# 1   Value Function Iteration

Consider the problem of a value maximizing perfectly competitive firm whose operating profit function at time t is given by

$$\pi_t = K_t^{\alpha}; \quad \alpha > 0 \tag{1}$$

where $K_t$ is the stock of capital at time $t$ and $\alpha$ is the elasticity of profit function w.r.t capital.
The firm faces a quadratic adjustment cost of capital given by

$$AC_t = \frac{\phi}{2} I_t^2$$

Dividends, $D_t$, are defined as operating profits net of adjustment costs and investment costs,

$$D_t = \pi_t - I_t - AC_t$$

Here we have normalized the price of capital to unity.  Assume that the firm lives for infinite periods, $t = 0, 1, 2, ...$, and future period values are discounted using the factor $0 < \beta < 1$. Furthermore, the stock of capital evolves as follows

$$K_{t+1} = (1 - \delta)K_t + I_t$$

where $0 < \delta < 1$ is the depreciation rate.

Write down the Bellman equation and solve for optimal investment policy using the method of Value Function Iteration.