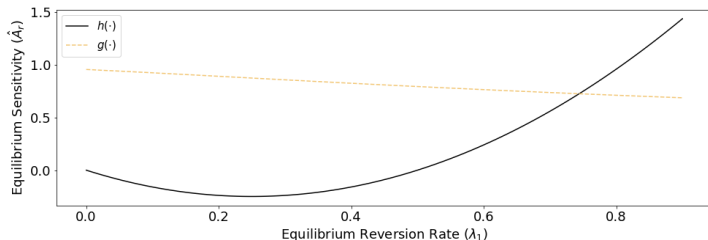
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- EH: two responses should be identical (only when  $a = 0$ )

# General Equilibrium

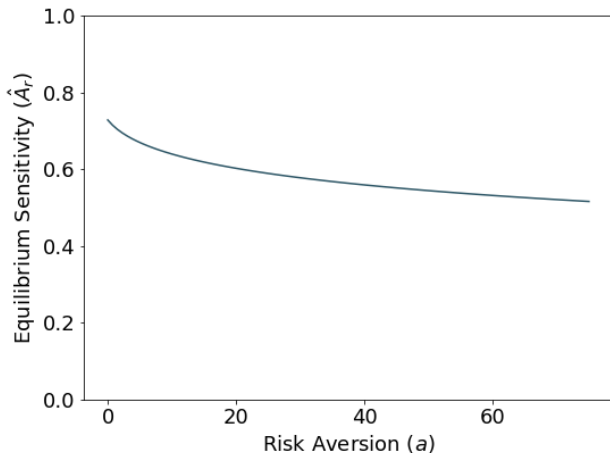


## Existence and Uniqueness

There exists a unique positive eigenvalue of  $\Upsilon$   $\lambda_1 > 0$  for which  $g(\lambda_1) = h(\lambda_1)$ , which fully characterizes the model equilibrium. Further, this implies  $0 < \hat{A}_r < 1$ .

# Conventional Policy and Financial Disruptions

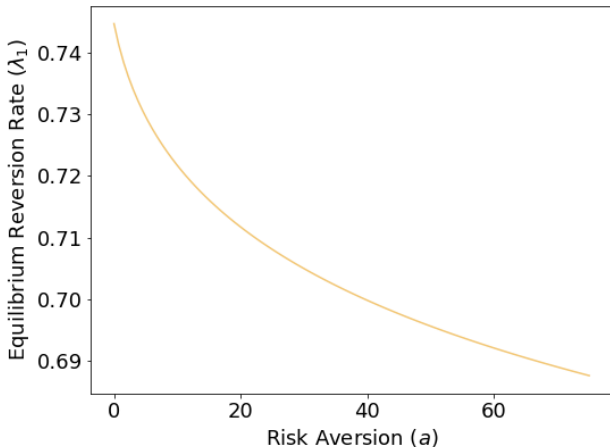
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Notes: equilibrium changes in sensitivity to the short rate  $\hat{A}_r$  as risk aversion  $a$  increases.

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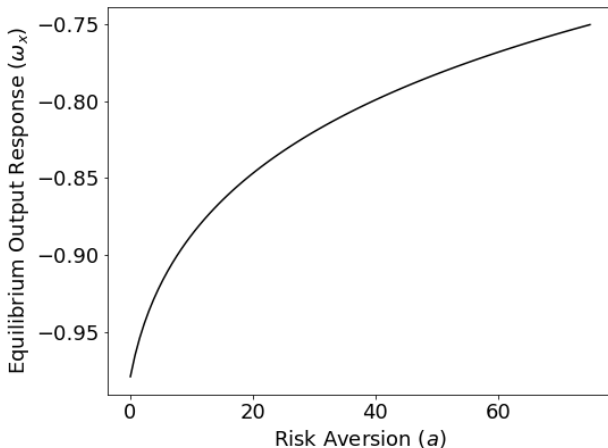
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Notes: equilibrium changes in monetary shock reversion  $\lambda_1$  as risk aversion  $a$  increases.

# Conventional Policy and Financial Disruptions

---



Notes: equilibrium changes in output response  $\omega_x$  to monetary shocks as risk aversion  $a$  increases.

# Policy Implications

---

- More aggressive response to output [\( \$\phi\_x\$  results\)](#)
- Higher inertia [\( \$\kappa\_r\$  results\)](#)
- Shifts in effective rate weights [\( \$\eta\(\tau\)\$  results\)](#)
- Forward guidance less effective as risk aversion increases [\(details\)](#)



## Modeling LSAPs

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- Suppose the central bank directly purchases bonds through open market operations
- Change to the demand shifter in PH demand

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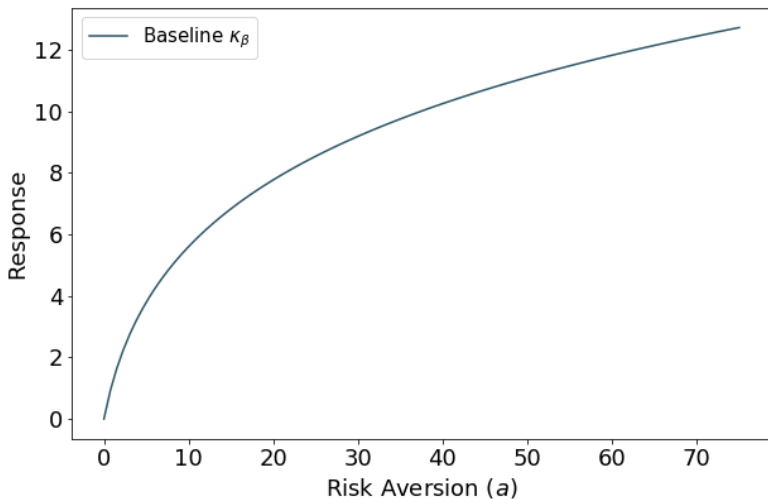
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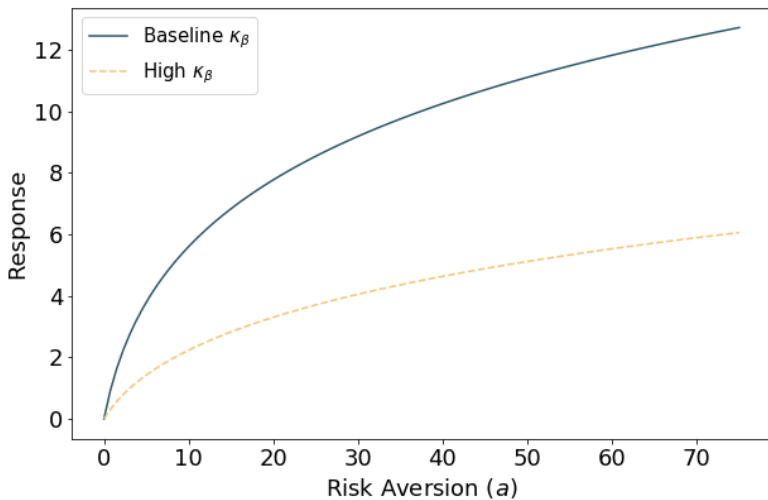
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- Results go through if determinacy condition is met:

$$\hat{A}_r > \frac{\delta}{\delta\phi_\pi + \rho\phi_x}$$

# Sticky Prices

---

- What about when prices are not fixed?

$$dx_t = \varsigma^{-1}(\tilde{r}_t - \pi_t - \bar{r}) dt$$

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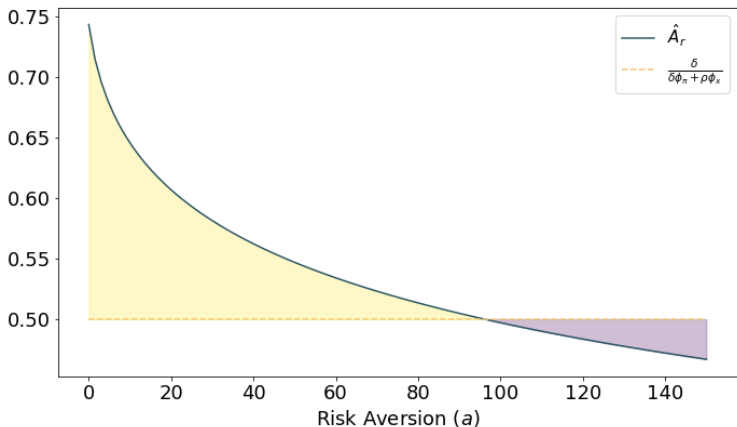
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- If  $\hat{A}_r = 1$  and  $\phi_x = 0$ , reduces to  $\phi_\pi > 1$

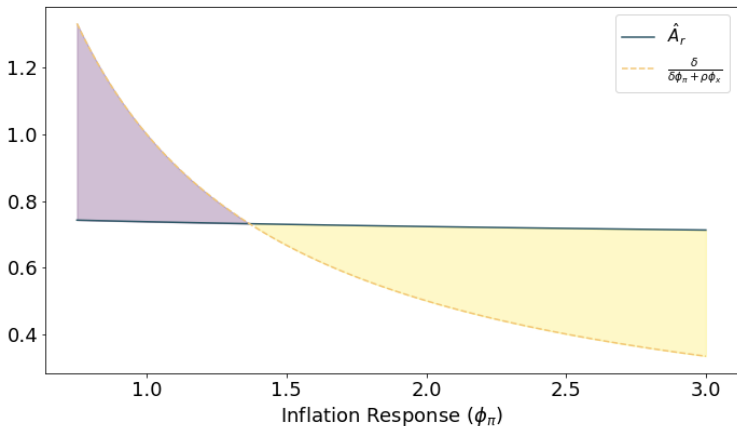
# Implications – Determinacy



Notes: determinacy condition as risk aversion  $a$  increases.

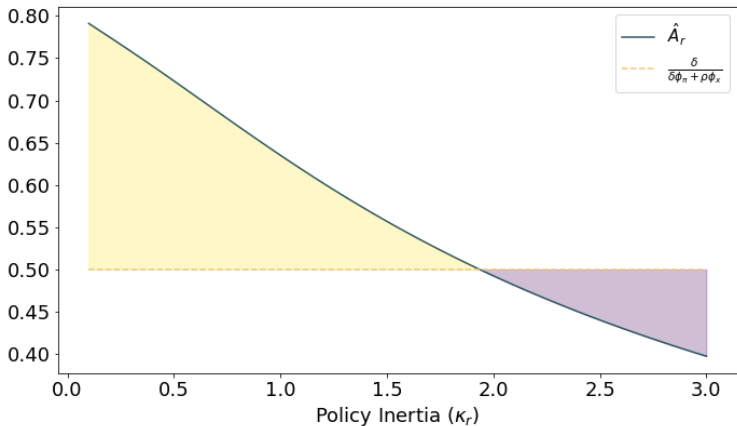
The model is determinate if the solid dark line lies above the dotted light line (light shaded region) and is indeterminate otherwise (dark shaded region).

# Implications – Determinacy



Notes: determinacy condition as central bank response to inflation  $\phi_\pi$  increases. The model is determinate if the solid dark line lies above the dotted light line (light shaded region) and is indeterminate otherwise (dark shaded region).

# Implications – Determinacy



Notes: determinacy condition as central bank inertia  $\kappa_r$  increases.

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# Quantitative (Generalized) Model

---

- Sticky price model with shocks

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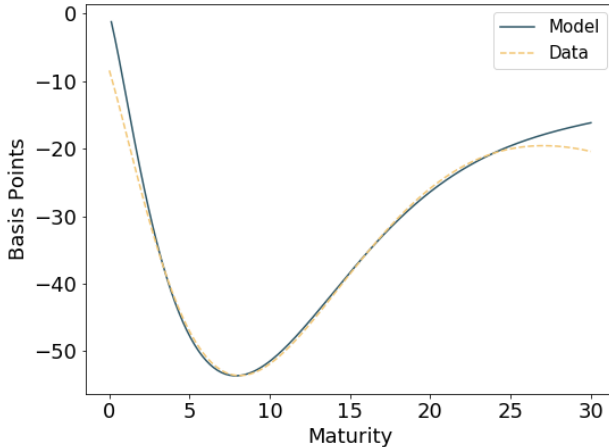
- Requires numerical solution methods

# Calibration

Table: Numerical Exercise Calibration

| Parameter                       | Value               | Description              | Target                                  |
|---------------------------------|---------------------|--------------------------|---|
| <i>Effective Borrowing Rate</i> |                     |                          |   |
| $\eta_1$                        | 1.7069              | Weight Scaling Factor    | Treasury Maturity Distribution          |
| <i>Macroeconomic Dynamics</i>   |                     |                          |   |
| $\rho$                          | 0.0400              | Discount Factor          | Long-Run Interest Rate                  |
| $\varsigma^{-1}$                | 1.0000              | Intertemporal Elasticity | Balanced Growth                         |
| $\kappa_r$                      | 0.9473              | Monetary Policy Inertia  | $\text{Cov}[r_t, r_{t-1}] = 3.5013$     |
| $\kappa_{z\pi}$                 | 0.5863              | Cost-Push Shock Inertia  | $\text{Cov}[\pi_t, \pi_{t-1}] = 0.9141$ |
| $\kappa_{zx}$                   | 0.2554              | Demand Shock Inertia     | $\text{Cov}[x_t, x_{t-1}] = 2.2908$     |
| $\phi_\pi$                      | 2.0420              | Inflation Taylor Coeff.  | $\text{Cov}[r_t, \pi_t] = 1.0006$       |
| $\phi_x$                        | 0.9709              | Output Taylor Coeff.     | $\text{Cov}[r_t, x_t] = 0.7722$         |
| $\delta$                        | 0.0459              | Nominal Rigidity         | $\text{Cov}[\pi_t, x_t] = -0.3015$      |
| $\sigma_r$                      | 0.0116              | Monetary Shock Vol.      | $\text{Var}[r_t] = 2.7066$              |
| $\sigma_{z\pi}$                 | 0.0068              | Cost-Push Shock Vol.     | $\text{Var}[\pi_t] = 0.5097$            |
| $\sigma_{zx}$                   | 0.0126              | Demand Shock Vol.        | $\text{Var}[x_t] = 1.5192$              |
| <i>Term Structure</i>           |                     |                          |   |
| $\theta_s(\tau)$                | $\delta(\tau - 2)$  | Short Factor Location    | LSAP Targets                            |
| $\theta_\ell(\tau)$             | $\delta(\tau - 10)$ | Long Factor Location     | LSAP Targets                            |
| $\alpha(\tau)$                  | 1.0000              | Habitat Elasticity       | Normalized                              |
| $\kappa_\beta$                  | 0.1710              | Habitat Factor Inertia   | QE1 Yield Curve Response                |
| $\sigma_{z\beta}$               | 0.0142              | Habitat Factor Vol.      | QE1 Yield Curve Response                |
| $a$                             | 1559.7              | Risk Aversion            | QE1 Yield Curve Response                |

# QE: Model vs. Data

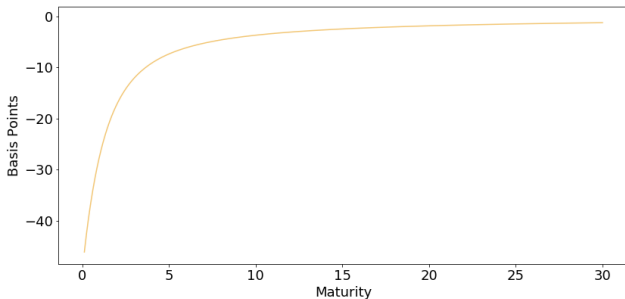


Notes: Yield curve response to the announcement of the initial round of QE on March 18, 2009 (light dotted line). The dark line corresponds to the yield curve response to a QE shock in the model. Source: Gurkaynak, Sack, and Wright (2007).  $\eta(\tau)$



# Yield Curve (Monetary Policy)

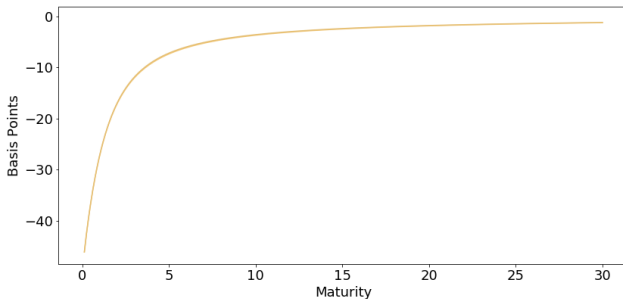
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Notes: yield curve response to a 50 b.p. monetary shock on impact, for different levels of risk aversion  $a$ . Darker lines correspond to higher levels of risk aversion.

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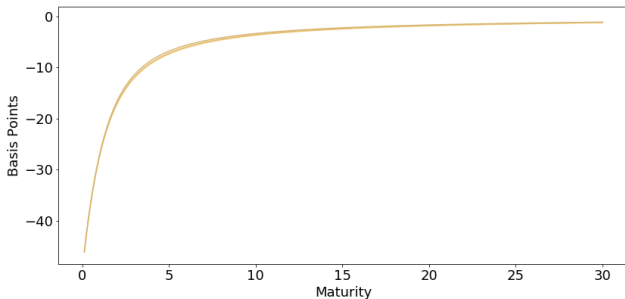
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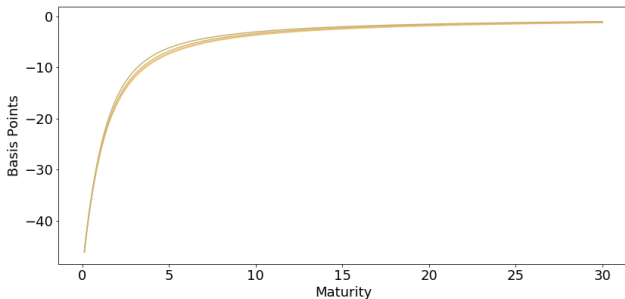
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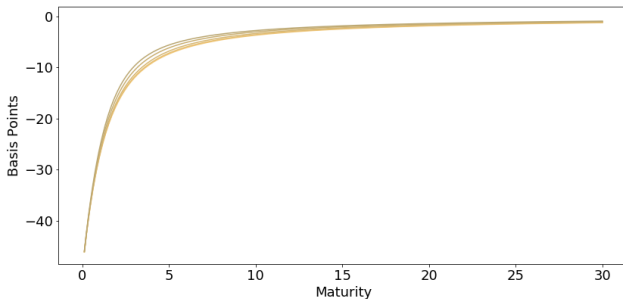
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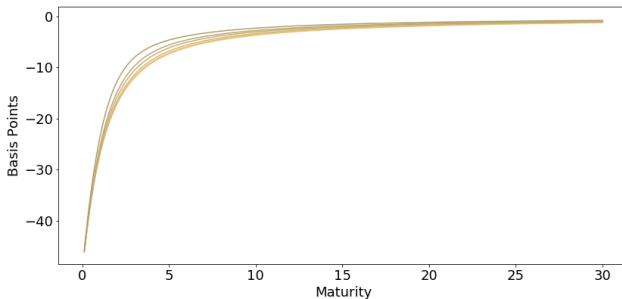
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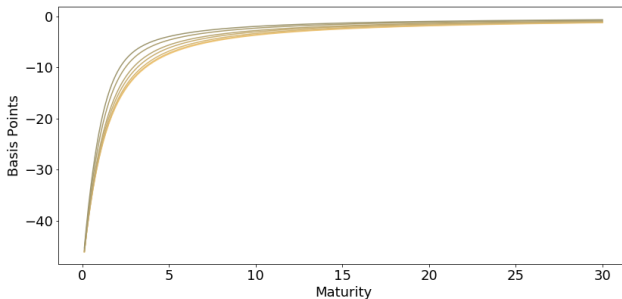
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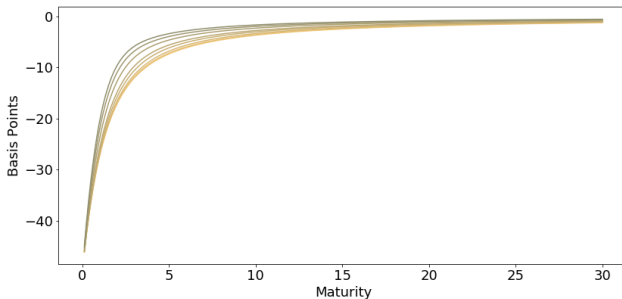
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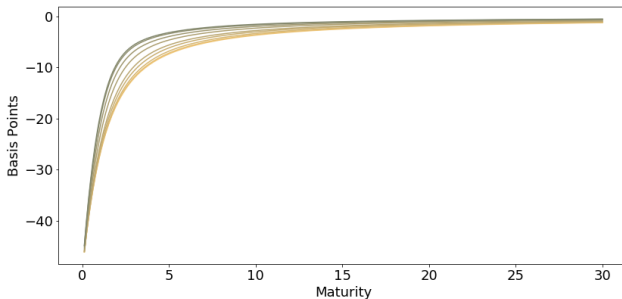


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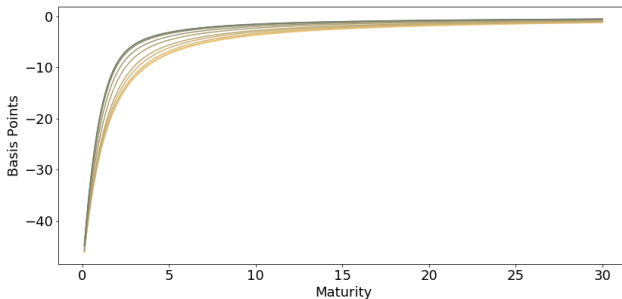
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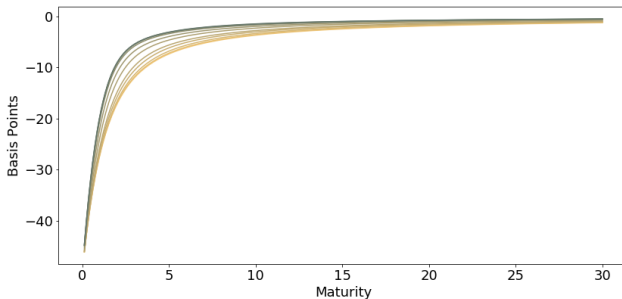
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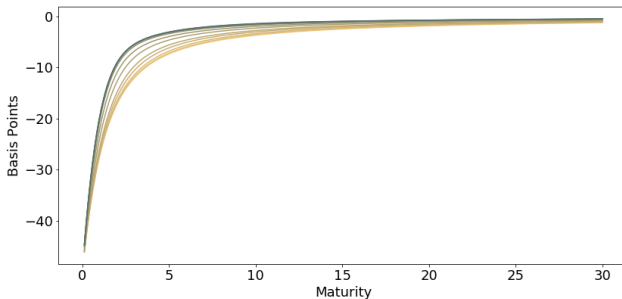
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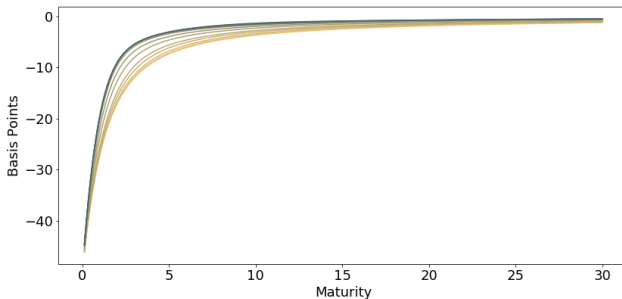
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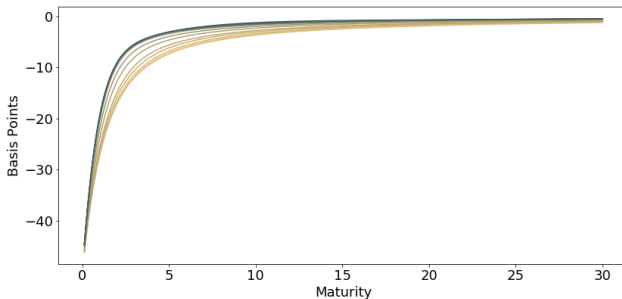
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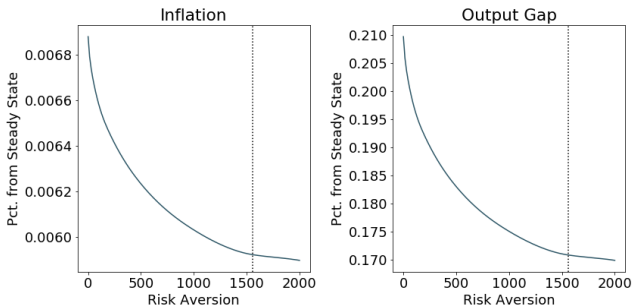
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# Aggregate Response (Monetary Policy)

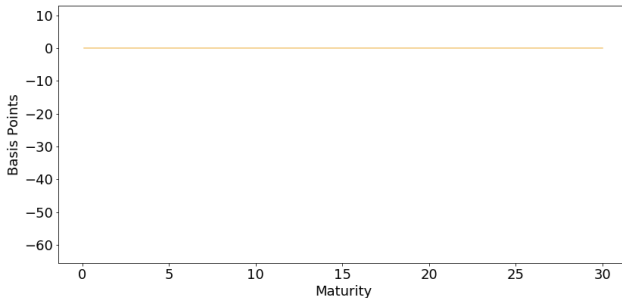
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Notes: inflation and output response a 50 b.p. monetary shock, for different levels of risk aversion  $a$ .

# Yield Curve (QE, long end)

---

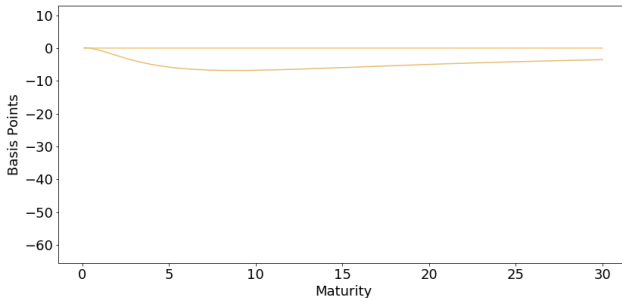


Notes: yield curve response to a “long” QE shock, for different levels of risk aversion  $\alpha$ . Darker lines correspond to higher levels of risk aversion.



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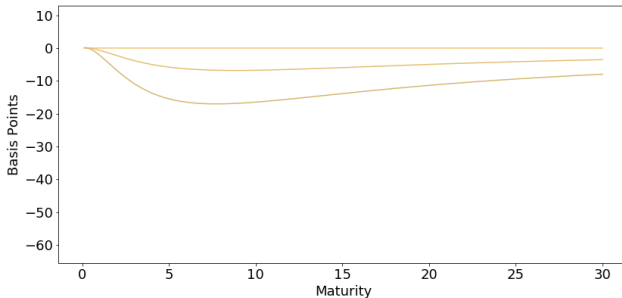
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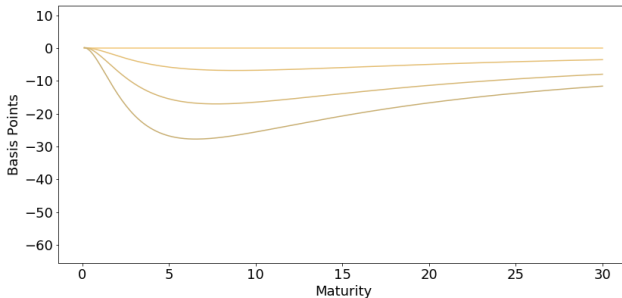
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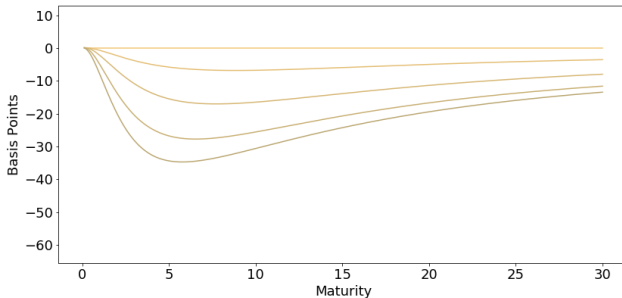
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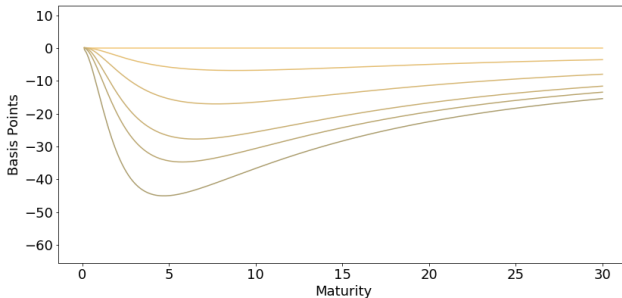
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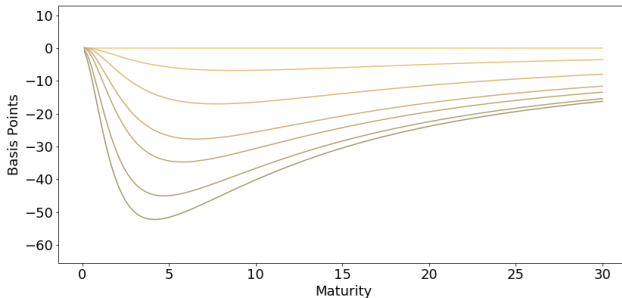
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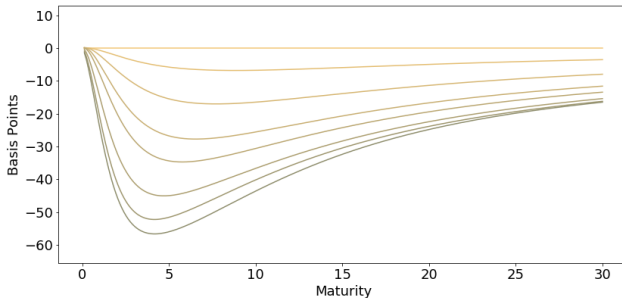
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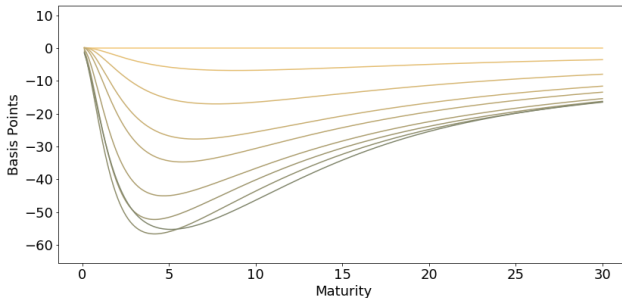
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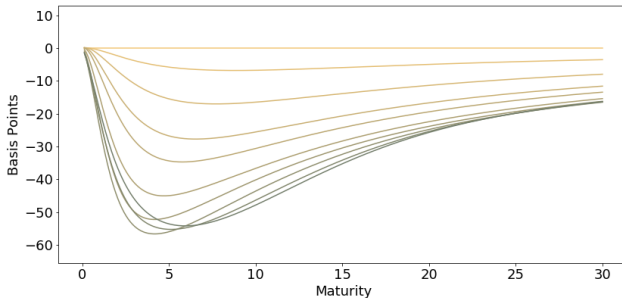


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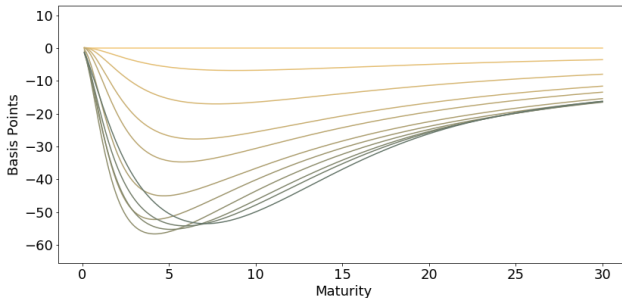
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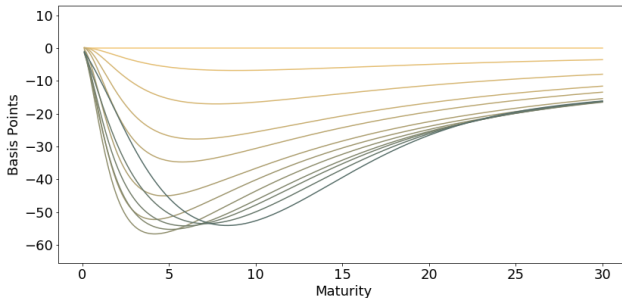
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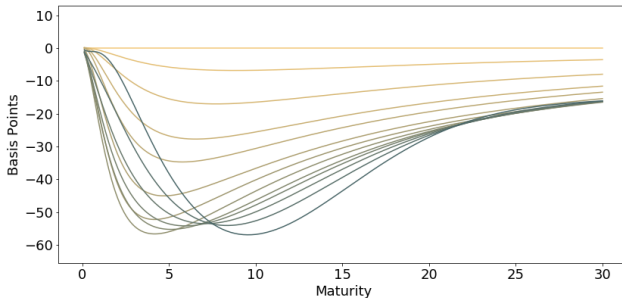
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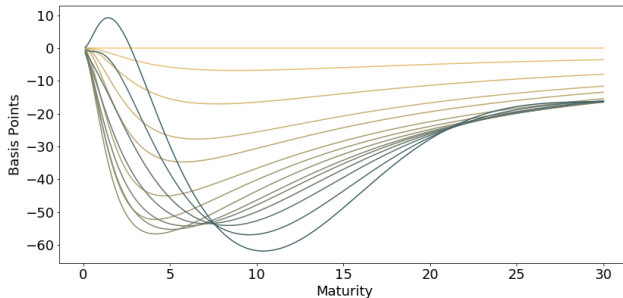
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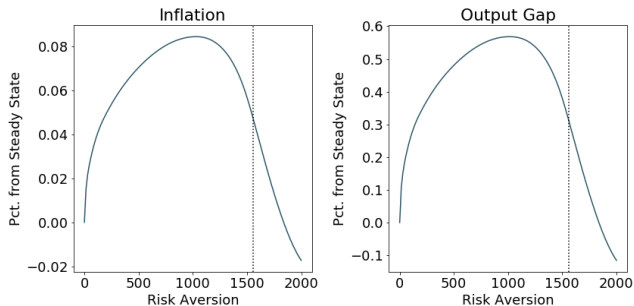
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# Aggregate Response (QE, long end)

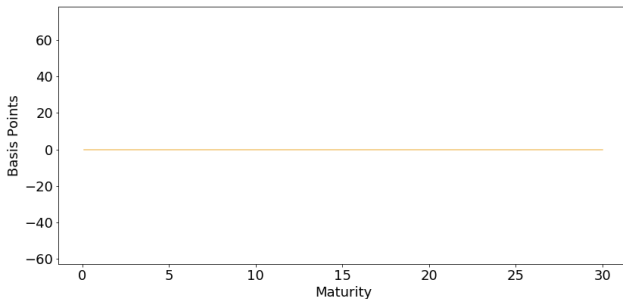
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Notes: inflation and output response to “long” QE shock on impact, for different levels of risk aversion  $a$ .

# Yield Curve (Operation Twist)

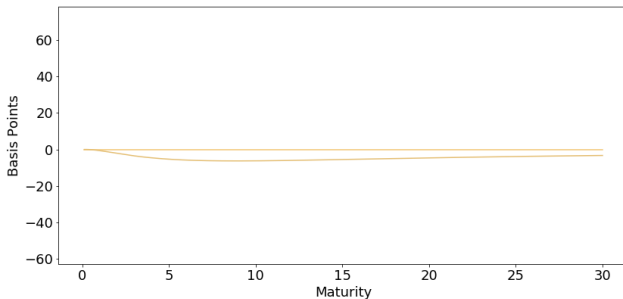
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Notes: yield curve response to an “Operation Twist” shock on impact, for different levels of risk aversion  $\alpha$ . Darker lines correspond to higher levels of risk aversion.

# Yield Curve (Operation Twist)

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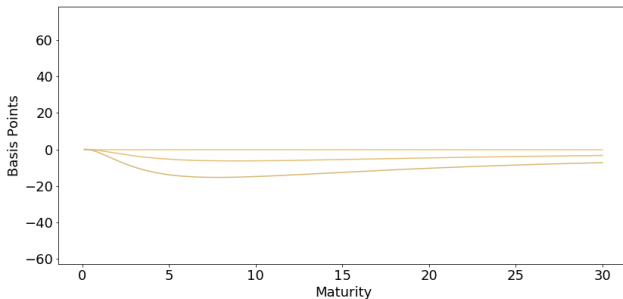


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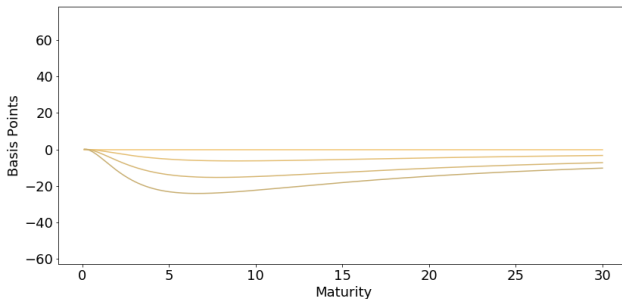
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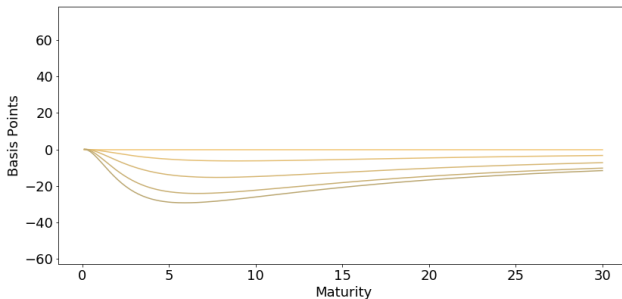
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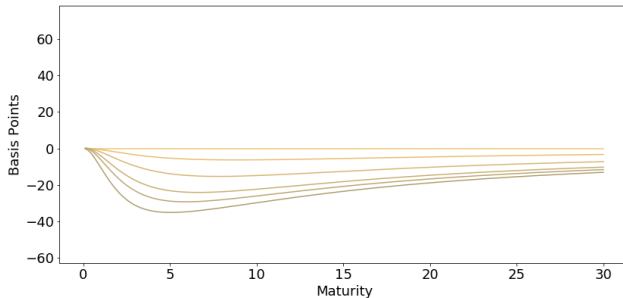
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# Yield Curve (Operation Twist)

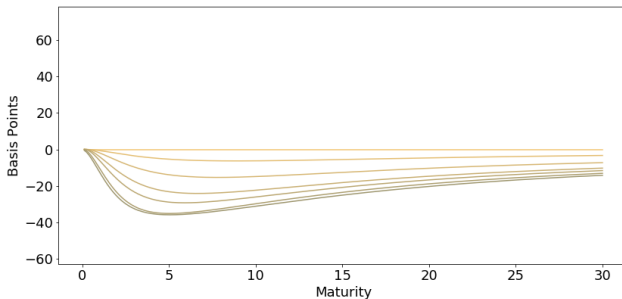
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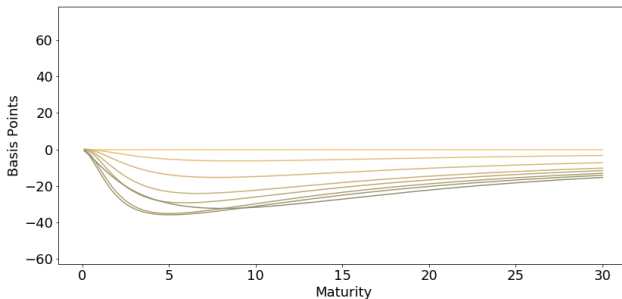
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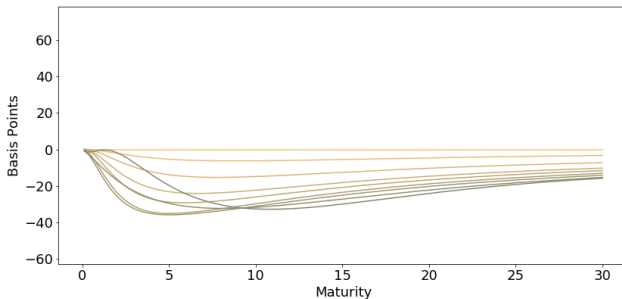
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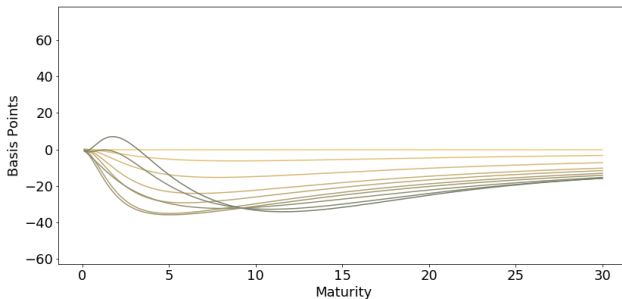
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# Yield Curve (Operation Twist)

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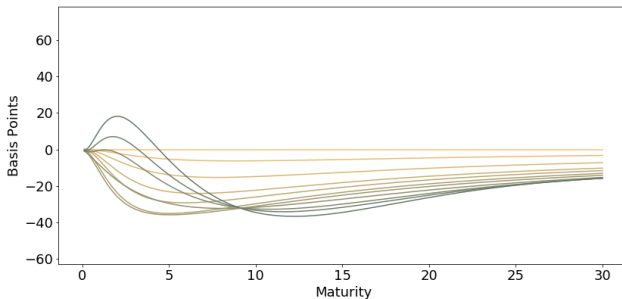


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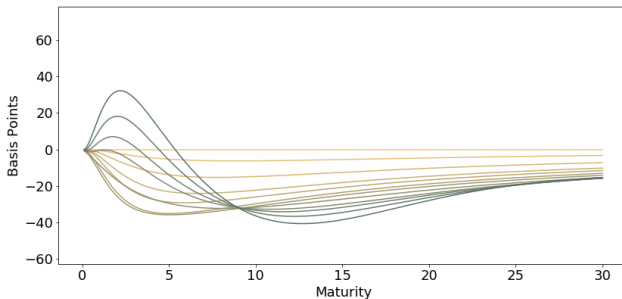
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# Yield Curve (Operation Twist)

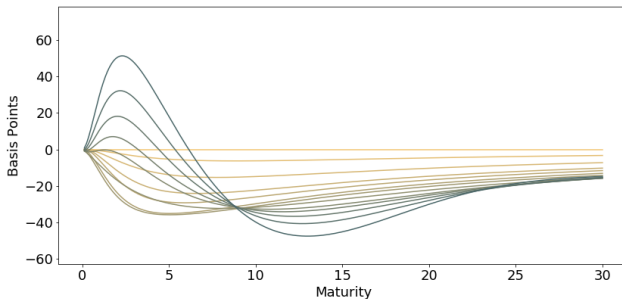
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# Yield Curve (Operation Twist)

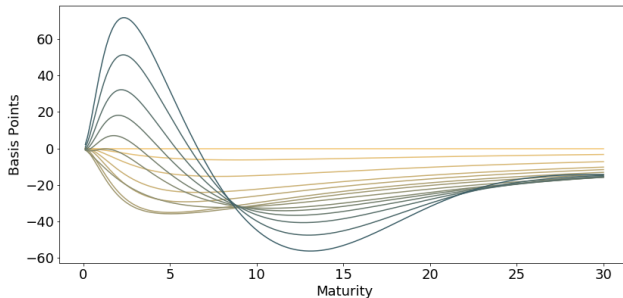
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# Yield Curve (Operation Twist)

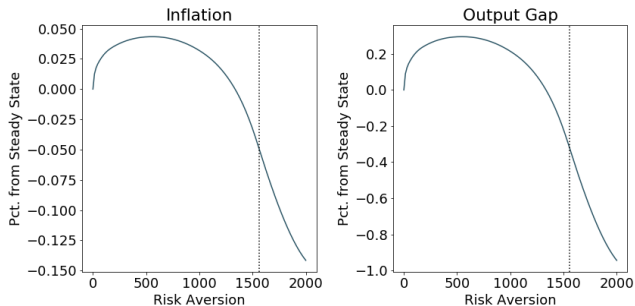
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Notes: yield curve response to an "Operation Twist" shock on impact, for different levels of risk aversion  $\alpha$ . Darker lines correspond to higher levels of risk aversion.

# Aggregate Response (Operation Twist)

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Notes: inflation and output response an “Operation Twist” shock, for different levels of risk aversion  $\alpha$ .

# Optimal Conventional Policy

---

- Can the planner improve outcomes?
- Loss function

$$E_0 \int_0^{\infty} e^{-\rho t} (w_{\pi} \pi_t^2 + w_x x_t^2) dt$$

# Optimal Conventional Policy

---

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# Optimal Conventional Policy

---

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- Loss function

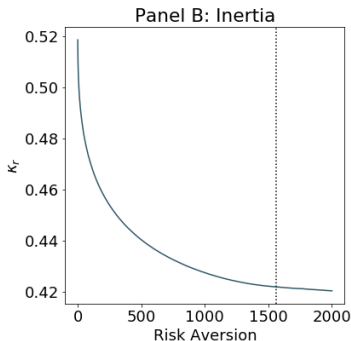
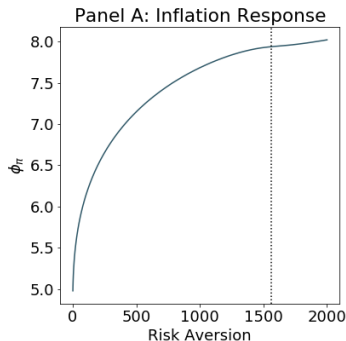
$$\min_{\phi_{\pi}, \kappa_r} E_0 \int_0^{\infty} e^{-\rho t} (w_{\pi} \pi_t^2 + w_x x_t^2) dt$$

- Optimal inflation response and inertia as financial disruptions increase conditional distribution



# Optimal Response: More Aggressive in Crises

---



Notes: optimal policy coefficients on inflation (Panel A) and inertia (Panel B) as risk aversion increases. Planner weights:  $w_\pi = 1$ ,  $w_x = 0.1$ .

# Stabilizing LSAPs

---

- Can LSAPs be used to ensure determinacy?
- Endogenous QE purchases:

$$d\beta_t = -\kappa_\beta \left( \beta_t - \phi_\pi^\beta \pi_t \right) dt$$

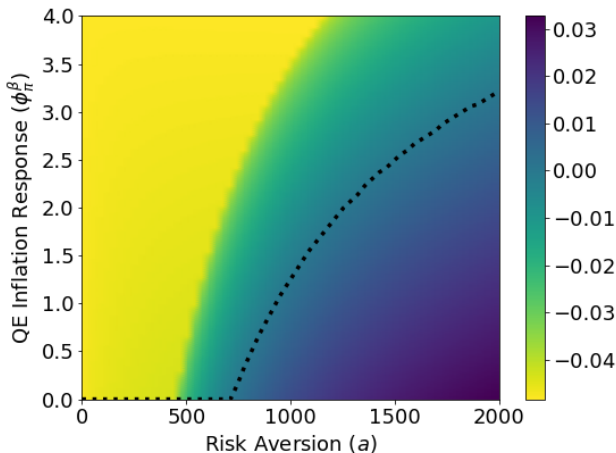
# Stabilizing LSAPs

---

- Can LSAPs be used to ensure determinacy?
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$$d\beta_t = -\kappa_\beta \left( \beta_t - \phi_\pi^\beta \pi_t \right) dt$$

# QE and Determinacy



Notes: determinacy conditions as a function of risk aversion (x-axis) and endogenous response of QE to inflation (y-axis). Darker colors correspond to larger values of the unstable eigenvalue. The dotted black line demarcates the region of determinacy.

## Concluding Remarks

---

- Develops a unified, parsimonious framework to study conventional and unconventional monetary policies
- Transmission depends crucially on the risk-bearing capacity of financial markets

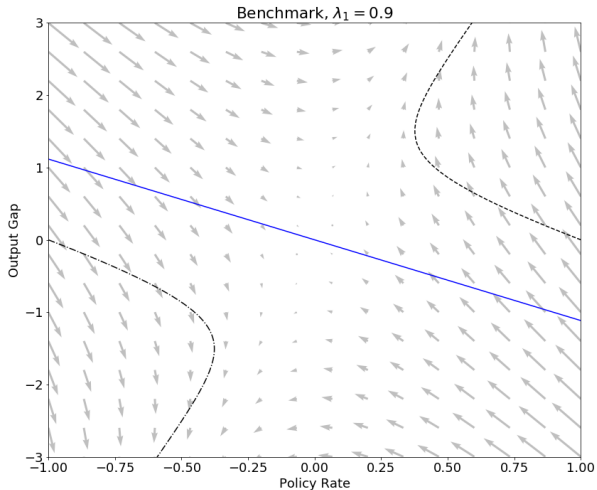
## Concluding Remarks

---

- Develops a unified, parsimonious framework to study conventional and unconventional monetary policies
- Transmission depends crucially on the risk-bearing capacity of financial markets
- Future work:
  - ▶ Macroprudential policies, default risk
  - ▶ Monetary policy in open economies
  - ▶ Debt management

## APPENDIX

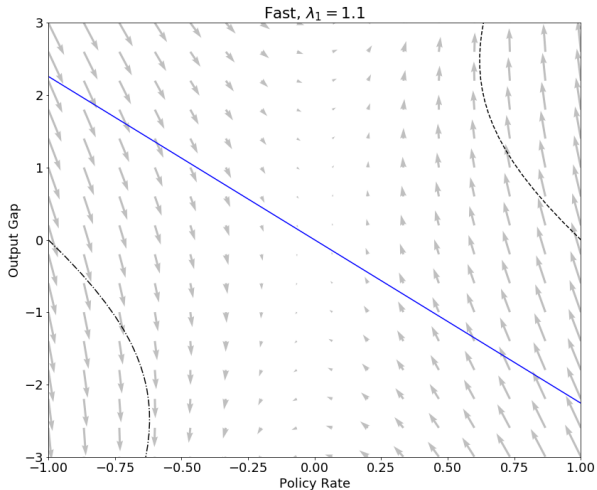
# Phase Diagrams



Notes: phase diagrams of the dynamics of output and the policy rate as the equilibrium mean reversion rate of shocks varies.

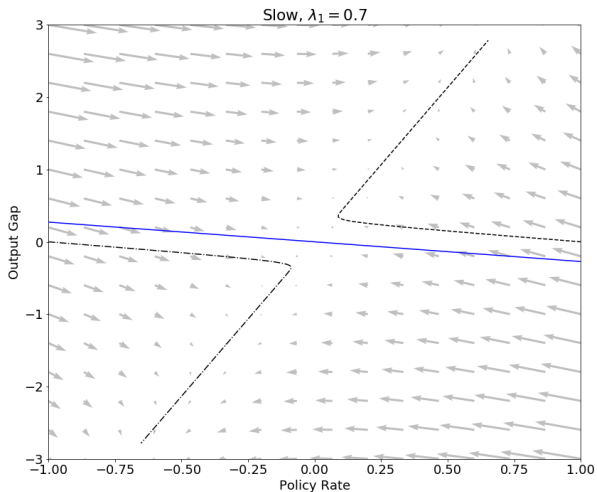


# Phase Diagrams



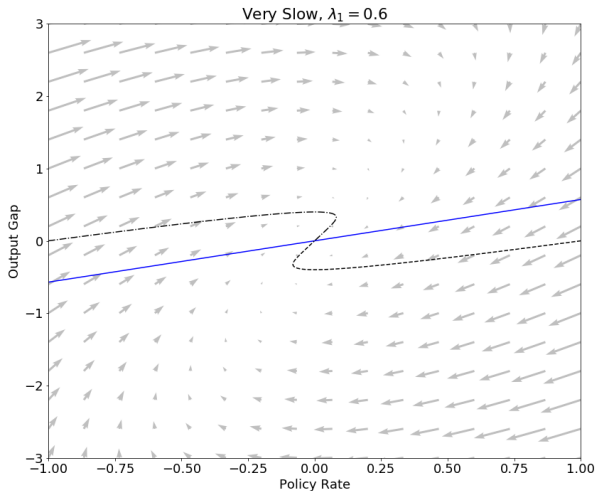
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# Phase Diagrams



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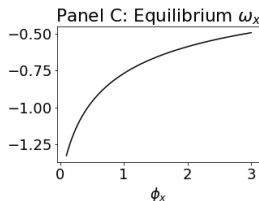
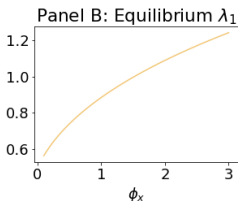
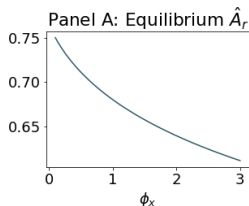
# Phase Diagrams



Notes: phase diagrams of the dynamics of output and the policy rate as the equilibrium mean reversion rate of shocks varies.

# Implications – Conventional Policy

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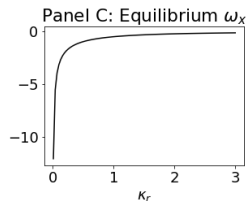
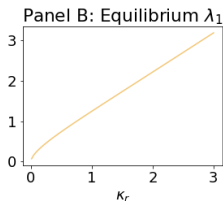
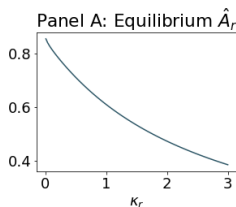


Notes: equilibrium changes in sensitivity to the short rate  $\hat{A}_r$  and monetary shock reversion  $\lambda_1$  as central bank response to output  $\phi_x$  increases.

[back](#)

# Implications – Conventional Policy

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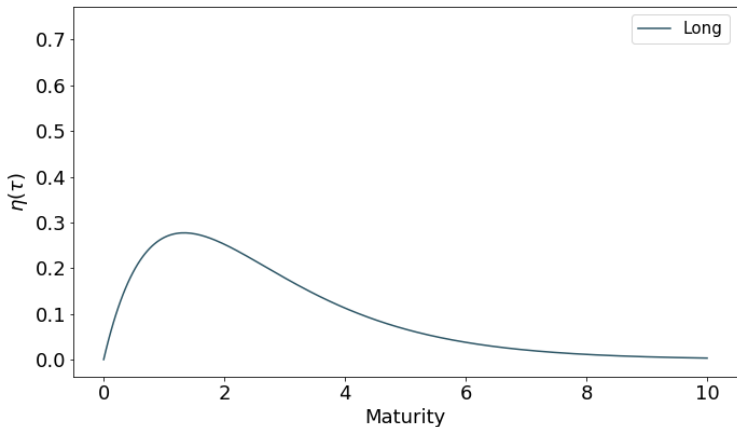


Notes: equilibrium changes in sensitivity to the short rate  $\hat{A}_r$  and monetary shock reversion  $\lambda_1$  as central bank inertia  $\kappa_r$  increases.

[back](#)

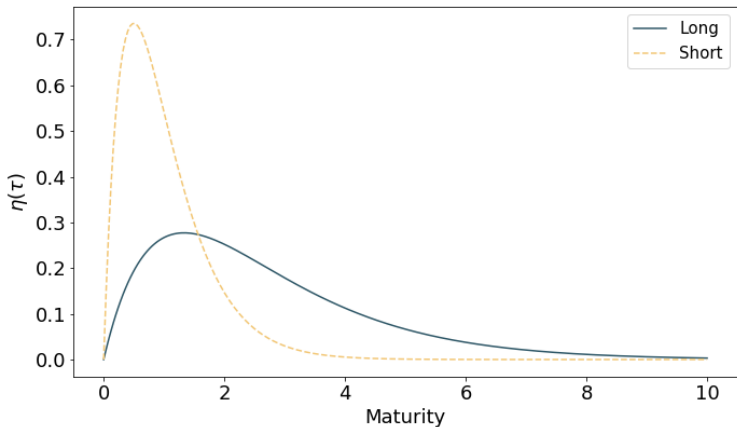
# Sensitivity to Long Rates

---



Notes: different weighting function  $\eta(\tau)$  in the determination of the effective borrowing rate  $\tilde{r}_t$ .

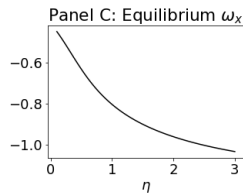
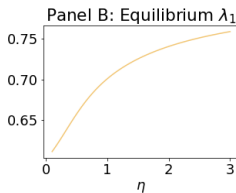
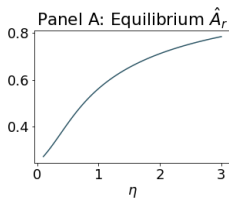
# Sensitivity to Long Rates



Notes: different weighting function  $\eta(\tau)$  in the determination of the effective borrowing rate  $\tilde{r}_t$ .

# Implications – Sensitivity to Long Rates

---



Notes: equilibrium changes in sensitivity to the short rate  $\hat{A}_r$  and monetary shock reversion  $\lambda_1$  as the weighting function  $\eta(\tau)$  shifts towards short-term bonds.

[back](#)



## Forward Guidance

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- Central bank announces a peg:  $r_0 = r^\diamond$  and

$$dr_t = \begin{cases} -\kappa_r^\diamond(r_t - r^\diamond)dt + \sigma_r^\diamond dB_{r,t} & \text{if } 0 < t < t^\diamond \\ -\kappa_r(r_t - \phi_x x_t - r^*)dt + \sigma_r dB_{r,t} & \text{if } t \geq t^\diamond \end{cases}$$

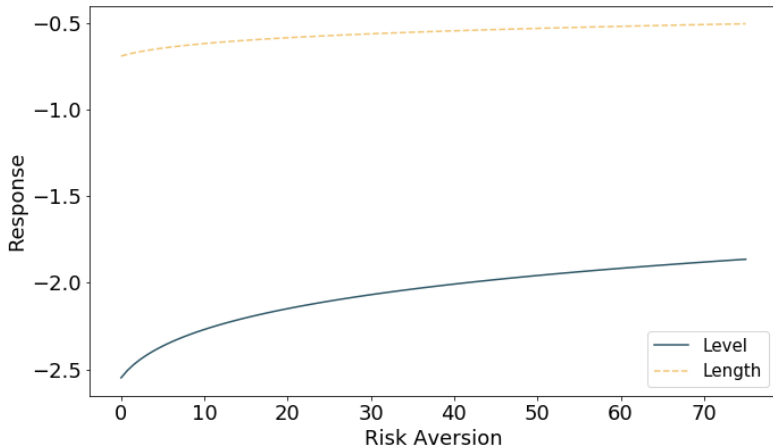
- Affine coefficient functions during peg:

$$\begin{aligned} -\log P_{t,\tau} &= A_r^\diamond(\tau)r_t + C^\diamond(\tau) \\ \implies \tilde{r}_t &= \hat{A}_r^\diamond r_t + \hat{C}^\diamond \end{aligned}$$

- Rational expectations dynamics for output:

$$\frac{\partial x_0}{\partial r^\diamond} = \omega_x - t^\diamond \varsigma^{-1} \hat{A}_r^\diamond, \quad \frac{\partial^2 x_0}{\partial r^\diamond \partial t^\diamond} = -\varsigma^{-1} \hat{A}_r^\diamond$$

# Response to Forward Guidance



Notes: plots of  $\frac{\partial x_0}{\partial r^\diamond}$  ("level") and  $\frac{\partial^2 x_0}{\partial r^\diamond \partial t^\diamond}$  ("length") as risk aversion increases.

[back](#)

# Long-Run Variance

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- State-space representation

$$d\mathbf{y}_t = -\Gamma (\mathbf{y}_t - \mathbf{y}^{SS}) dt + \mathbf{S} d\mathbf{B}_t, \quad \mathbf{x}_t = \Omega (\mathbf{y}_t - \mathbf{y}^{SS})$$

- Conditional distribution  $\mathbf{y}_t | \mathbf{y}_0 \sim N(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$  where

$$\boldsymbol{\mu}_t = \mathbf{y}^{SS} + e^{-\Gamma t}(\mathbf{y}_0 - \mathbf{y}^{SS}), \quad \boldsymbol{\Sigma}_t = \int_0^t e^{\Gamma(u-t)} \boldsymbol{\Sigma} e^{\Gamma^T(u-t)} du$$

- Present-discounted value

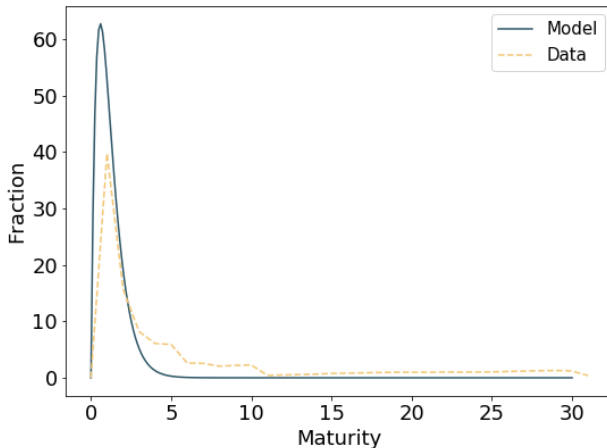
$$\begin{aligned} \tilde{\boldsymbol{\Sigma}}_{\infty} &\equiv \int_0^{\infty} e^{-\rho t} \boldsymbol{\Sigma}_t dt \\ \implies \text{vec } \tilde{\boldsymbol{\Sigma}}_{\infty} &= (\Gamma \oplus \Gamma)^{-1} (\rho \mathbf{I} + \Gamma \oplus \Gamma)^{-1} \text{vec } \boldsymbol{\Sigma} \end{aligned}$$

- Jump variables

$$\tilde{\boldsymbol{\Sigma}}_{\infty}^{\mathbf{x}} = \Omega \tilde{\boldsymbol{\Sigma}}_{\infty} \Omega^T$$

# Effective Borrowing Rate Weights

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Notes: average maturity distribution of outstanding Treasury debt (light dotted line). The dark line corresponds to the effective borrowing rate weights in the model. Source: FRED.