HW2

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2a. Derive the ADMM algorithm steps.

Given: $\underset{x \in \mathbb{R}^n}{\text{minimize}} \left\{ \frac{1}{2}\|x-y\|_2^2 + \alpha \sum_{i=1}^{n-1} |x_{i+1} - x_i| \right\}$, with ADMM form

$\underset{x \in \mathbb{R}^n}{\text{minimize}} \left\{ \frac{1}{2}\|x-y\|_2^2 + \alpha \|z\|_1 \right\}$ subject to $Fx - z = 0$.

Minimizing the augmented Lagrangian

$$\mathcal{L}(x, z, u) = \frac{1}{2}\|x-y\|_2^2 + \alpha\|z\|_1 + \frac{\rho}{2}\|Fx - z + u\|_2^2 - \frac{\rho}{2}\|u\|_2^2, \text{ where } \rho = 1 \text{ is given,}$$

gives ADMM algorithm

$$x^{(k+1)} = (I_n + \rho F^T F)^{-1} (I_n y + \rho F^T (z^{(k)} - u^{(k)})),$$

$$z^{(k+1)} = S_{\alpha/\rho}(Fx^{(k+1)} + u^{(k)}),$$

$$u^{(k+1)} = u^{(k)} + Fx^{(k+1)} - z^{(k+1)}.$$