Chapter 18, Problem 1.

Obtain the Fourier transform of the function in Fig. 18.26.

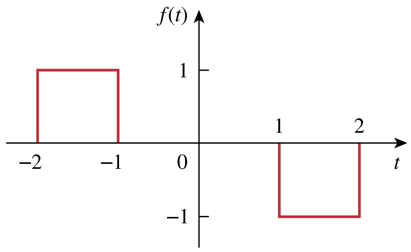


Figure 18.26
For Prob. 18.1.

Chapter 18, Solution 1.

$$f'(t) = \delta(t+2) - \delta(t+1) - \delta(t-1) + \delta(t-2)$$

$$j\omega F(\omega) = e^{j2\omega} - e^{j\omega} - e^{-j\omega} + e^{-j\omega 2}$$

$$= 2\cos 2\omega - 2\cos\omega$$

$$F(\omega) = \underline{\frac{\mathbf{2[\cos 2\omega - \cos\omega]}}{\mathbf{j\omega}}}$$

Chapter 18, Problem 2.

What is the Fourier transform of the triangular pulse in Fig. 18.27?

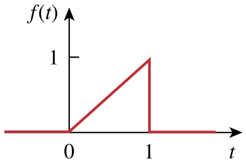
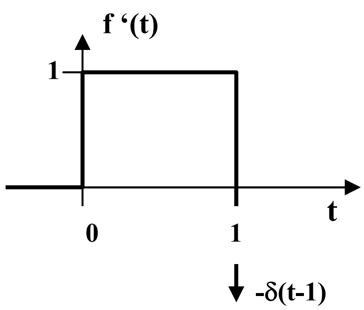
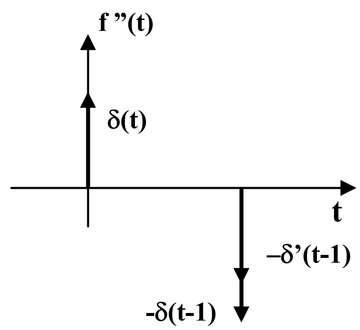


Figure 18.27
For Prob. 18.2.

Chapter 18, Solution 2.

$$f(t)=\begin{cases} t, & 0<t<1 \\ 0, & \text{otherwise} \end{cases}$$





$$f''(t) = \delta(t) - \delta(t-1) - \delta'(t-1)$$

Taking the Fourier transform gives

$$-\omega^2 F(\omega) = 1 - e^{-j\omega} - j\omega e^{-j\omega}$$

$$F(\omega) = \underline{\frac{\mathbf{(1+j\omega)e^{j\omega}-1}}{\mathbf{\omega^2}}}$$

or $F(\omega)=\int_0^1 t\,e^{-j\omega t}\,dt$

But $\int x\,e^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)+c$

$$F(\omega)=\frac{e^{-j\omega}}{(-j\omega)^2}(-j\omega t-1)\Big|_0^1 = \underline{\frac{\mathbf{1}}{\mathbf{\omega^2}}\left[\mathbf{(1+j\omega)e^{-j\omega}-1}\right]}$$

Chapter 18, Problem 3.

Calculate the Fourier transform of the signal in Fig. 18.28.

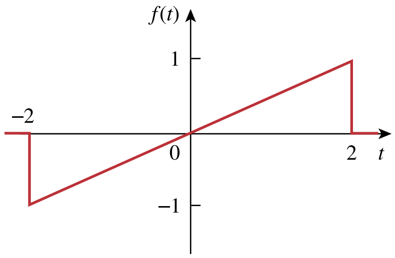


Figure 18.28
For Prob. 18.3.

Chapter 18, Solution 3.

$$f(t) = \frac{1}{2}t, \; -2 < t < 2, \qquad f'(t) = \frac{1}{2}, \; -2 < t < 2$$

$$F(\omega) = \int_{-2}^{2} \frac{1}{2} t\, e^{j\omega t} dt = \frac{e^{-j\omega t}}{2(-j\omega)^2}(-j\omega t - 1)\Big|_{-2}^{2}$$

$$= -\frac{1}{2\omega^2}\left[e^{-j\omega 2}(-j\omega 2 - 1) - e^{j\omega 2}(j\omega 2 - 1)\right]$$

$$= -\frac{1}{2\omega^2}\left[-j\omega 2\left(e^{-j\omega 2} + e^{j\omega 2}\right) + e^{j\omega 2} - e^{-j\omega 2}\right]$$

$$= -\frac{1}{2\omega^2}\left(-j\omega 4\cos 2\omega + j2\sin 2\omega\right)$$

$$\underline{F(\omega) = \frac{j}{\omega^2}(2\omega\cos 2\omega - \sin 2\omega)}$$

Chapter 18, Problem 4.

Find the Fourier transform of the waveform shown in Fig. 18.29.

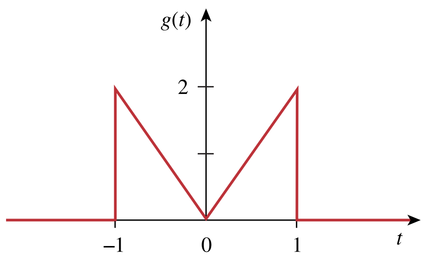
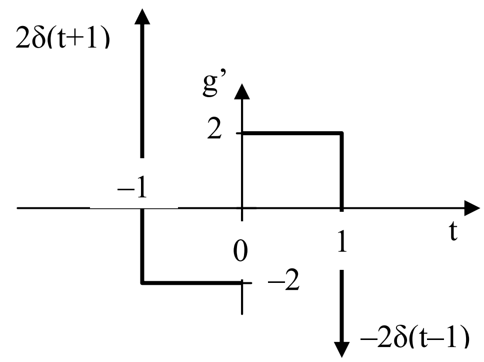
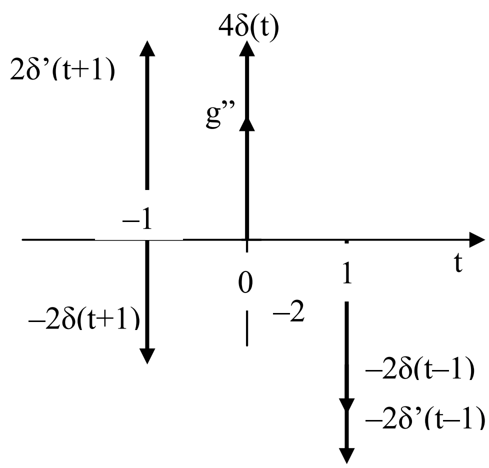


Figure 18.29
For Prob. 18.4.

Chapter 18, Solution 4.





$$g'' = -2\delta(t+1) + 2\delta'(t+1) + 4\delta(t) - 2\delta(t-1) - 2\delta'(t-1)$$

$$\begin{aligned}(j\omega)^2 G(\omega) &= -2e^{j\omega} + 2j\omega e^{j\omega} + 4 - 2e^{-j\omega} - 2j\omega e^{-j\omega} \\ &= -4\cos\omega - 4\omega\sin\omega + 4\end{aligned}$$

$$\underline{G(\omega) = \frac{4}{\omega^2}(\cos\omega + \omega\sin\omega - 1)}$$

Chapter 18, Problem 5.

Obtain the Fourier transform of the signal shown in Fig. 18.30.

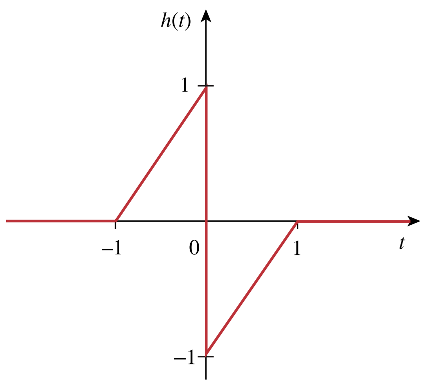
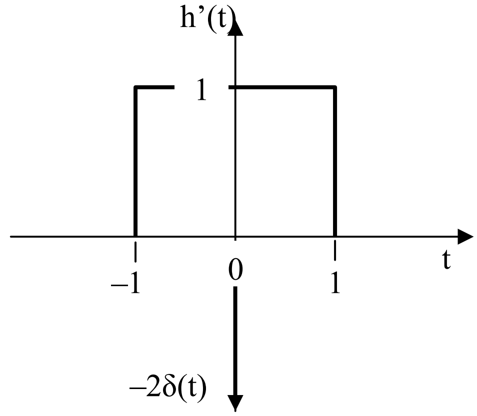
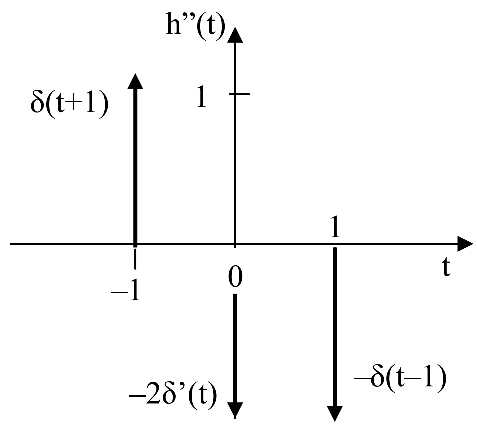


Figure 18.30
For Prob. 18.5.

Chapter 18, Solution 5.





$$h''(t) = \delta(t+1) - \delta(t-1) - 2\delta'(t)$$

$$(j\omega)^2 H(\omega) = e^{j\omega} - e^{-j\omega} - 2j\omega = 2j\sin\omega - 2j\omega$$

$$\underline{H(\omega) = \frac{2j}{\omega} - \frac{2j}{\omega^2}\sin\omega}$$

Chapter 18, Problem 6.

Find the Fourier transforms of both functions in Fig. 18.31 on the following page.

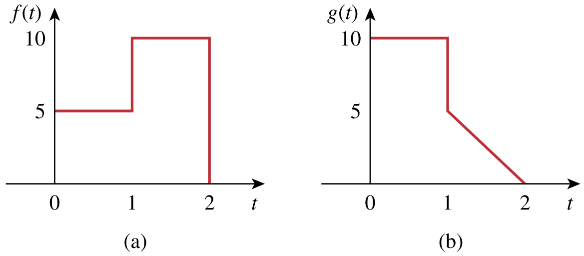
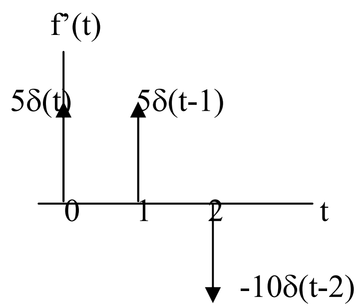


Figure 18.31
For Prob. 18.6.

Chapter 18, Solution 6.

(a) The derivative of f(t) is shown below.



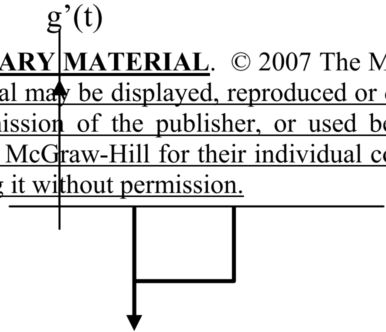
$$f\,'(t) = 5\delta(t) + 5\delta(t-1) - 10\delta(t-2)$$

Taking the Fourier transform of each term,

$$j\omega F(\omega) = 5 + 5e^{-j\omega} - 10e^{-j2\omega}$$

$$F(\omega) = \boxed{\frac{5 + 5e^{-j\omega} - 10e^{-j2\omega}}{j\omega}}$$

(b) The derivative of g(t) is shown below.



10δ(t)

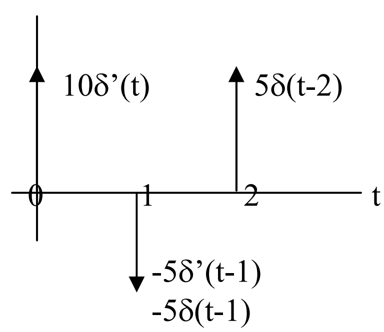
0 1 2

-5

-5δ(t-1)

The second derivative of g(t) is shown below.

g''(t)



$g''(t) = 10\delta'(t) - 5\delta'(t-1) - 5\delta(t-1) + 5\delta(t-2)$

Take the Fourier transform of each term.

$(j\omega)^2 G(j\omega) = 10j\omega - 5j\omega e^{-j\omega} - 5e^{-j\omega} + 5e^{-j2\omega}$ which leads to

$$G(j\omega) = \underline{\boldsymbol{(-10j\omega + 5j\omega e^{-j\omega} + 5e^{-j\omega} - 5e^{-j2\omega})/\omega^2}}$$

Chapter 18, Problem 7.

Find the Fourier transforms of the signals in Fig. 18.32.

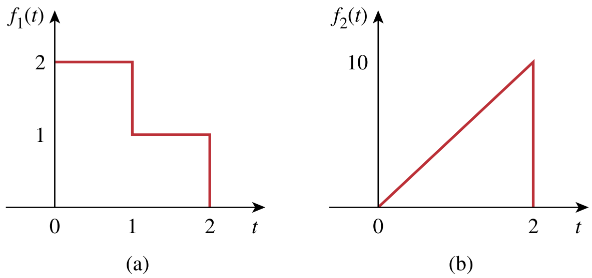
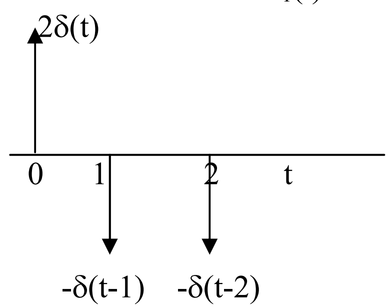


Figure 18.32
For Prob. 18.7.

Chapter 18, Solution 7.

(a) Take the derivative of $f_1(t)$ and obtain $f_1'(t)$ as shown below.



$$f_1^{'}(t) = 2\delta(t) - \delta'(t-1) - \delta(t-2)$$

Take the Fourier transform of each term,

$$j\omega F_1(\omega) = 2 - e^{-j\omega} - e^{-j2\omega}$$

$$\underline{F_1(\omega) = \frac{2 - e^{-j\omega} - e^{-j2\omega}}{j\omega}}$$

(b) $f_2(t) = 5t$

$$F_2(\omega) = \int_{-\infty}^{\infty} f_2(t) e^{-j\omega} dt = \int_0^2 5te^{-j\omega} dt = \frac{5}{(-j\omega)^2} e^{-j\omega t}(-j\omega - 1)\Big|_0^2$$

$$\underline{F_2(\omega) = \frac{5e^{-j2\omega}}{\omega^2}(1 + j\omega 2) - \frac{5}{\omega^2}}$$

Chapter 18, Problem 8.

Obtain the Fourier transforms of the signals shown in Fig. 18.33.

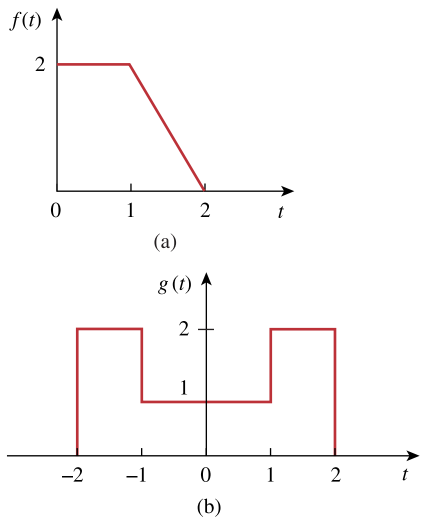


Figure 18.33
For Prob. 18.8.

Chapter 18, Solution 8.

(a)

$$F(\omega) = \int_0^1 2e^{-j\omega t}dt + \int_1^2 (4-2t)e^{-j\omega t}dt$$

$$= \frac{2}{-j\omega}e^{-j\omega t}\Big|_0^1 + \frac{4}{-j\omega}e^{-j\omega t}\Big|_1^2 - \frac{2}{-\omega^2}e^{-j\omega t}(-j\omega t - 1)\Big|_1^2$$

$$\underline{F(\omega) = \frac{2}{\omega^2} + \frac{2}{j\omega}e^{-j\omega} + \frac{2}{j\omega} - \frac{4}{j\omega}e^{-j2\omega} - \frac{2}{\omega^2}(1 + j2\omega)e^{-j2\omega}}$$

(b) g(t) = 2[u(t+2) – u(t-2)] - [u(t+1) – u(t-1)]

$$G(\omega) = \underline{\frac{4\sin 2\omega}{\omega} - \frac{2\sin\omega}{\omega}}$$

Chapter 18, Problem 9.

Determine the Fourier transforms of the signals in Fig. 18.34.

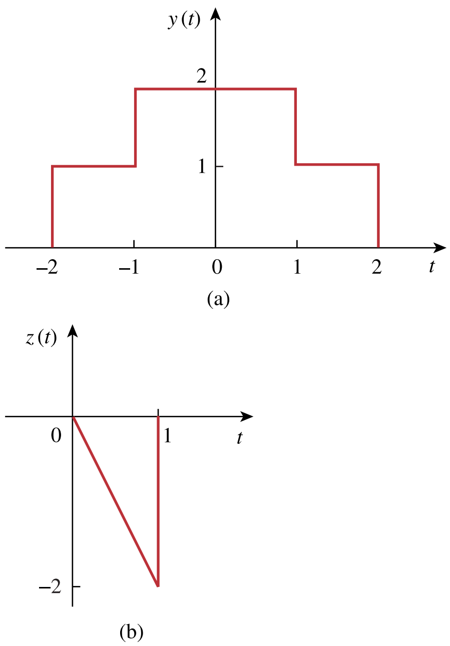


Figure 18.34
For Prob. 18.9.

Chapter 18, Solution 9.

(a) $y(t) = u(t+2) - u(t-2) + 2[\, u(t+1) - u(t-1)\,]$

$$Y(\omega) = \underline{\frac{2}{\omega}\sin 2\omega + \frac{4}{\omega}\sin\omega}$$

(b) $$Z(\omega) = \int_0^1 (-2t)e^{-j\omega t}dt = \frac{-2e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\Big|_0^1 = \underline{\frac{2}{\omega^2} - \frac{2e^{-j\omega}}{\omega^2}(1+j\omega)}$$

Chapter 18, Problem 10.

Obtain the Fourier transforms of the signals shown in Fig. 18.35.

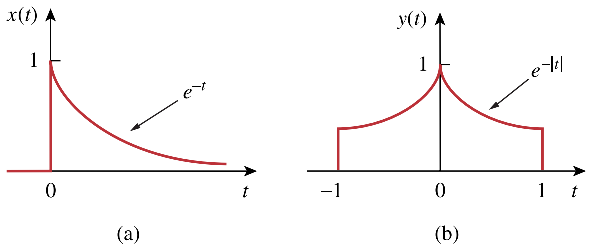


Figure 18.35
For Prob. 18.10.

Chapter 18, Solution 10.

(a) $x(t) = e^{2t}u(t)$

$X(\omega) = \underline{\mathbf{1/(-2 + j\omega)}}$

(b) $$e^{-(t)} = \begin{cases} e^{-t}, & t > 0 \\ e^{t}, & t < 0 \end{cases}$$

$$Y(\omega) = \int_{-1}^{1} y(t)e^{j\omega t}\,dt = \int_{-1}^{0} e^{t}\,e^{j\omega t}\,dt + \int_{0}^{1} e^{-t}\,e^{-j\omega t}\,dt$$

$$= \left.\frac{e^{(1-j\omega)t}}{1-j\omega}\right|_{-1}^{0} + \left.\frac{e^{-(1+j\omega)t}}{-(1+j\omega)}\right|_{0}^{1}$$

$$= \frac{2}{1+\omega^2} - e^{-1}\left[\frac{\cos\omega + j\sin\omega}{1-j\omega} + \frac{\cos\omega - j\sin\omega}{1+j\omega}\right]$$

$$Y(\omega) = \underline{\frac{\mathbf{2}}{\mathbf{1+\omega^2}}\left[\mathbf{1 - e^{-1}(\cos\omega - \omega\sin\omega)}\right]}$$

Chapter 18, Problem 11.

Find the Fourier transform of the “sine-wave pulse” shown in Fig. 18.36.

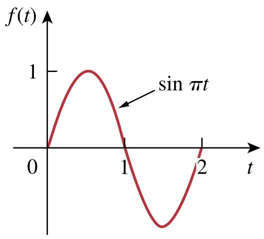


Figure 18.36
For Prob. 18.11.

Chapter 18, Solution 11.

$$f(t) = \sin \pi t\,[u(t) - u(t - 2)]$$

$$F(\omega) = \int_0^2 \sin \pi t\, e^{-j\omega t}\, dt = \frac{1}{2j}\int_0^2 \left(e^{j\pi t} - e^{-j\pi t}\right)e^{-j\omega t}\, dt$$

$$= \frac{1}{2j}\left[\int_0^2 (e^{+j(-\omega+\pi)t} + e^{-j(\omega+\pi)t})dt\right]$$

$$= \frac{1}{2j}\left[\frac{1}{-j(\omega-\pi)}e^{-j(\omega-\pi)t}\Big|_0^2 + \frac{e^{-j(\omega+\pi)t}}{-j(\omega+\pi)}\Big|_0^2\right]$$

$$= \frac{1}{2}\left(\frac{1-e^{-j2\omega}}{\pi-\omega} + \frac{1-e^{-j2\omega}}{\pi+\omega}\right)$$

$$= \frac{1}{2(\pi^2-\omega^2)}\left(2\pi + 2\pi e^{-j2\omega}\right)$$

$$F(\omega) = \underline{\boldsymbol{\frac{\pi}{\omega^2-\pi^2}\left(e^{-j\omega 2} - 1\right)}}$$

Chapter 18, Problem 12.

Find the Fourier transform of the following signals.

(a) $f_1(t) = e^{-3t}\sin(10t)u(t)$

(b) $f_2(t) = e^{-4t}\cos(10t)u(t)$

Chapter 18, Solution 12.

(a) $F_1(\omega) = \dfrac{10}{(3+j\omega)^2+100}$

(b) $F_2(\omega) = \dfrac{4+j\omega}{(4+j\omega)^2+100}$

Chapter 18, Problem 13.

Find the Fourier transform of the following signals:

(a) $f(t) = \cos(at - \pi/3)$, $\quad -\infty < t < \infty$

(b) $g(t) = u(t+1)\sin \pi t$, $\quad -\infty < t < \infty$

(c) $h(t) = (1 + A \sin at) \cos bt$, $\quad -\infty < t < \infty$, where A, a and b are constants

(d) $i(t) = 1 - t$, $\quad 0 < t < 4$

Chapter 18, Solution 13.

(a) We know that $F[\cos at] = \pi[\delta(\omega - a) + \delta(\omega + a)]$.

Using the time shifting property,

$$F[\cos a(t - \pi/3a)] = \pi e^{-j\omega\pi/3a}[\delta(\omega - a) + \delta(\omega + a)] = \underline{\pi e^{-j\pi/3}\delta(\omega - a) + \pi e^{j\pi/3}\delta(\omega + a)}$$

(b) $\sin \pi(t+1) = \sin \pi t \cos \pi + \cos \pi t \sin \pi = -\sin \pi t$

g(t) = -u(t+1) sin (t+1)

Let $x(t) = u(t)\sin t$, then $X(\omega) = \dfrac{1}{(j\omega)^2 + 1} = \dfrac{1}{1 - \omega^2}$

Using the time shifting property,

$$G(\omega) = -\frac{1}{1 - \omega^2} e^{j\omega} = \underline{\frac{e^{j\omega}}{\omega^2 - 1}}$$

(c) Let $y(t) = 1 + A\sin at$, then $Y(\omega) = 2\pi\delta(\omega) + j\pi A[\delta(\omega + a) - \delta(\omega - a)]$

$$h(t) = y(t) \cos bt$$

Using the modulation property,

$$H(\omega) = \frac{1}{2}[Y(\omega + b) + Y(\omega - b)]$$

$$H(\omega) = \underline{\pi[\delta(\omega + b) + \delta(\omega - b)] + \frac{j\pi A}{2}[\delta(\omega + a + b) - \delta(\omega - a + b) + \delta(\omega + a - b) - \delta(\omega - a - b)]}$$

(d) $I(\omega) = \displaystyle\int_0^4 (1 - t)e^{-j\omega t}dt = \frac{e^{-j\omega t}}{-j\omega} - \frac{e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\Big|_0^4 = \underline{\frac{1}{\omega^2} - \frac{e^{-j4\omega}}{j\omega} - \frac{e^{-j4\omega}}{\omega^2}(j4\omega + 1)}$

Chapter 18, Problem 14.

Find the Fourier transforms of these functions:

(a) $f(t) = e^{-t}\cos(3t + \pi)u(t)$

(b) $g(t) = \sin \pi t[u(t + 1) - u(t - 1)]$

(c) $h(t) = e^{-2t}\cos \pi t u(t - 1)$

(d) $p(t) = e^{-2t}\sin 4t u(-t)$

(e) $q(t) = 4\,\mathrm{sgn}(t - 2) + 3\delta(t) - 2u(t - 2)$

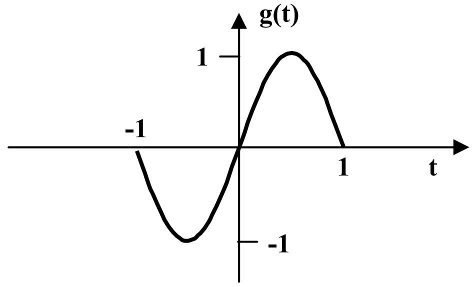
Chapter 18, Solution 14.

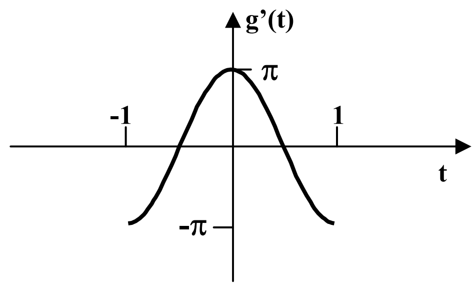
(a) $\cos(3t+\pi) = \cos 3t \cos\pi - \sin 3t \sin\pi = \cos 3t(-1) - \sin 3t(0) = -\cos(3t)$

$$f(t) = -e^{-t}\cos 3t\, u(t)$$

$$F(\omega) = \underline{\mathbf{\frac{-(1+j\omega)}{(1+j\omega)^2+9}}}$$

(b)





$$g'(t) = \pi\cos\pi t[u(t-1) - u(t-1)]$$
$$g''(t) = -\pi^2 g(t) - \pi\delta(t+1) + \pi\delta(t-1)$$
$$-\omega^2 G(\omega) = -\pi^2 G(\omega) - \pi e^{j\omega} + \pi e^{-j\omega}$$
$$(\pi^2 - \omega^2)G(\omega) = -\pi(e^{j\omega} - e^{-j\omega}) = -2j\pi\sin\omega$$
$$G(\omega) = \underline{\mathbf{\frac{2j\pi\sin\omega}{\omega^2 - \pi^2}}}$$

Alternatively, we compare this with Prob. 17.7

f(t) = g(t - 1)

$F(\omega) = G(\omega)e^{-j\omega}$

$$G(\omega) = F(\omega)e^{j\omega} = \frac{\pi}{\omega^2 - \pi^2}\left(e^{-j\omega} - e^{j\omega}\right)$$
$$= \frac{-j2\pi\sin\omega}{\omega^2 - \pi^2}$$
$$G(\omega) = \underline{\mathbf{\frac{2j\pi\sin\omega}{\pi^2 - \omega^2}}}$$

(c) $\cos\pi(t-1) = \cos\pi t\cos\pi + \sin\pi t\sin\pi = \cos\pi t(-1) + \sin\pi t(0) = -\cos\pi t$

Let $x(t) = e^{-2(t-1)}\cos\pi(t-1)u(t-1) = -e^{2}h(t)$

and $y(t) = e^{-2t}\cos(\pi t)u(t)$

$$Y(\omega) = \frac{2+j\omega}{(2+j\omega)^2 + \pi^2}$$

$$y(t) = x(t-1)$$

$$Y(\omega) = X(\omega)e^{-j\omega}$$

$$X(\omega) = \frac{(2+j\omega)e^{j\omega}}{(2+j\omega)^2 + \pi^2}$$

$$X(\omega) = -e^{2}H(\omega)$$

$$H(\omega) = -e^{-2}X(\omega)$$

$$= \underline{\boldsymbol{\frac{-(2+j\omega)e^{j\omega-2}}{(2+j\omega)^2 + \pi^2}}}$$

(d) Let $x(t) = e^{-2t}\sin(-4t)u(-t) = y(-t)$

$p(t) = -x(t)$

where $y(t) = e^{2t}\sin 4t\,u(t)$

$$Y(\omega) = \frac{2+j\omega}{(2+j\omega)^2 + 4^2}$$

$$X(\omega) = Y(-\omega) = \frac{2-j\omega}{(2-j\omega)^2 + 16}$$

$$p(\omega) = -X(\omega) = \underline{\boldsymbol{\frac{j\omega-2}{(j\omega-2)^2 + 16}}}$$

(e) $Q(\omega) = \dfrac{8}{j\omega}e^{-j\omega 2} + 3 - 2\left(\pi\delta(\omega) + \dfrac{1}{j\omega}\right)e^{-j\omega 2}$

$$Q(\omega) = \underline{\boldsymbol{\frac{6}{j\omega}e^{j\omega 2} + 3 - 2\pi\delta(\omega)e^{-j\omega 2}}}$$

Chapter 18, Problem 15.

Find the Fourier transforms of the following functions:

(a) $f(t) = \delta(t+3) - \delta(t-3)$

(b) $f(t) = \int_{-\infty}^{\infty} 2\delta(t-1)\,dt$

(c) $f(t) = \delta(3t) - \delta'(2t)$

Chapter 18, Solution 15.

(a) $F(\omega) = e^{j3\omega} - e^{-j\omega 3} = \underline{\mathbf{2j\sin 3\omega}}$

(b) Let $g(t) = 2\delta(t-1)$, $G(\omega) = 2e^{-j\omega}$

$$F(\omega) = F\left(\int_{-\infty}^{t} g(t)\,dt\right)$$

$$= \frac{G(\omega)}{j\omega} + \pi F(0)\delta(\omega)$$

$$= \frac{2e^{-j\omega}}{j\omega} + 2\pi\delta(-1)\delta(\omega)$$

$$= \underline{\frac{\mathbf{2e^{-j\omega}}}{\mathbf{j\omega}}}$$

(c) $F\left[\delta(2t)\right] = \frac{1}{2}\cdot 1$

$$F(\omega) = \frac{1}{3}\cdot 1 - \frac{1}{2}j\omega = \underline{\frac{\mathbf{1}}{\mathbf{3}} - \frac{\mathbf{j\omega}}{\mathbf{2}}}$$

Chapter 18, Problem 16.

* Determine the Fourier transforms of these functions:

(a) $f(t) = 4/t^2$ (b) $g(t) = 8/(4 + t^2)$

* An asterisk indicates a challenging problem.

Chapter 18, Solution 16.

(a) Using duality properly

$$|t| \rightarrow \frac{-2}{\omega^2}$$

$$\frac{-2}{t^2} \rightarrow 2\pi|\omega|$$

or
$$\frac{4}{t^2} \rightarrow -4\pi|\omega|$$

$$F(\omega) = F\left(\frac{4}{t^2}\right) = \underline{-\mathbf{4\pi|\omega|}}$$

(b)
$$e^{-|a|t} \longrightarrow \frac{2a}{a^2 + \omega^2}$$

$$\frac{2a}{a^2 + t^2} \longrightarrow 2\pi e^{-a|\omega|}$$

$$\frac{8}{a^2 + t^2} \longrightarrow 4\pi e^{-2|\omega|}$$

$$G(\omega) = F\left(\frac{8}{4 + t^2}\right) = \underline{\mathbf{4\pi e^{-2|\omega|}}}$$

Chapter 18, Problem 17.

Find the Fourier transforms of:

(a) $\cos 2tu(t)$ (b) $\sin 10tu(t)$

Chapter 18, Solution 17.

(a) Since $H(\omega) = \mathcal{F}\left(\cos\omega_0 t\ f(t)\right) = \frac{1}{2}\left[F(\omega+\omega_0) + F(\omega-\omega_0)\right]$

where $F(\omega) = \mathcal{F}\left[u(t)\right] = \pi\delta(\omega) + \frac{1}{j\omega}$, $\omega_0 = 2$

$$H(\omega) = \frac{1}{2}\left[\pi\delta(\omega+2) + \frac{1}{j(\omega+2)} + \pi\delta(\omega-2) + \frac{1}{j(\omega-2)}\right]$$

$$= \frac{\pi}{2}\left[\delta(\omega+2) + \delta(\omega-2)\right] - \frac{j}{2}\left[\frac{\omega+2+\omega-2}{(\omega+2)(\omega-2)}\right]$$

$$H(\omega) = \underline{\boldsymbol{\frac{\pi}{2}\left[\delta(\omega+2) + \delta(\omega-2)\right] - \frac{j\omega}{\omega^2-4}}}$$

(b) $G(\omega) = \mathcal{F}\left[\sin\omega_0 t\ f(t)\right] = \frac{j}{2}\left[F(\omega+\omega_0) - F(\omega-\omega_0)\right]$

where $F(\omega) = \mathcal{F}\left[u(t)\right] = \pi\delta(\omega) + \frac{1}{j\omega}$

$$G(\omega) = \frac{j}{2}\left[\pi\delta(\omega+10) + \frac{1}{j(\omega+10)} - \pi\delta(\omega-10) - \frac{1}{j(\omega-10)}\right]$$

$$= \frac{j\pi}{2}\left[\delta(\omega+10) - \delta(\omega-10)\right] + \frac{j}{2}\left[\frac{j}{\omega-10} - \frac{j}{\omega+10}\right]$$

$$= \underline{\boldsymbol{\frac{j\pi}{2}\left[\delta(\omega+10) - \delta(\omega-10)\right] - \frac{10}{\omega^2-100}}}$$

Chapter 18, Problem 18.

Given that $F(\omega)$ = F[$f(t)$], prove the following results, using the definition of Fourier transform:

(a) $F[f(t-t_0)] = e^{-j\omega t_0} F(\omega)$

(b) $F\left[\dfrac{df(t)}{dt}\right] = j\omega F(\omega)$

(c) $F[f(-t)] = F(-\omega)$

(d) $F[tf(t)] = j\dfrac{d}{d\omega}F(\omega)$

Chapter 18, Solution 18.

(a) $F[f(t-t_o)] = \int_{-\infty}^{\infty} f(t-t_o)e^{-j\omega t}dt$

Let $t - t_o = \lambda \longrightarrow t = \lambda + t_o, \quad dt = d\lambda$

$$F[f(t-t_o)] = \int_{-\infty}^{\infty} f(\lambda)e^{-j\omega\lambda}e^{-j\omega t_o}d\lambda = e^{-j\omega t_o}F(\omega)$$

(b) Given that $f(t) = F^{-1}[F(\omega)] = \dfrac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$

$$f'(t) = \frac{j\omega}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}dt = j\omega F^{-1}[F(\omega)]$$

or

$$F[f'(t)] = j\omega F(\omega)$$

(c) This is a special case of the time scaling property when a = –1. Hence,

$$F[f(-t)] = \frac{1}{|-1|}F(-\omega) = F(-\omega)$$

(d) $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$

Differentiating both sides respect to ω and multiplying by t yields

$$j\frac{dF(\omega)}{d\omega} = j\int_{-\infty}^{\infty}(-jt)f(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} tf(t)e^{-j\omega t}dt$$

Hence,

$$j\frac{dF(\omega)}{d\omega} = F[tf(t)]$$

Chapter 18, Problem 19.

Find the Fourier transform of
$f(t) = \cos 2\pi t[u(t) - u(t-1)]$

Chapter 18, Solution 19.

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{j\omega t}\,dt = \frac{1}{2}\int_0^1 \left(e^{j2\pi t} + e^{-j2\pi t}\right)e^{-j\omega t}\,dt$$

$$F(\omega) = \frac{1}{2}\int_0^1 \left[e^{-j(\omega+2\pi)t} + e^{-j(\omega-2\pi)t}\right]dt$$

$$= \frac{1}{2}\left[\frac{1}{-j(\omega+2\pi)}e^{-j(\omega+2\pi)t} + \frac{1}{-j(\omega-2\pi)}e^{-j(\omega-2\pi)t}\right]_0^1$$

$$= -\frac{1}{2}\left[\frac{e^{-j(\omega+2\pi)}-1}{j(\omega+2\pi)} + \frac{e^{-j(\omega-2\pi)}-1}{j(\omega-2\pi)}\right]$$

But $\quad e^{j2\pi} = \cos 2\pi + j\sin 2\pi = 1 = e^{-j2\pi}$

$$F(\omega) = -\frac{1}{2}\left(\frac{e^{-j\omega}-1}{j}\right)\left(\frac{1}{\omega+2\pi} + \frac{1}{\omega-2\pi}\right)$$

$$= \underline{\boldsymbol{\frac{j\omega}{\omega^2 - 4\pi^2}\left(e^{-j\omega} - 1\right)}}$$

Chapter 18, Problem 20.

(a) Show that a periodic signal with exponential Fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

has the Fourier transform

$$F(\omega) = \sum_{n=-\infty}^{\infty} c_n \delta(\omega - n\omega_0)$$

where $\omega_0 = 2\pi/T$.

(b) Find the Fourier transform of the signal in Fig. 18.37.

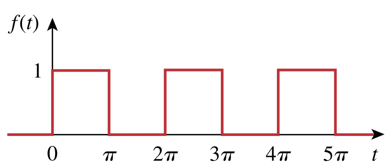


Figure 18.37
For Prob. 18.20(b).

Chapter 18, Solution 20.

(a) $F(c_n) = c_n\delta(\omega)$

$$F\left(c_n e^{jn\omega_o t}\right) = c_n\delta(\omega - n\omega_o)$$

$$F\left(\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}\right) = \underline{\boldsymbol{\sum_{n=-\infty}^{\infty} c_n\delta(\omega - n\omega_o)}}$$

(b) $T = 2\pi \longrightarrow \omega_o = \frac{2\pi}{T} = 1$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}\,dt = \frac{1}{2\pi}\left(\int_0^\pi 1\cdot e^{-jnt}\,dt + 0\right)$$

$$= \frac{1}{2\pi}\left(-\frac{1}{jn}e^{jnt}\Big|_0^\pi\right) = \frac{j}{2\pi n}\left(e^{-jn\pi} - 1\right)$$

But $e^{-jn\pi} = \cos n\pi + j\sin n\pi = \cos n\pi = (-1)^n$

$$c_n = \frac{j}{2n\pi}\left[(-1)^n - 1\right] = \begin{bmatrix} 0, & n=\text{even} \\ \frac{-j}{n\pi}, & n=\text{odd}, n\neq 0 \end{bmatrix}$$

for n = 0

$$c_n = \frac{1}{2\pi}\int_0^\pi 1\,dt = \frac{1}{2}$$

Hence

$$f(t) = \frac{1}{2} - \sum_{\substack{n=-\infty \\ n\neq 0 \\ n=\text{odd}}}^{\infty} \frac{j}{n\pi} e^{jnt}$$

$$F(\omega) = \underline{\boldsymbol{\frac{1}{2}\delta\omega - \sum_{\substack{n=-\infty \\ n\neq 0 \\ n=\text{odd}}}^{\infty} \frac{j}{n\pi}\delta(\omega - n)}}$$

Chapter 18, Problem 21.

Show that

$$\int_{-\infty}^{\infty} \left(\frac{\sin a\omega}{a\omega}\right)^2 d\omega = \frac{\pi}{a}$$

Hint: Use the fact that

$$F[u(t+a) - u(t-a)] = 2a\left(\frac{\sin a\omega}{a\omega}\right).$$

Chapter 18, Solution 21.

Using Parseval's theorem,

$$\int_{-\infty}^{\infty} f^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 \, d\omega$$

If $f(t) = u(t+a) - u(t+a)$, then

$$\int_{-\infty}^{\infty} f^2(t)dt = \int_{-a}^{a} (1)^2 dt = 2a = \frac{1}{2\pi}\int_{-\infty}^{\infty} 4a^2\left(\frac{\sin a\omega}{a\omega}\right)^2 d\omega$$

or

$$\int_{-\infty}^{\infty} \left(\frac{\sin a\omega}{a\omega}\right)^2 d\omega = \frac{4\pi a}{4a^2} = \frac{\pi}{a} \quad \text{as required.}$$

Chapter 18, Problem 22.

Prove that if $F(\omega)$ is the Fourier transform of $f(t)$,

$$F[f(t)\sin\omega_0 t] = \frac{j}{2}[F(\omega+\omega_0) - F(\omega-\omega_o)]$$

Chapter 18, Solution 22.

$$F[f(t)\sin\omega_o t] = \int_{-\infty}^{\infty} f(t)\frac{\left(e^{j\omega_o t} - e^{-j\omega_o t}\right)}{2j} e^{-j\omega t} dt$$

$$= \frac{1}{2j}\left[\int_{-\infty}^{\infty} f(t)e^{-j(\omega-\omega_o)t} dt - \int_{-\infty}^{\infty} e^{-j(\omega+\omega_o)t} dt\right]$$

$$= \underline{\frac{\mathbf{1}}{\mathbf{2j}}\left[\mathbf{F}(\omega-\omega_o) - \mathbf{F}(\omega+\omega_o)\right]}$$

Chapter 18, Problem 23.

If the Fourier transform of $f(t)$ is

$$F(\omega) = \frac{10}{(2+j\omega)(5+j\omega)}$$

determine the transforms of the following:

(a) $f(-3t)$ (b) $f(2t-1)$ (c) $f(t)\cos 2t$

(d) $\frac{d}{dt}f(t)$ (e) $\int_{-\infty}^{t} f(t)dt$

Chapter 18, Solution 23.

(a) f(3t) leads to $\frac{1}{3}\cdot\frac{10}{(2+j\omega/3)(5+j\omega/3)} = \frac{30}{(6+j\omega)(15+j\omega)}$

$$F\left[f(-3t)\right] = \underline{\mathbf{\frac{30}{(6-j\omega)(15-j\omega)}}}$$

(b) $f(2t) \longrightarrow \frac{1}{2}\cdot\frac{10}{(2+j\omega/2)(15+j\omega/2)} = \frac{20}{(4+j\omega)(10+j\omega)}$

$$f(2t-1) = f\,[2(t-1/2)] \longrightarrow \underline{\mathbf{\frac{20e^{-j\omega/2}}{(4+j\omega)(10+j\omega)}}}$$

(c) $f(t)\cos 2t \longrightarrow \frac{1}{2}F(\omega+2)+\frac{1}{2}F(\omega+2)$

$$= \underline{\mathbf{\frac{5}{[2+j(\omega+2)][5+j(\omega+2)]} + \frac{5}{[2+j(\omega-2)[5+j(\omega-2)]]}}}$$

(d) $F\left[f'(t)\right] = j\omega F(\omega) = \frac{j\omega 10}{(2+j\omega)(5+j\omega)}$

(e) $\int_{-\infty}^{t} f(t)dt \longrightarrow \frac{F(\omega)}{j(\omega)} + \pi F(0)\delta(\omega)$

$$= \frac{10}{j\omega(2+j\omega)(5+j\omega)} + \pi\delta(\omega)\frac{x10}{2x5}$$

$$= \underline{\mathbf{\frac{10}{j\omega(2+j\omega)(5+j\omega)} + \pi\delta(\omega)}}$$

Chapter 18, Problem 24.

Given that $F[f(t)] = (j / \omega)(e^{-j\omega} - 1)$, find the Fourier transforms of:

(a) $x(t) = f(t) + 3$

(b) $y(t) = f(t-2)$

(c) $h(t) = f'(t)$

(d) $g(t) = 4f\left(\frac{2}{3}t\right) + 10f\left(\frac{5}{3}t\right)$

Chapter 18, Solution 24.

(a) $X(\omega) = F(\omega) + F[3]$

$$= \underline{\mathbf{6\pi\delta(\omega) + \frac{j}{\omega}\left(e^{-j\omega} - 1\right)}}$$

(b) $y(t) = f(t-2)$

$$Y(\omega) = e^{-j\omega 2}F(\omega) = \underline{\mathbf{\frac{je^{-j2\omega}}{\omega}\left(e^{-j\omega} - 1\right)}}$$

(c) If $h(t) = f'(t)$

$$H(\omega) = j\omega F(\omega) = j\omega\frac{j}{\omega}\left(e^{-j\omega} - 1\right) = \underline{\mathbf{1 - e^{-j\omega}}}$$

(d) $g(t) = 4f\left(\frac{2}{3}t\right) + 10f\left(\frac{5}{3}t\right)$, $G(\omega) = 4x\frac{3}{2}F\left(\frac{3}{2}\omega\right) + 10x\frac{3}{5}F\left(\frac{3}{5}\omega\right)$

$$= 6 \cdot \frac{j}{\frac{3}{2}\omega}\left(e^{-j3\omega/2} - 1\right) + \frac{6j}{\frac{3}{5}\omega}\left(e^{-j3\omega/5} - 1\right)$$

$$= \underline{\mathbf{\frac{j4}{\omega}\left(e^{-j3\omega/2} - 1\right) + \frac{j10}{\omega}\left(e^{-j3\omega/5} - 1\right)}}$$

Chapter 18, Problem 25.

Obtain the inverse Fourier transform of the following signals.

(a) $F(\omega) = \dfrac{5}{j\omega - 2}$

(b) $H(\omega) = \dfrac{12}{\omega^2 + 4}$

(c) $X(\omega) = \dfrac{10}{(j\omega - 1)(j\omega - 2)}$

Chapter 18, Solution 25.

(a) $g(t) = \underline{5e^{2t}u(t)}$

(b) $h(t) = \underline{6e^{-2|t|}}$

(c) $X(\omega) = \dfrac{A}{s-1} + \dfrac{B}{s-2}, \qquad s = j\omega$

$$A = \frac{10}{1-2} = -10, \qquad B = \frac{10}{2-1} = 10$$

$$X(\omega) = \frac{-10}{j\omega - 1} + \frac{10}{j\omega - 2}$$

$$x(t) = \underline{-10e^{t}u(t) + 10e^{2t}u(t)}$$

Chapter 18, Problem 26.

Determine the inverse Fourier transforms of the following:

(a) $F(\omega) = \dfrac{e^{-j2\omega}}{1+j\omega}$

(b) $H(\omega) = \dfrac{1}{(j\omega+4)^2}$

(c) $G(\omega) = 2u(\omega+1) - 2u(\omega-1)$

Chapter 18, Solution 26.

(a) $\underline{f(t) = e^{-(t-2)}u(t)}$

(b) $\underline{h(t) = te^{-4t}u(t)}$

(c) If $x(t) = u(t+1) - u(t-1) \longrightarrow X(\omega) = 2\dfrac{\sin\omega}{\omega}$

By using duality property,

$$G(\omega) = 2u(\omega+1) - 2u(\omega-1) \longrightarrow \underline{g(t) = \frac{2\sin t}{\pi t}}$$

Chapter 18, Problem 27.

Find the inverse Fourier transforms of the following functions:

(a) $F(\omega) = \dfrac{100}{j\omega(j\omega+10)}$

(b) $G(\omega) = \dfrac{10j\omega}{(-j\omega+2)(j\omega+3)}$

(c) $H(\omega) = \dfrac{60}{-\omega^2 + j40\omega + 1300}$

(d) $Y(\omega) = \dfrac{\delta(\omega)}{(j\omega+1)(j\omega+2)}$

Chapter 18, Solution 27.

(a) Let $F(s) = \dfrac{100}{s(s+10)} = \dfrac{A}{s} + \dfrac{B}{s+10}, \quad s = j\omega$

$$A = \frac{100}{10} = 10, \ B = \frac{100}{-10} = -10$$

$$F(\omega) = \frac{10}{j\omega} - \frac{10}{j\omega+10}$$

$$f(t) \ = \ \underline{\mathbf{5sgn(t) - 10e^{-10t}u(t)}}$$

(b) $G(s) = \dfrac{10s}{(2-s)(3+s)} = \dfrac{A}{2-s} + \dfrac{B}{s+3}, \quad s = j\omega$

$$A = \frac{20}{5} = 4, \quad B = \frac{-30}{5} = -6$$

$$G(\omega) = \frac{4}{= -j\omega + 2} - \frac{6}{j\omega+3}$$

$$g(t) \ = \ \underline{\mathbf{4e^{2t}u(-t) - 6e^{-3t}u(t)}}$$

(c) $H(\omega) = \dfrac{60}{(j\omega)^2 + j40\omega + 1300} = \dfrac{60}{(j\omega+20)^2 + 900}$

$$h(t) \ = \ \underline{\mathbf{2e^{-20t}\sin(30t)u(t)}}$$

(d) $y(t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} \dfrac{\delta(\omega)e^{j\omega t}d\omega}{(2+j\omega)(j\omega+1)} = \dfrac{1}{2}\pi \cdot \dfrac{1}{2} = \underline{\dfrac{\mathbf{1}}{\mathbf{4}}\pi}$

Chapter 18, Problem 28.

Find the inverse Fourier transforms of:

(a) $\dfrac{\pi\delta(\omega)}{(5+j\omega)(2+j\omega)}$

(b) $\dfrac{10\delta(\omega+2)}{j\omega(j\omega+1)}$

(c) $\dfrac{20\delta(\omega-1)}{(2+j\omega)(3+j\omega)}$

(d) $\dfrac{5\pi\delta(\omega)}{5+j\omega}+\dfrac{5}{j\omega(5+j\omega)}$

Chapter 18, Solution 28.

(a) $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\pi\delta(\omega)\,e^{j\omega t}}{(5+j\omega)(2+j\omega)}\,d\omega$$

$$= \frac{1}{2}\,\frac{1}{(5)(2)} = \frac{1}{20} = \underline{\mathbf{0.05}}$$

(b) $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{10\delta(\omega+2)}{j\omega(j\omega+1)}e^{j\omega t}\,d\omega = \frac{10}{2\pi}\,\frac{e^{-j2t}}{(-j2)(-j2+1)}$$

$$= \frac{j5}{2\pi}\,\frac{e^{-j2t}}{1-j2} = \underline{\frac{\mathbf{(-2+j)e^{-j2t}}}{\mathbf{2\pi}}}$$

(c) $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{20\delta(\omega-1)e^{j\omega t}}{(2+j\omega)(3+5\omega)}\,d\omega = \frac{20}{2\pi}\,\frac{e^{jt}}{(2+j)(3+j)}$$

$$= \frac{20e^{jt}}{2\pi(5+5j)} = \underline{\frac{\mathbf{(1-j)e^{jt}}}{\pi}}$$

(d) Let $$F(\omega) = \frac{5\pi\delta(\omega)}{(5+j\omega)} + \frac{5}{j\omega(5+j\omega)} = F_1(\omega) + F_2(\omega)$$

$$f_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{5\pi\delta(\omega)}{5+j\omega}e^{j\omega t}\,d\omega = \frac{5\pi}{2\pi}\cdot\frac{1}{5} = 0.5$$

$$F_2(s) = \frac{5}{s(5+s)} = \frac{A}{s} + \frac{B}{s+5}\ ,\ A = 1, B = -1$$

$$F_2(\omega) = \frac{1}{j\omega} - \frac{1}{j\omega+5}$$

$$f_2(t) = \frac{1}{2}\mathrm{sgn}(t) - e^{-5t} = -\frac{1}{2} + u(t) - e^{5t}$$

$$f(t) = f_1(t) + f_2(t) = \underline{\mathbf{u(t) - e^{-5t}}}$$

Chapter 18, Problem 29.

* Determine the inverse Fourier transforms of:

(a) $F(\omega) = 4\delta(\omega + 3) + \delta(\omega) + 4\delta(\omega - 3)$
(b) $G(\omega) = 4u(\omega + 2) - 4u(\omega - 2)$
(c) $H(\omega) = 6 \cos 2\omega$

* An asterisk indicates a challenging problem.

Chapter 18, Solution 29.

(a) $$f(t) = F^{-1}[\delta(\omega)] + F^{-1}\left[4\delta(\omega+3) + 4\delta(\omega-3)\right]$$
$$= \frac{1}{2\pi} + \frac{4\cos 3t}{\pi} = \underline{\mathbf{\frac{1}{2\pi}\left(1 + 8\cos 3t\right)}}$$

(b) If $h(t) = u(t+2) - u(t-2)$

$$H(\omega) = \frac{2\sin 2\omega}{\omega}$$

$$G(\omega) = 4H(\omega) \longrightarrow g(t) = \frac{1}{2\pi} \cdot \frac{8\sin 2t}{t}$$

$$g(t) = \underline{\mathbf{\frac{4\sin 2t}{\pi t}}}$$

(c) Since

$$\cos(at) \quad \pi\delta(\omega + a) + \pi\delta(\omega - a)$$

Using the reversal property,

$$2\pi\cos 2\omega \leftrightarrow \pi\delta(t+2) + \pi\delta(t-2)$$

or $F^{-1}\left[6\cos 2\omega\right] = \underline{\mathbf{3\delta(t+2) + 3\delta(t-2)}}$

Chapter 18, Problem 30.

For a linear system with input $x(t)$ and output $y(t)$ find the impulse response for the following cases:

(a) $x(t) = e^{-at} u(t)$, $\quad y(t) = u(t) - u(-t)$

(b) $x(t) = e^{-t} u(t)$, $\quad y(t) = e^{-2t} u(t)$

(c) $x(t) = \delta(t)$, $\quad y(t) = e^{-at} \sin btu(t)$

Chapter 18, Solution 30.

(a) $\quad y(t) = \text{sgn}(t) \longrightarrow Y(\omega) = \dfrac{2}{j\omega}, \qquad X(\omega) = \dfrac{1}{a + j\omega}$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2(a + j\omega)}{j\omega} = 2 + \frac{2a}{j\omega} \longrightarrow \underline{h(t) = 2\delta(t) + a[u(t) - u(-t)]}$$

(b) $X(\omega) = \dfrac{1}{1 + j\omega}, \qquad Y(\omega) = \dfrac{1}{2 + j\omega}$

$$H(\omega) = \frac{1 + j\omega}{2 + j\omega} = 1 - \frac{1}{2 + j\omega} \longrightarrow \underline{h(t) = \delta(t) - e^{-2t} u(t)}$$

(c) In this case, by definition, $\underline{h(t) = \quad y(t) = e^{-at} \sin bt\, u(t)}$

Chapter 18, Problem 31.

Given a linear system with output $y(t)$ and impulse response $h(t)$, find the corresponding input $x(t)$ for the following cases:

(a) $y(t) = te^{-at}u(t), \quad h(t) = e^{-at}u(t)$

(b) $y(t) = u(t+1) - u(t-1), \quad h(t) = \delta(t)$

(c) $y(t) = e^{-at}u(t), \quad h(t) = \mathrm{sgn}(t)$

Chapter 18, Solution 31.

(a)

$$Y(\omega) = \frac{1}{(a+j\omega)^2}, \quad H(\omega) = \frac{1}{a+j\omega}$$

$$X(\omega) = \frac{Y(\omega)}{H(\omega)} = \frac{1}{a+j\omega} \longrightarrow \underline{x(t) = e^{-at}u(t)}$$

(b) By definition, $\underline{x(t) = y(t) = u(t+1) - u(t-1)}$

(c)

$$Y(\omega) = \frac{1}{(a+j\omega)}, \quad H(\omega) = \frac{2}{j\omega}$$

$$X(\omega) = \frac{Y(\omega)}{H(\omega)} = \frac{j\omega}{2(a+j\omega)} = \frac{1}{2} - \frac{a}{2(a+j\omega)} \longrightarrow \underline{x(t) = \frac{1}{2}\delta(t) - \frac{a}{2}e^{-at}u(t)}$$

Chapter 18, Problem 32.

* Determine the functions corresponding to the following Fourier transforms:

(a) $F_1(\omega) = \dfrac{e^{jw}}{-j\omega+1}$ (b) $F_2(\omega) = 2e^{|\omega|}$

(c) $F_3(\omega) = \dfrac{1}{(1+\omega^2)^2}$ (d) $F_4(\omega) = \dfrac{\delta(\omega)}{1+j2\omega}$

* An asterisk indicates a challenging problem.

Chapter 18, Solution 32.

(a) Since $\dfrac{e^{-j\omega}}{j\omega+1}$ $\qquad e^{-(t-1)}u(t-1)$

and $F(-\omega)$ $\qquad \cancel{f(-t)} \rightarrow$

$F_1(\omega) = \dfrac{e^{j\omega}}{-j\omega+1}$ $\qquad \cancel{f_1(t)} \rightarrow e^{-(-t-1)}u(-t-1)$

$f_1(t) = \underline{\mathbf{e^{(t+1)}u(-t-1)}}$

(b) From Section 17.3,

$$\frac{2}{t^2+1} \longrightarrow 2\pi e^{-|\omega|}$$

If $F_2(\omega) = 2e^{-|\omega|}$, then

$f_2(t) = \underline{\dfrac{\mathbf{2}}{\pi(\mathbf{t^2+1})}}$

(d) By partial fractions

$$F_3(\omega) = \frac{1}{(j\omega+1)^2(j\omega-1)^2} = \frac{\frac{1}{4}}{(j\omega+1)^2} + \frac{\frac{1}{4}}{(j\omega+1)} + \frac{\frac{1}{4}}{(j\omega-1)^2} - \frac{\frac{1}{4}}{j\omega-1}$$

Hence $f_3(t) = \frac{1}{4}\left(te^{-t} + e^{-t} + te^{t} - e^{t}\right)u(t)$

$= \underline{\frac{\mathbf{1}}{\mathbf{4}}\mathbf{(t+1)e^{-t}u(t)} + \frac{\mathbf{1}}{\mathbf{4}}\mathbf{(t-1)e^{t}u(t)}}$

(d) $f_4(t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} F_1(\omega)e^{j\omega t}\,d\omega = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} \dfrac{\delta(\omega)e^{j\omega t}}{1+j2\omega} = \underline{\dfrac{\mathbf{1}}{\mathbf{2\pi}}}$

Chapter 18, Problem 33.

* Find $f(t)$ if:

(a) $F(\omega) = 2\sin \pi\omega[u(\omega+1) - u(\omega-1)]$

(b) $F(\omega) = \dfrac{1}{\omega}(\sin 2\omega - \sin\omega) + \dfrac{j}{\omega}(\cos 2\omega - \cos\omega)$

* An asterisk indicates a challenging problem.

Chapter 18, Solution 33.

(a) Let $x(t) = 2\sin \pi t[u(t+1) - u(t-1)]$

From Problem 17.9(b),

$$X(\omega) = \frac{4j\pi\sin\omega}{\pi^2 - \omega^2}$$

Applying duality property,

$$f(t) = \frac{1}{2\pi}X(-t) = \frac{2j\sin(-t)}{\pi^2 - t^2}$$

$$f(t) = \underline{\frac{2j\sin t}{t^2 - \pi^2}}$$

(b) $F(\omega) = \dfrac{j}{\omega}(\cos 2\omega - j\sin 2\omega) - \dfrac{j}{\omega}(\cos\omega - j\sin\omega)$

$$= \frac{j}{\omega}\left(e^{j2\omega} - e^{-j\omega}\right) = \frac{e^{-j\omega}}{j\omega} - \frac{e^{j2\omega}}{j\omega}$$

$$f(t) = \frac{1}{2}\operatorname{sgn}(t-1) - \frac{1}{2}\operatorname{sgn}(t-2)$$

But $\operatorname{sgn}(t) = 2u(t) - 1$

$$f(t) = u(t-1) - \frac{1}{2} - u(t-2) + \frac{1}{2}$$

$$= \underline{\mathbf{u(t-1) - u(t-2)}}$$

Chapter 18, Problem 34.

Determine the signal $f(t)$ whose Fourier transform is shown in Fig. 18.38. (*Hint:* Use the duality property.)

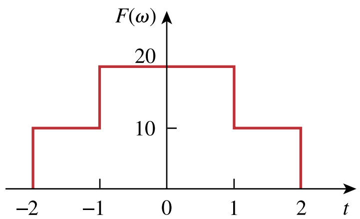


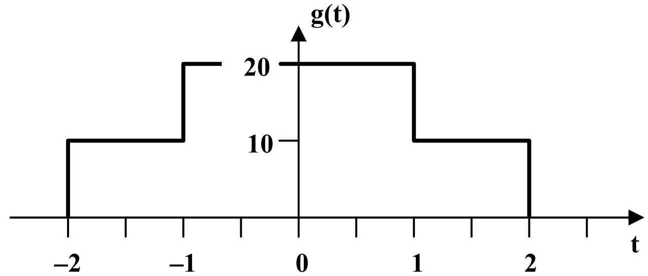
Figure 18.38
For Prob. 18.34.

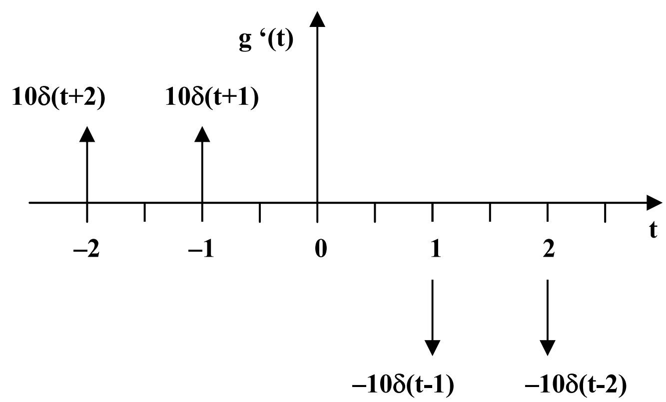
Chapter 18, Solution 34.

First, we find G(ω) for g(t) shown below.

$$g(t) = 10[u(t+2) - u(t-2)] + 10[u(t+1) - u(t-1)]$$
$$g'(t) = 10[\delta(t+2) - \delta(t-2)] + 10[\delta(t+1) - \delta(t-1)]$$

The Fourier transform of each term gives





$$j\omega G(\omega) = 10\left(e^{j\omega 2} - e^{-j\omega 2}\right) + 10\left(e^{j\omega} - e^{-j\omega}\right)$$
$$= 20j\sin 2\omega + 20j\sin\omega$$
$$G(\omega) = \frac{20\sin 2\omega}{\omega} + \frac{20\sin\omega}{\omega} = 40\ \mathrm{sinc}(2\omega) + 20\ \mathrm{sinc}(\omega)$$

Note that G(ω) = G(-ω).

$$F(\omega) = 2\pi G(-\omega)$$
$$f(t) = \frac{1}{2\pi} G(t)$$
$$= \underline{\mathbf{(20/\pi)sinc(2t) + (10/\pi)sinc(t)}}$$

Chapter 18, Problem 35.

A signal $f(t)$ has Fourier transform

$$F(\omega) = \frac{1}{2 + j\omega}$$

Determine the Fourier transform of the following signals:

(a) $x(t) = f(3t - 1)$

(b) $y(t) = f(t) \cos 5t$

(c) $z(t) = \frac{d}{dt} f(t)$

(d) $h(t) = f(t) * f(t)$

(e) $i(t) = tf(t)$

Chapter 18, Solution 35.

(a) $x(t) = f[3(t-1/3)]$. Using the scaling and time shifting properties,

$$X(\omega) = \frac{1}{3}\frac{1}{2 + j\omega/3}e^{-j\omega/3} = \underline{\frac{e^{-j\omega/3}}{(6 + j\omega)}}$$

(b) Using the modulation property,

$$Y(\omega) = \frac{1}{2}[F(\omega + 5) + F(\omega - 5)] = \underline{\frac{1}{2}\left[\frac{1}{2 + j(\omega + 5)} + \frac{1}{2 + j(\omega - 5)}\right]}$$

(c) $$Z(\omega) = j\omega F(\omega) = \underline{\frac{j\omega}{2 + j\omega}}$$

(d) $$H(\omega) = F(\omega)F(\omega) = \underline{\frac{1}{(2 + j\omega)^2}}$$

(e) $$I(\omega) = j\frac{d}{d\omega}F(\omega) = j\frac{(0 - j)}{(2 + j\omega)^2} = \underline{\frac{1}{(2 + j\omega)^2}}$$

Chapter 18, Problem 36.

The transfer function of a circuit is

$$H(\omega) = \frac{2}{j\omega + 2}$$

If the input signal to the circuit is $v_s(t) = e^{-4t}u(t)$ V find the output signal. Assume all initial conditions are zero.

Chapter 18, Solution 36.

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} \longrightarrow Y(\omega) = H(\omega)X(\omega)$$

$$x(t) = v_s(t) = e^{-4t}u(t) \longrightarrow X(\omega) = \frac{1}{4 + j\omega}$$

$$Y(\omega) = \frac{2}{(j\omega + 2)(4 + j\omega)} = \frac{2}{(s+2)(s+4)}, \qquad s = j\omega$$

$$Y(s) = \frac{A}{s+2} + \frac{B}{s+4}$$

$$A = \frac{2}{-2+4} = 1, \qquad B = \frac{2}{-4+2} = -1$$

$$Y(s) = \frac{1}{s+2} - \frac{1}{s+4}$$

$$y(t) = \underline{\left(e^{-2t} - e^{-4t}\right)u(t)}$$

Please note, the units are not known since the transfer function does not give them. If the transfer function was a voltage gain then the units on y(t) would be volts.

Chapter 18, Problem 37.

Find the transfer function $I_o(\omega)/I_s(\omega)$ for the circuit in Fig. 18.39.

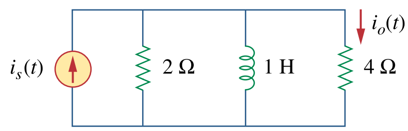


Figure 18.39
For Prob. 18.37.

Chapter 18, Solution 37.

$$2\|j\omega = \frac{j2\omega}{2+j\omega}$$

By current division,

$$H(\omega) = \frac{I_o(\omega)}{I_s(\omega)} = \frac{\dfrac{j2\omega}{2+j\omega}}{4+\dfrac{j2\omega}{2+j\omega}} = \frac{j2\omega}{j2\omega+8+j4\omega}$$

$$H(\omega) \;=\; \underline{\frac{\mathbf{j\omega}}{\mathbf{4+j3\omega}}}$$

Chapter 18, Problem 38.

Suppose $v_s(t) = u(t)$ for $t > 0$. Determine $i(t)$ in the circuit of Fig. 18.40, using the Fourier transform.

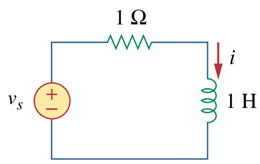


Figure 18.40
For Prob. 18.38.

Chapter 18, Solution 38.

$$V_s = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$I(\omega) = \frac{V_s}{1+j\omega} = \frac{1}{1+j\omega}\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)$$

Let $I(\omega) = I_1(\omega) + I_2(\omega) = \dfrac{\pi\delta(\omega)}{1+j\omega} + \dfrac{1}{j\omega(1+j\omega)}$

$$I_2(\omega) = \frac{1}{j\omega(1+j\omega)} = \frac{A}{s} + \frac{B}{s+1}, \qquad s = j\omega$$

where $A = \dfrac{1}{1} = 1, \qquad B = \dfrac{1}{-1} = -1$

$$I_2(\omega) = \frac{1}{j\omega} + \frac{-1}{j\omega+1} \quad \longrightarrow \quad i_2(t) = \frac{1}{2}\operatorname{sgn}(t) - e^{-t}$$

$$I_1(\omega) = \frac{\pi\delta(\omega)}{1+j\omega}$$

$$i_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\pi\delta(\omega)}{1+j\omega}e^{j\omega t}d\omega = \frac{1}{2}\frac{e^{j\omega t}}{1+j\omega}\bigg|_{\omega=0} = \frac{1}{2}$$

Hence,

$$i(t) = i_1(t) + i_2(t) = \underline{\frac{1}{2} + \frac{1}{2}\operatorname{sgn}(t) - e^{-t}}$$

Chapter 18, Problem 39.

Given the circuit in Fig. 18.41, with its excitation, determine the Fourier transform of $i(t)$.

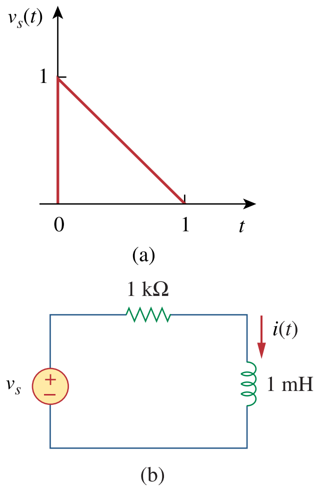


Figure 18.41
For Prob. 18.39.

Chapter 18, Solution 39.

$$V_s(\omega) = \int_{-\infty}^{\infty} (1-t)e^{-j\omega t}dt = \frac{1}{j\omega} + \frac{1}{\omega^2} - \frac{1}{\omega^2}e^{-j\omega}$$

$$I(\omega) = \frac{V_s(\omega)}{10^3 + j\omega x 10^{-3}} = \underline{\frac{10^3}{10^6 + j\omega}\left(\frac{1}{j\omega} + \frac{1}{\omega^2} - \frac{1}{\omega^2}e^{-j\omega}\right)}$$

Chapter 18, Problem 40.

Determine the current *i*(*t*) in the circuit of Fig. 18.42(b), given the voltage source shown in Fig. 18.42(a).

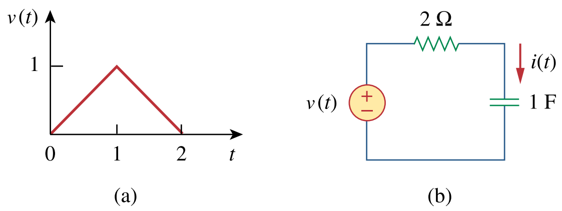


Figure 18.42
For Prob. 18.40.

Chapter 18, Solution 40.

$$\ddot{v}(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$$

$$-\omega^2 V(\omega) = 1 - 2e^{-j\omega} + e^{j\omega 2}$$

$$V(\omega) = \frac{1 - 2e^{-j\omega} + e^{-j\omega 2}}{-\omega^2}$$

Now $\quad Z(\omega) = 2 + \frac{1}{j\omega} = \frac{1 + j2\omega}{j\omega}$

$$I = \frac{V(\omega)}{Z(\omega)} = \frac{2e^{j\omega} - e^{j\omega 2} - 1}{\omega^2} \cdot \frac{j\omega}{1 + j2\omega}$$

$$= \frac{1}{j\omega(0.5 + j\omega)}\left(0.5 + 0.5e^{-j\omega 2} - e^{-j\omega}\right)$$

But $\quad \frac{1}{s(s+0.5)} = \frac{A}{s} + \frac{B}{s+0.5} \longrightarrow A = 2,\ B = -2$

$$I(\omega) = \frac{2}{j\omega}\left(0.5 + 0.5e^{j\omega 2} - e^{-j\omega}\right) - \frac{2}{0.5 + j\omega}\left(0.5 + 0.5e^{-j\omega 2} - e^{-j\omega}\right)$$

$$i(t) = \underline{\mathbf{\frac{1}{2}sgn(t) + \frac{1}{2}sgn(t-2) - sgn(t-1) - e^{-0.5t}u(t) - e^{-0.5(t-2)}u(t-2) - 2e^{-0.5(t-1)}u(t-1)}}$$

Chapter 18, Problem 41.

Determine the Fourier transform of $v(t)$ in the circuit shown in Fig. 18.43.

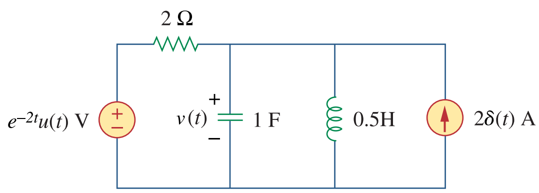
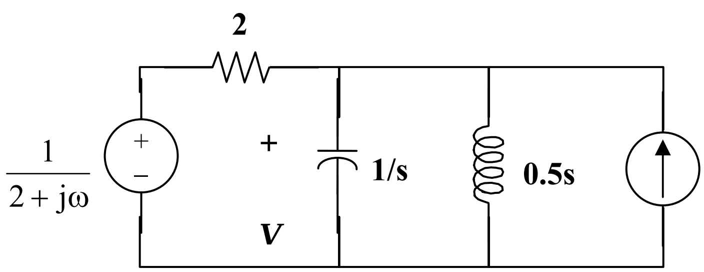


Figure 18.43
For Prob. 18.41.

Chapter 18, Solution 41.



$$\frac{V - \frac{1}{2+j\omega}}{2} + j\omega V + \frac{2V}{j\omega} - 2 = 0$$

$$\left(j\omega - 2\omega^2 + 4\right)V = j4\omega + \frac{j\omega}{2+j\omega} = \frac{-4\omega^2 + j9\omega}{2+j\omega}$$

$$\underline{V(\omega) = \frac{2j\omega(4.5 + j2\omega)}{(2+j\omega)(4 - 2\omega^2 + j\omega)}}$$

Chapter 18, Problem 42.

Obtain the current $i_o(t)$ in the circuit of Fig. 18.44.

(a) Let $i(t) = \text{sgn}(t)$ A.

(b) Let $i(t) = 4[\text{u}(t) - u(t-1)]$ A.

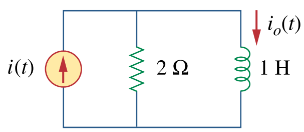


Figure 18.44
For Prob. 18.42.

Chapter 18, Solution 42.

By current division, $I_o = \dfrac{2}{2 + j\omega} \cdot I(\omega)$

(a) For $i(t) = 5\,\text{sgn}(t)$,

$$I(\omega) = \frac{10}{j\omega}$$

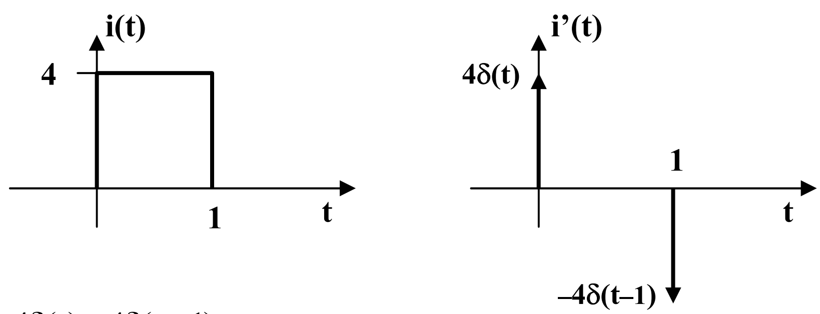
$$I_o = \frac{2}{2 + j\omega} \cdot \frac{10}{j\omega} = \frac{20}{j\omega(2 + j\omega)}$$

Let $I_o = \dfrac{20}{s(s + 2)} = \dfrac{A}{s} + \dfrac{B}{s + 2}$, $A = 10$, $B = -10$

$$I_o(\omega) = \frac{10}{j\omega} - \frac{10}{2 + j\omega}$$

$$i_o(t) = \underline{\mathbf{5\,sgn(t) - 10e^{-2t}u(t)\,A}}$$

(b)



$$i'(t) = 4\delta(t) - 4\delta(t - 1)$$

$$j\omega I(\omega) = 4 - 4e^{-j\omega}$$

$$I(\omega) = \frac{4\left(1 - e^{-j\omega}\right)}{j\omega}$$

$$I_o = \frac{8\left(1 - e^{-j\omega}\right)}{j\omega(2 + j\omega)} = 4\left(\frac{1}{j\omega} - \frac{1}{2 + j\omega}\right)\left(1 - e^{-j\omega}\right)$$

$$= \frac{4}{j\omega} - \frac{4}{2 + j\omega} - \frac{4e^{-j\omega}}{j\omega} + \frac{4e^{-j\omega}}{2 + j\omega}$$

$$i_o(t) = \underline{\mathbf{2\,sgn(t) - 2\,sgn(t - 1) - 4e^{-2t}u(t) + 4e^{-2(t-1)}u(t - 1)\,A}}$$

Chapter 18, Problem 43.

Find $v_o(t)$ in the circuit of Fig. 18.45, where $i_s = 5e^{-t}u(t)$ A.

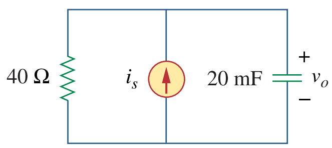


Figure 18.45
For Prob. 18.43.

Chapter 18, Solution 43.

$$20\text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j20x10^{-3}\omega} = \frac{50}{j\omega}, \qquad i_s = 5e^{-t} \longrightarrow I_s = \frac{5}{1+j\omega}$$

$$V_o = \frac{40}{40+\dfrac{50}{j\omega}} I_s \bullet \frac{50}{j\omega} = \frac{250}{(s+1)(s+1.25)}, \qquad s = j\omega$$

$$V_o = \frac{A}{s+1} + \frac{B}{s+1.25} = 1000\left[\frac{1}{s+1} - \frac{1}{s+1.25}\right]$$

$$v_o(t) = \underline{1000(e^{-1t} - e^{-1.25t})u(t)V}$$

Chapter 18, Problem 44.

If the rectangular pulse in Fig. 18.46(a) is applied to the circuit in Fig. 18.46(b), find v_o at $t = 1$ s.

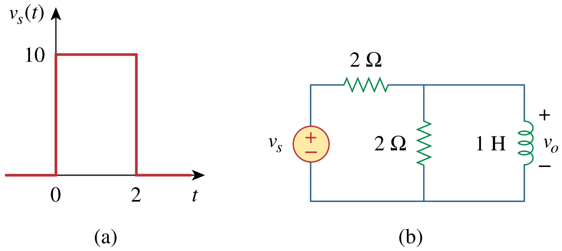
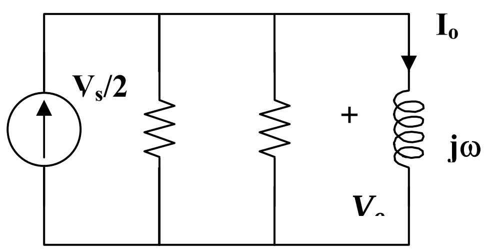


Figure 18.46
For Prob. 18.44.

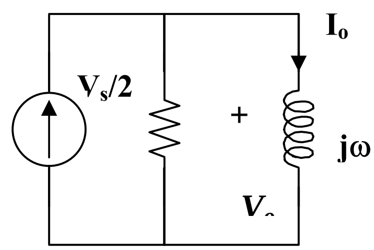
Chapter 18, Solution 44.

$$1\text{H} \longrightarrow j\omega$$

We transform the voltage source to a current source as shown in Fig. (a) and then combine the two parallel 2Ω resistors, as shown in Fig. (b).



(a)



(b)

$$2\|2 = 1\Omega,\ \ I_o = \frac{1}{1+j\omega}\cdot\frac{V_s}{2}$$

$$V_o = j\omega I_o = \frac{j\omega V_s}{2(1+j\omega)}$$

$$\ddot{v}_s(t) = 10\delta(t) - 10\delta(t-2)$$

$$j\omega V_s(\omega) = 10 - 10e^{-j2\omega}$$

$$V_s(\omega) = \frac{10\left(1-e^{-j2\omega}\right)}{j\omega}$$

$$\text{Hence}\ \ V_o = \frac{5\left(1-e^{-j2\omega}\right)}{1+j\omega} = \frac{5}{1+j\omega} - \frac{5}{1+j\omega}e^{-j2\omega}$$

$$v_o(t) = 5e^{-t}u(t) - 5e^{-(t-2)}u(t-2)$$

$$v_o(1) = 5e^{-1} - 0 = \underline{\mathbf{1.839\ V}}$$

Chapter 18, Problem 45.

Use the Fourier transform to find i(t) in the circuit of Fig. 18.47 if $v_s(t) = 10e^{-2t}\,u(t)$.

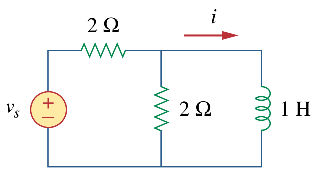
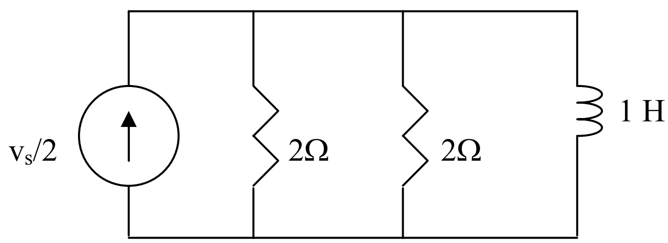


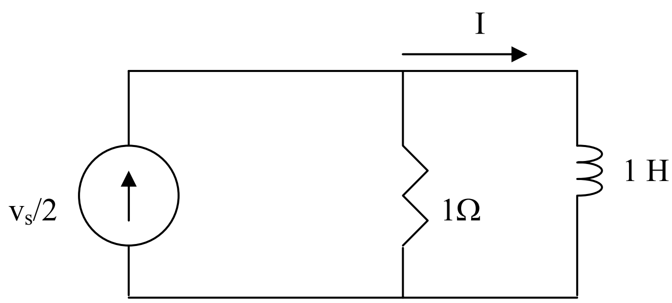
Figure 18.47
For Prob. 18.45.

Chapter 18, Solution 45.

We may convert the voltage source to a current source as shown below.



Combining the two 2-Ω resistors gives 1 Ω. The circuit now becomes that shown below.



$$I = \frac{1}{1+j\omega}\cdot\frac{V_s}{2} = \frac{1}{1+j\omega}\cdot\frac{5}{2+j\omega} = \frac{5}{(s+1)(s+2)}, \qquad s = j\omega$$

$$= \frac{A}{s+1} + \frac{B}{s+2}$$

where $A = 5/1 = 5, \quad B = 5/-1 = -5$

$$I = \frac{5}{s+1} - \frac{5}{s+2}$$

$$i(t) = \underline{5(e^{-t} - e^{-2t})u(t)}\text{ A}$$

Chapter 18, Problem 46.

Determine the Fourier transform of $i_o(t)$ in the circuit of Fig. 18.48.

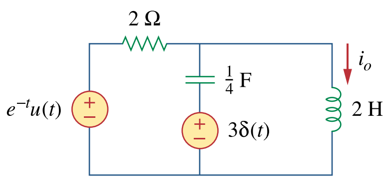


Figure 18.48
For Prob. 18.46.

Chapter 18, Solution 46.

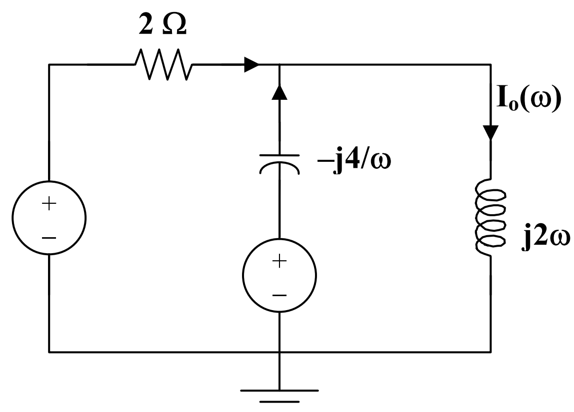
$$\frac{1}{4}\text{F} \longrightarrow \frac{1}{j\omega\frac{1}{4}} = \frac{-j4}{\omega}$$

$$2\text{H} \longrightarrow j\omega 2$$

$$3\delta(t) \longrightarrow 3$$

$$e^{-t}u(t) \longrightarrow \frac{1}{1+j\omega}$$

The circuit in the frequency domain is shown below:



At node V_o, KCL gives

$$\frac{\frac{1}{1+j\omega} - V_o}{2} + \frac{3 - V_o}{\frac{-j4}{\omega}} = \frac{V_o}{j2\omega}$$

$$\frac{2}{1+j\omega} - 2V_o + j\omega 3 - j\omega V_o = -\frac{j2V_o}{\omega}$$

$$V_o = \frac{\frac{2}{1+j\omega} + j\omega 3}{2 + j\omega - \frac{j2}{\omega}}$$

$$I_o(\omega) = \frac{V_o}{j2\omega} = \frac{\frac{2 + j\omega 3 - 3\omega^2}{1+j\omega}}{j2\omega\left(2 + j\omega - \frac{j2}{\omega}\right)}$$

$$I_o(\omega) = \underline{\frac{\mathbf{2 + j\omega^2 - 3\omega^2}}{\mathbf{4 - 6\omega^2 + j(8\omega - 2\omega^3)}}}$$

Chapter 18, Problem 47.

Find the voltage $v_o(t)$ in the circuit of Fig. 18.49. Let $i_s(t) = 8e^{-t}u(t)$ A.

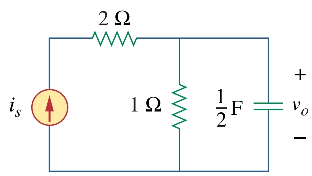


Figure 18.49
For Prob. 18.47.

Chapter 18, Solution 47.

$$\frac{1}{2}F \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{2}{j\omega}$$

$$I_o = \frac{1}{1+\dfrac{2}{j\omega}} I_s$$

$$V_o = \frac{2}{j\omega} I_o = \frac{\dfrac{2}{j\omega}}{1+\dfrac{2}{j\omega}} I_s = \frac{2}{2+j\omega}\frac{8}{1+j\omega}$$

$$= \frac{16}{(s+1)(s+2)}, \; s = j\omega$$

$$= \frac{A}{s+1} + \frac{B}{s+2}$$

where $A = 16/1 = 16, \quad B = 16/(-1) = -16$

Thus,

$$v_o(t) = \underline{\mathbf{16(e^{-t} - e^{-2t})u(t)\ V}}.$$

Chapter 18, Problem 48.

Find $i_o(t)$ in the op amp circuit of Fig. 18.50.

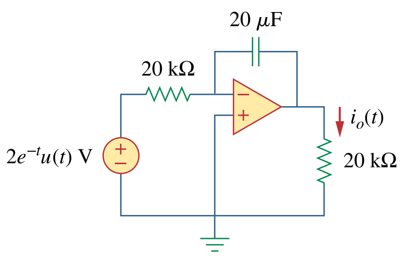


Figure 18.50
For Prob. 18.48.

Chapter 18, Solution 48.

$$0.2\text{F} \longrightarrow \frac{1}{j\omega C} = -\frac{j5}{\omega}$$

As an integrator,

$$RC = 20 \times 10^3 \times 20 \times 10^{-6} = 0.4$$

$$v_o = -\frac{1}{RC}\int_o^t v_i \, dt$$

$$V_o = -\frac{1}{RC}\left[\frac{V_i}{j\omega} + \pi V_i(0)\delta(\omega)\right]$$

$$= -\frac{1}{0.4}\left[\frac{2}{j\omega(2+j\omega)} + \pi\delta(\omega)\right]$$

$$I_o = \frac{V_o}{20}\text{mA} = -0.125\left[\frac{2}{j\omega(2+j\omega)} + \pi\delta(\omega)\right]$$

$$= -\frac{0.125}{j\omega} + \frac{0.125}{2+j\omega} - 0.125\pi\delta(\omega)$$

$$i_o(t) = -0.125\,\text{sgn}(t) + 0.125e^{-2t}u(t) - \frac{0.125}{2\pi}\int \pi\delta(\omega)e^{j\omega t}dt$$

$$= 0.125 + 0.25u(t) + 0.125e^{-2t}u(t) - \frac{0.125}{2}$$

$$i_o(t) = \underline{\mathbf{0.625 - 0.25u(t) + 0.125e^{-2t}u(t)\,mA}}$$

Chapter 18, Problem 49.

Use the Fourier transform method to obtain $v_o(t)$ in the circuit of Fig. 18.51.

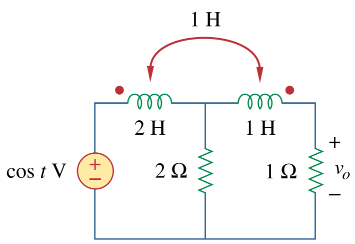
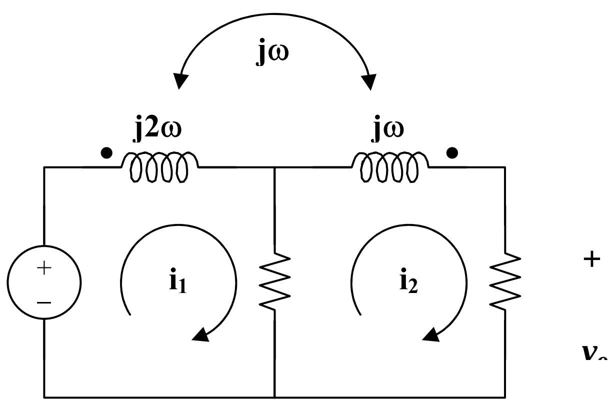


Figure 18.51
For Prob. 18.49.

Chapter 18, Solution 49.

Consider the circuit shown below:



$$V_s = \pi[\delta(\omega+1)+\delta(\omega-2)]$$

For mesh 1, $-V_s + (2+j2\omega)I_1 - 2I_2 - j\omega I_2 = 0$

$$V_s = 2(1+j\omega)I_1 - (2+j\omega)I_2 \qquad (1)$$

For mesh 2, $0 = (3+j\omega)I_2 - 2I_1 - j\omega I_1$

$$I_1 = \frac{(3+\omega)I_2}{(2+\omega)} \qquad (2)$$

Substituting (2) into (1) gives

$$V_s = 2\frac{2(1+j\omega)(3+j\omega)I_2}{2+j\omega} - (2+j\omega)I_2$$

$$V_s(2+\omega) = [2(3+j4\omega-\omega^2) - (4+j4\omega-\omega^2)]I_2$$

$$= I_2(2+j4\omega-\omega^2)$$

$$I_2 = \frac{(s+2)V_s}{s^2+4s+2},\ s = j\omega$$

$$V_o = I_2 = \frac{(j\omega+2)\pi[\delta(\omega+1)+\delta(\omega-1)]}{(j\omega)^2+j\omega 4+2}$$

$$v_o(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} v_o(\omega)e^{j\omega t}d\omega$$

$$= \int_{-\infty}^{\infty} \frac{\frac{1}{2}(j\omega+2)e^{j\omega t}\delta(\omega+1)d\omega}{(j\omega)^2+j\omega 4+2} + \frac{\frac{1}{2}(j\omega+2)e^{j\omega t}\delta(\omega-1)d\omega}{(j\omega)^2+j\omega 4+2}$$

$$= \frac{\frac{1}{2}(-j+2)e^{jt}}{-1-j4+2} + \frac{\frac{1}{2}(j+2)e^{jt}}{-1+j4+2}$$

$$v_o(t) = \frac{\frac{1}{2}(2-j)(1+j4)}{17}e^{jt} + \frac{\frac{1}{2}(2-j)(1-j4)e^{jt}}{17}$$

$$= \frac{1}{34}(6+j7)e^{jt} + \frac{1}{34}(6-j7)e^{jt}$$

$$= 0.271e^{-j(t-13.64°)} + 0.271e^{j(t-13.64°)}$$

$v_o(t) =$ $\underline{\mathbf{0.542\cos(t-13.64°)V}}$

Chapter 18, Problem 50.

Determine $v_o(t)$ in the transformer circuit of Fig. 18.52.

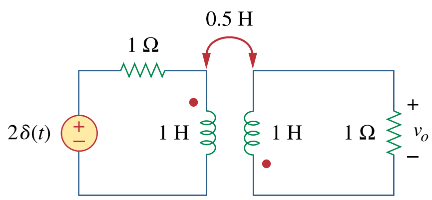
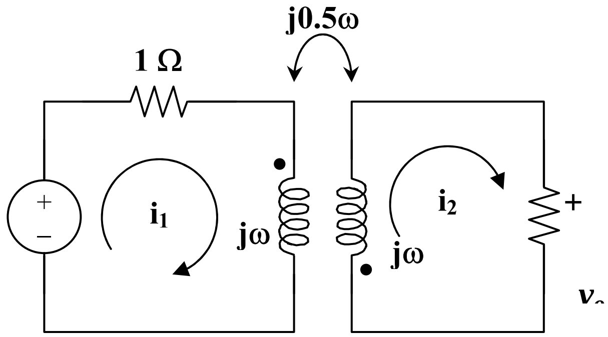


Figure 18.52
For Prob. 18.50.

Chapter 18, Solution 50.

Consider the circuit shown below:



For loop 1,

$$-2+(1+j\omega)I_1 + j0.5\omega I_2 = 0 \qquad (1)$$

For loop 2,

$$(1+j\omega)I_2 + j0.5\omega I_1 = 0 \qquad (2)$$

From (2),

$$I_1 = \frac{(1+j\omega)I_2}{-j0.5\omega} = -2\frac{(1+j\omega)I_2}{j\omega}$$

Substituting this into (1),

$$2 = \frac{-2(1+j\omega)I_2}{j\omega} + \frac{j\omega}{2}I_2$$

$$2j\omega = -\left(4+j4\omega-\frac{3}{2}\omega^2\right)I_2$$

$$I_2 = \frac{2j\omega}{4+j4\omega-1.5\omega^2}$$

$$V_o = I_2 = \frac{-2j\omega}{4+j4\omega+1.5(j\omega)^2}$$

$$V_o = \frac{\frac{4}{3}j\omega}{\frac{8}{3}+j\frac{8\omega}{3}+(j\omega)^2}$$

$$= \frac{-4\left(\frac{4}{3}+j\omega\right)}{\left(\frac{4}{3}+j\omega\right)^2+\left(\frac{\sqrt{8}}{3}\right)^2} + \frac{\frac{16}{3}}{\left(\frac{4}{3}+j\omega\right)^2+\left(\frac{\sqrt{8}}{3}\right)^2}$$

$$V_o(t) = \mathbf{-4e^{-4t/3}\cos\left(\frac{\sqrt{8}}{3}t\right)u(t)+5.657e^{-4t/3}\sin\left(\frac{\sqrt{8}}{3}t\right)u(t)\,V}$$

Chapter 18, Problem 51.

Find the energy dissipated by the resistor in the circuit of Fig. 18.53.

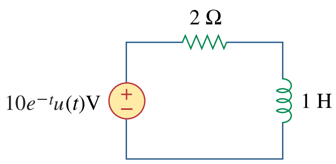


Figure 18.53
For Prob. 18.51.

Chapter 18, Solution 51.

In the frequency domain, the voltage across the 2-Ω resistor is

$$V(\omega)=\frac{2}{2+j\omega}V_s=\frac{2}{2+j\omega}\cdot\frac{10}{1+j\omega}=\frac{20}{(s+1)(s+2)},\qquad s=j\omega$$

$$V(s)=\frac{A}{s+1}+\frac{B}{s+2}$$

$$A=20/1=20,\qquad B=20/-1=-20$$

$$V(\omega)=\frac{20}{j\omega+1}-\frac{20}{j\omega+2}$$

$$v(t)=\left(20e^{-t}-20e^{-2t}\right)u(t)$$

$$W=\frac{1}{2}\int_0^\infty v^2(t)dt=0.5\int 400\left(e^{-2t}+e^{-4t}-3e^{-3t}\right)dt$$

$$=200\left(\frac{e^{-2t}}{-2}+\frac{e^{-4t}}{-4}-\frac{2e^{-3t}}{-3}\right)\Bigg|_0^\infty=\underline{\mathbf{16.667\ J}}.$$

Chapter 18, Problem 52.

For $F(\omega) = \dfrac{1}{3 + j\omega}$, find $J = \int_{-\infty}^{\infty} f^2(t)dt$.

Chapter 18, Solution 52.

$$J = 2\int_0^{\infty} f^2(t)\,dt = \frac{1}{\pi}\int_0^{\infty} |F(\omega)|^2\,d\omega$$

$$= \frac{1}{\pi}\int_0^{\infty} \frac{1}{9^2 + \omega^2}\,d\omega = \frac{1}{3\pi}\tan^{-1}(\omega/3)\Big|_0^{\infty} = \frac{1}{3\pi}\frac{\pi}{2} = \underline{\mathbf{(1/6)}}$$

Chapter 18, Problem 53.

If $f(t) = e^{-2|t|}$, find $J = \int_{-\infty}^{\infty} |F(\omega)|^2\, d\omega$.

Chapter 18, Solution 53.

If $f(t) = e^{-2|t|}$, find $J = \int_{-\infty}^{\infty} |F(\omega)|^2\, d\omega$.

$$J = \int_{-\infty}^{\infty} |F(\omega)|^2\,d\omega = 2\pi\int_{-\infty}^{\infty} f^2(t)\,dt$$

$$f(t) = \begin{vmatrix} e^{2t}, & t < 0 \\ e^{-2t}, & t > 0 \end{vmatrix}$$

$$J = 2\pi\left[\int_{-\infty}^{0} e^{4t}dt + \int_0^{\infty} e^{-4t}dt\right] = 2\pi\left[\frac{e^{4t}}{4}\Big|_{-\infty}^{0} + \frac{e^{-4t}}{-4}\Big|_0^{\infty}\right] = 2\pi[(1/4) + (1/4)] = \underline{\pi}$$

Chapter 18, Problem 54.

Given the signal $f(t) = 4e^{-t}\,u(t)$ what is the total energy in $f(t)$?

Chapter 18, Solution 54.

$$W_{1\Omega} = \int_{-\infty}^{\infty} f^2(t)\,dt = 16\int_0^{\infty} e^{-2t}dt = -8e^{-2t}\Big|_0^{\infty} = \underline{\mathbf{8\ J}}$$

Chapter 18, Problem 55.

Let $f(t) = 5e^{-(t-2)}\, u(t)$ and use it to find the total energy in $f(t)$.

Chapter 18, Solution 55.

$$f(t) = 5e^{2}e^{-t}u(t)$$

$$F(\omega) = 5e^{2}/(1 + j\omega), \quad |F(\omega)|^{2} = 25e^{4}/(1 + \omega^{2})$$

$$W_{1\Omega} = \frac{1}{\pi}\int_{0}^{\infty}|F(\omega)|^{2}\,d\omega = \frac{25e^{4}}{\pi}\int_{0}^{\infty}\frac{1}{1+\omega^{2}}\,d\omega = \frac{25e^{4}}{\pi}\tan^{-1}(\omega)\Big|_{0}^{\infty}$$

$$= 12.5e^{4} = \underline{\mathbf{682.5\ J}}$$

or $$W_{1\Omega} = \int_{-\infty}^{\infty} f^{2}(t)\,dt = 25e^{4}\int_{0}^{\infty} e^{-2t}dt = 12.5e^{4} = \underline{\mathbf{682.5\ J}}$$

Chapter 18, Problem 56.

The voltage across a 1-Ω resistor is $v(t) = te^{-2t}\,u(t)$ V. (a) What is the total energy absorbed by the resistor? (b) What fraction of this energy absorbed is in the frequency band $-2 \le \omega \le 2$?

Chapter 18, Solution 56.

(a) $$W = \int_{-\infty}^{\infty} V^{2}(t)dt = \int_{0}^{\infty} t^{2}e^{-4t}dt = \frac{e^{-4t}}{(-4)^{3}}(16t^{2}+8t+2)\Big|_{0}^{\infty} = \frac{2}{64} = \underline{0.0313\text{ J}}$$

(b) In the frequency domain,

$$V(\omega) = \frac{1}{(2+j\omega)^{2}}$$

$$|V(\omega)|^{2} = V(\omega)V^{*}(\omega) = \frac{1}{(4+j\omega)^{2}}$$

$$W_{o} = \frac{1}{2\pi}\int_{-2}^{2}|V(\omega)|^{2}d\omega = \frac{2}{2\pi}\int_{0}^{2}\frac{1}{(4+\omega^{2})^{2}}d\omega$$

$$= \frac{1}{\pi}\frac{1}{2\text{x}4}\left(\frac{\omega}{\omega^{2}+4} + 0.5\tan^{-1}(0.5\omega)\right)\Bigg|_{0}^{2} = \frac{1}{32\pi} + \frac{1}{64} = 0.0256$$

$$\text{Fraction} = \frac{W_{o}}{W} = \frac{0.0256}{0.0313} = \underline{81.79\%}$$

Chapter 18, Problem 57.

Let $i(t) = 2e^{t}\,u(-t)$A. Find the total energy carried by $i(t)$ and the percentage of the 1-Ω energy in the frequency range of $-5 < \omega < 5$ rad/s.

Chapter 18, Solution 57.

$$W_{1\Omega} = \int_{-\infty}^{\infty} i^2(t)\,dt = \int_{-\infty}^{0} 4e^{2t}dt = 2e^{2t}\Big|_{-\infty}^{0} = \underline{\mathbf{2\ J}}$$

or $\quad I(\omega) = 2/(1 - j\omega), \quad |I(\omega)|^2 = 4/(1 + \omega^2)$

$$W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |I(\omega)|^2 d\omega = \frac{4}{2\pi}\int_{-\infty}^{\infty} \frac{1}{(1+\omega^2)}d\omega = \frac{4}{\pi}\tan^{-1}(\omega)\Big|_{0}^{\infty} = \frac{4}{\pi}\frac{\pi}{2} = \underline{\mathbf{2\ J}}$$

In the frequency range, $-5 < \omega < 5$,

$$W = \frac{4}{\pi}\tan^{-1}\omega\Big|_{0}^{5} = \frac{4}{\pi}\tan^{-1}(5) = \frac{4}{\pi}(1.373) = 1.7487$$

$W/W_{1\Omega} = 1.7487/2 = 0.8743$ or $\underline{\mathbf{87.43\%}}$

Chapter 18, Problem 58.

e⊘d

An AM signal is specified by

$f(t) = 10(1 + 4\cos 200\,\pi t)\cos \pi \times 10^4\, t$

Determine the following:

(a) the carrier frequency,
(b) the lower sideband frequency,
(c) the upper sideband frequency.

Chapter 18, Solution 58.

$\omega_m = 200\pi = 2\pi f_m$ which leads to $f_m = 100$ Hz

(a) $\omega_c = \pi x 10^4 = 2\pi f_c$ which leads to $f_c = 10^4/2 =$ **<u>5 kHz</u>**

(b) lsb $= f_c - f_m = 5{,}000 - 100 =$ **<u>4,900 Hz</u>**

(c) usb $= f_c + f_m = 5{,}000 + 100 =$ **<u>5,100 Hz</u>**

Chapter 18, Problem 59.

For the linear system in Fig. 18.54, when the input voltage is $v_i(t) = 2\delta(t)$ V, the output is $v_0(t) = 10e^{-2t} - 6e^{-4t}$ V. Find the output when the input is $v_i(t) = 4e^{-t}u(t)$ V.

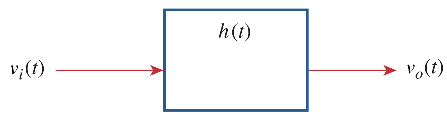


Figure 18.54
For Prob. 18.9.

Chapter 18, Solution 59.

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{\dfrac{10}{2+j\omega} - \dfrac{6}{4+j\omega}}{2} = \frac{5}{2+j\omega} - \frac{3}{4+j\omega}$$

$$V_o(\omega) = H(\omega)V_i(\omega) = \left(\frac{5}{2+j\omega} - \frac{3}{4+j\omega}\right)\frac{4}{1+j\omega}$$

$$= \frac{20}{(s+1)(s+2)} - \frac{12}{(s+1)(s+4)}, \qquad s = j\omega$$

Using partial fraction,

$$V_o(\omega) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+1} + \frac{D}{s+4} = \frac{16}{1+j\omega} - \frac{20}{2+j\omega} + \frac{4}{4+j\omega}$$

Thus,

$$v_o(t) = \underline{\left(16e^{-t} - 20e^{-2t} + 4e^{-4t}\right)u(t)\,V}$$

Chapter 18, Problem 60.

A band-limited signal has the following Fourier series representation:

$i_s(t) = 10 + 8\cos(2\pi t + 30^\circ) + 5\cos(4\pi t - 150^\circ)$mA

If the signal is applied to the circuit in Fig. 18.55, find $v(t)$.

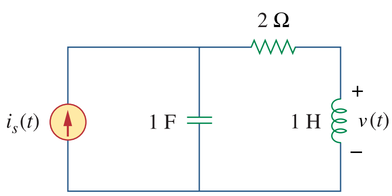
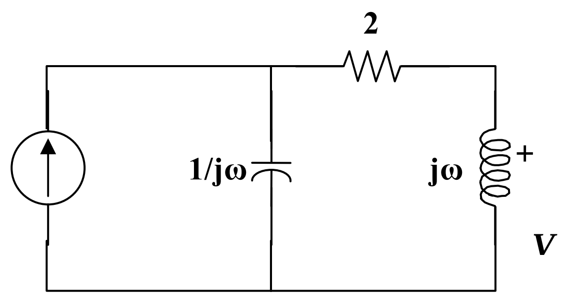


Figure 18.55
For Prob. 18.60.

Chapter 18, Solution 60.



$$V = j\omega I_s \frac{\frac{1}{j\omega}}{\frac{1}{j\omega} + 2 + j\omega} = \frac{j\omega I_s}{1 - \omega^2 + j2\omega}$$

Since the voltage appears across the inductor, there is no DC component.

$$V_1 = \frac{2\pi\angle 90°8}{1 - 4\pi^2 + j4\pi} = \frac{50.27\angle 90°}{-38.48 + j12.566} = 1.2418\angle -71.92°$$

$$V_2 = \frac{4\pi\angle 90°5}{1 - 16\pi^2 + j8\pi} = \frac{62.83\angle 90°}{-156.91 + j25.13} = 0.3954\angle -80.9°$$

$$v(t) = \underline{1.2418\cos(2\pi t - 41.92°) + 0.3954\cos(4\pi t + 129.1°)\,\text{mV}}$$

Chapter 18, Problem 61.

In a system, the input signal $x(t)$ is amplitude-modulated by $m(t) = 2 + \cos\omega_0 t$. The response $y(t) = m(t)x(t)$. Find $Y(\omega)$ in terms of $X(\omega)$.

Chapter 18, Solution 61.

$$y(t) = (2 + \cos\omega_o t)x(t)$$

We apply the Fourier Transform

$$Y(\omega) = \underline{\mathbf{2X(\omega) + 0.5X(\omega+\omega_o) + 0.5X(\omega-\omega_o)}}.$$

Chapter 18, Problem 62.

A voice signal occupying the frequency band of 0.4 to 3.5 kHz is used to amplitude-modulate a 10-MHz carrier. Determine the range of frequencies for the lower and upper sidebands.

Chapter 18, Solution 62.

For the lower sideband, the frequencies range from

10,000,000 – 3,500 Hz = **9,996,500 Hz** to
10,000,000 – 400 Hz = **9,999,600 Hz**

For the upper sideband, the frequencies range from

10,000,000 + 400 Hz = **10,000,400 Hz** to
10,000,000 + 3,500 Hz = **10,003,500 Hz**

Chapter 18, Problem 63.

For a given locality, calculate the number of stations allowable in the AM broadcasting band (540 to 1600 kHz) without interference with one another.

Chapter 18, Solution 63.

Since $f_n = 5$ kHz, $2f_n = 10$ kHz

i.e. the stations must be spaced 10 kHz apart to avoid interference.

$$\Delta f = 1600 - 540 = 1060 \text{ kHz}$$

The number of stations = Δf /10 kHz = **106 stations**

Chapter 18, Problem 64.

Repeat the previous problem for the FM broadcasting band (88 to 108 MHz), assuming that the carrier frequencies are spaced 200 kHz apart.

Chapter 18, Solution 64.

$\Delta f = 108 - 88 \text{ MHz} = 20 \text{ MHz}$

The number of stations $= 20 \text{ MHz}/0.2 \text{ MHz} =$ **<u>100 stations</u>**

Chapter 18, Problem 65.

The highest-frequency component of a voice signal is 3.4 kHz. What is the Nyquist rate of the sampler of the voice signal?

Chapter 18, Solution 65.

$\omega = 3.4 \text{ kHz}$

$f_s = 2\omega =$ **<u>6.8 kHz</u>**

Chapter 18, Problem 66.

A TV signal is band-limited to 4.5 MHz. If samples are to be reconstructed at a distant point, what is the maximum sampling interval allowable?

Chapter 18, Solution 66.

$\omega = 4.5 \text{ MHz}$

$f_c = 2\omega = 9 \text{ MHz}$

$T_s = 1/f_c = 1/(9\text{x}10^6) = 1.11\text{x}10^{-7} =$ **<u>111 ns</u>**

Chapter 18, Problem 67.

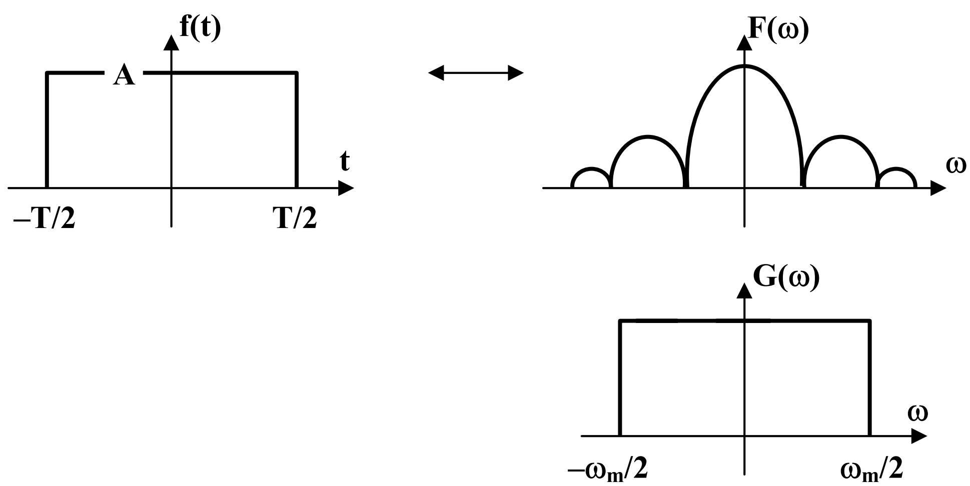
* Given a signal $g(t) = \text{sinc}(200\,\pi\,t)$ find the Nyquist rate and the Nyquist interval for the signal.

* An asterisk indicates a challenging problem.

Chapter 18, Solution 67.

We first find the Fourier transform of g(t). We use the results of Example 17.2 in conjunction with the duality property. Let Arect(t) be a rectangular pulse of height A and width T as shown below.

$$\text{Arect(t) transforms to Atsinc}(\omega^2/2)$$



According to the duality property,

$$A\tau\text{sinc}(\tau t/2) \quad \text{becomes} \quad 2\pi A\text{rect}(\tau)$$

$$g(t) = \text{sinc}(200\pi t) \ \text{becomes} \ 2\pi A\text{rect}(\tau)$$

where $A\tau = 1$ and $\tau/2 = 200\pi$ or $T = 400\pi$

i.e. the upper frequency $\omega_u = 400\pi = 2\pi f_u$ or $f_u = 200$ Hz

The Nyquist rate $= f_s =$ **<u>200 Hz</u>**

The Nyquist interval $= 1/f_s = 1/200 =$ **<u>5 ms</u>**

Chapter 18, Problem 68.

The voltage signal at the input of a filter is $v(t) = 50e^{-2|t|}$ V What percentage of the total 1-Ω energy content lies in the frequency range of $1 < \omega < 5$ rad/s?

Chapter 18, Solution 68.

The total energy is

$$W_T = \int_{-\infty}^{\infty} v^2(t)\,dt$$

Since v(t) is an even function,

$$W_T = \int_0^{\infty} 2500e^{-4t}dt = 5000\frac{e^{-4t}}{-4}\bigg|_0^{\infty} = 1250 \text{ J}$$

$V(\omega) = 50x4/(4 + \omega^2)$

$$W = \frac{1}{2\pi}\int_1^5 |V(\omega)|^2\,d\omega = \frac{1}{2\pi}\int_1^5 \frac{(200)^2}{(4+\omega^2)^2}d\omega$$

But $$\int \frac{1}{(a^2+x^2)^2}dx = \frac{1}{2a^2}\left[\frac{x}{x^2+a^2} + \frac{1}{a}\tan^{-1}(x/a)\right] + C$$

$$W = \frac{2x10^4}{\pi}\frac{1}{8}\left[\frac{\omega}{4+\omega^2} + \frac{1}{2}\tan^{-1}(\omega/2)\right]\Bigg|_1^5$$

$= (2500/\pi)[(5/29) + 0.5\tan^{-1}(5/2) - (1/5) - 0.5\tan^{-1}(1/2) = 267.19$

$W/W_T = 267.19/1250 = 0.2137$ or **21.37%**

Chapter 18, Problem 69.

A signal with Fourier transform

$$F(\omega) = \frac{20}{4 + j\omega}$$

is passed through a filter whose cutoff frequency is 2 rad/s (i.e., $0 < \omega < 2$). What fraction of the energy in the input signal is contained in the output signal?

Chapter 18, Solution 69.

The total energy is

$$W_T = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{400}{4^2 + \omega^2} d\omega$$

$$= \frac{400}{\pi}\left[(1/4)\tan^{-1}(\omega/4)\right]_0^{\infty} = \frac{100}{\pi}\frac{\pi}{2} = 50$$

$$W = \frac{1}{2\pi}\int_0^2 |F(\omega)|^2 d\omega = \frac{400}{2\pi}\left[(1/4)\tan^{-1}(\omega/4)\right]_0^2$$

$$= [100/(2\pi)]\tan^{-1}(2) = (50/\pi)(1.107) = 17.6187$$

$W/W_T = 17.6187/50 = 0.3524$ or **35.24%**