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Faculty of Informatics

An Introduction to Alpha Matting

Ji-Ho Cho, Margrit Gelautz, Christoph Rhemann

Vienna University of Technology

Institute for Software Technology and Interactive Systems, Interactive Media Systems Group

www.ims.tuwien.ac.at/IVAS

December 1, 2013

Outline

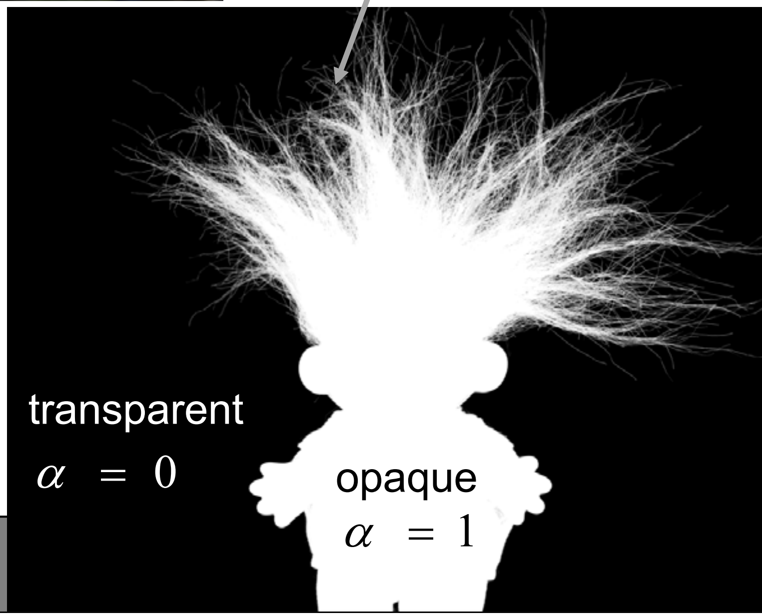
- Introduction
- Matting algorithms
  - Closed form matting
  - Robust and improved color sampling
- Alpha matting website (www.alphamatting.com)
- Conclusion

Introduction

Matting and compositing

Matting and compositing

Semi-transparent
$0 < \alpha < 1$



Matting: Precise extraction of foreground object

Compositing: Object is pasted onto new background

Compositing equation

- **Splitting up a natural image**

$$\boxed{I} = \boxed{\alpha} \cdot \boxed{F} + (1 - \boxed{\alpha}) \cdot \boxed{B}$$

Matting = inverse process of compositing:

Determine: **F**(oreground), **B**(ackground), **α** (matte)

Given: I(mage)

Why is matting difficult?

$$I^r = \alpha \times F^r + (1-\alpha) \times B^r$$
$$I^g = \alpha \times F^g + (1-\alpha) \times B^g$$
$$I^b = \alpha \times F^b + (1-\alpha) \times B^b$$

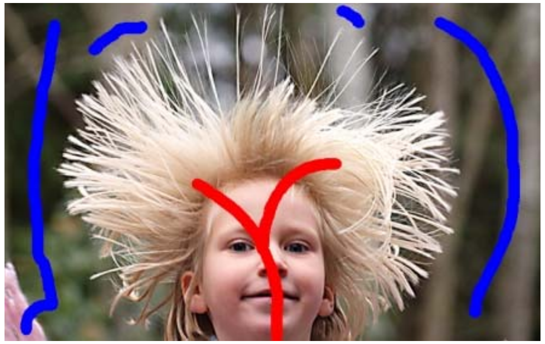
Under- constrained problem:
7 unknowns in only 3 equations

No unique solution!
Constraints are necessary

Constrain the matting problem

- **Two main ways to make the matting problem tractable**

Known background (Chroma keying)
- Image from Wikipedia



Scribble-based interface

Trimap

Alpha matte
(BMVC08)

User interaction (natural image matting)

Natural image matting

- **Sampling-based approaches**
  - Knockout, Bayesian
- **Propagation-based approaches**
  - Random walk, Poisson matting, <u>closed form matting</u>
- **Hybrid approaches**
  - <u>Robust color sampling</u>, <u>improved color sampling</u>, global sampling, shared sampling, comprehensive sampling, etc.

Introduction to Closed Form Matting

A. Levin D. Lischinski and Y. Weiss. A Closed Form Solution to Natural Image Matting. *Conference on Computer Vision and Pattern Recognition (CVPR)*, June 2006.

Closed form matting

Recall compositing equation: $I = \alpha F + (1 - \alpha) B$

- **Assumption:**
  - F&B are locally smooth
  - <u>F&B are constant over a small window *w*</u>

$$I_i = \alpha_i F_i + (1 - \alpha_i) B_i$$

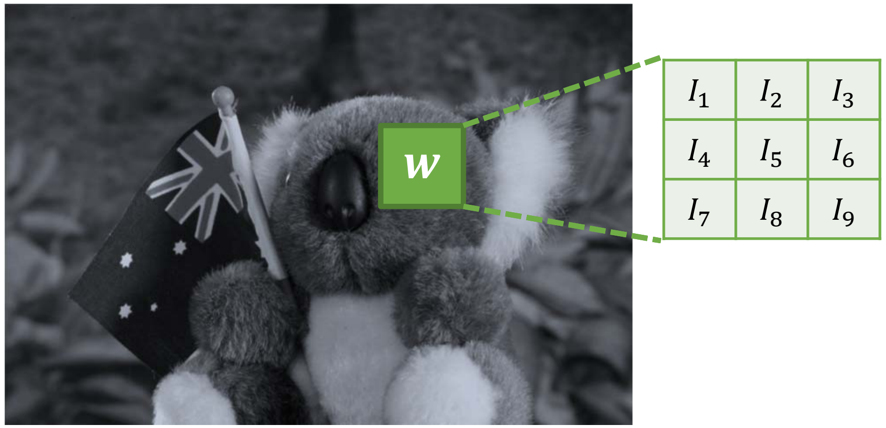
$$I_i = \alpha_i (F_i - B_i) + B_i$$

$$\alpha_i = \frac{I_i - B_i}{F_i - B_i}$$

$$\alpha_i \approx a I_i + b, \forall i \in w$$

$$a = \frac{1}{F - B}$$

$$b = -\frac{B}{F - B}$$



Closed form matting

- **Goal:**
  - Find ***α***, ***a***, and **b** minimizing the following equation:

$$J(\alpha,a,b)=\sum_{j\in I}\left(\sum_{i\in w_j}\boxed{(\alpha_i-a_jI_i-b_j)^2+\varepsilon a_j^2}\right)$$

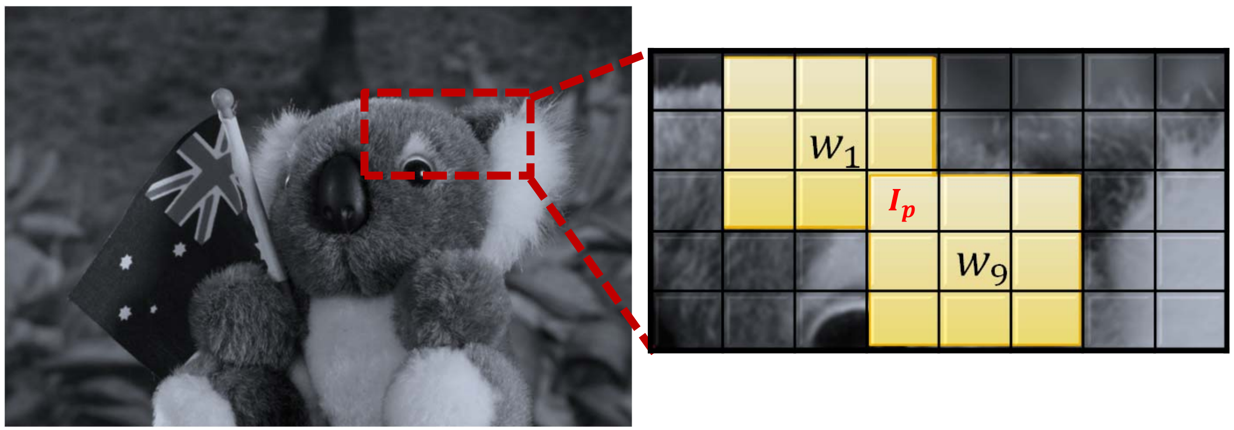
Regularization term for
- Numerical stability
- Smoother α matte

Overlapping window $\longrightarrow$ Propagates the information between neighboring pixels

Window of 3x3 pixels

$$\downarrow$$

$$(\alpha_p-a_1I_p-b_1)^2+\varepsilon a_1^2$$
$$\vdots$$
$$(\alpha_p-a_9I_p-b_9)^2+\varepsilon a_9^2$$



Currently, there are 3N unknowns for N number of pixels

$$J(\alpha, a, b) = \sum_{j \in I} \left(\sum_{i \in w_j} (\alpha_i - a_j I_i - b_j)^2 + \varepsilon a_j^2 \right)$$

$$J(\alpha) = \min_{a,b} J(\alpha, a, b)$$

Eliminating a, b, and obtain quadratic cost in $\boldsymbol{\alpha}$

$$J(\alpha) = \alpha^T L \alpha$$

$$L_{ij} = \sum_{k|(i,j) \in w_k} \left(\delta_{ij} - \frac{1}{|w_k|} \left(1 + \frac{1}{\frac{\varepsilon}{|w_k|} + \sigma_k^2} (I_i - \mu_k)(I_j - \mu_k) \right) \right)$$

Kronecker delta

Mean and variance of the intensities in the window

Moving to color images

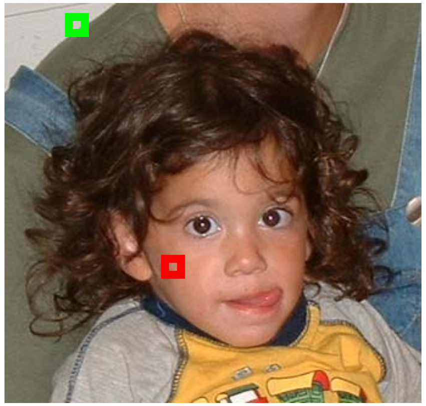
Recall linear model in gray scale: $\alpha_i \approx aI_i + b, \forall i \in w$

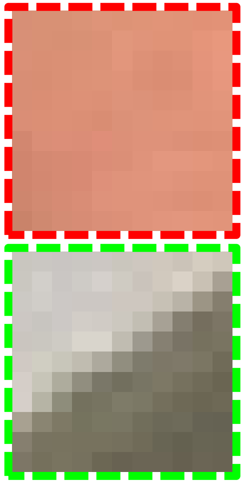
- **For every window *w***

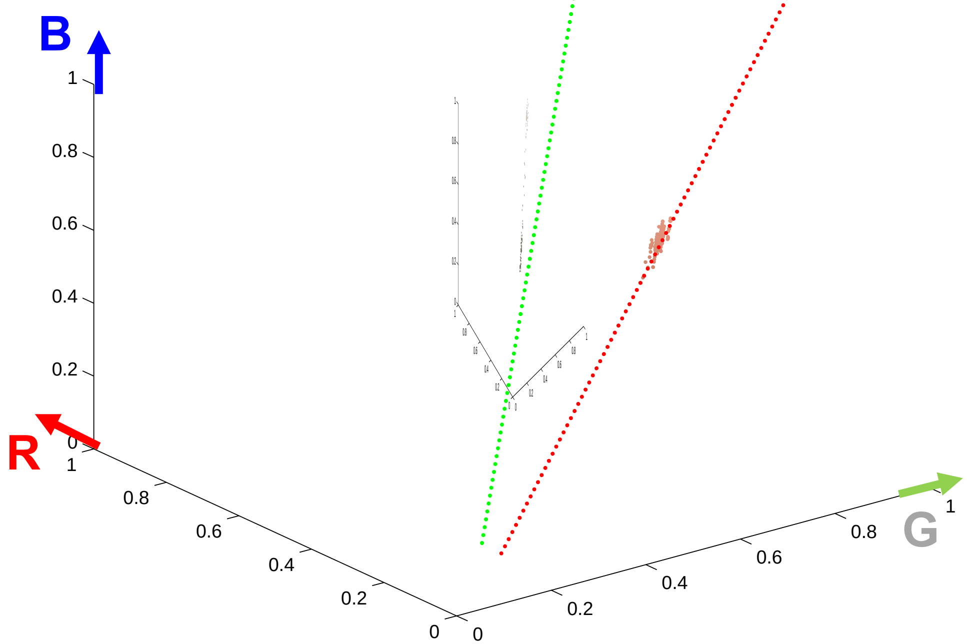
$$\alpha_i \approx \sum_c a^c I_i^c + b, \forall i \in w \longleftarrow \text{4D linear model}$$

Assumption: *F* and *B* in a local window lie on a single line in RGB color space

$$F_i = \beta_i^F F_1 + (1 - \beta_i^F) F_2 \quad B_i = \beta_i^B B_1 + (1 - \beta_i^B) B_2$$







Using 4D linear model, cost function for matting of RGB images can be defined:

$$J(\alpha, a, b) = \sum_{j \in I} \left(\sum_{i \in w_j} \left(\alpha_i - \sum_c a_j^c I_i^c + b_j \right)^2 + \varepsilon \sum_c a_j^{c^2} \right)$$

Similary to grayscale case, a^c and b can be eliminated from the cost function, then

$$J(\alpha) = \alpha^T L \alpha$$

\# of pixels of image

where L is an N x N matrix, whose $(i, j)^{\text{th}}$ element is

$$\sum_{k|(i,j) \in w_k} \left(\delta_{ij} - \frac{1}{|w_k|} \left(1 + (C_i - \mu_k)^T (\Sigma_k + \frac{\varepsilon}{|w_k|} I_3)^{-1} (C_j - \mu_k) \right) \right)$$

Matting Laplacian

- **Solving for α, based on user input**

Some large number

Specified alpha values for F and B

$$\alpha = \arg\min \alpha^T L\alpha + \lambda(\alpha^T - b_S^T)D_S(\alpha - b_S)$$

Diagonal matrix whose diagonal elements are 1 for F and B

by differentiating

$$(L + \lambda D_S)\alpha = \lambda b_S$$

image | Image with scribbles | D_s | b_s

Implementation (Matlab)

$$(L+\lambda D_S)\alpha = \lambda b_S$$

In Matlab, equation Ax=b can be solved using division operator x=A\b;

Matlab code available (http://people.csail.mit.edu/alevin/matting.tar.gz)

```
L=getLaplacian1(Image,user_input_map);
D_s=spdiags(user_input_map(:),0,img_size,img_size);
lambda=100;
alpha=(L+lambda*D_s)\(lambda*user_input_value(:));
```

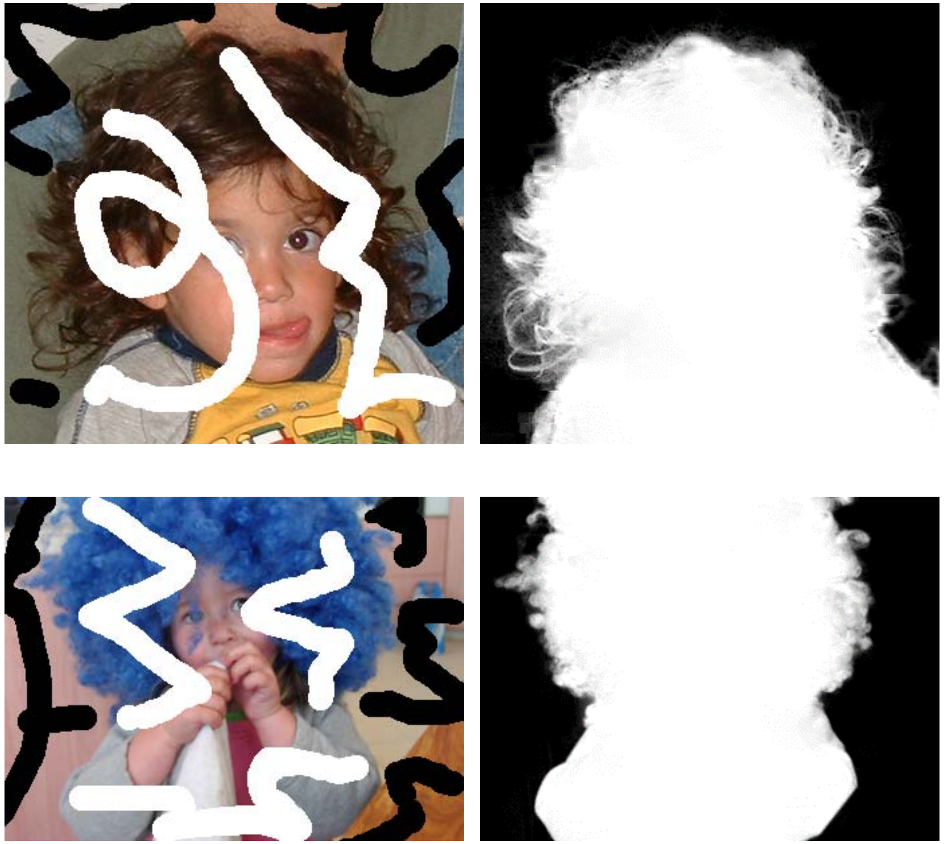
image

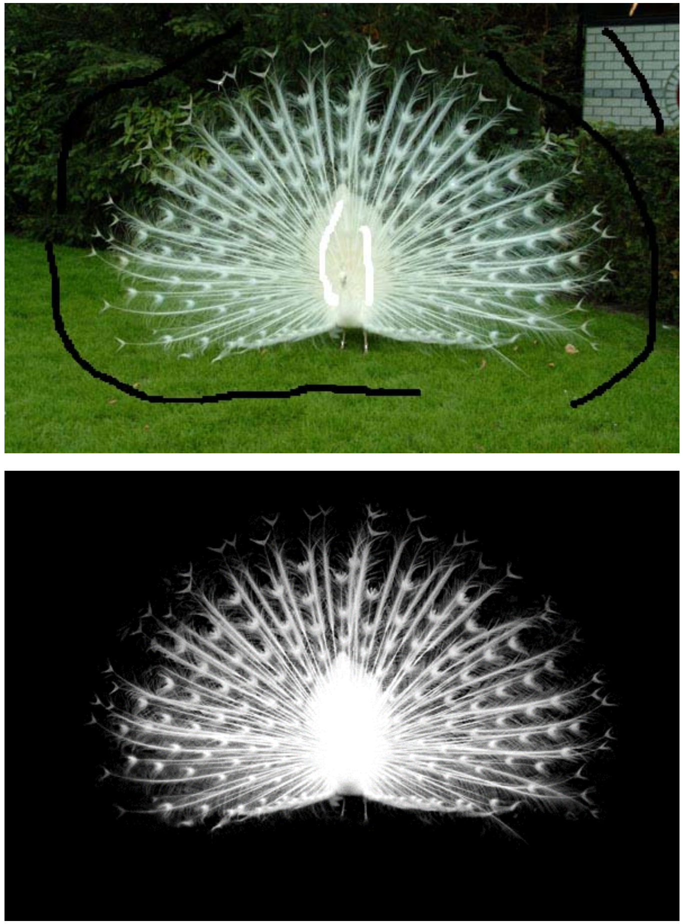
Image with scribbles

user_input_map

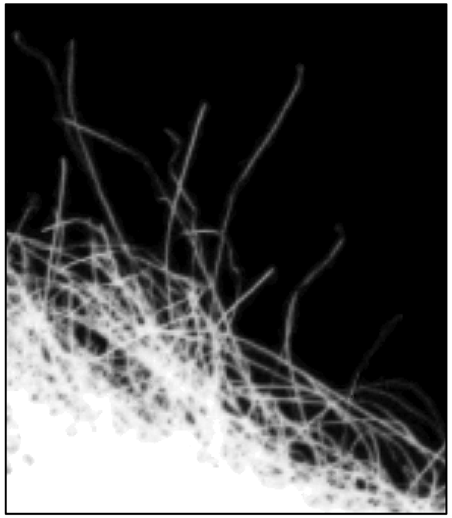
user_input_value

- **Results**



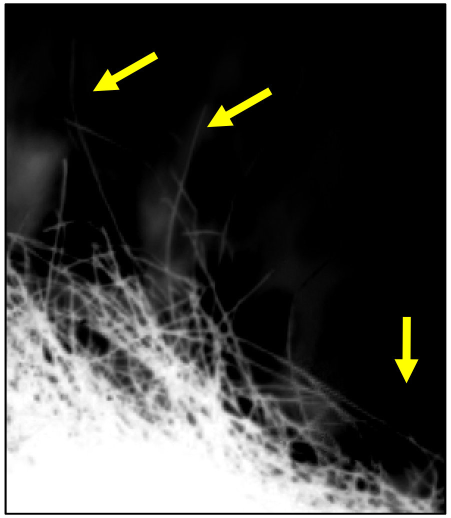


- **Result imperfect: Hairs cut off**



True solution

Input image + trimap



Result of closed form matting

Robust and improved color sampling

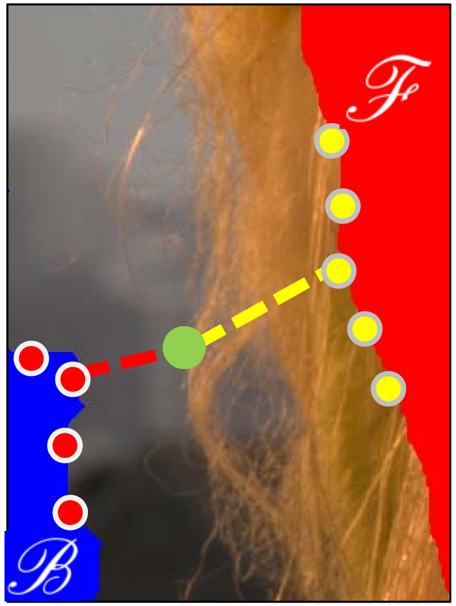
J. Wang, M. Cohen, Optimized Color Sampling for Robust Matting. *Conference on Computer Vision and Pattern Recognition (CVPR)*, 2007.

C. Rhemann, C. Rother, M. Gelautz, Improving Color Modeling for Alpha Matting. *British Machine Vision Conference (BMVC)*, 2008

- **Three steps:**

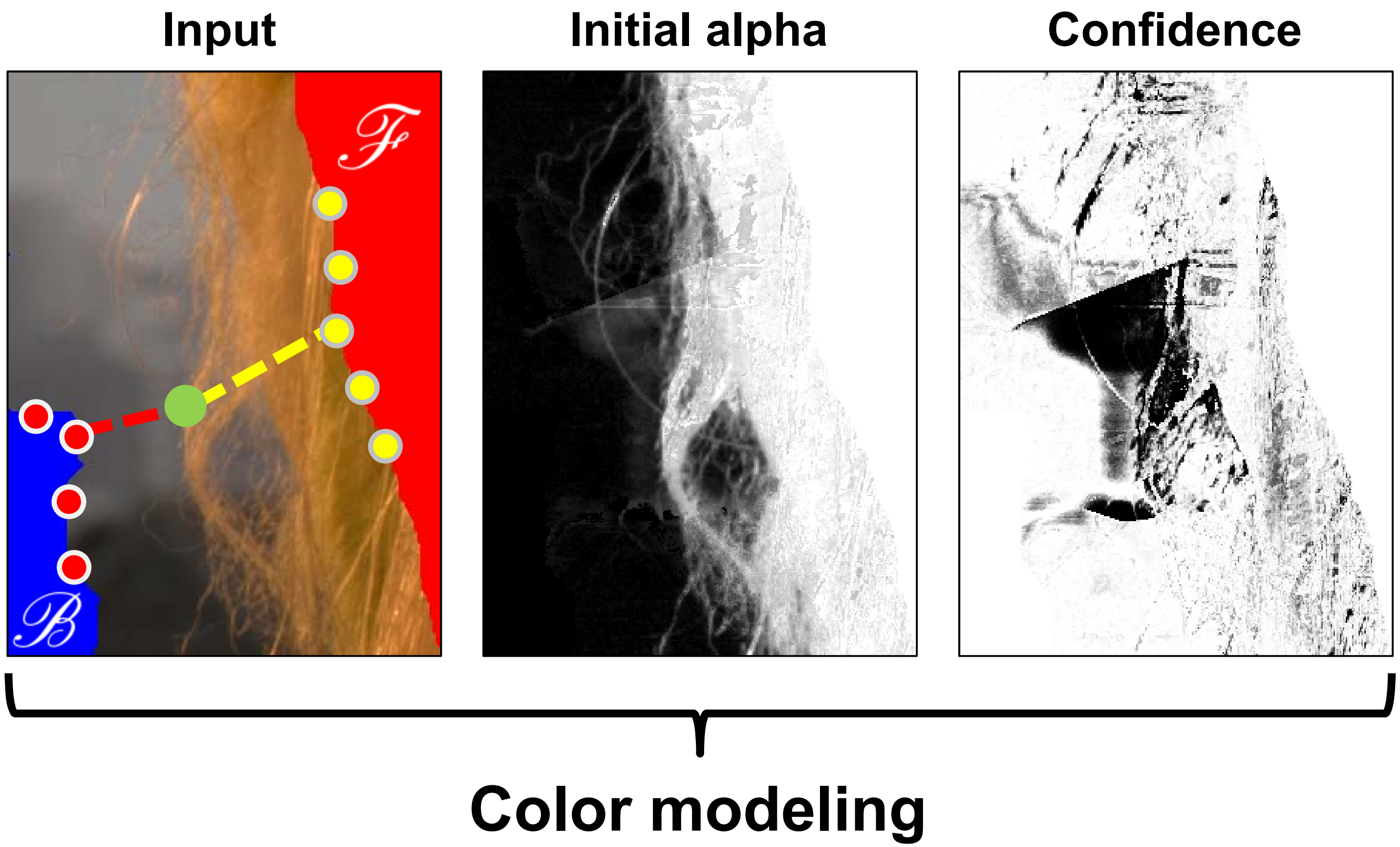
1. Collect potential set of fore- and background colors

Input



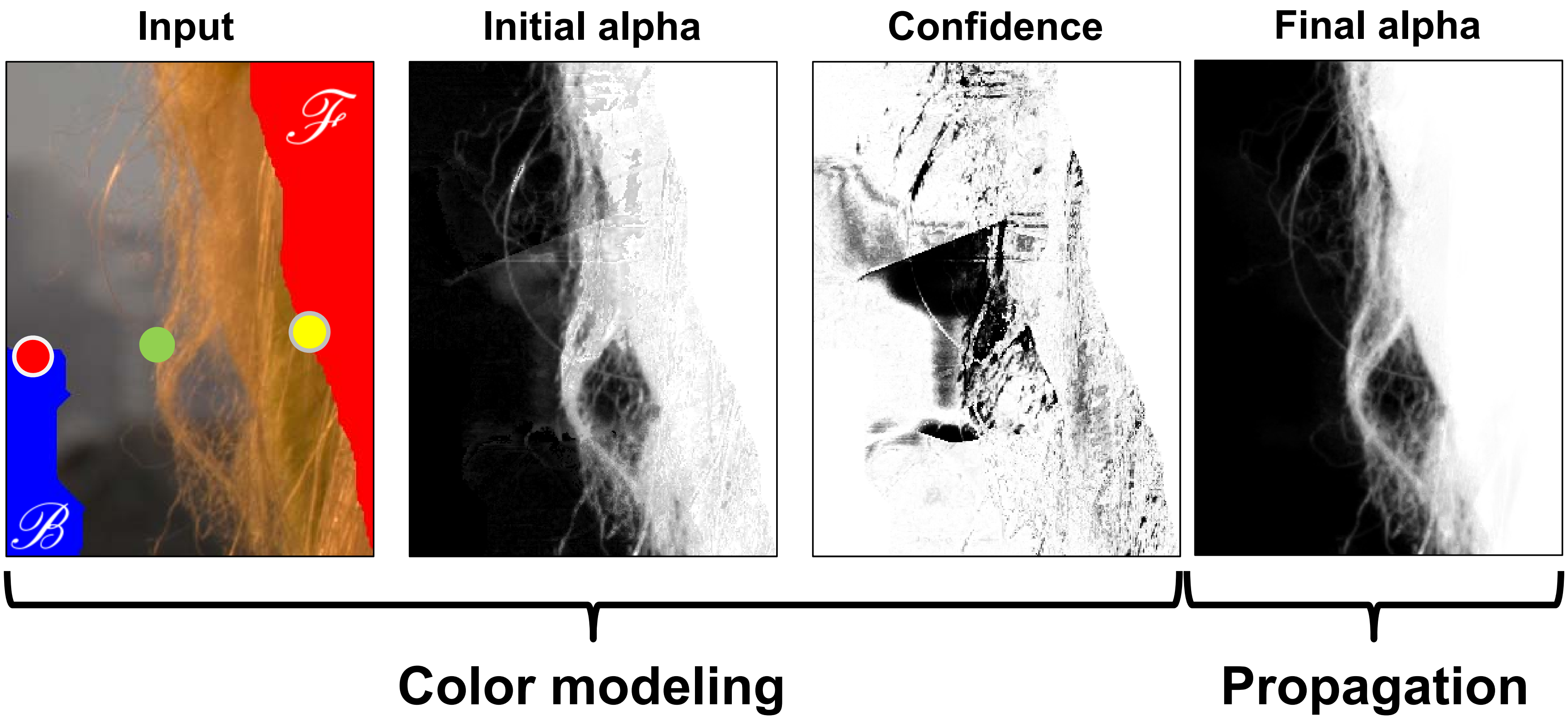
- **Three steps:**

  2. Select most confident samples from initial set



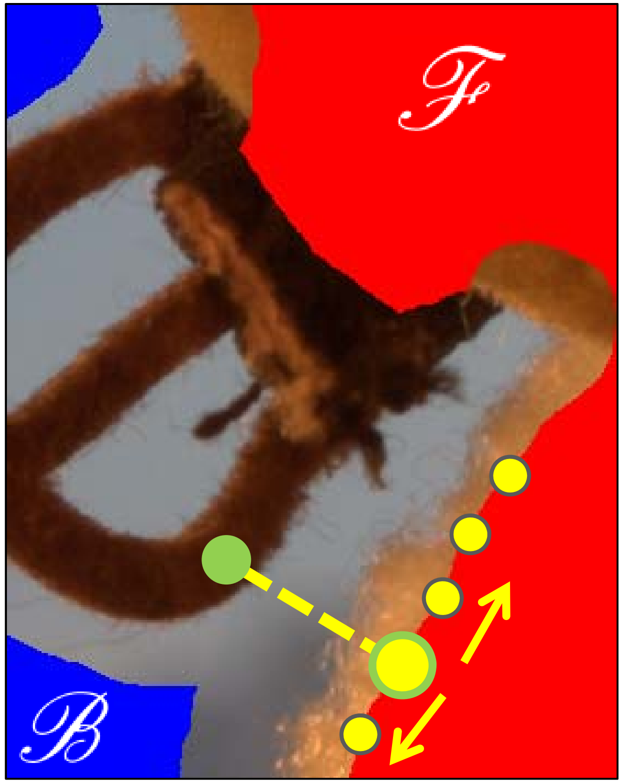
- **Three steps:**

  3. Encode smoothness assumption



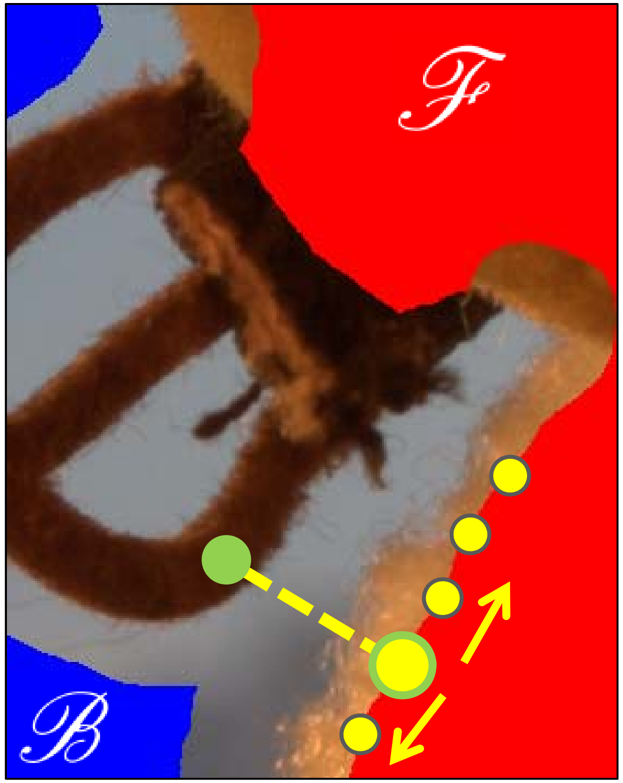
Robust color sampling

- **Find nearest neighbor in spatial domain, and then propagate the samples along the boundary**



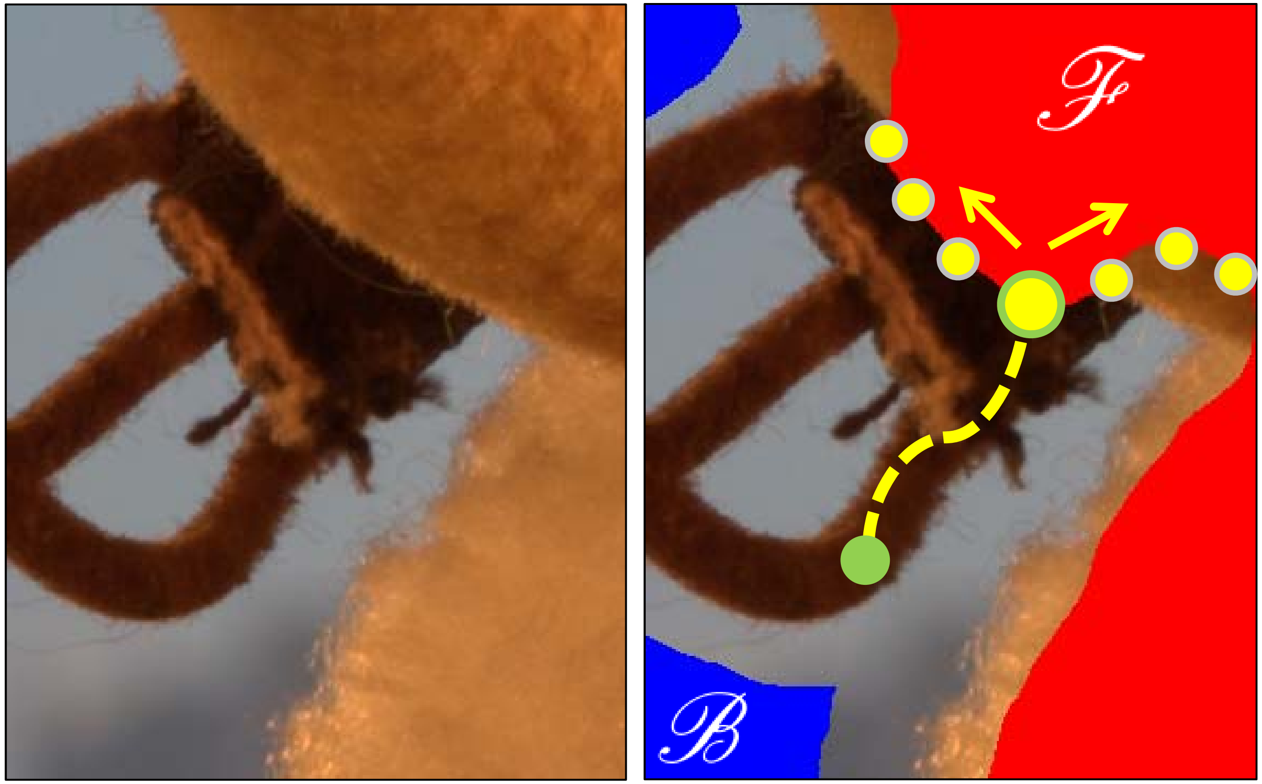
Robust color sampling

- **Find nearest neighbor in spatial domain, and then propagate the samples along the boundary**



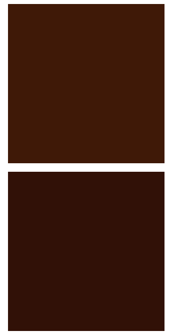
Improved color sampling

- **Geodesic sampling: Assume foreground spatially connected**

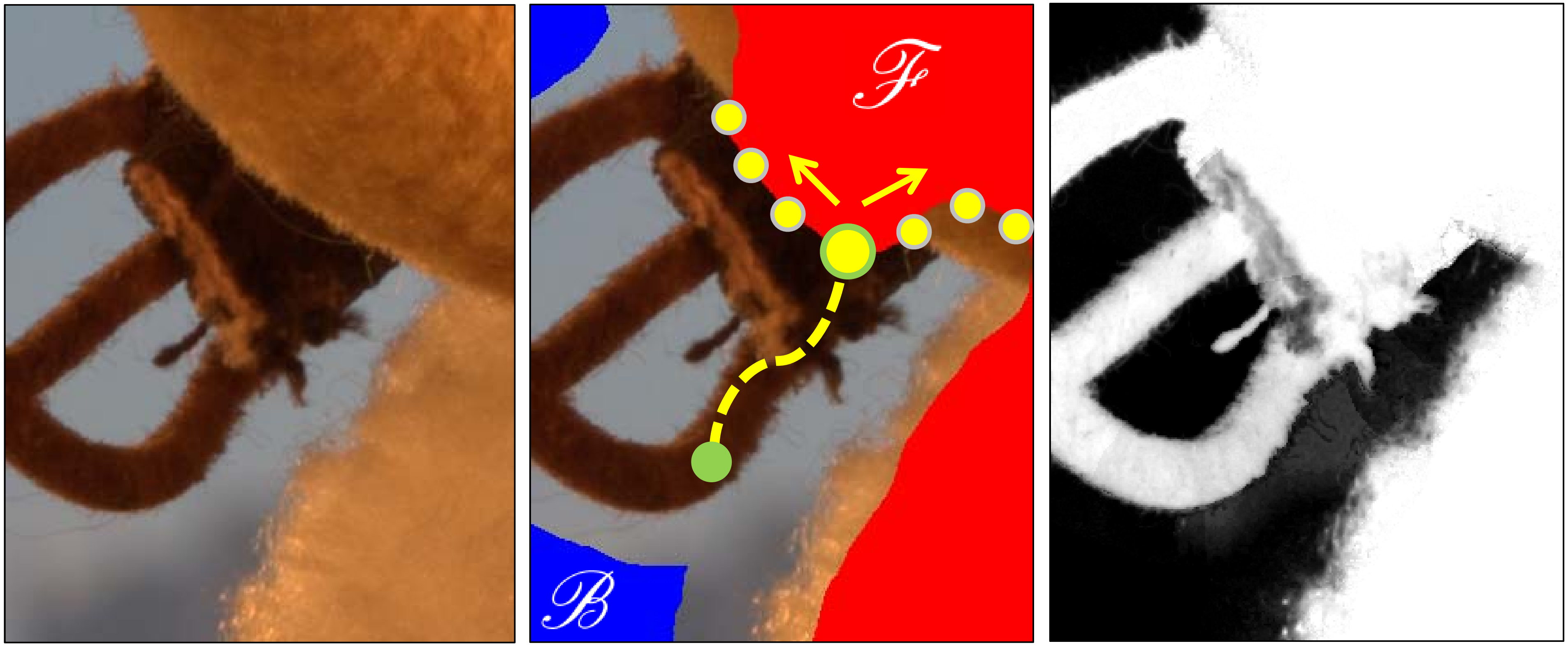


True fgd color

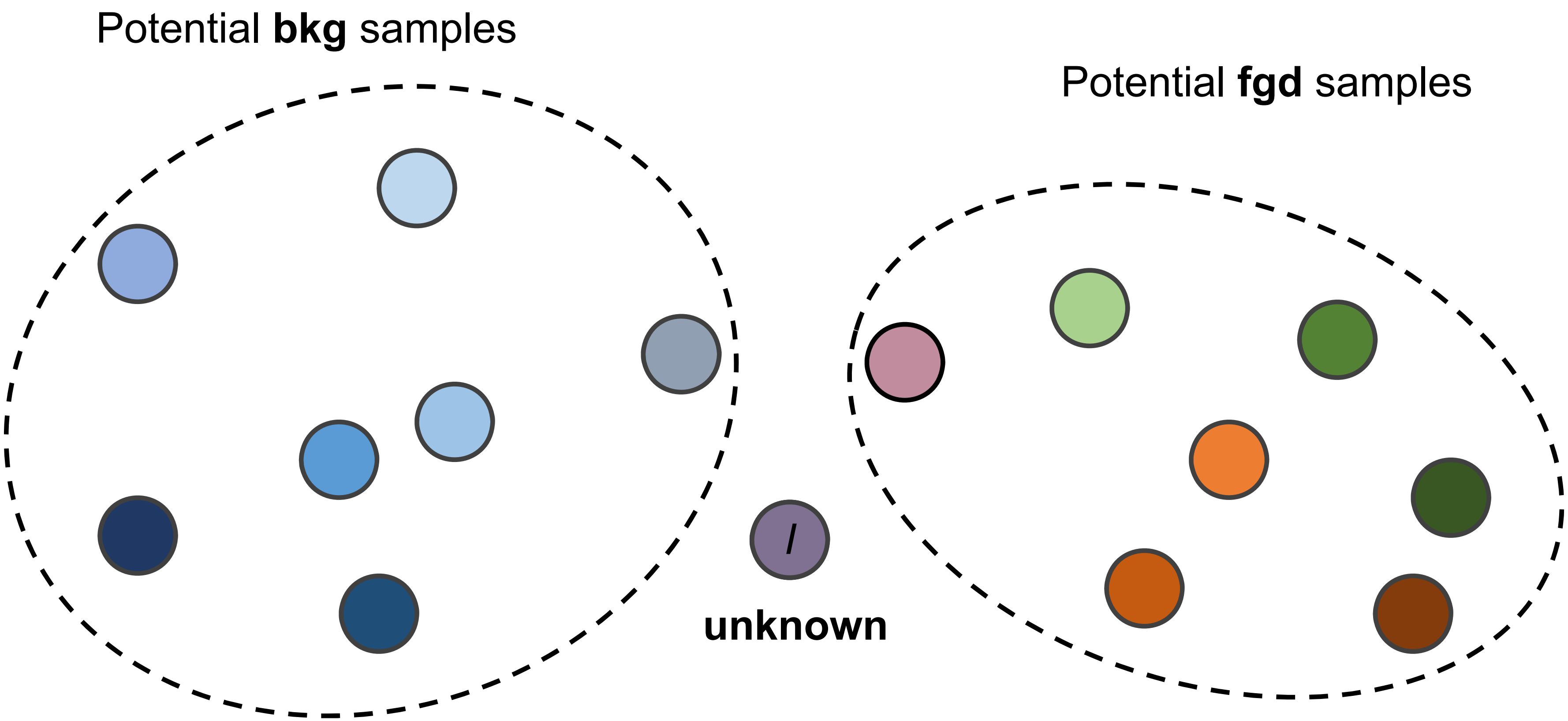
Sampled fgd color



- **Geodesic sampling: Assume foreground spatially connected**

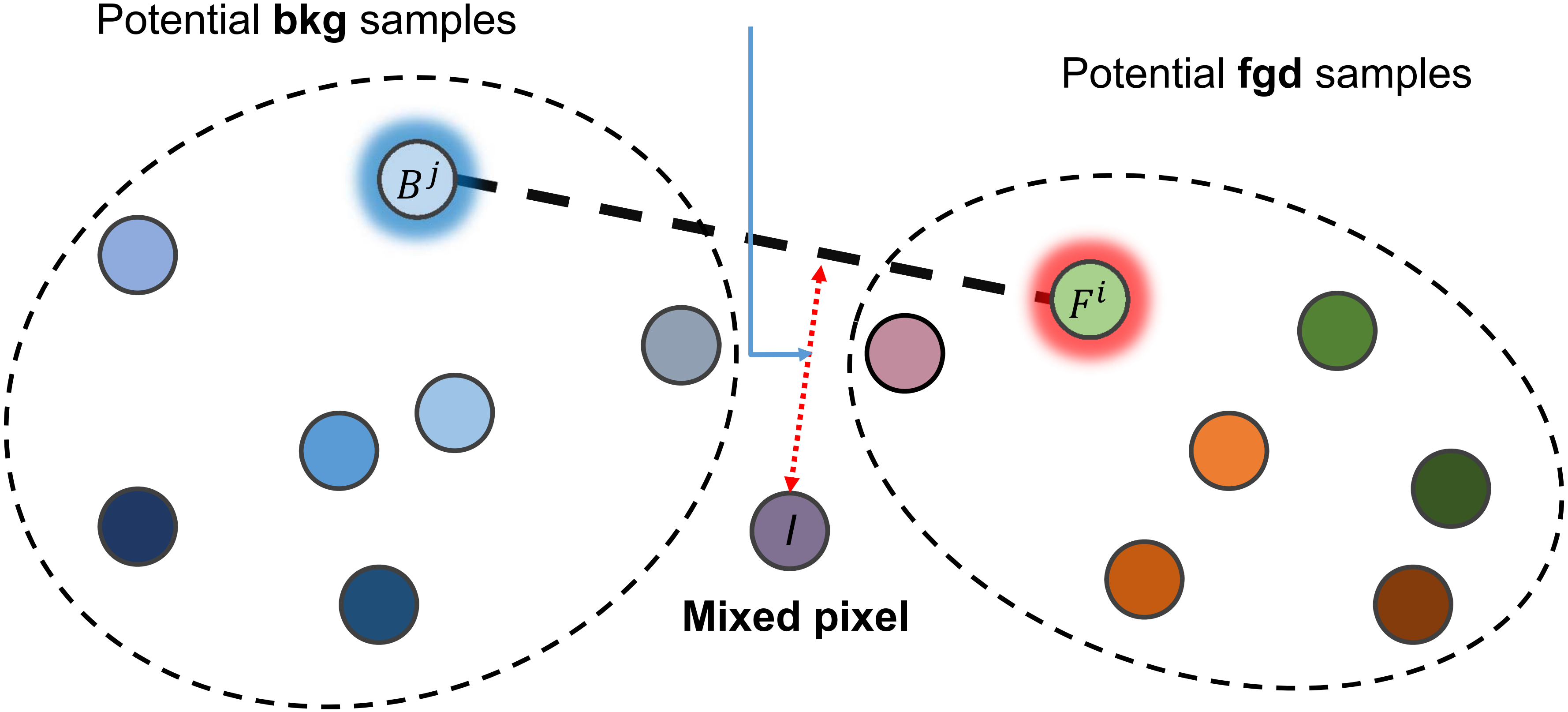


Step 2 - Selecting best samples



1. Linear model fit

$$E(F^i, B^j) = \left\| I - (\hat{\alpha} F^i + (1-\hat{\alpha}) B^j \right\|$$



1. Linear model fit

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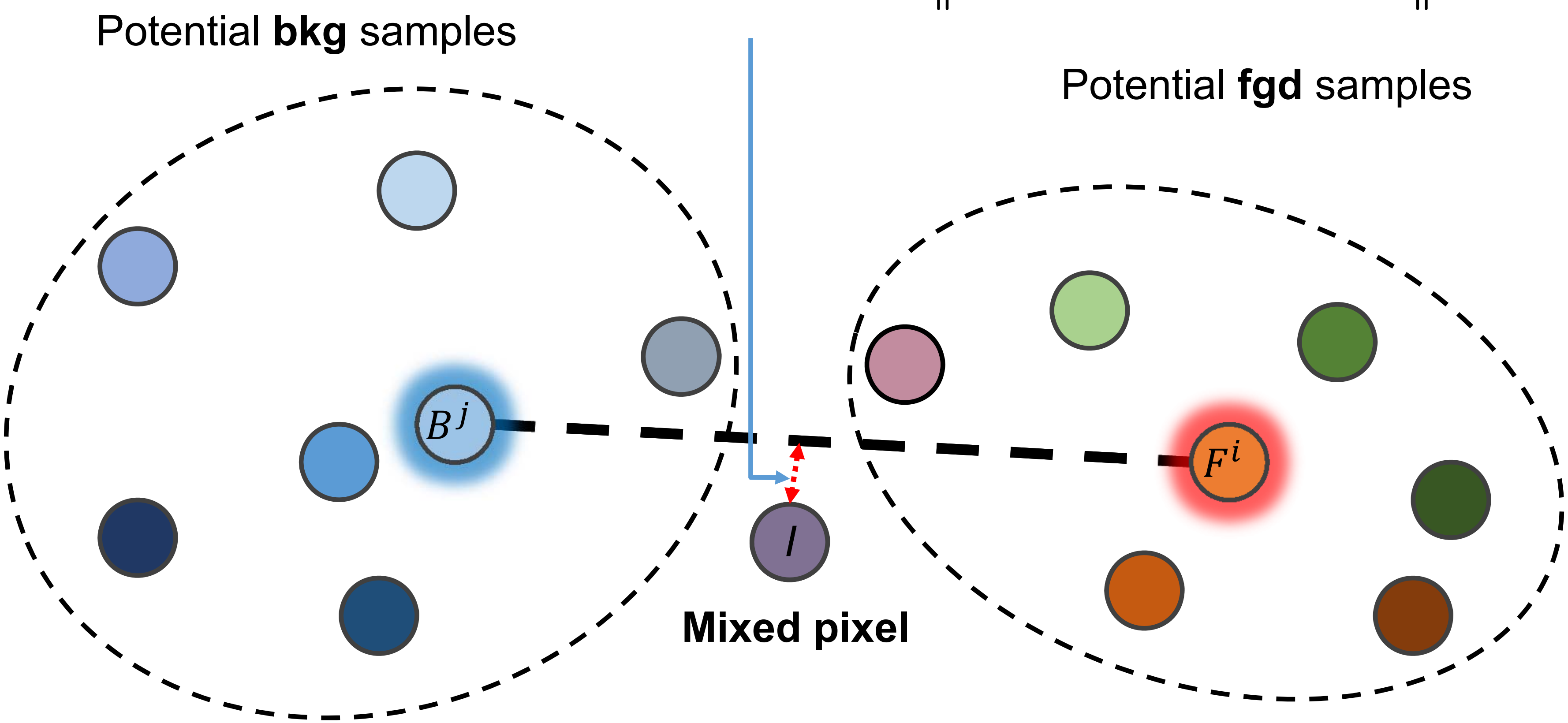
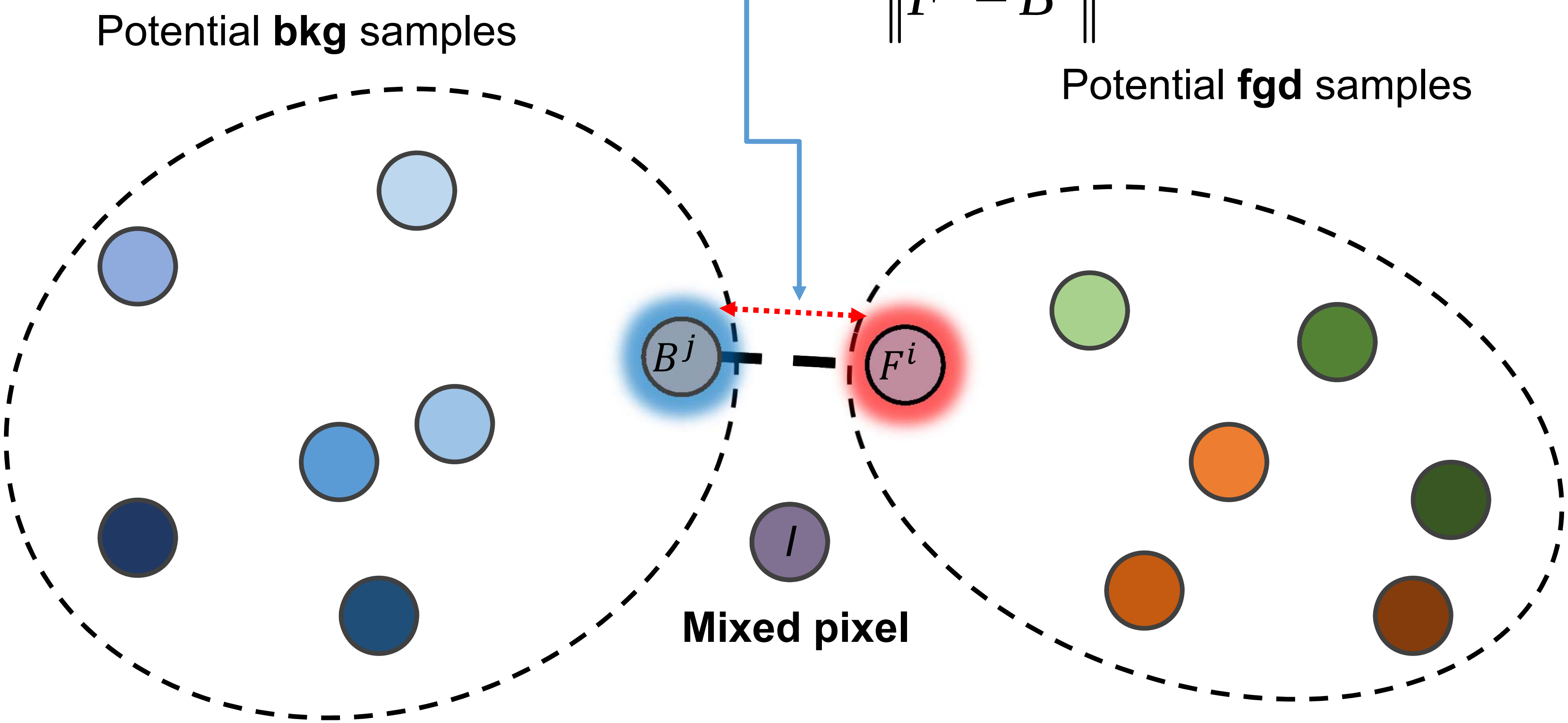


2. Robustness

$$R(F^i, B^j) = \frac{1}{\left\| F^i - B^j \right\|}$$

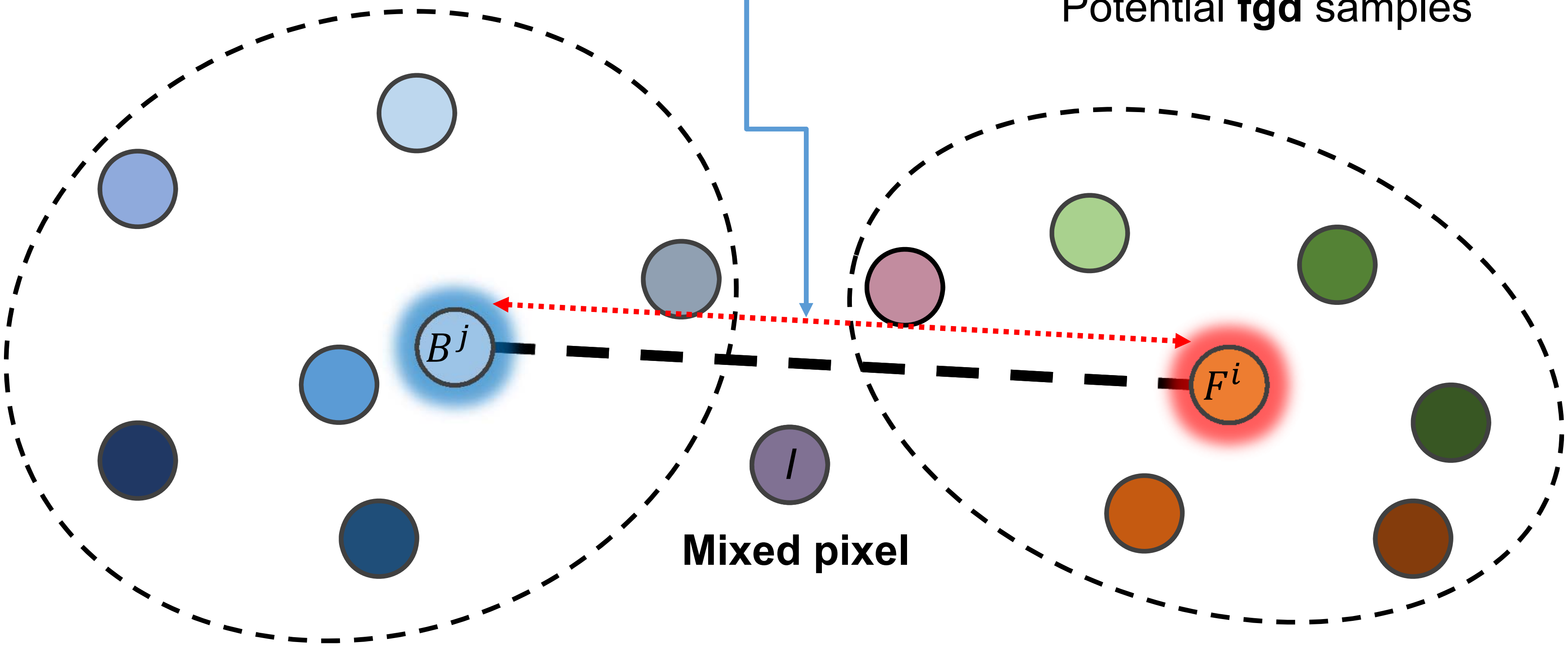


2. Robustness

$$R(F^i, B^j) = \frac{1}{\left\| F^i - B^j \right\|}$$

Potential **bkg** samples

Potential **fgd** samples



Mixed pixel

Step 2 - Selecting best samples

3. Color similarity

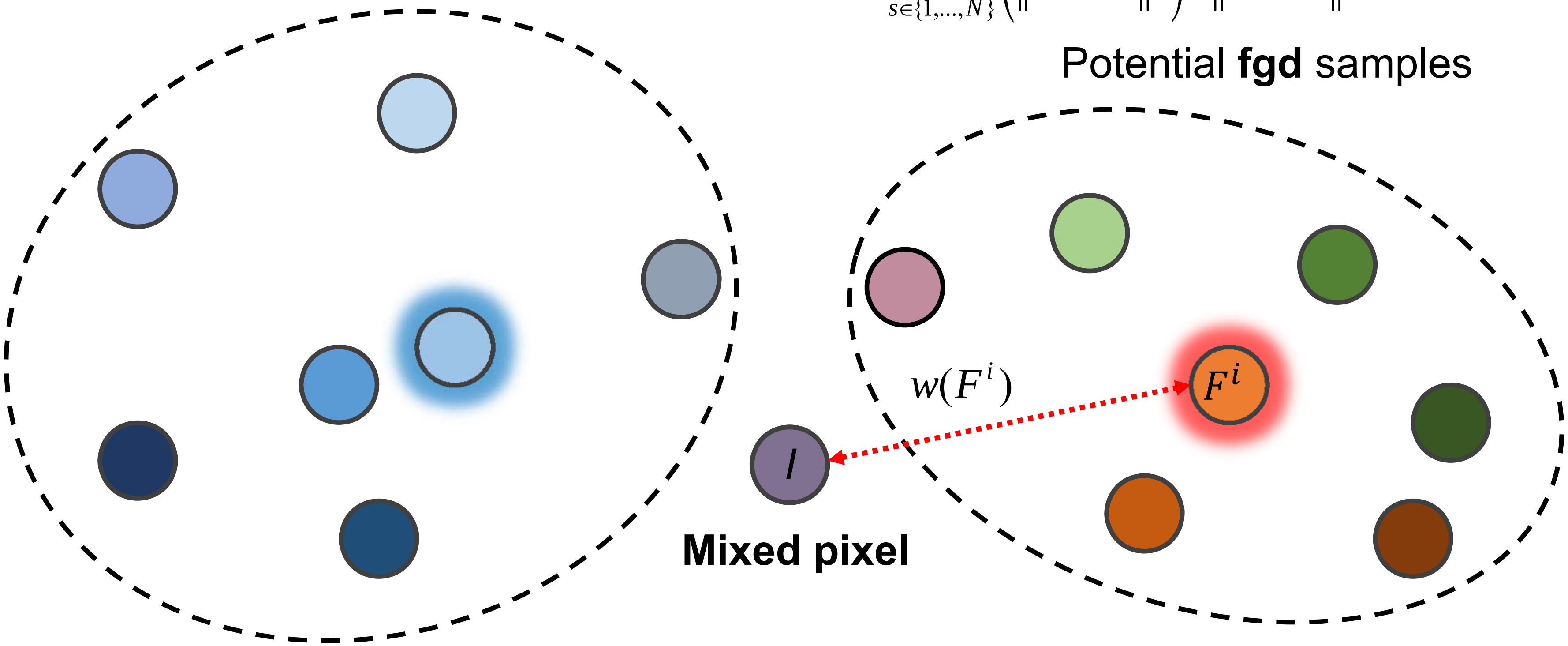
robust sampling

$$w(F^i) = 1.0 - \exp\{-\left\|F^i - I\right\|^2 / \min_{s\in\{1,\dots,N\}}\left(\left\|F^s - I\right\|^2\right)\}$$

improved sampling

$$w(F^i) = \exp\{-\max_{s\in\{1,\dots,N\}}\left(\left\|F^s - I\right\|^2\right) / \left\|F^i - I\right\|^2\}$$

Potential **bkg** samples



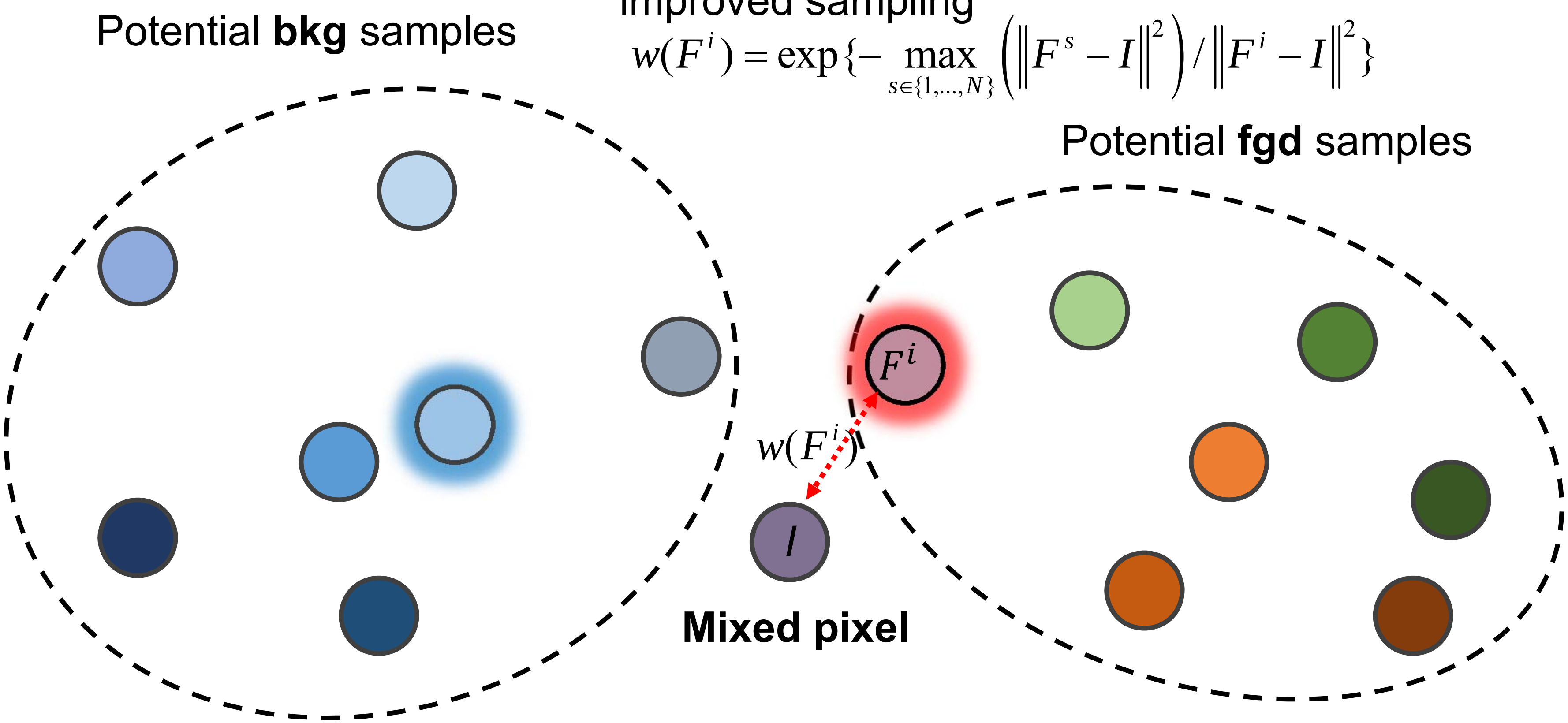
Step 2 - Selecting best samples

robust sampling

3. Color similarity $$w(F^i) = 1.0 - \exp\{-\left\|F^i - I\right\|^2 / \min_{s\in\{1,\ldots,N\}}\left(\left\|F^s - I\right\|^2\right)\}$$

improved sampling

$$w(F^i) = \exp\{-\max_{s\in\{1,\ldots,N\}}\left(\left\|F^s - I\right\|^2\right) / \left\|F^i - I\right\|^2\}$$



Step 2 - Selecting best samples

- **Three criteria for a confident pair:**

$$\hat{f}(F^i, B^j) = \exp\left\{\frac{E(F^i, B^j)^2 \cdot R(F^i, B^j)^2 \cdot w(F^i) \cdot w(B^j)}{\sigma^2}\right\}$$

1. Linear model fit $E(F^i, B^j) = \left\| I - (\hat{\alpha} F^i + (1-\hat{\alpha}) B^j \right\|$

2. Robustness $R(F^i, B^j) = \dfrac{1}{\left\| F^i - B^j \right\|}$

3. Color similarity

robust sampling

$$w(F^i) = 1.0 - \exp\{-\left\| F^i - I \right\|^2 / \min_{s \in \{1,\dots,N\}} \left(\left\| F^s - I \right\|^2 \right)\}$$

improved sampling

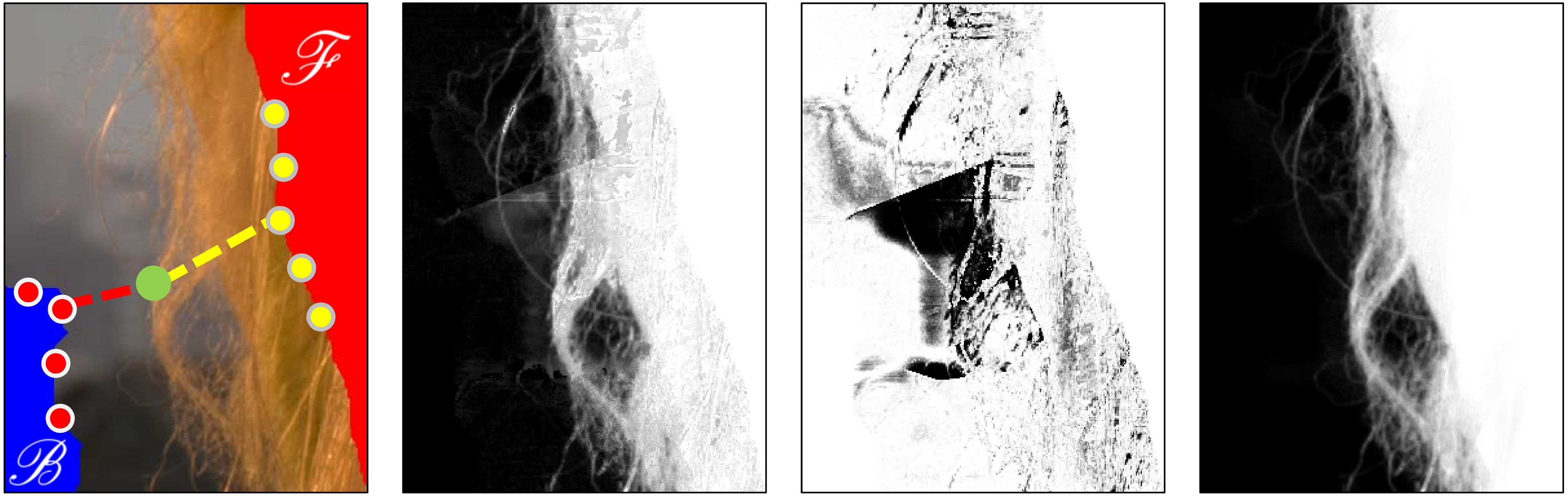
$$w(F^i) = \exp\{-\max_{s \in \{1,\dots,N\}} \left(\left\| F^s - I \right\|^2 \right) / \left\| F^i - I \right\|^2\}$$

Step 3 – solve final alpha

$$J(\alpha) = \alpha^T L \alpha + (\alpha - \hat{\alpha})^T \hat{\Gamma} (\alpha - \hat{\alpha})$$

Diagonal matrix whose diagonal elements are $\gamma \cdot \hat{f}$

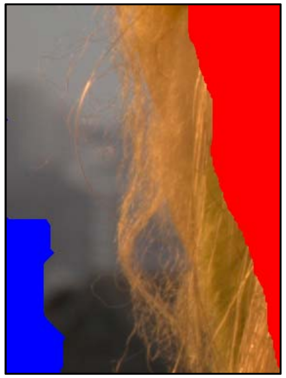
Input | **Initial alpha ($\hat{\alpha}$)** | **Confidence ($\hat{f}$)** | **Final alpha (α)**



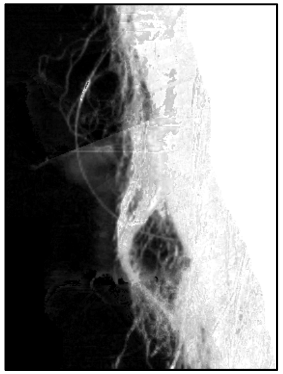
Color modeling | **Propagation**

$$J(\alpha) = \alpha^T L\alpha + (\alpha - \hat{\alpha})^T \hat{\Gamma}(\alpha - \hat{\alpha})$$

```
alpha_hat=getInitial_alpha(Image,user_input_map);
L=getLaplacian1(Image,user_input_map);
D=spdiags(user_input_map(:),0,img_size,img_size);
Gamma_hat=spdiags(confidence(:).*~user_input_map,0,img_size,img_size);
lambda=100;
K=lambda*D+gamma*Gamma_hat;
alpha=(L+K)\(K*alpha_hat(:));
```



image



Initial alpha ($\hat{\boldsymbol{\alpha}}$)

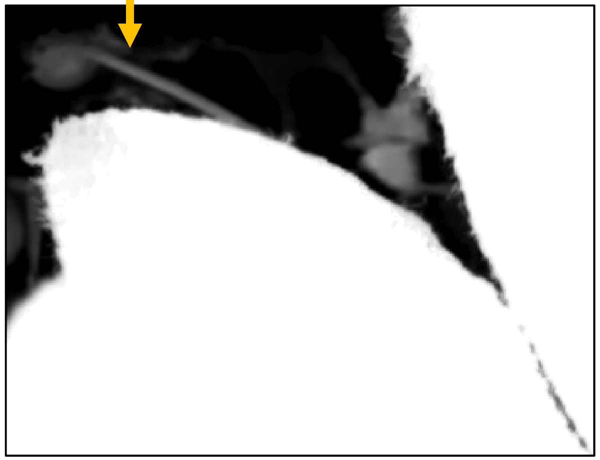
Confidence ($\hat{f}$)

Final alpha ($\boldsymbol{\alpha}$)

Matting results

Wrong sampling

Crop of input image



Robust matting

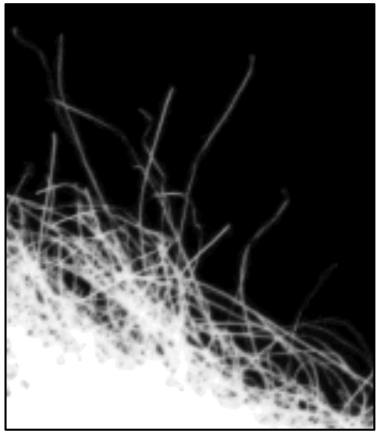
Closed form matting

Input image

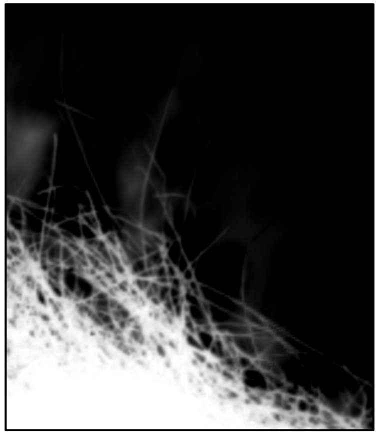
Improved color modeling

Ground truth

- Hair is better captured
- Many artifacts in the background


True solution


Input image + trimap


Closed form matting

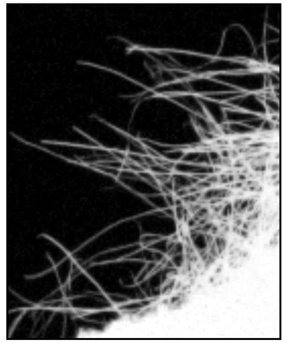

Robust matting

Alpha matting evaluation system

C. Rhemann, C. Rother, J. Wang, M. Gelautz, P. Kohli, and P. Rott, A Perceptually Motivated Online Benchmark for Image Matting. *IEEE CVPR*, June 2009.

- **For quantitative evalution**
  - **Ground truth dataset**
  - **Appropriate error metric**
  - **Online evaluation**

- 35 natural images



Input image Ground truth

Triangulation Matting [Smith and Blinn, 96]

- Photograph object against 2 different backgrounds

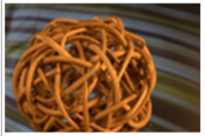
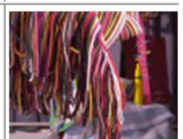
→ **True solution to matting problem**

Ground truth dataset

Test Dataset (Hidden ground-truth alpha)

Highly Transparent		Strongly Transparent		Medium Transparent		Little Transparent	
 Net*	 Plastic bag*	 Doll*	 Troll**	 Donkey**	 Elephant**	 Pineapple**	 Plant*

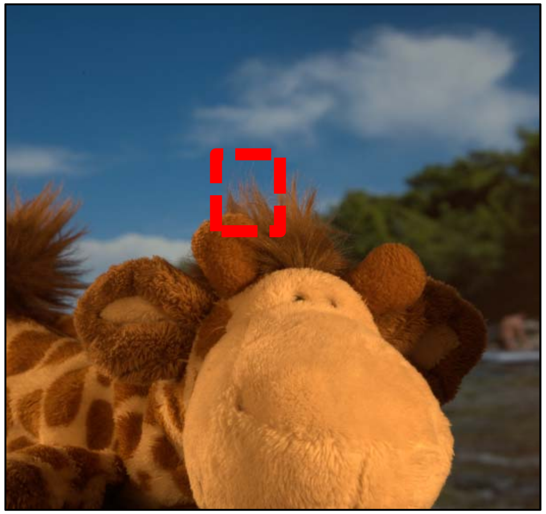
Training Dataset (With public available ground-truth alpha)

 GT01**	 GT02**	 GT03**	 GT04**	 GT05**	 GT06**	 GT07**	 GT08**	 GT09**
 GT10**	 GT11**	 GT12**	 GT13**	 GT14**	 GT15**	 GT16**	 GT17**	 GT18**
 GT19**	 GT20**	 GT21**	 GT22**	 GT23**	 GT24*	 GT25*	 GT26*	 GT27*

*Objects photographed in front of natural scenes.
**Objects photographed in front of a monitor showing natural images.

Perceptually motivated error functions

- **Motivation**
  - **Simple metrics not always correlated with visual quality**



Input image

Zoom in

Result 1
SAD: 1215

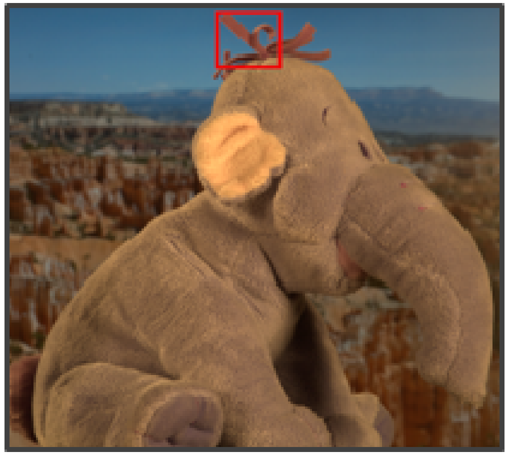
Result 2
SAD: 806

SAD = Sum of Absolute Differences

Perceptually motivated error functions

Develop error measures for two properties:

- **Connectivity** of foreground object
- **Gradient** of the alpha matte



Input image

Zoom in

Result 1
SAD: 312

Result 2
SAD: 83

Data and evaluation scripts online

Advantages:

- Investigate results
- Upload novel results

www.alphamatting.com

Image matting evaluation results

Competition: Low resolution High resolution

Error type: SAD MSE Gradient Connectivity

Sum of Absolute Differences	overall rank	avg. small rank	avg. large rank	avg. user rank	Troll (Strongly Transparent) Input small	Troll large	Troll user	Doll (Strongly Transparent) Input small	Doll large	Doll user	Donkey (Medium Transparent) Input small	Donkey large	Donkey user
Closed-Form Matting	1.3	1.4	1.4	1.3	12.7	21.9	17.2	5.9	8.5	8.6	4.7	6	4.3
Robust Matting	1.9	1.6	2.1	1.9	17.3	28.4	21.1	10.1	16.9	11.4	4.8	6.5	5
Random Walk Matting	3.3	3.5	3	3.5	17.9	20.3	19.4	11.3	15.6	11.8	5.8	7	6.3
Easy Matting	4	4	4.1	4	23.9	32.6	30	17.1	21.8	19.4	6.3	7.5	5.8
Bayesian Matting	4.5	4.5	4.6	4.5	30.3	42.4	33.4	19.2	25.8	18.4	10.8	12.4	10.8
Poisson Matting	5.9	6	5.8	5.9	51.8	56.2	52	28.3	43.5	30.7	12.1	13.7	9.2

Sum of Absolute Differences	Elephant (Medium Transparent) Input small	Elephant large	Elephant user	Plant (Little Transparent) Input small	Plant large	Plant user	Pineapple (Little Transparent) Input small	Pineapple large	Pineapple user	Plastic bag (Highly Transparent) Input small	Plastic bag large	Plastic bag user	Net (Highly Transparent) Input small	Net large	Net user
Closed-Form Matting	2.2	4.6	3.3	9.3	12.1	19.3	8.3	14.9	13.4	34.2	32.4	27.4	26.5	25.7	27.1
Robust Matting	2.8	7.3	4.4	7.3	14	18.1	6.8	14.6	10.6	22.7	26.1	32.1	24.4	37	38
Random Walk Matting	3.4	6.7	4.6	13.1	22.1	27.4	12.3	18	15.7	44.1	43.5	41	75.1	81.8	72.2
Easy Matting	4.7	10.5	5.6	12.1	15.7	22.9	11.2	17	14.8	49.5	49.6	46.2	77.8	108.6	109.2
Bayesian Matting	6.6	18.5	6.2	14.2	29.8	33.2	15.4	30.6	19.7	35.8	40.6	39.6	45.3	70.8	43.6
Poisson Matting	11.7	18.4	11.2	22.4	36.8	55.5	21.4	32.2	22.7	53.6	72.9	58.4	125.5	84.8	139.7

How to submit your algorithm

1. Download the evaluation datasets:
   - Make sure you pick the **low-resolution** dataset, since we currently only evaluate on this dataset.
   - The evaluation set consists of 8 input images with 3 different trimaps each.
2. Run your matting algorithm:
   - Use constant parameters across all images and trimaps.
   - Save the resulting alpha mattes with the following convention.

Trimap1
net.png
...
pineapple.png
Trimap2
net.png
...
Trimap3
net.png
...

3. Fina 3 trimap folders (that contain your results) in a single .zip ıbmit this file in the upload form on the website.
4. We ate your results and compile them in an online table. You see your results on a "private" page first, and we will only add your results to the public table after you give your consent.

Note that before making your results publically available, we will manually check your submission. This might take up to a few days but usually we will process your request within one working day.

Online submission form

Please enter the conference or journal where you are submitting.

Important notes:

- Processing your results will take at least **10 minutes**! Please be patient and avoid to repeatedly upload results or clicking the "Upload" button serval times. Try to upload early before your paper deadline and spread your uploads over time.

Upload form:

File upload

File upload: Choose File No file chosen

Publication info

Short name: Suggest a short name for your method

Authors: Fill in the name of the authors. Put "anonymous" for Anonymous submissions

Reference details: Fill in the publication details (i.e. paper title, conference name, year)

Email: Fill in a valid email address (please double check since we will send you a verification number, which is needed to complete the process).

Algorithmic details

Programming language: Fill in the programming language used (e.g. C++)

Processor type: Fill in the processor type used (e.g. Intel Core2 Quad)

Clock speed: Fill in the clock speed (in GHZ) of you test machine (e.g. 2.2)

Constant parameters used? Yes No You must use constant parameter settings

Upload File

After clicking the upload button, you will be promted to type in a verification code that you will receive via email.

Summary

- **Closed form matting**
  - **Closed form solution under local smoothness assumption**
  - **Analytically eliminate F, B and obtain quadratic cost $\alpha^T L\alpha$**
  - **Over-smooth result**
- **Robust and improved matting**
  - **Combine both sampling and propagation approaches**
  - **Very similar concept**
  - **Produced better results than closed form matting, but takes more time**
- **Alphamatting.com**
  - **Large set of ground truth data**
  - **Online benchmark**
  - **Perceptually motivated error measures**

Thank you

Acknowledgement

- **Ji-Ho Cho has been funded by the Austrian Science Fund (FWF) under project M1383-N23.**

FWF Der Wissenschaftsfonds.

- **The project has been funded by the Vienna Science and Technology Fund (WWTF) through project ICT08-019.**

WWTF VIENNA SCIENCE AND TECHNOLOGY FUND