Chapter 17, Problem 1.

Evaluate each of the following functions and see if it is periodic. If periodic, find its period.

(a) $f(t) = \cos \pi t + 2\cos 3\pi t + 3\cos 5\pi t$
(b) $y(t) = \sin t + 4\cos 2\pi t$
(c) $g(t) = \sin 3t \cos 4t$
(d) $h(t) = \cos^2 t$
(e) $z(t) = 4.2\sin(0.4\pi t + 10^\circ)$
$+ 0.8\sin(0.6\pi t + 50^\circ)$
(f) $p(t) = 10$
(g) $q(t) = e^{-\pi t}$

Chapter 17, Solution 1.

(a) This is **periodic** with $\omega = \pi$ which leads to $T = 2\pi/\omega =$ **2**.

(b) y(t) is **not periodic** although sin t and 4 cos 2πt are independently periodic.

(c) Since $\sin A \cos B = 0.5[\sin(A + B) + \sin(A - B)]$,
$$g(t) = \sin 3t \cos 4t = 0.5[\sin 7t + \sin(-t)] = -0.5\sin t + 0.5\sin 7t$$
which is harmonic or **periodic** with the fundamental frequency
$$\omega = 1 \text{ or } T = 2\pi/\omega = \mathbf{2\pi}.$$

(d) $h(t) = \cos^2 t = 0.5(1 + \cos 2t)$. Since the sum of a periodic function and a constant is also **periodic**, h(t) is periodic. $\omega = 2$ or $T = 2\pi/\omega = \underline{\pi}$.

(e) The frequency ratio 0.6|0.4 = 1.5 makes z(t) **periodic**.
$$\omega = 0.2\pi \text{ or } T = 2\pi/\omega = \mathbf{10}.$$

(f) p(t) = 10 is **not periodic**.

(g) g(t) is **not periodic**.

Chapter 17, Problem 2.

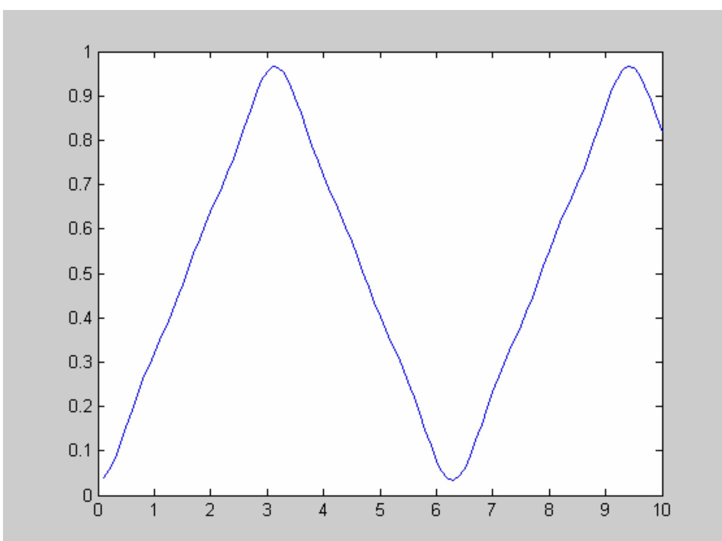
Using MATLAB, synthesize the periodic waveform for which the Fourier trigonometric Fourier series is

$$f(t) = \frac{1}{2} - \frac{4}{\pi^2}\left(\cos t + \frac{1}{9}\cos 3t + \frac{1}{25}\cos 5t + \cdots\right)$$

Chapter 17, Solution 2.

The function f(t) has a DC offset and is even. We use the following MATLAB code to plot f(t). The plot is shown below. If more terms are taken, the curve is clearly indicating a triangular wave shape which is easily represented with just the DC component and three, cosinusoidal terms of the expansion.

```
for n=1:100
   tn(n)=n/10;
   t=n/10;
   y1=cos(t);
   y2=(1/9)*cos(3*t);
   y3=(1/25)*cos(5*t);
   factor=4/(pi*pi);
   y(n)=0.5- factor*(y1+y2+y3);
end
plot(tn,y)
```



Chapter 17, Problem 3.

Give the Fourier coefficients a_0, a_n, and b_n of the waveform in Fig. 17.47. Plot the amplitude and phase spectra.

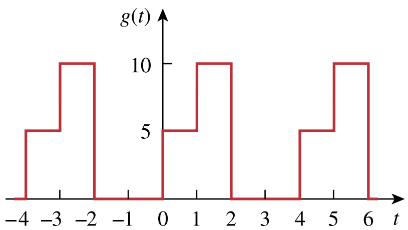


Figure 17.47
For Prob. 17.3.

Chapter 17, Solution 3.

$$T = 4, \ \omega_o = 2\pi/T = \pi/2$$

$$g(t) = \begin{cases} 5, & 0 < t < 1 \\ 10, & 1 < t < 2 \\ 0, & 2 < t < 4 \end{cases}$$

$$a_o = (1/T)\int_0^T g(t)dt = 0.25[\int_0^1 5dt + \int_1^2 10dt] = \underline{3.75}$$

$$a_n = (2/T)\int_0^T g(t)\cos(n\omega_o t)dt = (2/4)[\int_0^1 5\cos(\frac{n\pi}{2}t)dt + \int_1^2 10\cos(\frac{n\pi}{2}t)dt]$$

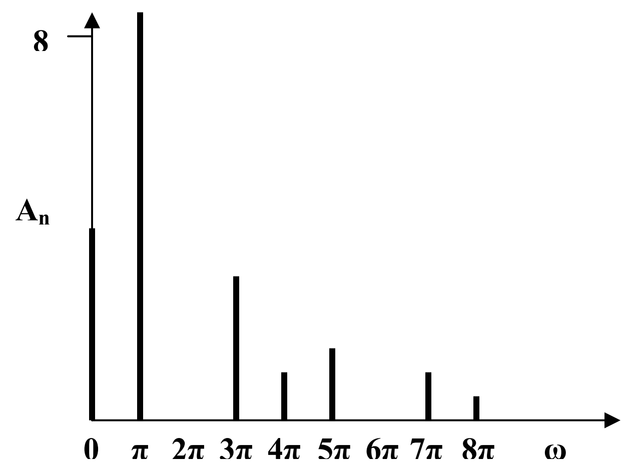
$$= 0.5[5\frac{2}{n\pi}\sin\frac{n\pi}{2}t\Big|_0^1 + 10\frac{2}{n\pi}\sin\frac{n\pi}{2}t\Big|_1^2] = (-1/(n\pi))5\sin(n\pi/2)$$

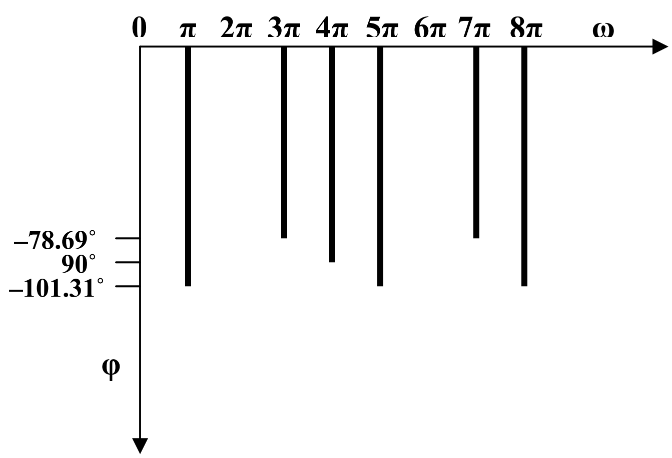
$$a_n = \begin{cases} \underline{\mathbf{(5/(n\pi))(-1)^{(n+1)/2},}} & \underline{\mathbf{n = odd}} \\ \underline{\mathbf{0,}} & \underline{\mathbf{n = even}} \end{cases}$$

$$b_n = (2/T)\int_0^T g(t)\sin(n\omega_o t)dt = (2/4)[\int_0^1 5\sin(\frac{n\pi}{2}t)dt + \int_1^2 10\sin(\frac{n\pi}{2}t)dt]$$

$$= 0.5[\frac{-2x5}{n\pi}\cos\frac{n\pi}{2}t\Big|_0^1 - \frac{2x10}{n\pi}\cos\frac{n\pi}{2}t\Big|_1^2] = \underline{\mathbf{(5/(n\pi))[3 - 2\cos n\pi + \cos(n\pi/2)]}}$$

n	a_n	b_n	A_n	phase
1	–1.59	7.95	8.11	–101.31
2	0	0	0	0
3	0.53	2.65	2.70	–78.69
4	0	0.80	0.80	–90
5	–0.32	1.59	1.62	–101.31
6	0	0	0	0
7	0.23	1.15	1.17	–78.69
8	0	0.40	0.40	–90





Chapter 17, Problem 4.

Find the Fourier series expansion of the backward sawtooth waveform of Fig. 17.48. Obtain the amplitude and phase spectra.

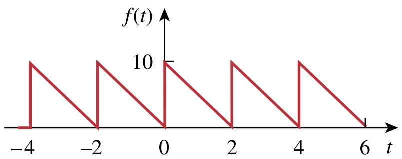


Figure 17.48
For Probs. 17.4 and 17.66.

Chapter 17, Solution 4.

$$f(t) = 10 - 5t, \quad 0 < t < 2, \quad T = 2, \quad \omega_o = 2\pi/T = \pi$$

$$a_o = (1/T)\int_0^T f(t)dt = (1/2)\int_0^2 (10-5t)dt = 0.5[10t - (5t^2/2)]\Big|_0^2 = 5$$

$$a_n = (2/T)\int_0^T f(t)\cos(n\omega_o t)dt = (2/2)\int_0^2 (10-5t)\cos(n\pi t)dt$$

$$= \int_0^2 (10)\cos(n\pi t)dt - \int_0^2 (5t)\cos(n\pi t)dt$$

$$= \frac{-5}{n^2\pi^2}\cos n\pi t\Big|_0^2 + \frac{5t}{n\pi}\sin n\pi t\Big|_0^2 = [-5/(n^2\pi^2)](\cos 2n\pi - 1) = 0$$

$$b_n = (2/2)\int_0^2 (10-5t)\sin(n\pi t)dt$$

$$= \int_0^2 (10)\sin(n\pi t)dt - \int_0^2 (5t)\sin(n\pi t)dt$$

$$= \frac{-5}{n^2\pi^2}\sin n\pi t\Big|_0^2 + \frac{5t}{n\pi}\cos n\pi t\Big|_0^2 = 0 + [10/(n\pi)](\cos 2n\pi) = 10/(n\pi)$$

Hence $\quad f(t) = \underline{\mathbf{5 + \frac{10}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(n\pi t)}}$

Chapter 17, Problem 5.

Obtain the Fourier series expansion for the waveform shown in Fig. 17.49.

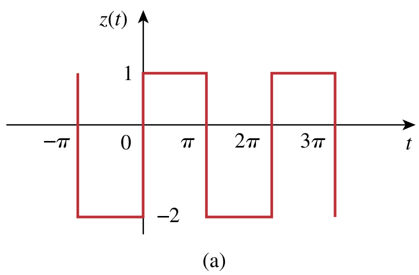


Figure 17.49
For Prob. 17.5.

Chapter 17, Solution 5.

$$T = 2\pi, \qquad \omega = 2\pi / T = 1$$

$$a_o = \frac{1}{T}\int_0^T z(t)dt = \frac{1}{2\pi}[1x\pi - 2x\pi] = -0.5$$

$$a_n = \frac{2}{T}\int_0^T z(t)\cos n\omega_o dt = \frac{1}{\pi}\int_0^{\pi} 1\cos ntdt - \frac{1}{\pi}\int_{\pi}^{2\pi} 2\cos ntdt = \frac{1}{n\pi}\sin ..nt \Big|_0^{\pi} - \frac{2}{n\pi}\sin nt \Big|_{\pi}^{2\pi} = 0$$

$$b_n = \frac{2}{T}\int_0^T z(t)\cos n\omega_o dt = \frac{1}{\pi}\int_0^{\pi} 1\sin ntdt - \frac{1}{\pi}\int_{\pi}^{2\pi} 2\sin ntdt = -\frac{1}{n\pi}\cos nt \Big|_0^{\pi} + \frac{2}{n\pi}\cos nt \Big|_{\pi}^{2\pi} = \begin{cases} \frac{6}{n\pi}, n = \text{odd} \\ 0, \quad n = \text{even} \end{cases}$$

Thus,

$$z(t) = -0.5 + \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{6}{n\pi}\sin nt$$

Chapter 17, Problem 6.

Find the trigonometric Fourier series for

$$f(t) = \begin{cases} 5, & 0 < t < \pi \\ 10, & \pi < t < 2\pi \end{cases} \quad \text{and} \quad f(t+2\pi) = f(t).$$

Chapter 17, Solution 6.

T=2π, ω_o=2π/T = 1

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2\pi}\left[\int_0^{\pi} 5dt + \int_{\pi}^{2\pi} 10dt\right] = \frac{1}{2\pi}(5\pi + 10\pi) = 7.5$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o t dt = \frac{2}{2\pi}\left[\int_0^{\pi} 5\cos nt dt + \int_{\pi}^{2\pi} 10\cos nt dt\right] = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o t dt = \frac{2}{2\pi}\left[\int_0^{\pi} 5\sin nt dt + \int_{\pi}^{2\pi} 10 sinnt dt\right] = \frac{1}{\pi}\left[-\frac{1}{n}\cos nt\Big|_0^{\pi} - \frac{1}{n}\cos nt\Big|_{\pi}^{2\pi}\right]$$

$$= \frac{5}{n\pi}\left[\cos \pi n - 1\right] = \begin{cases} -\dfrac{10}{n\pi}, & n = odd \\ 0, & n = even \end{cases}$$

Thus,

$$\underline{f(t) = 7.5 - \sum_{n=odd}^{\infty} \frac{10}{n\pi}\sin nt}$$

Chapter 17, Problem 7.

ML * Determine the Fourier series of the periodic function in Fig. 17.50.

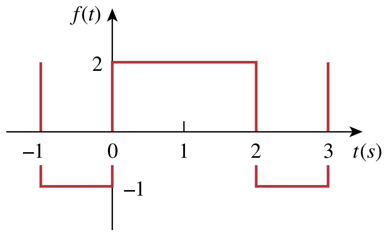


Figure 17.50
For Prob. 17.7.

* An asterisk indicates a challenging problem.

Chapter 17, Solution 7.

$T = 3, \quad \omega_o = 2\pi / T = 2\pi / 3$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{3}\left[\int_0^2 2dt + \int_2^3 (-1)dt\right] = \frac{1}{3}(4-1) = 1$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos\frac{2n\pi t}{3}dt = \frac{2}{3}\left[\int_0^2 2\cos\frac{2n\pi t}{3}dt + \int_2^3 (-1)\cos\frac{2n\pi t}{3}dt\right]$$

$$= \frac{2}{3}\left[2\frac{3}{2n\pi}\sin\frac{2n\pi t}{3}\Big|_0^2 - 1\frac{3}{2n\pi}\sin\frac{2n\pi t}{3}\Big|_2^3\right] = \frac{3}{n\pi}\sin\frac{4n\pi}{3}$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin\frac{2n\pi t}{3}dt = \frac{2}{3}\left[\int_0^2 2\sin\frac{2n\pi t}{3}dt + \int_2^3 (-1)sin\frac{2n\pi t}{3}dt\right]$$

$$= \frac{2}{3}\left[-2x\frac{3}{2n\pi}\cos\frac{2n\pi t}{3}\Big|_0^2 + \frac{3}{2n\pi}\cos\frac{2n\pi t}{3}\Big|_2^3\right] = \frac{3}{n\pi}(1 - 2\cos\frac{4n\pi}{3})$$

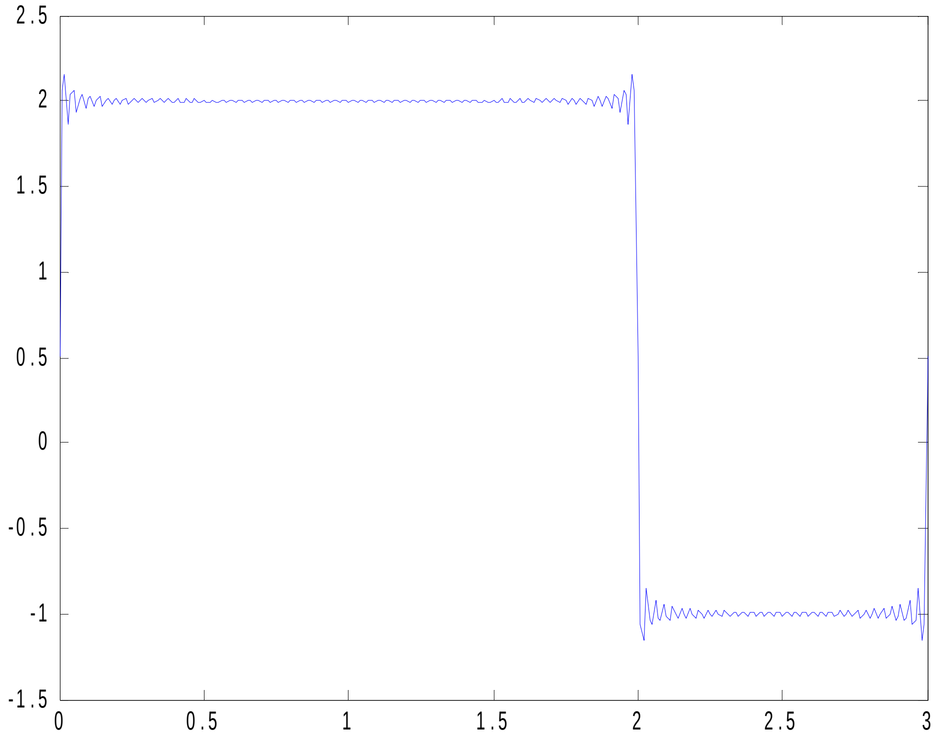
$$= \frac{1}{n\pi}\left(2 - 3\cos\frac{4n\pi}{3} + 1\right) = \frac{3}{n\pi}\left(1 - \cos\frac{4n\pi}{3}\right)$$

Hence,

$$\underline{f(t) = 1 + \sum_{n=0}^{\infty}\left[\frac{3}{n\pi}\sin\frac{4n\pi}{3}\cos\frac{2n\pi t}{3} + \frac{3}{n\pi}\left(1-\cos\frac{4n\pi}{3}\right)\sin\frac{2n\pi t}{3}\right]}$$

We can now use MATLAB to check our answer,

```
>> t=0:.01:3;
>> f=1*ones(size(t));
>> for n=1:1:99,
f=f+(3/(n*pi))*sin(4*n*pi/3)*cos(2*n*pi*t/3)+(3/(n*pi))*(1-
cos(4*n*pi/3))*sin(2*n*pi*t/3);
end
>> plot(t,f)
```



Clearly we have the same figure we started with!!

Chapter 17, Problem 8.

Obtain the exponential Fourier series of the function in Fig. 17.51.

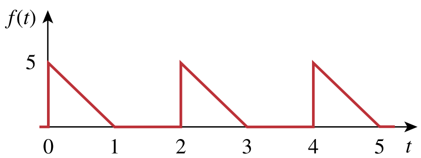


Figure 17.51
For Prob. 17.8.

Chapter 17, Solution 8.

$T = 2, \quad \omega_o = 2\pi / T = \pi$

$$f(t) = \begin{cases} 5(1-t), & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}dt = \frac{1}{2}\int_0^1 5(1-t)e^{-jn\pi t}dt$$

$$= \frac{5}{2}\int_0^1 e^{-jn\pi t}dt - \frac{5}{2}\int_0^1 te^{-jn\pi t}dt = \frac{5}{2}\frac{e^{-jn\pi t}}{-jn\pi}\Big|_0^1 - \frac{5}{2}\frac{e^{-jn\pi t}}{(-jn\pi)^2}(-jn\pi t - 1)\Big|_0^1$$

$$= \frac{5}{2}\frac{\left[e^{-jn\pi} - 1\right]}{-jn\pi} - \frac{5}{2}\frac{e^{-jn\pi}}{-n^2\pi^2}(-jn\pi - 1) + \frac{5}{2}\frac{(-1)}{-n^2\pi^2}$$

But $e^{-jn\pi} = \cos\pi n - j\sin n\pi = \cos n\pi + 0 = (-1)^n$

$$c_n = \frac{2.5[1-(-1)^n]}{jn\pi} - \frac{2.5(-1)^n[1 + jn\pi]}{n^2\pi^2} + \frac{2.5}{n^2\pi^2}$$

Chapter 17, Problem 9.

Determine the Fourier coefficients a_n and b_n of the first three harmonic terms of the rectified cosine wave in Fig. 17.52.

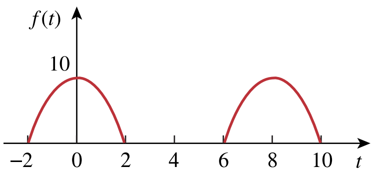


Figure 17.52
For Prob. 17.9.

Chapter 17, Solution 9.

f(t) is an even function, b_n=0.

$$T = 8, \qquad \omega = 2\pi / T = \pi / 4$$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{2}{8}\left[\int_0^2 10\cos \pi t/4 dt + 0\right] = \frac{10}{4}\left(\frac{4}{\pi}\right)\sin \pi t/4 \Big|_0^2 = \frac{10}{\pi} = 3.183$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_o dt = \frac{40}{8}\left[\int_0^2 10\cos \pi t/4 \cos n\pi t/4 dt + 0\right] = 5\int_0^2 \left[\cos \pi t(n+1)/4 + \cos \pi t(n-1)/4\right] dt$$

For n = 1,

$$a_1 = 5\int_0^2 [\cos \pi t/2 + 1]dt = 5\left[\frac{2}{\pi}\sin \pi t/2 dt + t\right]_0^2 = 10$$

For n>1,

$$a_n = \frac{20}{\pi(n+1)}\sin\frac{\pi(n+1)t}{4} + \frac{20}{\pi(n-1)}\sin\frac{\pi(n-1)}{4}\Bigg|_0^2 = \frac{20}{\pi(n+1)}\sin\frac{\pi(n+1)}{2} + \frac{20}{\pi(n-1)}\sin\frac{\pi(n-1)}{2}$$

$$a_2 = \frac{20}{3\pi}\sin 1.5\pi + \frac{20}{\pi}\sin \pi/2 = 4.244, \qquad a_3 = \frac{20}{4\pi}\sin 2\pi + \frac{10}{\pi}\sin \pi = 0$$

Thus,

$$\underline{a_0 = 3.183, \quad a_1 = 10, \quad a_2 = 4.244, \quad a_3 = 0, \quad b_1 = 0 = b_2 = b_3}$$

Chapter 17, Problem 10.

Find the exponential Fourier series for the waveform in Fig. 17.53.

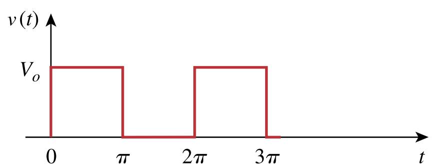


Figure 17.53
For Prob. 17.10.

Chapter 17, Solution 10.

$T = 2\pi, \qquad \omega_o = 2\pi / T = 1$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}dt = \frac{V_o}{2\pi}\int_0^{\pi}(1)e^{-jnt}dt = \frac{V_o}{2\pi}\frac{e^{=jnt}}{-jn}\Bigg|_0^{\pi}$$

$$= \frac{V_o}{2n\pi}\left[je^{-jn\pi} - j\right] = \frac{jV_o}{2n\pi}(\cos n\pi - 1)$$

$$\underline{f(t) = \sum_{n=-\infty}^{\infty}\frac{jV_o}{2n\pi}(\cos n\pi - 1)e^{jnt}}$$

Chapter 17, Problem 11.

Obtain the exponential Fourier series for the signal in Fig. 17.54.

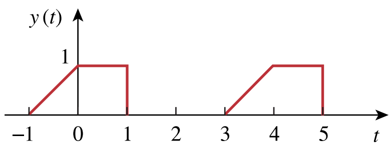


Figure 17.54
For Prob. 17.11.

Chapter 17, Solution 11.

$$T = 4, \qquad \omega_o = 2\pi/T = \pi/2$$

$$c_n = \frac{1}{T}\int_0^T y(t)e^{-jn\omega_o t}dt = \frac{1}{4}\left[\int_{-1}^{0}(t+1)e^{-jn\pi t/2}dt + \int_0^1 (1)e^{-jn\pi t/2}dt\right]$$

$$c_n = \frac{1}{4}\left[\frac{e^{-jn\pi t/2}}{-n^2\pi^2/4}(-jn\pi t/2 - 1) - \frac{2}{jn\pi}e^{-jn\pi t/2}\Big|_{-1}^{0} - \frac{2}{jn\pi}e^{-jn\pi t/2}\Big|_0^1\right]$$

$$= \frac{1}{4}\left[\frac{4}{n^2\pi^2} - \frac{2}{jn\pi} + \frac{4}{n^2\pi^2}e^{jn\pi/2}(jn\pi/2 - 1) + \frac{2}{jn\pi}e^{jn\pi/2} - \frac{2}{jn\pi}e^{-jn\pi/2} + \frac{2}{jn\pi}\right]$$

But

$$e^{jn\pi/2} = \cos n\pi/2 + j\sin n\pi/2 = j\sin n\pi/2, \qquad e^{-jn\pi/2} = \cos n\pi/2 - j\sin n\pi/2 = -j\sin n\pi/2$$

$$c_n = \frac{1}{n^2\pi^2}\left[1 + j(jn\pi/2 - 1)\sin n\pi/2 + n\pi\sin n\pi/2\right]$$

$$y(t) = \underline{\sum_{n=-\infty}^{\infty}\frac{1}{n^2\pi^2}\left[1 + j(jn\pi/2 - 1)\sin n\pi/2 + n\pi\sin n\pi/2\right]e^{jn\pi t/2}}$$

Chapter 17, Problem 12.

* A voltage source has a periodic waveform defined over its period as

$v(t) = t(2\pi - t)$ V, $\quad 0 < t < 2\pi$

Find the Fourier series for this voltage.

* An asterisk indicates a challenging problem.

Chapter 17, Solution 12.

A voltage source has a periodic waveform defined over its period as
$v(t) = t(2\pi - t)$ V, $\quad$ for all $0 < t < 2\pi$

Find the Fourier series for this voltage.

$v(t) = 2\pi t - t^2,\ 0 < t < 2\pi,\ T = 2\pi,\ \omega_o = 2\pi/T = 1$

$$a_o = (1/T)\int_0^T f(t)dt = \frac{1}{2\pi}\int_0^{2\pi}(2\pi t - t^2)dt = \frac{1}{2\pi}(\pi t^2 - t^3/3)\Big|_0^{2\pi} = \frac{4\pi^3}{2\pi}(1-2/3) = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{T}\int_0^T (2\pi t - t^2)\cos(nt)dt = \frac{1}{\pi}\left[\frac{2\pi}{n^2}\cos(nt) + \frac{2\pi t}{n}\sin(nt)\right]_0^{2\pi}$$

$$-\frac{1}{\pi n^3}\left[2nt\cos(nt) - 2\sin(nt) + n^2t^2\sin(nt)\right]_0^{2\pi}$$

$$= \frac{2}{n^2}(1-1) - \frac{1}{\pi n^3}4n\pi\cos(2\pi n) = \frac{-4}{n^2}$$

$$b_n = \frac{2}{T}\int_0^T (2nt - t^2)\sin(nt)dt = \frac{1}{\pi}\int (2nt - t^2)\sin(nt)dt$$

$$= \frac{2n}{\pi}\frac{1}{n^2}(\sin(nt) - nt\cos(nt))\Big|_0^{\pi} - \frac{1}{\pi n^3}(2nt\sin(nt) + 2\cos(nt) - n^2t^2\cos(nt))\Big|_0^{2\pi}$$

$$= \frac{-4\pi}{n} + \frac{4\pi}{n} = 0$$

Hence, $\quad f(t) = \underline{\boldsymbol{\frac{2\pi^2}{3} - \sum_{n=1}^{\infty}\frac{4}{n^2}\cos(nt)}}$

Chapter 17, Problem 13.

A periodic function is defined over its period as

$$h(t) = \begin{cases} 10\sin t, & 0 < t < \pi \\ 20\sin(t-\pi), & \pi < t < 2\pi \end{cases}$$

Find the Fourier series of $h(t)$.

Chapter 17, Solution 13.

$T = 2\pi, \ \omega_o = 1$

$$a_o = (1/T)\int_0^T h(t)dt = \frac{1}{2\pi}[\int_0^{\pi} 10\sin t\, dt + \int_{\pi}^{2\pi} 20\sin(t-\pi)\, dt]$$

$$= \frac{1}{2\pi}\left[-10\cos t\Big|_0^{\pi} - 20\cos(t-\pi)\Big|_{\pi}^{2\pi}\right] = \frac{30}{\pi}$$

$$a_n = (2/T)\int_0^T h(t)\cos(n\omega_o t)dt$$

$$= [2/(2\pi)]\left[\int_0^{\pi} 10\sin t\cos(nt)dt + \int_{\pi}^{2\pi} 20\sin(t-\pi)\cos(nt)dt\right]$$

Since $\sin A\cos B = 0.5[\sin(A+B) + \sin(A-B)]$
$\sin t\cos nt = 0.5[\sin((n+1)t) + \sin((1-n))t]$
$\sin(t-\pi) = \sin t\cos\pi - \cos t\sin\pi = -\sin t$
$\sin(t-\pi)\cos(nt) = -\sin(t)\cos(nt)$

$$a_n = \frac{1}{2\pi}\left[10\int_0^{\pi}[\sin([1+n]t) + \sin([1-n]t)]dt - 20\int_{\pi}^{2\pi}[\sin([1+n]t) + \sin([1-n]t)]dt\right]$$

$$= \frac{5}{\pi}\left[\left(-\frac{\cos([1+n]t)}{1+n} - \frac{\cos([1-n]t)}{1-n}\right)\Bigg|_0^{\pi} + \left(\frac{2\cos([1+n]t)}{1+n} + \frac{2\cos([1-n]t)}{1-n}\right)\Bigg|_{\pi}^{2\pi}\right]$$

$$a_n = \frac{5}{\pi}\left[\frac{3}{1+n} + \frac{3}{1-n} - \frac{3\cos([1+n]\pi)}{1+n} - \frac{3\cos([1-n]\pi)}{1-n}\right]$$

But, $[1/(1+n)] + [1/(1-n)] = 1/(1-n^2)$

$\cos([n-1]\pi) = \cos([n+1]\pi) = \cos\pi\cos n\pi - \sin\pi\sin n\pi = -\cos n\pi$

$a_n = (5/\pi)[(6/(1-n^2)) + (6\cos(n\pi)/(1-n^2))]$

$$= [30/(\pi(1-n^2))](1+\cos n\pi) = [-60/(\pi(n-1))],\ n = \text{even}$$
$$= 0,\quad n = \text{odd}$$

$$b_n = (2/T)\int_0^T h(t)\sin n\omega_o t\, dt$$

$$= [2/(2\pi)][\int_0^{\pi} 10\sin t\sin nt\, dt + \int_{\pi}^{2\pi} 20(-\sin t)\sin nt\, dt$$

But, $\sin A\sin B = 0.5[\cos(A-B) - \cos(A+B)]$

$\sin t\sin nt = 0.5[\cos([1-n]t) - \cos([1+n]t)]$

$$b_n = (5/\pi)\{[(\sin([1-n]t)/(1-n)) - (\sin([1+n]t)/(1+n)]\Big|_0^{\pi}$$
$$+ [(2\sin([1-n]t)/(1-n)) - (2\sin([1+n]t)/(1+n)]\Big|_{\pi}^{2\pi}\}$$

$$= \frac{5}{\pi}\left[-\frac{\sin([1-n]\pi)}{1-n} + \frac{\sin([1+n]\pi)}{1+n}\right] = 0$$

Thus, $$\underline{h(t) = \mathbf{\frac{30}{\pi} - \frac{60}{\pi}\sum_{k=1}^{\infty}\frac{\cos(2kt)}{(4k^2-1)}}}$$

Chapter 17, Problem 14.

Find the quadrature (cosine and sine) form of the Fourier series

$$f(t) = 2 + \sum_{n=1}^{\infty} \frac{10}{n^3+1} \cos\left(2nt + \frac{n\pi}{4}\right)$$

Chapter 17, Solution 14.

Since cos(A + B) = cos A cos B – sin A sin B.

$$f(t) \;=\; \underline{\mathbf{2 + \sum_{n=1}^{\infty}\left(\frac{10}{n^3+1}\cos(n\pi/4)\cos(2nt) - \frac{10}{n^3+1}\sin(n\pi/4)\sin(2nt)\right)}}$$

Chapter 17, Problem 15.

Express the Fourier series

$$f(t) = 10 + \sum_{n=1}^{\infty} \frac{4}{n^2+1}\cos 10nt + \frac{1}{n^3}\sin 10nt$$

(a) in a cosine and angle form.
(b) in a sine and angle form.

Chapter 17, Solution 15.

(a) $$D\cos\omega t + E\sin\omega t \;=\; A\cos(\omega t - \theta)$$

where $A = \sqrt{D^2 + E^2}$, $\theta = \tan^{-1}(E/D)$

$$A = \sqrt{\frac{16}{(n^2+1)^2} + \frac{1}{n^6}}, \quad \theta = \tan^{-1}((n^2+1)/(4n^3))$$

$$f(t) \;=\; \underline{\mathbf{10 + \sum_{n=1}^{\infty}\sqrt{\frac{16}{(n^2+1)^2} + \frac{1}{n^6}}\cos\left(10nt - \tan^{-1}\frac{n^2+1}{4n^3}\right)}}$$

(b) $$D\cos\omega t + E\sin\omega t \;=\; A\sin(\omega t + \theta)$$

where $A = \sqrt{D^2 + E^2}$, $\theta = \tan^{-1}(D/E)$

$$f(t) \;=\; \underline{\mathbf{10 + \sum_{n=1}^{\infty}\sqrt{\frac{16}{(n^2+1)^2} + \frac{1}{n^6}}\sin\left(10nt + \tan^{-1}\frac{4n^3}{n^2+1}\right)}}$$

Chapter 17, Problem 16.

The waveform in Fig. 17.55(a) has the following Fourier series:

$$v_1(t) = \frac{1}{2} - \frac{4}{\pi^2}\left(\cos \pi t + \frac{1}{9}\cos 3\pi t + \frac{1}{25}\cos 5\pi t + \cdots\right) \text{V}$$

Obtain the Fourier series of $v_2(t)$ in Fig. 17.55(b).

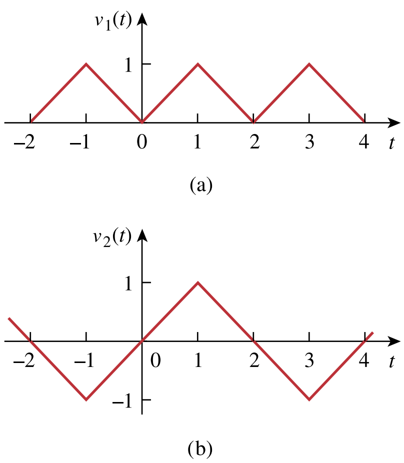
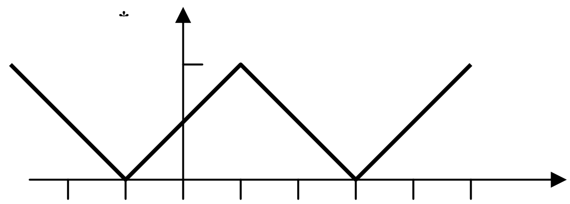


Figure 17.55
For Probs. 17.16 and 17.69.

Chapter 17, Solution 16.

If $v_2(t)$ is shifted by 1 along the vertical axis, we obtain $v_2^*(t)$ shown below, i.e. $v_2^*(t) = v_2(t) + 1$.



Comparing $v_2^*(t)$ with $v_1(t)$ shows that

$$v_2^*(t) = 2v_1((t + t_o)/2)$$

where $(t + t_o)/2 = 0$ at $t = -1$ or $t_o = 1$

Hence $$v_2^*(t) = 2v_1((t + 1)/2)$$

But $$v_2^*(t) = v_2(t) + 1$$

$$v_2(t) + 1 = 2v_1((t+1)/2)$$

$$v_2(t) = -1 + 2v_1((t+1)/2)$$

$$= -1 + 1 - \frac{8}{\pi^2}\left[\cos \pi\left(\frac{t+1}{2}\right) + \frac{1}{9}\cos 3\pi\left(\frac{t+1}{2}\right) + \frac{1}{25}\cos 5\pi\left(\frac{t+1}{2}\right) + \cdots\right]$$

$$v_2(t) = -\frac{8}{\pi^2}\left[\cos\left(\frac{\pi t}{2} + \frac{\pi}{2}\right) + \frac{1}{9}\cos\left(\frac{3\pi t}{2} + \frac{3\pi}{2}\right) + \frac{1}{25}\cos\left(\frac{5\pi t}{2} + \frac{5\pi}{2}\right) + \cdots\right]$$

$$v_2(t) = \underline{-\frac{\mathbf{8}}{\pi^2}\left[\mathbf{sin}\left(\frac{\pi \mathbf{t}}{\mathbf{2}}\right) + \frac{\mathbf{1}}{\mathbf{9}}\mathbf{sin}\left(\frac{\mathbf{3}\pi \mathbf{t}}{\mathbf{2}}\right) + \frac{\mathbf{1}}{\mathbf{25}}\mathbf{sin}\left(\frac{\mathbf{5}\pi \mathbf{t}}{\mathbf{2}}\right) + \cdots\right]}$$

Chapter 17, Problem 17.

Determine if these functions are even, odd, or neither.

(a) $1 + t$ (b) $t^2 - 1$ (c) $\cos n\pi t \sin n\pi t$
(d) $\sin^2 \pi t$ (e) e^{-t}

Chapter 17, Solution 17.

We replace t by –t in each case and see if the function remains unchanged.

(a) $1 - t$, **neither odd nor even.**

(b) $t^2 - 1$, **even**

(c) $\cos n\pi(-t) \sin n\pi(-t) = -\cos n\pi t \sin n\pi t$, **odd**

(d) $\sin^2 n(-t) = (-\sin \pi t)^2 = \sin^2 \pi t$, **even**

(e) e^t, **neither odd nor even**.

Chapter 17, Problem 18.

Determine the fundamental frequency and specify the type of symmetry present in the functions in Fig. 17.56.

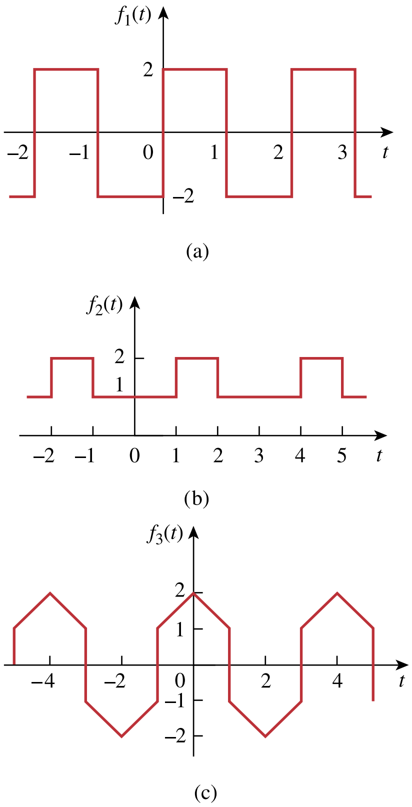


Figure 17.56
For Probs. 17.18 and 17.63.

Chapter 17, Solution 18.

(a) $T = 2$ leads to $\omega_o = 2\pi/T = \underline{\pi}$

$f_1(-t) = -f_1(t)$, showing that $f_1(t)$ is **<u>odd and half-wave symmetric</u>**.

(b) $T = 3$ leads to $\omega_o = 2\pi/3$

$f_2(t) = f_2(-t)$, showing that $f_2(t)$ is **<u>even</u>**.

(c) $T = 4$ leads to $\omega_o = \pi/2$

$f_3(t)$ is **<u>even and half-wave symmetric</u>**.

Chapter 17, Problem 19.

Obtain the Fourier series for the periodic waveform in Fig. 17.57.

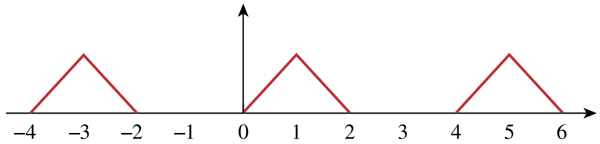


Figure 17.57
For Prob. 17.19.

Chapter 17, Solution 19.

$T = 4, \qquad \omega_o = 2\pi / T = \pi / 2$

$$f(t) = \begin{cases} 10t, & 0 < t < 1 \\ 10(2-t), & 1 < t < 2 \end{cases}$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{4}\int_0^1 10t\,dt + \frac{1}{4}\int_1^2 10(2-t)dt = \frac{1}{4}5t^2\Big|_0^1 + \frac{10}{4}(2t - \frac{t^2}{2})\Big|_1^2 = 2.5$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o t\,dt = \frac{2}{4}\int_0^1 10t\cos n\omega_o t\,dt + \frac{2}{4}\int_1^2 10(2-t)\cos n\omega_o t\,dt$$

$$= \frac{20}{n\omega_o}\cos n\omega_o t + \frac{t}{n\omega_o}\sin n\omega_o t\Big|_0^1 + \frac{10}{n\omega_o}\sin n\omega_o t\Big|_1^2 + \frac{5}{n^2\omega_o^2}\cos n\omega_o t + \frac{5t}{n\omega_o}\sin n\omega_o t\Big|_1^2$$

$$= \frac{20}{n\omega_o}(\cos n\pi/2 - 1) + \frac{1}{n\omega_o}\sin n\pi/2 + \frac{10}{n\omega_o}(\sin n\pi - \sin n\pi/2) + \frac{5}{n^2\pi^2/4}\cos n\pi$$

$$- \frac{5}{n^2\pi^2/4}\cos n\pi/2 + \frac{10}{n\omega_o}\sin n\pi - \frac{5}{n\pi/2}\sin n\pi/2$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o t\,dt = \frac{2}{4}\int_0^1 10t\sin n\omega_o t\,dt + \frac{2}{4}\int_1^2 10(2-t)\sin n\omega_o t\,dt$$

$$= \frac{5}{n\omega_o}\sin n\omega_o t\Big|_0^1 - \frac{10}{n\omega_o}\cos n\omega_o t\Big|_0^1 - \frac{5}{n^2\omega_o^2}\sin n\omega_o t\Big|_1^2 + \frac{t}{n\omega_o}\cos n\omega_o t\Big|_1^2$$

$$= \frac{5}{n^2\omega_o^2}\sin n\pi/2 - \frac{10}{n\omega_o}(\cos\pi n - \cos n\pi/2) - \frac{5}{n^2\omega_o^2}(\sin\pi n - \sin n\pi/2)$$

$$- \frac{2}{n\omega_o}\cos n\pi - \frac{\cos\pi n/2}{n\omega_o}$$

Chapter 17, Problem 20.

ML Find the Fourier series for the signal in Fig. 17.58. Evaluate $f(t)$ at $t = 2$ using the first three nonzero harmonics.

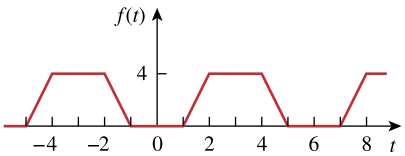


Figure 17.58
For Probs. 17.20 and 17.67.

Chapter 17, Solution 20.

This is an even function.

$$b_n = 0,\ T = 6,\ \omega = 2\pi/6 = \pi/3$$

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{6}\left[\int_1^2 (4t-4)dt \int_2^3 4\,dt\right]$$

$$= \frac{1}{3}\left[(2t^2 - 4t)\Big|_1^2 + 4(3-2)\right] = 2$$

$$a_n = \frac{4}{T}\int_0^{T/4} f(t)\cos(n\pi t/3)dt$$

$$= (4/6)[\int_1^2 (4t-4)\cos(n\pi t/3)dt + \int_2^3 4\cos(n\pi t/3)dt\,]$$

$$= \frac{16}{6}\left[\frac{9}{n^2\pi^2}\cos\left(\frac{n\pi t}{3}\right) + \frac{3t}{n\pi}\sin\left(\frac{n\pi t}{3}\right) - \frac{3}{n\pi}\sin\left(\frac{n\pi t}{3}\right)\right]_1^2 + \frac{16}{6}\left[\frac{3}{n\pi}\sin\left(\frac{n\pi t}{3}\right)\right]_2^3$$

$$= [24/(n^2\pi^2)][\cos(2n\pi/3) - \cos(n\pi/3)]$$

Thus $$f(t) = \underline{\mathbf{2 + \frac{24}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[\cos\left(\frac{2\pi n}{3}\right) - \cos\left(\frac{\pi n}{3}\right)\right]\cos\left(\frac{n\pi t}{3}\right)}}$$

At $t = 2$,

$$f(2) = 2 + (24/\pi^2)[(\cos(2\pi/3) - \cos(\pi/3))\cos(2\pi/3)$$

$$+ (1/4)(\cos(4\pi/3) - \cos(2\pi/3))\cos(4\pi/3)$$

$$+ (1/9)(\cos(2\pi) - \cos(\pi))\cos(2\pi) + \text{-----}]$$

$$= 2 + 2.432(0.5 + 0 + 0.2222 + \text{-----})$$

$f(2) = \underline{\mathbf{3.756}}$

Chapter 17, Problem 21.

Determine the trigonometric Fourier series of the signal in Fig. 17.59.

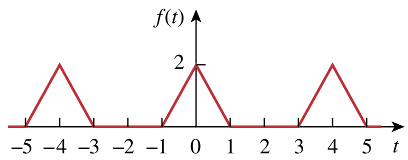


Figure 17.59
For Prob. 17.21.

Chapter 17, Solution 21.

This is an even function.

$b_n = 0,\ T = 4,\ \omega_o = 2\pi/T = \pi/2.$

$$f(t) = 2 - 2t, \qquad 0 < t < 1$$
$$= 0, \qquad 1 < t < 2$$

$$a_o = \frac{2}{4}\int_0^1 2(1-t)dt = \left[t - \frac{t^2}{2}\right]_0^1 = 0.5$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_o t)dt = \frac{4}{4}\int_0^1 2(1-t)\cos\left(\frac{n\pi t}{2}\right)dt$$

$$= [8/(\pi^2 n^2)][1 - \cos(n\pi/2)]$$

$$f(t) = \underline{\mathbf{\frac{1}{2} + \sum_{n=1}^{\infty}\frac{8}{n^2\pi^2}\left[1 - \cos\left(\frac{n\pi}{2}\right)\right]\cos\left(\frac{n\pi t}{2}\right)}}$$

Chapter 17, Problem 22.

Calculate the Fourier coefficients for the function in Fig. 17.60.

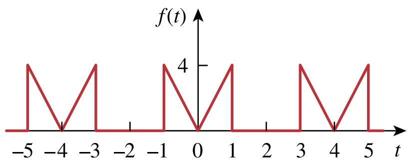


Figure 17.60
For Prob. 17.22.

Chapter 17, Solution 22.

Calculate the Fourier coefficients for the function in Fig. 16.54.

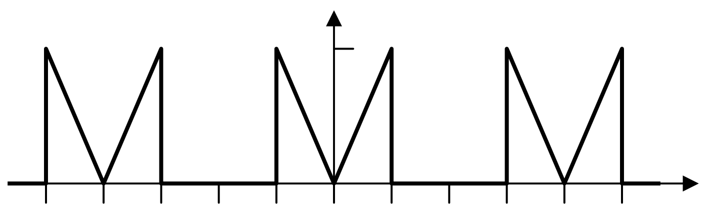


Figure 16.54 For Prob. 16.15

This is an even function, therefore $b_n = 0$. In addition, T=4 and $\omega_o = \pi/2$.

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{4}\int_0^1 4tdt = t^2\Big|_0^1 = \underline{\mathbf{1}}$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(\omega_o nt)dt = \frac{4}{4}\int_0^1 4t\cos(n\pi t/2)dt$$

$$= 4\left[\frac{4}{n^2\pi^2}\cos(n\pi t/2) + \frac{2t}{n\pi}\sin(n\pi t/2)\right]_0^1$$

$$a_n = \underline{\mathbf{\frac{16}{n^2\pi^2}(\cos(n\pi/2) - 1) + \frac{8}{n\pi}\sin(n\pi/2)}}$$

Chapter 17, Problem 23.

Find the Fourier series of the function shown in Fig. 17.61.

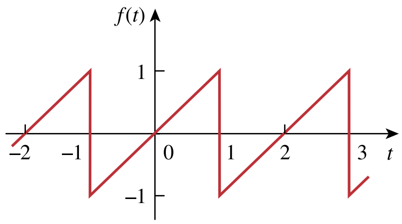


Figure 17.61
For Prob. 17.23.

Chapter 17, Solution 23.

f(t) is an odd function.

$$f(t) = t, \quad -1 < t < 1$$

$$a_o = 0 = a_n, \; T = 2, \; \omega_o = 2\pi/T = \pi$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_o t)dt = \frac{4}{2}\int_0^1 t\sin(n\pi t)dt$$

$$= \frac{2}{n^2\pi^2}\left[\sin(n\pi t) - n\pi t\cos(n\pi t)\right]_0^1$$

$$= -[2/(n\pi)]\cos(n\pi) = 2(-1)^{n+1}/(n\pi)$$

$$f(t) = \underline{\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin(n\pi t)}$$

Chapter 17, Problem 24.

In the periodic function of Fig. 17.62,

(a) find the trigonometric Fourier series coefficients a_2 and b_2,
(b) calculate the magnitude and phase of the component of $f(t)$ that has $\omega_n = 10$ rad/s,
(c) use the first four nonzero terms to estimate $f(\pi/2)$.
(d) show that

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \cdots$$

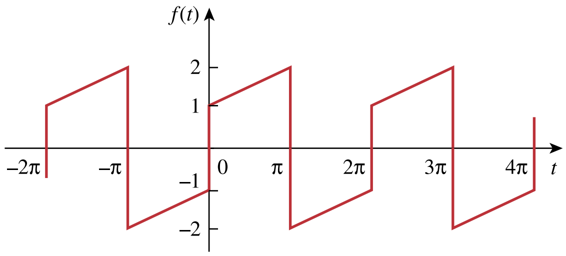


Figure 17.62
For Probs. 17.24 and 17.60.

Chapter 17, Solution 24.

(a) This is an odd function.

$$a_o = 0 = a_n,\ T = 2\pi,\ \omega_o = 2\pi/T = 1$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(\omega_o nt)dt$$

$$f(t) = 1 + t/\pi, \qquad 0 < t < \pi$$

$$b_n = \frac{4}{2\pi}\int_0^{\pi} (1 + t/\pi)\sin(nt)dt$$

$$= \frac{2}{\pi}\left[-\frac{1}{n}\cos(nt) + \frac{1}{n^2\pi}\sin(nt) - \frac{t}{n\pi}\cos(nt)\right]_0^{\pi}$$

$$= [2/(n\pi)][1 - 2\cos(n\pi)] = [2/(n\pi)][1 + 2(-1)^{n+1}]$$

$a_2 = \underline{\mathbf{0}}$, $b_2 = [2/(2\pi)][1 + 2(-1)] = -1/\pi = \underline{\mathbf{-0.3183}}$

(b) $\omega n = n\omega_o = 10$ or $n = 10$

$$a_{10} = 0,\ b_{10} = [2/(10\pi)][1 - \cos(10\pi)] = -1/(5\pi)$$

Thus the magnitude is $A_{10} = \sqrt{a_{10}^2 + b_{10}^2} = 1/(5\pi) = \underline{\mathbf{0.06366}}$

and the phase is $\phi_{10} = \tan^{-1}(b_n/a_n) = \underline{\mathbf{-90^\circ}}$

(c) $$f(t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}[1 - 2\cos(n\pi)]\sin(nt)\,\pi$$

$$f(\pi/2) = \sum_{n=1}^{\infty} \frac{2}{n\pi}[1 - 2\cos(n\pi)]\sin(n\pi/2)\,\pi$$

For $n = 1$, $f_1 = (2/\pi)(1 + 2) = 6/\pi$

For $n = 2$, $f_2 = 0$

For $n = 3$, $f_3 = [2/(3\pi)][1 - 2\cos(3\pi)]\sin(3\pi/2) = -6/(3\pi)$

For $n = 4$, $f_4 = 0$

For $n = 5$, $f_5 = 6/(5\pi)$, ----

Thus, $f(\pi/2) = 6/\pi - 6/(3\pi) + 6/(5\pi) - 6/(7\pi)$ ---------

$= (6/\pi)[1 - 1/3 + 1/5 - 1/7 +$ --------$]$

$f(\pi/2) \cong \underline{\mathbf{1.3824}}$

which is within 8% of the exact value of 1.5.

(d) From part (c)

$f(\pi/2) = 1.5 = (6/\pi)[1 - 1/3 + 1/5 - 1/7 + \text{ - - -}]$

$(3/2)(\pi/6) = [1 - 1/3 + 1/5 - 1/7 + \text{ - - -}]$

or $\pi/4 = \underline{\mathbf{1 - 1/3 + 1/5 - 1/7 + \text{ - - -}}}$

Chapter 17, Problem 25.

Determine the Fourier series representation of the function in Fig. 17.63.

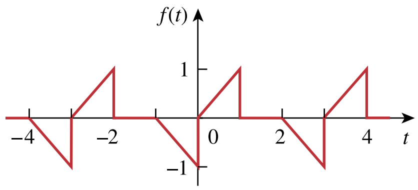


Figure 17.63
For Prob. 17.25.

Chapter 17, Solution 25.

This is a half-wave (odd) function since $f(t-T/2) = -f(t)$.

$a_o = 0$, $a_n = b_n = 0$ for n = even, $T = 3$, $\omega_o = 2\pi/3$.

For n = odd,

$$a_n = \frac{4}{3}\int_0^{1.5} f(t)\cos n\omega_0 t dt = \frac{4}{3}\int_0^{1} t\cos n\omega_0 t dt$$

$$= \frac{4}{3}\left[\frac{9}{4\pi^2 n^2}\cos\left(\frac{2\pi nt}{3}\right) + \frac{3t}{2\pi n}\sin\left(\frac{2\pi nt}{3}\right)\right]_0^1$$

$$= \left[\frac{3}{\pi^2 n^2}\left(\cos\left(\frac{2\pi n}{3}\right) - 1\right) + \frac{2}{\pi n}\sin\left(\frac{2\pi n}{3}\right)\right]$$

$$b_n = \frac{4}{3}\int_0^{1.5} f(t)\sin(n\omega_o t)dt = \frac{4}{3}\int_0^{1} t\sin(2\pi nt/3)dt$$

$$= \frac{4}{3}\left[\frac{9}{4\pi^2 n^2}\sin\left(\frac{2\pi nt}{3}\right) - \frac{3t}{2n\pi}\cos\left(\frac{2\pi nt}{3}\right)\right]_0^1$$

$$= \left[\frac{3}{\pi^2 n^2}\sin\left(\frac{2\pi n}{3}\right) - \frac{2}{\pi n}\cos\left(\frac{2\pi n}{3}\right)\right]$$

$$\underline{f(t) = \sum_{\substack{n=1 \\ n=odd}}^{\infty} \left\{\begin{array}{l}\left[\frac{3}{\pi^2 n^2}\left(\cos\left(\frac{2\pi n}{3}\right) - 1\right) + \frac{2}{\pi n}\sin\left(\frac{2\pi n}{3}\right)\right]\cos\left(\frac{2\pi nt}{3}\right) \\ + \left[\frac{3}{\pi^2 n^2}\sin\left(\frac{2\pi n}{3}\right) - \frac{2}{n\pi}\cos\left(\frac{2\pi n}{3}\right)\right]\sin\left(\frac{2\pi nt}{3}\right)\end{array}\right\}}$$

Chapter 17, Problem 26.

Find the Fourier series representation of the signal shown in Fig. 17.64.

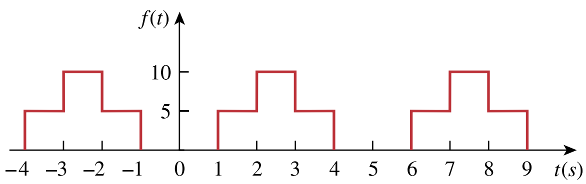


Figure 17.64
For Prob. 17.26.

Chapter 17, Solution 26.

$$T = 4, \ \omega_o = 2\pi/T = \pi/2$$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{4}\left[\int_0^1 1\,dt + \int_1^3 2\,dt + \int_3^4 1\,dt\right] = 1$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_o t)dt$$

$$a_n = \frac{2}{4}\left[\int_1^2 1\cos(n\pi t/2)dt + \int_2^3 2\cos(n\pi t/2)dt + \int_3^4 1\cos(n\pi t/2)dt\right]$$

$$= 2\left[\frac{2}{n\pi}\sin\frac{n\pi t}{2}\Big|_1^2 + \frac{4}{n\pi}\sin\frac{n\pi t}{2}\Big|_2^3 + \frac{2}{n\pi}\sin\frac{n\pi t}{2}\Big|_3^4\right]$$

$$= \frac{4}{n\pi}\left[\sin\frac{3n\pi}{2} - \sin\frac{n\pi}{2}\right]$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_o t)dt$$

$$= \frac{2}{4}\left[\int_1^2 1\sin\frac{n\pi t}{2}dt + \int_2^3 2\sin\frac{n\pi t}{2}dt + \int_3^4 1\sin\frac{n\pi t}{2}dt\right]$$

$$= 2\left[-\frac{2}{n\pi}\cos\frac{n\pi t}{2}\Big|_1^2 - \frac{4}{n\pi}\cos\frac{n\pi t}{2}\Big|_2^3 - \frac{2}{n\pi}\cos\frac{n\pi t}{2}\Big|_3^4\right]$$

$$= \frac{4}{n\pi}\left[\cos(n\pi) - 1\right]$$

Hence

$f(t) =$

$$\mathbf{1 + \sum_{n=1}^{\infty}\frac{4}{n\pi}\left[(\sin(3n\pi/2) - \sin(n\pi/2))\cos(n\pi t/2) + (\cos(n\pi) - 1)\sin(n\pi t/2)\right]}$$

Chapter 17, Problem 27.

For the waveform shown in Fig. 17.65 below,

(a) specify the type of symmetry it has,
(b) calculate a_3 and b_3,
(c) find the rms value using the first five nonzero harmonics.

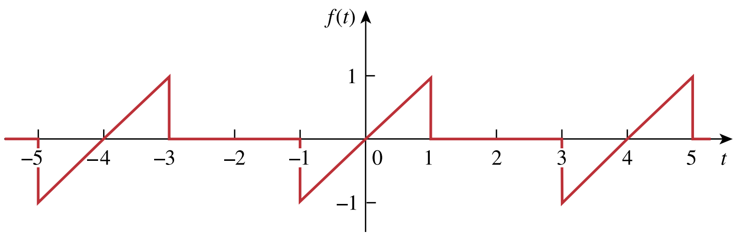


Figure 17.65
For Prob. 17.27.

Chapter 17, Solution 27.

(a) **odd** symmetry.

(b) $a_o = 0 = a_n,\ T = 4,\ \omega_o = 2\pi/T = \pi/2$

$$f(t) = t, \quad 0 < t < 1$$
$$= 0, \quad 1 < t < 2$$

$$b_n = \frac{4}{4}\int_0^1 t\sin\frac{n\pi t}{2}\,dt = \left[\frac{4}{n^2\pi^2}\sin\frac{n\pi t}{2} - \frac{2t}{n\pi}\cos\frac{n\pi t}{2}\right]_0^1$$

$$= \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2} - \frac{2}{n\pi}\cos\frac{n\pi}{2} - 0$$

$$= 4(-1)^{(n-1)/2}/(n^2\pi^2), \qquad n = \text{odd}$$

$$-2(-1)^{n/2}/(n\pi), \qquad n = \text{even}$$

$$a_3 = \underline{\mathbf{0}},\ b_3 = 4(-1)/(9\pi^2) = \underline{\mathbf{-0.04503}}$$

(c) $b_1 = 4/\pi^2,\ b_2 = 1/\pi,\ b_3 = -4/(9\pi^2),\ b_4 = -1/(2\pi),\ b_5 = 4/(25\pi^2)$

$$F_{rms} = \sqrt{a_o^2 + \frac{1}{2}\sum\left(a_n^2 + b_n^2\right)}$$

$$F_{rms}^2 = 0.5\Sigma b_n^2 = [1/(2\pi^2)][(16/\pi^2) + 1 + (16/(81\pi^2)) + (1/4) + (16/(625\pi^2))]$$

$$= (1/19.729)(2.6211 + 0.27 + 0.00259)$$

$$F_{rms} = \sqrt{0.14667} = \underline{\mathbf{0.383}}$$

Compare this with the exact value of $F_{rms} = \sqrt{\frac{2}{T}\int_0^1 t^2 dt} = \sqrt{1/6} = 0.4082$ or

(0.383/0.4082)x100 = 93.83%, close.

Chapter 17, Problem 28.

ML Obtain the trigonometric Fourier series for the voltage waveform shown in Fig. 17.66.

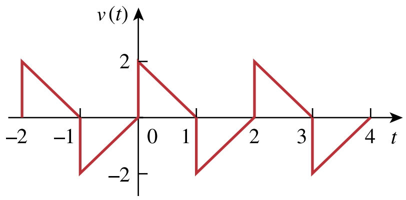


Figure 17.66
For Prob. 17.28.

Chapter 17, Solution 28.

This is half-wave symmetric since f(t – T/2) = –f(t).

$$a_o = 0,\ T = 2,\ \omega_o = 2\pi/2 = \pi$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_o t)dt = \frac{4}{2}\int_0^1 (2 - 2t)\cos(n\pi t)dt$$

$$= 4\left[\frac{1}{n\pi}\sin(n\pi t) - \frac{1}{n^2\pi^2}\cos(n\pi t) - \frac{t}{n\pi}\sin(n\pi t)\right]_0^1$$

$$= [4/(n^2\pi^2)][1 - \cos(n\pi)] = \begin{cases} 8/(n^2\pi^2), & n = \text{odd} \\ 0, & n = \text{even} \end{cases}$$

$$b_n = 4\int_0^1 (1 - t)\sin(n\pi t)dt$$

$$= 4\left[-\frac{1}{n\pi}\cos(n\pi t) - \frac{1}{n^2\pi^2}\sin(n\pi t) + \frac{t}{n\pi}\cos(n\pi t)\right]_0^1$$

$$= 4/(n\pi),\quad n = \text{odd}$$

$$f(t) = \underline{\sum_{k=1}^{\infty}\left(\frac{8}{n^2\pi^2}\cos(n\pi t) + \frac{4}{n\pi}\sin(n\pi t)\right)},\ \underline{n = 2k - 1}$$

Chapter 17, Problem 29.

Determine the Fourier series expansion of the sawtooth function in Fig. 17.67.

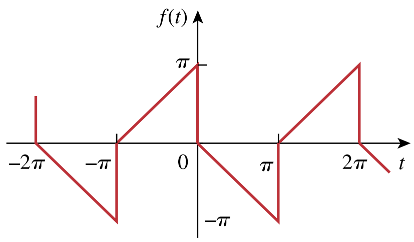


Figure 17.67
For Prob. 17.29.

Chapter 17, Solution 29.

This function is half-wave symmetric.

$$T = 2\pi,\ \omega_o = 2\pi/T = 1,\ f(t) = -t,\ 0 < t < \pi$$

For odd n,
$$a_n = \frac{2}{T}\int_0^{\pi}(-t)\cos(nt)dt = -\frac{2}{n^2\pi}\left[\cos(nt) + nt\sin(nt)\right]_0^{\pi} = 4/(n^2\pi)$$

$$b_n = \frac{2}{\pi}\int_0^{\pi}(-t)\sin(nt)dt = -\frac{2}{n^2\pi}\left[\sin(nt) - nt\cos(nt)\right]_0^{\pi} = -2/n$$

Thus,

$$f(t) = \underline{2\sum_{k=1}^{\infty}\left[\frac{\mathbf{2}}{\mathbf{n^2\pi}}\mathbf{\cos(nt)} - \frac{\mathbf{1}}{\mathbf{n}}\mathbf{\sin(nt)}\right]}, \qquad \underline{\mathbf{n = 2k - 1}}$$

Chapter 17, Problem 30.

(a) If $f(t)$ is an even function, show that

$$c_n = \frac{2}{T}\int_0^{T/2} f(t)\cos n\omega_o t \, dt$$

(b) If *f*(*t*) is an odd function, show that

$$c_n = \frac{j2}{T}\int_0^{T/2} f(t)\sin n\omega_o t \, dt$$

Chapter 17, Solution 30.

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_o t}dt = \frac{1}{T}\left[\int_{-T/2}^{T/2} f(t)\cos n\omega_o tdt - j\int_{-T/2}^{T/2} f(t)\sin n\omega_o tdt\right] \qquad (1)$$

(a) The second term on the right hand side vanishes if f(t) is even. Hence

$$c_n = \frac{2}{T}\int_0^{T/2} f(t)\cos n\omega_o tdt$$

(b) The first term on the right hand side of (1) vanishes if f(t) is odd. Hence,

$$c_n = -\frac{j2}{T}\int_0^{T/2} f(t)\sin n\omega_o tdt$$

Chapter 17, Problem 31.

Let a_n and b_n be the Fourier series coefficients of $f(t)$ and let ω_o be its fundamental frequency. Suppose $f(t)$ is time-scaled to give $h(t) = f(\alpha t)$. Express the $a_n^{'}$ and $b_n^{'}$, and $\omega_o^{'}$, of $h(t)$ in terms of a_n, b_n, and ω_o of $f(t)$.

Chapter 17, Solution 31.

If $h(t) = f(\alpha t)$, $\quad T' = T/\alpha \quad \longrightarrow \quad \omega_o' = \dfrac{2\pi}{T'} = \dfrac{2\pi}{T/\alpha} = \underline{\alpha\omega_o}$

$$a_n' = \frac{2}{T'}\int_0^{T'} h(t)\cos n\omega_o' t dt = \frac{2}{T'}\int_0^{T'} f(\alpha t)\cos n\omega_o' t dt$$

Let $\alpha t = \lambda,$, $\quad d t = d\lambda/\alpha, \quad \alpha T' = T$

$$a_n' = \frac{2\alpha}{T}\int_0^{T} f(\lambda)\cos n\omega_o \lambda d\lambda/\alpha = a_n$$

Similarly, $\quad \underline{b_n' = b_n}$

Chapter 17, Problem 32.

Find $i(t)$ in the circuit of Fig. 17.68 given that

$$i_s(t) = 1 + \sum_{n=1}^{\infty} \frac{1}{n^2} \cos 3nt \text{ A}$$

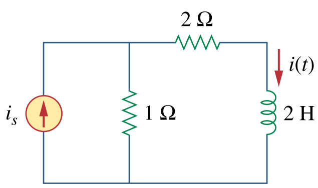


Figure 17.68
For Prob. 17.32.

Chapter 17, Solution 32.

When $i_s = 1$ (DC component)

$$i = 1/(1 + 2) = 1/3$$

For $n \geq 1$, $\omega_n = 3n$, $I_s = 1/n^2\angle 0°$

$$I = [1/(1 + 2 + j\omega_n^2)]I_s = I_s/(3 + j6n)$$

$$= \frac{\frac{1}{n^2}\angle 0°}{3\sqrt{1+4n^2}\angle \tan^{-1}(6n/3)} = \frac{1}{3n^2\sqrt{1+4n^2}}\angle -\tan(2n)$$

Thus,

$$i(t) = \underline{\mathbf{\frac{1}{3} + \sum_{n=1}^{\infty} \frac{1}{3n^2\sqrt{1 + 4n^2}} \cos(3n - \tan^{-1}(2n))}}$$

Chapter 17, Problem 33.

In the circuit shown in Fig. 17.69, the Fourier series expansion of $v_s(t)$ is

$$v_s(t) = 3 + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(n\pi t)$$

Find $v_o(t)$.

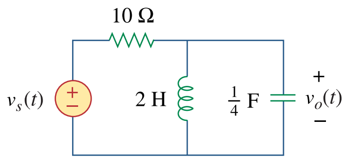


Figure 17.69
For Prob. 17.33.

Chapter 17, Solution 33.

For the DC case, the inductor acts like a short, $V_o = 0$.

For the AC case, we obtain the following:

$$\frac{V_o - V_s}{10} + \frac{V_o}{j2n\pi} + \frac{jn\pi V_o}{4} = 0$$

$$\left(1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)\right)V_o = V_s$$

$$V_o = \frac{V_s}{1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)}$$

$$A_n\angle\Theta_n = \frac{4}{n\pi}\frac{1}{1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)} = \frac{4}{n\pi + j(2.5n^2\pi^2 - 5)}$$

$$A_n = \frac{4}{\sqrt{n^2\pi^2 + (2.5n^2\pi^2 - 5)^2}}; \quad \Theta_n = -\tan^{-1}\left(\frac{2.5n^2\pi^2 - 5}{n\pi}\right)$$

$$v_o(t) = \underline{\sum_{n=1}^{\infty} A_n \sin(n\pi t + \Theta_n)\,V}$$

Chapter 17, Problem 34.

Obtain $v_o(t)$ in the network of Fig. 17.70 if

$$v(t) = \sum_{n=1}^{\infty} \frac{10}{n^2} \cos\left(nt + \frac{n\pi}{4} \right) \text{ V}$$

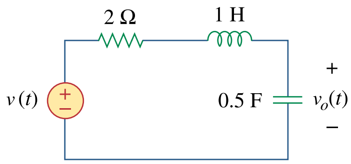
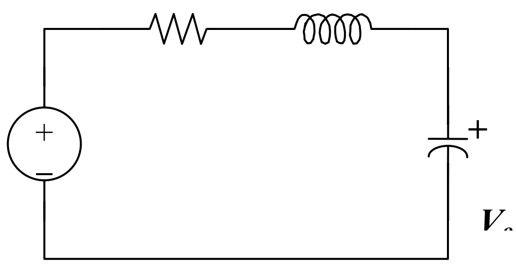


Figure 17.70
For Prob. 17.34.

Chapter 17, Solution 34.

For any n, V = $[10/n^2]\angle(n\pi/4)$, $\omega = n$.

1 H becomes $j\omega_n L = jn$ and 0.5 F becomes $1/(j\omega_n C) = -j2/n$



$$V_o = \{-j(2/n)/[2 + jn - j(2/n)]\}V = \{-j2/[2n + j(n^2 - 2)]\}[(10/n^2)\angle(n\pi/4)]$$

$$= \frac{20\angle((n\pi/4) - \pi/2)}{n^2\sqrt{4n^2 + (n^2 - 2)^2}\angle\tan^{-1}((n^2 - 2)/2n)}$$

$$= \frac{20}{n^2\sqrt{n^2 + 4}}\angle[(n\pi/4) - (\pi/2) - \tan^{-1}((n^2 - 2)/2n)]$$

$$v_o(t) = \underline{\sum_{n=1}^{\infty} \frac{20}{n^2\sqrt{n^2 + 4}} \cos\left(nt + \frac{n\pi}{4} - \frac{\pi}{2} - \tan^{-1}\frac{n^2 - 2}{2n} \right)}$$

Chapter 17, Problem 35.

If v_s in the circuit of Fig. 17.71 is the same as function $f_2(t)$ in Fig. 17.56(b), determine the dc component and the first three nonzero harmonics of $v_o(t)$.

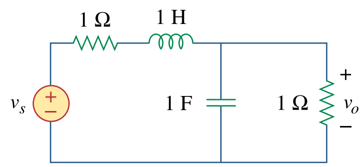


Figure 17.71
For Prob. 17.35.

Chapter 17, Solution 35.

If v_s in the circuit of Fig. 17.72 is the same as function $f_2(t)$ in Fig. 17.57(b), determine the dc component and the first three nonzero harmonics of $v_o(t)$.

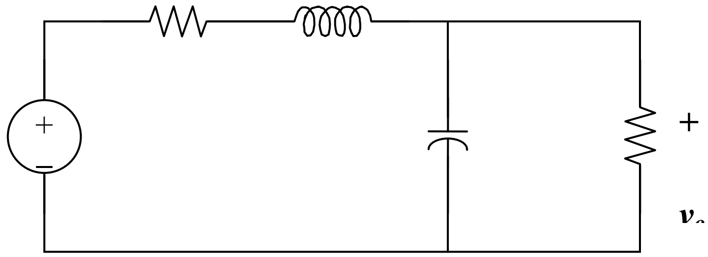


Figure 16.64 For Prob. 16.25

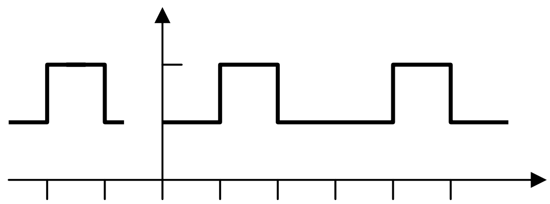


Figure 16.50(b) For Prob. 16.25

The signal is even, hence, $b_n = 0$. In addition, $T = 3$, $\omega_o = 2\pi/3$.

$$v_s(t) = \begin{cases} 1 & \text{for all } 0 < t < 1 \\ 2 & \text{for all } 1 < t < 1.5 \end{cases}$$

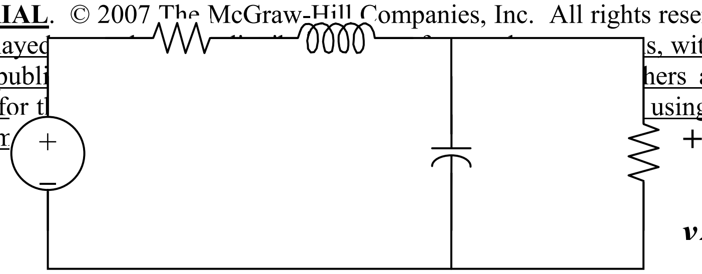
$$a_o = \frac{2}{3}\left[\int_0^1 1dt + \int_1^{1.5} 2dt\right] = \frac{4}{3}$$

$$a_n = \frac{4}{3}\left[\int_0^1 \cos(2n\pi t/3)dt + \int_1^{1.5} 2\cos(2n\pi t/3)dt\right]$$

$$= \frac{4}{3}\left[\frac{3}{2n\pi}\sin(2n\pi t/3)\Big|_0^1 + \frac{6}{2n\pi}\sin(2n\pi t/3)\Big|_1^{1.5}\right] = -\frac{2}{n\pi}\sin(2n\pi/3)$$

$$v_s(t) = \frac{4}{3} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(2n\pi/3)\cos(2n\pi t/3)$$

Now consider this circuit,



Let Z = [-j3/(2nπ)](1)/(1 – j3/(2nπ)) = -j3/(2nπ - j3)

Therefore, v_o = Zv_s/(Z + 1 + j2nπ/3). Simplifying, we get

$$v_o = \frac{-j9v_s}{12n\pi + j(4n^2\pi^2 - 18)}$$

For the dc case, n = 0 and v_s = ¾ V and v_o = v_s/2 = 3/8 V.

We can now solve for $v_o(t)$

$$v_o(t) = \underline{\left[\frac{3}{8} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{2n\pi t}{3} + \Theta_n\right)\right]\text{volts}}$$

$$\text{where } \underline{A_n = \frac{\frac{6}{n\pi}\sin(2n\pi/3)}{\sqrt{16n^2\pi^2 + \left(\frac{4n^2\pi^2}{3} - 6\right)^2}} \text{ and } \Theta_n = 90^o - \tan^{-1}\left(\frac{n\pi}{3} - \frac{3}{2n\pi}\right)}$$

where we can further simplify A_n to this, $\underline{A_n = \frac{9\sin(2n\pi/3)}{n\pi\sqrt{4n^4\pi^4 + 81}}}$

Chapter 17, Problem 36.

* Find the response i_o for the circuit in Fig. 17.72(a), where $v(t)$ is shown in Fig. 17.72(b).

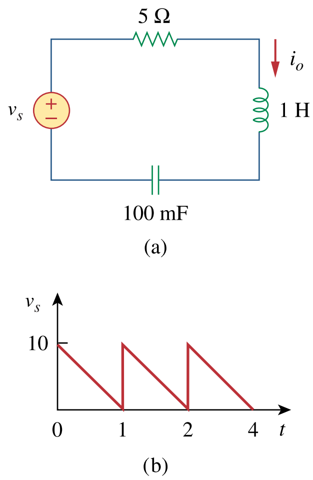


Figure 17.72
For Prob. 17.36.

* An asterisk indicates a challenging problem.

Chapter 17, Solution 36.

We first find the Fourier series expansion of v_s. $T = 1, \quad \omega_o = 2\pi / T = 2\pi$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\int_0^1 10(1-t)tdt = 10(t - \frac{t^2}{2})\Big|_1^0 = 5$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o tdt = 2\int_0^1 10(1-t)\cos 2n\pi tdt$$

$$= 20\left[\frac{1}{2\pi n}\sin 2n\pi t - \frac{1}{4n^2\pi^2}\cos 2n\pi t - \frac{t}{2n\pi}\sin 2n\pi t\right]\Bigg|_o^1 = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o tdt = \frac{2}{2}\int_0^1 10(1-t)t\sin n\omega_o tdt$$

$$= 20\left[-\frac{1}{2n\pi}\cos 2n\pi t - \frac{1}{4n^2\pi^2}\sin 2n\pi t + \frac{1}{2n\pi}\cos 2n\pi t\right]\Bigg|_0^1 = \frac{10}{n\pi}$$

$$v_s(t) = 5 + \sum_{n=1}^{\infty}\frac{10}{n\pi}\sin 2n\pi t$$

$$1H \longrightarrow j\omega_n L = j\omega_n$$

$$10mF \longrightarrow \frac{1}{j\omega_n C} = \frac{1}{j\omega_n 0.01} = \frac{-j100}{\omega_n}$$

$$I_o = \frac{V_s}{5 + j\omega_n - \dfrac{j100}{\omega_n}}$$

For dc component, $\omega_0 = 0$ which leads to $I_0 = 0$.
For the nth harmonic,

$$I_n = \frac{\dfrac{10}{n\pi}\angle 0°}{5 + j2n\pi - \dfrac{j100}{2n\pi}} = \frac{10}{5n\pi + j(2n^2\pi^2 - 50)} = A_n\angle\phi_n$$

where

$$A_n = \frac{10}{\sqrt{25n^2\pi^2 + (2n^2\pi^2 - 50)^2}}, \qquad \phi_n = -\tan^{-1}\frac{2n^2\pi^2 - 50}{5n\pi}$$

$$\underline{i_o(t) = \sum_{n=1}^{\infty} A_n \sin(2n\pi t + \phi_n)}$$

Chapter 17, Problem 37.

If the periodic current waveform in Fig. 17.73(a) is applied to the circuit in Fig. 17.73(b), find v_o.

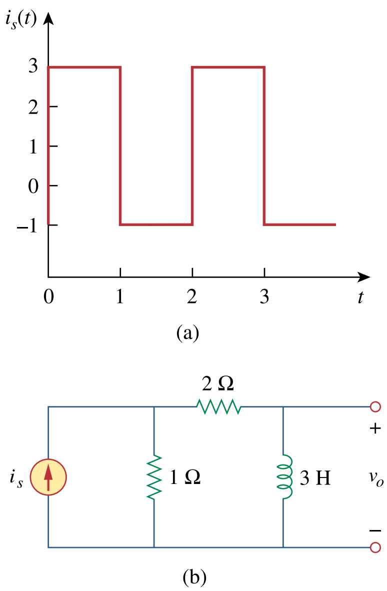


Figure 17.73
For Prob. 17.37.

Chapter 17, Solution 37.

We first need to express i_s in Fourier series. $T = 2$, $\omega_o = 2\pi / T = \pi$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\left[\int_0^1 3dt + \int_1^2 1dt\right] = \frac{1}{2}(3+1) = 2$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o t dt = \frac{2}{2}\left[\int_0^1 3\cos n\pi t dt + \int_1^2 \cos n\pi t dt\right] = \frac{3}{n\pi}\sin n\pi t\Big|_0^1 + \frac{1}{n\pi}\sin n\pi t\Big|_1^2 = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o t dt = \frac{2}{2}\left[\int_0^1 3\sin n\pi t dt + \int_1^2 \sin n\pi t dt\right] = \frac{-3}{n\pi}\cos n\pi t\Big|_0^1 + \frac{-1}{n\pi}\cos n\pi t\Big|_1^2 = \frac{2}{n\pi}(1-\cos n\pi)$$

$$i_s(t) = 2 + \sum_{n=1}^{\infty}\frac{2}{n\pi}(1-\cos n\pi)\sin n\pi t$$

By current division,

$$I_o = \frac{1}{1+2+j\omega_n L}I_s = \frac{I_s}{3+j3\omega_n}$$

$$V_o = j\omega_n L I_o = \frac{j\omega_n 3I_s}{3+j3\omega_n} = \frac{j\omega_n I_s}{1+j\omega_n}$$

For dc component (n=0), $V_o = 0$.
For the nth harmonic,

$$V_o = \frac{jn\pi}{1+jn\pi}\frac{2}{n\pi}(1-\cos n\pi)\angle -90^\circ = \frac{2(1-\cos n\pi)}{\sqrt{1+n^2\pi^2}}\angle(90^\circ - \tan^{-1} n\pi - 90^\circ)$$

$$\underline{v_o(t) = \sum_{n=1}^{\infty}\frac{2(1-\cos\pi n)}{\sqrt{1+n^2\pi^2}}\cos(n\pi t - \tan^{-1} n\pi)}$$

Chapter 17, Problem 38.

If the square wave shown in Fig. 17.74(a) is applied to the circuit in Fig. 17.74(b), find the Fourier series for $v_o(t)$.

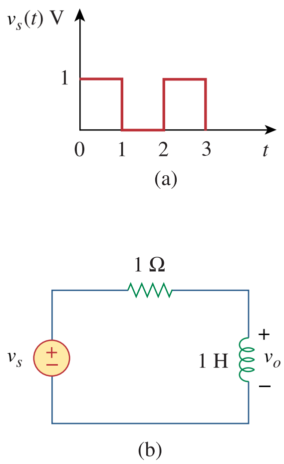


Figure 17.74
For Prob. 17.38.

Chapter 17, Solution 38.

$$v_s(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin n\pi t, \qquad n = 2k+1$$

$$V_o = \frac{j\omega_n}{1 + j\omega_n} V_s, \qquad \omega_n = n\pi$$

For dc, $\omega_n = 0, \quad V_s = 0.5, \quad V_o = 0$

For nth harmonic, $V_s = \frac{2}{n\pi}\angle -90^o$

$$V_o = \frac{n\pi\angle 90^o}{\sqrt{1+n^2\pi^2}\angle \tan^{-1} n\pi} \bullet \frac{2}{n\pi}\angle 90^o = \frac{2\angle -\tan^{-1} n\pi}{\sqrt{1+n^2\pi^2}}$$

$$v_o(t) = \underline{\sum_{k=1}^{\infty}\frac{2}{\sqrt{1+n^2\pi^2}}\cos(n\pi t - \tan^{-1} n\pi), \qquad n = 2k-1}$$

Chapter 17, Problem 39.

If the periodic voltage in Fig. 17.75(a) is applied to the circuit in Fig. 17.75(b), find $i_o(t)$.

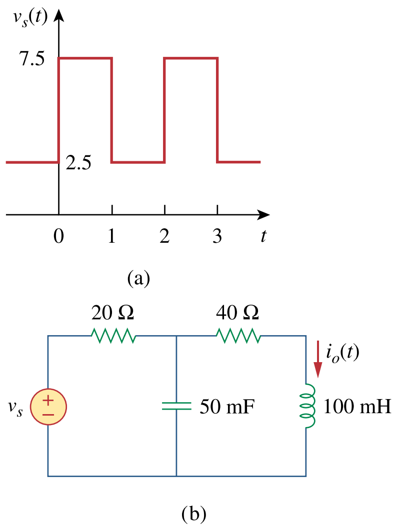


Figure 17.75
For Prob. 17.39.

Chapter 17, Solution 39.

Comparing $v_s(t)$ with f(t) in Figure 15.1, v_s is shifted by 2.5 and the magnitude is 5 times that of f(t).
Hence

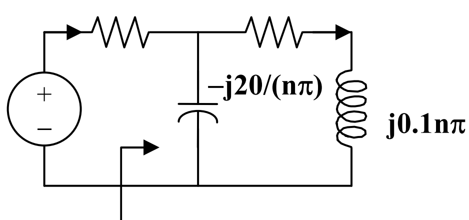
$$v_s(t) = 5 + \frac{10}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin(n\pi t), \qquad n = 2k - 1$$

$$T = 2, \ \omega_o = 2\pi//T = \pi, \ \omega_n = n\omega_o = n\pi$$

For the DC component, $\qquad i_o = 5/(20 + 40) = 1/12$

For the kth harmonic, $\qquad V_s = (10/(n\pi))\angle 0°$

$$100 \text{ mH becomes } j\omega_n L = jn\pi x0.1 = j0.1n\pi$$
$$50 \text{ mF becomes } 1/(j\omega_n C) = -j20/(n\pi)$$



Let $Z = -j20/(n\pi)||(40 + j0.1n\pi) = \dfrac{-\dfrac{j20}{n\pi}(40 + j0.1n\pi)}{-\dfrac{j20}{n\pi} + 40 + j0.1n\pi}$

$$= \frac{-j20(40 + j0.1n\pi}{-j20 + 40n\pi + j0.1n^2\pi^2} = \frac{2n\pi - j800}{40n\pi + j(0.1n^2\pi^2 - 20)}$$

$$Z_{in} = 20 + Z = \frac{802n\pi + j(2n^2\pi^2 - 1200)}{40n\pi + j(0.1n^2\pi^2 - 20)}$$

$$I = \frac{V_s}{Z_{in}} = \frac{400n\pi + j(n^2\pi^2 - 200)}{n\pi[802n\pi + j(2n^2\pi^2 - 1200)]}$$

$$I_o = \frac{-\frac{j20}{n\pi} I}{-\frac{j20}{n\pi} + (40 + j0.1n\pi)} = \frac{-j20I}{40n\pi + j(0.1n^2\pi^2 - 20)}$$

$$= \frac{-j200}{n\pi[802n\pi + j(2n^2\pi^2 - 1200)]}$$

$$= \frac{200\angle -90^\circ - \tan^{-1}\{(2n^2\pi^2 - 1200)/(802n\pi)\}}{n\pi\sqrt{(802)^2 + (2n^2\pi^2 - 1200)^2}}$$

Thus

$$\underline{i_o(t) = \frac{1}{20} + \frac{200}{\pi}\sum_{k=1}^{\infty} I_n \sin(n\pi t - \theta_n)}, \qquad \underline{n = 2k - 1}$$

where $\theta_n = 90^\circ + \tan^{-1}\dfrac{2n^2\pi^2 - 1200}{802n\pi}$

$$I_n = \frac{1}{n\sqrt{(804n\pi)^2 + (2n^2\pi^2 - 1200)}}$$

Chapter 17, Problem 40.

* The signal in Fig. 17.76(a) is applied to the circuit in Fig. 17.76(b). Find $v_o(t)$.

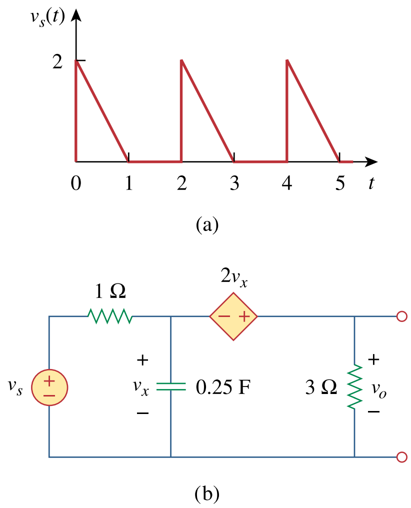


Figure 17.76
For Prob. 17.40.

* An asterisk indicates a challenging problem.

Chapter 17, Solution 40.

$$T = 2, \quad \omega_o = 2\pi/T = \pi$$

$$a_o = \frac{1}{T}\int_0^T v(t)dt = \frac{1}{2}\int_0^1 (2 - 2t)dt = \left[t - \frac{t^2}{2} \right]_0^1 = 1/2$$

$$a_n = \frac{2}{T}\int_0^T v(t)\cos(n\pi t)dt = \int_0^1 2(1-t)\cos(n\pi t)dt$$

$$= 2\left[\frac{1}{n\pi}\sin(n\pi t) - \frac{1}{n^2\pi^2}\cos(n\pi t) - \frac{t}{n\pi}\sin(n\pi t)\right]_0^1$$

$$= \frac{2}{n^2\pi^2}(1-\cos n\pi) = \begin{cases} 0, & n = \text{even} \\ \dfrac{4}{n^2\pi^2}, & n = \text{odd} \end{cases} = \frac{4}{\pi^2(2n-1)^2}$$

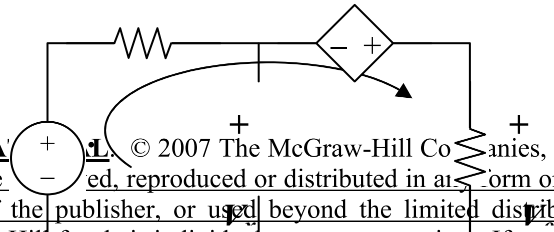
$$b_n = \frac{2}{T}\int_0^T v(t)\sin(n\pi t)dt = 2\int_0^1 (1-t)\sin(n\pi t)dt$$

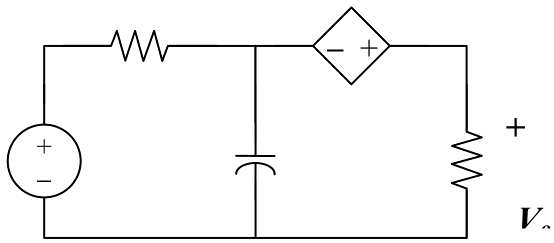
$$= 2\left[-\frac{1}{n\pi}\cos(n\pi t) - \frac{1}{n^2\pi^2}\sin(n\pi t) + \frac{t}{n\pi}\cos(n\pi t)\right]_0^1 = \frac{2}{n\pi}$$

$$v_s(t) = \frac{1}{2} + \sum A_n\cos(n\pi t - \varphi_n)$$

$$\text{where } \phi_n = \tan^{-1}\frac{\pi(2n-1)^2}{2n}, \quad A_n = \sqrt{\frac{4}{n^2\pi^2} + \frac{16}{\pi^4(2n-1)^4}}$$

For the DC component, $v_s = 1/2$. As shown in Figure (a), the capacitor acts like an open circuit.





Applying KVL to the circuit in Figure (a) gives

$$-0.5 - 2V_x + 4i = 0 \qquad (1)$$

But $$-0.5 + i + V_x = 0 \text{ or } -1 + 2V_x + 2i = 0 \qquad (2)$$

Adding (1) and (2), $-1.5 + 6i = 0$ or $i = 0.25$

$$V_o = 3i = 0.75$$

For the nth harmonic, we consider the circuit in Figure (b).

$$\omega_n = n\pi, \; V_s = A_n\angle{-\phi}, \; 1/(j\omega_n C) = -j4/(n\pi)$$

At the supernode,

$$(V_s - V_x)/1 = -[n\pi/(j4)]V_x + V_o/3$$

$$V_s = [1 + jn\pi/4]V_x + V_o/3 \qquad (3)$$

But $-V_x - 2V_x + V_o = 0$ or $V_o = 3V_x$

Substituting this into (3),

$$V_s = [1 + jn\pi/4]V_x + V_x = [2 + jn\pi/4]V_x$$

$$= (1/3)[2 + jn\pi/4]V_o = (1/12)[8 + jn\pi]V_o$$

$$V_o = 12V_s/(8 + jn\pi) = \frac{12A_n\angle -\phi}{\sqrt{64+n^2\pi^2}\angle\tan^{-1}(n\pi/8)}$$

$$V_o =$$
$$\frac{12}{\sqrt{64+n^2\pi^2}}\sqrt{\frac{4}{n^2\pi^2}+\frac{16}{\pi^4(2n-1)^4}}\angle[\tan^{-1}(n\pi/8) - \tan^{-1}(\pi(2n-1)/(2n))]$$

Thus

$$v_o(t) = \underline{\mathbf{\frac{3}{4} + \sum_{n=1}^{\infty} V_n \cos(n\pi t + \theta_n)}}$$

where
$$V_n = \underline{\mathbf{\frac{12}{\sqrt{64+n^2\pi^2}}\sqrt{\frac{4}{n^2\pi^2}+\frac{16}{\pi^4(2n-1)^4}}}}$$

$$\theta_n = \underline{\mathbf{\tan^{-1}(n\pi/8) - \tan^{-1}(\pi(2n-1)/(2n))}}$$

Chapter 17, Problem 41.

The full-wave rectified sinusoidal voltage in Fig. 17.77(a) is applied to the lowpass filter in Fig. 17.77(b). Obtain the output voltage $v_o(t)$ of the filter.

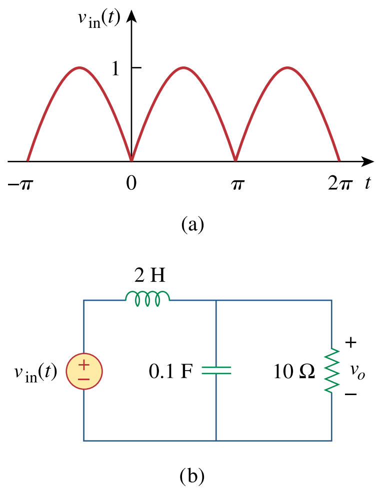


Figure 17.77
For Prob. 17.41.

Chapter 17, Solution 41.

For the full wave rectifier,

$$T = \pi, \ \omega_o = 2\pi/T = 2, \ \omega_n = n\omega_o = 2n$$

Hence

$$v_{in}(t) = \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}\cos(2nt)$$

For the DC component,

$$V_{in} = 2/\pi$$

The inductor acts like a short-circuit, while the capacitor acts like an open circuit.

$$V_o = V_{in} = 2/\pi$$

For the nth harmonic,

$$V_{in} = [-4/(\pi(4n^2 - 1))]\angle 0°$$

$$2\text{ H becomes } j\omega_n L = j4n$$

$$0.1\text{ F becomes } 1/(j\omega_n C) = -j5/n$$

$$Z = 10||(-j5/n) = -j10/(2n - j)$$

$$V_o = [Z/(Z + j4n)]V_{in} = -j10V_{in}/(4 + j(8n - 10))$$

$$= -\frac{j10}{4 + j(8n-10)}\left(-\frac{4\angle 0°}{\pi(4n^2-1)}\right)$$

$$= \frac{40\angle\{90° - \tan^{-1}(2n-2.5)\}}{\pi(4n^2-1)\sqrt{16+(8n-10)^2}}$$

Hence $\qquad v_o(t) = \underline{\mathbf{\frac{2}{\pi} + \sum_{n=1}^{\infty} A_n \cos(2nt + \theta_n)}}$

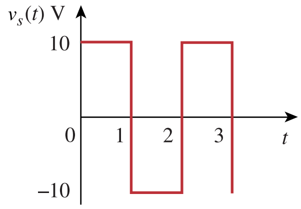
where

$$A_n = \underline{\mathbf{\frac{20}{\pi(4n^2 - 1)\sqrt{16n^2 - 40n + 29}}}}$$

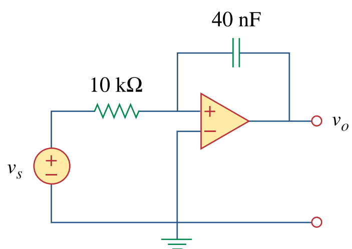
$$\theta_n = \underline{\mathbf{90° - \tan^{-1}(2n - 2.5)}}$$

Chapter 17, Problem 42.

The square wave in Fig. 17.78(a) is applied to the circuit in Fig. 17.78(b). Find the Fourier series of $v_o(t)$.



(a)



(b)

Figure 17.78
For Prob. 17.42.

Chapter 17, Solution 42.

$$v_s = 5 + \frac{20}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin n\pi t,\ \ n = 2k - 1$$

$$\frac{V_s - 0}{R} = j\omega_n C(0 - V_o) \quad \longrightarrow \quad V_o = \frac{j}{\omega_n RC} V_s, \quad \omega_n = n\omega_o = n\pi$$

For n = 0 (dc component), V_o=0.

For the nth harmonic,

$$V_o = \frac{1\angle 90^o}{n\pi RC} \frac{20}{n\pi} \angle -90^o = \frac{20}{n^2\pi^2 x10^4 x40x10^{-9}} = \frac{10^5}{2n^2\pi^2}$$

Hence,

$$\underline{v_o(t) = \frac{10^5}{2\pi^2} \sum_{k=1}^{\infty} \frac{1}{n^2} \cos n\pi t, \ n = 2k - 1}$$

Alternatively, we notice that this is an integrator so that

$$v_o(t) = -\frac{1}{RC}\int v_s dt = \underline{\frac{10^5}{2\pi^2} \sum_{k=1}^{\infty} \frac{1}{n^2} \cos n\pi t, \ n = 2k - 1}$$

Chapter 17, Problem 43.

The voltage across the terminals of a circuit is

$v(t) = 30 + 20\cos(60\pi t + 45°)$
$\quad + 10\cos(60\pi t - 45°)$ V

If the current entering the terminal at higher potential is
$i(t) = 6 + 4\cos(60\pi t + 10°)$
$\quad - 2\cos(120\pi t - 60°)$ A

find:

(a) the rms value of the voltage,
(b) the rms value of the current,
(c) the average power absorbed by the circuit.

Chapter 17, Solution 43.

(a) $$V_{rms} = \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)} = \sqrt{30^2 + \frac{1}{2}(20^2 + 10^2)} = \underline{\mathbf{33.91\ V}}$$

(b) $$I_{rms} = \sqrt{6^2 + \frac{1}{2}(4^2 + 2^2)} = \underline{\mathbf{6.782\ A}}$$

(c) $$P = V_{dc}I_{dc} + \frac{1}{2}\sum V_n I_n \cos(\Theta_n - \Phi_n)$$

$$= 30x6 + 0.5[20x4\cos(45^o\text{-}10^o) - 10x2\cos(\text{-}45^o\text{+}60^o)]$$

$$= 180 + 32.76 - 9.659 = \underline{\mathbf{203.1\ W}}$$

Chapter 17, Problem 44.

The voltage and current through an element are, respectively,

$v(t) = 30\cos(t + 25°) + 10\cos(2t + 35°)$
$\quad + 4\cos(3t - 10°)$ V

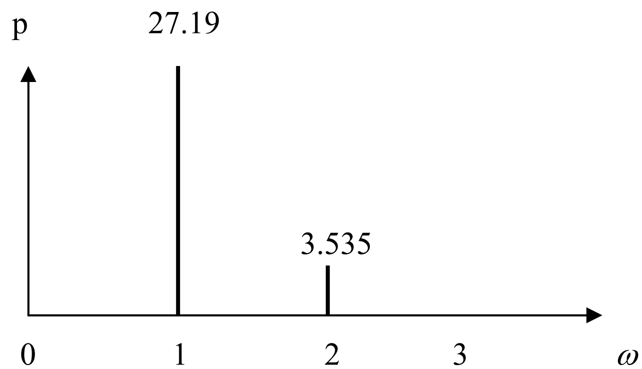
$i(t) = 2\cos t + \cos(2t + 10°)$ A

(a) Find the average power delivered to the element.
(b) Plot the power spectrum.

Chapter 17, Solution 44.

(a) $$p = vi = \frac{1}{2}\left[60\cos 25^o + 10\cos 45^o + 0\right] = 27.19 + 3.535 + 0 = \underline{30.73\ \text{W}}$$

(b) The power spectrum is shown below.



Chapter 17, Problem 45.

A series *RLC* circuit has $R = 10\,\Omega$, $L = 2$ mH, and $C = 40\ \mu$F. Determine the effective current and average power absorbed when the applied voltage is
$v(t) = 100\cos 1000t + 50\cos 2000t$
$+ 25\cos 3000t$ V

Chapter 17, Solution 45.

$\omega_n = 1000n$

$$j\omega_n L = j1000n\text{x}2\text{x}10^{-3} = j2n$$

$$1/(j\omega_n C) = -j/(1000n\text{x}40\text{x}10^{-6}) = -j25/n$$

$$Z = R + j\omega_n L + 1/(j\omega_n C) = 10 + j2n - j25/n$$

$$I = V/Z$$

For $n = 1$, $V_1 = 100$, $Z = 10 + j2 - j25 = 10 - j23$

$$I_1 = 100/(10 - j23) = 3.987\angle 73.89°$$

For $n = 2$, $V_2 = 50$, $Z = 10 + j4 - j12.5 = 10 - j8.5$

$$I_2 = 50/(10 - j8.5) = 3.81\angle 40.36°$$

For $n = 3$, $V_3 = 25$, $Z = 10 + j6 - j25/3 = 10 - j2.333$

$$I_3 = 25/(10 - j2.333) = 2.435\angle 13.13°$$

$$I_{rms} = \sqrt{0.5(3.987^2 + 3.81^2 + 2.435^2)} = \underline{\mathbf{4.263\ A}}$$

$$p = R(I_{rms})^2 = \underline{\mathbf{181.7W}}$$

Chapter 17, Problem 46.

Use *MATLAB* to plot the following sinusoids for $0 < t < 5$:

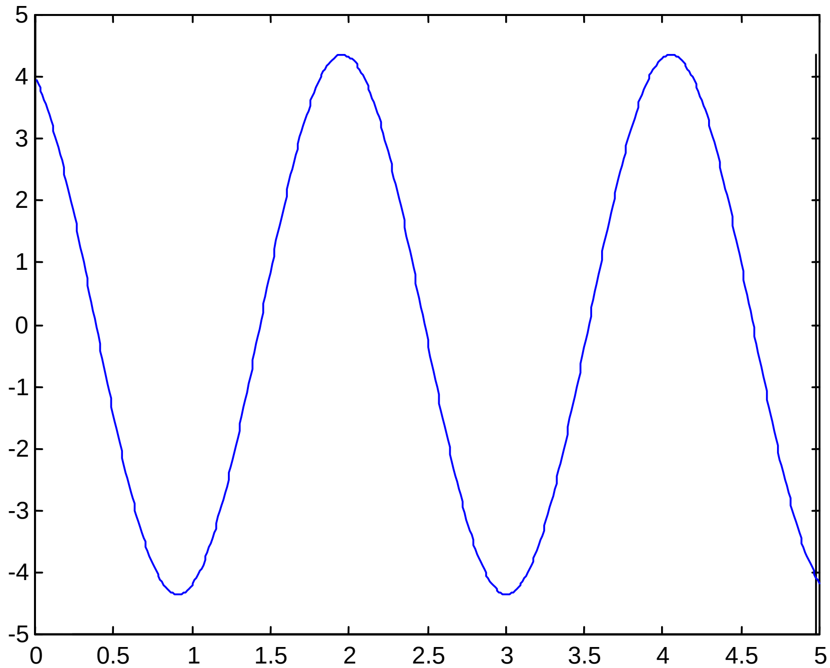
(a) $5 \cos 3t - 2 \cos(3t - \pi/3)$

(b) $8 \sin(\pi t + \pi/4) + 10 \cos(\pi t - \pi/8)$

Chapter 17, Solution 46.

(a)The MATLAB commands are:

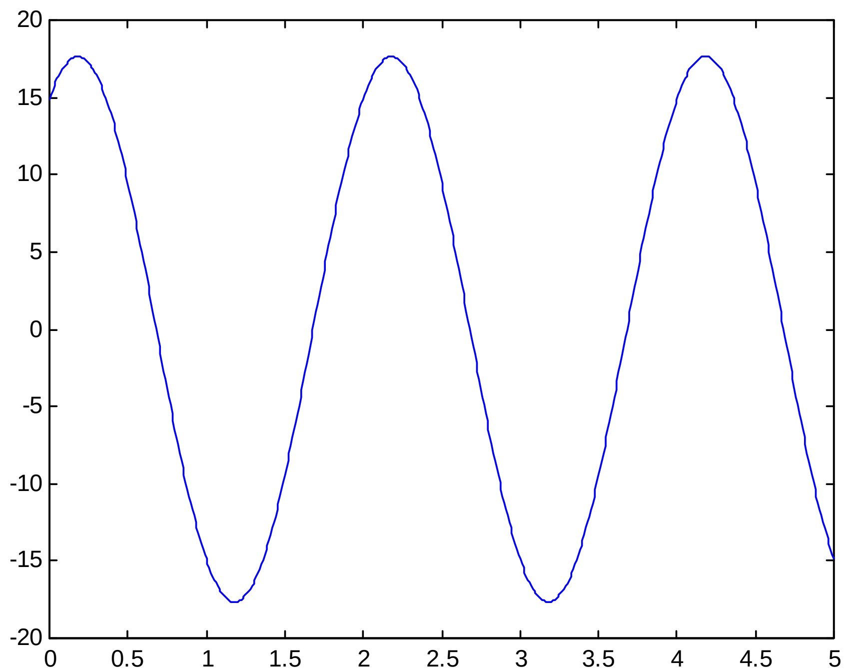
```
t=0:0.01:5;
y=5*cos(3*t) – 2*cos(3*t-pi/3);
plot(t,y)
```



(b) The MATLAB commands are:

```
t=0:0.01:5;
```

```
» x=8*sin(pi*t+pi/4)+10*cos(pi*t-pi/8);
» plot(t,x)
» plot(t,x)
```



Chapter 17, Problem 47.

The periodic current waveform in Fig. 17.79 is applied across a 2-kΩ resistor. Find the percentage of the total average power dissipation caused by the dc component.

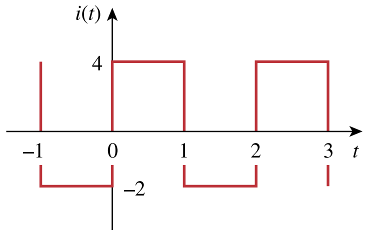


Figure 17.79
For Prob. 17.47.

Chapter 17, Solution 47.

$T = 2, \qquad \omega_o = 2\pi / T = \pi$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\left[\int_0^1 4dt + \int_1^2 (-2)dt\right] = \frac{1}{2}(4-2) = 1$$

$$P = Ri_{rms}^2 = \frac{R}{T}\int_0^T f^2(t)dt = \frac{R}{2}\left[\int_0^1 4^2 dt + \int_1^2 (-2)^2 dt\right] = 10R$$

The average power dissipation caused by the dc component is

$$P_0 = Ra_o^2 = R = \underline{10\%} \text{ of P}$$

Chapter 17, Problem 48.

For the circuit in Fig. 17.80,

$$i(t) = 20 + 16\cos(10t + 45°)$$
$$+ 12\cos(20t - 60°)\ \text{mA}$$

(a) find $v(t)$, and
(b) calculate the average power dissipated in the resistor.

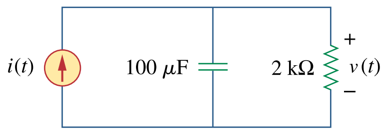


Figure 17.80
For Prob. 17.48.

Chapter 17, Solution 48.

(a) For the DC component, i(t) = 20 mA. The capacitor acts like an open circuit so that

$$v = Ri(t) = 2x10^3x20x10^{-3} = 40$$

For the AC component,

$$\omega_n = 10n,\ n = 1,2$$

$$1/(j\omega_n C) = -j/(10nx100x10^{-6}) = (-j/n)\ k\Omega$$

$$Z = 2||(-j/n) = 2(-j/n)/(2 - j/n) = -j2/(2n - j)$$

$$V = ZI = [-j2/(2n - j)]I$$

For n = 1, $V_1 = [-j2/(2 - j)]16\angle 45° = 14.311\angle -18.43°$ mV

For n = 2, $V_2 = [-j2/(4 - j)]12\angle -60° = 5.821\angle -135.96°$ mV

$v(t) =$ **<u>40 + 0.014311cos(10t – 18.43°) + 0.005821cos(20t – 135.96°) V</u>**

(b) $$p = V_{DC}I_{DC} + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \phi_n)$$

$$= 20x40 + 0.5x10x0.014311\cos(45° + 18.43°)$$
$$+ 0.5x12x0.005821\cos(-60° + 135.96°)$$

= **<u>800.1 mW</u>**

Chapter 17, Problem 49.

(a) For the periodic waveform in Prob. 17.5, find the rms value.

(b) Use the first five harmonic terms of the Fourier series in Prob. 17.5 to determine the effective value of the signal.

(c) Calculate the percentage error in the estimated rms value of $z(t)$ if

$$\%\ \text{error} = \left(\frac{\text{estimated value}}{\text{exact value}} - 1\right) \times 100$$

Chapter 17, Solution 49.

(a) $$Z^2_{rms} = \frac{1}{T}\int_0^T z^2(t)dt = \frac{1}{2\pi}\left[\int_0^{\pi} 1dt + \int_{\pi}^{2\pi} 4dt\right] = \frac{1}{2\pi}(5\pi) = 2.5$$

$$\underline{Z_{rms} = 1.581}$$

(b)

$$Z^2_{rms} = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2) = \frac{1}{4} + \frac{1}{2}\sum_{\substack{n=1 \\ n=odd}}^{\infty} \frac{36}{n^2\pi^2} = \frac{1}{4} + \frac{18}{\pi^2}\left(1 + 0 + \frac{1}{9} + 0 + \frac{1}{25} + \ldots\right) = 2.349$$

Z_{rms} = **$\underline{1.5326}$**

(c) $$\%\text{error} = \left(1 - \frac{1.5326}{1.581}\right)\text{x}100 = \underline{\mathbf{3.061\%}}$$

Chapter 17, Problem 50.

Obtain the exponential Fourier series for $f(t) = t$, $-1 < t < 1$, with $f(t + 2n) = f(t)$ for all integer values of n.

Chapter 17, Solution 50.

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_o nt}dt, \quad \omega_o = \frac{2n}{1} = \pi$$

$$= \frac{1}{2}\int_{-}^{1} te^{-jn\pi t}dt$$

Using integration by parts,

$$u = t \text{ and } du = dt$$

$$dv = e^{-jn\pi t}dt \text{ which leads to } v = -[1/(2jn\pi)]e^{-jn\pi t}$$

$$c_n = -\frac{t}{2jn\pi}e^{-jn\pi t}\Bigg|_{-1}^{1} + \frac{1}{2jn\pi}\int_{-1}^{1} e^{-jn\pi t}dt$$

$$= \frac{j}{n\pi}\left[e^{-jn\pi} + e^{jn\pi t}\right] + \frac{1}{2n^2\pi^2(-j)^2}e^{-jn\pi t}\Bigg|_{-1}^{1}$$

$$= [j/(n\pi)]\cos(n\pi) + [1/(2n^2\pi^2)](e^{-jn\pi} - e^{jn\pi})$$

$$c_n = \frac{j(-1)^n}{n\pi} + \frac{2j}{2n^2\pi^2}\sin(n\pi) = \frac{j(-1)^n}{n\pi}$$

Thus

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t} = \underline{\boldsymbol{\sum_{n=-\infty}^{\infty}(-1)^n \frac{j}{n\pi} e^{jn\pi t}}}$$

Chapter 17, Problem 51.

Given the periodic function

$f(t) = t^2, \qquad 0 < t < T$

obtain the exponential Fourier series for the special case $T = 2$.

Chapter 17, Solution 51.

$$T = 2, \qquad \omega_o = 2\pi / T = \pi$$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}dt = \frac{1}{2}\int_0^2 t^2 e^{-jn\pi t}dt = \frac{1}{2}\frac{e^{-jn\pi t}}{(-jn\pi)^3}\left(-n^2\pi^2 t^2 + 2jn\pi t + 2\right)\Big|_0^2$$

$$c_n = \frac{1}{j2n^3\pi^3}(-4n^2\pi^2 + j4n\pi) = \frac{2}{n^2\pi^2}(1 + jn\pi)$$

$$f(t) = \underline{\sum_{n=-\infty}^{\infty} \frac{2}{n^2\pi^2}(1 + jn\pi)e^{jn\pi t}}$$

Chapter 17, Problem 52.

Calculate the complex Fourier series for $f(t) = e^t$, $-\pi < t < \pi$, with $f(t + 2\pi n) = f(t)$ for all integer values of n.

Chapter 17, Solution 52.

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_o nt}dt, \quad \omega_o = \frac{2n}{1} = \pi$$

$$= \frac{1}{2}\int_{-}^{1} te^{-jn\pi t}dt$$

Using integration by parts,

$$u = t \text{ and } du = dt$$

$$dv = e^{-jn\pi t}dt \text{ which leads to } v = -[1/(2jn\pi)]e^{-jn\pi t}$$

$$c_n = -\frac{t}{2jn\pi}e^{-jn\pi t}\Big|_{-1}^{1} + \frac{1}{2jn\pi}\int_{-1}^{1} e^{-jn\pi t}dt$$

$$= \frac{j}{n\pi}\left[e^{-jn\pi} + e^{jn\pi t}\right] + \frac{1}{2n^2\pi^2(-j)^2}e^{-jn\pi t}\Big|_{-1}^{1}$$

$$= [j/(n\pi)]\cos(n\pi) + [1/(2n^2\pi^2)](e^{-jn\pi} - e^{jn\pi})$$

$$c_n = \frac{j(-1)^n}{n\pi} + \frac{2j}{2n^2\pi^2}\sin(n\pi) = \frac{j(-1)^n}{n\pi}$$

Thus

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t} = \underline{\boldsymbol{\sum_{n=-\infty}^{\infty} (-1)^n \frac{j}{n\pi} e^{jn\pi t}}}$$

Chapter 17, Problem 53.

Find the complex Fourier series for $f(t) = e^{-t}$, $0 < t < 1$, with $f(t + n) = f(t)$ for all integer values of n.

Chapter 17, Solution 53.

$$\omega_o = 2\pi/T = 2\pi$$

$$c_n = \int_0^T e^{-t} e^{-jn\omega_o t} dt = \int_0^1 e^{-(1+jn\omega_o)t} dt$$

$$= \frac{-1}{1 + j2\pi n} e^{-(1+j2n\pi)t}\Big|_0^1 = \frac{-1}{1 + j2n\pi}\left[e^{-(1+j2n\pi)} - 1\right]$$

$$= [1/(j2n\pi)][1 - e^{-1}(\cos(2\pi n) - j\sin(2n\pi))]$$

$$= (1 - e^{-1})/(1 + j2n\pi) = 0.6321/(1 + j2n\pi$$

$$f(t) = \underline{\sum_{n=-\infty}^{\infty} \frac{\mathbf{0.6321e^{j2n\pi t}}}{\mathbf{1 + j2n\pi}}}$$

Chapter 17, Problem 54.

Find the exponential Fourier series for the function in Fig. 17.81.

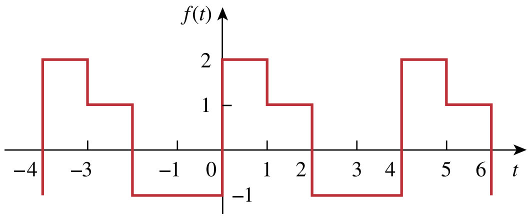


Figure 17.81
For Prob. 17.54.

Chapter 17, Solution 54.

$$T = 4, \quad \omega_o = 2\pi/T = \pi/2$$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_o nt}dt$$

$$= \frac{1}{4}\left[\int_0^1 2e^{-jn\pi t/2}dt + \int_1^2 1e^{-jn\pi t/2}dt - \int_2^4 1e^{-jn\pi t/2}dt\right]$$

$$= \frac{j}{2n\pi}\left[2e^{-jn\pi/2} - 2 + e^{-jn\pi} - e^{-jn\pi/2} - e^{-j2n\pi} + e^{-jn\pi}\right]$$

$$= \frac{j}{2n\pi}\left[3e^{-jn\pi/2} - 3 + 2e^{-jn\pi}\right]$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}$$

Chapter 17, Problem 55.

Obtain the exponential Fourier series expansion of the half-wave rectified sinusoidal current of Fig. 17.82.

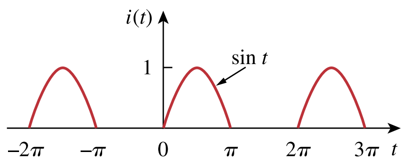


Figure 17.82
For Prob. 17.55.

Chapter 17, Solution 55.

$T = 2\pi, \ \omega_o = 2\pi/T = 1$

$$c_n = \frac{1}{T}\int_0^T i(t)e^{-jn\omega_o t}dt$$

But $$i(t) = \begin{cases} \sin(t), & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

$$c_n = \frac{1}{2\pi}\int_0^{\pi}\sin(t)e^{-jnt}dt = \frac{1}{2\pi}\int_0^{\pi}\frac{1}{2j}(e^{jt} - e^{-jt})e^{-jnt}dt$$

$$= \frac{1}{4\pi j}\left[\frac{e^{jt(1-n)}}{j(1-n)} + \frac{e^{-jt(1+n)}}{j(1+n)}\right]\Bigg|_0^{\pi}$$

$$= -\frac{1}{4\pi}\left[\frac{e^{j\pi(1-n)} - 1}{1-n} + \frac{e^{-j\pi(n+1)} - 1}{1+n}\right]$$

$$= \frac{1}{4\pi(n^2 - 1)}\left[e^{j\pi(1-n)} - 1 + ne^{j\pi(1-n)} - n + e^{-j\pi(1+n)} - 1 - ne^{-j\pi(1+n)} + n\right]$$

But $e^{j\pi} = \cos(\pi) + j\sin(\pi) = -1 = e^{-j\pi}$

$$c_n = \frac{1}{4\pi(n^2 - 1)}\left[-e^{-jn\pi} - e^{-jn\pi} - ne^{-jn\pi} + ne^{-jn\pi} - 2\right] = \frac{1 + e^{-jn\pi}}{2\pi(1 - n^2)}$$

Thus

$$\underline{i(t) = \sum_{n=-\infty}^{\infty}\frac{1 + e^{-jn\pi}}{2\pi(1 - n^2)}e^{jnt}}$$

Chapter 17, Problem 56.

The Fourier series trigonometric representation of a periodic function is

$$f(t) = 10 + \sum_{n=1}^{\infty} \left(\frac{1}{n^2+1} \cos n\pi t + \frac{n}{n^2+1} \sin n\pi t \right)$$

Find the exponential Fourier series representation of $f(t)$.

Chapter 17, Solution 56.

$c_o = a_o = 10, \ \omega_o = \pi$

$c_o = (a_n - jb_n)/2 = (1 - jn)/[2(n^2 + 1)]$

$$\underline{\mathbf{f(t) = 10 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{(1 - jn)}{2(n^2+1)} e^{jn\pi t}}}$$

Chapter 17, Problem 57.

The coefficients of the trigonometric Fourier series representation of a function are:

$b_n = 0, \ a_n = \dfrac{6}{n^3 - 2}, \ n = 0, 1, 2, \cdots$

If $\omega_n = 50n$, find the exponential Fourier series for the function.

Chapter 17, Solution 57.

$a_o = (6/{-2}) = -3 = c_o$

$c_n = 0.5(a_n - jb_n) = a_n/2 = 3/(n^3 - 2)$

$$\underline{\mathrm{f(t)} = \mathbf{-3 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{3}{n^3 - 2} e^{j50nt}}}$$

Chapter 17, Problem 58.

Find the exponential Fourier series of a function that has the following trigonometric Fourier series coefficients:

$$a_0 = \frac{\pi}{4}, \qquad b_n = \frac{(-1)^n}{n}, \qquad a_n = \frac{(-1)^n - 1}{\pi n^2}$$

Take $T = 2\pi$.

Chapter 17, Solution 58.

$c_n = (a_n - jb_n)/2$, $(-1)^n = \cos(n\pi)$, $\omega_o = 2\pi/T = 1$

$c_n = [(\cos(n\pi) - 1)/(2\pi n^2)] - j\cos(n\pi)/(2n)$

Thus

$$f(t) = \underline{\frac{\pi}{4} + \sum\left(\frac{\cos(n\pi) - 1}{2\pi n^2} - j\frac{\cos(n\pi)}{2n}\right)e^{jnt}}$$

Chapter 17, Problem 59.

The complex Fourier series of the function in Fig. 17.83(a) is

$$f(t) = \frac{1}{2} - \sum_{n=-\infty}^{\infty} \frac{je^{-j(2n+1)t}}{(2n+1)\pi}$$

Find the complex Fourier series of the function *h*(*t*) in Fig. 17.83(b).

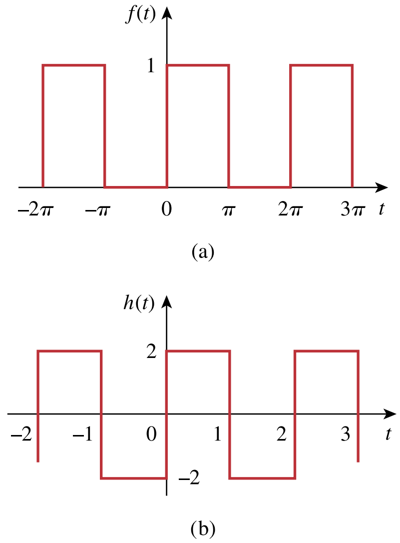


Figure 17.83
For Prob.17.59.

Chapter 17, Solution 59.

For f(t), $T = 2\pi$, $\omega_o = 2\pi/T = 1$.

a_o = DC component = $(1x\pi + 0)/2\pi = 0.5$

For h(t), $T = 2$, $\omega_o = 2\pi/T = \pi$.

$a_o = (2x1 - 2x1)/2 = 0$

Thus by replacing $\omega_o = 1$ with $\omega_o = \pi$ and multiplying the magnitude by four, we obtain

$$h(t) = -\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{j4e^{-j(2n+1)\pi t}}{(2n+1)\pi}$$

Chapter 17, Problem 60.

Obtain the complex Fourier coefficients of the signal in Fig. 17.62.

Chapter 17, Solution 60.

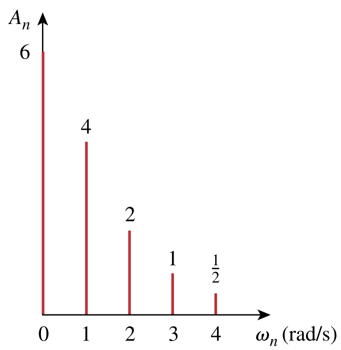
From Problem 17.24,

$$a_o = 0 = a_n, \; b_n = [2/(n\pi)][1 - 2\cos(n\pi)], \; c_o = \underline{\mathbf{0}}$$

$$c_n = (a_n - jb_n)/2 = \underline{\mathbf{[j/(n\pi)][2\cos(n\pi) - 1], \; n \neq 0}}.$$

Chapter 17, Problem 61.

The spectra of the Fourier series of a function are shown in Fig. 17.84. (a) Obtain the trigonometric Fourier series. (b) Calculate the rms value of the function.



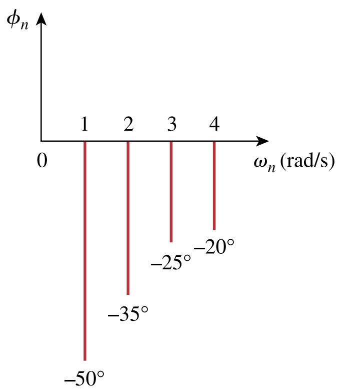


Figure 17.84
For Prob. 17.61.

Chapter 17, Solution 61.

(a) $$\omega_o = 1.$$

$$f(t) = a_o + \sum A_n \cos(n\omega_o t - \phi_n)$$

$$= 6 + 4\cos(t + 50°) + 2\cos(2t + 35°) + \cos(3t + 25°) + 0.5\cos(4t + 20°)$$

$$= 6 + 4\cos(t)\cos(50°) – 4\sin(t)\sin(50°) + 2\cos(2t)\cos(35°) – 2\sin(2t)\sin(35°) + \cos(3t)\cos(25°) – \sin(3t)\sin(25°) + 0.5\cos(4t)\cos(20°) – 0.5\sin(4t)\sin(20°)$$

$$= \underline{\mathbf{6 + 2.571\cos(t) – 3.73\sin(t) + 1.635\cos(2t) – 1.147\sin(2t) + 0.906\cos(3t) – 0.423\sin(3t) + 0.47\cos(4t) – 0.171\sin(4t)}}$$

(b)

$$f_{rms} = \sqrt{a_o^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2}$$

$$f_{rms}^2 = 6^2 + 0.5[4^2 + 2^2 + 1^2 + (0.5)^2] = 46.625$$

$$f_{rms} = \underline{\mathbf{6.828}}$$

Chapter 17, Problem 62.

The amplitude and phase spectra of a truncated Fourier series are shown in Fig. 17.85.

(a) Find an expression for the periodic voltage using the amplitude-phase form. See Eq. (17.10).

(b) Is the voltage an odd or even function of *t*?

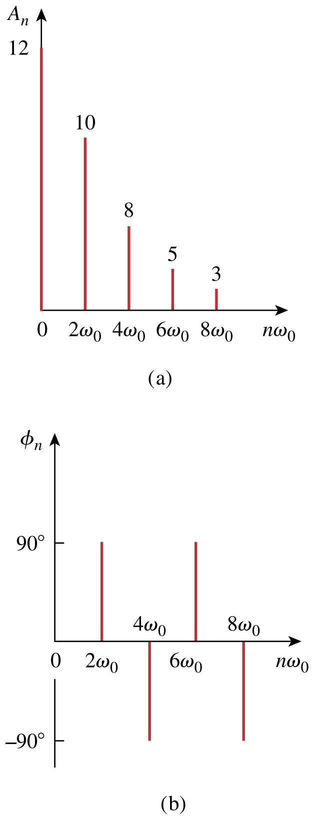


Figure 17.85
For Prob. 17.62.

Chapter 17, Solution 62.

(a)

$$f(t) = 12 + 10\cos(2\omega_o t + 90^o) + 8\cos(4\omega_o t - 90^o) + 5\cos(6\omega_o t + 90^o) + 3\cos(8\omega_o t - 90^o)$$

(b) f(t) is an even function of t.

Chapter 17, Problem 63.

Plot the amplitude spectrum for the signal $f_2(t)$ in Fig. 17.56(b). Consider the first five terms.

Chapter 17, Solution 63.

This is an even function.

$T = 3, \ \omega_o = 2\pi/3, \ b_n = 0.$

$$f(t) = \begin{vmatrix} 1, & 0 < t < 1 \\ 2, & 1 < t < 1.5 \end{vmatrix}$$

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{3}\left[\int_0^1 1\,dt + \int_1^{1.5} 2\,dt\right] = (2/3)[1 + 1] = 4/3$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_o t)dt = \frac{4}{3}\left[\int_0^1 1\cos(2n\pi t/3)dt + \int_1^{1.5} 2\cos(2n\pi t/3)dt\right]$$

$$= \frac{4}{3}\left[\frac{3}{2n\pi}\sin\left(\frac{2n\pi t}{3}\right)\Big|_0^1 + \frac{6}{2n\pi}\sin\left(\frac{2n\pi t}{3}\right)\Big|_1^{1.5}\right]$$

$$= [-2/(n\pi)]\sin(2n\pi/3)$$

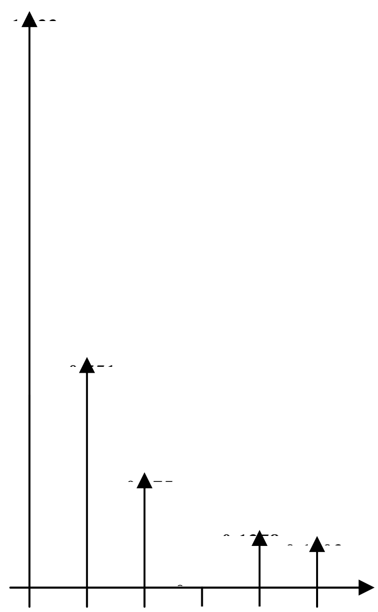
$$f_2(t) = \frac{4}{3} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{3n\pi}{3}\right)\cos\left(\frac{2n\pi t}{3}\right)$$

$a_o = 4/3 = 1.3333, \ \ \omega_o = 2\pi/3, \ a_n = -[2/(n\pi)]\sin(2n\pi t/3)$

$$A_n = \sqrt{a_n^2 + b_n^2} = \left|\frac{2}{n\pi}\sin\left(\frac{2n\pi}{3}\right)\right|$$

$$A_1 = 0.5513,\ A_2 = 0.2757,\ A_3 = 0,\ A_4 = 0.1375,\ A_5 = 0.1103$$

The amplitude spectra are shown below.



Chapter 17, Problem 64.

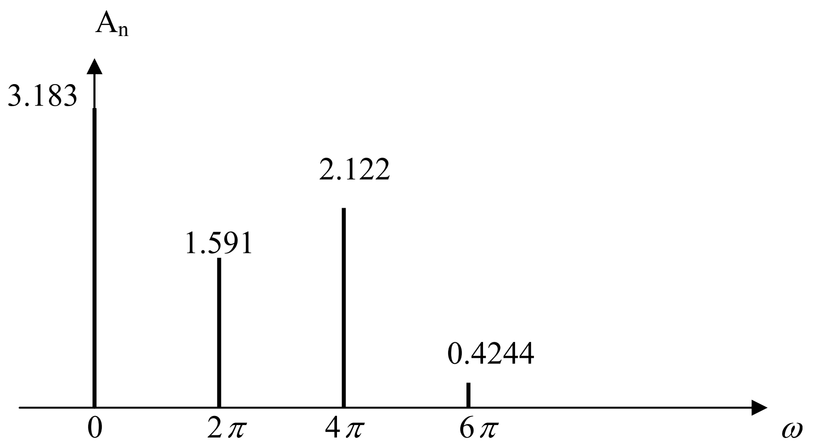
Given that

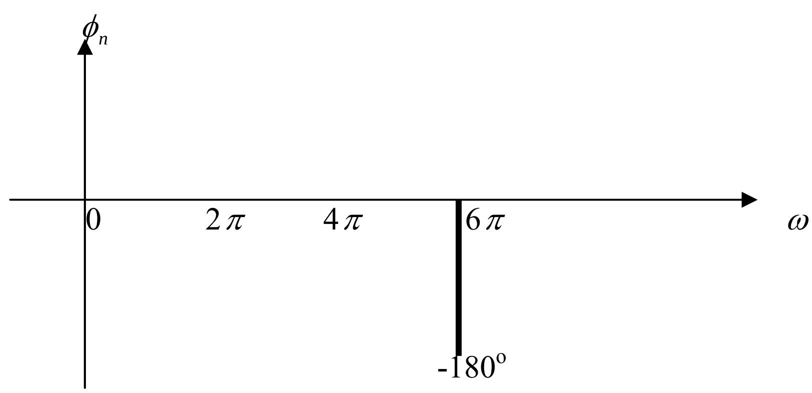
$$v(t) = \frac{10}{\pi}\left[1 + \frac{1}{2}\cos 2\pi t + \frac{2}{3}\cos 4\pi t - \frac{2}{15}\cos 6\pi t + \cdots\right]$$

draw the amplitude and phase spectra for $v(t)$.

Chapter 17, Solution 64.

The amplitude and phase spectra are shown below.





Chapter 17, Problem 65.

Given that

$$f(t) = \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \left(\frac{20}{n^2\pi^2} \cos 2nt - \frac{3}{n\pi} \sin 2nt \right)$$

plot the first five terms of the amplitude and phase spectra for the function.

Chapter 17, Solution 65.

$a_n = 20/(n^2\pi^2)$, $b_n = -3/(n\pi)$, $\omega_n = 2n$

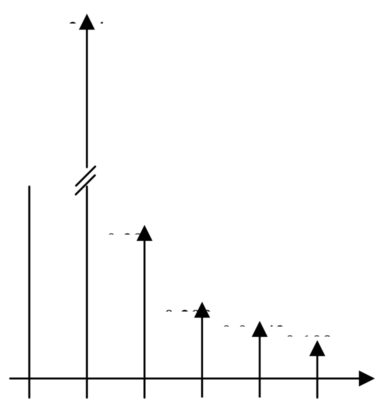
$$A_n = \sqrt{a_n^2 + b_n^2} = \sqrt{\frac{400}{n^4\pi^4} + \frac{9}{n^2\pi^2}}$$

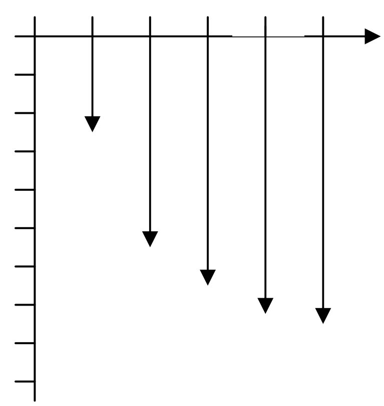
$$= \frac{3}{n\pi}\sqrt{1 + \frac{44.44}{n^2\pi^2}}, \quad n = 1, 3, 5, 7, 9, \text{ etc.}$$

n	A_n
1	2.24
3	0.39
5	0.208
7	0.143
9	0.109

$$\phi_n = \tan^{-1}(b_n/a_n) = \tan^{-1}\{[-3/(n\pi)][n^2\pi^2/20]\} = \tan^{-1}(-n\text{x}0.4712)$$

n	ϕ_n
1	–25.23°
3	–54.73°
5	–67°
7	–73.14°
9	–76.74°
∞	–90°



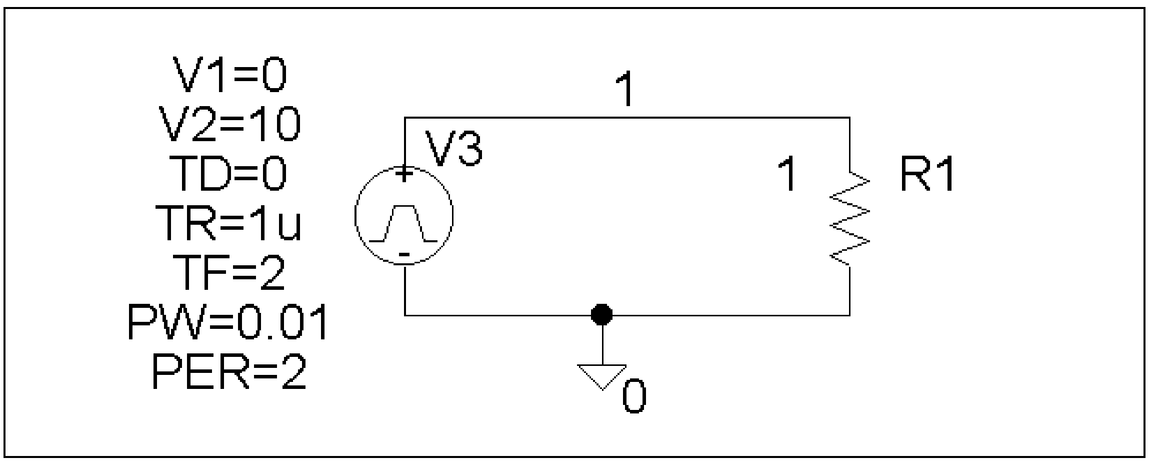


Chapter 17, Problem 66.

Determine the Fourier coefficients for the waveform in Fig. 17.48 using *PSpice*.

Chapter 17, Solution 66.

The schematic is shown below. The waveform is inputted using the attributes of VPULSE. In the Transient dialog box, we enter Print Step = 0.05, Final Time = 12, Center Frequency = 0.5, Output Vars = V(1) and click enable Fourier. After simulation, the output plot is shown below. The output file includes the following Fourier components.



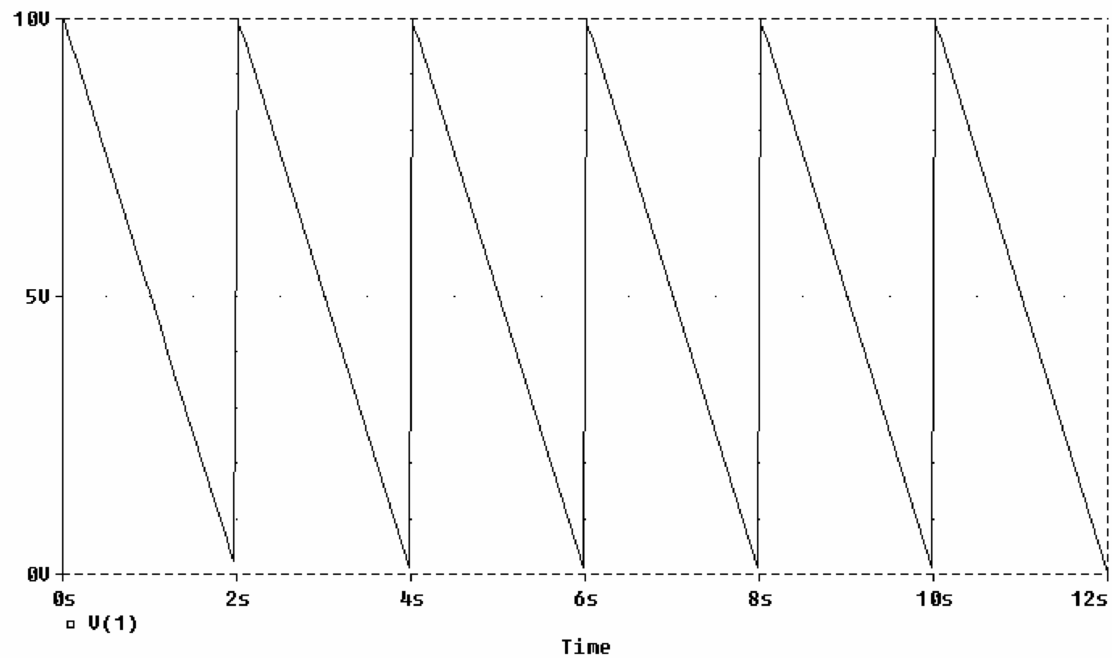
FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 5.099510E+00

HARMONIC FREQUENCY FOURIER NORMALIZED PHASE NORMALIZED

NO	(HZ)	COMPONENT	COMPONENT	(DEG)	PHASE (DEG)
1	5.000E-01	3.184E+00	1.000E+00	1.782E+00	0.000E+00
2	1.000E+00	1.593E+00	5.002E-01	3.564E+00	1.782E+00
3	1.500E+00	1.063E+00	3.338E-01	5.347E+00	3.564E+00
4	2.000E+00	7.978E-01	2.506E-01	7.129E+00	5.347E+00
5	2.500E+00	6.392E-01	2.008E-01	8.911E+00	7.129E+00
6	3.000E+00	5.336E-01	1.676E-01	1.069E+01	8.911E+00
7	3.500E+00	4.583E-01	1.440E-01	1.248E+01	1.069E+01
8	4.000E+00	4.020E-01	1.263E-01	1.426E+01	1.248E+01
9	4.500E+00	3.583E-01	1.126E-01	1.604E+01	1.426E+01

TOTAL HARMONIC DISTORTION = 7.363360E+01 PERCENT

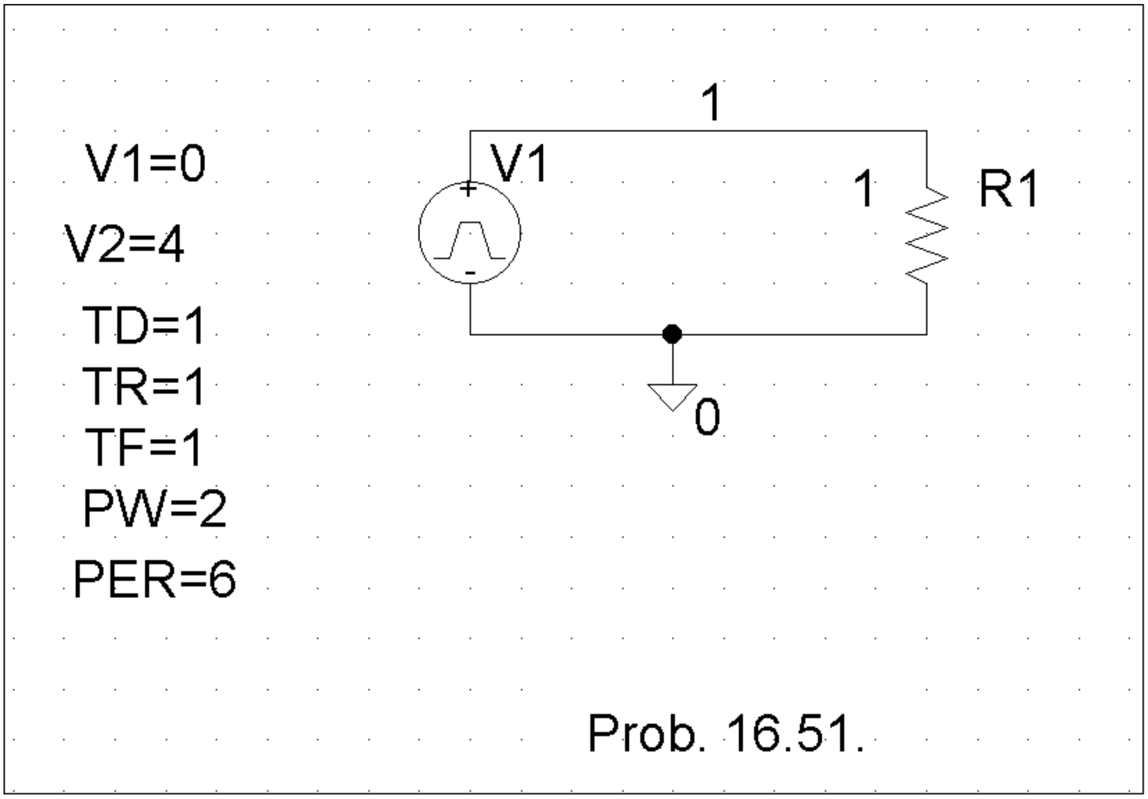


Chapter 17, Problem 67.

Calculate the Fourier coefficients of the signal in Fig. 17.58 using *PSpice*.

Chapter 17, Solution 67.

The Schematic is shown below. In the Transient dialog box, we type "Print step = 0.01s, Final time = 36s, Center frequency = 0.1667, Output vars = v(1)," and click Enable Fourier. After simulation, the output file includes the following Fourier components,



FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 2.000396E+00

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	1.667E-01	2.432E+00	1.000E+00	-8.996E+01	0.000E+00
2	3.334E-01	6.576E-04	2.705E-04	-8.932E+01	6.467E-01
3	5.001E-01	5.403E-01	2.222E-01	9.011E+01	1.801E+02
4	6.668E-01	3.343E-04	1.375E-04	9.134E+01	1.813E+02
5	8.335E-01	9.716E-02	3.996E-02	-8.982E+01	1.433E-01
6	1.000E+00	7.481E-06	3.076E-06	-9.000E+01	-3.581E-02
7	1.167E+00	4.968E-02	2.043E-02	-8.975E+01	2.173E-01
8	1.334E+00	1.613E-04	6.634E-05	-8.722E+01	2.748E+00
9	1.500E+00	6.002E-02	2.468E-02	9.032E+01	1.803E+02

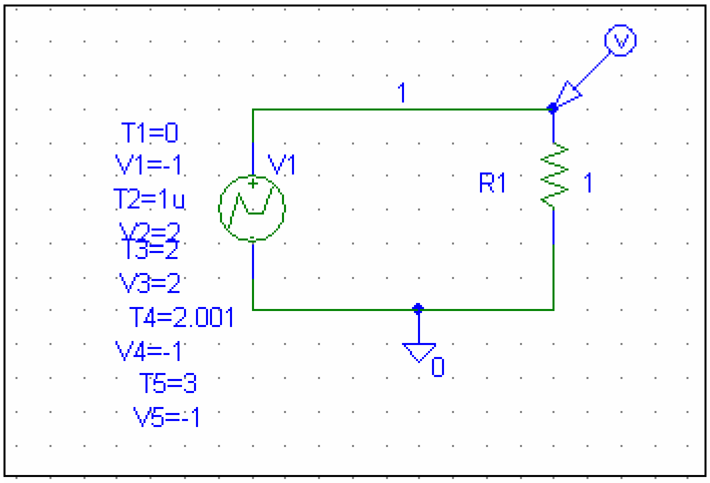
TOTAL HARMONIC DISTORTION = 2.280065E+01 PERCENT

Chapter 17, Problem 68.

Use *PSpice* to find the Fourier components of the signal in Prob. 17.7.

Chapter 17, Solution 68.

Since T=3, f =1/3 = 0.333 Hz. We use the schematic below.



We use VPWL to enter in the signal as shown. In the transient dialog box, we enable Fourier, select 15 for Final Time, 0.01s for Print Step, and 10ms for the Step Ceiling. When the file is saved and run, we obtain the Fourier coefficients as part of the output file as shown below.

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = -1.000000E+00

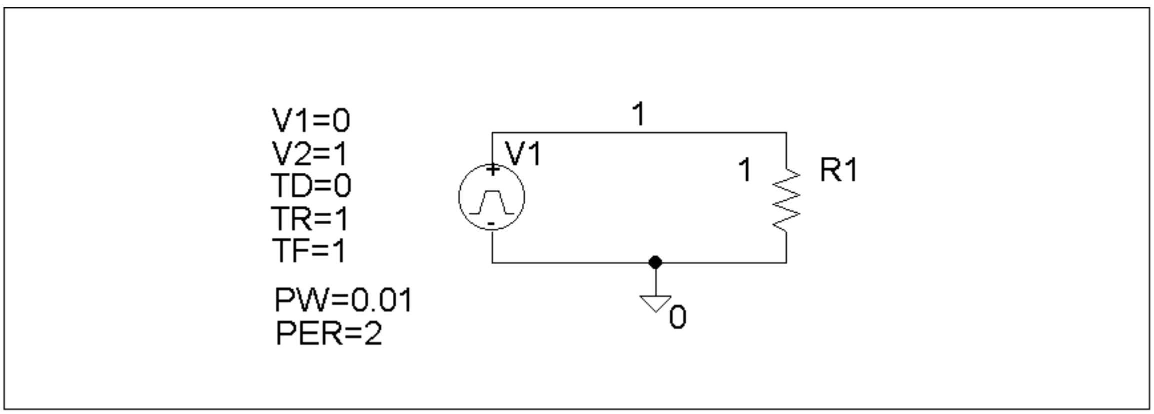
HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	3.330E-01	1.615E-16	1.000E+00	1.762E+02	0.000E+00
2	6.660E-01	5.133E-17	3.179E-01	2.999E+01	-3.224E+02
3	9.990E-01	6.243E-16	3.867E+00	6.687E+01	-4.617E+02
4	1.332E+00	1.869E-16	1.158E+00	7.806E+01	-6.267E+02
5	1.665E+00	6.806E-17	4.215E-01	1.404E+02	-7.406E+02
6	1.998E+00	1.949E-16	1.207E+00	-1.222E+02	-1.179E+03
7	2.331E+00	1.465E-16	9.070E-01	-4.333E+01	-1.277E+03
8	2.664E+00	3.015E-16	1.867E+00	-1.749E+02	-1.584E+03
9	2.997E+00	1.329E-16	8.233E-01	-9.565E+01	-1.681E+03

Chapter 17, Problem 69.

Use *PSpice* to obtain the Fourier coefficients of the waveform in Fig. 17.55(a).

Chapter 17, Solution 69.

The schematic is shown below. In the Transient dialog box, set Print Step = 0.05 s, Final Time = 120, Center Frequency = 0.5, Output Vars = V(1) and click enable Fourier. After simulation, we obtain V(1) as shown below. We also obtain an output file which includes the following Fourier components.

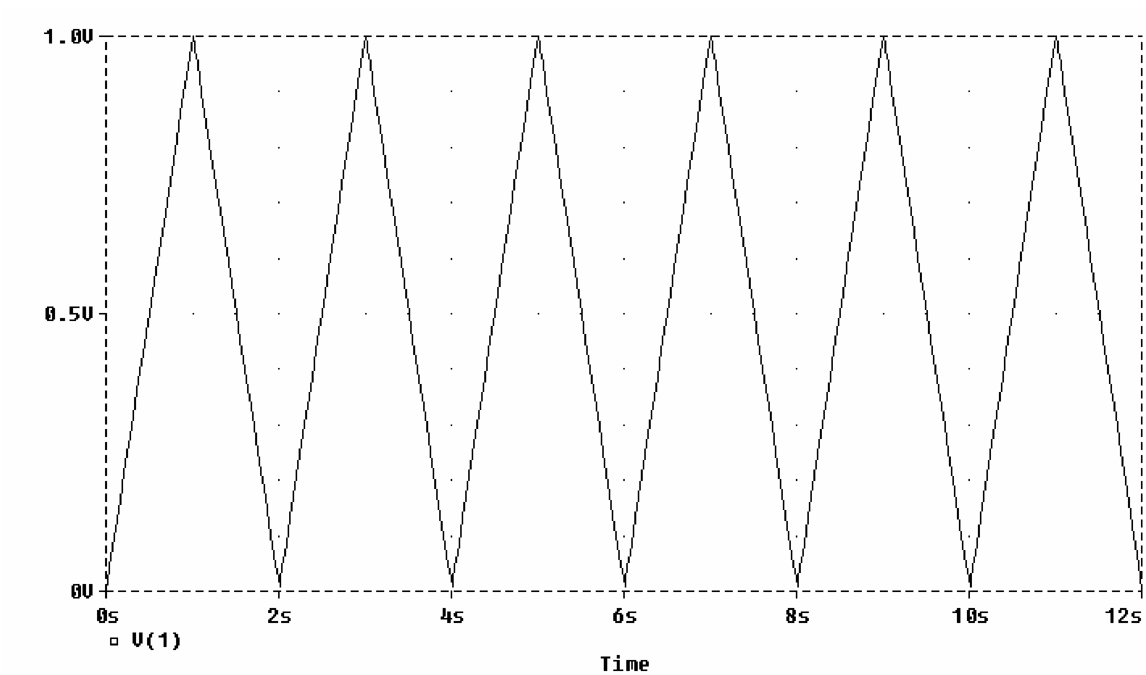


FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 5.048510E-01

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	4.056E-01	1.000E+00	-9.090E+01	0.000E+00
2	1.000E+00	2.977E-04	7.341E-04	-8.707E+01	3.833E+00
3	1.500E+00	4.531E-02	1.117E-01	-9.266E+01	-1.761E+00
4	2.000E+00	2.969E-04	7.320E-04	-8.414E+01	6.757E+00
5	2.500E+00	1.648E-02	4.064E-02	-9.432E+01	-3.417E+00
6	3.000E+00	2.955E-04	7.285E-04	-8.124E+01	9.659E+00
7	3.500E+00	8.535E-03	2.104E-02	-9.581E+01	-4.911E+00
8	4.000E+00	2.935E-04	7.238E-04	-7.836E+01	1.254E+01
9	4.500E+00	5.258E-03	1.296E-02	-9.710E+01	-6.197E+00

TOTAL HARMONIC DISTORTION = 1.214285E+01 PERCENT

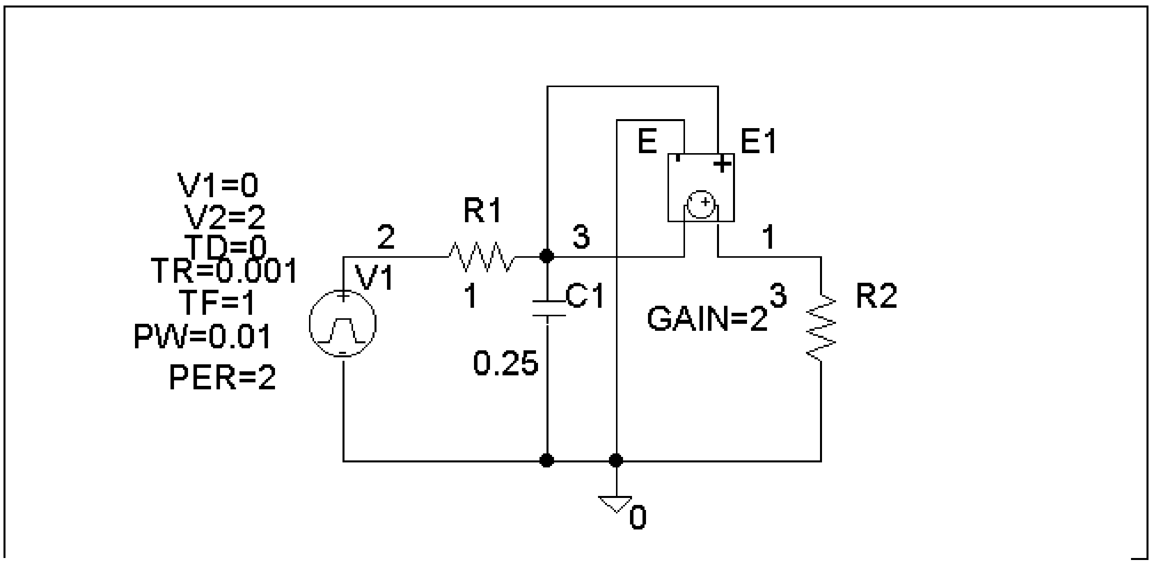


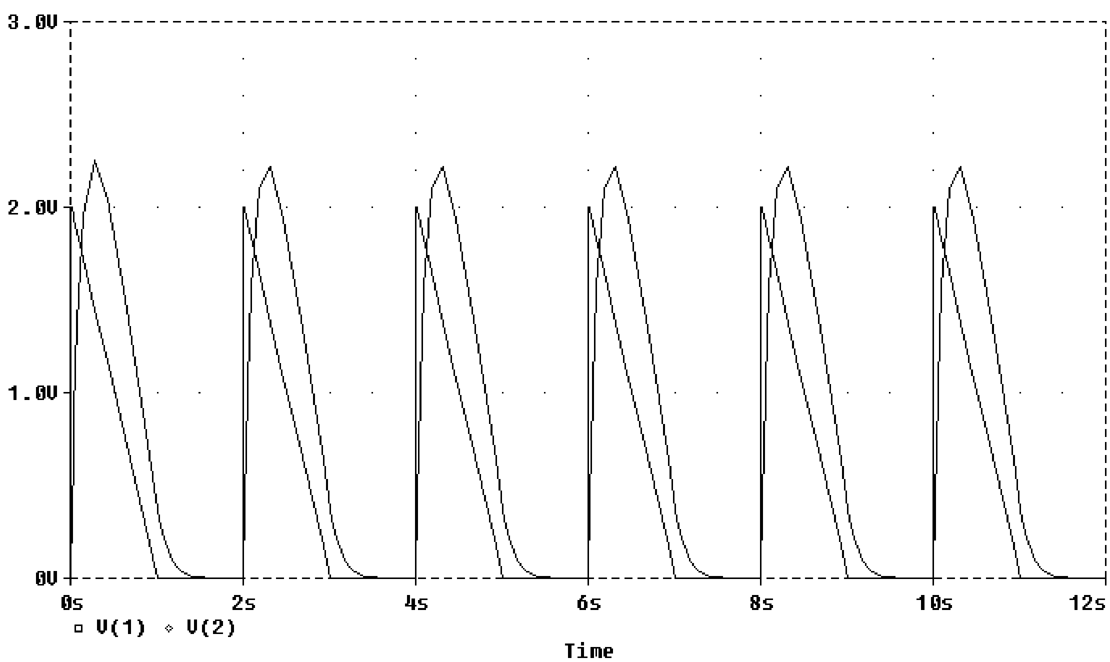
Chapter 17, Problem 70.

Rework Prob. 17.40 using *PSpice*.

Chapter 17, Solution 70.

The schematic is shown below. In the Transient dialog box, we set Print Step = 0.02 s, Final Step = 12 s, Center Frequency = 0.5, Output Vars = V(1) and V(2), and click enable Fourier. After simulation, we compare the output and output waveforms as shown. The output includes the following Fourier components.





FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 7.658051E-01

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	1.070E+00	1.000E+00	1.004E+01	0.000E+00
2	1.000E+00	3.758E-01	3.512E-01	-3.924E+01	-4.928E+01
3	1.500E+00	2.111E-01	1.973E-01	-3.985E+01	-4.990E+01
4	2.000E+00	1.247E-01	1.166E-01	-5.870E+01	-6.874E+01
5	2.500E+00	8.538E-02	7.980E-02	-5.680E+01	-6.685E+01
6	3.000E+00	6.139E-02	5.738E-02	-6.563E+01	-7.567E+01
7	3.500E+00	4.743E-02	4.433E-02	-6.520E+01	-7.524E+01
8	4.000E+00	3.711E-02	3.469E-02	-7.222E+01	-8.226E+01
9	4.500E+00	2.997E-02	2.802E-02	-7.088E+01	-8.092E+01

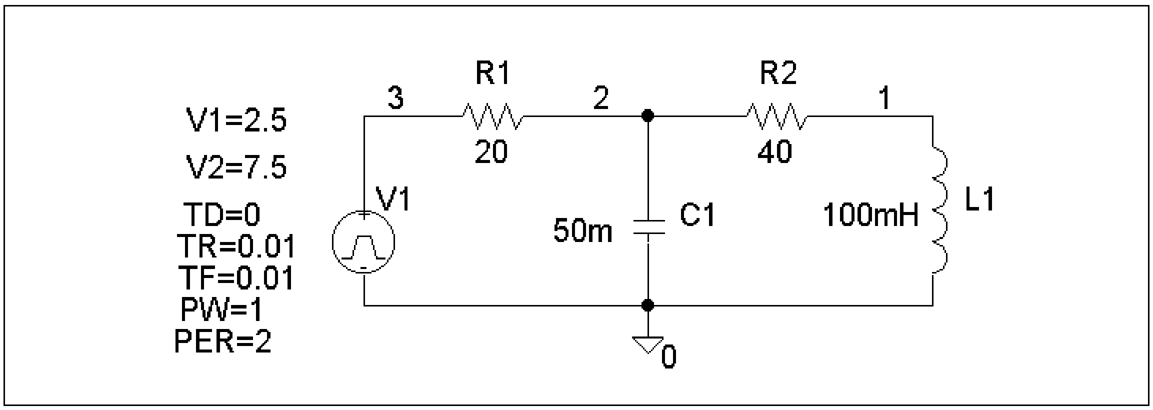
TOTAL HARMONIC DISTORTION = 4.352895E+01 PERCENT

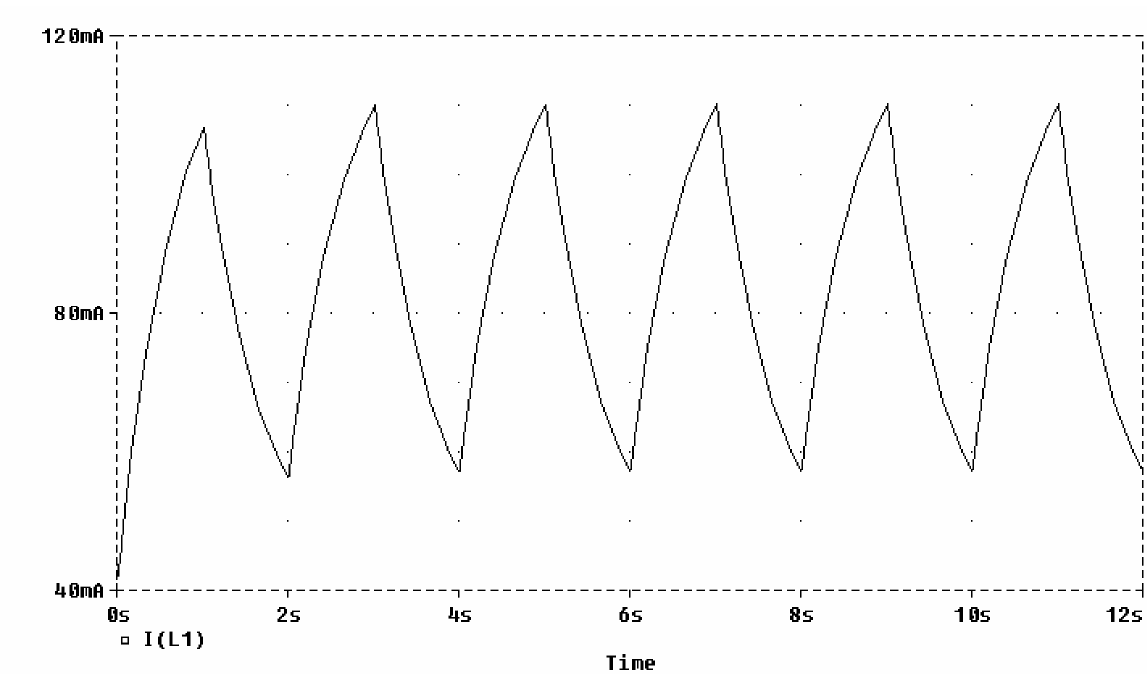
Chapter 17, Problem 71.

Use *PSpice* to solve Prob. 17.39.

Chapter 17, Solution 71.

The schematic is shown below. We set Print Step = 0.05, Final Time = 12 s, Center Frequency = 0.5, Output Vars = I(1), and click enable Fourier in the Transient dialog box. After simulation, the output waveform is as shown. The output file includes the following Fourier components.





FOURIER COMPONENTS OF TRANSIENT RESPONSE I(L_L1)

DC COMPONENT = 8.374999E-02

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	2.287E-02	1.000E+00	-6.749E+01	0.000E+00
2	1.000E+00	1.891E-04	8.268E-03	8.174E+00	7.566E+01
3	1.500E+00	2.748E-03	1.201E-01	-8.770E+01	-2.021E+01
4	2.000E+00	9.583E-05	4.190E-03	-1.844E+00	6.565E+01
5	2.500E+00	1.017E-03	4.446E-02	-9.455E+01	-2.706E+01
6	3.000E+00	6.366E-05	2.783E-03	-7.308E+00	6.018E+01
7	3.500E+00	5.937E-04	2.596E-02	-9.572E+01	-2.823E+01
8	4.000E+00	6.059E-05	2.649E-03	-2.808E+01	3.941E+01
9	4.500E+00	2.113E-04	9.240E-03	-1.214E+02	-5.387E+01

TOTAL HARMONIC DISTORTION = 1.314238E+01 PERCENT

Chapter 17, Problem 72.

The signal displayed by a medical device can be approximated by the waveform shown in Fig. 17.86. Find the Fourier series representation of the signal.

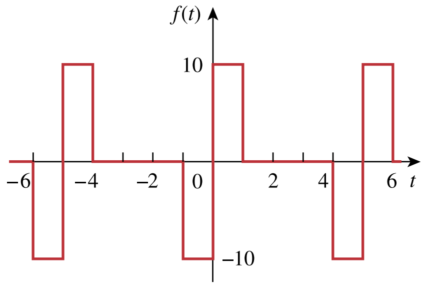


Figure 17.86
For Prob. 17.72.

Chapter 17, Solution 72.

$$T = 5, \ \omega_o = 2\pi/T = 2\pi/5$$

f(t) is an odd function. $a_o = 0 = a_n$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_o t)dt = \frac{4}{5}\int_0^{10} 10\sin(0.4n\pi t)dt$$

$$= -\frac{8x5}{2n\pi}\cos(0.4\pi nt)\Big|_0^1 = \frac{20}{n\pi}[1-\cos(0.4n\pi)]$$

$$f(t) = \underline{\mathbf{\frac{20}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}[1-\cos(0.4n\pi)]\sin(0.4n\pi t)}}$$

Chapter 17, Problem 73.

A spectrum analyzer indicates that a signal is made up of three components only: 640 kHz at 2 V, 644 kHz at 1 V, 636 kHz at 1 V. If the signal is applied across a 10-Ω resistor, what is the average power absorbed by the resistor?

Chapter 17, Solution 73.

$$p = \frac{V_{DC}^2}{R} + \frac{1}{2}\sum\frac{V_n^2}{R}$$

$$= 0 + 0.5[(2^2 + 1^2 + 1^2)/10] = \underline{\mathbf{300\ mW}}$$

Chapter 17, Problem 74.

A certain band-limited periodic current has only three frequencies in its Fourier series representation: dc, 50 Hz, and 100 Hz. The current may be represented as

$$i(t) = 4 + 6\sin 100\pi t + 8\cos 100\pi t$$
$$- 3\sin 200\pi t - 4\cos 200\pi t \text{ A}$$

(a) Express $i(t)$ in amplitude-phase form.
(b) If $i(t)$ flows through a 2-Ω resistor, how many watts of average power will be dissipated?

Chapter 17, Solution 74.

(a)

$$A_n = \sqrt{a_n^2 + b_n^2}, \qquad \phi = \tan^{-1}(b_n/a_n)$$

$$A_1 = \sqrt{6^2 + 8^2} = 10, \qquad \phi_1 = \tan^{-1}(6/8) = 36.87°$$

$$A_2 = \sqrt{3^2 + 4^2} = 5, \qquad \phi_2 = \tan^{-1}(3/4) = 36.87°$$

$$i(t) = \underline{\mathbf{\{4 + 10\cos(100\pi t - 36.87°) - 5\cos(200\pi t - 36.87°)\}\ A}}$$

(b)

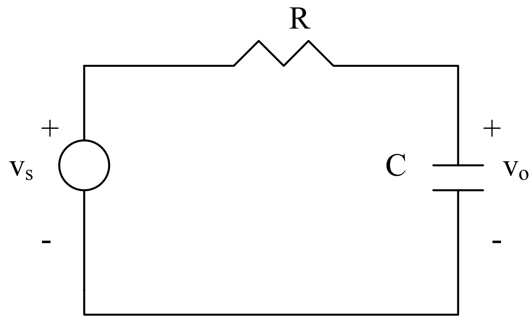
$$p = I_{DC}^2 R + 0.5\sum I_n^2 R$$
$$= 2[4^2 + 0.5(10^2 + 5^2)] = \underline{\mathbf{157\ W}}$$

Chapter 17, Problem 75.

e⃝d Design a lowpass RC filter with a resistance $R = 2\ k\Omega$. The input to the filter is a periodic rectangular pulse train (see Table 17.3) with A = 1 V, T = 10 ms, and τ = 1 ms. Select C such that the dc component of the output is 50 times greater than the fundamental component of the output.

Chapter 17, Solution 75.

The lowpass filter is shown below.



$$v_s = \frac{A\tau}{T} + \frac{2A}{T}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi\tau}{T}\cos n\omega_o t$$

$$V_o = \frac{\frac{1}{j\omega_n C}}{R + \frac{1}{j\omega_n C}} V_s = \frac{1}{1 + j\omega_n RC} V_s, \qquad \omega_n = n\omega_o = 2n\pi / T$$

For n=0, (dc component), $V_o = V_s = \dfrac{A\tau}{T}$ (1)

For the nth harmonic,

$$V_o = \frac{1}{\sqrt{1 + \omega^2{}_n R^2 C^2} \angle \tan^{-1}\omega_n RC} \bullet \frac{2A}{nT}\sin\frac{n\pi\tau}{T}\angle -90^o$$

When n=1, $|V_o| = \dfrac{2A}{T}\sin\dfrac{n\pi\tau}{T} \bullet \dfrac{1}{\sqrt{1 + \dfrac{4\pi^2}{T}R^2C^2}}$ (2)

From (1) and (2),

$$\frac{A\tau}{T} = 50\text{x}\frac{2A}{T}\sin\frac{\pi}{10}\frac{1}{\sqrt{1 + \frac{4\pi^2}{T}R^2C^2}} \longrightarrow \sqrt{1 + \frac{4\pi^2}{T}R^2C^2} = \frac{30.9}{\tau} = 3.09\text{x}10^4$$

$$1 + \frac{4\pi^2}{T}R^2C^2 = 10^{10} \longrightarrow C = \frac{T}{2\pi R}10^5 = \frac{10^{-2}\text{x}3.09\text{x}10^4}{4\pi\text{x}10^3} = \underline{24.59\text{ mF}}$$

Chapter 17, Problem 76.

A periodic signal given by $v_s(t) = 10\text{ V}$ for $0 < t < 1$ and 0 V for $1 < t < 2$ is applied to the highpass filter in Fig. 17.87. Determine the value of R such that the output signal $v_o(t)$ has an average power of at least 70 percent of the average power of the input signal.

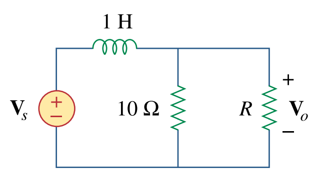


Figure 17.87
For Prob. 17.76.

Chapter 17, Solution 76.

$v_s(t)$ is the same as f(t) in Figure 16.1 except that the magnitude is multiplied by 10. Hence

$$v_o(t) = 5 + \frac{20}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin(n\pi t), \quad n = 2k - 1$$

$$T = 2, \ \omega_o = 2\pi/T = 2\pi, \ \omega_n = n\omega_o = 2n\pi$$

$$j\omega_n L = j2n\pi; \ Z = R\|10 = 10R/(10 + R)$$

$$V_o = ZV_s/(Z + j2n\pi) = [10R/(10R + j2n\pi(10 + R))]V_s$$

$$V_o = \frac{10R\angle - \tan^{-1}\{(n\pi/5R)(10 + R)\}}{\sqrt{100R^2 + 4n^2\pi^2(10 + R)^2}}V_s$$

$$V_s = [20/(n\pi)]\angle 0°$$

The source current I_s is

$$I_s = \frac{V_s}{Z + j2n\pi} = \frac{V_s}{\frac{10R}{10+R} + j2n\pi} = \frac{(10+R)\frac{20}{n\pi}}{10R + j2n\pi(10+R)}$$

$$= \frac{(10+R)\frac{20}{n\pi}\angle - \tan^{-1}\{(n\pi/3)(10+R)\}}{\sqrt{100R^2 + 4n^2\pi^2(10+R)^2}}$$

$$p_s = V_{DC}I_{DC} + \frac{1}{2}\sum V_{sn}I_{sn}\cos(\theta_n - \phi_n)$$

For the DC case, L acts like a short-circuit.

$$I_s = \frac{5}{\frac{10R}{10+R}} = \frac{5(10+R)}{10R}, \quad V_s = 5 = V_o$$

$$p_s = \frac{25(10+R)}{10R} + \frac{1}{2}\left[\left(\frac{20}{\pi}\right)^2 \frac{(10+R)\cos\left(\tan^{-1}\left(\frac{\pi}{5}(10+R)\right)\right)}{\sqrt{100R^2 + 4\pi^2(10+R)^2}}\right.$$

$$\left. + \left(\frac{10}{\pi}\right)^2 \frac{(10+R)^2\cos\left(\tan^{-1}\left(\frac{2\pi}{5}(10+R)\right)\right)}{\sqrt{100R^2 + 16\pi^2(10+R)^2}} + \cdots\right]$$

$$p_s = \frac{V_{DC}}{R} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{V_{on}}{R}$$

$$= \frac{25}{R} + \frac{1}{2}\left[\frac{100R}{100R^2 + 4\pi^2(10+R)^2} + \frac{100R}{100R^2 + 10\pi^2(10+R)^2} + \cdots\right]$$

We want $p_o = (70/100)\, p_s = 0.7p_s$. Due to the complexity of the terms, we consider only the DC component as an approximation. In fact the DC component has the largest share of the power for both input and output signals.

$$\frac{25}{R} = \frac{7}{10} \text{x} \frac{25(10+R)}{10R}$$

100 = 70 + 7R which leads to R = 30/7 = **4.286 Ω**

Chapter 17, Problem 77.

The voltage across a device is given by
$v(t) = -2 + 10\cos 4t + 8\cos 6t + 6\cos 8t$
$\quad - 5\sin 4t - 3\sin 6t - \sin 8t$ V

Find:

(a) the period of $v(t)$,
(b) the average value of $v(t)$,
(c) the effective value of $v(t)$.

Chapter 17, Solution 77.

(a) For the first two AC terms, the frequency ratio is 6/4 = 1.5 so that the highest common factor is 2. Hence $\omega_o = 2$.

$$T = 2\pi/\omega_o = 2\pi/2 = \underline{\pi}$$

(b) The average value is the DC component = $\underline{\mathbf{-2}}$

(c) $$V_{rms} = \sqrt{a_o + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)}$$

$$V_{rms}^2 = (-2)^2 + \frac{1}{2}(10^2 + 8^2 + 6^2 + 3^2 + 1^2) = 121.5$$

$$V_{rms} = \underline{\mathbf{11.02\ V}}$$

Chapter 17, Problem 78.

A certain band-limited periodic voltage has only three harmonics in its Fourier series representation. The harmonics have the following rms values: fundamental 40 V, third harmonic 20 V, fifth harmonic 10 V.

(a) If the voltage is applied across a 5-Ω resistor, find the average power dissipated by the resistor.

(b) If a dc component is added to the periodic voltage and the measured power dissipated increases by 5 percent, determine the value of the dc component added.

Chapter 17, Solution 78.

(a) $$p = \frac{V_{DC}^2}{R} + \frac{1}{2}\sum\frac{V_n^2}{R} = \frac{V_{DC}^2}{R} + \sum\frac{V_{n,rms}^2}{R}$$

$$= 0 + (40^2/5) + (20^2/5) + (10^2/5) = \underline{\textbf{420 W}}$$

(b) 5% increase = (5/100)420 = 21

$$p_{DC} = 21\text{ W} = \frac{V_{DC}^2}{R} \quad \text{which leads to} \quad V_{DC}^2 = 21R = 105$$

V_{DC} = **10.25 V**

Chapter 17, Problem 79.

Write a program to compute the Fourier coefficients (up to the 10th harmonic) of the square wave in Table 17.3 with $A = 10$ and $T = 2$.

Chapter 17, Solution 79.

From Table 17.3, it is evident that $a_n = 0$,

$$b_n = 4A/[\pi(2n - 1)], \quad A = 10.$$

A Fortran program to calculate b_n is shown below. The result is also shown.

```
C       FOR PROBLEM 17.79
              DIMENSION B(20)

              A = 10
              PIE = 3.142
              C = 4.*A/PIE
              DO 10 N = 1, 10
              B(N) = C/(2.*FLOAT(N) – 1.)
              PRINT *, N, B(N)
10            CONTINUE
              STOP
              END
```

n	b_n
1	12.731
2	4.243
3	2.546
4	1.8187
5	1.414
6	1.1573
7	0.9793
8	0.8487
9	0.7498
10	0.67

Chapter 17, Problem 80.

Write a computer program to calculate the exponential Fourier series of the half-wave rectified sinusoidal current of Fig. 17.82. Consider terms up to the 10th harmonic.

Chapter 17, Solution 80.

From Problem 17.55,

$$c_n = [1 + e^{-jn\pi}]/[2\pi(1 - n^2)]$$

This is calculated using the Fortran program shown below. The results are also shown.

```
C       FOR PROBLEM 17.80
             COMPLEX X, C(0:20)

             PIE  =  3.1415927
             A  =  2.0*PIE
             DO  10  N = 0, 10
             IF(N.EQ.1) GO TO 10
             X  =  CMPLX(0, PIE*FLOAT(N))
             C(N)  =  (1.0 + CEXP(–X))/(A*(1 – FLOAT(N*N)))
             PRINT *, N, C(N)
10           CONTINUE
             STOP
             END
```

n	c_n
0	$0.3188 + j0$
1	0
2	$-0.1061 + j0$
3	0
4	$-0.2121\text{x}10^{-1} + j0$
5	0
6	$-0.9095\text{x}10^{-2} + j0$
7	0
8	$-0.5052\text{x}10^{-2} + j0$
9	0
10	$-0.3215\text{x}10^{-2} + j0$

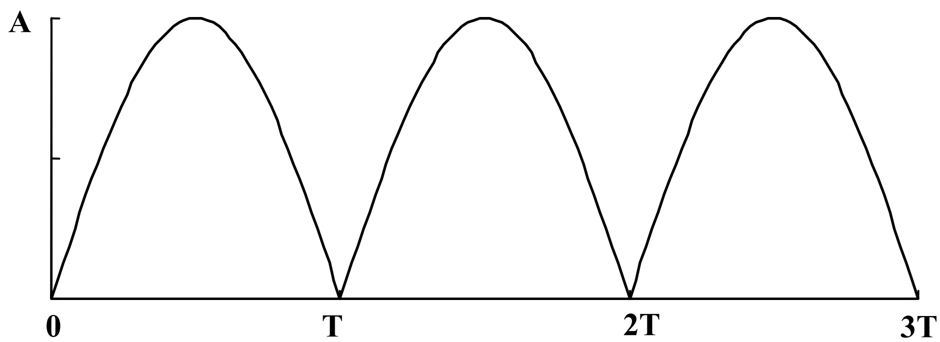
Chapter 17, Problem 81.

Consider the full-wave rectified sinusoidal current in Table 17.3. Assume that the current is passed through a 1-Ω resistor.

(a) Find the average power absorbed by the resistor.
(b) Obtain c_n for $n = 1, 2, 3,$ and 4.
(c) What fraction of the total power is carried by the dc component?
(d) What fraction of the total power is carried by the second harmonic ($n = 2$)?

Chapter 17, Solution 81.

(a)



$$f(t) = \frac{2A}{\pi} - \frac{4A}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}\cos(n\omega_o t)$$

The total average power is $p_{avg} = F_{rms}^2 R = F_{rms}^2$ since R = 1 ohm.

$$P_{avg} = F_{rms}^2 = \frac{1}{T}\int_0^T f^2(t)dt = \underline{\mathbf{0.5A^2}}$$

(b) From the Fourier series above

$|c_o| = 2A/\pi$, $|c_n| = |a_n|/2 = 2A/[\pi(4n^2 - 1)]$

n	ω_o	$\lvert c_n\rvert$	$\lvert c_o\rvert^2$ or $2\lvert c_n\rvert^2$	% power
0	0	$2A/\pi$	$4A^2/(\pi^2)$	81.1%
1	$2\omega_o$	$2A/(3\pi)$	$8A^2/(9\pi^2)$	18.01%
2	$4\omega_o$	$2A/(15\pi)$	$8A^2/(225\pi^2)$	0.72%
3	$6\omega_o$	$2A/(35\pi)$	$8A^2/(1225\pi^2)$	0.13%
4	$8\omega_o$	$2A/(63\pi)$	$8A^2/(3969\pi^2)$	0.04%

(c) **81.1%**

(d) **0.72%**

Chapter 17, Problem 82.

A band-limited voltage signal is found to have the complex Fourier coefficients presented in the table below. Calculate the average power that the signal would supply a 4-Ω resistor.

$n\omega_0$	$\lvert c_n \rvert$	θ_n
0	10.0	0º
ω	8.5	15º
2ω	4.2	30º
3ω	2.1	45º
4ω	0.5	60º
5ω	0.2	75º

Chapter 17, Solution 82.

$$P = \frac{V_{DC}^2}{R} + \frac{1}{2}\sum_{n=1}^{\infty} \frac{V_n^2}{R}$$

Assuming V is an amplitude-phase form of Fourier series. But

$$|A_n| = 2|C_n|, \ c_o = a_o$$

$$|A_n|^2 = 4|C_n|^2$$

Hence,

$$P = \frac{c_o^2}{R} + 2\sum_{n=1}^{\infty} \frac{c_n^2}{R}$$

Alternatively,

$$P = \frac{V_{rms}^2}{R}$$

where

$$V_{rms}^2 = a_o^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2 = c_o^2 + 2\sum_{n=1}^{\infty} c_n^2 = \sum_{n=-\infty}^{\infty} c_n^2$$

$$= 10^2 + 2(8.5^2 + 4.2^2 + 2.1^2 + 0.5^2 + 0.2^2)$$

$$= 100 + 2\text{x}94.57 = 289.14$$

$$P = 289.14/4 = \underline{\mathbf{72.3\ W}}$$