

The Edgeworth Box

230333 Microeconomics 3 (CentER) – Part II
Tilburg University

Introduction

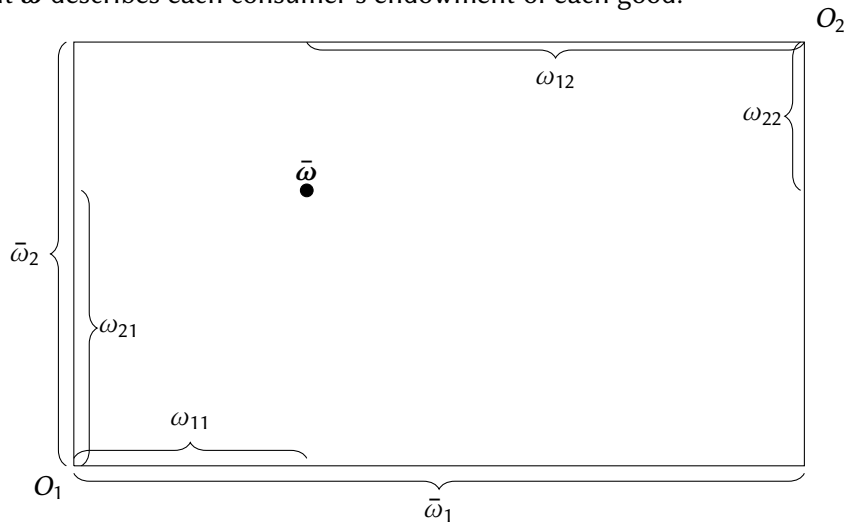
- ▶ We will now study how prices are determined in general equilibrium in an economy with 2 goods and 2 consumers.
- ▶ We will first study this graphically using a useful tool called the *Edgeworth Box*.
- ▶ Francis Ysidro Edgeworth was born in Edgeworthstown, Ireland in 1845.

Edgeworth Box: $I = L = 2$ with Pure Exchange

- ▶ Two consumers $i = 1, 2$ and two commodities $\ell = 1, 2$.
- ▶ Consumer i has preferences \succeq_i over bundles $\mathbf{x}_i = (x_{1i}, x_{2i})$.
- ▶ Consumer i has an endowment $\boldsymbol{\omega}_i = (\omega_{1i}, \omega_{2i})$.
- ▶ The total endowment in the economy is $\bar{\boldsymbol{\omega}} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$.
- ▶ There is one firm with a production set $Y_1 = \mathbb{R}_-^2$ (*free disposal*).
- ▶ An allocation $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2) \in \mathbb{R}_+^4$ is *feasible* if $x_{\ell 1} + x_{\ell 2} \leq \bar{\omega}_\ell, \forall \ell = 1, 2$.
- ▶ If $x_{\ell 1} + x_{\ell 2} = \bar{\omega}_\ell, \forall \ell$, an allocation is *nonwasteful* (no disposal).
- ▶ All nonwasteful allocations can be represented in an *Edgeworth box*.

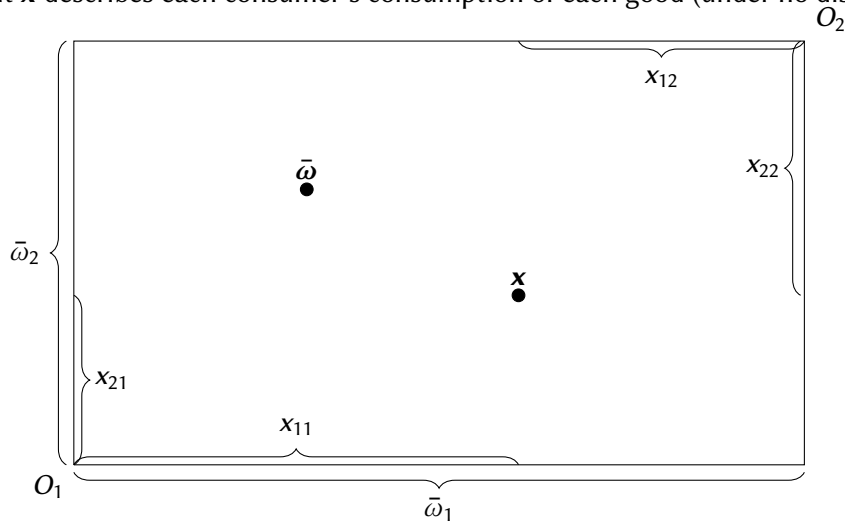
Edgeworth Box: Endowments

The point $\bar{\omega}$ describes each consumer's endowment of each good.



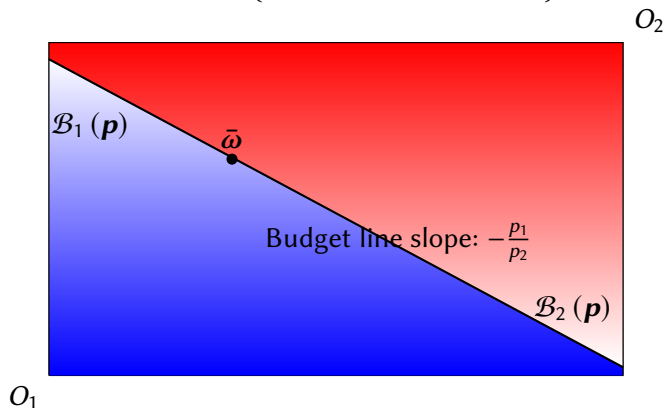
Edgeworth Box: Allocations

The point \mathbf{x} describes each consumer's consumption of each good (under no disposal):



Edgeworth Box: Budget Sets

A consumer's budget set is: $\mathcal{B}_i(\mathbf{p}) = \{\mathbf{x}_i \in \mathbb{R}_+^2 : \mathbf{p} \cdot \mathbf{x}_i \leq \mathbf{p} \cdot \boldsymbol{\omega}_i\}$

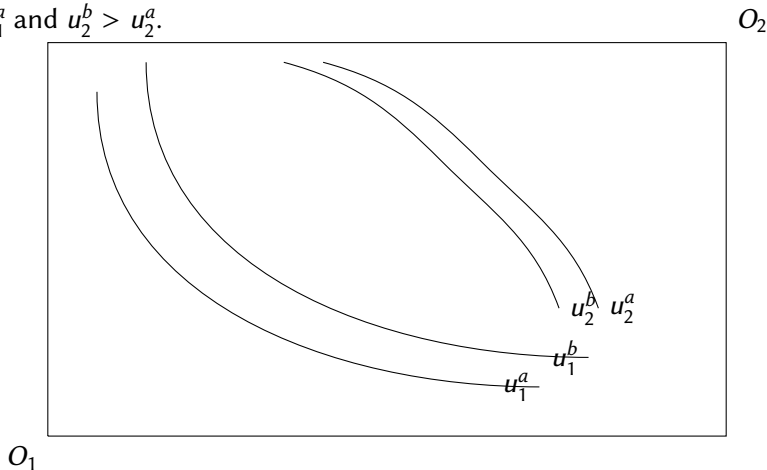


Only bundles on the budget line are affordable to both consumers simultaneously.

Edgeworth Box: Indifference Curves

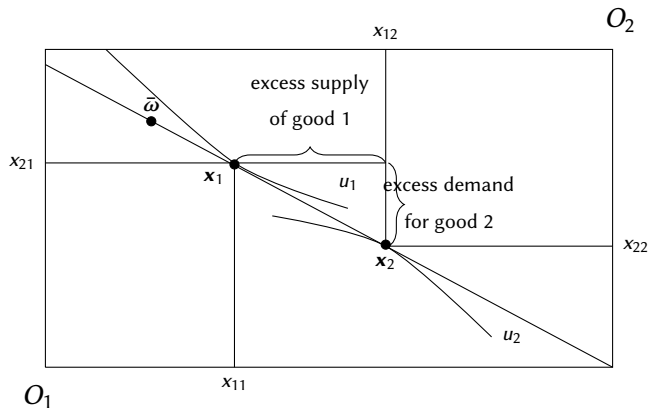
Example with strongly monotone, continuous and strictly convex preferences:

- ▶ Consumer 1 prefers bundles towards the north east.
- ▶ Consumer 2 prefers bundles towards the south west.
- ▶ $u_1^b > u_1^a$ and $u_2^b > u_2^a$.



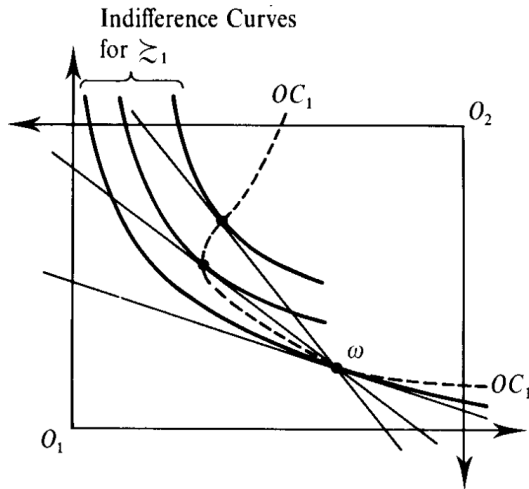
Edgeworth Box: Demand

- ▶ Consumer 1 is a net demander of good 1 and a net supplier of good 2.
- ▶ Consumer 2 is a net supplier of good 1 and a net demander of good 2.
- ▶ However, markets do not clear at these prices, as $x_{11} + x_{12} < \bar{\omega}_1$ and $x_{21} + x_{22} > \bar{\omega}_2$.



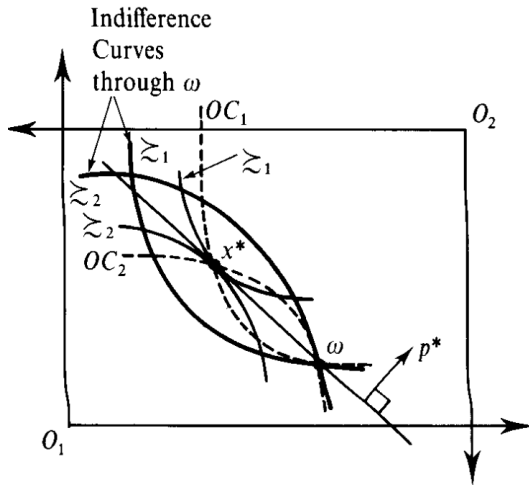
Edgeworth Box: Offer Curve

A consumer's *offer curve* traces out the consumer's demand at each price vector \mathbf{p} . Since ω_i is always affordable, it lies in the upper contour set of ω_i .



Edgeworth Box: Intersection of Offer Curves

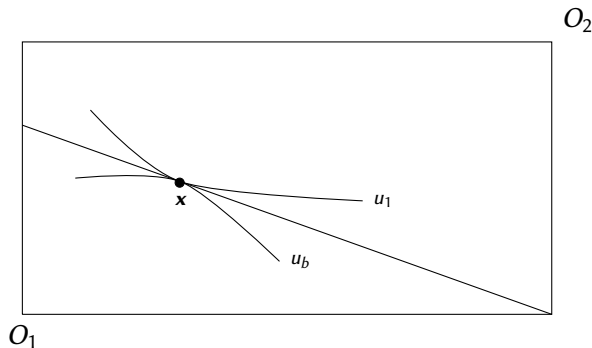
When both consumers' offer curves intersect, the total amount demanded equals the total endowment for each good: the market clears.



Edgeworth Box: Equilibrium

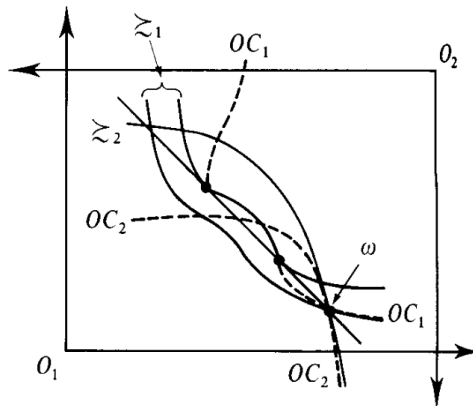
Definition

A *Walrasian equilibrium* for an Edgeworth box economy is a price vector \mathbf{p}^* and an allocation $\mathbf{x}^* = (\mathbf{x}_1^*, \mathbf{x}_2^*)$ in the Edgeworth box such that for $i = 1, 2$, $\mathbf{x}_i^* \succeq_i \mathbf{x}'_i$ for all $\mathbf{x}'_i \in \mathcal{B}_i(\mathbf{p}^*)$.



Nonexistence of Equilibria: Nonconvex Preferences

- Equilibria do not always exist:



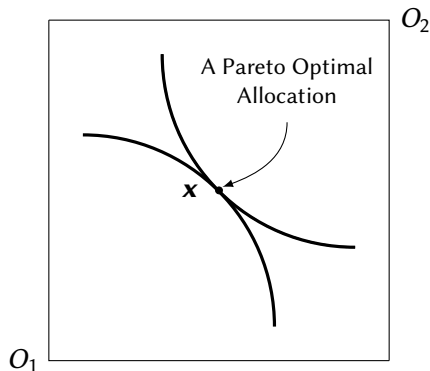
Source: Mas-Colell, A., et al. (1995) *Microeconomic Theory*

- The consumers' offer curves never intersect at any point where $\mathbf{x}_i \neq \boldsymbol{\omega}_i$.
- $\mathbf{x}_i = \boldsymbol{\omega}_i$ is also not an equilibrium.

Pareto Optimality

Definition

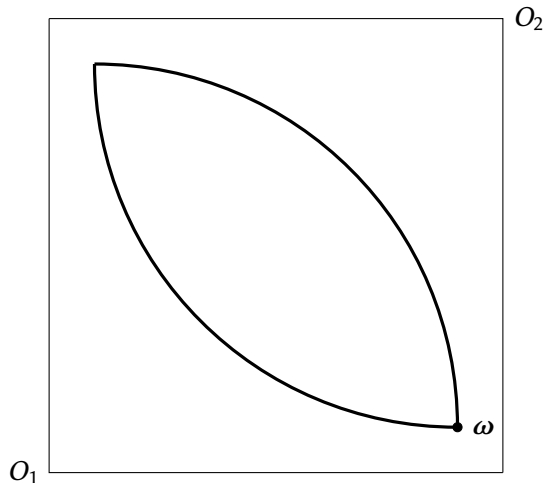
An allocation \mathbf{x} in the Edgeworth box is *Pareto optimal* if there is no other allocation \mathbf{x}' in the Edgeworth box with $\mathbf{x}'_i \geq \mathbf{x}_i$ for $i = 1, 2$ and $\mathbf{x}'_i > \mathbf{x}_i$ for some i .



With smooth indifference curves, interior Pareto optimal allocations occur at the tangency.

The Lens of Pareto Improvements on ω

$$\{(\mathbf{x}_1, \mathbf{x}_2) \in \mathbb{R}_+^4 : \mathbf{x}_1 \succeq_1 \omega_1 \text{ and } \mathbf{x}_2 \succeq_2 \omega_2 \text{ and } \mathbf{x}_1 + \mathbf{x}_2 = \omega_1 + \omega_2\}$$



The interior of the lens are all Pareto improvements on ω .

The Pareto Set

- ▶ The set of Pareto optimal allocations is called the Pareto set.
- ▶ In the pure exchange Edgeworth box, the Pareto set is:

$$\mathcal{P} = \left\{ (\mathbf{x}_1, \mathbf{x}_2) \in \mathbb{R}_+^4 : \nexists \mathbf{x}'_1, \mathbf{x}'_2 \text{ satisfying } \mathbf{x}'_1 + \mathbf{x}'_2 \leq \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2 \right. \\ \left. \text{and } \mathbf{x}'_i \succeq_i \mathbf{x}_i \forall i = 1, 2 \text{ and } \mathbf{x}'_i \succ_i \mathbf{x}_i \text{ for some } i \right\}$$

- ▶ With well-behaved preferences, the union of the locus of tangencies of the indifference curves and the origins make up the Pareto set.

The Contract Curve

- ▶ The Pareto set is the red and blue line.
- ▶ The contract curve, CC , is a subset of the Pareto set where the allocations are at least as good as the endowment for each consumer (red line):

$$CC = \{(\mathbf{x}_1, \mathbf{x}_2) \in \mathbb{R}_+^4 : \mathbf{x}_1 \succeq_1 \boldsymbol{\omega}_1 \text{ and } \mathbf{x}_2 \succeq_2 \boldsymbol{\omega}_2\} \cap \mathcal{P}$$

