

# Statistics 2 2025/26 Exam

## Introduction

You work for an airline considering opening a new route between the cities Dehli and Mumbai in India. You have been tasked with analyzing flight pricing among rival airlines on this route. You go to the websites of all the airlines that fly direct between those two cities and take note of the price of an economy class ticket for different dates: if you were to fly tomorrow, 2 days later, 3 days later and so on, until 49 days later.

You also take note of the airline, source city and destination city. You put all of these data into a spreadsheet. Here is a link to download the spreadsheet you created: [dehli\\_mumbai\\_flights.csv](#).

The definitions of all the variables are as follows:

- **flight**: The flight number.
- **airline**: The name of the airline service running the flight.
- **source\_city**: The city the flight is leaving from (either Dehli or Mumbai).
- **destination\_city**: The city the flight is flying to (either Dehli or Mumbai).
- **days\_left**: The number of days until the flight departs. If **days\_left** equals 1, it means the flight is leaving tomorrow. If it equals 2, it means the flight is leaving in 2 days, etc.
- **price**: The price of the flight converted to euro.

Your supervisor asks you to answer the following questions using your data.

## Question 1

What is the sample correlation between **days\_left** and **price**?

## Model 1

Estimate a linear regression model with **price** as the dependent variable and **days\_left** as the independent variable.

If you estimated the model correctly, you should have an estimated intercept of 56.03504.

### Question 2

What is the sample regression slope?

### Question 3

Use the model to estimate the mean price of flights departing in 50 days (i.e. `days_left` equal to 50).

Also obtain a 95% confidence interval for the mean price of flights departing in 50 days.

- Estimate of the mean price of flights departing in 50 days: \_\_\_\_\_
- Confidence interval lower bound: \_\_\_\_\_
- Confidence interval upper bound: \_\_\_\_\_

### Question 4

Plot the residuals (on the vertical axis) against `days_left` (on the horizontal axis). Choose the answer below which best interprets what this plot says about our model assumptions.

- All residuals are positive when `days_left` is less than 15. This evidence suggests that the assumption  $\mathbb{E}[\varepsilon_i|x_i] = 0$  for all  $x_i$  is satisfied.
- All residuals are negative when `days_left` is less than 15. This evidence suggests that the assumption  $\mathbb{E}[\varepsilon_i|x_i] = 0$  for all  $x_i$  is satisfied.
- All residuals are positive when `days_left` is less than 15. This evidence suggests that the assumption  $\mathbb{E}[\varepsilon_i|x_i] = 0$  for all  $x_i$  is violated.
- All residuals are negative when `days_left` is less than 15. This evidence suggests that the assumption  $\mathbb{E}[\varepsilon_i|x_i] = 0$  for all  $x_i$  is violated.

### Question 5

Perform a formal test for heteroskedasticity by regressing the square of the residuals on `days_left`.

Use a 5% significance level.

In this test, the null hypothesis is that the model exhibits which of the following:

- Homoskedasticity
- Heteroskedasticity
- Multiskedasticity
- Paraskedasticity

What is the  $p$ -value from this test? \_\_\_\_\_

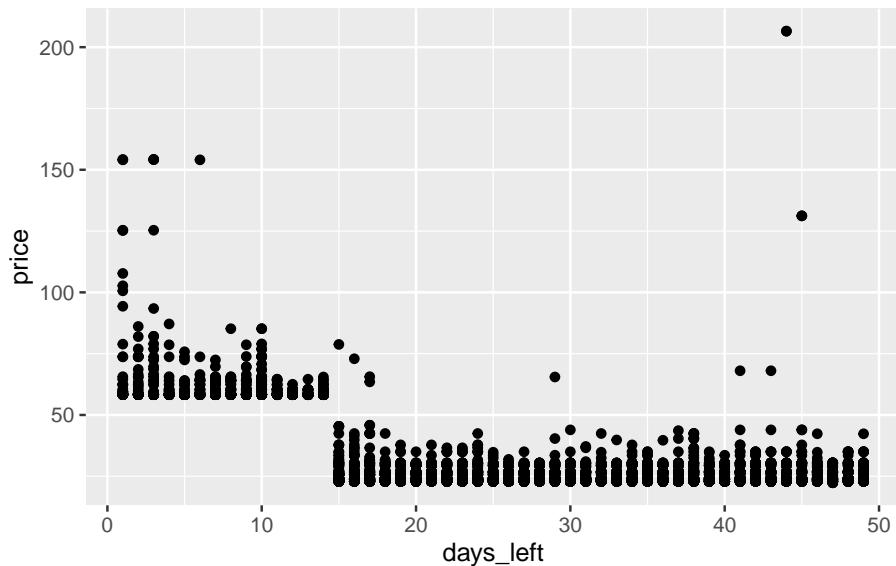
What is the conclusion? Choose one of the answers below:

- The model exhibits heteroskedasticity. Therefore the standard errors are not reliable.

- The model exhibits homoskedasticity. Therefore the standard errors are not reliable.
- The model exhibits heteroskedasticity. Therefore the standard errors are reliable.
- The model exhibits homoskedasticity. Therefore the standard errors are reliable.

## Model 2

A scatter plot of `days_left` and `price` is shown below:



Based on this, your supervisor suggests an alternative way of modeling `price` as a function of `days_left`. Your supervisor suggests to use a dummy variable that equals 1 if the flight is within 14 days (i.e. `days_left`  $\leq 14$ ) and 0 if the flight is more than 14 days away (i.e. `days_left`  $> 14$ ).

Create this variable with the command:

```
df$within14 <- df$days_left <= 14
```

Use this variable to estimate a linear regression model with `price` as the dependent variable and `within14` as the independent variable.

If you estimated the model correctly, your estimated intercept should equal 25.5823.

## Question 6

Compare the sum of squares due to regression ( $SSR$ ) between Model 1 (using

`days_left`) and Model 2 (using `within14`). Choose the answer below which best describes what you learn from this.

- Model 2 has a **higher  $SSR$**  than Model 1 and therefore Model 2 is preferred.
- Model 2 has a **lower  $SSR$**  than Model 1 and therefore Model 2 is preferred.
- Model 2 has a **higher  $SSR$**  than Model 1 and therefore Model 2 is **not** preferred.
- Model 2 has a **lower  $SSR$**  than Model 1 and therefore Model 2 is **not** preferred.
- The two models use different **independent** variables, and therefore we can't compare their  $SSRs$  in a meaningful way.
- The two models use different **dependent** variables, and therefore we can't compare their  $SSRs$  in a meaningful way.

### Question 7

Obtain a 95% confidence interval for the coefficient in front of the variable `within14`. Report the lower and upper bound.

### Model 3

Your supervisor is also interested in the average price differences between airlines.

To study this, estimate a linear regression model with `price` as the dependent variable and dummy variables for the airlines as the independent variables. Use "`Air_India`" as the base category, so the included variables should be dummy variables for "`AirAsia`", "`Go_FIRST`", "`Indigo`", "`SpiceJet`" and "`Vistara`".

In R, you can estimate this model using the `airline` variable as is (i.e. using the formula `price ~ airline`). It is not necessary to create the individual dummy variables. "`Air_India`" will automatically be chosen as the base category using this approach.

If you estimated the model correctly, the estimated regression intercept should equal 33.6899.

### Question 8

The estimated intercept of 33.6899 means that the average price of which airline equal to €33.68?

- Air India
- AirAsia
- Go First
- Indigo
- SpiceJet
- Vistara

### Question 9

According to the model, what is the average price of SpiceJet flights in the data?

### Question 10

Use the model to test the following claim at the 5% level:

“GO FIRST flights are more than €2 cheaper compared to Air India”

Perform this test by answering the questions below.

*Note:* If you estimated your regression model using the variable `airline` as is, and assigned it to the name `m3`, then you can extract the coefficient for GO FIRST with the following R command:

```
b_2 <- coef(summary(m3))["airlineGO_FIRST", "Estimate"]
```

- What is the null hypothesis?  $\beta_2 < / \leq / > / \geq / = / \neq$  \_\_\_\_\_  
(choose one comparison operator and fill in a value in the blank).
- What is the alternative hypothesis?  $\beta_2 < / \leq / > / \geq / = / \neq$  \_\_\_\_\_  
(choose one comparison operator and fill in a value in the blank).
- Under the null hypothesis, the test statistic  $T = (B_2 - b)/S_{B_2}$ , where  $b$  is the hinge, follows a  $t$  distribution with how many degrees of freedom?  
\_\_\_\_\_
- What is the value of the test statistic? \_\_\_\_\_
- What is the associated  $p$ -value? \_\_\_\_\_
- What is your conclusion? Choose an option below:
  - Reject H0: There is sufficient evidence for the claim.
  - Reject H0: There is not sufficient evidence for the claim.
  - Don't reject H0: There is sufficient evidence for the claim.
  - Don't reject H0: There is not sufficient evidence for the claim.

### Model 4

Estimate a model explaining `price` with the `within14` dummy variable AND dummy variables for the airlines (using "Air\_India" as the base category).

Like with the previous model, you can add these airline dummies to your model using the `airline` variable as is (which also automatically chooses "Air\_India" as the base category). You simply use the formula `price ~ within14 + airline`.

If you estimated the model correctly your estimated intercept should equal 24.2944.

### Question 11

Test the joint usefulness of the airline dummies in this model (variables 2-6) at the 5% level.

Choose one of the options in *italics* and fill in the blanks.

- The null hypothesis is that *all/at least one/none* of  $\beta_j = \underline{\hspace{2cm}}$  for  $j$  from  $\underline{\hspace{2cm}}$  to  $\underline{\hspace{2cm}}$ .
- The alternative hypothesis is that *all/at least one/none* of  $\beta_j \neq \underline{\hspace{2cm}}$  for the same  $j$ .

The test statistic is of the form:

$$\frac{\frac{SSE_r - a}{g-k}}{\frac{a}{n-k-1}}$$

What is the numeric value of  $a$  in the test?  $\underline{\hspace{2cm}}$

What is the numeric value of the test statistic?  $\underline{\hspace{2cm}}$

What is the critical value?  $\underline{\hspace{2cm}}$

Which of the 4 options below is the correct conclusion from the test?

- Reject H<sub>0</sub>. The variables are useful additions to the model.
- Reject H<sub>0</sub>. The variables are not useful additions to the model.
- Don't reject H<sub>0</sub>. The variables are useful additions to the model.
- Don't reject H<sub>0</sub>. The variables are not useful additions to the model.

### Model 5

Estimate a regression model explaining `price` with:

- A dummy variable that equals 1 if `days_left` is less than or equal to 14 and 0 otherwise.
- A dummy variable that equals 1 if `source_city` is Mumbai and 0 otherwise.
- The interaction of the above two variables.

*Note:* Using the variable `source_city` as is will automatically create a dummy variable that equals 1 if the source city is Mumbai.

If you estimated the model correctly, your estimated regression intercept should equal 26.2802.

### Question 12

Which variables are individually statistically significant at the 1% level?

**Question 13**

For flights departing from Delhi, on average how much more expensive are flights departing within 14 days compared to more than 14 days away (according to your estimated model)?

**Question 14**

For flights departing from Mumbai, on average how much more expensive are flights departing within 14 days compared to more than 14 days away (according to your estimated model)?

**Question 15**

Your supervisor asks you to add another dummy variable to this model for flights departing from Delhi. You don't think this is a great idea. Why?

Choose the option from the list below which best describes why we shouldn't add this dummy to the model.

- Including this dummy lowers the  $R^2$  of the model.
- Including this dummy causes a strict multicollinearity problem.
- Including this dummy lowers the significance level of the other variables.
- Including this dummy will make the model suffer from heteroskedasticity.
- Including this dummy will make the model suffer from serial correlation.