Fall 2010 - 91.503 - HomeWork Problems

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27.1-3, p. 791. Prove that a greedy scheduler achieves the following time-bound, which is slightly stronger than the bound proven in Theorem 27.1:

$$T_P \leq \frac{T_1 - T_\infty}{P} + T_\infty.$$

Proof. Observe that T_∞ already accounts for all the incomplete steps and the complete ones. The catch is that in a P-processor system a complete step allocates P threads. Recall further that $P \cdot T_P \geq T_1$: the total work done by P processors is at least as large as that done by one.

Instead of computing bounds on the number of complete steps and incomplete ones *separately* (as in Thm. 27.1), we will try to combine the arguments.

T_∞ is, by definition, the length of a longest path in the computation graph. At each vertex in this path, the scheduler allocates threads to processors: if only fewer than P threads are ready to execute, we have an **incomplete step**, otherwise a **complete one**. We add a twist: a complete step can allocate $kP + j$ strands with $0 \leq j < P$ and $k \geq 1$, so we think of such steps as one *exact* complete step ($j = 0$) followed by (possibly) multiple exact complete ones (if $P > 1$), or an incomplete ($0 < j < P$) one followed by (possibly) multiple exact complete ones ($P > 0$). We will call such first incomplete step a *pseudo*-incomplete one and such first complete one a *pseudo*-complete one. The total number of incomplete, pseudo-incomplete and pseudo-complete steps, according to this definition, is still $\leq T_\infty$, with execution time T_∞, since at no point we try to allocate more than P strands.

The number $T_1 - T_\infty$ counts all strands not belonging to this longest path. Each of them belongs to one of four sets: T_i, the set of strands executed during an incomplete step, T_{pi}, those executed during a pseudo-incomplete step, T_{pc}, those executed during a pseudo-complete step; and T_c, those executed during an allocation of P threads following the execution of a pseudo-incomplete or a pseudo-complete step.

We have $|T_i| + |T_{pi}| + |T_{pc}| \geq T_\infty$, since every strand of the longest path in the computation graph belongs to one of these three sets. This implies that the cardinality of T_c, the remaining number of strands, satisfies $|T_c| \leq T_1 - T_\infty$, and $|T_c|$ is a multiple of P by construction.

The total computation time on P processors, T_P, will be bounded above by

$$\frac{|T_c|}{P} + T_\infty \leq \frac{T_1 - T_\infty}{P} + T_\infty$$