Chapter 15, Problem 1.

Find the Laplace transform of:

(a) cosh *at* (b) sinh *at*

[*Hint:* $\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$, $\sinh\ x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$.]

Chapter 15, Solution 1.

(a) $$\cosh(at) = \frac{e^{at} + e^{-at}}{2}$$

$$\mathrm{L}\left[\cosh(at)\right] = \frac{1}{2}\left[\frac{1}{s-a} + \frac{1}{s+a}\right] = \underline{\frac{\mathbf{s}}{\mathbf{s^2 - a^2}}}$$

(b) $$\sinh(at) = \frac{e^{at} - e^{-at}}{2}$$

$$\mathrm{L}\left[\sinh(at)\right] = \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right] = \underline{\frac{\mathbf{a}}{\mathbf{s^2 - a^2}}}$$

Chapter 15, Problem 2.

Determine the Laplace transform of:

(a) $\cos(\omega t + \theta)$ (b) $\sin(\omega t + \theta)$

Chapter 15, Solution 2.

(a) $$f(t) = \cos(\omega t)\cos(\theta) - \sin(\omega t)\sin(\theta)$$

$$F(s) = \cos(\theta)\,\mathrm{L}\left[\cos(\omega t)\right] - \sin(\theta)\,\mathrm{L}\left[\sin(\omega t)\right]$$

$$F(s) = \underline{\frac{\mathbf{s\cos(\theta) - \omega\sin(\theta)}}{\mathbf{s^2 + \omega^2}}}$$

(b) $$f(t) = \sin(\omega t)\cos(\theta) + \cos(\omega t)\sin(\theta)$$

$$F(s) = \sin(\theta)\,\mathrm{L}\left[\cos(\omega t)\right] + \cos(\theta)\,\mathrm{L}\left[\sin(\omega t)\right]$$

$$F(s) = \underline{\frac{\mathbf{s\sin(\theta) - \omega\cos(\theta)}}{\mathbf{s^2 + \omega^2}}}$$

Chapter 15, Problem 3.

Obtain the Laplace transform of each of the following functions:

(a) $e^{-2t}\cos 3t u(t)$ (b) $e^{-2t}\sin 4t u(t)$

(c) $e^{-3t}\cosh 2t u(t)$ (d) $e^{-4t}\sinh t u(t)$

(e) $te^{-t}\sin 2t u(t)$

Chapter 15, Solution 3.

(a) $$L\left[e^{-2t}\cos(3t)u(t)\right] = \underline{\frac{\mathbf{s+2}}{\mathbf{(s+2)^2+9}}}$$

(b) $$L\left[e^{-2t}\sin(4t)u(t)\right] = \underline{\frac{\mathbf{4}}{\mathbf{(s+2)^2+16}}}$$

(c) Since $L\left[\cosh(at)\right] = \dfrac{s}{s^2 - a^2}$

$$L\left[e^{-3t}\cosh(2t)u(t)\right] = \underline{\frac{\mathbf{s+3}}{\mathbf{(s+3)^2-4}}}$$

(d) Since $L\left[\sinh(at)\right] = \dfrac{a}{s^2 - a^2}$

$$L\left[e^{-4t}\sinh(t)u(t)\right] = \underline{\frac{\mathbf{1}}{\mathbf{(s+4)^2-1}}}$$

(e) $$L\left[e^{-t}\sin(2t)\right] = \frac{2}{(s+1)^2+4}$$

If $f(t) \longleftrightarrow F(s)$

$t f(t) \longleftrightarrow \dfrac{-d}{ds}F(s)$

Thus, $$L\left[t e^{-t}\sin(2t)\right] = \frac{-d}{ds}\left[2\left((s+1)^2+4\right)^{-1}\right]$$

$$= \frac{2}{((s+1)^2+4)^2}\cdot 2(s+1)$$

$$L\left[t e^{-t}\sin(2t)\right] = \underline{\frac{\mathbf{4(s+1)}}{\mathbf{((s+1)^2+4)^2}}}$$

Chapter 15, Problem 4.

Find the Laplace transforms of the following:

(a) $g(t) = 6\cos(4t - 1)$

(b) $f(t) = 2tu(t) + 5e^{-3(t-2)}u(t-2)$

Chapter 15, Solution 4.

(a) $$G(s) = 6\frac{s}{s^2 + 4^2}e^{-s} = \underline{\frac{6se^{-s}}{s^2 + 16}}$$

(b) $$F(s) = \underline{\frac{2}{s^2} + 5\frac{e^{-2s}}{s+3}}$$

Chapter 15, Problem 5.

Find the Laplace transform of each of the following functions:

(a) $t^2 \cos(2t + 30°)u(t)$ (b) $3t^4 e^{-2t} u(t)$

(c) $2tu(t) - 4\frac{d}{dt}\delta(t)$ (d) $2e^{-(t-1)}u(t)$

(e) $5u(t/2)$ (f) $6e^{-t/3}u(t)$

(g) $\frac{d^n}{dt^n}\delta(t)$

Chapter 15, Solution 5.

(a)

$$\mathsf{L}\left[\cos(2t+30°)\right] = \frac{s\cos(30°) - 2\sin(30°)}{s^2+4}$$

$$\mathsf{L}\left[t^2\cos(2t+30°)\right] = \frac{d^2}{ds^2}\left[\frac{s\cos(30°)-1}{s^2+4}\right]$$

$$= \frac{d}{ds}\frac{d}{ds}\left[\left(\frac{\sqrt{3}}{2}s-1\right)(s^2+4)^{-1}\right]$$

$$= \frac{d}{ds}\left[\frac{\sqrt{3}}{2}(s^2+4)^{-1} - 2s\left(\frac{\sqrt{3}}{2}s-1\right)(s^2+4)^{-2}\right]$$

$$= \frac{\frac{\sqrt{3}}{2}(\text{-}\,2s)}{(s^2+4)^2} - \frac{2\left(\frac{\sqrt{3}}{2}s-1\right)}{(s^2+4)^2} - \frac{2s\left(\frac{\sqrt{3}}{2}\right)}{(s^2+4)^2} + \frac{(8s^2)\left(\frac{\sqrt{3}}{2}s-1\right)}{(s^2+4)^3}$$

$$= \frac{\text{-}\,\sqrt{3}s - \sqrt{3}s + 2 - \sqrt{3}s}{(s^2+4)^2} + \frac{(8s^2)\left(\frac{\sqrt{3}}{2}s-1\right)}{(s^2+4)^3}$$

$$= \frac{(\text{-}3\sqrt{3}s+2)(s^2+4)}{(s^2+4)^3} + \frac{4\sqrt{3}s^3 - 8s^2}{(s^2+4)^3}$$

$$\mathsf{L}\left[t^2\cos(2t+30°)\right] = \underline{\mathbf{\frac{8 - 12\sqrt{3}s - 6s^2 + \sqrt{3}s^3}{(s^2+4)^3}}}$$

(b) $\mathcal{L}\left[3t^4 e^{-2t}\right] = 3\cdot\dfrac{4!}{(s+2)^5} = \underline{\dfrac{72}{(s+2)^5}}$

(c) $\mathcal{L}\left[2t\,u(t) - 4\dfrac{d}{dt}\delta(t)\right] = \dfrac{2}{s^2} - 4(s\cdot 1 - 0) = \underline{\mathbf{\dfrac{2}{s^2} - 4s}}$

(d) $2e^{-(t-1)}\,u(t) = 2e^{-t}\,u(t)$

$\mathcal{L}\left[2e^{-(t-1)}\,u(t)\right] = \underline{\mathbf{\dfrac{2e}{s+1}}}$

(e) Using the scaling property,

$\mathcal{L}\left[5u(t/2)\right] = 5\cdot\dfrac{1}{1/2}\cdot\dfrac{1}{s/(1/2)} = 5\cdot 2\cdot\dfrac{1}{2s} = \underline{\mathbf{\dfrac{5}{s}}}$

(f) $\mathcal{L}\left[6e^{-t/3}\,u(t)\right] = \dfrac{6}{s+1/3} = \underline{\mathbf{\dfrac{18}{3s+1}}}$

(g) Let $f(t) = \delta(t)$. Then, $F(s) = 1$.

$\mathcal{L}\left[\dfrac{d^n}{dt^n}\delta(t)\right] = \mathcal{L}\left[\dfrac{d^n}{dt^n}f(t)\right] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \cdots$

$\mathcal{L}\left[\dfrac{d^n}{dt^n}\delta(t)\right] = \mathcal{L}\left[\dfrac{d^n}{dt^n}f(t)\right] = s^n\cdot 1 - s^{n-1}\cdot 0 - s^{n-2}\cdot 0 - \cdots$

$\mathcal{L}\left[\dfrac{d^n}{dt^n}\delta(t)\right] = \underline{\mathbf{s^n}}$

Chapter 15, Problem 6.

Find $F(s)$ given that

$$f(t) = \begin{cases} 2t, & 0 < t < 1 \\ t, & 1 < t < 2 \\ 0, & otherwise \end{cases}$$

Chapter 15, Solution 6.

$$F(s) = \int_0^\infty f(t)e^{-st}dt = \int_0^1 2te^{-st}dt + \int_1^2 2e^{-st}dt$$

$$2\frac{e^{-st}}{s^2}(-st-1)\Bigg|_0^1 + 2\frac{e^{-st}}{-s}\Bigg|_1^2 = \underline{\frac{2}{s^2}(1-e^{-s}-se^{-2s})}$$

Chapter 15, Problem 7.

Find the Laplace transform of the following signals:

(a) $f(t) = (2t+4)u(t)$

(b) $g(t) = (4+3e^{-2t})u(t)$

(c) $h(t) = (6\sin(3t) + 8\cos(3t))u(t)$

(d) $x(t) = (e^{-2t}\cosh(4t))u(t)$

Chapter 15, Solution 7.

(a) $\underline{F(s) = \frac{2}{s^2} + \frac{4}{s}}$

(b) $\underline{G(s) = \frac{4}{s} + \frac{3}{s+2}}$

(c) $\mathrm{H(s)} = 6\frac{3}{s^2+9} + 8\frac{s}{s^2+9} = \underline{\frac{8s+18}{s^2+9}}$

(d) From Problem 15.1,

$$L\{\cosh at\} = \frac{s}{s^2 - a^2}$$

$$X(s) = \frac{s+2}{(s+2)^2 - 4^2} = \underline{\frac{s+2}{s^2+4s-12}}$$

$$\underline{\text{(a) } \frac{2}{s^2} + \frac{4}{s}, \text{(b) } \frac{4}{s} + \frac{3}{s+2}, \text{(c) } \frac{8s+18}{s^2+9}, \text{(d) } \frac{s+2}{s^2+4s-12}}$$

Chapter 15, Problem 8.

Find the Laplace transform $F(s)$, given that $f(t)$ is:

(a) $2tu(t-4)$

(b) $5\cos(t)\delta(t-2)$

(c) $e^{-t}u(t-t)$

(d) $\sin(2t)u(t-\tau)$

Chapter 15, Solution 8.

(a) 2t=2(t-4) + 8

f(t) = 2tu(t-4) = 2(t-4)u(t-4) + 8u(t-4)

$$F(s) = \frac{2}{s^2}e^{-4s} + \frac{8}{s}e^{-4s} = \underline{\left(\frac{2}{s^2} + \frac{8}{s}\right)e^{-4s}}$$

(b) $$F(s) = \int_0^\infty f(t)e^{-st}dt = \int_0^\infty 5\cos t\delta(t-2)e^{-st}dt = 5\cos te^{-st}\Big|_{t=2} = \underline{\mathbf{5\cos(2)e^{-2s}}}$$

(c) $$e^{-t} = e^{-(t-\tau)}e^{-\tau}$$

$$f(t) = e^{-\tau}e^{-(t-\tau)}u(t-\tau)$$

$$F(s) = e^{-\tau}e^{-\tau s}\frac{1}{s+1} = \underline{\frac{e^{-\tau(s+1)}}{s+1}}$$

(d) $$\sin 2t = \sin[2(t-\tau) + 2\tau] = \sin 2(t-\tau)\cos 2\tau + \cos 2(t-\tau)\sin 2\tau$$

$$f(t) = \cos 2\tau \sin 2(t-\tau)u(t-\tau) + \sin 2\tau \cos 2(t-\tau)u(t-\tau)$$

$$F(s) = \underline{\cos 2\tau e^{-\tau s}\frac{2}{s^2+4} + \sin 2\tau e^{-\tau s}\frac{s}{s^2+4}}$$

Chapter 15, Problem 9.

Determine the Laplace transforms of these functions:

(a) $f(t) = (t-4)u(t-2)$

(b) $g(t) = 2e^{-4t}u(t-1)$

(c) $h(t) = 5\cos(2t-1)u(t)$

(d) $p(t) = 6[u(t-2) - u(t-4)]$

Chapter 15, Solution 9.

(a) $f(t) = (t-4)\,u(t-2) = (t-2)\,u(t-2) - 2\,u(t-2)$

$$\underline{F(s) = \frac{\mathbf{e^{-2s}}}{\mathbf{s^2}} - \frac{\mathbf{2e^{-2s}}}{\mathbf{s^2}}}$$

(b) $g(t) = 2\,e^{-4t}\,u(t-1) = 2\,e^{-4}\,e^{-4(t-1)}\,u(t-1)$

$$\underline{G(s) = \frac{\mathbf{2\,e^{-s}}}{\mathbf{e^4\,(s+4)}}}$$

(c) $h(t) = 5\cos(2t-1)\,u(t)$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$
$$\cos(2t-1) = \cos(2t)\cos(1) + \sin(2t)\sin(1)$$

$$h(t) = 5\cos(1)\cos(2t)\,u(t) + 5\sin(1)\sin(2t)\,u(t)$$

$$H(s) = 5\cos(1)\cdot\frac{s}{s^2+4} + 5\sin(1)\cdot\frac{2}{s^2+4}$$
$$\underline{H(s) = \frac{\mathbf{2.702\,s}}{\mathbf{s^2+4}} + \frac{\mathbf{8.415}}{\mathbf{s^2+4}}}$$

(d) $p(t) = 6u(t-2) - 6u(t-4)$

$$\underline{P(s) = \frac{\mathbf{6}}{\mathbf{s}}\mathbf{e^{-2s}} - \frac{\mathbf{6}}{\mathbf{s}}\mathbf{e^{-4s}}}$$

Chapter 15, Problem 10.

In two different ways, find the Laplace transform of

$$g(t)=\frac{d}{dt}\left(te^{-t}\cos t\right)$$

Chapter 15, Solution 10.

(a) By taking the derivative in the time domain,

$$g(t) = (-t\,e^{-t} + e^{-t})\cos(t) - t\,e^{-t}\sin(t)$$

$$g(t) = e^{-t}\cos(t) - t\,e^{-t}\cos(t) - t\,e^{-t}\sin(t)$$

$$G(s) = \frac{s+1}{(s+1)^2+1} + \frac{d}{ds}\left[\frac{s+1}{(s+1)^2+1}\right] + \frac{d}{ds}\left[\frac{1}{(s+1)^2+1}\right]$$

$$G(s) = \frac{s+1}{s^2+2s+2} - \frac{s^2+2s}{(s^2+2s+2)^2} - \frac{2s+2}{(s^2+2s+2)^2} = \underline{\mathbf{\frac{s^2(s+2)}{(s^2+2s+2)^2}}}$$

(b) By applying the time differentiation property,

$$G(s) = sF(s) - f(0)$$

where $f(t) = t\,e^{-t}\cos(t)$, $f(0) = 0$

$$G(s) = (s)\cdot\frac{-d}{ds}\left[\frac{s+1}{(s+1)^2+1}\right] = \frac{(s)(s^2+2s)}{(s^2+2s+2)^2} = \underline{\mathbf{\frac{s^2(s+2)}{(s^2+2s+2)^2}}}$$

Chapter 15, Problem 11.

Find $F(s)$ if:

(a) $f(t) = 6e^{-t}\cosh 2t$ (b) $f(t) = 3te^{-2t}\sinh 4t$

(c) $f(t) = 8e^{-3t}\cosh tu(t-2)$

Chapter 15, Solution 11.

(a) Since $\mathsf{L}\left[\cosh(at)\right] = \dfrac{s}{s^2 - a^2}$

$$F(s) = \frac{6(s+1)}{(s+1)^2 - 4} = \underline{\mathbf{\frac{6(s+1)}{s^2 + 2s - 3}}}$$

(b) Since $\mathsf{L}\left[\sinh(at)\right] = \dfrac{a}{s^2 - a^2}$

$$\mathsf{L}\left[3e^{-2t}\sinh(4t)\right] = \frac{(3)(4)}{(s+2)^2 - 16} = \frac{12}{s^2 + 4s - 12}$$

$$F(s) = \mathsf{L}\left[t \cdot 3e^{-2t}\sinh(4t)\right] = \frac{-d}{ds}\left[12(s^2 + 4s - 12)^{-1}\right]$$

$$F(s) = (12)(2s+4)(s^2 + 4s - 12)^{-2} = \underline{\mathbf{\frac{24(s+2)}{(s^2 + 4s - 12)^2}}}$$

(c) $\cosh(t) = \dfrac{1}{2}\cdot(e^t + e^{-t})$

$$f(t) = 8e^{-3t}\cdot\frac{1}{2}\cdot(e^t + e^{-t})u(t-2)$$

$$= 4e^{-2t}u(t-2) + 4e^{-4t}u(t-2)$$

$$= 4e^{-4}e^{-2(t-2)}u(t-2) + 4e^{-8}e^{-4(t-2)}u(t-2)$$

$$\mathsf{L}\left[4e^{-4}e^{-2(t-2)}u(t-2)\right] = 4e^{-4}e^{-2s}\cdot\mathsf{L}\left[e^{-2}u(t)\right]$$

$$\mathsf{L}\left[4e^{-4}e^{-2(t-2)}u(t-2)\right] = \frac{4e^{-(2s+4)}}{s+2}$$

Similarly, $\mathsf{L}\left[4e^{-8}e^{-4(t-2)}u(t-2)\right] = \dfrac{4e^{-(2s+8)}}{s+4}$

Therefore,

$$F(s) = \frac{4e^{-(2s+4)}}{s+2} + \frac{4e^{-(2s+8)}}{s+4} = \underline{\mathbf{\frac{e^{-(2s+6)}\left[(4e^2 + 4e^{-2})s + (16e^2 + 8e^{-2})\right]}{s^2 + 6s + 8}}}$$

Chapter 15, Problem 12.

If $g(t) = e^{-2t}\cos 4t$ find $G(s)$.

Chapter 15, Solution 12.

$$G(s) = \frac{s+2}{(s+2)^2 + 4^2} = \underline{\frac{s+2}{s^2+4s+20}}$$

Chapter 15, Problem 13.

Find the Laplace transform of the following functions:

(a) $t\cos t\, u(t)$ (b) $e^{-t}t\sin t\, u(t)$ (c) $\dfrac{\sin\beta t}{t}u(t)$

Chapter 15, Solution 13.

(a) $tf(t) \quad \longleftrightarrow \quad -\dfrac{d}{ds}F(s)$

If f(t) = cost, then $F(s) = \dfrac{s}{s^2+1}$ and $-\dfrac{d}{ds}F(s) = -\dfrac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}$

$$\underline{L(t\cos t) = \frac{s^2-1}{(s^2+1)^2}}$$

(b) Let f(t) = e^{-t} sin t.

$$F(s) = \frac{1}{(s+1)^2+1} = \frac{1}{s^2+2s+2}$$

$$\frac{dF}{ds} = \frac{(s^2+2s+2)(0) - (1)(2s+2)}{(s^2+2s+2)^2}$$

$$\underline{L(e^{-t}t\sin t) = -\frac{dF}{ds} = \frac{2(s+1)}{(s^2+2s+2)^2}}$$

(c) $\dfrac{f(t)}{t} \quad \longleftrightarrow \quad \int_s^\infty F(s)ds$

Let $f(t) = \sin\beta t$, then $F(s) = \dfrac{\beta}{s^2+\beta^2}$

$$L\left[\frac{\sin\beta t}{t}\right] = \int_s^\infty \frac{\beta}{s^2+\beta^2}ds = \beta\frac{1}{\beta}\tan^{-1}\frac{s}{\beta}\Big|_s^\infty = \frac{\pi}{2} - \tan^{-1}\frac{s}{\beta} = \underline{\tan^{-1}\frac{\beta}{s}}$$

Chapter 15, Problem 14.

Find the Laplace transform of the signal in Fig. 15.26.

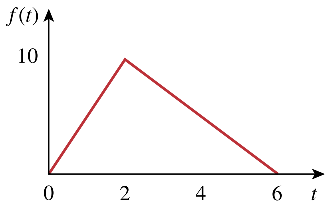
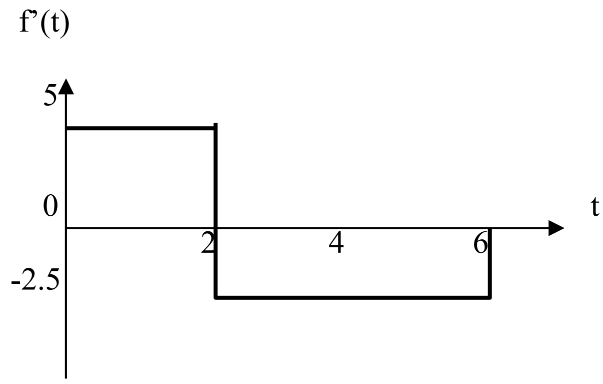


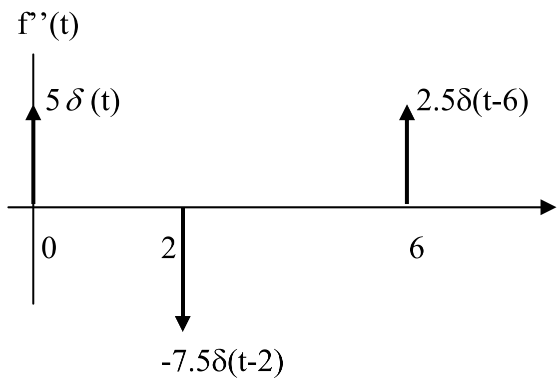
Figure 15.26

For Prob. 15.14.

Chapter 15, Solution 14.

Taking the derivative of f(t) twice, we obtain the figures below.





$$f'' = 5\delta(t) - 7.5\delta(t-2) + 2.5\delta(t-6)$$

Taking the Laplace transform of each term,

$$s^2F(s) = 5 - 7.5e^{-2s} + 2.5e^{-6s} \text{ or } F(s) = \underline{\frac{5}{s} - 7.5\frac{e^{-2s}}{s^2} + 2.5\frac{e^{-6s}}{s^2}}$$

Please note that we can obtain the same answer by representing the function as,

$$f(t) = 5tu(t) - 7.5u(t-2) + 2.5u(t-6).$$

Chapter 15, Problem 15.

Determine the Laplace transform of the function in Fig. 15.27.

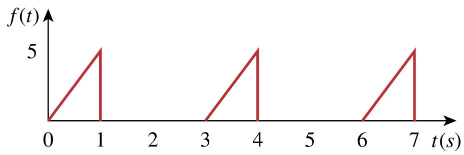


Figure 15.27

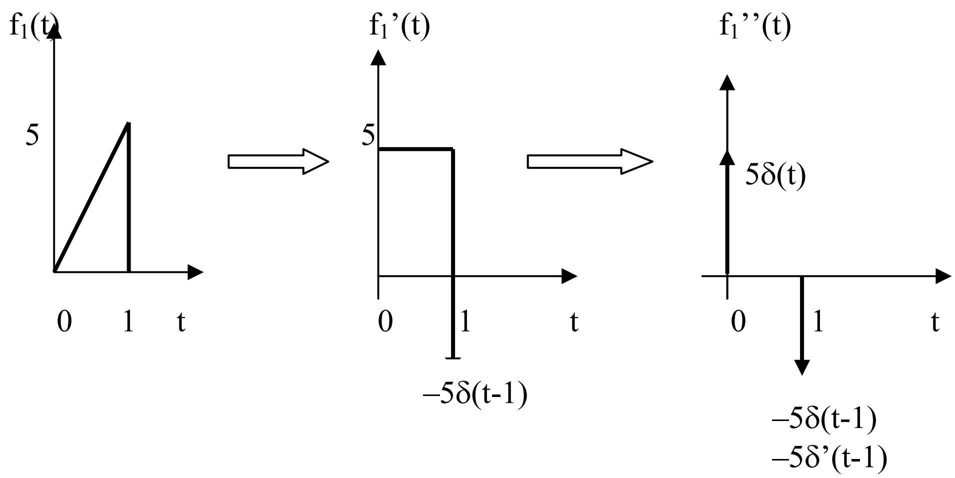
For Prob. 15.15.

Chapter 15, Solution 15.

This is a periodic function with T=3.

$$F(s) = \frac{F_1(s)}{1 - e^{-3s}}$$

To get $F_1(s)$, we consider f(t) over one period.



$f_1'' = 5\delta(t) - 5\delta(t-1) - 5\delta'(t-1)$

Taking the Laplace transform of each term,

$s^2 F_1(s) = 5 - 5e^{-s} - 5se^{-s}$ or $F_1(s) = 5(1 - e^{-s} - se^{-s})/s^2$

Hence,

$$F(s) = 5\frac{1 - e^{-s} - se^{-s}}{s^2(1 - e^{-3s})}$$

Alternatively, we can obtain the same answer by noting that $f_1(t) = 5tu(t) - 5tu(t-1) - 5u(t-1)$.

Chapter 15, Problem 16.

Obtain the Laplace transform of $f(t)$ in Fig. 15.28.

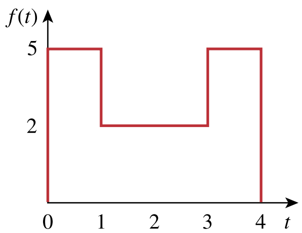


Figure 15.28

For Prob. 15.16.

Chapter 15, Solution 16.

$$f(t) = 5u(t) - 3u(t-1) + 3u(t-3) - 5u(t-4)$$

$$F(s) = \underline{\frac{1}{s}\left[5 - 3e^{-s} + 3e^{-3s} - 5e^{-4s}\right]}$$

Chapter 15, Problem 17.

Find the Laplace transform of $f(t)$ shown in Fig. 15.29.

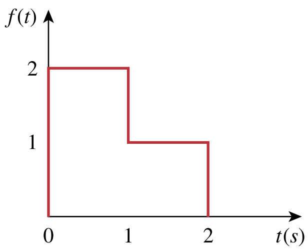
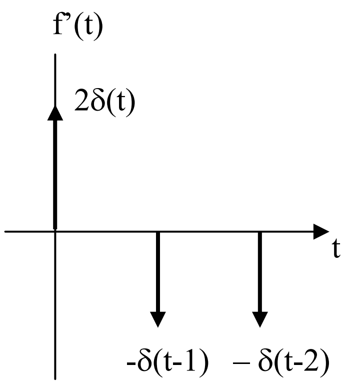


Figure 15.29

For Prob. 15.17.

Chapter 15, Solution 17.

Taking the derivative of f(t) gives f'(t) as shown below.



$$f'(t) = 2\delta(t) - \delta(t-1) - \delta(t-2)$$

Taking the Laplace transform of each term,

$$sF(s) = 2 - e^{-s} - e^{-2s}$$ which leads to

$$F(s) = \underline{\mathbf{[2 - e^{-s} - e^{-2s}]/s}}$$

We can also obtain the same answer noting that $f(t) = 2u(t) - u(t-1) - u(t-2)$.

Chapter 15, Problem 18.

Obtain the Laplace transforms of the functions in Fig. 15.30.

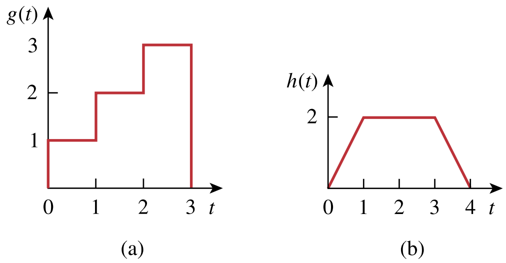


Figure 15.30

For Prob. 15.18.

Chapter 15, Solution 18.

(a)

$$g(t) = u(t) - u(t-1) + 2\left[u(t-1) - u(t-2)\right] + 3\left[u(t-2) - u(t-3)\right]$$
$$= u(t) + u(t-1) + u(t-2) - 3u(t-3)$$

$$G(s) = \underline{\mathbf{\frac{1}{s}(1 + e^{-s} + e^{-2s} - 3e^{-3s})}}$$

(b)

$$h(t) = 2t\left[u(t) - u(t-1)\right] + 2\left[u(t-1) - u(t-3)\right] + (8-2t)\left[u(t-3) - u(t-4)\right]$$

$$= 2t\,u(t) - 2(t-1)u(t-1) - 2u(t-1) + 2u(t-1) - 2u(t-3) - 2(t-3)u(t-3) + 2u(t-3) + 2(t-4)u(t-4)$$

$$= 2t\,u(t) - 2(t-1)u(t-1) - 2(t-3)u(t-3) + 2(t-4)u(t-4)$$

$$H(s) = \frac{2}{s^2}(1 - e^{-s}) - \frac{2}{s^2}e^{-3s} + \frac{2}{s^2}e^{-4s} = \underline{\mathbf{\frac{2}{s^2}(1 - e^{-s} - e^{-3s} + e^{-4s})}}$$

Chapter 15, Problem 19.

Calculate the Laplace transform of the train of unit impulses in Fig. 15.31.

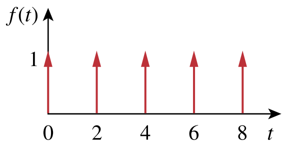


Figure 15.31

For Prob. 15.19.

Chapter 15, Solution 19.

$$\text{Since } \mathcal{L}\left[\delta(t)\right] = 1 \text{ and } T = 2, \quad F(s) = \underline{\mathbf{\frac{1}{1 - e^{-2s}}}}$$

Chapter 15, Problem 20.

The periodic function shown in Fig. 15.32 is defined over its period as

$$g(t)\begin{cases} \sin \pi t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

Find $G(s)$

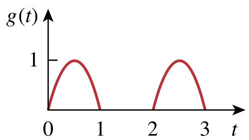


Figure 15.32

For Prob. 15.20.

Chapter 15, Solution 20.

$$\text{Let} \quad g_1(t) = \sin(\pi t), \quad 0 < t < 1$$
$$= \sin(\pi t)\left[u(t) - u(t-1)\right]$$
$$= \sin(\pi t)\,u(t) - \sin(\pi t)\,u(t-1)$$

Note that $\sin(\pi(t-1)) = \sin(\pi t - \pi) = -\sin(\pi t)$.

So, $\quad g_1(t) = \sin(\pi t)\,u(t) + \sin(\pi(t-1))\,u(t-1)$

$$G_1(s) = \frac{\pi}{s^2 + \pi^2}(1 + e^{-s})$$

$$G(s) = \frac{G_1(s)}{1 - e^{-2s}} = \underline{\mathbf{\frac{\pi(1 + e^{-s})}{(s^2 + \pi^2)(1 - e^{-2s})}}}$$

Chapter 15, Problem 21.

Obtain the Laplace transform of the periodic waveform in Fig. 15.33.

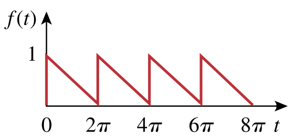


Figure 15.33

For Prob. 15.21.

Chapter 15, Solution 21.

$$T = 2\pi$$

Let $$f_1(t) = \left(1 - \frac{t}{2\pi}\right)\left[u(t) - u(t-2\pi)\right]$$

$$f_1(t) = u(t) - \frac{t}{2\pi}u(t) + \frac{1}{2\pi}(t-2\pi)\,u(t-2\pi)$$

$$F_1(s) = \frac{1}{s} - \frac{1}{2\pi s^2} + \frac{e^{-2\pi s}}{2\pi s^2} = \frac{2\pi\, s + \left[-1 + e^{-2\pi s}\right]}{2\pi s^2}$$

$$F(s) = \frac{F_1(s)}{1 - e^{-Ts}} = \underline{\frac{2\pi s - 1 + e^{-2\pi s}}{2\pi s^2\,(1 - e^{-2\pi s})}}$$

Chapter 15, Problem 22.

Find the Laplace transforms of the functions in Fig. 15.34.

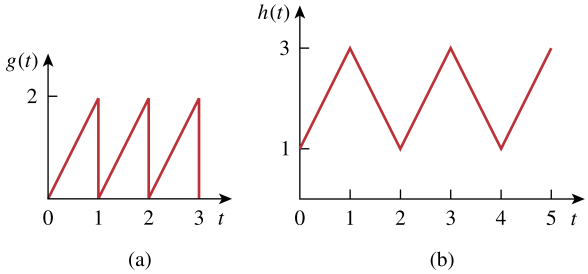


Figure 15.34

For Prob. 15.22.

Chapter 15, Solution 22.

(a) Let $g_1(t) = 2t, \quad 0 < t < 1$

$$= 2t\left[u(t) - u(t-1)\right]$$

$$= 2t\,u(t) - 2(t-1)u(t-1) + 2u(t-1)$$

$$G_1(s) = \frac{2}{s^2} - \frac{2e^{-s}}{s^2} + \frac{2}{s}e^{-s}$$

$$G(s) = \frac{G_1(s)}{1 - e^{-sT}}, \quad T = 1$$

$$G(s) = \underline{\mathbf{\frac{2(1 - e^{-s} + s e^{-s})}{s^2(1 - e^{-s})}}}$$

(b) Let $h = h_0 + u(t)$, where h_0 is the periodic triangular wave.

Let h_1 be h_0 within its first period, i.e.

$$h_1(t) = \begin{cases} 2t & 0 < t < 1 \\ 4 - 2t & 1 < t < 2 \end{cases}$$

$$h_1(t) = 2t\,u(t) - 2t\,u(t-1) + 4u(t-1) - 2t\,u(t-1) - 2(t-2)u(t-2)$$

$$h_1(t) = 2t\,u(t) - 4(t-1)u(t-1) - 2(t-2)u(t-2)$$

$$H_1(s) = \frac{2}{s^2} - \frac{4}{s^2}e^{-s} - \frac{2e^{-2s}}{s^2} = \frac{2}{s^2}(1 - e^{-s})^2$$

$$H_0(s) = \frac{2}{s^2}\frac{(1 - e^{-s})^2}{(1 - e^{-2s})}$$

$$H(s) = \underline{\mathbf{\frac{1}{s} + \frac{2}{s^2}\frac{(1 - e^{-s})^2}{(1 - e^{-2s})}}}$$

Chapter 15, Problem 23.

Determine the Laplace transforms of the periodic functions in Fig. 15.35.

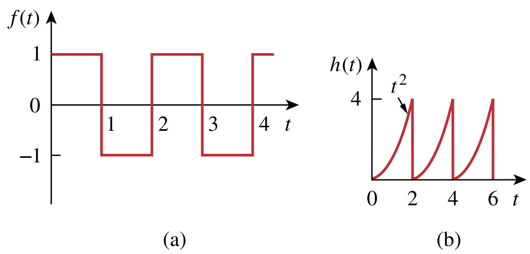


Figure 15.35

For Prob. 15.23.

Chapter 15, Solution 23.

(a) Let $f_1(t) = \begin{cases} 1 & 0 < t < 1 \\ -1 & 1 < t < 2 \end{cases}$

$$f_1(t) = [u(t) - u(t-1)] - [u(t-1) - u(t-2)]$$
$$f_1(t) = u(t) - 2u(t-1) + u(t-2)$$

$$F_1(s) = \frac{1}{s}(1 - 2e^{-s} + e^{-2s}) = \frac{1}{s}(1-e^{-s})^2$$

$$F(s) = \frac{F_1(s)}{(1-e^{-sT})}, \quad T = 2$$

$$F(s) = \underline{\boldsymbol{\frac{(1-e^{-s})^2}{s(1-e^{-2s})}}}$$

(b) Let $h_1(t) = t^2[u(t) - u(t-2)] = t^2u(t) - t^2u(t-2)$

$$h_1(t) = t^2u(t) - (t-2)^2u(t-2) - 4(t-2)u(t-2) - 4u(t-2)$$

$$H_1(s) = \frac{2}{s^3}(1-e^{-2s}) - \frac{4}{s^2}e^{-2s} - \frac{4}{s}e^{-2s}$$

$$H(s) = \frac{H_1(s)}{(1-e^{-Ts})}, \quad T = 2$$

$$H(s) = \underline{\boldsymbol{\frac{2(1-e^{-2s}) - 4se^{-2s}(s+s^2)}{s^3(1-e^{-2s})}}}$$

Chapter 15, Problem 24.

Given that

$$F(s) = \frac{s^2 + 10s + 6}{s(s+1)^2(s+3)}$$

Evaluate $f(0)$ and $f(\infty)$ if they exist.

Chapter 15, Solution 24.

$$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s^2 + 10s + 6}{(s+1)^2(s+2)} = \lim_{s\to\infty} \frac{1/s + 10/s^2 + 6/s^3}{(1+1/s)(1+2/s)} = \frac{0}{1} = \underline{0}$$

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{s^2 + 10s + 6}{(s+1)^2(s+2)} = \frac{6}{(1)(2)} = \underline{3}$$

Chapter 15, Problem 25.

Let

$$F(s) = \frac{5(s+1)}{(s+2)(s+3)}$$

(a) Use the initial and final value theorems to find $f(0)$ and $f(\infty)$.

(b) Verify your answer in part (a) by finding $f(t)$, using partial fractions.

Chapter 15, Solution 25.

(a) $$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{5s(s+1)}{(s+2)(s+3)} = \lim_{s\to\infty} \frac{5(1+1/s)}{(1+2/s)(1+3/s)} = \underline{5}$$

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{5s(s+1)}{(s+2)(s+3)} = \underline{0}$$

(b) $$F(s) = \frac{5(s+1)}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A = \frac{5(-1)}{1} = -5, \qquad B = \frac{5(-2)}{-1} = 10$$

$$F(s) = \frac{-5}{s+2} + \frac{10}{s+3} \longrightarrow f(t) = -5e^{-2t} + 10e^{-3t}$$

$f(0) = -5 + 10 = \underline{\mathbf{5}}$

$f(\infty) = -0 + 0 = \underline{\mathbf{0.}}$

Chapter 15, Problem 26.

Determine the initial and final values of $f(t)$, if they exist, given that:

(a) $F(s)=\dfrac{s^2+3}{s^3+4s^2+6}$

(b) $F(s)=\dfrac{s^2-2s+1}{(s-2)(s^2+2s+4)}$

Chapter 15, Solution 26.

(a) $$f(0)=\lim_{s\to\infty} sF(s)=\lim_{s\to\infty}\frac{s^3+3s}{s^3+4s^2+6}=\underline{\mathbf{1}}$$

Two poles are not in the left-half plane.
$f(\infty)$ **does not exist**

(b) $$f(0)=\lim_{s\to\infty} sF(s)=\lim_{s\to\infty}\frac{s^3-2s^2+s}{(s-2)(s^2+2s+4)}$$

$$=\lim_{s\to\infty}\frac{1-\frac{2}{s}+\frac{1}{s^2}}{\left(1-\frac{2}{s}\right)\left(1+\frac{2}{s}+\frac{4}{s^2}\right)}=\underline{\mathbf{1}}$$

One pole is not in the left-half plane.
$f(\infty)$ **does not exist**

Chapter 15, Problem 27.

Determine the inverse Laplace transform of each of the following functions:

(a) $F(s)=\dfrac{1}{s}+\dfrac{2}{s+1}$

(b) $G(s)=\dfrac{3s+1}{s+4}$

(c) $H(s)=\dfrac{4}{(s+1)(s+3)}$

(d) $J(s)=\dfrac{12}{(s+2)^2(s+4)}$

Chapter 15, Solution 27.

(a) $f(t) = u(t) + 2e^{-t}u(t)$

(b) $G(s) = \dfrac{3(s+4) - 11}{s+4} = 3 - \dfrac{11}{s+4}$

$g(t) = \underline{3\delta(t) - 11e^{-4t}u(t)}$

(c) $H(s) = \dfrac{4}{(s+1)(s+3)} = \dfrac{A}{s+1} + \dfrac{B}{s+3}$

$A = 2, \qquad B = -2$

$H(s) = \dfrac{2}{s+1} - \dfrac{2}{s+3}$

$h(t) = \underline{\mathbf{2e^{-t} - 2e^{-3t}}}\ \underline{\mathbf{u(t)}}$

(d) $J(s) = \dfrac{12}{(s+2)^2(s+4)} = \dfrac{A}{s+2} + \dfrac{B}{(s+2)^2} + \dfrac{C}{s+4}$

$B = \dfrac{12}{2} = 6, \quad C = \dfrac{12}{(-2)^2} = 3$

$12 = A(s+2)(s+4) + B(s+4) + C(s+2)^2$

Equating coefficients :

$s^2: \quad 0 = A + C \longrightarrow A = -C = -3$

$s^1: \quad 0 = 6A + B + 4C = 2A + B \longrightarrow B = -2A = 6$

$s^0: \quad 12 = 8A + 4B + 4C = -24 + 24 + 12 = 12$

$J(s) = \dfrac{-3}{s+2} + \dfrac{6}{(s+2)^2} + \dfrac{3}{s+4}$

$j(t) = \underline{\mathbf{3e^{-4t} - 3e^{-2t} + 6te^{-2t}}}\ \underline{\mathbf{u(t)}}$

Chapter 15, Problem 28.

Find the inverse Laplace transform of the following functions:

(a) $F(s) = \dfrac{20(s+2)}{s(s^2+6s+25)}$

(b) $P(s) = \dfrac{6s^2+36s+20}{(s+1)(s+2)(s+3)}$

Chapter 15, Solution 28.

(a) $F(s) = \dfrac{20(s+2)}{s(s^2+6s+25)} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+6s+25}$

$20(s+2) = A(s^2+6s+25s) + Bs^2 + Cs$

Equating components,

s^2 :	0 = A + B or B= - A
s:	20 = 6A + C
constant:	40 – 25 A or A = 8/5, B = -8/5, C= 20 – 6A= 52/5

$$F(s) = \frac{8}{5s} + \frac{-\frac{8}{5}s + \frac{52}{5}}{(s+3)^2+4^2} = \frac{8}{5s} + \frac{-\frac{8}{5}(s+3) + \frac{24}{5} + \frac{52}{5}}{(s+3)^2+4^2}$$

$$f(t) = \underline{\frac{8}{5}u(t) - \frac{8}{5}e^{-3t}\cos 4t + \frac{19}{5}e^{-3t}\sin 4t}$$

(b) $P(s) = \dfrac{6s^2+36s+20}{(s+1)(s+2)(s+3)} = \dfrac{A}{s+1} + \dfrac{B}{s+2} + \dfrac{C}{s+3}$

$$A = \frac{6-36+20}{(-1+2)(-1+3)} = -5$$

$$B = \frac{24-72+20}{(-1)(1)} = 28$$

$$C = \frac{54-108+20}{(-2)(-1)} = -17$$

$$P(s) = \frac{-5}{s+1} + \frac{28}{s+2} - \frac{17}{s+3}$$

$$p(t) = \underline{(-5e^{-t} + 28e^{-2t} - 17e^{-3t})u(t)}$$

Chapter 15, Problem 29.

Find the inverse Laplace transform of:

$$V(s) = \frac{2s + 26}{s\left(s^2 + 4s + 13\right)}$$

Chapter 15, Solution 29.

$$V(s) = \frac{2}{s} + \frac{As + B}{(s+2)^2 + 3^2}; \; 2s^2 + 8s + 26 + As^2 + Bs = 2s + 26 \rightarrow A = -2 \text{ and } B = -6$$

$$V(s) = \frac{2}{s} - \frac{2(s+2)}{(s+2)^2 + 3^2} - \frac{2}{3}\frac{3}{(s+2)^2 + 3^2}$$

$$v(t) = \underline{(2 - 2e^{-2t}\cos 3t - \frac{2}{3}e^{-2t}\sin 3t)u(t), \;\; t \geq 0}$$

Chapter 15, Problem 30.

Find the inverse Laplace transform of:

(a) $F_1(s) = \dfrac{6s^2 + 8s + 3}{s(s^2 + 2s + 5)}$

(b) $F_2(s) = \dfrac{s^2 + 5s + 6}{(s+1)^2(s+4)}$

(c) $F_3(s) = \dfrac{10}{(s+1)(s^2 + 4s + 8)}$

Chapter 15, Solution 30.

(a) $$F_1(s) = \frac{6s^2 + 8s + 3}{s(s^2 + 2s + 5)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 2s + 5}$$

$$6s^2 + 8s + 3 = A(s^2 + 2s + 5) + Bs^2 + Cs$$

We equate coefficients.

s^2 : 6 = A + B

s: 8= 2A + C

constant: 3=5A or A=3/5

B=6-A = 27/5, C=8-2A = 34/5

$$F_1(s) = \frac{3/5}{s} + \frac{27s/5 + 34/5}{s^2 + 2s + 5} = \frac{3/5}{s} + \frac{27(s+1)/5 + 7/5}{(s+1)^2 + 2^2}$$

$$f_1(t) = \underline{\left[\frac{3}{5} + \frac{27}{5}e^{-t}\cos 2t + \frac{7}{10}e^{-t}\sin 2t\right]u(t)}$$

(b) $F_2(s) = \dfrac{s^2+5s+6}{(s+1)^2(s+4)} = \dfrac{A}{s+1} + \dfrac{B}{(s+1)^2} + \dfrac{C}{s+4}$

$s^2+5s+6 = A(s+1)(s+4) + B(s+4) + C(s+1)^2$

Equating coefficients,

s^2 : 1=A+C
s: 5=5A+B+2C
constant: 6=4A+4B+C

Solving these gives

A=7/9, B= 2/3, C=2/9

$$F_2(s) = \frac{7/9}{s+1} + \frac{2/3}{(s+1)^2} + \frac{2/9}{s+4}$$

$$f_2(t) = \underline{\left[\frac{7}{9}e^{-t} + \frac{2}{3}te^{-t} + \frac{2}{9}e^{-4t}\right]u(t)}$$

(c) $F_3(s) = \dfrac{10}{(s+1)(s^2+4s+8)} = \dfrac{A}{s+1} + \dfrac{Bs+C}{s^2+4s+8}$

$10 = A(s^2+4s+8) + B(s^2+s) + C(s+1)$

s^2 : 0 = A + B or B = -A
s: 0=4A+ B + C
constant: 10=8A+C

Solving these yields

A=2, B= -2, C= -6

$$F_3(s) = \frac{2}{s+1} + \frac{-2s-6}{s^2+4s+8} = \frac{2}{s+1} - \frac{2(s+1)}{(s+1)^2+2^2} - \frac{4}{(s+1)^2+2^2}$$

$f_3(t) =$ $\underline{\mathbf{(2e^{-t} - 2e^{-t}\cos(2t) - 2e^{-t}\sin(2t))u(t)}}$.

Chapter 15, Problem 31.

Find $f(t)$ for each $F(s)$:

(a) $\dfrac{10s}{(s+1)(s+2)(s+3)}$

(b) $\dfrac{2s^2+4s+1}{(s+1)(s+2)^3}$

(c) $\dfrac{s+1}{(s+2)(s^2+2s+5)}$

Chapter 15, Solution 31.

(a) $$F(s) = \frac{10s}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = F(s)\,(s+1)\Big|_{s=-1} = \frac{-10}{2} = -5$$

$$B = F(s)\,(s+2)\Big|_{s=-2} = \frac{-20}{-1} = 20$$

$$C = F(s)\,(s+3)\Big|_{s=-3} = \frac{-30}{2} = -15$$

$$F(s) = \frac{-5}{s+1} + \frac{20}{s+2} - \frac{15}{s+3}$$

$$f(t) = \underline{(-5e^{-t} + 20e^{-2t} - 15e^{-3t})u(t)}$$

(b) $$F(s) = \frac{2s^2+4s+1}{(s+1)(s+2)^3} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2} + \frac{D}{(s+2)^3}$$

$$A = F(s)\,(s+1)\Big|_{s=-1} = -1$$

$$D = F(s)\,(s+2)^3\Big|_{s=-2} = -1$$

$$2s^2+4s+1 = A(s+2)(s^2+4s+4) + B(s+1)(s^2+4s+4) + C(s+1)(s+2) + D(s+1)$$

Equating coefficients :

s^3: $0 = A + B \longrightarrow B = -A = 1$

s^2: $2 = 6A + 5B + C = A + C \longrightarrow C = 2 - A = 3$

s^1: $4 = 12A + 8B + 3C + D = 4A + 3C + D$

$4 = 6 + A + D \longrightarrow D = -2 - A = -1$

s^0: $1 = 8A + 4B + 2C + D = 4A + 2C + D = -4 + 6 - 1 = 1$

$$F(s) = \frac{-1}{s+1} + \frac{1}{s+2} + \frac{3}{(s+2)^2} - \frac{1}{(s+2)^3}$$

$$f(t) = -e^{-t} + e^{-2t} + 3t\,e^{-2t} - \frac{t^2}{2}e^{-2t}$$

$$\underline{f(t) = \left(-e^{-t} + \left(1 + 3t - \frac{t^2}{2}\right)e^{-2t}\right)u(t)}$$

(c) $$F(s) = \frac{s+1}{(s+2)(s^2+2s+5)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+2s+5}$$

$$A = F(s)\,(s+2)\Big|_{s=-2} = \frac{-1}{5}$$

$$s+1 = A(s^2+2s+5) + B(s^2+2s) + C(s+2)$$

Equating coefficients :

s^2: $0 = A + B \longrightarrow B = -A = \frac{1}{5}$

s^1: $1 = 2A + 2B + C = 0 + C \longrightarrow C = 1$

s^0: $1 = 5A + 2C = -1 + 2 = 1$

$$F(s) = \frac{-1/5}{s+2} + \frac{1/5 \cdot s + 1}{(s+1)^2 + 2^2} = \frac{-1/5}{s+2} + \frac{1/5(s+1)}{(s+1)^2+2^2} + \frac{4/5}{(s+1)^2+2^2}$$

$$\underline{f(t) = (-0.2e^{-2t} + 0.2e^{-t}\cos(2t) + 0.4e^{-t}\sin(2t))u(t)}$$

Chapter 15, Problem 32.

Determine the inverse Laplace transform of each of the following functions:

(a) $\dfrac{8(s+1)(s+3)}{s(s+2)(s+4)}$ (b) $\dfrac{s^2-2s+4}{(s+1)(s+2)^2}$ (c) $\dfrac{s^2+1}{(s+3)(s^2+4s+5)}$

Chapter 15, Solution 32.

(a) $$F(s) = \frac{8(s+1)(s+3)}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$A = F(s)\,s\Big|_{s=0} = \frac{(8)(3)}{(2)(4)} = 3$$

$$B = F(s)\,(s+2)\Big|_{s=-2} = \frac{(8)(-1)}{(-4)} = 2$$

$$C = F(s)\,(s+4)\Big|_{s=-4} = \frac{(8)(-1)(-3)}{(-4)(-2)} = 3$$

$$F(s) = \frac{3}{s} + \frac{2}{s+2} + \frac{3}{s+4}$$

$$f(t) = \underline{\mathbf{3u(t) + 2e^{-2t} + 3e^{-4t}}}$$

(b) $$F(s) = \frac{s^2-2s+4}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$s^2 - 2s + 4 = A(s^2+4s+4) + B(s^2+3s+2) + C(s+1)$$

Equating coefficients :

s^2: $1 = A + B \longrightarrow B = 1 - A$

s^1: $-2 = 4A + 3B + C = 3 + A + C$

s^0: $4 = 4A + 2B + C = -B - 2 \longrightarrow B = -6$

$A = 1 - B = 7$ $\qquad$ $C = -5 - A = -12$

$$F(s) = \frac{7}{s+1} - \frac{6}{s+2} - \frac{12}{(s+2)^2}$$

$$f(t) = \underline{\mathbf{7e^{-t} - 6(1+2t)e^{-2t}}}$$

(c) $$F(s) = \frac{s^2+1}{(s+3)(s^2+4s+5)} = \frac{A}{s+3} + \frac{Bs+C}{s^2+4s+5}$$

$$s^2+1 = A(s^2+4s+5) + B(s^2+3s) + C(s+3)$$

Equating coefficients :

s^2 : $1 = A + B \longrightarrow B = 1 - A$

s^1 : $0 = 4A + 3B + C = 3 + A + C \longrightarrow A + C = -3$

s^0 : $1 = 5A + 3C = -9 + 2A \longrightarrow A = 5$

$B = 1 - A = -4$ $\qquad$ $C = -A - 3 = -8$

$$F(s) = \frac{5}{s+3} - \frac{4s+8}{(s+2)^2+1} = \frac{5}{s+3} - \frac{4(s+2)}{(s+2)^2+1}$$

$$f(t) = \underline{\mathbf{5e^{-3t} - 4e^{-2t}\cos(t)}}$$

Chapter 15, Problem 33.

Calculate the inverse Laplace transform of:

(a) $\dfrac{6(s-1)}{s^4-1}$ (b) $\dfrac{se^{-\pi s}}{s^2+1}$ (c) $\dfrac{8}{s(s+1)^3}$

Chapter 15, Solution 33.

(a) $$F(s)=\frac{6(s-1)}{s^4-1}=\frac{6}{(s^2+1)(s+1)}=\frac{As+B}{s^2+1}+\frac{C}{s+1}$$

$$6=A(s^2+s)+B(s+1)+C(s^2+1)$$

Equating coefficients :

s^2: $0=A+C \longrightarrow A=-C$

s^1: $0=A+B \longrightarrow B=-A=C$

s^0: $6=B+C=2B \longrightarrow B=3$

$A=-3,\quad B=3,\quad C=3$

$$F(s)=\frac{3}{s+1}+\frac{-3s+3}{s^2+1}=\frac{3}{s+1}+\frac{-3s}{s^2+1}+\frac{3}{s^2+1}$$

$$f(t)=\underline{(3e^{-t}+3\sin(t)-3\cos(t))u(t)}$$

(b) $$F(s)=\frac{se^{-\pi s}}{s^2+1}$$

$$f(t)=\underline{\mathbf{\cos(t-\pi)u(t-\pi)}}$$

(c) $$F(s)=\frac{8}{s(s+1)^3}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{(s+1)^2}+\frac{D}{(s+1)^3}$$

$A=8,\quad D=-8$

$$8=A(s^3+3s^2+3s+1)+B(s^3+2s^2+s)+C(s^2+s)+Ds$$

Equating coefficients :

s^3: $0=A+B \longrightarrow B=-A$

s^2: $0=3A+2B+C=A+C \longrightarrow C=-A=B$

s^1: $0=3A+B+C+D=A+D \longrightarrow D=-A$

s^0: $A=8,\quad B=-8,\quad C=-8,\quad D=-8$

$$F(s)=\frac{8}{s}-\frac{8}{s+1}-\frac{8}{(s+1)^2}-\frac{8}{(s+1)^3}$$

$$f(t)=\underline{\mathbf{8\left[1-e^{-t}-te^{-t}-0.5t^2e^{-t}\right]u(t)}}$$

Chapter 15, Problem 34.

Find the time functions that have the following Laplace transforms:

(a) $F(s) = 10 + \dfrac{s^2+1}{s^2+4}$ (b) $G(s) = \dfrac{e^{-s}+4e^{-2s}}{s^2+6s+8}$ (c) $H(s) = \dfrac{(s+1)e^{-2s}}{s(s+3)(s+4)}$

Chapter 15, Solution 34.

(a) $$F(s) = 10 + \frac{s^2+4-3}{s^2+4} = 11 - \frac{3}{s^2+4}$$

$$f(t) = \underline{\mathbf{11\delta(t) - 1.5\sin(2t)}}$$

(b) $$G(s) = \frac{e^{-s} + 4e^{-2s}}{(s+2)(s+4)}$$

Let $$\frac{1}{(s+2)(s+4)} = \frac{A}{s+2} + \frac{B}{s+4}$$

$A = 1/2$ $B = 1/2$

$$G(s) = \frac{e^{-s}}{2}\left(\frac{1}{s+2} + \frac{1}{s+4}\right) + 2e^{-2s}\left(\frac{1}{s+2} + \frac{1}{s+4}\right)$$

$$g(t) = \underline{\mathbf{0.5\left[e^{-2(t-1)} - e^{-4(t-1)}\right]u(t-1) + 2\left[e^{-2(t-2)} - e^{-4(t-2)}\right]u(t-2)}}$$

(c) Let $$\frac{s+1}{s(s+3)(s+4)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+4}$$

$A = 1/12$, $B = 2/3$, $C = -3/4$

$$H(s) = \left(\frac{1}{12}\cdot\frac{1}{s} + \frac{2/3}{s+3} - \frac{3/4}{s+4}\right)e^{-2s}$$

$$h(t) = \underline{\mathbf{\left(\frac{1}{12} + \frac{2}{3}e^{-3(t-2)} - \frac{3}{4}e^{-4(t-2)}\right)u(t-2)}}$$

Chapter 15, Problem 35.

Obtain *f*(*t*) for the following transforms:

(a) $F(s)=\dfrac{(s+3)e^{-6s}}{(s+1)(s+2)}$ (b) $F(s)=\dfrac{4-e^{-2s}}{s^2+5s+4}$ (c) $F(s)=\dfrac{se^{-s}}{(s+3)(s^2+4)}$

Chapter 15, Solution 35.

(a) Let $G(s)=\dfrac{s+3}{(s+1)(s+2)}=\dfrac{A}{s+1}+\dfrac{B}{s+2}$

$A=2, \qquad B=-1$

$$G(s)=\frac{2}{s+1}-\frac{1}{s+2} \longrightarrow g(t)=2e^{-t}-e^{-2t}$$

$$F(s)=e^{-6s}G(s) \longrightarrow f(t)=g(t-6)u(t-6)$$

$$f(t)=\underline{\mathbf{\left[2e^{-(t-6)}-e^{-2(t-6)}\right]u(t-6)}}$$

(b) Let $G(s)=\dfrac{1}{(s+1)(s+4)}=\dfrac{A}{s+1}+\dfrac{B}{s+4}$

$A=1/3, \qquad B=-1/3$

$$G(s)=\frac{1}{3(s+1)}-\frac{1}{3(s+4)}$$

$$g(t)=\frac{1}{3}\left[e^{-t}-e^{-4t}\right]$$

$$F(s)=4G(s)-e^{-2t}G(s)$$

$$f(t)=4g(t)u(t)-g(t-2)u(t-2)$$

$$f(t)=\underline{\mathbf{\frac{4}{3}\left[e^{-t}-e^{-4t}\right]u(t)-\frac{1}{3}\left[e^{-(t-2)}-e^{-4(t-2)}\right]u(t-2)}}$$

(c) Let $G(s) = \dfrac{s}{(s+3)(s^2+4)} = \dfrac{A}{s+3} + \dfrac{Bs+C}{s^2+4}$

$A = -3/13$

$s = A(s^2+4) + B(s^2+3s) + C(s+3)$

Equating coefficients :

$s^2:\quad 0 = A + B \longrightarrow B = -A$

$s^1:\quad 1 = 3B + C$

$s^0:\quad 0 = 4A + 3C$

$A = -3/13, \quad B = 3/13, \quad C = 4/13$

$$13\,G(s) = \frac{-3}{s+3} + \frac{3s+4}{s^2+4}$$

$$13\,g(t) = -3e^{-3t} + 3\cos(2t) + 2\sin(2t)$$

$$F(s) = e^{-s}\,G(s)$$

$$f(t) = g(t-1)\,u(t-1)$$

$$f(t) = \underline{\boldsymbol{\frac{1}{13}\left[-3e^{-3(t-1)} + 3\cos(2(t-1)) + 2\sin(2(t-1))\right]u(t-1)}}$$

Chapter 15, Problem 36.

Obtain the inverse Laplace transforms of the following functions:

(a) $X(s) = \dfrac{1}{s^2(s+2)(s+3)}$

(b) $Y(s) = \dfrac{1}{s(s+1)^2}$

(c) $Z(s) = \dfrac{1}{s(s+1)(s^2+6s+10)}$

Chapter 15, Solution 36.

(a) $$X(s) = \frac{1}{s^2(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2} + \frac{D}{s+3}$$

$B = 1/6, \qquad C = 1/4, \qquad D = -1/9$

$$1 = A(s^3 + 5s^2 + 6s) + B(s^2 + 5s + 6) + C(s^3 + 3s^2) + D(s^3 + 2s^2)$$

Equating coefficients :

s^3: $\quad 0 = A + C + D$

s^2: $\quad 0 = 5A + B + 3C + 2D = 3A + B + C$

s^1: $\quad 0 = 6A + 5B$

s^0: $\quad 1 = 6B \longrightarrow B = 1/6$

$A = -5/6\,B = -5/36$

$$X(s) = \frac{-5/36}{s} + \frac{1/6}{s^2} + \frac{1/4}{s+2} - \frac{1/9}{s+3}$$

$$x(t) = \underline{\mathbf{\frac{-5}{36}u(t) + \frac{1}{6}t + \frac{1}{4}e^{-2t} - \frac{1}{9}e^{-3t}}}$$

(b) $$Y(s) = \frac{1}{s(s+1)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

$A = 1, \qquad C = -1$

$$1 = A(s^2 + 2s + 1) + B(s^2 + s) + Cs$$

Equating coefficients :

$s^2: \quad 0 = A + B \longrightarrow B = -A$

$s^1: \quad 0 = 2A + B + C = A + C \longrightarrow C = -A$

$s^0: \quad 1 = A, \quad B = -1, \quad C = -1$

$$Y(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}$$

$$y(t) = \underline{\mathbf{u(t) - e^{-t} - t\,e^{-t}}}$$

(c) $$Z(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs + D}{s^2 + 6s + 10}$$

$A = 1/10, \qquad B = -1/5$

$$1 = A(s^3 + 7s^2 + 16s + 10) + B(s^3 + 6s^2 + 10s) + C(s^3 + s^2) + D(s^2 + s)$$

Equating coefficients :

$s^3: \quad 0 = A + B + C$

$s^2: \quad 0 = 7A + 6B + C + D = 6A + 5B + D$

$s^1: \quad 0 = 16A + 10B + D = 10A + 5B \longrightarrow B = -2A$

$s^0: \quad 1 = 10A \longrightarrow A = 1/10$

$A = 1/10, \qquad B = -2A = -1/5, \qquad C = A = 1/10, \qquad D = 4A = \frac{4}{10}$

$$10\,Z(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{s+4}{s^2 + 6s + 10}$$

$$10\,Z(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{s+3}{(s+3)^2 + 1} + \frac{1}{(s+3)^2 + 1}$$

$$z(t) = \underline{\mathbf{0.1\left[\,1 - 2e^{-t} + e^{-3t}\cos(t) + e^{-3t}\sin(t)\right]\,u(t)}}$$

Chapter 15, Problem 37.

Find the inverse Laplace transform of:

(a) $H(s) = \dfrac{s+4}{s(s+2)}$

(b) $G(s) = \dfrac{s^2 + 4s + 5}{(s+3)(s^2 + 2s + 2)}$

(c) $F(s) = \dfrac{e^{-4s}}{s+2}$

(d) $D(s) = \dfrac{10s}{(s^2 + 1)(s^2 + 4)}$

Chapter 15, Solution 37.

(a) $H(s) = \dfrac{s+4}{s(s+2)} = \dfrac{A}{s} + \dfrac{B}{s+2}$

s+4 =A(s+2) + Bs
Equating coefficients,
s: 1 = A + B
constant: 4= 2 A $\longrightarrow$ A =2, B=1-A = -1

$$H(s) = \frac{2}{s} - \frac{1}{s+2}$$
$$h(t) = 2u(t) - e^{-2t}u(t) = \underline{(2 - e^{-2t})u(t)}$$

(b) $G(s) = \dfrac{A}{s+3} + \dfrac{Bs+C}{s^2+2s+2}$

$$s^2+4s+5 = (Bs+C)(s+3) + A(s^2+2s+2)$$

Equating coefficients,

s^2:	1= B + A	(1)
s:	4 = 3B + C + 2A	(2)
Constant:	5 =3C + 2A	(3)

Solving (1) to (3) gives

$$A = \frac{2}{5}, \quad B = \frac{3}{5}, \quad C = \frac{7}{5}$$

$$G(s) = \frac{0.4}{s+3} + \frac{0.6s+1.4}{s^2+2s+2} = \frac{0.4}{s+3} + \frac{0.6(s+1)+0.8}{(s+1)^2+1}$$

$g(t) = \underline{0.4e^{-3t} + 0.6e^{-t}\cos t + 0.8e^{-t}\sin t}$

(c) $f(t) = \underline{e^{-2(t-4)}u(t-4)}$

(d) $D(s) = \dfrac{10s}{(s^2+1)(s^2+4)} = \dfrac{As+B}{s^2+1} + \dfrac{Cs+D}{s^2+4}$

$$10s = (s^2+4)(As+B) + (s^2+1)(Cs+D)$$

Equating coefficients,

s^3 :	0 = A + C
s^2 :	0 = B + D
s:	10 = 4A + C
constant:	0 = 4B+D

Solving these leads to

A = -10/3, B = 0, C = -10/3, D = 0

$$D(s) == \frac{10s/3}{s^2+1} - \frac{10s/3}{s^2+4}$$

$$d(t) = \underline{\frac{10}{3}\cos t - \frac{10}{3}\cos 2t}$$

Chapter 15, Problem 38.

Find $f(t)$ given that:

(a) $F(s)=\dfrac{s^2+4s}{s^2+10s+26}$

(b) $F(s)=\dfrac{5s^2+7s+29}{s(s^2+4s+29)}$

Chapter 15, Solution 38.

(a) $$F(s)=\frac{s^2+4s}{s^2+10s+26}=\frac{s^2+10s+26-6s-26}{s^2+10s+26}$$

$$F(s)=1-\frac{6s+26}{s^2+10s+26}$$

$$F(s)=1-\frac{6(s+5)}{(s+5)^2+1^2}+\frac{4}{(s+5)^2+1^2}$$

$$f(t)=\underline{\boldsymbol{\delta(t)-6e^{-t}\cos(5t)+4e^{-t}\sin(5t)}}$$

(b) $$F(s)=\frac{5s^2+7s+29}{s(s^2+4s+29)}=\frac{A}{s}+\frac{Bs+C}{s^2+4s+29}$$

$$5s^2+7s+29=A(s^2+4s+29)+Bs^2+Cs$$

Equating coefficients :

s^0: $29=29A \longrightarrow A=1$

s^1: $7=4A+C \longrightarrow C=7-4A=3$

s^2: $5=A+B \longrightarrow B=5-A=4$

$A=1,\qquad B=4,\qquad C=3$

$$F(s)=\frac{1}{s}+\frac{4s+3}{s^2+4s+29}=\frac{1}{s}+\frac{4(s+2)}{(s+2)^2+5^2}-\frac{5}{(s+2)^2+5^2}$$

$$f(t)=\underline{\boldsymbol{u(t)+4e^{-2t}\cos(5t)-e^{-2t}\sin(5t)}}$$

Chapter 15, Problem 39.

*Determine *f*(*t*) if:

(a) $F(s) = \dfrac{2s^3 + 4s^2 + 1}{(s^2 + 2s + 17)(s^2 + 4s + 20)}$

(b) $F(s) = \dfrac{s^2 + 4}{(s^2 + 9)(s^2 + 6s + 3)}$

* An asterisk indicates a challenging problem.

Chapter 15, Solution 39.

(a) $$F(s) = \frac{2s^3 + 4s^2 + 1}{(s^2 + 2s + 17)(s^2 + 4s + 20)} = \frac{As + B}{s^2 + 2s + 17} + \frac{Cs + D}{s^2 + 4s + 20}$$

$$s^3 + 4s^2 + 1 = A(s^3 + 4s^2 + 20s) + B(s^2 + 4s + 20)$$
$$+ C(s^3 + 2s^2 + 17s) + D(s^2 + 2s + 17)$$

Equating coefficients :

s^3: $2 = A + C$

s^2: $4 = 4A + B + 2C + D$

s^1: $0 = 20A + 4B + 17C + 2D$

s^0: $1 = 20B + 17D$

Solving these equations (Matlab works well with 4 unknowns),

$A = -1.6$, $B = -17.8$, $C = 3.6$, $D = 21$

$$F(s) = \frac{-1.6s - 17.8}{s^2 + 2s + 17} + \frac{3.6s + 21}{s^2 + 4s + 20}$$

$$F(s) = \frac{(-1.6)(s+1)}{(s+1)^2 + 4^2} + \frac{(-4.05)(4)}{(s+1)^2 + 4^2} + \frac{(3.6)(s+2)}{(s+2)^2 + 4^2} + \frac{(3.45)(4)}{(s+2)^2 + 4^2}$$

$$f(t) = \underline{\mathbf{-1.6e^{-t}\cos(4t) - 4.05e^{-t}\sin(4t) + 3.6e^{-2t}\cos(4t) + 3.45e^{-2t}\sin(4t)}}$$

(b) $$F(s)=\frac{s^2+4}{(s^2+9)(s^2+6s+3)}=\frac{As+B}{s^2+9}+\frac{Cs+D}{s^2+6s+3}$$

$$s^2+4=A(s^3+6s^2+3s)+B(s^2+6s+3)+C(s^3+9s)+D(s^2+9)$$

Equating coefficients :

s^3: $0=A+C \longrightarrow C=-A$

s^2: $1=6A+B+D$

s^1: $0=3A+6B+9C=6B+6C \longrightarrow B=-C=A$

s^0: $4=3B+9D$

Solving these equations,

$A=1/12,\quad B=1/12,\quad C=-1/12,\quad D=5/12$

$$12F(s)=\frac{s+1}{s^2+9}+\frac{-s+5}{s^2+6s+3}$$

$$s^2+6s+3=0 \longrightarrow \frac{-6\pm\sqrt{36-12}}{2}=-0.551,-5.449$$

Let $$G(s)=\frac{-s+5}{s^2+6s+3}=\frac{E}{s+0.551}+\frac{F}{s+5.449}$$

$$E=\left.\frac{-s+5}{s+5.449}\right|_{s=-0.551}=1.133$$

$$F=\left.\frac{-s+5}{s+0.551}\right|_{s=-5.449}=-2.133$$

$$G(s)=\frac{1.133}{s+0.551}-\frac{2.133}{s+5.449}$$

$$12F(s)=\frac{s}{s^2+3^2}+\frac{1}{3}\cdot\frac{3}{s^2+3^2}+\frac{1.133}{s+0.551}-\frac{2.133}{s+5.449}$$

$$f(t)=\underline{\mathbf{0.08333\cos(3t)+0.02778\sin(3t)+0.0944e^{-0.551t}-0.1778e^{-5.449t}}}$$

Chapter 15, Problem 40.

Show that

$$L^{-1}\left[\frac{4s^2+7s+13}{(s+2)(s^2+2s+5)}\right]=\left[\sqrt{2}e^{-t}\cos(2t+45°)+3e^{-2t}\right]u(t)$$

Chapter 15, Solution 40.

$$\text{Let } H(s) = \left[\frac{4s^2+7s+13}{(s+2)(s^2+2s+5)}\right]=\frac{A}{s+2}+\frac{Bs+C}{s^2+2s+5}$$

$$4s^2+7s+13 = A(s^2+2s+5)+B(s^2+2s)+C(s+2)$$

Equating coefficients gives:

$$s^2: \quad 4 = A+B$$

$$s: \quad 7 = 2A+2B+C \longrightarrow C=-1$$

$$\text{constant}: \quad 13 = 5A+2C \longrightarrow 5A = 15 \text{ or } A=3,\ B=1$$

$$H(s)=\frac{3}{s+2}+\frac{s-1}{s^2+2s+5}=\frac{3}{s+2}+\frac{(s+1)-2}{(s+1)^2+2^2}$$

Hence,

$$h(t) = 3e^{-2t}+e^{-t}\cos 2t - e^{-t}\sin 2t = 3e^{-2t}+e^{-t}(A\cos\alpha\cos 2t - A\sin\alpha\sin 2t)$$

$$\text{where } A\cos\alpha = 1, \qquad A\sin\alpha = 1 \longrightarrow A=\sqrt{2}, \qquad \alpha = 45^{o}$$

Thus,

$$h(t) = \underline{\left[\sqrt{2}e^{-t}\cos(2t+45^{o})+3e^{-2t}\right]u(t)}$$

Chapter 15, Problem 41.

* Let $x(t)$ and $y(t)$ be as shown in Fig. 15.36. Find $z(t) = x(t) * y(t)$.

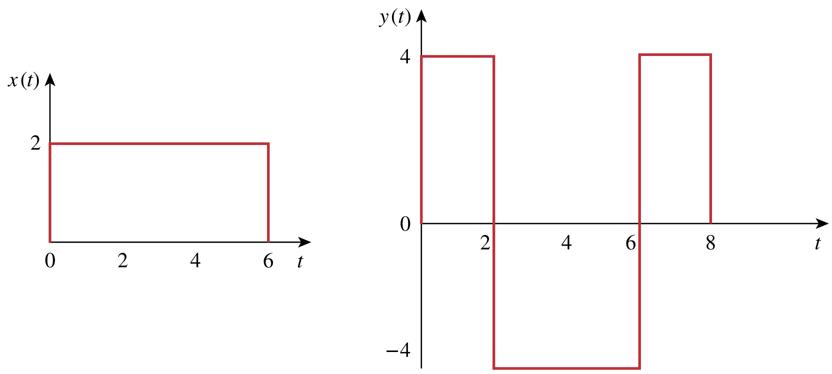


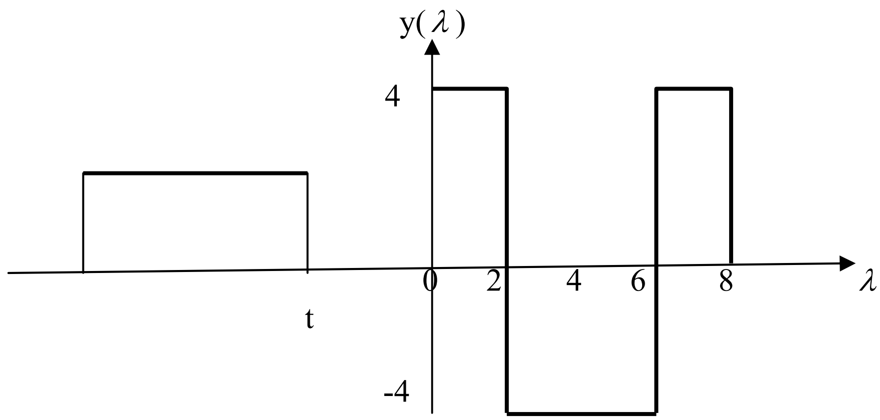
Figure 15.36

For Prob. 15.41.

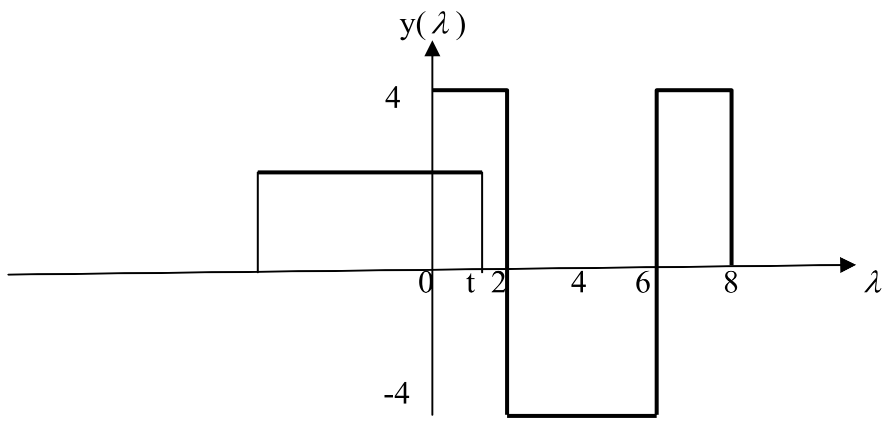
* An asterisk indicates a challenging problem.

Chapter 15, Solution 41.

We fold x(t) and slide on y(t). For t<0, no overlapping as shown below. x(t) =0.

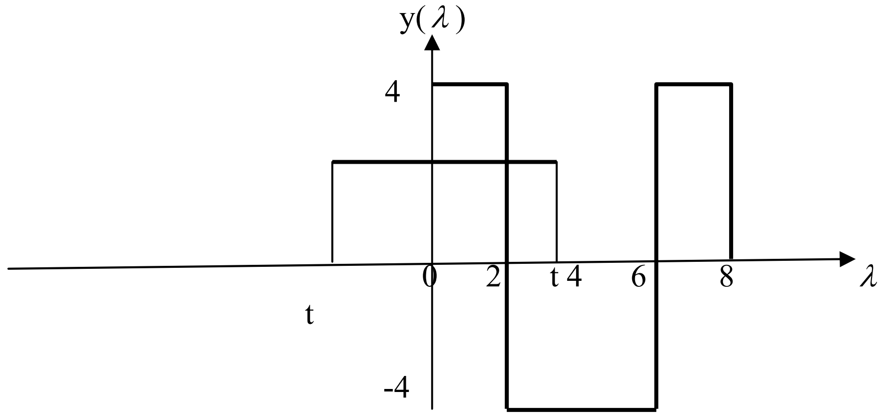


For 0 < t < 2, there is overlapping, as shown below.



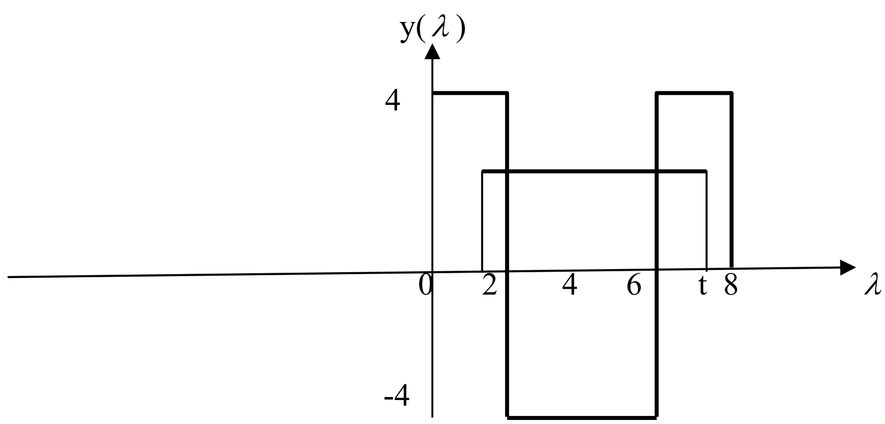
$$z(t) = \int_0^t (2)(4)dt = 8t$$

For $2 < t < 6$, the two functions overlap, as shown below.



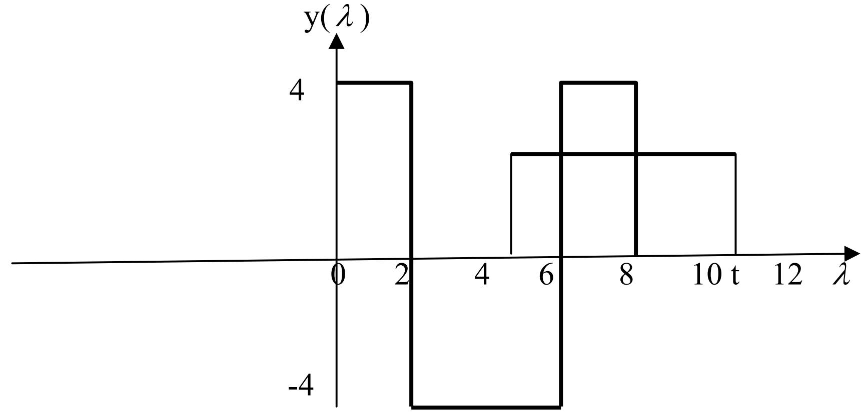
$$z(t) = \int_0^2 (2)(4)d\lambda + \int_0^t (2)(-4)d\lambda = 16 - 8t$$

For 6<t<8, they overlap as shown below.



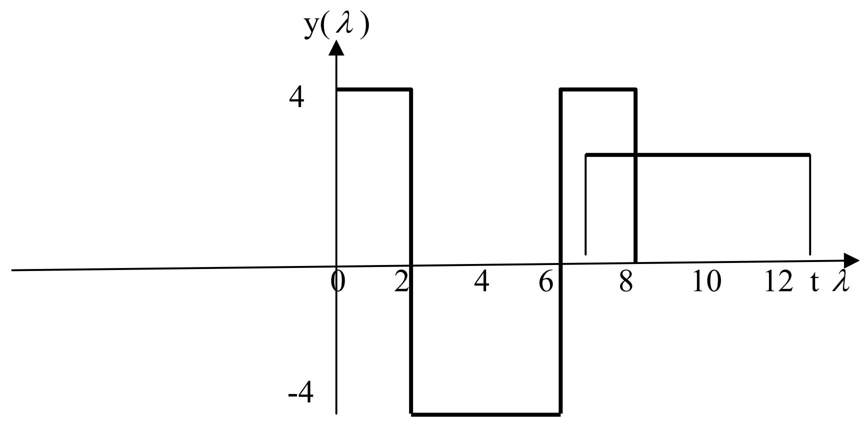
$$z(t) = \int_{t-6}^{2} (2)(4)d\lambda + \int_2^6 (2)(-4)d\lambda + \int_6^t (2)(4)d\lambda = 8\lambda\Big|_{t-6}^{2} - 8\lambda\Big|_2^6 + 8\lambda\Big|_6^t = -16$$

For 8< t <12, they overlap as shown below.



$$z(t) = \int_{t-6}^{6} (2)(-4)d\lambda + \int_{6}^{8} (2)(4)d\lambda = -8\lambda\Big|_{t-6}^{6} + 8\lambda\Big|_{6}^{8} = 8t - 80$$

For 12 < t < 14, they overlap as shown below.



$$z(t) = \int_{t-6}^{8} (2)(4)d\lambda = 8\lambda\Big|_{t-6}^{8} = 112 - 8t$$

Hence,

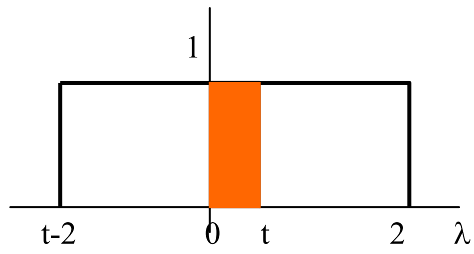
$$z(t) = \begin{cases} 8t, & 0<t<2 \\ 16-8t, & 2<t<6 \\ -16, & 6<t<8 \\ 8t-80, & 8<t<12 \\ 112-8t, & 12<t<14 \\ 0, & \text{otherwise}. \end{cases}$$

Chapter 15, Problem 42.

Suppose that $f(t) = u(t) - u(t-2)$. Determine $f(t) * f(t)$.

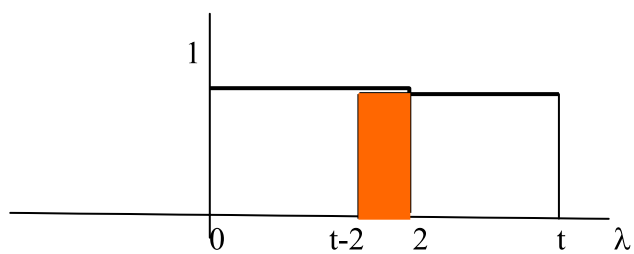
Chapter 15, Solution 42.

For 0<t<2, the signals overlap as shown below.



$$y(t) = f(t) * f(t) = \int_0^t (1)(1) d\lambda = t$$

For $2 < t < 4$, they overlap as shown below.



$$y(t) = \int_{t-2}^{2} (1)(1) d\lambda = t \Big|_{t-2}^{2} = 4 - t$$

Thus,

$$y(t) = \begin{cases} t, & 0 < t < 2 \\ 4 - t, & 2 < t < 4 \\ 0, & \text{otherwise} \end{cases}$$

Chapter 15, Problem 43.

Find $y(t) = x(t) * h(t)$ for each paired $x(t)$ and $h(t)$ in Fig. 15.37.

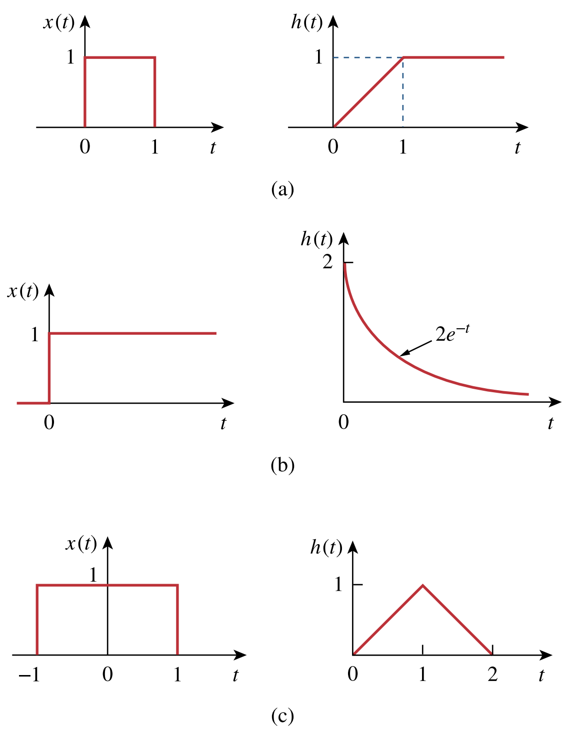


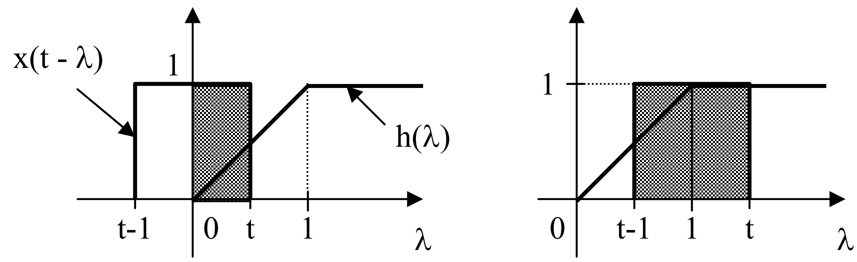
Figure 15.37

For Prob. 15.43.

Chapter 15, Solution 43.

(a) For $0 < t < 1$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (a).

$$y(t) = x(t) * h(t) = \int_0^t (1)(\lambda)\, d\lambda = \frac{\lambda^2}{2}\Big|_0^t = \frac{t^2}{2}$$



(a) **(b)**

For $1 < t < 2$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (b).

$$y(t) = \int_{t-1}^1 (1)(\lambda)\, d\lambda + \int_1^t (1)(1)\, d\lambda = \frac{\lambda^2}{2}\Big|_{t-1}^1 + \lambda\Big|_1^t = \frac{-1}{2}t^2 + 2t - 1$$

For $t > 2$, there is a complete overlap so that

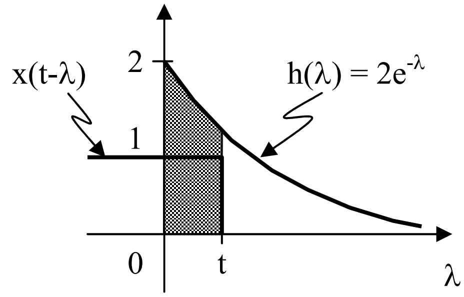
$$y(t) = \int_{t-1}^t (1)(1)\, d\lambda = \lambda\Big|_{t-1}^t = t - (t-1) = 1$$

Therefore,

$$y(t) = \begin{cases} \mathbf{t^2/2,} & \mathbf{0 < t < 1} \\ \mathbf{-(t^2/2) + 2t - 1,} & \mathbf{1 < t < 2} \\ \mathbf{1,} & \mathbf{t > 2} \\ \mathbf{0,} & \mathbf{otherwise} \end{cases}$$

(b) For $t > 0$, the two functions overlap as shown in Fig. (c).

$$y(t) = x(t) * h(t) = \int_0^t (1)\, 2\, e^{-\lambda}\, d\lambda = -2\, e^{-\lambda} \Big|_0^t$$



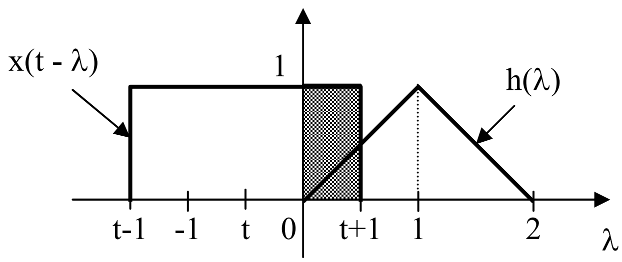
(c)

Therefore,

$$y(t) = \underline{\mathbf{2(1 - e^{-t}), \quad t > 0}}$$

(c) For $-1 < t < 0$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (d).

$$y(t) = x(t) * h(t) = \int_0^{t+1} (1)(\lambda)\, d\lambda = \frac{\lambda^2}{2}\Big|_0^{t+1} = \frac{1}{2}(t+1)^2$$

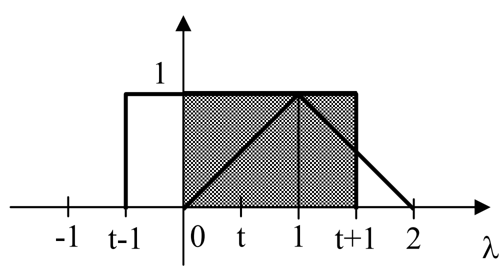


(d)

For $0 < t < 1$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (e).

$$y(t) = \int_0^1 (1)(\lambda)\, d\lambda + \int_1^{t+1} (1)(2-\lambda)\, d\lambda$$

$$y(t) = \frac{\lambda^2}{2}\Big|_0^1 + \left(2\lambda - \frac{\lambda^2}{2}\right)\Big|_1^{t+1} = \frac{-1}{2} t^2 + t + \frac{1}{2}$$

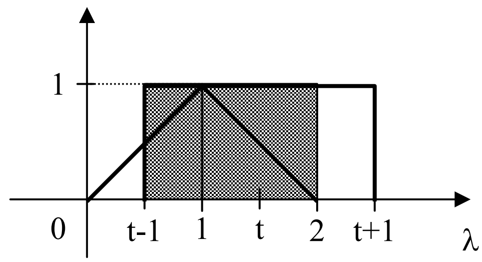


(e)

For $1 < t < 2$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (f).

$$y(t) = \int_{t-1}^{1} (1)(\lambda)\, d\lambda + \int_{1}^{2} (1)(2-\lambda)\, d\lambda$$

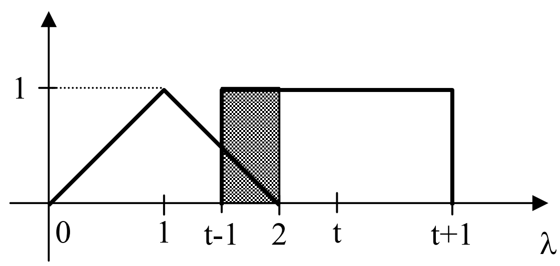
$$y(t) = \frac{\lambda^2}{2}\Big|_{t-1}^{1} + \left(2\lambda - \frac{\lambda^2}{2}\right)\Big|_{1}^{2} = \frac{-1}{2}t^2 + t + \frac{1}{2}$$



(f)

For $2 < t < 3$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (g).

$$y(t) = \int_{t-1}^{2} (1)(2-\lambda)\, d\lambda = \left(2\lambda - \frac{\lambda^2}{2}\right)\Big|_{t-1}^{2} = \frac{9}{2} - 3t + \frac{1}{2}t^2$$



(g)

Therefore,

$$y(t) = \begin{cases} \mathbf{(t^2/2) + t + 1/2}, & \mathbf{-1 < t < 0} \\ \mathbf{-(t^2/2) + t + 1/2}, & \mathbf{0 < t < 2} \\ \mathbf{(t^2/2) - 3t + 9/2}, & \mathbf{2 < t < 3} \\ \mathbf{0}, & \textbf{otherwise} \end{cases}$$

Chapter 15, Problem 44.

Obtain the convolution of the pairs of signals in Fig. 15.38.

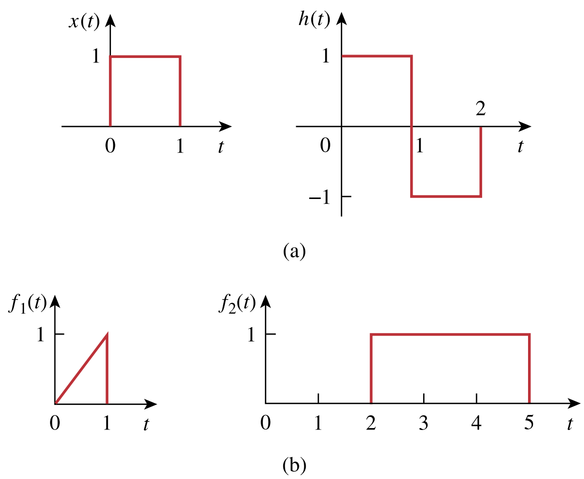


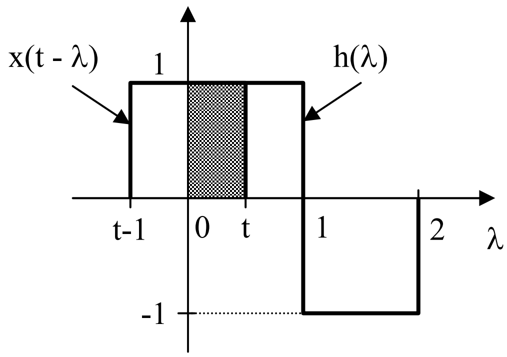
Figure 15.38

For Prob. 15.44.

Chapter 15, Solution 44.

(a) For $0 < t < 1$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (a).

$$y(t) = x(t) * h(t) = \int_0^t (1)(1)\, d\lambda = t$$



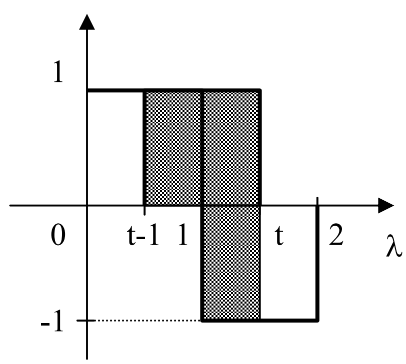
(a)

For $1 < t < 2$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (b).

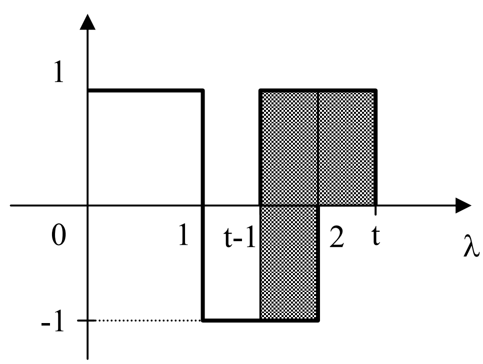
$$y(t) = \int_{t-1}^{1} (1)(1)\, d\lambda + \int_1^t (-1)(1)\, d\lambda = \lambda\Big|_{t-1}^{1} - \lambda\Big|_1^t = 3 - 2t$$

For $2 < t < 3$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (c).

$$y(t) = \int_{t-1}^{2} (1)(-1)\, d\lambda = -\lambda\Big|_{t-1}^{2} = t - 3$$



(b)



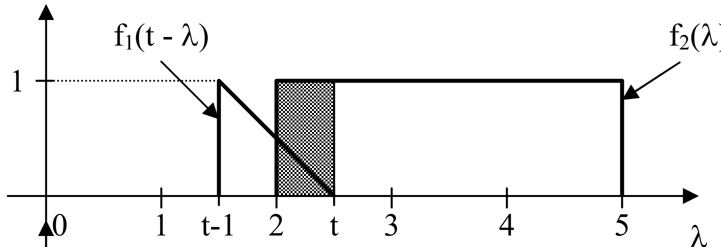
(c)

Therefore,

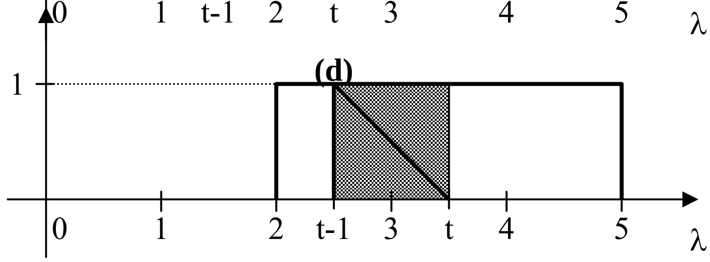
$$y(t) = \begin{cases} \mathbf{t}, & \mathbf{0 < t < 1} \\ \mathbf{3 - 2t}, & \mathbf{1 < t < 2} \\ \mathbf{t - 3}, & \mathbf{2 < t < 3} \\ \mathbf{0}, & \textbf{otherwise} \end{cases}$$

(b) For $t < 2$, there is no overlap. For $2 < t < 3$, $f_1(t-\lambda)$ and $f_2(\lambda)$ overlap, as shown in Fig. (d).

$$y(t) = f_1(t) * f_2(t) = \int_2^t (1)(t-\lambda)\, d\lambda$$



(d)



(e)

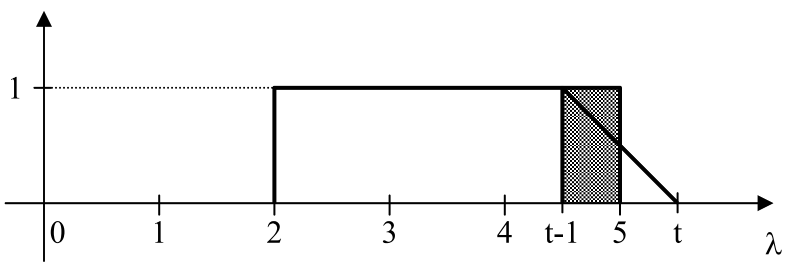
$$= \left(\lambda t - \frac{\lambda^2}{2} \right) \Big|_2^t = \frac{t^2}{2} - 2t + 2$$

For $3 < t < 5$, $f_1(t-\lambda)$ and $f_2(\lambda)$ overlap as shown in Fig. (e).

$$y(t) = \int_{t-1}^t (1)(t-\lambda)\, d\lambda = \left(\lambda t - \frac{\lambda^2}{2} \right) \Big|_{t-1}^t = \frac{1}{2}$$

For $5 < t < 6$, the functions overlap as shown in Fig. (f).

$$y(t) = \int_{t-1}^5 (1)(t-\lambda)\, d\lambda = \left(\lambda t - \frac{\lambda^2}{2} \right) \Big|_{t-1}^5 = \frac{-1}{2} t^2 + 5t - 12$$



(f)

Therefore, $$y(t) = \begin{cases} \mathbf{(t^2/2) - 2t + 2,} & \mathbf{2 < t < 3} \\ \mathbf{1/2,} & \mathbf{3 < t < 5} \\ \mathbf{-(t^2/2) + 5t - 12,} & \mathbf{5 < t < 6} \\ \mathbf{0,} & \mathbf{otherwise} \end{cases}$$

Chapter 15, Problem 45.

Given $h(t) = 4e^{-2t}u(t)$ and $x(t) = \delta(t) - 2e^{-2t}u(t)$, find $y(t) = x(t) * h(t)$.

Chapter 15, Solution 45.

$$y(t) = h(t) * x(t) = \left[4e^{-2t}u(t)\right] * \left[\delta(t) - 2e^{-2t}u(t)\right]$$
$$= 4e^{-2t}u(t) * \delta(t) - 4e^{-2t}u(t) * 2e^{-2t}u(t) = 4e^{-2t}u(t) - 8e^{-2t}\int_0^t e^o d\lambda$$
$$= \underline{4e^{-2t}u(t) - 8te^{-2t}u(t)}$$

Chapter 15, Problem 46.

Given the following functions

$x(t) = 2\delta(t)$, $\quad y(t) = 4u(t)$, $\quad z(t) = e^{-2t}u(t)$,

evaluate the following convolution operations.

(a) $x(t) * y(t)$
(b) $x(t) * z(t)$
(c) $y(t) * z(t)$
(d) $y(t) * [y(t) + z(t)]$

Chapter 15, Solution 46.

(a) $x(t) * y(t) = 2\delta(t) * 4u(t) = \underline{8u(t)}$

(b) $x(t) * z(t) = 2\delta(t) * e^{-2t}u(t) = \underline{2e^{-2t}u(t)}$

(c) $y(t) * z(t) = 4u(t) * e^{-2t}u(t) = 4\int_0^t e^{-2\lambda} d\lambda = \left.\frac{4e^{-2\lambda}}{-2}\right|_0^t = \underline{2(1 - e^{-2t})}$

(d) $y(t) * [y(t) + z(t)] = 4u(t) * [4u(t) + e^{-2t}u(t)] = 4\int [4u(\lambda) + e^{-2\lambda}u(\lambda)] d\lambda$

$$= 4\int_0^t [4 + e^{-2\lambda}] d\lambda = 4[4t + \left.\frac{e^{-2\lambda}}{-2}]\right|_0^t = \underline{16t - 2e^{-2t} + 2}$$

Chapter 15, Problem 47.

A system has the transfer function

$$H(s) = \frac{s}{(s+1)(s+2)}$$

(a) Find the impulse response of the system.
(b) Determine the output $y(t)$, given that the input is $x(t) = u(t)$

Chapter 15, Solution 47.

(a) $H(s) = \dfrac{s}{(s+1)(s+2)} = \dfrac{A}{s+1} + \dfrac{B}{s+2}$

s=A(s+2) + B(s+1)

We equate the coefficients.

s: 1= A+B
constant: 0 =2A +B

Solving these, A = -1, B= 2.

$$H(s) = \frac{-1}{s+1} + \frac{2}{s+2}$$

$$h(t) = \underline{(-e^{-t} + 2e^{-2t})u(t)}$$

(b) $H(s) = \dfrac{Y(s)}{X(s)} \longrightarrow Y(s) = H(s)X(s) = \dfrac{s}{(s+1)(s+2)}\dfrac{1}{s}$

$$Y(s) = \frac{1}{(s+1)(s+2)} = \frac{C}{s+1} + \frac{D}{s+2}$$

C=1 and D=-1 so that

$$Y(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$$y(t) = \underline{(e^{-t} - e^{-2t})u(t)}$$

Chapter 15, Problem 48.

Find $f(t)$ using convolution given that:

(a) $F(s) = \dfrac{4}{\left(s^2 + 2s + 5\right)^2}$

(b) $F(s) = \dfrac{2s}{(s+1)\left(s^2 + 4\right)}$

Chapter 15, Solution 48.

(a) Let $G(s) = \dfrac{2}{s^2 + 2s + 5} = \dfrac{2}{(s+1)^2 + 2^2}$

$$g(t) = e^{-t}\sin(2t)$$

$$F(s) = G(s)\,G(s)$$

$$f(t) = L^{-1}\left[\,G(s)\,G(s)\right] = \int_0^t g(\lambda)\,g(t-\lambda)\,d\lambda$$

$$f(t) = \int_0^t e^{-\lambda}\sin(2\lambda)\,e^{-(t-\lambda)}\sin(2(t-\lambda))\,d\lambda$$

$$\sin(A)\sin(B) = \frac{1}{2}\left[\,\cos(A-B) - \cos(A+B)\right]$$

$$f(t) = \frac{1}{2}e^{-t}\int_0^t e^{-\lambda}\left[\,\cos(2t) - \cos(2(t-2\lambda))\right]d\lambda$$

$$f(t) = \frac{e^{-t}}{2}\cos(2t)\int_0^t e^{-2\lambda}\,d\lambda - \frac{e^{-t}}{2}\int_0^t e^{-2\lambda}\cos(2t-4\lambda)\,d\lambda$$

$$f(t) = \frac{e^{-t}}{2}\cos(2t)\cdot\frac{e^{-2\lambda}}{-2}\Big|_0^t - \frac{e^{-t}}{2}\int_0^t e^{-2\lambda}\left[\,\cos(2t)\cos(4\lambda) + \sin(2t)\sin(4\lambda)\right]d\lambda$$

$$f(t) = \frac{1}{4}e^{-t}\cos(2t)\,(-e^{-2t}+1) - \frac{e^{-t}}{2}\cos(2t)\int_0^t e^{-2\lambda}\cos(4\lambda)\,d\lambda$$
$$-\frac{e^{-t}}{2}\sin(2t)\int_0^t e^{-2\lambda}\sin(4\lambda)\,d\lambda$$

$$f(t) = \frac{1}{4}e^{-t}\cos(2t)\,(1-e^{-2t})$$
$$-\frac{e^{-t}}{2}\cos(2t)\left[\,\frac{e^{-2\lambda}}{4+16}\left(-2\cos(4\lambda) - 4\sin(4\lambda)\right)\right]\Big|_0^t$$
$$-\frac{e^{-t}}{2}\sin(2t)\left[\,\frac{e^{-2\lambda}}{4+16}\left(-2\sin(4\lambda) + 4\cos(4\lambda)\right)\right]\Big|_0^t$$

$$f(t) = \underline{\mathbf{\frac{e^{-t}}{2}\cos(2t) - \frac{e^{-3t}}{4}\cos(2t) - \frac{e^{-t}}{20}\cos(2t) + \frac{e^{-3t}}{20}\cos(2t)\cos(4t)}}$$
$$\underline{\mathbf{+\frac{e^{-3t}}{10}\cos(2t)\sin(4t) + \frac{e^{-t}}{10}\sin(2t)}}$$
$$\underline{\mathbf{+\frac{e^{-t}}{20}\sin(2t)\sin(4t) - \frac{e^{-t}}{10}\sin(2t)\cos(4t)}}$$

(b) Let $X(s) = \dfrac{2}{s+1}$, $\quad Y(s) = \dfrac{s}{s+4}$

$x(t) = 2\,e^{-t}\,u(t)$, $\quad y(t) = \cos(2t)\,u(t)$

$F(s) = X(s)\,Y(s)$

$$f(t) = L^{-1}\left[\,X(s)\,Y(s)\right] = \int_0^{\infty} y(\lambda)\,x(t-\lambda)\,d\lambda$$

$$f(t) = \int_0^{t} \cos(2\lambda)\cdot 2\,e^{-(t-\lambda)}\,d\lambda$$

$$f(t) = 2\,e^{-t}\cdot\frac{e^{\lambda}}{1+4}\left(\cos(2\lambda) + 2\sin(2\lambda)\right)\Big|_0^t$$

$$f(t) = \frac{2}{5}e^{-t}\left[\,e^{t}\left(\cos(2t) + 2\sin(2t) - 1\right)\right]$$

$$f(t) = \underline{\mathbf{\frac{2}{5}\cos(2t) + \frac{4}{5}\sin(2t) - \frac{2}{5}e^{-t}}}$$

Chapter 15, Problem 49.

* Use the convolution integral to find:

(a) $t * e^{at} u(t)$

(b) $\cos(t) * \cos(t) u(t)$

* An asterisk indicates a challenging problem.

Chapter 15, Solution 49.

(a) $t*e^{\alpha t}u(t) =$

$$\int_0^t e^{a\lambda}(t-\lambda)d\lambda = t\frac{e^{a\lambda}}{a}\Bigg|_0^t - \frac{e^{a\lambda}}{a^2}(a\lambda - 1)\Bigg|_0^t = \underline{\frac{t}{a}(e^{at}-1) - \frac{1}{a^2} - \frac{e^{at}}{a^2}(at-1)}$$

(b) $$\cos t * \cos t u(t) = \int_0^t \cos\lambda \cos(t-\lambda)d\lambda = \int_0^t \{\cos t \cos\lambda \cos\lambda + \sin t \sin\lambda \cos\lambda\}d\lambda$$

$$= \left[\cos t \int_0^t \frac{1}{2}[1+\cos 2\lambda]d\lambda + \sin t \int_0^t \cos\lambda d(-\cos\lambda)\right] = \left[\frac{1}{2}\cos t[\lambda + \frac{\sin 2\lambda}{2}]\Bigg|_0^t - \sin t \frac{\cos\lambda}{2}\Bigg|_0^t\right]$$

= **<u>0.5cos(t)(t+0.5sin(2t)) – 0.5sin(t)(cos(t) – 1)</u>**.

Chapter 15, Problem 50.

Use the Laplace transform to solve the differential equation

$$\frac{d^2v(t)}{dt^2} + 2\frac{dv(t)}{dt} + 10v(t) = 3\cos 2t$$

subject to $v(0) = 1, dv(0)/dt = -2$.

Chapter 15, Solution 50.

Take the Laplace transform of each term.

$$\left[\,s^2\,V(s) - s\,v(0) - v'(0)\right] + 2\left[\,s\,V(s) - v(0)\right] + 10\,V(s) = \frac{3s}{s^2+4}$$

$$s^2\,V(s) - s + 2 + 2s\,V(s) - 2 + 10\,V(s) = \frac{3s}{s^2+4}$$

$$(s^2 + 2s + 10)\,V(s) = s + \frac{3s}{s^2+4} = \frac{s^3+7s}{s^2+4}$$

$$V(s) = \frac{s^3+7s}{(s^2+4)(s^2+2s+10)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+2s+10}$$

$$s^3 + 7s = A\,(s^3 + 2s^2 + 10s) + B\,(s^2 + 2s + 10) + C\,(s^3 + 4s) + D\,(s^2 + 4)$$

Equating coefficients :

$$s^3: \quad 1 = A + C \longrightarrow C = 1 - A$$

$$s^2: \quad 0 = 2A + B + D$$

$$s^1: \quad 7 = 10A + 2B + 4C = 6A + 2B + 4$$

$$s^0: \quad 0 = 10B + 4D \longrightarrow D = \text{-}2.5B$$

Solving these equations yields

$$A = \frac{9}{26}, \qquad B = \frac{12}{26}, \qquad C = \frac{17}{26}, \qquad D = \frac{-30}{26}$$

$$V(s) = \frac{1}{26}\left[\frac{9s+12}{s^2+4} + \frac{17s-30}{s^2+2s+10}\right]$$

$$V(s) = \frac{1}{26}\left[\frac{9s}{s^2+4} + 6\cdot\frac{2}{s^2+4} + 17\cdot\frac{s+1}{(s+1)^2+3^2} - \frac{47}{(s+1)^2+3^2}\right]$$

$$v(t) = \underline{\mathbf{\frac{9}{26}\cos(2t) + \frac{6}{26}\sin(2t) + \frac{17}{26}e^{-t}\cos(3t) - \frac{47}{78}e^{-t}\sin(3t)}}$$

Chapter 15, Problem 51.

Given that $v(0) = 2$ and $dv(0)/dt = 4$, solve

$$\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 6v = 10e^{-t}u(t)$$

Chapter 15, Solution 51.

Taking the Laplace transform of the differential equation yields

$$\left[s^2V(s) - sv(0) - v'(0)\right] + 5\left[sV(s) - v(0)\right] + 6V(s) = \frac{10}{s+1}$$

or $\left(s^2 + 5s + 6\right)V(s) - 2s - 4 - 10 = \dfrac{10}{s+1} \quad \longrightarrow \quad V(s) = \dfrac{2s^2 + 16s + 24}{(s+1)(s+2)(s+3)}$

Let $V(s) = \dfrac{A}{s+1} + \dfrac{B}{s+2} + \dfrac{C}{s+3}$, $\quad A = 5, \quad B = 0, \quad C = -3$

Hence,

$$v(t) = \underline{(5e^{-t} - 3e^{-3t})u(t)}$$

Chapter 15, Problem 52.

Use the Laplace transform to find $i(t)$ for $t > 0$ if

$$\frac{d^2 i}{dt^2} + 3\frac{di}{dt} + 2i + \delta(t) = 0,$$

$$i(0) = 0, \qquad i'(0) = 3$$

Chapter 15, Solution 52.

Take the Laplace transform of each term.

$$\left[s^2\, I(s) - s\, i(0) - i'(0) \right] + 3\left[s\, I(s) - i(0) \right] + 2\, I(s) + 1 = 0$$

$$(s^2 + 3s + 2)\, I(s) - s - 3 - 3 + 1 = 0$$

$$I(s) = \frac{s+5}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = 4, \qquad B = -3$$

$$I(s) = \frac{4}{s+1} - \frac{3}{s+2}$$

$$i(t) = \underline{\mathbf{(4e^{-t} - 3e^{-2t})u(t)}}$$

Chapter 15, Problem 53.

* Use Laplace transforms to solve for $x(t)$ in

$$x(t) = \cos t + \int_0^t e^{\lambda - t} x(\lambda)\, d\lambda$$

* An asterisk indicates a challenging problem.

Chapter 15, Solution 53.

Transform each term.

We begin by noting that the integral term can be rewritten as,

$\int_0^t x(\lambda)e^{-(t-\lambda)}d\lambda$ which is convolution and can be written as $e^{-t}*x(t)$.

Now, transforming each term produces,

$$X(s) = \frac{s}{s^2+1} + \frac{1}{s+1}X(s) \to \left(\frac{s+1-1}{s+1}\right)X(s) = \frac{s}{s^2+1}$$

$$X(s) = \frac{s+1}{s^2+1} = \frac{s}{s^2+1} + \frac{1}{s^2+1}$$

x(t) = **<u>cos(t) + sin(t)</u>**.

If partial fraction expansion is used we obtain,

x(t) = **<u>1.4141cos(t–45˚)</u>**.

This is the same answer and can be proven by using trigonometric identities.

Chapter 15, Problem 54.

Using the Laplace transform, solve the following differential equation for

$$\frac{d^2 i}{dt^2} + 4\frac{di}{dt} + 5i = 2e^{-2t}$$

Subject to $i(0) = 0, i'(0) = 2$.

Chapter 15, Solution 54.

Taking the Laplace transform of each term gives

$$\left[s^2 I(s) - si(0) - i'(0)\right] + 4\left[sI(s) - i(0)\right] + 5I(s) = \frac{2}{s+2}$$

$$\left[s^2 I(s) - 0 - 2\right] + 4\left[sI(s) - 0\right] + 5I(s) = \frac{2}{s+2}$$

$$I(s)(s^2 + 4s + 5) = \frac{2}{s+2} + 2 = \frac{2s+6}{s+2}$$

$$I(s) = \frac{2s+6}{(s+2)(s^2+4s+5)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+4s+5}$$

$$2s + 6 = A(s^2 + 4s + 5) + B(s^2 + 2s) + C(s+2)$$

We equate the coefficients.

s^2 : 0 = A+ B
s: 2= 4A + 2B + C
constant: 6 = 5A + 2C

Solving these gives
A = 2, B= -2, C = -2

$$I(s) = \frac{2}{s+2} - \frac{2s+2}{s^2+4s+5} = \frac{2}{s+2} - \frac{2(s+2)}{(s+2)^2+1} + \frac{2}{(s+2)^2+1}$$

Taking the inverse Laplace transform leads to:

$$i(t) = \left(2e^{-2t} - 2e^{-2t}\cos t + 2e^{-2t}\sin t\right)u(t) = \underline{2e^{-2t}(1 - \cos t + \sin t)u(t)}$$

Chapter 15, Problem 55.

Solve for $y(t)$ in the following differential equation if the initial conditions are zero.

$$\frac{d^3y}{dt^3}+6\frac{d^2y}{dt^2}+8\frac{dy}{dt}e^{-t}\cos 2t$$

Chapter 15, Solution 55.

Take the Laplace transform of each term.

$$\left[s^3\,Y(s)-s^2\,y(0)-s\,y'(0)-y''(0)\right]+6\left[s^2\,Y(s)-s\,y(0)-y'(0)\right]$$
$$+8\left[s\,Y(s)-y(0)\right]=\frac{s+1}{(s+1)^2+2^2}$$

Setting the initial conditions to zero gives

$$(s^3+6s^2+8s)\,Y(s)=\frac{s+1}{s^2+2s+5}$$

$$Y(s)=\frac{(s+1)}{s(s+2)(s+4)(s^2+2s+5)}=\frac{A}{s}+\frac{B}{s+2}+\frac{C}{s+4}+\frac{Ds+E}{s^2+2s+5}$$

$$A=\frac{1}{40},\qquad B=\frac{1}{20},\qquad C=\frac{-3}{104},\qquad D=\frac{-3}{65},\qquad E=\frac{-7}{65}$$

$$Y(s)=\frac{1}{40}\cdot\frac{1}{s}+\frac{1}{20}\cdot\frac{1}{s+2}-\frac{3}{104}\cdot\frac{1}{s+4}-\frac{1}{65}\cdot\frac{3s+7}{(s+1)^2+2^2}$$

$$Y(s)=\frac{1}{40}\cdot\frac{1}{s}+\frac{1}{20}\cdot\frac{1}{s+2}-\frac{3}{104}\cdot\frac{1}{s+4}-\frac{1}{65}\cdot\frac{3(s+1)}{(s+1)^2+2^2}-\frac{1}{65}\cdot\frac{4}{(s+1)^2+2^2}$$

$$y(t)=\underline{\boldsymbol{\frac{1}{40}u(t)+\frac{1}{20}e^{-2t}-\frac{3}{104}e^{-4t}-\frac{3}{65}e^{-t}\cos(2t)-\frac{2}{65}e^{-t}\sin(2t)}}$$

Chapter 15, Problem 56.

Solve for $v(t)$ in the integrodifferential equation

$$4\frac{dv}{dt} + 12\int_{-\infty}^{t} v\,dt = 0$$

Given that $v(0) = 2$.

Chapter 15, Solution 56.

Taking the Laplace transform of each term we get:

$$4\left[\,s\,V(s) - v(0)\right] + \frac{12}{s}V(s) = 0$$

$$\left[\,4s + \frac{12}{s}\right]V(s) = 8$$

$$V(s) = \frac{8s}{4s^2 + 12} = \frac{2s}{s^2 + 3}$$

$$v(t) = \underline{\mathbf{2\cos\left(\sqrt{3}t\right)}}$$

Chapter 15, Problem 57.

Solve the following integrodifferential equation using the Laplace transform method:

$$\frac{dy(t)}{dt} + 9\int_0^t y(\tau)d\tau = \cos\ 2t, \qquad y(0) = 1$$

Chapter 15, Solution 57.

Take the Laplace transform of each term.

$$\left[\, s\,Y(s) - y(0)\right] + \frac{9}{s}Y(s) = \frac{s}{s^2+4}$$

$$\left(\frac{s^2+9}{s}\right)Y(s) = 1 + \frac{s}{s^2+4} = \frac{s^2+s+4}{s^2+4}$$

$$Y(s) = \frac{s^3+s^2+4s}{(s^2+4)(s^2+9)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+9}$$

$$s^3+s^2+4s = A\,(s^3+9s) + B\,(s^2+9) + C\,(s^3+4s) + D\,(s^2+4)$$

Equating coefficients :

s^0 : $\quad 0 = 9B + 4D$

s^1 : $\quad 4 = 9A + 4C$

s^2 : $\quad 1 = B + D$

s^3 : $\quad 1 = A + C$

Solving these equations gives

$$A = 0, \qquad B = -4/5, \qquad C = 1, \qquad D = 9/5$$

$$Y(s) = \frac{-4/5}{s^2+4} + \frac{s+9/5}{s^2+9} = \frac{-4/5}{s^2+4} + \frac{s}{s^2+9} + \frac{9/5}{s^2+9}$$

$$y(t) = \underline{\mathbf{-0.4\sin(2t) + \cos(3t) + 0.6\sin(3t)}}$$

Chapter 15, Problem 58.

Given that

$$\frac{dv}{dt} + 2v + 5\int_0^t v(\lambda)d\lambda = 4u(t)$$

with $v(0) = -1$, determine $v(t)$ for $t > 0$.

Chapter 15, Solution 58.

We take the Laplace transform of each term.

$$[sV(s) - v(0)] + 2V(s) + \frac{5}{s}V(s) = \frac{4}{s}$$

$$[sV(s) + 1] + 2V(s) + \frac{5}{s}V(s) = \frac{4}{s} \qquad \longrightarrow \qquad V(s) = \frac{4-s}{s^2 + 2s + 5}$$

$$V(s) = \frac{-(s+1)+5}{(s+1)^2 + 2^2} = \frac{-(s+1)}{(s+1)^2 + 2^2} + \frac{5}{2}\frac{2}{(s+1)^2 + 2^2}$$

$$v(t) = \underline{(-e^{-t}\cos 2t + 2.5e^{-t}\sin 2t)u(t)}$$

Chapter 15, Problem 59.

Solve the integrodifferential equation

$$\frac{dy}{dt}+4y+3\int_0^t y\,dt=6e^{-2t}, \qquad y(0)=-1$$

Chapter 15, Solution 59.

Take the Laplace transform of each term of the integrodifferential equation.

$$\left[\,s\,Y(s)-y(0)\right]+4\,Y(s)+\frac{3}{s}Y(s)=\frac{6}{s+2}$$

$$(s^2+4s+3)\,Y(s)=s\left(\frac{6}{s+2}-1\right)$$

$$Y(s)=\frac{s\,(4-s)}{(s+2)(s^2+4s+3)}=\frac{(4-s)\,s}{(s+1)(s+2)(s+3)}$$

$$Y(s)=\frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{s+3}$$

A = –2.5, B = 12, C = -10.5

$$Y(s)=\frac{-2.5}{s+1}+\frac{12}{s+2}-\frac{10.5}{s+3}$$

$$y(t)=\underline{\mathbf{-2.5e^{-t}+12e^{-2t}-10.5e^{-3t}}}$$

Chapter 15, Problem 60.

Solve the following integrodifferential equation

$$2\frac{dx}{dt} + 5x + 3\int_0^t x\, dt + 4 = \sin\, 4t\,, \qquad x(0) = 1$$

Chapter 15, Solution 60.

Take the Laplace transform of each term of the integrodifferential equation.

$$2\left[\,\mathrm{s\,X(s) - x(0)}\right] + 5\,\mathrm{X(s)} + \frac{3}{\mathrm{s}}\mathrm{X(s)} + \frac{4}{\mathrm{s}} = \frac{4}{\mathrm{s}^2 + 16}$$

$$(2\mathrm{s}^2 + 5\mathrm{s} + 3)\,\mathrm{X(s)} = 2\mathrm{s} - 4 + \frac{4\mathrm{s}}{\mathrm{s}^2 + 16} = \frac{2\mathrm{s}^3 - 4\mathrm{s}^2 + 36\mathrm{s} - 64}{\mathrm{s}^2 + 16}$$

$$\mathrm{X(s)} = \frac{2\mathrm{s}^3 - 4\mathrm{s}^2 + 36\mathrm{s} - 64}{(2\mathrm{s}^2 + 5\mathrm{s} + 3)(\mathrm{s}^2 + 16)} = \frac{\mathrm{s}^3 - 2\mathrm{s}^2 + 18\mathrm{s} - 32}{(\mathrm{s}+1)(\mathrm{s}+1.5)(\mathrm{s}^2 + 16)}$$

$$\mathrm{X(s)} = \frac{\mathrm{A}}{\mathrm{s}+1} + \frac{\mathrm{B}}{\mathrm{s}+1.5} + \frac{\mathrm{Cs} + \mathrm{D}}{\mathrm{s}^2 + 16}$$

$$\mathrm{A} = (\mathrm{s}+1)\,\mathrm{X(s)}\Big|_{\mathrm{s=-1}} = \text{-}6.235$$

$$\mathrm{B} = (\mathrm{s}+1.5)\,\mathrm{X(s)}\Big|_{\mathrm{s=-1.5}} = 7.329$$

When $\mathrm{s} = 0$,

$$\frac{-32}{(1.5)(16)} = \mathrm{A} + \frac{\mathrm{B}}{1.5} + \frac{\mathrm{D}}{16} \longrightarrow \mathrm{D} = 0.2579$$

$$\mathrm{s}^3 - 2\mathrm{s}^2 + 18\mathrm{s} - 32 = \mathrm{A}\,(\mathrm{s}^3 + 1.5\mathrm{s}^2 + 16\mathrm{s} + 24) + \mathrm{B}\,(\mathrm{s}^3 + \mathrm{s}^2 + 16\mathrm{s} + 16)$$
$$+\,\mathrm{C}\,(\mathrm{s}^3 + 2.5\mathrm{s}^2 + 1.5\mathrm{s}) + \mathrm{D}\,(\mathrm{s}^2 + 2.5\mathrm{s} + 1.5)$$

Equating coefficients of the s^3 terms,

$$1 = \mathrm{A} + \mathrm{B} + \mathrm{C} \longrightarrow \mathrm{C} = \text{-}0.0935$$

$$\mathrm{X(s)} = \frac{-6.235}{\mathrm{s}+1} + \frac{7.329}{\mathrm{s}+1.5} + \frac{-0.0935\mathrm{s} + 0.2579}{\mathrm{s}^2 + 16}$$

$$\mathrm{x(t)} = \underline{\mathbf{-\,6.235\,e^{-t} + 7.329\,e^{-1.5t} - 0.0935\cos(4t) + 0.0645\sin(4t)}}$$