EGN 3443
HOMEWORK 2
(Due on June 21st, Thursday)

1. **(1.2 point)** The probability density function of the time customers arrive at a terminal (in minutes after 8:00 am) is $f(x)= e^{-x/10}/10$ for $0 < x$. Determine the probability that
   a. The first customer arrives by 9:00am
   b. The first customer arrives between 8:15am and 8:30am
   c. Two or more customers arrive before 8:40am among five who arrive at the terminal. Assume customer arrivals are independent.
   d. Determine the cumulative distribution function and use the cumulative distribution function to determine the probability that the first customer arrives between 8:15am and 8:30am.

2. **(1.5 points)** In commuting to work I must first get on a bus near my house and then transfer to a second bus. If the waiting time (in minutes) at each stop has a uniform distribution with A=0 and B=5, then it can be shown that my total waiting time Y has the pdf

$$f(y) = \begin{cases} \left(\frac{1}{25}\right)y & 0 \le y < 5 \\ \frac{2}{5} - \left(\frac{1}{25}\right)y & 5 \le y \le 10 \\ 0 & y < 0 or y > 10 \end{cases}$$

   a. Sketch a graph of pdf of Y
   b. Verify that $\int_{-\infty}^{\infty} f(y)\ dy = 1$
   c. What is the probability that total waiting time is at most 3 min?
   d. What is the probability that total waiting time is between 3 and 8 min?
   e. What is the probability that total waiting time is either less than 2 min or <u>more</u> than 6 min?

3. **(0.5 point)** An ecologist wishes to mark off a circular sampling region having radius 10 meters. However the radius of the resulting region is actually a random variable R with pdf

$$f(r) = \begin{cases} \left(\frac{3}{4}\right)[1-(10-r)^2] & 9 \le r \le 11 \\ 0 & otherwise \end{cases}$$

What is the expected <u>area</u> of the resulting region?

4. **(0.6 point)** The line width for semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer.
   a. What is the probability that the line width is greater than 0.62 micrometer?
   b. What is the probability that the line width is between 0.47 and 0.63 micrometer?
   c. The line width of 90% of samples is below what value?

5. **(0.4 point)** There are 49.7 million people with some type of long-lasting condition or disability living in the US in 2000. This represented 19.3 percent of the majority of the civilians aged five or over (http://factfinder.census.gov). A sample of 1000 persons is selected at random.
   a. Approximate the probability that more than 200 persons in the sample have a disability.
   b. Approximate the probability that between 180 and 300 people in the sample have a disability.

6. **(0.4 point)** Suppose that the number of asbestos particles in a sample of 1 squared centimeter of a dust is a Poisson random variable with a mean of 1000. What is the probability that 10 squared centimeters of dust contains more than 10,000 particles?

7. **(0.8 point)** In the transmission of digital information, the probability that a bit has high, moderate, and low distortion is 0.01, 0.04, and 0.95 respectively. Suppose that three bits are transmitted and that the amount of distortion of each bit is assumed to be independent. Let X and Y denote the number of bits with high and moderate distortion out of the three transmitted, respectively. Determine the following:
   a. The probability that two bits have high distortion and one has moderate distortion
   b. The probability that all three bits have low distortion
   c. The probability distribution, mean and variance of X
   d. The conditional probability distribution, conditional mean and conditional variance of X given that Y = 2

8. **(0.6 point)** Determine the covariance and the correlation for the following joint probability distribution.

x	1	1	2	4
y	3	4	5	6
$f_{XY}(x, y)$	1/8	1/4	1/2	1/8

Good Luck.