Chapter 16, Problem 1.

Determine $i(t)$ in the circuit of Fig. 16.35 by means of the Laplace transform.

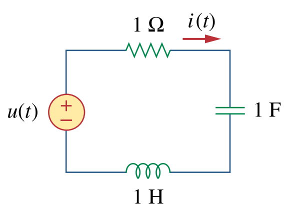
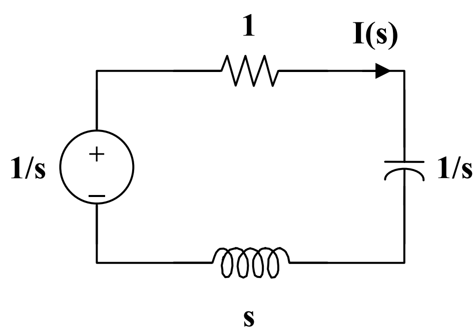


Figure 16.35
For Prob. 16.1.

Chapter 16, Solution 1.

Consider the s-domain form of the circuit which is shown below.



$$I(s) = \frac{1/s}{1+s+1/s} = \frac{1}{s^2+s+1} = \frac{1}{(s+1/2)^2+(\sqrt{3}/2)^2}$$

$$i(t) = \frac{2}{\sqrt{3}} e^{-t/2} \sin\left(\frac{\sqrt{3}}{2} t\right)$$

$$i(t) = \underline{\mathbf{1.155 e^{-0.5t} \sin(0.866t)\ A}}$$

Chapter 16, Problem 2.

Find v_x in the circuit shown in Fig. 16.36 given $v_s = 4u(t)$V.

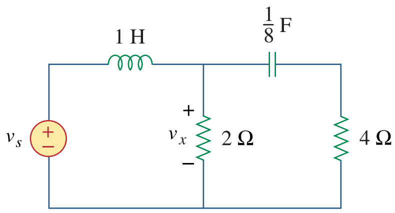
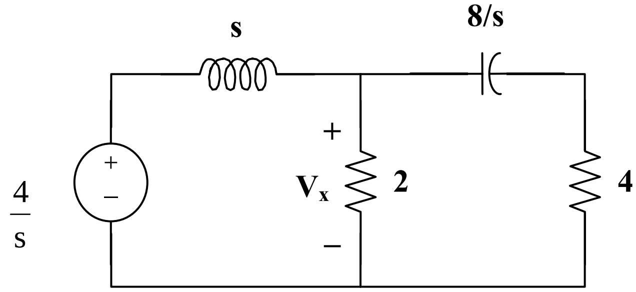


Figure 16.36
For Prob. 16.2.

Chapter 16, Solution 2.



$$\frac{V_x - \frac{4}{s}}{s} + \frac{V_x - 0}{2} + \frac{V_x - 0}{4 + \frac{8}{s}} = 0$$

$$V_x(4s+8) - \frac{(16s+32)}{s} + (2s^2+4s)V_x + s^2V_x = 0$$

$$V_x(3s^2+8s+8) = \frac{16s+32}{s}$$

$$V_x = -16\frac{s+2}{s(3s^2+8s+8)} = -16\left(\frac{0.25}{s} + \frac{-0.125}{s+\frac{4}{3}+j\frac{\sqrt{8}}{3}} + \frac{-0.125}{s+\frac{4}{3}-j\frac{\sqrt{8}}{3}}\right)$$

$$v_x = \underline{(-4 + 2e^{-(1.3333+j0.9428)t} + 2e^{-(1.3333-j0.9428)t})u(t)\,V}$$

$$v_x = \underline{4u(t) - e^{-4t/3}\cos\left(\frac{2\sqrt{2}}{3}t\right) - \frac{6}{\sqrt{2}}e^{-4t/3}\sin\left(\frac{2\sqrt{2}}{3}t\right)\ V}$$

Chapter 16, Problem 3.

Find $i(t)$ for $t > 0$ for the circuit in Fig. 16.37. Assume $i_s = 4u(t) + 2\delta(t)$mA. (Hint: Can we use superposition to help solve this problem?)

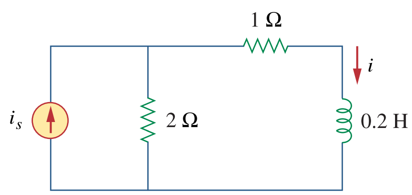
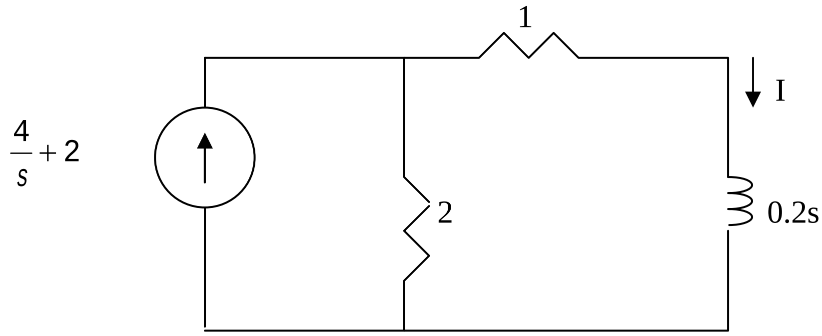


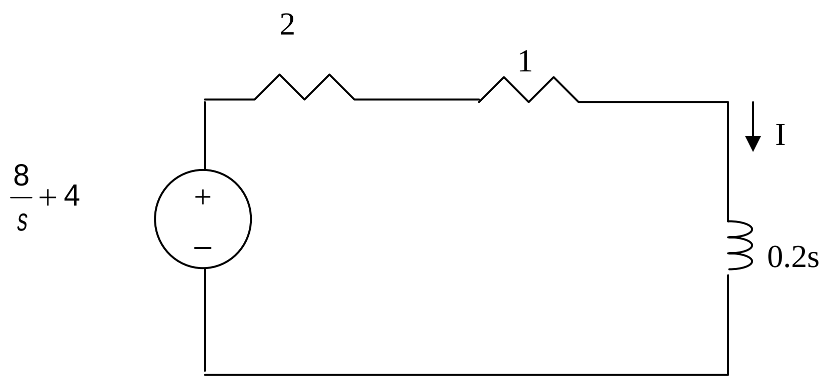
Figure 16.37
For Prob. 16.3.

Chapter 16, Solution 3.

In the s-domain, the circuit becomes that shown below.



We transform the current source to a voltage source and obtain the circuit shown below.



$$I = \frac{\frac{8}{s}+4}{3+0.2s} = \frac{20s+40}{s(s+15)} = \frac{A}{s} + \frac{B}{s+15}$$

$$A = \frac{40}{15} = \frac{8}{3}, \qquad B = \frac{-15x20+40}{-15} = \frac{52}{3}$$

$$I = \frac{8/3}{s} + \frac{52/3}{s+15}$$

$$i(t) = \underline{\left[\frac{8}{3} + \frac{52}{3}e^{-15t}\right]u(t)}$$

Chapter 16, Problem 4.

The capacitor in the circuit of Fig. 16.38 is initially uncharged. Find $v_0(t)$ for $t > 0$.

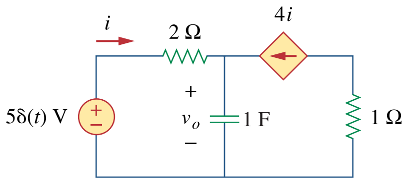
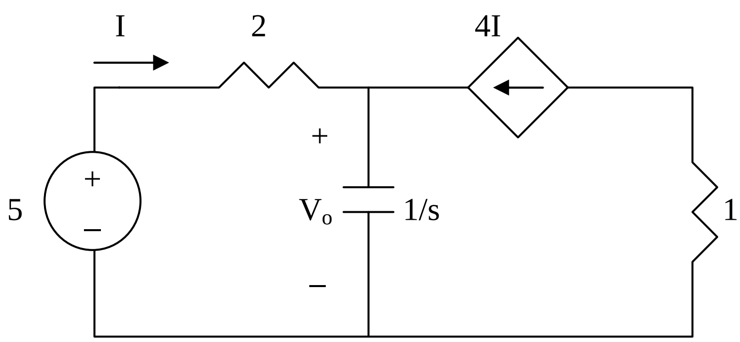


Figure 16.38
For Prob. 16.4.

Chapter 16, Solution 4.

The circuit in the s-domain is shown below.



$$I + 4I = \frac{V_o}{1/s} \quad \longrightarrow \quad 5I = sV_o$$

But $I = \dfrac{5 - V_o}{2}$

$$5\left(\frac{5 - V_o}{2}\right) = sV_o \quad \longrightarrow \quad V_o = \frac{12.5}{s + 5/2}$$

$$v_o(t) = \underline{12.5e^{-2.5t}\ \text{V}}$$

Chapter 16, Problem 5.

If $i_s(t) = e^{-2t}u(t)$ A in the circuit shown in Fig. 16.39, find the value of $i_0(t)$.

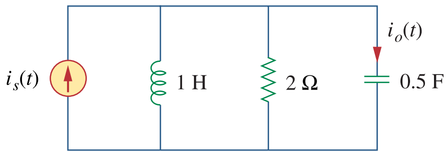
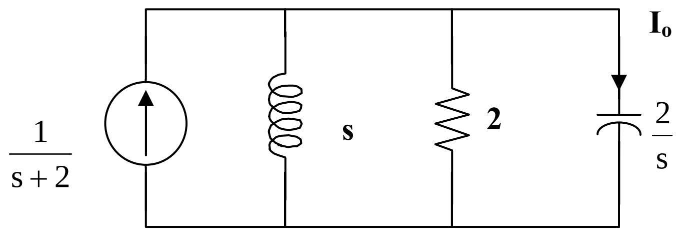


Figure 16.39
For Prob. 16.5.

Chapter 16, Solution 5.



$$V = \frac{1}{s+2}\left(\frac{1}{\frac{1}{s}+\frac{1}{2}+\frac{s}{2}}\right) = \frac{1}{s+2}\left(\frac{2s}{s^2+s+2}\right) = \frac{2s}{(s+2)(s+0.5+j1.3229)(s+0.5-j1.3229)}$$

$$I_o = \frac{Vs}{2} = \frac{s^2}{(s+2)(s+0.5+j1.3229)(s+0.5-j1.3229)}$$

$$= \frac{1}{s+2} + \frac{\frac{(-0.5-j1.3229)^2}{(1.5-j1.3229)(-j2.646)}}{s+0.5+j1.3229} + \frac{\frac{(-0.5+j1.3229)^2}{(1.5+j1.3229)(+j2.646)}}{s+0.5-j1.3229}$$

$$\underline{i_o(t) = \left(e^{-2t} + 0.3779e^{-90°}e^{-t/2}e^{-j1.3229t} + 0.3779e^{90°}e^{-t/2}e^{j1.3229t}\right)u(t)\,A}$$

or

$$\underline{\quad = \left(e^{-2t} - 0.7559\sin 1.3229t\right)u(t)\,A}$$

$$\text{or } i_o(t) = \left(e^{-2t} - \frac{2}{\sqrt{7}}\sin\left(\frac{\sqrt{7}}{2}t\right)\right)u(t)A$$

Chapter 16, Problem 6.

Find $v(t)$, $t > 0$ in the circuit of Fig. 16.40. Let $v_s = 20$ V.

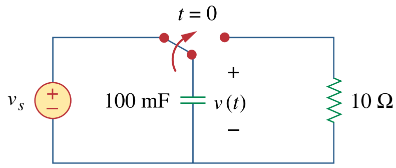
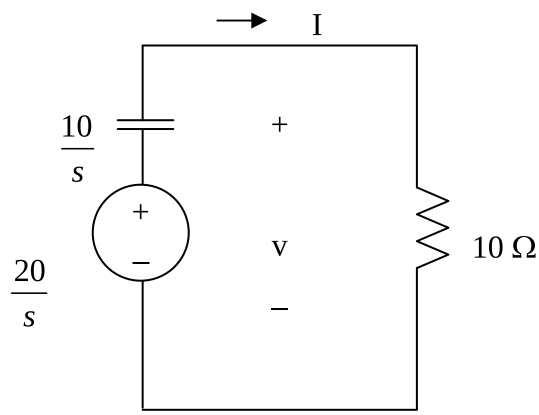


Figure 16.40
For Prob. 16.6.

Chapter 16, Solution 6.

For t<0, v(0) = v_s = 20 V
For t>0, the circuit in the s-domain is as shown below.



$$100mF = 0.1F \longrightarrow \frac{1}{sC} = \frac{10}{s}$$

$$I = \frac{20/s}{10 + 10/s} = \frac{2}{s+1}$$

$$V = 10I = \frac{20}{s+1}$$

$$v(t) = \underline{20e^{-t}u(t)}$$

Chapter 16, Problem 7.

Find $v_0(t)$, for all $t > 0$, in the circuit of Fig. 16.41.

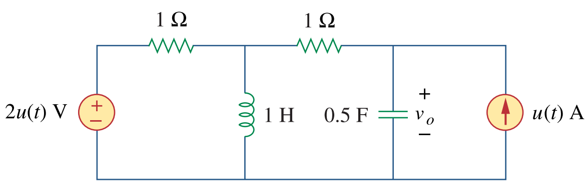
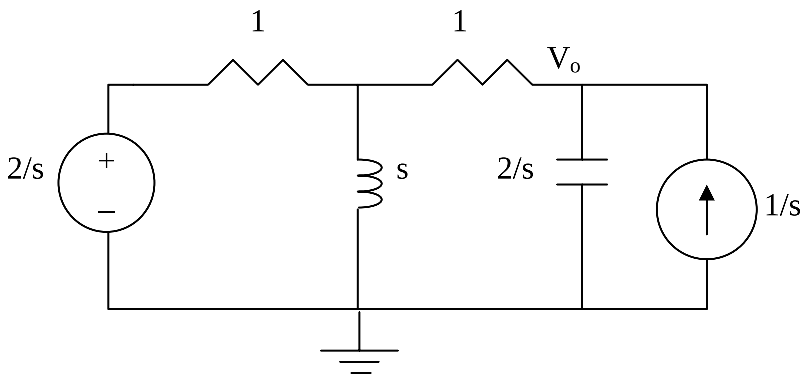


Figure 16.41
For Prob. 16.7.

Chapter 16, Solution 7.

The circuit in the s-domain is shown below. Please note, $i_L(0) = 0$ and $v_o(0) = o$ because both sources were equal to zero for all t<0.



At node 1

$$\frac{2/s - V_1}{1} = \frac{V_1}{s} + \frac{V_1 - V_o}{1} \quad \longrightarrow \quad \frac{2}{s} = V_1(2 + 1/s) - V_o \qquad (1)$$

At node O,

$$\frac{V_1 - V_o}{1} + \frac{1}{s} = \frac{V_o}{2/s} = \frac{s}{2}V_o \quad \longrightarrow \quad V_1 = (1 + s/2)V_o - 1/s \qquad (2)$$

Substituting (2) into (1) gives

$$2/s = (2 + 1/s)(1 + s/2)V_o - \frac{1}{s}(2 + \frac{1}{s}) - V_o$$

$$V_o = \frac{(4s+1)}{s(s^2 + 1.5s + 1)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 1.5s + 1}$$

$$4s + 1 = A(s^2 + 1.5s + 1) + Bs^2 + Cs$$

We equate coefficients.

s^2 : 0 = A+ B or B = - A

s: 4=1.5A + C

constant: 1 = A, B=-1, C = 4-1.5A = 2.5

$$V_o = \frac{1}{s} + \frac{-s + 2.5}{s^2 + 1.5s + 1} = \frac{1}{s} - \frac{s + 3/4}{(s + 3/4)^2 + \left(\frac{\sqrt{7}}{4}\right)^2} + \frac{\frac{3.25}{\frac{\sqrt{7}}{4}} x \frac{\sqrt{7}}{4}}{(s + 3/4)^2 + \left(\frac{\sqrt{7}}{4}\right)^2}$$

$$v(t) = \underline{u(t) - e^{-3t/4}\cos\frac{\sqrt{7}}{4}t + 4.9135e^{-3t/4}\sin\frac{\sqrt{7}}{4}t}$$

Chapter 16, Problem 8.

If $v_0(0)$ = -1V,obtain $v_0(t)$ in the circuit of Fig. 16.42.

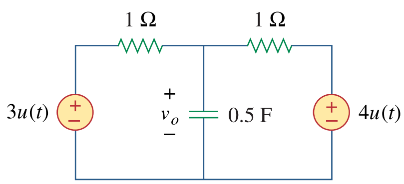
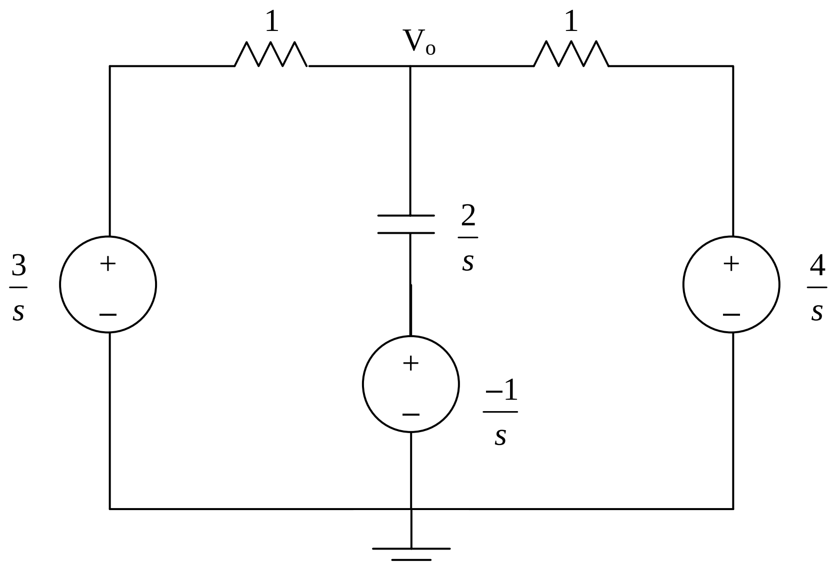


Figure 16.42
For Prob. 16.8.

Chapter 16, Solution 8.

$$\frac{1}{2}F \longrightarrow \frac{1}{sC} = \frac{2}{s}$$

We analyze the circuit in the s-domain as shown below. We apply nodal analysis.



$$\frac{\frac{3}{s}-V_o}{1}+\frac{-\frac{1}{s}-V_o}{\frac{2}{s}}+\frac{\frac{4}{s}-V_o}{1}=0 \longrightarrow V_0=\frac{14-s}{s(s+4)}$$

$$V_o=\frac{A}{s}+\frac{B}{s+4}$$

$$A=\frac{14}{4}=7/2, \qquad B=\frac{18}{-4}=-9/2$$

$$V_o=\frac{7/2}{s}-\frac{9/2}{s+4}$$

$$v_o(t)=\underline{\left(\frac{7}{2}-\frac{9}{2}e^{-4t}\right)u(t)}$$

Chapter 16, Problem 9.

Find the input impedance $Z_{in}(s)$ of each of the circuits in Fig. 16.43.

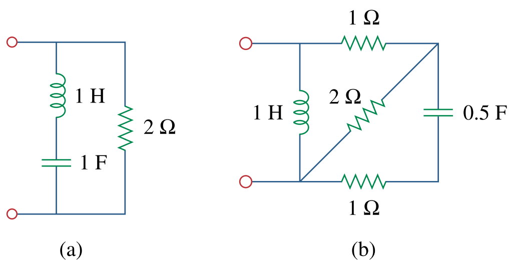
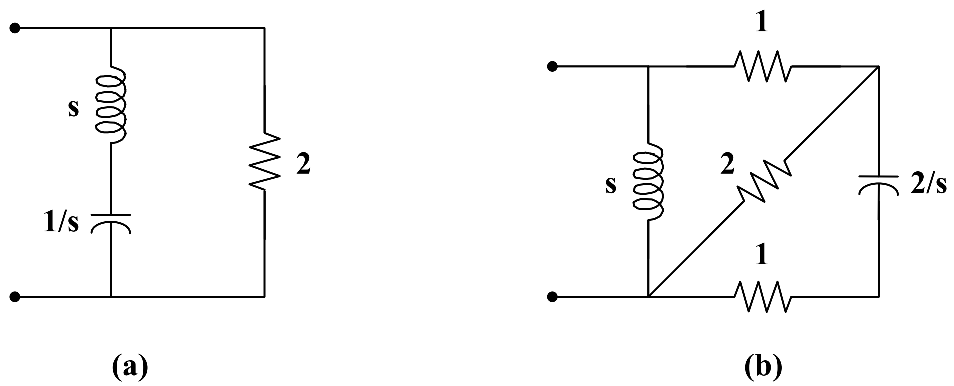


Figure 16.43
For Prob. 16.9.

Chapter 16, Solution 9.

(a) The s-domain form of the circuit is shown in Fig. (a).

$$Z_{in} = 2 \,||\, (s + 1/s) = \frac{2(s+1/s)}{2+s+1/s} = \underline{\mathbf{\frac{2(s^2+1)}{s^2+2s+1}}}$$



(b) The s-domain equivalent circuit is shown in Fig. (b).

$$2 \,||\, (1 + 2/s) = \frac{2(1+2/s)}{3+2/s} = \frac{2(s+2)}{3s+2}$$

$$1 + 2 \,||\, (1 + 2/s) = \frac{5s+6}{3s+2}$$

$$Z_{in} = s \,||\, \left(\frac{5s+6}{3s+2}\right) = \frac{s \cdot \left(\frac{5s+6}{3s+2}\right)}{s + \left(\frac{5s+6}{3s+2}\right)} = \underline{\mathbf{\frac{s(5s+6)}{3s^2+7s+6}}}$$

Chapter 16, Problem 10.

Use Thevenin's theorem to determine $v_0(t)$, $t > 0$ in the circuit of Fig. 16.44.

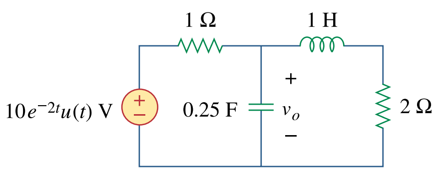


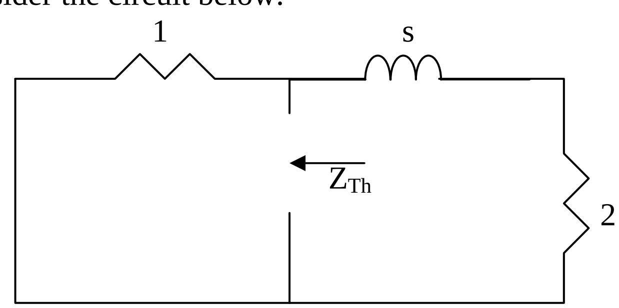
Figure 16.44
For Prob. 16.10.

Chapter 16, Solution 10.

$1H \longrightarrow 1s$ and $i_L(0) = 0$ (the sources is zero for all t<0).

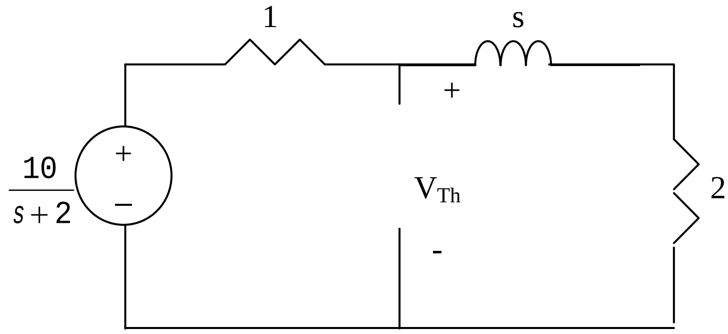
$\frac{1}{4}F \longrightarrow \frac{1}{sC} = \frac{4}{s}$ and $v_C(0) = 0$ (again, there are no source contributions for all t<0).

To find Z_{Th}, consider the circuit below.



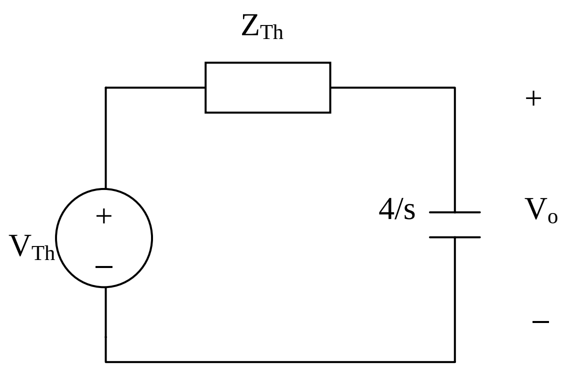
$$Z_{Th} = 1//(s+2) = \frac{s+2}{s+3}$$

To find V_{Th}, consider the circuit below.



$$V_{Th} = \frac{s+2}{s+3} \cdot \frac{10}{s+2} = \frac{10}{s+3}$$

The Thevenin equivalent circuit is shown below



$$V_o = \frac{\frac{4}{s}}{\frac{4}{s} + Z_{Th}} V_{Th} = \frac{\frac{4}{s}}{\frac{4}{s} + \frac{s+2}{s+3}} \cdot \frac{10}{s+3} = \frac{40}{s^2 + 6s + 12} = \frac{\frac{40}{\sqrt{3}}\sqrt{3}}{(s+3)^3 + (\sqrt{3})^2}.$$

$$v_o(t) = \underline{23.094 e^{-3t} \sin\sqrt{3}t}$$

Chapter 16, Problem 11.

Solve for the mesh currents in the circuit of Fig. 16.45. You may leave your results in the *s*-domain.

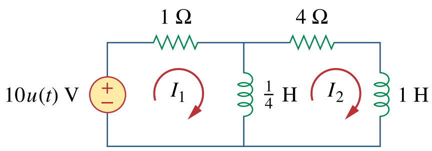
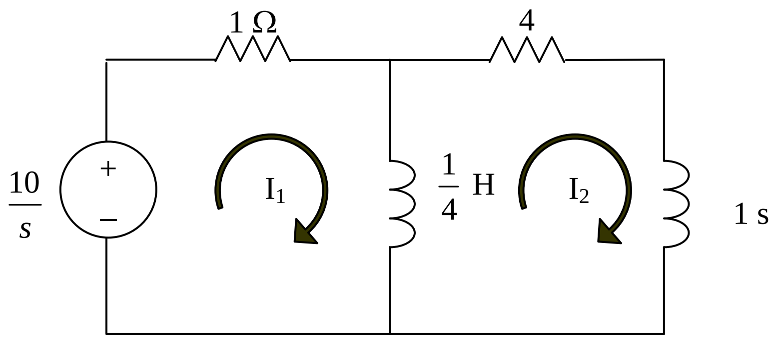


Figure 16.45
For Prob. 16.11.

Chapter 16, Solution 11.

In the s-domain, the circuit is as shown below.



$$\frac{10}{s} = (1+\frac{s}{4})I_1 - \frac{1}{4}sI_2 \qquad (1)$$

$$-\frac{1}{4}sI_1 + I_2(4+\frac{5}{4}s) = 0 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} \frac{10}{s} \\ 0 \end{bmatrix} = \begin{bmatrix} 1+\frac{s}{4} & -\frac{1}{4}s \\ -\frac{1}{4}s & 4+\frac{5}{4}s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \frac{1}{4}s^2 + \frac{9}{4}s + 4$$

$$\Delta_1 = \begin{vmatrix} \frac{10}{s} & -\frac{1}{4}s \\ 0 & 4+\frac{5}{4}s \end{vmatrix} = \frac{40}{s} + \frac{50}{4}$$

$$\Delta_2 = \begin{vmatrix} 1+\frac{s}{4} & \frac{10}{s} \\ -\frac{1}{4}s & 0 \end{vmatrix} = \frac{5}{2}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{\frac{40}{s} + \frac{25}{2}}{0.25s^2 + 2.25s + 4} = \underline{\frac{50s + 160}{s(s^2 + 9s + 16)}}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{2.5}{0.25s^2 + 2.25s + 4} = \underline{\frac{10}{s^2 + 9s + 16}}$$

Chapter 16, Problem 12.

Find $v_o(t)$ in the circuit of Fig. 16.46.

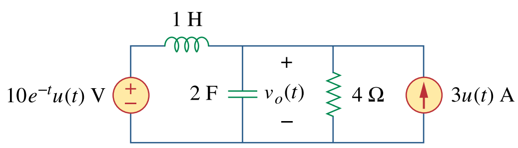
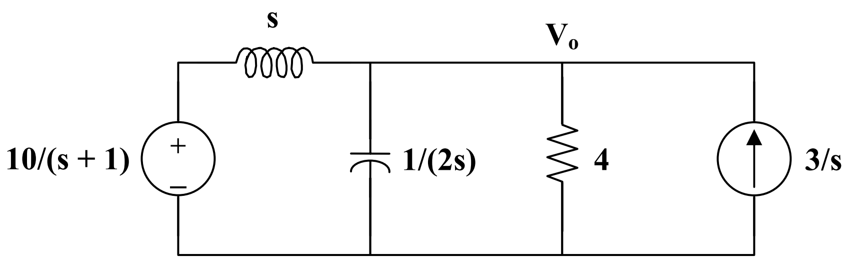


Figure 16.46
For Prob. 16.12.

Chapter 16, Solution 12.

We apply nodal analysis to the s-domain form of the circuit below.



$$\frac{\frac{10}{s+1}-V_o}{s}+\frac{3}{s}=\frac{V_o}{4}+2sV_o$$

$$(1+0.25s+s^2)\,V_o = \frac{10}{s+1}+15=\frac{10+15s+15}{s+1}$$

$$V_o = \frac{15s+25}{(s+1)(s^2+0.25s+1)}=\frac{A}{s+1}+\frac{Bs+C}{s^2+0.25s+1}$$

$$A=(s+1)\,V_o\big|_{s=-1}=\frac{40}{7}$$

$$15s+25=A\,(s^2+0.25s+1)+B\,(s^2+s)+C\,(s+1)$$

Equating coefficients :

s^2: $\quad 0=A+B \longrightarrow B=-A$

s^1: $\quad 15=0.25A+B+C=-0.75A+C$

s^0: $\quad 25=A+C$

$A=40/7, \qquad B=-40/7, \qquad C=135/7$

$$V_o=\frac{\frac{40}{7}}{s+1}+\frac{\frac{-40}{7}s+\frac{135}{7}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}}=\frac{40}{7}\frac{1}{s+1}-\frac{40}{7}\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}}+\left(\frac{155}{7}\cdot\frac{2}{\sqrt{3}}\right)\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$v_o(t)=\frac{40}{7}e^{-t}-\frac{40}{7}e^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t\right)+\frac{(155)(2)}{(7)(\sqrt{3})}e^{-t/2}\sin\left(\frac{\sqrt{3}}{2}t\right)$$

$$v_o(t)=\underline{\mathbf{5.714\,e^{-t}-5.714\,e^{-t/2}\cos(0.866t)+25.57\,e^{-t/2}\sin(0.866t)\ V}}$$

Chapter 16, Problem 13.

Determine $i_0(t)$ in the circuit of Fig. 16.47.

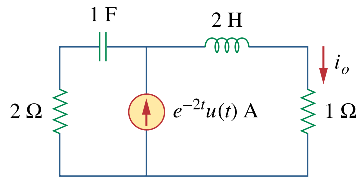
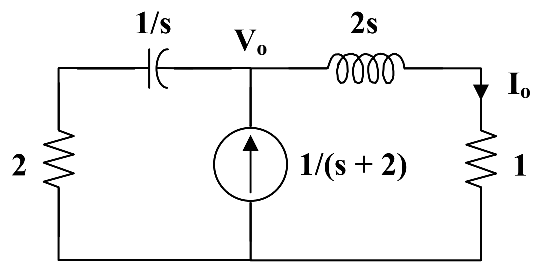


Figure 16.47
For Prob. 16.13.

Chapter 16, Solution 13.

Consider the following circuit.



Applying KCL at node o,

$$\frac{1}{s+2} = \frac{V_o}{2s+1} + \frac{V_o}{2+1/s} = \frac{s+1}{2s+1} V_o$$

$$V_o = \frac{2s+1}{(s+1)(s+2)}$$

$$I_o = \frac{V_o}{2s+1} = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = 1, \qquad B = -1$$

$$I_o = \frac{1}{s+1} - \frac{1}{s+2}$$

$$i_o(t) = \underline{\left(e^{-t} - e^{-2t}\right) u(t) \text{ A}}$$

Chapter 16, Problem 14.

* Determine $i_0(t)$ in the network shown in Fig. 16.48.

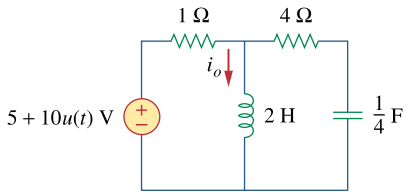
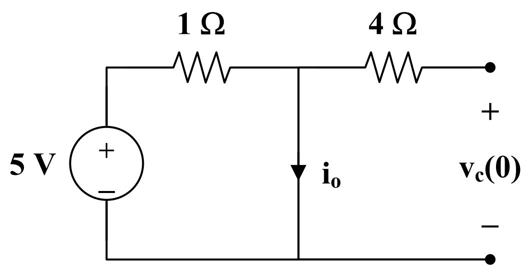


Figure 16.48
For Prob. 16.14.

* An asterisk indicates a challenging problem.

Chapter 16, Solution 14.

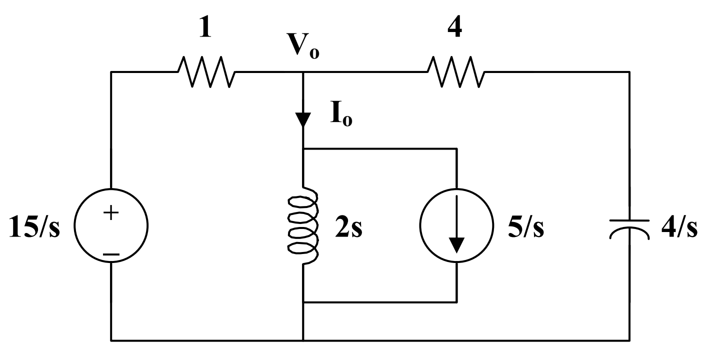
We first find the initial conditions from the circuit in Fig. (a).



(a)

$$i_o(0^-) = 5\ \text{A}\,,\quad v_c(0^-) = 0\ \text{V}$$

We now incorporate these conditions in the s-domain circuit as shown in Fig.(b).



(b)

At node o,

$$\frac{V_o - 15/s}{1} + \frac{V_o}{2s} + \frac{5}{s} + \frac{V_o - 0}{4 + 4/s} = 0$$

$$\frac{15}{s} - \frac{5}{s} = \left(1 + \frac{1}{2s} + \frac{s}{4(s+1)}\right) V_o$$

$$\frac{10}{s} = \frac{4s^2 + 4s + 2s + 2 + s^2}{4s(s+1)} V_o = \frac{5s^2 + 6s + 2}{4s(s+1)} V_o$$

$$V_o = \frac{40(s+1)}{5s^2 + 6s + 2}$$

$$I_o = \frac{V_o}{2s} + \frac{5}{s} = \frac{4(s+1)}{s(s^2 + 1.2s + 0.4)} + \frac{5}{s}$$

$$I_o = \frac{5}{s} + \frac{A}{s} + \frac{Bs + C}{s^2 + 1.2s + 0.4}$$

$$4(s+1) = A(s^2 + 1.2s + 0.4) + Bs^s + Cs$$

Equating coefficients :

s^0 : $\quad 4 = 0.4A \longrightarrow A = 10$

s^1: $\quad 4 = 1.2A + C \longrightarrow C = \text{-}1.2A + 4 = \text{-}8$

s^2 : $\quad 0 = A + B \longrightarrow B = \text{-}A = \text{-}10$

$$I_o = \frac{5}{s} + \frac{10}{s} - \frac{10s + 8}{s^2 + 1.2s + 0.4}$$

$$I_o = \frac{15}{s} - \frac{10(s+0.6)}{(s+0.6)^2 + 0.2^2} - \frac{10(0.2)}{(s+0.6)^2 + 0.2^2}$$

$$i_o(t) = \underline{\left[\mathbf{15 - 10\,e^{-0.6t}\left(\cos(0.2\,t) - \sin(0.2\,t)\right)}\right]\ \mathbf{u(t)\ A}}$$

Chapter 16, Problem 15.

Find $V_x(s)$ in the circuit shown in Fig. 16.49.

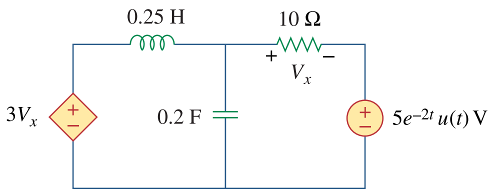
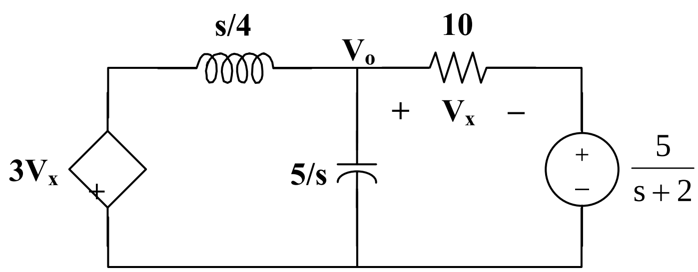


Figure 16.49
For Prob. 16.15.

Chapter 16, Solution 15.
First we need to transform the circuit into the s-domain.



$$\frac{V_o - 3V_x}{s/4} + \frac{V_o - 0}{5/s} + \frac{V_o - \frac{5}{s+2}}{10} = 0$$

$$40V_o - 120V_x + 2s^2V_o + sV_o - \frac{5s}{s+2} = 0 = (2s^2 + s + 40)V_o - 120V_x - \frac{5s}{s+2}$$

$$\text{But, } V_x = V_o - \frac{5}{s+2} \rightarrow V_o = V_x + \frac{5}{s+2}$$

We can now solve for V_x.

$$(2s^2 + s + 40)\left(V_x + \frac{5}{s+2}\right) - 120V_x - \frac{5s}{s+2} = 0$$

$$2(s^2 + 0.5s - 40)V_x = -10\frac{(s^2 + 20)}{s+2}$$

$$V_x = -5\frac{(s^2 + 20)}{(s+2)(s^2 + 0.5s - 40)}$$

Chapter 16, Problem 16.

* Find $i_0(t)$ for $t > 0$ in the circuit of Fig. 16.50.

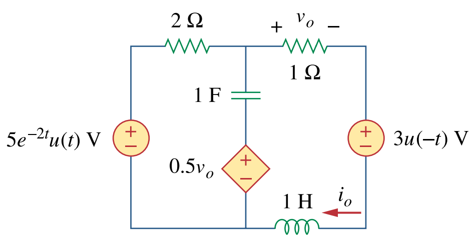
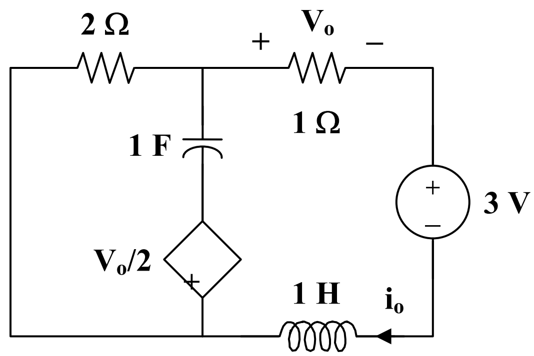


Figure 16.50
For Prob. 16.16.

* An asterisk indicates a challenging problem.

Chapter 16, Solution 16.

We first need to find the initial conditions. For $t < 0$, the circuit is shown in Fig. (a).



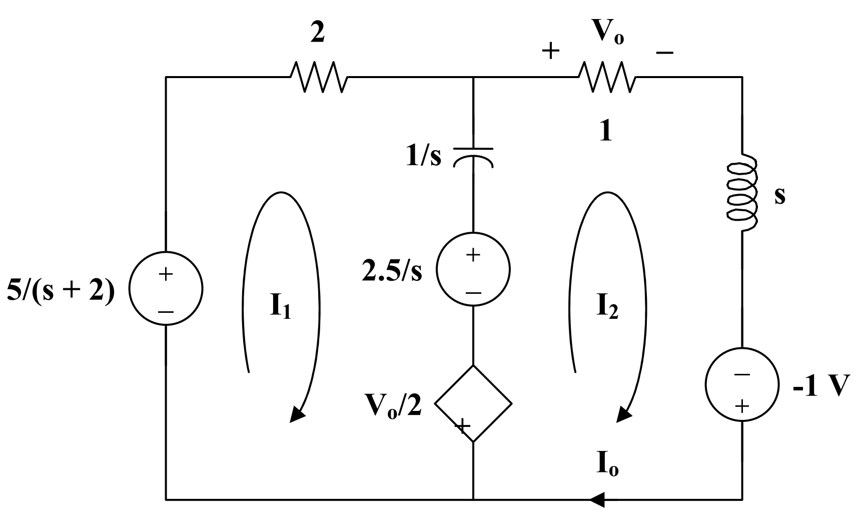
(a)

To dc, the capacitor acts like an open circuit and the inductor acts like a short circuit. Hence,

$$i_L(0) = i_o = \frac{-3}{3} = -1\text{ A}, \qquad v_o = -1\text{ V}$$

$$v_c(0) = -(2)(-1) - \left(\frac{-1}{2}\right) = 2.5\text{ V}$$

We now incorporate the initial conditions for $t > 0$ as shown in Fig. (b).



(b)

For mesh 1,

$$\frac{-5}{s+2} + \left(2 + \frac{1}{s}\right)I_1 - \frac{1}{s}I_2 + \frac{2.5}{s} + \frac{V_o}{2} = 0$$

But, $\quad V_o = I_o = I_2$

$$\left(2+\frac{1}{s}\right)I_1 + \left(\frac{1}{2}-\frac{1}{s}\right)I_2 = \frac{5}{s+2} - \frac{2.5}{s} \qquad (1)$$

For mesh 2,

$$\left(1+s+\frac{1}{s}\right)I_2 - \frac{1}{s}I_1 + 1 - \frac{V_o}{2} - \frac{2.5}{s} = 0$$

$$-\frac{1}{s}I_1 + \left(\frac{1}{2}+s+\frac{1}{s}\right)I_2 = \frac{2.5}{s} - 1 \qquad (2)$$

Put (1) and (2) in matrix form.

$$\begin{bmatrix} 2+\frac{1}{s} & \frac{1}{2}-\frac{1}{s} \\ \frac{-1}{s} & \frac{1}{2}+s+\frac{1}{s} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{5}{s+2} - \frac{2.5}{s} \\ \frac{2.5}{s} - 1 \end{bmatrix}$$

$$\Delta = 2s + 2 + \frac{3}{s}, \qquad \Delta_2 = -2 + \frac{4}{s} + \frac{5}{s(s+2)}$$

$$I_o = I_2 = \frac{\Delta_2}{\Delta} = \frac{-2s^2+13}{(s+2)(2s^2+2s+3)} = \frac{A}{s+2} + \frac{Bs+C}{2s^2+2s+3}$$

$$-2s^2 + 13 = A(2s^2+2s+3) + B(s^2+2s) + C(s+2)$$

Equating coefficients :

s^2: $\quad -2 = 2A + B$

s^1: $\quad 0 = 2A + 2B + C$

s^0: $\quad 13 = 3A + 2C$

Solving these equations leads to

$A = 0.7143, \quad B = -3.429, \quad C = 5.429$

$$I_o = \frac{0.7143}{s+2} - \frac{3.429s - 5.429}{2s^2+2s+3} = \frac{0.7143}{s+2} - \frac{1.7145s - 2.714}{s^2+s+1.5}$$

$$I_o = \frac{0.7143}{s+2} - \frac{1.7145(s+0.5)}{(s+0.5)^2+1.25} + \frac{(3.194)(\sqrt{1.25})}{(s+0.5)^2+1.25}$$

$$i_o(t) = \underline{\left[\mathbf{0.7143\,e^{-2t} - 1.7145\,e^{-0.5t}\cos(1.25t) + 3.194\,e^{-0.5t}\sin(1.25t)}\right]\mathbf{u(t)\ A}}$$

Chapter 16, Problem 17.

Calculate $i_0(t)$ for $t > 0$ in the network of Fig. 16.51.

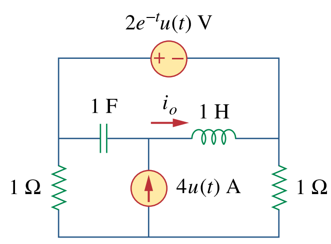
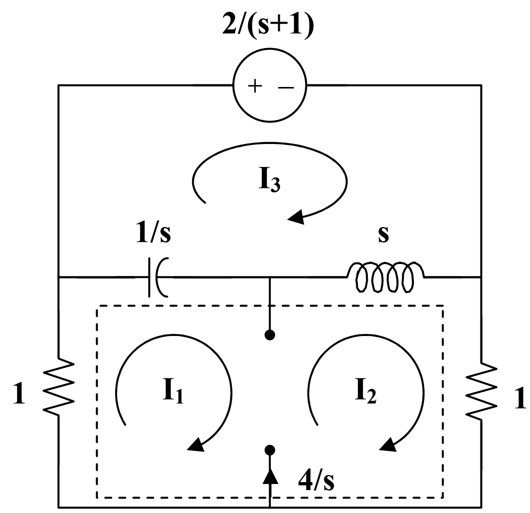


Figure 16.51
For Prob. 16.17.

Chapter 16, Solution 17.

We apply mesh analysis to the s-domain form of the circuit as shown below.



For mesh 3,

$$\frac{2}{s+1} + \left(s + \frac{1}{s}\right) I_3 - \frac{1}{s} I_1 - s I_2 = 0 \qquad (1)$$

For the supermesh,

$$\left(1 + \frac{1}{s}\right) I_1 + (1+s) I_2 - \left(\frac{1}{s} + s\right) I_3 = 0 \qquad (2)$$

Adding (1) and (2) we get, $I_1 + I_2 = –2/(s+1)$ (3)

But $–I_1 + I_2 = 4/s$ (4)

Adding (3) and (4) we get, $I_2 = (2/s) – 1/(s+1)$ (5)

Substituting (5) into (4) yields, $I_1 = –(2/s) – (1/(s+1))$ (6)

Substituting (5) and (6) into (1) we get,

$$\frac{2}{s^2}+\frac{1}{s(s+1)}-2+\frac{s}{s+1}+\left(\frac{s^2+1}{s}\right)I_3=-\frac{2}{s+1}$$

$$I_3=-\frac{2}{s}+\frac{1.5-0.5j}{s+j}+\frac{1.5+0.5j}{s-j}$$

Substituting (3) into (1) and (2) leads to

$$-\left(s+\frac{1}{s}\right)I_2+\left(s+\frac{1}{s}\right)I_3=\frac{2(-s^2+2s+2)}{s^2(s+1)} \qquad (4)$$

$$\left(2+s+\frac{1}{s}\right)I_2-\left(s+\frac{1}{s}\right)I_3=-\frac{4(s+1)}{s^2} \qquad (5)$$

We can now solve for I_o.

$$I_o = I_2 – I_3 = (4/s) – (1/(s+1)) + ((–1.5+0.5j)/(s+j)) + ((–1.5–0.5)/(s–j))$$

or

$$i_o(t) = \underline{\mathbf{[4 – e^{–t} + 1.5811e^{–jt+161.57^\circ} + 1.5811e^{jt–161.57^\circ}]u(t)A}}$$

This is a challenging problem. I did check it with using a Thevenin equivalent circuit and got the same exact answer.

Chapter 16, Problem 18.

(a) Find the Laplace transform of the voltage shown in Fig. 16.52(a). (b) Using that value of $v_s(t)$ in the circuit shown in Fig. 16.52(b), find the value of $v_0(t)$.

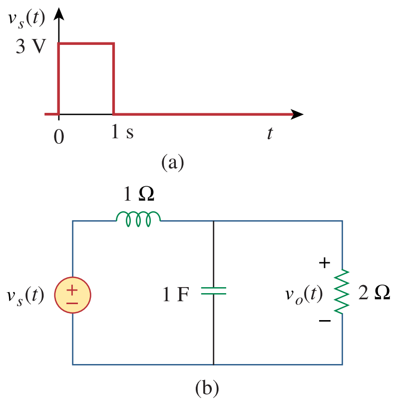
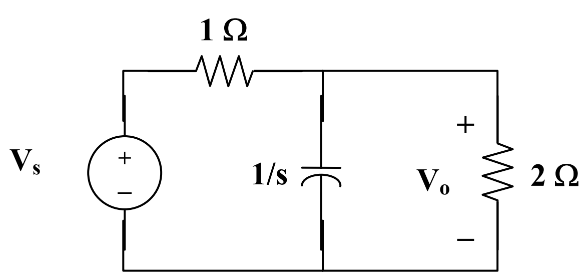


Figure 16.52
For Prob. 16.18.

Chapter 16, Solution 18.

$v_s(t) = 3u(t) – 3u(t–1)$ or $V_s = \frac{3}{s} - \frac{e^{-s}}{s} = \frac{3}{s}(1 - e^{-s})$



$$\frac{V_o - V_s}{1} + sV_o + \frac{V_o}{2} = 0 \rightarrow (s + 1.5)V_o = V_s$$

$$V_o = \frac{3}{s(s+1.5)}(1 - e^{-s}) = \left(\frac{2}{s} - \frac{2}{s+1.5}\right)(1 - e^{-s})$$

$$v_o(t) = \underline{[(2 - 2e^{-1.5t})u(t) - (2 - 2e^{-1.5(t-1)})u(t-1)]\,V}$$

Chapter 16, Problem 19.

In the circuit of Fig. 16.53, let $i(0) = 1$ A, $v_0(0)$ and $v_s = 4e^{-2t}u(t)$ V. Find $v_0(t)$ for $t > 0$.

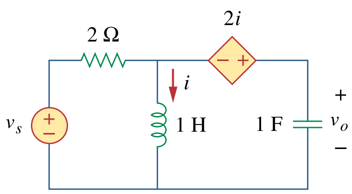
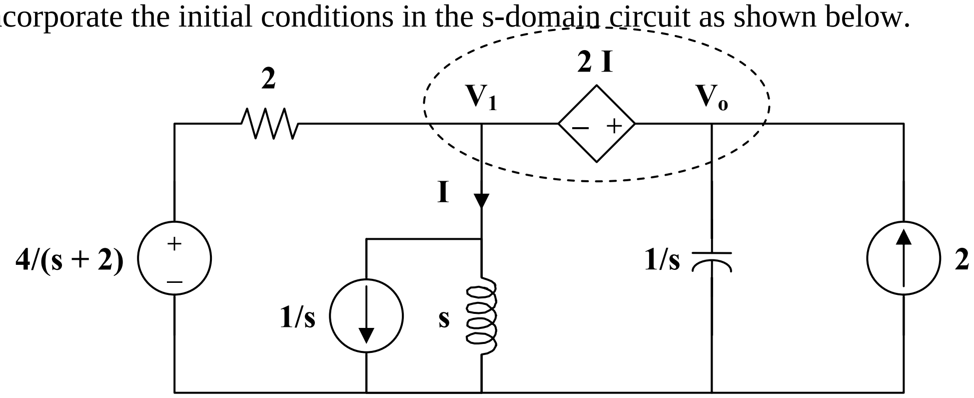


Figure 16.53
For Prob. 16.19.

Chapter 16, Solution 19.

We incorporate the initial conditions in the s-domain circuit as shown below.



At the supernode,

$$\frac{(4/(s+2))-V_1}{2}+2=\frac{V_1}{s}+\frac{1}{s}+sV_o$$

$$\frac{2}{s+2}+2=\left(\frac{1}{2}+\frac{1}{s}\right)V_1+\frac{1}{s}+s\,V_o \qquad (1)$$

But $\quad V_o = V_1 + 2I \quad$ and $\quad I=\dfrac{V_1+1}{s}$

$$V_o = V_1+\frac{2(V_1+1)}{s} \longrightarrow V_1=\frac{V_o-2/s}{(s+2)/s}=\frac{s\,V_o-2}{s+2} \qquad (2)$$

Substituting (2) into (1)

$$\frac{2}{s+2}+2-\frac{1}{s}=\left(\frac{s+2}{2s}\right)\left[\left(\frac{s}{s+2}\right)V_o-\frac{2}{s+2}\right]+s\,V_o$$

$$\frac{2}{s+2}+2-\frac{1}{s}+\frac{1}{s}=\left[\left(\frac{1}{2}\right)+s\right]V_o$$

$$\frac{2s+4+2}{(s+2)}=\frac{2s+6}{s+2}=(s+1/2)V_o$$

$$V_o=\frac{2s+6}{(s+2)(s+1/2)}=\frac{A}{s+1/2}+\frac{B}{s+2}$$

$$A=(-1+6)/(-0.5+2)=3.333,\quad B=(-4+6)/(-2+1/2)=-1.3333$$

$$V_o=\frac{3.333}{s+1/2}-\frac{1.3333}{s+2}$$

Therefore,

$v_o(t)=$ **$(3.333e^{-t/2}-1.3333e^{-2t})u(t)$ V**

Chapter 16, Problem 20.

Find $v_0(t)$ in the circuit of Fig. 16.54 if $v_x(0) = 2$ V and $i(0) = 1$A.

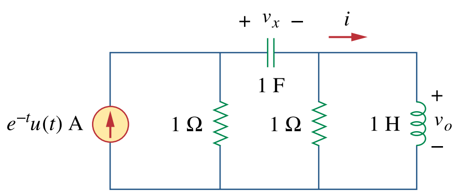
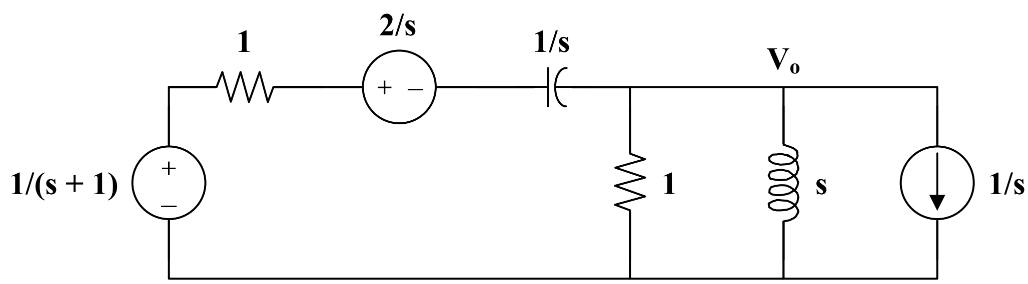


Figure 16.54
For Prob. 16.20.

Chapter 16, Solution 20.

We incorporate the initial conditions and transform the current source to a voltage source as shown.



At the main non-reference node, KCL gives

$$\frac{1/(s+1) - 2/s - V_o}{1 + 1/s} = \frac{V_o}{1} + \frac{V_o}{s} + \frac{1}{s}$$

$$\frac{s}{s+1} - 2 - s\,V_o = (s+1)(1+1/s)\,V_o + \frac{s+1}{s}$$

$$\frac{s}{s+1} - \frac{s+1}{s} - 2 = (2s + 2 + 1/s)\,V_o$$

$$V_o = \frac{-2s^2 - 4s - 1}{(s+1)(2s^2 + 2s + 1)}$$

$$V_o = \frac{-s - 2s - 0.5}{(s+1)(s^2 + s + 0.5)} = \frac{A}{s+1} + \frac{Bs + C}{s^2 + s + 0.5}$$

$$A = (s+1)\,V_o\Big|_{s=-1} = 1$$

$$-s^2 - 2s - 0.5 = A\,(s^2 + s + 0.5) + B\,(s^2 + s) + C\,(s+1)$$

Equating coefficients :

$$s^2: \qquad -1 = A + B \quad \longrightarrow \quad B = -2$$

$$s^1: \qquad -2 = A + B + C \quad \longrightarrow \quad C = -1$$

$$s^0: \qquad -0.5 = 0.5A + C = 0.5 - 1 = -0.5$$

$$V_o = \frac{1}{s+1} - \frac{2s+1}{s^2 + s + 0.5} = \frac{1}{s+1} - \frac{2\,(s+0.5)}{(s+0.5)^2 + (0.5)^2}$$

$$v_o(t) = \underline{\left[\mathbf{e^{-t} - 2\,e^{-t/2}\cos(t/2)}\right]\mathbf{u(t)\ V}}$$

Chapter 16, Problem 21.

Find the voltage $v_0(t)$ in the circuit of Fig. 16.55 by means of the Laplace transform.

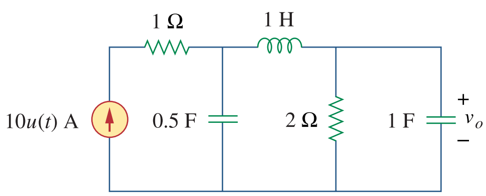
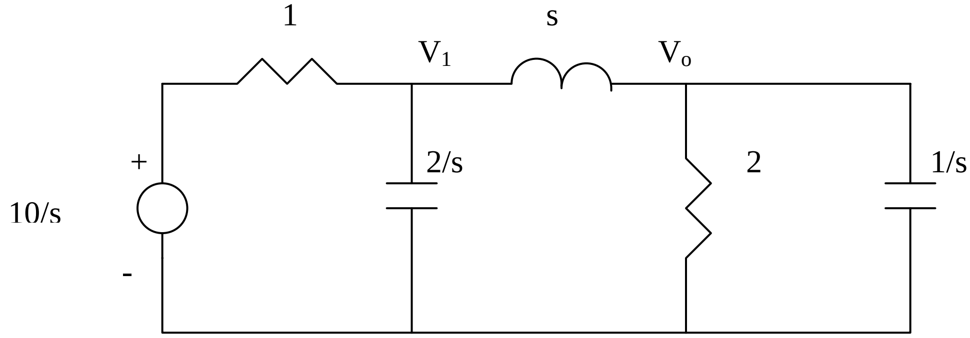


Figure 16.55
For Prob. 16.21.

Chapter 16, Solution 21.

The s-domain version of the circuit is shown below.



At node 1,

$$\frac{\frac{10}{s}-V_1}{1}=\frac{V_1-V_o}{s}+\frac{s}{2}V_o \longrightarrow 10=(s+1)V_1+(\frac{s^2}{2}-1)V_o \qquad (1)$$

At node 2,

$$\frac{V_1-V_o}{s}=\frac{V_o}{2}+sV_o \longrightarrow V_1=V_o(\frac{s}{2}+s^2+1) \qquad (2)$$

Substituting (2) into (1) gives

$$10=(s+1)(s^2+s/2+1)V_o+(\frac{s^2}{2}-1)V_o=s(s^2+2s+1.5)V_o$$

$$V_o=\frac{10}{s(s^2+2s+1.5)}=\frac{A}{s}+\frac{Bs+C}{s^2+2s+1.5}$$

$$10=A(s^2+2s+1.5)+Bs^2+Cs$$

$$s^2: \qquad 0=A+B$$

$$s: \qquad 0=2A+C$$

$$\text{constant}: \qquad 10=1.5A \longrightarrow A=20/3,\ \text{B = -20/3},\ \text{C = -40/3}$$

$$V_o=\frac{20}{3}\left[\frac{1}{s}-\frac{s+2}{s^2+2s+1.5}\right]=\frac{20}{3}\left[\frac{1}{s}-\frac{s+1}{(s+1)^2+0.7071^2}-1.414\frac{0.7071}{(s+1)^2+0.7071^2}\right]$$

Taking the inverse Laplace tranform finally yields

$$\underline{v_o(t)=\frac{20}{3}\left[1-e^{-t}\cos 0.7071t-1.414e^{-t}\sin 0.7071t\right]u(t)\,V}$$

Chapter 16, Problem 22.

Find the node voltages v_1 and v_2 in the circuit of Fig. 16.56 using the Laplace transform technique. Assume that $i_s = 12e^{-t}u(t)$ A and that all initial conditions are zero.

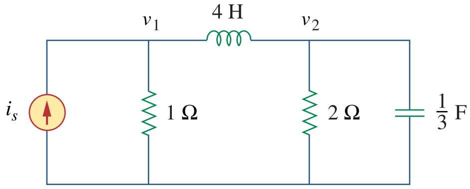
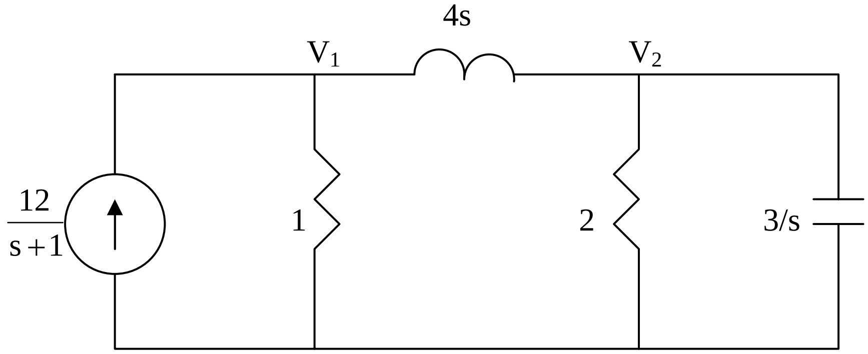


Figure 16.56
For Prob. 16.22.

Chapter 16, Solution 22.
The s-domain version of the circuit is shown below.



At node 1,

$$\frac{12}{s+1} = \frac{V_1}{1} + \frac{V_1 - V_2}{4s} \longrightarrow \frac{12}{s+1} = V_1\left(1+\frac{1}{4s}\right) - \frac{V_2}{4s} \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{4s} = \frac{V_2}{2} + \frac{s}{3}V_2 \longrightarrow V_1 = V_2\left(\frac{4}{3}s^2 + 2s + 1\right) \qquad (2)$$

Substituting (2) into (1),

$$\frac{12}{s+1} = V_2\left[\left(\frac{4}{3}s^2 + 2s + 1\right)\left(1+\frac{1}{4s}\right) - \frac{1}{4s}\right] = \left(\frac{4}{3}s^2 + \frac{7}{3}s + \frac{3}{2}\right)V_2$$

$$V_2 = \frac{9}{(s+1)(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{A}{(s+1)} + \frac{Bs + C}{(s^2 + \frac{7}{4}s + \frac{9}{8})}$$

$$9 = A(s^2 + \frac{7}{4}s + \frac{9}{8}) + B(s^2 + s) + C(s+1)$$

Equating coefficients:

$$s^2: \qquad 0 = A + B$$

$$s: \qquad 0 = \frac{7}{4}A + B + C = \frac{3}{4}A + C \quad \longrightarrow \quad C = -\frac{3}{4}A$$

$$\text{constant}: \qquad 9 = \frac{9}{8}A + C = \frac{3}{8}A \quad \longrightarrow \quad A = 24,\ B = \text{-}24,\ C = \text{-}18$$

$$V_2 = \frac{24}{(s+1)} - \frac{24s+18}{(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{24}{(s+1)} - \frac{24(s+7/8)}{(s+\frac{7}{8})^2 + \frac{23}{64}} + \frac{3}{(s+\frac{7}{8})^2 + \frac{23}{64}}$$

Taking the inverse of this produces:

$$\underline{v_2(t) = \left[24e^{-t} - 24e^{-0.875t}\cos(0.5995t) + 5.004e^{-0.875t}\sin(0.5995t)\right]u(t)}$$

Similarly,

$$V_1 = \frac{9\left(\frac{4}{3}s^2 + 2s + 1\right)}{(s+1)(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{D}{(s+1)} + \frac{Es+F}{(s^2 + \frac{7}{4}s + \frac{9}{8})}$$

$$9\left(\frac{4}{3}s^2 + 2s + 1\right) = D(s^2 + \frac{7}{4}s + \frac{9}{8}) + E(s^2 + s) + F(s+1)$$

Equating coefficients:

$$s^2: \qquad 12 = D + E$$

$$s: \qquad 18 = \frac{7}{4}D + E + F \text{ or } 6 = \frac{3}{4}D + F \quad \longrightarrow \quad F = 6 - \frac{3}{4}D$$

$$\text{constant}: \qquad 9 = \frac{9}{8}D + F \text{ or } 3 = \frac{3}{8}D \quad \longrightarrow \quad D = 8,\ E = 4,\ F = 0$$

$$V_1 = \frac{8}{(s+1)} + \frac{4s}{(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{8}{(s+1)} + \frac{4(s+7/8)}{(s+\frac{7}{8})^2 + \frac{23}{64}} - \frac{7/2}{(s+\frac{7}{8})^2 + \frac{23}{64}}$$

Thus,

$$\underline{v_1(t) = \left[8e^{-t} + 4e^{-0.875t}\cos(0.5995t) - 5.838e^{-0.875t}\sin(0.5995t)\right]u(t)}$$

Chapter 16, Problem 23.

Consider the parallel *RLC* circuit of Fig. 16.57. Find $v(t)$ and $i(t)$ given that $v(0) = 5$ and $i(0) = -2$ A.

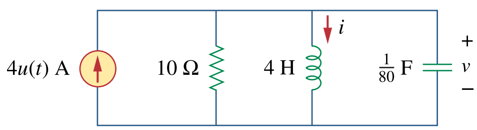
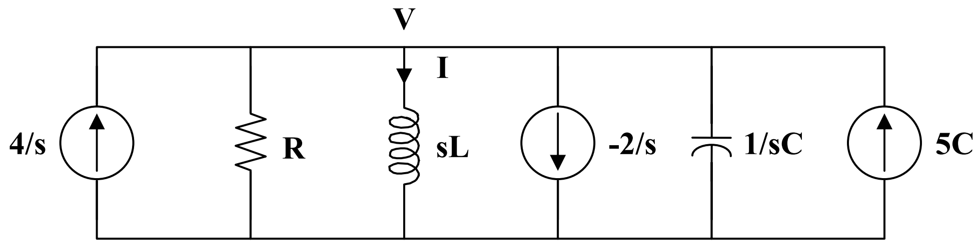


Figure 16.57
For Prob. 16.23.

Chapter 16, Solution 23.

The s-domain form of the circuit with the initial conditions is shown below.



At the non-reference node,

$$\frac{4}{s}+\frac{2}{s}+5C=\frac{V}{R}+\frac{V}{sL}+sCV$$

$$\frac{6+5sC}{s}=\frac{CV}{s}\left(s^2+\frac{s}{RC}+\frac{1}{LC}\right)$$

$$V=\frac{5s+6/C}{s^2+s/RC+1/LC}$$

But $\quad \dfrac{1}{RC}=\dfrac{1}{10/80}=8, \qquad \dfrac{1}{LC}=\dfrac{1}{4/80}=20$

$$V=\frac{5s+480}{s^2+8s+20}=\frac{5(s+4)}{(s+4)^2+2^2}+\frac{(230)(2)}{(s+4)^2+2^2}$$

$$v(t)=\underline{(5e^{-4t}\cos(2t)+230e^{-4t}\sin(2t))u(t)\ \mathrm{V}}$$

$$I=\frac{V}{sL}=\frac{5s+480}{4s(s^2+8s+20)}$$

$$I=\frac{1.25s+120}{s(s^2+8s+20)}=\frac{A}{s}+\frac{Bs+C}{s^2+8s+20}$$

$A = 6, \qquad B = -6, \qquad C = -46.75$

$$I=\frac{6}{s}-\frac{6s+46.75}{s^2+8s+20}=\frac{6}{s}-\frac{6(s+4)}{(s+4)^2+2^2}-\frac{(11.375)(2)}{(s+4)^2+2^2}$$

$$i(t)=\underline{(6-6e^{-4t}\cos(2t)-11.375e^{-4t}\sin(2t))u(t),\quad t>0}$$

Chapter 16, Problem 24.

The switch in Fig. 16.58 moves from position 1 to position 2 at $t = 0$. Find $v(t)$, for all $t > 0$.

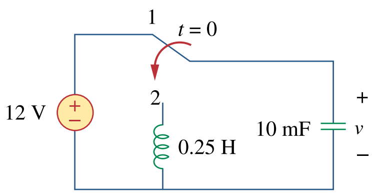
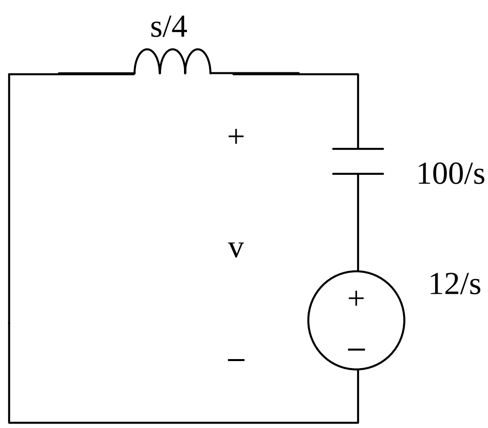


Figure 16.58
For Prob. 16.24.

Chapter 16, Solution 24.

When the switch is position 1, v(0)=12, and $i_L(0) = 0$. When the switch is in position 2, we have the circuit as shown below.



$$10mF = 0.01F \quad \longrightarrow \quad \frac{1}{sC} = \frac{100}{s}$$

$$I = \frac{12/s}{s/4 + 100/s} = \frac{48}{s^2 + 400}, \qquad V = sLI = \frac{s}{4}I = \frac{12s}{s^2+400}$$

$$v(t) = \underline{12\cos 20t, \quad t > 0}$$

Chapter 16, Problem 25.

For the *RLC* circuit shown in Fig. 16.59, find the complete response if $v(0) = 2$ V when the switch is closed.

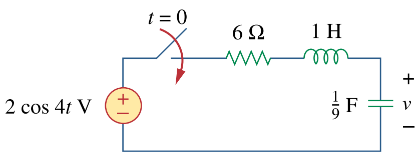
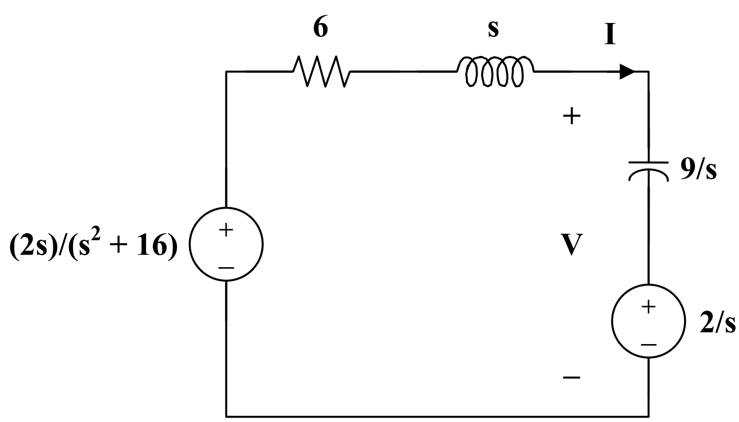


Figure 16.59
For Prob. 16.25.

Chapter 16, Solution 25.

For $t > 0$, the circuit in the s-domain is shown below.



Applying KVL,

$$\frac{-2s}{s^2+16}+\left(6+s+\frac{9}{s}\right)I+\frac{2}{s}=0$$

$$I=\frac{-32}{(s^2+6s+9)(s^2+16)}$$

$$V=\frac{9}{s}I+\frac{2}{s}=\frac{2}{s}+\frac{-288}{s(s+3)^2(s^2+16)}$$

$$=\frac{2}{s}+\frac{A}{s}+\frac{B}{s+3}+\frac{C}{(s+3)^2}+\frac{Ds+E}{s^2+16}$$

$$\begin{aligned}-288 = {} & A(s^4+6s^3+25s^2+96s+144)+B(s^4+3s^3+16s^2+48s)\\ & +C(s^3+16s)+D(s^4+6s^3+9s^2)+E(s^3+6s^2+9s)\end{aligned}$$

Equating coefficients :

$s^0:\quad -288=144A \qquad (1)$

$s^1:\quad 0=96A+48B+16C+9E \qquad (2)$

$s^2:\quad 0=25A+16B+9D+6E \qquad (3)$

$s^3:\quad 0=6A+3B+C+6D+E \qquad (4)$

$s^4:\quad 0=A+B+D \qquad (5)$

Solving equations (1), (2), (3), (4) and (5) gives

$A=-2,\quad B=2.202,\quad C=3.84,\quad D=-0.202,\quad E=2.766$

$$V(s)=\frac{2.202}{s+3}+\frac{3.84}{(s+3)^2}-\frac{0.202s}{s^2+16}+\frac{(0.6915)(4)}{s^2+16}$$

$v(t) =$ <u>**$\{2.202e^{-3t} + 3.84te^{-3t} - 0.202\cos(4t) + 0.6915\sin(4t)\}u(t)$ V**</u>

Chapter 16, Problem 26.

For the op amp circuit in Fig. 16.60, find $v_0(t)$ for $t > 0$. Take $v_s = 3e^{-5t}u(t)$ V.

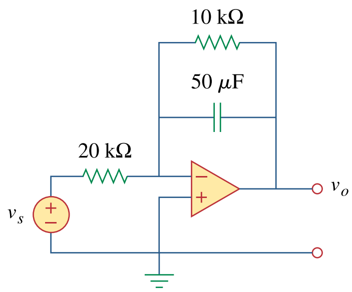
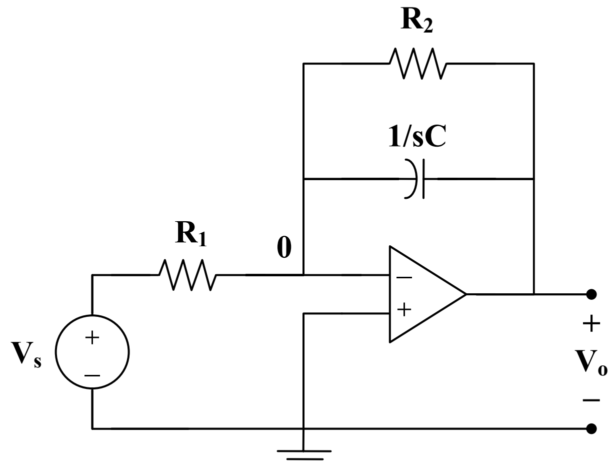


Figure 16.60
For Prob. 16.26.

Chapter 16, Solution 26.

Consider the op-amp circuit below.



At node 0,

$$\frac{V_s - 0}{R_1} = \frac{0 - V_o}{R_2} + (0 - V_o)sC$$

$$V_s = R_1\left(\frac{1}{R_2} + sC\right)(-V_o)$$

$$\frac{V_o}{V_s} = \frac{-1}{sR_1C + R_1/R_2}$$

But $\quad \frac{R_1}{R_2} = \frac{20}{10} = 2\,, \qquad R_1C = (20\times10^3)(50\times10^{-6}) = 1$

So, $\quad \frac{V_o}{V_s} = \frac{-1}{s+2}$

$$V_s = 3e^{-5t} \quad \longrightarrow \quad V_s = 3/(s+5)$$

$$V_o = \frac{-3}{(s+2)(s+5)}$$

$$-V_o = \frac{3}{(s+2)(s+5)} = \frac{A}{s+2} + \frac{B}{s+5}$$

$$A = 1, \qquad B = -1$$

$$V_o = \frac{1}{s+5} - \frac{1}{s+2}$$

$$v_o(t) = \underline{\left(\mathbf{e^{-5t} - e^{-2t}}\right)\mathbf{u(t)}}$$

Chapter 16, Problem 27.

Find $I_1(s)$ and $I_2(s)$ in the circuit of Fig. 16.61.

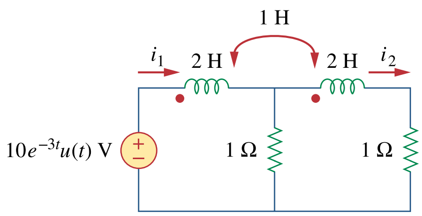
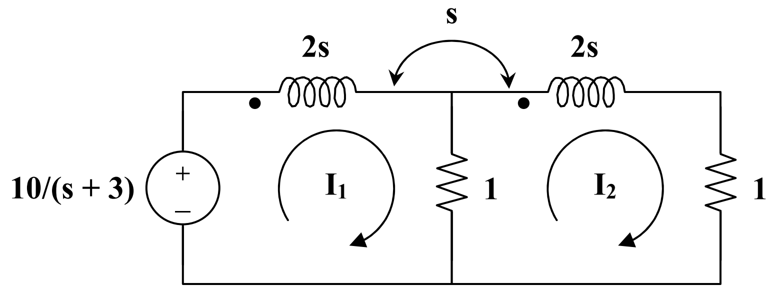


Figure 16.61
For Prob. 16.27.

Chapter 16, Solution 27.

Consider the following circuit.



For mesh 1,

$$\frac{10}{s+3} = (1+2s)\,I_1 - I_2 - s\,I_2$$

$$\frac{10}{s+3} = (1+2s)\,I_1 - (1+s)\,I_2 \qquad (1)$$

For mesh 2,

$$0 = (2+2s)\,I_2 - I_1 - s\,I_1$$

$$0 = -(1+s)\,I_1 + 2\,(s+1)\,I_2 \qquad (2)$$

(1) and (2) in matrix form,

$$\begin{bmatrix} 10/(s+3) \\ 0 \end{bmatrix} = \begin{bmatrix} 2s+1 & -(s+1) \\ -(s+1) & 2\,(s+1) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = 3s^2 + 4s + 1$$

$$\Delta_1 = \frac{20\,(s+1)}{s+3}$$

$$\Delta_2 = \frac{10\,(s+1)}{s+3}$$

Thus

$$I_1 = \frac{\Delta_1}{\Delta} = \underline{\mathbf{\frac{20\,(s+1)}{(s+3)(3s^2+4s+1)}}}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \underline{\mathbf{\frac{10\,(s+1)}{(s+3)(3s^2+4s+1)}}} = \frac{I_1}{2}$$

Chapter 16, Problem 28.

For the circuit in Fig. 16.62, find $v_0(t)$ for $t > 0$.

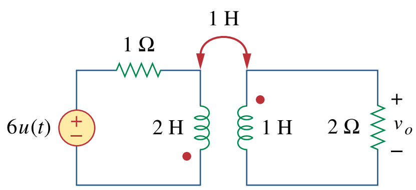
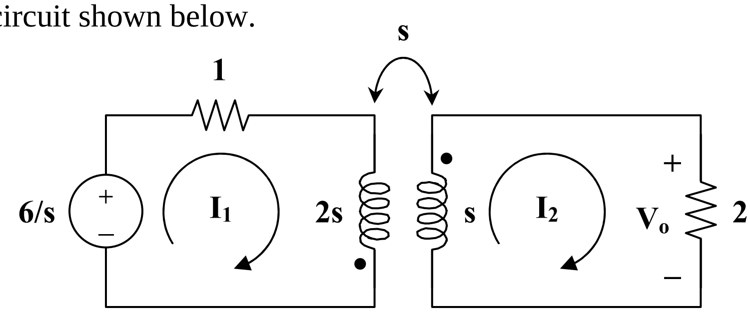


Figure 16.62
For Prob. 16.28.

Chapter 16, Solution 28.

Consider the circuit shown below.



For mesh 1,

$$\frac{6}{s} = (1+2s)\,I_1 + s\,I_2 \qquad (1)$$

For mesh 2,

$$0 = s\,I_1 + (2+s)\,I_2$$

$$I_1 = -\left(1+\frac{2}{s}\right)I_2 \qquad (2)$$

Substituting (2) into (1) gives

$$\frac{6}{s} = -(1+2s)\left(1+\frac{2}{s}\right)I_2 + s\,I_2 = \frac{-(s^2+5s+2)}{s}\,I_2$$

or $\qquad I_2 = \dfrac{-6}{s^2+5s+2}$

$$V_o = 2\,I_2 = \frac{-12}{s^2+5s+2} = \frac{-12}{(s+0.438)(s+4.561)}$$

Since the roots of $s^2+5s+2=0$ are -0.438 and -4.561,

$$V_o = \frac{A}{s+0.438} + \frac{B}{s+4.561}$$

$$A = \frac{-12}{4.123} = -2.91, \qquad B = \frac{-12}{-4.123} = 2.91$$

$$V_o(s) = \frac{-2.91}{s+0.438} + \frac{2.91}{s+4.561}$$

$$v_o(t) = \underline{\mathbf{2.91\left[e^{-4.561t} - e^{0.438t}\right]u(t)\ V}}$$

Chapter 16, Problem 29.

For the ideal transformer circuit in Fig. 16.63, determine $i_0(t)$.

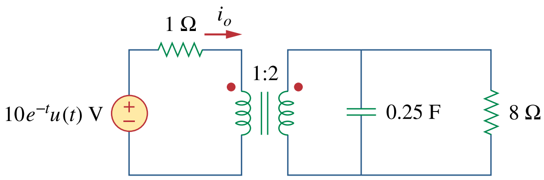
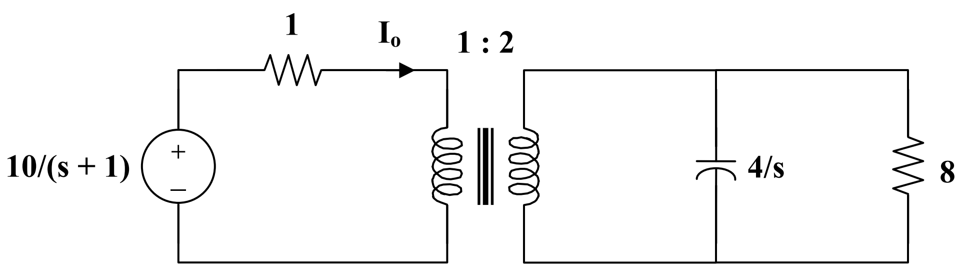


Figure 16.63
For Prob. 16.29.

Chapter 16, Solution 29.
Consider the following circuit.



Let $\quad Z_L = 8 \| \dfrac{4}{s} = \dfrac{(8)(4/s)}{8+4/s} = \dfrac{8}{2s+1}$

When this is reflected to the primary side,

$$Z_{in} = 1 + \frac{Z_L}{n^2}, \quad n = 2$$

$$Z_{in} = 1 + \frac{2}{2s+1} = \frac{2s+3}{2s+1}$$

$$I_o = \frac{10}{s+1}\cdot\frac{1}{Z_{in}} = \frac{10}{s+1}\cdot\frac{2s+1}{2s+3}$$

$$I_o = \frac{10s+5}{(s+1)(s+1.5)} = \frac{A}{s+1} + \frac{B}{s+1.5}$$

$$A = -10, \qquad B = 20$$

$$I_o(s) = \frac{-10}{s+1} + \frac{20}{s+1.5}$$

$$i_o(t) = \underline{\mathbf{10\left[2e^{-1.5t} - e^{-t}\right]u(t)\ A}}$$

Chapter 16, Problem 30.

The transfer function of a system is

$$H(s) = \frac{s^2}{3s+1}$$

Find the output when the system has an input of $4e^{-t/3}u(t)$.

Chapter 16, Solution 30.

$$Y(s) = H(s)\,X(s)\,, \qquad X(s) = \frac{4}{s+1/3} = \frac{12}{3s+1}$$

$$Y(s) = \frac{12\,s^2}{(3s+1)^2} = \frac{4}{3} - \frac{8s+4/3}{(3s+1)^2}$$

$$Y(s) = \frac{4}{3} - \frac{8}{9}\cdot\frac{s}{(s+1/3)^2} - \frac{4}{27}\cdot\frac{1}{(s+1/3)^2}$$

$$\text{Let }\ G(s) = \frac{-8}{9}\cdot\frac{s}{(s+1/3)^2}$$

Using the time differentiation property,

$$g(t) = \frac{-8}{9}\cdot\frac{d}{dt}(t\,e^{-t/3}) = \frac{-8}{9}\left(\frac{-1}{3}t\,e^{-t/3} + e^{-t/3}\right)$$

$$g(t) = \frac{8}{27}t\,e^{-t/3} - \frac{8}{9}e^{-t/3}$$

Hence,

$$y(t) = \frac{4}{3}u(t) + \frac{8}{27}t\,e^{-t/3} - \frac{8}{9}e^{-t/3} - \frac{4}{27}t\,e^{-t/3}$$

$$y(t) = \underline{\boldsymbol{\frac{4}{3}u(t) - \frac{8}{9}e^{-t/3} + \frac{4}{27}t\,e^{-t/3}}}$$

Chapter 16, Problem 31.

When the input to a system is a unit step function, the response is 10 cos 2*tu*(*t*). Obtain the transfer function of the system.

Chapter 16, Solution 31.

$$x(t) = u(t) \longrightarrow X(s) = \frac{1}{s}$$

$$y(t) = 10\cos(2t) \longrightarrow Y(s) = \frac{10s}{s^2 + 4}$$

$$H(s) = \frac{Y(s)}{X(s)} = \underline{\mathbf{\frac{10s^2}{s^2 + 4}}}$$

Chapter 16, Problem 32.

A circuit is known to have its transfer function as

$H(s) = \dfrac{s+3}{s^2+4s+5}$

Find its output when:

(a) the input is a unit step function

(b) the input is $6te^{-2t}\,u(t)$.

Chapter 16, Solution 32.

(a) $Y(s) = H(s)\,X(s)$

$$= \frac{s+3}{s^2+4s+5}\cdot\frac{1}{s}$$
$$= \frac{s+3}{s\,(s^2+4s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+5}$$

$$s+3 = A\,(s^2+4s+5) + Bs^2 + Cs$$

Equating coefficients :

$s^0:\quad 3 = 5A \longrightarrow A = 3/5$

$s^1:\quad 1 = 4A + C \longrightarrow C = 1 - 4A = \text{-}\,7/5$

$s^2:\quad 0 = A + B \longrightarrow B = \text{-}A = \text{-}\,3/5$

$$Y(s) = \frac{3/5}{s} - \frac{1}{5}\cdot\frac{3s+7}{s^2+4s+5}$$
$$Y(s) = \frac{0.6}{s} - \frac{1}{5}\cdot\frac{3(s+2)+1}{(s+2)^2+1}$$

$$y(t) = \underline{\left[\mathbf{0.6 - 0.6\,e^{-2t}\cos(t) - 0.2\,e^{-2t}\sin(t)}\right]\mathbf{u(t)}}$$

(b) $x(t) = 6\,t\,e^{-2t} \longrightarrow X(s) = \dfrac{6}{(s+2)^2}$

$$Y(s) = H(s)\,X(s) = \frac{s+3}{s^2+4s+5} \cdot \frac{6}{(s+2)^2}$$

$$Y(s) = \frac{6(s+3)}{(s+2)^2(s^2+4s+5)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{Cs+D}{s^2+4s+5}$$

Equating coefficients :

$s^3:\quad 0 = A + C \longrightarrow C = -A \qquad (1)$

$s^2:\quad 0 = 6A + B + 4C + D = 2A + B + D \qquad (2)$

$s^1:\quad 6 = 13A + 4B + 4C + 4D = 9A + 4B + 4D \qquad (3)$

$s^0:\quad 18 = 10A + 5B + 4D = 2A + B \qquad (4)$

Solving (1), (2), (3), and (4) gives

$A = 6, \qquad B = 6, \qquad C = -6, \qquad D = -18$

$$Y(s) = \frac{6}{s+2} + \frac{6}{(s+2)^2} - \frac{6s+18}{(s+2)^2+1}$$

$$Y(s) = \frac{6}{s+2} + \frac{6}{(s+2)^2} - \frac{6(s+2)}{(s+2)^2+1} - \frac{6}{(s+2)^2+1}$$

$$y(t) = \underline{\left[\mathbf{6\,e^{-2t} + 6\,t\,e^{-2t} - 6\,e^{-2t}\cos(t) - 6\,e^{-2t}\sin(t)}\right]\mathbf{u(t)}}$$

Chapter 16, Problem 33.

When a unit step is applied to a system at t = 0 its response is

$$y(t) = \left[4+\frac{1}{2}e^{-3t} - e^{-2t}(2\cos 4t + 3\sin 4t)\right]u(t)$$

What is the transfer function of the system?

Chapter 16, Solution 33.

$$\mathrm{H(s)} = \frac{\mathrm{Y(s)}}{\mathrm{X(s)}}, \qquad \mathrm{X(s)} = \frac{1}{\mathrm{s}}$$

$$\mathrm{Y(s)} = \frac{4}{\mathrm{s}} + \frac{1}{2(\mathrm{s}+3)} - \frac{2\mathrm{s}}{(\mathrm{s}+2)^2+16} - \frac{(3)(4)}{(\mathrm{s}+2)^2+16}$$

$$\mathrm{H(s)} = \mathrm{s\,Y(s)} = \underline{4 + \frac{\mathrm{s}}{2(\mathrm{s}+3)} - \frac{2\mathrm{s(s+2)}}{\mathrm{s}^2+4\mathrm{s}+20} - \frac{12\mathrm{s}}{\mathrm{s}^2+4\mathrm{s}+20}}$$

Chapter 16, Problem 34.

For the circuit in Fig. 16.64, find $H(s) = V_0(s)/V_s(s)$. Assume zero initial conditions.

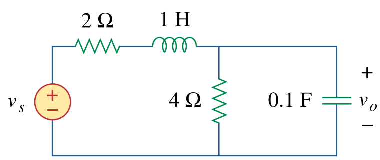
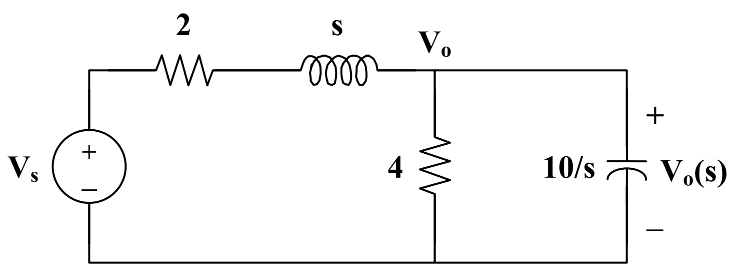


Figure 16.64
For Prob. 16.34.

Chapter 16, Solution 34.

Consider the following circuit.



Using nodal analysis,

$$\frac{V_s - V_o}{s+2} = \frac{V_o}{4} + \frac{V_o}{10/s}$$

$$V_s = (s+2)\left(\frac{1}{s+2} + \frac{1}{4} + \frac{s}{10}\right)V_o = \left(1 + \frac{1}{4}(s+2) + \frac{1}{10}(s^2 + 2s)\right)V_o$$

$$V_s = \frac{1}{20}\left(2s^2 + 9s + 30\right)V_o$$

$$\frac{V_o}{V_s} = \underline{\mathbf{\frac{20}{2s^2 + 9s + 30}}}$$

Chapter 16, Problem 35.

Obtain the transfer function $H(s) = V_0/V_s$ for the circuit of Fig. 16.65.

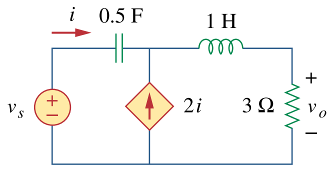
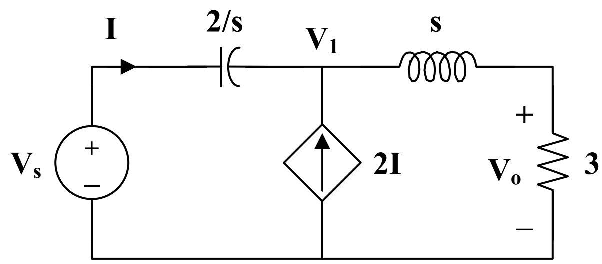


Figure 16.65
For Prob. 16.35.

Chapter 16, Solution 35.
Consider the following circuit.



At node 1,

$$2I + I = \frac{V_1}{s+3}, \qquad \text{where} \quad I = \frac{V_s - V_1}{2/s}$$

$$3 \cdot \frac{V_s - V_1}{2/s} = \frac{V_1}{s+3}$$

$$\frac{V_1}{s+3} = \frac{3s}{2} V_s - \frac{3s}{2} V_1$$

$$\left(\frac{1}{s+3} + \frac{3s}{2} \right) V_1 = \frac{3s}{2} V_s$$

$$V_1 = \frac{3s\,(s+3)}{3s^2 + 9s + 2} V_s$$

$$V_o = \frac{3}{s+3} V_1 = \frac{9s}{3s^2 + 9s + 2} V_s$$

$$H(s) = \frac{V_o}{V_s} = \underline{\frac{\mathbf{9s}}{\mathbf{3s^2 + 9s + 2}}}$$

Chapter 16, Problem 36.

The transfer function of a certain circuit is

$$H(s) = \frac{5}{s+1} - \frac{3}{s+2} + \frac{6}{s+4}$$

Find the impulse response of the circuit.

Chapter 16, Solution 36.

Taking the inverse Laplace transform of each term gives

$$h(t) = \underline{\left(5e^{-t} - 3e^{-2t} + 6e^{-4t}\right)u(t)}$$

Chapter 16, Problem 37.

For the circuit in Fig. 16.66, find:

(a) I_1/V_s (b) I_2/V_x

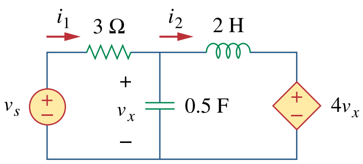
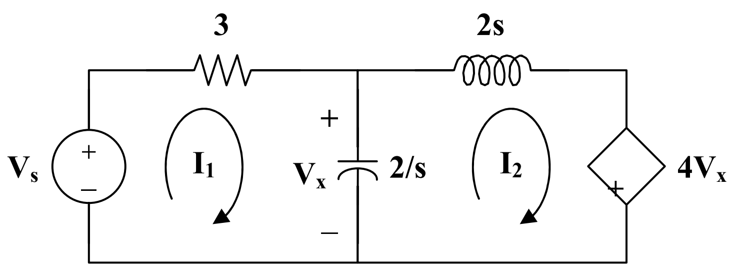


Figure 16.66
For Prob. 16.37.

Chapter 16, Solution 37.

(a) Consider the circuit shown below.



For loop 1,

$$V_s = \left(3 + \frac{2}{s}\right)I_1 - \frac{2}{s}I_2 \qquad (1)$$

For loop 2,

$$4V_x + \left(2s + \frac{2}{s}\right)I_2 - \frac{2}{s}I_1 = 0$$

But, $V_x = (I_1 - I_2)\left(\frac{2}{s}\right)$

So, $\frac{8}{s}(I_1 - I_2) + \left(2s + \frac{2}{s}\right)I_2 - \frac{2}{s}I_1 = 0$

$$0 = \frac{-6}{s}I_1 + \left(\frac{6}{s} - 2s\right)I_2 \qquad (2)$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} V_s \\ 0 \end{bmatrix} = \begin{bmatrix} 3+2/s & -2/s \\ -6/s & 6/s-2s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \left(3+\frac{2}{s}\right)\left(\frac{6}{s}-2s\right)-\left(\frac{6}{s}\right)\left(\frac{2}{s}\right)$$

$$\Delta = \frac{18}{s}-6s-4$$

$$\Delta_1 = \left(\frac{6}{s}-2s\right)V_s, \qquad \Delta_2 = \frac{6}{s}V_s$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{(6/s-2s)}{18/s-4-6s}V_s$$

$$\frac{I_1}{V_s} = \frac{3/s-s}{9/s-2-3} = \underline{\mathbf{\frac{s^2-3}{3s^2+2s-9}}}$$

(b) $$I_2 = \frac{\Delta_2}{\Delta}$$

$$V_x = \frac{2}{s}(I_1-I_2) = \frac{2}{s}\left(\frac{\Delta_1-\Delta_2}{\Delta}\right)$$

$$V_x = \frac{2/s\,V_s\,(6/s-2s-6/s)}{\Delta} = \frac{-4V_s}{\Delta}$$

$$\frac{I_2}{V_x} = \frac{6/s\,V_s}{-4V_s} = \underline{\mathbf{\frac{-3}{2s}}}$$

Chapter 16, Problem 38.

Refer to the network in Fig. 16.67. Find the following transfer functions:

(a) $H_1(s) = V_0(s)/V_s(s)$

(b) $H_2(s) = V_0(s)/I_s(s)$

(c) $H_3(s) = I_0(s)/I_s(s)$

(d) $H_4(s) = I_0(s)/V_s(s)$

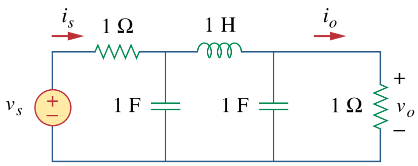
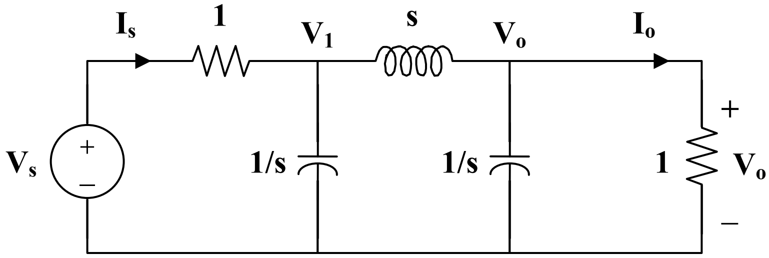


Figure 16.67
For Prob. 16.38.

Chapter 16, Solution 38.

(a) Consider the following circuit.



At node 1,

$$\frac{V_s - V_1}{1} = s\,V_1 + \frac{V_1 - V_o}{s}$$

$$V_s = \left(1 + s + \frac{1}{s}\right)V_1 - \frac{1}{s}V_o \qquad (1)$$

At node o,

$$\frac{V_1 - V_o}{s} = s\,V_o + V_o = (s+1)\,V_o$$

$$V_1 = (s^2 + s + 1)\,V_o \qquad (2)$$

Substituting (2) into (1)

$$V_s = (s + 1 + 1/s)(s^2 + s + 1)V_o - 1/s\,V_o$$

$$V_s = (s^3 + 2s^2 + 3s + 2)V_o$$

$$H_1(s) = \frac{V_o}{V_s} = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + 2s^2 + 3s + 2}}}$$

(b)
$$I_s = V_s - V_1 = (s^3 + 2s^2 + 3s + 2)V_o - (s^2 + s + 1)V_o$$

$$I_s = (s^3 + s^2 + 2s + 1)V_o$$

$$H_2(s) = \frac{V_o}{I_s} = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + s^2 + 2s + 1}}}$$

(c)
$$I_o = \frac{V_o}{1}$$

$$H_3(s) = \frac{I_o}{I_s} = \frac{V_o}{I_s} = H_2(s) = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + s^2 + 2s + 1}}}$$

(d)
$$H_4(s) = \frac{I_o}{V_s} = \frac{V_o}{V_s} = H_1(s) = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + 2s^2 + 3s + 2}}}$$

Chapter 16, Problem 39.

Calculate the gain $H(s) = V_0/V_s$ in the op amp circuit of Fig. 16.68.

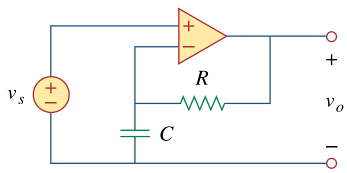
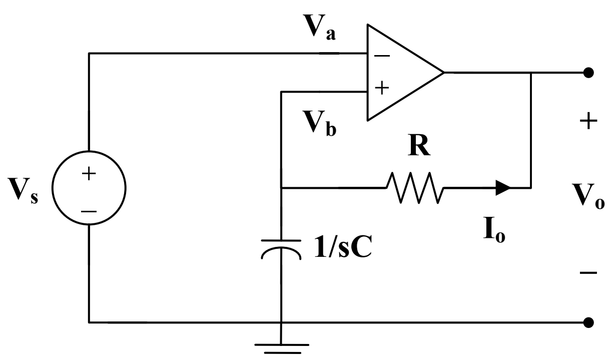


Figure 16.68
For Prob. 16.39.

Chapter 16, Solution 39.

Consider the circuit below.



Since no current enters the op amp, I_o flows through both R and C.

$$V_o = -I_o\left(R + \frac{1}{sC}\right)$$

$$V_a = V_b = V_s = \frac{-I_o}{sC}$$

$$H(s) = \frac{V_o}{V_s} = \frac{R + 1/sC}{1/sC} = \underline{\mathbf{sRC + 1}}$$

Chapter 16, Problem 40.

Refer to the *RL* circuit in Fig. 16.69. Find:

(a) the impulse response $h(t)$ of the circuit.
(b) the unit step response of the circuit.

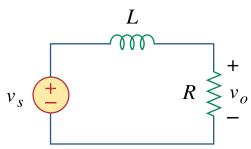


Figure 16.69
For Prob. 16.40.

Chapter 16, Solution 40.

(a) $$H(s) = \frac{V_o}{V_s} = \frac{R}{R + sL} = \frac{R/L}{s + R/L}$$

$$h(t) = \underline{\mathbf{\frac{R}{L} e^{-Rt/L}\ u(t)}}$$

(b) $$v_s(t) = u(t) \longrightarrow V_s(s) = 1/s$$

$$V_o = \frac{R/L}{s + R/L} V_s = \frac{R/L}{s(s + R/L)} = \frac{A}{s} + \frac{B}{s + R/L}$$

$$A = 1, \qquad B = -1$$

$$V_o = \frac{1}{s} - \frac{1}{s + R/L}$$

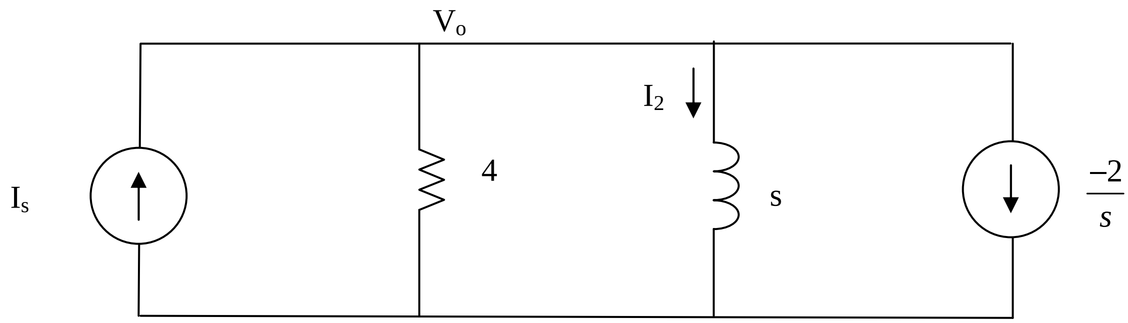
$$v_o(t) = u(t) - e^{-Rt/L}\ u(t) = \underline{\mathbf{(1 - e^{-Rt/L})\,u(t)}}$$

Chapter 16, Problem 41.

A parallel *RL* circuit has $R = 4\,\Omega$ and $L = 1$ H. The input to the circuit is $i_s(t) = 2e^{-t} u(t)$A. Find the inductor current $i_L(t)$ for all $t > 0$ and assume that $i_L(0) = -2$ A.

Chapter 16, Solution 41.

Consider the circuit as shown below.



$$I_s = \frac{V_o}{4} + \frac{V_o}{s} - \frac{2}{s}$$

But $\quad I_S = \dfrac{2}{s+1}$

$$\frac{2}{s+1} = V_o\left(\frac{1}{4} + \frac{1}{s}\right) - \frac{2}{s} \qquad \longrightarrow \qquad V_o\left(\frac{s+4}{4s}\right) = \frac{2}{s+1} + \frac{2}{s} = \frac{4s+2}{s(s+1)}$$

$$V_o = \frac{8(2s+1)}{(s+1)(s+4)}$$

$$I_L = \frac{V_o}{s} = \frac{8(2s+1)}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}$$

$$A = \frac{8(1)}{(1)(4)} = 2, \qquad B = \frac{8(-2+1)}{(-1)(2)} = 8/3, \qquad C = \frac{8(-8+1)}{(-4)(-3)} = -14/3$$

$$I_L = \frac{V_o}{s} = \frac{2}{s} + \frac{8/3}{s+1} + \frac{-14/3}{s+4}$$

$$i_L(t) = \underline{\left(2 + \frac{8}{3}e^{-t} - \frac{14}{3}e^{-4t}\right)u(t)} \;=\; \underline{\left(2 + \frac{8}{3}e^{-t} - \frac{14}{3}e^{-4t}\right)u(t)\text{A}}$$

Chapter 16, Problem 42.

A circuit has a transfer function

H(s) = $\dfrac{s+4}{(s+1)(s+2)^2}$

Find the impulse response.

Chapter 16, Solution 42.

$$H(s)=\frac{s+4}{(s+1)(s+2)^2}=\frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{(s+2)^2}$$
$$s+4=A(s+2)^2+B(s+1)(s+2)+C(s+1)=A(s^2+2s+4)+B(s^2+3s+2)+C(s+1)$$

We equate coefficients.

s^2 :	0=A+B or B=-A
s:	1=4A+3B+C=B+C
constant:	4=4A+2B+C =2A+C

Solving these gives A=3, B=-3, C=-2

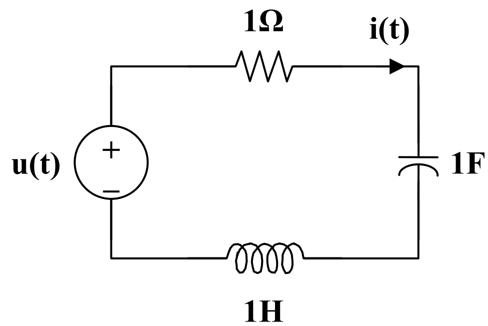
$$H(s)=\frac{3}{s+1}-\frac{3}{s+2}-\frac{2}{(s+2)^2}$$

$$h(t)=\underline{(3e^{-t}-3e^{-2t}-2te^{-2t})u(t)}$$

Chapter 16, Problem 43.

Develop the state equations for Prob. 16.1.

Chapter 16, Solution 43.



First select the inductor current i_L and the capacitor voltage v_C to be the state variables.

Applying KVL we get:

$$-u(t)+i+v_C+i'=0;\ \ i=v_C'$$

Thus,

$$v_C' = i$$
$$i' = -v_C - i + u(t)$$

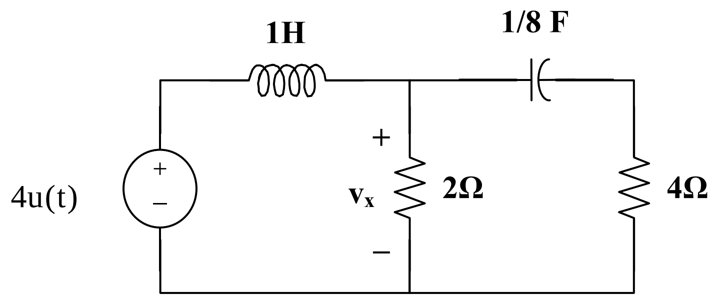
Finally we get,

$$\begin{bmatrix} v_C' \\ i' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} v_C \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t);\ i(t) = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} v_C \\ i \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}u(t)$$

Chapter 16, Problem 44.

Develop the state equations for Prob. 16.2.

Chapter 16, Solution 44.



First select the inductor current i_L and the capacitor voltage v_C to be the state variables.

Applying KCL we get:

$$-i_L + \frac{v_x}{2} + \frac{v_C'}{8} = 0; \quad \text{or} \quad v_C' = 8i_L - 4v_x$$

$$i_L' = 4u(t) - v_x$$

$$v_x = v_C + 4\frac{v_C'}{8} = v_C + \frac{v_C'}{2} = v_C + 4i_L - 2v_x; \text{ or } v_x = 0.3333v_C + 1.3333i_L$$

$$v_C' = 8i_L - 1.3333v_C - 5.333i_L = -1.3333v_C + 2.666i_L$$

$$i_L' = 4u(t) - 0.3333v_C - 1.3333i_L$$

Now we can write the state equations.

$$\begin{bmatrix} v_C' \\ i_L' \end{bmatrix} = \underline{\begin{bmatrix} -1.3333 & 2.666 \\ -0.3333 & -1.3333 \end{bmatrix}\begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} u(t)}; \; v_x = \underline{\begin{bmatrix} 0.3333 \\ 1.3333 \end{bmatrix}\begin{bmatrix} v_C \\ i_L \end{bmatrix}}$$

Chapter 16, Problem 45.

Develop the state equations for the circuit shown in Fig. 16.70.

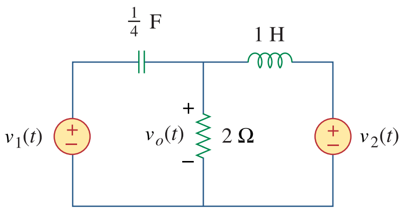
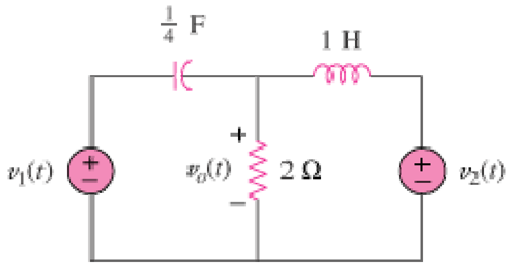


Figure 16.70
For Prob. 16.45.

Chapter 16, Solution 45.



First select the inductor current i_L (current flowing left to right) and the capacitor voltage v_C (voltage positive on the left and negative on the right) to be the state variables.

Applying KCL we get:

$$-\frac{v_C'}{4}+\frac{v_o}{2}+i_L=0 \text{ or } v_C'=4i_L+2v_o$$

$$i_L'=v_o-v_2$$

$$v_o=-v_C+v_1$$

$$v_C'=4i_L-2v_C+2v_1$$

$$i_L'=-v_C+v_1-v_2$$

$$\begin{bmatrix} i_L' \\ v_C' \end{bmatrix}=\underline{\begin{bmatrix} 0 & -1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} i_L \\ v_C \end{bmatrix}+\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}};\ v_o(t)=\underline{\begin{bmatrix} 0 & -1 \end{bmatrix}\begin{bmatrix} i_L \\ v_C \end{bmatrix}+\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}}$$

Chapter 16, Problem 46.

Develop the state equations for the circuit shown in Fig. 16.71.

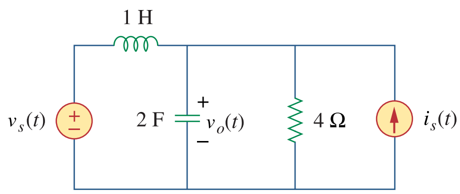
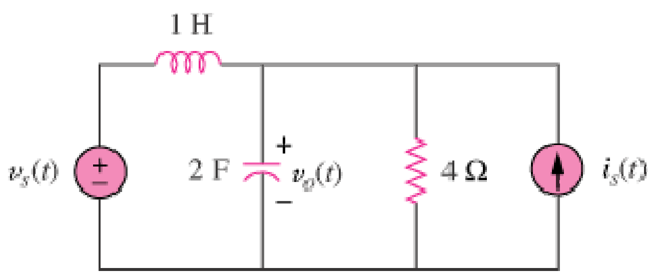


Figure 16.71
For Prob. 16.46.

Chapter 16, Solution 46.



First select the inductor current i_L (left to right) and the capacitor voltage v_C to be the state variables.

Letting $v_o = v_C$ and applying KCL we get:

$$-i_L + v_C' + \frac{v_C}{4} - i_s = 0 \text{ or } v_C' = -0.25v_C + i_L + i_s$$

$$i_L' = -v_C + v_s$$

Thus,

$$\begin{bmatrix} v_C' \\ i_L' \end{bmatrix} = \underline{\begin{bmatrix} -0.25 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} v_C' \\ i_L' \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} v_s \\ i_s \end{bmatrix}}; \; v_o(t) = \underline{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_s \\ i_s \end{bmatrix}}$$

Chapter 16, Problem 47.

Develop the state equations for the circuit shown in Fig. 16.72.

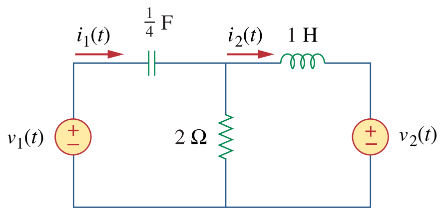
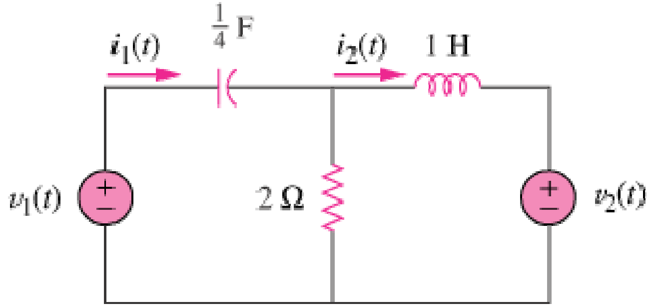


Figure 16.72
For Prob. 16.47.

Chapter 16, Solution 47.



First select the inductor current i_L (left to right) and the capacitor voltage v_C (+ on the left) to be the state variables.

Letting $i_1 = \frac{v_C'}{4}$ and $i_2 = i_L$ and applying KVL we get:

Loop 1:

$$-v_1 + v_C + 2\left(\frac{v_C'}{4} - i_L\right) = 0 \text{ or } v_C' = 4i_L - 2v_C + 2v_1$$

Loop 2:

$$2\left(i_L - \frac{v_C'}{4}\right) + i_L' + v_2 = 0 \text{ or}$$

$$i_L' = -2i_L + \frac{4i_L - 2v_C + 2v_1}{2} - v_2 = -v_C + v_1 - v_2$$

$$i_1 = \frac{4i_L - 2v_C + 2v_1}{4} = i_L - 0.5v_C + 0.5v_1$$

$$\begin{bmatrix} i_L' \\ v_C' \end{bmatrix} = \underline{\begin{bmatrix} 0 & -1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}}; \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} = \underline{\begin{bmatrix} 1 & -0.5 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}}$$

Chapter 16, Problem 48.

Develop the state equations for the following differential equation.

$$\frac{d^2y(t)}{dt^2} + \frac{4dy(t)}{dt} + 3y(t) = z(t)$$

Chapter 16, Solution 48.

Let $x_1 = y(t)$. Thus, $x_1' = y' = x_2$ and $x_2' = y'' = -3x_1 - 4x_2 + z(t)$

This gives our state equations.

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \underline{\begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} z(t)}; \quad \underline{y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} z(t)}$$

Chapter 16, Problem 49.

* Develop the state equations for the following differential equation.

$$\frac{d^2y(t)}{dt^2} + \frac{5dy(t)}{dt} + 6y(t) = \frac{dz(t)}{dt}\ z(t)$$

* An asterisk indicates a challenging problem.

Chapter 16, Solution 49.

Let $x_1 = y(t)$ and $x_2 = x_1' - z = y' - z$ or $y' = x_2 + z$

Thus,

$$x_2' = y'' - z' = -6x_1 - 5(x_2 + z) + z' + 2z - z' = -6x_1 - 5x_2 - 3z$$

This now leads to our state equations,

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \underline{\begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix} z(t)}; \quad \underline{y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} z(t)}$$

Chapter 16, Problem 50.

* Develop the state equations for the following differential equation.

$$\frac{d^3y(t)}{dt^3} + \frac{6d^2y(t)}{dt^2} + \frac{11dy(t)}{dt} + 6y(t) = z(t)$$

* An asterisk indicates a challenging problem.

Chapter 16, Solution 50.

Let $x_1 = y(t)$, $x_2 = x_1'$, and $x_3 = x_2'$.

Thus,

$$x_3'' = -6x_1 - 11x_2 - 6x_3 + z(t)$$

We can now write our state equations.

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \underline{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} z(t);}\ \ y(t) = \underline{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} z(t)}$$

Chapter 16, Problem 51.

* Given the following state equation, solve for $y(t)$:

$$\dot{\mathbf{x}} = \begin{bmatrix} -4 & 4 \\ -2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t)$$

$$\mathbf{y}(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

* An asterisk indicates a challenging problem.

Chapter 16, Solution 51.

We transform the state equations into the s-domain and solve using Laplace transforms.

$$sX(s) - x(0) = AX(s) + B\left(\frac{1}{s}\right)$$

Assume the initial conditions are zero.

$$(sI - A)X(s) = B\left(\frac{1}{s}\right)$$

$$X(s) = \begin{bmatrix} s+4 & -4 \\ 2 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 2 \end{bmatrix}\left(\frac{1}{s}\right) = \frac{1}{s^2 + 4s + 8}\begin{bmatrix} s & 4 \\ 2 & s+4 \end{bmatrix}\begin{bmatrix} 0 \\ (2/s) \end{bmatrix}$$

$$Y(s) = X_1(s) = \frac{8}{s(s^2 + 4s + 8)} = \frac{1}{s} + \frac{-s-4}{s^2 + 4s + 8}$$

$$= \frac{1}{s} + \frac{-s-4}{(s+2)^2 + 2^2} = \frac{1}{s} + \frac{-(s+2)}{(s+2)^2 + 2^2} + \frac{-2}{(s+2)^2 + 2^2}$$

$$\mathbf{y(t)} = \underline{\left(1 - e^{-2t}(\cos 2t + \sin 2t)\right)u(t)}$$

Chapter 16, Problem 52.

* Given the following state equation, solve for $y_1(t)$.

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & -1 \\ 2 & -4 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} u(t) \\ 2u(t) \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} -2 & -2 \\ 1 & -0 \end{bmatrix} x + \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u(t) \\ 2u(t) \end{bmatrix}$$

* An asterisk indicates a challenging problem.

Chapter 16, Solution 52.

Assume that the initial conditions are zero. Using Laplace transforms we get,

$$X(s) = \begin{bmatrix} s+2 & 1 \\ -2 & s+4 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1/s \\ 2/s \end{bmatrix} = \frac{1}{s^2+6s+10} \begin{bmatrix} s+4 & -1 \\ 2 & s+2 \end{bmatrix} \begin{bmatrix} 3/s \\ 4/s \end{bmatrix}$$

$$X_1 = \frac{3s+8}{s((s+3)^2+1^2)} = \frac{0.8}{s} + \frac{-0.8s-1.8}{(s+3)^2+1^2}$$

$$= \frac{0.8}{s} - 0.8\frac{s+3}{(s+3)^2+1^2} + .6\frac{1}{(s+3)^2+1^2}$$

$$x_1(t) = (0.8 - 0.8e^{-3t}\cos t + 0.6e^{-3t}\sin t)u(t)$$

$$X_2 = \frac{4s+14}{s((s+3)^2+1^2} = \frac{1.4}{s} + \frac{-1.4s-4.4}{(s+3)^2+1^2}$$

$$= \frac{1.4}{s} - 1.4\frac{s+3}{(s+3)^2+1^2} - 0.2\frac{1}{(s+3)^2+1^2}$$

$$x_2(t) = (1.4 - 1.4e^{-3t}\cos t - 0.2e^{-3t}\sin t)u(t)$$

$$y_1(t) = -2x_1(t) - 2x_2(t) + 2u(t)$$
$$= \underline{(-2.4 + 4.4e^{-3t}\cos t - 0.8e^{-3t}\sin t)u(t)}$$

$$y_2(t) = x_1(t) - 2u(t) = \underline{(-1.2 - 0.8e^{-3t}\cos t + 0.6e^{-3t}\sin t)u(t)}$$

Chapter 16, Problem 53.

Show that the parallel *RLC* circuit shown in Fig. 16.73 is stable.

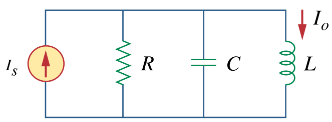


Figure 16.73
For Prob. 16.53.

Chapter 16, Solution 53.

If V_o is the voltage across R, applying KCL at the non-reference node gives

$$I_s = \frac{V_o}{R} + sCV_o + \frac{V_o}{sL} = \left(\frac{1}{R} + sC + \frac{1}{sL}\right)V_o$$

$$V_o = \frac{I_s}{\frac{1}{R} + sC + \frac{1}{sL}} = \frac{sRL\,I_s}{sL + R + s^2RLC}$$

$$I_o = \frac{V_o}{R} = \frac{sL\,I_s}{s^2RLC + sL + R}$$

$$H(s) = \frac{I_o}{I_s} = \frac{sL}{s^2RLC + sL + R} = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

The roots

$$s_{1,2} = \frac{-1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} - \frac{1}{LC}}$$

both lie in the left half plane since R, L, and C are positive quantities.

Thus, <u>**the circuit is stable**</u>.

Chapter 16, Problem 54.

A system is formed by cascading two systems as shown in Fig. 16.74. Given that the impulse response of the systems are

$h_1(t) = 3e^{-t}\,u(t)$, $\qquad h_2(t) = e^{-4t}\,u(t)$

(a) Obtain the impulse response of the overall system.
(b) Check if the overall system is stable.

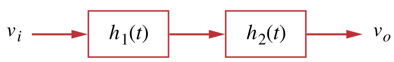


Figure 16.74
For Prob. 16.54.

Chapter 16, Solution 54.

(a) $$H_1(s) = \frac{3}{s+1}, \qquad H_2(s) = \frac{1}{s+4}$$

$$H(s) = H_1(s)\,H_2(s) = \frac{3}{(s+1)(s+4)}$$

$$h(t) = L^{-1}\left[H(s)\right] = L^{-1}\left[\frac{A}{s+1} + \frac{B}{s+4}\right]$$

$$A = 1, \qquad B = -1$$

$$h(t) = \underline{\mathbf{(e^{-t} - e^{-4t})\,u(t)}}$$

(b) Since the poles of H(s) all lie in the left half s-plane, **the system is stable**.

Chapter 16, Problem 55.

Determine whether the op amp circuit in Fig. 16.75 is stable.

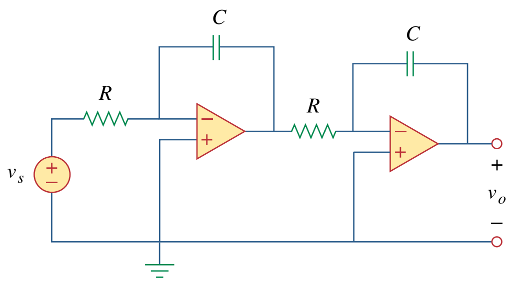


Figure 16.75
For Prob. 16.55.

Chapter 16, Solution 55.

Let V_{o1} be the voltage at the output of the first op amp.

$$\frac{V_{o1}}{V_s} = \frac{-1/sC}{R} = \frac{-1}{sRC}, \qquad \frac{V_o}{V_{o1}} = \frac{-1}{sRC}$$

$$H(s) = \frac{V_o}{V_s} = \frac{1}{s^2R^2C^2}$$

$$h(t) = \frac{t}{R^2C^2}$$

$\lim\limits_{t\to\infty} h(t) = \infty$, i.e. the output is unbounded.

Hence, **<u>the circuit is unstable</u>**.

Chapter 16, Problem 56.

It is desired to realize the transfer function

$$\frac{V_2(s)}{V_1(s)} = \frac{2s}{s^2 + 2s + 6}$$

using the circuit in Fig. 16.76. Choose $R = 1\ \text{k}\Omega$ and find L and C.

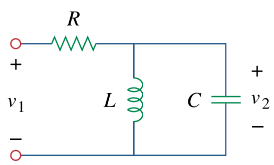


Figure 16.76
For Prob. 16.56.

Chapter 16, Solution 56.

$$\text{sL} \,\|\, \frac{1}{\text{sC}} = \frac{\text{sL}\cdot\frac{1}{\text{sC}}}{\text{sL} + \frac{1}{\text{sC}}} = \frac{\text{sL}}{1 + \text{s}^2\text{LC}}$$

$$\frac{\text{V}_2}{\text{V}_1} = \frac{\frac{\text{sL}}{1+\text{s}^2\text{LC}}}{\text{R} + \frac{\text{sL}}{1+\text{s}^2\text{LC}}} = \frac{\text{sL}}{\text{s}^2\text{RLC} + \text{sL} + \text{R}}$$

$$\frac{\text{V}_2}{\text{V}_1} = \frac{\text{s}\cdot\frac{1}{\text{RC}}}{\text{s}^2 + \text{s}\cdot\frac{1}{\text{RC}} + \frac{1}{\text{LC}}}$$

Comparing this with the given transfer function,

$$2 = \frac{1}{\text{RC}}, \qquad 6 = \frac{1}{\text{LC}}$$

If $\text{R} = 1\,\text{k}\Omega$, $\quad \text{C} = \frac{1}{2\text{R}} = \underline{\mathbf{500\ \mu F}}$

$$\text{L} = \frac{1}{6\text{C}} = \underline{\mathbf{333.3\ H}}$$

Chapter 16, Problem 57.

Design an op amp circuit, using Fig. 16.77, that will realize the following transfer function:

$$\frac{V_0(s)}{V_i(s)} = -\frac{s+1{,}000}{2(s+4{,}000)}$$

Choose C_1=10 μF; determine R_1, R_2, and C_2

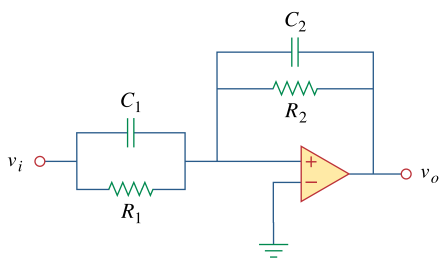
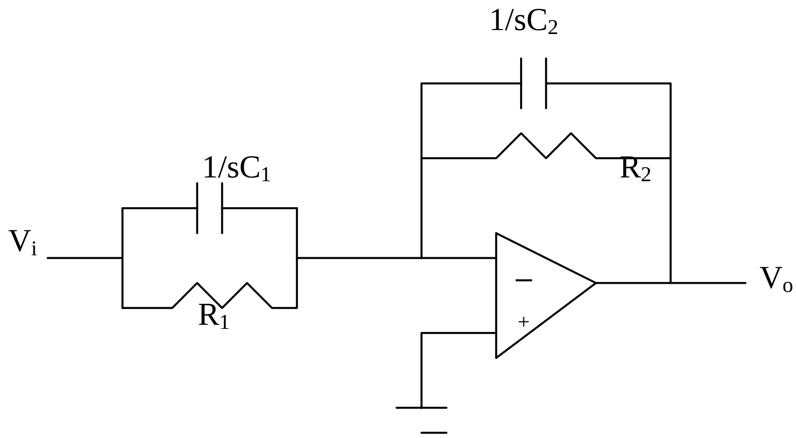


Figure 16.77
For Prob. 16.57.

Chapter 16, Solution 57.

The circuit is transformed in the s-domain as shown below.



Let $Z_1 = R_1 // \frac{1}{sC_1} = \frac{R_1 \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{1 + sR_1C_1}$

$$Z_2 = R_2 // \frac{1}{sC_2} = \frac{R_2 \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{1 + sR_2C_2}$$

This is an inverting amplifier.

$$H(s) = V_o / V_i = -\frac{Z_2}{Z_1} = \frac{-\frac{R_2}{1+sR_2C_2}}{\frac{R_1}{1+sR_1C_1}} = -\frac{R_2}{R_1}\frac{R_1C_1}{R_2C_2}\left[\frac{s+\frac{1}{R_1C_1}}{s+\frac{1}{R_2C_2}}\right] = \frac{-C_1}{C_2}\left[\frac{s+\frac{1}{R_1C_1}}{s+\frac{1}{R_2C_2}}\right]$$

Comparing this with

$$H(s) = -\frac{(s+1000)}{2(s+4000)}$$

we obtain:

$$\frac{C_1}{C_2} = 1/2 \quad \longrightarrow \quad C_2 = 2C_1 = \underline{20\mu F}$$

$$\frac{1}{R_1C_1} = 1000 \quad \longrightarrow \quad R_1 = \frac{1}{1000C_1} = \frac{1}{10^3 x 10 x 10^{-6}} = \underline{100\Omega}$$

$$\frac{1}{R_2C_2} = 4000 \quad \longrightarrow \quad R_2 = \frac{1}{4000C_2} = \frac{1}{4x10^3 x 20x10^{-6}} = \underline{\mathbf{12.5\Omega}}$$

Chapter 16, Problem 58.

Realize the transfer function

$$\frac{V_0(s)}{V_s(s)} = \frac{s}{s+10}$$

using the circuit in Fig. 16.78. Let $Y_1 = sC_1$, $Y_2 = 1/R_1$, $Y_3 = sC_2$. Choose $R_1 = 1\text{k}\,\Omega$ and determine C_1 and C_2.

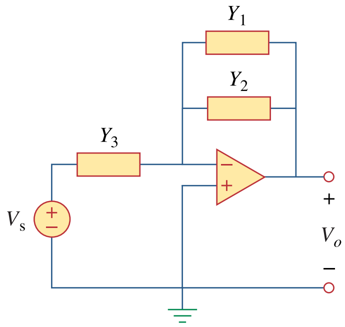


Figure 16.78
For Prob. 16.58.

Chapter 16, Solution 58.

We apply KCL at the noninverting terminal at the op amp.

$$(V_s - 0)\,Y_3 = (0 - V_o)(Y_1 - Y_2)$$

$$Y_3 V_s = -(Y_1 + Y_2)V_o$$

$$\frac{V_o}{V_s} = \frac{-Y_3}{Y_1 + Y_2}$$

Let $\quad Y_1 = sC_1, \qquad Y_2 = 1/R_1, \qquad Y_3 = sC_2$

$$\frac{V_o}{V_s} = \frac{-sC_2}{sC_1 + 1/R_1} = \frac{-sC_2/C_1}{s + 1/R_1 C_1}$$

Comparing this with the given transfer function,

$$\frac{C_2}{C_1} = 1, \qquad \frac{1}{R_1 C_1} = 10$$

If $R_1 = 1\,\text{k}\Omega$,

$$C_1 = C_2 = \frac{1}{10^4} = \underline{\mathbf{100\ \mu F}}$$

Chapter 16, Problem 59.

Synthesize the transfer function

$$\frac{V_0(s)}{V_{in}(s)} = \frac{10^6}{s^2 + 100s + 10^6}$$

using the topology of Fig. 16.79. Let $Y_1 = 1/R_1$, $Y_2 = 1/R_2$, $Y_3 = sC_1$, $Y_4\ sC_2$. Choose $R_1 = 1\text{k}\Omega$ and determine C_1, C_2, and R_2.

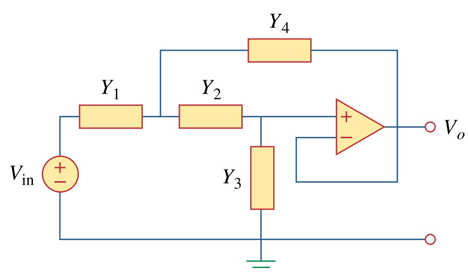
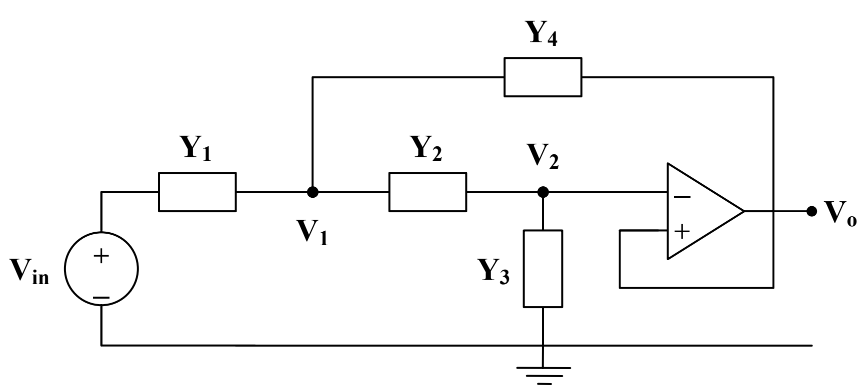


Figure 16.79
For Prob. 16.59.

Chapter 16, Solution 59.

Consider the circuit shown below. We notice that $V_3 = V_o$ and $V_2 = V_3 = V_o$.



At node 1,

$$(V_{in} - V_1)Y_1 = (V_1 - V_o)Y_2 + (V_1 - V_o)Y_4$$

$$V_{in}\,Y_1 = V_1(Y_1 + Y_2 + Y_4) - V_o\,(Y_2 + Y_4) \qquad (1)$$

At node 2,

$$(V_1 - V_o)Y_2 = (V_o - 0)Y_3$$

$$V_1\,Y_2 = (Y_2 + Y_3)V_o$$

$$V_1 = \frac{Y_2 + Y_3}{Y_2}V_o \qquad (2)$$

Substituting (2) into (1),

$$V_{in}\,Y_1 = \frac{Y_2 + Y_3}{Y_2}\cdot(Y_1 + Y_2 + Y_4)V_o - V_o\,(Y_2 + Y_4)$$

$$V_{in}Y_1Y_2 = V_o(Y_1Y_2 + Y_2^2 + Y_2Y_4 + Y_1Y_3 + Y_2Y_3 + Y_3Y_4 - Y_2^2 - Y_2Y_4)$$

$$\frac{V_o}{V_{in}} = \frac{Y_1Y_2}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3 + Y_3Y_4}$$

Y_1 and Y_2 must be resistive, while Y_3 and Y_4 must be capacitive.

Let $\quad Y_1 = \dfrac{1}{R_1}, \qquad Y_2 = \dfrac{1}{R_2}, \qquad Y_3 = sC_1, \qquad Y_4 = sC_2$

$$\frac{V_o}{V_{in}} = \frac{\dfrac{1}{R_1R_2}}{\dfrac{1}{R_1R_2} + \dfrac{sC_1}{R_1} + \dfrac{sC_1}{R_2} + s^2C_1C_2}$$

$$\frac{V_o}{V_{in}} = \frac{\dfrac{1}{R_1R_2C_1C_2}}{s^2 + s\cdot\left(\dfrac{R_1 + R_2}{R_1R_2C_2}\right) + \dfrac{1}{R_1R_2C_1C_2}}$$

Choose $R_1 = 1\,\text{k}\Omega$, then

$$\frac{1}{R_1R_2C_1C_2} = 10^6 \qquad \text{and} \qquad \frac{R_1 + R_2}{R_1R_2C_2} = 100$$

We have three equations and four unknowns. Thus, there is a family of solutions. One such solution is

$$R_2 = \underline{\mathbf{1\,k\Omega}}, \qquad C_1 = \underline{\mathbf{50\,nF}}, \qquad C_2 = \underline{\mathbf{20\,\mu F}}$$

Chapter 16, Problem 60.

Obtain the transfer function of the op amp circuit in Fig. 16.80 in the form of

$$\frac{V_0(s)}{V_i(s)} = \frac{as}{s^2 + bs + c}$$

where a, b, and c are constants. Determine the constants.

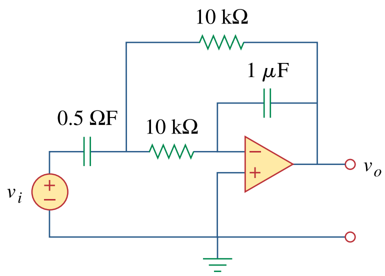
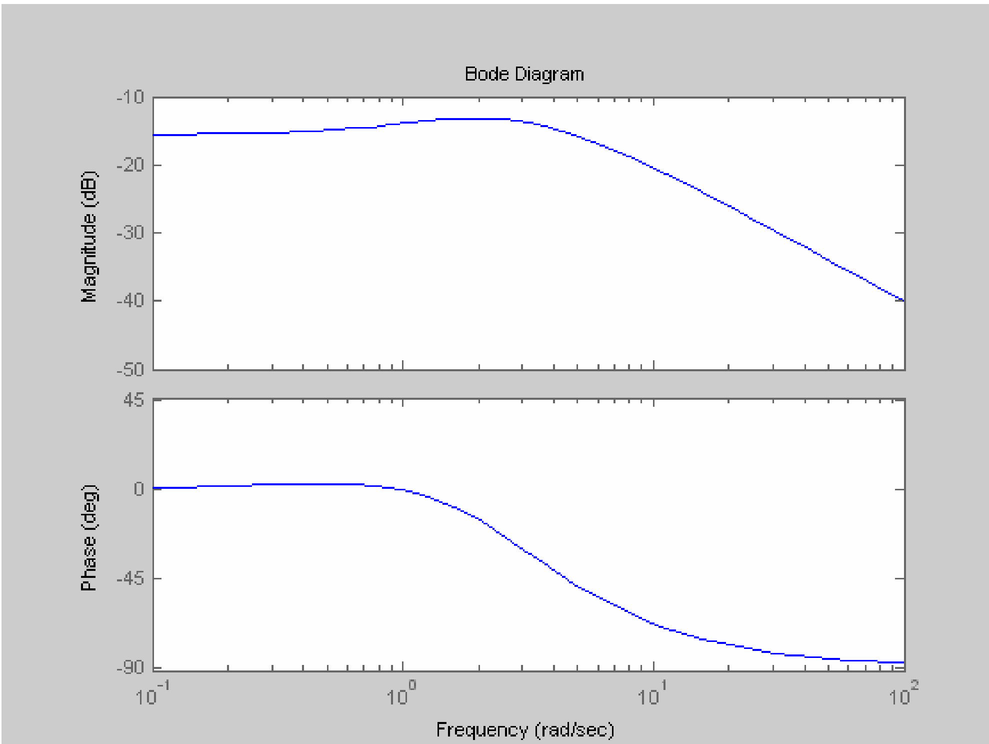


Figure 16.80
For Prob. 16.67.

Chapter 16, Solution 60.

With the following MATLAB codes, the Bode plots are generated as shown below.

num=[1 1];
den= [1 5 6];
bode(num,den);



Chapter 16, Problem 61.

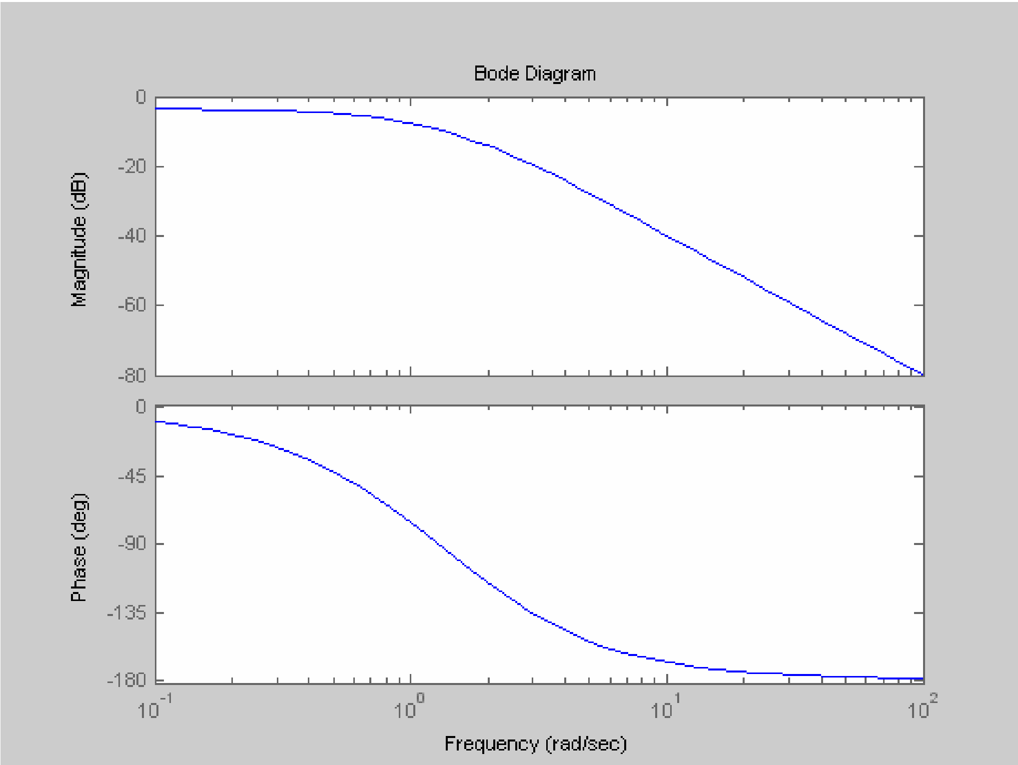
A certain network has an input admittance $Y(s)$. The admittance has a pole at $s = -3$, a zero at $s = -1$, and $Y(\infty) = 0.25$ S.

(a) Find $Y(s)$.
(b) An 8-V battery is connected to the network via a switch. If the switch is closed at $t = 0$, find the current $i(t)$ through $Y(s)$ using the Laplace transform.

Chapter 16, Solution 61.

We use the following codes to obtain the Bode plots below.

```
num=[1 4];
den= [1 6  11  6];
bode(num,den);
```



Chapter 16, Problem 62.

A gyrator is a device for simulating an inductor in a network. A basic gyrator circuit is shown in Fig. 16.81. By finding $V_i(s)/I_0(s)$, show that the inductance produced by the gyrator is $L = CR^2$.

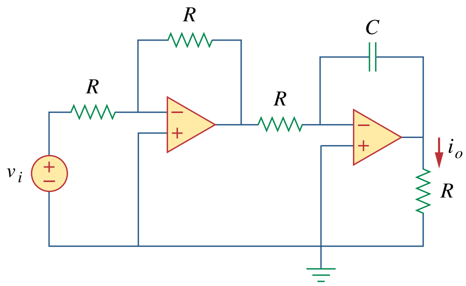


Figure 16.81
For Prob. 16.69.

Chapter 16, Solution 62.
The following codes are used to obtain the Bode plots below.

```
num=[1 1];
den= [1 0.5  1];
bode(num,den);
```