Competitive-programming

Algoritmos e ideias de Programação Competitiva

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Flags for compilation:

```
g++ -Wall -Wextra -Wshadow -ggdb3 -D_GLIBCXX_ASSERTIONS -fmax-errors=2 -std=c++17 -O3
test.cpp -o test
```

Linux Alias

```
alias comp='g++ -std=c++17 -g -O2 -Wall -Wextra -Wconversion -Wshadow -
D_GLIBCXX_ASSERTIONS -fsanitize=address,undefined -fno-sanitize-recover -ggdb -o test'
```

Template:

```
// Needed
#include <bits/stdc++.h>
using namespace std;
#define sws cin.tie(0)->sync_with_stdio(0)

// Life Quality
#define endl '\n'
#define ll long long
#define vll vector<ll>
#define pb push_back
#define ld long double
#define vld vector<ld>
#define pll pair<ll, ll>
#define vpll vector<pll>
#define ff first
#define ss second
#define tlll tuple<ll, ll, ll>

// Utility
#define teto(a, b) ((a+b-1)/(b))
#define LSB(i) ((i) & -(i))
#define MSB(i) (32 - __builtin_clz(i)) // or 64 - clzll
#define BITS(i) __builtin_popcount(i) // count bits
```

```
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());

// Debugging
#define db(a) cerr << " [ " << #a << " = " << a << " ] " << endl;
#define debug(a...) cerr<<#a<<": ";for(auto &b:a)cerr<<b<<" ";cerr<<endl;
template <typename... A> void dbg(A const&... a) { ((cerr << "{" << a << "} "), ...);
cerr << endl; }

// Constants
const int MAX = 2e5+10;
const long long MOD = 1e9+7;
const int INF = 0x3f3f3f3f;
const long long LLINF = 0x3f3f3f3f3f3f3f3f;
const long double EPS = 1e-7;
const long double PI = acos(-1);

int32_t main(){ sws;

}
```

DP

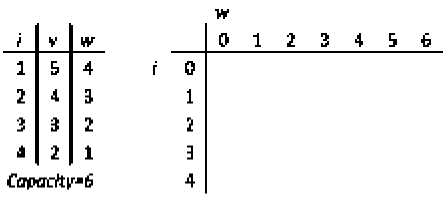
LIS (Longest Increasing Sequence)

Strictly Increasing: ans_i < ans_(i+1)

Requires a vector *x* with size *n*

```
vll d(n+1, LLINF);
d[0] = -LLINF;
for(ll i=0; i<n; i++){
    ll idx = upper_bound(d.begin(), d.end(), x[i]) - d.begin();
    if (d[idx-1] < x[i])
        d[idx] = min(d[idx], x[i]);
}
ll lis = (lower_bound(d.begin(), d.end(), LLINF) - d.begin() - 1);
```

Knapsack



- Use int instead of long long for 10^8 size matrix

```
    int n; cin >> n; // quantity of items to be chosen
    int x; cin >> x; // maximum capacity or weight
    vector<int> cost(n+1);
```

```
    vector<int> value(n+1);
    for(int i=1; i<=n; i++) cin >> cost[i];
    for(int i=1; i<=n; i++) cin >> value[i];

    vector<vector<int>> dp(n+1, vector<int>(x+1, 0));

    for(int i=1; i<=n; i++){
        for(int j=1; j<=x; j++){
            // same answer as if using -1 total capacity (n pega)
            dp[i][j] = max(dp[i][j], dp[i-1][j]);
            // use the item with index i (pega)
            if (j-cost[i] >= 0)
                dp[i][j] = max(dp[i][j], dp[i-1][j-cost[i]] + value[i]);
        }
    }

    cout << dp[n][x] << endl;
```

Digit DP

Use each digit position as state and also is the considered number is already smaller than the reference. The rest of the states are defined by the problem

Example1:

Calculate the quantity of numbers with no consective equal digits

```
string s; // number
ll tab[20][2][2][2][20];

// * returns the qtd of numbers with no consective equal digits
ll dp(ll i, bool smaller, bool consec, bool significantDigit, ll lastDigit){
    if (i >= (ll) s.size()) {
        if (consec) return 0;
        else return 1;
    }

    if (tab[i][smaller][consec][significantDigit][lastDigit] != -1)
        return tab[i][smaller][consec][significantDigit][lastDigit];

    ll limit = (s[i] - '0');
    ll ans = 0;

    for(ll a=0; a<=9; a++){
        bool tmp = consec;
        bool tmp2 = significantDigit; // avoid left zeros: 00001

        if (a > 0) tmp2 = 1;
        if (a == lastDigit and significantDigit) tmp = 1;

        if (smaller){
            ans += dp(i+1, 1, tmp, tmp2, a);
        }
        else if (a < limit){
            ans += dp(i+1, 1, tmp, tmp2, a);
```

```
        }
        else if (a == limit){
            ans += dp(i+1, 0, tmp, tmp2, a);
        }
    }
    return tab[i][smaller][consec][significantDigit][lastDigit] = ans;
}


int32_t main(void){ sws;
    ll a, b; cin >> a >> b;

    memset(tab, -1, sizeof(tab));
    s = to_string(b);
    ll ansr = dp(0, 0, 0, 0, 15); // 15 is simply a not valid number

    memset(tab, -1, sizeof(tab));
    s = to_string(a-1);
    ll ansl = dp(0, 0, 0, 0, 15);

    cout << ansr - ansl << endl;
}
```

Example2:

Classy numbers are the numbers than contains no more than 3 non-zero digit

```
string s;
ll tab[20][2][5];

// * returns qtd of classy numbers
ll dp(ll i, bool smaller, ll dnn){
    if (dnn > 3) return 0;
    if (i >= s.size()) return 1;

    if (tab[i][smaller][dnn] != -1) return tab[i][smaller][dnn];

    ll limit = (s[i] - '0');
    ll ans = 0;

    for(ll a=0; a<=9; a++){
        ll dnn2 = dnn;
        if (a > 0) dnn2 += 1;

        if (smaller){
            ans += dp(i+1, 1, dnn2);
        }
        else if (a < limit){
            ans += dp(i+1, 1, dnn2);
        }
        else if (a == limit){
            ans += dp(i+1, 0, dnn2);
        }
    }
    return tab[i][smaller][dnn] = ans;
```

```
}

int32_t main(void){ sws;
    ll t; cin >> t;
    while(t--){
        ll l, r;
        cin >> l >> r;

        memset(tab, -1, sizeof(tab));
        s = to_string(r);
        ll ansr = dp(0, 0, 0);

        memset(tab, -1, sizeof(tab));
        s = to_string(l-1);
        ll ansl = dp(0, 0, 0);

        cout << ansr - ansl << endl;
    }
}
```

Bitmask DP

use a bitmask of chosen itens to be a state of the DP

Example:

https://cses.fi/problemset/task/1653/

```
int32_t main(){
    ll n, x; cin >> n >> x;
    vll a(n);

    for(ll i=0; i<n; i++) cin >> a[i];

    // dp[bitmask of selected people] -> {elevator rides, weight occupied}
    vpll dp( (1 << n) , {INF, INF});
    dp[0] = {1, 0};

    for(ll mask=0; mask< (1<<n); mask++) {
        for(ll j=0; j<n; j++) {
            if (mask & (1 << j)) {
                ll bit = mask ^ (1 << j);

                // there is room for one more weight
                if (dp[bit].ss + a[j] <= x)
                    dp[mask] = min( dp[mask], {dp[bit].ff, dp[bit].ss + a[j]} );

                // add an elevator ride, and create a new one with just one person
                else
                    dp[mask] = min( dp[mask], {dp[bit].ff + 1, a[j]} );
            }
        }
    }
    cout << dp[(1 << n) - 1].ff << endl;
}
```

Broken Profile

Solves problem where is needed to count the ways of filling a *n* x *m* grid with dominos/tilings of specific size.

Example: Fill *n* x *m* with 2x1 dominos or 1x2 dominos -> https://cses.fi/problemset/task/2181

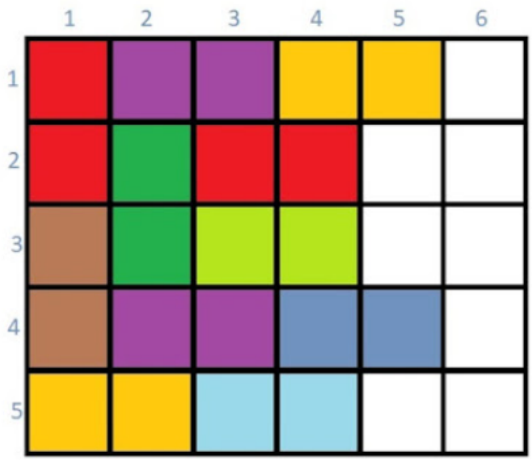
1 ≤ n ≤ 10 (rows) 1 ≤ m ≤ 1000 (collums)

States

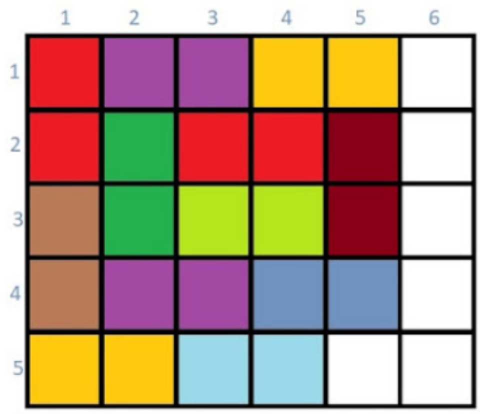
dp[j][p] -> numbers of ways of filling first *j* columns completely with dominoes (without leaving any block as empty) and leaving the profile *p* for the *j+1* collum.

j = collums completely filled. *p* = bitmask representation of the "profile" for the j+1 collum.

Note that, the (j+1) th column should not contain a complete domino (in the vertical), those types will be included in the dp transition from j+1 to j+2.



Valid filling of first 4 columns leaving the 5th column with profile 9 i.e. 01001



Invalid filling that cannot be counted into dp[4][15] as the brown domino is not used in filling any of the first four columns.
Note: 15 represents 01111 i.e. the profile of 5th columns

Transitions

Initial States:

```
dp[i = 0][p = 0] = 1
dp[i = 0][p != 0] = 0
```

From a specific **dp[j][q]**, with profile *q* for the *j-th* collum and all the collums before filled, it's possible to generate several **dp[j+1][p]** just by adding vertical and horizontal tiles.

Final Answer: dp[m][0]

```
// grid size
ll dp[1010][1 << 11];
ll n, m;
// n is the num of rows, m is the num of collums

// check if i'th bit of q is occupied
```

```cpp
bool occupied(ll i, ll q) {
    return q & (1 << (i-1));
}

void solveBlock(ll i, ll j, ll p, ll q) {
    // from profile q -> generate profile p
    // OBS: q represents the profile of the j col, and p represents the profile of the
j+1 col when j is filled;
    // i <= 10 (row); j <= 1000 (col)

    // found a new way to fill j, add this possibility
    if (i == n+1) {
        dp[j+1][p] = (dp[j+1][p] + dp[j][q]) % MOD;
        return;
    }

    // skip occupied block
    if ( occupied(i, q) ) {
        solveBlock(i+1, j, p, q);
        return;
    }

    // insert vertical tile
    if(i+1 <= n and !occupied(i+1, q)){
        solveBlock(i+2, j, p, q);
    }

    // insert horizontal tile
    if (j+1 <= m) {
        solveBlock(i+1, j, p^(1<<(i-1)), q);
    }
}

int32_t main(){ sws;
    cin >> n >> m;
    memset(dp, 0, sizeof(dp));
    dp[0][0] = 1; // Initial Condition
    for(ll j=0; j<m; j++) { // each collum
        for(ll q=0; q < (1 << n); q++){ // each collum profile
            solveBlock(1, j, 0, q);
        }
    }
    cout << dp[m][0] << endl;
}
```

Graph

Topological Sort

Sort a directed graph with no cycles in an order which each source of an edge is visited before the sink of this edge.

Cannot have cycles, because it would create a contradition of which vertices whould come before.

It can be done with a DFS, appending in the reverse order of transversal.

```
vector<vll> g(MAX, vll());
vector<bool> vis;
vll topological;

void dfs(ll u) {
    vis[u] = 1;
    for(auto v : g[u]) if (!vis[v]) dfs(v);
    topological.pb(u);
}

// 1 - indexed
void topological_sort(ll n) {
    vis.assign(n+1, 0);
    topological.clear();
    for(ll i=1; i<=n; i++) if (!vis[i]) dfs(i);
    reverse(topological.begin(), topological.end());
}
```

DFS Tree

A *Back Edge* existence means that there is a cycle.

```
bool visited[MAX];
vector<vll> g(MAX, vll());
map<ll, ll> spanEdges;
map<ll, ll> backEdges; // children to parent
ll h[MAX];
ll p[MAX];

void dfs(ll u=1, ll parent=0, ll layer=1){
    if (visited[u]) return;
    visited[u] = 1;
    h[u] = layer;
    for(auto v : g[u]){
        if (v == parent) spanEdges[u] = v;
        else if (visited[v] and h[v] < h[u]) backEdges[u] = v;
        else dfs(v, u, layer+1);
    }
}
```

Bridges (Cut Edges)

Also called *isthmus* or *cut arc*.

Theory: After constructing a DFS Tree, an edge (u, v) is a bridge if and only if there is no back-edge from *v, or a descendent of v*, to *u, or an ancestor of u*.

To do this efficiently, it's used *tin[i] (entry time of node **i**)* and *low[i] (minimum entry time of all nodes that can be reached from node **i**)*.

```
vector<vll> g(MAX, vll());
bool vis[MAX];
ll tin[MAX], low[MAX];
ll timer;
vpll bridges;

void dfs(ll u, ll p = -1){
    vis[u] = 1;
    tin[u] = low[u] = timer++;
    for(auto v : g[u]) if (v != p) {
        if (vis[v]) low[u] = min(low[u], tin[v]);
        else {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] > tin[u])
                bridges.pb( {u, v} );
        }
    }
}

void find_bridges(ll n) {
    timer = 1;
    memset(vis, 0, sizeof(vis));
    memset(tin, 0, sizeof(tin));
    memset(low, 0, sizeof(low));
    for(ll i=1; i<=n; i++) if (!vis[i]) dfs(i);
}
```

Articulation Points and Bridges

Finds all Cut-Vertices and Cut-Edges in a single dfs tranversal O(V+E)

Maybe is working, maybe it's not, needs testing for exquisite graphs, like cliques

```
vector<vll> g(MAX, vll());
vll tin(MAX, -1), low(MAX, 0);
// tin[] = the first time a node is visited ("time in")
// if tin[u] != -1, u was visited
// low[] = lowest first_time of any node reachable by the current node

ll root = -1, rootChildren = 0, timer = 0;
// root = the root of a dfs transversal, rootChildren = number of direct descedentes of
the root

vector<bool> isArticulation(MAX, 0); // this vector exists, because we can define several
time if a node is a cut vertice
vll articulations; // cut vertices
vpll bridges; // cut edges

void dfs(ll u, ll p) {
    low[u] = tin[u] = timer++;

    for(auto v : g[u]) if (v != p) {
        if (tin[v] == -1) { // not visited
            if (u == root) rootChildren += 1;
```

```
            dfs(v, u);

            if (low[v] >= tin[u]) isArticulation[u] = 1;
            if (low[v] > tin[u]) bridges.pb({u, v});
        }

        low[u] = min(low[u], low[v]);
    }
}

void findBridgesAndPoints(ll n) {
    timer = 0;
    for(ll i=1; i<=n; i++) if (tin[i] == -1) {
        root = i; rootChildren = 0;
        dfs(i, -1);
        if (rootChildren > 1) isArticulation[i] = 1;
    }
    for(ll i=1; i<=n; i++) if (isArticulation[i]) articulations.pb(i);
}
```

Euler Path

Definitions:

An **Eulerian Path** or **Eulerian Trail** (*Caminho Euleriano*) consists of a path that transverses all **Edges**.

A special case is the closed path, which is an **Eulerian Circuit** or **Eulerian Cycle** (*Circuito/Ciclo Euleriano*). A graph is considered *eulerian* (**Eulerian Graph**) if it has an Eulerian Circuit.

Similarly, a **Hamiltonian Path** consists of a path that transverses all **Vertices**.

Conditions for Eulerian Path existence

To check if it is possible, there is a need for connectivity:

connectivity, all vertices (that contains at least 1 edge) are connected. But there is no need for it to be strongly connected. To check connectivity, you can consider a directed graph as undirected and do a dfs.

and also:

What conditions are required for a valid Eulerian Path/Circuit?

That depends on what kind of graph you're dealing with. Altogether there are four flavors of the Euler path/circuit problem we care about:

	Eulerian Circuit	Eulerian Path
Undirected Graph	Every vertex has an even degree.	Either every vertex has even degree or exactly two vertices have odd degree.
Directed Graph	Every vertex has equal indegree and outdegree	At most one vertex has (outdegree) - (indegree) = 1 and at most one vertex has (indegree) - (outdegree) = 1 and all other vertices have equal in and out degrees.

Hierholzer Algorithm

Find a **Eulerian Path/Circuit** with a linear complexity of *O(Edges)*.

Using an *ordered set* on **Undirected Graphs** increases complexity by *log2(Edges)*. This can be optimized using a *list* with references to each bidirectional edge so that any reversed edge can be erased in *O(1)*.

Example 1:

Generating an **Eulerian Path** with Hierholzer in a *Directed Graph*, starting on node *1* and ending on node *n*.

https://cses.fi/problemset/task/1693

```
vector<vll> g(MAX, vll());
vector<vll> ug(MAX, vll()); // undirected graph

vll inDegree(MAX, 0);
vll outDegree(MAX, 0);

vector<bool> vis(MAX, 0);

ll dfsConnected(ll u) {
    ll total = 1; vis[u] = 1;
    for(auto v : ug[u]) if (!vis[v]) {
        total += dfsConnected(v);
    }
```

```cpp
    return total;
}

// O(n) -> O(Vertices)
bool checkPossiblePath(ll start, ll end, ll n, ll nodes) {

    // check connectivity
    vis.assign(n+1, 0);
    ll connectedNodes = dfsConnected(1);
    if (connectedNodes != nodes) return 0;

    // check degrees
    for(ll i=1; i<=n; i++) {
        if (i == start) { // start node needs to have 1 more outDegree than inDegree
            if (inDegree[i]+1 != outDegree[i]) return 0;
        }
        else if (i == end) { // end node needs to have 1 more inDegree than outDegree
            if (inDegree[i] != outDegree[i]+1) return 0;
        }
        else {
            if (inDegree[i] != outDegree[i]) return 0;
        }
    }

    return 1;
}

// O(m) -> O(Edges)
// Hierholzer function can be used directly if there is already a garanted existance of
an eulerian path/circuit.
vll hierholzer(ll start, ll n) { // generate an eulerian path, assuming there is only 1
end node
    vll ans, pilha, idx(n+1, 0);

    pilha.pb(start);
    while(!pilha.empty()) {
        ll u = pilha.back();
        if (idx[u] < (ll) g[u].size()) {
            pilha.pb( g[u][idx[u]] );
            idx[u] += 1;
        }
        else { // no more outEdge from node u, backtracking
            ans.pb(u);
            pilha.pop_back();
        }
    }
    reverse(ans.begin(), ans.end());
    return ans;
}

int32_t main(){ sws;
    ll n, m; cin >> n >> m;

    // OBS: some nodes are isolated and don't contribute to the eulerian path
    ll participantNodes = 0;

    for(ll i=0; i<m; i++) {
        ll a, b; cin >> a >> b;
```

```
        g[a].pb(b);
        ug[a].pb(b); ug[b].pb(a);

        outDegree[a] += 1;
        inDegree[b] += 1;

        if (!vis[a]) {
            vis[a] = 1;
            participantNodes += 1;
        }
        if (!vis[b]) {
            vis[b] = 1;
            participantNodes += 1;
        }
    }

    if ( !checkPossiblePath(1, n, n, participantNodes) ) {
        cout << "IMPOSSIBLE" << endl;
        return 0;
    }

    for(auto elem : hierholzer(1, n)) cout << elem << ' ';
    cout << endl;
}
```

Example 2:

Generating an **Eulerian Circuit** with Hierholzer in an *Undirected Graph*, starting on node *1* and also ending on node *1*.

https://cses.fi/problemset/task/1691

```
// adding log2(m) complexity due to ordered_set structure required for not using a same
bidirectional edge twice
#include <bits/extc++.h>
using namespace __gnu_pbds;
template <class T> using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
tree_order_statistics_node_update>;

vector<ordered_set<ll>> g(MAX, ordered_set<ll>()); // undirected graph

vll degree(MAX, 0);

vector<bool> vis(MAX, 0);

ll dfsConnected(ll u) {
    ll total = 1; vis[u] = 1;
    for(auto v : g[u]) if (!vis[v]) {
        total += dfsConnected(v);
    }
    return total;
}

// O(n log2(m)) -> O(Vertices * log2(Edges))
bool checkPossiblePath(ll n, ll nodes) {
```

```
    // check connectivity
    vis.assign(n+1, 0);
    ll connectedNodes = dfsConnected(1);
    if (connectedNodes != nodes) return 0;

    // check degrees
    for(ll i=1; i<=n; i++) {
        // all degrees need to be even
        if (degree[i] % 2 == 1) return 0;
    }

    return 1;
}

// O(m * log2(m)) -> O(Edges * log2(m))
// Hierholzer function can be used directly if there is already a garanted existance of
an eulerian path/circuit.
vll hierholzer(ll start, ll n) { // generate an eulerian path, assuming there is only 1
end node
    vll ans, pilha, idx(n+1, 0);

    pilha.pb(start);
    while(!pilha.empty()) {
        ll u = pilha.back();
        if (idx[u] < (ll) g[u].size()) {
            ll v = *(g[u].find_by_order(idx[u]));

            pilha.pb( v );
            g[v].erase(u);

            idx[u] += 1;
        }
        else { // no more outEdge from node u, backtracking
            ans.pb(u);
            pilha.pop_back();
        }
    }
    reverse(ans.begin(), ans.end());
    return ans;
}

int32_t main(){ sws;
    ll n, m; cin >> n >> m;

    // OBS: some nodes are isolated and don't contribute to the eulerian circuit
    ll participantNodes = 0;

    for(ll i=0; i<m; i++) {
        ll a, b; cin >> a >> b;

        g[a].insert(b);
        g[b].insert(a);

        degree[a] += 1;
        degree[b] += 1;

        if (!vis[a]) {
```

```
            vis[a] = 1;
            participantNodes += 1;
        }
        if (!vis[b]) {
            vis[b] = 1;
            participantNodes += 1;
        }
    }

    if ( !checkPossiblePath(n, participantNodes) ) {
        cout << "IMPOSSIBLE" << endl;
        return 0;
    }

    for(auto elem : hierholzer(1, n)) cout << elem << ' ';
    cout << endl;
}
```

Minimum Spanning Tree

MST minimizes the maximum edge of a tree (considering all possible trees).

Kruskal's Algorithm

Sort all edges in crescent order by weight, include all edges which joins two disconnected trees. In case of tie, choose whichever. Don't include edges that will join a already connected part of the tree.

```
// use DSU struct
struct DSU{};

set<tlll> edges;

int32_t main(){ sws;
    ll n, m; cin >> n >> m;
    DSU dsu(n+1);
    for(ll i=0; i<m; i++) {
        ll u, v, w; cin >> u >> v >> w;
        edges.insert({w, u, v});
    }
    ll minCost = 0;
    for(auto [w, u, v] : edges) {
        if (dsu.find(u) != dsu.find(v)) {
            dsu.join(u, v);
            minCost += w;
        }
    }
    cout << minCost << endl;
}
``



## Strongly Connected Components

### Kosaraju

```

```
Used for **Finding Strongly Connected Somponents** (SCCs) in a *directed graph*
(digraph).

**Complexity** O(1) -> O(V + E), linear on number of edges and vertices.

**Remember** to also construct the inverse graph (*gi*).

```cpp
vector<vll> g(MAX, vll());
vector<vll> gi(MAX, vll()); // inverted edges
bool vis[MAX]; // visited vertice?
ll component[MAX]; // connected component of each vertice
stack<ll> pilha; // for inverting order of transversal

void dfs(ll u) {
    vis[u] = 1;
    for(auto v : g[u]) if (!vis[v]) dfs(v);
    pilha.push(u);
}

void dfs2(ll u, ll c) {
    vis[u] = 1; component[u] = c;
    for(auto v : gi[u]) if (!vis[v]) dfs2(v, c);
}

// 1 - idx
void kosaraju(ll n){
    memset(vis, 0, sizeof(vis));
    for(ll i=1; i<=n; i++) if (!vis[i]) dfs(i);

    memset(vis, 0, sizeof(vis));
    memset(component, 0, sizeof(component));

    while(!pilha.empty()) {
        ll u = pilha.top(); pilha.pop();
        if (!vis[u]) dfs2(u, u);
    }
}
```

Can be extended to generate a Condensation Graph

AKA: condensate/convert all SCC's into single vertices and create a new graph

```
vector<vll> gc(MAX, vll()); // Condensation Graph

void condensate(ll n){
    for(ll u=1; u<=n; u++)
        for(auto v : g[u]) if (component[v] != component[u])
            gc[ component[u] ].pb( component[v] );
}
```

Single-Source Shortest Paths (SSSP)

Bellman-Ford for shortest paths
```

Supports Negative edges!

Solves: Finds all shortest paths from a initial node *x* to every other node

Complexity: O(n * m) = O(vertices * edges) -> O(n^2) "quadratic"

Conjecture: After **at most** *n-1 (Vertices-1) iterations*, all shortest paths will be found.

```
#define tlll tuple<ll, ll, ll>
vector<tlll> edges(MAX, tlll() );
vll d(MAX, INF);

void BellmanFord(ll x, ll n) {
    d[x] = 0;
    for(ll i=0; i<n-1; i++) { // n-1 iterations will suffice
        for(auto [u, v, w] : edges) if (d[u] + w < d[v]) {
              d[v] = d[u] + w;
        }
    }
}
```

Variation of Bellman-Ford to find a negative cycle

Iterate *n* (number of Vertices) times and if in the last iteration a distance if reduced, it means that there is a negative cycle. Save this last node, whose distance was reduced, and, which a parent array, reconstruct the negative cycle.

```
#define tlll tuple<ll, ll, ll>
vector<tlll> edges;
vll d(5050, INF);
vll p(5050, -1);

// modification of bellman-ford algorithm to detect negative cycle
void BellmanFord_Cycle(ll start, ll n){ // O (Vertices * Edges)
    d[start] = 0;
    ll x = -1; // possible node inside a negative cycle

    for(ll i=0; i<n; i++) { // n-iterations to find a cycle in the last iteration
        x = -1; // default value
        for(auto [u, v, w] : edges) if (d[u] + w < d[v]) {
            d[v] = d[u] + w;
            p[v] = u;
            x = v;
        }
    }

    if (x != -1) { // Negative cycle found
        for(ll i=0; i<n; i++) x = p[x]; // set x to a node, contained in a cycle in p[]

        vll cycle = {x};
        for(ll tmp = p[x]; tmp != x; tmp = p[tmp]) cycle.pb(tmp);
        cycle.pb(x);
        reverse(cycle.begin(), cycle.end());

        //output
```

```cpp
            for(auto elem : cycle) cout << elem << ' ';
            cout << endl;
            return;
        }
        // No Negative cycles
        return;
    }
```

Dijkstra

Only Works for Non-Negative Weighted Graph

Complexity: O((V+E)log(V)) -> O(n log n)

```cpp
priority_queue<pll, vpll, greater<pll>> pq;
vector<vpll> g(MAX, vpll());
vll d(MAX, INF);

void dijkstra(ll start){
    pq.push({0, start});
    d[start] = 0;

    while( !pq.empty() ){
        auto [p1, u] = pq.top(); pq.pop();
        if (p1 > d[u]) continue;
        for(auto [v, p2] : g[u]){
            if (d[u] + p2 < d[v]){
                d[v] = d[u] + p2;
                pq.push({d[v], v});
            }
        }
    }
}
```

OBS Dijkstra can be modified for the opposite operation: *longest paths*.

Modified Dijkstra for K-Shortest Paths

```cpp
priority_queue<pll, vpll, greater<pll>> pq;
vector<vpll> g(MAX, vpll());
vll cnt(MAX, 0);

// modified Dijkstra for K-Shortest Paths (not necessarily the same distance)
vll dijkstraKSP(ll start, ll end, ll k){ // O(K * M) = O(K * Edges)

    vll ans;
    pq.push({0, start});

    while( cnt[end] < k ){
        auto [dis, u] = pq.top(); pq.pop();

        if (cnt[u] == k) continue;
        cnt[u] += 1;
```

```
        if (u == end) { // found a shortest path
            ans.pb(dis); // adding the distance of this path
        }

        for(auto [v, w] : g[u]){
            pq.push({dis+w, v});
        }
    }

    return ans; // not ordered!
}
```

Extended Dijkstra

Besides the **Shortest Path Distance**,

Also *Computes*:

- **how many shortest paths**;
- **what is the minimum number of edges transversed in any shortest path**;
- **what is the maximum number of edges transversed in any shortest path**;

https://cses.fi/problemset/task/1202

```
priority_queue<pll, vector<pll>, greater<pll>> pq;
vector<vpll> g(MAX, vpll());
vll d(MAX, LLINF);
vll ways(MAX, 0);
vll mn(MAX, LLINF);
vll mx(MAX, -LLINF);

void dijkstra(ll start){
    pq.push({0, start});

    ways[start] = 1;
    d[start] = 0, mn[start] = 0, mx[start] = 0;

    while( !pq.empty() ){
        auto [p1, u] = pq.top(); pq.pop();

        if (p1 > d[u]) continue;

        for(auto [v, p2] : g[u]){
            // reset info, shorter path found, previous ones are discarted
            if (d[u] + p2 < d[v]){

                ways[v] = ways[u];
                mn[v] = mn[u]+1;
                mx[v] = mx[u]+1;
                d[v] = d[u] + p2;

                pq.push({d[v], v});

            }
```

```
            // same distance, another path, update info
            else if (d[u] + p2 == d[v]) {
                ways[v] = (ways[v] + ways[u]) % MOD;
                mn[v] = min(mn[v], mn[u]+1);
                mx[v] = max(mx[v], mx[u]+1);
            }
        }
    }
}
```

All-Pairs Shortest Paths (APSP)

Floyd Warshall

Complexity: O(V^3) *Suports negative edges*

```
// N < sqr3(1e8) = 460
ll N = 200;

// d[u][v] = INF (no edge)
vector<vll> d(N+1, vll(N+1, INF));

void floyd_warshall() { // O(N^3)
    for(ll i=1; i<=N; i++) d[i][i] = 0;

    for(ll aux=1; aux<=N; aux++)
        for(ll u=1; u<=N; u++)
            for(ll v=1; v<=N; v++)
                if (d[u][aux] < INF and d[v][aux] < INF)
                    d[u][v] = min(d[u][v], d[u][aux] + d[v][aux]);
}
```

Euler Tour Technique (ETT)

AKA: Preorder time , DFS time.

Flattening a tree into an array to easily query and update subtrees. This is achieved by doing a *Pre Order Tree Transversal:* (childs -> node), a simple *dfs* marking *entry times* and *leaving times*.

Creates an array that can have some properties, like all child vetices are ordered after their respective roots.

```
vector<vector<int>> g(MAX, vector<int>());
int timer = 1; // to make a 1-indexed array
int st[MAX]; // L index
int en[MAX]; // R index

void dfs_time(int u, int p) {
    st[u] = timer++;
    for (int v : g[u]) if (v != p) {
        dfs_time(v, u);
    }
    en[u] = timer-1;
}
```

Problems

https://cses.fi/problemset/task/1138 -> change value of node and calculate sum of the path to root of a tree

Cycles

Find a Cycle

vis[] array stores the current state of a node: **-1** -> not visited **0** -> explored, not ended (still need to end edge transversals) **1** -> visited, totally explored (no more edges to transverse)

p[] array stores the descedent of each node, to reconstruct cycle components

```
vector<vll> g(MAX, vll());
vll vis(MAX, -1);
vll p(MAX, -1);
ll cycle_end, cycle_start;

bool dfs(ll u) {
    vis[u] = 0;
    for(auto v : g[u]) if (vis[v] != 1) {
        if (vis[v] == 0){
            cycle_end = u;
            cycle_start = v;
            return 1;
        }
        p[v] = u;
        if (dfs(v)) return 1;
    }
    vis[u] = 1;
    return 0;
}

bool find_first_cycle(ll n) {
    for(ll i=1; i<=n; i++) if (vis[i] == -1) {
        if (dfs(i)){
            stack<ll> ans;
            ans.push(cycle_start);

            ll j = cycle_end;
            while(j != cycle_start){
                ans.push(j);
                j = p[j];
            }
            ans.push(cycle_start);

            cout << ans.size() << endl;
            while(!ans.empty()){
                cout << ans.top() << ' ';
                ans.pop();
            }
            cout << endl;
            return 1;
        }
```

```
    }
    return 0
}
```

2-SAT (2-satisfiability)

SAT (Boolean satisfiability problem) is NP-Complete.

2-SAT is a restriction of the SAT problem, in 2-SAT every clause has exactly two literals.

Can be solved with graphs in *O(Vertices + Edges)*.

```
// 0-idx graph !!!!
struct TwoSat {
    ll N; // needs to be the twice of the number of variables
    // node with idx 2x => variable x
    // node with idx 2x+1 => negation of variable x

    vector<vll> g, gt;
    // g = graph; gt = transposed graph (all edges are inverted)

    TwoSat(ll n) { // number of variables
        N = 2*n;
        g.assign(N, vll());
        gt.assign(N, vll());
    }

    vector<bool> used;
    vll order, comp;
    vector<bool> assignment;
    //  assignment[x] == 1 -> x was assigned
    //  assignment[x] == 0 -> !x was assigned

    // dfs1 and dfs2 are part of kosaraju algorithm
    void dfs1(ll u) {
        used[u] = true;
        for (ll v : g[u]) if (!used[v]) dfs1(v);
        order.pb(u); // topological order
    }

    void dfs2(ll u, ll timer) {
        comp[u] = timer;
        for (ll v : gt[u]) if (comp[v] == -1) dfs2(v, timer);
    }

    bool solve_2SAT() {
        order.clear();
        used.assign(N, false);
        for (ll i = 0; i < N; i++) if (!used[i]) dfs1(i);

        comp.assign(N, -1);
        for (ll i = 0, j = 0; i < N; i++) {
            ll u = order[N - i - 1]; // reverse order
            if (comp[u] == -1) dfs2(u, j++);
        }
```

```
        assignment.assign(N/2, false);
        for (ll i = 0; i < N; i += 2) {
            if (comp[i] == comp[i + 1]) return false; // x and !x contradiction
            assignment[i / 2] = comp[i] > comp[i + 1];
        }
        return true;
    }

    void add_disjunction(ll a, bool flagA, ll b, bool flagB) {
        // disjunction of (a, b) => if one of the two variables is false, then the other
one must be true
        // a and b can't be false at the same time

        // flagA and flagB represents whether a and b are negated
        // flagA == 1 => a ; flagA == 0 => !a
        // flagB == 1 => b ; flagB == 0 => !b
        a = 2*a ^ (!flagA);
        b = 2*b ^ (!flagB);
        ll neg_a = a ^ 1;
        ll neg_b = b ^ 1;
        g[neg_a].pb(b);
        g[neg_b].pb(a);
        gt[b].pb(neg_a);
        gt[a].pb(neg_b);
    }
};
```

Example of Application:

https://cses.fi/problemset/task/1684/ (Giant Pizza)

```
int32_t main(){ sws;
    ll m, n; cin >> m >> n;

    TwoSat twoSat(n);

    for(ll i=0; i<m; i++) {
        char charA, charB;
        ll a, b;
        cin >> charA >> a >> charB >> b;
        // at least one => (!a 'disjoint' !b)
        bool na = (charA == '-');
        bool nb = (charB == '-');
        twoSat.add_disjunction(a-1, na, b-1, nb);
    }

    if (!twoSat.solve_2SAT()) cout << "IMPOSSIBLE" << endl;
    else {
        for(ll i=0; i<n; i++) {
            if (twoSat.assignment[i]) cout << "+ ";
            else cout << "- ";
        }
        cout << endl;
```

```
    }
}
```

Geometry

Point Struct for 2D

ld behaviour not tested!

```
typedef ll unit;
bool eq(unit a, unit b){ return abs(a - b) <= EPS; }

struct P {
    unit x, y;
    P(unit xx=0, unit yy=0): x(xx), y(yy){}
    P operator +(const P& b) const {
        return P{x + b.x, y + b.y};
    }
    P operator -(const P& b) const {
        return P{x - b.x, y - b.y};
    }
    P operator *(unit t) const {
        return {x*t, y*t};
    }
    P operator /(unit t) const {
        return {x/t, y/t};
    }
    unit operator *(const P& b) const {
        return x*b.x + y*b.y;
    }
    unit operator ^(const P& b) const {
        return x*b.y - y*b.x;
    }
    bool operator <(const P& b) const {
        return (eq(x, b.x) ? y < b.y : x < b.x);
    }
    bool operator ==(const P& b) const {
        return eq(x, b.x) and eq(y, b.y);
    }
    unit dist(P b) {
        return ((x-b.x)*(x-b.x) + (y-b.y)*(y-b.y));
    }
    unit dot(const P& b, const P& c) const{
        return (b-*this) * (c-*this);
    }
    unit cross(const P& b, const P& c) const{
        return (b-*this) ^ (c-*this);
    }
};
```

Minimum pair distance

Using **Divide and conquer**, it's possible to split the vector of points into two parts and solve each one separately. When merging, it's sufficient to compare only the *6* closest points for each point which is inside a delimited section. This section is defined by all points between *median.x-d* and *median.x+d*. *d* is the minimum distance of the two parts.

```
// xs = points sorted by X; ys = points sorted by Y
ll solve(vector<P> xs, vector<P> ys){ // -> O(n Log2(n) )
    ll n = xs.size();

    // Base case, brute force
    if(n <= 3){
        ll d = xs[0].dist(xs[1]);
        for(ll i=0; i<n; i++)
            for(ll j=i+1; j<n; j++)
                d = min(d, xs[i].dist(xs[j]));
        return d;
    }

    // Divide
    ll mid = n/2;
    P median = xs[mid];
    vector<P> xsl(xs.begin(), xs.begin() + mid);
    vector<P> xsr(xs.begin() + mid, xs.end());

    vector<P> ysl, ysr;
    for(auto p : ys){
        if(p.x <= median.x)
            ysl.push_back(p);
        else
            ysr.push_back(p);
    }

    ll dl = solve(xsl, ysl);
    ll dr = solve(xsr, ysr);

    // Merge !!!
    ll d = min(dl, dr);

    vector<P> possible;
    for(auto p : ys){
        if(median.x-d < p.x and p.x < median.x+d)
            possible.push_back(p);
    }

    ll m = possible.size();
    for(ll i=0; i<m; i++){
        for(ll j=1; (j<=6 and j+i<m); j++){
            d = min(d, possible[i].dist(possible[i+j]));
        }
    }

    return d;
}

bool cmp_by_Y(P a, P b) {
    return (eq(a.y, b.y) ? a.x < b.x : a.y < b.y);
}
```

Basics

Por definição, o produto escalar define o cosseno entre dois vetores:

cos(a, b) = (a * b) / (||a|| * ||b||)

a * b = cos(a, b) * (||a|| * ||b||)

O sinal do produto vetorial de A com B indica a relação espacial entre os vetores A e B.

cross(a, b) > 0 -> ***B*** está a esquerda de ***A***.

cross(a, b) = 0 -> ***B*** é colinear ao ***A***.

cross(a, b) > 0 -> ***B*** está a direita de ***A***.

A magnitude do produto vetorial de A com B é a área do paralelogramo formado por A e B. Logo, a metade é a área do triângulo formado por A e B.

Área de qualquer polígono, convexo ou não.

Definindo um vértice como 0, e enumerando os demais de [1 a N), calcula-se a área do polígono como o somatório da metade de todos os produtos vetorias entre o 0 e os demais.

```
For i in [1, N) :
    Area += v0 ^ vi
Area = abs(Area)
```

Lembre-se de pegar o módulo da área para ignorar o sentido escolhido.

Lattice Points

Boundary points: Use gcd(abs(a.x, b.x), abs(a.y, b.y)) for each pair of adjacent points.

Interior points: Use Pick's Theorem:

```
Area_of_polygon = interior_points + boundary_points/2 - 1
```

Andrew's monotone chain convex hull algorithm

Complexity: O(n * log (n))

```
vector<P> convex_hull(vector<P>& v){
    vector<P> hull;
    sort(v.begin(), v.end()); // sort by x, then by y
    for(ll rep=0; rep<2; rep++){ // top part, then, bottom part
        ll old_size = hull.size();
        for(P next : v){
            while(hull.size() - old_size >= 2){
                P prev = hull.end()[-2]; // hull[size - 2]
```

```
                P mid = hull.end()[-1]; // hull[size - 1]
                if(prev.cross(mid, next) <= 0) // 0 collinear, <0 mid on the left of next
                    break;
                hull.pop_back(); // stack behaviour
            }
            hull.push_back(next);
        }
        hull.pop_back();
        reverse(v.begin(), v.end());
    }
    return hull;
}
```

Flow

Fluxo

```
const ll N = 505; // number of nodes, including sink and source

struct Dinic {  // O( Vertices^2 * Edges)
    struct Edge {
        ll from, to, flow, cap;
    };
    vector<Edge> edges;

    vector<ll> g[N];
    ll ne = 0, lvl[N], vis[N], pass;
    ll qu[N], px[N], qt;

    ll run(ll s, ll sink, ll minE) {
        if (s == sink) return minE;
        ll ans = 0;
        for(; px[s] < (int)g[s].size(); px[s]++){
            ll e = g[s][ px[s] ];
            auto &v = edges[e], &rev = edges[e^1];
            if( lvl[v.to] != lvl[s]+1 || v.flow >= v.cap) continue;
            ll tmp = run(v.to, sink, min(minE, v.cap - v.flow));
            v.flow += tmp, rev.flow -= tmp;
            ans += tmp, minE -= tmp;
            if (minE == 0) break;
        }
        return ans;
    }

    bool bfs(ll source, ll sink) {
        qt = 0;
        qu[qt++] = source;
        lvl[source] = 1;
        vis[source] = ++pass;
        for(ll i=0; i<qt; i++) {
            ll u = qu[i];
            px[u] = 0;
            if (u == sink) return 1;
```

```
            for(auto& ed :g[u]) {
                auto v = edges[ed];
                if (v.flow >= v.cap || vis[v.to] == pass) continue;
                vis[v.to] = pass;
                lvl[v.to] = lvl[u]+1;
                qu[qt++] = v.to;
            }
        }
        return false;
    }

    ll flow(ll source, ll sink) { // max_flow
        reset_flow();
        ll ans = 0;
        while(bfs(source, sink))
            ans += run(source, sink, LLINF);
        return ans;
    }

    void addEdge(ll u, ll v, ll c, ll rc = 0) { // c = capacity, rc = retro-capacity;
        Edge e = {u, v, 0, c};
        edges.pb(e);
        g[u].pb(ne++);
        e = {v, u, 0, rc};
        edges.pb(e);
        g[v].pb(ne++);
    }

    void reset_flow() {
        for (ll i=0; i<ne; i++) edges[i].flow = 0;
        memset(lvl, 0, sizeof(lvl));
        memset(vis, 0, sizeof(vis));
        memset(qu, 0, sizeof(qu));
        memset(px, 0, sizeof(px));
        qt = 0; pass = 0;
    }

    vector<pll> cut() { // OBS: cut set cost is equal to max flow (number of edges)
        // the cut set is the set of edges that, if removed, will disrupt flow and make
it 0.
        vector<pll> cuts;
        for (auto [from, to, flow, cap]: edges)
            if (flow == cap and vis[from] == pass and vis[to] < pass and cap > 0)
                cuts.pb({from, to});
        return cuts;
    }
};
```

How to use?

Set an unique id for all nodes

Remember to include the sink vertex and the source vertex. Usually *n+1* and *n+2*, *n* = max number of normal vertices

use **dinic.addEdge** to add edges -> (from, to, normal way capacity, retro-capacity) use **dinic.flow(source_id, sink_id)** to receive maximum flow from source to sink through the network

Minimum Cut

Another problem solved by network flow is the **minimum cut**.

Let’s define an **s-t cut C** = *(S-component, T-component)* as a partition of *V* ∈ *G* such that source *s* ∈ *S-component* and sink *t* ∈ *T-component*. Let’s also define a cut-set of C to be the set {(u, v) ∈ E | u ∈ S-component, v ∈ T-component} such that if all edges in the cut-set of C are removed, the Max Flow from s to t is 0 (i.e., s and t are disconnected). The cost of an s-t cut C is defined by the sum of the capacities of the edges in the cut-set of C.

The by-product of computing Max Flow is Min Cut! After Max Flow algorithm stops, we run graph traversal (DFS/BFS) from source s again. All reachable vertices from source s using positive weighted edges in the residual graph belong to the S-component. All other unreachable vertices belong to the T-component. All edges connecting the S-component to the T-component belong to the cut-set of C. The Min Cut value is equal to the Max Flow value mf. This is the minimum over all possible s-t cuts values.

Example:

https://cses.fi/problemset/task/1695/

```
int32_t main(){ sws;
    ll n, m; cin >> n >> m;
    Dinic dinic;
    for(ll i=0; i<m; i++) {
        ll u, v; cin >> u >> v;
        dinic.addEdge(u, v, 1, 1);
    }
    dinic.flow(1, n);
    vpll ans = dinic.cut();
    cout << ans.size() << endl;
    for(auto [u, v] : ans) cout << u << ' ' << v << endl;
}
```

Matching

A perfect matching includes all vertices from the bipartite graph L and R.

A maximum matching has the maximum cadinality. A perfect matching is a maximum matching. But the opposite is not necessarity true.

It's possible to access *dinic.edges*, which is a vector that contains all edges and also its respective attributes, like the .flow passing through each edge. Remember to consider that negative flow exist for reverse edges.

This can be used to **matching problems** with a bipartite graph and *1 capacity* for example.

Searching-Sorting

Policy Based Data Structures (PBDS)

Ordered Set

```
// * Ordered Set and Map
// find_by_order(i) -> iterator to elem with index i; O(log(N))
// order_of_key(i) -> index of key; O(log(N))

#include <bits/extc++.h>
using namespace __gnu_pbds;
template <class T> using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
tree_order_statistics_node_update>;
```

Ordered Map

```
// * Ordered Set and Map
// find_by_order(i) -> O(log(N))
// order_of_key(i) -> O(log(N))

#include <bits/extc++.h>
using namespace __gnu_pbds;
template <class K, class V> using ordered_map = tree<K, V, less<K>, rb_tree_tag,
tree_order_statistics_node_update>;
```

Ordered Multiset

Ordered Set pode ser tornar um multiset se utilizar um pair do valor com um index distinto. pll{val, t}, 1 <= t <= n

Observação:

O set não precisa conter a chave sendo buscada pelo *order_of_key()*.

order_of_key() returns index starting from 0; [0, n)

Problemas

Consegue computar em O(log(N)), quantos elementos são menores que K, utilizando o index.

Merge sort

Merge Sort with number of inversions counter.

Directly updates the *v* vector. Also return the number of swaps (inversions).

O(N log(N))

```
// O(N)
ll merge(vll &v, ll l, ll r) {
    ll i = l, mid = (l+r)/2, j = mid+1, swaps = 0;
    vll ans;

    while(i <= mid or j <= r) {

        if(j > r or (v[i] <= v[j] and i <= mid)) {
            ans.pb(v[i]);
```

```
            i += 1;
        }

        else if(i > mid or (v[j] < v[i] and j <= r)){
            ans.pb(v[j]);
            j += 1;
            swaps += (mid-1)+1; // mid-i+1 = elements remaining in the left subarray
(same number of elements that will be swaped to the right)
        }
    }

    for(ll k=l; k<=r; k++) v[k] = ans[k-l];

    return swaps;
}

// O(N log2(N))
ll merge_sort(vll &v, ll l, ll r){
    if(l == r) return 0;

    ll mid = (l+r)/2, swaps = 0;
    swaps += merge_sort(v, l, mid);
    swaps += merge_sort(v, mid+1, r);
    swaps += merge(v, l, r);

    return swaps;
}
```

Ternary Search

Complexity: O(log(n))

Float and Min Version: *Requires EPS!*, precision usually defined in the question text

```
ld f(ld d){
    // function here
}

// for min value
ld ternary_search(ld l, ld r){
    while(r - l > EPS){
        // divide into 3 equal parts and eliminate one side
        ld m1 = l + (r - l) / 3;
        ld m2 = r - (r - l) / 3;
        if (f(m1) < f(m2)){
            r = m2;
        }
        else {
            l = m1;
        }
    }
    return f(l);
}
```

Integer and Max Version: (probably working xD)

```
ll f(ll idx) {
    // function here
}

// for max value, using integer idx
ll ternary_search(ll l, ll r) {
    while(l <= r) {
        // divide into 3 equal parts and eliminate one side
        ll m1 = l + (r-l)/3;
        ll m2 = r - (r-l)/3;
        if(f(m1) < f(m2)) {
            l = m1+1;
        }
        else {
            r = m2-1;
        }
    }
    return f(l);
}
```

Binary search

Finds the first element that changes value in any monotonic function

Maximum

Monotonically Decreasing [1, 1, 1, 1, 0, 0, 0, 0]

```
bool f(ll a){
    // Add desired function here
    return true;
}

ll search(ll l=0, ll r=1e9, ll ans=0){
    while(l <= r) { // [l, r]
        ll mid = (l+r)/2;
        if(f(mid)) { // (mid, r]
            ans = mid;
            l = mid+1;
        }
        else { // [l; mid)
            r = mid-1;
        }
    }
    return ans;
}
```

Minimum

Monotonically Increasing [0, 0, 0, 0, 1, 1, 1, 1]

```cpp
bool f(ll mid){
    // Add desired function here
    return true;
}

ll bSearch(ll l=0, ll r=1e9, ll ans=0){
    while(l <= r) { // [l, r]
        ll mid = (l+r)/2;
        if (f(mid) ) { // [l, mid)
            ans = mid;
            r = mid-1;
        }
        else { // (mid, r]
            l = mid+1;
        }
    }
    return ans;
}
```

Strings

Z-function

Suppose we are given a string s of length n. The Z-function for this string is an array of length n where the i-th element is equal to the greatest number of characters starting from the position i that coincide with the first characters of s.

The first element of the Z-function, *z[0]*, is generally not well defined. This implementation assumes it as *z[0] = 0*. But it can also be interpreted as *z[0] = n* (all characters coincide).

```cpp
vector<ll> z_function(string s) { // O(n)
    ll n = (ll) s.length();
    vll z(n);
    for (ll i=1, l=0, r=0; i<n; i++) {
        if (i <= r) z[i] = min(r - i + 1, z[i - l]);

        while (i + z[i] < n and s[z[i]] == s[i + z[i]]) z[i]++;

        if (r < i + z[i] - 1) l = i, r = i + z[i] - 1;
    }
    return z;
}
```

Solves

- Find occurrences of pattern string (*pattern*) in the main string (*str*):

```cpp
int32_t main() { sws;
    string str, pattern; cin >> str >> pattern;
```

```
    string s = pattern + '$' + str;
    vll z = z_function(s);
    ll ans = 0;
    ll n = pattern.size();
    for(ll i=0; i< (ll) str.size(); i++){
        if( z[i + n + 1] == n)
            ans += 1;
    }
    cout << ans << endl;
}
```

- Find all border lengths of a given string.

OBS: A border of a string is a prefix that is also a suffix of the string but not the whole string. For example, the borders of *abcababcab* are *ab* and *abcab*.

Works because *z[i] == j* is the condition when the common characters of z[i] reaches the end of the string. For example:

abcababcab

z[8] = 2

ab is the border;

```
int32_t main(){ sws;
    string s; cin >> s;
    vll z = z_function(s);
    ll n = s.length();
    for(ll i=n-1, j=1; i>=0; i--) {
        if (z[i] == j) cout << j << ' ';
        j += 1;
    }
    cout << endl;
}
```

- Find all period lengths of a string.

OBS: A period of a string is a prefix that can be used to generate the whole string by repeating the prefix. The last repetition may be partial. For example, the periods of *abcabca* are *abc*, *abcabc* and *abcabca*.

Works because *z[i] + i >= n* is the condition when the common characters of z[i] in addition to the elements already passed, exceeds or is equal to the end of the string. For example:

abaababaab

z[8] = 2

abaababa is the period; the remaining (z[i] characters) are a prefix of the period; and when all these characters are combined, it can form the string (which has *n* characters).

```
int32_t main(){ sws;
    string s; cin >> s;
    vll z = z_function(s);
    ll n = s.length();
```

```
    for(ll i=1; i<n; i++) {
        if (z[i] + i >= n) {
            cout << i << ' ';
        }
    }
    cout << n << endl;
}
```

SUFFIX ARRAY

Complexity: O(n * log (n))

Returns: An array with size *n*, whose values are the indexes from the longest substring (0) to the smallest substring (n) after ordering it lexicographically. Example:

```
Let the given string be "banana".

0 banana                          5 a
1 anana     Sort the Suffixes     3 ana
2 nana      ---------------->     1 anana
3 ana        alphabetically       0 banana
4 na                              4 na
5 a                               2 nana

So the suffix array for "banana" is {5, 3, 1, 0, 4, 2}
```

Solves: Finding the number of all distint substrings of a string. Done by adding all sizes of the substrings (size[i] = *total_size* - sa[i]) and subtracting all lcp's.

```
vector<int> suffix_array(string s) {
    s += "$";
    int n = s.size(), N = max(n, 260);
    vector<int> sa(n), ra(n);
    for (int i = 0; i < n; i++) sa[i] = i, ra[i] = s[i];

    for (int k = 0; k < n; k ? k *= 2 : k++) {
        vector<int> nsa(sa), nra(n), cnt(N);

        for (int i = 0; i < n; i++) nsa[i] = (nsa[i]-k+n)%n, cnt[ra[i]]++;
        for (int i = 1; i < N; i++) cnt[i] += cnt[i-1];
        for (int i = n-1; i+1; i--) sa[--cnt[ra[nsa[i]]]] = nsa[i];

        for (int i = 1, r = 0; i < n; i++) nra[sa[i]] = r += ra[sa[i]] !=
            ra[sa[i-1]] or ra[(sa[i]+k)%n] != ra[(sa[i-1]+k)%n];
        ra = nra;
        if (ra[sa[n-1]] == n-1) break;
    }
    return vector<int>(sa.begin()+1, sa.end());
}
```

KASAI's ALGORITHM FOR LCP (longest common prefix)

Complexity: O(log (n))

Returns: An array of size *n* (like the suffix array), whose values indicates the lenght of the longest common prefix beetwen *sa[i]* and *sa[i+1]*

```
vector<int> kasai(string s, vector<int> sa) {
    int n = s.size(), k = 0;
    vector<int> ra(n), lcp(n);
    for (int i = 0; i < n; i++) ra[sa[i]] = i;

    for (int i = 0; i < n; i++, k -= !!k) {
        if (ra[i] == n-1) { k = 0; continue; }
        int j = sa[ra[i]+1];
        while (i+k < n and j+k < n and s[i+k] == s[j+k]) k++;
        lcp[ra[i]] = k;
    }
    return lcp;
}
```

Hashing

May have complications with time limit, should avoid hashing

```
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());

// hash(s) -> s[0]*P^n + s[1]*P^(n-1) + ... + s[n-1]*P + s[n]
struct Hashing {
    vector<ll> mods = { // multiple mods to surpass colision
        1000000009,1000000021,1000000033,
        1000000087,1000000093,1000000097,
        1000000103,1000000123,1000000181,
        1000000207,1000000223,1000000241,
        1000000271,1000000289,1000000297
    };

    ll num; // number of mods used (3 is a good number)
    ll P = 131; // more than ascii
    vector<unordered_set<ll>> cnt;
    vector<vector<ll>> h, p;

    Hashing(string &s, ll n = 3) : num(n) { // O(len(s) * num)
        shuffle(mods.begin(), mods.end(), rng);
        ll len = (ll) s.size();
        cnt.assign(n, unordered_set<ll>());
        h.assign(n, vector<ll>(len));
        p.assign(n, vector<ll>(len));

        for(ll i=0; i<num; i++) {
            p[i][0] = 1, h[i][0] = s[0];
            for(ll j=1; j<len; j++) {
                p[i][j] = (p[i][j-1] * P) % mods[i];
                h[i][j] = (h[i][j-1] * P + s[j]) % mods[i];
            }
        }
```

```
    }

    // hash of s[l...r]
    vector<ll> get(ll l, ll r) {  // O(num)
        vector<ll> vec(num);
        for(ll i=0; i<num; i++) {
            vec[i] = h[i][r];
            if (l > 0)
                vec[i] = vec[i] - (h[i][l-1] * p[i][r-l+1]) % mods[i];
            if (vec[i] < 0)
                vec[i] += mods[i];
        }
        return vec;
    }

    // hash of t
    vector<ll> get(string &t) { // O(len(t) * num)
        ll len = (ll) t.size();
        vector<ll> vec(num);
        for(ll i=0; i<num; i++) {
            ll pow = 1, hash = t[0];
            for(ll j=1; j<len; j++) {
                pow = (pow * P) % mods[i];
                hash = (hash * P  + t[j]) % mods[i];
            }
            vec[i] = hash;
        }
        return vec;
    }

    void add(vector<ll> &vec) { // O(num)
        for(ll i=0; i<num; i++)
            cnt[i].insert(vec[i]);
    }

    bool check(vector<ll> &vec) { // O(num)
        for(ll i=0; i<num; i++)
            if (!cnt[i].count(vec[i])) return 0;
        return 1;
    }
};
```

Booth's Algorithm

An efficient algorithm which uses a modified version of KMP to compute the **least amount of rotation needed** to reach the **lexicographically minimal string rotation**.

A *rotation* of a string can be generated by moving characters one after another from beginning to end. For example, the rotations of *acab* are *acab*, *caba*, *abac*, and *baca*.

```
// Booth Algorithm
ll least_rotation(string s) { // O(n)
    ll n = s.length();
    vll f(2*n, -1);
    ll k = 0;
```

```
        for(ll j=1; j<2*n; j++) {
            ll i = f[j-k-1];
            while(i != -1 and s[j % n] != s[(k+i+1) % n] ) {
                if (s[j % n] < s[(k+i+1) % n])
                    k = j - i - 1;
                i = f[i];
            }
            if (i == -1 and s[j % n] != s[(k+i+1) % n] ) {
                if (s[j % n] < s[(k+i+1) % n])
                    k = j;
                f[j - k] = -1;
            }
            else
                f[j - k] = i + 1;
        }
        return k;
    }

    int32_t main(){ sws;
        string s; cin >> s;
        ll n = s.length();
        ll ans_idx = least_rotation(s);
        string tmp = s + s;
        cout << tmp.substr(ans_idx, n) << endl;
    }
```

Aho Cosarick

find first occurences of match for each pattern

```
namespace aho {
    map<char, int> to[MAX];
    int link[MAX], idx = 0, term[MAX], exit[MAX], sobe[MAX];
    vector<int> word[MAX];
    bool vis[MAX]; // avoids recalculation
    int match[MAX];
    // idx+1 of the last char of the pattern match
    // idx = 0 -> no match

    void add(string& s, int id) {
        int at = 0;
        for (char c : s) {
            if (to[at].count(c)) at = to[at][c];
            else at = to[at][c] = ++idx;
        }
        term[at]++, sobe[at]++;
        word[at].pb(id);
    }

    void build() {
        queue<int> q;
        q.push(0);
        link[0] = exit[0] = -1;
        while (q.size()) {
            int i = q.front(); q.pop();
```

```cpp
            for (auto [c, j] : to[i]) {
                int l = link[i];
                while (l != -1 and !to[l].count(c)) l = link[l];
                link[j] = l == -1 ? 0 : to[l][c];
                exit[j] = term[link[j]] ? link[j] : exit[link[j]];
                if (exit[j] != -1) sobe[j] += sobe[exit[j]];
                q.push(j);
            }
        }
    }

    ll query(string& s) { // returns number of matches
        memset(vis, 0, sizeof(vis));
        int at = 0, ans = 0;
        int n = (int) s.size();
        for (int i=0; i<n; i++){
            char c = s[i];
            while (at != -1 and !to[at].count(c)) at = link[at];
            at = at == -1 ? 0 : to[at][c];

            int tmp = at;
            while ((tmp > 0 and !vis[tmp])) {
                vis[tmp] = 1;

                for(auto id : word[tmp])
                    match[id] = i+1;

                tmp = exit[tmp];
            }

            ans += sobe[at];
        }
        return ans;
    }
}

void solve() {
    string s; cin >> s;
    int n; cin >> n;
    vector<string> p(n);
    for(int i=0; i<n; i++) {
        cin >> p[i];
        aho::add(p[i], i);
    }
    aho::build();
    aho::query(s);
    for(int i=0; i<n; i++) {
        int pos = aho::match[i];
        if (pos == 0)
            cout << -1 << endl;
        else
            cout << pos - p[i].size() + 1 << endl;
    }
}
```

Count how many matches per pattern

```
namespace aho {
    map<char, int> to[MAX];
    int link[MAX], idx = 0, term[MAX], exit[MAX], psum[MAX];
    vector<int> word[MAX];
    int lazy[MAX], match[MAX];

    void add(string& s, int id) {
        int at = 0;
        for (char c : s) {
            if (to[at].count(c)) at = to[at][c];
            else at = to[at][c] = ++idx;
        }
        term[at]++, psum[at]++;
        word[at].pb(id);
    }

    void build() {
        queue<int> q;
        q.push(0);
        link[0] = exit[0] = -1;
        while (q.size()) {
            int u = q.front(); q.pop();
            for (auto [c, v] : to[u]) {
                int l = link[u];
                while (l != -1 and !to[l].count(c)) l = link[l];
                link[v] = l == -1 ? 0 : to[l][c];
                exit[v] = term[link[v]] ? link[v] : exit[link[v]];
                if (exit[v] != -1) psum[v] += psum[exit[v]];
                q.push(v);
            }
        }
    }

    int query(string& s) {
        memset(lazy, 0, sizeof(lazy));
        memset(match, 0, sizeof(match));

        int at = 0, ans = 0;
        int n = (int) s.size();
        for (int i=0; i<n; i++){
            char c = s[i];
            while (at != -1 and !to[at].count(c)) at = link[at];
            at = at == -1 ? 0 : to[at][c];

            if (term[at]) lazy[at] += 1;
            else if (exit[at] != -1) lazy[exit[at]] += 1;

            ans += psum[at];
        }

        queue<int> q; q.push(0);
        stack<int> st;
        while(!q.empty()) {
            int u = q.front(); q.pop();
            st.push(u);
            for(auto [c, v] : to[u])
```

```
                q.push(v);
        }

        while(!st.empty()) {
            int i = st.top(); st.pop();
            if (lazy[i]) {
                for(auto id : word[i]) {
                    match[id] += lazy[i];
                }
                if (exit[i] != -1) lazy[exit[i]] += lazy[i];
            }
        }

        return ans;
    }
}

void solve() {
    string s; cin >> s;
    int n; cin >> n;
    vector<string> p(n);
    for(int i=0; i<n; i++) {
        cin >> p[i];
        aho::add(p[i], i);
    }
    aho::build();
    aho::query(s);
    for(int i=0; i<n; i++) {
        cout << aho::match[i] << endl;
    }
}
```

Tree

Binary lifting

Solves: LCA, O(log) travelling in a tree

OBS: log2(1e5) ~= 17; log2(1e9) ~= 30 ; log2(1e18) ~= 60

Use for deep trees: LLOGMAX = 62;

```
const ll LOGMAX = 32;

vector<vll> g(MAX, vll());
ll depth[MAX]; // depth[1] = 0
ll jump[MAX][LOGMAX]; // jump[v][k] -> 2^k antecessor of v
// 1 points to 0 and 0 is the end point loop
ll N; // quantity of vertices of the tree

void binary_lifting(ll u = 1, ll p = -1){ // DFS, O(N)
    for(auto v : g[u]) if (v != p){
        depth[v] = depth[u] + 1;
```

```
        jump[v][0] = u;
        for(ll k=1; k < LOGMAX; k++)
            jump[v][k] = jump[ jump[v][k-1] ][k-1];
        binary_lifting(v, u);
    }
}

ll go(ll v, ll dist){ // O(Log(N))
    for(ll k = LOGMAX-1; k >= 0; k--)
        if (dist & (1 << k))
            v = jump[v][k];
    return v;
}

ll lca(ll a, ll b){  // O(Log(N))
    if (depth[a] < depth[b]) swap(a, b);

    a = go(a, depth[a] - depth[b]);
    if (a == b) return a;

    for(ll k = LOGMAX-1; k >= 0; k--){
        if (jump[a][k] != jump[b][k]){
            a = jump[a][k];
            b = jump[b][k];
        }
    }
    return jump[a][0];
}


int32_t main(){ sws;
    ll n; cin >> n;
    N = n;

    binary_lifting();
}
```

Find the Diameter

From any node X find a node A which is the farthest away from X. Then, from node A, find a node B which is the farthest away from A.

Path from (A - B) is a diameter.

It can be proven by drawing a diameter line. If any node is further than any of the diameter extremities, then it should be switched (so the first line wasn't a diameter at all).

From any node, the fasthest node is a diameter extremity. Then from this extremity, the fasthest node is the other diameter extremity.

```
vector<vll> g(MAX, vll());

pll dfs(ll u, ll p){
    pll ans = {u, 0};
```

```cpp
    for(auto v : g[u]) if (v != p) {
        auto [node, comp] = dfs(v, u);
        if (comp+1 > ans.ss){
            ans = {node, comp+1};
        }
    }
    return ans;
}

int32_t main(){sws;
    ll n; cin >> n;
    for(ll i=1; i<n; i++){
        ll a, b; cin >> a >> b;
        g[a].pb(b);
        g[b].pb(a);
    }
    ll ans1 = dfs(1, 1).ff;
    cout << dfs(ans1 , ans1).ss << endl;
}
```

Find the lenght of the longest path from all nodes

It can be proven that to any node X, the maximum distance is either dist(X, A) or dist(X, B), which are the extremities of a diameter.

```cpp
vector<vll> g(MAX, vll());
vll distA(MAX, 0);
vll distB(MAX, 0);

pll dfs(ll u, ll p, ll op) {
    pll ans = {u, 0};
    for(auto v : g[u]) if (v != p) {
        if (op == 1) distA[v] = distA[u]+1;
        else if (op == 2) distB[v] = distB[u]+1;

        auto [node, length] = dfs(v, u, op);

        if (length + 1 > ans.ss)
            ans = {node, length+1};
    }
    return ans;
}

int32_t main() { sws;
    ll n; cin >> n;
    for(ll i=0; i<n-1; i++) {
        ll u, v; cin >> u >> v;
        g[u].pb(v);
        g[v].pb(u);
    }

    auto [nodeA, _t1] = dfs(1, -1, 0);
    auto [nodeB, _t2] = dfs(nodeA, -1, 1);
    dfs(nodeB, -1, 2);
```

```
    for(ll i=1; i<=n; i++) {
        cout << max(distA[i], distB[i]) << ' ';
    }
    cout << endl;
}
```

Heavy Light Decomposition (WIP)

Features:

- Update all nodes along the path from node *x* to node *y*.
- Find the sum, maximum, minimum (or any other operation that satisfies the associative property) along the path from node *x* to node *y*.

Each query takes *O(log(N))* time. So the total complexity should be *O(Q log(N))*

Definitions:

- A heavy child of a node is the child with the largest subtree size rooted at the child.
- A light child of a node is any child that is not a heavy child.
- A heavy edge connects a node to its heavy child.
- A light edge connects a node to any of its light children.
- A heavy path is the path formed by a collection heavy edges.
- A light path is the path formed by a collection light edges.

```
ll N, v[MAX]; // Set N = n !
vector<vll> g(MAX, vll());

ll sz[MAX], p[MAX], dep[MAX], id[MAX], tp[MAX];
ll st[1 << 19];

void update_node(ll idx, ll val) { // O(Log^2(N))
    st[idx += N] = val;
    for (idx /= 2; idx; idx /= 2)
        st[idx] = max(st[2 * idx], st[2 * idx + 1]);
}

ll query(ll lo, ll hi) {
    ll ra = 0, rb = 0;
    for (lo += N, hi += N + 1; lo < hi; lo /= 2, hi /= 2) {
        if (lo & 1)
            ra = max(ra, st[lo++]);
        if (hi & 1)
            rb = max(rb, st[--hi]);
    }
    return max(ra, rb);
}

ll dfs_sz(ll cur, ll par) {
    sz[cur] = 1;
    p[cur] = par;
    for(ll chi : g[cur]) {
        if(chi == par) continue;
        dep[chi] = dep[cur] + 1;
```

```cpp
        p[chi] = cur;
        sz[cur] += dfs_sz(chi, cur);
    }
    return sz[cur];
}

ll ct = 1; // counter
void dfs_hld(ll cur, ll par, ll top) {
    id[cur] = ct++;
    tp[cur] = top;
    update_node(id[cur], v[cur]);
    ll h_chi = -1, h_sz = -1;
    for(ll chi : g[cur]) {
        if(chi == par) continue;
        if(sz[chi] > h_sz) {
            h_sz = sz[chi];
            h_chi = chi;
        }
    }
    if(h_chi == -1) return;
    dfs_hld(h_chi, cur, top);
    for(ll chi : g[cur]) {
        if(chi == par || chi == h_chi) continue;
        dfs_hld(chi, cur, chi);
    }
}

// returns the max_value of a node in the path from X to Y
ll path(ll x, ll y){ // O(log^2(N))
    ll ret = 0;
    while(tp[x] != tp[y]){
        if(dep[tp[x]] < dep[tp[y]])swap(x,y);
        ret = max(ret, query(id[tp[x]],id[x]));
        x = p[tp[x]];
    }
    if(dep[x] > dep[y])swap(x,y);
    ret = max(ret, query(id[x],id[y]));
    return ret;
}


int32_t main(){ sws;
    ll n, q; cin >> n >> q;
    N = n;
    for(ll i=1; i<=n; i++) cin >> v[i];
    for(ll i=2; i<=n; i++) {
        ll a, b; cin >> a >> b;
        g[a].pb(b);
        g[b].pb(a);
    }
    dfs_sz(1, 1);
    dfs_hld(1, 1, 1);
    while(q--) {
        ll t; cin >> t;
        if (t == 1) {
            ll s, x; cin >> s >> x;
            v[s] = x;
            update_node(id[s], v[s]);
```

```
        }
        else {
            ll a, b; cin >> a >> b;
            cout << path(a, b) << endl;
        }
    }
}
```

Find the Centroid of a Tree

A centroid of a tree is defined as a node such that when the tree is rooted at it, no other nodes have a subtree of size greater than *N/2*.

We can find a centroid in a tree by starting at the root. Each step, loop through all of its children. If all of its children have subtree size less than or equal to *N/2*, then it is a centroid. Otherwise, move to the child with a subtree size that is more than *N/2* and repeat until you find a centroid.

```
vector<vll> g(MAX, vll());
vll subtreeSize(MAX, 1);
ll N; // <- initialize N = n !!

void getSizes(ll u = 1, ll p = -1) {
    for(auto v : g[u]) if (v != p) {
        getSizes(v, u);
        subtreeSize[u] += subtreeSize[v];
    }
}

ll centroid(ll u = 1, ll p = -1) {
    for(auto v : g[u]) if (v != p) {
        if (subtreeSize[v] * 2 > N) return centroid(v, u);
    }
    return u;
}
```

Math

Matrix

```
struct Matrix{
    vector<vll> M, IND;

    Matrix(vector<vector<int>> mat){
        M = mat;
    }

    Matrix(int row, int col, bool ind=0){
        M = vector<vector<int>>(row, vector<int>(col, 0));
        if(ind){
            vector<int> aux(row, 0);
```

```
            for(int i=0; i<row; i++){
                aux[i] = 1;
                IND.push_back(aux);
                aux[i] = 0;
            }
        }
    }

    Matrix operator +(const Matrix &B) const{ // A+B (sizeof(A) == sizeof(B))
        vector<vector<int>> ans(M.size(), vector<int>(M[0].size(), 0));
        for(int i=0; i<(int)M.size(); i++){
            for(int j=0; j<(int)M[i].size(); j++){
                ans[i][j] = M[i][j] + B.M[i][j];
            }
        }
        return ans;
    }

    Matrix operator *(const Matrix &B) const{ // A*B (A.column == B.row)
        vector<vector<int>> ans;
        for(int i=0; i<(int)M.size(); i++){
            vector<int> aux;
            for(int j=0; j<(int)M[i].size(); j++){
                int sum=0;
                for(int k=0; k<(int)B.M.size(); k++){
                    sum = sum + (M[i][k]*B.M[k][j]);
                }
                aux.push_back(sum);
            }
            ans.push_back(aux);
        }
        return ans;
    }

    Matrix operator ^(const int n) const{ // Need identity Matrix
        if (n == 0) return IND;
        if (n == 1) return (*this);
        Matrix aux = (*this) ^ (n/2);
        aux = aux * aux;
        if(n % 2 == 0)
            return aux;
        else{
            return (*this) * aux;
        }
    }
};
```

Another Version with LL and MOD

```
struct Matrix {
    vector<vector<ll>> M;

    Matrix(vector<vector<ll>> mat) {
        M = mat;
    }
```

```cpp
    // identity == 0 => Empty matrix constructor
    // identity == 1 => Generates a Identity Matrix (row == col)
    Matrix(ll row, ll col, bool identity = 0){
        M.assign(row, vector<ll>(col, 0));
        if (identity)
            for(ll i=0; i<row; i++) M[i][i] = 1;
    }

    // A+B  (sizeof(A) == sizeof(B))
    Matrix operator +(const Matrix &B) const{
        ll row = M.size(); ll col = M[0].size();
        Matrix ans(row, col);

        for(ll i=0; i<row; i++){
            for(ll j=0; j<col; j++){
                ans.M[i][j] = ( M[i][j] + B.M[i][j] ) % MOD;
            }
        }

        return ans;
    }

    // A*B  (A.column == B.row)
    Matrix operator *(const Matrix &B) const{
        ll rowA = M.size();
        ll colA; ll rowB = colA = M[0].size();

        Matrix ans(rowB, colA);

        for(ll i=0; i<rowA; i++){
            for(ll j=0; j<colA; j++){
                for(ll k=0; k<rowB; k++){
                    ans.M[i][j] += (M[i][k] * B.M[k][j]) % MOD;
                    ans.M[i][j] %= MOD;
                }
            }
        }

        return ans;
    }

    Matrix operator ^(ll n) const{ // row == col
        ll sz = M.size();

        Matrix ans(sz, sz, 1); // initialized as identity
        Matrix tmp(M);

        while(n) {
            if (n & 1) ans = (ans * tmp);
            tmp = (tmp * tmp);
            n >>= 1;
        }

        return ans;
    }
};
```

Usage

For faster linear recurrence computation with matrix exponentiation.

Base * Operator^(n) = Result[n]

Example:

Recorrence: dp[i] = dp[i-1] + dp[i-2] + dp[i-3] + dp[i-4] + dp[i-5] + dp[i-6]

Base Matrix [dp[5], dp[4], dp[3], dp[2], dp[1], dp[0]]

- Operator Matrix ^ 1 [1, 1, 0, 0, 0, 0] [1, 0, 1, 0, 0, 0] [1, 0, 0, 1, 0, 0] [1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 0, 1] [1, 0, 0, 0, 0, 0]

= Result Matrix [dp[n+5], dp[n+4], dp[n+3], dp[n+2], dp[n+1], dp[n]]

```
int32_t main(){ sws;
    ll n; cin >> n;
    Matrix op(6, 6, 1);
    op.M[0] = {1, 1, 0, 0, 0, 0};
    op.M[1] = {1, 0, 1, 0, 0, 0};
    op.M[2] = {1, 0, 0, 1, 0, 0};
    op.M[3] = {1, 0, 0, 0, 1, 0};
    op.M[4] = {1, 0, 0, 0, 0, 1};
    op.M[5] = {1, 0, 0, 0, 0, 0};

    Matrix base(vector(1, vll({16, 8, 4, 2, 1, 1})));
    if (n <= 5) cout << base.M[0][5-n] << endl;
    else {
        op = op^(n-5);
        Matrix ans = base * op;
        cout << ans.M[0][0] << endl;
    }
}
```

Factorization

Trial Division with precomputed primes

Complexity: O(sqrt(N))

Returns: a vector containing all the primes that divides *N* (There can be multiples instances of a prime and it is ordered)

```
// import this and create vector<int> prime
void sieve(ll n){}

vector<int> factorization(int n){ // O(sqrt(n))
    vector<int> factors;

    for(int p : prime){
        if (p*p > n) break;
```

```
        while(n % p == 0){
            factors.pb(p);
            n /= p;
        }
    }

    if (n > 1) factors.pb(n);

    return factors;
}
```

Using Smallest Prime technique

Requires Values less than 1e8

Complexity: O(log2(N))

```
vector<int> prime;
bool is_composite[MAX]; // can be 1e7
ll sp[MAX]; // smallest prime

void sieve (int n) { // O(n)
    memset(is_composite, 0, sizeof(is_composite));

    for (int i = 2; i <= n; i++) {
        if (!is_composite[i]) {
            prime.pb(i);
            sp[i] = i;
        }
        for (int j = 0; j < (int) prime.size () && i * prime[j] <= n; j++) {
            is_composite[i * prime[j]] = true;
            sp[i * prime[j]] = prime[j];
            if (i % prime[j] == 0) break;
        }
    }
}

vll factorization(ll a) { // Log2(a)
    vll factors;
    while(a > 1) {
        factors.pb(sp[a]);
        a /= sp[a];
    }
    return factors;
}
```

Pollard Rho

Complexity: better than O(sqrt(N)) 😄

Returns: a vector containing all the primes that divides *N* (There can be multiples instances of a prime and it is *not* ordered)

```cpp
ll mul(ll a, ll b, ll m) {
    ll ret = a*b - (ll)((ld)1/m*a*b+0.5)*m;
    return ret < 0 ? ret+m : ret;
}

ll pow(ll a, ll b, ll m) {
    ll ans = 1;
    for (; b > 0; b /= 2ll, a = mul(a, a, m)) {
        if (b % 2ll == 1)
            ans = mul(ans, a, m);
    }
    return ans;
}

bool prime(ll n) {
    if (n < 2) return 0;
    if (n <= 3) return 1;
    if (n % 2 == 0) return 0;

    ll r = __builtin_ctzll(n - 1), d = n >> r;
    for (int a : {2, 325, 9375, 28178, 450775, 9780504, 795265022}) {
        ll x = pow(a, d, n);
        if (x == 1 or x == n - 1 or a % n == 0) continue;

        for (int j = 0; j < r - 1; j++) {
            x = mul(x, x, n);
            if (x == n - 1) break;
        }
        if (x != n - 1) return 0;
    }
    return 1;
}

ll rho(ll n) {
    if (n == 1 or prime(n)) return n;
    auto f = [n](ll x) {return mul(x, x, n) + 1;};

    ll x = 0, y = 0, t = 30, prd = 2, x0 = 1, q;
    while (t % 40 != 0 or gcd(prd, n) == 1) {
        if (x==y) x = ++x0, y = f(x);
        q = mul(prd, abs(x-y), n);
        if (q != 0) prd = q;
        x = f(x), y = f(f(y)), t++;
    }
    return gcd(prd, n);
}

vector<ll> fact(ll n) {
    if (n == 1) return {};
    if (prime(n)) return {n};
    ll d = rho(n);
    vector<ll> l = fact(d), r = fact(n / d);
    l.insert(l.end(), r.begin(), r.end());
    return l;
}
```

Xor Basis

```
struct XorBasis {
    vector<ll> basis;

    ll reduce(ll vec) {
        for(auto b : basis) vec = min(vec, vec^b);
        return vec;
    }

    void add(ll vec) {
        ll val = reduce(vec);
        if (val) B.pb(val);
    }
};
```

Extended:

```
struct XorBasis {
    vector<ll> basis;
    ll mx = 0;

    ll reduce(ll vec) {
        for(auto b : basis) vec = min(vec, vec^b);
        return vec;
    }

    void add(ll vec) {
        ll val = reduce(vec);
        if (val) {
            basis.pb(val);
            mx = max(mx, mx^val);
        }
    }

    ll dim() {
        return basis.size();
    }

    void jordan() {
        sort(basis.begin(), basis.end(), greater<ll>());
        for(ll i=1; i<(ll)basis.size(); i++) {
            for(ll j=0; j<i; j++) {
                basis[j] = min(basis[j], basis[j]^basis[i]);
            }
        }
    }
};
```

Common problems:

- Find if a vector can be formed by the basis (if(reduce(val)))
- Find how many linear combinations form a vector (ans = 2^(dim(kernel)))

- Find the maximum vector that can be formed (mx = max(mx, mx^b))

Operations with Combinatorics

Also contains combinatorics operations

```
struct OpMOD{
    vll fact, ifact;

    OpMOD () {}

    // overloaded constructor that computes factorials
    OpMOD(ll n){ // from fact[0] to fact[n]; O(n)
        fact.assign(n+1 , 1);
        for(ll i=2; i<=n; i++) fact[i] = mul(fact[i-1], i);

        ifact.assign(n+1, 1);
        ifact[n] = inv(fact[n]);
        for(ll i=n-1; i>=0; i--) ifact[i] = mul(ifact[i+1], i+1);
    }

    ll add(ll a, ll b){
        return ( (a%MOD) + (b%MOD) ) % MOD;
    }

    ll sub(ll a, ll b){
        return ( ((a%MOD) - (b%MOD)) + MOD ) % MOD;
    }

    ll mul(ll a, ll b){
        return ( (a%MOD) * (b%MOD) ) % MOD;
    }

    ll fast_exp(ll n, ll i){ // n ** i
        if (i == 0) return 1;
        if (i == 1) return n;
        ll tmp = fast_exp(n, i/2);
        if (i % 2 == 0) return mul(tmp, tmp);
        else return mul( mul(tmp, tmp), n );
    }

    ll inv(ll n){
        return fast_exp(n, MOD-2);
    }

    ll div(ll a, ll b){
        return mul(a, inv(b));
    }

    // n! / (n! (n-k)! )
    ll combination(ll n, ll k){ // "Combinação/Binomio de Newton"
        if(k > n) return 0;
        return mul( mul(fact[n], ifact[k]) , ifact[n-k]);
    }

    // n! / (n-k)!
```

```
    ll disposition(ll n, ll k){ // "Arranjo Simples"
        if(k > n) return 0;
        return mul(fact[n], ifact[n-k]);
    }

    // n!
    ll permutation(ll n){ // "Permutação Simples"
        return fact[n];
    }

    // n! / (k1! k2! k3!)
    ll permutationRepetition(ll n, vll x) { // "Permutação com Repetição"
        ll tmp = fact[n];
        for(auto k : x) tmp = mul(tmp, ifact[k]);
        return tmp;
    }

    // (n+m-1)! / ((n-1)! (m!))
    ll starBars(ll n, ll m) { // "pontos e virgulas"
        // n Groups -> n-1 Bars
        // m Stars
        return combination(n+m-1, m);
    }

    // !n = (n-1) * ( !(n-1) + !(n-2) )
    vll subfactorial; // derangements
    void computeSubfactorials(ll n) {
        subfactorial.assign(n+1, 0);
        subfactorial[0] = 1;
        // !0 = 1
        // !1 = 0
        for(ll i=2; i<=n; i++) {
            subfactorial[i] = mul( (i-1) , add(subfactorial[i-1], subfactorial[i-2]) );
        }
    }
};

// remember to pass a number delimeter (n) to precompute factorials
OpMOD op;
```

Overloading Operations Struct

```
const int MOD = 1e9+7;

struct intM{
    long long val = 0;

    intM(long long n=0){
        val = n%MOD;
        if (val < 0) val += MOD;
    }

    bool operator ==(const intM& b) const{
        return (val == b.val);
    }
```

```
    intM operator +(const intM& b) const{
        return (val + b.val) % MOD;
    }

    intM operator -(const intM& b) const{
        return (val - b.val + MOD) % MOD;
    }

    intM operator *(const intM& b) const{
        return (val*b.val) % MOD;
    }

    intM operator ^(const intM& b) const{ // fast exp [(val^b) mod M];
        if (b == 0) return 1;
        if (b == 1) return (*this);
        intM tmp = (*this)^(b.val/2); // diria que não vale a pena definir "/", "/" já é a multiplicação pelo inv
        if (b.val % 2 == 0) return tmp*tmp; // diria que não vale a pena definir "%", para não confidir com o %MOD
        else return tmp * tmp * (*this);
    }

    intM operator /(const intM& b) const{
        return (*this) * (b ^ (MOD-2));
    }
};
```

Sieve of Eratóstenes

```
// O (N log^2(N) ) -> Teorema de Merten

vll primes {2, 3};
set<ll> isPrime = {2, 3};
void eratostenes(ll n){
    bitset<MAX> sieve;
    sieve.set();
    for(ll i=5, step=2; i<=n; i+=step, step = 6 - step){
        if(sieve[i]){ // i is prime
            primes.push_back(i);
            isPrime.insert(i);
            for(ll j= i*i; j<=n; j += 2*i) // sieving all odd multiples of i >= i*i
                sieve[j] = false;
        }
    }
}
```

Linear Sieve

Computes all primes and composites between [2, n] in O(n) time.

Note that every composite q must have at least one prime factor, so we can pick the smallest prime factor p, and let the rest of the part be i, i.e. $q = ip$.

Since p is the smallest prime factor, we have $i \geq p$ (this garantees that p will already exist in the vector when reached by i)

Also, no prime less than p can divide i (break point).

Now let us take a look at the code we have a moment ago. This way, it's possible to pick out each composite exactly once.

```
vector<int> prime;
bool is_composite[MAX]; // can be 1e7

void sieve (int n) { // O(n)
    memset(is_composite, 0, sizeof(is_composite));

    for (int i = 2; i <= n; i++) {
        if (!is_composite[i]) prime.pb(i);
        for (int j = 0; j < (int) prime.size () && i * prime[j] <= n; j++) {
            is_composite[i * prime[j]] = true;
            if (i % prime[j] == 0) break;
        }
    }
}
```

Basic Knowledge

```
"a is divisible by b" or
"a is a multiple of b" or
"b is a divisor of a" or
"b is a factor of a" or
"b divides a" ( b|a )
=> a % b == 0

"a1, a2 are divisible by b"
=> gcd(a1, a2) % b == 0

"a is divisible by b1 and b2"
=> a % lcm(b1, b2) == 0

"a is divisible by b and b is divisible by c"
=> a % b == 0
=> b % c == 0
=> a % c == 0 (transitivity)
```

Greatest Common Divisor (GCD)

```
gcd(a) = a
gcd(a, b, c) = gcd( gcd(a, b), c )
gcd(a, b) = (a*b) / lcm(a, b)
```

Least Commom Multiple (LCM)

```
lcm(a) = a
lcm(a, b, c) = lcm( lcm(a, b), c )
lcm(a, b) = (a*b) / gcd(a, b)
```

Observation

std-c++17 implements gcd() function, which works correcly for negative numbers as well.

For negatives numbers, the following is true:

```
gcd(a,b) = gcd(-a,-b) = gcd(-a,b) = gcd(a,-b)
```

Closed Formulas related to divisors of a number

Let **n** be a number represented by it's prime factors p_i and respective exponents e_i:

d(n) = k = t = number of divisors

Quantidade Soma Produto

Lucas' Theorem

By definition, *n choose k* **($C ^n_k$)** is equal to:

```
n! / (k! * (n-k)!), 0 <= k <= n
0, otherwise


C(n, k) mod p = C(n_i, k_i) * C(n_i-1, k_i-1) * ... * C(n_0, k_0) mod p
```

Whereas:

n_i and *k_i* are the i-th digit of their respective numbers written in base *p*. All terms need to smaller than *p* by definition.

Example:

```
10 in base 3 = 1*3^2 + 0*3^1 + 1*3^0
n_2 = 1
n_1 = 0
n_0 = 1
```

Series' Theory

Closed formulas for some sequences

Natural Number Summation (PA):

$ 1 + 2 + 3 + 4 + 5 + ... + n-1 + n $

$ = \sum_{i=1}^n i $

= $ \frac{ n(n+1) }{ 2 } $

Natural Number Quadratic Summation:

$ 1 + 4 + 9 + 16 + 25 + ... + (n-1)^2 + n^2 $

$ = \sum_{i=1}^n i^2 $

= $ \frac{ n(n+1)(2n+1) }{ 6 } $

Triangular Numbers Summation:

$ 1 + 3 + 6 + 10 + 15 + ... + \frac{(n-1)(n)}{2} + \frac{(n)(n+1)}{2} $

$ = \sum_{i=1}^n \frac{i(i+1)}{2} = \frac{1}{2}(\sum_{i=1}^n i^2 + \sum_{i=1}^n i) $

$ = \frac{1}{2} (\frac{ n(n+1) }{ 2 } + \frac{ n(n+1)(2n+1) }{ 6 }) $

Extended Euclidian Algorithm

Computes the coeficients of this diofantine equation:

$ax + by$ = gcd(a, b)

Can be used to find the inverse multiplicative of a number if gcd(a, mod) == 1

a * x + m * y = gcd(a, m) a * x = 1 (mod m)

```
// a*x + b*y = gcd(a, b)
ll extended_euclid(ll a, ll b, ll &x, ll &y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    ll x1, y1;
    ll g = extended_euclid(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a/b);
    return g;
}
```

Combinatorics Theory

Stars and Bars

Also called "sticks and stones", "balls and bars", and "dots and dividers"

$ x_1 + x_2 + ... + x_n = m $

Example: (n = 3, m = 7)

★ ★ ★ ★ | ★ | ★ ★

n Groups; *n-1* Bars; *m* Stars;

Solution

C(n+m-1, n-1) = (n+m-1)! / ((n-1)! (m)!)

Proof

Elements = Bars + Stars = (n-1) + m = n+m-1; Repetition of Bars = n-1 Repetition of Stars = m

Therefore, it's a simple *permutation with repetition*.

$ P^{(n+m-1)}_{(n-1,m)} = C (n+m-1, m)$

Derangement

In combinatorial mathematics, a derangement is a permutation of the elements of a set, such that no element appears in its original position. In other words, a derangement is a permutation that has no fixed points.

Counting derangements

The number of derangements of a set of size n is known as the subfactorial of n or the n-th derangement number or n-th de Montmort number.

A subfactorial is noted as:

!n = (n-1) * (!(n-1) + !(n-2)), for n >= 2.

!1 = 0 *!0* = 1

Burside Lemma

necklaces with *n* pearls and *k* colors:

$\frac{1}{n} \sum_{i=1}^n k^{\gcd(i, n)}$

Misc

Random Numbers Generator

HOW TO USE :

```
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());

// to shuffle a vector
vector<int> vec;
shuffle(vec.begin(), vec.end(), rng);

// to simply generate a unsigned 32 bit number
unsigned int num = rng();

// to limit the number to the range [0, n[
unsigned int num = rng() % n;
```

```
// to limit the number to the range [1, n]
unsigned int num = rng() % n + 1;
```

For 64-bit numbers: mt19937_64

Getline

```
int32_t main() {
    ll n; cin >> n >> ws; // ws is input manipulator to retrieve the whitespace character
    string line;
    getline(cin, line); // the second line is therefore stored in the object "line". The default delimiter \n is not stored.
}
```

NOTE: When consuming whitespace-delimited input (e.g. int n; std::cin >> n;) any whitespace that follows, including a newline character, will be left on the input stream. Then when switching to line-oriented input, the first line retrieved with getline will be just that whitespace. In the likely case that this is unwanted behaviour.

Game-Theory

The Nim game consists of N piles containing K_i > 0 stones in each pile. Each turn, a player selects a pile and removes at least one stone from this pile. This is replayed in turns until the last player removes the last stone. In the classical version, this player who removed the last stone wins.

Proof of impartiality

Suppose the pile sizes are $a_1, a_2, \ldots, a_n$ before a move, and $b_1, b_2, \ldots, b_n$ after a move. Suppose that the move is on pile k; then for all $i \neq k$, $a_i = b_i$. Let $s = a_1 \oplus a_2 \oplus \ldots \oplus a_n$ and $t = b_1 \oplus b_2 \oplus \ldots \oplus b_n$. We have

$$\begin{aligned} t &= 0 \oplus t \\ &= (s \oplus s) \oplus t \\ &= s \oplus (s \oplus t) \\ &= s \oplus \left((a_1 \oplus a_2 \oplus \ldots \oplus a_n) \oplus (b_1 \oplus b_2 \oplus \ldots \oplus b_n)\right) \\ &= s \oplus \left((a_1 \oplus b_1) \oplus (a_2 \oplus b_2) \oplus \ldots \oplus (a_n \oplus b_n)\right) \\ &= s \oplus \left(0 \oplus 0 \oplus \ldots \oplus 0 \oplus (a_k \oplus b_k) \oplus 0 \oplus \ldots \oplus 0\right) \\ &= s \oplus (a_k \oplus b_k). \end{aligned}$$

Now we will prove two results.

Result 1: If $s = 0$, then $t \neq 0$. If the nim-sum of the original sizes is zero, then the moving player is losing (they must make the nim-sum nonzero).

We claim that $a_k \oplus b_k \neq 0$. Indeed, suppose it is, then

$$\begin{aligned} a_k &= a_k \oplus 0 \\ &= a_k \oplus (a_k \oplus b_k) \\ &= (a_k \oplus a_k) \oplus b_k \\ &= b_k. \end{aligned}$$

Thus $a_k = b_k$. But this contradicts the fact that the moving player moved on pile b_k, and thus must make the size different.

Thus, since $a_k \oplus b_k \neq 0$, we have

$$\begin{aligned} t &= s \oplus (a_k \oplus b_k) \\ &= 0 \oplus (a_k \oplus b_k) \\ &= a_k \oplus b_k \\ &\neq 0. \end{aligned}$$

Result 2: If $s \neq 0$, it's possible to make $t = 0$. If the nim-sum of the original sizes is not zero, the moving player is winning (they can make the nim-sum zero).

Consider the largest power of 2, 2^k, not greater than s. There must be at least one a_i such that it also contains 2^k, otherwise 2^k cannot appear in s. Now, take $b_i = s \oplus a_i$. The value b_i decreases by 2^k, and increases by at most $2^{k-1} + 2^{k-2} + \cdots + 2^0 = 2^k - 1$ (each remaining powers of 2 making up s adds to the value; for example $s = 2^2 + 2^1 + 2^0$ and $a_i = 2^3 + 2^2$ gives $b_i = 2^3 + 2^1 + 2^0$), so $b_i < a_i$. Moreover,

$$\begin{aligned} t &= s \oplus (a_i \oplus b_i) \\ &= s \oplus (a_i \oplus (s \oplus a_i)) \\ &= (s \oplus s) \oplus (a_i \oplus a_i) \\ &= 0. \end{aligned}$$

This proves the theorem. □

Structures

2-Dimensional Binary Indexed Tree

```
// 1-Indexed
struct FT2D {
    ll n, m;
    vector<vll> bit;

    FT2D(ll nn, ll mm) : n(nn), m(mm) {
        bit.assign(nn+1, vll(mm+1, 0));
    }

    void add(ll x, ll y, ll delta) {
        for(ll i=x; i<=n; i += i & -i)
            for(ll j=y; j<=m; j += j & -j)
                bit[i][j] += delta;
```

```
    }

    ll sum(ll x, ll y) {
        ll ans = 0;
        for(ll i=x; i>=1; i -= i & -i)
            for(ll j=y; j>=1; j -= j & -j)
                ans += bit[i][j];
        return ans;
    }

    ll query(ll x1, ll y1, ll x2, ll y2) {
        x1--; y1--; // if (x1, y1) are originally inclusive
        return sum(x2, y2) - sum(x2, y1) - sum(x1, y2) + sum(x1, y1);
    }
};
```

TRIE

AKA: prefix tree, digital tree

MAX Should be the number of maximum nodes to be created.

```
struct Trie{
    ll trie[MAX][26];
    bool isWordEnd[MAX];
    ll nxt = 1, wordsCnt = 0;

    void add(string s){ // O(n)
        ll node = 0;
        for(auto c: s) {
            if(trie[node][c-'a'] == 0)
                node = trie[node][c-'a'] = nxt++;
            else
                node = trie[node][c-'a'];
        }
        if(!isWordEnd[node]){
            isWordEnd[node] = true;
            wordsCnt++;
        }
    }

    bool find(string s, bool remove=false){ // O(n)
        ll node = 0;
        for(auto c: s) {
            if(trie[node][c-'a'] == 0)
                return false;
            else
                node = trie[node][c-'a'];
        }
        if(remove and isWordEnd[node]){
            isWordEnd[node] = false;
            wordsCnt--;
        }
        return isWordEnd[node];
```

```
    }
};
```

BIT (Fenwick Tree or Binary indexed tree)

OBS: BIT cannot support min/max queries, because it's mandatory to have an inverse operation.

Let's define **g(i)** as the number acquired after removing the **LSB(i)** of **i**:

```
g(i) = i - LSB(i) = i - (i & (-i))
```

Then, each value of the **Bit vector** will be resposible to store the range value of:

```
(g(i), i]
```

Therefore, to retrieve the range value of an arbitrary value **x**, it's only necessary to merge:

```
Bit[i] + Bit[g(i)] + Bit[g(g(i))] + ...
```

In the 1-Indexed implementation, Bit[0] is undefined and not used.

Complexity O(log(n)): add, sum, query

1-indexed Implementation

```
struct FT {
    vll bit;
    ll n;

    // constructor (all zeros)
    FT(ll sz) : n(sz) {
        bit.assign(sz + 1, 0);
    }

    void add(ll idx, ll delta) { // add delta to current value
        for (; idx <= n; idx += idx & -idx)
            bit[idx] += delta;
    }

    // sum from 1 to idx [inclusive]
    // idx is also 1-idx, obviously
    ll sum(ll idx) {
        ll ans = 0;
        for (; idx >= 1; idx -= idx & -idx) // LSB
            ans += bit[idx];
        return ans;
    }

    ll query(ll l, ll r) { // sum of [l, r]
```

```
        return sum(r) - sum(l - 1);
    }
};
```

Disjoint Set Union

```
struct DSU{
    vll group, card;
    DSU (ll n){
        n += 1; // 0-idx -> 1-idx
        group = vll(n);
        iota(group.begin(), group.end(), 0);
        card = vll(n, 1);
    }
    ll find(ll i){
        return (i == group[i]) ? i : (group[i] = find(group[i]));
    }
    void join(ll a ,ll b){
        a = find(a);
        b = find(b);
        if (a == b) return;
        if (card[a] < card[b]) swap(a, b);
        card[a] += card[b];
        group[b] = a;
    }
};
```

Avisos:

Possui a optimização de *Compressão* e *Balanceamento*

Both are: *O(a(N)) ~ O(1)*:

find(i): finds the representative of an element and returns it

join(a, b): finds both representatives and unites them, remaining only one for all. No return value

---

Segtree

Recursive Classic Segtree

Data structure that creates parent vertices for a linear array to do faster computation with binary agregation.

Diagrama

Clearer version (min-seg)

```
// 1 indexed segtree for minimum
ll L=1, R;
struct Segtree {
    struct Node {
```

```
        ll mn;
    };

    vector<Node> tree;
    vll v;

    Segtree(ll n) {
        v.assign(n+1, 0);
        tree.assign(4*(n+1), Node{});
        R = n;
    }

    Node merge(Node a, Node b) {
        Node tmp;
        // merge operaton:
        tmp.mn = min(a.mn, b.mn);
        //
        return tmp;
    }

    void build( ll l=L, ll r=R, ll i=1 ) {
        if (l == r) {
            Node tmp;
            // leaf element:
            tmp.mn = v[l];
            //
            tree[i] = tmp;
        }
        else {
            ll mid = (l+r)/2;
            build(l, mid, 2*i);
            build(mid+1, r, 2*i+1);
            tree[i] = merge(tree[2*i], tree[2*i+1]);
        }
    }

    void point_update(ll idx=1, ll val=0, ll l=L, ll r=R, ll i=1 ) {
        if (l == r) {
            // update operation:
            Node tmp{val};
            //
            tree[i] = tmp;
        }
        else {
            ll mid = (l+r)/2;
            if (idx <= mid) point_update(idx, val, l, mid, 2*i);
            else point_update(idx, val, mid+1, r, 2*i+1);
            tree[i] = merge(tree[2*i], tree[2*i+1]);
        }
    }

    Node range_query(ll left=L, ll right=R, ll l=L, ll r=R, ll i=1) {
        // left/right are the range limits for the update query
        // l / r are the variables used for the vertex limits
        if (right < l or r < left){
            // null element
            Node tmp{INF};
            //
```

```
            return tmp;
        }
        else if (left <= l and r <= right) return tree[i];
        else{
            ll mid = (l+r)/2;
            Node ansl = range_query(left, right, l, mid, 2*i);
            Node ansr = range_query(left, right, mid+1, r, 2*i+1);
            return merge(ansl, ansr);
            }
    }
};
```

Even more polished (sum-seg):

```
// 1 indexed segtree for sum
ll L=1, R;
struct Segtree {
    struct Node {
        // null element:
        ll ps = 0;
    };

    vector<Node> tree;
    vll v;

    Segtree(ll n) {
        v.assign(n+1, 0);
        tree.assign(4*(n+1), Node{});
        R = n;
    }

    Node merge(Node a, Node b) {
        return Node{
            // merge operaton:
            a.ps + b.ps
        };
    }

    void build( ll l=L, ll r=R, ll i=1 ) {
        if (l == r) {
            tree[i] = Node {
                // leaf element:
                v[l]
            };
        }
        else {
            ll mid = (l+r)/2;
            build(l, mid, 2*i);
            build(mid+1, r, 2*i+1);
            tree[i] = merge(tree[2*i], tree[2*i+1]);
        }
    }

    void update(ll idx=1, ll val=0, ll l=L, ll r=R, ll i=1 ) {
        if (l == r) {
```

```
            tree[i] = Node{
                // update operation:
                val
            };
        }
        else {
            ll mid = (l+r)/2;
            if (idx <= mid) update(idx, val, l, mid, 2*i);
            else update(idx, val, mid+1, r, 2*i+1);
            tree[i] = merge(tree[2*i], tree[2*i+1]);
        }
    }

    Node query(ll left=L, ll right=R, ll l=L, ll r=R, ll i=1) {
        // left/right are the range limits for the update query
        // l / r are the variables used for the vertex limits
        if (right < l or r < left){
            // null element:
            return Node{};
        }
        else if (left <= l and r <= right) return tree[i];
        else{
            int mid = (l+r)/2;
            return merge(
                query(left, right, l, mid, 2*i),
                query(left, right, mid+1, r, 2*i+1)
            );
        }
    }
};
```

Details

- **0 or 1-indexed**, depends on the arguments used as default value
- Uses a **struct node** to define node/vertex properties. *Default:* psum
- Uses a **merge function** to define how to join nodes

Parameters

- **left** and **right**: parameters that are the range limits for the range query
- **l** and **r**: are auxilary variables used for delimiting a vertex boundaries
- **idx**: index of the leaf node that will be updated
- **val**: value that will be inserted to the idx node

Atributes

- **v**: vector that are used for leaf nodes
- **Tree**: node array

Methods

O(n):

- **build(l, r, i)**: From **v** vector, constructs Segtree.

O(log(N))

- **point_update(idx, l, r, i, val)**: updates leaf node with *idx* index to *val* value. No return value
- **range_query(left, right, l, r, i)**: does a range query from *left* to *right* (inclusive) and returns a node with the result

Problems

- Range Sum Query, point update
- Range Max/Min Query, point update
- Range Xor Query, point update

Inverted Segtree

Range_increase -> using delta encoding

Point_update -> adding all values during transversal

```
int L = 1, N; // L = 1 = left limit; N = right limit
class SegmentTree {
    public:
        struct node{
            int psum;
        };

        node tree[4*MAX];
        int v[MAX];

        // requires minimum index and maximum index
        SegmentTree() {
            memset(v, 0, sizeof(v));
        }

        node merge(node a, node b){
            node tmp;
            // merge operaton:
            tmp.psum = a.psum + b.psum;
            //
            return tmp;
        }

        void build(int l=L, int r=N, int i=1) {
            if (l == r){
                node tmp;
                // leaf element
                tmp.psum = v[l];
                //
                tree[i] = tmp;
            }
            else{
                int mid = (l+r)/2;
                build(l, mid, 2*i);
                build(mid+1, r, 2*i+1);
                tree[i] = node{0};
            }
```

```
        }
        node point_query(int idx=1, int l=L, int r=N, int i=1){
            if (l == r){
                return tree[i];
            }
            else{
                int mid = (l+r)/2;
                if (idx <= mid)
                    return merge(tree[i], point_query(idx, l, mid, 2*i));
                else
                    return merge(tree[i], point_query(idx, mid+1, r, 2*i+1));
            }
        }
        void range_increase(int val, int left=L, int right=N, int l=L, int r=N, int i=1){
            // left/right are the range limits for the update query
            // l / r are the variables used for the vertex limits
            if (right < l or r < left){
                return;
            }
            else if (left <= l and r <= right){
                tree[i] = merge(tree[i], node{val});
            }
            else{
                int mid = (l+r)/2;
                range_increase(val, left, right, l, mid, 2*i);
                range_increase(val, left, right, mid+1, r, 2*i+1);
            }
        }
};
```

Implicit Segtree or Sparse Segtree

Creates vertices only when needed. Uncreated vertices are considered to have default values.

TODO Needs testing!

```
// Remember to set R value !!
ll L=1, R;
struct SegImplicit {
    struct Node{
        ll ps = 0, Lnode = 0, Rnode = 0;
    };
    ll idx = 2; // 1-> root / 0-> zero element
    Node tree[4*MAX];

    ll merge(Node a, Node b){
        return a.ps + b.ps;
    }

    void increase(ll pos, ll x, ll l=L, ll r=R, ll i=1) {
        if (l == r) {
            tree[i].ps += x;
            return;
        }
```

```
        ll mid = (l+r)/2;

        if (pos <= mid) {
            if (tree[i].Lnode == 0) tree[i].Lnode = idx++; // new vertex
            increase(pos, x, l, mid, tree[i].Lnode);
        }
        else {
            if (tree[i].Rnode == 0) tree[i].Rnode = idx++;
            increase(pos, x, mid+1, r, tree[i].Rnode);
        }

        tree[i].ps = merge(tree[ tree[i].Lnode ], tree[ tree[i].Rnode ]);
    }

    Node query(ll left, ll right, ll l=L, ll r=R, ll i=1) {
        if (right < l or r < left)
            return Node{};

        if (left <= l and r <= right)
            return tree[i];

        ll mid = (l+r)/2;
        Node ansl, ansr;
        if (tree[i].Lnode != 0) ansl = query(left, right, l, mid, tree[i].Lnode);
        if (tree[i].Rnode != 0) ansr = query(left, right, mid+1, r, tree[i].Rnode);

        return Node{merge(ansl, ansr), 0, 0};
    }
};
```

Concepts

- Uses *2n* memory.
- Notation is *node_index*: each segment is **[left border included, right excluded)**
- Leaves are stored in continuous nodes with indices **[n, 2n)**.
- *build()* is done bottom up, from bigger indices (leaves) to index 1 (highest node)
- updating parent nodes is done bottom up.
- TLDR: **witchcraft**

Iterative P-sum Classic Segtree with MOD

```
struct Segtree{
    vector<ll> t;
    int n;

    Segtree(int n){
        this->n = n;
        t.assign(2*n, 0);
    }
```

```
    ll merge(ll a, ll b){
        return (a + b) % MOD;
    }

    void build(){
        for(int i=n-1; i>0; i--)
            t[i]=merge(t[i<<1], t[i<<1|1]);
    }

    ll query(int l, int r){ // [l, r]
        ll resl=0, resr=0;
        for(l+=n, r+=n+1; l<r; l>>=1, r>>=1){
            if(l&1) resl = merge(resl, t[l++]);
            if(r&1) resr = merge(t[--r], resr);
        }
        return merge(resl, resr);
    }

    void update(int p, ll value){
        p+=n;
        for(t[p]=(t[p] + value)%MOD; p >>= 1;)
            t[p] = merge(t[p<<1], t[p<<1|1]);
    }

};
```

Iterative Range-Increment Range-Maximum (Lazy)

!! needs testing !!

```
int h = sizeof(int) * 8 - __builtin_clz(n);
int d[N];

const int N = 1e5;  // limit for array size
int n;  // array size
int t[2 * N];


void apply(int p, int value) {
  t[p] += value;
  if (p < n) d[p] += value;
}

void build(int p) {
  while (p > 1) p >>= 1, t[p] = max(t[p<<1], t[p<<1|1]) + d[p];
}

void push(int p) {
  for (int s = h; s > 0; --s) {
    int i = p >> s;
    if (d[i] != 0) {
      apply(i<<1, d[i]);
      apply(i<<1|1, d[i]);
      d[i] = 0;
    }
```

```
    }
  }

  void inc(int l, int r, int value) {
    l += n, r += n;
    int l0 = l, r0 = r;
    for (; l < r; l >>= 1, r >>= 1) {
      if (l&1) apply(l++, value);
      if (r&1) apply(--r, value);
    }
    build(l0);
    build(r0 - 1);
  }

  int query(int l, int r) {
    l += n, r += n;
    push(l);
    push(r - 1);
    int res = -2e9;
    for (; l < r; l >>= 1, r >>= 1) {
      if (l&1) res = max(res, t[l++]);
      if (r&1) res = max(t[--r], res);
    }
    return res;
  }
```

Recursive Segtree with Lazy propagation

Needs refactoring 😦 to leave only 1 version

Most recent version (Max range query, increase range update)

```
ll L = 1, R; // construct Seg with R value and build() afterwards
struct SegtreeLazy{

    struct Node {
        ll mx = -LLINF;
    };

    Node merge(Node a, Node b) {
        return Node{
            max(a.mx, b.mx)
        };
    }

    vector<ll> v, lazy;
    vector<Node> tree;

    SegtreeLazy(ll n) {
        R = n;
        tree.assign(4*(R-L+2), Node{});
        lazy.assign(4*(R-L+2), 0);
        v.assign((R-L+2), 0);
    }
```

```
void build(ll l=L, ll r=R, ll i=1) {
    if (l == r) {
        tree[i] = Node{
            v[l]
        };
    }
    else{
        ll mid = (l+r)/2;
        build(l, mid, 2*i);
        build(mid+1, r, 2*i+1);
        tree[i] = merge(tree[2*i], tree[2*i+1]);
    }
    lazy[i] = 0;
}

void propagate(ll l, ll r, ll i){
    if(lazy[i]) {
        tree[i].mx += lazy[i];

        if(l != r){
            lazy[2*i] = lazy[2*i] + lazy[i];
            lazy[2*i+1] = lazy[2*i+1] + lazy[i];
        }

        lazy[i] = 0;
    }
}

// [left, right] = (selected interval for the query)
// l, r = the variables used for the nodes boundaries

// increase function adds 'val' to [left, right]
void increase(ll left, ll right, ll val, ll l=L, ll r=R, ll i=1){
    propagate(l, r, i);

    if (right < l or r < left) return;

    else if (left <= l and r <= right){
        lazy[i] = lazy[i] + val;
        propagate(l, r, i);
    }

    else{
        ll mid = (l+r)/2;
        increase(left, right, val, l, mid, 2*i);
        increase(left, right, val, mid+1, r, 2*i+1);
        tree[i] = merge(tree[2*i], tree[2*i+1]);
    }
}

Node query(ll left, ll right, ll l=L, ll r=R, ll i=1){
    propagate(l, r, i);

    if (right < l or r < left) return Node{};

    else if (left <= l and r <= right) return tree[i];

    else{
```

```
            ll mid = (l+r)/2;
            return merge(
                query(left, right, l, mid, 2*i),
                query(left, right, mid+1, r, 2*i+1)
            );
        }
    }
};
```

Sum range query, increase range update

```
ll L = 1, R;
struct SegtreeLazy{
    vector<ll> tree, lazy, v;

    SegtreeLazy() {
        tree.assign(4*(R-L+2), 0);
        lazy.assign(4*(R-L+2), 0);
        v.assign((R-L+2), 0);
    }

    void build(ll l=L, ll r=R, ll i=1) {
        if (l == r) tree[i] = v[l];
        else{
            ll mid = (l+r)/2;
            build(l, mid, 2*i);
            build(mid+1, r, 2*i+1);
            tree[i] = tree[2*i] + tree[2*i+1];
        }
        lazy[i] = 0;
    }

    void propagate(ll l, ll r, ll i){
        if(lazy[i]) {
            tree[i] += lazy[i] * (r-l+1);
            if(l != r){
                lazy[2*i] += lazy[i];
                lazy[2*i+1] += lazy[i];
            }
            else v[l] += lazy[i]; // update array
            lazy[i] = 0;
        }
    }

    // [left, right] = (selected interval for the query)
    // l, r = the variables used for the vertex limits

    // increase function adds 'val' to [left, right]
    void increase(ll left=L, ll right=R, ll val=0, ll l=L, ll r=R, ll i=1){
        propagate(l, r, i);

        if (right < l or r < left) return;

        else if (left <= l and r <= right){
            lazy[i] += val;
```

```
            propagate(l, r, i);
        }

        else{
            ll mid = (l+r)/2;
            increase(left, right, val, l, mid, 2*i);
            increase(left, right, val, mid+1, r, 2*i+1);
            tree[i] = tree[2*i] + tree[2*i+1];
        }
    }

    ll query(ll left=L, ll right=R, ll l=L, ll r=R, ll i=1){
        propagate(l, r, i);

        if (right < l or r < left) return 0;

        else if (left <= l and r <= right) return tree[i];

        else{
            ll mid = (l+r)/2;
            return (
                query(left, right, l, mid, 2*i) +
                query(left, right, mid+1, r, 2*i+1)
            );
        }
    }
};
```

Range Minimum Query, Update (Assignment) Query

```
ll L = 1, R;
struct SegtreeLazy{
    vll tree, lazy, v;

    SegtreeLazy() {
        tree.assign(4*(R-L+2), 0);
        lazy.assign(4*(R-L+2), 0);
        v.assign((R-L+2), 0);
    }

    void build(ll l=L, ll r=R, ll i=1) {
        if (l == r) tree[i] = v[l];
        else{
            ll mid = (l+r)/2;
            build(l, mid, 2*i);
            build(mid+1, r, 2*i+1);
            tree[i] = min(tree[2*i], tree[2*i+1]);
        }
        lazy[i] = LLINF; // min query default value
    }

    void propagate(ll l, ll r, ll i){
        if(lazy[i] != LLINF) { // need to propagate lazy
            tree[i] = lazy[i];
            if(l != r)
```

```
                lazy[2*i] = lazy[2*i+1] = lazy[i];
            else
                v[l] = lazy[i]; // update 'v' vector
            lazy[i] = LLINF;
        }
    }

    // [left, right] = (selected interval for the query)
    // l, r = the variables used for the vertex limits

    // update function changes all elements in [left, right] to val
    void update(ll left=L, ll right=R, ll val=0, ll l=L, ll r=R, ll i=1){
        propagate(l, r, i);

        if (right < l or r < left) return;

        else if (left <= l and r <= right){
            lazy[i] = val;
            propagate(l, r, i);
        }

        else{
            ll mid = (l+r)/2;
            update(left, right, val, l, mid, 2*i);
            update(left, right, val, mid+1, r, 2*i+1);
            tree[i] = min(tree[2*i], tree[2*i+1]);
        }
    }

    ll query(ll left=L, ll right=R, ll l=L, ll r=R, ll i=1){
        propagate(l, r, i);

        if (right < l or r < left) return LLINF;

        else if (left <= l and r <= right) return tree[i];

        else{
            ll mid = (l+r)/2;
            return min(
                query(left, right, l, mid, 2*i),
                query(left, right, mid+1, r, 2*i+1)
            );
        }
    }
};
```

For MAX Query

Use the same code as min segtree, change:

min() -> **max() LLINF** -> **-LLINF**

Complex Lazy Problems

Requirements: to be able to *propagate/push* the lazy stored updates. In other words, the property of **Aggregation:** to transfer the data saved in *lazy[i]* to *tree[i]* and also the property of **Composition:** to push the updates to the children (*lazy[i]* to *lazy[2*i]* and *lazy[i]* to *lazy[2*i+1]*).

Example1:

range increase query by x range sum of the squares = a[l]^2 + ... + a[r]^2

```
// import this struct
struct intM{};

ll L = 1, R; // Declare R = n and also use build() afterwards -_- macake
struct SegtreeLazy{

    struct Node {
        intM val;
        intM sqr;
    };

    Node merge(Node a, Node b) {
        return Node{
            a.val + b.val,
            a.sqr + b.sqr
        };
    }

    vector<intM> v, lazy;
    vector<Node> tree;

    SegtreeLazy() {
        tree.assign(4*(R-L+2), Node{});
        lazy.assign(4*(R-L+2), intM{});
        v.assign((R-L+2), intM{});
    }

    void build(ll l=L, ll r=R, ll i=1) {
        if (l == r) {
            tree[i] = Node{
                v[l],
                v[l] * v[l]
            };
        }
        else{
            ll mid = (l+r)/2;
            build(l, mid, 2*i);
            build(mid+1, r, 2*i+1);
            tree[i] = merge(tree[2*i], tree[2*i+1]);
        }
        lazy[i] = intM{};
    }

    void propagate(ll l, ll r, ll i){
        if(lazy[i].val) {
            tree[i].sqr = tree[i].sqr + (intM(2) * lazy[i] * tree[i].val);
            tree[i].sqr = tree[i].sqr + (lazy[i] * lazy[i] * intM(r-l+1));

            tree[i].val = tree[i].val + (lazy[i] * intM(r-l+1));
```

```
            if(l != r){
                lazy[2*i] = lazy[2*i] + lazy[i];
                lazy[2*i+1] = lazy[2*i+1] + lazy[i];
            }

            lazy[i] = intM{};
        }
    }

    // [left, right] = (selected interval for the query)
    // l, r = the variables used for the vertex limits

    // increase function adds 'val' to [left, right]
    void increase(ll left=L, ll right=R, ll val=0, ll l=L, ll r=R, ll i=1){
        propagate(l, r, i);

        if (right < l or r < left) return;

        else if (left <= l and r <= right){
            lazy[i] = lazy[i] + intM(val);
            propagate(l, r, i);
        }

        else{
            ll mid = (l+r)/2;
            increase(left, right, val, l, mid, 2*i);
            increase(left, right, val, mid+1, r, 2*i+1);
            tree[i] = merge(tree[2*i], tree[2*i+1]);
        }
    }

    Node query(ll left=L, ll right=R, ll l=L, ll r=R, ll i=1){
        propagate(l, r, i);

        if (right < l or r < left) return Node{};

        else if (left <= l and r <= right) return tree[i];

        else{
            ll mid = (l+r)/2;
            return merge(
                query(left, right, l, mid, 2*i),
                query(left, right, mid+1, r, 2*i+1)
            );
        }
    }
};

int32_t main(){ sws;
    ll n, q; cin >> n >> q;
    R = n;

    SegtreeLazy st;

    for(ll i=1; i<=n; i++) {
        ll x; cin >> x;
        st.v[i] = intM(x);
    }
```

```
    st.build();

    while(q--) {
        char c; cin >> c;
        if (c == 'u') {
            ll l, r, x; cin >> l >> r >> x;
            st.increase(l, r, x);
        }
        else {
            ll l, r; cin >> l >> r;
            cout << st.query(l, r).sqr.val << endl;
        }
    }
}
```