Solutions to Theory Part

Question 1: Color Theory

Solution:

1) Under normal conditions (white light) each square absorbs some of the spectrum from the white light and reflects the rest, which gives it its color. The absorption property of the material remains same even if the incident light spectrum changes. If the wavelengths aren't present in the shining light, then there will be nothing to reflect and it will appear darker.

2) It is harder to solve under a red light.

Question 2: Color Theory *(10 points)*

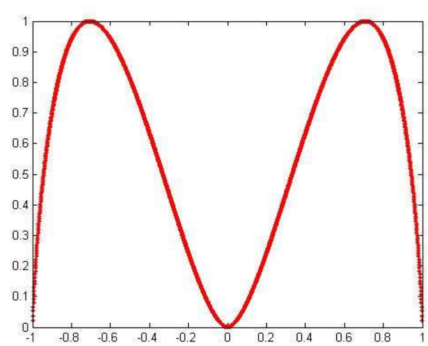
- Proof:
  $x = X/(X+Y+Z)$; $y = Y/(X+Y+Z)$; $z = Z/(X+Y+Z)$ and $x+y+z = 1$;

$$
\begin{aligned}
[\,(1-x-y)/y\,]\,Y &= (\,(1 - X/(X+Y+Z) - Y/(X+Y+Z)\,) / (Y/(X+Y+Z)\,)\,) * Y \\
&= (\,(Z/(X+Y+Z)\,) / (Y/(X+Y+Z)\,)\,) * Y \\
&= (\,Z / Y\,) * Y \\
&= Z
\end{aligned}
$$

- This is not a good algorithm. There are colors in one gamut not in the other. All these colors will probably map to just one or a few colors. If the image contains this color variation, it will look constant when printed
- It will work better for constant color tones, for reasons explained above
- To create a better mapping, we can define a continuous "elastic" transformation from one gamut triangle to another. This can be specified in various way, normally a 3x3 matrix

Question 3: Entropy Coding

H = -[xk log(xk) + (1-xk) log (1-xk)]



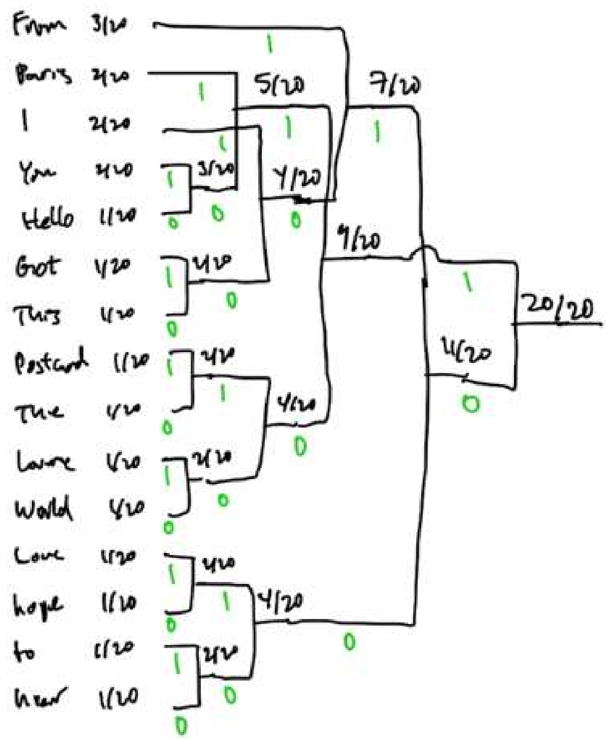
- From the plot below for k=2, H has a min at -1, 0 and 1
- Generalized formula: Min – for odd k, min is 0 and 1, for even k min is -1, 0 and 1

- From the plot below for k=2, H has a max at –sqrt(1/2) and +sqrt(1/2)
- Generalized formula – Max – for odd k, min is (1/2)^(1/k), for even k it is +(1/2)^(1/k) and –(1/2)^(1/k)

Queston 4: Huffman coding

a) Word, probability, code

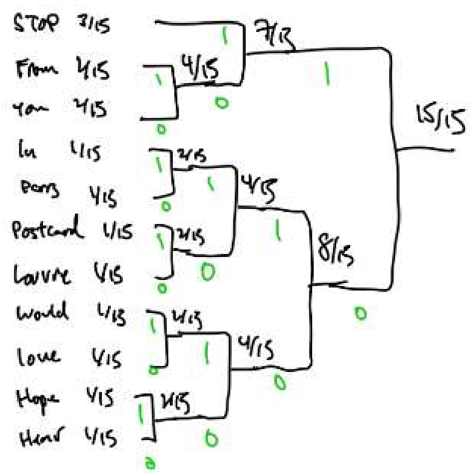
From: 3/20, 011
Paris: 2/20, 111
I: 2/20, 0101
You: 2/20, 1101
Hello: 1/20, 1100
Got: 1/20, 01001
This: 1/20, 01000
Postcard: 1/20, 1011
The: 1/20, 1010
Louvre: 1/20, 1001
Would: 1/20, 1000
Love: 1/20, 0011
Hope: 1/20, 0010
To: 1/20, 0001
Hear: 1/20, 0000



The average code length is 1/20 (3*3 + 2*3 + 2*4 + 2*4 + 1*4 + 1*5 + 1*5 + 1*4 + 1*4 + 1*4 + 1*4 + 1*4 + 1*4 + 1*4 + 1*4) = 3.85

- Word, probability, code

STOP: 3/15, 11
From: 2/15, 101
You: 2/15, 100
In: 1/15, 0111
Paris: 1/15, 0110
Postcard: 1/15, 0101
Louvre: 1/15, 0100
Would: 1/15, 0011
Love: 1/15, 0010
Hope: 1/15, 0001
Hear: 1/15, 0000



The average code length is 1/15 (3*2 + 2*3 + 2*3 + 1*4 + 1*4+ 1*4 + 1*4+ 1*4 + 1*4 + 1*4 + 1*4) = 3.33.

c) The entropy of the first message is 3.78. (by Shannon entropy equation). The entropy of the second is 3.32. So, the first message contains more information. The difference likely comes from the small loss of detail from cutting words out of the original message—for example “Hello from Paris!” conveys a different message from “In Paris.” Similarly for “I got this postcard from the Louvre” becoming “From Louvre” (who got the postcard?).