Ca1 cheat sheet

Thermodynamics & Heat Transfer (Nanyang Technological University)

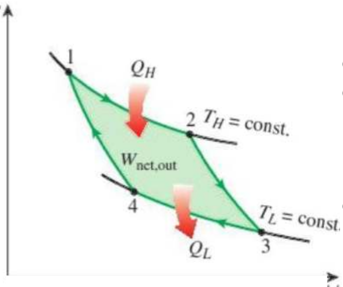
$$\text{Thermal Efficiency} = \frac{\text{Net work output}}{\text{Total heat input}}$$

$$\text{Coefficient of Performance} = \frac{\text{Desired output}}{\text{Required input}}$$

$$\eta_{th} = \frac{W_{net,out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} \qquad \mathrm{COP_R} = \frac{Q_L}{W_{net,in}} \qquad \mathrm{COP_{HP}} = \frac{Q_H}{W_{net,in}}$$

Energy balance: $W_{net,in} = Q_H - Q_L$ Energy balance: $W_{net,in} = Q_H - ($

$$\eta_{th} = \frac{W_{net,out}}{Q_H} = \boxed{1 - \frac{Q_L}{Q_H}} \qquad \boxed{\mathrm{COP_R} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{Q_H/Q_L - 1}} \qquad \boxed{\mathrm{COP_{HP}} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - Q_L/Q_H}}$$



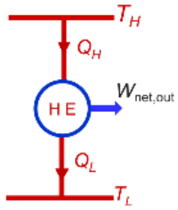
1. Closed Systems

- Fixed mass
- Energy can be transferred through boundary by heat or work
- Boundary can be movable

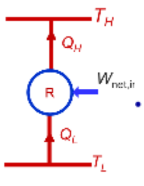
Isolated

No mass flow, no energy transferred through boundary

Heat Engine:



Reverse Heat Engine:



Refrigerator/Heat Pur

- Coefficient of performance for a Carnot refrigerator:

$$\boxed{\mathrm{COP_{R,rev}} = \frac{1}{T_H/T_L - 1}}$$

- Coefficient of performance for a Carnot heat pump:

$$\boxed{\mathrm{COP_{HP,rev}} = \frac{1}{1 - T_L/T_H}}$$

$$w_{rev} = -v(P_2 - P_1)$$

$$w_{poly} = -\frac{nR_{sp}(T_2 - T_1)}{n-1} = -\frac{nR_{sp}T_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

$$w_{s=cnst} = -\frac{kR_{sp}(T_2 - T_1)}{k-1} = \frac{kR_{sp}T_1}{1-k}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right]$$

$$w_{T=cnst} = -R_{sp}T\ln\frac{P_2}{P_1}$$

$$\Delta S = S_2 - S_1 = m(s_2 - s_1)$$

$$s = s_f + x \cdot s_{fg} \qquad Q - W = \Delta U$$

$$S_{gen} \begin{cases} > 0 & \text{irreversible process} \\ = 0 & \text{reversible process} \\ < 0 & \text{impossible process} \end{cases}$$

$$\dot{W} = \dot{m}(h_1 - h_{2a})$$

$$P_x = \sqrt{P_1 P_2} \Rightarrow \frac{P_x}{P_1} = \frac{P_2}{P_x}$$

$$\eta_T = \frac{\text{Actual work}}{\text{Isentropic work}} = \frac{w_a}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

$$s_2 - s_1 = c_{avg}\ln\frac{T_2}{T_1} \qquad Q = m(h_2 - h_1)$$

$$\boxed{\left(\frac{T_2}{T_1}\right)_{isen} = \left(\frac{v_1}{v_2}\right)^{k-1}}$$

$$\Delta S_{system} = S_{in} - S_{out} + S_{gen}$$

$$\boxed{s_2 - s_1 = c_{p,avg}\ln\frac{T_2}{T_1} - R_{sp}\ln\frac{P_2}{P_1}}$$

$$\boxed{\left(\frac{T_2}{T_1}\right)_{isen} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}}$$

) The iron casting block is a closed system.

$$\Delta S_{sys} = m(s_2 - s_1) = mc_{avg}\ln\frac{T_2}{T_1}$$

$$\boxed{\left(\frac{P_2}{P_1}\right)_{isen} = \left(\frac{v_1}{v_2}\right)^{k}}$$

$$\Delta S = \frac{Q}{T_0} \quad (\text{kJ/K}) \quad Q = mc_{avg}(T_2 - T_1)$$

Specific/absolute humidity (ω) – ratio of mass of water vapour to mass of dry air:

$$\omega = \frac{m_v}{m_a} \qquad \text{Humidity ratio}$$

$$\omega = \frac{m_v}{m_a} = \frac{(P_vV)/(R_vT)}{(P_aV)/(R_aT)} = \frac{P_v/R_v}{P_a/R_a} = \frac{R_a}{R_v}\cdot\frac{P_v}{P_a} = 0.622\frac{P_v}{P_a}$$

$$\because P = P_a + P_v \qquad \rightarrow \omega = \frac{0.622P_v}{P - P_v}$$

$$\dot{W} = \dot{m}(h_2 - h_1)$$

Work output per unit mass:

$$\dot{w} = h_1 - h_2$$

Air enters an adiabatic nozzle at 400 kPa and 547 °C with low velocity and exits at 240 m/s. If the isentropic efficiency of the nozzle is 90%, determine the exit temperature and pressure of the air. Utilize the properties of air at 800 K when applicable.

(7-120) [793.8 K, 348 kPa]

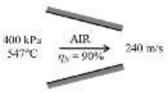
Assumptions: Air as an ideal gas, constant specific heats

Steady state; steady flow

Adiabatic

Negligible P.E

Average temperature at 800 K



Properties: $T_{avg} = 800$ K, $c_p = 1.099$ kJ/kg·K, $k = 1.354$

Energy balance for actual exit velocity

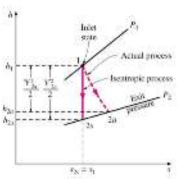
$$h_1 + \frac{v_1^2}{2} = h_{2a} + \frac{v_{2a}^2}{2}$$

$$2(h_1 - h_{2a}) = v_{2a}^2$$

$$2c_p(T_1 - T_{2a}) = v_{2a}^2$$

$$T_{2a} = T_1 - \frac{v_{2a}^2}{2c_p}$$

$$= (547 + 273) - \frac{240^2}{2 \times 1.099 \times 10^3} = 793.8\ \text{K}$$



Isentropic efficiency

$$\eta_N = \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{c_p(T_1 - T_{2a})}{c_p(T_1 - T_{2s})}$$

$$T_{2s} = T_1 - \frac{T_1 - T_{2a}}{\eta_N}$$

$$= 820 - \frac{820 - 793.8}{0.9} = 790.9\ \text{K}$$

Exit pressure is similar to isentropic exit pressure

$$\left(\frac{T_2}{T_1}\right)_{isen} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{T_{2s}}{T_1}\right)^{k/(k-1)}$$

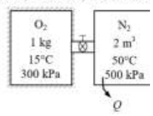
$$P_2 = P_1\left(\frac{T_{2s}}{T_1}\right)^{k/(k-1)}$$

$$= 400\left(\frac{790.9}{820}\right)^{1.354/0.354} = 348.4\ \text{kPa}$$

An insulated tank that contains 1 kg of O_2 at 15°C and 300 kPa is connected to a 2-m uninsulated tank that contains N_2 at 50°C and 500 kPa. The valve connecting the tw tanks is opened, and the two gases form a homogeneous mixture at 25°C. Determine (a) th final pressure in the tank, (b) the heat transfer to the surroundings, and (c) the entrop generated during this process. Assume the surrounding temperature to be T_o = 25°C.

[Ans: 444.6 kPa; 187.2 kJ; 0.962 kJ/K

Assumptions: All gases and mixtures as ideal gases



Gas constants: $R_{O_2} = 0.2598\ \text{kJ/kg}\cdot\text{K}$

$R_{N_2} = 0.2968\ \text{kJ/kg}\cdot\text{K}$

Volume of O_2 tank:

$$V_{tank,O_2} = \left(\frac{mR_{sp}T_1}{P_1}\right)_{O_2}$$

$$= \frac{1 \times 0.2598 \times (273 + 15)}{300} = 0.249\ \text{m}^3$$

Mass of N_2:

$$m_{N_2} = \left(\frac{P_1V_1}{R_{sp}T_1}\right)_{N_2}$$

$$= \frac{500 \times 2}{0.2968 \times (273 + 50)} = 10.43\ \text{kg}$$

Total volume of mixture after mixing = total volume of both tanks

$$V_m = V_{tank,O_2} + V_{tank,N_2} = 2.249\ \text{m}^3$$

Mol count of each component:

$$N_{O_2} = \frac{m_{O_2}}{M_{O_2}} = \frac{1}{31.999} = 0.0313\ \text{kmol}$$

$$N_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{10.43}{28.013} = 0.372\ \text{kmol}$$

Total number of mol in mixture:

$$N_m = N_{O_2} + N_{N_2} = 0.4033\ \text{kmol}$$

a) Final pressure after mixing:

$$P_m = \frac{N_mR_uT_m}{V_m}$$

$$= \frac{0.4033 \times 8.314 \times (25 + 273)}{2.249} = 444.3\ \text{kPa}$$

b) The whole system (both tanks) is a closed system. Energy balance for the syst

$$Q - \cancel{W} = \Delta E_{sys}$$

$$Q = (mc_v\Delta T)_{O_2} + (mc_v\Delta T)_{N_2}$$

$$Q = m_{O_2}c_{v,O_2}(T_m - T_{1,O_2}) + m_{N_2}c_{v,N_2}(T_m - T_{1,N_2})$$

$$= 0.658(25 - 15) + 10.43 \times 0.743(25 - 50) = -187.2\ \text{kJ}$$

c) Mole fraction of each gas:

$$y_{O_2} = \frac{N_{O_2}}{N_m} = \frac{0.0313}{0.4033} = 0.0776$$

$$y_{N_2} = \frac{N_{N_2}}{N_m} = \frac{0.372}{0.4033} = 0.9224$$

Partial pressure of each gas (the alternative method is to calculate the partial pressure using ideal gas equations $P_iV_m = m_iR_iT_m$):

$$\frac{P_{O_2}}{P_m} = y_{O_2}$$

$$P_{O_2} = y_{O_2}P_m = 0.0776 \times 444.3 = 34.48\ \text{kPa}$$

$$P_{N_2} = y_{N_2}P_m = 0.9224 \times 444.3 = 409.82\ \text{kPa}$$

Entropy change of system (both tanks), using the 2nd Tds equation: (the 1st Tds equatio is also usable, rather than calculating P_2 for each gas shown above, calculate $v_1 = m/V_1$ and $v_2 = m/V_m$ for each of the gases)

$$\Delta S_{sys} = \Delta S_{O_2} + \Delta S_{N_2}$$

$$= m_{O_2}\left(c_p\ln\frac{T_2}{T_1} - R_{sp}\ln\frac{P_2}{P_1}\right)_{O_2} + m_{N_2}\left(c_p\ln\frac{T_2}{T_1} - R_{sp}\ln\frac{P_2}{P_1}\right)_{N_2}$$

$$= 0.918\ln\frac{(25 + 273)}{(15 + 273)} - 0.2598\ln\frac{34.48}{300}$$

$$+ 10.43\left(1.039\ln\frac{(25 + 273)}{(50 + 273)} - 0.2968\ln\frac{409.82}{500}\right)$$

$$= 0.593 - 0.2573 = 0.3357\ \text{kJ/K}$$

Take the system and its surroundings as an isolated system:

$$\Delta S_{surr} = \frac{Q}{T_{surr}}$$

$$= \frac{187.2}{25 + 273} = 0.6282\ \text{kJ/K}$$

$$S_{gen} = \Delta S_{iso}$$

$$= \Delta S_{sys} + \Delta S_{surr}$$

$$= 0.3357 + 0.6282 = 0.964\ \text{kW/K}$$

1. A completely reversible refrigerator is removing heat from a 4°C (T_L) refrigerated space at a rate of 5400 kJ/h (Q_L) and rejecting the waste heat to the environment at 27°C (T_H). What is the power consumption of the refrigerator (W_{in})?

$$\mathrm{COP_R} = \frac{1}{\frac{T_H}{T_L} - 1}$$

$$W_{in} = \frac{Q_L}{\mathrm{COP_R}}_{\#}$$

2. What is the power consumed (W) for a reversible and adiabatic compression of 2 kg/s ($\dot{m}$) of oxygen from 100 kPa (P_1) and 50°C (T_1) to 1.2 MPa (P_2) and 355°C (T_2)? Assume an average temperature of 500 K.

reversible and adiabatic = isentropic, use $n = k$

$$W = \dot{m}\frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]_{\#} = \dot{m}c_{p,avg}(T_2 - T_1)_{\#}$$

3. 250 ml (V) of hot tea (ρ = 1000 kg/m3, c = 4.18 kJ/kg·K) cools from 80°C (T_1) to 27°C (T_2). What is the entropy change (ΔS) of this process?

$$m = \rho V$$

$$\Delta S = mc\ln\frac{T_2}{T_1}_{\#}$$

4. A tank is filled with saturated moist air at 30°C (T) and the total pressure in the tank is 150 kPa (P). If the mass of water vapour in the tank is 800 g (m_v), calculate the mass of dry air (m_a).

saturated moist air, $\phi = 100\%$

$$P_g = P_{sat@T}$$

$$\omega = 0.622\frac{\phi P_g}{P - \phi P_g} = \frac{m_v}{m_a}$$

$$m_a = \frac{m_v}{\omega}_{\#}$$

5. An ideal gas mixture consists of 2.5 kmol (N_{N2}) of nitrogen and 4.2 kmol (N_{O2}) of oxygen. Determine the specific heat at constant pressure ($c_{p,m}$) of the mixture at a temperature of 350 K.

$$m_{O_2} = N_{O_2}M_{O_2}$$
$$m_{N_2} = N_{N_2}M_{N_2}$$
$$\mathrm{mf}_{O_2} = \frac{m_{O_2}}{m_{O_2} + m_{N_2}}$$
$$\mathrm{mf}_{N_2} = 1 - \mathrm{mf}_{O_2}$$
$$c_{p,m} = \mathrm{mf}_{O_2} \times c_{p,O_2} + \mathrm{mf}_{N_2} \times c_{p,N_2}{}_{\#}$$

Using the mole fraction version to obtain specific heat per kmol is also acceptable but the answer and values used must be in terms of kJ/kmol·K.

$$y_{O_2} = \frac{N_{O_2}}{N_{O_2} + N_{N_2}}$$
$$y_{N_2} = 1 - y_{O_2}$$
$$\bar{c}_{p,m} = y_{O_2} \times \bar{c}_{p,O_2} + y_{N_2} \times \bar{c}_{p,N_2}{}_{\#}$$

6. Carbon dioxide is compressed steadily at a rate of 1.5 kg/s ($\dot{m}$) by an adiabatic compressor from 100 kPa (P_1) and 30°C (T_1) to 500 kPa (P_2). If the compressor consumes 185 kW (W_a) of power during operation, calculate its isentropic efficiency (η_c). Assume an average temperature of 400 K.

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$W_{isen} = -\dot{m}\frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right], n = k$$

$$\eta_c = \frac{W_{isen}}{W_a}_{\#} = \frac{\dot{m}c_p(T_{2s} - T_1)}{W_a}_{\#}$$

7. An inventor has developed a new heat engine that taps into a geothermal energy source at 150°C (T_H) while rejecting the waste heat to the environment at 10°C (T_L). It is claimed that the engine produces 2 kW (W_{out}) of power with a heat rejection rate of 12000 kJ/h (Q_L). Is this claim valid?

$$\eta_{TH} = \frac{W_{out}}{Q_H} = \frac{W_{out}}{W_{out} + Q_L}$$

$$\eta_{TH,\mathrm{Carnot}} = 1 - \frac{T_L}{T_H}$$

if $\eta_{TH} \le \eta_{TH,\mathrm{Carnot}}$, claim is valid, otherwise, claim is invalid

Alternatively, comparisons between power produced and heat rejection rates are acceptable as well.

8. 3.3 kg (m) of saturated liquid R134a at 600 kPa (P_1) evaporates into vapour at 100 kPa (P_2) and 30°C (T_2). Calculate the entropy change of R134a (ΔS).

$s_1 = s_f$ at pressure, P_1 (saturated liquid)

s_2 = properties at T_2, P_2 (superheated vapour)

$$\Delta S = m(s_2 - s_1)_{\#}$$

Long cylindrical steel rods (ρ = 7833 kg/m³) and c_p = 0.465 kJ/kg °C) of 10 cm diameter are heat treated by drawing them at a speed of 3 m/min through a 7 m long oven maintained at 900°C. If the rods enter the oven at 30°C and leave at 700°C, determine (a) the rate of heat transfer to the rods in the oven and (b) the rate of entropy generation with the heat transfer process.

(7-139) [(a) 958.5 kW, (b) 0.85 kW/K]

Assumptions: Steady-state

Constant specific heat

Negligible KE and PE change

Rods are moving continuously through the oven at 3 m/min.

Analyse the rod using a period of 1 min interval.

Mass of rod heated per min:

$$m = \rho V = \rho \times \pi\frac{D^2}{4} \cdot L$$
$$= 7833 \times \pi \times \frac{0.1^2}{4} \times 3 = 184.56\ \mathrm{kg/min}$$

a) Energy balance:

$$Q - W = \Delta E$$
$$Q = mc(T_2 - T_1)$$
$$= 184.56 \times 0.465 \times (700 - 30) = 57500\ \mathrm{kJ/min}$$
$$= 958.3\ \mathrm{kW}$$

b) For a 1 min interval,

Rate of entropy change for iron rod:

$$\dot{S}_{rod} = \dot{m}c\ln\frac{T_2}{T_1}$$
$$= 184.56 \times 0.465\ln\frac{700 + 273}{30 + 273} = 100.122\ \mathrm{kJ/K \cdot min}$$

Take the surroundings of the rod in the oven as a thermal energy reservoir $T_\circ$ = 900°C

Rate of entropy change inside the oven

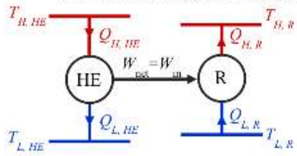
$$\dot{S}_{oven} = \frac{\dot{Q}}{T_{oven}}$$
$$= \frac{-57500}{900 + 273} = -49.02\ \mathrm{kJ/K \cdot min}$$

Entropy generation:

$$\dot{S}_{gen} = \dot{S}_{tot} = \dot{S}_{rod} + \dot{S}_{oven}$$
$$= 100.122 - 49.02 = 51.1\ \mathrm{kJ/K \cdot min}$$
$$= 0.852\ \mathrm{kW/K}$$

A Carnot heat engine receives heat from a reservoir at 927 °C at a rate of 740 kJ/min and rejects the waste heat to the ambient air at 27 °C. The entire work output of the heat engine is used to drive a refrigerator that removes heat from the refrigerated space at -7 °C and transfers it to the same ambient at 27 °C. Determine (a) the maximum rate of heat removal from the refrigerated space and (b) the total rate of heat rejection to the ambient air.

(6-103 equivalent)[Ans: (a) 4342 kJ/min, (b) 5082 kJ/min]



Assumption: Steady-state

a) Max. rate of heat removal = $Q_{L,R}$ at max. refrigerator COP

Max. refrigerator COP = Carnot refrigerator

$$\mathrm{COP}_{R,rev} = \frac{\text{Desired Output}}{\text{Required Input}}$$

$$\mathrm{COP}_{R,rev} = \frac{Q_{L,R}}{W_{in}} = \frac{1}{Q_{H,R}/Q_{L,R} - 1}$$
$$= \frac{1}{T_{H,R}/T_{L,R} - 1} = \frac{1}{(27 + 273)/(-7 + 273) - 1}$$
$$= 7.82$$

$$Q_{L,R} = \mathrm{COP}_{R,rev} \times W_{in}$$

Need to find W_{in}, which is equal to W_{net} from heat engine

For Carnot heat engine:

$$\eta_{th,rev} = \frac{W_{net}}{Q_{H,HE}} = 1 - \frac{T_{L,HE}}{T_{H,HE}}$$
$$= 1 - \frac{27 + 273}{927 + 273} = 0.75$$

$$\dot{W}_{net} = \eta_{th,rev} \times \dot{Q}_{H,HE}$$
$$= 0.75 \times 740 = 555\ \mathrm{kJ/min}$$

$$\therefore \dot{Q}_{L,R} = \mathrm{COP}_{\mathrm{R,carnot}} \times \dot{W}_{net}$$
$$= 7.82 \times 555 = 4340\ \mathrm{kJ/min}$$

b) Total heat rejection to ambient = $Q_{L,HE} + Q_{H,R}$

Energy balance for Heat Engine:

$$\dot{W}_{net} = \dot{Q}_{H,HE} - \dot{Q}_{L,HE}$$

$$\dot{Q}_{L,HE} = \dot{Q}_{H,HE} - \dot{W}_{net}$$
$$= 740 - 555 = 185\ \mathrm{kJ/min}$$

Energy balance for Refrigerator

$$\dot{W}_{in} = \dot{Q}_{H,R} - \dot{Q}_{L,R}$$

$$\dot{Q}_{H,R} = \dot{Q}_{L,R} + \dot{W}_{in}$$
$$= 4340 + 555 = 4895\ \mathrm{kJ/min}$$

Total heat rejection rate Q_T:

$$\dot{Q}_T = \dot{Q}_{L,HE} + \dot{Q}_{H,R}$$
$$= 185 + 4895 = 5080\ \mathrm{kJ/min}$$

Alternatively, the thermodynamic temperature scale relation can be used to evaluate heat rejected to ambient since both are Carnot devices:

$$\frac{T_{H,HE}}{T_{L,HE}} = \frac{Q_{H,HE}}{Q_{L,HE}}$$

$$\rightarrow \dot{Q}_{L,HE} = \dot{Q}_{H,HE} \cdot \frac{T_{H,HE}}{T_{L,HE}}$$

$$\frac{T_{L,R}}{T_{H,R}} = \frac{Q_{L,R}}{Q_{H,R}}$$

$$\rightarrow \dot{Q}_{H,R} = \dot{Q}_{L,R} \cdot \frac{T_{L,R}}{T_{H,R}}$$