Chapter 13, Problem 1.

For the three coupled coils in Fig. 13.72, calculate the total inductance.

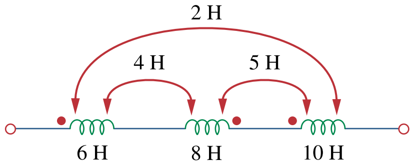


Figure 13.72
For Prob. 13.1.

Chapter 13, Solution 1.

For coil 1, $L_1 - M_{12} + M_{13} = 6 - 4 + 2 = 4$

For coil 2, $L_2 - M_{21} - M_{23} = 8 - 4 - 5 = -1$

For coil 3, $L_3 + M_{31} - M_{32} = 10 + 2 - 5 = 7$

$$L_T = 4 - 1 + 7 = 10\text{H}$$

or $$L_T = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} + 2M_{12}$$

$L_T = 6 + 8 + 10 =$ **<u>10H</u>**

Chapter 13, Problem 2.

Determine the inductance of the three series-connected inductors of Fig. 13.73.

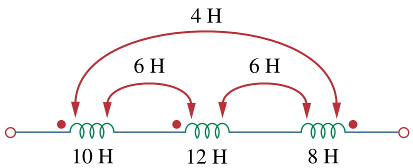


Figure 13.73
For Prob. 13.2.

Chapter 13, Solution 2.

$L = L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} - 2M_{31}$

$= 10 + 12 + 8 + 2\text{x}6 - 2\text{x}6 - 2\text{x}4$

$=$ **<u>22H</u>**

Chapter 13, Problem 3.

Two coils connected in series-aiding fashion have a total inductance of 250 mH. When connected in a series-opposing configuration, the coils have a total inductance of 150 mH. If the inductance of one coil (L_1) is three times the other, find L_1, L_2, and M. What is the coupling coefficient?

Chapter 13, Solution 3.

$L_1 + L_2 + 2M = 250$ mH (1)

$L_1 + L_2 - 2M = 150$ mH (2)

Adding (1) and (2),

$$2L_1 + 2L_2 = 400 \text{ mH}$$

But, $L_1 = 3L_{2,}$, or $8L_2 + 400$, and $L_2 = \underline{\mathbf{50\ mH}}$

$$L_1 = 3L_2 = \underline{\mathbf{150\ mH}}$$

From (2), $150 + 50 - 2M = 150$ leads to $M = \underline{\mathbf{25\ mH}}$

$$k = M/\sqrt{L_1 L_2} = 25/\sqrt{50x150} = \underline{\mathbf{0.2887}}$$

Chapter 13, Problem 4.

(a) For the coupled coils in Fig. 13.74(a), show that

$$L_{eq} = L_1 + L_2 + 2M$$

(b) For the coupled coils in Fig. 13.74(b), show that

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

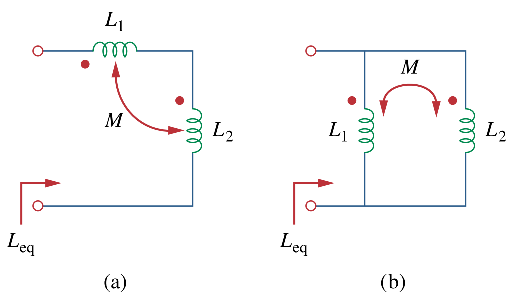
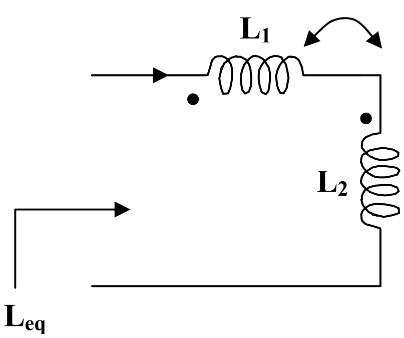


Figure 13.74
For Prob. 13.4.

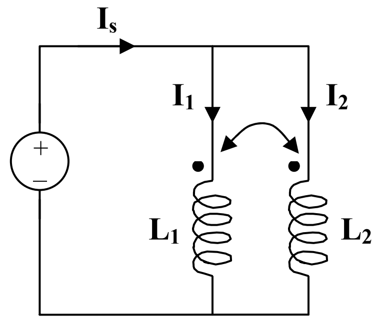
Chapter 13, Solution 4.

(a) For the series connection shown in Figure (a), the current I enters each coil from its dotted terminal. Therefore, the mutually induced voltages have the same sign as the self-induced voltages. Thus,

$$L_{eq} = \underline{\mathbf{L_1 + L_2 + 2M}}$$



(a)



(b)

(b) For the parallel coil, consider Figure (b).

$$I_s = I_1 + I_2 \quad \text{and} \quad Z_{eq} = V_s/I_s$$

Applying KVL to each branch gives,

$$V_s = j\omega L_1 I_1 + j\omega M I_2 \qquad (1)$$

$$V_s = j\omega M I_1 + j\omega\, L_2 I_2 \qquad (2)$$

or

$$\begin{bmatrix} V_s \\ V_s \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = -\omega^2 L_1 L_2 + \omega^2 M^2, \quad \Delta_1 = j\omega V_s(L_2 - M), \quad \Delta_2 = j\omega V_s(L_1 - M)$$

$$I_1 = \Delta_1/\Delta, \quad \text{and} \quad I_2 = \Delta_2/\Delta$$

$$I_s = I_1 + I_2 = (\Delta_1 + \Delta_2)/\Delta = j\omega(L_1 + L_2 - 2M)V_s/(-\omega^2(L_1L_2 - M^2))$$
$$= (L_1 + L_2 - 2M)V_s/(j\omega(L_1L_2 - M^2))$$

$$Z_{eq} = V_s/I_s = j\omega(L_1L_2 - M^2)/(L_1 + L_2 - 2M) = j\omega L_{eq}$$

i.e., $$L_{eq} = \underline{\mathbf{(L_1L_2 - M^2)/(L_1 + L_2 - 2M)}}$$

Chapter 13, Problem 5.

Two coils are mutually coupled, with L_1 = 25 mH, L_2 = 60 mH, and k = 0.5. Calculate the maximum possible equivalent inductance if:

(a) the two coils are connected in series

(b) the coils are connected in parallel

Chapter 13, Solution 5.

(a) If the coils are connected in series,

$$L = L_1 + L_2 + 2M = 25 + 60 + 2(0.5)\sqrt{25x60} = \underline{\mathbf{123.7\ mH}}$$

(b) If they are connected in parallel,

$$L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{25x60 - 19.36^2}{25 + 60 - 2x19.36}\ mH = \underline{\mathbf{24.31\ mH}}$$

Chapter 13, Problem 6.

The coils in Fig. 13.75 have $L_1 = 40$ mH, $L_2 = 5$ mH, and coupling coefficient $k = 0.6$. Find $i_1(t)$ and $v_2(t)$, given that $v_1(t) = 10 \cos \omega t$ and $i_2(t) = 2 \sin \omega t$, $\omega = 2000$ rad/s.

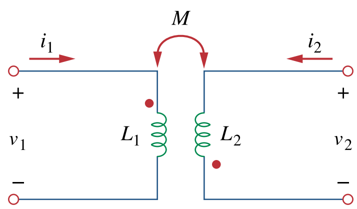


Figure 13.75
For Prob. 13.6.

Chapter 13, Solution 6.

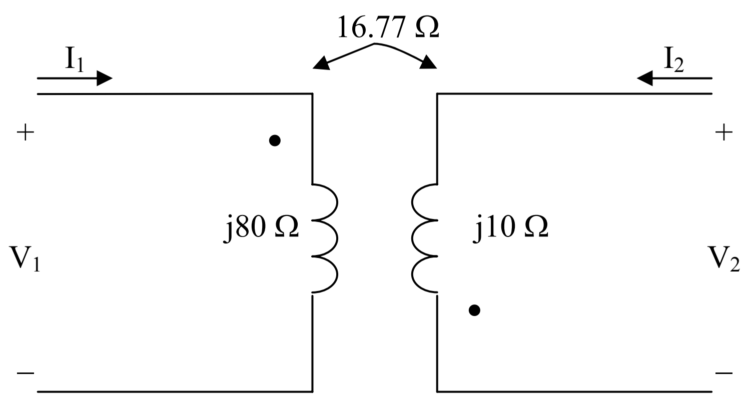
$M = k\sqrt{L_1 L_2} = 0.6\sqrt{40x5} = 8.4853 \text{ mH}$

$40mH \longrightarrow j\omega L = j2000x40x10^{-3} = j80$

$5mH \longrightarrow j\omega L = j2000x5x10^{-3} = j10$

$8.4853mH \longrightarrow j\omega M = j2000x8.4853x10^{-3} = j16.97$

We analyze the circuit below.



$$V_1 = j80I_1 - j16.97I_2 \qquad (1)$$
$$V_2 = -16.97I_1 + j10I_2 \qquad (2)$$

But $V_1 = 10 < 0^o$ and $I_2 = 2 < -90^o = -j2$. Substituting these in eq.(1) gives

$$I_1 = \frac{V_1 + j16.97I_2}{j80} = \frac{10 + j16.97x(-j2)}{j80} = 0.5493 < -90^o$$

$i_1(t) = \underline{0.5493 \sin \omega t \text{ A}}$

From (2),

$$V_2 = -16.97x(-0.j5493) + j10x(-j2) = 20 + j9.3216 = 22.0656 < 24.99^o$$

$v_2(t) = \underline{22.065 \cos(\omega t + 25^o) \text{ V}}$

Chapter 13, Problem 7.

For the circuit in Fig. 13.76, find V_o.

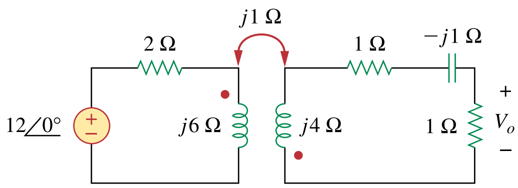
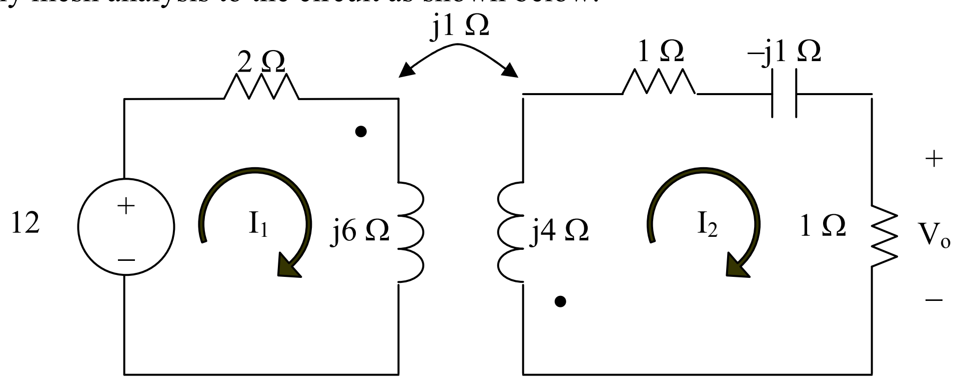


Figure 13.76
For Prob. 13.7.

Chapter 13, Solution 7.

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$12 = I_1(2 + j6) + jI_2 \qquad (1)$$

For mesh 2,

$$0 = jI_1 + (2 - j1 + j4)I_2$$

or

$$0 = jI_1 + (2 + j3)I_2 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 12 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 + j6 & j \\ j & 2 + j3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$I_2 = -0.4381 + j0.3164$$

$V_o = I_2 x1 =$ **$\underline{540.5\angle 144.16^\circ \text{ mV}}$**.

Chapter 13, Problem 8.

Find $v(t)$ for the circuit in Fig. 13.77.

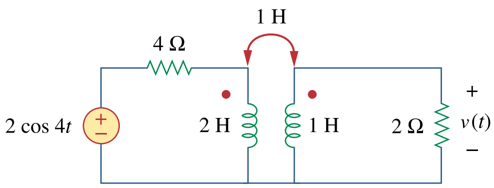


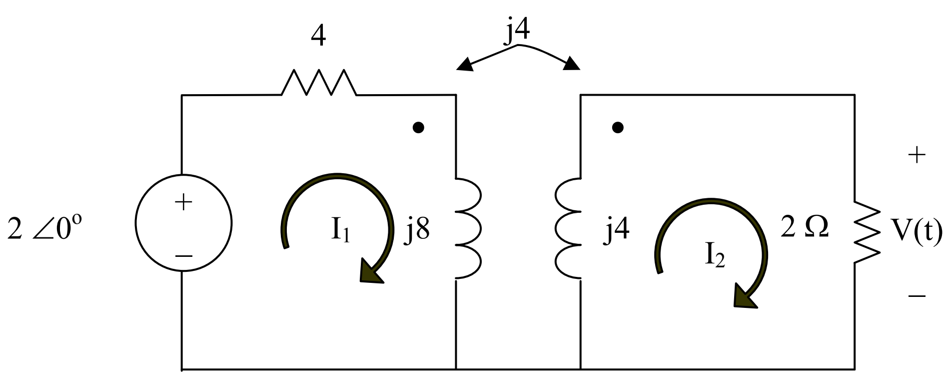
Figure 13.77
For Prob. 13.8.

Chapter 13, Solution 8.

$2H \longrightarrow \quad j\omega L = j4x2 = j8$

$1H \longrightarrow \quad j\omega L = j4x1 = j4$

Consider the circuit below.



$$2 = (4 + j8)I_1 - j4I_2 \qquad (1)$$
$$0 = -j4I_1 + (2 + j4)I_2 \qquad (2)$$

In matrix form, these equations become

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 + j8 & -j4 \\ -j4 & 2 + j4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this leads to

$I_2 = 0.2353 - j0.0588$

$V = 2I_2 = 0.4851 <-14.04^o$

Thus,

$$v(t) = \underline{0.4851\cos(4t - 14.04^o)\text{ V}}$$

Chapter 13, Problem 9.

Find $\mathbf{V}_x$ in the network shown in Fig. 13.78.

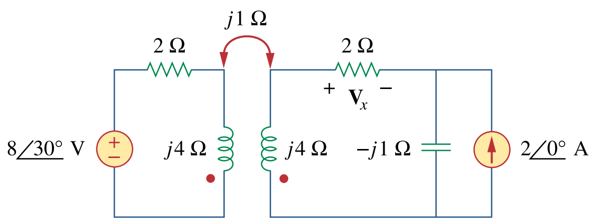
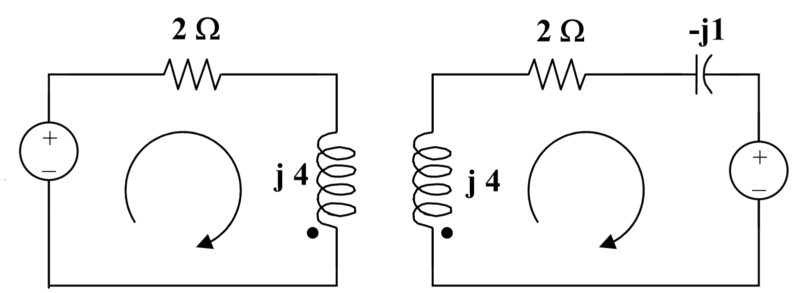


Figure 13.78
For Prob. 13.9.

Chapter 13, Solution 9.

Consider the circuit below.



For loop 1,

$$8\angle 30° = (2 + j4)I_1 - jI_2 \qquad (1)$$

For loop 2,

$$((j4 + 2 - j)I_2 - jI_1 + (-j2) = 0$$

or

$$I_1 = (3 - j2)i_2 - 2 \qquad (2)$$

Substituting (2) into (1),

$$8\angle 30° + (2 + j4)2 = (14 + j7)I_2$$

$$I_2 = (10.928 + j12)/(14 + j7) = 1.037\angle 21.12°$$

$$V_x = 2I_2 = \underline{\mathbf{2.074\angle 21.12°}}$$

Chapter 13, Problem 10.

Find v_o in the circuit of Fig. 13.79.

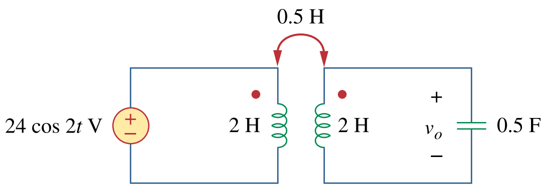
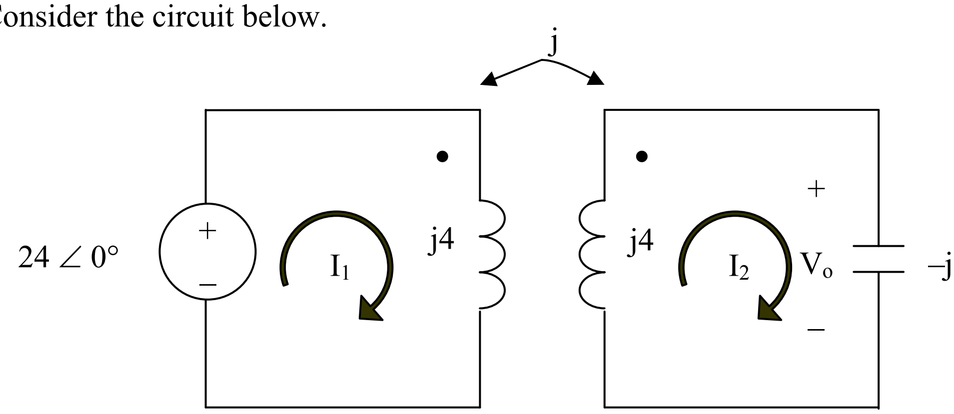


Figure 13.79
For Prob. 13.10.

Chapter 13, Solution 10.

$$2H \longrightarrow j\omega L = j2x2 = j4$$
$$0.5H \longrightarrow j\omega L = j2x0.5 = j$$
$$\frac{1}{2}F \longrightarrow {}^{1}\!/_{j\omega C} = \frac{1}{j2x1/2} = -j$$

Consider the circuit below.



$$24 = j4I_1 - jI_2 \qquad (1)$$
$$0 = -jI_1 + (j4 - j)I_2 \longrightarrow 0 = -I_1 + 3I_2 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 24 \\ 0 \end{bmatrix} = \begin{bmatrix} j4 & -j \\ -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this,

$$I_2 = -j2.1818, \qquad V_o = -jI_2 = -2.1818$$

v_o = **<u>–2.1818cos2t V</u>**

Chapter 13, Problem 11.

ML Use mesh analysis to find i_x in Fig. 13.80, where
$i_s = 4\cos(600t)$ A and $v_s = 110\cos(600t + 30°)$

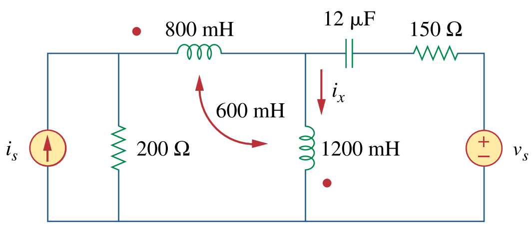


Figure 13.80
For Prob. 13.11.

Chapter 13, Solution 11.

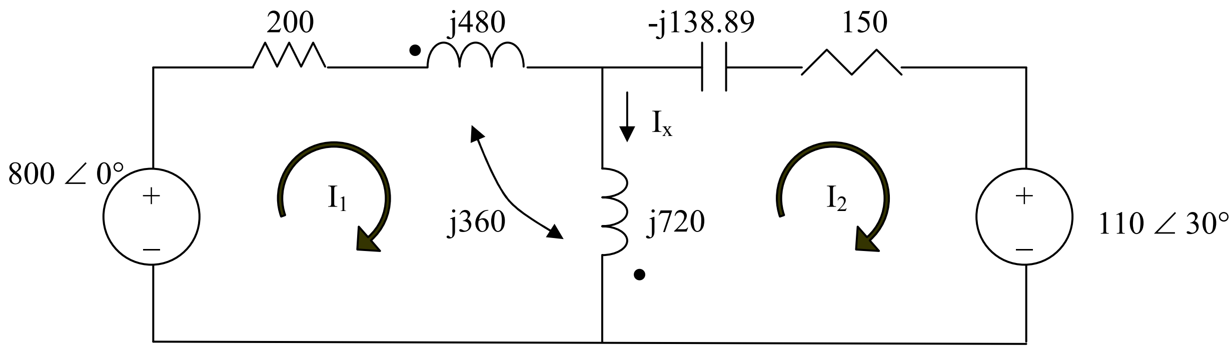
$$800mH \longrightarrow j\omega L = j600x800x10^{-3} = j480$$

$$600mH \longrightarrow j\omega L = j600x600x10^{-3} = j360$$

$$1200mH \longrightarrow j\omega L = j600x1200x10^{-3} = j720$$

$$12\mu F \rightarrow \frac{1}{j\omega C} = \frac{-j}{600x12x10^{-6}} = -j138.89$$

After transforming the current source to a voltage source, we get the circuit shown below.



For mesh 1,

$$800 = (200 + j480 + j720)I_1 + j360I_2 - j720I_2$$

or

$$800 = (200 + j1200)I_1 - j360I_2 \qquad (1)$$

For mesh 2,

$$110\angle 30^\circ + 150–j138.89+j720)I_2 + j360I_1 = 0$$

or

$$-95.2628 - j55 = -j360I_1 + (150 + j581.1)I_2 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 800 \\ -95.2628 - j55 \end{bmatrix} = \begin{bmatrix} 200 + j1200 & -j360 \\ -j360 & 150 + j581.1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this using MATLAB leads to:

```
>> Z = [(200+1200i),-360i;-360i,(150+581.1i)]
Z =
  1.0e+003 *
   0.2000 + 1.2000i        0 - 0.3600i
        0 - 0.3600i   0.1500 + 0.5811i
>> V = [800;(-95.26-55i)]
V =
  1.0e+002 *
   8.0000
  -0.9526 - 0.5500i
>> I = inv(Z)*V
I =
   0.1390 - 0.7242i
   0.0609 - 0.2690i
```

$I_x = I_1 – I_2 = 0.0781 – j0.4552 = 0.4619\angle –80.26^\circ$.

Hence, i_x = **461.9cos(600t–80.26˚) mA**.

Chapter 13, Problem 12.

Determine the equivalent L_{eq} in the circuit of Fig. 13.81.

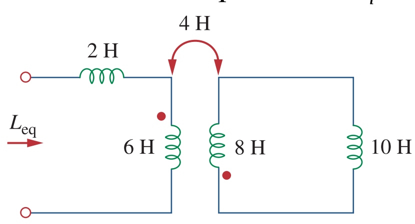
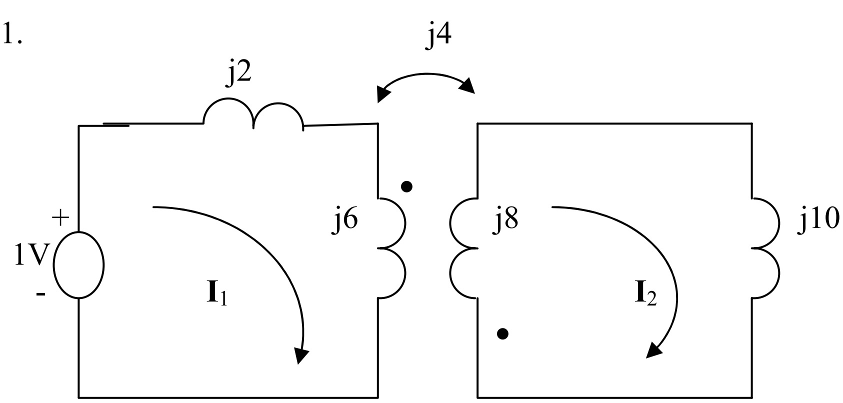


Figure 13.81
For Prob. 13.12.

Chapter 13, Solution 12.

Let $\omega = 1$.



Applying KVL to the loops,

$$1 = j8I_1 + j4I_2 \qquad (1)$$
$$0 = j4I_1 + j18I_2 \qquad (2)$$

Solving (1) and (2) gives $\mathbf{I}_1$ = -j0.1406. Thus

$$Z = \frac{1}{I_1} = jL_{eq} \quad \longrightarrow \quad L_{eq} = \frac{1}{jI_1} = \underline{7.111\ \text{H}}$$

We can also use the equivalent T-section for the transform to find the equivalent inductance.

Chapter 13, Problem 13.

For the circuit in Fig. 13.82, determine the impedance seen by the source.

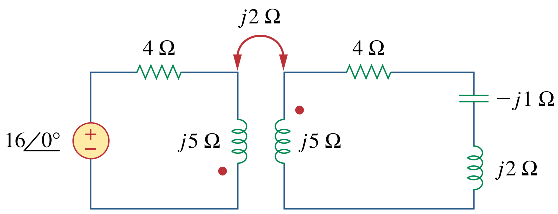


Figure 13.82
For Prob. 13.13.

Chapter 13, Solution 13.

$$Z_{in} = 4 + j(2+5) + \frac{4}{j5 + 4 - j + j2} = 4 + j7 + \frac{4}{4 + j6} = \underline{\mathbf{4.308+j6.538\ \Omega}}.$$

Chapter 13, Problem 14.

Obtain the Thevenin equivalent circuit for the circuit in Fig. 13.83 at terminals *a*-*b*.

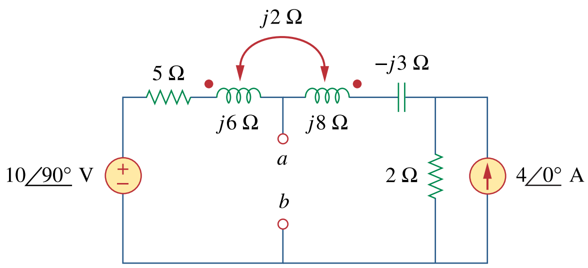
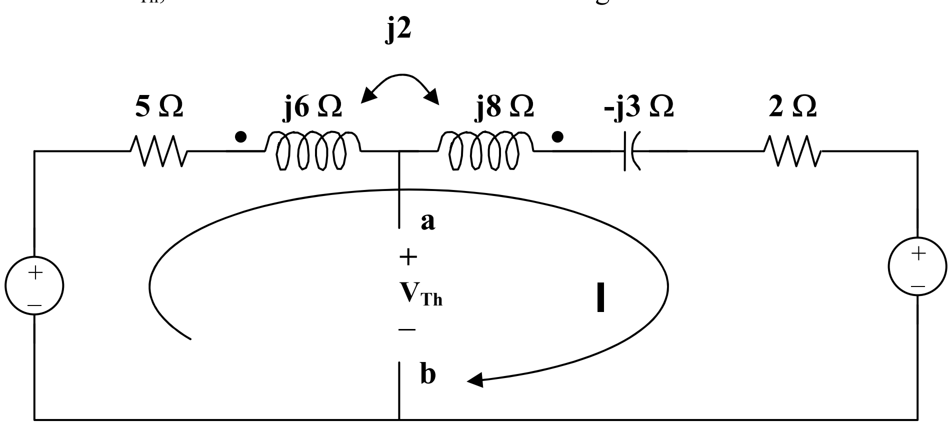


Figure 13.83
For Prob. 13.14.

Chapter 13, Solution 14.

To obtain V_{Th}, convert the current source to a voltage source as shown below.



Note that the two coils are connected series aiding.

$$\omega L \;=\; \omega L_1 + \omega L_2 - 2\omega M$$

$$j\omega L \;=\; j6 + j8 - j4 \;=\; j10$$

Thus,

$$-j10 + (5 + j10 - j3 + 2)I + 8 \;=\; 0$$

$$I \;=\; (-8 + j10)/\,(7 + j7)$$

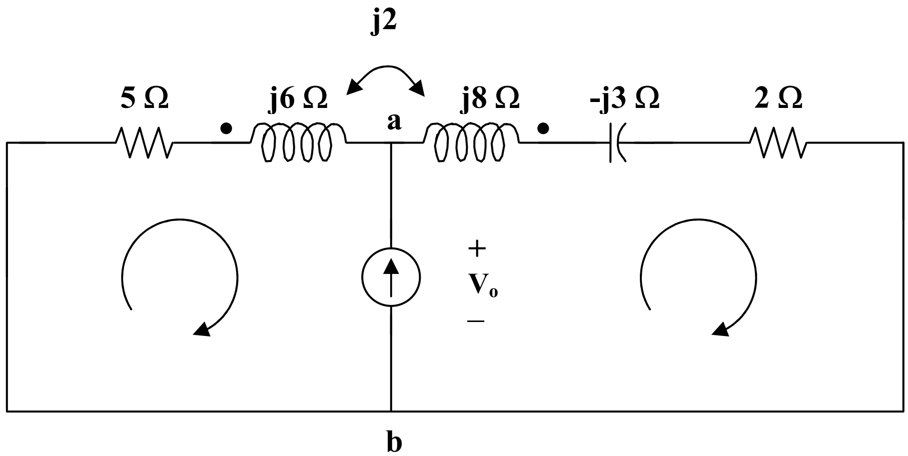
But,

$$-j10 + (5 + j6)I - j2I + V_{Th} \;=\; 0$$

$$V_{Th} \;=\; j10 - (5 + j4)I \;=\; j10 - (5 + j4)(-8 + j10)/(7 + j7)$$

$$V_{Th} \;=\; \underline{\mathbf{5.349\angle 34.11^\circ}}$$

To obtain Z_{Th}, we set all the sources to zero and insert a 1-A current source at the terminals a–b as shown below.



Clearly, we now have only a super mesh to analyze.

$$(5 + j6)I_1 - j2I_2 + (2 + j8 - j3)I_2 - j2I_1 = 0$$

$$(5 + j4)I_1 + (2 + j3)I_2 = 0 \qquad (1)$$

But, $$I_2 - I_1 = 1 \text{ or } I_2 = I_1 - 1 \qquad (2)$$

Substituting (2) into (1), $$(5 + j4)I_1 + (2 + j3)(1 + I_1) = 0$$

$$I_1 = -(2 + j3)/(7 + j7)$$

Now, $$((5 + j6)I_1 - j2I_1 + V_o = 0$$

$$V_o = -(5 + j4)I_1 = (5 + j4)(2 + j3)/(7 + j7) = (-2 + j23)/(7 + j7) = 2.332\angle 50°$$

$$Z_{Th} = V_o/1 = \underline{\mathbf{2.332\angle 50° \text{ ohms}}}$$

Chapter 13, Problem 15.

Find the Norton equivalent for the circuit in Fig. 13.84 at terminals *a*-*b*.

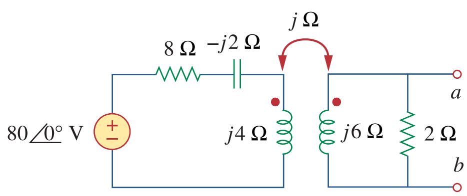
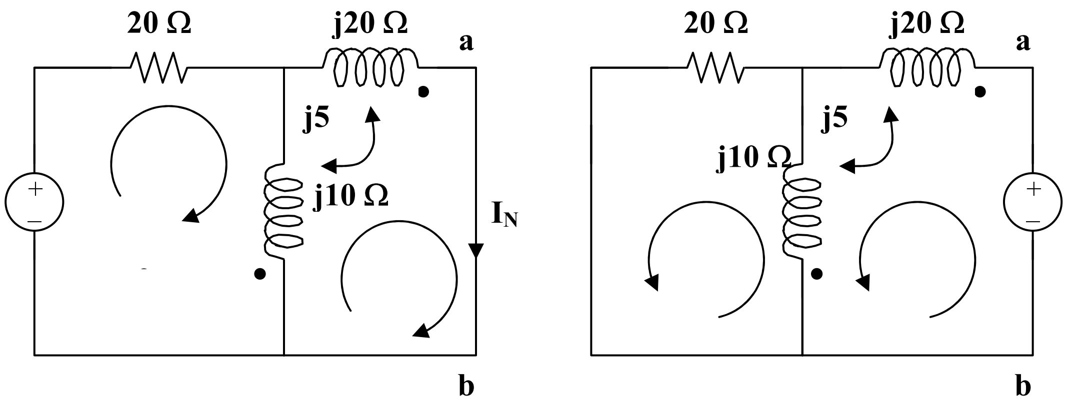


Figure 13.84
For Prob. 13.15.

Chapter 13, Solution 15.

To obtain I_N, short-circuit a–b as shown in Figure (a).



(a) (b)

For mesh 1,

$$60\angle 30° = (20 + j10)I_1 + j5I_2 - j10I_2$$

or $$12\angle 30° = (4 + j2)I_1 - jI_2 \qquad (1)$$

For mesh 2,

$$0 = (j20 + j10)I_2 + j5I_1 - j10I_1$$

or $$I_1 = 6I_2 \qquad (2)$$

Substituting (2) into (1), $$12\angle 30° = (24 + j11)I_2$$

$$I_N = I_2 = 12\angle 30°/(24 + j11) = \underline{\mathbf{1.404\angle 9.44° \ A}}$$

To find Z_N, we set all the sources to zero and insert a 1-volt voltage source at the a–b terminals as shown in Figure (b).

For mesh 1, $$1 = I_1(j10 + j20 - j5x2) + j5I_2 - j10I_2$$

$$1 = j20I_1 - j5I_2 \qquad (3)$$

For mesh 2, $$0 = (20 + j10)I_2 + j5I_1 - j10I_1 \text{ or } (4 + j2)I_2 - jI_1 = 0$$

or $$I_2 = jI_1/(4 + j2) \qquad (4)$$

Substituting (4) into (3), $$1 = j20I_1 - j(j5)I_1/(4 + j2) = (1 + j19.5)I_1$$

$$I_1 = 1/(-1 + j20.5)$$

$$Z_N = 1/I_1 = \underline{\mathbf{(1 + j19.5) \ ohms}}$$

Chapter 13, Problem 16.

Obtain the Norton equivalent at terminals *a*-*b* of the circuit in Fig. 13.85.

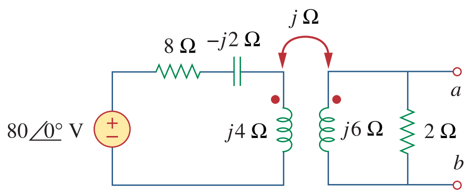
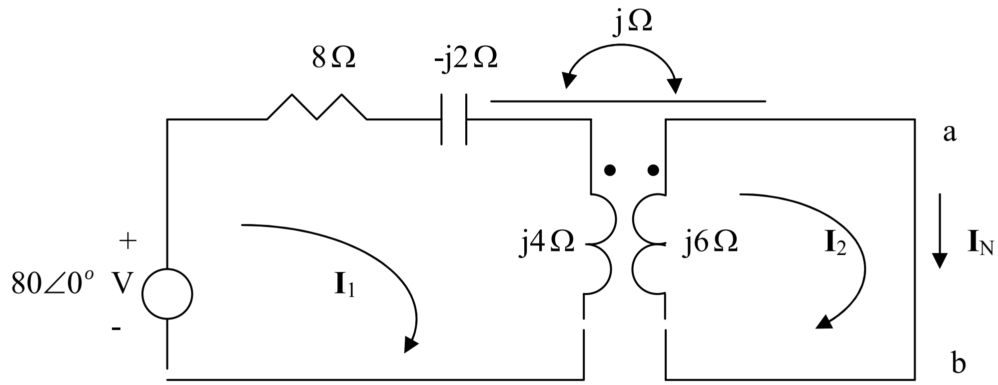


Figure 13.85
For Prob. 13.16.

Chapter 13, Solution 16.

To find $\mathbf{I}_N$, we short-circuit a-b.

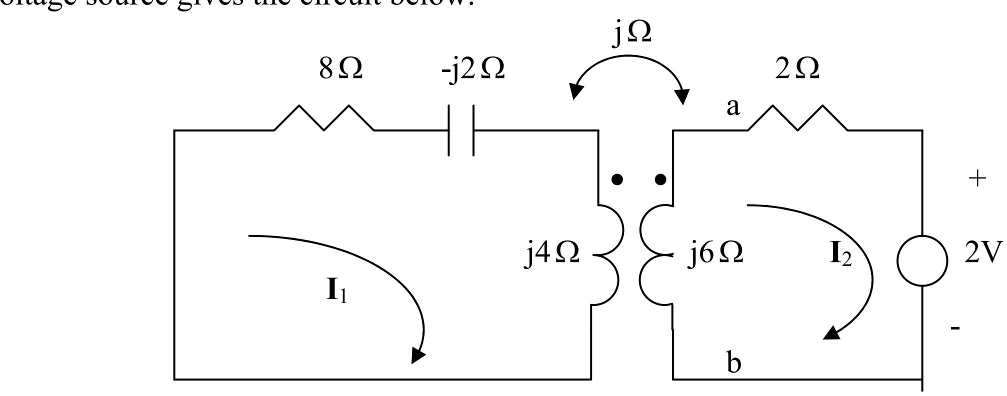


$$-80+(8-j2+j4)I_1-jI_2=0 \longrightarrow (8+j2)I_1-jI_2=80 \qquad (1)$$
$$j6I_2-jI_1=0 \longrightarrow I_1=6I_2 \qquad (2)$$

Solving (1) and (2) leads to

$$I_N=I_2=\frac{80}{48+j11}=1.584-j0.362=\underline{1.6246\angle-12.91^o\ \text{A}}$$

To find Z_N, insert a 1-A current source at terminals a-b. Transforming the current source to voltage source gives the circuit below.



$$0=(8+j2)I_1-jI_2 \longrightarrow I_1=\frac{jI_2}{8+j2} \qquad (3)$$
$$2+(2+j6)I_2-jI_1=0 \qquad (4)$$

Solving (3) and (4) leads to $\mathbf{I}_2$ = -0.1055 +j0.2975, $\mathbf{V}_{ab}$=-j6$\mathbf{I}_2$ = 1.7853 +0.6332

$$Z_N=\frac{V_{ab}}{1}=\underline{1.894\angle 19.53^o\,\Omega}$$

Chapter 13, Problem 17.

ML In the circuit of Fig. 13.86, Z_L is a 15-mH inductor having an impedance of $j40\ \Omega$. Determine Z_{in} when $k = 0.6$.

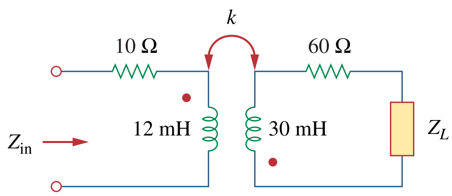


Figure 13.86
For Prob. 13.17.

Chapter 13, Solution 17.

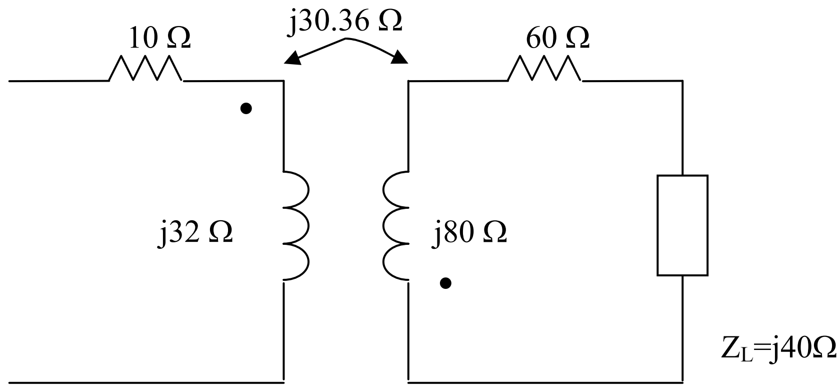
$$j\omega L = j40 \longrightarrow \omega = \frac{40}{L} = \frac{40}{15x10^{-3}} = 2667 \text{ rad/s}$$

$$M = k\sqrt{L_1L_2} = 0.6\sqrt{12x10^{-3}x30x10^{-3}} = 11.384 \text{ mH}$$

If 15 mH ⟶ 40 Ω

Then 12 mH ⟶ 32 Ω
30 mH ⟶ 80 Ω
11.384 mH ⟶ 30.36 Ω

The circuit becomes that shown below.



$$Z_{in} = 10 + j32 + \frac{\omega^2 M^2}{j80 + 60 + j40} = 10 + j32 + \frac{(30.36)^2}{60 + j120} = \underline{\mathbf{13.073 + j25.86\ \Omega}}.$$

Chapter 13, Problem 18.

Find the Thevenin equivalent to the left of the load **Z** in the circuit of Fig. 13.87.

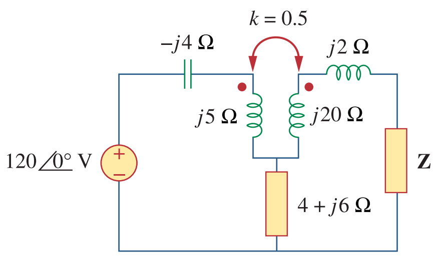


Figure 13.87
For Prob. 13.18.

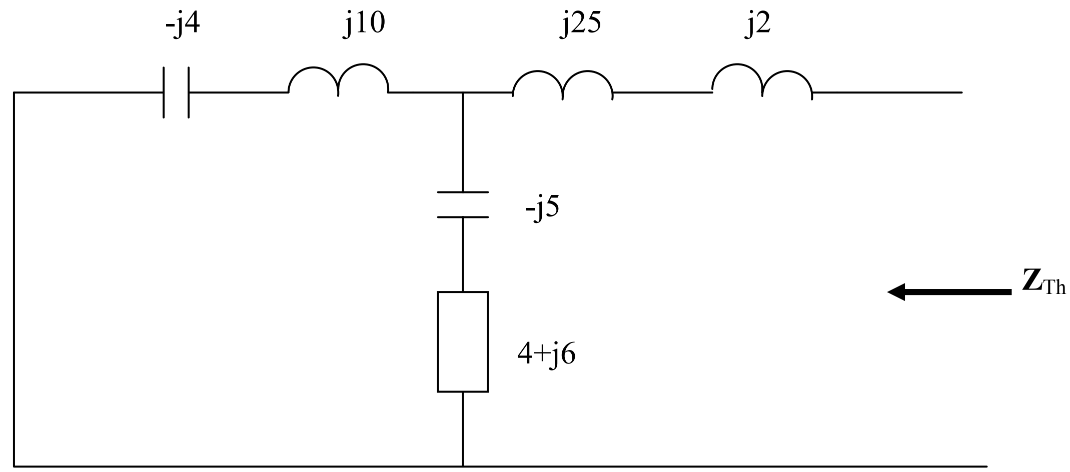
Chapter 13, Solution 18.

Let $\omega = 1$. $L_1 = 5, L_2 = 20, M = k\sqrt{L_1 L_2} = 0.5x10 = 5$

We replace the transformer by its equivalent T-section.

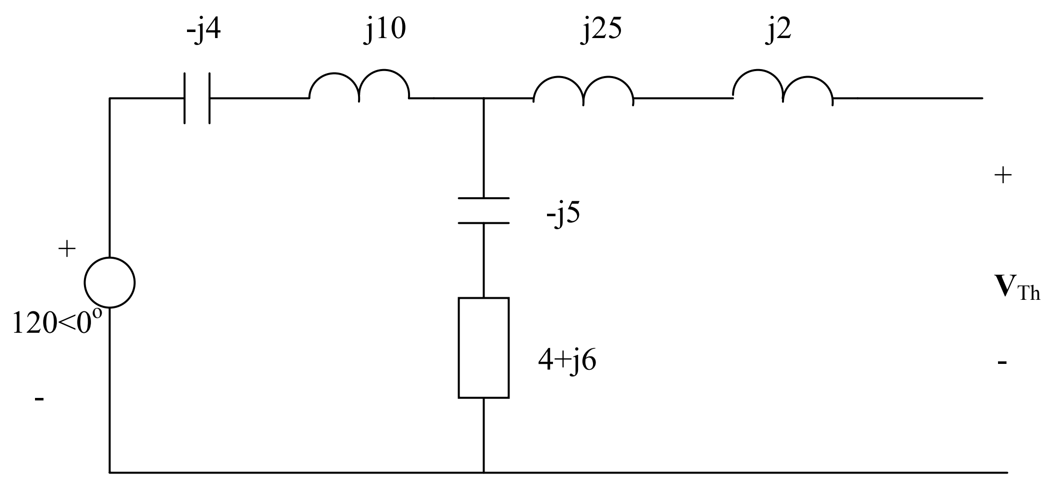
$L_a = L_1 - (-M) = 5 + 5 = 10, \quad L_b = L_1 + M = 20 + 5 = 25, \quad L_c = -M = -5$

We find $\mathbf{Z}_{Th}$ using the circuit below.



$$Z_{Th} = j27 + (4+j)//(j6) = j27 + \frac{j6(4+j)}{4+j7} = \underline{2.215 + j29.12\Omega}$$

We find $\mathbf{V}_{Th}$ by looking at the circuit below.



$$V_{Th} = \frac{4+j}{4+j+j6}(120) = \underline{61.37\angle -46.22^o \text{ V}}$$

Chapter 13, Problem 19.

Determine an equivalent T-section that can be used to replace the transformer in Fig. 13.88.

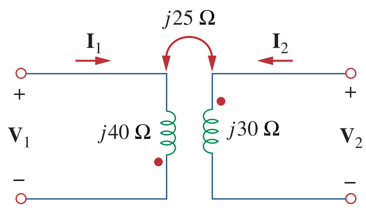


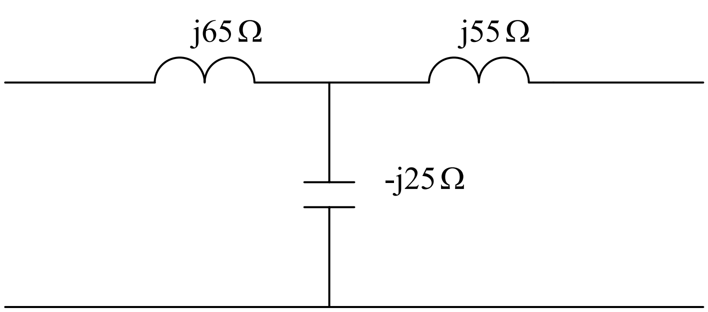
Figure 13.88
For Prob. 13.19.

Chapter 13, Solution 19.

Let $\omega = 1$. $\quad L_a = L_1 - (-M) = 40 + 25 = 65\text{ H}$

$$L_b = L_2 + M = 30 + 25 = 55\text{ H}, \qquad L_C = -M = -25$$

Thus, the T-section is as shown below.



Chapter 13, Problem 20.

ps ML Determine currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 13.89. Find the energy stored in the coupled coils at $t = 2$ ms. Take $\omega = 1{,}000$ rad/s.

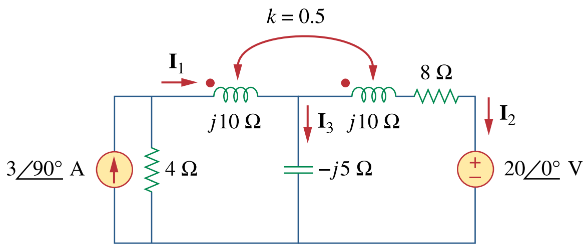
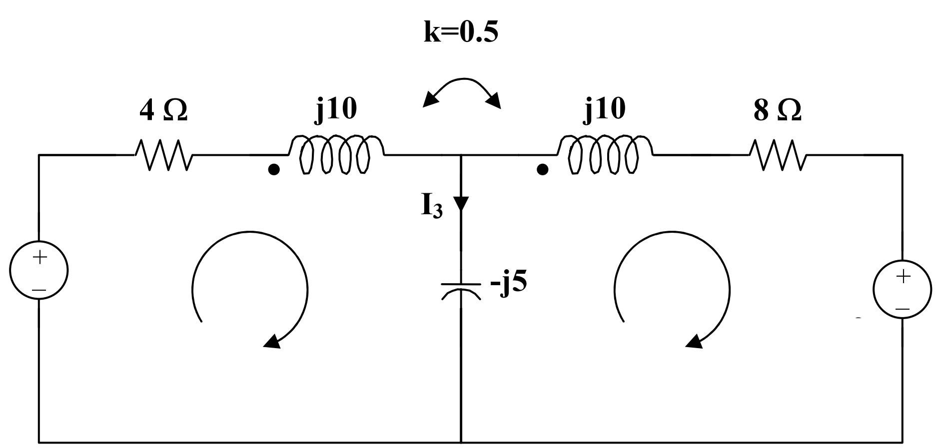


Figure 13.89
For Prob. 13.20.

Chapter 13, Solution 20.

Transform the current source to a voltage source as shown below.



$$k = M/\sqrt{L_1 L_2} \quad \text{or} \quad M = k\sqrt{L_1 L_2}$$

$$\omega M = k\sqrt{\omega L_1 \omega L_2} = 0.5(10) = 5$$

For mesh 1, $\quad j12 = (4 + j10 - j5)I_1 + j5I_2 + j5I_2 = (4 + j5)I_1 + j10I_2 \qquad (1)$

For mesh 2, $\quad 0 = 20 + (8 + j10 - j5)I_2 + j5I_1 + j5I_1$

$$-20 = +j10I_1 + (8 + j5)I_2 \qquad (2)$$

From (1) and (2), $\begin{bmatrix} j12 \\ 20 \end{bmatrix} = \begin{bmatrix} 4 + j5 & +j10 \\ +j10 & 8 + j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\Delta = 107 + j60, \quad \Delta_1 = -60 - j296, \quad \Delta_2 = 40 - j100$$

$$I_1 = \Delta_1/\Delta = \underline{\mathbf{2.462\angle 72.18° \ A}}$$

$$I_2 = \Delta_2/\Delta = \underline{\mathbf{0.878\angle -97.48° \ A}}$$

$$I_3 = I_1 - I_2 = \underline{\mathbf{3.329\angle 74.89° \ A}}$$

$$i_1 = 2.462 \cos(1000t + 72.18°) \text{ A}$$

$$i_2 = 0.878 \cos(1000t - 97.48°) \text{ A}$$

At $t = 2$ ms, $1000t = 2 \text{ rad} = 114.6°$

$$i_1 = 0.9736\cos(114.6° + 143.09°) = -2.445$$

$$i_2 = 2.53\cos(114.6° + 153.61°) = -0.8391$$

The total energy stored in the coupled coils is

$$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 - Mi_1i_2$$

Since $\omega L_1 = 10$ and $\omega = 1000$, $L_1 = L_2 = 10$ mH, $M = 0.5L_1 = 5$mH

$$w = 0.5(10)(-2.445)^2 + 0.5(10)(-0.8391)^2 - 5(-2.445)(-0.8391)$$

w = <u>43.67 mJ</u>

Chapter 13, Problem 21.

PS ML Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.90. Calculate the power absorbed by the 4-Ω resistor.

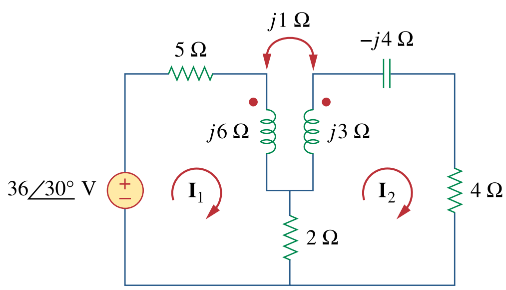


Figure 13.90
For Prob. 13.21.

Chapter 13, Solution 21.

For mesh 1, $36\angle 30° = (7 + j6)I_1 - (2 + j)I_2$ (1)

For mesh 2, $0 = (6 + j3 - j4)I_2 - 2I_1 - jI_1 = -(2 + j)I_1 + (6 - j)I_2$ (2)

Placing (1) and (2) into matrix form, $\begin{bmatrix} 36\angle 30° \\ 0 \end{bmatrix} = \begin{bmatrix} 7 + j6 & -2 - j \\ -2 - j & 6 - j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\Delta = 45 + j25 = 51.48\angle 29.05°, \quad \Delta_1 = (6 - j)36\angle 30° = 219\angle 20.54°$$

$\Delta_2 = (2 + j)36\angle 30° = 80.5\angle 56.57°$, $I_1 = \Delta_1/\Delta =$ **$\underline{4.254\angle -8.51°\ \text{A}}$**, $I_2 = \Delta_2/\Delta =$ **$\underline{1.5637\angle 27.52°\ \text{A}}$**

Power absorbed by the 4-ohm resistor,

$$= 0.5(I_2)^2 4 = 2(1.5637)^2 = \underline{\textbf{4.89 watts}}$$

Chapter 13, Problem 22.

* Find current $\mathbf{I}_o$ in the circuit of Fig. 13.91.

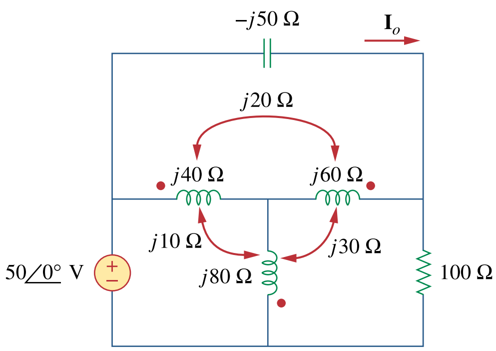
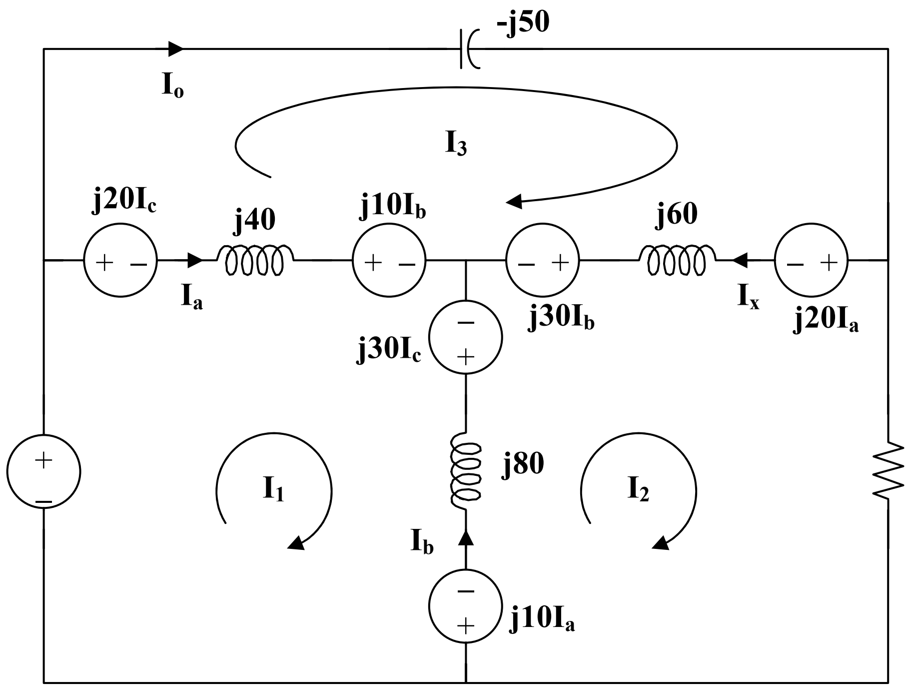


Figure 13.91
For Prob. 13.22.

* An asterisk indicates a challenging problem.

Chapter 13, Solution 22.

With more complex mutually coupled circuits, it may be easier to show the effects of the coupling as sources in terms of currents that enter or leave the dot side of the coil. Figure 13.85 then becomes,



Note the following,

$$I_a = I_1 - I_3$$
$$I_b = I_2 - I_1$$
$$I_c = I_3 - I_2$$

$$\text{and} \quad I_o = I_3$$

Now all we need to do is to write the mesh equations and to solve for I_o.

Loop # 1,

$$-50 + j20(I_3 - I_2)\, j\, 40(I_1 - I_3) + j10(I_2 - I_1) - j30(I_3 - I_2) + j80(I_1 - I_2) - j10(I_1 - I_3) = 0$$

$$j100I_1 - j60I_2 - j40I_3 = 50$$

Multiplying everything by (1/j10) yields $10I_1 - 6I_2 - 4I_3 = -j5$ (1)

Loop # 2,

$j10(I_1 - I_3) + j80(I_2 - I_1) + j30(I_3 - I_2) - j30(I_2 - I_1) + j60(I_2 - I_3) - j20(I_1 - I_3) + 100I_2 = 0$

$$-j60I_1 + (100 + j80)I_2 - j20I_3 = 0 \qquad (2)$$

Loop # 3,

$-j50I_3 + j20(I_1 - I_3) + j60(I_3 - I_2) + j30(I_2 - I_1) - j10(I_2 - I_1) + j40(I_3 - I_1) - j20(I_3 - I_2) = 0$

$$-j40I_1 - j20I_2 + j10I_3 = 0$$

Multiplying by (1/j10) yields, $-4I_1 - 2I_2 + I_3 = 0$ (3)

Multiplying (2) by (1/j20) yields $-3I_1 + (4 - j5)I_2 - I_3 = 0$ (4)

Multiplying (3) by (1/4) yields $-I_1 - 0.5I_2 - 0.25I_3 = 0$ (5)

Multiplying (4) by (-1/3) yields $I_1 - ((4/3) - j(5/3))I_2 + (1/3)I_3 = -j0.5$ (7)

Multiplying [(6)+(5)] by 12 yields $(-22 + j20)I_2 + 7I_3 = 0$ (8)

Multiplying [(5)+(7)] by 20 yields $-22I_2 - 3I_3 = -j10$ (9)

(8) leads to $I_2 = -7I_3/(-22 + j20) = 0.2355\angle 42.3^\circ = (0.17418 + j0.15849)I_3$ (10)

(9) leads to $I_3 = (j10 - 22I_2)/3$, substituting (1) into this equation produces,

$$I_3 = j3.333 + (-1.2273 - j1.1623)I_3$$

or $I_3 = I_o = \underline{\mathbf{1.3040\angle 63^\circ \text{ amp.}}}$

Chapter 13, Problem 23.

If $M = 0.2$ H and $v_s = 12 \cos 10t$ V in the circuit of Fig. 13.92, find i_1 and i_2 Calculate the energy stored in the coupled coils at $t = 15$ ms.

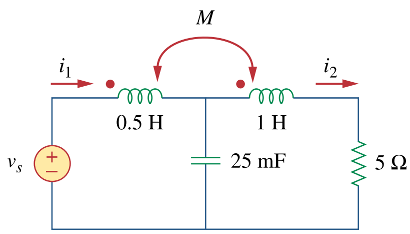


Figure 13.92
For Prob. 13.23.

Chapter 13, Solution 23.

$$\omega = 10$$

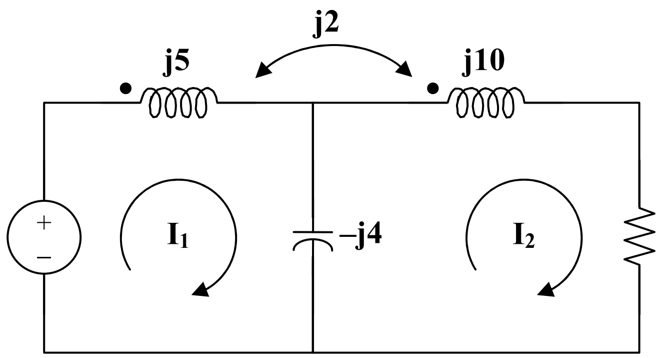
$$0.5 \text{ H converts to } j\omega L_1 = j5 \text{ ohms}$$

$$1 \text{ H converts to } j\omega L_2 = j10 \text{ ohms}$$

$$0.2 \text{ H converts to } j\omega M = j2 \text{ ohms}$$

$$25 \text{ mF converts to } 1/(j\omega C) = 1/(10\text{x}25\text{x}10^{-3}) = -j4 \text{ ohms}$$

The frequency-domain equivalent circuit is shown below.



For mesh 1, $12 = (j5 - j4)I_1 + j2I_2 - (-j4)I_2$

$$-j12 = I_1 + 6I_2 \qquad (1)$$

For mesh 2, $0 = (5 + j10)I_2 + j2I_1 - (-j4)I_1$

$$0 = (5 + j10)I_2 + j6I_1 \qquad (2)$$

From (1), $I_1 = -j12 - 6I_2$

Substituting this into (2) produces,

$$I_2 = 72/(-5 + j26) = 2.7194\angle{-100.89°}$$

$$I_1 = -j12 - 6\, I_2 = -j12 - 163.17\angle{-100.89} = 5.068\angle{52.54°}$$

Hence,

$i_1 = \underline{\mathbf{5.068\cos(10t + 52.54°)\ A}}$, $i_2 = \underline{\mathbf{2.719\cos(10t - 100.89°)\ A}}$.

At $t = 15$ ms, $10t = 10\text{x}15\text{x}10^{-3}\ 0.15 \text{ rad} = 8.59°$

$$i_1 = 5.068\cos(61.13°) = 2.446$$

$$i_2 = 2.719\cos(-92.3°) = -0.1089$$

$$w = 0.5(5)(2.446)^2 + 0.5(1)(-0.1089)^2 - (0.2)(2.446)(-0.1089) = \underline{\mathbf{15.02\ J}}$$

Chapter 13, Problem 24.

In the circuit of Fig. 13.93,

(a) find the coupling coefficient,
(b) calculate v_o,
(c) determine the energy stored in the coupled inductors at $t = 2$ s.

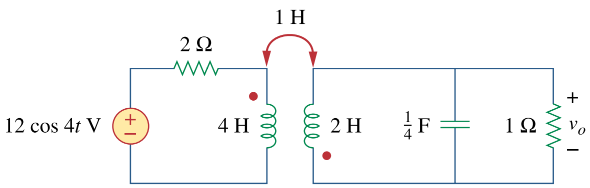


Figure 13.93
For Prob. 13.24.

Chapter 13, Solution 24.

(a) $k = M/\sqrt{L_1L_2} = 1/\sqrt{4x2} = \underline{\mathbf{0.3535}}$

(b) $\omega = 4$

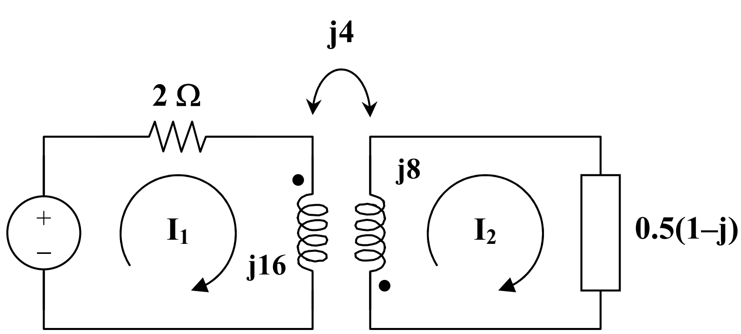
1/4 F leads to $1/(j\omega C) = -j/(4x0.25) = -j$

$1||(-j) = -j/(1-j) = 0.5(1-j)$

1 H produces $j\omega M = j4$

4 H produces j16

2 H becomes j8



$$12 = (2 + j16)I_1 + j4I_2$$

$$\text{or} \qquad 6 = (1 + j8)I_1 + j2I_2 \qquad (1)$$

$$0 = (j8 + 0.5 - j0.5)I_2 + j4I_1 \ \text{ or } \ I_1 = (0.5 + j7.5)I_2/(-j4) \qquad (2)$$

Substituting (2) into (1),

$$24 = (-11.5 - j51.5)I_2 \ \text{ or } \ I_2 = -24/(11.5 + j51.5) = -0.455\angle -77.41°$$

$$V_o = I_2(0.5)(1 - j) = 0.3217\angle 57.59°$$

$$v_o = \underline{\mathbf{321.7cos(4t + 57.6°) \ mV}}$$

(c) From (2), $I_1 = (0.5 + j7.5)I_2/(-j4) = 0.855\angle -81.21°$

$$i_1 = 0.885\cos(4t - 81.21°) \text{ A}, \ i_2 = -0.455\cos(4t - 77.41°) \text{ A}$$

At t = 2s,

$$4t = 8 \text{ rad} = 98.37°$$

$$i_1 = 0.885\cos(98.37° - 81.21°) = 0.8169$$

$$i_2 = -0.455\cos(98.37° - 77.41°) = -0.4249$$

$$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 + Mi_1i_2$$

$$= 0.5(4)(0.8169)^2 + 0.5(2)(-.4249)^2 + (1)(0.1869)(-0.4249) = \underline{\mathbf{1.168 \ J}}$$

Chapter 13, Problem 25.

For the network in Fig. 13.94, find $\mathbf{Z}_{ab}$ and $\mathbf{I}_o$.

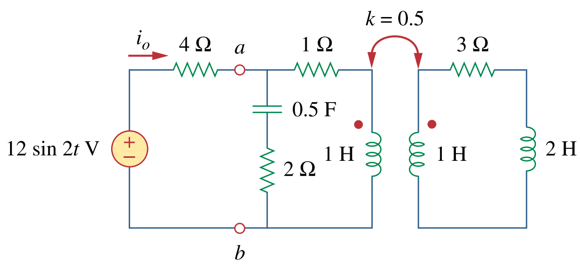


Figure 13.94
For Prob. 13.25.

Chapter 13, Solution 25.

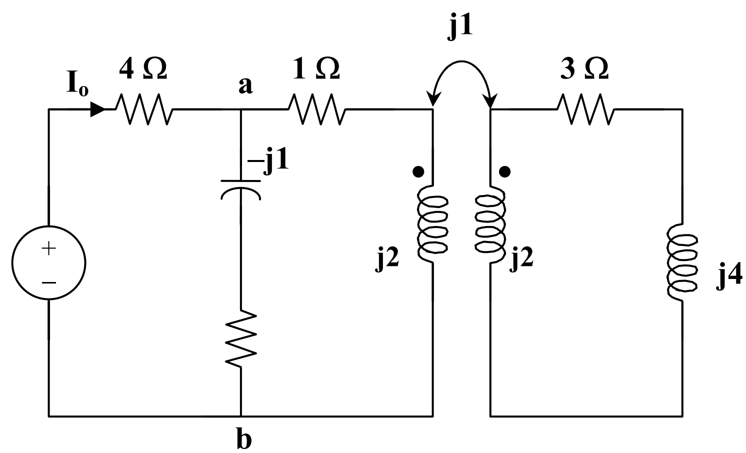
$$m = k\sqrt{L_1 L_2} = 0.5 \text{ H}$$

We transform the circuit to frequency domain as shown below.

$$12\sin 2t \text{ converts to } 12\angle 0°, \ \omega = 2$$

$$0.5 \text{ F converts to } 1/(j\omega C) = -j$$

$$2 \text{ H becomes } j\omega L = j4$$



Applying the concept of reflected impedance,

$$Z_{ab} = (2 - j)||(1 + j2 + (1)^2/(j2 + 3 + j4))$$

$$= (2 - j)||(1 + j2 + (3/45) - j6/45)$$

$$= (2 - j)||(1 + j2 + (3/45) - j6/45)$$

$$= (2 - j)||(1.0667 + j1.8667)$$

$$=(2 - j)(1.0667 + j1.8667)/(3.0667 + j0.8667) = 1.5085\angle 17.9° \text{ ohms}$$

$$I_o = 12\angle 0°/(Z_{ab} + 4) = 12/(5.4355 + j0.4636) = 2.2\angle{-4.88°}$$

$$i_o = \underline{\mathbf{2.2\sin(2t - 4.88°)\ A}}$$

Chapter 13, Problem 26.

PS ML Find $\mathbf{I}_o$ in the circuit of Fig. 13.95. Switch the dot on the winding on the right and calculate $\mathbf{I}_o$ again.

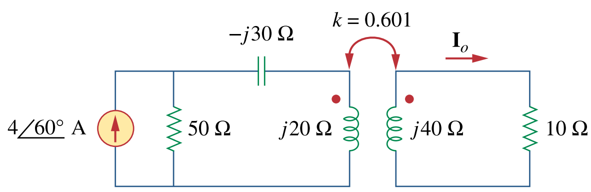


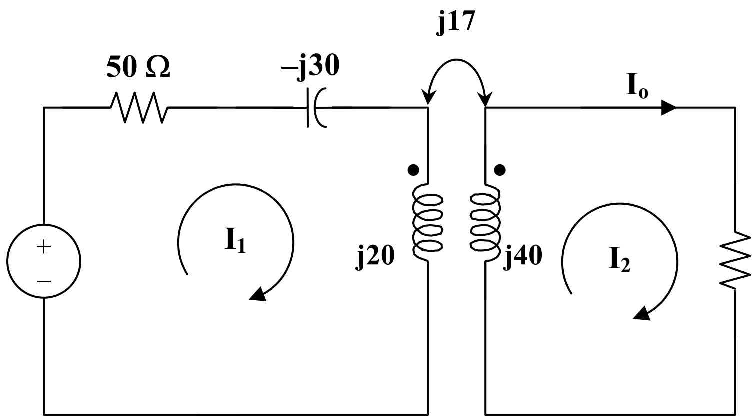
Figure 13.95
For Prob. 13.26.

Chapter 13, Solution 26.

$$M = k\sqrt{L_1 L_2}$$

$$\omega M = k\sqrt{\omega L_1 \omega L_2} = 0.6\sqrt{20x40} = 17$$

The frequency-domain equivalent circuit is shown below.



For mesh 1, $200\angle 60° = (50 - j30 + j20)I_1 + j17I_2 = (50 - j10)I_1 + j17I_2$ (1)

For mesh 2, $0 = (10 + j40)I_2 + j17I_1$ (2)

In matrix form,

$$\begin{bmatrix} 200\angle 60° \\ 0 \end{bmatrix} = \begin{bmatrix} 50 - j10 & j17 \\ j17 & 10 + j40 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = 900 + j100,\ \Delta_1 = 2000\angle 60°(1 + j4) = 8246.2\angle 136°,\ \Delta_2 = 3400\angle -30°$$

$$I_2 = \Delta_2/\Delta = 3.755\angle -36.34°$$

$$I_o = I_2 = \underline{\mathbf{3.755\angle -36.34°\ A}}$$

Switching the dot on the winding on the right only reverses the direction of I_o. This can be seen by looking at the resulting value of Δ_2 which now becomes $3400\angle 150°$. Thus,

$$I_o = \underline{\mathbf{3.755\angle 143.66°\ A}}$$

Chapter 13, Problem 27.

Find the average power delivered to the 50-Ω resistor in the circuit of Fig. 13.96.

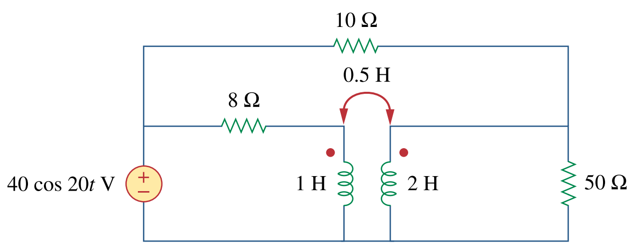
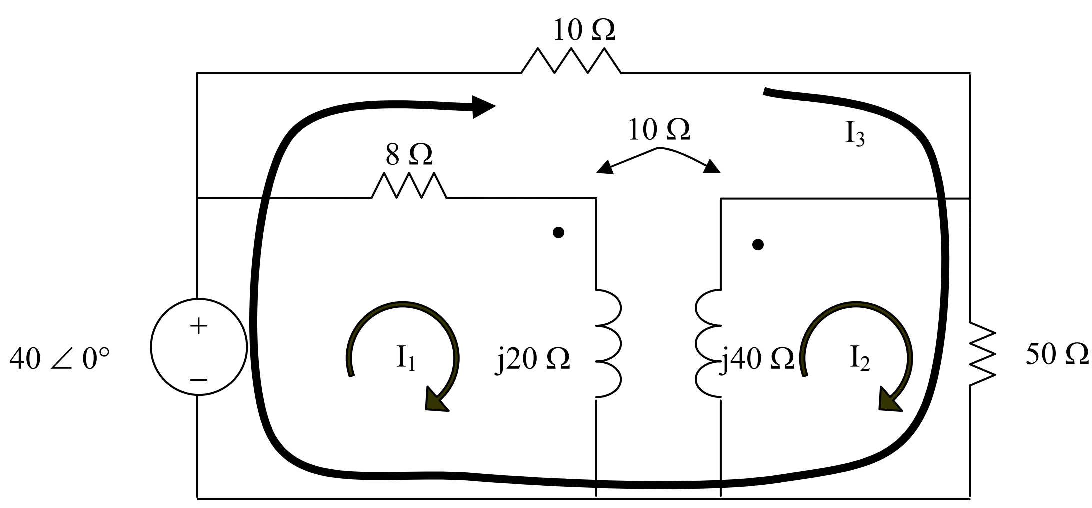


Figure 13.96
For Prob. 13.27.

Chapter 13, Solution 27.

$$1H \longrightarrow j\omega L = j20$$
$$2H \longrightarrow j\omega L = j40$$
$$0.5H \longrightarrow j\omega L = j10$$

We apply mesh analysis to the circuit as shown below.



To make the problem easier to solve, let us have I_3 flow around the outside loop as shown.

For mesh 1,

$$(8+j20)I_1 - j10I_2 = 40 \qquad (1)$$

For mesh 2,

$$-j10I_1 + (50+j40)I_2 + 50I_3 = 0 \qquad (2)$$

For mesh 3,

$$-40 + 50I_2 + 60I_3 = 0 \qquad (3)$$

In matrix form, (1) to (3) become

$$\begin{bmatrix} 8+j20 & -j10 & 0 \\ -j10 & 50+j40 & 50 \\ 0 & 50 & 60 \end{bmatrix} I = \begin{bmatrix} 40 \\ 0 \\ 0 \end{bmatrix}$$

```
>> Z=[(8+20i),-10i,0;-10i,(50+40i),50;0,50,60]
Z =
   8.0000 +20.0000i        0 -10.0000i        0
        0 -10.0000i  50.0000 +40.0000i  50.0000
        0            50.0000            60.0000

>> V=[40;0;0]
V =
    40
     0
     0
>> I=inv(Z)*V
I =
   0.8896 - 1.8427i
   0.3051 - 0.3971i
  -0.2543 + 0.3309i
```

Solving this leads to $I_{50} = I_2 + I_3 = 0.0508 - j0.0662 = 0.08345\angle -52.5^\circ$ or $I_{50rms} = 0.08345/1.4142 = 0.059$.

The power delivered to the 50-Ω resistor is

$$P = (I_{50rms})^2R = (0.059)^2 50 = \underline{\mathbf{174.05\ mW}}.$$

Chapter 13, Problem 28.

ML In the circuit of Fig. 13.97, find the value of *X* that will give maximum power transfer to the 20-Ω load.

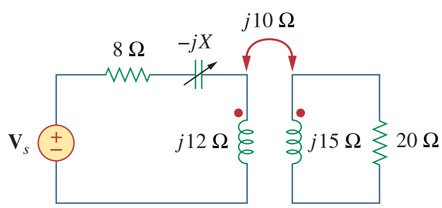
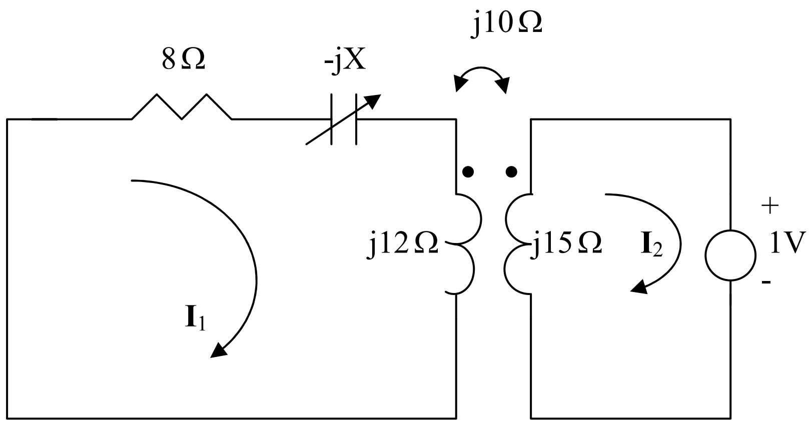


Figure 13.97
For Prob. 13.28.

Chapter 13, Solution 28.

We find Z_{Th} by replacing the 20-ohm load with a unit source as shown below.



For mesh 1, $$0 = (8 - jX + j12)I_1 - j10I_2 \quad (1)$$

For mesh 2,

$$1 + j15I_2 - j10I_1 = 0 \quad \longrightarrow \quad I_1 = 1.5I_2 - 0.1j \quad (2)$$

Substituting (2) into (1) leads to

$$I_2 = \frac{-1.2 + j0.8 + 0.1X}{12 + j8 - j1.5X}$$

$$Z_{Th} = \frac{1}{-I_2} = \frac{12 + j8 - j1.5X}{1.2 - j0.8 - 0.1X}$$

$$|Z_{Th}| = 20 = \frac{\sqrt{12^2 + (8 - 1.5X)^2}}{\sqrt{(1.2 - 0.1X)^2 + 0.8^2}} \quad \longrightarrow \quad 0 = 1.75X^2 + 72X - 624$$

Solving the quadratic equation yields <u>X = 6.425</u>

Chapter 13, Problem 29.

In the circuit of Fig. 13.98, find the value of the coupling coefficient k that will make the 10-Ω resistor dissipate 320 W. For this value of k, find the energy stored in the coupled coils at $t = 1.5$ s.

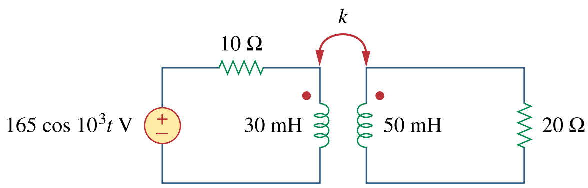


Figure 13.98
For Prob. 13.29.

Chapter 13, Solution 29.

$$30 \text{ mH becomes } j\omega L = j30x10^{-3}x10^{3} = j30$$

$$50 \text{ mH becomes } j50$$

$$\text{Let } X = \omega M$$

Using the concept of reflected impedance,

$$Z_{in} = 10 + j30 + X^2/(20 + j50)$$

$$I_1 = V/Z_{in} = 165/(10 + j30 + X^2/(20 + j50))$$

$$p = 0.5|I_1|^2(10) = 320 \text{ leads to } |I_1|^2 = 64 \text{ or } |I_1| = 8$$

$$8 = |165(20 + j50)/(X^2 + (10 + j30)(20 + j50))|$$

$$= |165(20 + j50)/(X^2 - 1300 + j1100)|$$

or $$64 = 27225(400 + 2500)/((X^2 - 1300)^2 + 1{,}210{,}000)$$

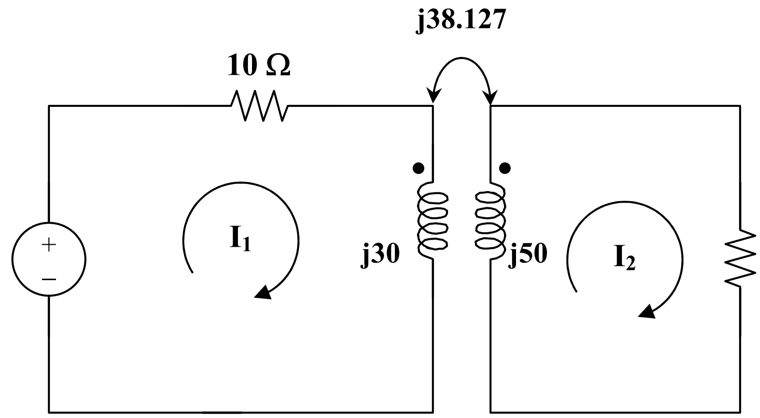
$$(X^2 - 1300)^2 + 1{,}210{,}000 = 1{,}233{,}633$$

$$X = 33.86 \text{ or } 38.13$$

$$\text{If } X = 38.127 = \omega M$$

$$M = 38.127 \text{ mH}$$

$$k = M/\sqrt{L_1L_2} = 38.127/\sqrt{30x50} = \underline{\mathbf{0.984}}$$



$$165 = (10 + j30)I_1 - j38.127I_2 \qquad (1)$$

$$0 = (20 + j50)I_2 - j38.127I_1 \qquad (2)$$

In matrix form,
$$\begin{bmatrix} 165 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 + j30 & -j38.127 \\ -j38.127 & 20 + j50 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = 154 + j1100 = 1110.73\angle 82.03°, \ \Delta_1 = 888.5\angle 68.2°, \ \Delta_2 = j6291$$

$$I_1 = \Delta_1/\Delta = 8\angle{-13.81°}, \ I_2 = \Delta_2/\Delta = 5.664\angle 7.97°$$

$$i_1 = 8\cos(1000t - 13.83°), \ i_2 = 5.664\cos(1000t + 7.97°)$$

$$\text{At } t = 1.5 \text{ ms}, \ 1000t = 1.5 \text{ rad} = 85.94°$$

$$i_1 = 8\cos(85.94° - 13.83°) = 2.457$$

$$i_2 = 5.664\cos(85.94° + 7.97°) = -0.3862$$

$$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 + Mi_1i_2$$

$$= 0.5(30)(2.547)^2 + 0.5(50)(-0.3862)^2 - 38.127(2.547)(-0.3862)$$

$$= \underline{\mathbf{130.51\ mJ}}$$

Chapter 13, Problem 30.

(a) Find the input impedance of the circuit in Fig. 13.99 using the concept of reflected impedance.
(b) Obtain the input impedance by replacing the linear transformer by its T equivalent.

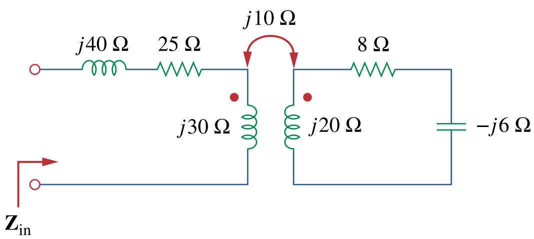


Figure 13.99
For Prob. 13.30.

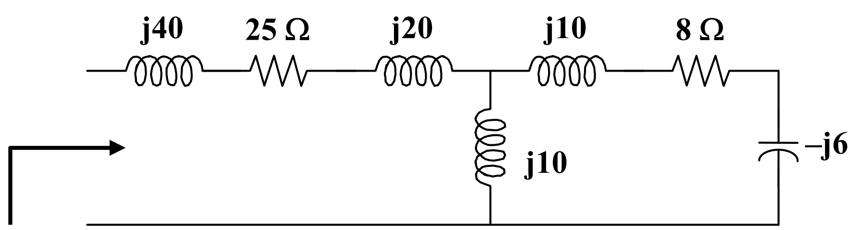
Chapter 13, Solution 30.

(a) $$Z_{in} = j40 + 25 + j30 + (10)^2/(8 + j20 - j6)$$

$$= 25 + j70 + 100/(8 + j14) = \underline{\mathbf{(28.08 + j64.62)\ ohms}}$$

(b) $$j\omega L_a = j30 - j10 = j20,\ j\omega L_b = j20 - j10 = j10,\ j\omega L_c = j10$$

Thus the Thevenin Equivalent of the linear transformer is shown below.



$$Z_{in} = j40 + 25 + j20 + j10||(8 + j4) = 25 + j60 + j10(8 + j4)/(8 + j14)$$

$$= \underline{\mathbf{(28.08 + j64.62)\ ohms}}$$

Chapter 13, Problem 31.

For the circuit in Fig. 13.100, find:

(a) the *T*-equivalent circuit,
(b) the Π-equivalent circuit.

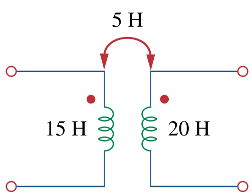


Figure 13.100
For Prob. 13.31.

Chapter 13, Solution 31.

(a)
$$L_a = L_1 - M = \underline{\mathbf{10\ H}}$$

$$L_b = L_2 - M = \underline{\mathbf{15\ H}}$$

$$L_c = M = \underline{\mathbf{5\ H}}$$

(b)
$$L_1L_2 - M^2 = 300 - 25 = 275$$

$$L_A = (L_1L_2 - M^2)/(L_1 - M) = 275/15 = \underline{\mathbf{18.33\ H}}$$

$$L_B = (L_1L_2 - M^2)/(L_1 - M) = 275/10 = \underline{\mathbf{27.5\ H}}$$

$$L_C = (L_1L_2 - M^2)/M = 275/5 = \underline{\mathbf{55\ H}}$$

Chapter 13, Problem 32.

* Two linear transformers are cascaded as shown in Fig. 13.101. Show that

$$\mathbf{Z}_{\text{in}} = \frac{\omega^2 R(L_a^2 + L_a L_b - M_a^2 + j\omega^3(L_a^2 L_b + L_a L_b^2 - L_a M_b^2 - L_b M_a^2}{\omega^2 (L_a L_b + L_b^2 - M_b^2) - j\omega R(L_a + L_b)}$$

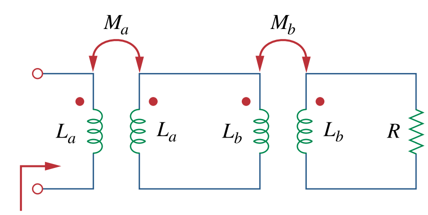
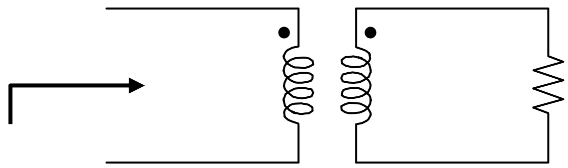


Figure 13.101
For Prob. 13.32.

* An asterisk indicates a challenging problem.

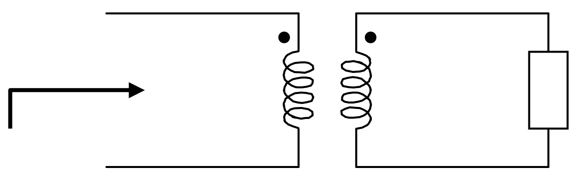
Chapter 13, Solution 32.

We first find Z_{in} for the second stage using the concept of reflected impedance.



$$Z_{in}' = j\omega L_b + \omega^2 M_b^2/(R + j\omega L_b) = (j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)/(R + j\omega L_b) \qquad (1)$$

For the first stage, we have the circuit below.



$$Z_{in} = j\omega L_a + \omega^2 M_a^2/(j\omega L_a + Z_{in})$$

$$= (-\omega^2 L_a^2 + \omega^2 M_a^2 + j\omega L_a Z_{in})/(j\omega L_a + Z_{in}) \qquad (2)$$

Substituting (1) into (2) gives,

$$= \frac{-\omega^2 L_a^2 + \omega^2 M_a^2 + j\omega L_a \dfrac{(j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)}{R + j\omega L_b}}{j\omega L_a + \dfrac{j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2}{R + j\omega L_b}}$$

$$= \frac{-R\omega^2 L_a^2 + \omega^2 M_a^2 R - j\omega^3 L_b L_a + j\omega^3 L_b M_a^2 + j\omega L_a(j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)}{j\omega RLa - \omega^2 L_a L_b + j\omega L_b R - \omega^2 L_a^2 + \omega^2 M_b^2}$$

$$Z_{in} = \frac{\boldsymbol{\omega^2 R(L_a^2 + L_a L_b - M_a^2) + j\omega^3(L_a^2 L_b + L_a L_b^2 - L_a M_b^2 - L_b M_a^2)}}{\boldsymbol{\omega^2(L_a L_b + L_b^2 - M_b^2) - j\omega R(L_a + L_b)}}$$

Chapter 13, Problem 33.

ML Determine the input impedance of the air-core transformer circuit of Fig. 13.102.

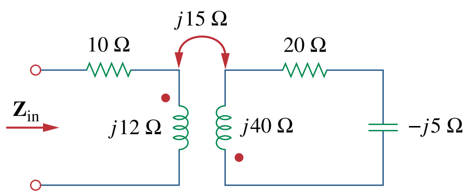


Figure 13.102
For Prob. 13.33.

Chapter 13, Solution 33.

$$Z_{in} = 10 + j12 + (15)^2/(20 + j\,40 - j5) = 10 + j12 + 225/(20 + j35)$$

$$= 10 + j12 + 225(20 - j35)/(400 + 1225)$$

$$= \underline{(12.769 + j7.154)\ \text{ohms}}$$

Chapter 13, Problem 34.

ML Find the input impedance of the circuit in Fig. 13.103.

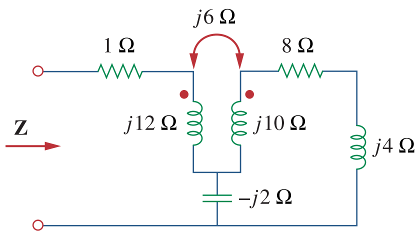
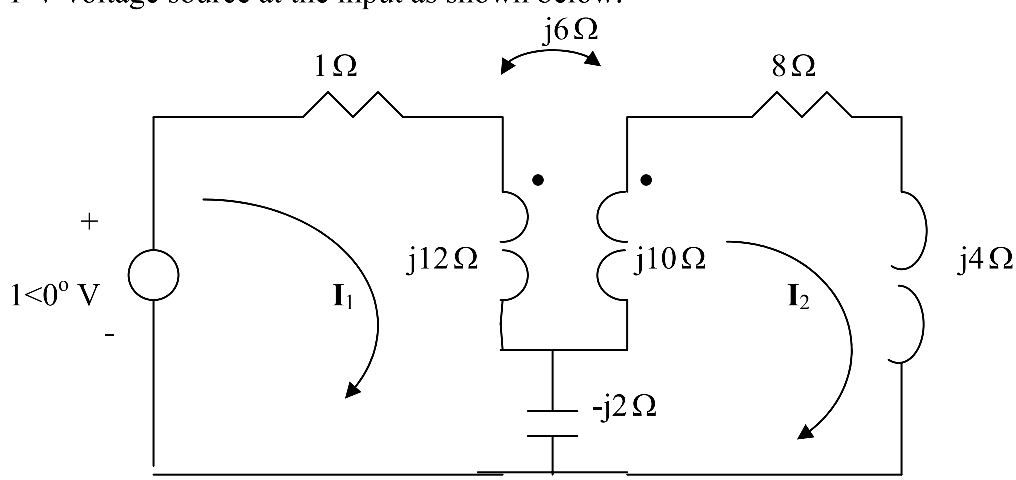


Figure 13.103
For Prob. 13.34.

Chapter 13, Solution 34.

Insert a 1-V voltage source at the input as shown below.



For loop 1,

$$1 = (1 + j10)I_1 - j4I_2 \qquad (1)$$

For loop 2,

$$0 = (8 + j4 + j10 - j2)I_2 + j2I_1 - j6I_1 \quad \longrightarrow \quad 0 = -jI_1 + (2 + j3)I_2 \qquad (2)$$

Solving (1) and (2) leads to $\mathbf{I}_1$=0.019 –j0.1068

$$Z = \frac{1}{I_1} = 1.6154 + j9.077 = \underline{9.219\angle 79.91^o\,\Omega}$$

Alternatively, an easier way to obtain **Z** is to replace the transformer with its equivalent T circuit and use series/parallel impedance combinations. This leads to exactly the same result.

Chapter 13, Problem 35.

PS ML * Find currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 13.104.

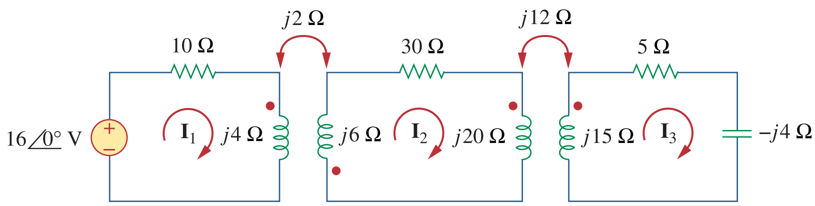


Figure 13.104
For Prob. 13.35.

* An asterisk indicates a challenging problem.

Chapter 13, Solution 35.

For mesh 1,

$$16 = (10 + j4)I_1 + j2I_2 \qquad (1)$$

For mesh 2,
$$0 = j2I_1 + (30 + j26)I_2 - j12I_3 \qquad (2)$$

For mesh 3,
$$0 = -j12I_2 + (5 + j11)I_3 \qquad (3)$$

We may use MATLAB to solve (1) to (3) and obtain

$$I_1 = 1.3736 - j0.5385 = \underline{1.4754\angle -21.41^o \ \ A}$$
$$I_2 = -0.0547 - j0.0549 = \underline{0.0775\angle -134.85^o \ \ A}$$
$$I_3 = -0.0268 - j0.0721 = \underline{0.077\angle -110.41^o \ \ A}$$

Chapter 13, Problem 36.

As done in Fig. 13.32, obtain the relationships between terminal voltages and currents for each of the ideal transformers in Fig. 13.105.

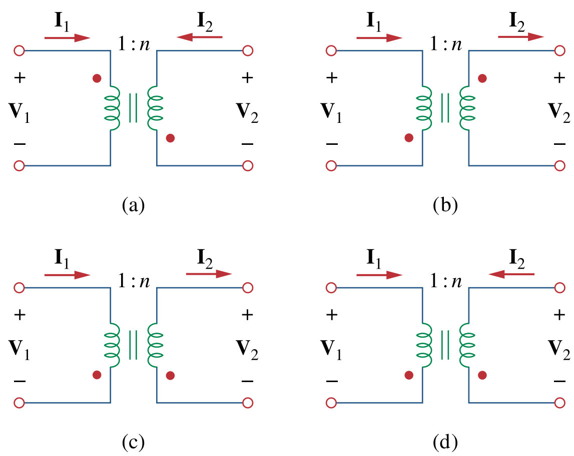


Figure 13.105
For Prob. 13.36.

Chapter 13, Solution 36.

Following the two rules in section 13.5, we obtain the following:

(a) $V_2/V_1 = \underline{\mathbf{-n}}$, $I_2/I_1 = \underline{\mathbf{-1/n}}$ $(n = V_2/V_1)$

(b) $V_2/V_1 = \underline{\mathbf{-n}}$, $I_2/I_1 = \underline{\mathbf{-1/n}}$

(c) $V_2/V_1 = \underline{\mathbf{n}}$, $I_2/I_1 = \underline{\mathbf{1/n}}$

(d) $V_2/V_1 = \underline{\mathbf{n}}$, $I_2/I_1 = \underline{\mathbf{-1/n}}$

Chapter 13, Problem 37.

A 480/2,400-V rms step-up ideal transformer delivers 50 kW to a resistive load. Calculate:

(a) the turns ratio
(b) the primary current
(c) the secondary current

Chapter 13, Solution 37.

(a) $n = \dfrac{V_2}{V_1} = \dfrac{2400}{480} = \underline{5}$

(b) $S_1 = I_1 V_1 = S_2 = I_2 V_2 = 50{,}000 \quad \longrightarrow \quad I_1 = \dfrac{50{,}000}{480} = \underline{104.17\text{ A}}$

(c) $I_2 = \dfrac{50{,}000}{2400} = \underline{20.83\text{ A}}$

Chapter 13, Problem 38.

A 4-kVA, 2,300/230-V rms transformer has an equivalent impedance of $2\angle 10^\circ\ \Omega$ on the primary side. If the transformer is connected to a load with 0.6 power factor leading, calculate the input impedance.

Chapter 13, Solution 38.

$Z_{in} = Z_p + Z_L/n^2, \quad n = v_2/v_1 = 230/2300 = 0.1$

$v_2 = 230\text{ V}, \quad s_2 = v_2 I_2^*$

$I_2^* = s_2/v_2 = 17.391\angle{-53.13^\circ} \text{ or } I_2 = 17.391\angle 53.13^\circ\text{ A}$

$Z_L = v_2/I_2 = 230\angle 0^\circ/17.391\angle 53.13^\circ = 13.235\angle{-53.13^\circ}$

$Z_{in} = 2\angle 10^\circ + 1323.5\angle{-53.13^\circ}$

$= 1.97 + j0.3473 + 794.1 - j1058.8$

$Z_{in} = \underline{\mathbf{1.324\angle{-53.05^\circ}\ kohms}}$

Chapter 13, Problem 39.

A 1,200/240-V rms transformer has impedance $60\angle -30^\circ\,\Omega$ on the high-voltage side. If the transformer is connected to a $0.8\angle 10^\circ\text{-}\Omega$ load on the low-voltage side, determine the primary and secondary currents when the transformer is connected to 1200 V rms.

Chapter 13, Solution 39.

Referred to the high-voltage side,

$$Z_L = (1200/240)^2(0.8\angle 10^\circ) = 20\angle 10^\circ$$

$$Z_{in} = 60\angle -30^\circ + 20\angle 10^\circ = 76.4122\angle -20.31^\circ$$

$$I_1 = 1200/Z_{in} = 1200/76.4122\angle -20.31^\circ = \underline{\mathbf{15.7\angle 20.31^\circ\ A}}$$

$$\text{Since } S = I_1 v_1 = I_2 v_2,\ I_2 = I_1 v_1/v_2$$

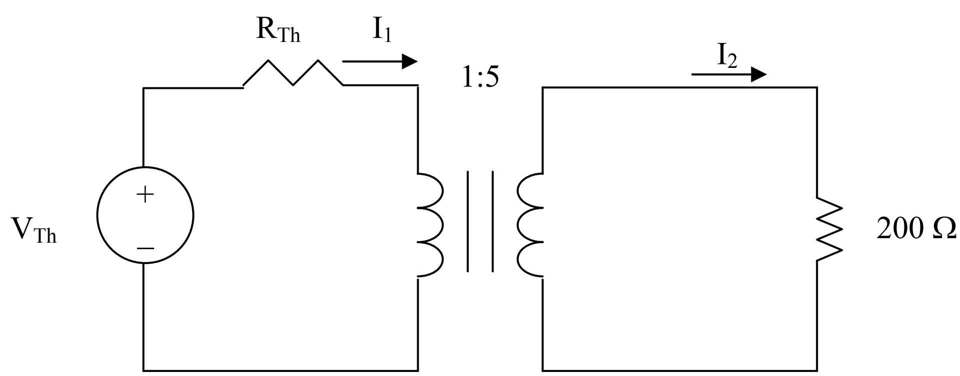
$$= (1200/240)(15.7\angle 20.31^\circ) = \underline{\mathbf{78.5\angle 20.31^\circ\ A}}$$

Chapter 13, Problem 40.

The primary of an ideal transformer with a turns ratio of 5 is connected to a voltage source with Thevenin parameters $v_{Th} = 10 \cos 2000t$ V and $R_{Th} = 100\,\Omega$ Determine the average power delivered to a 200-Ω load connected across the secondary winding.

Chapter 13, Solution 40.

Consider the circuit as shown below.



We reflect the 200-Ω load to the primary side.

$$Z_p = 100 + \frac{200}{5^2} = 108$$

$$I_1 = \frac{10}{108}, \qquad I_2 = \frac{I_1}{n} = 2/108$$

$$P = \frac{1}{2} | I_2 |^2 R_L = \frac{1}{2} (\frac{2}{108})^2 (200) = \underline{34.3 \text{ mW}}$$

Chapter 13, Problem 41.

Determine $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.106.

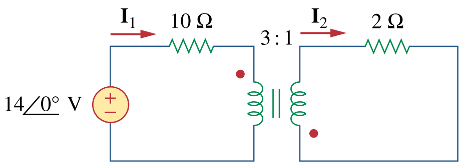


Figure 13.106
For Prob. 13.41.

Chapter 13, Solution 41.

We reflect the 2-ohm resistor to the primary side.

$$Z_{in} = 10 + 2/n^2, \quad n = -1/3$$

Since both I_1 and I_2 enter the dotted terminals, $\quad Z_{in} = 10 + 18 = 28$ ohms

$$I_1 = 14\angle 0°/28 = \underline{\mathbf{0.5\ A}} \text{ and } I_2 = I_1/n = 0.5/(-1/3) = \underline{\mathbf{-1.5\ A}}$$

Chapter 13, Problem 42.

For the circuit in Fig. 13.107, determine the power absorbed by the 2-Ω resistor. Assume the 80 V is an rms value.

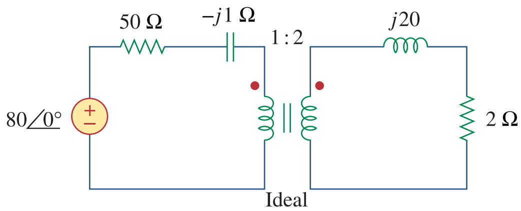
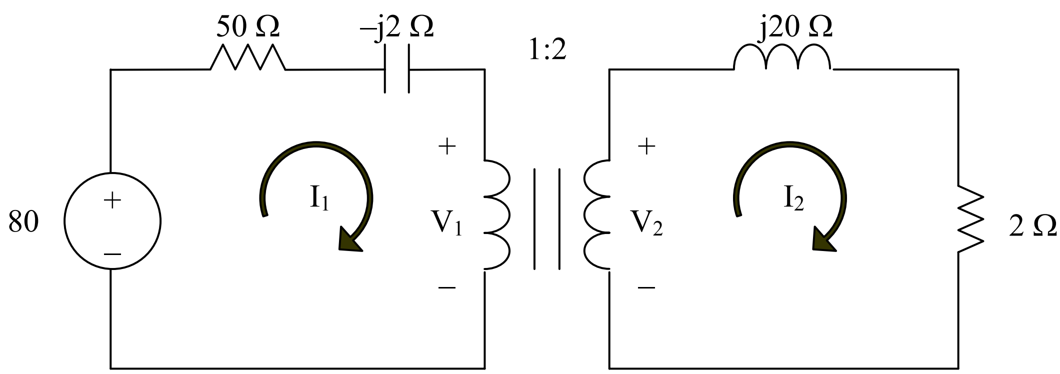


Figure 13.107
For Prob. 13.42.

Chapter 13, Solution 42.

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$80 = (50 - j2)I_1 + V_1 \qquad (1)$$

For mesh 2,

$$-V_2 + (2 - j20)I_2 = 0 \qquad (2)$$

At the transformer terminals,

$$V_2 = 2V_1 \qquad (3)$$

$$I_1 = 2I_2 \qquad (4)$$

From (1) to (4),

$$\begin{bmatrix} (50-j2) & 0 & 1 & 0 \\ 0 & (2-j20) & 0 & 1 \\ 0 & 0 & 2 & -1 \\ 1 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this with MATLAB gives

$I_2 = 0.8051 – j0.0488 = 0.8056\angle –3.47^\circ$.

The power absorbed by the 2-Ω resistor is

$$P = |I_2|^2 R = (0.8056)^2 2 = \underline{\mathbf{1.3012\ W}}.$$

Chapter 13, Problem 43.

Obtain $\mathbf{V}_1$ and $\mathbf{V}_2$ in the ideal transformer circuit of Fig. 13.108.

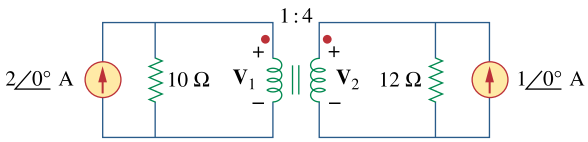
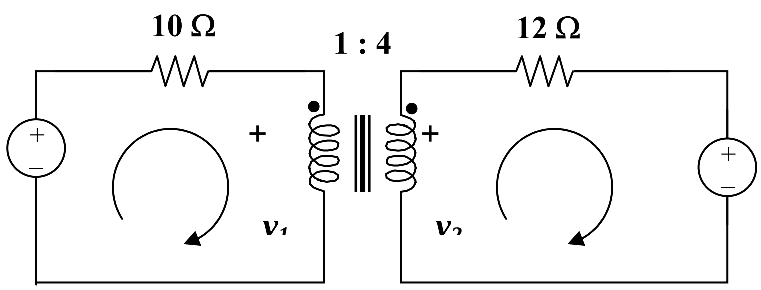


Figure 13.108
For Prob. 13.43.

Chapter 13, Solution 43.

Transform the two current sources to voltage sources, as shown below.



Using mesh analysis, $\quad -20 + 10I_1 + v_1 = 0$

$$20 = v_1 + 10I_1 \qquad (1)$$

$$12 + 12I_2 - v_2 = 0 \quad \text{or} \quad 12 = v_2 - 12I_2 \qquad (2)$$

At the transformer terminal, $\quad v_2 = nv_1 = 4v_1 \qquad (3)$

$$I_1 = nI_2 = 4I_2 \qquad (4)$$

Substituting (3) and (4) into (1) and (2), we get,

$$20 = v_1 + 40I_2 \qquad (5)$$

$$12 = 4v_1 - 12I_2 \qquad (6)$$

Solving (5) and (6) gives $\quad v_1 = \underline{\mathbf{4.186\ V}}$ and $v_2 = 4v = \underline{\mathbf{16.744\ V}}$

Chapter 13, Problem 44.

*In the ideal transformer circuit of Fig. 13.109, find $i_1(t)$ and $i_2(t)$.

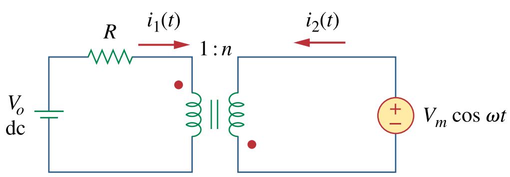


Figure 13.109
For Prob. 13.44.

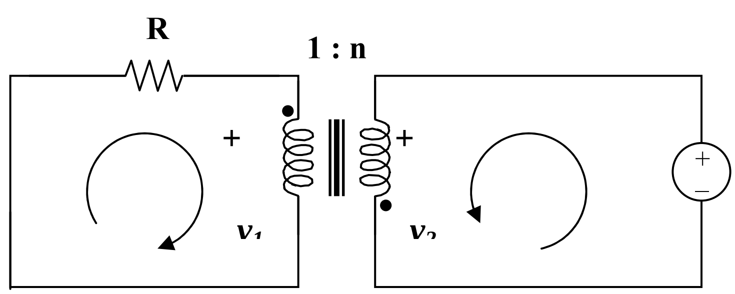
* An asterisk indicates a challenging problem.

Chapter 13, Solution 44.

We can apply the superposition theorem. Let $i_1 = i_1' + i_1''$ and $i_2 = i_2' + i_2''$ where the single prime is due to the DC source and the double prime is due to the AC source. Since we are looking for the steady-state values of i_1 and i_2,

$$i_1' = i_2' = 0.$$

For the AC source, consider the circuit below.



$$v_2/v_1 = -n, \quad I_2''/I_1'' = -1/n$$

But $v_2 = v_m$, $v_1 = -v_m/n$ or $I_1'' = v_m/(Rn)$

$$I_2'' = -I_1''/n = -v_m/(Rn^2)$$

Hence, $i_1(t) = \underline{\mathbf{(v_m/Rn)\cos\omega t\ A}}$, and $i_2(t) = \underline{\mathbf{(-v_m/(n^2R))\cos\omega t\ A}}$

Chapter 13, Problem 45.

For the circuit shown in Fig. 13.110, find the value of the average power absorbed by the 8-Ω resistor.

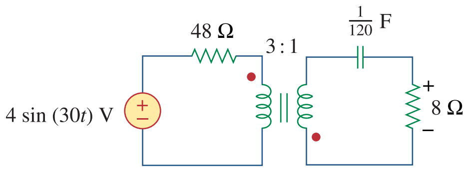
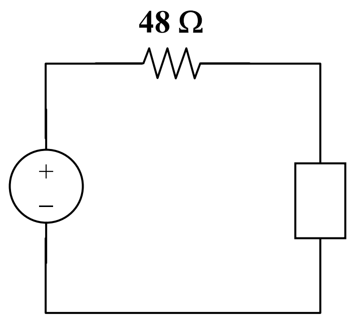


Figure 13.110
For Prob. 13.45.

Chapter 13, Solution 45.



$$Z_L = 8 - \frac{j}{\omega C} = 8 - j4, \quad n = 1/3$$

$$Z = \frac{Z_L}{n^2} = 9Z_L = 72 - j36$$

$$I = \frac{4\angle -90°}{48 + 72 - j36} = \frac{4\angle -90°}{125.28\angle -16.7°} = 0.03193\angle -73.3°$$

We now have some choices, we can go ahead and calculate the current in the second loop and calculate the power delivered to the 8-ohm resistor directly or we can merely say that the power delivered to the equivalent resistor in the primary side must be the same as the power delivered to the 8-ohm resistor. Therefore,

$$P_{8\Omega} = \left|\frac{I}{2}\right|^2 72 = 0.5098\text{x}10^{-3} 72 = \underline{\mathbf{36.71\ mW}}$$

The student is encouraged to calculate the current in the secondary and calculate the power delivered to the 8-ohm resistor to verify that the above is correct.

Chapter 13, Problem 46.

(a) Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.111 below.
(b) Switch the dot on one of the windings. Find $\mathbf{I}_1$ and $\mathbf{I}_2$ again.

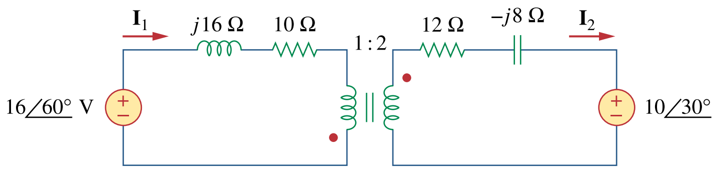
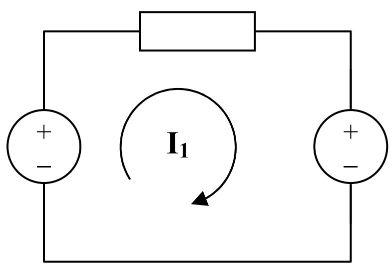


Figure 13.111
For Prob. 13.46.

Chapter 13, Solution 46.

(a) Reflecting the secondary circuit to the primary, we have the circuit shown below.



$$Z_{in} = 10 + j16 + (1/4)(12 - j8) = 13 + j14$$

$$-16\angle 60° + Z_{in}I_1 - 5\angle 30° = 0 \text{ or } I_1 = (16\angle 60° + 5\angle 30°)/(13 + j14)$$

Hence, $I_1 = \underline{\mathbf{1.072\angle 5.88°\ A}}$, and $I_2 = -0.5I_1 = \underline{\mathbf{0.536\angle 185.88°\ A}}$

(b) Switching a dot will not effect Z_{in} but will effect I_1 and I_2.

$$I_1 = (16\angle 60° - 5\angle 30°)/(13 + j14) = \underline{\mathbf{0.625\ \angle 25\ A}}$$

and $I_2 = 0.5I_1 = \underline{\mathbf{0.3125\angle 25°\ A}}$

Chapter 13, Problem 47.

Find $v(t)$ for the circuit in Fig. 13.112.

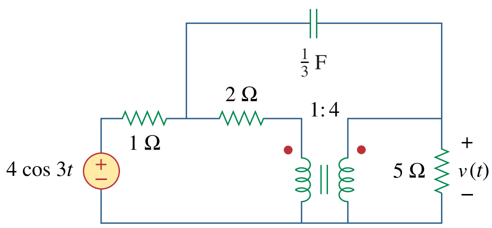
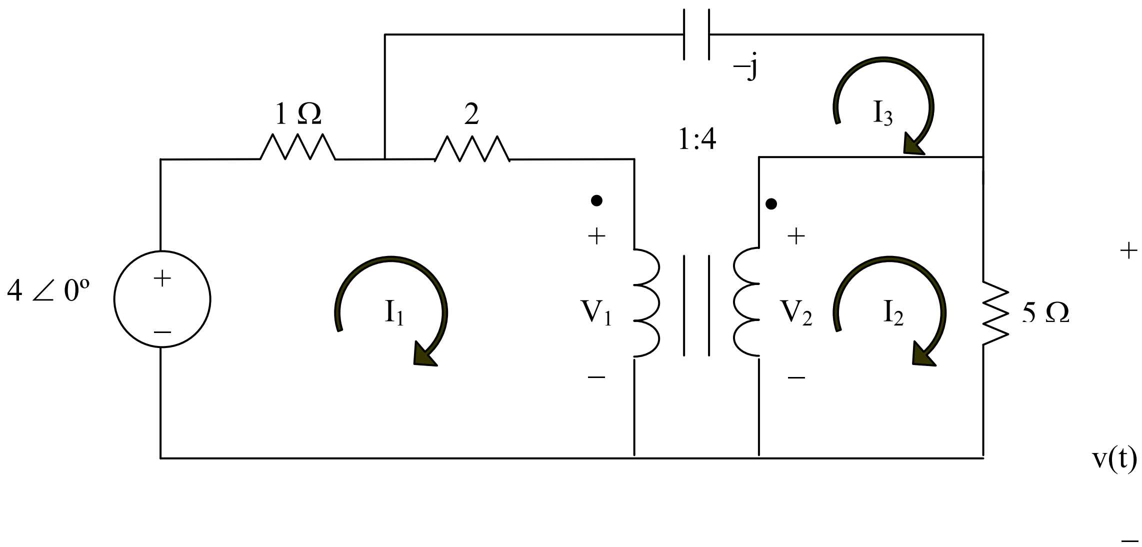


Figure 13.112
For Prob. 13.47.

Chapter 13, Solution 47.

$$1F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j3x1/3} = -j1$$

Consider the circuit shown below.



For mesh 1,

$3I_1 - 2I_3 + V_1 = 4$ (1)

For mesh 2,

$5I_2 - V_2 = 0$ (2)

For mesh 3,

$-2I_1 (2-j)I_3 - V_1 + V_2 = 0$ (3)

At the terminals of the transformer,

$V_2 = nV_1 = 4V_1$ (4)

$I_1 = nI_2 = 4I_2$ (5)

In matrix form,

$$\begin{bmatrix} 3 & 0 & -2 & 1 & 0 \\ 0 & 5 & 0 & 0 & -1 \\ -2 & 0 & 2-j & -1 & 1 \\ 0 & 0 & 0 & -4 & 1 \\ 1 & -4 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this using MATLAB yields

```
A = [3,0,-2,1,0;0,5,0,0,-1;-2,0,(2-i),-1,1;0,0,0,-4,1;1,-4,0,0,0]
U = [4;0;0;0;0]
X = inv(A)*U
>> A = [3,0,-2,1,0;0,5,0,0,-1;-2,0,(2-i),-1,1;0,0,0,-4,1;1,-4,0,0,0]

A =
  Columns 1 through 4

   3.0000             0            -2.0000             1.0000
        0        5.0000                  0                  0
  -2.0000             0        2.0000 - 1.0000i     -1.0000
        0             0             0            -4.0000
   1.0000       -4.0000                  0                  0

  Column 5

        0
  -1.0000
   1.0000
   1.0000
        0

>> U = [4;0;0;0;0]

U =
     4
     0
     0
     0
     0

>> X = inv(A)*U

X =
   1.5774 + 0.2722i
   0.3943 + 0.0681i
   0.6125 + 0.4509i
   0.4929 + 0.0851i
   1.9717 + 0.3403i
```

$I_2 = 0.3943+j0.681 = 0.7869\angle 59.93^\circ$ but $V = 5I_2 = 3.934\angle 59.93^\circ$.

$v(t) =$ **$3.934\cos(3t+59.93^\circ)$ V**

Chapter 13, Problem 48.

Find $\mathbf{I}_x$ in the ideal transformer circuit of Fig. 13.113.

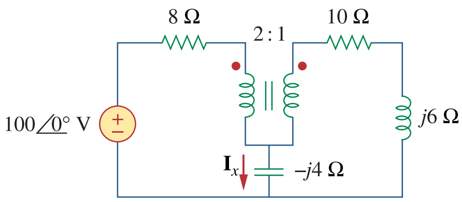
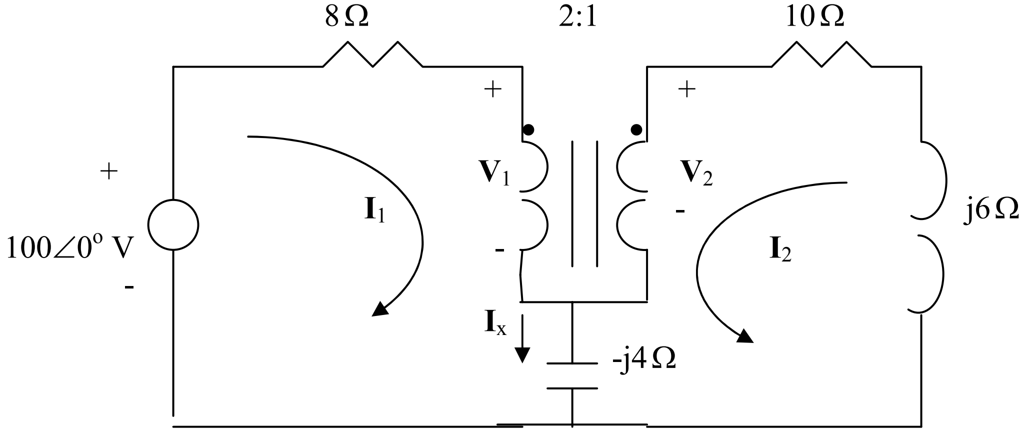


Figure 13.113
For Prob. 13.48.

Chapter 13, Solution 48.

We apply mesh analysis.



$$100 = (8 - j4)I_1 - j4I_2 + V_1 \qquad (1)$$

$$0 = (10 + j2)I_2 - j4I_1 + V_2 \qquad (2)$$

But

$$\frac{V_2}{V_1} = n = \frac{1}{2} \quad \longrightarrow \quad V_1 = 2V_2 \qquad (3)$$

$$\frac{I_2}{I_1} = -\frac{1}{n} = -2 \quad \longrightarrow \quad I_1 = -0.5I_2 \qquad (4)$$

Substituting (3) and (4) into (1) and (2), we obtain

$$100 = (-4 - j2)I_2 + 2V_2 \qquad (1)\text{a}$$
$$0 = (10 + j4)I_2 + V_2 \qquad (2)\text{a}$$

Solving (1)a and (2)a leads to $\mathbf{I}_2$ = -3.5503 +j1.4793

$$I_x = I_1 + I_2 = 0.5I_2 = \underline{1.923\angle 157.4^o \text{ A}}$$

Chapter 13, Problem 49.

Find current i_x in the ideal transformer circuit shown in Fig. 13.114.

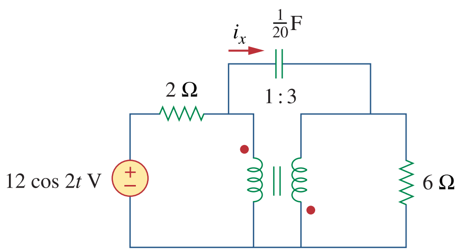
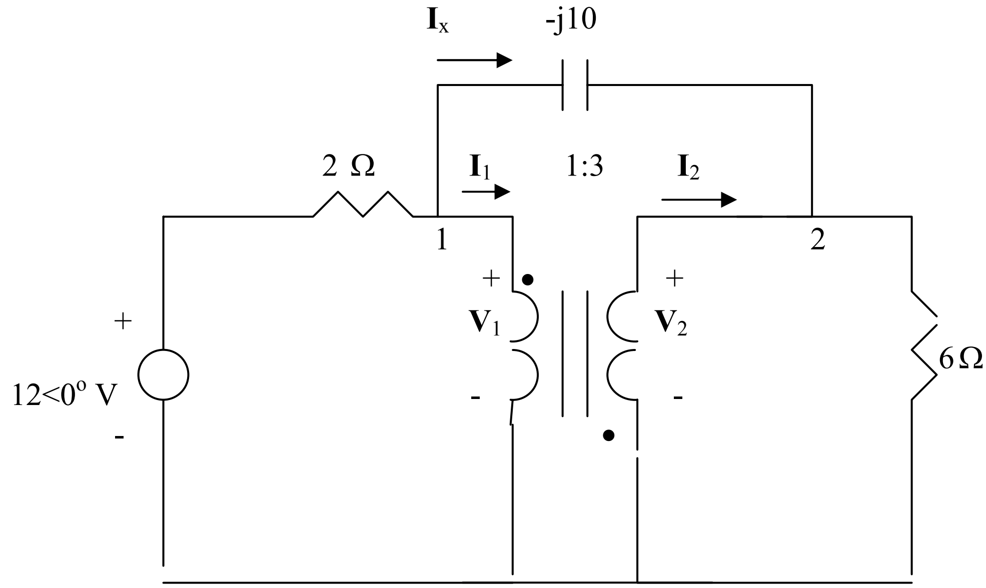


Figure 13.114
For Prob. 13.49.

Chapter 13, Solution 49.

$\omega = 2, \quad \frac{1}{20}\text{F} \quad \longrightarrow \quad \frac{1}{j\omega C} = -j10$



At node 1,

$$\frac{12 - V_1}{2} = \frac{V_1 - V_2}{-j10} + I_1 \quad \longrightarrow \quad 12 = 2I_1 + V_1(1 + j0.2) - j0.2V_2 \qquad (1)$$

At node 2,

$$I_2 + \frac{V_1 - V_2}{-j10} = \frac{V_2}{6} \quad \longrightarrow \quad 0 = 6I_2 + j0.6V_1 - (1 + j0.6)V_2 \qquad (2)$$

At the terminals of the transformer, $V_2 = -3V_1, \quad I_2 = -\frac{1}{3}I_1$

Substituting these in (1) and (2),

$$12 = -6I_2 + V_1(1 + j0.8), \qquad 0 = 6I_2 + V_1(3 + j2.4)$$

Adding these gives $\mathbf{V}_1$=1.829 –j1.463 and

$$I_x = \frac{V_1 - V_2}{-j10} = \frac{4V_1}{-j10} = 0.937\angle 51.34^o$$

$$\underline{i_x = 0.937\cos(2t + 51.34^\circ)\ \text{A}}$$

Chapter 13, Problem 50.

ML Calculate the input impedance for the network in Fig. 13.115.

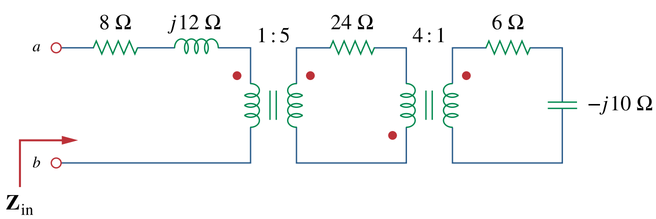


Figure 13.115
For Prob. 13.50.

Chapter 13, Solution 50.

The value of Z_{in} is not effected by the location of the dots since n^2 is involved.

$Z_{in}' = (6 - j10)/(n')^2, \ n' = 1/4$

$Z_{in}' = 16(6 - j10) = 96 - j160$

$Z_{in} = 8 + j12 + (Z_{in}' + 24)/n^2, \quad n = 5$

$Z_{in} = 8 + j12 + (120 - j160)/25 = 8 + j12 + 4.8 - j6.4$

Z_{in} = <u>**(12.8 + j5.6) ohms**</u>

Chapter 13, Problem 51.

Use the concept of reflected impedance to find the input impedance and current $\mathbf{I}_1$ in Fig. 13.116.

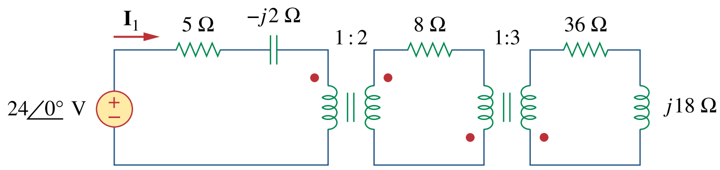


Figure 13.116
For Prob. 13.51.

Chapter 13, Solution 51.

Let $Z_3 = 36 + j18$, where Z_3 is reflected to the middle circuit.

$Z_R' = Z_L/n^2 = (12 + j2)/4 = 3 + j0.5$

$Z_{in} = 5 - j2 + Z_R' =$ **$(8 - j1.5)$ ohms**

$I_1 = 24\angle 0°/Z_{Th} = 24\angle 0°/(8 - j1.5) = 24\angle 0°/8.14\angle -10.62° =$ **$8.95\angle 10.62°$ A**

Chapter 13, Problem 52.

For the circuit in Fig. 13.117, determine the turns ratio *n* that will cause maximum average power transfer to the load. Calculate that maximum average power.

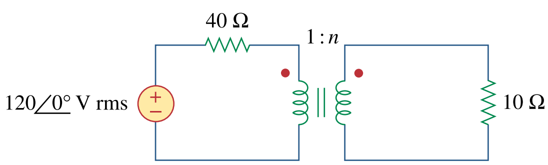


Figure 13.117
For Prob. 13.52.

Chapter 13, Solution 52.

For maximum power transfer,

$40 = Z_L/n^2 = 10/n^2$ or $n^2 = 10/40$ which yields $n = 1/2 = 0.5$

$I = 120/(40 + 40) = 3/2$

$p = I^2R = (9/4)x40 =$ **90 watts**.

Chapter 13, Problem 53.

ML Refer to the network in Fig. 13.118.

(a) Find *n* for maximum power supplied to the 200-Ω load.
(b) Determine the power in the 200-Ω load if *n* = 10.

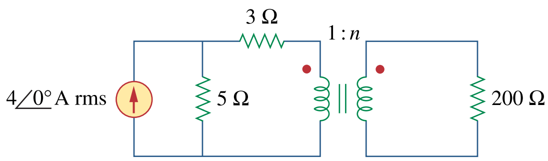
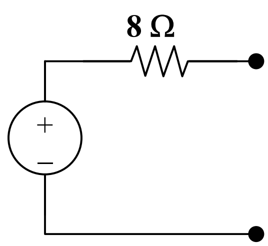


Figure 13.118
For Prob. 13.53.

Chapter 13, Solution 53.

(a) The Thevenin equivalent to the left of the transformer is shown below.



The reflected load impedance is $Z_L' = Z_L/n^2 = 200/n^2$.

For maximum power transfer, $8 = 200/n^2$ produces $n = $ **5**.

(b) If $n = 10$, $Z_L' = 200/10 = 2$ and $I = 20/(8 + 2) = 2$

$p = I^2 Z_L' = (2)^2(2) =$ **8 watts**.

Chapter 13, Problem 54.

A transformer is used to match an amplifier with an 8-Ω load as shown in Fig. 13.119. The Thevenin equivalent of the amplifier is: V_{Th} = 10 V, Z_{Th} = 128 Ω.

(a) Find the required turns ratio for maximum energy power transfer.
(b) Determine the primary and secondary currents.
(c) Calculate the primary and secondary voltages.

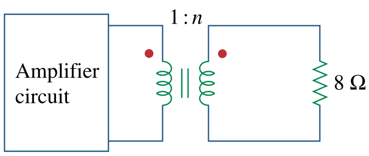
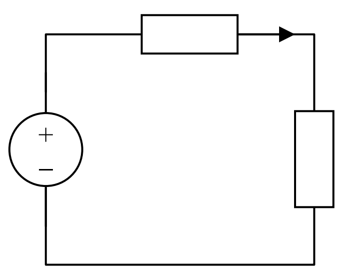


Figure 13.119
For Prob. 13.54.

Chapter 13, Solution 54.

(a)



For maximum power transfer,

$$Z_{Th} = Z_L/n^2, \text{ or } n^2 = Z_L/Z_{Th} = 8/128$$

$$n = \underline{\mathbf{0.25}}$$

(b) $\quad I_1 = V_{Th}/(Z_{Th} + Z_L/n^2) = 10/(128 + 128) = \underline{\mathbf{39.06\text{ mA}}}$

(c) $\quad v_2 = I_2 Z_L = 156.24\text{x}8\text{ mV} = 1.25\text{ V}$

But $\quad v_2 = nv_1$ therefore $v_1 = v_2/n = 4(1.25) = \underline{\mathbf{5\text{ V}}}$

Chapter 13, Problem 55.

ML For the circuit in Fig. 13.120, calculate the equivalent resistance.

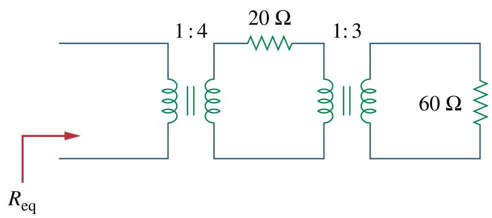


Figure 13.120
For Prob. 13.55.

Chapter 13, Solution 55.

We first reflect the 60-Ω resistance to the middle circuit.

$$Z_L^{'} = 20 + \frac{60}{3^2} = 26.67\Omega$$

We now reflect this to the primary side.

$$Z_L = \frac{Z_L^{'}}{4^2} = \frac{26.67}{16} = \underline{\mathbf{1.6669\ \Omega}}$$

Chapter 13, Problem 56.

Find the power absorbed by the 10-Ω resistor in the ideal transformer circuit of Fig. 13.121.

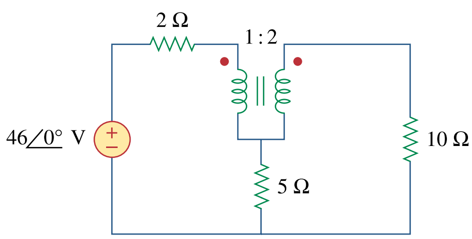
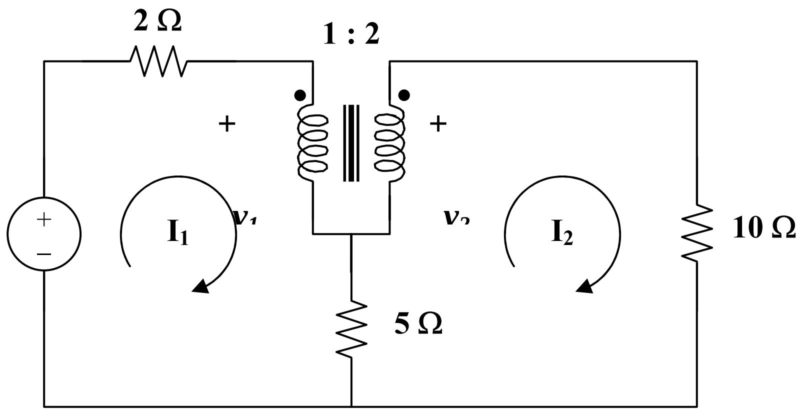


Figure 13.121
For Prob. 13.56.

Chapter 13, Solution 56.

We apply mesh analysis to the circuit as shown below.



For mesh 1, $$46 = 7I_1 - 5I_2 + v_1 \quad (1)$$

For mesh 2, $$v_2 = 15I_2 - 5I_1 \quad (2)$$

At the terminals of the transformer,

$$v_2 = nv_1 = 2v_1 \quad (3)$$

$$I_1 = nI_2 = 2I_2 \quad (4)$$

Substituting (3) and (4) into (1) and (2),

$$46 = 9I_2 + v_1 \quad (5)$$

$$v_1 = 2.5I_2 \quad (6)$$

Combining (5) and (6), $$46 = 11.5I_2 \text{ or } I_2 = 4$$

$$P_{10} = 0.5I_2^2(10) = \underline{\mathbf{80\ watts}}.$$

Chapter 13, Problem 57.

For the ideal transformer circuit of Fig. 13.122 below, find:

(a) $\mathbf{I}_1$ and $\mathbf{I}_2$,
(b) $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_o$,
(c) the complex power supplied by the source.

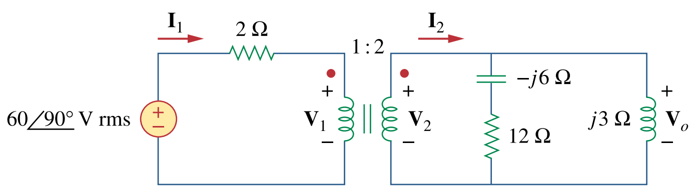


Figure 13.122
For Prob. 13.57.

Chapter 13, Solution 57.

(a) $\quad Z_L = j3\|(12 - j6) = j3(12 - j6)/(12 - j3) = (12 + j54)/17$

Reflecting this to the primary side gives

$$Z_{in} = 2 + Z_L/n^2 = 2 + (3 + j13.5)/17 = 2.3168\angle 20.04°$$

$$I_1 = v_s/Z_{in} = 60\angle 90°/2.3168\angle 20.04° = \underline{\mathbf{25.9\angle 69.96°\ A(rms)}}$$

$$I_2 = I_1/n = \underline{\mathbf{12.95\angle 69.96°\ A(rms)}}$$

(b) $\quad 60\angle 90° = 2I_1 + v_1 \text{ or } v_1 = j60 - 2I_1 = j60 - 51.8\angle 69.96°$

$$v_1 = \underline{\mathbf{21.06\angle 147.44°\ V(rms)}}$$

$$v_2 = nv_1 = \underline{\mathbf{42.12\angle 147.44°\ V(rms)}}$$

$$v_o = v_2 = \underline{\mathbf{42.12\angle 147.44°\ V(rms)}}$$

(c) $\quad S = v_s I_1^* = (60\angle 90°)(25.9\angle -69.96°) = \underline{\mathbf{1554\angle 20.04°\ VA}}$

Chapter 13, Problem 58.

Determine the average power absorbed by each resistor in the circuit of Fig. 13.123.

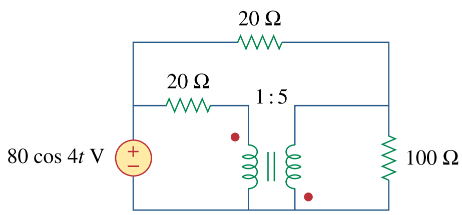
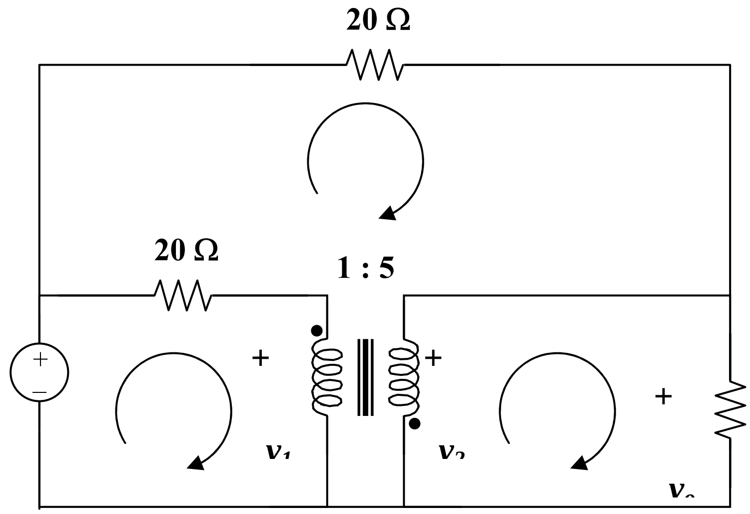


Figure 13.123
For Prob. 13.58.

Chapter 13, Solution 58.

Consider the circuit below.



For mesh1, $80 = 20I_1 - 20I_3 + v_1$ (1)

For mesh 2, $v_2 = 100I_2$ (2)

For mesh 3, $0 = 40I_3 - 20I_1$ which leads to $I_1 = 2I_3$ (3)

At the transformer terminals, $v_2 = -nv_1 = -5v_1$ (4)

$I_1 = -nI_2 = -5I_2$ (5)

From (2) and (4), $-5v_1 = 100I_2$ or $v_1 = -20I_2$ (6)

Substituting (3), (5), and (6) into (1),

$$4 = I_1 - I_2 - I_3 = I_1 - (I_1/(-5)) - I_1/2 = (7/10)I_1$$

$$I_1 = 40/7,\ I_2 = -8/7,\ I_3 = 20/7$$

p_{20}(the one between 1 and 3) $= 0.5(20)(I_1 - I_3)^2 = 10(20/7)^2 =$ **81.63 watts**

p_{20}(at the top of the circuit) $= 0.5(20)I_3^2 =$ **81.63 watts**

$p_{100} = 0.5(100)I_2^2 =$ **65.31 watts**

Chapter 13, Problem 59.

PS ML In the circuit of Fig. 13.124, let $v_s = 40 \cos 1000t$. Find the average power delivered to each resistor.

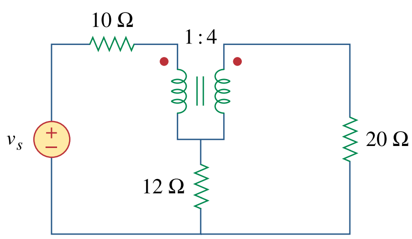
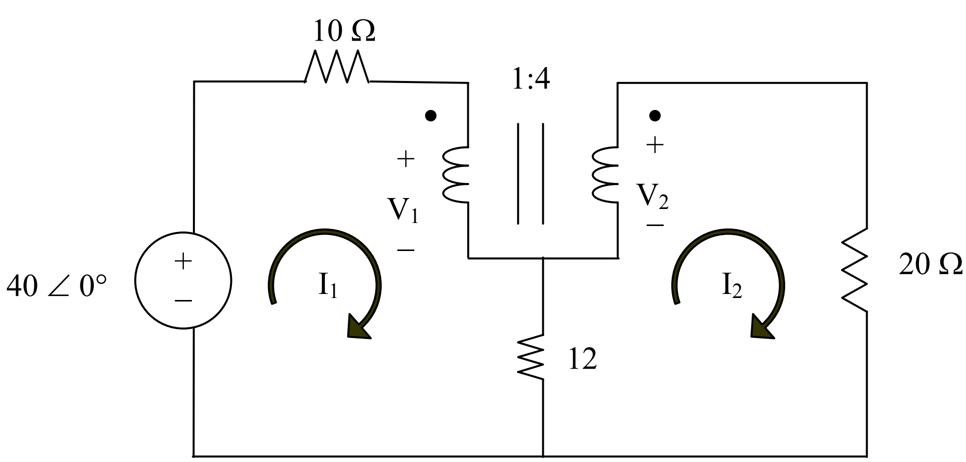


Figure 13.124
For Prob. 13.59.

Chapter 13, Solution 59.

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$-40 + 22I_1 - 12I_2 + V_1 = 0 \quad (1)$$

For mesh 2,

$$-12I_1 + 32I_2 - V_2 = 0 \quad (2)$$

At the transformer terminals,

$$-4V_1 + V_2 = 0 \quad (3)$$

$$I_1 - 4I_2 = 0 \quad (4)$$

Putting (1), (2), (3), and (4) in matrix form, we obtain

$$\begin{bmatrix} 22 & -12 & 1 & 0 \\ -12 & 32 & 0 & -1 \\ 0 & 0 & -4 & 1 \\ 1 & -4 & 0 & 0 \end{bmatrix} I = \begin{bmatrix} 40 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
>> A=[22,-12,1,0;-12,32,0,-1;0,0,-4,1;1,-4,0,0]
A =
   22  -12    1    0
  -12   32    0   -1
    0    0   -4    1
    1   -4    0    0
>> U=[40;0;0;0]
U =
   40
    0
    0
    0
>> X=inv(A)*U
X =
   2.2222
   0.5556
  -2.2222
  -8.8889
```

For 10-Ω resistor,

$P_{10} = [(2.222)^2/2]10 =$ **<u>24.69 W</u>**

For 12-Ω resistor,

$P_{12} = [(2.222–0.5556)^2/2]12 =$ **<u>16.661 W</u>**

For 20-Ω resistor,

$P_{20} = [(0.5556)^2/2]20 =$ **<u>3.087 W</u>**.

Chapter 13, Problem 60.

PS ML Refer to the circuit in Fig. 13.125 on the following page.

(a) Find currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.
(b) Find the power dissipated in the 40-Ω resistor.

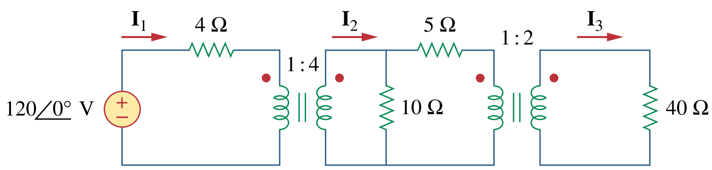


Figure 13.125
For Prob. 13.60.

Chapter 13, Solution 60.

(a) Transferring the 40-ohm load to the middle circuit,

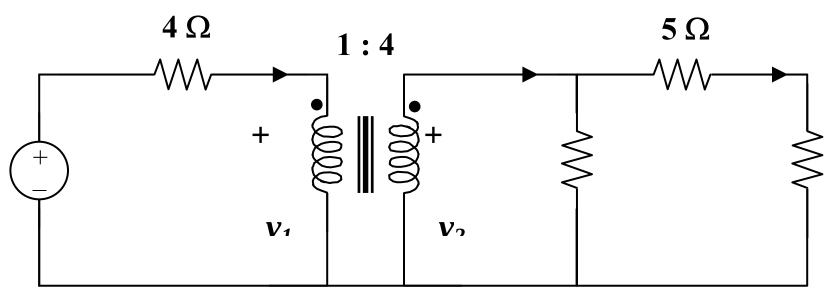
$$Z_L' = 40/(n')^2 = 10 \text{ ohms where } n' = 2$$

$$10||(5 + 10) = 6 \text{ ohms}$$

We transfer this to the primary side.

$$Z_{in} = 4 + 6/n^2 = 4 + 96 = 100 \text{ ohms, where } n = 0.25$$

$I_1 = 120/100 = \underline{\mathbf{1.2\ A}}$ and $I_2 = I_1/n = \underline{\mathbf{4.8\ A}}$



Using current division, $I_2' = (10/25)I_2 = 1.92$ and $I_3 = I_2'/n' = \underline{\mathbf{0.96\ A}}$

(b) $p = 0.5(I_3)^2(40) = \underline{\mathbf{18.432\ watts}}$

Chapter 13, Problem 61.

* For the circuit in Fig. 13.126, find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{V}_o$.

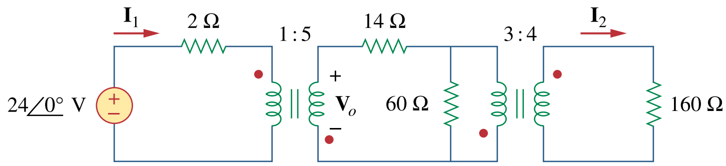


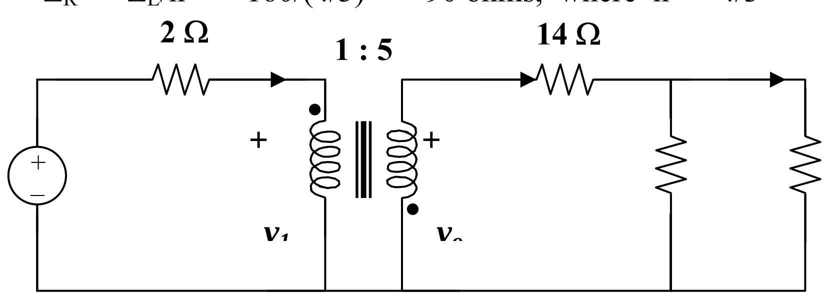
Figure 13.126
For Prob. 13.61.

* An asterisk indicates a challenging problem.

Chapter 13, Solution 61.

We reflect the 160-ohm load to the middle circuit.

$$Z_R = Z_L/n^2 = 160/(4/3)^2 = 90 \text{ ohms, where } n = 4/3$$



$$14 + 60||90 = 14 + 36 = 50 \text{ ohms}$$

We reflect this to the primary side.

$$Z_R' = Z_L'/(n')^2 = 50/5^2 = 2 \text{ ohms when } n' = 5$$

$$I_1 = 24/(2 + 2) = \underline{\mathbf{6A}}$$

$$24 = 2I_1 + v_1 \text{ or } v_1 = 24 - 2I_1 = 12 \text{ V}$$

$$v_o = -nv_1 = \underline{\mathbf{-60\ V}},\ I_o = -I_1/n_1 = -6/5 = -1.2$$

$$I_o' = [60/(60 + 90)]I_o = -0.48\text{A}$$

$$I_2 = -I_o'/n = 0.48/(4/3) = \underline{\mathbf{0.36\ A}}$$

Chapter 13, Problem 62.

For the network in Fig. 13.127, find

(a) the complex power supplied by the source,
(b) the average power delivered to the 18-Ω resistor.

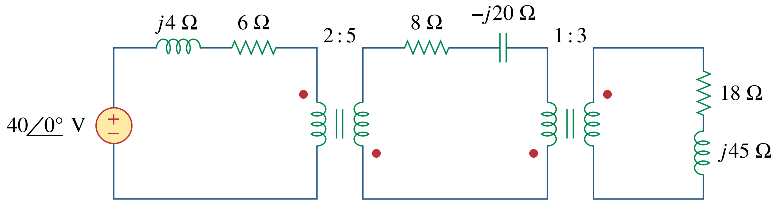


Figure 13.127
For Prob. 13.62.

Chapter 13, Solution 62.

(a) Reflect the load to the middle circuit.

$Z_L' = 8 - j20 + (18 + j45)/3^2 = 10 - j15$

We now reflect this to the primary circuit so that

$Z_{in} = 6 + j4 + (10 - j15)/n^2 = 7.6 + j1.6 = 7.767\angle 11.89°$, where $n = 5/2 = 2.5$

$I_1 = 40/Z_{in} = 40/7.767\angle 11.89° = 5.15\angle{-11.89°}$

$S = 0.5v_sI_1^* = (20\angle 0°)(5.15\angle 11.89°) = \underline{\mathbf{103\angle 11.89°\ VA}}$

(b) $I_2 = -I_1/n, \quad n = 2.5$

$I_3 = -I_2/n', \quad n = 3$

$I_3 = I_1/(nn') = 5.15\angle{-11.89°}/(2.5x3) = 0.6867\angle{-11.89°}$

$p = 0.5|I_2|^2(18) = 9(0.6867)^2 = \underline{\mathbf{4.244\ watts}}$

Chapter 13, Problem 63.

ML Find the mesh currents in the circuit of Fig. 13.128

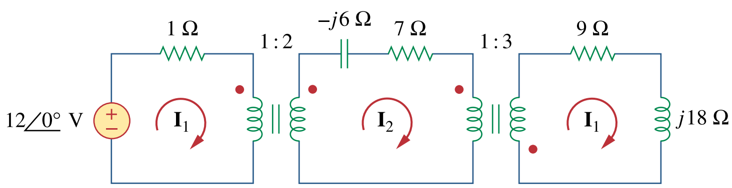


Figure 13.128
For Prob. 13.63.

Chapter 13, Solution 63.

Reflecting the (9 + j18)-ohm load to the middle circuit gives,

$Z_{in}' = 7 - j6 + (9 + j18)/(n')^2 = 7 - j6 + 1 + j2 = 8 - j4$ when $n' = 3$

Reflecting this to the primary side,

$Z_{in} = 1 + Z_{in}'/n^2 = 1 + 2 - j = 3 - j$, where $n = 2$

$I_1 = 12\angle 0°/(3 - j) = 12/3.162\angle -18.43° =$ **$\underline{3.795\angle 18.43A}$**

$I_2 = I_1/n =$ **$\underline{1.8975\angle 18.43°\ A}$**

$I_3 = -I_2/n^2 =$ **$\underline{632.5\angle 161.57°\ mA}$**

Chapter 13, Problem 64.

PS ML For the circuit in Fig. 13.129, find the turns ratio so that the maximum power is delivered to the 30-kΩ resistor.

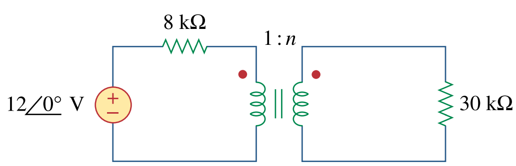
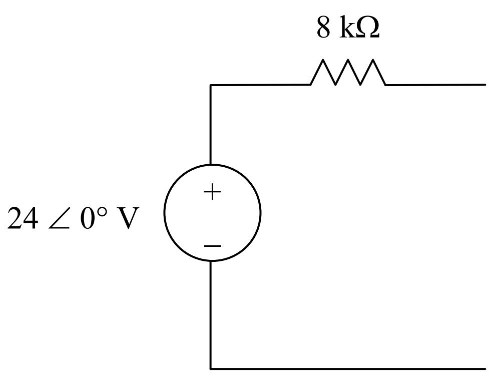


Figure 13.129
For Prob. 13.64.

Chapter 13, Solution 64.

The Thevenin equivalent to the left of the transformer is shown below.



The reflected load impedance is

$$Z_L' = \frac{Z_L}{n^2} = \frac{30k}{n^2}$$

For maximum power transfer,

$$8k\Omega = \frac{30k\Omega}{n^2} \longrightarrow n^2 = 30/8 = 3.75$$

n =**<u>1.9365</u>**

Chapter 13, Problem 65.

* Calculate the average power dissipated by the 20-Ω resistor in Fig. 13.130.

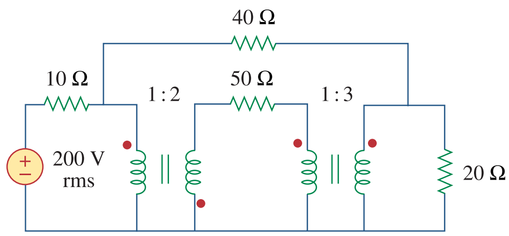
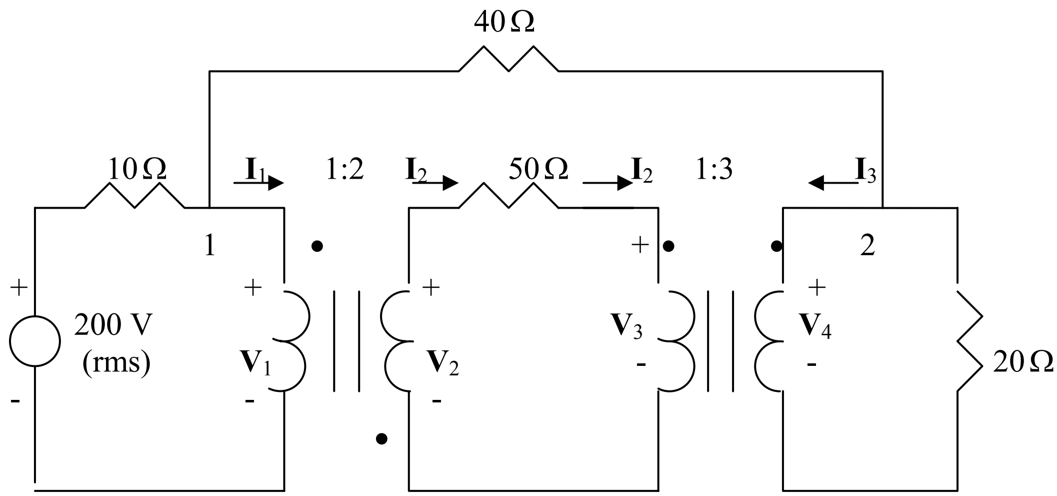


Figure 13.130
For Prob. 13.65.

* An asterisk indicates a challenging problem.

Chapter 13, Solution 65.



At node 1,

$$\frac{200 - V_1}{10} = \frac{V_1 - V_4}{40} + I_1 \quad \longrightarrow \quad 200 = 1.25V_1 - 0.25V_4 + 10I_1 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_4}{40} = \frac{V_4}{20} + I_3 \quad \longrightarrow \quad V_1 = 3V_4 + 40I_3 \qquad (2)$$

At the terminals of the first transformer,

$$\frac{V_2}{V_1} = -2 \quad \longrightarrow \quad V_2 = -2V_1 \qquad (3)$$

$$\frac{I_2}{I_1} = -1/2 \quad \longrightarrow \quad I_1 = -2I_2 \qquad (4)$$

For the middle loop,

$$-V_2 + 50I_2 + V_3 = 0 \quad \longrightarrow \quad V_3 = V_2 - 50I_2 \qquad (5)$$

At the terminals of the second transformer,

$$\frac{V_4}{V_3} = 3 \quad \longrightarrow \quad V_4 = 3V_3 \tag{6}$$

$$\frac{I_3}{I_2} = -1/3 \quad \longrightarrow \quad I_2 = -3I_3 \tag{7}$$

We have seven equations and seven unknowns. Combining (1) and (2) leads to

$$200 = 3.5V_4 + 10I_1 + 50I_3$$

But from (4) and (7), $I_1 = -2I_2 = -2(-3I_3) = 6I_3$. Hence

$$200 = 3.5V_4 + 110I_3 \tag{8}$$

From (5), (6), (3), and (7),

$$V_4 = 3(V_2 - 50I_2) = 3V_2 - 150I_2 = -6V_1 + 450I_3$$

Substituting for V_1 in (2) gives

$$V_4 = -6(3V_4 + 40I_3) + 450I_3 \quad \longrightarrow \quad I_3 = \frac{19}{210}V_4 \tag{9}$$

Substituting (9) into (8) yields

$$200 = 13.452V_4 \quad \longrightarrow \quad V_4 = 14.87$$

$$P = \frac{V^2{}_4}{20} = \underline{11.05\ \text{W}}$$

Chapter 13, Problem 66.

An ideal autotransformer with a 1:4 step-up turns ratio has its secondary connected to a 120-Ω load and the primary to a 420-V source. Determine the primary current.

Chapter 13, Solution 66.

$$v_1 = 420 \text{ V} \quad (1)$$

$$v_2 = 120 I_2 \quad (2)$$

$$v_1/v_2 = 1/4 \text{ or } v_2 = 4v_1 \quad (3)$$

$$I_1/I_2 = 4 \text{ or } I_1 = 4\, I_2 \quad (4)$$

Combining (2) and (4),

$$v_2 = 120[(1/4)I_1] = 30\, I_1$$

$$4v_1 = 30 I_1$$

$$4(420) = 1680 = 30 I_1 \text{ or } I_1 = \underline{\mathbf{56\ A}}$$

Chapter 13, Problem 67.

An autotransformer with a 40 percent tap is supplied by a 400-V, 60-Hz source and is used for step-down operation. A 5-kVA load operating at unity power factor is connected to the secondary terminals. Find:

(a) the secondary voltage
(b) the secondary current
(c) the primary current

Chapter 13, Solution 67.

(a) $$\frac{V_1}{V_2} = \frac{N_1 + N_2}{N_2} = \frac{1}{0.4} \longrightarrow V_2 = 0.4V_1 = 0.4\text{x}400 = \underline{160\text{ V}}$$

(b) $$S_2 = I_2 V_2 = 5{,}000 \longrightarrow I_2 = \frac{5000}{160} = \underline{31.25\text{ A}}$$

(c) $$S_2 = S_1 = I_1 V_1 = 5{,}000 \longrightarrow I_1 = \frac{5000}{400} = \underline{12.5\text{ A}}$$

Chapter 13, Problem 68.

ML In the ideal autotransformer of Fig. 13.131, calculate $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_o$ Find the average power delivered to the load.

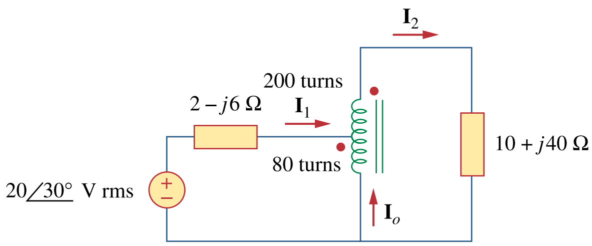
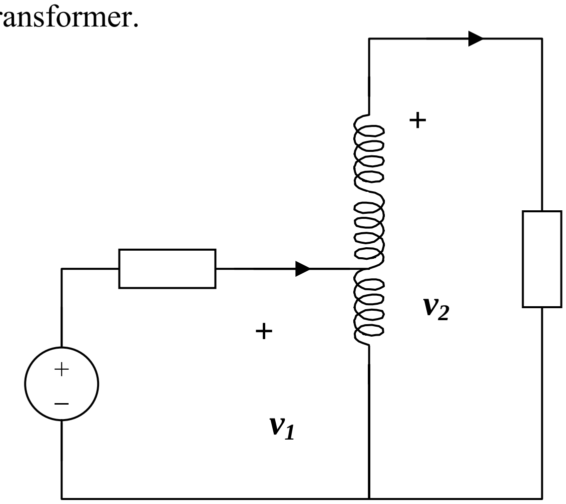


Figure 13.131
For Prob. 13.68.

Chapter 13, Solution 68.

This is a step-up transformer.



For the primary circuit, $20\angle 30° = (2 - j6)I_1 + v_1$ (1)

For the secondary circuit, $v_2 = (10 + j40)I_2$ (2)

At the autotransformer terminals,

$$v_1/v_2 = N_1/(N_1 + N_2) = 200/280 = 5/7,$$

thus $v_2 = 7v_1/5$ (3)

Also, $I_1/I_2 = 7/5$ or $I_2 = 5I_1/7$ (4)

Substituting (3) and (4) into (2), $v_1 = (10 + j40)25I_1/49$

Substituting that into (1) gives $20\angle 30° = (7.102 + j14.408)I_1$

$I_1 = 20\angle 30°/16.063\angle 63.76° =$ $\underline{\mathbf{1.245\angle -33.76° \text{ A}}}$

$I_2 = 5I_1/7 =$ $\underline{\mathbf{0.8893\angle -33.76° \text{ A}}}$

$I_o = I_1 - I_2 = [(5/7) - 1]I_1 = -2I_1/7 =$ $\underline{\mathbf{0.3557\angle 146.2° \text{ A}}}$

$p = |I_2|^2R = (0.8893)^2(10) =$ **<u>7.51 watts</u>**

Chapter 13, Problem 69.

* In the circuit of Fig. 13.132, $\mathbf{Z}_L$ is adjusted until maximum average power is delivered to $\mathbf{Z}_L$. Find $\mathbf{Z}_L$ and the maximum average power transferred to it. Take $N_1 = 600$ turns and $N_2 = 200$ turns.

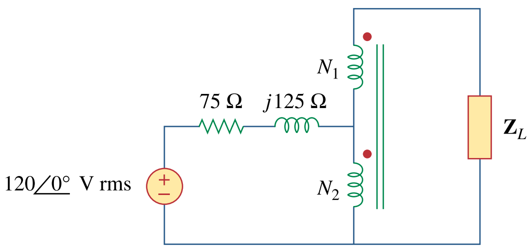
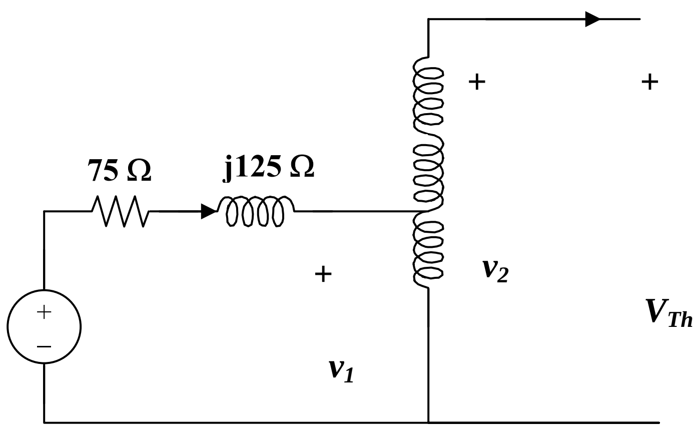


Figure 13.132
For Prob. 13.69.

* An asterisk indicates a challenging problem.

Chapter 13, Solution 69.

We can find the Thevenin equivalent.

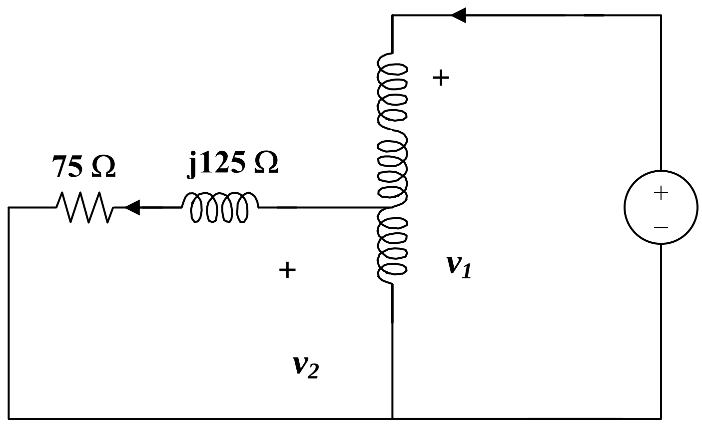


$I_1 = I_2 = 0$

As a step up transformer, $v_1/v_2 = N_1/(N_1 + N_2) = 600/800 = 3/4$

$$v_2 = 4v_1/3 = 4(120)/3 = 160\angle 0° \text{ rms} = V_{Th}.$$

To find Z_{Th}, connect a 1-V source at the secondary terminals. We now have a step-down transformer.



$$v_1 = 1\text{V}, \ v_2 = I_2(75 + j125)$$

But $v_1/v_2 = (N_1 + N_2)/N_1 = 800/200$ which leads to $v_1 = 4v_2 = 1$

and $v_2 = 0.25$

$$I_1/I_2 = 200/800 = 1/4 \text{ which leads to } I_2 = 4I_1$$

Hence $0.25 = 4I_1(75 + j125)$ or $I_1 = 1/[16(75 + j125)$

$Z_{Th} = 1/I_1 = 16(75 + j125)$

Therefore, $Z_L = Z_{Th}^* =$ **(1.2 – j2) kΩ**

Since V_{Th} is rms, $p = (|V_{Th}|/2)^2/R_L = (80)^2/1200 =$ **5.333 watts**

Chapter 13, Problem 70.

ML In the ideal transformer circuit shown in Fig. 13.133, determine the average power delivered to the load.

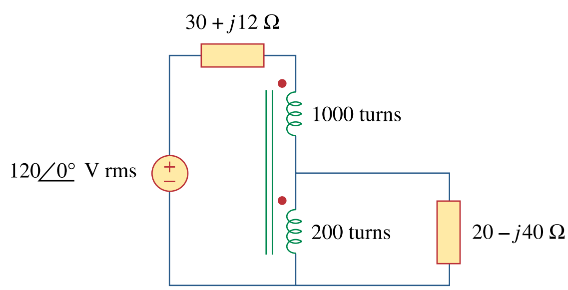
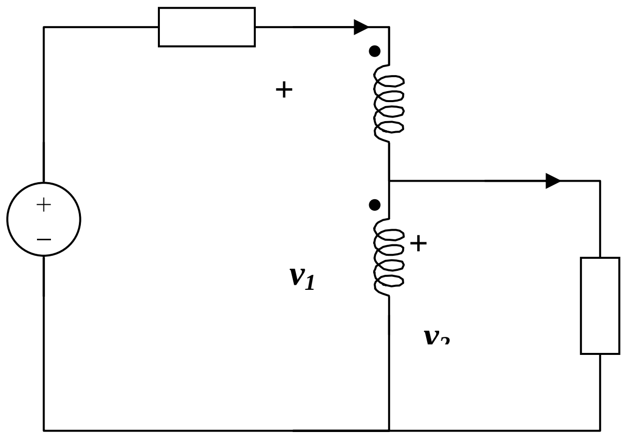


Figure 13.133
For Prob. 13.70.

Chapter 13, Solution 70.

This is a step-down transformer.



$I_1/I_2 = N_2/(N_1 + N_2) = 200/1200 = 1/6$, or $I_1 = I_2/6$ (1)

$v_1/v_2 = (N_2 + N_2)/N_2 = 6$, or $v_1 = 6v_2$ (2)

For the primary loop, $120 = (30 + j12)I_1 + v_1$ (3)

For the secondary loop, $v_2 = (20 – j40)I_2$ (4)

Substituting (1) and (2) into (3),

$$120 = (30 + j12)(I_2/6) + 6v_2$$

and substituting (4) into this yields

$$120 = (49 – j38)I_2 \text{ or } I_2 = 1.935\angle 37.79°$$

$p = |I_2|^2(20) =$ **74.9 watts**.

Chapter 13, Problem 71.

In the autotransformer circuit in Fig. 13.134, show that

$$\mathbf{Z}_{\text{in}} = \left(1 + \frac{N_1}{N_2}\right)^2 \mathbf{Z}_L$$

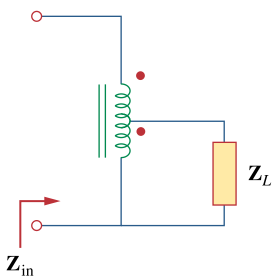


Figure 13.134
For Prob. 13.71.

Chapter 13, Solution 71.

$$Z_{in} = V_1/I_1$$

But $\qquad V_1I_1 = V_2I_2$, or $V_2 = I_2Z_L$ and $I_1/I_2 = N_2/(N_1 + N_2)$

Hence $\qquad V_1 = V_2I_2/I_1 = Z_L(I_2/I_1)I_2 = Z_L(I_2/I_1)^2I_1$

$$V_1/I_1 = Z_L[(N_1 + N_2)/N_2]^2$$

$$Z_{in} = \underline{\mathbf{[1 + (N_1/N_2)]^2 Z_L}}$$

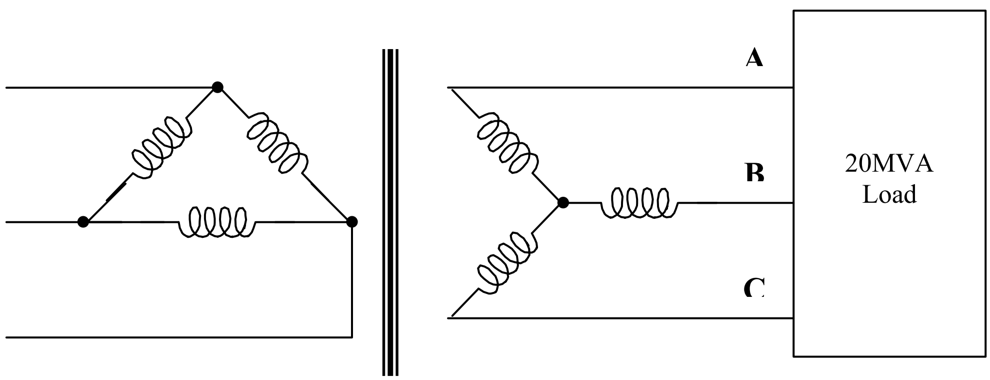
Chapter 13, Problem 72.

In order to meet an emergency, three single-phase transformers with 12,470/7,200 V rms are connected in Δ-Y to form a three-phase transformer which is fed by a 12,470-V transmission line. If the transformer supplies 60 MVA to a load, find:

(a) the turns ratio for each transformer,
(b) the currents in the primary and secondary windings of the transformer,
(c) the incoming and outgoing transmission line currents.

Chapter 13, Solution 72.

(a) Consider just one phase at a time.



$$n = V_L/\sqrt{3}V_{Lp} = 7200/(12470\sqrt{3}) = \underline{\mathbf{1/3}}$$

(b) The load carried by each transformer is 60/3 = 20 MVA.

Hence I_{Lp} = 20 MVA/12.47 k = **1604 A**

I_{Ls} = 20 MVA/7.2 k = **2778 A**

(c) The current in incoming line a, b, c is

$$\sqrt{3}I_{Lp} = \sqrt{3}\text{x}1603.85 = \underline{\mathbf{2778\ A}}$$

Current in each outgoing line A, B, C is

$$2778/(n\sqrt{3}) = \underline{\mathbf{4812\ A}}$$

Chapter 13, Problem 73.

ML Figure 13.135 on the following page shows a three-phase transformer that supplies a Y-connected load.

(a) Identify the transformer connection.
(b) Calculate currents $\mathbf{I}_2$ and $\mathbf{I}_c$.
(c) Find the average power absorbed by the load.

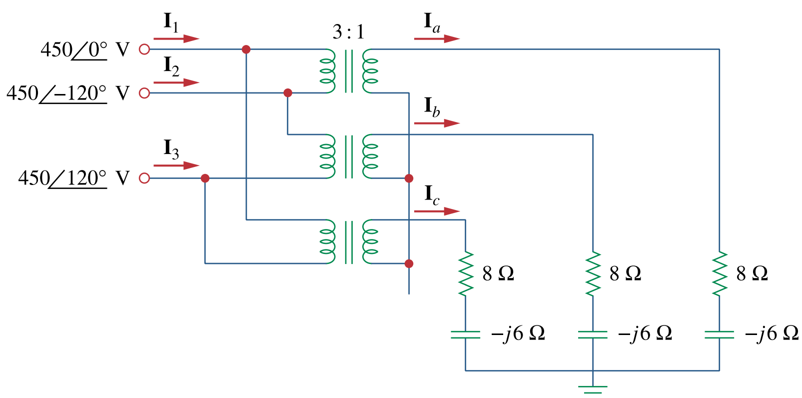


Figure 13.135
For Prob. 13.73.

Chapter 13, Solution 73.

(a) This is a **<u>three-phase Δ-Y transformer</u>**.

(b) $V_{Ls} = nv_{Lp}/\sqrt{3} = 450/(3\sqrt{3}) = 86.6$ V, where $n = 1/3$

As a Y-Y system, we can use per phase equivalent circuit.

$$I_a = V_{an}/Z_Y = 86.6\angle 0°/(8 - j6) = 8.66\angle 36.87°$$

$$I_c = I_a\angle 120° = \underline{\mathbf{8.66\angle 156.87°\ A}}$$

$$I_{Lp} = n\sqrt{3}\, I_{Ls}$$

$$I_1 = (1/3)\sqrt{3}\,(8.66\angle 36.87°) = 5\angle 36.87°$$

$$I_2 = I_1\angle -120° = \underline{\mathbf{5\angle -83.13°\ A}}$$

(c) $p = 3|I_a|^2(8) = 3(8.66)^2(8) = \underline{\mathbf{1.8\ kw}}$.

Chapter 13, Problem 74.

Consider the three-phase transformer shown in Fig. 13.136. The primary is fed by a three-phase source with line voltage of 2.4 kV rms, while the secondary supplies a three-phase 120-kW balanced load at pf of 0.8. Determine:

(a) the type of transformer connections,
(b) the values of I_{LS} and I_{PS},
(c) the values of I_{LP} and I_{PP},
(d) the kVA rating of each phase of the transformer.

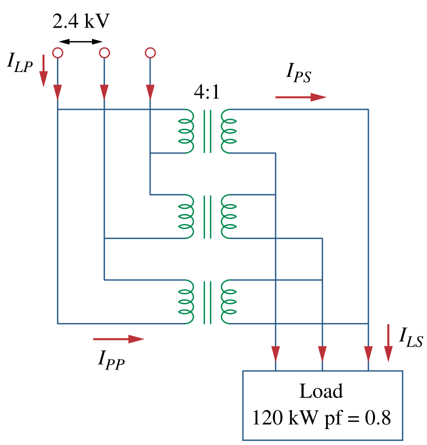


Figure 13.136
For Prob. 13.74.

Chapter 13, Solution 74.

(a) This is a **Δ-Δ connection**.

(b) The easy way is to consider just one phase.

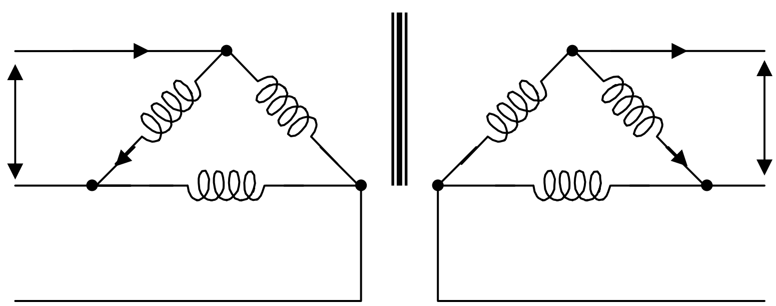
$$1{:}n = 4{:}1 \text{ or } n = 1/4$$

$n = V_2/V_1$ which leads to $V_2 = nV_1 = 0.25(2400) = 600$

$$\text{i.e. } V_{Lp} = 2400 \text{ V and } V_{Ls} = 600 \text{ V}$$

$S = p/\cos\theta = 120/0.8 \text{ kVA} = 150 \text{ kVA}$

$$p_L = p/3 = 120/3 = 40 \text{ kw}$$



But $\quad p_{Ls} = V_{ps}I_{ps}$

For the Δ-load, $\quad I_L = \sqrt{3}\, I_p$ and $V_L = V_p$

Hence, $\quad I_{ps} = 40{,}000/600 = 66.67 \text{ A}$

$$I_{Ls} = \sqrt{3}\, I_{ps} = \sqrt{3}\text{x}66.67 = \mathbf{115.48\ A}$$

(c) Similarly, for the primary side

$$p_{pp} = V_{pp}I_{pp} = p_{ps} \text{ or } I_{pp} = 40{,}000/2400 = \mathbf{16.667\ A}$$

$$\text{and} \quad I_{Lp} = \sqrt{3}\, I_p = \mathbf{28.87\ A}$$

(d) Since $S = 150$ kVA therefore $S_p = S/3 = \mathbf{50\ kVA}$

Chapter 13, Problem 75.

A balanced three-phase transformer bank with the Δ-Y connection depicted in Fig. 13.137 is used to step down line voltages from 4,500 V rms to 900 V rms. If the transformer feeds a 120-kVA load, find:

(a) the turns ratio for the transformer,
(b) the line currents at the primary and secondary sides.

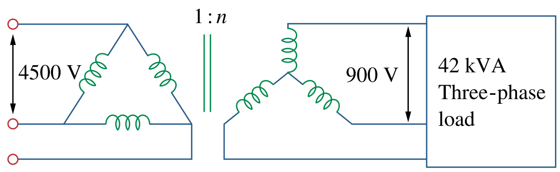


Figure 13.137
For Prob. 13.75.

Chapter 13, Solution 75.

(a) $n = V_{Ls}/(\sqrt{3}\, V_{Lp}) = 900/(4500\sqrt{3}) = \underline{\mathbf{0.11547}}$

(b) $S = \sqrt{3}\, V_{Ls} I_{Ls}$ or $I_{Ls} = 120{,}000/(900\sqrt{3}) = \underline{\mathbf{76.98\ A}}$

$I_{Ls} = I_{Lp}/(n\sqrt{3}) = 76.98/(2.887\sqrt{3}) = \underline{\mathbf{15.395\ A}}$

Chapter 13, Problem 76.

A Y-Δ three-phase transformer is connected to a 60-kVA load with 0.85 power factor (leading) through a feeder whose impedance is $0.05 + j0.1\ \Omega$ per phase, as shown in Fig. 13.138. Find the magnitude of:

(a) the line current at the load,
(b) the line voltage at the secondary side of the transformer,
(c) the line current at the primary side of the transformer.

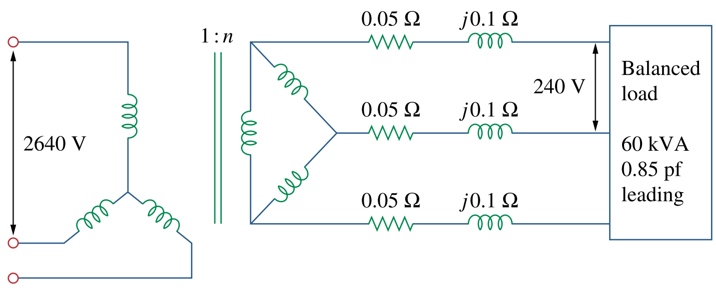


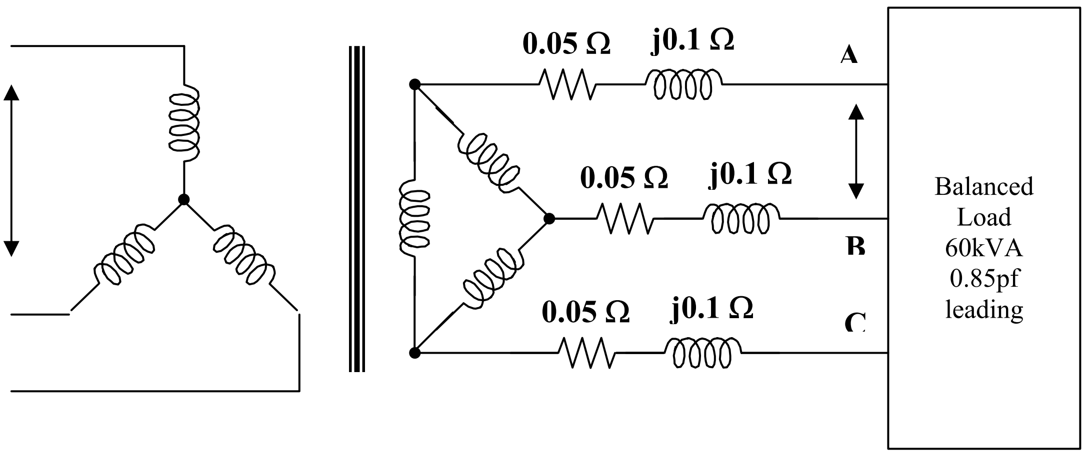
Figure 13.138
For Prob. 13.76.

Chapter 13, Solution 76.

(a) At the load, $V_L = 240 \text{ V} = V_{AB}$

$$V_{AN} = V_L/\sqrt{3} = 138.56 \text{ V}$$

Since $S = \sqrt{3}V_L I_L$ then $I_L = 60{,}000/(240\sqrt{3}) = \underline{\mathbf{144.34 \text{ A}}}$



(b) Let $V_{AN} = |V_{AN}|\angle 0° = 138.56\angle 0°$

$\cos\theta = \text{pf} = 0.85$ or $\theta = 31.79°$

$I_{AA'} = I_L\angle\theta = 144.34\angle 31.79°$

$$V_{A'N'} = ZI_{AA'} + V_{AN}$$

$$= 138.56\angle 0° + (0.05 + j0.1)(144.34\angle 31.79°)$$

$$= 138.03\angle 6.69°$$

$$V_{Ls} = V_{A'N'}\sqrt{3} = 137.8\sqrt{3} = \underline{\mathbf{238.7 \text{ V}}}$$

(c) For Y-Δ connections,

$$n = \sqrt{3}V_{Ls}/V_{ps} = \sqrt{3}\text{x}238.7/2640 = 0.1569$$

$$f_{Lp} = nI_{Ls}/\sqrt{3} = 0.1569\text{x}144.34/\sqrt{3} = \underline{\mathbf{13.05 \text{ A}}}$$

Chapter 13, Problem 77.

The three-phase system of a town distributes power with a line voltage of 13.2 kV. A pole transformer connected to single wire and ground steps down the high-voltage wire to 120 V rms and serves a house as shown in Fig. 13.139.

(a) Calculate the turns ratio of the pole transformer to get 120 V.
(b) Determine how much current a 100-W lamp connected to the 120-V hot line draws from the high-voltage line.

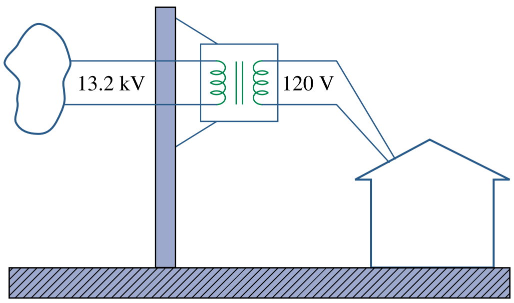


Figure 13.139
For Prob. 13.77.

Chapter 13, Solution 77.

(a) This is a single phase transformer. $V_1 = 13.2$ kV, $V_2 = 120$ V

$$n = V_2/V_1 = 120/13{,}200 = 1/110, \text{ therefore } n = \underline{\mathbf{1/110}}$$

or 110 turns on the primary to every turn on the secondary.

(b) $P = VI$ or $I = P/V = 100/120 = 0.8333$ A

$$I_1 = nI_2 = 0.8333/110 = \underline{\mathbf{7.576\ mA}}$$

Chapter 13, Problem 78.

Use *PSpice* to determine the mesh currents in the circuit of Fig. 13.140. Take ω = 1 rad/s.

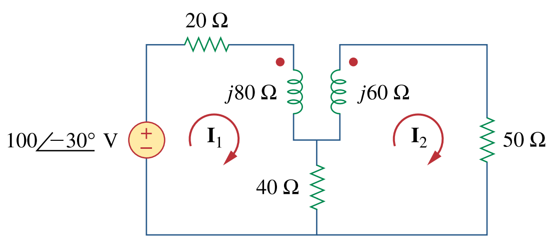


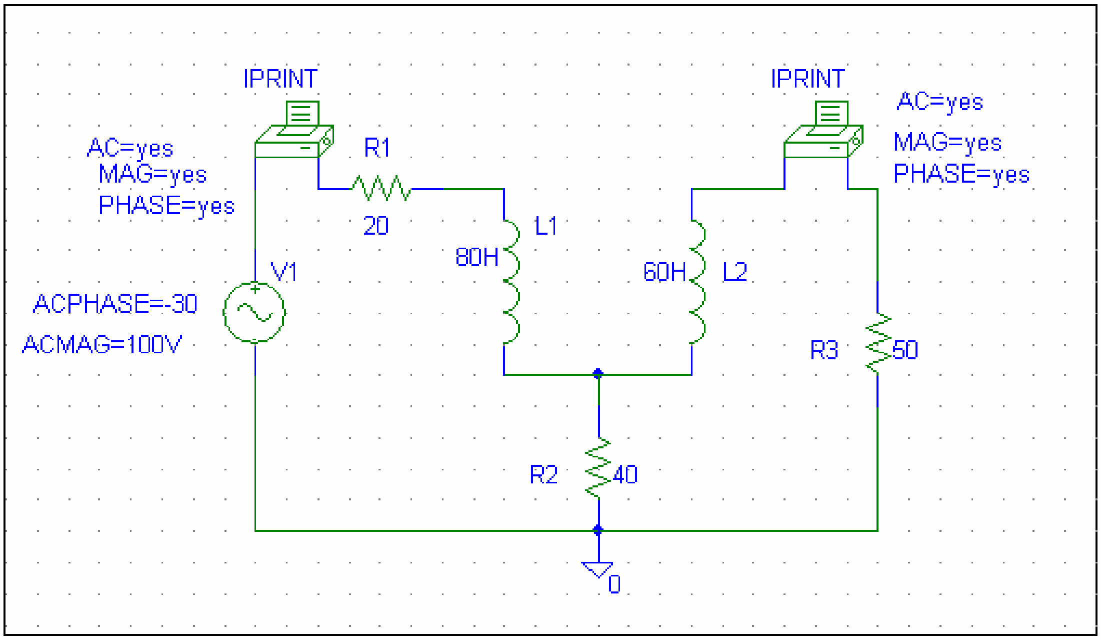
Figure 13.140
For Prob. 13.78.

Chapter 13, Solution 78.

We convert the reactances to their inductive values.

$$X = \omega L \qquad \longrightarrow \qquad L = \frac{X}{\omega}$$

The schematic is as shown below.



When the circuit is simulated, the output file contains

FREQ IM(V_PRINT1)IP(V_PRINT1)

1.592E-01 9.971E-01 -9.161E+01

FREQ IM(V_PRINT2)IP(V_PRINT2)

1.592E-01 3.687E-01 -1.253E+02

From this, we obtain

I_1 = **997.1∠–91.61° mA**, I_2 = **368.7∠–135.3° mA**.

Chapter 13, Problem 79.

Use *PSpice* to find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 13.141.

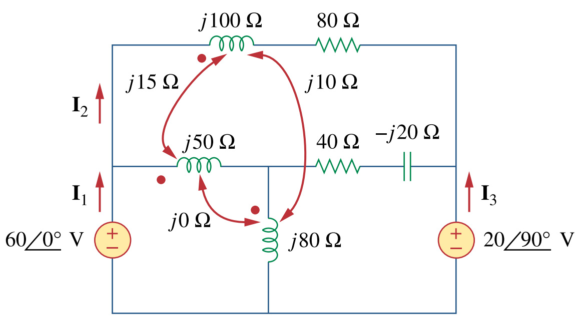


Figure 13.141
For Prob. 13.79.

Chapter 13, Solution 79.

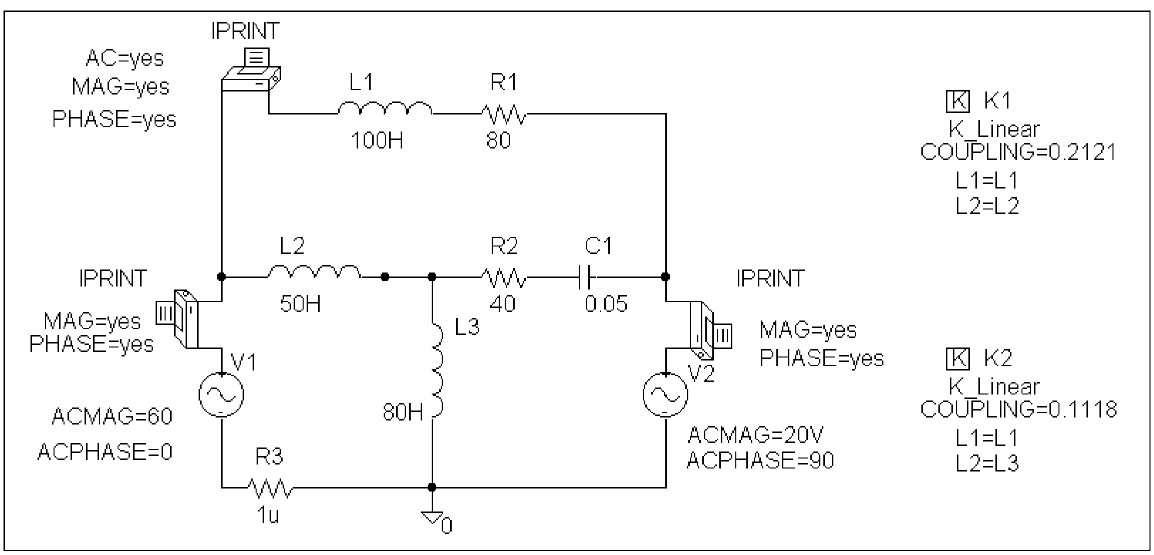
The schematic is shown below.

$$k_1 = 15/\sqrt{5000} = 0.2121, \; k_2 = 10/\sqrt{8000} = 0.1118$$

In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After the circuit is saved and simulated, the output includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	4.068 E–01	–7.786 E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	1.306 E+00	–6.801 E+01
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	1.336 E+00	–5.492 E+01

Thus, I_1 = $\underline{\mathbf{1.306\angle{-68.01}^\circ\ A}}$, I_2 = $\underline{\mathbf{406.8\angle{-77.86}^\circ\ mA}}$, I_3 = $\underline{\mathbf{1.336\angle{-54.92}^\circ\ A}}$



Chapter 13, Problem 80.

Rework Prob. 13.22 using *PSpice*.

Chapter 13, Solution 80.

The schematic is shown below.

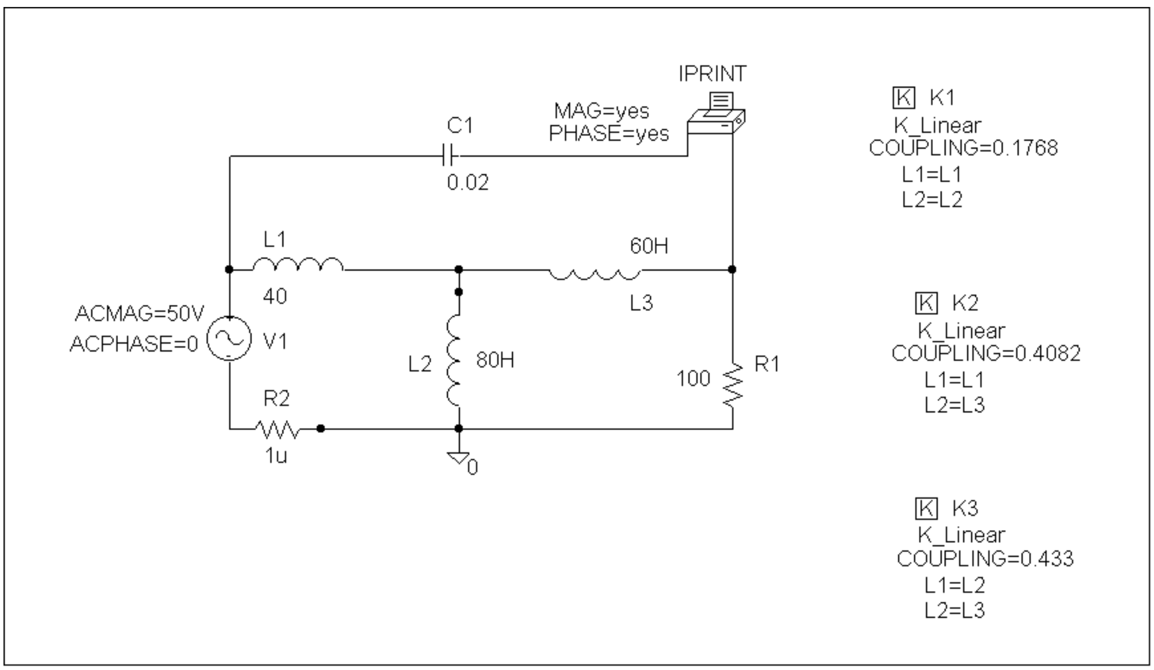
$$k_1 = 10/\sqrt{40x80} = 0.1768,\ k_2 = 20/\sqrt{40x60} = 0.482$$

$$k_3 = 30/\sqrt{80x60} = 0.433$$

In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After the simulation, we obtain the output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.304 E+00	6.292 E+01

i.e. I_o = **$1.304\angle 62.92°$ A**



Chapter 13, Problem 81.

Use *PSpice* to find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 13.142.

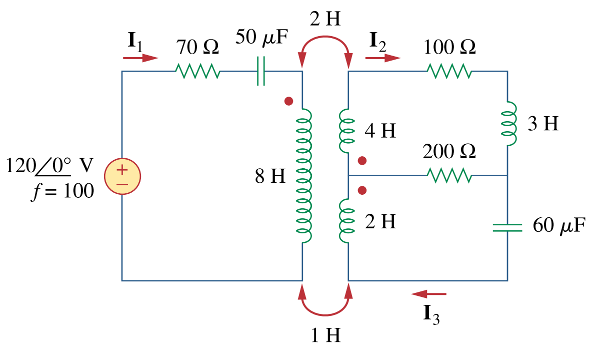


Figure 13.142
For Prob. 13.81.

Chapter 13, Solution 81.

The schematic is shown below.

$$k_1 = 2/\sqrt{4x8} = 0.3535, \; k_2 = 1/\sqrt{2x8} = 0.25$$

In the AC Sweep box, we let Total Pts = 1, Start Freq = 100, and End Freq = 100. After simulation, the output file includes

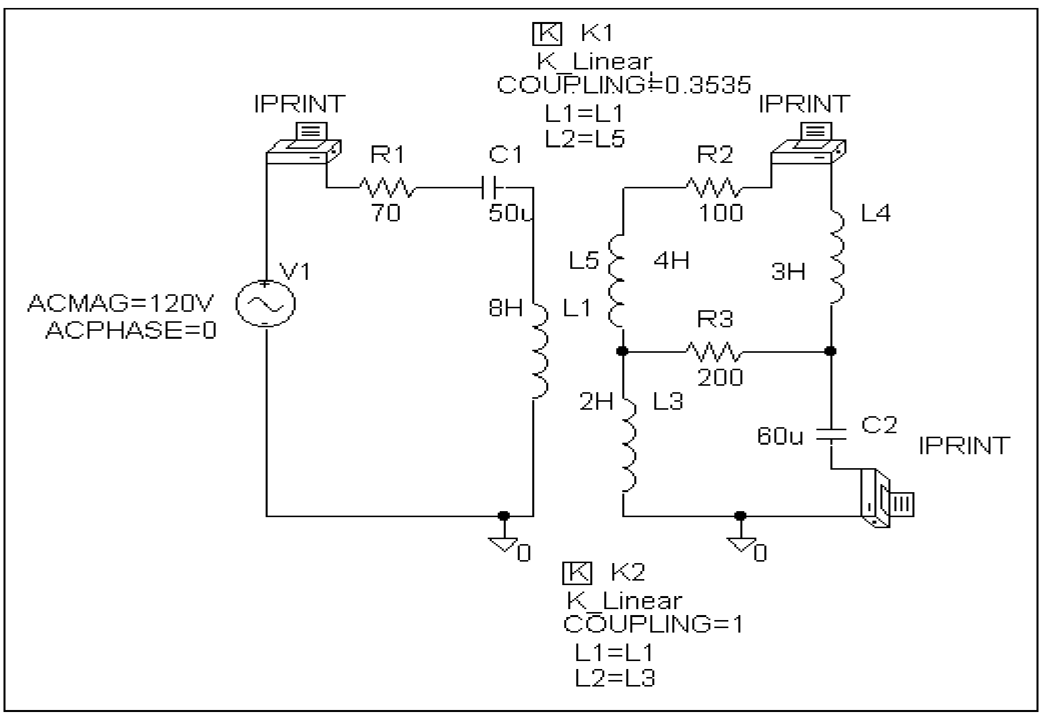
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.000 E+02	1.0448 E–01	1.396 E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.000 E+02	2.954 E–02	–1.438 E+02

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.000 E+02	2.088 E–01	2.440 E+01

i.e. I_1 = **<u>104.5∠13.96° mA</u>**, I_2 = **<u>29.54∠–143.8° mA</u>**,

I_3 = **<u>208.8∠24.4° mA</u>**.



Chapter 13, Problem 82.

Use *PSpice* to find $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{I}_o$ in the circuit of Fig. 13.143.

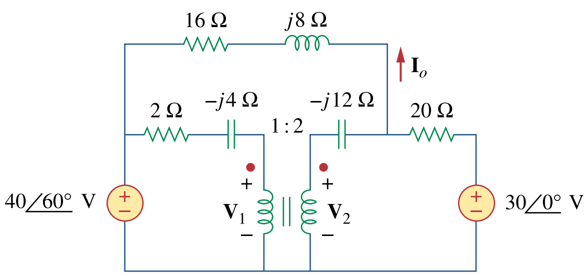


Figure 13.143
For Prob. 13.82.

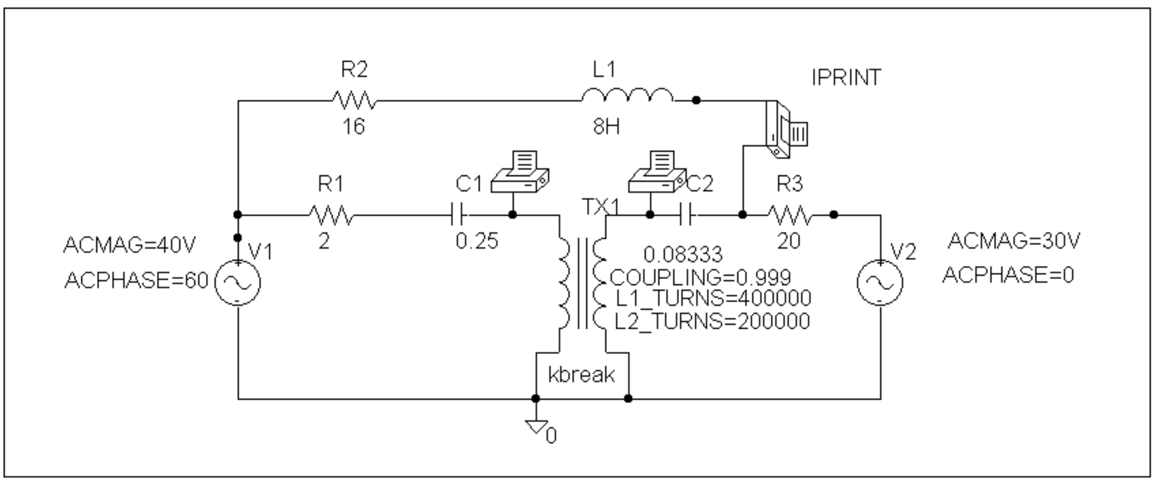
Chapter 13, Solution 82.

The schematic is shown below. In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain the output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.955 E+01	8.332 E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	6.847 E+01	4.640 E+01
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	4.434 E–01	–9.260 E+01

i.e. $V_1 = \underline{\mathbf{19.55\angle 83.32°\ V}}$, $V_2 = \underline{\mathbf{68.47\angle 46.4°\ V}}$,

$I_o = \underline{\mathbf{443.4\angle -92.6°\ mA}}$.



Chapter 13, Problem 83.

Find $\mathbf{I}_x$ and $\mathbf{V}_x$ in the circuit of Fig. 13.144 using *PSpice*.

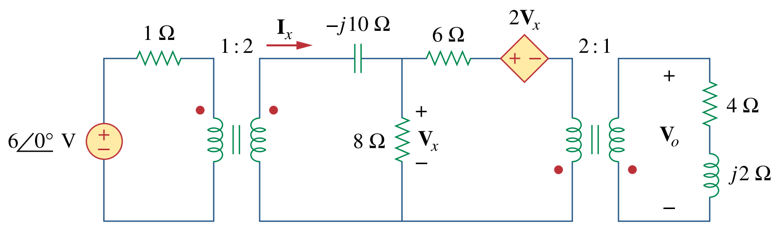


Figure 13.144
For Prob. 13.83.

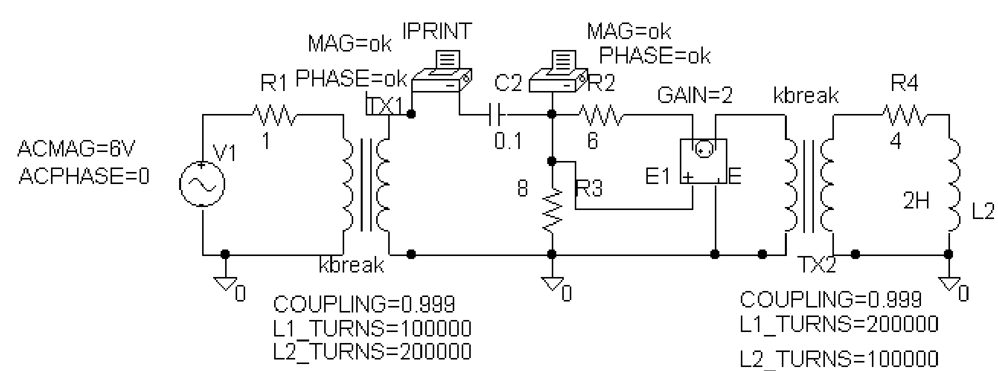
Chapter 13, Solution 83.

The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.080 E+00	3.391 E+01

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	1.514 E+01	–3.421 E+01

i.e. $\quad i_X = \underline{\mathbf{1.08\angle 33.91^\circ\ A}}$, $V_x = \underline{\mathbf{15.14\angle{-34.21^\circ}\ V}}$.



Chapter 13, Problem 84.

Determine $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the ideal transformer circuit of Fig. 13.145 using *PSpice*.

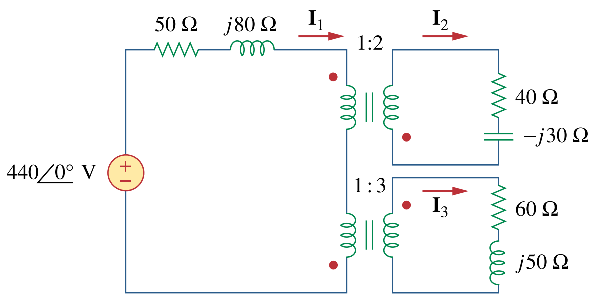


Figure 13.145
For Prob. 13.84.

Chapter 13, Solution 84.

The schematic is shown below. we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

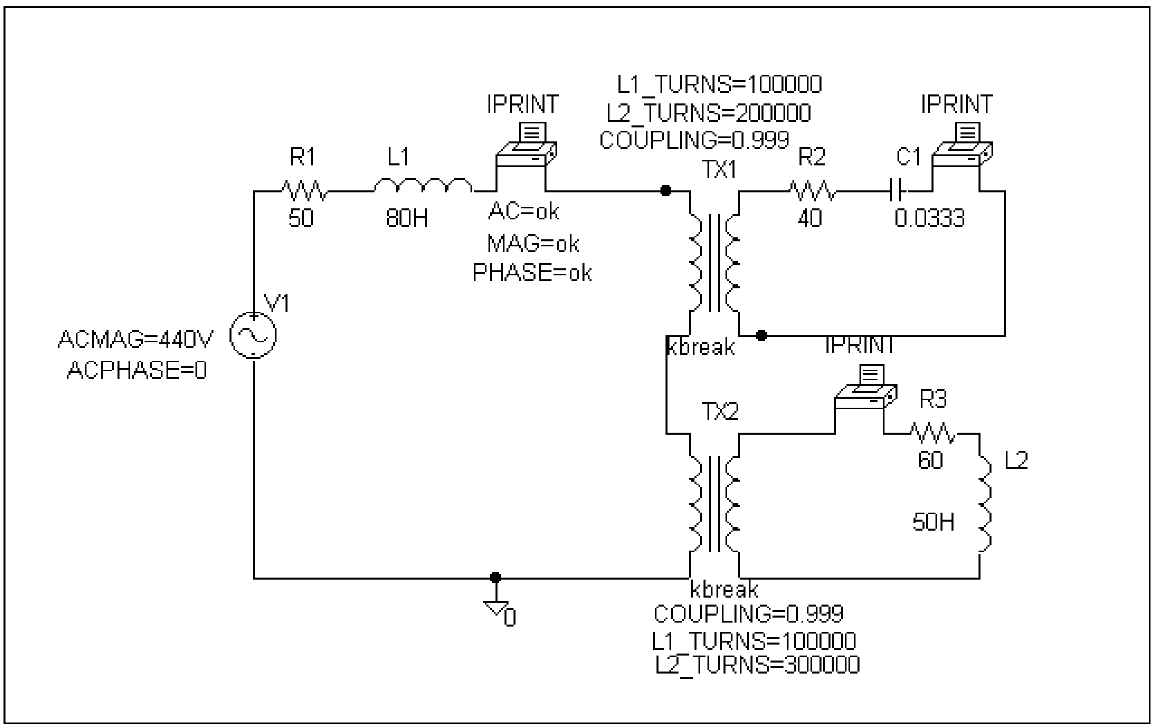
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	4.028 E+00	–5.238 E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	2.019 E+00	–5.211 E+01

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	1.338 E+00	–5.220 E+01

i.e. $I_1 = \underline{\mathbf{4.028\angle{-52.38^\circ}\ A}}$, $I_2 = \underline{\mathbf{2.019\angle{-52.11^\circ}\ A}}$,

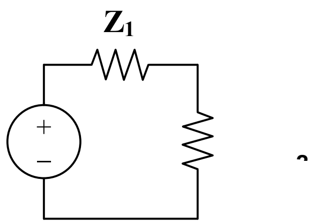
$I_3 = \underline{\mathbf{1.338\angle{-52.2^\circ}\ A}}$.



Chapter 13, Problem 85.

A stereo amplifier circuit with an output impedance of 7.2 kΩ is to be matched to a speaker with an input impedance of 8 Ω by a transformer whose primary side has 3,000 turns. Calculate the number of turns required on the secondary side.

Chapter 13, Solution 85.



For maximum power transfer,

$$Z_1 = Z_L/n^2 \text{ or } n^2 = Z_L/Z_1 = 8/7200 = 1/900$$

$$n = 1/30 = N_2/N_1. \text{ Thus } N_2 = N_1/30 = 3000/30 = \underline{\textbf{100 turns}}.$$

Chapter 13, Problem 86.

A transformer having 2,400 turns on the primary and 48 turns on the secondary is used as an impedance-matching device. What is the reflected value of a 3-Ω load connected to the secondary?

Chapter 13, Solution 86.

$$n = N_2/N_1 = 48/2400 = 1/50$$

$$Z_{Th} = Z_L/n^2 = 3/(1/50)^2 = \underline{\textbf{7.5 k}\Omega}$$

Chapter 13, Problem 87.

e2d A radio receiver has an input resistance of $300\,\Omega$. When it is connected directly to an antenna system with a characteristic impedance of $75\ \Omega$, an impedance mismatch occurs. By inserting an impedance-matching transformer ahead of the receiver, maximum power can be realized. Calculate the required turns ratio.

Chapter 13, Solution 87.

$$Z_{Th} = Z_L/n^2 \quad \text{or} \quad n = \sqrt{Z_L / Z_{Th}} = \sqrt{75/300} = \underline{\mathbf{0.5}}$$

Chapter 13, Problem 88.

A step-down power transformer with a turns ratio of $n = 0.1$ supplies 12.6 V rms to a resistive load. If the primary current is 2.5 A rms, how much power is delivered to the load?

Chapter 13, Solution 88.

$$n = V_2/V_1 = I_1/I_2 \quad \text{or} \quad I_2 = I_1/n = 2.5/0.1 = 25 \text{ A}$$

$$p = IV = 25\text{x}12.6 = \underline{\mathbf{315\ watts}}$$

Chapter 13, Problem 89.

A 240/120-V rms power transformer is rated at 10 kVA. Determine the turns ratio, the primary current, and the secondary current.

Chapter 13, Solution 89.

$$n = V_2/V_1 = 120/240 = 0.5$$

$$S = I_1V_1 \quad \text{or} \quad I_1 = S/V_1 = 10\text{x}10^3/240 = 41.67 \text{ A}$$

$$S = I_2V_2 \quad \text{or} \quad I_2 = S/V_2 = 10^4/120 = 83.33 \text{ A}$$

Chapter 13, Problem 90.

A 4-kVA, 2,400/240-V rms transformer has 250 turns on the primary side. Calculate:

(a) the turns ratio,
(b) the number of turns on the secondary side,
(c) the primary and secondary currents.

Chapter 13, Solution 90.

(a) $n = V_2/V_1 = 240/2400 = \underline{\mathbf{0.1}}$

(b) $n = N_2/N_1$ or $N_2 = nN_1 = 0.1(250) = \underline{\mathbf{25\ turns}}$

(c) $S = I_1V_1$ or $I_1 = S/V_1 = 4x10^3/2400 = \underline{\mathbf{1.6667\ A}}$

$S = I_2V_2$ or $I_2 = S/V_2 = 4x10^4/240 = \underline{\mathbf{16.667\ A}}$

Chapter 13, Problem 91.

A 25,000/240-V rms distribution transformer has a primary current rating of 75 A.

(a) Find the transformer kVA rating.
(b) Calculate the secondary current.

Chapter 13, Solution 91.

(a) The kVA rating is $S = VI = 25{,}000x75 = \underline{\mathbf{1875\ kVA}}$

(b) Since $S_1 = S_2 = V_2I_2$ and $I_2 = 1875x10^3/240 = \underline{\mathbf{7812\ A}}$

Chapter 13, Problem 92.

A 4,800-V rms transmission line feeds a distribution transformer with 1,200 turns on the primary and 28 turns on the secondary. When a 10-Ω load is connected across the secondary, find:

(a) the secondary voltage,
(b) the primary and secondary currents,
(c) the power supplied to the load.

Chapter 13, Solution 92.

(a) $V_2/V_1 = N_2/N_1 = n, \quad V_2 = (N_2/N_1)V_1 = (28/1200)4800 = \underline{\mathbf{112\ V}}$

(b) $I_2 = V_2/R = 112/10 = \underline{\mathbf{11.2\ A}}$ and $I_1 = nI_2, \ n = 28/1200$

$I_1 = (28/1200)11.2 = \underline{\mathbf{261.3\ mA}}$

(c) $p = |I_2|^2R = (11.2)^2(10) = \underline{\mathbf{1254\ watts}}$.

Chapter 13, Problem 93.

A four-winding transformer (Fig. 13.146) is often used in equipment (e.g., PCs, VCRs) that may be operated from either 110 V or 220 V. This makes the equipment suitable for both domestic and foreign use. Show which connections are necessary to provide:

(a) an output of 14 V with an input of 110 V,
(b) an output of 50 V with an input of 220 V.

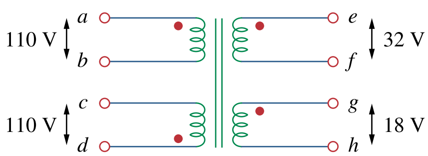
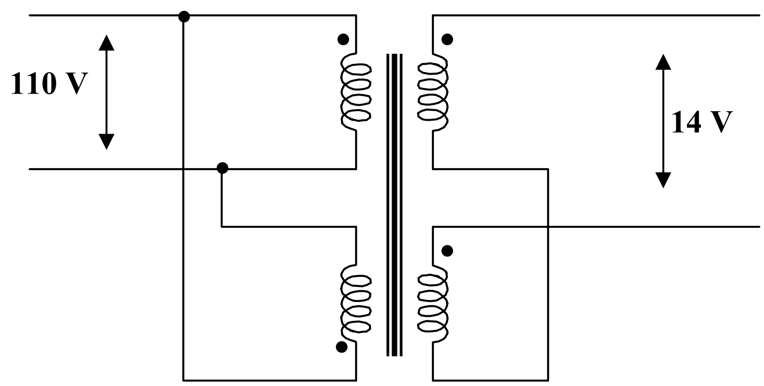


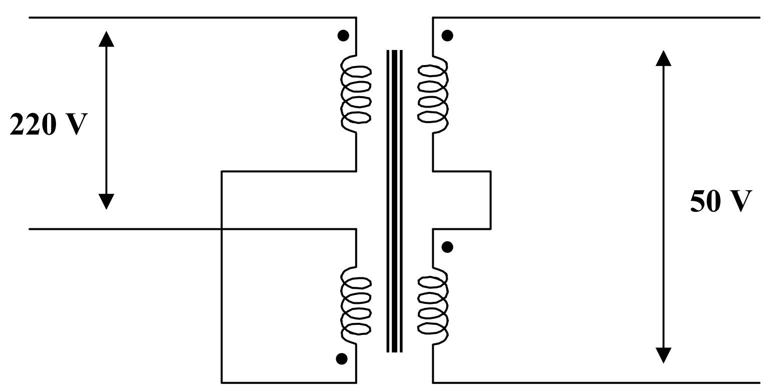
Figure 13.146
For Prob. 13.93.

Chapter 13, Solution 93.

(a) For an input of 110 V, the primary winding must be connected in parallel, with series aiding on the secondary. The coils must be series opposing to give 14 V. Thus, the connections are shown below.



(b) To get 220 V on the primary side, the coils are connected in series, with series aiding on the secondary side. The coils must be connected series aiding to give 50 V. Thus, the connections are shown below.



Chapter 13, Problem 94.

* A 440/110-V ideal transformer can be connected to become a 550/440-V ideal autotransformer. There are four possible connections, two of which are wrong. Find the output voltage of:

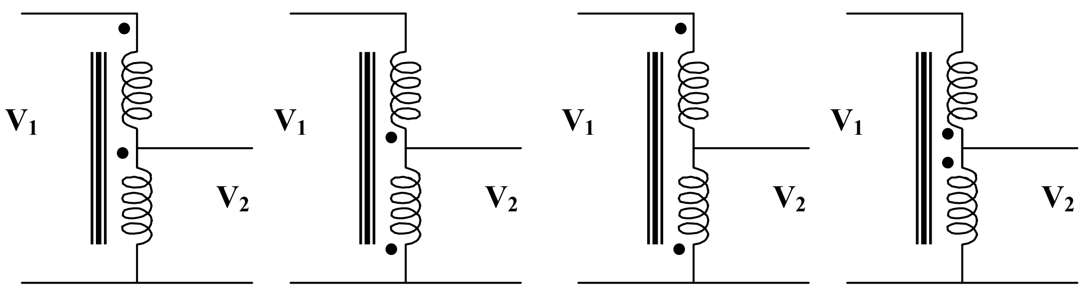
(a) a wrong connection,
(b) the right connection.

* An asterisk indicates a challenging problem.

Chapter 13, Solution 94.

$$V_2/V_1 = 110/440 = 1/4 = I_1/I_2$$

There are four ways of hooking up the transformer as an auto-transformer. However it is clear that there are only two outcomes.



(1) **(2)** **(3)** **(4)**

(1) and (2) produce the same results and (3) and (4) also produce the same results. Therefore, we will only consider Figure (1) and (3).

(a) For Figure (3), $V_1/V_2 = 550/V_2 = (440 - 110)/440 = 330/440$

Thus, $V_2 = 550\text{x}440/330 =$ **<u>733.4 V (not the desired result)</u>**

(b) For Figure (1), $V_1/V_2 = 550/V_2 = (440 + 110)/440 = 550/440$

Thus, $V_2 = 550\text{x}440/550 =$ **<u>440 V (the desired result)</u>**

Chapter 13, Problem 95.

Ten bulbs in parallel are supplied by a 7,200/120-V transformer as shown in Fig. 13.147, where the bulbs are modeled by the 144-Ω resistors. Find:

(a) the turns ratio *n*,

(b) the current through the primary winding.

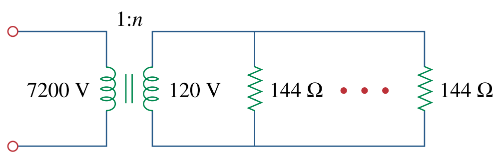


Figure 13.147
For Prob. 13.95.

Chapter 13, Solution 95.

(a) $$n = V_s/V_p = 120/7200 = \underline{\mathbf{1/60}}$$

(b) $$I_s = 10x120/144 = 1200/144$$

$$S = V_pI_p = V_sI_s$$

$$I_p = V_sI_s/V_p = (1/60)x1200/144 = \underline{\mathbf{139\ mA}}$$