Chapter 12, Problem 1.

If $\mathbf{V}_{ab}$ = 400 V in a balanced Y-connected three-phase generator, find the phase voltages, assuming the phase sequence is:

(a) *abc* (b) *acb*

Chapter 12, Solution 1.

(a) If $\mathbf{V}_{ab} = 400$, then

$$\mathbf{V}_{an} = \frac{400}{\sqrt{3}}\angle\text{-}30° = \underline{\mathbf{231\angle\text{-}30°\ V}}$$

$$\mathbf{V}_{bn} = \underline{\mathbf{231\angle\text{-}150°\ V}}$$

$$\mathbf{V}_{cn} = \underline{\mathbf{231\angle\text{-}270°\ V}}$$

(b) For the acb sequence,

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\angle 0° - V_p\angle 120°$$

$$\mathbf{V}_{ab} = V_p\left(1 + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = V_p\sqrt{3}\angle\text{-}30°$$

i.e. in the acb sequence, $\mathbf{V}_{ab}$ lags $\mathbf{V}_{an}$ by 30°.

Hence, if $\mathbf{V}_{ab} = 400$, then

$$\mathbf{V}_{an} = \frac{400}{\sqrt{3}}\angle 30° = \underline{\mathbf{231\angle 30°\ V}}$$

$$\mathbf{V}_{bn} = \underline{\mathbf{231\angle 150°\ V}}$$

$$\mathbf{V}_{cn} = \underline{\mathbf{231\angle\text{-}90°\ V}}$$

Chapter 12, Problem 2.

What is the phase sequence of a balanced three-phase circuit for which $\mathbf{V}_{an} = 160\angle 30°$ V and $\mathbf{V}_{cn} = 160\angle -90°$ V? Find $\mathbf{V}_{bn}$.

Chapter 12, Solution 2.

Since phase c lags phase a by 120°, this is an **acb sequence**.

$$\mathbf{V}_{bn} = 160\angle(30° + 120°) = \underline{\mathbf{160\angle 150°\ V}}$$

Chapter 12, Problem 3.

Determine the phase sequence of a balanced three-phase circuit in which $\mathbf{V}_{bn} = 208\angle 130°$ V and $\mathbf{V}_{cn} = 208\angle 10°$ V. Obtain $\mathbf{V}_{an}$.

Chapter 12, Solution 3.

Since $\mathbf{V}_{\text{bn}}$ leads $\mathbf{V}_{\text{cn}}$ by 120°, this is an **abc sequence**.

$$\mathbf{V}_{\text{an}} = 208\angle(130° + 120°) = \underline{\mathbf{208\angle 250°\ V}}$$

Chapter 12, Problem 4.

A three-phase system with *abc* sequence and $\mathbf{V}_L = 200$ V feeds a Y-connected load with $Z_L = 40\angle 30°\Omega$. Find the line currents.

Chapter 12, Solution 4.

$$V_L = 200 = \sqrt{3}V_p \qquad \longrightarrow \qquad V_p = \frac{200}{\sqrt{3}}$$

$$I_a = \frac{V_{an}}{Z_Y} = \frac{200 < 0^o}{\sqrt{3}x40 < 30^o} = \underline{2.887 < -30^o\ \text{A}}$$

$$I_b = I_a < -120^o = \underline{2.887 < -150^o\ \text{A}}$$

$$I_c = I_a < +120^o = \underline{2.887 < 90^o\ \text{A}}$$

Chapter 12, Problem 5.

For a Y-connected load, the time-domain expressions for three line-to-neutral voltages at the terminals are:

$v_{AN} = 150 \cos(\omega t + 32^o)$ V

$v_{BN} = 150 \cos(\omega t - 88^o)$ V

$v_{CN} = 150 \cos(\omega t + 152^o)$ V

Write the time-domain expressions for the line-to-line voltages v_{AN}, v_{BC}, and v_{CA} .

Chapter 12, Solution 5.

$$V_{AB} = \sqrt{3}V_p < 30^o = \sqrt{3}x150 < 32^o + 30^o = 260 < 62^o$$

Thus,

$$v_{AB} = \underline{260\cos(\omega t + 62^o)\ \text{V}}$$

Using abc sequence,

$$v_{BC} = \underline{260\cos(\omega t - 58^o)\ \text{V}}$$

$$v_{CA} = \underline{260\cos(\omega t + 182^o)\ \text{V}}$$

Chapter 12, Problem 6.

For the Y-Y circuit of Fig. 12.41, find the line currents, the line voltages, and the load voltages.

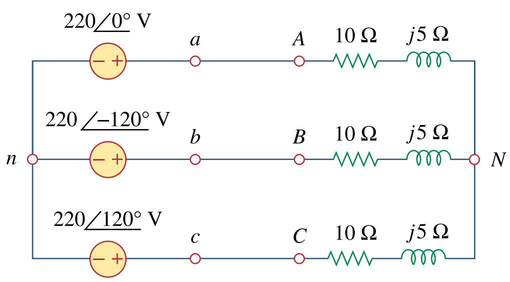


Figure 12.41
For Prob. 12.6.

Chapter 12, Solution 6.

$$\mathbf{Z}_Y = 10 + j5 = 11.18\angle 26.56°$$

The line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{220\angle 0°}{11.18\angle 26.56°} = \underline{\mathbf{19.68\angle -26.56°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{19.68\angle -146.56°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{19.68\angle 93.44°\ A}}$$

The line voltages are

$$\mathbf{V}_{ab} = 220\sqrt{3}\angle 30° = \underline{\mathbf{381\angle 30°\ V}}$$

$$\mathbf{V}_{bc} = \underline{\mathbf{381\angle -90°\ V}}$$

$$\mathbf{V}_{ca} = \underline{\mathbf{381\angle -210°\ V}}$$

The load voltages are

$$\mathbf{V}_{AN} = \mathbf{I}_a\mathbf{Z}_Y = \mathbf{V}_{an} = \underline{\mathbf{220\angle 0°\ V}}$$

$$\mathbf{V}_{BN} = \mathbf{V}_{bn} = \underline{\mathbf{220\angle -120°\ V}}$$

$$\mathbf{V}_{CN} = \mathbf{V}_{cn} = \underline{\mathbf{220\angle 120°\ V}}$$

Chapter 12, Problem 7.

Obtain the line currents in the three-phase circuit of Fig. 12.42 on the next page.

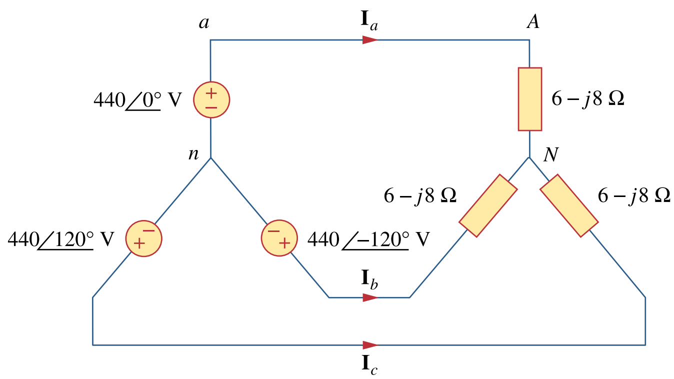
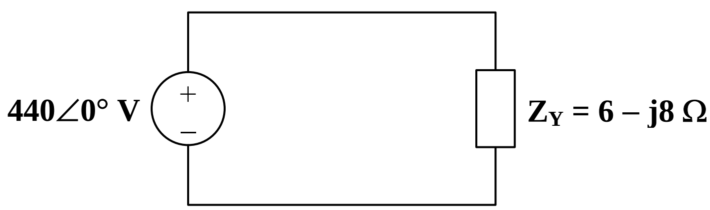


Figure 12.42
For Prob. 12.7.

Chapter 12, Solution 7.

This is a balanced Y-Y system.



Using the per-phase circuit shown above,

$$\mathbf{I}_a = \frac{440\angle 0^\circ}{6 - j8} = \underline{\mathbf{44\angle 53.13^\circ\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ = \underline{\mathbf{44\angle -66.87^\circ\ A}}$$

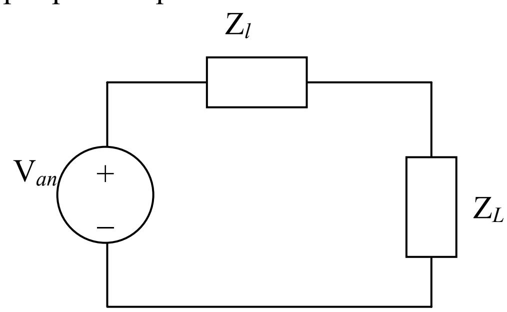
$$\mathbf{I}_c = \mathbf{I}_a \angle 120^\circ = \underline{\mathbf{44\angle 173.13^\circ\ A}}$$

Chapter 12, Problem 8.

In a balanced three-phase Y-Y system, the source is an *abc* sequence of voltages and $\mathbf{V}_{an}$ = 100 $\angle 20°$ V rms. The line impedance per phase is $0.6 + j1.2\ \Omega$, while the per-phase impedance of the load is $10 + j14\ \Omega$. Calculate the line currents and the load voltages.

Chapter 12, Solution 8.

Consider the per phase equivalent circuit shown below.



$$I_a = \frac{V_{an}}{Z_L + Z_\ell} = \frac{100 < 20^o}{10.6 + j15.2} = \ \mathbf{\underline{5.396\angle{-35.1}^\circ\ A}}$$

$$I_b = I_a < -120^o = \ \mathbf{\underline{5.396\angle{-155.1}^\circ\ A}}$$

$$I_c = I_a < +120^o = \ \mathbf{\underline{5.396\angle 84.9^\circ\ A}}$$

$$V_{La} = I_a Z_L = (4.4141 - j3.1033)(10 + j14) = \ \mathbf{\underline{92.83\angle 19.35^\circ\ A}}$$

$$V_{Lb} = V_{La} < -120^o = \ \mathbf{\underline{92.83\angle{-100.65}^\circ\ A}}$$

$$V_{Lc} = V_{La} < +120^o = \ \mathbf{\underline{92.83\angle 139.35^\circ\ A}}$$

Chapter 12, Problem 9.

A balanced Y-Y four-wire system has phase voltages

$\mathbf{V}_{an} = 120\angle 0°$ $\quad\quad$ $\mathbf{V}_{bn} = 120\angle -120°$

$\mathbf{V}_{cn} = 120\angle 120°$ V

The load impedance per phase is $19 + j13\ \Omega$, and the line impedance per phase is $1 + j2\ \Omega$. Solve for the line currents and neutral current.

Chapter 12, Solution 9.

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_L + \mathbf{Z}_Y} = \frac{120\angle 0°}{20 + j15} = \underline{\mathbf{4.8\angle\text{-}36.87°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle\text{-}120° = \underline{\mathbf{4.8\angle\text{-}156.87°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{4.8\angle 83.13°\ A}}$$

As a balanced system, $\mathbf{I}_n = \underline{\mathbf{0\ A}}$

Chapter 12, Problem 10.

For the circuit in Fig. 12.43, determine the current in the neutral line.

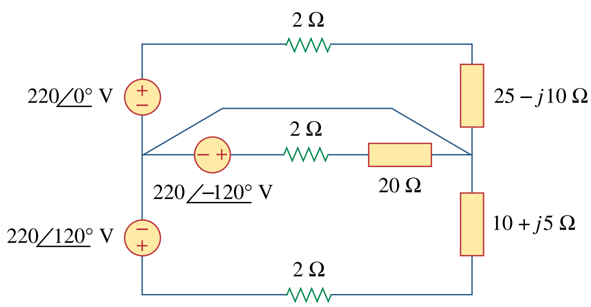


Figure 12.43
For Prob. 12.10.

Chapter 12, Solution 10.

Since the neutral line is present, we can solve this problem on a per-phase basis.

For phase a,

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_A + 2} = \frac{220\angle 0°}{27 - j10} = \frac{220}{28.79\angle -20.32°} = 7.642\angle 20.32°$$

For phase b,

$$\mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_B + 2} = \frac{220\angle -120°}{22} = 10\angle -120°$$

For phase c,

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_C + 2} = \frac{220\angle 120°}{12 + j5} = \frac{220\angle 120°}{13\angle 22.62°} = 16.923\angle 97.38°$$

The current in the neutral line is

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) \text{ or } -\mathbf{I}_n = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c$$

$$-\mathbf{I}_n = (7.166 + j2.654) + (-5 - j8.667) + (-2.173 + j16.783)$$

$$\mathbf{I}_n = 0.007 - j10.77 = \underline{\mathbf{10.77\angle 90°A}}$$

Chapter 12, Problem 11.

In the Y-Δ system shown in Fig. 12.44, the source is a positive sequence with $\mathbf{V}_{an} = 120\ \angle 0°$ V and phase impedance $\mathbf{Z}_p = 2 - j3\ \Omega$. Calculate the line voltage $\mathbf{V}_L$ and the line current $\mathbf{I}_L$.

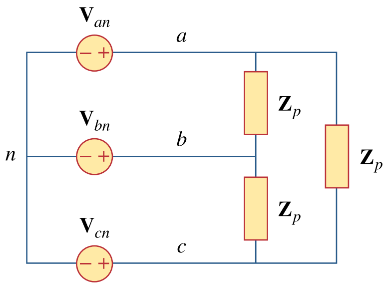


Figure 12.44
For Prob. 12.11.

Chapter 12, Solution 11.

$$V_{AB} = V_{ab} = \sqrt{3}V_p < 30^o = \sqrt{3}(120) < 30^o$$

$$V_L = |V_{ab}| = \sqrt{3}x120 = \underline{207.85\ \text{V}}$$

$$I_{AB} = \frac{V_{AB}}{Z_A} = \frac{\sqrt{3}V_p < 30^o}{2 - j3}$$

$$I_a = I_{AB}\sqrt{3} < -30^o = \frac{3V_p < 0^o}{2 - j3} = \frac{3x120}{2 - j3} = 55.385 + j83.07$$

$$I_L = |I_a| = \underline{99.846\ \ \text{A}}$$

Chapter 12, Problem 12.

Solve for the line currents in the Y-Δ circuit of Fig. 12.45. Take $\mathbf{Z}_\Delta = 60\angle 45°\Omega$.

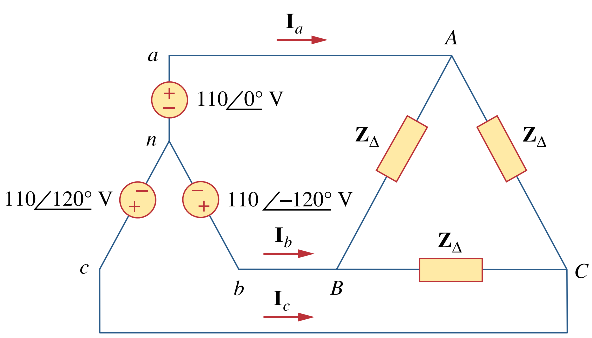
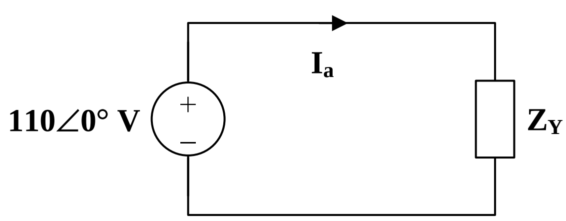


Figure 12.45
For Prob. 12.12.

Chapter 12, Solution 12.

Convert the delta-load to a wye-load and apply per-phase analysis.



$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 20\angle 45°\,\Omega$$

$$\mathbf{I}_a = \frac{110\angle 0°}{20\angle 45°} = \underline{\mathbf{5.5\angle -45°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{5.5\angle -165°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{5.5\angle 75°\ A}}$$

Chapter 12, Problem 13.

In the balanced three-phase Y-Δ system in Fig. 12.46, find the line current I_L and the average power delivered to the load.

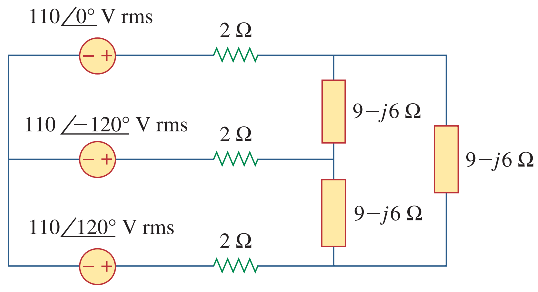
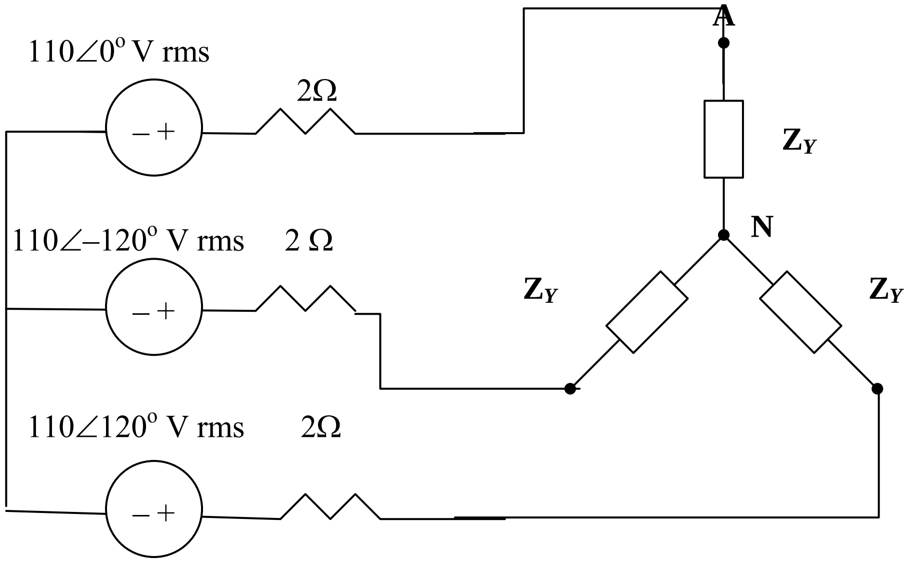


Figure 12.46
For Prob. 12.13.

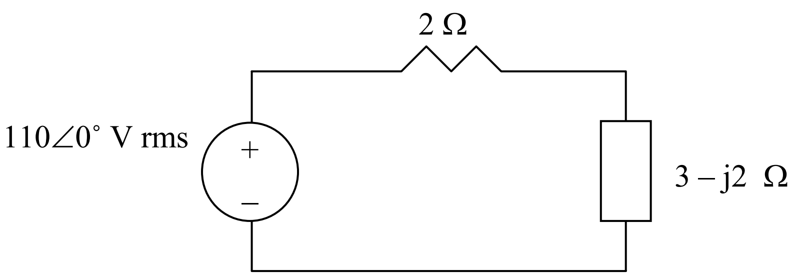
Chapter 12, Solution 13.

Convert the delta load to wye as shown below.



$$Z_Y = \frac{1}{3} Z_\Delta = 3 - j2\ \Omega$$

We consider the single phase equivalent shown below.



$$I_a = \frac{110}{2+3-j2} = 20.4265 < 21.8^o$$

$$I_L = |I_a| = \underline{20.43\ \text{A}}$$

$S = 3|I_a|^2 Z_Y = 3(20.43)^2(3–j2) = 4514\angle–33.96^\circ = 3744 – j2522$

P = Re(S) = **<u>3744 W</u>**.

Chapter 12, Problem 14.

Obtain the line currents in the three-phase circuit of Fig. 12.47 on the next page.

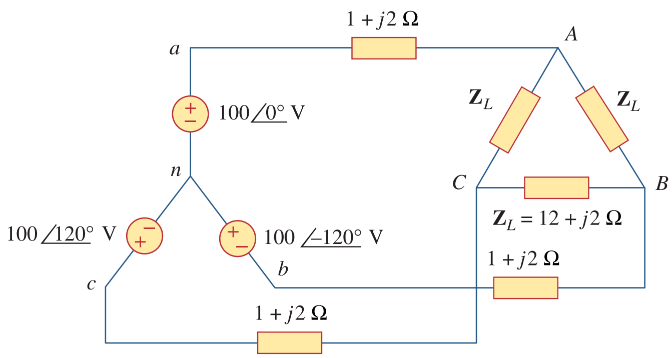
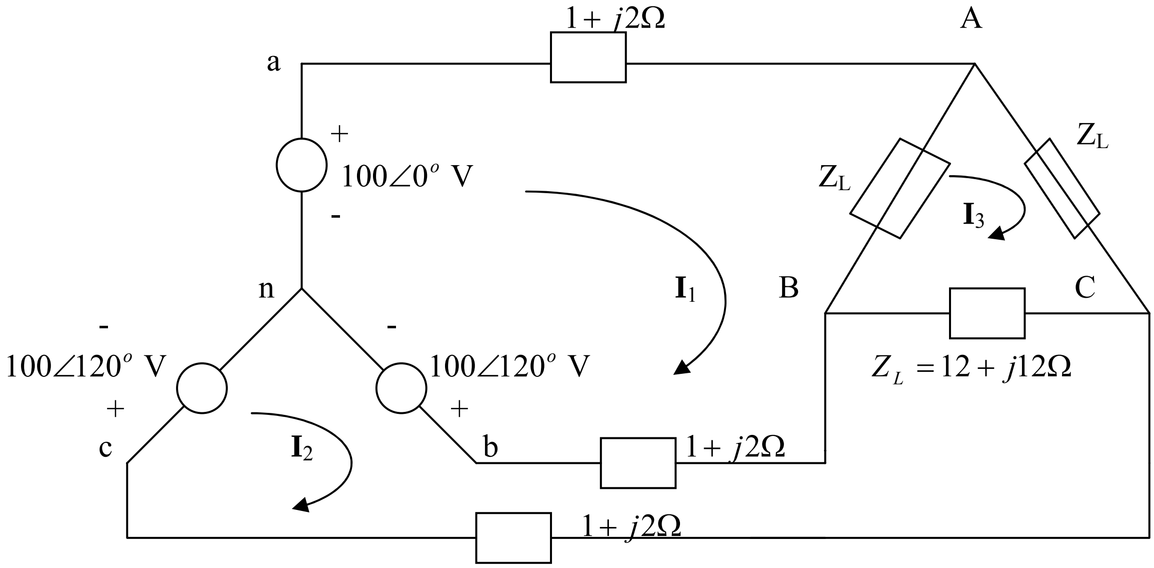


Figure 12.47
For Prob. 12.14.

Chapter 12, Solution 14.

We apply mesh analysis.



For mesh,

$$-100+100\angle 120^o + I_1(14+j16)-(1+j2)I_2-(12+j12)I_3 = 0$$

or

$$(14+j16)I_1-(1+j2)I_2-(12+j12)I_3 = 100+50-j86.6 = 150-j86.6 \qquad (1)$$

For mesh 2,

$$100\angle 120^o - 100\angle -120^o - I_1(1+j2)-(12+j12)I_3+(14+j16)I_2 = 0$$

or

$$-(1+j2)I_1+(14+j16)I_2-(12+j12)I_3 = -50-j86.6+50-j86.6 = -j173.2 \quad (2)$$

For mesh 3,

$$-(12+j12)I_1-(12+j12)I_2+(36+j36)I_3 = 0 \qquad (3)$$

Solving (1) to (3) gives

$$I_1 = -3.161-j19.3, \qquad I_2 = -10.098-j16.749, \qquad I_3 = -4.4197-j12.016$$

$$I_{aA} = I_1 = \underline{19.58\angle -99.3^o \text{ A}}$$

$$I_{bB} = I_2 - I_1 = \underline{7.392\angle 159.8^o \text{ A}}$$

$$I_{cC} = -I_2 = \underline{19.56\angle 58.91^o \text{ A}}$$

Chapter 12, Problem 15.

The circuit in Fig. 12.48 is excited by a balanced three-phase source with a line voltage of 210 V. If $\mathbf{Z}_l = 1 + j1\ \Omega$, $\mathbf{Z}_\Delta = 24 - j30\Omega$, and $\mathbf{Z}_Y = 12 + j5\ \Omega$, determine the magnitude of the line current of the combined loads.

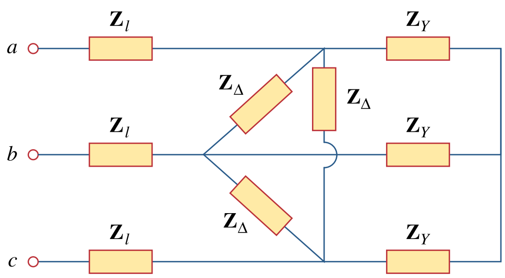


Figure 12.48
For Prob. 12.15.

Chapter 12, Solution 15.

Convert the delta load, $\mathbf{Z}_\Delta$, to its equivalent wye load.

$$\mathbf{Z}_{Ye} = \frac{\mathbf{Z}_\Delta}{3} = 8 - j10$$

$$\mathbf{Z}_p = \mathbf{Z}_Y \parallel \mathbf{Z}_{Ye} = \frac{(12 + j5)(8 - j10)}{20 - j5} = 8.076\angle -14.68^\circ$$

$$\mathbf{Z}_p = 7.812 - j2.047$$

$$\mathbf{Z}_T = \mathbf{Z}_p + \mathbf{Z}_L = 8.812 - j1.047$$

$$\mathbf{Z}_T = 8.874\angle -6.78^\circ$$

We now use the per-phase equivalent circuit.

$$\mathbf{I}_a = \frac{V_p}{\mathbf{Z}_p + \mathbf{Z}_L}, \qquad \text{where} \quad V_p = \frac{210}{\sqrt{3}}$$

$$\mathbf{I}_a = \frac{210}{\sqrt{3}\,(8.874\angle -6.78^\circ)} = 13.66\angle 6.78^\circ$$

$$\mathbf{I}_L = \left|\mathbf{I}_a\right| = \underline{\mathbf{13.66\ A}}$$

Chapter 12, Problem 16.

A balanced delta-connected load has a phase current $\mathbf{I}_{AC} = 10\ \angle -30°$ A.

(a) Determine the three line currents assuming that the circuit operates in the positive phase sequence.

(b) Calculate the load impedance if the line voltage is $\mathbf{V}_{AB} = 110\ \angle 0°$ V.

Chapter 12, Solution 16.

(a) $\mathbf{I}_{CA} = -\mathbf{I}_{AC} = 10\angle(-30° + 180°) = 10\angle 150°$

This implies that

$$\mathbf{I}_{AB} = 10\angle 30°$$
$$\mathbf{I}_{BC} = 10\angle -90°$$

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30° = \underline{\mathbf{17.32\angle 0°\ A}}$$
$$\mathbf{I}_b = \underline{\mathbf{17.32\angle -120°\ A}}$$
$$\mathbf{I}_c = \underline{\mathbf{17.32\angle 120°\ A}}$$

(b) $$\mathbf{Z}_\Delta = \frac{\mathbf{V}_{AB}}{\mathbf{I}_{AB}} = \frac{110\angle 0°}{10\angle 30°} = \underline{\mathbf{11\angle -30°\ \Omega}}$$

Chapter 12, Problem 17.

A balanced delta-connected load has line current $\mathbf{I}_a = 10\ \angle -25°$ A. Find the phase currents $\mathbf{I}_{AB}$, $\mathbf{I}_{BC}$, and $\mathbf{I}_{CA}$.

Chapter 12, Solution 17.

$$I_a = I_{AB}\sqrt{3} < -30^o \quad \longrightarrow \quad I_{AB} = \frac{I_a}{\sqrt{3} < -30^o} = \frac{10}{\sqrt{3}} < -25^o + 30^o = \underline{5.773 < 5^o\ \ A}$$

$$I_{BC} = I_{AB} < -120^o = \underline{5.775 < -115^o\ A}$$

$$I_{CA} = I_{AB} < +120^o = \underline{5.775 < 125^o\ A}$$

Chapter 12, Problem 18.

If $\mathbf{V}_{an} = 440\ \angle 60°$ V in the network of Fig. 12.49, find the load phase currents $\mathbf{I}_{AB}$, $\mathbf{I}_{BC}$, and $\mathbf{I}_{CA}$.

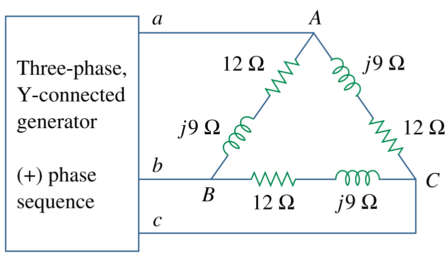


Figure 12.49
For Prob. 12.18.

Chapter 12, Solution 18.

$$\mathbf{V}_{AB} = \mathbf{V}_{an}\sqrt{3}\angle 30° = (440\angle 60°)(\sqrt{3}\angle 30°) = 762.1\angle 90°$$

$$\mathbf{Z}_{\Delta} = 12 + j9 = 15\angle 36.87°$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{762.1\angle 90°}{15\angle 36.87°} = \underline{\mathbf{50.81\angle 53.13°\ A}}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \underline{\mathbf{50.81\angle -66.87°\ A}}$$
$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \underline{\mathbf{50.81\angle 173.13°\ A}}$$

Chapter 12, Problem 19.

For the Δ-Δ circuit of Fig. 12.50, calculate the phase and line currents.

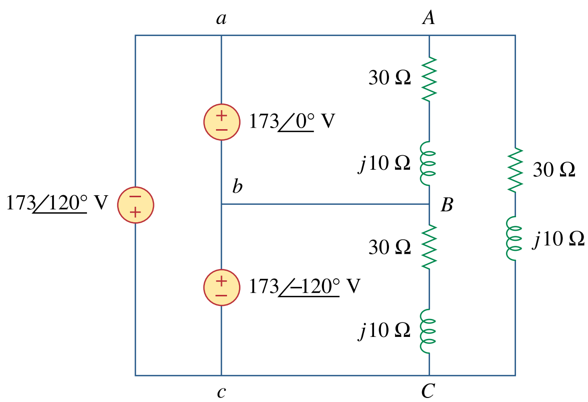


Figure 12.50
For Prob. 12.19.

Chapter 12, Solution 19.

$$\mathbf{Z}_\Delta = 30 + j10 = 31.62\angle 18.43°$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = \frac{173\angle 0°}{31.62\angle 18.43°} = \underline{\mathbf{5.47\angle -18.43°\ A}}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \underline{\mathbf{5.47\angle -138.43°\ A}}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \underline{\mathbf{5.47\angle 101.57°\ A}}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}\sqrt{3}\angle -30°$$

$$\mathbf{I}_a = 5.47\sqrt{3}\angle -48.43° = \underline{\mathbf{9.474\angle -48.43°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{9.474\angle -168.43°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{9.474\angle 71.57°\ A}}$$

Chapter 12, Problem 20.

Refer to the Δ-Δ circuit in Fig. 12.51. Find the line and phase currents. Assume that the load impedance is $Z_L = 12 + j9\ \Omega$ per phase.

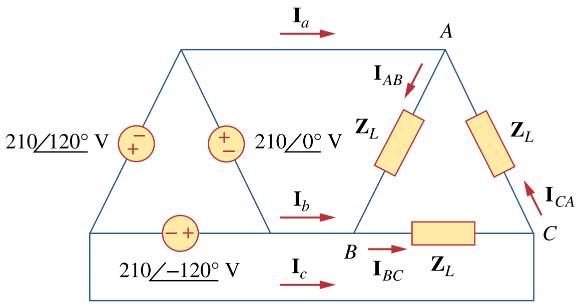


Figure 12.51
For Prob. 12.20.

Chapter 12, Solution 20.

$$\mathbf{Z}_\Delta = 12 + j9 = 15\angle 36.87°$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{210\angle 0°}{15\angle 36.87°} = \underline{\mathbf{14\angle -36.87°\ A}}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \underline{\mathbf{14\angle -156.87°\ A}}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \underline{\mathbf{14\angle 83.13°\ A}}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30° = \underline{\mathbf{24.25\angle -66.87°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{24.25\angle -186.87°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{24.25\angle 53.13°\ A}}$$

Chapter 12, Problem 21.

Three 230-V generators form a delta-connected source that is connected to a balanced delta-connected load of $\mathbf{Z}_L = 10 + j8\ \Omega$ per phase as shown in Fig. 12.52.

(a) Determine the value of $\mathbf{I}_{AC}$.
(b) What is the value of $\mathbf{I}_b$?

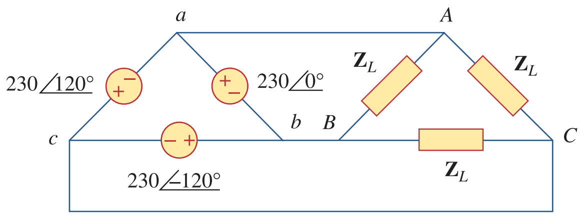


Figure 12.52
For Prob. 12.21.

Chapter 12, Solution 21.

(a) $$I_{AC} = \frac{-230\angle 120°}{10 + j8} = \frac{-230\angle 120°}{12.806\angle 38.66°} = \underline{17.96\angle -98.66°\ A}(\text{rms})$$

17.96∠–98.66˚ A rms

(b) $$I_{bB} = I_{BC} + I_{BA} = I_{BC} - I_{AB} = \frac{230\angle -120}{10 + j8} - \frac{230\angle 0°}{10 + j8}$$

$$= 17.96\angle -158.66° - 17.96\angle -38.66°$$

$$= -16.729 - j6.536 - 14.024 + j11.220 = -30.75 + j4.684$$

$$= \underline{31.10\angle 171.34°\ A}$$

31.1∠171.34˚ A rms

Chapter 12, Problem 22.

Find the line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ in the three-phase network of Fig. 12.53 below. Take $\mathbf{Z}_\Delta = 12 - j15\Omega$, $\mathbf{Z}_Y = 4 + j6\ \Omega$, and $\mathbf{Z}_l = 2\ \Omega$.

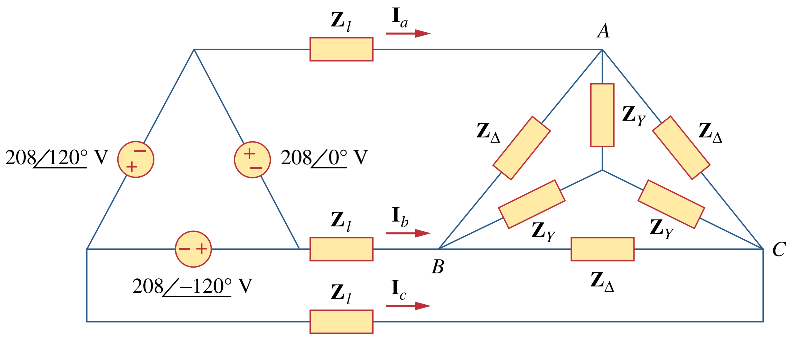


Figure 12.53
For Prob. 12.22.

Chapter 12, Solution 22.

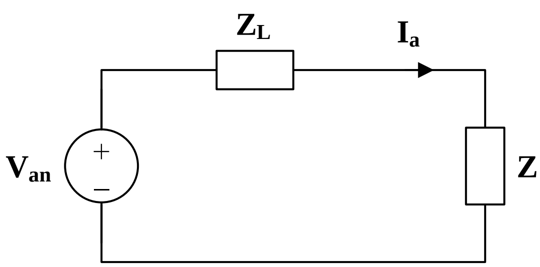
Convert the Δ-connected source to a Y-connected source.

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle -30° = \frac{208}{\sqrt{3}}\angle -30° = 120\angle -30°$$

Convert the Δ-connected load to a Y-connected load.

$$\mathbf{Z} = \mathbf{Z}_Y \,||\, \frac{\mathbf{Z}_\Delta}{3} = (4 + j6)\,||\,(4 - j5) = \frac{(4 + j6)(4 - j5)}{8 + j}$$

$$\mathbf{Z} = 5.723 - j0.2153$$



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_L + \mathbf{Z}} = \frac{120\angle -30°}{7.723 - j0.2153} = \underline{\mathbf{15.53\angle -28.4°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{15.53\angle -148.4°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{15.53\angle 91.6°\ A}}$$

Chapter 12, Problem 23.

A three-phase balanced system with a line voltage of 202 V rms feeds a delta-connected load with $\mathbf{Z}_p = 25\ \angle 60°\Omega$.

(a) Find the line current.
(b) Determine the total power supplied to the load using two wattmeters connected to the *A* and *C* lines.

Chapter 12, Solution 23.

(a) $$I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{202}{25\angle 60^o}$$

$$I_a = I_{AB}\sqrt{3}\angle -30^o = \frac{202\sqrt{3}\angle -30^o}{25\angle 60^o} = 13.995\angle -90^o$$

$$I_L = |I_a| = \underline{13.995 A}$$

(b)

$$P = P_1 + P_2 = \sqrt{3} V_L I_L \cos\theta = \sqrt{3}(202)\left(\frac{202\sqrt{3}}{25}\right)\cos 60^o = \underline{2.448\ \text{kW}}$$

Chapter 12, Problem 24.

A balanced delta-connected source has phase voltage $\mathbf{V}_{ab} = 416\ \angle 30°$ V and a positive phase sequence. If this is connected to a balanced delta-connected load, find the line and phase currents. Take the load impedance per phase as $60\ \angle 30° \Omega$ and line impedance per phase as $1 + j1\ \Omega$.

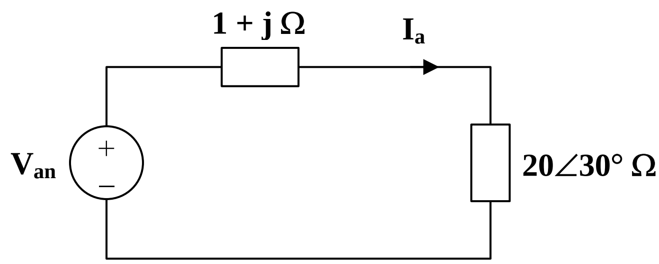
Chapter 12, Solution 24.

Convert both the source and the load to their wye equivalents.

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 20\angle 30° = 17.32 + j10$$

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}}\angle -30° = 240.2\angle 0°$$

We now use per-phase analysis.



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{(1+j)+(17.32+j10)} = \frac{240.2}{21.37\angle 31°} = \underline{\mathbf{11.24\angle -31°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{11.24\angle -151°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{11.24\angle 89°\ A}}$$

But $\quad \mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30°$

$$\mathbf{I}_{AB} = \frac{11.24\angle -31°}{\sqrt{3}\angle -30°} = \underline{\mathbf{6.489\angle -1°\ A}}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \underline{\mathbf{6.489\angle -121°\ A}}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \underline{\mathbf{6.489\angle 119°\ A}}$$

Chapter 12, Problem 25.

In the circuit of Fig. 12.54, if $\mathbf{V}_{ab} = 440\ \angle 10°$, $\mathbf{V}_{bc} = 440\ \angle 250°$, $\mathbf{V}_{ca} = 440\ \angle 130°$ V, find the line currents.

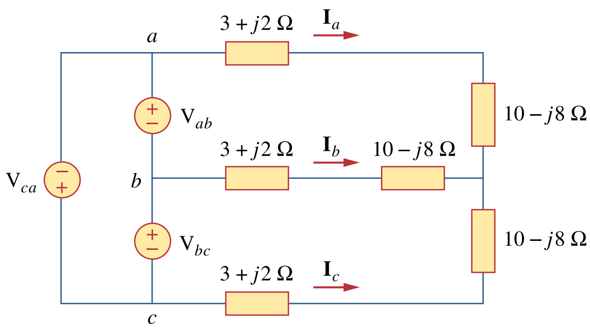


Figure 12.54
For Prob. 12.25.

Chapter 12, Solution 25.

Convert the delta-connected source to an equivalent wye-connected source and consider the single-phase equivalent.

$$\mathbf{I}_a = \frac{440\angle(10° - 30°)}{\sqrt{3}\,\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = 3 + j2 + 10 - j8 = 13 - j6 = 14.32\angle -24°.78°$

$$\mathbf{I}_a = \frac{440\angle -20°}{\sqrt{3}\,(14.32\angle -24.78°)} = \underline{\mathbf{17.74\angle 4.78°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{17.74\angle -115.22°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{17.74\angle 124.78°\ A}}$$

Chapter 12, Problem 26.

For the balanced circuit in Fig. 12.55, $\mathbf{V}_{ab} = 125\angle 0°$ V. Find the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

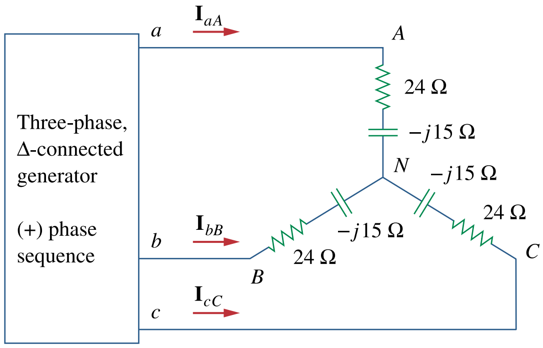


Figure 12.55
For Prob. 12.26.

Chapter 12, Solution 26.

Transform the source to its wye equivalent.

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle -30° = 72.17\angle -30°$$

Now, use the per-phase equivalent circuit.

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}}, \qquad \mathbf{Z} = 24 - j15 = 28.3\angle -32°$$

$$\mathbf{I}_{aA} = \frac{72.17\angle -30°}{28.3\angle -32°} = \underline{\mathbf{2.55\angle 2°\ A}}$$

$$\mathbf{I}_{bB} = \mathbf{I}_{aA}\angle -120° = \underline{\mathbf{2.55\angle -118°\ A}}$$

$$\mathbf{I}_{cC} = \mathbf{I}_{aA}\angle 120° = \underline{\mathbf{2.55\angle 122°\ A}}$$

Chapter 12, Problem 27.

ps A Δ-connected source supplies power to a Y-connected load in a three-phase balanced system. Given that the line impedance is $2 + j1\ \Omega$ per phase while the load impedance is $6 + j4\ \Omega$ per phase, find the magnitude of the line voltage at the load. Assume the source phase voltage $\mathbf{V}_{ab} = 208\ \angle 0°$ V rms.

Chapter 12, Solution 27.

Since Z_L and Z_ℓ are in series, we can lump them together so that

$$Z_Y = 2 + j + 6 + j4 = 8 + j5$$

$$I_a = \frac{\frac{V_P}{\sqrt{3}} < -30^o}{Z_Y} = \frac{208 < -30^o}{\sqrt{3}(8 + j5)}$$

$$V_L = (6 + j4)I_a = \frac{208(0.866 - j0.5)(6 + j4)}{\sqrt{3}(8 + j5)} = 80.81 - j43.54$$

$|V_L| =$ **<u>91.79 V</u>**

Chapter 12, Problem 28.

The line-to-line voltages in a Y-load have a magnitude of 440 V and are in the positive sequence at 60 Hz. If the loads are balanced with $Z_1 = Z_2 = Z_3 = 25\angle 30°$, find all line currents and phase voltages.

Chapter 12, Solution 28.

$V_L = |V_{ab}| = 440 = \sqrt{3}V_P$ or $V_P = 440/1.7321 = 254$

For reference, let V_{AN} = **<u>254∠0˚ V</u>** which leads to
V_{BN} = **<u>254∠–120˚ V</u>** and V_{CN} = **<u>254∠120˚ V</u>**.

The line currents are found as follows,

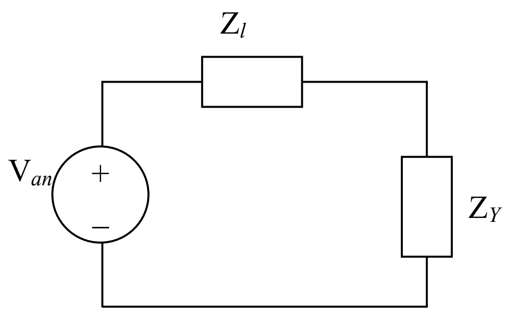
$I_a = V_{AN}/Z_Y = 254/25\angle 30^\circ$ = **<u>10.16∠–30˚ A</u>**.
This leads to, I_b = **<u>10.16∠–150˚ A</u>** and I_c = **<u>10.16∠90˚ A</u>**.

Chapter 12, Problem 29.

A balanced three-phase Y-Δ system has $\mathbf{V}_{an} = 120\ \angle 0°$ V rms and $\mathbf{Z}_\Delta = 51 + j45\Omega$. If the line impedance per phase is $0.4 + j1.2\ \Omega$, find the total complex power delivered to the load.

Chapter 12, Solution 29.

We can replace the delta load with a wye load, $Z_Y = Z_\Delta/3 = 17+j15\Omega$.
The per-phase equivalent circuit is shown below.



$$I_a = \frac{V_{an}}{Z_Y + Z_\ell} = \frac{120}{17 + j15 + 0.4 + j1.2} = 5.0475\angle{-42.96^\circ}$$

$$S = 3S_p = 3|I_a|^2 Z_Y = 3(5.0475)^2(17 + j15) = \underline{1.3 + j1.1465\ \text{kVA}}$$

Chapter 12, Problem 30.

In Fig. 12.56, the rms value of the line voltage is 208 V. Find the average power delivered to the load.

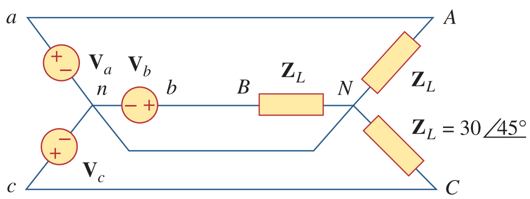
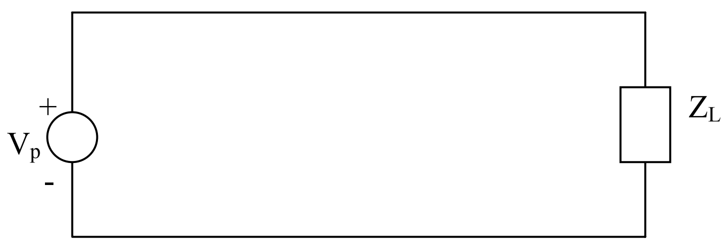


Figure 12.56
For Prob. 12.30.

Chapter 12, Solution 30.

Since this a balanced system, we can replace it by a per-phase equivalent, as shown below.



$$\overline{S} = 3\overline{S}_p = \frac{3V^2_p}{Z^*_p}, \qquad V_p = \frac{V_L}{\sqrt{3}}$$

$$\overline{S} = \frac{V^2_L}{Z^*_p} = \frac{(208)^2}{30\angle -45^o} = 1.4421\angle 45^o \text{ kVA}$$

$$P = S\cos\theta = \underline{1.02 \text{ kW}}$$

Chapter 12, Problem 31.

A balanced delta-connected load is supplied by a 60-Hz three-phase source with a line voltage of 240 V. Each load phase draws 6 kW at a lagging power factor of 0.8. Find:

(a) the load impedance per phase
(b) the line current
(c) the value of capacitance needed to be connected in parallel with each load phase to minimize the current from the source

Chapter 12, Solution 31.

(a)

$$P_p = 6{,}000, \qquad \cos\theta = 0.8, \qquad S_p = \frac{P_P}{\cos\theta} = 6/0.8 = 7.5\,\text{kVA}$$

$$Q_p = S_P \sin\theta = 4.5\,\text{kVAR}$$

$$\overline{S} = 3\overline{S}_p = 3(6 + j4.5) = 18 + j13.5\,\text{kVA}$$

For delta-connected load, $V_p = V_L = 240$ (rms). But

$$\overline{S} = \frac{3V^2{}_p}{Z^*{}_p} \quad \longrightarrow \quad Z^*{}_p = \frac{3V^2{}_p}{S} = \frac{3(240)^2}{(18 + j13.5)x10^3}, \qquad \underline{Z_P = 6.144 + j4.608\Omega}$$

(b) $$P_p = \sqrt{3}V_L I_L \cos\theta \quad \longrightarrow \quad I_L = \frac{6000}{\sqrt{3}x240x0.8} = \underline{18.04\text{ A}}$$

(c) We find C to bring the power factor to unity

$$Q_c = Q_p = 4.5\,\text{kVA} \quad \longrightarrow \quad C = \frac{Q_c}{\omega V^2{}_{rms}} = \frac{4500}{2\pi x60x240^2} = \underline{207.2\,\mu\text{F}}$$

Chapter 12, Problem 32.

A balanced Y-load is connected to a 60-Hz three-phase source with $\mathbf{V}_{ab} = 240\ \angle 0°$ V. The load has pf = 0.5 lagging and each phase draws 5 kW. (a) Determine the load impedance $\mathbf{Z}_Y$. (b) Find $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$.

Chapter 12, Solution 32.

(a) $|V_{ab}| = \sqrt{3}V_p = 240 \quad\longrightarrow\quad V_p = \dfrac{240}{\sqrt{3}} = 138.56$

$$V_{an} = V_p < -30^o$$

$$pf = 0.5 = \cos\theta \quad\longrightarrow\quad \theta = 60^o$$

$$P = S\cos\theta \quad\longrightarrow\quad S = \frac{P}{\cos\theta} = \frac{5}{0.5} = 10 \text{ kVA}$$

$$Q = S\sin\theta = 10\sin 60 = 8.66$$

$$S_p = 5 + j8.66 \text{ kVA}$$

But

$$S_P = \frac{V_p^2}{Z_p^*} \quad\longrightarrow\quad Z_p^* = \frac{V_p^2}{S_p} = \frac{138.56^2}{(5 + j8.66)x10^3} = 0.96 - j1.663$$

$Z_p = \underline{0.96 + j1.663\ \ \Omega}$

(b) $I_a = \dfrac{V_{an}}{Z_Y} = \dfrac{138.56 < -30^o}{0.96 + j1.6627} = \underline{72.17 < -90^o\ \text{A}}$

$$I_b = I_a < -120^o = \underline{72.17 < -210^o\ \text{A}}$$

$$I_c = I_a < +120^o = \underline{72.17 < 30^o\ \text{A}}$$

Chapter 12, Problem 33.

A three-phase source delivers 4800 VA to a wye-connected load with a phase voltage of 208 V and a power factor of 0.9 lagging. Calculate the source line current and the source line voltage.

Chapter 12, Solution 33.

$$\mathbf{S} = \sqrt{3}\, V_L I_L \angle\theta$$

$$S = |\mathbf{S}| = \sqrt{3}\, V_L I_L$$

For a Y-connected load,

$$I_L = I_p, \qquad V_L = \sqrt{3}\, V_p$$

$$S = 3 V_p I_p$$

$$I_L = I_p = \frac{S}{3V_p} = \frac{4800}{(3)(208)} = \underline{\mathbf{7.69\ A}}$$

$$V_L = \sqrt{3}\, V_p = \sqrt{3} \times 208 = \underline{\mathbf{360.3\ V}}$$

Chapter 12, Problem 34.

A balanced wye-connected load with a phase impedance of $10 - j16\ \Omega$ is connected to a balanced three-phase generator with a line voltage of 220 V. Determine the line current and the complex power absorbed by the load.

Chapter 12, Solution 34.

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}}$$

$$\mathbf{I}_a = \frac{V_p}{\mathbf{Z}_Y} = \frac{220}{\sqrt{3}(10 - j16)} = \frac{127.02}{18.868\angle -58^\circ} = 6.732\angle 58^\circ$$

$$I_L = I_p = \underline{\mathbf{6.732A}}$$

$$\mathbf{S} = \sqrt{3}\, V_L I_L \angle\theta = \sqrt{3} \times 220 \times 6.732\angle -58^\circ = 2565\angle -58^\circ$$

$$\mathbf{S} = \underline{\mathbf{1359.2 - j2175\ VA}}$$

Chapter 12, Problem 35.

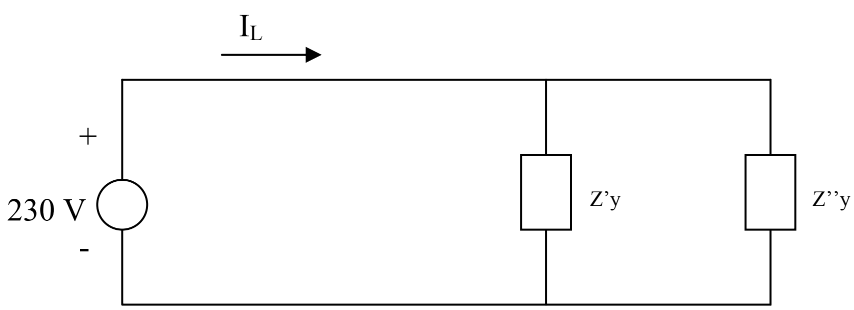
Three equal impedances, $60 + j30\ \Omega$ each, are delta-connected to a 230-V rms, three-phase circuit. Another three equal impedances, $40 + j10\ \Omega$ each, are wye-connected across the same circuit at the same points. Determine:

(a) the line current
(b) the total complex power supplied to the two loads
(c) the power factor of the two loads combined

Chapter 12, Solution 35.

(a) This is a balanced three-phase system and we can use per phase equivalent circuit. The delta-connected load is converted to its wye-connected equivalent

$$Z''_y = \frac{1}{3}Z_\Delta = (60 + j30)/3 = 20 + j10$$



$$Z_y = Z'_y \,//\, Z''_y = (40 + j10)//(20 + j10) = 13.5 + j5.5$$

$$I_L = \frac{230}{13.5 + j5.5} = \underline{14.61 - j5.953\text{ A}}$$

(b) $\overline{S} = V_s I^*_L = \underline{3.361 + j1.368\ \text{kVA}}$

(c) pf = P/S = 0.9261

Chapter 12, Problem 36.

A 4200-V, three-phase transmission line has an impedance of $4 + j10\ \Omega$ per phase. If it supplies a load of 1 MVA at 0.75 power factor (lagging), find:

(a) the complex power
(b) the power loss in the line
(c) the voltage at the sending end

Chapter 12, Solution 36.

(a) $S = 1\,[0.75 + \sin(\cos^{-1}0.75)\,] =$ $\underline{\mathbf{0.75 + j0.6614\ \ MVA}}$

(b) $\overline{S} = 3V_p I^*{}_p \longrightarrow I^*{}_p = \dfrac{S}{3V_p} = \dfrac{(0.75 + j0.6614)x10^6}{3x4200} = 59.52 + j52.49$

$$P_L = |\,I_p\,|^2\ R_l = (79.36)^2(4) = \underline{25.19\,\text{kW}}$$

(c) $V_s = V_L + I_p(4 + j) = 4.4381 - j0.21\,\text{kV} = \underline{4.443\angle\text{-}2.709^\circ\ \text{kV}}$

Chapter 12, Problem 37.

The total power measured in a three-phase system feeding a balanced wye-connected load is 12 kW at a power factor of 0.6 leading. If the line voltage is 208 V, calculate the line current I_L and the load impedance $\mathbf{Z}_Y$.

Chapter 12, Solution 37.

$$S = \frac{P}{pf} = \frac{12}{0.6} = 20$$

$$\mathbf{S} = S\angle\theta = 20\angle\theta = 12 - j16 \text{ kVA}$$

But $\quad \mathbf{S} = \sqrt{3}\, V_L I_L \angle\theta$

$$I_L = \frac{20 \times 10^3}{\sqrt{3} \times 208} = \underline{\mathbf{55.51\ A}}$$

$$\mathbf{S} = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_p$$

For a Y-connected load, $I_L = I_p$.

$$\mathbf{Z}_p = \frac{\mathbf{S}}{3\left|I_L\right|^2} = \frac{(12 - j16) \times 10^3}{(3)(55.51)^2}$$

$$\mathbf{Z}_p = \underline{\mathbf{1.298 - j1.731\ \Omega}}$$

Chapter 12, Problem 38.

Given the circuit in Fig. 12.57 below, find the total complex power absorbed by the load.

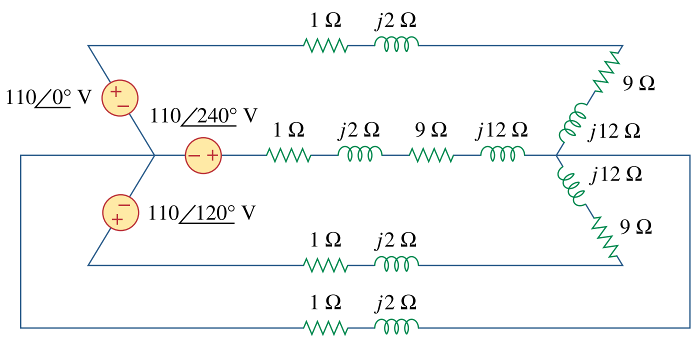


Figure 12.57
For Prob. 12.38.

Chapter 12, Solution 38.

As a balanced three-phase system, we can use the per-phase equivalent shown below.

$$\mathbf{I}_a = \frac{110\angle 0°}{(1+j2)+(9+j12)} = \frac{110\angle 0°}{10+j14}$$

$$\mathbf{S}_p = \frac{1}{2}\left|\mathbf{I}_a\right|^2 \mathbf{Z}_Y = \frac{1}{2}\cdot\frac{(110)^2}{(10^2+14^2)}\cdot(9+j12)$$

The complex power is

$$\mathbf{S} = 3\mathbf{S}_p = \frac{3}{2}\cdot\frac{(110)^2}{296}\cdot(9+j12)$$

$$\mathbf{S} = \underline{\mathbf{551.86 + j735.81\ VA}}$$

Chapter 12, Problem 39.

Find the real power absorbed by the load in Fig. 12.58.

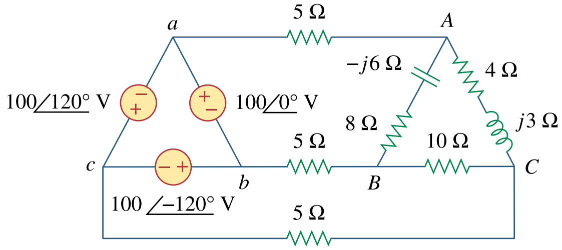
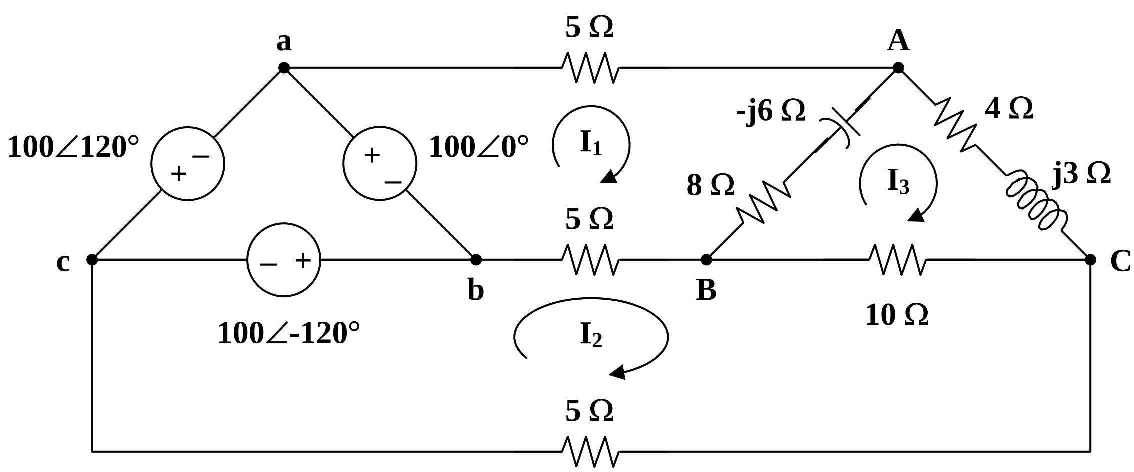


Figure 12.58
For Prob. 12.39.

Chapter 12, Solution 39.

Consider the system shown below.



For mesh 1,

$$100 = (18 - j6)\mathbf{I}_1 - 5\mathbf{I}_2 - (8 - j6)\mathbf{I}_3 \quad (1)$$

For mesh 2,

$$100\angle\text{-}120° = 20\mathbf{I}_2 - 5\mathbf{I}_1 - 10\mathbf{I}_3$$

$$20\angle\text{-}120° = \text{-}\mathbf{I}_1 + 4\mathbf{I}_2 - 2\mathbf{I}_3 \quad (2)$$

For mesh 3,

$$0 = \text{-}(8 - j6)\mathbf{I}_1 - 10\mathbf{I}_2 + (22 - j3)\mathbf{I}_3 \quad (3)$$

To eliminate $\mathbf{I}_2$, start by multiplying (1) by 2,

$$200 = (36 - j12)\mathbf{I}_1 - 10\mathbf{I}_2 - (16 - j12)\mathbf{I}_3 \quad (4)$$

Subtracting (3) from (4),

$$200 = (44 - j18)\mathbf{I}_1 - (38 - j15)\mathbf{I}_3 \qquad (5)$$

Multiplying (2) by $5/4$,

$$25\angle\text{-}120° = \text{-}1.25\mathbf{I}_1 + 5\mathbf{I}_2 - 2.5\mathbf{I}_3 \qquad (6)$$

Adding (1) and (6),

$$87.5 - j21.65 = (16.75 - j6)\mathbf{I}_1 - (10.5 - j6)\mathbf{I}_3 \qquad (7)$$

In matrix form, (5) and (7) become

$$\begin{bmatrix} 200 \\ 87.5 - j12.65 \end{bmatrix} = \begin{bmatrix} 44 - j18 & \text{-}38 + j15 \\ 16.75 - j6 & \text{-}10.5 + j6 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_3 \end{bmatrix}$$

$$\Delta = 192.5 - j26.25, \quad \Delta_1 = 900.25 - j935.2, \quad \Delta_3 = 110.3 - j1327.6$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{1298.1\angle\text{-}46.09°}{194.28\angle\text{-}7.76°} = 6.682\angle\text{-}38.33° = 5.242 - j4.144$$

$$\mathbf{I}_3 = \frac{\Delta_3}{\Delta} = \frac{1332.2\angle\text{-}85.25°}{194.28\angle\text{-}7.76°} = 6.857\angle\text{-}77.49° = 1.485 - j6.694$$

We obtain $\mathbf{I}_2$ from (6),

$$\mathbf{I}_2 = 5\angle\text{-}120° + \frac{1}{4}\mathbf{I}_1 + \frac{1}{2}\mathbf{I}_3$$
$$\mathbf{I}_2 = (\text{-}2.5 - j4.33) + (1.3104 - j1.0359) + (0.7425 - j3.347)$$
$$\mathbf{I}_2 = \text{-}0.4471 - j8.713$$

The average power absorbed by the 8-Ω resistor is

$$P_1 = \left|\mathbf{I}_1 - \mathbf{I}_3\right|^2 (8) = \left|3.756 + j2.551\right|^2 (8) = 164.89 \text{ W}$$

The average power absorbed by the 4-Ω resistor is

$$P_2 = \left|\mathbf{I}_3\right|^2 (4) = (6.8571)^2 (4) = 188.1 \text{ W}$$

The average power absorbed by the 10-Ω resistor is

$$P_3 = \left|\mathbf{I}_2 - \mathbf{I}_3\right|^2 (10) = \left|\text{-}1.9321 - j2.019\right|^2 (10) = 78.12 \text{ W}$$

Thus, the total real power absorbed by the load is

$$P = P_1 + P_2 + P_3 = \underline{\mathbf{431.1\ W}}$$

Chapter 12, Problem 40.

For the three-phase circuit in Fig. 12.59, find the average power absorbed by the delta-connected load with $\mathbf{Z}_{\Delta} = 21 + j24\Omega$.

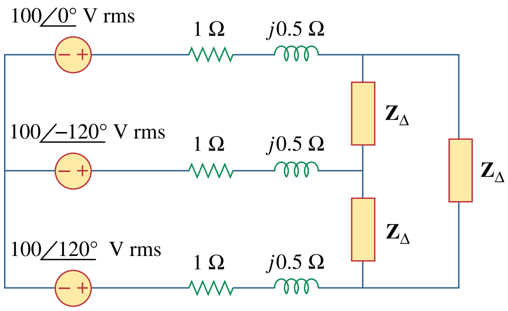


Figure 12.59
For Prob. 12.40.

Chapter 12, Solution 40.

Transform the delta-connected load to its wye equivalent.

$$\mathbf{Z}_{\mathrm{Y}} = \frac{\mathbf{Z}_{\Delta}}{3} = 7 + \mathrm{j}8$$

Using the per-phase equivalent circuit above,

$$\mathbf{I}_{\mathrm{a}} = \frac{100\angle 0°}{(1 + \mathrm{j}0.5) + (7 + \mathrm{j}8)} = 8.567\angle -46.75°$$

For a wye-connected load,

$$\mathrm{I}_{\mathrm{p}} = \mathrm{I}_{\mathrm{a}} = \left|\mathbf{I}_{\mathrm{a}}\right| = 8.567$$

$$\mathbf{S} = 3\left|\mathbf{I}_{\mathrm{p}}\right|^2 \mathbf{Z}_{\mathrm{p}} = (3)(8.567)^2(7 + \mathrm{j}8)$$

$$\mathrm{P} = \mathrm{Re}(\mathbf{S}) = (3)(8.567)^2(7) = \underline{\mathbf{1.541\ kW}}$$

Chapter 12, Problem 41.

A balanced delta-connected load draws 5 kW at a power factor of 0.8 lagging. If the three-phase system has an effective line voltage of 400 V, find the line current.

Chapter 12, Solution 41.

$$S = \frac{P}{pf} = \frac{5\text{ kW}}{0.8} = 6.25\text{ kVA}$$

But $\quad S = \sqrt{3}\, V_L I_L$

$$I_L = \frac{S}{\sqrt{3}\, V_L} = \frac{6.25\times10^3}{\sqrt{3}\times400} = \underline{\mathbf{9.021\ A}}$$

Chapter 12, Problem 42.

A balanced three-phase generator delivers 7.2 kW to a wye-connected load with impedance $30 - j40\,\Omega$ per phase. Find the line current I_L and the line voltage V_L.

Chapter 12, Solution 42.

The load determines the power factor.

$$\tan\theta = \frac{40}{30} = 1.333 \quad\longrightarrow\quad \theta = 53.13°$$

$$pf = \cos\theta = 0.6 \quad \text{(leading)}$$

$$\mathbf{S} = 7.2 - j\left(\frac{7.2}{0.6}\right)(0.8) = 7.2 - j9.6\text{ kVA}$$

But $\quad \mathbf{S} = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_p$

$$\left|\mathbf{I}_p\right|^2 = \frac{\mathbf{S}}{3\mathbf{Z}_p} = \frac{(7.2 - j9.6)\times10^3}{(3)(30 - j40)} = 80$$

$$I_p = 8.944\text{ A}$$

$$I_L = I_p = \underline{\mathbf{8.944\ A}}$$

$$V_L = \frac{S}{\sqrt{3}\, I_L} = \frac{12\times10^3}{\sqrt{3}\,(8.944)} = \underline{\mathbf{774.6\ V}}$$

Chapter 12, Problem 43.

Refer to Fig. 12.48. Obtain the complex power absorbed by the combined loads.

Chapter 12, Solution 43.

$$\mathbf{S} = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_p, \qquad I_p = I_L \text{ for Y-connected loads}$$

$$\mathbf{S} = (3)(13.66)^2(7.812 - j2.047)$$

$$\mathrm{S} = \underline{\mathbf{4.373 - j1.145\ kVA}}$$

Chapter 12, Problem 44.

A three-phase line has an impedance of $1 + j3\,\Omega$ per phase. The line feeds a balanced delta-connected load, which absorbs a total complex power of $12 + j5$ k VA. If the line voltage at the load end has a magnitude of 240 V, calculate the magnitude of the line voltage at the source end and the source power factor.

Chapter 12, Solution 44.

For a Δ-connected load,

$$V_p = V_L, \qquad I_L = \sqrt{3}\,I_p$$

$$S = \sqrt{3}\,V_L I_L$$

$$I_L = \frac{S}{\sqrt{3}\,V_L} = \frac{\sqrt{(12^2 + 5^2)} \times 10^3}{\sqrt{3}\,(240)} = 31.273$$

At the source,

$$\mathbf{V}_L' = \mathbf{V}_L + \mathbf{I}_L\,\mathbf{Z}_L$$

$$\mathbf{V}_L' = 240\angle 0° + (31.273)(1 + j3)$$

$$\mathbf{V}_L' = 271.273 + j93.819$$

$$\left|\mathbf{V}_L'\right| = \underline{\mathbf{287.04\ V}}$$

Also, at the source,

$$\mathbf{S}' = \sqrt{3}\mathbf{V}_L'\mathbf{I}_L^*$$

$$\mathbf{S}' = \sqrt{3}\,(271.273 + j93.819)(31.273)$$

$$\theta = \tan^{-1}\left(\frac{93.819}{271.273}\right) = 19.078$$

$$\mathrm{pf} = \cos\theta = \underline{\mathbf{0.9451}}$$

Chapter 12, Problem 45.

A balanced wye-connected load is connected to the generator by a balanced transmission line with an impedance of $0.5 + j2\,\Omega$ per phase. If the load is rated at 450 kW, 0.708 power factor lagging, 440-V line voltage, find the line voltage at the generator.

Chapter 12, Solution 45.

$$\mathbf{S} = \sqrt{3}\,V_L I_L \angle\theta$$

$$I_L = \frac{|\mathbf{S}|\angle -\theta}{\sqrt{3}\,V_L}, \qquad |\mathbf{S}| = \frac{P}{pf} = \frac{450\times 10^3}{0.708} = 635.6 \text{ kVA}$$

$$\mathbf{I}_L = \frac{(635.6)\angle -\theta}{\sqrt{3}\times 440} = 834\angle -45^\circ \text{ A}$$

At the source,

$$\mathbf{V}_L = 440\angle 0^\circ + \mathbf{I}_L\,(0.5 + j2)$$
$$\mathbf{V}_L = 440 + (834\angle -45^\circ)(2.062\angle 76^\circ)$$
$$\mathbf{V}_L = 440 + 1719.7\angle 31^\circ$$
$$\mathbf{V}_L = 1914.1 + j885.7$$
$$\mathbf{V}_L = \underline{2.109\angle 24.83^\circ \text{ kV}}$$

Note, this is not normally experienced in practice since transformers are use which can significantly reduce line losses.

Chapter 12, Problem 46.

A three-phase load consists of three 100-Ω resistors that can be wye- or delta-connected. Determine which connection will absorb the most average power from a three-phase source with a line voltage of 110 V. Assume zero line impedance.

Chapter 12, Solution 46.

For the wye-connected load,

$$I_L = I_p, \qquad V_L = \sqrt{3}\, V_p \qquad\qquad I_p = V_p / \mathbf{Z}$$

$$\mathbf{S} = 3\mathbf{V}_p \mathbf{I}_p^* = \frac{3\left|\mathbf{V}_p\right|^2}{\mathbf{Z}^*} = \frac{3\left|\mathbf{V}_L/\sqrt{3}\right|^2}{\mathbf{Z}^*}$$

$$\mathbf{S} = \frac{\left|\mathbf{V}_L\right|^2}{\mathbf{Z}^*} = \frac{(110)^2}{100} = 121\text{ W}$$

For the delta-connected load,

$$V_p = V_L, \qquad I_L = \sqrt{3}\, I_p, \qquad I_p = V_p / \mathbf{Z}$$

$$\mathbf{S} = 3\mathbf{V}_p \mathbf{I}_p^* = \frac{3\left|\mathbf{V}_p\right|^2}{\mathbf{Z}^*} = \frac{3\left|\mathbf{V}_L\right|^2}{\mathbf{Z}^*}$$

$$\mathbf{S} = \frac{(3)(110)^2}{100} = 363\text{ W}$$

This shows that the **delta-connected load** will deliver three times more average power than the wye-connected load. This is also evident from $\mathbf{Z}_Y = \dfrac{\mathbf{Z}_\Delta}{3}$.

Chapter 12, Problem 47.

The following three parallel-connected three-phase loads are fed by a balanced three-phase source:
Load 1: 250 kVA, 0.8 pf lagging
Load 2: 300 kVA, 0.95 pf leading
Load 3: 450 kVA, unity pf
If the line voltage is 13.8 kV, calculate the line current and the power factor of the source. Assume that the line impedance is zero.

Chapter 12, Solution 47.

$$\text{pf} = 0.8 \quad (\text{lagging}) \quad \longrightarrow \quad \theta = \cos^{-1}(0.8) = 36.87°$$
$$\mathbf{S}_1 = 250\angle 36.87° = 200 + j150 \text{ kVA}$$

$$\text{pf} = 0.95 \quad (\text{leading}) \quad \longrightarrow \quad \theta = \cos^{-1}(0.95) = -18.19°$$
$$\mathbf{S}_2 = 300\angle -18.19° = 285 - j93.65 \text{ kVA}$$

$$\text{pf} = 1.0 \quad \longrightarrow \quad \theta = \cos^{-1}(1) = 0°$$
$$\mathbf{S}_3 = 450 \text{ kVA}$$

$$\mathbf{S}_T = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = 935 + j56.35 = 936.7\angle 3.45° \text{ kVA}$$

$$\left|\mathbf{S}_T\right| = \sqrt{3}\,V_L I_L$$

$$I_L = \frac{936.7\times 10^3}{\sqrt{3}\,(13.8\times 10^3)} = \underline{\mathbf{39.19\ A\ rms}}$$

$$\text{pf} = \cos\theta = \cos(3.45°) = \underline{\mathbf{0.9982 \quad (lagging)}}$$

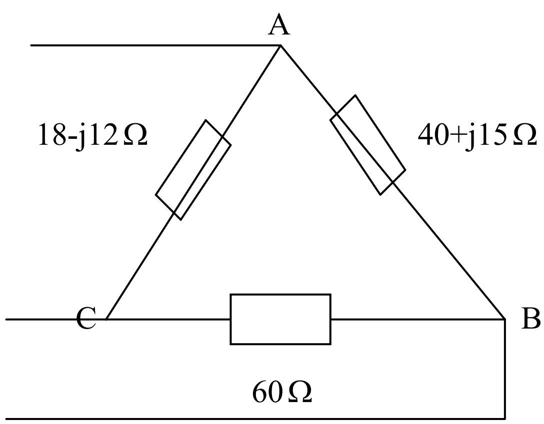
Chapter 12, Problem 48.

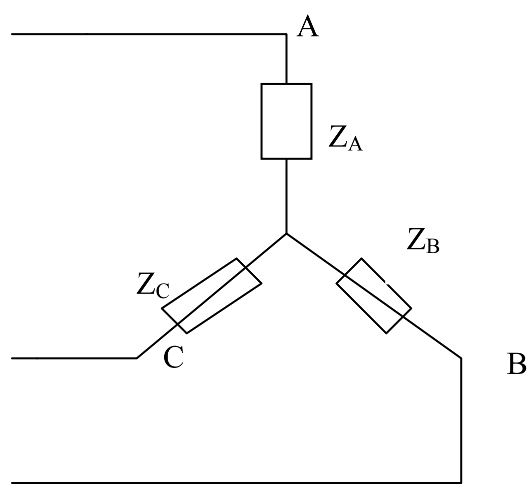
A balanced, positive-sequence wye-connected source has $\mathbf{V}_{an} = 240\ \angle 0°$ V rms and supplies an unbalanced delta-connected load via a transmission line with impedance $2 + j3\,\Omega$ per phase.

(a) Calculate the line currents if $\mathbf{Z}_{AB} = 40 + j15\,\Omega$, $\mathbf{Z}_{BC} = 60\,\Omega$, $\mathbf{Z}_{CA} = 18 - j12\,\Omega$.
(b) Find the complex power supplied by the source.

Chapter 12, Solution 48.

(a) We first convert the delta load to its equivalent wye load, as shown below.



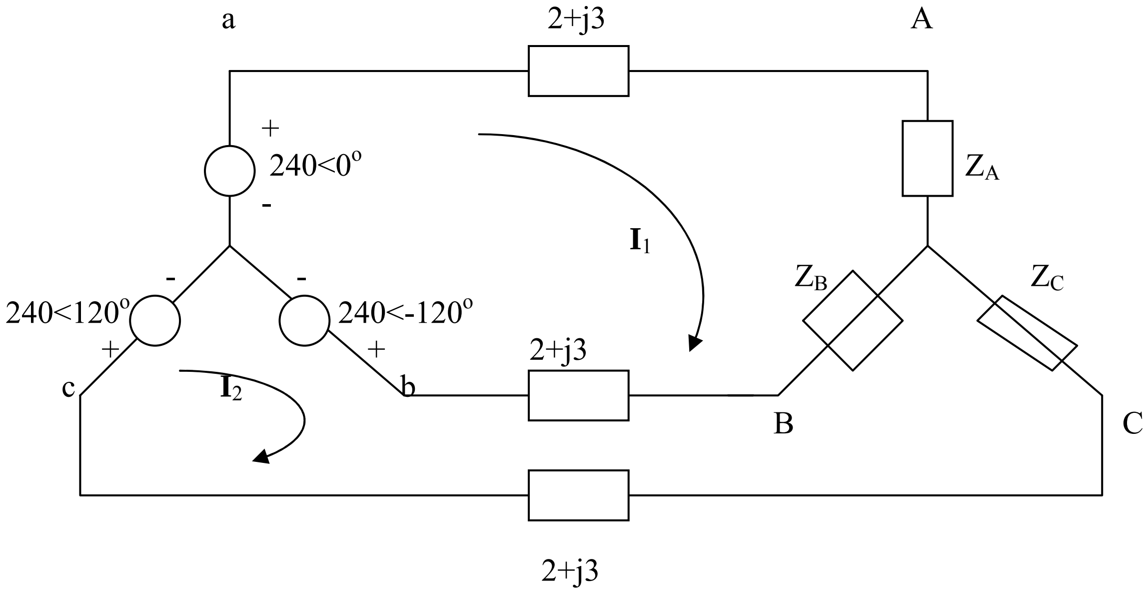


$$Z_A = \frac{(40 + j15)(18 - j12)}{118 + j3} = 7.577 - j1.923$$

$$Z_B = \frac{60(40 + j15).}{118 + j3} = 20.52 + j7.105$$

$$Z_C = \frac{60(18 - j12)}{118 + j3} = 8.992 - j6.3303$$

The system becomes that shown below.



We apply KVL to the loops. For mesh 1,

$$-240+240\angle-120^o+I_1(2Z_l+Z_A+Z_B)-I_2(Z_B+Z_l)=0$$

or

$$(32.097+j11.13)I_1-(22.52+j10.105)I_2=360+j207.85 \qquad (1)$$

For mesh 2,

$$240\angle120^o-240\angle-120^o-I_1(Z_B+Z_l)+I_2(2Z_l+Z_B+Z_C)=0$$

or

$$-(22.52+j10.105)I_1+(33.51+j6.775)I_2=-j415.69 \qquad (2)$$

Solving (1) and (2) gives

$I_1=23.75-j5.328, \qquad I_2=15.165-j11.89$

$I_{aA}=I_1=\underline{24.34\angle-12.64^o\text{ A}}, \qquad I_{bB}=I_2-I_1=\underline{10.81\angle-142.6^o\text{ A}}$

$I_{cC}=-I_2=\underline{19.27\angle141.9^o\text{ A}}$

(b) $\overline{S}_a=(240\angle0^o)(24.34\angle12.64^o)=5841.6\angle12.64^o$

$\overline{S}_b=(240\angle-120^o)(10.81\angle142.6^o)=2594.4\angle22.6^o$

$\overline{S}_b=(240\angle120^o)(19.27\angle-141.9^o)=4624.8\angle-21.9^o$

$\overline{S}=\overline{S}_a+\overline{S}_b+\overline{S}_c=12.386+j0.55\text{ kVA}=\underline{12.4\angle2.54^o\text{ kVA}}$

Chapter 12, Problem 49.

Each phase load consists of a 20-Ω resistor and a 10-Ω inductive reactance. With a line voltage of 220 V rms, calculate the average power taken by the load if:

(a) the three-phase loads are delta-connected
(b) the loads are wye-connected

Chapter 12, Solution 49.

(a) For the delta-connected load, $Z_p = 20 + j10\Omega, \quad V_p = V_L = 220 \text{ (rms)},$

$$S = \frac{3V^2{}_p}{Z^*{}_p} = \frac{3x220^2}{(20 - j10)} = 5808 + j2904 = \underline{6.943\angle 26.56^o \text{ kVA}}$$

or **<u>5.808kW</u>**

(b) For the wye-connected load, $Z_p = 20 + j10\Omega, \quad V_p = V_L / \sqrt{3},$

$$S = \frac{3V^2{}_p}{Z^*{}_p} = \frac{3x220^2}{3(20 - j10)} = \underline{2.164\angle 26.56^o \text{ kVA}} \text{ or } \mathbf{\underline{1.9356 \text{ kW}}}$$

Chapter 12, Problem 50.

A balanced three-phase source with $\mathbf{V}_L$ = 240 V rms is supplying 8 kVA at 0.6 power factor lagging to two wye-connected parallel loads. If one load draws 3 kW at unity power factor, calculate the impedance per phase of the second load.

Chapter 12, Solution 50.

$$\overline{S} = \overline{S}_1 + \overline{S}_2 = 8(0.6 + j0.8) = 4.8 + j6.4 \text{ kVA}, \qquad \overline{S}_1 = 3 \text{ kVA}$$

Hence,

$$\overline{S}_2 = \overline{S} - \overline{S}_1 = 1.8 + j6.4 \text{ kVA}$$

But $\overline{S}_2 = \dfrac{3V^2{}_p}{Z^*{}_p}, \quad V_p = \dfrac{V_L}{\sqrt{3}} \longrightarrow \overline{S}_2 = \dfrac{.V^2{}_L}{Z^*{}_p}$

$$Z^*{}_p = \frac{V^*{}_L}{\overline{S}_2} = \frac{240^2}{(1.8 + j6.4)x10^3} \longrightarrow \underline{Z_p = 2.346 + j8.34\Omega}$$

Chapter 12, Problem 51.

Consider the Δ-Δ system shown in Fig. 12.60. Take $\mathbf{Z}_1 = 8 + j6\,\Omega$, $\mathbf{Z}_2 = 4.2 - j2.2\,\Omega$, $\mathbf{Z}_3 = 10 + j0\,\Omega$.

(a) Find the phase current $\mathbf{I}_{AB}$, $\mathbf{I}_{BC}$, $\mathbf{I}_{CA}$.
(b) Calculate line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

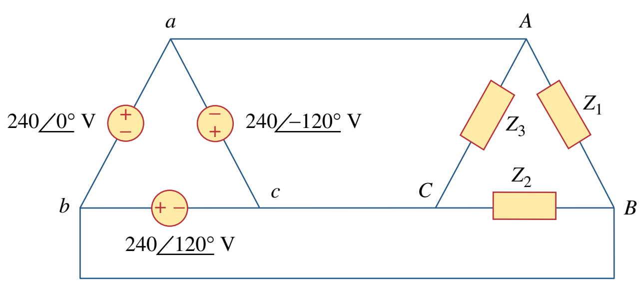


Figure 12.60
For Prob. 12.51.

Chapter 12, Solution 51.

This is an unbalanced system.

$$I_{AB} = \frac{240 < 0^o}{Z_1} = \frac{240 < 0^o}{8 + j6} = \underline{19.2\text{-}j14.4 \ \text{A}} = \underline{\mathbf{19.2–j14.4\ A}}$$

$$\mathrm{I}_{\mathrm{BC}} = \frac{240\angle 120^\circ}{\mathrm{Z}_2} = \frac{240\angle 120^\circ}{4.7413\angle -27.65} = 50.62\angle 147.65^\circ = \underline{\mathbf{–42.76+j27.09\ A}}$$

$$\mathrm{I}_{\mathrm{CA}} = \frac{240\angle -120^\circ}{\mathrm{Z}_3} = \frac{240\angle -120^\circ}{10} = \underline{\mathbf{–12–j20.78\ A}}$$

At node A,

$$I_{aA} = I_{AB} - I_{CA} = (19.2 - j14.4) - (-12 - j20.78) = \underline{31.2 + j6.38 \ \text{A}} = \underline{\mathbf{31.2+j6.38\ A}}$$

$$\mathrm{I}_{bB} = I_{BC} - I_{AB} = (-42.76 + j27.08) - (19.2 - j14.4) = \underline{-61.96 + j41.48 \ \text{A}}$$
$$= \underline{\mathbf{–61.96+j41.48\ A}}$$

$$\mathrm{I}_{cC} = I_{CA} - I_{BC} = (-12 - j20.78) - (-42.76 + j27.08) = \underline{30.76 - j47.86 \ \text{A}}$$
$$= \underline{\mathbf{30.76–j47.86\ A}}$$

Chapter 12, Problem 52.

A four-wire wye-wye circuit has

$\mathbf{V}_{an} = 120\angle 120°$, $\mathbf{V}_{bn} = 120\ \angle 0°$

$\mathbf{V}_{cn} = 120\ \angle -120°$ V

If the impedances are

$\mathbf{Z}_{AN} = 20\ \angle 60°$, $\mathbf{Z}_{BN} = 30\ \angle 0°$

$\mathbf{Z}_{cn} = 40\ \angle 30° \Omega$

find the current in the neutral line.

Chapter 12, Solution 52.

Since the neutral line is present, we can solve this problem on a per-phase basis.

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_{AN}} = \frac{120\angle 120°}{20\angle 60°} = 6\angle 60°$$

$$\mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_{BN}} = \frac{120\angle 0°}{30\angle 0°} = 4\angle 0°$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_{CN}} = \frac{120\angle -120°}{40\angle 30°} = 3\angle -150°$$

Thus,

$$-\mathbf{I}_n = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c$$

$$-\mathbf{I}_n = 6\angle 60° + 4\angle 0° + 3\angle -150°$$

$$-\mathbf{I}_n = (3 + j5.196) + (4) + (-2.598 - j1.5)$$

$$-\mathbf{I}_n = 4.405 + j3.696 = 5.75\angle 40°$$

$$\mathbf{I}_n = \underline{\mathbf{5.75\angle 220°\ A}}$$

Chapter 12, Problem 53.

In the Y-Y system shown in Fig. 12.61, loads connected to the source are unbalanced. (a) Calculate $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$. (b) Find the total power delivered to the load. Take $\mathbf{V}_p = 240$ V rms.

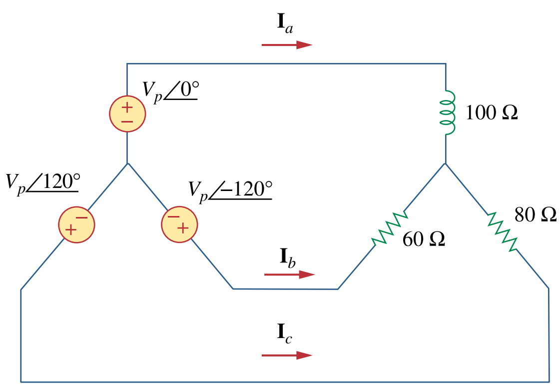
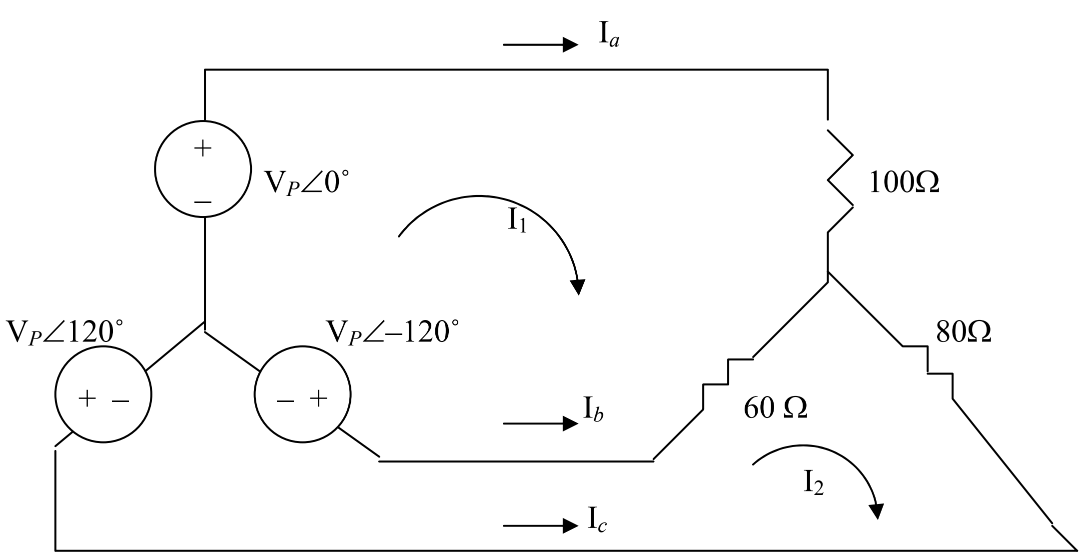


Figure 12.61
For Prob. 12.53.

Chapter 12, Solution 53.

Applying mesh analysis as shown below, we get.



$$240 < -120^o - 240 < 0^o + I_1 x160 - 60I_2 = 0 \quad \longrightarrow \quad 160I_1 - 60I_2 = 360 + j207.84 \quad (1)$$

$$240 < 120^o - 240 < -120^o + 140I_2 - 60I_1 = 0 \quad \longrightarrow \quad 140I_2 - 60I_1 = -j415.7 \quad (2)$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} 160 & -60 \\ -60 & 140 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 360 + j207.84 \\ -j415.7 \end{bmatrix}$$

Using MATLAB, we get,

```
>> Z=[160,-60;-60,140]
Z =
   160   -60
   -60   140
>> V=[(360+207.8i);-415.7i]
V =
  1.0e+002 *
   3.6000 + 2.0780i
        0 - 4.1570i
>> I=inv(Z)*V
I =
   2.6809 + 0.2207i
   1.1489 - 2.8747i
```

I_1 = 2.681+j0.2207 and I_2 = 1.1489–j2.875

$I_a = I_1 =$ **$2.69\angle 4.71^\circ$ A**

$I_b = I_2 - I_1 = -1.5321 - j3.096 =$ **$3.454\angle -116.33^\circ$ A**

$I_c = -I_2 =$ **$3.096\angle 111.78^\circ$ A**

$$S_a = |I_a|^2 Z_a = (2.69)^2 x100 = 723.61 \text{ W}$$
$$S_b = |I_b|^2 Z_b = (3.454)^2 x60 = 715.81 \text{W}$$
$$S_c = |I_c|^2 Z_c = (3.0957)^2 x80 = 766.67 \text{ W}$$

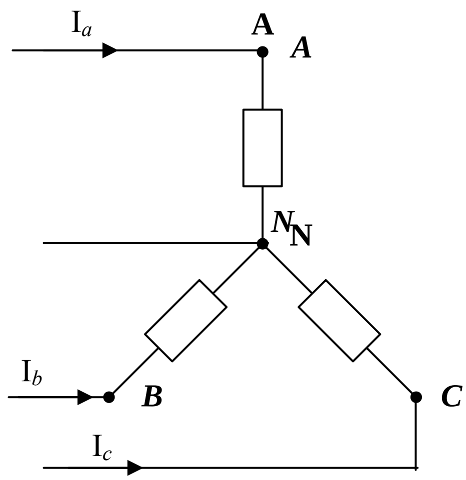
$P = P_a + P_b + P_c =$ **2.205 kW**

Chapter 12, Problem 54.

A balanced three-phase Y-source with $\mathbf{V}_P = 210$ V rms drives a Y-connected three-phase load with phase impedance $\mathbf{Z}_A = 80\,\Omega$, $\mathbf{Z}_B = 60 + j90\,\Omega$, and $\mathbf{Z}_C = j80\,\Omega$. Calculate the line currents and total complex power delivered to the load. Assume that the neutrals are connected.

Chapter 12, Solution 54.

Consider the load as shown below.



$$I_a = \frac{210 < 0^o}{80} = \underline{2.625\text{ A}}$$

$$\mathrm{I_b} = \frac{210\angle 0°}{60 + j90} = \frac{210}{108.17\angle 56.31°} = \underline{\mathbf{1.9414\angle{-56.31}^\circ\ A}}$$

$$I_c = \frac{210 < 0^o}{j80} = \underline{2.625 < -90^o\text{ A}}$$

$$S_a = VI_a^* = 210x2.625 = 551.25$$

$$S_b = VI_b^* = \frac{|V|^2}{Z_b^*} = \frac{210^2}{60 - j90} = 226.15 + j339.2$$

$$S_c = \frac{|V|^2}{Z_c^*} = \frac{210^2}{-j80} = j551.25$$

$$S = S_a + S_b + S_c = \underline{777.4 + j890.45\text{ VA}}$$

Chapter 12, Problem 55.

A three-phase supply, with the line voltage 240 V rms positively phased, has an unbalanced delta-connected load as shown in Fig. 12.62. Find the phase currents and the total complex power.

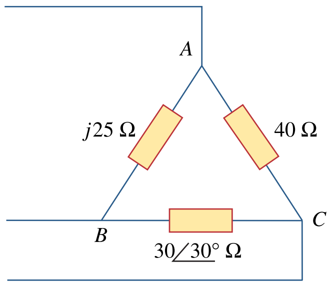


Figure 12.62
For Prob. 12.55.

Chapter 12, Solution 55.

The phase currents are:

$$I_{AB} = 240/j25 = \underline{\mathbf{9.6\angle{-90^\circ}\ A}}$$

$$I_{CA} = 240\angle 120^\circ/40 = \underline{\mathbf{6\angle 120^\circ\ A}}$$

$$I_{BC} = 240\angle{-120^\circ}/30\angle 30^\circ = \underline{\mathbf{8\angle{-150^\circ}\ A}}$$

The complex power in each phase is:

$$S_{AB} = |I_{AB}|^2 Z_{AB} = (9.6)^2 j25 = j2304$$

$$S_{AC} = |I_{AC}|^2 Z_{AC} = (6)^2 40 < 0^o = 1440$$

$$S_{BC} = |I_{BC}|^2 Z_{BC} = (8)^2 30 < 30^o = 1662.77 + j960$$

The total complex power is,

$$S = S_{AB} + S_{AC} + S_{BC} = \underline{3102.77 + j3264 \text{ VA}} = \underline{\mathbf{3.103+j3.264\ kVA}}$$

Chapter 12, Problem 56.

Refer to the unbalanced circuit of Fig. 12.63. Calculate:

(a) the line currents
(b) the real power absorbed by the load
(c) the total complex power supplied by the source

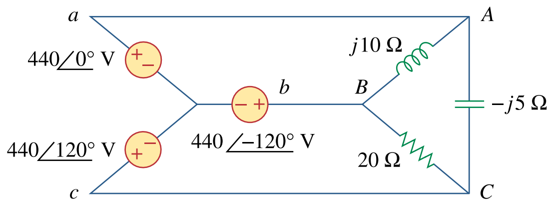
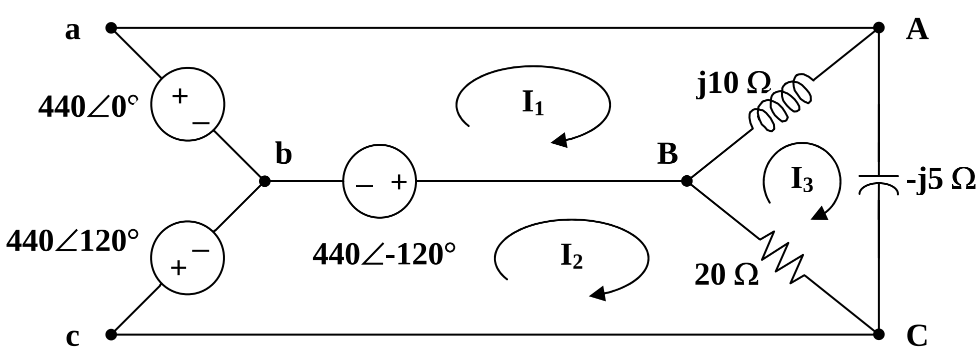


Figure 12.63
For Prob. 12.56.

Chapter 12, Solution 56.

(a) Consider the circuit below.



For mesh 1,

$$440\angle\text{-}120° - 440\angle 0° + j10(\mathbf{I}_1 - \mathbf{I}_3) = 0$$

$$\mathbf{I}_1 - \mathbf{I}_3 = \frac{(440)(1.5 + j0.866)}{j10} = 76.21\angle\text{-}60° \qquad (1)$$

For mesh 2,

$$440\angle 120° - 440\angle\text{-}120° + 20(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$\mathbf{I}_3 - \mathbf{I}_2 = \frac{(440)(j1.732)}{20} = j38.1 \qquad (2)$$

For mesh 3,

$$j10(\mathbf{I}_3 - \mathbf{I}_1) + 20(\mathbf{I}_3 - \mathbf{I}_2) - j5\mathbf{I}_3 = 0$$

Substituting (1) and (2) into the equation for mesh 3 gives,

$$\mathbf{I}_3 = \frac{(440)(\text{-}1.5 + j0.866)}{j5} = 152.42\angle 60° \qquad (3)$$

From (1),

$$\mathbf{I}_1 = \mathbf{I}_3 + 76.21\angle\text{-}60° = 114.315 + j66 = 132\angle 30°$$

From (2),

$$\mathbf{I}_2 = \mathbf{I}_3 - j38.1 = 76.21 + j93.9 = 120.93\angle 50.94°$$

$$\mathbf{I}_a = \mathbf{I}_1 = \underline{\mathbf{132\angle 30°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_1 = \text{-}38.105 + j27.9 = \underline{\mathbf{47.23\angle 143.8°\ A}}$$

$$\mathbf{I}_c = \text{-}\mathbf{I}_2 = \underline{\mathbf{120.9\angle 230.9°\ A}}$$

(b)

$$\mathbf{S}_{AB} = \left|\mathbf{I}_1 - \mathbf{I}_3\right|^2(j10) = j58.08\text{ kVA}$$

$$\mathbf{S}_{BC} = \left|\mathbf{I}_2 - \mathbf{I}_3\right|^2(20) = 29.04\text{ kVA}$$

$$\mathbf{S}_{CA} = \left|\mathbf{I}_3\right|^2(\text{-}j5) = (152.42)^2(\text{-}j5) = \text{-}j116.16\text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_{AB} + \mathbf{S}_{BC} + \mathbf{S}_{CA} = 29.04 - j58.08\text{ kVA}$$

Real power absorbed = $\underline{\mathbf{29.04\ kW}}$

(c) Total complex supplied by the source is

$$\mathbf{S} = \underline{\mathbf{29.04 - j58.08\ kVA}}$$

Chapter 12, Problem 57.

Determine the line currents for the three-phase circuit of Fig. 12.64. Let $\mathbf{V}_a = 110\angle 0°$, $\mathbf{V}_b = 110\angle -120°$, $\mathbf{V}_c = 110\angle 120°$ V.

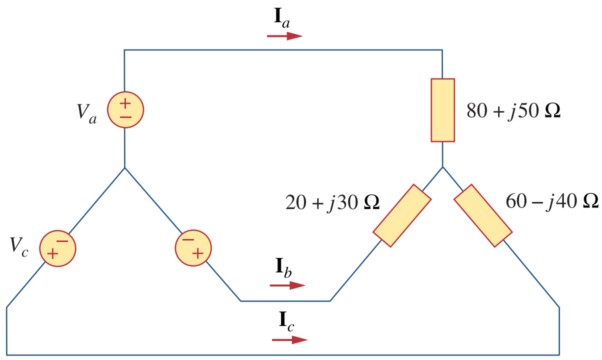
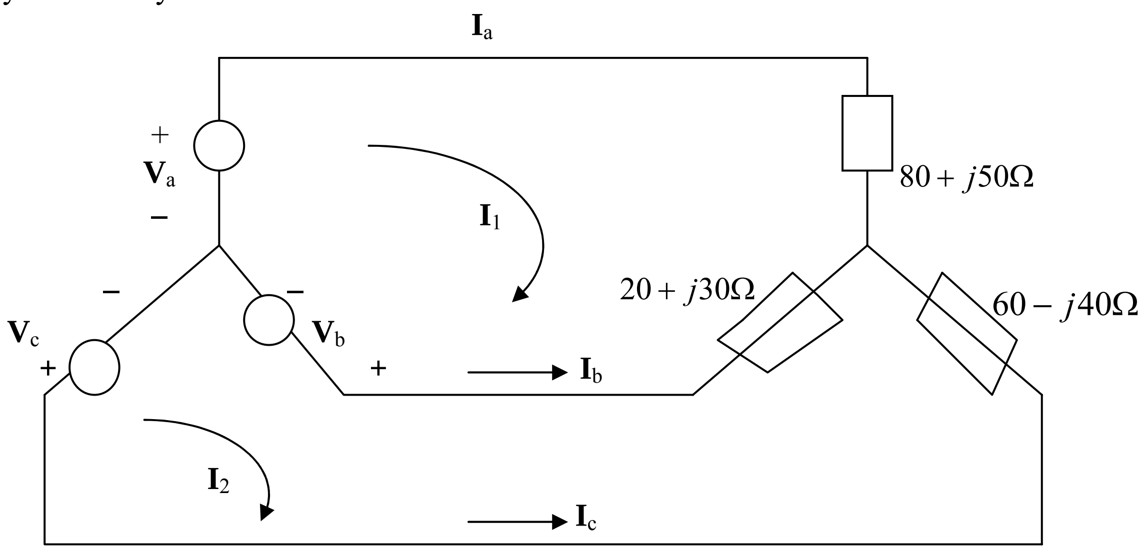


Figure 12.64
For Prob. 12.57.

Chapter 12, Solution 57.
We apply mesh analysis to the circuit shown below.



$$(100 + j80)I_1 - (20 + j30)I_2 = V_a - V_b = 165 + j95.263 \qquad (1)$$
$$-(20 + j30)I_1 + (80 - j10)I_2 = V_b - V_c = -j190.53 \qquad (2)$$

Solving (1) and (2) gives $I_1 = 1.8616 - j0.6084$, $I_2 = 0.9088 - j1.722$.

$I_a = I_1 = \underline{1.9585\angle -18.1^o \text{ A}}$, $I_b = I_2 - I_1 = -0.528 - j1.1136 = \underline{1.4656\angle -130.55^o \text{ A}}$

$I_c = -I_2 = \underline{1.947\angle 117.8^o \text{ A}}$

Chapter 12, Problem 58.

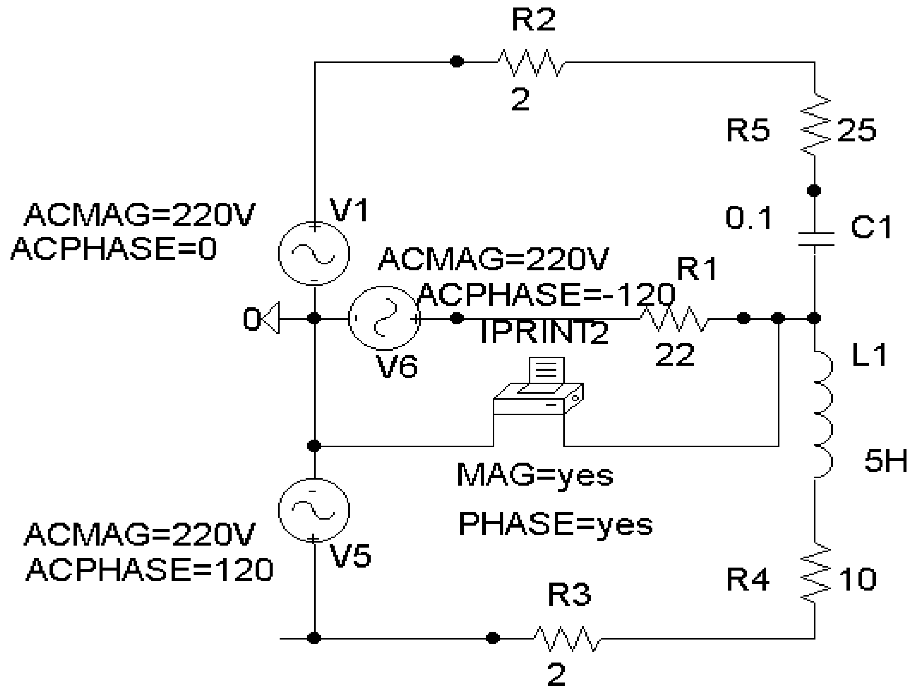
Solve Prob. 12.10 using *PSpice*.

Chapter 12, Solution 58.

The schematic is shown below. IPRINT is inserted in the neutral line to measure the current through the line. In the AC Sweep box, we select Total Ptss = 1, Start Freq. = 0.1592, and End Freq. = 0.1592. After simulation, the output file includes

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
1.592 E–01	1.078 E+01	–8.997 E+01

i.e. I_n = **$10.78\angle{-89.97°}$ A**



Chapter 12, Problem 59.

The source in Fig. 12.65 is balanced and exhibits a positive phase sequence. If f = 60 Hz, use *PSpice* to find $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, and $\mathbf{V}_{CN}$.

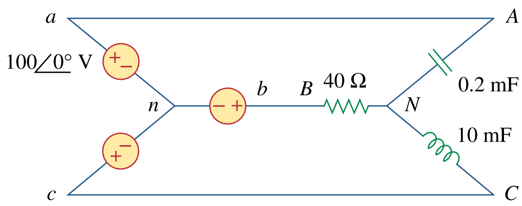


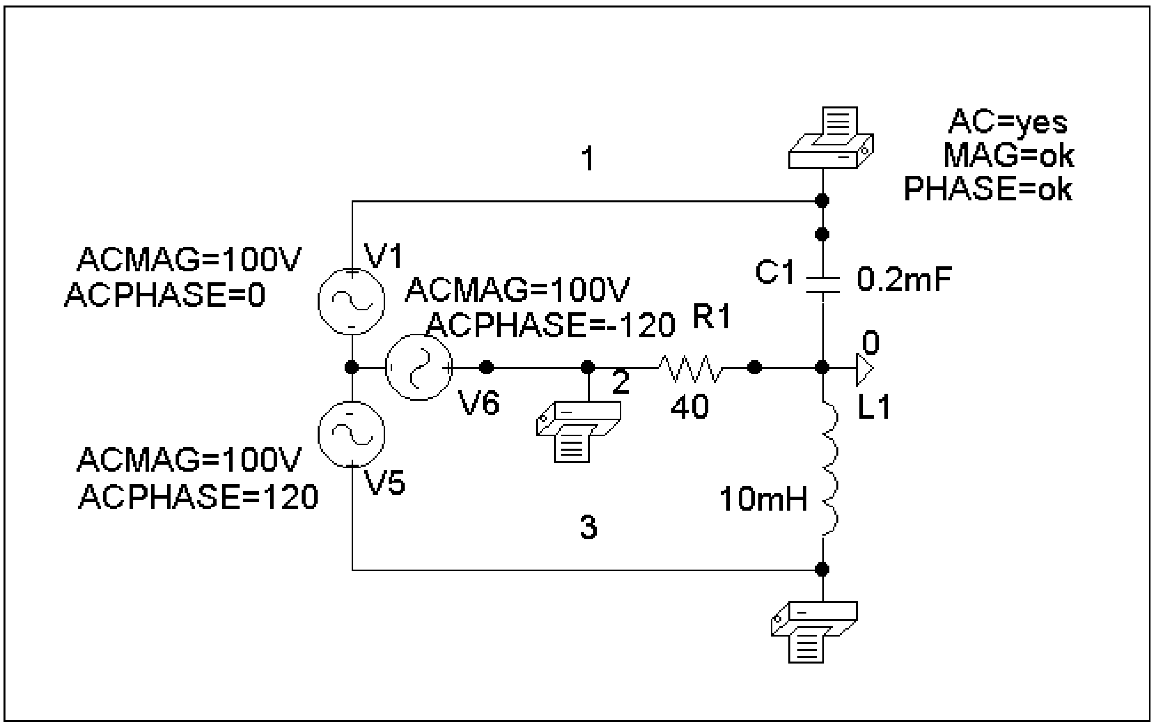
Figure 12.65
For Prob. 12.59.

Chapter 12, Solution 59.

The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 60, and End Freq = 60. After simulation, we obtain an output file which includes

FREQ	VM(1)	VP(1)
6.000 E+01	2.206 E+02	–3.456 E+01
FREQ	VM(2)	VP(2)
6.000 E+01	2.141 E+02	–8.149 E+01
FREQ	VM(3)	VP(3)
6.000 E+01	4.991 E+01	–5.059 E+01

i.e. V_{AN} = $\underline{\mathbf{220.6\angle -34.56°}}$, V_{BN} = $\underline{\mathbf{214.1\angle -81.49°}}$, V_{CN} = $\underline{\mathbf{49.91\angle -50.59°\ V}}$



Chapter 12, Problem 60.

Use *PSpice* to determine $\mathbf{I}_o$ in the single-phase, three-wire circuit of Fig. 12.66. Let $\mathbf{Z}_1 = 15 - j10\,\Omega$, $\mathbf{Z}_2 = 30 + j20\,\Omega$, and $\mathbf{Z}_3 = 12 + j5\,\Omega$.

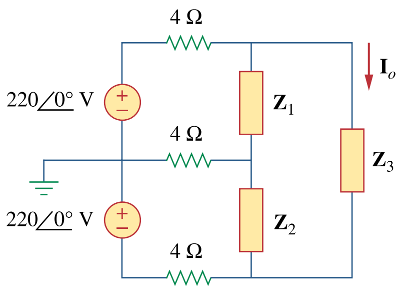


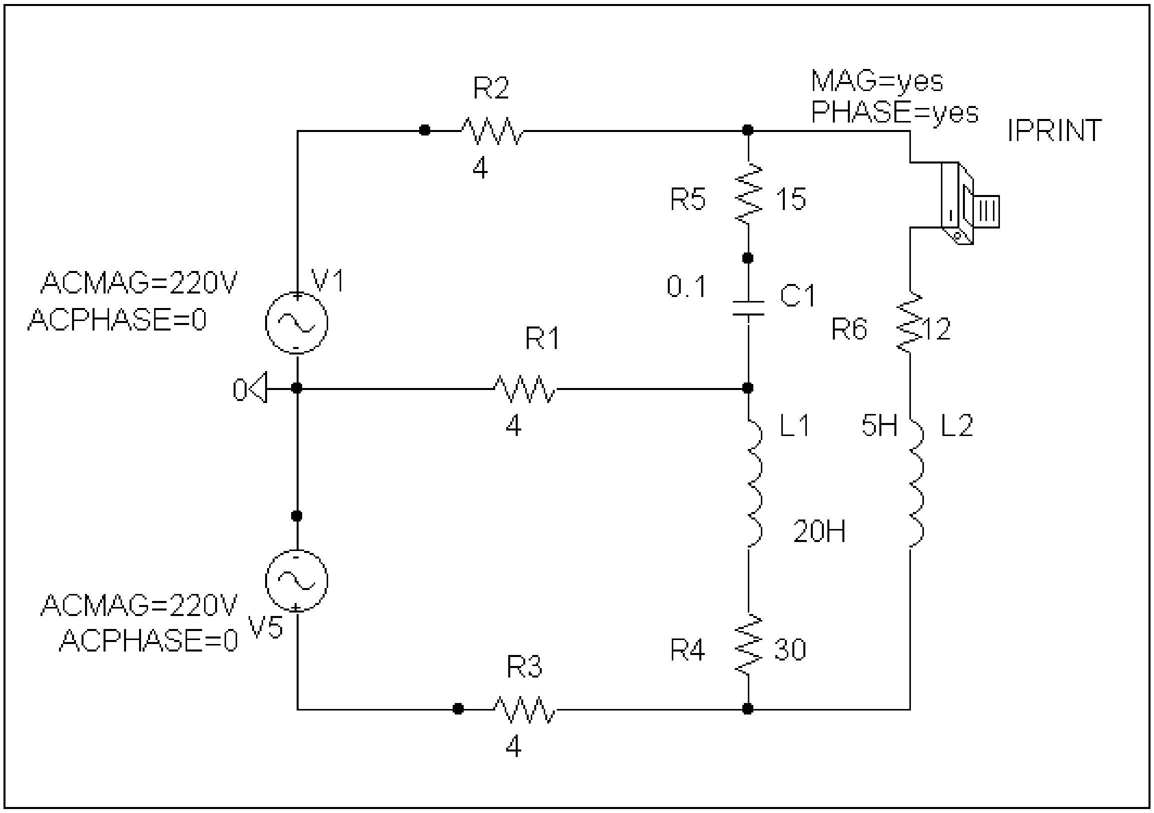
Figure 12.66
For Prob. 12.60.

Chapter 12, Solution 60.

The schematic is shown below. IPRINT is inserted to give I_o. We select Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592 in the AC Sweep box. Upon simulation, the output file includes

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
1.592 E–01	1.421 E+00	–1.355 E+02

from which, I_o = **$1.421\angle{-135.5°}$ A**



Chapter 12, Problem 61.

Given the circuit in Fig. 12.67, use *PSpice* to determine currents $\mathbf{I}_{aA}$ and voltage $\mathbf{V}_{BN}$.

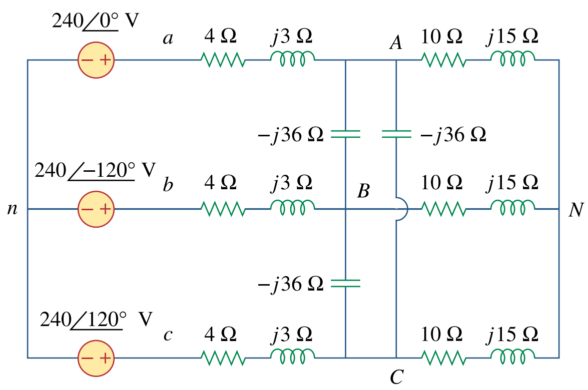


Figure 12.67
For Prob. 12.61.

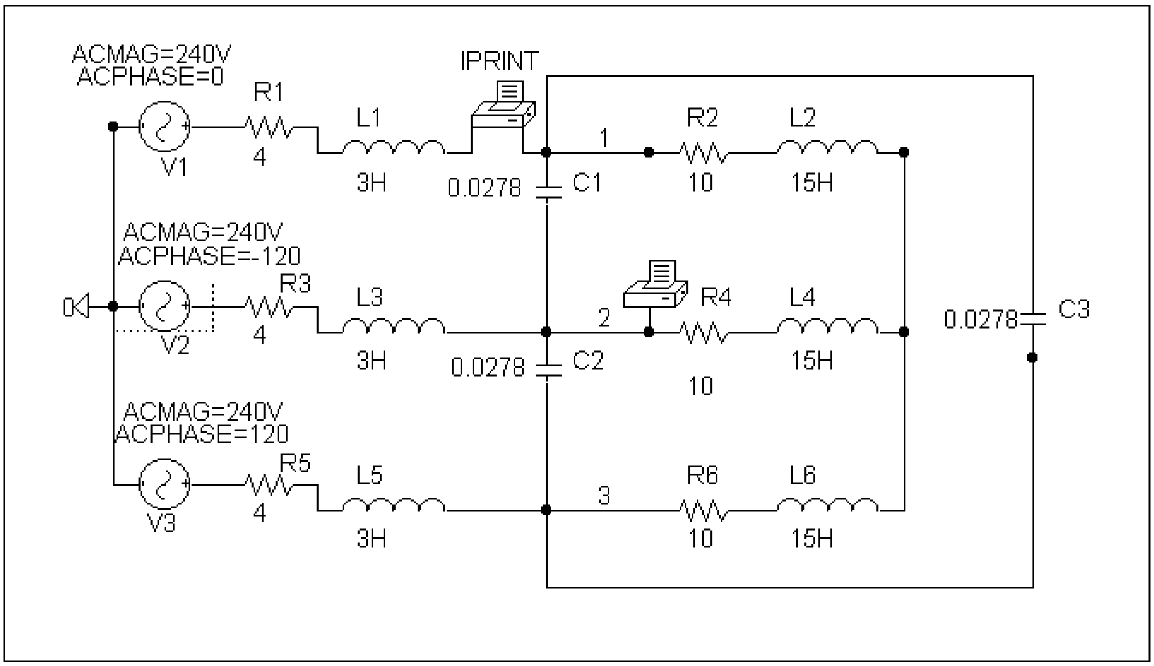
Chapter 12, Solution 61.

The schematic is shown below. Pseudocomponents IPRINT and PRINT are inserted to measure I_{aA} and V_{BN}. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. Once the circuit is simulated, we get an output file which includes

FREQ	VM(2)	VP(2)
1.592 E–01	2.308 E+02	–1.334 E+02
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	1.115 E+01	3.699 E+01

from which

$$I_{aA} = \underline{\mathbf{11.15\angle 37^\circ\ A}}, \quad V_{BN} = \underline{\mathbf{230.8\angle -133.4^\circ\ V}}$$



Chapter 12, Problem 62.

The circuit in Fig. 12.68 operates at 60 Hz. Use *PSpice* to find the source current $\mathbf{I}_{ab}$ and the line current $\mathbf{I}_{bB}$.

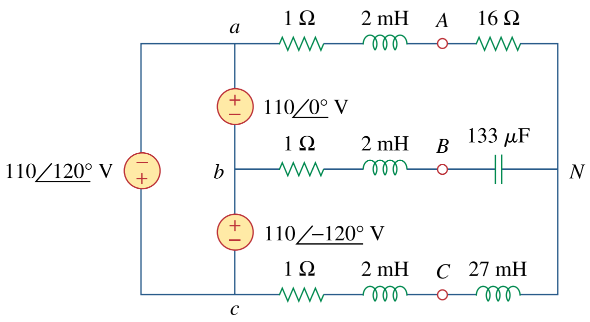


Figure 12.68
For Prob. 12.62.

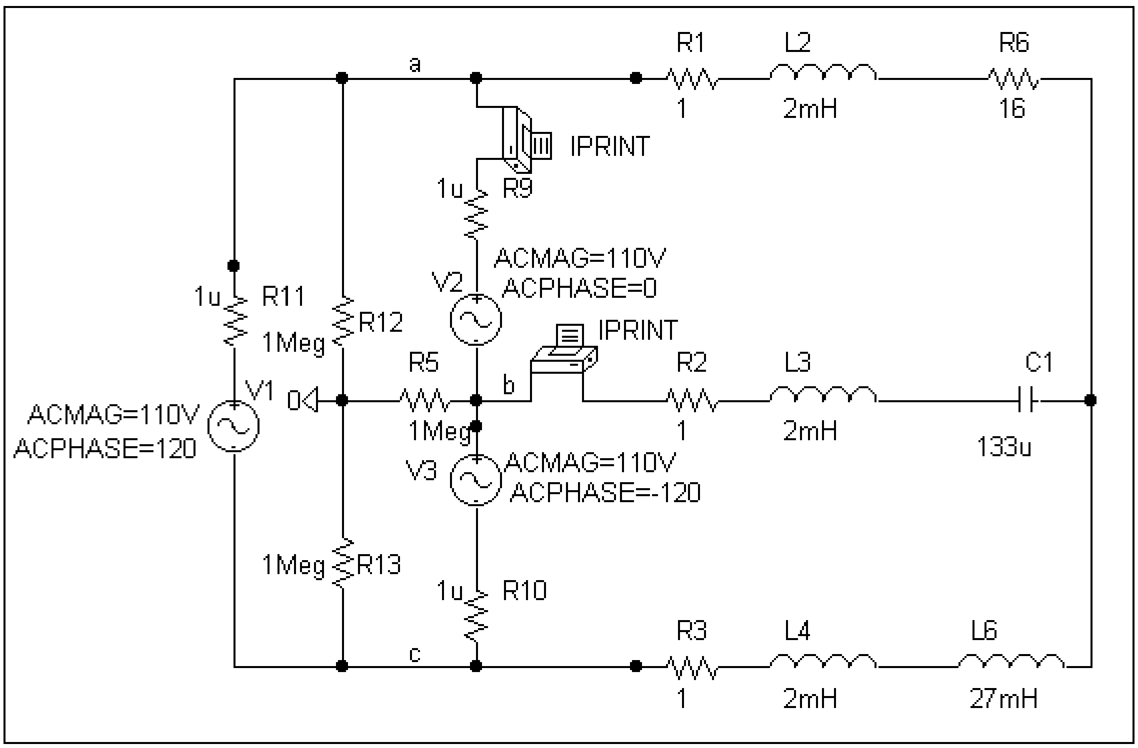
Chapter 12, Solution 62.

Because of the delta-connected source involved, we follow Example 12.12. In the AC Sweep box, we type Total Pts = 1, Start Freq = 60, and End Freq = 60. After simulation, the output file includes

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
6.000 E+01	5.960 E+00	–9.141 E+01
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
6.000 E+01	7.333 E+07	1.200 E+02

From which

$$I_{ab} = \underline{\mathbf{7.333x10^7\angle 120° \ A}}, \quad I_{bB} = \underline{\mathbf{5.96\angle -91.41° \ A}}$$



Chapter 12, Problem 63.

Use *PSpice* to find currents $\mathbf{I}_{aA}$ and $\mathbf{I}_{AC}$ in the unbalanced three-phase system shown in Fig. 12.69. Let

$\mathbf{Z}_l = 2 + j, \quad \mathbf{Z}_1 = 40 + j20\,\Omega,$

$\mathbf{Z}_2 = 50 - j30\,\Omega, \ \mathbf{Z}_3 = 25\,\Omega$

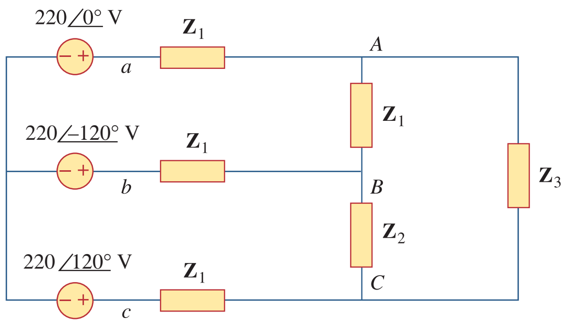
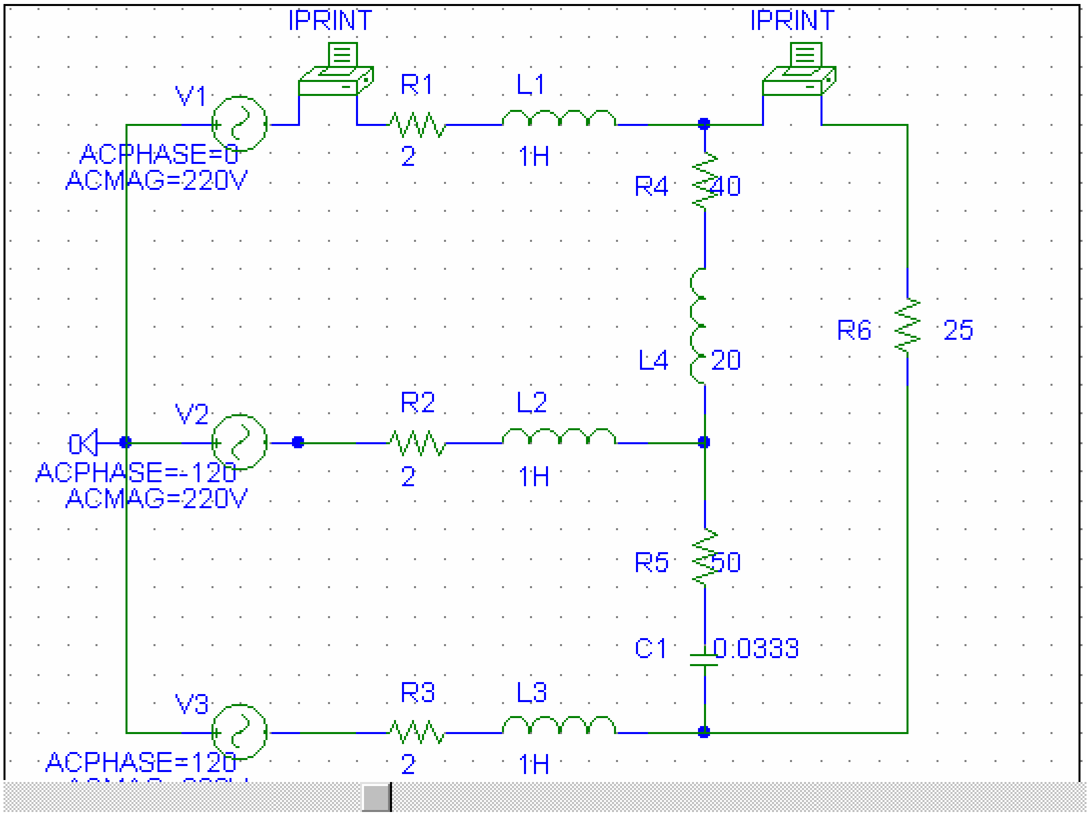


Figure 12.69
For Prob. 12.63.

Chapter 12, Solution 63.

Let $\omega = 1$ so that $L = X/\omega = 20\text{ H}$, and $C = \dfrac{1}{\omega X} = 0.0333\text{ F}$

The schematic is shown below.



When the file is saved and run, we obtain an output file which includes the following:

```
FREQ          IM(V_PRINT1)IP(V_PRINT1)

 1.592E-01    1.867E+01    1.589E+02

FREQ          IM(V_PRINT2)IP(V_PRINT2)

 1.592E-01    1.238E+01    1.441E+02
```

From the output file, the required currents are:

$$\underline{I_{aA} = 18.67\angle 158.9^o \text{ A}, \quad I_{AC} = 12.38\angle 144.1^o \text{ A}}$$

Chapter 12, Problem 64.

For the circuit in Fig. 12.58, use *PSpice* to find the line currents and the phase currents.

Chapter 12, Solution 64.

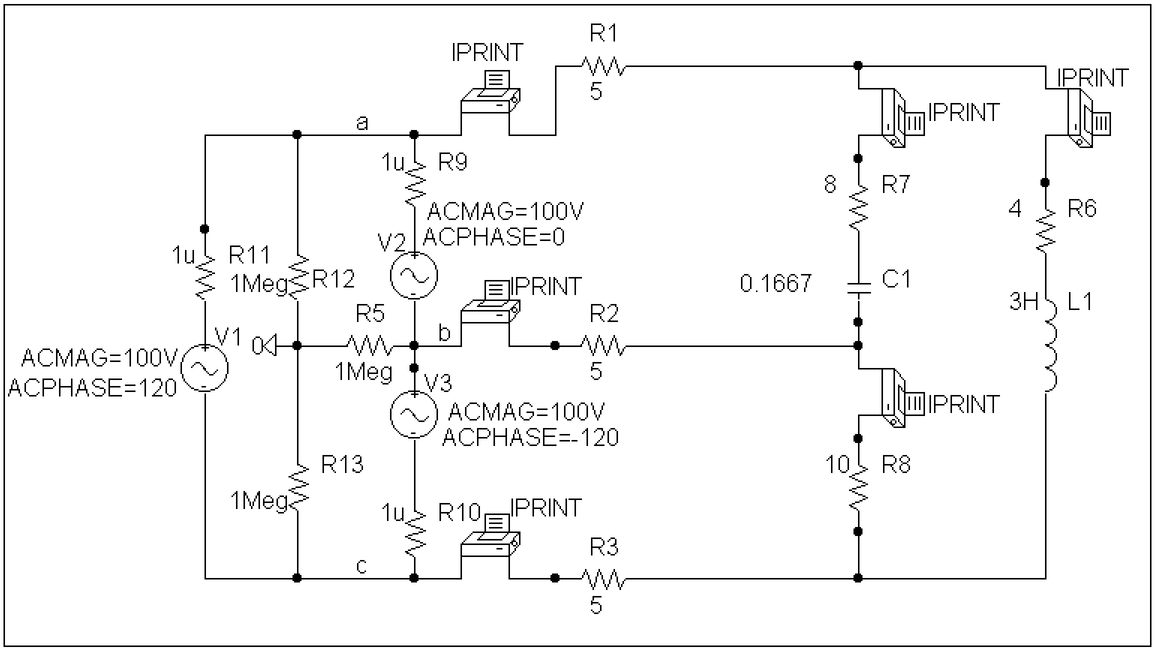
We follow Example 12.12. In the AC Sweep box we type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation the output file includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	4.710 E+00	7.138 E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	6.781 E+07	–1.426 E+02
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	3.898 E+00	–5.076 E+00
FREQ	IM(V_PRINT4)	IP(V_PRINT4)
1.592 E–01	3.547 E+00	6.157 E+01
FREQ	IM(V_PRINT5)	IP(V_PRINT5)
1.592 E–01	1.357 E+00	9.781 E+01
FREQ	IM(V_PRINT6)	IP(V_PRINT6)
1.592 E–01	3.831 E+00	–1.649 E+02

from this we obtain

$$I_{aA} = \underline{\mathbf{4.71\angle 71.38^\circ\ A}},\quad I_{bB} = \underline{\mathbf{6.781\angle -142.6^\circ\ A}},\quad I_{cC} = \underline{\mathbf{3.898\angle -5.08^\circ\ A}}$$

$$I_{AB} = \underline{\mathbf{3.547\angle 61.57^\circ\ A}},\ I_{AC} = \underline{\mathbf{1.357\angle 97.81^\circ\ A}},\quad I_{BC} = \underline{\mathbf{3.831\angle -164.9^\circ\ A}}$$



IPRINT
R1
5
a
1u
R9
ACMAG=100V
ACPHASE=0
V2
1u
R11
1Meg
R12
R5
b
0
V1
ACMAG=100V
ACPHASE=120
1Meg
V3
ACMAG=100V
ACPHASE=-120
R13
1Meg
1u
R10
IPRINT
R3
5
c
IPRINT
R2
5
IPRINT
8
R7
0.1667
C1
IPRINT
10
R8
IPRINT
4
R6
3H
L1

Chapter 12, Problem 65.

A balanced three-phase circuit is shown in Fig. 12.70 on the next page. Use *PSpice* to find the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

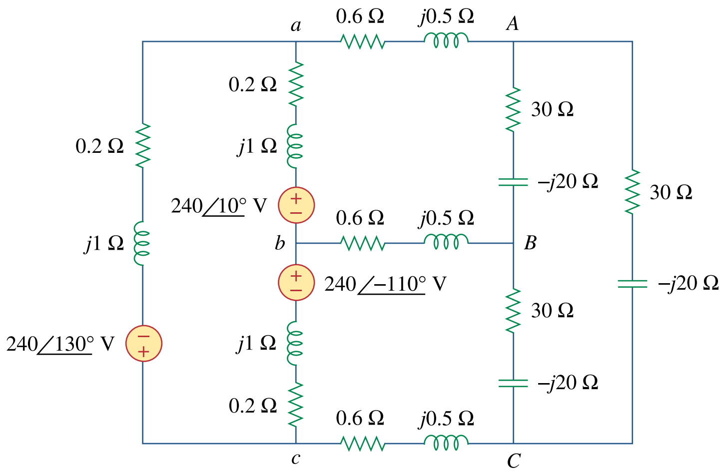


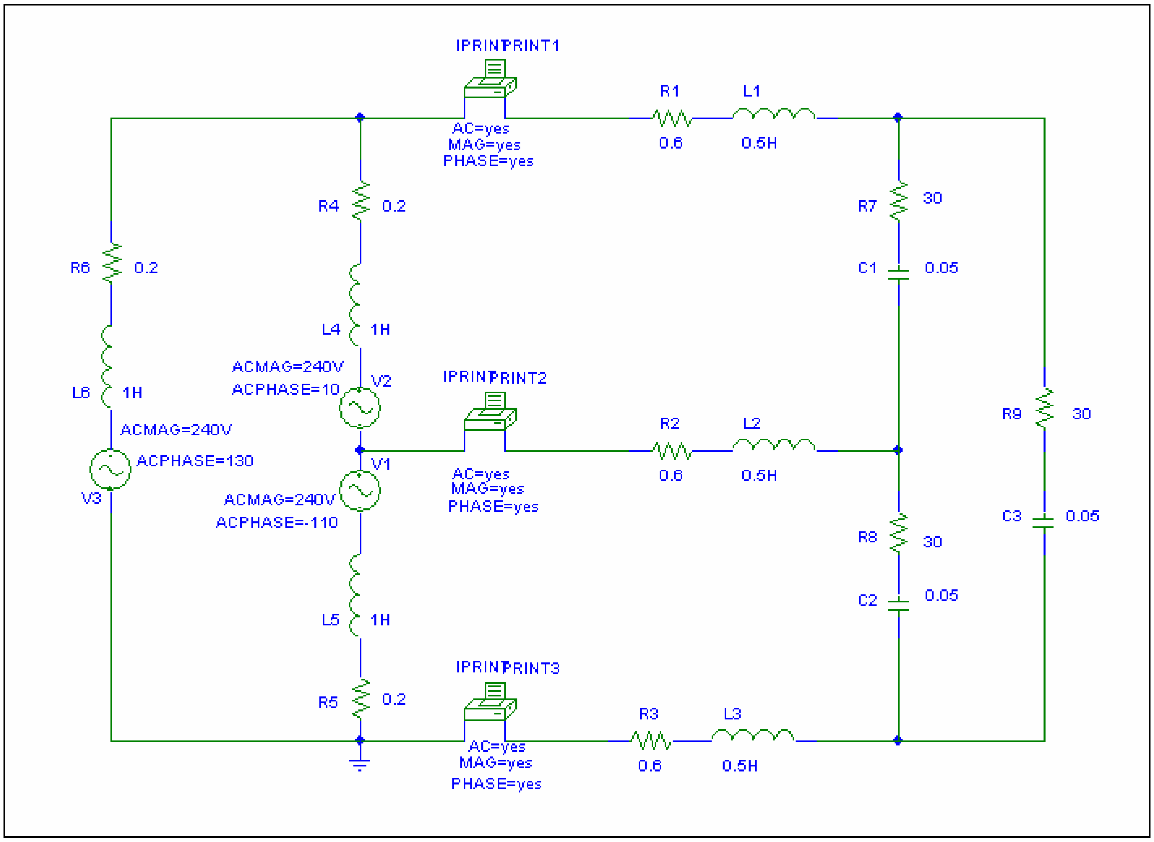
Figure 12.70
For Prob. 12.65.

Chapter 12, Solution 65.

Due to the delta-connected source, we follow Example 12.12. We type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. The schematic is shown below. After it is saved and simulated, we obtain an output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592E-01	1.140E+01	8.664E+00
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592E-01	1.140E+01	-1.113E+02
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592E-01	1.140E+01	1.287E+02

Thus, $I_{aA} = \underline{\mathbf{11.02\angle 12^\circ\ A}}$, $I_{bB} = \underline{\mathbf{11.02\angle -108^\circ\ A}}$, $I_{cC} = \underline{\mathbf{11.02\angle 132^\circ\ A}}$



Since this is a balanced circuit, we can perform a quick check. The load resistance is large compared to the line and source impedances so we will ignore them (although it would not be difficult to include them).

Converting the sources to a Y configuration we get:

$V_{an} = 138.56 \angle{-20^\circ}$ Vrms

and

$Z_Y = 10 - j6.667 = 12.019\angle{-33.69^\circ}$

Now we can calculate,

$I_{aA} = (138.56 \angle{-20^\circ})/(12.019\angle{-33.69^\circ}) = 11.528\angle 13.69^\circ$

Clearly, we have a good approximation which is very close to what we really have.

Chapter 12, Problem 66.

A three-phase, four-wire system operating with a 208-V line voltage is shown in Fig. 12.71. The source voltages are balanced. The power absorbed by the resistive wye-connected load is measured by the three-wattmeter method. Calculate:

(a) the voltage to neutral
(b) the currents $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, and $\mathbf{I}_n$
(c) the readings of the wattmeters
(d) the total power absorbed by the load

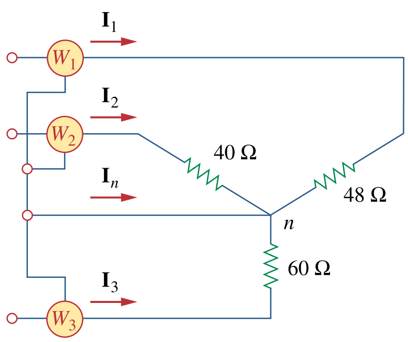


Figure 12.71
For Prob. 12.66.

Chapter 12, Solution 66.

(a) $V_p = \frac{V_L}{\sqrt{3}} = \frac{208}{\sqrt{3}} = \underline{\mathbf{120\ V}}$

(b) Because the load is unbalanced, we have an unbalanced three-phase system. Assuming an abc sequence,

$$\mathbf{I}_1 = \frac{120\angle 0°}{48} = 2.5\angle 0°\ \text{A}$$

$$\mathbf{I}_2 = \frac{120\angle -120°}{40} = 3\angle -120°\ \text{A}$$

$$\mathbf{I}_3 = \frac{120\angle 120°}{60} = 2\angle 120°\ \text{A}$$

$$-\mathbf{I}_N = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 2.5 + (3)\left(-0.5 - j\frac{\sqrt{3}}{2}\right) + (2)\left(-0.5 + j\frac{\sqrt{3}}{2}\right)$$

$$\mathbf{I}_N = j\frac{\sqrt{3}}{2} = j0.866 = 0.866\angle 90°\ \text{A}$$

Hence,

$I_1 = \underline{\mathbf{2.5\ A}}$, $I_2 = \underline{\mathbf{3\ A}}$, $I_3 = \underline{\mathbf{2\ A}}$, $I_N = \underline{\mathbf{0.866\ A}}$

(c) $P_1 = I_1^2 R_1 = (2.5)^2(48) = \underline{\mathbf{300\ W}}$

$P_2 = I_2^2 R_2 = (3)^2(40) = \underline{\mathbf{360\ W}}$

$P_3 = I_3^2 R_3 = (2)^2(60) = \underline{\mathbf{240\ W}}$

(d) $P_T = P_1 + P_2 + P_3 = \underline{\mathbf{900\ W}}$

Chapter 12, Problem 67.

eⒹd * As shown in Fig. 12.72, a three-phase four-wire line with a phase voltage of 120 V rms and positive phase sequence supplies a balanced motor load at 260 kVA at 0.85 pf lagging. The motor load is connected to the three main lines marked *a*, *b*, and *c*. In addition, incandescent lamps (unity pf) are connected as follows: 24 kW from line *c* to the neutral, 15 kW from line *b* to the neutral, and 9 kW from line *c* to the neutral.

(a) If three wattmeters are arranged to measure the power in each line, calculate the reading of each meter.
(b) Find the magnitude of the current in the neutral line.

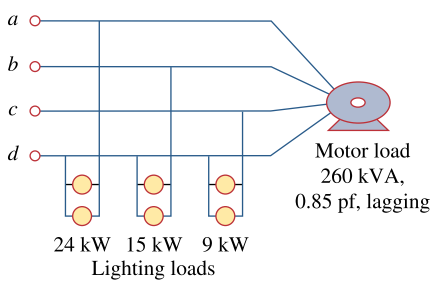


Figure 12.72
For Prob. 12.67.

* An asterisk indicates a challenging problem.

Chapter 12, Solution 67.

(a) The power to the motor is

$$P_T = S\cos\theta = (260)(0.85) = 221 \text{ kW}$$

The motor power per phase is

$$P_p = \frac{1}{3}P_T = 73.67 \text{ kW}$$

Hence, the wattmeter readings are as follows:

$$W_a = 73.67 + 24 = \underline{\mathbf{97.67\ kW}}$$

$$W_b = 73.67 + 15 = \underline{\mathbf{88.67\ kW}}$$

$$W_c = 73.67 + 9 = \underline{\mathbf{82.67\ kW}}$$

(b) The motor load is balanced so that $I_N = 0$.

For the lighting loads,

$$I_a = \frac{24{,}000}{120} = 200 \text{ A}$$

$$I_b = \frac{15{,}000}{120} = 125 \text{ A}$$

$$I_c = \frac{9{,}000}{120} = 75 \text{ A}$$

If we let

$$\mathbf{I}_a = I_a\angle 0° = 200\angle 0° \text{ A}$$

$$\mathbf{I}_b = 125\angle -120° \text{ A}$$

$$\mathbf{I}_c = 75\angle 120° \text{ A}$$

Then,

$$-\mathbf{I}_N = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c$$

$$-\mathbf{I}_N = 200 + (125)\left(-0.5 - j\frac{\sqrt{3}}{2}\right) + (75)\left(-0.5 + j\frac{\sqrt{3}}{2}\right)$$

$$-\mathbf{I}_N = 100 - j43.3 \text{ A}$$

$$\left|\mathbf{I}_N\right| = \underline{\mathbf{108.97\ A}}$$

Chapter 12, Problem 68.

Meter readings for a three-phase wye-connected alternator supplying power to a motor indicate that the line voltages are 330 V, the line currents are 8.4 A, and the total line power is 4.5 kW. Find:

(a) the load in VA
(b) the load pf
(c) the phase current
(d) the phase voltage

Chapter 12, Solution 68.

(a) $S = \sqrt{3}\, V_L I_L = \sqrt{3}(330)(8.4) = \underline{\mathbf{4801\ VA}}$

(b) $P = S\cos\theta \longrightarrow \text{pf} = \cos\theta = \dfrac{P}{S}$

$$\text{pf} = \frac{4500}{4801.24} = \underline{\mathbf{0.9372}}$$

(c) For a wye-connected load,

$$I_p = I_L = \underline{\mathbf{8.4\ A}}$$

(d) $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{330}{\sqrt{3}} = \underline{\mathbf{190.53\ V}}$

Chapter 12, Problem 69.

A certain store contains three balanced three-phase loads. The three loads are:

Load 1: 16 kVA at 0.85 pf lagging

Load 2: 12 kVA at 0.6 pf lagging

Load 3: 8 kW at unity pf

The line voltage at the load is 208 V rms at 60 Hz, and the line impedance is $0.4 + j0.8\,\Omega$. Determine the line current and the complex power delivered to the loads.

Chapter 12, Solution 69.

For load 1,

$$\overline{S}_1 = S_1 \cos\theta_1 + jS_1 \sin\theta_1$$

$$pf = 0.85 = \cos\theta_1 \quad \longrightarrow \quad \theta_1 = 31.79^o$$

$$\overline{S}_1 = 13.6 + j8.43 \quad \text{kVA}$$

For load 2,

$$\overline{S_2} = 12x0.6 + j12x0.8 = 7.2 + j9.6 \ \text{ kVA}$$

For load 3,

$$\overline{S}_3 = 8 + j0 \quad \text{kVA}$$

$\overline{S} = \overline{S}_1 + \overline{S_2} + \overline{S_3} = 28.8 + j18.03 \ = $ **<u>28.8+j18.03 kVA</u>**

But $\mathbf{S_P = V_P I_P^*}$ with $I_P = I_L$

$$I_L^* = \frac{S_P}{V_P} = \frac{(28800 + j18030)}{3x120.08}$$

$I_L = 79.95 - j50.05 \ =$ **<u>94.32∠–32.05˚ A</u>**. Note, this is relative to 120.08∠0˚ V. If we assume a positive phase rotation and $V_{ab} = 208\angle 0^\circ$, then $V_{an} = 120.08\angle{-30^\circ}$ which yields I_a = **<u>94.32∠–62.05˚ A</u>**, I_b = **<u>94.32∠177.95˚ A</u>**, I_c = **<u>94.32∠57.95˚ A</u>**.

Chapter 12, Problem 70.

The two-wattmeter method gives $P_1 = 1200$ W and $P_2 = –400$ W for a three-phase motor running on a 240-V line. Assume that the motor load is wye-connected and that it draws a line current of 6 A. Calculate the pf of the motor and its phase impedance.

Chapter 12, Solution 70.

$$P_T = P_1 + P_2 = 1200 - 400 = 800$$

$$Q_T = P_2 - P_1 = -400 - 1200 = -1600$$

$$\tan\theta = \frac{Q_T}{P_T} = \frac{-1600}{800} = -2 \quad \longrightarrow \quad \theta = -63.43°$$

$$\text{pf} = \cos\theta = \underline{\mathbf{0.4472 \quad (leading)}}$$

$$Z_p = \frac{V_L}{I_L} = \frac{240}{6} = 40$$

$$\mathbf{Z}_p = \underline{\mathbf{40\angle -63.43°\ \Omega}}$$

Chapter 12, Problem 71.

In Fig. 12.73, two wattmeters are properly connected to the unbalanced load supplied by a balanced source such that $\mathbf{V}_{ab} = 208\angle 0°$ V with positive phase sequence.

(a) Determine the reading of each wattmeter.
(b) Calculate the total apparent power absorbed by the load.

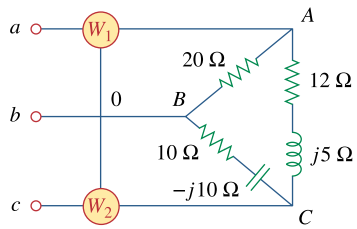


Figure 12.73
For Prob. 12.71.

Chapter 12, Solution 71.

(a) If $\mathbf{V}_{ab} = 208\angle 0°$, $\mathbf{V}_{bc} = 208\angle\text{-}120°$, $\mathbf{V}_{ca} = 208\angle 120°$,

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{Ab}} = \frac{208\angle 0°}{20} = 10.4\angle 0°$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_{BC}} = \frac{208\angle\text{-}120°}{10\sqrt{2}\,\angle\text{-}45°} = 14.708\angle\text{-}75°$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_{CA}} = \frac{208\angle 120°}{13\angle 22.62°} = 16\angle 97.38°$$

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = 10.4\angle 0° - 16\angle 97.38°$$

$$\mathbf{I}_{aA} = 10.4 + 2.055 - j15.867$$

$$\mathbf{I}_{aA} = 20.171\angle\text{-}51.87°$$

$$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC} = 16\angle 97.83° - 14.708\angle\text{-}75°$$

$$\mathbf{I}_{cC} = 30.64\angle 101.03°$$

$$P_1 = |\mathbf{V}_{ab}||\mathbf{I}_{aA}|\cos(\theta_{V_{ab}} - \theta_{I_{aA}})$$

$$P_1 = (208)(20.171)\cos(0° + 51.87°) = \underline{\mathbf{2590\ W}}$$

$$P_2 = |\mathbf{V}_{cb}||\mathbf{I}_{cC}|\cos(\theta_{V_{cb}} - \theta_{I_{cC}})$$

But $\mathbf{V}_{cb} = \text{-}\mathbf{V}_{bc} = 208\angle 60°$

$$P_2 = (208)(30.64)\cos(60° - 101.03°) = \underline{\mathbf{4808\ W}}$$

(b)

$$P_T = P_1 + P_2 = 7398.17 \text{ W}$$

$$Q_T = \sqrt{3}\,(P_2 - P_1) = 3840.25 \text{ VAR}$$

$$\mathbf{S}_T = P_T + jQ_T = 7398.17 + j3840.25 \text{ VA}$$

$$S_T = |\mathbf{S}_T| = \underline{\mathbf{8335\ VA}}$$

Chapter 12, Problem 72.

If wattmeters W_1 and W_2 are properly connected respectively between lines *a* and *b* and lines *b* and *c* to measure the power absorbed by the delta-connected load in Fig. 12.44, predict their readings.

Chapter 12, Solution 72.

From Problem 12.11,

$$\mathbf{V}_{AB} = 220\angle 130° \text{ V} \quad \text{and} \quad \mathbf{I}_{aA} = 30\angle 180° \text{ A}$$

$$P_1 = (220)(30)\cos(130° - 180°) = \underline{\mathbf{4242\ W}}$$

$$\mathbf{V}_{CB} = -\mathbf{V}_{BC} = 220\angle 190°$$
$$\mathbf{I}_{cC} = 30\angle -60°$$

$$P_2 = (220)(30)\cos(190° + 60°) = \underline{\mathbf{-\ 2257\ W}}$$

Chapter 12, Problem 73.

For the circuit displayed in Fig. 12.74, find the wattmeter readings.

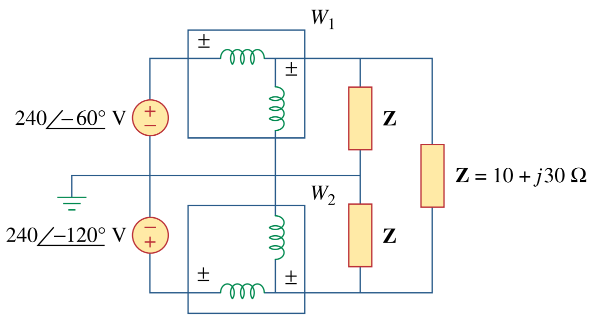
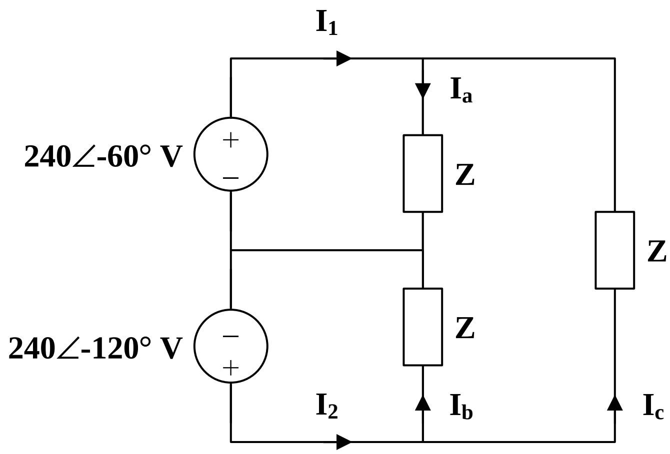


Figure 12.74
For Prob. 12.73.

Chapter 12, Solution 73.

Consider the circuit as shown below.



$$\mathbf{Z} = 10 + j30 = 31.62\angle 71.57°$$

$$\mathbf{I}_a = \frac{240\angle -60°}{31.62\angle 71.57°} = 7.59\angle -131.57°$$

$$\mathbf{I}_b = \frac{240\angle -120°}{31.62\angle 71.57°} = 7.59\angle -191.57°$$

$$\mathbf{I}_c\,\mathbf{Z} + 240\angle -60° - 240\angle -120° = 0$$

$$\mathbf{I}_c = \frac{-240}{31.62\angle 71.57°} = 7.59\angle 108.43°$$

$$\mathbf{I}_1 = \mathbf{I}_a - \mathbf{I}_c = 13.146\angle -101.57°$$

$$\mathbf{I}_2 = \mathbf{I}_b + \mathbf{I}_c = 13.146\angle 138.43°$$

$$P_1 = \text{Re}\left[\mathbf{V}_1\,\mathbf{I}_1^*\right] = \text{Re}\left[(240\angle -60°)(13.146\angle 101.57°)\right] = \underline{\mathbf{2360\ W}}$$

$$P_2 = \text{Re}\left[\mathbf{V}_2\,\mathbf{I}_2^*\right] = \text{Re}\left[(240\angle -120°)(13.146\angle -138.43°)\right] = \underline{\mathbf{-\ 632.8\ W}}$$

Chapter 12, Problem 74.

Predict the wattmeter readings for the circuit in Fig. 12.75.

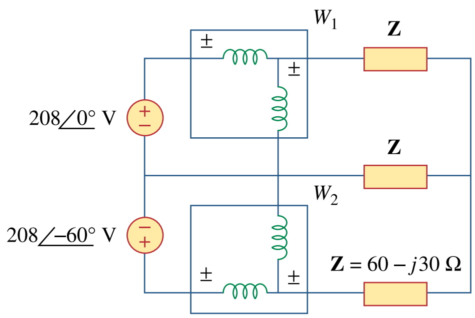
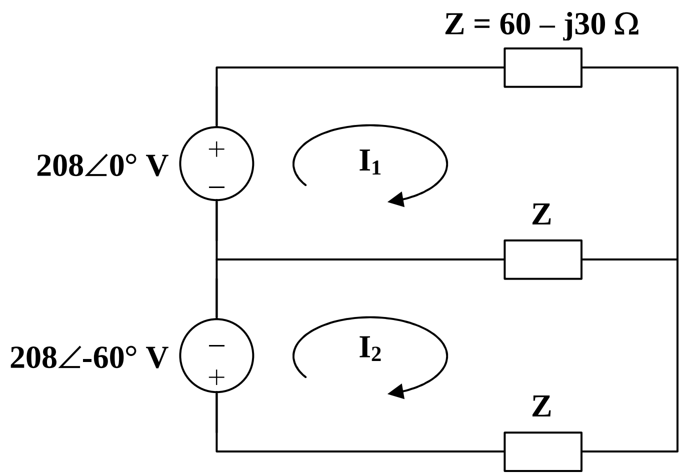


Figure 12.75
For Prob. 12.74.

Chapter 12, Solution 74.

Consider the circuit shown below.



For mesh 1,

$$208 = 2\mathbf{Z}\mathbf{I}_1 - \mathbf{Z}\mathbf{I}_2$$

For mesh 2,

$$-208\angle -60° = -\mathbf{Z}\mathbf{I}_1 + 2\mathbf{Z}\mathbf{I}_2$$

In matrix form,

$$\begin{bmatrix} 208 \\ -208\angle -60° \end{bmatrix} = \begin{bmatrix} 2\mathbf{Z} & -\mathbf{Z} \\ -\mathbf{Z} & 2\mathbf{Z} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = 3\mathbf{Z}^2, \quad \Delta_1 = (208)(1.5 + j0.866)\mathbf{Z}, \quad \Delta_2 = (208)(j1.732)\mathbf{Z}$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{(208)(1.5 + j0.866)}{(3)(60 - j30)} = 1.789\angle 56.56°$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{(208)(j1.732)}{(3)(60 - j30)} = 1.79\angle 116.56°$$

$$P_1 = \text{Re}\left[\mathbf{V}_1\mathbf{I}_1^*\right] = \text{Re}\left[(208)(1.789\angle -56.56°)\right] = \underline{\mathbf{208.98\ W}}$$

$$P_2 = \text{Re}\left[\mathbf{V}_2(-\mathbf{I}_2)^*\right] = \text{Re}\left[(208\angle -60°))(1.79\angle 63.44°)\right] = \underline{\mathbf{371.65\ W}}$$

Chapter 12, Problem 75.

A man has a body resistance of $600\,\Omega$. How much current flows through his ungrounded body:

(a) when he touches the terminals of a 12-V autobattery?
(b) when he sticks his finger into a 120-V light socket?

Chapter 12, Solution 75.

(a) $$I = \frac{V}{R} = \frac{12}{600} = \underline{\mathbf{20\ mA}}$$

(b) $$I = \frac{V}{R} = \frac{120}{600} = \underline{\mathbf{200\ mA}}$$

Chapter 12, Problem 76.

e2d Show that the I^2R losses will be higher for a 120-V appliance than for a 240-V appliance if both have the same power rating.

Chapter 12, Solution 76.

If both appliances have the same power rating, P,

$$I = \frac{P}{V_s}$$

For the 120-V appliance, $I_1 = \frac{P}{120}$.

For the 240-V appliance, $I_2 = \frac{P}{240}$.

$$\text{Power loss} = I^2 R = \begin{cases} \dfrac{P^2 R}{120^2} & \text{for the 120-V appliance} \\ \dfrac{P^2 R}{240^2} & \text{for the 240-V appliance} \end{cases}$$

Since $\frac{1}{120^2} > \frac{1}{240^2}$, **the losses in the 120-V appliance are higher**.

Chapter 12, Problem 77.

A three-phase generator supplied 3.6 kVA at a power factor of 0.85 lagging. If 2500 W are delivered to the load and line losses are 80 W per phase, what are the losses in the generator?

Chapter 12, Solution 77.

$$P_g = P_T - P_{load} - P_{line}, \qquad pf = 0.85$$

But $\quad P_T = 3600\cos\theta = 3600 \times pf = 3060$

$$P_g = 3060 - 2500 - (3)(80) = \underline{\mathbf{320\ W}}$$

Chapter 12, Problem 78.

A three-phase 440-V, 51-kW, 60-kVA inductive load operates at 60 Hz and is wye-connected. It is desired to correct the power factor to 0.95 lagging. What value of capacitor should be placed in parallel with each load impedance?

Chapter 12, Solution 78.

$$\cos\theta_1 = \frac{51}{60} = 0.85 \longrightarrow \theta_1 = 31.79°$$

$$Q_1 = S_1 \sin\theta_1 = (60)(0.5268) = 31.61\ \text{kVAR}$$

$$P_2 = P_1 = 51\ \text{kW}$$

$$\cos\theta_2 = 0.95 \longrightarrow \theta_2 = 18.19°$$

$$S_2 = \frac{P_2}{\cos\theta_2} = 53.68\ \text{kVA}$$

$$Q_2 = S_2 \sin\theta_2 = 16.759\ \text{kVAR}$$

$$Q_c = Q_1 - Q_2 = 3.61 - 16.759 = 14.851\ \text{kVAR}$$

For each load,

$$Q_{c1} = \frac{Q_c}{3} = 4.95\ \text{kVAR}$$

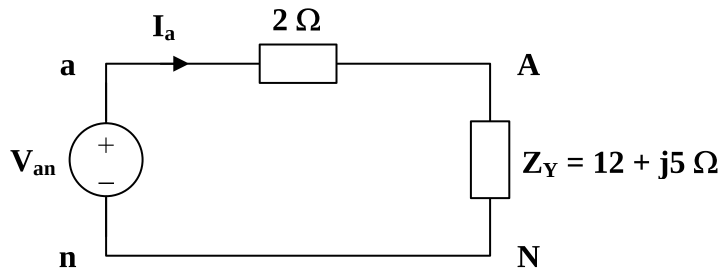
$$C = \frac{Q_{c1}}{\omega V^2} = \frac{4950}{(2\pi)(60)(440)^2} = \underline{\mathbf{67.82\ \mu F}}$$

Chapter 12, Problem 79.

A balanced three-phase generator has an *abc* phase sequence with phase voltage $\mathbf{V}_{an} = 255\angle 0°$ V. The generator feeds an induction motor which may be represented by a balanced Y-connected load with an impedance of $12 + j5\,\Omega$ per phase. Find the line currents and the load voltages. Assume a line impedance of $2\ \Omega$ per phase.

Chapter 12, Solution 79.

Consider the per-phase equivalent circuit below.



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y + 2} = \frac{255\angle 0°}{14 + j5} = \underline{\mathbf{17.15\angle -19.65°\ A}}$$

Thus,

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{17.15\angle -139.65°\ A}}$$
$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{17.15\angle 100.35°\ A}}$$

$$\mathbf{V}_{AN} = \mathbf{I}_a\,\mathbf{Z}_Y = (17.15\angle -19.65°)(13\angle 22.62°) = \underline{\mathbf{223\angle 2.97°\ V}}$$

Thus,

$$\mathbf{V}_{BN} = \mathbf{V}_{AN}\angle -120° = \underline{223\angle -117.03°\ \text{V}}$$
$$\mathbf{V}_{CN} = \mathbf{V}_{AN}\angle 120° = \underline{\mathbf{223\angle 122.97°\ V}}$$

Chapter 12, Problem 80.

A balanced three-phase source furnishes power to the following three loads:

Load 1: 6 kVA at 0.83 pf lagging
Load 2: unknown
Load 3: 8 kW at 0.7071 pf leading

If the line current is 84.6 A rms, the line voltage at the load is 208 V rms, and the combined load has a 0.8 pf lagging, determine the unknown load.

Chapter 12, Solution 80.

$$S = S_1 + S_2 + S_3 = 6[0.83 + j\sin(\cos^{-1} 0.83)] + S_2 + 8(0.7071 - j0.7071)$$
$$S = 10.6368 - j2.31 + S_2 \text{ kVA} \qquad (1)$$

But
$$S = \sqrt{3} V_L I_L \angle\theta = \sqrt{3}(208)(84.6)(0.8 + j0.6) \text{ VA} = 24.383 + \text{j}18.287 \text{ kVA} \qquad (2)$$

From (1) and (2),
$$S_2 = 13.746 + j20.6 = 24.76\angle 56.28 \text{ kVA}$$
Thus, the unknown load is <u>24.76 kVA at 0.5551 pf lagging</u>.

Chapter 12, Problem 81.

A professional center is supplied by a balanced three-phase source. The center has four balanced three-phase loads as follows:

Load 1: 150 kVA at 0.8 pf leading

Load 2: 100 kW at unity pf

Load 3: 200 kVA at 0.6 pf lagging

Load 4: 80 kW and 95 kVAR (inductive)

If the line impedance is $0.02 + j0.05\ \Omega$ per phase and the line voltage at the loads is 480 V, find the magnitude of the line voltage at the source.

Chapter 12, Solution 81.

$$\text{pf} = 0.8 \quad (\text{leading}) \longrightarrow \theta_1 = -36.87°$$
$$\mathbf{S}_1 = 150\angle -36.87° \text{ kVA}$$

$$\text{pf} = 1.0 \longrightarrow \theta_2 = 0°$$
$$\mathbf{S}_2 = 100\angle 0° \text{ kVA}$$

$$\text{pf} = 0.6 \quad (\text{lagging}) \longrightarrow \theta_3 = 53.13°$$
$$\mathbf{S}_3 = 200\angle 53.13° \text{ kVA}$$

$$\mathbf{S}_4 = 80 + j95 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4$$
$$\mathbf{S} = 420 + j165 = 451.2\angle 21.45° \text{ kVA}$$

$$S = \sqrt{3}\, V_L I_L$$
$$I_L = \frac{S}{\sqrt{3}\, V_L} = \frac{451.2 \times 10^3}{\sqrt{3} \times 480} = 542.7 \text{ A}$$

For the line,

$$\mathbf{S}_L = 3 I_L^2\, \mathbf{Z}_L = (3)(542.7)^2 (0.02 + j0.05)$$
$$\mathbf{S}_L = 17.67 + j44.18 \text{ kVA}$$

At the source,

$$\mathbf{S}_T = \mathbf{S} + \mathbf{S}_L = 437.7 + j209.2$$
$$\mathbf{S}_T = 485.1\angle 25.55° \text{ kVA}$$

$$V_T = \frac{S_T}{\sqrt{3}\, I_L} = \frac{485.1 \times 10^3}{\sqrt{3} \times 542.7} = \underline{\mathbf{516\ V}}$$

Chapter 12, Problem 82.

A balanced three-phase system has a distribution wire with impedance $2 + j6\,\Omega$ per phase. The system supplies two three-phase loads that are connected in parallel. The first is a balanced wye-connected load that absorbs 400 kVA at a power factor of 0.8 lagging. The second load is a balanced delta-connected load with impedance of $10 + j8\,\Omega$ per phase. If the magnitude of the line voltage at the loads is 2400 V rms, calculate the magnitude of the line voltage at the source and the total complex power supplied to the two loads.

Chapter 12, Solution 82.

$$\overline{S}_1 = 400(0.8 + j0.6) = 320 + j240\,\text{kVA}, \qquad \overline{S}_2 = 3\frac{V^2_p}{Z^*_p}$$

For the delta-connected load, $V_L = V_p$

$$\overline{S}_2 = 3x\frac{(2400)^2}{10 - j8} = 1053.7 + j842.93\,\text{kVA}$$

$$\overline{S} = \overline{S}_1 + \overline{S}_2 = 1.3737 + j1.0829\,\text{MVA}$$

Let $I = I_1 + I_2$ be the total line current. For I_1,

$$S_1 = 3V_p I^*_1, \qquad V_p = \frac{V_L}{\sqrt{3}}$$

$$I^*_1 = \frac{S_1}{\sqrt{3}V_L} = \frac{(320 + j240)x10^3}{\sqrt{3}(2400)}, \qquad I_1 = 76.98 - j57.735$$

For I_2, convert the load to wye.

$$I_2 = I_p\sqrt{3}\angle -30^o = \frac{2400}{10 + j8}\sqrt{3}\angle -30^o = 273.1 - j289.76$$

$$I = I_1 + I_2 = 350 - j347.5$$

$$V_s = V_L + V_{line} = 2400 + I(3 + j6) = 5.185 + j1.405\,\text{kV} \qquad \longrightarrow \qquad |V_s| = \underline{5.372\,\text{kV}}$$

Chapter 12, Problem 83.

A commercially available three-phase inductive motor operates at a full load of 120 hp (1 hp = 746 W) at 95 percent efficiency at a lagging power factor of 0.707. The motor is connected in parallel to a 80-kW balanced three-phase heater at unity power factor. If the magnitude of the line voltage is 480 V rms, calculate the line current.

Chapter 12, Solution 83.

$$S_1 = 120x746x0.95(0.707 + j0.707) = 60.135 + j60.135 \text{ kVA}, \qquad \text{S}_2 = 80 \text{ kVA}$$

$$S = S_1 + S_2 = 140.135 + j60.135 \text{ kVA}$$

But $|S| = \sqrt{3} V_L I_L \quad \longrightarrow \quad I_L = \dfrac{|S|}{\sqrt{3} V_L} = \dfrac{152.49x10^3}{\sqrt{3}x480} = \underline{183.42 \text{ A}}$

Chapter 12, Problem 84.

* Figure 12.76 displays a three-phase delta-connected motor load which is connected to a line voltage of 440 V and draws 4 kVA at a power factor of 72 percent lagging. In addition, a single 1.8 kVAR capacitor is connected between lines *a* and *b*, while a 800-W lighting load is connected between line *c* and neutral. Assuming the *abc* sequence and taking $\mathbf{V}_{an} = V_p\angle 0°$, find the magnitude and phase angle of currents $\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$, and $\mathbf{I}_n$.

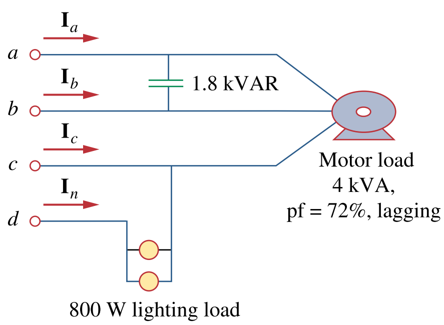


Figure 12.76
For Prob. 12.84.

* An asterisk indicates a challenging problem.

Chapter 12, Solution 84.

We first find the magnitude of the various currents.

For the motor,

$$I_L = \frac{S}{\sqrt{3}\,V_L} = \frac{4000}{440\sqrt{3}} = 5.248\text{ A}$$

For the capacitor,

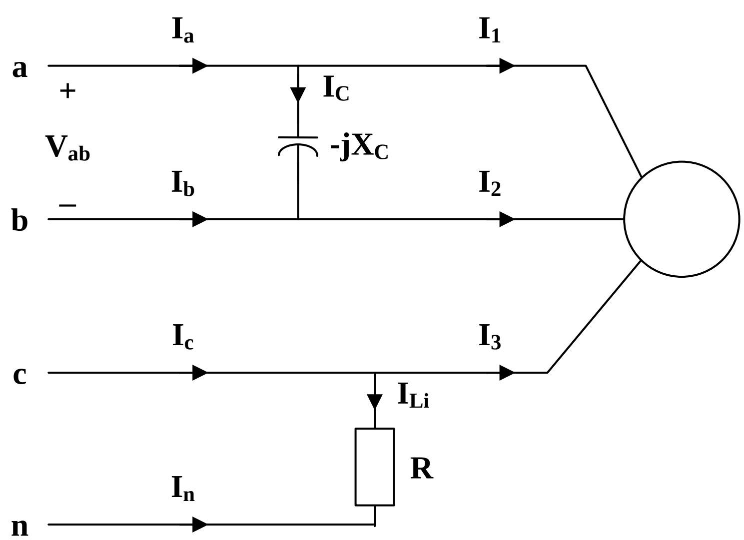
$$I_C = \frac{Q_c}{V_L} = \frac{1800}{440} = 4.091\text{ A}$$

For the lighting,

$$V_p = \frac{440}{\sqrt{3}} = 254\text{ V}$$

$$I_{Li} = \frac{P_{Li}}{V_p} = \frac{800}{254} = 3.15\text{ A}$$

Consider the figure below.



If $\mathbf{V}_{an} = V_p \angle 0°$,

$$\mathbf{V}_{ab} = \sqrt{3}\, V_p \angle 30°$$
$$\mathbf{V}_{cn} = V_p \angle 120°$$

$$\mathbf{I}_C = \frac{\mathbf{V}_{ab}}{-jX_C} = 4.091\angle 120°$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = 4.091\angle(\theta + 30°)$$

where $\theta = \cos^{-1}(0.72) = 43.95°$

$$\mathbf{I}_1 = 5.249\angle 73.95°$$

$$\mathbf{I}_2 = 5.249\angle -46.05°$$

$$\mathbf{I}_3 = 5.249\angle 193.95°$$

$$\mathbf{I}_{Li} = \frac{\mathbf{V}_{cn}}{R} = 3.15\angle 120°$$

Thus,

$$\mathbf{I}_a = \mathbf{I}_1 + \mathbf{I}_C = 5.249\angle 73.95° + 4.091\angle 120°$$
$$\mathbf{I}_a = \underline{\mathbf{8.608\angle 93.96°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_C = 5.249\angle -46.05° - 4.091\angle 120°$$
$$\mathbf{I}_b = \underline{\mathbf{9.271\angle -52.16°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_3 + \mathbf{I}_{Li} = 5.249\angle 193.95° + 3.15\angle 120°$$
$$\mathbf{I}_c = \underline{\mathbf{6.827\angle 167.6°\ A}}$$

$$\mathbf{I}_n = -\mathbf{I}_{Li} = \underline{\mathbf{3.15\angle -60°\ A}}$$

Chapter 12, Problem 85.

Design a three-phase heater with suitable symmetric loads using wye-connected pure resistance. Assume that the heater is supplied by a 240-V line voltage and is to give 27 kW of heat.

Chapter 12, Solution 85.

Let $Z_Y = R$

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{240}{\sqrt{3}} = 138.56 \text{ V}$$

$$P = V_p I_p = \frac{27}{2} = 9 \text{ kW} = \frac{V_p^2}{R}$$

$$R = \frac{V_p^2}{P} = \frac{(138.56)^2}{9000} = 2.133\,\Omega$$

Thus, $Z_Y = \underline{\mathbf{2.133\,\Omega}}$

Chapter 12, Problem 86.

For the single-phase three-wire system in Fig. 12.77, find currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{nN}$.

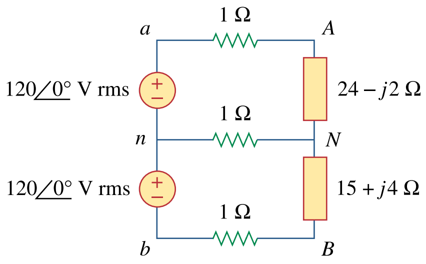
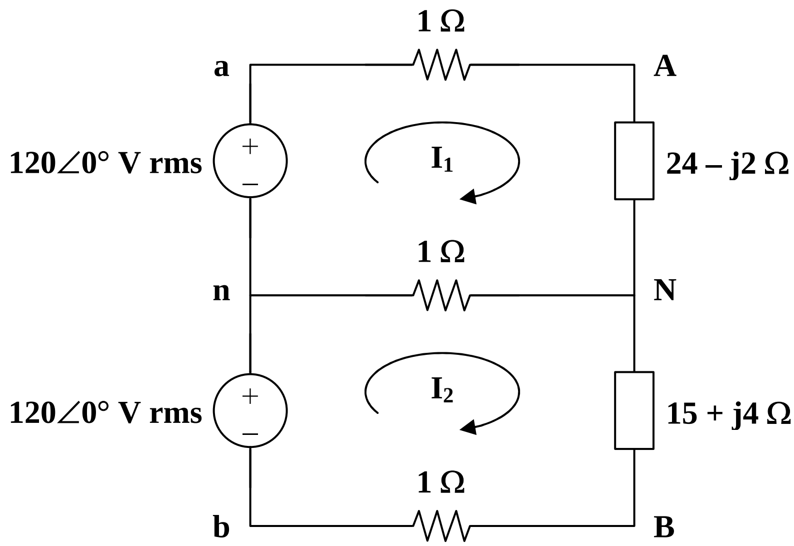


Figure 12.77
For Prob. 12.86.

Chapter 12, Solution 86.

Consider the circuit shown below.



For the two meshes,

$$120 = (26 - j2)\mathbf{I}_1 - \mathbf{I}_2 \qquad (1)$$

$$120 = (17 + j4)\mathbf{I}_2 - \mathbf{I}_1 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 120 \\ 120 \end{bmatrix} = \begin{bmatrix} 26 - j2 & -1 \\ -1 & 17 + j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = 449 + j70\,, \qquad \Delta_1 = (120)(18 + j4)\,, \qquad \Delta_2 = (120)(27 - j2)$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{120 \times 18.44\angle 12.53°}{454.42\angle 8.86°} = 4.87\angle 3.67°$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{120 \times 27.07\angle -4.24°}{454.42\angle 8.86°} = 7.15\angle -13.1°$$

$$\mathbf{I}_{aA} = \mathbf{I}_1 = \underline{\mathbf{4.87\angle 3.67°\ A}}$$

$$\mathbf{I}_{bB} = -\mathbf{I}_2 = \underline{\mathbf{7.15\angle 166.9°\ A}}$$

$$\mathbf{I}_{nN} = \mathbf{I}_2 - \mathbf{I}_1 = \frac{\Delta_2 - \Delta_1}{\Delta}$$

$$\mathbf{I}_{nN} = \frac{(120)(9 - j6)}{449 + j70} = \underline{\mathbf{2.856\angle -42.55°\ A}}$$

Chapter 12, Problem 87.

Consider the single-phase three-wire system shown in Fig. 12.78. Find the current in the neutral wire and the complex power supplied by each source. Take $\mathbf{V}_s$ as a $115\angle 0°$-V, 60-Hz source.

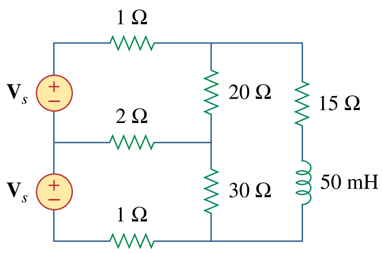
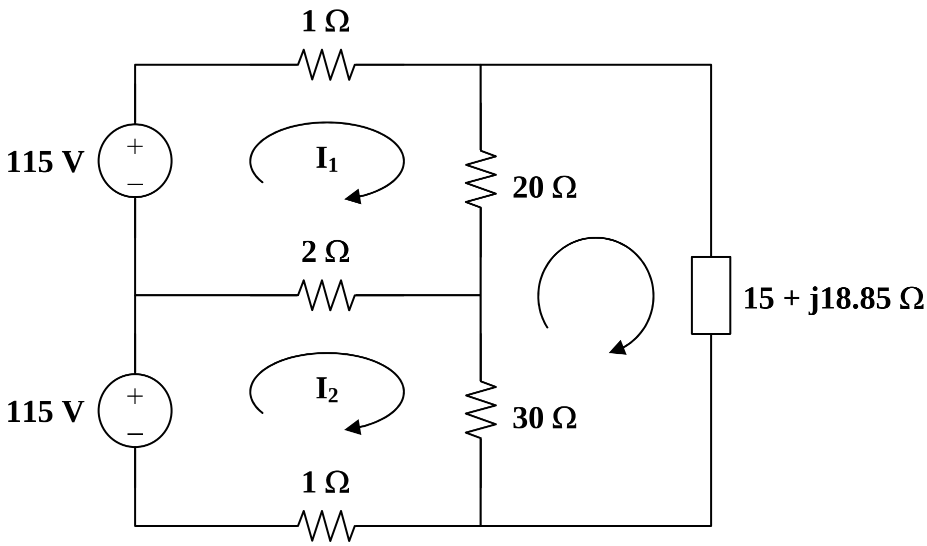


Figure 12.78
For Prob. 12.87.

Chapter 12, Solution 87.

$$L = 50 \text{ mH} \longrightarrow j\omega L = j(2\pi)(60)(5010^{-3}) = j18.85$$

Consider the circuit below.



Applying KVl to the three meshes, we obtain

$$23\mathbf{I}_1 - 2\mathbf{I}_2 - 20\mathbf{I}_3 = 115 \qquad (1)$$

$$-2\mathbf{I}_1 + 33\mathbf{I}_2 - 30\mathbf{I}_3 = 115 \qquad (2)$$

$$-20\mathbf{I}_1 - 30\mathbf{I}_2 + (65 + j18.85)\mathbf{I}_3 = 0 \qquad (3)$$

In matrix form,

$$\begin{bmatrix} 23 & -2 & -20 \\ -2 & 33 & -30 \\ -20 & -30 & 65 + j18.85 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 115 \\ 115 \\ 0 \end{bmatrix}$$

$$\Delta = 12{,}775 + j14{,}232, \qquad \Delta_1 = (115)(1975 + j659.8)$$

$$\Delta_2 = (115)(1825 + j471.3), \qquad \Delta_3 = (115)(1450)$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{115 \times 2082\angle 18.47°}{19214\angle 48.09°} = 12.52\angle -29.62°$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{115 \times 1884.9\angle 14.48°}{19124\angle 48.09°} = 11.33\angle -33.61°$$

$$\mathbf{I}_n = \mathbf{I}_2 - \mathbf{I}_1 = \frac{\Delta_2 - \Delta_1}{\Delta} = \frac{(115)(-150 - j188.5)}{12{,}775 + j14{,}231.75} = \underline{\mathbf{1.448\angle -176.6°\ A}}$$

$$\mathbf{S_1 = V_1 I_1^* = (115)(12.52\angle 29.62°) = \underline{1252 + j711.6\ VA}}$$

$$\mathbf{S}_2 = \mathbf{V}_2\mathbf{I}_2^* = (115)(1.33\angle 33.61°) = \underline{\mathbf{1085 + j721.2\ VA}}$$