Formula Sheet

$\sin^2 x + \cos^2 x = 1$

Integrals

$\sin^2 x = \cos^2(x))$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \text{ for } n \neq -1$$

$$\int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int \sin(x) \, dx = -\cos(x) + C$$

$$\int \cos(x) \, dx = \sin(x) + C$$

$$\int \tan(x) \, dx = -\ln|\cos(x)| + C$$

$$\int \cot(x) \, dx = \ln|\sin(x)| + C$$

$$\int \sec(x) \, dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int \csc(x) \, dx = -\ln|\csc(x) + \cot(x)| + C$$

$$\int \sec(x)\tan(x) \, dx = \sec(x) + C$$

$$\int \sec^2(x) \, dx = \tan(x) + C$$

$$\int \csc(x)\cot(x) \, dx = -\csc(x) + C$$

$$\int \csc^2(x) \, dx = -\cot(x) + C$$

$$\int a^x \, dx = \frac{a^x}{\ln(a)} + C$$

$$\int \frac{1}{1+x^2} \, dx = \tan^{-1}(x) + C$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1}(x) + C$$

Trigonometric Formulas

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$\sec^2(x) - 1 = \tan^2(x)$$

$$\sin(x)\cos(y) = \tfrac{1}{2}\left(\sin(x-y) + \sin(x+y)\right)$$

$$\sin(x)\sin(y) = \tfrac{1}{2}\left(\cos(x-y) - \cos(x+y)\right)$$

$$\cos(x)\cos(y) = \tfrac{1}{2}\left(\cos(x-y) + \cos(x+y)\right)$$

$\frac{d}{dx} = \arctan(x) = \frac{1}{x^2+1}$

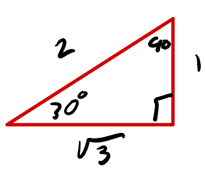
$\frac{d}{dx} \cot = -\csc^2 x$

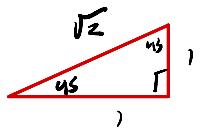
$\frac{d}{dx} \csc = -\cot x \csc x$

$1 + \cot^2 x = \csc^2 x$

$\frac{d}{dx} = \arcsin = \sqrt{\frac{1}{1-x^2}}$

$\frac{\sqrt{3}}{2}$





$\tan(\pi/4)$

$\frac{1}{1} = .$

Trig Integrals Cheat Sheet:

if odd+even:

- take one off odd power, rewrite odd with the other trig function
- u-sub on odd

if only one power/even:

half angle formula → reduce the power

if both are even:

use half/angle again

tan stuff:

if sec is even:

save $\sec^2 x$ and use $\sec^2 x = 1+\tan^2 x$

$u = \tan$

if tan is odd:

save $\sec x \tan x$ and use $\tan^2 x = \sec^2 x - 1$

$u = \sec x$

if both even:

separate $\sec x \tan x$

Half angles:

$$\sin^2 x = \frac{1}{2}(1-\cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1+\cos 2x)$$

$$\sin x \cos x = \frac{1}{2}\sin 2x$$

tan stuff:

$$\tan^2 x + 1 = \sec^2 x \quad \sec^2 x - \tan^2 x = 1$$

$$u = \sec(x),\ du = \sec\tan x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\int \tan^5 \sec^3 d\theta$$

$$\int \tan^4 \tan \sec^3 d\theta$$

$$\int (\sec^2 x - 1)^2 \quad \sec\tan d\theta$$

u-sub

$$\int \tan^2 t \sec^4 t$$

$$\tan^2 t \cdot \sec^2 t \cdot \sec^2 t$$

$$\int \tan^2 t\,(1+\tan^2 t)\cdot \sec^2 t\,dt$$

$u = \tan$

TRIG SUB

1) Identify radical
2) choose appropriate substitution

$$\sqrt{a^2 - x^2} = a\sin\theta$$

$$\sqrt{a^2 + x^2} = a\tan\theta$$

$$\sqrt{x^2 - a^2} = a\sec\theta$$

3) Substitute the above for x in the integral
4) Simplify with identities + Algebra (express in terms of θ)
5) integrate
6) Substitute all θ with x using identities

equation of plane

$$\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \cdot \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix} = 0$$

eqn. of line

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}}_{\text{point}} + t\underbrace{\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}}_{\text{dir. vector}}$$

Partial Fractions

1) factor the denominator
2) write the decomposition
3) determine constants by equating the original function to the partial fraction and solving
4) simplify and combine like terms

if the numerator is higher power - long division

$$\int \frac{1}{(x^2-4)}$$

$$\int \frac{1}{(x+2)(x-2)}$$

$$\int \frac{A}{x+2} + \int \frac{B}{x-2}$$

$$\frac{A(x-2) + B(x+2)}{(x+2)(x-2)} = \frac{1}{x^2-4}$$

$x = 2 \quad B = \frac{1}{4}$

$x = -2 \quad A = -\frac{1}{4}$

$$\int -\frac{1}{4}\cdot\frac{1}{x-2} + \frac{1}{4}\cdot\frac{1}{x+2}$$

$$= -\frac{1}{4}\ln|x-2| + \frac{1}{4}\ln|x+2| + C$$

taylor polynomials:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Dot Product!

$$\vec{a}\cdot\vec{b} = |a||b|\cos\theta$$

$$\vec{a}\cdot\vec{b} \quad \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}\cdot\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = a_1 b_1 + a_2 b_2$$

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2} \leftarrow \sqrt{\dots}$$

$$\text{Proj}_a B = \frac{a \cdot b}{|b|^2} \times b$$