Physically-based simulation in Computer Graphics

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Disney Research

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Serie 1

Exercise 1

Backward Euler update formula derivation

General update formula

$$y_{k+1} = y_k + \mathrm{d}t \cdot f(t_{k+1}, yk + 1) \tag{1}$$

In concrete terms for the equations of motion, it is

$$x_{k+1} = x_k + \mathrm{d}t \cdot v_{k+1} \tag{2a}$$

$$v_{k+1} = v_k + \mathrm{d}t \cdot \left(\frac{k(-x_{k+1} - L)}{m} - \frac{d \cdot v_{k+1}}{m} - g \right) \tag{2b}$$

Insert (2a) into (2b) and reorder.

$$\begin{aligned} v_{k+1} &= v_k + \mathrm{d}t \cdot \left(\frac{k(-x_{k+1} - L)}{m} - \frac{d \cdot v_{k+1}}{m} - g \right) && \text{(3a)} \\ &= \mathrm{d}t \left(-\frac{k}{m}\mathrm{d}t - \frac{d}{m} \right) v_{k+1} + v_k + \mathrm{d}t \left(\frac{k}{m}(-x_k - L) - g \right) && \text{(3b)} \\ &= \frac{v_k - \mathrm{d}t \left(\frac{k}{m}(x_k + L) - g \right)}{1 + \mathrm{d}t \left(\frac{k \cdot \mathrm{d}t + d}{m} \right)} && \text{(3c)} \end{aligned}$$

Exercise 2

2.1. Analytic solution

First lets do a sketch as seen in figure 0.1 to omit any misunderstanding of the problem. We would like to find the constants c_1, c_2 of the equation

$$y(t) = c_1 e^{\alpha t} \cos(\beta t) + c_2 e^{\alpha t} \sin(\beta t) - L - \frac{mg}{k}, \tag{4}$$

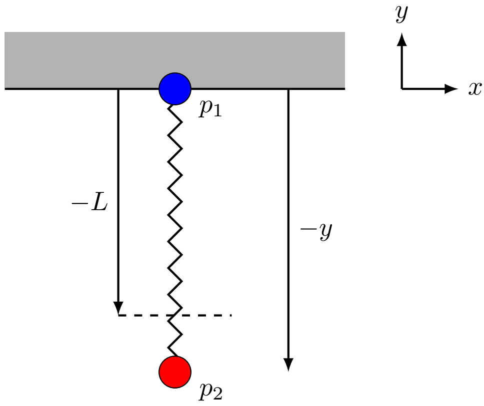


Figure 0.1: Sketch of the spring problem, p_2 being not in rest state.

where the constraints are the rest state initial conditions, which means there is no energy in the system except gravitational energy at $t = 0$. This has the same physical meaning as the spring having its rest length L [1] and no kinetic energy at $t = 0$. This can be expressed as

$$y(t = 0) = -L, \tag{5}$$

and

$$\dot{y}(t = 0) = 0. \tag{6}$$

The position of p_2 computed with the given solution Eq. 4 is

$$y(t = 0) = c_1 - L - \frac{mg}{k}. \tag{7}$$

Comparing Eq. 5 and Eq. 7 gives us

$$c_1 = \frac{mg}{k}. \tag{8}$$

To get c_2 we need to compute $\dot{y}(t = 0)$ from Eq. 4

$$\dot{y}(t = 0) = \left(c_1\alpha + c_2\beta - L - \frac{mg}{k}\right), \tag{9}$$

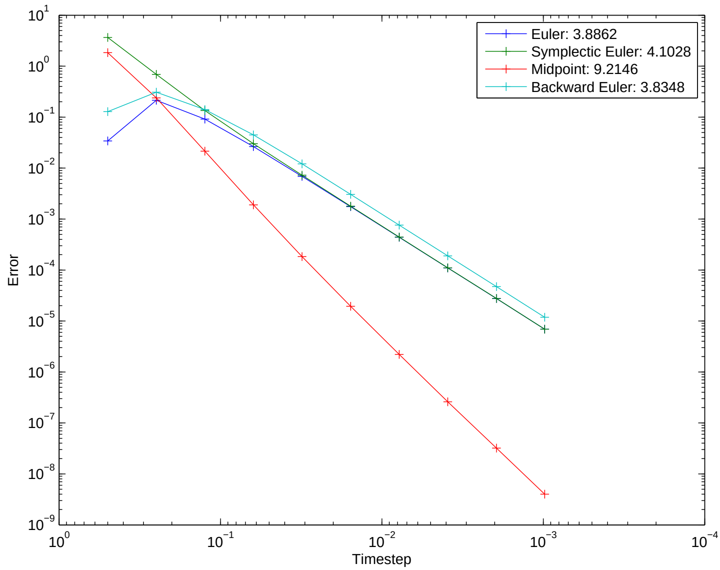
where $c_1 = mg/k$ leads to

$$\dot{y}(t = 0) = \frac{mg}{k}\alpha + c_2\beta - L - \frac{mg}{k}. \tag{10}$$

Using the constraint in Eq. 6 finally gives us

$$c_2 = \frac{mg}{\beta k}(1 - \alpha) + \frac{L}{\beta}. \tag{11}$$

2.2. Error convergence analysis



(a) damping factor $d = 0.01$

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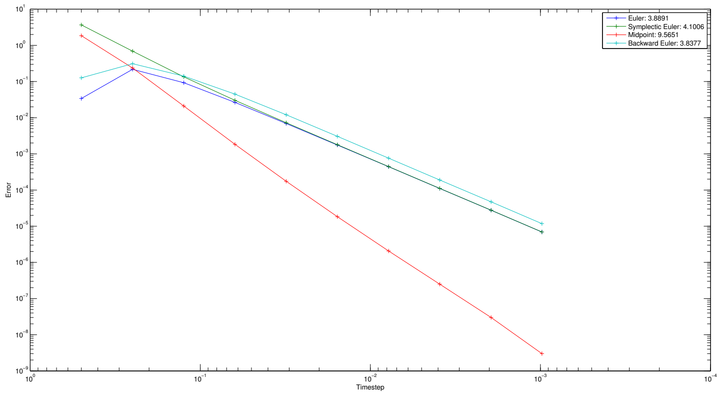
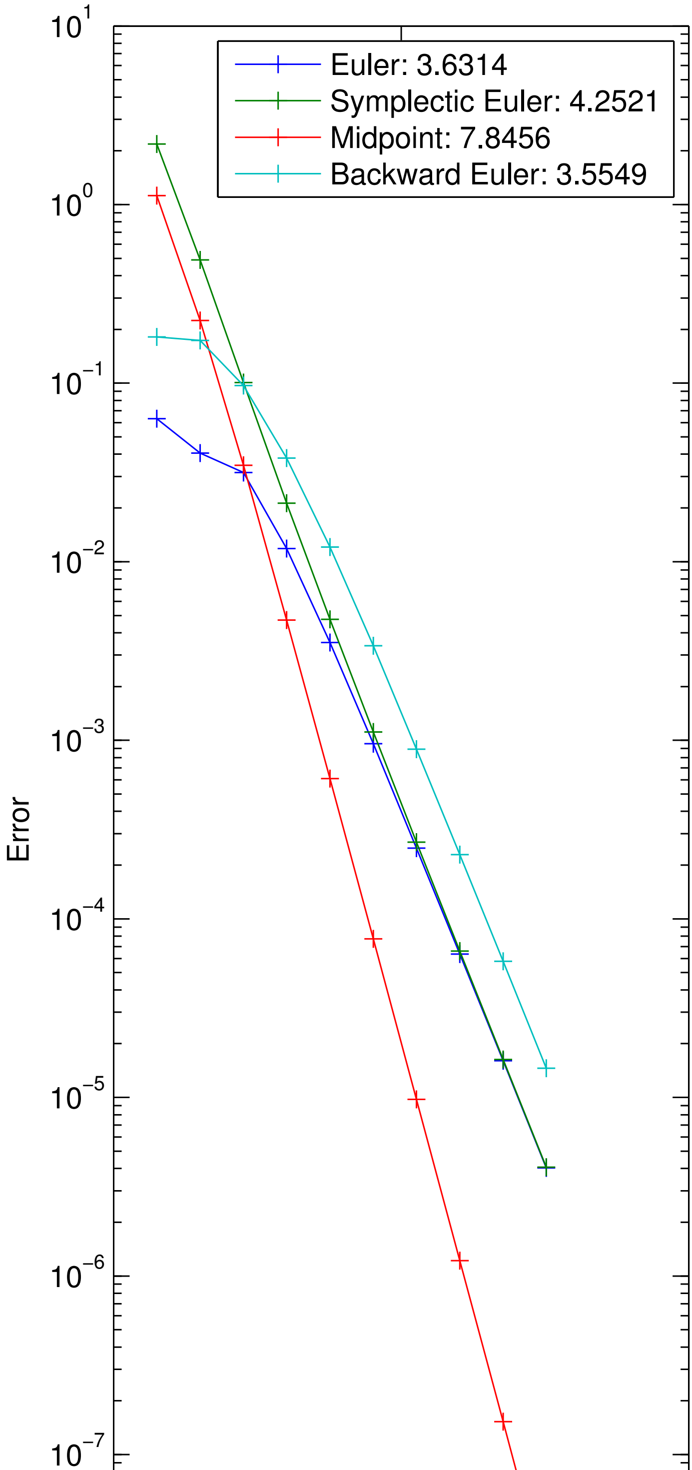
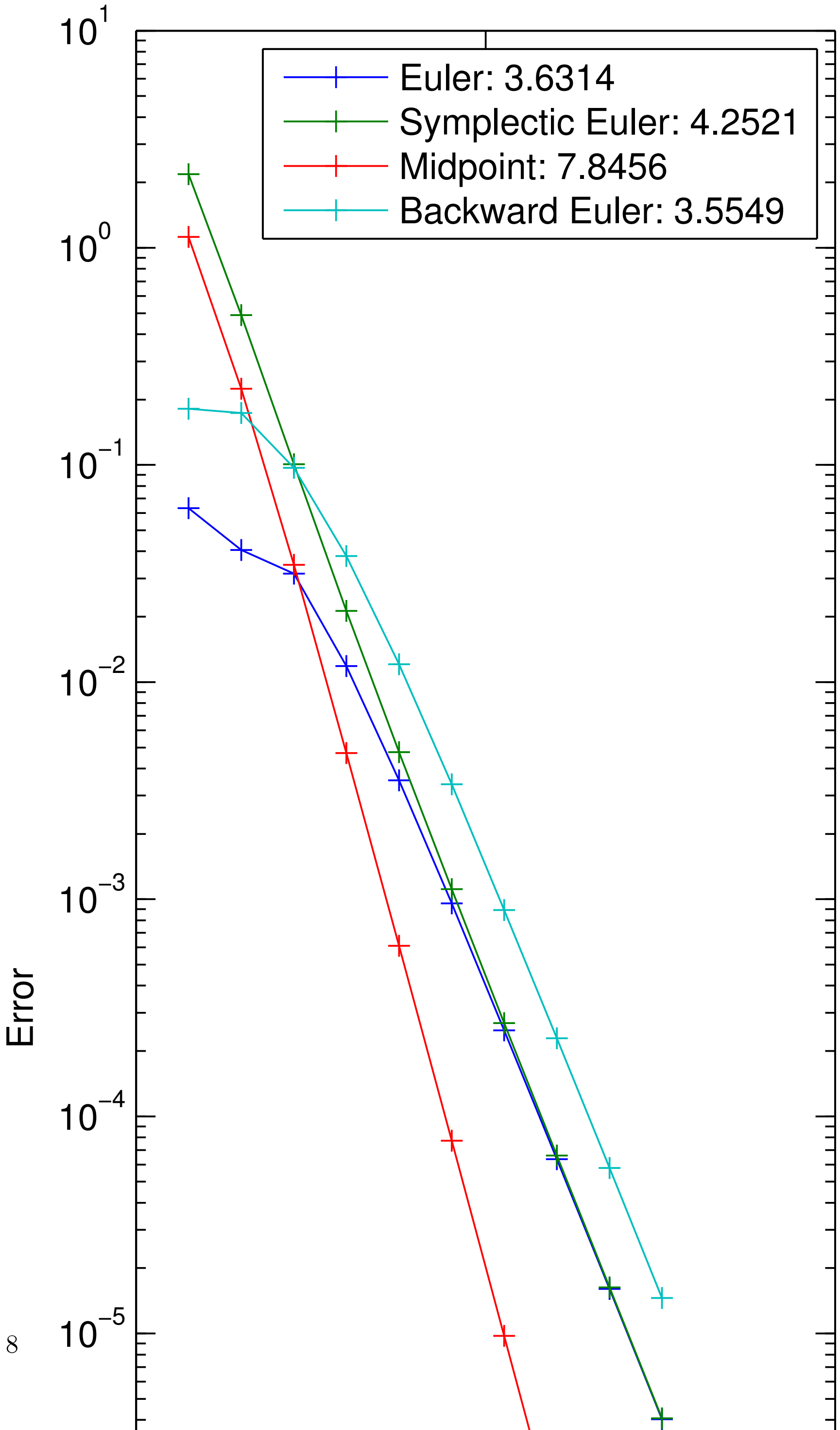


(b) damping factor $d = 0$


Euler: 3.6314
Symplectic Euler: 4.2521
Midpoint: 7.8456
Backward Euler: 3.5549
Error
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}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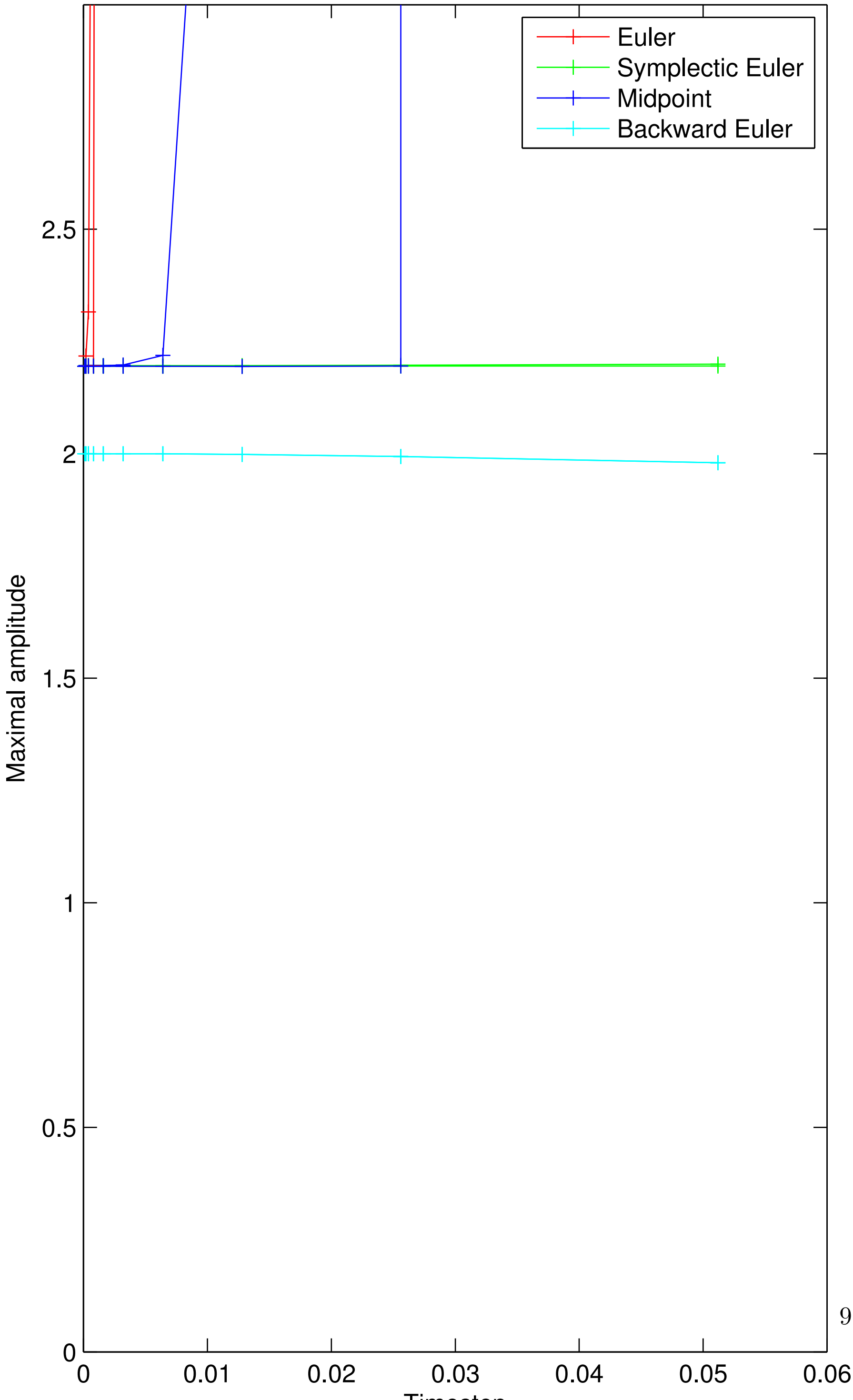
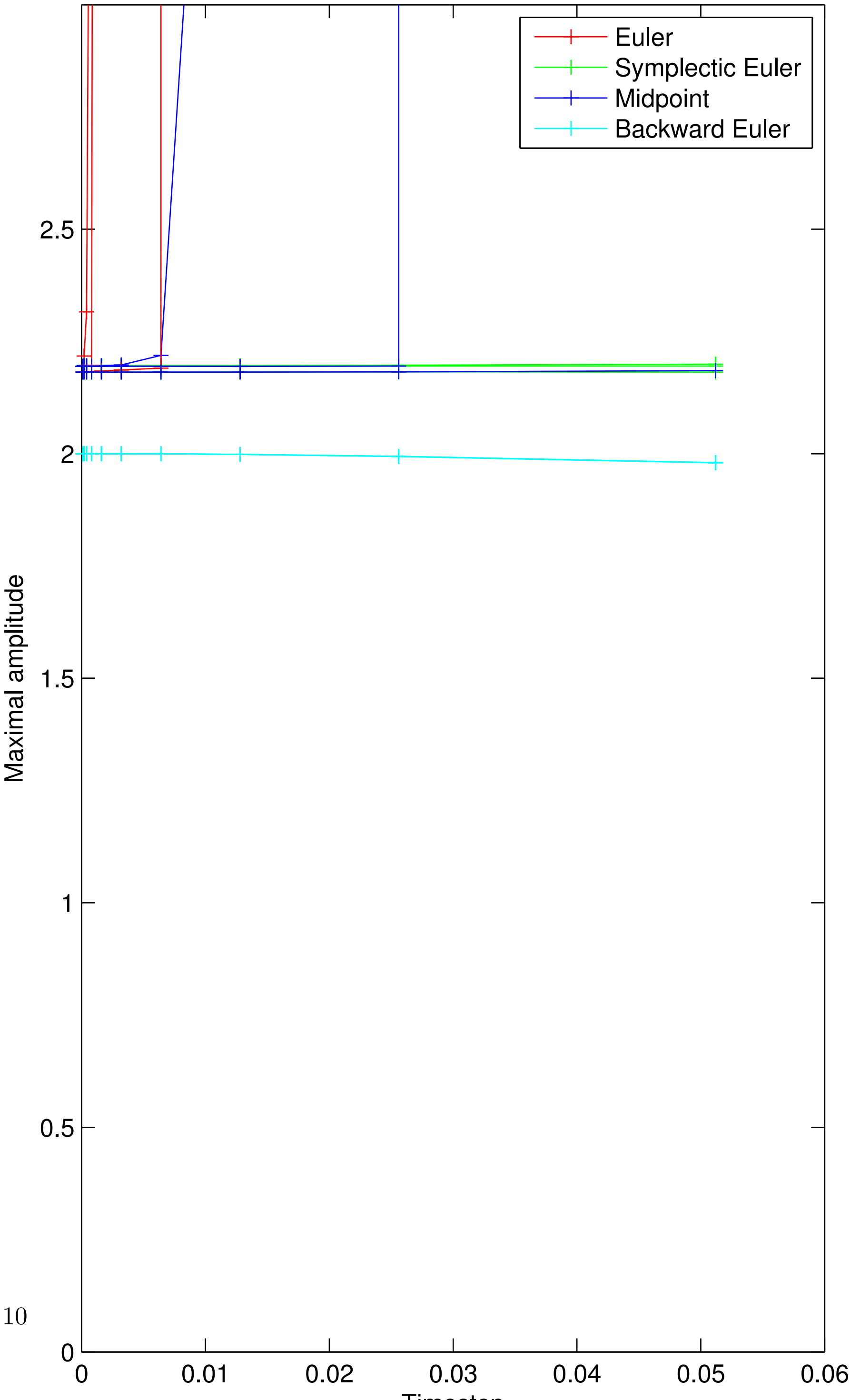


Euler
Symplectic Euler
Midpoint
Backward Euler
Maximal amplitude
2.5
2
1.5
1
0.5
0
0.01
0.02
0.03
0.04
0.05
0.06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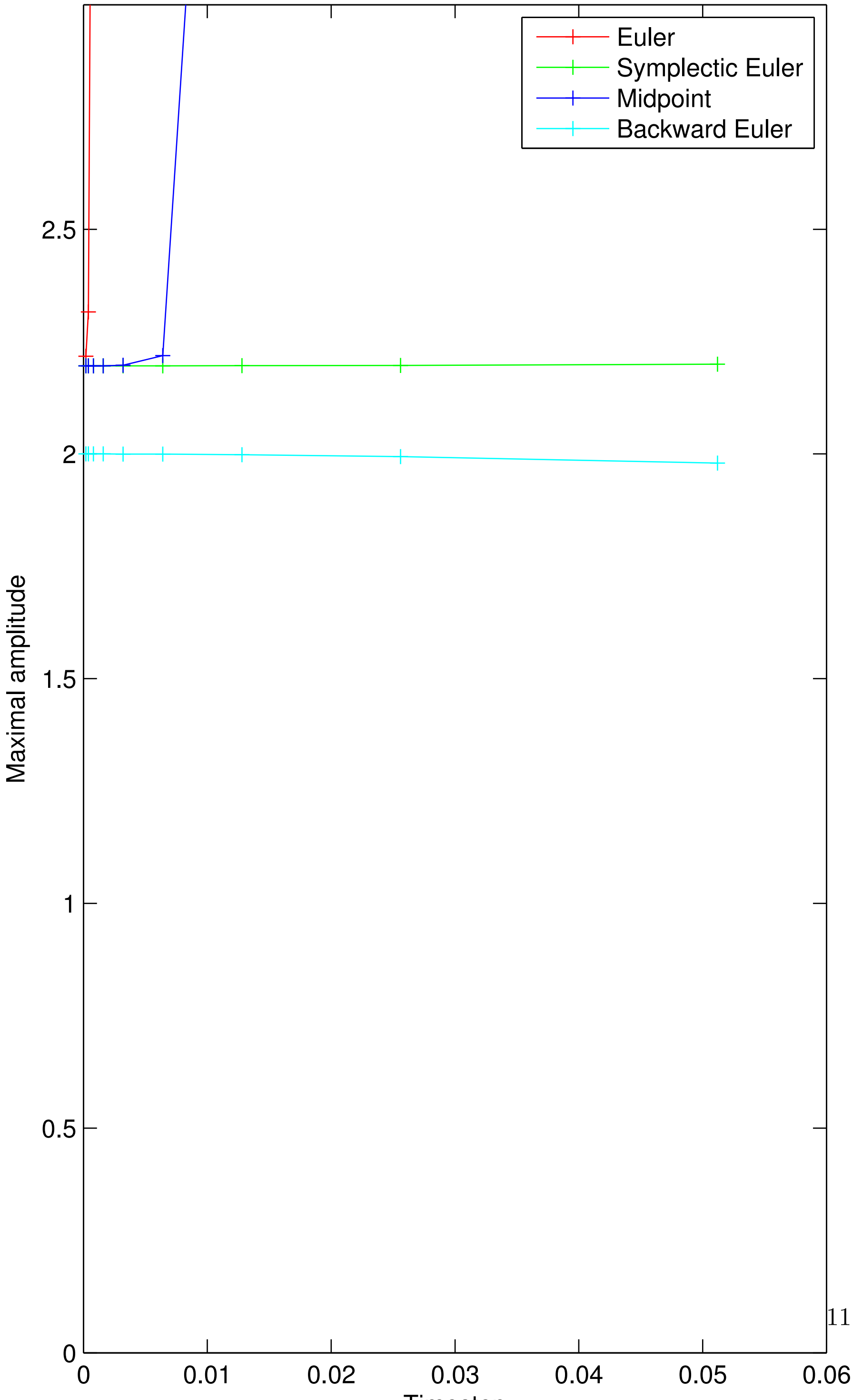
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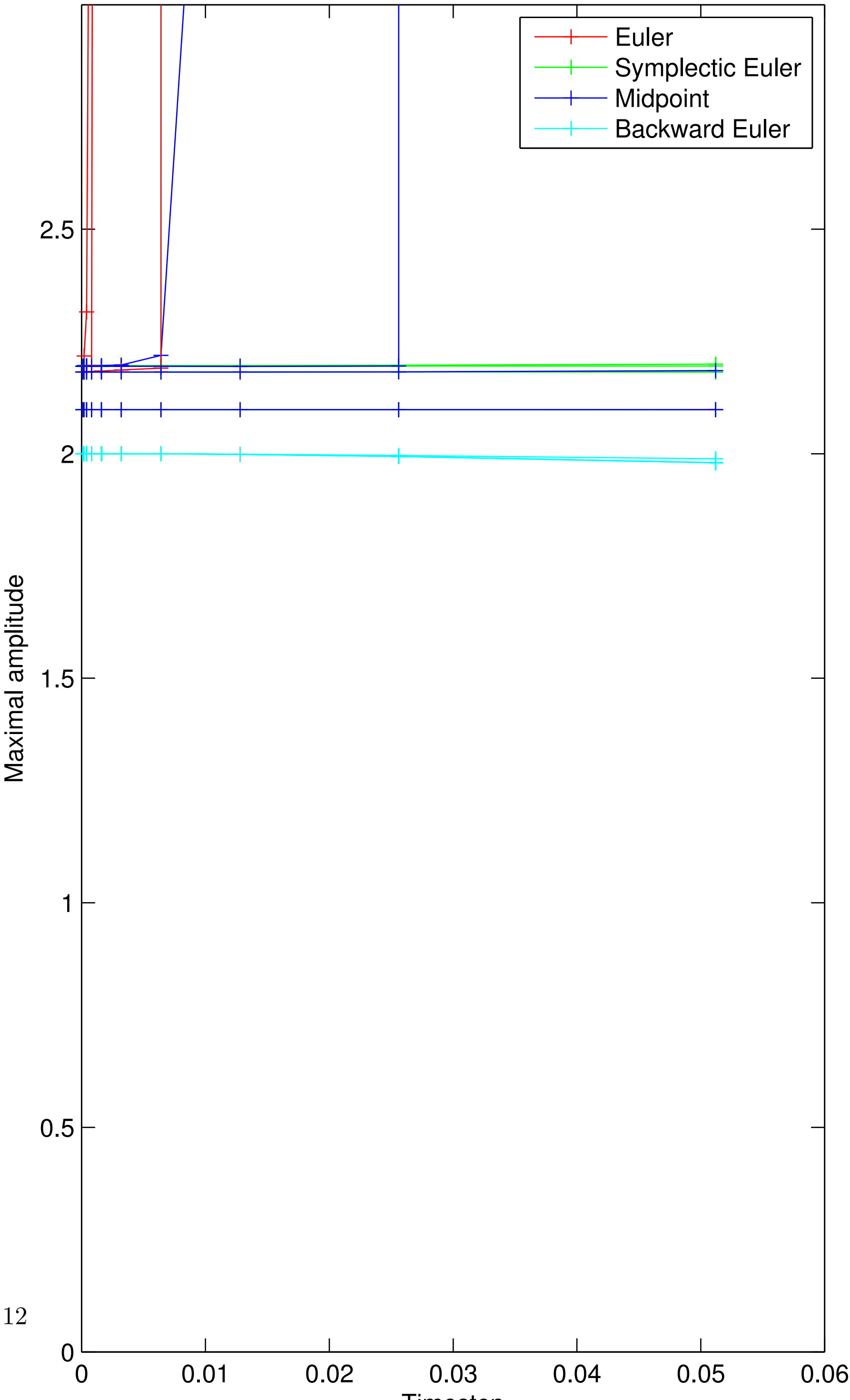

Euler
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