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2024-02-06

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# Contest (1)

template.cpp33 lines

```
#include <bits/stdc++.h>
using namespace std;
#define sws cin.tie(0)->sync_with_stdio(0)

#define endl '\n'
#define ll long long
#define ld long double
#define pb push_back
#define ff first
#define ss second
#define pll pair<ll, ll>
#define vll vector<ll>

#define teto(a, b) ((a+b-1)/(b))
#define LSB(i) ((i) & -(i))
#define MSB(i) (32 - __builtin_clz(i)) //64 - clzll
#define BITS(i) __builtin_popcountll(i) //count set bits

mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());

#define debug(a...) cerr<<#a<<" ";for(auto b:a)cerr<<b<<" ";
cerr<<endl;
template<typename... A> void dbg(A const&... a){(cerr<<"{"<<a
<<" "}, ...);cerr<<endl;}

const int MAX = 3e5+10;
const int INF = INT32_MAX;
const long long MOD = 1e9+7;
const long long LLINF = INT64_MAX;
const long double EPS = 1e-7;
const long double PI = acos(-1);

int32_t main(){ sws;

}
```

.bashrc1 lines

```
alias comp='g++ -std=c++17 -g3 -ggdb3 -O3 -Wall -Wextra -
fsanitize=address,undefined -Wshadow -Wconversion -
D_GLIBCXX_ASSERTIONS -o test'
```

hash.sh3 lines

```
# Hashes a file, ignoring all whitespace and comments. Use for
# verifying that code was correctly typed. CTRL+D to send EOF
cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum | cut -c
-6
```

troubleshoot.txt52 lines

```
Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a teammate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your teammates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?
```

# Mathematics (2)

## 2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by  $x = -b/2a$ .

$$\begin{matrix} ax + by = e \\ cx + dy = f \end{matrix} \Rightarrow \begin{matrix} x = \frac{ed - bf}{ad - bc} \\ y = \frac{af - ec}{ad - bc} \end{matrix}$$

In general, given an equation  $Ax = b$ , the solution to a variable  $x_i$  is given by

$$x_i = \frac{\det A'_i}{\det A}$$

where  $A'_i$  is  $A$  with the  $i$ 'th column replaced by  $b$ .

## 2.2 Recurrences

If  $a_n = c_1a_{n-1} + \dots + c_ka_{n-k}$ , and  $r_1, \dots, r_k$  are distinct roots of  $x^k - c_1x^{k-1} - \dots - c_k$ , there are  $d_1, \dots, d_k$  s.t.

$$a_n = d_1r_1^n + \dots + d_kr_k^n.$$

Non-distinct roots  $r$  become polynomial factors, e.g.  $a_n = (d_1n + d_2)r^n$ .

## 2.3 Trigonometry

$$\sin(v + w) = \sin v \cos w + \cos v \sin w$$

$$\cos(v + w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$

$$\sin v + \sin w = 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2}$$

$$\cos v + \cos w = 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}$$

$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where  $V, W$  are lengths of sides opposite angles  $v, w$ .

$$a \cos x + b \sin x = r \cos(x - \phi)$$

$$a \sin x + b \cos x = r \sin(x + \phi)$$

where  $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$ .

2.4 Geometry

2.4.1 Triangles

Side lengths:  $a, b, c$

Semiperimeter:  $p = \frac{a + b + c}{2}$

Area:  $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius:  $R = \frac{abc}{4A}$

Inradius:  $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles):

$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$s_a = \sqrt{bc \left[ 1 - \left( \frac{a}{b + c} \right)^2 \right]}$

Law of sines:  $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$

Law of cosines:  $a^2 = b^2 + c^2 - 2bc \cos \alpha$

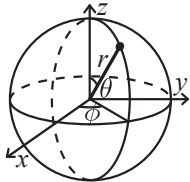
2.4.2 Quadrilaterals

With side lengths  $a, b, c, d$ , diagonals  $e, f$ , diagonals angle  $\theta$ , area  $A$  and magic flux  $F = b^2 + d^2 - a^2 - c^2$ :

$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$

2.4.3 Spherical coordinates

For cyclic quadrilaterals the sum of opposite angles is  $180^\circ$ ,  $ef = ac + bd$ , and  $A = \sqrt{(p - a)(p - b)(p - c)(p - d)}$ .



$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \arccos(z / \sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \operatorname{atan2}(y, x) \end{aligned}$$

2.5 Derivatives/Integrals

$$\begin{aligned} \frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1 - x^2}} & \frac{d}{dx} \arccos x &= -\frac{1}{\sqrt{1 - x^2}} \\ \frac{d}{dx} \tan x &= 1 + \tan^2 x & \frac{d}{dx} \arctan x &= \frac{1}{1 + x^2} \\ \int \tan ax &= -\frac{\ln |\cos ax|}{a} & \int x \sin ax &= \frac{\sin ax - ax \cos ax}{a^2} \\ \int e^{-x^2} &= \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) & \int x e^{ax} dx &= \frac{e^{ax}}{a^2} (ax - 1) \end{aligned}$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.6 Sums

$$c^a + c^{a+1} + \dots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$\begin{aligned} 1 + 2 + 3 + \dots + n &= \frac{n(n + 1)}{2} \\ 1^2 + 2^2 + 3^2 + \dots + n^2 &= \frac{n(2n + 1)(n + 1)}{6} \\ 1^3 + 2^3 + 3^3 + \dots + n^3 &= \frac{n^2(n + 1)^2}{4} \\ 1^4 + 2^4 + 3^4 + \dots + n^4 &= \frac{n(n + 1)(2n + 1)(3n^2 + 3n - 1)}{30} \end{aligned}$$

2.7 Series

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty) \\ \ln(1 + x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1) \\ \sqrt{1 + x} &= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1) \\ \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty) \\ \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty) \end{aligned}$$

2.8 Probability theory

Let  $X$  be a discrete random variable with probability  $p_X(x)$  of assuming the value  $x$ . It will then have an expected value (mean)  $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$  and variance  $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$  where  $\sigma$  is the standard deviation. If  $X$  is instead continuous it will have a probability density function  $f_X(x)$  and the sums above will instead be integrals with  $p_X(x)$  replaced by  $f_X(x)$ .

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent  $X$  and  $Y$ ,

$$V(aX + bY) = a^2 V(X) + b^2 V(Y).$$

2.8.1 Discrete distributions

Binomial distribution

The number of successes in  $n$  independent yes/no experiments, each which yields success with probability  $p$  is  $\operatorname{Bin}(n, p)$ ,  $n = 1, 2, \dots$ ,  $0 \leq p \leq 1$ .

$$p(k) = \binom{n}{k} p^k (1 - p)^{n - k}$$

$$\mu = np, \sigma^2 = np(1 - p)$$

$\operatorname{Bin}(n, p)$  is approximately  $\operatorname{Po}(np)$  for small  $p$ .

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability  $p$  is  $\operatorname{Fs}(p)$ ,  $0 \leq p \leq 1$ .

$$p(k) = p(1 - p)^{k - 1}, k = 1, 2, \dots$$

$$\mu = \frac{1}{p}, \sigma^2 = \frac{1 - p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time  $t$  if these events occur with a known average rate  $\kappa$  and independently of the time since the last event is  $\operatorname{Po}(\lambda)$ ,  $\lambda = t\kappa$ .

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, 2, \dots$$

$$\mu = \lambda, \sigma^2 = \lambda$$

2.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between  $a$  and  $b$  and 0 elsewhere it is  $U(a, b)$ ,  $a < b$ .

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$
$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is  $\text{Exp}(\lambda)$ ,  $\lambda > 0$ .

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$
$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean  $\mu$  and variance  $\sigma^2$  are well described by  $\mathcal{N}(\mu, \sigma^2)$ ,  $\sigma > 0$ .

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If  $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$  and  $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$  then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.9 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let  $X_1, X_2, \dots$  be a sequence of random variables generated by the Markov process. Then there is a transition matrix  $\mathbf{P} = (p_{ij})$ , with  $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$ , and  $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$  is the probability distribution for  $X_n$  (i.e.,  $p_i^{(n)} = \Pr(X_n = i)$ ), where  $\mathbf{p}^{(0)}$  is the initial distribution.

$\pi$  is a stationary distribution if  $\pi = \pi \mathbf{P}$ . If the Markov chain is *irreducible* (it is possible to get to any state from any state), then  $\pi_i = \frac{1}{\mathbb{E}(T_i)}$  where  $\mathbb{E}(T_i)$  is the expected time between two visits in state  $i$ .  $\pi_j / \pi_i$  is the expected number of visits in state  $j$  between two visits in state  $i$ .

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors,  $\pi_i$  is proportional to node  $i$ 's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1).  $\lim_{k \rightarrow \infty} \mathbf{P}^k = \mathbf{1}\pi$ .

A Markov chain is an A-chain if the states can be partitioned into two sets  $\mathbf{A}$  and  $\mathbf{G}$ , such that all states in  $\mathbf{A}$  are absorbing ( $p_{ii} = 1$ ), and all states in  $\mathbf{G}$  leads to an absorbing state in  $\mathbf{A}$ . The probability for absorption in state  $i \in \mathbf{A}$ , when the initial state is  $j$ , is  $a_{ij} = p_{ij} + \sum_{k \in \mathbf{G}} a_{ik} p_{kj}$ . The expected time until absorption, when the initial state is  $i$ , is  $t_i = 1 + \sum_{k \in \mathbf{G}} p_{ki} t_k$ .

Data structures (3)

3.1 Disjoint Set Union

There are two optional improvements: -Tree Balancing -Path Compression

If one improvement is used, the time complexity will become  $O(\log N)$

If both are used,  $O(\alpha) \approx O(5)$

```
dsu.cpp
Description: Disjoint Set Union with path compression and tree balancing
Time:  $\mathcal{O}(\alpha)$ 
0479c4, 22 lines
struct DSU{
    vll group, card;
    DSU (ll n){
        n += 1; // 0-idx -> 1-idx
        group = vll(n);
        iota(group.begin(), group.end(), 0);
        card = vll(n, 1);
    }
    ll find(ll i){
        return (i == group[i]) ? i : (group[i] = find(group[i]));
    }
    // returns false if a and b are already in the same component
    bool join(ll a ,ll b){
        a = find(a);
        b = find(b);
        if (a == b) return false;
        if (card[a] < card[b]) swap(a, b);
        card[a] += card[b];
        group[b] = a;
        return true;
    }
};
```

Numerical (4)

Number theory (5)

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

$n$	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
$n$	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
$n$	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

Graph (7)

7.1 Fundamentals

7.2 Network flow

7.3 Matching

7.4 DFS algorithms

7.5 Coloring

7.6 Undirected Graph

Bridges and Articulation Points are concepts for undirected graphs!

7.6.1 Bridges (Cut Edges)

Also called **isthmus** or **cut arc**.

A back-edge is never a bridge!

A **lowlink** for a vertice  $U$  is the closest vertice to the root reachable using only span edges and a *single* back-edge, starting in the subtree of  $U$ .

After constructing a DFS Tree, an edge (u, v) is a bridge  $\iff$  there is no back-edge from  $v$  (or a descendent of  $v$ ) to  $u$  (or an ancestor of  $u$ )

To do this efficiently, it's used  $tin[i]$  (entry time of node  $i$ ) and  $low[i]$  (minimum entry time considering all nodes that can be reached from node  $i$ ).

In another words, a edge (u, v) is a bridge  $\iff$  the  $low[v] \nlessdot tin[u]$ .

```
bridges.cpp
Description: Using the concepts of entry time (tin) and lowlink (low), an edge is a bridge if, and only if,  $low[v] > tin[u]$ 
Time:  $\mathcal{O}(V + E)$ 
87e0d3, 25 lines
vector<vll> g(MAX);
ll timer = 1;
```

```
11 tin[MAX], low[MAX];
vector<pll> bridges;

void dfs(11 u, 11 p = -1){
    tin[u] = low[u] = timer++;
    for(auto v : g[u]) if (v != p) {
        if (tin[v]) // v was visited ({u,v} is a back-edge)
            // considering a single back-edge:
            low[u] = min(low[u], tin[v]);
        else { // v wasn't visited ({u, v} is a span-edge)
            dfs(v, u);
            // after low[v] was computed by dfs(v, u):
            low[u] = min(low[u], low[v]);
            if (low[v] > tin[u])
                bridges.pb({u, v});
        }
    }
}

void findBridges(11 n) {
    for(11 i=1; i<=n; i++) if (!tin[i])
        dfs(i);
}
```

7.6.2 Bridge Tree

After merging *vertices* of a **2-edge connected component** into single vertices, and leaving only bridges, one can generate a Bridge Tree.

Every **2-edge connected component** has following properties:

- For each pair of vertices A, B inside the same component, there are at least 2 distinct paths from A to B (which may repeat vertices).

bridgeTree.cpp

**Description:** After finding bridges, set an component id for each vertice, then merge vertices that are in the same 2-edge connected component  
**Time:**  $\mathcal{O}(V + E)$

```
6d00bd, 47 lines
// g: u -> {v, edge id}
vector<vector<pll>> g(MAX);
vector<vll> gc(MAX);
11 timer = 1;
11 tin[MAX], low[MAX], comp[MAX];
bool isBridge[MAX];

void dfs(11 u, 11 p = -1) {
    tin[u] = low[u] = timer++;
    for(auto [v, id] : g[u]) if (v != p) {
        if (tin[v])
            low[u] = min(low[u], tin[v]);
        else {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] > tin[u])
                isBridge[id] = 1;
        }
    }
}

void dfs2(11 u, 11 c, 11 p = -1) {
    comp[u] = c;
    for(auto [v, id] : g[u]) if (v != p) {
        if (isBridge[id]) continue;
        if (!comp[v]) dfs2(v, c, u);
    }
}
```

```
}
}

void bridgeTree(11 n) {
    // find bridges
    for(11 i=1; i<=n; i++) if (!tin[i])
        dfs(i);

    // find components
    for(11 i=1; i<=n; i++) if (!comp[i])
        dfs2(i, i);

    // condensate into a TREE (or TREES if disconnected)
    for(11 u=1; u<=n; u++) {
        for(auto [v, id] : g[u]) {
            if (comp[u] != comp[v]) {
                gc[comp[u]].pb(comp[v]);
            }
        }
    }
}
```

7.6.3 Articulation Points

One Vertice in a graph is considered a Articulation Points or Cut Vertice if its removal in the graph will generate more disconnected components

articulation.cpp

**Description:** if low[v] >= tin[u], u is an articulation points The root is a corner case  
**Time:**  $\mathcal{O}(V + E)$

```
8707a0, 29 lines
vector<vll> g(MAX);
11 timer = 1;
11 low[MAX], tin[MAX], isAP[MAX];
// when vertex i is removed from graph
// isAP[i] is the quantity of new disjoint components created
// isAP[i] >= 1 {i is a Articulation Point}
void dfs(11 u, 11 p = -1) {
    low[u] = tin[u] = timer++;

    for(auto v : g[u]) if (v != p) {
        if (tin[v]) // visited
            low[u] = min(low[u], tin[v]);
        else { // not visited
            dfs(v, u);
            low[u] = min(low[u], low[v]);

            if (low[v] >= tin[u])
                isAP[u]++;
        }
    }

    // corner case: root
    if (p == -1 and isAP[u]) isAP[u]--;
}

void findAP(11 n) {
    for(11 i=1; i<=n; i++) if (!tin[i])
        dfs(i);
}
```

7.6.4 Block Cut Tree

After merging *edges* of a **2-vertex connected component** into single vertices, one can obtain a block cut tree.

2-vertex connected components are also called as biconnected component

Every bridge by itself is a biconnected component

Each edge in the block-cut tree connects exactly an Articulation Point and a biconnected component (bipartite graph)

Each biconnected component has the following properties:

- For each pair of edges, there is a cycle that contains both edges
- For each pair of vertices A, B inside the same connected component, there are at least 2 distinct paths from A to B (which do not repeat vertices).

blockCutTree.cpp

**Description:** After Merging 2-Vertex Connected Components, one can generate a block cut tree  
**Time:**  $\mathcal{O}(V + E)$

```
f752d5, 100 lines
// Block-Cut Tree (bruno monteiro)
//
// Cria a block-cut tree, uma arvore com os blocos
// e os pontos de articulacao
// Blocos sao as componentes 2-vertice-conexas maximais
// Uma 2-coloracao da arvore eh tal que uma cor sao
// os componentes, e a outra cor sao os pontos de articulacao
//
// Funciona para grafo nao conexo
//
// isAP[i] responde o numero de novas componentes conexas
// criadas apos a remocao de i do grafo g
// Se isAP[i] >= 1, i eh ponto de articulacao
//
// Para todo i < blocks.size()
// blocks[i] eh uma componente 2-vertice-conexa maximal
// blockEdges[i] sao as arestas do bloco i
//
// tree eh a arvore block-cut-tree
// tree[i] eh um vertice da arvore que corresponde ao bloco i
//
// comp[i] responde a qual vertice da arvore vertice i pertence
//
// Arvore tem no maximo 2n vertices
//
// O(n+m)

// 0-idx graph!!!
vector<vll> g(MAX), tree, blocks; // 2-vertex-connected-
COMpOnentS
vector<vector<pll>> blockEdges;
stack<11> st; // st for vertices,
stack<pll> st2; // st2 for edges
vector<11> low, tin, comp, isAP;
11 timer = 1;

void dfs(11 u, 11 p = -1) {
    low[u] = tin[u] = timer++;

    st.push(u);

    // add only back-edges to stack
    if (p != -1) st2.push({u, p});
    for(auto v : g[u]) if (v != p) {
        if (tin[v] != -1) // visited
```

```
        st2.push({u, v});
    }

    for(auto v : g[u]) if (v != p) {
        if (tin[v] != -1) // visited
            low[u] = min(low[u], tin[v]);
        else { // not visited
            dfs(v, u);
            low[u] = min(low[u], low[v]);

            if (low[v] >= tin[u]) {
                isAP[u] += 1;

                blocks.pb(vll(1, u));
                while(blocks.back().back() != v)
                    blocks.back().pb(st.top()), st.pop();

                blockEdges.pb(vector<p11>(1, st2.top())), st2.pop();
                while(blockEdges.back().back() != pair<11, 11>(v, u))
                    blockEdges.back().pb(st2.top()), st2.pop();
            }
        }
    }

    // corner case: root
    if (p == -1 and isAP[u]) isAP[u]--;
}

void blockCutTree(11 n) {

    // initialize vectors and reset
    tree.clear(), blocks.clear(), blockEdges.clear();
    st = stack<11>(), st2 = stack<p11>();
    tin.assign(n, -1);
    low.assign(n, 0), comp.assign(n, 0), isAP.assign(n, 0);
    timer = 1;

    // find Articulation Points
    for(11 i=0; i<n; i++) if (tin[i] == -1)
        dfs(i);

    // set component id for APs
    tree.assign(blocks.size(), v11());
    for(11 i=0; i<n; i++) if (isAP[i])
        comp[i] = tree.size(), tree.pb(v11());

    // set component id for non-APs and construct tree
    for(11 u=0; u<(11)blocks.size(); u++) {
        for(auto v : blocks[u]) {
            if (!isAP[v])
                comp[v] = u;
            else
                tree[u].pb(comp[v]), tree[comp[v]].pb(u);
        }
    }
}
```

7.6.5 Minimum Spanning Tree

A minimum spanning tree (MST) or minimum weight spanning tree is a subset of the edges of a connected, edge-weighted undirected graph that connects all the vertices together, without any cycles and with the minimum possible total edge weight. That is, it is a spanning tree whose sum of edge weights is as small as possible.

```
kruskal.cpp
Description: Sort all edges in crescent order by weight, include all edges
which joins two disconnected trees. In case of tie, choose whichever. Dont
include edges that will join a already connected part of the tree.
Time: O(E log Eα)
206ba3, 21 lines

// use DSU struct
struct DSU{};

set<array<11, 3>> edges;

int32_t main(){ sws;
    11 n, m; cin >> n >> m;
    DSU dsu(n+1);
    for(11 i=0; i<m; i++) {
        11 u, v, w; cin >> u >> v >> w;
        edges.insert({w, u, v});
    }
    11 minCost = 0;
    for(auto [w, u, v] : edges) {
        if (dsu.find(u) != dsu.find(v)) {
            dsu.join(u, v);
            minCost += w;
        }
    }
    cout << minCost << endl;
}
```

7.7 Directed Graph

7.7.1 Topological Sort

Sort a directed graph with no cycles (DAG) in an order which each source of an edge is visited before the sink of this edge.

Cannot have cycles, because it would create a contradiction of which vertices whould come before.

It can be done with a DFS, appending in the reverse order of transversal. Also a stack can be used to reverse order

```
toposort.cpp
Description: Using DFS pos order transversal and inverting the order, one
can obtain the topological order
Time: O(V + E)
75f781, 17 lines

vector<v11> g(MAX, v11());
vector<bool> vis;
v11 topological;

void dfs(11 u) {
    vis[u] = 1;
    for(auto v : g[u]) if (!vis[v]) dfs(v);
    topological.pb(u);
}

// 1 - indexed
void topological_sort(11 n) {
    vis.assign(n+1, 0);
    topological.clear();
    for(11 i=1; i<=n; i++) if (!vis[i]) dfs(i);
    reverse(topological.begin(), topological.end());
}
```

7.7.2 Kosaraju

A Strongly Connected Component is a maximal subgraph in which every vertex is reachable from any vertex inside this same subgraph.

A important *property* is that the inverted graph or transposed graph has the same SCCs as the original graph.

```
kosaraju.cpp
Description: By using the fact that the inverted graph has the same SCCs,
just do a DFS twice to find all SCCs. A condensated graph can be created
if wished. The condensated graph is a DAG!!
Time: O(V + E)
381904, 45 lines

struct Kosaraju {
    11 n;
    vector<v11> g, gi, gc;
    vector<bool> vis;
    vector<11> comp;
    stack<11, v11> st;

    void dfs(11 u) { // g
        vis[u] = 1;
        for(auto v : g[u]) if (!vis[v]) dfs(v);
        st.push(u);
    }

    void dfs2(11 u, 11 c) { // gi
        comp[u] = c;
        for(auto v : gi[u]) if (comp[v] == -1) dfs2(v, c);
    }

    Kosaraju(vector<v11> &g_)
        : g(g_), n(g_.size()-1) { // 1-idx

        gi.assign(n+1, v11());
        for(11 i=1; i<=n; i++) {
            for(auto j : g[i])
                gi[j].pb(i);
        }

        gc.assign(n+1, v11());
        vis.assign(n+1, 0);
        comp.assign(n+1, -1);
        st = stack<11, v11>();

        for(11 i=1; i<=n; i++) if (!vis[i]) dfs(i);

        while(!st.empty()) {
            auto u = st.top(); st.pop();
            if (comp[u] == -1) dfs2(u, u);
        }

        for(11 u=1; u<=n; u++)
            for(auto v : g[u])
                if (comp[u] != comp[v])
                    gc[comp[u]].pb(comp[v]);
    }
}
```

7.7.3 2-SAT

$\text{SAT}$  (Boolean satisfiability problem) is NP-Complete.

2-SAT is a restriction of the SAT problem, in 2-SAT every clause has exactly two variables:  $(X_1 \vee X_2) \wedge (X_2 \vee X_3)$

Every restriction or implication are represented in the graph as directed edges.

The algorithm uses kosaraju to check if any  $(X$  and  $\neg X)$  are in the same Strongly Connected Component (which implies that the problem is impossible).

If it doesn't, there is at least one solution, which can be generated using the topological sort of the same kosaraju (opting for the variables that appers latter in the sorted order)

## 2sat.cpp

**Description:** Kosaraju to find if there are SCCs. If there are not cycles, use toposort to choose states

**Time:**  $\mathcal{O}(V + E)$  87417c, 83 lines

```
// 0-idx graph !!!!
struct TwoSat {
    ll N; // needs to be the twice of the number of variables
    // node with idx 2x => variable x
    // node with idx 2x+1 => variable !x

    vector<vll> g, gi;
    // g = graph; gi = transposed graph (all edges are inverted)

    TwoSat(ll n) { // number of variables (add +1 faor 1-idx)
        N = 2*n;
        g.assign(N, vll());
        gi.assign(N, vll());
    }

    ll idx; // component idx
    vector<ll> comp, order; // topological order (reversed)
    vector<bool> vis, chosen;
    // chosen[x] == 0 -> x was assigned
    // chosen[x] == 1 -> !x was assigned

    // dfs and dfs2 are part of kosaraju algorithm
    void dfs(ll u) {
        vis[u] = 1;
        for (ll v : g[u]) if (!vis[v]) dfs(v);
        order.pb(u);
    }

    void dfs2(ll u, ll c) {
        comp[u] = c;
        for (ll v : gi[u]) if (comp[v] == -1) dfs2(v, c);
    }

    bool solve() {
        vis.assign(N, 0);
        order = vector<ll>();
        for (ll i = 0; i < N; i++) if (!vis[i]) dfs(i);

        comp.assign(N, -1); // comp = 0 can exist
        idx = 1;
        for (ll i = (ll)order.size()-1; i >= 0; i--) {
            ll u = order[i];
            if (comp[u] == -1) dfs2(u, idx++);
        }

        chosen.assign(N/2, 0);
        for (ll i = 0; i < N; i += 2) {
            // x and !x in the same component => contradiction
            if (comp[i] == comp[i+1]) return false;
        }
    }
};
```

```
        chosen[i/2] = comp[i] < comp[i+1]; // choose latter node
    }
    return true;
}

// a (with flagA) implies => b (with flagB)
void add(ll a, bool fa, ll b, bool fb) {
    // {fa == 0} => a
    // {fa == 1} => !a
    a = 2*a + fa;
    b = 2*b + fb;
    g[a].pb(b);
    gi[b].pb(a);
}

// force a state for a certain variable (must be true)
void force(ll a, bool fa) {
    add(a, fa^1, a, fa);
}

// xor operation: one must exist, and only one can exist
void exclusive(ll a, bool fa, ll b, bool fb) {
    add(a, fa^0, b, fb^1);
    add(a, fa^1, b, fb^0);
    add(b, fb^0, a, fa^1);
    add(b, fb^1, a, fa^0);
}

// nand operation: no more than one can exist
void nand(ll a, bool fa, ll b, bool fb) {
    add(a, fa^0, b, fb^1);
    add(b, fb^0, a, fa^1);
}

};
```

## 7.8 Trees

### lca.cpp

**Description:** Solves LCA for trees

**Time:**  $\mathcal{O}(N \log(N))$  to build,  $\mathcal{O}(\log(N))$  per query 7afc1a, 54 lines

```
struct BinaryLifting {
    ll n, logN = 20; // ~1e6
    vector<vll> g;
    vector<ll> depth;
    vector<vll> up;

    BinaryLifting(vector<vll> &g_)
        : g(g_), n(g_.size() + 1) { // 1-idx
        depth.assign(n, 0);

        while((1 << logN) < n) logN++;
        up.assign(n, vll(logN, 0));
        build();
    }

    void build(ll u = 1, ll p = -1) {
        for (ll i = 1; i < logN; i++) {
            up[u][i] = up[ up[u][i-1] ][i-1];
        }

        for (auto v : g[u]) if (v != p) {
            up[v][0] = u;
            depth[v] = depth[u] + 1;
            build(v, u);
        }
    }

    ll go(ll u, ll dist) { // O(log(n))
```

```
        for (ll i = logN-1; i >= 0; i--) { // bigger jumps first
            if (dist & (1LL << i)) {
                u = up[u][i];
            }
        }
        return u;
    }

    ll lca(ll a, ll b) { // O(log(n))
        if (depth[a] < depth[b]) swap(a, b);
        a = go(a, depth[a] - depth[b]);
        if (a == b) return a;

        for (ll i = logN-1; i >= 0; i--) {
            if (up[a][i] != up[b][i]) {
                a = up[a][i];
                b = up[b][i];
            }
        }
        return up[a][0];
    }

    ll lca(ll a, ll b, ll root) { // lca(a, b) when tree is rooted at 'root'
        return lca(a, b)^lca(b, root)^lca(a, root); //magic
    }
};
```

### queryTree.cpp

**Description:** Binary Lifting for min, max weight present in a simple path

**Time:**  $\mathcal{O}(N \log(N))$  to build;  $\mathcal{O}(\log(N))$  per query 75ba37, 67 lines

```
struct BinaryLifting {
    ll n, logN = 20; // ~1e6
    vector<vpll> g;
    vector<ll> depth;
    vector<vll> up, mx, mn;

    BinaryLifting(vector<vpll> &g_)
        : g(g_), n(g_.size() + 1) { // 1-idx
        depth.assign(n, 0);

        while((1 << logN) < n) logN++;
        up.assign(n, vll(logN, 0));
        mx.assign(n, vll(logN, -INF));
        mn.assign(n, vll(logN, INF));
        build();
    }

    void build(ll u = 1, ll p = -1) {

        for (ll i = 1; i < logN; i++) {
            mx[u][i] = max(mx[u][i-1], mx[ up[u][i-1] ][i-1]);
            mn[u][i] = min(mn[u][i-1], mn[ up[u][i-1] ][i-1]);
            up[u][i] = up[ up[u][i-1] ][i-1];
        }

        for (auto [v, w] : g[u]) if (v != p) {
            mx[v][0] = mn[v][0] = w;
            up[v][0] = u;
            depth[v] = depth[u] + 1;
            build(v, u);
        }
    }

    array<ll, 3> go(ll u, ll dist) { // O(log(n))
        ll mxval = -INF, mnval = INF;
        for (ll i = logN-1; i >= 0; i--) { // bigger jumps first
            if (dist & (1LL << i)) {
```



```

        mxval = max(mxval, mx[u][i]);
        mnval = min(mnval, mn[u][i]);
        u = up[u][i];
    }
}
return {u, mxval, mnval};
}

array<ll, 3> query(ll u, ll v) { // O(log(n))
    if (depth[u] < depth[v]) swap(u, v);

    auto [a, mxval, mnval] = go(u, depth[u] - depth[v]);
    ll b = v;

    if (a == b) return {a, mxval, mnval};

    for (ll i=logN-1; i>=0; i--) {
        if (up[a][i] != up[b][i]) {
            mxval = max({mxval, mx[a][i], mx[b][i]});
            mnval = min({mnval, mn[a][i], mn[b][i]});
            a = up[a][i];
            b = up[b][i];
        }
    }

    mxval = max({mxval, mx[a][0], mx[b][0]});
    mnval = min({mnval, mn[a][0], mn[b][0]});
    return {up[a][0], mxval, mnval};
}
};
```

- 7.9 Math
- Geometry (8)
- Strings (9)
- Miscellaneous (10)



# Techniques (A)

techniques.txt	159 lines
Recursion	
Divide and conquer	
Finding interesting points in N log N	
Algorithm analysis	
Master theorem	
Amortized time complexity	
Greedy algorithm	
Scheduling	
Max contiguous subvector sum	
Invariants	
Huffman encoding	
Graph theory	
Dynamic graphs (extra book-keeping)	
Breadth first search	
Depth first search	
* Normal trees / DFS trees	
Dijkstra's algorithm	
MST: Prim's algorithm	
Bellman-Ford	
Konig's theorem and vertex cover	
Min-cost max flow	
Lovasz toggle	
Matrix tree theorem	
Maximal matching, general graphs	
Hopcroft-Karp	
Hall's marriage theorem	
Graphical sequences	
Floyd-Warshall	
Euler cycles	
Flow networks	
* Augmenting paths	
* Edmonds-Karp	
Bipartite matching	
Min. path cover	
Topological sorting	
Strongly connected components	
2-SAT	
Cut vertices, cut-edges and biconnected components	
Edge coloring	
* Trees	
Vertex coloring	
* Bipartite graphs (=> trees)	
* 3^n (special case of set cover)	
Diameter and centroid	
K'th shortest path	
Shortest cycle	
Dynamic programming	
Knapsack	
Coin change	
Longest common subsequence	
Longest increasing subsequence	
Number of paths in a dag	
Shortest path in a dag	
Dynprog over intervals	
Dynprog over subsets	
Dynprog over probabilities	
Dynprog over trees	
3^n set cover	
Divide and conquer	
Knuth optimization	
Convex hull optimizations	
RMQ (sparse table a.k.a 2^k-jumps)	
Bitonic cycle	
Log partitioning (loop over most restricted)	
Combinatorics	

Computation of binomial coefficients
Pigeon-hole principle
Inclusion/exclusion
Catalan number
Pick's theorem
Number theory
Integer parts
Divisibility
Euclidean algorithm
Modular arithmetic
* Modular multiplication
* Modular inverses
* Modular exponentiation by squaring
Chinese remainder theorem
Fermat's little theorem
Euler's theorem
Phi function
Frobenius number
Quadratic reciprocity
Pollard-Rho
Miller-Rabin
Hensel lifting
Vieta root jumping
Game theory
Combinatorial games
Game trees
Mini-max
Nim
Games on graphs
Games on graphs with loops
Grundy numbers
Bipartite games without repetition
General games without repetition
Alpha-beta pruning
Probability theory
Optimization
Binary search
Ternary search
Unimodality and convex functions
Binary search on derivative
Numerical methods
Numeric integration
Newton's method
Root-finding with binary/ternary search
Golden section search
Matrices
Gaussian elimination
Exponentiation by squaring
Sorting
Radix sort
Geometry
Coordinates and vectors
* Cross product
* Scalar product
Convex hull
Polygon cut
Closest pair
Coordinate-compression
Quadtrees
KD-trees
All segment-segment intersection
Sweeping
Discretization (convert to events and sweep)
Angle sweeping
Line sweeping
Discrete second derivatives
Strings
Longest common substring
Palindrome subsequences

Knuth-Morris-Pratt
Tries
Rolling polynomial hashes
Suffix array
Suffix tree
Aho-Corasick
Manacher's algorithm
Letter position lists
Combinatorial search
Meet in the middle
Brute-force with pruning
Best-first (A*)
Bidirectional search
Iterative deepening DFS / A*
Data structures
LCA (2^k-jumps in trees in general)
Pull/push-technique on trees
Heavy-light decomposition
Centroid decomposition
Lazy propagation
Self-balancing trees
Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)
Monotone queues / monotone stacks / sliding queues
Sliding queue using 2 stacks
Persistent segment tree