Chapter 7, Problem 1.

In the circuit shown in Fig. 7.81

$$v(t) = 56e^{-200t}\,\text{V},\ t > 0$$
$$i(t) = 8e^{-200t}\,\text{mA},\ t > 0$$

(a) Find the values of *R* and *C*.
(b) Calculate the time constant τ .
(c) Determine the time required for the voltage to decay half its initial value at $t = 0$.

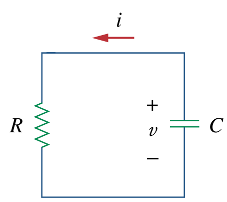


Figure 7.81
For Prob. 7.1

Chapter 7, Solution 1.

(a) τ=RC = 1/200

For the resistor, V=iR= $56e^{-200t} = 8Re^{-200t}x10^{-3} \longrightarrow R = \frac{56}{8} = \underline{7\ k\Omega}$

$$C = \frac{1}{200R} = \frac{1}{200X7X10^3} = \underline{0.7143\mu F}$$

(b) τ =1/200= **5 ms**
(c) If value of the voltage at = 0 is 56 .

$$\frac{1}{2}x56 = 56e^{-200t} \longrightarrow e^{200t} = 2$$

$$200t_o = \ln 2 \longrightarrow t_o = \frac{1}{200}\ln 2 = \underline{3.466\ ms}$$

Chapter 7, Problem 2.

Find the time constant for the *RC* circuit in Fig. 7.82.

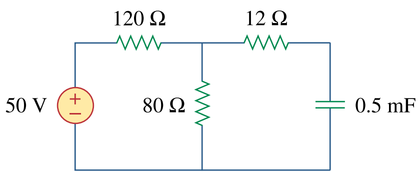


Figure 7.82
For Prob. 7.2.

Chapter 7, Solution 2.

$\tau = R_{th}C$

where R_{th} is the Thevenin equivalent at the capacitor terminals.

$$R_{th} = 120 \,||\, 80 + 12 = 60\ \Omega$$

$$\tau = 60 \times 0.5 \times 10^{-3} = \underline{\mathbf{30\ ms}}$$

Chapter 7, Problem 3.

Determine the time constant for the circuit in Fig. 7.83.

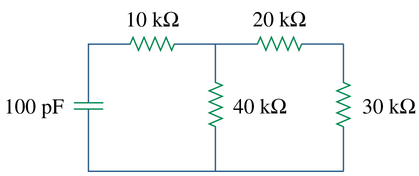


Figure 7.83
For Prob. 7.3.

Chapter 7, Solution 3.

R = 10 +20//(20+30) =10 + 40x50/(40 + 50)=32.22 kΩ

$$\tau = RC = 32.22X10^3 X100X10^{-12} = \underline{3.222\ \mu S}$$

Chapter 7, Problem 4.

The switch in Fig. 7.84 moves instantaneously from A to B at $t = 0$. Find v for $t > 0$.

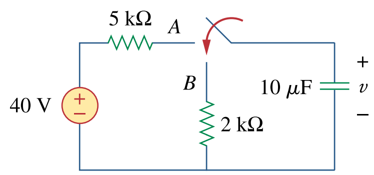


Figure 7.84
For Prob. 7.4.

Chapter 7, Solution 4.

For $t<0$, $v(0^-)=40$ V.

For $t>0$. we have a source-free RC circuit.

$$\tau = RC = 2x10^3 x10x10^{-6} = 0.02$$

$$v(t) = v(0)e^{-t/\tau} = \underline{40e^{-50t} \text{ V}}$$

Chapter 7, Problem 5.

For the circuit shown in Fig. 7.85, find $i(t)$, $t > 0$.

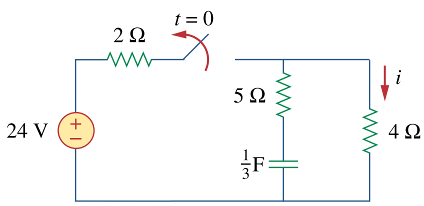


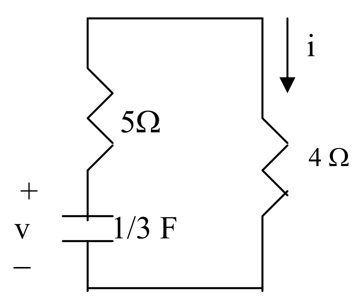
Figure 7.85
For Prob. 7.5.

Chapter 7, Solution 5.

Let v be the voltage across the capacitor.
For t <0,

$$v(0^-) = \frac{4}{2+4}(24) = 16 \text{ V}$$

For t >0, we have a source-free RC circuit as shown below.



$$\tau = RC = (4+5)\frac{1}{3} = 3s$$

$$v(t) = v(0)e^{-t/\tau} = 16e^{-t/3}$$

$$i(t) = -C\frac{dv}{dt} = -\frac{1}{3}(-\frac{1}{3})16e^{-t/3} = \underline{1.778e^{-t/3} \text{ A}}$$

Chapter 7, Problem 6.

The switch in Fig. 7.86 has been closed for a long time, and it opens at $t = 0$. Find $v(t)$ for $t \geq 0$.

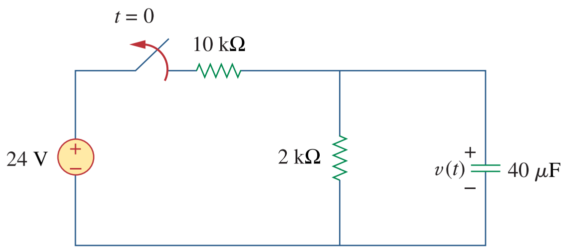


Figure 7.86
For Prob. 7.6.

Chapter 7, Solution 6.

$$v_o = v(0) = \frac{2}{10+2}(24) = 4\,\mathrm{V}$$

$$v(t) = v_o e^{-t/\tau}, \quad \tau = RC = 40x10^{-6} x2x10^{3} = \frac{2}{25}$$

$$v(t) = \underline{4e^{-12.5t}\mathrm{V}}$$

Chapter 7, Problem 7.

Assuming that the switch in Fig. 7.87 has been in position *A* for a long time and is moved to position *B* at $t = 0$, find $v_0(t)$ for $t \geq 0$.

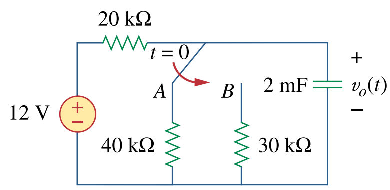


Figure 7.87
For Prob. 7.7.

Chapter 7, Solution 7.

When the switch is at position A, the circuit reaches steady state. By voltage division,

$$v_o(0) = \frac{40}{40+20}(12V) = 8V$$

When the switch is at position B, the circuit reaches steady state. By voltage division,

$$v_o(\infty) = \frac{30}{30+20}(12V) = 7.2V$$

$$R_{Th} = 20k // 30k = \frac{20x30}{50} = 12k\Omega$$

$$\tau = R_{Th}C = 12x10^3 x2x10^{-3} = 24s$$

$$v_o(t) = v_o(\infty) + [v_o(0) - v_o(\infty)]e^{-t/\tau} = 7.2 + (8 - 7.2)e^{-t/24} = \underline{7.2 + 0.8e^{-t/24}\ \ \text{V}}$$

Chapter 7, Problem 8.

For the circuit in Fig. 7.88, if

$v = 10e^{-4t}$ V and $i = 0.2e - 4t$ A, $t > 0$

(a) Find R and C.
(b) Determine the time constant.
(c) Calculate the initial energy in the capacitor.
(d) -Obtain the time it takes to dissipate 50 percent of the initial energy.

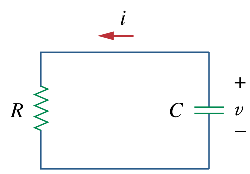


Figure 7.88
For Prob. 7.8.

Chapter 7, Solution 8.

(a) $\tau = RC = \frac{1}{4}$

$-i = C\frac{dv}{dt}$

$-0.2e^{-4t} = C(10)(-4)e^{-4t} \longrightarrow C = \underline{\mathbf{5\ mF}}$

$R = \frac{1}{4C} = \underline{\mathbf{50\ \Omega}}$

(b) $\tau = RC = \frac{1}{4} = \underline{\mathbf{0.25\ s}}$

(c) $w_C(0) = \frac{1}{2}CV_0^2 = \frac{1}{2}(5 \times 10^{-3})(100) = \underline{\mathbf{250\ mJ}}$

(d) $w_R = \frac{1}{2} \times \frac{1}{2}CV_0^2 = \frac{1}{2}CV_0^2\left(1 - e^{-2t_0/\tau}\right)$

$0.5 = 1 - e^{-8t_0} \longrightarrow e^{-8t_0} = \frac{1}{2}$

or $e^{8t_0} = 2$

$t_0 = \frac{1}{8}\ln(2) = \underline{\mathbf{86.6\ ms}}$

Chapter 7, Problem 9.

The switch in Fig. 7.89 opens at $t = 0$. Find v_0 for $t > 0$

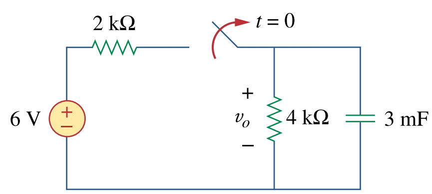


Figure 7.89
For Prob. 7.9.

Chapter 7, Solution 9.

For t < 0, the switch is closed so that

$$v_o(0) = \frac{4}{2+4}(6) = 4 \text{ V}$$

For t >0, we have a source-free RC circuit.

$$\tau = RC = 3x10^{-3}x4x10^{3} = 12s$$

$$v_o(t) = v_o(0)e^{-t/\tau} = \underline{4e^{-t/12} \text{ V}}$$

Chapter 7, Problem 10.

For the circuit in Fig. 7.90, find $v_0(t)$ for $t > 0$. Determine the time necessary for the capacitor voltage to decay to one-third of its value at $t = 0$.

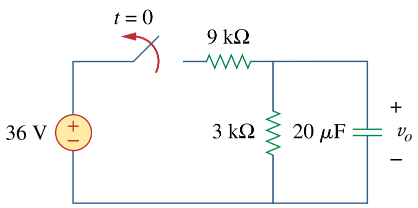


Figure 7.90
For Prob. 7.10.

Chapter 7, Solution 10.

For t<0, $$v(0^-) = \frac{3}{3+9}(36V) = \underline{9\ V}$$

For t>0, we have a source-free RC circuit

$$\tau = RC = 3x10^3 x20x10^{-6} = 0.06s$$

$$v_o(t) = \underline{\mathbf{9e^{-16.667t}\ V}}$$

Let the time be t_o.

$3 = 9e^{-16.667to}$ or $e^{16.667to} = 9/3 = 3$

$$t_o = \ln(3)/16.667 = \underline{\mathbf{65.92\ ms}}.$$

Chapter 7, Problem 11.

For the circuit in Fig. 7.91, find i_0 for $t > 0$.

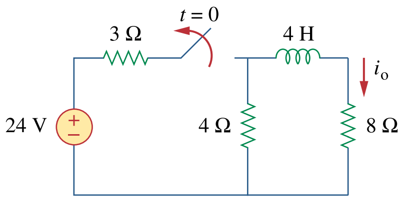
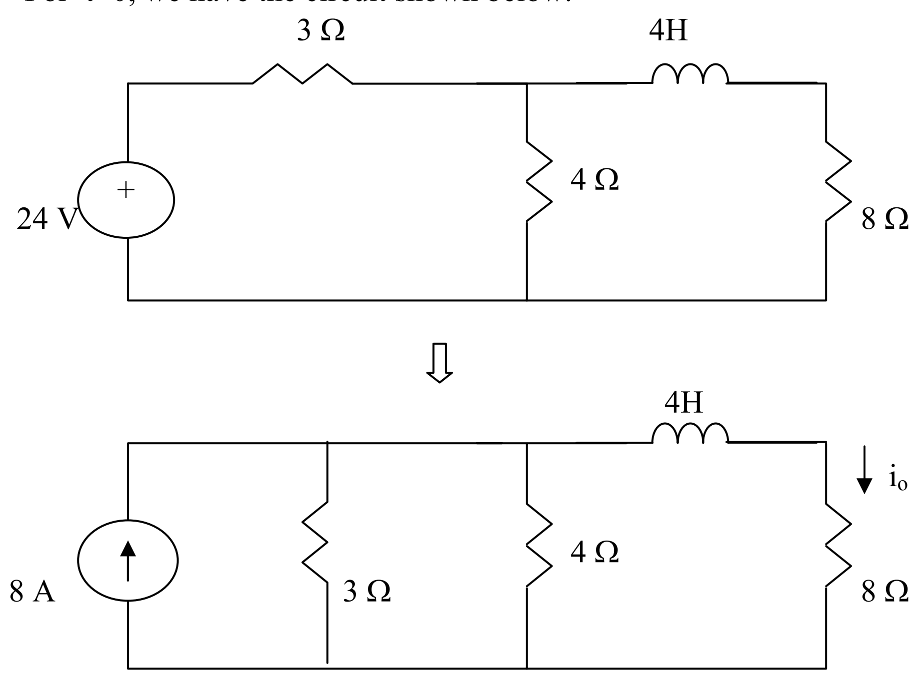


Figure 7.91
For Prob. 7.11.

Chapter 7, Solution 11.

For t<0, we have the circuit shown below.



3//4= 4x3/7=1.7143

$$i_o(0^-) = \frac{1.7143}{1.7143 + 8}(8) = 1.4118 \text{ A}$$

For t >0, we have a source-free RL circuit.

$$\tau = \frac{L}{R} = \frac{4}{4+8} = 1/3$$

$$i_o(t) = i_o(0)e^{-t/\tau} = \underline{1.4118e^{-3t} \text{ A}}$$

Chapter 7, Problem 12.

The switch in the circuit of Fig. 7.92 has been closed for a long time. At $t = 0$ the switch is opened. Calculate $i(t)$ for $t > 0$.

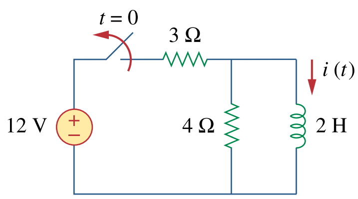
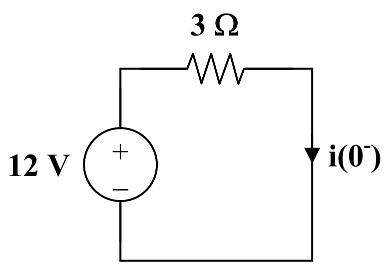


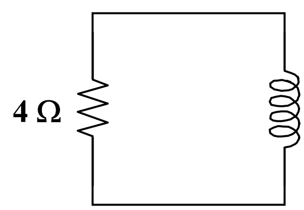
Figure 7.92
For Prob. 7.12.

Chapter 7, Solution 12.

When t < 0, the switch is closed and the inductor acts like a short circuit to dc. The 4 Ω resistor is short-circuited so that the resulting circuit is as shown in Fig. (a).



(a)



(b)

$$i(0^-) = \frac{12}{3} = 4 \text{ A}$$

Since the current through an inductor cannot change abruptly,

$$i(0) = i(0^-) = i(0^+) = 4 \text{ A}$$

When t > 0, the voltage source is cut off and we have the RL circuit in Fig. (b).

$$\tau = \frac{L}{R} = \frac{2}{4} = 0.5$$

Hence,

$$i(t) = i(0)e^{-t/\tau} = \underline{\mathbf{4e^{-2t}\ A}}$$

Chapter 7, Problem 13.

In the circuit of Fig. 7.93,

$$v(t) = 20e^{-10^3 t} \text{ V}, \qquad t > 0$$

$$i(t) = 4e^{-10^3 t} \text{ mA}, \qquad t > 0$$

(a) Find *R*, *L*, and τ.
(b) Calculate the energy dissipated in the resistance for $0 < t < 0.5$ ms.

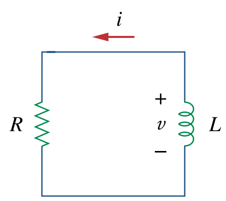


Figure 7.93
For Prob. 7.13.

Chapter 7, Solution 13.

(a) $\tau = \dfrac{1}{10^3} = \underline{1ms}$

$v = iR \longrightarrow 20e^{-1000t} = Rx4e^{-1000t}x10^{-3}$
From this, R = 20/4 kΩ= <u>5 kΩ</u>
But $\tau = \dfrac{L}{R} = 1/10^3 \qquad \longrightarrow \qquad L = \dfrac{5x1000}{1000} = \underline{5H}$

(b) The energy dissipated in the resistor is

$$w = \int_0^t p\,dt = \int_0^t 80x10^{-3} e^{-2x10^3} dt = -\frac{80x10^{-3}}{2x10^3} e^{-2x10^3 t} \Bigg|_0^{0.5x10^{-3}}$$

$$= 40(1 - e^{-1})\mu J = \underline{25.28\mu J}$$

Chapter 7, Problem 14.

Calculate the time constant of the circuit in Fig. 7.94.

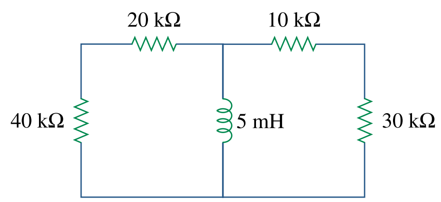


Figure 7.94
For Prob. 7.14.

Chapter 7, Solution 14.

$$R_{Th} = (40+20)//(10+30) = \frac{60x40}{100} = 24k\Omega$$

$$\tau = L/R = \frac{5x10^{-3}}{24x10^{3}} = \underline{0.2083\mu s}$$

Chapter 7, Problem 15.

Find the time constant for each of the circuits in Fig. 7.95.

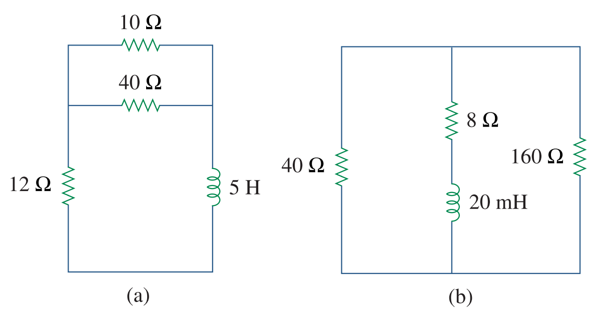


Figure 7.95
For Prob. 7.15.

Chapter 7, Solution 15

(a) $R_{Th} = 12 + 10//40 = 20\Omega, \qquad \tau = \frac{L}{R_{Th}} = 5/20 = \underline{0.25s}$

(b) $R_{Th} = 40//160 + 8 = 40\Omega, \qquad \tau = \frac{L}{R_{Th}} = (20x10^{-3})/40 = \underline{0.5\text{ ms}}$

Chapter 7, Problem 16.

Determine the time constant for each of the circuits in Fig. 7.96.

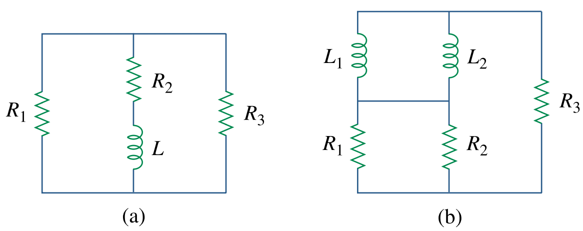


Figure 7.96
For Prob. 7.16.

Chapter 7, Solution 16.

$$\tau = \frac{L_{eq}}{R_{eq}}$$

(a) $L_{eq} = L$ and $R_{eq} = R_2 + \dfrac{R_1 R_3}{R_1 + R_3} = \dfrac{R_2(R_1 + R_3) + R_1 R_3}{R_1 + R_3}$

$$\tau = \underline{\mathbf{\frac{L(R_1 + R_3)}{R_2(R_1 + R_3) + R_1 R_3}}}$$

(b) where $L_{eq} = \dfrac{L_1 L_2}{L_1 + L_2}$ and $R_{eq} = R_3 + \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{R_3(R_1 + R_2) + R_1 R_2}{R_1 + R_2}$

$$\tau = \underline{\mathbf{\frac{L_1 L_2 (R_1 + R_2)}{(L_1 + L_2)(R_3(R_1 + R_2) + R_1 R_2)}}}$$

Chapter 7, Problem 17.

Consider the circuit of Fig. 7.97. Find $v_0(t)$ if $i(0) = 2$ A and $v(t) = 0$.

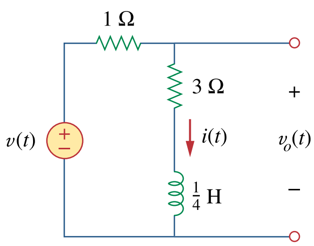


Figure 7.97
For Prob. 7.17.

Chapter 7, Solution 17.

$$i(t) = i(0)\,e^{-t/\tau}, \qquad \tau = \frac{L}{R_{eq}} = \frac{1/4}{4} = \frac{1}{16}$$

$$i(t) = 2\,e^{-16t}$$

$$v_o(t) = 3i + L\frac{di}{dt} = 6\,e^{-16t} + (1/4)(-16)\,2\,e^{-16t}$$

$$v_o(t) = \underline{-2\,e^{-16t}\;u(t)V}$$

Chapter 7, Problem 18.

For the circuit in Fig. 7.98, determine $v_0(t)$ when $i(0) = 1$ A and $v(t) = 0$.

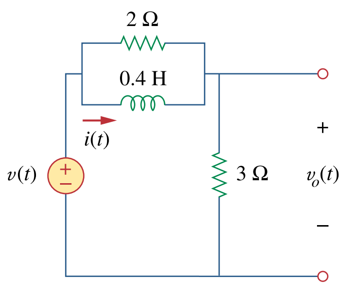
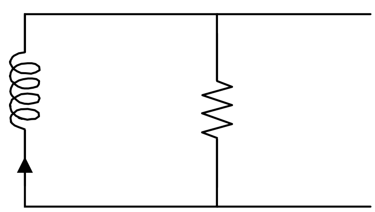


Figure 7.98
For Prob. 7.18.

Chapter 7, Solution 18.

If $v(t) = 0$, the circuit can be redrawn as shown below.



$$R_{eq} = 2 \,||\, 3 = \frac{6}{5}, \qquad \tau = \frac{L}{R} = \frac{2}{5} \times \frac{5}{6} = \frac{1}{3}$$

$$i(t) = i(0)\,e^{-t/\tau} = e^{-3t}$$

$$v_o(t) = -L\frac{di}{dt} = \frac{-2}{5}(-3)e^{-3t} = \underline{\mathbf{1.2\,e^{-3t}\ V}}$$

Chapter 7, Problem 19.

In the circuit of Fig. 7.99, find $i(t)$ for $t > 0$ if $i(0) = 2$ A.

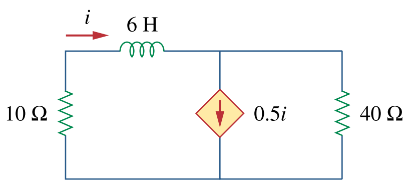
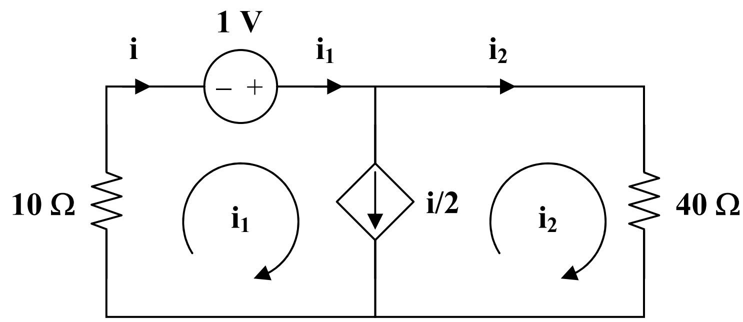


Figure 7.99
For Prob. 7.19.

Chapter 7, Solution 19.



To find R_{th} we replace the inductor by a 1-V voltage source as shown above.

$$10i_1 - 1 + 40i_2 = 0$$

But $\quad i = i_2 + i/2 \quad$ and $\quad i = i_1$

i.e. $\quad i_1 = 2i_2 = i$

$$10i - 1 + 20i = 0 \longrightarrow i = \frac{1}{30}$$

$$R_{th} = \frac{1}{i} = 30\,\Omega$$

$$\tau = \frac{L}{R_{th}} = \frac{6}{30} = 0.2 \text{ s}$$

$$i(t) = \underline{2e^{-5t}u(t) \text{ A}}$$

Chapter 7, Problem 20.

For the circuit in Fig. 7.100,

$v = 120e^{-50t}$ V

and

$i = 30e^{-50t}$ A, $t > 0$

(a) Find L and R.
(b) Determine the time constant.
(c) Calculate the initial energy in the inductor.
(d) What fraction of the initial energy is dissipated in 10 ms?

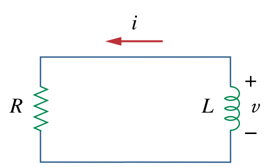


Figure 7.100
For Prob. 7.20.

Chapter 7, Solution 20.

(a) $\tau = \frac{L}{R} = \frac{1}{50} \longrightarrow R = 50L$

$-v = L\frac{di}{dt}$

$-120e^{-50t} = L(30)(-50)e^{-50t} \longrightarrow L = \underline{\textbf{80 mH}}$

$R = 50L = \underline{\textbf{4 }\Omega}$

(b) $\tau = \frac{L}{R} = \frac{1}{50} = \underline{\textbf{20 ms}}$

(c) $w = \frac{1}{2}Li^2(0) = \frac{1}{2}(0.08)(30)^2 = \underline{\textbf{36J}}$

The value of the energy remaining at 10 ms is given by:

$w_{10} = 0.04(30e^{-0.5})^2 = 0.04(18.196)^2 = 13.24$J.

So, the fraction of the energy dissipated in the first 10 ms is given by:

(36–13.24)/36 = **0.6322** or **63.2%**.

Chapter 7, Problem 21.

In the circuit of Fig. 7.101, find the value of R for which the steady-state energy stored in the inductor will be 1 J.

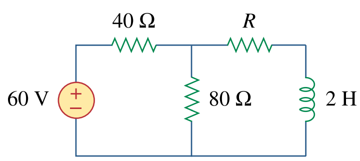
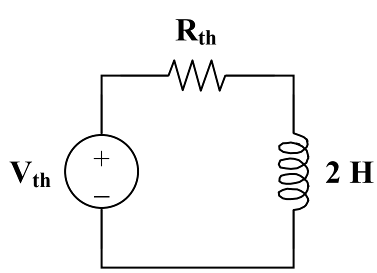


Figure 7.101
For Prob. 7.21.

Chapter 7, Solution 21.

The circuit can be replaced by its Thevenin equivalent shown below.



$$V_{th} = \frac{80}{80+40}(60) = 40 \text{ V}$$

$$R_{th} = 40 \,||\, 80 + R = \frac{80}{3} + R$$

$$I = i(0) = i(\infty) = \frac{V_{th}}{R_{th}} = \frac{40}{80/3 + R}$$

$$w = \frac{1}{2} L I^2 = \frac{1}{2}(2)\left(\frac{40}{R + 80/3}\right)^2 = 1$$

$$\frac{40}{R + 80/3} = 1 \longrightarrow R = \frac{40}{3}$$

$$R = \underline{13.333\ \Omega}$$

Chapter 7, Problem 22.

Find $i(t)$ and $v(t)$ for $t > 0$ in the circuit of Fig. 7.102 if $i(0) = 10$ A.

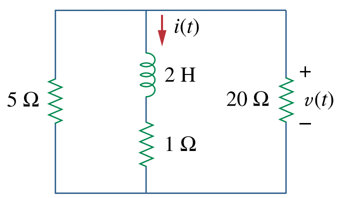


Figure 7.102
For Prob. 7.22.

Chapter 7, Solution 22.

$$i(t) = i(0)\, e^{-t/\tau}, \qquad \tau = \frac{L}{R_{eq}}$$

$$R_{eq} = 5 \,||\, 20 + 1 = 5\,\Omega, \qquad \tau = \frac{2}{5}$$

$$i(t) = \underline{\mathbf{10\, e^{-2.5t}\ A}}$$

Using current division, the current through the 20 ohm resistor is

$$i_o = \frac{5}{5+20}(-i) = \frac{-i}{5} = -2\,e^{-2.5t}$$

$$v(t) = 20\, i_o = \underline{\mathbf{-40\, e^{-2.5t}\ V}}$$

Chapter 7, Problem 23.

Consider the circuit in Fig. 7.103. Given that $v_0(0) = 2$ V, find v_0 and v_x for $t > 0$.

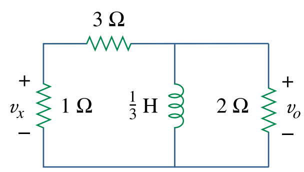


Figure 7.103
For Prob. 7.23.

Chapter 7, Solution 23.

Since the 2 Ω resistor, 1/3 H inductor, and the (3+1) Ω resistor are in parallel, they always have the same voltage.

$$-i = \frac{2}{2} + \frac{2}{3+1} = 1.5 \longrightarrow i(0) = -1.5$$

The Thevenin resistance R_{th} at the inductor's terminals is

$$R_{th} = 2 \,\|\, (3+1) = \frac{4}{3}, \qquad \tau = \frac{L}{R_{th}} = \frac{1/3}{4/3} = \frac{1}{4}$$

$$i(t) = i(0)\, e^{-t/\tau} = -1.5\, e^{-4t}, \quad t > 0$$

$$v_L = v_o = L\frac{di}{dt} = -1.5(-4)(1/3)\, e^{-4t}$$

$$v_o = \underline{\mathbf{2\, e^{-4t}\ V, \quad t > 0}}$$

$$v_x = \frac{1}{3+1} v_L = \underline{\mathbf{0.5\, e^{-4t}\ V, \quad t > 0}}$$

Chapter 7, Problem 24.

Express the following signals in terms of singularity functions.

(a) $v(t) = \begin{cases} 0, & t < 0 \\ -5, & t > 0 \end{cases}$

(b) $i(t) = \begin{cases} 0, & t < 1 \\ -10, & 1 < t < 3 \\ 10, & 3 < t < 5 \\ 0, & t < 5 \end{cases}$

(c) $x(t) = \begin{cases} t-1 & 1 < t < 2 \\ 1, & 2 < t < 3 \\ 4-t & 3 < t < 4 \\ 0, & \text{Otherwise} \end{cases}$

(d) $y(t) = \begin{cases} 2, & t < 0 \\ -5, & 0 < t < 1 \\ 0, & t < 1 \end{cases}$

Chapter 7, Solution 24.

(a) $v(t) = \underline{\mathbf{-5\,u(t)}}$

(b) $i(t) = -10[u(t) - u(t-3)] + 10[u(t-3) - u(t-5)]$

$= \underline{\mathbf{-10\,u(t) + 20\,u(t-3) - 10\,u(t-5)}}$

(c) $x(t) = (t-1)[u(t-1) - u(t-2)] + [u(t-2) - u(t-3)]$
$+ (4-t)[u(t-3) - u(t-4)]$
$= (t-1)\,u(t-1) - (t-2)\,u(t-2) - (t-3)\,u(t-3) + (t-4)\,u(t-4)$
$= \underline{\mathbf{r(t-1) - r(t-2) - r(t-3) + r(t-4)}}$

(d) $y(t) = 2\,u(-t) - 5[u(t) - u(t-1)]$
$= \underline{\mathbf{2\,u(-t) - 5\,u(t) + 5\,u(t-1)}}$

Chapter 7, Problem 25.

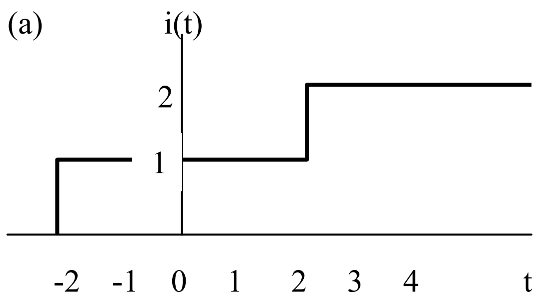
Sketch each of the following waveforms.

(a) $i(t) = u(t - 2) + u(t + 2)$
(b) $v(t) = r(t) - r(t - 3) + 4u(t - 5) - 8u(t - 8)$

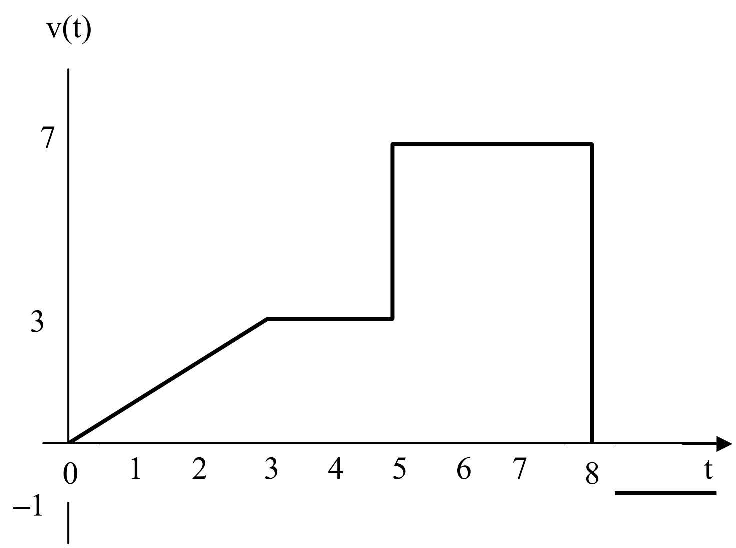
Chapter 7, Solution 25.

The waveforms are sketched below.

(a)



(b)



Chapter 7, Problem 26.

Express the signals in Fig. 7.104 in terms of singularity functions.

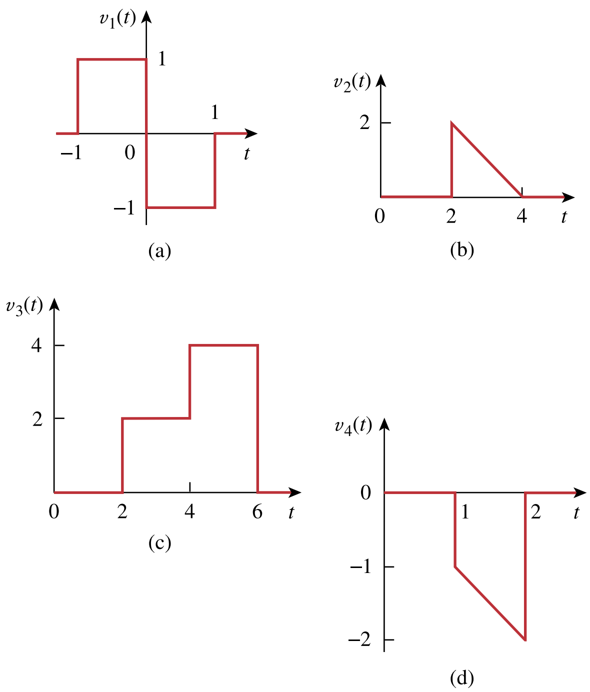


Figure 7.104
For Prob. 7.26.

Chapter 7, Solution 26.

(a)
$$v_1(t) = u(t+1) - u(t) + \left[u(t-1) - u(t) \right]$$
$$v_1(t) = \underline{\mathbf{u(t+1) - 2\,u(t) + u(t-1)}}$$

(b)
$$v_2(t) = (4-t)\left[u(t-2) - u(t-4) \right]$$
$$v_2(t) = \text{-}(t-4)\,u(t-2) + (t-4)\,u(t-4)$$
$$v_2(t) = \underline{\mathbf{2\,u(t-2) - r(t-2) + r(t-4)}}$$

(c)
$$v_3(t) = 2\left[u(t-2) - u(t-4) \right] + 4\left[u(t-4) - u(t-6) \right]$$
$$v_3(t) = \underline{\mathbf{2\,u(t-2) + 2\,u(t-4) - 4\,u(t-6)}}$$

(d)
$$v_4(t) = \text{-}t\left[u(t-1) - u(t-2) \right] = \text{-}t\,u(t-1) + t\,u(t-2)$$
$$v_4(t) = (\text{-}t + 1 - 1)\,u(t-1) + (t-2+2)\,u(t-2)$$
$$v_4(t) = \underline{\mathbf{\text{-}\,r(t-1) - u(t-1) + r(t-2) + 2\,u(t-2)}}$$

Chapter 7, Problem 27.

Express *v*(*t*) in Fig. 7.105 in terms of step functions.

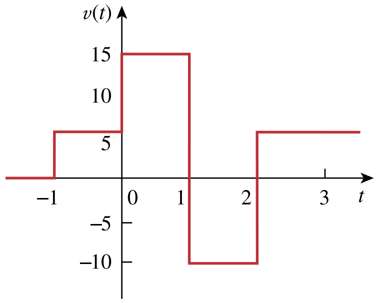


Figure 7.105
For Prob. 7.27.

Chapter 7, Solution 27.

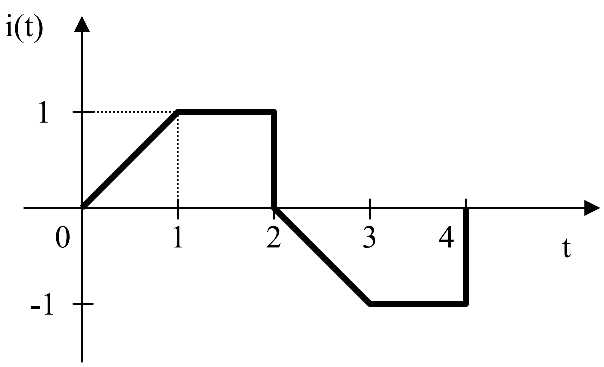
$$v(t) = \underline{\mathbf{5u(t+1)+10u(t)-25u(t-1)+15u(t-2)V}}$$

Chapter 7, Problem 28.

Sketch the waveform represented by
$i(t) = r(t) - r(t-1) - u(t-2) - r(t-2)$
$\qquad + r(t-3) + u(t-4)$

Chapter 7, Solution 28.

i(*t*) is sketched below.



Chapter 7, Problem 29.

Sketch the following functions:

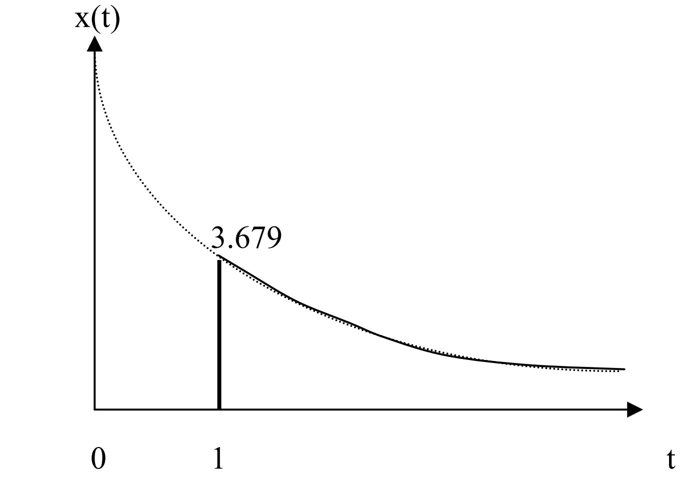
(a) $x(t) = 10e^{-t}\, u(t\text{-}1)$

(b) $y(t) = 10e^{-(t-1)}\, u(t)$

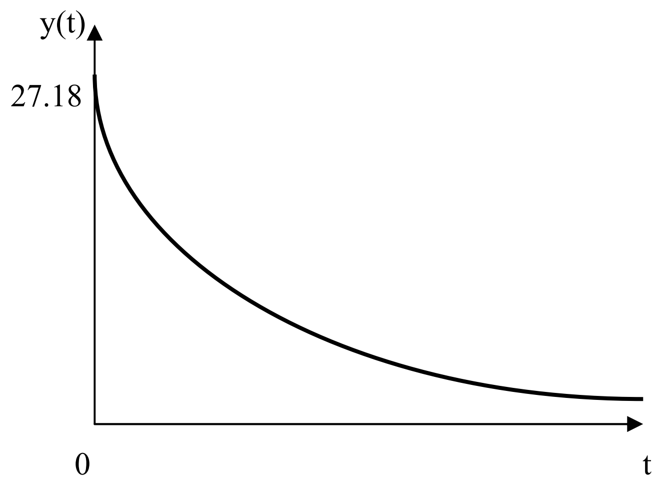
(c) $z(t) = \cos 4t\, \delta\, (t - 1)$

Chapter 7, Solution 29

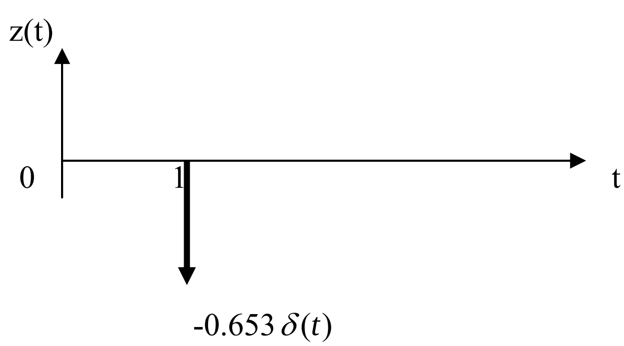
(a)



(b)



(c) $z(t) = \cos 4t\delta(t-1) = \cos 4\delta(t-1) = -0.6536\delta(t-1)$, which is sketched below.



Chapter 7, Problem 30.

Evaluate the following integrals involving the impulse functions:

(a) $\int_{-\infty}^{\infty} 4t^2 \delta(t-1)dt$

(b) $\int_{-\infty}^{\infty} 4t^2 \cos 2\pi t\, \delta(t-0.5)dt$

Chapter 7, Solution 30.

(a) $$\int_{-\infty}^{\infty} 4t^2\,\delta(t-1)\,dt = 4t^2\big|_{t=1} = \underline{\mathbf{4}}$$

(b) $$\int_{-\infty}^{\infty} 4t^2\cos(2\pi t)\,\delta(t-0.5)\,dt = 4t^2\cos(2\pi t)\big|_{t=0.5} = \cos\pi = \underline{\mathbf{-1}}$$

Chapter 7, Problem 31.

Evaluate the following integrals:

(a) $\int_{-\infty}^{\infty} e^{-4r^2} \delta(t-2)dt$

(b) $\int_{-\infty}^{\infty} [5\delta(t) + e^{-t}\,\delta(t) + \cos 2\pi t\,\delta(t)]dt$

Chapter 7, Solution 31.

(a) $$\int_{-\infty}^{\infty}\left[e^{-4t^2}\,\delta(t-2)\right]dt = e^{-4t^2}\Big|_{t=2} = e^{-16} = \underline{\mathbf{112\times10^{-9}}}$$

(b) $$\int_{-\infty}^{\infty}\left[5\delta(t) + e^{-t}\,\delta(t) + \cos 2\pi t\,\delta(t)\right]dt = \left(5 + e^{-t} + \cos(2\pi t)\right)\Big|_{t=0} = 5+1+1 = \underline{7}$$

Chapter 7, Problem 32.

Evaluate the following integrals:

(a) $\int_{l}^{t} u(\lambda)\,d\lambda$

(b) $\int_{0}^{4} r(t-1)dt$

(c) $\int_{1}^{5} (t-6)^2\,\delta(t-2)dt$

Chapter 7, Solution 32.

(a) $\int_{1}^{t} u(\lambda)d\lambda = \int_{1}^{t} 1d\lambda = \lambda \Big|_{1}^{t} = \underline{t-1}$

(b) $\int_{0}^{4} r(t-1)dt = \int_{0}^{1} 0dt + \int_{1}^{4} (t-1)dt = \frac{t^2}{2} - t\Big|_{1}^{4} = \underline{4.5}$

(c) $\int_{1}^{5} (t-6)^2\delta(t-2)dt = (t-6)^2\Big|_{t=2} = \underline{16}$

Chapter 7, Problem 33.

The voltage across a 10-mH inductor is $20\,\delta\,(t\text{ -}2)$ mV. Find the inductor current, assuming that the inductor is initially uncharged.

Chapter 7, Solution 33.

$$\mathrm{i(t)} = \frac{1}{\mathrm{L}}\int_0^{\mathrm{t}} \mathrm{v(t)\ dt + i(0)}$$

$$\mathrm{i(t)} = \frac{10^{-3}}{10\times 10^{-3}}\int_0^{\mathrm{t}} 20\delta(\mathrm{t}-2)\ \mathrm{dt} + 0$$

$$\mathrm{i(t)} = \underline{\mathbf{2\,u(t-2)\ A}}$$

Chapter 7, Problem 34.

Evaluate the following derivatives:

(a) $\frac{d}{dt}[u(t-1)\,u(t+1)]$

(b) $\frac{d}{dt}[r(t-6)\,u(t-2)]$

(c) $\frac{d}{dt}[\sin 4t u(t-31)]$

Chapter 7, Solution 34.

(a)
$$\frac{d}{dt}\left[u(t-1)\,u(t+1)\right] = \delta(t-1)u(t+1) +$$
$$u(t-1)\delta(t+1) = \delta(t-1)\bullet 1 + 0\bullet\delta(t+1) = \underline{\delta(t-1)}$$

(b)
$$\frac{d}{dt}\left[r(t-6)\,u(t-2)\right] = u(t-6)u(t-2) +$$
$$r(t-6)\delta(t-2) = u(t-6)\bullet 1 + 0\bullet\delta(t-2) = \underline{u(t-6)}$$

(c)
$$\frac{d}{dt}\left[\sin 4t\; u(t-3)\right] = 4\cos 4t\; u(t-3) + \sin 4t\delta(t-3)$$
$$= 4\cos 4t\; u(t-3) + \sin 4x3\delta(t-3)$$
$$= \underline{4\cos 4t\; u(t-3) - 0.5366\delta(t-3)}$$

Chapter 7, Problem 35.

Find the solution to the following differential equations:

(a) $\frac{dv}{dt} + 2v = 0, \qquad v(0) = -1\text{ V}$

(b) $2\frac{di}{dt} + 3i = 0, \qquad i(0) = 2$

Chapter 7, Solution 35.

(a)

$$v = Ae^{-2t}, \qquad v(0) = A = -1$$
$$v = \underline{-e^{-2t}}\ \underline{\mathbf{u(t)V}}$$

(b)

$$i = Ae^{3t/2}, \qquad i(0) = A = 2$$
$$i(t) = \underline{2e^{1.5t}}\ \underline{\mathbf{u(t)A}}$$

Chapter 7, Problem 36.

Solve for v in the following differential equations, subject to the stated initial condition.

(a) $dv / dt + v = u(t)$, $v(0) = 0$
(b) $2\, dv / dt - v = 3u(t)$, $v(0) = -6$

Chapter 7, Solution 36.

(a) $v(t) = A + Be^{-t}, \quad t > 0$

$A = 1, \qquad v(0) = 0 = 1 + B \qquad \text{or} \qquad B = -1$

$v(t) = \underline{\mathbf{1 - e^{-t}\ V, \quad t > 0}}$

(b) $v(t) = A + Be^{t/2}, \quad t > 0$

$A = -3, \qquad v(0) = -6 = -3 + B \qquad \text{or} \qquad B = -3$

$v(t) = \underline{\mathbf{-3\left(1 + e^{t/2}\right) V, \quad t > 0}}$

Chapter 7, Problem 37.

A circuit is described by

$$4\frac{dv}{dt} + v = 10$$

(a) What is the time constant of the circuit?
(b) What is $v(\infty)$ the final value of v?
(c) If $v(0) = 2$ find $v(t)$ for $t \geq 0$.

Chapter 7, Solution 37.

Let $v = v_h + v_p$, $v_p = 10$.

$$\dot{v}_h + \frac{1}{4}v_h = 0 \longrightarrow v_h = Ae^{-t/4}$$

$$v = 10 + Ae^{-0.25t}$$

$$v(0) = 2 = 10 + A \longrightarrow A = -8$$

$$v = 10 - 8e^{-0.25t}$$

(a) $\tau = \underline{4s}$

(b) $v(\infty) = \underline{10}\ \underline{\mathbf{V}}$

(c) $\underline{v = 10 - 8e^{-0.25t}}\ \underline{\mathbf{u(t)V}}$

Chapter 7, Problem 38.

A circuit is described by

$$\frac{di}{dt} + 3i = 2u(t)$$

Find $i(t)$ for $t > 0$ given that $i(0) = 0$.

Chapter 7, Solution 38

Let $i = i_p + i_h$

$$\dot{i}_h + 3i_h = 0 \longrightarrow i_h = Ae^{-3t}u(t)$$

Let $i_p = ku(t), \quad \dot{i}_p = 0, \quad 3ku(t) = 2u(t) \longrightarrow k = \frac{2}{3}$

$$i_p = \frac{2}{3}u(t)$$

$$i = (Ae^{-3t} + \frac{2}{3})u(t)$$

If $i(0) = 0$, then $A + 2/3 = 0$, i.e. $A = -2/3$. Thus

$$\underline{i = \frac{2}{3}(1 - e^{-3t})u(t)}$$

Chapter 7, Problem 39.

Calculate the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.106.

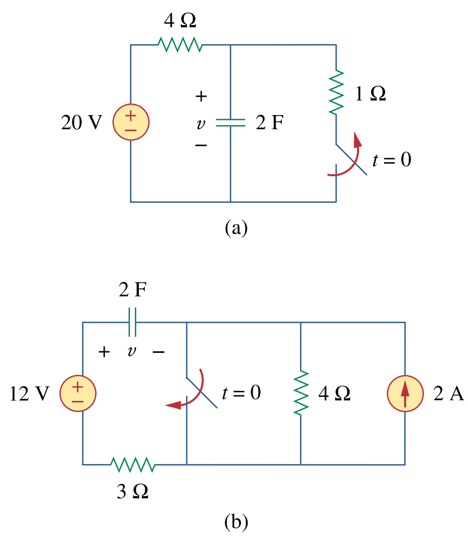


Figure 7.106
For Prob. 7.39.

Chapter 7, Solution 39.

(a) Before t = 0,

$$v(t) = \frac{1}{4+1}(20) = \underline{\mathbf{4\ V}}$$

After t = 0,

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$$

$$\tau = RC = (4)(2) = 8, \quad v(0) = 4, \qquad v(\infty) = 20$$

$$v(t) = 20 + (4-20)\,e^{-t/8}$$

$$v(t) = \underline{20 - 16\,e^{-t/8}}\ V$$

(b) Before t = 0, $v = v_1 + v_2$, where v_1 is due to the 12-V source and v_2 is due to the 2-A source.

$$v_1 = 12\ V$$

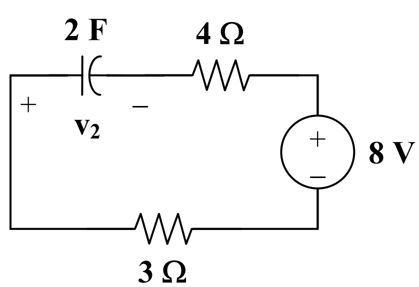
To get v_2, transform the current source as shown in Fig. (a).

$$v_2 = -8\ V$$

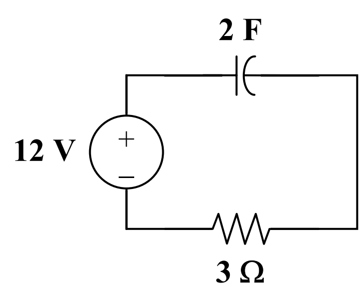
Thus,

$$v = 12 - 8 = \underline{\mathbf{4\ V}}$$

After t = 0, the circuit becomes that shown in Fig. (b).



(a)



(b)

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$$

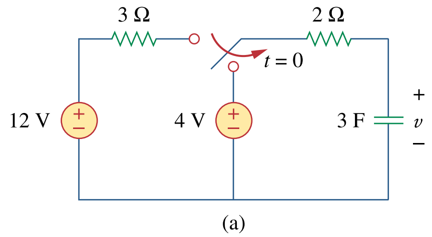
$$v(\infty) = 12, \qquad v(0) = 4, \qquad \tau = RC = (2)(3) = 6$$

$$v(t) = 12 + (4-12)\,e^{-t/6}$$

$$v(t) = \underline{\mathbf{12 - 8\,e^{-t/6}\ V}}$$

Chapter 7, Problem 40.

Find the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.107.



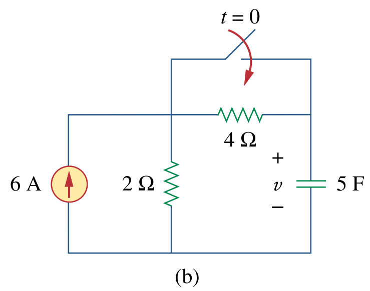


Figure 7.107
For Prob. 7.40.

Chapter 7, Solution 40.

(a) Before t = 0, $v = \underline{\mathbf{12\ V}}$.

After t = 0, $v(t) = v(\infty) + \left[v(0) - v(\infty)\right] e^{-t/\tau}$

$$v(\infty) = 4, \qquad v(0) = 12, \qquad \tau = RC = (2)(3) = 6$$

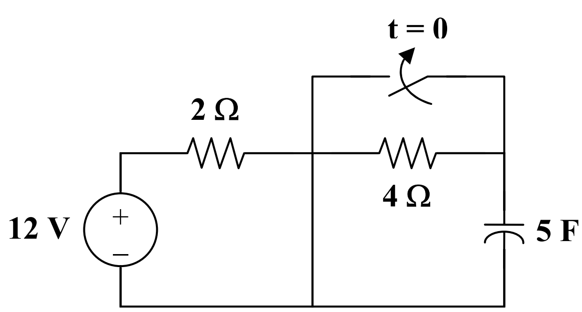
$$v(t) = 4 + (12 - 4)\, e^{-t/6}$$

$$v(t) = \underline{\mathbf{4 + 8e^{-t/6}\ V}}$$

(b) Before t = 0, $v = \underline{\mathbf{12\ V}}$.

After t = 0, $v(t) = v(\infty) + \left[v(0) - v(\infty)\right] e^{-t/\tau}$

After transforming the current source, the circuit is shown below.



$$v(0) = 12, \qquad v(\infty) = 12, \qquad \tau = RC = (2)(5) = 10$$

$v = \underline{\mathbf{12\ V}}$

Chapter 7, Problem 41.

For the circuit in Fig. 7.108, find $v(t)$ for $t > 0$.

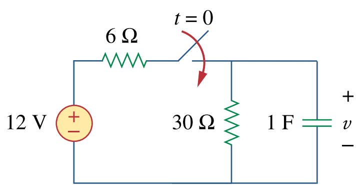


Figure 7.108
For Prob. 7.41.

Chapter 7, Solution 41.

$$v(0) = 0, \qquad v(\infty) = \frac{30}{36}(12) = 10$$

$$R_{eq}C = (6 \parallel 30)(1) = \frac{(6)(30)}{36} = 5$$

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(t) = 10 + (0 - 10)\, e^{-t/5}$$

$$v(t) = \underline{10(1 - e^{-0.2t})\, u(t) V}$$

Chapter 7, Problem 42.

(a) If the switch in Fig. 7.109 has been open for a long time and is closed at $t = 0$, find $v_o(t)$.

(b) Suppose that the switch has been closed for a long time and is opened at $t = 0$. Find $v_o(t)$.

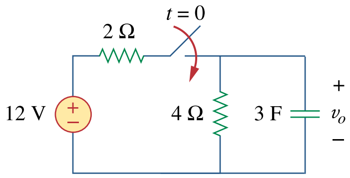


Figure 7.109
For Prob. 7.42.

Chapter 7, Solution 42.

(a)

$$v_o(t) = v_o(\infty) + \left[v_o(0) - v_o(\infty) \right] e^{-t/\tau}$$

$$v_o(0) = 0, \qquad v_o(\infty) = \frac{4}{4+2}(12) = 8$$

$$\tau = R_{eq}C_{eq}, \qquad R_{eq} = 2 \,||\, 4 = \frac{4}{3}$$

$$\tau = \frac{4}{3}(3) = 4$$

$$v_o(t) = 8 - 8e^{-t/4}$$

$$v_o(t) = \underline{\mathbf{8(1 - e^{-0.25t})\ V}}$$

(b) For this case, $v_o(\infty) = 0$ so that

$$v_o(t) = v_o(0)e^{-t/\tau}$$

$$v_o(0) = \frac{4}{4+2}(12) = 8, \qquad \tau = RC = (4)(3) = 12$$

$$v_o(t) = \underline{\mathbf{8e^{-t/12}\ V}}$$

Chapter 7, Problem 43.

Consider the circuit in Fig. 7.110. Find $i(t)$ for $t < 0$ and $t > 0$.

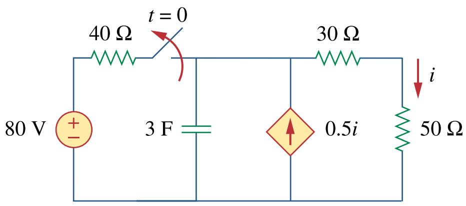
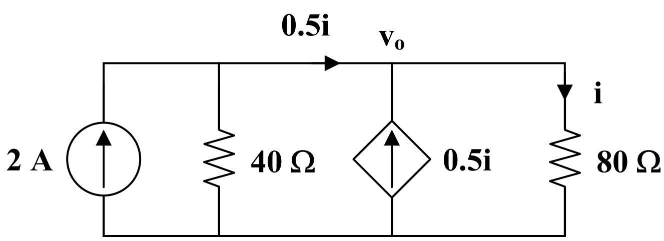


Figure 7.110
For Prob. 7.43.

Chapter 7, Solution 43.

Before t = 0, the circuit has reached steady state so that the capacitor acts like an open circuit. The circuit is equivalent to that shown in Fig. (a) after transforming the voltage source.



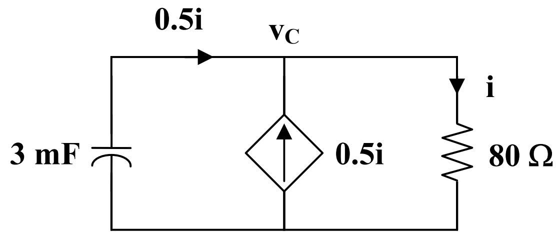
(a)

$$0.5i = 2 - \frac{v_o}{40}, \qquad i = \frac{v_o}{80}$$

Hence, $\frac{1}{2}\frac{v_o}{80} = 2 - \frac{v_o}{40} \longrightarrow v_o = \frac{320}{5} = 64$

$$i = \frac{v_o}{80} = \underline{\mathbf{0.8\ A}}$$

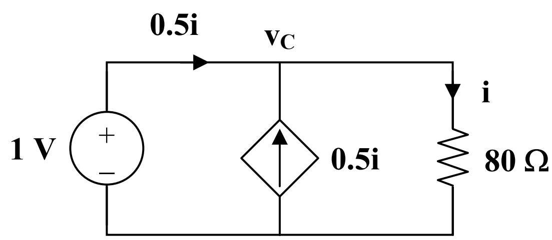
After t = 0, the circuit is as shown in Fig. (b).



(b)

$$v_C(t) = v_C(0)\,e^{-t/\tau}, \qquad \tau = R_{th}C$$

To find R_{th}, we replace the capacitor with a 1-V voltage source as shown in Fig. (c).



(c)

$$i = \frac{v_C}{80} = \frac{1}{80}, \qquad i_o = 0.5\,i = \frac{0.5}{80}$$

$$R_{th} = \frac{1}{i_o} = \frac{80}{0.5} = 160\,\Omega, \qquad \tau = R_{th}C = 480$$

$$v_C(0) = 64\text{ V}$$

$$v_C(t) = 64\,e^{-t/480}$$

$$0.5\,i = -i_C = -C\frac{dv_C}{dt} = -3\left(\frac{1}{480}\right)64\,e^{-t/480}$$

$$i(t) = \underline{0.8\,e^{-t/480}\ u(t)\text{A}}$$

Chapter 7, Problem 44.

The switch in Fig. 7.111 has been in position *a* for a long time. At $t = 0$ it moves to position *b*. Calculate $i(t)$ for all $t > 0$.

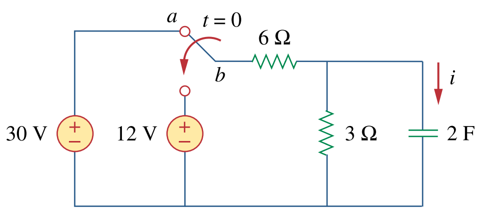


Figure 7.111
For Prob. 7.44.

Chapter 7, Solution 44.

$$R_{eq} = 6 \,||\, 3 = 2\,\Omega, \qquad \tau = RC = 4$$
$$v(t) = v(\infty) + \left[\, v(0) - v(\infty) \right] e^{-t/\tau}$$

Using voltage division,

$$v(0) = \frac{3}{3+6}(30) = 10 \text{ V}, \qquad v(\infty) = \frac{3}{3+6}(12) = 4 \text{ V}$$

Thus,

$$v(t) = 4 + (10 - 4)\, e^{-t/4} = 4 + 6\, e^{-t/4}$$
$$i(t) = C\frac{dv}{dt} = (2)(6)\left(\frac{-1}{4}\right)e^{-t/4} = \underline{\mathbf{-3\,e^{-0.25t}\ A}}$$

Chapter 7, Problem 45.

Find v_o in the circuit of Fig. 7.112 when $v_s = 6u(t)$. Assume that $v_o(0) = 1$ V.

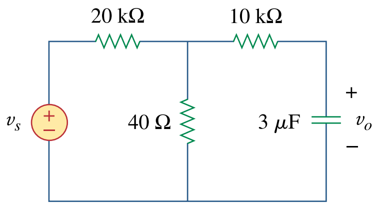
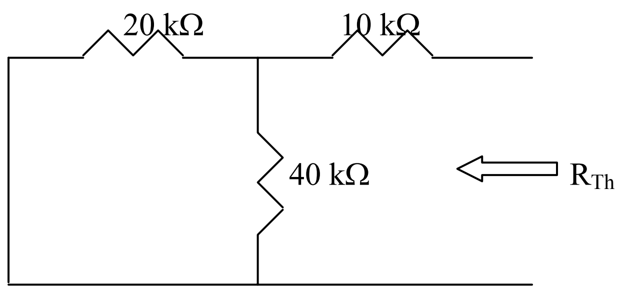


Figure 7.112
For Prob. 7.45.

Chapter 7, Solution 45.

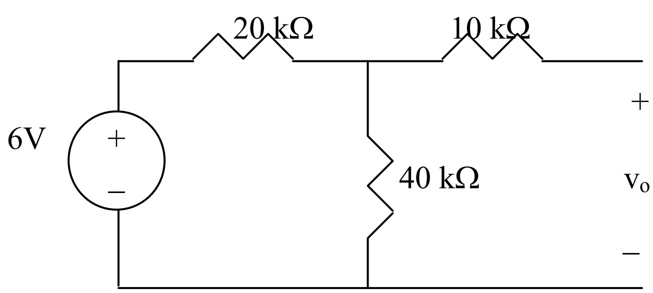
To find R_{Th}, consider the circuit shown below.



$$R_{Th} = 10 + 20 // 40 = 10 + \frac{20x40}{60} = \frac{70}{3} k\Omega$$

$$\tau = R_{Th}C = \frac{70}{3} x10^3 x3x10^{-6} = 0.07$$

To find $v_o(\infty)$, consider the circuit below.



$$v_o(\infty) = \frac{40}{40+20}(6V) = 4V$$

$$v_o(t) = v_o(\infty) + [v_o(0) - v_o(\infty)]e^{-t/\tau} = 4 + (1-4)e^{-t/0.07} = \underline{4 - 3e^{-14.286t} \quad V} \; \mathbf{\underline{u(t)}}$$

Chapter 7, Problem 46.

For the circuit in Fig. 7.113, $i_s(t) = 5u(t)$ Find $v(t)$.

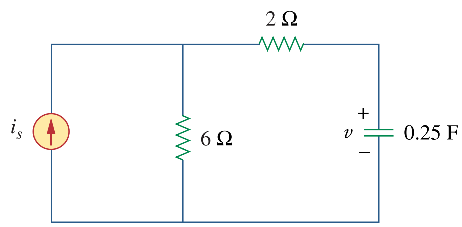


Figure 7.113
For Prob. 7.46.

Chapter 7, Solution 46.

$\tau = R_{Th}C = (2+6)x0.25 = 2s, \qquad v(0) = 0, \qquad v(\infty) = 6i_s = 6x5 = 30$

$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} = \underline{30(1 - e^{-t/2})}$ V

Chapter 7, Problem 47.

Determine $v(t)$ for $t > 0$ in the circuit of Fig. 7.114 if $v(0) = 0$.

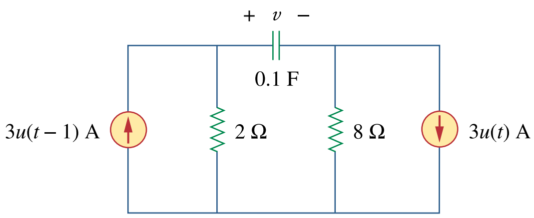


Figure 7.114
For Prob. 7.47.

Chapter 7, Solution 47.

For $t < 0,\ u(t) = 0, \quad u(t-1) = 0, \quad v(0) = 0$

For $0 < t < 1, \quad \tau = RC = (2+8)(0.1) = 1$

$$v(0) = 0, \qquad v(\infty) = (8)(3) = 24$$
$$v(t) = v(\infty) + [\,v(0) - v(\infty)\,]\, e^{-t/\tau}$$
$$v(t) = 24\left(1 - e^{-t}\right)$$

For $t > 1, \qquad v(1) = 24\left(1 - e^{-1}\right) = 15.17$

$$-6 + v(\infty) - 24 = 0 \longrightarrow v(\infty) = 30$$
$$v(t) = 30 + (15.17 - 30)\, e^{-(t-1)}$$
$$v(t) = 30 - 14.83\, e^{-(t-1)}$$

Thus,

$$v(t) = \begin{cases} \mathbf{24\left(1 - e^{-t}\right) V,} & \mathbf{0 < t < 1} \\ \mathbf{30 - 14.83\, e^{-(t-1)}\ V,} & \mathbf{t > 1} \end{cases}$$

Chapter 7, Problem 48.

Find $v(t)$ and $i(t)$ in the circuit of Fig. 7.115.

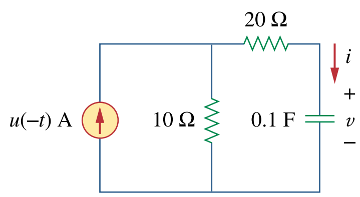


Figure 7.115
For Prob. 7.48.

Chapter 7, Solution 48.

For $t < 0$, $\quad u(-t) = 1, \quad v(0) = 10 \text{ V}$

For $t > 0$, $\quad u(-t) = 0, \quad v(\infty) = 0$

$$R_{th} = 20 + 10 = 30, \quad \tau = R_{th}C = (30)(0.1) = 3$$

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

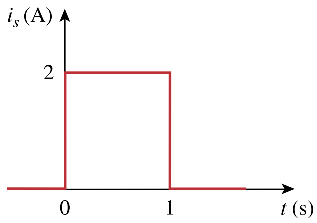
$$v(t) = \underline{\mathbf{10\,e^{-t/3}\ V}}$$

$$i(t) = C\frac{dv}{dt} = (0.1)\left(\frac{-1}{3}\right)10\,e^{-t/3}$$

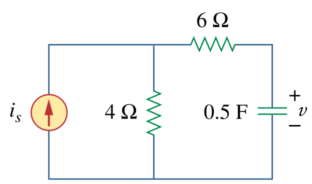
$$i(t) = \underline{\mathbf{\frac{-1}{3}e^{-t/3}\ A}}$$

Chapter 7, Problem 49.

If the waveform in Fig. 7.116(a) is applied to the circuit of Fig. 7.116(b), find $v(t)$. Assume $v(0) = 0$.



(a)



(b)

Figure 7.116
For Prob. 7.49 and Review Question 7.10.

Chapter 7, Solution 49.

For $0 < t < 1$, $\quad v(0) = 0, \qquad v(\infty) = (2)(4) = 8$

$$R_{eq} = 4 + 6 = 10, \qquad \tau = R_{eq}C = (10)(0.5) = 5$$

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right]\, e^{-t/\tau}$$

$$v(t) = 8\left(1 - e^{-t/5}\right)\ \text{V}$$

For $t > 1$, $\qquad v(1) = 8\left(1 - e^{-0.2}\right) = 1.45, \qquad v(\infty) = 0$

$$v(t) = v(\infty) + \left[\, v(1) - v(\infty)\right]\, e^{-(t-1)/\tau}$$

$$v(t) = 1.45\, e^{-(t-1)/5}\ \text{V}$$

Thus,

$$v(t) = \underline{\begin{cases} \mathbf{8\left(1 - e^{-t/5}\right)\,V}, & \mathbf{0 < t < 1} \\ \mathbf{1.45\, e^{-(t-1)/5}\ V}, & \mathbf{t > 1} \end{cases}}$$

Chapter 7, Problem 50.

* In the circuit of Fig. 7.117, find i_x for $t > 0$. Let $R_1 = R_2 = 1\text{k}\Omega$, $R_3 = 2\text{k}\Omega$, and $C =$ 0.25 mF.

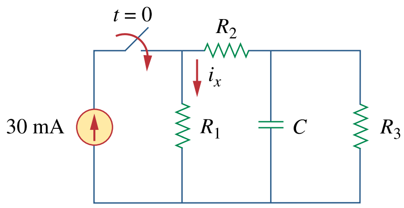


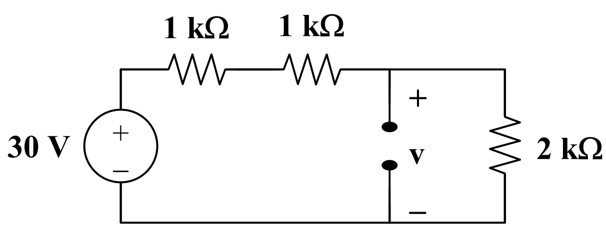
Figure 7.117
For Prob. 7.50.

Chapter 7, Solution 50.

For the capacitor voltage,

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$
$$v(0) = 0$$

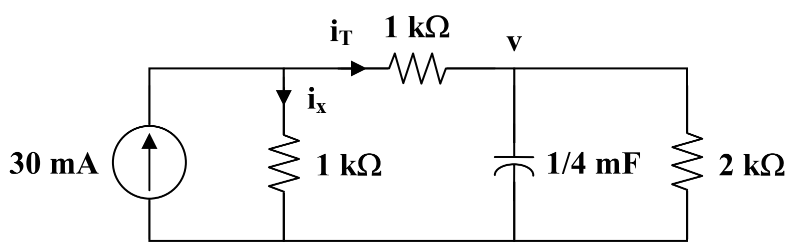
For t > 0, we transform the current source to a voltage source as shown in Fig. (a).



(a)

$$v(\infty) = \frac{2}{2+1+1}(30) = 15 \text{ V}$$
$$R_{th} = (1+1) \,\|\, 2 = 1 \text{ k}\Omega$$
$$\tau = R_{th} C = 10^3 \times \frac{1}{4} \times 10^{-3} = \frac{1}{4}$$
$$v(t) = 15\left(1 - e^{-4t}\right), \quad t > 0$$

We now obtain i_x from v(t). Consider Fig. (b).



(b)

$$i_x = 30 \text{ mA} - i_T$$

But $\quad i_T = \dfrac{v}{R_3} + C\dfrac{dv}{dt}$

$$i_T(t) = 7.5\left(1 - e^{-4t}\right) \text{ mA} + \frac{1}{4} \times 10^{-3} \,(-15)(-4)e^{-4t} \text{ A}$$
$$i_T(t) = 7.5\left(1 + e^{-4t}\right) \text{ mA}$$

Thus,

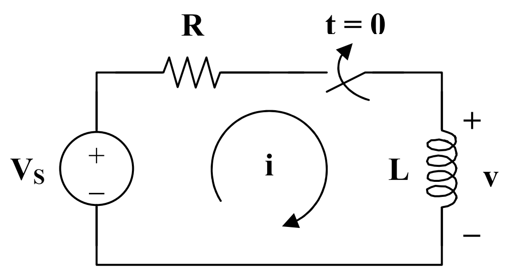
$$i_x(t) = 30 - 7.5 - 7.5e^{-4t} \text{ mA}$$
$$i_x(t) = \mathbf{7.5\left(3 - e^{-4t}\right) mA, \quad t > 0}$$

Chapter 7, Problem 51.

Rather than applying the short-cut technique used in Section 7.6, use KVL to obtain Eq. (7.60).

Chapter 7, Solution 51.

Consider the circuit below.



After the switch is closed, applying KVL gives

$$V_S = Ri + L\frac{di}{dt}$$

or $$L\frac{di}{dt} = \text{-}R\left(i - \frac{V_S}{R}\right)$$

$$\frac{di}{i - V_S/R} = \frac{\text{-}R}{L}dt$$

Integrating both sides,

$$\ln\left(i - \frac{V_S}{R}\right)\Big|_{I_0}^{i(t)} = \frac{\text{-}R}{L}t$$

$$\ln\left(\frac{i - V_S/R}{I_0 - V_S/R}\right) = \frac{\text{-}t}{\tau}$$

or $$\frac{i - V_S/R}{I_0 - V_S/R} = e^{-t/\tau}$$

$$\underline{\mathbf{i(t)} = \frac{\mathbf{V_S}}{\mathbf{R}} + \left(\mathbf{I_0} - \frac{\mathbf{V_S}}{\mathbf{R}}\right)\mathbf{e^{-t/\tau}}}$$

which is the same as Eq. (7.60).

Chapter 7, Problem 52.

For the circuit in Fig. 7.118, find $i(t)$ for $t > 0$.

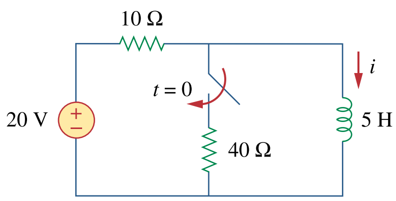


Figure 7.118
For Prob. 7.52.

Chapter 7, Solution 52.

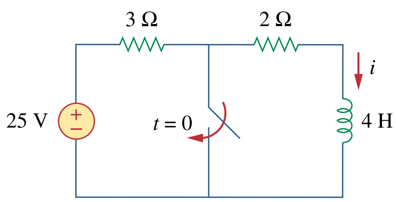
$$i(0) = \frac{20}{10} = 2\text{ A}, \qquad i(\infty) = 2\text{ A}$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

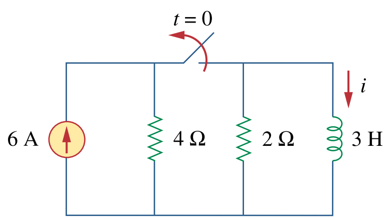
$$i(t) = \underline{\mathbf{2\ A}}$$

Chapter 7, Problem 53.

Determine the inductor current $i(t)$ for both $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.119.



(a)



(b)

Figure 7.119
For Prob. 7.53.

Chapter 7, Solution 53.

(a) Before t = 0, $i = \dfrac{25}{3+2} = \underline{\mathbf{5\ A}}$

After t = 0, $i(t) = i(0)e^{-t/\tau}$

$$\tau = \frac{L}{R} = \frac{4}{2} = 2, \qquad i(0) = 5$$

$$i(t) = \underline{5e^{-t/2}\ u(t)A}$$

(b) Before t = 0, the inductor acts as a short circuit so that the 2 Ω and 4 Ω resistors are short-circuited.

$$i(t) = \underline{\mathbf{6\ A}}$$

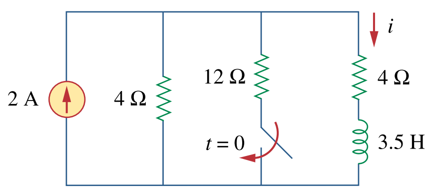
After t = 0, we have an RL circuit.

$$i(t) = i(0)e^{-t/\tau}, \qquad \tau = \frac{L}{R} = \frac{3}{2}$$

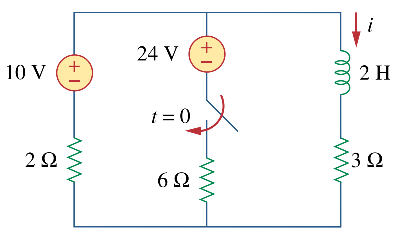
$$i(t) = \underline{6e^{-2t/3}\ u(t)A}$$

Chapter 7, Problem 54.

Obtain the inductor current for both $t < 0$ and $t > 0$ in each of the circuits in Fig. 7.120.



(a)



(b)

Figure 7.120
For Prob. 7.54.

Chapter 7, Solution 54.

(a) Before t = 0, i is obtained by current division or

$$i(t) = \frac{4}{4+4}(2) = \underline{\mathbf{1\ A}}$$

After t = 0,

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

$$\tau = \frac{L}{R_{eq}}, \qquad R_{eq} = 4 + 4 \,||\, 12 = 7\ \Omega$$

$$\tau = \frac{3.5}{7} = \frac{1}{2}$$

$$i(0) = 1, \qquad i(\infty) = \frac{4 \,||\, 12}{4 + 4 \,||\, 12}(2) = \frac{3}{4+3}(2) = \frac{6}{7}$$

$$i(t) = \frac{6}{7} + \left(1 - \frac{6}{7}\right)e^{-2t}$$

$$i(t) = \underline{\mathbf{\frac{1}{7}\left(6 - e^{-2t}\right)\ A}}$$

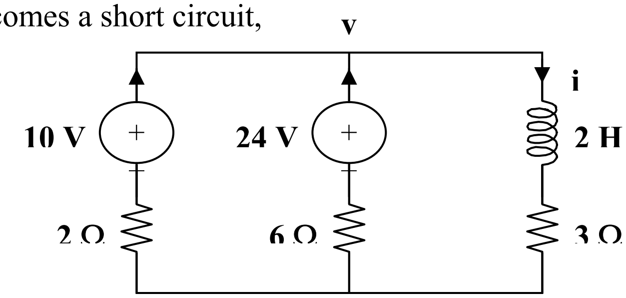
(b) Before t = 0, $i(t) = \frac{10}{2+3} = \underline{\mathbf{2\ A}}$

After t = 0, $R_{eq} = 3 + 6 \,||\, 2 = 4.5$

$$\tau = \frac{L}{R_{eq}} = \frac{2}{4.5} = \frac{4}{9}$$

$$i(0) = 2$$

To find $i(\infty)$, consider the circuit below, at t = when the inductor becomes a short circuit,



$$\frac{10 - v}{2} + \frac{24 - v}{6} = \frac{v}{3} \longrightarrow v = 9$$

$$i(\infty) = \frac{v}{3} = 3\ A$$

$$i(t) = 3 + (2 - 3)e^{-9t/4}$$

$$i(t) = \underline{\mathbf{3 - e^{-9t/4}\ A}}$$

Chapter 7, Problem 55.

Find $v(t)$ for $t < 0$ and $t > 0$ in the circuit of Fig. 7.121.

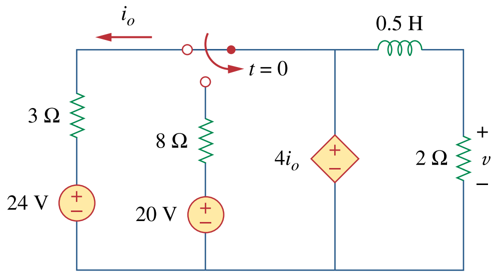
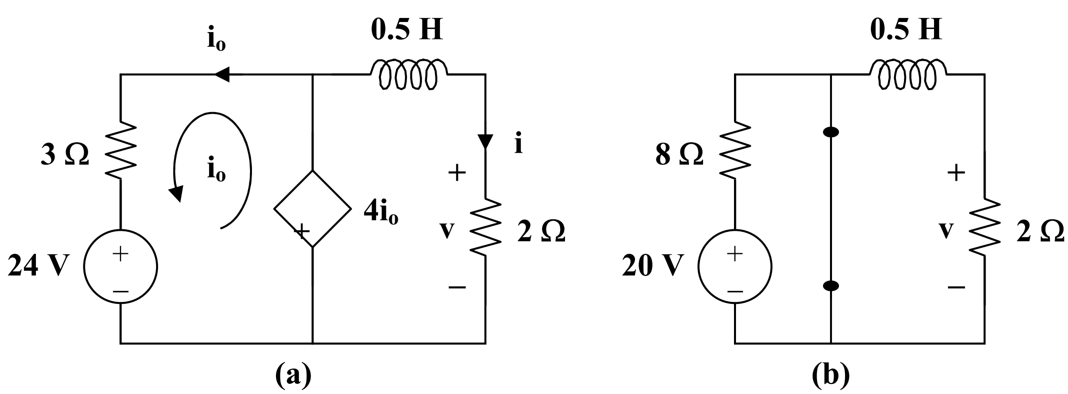


Figure 7.121
For Prob. 7.55.

Chapter 7, Solution 55.

For t < 0, consider the circuit shown in Fig. (a).



$$3i_o + 24 - 4i_o = 0 \longrightarrow i_o = 24$$

$$\underline{v(t) = 4i_o = \mathbf{96\ V}} \qquad i = \frac{v}{2} = 48\ \text{A}$$

For t > 0, consider the circuit in Fig. (b).

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(0) = 48, \qquad i(\infty) = 0$$

$$R_{th} = 2\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{0.5}{2} = \frac{1}{4}$$

$$i(t) = (48)e^{-4t}$$

$$v(t) = 2i(t) = \underline{96e^{-4t}\ u(t)V}$$

Chapter 7, Problem 56.

For the network shown in Fig. 7.122, find $v(t)$ for $t > 0$.

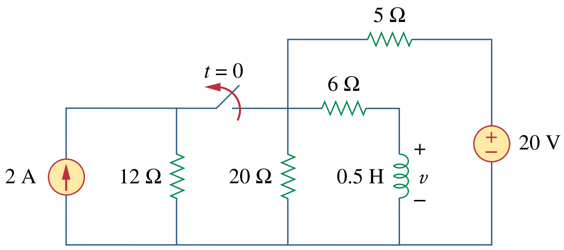


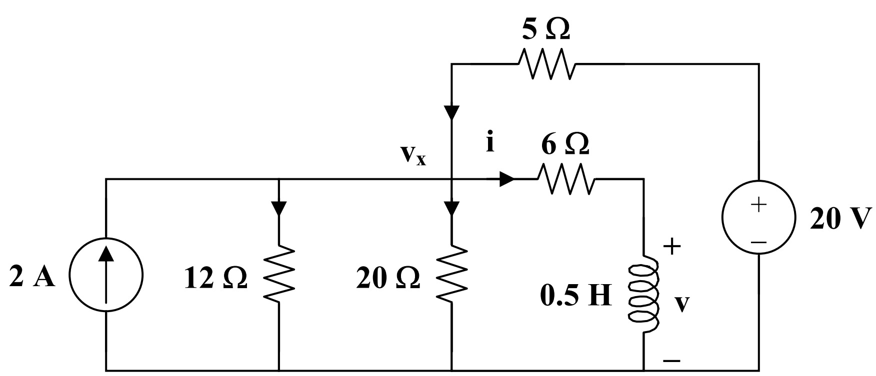
Figure 7.122
For Prob. 7.56.

Chapter 7, Solution 56.

$$R_{eq} = 6 + 20 \,||\, 5 = 10\,\Omega, \qquad \tau = \frac{L}{R} = 0.05$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty) \right] e^{-t/\tau}$$

i(0) is found by applying nodal analysis to the following circuit.



$$2 + \frac{20 - v_x}{5} = \frac{v_x}{12} + \frac{v_x}{20} + \frac{v_x}{6} \longrightarrow v_x = 12$$

$$i(0) = \frac{v_x}{6} = 2\text{ A}$$

Since $20 \,||\, 5 = 4$,

$$i(\infty) = \frac{4}{4+6}(4) = 1.6$$

$$i(t) = 1.6 + (2 - 1.6)e^{-t/0.05} = 1.6 + 0.4e^{-20t}$$

$$v(t) = L\frac{di}{dt} = \frac{1}{2}(0.4)(-20)e^{-20t}$$

$$v(t) = \underline{\mathbf{-4e^{-20t}\ V}}$$

Chapter 7, Problem 57.

* Find $i_1(t)$ and $i_2(t)$ for $t > 0$ in the circuit of Fig. 7.123.

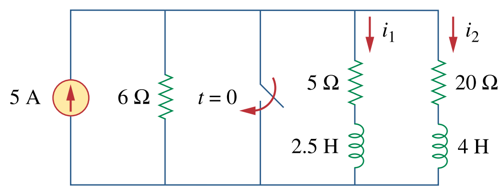
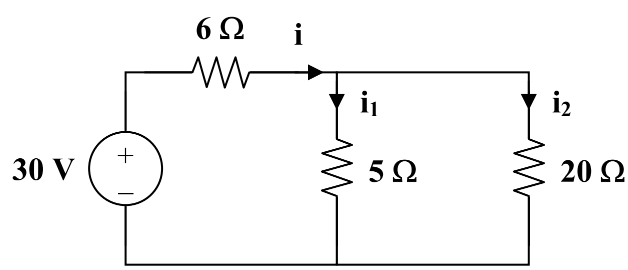


Figure 7.123
For Prob. 7.57.

* An asterisk indicates a challenging problem.

Chapter 7, Solution 57.

At $t = 0^-$, the circuit has reached steady state so that the inductors act like short circuits.



$$i = \frac{30}{6 + 5 \,||\, 20} = \frac{30}{10} = 3, \qquad i_1 = \frac{20}{25}(3) = 2.4, \qquad i_2 = 0.6$$

$$i_1(0) = 2.4 \text{ A}, \qquad i_2(0) = 0.6 \text{ A}$$

For $t > 0$, the switch is closed so that the energies in L_1 and L_2 flow through the closed switch and become dissipated in the 5 Ω and 20 Ω resistors.

$$i_1(t) = i_1(0)e^{-t/\tau_1}, \qquad \tau_1 = \frac{L_1}{R_1} = \frac{2.5}{5} = \frac{1}{2}$$

$$i_1(t) = \underline{2.4e^{-2t}\ u(t)\text{A}}$$

$$i_2(t) = i_2(0)e^{-t/\tau_2}, \qquad \tau_2 = \frac{L_2}{R_2} = \frac{4}{20} = \frac{1}{5}$$

$$i_2(t) = \underline{0.6e^{-5t}\ u(t)\text{A}}$$

Chapter 7, Problem 58.

Rework Prob. 7.17 if $i(0) = 10$ A and $v(t) = 20u(t)$ V.

Chapter 7, Solution 58.

For t < 0, $\quad v_o(t) = 0$

For t > 0, $\quad i(0) = 10, \quad i(\infty) = \dfrac{20}{1+3} = 5$

$$R_{th} = 1 + 3 = 4\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{1/4}{4} = \frac{1}{16}$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = 5\left(1 + e^{-16t}\right) \text{ A}$$

$$v_o(t) = 3i + L\frac{di}{dt} = 15\left(1 + e^{-16t}\right) + \frac{1}{4}(-16)(5)e^{-16t}$$

$$v_o(t) = \underline{\mathbf{15 - 5e^{-16t}\ V}}$$

Chapter 7, Problem 59.

Determine the step response $v_0(t)$ to v_s in the circuit of Fig. 7.124.

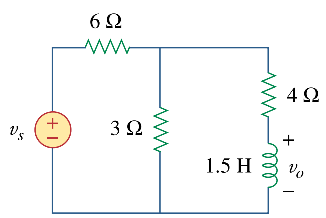


Figure 7.124
For Prob. 7.59.

Chapter 7, Solution 59.

Let I be the current through the inductor.

For t < 0, $\quad v_s = 0, \quad i(0) = 0$

For t > 0, $\quad R_{eq} = 4 + 6 \,||\, 3 = 6, \quad \tau = \dfrac{L}{R_{eq}} = \dfrac{1.5}{6} = 0.25$

$$i(\infty) = \frac{2}{2+4}(3) = 1$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = 1 - e^{-4t}$$

$$v_o(t) = L\frac{di}{dt} = (1.5)(-4)(-e^{-4t})$$

$$v_o(t) = \underline{6e^{-4t}u(t)\ V}$$

Chapter 7, Problem 60.

Find $v(t)$ for $t > 0$ in the circuit of Fig. 7.125 if the initial current in the inductor is zero.

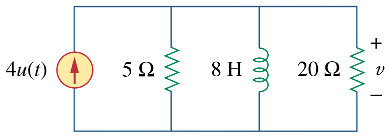


Figure 7.125
For Prob. 7.60.

Chapter 7, Solution 60.

Let I be the inductor current.

For $t < 0$, $\quad u(t) = 0 \longrightarrow i(0) = 0$

For $t > 0$, $\quad R_{eq} = 5 \,||\, 20 = 4\,\Omega, \qquad \tau = \dfrac{L}{R_{eq}} = \dfrac{8}{4} = 2$

$$i(\infty) = 4$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = 4\left(1 - e^{-t/2}\right)$$

$$v(t) = L\frac{di}{dt} = (8)(-4)\left(\frac{-1}{2}\right)e^{-t/2}$$

$$v(t) = \underline{\mathbf{16\,e^{-0.5t}\ V}}$$

Chapter 7, Problem 61.

In the circuit of Fig. 7.126, i_s changes from 5 A to 10 A at $t = 0$ that is, $i_s = 5u(-t) + 10u(t)$ Find v and i.

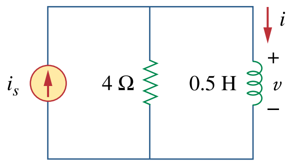
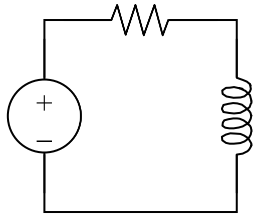


Figure 7.126
For Prob. 7.61.

Chapter 7, Solution 61.

The current source is transformed as shown below.



$$\tau = \frac{L}{R} = \frac{1/2}{4} = \frac{1}{8}, \qquad i(0) = 5, \qquad i(\infty) = 10$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = \underline{10 - 5e^{-8t}\ u(t)A}$$

$$v(t) = L\frac{di}{dt} = \left(\frac{1}{2}\right)(-5)(-8)\,e^{-8t}$$

$$v(t) = \underline{20e^{-8t}u(t)\ V}$$

Chapter 7, Problem 62.

For the circuit in Fig. 7.127, calculate $i(t)$ if $i(0) = 0$.

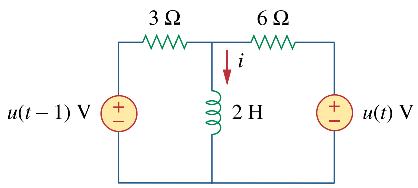


Figure 7.127
For Prob. 7.62.

Chapter 7, Solution 62.

$$\tau = \frac{L}{R_{eq}} = \frac{2}{3 \,||\, 6} = 1$$

For $0 < t < 1$, $u(t-1) = 0$ so that

$$i(0) = 0, \qquad i(\infty) = \frac{1}{6}$$

$$i(t) = \frac{1}{6}\left(1 - e^{-t}\right)$$

For $t > 1$, $\quad i(1) = \frac{1}{6}\left(1 - e^{-1}\right) = 0.1054$

$$i(\infty) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

$$i(t) = 0.5 + (0.1054 - 0.5)e^{-(t-1)}$$

$$i(t) = 0.5 - 0.3946e^{-(t-1)}$$

Thus,

$$\underline{i(t) = \begin{cases} \mathbf{\frac{1}{6}\left(1 - e^{-t}\right) A} & \mathbf{0 < t < 1} \\ \mathbf{0.5 - 0.3946e^{-(t-1)}\ A} & \mathbf{t > 1} \end{cases}}$$

Chapter 7, Problem 63.

Obtain $v(t)$ and $i(t)$ in the circuit of Fig. 7.128.

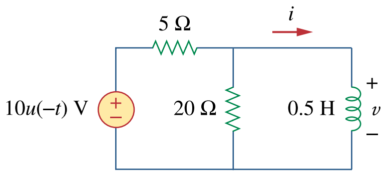


Figure 7.128
For Prob. 7.63.

Chapter 7, Solution 63.

For $t < 0$, $\quad u(-t) = 1, \quad i(0) = \frac{10}{5} = 2$

For $t > 0$, $\quad u(-t) = 0, \quad i(\infty) = 0$

$$R_{th} = 5 \,||\, 20 = 4\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{0.5}{4} = \frac{1}{8}$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = \underline{2e^{-8t}\, u(t) A}$$

$$v(t) = L\frac{di}{dt} = \left(\frac{1}{2}\right)(-8)(2)e^{-8t}$$

$$v(t) = \underline{-8e^{-8t} u(t)\ V}$$

Chapter 7, Problem 64.

Find $v_0(t)$ for $t > 0$ in the circuit of Fig. 7.129.

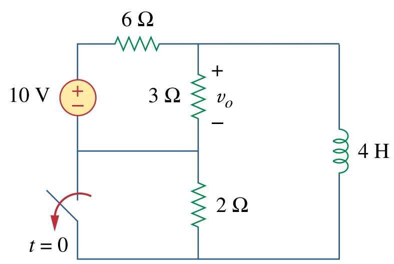
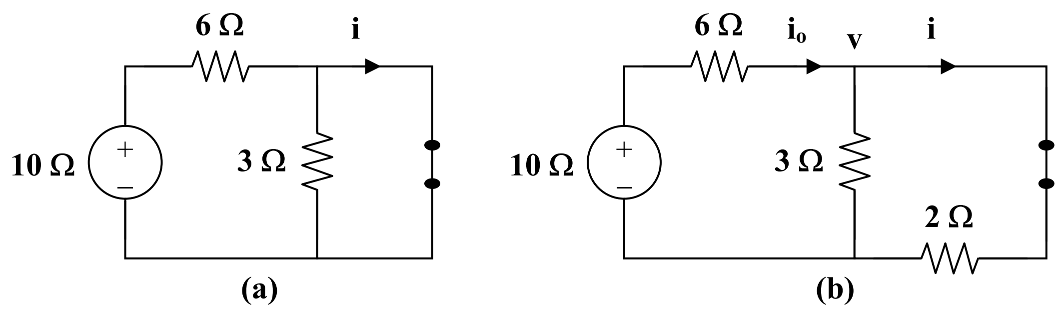


Figure 7.129
For Prob. 7.64.

Chapter 7, Solution 64.

Let i be the inductor current.
For t < 0, the inductor acts like a short circuit and the 3 Ω resistor is short-circuited so that the equivalent circuit is shown in Fig. (a).



$$i = i(0) = \frac{10}{6} = 1.667 \text{ A}$$

For t > 0, $\qquad R_{th} = 2 + 3 \,||\, 6 = 4\,\Omega$, $\qquad \tau = \frac{L}{R_{th}} = \frac{4}{4} = 1$

To find $i(\infty)$, consider the circuit in Fig. (b).

$$\frac{10 - v}{6} = \frac{v}{3} + \frac{v}{2} \longrightarrow v = \frac{10}{6}$$

$$i = i(\infty) = \frac{v}{2} = \frac{5}{6}$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = \frac{5}{6} + \left(\frac{10}{6} - \frac{5}{6}\right) e^{-t} = \frac{5}{6}\left(1 + e^{-t}\right) A$$

v_o is the voltage across the 4 H inductor and the 2 Ω resistor

$$v_o(t) = 2i + L\frac{di}{dt} = \frac{10}{6} + \frac{10}{6} e^{-t} + (4)\left(\frac{5}{6}\right)(-1)\, e^{-t} = \frac{10}{6} - \frac{10}{6} e^{-t}$$

$$v_o(t) = \underline{1.6667\left(1 - e^{-t}\right) V}$$

Chapter 7, Problem 65.

If the input pulse in Fig. 7.130(a) is applied to the circuit in Fig. 7.130(b), determine the response $i(t)$.

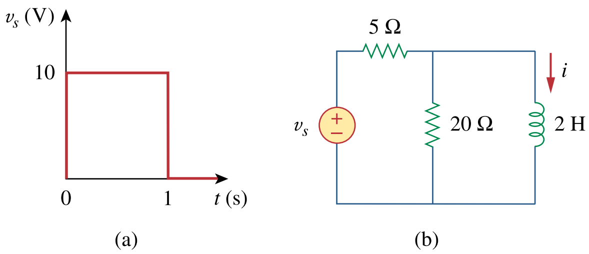
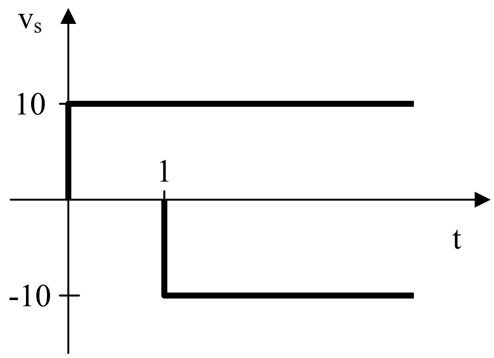


Figure 7.130
For Prob. 7.65.

Chapter 7, Solution 65.

Since $v_s = 10\left[\,u(t) - u(t-1)\right]$, this is the same as saying that a 10 V source is turned on at t = 0 and a -10 V source is turned on later at t = 1. This is shown in the figure below.



For $0 < t < 1$, $\quad i(0) = 0$, $\qquad i(\infty) = \dfrac{10}{5} = 2$

$$R_{th} = 5 \,||\, 20 = 4, \qquad \tau = \frac{L}{R_{th}} = \frac{2}{4} = \frac{1}{2}$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$
$$i(t) = 2\left(1 - e^{-2t}\right) \text{ A}$$
$$i(1) = 2\left(1 - e^{-2}\right) = 1.729$$

For $t > 1$, $\qquad i(\infty) = 0 \qquad$ since $v_s = 0$

$$i(t) = i(1)\, e^{-(t-1)/\tau}$$
$$i(t) = 1.729\, e^{-2(t-1)} \text{ A}$$

Thus,

$$i(t) = \begin{cases} \mathbf{2\left(1 - e^{-2t}\right) A} & \mathbf{0 < t < 1} \\ \mathbf{1.729\, e^{-2(t-1)} \ A} & \mathbf{t > 1} \end{cases}$$

Chapter 7, Problem 66.

For the op amp circuit of Fig. 7.131, find v_0. Assume that v_s changes abruptly from 0 to 1 V at $t = 0$.

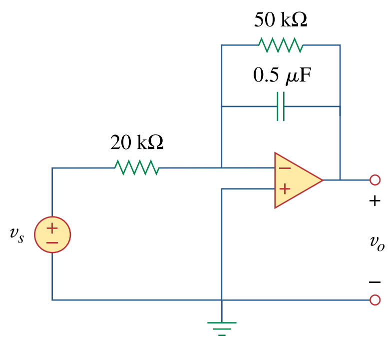


Figure 7.131
For Prob. 7.66.

Chapter 7, Solution 66.

For t<0-, v_s =0 so that $v_o(0)$=0
Let v be the capacitor voltage
For t>0, v_s =1. At steady state, the capacitor acts like an open circuit so that we have an inverting amplifier

$$v_o(\infty) = -(50k/20k)(1V) = -2.5 \text{ V}$$

$$\tau = RC = 50x10^3 x0.5x10^{-6} = 25 \text{ ms}$$

$$v_o(t) = v_o(\infty) + (v_o(0) - v_o(\infty))e^{-t/0.025} = \underline{\mathbf{2.5(e^{-40t} - 1)\ V}}.$$

Chapter 7, Problem 67.

If $v(0) = 5$ V, find $v_0(t)$ for $t > 0$ in the op amp circuit of Fig. 7.132. Let $R = 10\text{k}\Omega$ and $C = 1\ \mu\text{F}$.

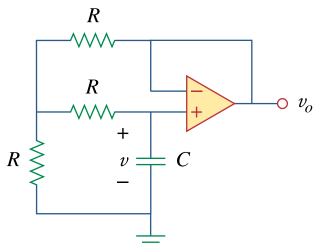
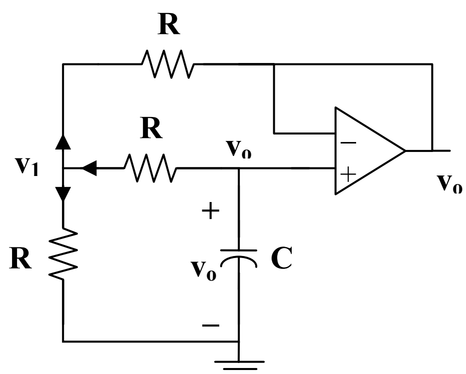


Figure 7.132
For Prob. 7.67.

Chapter 7, Solution 67.

The op amp is a voltage follower so that $v_o = v$ as shown below.



At node 1,

$$\frac{v_o - v_1}{R} = \frac{v_1 - 0}{R} + \frac{v_1 - v_o}{R} \longrightarrow v_1 = \frac{2}{3}v_o$$

At the noninverting terminal,

$$C\frac{dv_o}{dt} + \frac{v_o - v_1}{R} = 0$$

$$-RC\frac{dv_o}{dt} = v_o - v_1 = v_o - \frac{2}{3}v_o = \frac{1}{3}v_o$$

$$\frac{dv_o}{dt} = -\frac{v_o}{3RC}$$

$$v_o(t) = V_T\, e^{-t/3RC}$$

$$V_T = v_o(0) = 5\ \text{V}, \qquad \tau = 3RC = (3)(10\times10^3)(1\times10^{-6}) = \frac{3}{100}$$

$$v_o(t) = \underline{5\,e^{-100t/3}\ u(t)\text{V}}$$

Chapter 7, Problem 68.

Obtain v_0 for $t > 0$ in the circuit of Fig. 7.133.

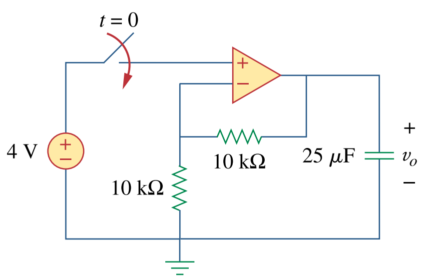


Figure 7.133
For Prob. 7.68.

Chapter 7, Solution 68.

This is a very interesting problem and has both an important ideal solution as well as an important practical solution. Let us look at the ideal solution first. Just before the switch closes, the value of the voltage across the capacitor is zero which means that the voltage at both terminals input of the op amp are each zero. As soon as the switch closes, the output tries to go to a voltage such that the input to the op amp both go to 4 volts. The ideal op amp puts out whatever current is necessary to reach this condition. An infinite (impulse) current is necessary if the voltage across the capacitor is to go to 8 volts in zero time (8 volts across the capacitor will result in 4 volts appearing at the negative terminal of the op amp). So v_o will be equal to **8 volts** for all t > 0.

What happens in a real circuit? Essentially, the output of the amplifier portion of the op amp goes to whatever its maximum value can be. Then this maximum voltage appears across the output resistance of the op amp and the capacitor that is in series with it. This results in an exponential rise in the capacitor voltage to the steady-state value of 8 volts.

$$vC(t) = V_{\text{op amp max}}(1 - e^{-t/(R_{out}C)}) \text{ volts, for all values of vC less than 8 V,}$$

$$= 8 \text{ V when t is large enough so that the 8 V is reached.}$$

Chapter 7, Problem 69.

For the op amp circuit in Fig. 7.134, find $v_0(t)$ for $t > 0$.

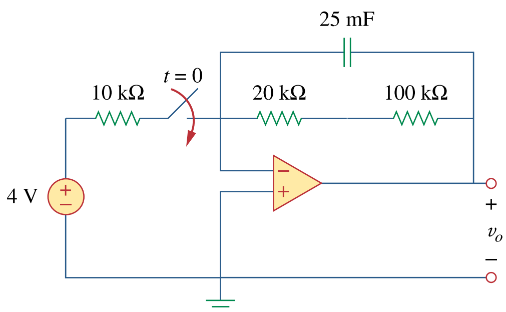


Figure 7.134
For Prob. 7.69.

Chapter 7, Solution 69.

Let v_x be the capacitor voltage.
For $t < 0$, $\quad v_x(0) = 0$

For $t > 0$, the 20 kΩ and 100 kΩ resistors are in series and together, they are in parallel with the capacitor since no current enters the op amp terminals.
As $t \to \infty$, the capacitor acts like an open circuit so that

$$v_o(\infty) = \frac{-4}{10}(20 + 100) = -48$$

$$R_{th} = 20 + 100 = 120\ \text{k}\Omega, \qquad \tau = R_{th}C = (120 \times 10^3)(25 \times 10^{-3}) = 3000$$

$$v_o(t) = v_o(\infty) + \left[v_o(0) - v_o(\infty)\right] e^{-t/\tau}$$

$$v_o(t) = -48\left(1 - e^{-t/3000}\right)\text{V} = \mathbf{48(e^{-t/3000} - 1)u(t)V}$$

Chapter 7, Problem 70.

Determine v_0 for $t > 0$ when $v_s = 20$ mV in the op amp circuit of Fig. 7.135.

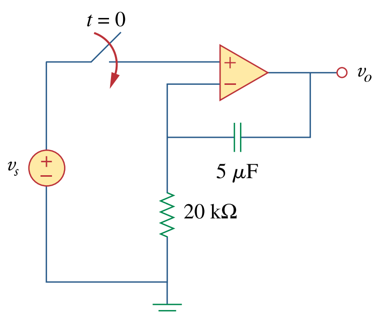


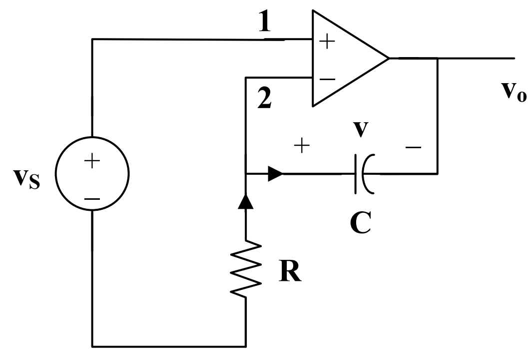
Figure 7.135
For Prob. 7.70.

Chapter 7, Solution 70.

Let v = capacitor voltage.

For t < 0, the switch is open and $v(0) = 0$.

For t > 0, the switch is closed and the circuit becomes as shown below.



$$v_1 = v_2 = v_s \qquad (1)$$

$$\frac{0 - v_s}{R} = C\frac{dv}{dt} \qquad (2)$$

where $v = v_s - v_o \quad \longrightarrow \quad v_o = v_s - v \qquad (3)$

From (1),

$$\frac{dv}{dt} = \frac{v_s}{RC} = 0$$

$$v = \frac{-1}{RC}\int v_s \, dt + v(0) = \frac{-t\,v_s}{RC}$$

Since v is constant,

$$RC = (20\times10^3)(5\times10^{-6}) = 0.1$$

$$v = \frac{-20t}{0.1}\text{ mV} = -200t \text{ mV}$$

From (3),

$$v_o = v_s - v = 20 + 200t$$

$$v_o = \underline{\mathbf{20(1+10t)\ mV}}$$

Chapter 7, Problem 71.

For the op amp circuit in Fig. 7.136, suppose $v_0 = 0$ and $v_s = 3$ V. Find $v(t)$ for $t > 0$.

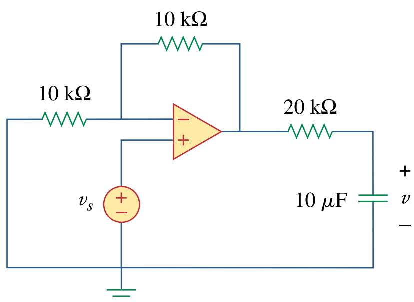
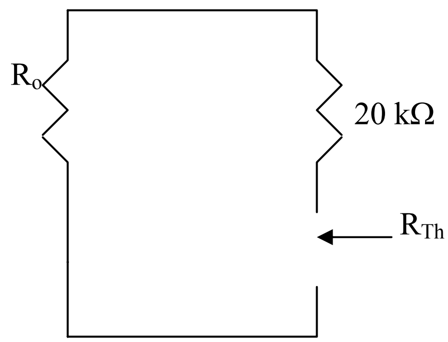


Figure 7.136
For Prob. 7.71.

Chapter 7, Solution 71.

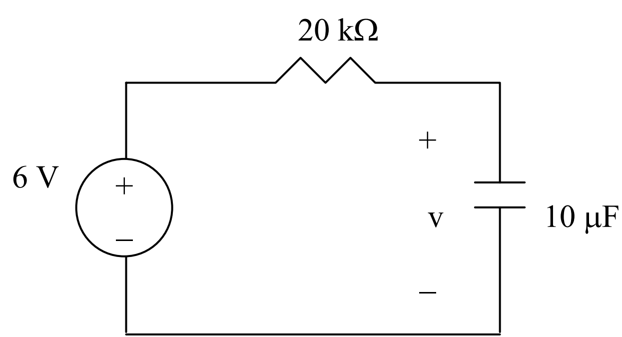
We temporarily remove the capacitor and find the Thevenin equivalent at its terminals. To find R_{Th}, we consider the circuit below.



Since we are assuming an ideal op amp, $R_o = 0$ and R_{Th}=20kΩ . The op amp circuit is a noninverting amplifier. Hence,

$$V_{Th} = (1 + \frac{10}{10})v_s = 2v_s = 6V$$

The Thevenin equivalent is shown below.



Thus,

$$v(t) = 6(1 - e^{-t/\tau}), t > 0$$

where $\tau = R_{TH}C = 20x10^{-3}x10x10^{-6} = 0.2$

$$\underline{v(t) = 6(1 - e^{-5t}), t > 0} \ \mathbf{\underline{V}}$$

Chapter 7, Problem 72.

Find i_0 in the op amp circuit in Fig. 7.137. Assume that $v(0)$ = -2 V, $R = 10\ \text{k}\Omega$, and $C = 10\ \mu\text{F}$.

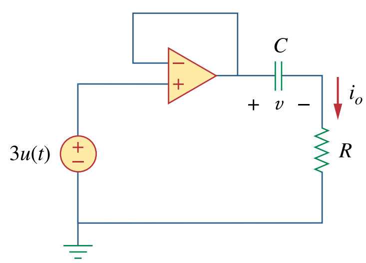
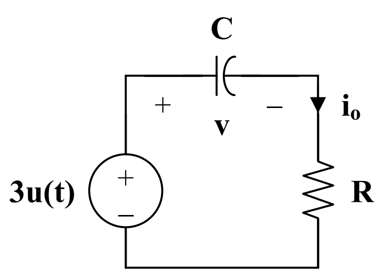


Figure 7.137
For Prob. 7.72.

Chapter 7, Solution 72.

The op amp acts as an emitter follower so that the Thevenin equivalent circuit is shown below.



Hence,

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty) \right] e^{-t/\tau}$$
$$v(0) = \text{-2 V}, \quad v(\infty) = 3\ \text{V}, \quad \tau = RC = (10\times10^{3})(10\times10^{-6}) = 0.1$$
$$v(t) = 3 + (\text{-}2 - 3)\,e^{-10t} = 3 - 5\,e^{-10t}$$

$$i_o = C\frac{dv}{dt} = (10\times10^{-6})(\text{-}5)(\text{-}10)\,e^{-10t}$$
$$i_o = \underline{\mathbf{0.5\,e^{-10t}\ mA, \quad t > 0}}$$

Chapter 7, Problem 73.

For the op amp circuit in Fig. 7.138, let $R_1 = 10\ \text{k}\Omega$, $R_f = 20\ \text{k}\Omega$, $C = 20\ \mu\text{F}$, and $v(0) = 1$ V. Find v_0 .

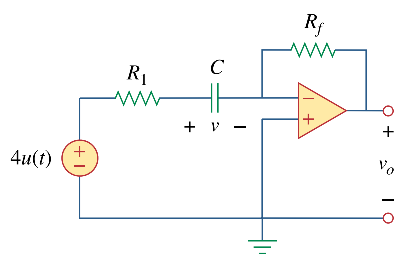
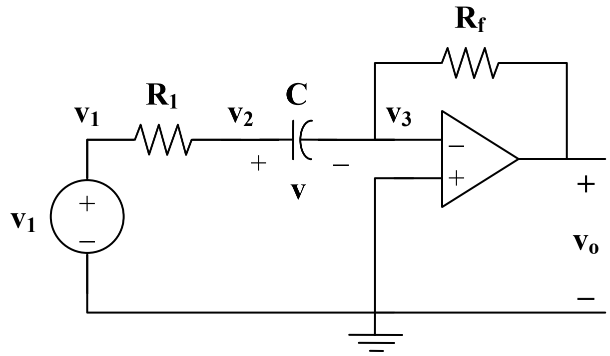


Figure 7.138
For Prob. 7.73.

Chapter 7, Solution 73.

Consider the circuit below.



At node 2,

$$\frac{v_1 - v_2}{R_1} = C\frac{dv}{dt} \qquad (1)$$

At node 3,

$$C\frac{dv}{dt} = \frac{v_3 - v_o}{R_f} \qquad (2)$$

But $v_3 = 0$ and $v = v_2 - v_3 = v_2$. Hence, (1) becomes

$$\frac{v_1 - v}{R_1} = C\frac{dv}{dt}$$

$$v_1 - v = R_1 C\frac{dv}{dt}$$

or $$\frac{dv}{dt} + \frac{v}{R_1 C} = \frac{v_1}{R_1 C}$$

which is similar to Eq. (7.42). Hence,

$$v(t) = \begin{cases} v_T & t < 0 \\ v_1 + (v_T - v_1)e^{-t/\tau} & t > 0 \end{cases}$$

where $v_T = v(0) = 1$ and $v_1 = 4$

$$\tau = R_1 C = (10\times 10^3)(20\times 10^{-6}) = 0.2$$

$$v(t) = \begin{cases} 1 & t < 0 \\ 4 - 3e^{-5t} & t > 0 \end{cases}$$

From (2),

$$v_o = -R_f C\frac{dv}{dt} = (20\times 10^3)(20\times 10^{-6})(15e^{-5t})$$

$$v_o = -6e^{-5t}, \quad t > 0$$

$$v_o = \underline{\mathbf{-6e^{-5t}\,u(t)\;V}}$$

Chapter 7, Problem 74.

Determine $v_0(t)$ for $t > 0$ in the circuit of Fig. 7.139. Let $i_s = 10u(t)\ \mu$A and assume that the capacitor is initially uncharged.

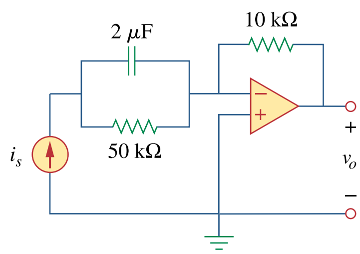
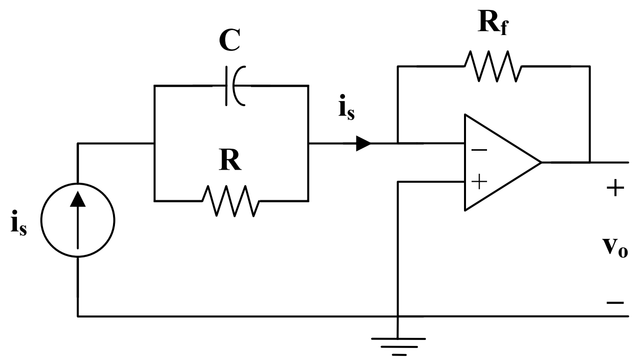


Figure 7.139
For Prob. 7.74.

Chapter 7, Solution 74.

Let v = capacitor voltage. For $t < 0$, $v(0) = 0$



For $t > 0$, $i_s = 10\ \mu A$. Consider the circuit below.

$$i_s = C\frac{dv}{dt} + \frac{v}{R} \qquad (1)$$

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau} \qquad (2)$$

It is evident from the circuit that

$$\tau = RC = (2 \times 10^{-6})(50 \times 10^{3}) = 0.1$$

At steady state, the capacitor acts like an open circuit so that i_s passes through R. Hence,

$$v(\infty) = i_s R = (10 \times 10^{-6})(50 \times 10^{3}) = 0.5\ V$$

Then,

$$v(t) = 0.5\left(1 - e^{-10t}\right)\ V \qquad (3)$$

But $$i_s = \frac{0 - v_o}{R_f} \longrightarrow v_o = -i_s R_f \qquad (4)$$

Combining (1), (3), and (4), we obtain

$$v_o = \frac{-R_f}{R} v - R_f C \frac{dv}{dt}$$

$$v_o = \frac{-1}{5} v - (10 \times 10^{3})(2 \times 10^{-6})\frac{dv}{dt}$$

$$v_o = -0.1 + 0.1 e^{-10t} - (2 \times 10^{-2})(0.5)\left(-10 e^{-10t}\right)$$

$$v_o = 0.2 e^{-10t} - 0.1$$

$$v_o = \underline{\mathbf{0.1\left(2 e^{-10t} - 1\right) V}}$$

Chapter 7, Problem 75.

In the circuit of Fig. 7.140, find v_0 and i_0, given that $v_s = 4u(t)$ V and $v(0) = 1$ V.

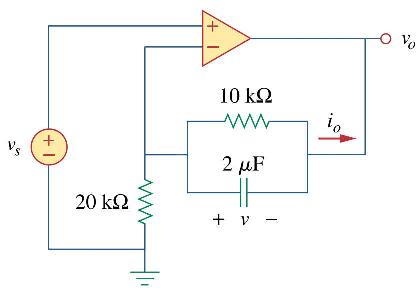


Figure 7.140
For Prob. 7.75.

Chapter 7, Solution 75.

Let v_1 = voltage at the noninverting terminal.

Let v_2 = voltage at the inverting terminal.

For t > 0, $\quad v_1 = v_2 = v_s = 4$

$$\frac{0 - v_s}{R_1} = i_o, \quad R_1 = 20\ \text{k}\Omega$$

$$v_o = -i_o R \qquad (1)$$

Also, $i_o = \dfrac{v}{R_2} + C\dfrac{dv}{dt}, \qquad R_2 = 10\ \text{k}\Omega, \quad C = 2\ \mu\text{F}$

i.e. $\dfrac{-v_s}{R_1} = \dfrac{v}{R_2} + C\dfrac{dv}{dt} \qquad (2)$

This is a step response.

$$v(t) = v(\infty) + \left[v(0) - v(\infty)\right] e^{-t/\tau}, \qquad v(0) = 1$$

where $\tau = R_2 C = (10\times 10^3)(2\times 10^{-6}) = \dfrac{1}{50}$

At steady state, the capacitor acts like an open circuit so that i_o passes through R_2. Hence, as $t \to \infty$

$$\frac{-v_s}{R_1} = i_o = \frac{v(\infty)}{R_2}$$

i.e. $v(\infty) = \dfrac{-R_2}{R_1} v_s = \dfrac{-10}{20}(4) = -2$

$$v(t) = -2 + (1+2)\,e^{-50t}$$

$$v(t) = -2 + 3\,e^{-50t}$$

But $\quad v = v_s - v_o$

or $\quad v_o = v_s - v = 4 + 2 - 3\,e^{-50t}$

$$v_o = \underline{6 - 3\,e^{-50t}\ u(t)\text{V}}$$

$$i_o = \frac{-v_s}{R_1} = \frac{-4}{20k} = -0.2\ \text{mA}$$

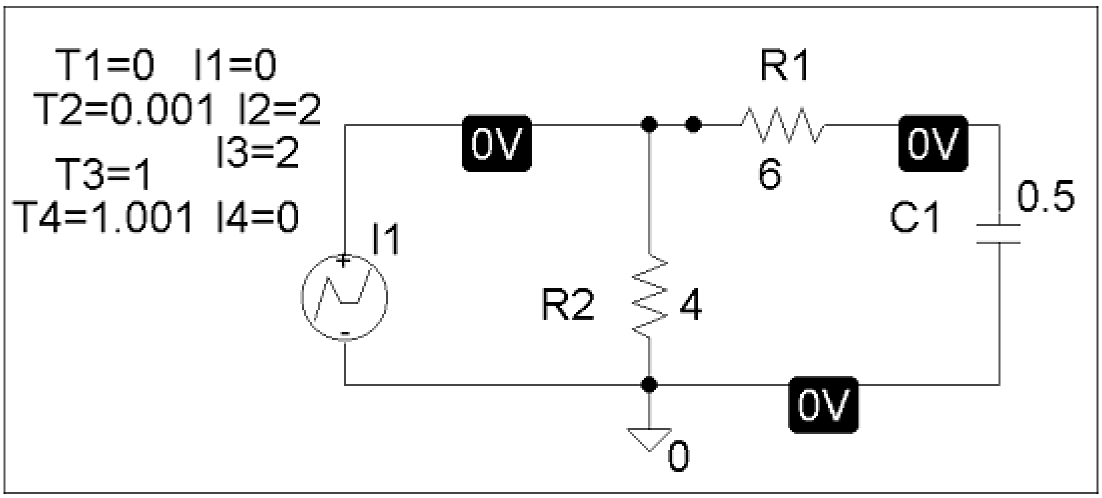
or $\quad i_o = \dfrac{v}{R_2} + C\dfrac{dv}{dt} = \underline{\mathbf{-0.2\ mA}}$

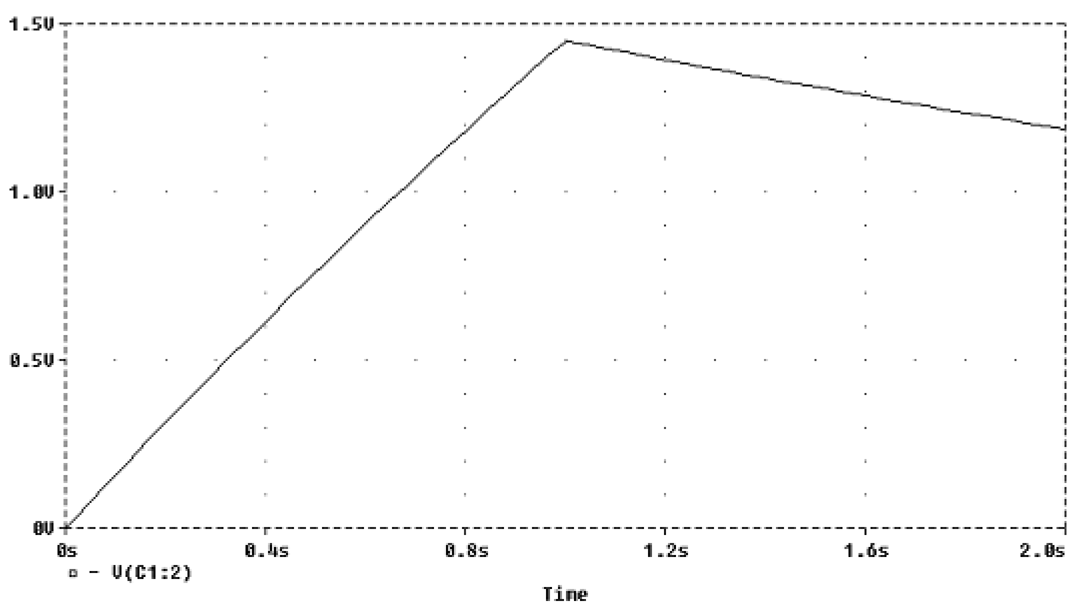
Chapter 7, Problem 76.

Repeat Prob. 7.49 using *PSpice*.

Chapter 7, Solution 76.

The schematic is shown below. For the pulse, we use IPWL and enter the corresponding values as attributes as shown. By selecting Analysis/Setup/Transient, we let Print Step = 25 ms and Final Step = 2 s since the width of the input pulse is 1 s. After saving and simulating the circuit, we select Trace/Add and display –V(C1:2). The plot of V(t) is shown below.





Chapter 7, Problem 77.

The switch in Fig. 7.141 opens at $t = 0$. Use *PSpice* to determine $v(t)$ for $t > 0$.

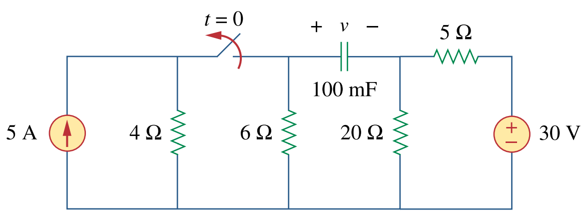
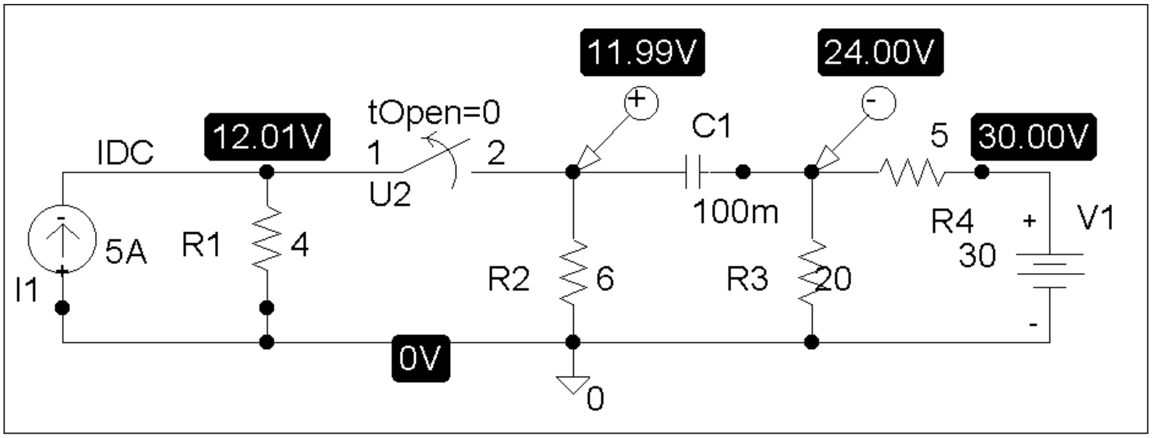
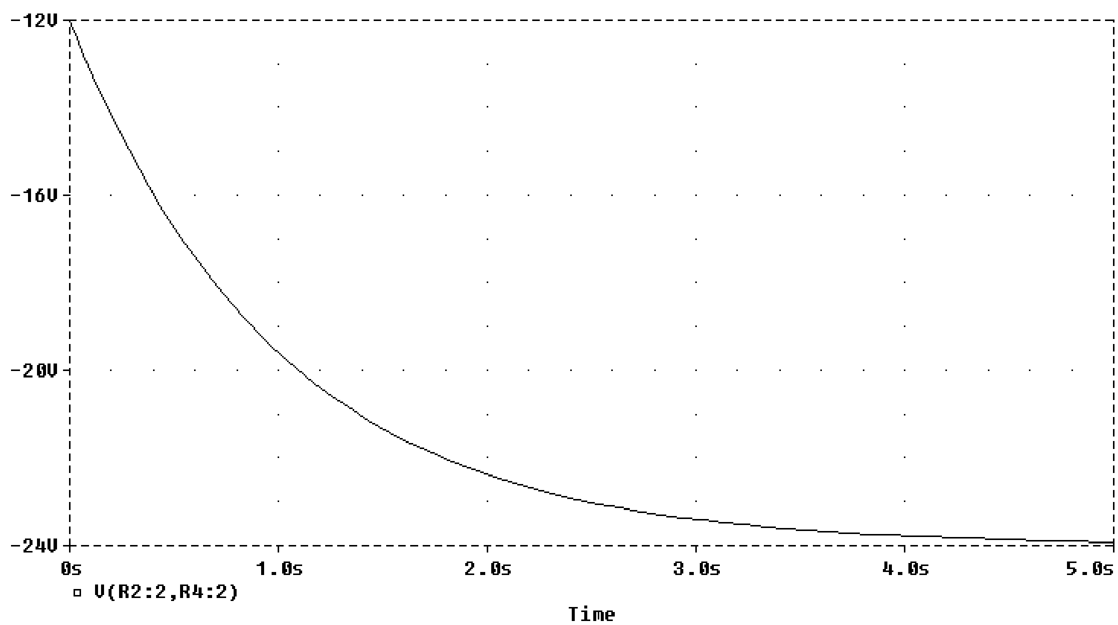


Figure 7.141
For Prob. 7.77.

Chapter 7, Solution 77.

The schematic is shown below. We click Marker and insert Mark Voltage Differential at the terminals of the capacitor to display V after simulation. The plot of V is shown below. Note from the plot that V(0) = 12 V and V(∞) = -24 V which are correct.





Chapter 7, Problem 78.

The switch in Fig. 7.142 moves from position *a* to *b* at $t = 0$. Use *PSpice* to find $i(t)$ for $t > 0$.

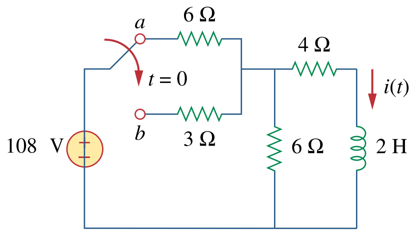


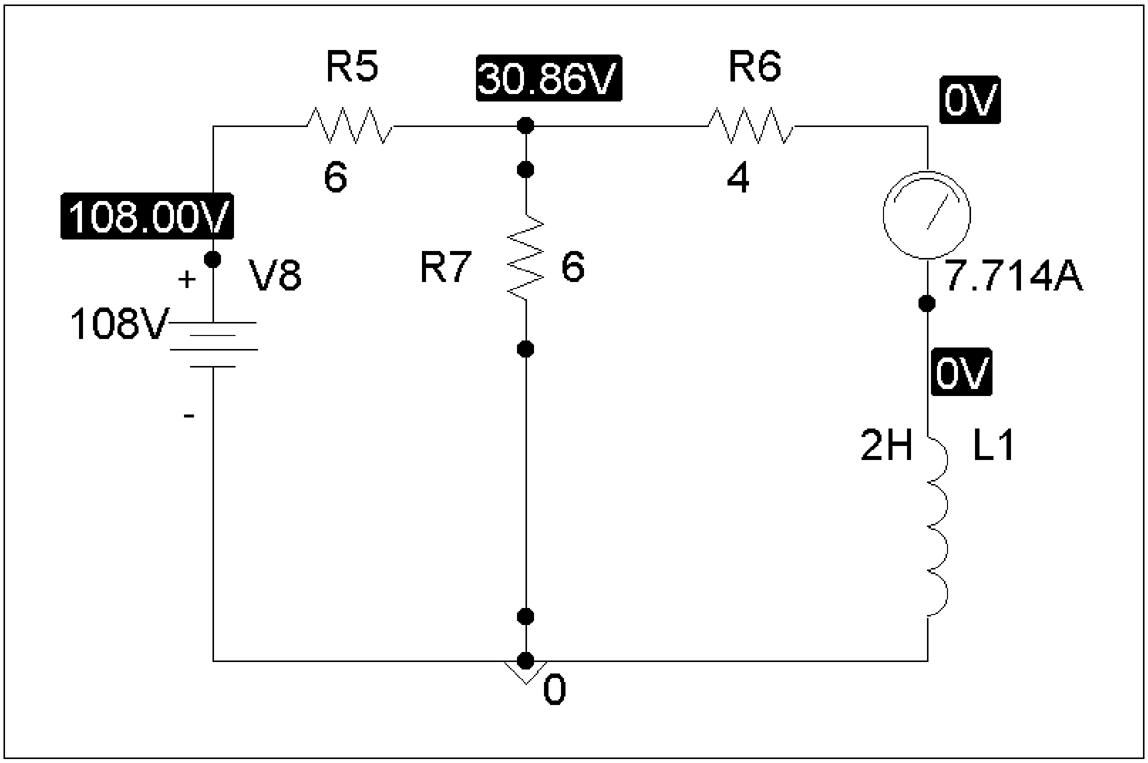
Figure 7.142
For Prob. 7.78.

Chapter 7, Solution 78.

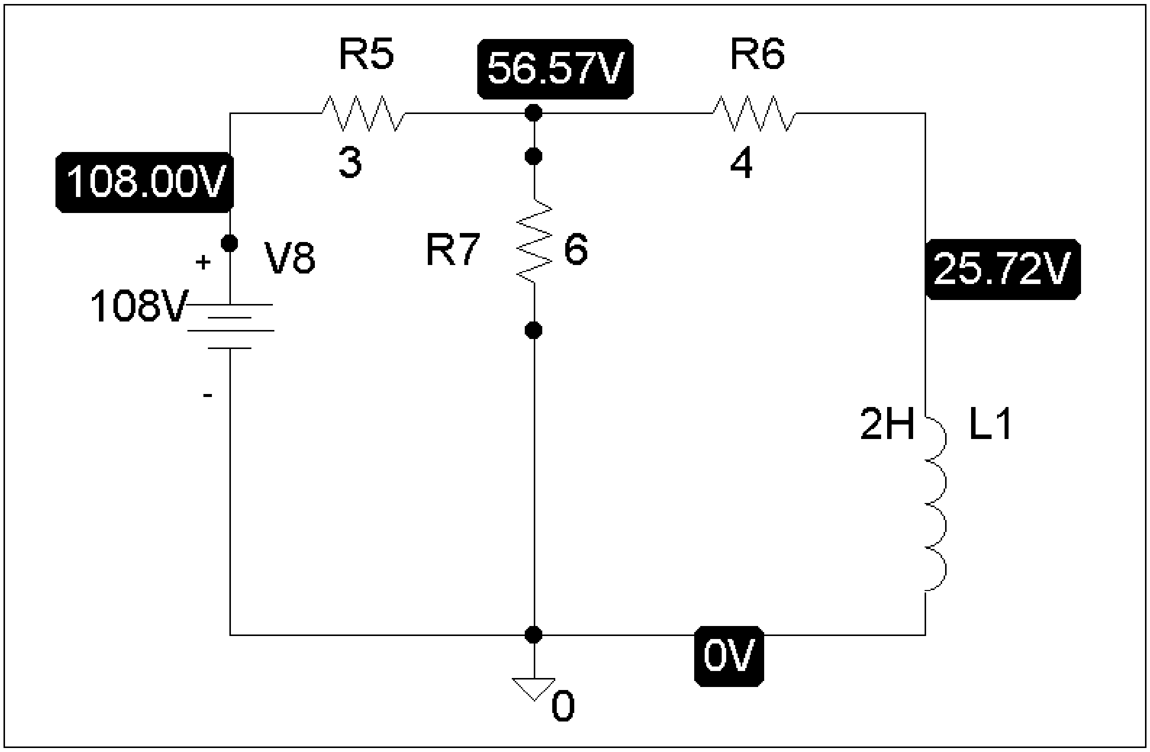
(a) When the switch is in position (a), the schematic is shown below. We insert IPROBE to display i. After simulation, we obtain,

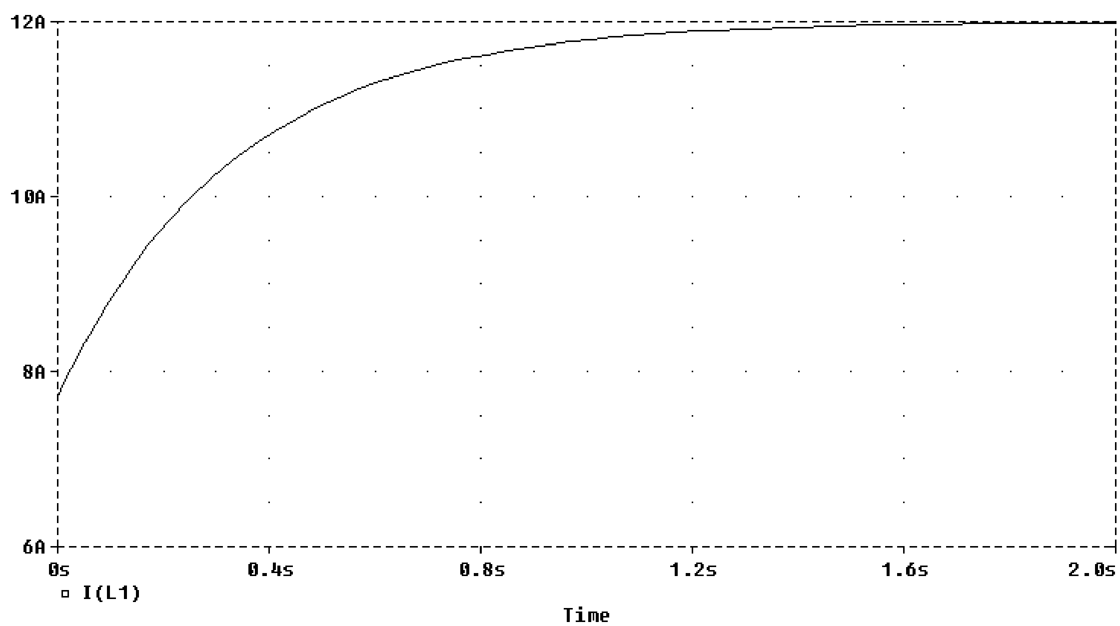
$$i(0) = 7.714 \text{ A}$$

from the display of IPROBE.



(b) When the switch is in position (b), the schematic is as shown below. For inductor I1, we let IC = 7.714. By clicking Analysis/Setup/Transient, we let Print Step = 25 ms and Final Step = 2 s. After Simulation, we click Trace/Add in the probe menu and display I(L1) as shown below. Note that $i(\infty) = 12\text{A}$, which is correct.


R5
56.57V
R6
3
4
108.00V
+
V8
R7
6
108V
25.72V
-
2H
L1
0
0V


12A
10A
8A
6A
0s
0.4s
0.8s
1.2s
1.6s
2.0s
I(L1)
Time

Chapter 7, Problem 79.

In the circuit of Fig. 7.143, the switch has been in position *a* for a long time but moves instantaneously to position *b* at $t = 0$ Determine $i_0(t)$.

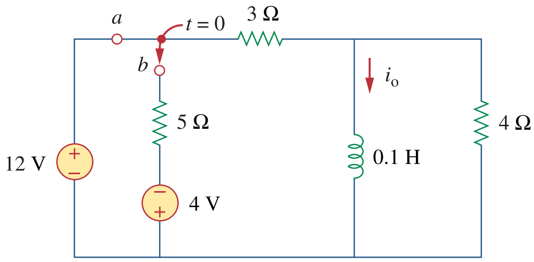


Figure 7.143
For Prob. 7.79.

Chapter 7, Solution 79.

When the switch is in position 1, $i_o(0) = 12/3 = 4A$. When the switch is in position 2,

$$i_o(\infty) = -\frac{4}{5+3} = -0.5 \text{ A}, \qquad R_{Th} = (3+5)//4 = 8/3, \qquad \tau = \frac{L}{R_{Th}} = 3/80$$

$$i_o(t) = i_o(\infty) + [i_o(0) - i_o(\infty)]e^{-t/\tau} = \underline{-0.5 + 4.5e^{-80t/3}}\ u(t)\text{A}$$

Chapter 7, Problem 80.

In the circuit of Fig. 7.144, assume that the switch has been in position *a* for a long time, find:

(a) $i_1(0)$, $i_2(0)$, and $v_0(0)$

(b) $i_L(t)$

(c) $i_1(\infty)$, $i_2(\infty)$, and $v_0(\infty)$.

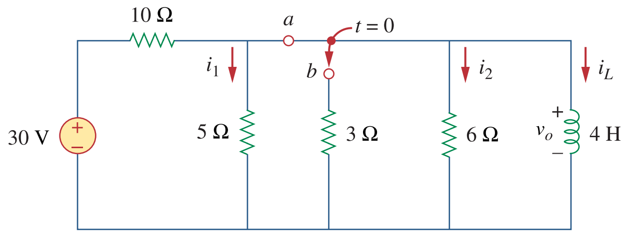


Figure 7.144
For Prob. 7.80.

Chapter 7, Solution 80.

(a) When the switch is in position A, the 5-ohm and 6-ohm resistors are short-circuited so that

$$\underline{i_1(0) = \quad i_2(0) = v_o(0) = 0}$$

but the current through the 4-H inductor is $i_L(0)$ =30/10 = 3A.

(b) When the switch is in position B,

$$R_{Th} = 3//6 = 2\Omega, \qquad \tau = \frac{L}{R_{Th}} = 4/2 = 2\,\text{sec}$$

$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-t/\tau} = 0 + 3e^{-t/2} = \underline{3e^{-t/2}\ \text{A}}$$

(c) $i_1(\infty) = \dfrac{30}{10+5} = \underline{2\ \text{A}}, \qquad i_2(\infty) = -\dfrac{3}{9} i_L(\infty) = \underline{0\ \text{A}}$

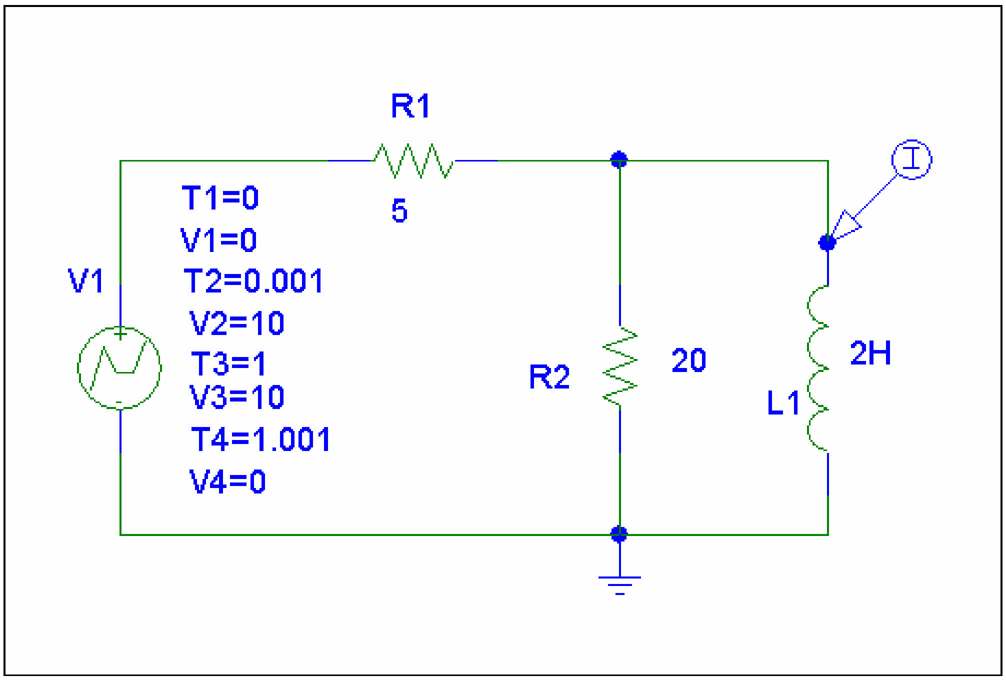
$$v_o(t) = L\frac{di_L}{dt} \quad \longrightarrow \quad \underline{v_o(\infty) = 0\ \text{V}}$$

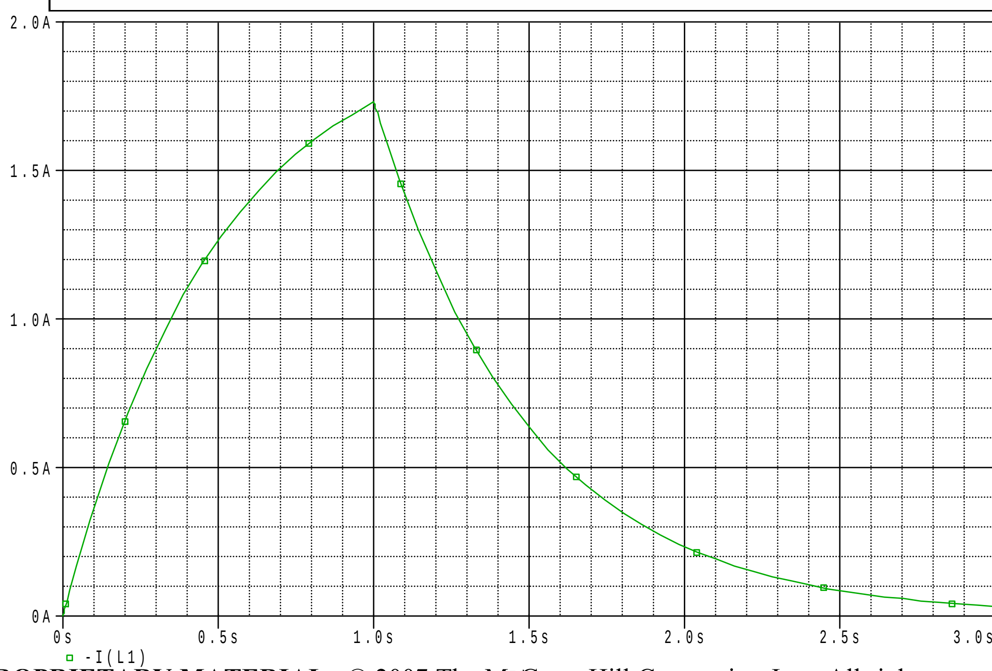
Chapter 7, Problem 81.

Repeat Prob. 7.65 using *PSpice*.

Chapter 7, Solution 81.

The schematic is shown below. We use VPWL for the pulse and specify the attributes as shown. In the Analysis/Setup/Transient menu, we select Print Step = 25 ms and final Step = 3 S. By inserting a current marker at one terminal of LI, we automatically obtain the plot of i after simulation as shown below.





Chapter 7, Problem 82.

In designing a signal-switching circuit, it was found that a 100-μF capacitor was needed for a time constant of 3 ms. What value resistor is necessary for the circuit?

Chapter 7, Solution 82.

$$\tau = \mathrm{RC} \longrightarrow \mathrm{R} = \frac{\tau}{\mathrm{C}} = \frac{3\times10^{-3}}{100\times10^{-6}} = \underline{\mathbf{30\ \Omega}}$$

Chapter 7, Problem 83.

e⊘d

An *RC* circuit consists of a series connection of a 120-V source, a switch, a 34-MΩ resistor, and a 15-μF capacitor. The circuit is used in estimating the speed of a horse running a 4-km racetrack. The switch closes when the horse begins and opens when the horse crosses the finish line. Assuming that the capacitor charges to 85.6 V, calculate the speed of the horse.

Chapter 7, Solution 83.

$$v(\infty) = 120, \qquad v(0) = 0, \qquad \tau = RC = 34x10^6\,x15x10^{-6} = 510s$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \qquad \longrightarrow \qquad 85.6 = 120(1 - e^{-t/510})$$

Solving for t gives

$$t = 510\ln 3.488 = 637.16s$$

speed = 4000m/637.16s = **<u>6.278m/s</u>**

Chapter 7, Problem 84.

The resistance of a 160-mH coil is 8Ω. Find the time required for the current to build up to 60 percent of its final value when voltage is applied to the coil.

Chapter 7, Solution 84.

Let I_o be the final value of the current. Then

$$i(t) = I_o(1 - e^{-t/\tau}), \qquad \tau = R/L = 0.16/8 = 1/50$$

$$0.6I_o = I_o(1 - e^{-50t}) \qquad \longrightarrow \qquad t = \frac{1}{50}\ln\frac{1}{0.4} = \underline{18.33\text{ ms}}.$$

Chapter 7, Problem 85.

A simple relaxation oscillator circuit is shown in Fig. 7.145. The neon lamp fires when its voltage reaches 75 V and turns off when its voltage drops to 30 V. Its resistance is 120 Ω when on and infinitely high when off.

(a) For how long is the lamp on each time the capacitor discharges?
(b) What is the time interval between light flashes?

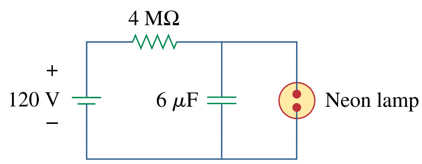


Figure 7.145
For Prob. 7.85.

Chapter 7, Solution 85.

(a) The light is on from 75 volts until 30 volts. During that time we essentially have a 120-ohm resistor in parallel with a 6-μF capacitor.

$v(0) = 75,\ v(\infty) = 0,\ \tau = 120\text{x}6\text{x}10^{-6} = 0.72 \text{ ms}$

$v(t_1) = 75\,e^{-t_1/\tau} = 30$ which leads to $t_1 = -0.72\ln(0.4)$ ms = **<u>659.7 μs</u>** of lamp on time.

(b) $\tau = RC = (4\times10^{6})(6\times10^{-6}) = 24 \text{ s}$

Since $v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$

$$v(t_1) - v(\infty) = \left[\, v(0) - v(\infty)\right] e^{-t_1/\tau} \qquad (1)$$

$$v(t_2) - v(\infty) = \left[\, v(0) - v(\infty)\right] e^{-t_2/\tau} \qquad (2)$$

Dividing (1) by (2),

$$\frac{v(t_1) - v(\infty)}{v(t_2) - v(\infty)} = e^{(t_2 - t_1)/\tau}$$

$$t_0 = t_2 - t_1 = \tau \ln\left(\frac{v(t_1) - v(\infty)}{v(t_2) - v(\infty)}\right)$$

$$t_0 = 24\ln\left(\frac{75-120}{30-120}\right) = 24\ln(2) = \underline{\mathbf{16.636\ s}}$$

Chapter 7, Problem 86.

Figure 7.146 shows a circuit for setting the length of time voltage is applied to the electrodes of a welding machine. The time is taken as how long it takes the capacitor to charge from 0 to 8 V. What is the time range covered by the variable resistor?

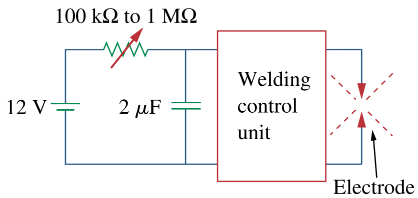


Figure 7.146
For Prob. 7.86.

Chapter 7, Solution 86.

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(\infty) = 12, \qquad v(0) = 0$$

$$v(t) = 12\left(1 - e^{-t/\tau}\right)$$

$$v(t_0) = 8 = 12\left(1 - e^{-t_0/\tau}\right)$$

$$\frac{8}{12} = 1 - e^{-t_0/\tau} \longrightarrow e^{-t_0/\tau} = \frac{1}{3}$$

$$t_0 = \tau \ln(3)$$

For $R = 100\ \text{k}\Omega$,

$$\tau = RC = (100 \times 10^3)(2 \times 10^{-6}) = 0.2\ \text{s}$$

$$t_0 = 0.2 \ln(3) = 0.2197\ \text{s}$$

For $R = 1\ \text{M}\Omega$,

$$\tau = RC = (1 \times 10^6)(2 \times 10^{-6}) = 2\ \text{s}$$

$$t_0 = 2 \ln(3) = 2.197\ \text{s}$$

Thus,

$$\underline{\mathbf{0.2197\ s} < \mathbf{t_0} < \mathbf{2.197\ s}}$$

Chapter 7, Problem 87.

A 120-V dc generator energizes a motor whose coil has an inductance of 50 H and a resistance of $100\,\Omega$. A field discharge resistor of $400\,\Omega$ is connected in parallel with the motor to avoid damage to the motor, as shown in Fig. 7.147. The system is at steady state. Find the current through the discharge resistor 100 ms after the breaker is tripped.

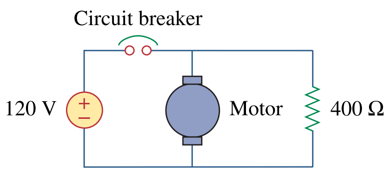


Figure 7.147
For Prob. 7.87.

Chapter 7, Solution 87.

Let i be the inductor current.

For $t < 0$, $\quad i(0^-) = \dfrac{120}{100} = 1.2 \text{ A}$

For $t > 0$, we have an RL circuit

$$\tau = \frac{L}{R} = \frac{50}{100+400} = 0.1, \qquad i(\infty) = 0$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = 1.2\, e^{-10t}$$

At $t = 100$ ms $= 0.1$ s,

$$i(0.1) = 1.2\, e^{-1} = \underline{\mathbf{441 mA}}$$

which is the same as the current through the resistor.

Chapter 7, Problem 88.

The circuit in Fig. 7.148(a) can be designed as an approximate differentiator or an integrator, depending on whether the output is taken across the resistor or the capacitor, and also on the time constant τ = RC of the circuit and the width T of the input pulse in Fig. 7.148(b). The circuit is a differentiator if $\tau \ll T$, say $\tau < 0.1T$, or an integrator if $\tau \gg T$, say $\tau > 10T$.

(a) What is the minimum pulse width that will allow a differentiator output to appear across the capacitor?

(b) If the output is to be an integrated form of the input, what is the maximum value the pulse width can assume?

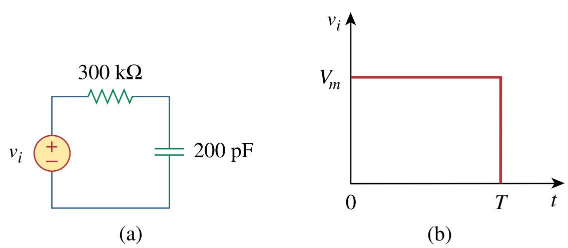


Figure 7.148
For Prob. 7.88.

Chapter 7, Solution 88.

(a) $\tau = RC = (300\times10^{3})(200\times10^{-12}) = 60\ \mu s$

As a differentiator,

$$T > 10\tau = 600\ \mu s = 0.6\ ms$$

i.e. $T_{min} = \underline{\mathbf{0.6\ ms}}$

(b) $\tau = RC = 60\ \mu s$

As an integrator,

$$T < 0.1\tau = 6\ \mu s$$

i.e. $T_{max} = \underline{\mathbf{6\ \mu s}}$

Chapter 7, Problem 89.

An *RL* circuit may be used as a differentiator if the output is taken across the inductor and $\tau \ll T$ (say $\tau < 0.1T$), where T is the width of the input pulse. If R is fixed at 200 kΩ determine the maximum value of L required to differentiate a pulse with $T = 10\ \mu$s.

Chapter 7, Solution 89.

Since $\tau < 0.1\mathrm{T} = 1\ \mu\mathrm{s}$

$$\frac{L}{R} < 1\ \mu\mathrm{s}$$

$$L < R \times 10^{-6} = (200 \times 10^{3})(1 \times 10^{-6})$$

$\underline{\textbf{L < 200 mH}}$

Chapter 7, Problem 90.

eⒼd

An attenuator probe employed with oscilloscopes was designed to reduce the magnitude of the input voltage v_i by a factor of 10. As shown in Fig. 7.149, the oscilloscope has internal resistance R_s and capacitance C_s while the probe has an internal resistance R_p. If R_p is fixed at 6 $M\Omega$ find R_s and C_s for the circuit to have a time constant of 15 μs.

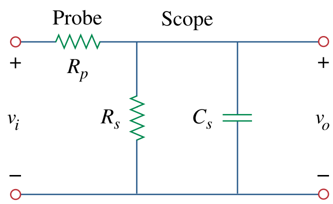
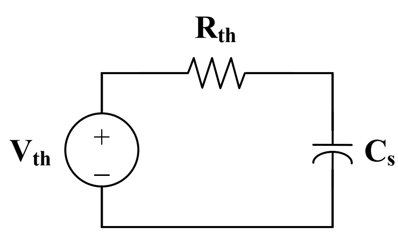


Figure 7.149
For Prob. 7.90.

Chapter 7, Solution 90.

We determine the Thevenin equivalent circuit for the capacitor C_s.

$$V_{th} = \frac{R_s}{R_s + R_p} V_i, \qquad R_{th} = R_s \parallel R_p$$



The Thevenin equivalent is an RC circuit. Since

$$V_{th} = \frac{1}{10} V_i \longrightarrow \frac{1}{10} = \frac{R_s}{R_s + R_p}$$

$$R_s = \frac{1}{9} R_p = \frac{6}{9} = \underline{\mathbf{\frac{2}{3}\ M\Omega}}$$

Also,

$$\tau = R_{th} C_s = 15\ \mu s$$

where $R_{th} = R_p \parallel R_s = \dfrac{6(2/3)}{6 + 2/3} = 0.6\ M\Omega$

$$C_s = \frac{\tau}{R_{th}} = \frac{15 \times 10^{-6}}{0.6 \times 10^{6}} = \underline{\mathbf{25\ pF}}$$

Chapter 7, Problem 91.

The circuit in Fig. 7.150 is used by a biology student to study "frog kick." She noticed that the frog kicked a little when the switch was closed but kicked violently for 5 s when the switch was opened. Model the frog as a resistor and calculate its resistance. Assume that it takes 10 mA for the frog to kick violently.

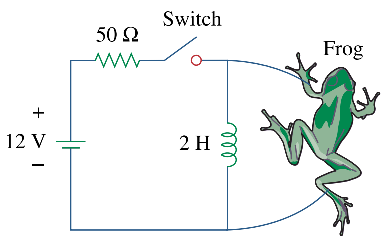


Figure 7.150
For Prob. 7.91.

Chapter 7, Solution 91.

$$i_o(0) = \frac{12}{50} = 240 \text{ mA}, \qquad i(\infty) = 0$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = 240 e^{-t/\tau}$$

$$\tau = \frac{L}{R} = \frac{2}{R}$$

$$i(t_0) = 10 = 240 e^{-t_0/\tau}$$

$$e^{t_0/\tau} = 24 \longrightarrow t_0 = \tau \ln(24)$$

$$\tau = \frac{t_0}{\ln(24)} = \frac{5}{\ln(24)} = 1.573 = \frac{2}{R}$$

$$R = \frac{2}{1.573} = \underline{\mathbf{1.271\ \Omega}}$$

Chapter 7, Problem 92.

To move a spot of a cathode-ray tube across the screen requires a linear increase in the voltage across the deflection plates, as shown in Fig. 7.151. Given that the capacitance of the plates is 4 nF, sketch the current flowing through the plates.

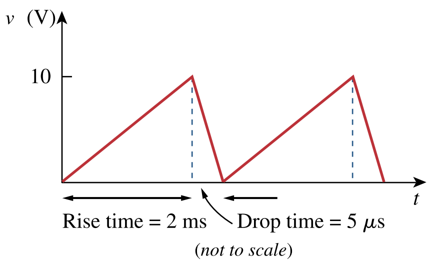


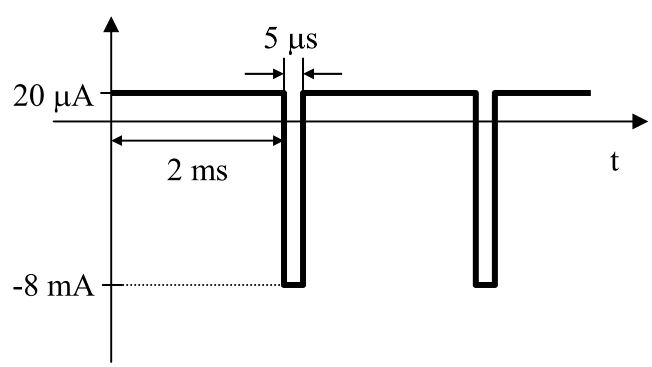
Figure 7.151
For Prob. 7.92.

Chapter 7, Solution 92.

$$i = C\frac{dv}{dt} = 4\times10^{-9}\cdot\begin{cases}\dfrac{10}{2\times10^{-3}} & 0 < t < t_R \\ \dfrac{-10}{5\times10^{-6}} & t_R < t < t_D\end{cases}$$

$$i(t) = \begin{cases}20\ \mu A & 0 < t < 2\ ms \\ -8\ mA & 2\ ms < t < 2\ ms + 5\ \mu s\end{cases}$$

which is sketched below.



(not to scale)