# Externalities and Public Goods

230333 Microeconomics 3 (CentER) – Part II
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# Introduction

- Here we consider two types of *market failures*, where competitive equilibria may fail to be Pareto efficient.
- If a consumer's preferences depends on the consumption of other consumers and/or a firm's output depends on other firms' production, there may be *externalities*.
  - These consumers and firms do not internalize the preferences of others, and thus competitive equilibria may not be efficient.
- Non-rival and non-excludable goods are called *public goods*.
  - Under private provision, these will be underprovided due to the *free-rider problem*.
- Rival and non-excludable goods are *common goods*.
  - This can lead to the *tragedy of the commons* (to be studied in the assignment).

# Definition of an Externality

**Definition**

An *externality* is present whenever the well-being of a consumer or the production possibilities of a firm are directly affected by the actions of another agent in the economy.

Examples:

- Positive consumption externality: Getting the flu vaccine.
- Negative consumption externality: Smoking in a restaurant.
- Positive production externality: Apiculture (on horticulture).
- Negative production externality: Pollution from a factory.

# Simple Bilateral Externalities

- We consider a partial equilibrium model and two consumers out of the overall economy with wealth levels $w_i$, $i = 1, 2$.
- Each consumer has preferences over the $L$ traded goods *and* some action $h \in \mathbb{R}_+$ taken by consumer 1.
- Consumer $i$'s utility function is $u_i(\mathbf{x}_i, h)$ where $\frac{\partial u_2(\mathbf{x}_i, h)}{\partial h} \neq 0$.
- If $\frac{\partial u_2(\mathbf{x}_2, h)}{\partial h} > 0$, we have a positive consumption externality.
- If $\frac{\partial u_2(\mathbf{x}_2, h)}{\partial h} < 0$, we have a negative consumption externality.

# Simple Bilateral Externalities

- We assume utility $u_i(\boldsymbol{x}_i, h) = g_i(\boldsymbol{x}_{-1i}, h) + x_{i1}$ is quasilinear with respect to the numeraire commodity (good 1).
- Demand for goods $\ell > 1$ is then $\boldsymbol{x}_{-1i}(\boldsymbol{p}, h)$, which is independent of wealth $w_i$.
- Define:
$$\phi_i(\boldsymbol{p}, h) = g_i(\boldsymbol{x}_{-1i}(\boldsymbol{p}, h), h) - \boldsymbol{p} \cdot \boldsymbol{x}_{-1i}(\boldsymbol{p}, h)$$
- Then each consumer's indirect utility function takes the form:
$$\begin{aligned} v_i(\boldsymbol{p}, w_i, h) &= g_i(\boldsymbol{x}_{-1i}(\boldsymbol{p}, h), h) + x_{i1}(\boldsymbol{p}, w_i) \\ &= g_i(\boldsymbol{x}_{-1i}(\boldsymbol{p}, h), h) + w_i - \boldsymbol{p} \cdot \boldsymbol{x}_{-1i}(\boldsymbol{p}, h) \\ &= \phi_i(\boldsymbol{p}, h) + w_i \end{aligned}$$
- We will suppress the argument $\boldsymbol{p}$ because we only consider changes in $h$.

# Competitive Outcome

- $\phi_2'(h) > 0$ if positive externality
- $\phi_2'(h) < 0$ if negative externality.
- Assume $\phi_i''(h) < 0$ for $i = 1, 2$.
- Consumer 1's problem is:

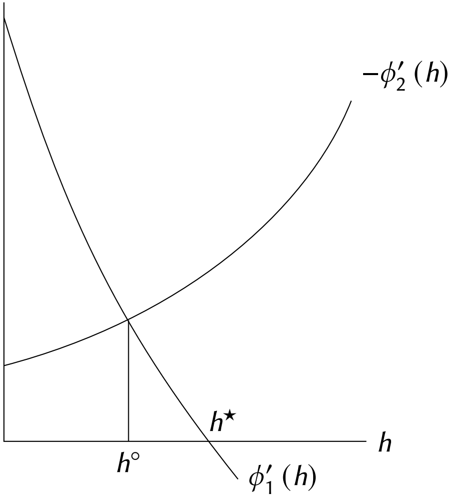
$$\max_{h \geq 0} \phi_1(h) + w_1$$

- In a competitive outcome, consumer 1 optimally sets $h^\star$ such that $\phi_1'\left(h^\star\right) \leq 0$, with equality if $h^\star > 0$.

# Non-Optimality of the Competitive Outcome

- A social planner solves:
$$\max_{h \geq 0} \ \phi_1(h) + \phi_2(h)$$
- The first-order condition is $\phi_1'(h^\circ) \leq -\phi_2'(h^\circ)$, with equality if $h^\circ > 0$.
- In the competitive outcome, $h^\star$ is such that $\phi_1'\left(h^\star\right) \leq 0$, with equality if $h^\star > 0$.
- When externalties are present ($\phi_2'(h) \neq 0$), the competitive outcome is not optimal unless $h^\star = h^\circ = 0$.
- In interior solutions:
  - If there is a negative externality: $h^\star > h^\circ$.
  - If there is a positive externality: $h^\star < h^\circ$.

# Negative Externality



# Pigouvian Taxation On Negative Externalities

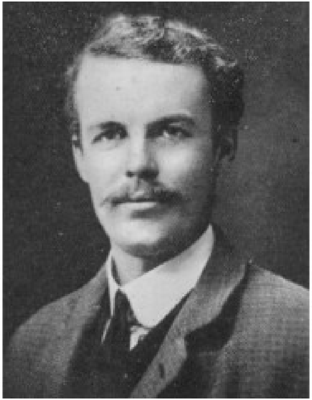
- Suppose there is a tax $t_h$ on each unit of $h$.
- Consumer 1's problem becomes:

$$\max_{h \geq 0} \ \phi_1(h) - t_h h$$

- This has the FOC: $\phi_1'(h) \leq t_h$, with equality if $h > 0$.
- Setting a tax $t_h = -\phi_2'(h^\circ)$ will achieve the optimal outcome.
- This tax makes consumer 1 internalize the externality.

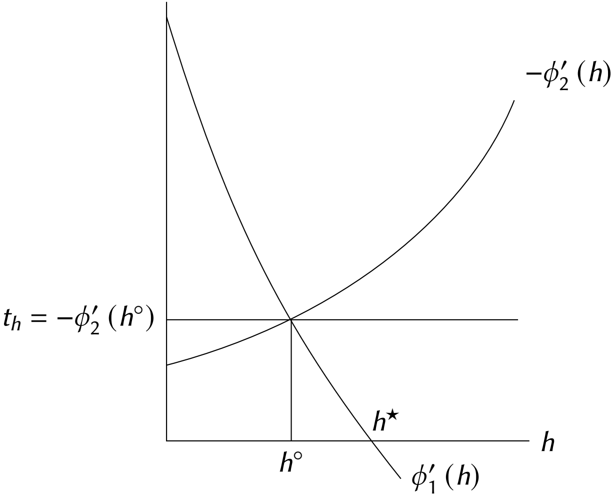
# Arthur Cecil Pigou (1877-1959)

- Born on the Isle of Wight and educated in Cambridge.
- In *The Economics of Welfare*, he introduced the concept of an externality and proposed Pigouvian taxation/subsidies to correct them.



# Negative Externality

Setting a tax $t_h = -\phi_2'(h^\circ)$ will achieve the optimal outcome:



# Pigouvian Taxation On Negative Externalities

- The same result could be achieved by instead by offering a per-unit subsidy for the reduction of $h$ below $h^\star$.
- The consumer's problem is then:

$$\phi_1(h) + s_h\left(h^\star - h\right)$$

- This has the FOC: $\phi_1'(h) \leq s_h$, with equality if $h > 0$.
- $s_h = -\phi_2'(h^\circ)$ will result in the optimal outcome.
- This is equivalent to a tax on $h$ with a lump sum transfer $s_h h^\star$.

# Coase Theorem

- In reality, Pigouvian taxation requires a lot of information.
  - It requires knowledge of all consumers' utility functions with respect to the externality.

**Coase Theorem**

If trade of the externality can occur, then bargaining will lead to an efficient outcome, no matter how property rights are allocated.

- Suppose $\phi_2'(h) < 0$ and we give consumer 2 enforceable property rights for an externality-free environment.
- Consumer 1 must bargain with consumer 2 to be able to consume $h$.

# Coase Theorem

- Assume bargaining is a take-it-or-leave-it offer where consumer 1 pays $T$ in return for the right to consume $h$ units.
- Consumer 2's problem is then:

$$\max_{h \geq 0, T} \; \phi_2(h) + T \text{ subject to } \phi_1(h) - T \geq \phi_1(0)$$

- At the optimum, the constraint binds. Substituting yields the social planner's problem:

$$\max_{h \geq 0} \; \phi_2(h) + \phi_1(h) - \phi_1(0)$$

- If consumer 1 owns the property rights, the same outcome will occur, with instead consumer 2 paying consumer 1 to consume less.
- The distribution of surplus depends on the distribution of property rights and the bargaining procedure.

# Public Goods

**Definition**

A *public good* is a commodity for which use of a unit of the good by one agent does not preclude its use by other agents.

- A public good is *nondepletable* and *nonexcludable*.
- Consider again a market with $L$ traded, private goods and one public good.
- Assume the price or quantity of the public good doesn't affect prices of the other goods (partial equilibrium).
- The derived utility for consumer $i$ is then $\phi_i(x)$, where $\phi_i'(x) > 0$ and $\phi_i''(x) < 0$, $\forall i$ and $\forall x \geq 0$.
- The cost of supplying the public good is $c(q)$, with $c'(q) > 0$ and $c''(q) > 0$, $\forall q \geq 0$.

# Pareto Optimal Allocation of the Public Good

- The social planner solves:

$$\max_{q \geq 0} \sum_{i=1}^{I} \phi_i (q) - c (q)$$

- The FOC is $\sum_{i=1}^{I} \phi_i' (q^\circ) \leq c' (q^\circ)$ with equality if $q^\circ > 0$.
- In an interior solution, the sum of the marginal benefits equals the marginal cost.

# Inefficiency of Private Provision of the Public Good

- Suppose the equilibrium price for the public good is $p^\star$.
- Given $p^\star$, each individual solves:

$$\max_{x_i \geq 0} \; \phi_i \left( x_i + \sum_{k \neq i} x_k^\star \right) - p^\star x_i$$

- The FOC is $\phi_i' \left( x_i^\star + \sum_{k \neq i} x_k^\star \right) \leq p^\star$, with equality if $x_i^\star > 0$.
- The firm's FOC is $p^\star \leq c' \left( q^\star \right)$, with equality if $q^\star > 0$.
- In equilibrium, markets clear so $\sum_{i=1}^{I} x_i^\star = q^\star$

# Inefficiency of Private Provision of the Public Good

- The equilibrium conditions again are:
  - $\phi_i'\left(x_i^\star + \sum_{k\neq i} x_k^\star\right) \leq p^\star$, with equality if $x_i^\star > 0$.
  - $p^\star \leq c'\left(q^\star\right)$, with equality if $q^\star > 0$.
  - $\sum_{i=1}^{I} x_i^\star = q^\star$.
- Therefore we can write:

$$\sum_{i=1}^{I} \mathbb{1}\left\{x_i^\star > 0\right\}\left[\phi_i'\left(q^\star\right) - c'\left(q^\star\right)\right] = 0$$

- If $I > 1$ and if $x_i^\star > 0$ for at least one $i$:

$$\sum_{i=1}^{I} \phi_i'\left(q^\star\right) > c'\left(q^\star\right)$$

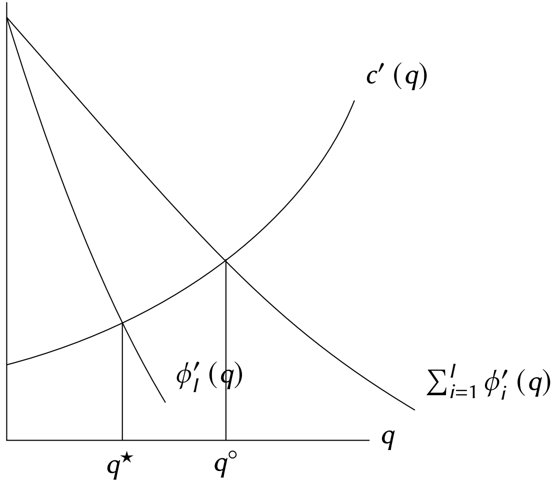
- If $q^\circ > 0$, then the planner sets $\sum_{i=1}^{I} \phi_i'\left(q^\circ\right) = c'\left(q^\circ\right)$.
- Since $\sum_{i=1}^{I} \phi_i'\left(q\right) - c'\left(q\right)$ is strictly decreasing in $q$, $q^\star < q^\circ$.
- There is underprovision of the public good.

# Free-Rider Problem

- The consumption of the public good by one consumer has a positive externality on other consumers.
- Each consumer's incentive is to enjoy the benefits provided by others but not provide it themselves.
  - This is the *free-rider problem*.
- Suppose $\phi_1'(x) < \cdots < \phi_I'(x), \forall x \geq 0$.
- Then the FOC can only hold with equality for one consumer.
- $q^\star$ in this case satisfies $\phi_I'\left(q^\star\right) = c'\left(q^\star\right)$ if $\phi_I'(0) > c'(0)$.

# Free-Rider Problem

If $\phi_1'(x) < \cdots < \phi_I'(x), \forall x \geq 0$ and $\phi_I'(0) > c'(0)$:



# Solving the Free-Rider Problem

- Typically public goods are instead provided by the government.
- A price-based solution involves subsidizing the purchase of the public good with $s_i = \sum_{k \neq i} \phi_k' (q^\circ)$ for each $i$.
- Consumer $i$'s problem becomes:

$$\max_{x_i \geq 0} \phi_i \left( x_i + \sum_{k \neq i} x_k \right) + \underbrace{\sum_{k \neq i} \phi_k' (q^\circ)}_{=s_i} x_i - p x_i$$

  which, if $q^\circ > 0$, has first-order conditions:

$$\phi_i' (q^\star) + \sum_{k \neq i} \phi_k' (q^\circ) - \underbrace{p}_{=c'(q^\star)} = 0$$

- This is the same as from the planner's problem, so $q^\star = q^\circ$.

# Multilateral Externalities

- Often externalities are felt by multiple parties.
- We will study two types:
  - *Depletable externalities*: one consumer's experience of the externality reduces the amount needed to be felt by others.
    - For example, dumping waste on private property.
  - *Nondepletable externalities*: All consumers experience the same total level of the externality.
    - For example, air pollution.

# Multilateral Externalities

- There are $J$ firms generating the externalities and $I$ consumers.
- A firm's derived profit function as a function of its generated externality $h_j \geq 0$ is $\pi_j(h_j)$ with $\pi_j''(h_j) < 0$.
- Firms choose $h_j^\star$ satisfying $\pi_j'\left(h_j^\star\right) \leq 0$, with equality if $h_j^\star > 0$.
- Consumers have quasilinear preferences over the $L$ goods and their perceived externality.
- A consumer's derived utility function is $\phi_i\left(\widetilde{h}_i\right)$, with $\phi_i''\left(\widetilde{h}_i\right) < 0$.
- With depletable externalities, $\sum_{i=1}^{I} \widetilde{h}_i = \sum_{j=1}^{J} h_j$.
- With nondepletable externalities, $\widetilde{h}_i = \sum_{j=1}^{J} h_j$ for all $i$.

# Depletable Externalities

- The Pareto efficient allocation is the argument that solves:

$$\max_{\substack{(h_1,\ldots,h_J)\geq\mathbf{0},\\ (\widetilde{h}_1,\ldots,\widetilde{h}_I)\geq\mathbf{0}}} \sum_{i=1}^{I}\phi_i\left(\widetilde{h}_i\right)+\sum_{j=1}^{J}\pi_j\left(h_j\right) \text{ subject to } \sum_{j=1}^{J}h_j=\sum_{i=1}^{I}\widetilde{h}_i$$

- The FOCs are $\phi_i'\left(\widetilde{h}_i^\circ\right)\leq\mu$, with equality if $\widetilde{h}_i^\circ>0$, $\forall i$ and $\mu\leq-\pi_j'\left(h_j^\circ\right)$, with equality if $h_j^\circ>0$, $\forall j$.
- These are analogous to the conditions in the First Welfare Theorem:
  - If markets are competitive and property rights over the externality are well-defined and enforceable, then the Pareto optimal allocation can be achieved.

# Nondepletable Externalities

- With nondepletable externalities, the Pareto efficient allocation is the maximizer in the problem:

$$\max_{(h_1,\dots,h_J)\geq \mathbf{0}} \sum_{i=1}^{I} \phi_i \left( \sum_{j=1}^{J} h_j \right) + \sum_{j=1}^{J} \pi_j \left( h_j \right)$$

- The FOCs are $\sum_{i=1}^{I} \phi_i' \left( \sum_{j=1}^{J} h_j^\circ \right) \leq -\pi_j' \left( h_j^\circ \right)$, with equality if $h_j^\circ > 0$, for all $j$.
- In the competitive outcome, each firm chooses $h_j^\star$ to satisfy $\pi_j' \left( h_j^\star \right) \leq 0$, with equality if $h_j^\star > 0$.
- With a Pigouvian tax of $t_h = -\sum_{i=1}^{I} \phi_i' \left( \sum_{j=1}^{J} h_j^\circ \right)$ on each unit of the externality, however, we can achieve the Pareto optimal outcome.

# Tradeable Permits

- The social planner can also achieve the optimal level of the externality $h^\circ = \sum_{j=1}^{J} h_j^\circ$ with tradeable permits.
- The social planner distributes permits to each firm to produce up to $\bar{h}_j$ units of the externality, where $\sum_{j=1}^{J} \bar{h}_j = h^\circ$.
- Firms are allowed to buy and sell the externality in a centralized market at price $p_h^\star$.
- Each firm will choose $h_j$ to solve $\max\limits_{h_j \geq 0} \; \pi_j\left(h_j\right) - p_h^\star\left(h_j - \bar{h}_j\right)$.
- The FOCs are $\pi_j'\left(h_j^\star\right) \leq p_h^\star$, with equality if $h_j^\star > 0$.
- By market clearing, we have $\sum_{j=1}^{J} h_j^\star = h^\circ$.
- The equilibrium price will be $p_h^\star = -\sum_{i=1}^{I} \phi_i'\left(h^\circ\right)$.
- Here, the planner only needs to know $h^\circ$, and not each individual's utility function or each firm's profit function.