MA2011 MECHATRONICS SYSTEMS INTERFACING

Tutorial 6

Prof. Cai Yiyu

College of Engineering
School of Mechanical and Aerospace Engineering

A/D CONVERSION

&

D/A CONVERSION

Q1

Specify an appropriate ±5V n-bit A/D converter (8- or 12-bit), sample rate (up to 100 Hz) and signal conditioning to convert the following analog signal into digital series. Estimate the relative quantization error in quantizing the specified input voltage:

$$E(t) = 1.5 \sin \pi t + 20 \sin 32\pi t - 3 \sin (60\pi t + \pi/4) \text{ V}$$

(Many possible solutions.)

ANSWER TO Q1

Input vs output

KNOWN: A/D converter: $n = 8$ or 12; $V_{max} = 5$ V; $V_{min} = -5$ V

$0 < f_s \leq 100$ Hz

Estimate the maximum Quantifying Error for your choice

ANSWER TO Q1

Analysis

$$E(t) = 1.5 \sin \pi t + 20 \sin 32\pi t - 3 \sin (60\pi t + \pi/4) \text{ V}$$

This input signal contains amplitudes C_1, C_2 and C_3 with frequencies of $f_1 = 0.5$, $f_2 = 16$ and $f_3 = 30$ Hz, respectively.

ANSWER TO Q1

Shannon-Nyquist Theorem

to be able to reconstruct the signal correctly, sampling theorem must be met:

$$f_s > 2f_{max}$$

The maximum frequency in the signal, f_{max}, is 30 Hz. So we must select $f_s > 60$ Hz.

ANSWER TO Q1

Accuracy

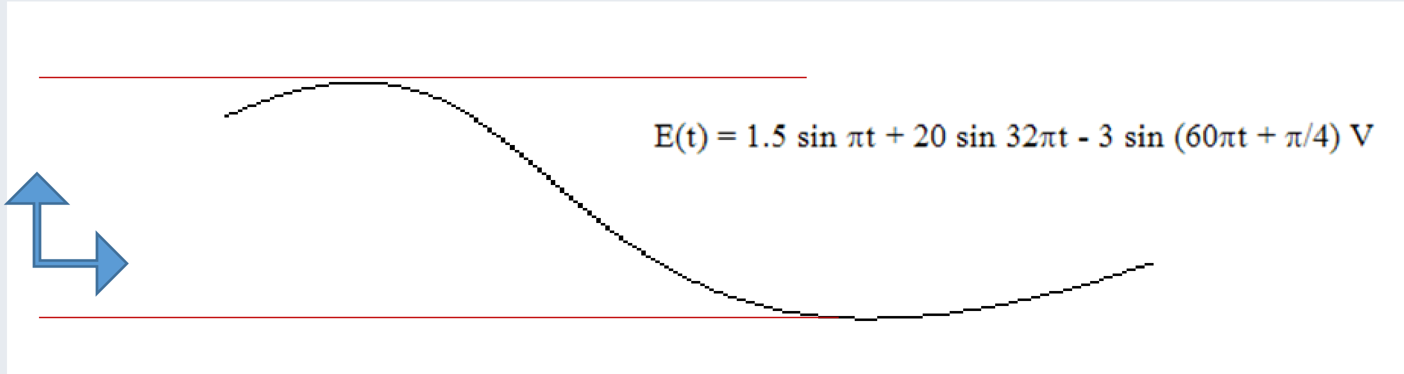
The choice of number of bits for the A/D converter depends on the required accuracy as well as the accuracy of the transducer. For 8-bit A/D converter,

$$Q = \frac{10\ \text{V}}{2^8} = 39.1\ \text{mV}$$

For 12-bit A/D converter, $Q = \dfrac{10}{2^{12}} = 2.44\ \text{mV}$

ANSWER TO Q1

Finding the Amplitude Range



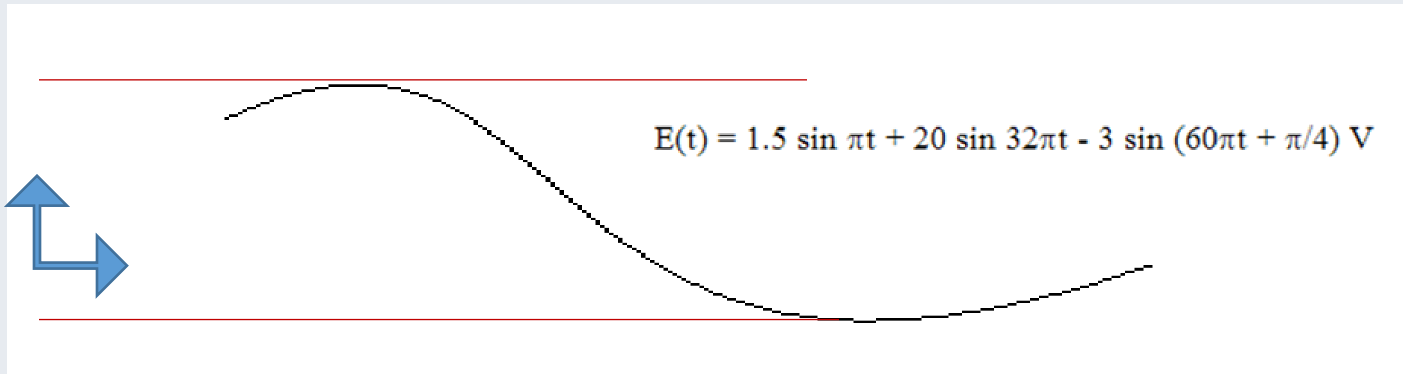
Amplitude range of the signal E(t) can only be found from the time domain plot or by differentiating the equation.

No unique solution

SOLUTION:

This design problem has an open-ended solution path. To demonstrate, one possible solution is presented.

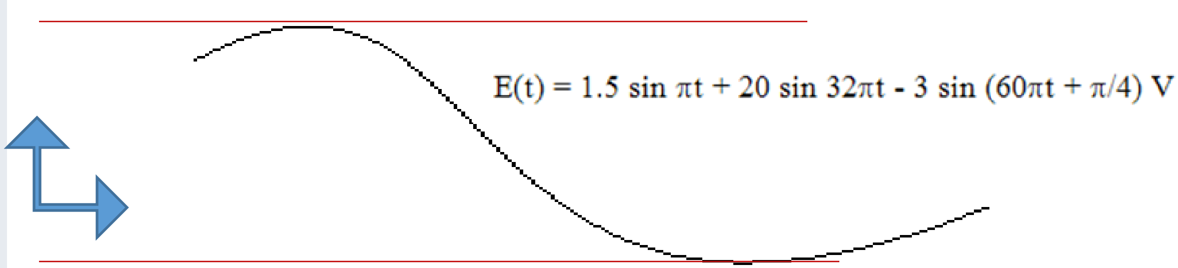
Estimation of Amplitude Range



However, we know that it will never exceed

$$\pm(1.5 + 20 + 3) = \pm 24.5 \text{ V}.$$

Estimated Gain



$$V_{out} = A_v V_{in}$$

So, we need a gain of ±5V / ±24.5 V = about 0.2.

ANSWER TO Q1

Error

The relative quantization error is given by

$$\frac{Q}{GA} \text{ X100\%}$$

G=0.2
Q(8-bits)=39.1mV
Q(12-bits)=2.44mV
Max Amplitudes A=?

The Detail

For each frequency component and the signal, this is listed as:

Component	Frequency	Max. Amplitude	Relative Quantization Error $\frac{Q}{0.2A}*100\%$	
			8-bits	12-bits
1.5sinπt	0.5	**1.5**	39.1mV/0.2/1.5*100%=13%	2.44mV/0.2/1.5*100%=**0.81%**
20sin32πt	16	**20**	39.1mV/0.2/20*100%=9.8%	2.44mV/0.2/20*100%=**0.06%**
3sin(60πt+π/4)	30	**3**	39.1mV/0.2/3*100%=6.5%	2.44mV/0.2/3*100%=**0.41%**
E(t)		<+**24.5**	39.1mV/0.2/24.5*100%=0.8%	2.44mV/0.2/24.5*100%=**0.05%**

As the relative quantization error for both 8-bit and 12-bit A/D converters are less than 1% for E(t), either is suitable for most applications. If the accuracies for the low-level components are important, then the 12-bit A/D converter is required.

Q2

Static pressures are to be measured at eight locations (at 8-cylinder points) under the hood of a NASCAR race car (with a V-8 engine). The pressure transducers to be used have an output span of ±1 V for an input span of ±25 cm H_2O. The signals are measured and recorded on a portable DAS/computer, which uses a 10-bit, ±5V A/D converter. Pressure needs to be resolved to within 0.25 cm H_2O. The dynamic content of the signal is important and has a fundamental period of about 0.5 s. Suggest an appropriate sample rate and signal conditioning (i.e. amplifier gain G and anti-aliasing filter cut off frequency f_c) for this application.

DAS=Direct-attached storage

(Many possible solutions.)

Input vs output

KNOWN: Transducer: ±1 V output; ±25 cm H_2O input

DAS (ADC): $n = 10$; $V_{max} = 5$ V; $V_{min} = -5$ V

FIND: f_s, f_c, G.

Race Car

Input span: ± 25 cm H_2O (Pressure Transducer)
Output span: ± 1 V (Pressure Transducer)
ADC: 10-bit, -5 V Range, 0.25 cm H_2O

ANSWER TO Q2

Sensitivity K

The transducer sensitivity

$\mathbf{K} = \Delta V_{out}/\Delta V_{in}$ =(1-(-1))V/(25-(-25)) cmH_2O=2V/50cmH_2O

=0.04V/cmH_2O

Sensitivity K meets problem constraint (10-bit ADC)?

The A/D resolution is:

$$Q = \frac{10V}{2^{10}} = 0.00976\ V.$$

Pressure needs within 0.25 cm H_2O

This can be expressed as

$$Q = \frac{0.00976V}{0.04V/cm\ H_2O} = 0.244\ cm\ H_2O < 0.25\ cm\ H_2O$$

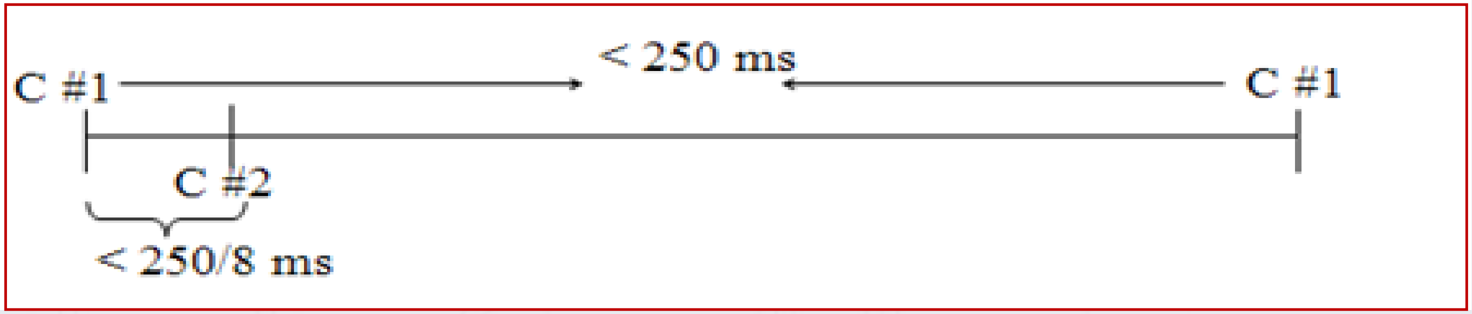
Amplifier (Gain 5)

The sensitivity meets problem constraints. However, an analog amplifier between the transducer and A/D converter with a gain of G = 5 will take full advantage of the A/D range and improve resolution:

$$Q = \frac{0.00976V}{5 \times 0.04V / cmH_2O} = 0.0488\ cm\ H_2O$$

ANSWER TO Q2

For one pressure sensor, the fundamental period of the signal is about 0.5s (500ms), or a frequency of 2Hz. The maximum sampling period is 250ms or the minimum sampling frequency is greater than 4 Hz.



Sampling

Since the car is powered by a high performance V-8 engine, there are 8 cylinders and there are 8 pressure signals to sense, one for each cylinder. Hence, the DAS has to sample at least up to 8 times faster than maximum sampling period required for 1 pressure sensor i.e. 250/8 [ms] or a minimum frequency of 32 Hz. $\Delta max = 250/8$ ms. $f_{min} = 8000/250 = 32$Hz

Q3

The range of a signal is between ±5 V and it is required to make measurements with a quantization size no more than 5 mV. What is the minimum resolution of an ADC needed?

ANSWER TO Q3

Since

$$\frac{5-(-5)}{2^N} = Q \leq 0.005$$

$$\Rightarrow 2^N \geq \frac{10}{0.005} = 2000$$

If N = 11, 2^N = 2048 and if N = 12, 2^N = 4096. Although a 11-bit ADC will suffice, it is not commercially available.

Hence a <u>**12-bit ADC**</u> will be needed.

REVISION

One Objective	Two Ideas	Three Techniques
Four Examples	Five Misunderstandings	Six & Seven Tutorials & Project

REVISION: ONE OBJECTIVE OF DIGITIZATION

Periodic Signal → Mathematic Representation → Sampling (Op-amp, Method) → **Digital Signal**

1. Fourier Series Representation (Theory)

- Frequency domain analysis

2. Shannon-Nyquist Theorem

- Aliasing, 2-phase filtering

REVISION: THREE TECHNIQUES

1: Frequency Domain Analysis

Spike plotting

2: First Order System

Dynamic Error

3:Sampling & Aliasing

Two-phase Filtering

REVISION: THREE TECHNIQUES

1: Frequency Domain Analysis

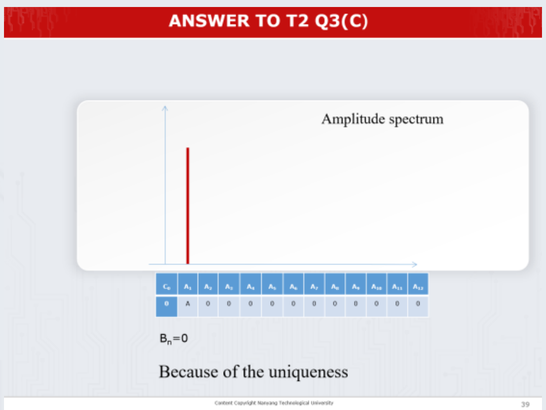
Spike plotting

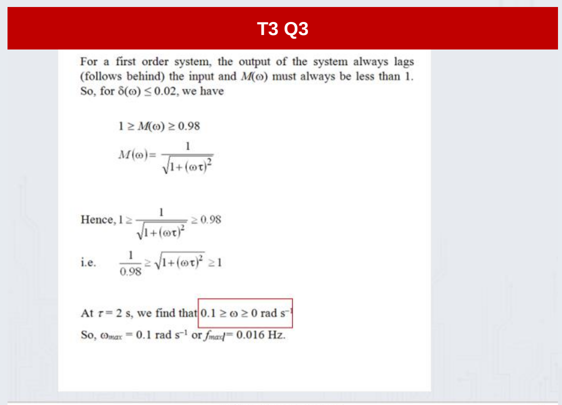
2: First Order System

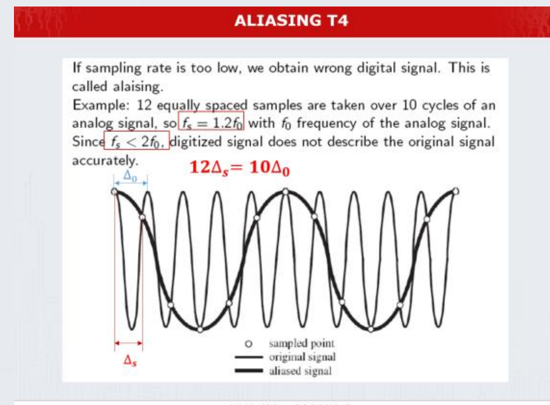
Dynamic Error

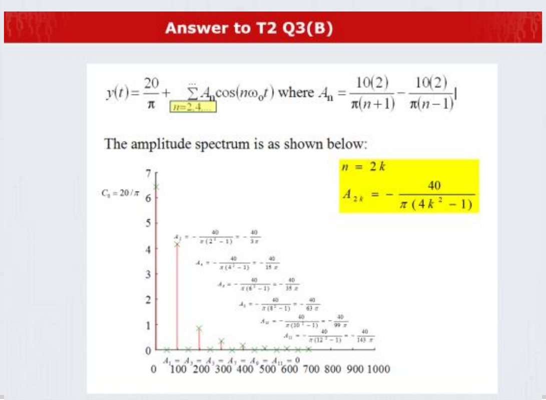
3:Sampling & Aliasing

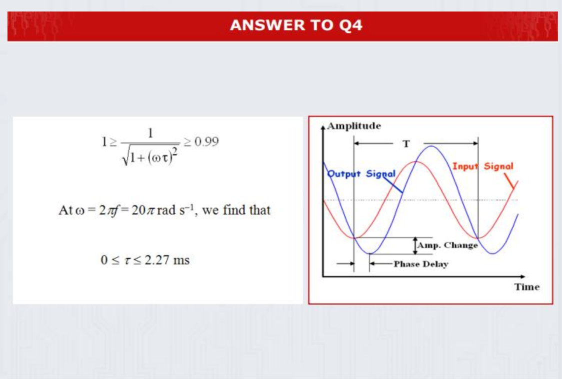
Two-phase Filtering

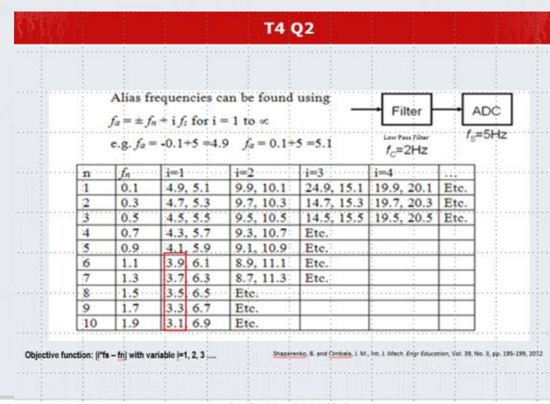








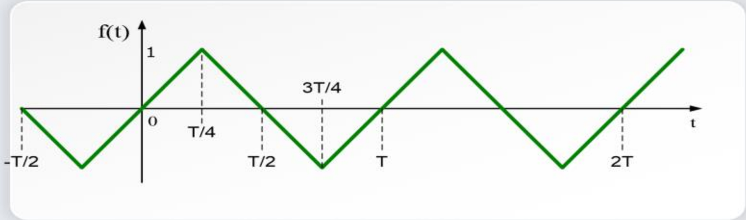




REVISION: FOUR EXAMPLES

T2 Q2

f(t) is a function defined as follows.



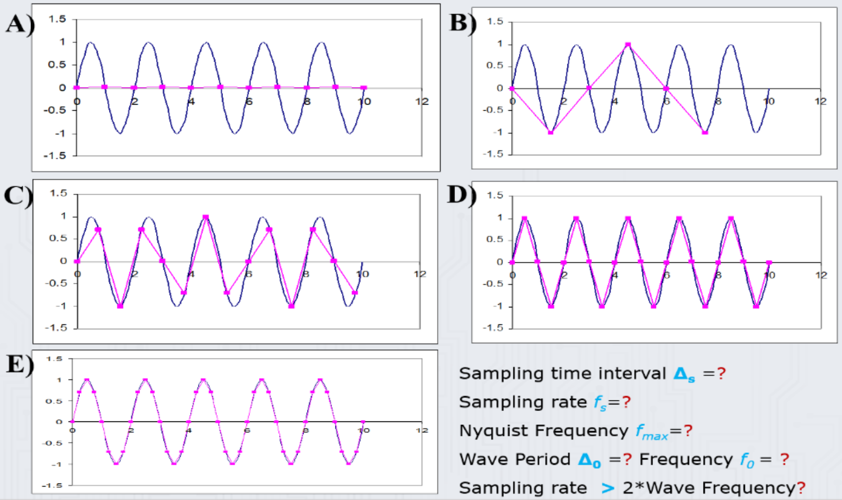
(a) Is it a periodic function? If yes, what is the period, and write the function of the waveform defined at [0, T]
(b) Is it a symmetric function?
(c) Can the waveform be represented by Fourier Series? If yes, what are the DC C_0, A_n and B_n of the waveform and the Fourier Series?
(d) What are the peak amplitude, and peak-to-peak amplitude?
(e) If the peak amplitude is changed to A, what will be the function of f(t) and its Fourier Series Representation?

T3 Q2

During a step function calibration, a first-order instrument is exposed to a step change of 100 units. If after 1.2 s the instrument indicates 80 units, estimate the instrument time constant. Estimate the error in the indicated value after 1.5 s. Assume $X_{out}(0)$ = 0 units and K = 1 unit/unit.

c τ = 0.75 s; error at 1.5 s = 13.4 units

T4 Q1



Sampling time interval Δ_s =?

Sampling rate f_s=?

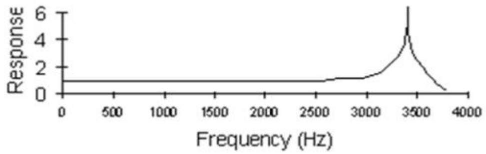
Nyquist Frequency f_{max}=?

Wave Period Δ_0 =? Frequency f_0 = ?

Sampling rate > 2*Wave Frequency?

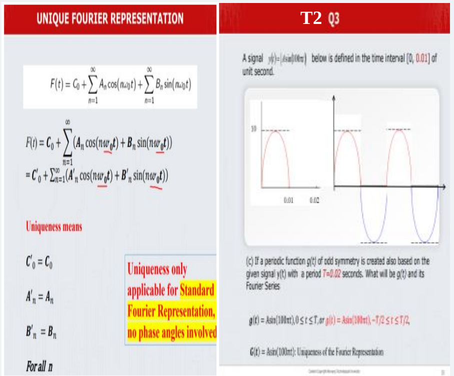
T5 Q3

3. An accelerometer that is used to measure the vibration of a machine has a frequency response as shown in the figure below:

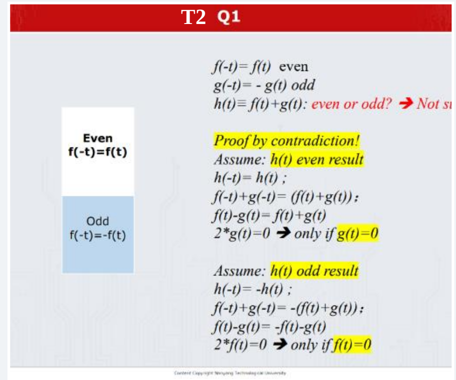


The vibration signal is filtered to ensure that the vibration signal beyond 2.5 kHz does not affect the recording of the measurements.

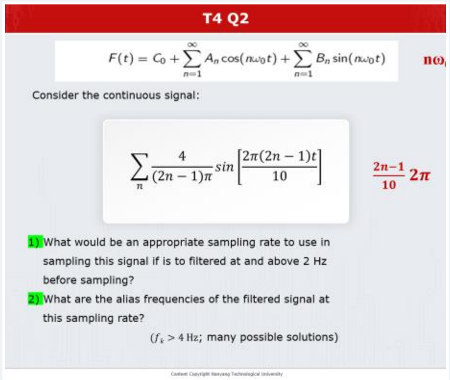
REVISION: FIVE MISUNDERSTANDINGS



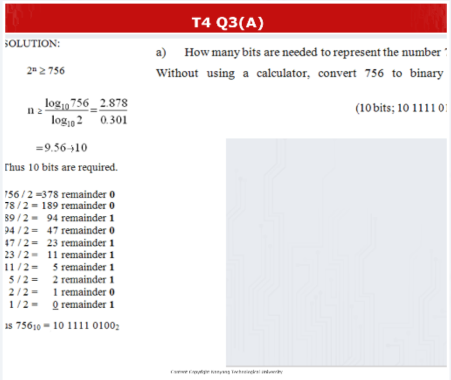
1. *Fourier Representation is always time consuming*



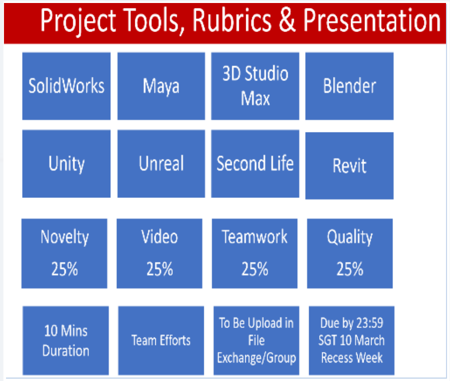
2. *Proof vs Proof by contradiction*



3. Fourier Series must be in (nw_0t) form



4. Binary conversion must have a calculator



Digital twin has nothing to do with mechatronics

REVISION

Tutorial	Content
# 6	A/D & D/A Conversion
# 5	Amplifiers
# 4	Sampling Quantizing Coding
# 3	Zero-order system First-order system Second-order system
# 2	Fourier Series
# 1	Characteristics of Measurement System

REVISION

Lecture	Content	Fundamental
# 6	A/D & D/A Conversions	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series
# 5	Amplifiers	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series
# 4	Sampling Quantizing Coding	Shannon-Nyquist Theorem Fourier Series
# 3	Zero-order system First-order system Second-order system	Ordinary Differential Equations Fourier Series
# 2	Fourier Series	Fourier Series
# 1	Characteristics of Measurement System	Concepts

REVISION

Lecture	Content	Fundamental	Application
# 6	A/D & D/A Conversions	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series	A/D Converter D/A Converter Design
# 5	Amplifiers	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series	Amplifier Design
# 4	Sampling Quantizing Coding	Shannon-Nyquist Theorem Fourier Series	A/D & D/A Conversion
# 3	Zero-order system First-order system Second-order system	Ordinary Differential Equations Fourier Series	Amplifier Design, ADC, DAC
# 2	Fourier Series	Fourier Series	Amplifier Design, ADC, DAC
# 1	Characteristics of Measurement System	Concepts	

REVISION

Lecture	Content	Fundamental	Application	Difficult Level
# 6	A/D Conversion	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series	A/D Converter D/A Converter Design	***
# 5	Amplifiers	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series	Amplifier Design	***
# 4	Sampling Quantizing Coding	Shannon-Nyquist Theorem Fourier Series		***
# 3	Zero-order system First-order system Second-order system	Ordinary Differential Equations Fourier Series		*****
# 2	Fourier Series	Fourier Series		****
# 1	Resistors, Capacitors, …	Mechatronics Concepts		*

REVISION

Lecture	Content	Fundamental	Not Examinable
# 6	A/D & D/A Conversions	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series	
# 5	Amplifiers	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series	
# 4	Sampling Quantizing Coding	Shannon-Nyquist Theorem Fourier Series	Mathematics Proof of Shannon-Nyquist Theorem
# 3	Zero-order system First-order system Second-order system	Ordinary Differential Equations Fourier Series	Mathematics Proof of Ordinary Different Equation
# 2	Fourier Series	Fourier Series	Mathematics Proof of Fourier Series Representation
# 1	Characteristics of Measurement System	Mechatronics Concepts	