ECE 490: Introduction to Optimization **Spring 2017**

Project III: Method of Multipliers

R. Srikant **Due April 25**

Consider a linear equality constrained optimization problem of the form

$$\begin{aligned} \min \quad & x^T Q x \\ \text{s.t.} \quad & Ax = b \end{aligned}$$

where $Q > 0$ and A is a $m \times n$ matrix with $m < n$, i.e. $Ax = b$ is underdetermined.

The goal of this programming assignment is to implement the method of multipliers (known as the Augmented Lagrangian Method, see Section 4.2.2 of Bertsekas) to solve the above problem. Write a python function that will take as input the matrices Q and A, the vector b, and a constant ϵ (this ϵ will be used to define a stopping criterion for the algorithm). Use the stopping condition $\left\|Ax^k - b\right\| \leq \epsilon$. Note that the stopping conditions states that the algorithm is terminated when the Lagrange multipliers do not change significantly. The output should be:

- The optimal x, found using the method of multipliers.
- The number of iterations taken.
- Plot the relative error $\dfrac{\left\|x^* - x^k\right\|}{\|x^*\|}$ versus the iteration number k, for different choices of the sequence $\{c_k\}$. Here, x^* is the optimal solution.

Notes:

- There is no universally acceptable way to choose the sequence $\{c_k\}$, please experiment with various choices. Page 414 of Bertsekas provides some guidance for choosing $\{c_k\}$. You do not have to explain why you chose a particular sequence $\{c_k\}$, but please make sure that the sequence works for the class of problems mentioned below.
- You can experiment with the set of Q, A, and b, posted on the class webpage. You will be provided with different sets when you meet with the TA.