Statistics 2 2025/26 Exam Solutions

Introduction

You work for an airline considering opening a new route between the cities Dehli and Mumbai in India. You have been tasked with analyzing flight pricing among rival airlines on this route. You go to the websites of all the airlines that fly direct between those two cities and take note of the price of an economy class ticket for different dates: if you were to fly tomorrow, 2 days later, 3 days later and so on, until 49 days later.

You also take note of the airline, source city and destination city. You put all of these data into a spreadsheet. Here is a link to download the spreadsheet you created: `dehli_mumbai_flights.csv`.

The definitions of all the variables are as follows:

- `flight`: The flight number.
- `airline`: The name of the airline service running the flight.
- `source_city`: The city the flight is leaving from (either Dehli or Mumbai).
- `destination_city`: The city the flight is flying to (either Dehli or Mumbai).
- `days_left`: The number of days until the flight departs. If `days_left` equals 1, it means the flight is leaving tomorrow. If it equals 2, it means the flight is leaving in 2 days, etc.
- `price`: The price of the flight converted to euro.

Your supervisor asks you to answer the following questions using your data.

* Question 1

What is the sample correlation between `days_left` and `price`?

Answer:

```
df <- read.csv("delhi_mumbai_flights.csv")
cor(df$days_left, df$price)
```

```
[1] -0.6908118
```

Model 1

Estimate a linear regression model with `price` as the dependent variable and `days_left` as the independent variable.

If you estimated the model correctly, you should have an estimated intercept of 56.03504.

```r
m1 <- lm(price ~ days_left, data = df)
summary(m1)
```

```

Call:
lm(formula = price ~ days_left, data = df)

Residuals:
    Min      1Q  Median      3Q     Max 
-20.460  -8.810   0.598   8.037 187.558 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 56.03504    0.35869  156.22   <2e-16 ***
days_left   -0.84233    0.01226  -68.69   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11.88 on 5168 degrees of freedom
Multiple R-squared:  0.4772,	Adjusted R-squared:  0.4771 
F-statistic:  4718 on 1 and 5168 DF,  p-value: < 2.2e-16
```

* Question 2

What is the sample regression slope?

Answer:

```r
coef(m1)[2]
```

```
 days_left 
-0.8423306 
```

* Question 3

Use the model to estimate the mean price of flights departing in 50 days (i.e. `days_left` equal to 50).

Also obtain a 95% confidence interval for the mean price of flights departing in 50 days.

- Estimate of the mean price of flights departing in 50 days: ___________
- Confidence interval lower bound: ___________
- Confidence interval upper bound: ___________

Answer:

```
df_p <- data.frame(days_left = 50)
predict(m1, df_p, interval = "confidence", level = 0.95)
```

```
       fit      lwr      upr
1 13.91851 13.25596 14.58106
```

The prediction is under `fit`, the lower bound of the confidence interval is under `lwr`, and the upper bound of the confidence interval is under `upr`.

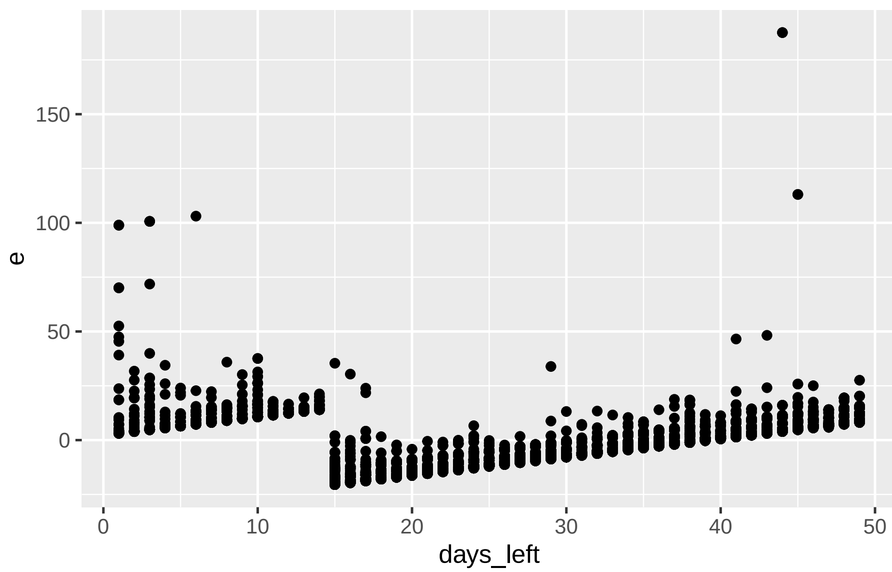
* Question 4

Plot the residuals (on the vertical axis) against `days_left` (on the horizontal axis). Choose the answer below which best interprets what this plot says about our model assumptions.

- All residuals are positive when `days_left` is less than 15. This evidence suggests that the assumption $\mathbb{E}[\varepsilon_i|x_i] = 0$ for all x_i is satisfied.
- All residuals are negative when `days_left` is less than 15. This evidence suggests that the assumption $\mathbb{E}[\varepsilon_i|x_i] = 0$ for all x_i is satisfied.
- All residuals are positive when `days_left` is less than 15. This evidence suggests that the assumption $\mathbb{E}[\varepsilon_i|x_i] = 0$ for all x_i is violated.
- All residuals are negative when `days_left` is less than 15. This evidence suggests that the assumption $\mathbb{E}[\varepsilon_i|x_i] = 0$ for all x_i is violated.

Answer:

```
df$e <- m1$residuals
ggplot(df, aes(days_left, e)) + geom_point()
```



All residuals are positive when `days_left` is less than 15. This evidence suggests that the assumption $\mathbb{E}[\varepsilon_i | x_i] = 0$ for all x_i is violated.

* Question 5

Perform a formal test for heteroskedasticity by regressing the square of the residuals on `days_left`.

Use a 5% significance level.

In this test, the null hypothesis is that the model exhibits which of the following:

- Homoskedasticity
- Heteroskedasticity
- Multiskedasticity
- Paraskedasticity

What is the p-value from this test? ___________

What is the conclusion? Choose one of the answers below:

- The model exhibits heteroskedasticity. Therefore the standard errors are not reliable.
- The model exhibits homoskedasticity. Therefore the standard errors are not reliable.
- The model exhibits heteroskedasticity. Therefore the standard errors are reliable.
- The model exhibits homoskedasticity. Therefore the standard errors are reliable.

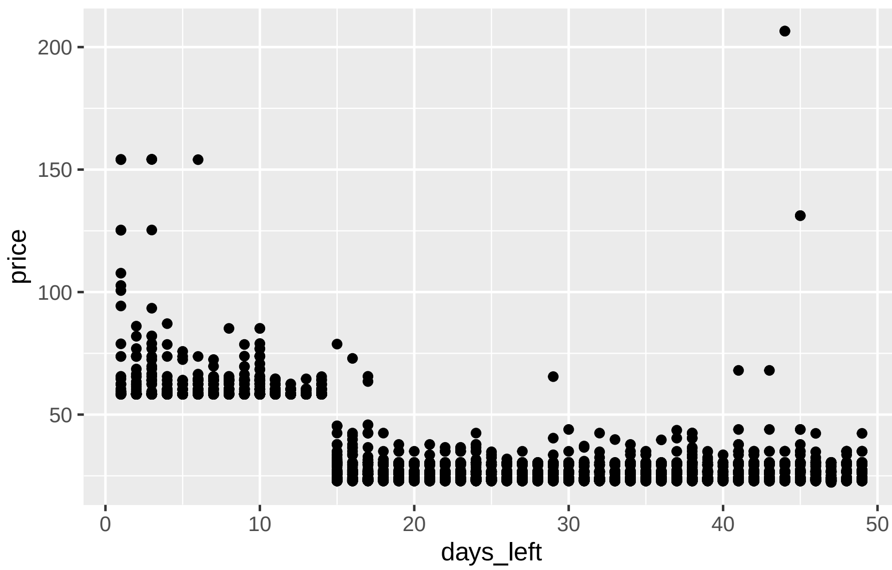
Answer:

```
df$e2 <- m1$residuals^2
m_aux <- lm(e2 ~ days_left, data = df)
coef(summary(m_aux))["days_left", "Pr(>|t|)"]
```

```
[1] 0.00008592984
```

Model 2

A scatter plot of `days_left` and `price` is shown below:



Based on this, your supervisor suggests an alternative way of modeling `price` as a function of `days_left`. Your supervisor suggests to use a dummy variable that equals 1 if the flight is within 14 days (i.e. `days_left` ≤ 14) and 0 if the flight is more than 14 days away (i.e. `days_left` > 14).

Create this variable with the command:

```
df$within14 <- df$days_left <= 14
```

Use this variable to estimate a linear regression model with `price` as the dependent variable and `within14` as the independent variable.

If you estimated the model correctly, your estimated intercept should equal 25.5823.

```r
df$within14 <- df$days_left <= 14
m2 <- lm(price ~ within14, data = df)
summary(m2)
```

```

Call:
lm(formula = price ~ within14, data = df)

Residuals:
    Min      1Q  Median      3Q     Max 
 -3.212  -2.052  -1.932   0.898 180.948 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)   25.5823     0.1077   237.5   <2e-16 ***
within14TRUE  34.7598     0.2168   160.4   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.722 on 5168 degrees of freedom
Multiple R-squared:  0.8326,    Adjusted R-squared:  0.8326 
F-statistic: 2.571e+04 on 1 and 5168 DF,  p-value: < 2.2e-16
```

* Question 6

Compare the sum of squares due to regression (SSR) between Model 1 (using `days_left`) and Model 2 (using `within14`). Choose the answer below which best describes what you learn from this.

- Model 2 has a **higher** SSR than Model 1 and therefore Model 2 is preferred.
- Model 2 has a **lower** SSR than Model 1 and therefore Model 2 is preferred.
- Model 2 has a **higher** SSR than Model 1 and therefore Model 2 is **not** preferred.
- Model 2 has a **lower** SSR than Model 1 and therefore Model 2 is **not** preferred.
- The two models use different **independent** variables, and therefore we can't compare their SSRs in a meaningful way.
- The two models use different **dependent** variables, and therefore we can't compare their SSRs in a meaningful way.

Answer:

```r
m0 <- lm(price ~ 1, data = df)
anova(m0, m1)
```

```
Analysis of Variance Table

Model 1: price ~ 1
Model 2: price ~ days_left
  Res.Df     RSS Df Sum of Sq      F    Pr(>F)    
1   5169 1395330                                  
2   5168  729449  1    665881 4717.6 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(m0, m2)
```

```
Analysis of Variance Table

Model 1: price ~ 1
Model 2: price ~ within14
  Res.Df     RSS Df Sum of Sq     F    Pr(>F)    
1   5169 1395330                                 
2   5168  233510  1   1161820 25713 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Model 2 has a **higher** SSR than Model 1 and is therefore preferred.

* Question 7

Obtain a 95% confidence interval for the coefficient in front of the variable `within14`. Report the lower and upper bound.

Answer:

```r
confint(m2, level = 0.95)
```

```
                2.5 %   97.5 %
(Intercept)  25.37107 25.79348
within14TRUE 34.33482 35.18474
```

The lower bound is 34.33482 and the upper bound is 35.18474.

Model 3

Your supervisor is also interested in the average price differences between airlines.

To study this, estimate a linear regression model with `price` as the dependent variable and dummy variables for the airlines as the independent variables. Use `"Air_India"` as the base category, so the included variables should be dummy variables for `"AirAsia"`, `"Go_FIRST"`, `"Indigo"`, `"SpiceJet"` and `"Vistara"`.

In R, you can estimate this model using the `airline` variable as is (i.e. using the formula `price ~ airline`). It is not necessary to create the individual dummy variables. `"Air_India"` will automatically be chosen as the base category using this approach.

If you estimated the model correctly, the estimated regression intercept should equal 33.6899.

```
m3 <- lm(price ~ airline, data = df)
summary(m3)
```

```

Call:
lm(formula = price ~ airline, data = df)

Residuals:
    Min      1Q  Median      3Q     Max 
-14.914 -10.963  -7.797  13.400 171.643 

Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept)      33.6899     0.6502  51.813  < 2e-16 ***
airlineAirAsia    0.2583     1.3884   0.186 0.852394    
airlineGO_FIRST  -2.9532     0.8189  -3.606 0.000313 ***
airlineIndigo     1.1968     0.7837   1.527 0.126772    
airlineSpiceJet   3.5940     0.9859   3.645 0.000270 ***
airlineVistara    1.5533     0.7820   1.986 0.047062 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 16.32 on 5164 degrees of freedom
Multiple R-squared:  0.01424,	Adjusted R-squared:  0.01328 
F-statistic: 14.92 on 5 and 5164 DF,  p-value: 0.00000000000001447
```

* Question 8

The estimated intercept of 33.6899 means that the average price of which airline equal to €33.68?

- Air India
- AirAsia
- Go First
- Indigo
- SpiceJet
- Vistara

Answer:

This is the average price of the base category, which is Air India.

* Question 9

According to the model, what is the average price of SpiceJet flights in the data?

Answer:

We add the estimated intercept and the coefficient on SpiceJet:

```
coef(m3)[1] + coef(m3)[5]
```

```
(Intercept)
    37.2839
```

* Question 10

Use the model to test the following claim at the 5% level:

"GO FIRST flights are more than €2 cheaper compared to Air India"

Perform this test by answering the questions below.

Note: If you estimated your regression model using the variable `airline` as is, and assigned it to the name `m3`, then you can extract the coefficient for GO FIRST with the following R command:

```
b_2 <- coef(summary(m3))["airlineGO_FIRST", "Estimate"]
```

- What is the null hypothesis? β_2 $<$ / $\leq$ / $>$ / $\geq$ / $=$ / $\neq$ ____________ (choose one comparison operator and fill in a value in the blank).
- What is the alternative hypothesis? β_2 $<$ / $\leq$ / $>$ / $\geq$ / $=$ / $\neq$ ____________ (choose one comparison operator and fill in a value in the blank).
- Under the null hypothesis, the test statistic $T = (B_2 - b)/S_{B_2}$, where b is the hinge, follows a t distribution with how many degrees of freedom? ____________
- What is the value of the test statistic? ____________
- What is the associated p-value? ____________
- What is your conclusion? Choose an option below:
  - Reject H0: There is sufficient evidence for the claim.
  - Reject H0: There is not sufficient evidence for the claim.
  - Don't reject H0: There is sufficient evidence for the claim.
  - Don't reject H0: There is not sufficient evidence for the claim.

Answer:

- H_0: $\beta_1 \geq -2$.

- H_1: $\beta_1 < -2$.
- Under H_0, $T = (B_2 - b)/S_{B_2}$, where b is the hinge, follows a t distribution with $n - k - 1 = 5164$ degrees of freedom.
- The value of the test statistic can be computed with:

```
b_2 <- coef(summary(m3))["airlineGO_FIRST", "Estimate"]
s_b_2 <- coef(summary(m3))["airlineGO_FIRST", "Std. Error"]
t <- (b_2 + 2) / s_b_2
t
```

```
[1] -1.163996
```

- The p-value for this lower tail test can be computed with:

```
pt(t, m3$df.residual)
```

```
[1] 0.1222397
```

- The conclusion is: Don't reject H0: There is not sufficient evidence for the claim.

Model 4

Estimate a model explaining `price` with the `within14` dummy variable AND dummy variables for the airlines (using `"Air_India"` as the base category).

Like with the previous model, you can add these airline dummies to your model using the `airline` variable as is (which also automatically chooses `"Air_India"` as the base category). You simply use the formula `price ~ within14 + airline`.

If you estimated the model correctly your estimated intercept should equal 24.2944.

```
m4 <- lm(price ~ within14 + airline, data = df)
print(summary(m4))
```

```

Call:
lm(formula = price ~ within14 + airline, data = df)

Residuals:
    Min      1Q  Median      3Q     Max 
 -4.368  -2.543  -0.458   0.466 179.222 

Coefficients:
                Estimate Std. Error t value     Pr(>|t|)
```

```
(Intercept)      24.2944     0.2685  90.466      < 2e-16 ***
within14TRUE     34.8186     0.2136 163.011      < 2e-16 ***
airlineAirAsia    2.3754     0.5602   4.240 0.0000227248 ***
airlineGO_FIRST  -0.5214     0.3307  -1.577        0.115
airlineIndigo     3.0133     0.3163   9.526      < 2e-16 ***
airlineSpiceJet   0.2107     0.3982   0.529        0.597
airlineVistara    1.7197     0.3155   5.452 0.0000000523 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.583 on 5163 degrees of freedom
Multiple R-squared:  0.8396,    Adjusted R-squared:  0.8394
F-statistic:  4505 on 6 and 5163 DF,  p-value: < 2.2e-16
```

* Question 11

Test the joint usefulness of the airline dummies in this model (variables 2-6) at the 5% level.

Choose one of the options in *italics* and fill in the blanks.

- The null hypothesis is that *all/at least one/none* of $\beta_j =$ ______ for j from ______ to ______.
- The alternative hypothesis is that *all/at least one/none* of $\beta_j \neq$ ______ for the same j.

The test statistic is of the form:

$$\frac{\frac{SSE_r - a}{g-k}}{\frac{a}{n-k-1}}$$

What is the numeric value of a in the test? ______

What is the numeric value of the test statistic? ______

What is the critical value? ______

Which of the 4 options below is the correct conclusion from the test?

- Reject H0. The variables are useful additions to the model.
- Reject H0. The variables are not useful additions to the model.
- Don't reject H0. The variables are useful additions to the model.
- Don't reject H0. The variables are not useful additions to the model.

Answer:

We can perform the partial F test by using the `anova()` function with the reduced model (price explained by `within14` only, which is model `m2` above) and the complete model (price explained by `within14` and `airline`, which is model `m4` above).

```r
anova(m2, m4)
```

```
Analysis of Variance Table

Model 1: price ~ within14
Model 2: price ~ within14 + airline
  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1   5168 233510
2   5163 223772  5    9738.7 44.939 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We now have everything to answer the questions:

- H_0: All $\beta_j = 0$ for j from 2 to 6.
- H_1: At least one $\beta_j \neq 0$ for the same j.
- a is SSE_c, the sum of squared errors in the complete model. This is 223772 from the `anova()` output.
- Value of the test statistic: 44.939.
- Critical value with $k-g=5$ numerator and $n-k-1=5163$ denominator degrees of freedom:

```r
qf(0.95, 5, 5163)
```

```
[1] 2.215831
```

- Conclusion: Reject H0. The variables are useful additions to the model (*reason:* the test statistic 44.939 is larger than the critical value of 2.215831).

Model 5

Estimate a regression model explaining `price` with:

- A dummy variable that equals 1 if `days_left` is less than or equal to 14 and 0 otherwise.
- A dummy variable that equals 1 if `source_city` is Mumbai and 0 otherwise.
- The interaction of the above two variables.

Note: Using the variable `source_city` as is will automatically create a dummy variable that equals 1 if the source city is Mumbai.

If you estimated the model correctly, your estimated regression intercept should equal 26.2802.

```r
m5 <- lm(price ~ within14 * source_city, data = df)
summary(m5)
```

```
Call:
lm(formula = price ~ within14 * source_city, data = df)

Residuals:
    Min      1Q  Median      3Q     Max 
 -3.910  -2.171  -1.945   0.200 181.645 

Coefficients:
                               Estimate Std. Error t value        Pr(>|t|)    
(Intercept)                     26.2802     0.1518 173.121         < 2e-16 ***
within14TRUE                    34.3006     0.3038 112.891         < 2e-16 ***
source_cityMumbai               -1.3947     0.2146  -6.499 0.0000000000883 ***
within14TRUE:source_cityMumbai   0.9109     0.4318   2.109           0.035 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.695 on 5166 degrees of freedom
Multiple R-squared:  0.8341,	Adjusted R-squared:  0.834 
F-statistic:  8655 on 3 and 5166 DF,  p-value: < 2.2e-16
```

- Question 12

Which variables are individually statistically significant at the 1% level?

Answer:

We can check the p-values from the summary output. The first two variables are significant at the 1% level but the interaction term is not. We can see this based on the number of stars (at least 2 stars means significant at the 1% level).

- Question 13

For flights departing from Delhi, on average how much more expensive are flights departing within 14 days compared to more than 14 days away (according to your estimated model)?

Answer:

If we let x_{i1} denote if the flight departs within 14 days and x_{i2} denote if the flight departs from Mumbai, we can write our model as:

$$\mathbb{E}\left[price_i|x_{i1},x_{i2}\right]=\beta_0+\beta_1x_{i1}+\beta_2x_{i2}+\beta_3x_{i1}x_{i2}$$

For flights departing from Delhi, we have:

$$\mathbb{E}\left[price_i|x_{i1},x_{i2}=0\right]=\beta_0+\beta_1x_{i1}$$

For flights within 14 days from Delhi:

$$\mathbb{E}\left[price_i | x_{i1} = 1, x_{i2} = 0\right] = \beta_0 + \beta_1$$

For flights more than 14 days away from Delhi:

$$\mathbb{E}\left[price_i | x_{i1} = 0, x_{i2} = 0\right] = \beta_0$$

The difference is β_1. So the answer is our estimate of this:

```
coef(m5)[2]
```

```
within14TRUE
    34.30058
```

* Question 14

For flights departing from Mumbai, on average how much more expensive are flights departing within 14 days compared to more than 14 days away (according to your estimated model)?

Answer:

This uses the notation in the answer from the previous question.

For flights departing from Mumbai, we have:

$$\mathbb{E}\left[price_i | x_{i1}, x_{i2} = 1\right] = \beta_0 + \beta_1 x_{i1} + \beta_2 + \beta_3 x_{i1}$$

We can simplify this to:

$$\mathbb{E}\left[price_i | x_{i1}, x_{i2} = 1\right] = \beta_0 + \beta_2 + (\beta_1 + \beta_3) x_{i1}$$

For flights within 14 days from Delhi:

$$\mathbb{E}\left[price_i | x_{i1} = 1, x_{i2} = 1\right] = \beta_0 + \beta_2 + \beta_1 + \beta_3$$

For flights more than 14 days away from Delhi:

$$\mathbb{E}\left[price_i | x_{i1} = 0, x_{i2} = 1\right] = \beta_0 + \beta_2$$

The difference is $\beta_1 + \beta_3$. So the answer is our estimate of these terms added together:

```
coef(m5)[2] + coef(m5)[4]
```

```
within14TRUE
    35.21146
```

* Question 15

Your supervisor asks you to add another dummy variable to this model for flights departing from Delhi. You don't think this is a great idea. Why?

Choose the option from the list below which best describes why we shouldn't add this dummy to the model.

- Including this dummy lowers the R^2 of the model.
- Including this dummy causes a strict multicolinearity problem.
- Including this dummy lowers the significance level of the other variables.
- Including this dummy will make the model suffer from heteroskedasticity.
- Including this dummy will make the model suffer from serial correlation.

Answer: Including this dummy causes a strict multicolinearity problem.