Trigonometrische und Hyperbolische Funktionen

Beziehung zur Exponentialfunktion

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2} \qquad \cosh(x) = \frac{e^{x} + e^{-x}}{2} \qquad \operatorname{arcosh}(y) = \ln\left(y + \sqrt{y^2 - 1}\right)$$

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i} \qquad \sinh(x) = \frac{e^{x} - e^{-x}}{2} \qquad \operatorname{arsinh}(y) = \ln\left(y + \sqrt{y^2 + 1}\right)$$

$$\tan(x) = \frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}} \cdot \frac{1}{i} \qquad \tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \qquad \operatorname{artanh}(y) = \frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)$$

Reihenentwicklungen

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} - + \dots = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{x^{2k}}{(2k)!}$$

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - + \dots = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{x^{2k+1}}{(2k+1)!}$$

$$\cosh x = 1 + \frac{x^2}{2} + \frac{x^4}{24} + \dots = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$$

$$\sinh x = x + \frac{x^3}{6} + \frac{x^5}{120} + \dots = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$$

$$\ln(1+x) = x - \frac{x}{2} + \frac{x^2}{3} - + \dots = \sum_{k=0}^{\infty} (-1)^{k-1} \cdot \frac{x^k}{k}$$

$$\ln(x) = (x-1) - \frac{(x-1)}{2} + \frac{(x-1)^2}{3} - + \dots = \sum_{k=0}^{\infty} (-1)^{k-1} \cdot \frac{(x-1)^k}{k}$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - + \dots = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{x^{2k+1}}{2k+1}$$

$$\operatorname{artanh} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{2k+1}$$

Additionstheoreme

$$e^{x+y} = e^x \cdot e^y \qquad \ln(xy) = \ln(x) + \ln(y)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y) \qquad \sinh(x+y) = \sinh(x)\cosh(y) + \cosh(x)\sinh(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \qquad \cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$$

$$\sin(2x) = 2\sin(x)\cos(x) \qquad \sinh(2x) = 2\sinh(x)\cosh(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) \qquad \cosh(2x) = \cosh^2(x) + \sinh^2(x)$$

$$\cos^2(x) + \sin^2(x) = 1 \qquad \cosh^2(x) - \sinh^2(x) = 1$$

$$\sin^2(x) = -\frac{\cos(2x)}{2} + \frac{1}{2} \qquad \sinh^2(x) = \frac{\cosh(2x)}{2} - \frac{1}{2}$$

$$\cos^2(x) = \frac{\cos(2x)}{2} + \frac{1}{2} \qquad \cosh^2(x) = \frac{\cosh(2x)}{2} + \frac{1}{2}$$

Genaue Funktionswerte

α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$-$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

Die Umkehrfunktion arcsin lässt sich ablesen, indem man zuerst den Eingabewert in der Zeile $\sin\alpha$ sucht, der Funktionswert ist dann der zugehörige Wert in der α-Zeile. $\arcsin(1/2)$ ist z.B. $\pi/6$. Analog mit $\arccos, \arctan$