

Chapter 9, Problem 1.

Given the sinusoidal voltage $v(t) = 50 \cos(30t + 10^\circ)$ V, find: (a) the amplitude V_m , (b) the period T , (c) the frequency f , and (d) $v(t)$ at $t = 10$ ms.

Chapter 9, Solution 1.

- (a) $V_m = \underline{50 \text{ V}}$.
- (b) Period $T = \frac{2\pi}{\omega} = \frac{2\pi}{30} = 0.2094 \text{ s} = \underline{209.4 \text{ ms}}$
- (c) Frequency $f = \omega/(2\pi) = 30/(2\pi) = \underline{4.775 \text{ Hz}}$.
- (d) At $t = 10 \text{ ms}$, $v(0.01) = 50 \cos(30 \times 0.01 \text{ rad} + 10^\circ)$
 $= 50 \cos(1.72^\circ + 10^\circ) = \underline{44.48 \text{ V}}$ and $\omega t = \underline{0.3 \text{ rad}}$.

Chapter 9, Problem 2.

A current source in a linear circuit has

$$i_s = 8 \cos(500\pi t - 25^\circ) \text{ A}$$

- (a) What is the amplitude of the current?
- (b) What is the angular frequency?
- (c) Find the frequency of the current.
- (d) Calculate i_s at $t = 2 \text{ ms}$.

Chapter 9, Solution 2.

- (a) amplitude = **8 A**
- (b) $\omega = 500\pi = \underline{1570.8 \text{ rad/s}}$
- (c) $f = \frac{\omega}{2\pi} = \underline{250 \text{ Hz}}$
- (d) $I_s = 8 \angle -25^\circ \text{ A}$
 $I_s(2 \text{ ms}) = 8 \cos((500\pi)(2 \times 10^{-3}) - 25^\circ)$
 $= 8 \cos(\pi - 25^\circ) = 8 \cos(155^\circ)$
 $= \underline{-7.25 \text{ A}}$

Chapter 9, Problem 3.

Express the following functions in cosine form:

(a) $4 \sin(\omega t - 30^\circ)$

(b) $-2 \sin 6t$

(c) $-10 \sin(\omega t + 20^\circ)$

Chapter 9, Solution 3.

(a) $4 \sin(\omega t - 30^\circ) = 4 \cos(\omega t - 30^\circ - 90^\circ) = \underline{4 \cos(\omega t - 120^\circ)}$

(b) $-2 \sin(6t) = \underline{2 \cos(6t + 90^\circ)}$

(c) $-10 \sin(\omega t + 20^\circ) = 10 \cos(\omega t + 20^\circ + 90^\circ) = \underline{10 \cos(\omega t + 110^\circ)}$

Chapter 9, Problem 4.

(a) Express $v = 8 \cos(7t + 15^\circ)$ in sine form.

(b) Convert $i = -10 \sin(3t - 85^\circ)$ to cosine form.

Chapter 9, Solution 4.

(a) $v = 8 \cos(7t + 15^\circ) = 8 \sin(7t + 15^\circ + 90^\circ) = \underline{8 \sin(7t + 105^\circ)}$

(b) $i = -10 \sin(3t - 85^\circ) = 10 \cos(3t - 85^\circ + 90^\circ) = \underline{10 \cos(3t + 5^\circ)}$

Chapter 9, Problem 5.

Given $v_1 = 20 \sin(\omega t + 60^\circ)$ and $v_2 = 60 \cos(\omega t - 10^\circ)$ determine the phase angle between the two sinusoids and which one lags the other.

Chapter 9, Solution 5.

$$v_1 = 20 \sin(\omega t + 60^\circ) = 20 \cos(\omega t + 60^\circ - 90^\circ) = 20 \cos(\omega t - 30^\circ)$$

$$v_2 = 60 \cos(\omega t - 10^\circ)$$

This indicates that the phase angle between the two signals is 20° and that v₁ lags v₂.

Chapter 9, Problem 6.

For the following pairs of sinusoids, determine which one leads and by how much.

(a) $v(t) = 10 \cos(4t - 60^\circ)$ and $i(t) = 4 \sin(4t + 50^\circ)$

(b) $v_1(t) = 4 \cos(377t + 10^\circ)$ and $v_2(t) = -20 \cos 377t$

(c) $x(t) = 13 \cos 2t + 5 \sin 2t$ and $y(t) = 15 \cos(2t - 11.8^\circ)$

Chapter 9, Solution 6.

(a) $v(t) = 10 \cos(4t - 60^\circ)$

$$i(t) = 4 \sin(4t + 50^\circ) = 4 \cos(4t + 50^\circ - 90^\circ) = 4 \cos(4t - 40^\circ)$$

Thus, **$i(t)$ leads $v(t)$ by 20° .**

(b) $v_1(t) = 4 \cos(377t + 10^\circ)$

$$v_2(t) = -20 \cos(377t) = 20 \cos(377t + 180^\circ)$$

Thus, **$v_2(t)$ leads $v_1(t)$ by 170° .**

(c) $x(t) = 13 \cos(2t) + 5 \sin(2t) = 13 \cos(2t) + 5 \cos(2t - 90^\circ)$

$$\mathbf{X} = 13\angle 0^\circ + 5\angle -90^\circ = 13 - j5 = 13.928\angle -21.04^\circ$$

$$x(t) = 13.928 \cos(2t - 21.04^\circ)$$

$$y(t) = 15 \cos(2t - 11.8^\circ)$$

$$\text{phase difference} = -11.8^\circ + 21.04^\circ = 9.24^\circ$$

Thus, **$y(t)$ leads $x(t)$ by 9.24° .**

Chapter 9, Problem 7.

If $f(\phi) = \cos \phi + j \sin \phi$, show that $f(\phi) = e^{j\phi}$.

Chapter 9, Solution 7.

$$\text{If } f(\phi) = \cos \phi + j \sin \phi,$$

$$\frac{df}{d\phi} = -\sin \phi + j \cos \phi = j(\cos \phi + j \sin \phi) = j f(\phi)$$

$$\frac{df}{f} = j d\phi$$

Integrating both sides

$$\ln f = j\phi + \ln A$$

$$f = A e^{j\phi} = \cos \phi + j \sin \phi$$

$$f(0) = A = 1$$

$$\text{i.e. } \mathbf{f(\phi) = e^{j\phi} = \cos \phi + j \sin \phi}$$

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Chapter 9, Problem 8.

Calculate these complex numbers and express your results in rectangular form:

$$(a) \frac{15\angle 45^\circ}{3-j4} + j2$$

$$(b) \frac{8\angle -20^\circ}{(2+j)(3-j4)} + \frac{10}{-5+j12}$$

$$(c) 10 + (8\angle 50^\circ)(5-j12)$$

Chapter 9, Solution 8.

$$\begin{aligned}(a) \quad \frac{15\angle 45^\circ}{3-j4} + j2 &= \frac{15\angle 45^\circ}{5\angle -53.13^\circ} + j2 \\ &= 3\angle 98.13^\circ + j2 \\ &= -0.4245 + j2.97 + j2 \\ &= \underline{\underline{-0.4243 + j4.97}}\end{aligned}$$

$$\begin{aligned}(b) \quad (2+j)(3-j4) &= 6-j8+j3+4 = 10-j5 = 11.18\angle -26.57^\circ \\ \frac{8\angle -20^\circ}{(2+j)(3-j4)} + \frac{10}{-5+j12} &= \frac{8\angle -20^\circ}{11.18\angle -26.57^\circ} + \frac{(-5-j12)(10)}{25+144} \\ &= 0.7156\angle 6.57^\circ - 0.2958 - j0.71 \\ &= 0.7109 + j0.08188 - 0.2958 - j0.71 \\ &= \underline{\underline{0.4151 - j0.6281}}\end{aligned}$$

$$\begin{aligned}(c) \quad 10 + (8\angle 50^\circ)(13\angle -68.38^\circ) &= 10 + 104\angle -17.38^\circ \\ &= \underline{\underline{109.25 - j31.07}}\end{aligned}$$

Chapter 9, Problem 9.

Evaluate the following complex numbers and leave your results in polar form:

$$(a) 5 \angle 30^\circ \left(6 - j8 + \frac{3 \angle 60^\circ}{2 + j} \right)$$

$$(b) \frac{(10 \angle 60^\circ)(35 \angle -50^\circ)}{(2 + j6) - (5 + j)}$$

Chapter 9, Solution 9.

$$(a) \quad (5 \angle 30^\circ)(6 - j8 + 1.1197 + j0.7392) = (5 \angle 30^\circ)(7.13 - j7.261) \\ = (5 \angle 30^\circ)(10.176 \angle -45.52^\circ) =$$

$$\underline{50.88 \angle -15.52^\circ}.$$

$$(b) \quad \frac{(10 \angle 60^\circ)(35 \angle -50^\circ)}{(-3 + j5) = (5.83 \angle 120.96^\circ)} = \underline{60.02 \angle -110.96^\circ}.$$

Chapter 9, Problem 10.

Given that $z_1 = 6 - j8$, $z_2 = 10 \angle -30^\circ$, and $z_3 = 8e^{-j120^\circ}$, find:

$$(a) z_1 + z_2 + z_3$$

$$(b) \frac{z_1 z_2}{z_3}$$

Chapter 9, Solution 10.

$$(a) \quad z_1 = 6 - j8, \quad z_2 = 8.66 - j5, \quad \text{and} \quad z_3 = -4 - j6.9282 \\ z_1 + z_2 + z_3 = \underline{10.66 - j19.93}$$

$$(b) \quad \frac{z_1 z_2}{z_3} = \underline{9.999 + j7.499}$$

Chapter 9, Problem 11.

Find the phasors corresponding to the following signals:

(a) $v(t) = 21 \cos(4t - 15^\circ) \text{ V}$

(b) $i(t) = -8 \sin(10t + 70^\circ) \text{ mA}$

(c) $v(t) = 120 \sin(10t - 50^\circ) \text{ V}$

(d) $i(t) = -60 \cos(30t + 10^\circ) \text{ mA}$

Chapter 9, Solution 11.

(a) $V = \underline{21 \angle -15^\circ} \text{ V}$

(b) $i(t) = 8 \sin(10t + 70^\circ + 180^\circ) = 8 \cos(10t + 70^\circ + 180^\circ - 90^\circ) = 8 \cos(10t + 160^\circ)$

$$I = \underline{8 \angle 160^\circ} \text{ mA}$$

(c) $v(t) = 120 \sin(10^3 t - 50^\circ) = 120 \cos(10^3 t - 50^\circ - 90^\circ)$

$$V = \underline{120 \angle -140^\circ} \text{ V}$$

(d) $i(t) = -60 \cos(30t + 10^\circ) = 60 \cos(30t + 10^\circ + 180^\circ)$

$$I = \underline{60 \angle 190^\circ} \text{ mA}$$

Chapter 9, Problem 12.

Let $\mathbf{X} = 8 \angle 40^\circ$ and $\mathbf{Y} = 10 \angle -30^\circ$. Evaluate the following quantities and express your results in polar form:

(a) $(\mathbf{X} + \mathbf{Y})\mathbf{X}^*$ (b) $(\mathbf{X} - \mathbf{Y})^*$ (c) $(\mathbf{X} + \mathbf{Y})/\mathbf{X}$

Chapter 9, Solution 12.

Let $\mathbf{X} = 8 \angle 40^\circ$ and $\mathbf{Y} = 10 \angle -30^\circ$. Evaluate the following quantities and express your results in polar form.

$$\begin{aligned} &(\mathbf{X} + \mathbf{Y})/\mathbf{X}^* \\ &(\mathbf{X} - \mathbf{Y})^* \\ &(\mathbf{X} + \mathbf{Y})/\mathbf{X} \end{aligned}$$

$$\mathbf{X} = 6.128 + j5.142; \quad \mathbf{Y} = 8.66 - j5$$

$$\begin{aligned} \text{(a)} \quad (\mathbf{X} + \mathbf{Y})\mathbf{X}^* &= (14.788 + j0.142)(8 \angle -40^\circ) \\ &= (14.789 \angle 0.55^\circ)(8 \angle -40^\circ) = 118.31 \angle -39.45^\circ \\ &= \underline{\underline{91.36 - j75.17}} \end{aligned}$$

$$\text{(b)} \quad (\mathbf{X} - \mathbf{Y})^* = (-2.532 + j10.142)^* = \underline{\underline{-2.532 - j10.142}} = 10.453 \angle -104.02^\circ$$

$$\begin{aligned} \text{(c)} \quad (\mathbf{X} + \mathbf{Y})/\mathbf{X} &= (14.789 \angle 0.55^\circ)/(8 \angle 40^\circ) = 1.8486 \angle -39.45^\circ \\ &= \underline{\underline{1.4275 - j1.1746}} \end{aligned}$$

Chapter 9, Problem 13.

Evaluate the following complex numbers:

$$(a) \frac{2+j3}{1-j6} + \frac{7-j8}{-5+j11}$$

$$(b) \frac{(5\angle 10^\circ)(10\angle -40^\circ)}{(4\angle -80^\circ)(-6\angle 50^\circ)}$$

$$(c) \begin{vmatrix} 2+j3 & -j2 \\ -j2 & 8-j5 \end{vmatrix}$$

Chapter 9, Solution 13.

$$(a) (-0.4324 + j0.4054) + (-0.8425 - j0.2534) = \underline{-1.2749 + j0.1520}$$

$$(b) \frac{50\angle -30^\circ}{24\angle 150^\circ} = \underline{-2.0833} = \underline{\mathbf{-2.083}}$$

$$(c) (2+j3)(8-j5) - (-4) = \underline{\mathbf{35+j14}}$$

Chapter 9, Problem 14.

Simplify the following expressions:

$$(a) \frac{(5-j6)-(2+j8)}{(-3+j4)(5-j)+(4-j6)}$$

$$(b) \frac{(240\angle 75^\circ + 160\angle -30^\circ)(60-j80)}{(67+j84)(20\angle 32^\circ)}$$

$$(c) \left(\frac{10+j20}{3+j4} \right)^2 \sqrt{(10+j5)(16-j120)}$$

Chapter 9, Solution 14.

$$(a) \frac{3-j14}{-7+j17} = \frac{14.318\angle -77.91^\circ}{18.385\angle 112.38^\circ} = 0.7788\angle 169.71^\circ = \underline{-0.7663 + j0.13912}$$

$$(b) \frac{(62.116 + j231.82 + 138.56 - j80)(60-j80)}{(67+j84)(16.96 + j10.5983)} = \frac{24186 - 6944.9}{246.06 + j2134.7} = \underline{-1.922 - j11.55}$$

$$(c) (-2+j4)^2 \sqrt{(260-j120)} = (20\angle -126.86^\circ)(16.923\angle -12.38^\circ) = \underline{338.46\angle -139.24^\circ} = \underline{-256.4 - j221}$$

Chapter 9, Problem 15.

Evaluate these determinants:

$$(a) \begin{vmatrix} 10 + j6 & 2 - j3 \\ -5 & -1 + j \end{vmatrix}$$

$$(b) \begin{vmatrix} 20\angle -30^\circ & -4\angle -10^\circ \\ 16\angle 0^\circ & 3\angle 45^\circ \end{vmatrix}$$

$$(c) \begin{vmatrix} 1 - j & -j & 0 \\ j & 1 & -j \\ 1 & j & 1 + j \end{vmatrix}$$

Chapter 9, Solution 15.

$$(a) \begin{vmatrix} 10 + j6 & 2 - j3 \\ -5 & -1 + j \end{vmatrix} = -10 - j6 + j10 - 6 + 10 - j15 \\ = \underline{\underline{-6 - j11}}$$

$$(b) \begin{vmatrix} 20\angle -30^\circ & -4\angle -10^\circ \\ 16\angle 0^\circ & 3\angle 45^\circ \end{vmatrix} = 60\angle 15^\circ + 64\angle -10^\circ \\ = 57.96 + j15.529 + 63.03 - j11.114 \\ = \underline{\underline{120.99 + j4.415}}$$

$$(c) \begin{vmatrix} 1 - j & -j & 0 \\ j & 1 & -j \\ 1 & j & 1 + j \end{vmatrix} = 1 + 1 + 0 - 1 - 0 + j^2(1 - j) + j^2(1 + j) \\ = 1 - 1(1 - j + 1 + j) \\ = 1 - 2 = \underline{\underline{-1}}$$

Chapter 9, Problem 16.

Transform the following sinusoids to phasors:

(a) $-10 \cos(4t + 75^\circ)$

(b) $5 \sin(20t - 10^\circ)$

(c) $4 \cos 2t + 3 \sin 2t$

Chapter 9, Solution 16.

(a) $-10 \cos(4t + 75^\circ) = 10 \cos(4t + 75^\circ - 180^\circ)$
 $= 10 \cos(4t - 105^\circ)$

The phasor form is $10\angle-105^\circ$

(b) $5 \sin(20t - 10^\circ) = 5 \cos(20t - 10^\circ - 90^\circ)$
 $= 5 \cos(20t - 100^\circ)$

The phasor form is $5\angle-100^\circ$

(c) $4 \cos(2t) + 3 \sin(2t) = 4 \cos(2t) + 3 \cos(2t - 90^\circ)$

The phasor form is $4\angle 0^\circ + 3\angle-90^\circ = 4 - j3 = \underline{5\angle-36.87^\circ}$

Chapter 9, Problem 17.

Two voltages v_1 and v_2 appear in series so that their sum is $v = v_1 + v_2$. If $v_1 = 10 \cos(50t - \pi/3)$ V and $v_2 = 12 \cos(50t + 30^\circ)$ V, find v .

Chapter 9, Solution 17.

$$V = V_1 + V_2 = 10 \angle -60^\circ + 12 \angle 30^\circ = 5 - j8.66 + 10.392 + j6 = 15.62 \angle -9.805^\circ$$

$$v = \underline{15.62 \cos(50t - 9.805^\circ)} \text{ V} = \underline{15.62 \cos(50t - 9.8^\circ)} \text{ V}$$

Chapter 9, Problem 18.

Obtain the sinusoids corresponding to each of the following phasors:

(a) $\mathbf{V}_1 = 60 \angle 15^\circ \text{ V}, \omega = 1$

(b) $\mathbf{V}_2 = 6 + j8 \text{ V}, \omega = 40$

(c) $\mathbf{I}_1 = 2.8e^{-j\pi/3} \text{ A}, \omega = 377$

(d) $\mathbf{I}_2 = -0.5 - j1.2 \text{ A}, \omega = 10^3$

Chapter 9, Solution 18.

(a) $v_1(t) = \underline{60 \cos(t + 15^\circ)}$

(b) $\mathbf{V}_2 = 6 + j8 = 10 \angle 53.13^\circ$
 $v_2(t) = \underline{10 \cos(40t + 53.13^\circ)}$

(c) $i_1(t) = \underline{2.8 \cos(377t - \pi/3)}$

(d) $\mathbf{I}_2 = -0.5 - j1.2 = 1.3 \angle 247.4^\circ$
 $i_2(t) = \underline{1.3 \cos(10^3t + 247.4^\circ)}$

Chapter 9, Problem 19.

Using phasors, find:

(a) $3\cos(20t + 10^\circ) - 5\cos(20t - 30^\circ)$

(b) $40\sin 50t + 30\cos(50t - 45^\circ)$

(c) $20\sin 400t + 10\cos(400t + 60^\circ) - 5\sin(400t - 20^\circ)$

Chapter 9, Solution 19.

$$\begin{aligned}\text{(a)} \quad 3\angle 10^\circ - 5\angle -30^\circ &= 2.954 + j0.5209 - 4.33 + j2.5 \\ &= -1.376 + j3.021 \\ &= 3.32\angle 114.49^\circ\end{aligned}$$

$$\text{Therefore, } 3\cos(20t + 10^\circ) - 5\cos(20t - 30^\circ) = \underline{\underline{3.32\cos(20t + 114.49^\circ)}}$$

$$\begin{aligned}\text{(b)} \quad 40\angle -90^\circ + 30\angle -45^\circ &= -j40 + 21.21 - j21.21 \\ &= 21.21 - j61.21 \\ &= 64.78\angle -70.89^\circ\end{aligned}$$

$$\text{Therefore, } 40\sin(50t) + 30\cos(50t - 45^\circ) = \underline{\underline{64.78\cos(50t - 70.89^\circ)}}$$

$$\begin{aligned}\text{(c)} \quad \text{Using } \sin\alpha &= \cos(\alpha - 90^\circ), \\ 20\angle -90^\circ + 10\angle 60^\circ - 5\angle -110^\circ &= -j20 + 5 + j8.66 + 1.7101 + j4.699 \\ &= 6.7101 - j6.641 \\ &= 9.44\angle -44.7^\circ\end{aligned}$$

$$\begin{aligned}\text{Therefore, } 20\sin(400t) + 10\cos(400t + 60^\circ) - 5\sin(400t - 20^\circ) \\ = \underline{\underline{9.44\cos(400t - 44.7^\circ)}}$$

Chapter 9, Problem 20.

A linear network has a current input $4\cos(\omega t + 20^\circ)\text{A}$ and a voltage output $10\cos(\omega t + 110^\circ)\text{V}$. Determine the associated impedance.

Chapter 9, Solution 20.

$$I = 4\angle 20^\circ, \quad V = 10\angle 110^\circ$$

$$Z = \frac{V}{I} = \frac{10\angle 110^\circ}{4\angle 20^\circ} = 2.5\angle 90^\circ = \underline{\underline{j2.5\ \Omega}}$$

Chapter 9, Problem 21.

Simplify the following:

(a) $f(t) = 5 \cos(2t + 15^\circ) - 4 \sin(2t - 30^\circ)$

(b) $g(t) = 8 \sin t + 4 \cos(t + 50^\circ)$

(c) $h(t) = \int_0^t (10 \cos 40t + 50 \sin 40t) dt$

Chapter 9, Solution 21.

(a) $F = 5 \angle 15^\circ - 4 \angle -30^\circ - 90^\circ = 6.8296 + j4.758 = 8.3236 \angle 34.86^\circ$

$$\underline{f(t) = 8.324 \cos(30t + 34.86^\circ)}$$

(b) $G = 8 \angle -90^\circ + 4 \angle 50^\circ = 2.571 - j4.9358 = 5.565 \angle -62.49^\circ$

$$\underline{g(t) = 5.565 \cos(t - 62.49^\circ)}$$

(c) $H = \frac{1}{j\omega} (10 \angle 0^\circ + 50 \angle -90^\circ), \quad \omega = 40$

i.e. $H = 0.25 \angle -90^\circ + 1.25 \angle -180^\circ = -j0.25 - 1.25 = 1.2748 \angle -168.69^\circ$

$$\underline{h(t) = 1.2748 \cos(40t - 168.69^\circ)}$$

Chapter 9, Problem 22.An alternating voltage is given by $v(t) = 20 \cos(5t - 30^\circ)$ V. Use phasors to find

$$10v(t) + 4 \frac{dv}{dt} - 2 \int_{-\infty}^t v(t) dt$$

Assume that the value of the integral is zero at $t = -\infty$.**Chapter 9, Solution 22.**

Let $f(t) = 10v(t) + 4 \frac{dv}{dt} - 2 \int_{-\infty}^t v(t) dt$

$$F = 10V + j\omega 4V - \frac{2V}{j\omega}, \quad \omega = 5, \quad V = 20 \angle -30^\circ$$

$$F = 10V + j20V - j0.4V = (10 + j20.4)(17.32 - j10) = 454.4 \angle 33.89^\circ$$

$$\underline{f(t) = 454.4 \cos(5t + 33.89^\circ)}$$

Chapter 9, Problem 23.

Apply phasor analysis to evaluate the following.

(a) $v = 50 \cos(\omega t + 30^\circ) + 30 \cos(\omega t + 90^\circ) \text{ V}$

(b) $i = 15 \cos(\omega t + 45^\circ) - 10 \sin(\omega t + 45^\circ) \text{ A}$

Chapter 9, Solution 23.

(a) $V = 50 \angle 30^\circ + 30 \angle 90^\circ = 43.3 + j25 - j30 = 43.588 \angle -6.587^\circ$

$$v = \underline{43.588 \cos(\omega t - 6.587^\circ)} \quad V = \underline{43.49 \cos(\omega t - 6.59^\circ)} \text{ V}$$

(b) $I = 15 \angle 45^\circ - 10 \angle 45^\circ - 90^\circ = (10.607 + j10.607) - (7.071 - j7.071) = 18.028 \angle 78.69^\circ$

$$i = \underline{18.028 \cos(\omega t + 78.69^\circ)} \quad A = \underline{18.028 \cos(\omega t + 78.69^\circ)} \text{ A}$$

Chapter 9, Problem 24.

Find $v(t)$ in the following integrodifferential equations using the phasor approach:

(a) $v(t) + \int v \, dt = 10 \cos t$

(b) $\frac{dv}{dt} + 5v(t) + 4 \int v \, dt = 20 \sin(4t + 10^\circ)$

Chapter 9, Solution 24.

(a)

$$V + \frac{V}{j\omega} = 10 \angle 0^\circ, \quad \omega = 1$$

$$V(1 - j) = 10$$

$$V = \frac{10}{1 - j} = 5 + j5 = 7.071 \angle 45^\circ$$

Therefore, $v(t) = \underline{7.071 \cos(t + 45^\circ)}$

(b)

$$j\omega V + 5V + \frac{4V}{j\omega} = 20 \angle (10^\circ - 90^\circ), \quad \omega = 4$$

$$V \left(j4 + 5 + \frac{4}{j4} \right) = 20 \angle -80^\circ$$

$$V = \frac{20 \angle -80^\circ}{5 + j3} = 3.43 \angle -110.96^\circ$$

Therefore, $v(t) = \underline{3.43 \cos(4t - 110.96^\circ)}$

Chapter 9, Problem 25.

Using phasors, determine $i(t)$ in the following equations:

$$(a) \ 2 \frac{di}{dt} + 3i(t) = 4 \cos(2t - 45^\circ)$$

$$(b) \ 10 \int i \, dt + \frac{di}{dt} + 6i(t) = 5 \cos(5t + 22^\circ)$$

Chapter 9, Solution 25.

(a)

$$2j\omega \mathbf{I} + 3\mathbf{I} = 4\angle -45^\circ, \quad \omega = 2$$

$$\mathbf{I}(3 + j4) = 4\angle -45^\circ$$

$$\mathbf{I} = \frac{4\angle -45^\circ}{3 + j4} = \frac{4\angle -45^\circ}{5\angle 53.13^\circ} = 0.8\angle -98.13^\circ$$

$$\text{Therefore, } i(t) = \underline{\underline{0.8 \cos(2t - 98.13^\circ)}}$$

(b)

$$10 \frac{\mathbf{I}}{j\omega} + j\omega \mathbf{I} + 6\mathbf{I} = 5\angle 22^\circ, \quad \omega = 5$$

$$(-j2 + j5 + 6)\mathbf{I} = 5\angle 22^\circ$$

$$\mathbf{I} = \frac{5\angle 22^\circ}{6 + j3} = \frac{5\angle 22^\circ}{6.708\angle 26.56^\circ} = 0.745\angle -4.56^\circ$$

$$\text{Therefore, } i(t) = \underline{\underline{0.745 \cos(5t - 4.56^\circ)}}$$

Chapter 9, Problem 26.

The loop equation for a series RLC circuit gives

$$\frac{di}{dt} + 2i + \int_{-\infty}^t i \, dt = \cos 2t$$

Assuming that the value of the integral at $t = -\infty$ is zero, find $i(t)$ using the phasor method.

Chapter 9, Solution 26.

$$j\omega \mathbf{I} + 2\mathbf{I} + \frac{\mathbf{I}}{j\omega} = 1\angle 0^\circ, \quad \omega = 2$$

$$\mathbf{I} \left(j2 + 2 + \frac{1}{j2} \right) = 1$$

$$\mathbf{I} = \frac{1}{2 + j1.5} = 0.4\angle -36.87^\circ$$

$$\text{Therefore, } i(t) = \underline{\underline{0.4 \cos(2t - 36.87^\circ)}}$$

Chapter 9, Problem 27.

A parallel RLC circuit has the node equation

$$\frac{dv}{dt} = 50v + 100 \int v dt = 110 \cos(377t - 10^\circ)$$

Determine $v(t)$ using the phasor method. You may assume that the value of the integral at $t = -\infty$ is zero.

Chapter 9, Solution 27.

$$\begin{aligned} j\omega V + 50V + 100 \frac{V}{j\omega} &= 110 \angle -10^\circ, \quad \omega = 377 \\ V \left(j377 + 50 - \frac{j100}{377} \right) &= 110 \angle -10^\circ \\ V (380.6 \angle 82.45^\circ) &= 110 \angle -10^\circ \\ V &= 0.289 \angle -92.45^\circ \end{aligned}$$

Therefore, $v(t) = \underline{\underline{0.289 \cos(377t - 92.45^\circ)}}$.

Chapter 9, Problem 28.

Determine the current that flows through an $8\text{-}\Omega$ resistor connected to a voltage source $v_s = 110 \cos 377t$ V.

Chapter 9, Solution 28.

$$i(t) = \frac{v_s(t)}{R} = \frac{110 \cos(377t)}{8} = \underline{\underline{13.75 \cos(377t) \text{ A}}}$$

Chapter 9, Problem 29.

What is the instantaneous voltage across a $2\text{-}\mu\text{F}$ capacitor when the current through it is $i = 4 \sin(10^6 t + 25^\circ)$ A?

Chapter 9, Solution 29.

$$\mathbf{Z} = \frac{1}{j\omega C} = \frac{1}{j(10^6)(2 \times 10^{-6})} = -j0.5$$

$$\mathbf{V} = \mathbf{IZ} = (4 \angle 25^\circ)(0.5 \angle -90^\circ) = 2 \angle -65^\circ$$

Therefore $v(t) = \underline{2 \sin(10^6 t - 65^\circ) \text{ V.}}$

Chapter 9, Problem 30.

A voltage $v(t) = 100 \cos(60t + 20^\circ)$ V is applied to a parallel combination of a $40\text{-k}\Omega$ resistor and a $50\text{-}\mu\text{F}$ capacitor. Find the steady-state currents through the resistor and the capacitor.

Chapter 9, Solution 30.

Since R and C are in parallel, they have the same voltage across them. For the resistor,

$$V = I_R R \longrightarrow I_R = V / R = \frac{100 \angle 20^\circ}{40k} = 2.5 \angle 20^\circ \text{ mA}$$
$$i_R = \underline{2.5 \cos(60t + 20^\circ) \text{ mA}}$$

For the capacitor,

$$i_C = C \frac{dv}{dt} = 50 \times 10^{-6} (-60) \times 100 \sin(60t + 20^\circ) = \underline{-300 \sin(60t + 20^\circ) \text{ mA}}$$

Chapter 9, Problem 31.

A series RLC circuit has $R = 80\ \Omega$, $L = 240\text{ mH}$, and $C = 5\text{ mF}$. If the input voltage is $v(t) = 10 \cos 2t$ find the current flowing through the circuit.

Chapter 9, Solution 31.

$$L = 240\text{ mH} \longrightarrow j\omega L = j2 \times 240 \times 10^{-3} = j0.48$$

$$C = 5\text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j2 \times 5 \times 10^{-3}} = -j100$$

$$Z = 80 + j0.48 - j100 = 80 - j99.52$$

$$I = \frac{V}{Z} = \frac{10 \angle 0^\circ}{80 - j99.52} = 0.0783 \angle 51.206^\circ$$

$$i(t) = \underline{78.3 \cos(2t + 51.206^\circ) \text{ mA}} = \underline{78.3 \cos(2t + 51.26^\circ) \text{ mA}}$$

Chapter 9, Problem 32.

For the network in Fig. 9.40, find the load current \mathbf{I}_L .

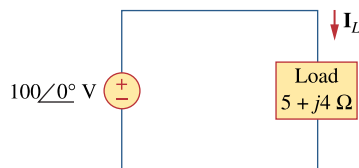


Figure 9.40

For Prob. 9.32.

Chapter 9, Solution 32.

$$I = \frac{V}{Z} = \frac{100 \angle 0^\circ}{5 + j4} = 12.195 - j9.756 = \underline{15.62 \angle -38.66^\circ \text{ A}}$$

Chapter 9, Problem 33.

A series RL circuit is connected to a 110-V ac source. If the voltage across the resistor is 85 V, find the voltage across the inductor.

Chapter 9, Solution 33.

$$\begin{aligned}110 &= \sqrt{v_R^2 + v_L^2} \\v_L &= \sqrt{110^2 - v_R^2} \\v_L &= \sqrt{110^2 - 85^2} = \underline{\underline{69.82 \text{ V}}}\end{aligned}$$

Chapter 9, Problem 34.

What value of ω will cause the forced response v_o in Fig. 9.41 to be zero?

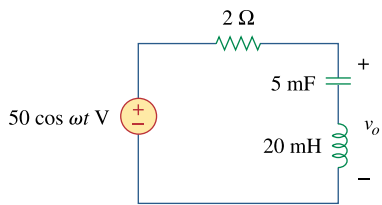


Figure 9.41
For Prob. 9.34.

Chapter 9, Solution 34.

$$\begin{aligned}v_o = 0 \text{ if } \omega L &= \frac{1}{\omega C} \longrightarrow \omega = \frac{1}{\sqrt{LC}} \\ \omega &= \frac{1}{\sqrt{(5 \times 10^{-3})(20 \times 10^{-3})}} = \underline{\underline{100 \text{ rad/s}}}\end{aligned}$$

Chapter 9, Problem 35.

Find current i in the circuit of Fig. 9.42, when $v_s(t) = 50 \cos 200t$ V.

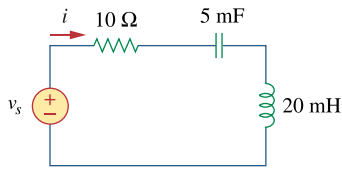


Figure 9.42
For Prob. 9.35.

Chapter 9, Solution 35.

$$v_s(t) = 50 \cos 200t \quad \longrightarrow \quad V_s = 50 \angle 0^\circ, \omega = 200$$

$$5mF \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j200 \times 5 \times 10^{-3}} = -j$$

$$20mH \quad \longrightarrow \quad j\omega L = j20 \times 10^{-3} \times 200 = j4$$

$$Z_{in} = 10 - j + j4 = 10 + j3$$

$$I = \frac{V_s}{Z_{in}} = \frac{50 \angle 0^\circ}{10 + j3} = 4.789 \angle -16.7^\circ$$

$$i(t) = \underline{4.789 \cos(200t - 16.7^\circ) \text{ A}}$$

Chapter 9, Problem 36.

In the circuit of Fig. 9.43, determine i . Let $v_s = 60 \cos(200t - 10^\circ) \text{ V}$.

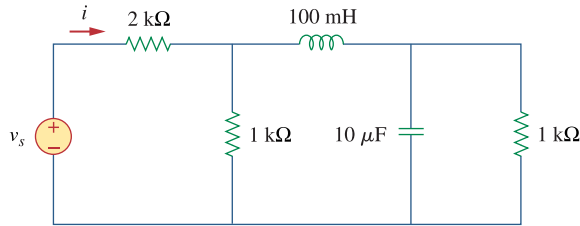


Figure 9.43

For Prob. 9.36.

Chapter 9, Solution 36.

Let Z be the input impedance at the source.

$$100 \text{ mH} \longrightarrow j\omega L = j200 \times 100 \times 10^{-3} = j20$$

$$10 \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10 \times 10^{-6} \times 200} = -j500$$

$$1000 // -j500 = 200 - j400$$

$$1000 // (j20 + 200 - j400) = 242.62 - j239.84$$

$$Z = 2242.62 - j239.84 = 2255 \angle -6.104^\circ$$

$$I = \frac{60 \angle -10^\circ}{2255 \angle -6.104^\circ} = 26.61 \angle -3.896^\circ \text{ mA}$$

$$i = \underline{266.1 \cos(200t - 3.896^\circ) \text{ mA}}$$

Chapter 9, Problem 37.

Determine the admittance \mathbf{Y} for the circuit in Fig. 9.44.

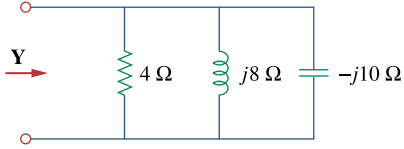


Figure 9.44
For Prob. 9.37.

Chapter 9, Solution 37.

$$Y = \frac{1}{4} + \frac{1}{j8} + \frac{1}{-j10} = \underline{0.25 - j0.025\ \text{S}} = \underline{\underline{250 - j25\ \text{mS}}}$$

Chapter 9, Problem 38.

Find $i(t)$ and $v(t)$ in each of the circuits of Fig. 9.45.

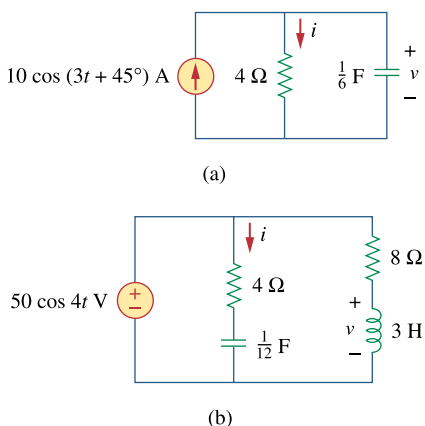


Figure 9.45

For Prob. 9.38.

Chapter 9, Solution 38.

$$(a) \quad \frac{1}{6} \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(1/6)} = -j2$$

$$\mathbf{I} = \frac{-j2}{4 - j2} (10 \angle 45^\circ) = 4.472 \angle -18.43^\circ$$

$$\text{Hence, } i(t) = \underline{\underline{4.472 \cos(3t - 18.43^\circ) \text{ A}}}$$

$$\mathbf{V} = 4\mathbf{I} = (4)(4.472 \angle -18.43^\circ) = 17.89 \angle -18.43^\circ$$

$$\text{Hence, } v(t) = \underline{\underline{17.89 \cos(3t - 18.43^\circ) \text{ V}}}$$

$$(b) \quad \frac{1}{12} \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4)(1/12)} = -j3$$

$$3 \text{ H} \longrightarrow j\omega L = j(4)(3) = j12$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50 \angle 0^\circ}{4 - j3} = 10 \angle 36.87^\circ$$

$$\text{Hence, } i(t) = \underline{\underline{10 \cos(4t + 36.87^\circ) \text{ A}}}$$

$$\mathbf{V} = \frac{j12}{8 + j12} (50 \angle 0^\circ) = 41.6 \angle 33.69^\circ$$

$$\text{Hence, } v(t) = \underline{\underline{41.6 \cos(4t + 33.69^\circ) \text{ V}}}$$

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Chapter 9, Problem 39.

For the circuit shown in Fig. 9.46, find z_{eq} and use that to find current \mathbf{I} . Let $\omega = 10$ rad/s.

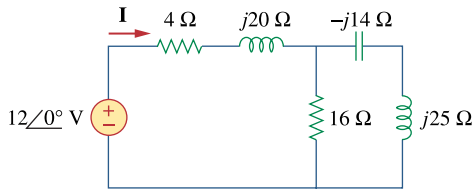


Figure 9.46
For Prob. 9.39.

Chapter 9, Solution 39.

$$Z_{eq} = 4 + j20 + 10 // (-j14 + j25) = \underline{9.135 + j27.47\ \Omega}$$

$$I = \frac{V}{Z_{eq}} = \frac{12}{9.135 + j27.47} = 0.4145 \angle -71.605^\circ$$

$$i(t) = \underline{0.4145 \cos(10t - 71.605^\circ)\ \text{A}} = \underline{414.5 \cos(10t - 71.6^\circ)\ \text{mA}}$$

Chapter 9, Problem 40.

In the circuit of Fig. 9.47, find i_o when:

- (a) $\omega = 1$ rad/s (b) $\omega = 5$ rad/s
(c) $\omega = 10$ rad/s

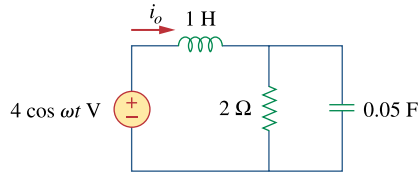


Figure 9.47
For Prob. 9.40.

Chapter 9, Solution 40.

(a) For $\omega = 1$,

$$1 \text{ H} \longrightarrow j\omega L = j(1)(1) = j$$

$$0.05 \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1)(0.05)} = -j20$$

$$\mathbf{Z} = j + 2 \parallel (-j20) = j + \frac{-j40}{2 - j20} = 1.98 + j0.802$$

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{4\angle 0^\circ}{1.98 + j0.802} = \frac{4\angle 0^\circ}{2.136\angle 22.05^\circ} = 1.872\angle -22.05^\circ$$

$$\text{Hence, } i_o(t) = \underline{\underline{1.872 \cos(t - 22.05^\circ) \text{ A}}}$$

(b) For $\omega = 5$,

$$1 \text{ H} \longrightarrow j\omega L = j(5)(1) = j5$$

$$0.05 \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(5)(0.05)} = -j4$$

$$\mathbf{Z} = j5 + 2 \parallel (-j4) = j5 + \frac{-j4}{1 - j2} = 1.6 + j4.2$$

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{4\angle 0^\circ}{1.6 + j4} = \frac{4\angle 0^\circ}{4.494\angle 69.14^\circ} = 0.89\angle -69.14^\circ$$

$$\text{Hence, } i_o(t) = \underline{\underline{0.89 \cos(5t - 69.14^\circ) \text{ A}}}$$

(c) For $\omega = 10$,

$$1 \text{ H} \longrightarrow j\omega L = j(10)(1) = j10$$

$$0.05 \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(0.05)} = -j2$$

$$\mathbf{Z} = j10 + 2 \parallel (-j2) = j10 + \frac{-j4}{2 - j2} = 1 + j9$$

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{4\angle 0^\circ}{1 + j9} = \frac{4\angle 0^\circ}{9.055\angle 83.66^\circ} = 0.4417\angle -83.66^\circ$$

$$\text{Hence, } i_o(t) = \underline{\underline{0.4417 \cos(10t - 83.66^\circ) \text{ A}}}$$

Chapter 9, Problem 41.

Find $v(t)$ in the RLC circuit of Fig. 9.48.

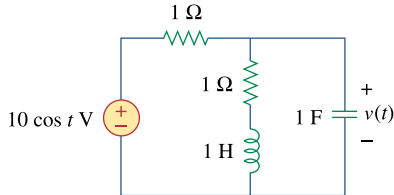


Figure 9.48
For Prob. 9.41.

Chapter 9, Solution 41.

$$\omega = 1,$$

$$1 \text{ H} \longrightarrow j\omega L = j(1)(1) = j$$

$$1 \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1)(1)} = -j$$

$$\mathbf{Z} = 1 + (1 + j) \parallel (-j) = 1 + \frac{-j + 1}{1} = 2 - j$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10}{2 - j}, \quad \mathbf{I}_c = (1 + j)\mathbf{I}$$

$$\mathbf{V} = (-j)(1 + j)\mathbf{I} = (1 - j)\mathbf{I} = \frac{(1 - j)(10)}{2 - j} = 6.325 \angle -18.43^\circ$$

$$\text{Thus,} \quad v(t) = \underline{\underline{6.325 \cos(t - 18.43^\circ) \text{ V}}}$$

Chapter 9, Problem 42.

Calculate $v_o(t)$ in the circuit of Fig. 9.49.

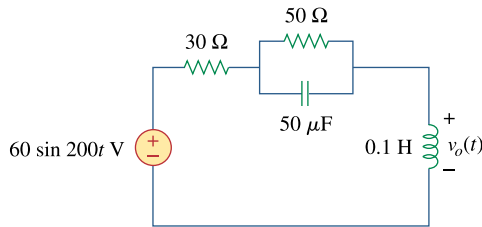


Figure 9.49
For Prob. 9.42.

Chapter 9, Solution 42.

$$\omega = 200$$
$$50 \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(200)(50 \times 10^{-6})} = -j100$$

$$0.1 \text{ H} \longrightarrow j\omega L = j(200)(0.1) = j20$$

$$50 \parallel -j100 = \frac{(50)(-j100)}{50 - j100} = \frac{-j100}{1 - j2} = 40 - j20$$

$$\mathbf{V}_o = \frac{j20}{j20 + 30 + 40 - j20} (60 \angle 0^\circ) = \frac{j20}{70} (60 \angle 0^\circ) = 17.14 \angle 90^\circ$$

Thus, $v_o(t) = \underline{\underline{17.14 \sin(200t + 90^\circ) \text{ V}}}$

or $v_o(t) = \underline{\underline{17.14 \cos(200t) \text{ V}}}$

Chapter 9, Problem 43.

Find current I_o in the circuit shown in Fig. 9.50.

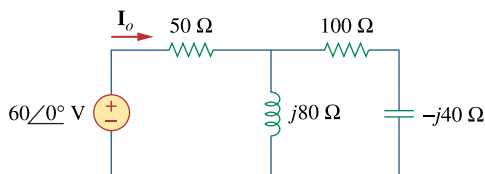


Figure 9.50

For Prob. 9.43.

Chapter 9, Solution 43.

$$Z_{in} = 50 + j80 \parallel (100 - j40) = 50 + \frac{j80(100 - j40)}{100 + j40} = 105.71 + j57.93$$

$$I_o = \frac{60 \angle 0^\circ}{Z_{in}} = 0.4377 - 0.2411j = 0.4997 \angle -28.85^\circ \text{ A} = \underline{\underline{499.7 \angle -28.85^\circ \text{ mA}}}$$

Chapter 9, Problem 44.

Calculate $i(t)$ in the circuit of Fig. 9.51.

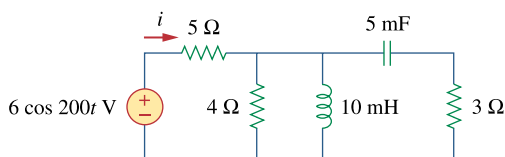


Figure 9.51

For prob. 9.44.

Chapter 9, Solution 44.

$$\omega = 200$$

$$10 \text{ mH} \longrightarrow j\omega L = j(200)(10 \times 10^{-3}) = j2$$

$$5 \text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(200)(5 \times 10^{-3})} = -j$$

$$Y = \frac{1}{4} + \frac{1}{j2} + \frac{1}{3 - j} = 0.25 - j0.5 + \frac{3 + j}{10} = 0.55 - j0.4$$

$$Z = \frac{1}{Y} = \frac{1}{0.55 - j0.4} = 1.1892 + j0.865$$

$$I = \frac{6 \angle 0^\circ}{5 + Z} = \frac{6 \angle 0^\circ}{6.1892 + j0.865} = 0.96 \angle -7.956^\circ$$

$$\text{Thus, } i(t) = \underline{\underline{0.96 \cos(200t - 7.956^\circ) \text{ A}}}$$

Chapter 9, Problem 45.



Find current \mathbf{I}_o in the network of Fig. 9.52.

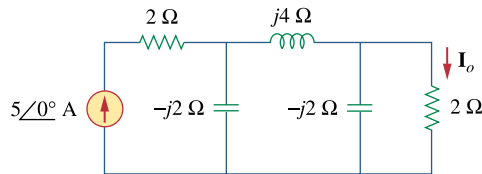
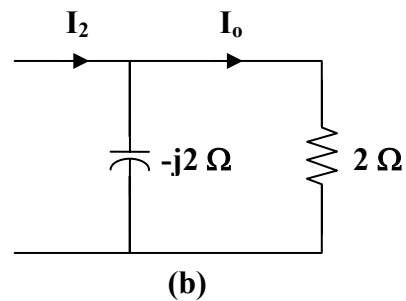
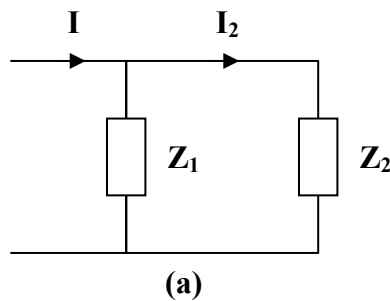


Figure 9.52

For Prob. 9.45.

Chapter 9, Solution 45.

We obtain \mathbf{I}_o by applying the principle of current division twice.



$$\mathbf{Z}_1 = -j2, \quad \mathbf{Z}_2 = j4 + (-j2) \parallel 2 = j4 + \frac{-j4}{2 - j2} = 1 + j3$$

$$\mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} \mathbf{I} = \frac{-j2}{-j2 + 1 + j3} (5 \angle 0^\circ) = \frac{-j10}{1 + j}$$

$$\mathbf{I}_o = \frac{-j2}{2 - j2} \mathbf{I}_2 = \left(\frac{-j}{1 - j} \right) \left(\frac{-j10}{1 + j} \right) = \frac{-10}{1 + 1} = \underline{\underline{-5 \text{ A}}}$$

Chapter 9, Problem 46.



If $i_s = 5 \cos(10t + 40^\circ)$ A in the circuit of Fig. 9.53, find i_o .

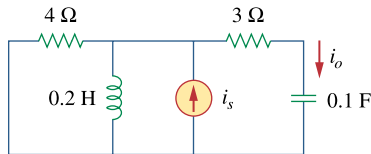


Figure 9.53

For Prob. 9.46.

Chapter 9, Solution 46.

$$i_s = 5 \cos(10t + 40^\circ) \longrightarrow \mathbf{I}_s = 5 \angle 40^\circ$$

$$0.1 \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(0.1)} = -j$$

$$0.2 \text{ H} \longrightarrow j\omega L = j(10)(0.2) = j2$$

$$\text{Let} \quad \mathbf{Z}_1 = 4 \parallel j2 = \frac{j8}{4 + j2} = 0.8 + j1.6, \quad \mathbf{Z}_2 = 3 - j$$

$$\mathbf{I}_o = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} \mathbf{I}_s = \frac{0.8 + j1.6}{3.8 + j0.6} (5 \angle 40^\circ)$$

$$\mathbf{I}_o = \frac{(1.789 \angle 63.43^\circ)(5 \angle 40^\circ)}{3.847 \angle 8.97^\circ} = 2.325 \angle 94.46^\circ$$

$$\text{Thus, } i_o(t) = \underline{\underline{2.325 \cos(10t + 94.46^\circ) \text{ A}}}$$

Chapter 9, Problem 47.

In the circuit of Fig. 9.54, determine the value of $i_s(t)$.

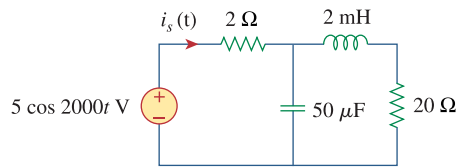
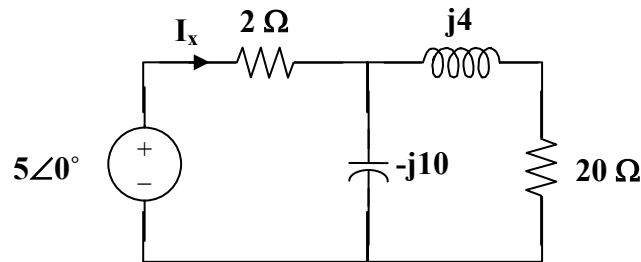


Figure 9.54
For Prob. 9.47.

Chapter 9, Solution 47.

First, we convert the circuit into the frequency domain.



$$I_x = \frac{5}{2 + \frac{-j10(20 + j4)}{-j10 + 20 + j4}} = \frac{5}{2 + 4.588 - j8.626} = \frac{5}{10.854 \angle -52.63^\circ} = 0.4607 \angle 52.63^\circ$$

$$i_s(t) = \underline{\underline{460.7 \cos(2000t + 52.63^\circ) \text{ mA}}}$$

Chapter 9, Problem 48.



Given that $v_s(t) = 20 \sin(100t - 40^\circ)$ in Fig. 9.55, determine $i_x(t)$.

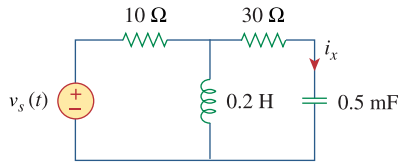
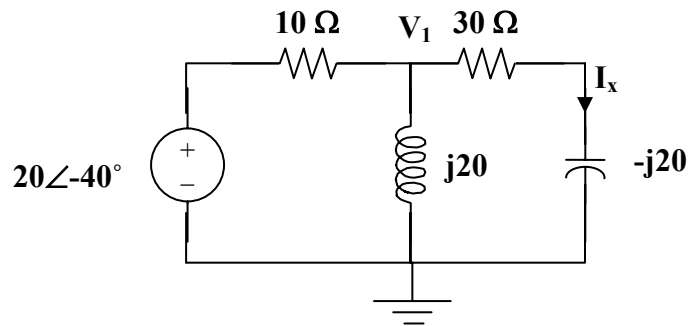


Figure 9.55

For Prob. 9.48.

Chapter 9, Solution 48.

Converting the circuit to the frequency domain, we get:



We can solve this using nodal analysis.

$$\frac{V_1 - 20\angle -40^\circ}{10} + \frac{V_1 - 0}{j20} + \frac{V_1 - 0}{30 - j20} = 0$$

$$V_1(0.1 - j0.05 + 0.02307 + j0.01538) = 2\angle -40^\circ$$

$$V_1 = \frac{2\angle 40^\circ}{0.12307 - j0.03462} = 15.643\angle -24.29^\circ$$

$$I_x = \frac{15.643\angle -24.29^\circ}{30 - j20} = 0.4338\angle 9.4^\circ$$

$$i_x = \underline{0.4338\sin(100t + 9.4^\circ) \text{ A}}$$

Chapter 9, Problem 49.

Find $v_s(t)$ in the circuit of Fig. 9.56 if the current i_x through the $1\text{-}\Omega$ resistor is $0.5 \sin 200t$ A.

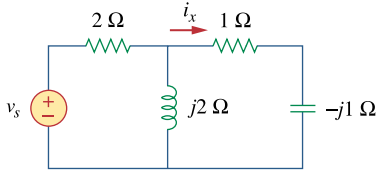
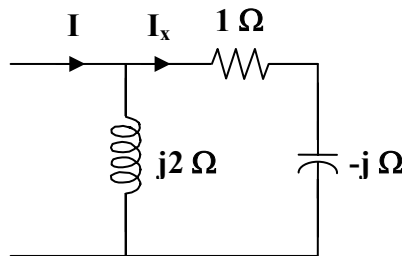


Figure 9.56

For Prob. 9.49.

Chapter 9, Solution 49.

$$\mathbf{Z}_T = 2 + j2 \parallel (1 - j) = 2 + \frac{(j2)(1 - j)}{1 + j} = 4$$



$$\mathbf{I}_x = \frac{j2}{j2 + 1 - j} \mathbf{I} = \frac{j2}{1 + j} \mathbf{I}, \quad \text{where } \mathbf{I}_x = 0.5 \angle 0^\circ = \frac{1}{2}$$

$$\mathbf{I} = \frac{1 + j}{j2} \mathbf{I}_x = \frac{1 + j}{j4}$$

$$\mathbf{V}_s = \mathbf{I} \mathbf{Z}_T = \frac{1 + j}{j4} (4) = \frac{1 + j}{j} = 1 - j = 1.414 \angle -45^\circ$$

$$v_s(t) = \underline{\underline{1.414 \sin(200t - 45^\circ) \text{ V}}}$$

Chapter 9, Problem 50.

Determine v_x in the circuit of Fig. 9.57. Let $i_s(t) = 5 \cos(100t + 40^\circ) \text{ A}$.

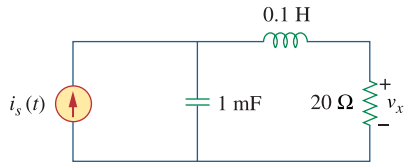
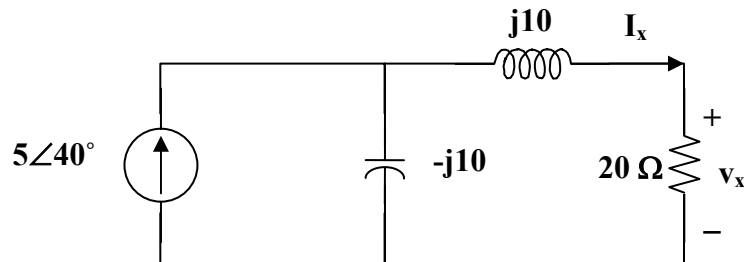


Figure 9.57
For Prob. 9.50.

Chapter 9, Solution 50.

Since $\omega = 100$, the inductor $= j100 \times 0.1 = j10 \Omega$ and the capacitor $= 1/(j100 \times 10^{-3}) = -j10 \Omega$.



Using the current dividing rule:

$$I_x = \frac{-j10}{-j10 + 20 + j10} 5 \angle 40^\circ = -j2.5 \angle 40^\circ = 2.5 \angle -50^\circ$$

$$V_x = 20 I_x = 50 \angle -50^\circ$$

$$v_x = \underline{50 \cos(100t - 50^\circ) \text{ V}}$$

Chapter 9, Problem 51.

If the voltage v_o across the $2\text{-}\Omega$ resistor in the circuit of Fig. 9.58 is $10 \cos 2t \text{ V}$, obtain i_s .

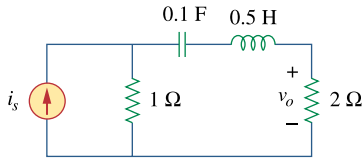


Figure 9.58

For Prob. 9.51.

Chapter 9, Solution 51.

$$0.1\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(0.1)} = -j5$$

$$0.5\text{ H} \longrightarrow j\omega L = j(2)(0.5) = j$$

The current \mathbf{I} through the $2\text{-}\Omega$ resistor is

$$\mathbf{I} = \frac{1}{1 - j5 + j + 2} \mathbf{I}_s = \frac{\mathbf{I}_s}{3 - j4},$$

$$\mathbf{I}_s = (5)(3 - j4) = 25 \angle -53.13^\circ$$

$$\text{where } \mathbf{I} = \frac{10}{2} \angle 0^\circ = 5$$

Therefore,

$$i_s(t) = \underline{\underline{25 \cos(2t - 53.13^\circ) \text{ A}}}$$

Chapter 9, Problem 52.

If $V_o = 8 \angle 30^\circ \text{ V}$ in the circuit of Fig. 9.59, find I_s .

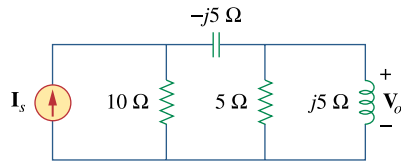


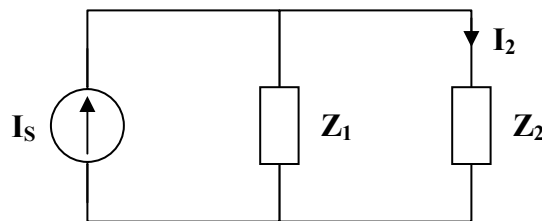
Figure 9.59

For Prob. 9.52.

Chapter 9, Solution 52.

$$5 \parallel j5 = \frac{j25}{5 + j5} = \frac{j5}{1 + j} = 2.5 + j2.5$$

$$Z_1 = 10, \quad Z_2 = -j5 + 2.5 + j2.5 = 2.5 - j2.5$$



$$I_2 = \frac{Z_1}{Z_1 + Z_2} I_s = \frac{10}{12.5 - j2.5} I_s = \frac{4}{5 - j} I_s$$

$$V_o = I_2 (2.5 + j2.5)$$

$$8 \angle 30^\circ = \left(\frac{4}{5 - j} \right) I_s (2.5)(1 + j) = \frac{10(1 + j)}{5 - j} I_s$$

$$I_s = \frac{(8 \angle 30^\circ)(5 - j)}{10(1 + j)} = \underline{\underline{2.884 \angle -26.31^\circ \text{ A}}}$$

Chapter 9, Problem 53.



Find I_o in the circuit of Fig. 9.60.

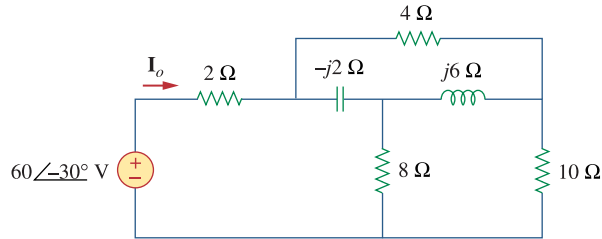
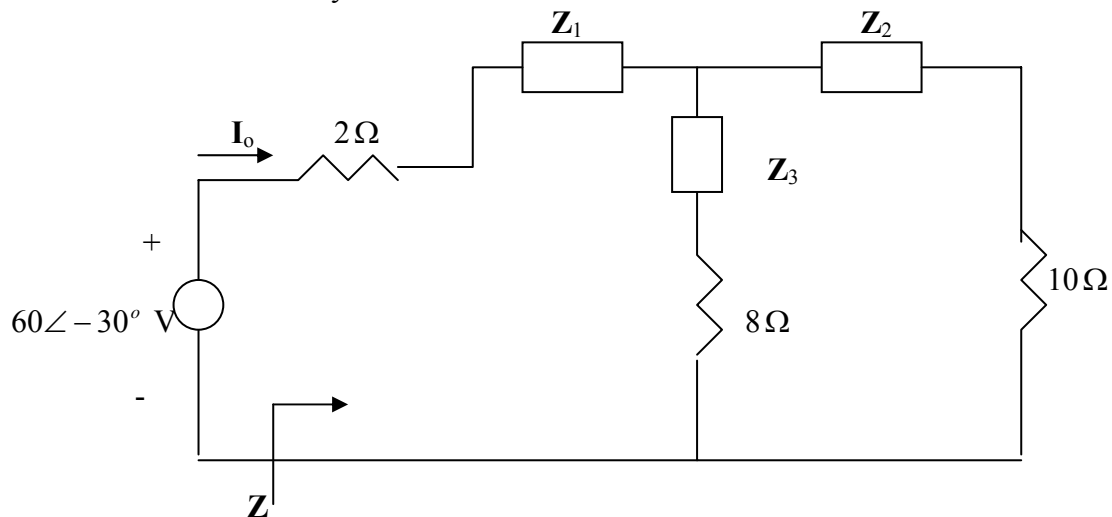


Figure 9.60

For Prob. 9.53.

Chapter 9, Solution 53.

Convert the delta to wye subnetwork as shown below.



$$Z_1 = \frac{-j2 \times 4}{4 + j4} = \frac{8 \angle -90^\circ}{5.6569 \angle 45^\circ} = -1 - j1, \quad Z_2 = \frac{j6 \times 4}{4 + j4} = 3 + j3,$$

$$Z_3 = \frac{12}{4 + j4} = 1.5 - j1.5$$

$$(Z_3 + 8) \parallel (Z_2 + 10) = (9.5 - j1.5) \parallel (13 + j3) = 5.691 \angle 0.21^\circ = 5.691 + j0.02086$$

$$Z = 2 + Z_1 + 5.691 + j0.02086 = 6.691 - j0.9791$$

$$I_o = \frac{60 \angle -30^\circ}{Z} = \frac{60 \angle -30^\circ}{6.7623 \angle -8.33^\circ} = 8.873 \angle -21.67^\circ \text{ A}$$

Chapter 9, Problem 54.



In the circuit of Fig. 9.61, find \mathbf{V}_s if $\mathbf{I}_o = 2 \angle 0^\circ \text{ A}$.

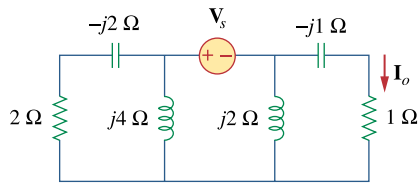
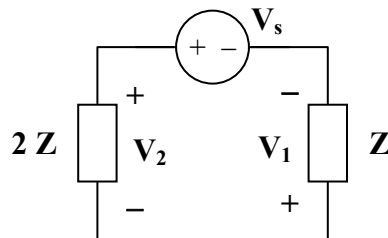


Figure 9.61

For Prob. 9.54.

Chapter 9, Solution 54.

Since the left portion of the circuit is twice as large as the right portion, the equivalent circuit is shown below.



$$\mathbf{V}_1 = \mathbf{I}_o(1 - j) = 2(1 - j)$$

$$\mathbf{V}_2 = 2\mathbf{V}_1 = 4(1 - j)$$

$$\mathbf{V}_s = -\mathbf{V}_1 - \mathbf{V}_2 = -6(1 - j)$$

$$\mathbf{V}_s = \underline{\underline{8.485 \angle -135^\circ \text{ V}}}$$

Chapter 9, Problem 55.



* Find \mathbf{Z} in the network of Fig. 9.62, given that $\mathbf{V}_o = 4 \angle 0^\circ \text{ V}$.

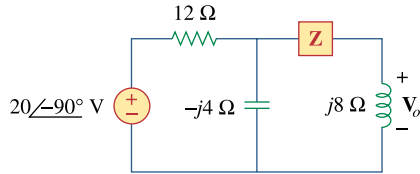
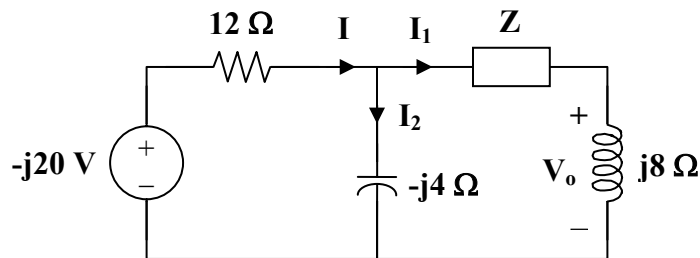


Figure 9.62

For Prob. 9.55.

* An asterisk indicates a challenging problem.

Chapter 9, Solution 55.



$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{j8} = \frac{4}{j8} = -j0.5$$

$$\mathbf{I}_2 = \frac{\mathbf{I}_1(\mathbf{Z} + j8)}{-j4} = \frac{(-j0.5)(\mathbf{Z} + j8)}{-j4} = \frac{\mathbf{Z}}{8} + j$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = -j0.5 + \frac{\mathbf{Z}}{8} + j = \frac{\mathbf{Z}}{8} + j0.5$$

$$-j20 = 12\mathbf{I} + \mathbf{I}_1(\mathbf{Z} + j8)$$

$$-j20 = 12\left(\frac{\mathbf{Z}}{8} + \frac{j}{2}\right) + \frac{-j}{2}(\mathbf{Z} + j8)$$

$$-4 - j26 = \mathbf{Z}\left(\frac{3}{2} - j\frac{1}{2}\right)$$

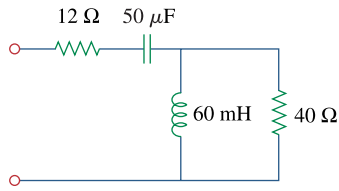
$$\mathbf{Z} = \frac{-4 - j26}{\frac{3}{2} - j\frac{1}{2}} = \frac{26.31 \angle 261.25^\circ}{1.5811 \angle -18.43^\circ} = 16.64 \angle 279.68^\circ$$

$$\mathbf{Z} = \underline{\underline{2.798 - j16.403 \, \Omega}}$$

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Chapter 9, Problem 56.

At $\omega = 377$ rad/s, find the input impedance of the circuit shown in Fig. 9.63.

**Figure 9.63**

For Prob. 9.56.

Chapter 9, Solution 56.

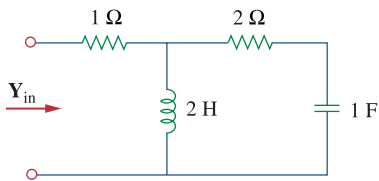
$$50\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j377 \times 50 \times 10^{-6}} = -j53.05$$

$$60mH \longrightarrow j\omega L = j377 \times 60 \times 10^{-3} = j22.62$$

$$Z_{in} = 12 - j53.05 + j22.62 // 40 = \underline{21.692 - j35.91 \Omega}$$

Chapter 9, Problem 57.

At $\omega = 1$ rad/s, obtain the input admittance in the circuit of Fig. 9.64.

**Figure 9.64**

For Prob. 9.57.

Chapter 9, Solution 57.

$$2H \longrightarrow j\omega L = j2$$

$$1F \longrightarrow \frac{1}{j\omega C} = -j$$

$$Z = 1 + j2 // (2 - j) = 1 + \frac{j2(2 - j)}{j2 + 2 - j} = 2.6 + j1.2$$

$$Y = 1/Z = \underline{0.3171 - j0.1463 \text{ S}}$$

Chapter 9, Problem 58.

Find the equivalent impedance in Fig. 9.65 at $\omega = 10$ krad/s.

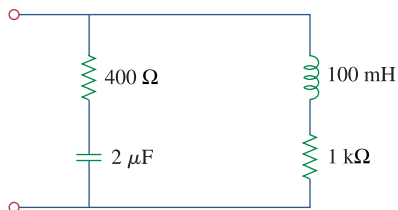


Figure 9.65

For Prob. 9.58.

Chapter 9, Solution 58.

$$2\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10^4 \times 2 \times 10^{-6}} = -j50$$

$$100\text{mH} \longrightarrow j\omega L = j10^4 \times 100 \times 10^{-3} = j1000$$

$$Z_{in} = (400 - j50) \parallel (1000 + j1000) = \frac{(400 - j50)(1000 + j1000)}{1400 + j950} = \underline{336.24 + j21.83 \Omega}$$

Chapter 9, Problem 59.

For the network in Fig. 9.66, find Z_{in} . Let $\omega = 10$ rad/s.

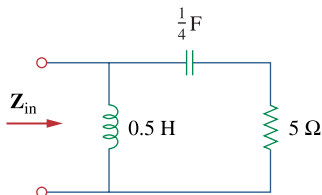


Figure 9.66

For Prob. 9.59.

Chapter 9, Solution 59.

$$0.25F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10 \times 0.25} = -j0.4$$

$$0.5H \longrightarrow j\omega L = j10 \times 0.5 = j5$$

$$Z_{in} = j5 \parallel (5 - j0.4) = \frac{(5 \angle 90^\circ)(5.016 \angle -4.57^\circ)}{6.794 \angle 42.61^\circ} = 3.691 \angle 42.82^\circ$$

$$= \underline{2.707 + j2.509 \Omega}.$$

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Chapter 9, Problem 60.

Obtain Z_{in} for the circuit in Fig. 9.67.

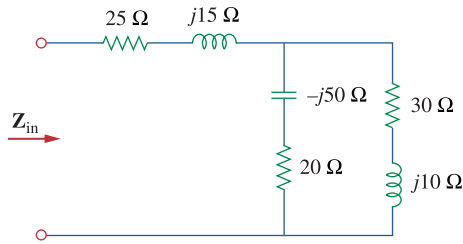


Figure 9.67

For Prob. 9.60.

Chapter 9, Solution 60.

$$Z = (25 + j15) + (20 - j50) // (30 + j10) = 25 + j15 + 26.097 - j5.122 = \underline{51.1 + j9.878\Omega}$$

Chapter 9, Problem 61.

Find Z_{eq} in the circuit of Fig. 9.68.

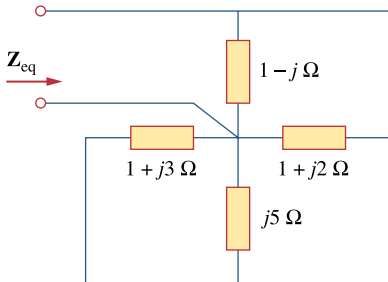


Figure 9.68

For Prob. 9.61.

Chapter 9, Solution 61.

All of the impedances are in parallel.

$$\frac{1}{Z_{eq}} = \frac{1}{1-j} + \frac{1}{1+j2} + \frac{1}{j5} + \frac{1}{1+j3}$$

$$\frac{1}{Z_{eq}} = (0.5 + j0.5) + (0.2 - j0.4) + (-j0.2) + (0.1 - j0.3) = 0.8 - j0.4$$

$$Z_{eq} = \frac{1}{0.8 - j0.4} = \underline{1 + j0.5\Omega}$$

Chapter 9, Problem 62.

For the circuit in Fig. 9.69, find the input impedance Z_{in} at 10 krad/s.

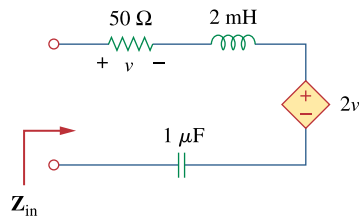
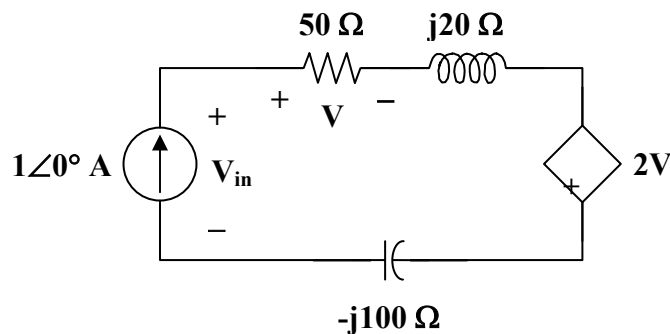


Figure 9.69
For Prob. 9.62.

Chapter 9, Solution 62.

$$2 \text{ mH} \longrightarrow j\omega L = j(10 \times 10^3)(2 \times 10^{-3}) = j20$$

$$1 \text{ } \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10 \times 10^3)(1 \times 10^{-6})} = -j100$$



$$V = (1 \angle 0^\circ)(50) = 50$$

$$V_{in} = (1 \angle 0^\circ)(50 + j20 - j100) + (2)(50)$$

$$V_{in} = 50 - j80 + 100 = 150 - j80$$

$$Z_{in} = \frac{V_{in}}{1 \angle 0^\circ} = \underline{\underline{150 - j80 \text{ } \Omega}}$$

Chapter 9, Problem 63.



For the circuit in Fig. 9.70, find the value of Z_T .

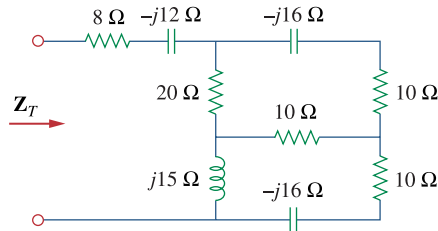


Figure 9.70

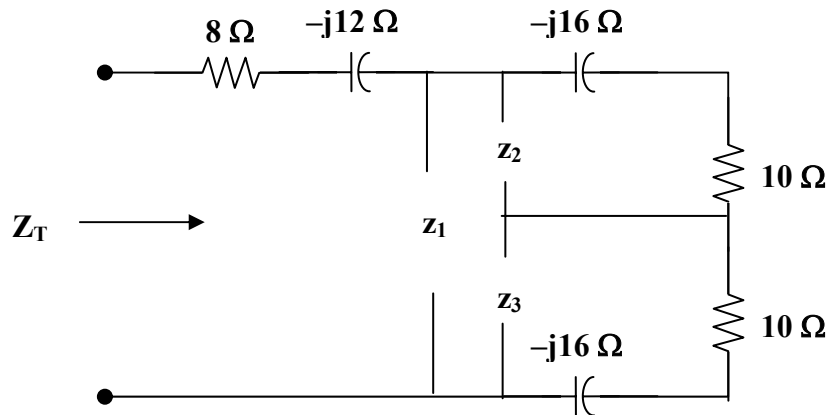
For Prob. 9.63.

Chapter 9, Solution 63.

First, replace the wye composed of the 20-ohm, 10-ohm, and $j15$ -ohm impedances with the corresponding delta.

$$z_1 = \frac{200 + j150 + j300}{10} = 20 + j45$$

$$z_2 = \frac{200 + j450}{j15} = 30 - j13.333, \quad z_3 = \frac{200 + j450}{20} = 10 + j22.5$$



Now all we need to do is to combine impedances.

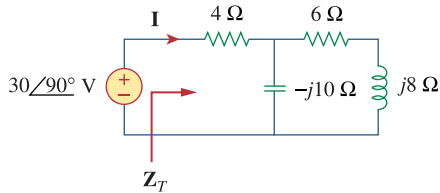
$$z_2 \parallel (10 - j16) = \frac{(30 - j13.333)(10 - j16)}{40 - j29.33} = 8.721 - j8.938$$

$$z_3 \parallel (10 - j16) = 21.70 - j3.821$$

$$Z_T = 8 - j12 + z_1 \parallel (8.721 - j8.938 + 21.7 - j3.821) = \underline{34.69 - j6.93 \Omega}$$

Chapter 9, Problem 64.

Find \mathbf{Z}_T and \mathbf{I} in the circuit of Fig. 9.71.

**Figure 9.71**

For Prob. 9.64.

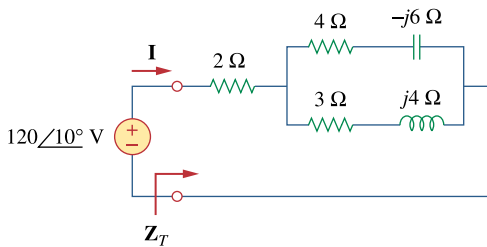
Chapter 9, Solution 64.

$$\mathbf{Z}_T = 4 + \frac{-j10(6 + j8)}{6 - j2} = \underline{19 - j5\Omega}$$

$$\mathbf{I} = \frac{30\angle 90^\circ}{\mathbf{Z}_T} = -0.3866 + j1.4767 = \underline{1.527\angle 104.7^\circ \text{ A}}$$

Chapter 9, Problem 65.

Determine \mathbf{Z}_T and \mathbf{I} for the circuit in Fig. 9.72.

**Figure 9.72**

For Prob. 9.65.

Chapter 9, Solution 65.

$$\mathbf{Z}_T = 2 + (4 - j6) \parallel (3 + j4)$$

$$\mathbf{Z}_T = 2 + \frac{(4 - j6)(3 + j4)}{7 - j2}$$

$$\mathbf{Z}_T = \underline{6.83 + j1.094\Omega} = 6.917\angle 9.1^\circ \Omega$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{120\angle 10^\circ}{6.917\angle 9.1^\circ} = \underline{17.35\angle 0.9^\circ \text{ A}}$$

Chapter 9, Problem 66.

For the circuit in Fig. 9.73, calculate \mathbf{Z}_T and \mathbf{V}_{ab} .

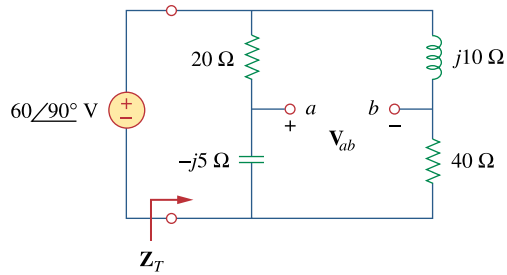


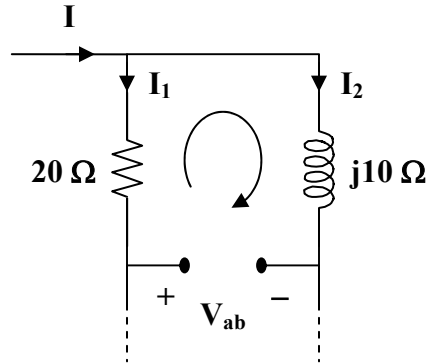
Figure 9.73
For Prob. 9.66.

Chapter 9, Solution 66.

$$\mathbf{Z}_T = (20 - j5) \parallel (40 + j10) = \frac{(20 - j5)(40 + j10)}{60 + j5} = \frac{170}{145}(12 - j)$$

$$\mathbf{Z}_T = \underline{\underline{14.069 - j1.172 \, \Omega}} = 14.118 \angle -4.76^\circ$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{60 \angle 90^\circ}{14.118 \angle -4.76^\circ} = 4.25 \angle 94.76^\circ$$



$$\mathbf{I}_1 = \frac{40 + j10}{60 + j5} \mathbf{I} = \frac{8 + j2}{12 + j} \mathbf{I}$$

$$\mathbf{I}_2 = \frac{20 - j5}{60 + j5} \mathbf{I} = \frac{4 - j}{12 + j} \mathbf{I}$$

$$\mathbf{V}_{ab} = -20\mathbf{I}_1 + j10\mathbf{I}_2$$

$$\mathbf{V}_{ab} = \frac{-(160 + j40)}{12 + j} \mathbf{I} + \frac{10 + j40}{12 + j} \mathbf{I}$$

$$\mathbf{V}_{ab} = \frac{-150}{12 + j} \mathbf{I} = \frac{(-12 + j)(150)}{145} \mathbf{I}$$

$$\mathbf{V}_{ab} = (12.457 \angle 175.24^\circ)(4.25 \angle 97.76^\circ)$$

$$\mathbf{V}_{ab} = \underline{\underline{52.94 \angle 273^\circ \text{ V}}}$$

Chapter 9, Problem 67.

At $\omega = 10^3$ rad/s find the input admittance of each of the circuits in Fig. 9.74.

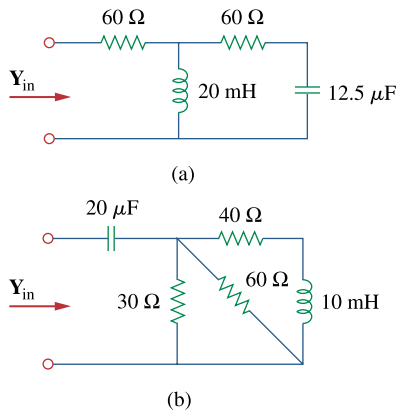


Figure 9.74

For Prob. 9.67.

Chapter 9, Solution 67.

$$\begin{aligned} \text{(a)} \quad 20 \text{ mH} &\longrightarrow j\omega L = j(10^3)(20 \times 10^{-3}) = j20 \\ 12.5 \mu\text{F} &\longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(12.5 \times 10^{-6})} = -j80 \end{aligned}$$

$$Z_{\text{in}} = 60 + j20 \parallel (60 - j80)$$

$$Z_{\text{in}} = 60 + \frac{(j20)(60 - j80)}{60 - j60}$$

$$Z_{\text{in}} = 63.33 + j23.33 = 67.494 \angle 20.22^\circ$$

$$Y_{\text{in}} = \frac{1}{Z_{\text{in}}} = \underline{\underline{14.8 \angle -20.22^\circ \text{ mS}}}$$

$$\begin{aligned} \text{(b)} \quad 10 \text{ mH} &\longrightarrow j\omega L = j(10^3)(10 \times 10^{-3}) = j10 \\ 20 \mu\text{F} &\longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(20 \times 10^{-6})} = -j50 \end{aligned}$$

$$30 \parallel 60 = 20$$

$$Z_{\text{in}} = -j50 + 20 \parallel (40 + j10)$$

$$Z_{\text{in}} = -j50 + \frac{(20)(40 + j10)}{60 + j10}$$

$$Z_{\text{in}} = 13.5 - j48.92 = 50.75 \angle -74.56^\circ$$

$$Y_{\text{in}} = \frac{1}{Z_{\text{in}}} = \underline{\underline{19.7 \angle 74.56^\circ \text{ mS}}} = 5.24 + j18.99 \text{ mS}$$

Chapter 9, Problem 68.

Determine Y_{eq} for the circuit in Fig. 9.75.

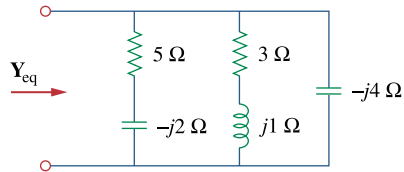


Figure 9.75
For Prob. 9.68.

Chapter 9, Solution 68.

$$Y_{eq} = \frac{1}{5 - j2} + \frac{1}{3 + j} + \frac{1}{-j4}$$

$$Y_{eq} = (0.1724 + j0.069) + (0.3 - j0.1) + (j0.25)$$

$$Y_{eq} = \underline{\underline{0.4724 + j0.219\text{ S}}}$$

Chapter 9, Problem 69.

Find the equivalent admittance Y_{eq} of the circuit in Fig. 9.76.

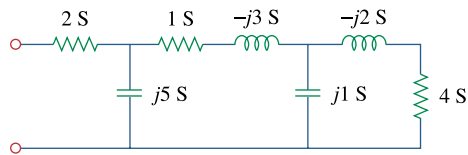


Figure 9.76

For Prob. 9.69.

Chapter 9, Solution 69.

$$\frac{1}{Y_o} = \frac{1}{4} + \frac{1}{-j2} = \frac{1}{4}(1 + j2)$$

$$Y_o = \frac{4}{1 + j2} = \frac{(4)(1 - j2)}{5} = 0.8 - j1.6$$

$$Y_o + j = 0.8 - j0.6$$

$$\frac{1}{Y_o'} = \frac{1}{1} + \frac{1}{-j3} + \frac{1}{0.8 - j0.6} = (1) + (j0.333) + (0.8 + j0.6)$$

$$\frac{1}{Y_o'} = 1.8 + j0.933 = 2.028 \angle 27.41^\circ$$

$$Y_o' = 0.4932 \angle -27.41^\circ = 0.4378 - j0.2271$$

$$Y_o' + j5 = 0.4378 + j4.773$$

$$\frac{1}{Y_{eq}} = \frac{1}{2} + \frac{1}{0.4378 + j4.773} = 0.5 + \frac{0.4378 - j4.773}{22.97}$$

$$\frac{1}{Y_{eq}} = 0.5191 - j0.2078$$

$$Y_{eq} = \frac{0.5191 - j0.2078}{0.3126} = \underline{\underline{1.661 + j0.6647 \text{ S}}}$$

Chapter 9, Problem 70.



Find the equivalent impedance of the circuit in Fig. 9.77.

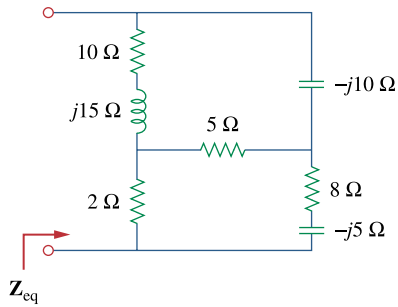
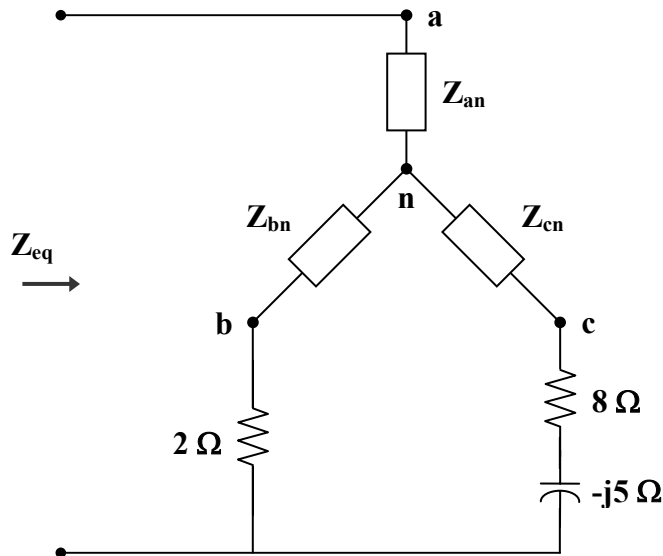


Figure 9.77
For Prob. 9.70.

Chapter 9, Solution 70.

Make a delta-to-wye transformation as shown in the figure below.



$$Z_{an} = \frac{(-j10)(10 + j15)}{5 - j10 + 10 + j15} = \frac{(10)(15 - j10)}{15 + j5} = 7 - j9$$

$$Z_{bn} = \frac{(5)(10 + j15)}{15 + j5} = 4.5 + j3.5$$

$$Z_{cn} = \frac{(5)(-j10)}{15 + j5} = -1 - j3$$

$$Z_{eq} = Z_{an} + (Z_{bn} + 2) \parallel (Z_{cn} + 8 - j5)$$

$$Z_{eq} = 7 - j9 + (6.5 + j3.5) \parallel (7 - j8)$$

$$Z_{eq} = 7 - j9 + \frac{(6.5 + j3.5)(7 - j8)}{13.5 - j4.5}$$

$$Z_{eq} = 7 - j9 + 5.511 - j0.2$$

$$Z_{eq} = 12.51 - j9.2 = \underline{\underline{15.53 \angle -36.33^\circ \Omega}}$$

Chapter 9, Problem 71.



Obtain the equivalent impedance of the circuit in Fig. 9.78.

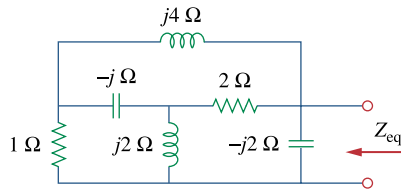
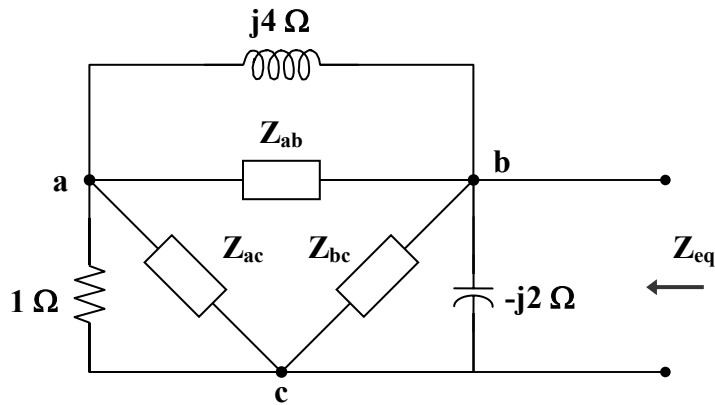


Figure 9.78

For Prob. 9.71.

Chapter 9, Solution 71.

We apply a wye-to-delta transformation.



$$Z_{ab} = \frac{2 - j2 + j4}{j2} = \frac{2 + j2}{j2} = 1 - j$$

$$Z_{ac} = \frac{2 + j2}{2} = 1 + j$$

$$Z_{bc} = \frac{2 + j2}{-j} = -2 + j2$$

$$j4 \parallel Z_{ab} = j4 \parallel (1 - j) = \frac{(j4)(1 - j)}{1 + j3} = 1.6 - j0.8$$

$$1 \parallel Z_{ac} = 1 \parallel (1 + j) = \frac{(1)(1 + j)}{2 + j} = 0.6 + j0.2$$

$$j4 \parallel Z_{ab} + 1 \parallel Z_{ac} = 2.2 - j0.6$$

$$\frac{1}{Z_{eq}} = \frac{1}{-j2} + \frac{1}{-2 + j2} + \frac{1}{2.2 - j0.6}$$

$$= j0.5 - 0.25 - j0.25 + 0.4231 + j0.1154$$

$$= 0.173 + j0.3654 = 0.4043 \angle 64.66^\circ$$

$$Z_{eq} = 2.473 \angle -64.66^\circ \Omega = \underline{\underline{1.058 - j2.235 \Omega}}$$

Chapter 9, Problem 72.



Calculate the value of \mathbf{Z}_{ab} in the network of Fig. 9.79.

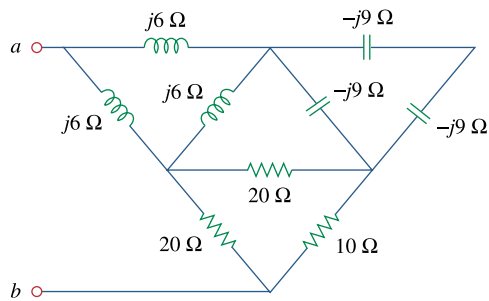
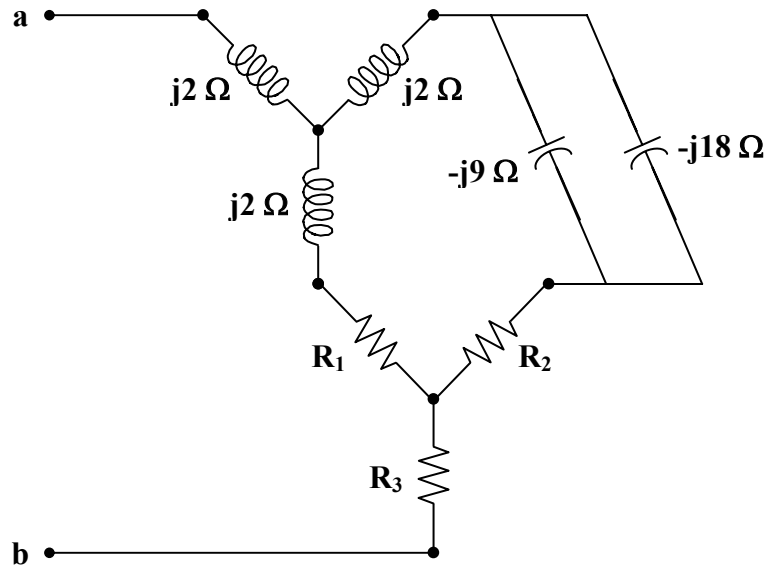


Figure 9.79

For Prob. 9.72.

Chapter 9, Solution 72.

Transform the delta connections to wye connections as shown below.



$$-j9 \parallel -j18 = -j6,$$

$$R_1 = \frac{(20)(20)}{20 + 20 + 10} = 8 \Omega,$$

$$R_2 = \frac{(20)(10)}{50} = 4 \Omega,$$

$$R_3 = \frac{(20)(10)}{50} = 4 \Omega$$

$$Z_{ab} = j2 + (j2 + 8) \parallel (j2 - j6 + 4) + 4$$

$$Z_{ab} = 4 + j2 + (8 + j2) \parallel (4 - j4)$$

$$Z_{ab} = 4 + j2 + \frac{(8 + j2)(4 - j4)}{12 - j2}$$

$$Z_{ab} = 4 + j2 + 3.567 - j1.4054$$

$$Z_{ab} = \underline{\underline{7.567 + j0.5946 \Omega}}$$

Chapter 9, Problem 73.



Determine the equivalent impedance of the circuit in Fig. 9.80.

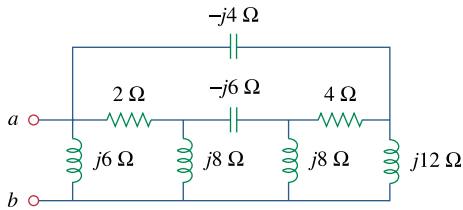
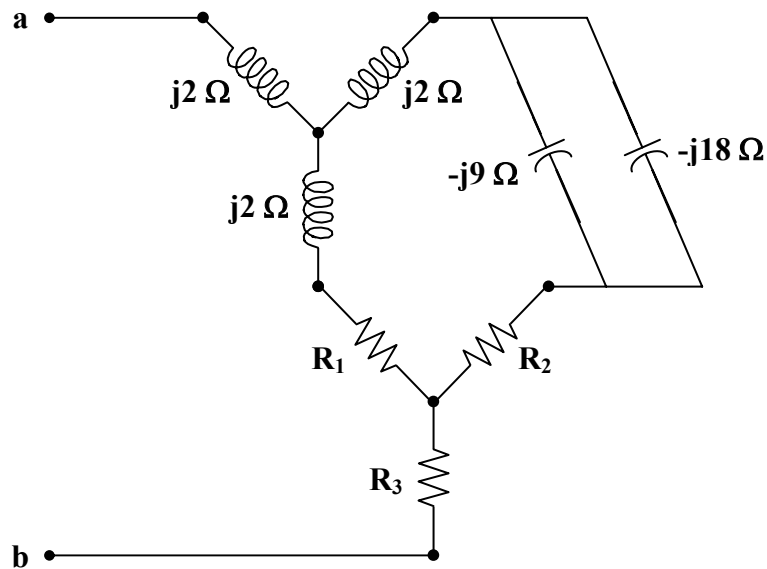


Figure 9.80

For Prob. 9.73.

Chapter 9, Solution 73.

Transform the delta connection to a wye connection as in Fig. (a) and then transform the wye connection to a delta connection as in Fig. (b).



$$\mathbf{Z}_1 = \frac{(j8)(-j6)}{j8 + j8 - j6} = \frac{48}{j10} = -j4.8$$

$$\mathbf{Z}_2 = \mathbf{Z}_1 = -j4.8$$

$$\mathbf{Z}_3 = \frac{(j8)(j8)}{j10} = \frac{-64}{j10} = j6.4$$

$$(2 + \mathbf{Z}_1)(4 + \mathbf{Z}_2) + (4 + \mathbf{Z}_2)(\mathbf{Z}_3) + (2 + \mathbf{Z}_1)(\mathbf{Z}_3) = \\ (2 - j4.8)(4 - j4.8) + (4 - j4.8)(j6.4) + (2 - j4.8)(j6.4) = 46.4 + j9.6$$

$$\mathbf{Z}_a = \frac{46.4 + j9.6}{j6.4} = 1.5 - j7.25$$

$$\mathbf{Z}_b = \frac{46.4 + j9.6}{4 - j4.8} = 3.574 + j6.688$$

$$\mathbf{Z}_c = \frac{46.4 + j9.6}{2 - j4.8} = 1.727 + j8.945$$

$$j6 \parallel \mathbf{Z}_b = \frac{(6 \angle 90^\circ)(7.583 \angle 61.88^\circ)}{3.574 + j12.688} = 0.7407 + j3.3716$$

$$-j4 \parallel \mathbf{Z}_a = \frac{(-j4)(1.5 - j7.25)}{1.5 - j11.25} = 0.186 - j2.602$$

$$j12 \parallel \mathbf{Z}_c = \frac{(12 \angle 90^\circ)(9.11 \angle 79.07^\circ)}{1.727 + j20.945} = 0.5634 + j5.1693$$

$$\mathbf{Z}_{eq} = (j6 \parallel \mathbf{Z}_b) \parallel (-j4 \parallel \mathbf{Z}_a + j12 \parallel \mathbf{Z}_c)$$

$$\mathbf{Z}_{eq} = (0.7407 + j3.3716) \parallel (0.7494 + j2.5673)$$

$$\mathbf{Z}_{eq} = 1.508 \angle 75.42^\circ \Omega = \underline{\underline{0.3796 + j1.46 \Omega}}$$

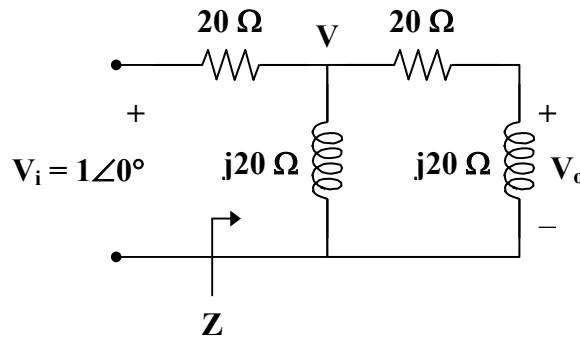
Chapter 9, Problem 74.



Design an RL circuit to provide a 90° leading phase shift.

Chapter 9, Solution 74.

One such RL circuit is shown below.



We now want to show that this circuit will produce a 90° phase shift.

$$\mathbf{Z} = j20 \parallel (20 + j20) = \frac{(j20)(20 + j20)}{20 + j40} = \frac{-20 + j20}{1 + j2} = 4(1 + j3)$$

$$\mathbf{V} = \frac{\mathbf{Z}}{\mathbf{Z} + 20} \mathbf{V}_i = \frac{4 + j12}{24 + j12} (1 \angle 0^\circ) = \frac{1 + j3}{6 + j3} = \frac{1}{3}(1 + j)$$

$$\mathbf{V}_o = \frac{j20}{20 + j20} \mathbf{V} = \left(\frac{j}{1 + j} \right) \left(\frac{1}{3}(1 + j) \right) = \frac{j}{3} = 0.3333 \angle 90^\circ$$

This shows that the output leads the input by 90° .

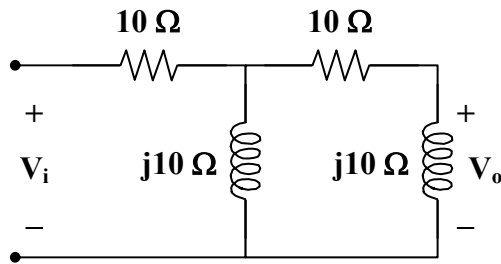
Chapter 9, Problem 75.



Design a circuit that will transform a sinusoidal voltage input to a cosinusoidal voltage output.

Chapter 9, Solution 75.

Since $\cos(\omega t) = \sin(\omega t + 90^\circ)$, we need a phase shift circuit that will cause the output to lead the input by 90° . **This is achieved by the RL circuit shown below, as explained in the previous problem.**



This can also be obtained by an RC circuit.

Chapter 9, Problem 76.



For the following pairs of signals, determine if v_1 leads or lags v_2 and by how much.

(a) $v_1 = 10 \cos(5t - 20^\circ)$, $v_2 = 8 \sin 5t$

(b) $v_1 = 19 \cos(2t - 90^\circ)$, $v_2 = 6 \sin 2t$

(c) $v_1 = -4 \cos 10t$, $v_2 = 15 \sin 10t$

Chapter 9, Solution 76.

(a) $v_2 = 8 \sin 5t = 8 \cos(5t - 90^\circ)$
 v_1 leads v_2 by 70° .

(b) $v_2 = 6 \sin 2t = 6 \cos(2t - 90^\circ)$
 v_1 leads v_2 by 180° .

(c) $v_1 = -4 \cos 10t = 4 \cos(10t + 180^\circ)$
 $v_2 = 15 \sin 10t = 15 \cos(10t - 90^\circ)$
 v_1 leads v_2 by 270° .

Chapter 9, Problem 77.

Refer to the RC circuit in Fig. 9.81.

- (a) Calculate the phase shift at 2 MHz.
(b) Find the frequency where the phase shift is 45° .

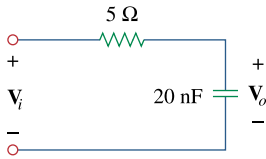


Figure 9.81

For Prob. 9.77.

Chapter 9, Solution 77.

$$(a) \quad V_o = \frac{-jX_c}{R - jX_c} V_i$$

$$\text{where } X_c = \frac{1}{\omega C} = \frac{1}{(2\pi)(2 \times 10^6)(20 \times 10^{-9})} = 3.979$$

$$\frac{V_o}{V_i} = \frac{-j3.979}{5 - j3.979} = \frac{3.979}{\sqrt{5^2 + 3.979^2}} \angle (-90^\circ + \tan^{-1}(3.979/5))$$

$$\frac{V_o}{V_i} = \frac{3.979}{\sqrt{25 + 15.83}} \angle (-90^\circ - 38.51^\circ)$$

$$\frac{V_o}{V_i} = 0.6227 \angle -51.49^\circ$$

Therefore, the phase shift is **51.49° lagging**

$$(b) \quad \theta = -45^\circ = -90^\circ + \tan^{-1}(X_c/R)$$

$$45^\circ = \tan^{-1}(X_c/R) \longrightarrow R = X_c = \frac{1}{\omega C}$$

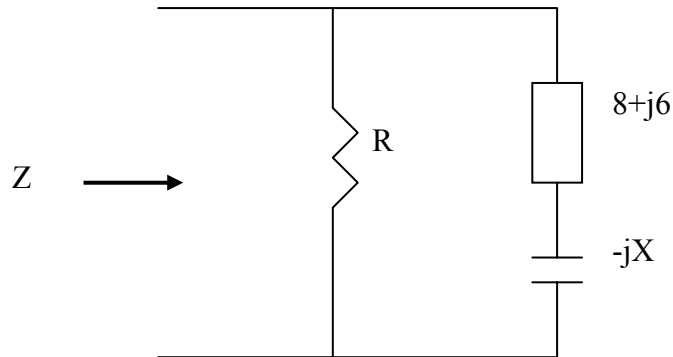
$$\omega = 2\pi f = \frac{1}{RC}$$

$$f = \frac{1}{2\pi RC} = \frac{1}{(2\pi)(5)(20 \times 10^{-9})} = \mathbf{1.5915 \text{ MHz}}$$

Chapter 9, Problem 78.

A coil with impedance $8 + j6 \, \Omega$ is connected in series with a capacitive reactance X . The series combination is connected in parallel with a resistor R . Given that the equivalent impedance of the resulting circuit is $5 \angle 0^\circ \, \Omega$ find the value of R and X .

Chapter 9, Solution 78.



$$Z = R // [8 + j(6 - X)] = \frac{R[8 + j(6 - X)]}{R + 8 + j(6 - X)} = 5$$

$$\text{i.e. } 8R + j6R - jXR = 5R + 40 + j30 - j5X$$

Equating real and imaginary parts:

$$8R = 5R + 40 \quad \text{which leads to} \quad \mathbf{R = \underline{13.333\Omega}}$$

$$6R - XR = 30 - 5X \quad \text{which leads to} \quad \mathbf{X = \underline{6\Omega}}$$

Chapter 9, Problem 79.

- (a) Calculate the phase shift of the circuit in Fig. 9.82.
- (b) State whether the phase shift is leading or lagging (output with respect to input).
- (c) Determine the magnitude of the output when the input is 120 V.

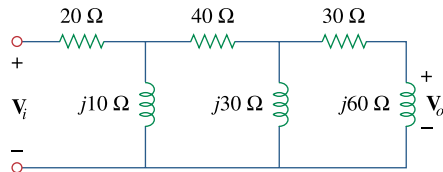
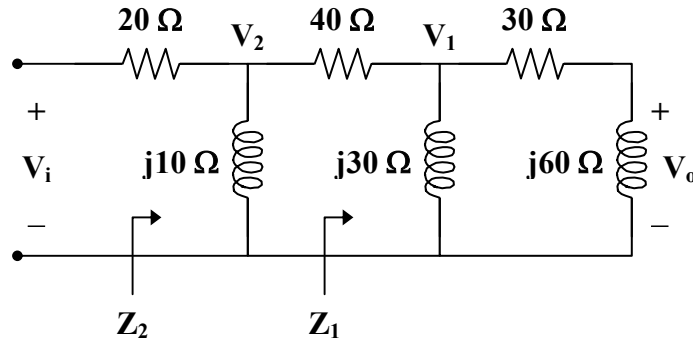


Figure 9.82
For Prob. 9.79.

Chapter 9, Solution 79.

- (a) Consider the circuit as shown.



$$Z_1 = j30 \parallel (30 + j60) = \frac{(j30)(30 + j60)}{30 + j90} = 3 + j21$$

$$Z_2 = j10 \parallel (40 + Z_1) = \frac{(j10)(43 + j21)}{43 + j31} = 1.535 + j8.896 = 9.028 \angle 80.21^\circ$$

Let $V_i = 1 \angle 0^\circ$.

$$V_2 = \frac{Z_2}{Z_2 + 20} V_i = \frac{(9.028 \angle 80.21^\circ)(1 \angle 0^\circ)}{21.535 + j8.896}$$

$$V_2 = 0.3875 \angle 57.77^\circ$$

$$V_1 = \frac{Z_1}{Z_1 + 40} V_2 = \frac{3 + j21}{43 + j21} V_2 = \frac{(21.213 \angle 81.87^\circ)(0.3875 \angle 57.77^\circ)}{47.85 \angle 26.03^\circ}$$

$$V_1 = 0.1718 \angle 113.61^\circ$$

$$V_o = \frac{j60}{30 + j60} V_1 = \frac{j2}{1 + j2} V_1 = \frac{2}{5} (2 + j) V_1$$

$$V_o = (0.8944 \angle 26.56^\circ)(0.1718 \angle 113.6^\circ)$$

$$V_o = 0.1536 \angle 140.2^\circ$$

Therefore, the phase shift is **140.2°**

- (b) The phase shift is **leading**.

- (c) If $V_i = 120 \text{ V}$, then

$$V_o = (120)(0.1536 \angle 140.2^\circ) = 18.43 \angle 140.2^\circ \text{ V}$$

and the magnitude is **18.43 V**.

Chapter 9, Problem 80.

Consider the phase-shifting circuit in Fig. 9.83. Let $V_i = 120$ V operating at 60 Hz. Find:

- (a) V_o when R is maximum
- (b) V_o when R is minimum
- (c) the value of R that will produce a phase shift of 45°

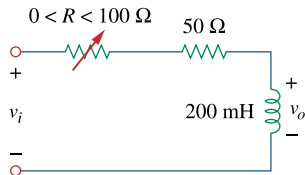


Figure 9.83
For Prob. 9.80.

Chapter 9, Solution 80.

$$200 \text{ mH} \longrightarrow j\omega L = j(2\pi)(60)(200 \times 10^{-3}) = j75.4 \Omega$$

$$V_o = \frac{j75.4}{R + 50 + j75.4} V_i = \frac{j75.4}{R + 50 + j75.4} (120 \angle 0^\circ)$$

- (a) When $R = 100 \Omega$,

$$V_o = \frac{j75.4}{150 + j75.4} (120 \angle 0^\circ) = \frac{(75.4 \angle 90^\circ)(120 \angle 0^\circ)}{167.88 \angle 26.69^\circ}$$
$$V_o = \underline{\underline{53.89 \angle 63.31^\circ \text{ V}}}$$

- (b) When $R = 0 \Omega$,

$$V_o = \frac{j75.4}{50 + j75.4} (120 \angle 0^\circ) = \frac{(75.4 \angle 90^\circ)(120 \angle 0^\circ)}{90.47 \angle 56.45^\circ}$$
$$V_o = \underline{\underline{100 \angle 33.55^\circ \text{ V}}}$$

- (c) To produce a phase shift of 45° , the phase of $V_o = 90^\circ + 0^\circ - \alpha = 45^\circ$.

$$\text{Hence, } \alpha = \text{phase of } (R + 50 + j75.4) = 45^\circ.$$

$$\text{For } \alpha \text{ to be } 45^\circ, \quad R + 50 = 75.4$$

$$\text{Therefore,} \quad R = \underline{\underline{25.4 \Omega}}$$

Chapter 9, Problem 81.

The ac bridge in Fig. 9.37 is balanced when $R_1 = 400 \, \Omega$, $R_2 = 600 \, \Omega$, $R_3 = 1.2 \text{ k}\Omega$, and $C_2 = 0.3 \, \mu\text{F}$. Find R_x and C_x . Assume R_2 and C_2 are in series.

Chapter 9, Solution 81.

$$\text{Let } Z_1 = R_1, \quad Z_2 = R_2 + \frac{1}{j\omega C_2}, \quad Z_3 = R_3, \text{ and } Z_x = R_x + \frac{1}{j\omega C_x}.$$

$$Z_x = \frac{Z_3}{Z_1} Z_2$$

$$R_x + \frac{1}{j\omega C_x} = \frac{R_3}{R_1} \left(R_2 + \frac{1}{j\omega C_2} \right)$$

$$R_x = \frac{R_3}{R_1} R_2 = \frac{1200}{400} (600) = \underline{\underline{1.8 \text{ k}\Omega}}$$

$$\frac{1}{C_x} = \left(\frac{R_3}{R_1} \right) \left(\frac{1}{C_2} \right) \longrightarrow C_x = \frac{R_1}{R_3} C_2 = \left(\frac{400}{1200} \right) (0.3 \times 10^{-6}) = \underline{\underline{0.1 \, \mu\text{F}}}$$

Chapter 9, Problem 82.

A capacitance bridge balances when $R_1 = 100 \, \Omega$, and $R_2 = 2 \text{ k}\Omega$ and $C_s = 40 \, \mu\text{F}$. What is C_x the capacitance of the capacitor under test?

Chapter 9, Solution 82.

$$C_x = \frac{R_1}{R_2} C_s = \left(\frac{100}{2000} \right) (40 \times 10^{-6}) = \underline{\underline{2 \, \mu\text{F}}}$$

Chapter 9, Problem 83.

An inductive bridge balances when $R_1 = 1.2 \text{ k}\Omega$, $R_2 = 500 \, \Omega$, and $L_s = 250 \text{ mH}$. What is the value of L_x , the inductance of the inductor under test?

Chapter 9, Solution 83.

$$L_x = \frac{R_2}{R_1} L_s = \left(\frac{500}{1200} \right) (250 \times 10^{-3}) = \underline{\underline{104.17 \text{ mH}}}$$

Chapter 9, Problem 84.

The ac bridge shown in Fig. 9.84 is known as a *Maxwell bridge* and is used for accurate measurement of inductance and resistance of a coil in terms of a standard capacitance C_s . Show that when the bridge is balanced,

$$L_x = R_2 R_3 C_s \quad \text{and} \quad R_x = \frac{R_2}{R_1} R_3$$

Find L_x and R_x for $R_1 = 40\text{k}\Omega$, $R_2 = 1.6\text{k}\Omega$, $R_3 = 4\text{k}\Omega$, and $C_s = 0.45\ \mu\text{F}$.

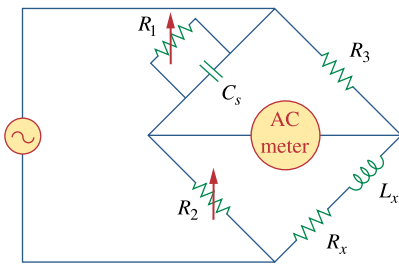


Figure 9.84

Maxwell bridge; For Prob. 9.84.

Chapter 9, Solution 84.

$$\text{Let } \mathbf{Z}_1 = R_1 \parallel \frac{1}{j\omega C_s}, \quad \mathbf{Z}_2 = R_2, \quad \mathbf{Z}_3 = R_3, \text{ and } \mathbf{Z}_x = R_x + j\omega L_x.$$

$$\mathbf{Z}_1 = \frac{\frac{R_1}{j\omega C_s}}{R_1 + \frac{1}{j\omega C_s}} = \frac{R_1}{j\omega R_1 C_s + 1}$$

$$\text{Since } \mathbf{Z}_x = \frac{\mathbf{Z}_3}{\mathbf{Z}_1} \mathbf{Z}_2,$$

$$R_x + j\omega L_x = R_2 R_3 \frac{j\omega R_1 C_s + 1}{R_1} = \frac{R_2 R_3}{R_1} (1 + j\omega R_1 C_s)$$

Equating the real and imaginary components,

$$\underline{\mathbf{R}_x = \frac{\mathbf{R}_2 \mathbf{R}_3}{\mathbf{R}_1}}$$

$$\omega L_x = \frac{R_2 R_3}{R_1} (\omega R_1 C_s) \text{ implies that}$$

$$\underline{\mathbf{L}_x = \mathbf{R}_2 \mathbf{R}_3 \mathbf{C}_s}$$

Given that $R_1 = 40 \text{ k}\Omega$, $R_2 = 1.6 \text{ k}\Omega$, $R_3 = 4 \text{ k}\Omega$, and $C_s = 0.45 \text{ }\mu\text{F}$

$$R_x = \frac{R_2 R_3}{R_1} = \frac{(1.6)(4)}{40} \text{ k}\Omega = 0.16 \text{ k}\Omega = \underline{\underline{160 \text{ }\Omega}}$$

$$L_x = R_2 R_3 C_s = (1.6)(4)(0.45) = \underline{\underline{2.88 \text{ H}}}$$

Chapter 9, Problem 85.

The ac bridge circuit of Fig. 9.85 is called a *Wien bridge*. It is used for measuring the frequency of a source. Show that when the bridge is balanced,

$$f = \frac{1}{2\pi\sqrt{R_2 R_4 C_2 C_4}}$$

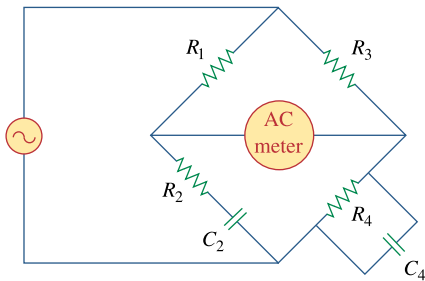


Figure 9.85
Wein bridge; For Prob. 9.85.

Chapter 9, Solution 85.

$$\text{Let } \mathbf{Z}_1 = R_1, \quad \mathbf{Z}_2 = R_2 + \frac{1}{j\omega C_2}, \quad \mathbf{Z}_3 = R_3, \text{ and } \mathbf{Z}_4 = R_4 \parallel \frac{1}{j\omega C_4}.$$

$$\mathbf{Z}_4 = \frac{R_4}{j\omega R_4 C_4 + 1} = \frac{-jR_4}{\omega R_4 C_4 - j}$$

$$\text{Since } \mathbf{Z}_4 = \frac{\mathbf{Z}_3}{\mathbf{Z}_1} \mathbf{Z}_2 \longrightarrow \mathbf{Z}_1 \mathbf{Z}_4 = \mathbf{Z}_2 \mathbf{Z}_3,$$

$$\begin{aligned} \frac{-jR_4 R_1}{\omega R_4 C_4 - j} &= R_3 \left(R_2 - \frac{j}{\omega C_2} \right) \\ \frac{-jR_4 R_1 (\omega R_4 C_4 + j)}{\omega^2 R_4^2 C_4^2 + 1} &= R_3 R_2 - \frac{jR_3}{\omega C_2} \end{aligned}$$

Equating the real and imaginary components,

$$\frac{R_1 R_4}{\omega^2 R_4^2 C_4^2 + 1} = R_2 R_3 \quad (1)$$

$$\frac{\omega R_1 R_4^2 C_4}{\omega^2 R_4^2 C_4^2 + 1} = \frac{R_3}{\omega C_2} \quad (2)$$

Dividing (1) by (2),

$$\begin{aligned} \frac{1}{\omega R_4 C_4} &= \omega R_2 C_2 \\ \omega^2 &= \frac{1}{R_2 C_2 R_4 C_4} \\ \omega &= 2\pi f = \frac{1}{\sqrt{R_2 C_2 R_4 C_4}} \\ \mathbf{f} &= \frac{1}{2\pi \sqrt{R_2 R_4 C_2 C_4}} \end{aligned}$$

Chapter 9, Problem 86.

The circuit shown in Fig. 9.86 is used in a television receiver. What is the total impedance of this circuit?

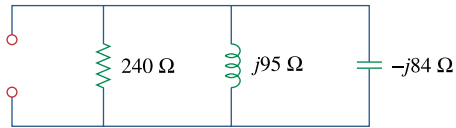


Figure 9.86

For Prob. 9.86.

Chapter 9, Solution 86.

$$Y = \frac{1}{240} + \frac{1}{j95} + \frac{1}{-j84}$$

$$Y = 4.1667 \times 10^{-3} - j0.01053 + j0.0119$$

$$Z = \frac{1}{Y} = \frac{1000}{4.1667 + j1.37} = \frac{1000}{4.3861 \angle 18.2^\circ}$$

$$Z = \underline{\underline{228 \angle -18.2^\circ\ \Omega}}$$

Chapter 9, Problem 87.

The network in Fig. 9.87 is part of the schematic describing an industrial electronic sensing device. What is the total impedance of the circuit at 2 kHz?

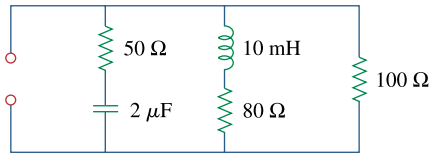


Figure 9.87

For Prob. 9.87.

Chapter 9, Solution 87.

$$\mathbf{Z}_1 = 50 + \frac{1}{j\omega C} = 50 + \frac{-j}{(2\pi)(2 \times 10^3)(2 \times 10^{-6})}$$

$$\mathbf{Z}_1 = 50 - j39.79$$

$$\mathbf{Z}_2 = 80 + j\omega L = 80 + j(2\pi)(2 \times 10^3)(10 \times 10^{-3})$$

$$\mathbf{Z}_2 = 80 + j125.66$$

$$\mathbf{Z}_3 = 100$$

$$\frac{1}{\mathbf{Z}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}$$

$$\frac{1}{\mathbf{Z}} = \frac{1}{100} + \frac{1}{50 - j39.79} + \frac{1}{80 + j125.66}$$

$$\frac{1}{\mathbf{Z}} = 10^{-3} (10 + 12.24 + j9.745 + 3.605 - j5.663)$$

$$= (25.85 + j4.082) \times 10^{-3}$$

$$= 26.17 \times 10^{-3} \angle 8.97^\circ$$

$$\mathbf{Z} = \underline{\underline{38.21 \angle -8.97^\circ \Omega}}$$

Chapter 9, Problem 88.

A series audio circuit is shown in Fig. 9.88.

(a) What is the impedance of the circuit?

(b) If the frequency were halved, what would be the impedance of the circuit?

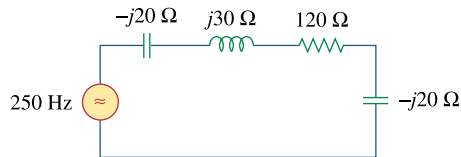


Figure 9.88

For Prob. 9.88.

Chapter 9, Solution 88.

(a) $Z = -j20 + j30 + 120 - j20$

$$Z = \underline{120 - j10 \, \Omega}$$

(b) If the frequency were halved, $\frac{1}{\omega C} = \frac{1}{2\pi f C}$ would cause the capacitive impedance to double, while $\omega L = 2\pi f L$ would cause the inductive impedance to halve. Thus,

$$Z = -j40 + j15 + 120 - j40$$

$$Z = \underline{120 - j65 \, \Omega}$$

Chapter 9, Problem 89.

An industrial load is modeled as a series combination of a capacitance and a resistance as shown in Fig. 9.89. Calculate the value of an inductance L across the series combination so that the net impedance is resistive at a frequency of 50 kHz.

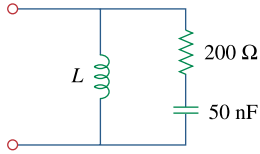


Figure 9.89

For Prob. 9.89.

Chapter 9, Solution 89.

$$\begin{aligned} \mathbf{Z}_{\text{in}} &= j\omega L \parallel \left(R + \frac{1}{j\omega C} \right) \\ \mathbf{Z}_{\text{in}} &= \frac{j\omega L \left(R + \frac{1}{j\omega C} \right)}{R + j\omega L + \frac{1}{j\omega C}} = \frac{\frac{L}{C} + j\omega L R}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \\ \mathbf{Z}_{\text{in}} &= \frac{\left(\frac{L}{C} + j\omega L R\right)\left(R - j\left(\omega L - \frac{1}{\omega C}\right)\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \end{aligned}$$

To have a resistive impedance, $\text{Im}(\mathbf{Z}_{\text{in}}) = 0$. Hence,

$$\omega L R^2 - \left(\frac{L}{C}\right)\left(\omega L - \frac{1}{\omega C}\right) = 0$$

$$\omega R^2 C = \omega L - \frac{1}{\omega C}$$

$$\omega^2 R^2 C^2 = \omega^2 LC - 1$$

$$L = \frac{\omega^2 R^2 C^2 + 1}{\omega^2 C}$$

Now we can solve for L.

$$L = R^2 C + 1/(\omega^2 C)$$

$$\begin{aligned} &= (200^2)(50 \times 10^{-9}) + 1/((2\pi \times 50,000)^2(50 \times 10^{-9})) \\ &= 2 \times 10^{-3} + 0.2026 \times 10^{-3} = \underline{\underline{2.203 \text{ mH}}} \end{aligned}$$

Checking, converting the series resistor and capacitor into a parallel combination, gives 220.3Ω in parallel with $-j691.9\Omega$. The value of the parallel inductance is $\omega L = 2\pi \times 50,000 \times 2.203 \times 10^{-3} = 692.1\Omega$ which we need to have if we are to cancel the effect of the capacitance. The answer checks.

Chapter 9, Problem 90.

An industrial coil is modeled as a series combination of an inductance L and resistance R , as shown in Fig. 9.90. Since an ac voltmeter measures only the magnitude of a sinusoid, the following measurements are taken at 60 Hz when the circuit operates in the steady state:

$$|V_s| = 145 \text{ V}, \quad |V_1| = 50 \text{ V}, \quad |V_o| = 110 \text{ V}$$

Use these measurements to determine the values of L and R .

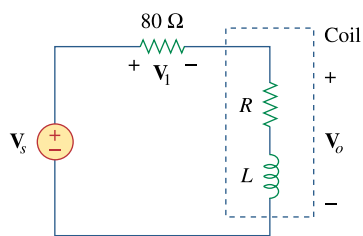


Figure 9.90
For Prob. 9.90.

Chapter 9, Solution 90.

Let $V_s = 145 \angle 0^\circ$, $X = \omega L = (2\pi)(60)L = 377L$

$$I = \frac{V_s}{80 + R + jX} = \frac{145 \angle 0^\circ}{80 + R + jX}$$

$$V_1 = 80I = \frac{(80)(145)}{80 + R + jX}$$

$$50 = \left| \frac{(80)(145)}{80 + R + jX} \right| \quad (1)$$

$$V_o = (R + jX)I = \frac{(R + jX)(145 \angle 0^\circ)}{80 + R + jX}$$

$$110 = \left| \frac{(R + jX)(145)}{80 + R + jX} \right| \quad (2)$$

From (1) and (2),

$$\begin{aligned} \frac{50}{110} &= \frac{80}{|R + jX|} \\ |R + jX| &= (80) \left(\frac{11}{5} \right) \\ R^2 + X^2 &= 30976 \end{aligned} \quad (3)$$

From (1),

$$\begin{aligned} |80 + R + jX| &= \frac{(80)(145)}{50} = 232 \\ 6400 + 160R + R^2 + X^2 &= 53824 \\ 160R + R^2 + X^2 &= 47424 \end{aligned} \quad (4)$$

Subtracting (3) from (4),

$$160R = 16448 \longrightarrow R = \underline{\underline{102.8 \, \Omega}}$$

From (3),

$$\begin{aligned} X^2 &= 30976 - 10568 = 20408 \\ X &= 142.86 = 377L \longrightarrow L = \underline{\underline{0.3789 \, H}} \end{aligned}$$

Chapter 9, Problem 91.

Figure 9.91 shows a parallel combination of an inductance and a resistance. If it is desired to connect a capacitor in series with the parallel combination such that the net impedance is resistive at 10 MHz, what is the required value of C ?

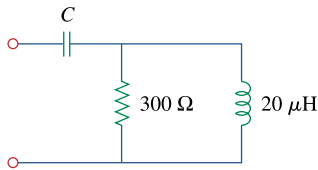


Figure 9.91

For Prob. 9.91.

Chapter 9, Solution 91.

$$\begin{aligned} \mathbf{Z}_{\text{in}} &= \frac{1}{j\omega C} + R \parallel j\omega L \\ \mathbf{Z}_{\text{in}} &= \frac{-j}{\omega C} + \frac{j\omega LR}{R + j\omega L} \\ &= \frac{-j}{\omega C} + \frac{\omega^2 L^2 R + j\omega LR^2}{R^2 + \omega^2 L^2} \end{aligned}$$

To have a resistive impedance, $\text{Im}(\mathbf{Z}_{\text{in}}) = 0$.

Hence,

$$\begin{aligned} \frac{-1}{\omega C} + \frac{\omega LR^2}{R^2 + \omega^2 L^2} &= 0 \\ \frac{1}{\omega C} &= \frac{\omega LR^2}{R^2 + \omega^2 L^2} \\ C &= \frac{R^2 + \omega^2 L^2}{\omega^2 LR^2} \end{aligned}$$

where $\omega = 2\pi f = 2\pi \times 10^7$

$$\begin{aligned} C &= \frac{9 \times 10^4 + (4\pi^2 \times 10^{14})(400 \times 10^{-12})}{(4\pi^2 \times 10^{14})(20 \times 10^{-6})(9 \times 10^4)} \\ C &= \frac{9 + 16\pi^2}{72\pi^2} \text{ nF} \\ C &= \underline{\underline{235 \text{ pF}}} \end{aligned}$$

Chapter 9, Problem 92.

A transmission line has a series impedance of $\mathbf{Z} = 100 \angle 75^\circ \Omega$ and a shunt admittance of $\mathbf{Y} = 450 \angle 48^\circ \mu\text{S}$. Find: (a) the characteristic impedance $\mathbf{Z}_o = \sqrt{\mathbf{Z}/\mathbf{Y}}$
(b) the propagation constant $\gamma = \sqrt{\mathbf{ZY}}$.

Chapter 9, Solution 92.

$$(a) \quad \mathbf{Z}_o = \sqrt{\frac{\mathbf{Z}}{\mathbf{Y}}} = \sqrt{\frac{100 \angle 75^\circ}{450 \angle 48^\circ \times 10^{-6}}} = \underline{471.4 \angle 13.5^\circ \Omega}$$

$$(b) \quad \gamma = \sqrt{\mathbf{ZY}} = \sqrt{100 \angle 75^\circ \times 450 \angle 48^\circ \times 10^{-6}} = \underline{0.2121 \angle 61.5^\circ}$$

Chapter 9, Problem 93.

A power transmission system is modeled as shown in Fig. 9.92. Given the following;

Source voltage $\mathbf{V}_s = 115 \angle 0^\circ \text{ V}$,
Source impedance $\mathbf{Z}_s = 1 + j0.5 \Omega$,
Line impedance $\mathbf{Z}_\ell = 0.4 + j0.3 \Omega$,
Load impedance $\mathbf{Z}_L = 23.2 + j18.9 \Omega$,
find the load current \mathbf{I}_L .

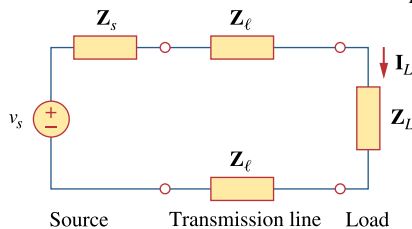


Figure 9.92

For Prob. 9.93.

Chapter 9, Solution 93.

$$\mathbf{Z} = \mathbf{Z}_s + 2\mathbf{Z}_\ell + \mathbf{Z}_L$$

$$\mathbf{Z} = (1 + 0.8 + 23.2) + j(0.5 + 0.6 + 18.9)$$

$$\mathbf{Z} = 25 + j20$$

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{115 \angle 0^\circ}{32.02 \angle 38.66^\circ}$$

$$\mathbf{I}_L = \underline{3.592 \angle -38.66^\circ \text{ A}}$$