



Fixed Costs and Irreversibility

Investment, Finance and Asset Prices ECON5068

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Lecture Overview

- **Motivation - Inaction**
- **Evidence**
- **Fixed Costs of Adjusting**
- **Irreversibility/Specificity of Installation and “Stranded Assets”**
- Reading:
 1. Cooper and Haltiwanger (2006),
On the Nature of Capital Adjustment Costs,
Review of Economic Studies.

Other Costs of Investing

- key assumption in Q model is that **adjustment costs are convex**.
- For example, $AC_t = \frac{\phi}{2} I_t^2$, a quadratic function is a common assumption.
- This assumption implies that the **investment rule is continuous and smooth**
- **Costs change smoothly with the size of investment**
 1. Since the adjustment function is convex, firms want to avoid large changes in investment as the cost rises at an increasing rate.
- **Is this the only cost faced by firms when growing their asset base?**

Evidence in the data

Costs Firms Face

- Think about investment at a factory or plant, expanding capacity
- In addition to the standard convex cost (a growing pains/tiredness proxy), firms/plants face:
 1. Some projects are **very hard to get started**. Fixed cost of any action
 2. **Disruption** costs from replacing existing capital.
 3. Costly **learning** as the structure of production might have changed.
 4. Delivery **lags** and **time to build delays**.
 5. **Irreversibility** of projects due to lack of secondary market for capital goods. Once installed, there is no turning back.

Lumpiness - Evidence from Data

- Micro evidence from plant-level data indicate investment is **lumpy**.

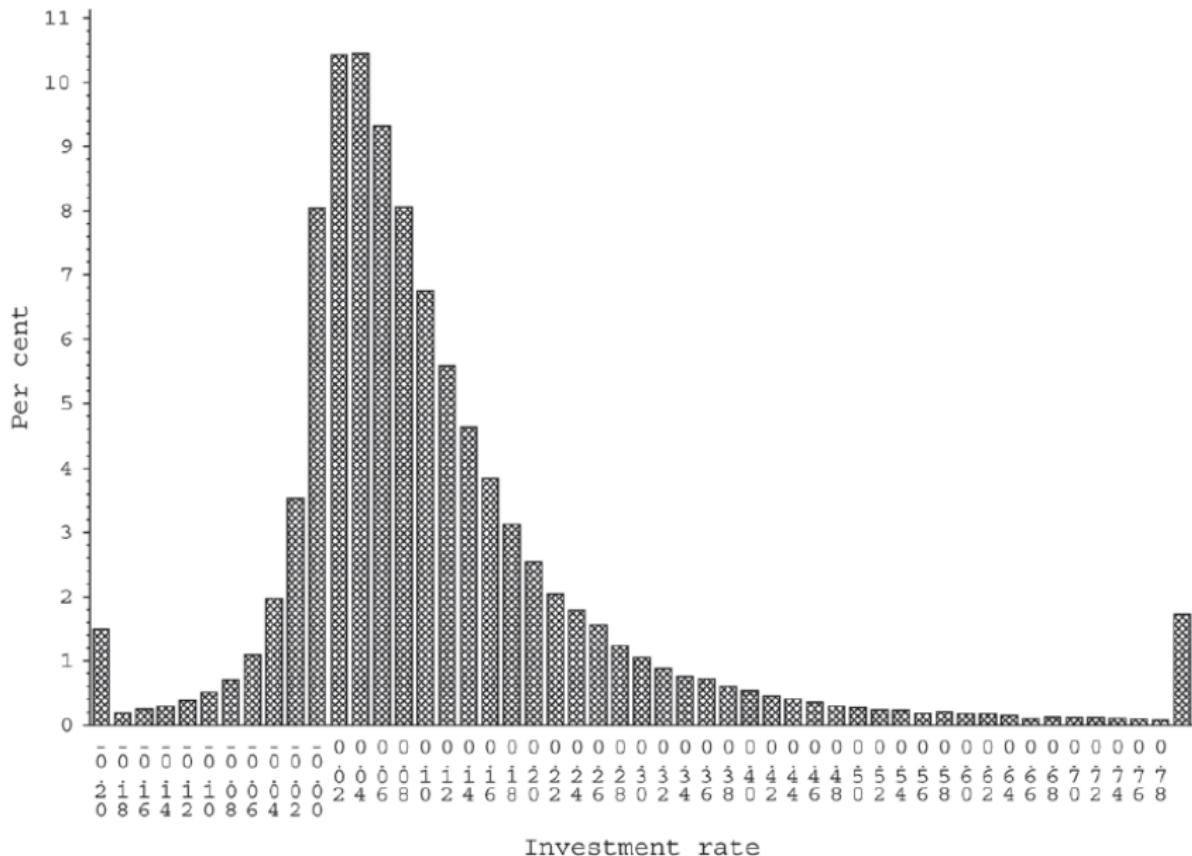
Investment is LUMPY

Firm-level investment is characterised by

- Long periods of inaction ($I \approx 0$)
- Bursts of large, infrequent investment activity.
- These spells can be several years, once started
- Lumpy: Not Continuous, not smooth. Clustered or bunched activity.

- This evidence suggests **convex adjustment costs may not be the whole story** in terms of capital adjustment.

Investment Rate (I/K) Distribution



Capital Adjustment: Cooper & Haltiwanger (2006)

Table 1: Summary Statistics: U.S. Manufacturing Plant-Level Investment

Variable	Mean	(Std. Dev.)
Investment Activity		
Average investment rate	12.2%	(0.10)
Inaction rate ($\text{abs}(I/K) < 1\%$)	8.1%	(0.08)
Negative investment rate	10.4%	(0.09)
Lumpy Adjustment		
Spike rate: positive, $(I/K) > 20\%$	18.6%	(0.12)
Spike rate: negative, $(I/K) < -20\%$	1.8%	(0.04)
Dynamics		
Serial correlation of investment rates	0.058	(0.003)
Correlation: profit shocks & investment	0.143	(0.003)

Source: Longitudinal Research Database (LRD), U.S. manufacturing plants

Nonconvex Costs - Evidence from Data

- Source: Longitudinal Research Database
- 7000 large manufacturing plants that were continually in operation between
- Years: 1972 and 1988.
- This is the **highest level of disaggregation**, since a firm is a collection of plants (factories/sites) and branches.
- First, notice the **shape of the distribution**: not symmetric, rather skewed to the right
- If these data were generated from a model with convex adjustment cost \Rightarrow distribution **would be symmetric**/ (double) bell shaped.

Nonconvex Costs - Evidence from Data

- Second, there are **periods of investment inaction**, 8% of observations, plants do nothing.
- Again, with convex adjustment costs only there will be no periods of inaction
 1. Plant will be doing a little bit of investment in all periods.
- Third, there is asymmetry: **80% of observations entail positive investment rate**, while only 10% of observations entail negative investment rate
 1. Suggests firms really dislike selling capital.

Nonconvex Costs - Modelling Non-Convex Costs

- The message from this data is that firms likely face **more than just convex adjustment costs**.
- Firms face **irreversibility** of investments - “the toothpaste cannot go back in the tube / the toast cannot be unburnt”
- **Fixed costs** of adjustment (in combination with convex adjustment costs) can explain these patterns of adjustment much better than convex costs alone. “**In for a penny, in for a pound**

Fixed Costs and the Lumpy Adjustment Model

Non Convex Costs

- Workhorse model of firm investment
- **Cooper and Haltiwanger** consider a dynamic programming problem specified at the plant level as:

$$V(A, K, p) = \max \{ V^i(A, K, p), V^a(A, K, p) \} \quad \text{for all } (A, K, p) \quad (1)$$

- where V^a is the value of the firm when it invests actively (a for **adjust**)
- V^i is the value of **inactivity** (no investments).
- The optimal value of the firm is the **maximum** of these two options.

Nonconvex Costs

- These options in turn are defined by **Value of Inaction**:

$$V^i(A, K, p) = \pi(A, K) + \beta E_{A', p' | A, p} V(A', (1 - \delta)K, p') \quad (2)$$

- and **Value of Adjustment**:

$$\begin{aligned} V^a(A, K, p) = \max_{K'} & \left\{ \lambda \cdot \pi(A, K) - F \cdot K - p(K' - (1 - \delta)K) \right. \\ & \left. + \beta E_{A', p' | A, p} V(A', K', p') \right\} \end{aligned} \quad (3)$$

Featuring two nonconvex costs:

1. adjustment disruption costs λ , some share of foregone profits
2. fixed costs of adjustment FK , which scale with firm size

1. Disruption of usual operations

- Two *costs* that are **independent of the level of investment activity**.
 - The first is a loss of profit = $(1 - \lambda)\pi(A, K)$
 - Opportunity cost of investment, plant must be shut down during a period of investment activity.
-
- **Byres Road** in 2025: large disruption to install new paving
 - **UofG Campus** in 2025: some paths blocked by construction
 - Doesn't matter if Keystone Building is 5 floors or 50

2. Fixed Costs

Fixed Costs: “in for a penny, in for a pound”

- The 2nd nonconvex cost is F (or as a percent of capital lost, FK).
- **Fixed Costs** are triggered for **any adjustment, independent of size**

$$FC_t = \begin{cases} 0 & \text{if } \mathcal{I}_t = 0 \\ F \times K_t & \text{if } \mathcal{I}_t \neq 0 \end{cases}$$

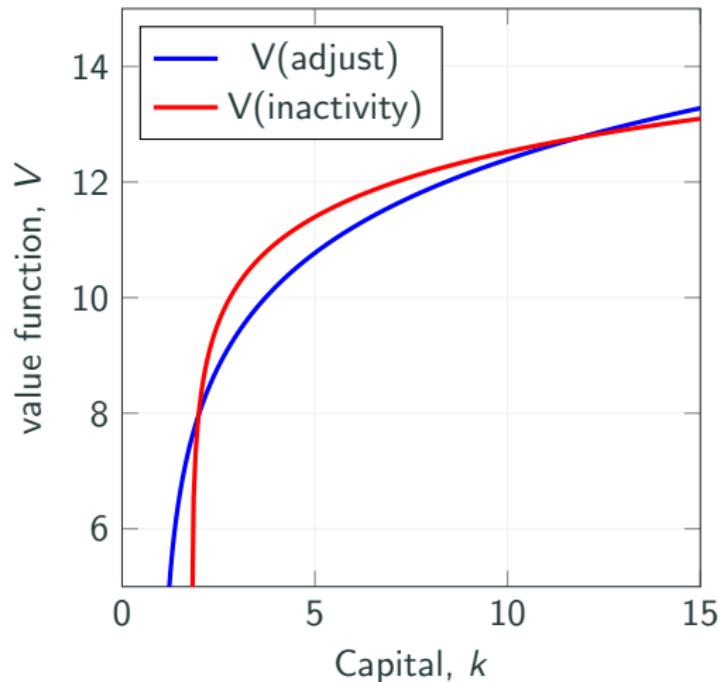
- The inclusion of K : while **independent of the current level of investment activity**, may have some **scale aspects to them**.
- "Getting started (expanding) to any degree is hard"

Overall the firm pays $[F \cdot K + (1 - \lambda) \cdot \pi(A, K)]$ to invest, regardless of size of investment!

Economic Behaviour with Fixed Costs

- How does the policy function for investment look in this case?
- Compare relative to the function obtained under a convex cost only?
- The **key feature** of the investment function is that it will entail **jumps**. How?
- Invest **when it is important** (low K), **invest in bursts** to avoid paying the fixed cost and disruption more than once.

Value Functions



The Economics of Inaction

- **Fixed Costs:** don't vary with size of adjustment, create regions of **inaction**
- Close to the optimal choice, the improvement of adjustment is **small**
- We can show the **Gains-of-Adjustment** curve is **U-shaped** and passes through zero twice:
 - Where the desired adjustment is small, it is not worth doing
 - Where the desired adjustment is large, gains of adjusting are large

Gains from Adjusting with Fixed Costs

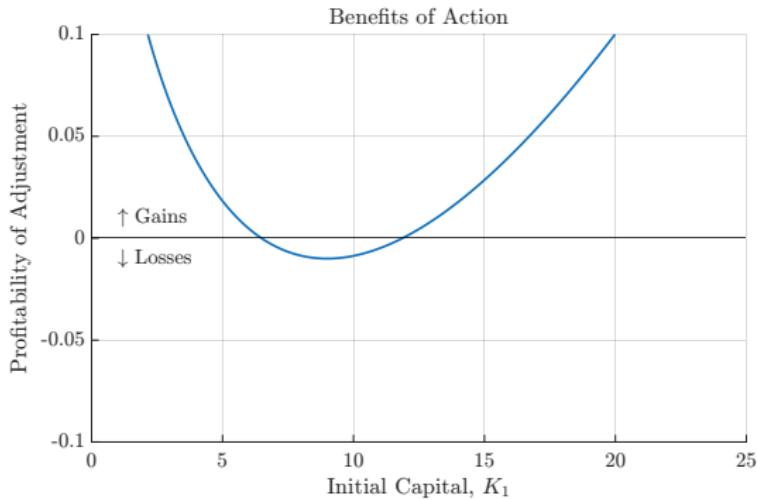


Figure 2: Gains from Action: $V^{Adjust} - V^{inaction}$

1

¹I have used a 2-period model to get clean results. The intuition generalises, but proofs are a nightmare for $T > 2$ for reasons beyond the scope of this course

Inaction Region

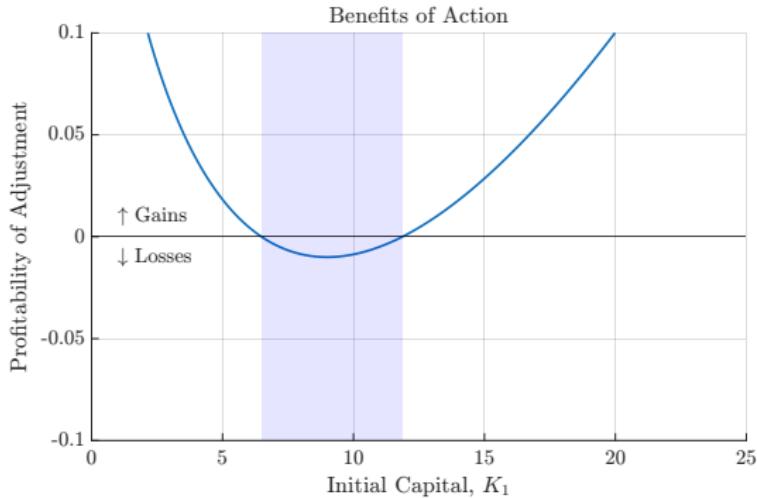


Figure 3: Inaction Region highlighted

- Small changes are not worth it: if $K \in [K_L, K_U]$, adjustment creates losses
- **Behavioural Rule:** Adjust when: $V^A \geq V^I$
- equivalent to $K \leq K_L$ or $K \geq K_U$

Optimal Choices and Regions of Inaction

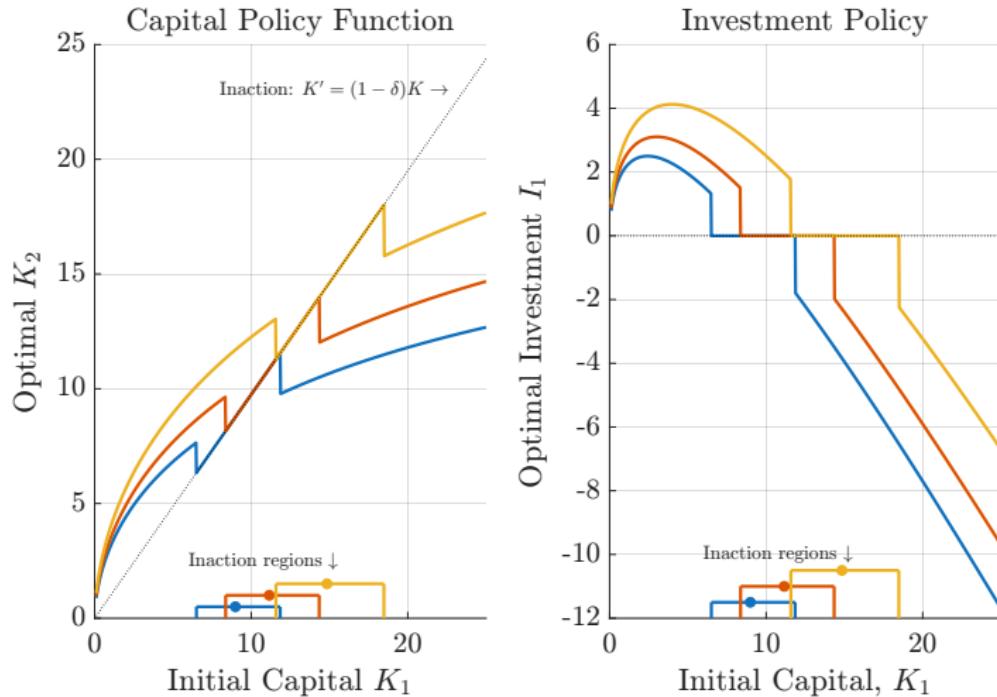
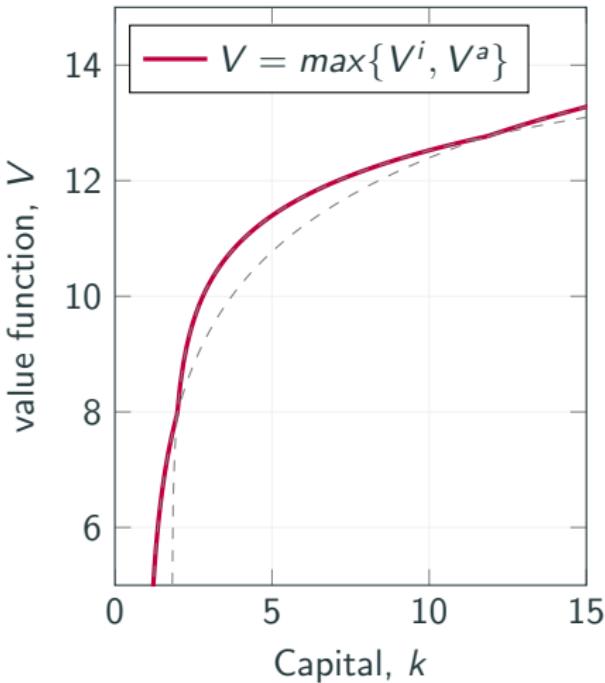
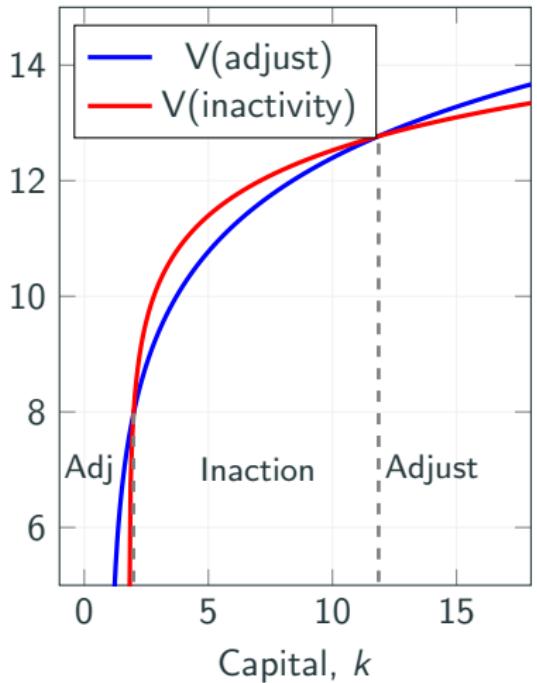


Figure 4: Capital and Investment Policy Functions;
Brackets mark inaction region, dot marks minimum of gains

The Value of Action and Inaction



Irreversibility

British English has a lot of phrases about no going back

- You can't put the toothpaste back in the tube.
- **Can't unburn the toast.**
- Point of no return.
- The cat's out of the bag.
- That ship has sailed.
- Crying over spilt milk.
- Locking the barn door after the horse has bolted.
- That bell can't be unrung.
- You can't unscramble an egg.
- The die is cast.
- Crossing the Rubicon.
- What's done is done.
- I've burned my bridges.
- That's water under the bridge.
- The bird has flown.
- You can't put the genie back in the bottle.
- The bolt is shot.

Everyone has a lot of idioms about irreversibility

(As far as I can tell) every language has rich idioms to describe irreversibility of life:

- 覆水难收 spilled water cannot be gathered
- 木已成舟 the wood has been made into a boat
- Are there more examples from your language(s)?

Costly Mistakes:

- **undoable mistakes and regret** are universal human experiences
- These idioms contain knowledge from generations of **costly mistakes**
- Society can't teach everyone economics
- Culture can give them **memorable phrases that approximate optimal behaviour under irreversibility.**

Irreversibility as Resale price wedges

Wedding Dresses and Ferraris

- (e.g.) When I drive my new Ferrari off the dealership, its market value drops considerably ($p_s < 1$)

Table 2: Hypothetical Price Wedge Between New and Resale Value Across Asset Classes

Asset	New Price	Resale (unused)	Wedge
Cars	100%	70–80%	20–30%
Diamonds	100%	30–50%	50–70%
Wedding dress	100%	20–40%	60–80%
Books	100%	50%	50%
Houses	100%	95–98%	2–5%
Land	100%	~100%	~0%
Stocks	100%	~100%	~0%

Irreversibility as Resale price wedges

A lot of assets / investments have basically **no secondary market for resale**

- **Firm-Specific/Customized Capital**
- Toyota's body-shape moulds and dies cannot be sold to Honda
- Supply chains are highly customised
- Vulnerable to shocks
- Entire Toyota production shut down after 2011 Earthquake. Small affected area, but very industrialised. One supplier of a single compound in paint shut down.
- A degree:
- Time (and money spent) cannot be reversed
- human capital: the degree diploma and skills/credentials can't be sold to someone else
- (but you can rent your labour and skills)

So Installations of new capital are often hard to undo

- Extractions or **retirements of capital might be hard or impossible** (it might destroy the capital)
- Even afterwards, there might be **no willing buyer for the used capital**

Economists use two terms for these **difficulties**:

- **Irreversibility:** it is hard to undo the installation process
- **Specificity:** the capital is specific to the firm, the capital has low scrap value (no buyer or unusable)

Examples:

- Hydroelectric Dam: can't move the river!
- Location-specific Oil pipeline
- Nuclear power plant: Very hard to retire and demolish
- Supply chains are often highly customised to buyers processes (Automobiles).

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Irreversibility in the model as a price-gap: $1 \geq p_s$

- We have not distinguished between **buying and selling price** of capital.
- There are several frictions present in the market for used capital that makes them imperfectly suitable for uses at other production sites.
- To allow for this **partial irreversibility**, we alter our optimization problem to **distinguish the buying and selling prices** of capital.
- Full irreversibility of installations can be captured as $p_s = 0$

Irreversibility in the model

- The value function now has three options in the max operator:

$$V(A, K) = \max \left\{ V^b(A, K), V^s(A, K), V^i(A, K) \right\} \quad (4)$$

- for all (A, K) : **B**uying / **S**elling / **I**naction

Irreversibility in the model

- These options are then given by value functions for buying (selling) and choosing investment (retirement) of capital

$$\text{buying: } V^b(A, K) = \max_I \{ \pi(A, K) - I + \beta E_{A'|A} V(A', K(1 - \delta) + I) \} \quad (5)$$

$$\text{selling: } V^s(A, K) = \max_R \{ \pi(A, K) + p^s R + \beta E_{A'|A} V(A', K(1 - \delta) - R) \} \quad (6)$$

- and

$$\text{inaction: } V^i(A, K) = \pi(A, K) + \beta E_{A'|A} V(A', K(1 - \delta)) \quad (7)$$

Irreversibility in the model

- Under the buy option, the plant obtains capital at a cost normalized to one.
- Under the sell option, the plant retires R units of capital at a price p^s .
- The third option is inaction so that the capital stock depreciates at a rate of δ .
- Intuitively, the gap between the buying and selling price of capital will produce inaction.
- The sell-buy back round trip is expensive. The costs of inaction might be smaller.

Inaction towards small negative shocks

- Suppose that there is an adverse shock to the profitability of the plant.
- If this shock was known to be temporary, then selling capital and repurchasing it in the near future.
- ... would not be profitable for the plant as long as $p_s < 1$.²
- Thus the manager of the plant does nothing, that is we observe inaction.
- Under the convex cost case, (with $p_s = 1$) an adverse shock to profitability would imply downsizing (selling capital).

- **Car rental companies sold all their cars** in Mar-Apr 2020, then couldn't find any new vehicles to buy in 2021: shortage of rentals, forgone profits! (obviously survival was more important, but highlights Sell-Buy-Back-Later dynamics are usually bad)

²Check appendix for derivation: old friend q makes an appearance again

Real World Applications of Irreversibility

In Today's Context

- How much of today's capex by large US firms is firm-specific?
- Large AI / datacentre spending – stranded assets?
- Secondary market if market collapses?
- 20th century: Detroit and fall of auto sector
- 20th century: Glasgow: Canals, Railways, Shipbuilding!

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Specificity & Irreversibility: Glasgow Shipbuilding

Asset Specificity:

- Dry docks built for River Clyde
- Giant cranes (no alternative use)
- Specialized workforce

Complete Irreversibility:

- Can't relocate infrastructure
- No 2nd market for ship cranes
- Large exit/disassembly costs



Why Specificity & Irreversibility Matter: Borrowing

The Glasgow Shipbuilding Death Spiral/Feedback Loop:

1. Global competition increases → Cash flow drops (z_L)
2. Irreversible investments → Can't liquidate assets for cash
3. Specific assets → Can't pledge as collateral for loans (P_s is very low)

$$B < B_{max} = (\text{e.g.}) \ 50\% \times P_s(1 - \delta)K = 50\% \times \text{resale value}$$

4. Can't finance modernization → Fall further behind (accelerator!)
5. More cash flow declines → Eventually exit

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Conclusion 1: Fixed Costs

- How important are fixed costs? **Very!**
- We've seen that fixed costs create an area of small adjustments which, if done, would lead to losses
- in the case of potential losses, the decision is simple: **inaction!**
- **Economic Behaviour:** Fixed Costs create a range of inaction, with bounds $[U, L]$
- many aspects of economics, finance, life(!) feature fixed costs of switching from one mode to another
 - relocation to new city (constant changing opportunity, move infrequently)
 - retraining
 - mental context switching
 - cooking one meal versus batching
 - only check emails a few times a day (constant arrivals)

Conclusion 2: Irreversibility and Specificity

- How important is investment irreversibility? **Very!**
- When capital goods are **highly specialized** on industry specific, firms may find that **reversing an investment** decision is impossible or costly
- **The Ferrari / Wedding Dress effect:** difference between the purchase price and resale price of capital.
- Irreversibility may generate a “**reluctance to invest**”.
- **Low Borrowing Capacity:** can reduce collateral value of assets (borrowing constrained)
- **Can harm future flexibility** to respond to changing business conditions. Flexibility is also an asset (next week!)

Appendix

Optional Appendix: Partial Irreversibility and Inaction

We would want to sell when the value of selling is higher than of inaction:

$$V^i = \pi(A_{low}, K) + \beta E_{A'|A} V(A', K(1 - \delta)) \quad (8)$$

$$V^s = \pi(A_{low}, K) + 0.6R + \beta E_{A'|A} V(A', K(1 - \delta) - R) \quad (9)$$

Let's say profits in both scenarios are the same today $\pi(A_{low}, K)$, what matters is continuation values. We prefer to keep the capital and not to sell when $V^i > V^s$, for some retirement of capital R

$$\beta E_{A'|A} [V(A', K(1 - \delta)) - V(A', K(1 - \delta) - R)] > 0.6R \quad (10)$$

We produce and then decide whether to sell or not.

As R gets very small, we can substitute with marginal value V_K , recall the meaning of this term from the Tobin unit.

$$q^{IRR} = \beta E_{A'|A}[V_K(A', K(1 - \delta))] > p_s = 0.6 \quad (11)$$

profits over useful lifetime of capital > revenue raised today by selling (12)

Shadow Price > Market Price for resold capital (13)

This rule says keep your capital if the expected continuation value is sufficiently high compared to sale value. This depends on the chances conditions $A'|A$ improve, how durable is the asset δ , market prices for used assets etc etc.

$$q \geq p_s$$

So the **inaction set** is pinned down by the range = $\mathbb{I}\{(A, K) : p_b > q > p_s\}$