

Backward Euler update formula derivation

General update formula

$$y_{k+1} = y_k + dt \cdot f(t_{k+1}, y_k + 1) \quad (1)$$

In concrete terms for the equations of motion, it is

$$x_{k+1} = x_k + dt \cdot v_{k+1} \quad (2a)$$

$$v_{k+1} = v_k + dt \cdot \left(\frac{k(-x_{k+1} - L)}{m} - \frac{d \cdot v_{k+1}}{m} - g \right) \quad (2b)$$

Insert (2a) into (2b) and reorder.

$$v_{k+1} = v_k + dt \cdot \left(\frac{k(-x_{k+1} - L)}{m} - \frac{d \cdot v_{k+1}}{m} - g \right) \quad (3a)$$

$$= dt \left(-\frac{k}{m} dt - \frac{d}{m} \right) v_{k+1} + v_k + dt \left(\frac{k}{m} (-x_k - L) - g \right) \quad (3b)$$

$$= \frac{v_k - dt \left(\frac{k}{m} (x_k + L) - g \right)}{1 + dt \left(\frac{k \cdot dt + d}{m} \right)} \quad (3c)$$