Homework 11

DUE: Wednesday, 1. Dec. 2021

Exercise 1. Let $\mathsf{C}, \mathsf{D} \in \mathsf{Cat}$. Show that $N(\mathsf{C} \star \mathsf{D}) \cong N(\mathsf{C}) \star N(\mathsf{D})$. Show that $\Delta^n \star \Delta^k \cong \Delta^{n+k+1}$.

Exercise 2. Note that for $K, X \in \mathsf{Set}_\Delta$, there is a canonical inclusion of simplicial sets $K \hookrightarrow K \star X$. Show that the assignment

$$\mathsf{Set}_\Delta \longrightarrow (\mathsf{Set}_\Delta)_{K/}$$
$$X \longmapsto K \hookrightarrow K \star X$$

defines a functor, and that this functor preserves colimits.

Exercise 3. Show that for any simplicial sets $X, Y \in \mathsf{Set}_\Delta$, there is an isomorphism $(X \star Y)^{\mathrm{op}} \cong Y^{\mathrm{op}} \star X^{\mathrm{op}}$. Let $\mathcal{C}$ be a quasi-category, and $f : K \to \mathcal{C}$ a morphism of simplicial sets. Show that $(\mathcal{C}_{f/})^{\mathrm{op}} \cong (\mathcal{C}^{\mathrm{op}})_{/f}$.

Exercise 4. Given a map $p : X \to Y$ of simplicial sets, define the *fibre* of p over $y \in Y_0$ to be the pullback

$$
\begin{array}{ccc}
X_y & \longrightarrow & X \\
\downarrow & & \downarrow \\
\Delta^0 & \xrightarrow{\{y\}} & Y
\end{array}
$$

Let $\mathcal{C}$ be a quasi-category, and $x, y \in \mathcal{C}$, what information is necessary to specify a simplex in the fibre of $\mathcal{C}_{/y} \to \mathcal{C}$ over x? How would you interpret this information?