Physically-based simulation in Computer Graphics

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Disney Research

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Contents

Serie 1

First lets do a sketch as seen in figure 0.1 to omit any misunderstanding of the problem. We would like to find the constants c_1, c_2 of the equation

$$y(t) = c_1 e^{\alpha t} \cos(\beta t) + c_2 e^{\alpha t} \sin(\beta t) - L - \frac{mg}{k}, \tag{1}$$

where the constraints are the rest state initial conditions, which means there is no energy in the system except gravitational energy at $t = 0$. This has the same physical meaning as the spring having its rest length L [1] and no kinetic energy at $t = 0$. This can be expressed as

$$y(t = 0) = -L, \tag{2}$$

and

$$\dot{y}(t = 0) = 0. \tag{3}$$

The position of p_2 computed with the given solution Eq. 1 is

$$y(t = 0) = c_1 - L - \frac{mg}{k}. \tag{4}$$

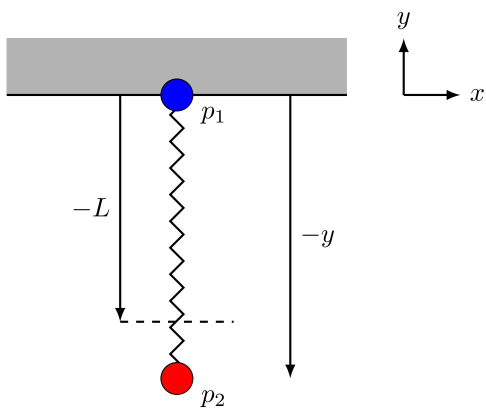


Figure 0.1: Sketch of the spring problem, p_2 being not in rest state.

Comparing Eq. 2 and Eq. 4 gives us

$$c_1 = \frac{mg}{k}. \tag{5}$$

To get c_2 we need to compute $\dot{y}(t=0)$ from Eq. 1

$$\dot{y}(t=0) = \left(c_1\alpha + c_2\beta - L - \frac{mg}{k}\right), \tag{6}$$

where $c_1 = mg/k$ leads to

$$\dot{y}(t=0) = \frac{mg}{k}\alpha + c_2\beta - L - \frac{mg}{k}. \tag{7}$$

Using the constraint in Eq. 3 finally gives us

$$c_2 = \frac{mg}{\beta k}(1-\alpha) + \frac{L}{\beta}. \tag{8}$$

¡++¿

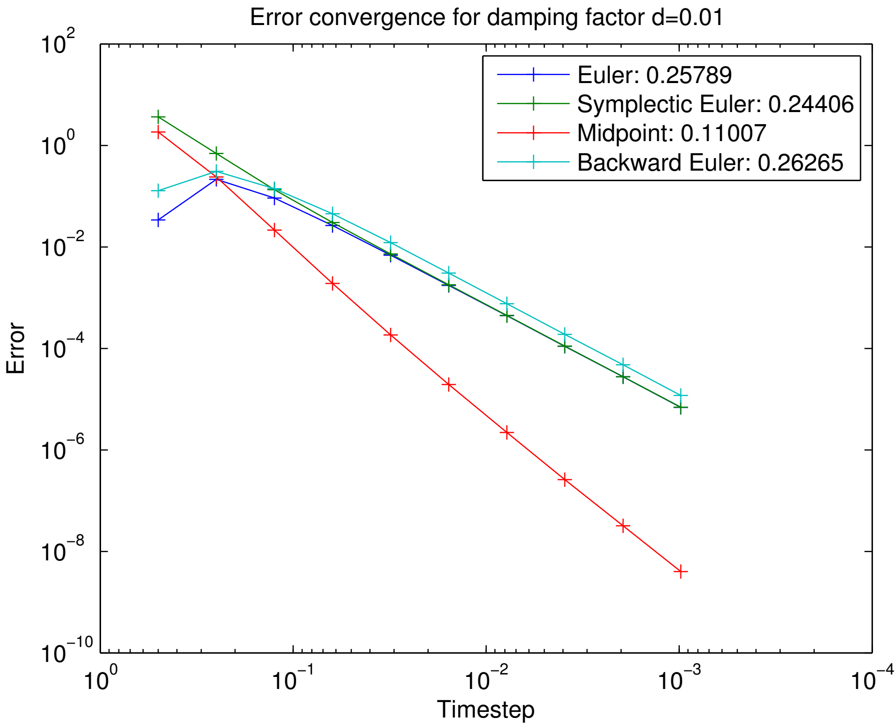

Error convergence for damping factor d=0.01
Euler: 0.25789
Symplectic Euler: 0.24406
Midpoint: 0.11007
Backward Euler: 0.26265
Error
Timestep
10^2
10^0
10^-2
10^-4
10^-6
10^-8
10^-10
10^0
10^-1
10^-2
10^-3
10^-4

Figure 0.2: ¡++¿

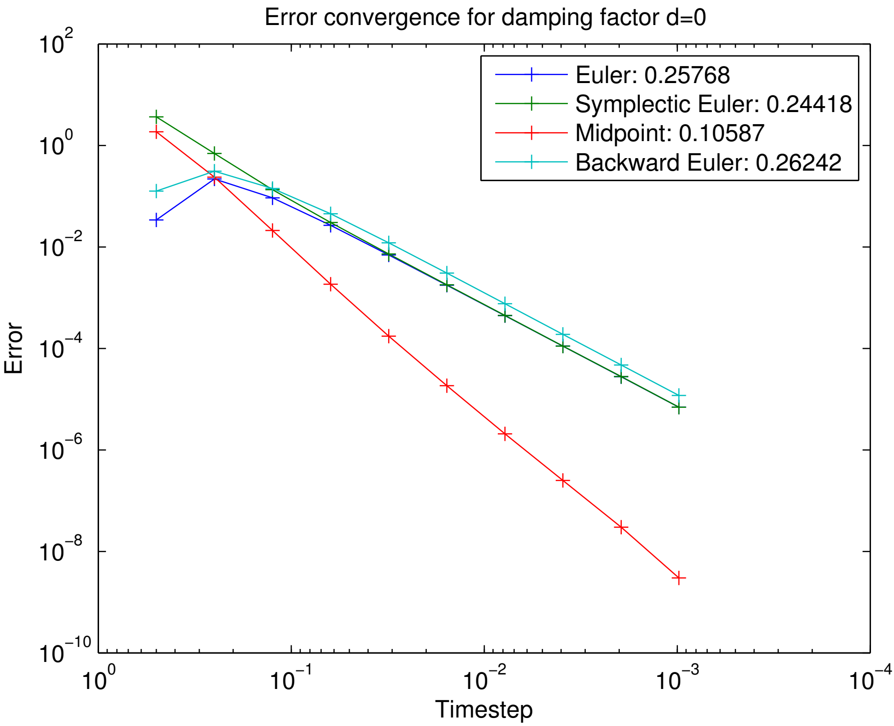
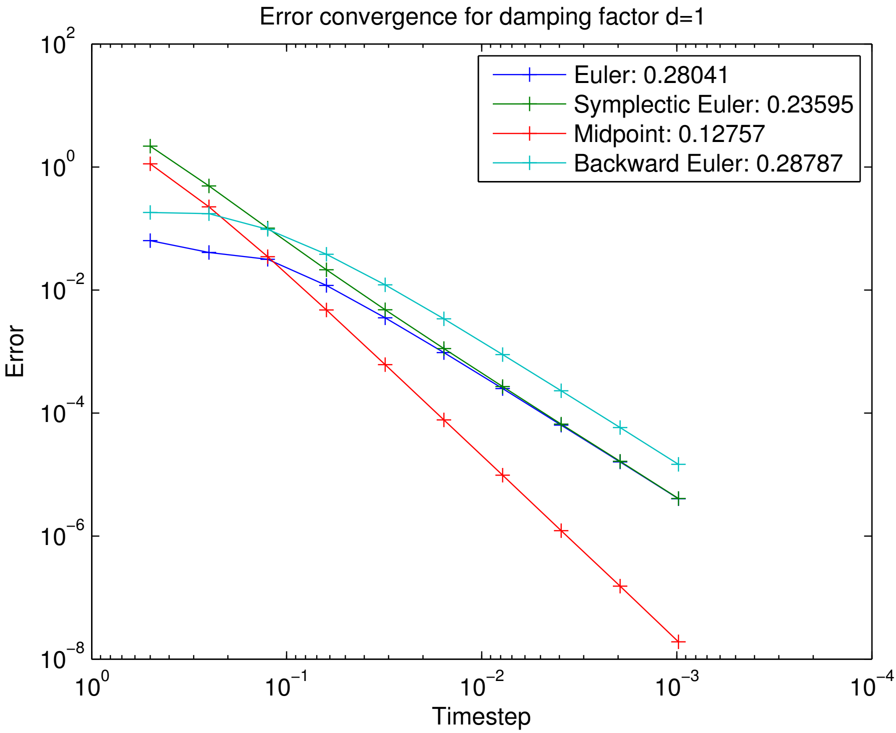

Error convergence for damping factor d=0
Euler: 0.25768
Symplectic Euler: 0.24418
Midpoint: 0.10587
Backward Euler: 0.26242
Error
Timestep
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}

Figure 0.3


Error convergence for damping factor d=1
Euler: 0.28041
Symplectic Euler: 0.23595
Midpoint: 0.12757
Backward Euler: 0.28787
Error
Timestep
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10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}

Figure 0.4

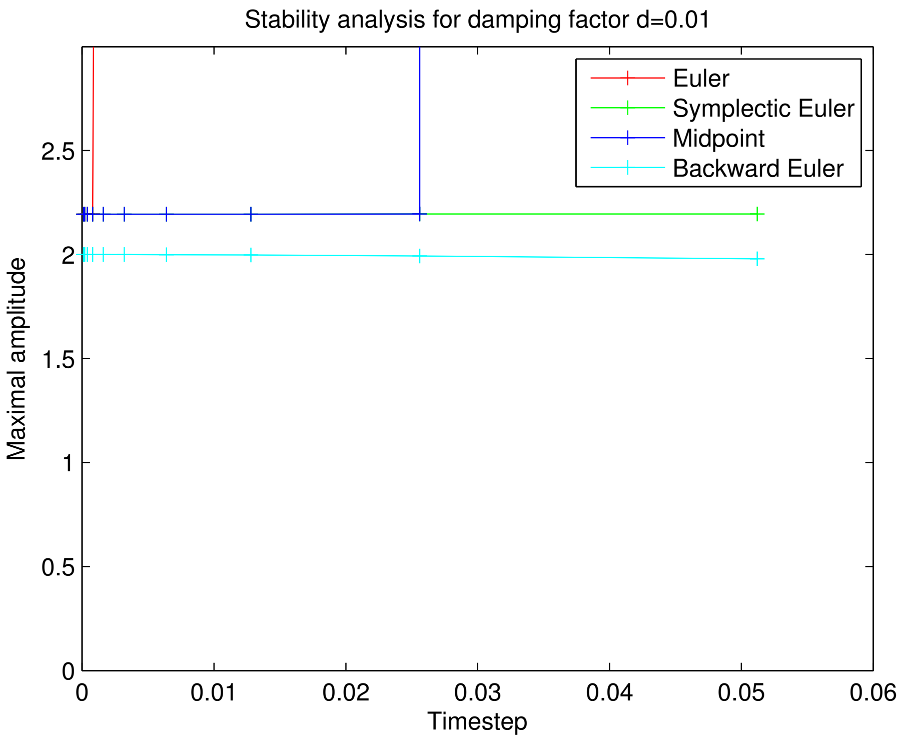
Stability analysis for damping factor d=0.01
Euler
Symplectic Euler
Midpoint
Backward Euler
Maximal amplitude
Timestep
0
0.5
1
1.5
2
2.5
0
0.01
0.02
0.03
0.04
0.05
0.06

Figure 0.5

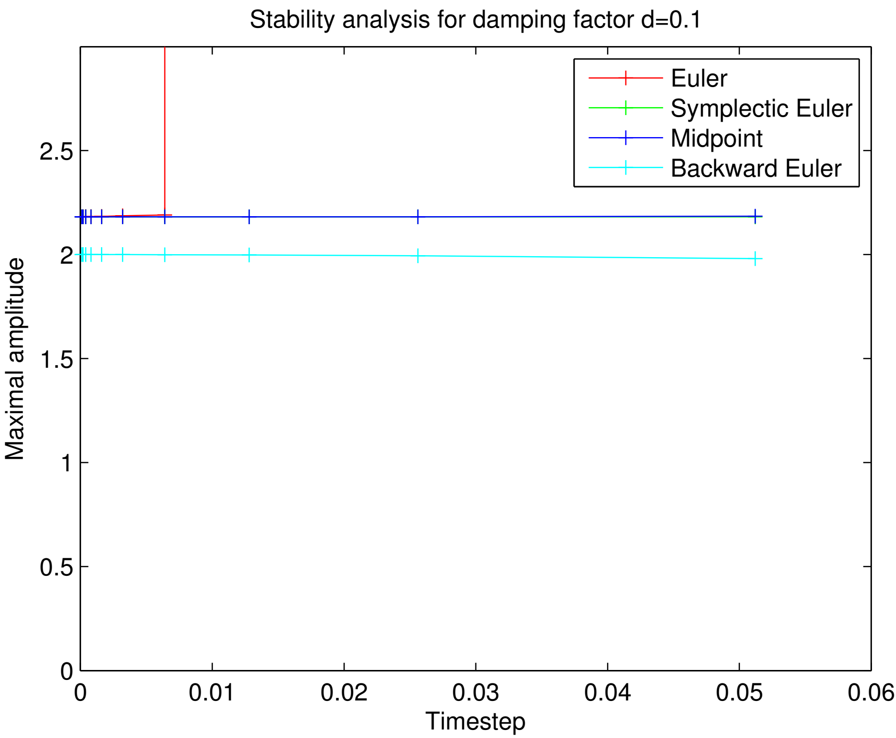
Stability analysis for damping factor d=0.1
Euler
Symplectic Euler
Midpoint
Backward Euler
Maximal amplitude
0
0.5
1
1.5
2
2.5
0
0.01
0.02
0.03
0.04
0.05
0.06
Timestep

Figure 0.6

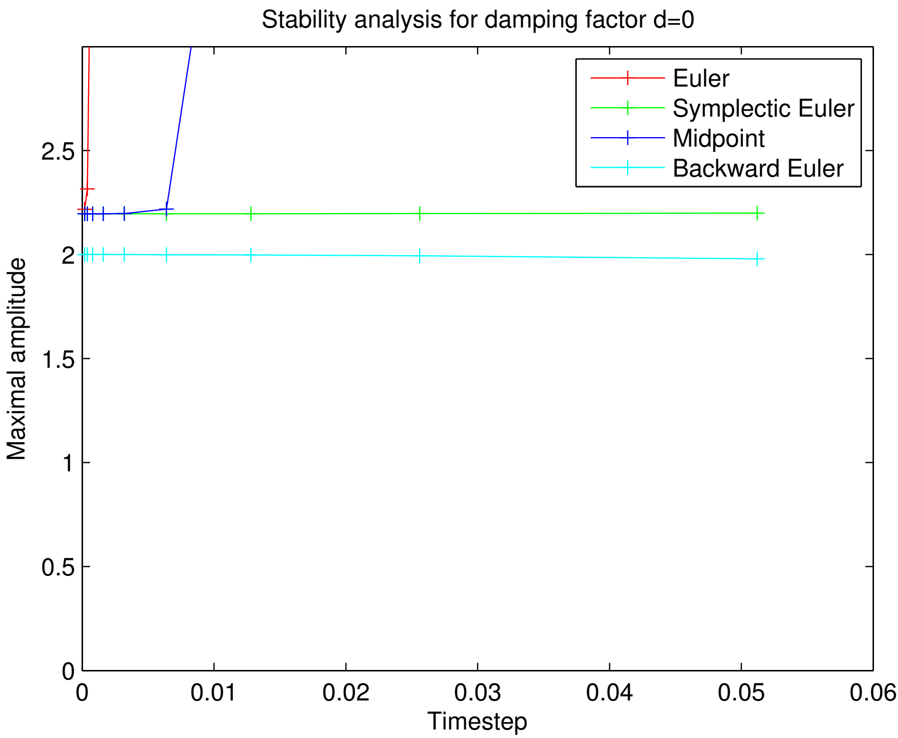
Stability analysis for damping factor d=0
Euler
Symplectic Euler
Midpoint
Backward Euler
Maximal amplitude
Timestep
0
0.5
1
1.5
2
2.5
0.01
0.02
0.03
0.04
0.05
0.06

Figure 0.7

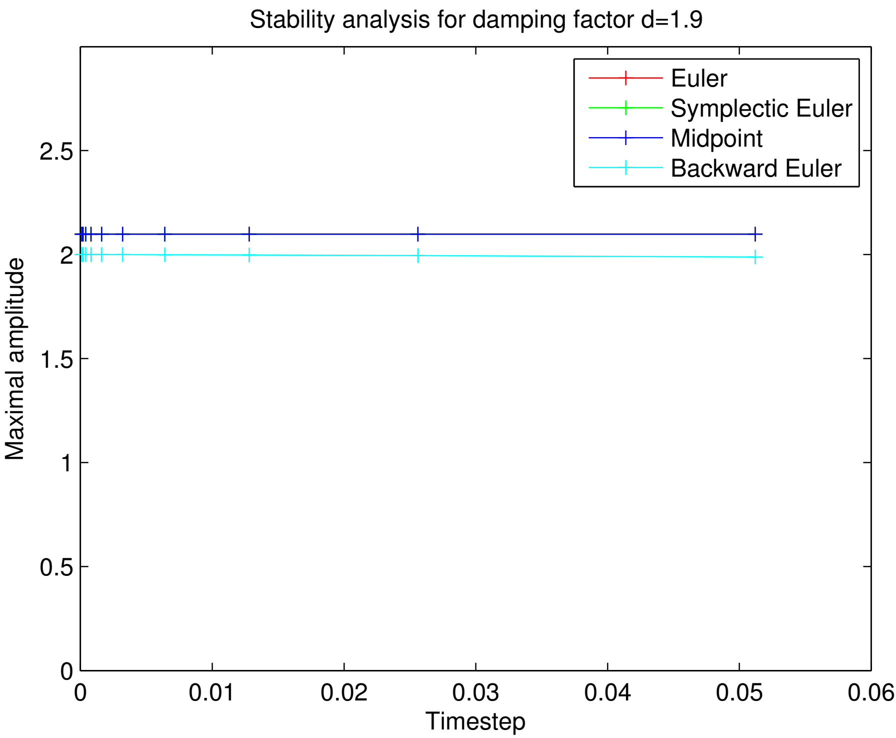
Stability analysis for damping factor d=1.9
Euler
Symplectic Euler
Midpoint
Backward Euler
Maximal amplitude
Timestep
0
0.5
1
1.5
2
2.5
0
0.01
0.02
0.03
0.04
0.05
0.06

Figure 0.8