Physically-based simulation in Computer Graphics

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October 2, 2013

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Serie 1

First lets do a sketch as seen in figure 0.1 to omit any misunderstanding of the problem. We would like to find the constants c_1, c_2 of the equation

$$y(t) = c_1 e^{\alpha t}\cos(\beta t) + c_2 e^{\alpha t}\sin(\beta t) - L - \frac{mg}{k}, \tag{1}$$

where the constraints are the rest state initial conditions, which means there is no energy in the system except gravitational energy at $t = 0$. This has the same physical meaning as the spring having its rest length L [1] and no kinetic energy at $t = 0$. This can be expressed as

$$y(t = 0) = -L, \tag{2}$$

and

$$\dot{y}(t = 0) = 0. \tag{3}$$

The position of p_2 computed with the given solution Eq. 1 is

$$y(t = 0) = c_1 - L - \frac{mg}{k}. \tag{4}$$

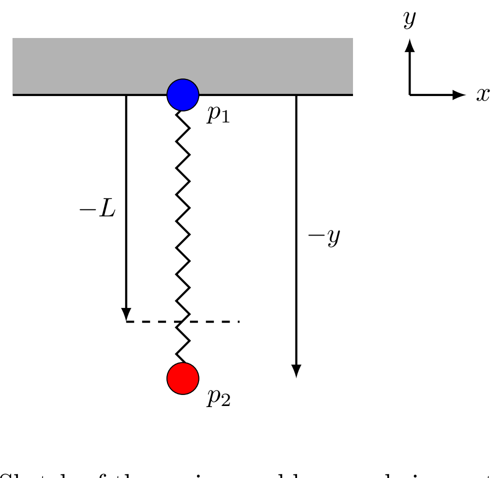

Figure 0.1: Sketch of the spring problem, p_2 being not in rest state.

Comparing Eq. 2 and Eq. 4 gives us

$$c_1 = \frac{mg}{k}. \tag{5}$$

To get c_2 we need to compute $\dot{y}(t=0)$ from Eq. 1

$$\dot{y}(t=0) = \left(c_1\alpha + c_2\beta - L - \frac{mg}{k}\right), \tag{6}$$

where $c_1 = mg/k$ leads to

$$\dot{y}(t=0) = \frac{mg}{k}\alpha + c_2\beta - L - \frac{mg}{k}. \tag{7}$$

Using the constraint in Eq. 3 finally gives us

$$c_2 = \frac{mg}{\beta k}(1-\alpha) + \frac{L}{\beta}. \tag{8}$$