NANYANG TECHNOLOGICAL UNIVERSITY

MA2011 MECHATRONICS SYSTEMS INTERFACING

Lecture 4

Prof. Cai Yiyu

College of Engineering
School of Mechanical and Aerospace Engineering

RECAP OF LECTURE 3

RECAP OF LECTURE 3

Ordinary Differential Equations

Linear System, Homogenous Equation, and Characteristic Equation

Zero-order, First-order, and Second-order Systems

Solutions, and Analysis

System Modelling and Analogies

Electrical, Mechanical, Hydraulic Systems, etc

CLAUDE ELWOOD SHANNON

CLAUDE ELWOOD SHANNON

American mathematician
Electronic engineer
Cryptographer
Father of Information Theory

Founded both digital computer
and digital circuit design theory
Founded Information Theory

Claude Elwood Shannon
(1916 – 2001)

CLAUDE ELWOOD SHANNON

When	Where	What
1932	University of Michigan	Took a course that introduced him to the work of George Boole
1936	University of Michigan	Received bachelor's degree in electrical engineering and bachelor's degree in mathematics
1937(21 year old)	MIT	The most important master's thesis of all time: A Symbolic Analysis of Relay and Switching Circuits Founded both digital computer and digital circuit design theory
1940	MIT	Ph D Thesis: An Algebra for Theoretical Genetics
World War II		Contributed to the field of cryptanalysis for national defense
1948	Princeton	Founded Information Theory with one landmark paper that he published in.

CLAUDE ELWOOD SHANNON

U Michigan
(George Boole)

Bell
(Turing)

MIT
(Vannevar Bush)

Princeton
(Von Neumann)

PART 4: SAMPLING

PART 4.1 BACKGROUND & OBJECTIVES

OBJECTIVES

1. How to properly sample a signal for digital processing
2. How digital data are coded
3. Related components of an A/D converter

DATA ACQUISITION

Real world or environment

- Mechanical
- Electrical
- Thermal
- Hydraulic
- …….

→ **Data acquisition** →

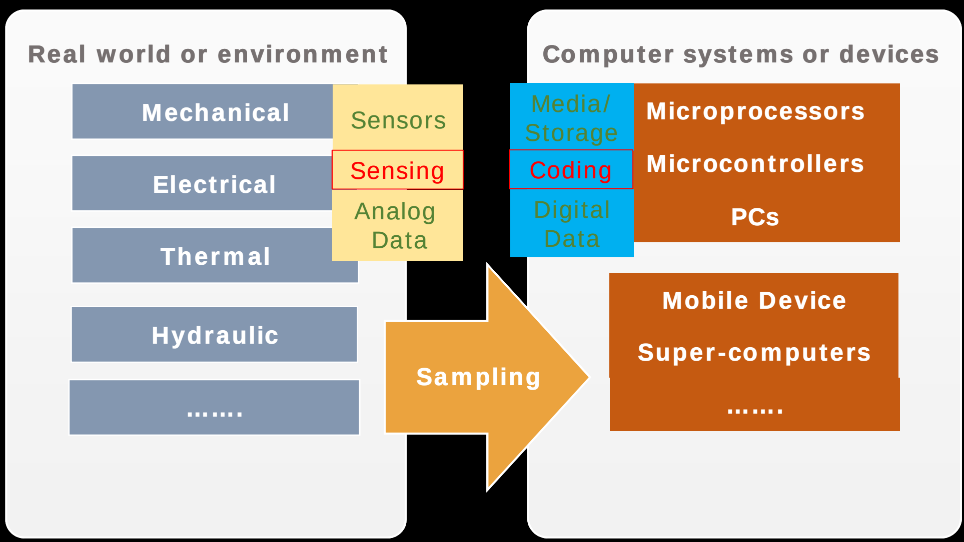
Computer systems or devices

- Microprocessors
- Microcontrollers
- PCs
- Super-computers
- Mobile Device
- …….

INTRODUCTION– DATA ACQUISITION

Data acquisition

SAMPLING



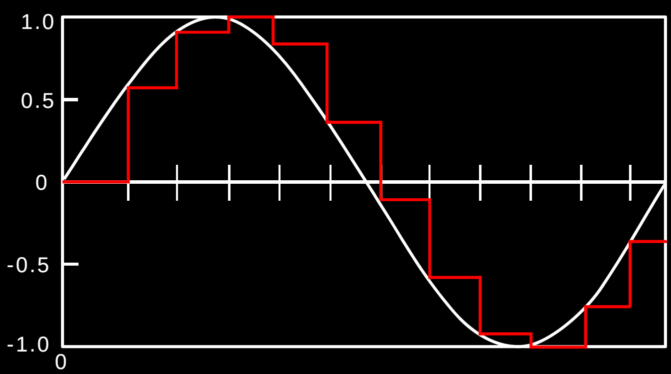
DIGITAL DATA ACQUISITION

1. More compact storage of the data using different media (magnetic, optical, flash memory, or even cloud,…)
2. More accurate data
3. Gathering large or mega data set
4. Allow the use of real-time control system
5. Enabling data process(online, offline, or even long after the events)

WAVEFORM SAMPLING

Objective

To represent waveforms on digital computers, we need to digitize or sample the waveform.

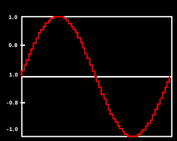


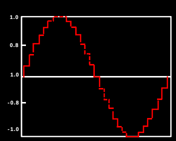
SAMPLING RATE

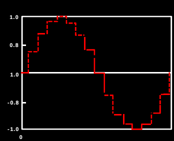
Sampling: Higher rate or low rate

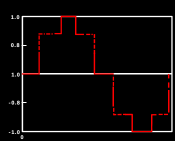
Higher sampling rates allow the waveform to be more accurately represented.

Low sampling rates may lead the waveform to be less accurately represented.

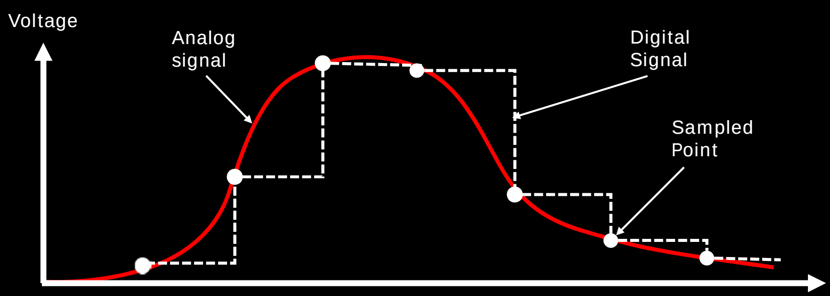

64 samples/period


32 samples/period


16 samples/period


8 samples/period

ANALOG SIGNAL VS. DIGITAL SIGNAL

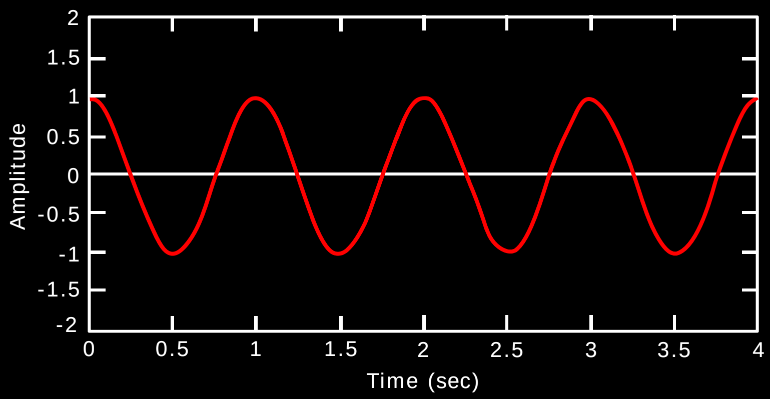


Analog Signal	Digital Signal
Continuous	Discrete
Generated via analog devices	Sampled in a fixed interval
Not coded	Coded value
Original signal	Sequential data array

PART 4.2 SHANNON & NYQUIST THEOREM

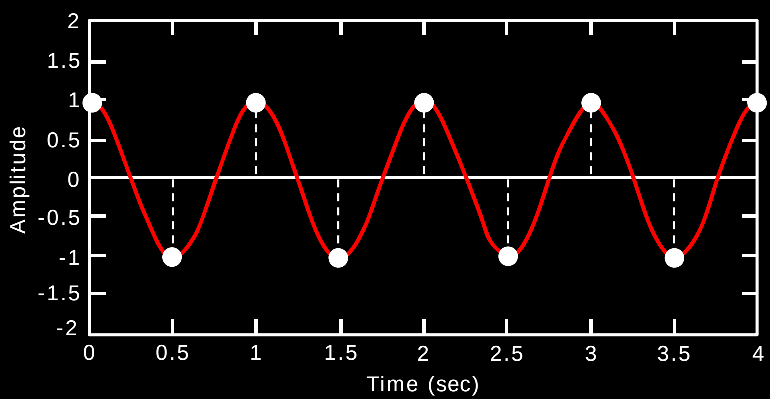
SHANNON & NYQUIST THEOREM

Example 1: Single sine wave at a frequency=1Hz (1 period per second)



SHANNON & NYQUIST THEOREM

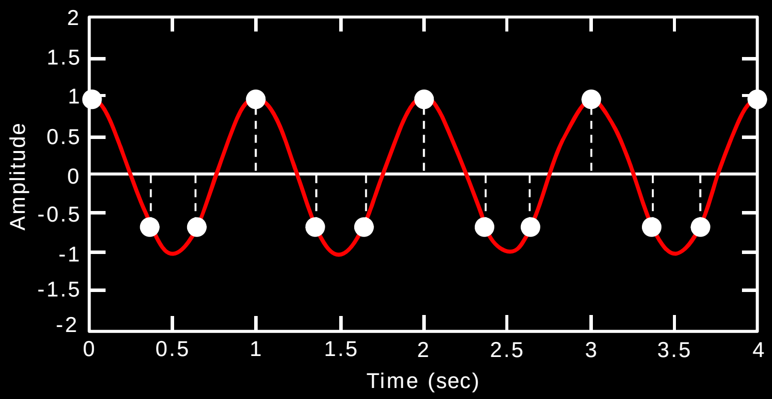
Example 1: Single sine waveform at a frequency=1Hz (1 period per second)



Sample rate: 2Hz (2 points per second)
Result: Sufficient to capture each peak and trough of the signal.

SHANNON & NYQUIST THEOREM

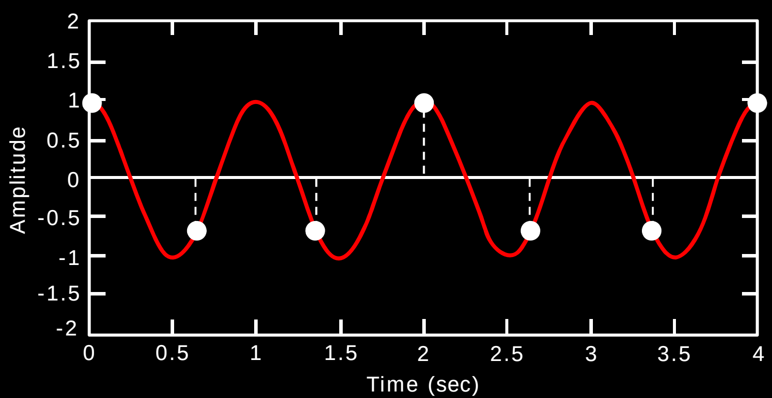
Example 1: Single sine wave at a frequency=1Hz (1 period per second)



Sample rate: 3Hz (3 points per second)
Result: More than enough samples to capture the variations in the signal.

SHANNON & NYQUIST THEOREM

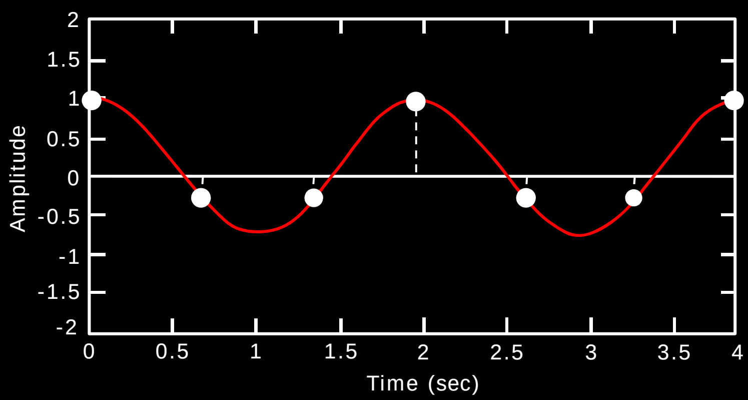
Example 1: Single sine wave at a frequency=1Hz (1 period per second)



Sample rate: 1.5Hz (3 points per 2 second)
Result: Not enough samples to capture all the peaks and troughs in the signal.

SHANNON & NYQUIST THEOREM

Example 1: Single sine wave at a frequency=1Hz (1 period per second)



Sample Rate: 1.5Hz (3 points per 2 second)
Result: Not enough samples to capture all the peaks and troughs in the signal.
Not only losing information, but also misleading the understanding of the signal.

SHANNON & NYQUIST THEOREM

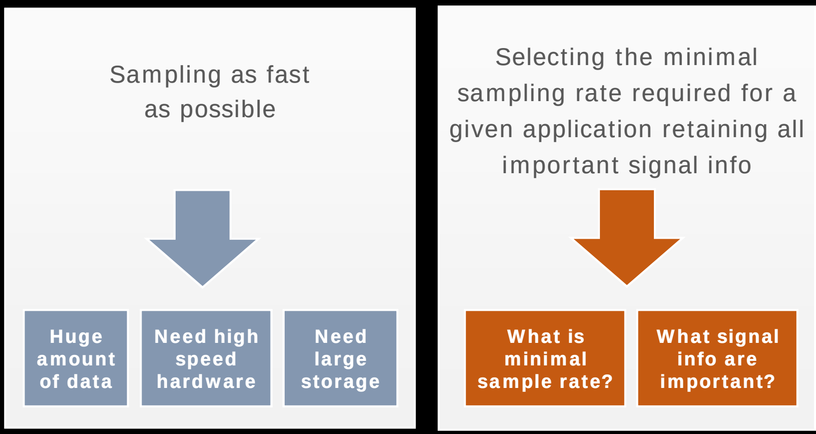
SAMPLING IS IMPORTANT!

What is the proper sample rate?
Shannon-Nyquist Theorem

How to deal with any periodic waveform?
Fourier Series Representation

SHANNON & NYQUIST THEOREM

SAMPLING GOOD OR BAD?



SHANNON & NYQUIST THEOREM

SHANNON & NYQUIST THEOREM

- We need to sample a digital signal at a rate more than two times the **maximum frequency (f_{max}) component** in the signal to retain all frequency components

- In other words, to faithfully represent the analogue signal, the digital samples must be taken at a frequency *fs*, such that

$$f_s > 2f_{max}$$

  f_s is called **sampling rate** (not sampling frequency), and f_{max} **is called Nyquist frequency**

- If we approximate a signal by a truncated Fourier series (**N** terms), the maximum frequency component is the **highest** harmonic frequency. Then the time interval between the digital samples is

$$\Delta t = 1/\, f_s$$

SHANNON & NYQUIST THEOREM

SHANNON & NYQUIST THEOREM

$$F(t) = \sum_{n=0}^{N} C_n \cos(n\omega_0 t + \varphi_n)$$

$$F(t) = \sum_{n=-N}^{N} \boldsymbol{D}_n e^{jn\omega_0 t}$$

Maximum frequency component: N

Sampling rate: f_s

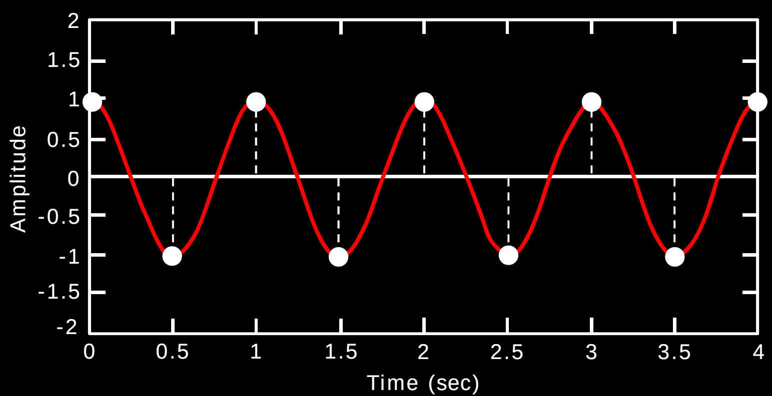
Nyquist frequency: f_{max}

Shannon-Nyquist Theorem: $f_s > 2f_{max}$

Time interval between the digital samples: $\Delta t = 1/\ f_s$

SHANNON & NYQUIST THEOREM

Example 1: Single sine waveform at a frequency=1Hz (1 period per second)



Maximum frequency component: $N = 1$

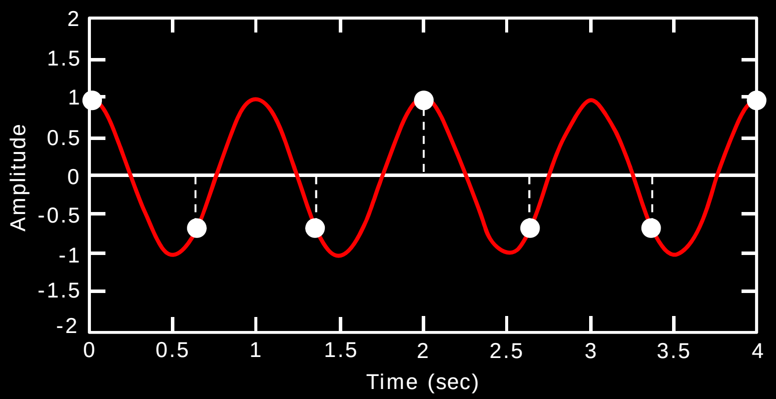
Nyquist frequency: $f_{max} = 1\text{Hz}$

Shannon-Nyquist Theorem: $f_s = 2\text{Hz} > 2f_{max} = 2 * 1\text{Hz}$

Time interval between the digital samples: $\Delta t = 1/2$ sec

SHANNON & NYQUIST THEOREM

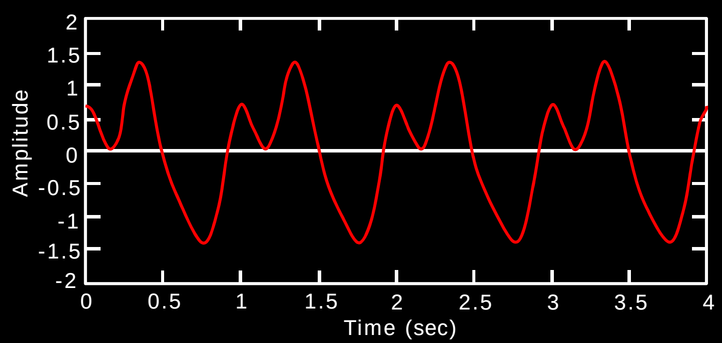
Example 1: Single sine waveform at a frequency=1Hz (1 period per second)



Maximum frequency component: $\mathrm{N} = 1$
Nyquist frequency: $f_{max} = 1\mathrm{Hz}$
Shannon-Nyquist Theorem: $f_s = 1.5\mathrm{Hz} > 2f_{max} = 2{*}\ 1\mathrm{Hz}$
Time interval between the digital samples: $\Delta\mathrm{t} = 2/3\ \mathrm{sec}$

SHANNON & NYQUIST THEOREM

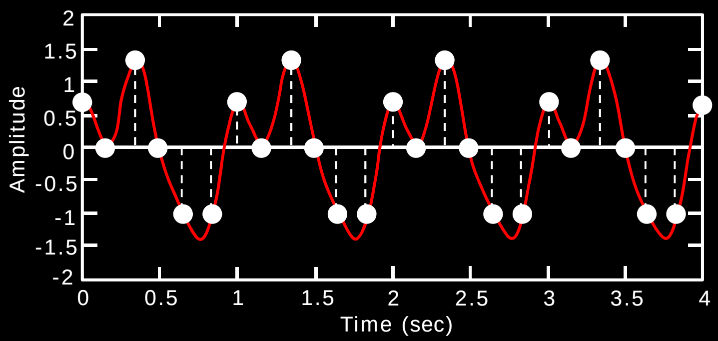
Example 2: Adding three sinewaves at frequencies of 1Hz, 2Hz, and 3Hz



Maximum frequency component: $\mathrm{N} = 3$
Nyquist frequency: $f_{max} = 3\mathrm{Hz}$
Shannon-Nyquist Theorem: $f_s > 2f_{max}$
Time interval between the digital samples: $\Delta\mathrm{t} = ?$

SHANNON & NYQUIST THEOREM

Example 2: Adding three sinewaves at frequencies of 1Hz, 2Hz, and 3Hz

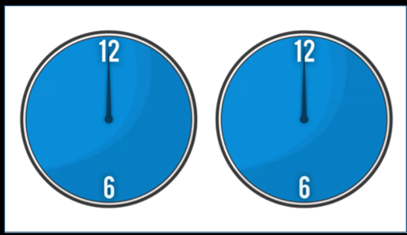


Maximum frequency component: $\mathrm{N} = 3$
Nyquist frequency: $f_{max} = 3\mathrm{Hz}$
Shannon-Nyquist Theorem: $f_s > 2f_{max} = 6$
Time interval between the digital samples: $\Delta \mathrm{t} = 1/6$

PART 4.3 ALIASING

ALIASING

Consider the revolution of a hand of a clock. The second hand of a clock has a period of 60s.



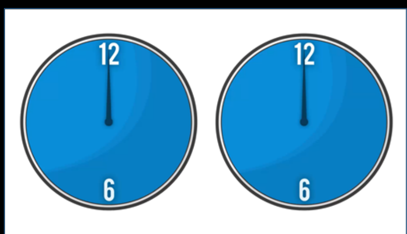
$T = 60s$
$f_{max} = 1/60Hz$

SAMPLING THEOREM

SAMPLING OF A CLOCK WITH ONLY ONE HAND

Samples can mean that the clock is moving either forward or backward. (12-6-12-6-12)

(a). Sampling at Nyquist rate: $T_s = T\frac{1}{2}$



ALIASING

Consider the revolution of a hand of a clock. The second hand of a clock has a period of 60s.

$T = 60s$

$f_{max} = 1/60Hz$

SAMPLING THEOREM

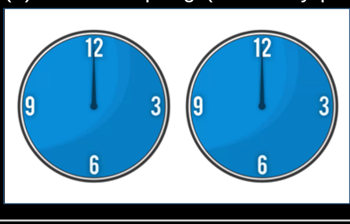
SAMPLING OF A CLOCK WITH ONLY ONE HAND

Samples show clock is moving forward. (12-3-6-9-12)

(b). Oversampling (above Nyquist rate): $T_s = T\frac{1}{4}$

Samples show clock is moving backward. (12-9-6-3-12)

(c). Oversampling (below Nyquist rate): $T_s = T\frac{3}{4}$



ALIASING

According to the Shannon-Nyquist theorem, we need to sample at least 2 times of the Nyquist frequency, or sample the hand no more than every 30s ($f_s > 2f_{max} = 1/30Hz$ or $T_s < T/2 = 30s$).

Aliasing occurs: When the sample points, in order, are 12, 6, 12, 6, 12, and 6, the receiver of the samples cannot tell if the clock is moving forward or backward.

Or

Samples can mean that the clock is moving either forward or backward: (12-6-12-6-12)

(a). **Sampling rate at** $f_s > 2f_{max} = 1/30Hz : T_s < T/2 = 30s$
Nyquist frequency = $f_{max} = 1/60Hz$

ALIASING

In part(b), we sample at 4 times of the Nyquist frequency

($f_s = 4^* f_{max} > 2^* f_{max}$) or every 15s.

The sample points are 12, 3, 6, 9, and 12.

The clock is moving forward.

Samples show clock is moving forward:(12-3-6-9-12)

(b). Oversampling (above 2 times Nyquis frequency):
$f_s = Hz > 2^* Nyquistfrequency = 2f_{max} = 1/30Hz$, or $T_s = T/4$

ALIASING

In part(c), we sample below the Nyquist rate

$(f_s < 2f_{max}\ or\ T_s > T/2)$.

The sample points are 12, 9, 6, 3, and 12.

Although the clock is moving forward, the receiver thinks that the clock is moving backward.

Samples show clock is moving backward:(12-9-6-3-12)

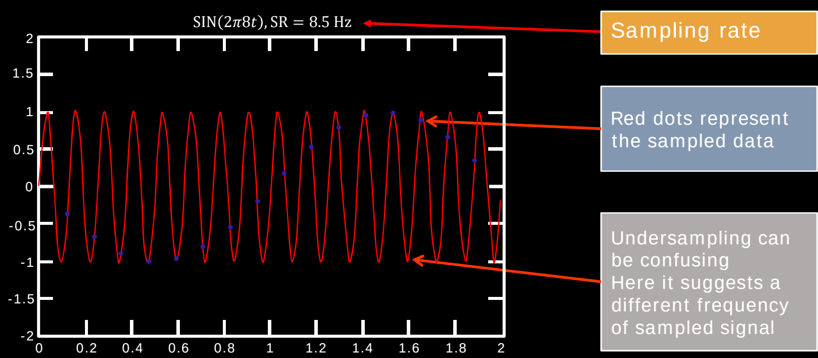
(c). Undersampling(below 2*Nyquist frequency):

$$T_s = \frac{3}{4}T = 45sec, f_s = 1/45Hz = 4/3^* f_{max}(1/60Hz) < 2f_{max} = 1/30Hz$$

QUANTISING & CODING

AN UNDERSAMPLED SIGNAL

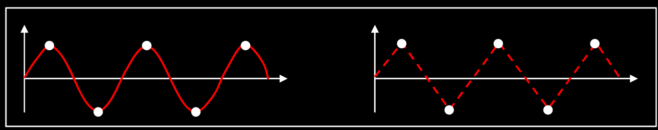
Here sampling rate is 8.5 hz and the frequency is 8 hz



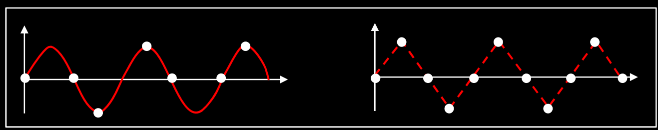
Under-sampled signal can confuse you about its frequency when reconstructed. Because we used to small frequency of sampling. Nyquist teaches us what should be a good frequency.

QUANTISING & CODING

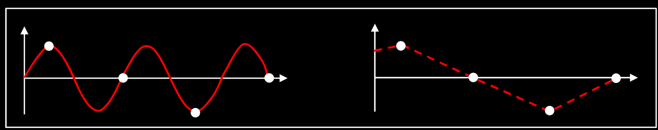
RECOVERY OF A SAMPLED SINE WAVE FOR DIFFERENT SAMPLING RATES



a. Nyquist rate sampling: $f_s = 2f$



b. Oversampling: $f_s = 4f$



c. Undersampling: $f_s = f$

ALIASING

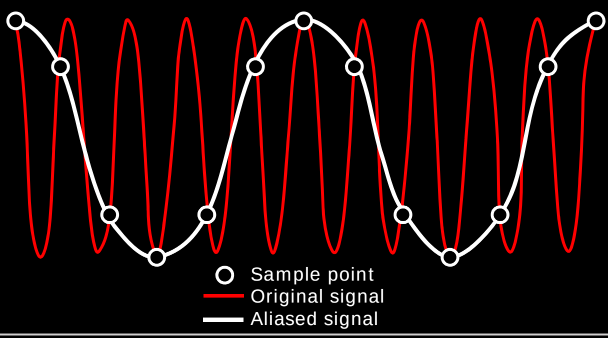
ALIASING

- If a signal is sample at less than Nyquist frequency $2f_{max}$, aliasing can result or obtain wrong digital signal.

ALIASING

If sampling rate is too low, we obtain wrong digital signal. This is called aliasing.

Example 1: 12 equally samples are taken over 10 cycles of an analog signal, so $f_s = 1.2f_0$ with f_0 frequency of the analog signal. Since $f_s < 2f_0$, digitized signal does not describe the original signal accurately.



ALIASING

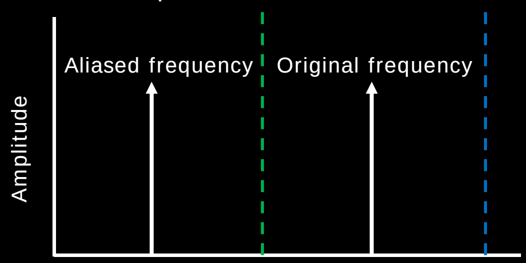
Aliasing frequency (f_a),

Shannon-Nyquist Theorem $f_s > 2^* f_{max}$ (two times of the Nyquist frequency),

Original frequency (f_n), and

Sampling rate (f_s)

An example:



$f_a = 30MHz$ $f_{max} = 50MHz$ $f_n = 70MHz$ $f_s = 100MS/s$

ALIASING

The frequency of the aliased signal (f_a) can be found from the following simple equation:

$$f_a = abs({f_s}^{*} i - f_n)$$

Where i is the closest integer multiple of the sampling rate(f_s) to the signal being aliased (f_n).

ALIASING

The frequency of the aliased signal (f_a) can be found from the following simple equation:

$$f_a = abs({f_s}^{*} i - f_n)$$

Where i is the closest integer multiple of the sampling rate(f_s) to the signal being aliased (f_n).

Example 2:

If the signal is of $\boldsymbol{f_n = 21\text{Hz}}$ and is sampled with $\boldsymbol{f_s = 10\text{Hz}}$, then the aliased frequency would be abs$(i * f_s - f_n) = abs(2 * 10 - 21) = 1\text{Hz}$.

ALIASING

Although sampling at twice the **Nyquist frequency** will ensure that you measure the correct frequency of your signal, it will not be sufficient to capture the shape of the waveform.

If the shape of the waveform is desired, you should sample at a rate approximately **10** time the **Nyquist frequency.**

PART 4.4 APPLICATIONS

APPLICATIONS

Example 3: Range of human hearing

- 20 – 20,000Hz
- We lose high frequency response with age
- Women generally have better response than men
- To reproduce 20KHz requires a sampling rate of 40KHz
- **Below** the sampling rate 40KHz will introduce aliasing according to Shannon-Nyquist Theorem

APPLICATIONS

Example 4: Digital voice telephone transmission

- Voice data for telephony purposes is limited to frequencies less than 4,000Hz
- According to Shannon-Nyquist Theorem, it would take 8,000 samples(2 times 4,000) to capture a 4,000Hz signal perfectly
- Generally, one byte is recorded per sample(256 levels). One byte is eight bits of binary data
- (8 bits * 8,000 samples per second = 64K bps) over a circuit

PART 4.5 QUANTISING & CODING

QUANTISING & CODING

DIGITISING

Digitising Sound

011100011101

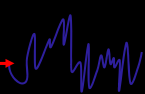
Waveform
(analogue sound)

Computer
Sound Card

Steam of zeros and ones
(digital sound)

Playing back the digital sound file

011100011101



Steam of zeros and ones
(digital sound)

Computer
Sound Card

Waveform
(analogue sound)

QUANTISING & CODING

KEY PARAMETERS

- Sampling frequency
- E.g., 11.025KHz or 22.05KHz or 44.1KHz
- Number of bits per sample
- E.g., 8 bits(256 levels) or 16 bits(65,536 levels)

QUANTISING & CODING

PULSE CODE MODULATION(PCM)

- PCM consists of three steps to digitize an analogue signal:

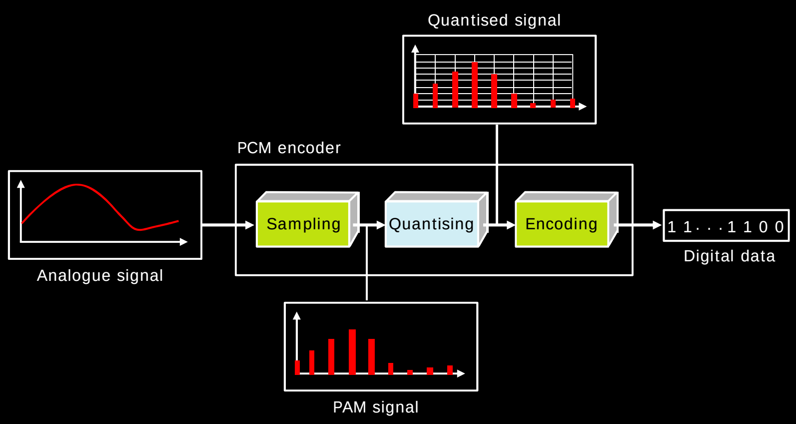
  1) Sampling

  2) Quantisation

  3) Binary encoding
- Before we sample, we have to filter the signal to limit the maximum frequency of the signal as it affects the sampling rate
- Filtering should ensure that we do not distort the signal, ie. remove high frequency components that affect the signal shape

QUANTISING & CODING

COMPONENTS OF PCM ENCODER



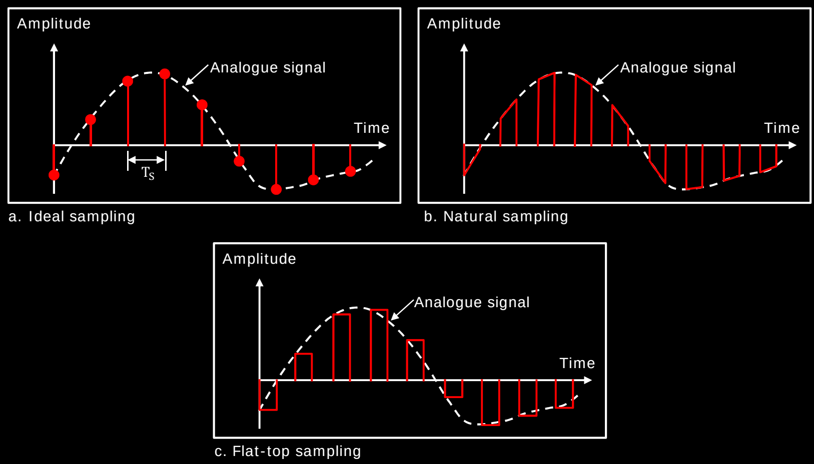
QUANTISING & CODING

SAMPLING METHODS & PAM

- Analogue signal is sampled every T_s secs
- T_S is referred to as the sampling interval
- $f_s = 1/T_s$ is called the sampling rate or sampling frequency
- There are 3 sampling methods:
  1) Ideal- An impulse at each sampling instant
  2) Natural- A pulse of short width with varying amplitude
  3) Flattop- Sample and hold, like natural but with single amplitude value
- The process is referred to as pulse amplitude modulation(PAM) and the outcome is a signal with analogue(non integer) values

QUANTISING & CODING

THREE DIFFERENT SAMPLING METHODS FOR PCM



QUANTISATION

Sampling results in a series of pulses of varying amplitude values ranging between two limits: a min and a max.

The amplitude values are finite between the two limits.

We need to map the finite amplitude values onto a finite set of known values.

This is achieved by dividing the distance between min and max into L zones, each of height Δ.

$$\Delta = (\max - \min)/L$$

ANALOGUE QUANTISATION SIZE OR CODE WIDTH Q

An example: $Q=(V_{max}-V_{min})/N$

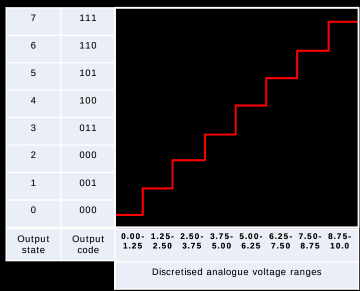
Given $N=8$, $V_{max}=10$ V, $V_{min}=0$;

Analogue Quantisation Size or Code Wide

$Q=(V_{max}-V_{min})/N=(10-0)/\ 8=1.25V$

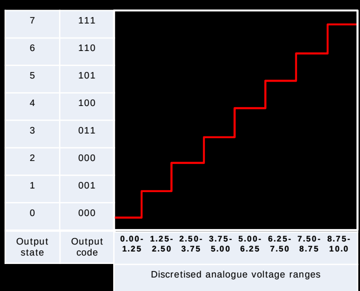
This means that the amplitude of the digitised signal has an error of at most 1.25V

Therefore, the A/D converter can only resolve a voltage within 1.25V of the exact analogue voltage



QUANTISING VS. CODING

- Quantising is the transformation of a continuous analogue input into a set of discrete output states
- Coding is the assignment of a digital code word or number to each output states



Notes:

- Each output state covers a subrange of the overall voltage range
- The step-stair signal represents the states of a digital signal generated by sampling a linear ramp of an analogue signal occurring over the voltage range
- The figure shows how a continuous voltage range is divided into discrete output states, each of which is assigned a unique code

ANALOGUE-TO-DIGITAL (A/D) CONVERTER

Is an electronic device that converts an analog voltage to a digital code

The output if the A/ D converter can be directly interfaced to a digital devices(microcontroller and computer)

The resolution of an A/D converter is the number of bits used to digitally approximate the analog value of the input.

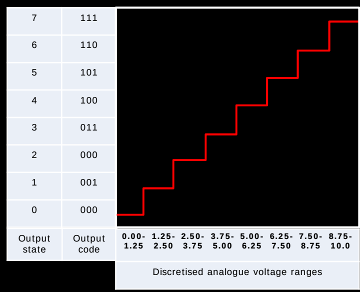
The number of possible states N is equal to the number of bit combinations that can be output from the converter:

$$N=2^n$$

Where n is the number of bits

Most of the commercial A/D converters are with 8/10/12-bit device or 256(2^8), 1024(2^{10}) or 4096(2^{12}).

n=3
$N=2^3=8$
The first column: 8 output states(0, 1,2, 3,4,5,6,7)
The second column: 8 corresponding code word



MIDPOINTS

- The midpoint of each zone is assigned a value from 0 to L-1(resulting in L values)
- Each sample falling in a zone is then approximated to the value of the midpoint

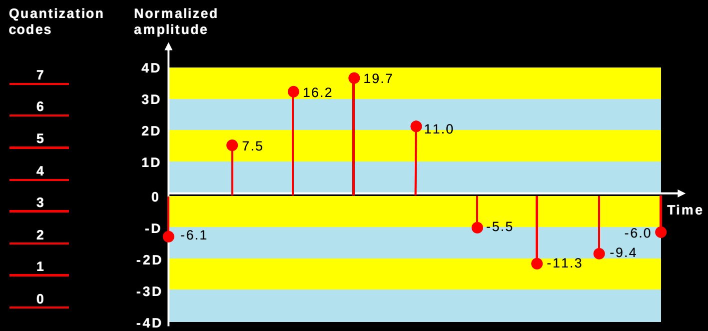
QUANTISING ZONES & MID POINTS

- Assume we have a voltage signal with amplitutes v_{min}= -20V and v_{max}=+20v.
- We want to use l=8 quantization levels.
- Zone width Δ=(20 – (-20))/ 8 = 5
- The 8 zones are: -20 to -15, -15 to -10, -10 to -5, -5 to 0, 0 to +5, +5 to +10, +10 to +15, +15 to +20
- The midpoints are: -17.5, -12.5, -7.5, -2.5, 2.5, 7.5, 12.5, 17.5

ASSIGINING CODES TO ZONES

- Each zone is then assigned a binary code
- The number of bits required to encode the zones, or the number of bits per sample as it is commonly referred to, is obtained as follows: $n_b = \log_2 L$
- Given our example, n_b= 3
- The 8 zone(or level) codes are therefore: 000, 001, 010, 011, 100, 101, 110, and 111
- Assigning codes to zones:
  - 000 will refer to zone -20 to -15
  - 001 to zone -15 to -10, etc.

QUANTIZATION AND ENCODING OF A SAMPLED SIGNAL

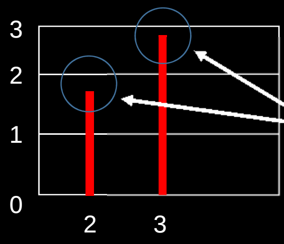


Normalized PAM values	-1.22	1.50	3.24	3.94	2.20	-1.10	-2.26	-1.88	-1.20
Normalized quantized values	-1.50	1.50	3.50	3.50	2.50	-1.50	-2.50	-1.50	-1.50
Normalized error	-0.38	0	+0.26	-0.44	+0.30	-0.40	-0.24	+0.38	-0.30
Quantization code	2	5	7	7	6	2	1	2	2
Encoded words	010	101	111	111	160	010	001	010	010

QUANTISING ERROR

- When a signal is quantized, we introduce an error- The coded signal is an approximation of the actual amplitude value
- The difference between actual and coded value(midpoint) is referred to as the quantization error
- The more zones, the smaller Δ which results in smaller errors
- But, the more zones the more bits required to encode the samples-> higher bit rate

QUANTISING ERROR

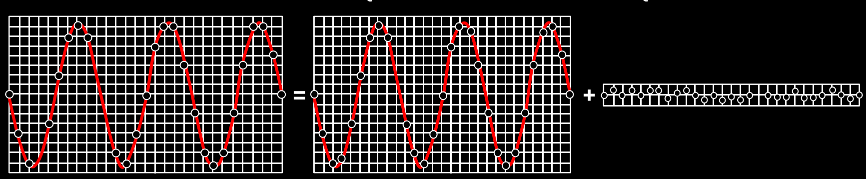


Approximation or Quantising error

Greater error = more noise

QUANTISING ERROR

Ideal Waveform = Quantized Waveform + Quantization Errors



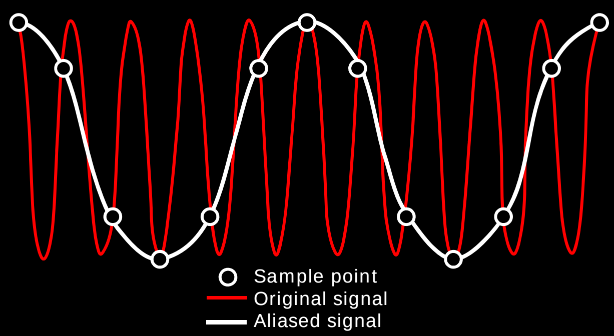
QUANTISING & CODING

A YouTube video– Music Sampling Rate and Resolution Effects

http://www.youtube.com/watch?v=tmzbFkzImQ4

MUSIC & FOURIER SERIES REPRESENTATION

ALIASING – A VIDEO



http://www.youtube.com/watch?v=Fy9dJgGCWZI