Chapter 6, Problem 1.

If the voltage across a 5-F capacitor is $2te^{-3t}$ V, find the current and the power.

Chapter 6, Solution 1.

$$i = C\frac{dv}{dt} = 5\left(2e^{-3t} - 6te^{-3t}\right) = \underline{10(1 - 3t)e^{-3t}\ A}$$

$$p = vi = 10(1\text{-}3t)e^{-3t} \cdot 2t\, e^{-3t} = \underline{20t(1 - 3t)e^{-6t}\ W}$$

Chapter 6, Problem 2.

A 20-μF capacitor has energy $w(t) = 10\cos^2 377t$ J. Determine the current through the capacitor.

Chapter 6, Solution 2.

$$w = \frac{1}{2}Cv^2 \quad \longrightarrow \quad v^2 = \frac{2W}{C} = \frac{20\cos^2 377t}{20x10^{-6}} = 10^6 \cos^2 377t$$

$v = \pm 10^3\cos(377t)$ V, let us assume the $v = +\cos(377t)$ mV, this then leads to,

$$i = C(dv/dt) = 20x10^{-6}(-377\sin(377t)10^{-3}) = \underline{\mathbf{-7.54\sin(377t)\ A}}.$$

Please note that if we had chosen the negative value for v, then i would have been positive.

Chapter 6, Problem 3.

In 5 s, the voltage across a 40-mF capacitor changes from 160 V to 220 V. Calculate the average current through the capacitor.

Chapter 6, Solution 3.

$$i = C\frac{dv}{dt} = 40x10^{-3}\frac{220 - 160}{5} = \underline{\mathbf{480\ mA}}$$

Chapter 6, Problem 4.

A current of 6 sin 4*t* A flows through a 2-F capacitor. Find the voltage *v(t)* across the capacitor given that *v*(0) = 1 V.

Chapter 6, Solution 4.

$$v = \frac{1}{C}\int_o^t i dt + v(0)$$

$$= \frac{1}{2}\int_0^t 6\sin 4t dt + 1 = \left(-\frac{3}{4}\cos 4t\right)\Bigg|_0^t + 1 = -0.75\cos 4t + 0.75 + 1$$

$$= \underline{\mathbf{1.75 - 0.75 \cos 4t\ V}}$$

Chapter 6, Problem 5.

The voltage across a 4-μF capacitor is shown in Fig. 6.45. Find the current waveform.

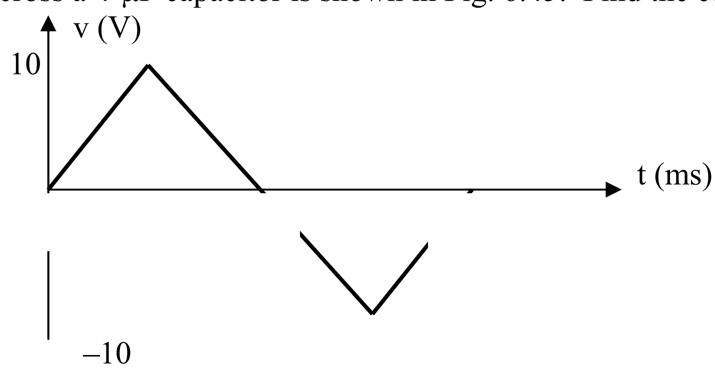


Figure 6.45 For Prob. 6.5.

Chapter 6, Solution 5.

$$v = \begin{cases} 5000t, & 0 < t < 2ms \\ 20 - 5000t, & 2 < t < 6ms \\ -40 + 5000t, & 6 < t < 8ms \end{cases}$$

$$i = C\frac{dv}{dt} = \frac{4x10^{-6}}{10^{-3}}\begin{cases} 5, & 0 < t < 2ms \\ -5, & 2 < t < 6ms \\ 5, & 6 < t < 8ms \end{cases} = \underline{\begin{cases} 20\ mA, & 0 < t < 2ms \\ -20\ mA, & 2 < t < 6ms \\ 20\ mA, & 6 < t < 8ms \end{cases}}$$

Chapter 6, Problem 6.

The voltage waveform in Fig. 6.46 is applied across a 30-μF capacitor. Draw the current waveform through it.

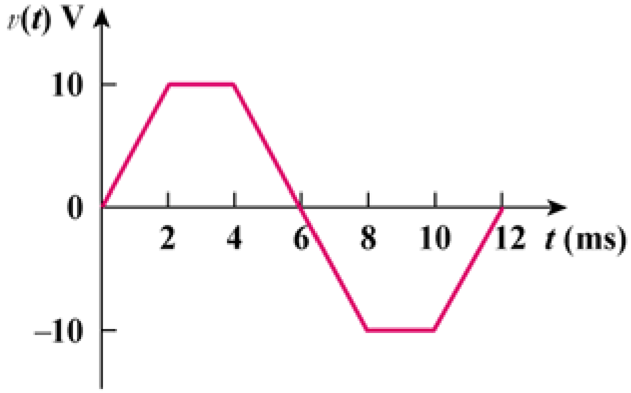


Figure 6.46

Chapter 6, Solution 6.

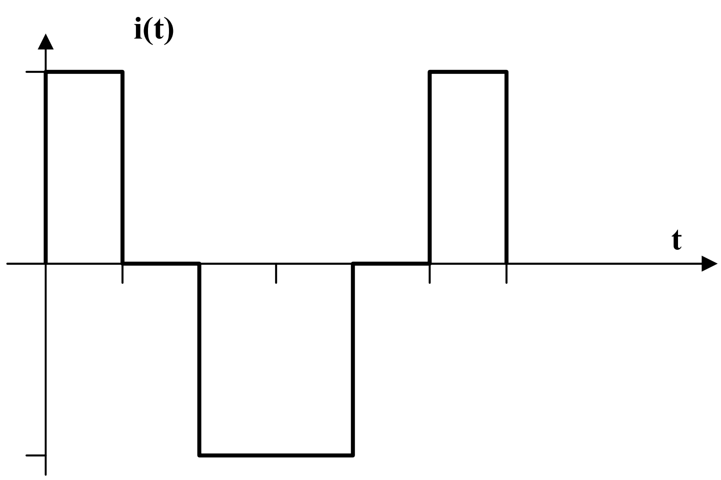
$i = C\dfrac{dv}{dt} = 30x10^{-6}$ x slope of the waveform.

For example, for $0 < t < 2$,

$$\frac{dv}{dt} = \frac{10}{2x10^{-3}}$$

$$i = C\frac{dv}{dt} = 30x10^{-6} x \frac{10}{2x10^{-3}} = 150mA$$

Thus the current i is sketched below.



Chapter 6, Problem 7.

At t=0, the voltage across a 50-mF capacitor is 10 V. Calculate the voltage across the capacitor for $t > 0$ when current 4t mA flows through it.

Chapter 6, Solution 7.

$$v = \frac{1}{C}\int i dt + v(t_o) = \frac{1}{50x10^{-3}}\int_o^t 4tx10^{-3} dt + 10$$

$$= \frac{2t^2}{50} + 10 = \underline{\mathbf{0.04t^2 + 10\ V}}$$

Chapter 6, Problem 8.

A 4-mF capacitor has the terminal voltage

$$v = \begin{cases} 50\text{ V}, & t \le 0 \\ \text{Ae}^{-100t} + \text{Be}^{-600t}\text{ V}, & t \ge 0 \end{cases}$$

If the capacitor has initial current of 2A, find:

(a) the constants A and B,
(b) the energy stored in the capacitor at $t = 0$,
(c) the capacitor current for $t > 0$.

Chapter 6, Solution 8.

(a) $i = C\dfrac{dv}{dt} = -100ACe^{-100t} - 600BCe^{-600t}$ (1)

$i(0) = 2 = -100AC - 600BC \longrightarrow 5 = -A - 6B$ (2)

$v(0^+) = v(0^-) \longrightarrow 50 = A + B$ (3)

Solving (2) and (3) leads to
<u>A=61, B=-11</u>

(b) $\text{Energy} = \frac{1}{2}Cv^2(0) = \frac{1}{2}x4x10^{-3}x2500 = \underline{5\ J}$

(c) From (1),

$$i = -100x61x4x10^{-3}e^{-100t} - 600x11x4x10^{-3}e^{-600t} = \underline{-24.4e^{-100t} - 26.4e^{-600t}\ \text{A}}$$

Chapter 6, Problem 9.

The current through a 0.5-F capacitor is $6(1-e^{-t})$A.
Determine the voltage and power at t=2 s. Assume v(0) = 0.

Chapter 6, Solution 9.

$$v(t) = \frac{1}{1/2}\int_o^t 6\left(1 - e^{-t}\right)dt + 0 = 12\left(t + e^{-t}\right)\Big|_0^t V = 12(t + e^{-t}) - 12$$

$v(2) = 12(2 + e^{-2}) - 12 =$ **<u>13.624 V</u>**

$p = iv = [12\ (t + e^{-t}) - 12]6(1-e^{-t})$

$p(2) = [12\ (2 + e^{-2}) - 12]6(1-e^{-2})\ =$ **<u>70.66 W</u>**

Chapter 6, Problem 10.

The voltage across a 2-mF capacitor is shown in Fig. 6.47. Determine the current through the capacitor.

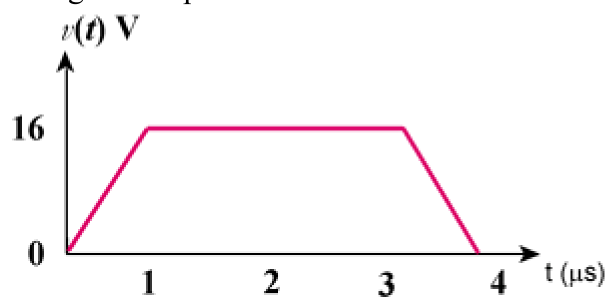


Figure 6.47

Chapter 6, Solution 10

$$i = C\frac{dv}{dt} = 2x10^{-3}\frac{dv}{dt}$$

$$v = \begin{cases} 16t, & 0 < t < 1\mu s \\ 16, & 1 < t < 3\ \mu s \\ 64 - 16t, & 3 < t < 4\mu s \end{cases}$$

$$\frac{dv}{dt} = \begin{cases} 16x10^6, & 0 < t < 1\mu s \\ 0, & 1 < t < 3\ \mu s \\ -16x10^6, & 3 < t < 4\mu s \end{cases}$$

$$\underline{i(t) = \begin{cases} 32\text{ kA}, & 0 < t < 1\mu s \\ 0, & 1 < t < 3\ \mu s \\ -32\text{ kA}, & 3 < t < 4\mu s \end{cases}}$$

Chapter 6, Problem 11.

3. A 4-mF capacitor has the current waveform shown in Fig. 6.48. Assuming that v(0)=10V, sketch the voltage waveform v(t).

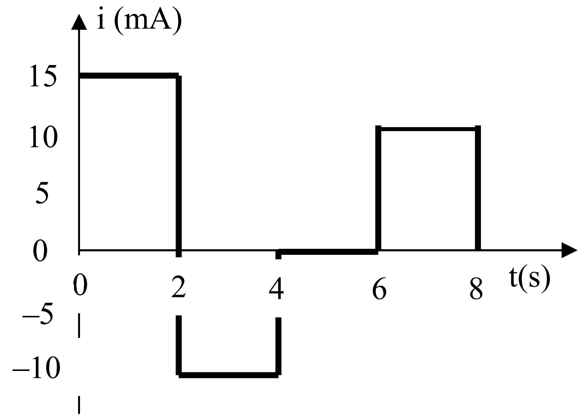


Figure 6.48 For Prob. 6.11.

Chapter 6, Solution 11.

$$v = \frac{1}{C}\int_0^t i dt + v(0) = 10 + \frac{1}{4x10^{-3}}\int_0^t i(t)dt$$

For 0<t <2, i(t)=15mA, V(t)= 10+ $v = 10 + \frac{10^3}{4x10^{-3}}\int_0^t 15dt = 10 + 3.76t$

v(2) = 10+7.5 =17.5

For 2 < t <4, i(t) = –10 mA

$$v(t) = \frac{1}{4x10^{-3}}\int_2^t i(t)dt + v(2) = -\frac{10x10^{-3}}{4x10^{-3}}\int_2^t dt + 17.5 = 22.5 + 2.5t$$

v(4)=22.5-2.5x4 =12.5

For 4<t<6, i(t) = 0, $v(t) = \frac{1}{4x10^{-3}}\int_2^t 0dt + v(4) = 12.5$

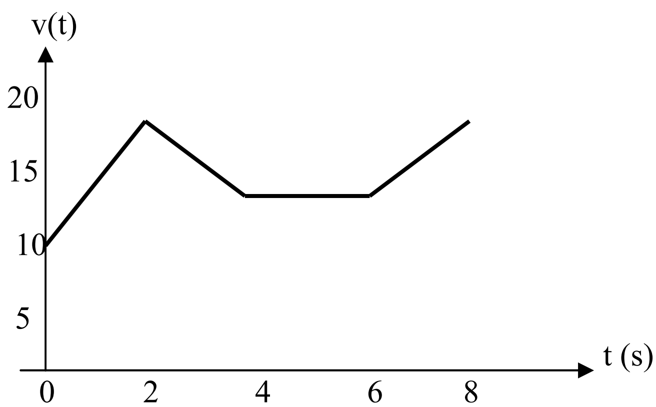
For 6<t<8, i(t) = 10 mA

$$v(t) = \frac{10x10^3}{4x10^{-3}}\int_4^t dt + v(6) = 2.5(t-6) + 12.5 = 2.5t - 2.5$$

Hence,

$$v(t) = \begin{cases} 10 + 3.75t\,V, & 0 < t < 2s \\ 22.5 - 2.5t\,V, & 2 < t < 4s \\ 12.5\,V, & 4 < t < 6s \\ 2.5t - 2.5\,V, & 6 < t < 8s \end{cases}$$

which is sketched below.



Chapter 6, Problem 12.

A voltage of $6e^{-2000t}$ V appears across a parallel combination of a 100-mF capacitor and a 12-Ω resistor. Calculate the power absorbed by the parallel combination.

Chapter 6, Solution 12.

$$i_R = \frac{v}{R} = \frac{6}{12}e^{-2000t} = 0.5e^{-2000t}$$

$$i_c = C\frac{dv}{dt} = 100x10^{-3}x6(-2000)e^{-2000t} = -1200e^{-2000t}$$

$$i = i_R + i_C = -1199.5e^{-2000t}$$

$$p = vi = \underline{-7197e^{-4000t}\ \text{W}}$$

Chapter 6, Problem 13.

Find the voltage across the capacitors in the circuit of Fig. 6.49 under dc conditions.

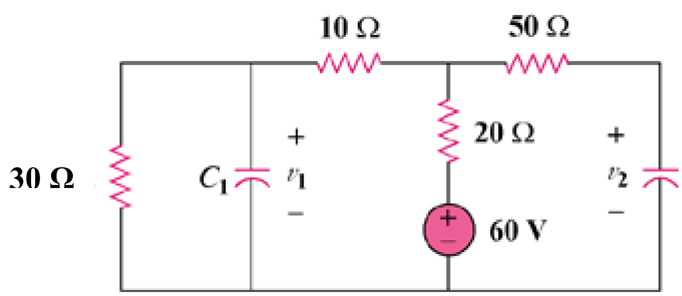
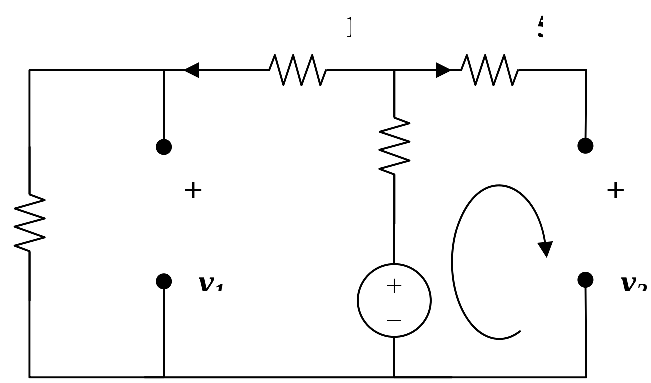


Figure 6.49

Chapter 6, Solution 13.

Under dc conditions, the circuit becomes that shown below:



$i_2 = 0$, $i_1 = 60/(30+10+20) = 1A$

$v_1 = 30i_1 = 30V$, $v_2 = 60–20i_1 = 40V$

Thus, $\underline{v_1 = 30V, v_2 = 40V}$

Chapter 6, Problem 14.

Series-connected 20-pF and 60-pF capacitors are placed in parallel with series-connected 30-pF and 70-pF capacitors. Determine the equivalent capacitance.

Chapter 6, Solution 14.

20 pF is in series with 60pF = 20*60/80=15 pF
30-pF is in series with 70pF = 30x70/100=21pF
15pF is in parallel with 21pF = 15+21 = **36 pF**

Chapter 6, Problem 15.

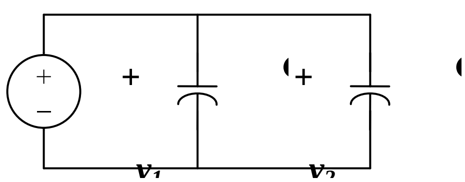
Two capacitors (20 μF and 30 μF) are connected to a 100-V source. Find the energy stored in each capacitor if they are connected in:

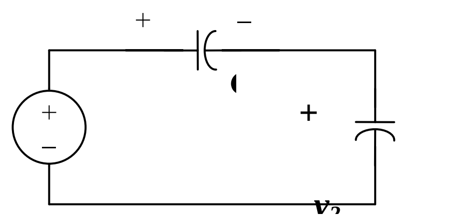
(a) parallel (b) series

Chapter 6, Solution 15.

In parallel, as in Fig. (a),

$v_1 = v_2 = 100$





$$w_{20} = \frac{1}{2}Cv^2 = \frac{1}{2}x20x10^{-6}x100^2 = \underline{\mathbf{100\ mJ}}$$

$$w_{30} = \frac{1}{2}x30x10^{-6}x100^2 = \underline{\mathbf{150\ mJ}}$$

(b) When they are connected in series as in Fig. (b):

$$v_1 = \frac{C_2}{C_1 + C_2}V = \frac{30}{50}x100 = 60, \ v_2 = 40$$

$$w_{20} = \frac{1}{2}x30x10^{-6}x60^2 = \underline{\mathbf{36\ mJ}}$$

$$w_{30} = \frac{1}{2}x30x10^{-6}x40^2 = \underline{\mathbf{24\ mJ}}$$

Chapter 6, Problem 16.

The equivalent capacitance at terminals *a*-*b* in the circuit in Fig. 6.50 is 30 μF. Calculate the value of *C*.

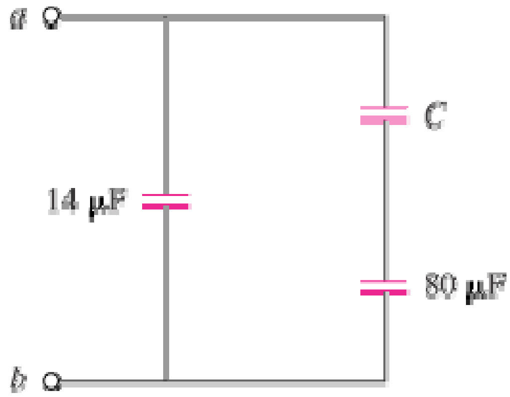


Figure 6.50

Chapter 6, Solution 16

$$C_{eq} = 14 + \frac{Cx80}{C+80} = 30 \quad \longrightarrow \quad \underline{C = 20\ \mu\text{F}}$$

Chapter 6, Problem 17.

Determine the equivalent capacitance for each of the circuits in Fig. 6.51.

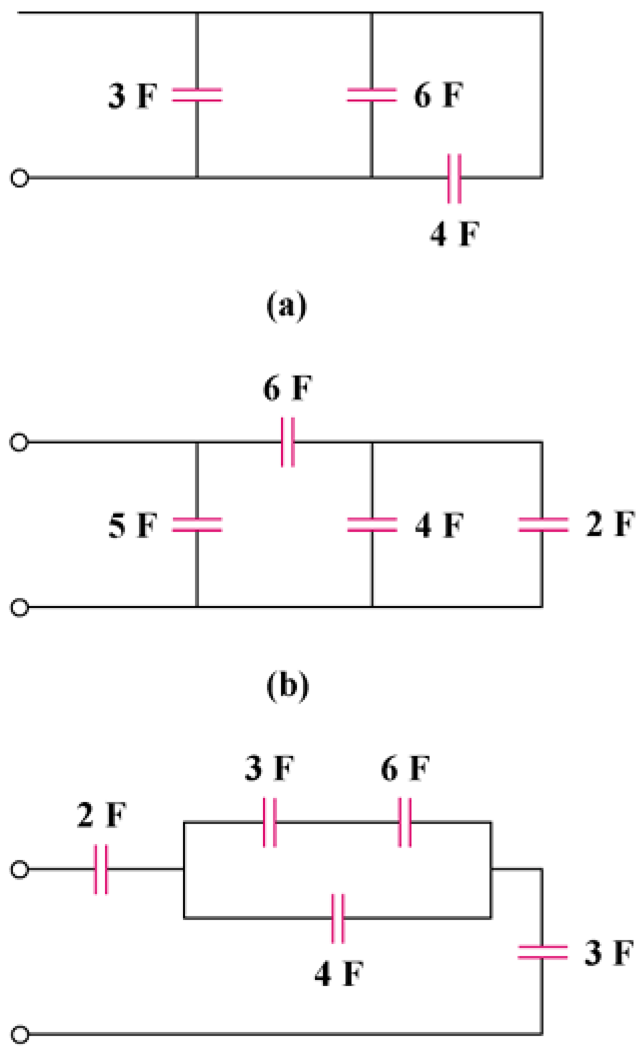


Figure 6.51

Chapter 6, Solution 17.

(a) 4F in series with 12F = 4 x 12/(16) = 3F
3F in parallel with 6F and 3F = 3+6+3 = 12F
4F in series with 12F = 3F
i.e. C_{eq} = **3F**

(b) C_{eq} = 5 + [6x(4 + 2)/(6+4+2)] = 5 + (36/12) = 5 + 3 = **8F**

(c) 3F in series with 6F = (3 x 6)/9 = 2F

$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{6} + \frac{1}{3} = 1$$

C_{eq} = **1F**

Chapter 6, Problem 18.

Find C_{eq} in the circuit of Fig. 6.52 if all capacitors are 4 μF

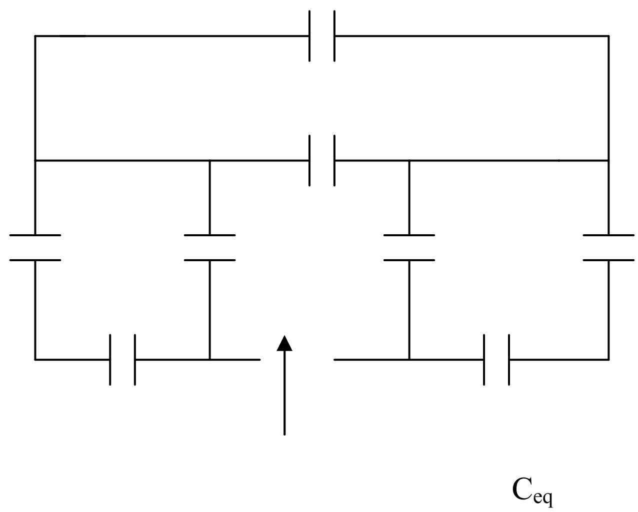
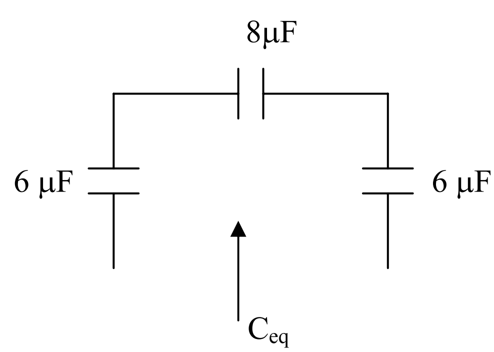


Figure 6.52 For Prob. 6.18.

Chapter 6, Solution 18.

4 μF in parallel with 4 μF = 8μF
4 μF in series with 4 μF = 2 μF
2 μF in parallel with 4 μF = 6 μF
Hence, the circuit is reduced to that shown below.



$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{6} + \frac{1}{8} = 0.4583 \longrightarrow C_{eq} = \underline{2.1818\ \mu F}$$

Chapter 6, Problem 19.

Find the equivalent capacitance between terminals *a* and *b* in the circuit of Fig. 6.53. All capacitances are in μF.

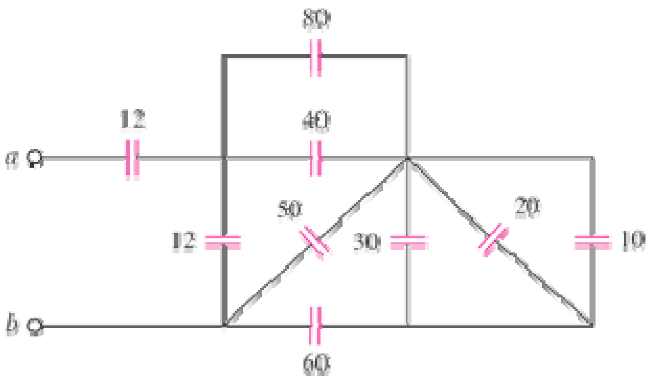
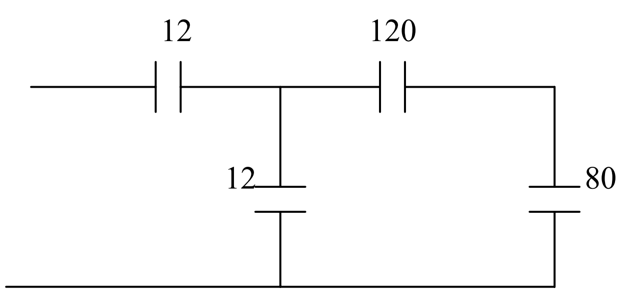


Figure 6.53

Chapter 6, Solution 19.

We combine 10-, 20-, and 30- μF capacitors in parallel to get 60 μF. The 60 - μF capacitor in series with another 60- μF capacitor gives 30 μF.
30 + 50 = 80 μF, 80 + 40 = 120 μF
The circuit is reduced to that shown below.



120- μF capacitor in series with 80 μF gives (80x120)/200 = 48
48 + 12 = 60
60- μF capacitor in series with 12 μF gives (60x12)/72 = **10 μ F**

Chapter 6, Problem 20.

Find the equivalent capacitance at terminals a-b of the circuit in Fig. 6.54.

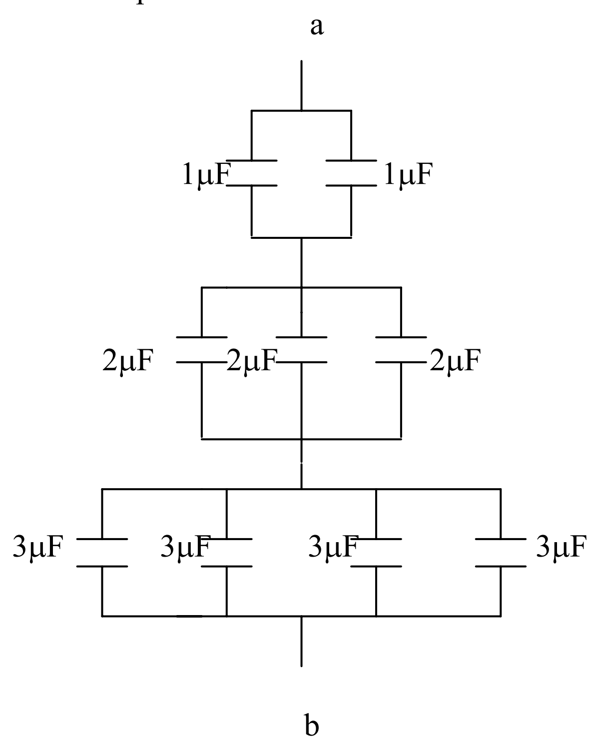
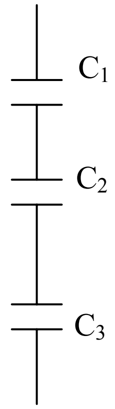


Figure 6.54 For Prob. 6.20.

Chapter 6, Solution 20.

Consider the circuit shown below.



$C_1 = 1 + 1 = 2\mu F$

$C_2 = 2 + 2 + 2 = 6\mu F$

$C_3 = 4x3 = 12\mu F$

$1/C_{eq} = (1/C_1) + (1/C_2) + (1/C_3) = 0.5 + 0.16667 + 0.08333 = 0.75x10^6$

C_{eq} = **1.3333 μF**.

Chapter 6, Problem 21.

Determine the equivalent capacitance at terminals ***a*** - ***b*** of the circuit in Fig. 6.55.

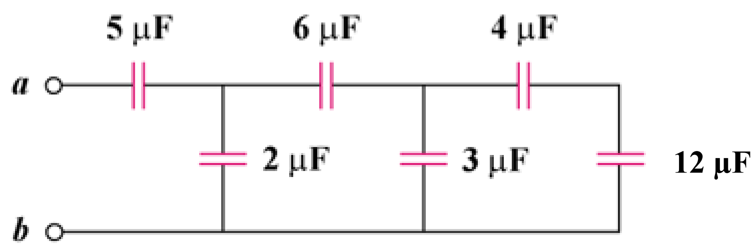


Figure 6.55

Chapter 6, Solution 21.

4μF in series with 12μF = (4x12)/16 = 3μF
3μF in parallel with 3μF = 6μF
6μF in series with 6μF = 3μF
3μF in parallel with 2μF = 5μF
5μF in series with 5μF = 2.5μF

Hence C_{eq} = **2.5μF**

Chapter 6, Problem 22.

Obtain the equivalent capacitance of the circuit in Fig. 6.56.

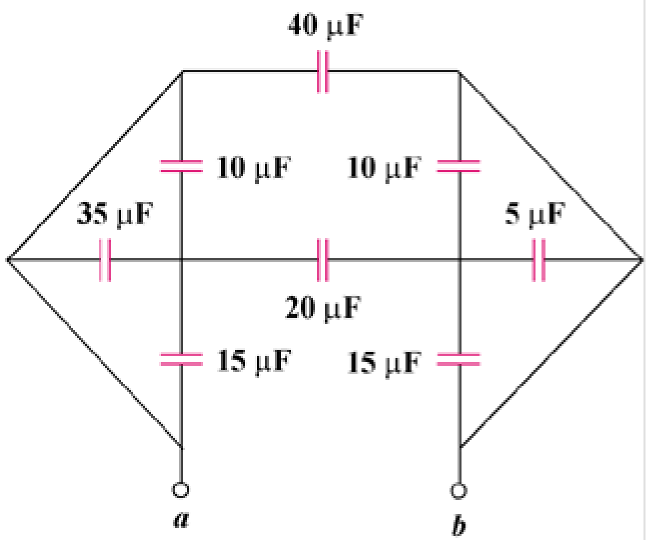
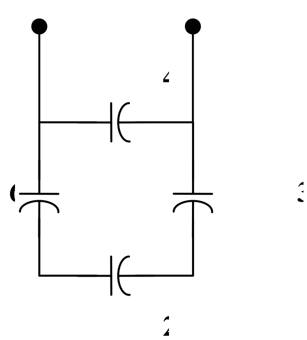


Figure 6.56

Chapter 6, Solution 22.

Combining the capacitors in parallel, we obtain the equivalent circuit shown below:



Combining the capacitors in series gives C^1_{eq}, where

$$\frac{1}{C^1_{eq}} = \frac{1}{60} + \frac{1}{20} + \frac{1}{30} = \frac{1}{10} \longrightarrow C^1_{eq} = 10\mu F$$

Thus

$$C_{eq} = 10 + 40 = \underline{\mathbf{50\ \mu F}}$$

Chapter 6, Problem 23.

For the circuit in Fig. 6.57, determine:

(a) the voltage across each capacitor,
(b) the energy stored in each capacitor.

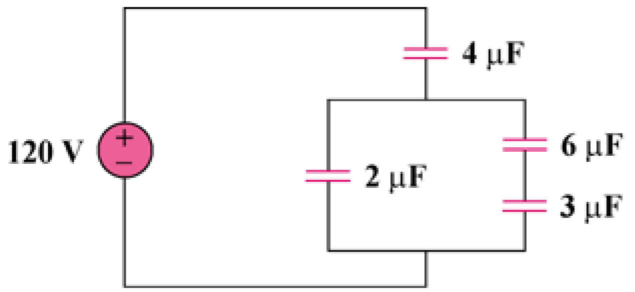


Figure 6.57

Chapter 6, Solution 23.

(a) 3μF is in series with 6μF 3x6/(9) = 2μF

$v_{4\mu F}$ = 1/2 x 120 = **60V**

$v_{2\mu F}$ = **60V**

$v_{6\mu F} = \dfrac{3}{6+3}(60) = \mathbf{20V}$

$v_{3\mu F}$ = 60 - 20 = **40V**

(b) Hence w = 1/2 Cv^2

$w_{4\mu F}$ = 1/2 x 4 x 10^{-6} x 3600 = **7.2mJ**

$w_{2\mu F}$ = 1/2 x 2 x 10^{-6} x 3600 = **3.6mJ**

$w_{6\mu F}$ = 1/2 x 6 x 10^{-6} x 400 = **1.2mJ**

$w_{3\mu F}$ = 1/2 x 3 x 10^{-6} x 1600 = **2.4mJ**

Chapter 6, Problem 24.

Repeat Prob. 6.23 for the circuit in Fig. 6.58.

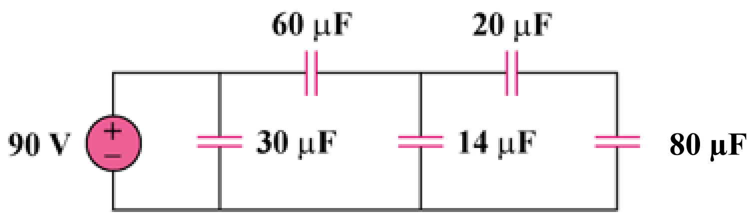


Figure 6.58

Chapter 6, Solution 24.

20μF is series with 80μF = 20x80/(100) = 16μF

14μF is parallel with 16μF = 30μF

(a) $v_{30\mu F} = \underline{\mathbf{90V}}$

$v_{60\mu F} = \underline{\mathbf{30V}}$

$v_{14\mu F} = \underline{\mathbf{60V}}$

$$v_{20\mu F} = \frac{80}{20+80} x60 = \underline{\mathbf{48V}}$$

$v_{80\mu F} = 60 - 48 = \underline{\mathbf{12V}}$

(b) Since $w = \frac{1}{2}Cv^2$

$w_{30\mu F} = 1/2 \text{ x } 30 \text{ x } 10^{-6} \text{ x } 8100 = \underline{\mathbf{121.5mJ}}$

$w_{60\mu F} = 1/2 \text{ x } 60 \text{ x } 10^{-6} \text{ x } 900 = \underline{\mathbf{27mJ}}$

$w_{14\mu F} = 1/2 \text{ x } 14 \text{ x } 10^{-6} \text{ x } 3600 = \underline{\mathbf{25.2mJ}}$

$w_{20\mu F} = 1/2 \text{ x } 20 \text{ x } 10^{-6} \text{ x } (48)^2 = \underline{\mathbf{23.04mJ}}$

$w_{80\mu F} = 1/2 \text{ x } 80 \text{ x } 10^{-6} \text{ x } 144 = \underline{\mathbf{5.76mJ}}$

Chapter 6, Problem 25.

(a) Show that the voltage-division rule for two capacitors in series as in Fig. 6.59(a) is

$$v_1 = \frac{C_2}{C_1 + C_2} v_s \ , \qquad v_2 = \frac{C_1}{C_1 + C_2} v_s$$

assuming that the initial conditions are zero.

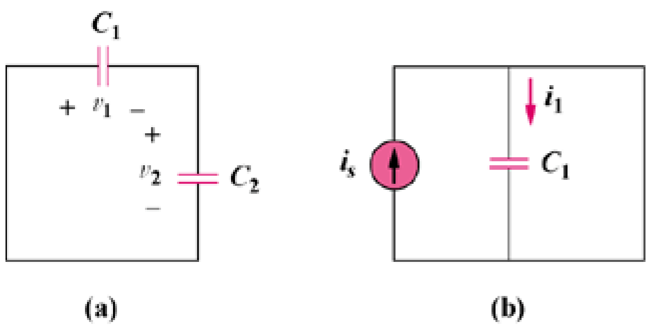


Figure 6.59

(b) For two capacitors in parallel as in Fig. 6.59(b), show that the current-division rule is

$$i_1 = \frac{C_1}{C_1 + C_2} i_s \ , \qquad i_2 = \frac{C_2}{C_1 + C_2} i_s$$

assuming that the initial conditions are zero.

Chapter 6, Solution 25.

(a) For the capacitors in series,

$$Q_1 = Q_2 \longrightarrow C_1v_1 = C_2v_2 \longrightarrow \frac{v_1}{v_2} = \frac{C_2}{C_1}$$

$$v_s = v_1 + v_2 = \frac{C_2}{C_1}v_2 + v_2 = \frac{C_1 + C_2}{C_1}v_2 \longrightarrow \underline{v_2 = \frac{C_1}{C_1 + C_2}v_s}$$

Similarly, $\underline{v_1 = \frac{C_2}{C_1 + C_2}v_s}$

(b) For capacitors in parallel

$$v_1 = v_2 = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$Q_s = Q_1 + Q_2 = \frac{C_1}{C_2}Q_2 + Q_2 = \frac{C_1 + C_2}{C_2}Q_2$$

or

$$Q_2 = \frac{C_2}{C_1 + C_2}$$

$$Q_1 = \frac{C_1}{C_1 + C_2}Q_s$$

$$i = \frac{dQ}{dt} \longrightarrow \underline{i_1 = \frac{C_1}{C_1 + C_2}i_s}, \quad \underline{i_2 = \frac{C_2}{C_1 + C_2}i_s}$$

Chapter 6, Problem 26.

Three capacitors, $\boldsymbol{C}_1 = 5\ \mu F$, $\boldsymbol{C}_2 = 10\ \mu F$, and $\boldsymbol{C}_3 = 20\ \mu F$, are connected in parallel across a 150-V source. Determine:

(a) the total capacitance,
(b) the charge on each capacitor,
(c) the total energy stored in the parallel combination.

Chapter 6, Solution 26.

(a) $C_{eq} = C_1 + C_2 + C_3 = \underline{\mathbf{35\mu F}}$

(b) $Q_1 = C_1 v = 5 \text{ x } 150\mu C = \underline{\mathbf{0.75mC}}$
$Q_2 = C_2 v = 10 \text{ x } 150\mu C = \underline{\mathbf{1.5mC}}$
$Q_3 = C_3 v = 20 \text{ x } 150 = \underline{\mathbf{3mC}}$

(c) $w = \frac{1}{2} C_{eq} v^2 = \frac{1}{2} x35x150^2 \mu J = \underline{\mathbf{393.8mJ}}$

Chapter 6, Problem 27.

Given that four 4-μF capacitors can be connected in series and in parallel, find the minimum and maximum values that can be obtained by such series/parallel combinations.

Chapter 6, Solution 27.

If they are all connected in parallel, we get $C_T = 4x4\mu F = 16\mu F$
If they are all connected in series, we get

$$\frac{1}{C_T} = \frac{4}{4\mu F} \longrightarrow C_T = 1\mu F$$

All other combinations fall within these two extreme cases. Hence,

$\underline{C_{\min} = 1\mu F, \quad C_{\max} = 16\mu F}$

Chapter 6, Problem 28.

Obtain the equivalent capacitance of the network shown in Fig. 6.58.

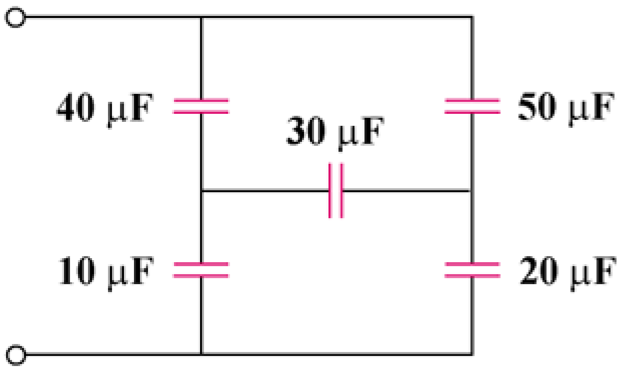
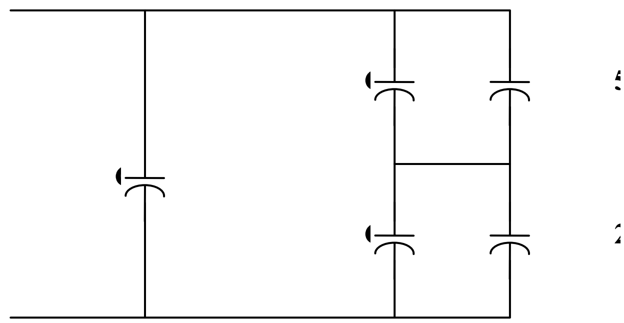


Figure 6.58

Chapter 6, Solution 28.

We may treat this like a resistive circuit and apply delta-wye transformation, except that R is replaced by 1/C.



$$\frac{1}{C_a} = \frac{\left(\frac{1}{10}\right)\left(\frac{1}{40}\right) + \left(\frac{1}{10}\right)\left(\frac{1}{30}\right) + \left(\frac{1}{30}\right)\left(\frac{1}{40}\right)}{\frac{1}{30}}$$

$$= \frac{3}{40} + \frac{1}{10} + \frac{1}{40} = \frac{2}{10}$$

$C_a = 5\mu F$

$$\frac{1}{C_b} = \frac{\frac{1}{400} + \frac{1}{300} + \frac{1}{1200}}{\frac{1}{10}} = \frac{2}{30}$$

$C_b = 15\mu F$

$$\frac{1}{C_c} = \frac{\frac{1}{400} + \frac{1}{300} + \frac{1}{1200}}{\frac{1}{40}} = \frac{4}{15}$$

$C_c = 3.75\mu F$

C_b in parallel with $50\mu F = 50 + 15 = 65\mu F$

C_c in series with $20\mu F = 23.75\mu F$

$65\mu F$ in series with $23.75\mu F = \dfrac{65 x 23.75}{88.75} = 17.39\mu F$

$17.39\mu F$ in parallel with $C_a = 17.39 + 5 = 22.39\mu F$

Hence $C_{eq} =$ **<u>22.39μF</u>**

Chapter 6, Problem 29.

Determine C_{eq} for each circuit in Fig. 6.61.

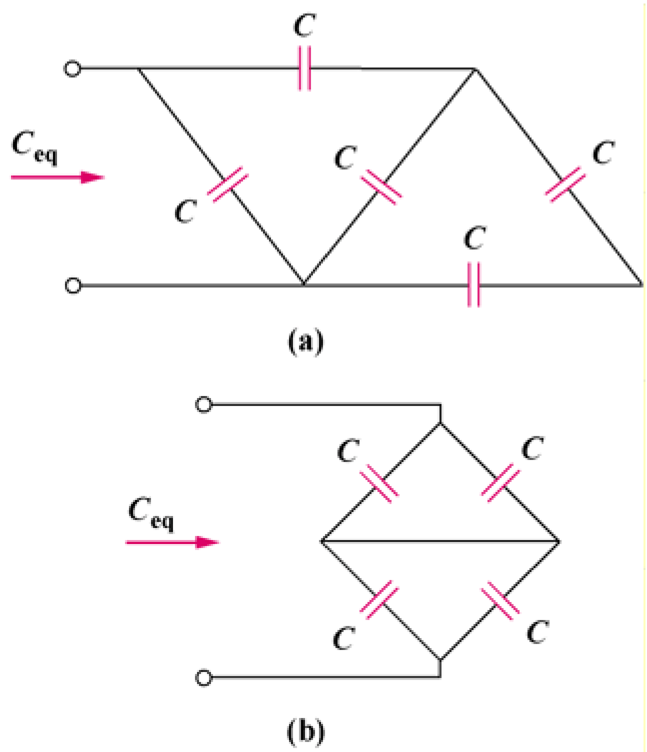


Figure 6.61

Chapter 6, Solution 29.

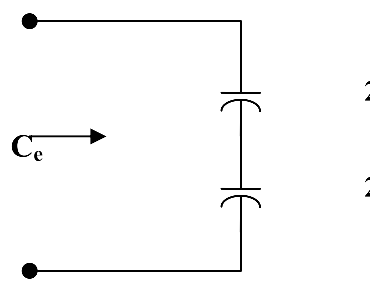
(a) C in series with C = C/(2)

C/2 in parallel with C = 3C/2

$$\frac{3C}{2} \text{ in series with } C = \frac{Cx\,\frac{3C}{2}}{5\frac{C}{2}} = \frac{3C}{5}$$

$$3\frac{C}{5} \text{ in parallel with } C = C + 3\frac{C}{5} = \underline{\mathbf{1.6\ C}}$$

(b)



$$\frac{1}{C_{eq}} = \frac{1}{2C} + \frac{1}{2C} = \frac{1}{C}$$

C_{eq} = <u>**1 C**</u>

Chapter 6, Problem 30.

Assuming that the capacitors are initially uncharged, find $\boldsymbol{v_o(t)}$ in the circuit in Fig. 6.62.

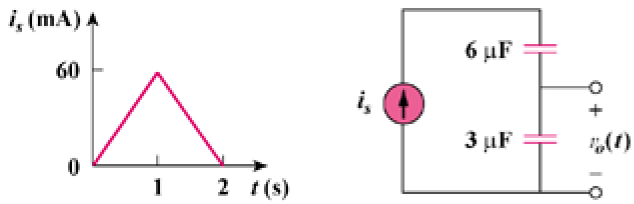


Figure 6.62

Chapter 6, Solution 30.

$$v_o = \frac{1}{C}\int_o^t idt + i(0)$$

For $0 < t < 1$, $i = 60t$ mA,

$$v_o = \frac{10^{-3}}{3x10^{-6}}\int_o^t 60tdt + 0 = 10t^2 kV$$

$v_o(1) = 10kV$

For $1 < t < 2$, $i = 120 - 60t$ mA,

$$v_o = \frac{10^{-3}}{3x10^{-6}}\int_1^t (120 - 60t)dt + v_o(1)$$

$$= [40t - 10t^2]\Big|_1^t + 10kV$$

$$= 40t - 10t^2 - 20$$

$$v_o(t) = \begin{bmatrix} 10t^2 kV, & 0 < t < 1 \\ 40t - 10t^2 - 20kV, & 1 < t < 2 \end{bmatrix}$$

Chapter 6, Problem 31.

If $\boldsymbol{v}(0)=0$, find $\boldsymbol{v}(\boldsymbol{t})$, $\boldsymbol{i_1}(\boldsymbol{t})$, and $\boldsymbol{i_2}(\boldsymbol{t})$ in the circuit in Fig. 6.63.

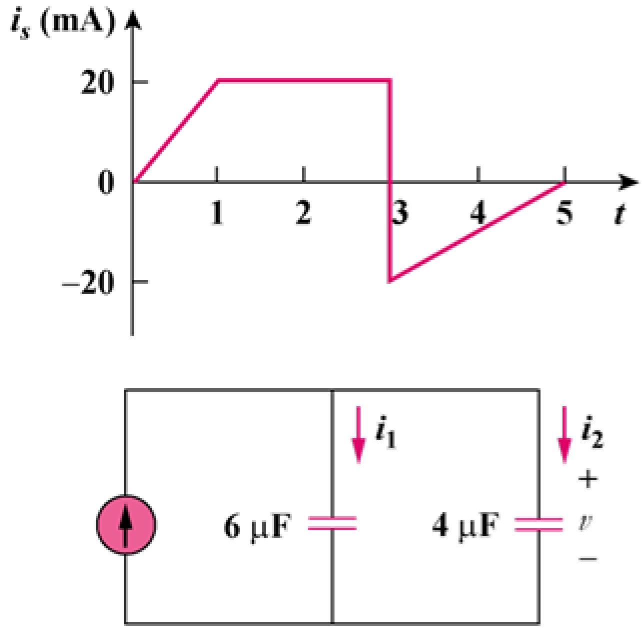


Figure 6.63

Chapter 6, Solution 31.

$$i_s(t) = \begin{cases} 20tmA, & 0 < t < 1 \\ 20mA, & 1 < t < 3 \\ -50 + 10t, & 3 < t < 5 \end{cases}$$

$C_{eq} = 4 + 6 = 10\mu F$

$$v = \frac{1}{C_{eq}} \int_o^t idt + v(0)$$

For $0 < t < 1$,

$$v = \frac{10^{-3}}{10x10^{-6}} \int_o^t 20t\,dt + 0 = t^2 \text{ kV}$$

For $1 < t < 3$,

$$v = \frac{10^3}{10} \int_1^t 20dt + v(1) = 2(t-1) + 1kV$$

$$= 2t - 1kV$$

For $3 < t < 5$,

$$v = \frac{10^3}{10} \int_3^t 10(t-5)dt + v(3)$$

$$= \frac{t^2}{2} - 5t \Big|_3^t + 5kV = \frac{t^2}{2} - 5t + 15.5kV$$

$$v(t) = \begin{cases} t^2 kV, & 0 < t < 1s \\ 2t - 1kV, & 1 < t < 3s \\ \frac{t^2}{2} - 5t + 15.5kV, & 3 < t < 5s \end{cases}$$

$$i_1 = C_1 \frac{dv}{dt} = 6x10^{-6} \frac{dv}{dt}$$

$$= \begin{cases} 12tmA, & 0 < t < 1s \\ 12mA, & 1 < t < 3s \\ 6t - 30mA, & 3 < t < 5s \end{cases}$$

$$i_2 = C_2 \frac{dv}{dt} = 4x10^{-6} \frac{dv}{dt}$$

$$= \begin{cases} 8tmA, & 0 < t < 1s \\ 8mA, & 1 < t < 3s \\ 4t - 20mA, & 3 < t < 5s \end{cases}$$

Chapter 6, Problem 32.

In the circuit in Fig. 6.64, let $i_s = 30e^{-2t}$ mA and $v_1(0) = 50$ V, $v_2(0) = 20$ V. Determine: (a) $v_1(t)$ and $v_2(t)$, (b) the energy in each capacitor at $t = 0.5$ s.

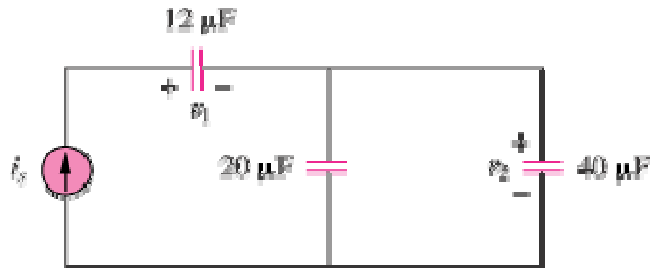


Figure 6.64

Chapter 6, Solution 32.

(a) $C_{eq} = (12x60)/72 = 10\ \mu F$

$$v_1 = \frac{10^{-3}}{12x10^{-6}}\int_0^t 30e^{-2t}dt + v_1(0) = \underline{-1250e^{-2t}\ \Big|_0^t} + 50 = \underline{-1250e^{-2t} + 1300}V$$

$$v_2 = \frac{10^{-3}}{60x10^{-6}}\int_0^t 30e^{-2t}dt + v_2(0) = \underline{250e^{-2t}\ \Big|_0^t} + 20 = \underline{-250e^{-2t} + 270V}$$

(b) At t=0.5s,

$$v_1 = -1250e^{-1} + 1300 = 840.2, \qquad v_2 = -250e^{-1} + 270 = 178.03$$

$$w_{12\mu F} = \frac{1}{2}x12x10^{-6}x(840.15)^2 = \underline{4.235\ \ J}$$

$$w_{20\mu F} = \frac{1}{2}x20x10^{-6}x(178.03)^2 = \underline{0.3169\ \ J}$$

$$w_{40\mu F} = \frac{1}{2}x40x10^{-6}x(178.03)^2 = \underline{0.6339\ J}$$

Chapter 6, Problem 33.

Obtain the Thèvenin equivalent at the terminals, *a*-*b*, of the circuit shown in Fig. 6.65. Please note that Thèvenin equivalent circuits do not generally exist for circuits involving capacitors and resistors. This is a special case where the Thèvenin equivalent circuit does exist.

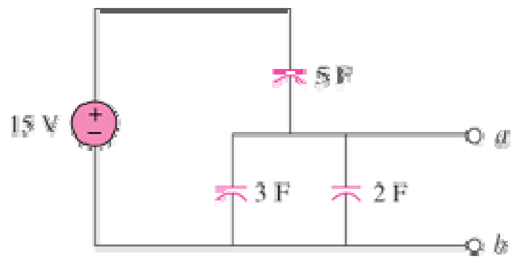


Figure 6.65

Chapter 6, Solution 33

Because this is a totally capacitive circuit, we can combine all the capacitors using the property that capacitors in parallel can be combined by just adding their values and we combine capacitors in series by adding their reciprocals. However, for this circuit we only have the three capacitors in parallel.

3 F + 2 F = 5 F (we need this to be able to calculate the voltage)

$C_{Th} = C_{eq} = 5+5 = 10$ F

The voltage will divide equally across the two 5 F capacitors. Therefore, we get:

V_{Th} = **7.5 V**, C_{Th} = **10 F**

Chapter 6, Problem 34.

The current through a 10-mH inductor is $6e^{-t/2}$ A. Find the voltage and the power at $t = 3$ s.

Chapter 6, Solution 34.

$i = 6e^{-t/2}$

$$v = L\frac{di}{dt} = 10x10^{-3}(6)\left(\frac{1}{2}\right)e^{-t/2}$$

$$= -30e^{-t/2} \text{ mV}$$

$v(3) = -30e^{-3/2}$ mV = **–6.694 mV**

$p = vi = -180e^{-t}$ mW

$p(3) = -180e^{-3}$ mW = **–8.962 mW**

Chapter 6, Problem 35.

An inductor has a linear change in current from 50 mA to 100 mA in 2 ms and induces a voltage of 160 mV. Calculate the value of the inductor.

Chapter 6, Solution 35.

$$v = L\frac{di}{dt} \longrightarrow L = \frac{v}{di/dt} = \frac{160x10^{-3}}{\dfrac{(100-50)x10^{-3}}{2x10^{-3}}} = \underline{6.4\ \text{mH}}$$

Chapter 6, Problem 36.

The current through a 12-mH inductor is $i(t) = 30te^{-2t}$ A, $t \geq 0$. Determine: (a) the voltage across the inductor, (b) the power being delivered to the inductor at t = 1 s, (c) the energy stored in the inductor at t = 1 s.

Chapter 6, Solution 36.

(a) $v = L\frac{di}{dt} = 12x10^{-3}(30e^{-2t} - 60te^{-2t}) = \underline{(0.36 - 0.72t)e^{-2t}\ \text{V}}$

(b) $p = vi = (0.36 - 0.72x1)e^{-2}x30x1e^{-2} = 0.36x30e^{-4} = \underline{-0.1978\ \text{W}}$

(c) $\text{w} = \frac{1}{2}\text{Li}^2 = 0.5\text{x}12\text{x}10^{-3}(30\text{x}1\text{xe}^{-2})^2 =$ **<u>98.9 mJ</u>**.

Chapter 6, Problem 37.

The current through a 12-mH inductor is 4 sin 100***t*** A. Find the voltage, and also the energy stored in the inductor for $0 < t < \pi/200$ s.

Chapter 6, Solution 37.

$$\text{v} = \text{L}\frac{\text{di}}{\text{dt}} = 12\text{x}10^{-3}\,\text{x}4(100)\cos 100\text{t}$$

$$= \mathbf{\underline{4.8\ cos\ 100t\ V}}$$

p = vi = 4.8 x 4 sin 100t cos 100t = 9.6 sin 200t

$$\text{w} = \int_{\text{o}}^{\text{t}} \text{pdt} = \int_{\text{o}}^{11/200} 9.6\sin 200\text{t}$$

$$= -\frac{9.6}{200}\cos 200\text{t}\Big|_{\text{o}}^{11/200}\ \text{J}$$

$$= -48(\cos\pi - 1)\text{mJ} = \mathbf{\underline{96\ mJ}}$$

Please note that this problem could have also been done by using (½)Li^2.

Chapter 6, Problem 38.

The current through a 40-mH inductor is

$$i(t) = \begin{cases} 0, & t < 0 \\ te^{-2t}\ \text{A}, & t > 0 \end{cases}$$

Find the voltage $\boldsymbol{v(t)}$.

Chapter 6, Solution 38.

$$\mathrm{v} = \mathrm{L}\frac{\mathrm{di}}{\mathrm{dt}} = 40\mathrm{x}10^{-3}\left(\mathrm{e}^{-2t} - 2t\mathrm{e}^{-2t}\right)\mathrm{dt}$$

$$= \underline{40(1-2t)e^{-2t}\mathrm{mV}, t > 0}$$

Chapter 6, Problem 39.

The voltage across a 200-mH inductor is given by

$v(t) = 3t^2 + 2t + 4$ V for $t > 0$.

Determine the current $i(t)$ through the inductor. Assume that $i(0) = 1$ A.

Chapter 6, Solution 39

$$\mathrm{v} = \mathrm{L}\frac{\mathrm{di}}{\mathrm{dt}} \longrightarrow \mathrm{i} = \frac{1}{\mathrm{L}}\int_0^t \mathrm{i}\,\mathrm{dt} + \mathrm{i}(0)$$

$$\mathrm{i} = \frac{1}{200\mathrm{x}10^{-3}}\int_0^t (3t^2 + 2t + 4)\mathrm{dt} + 1$$

$$= 5(t^3 + t^2 + 4t)\Big|_0^t + 1$$

i(t) = $\underline{\mathbf{5t^3 + 5t^2 + 20t + 1\ A}}$

Chapter 6, Problem 40.

The current through a 5-mH inductor is shown in Fig. 6.66. Determine the voltage across the inductor at t=1,3, and 5ms.

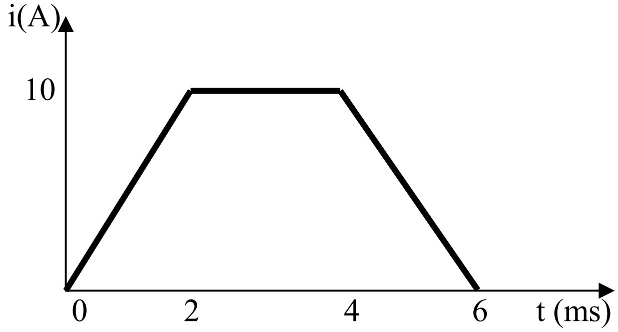


Figure 6.66 For Prob. 6.40.

Chapter 6, Solution 40.

$$i = \begin{cases} 5t, & 0 < t < 2ms \\ 10, & 2 < t < 4ms \\ 30 - 5t, & 4 < t < 6ms \end{cases}$$

$$v = L\frac{di}{dt} = \frac{5x10^{-3}}{10^{-3}}\begin{cases} 5, & 0 < t < 2ms \\ 0, & 2 < t < 4ms \\ -5, & 4 < t < 6ms \end{cases} = \begin{cases} 25, & 0 < t < 2ms \\ 0, & 2 < t < 4ms \\ -25, & 4 < t < 6ms \end{cases}$$

At t=1ms, v=25 V
At t=3ms, v=0 V
At t=5ms, v=-25 V

Chapter 6, Problem 41.

The voltage across a 2-H inductor is $20(1 - e^{-2t})$ V. If the initial current through the inductor is 0.3 A, find the current and the energy stored in the inductor at $t = 1$ s.

Chapter 6, Solution 41.

$$i = \frac{1}{L}\int_0^t v dt + C = \left(\frac{1}{2}\right)\int_o^t 20\left(1 - e^{-2t}\right)dt + C$$

$$= 10\left(t + \frac{1}{2}e^{-2t}\right)\Big|_o^t + C = 10t + 5e^{-2t} - 4.7\text{A}$$

Note, we get C = –4.7 from the initial condition for i needing to be 0.3 A.

We can check our results be solving for v = Ldi/dt.

$v = 2(10 - 10e^{-2t})$V which is what we started with.

At t = 1 s, $i = 10 + 5e^{-2} - 4.7 = 10 + 0.6767 - 4.7 =$ **5.977 A**

$w = \frac{1}{2}L\,i^2 =$ **35.72J**

Chapter 6, Problem 42.

If the voltage waveform in Fig. 6.67 is applied across the terminals of a 5-H inductor, calculate the current through the inductor. Assume ***i*(0)** = -1 A.

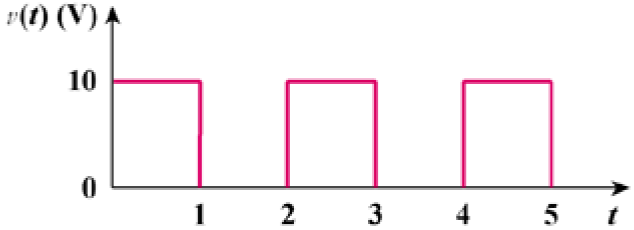


Figure 6.67

Chapter 6, Solution 42.

$$i = \frac{1}{L}\int_o^t vdt + i(0) = \frac{1}{5}\int_o^t v(t)dt - 1$$

For $0 < t < 1, \ i = \frac{10}{5}\int_0^t dt - 1 = 2t - 1 \text{ A}$

For $1 < t < 2, \quad i = 0 + i(1) = 1\text{A}$

For $2 < t < 3, \quad i = \frac{1}{5}\int 10dt + i(2) = 2t\Big|_t^2 + 1$
$= 2t - 3$ A

For $3 < t < 4, \quad i = 0 + i(3) = 3$ A

For $4 < t < 5, \quad i = \frac{1}{5}\int_4^t 10dt + i(4) = 2t\Big|_4^t + 3$
$= 2t - 5$ A

$$\text{Thus,}\quad i(t) = \begin{cases} 2t - 1\text{A}, & 0 < t < 1 \\ 1\text{A}, & 1 < t < 2 \\ 2t - 3\text{A}, & 2 < t < 3 \\ 3\text{A}, & 3 < t < 4 \\ 2t - 5, & 4 < t < 5 \end{cases}$$

Chapter 6, Problem 43.

The current in an 80-mH inductor increases from 0 to 60 mA.
How much energy is stored in the inductor?

Chapter 6, Solution 43.

$$w = L\int_{-\infty}^{t} i\,dt = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty)$$
$$= \frac{1}{2}x80x10^{-3}x\left(60x10^{-3}\right)^2 - 0$$
$$= \underline{\mathbf{144\ \mu J}}$$

***Chapter 6, Problem 44.**

A 100-mH inductor is connected in parallel with a 2-kΩ resistor. The current through the inductor is $i(t) = 50e^{-400t}$ mA. (a) Find the voltage v_L across the inductor. (b) Find the voltage v_R across the resistor. (c) Is $v_R(t) + v_L(t) = 0$? (d) Calculate the energy in the inductor at t=0.

Chapter 6, Solution 44.

(a) $v_L = L\frac{di}{dt} = 100x10^{-3}(-400)x50x10^{-3}e^{-400t} = \underline{-2e^{-400t}\ \text{V}}$

(b) Since R and L are in parallel, $v_R = v_L = \underline{-2e^{-400t}\ \text{V}}$

(c) **<u>No</u>**

(d) $w = \frac{1}{2}Li^2 = 0.5x100x10^{-3}(0.05)^2 =$ **<u>125 μJ</u>**.

Chapter 6, Problem 45.

If the voltage waveform in Fig. 6.68 is applied to a 10-mH inductor, find the inductor current ***i*(*t*)**. Assume ***i*(0)** = 0.

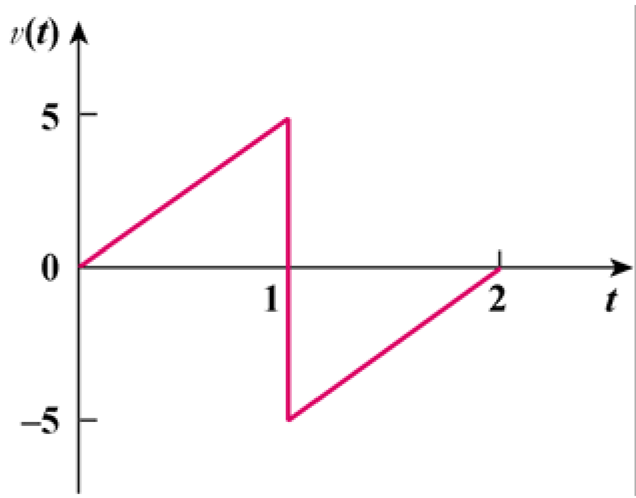


Figure 6.68

Chapter 6, Solution 45.

$$i(t) = \frac{1}{L}\int_o^t v(t) + i(0)$$

For $0 < t < 1$, $v = 5t$

$$i = \frac{1}{10x10^{-3}}\int_o^t 5t\,dt + 0$$

$$= 0.25t^2 \text{ kA}$$

For $1 < t < 2$, $v = -10 + 5t$

$$i = \frac{1}{10x10^{-3}}\int_1^t (-10 + 5t)dt + i(1)$$

$$= \int_1^t (0.5t - 1)dt + 0.25kA$$

$$= 1 - t + 0.25t^2 \text{ kA}$$

$$i(t) = \begin{bmatrix} 0.25t^2 kA, & 0 < t < 1s \\ 1 - t + 0.25t^2 kA, & 1 < t < 2s \end{bmatrix}$$

Chapter 6, Problem 46.

Find v_C, i_L, and the energy stored in the capacitor and inductor in the circuit of Fig. 6.69 under dc conditions.

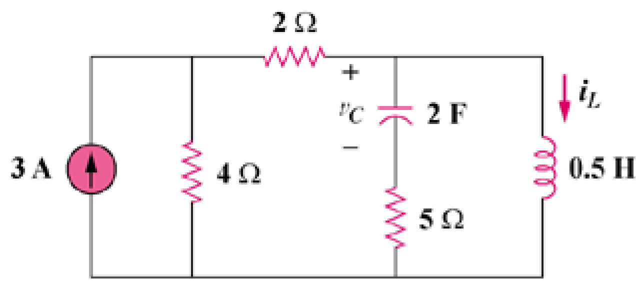
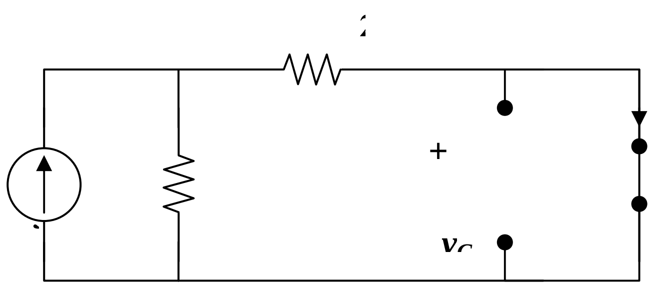


Figure 6.69

Chapter 6, Solution 46.

Under dc conditions, the circuit is as shown below:



By current division,

$$i_L = \frac{4}{4+2}(3) = \underline{\mathbf{2A}}, \quad v_c = \underline{\mathbf{0V}}$$

$$w_L = \frac{1}{2} L\, i_L^2 = \frac{1}{2}\left(\frac{1}{2}\right)(2)^2 = \underline{\mathbf{1J}}$$

$$w_c = \frac{1}{2} C\, v_c^2 = \frac{1}{2}(2)(v) = \underline{\mathbf{0J}}$$

Chapter 6, Problem 47.

For the circuit in Fig. 6.70, calculate the value of ***R*** that will make the energy stored in the capacitor the same as that stored in the inductor under dc conditions.

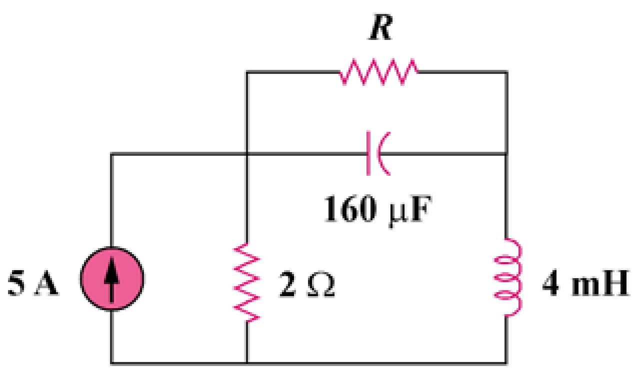
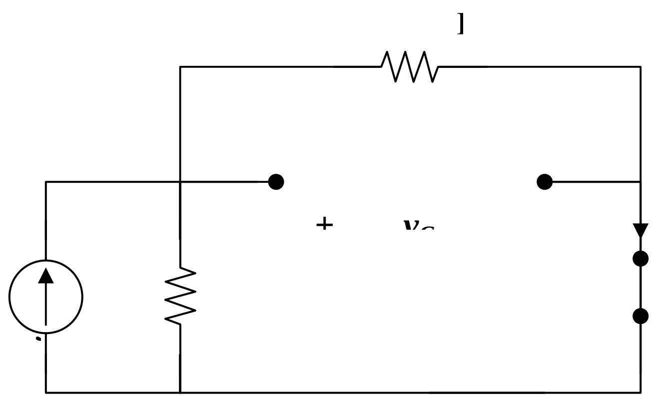


Figure 6.70

Chapter 6, Solution 47.

Under dc conditions, the circuit is equivalent to that shown below:



$$i_L = \frac{2}{R+2}(5) = \frac{10}{R+2}, \quad v_c = Ri_L = \frac{10R}{R+2}$$

$$w_c = \frac{1}{2}Cv_c^2 = 80x10^{-6}x\frac{100R^2}{(R+2)^2}$$

$$w_L = \frac{1}{2}Li_l^2 = 2x10^{-3}x\frac{100}{(R+2)^2}$$

If $w_c = w_L$,

$$80x10^{-6}x\frac{100R^2}{(R+2)^2} = \frac{2x10^{-3}x100}{(R+2)^2} \longrightarrow 80 \text{ x } 10^{-3}R^2 = 2$$

R = <u>**5Ω**</u>

Chapter 6, Problem 48.

Under steady-state dc conditions, find i and v in the circuit in Fig. 6.71.

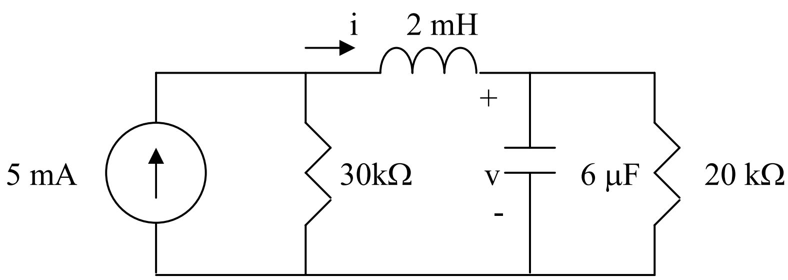
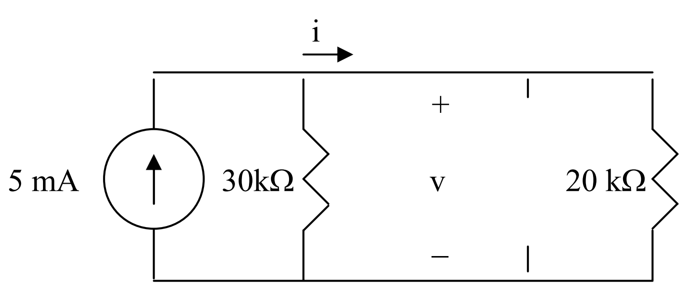


Figure 6.71 For Prob. 6.48.

Chapter 6, Solution 48.

Under steady-state, the inductor acts like a short-circuit, while the capacitor acts like an open circuit as shown below.



Using current division,

$$i = \frac{30k}{30k + 20k}(5mA) = \underline{3\ \text{mA}}$$

v = 20ki = **<u>60 V</u>**

Chapter 6, Problem 49.

Find the equivalent inductance of the circuit in Fig. 6.72. Assume all inductors are 10 mH.

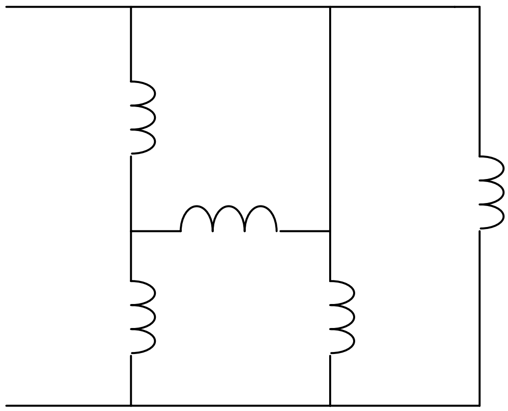
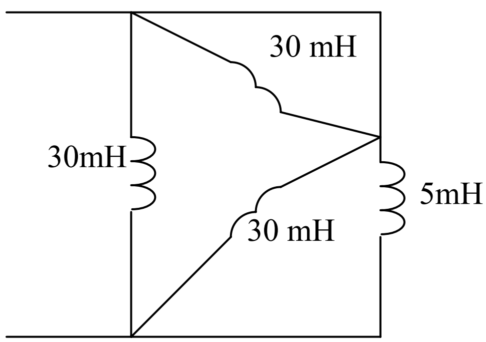


Figure 6.72 For Prob. 6.49.

Chapter 6, Solution 49.

Converting the wye-subnetwork to its equivalent delta gives the circuit below.



30//0 = 0, 30//5 = 30x5/35=4.286

$$L_{eq} = 30 // 4.286 = \frac{30x4.286}{34.286} = \underline{3.75 \text{ mH}}$$

Chapter 6, Problem 50.

An energy-storage network consists of series-connected 16-mH and 14-mH inductors in parallel with a series connected 24-mH and 36-mH inductors. Calculate the equivalent inductance.

Chapter 6, Solution 50.

16mH in series with 14 mH = 16+14=30 mH
24 mH in series with 36 mH = 24+36=60 mH
30mH in parallel with 60 mH = 30x60/90 = **20 mH**

Chapter 6, Problem 51.

Determine L_{eq} at terminals ***a-b*** of the circuit in Fig. 6.73.

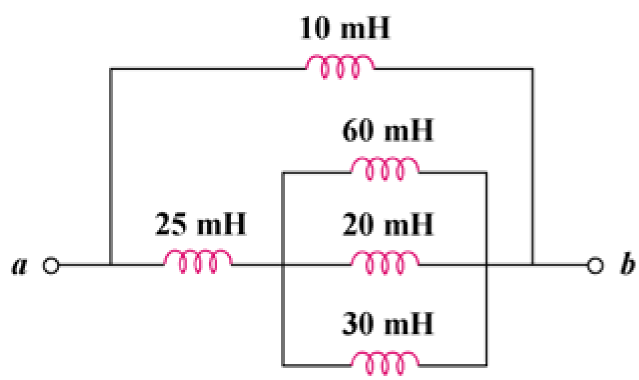


Figure 6.73

Chapter 6, Solution 51.

$$\frac{1}{L} = \frac{1}{60} + \frac{1}{20} + \frac{1}{30} = \frac{1}{10} \qquad L = 10 \text{ mH}$$

$$L_{eq} = 10 \| (25 + 10) = \frac{10x35}{45}$$

$$= \underline{\mathbf{7.778\ mH}}$$

Chapter 6, Problem 52.

Find L_{eq} in the circuit of Fig. 6.74.

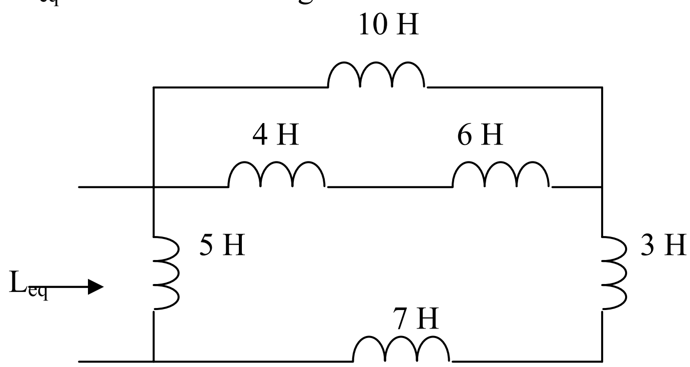


Figure 6.74 For Prob. 6.52.

Chapter 6, Solution 52.

$$L_{eq} = 5//(7+3+10//(4+6)) == 5//(7+3+5)) = \frac{5x15}{20} = \underline{3.75\ \ H}$$

Chapter 6, Problem 53.

Find $\boldsymbol{L}_{\mathbf{eq}}$ at the terminals of the circuit in Fig. 6.75.

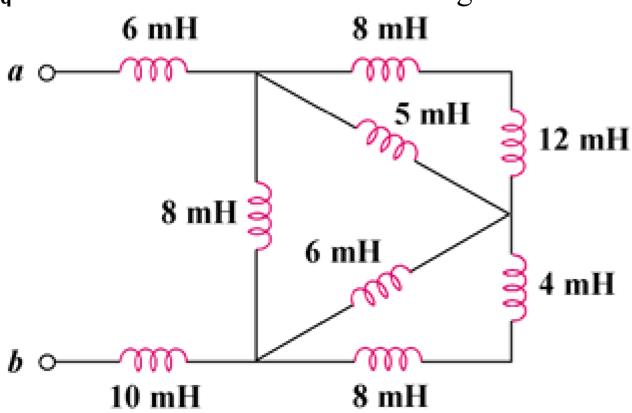


Figure 6.75

Chapter 6, Solution 53.

$$L_{eq} = 6 + 10 + 8\|[5\|(8+12) + 6\|(8+4)]$$

$$= 16 + 8\|(4+4) = 16 + 4$$

L_{eq} = **20 mH**

Chapter 6, Problem 54.

Find the equivalent inductance looking into the terminals of the circuit in Fig. 6.76.

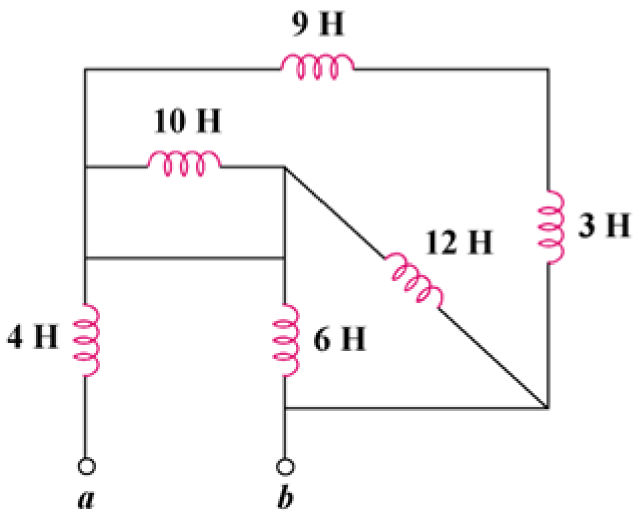


Figure 6.76

Chapter 6, Solution 54.

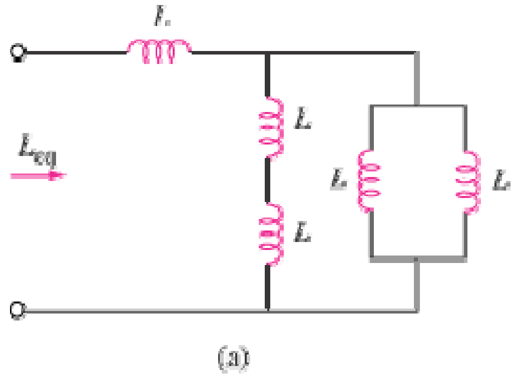
$$L_{eq} = 4 + (9+3)\|(10\|0 + 6\|12)$$

$$= 4 + 12\|(0+4) = 4 + 3$$

L_{eq} = **<u>7H</u>**

Chapter 6, Problem 55.

Find L_{eq} in each of the circuits of Fig. 6.77.



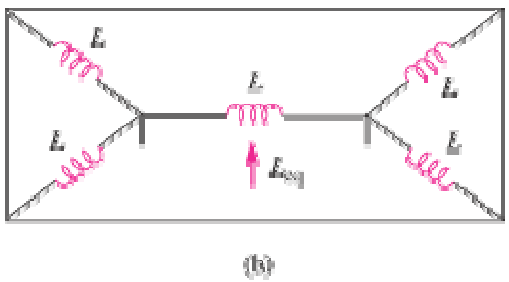


Figure 6.77

Chapter 6, Solution 55.

(a) L//L = 0.5L, L + L = 2L

$$L_{eq} = L + 2L // 0.5L = L + \frac{2Lx0.5L}{2L + 0.5L} = \underline{1.4L} = \underline{\mathbf{1.4\ L}}.$$

(b) L//L = 0.5L, L//L + L//L = L

$$L_{eq} = L//L = \underline{\mathbf{500\ mL}}$$

Chapter 6, Problem 56.

Find $\boldsymbol{L_{eq}}$ in the circuit in Fig. 6.78.

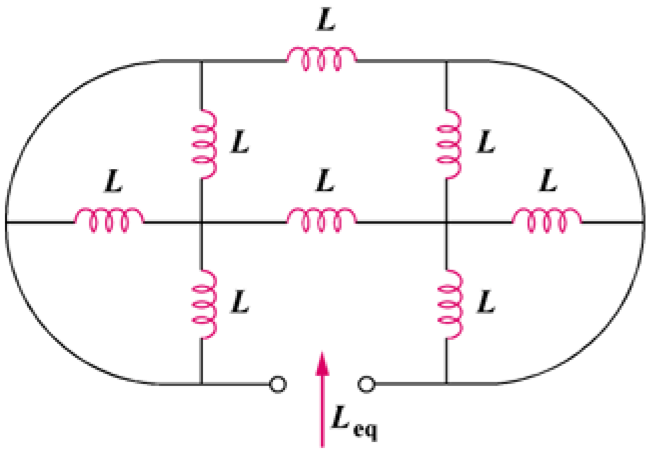
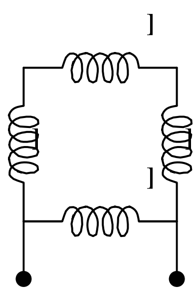


Figure 6.78

Chapter 6, Solution 56.

$$L\|L\|L = \frac{1}{\frac{3}{L}} = \frac{L}{3}$$

Hence the given circuit is equivalent to that shown below:



$$L_{eq} = L \,\Big\|\, \left(L + \frac{2}{3}L \right) = \frac{L x \frac{5}{3}L}{L + \frac{5}{3}L} = \underline{\frac{5}{8}L}$$

Chapter 6, Problem 57.

Determine the L_{eq} that can be used to represent the inductive network of Fig. 6.79 at the terminals.

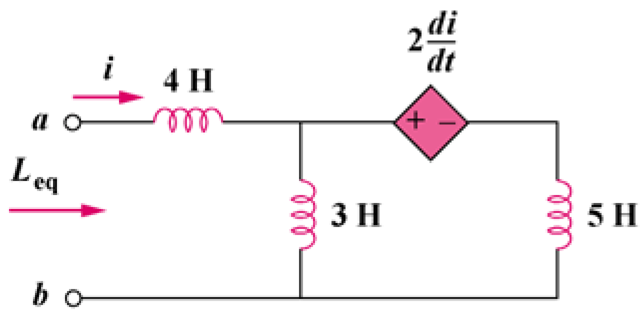


Figure 6.79

Chapter 6, Solution 57.

$$\text{Let } v = L_{eq}\frac{di}{dt} \qquad (1)$$

$$v = v_1 + v_2 = 4\frac{di}{dt} + v_2 \qquad (2)$$

$$i = i_1 + i_2 \longrightarrow i_2 = i - i_1 \qquad (3)$$

$$v_2 = 3\frac{di_1}{dt} \text{ or } \frac{di_1}{dt} = \frac{v_2}{3} \qquad (4)$$

and

$$-v_2 + 2\frac{di}{dt} + 5\frac{di_2}{dt} = 0$$

$$v_2 = 2\frac{di}{dt} + 5\frac{di_2}{dt} \qquad (5)$$

Incorporating (3) and (4) into (5),

$$v_2 = 2\frac{di}{dt} + 5\frac{di}{dt} - 5\frac{di_1}{dt} = 7\frac{di}{dt} - 5\frac{v_2}{3}$$

$$v_2\left(1 + \frac{5}{3}\right) = 7\frac{di}{dt}$$

$$v_2 = \frac{21}{8}\frac{di}{dt}$$

Substituting this into (2) gives

$$v = 4\frac{di}{dt} + \frac{21}{8}\frac{di}{dt}$$

$$= \frac{53}{8}\frac{di}{dt}$$

Comparing this with (1),

$$L_{eq} = \frac{53}{8} = \underline{\mathbf{6.625\ H}}$$

Chapter 6, Problem 58.

The current waveform in Fig. 6.80 flows through a 3-H inductor. Sketch the voltage across the inductor over the interval $0 < t < 6$ s.

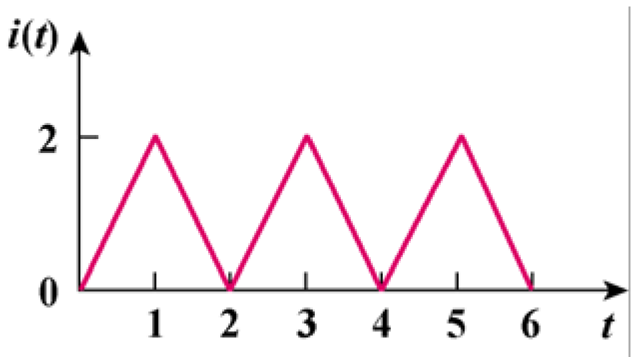
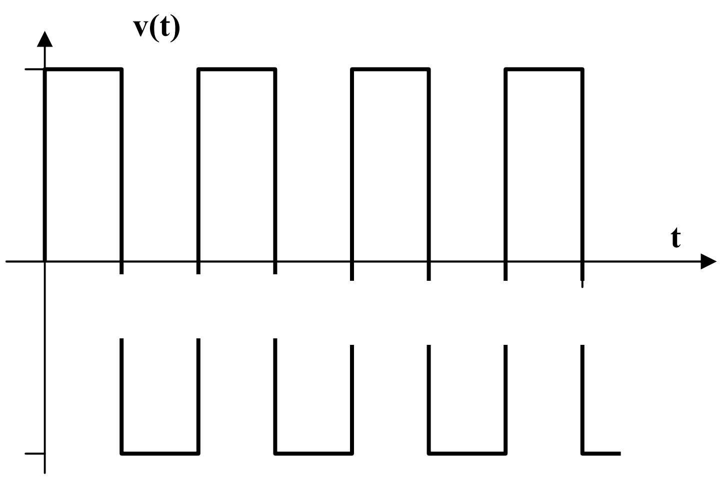


Figure 6.80

Chapter 6, Solution 58.

$$v = L\frac{di}{dt} = 3\frac{di}{dt} = 3 \text{ x slope of } i(t).$$

Thus v is sketched below:



Chapter 6, Problem 59.

(a) For two inductors in series as in Fig. 6.81(a), show that the current-division principle is

$$v_1 = \frac{L_1}{L_1 + L_2} v_s, \qquad v_2 = \frac{L_2}{L_1 + L_2} v_s$$

assuming that the initial conditions are zero.

(b) For two inductors in parallel as in Fig. 6.81(b), show that the current-division principle is

$$i_1 = \frac{L_2}{L_1 + L_2} i_s, \qquad i_2 = \frac{L_1}{L_1 + L_2} i_s$$

assuming that the initial conditions are zero.

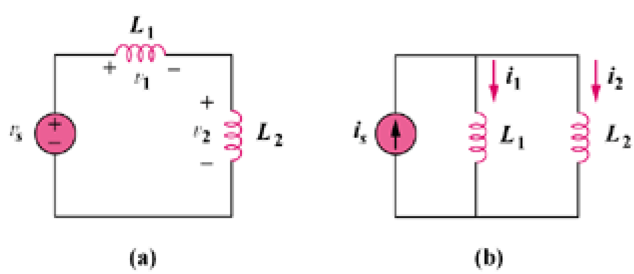


Figure 6.81

Chapter 6, Solution 59.

(a) $v_s = (L_1 + L_2)\dfrac{di}{dt}$

$$\frac{di}{dt} = \frac{v_s}{L_1 + L_2}$$

$$v_1 = L_1\frac{di}{dt},\ v_2 = L_2\frac{di}{dt}$$

$$\underline{v_1 = \frac{L_1}{L_1 + L_2}v_s,\ v_L = \frac{L_2}{L_1 + L_2}v_s}$$

(b) $v_i = v_2 = L_1\dfrac{di_1}{dt} = L_2\dfrac{di_2}{dt}$

$$i_s = i_1 + i_2$$

$$\frac{di_s}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = \frac{v}{L_1} + \frac{v}{L_2} = v\frac{(L_1 + L_2)}{L_1L_2}$$

$$i_1 = \frac{1}{L_1}\int vdt = \frac{1}{L_1}\int\frac{L_1L_2}{L_1 + L_2}\frac{di_s}{dt}dt = \underline{\frac{L_2}{L_1 + L_2}i_s}$$

$$i_2 = \frac{1}{L_2}\int vdt = \frac{1}{L_2}\int\frac{L_1L_2}{L_1 + L_2}\frac{di_s}{dt}dt = \underline{\frac{L_1}{L_1 + L_2}i_s}$$

Chapter 6, Problem 60.

In the circuit of Fig. 6.82, $i_o(0) = 2$ A. Determine $i_o(t)$ and $v_o(t)$ for $t > 0$.

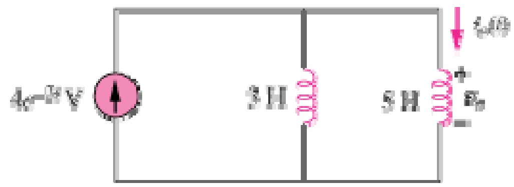


Figure 6.82

Chapter 6, Solution 60

$$L_{eq} = 3 // 5 = \frac{15}{8}$$

$$v_o = L_{eq}\frac{di}{dt} = \frac{15}{8}\frac{d}{dt}\left(4e^{-2t}\right) = \underline{-15e^{-2t}}$$

$$i_o = \frac{I}{L}\int_0^t v_o(t)dt + i_o(0) = 2 + \frac{1}{5}\int_0^t (-15)e^{-2t}dt = 2 + 1.5e^{-2t}\Big|_0^t = \underline{0.5 + 1.5e^{-2t}}\text{ A}$$

Chapter 6, Problem 61.

Consider the circuit in Fig. 6.83. Find: (a) L_{eq}, $i_1(t)$ and $i_2(t)$ if $i_s = 3e^{-t}$ mA, (b) $v_o(t)$, (c) energy stored in the 20-mH inductor at t=1s.

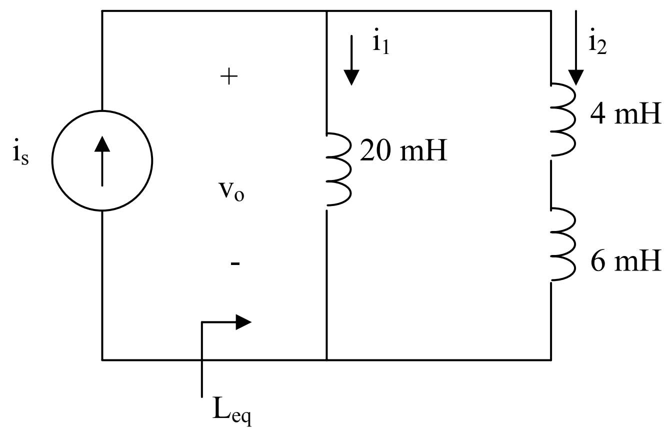


Figure 6.83 For Prob. 6.61.

Chapter 6, Solution 61.

(a) $L_{eq} = 20//(4+6) = 20x10/30 = \underline{6.667 \text{ mH}}$

Using current division,

$$i_1(t) = \frac{10}{10+20} i_s = \underline{e^{-t} \text{ mA}}$$

$$i_2(t) = \underline{2e^{-t} \text{ mA}}$$

(b) $v_o = L_{eq} \dfrac{di_s}{dt} = \dfrac{20}{3} x10^{-3}(-3e^{-t} x10^{-3}) = \underline{-20e^{-t} \ \mu\text{V}}$

(c) $w = \dfrac{1}{2} L i_1^2 = \dfrac{1}{2} x20x10^{-3} x e^{-2} x10^{-6} = \underline{1.3534 \text{ nJ}}$

Chapter 6, Problem 62.

Consider the circuit in Fig. 6.84. Given that $v(t) = 12e^{-3t}$ mV for $t > 0$ and $i_1(0) = -10$ mA, find: (a) $i_2(0)$, (b) $i_1(t)$ and $i_2(t)$.

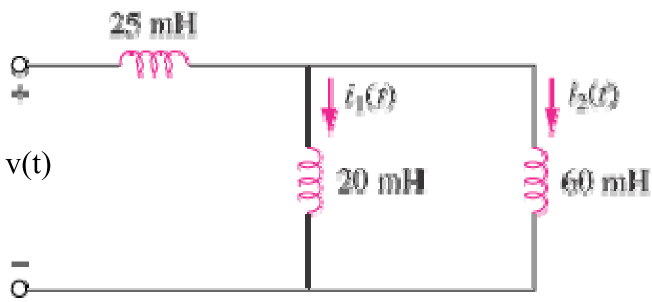


Figure 6.84

Chapter 6, Solution 62.

(a) $L_{eq} = 25 + 20 // 60 = 25 + \dfrac{20x60}{80} = 40 \text{ mH}$

$$v = L_{eq}\frac{di}{dt} \longrightarrow i = \frac{1}{L_{eq}}\int v(t)dt + i(0) = \frac{10^{-3}}{40x10^{-3}}\int_0^t 12e^{-3t}dt + i(0) = -0.1(e^{-3t} - 1) + i(0)$$

Using current division and the fact that all the currents were zero when the circuit was put together, we get,

$$i_1 = \frac{60}{80}i = \frac{3}{4}i, \quad i_2 = \frac{1}{4}i$$

$$i_1(0) = \frac{3}{4}i(0) \longrightarrow 0.75i(0) = -0.01 \longrightarrow i(0) = -0.01333$$

$$i_2 = \frac{1}{4}(-0.1e^{-3t} + 0.08667) \text{ A} = \text{-}25\text{e}^{-3t} + 21.67 \text{ mA}$$

$$i_2(0) = -25 + 21.67 = \underline{-3.33 \text{ mA}}$$

(b) $i_1 = \dfrac{3}{4}(-0.1e^{-3t} + 0.08667) \text{ A} = \underline{\text{-}75\text{e}^{-3t} + 65 \text{ mA}}$

$i_2 = \underline{\text{-}25\text{e}^{-3t} + 21.67 \text{ mA}}$

Chapter 6, Problem 63.

In the circuit in Fig. 6.85, sketch v_o.

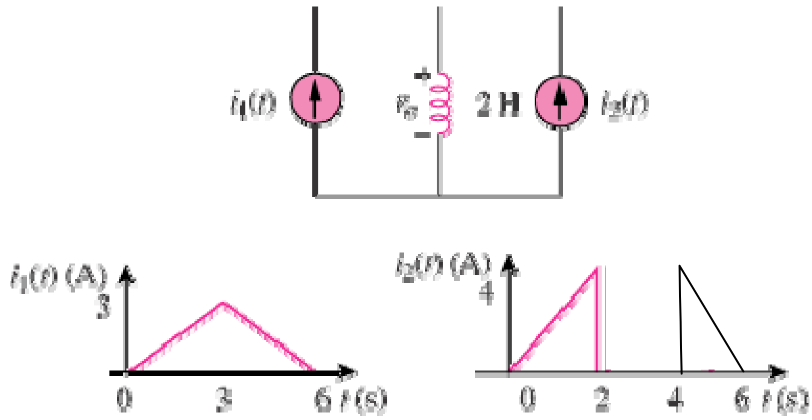


Figure 6.85

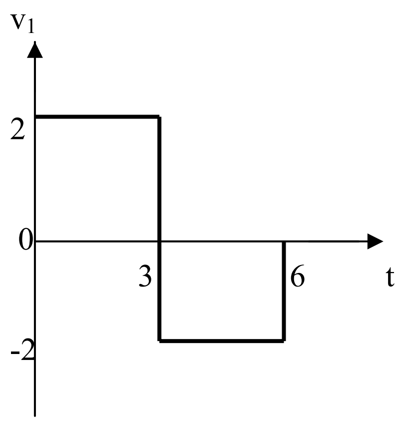
Chapter 6, Solution 63.

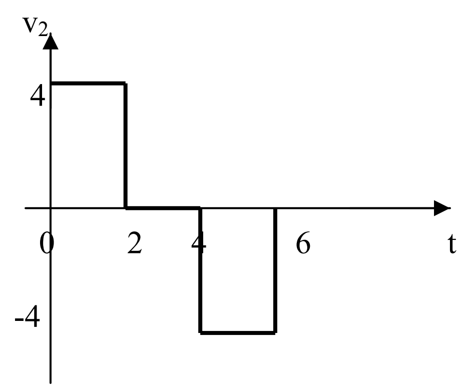
We apply superposition principle and let
$v_o = v_1 + v_2$

where v_1 and v_2 are due to i_1 and i_2 respectively.

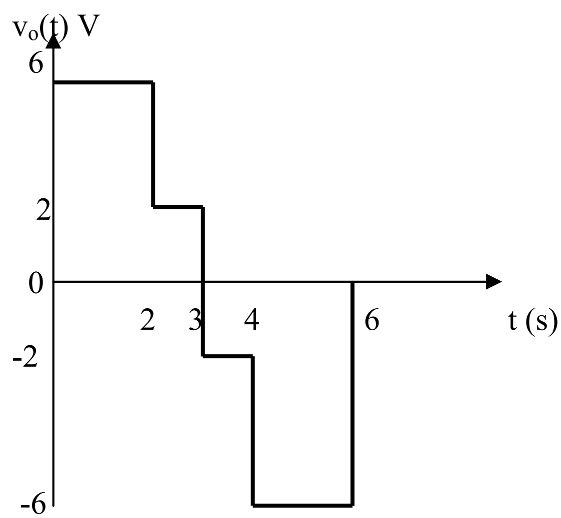
$$v_1 = L\frac{di_1}{dt} = 2\frac{di_1}{dt} = \begin{cases} 2, & 0 < t < 3 \\ -2, & 3 < t < 6 \end{cases}$$

$$v_2 = L\frac{di_2}{dt} = 2\frac{di_2}{dt} = \begin{cases} 4, & 0 < t < 2 \\ 0, & 2 < t < 4 \\ -4, & 4 < t < 6 \end{cases}$$





Adding v_1 and v_2 gives v_o, which is shown below.



Chapter 6, Problem 64.

The switch in Fig. 6.86 has been in position *A* for a long time. At $t = 0$, the switch moves from position *A* to *B*. The switch is a make-before-break type so that there is no interruption in the inductor current. Find:
(a) $i(t)$ for $t > 0$,
(b) *v just after* the switch has been moved to position *B*,
(c) $v(t)$ long after the switch is in position *B*.

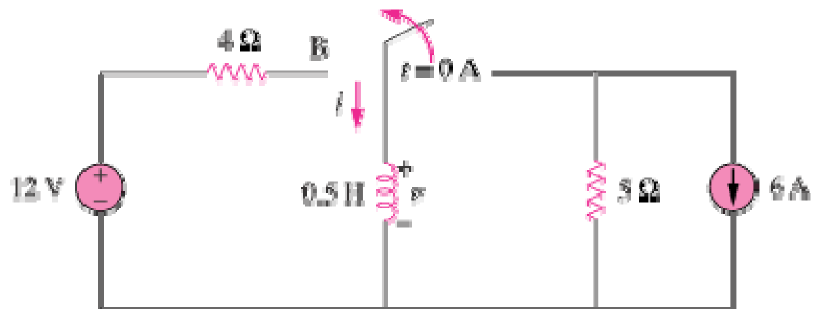


Figure 6.86

Chapter 6, Solution 64.

(a) When the switch is in position A,
i= –6 = i(0)
When the switch is in position B,
$i(\infty) = 12/4 = 3, \qquad \tau = L/R = 1/8$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/t} = \underline{3 - 9e^{-8t} \ \text{A}}$$

(b) -12 + 4i(0) + v=0, i.e. v=12 – 4i(0) = 36 V

(c) At steady state, the inductor becomes a short circuit so that
v= 0 V

Chapter 6, Problem 65.

The inductors in Fig. 6.87 are initially charged and are connected to the black box at $t = 0$. If $i_1(0) = 4$ A, $i_2(0) = -2$ A, and $v(t) = 50e^{-200t}$ mV, $t \geq 0$\$, find:

(a). the energy initially stored in each inductor,
(b). the total energy delivered to the black box from $t = 0$ to $t = \infty$,
(c). $i_1(t)$ and $i_2(t)$, $t \geq 0$,
(d). $i(t)$, $t \geq 0$.

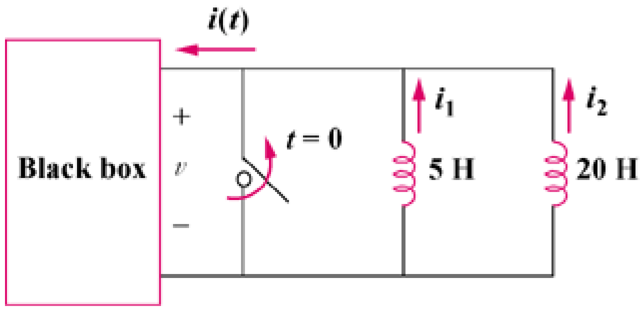


Figure 6.87

Chapter 6, Solution 65.

(a) $w_5 = \frac{1}{2}L_1 i_1^2 = \frac{1}{2} x 5 x (4)^2 = \underline{\textbf{40 J}}$

$w_{20} = \frac{1}{2}(20)(-2)^2 = \underline{\textbf{40 J}}$

(b) $w = w_5 + w_{20} = \underline{\textbf{80 J}}$

(c) $i_1 = \frac{1}{L_1}\int_0^t -50e^{-200t}dt + i_1(0) = \frac{1}{5}\left(\frac{1}{200}\right)\left(50e^{-200t}x10^{-3}\right)\Big|_0^t + 4$

$= \underline{\mathbf{5x10^{-5}(e^{-200t} - 1) + 4\ A}}$

$i_2 = \frac{1}{L_2}\int_0^t -50e^{-200t}dt + i_2(0) = \frac{1}{20}\left(\frac{1}{200}\right)\left(50e^{-200t}x10^{-3}\right)\Big|_0^t - 2$

$= \underline{\mathbf{1.25x10^{-5}\ (e^{-200t} - 1) - 2\ A}}$

(d) $i = i_1 + i_2 = \underline{\mathbf{6.25x10^{-5}\ (e^{-200t} - 1) + 2\ A}}$

Chapter 6, Problem 66.

The current i(t) through a 20-mH inductor is equal, in magnitude, to the voltage across it for all values of time. If i(0) =2 A, find i(t).

Chapter 6, Solution 66.

If v=i, then

$$i = L\frac{di}{dt} \longrightarrow \frac{dt}{L} = \frac{di}{i}$$

Integrating this gives

$$\frac{t}{L} = \ln(i) - \ln(C_o) = \ln\left(\frac{i}{C_o}\right) \rightarrow i = C_o e^{t/L}$$

$i(0) = 2 = C_o$

$$i(t) = 2e^{t/0.02} = \underline{\mathbf{2e^{50t}\ A}}.$$

Chapter 6, Problem 67.

An op amp integrator has R= 50 kΩ and C = 0.04 μF. If the input voltage is v_i = 10 sin 50t mV, obtain the output voltage.

Chapter 6, Solution 67.

$$v_o = -\frac{1}{RC}\int v_i\, dt, \text{ RC} = 50 \times 10^3 \times 0.04 \times 10^{-6} = 2 \times 10^{-3}$$

$$v_o = \frac{-10^3}{2}\int 10\sin 50t\ dt$$

v_o = **<u>100 cos 50t mV</u>**

Chapter 6, Problem 68.

A 10-V dc voltage is applied to an integrator with R = 50 kΩ, C = 100 μF at t = 0. How long will it take for the op amp to saturate if the saturation voltages are +12 V and -12 V? Assume that the initial capacitor voltage was zero.

Chapter 6, Solution 68.

$$v_o = -\frac{1}{RC}\int v_i\, dt + v(0), \text{ RC} = 50 \times 10^3 \times 100 \times 10^{-6} = 5$$

$$v_o = -\frac{1}{5}\int_o^t 10dt + 0 = -2t$$

The op amp will saturate at $v_o = \pm 12$

$$-12 = -2t \longrightarrow \underline{\mathbf{t = 6s}}$$

Chapter 6, Problem 69.

An op amp integrator with $R = 4$ MΩ and $C = 1$ μF has the input waveform shown in Fig. 6.88. Plot the output waveform.

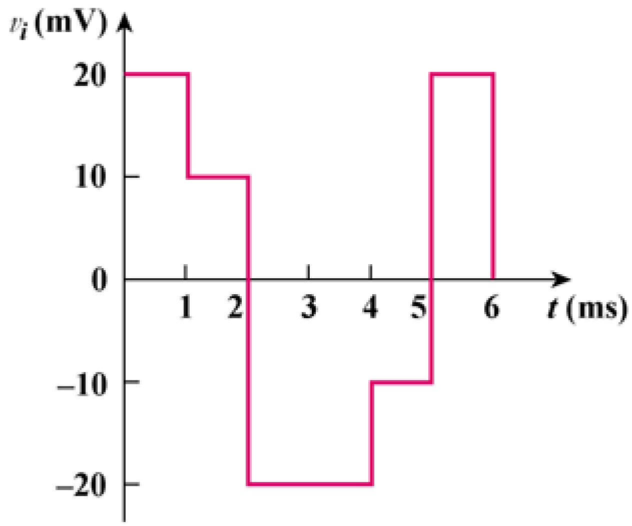


Figure 6.88

Chapter 6, Solution 69.

$RC = 4 \times 10^6 \times 1 \times 10^{-6} = 4$

$$v_o = -\frac{1}{RC}\int v_i dt = -\frac{1}{4}\int v_i dt$$

For $0 < t < 1$, $v_i = 20$, $v_o = -\frac{1}{4}\int_o^t 20dt = -5t$ mV

For $1 < t < 2$, $v_i = 10$, $v_o = -\frac{1}{4}\int_1^t 10dt + v(1) = -2.5(t-1) - 5$

$= -2.5t - 2.5$mV

For $2 < t < 4$, $v_i = -20$, $v_o = +\frac{1}{4}\int_2^t 20dt + v(2) = 5(t-2) - 7.5$

$= 5t - 17.5$ mV

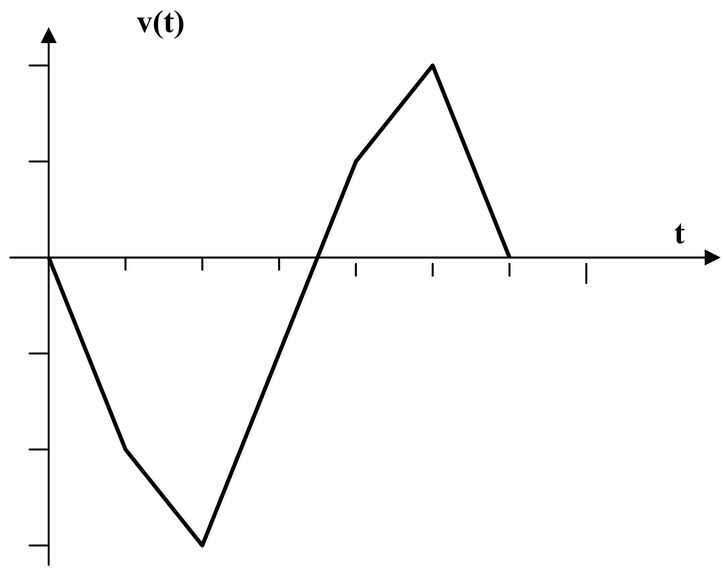
For $4 < t < 5$m, $v_i = -10$, $v_o = \frac{1}{4}\int_4^t 10dt + v(4) = 2.5(t-4) + 2.5$

$= 2.5t - 7.5$ mV

For $5 < t < 6$, $v_i = 20$, $v_o = -\frac{1}{4}\int_5^t 20dt + v(5) = -5(t-5) + 5$

$= -5t + 30$ mV

Thus $v_o(t)$ is as shown below:



Chapter 6, Problem 70.

Using a single op amp, a capacitor, and resistors of 100 kΩ or less, design a circuit to implement

$$v_0 = -50\int_0^t v_i(t)\,dt$$

Assume $v_o = 0$ at $t = 0$.

Chapter 6, Solution 70.
One possibility is as follows:

$$\frac{1}{RC} = 50$$

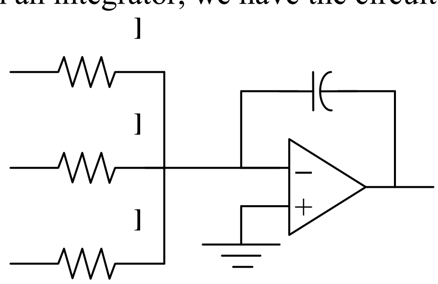
Let R = 100 kΩ, $C = \dfrac{1}{50x100x10^3} = 0.2\mu F$

Chapter 6, Problem 71.
Show how you would use a single op amp to generate

$$v_0 = -\int (v_1 + 4v_2 + 10v_3)\,dt$$

If the integrating capacitor is $C = 2\ \mu F$, obtain other component values.

Chapter 6, Solution 71.
By combining a summer with an integrator, we have the circuit below:



$$v_o = -\frac{1}{R_1C}\int v_1 dt - \frac{1}{R_2C}\int v_2 dt - \frac{1}{R_2C}\int v_2 dt$$

For the given problem, C = **2μF**,

$R_1C = 1 \longrightarrow R_1 = 1/(C) = 10^6/(2) =$ **500 kΩ**

$R_2C = 1/(4) \longrightarrow R_2 = 1/(4C) = 500k\Omega/(4) =$ **125 kΩ**

$R_3C = 1/(10) \longrightarrow R_3 = 1/(10C) =$ **50 kΩ**

Chapter 6, Problem 72.

At $t = 1.5$ ms, calculate v_o due to the cascaded integrators in Fig. 6.89. Assume that the integrators are reset to 0 V at $t = 0$.

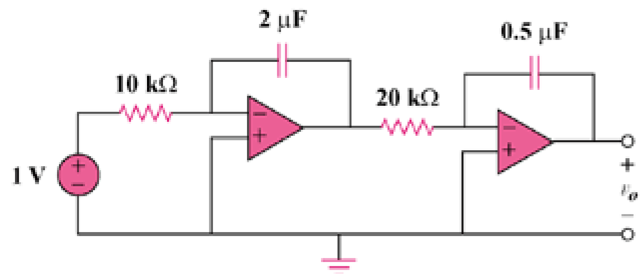


Figure 6.89

Chapter 6, Solution 72.

The output of the first op amp is

$$v_1 = -\frac{1}{RC}\int v_i\,dt = -\frac{1}{10x10^3 x2x10^{-6}}\int_o^t v_i dt = -\frac{100t}{2}$$

$$= -50t$$

$$v_o = -\frac{1}{RC}\int v_i\,dt = -\frac{1}{20x10^3 x0.5x10^{-6}}\int_o^t (-50t)dt$$

$$= 2500t^2$$

At t = 1.5ms,

$v_o = 2500(1.5)^2 x10^{-6}$ = **5.625 mV**

Chapter 6, Problem 73.

Show that the circuit in Fig. 6.90 is a noninverting integrator.

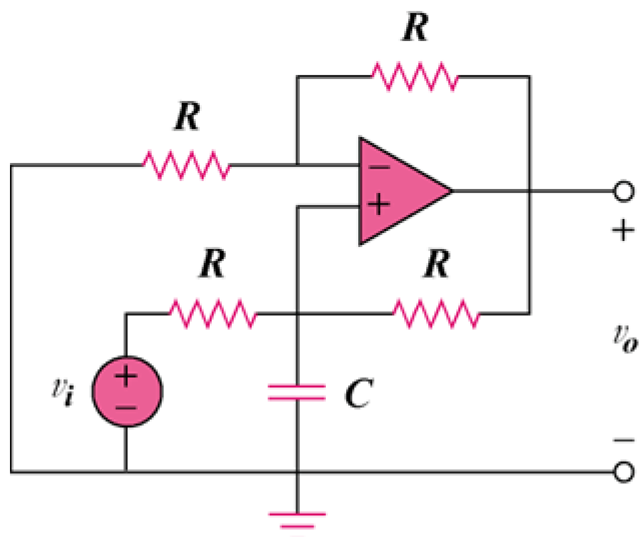


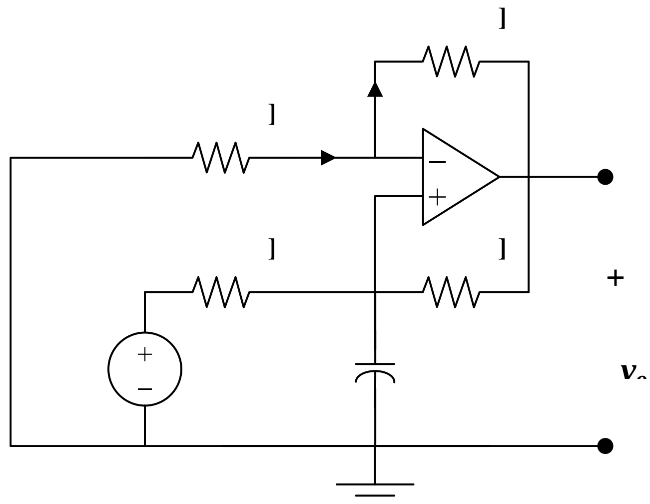
Figure 6.90

Chapter 6, Solution 73.

Consider the op amp as shown below:

Let $v_a = v_b = v$

At node a, $\frac{0 - v}{R} = \frac{v - v_o}{R} \longrightarrow 2v - v_o = 0$ (1)



At node b, $\frac{v_i - v}{R} = \frac{v - v_o}{R} + C\frac{dv}{dt}$

$$v_i = 2v - v_o + RC\frac{dv}{dt} \quad (2)$$

Combining (1) and (2),

$$v_i = v_o - v_o + \frac{RC}{2}\frac{dv_o}{dt}$$

or

$$v_o = \frac{2}{RC}\int v_i \, dt$$

showing that the circuit is a noninverting integrator.

Chapter 6, Problem 74.

The triangular waveform in Fig. 6.91(a) is applied to the input of the op amp differentiator in Fig. 6.91(b). Plot the output.

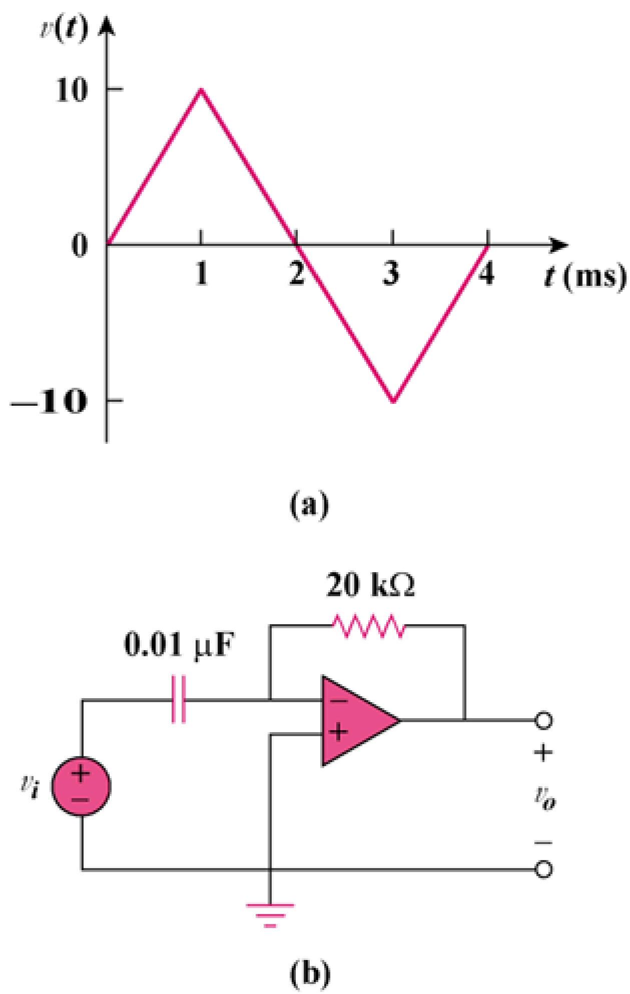


Figure 6.91

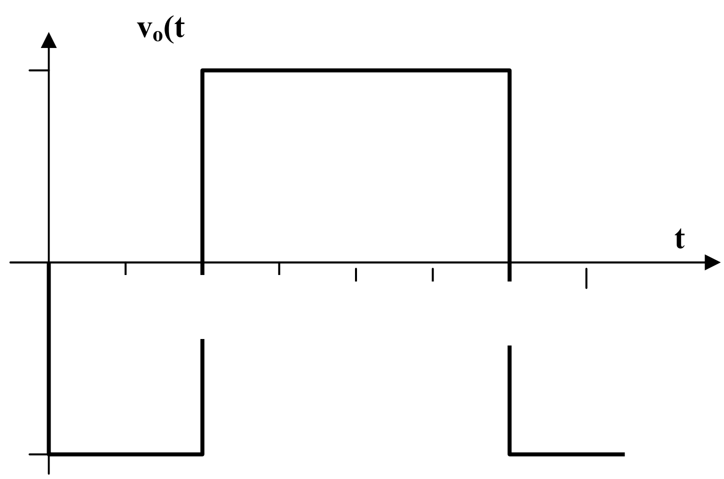
Chapter 6, Solution 74.

RC = 0.01 x 20 x 10^-3 sec

$$v_o = -RC\frac{dv_i}{dt} = -0.2\frac{dv}{dt}\,m\sec$$

$$v_o = \begin{cases} -2V, & 0 < t < 1 \\ 2V, & 1 < t < 3 \\ -2V, & 3 < t < 4 \end{cases}$$

Thus $v_o(t)$ is as sketched below:



Chapter 6, Problem 75.

An op amp differentiator has $\boldsymbol{R}$= 250 kΩ and $\boldsymbol{C}$ = 10 μF.
The input voltage is a ramp $\boldsymbol{r(t)}$ = 12 $\boldsymbol{t}$ mV. Find the output voltage.

Chapter 6, Solution 75.

$$v_0 = -RC\frac{dv_i}{dt}, \quad RC = 250x10^3 x10x10^{-6} = 2.5$$

$$v_o = -2.5\frac{d}{dt}(12t) = \underline{\boldsymbol{-30\ mV}}$$

Chapter 6, Problem 76.

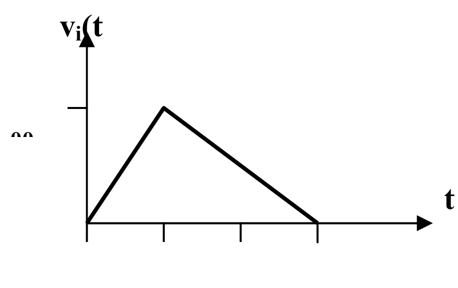
A voltage waveform has the following characteristics: a positive slope of 20 V/s for 5 ms followed by a negative slope of 10 V/s for 10 ms. If the waveform is applied to a differentiator with $\boldsymbol{R}$ = 50 kΩ, $\boldsymbol{C}$ = 10 μF, sketch the output voltage waveform.

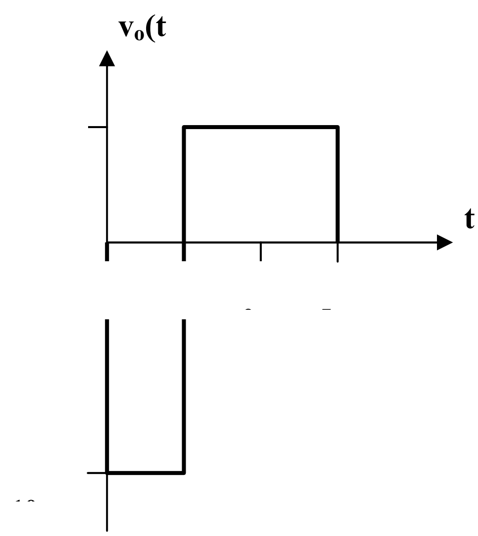
Chapter 6, Solution 76.

$$v_o = -RC\frac{dv_i}{dt}, \quad RC = 50 \text{ x } 10^3 \text{ x } 10 \text{ x } 10^{-6} = 0.5$$

$$v_o = -0.5\frac{dv_i}{dt} = \begin{bmatrix} -10, & 0 < t < 5 \\ 5, & 5 < t < 15 \end{bmatrix}$$

The input is sketched in Fig. (a), while the output is sketched in Fig. (b).





Chapter 6, Problem 77.

The output v_o of the op amp circuit of Fig. 6.92(a) is shown in Fig. 6.92(b). Let $Ri = R_f = 1$ MΩ and $C = 1$ μF. Determine the input voltage waveform and sketch it.

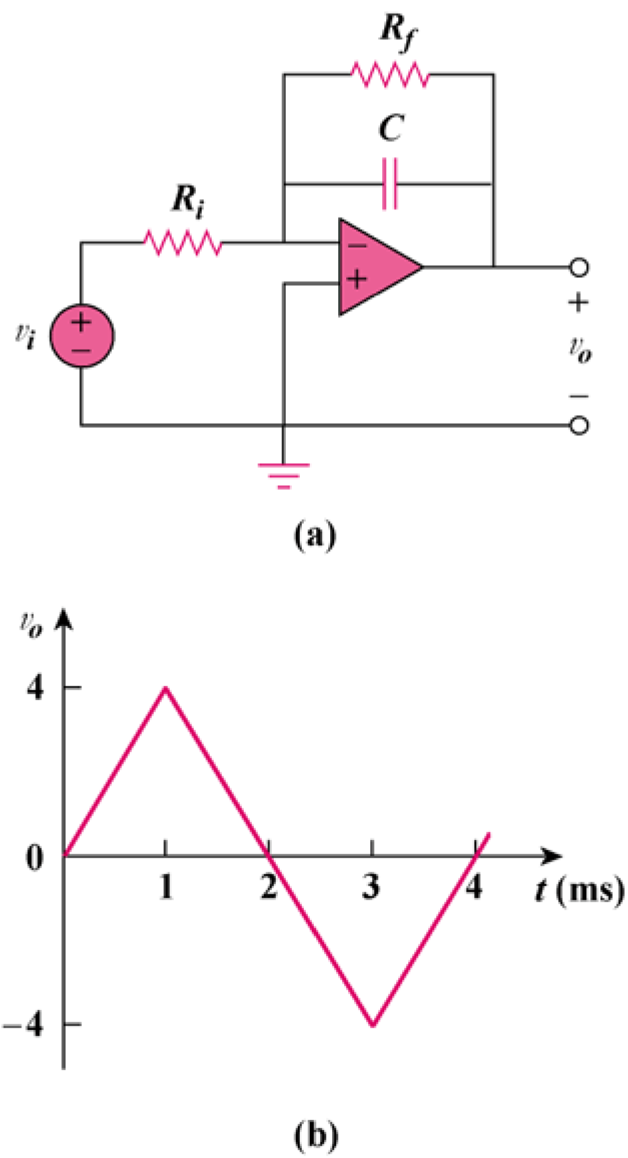


Figure 6.92

Chapter 6, Solution 77.

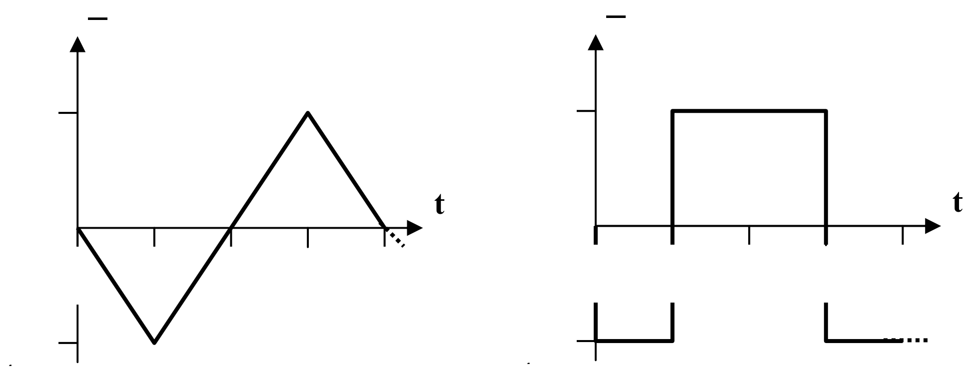
$i = i_R + i_C$

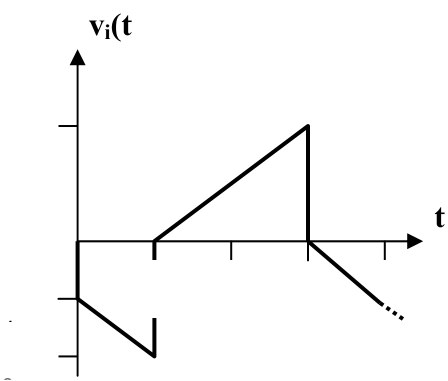
$$\frac{v_i - 0}{R} = \frac{0 - v_0}{R_F} + C\frac{d}{dt}(0 - v_o)$$

$$R_F C = 10^6 x 10^{-6} = 1$$

Hence $v_i = -\left(v_o + \frac{dv_o}{dt}\right)$

Thus v_i is obtained from v_o as shown below:





Chapter 6, Problem 78.

Design an analog computer to simulate

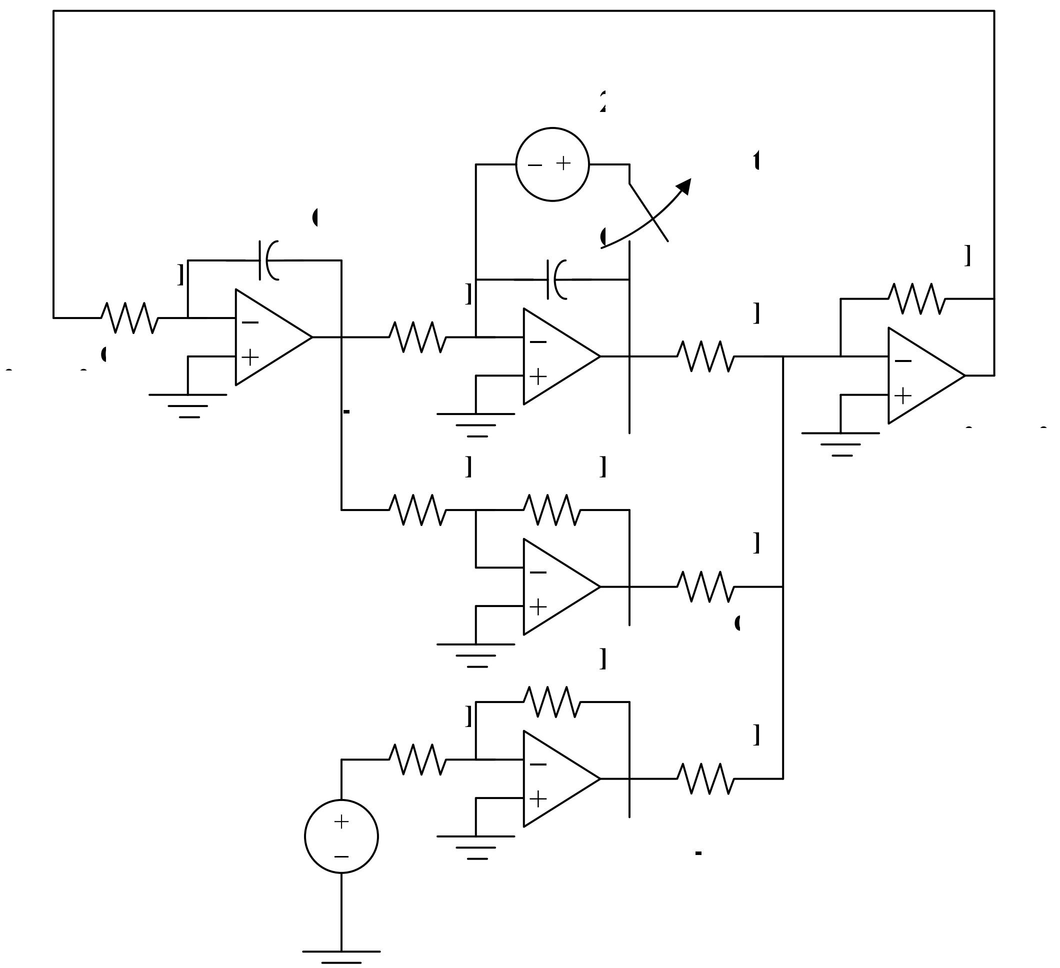
$$\frac{d^2v_0}{dt^2} + 2\frac{dv_0}{dt} + v_0 = 10\sin 2t$$

where $v_0(0) = 2$ and $v'_0(0) = 0$.

Chapter 6, Solution 78.

$$\frac{d^2v_o}{dt} = 10\sin 2t - \frac{2dv_o}{dt} - v_o$$

Thus, by combining integrators with a summer, we obtain the appropriate analog computer as shown below:



Chapter 6, Problem 79.

Design an analog computer circuit to solve the following ordinary differential equation.

$$\frac{dy(t)}{dt} + 4y(t) = f(t)$$

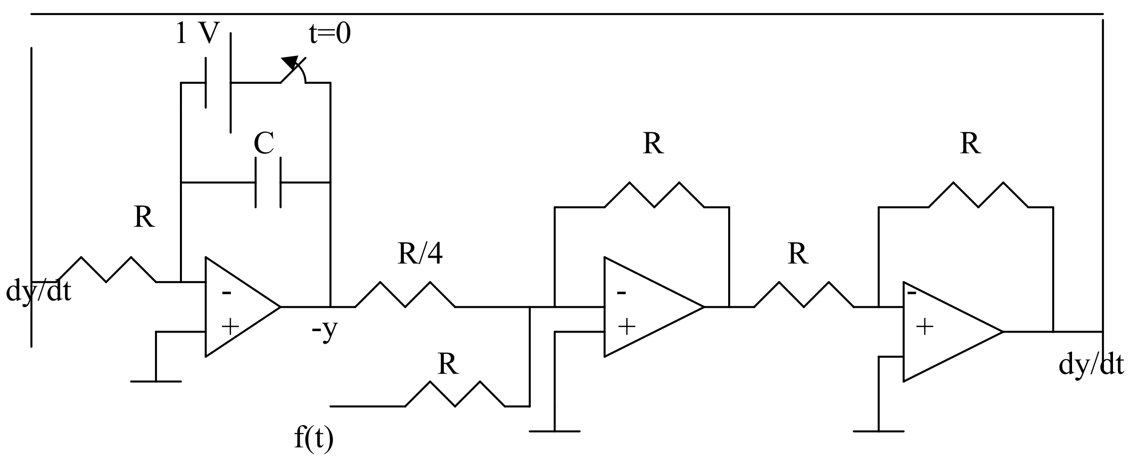
where $y(0) = 1$ V.

Chapter 6, Solution 79.

We can write the equation as

$$\frac{dy}{dt} = f(t) - 4y(t)$$

which is implemented by the circuit below.



Chapter 6, Problem 80.

Figure 6.93 presents an analog computer designed to solve a differential equation. Assuming *f*(*t*) is known, set up the equation for *f*(*t*).

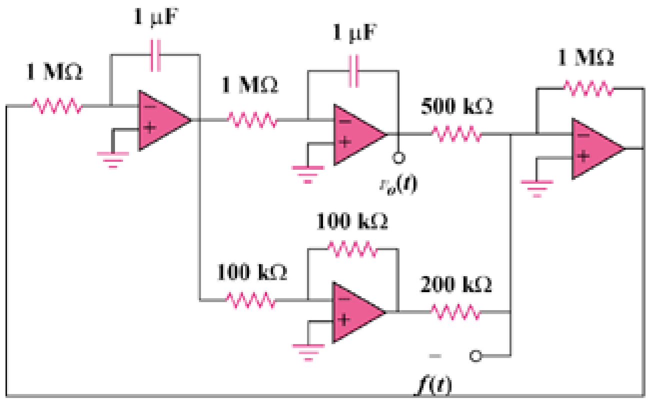


Figure 6.93

Chapter 6, Solution 80.

From the given circuit,

$$\frac{d^2v_o}{dt^2} = f(t) - \frac{1000k\Omega}{5000k\Omega}v_o - \frac{1000k\Omega}{200k\Omega}\frac{dv_o}{dt}$$

or

$$\underline{\frac{d^2v_o}{dt^2} + 5\frac{dv_o}{dt} + 2v_o = f(t)}$$

Chapter 6, Problem 81.

Design an analog computer to simulate the following equation:

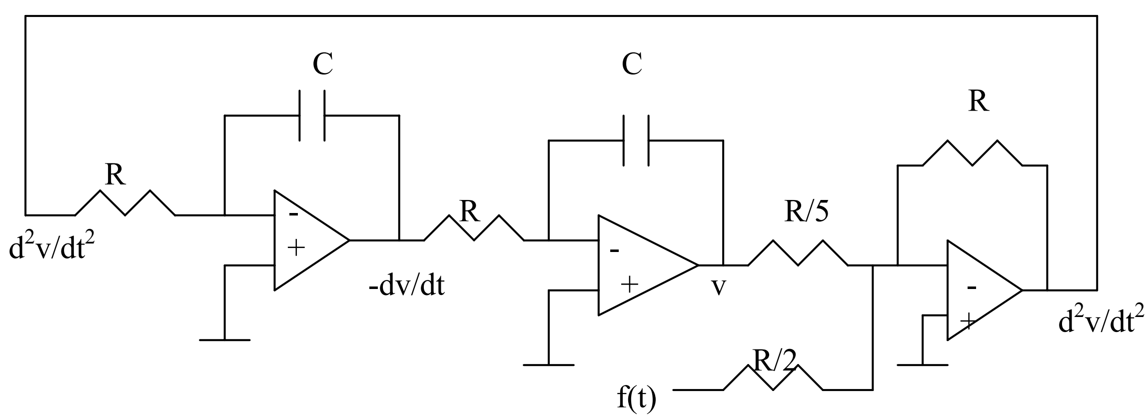
$$\frac{d^2v}{dt^2} + 5v = -2f(t)$$

Chapter 6, Solution 81

We can write the equation as

$$\frac{d^2v}{dt^2} = -5v - 2f(t)$$

which is implemented by the circuit below.



Chapter 6, Problem 82.

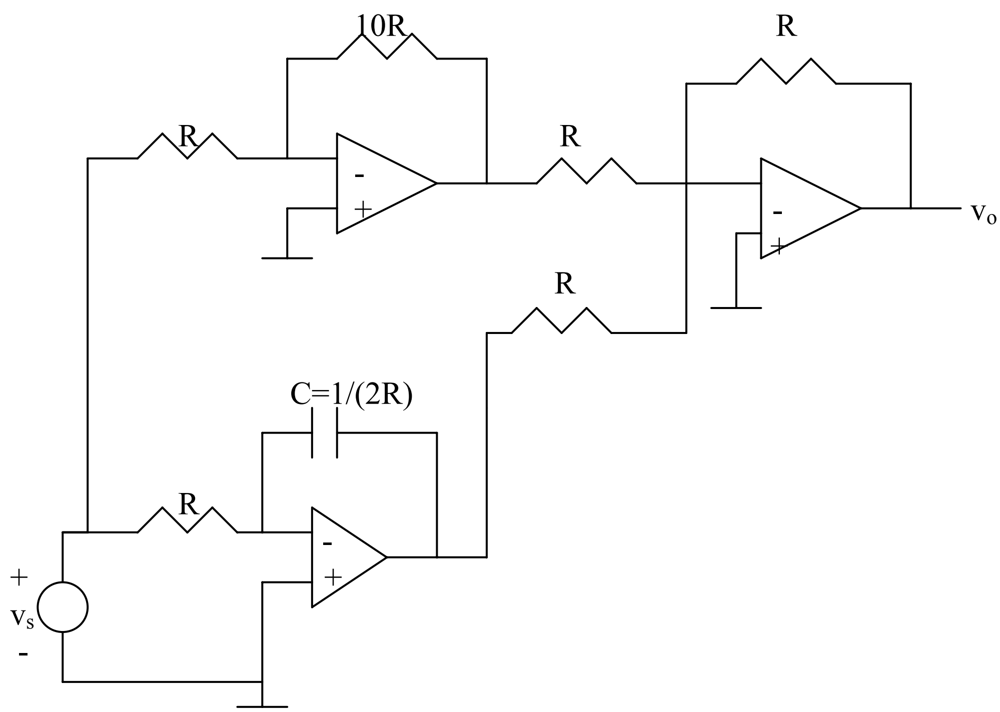
Design an an op amp circuit such that:

$$v_0 = 10v_s + 2\int v_s dt$$

where v_s and v_0 are the input voltage and output voltage respectively.

Chapter 6, Solution 82

The circuit consists of a summer, an inverter, and an integrator. Such circuit is shown below.

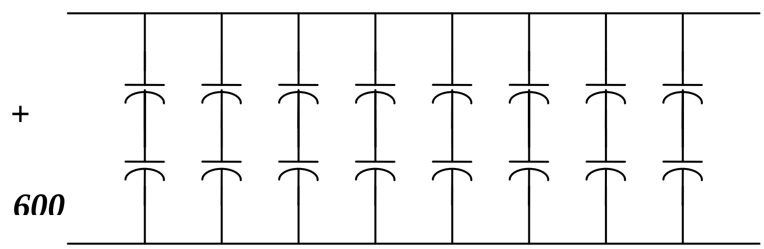


Chapter 6, Problem 83.

Your laboratory has available a large number of 10-μF capacitors rated at 300 V. To design a capacitor bank of 40-μF rated at 600 V, how many 10-μF capacitors are needed and how would you connect them?

Chapter 6, Solution 83.

Since two 10μF capacitors in series gives 5μF, rated at 600V, it requires 8 groups in parallel with each group consisting of two capacitors in series, as shown below:



Answer: **<u>8 groups in parallel with each group made up of 2 capacitors in series.</u>**

Chapter 6, Problem 84.

An 8-mH inductor is used in a fusion power experiment. If the current through the inductor is $i(t) = 5\sin^2 \pi t$ mA, t >0, find the power being delivered to the inductor and the energy stored in it at t=0.5s.

Chapter 6, Solution 84.

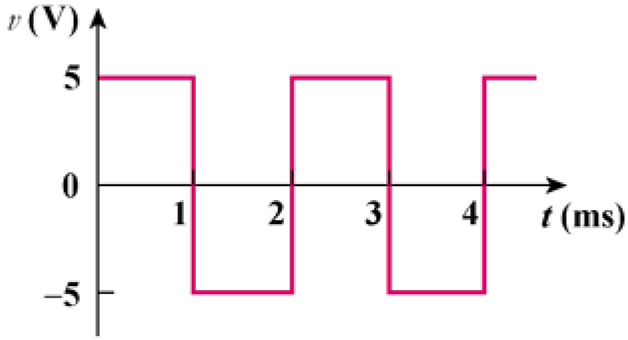
$$v = L(di/dt) = 8x10^{-3}x5x2\pi\sin(\pi t)\cos(\pi t)10^{-3} = 40\pi\sin(2\pi t)\ \mu V$$

$$p = vi = 40\pi\sin(2\pi t)5\sin^2(\pi t)10^{-9}\ W,\ \text{at t=0 p} = \underline{\mathbf{0W}}$$

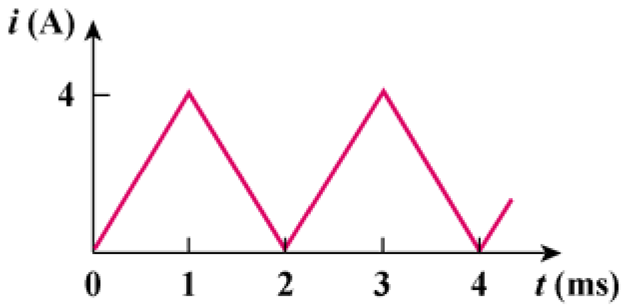
$$w = \frac{1}{2}Li^2 = \frac{1}{2}x8x10^{-3}x[5\sin^2(\pi/2)x10^{-3}]^2 = 4x25x10^{-9} = \underline{100\ \text{nJ}}$$

Chapter 6, Problem 85.

A square-wave generator produces the voltage waveform shown in Fig. 6.94(a). What kind of a circuit component is needed to convert the voltage waveform to the triangular current waveform shown in Fig. 6.94(b)? Calculate the value of the component, assuming that it is initially uncharged.



(a)



(b)

Figure 6.94

Chapter 6, Solution 85.

It is evident that differentiating i will give a waveform similar to v. Hence,

$$v = L\frac{di}{dt}$$

$$i = \begin{cases} 4t, 0 < t < 1ms \\ 8 - 4t, 1 < t < 2ms \end{cases}$$

$$v = L\frac{di}{dt} = \begin{cases} 4000L, 0 < t < 1ms \\ -4000L, 1 < t < 2ms \end{cases}$$

But, $$v = \begin{cases} 5V, 0 < t < 1ms \\ -5V, 1 < t < 2ms \end{cases}$$

Thus, $4000L = 5 \longrightarrow L = 1.25$ mH in a **<u>1.25 mH inductor</u>**

Chapter 6, Problem 86.

An electric motor can be modeled as a series combination of a 12-Ω resistor and 200-mH inductor. If a current $i(t) = 2te^{-10t}$A flows through the series combination, find the voltage across the combination.

Chapter 6, Solution 86.

$$v = v_R + v_L = Ri + L\frac{di}{dt} = 12x2te^{-10t} + 200x10^{-3}x(-20te^{-10t} + 2e^{-10t}) = \underline{(0.4 - 20t)e^{-10t}\ \text{V}}$$