NANYANG TECHNOLOGICAL UNIVERSITY

MA2011 MECHATRONICS SYSTEMS INTERFACING

Tutorial 1

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AMPLITUDE LINEARITY

Magical α

The output always changes with the same factor times by the change in the input, i.e.,

$$V_{out}(t) - V_{out}(0) = a(V_{in}(t) - V_{in}(0))$$

where a is the proportional constant (gain).

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AMPLITUDE LINEARITY

Ratio

The output always changes with the same factor times by the change in the input, i.e.,

$$(V_{out}(t) - V_{out}(0)) / (V_{in}(t) - V_{in}(0)) = \alpha$$

where α is the proportional constant (gain).

Q1

Sharp Eyes

If the following input (V_{in}) and output (V_{out}) relationships exist for different measurement systems, indicate whether each is linear or nonlinear and explain why:

a) $V_{out}(t) = 5V_{in}(t);$

b) $V_{out}(t)/V_{in}(t) = 5t;$

c) $V_{out}(t) = V_{in}(t) + 5;$

d) $V_{out}(t) = V_{in}(t) + V_{in}(t);$

e) $V_{out}(t) = V_{in}(t) * V_{in}(t);$

f) $V_{out}(t) = V_{in}(t) + 10t;$

g) $V_{out}(t) = V_{in}(t) + \sin(5)$

h) If $(V_{out}(t) - V_{out}(0)) = \alpha(V_{in}(t) - V_{in}(0))$, what will be the relation between $W_{out}(t) = \beta V_{out}(t) + C$ and $V_{in}(t)$?

ANSWER TO Q1

a) $V_{out}(t) = 5V_{in}(t)$: *Linear or Non-linear?*

Answer: $\alpha = 5$, ➔ *Linear*

Question: what are $V_{out}(0)$ and $V_{in}(0)$?
Answer: Do not really matter:
$V_{out}(t) - V_{out}(0) = 5V_{in}(t) - 5V_{in}(0)$
$= 5(V_{in}(t) - V_{in}(0))$

➔ *Linear*

ANSWER TO Q1

b) $V_{out}(t)/V_{in}(t) = 5t$: *Linear or Non-linear?*

Do we have an $\boldsymbol{\alpha}$ *existing?*

Answer: $V_{out}(t) - V_{out}(0) = 5t\,(V_{in}(t) - V_{in}(0))$

$\alpha = 5t$ *is a function of t, not a constant* ➔ *Non-linear*

Note: Any constant $C \neq 0$, $V_{out}(t)/V_{in}(t) = Ct$ ➔ *Non-linear*

ANSWER TO Q1

c) $V_{out}(t) = V_{in}(t) + 5;$

Answer: $V_{out}(t)-V_{out}(0) = V_{in}(t) +5 – (V_{in}(0)+5)= 1*(V_{in}(t) – V_{in}(0))$

$\alpha = 1$ *is a constant* ➔ *Linear*

Note: Any constant C, $V_{out}(t) = V_{in}(t)+ C$ ➔ *Linear*

ANSWER TO Q1

d) $V_{out}(t) = V_{in}(t) + V_{in}(t)$;

Answer: $V_{out}(t)-V_{out}(0) = V_{in}(t) + V_{in}(t) – (V_{in}(0)+V_{in}(0))= 2*(V_{in}(t) – V_{in}(0))$

$\alpha = 2$ *is a constant* ➔ *Linear*

Note: Any constant C, $V_{out}(t) = CV_{in}(t)$ ➔ *Linear*

$V_{out}(t) = V_{in}(t)+ V_{in}(t)$, ➔ *Linear*

ANSWER TO Q1

e) $V_{out}(t) = V_{in}(t) * V_{in}(t)$;

Answer: $V_{out}(t)-V_{out}(0) = V_{in}(t) * V_{in}(t) – V_{in}(0)*V_{in}(0)$

Proof by contradiction

Assume, we have a constant α so that ➔ *Mostly Non-linear*

$V_{out}(t)-V_{out}(0) = a *(V_{in}(t) – V_{in}(0))$

$V_{in}(t) * V_{in}(t)- V_{in}(0) * V_{in}(0) = a *(V_{in}(t) – V_{in}(0))$

$V_{in}(t) * V_{in}(t)- a V_{in}(t) + (aV_{in}(0)-V_{in}(0) * V_{in}(0))=0$

$D=a*a-4*(aV_{in}(0)-V_{in}(0) * V_{in}(0))$

$V_{in}(t)=(a \pm sqr(D))/2$

ANSWER TO Q1

Note: Any constant C, $V_{out}(t) =V_{in}(t)+ C$ ➔ *Linear*

f) $V_{out}(t) = V_{in}(t) + 10t$;

Answer: $V_{out}(t)-V_{out}(0) = V_{in}(t) +10t –(V_{in}(0) +10*0)= (V_{in}(t) –V_{in}(0)) +10t$

Usually, we do not have a constant α ➔ *Non-linear*

Note: *Any constant C,* $V_{out}(t) =V_{in}(t)+Ct$ ➔ *Non-linear*

Assume, we have a constant α so that

$V_{out}(t)-V_{out}(0) = a *(V_{in}(t) – V_{in}(0))$

$V_{in}(t) + Ct - V_{in}(0)- C0 = a *(V_{in}(t) – V_{in}(0))$

$V_{in}(t) *(1- a) =V_{in}(0) + C0- Ct - aV_{in}(0)$

$V_{in}(t)=(V_{in}(0) + C0- Ct - aV_{in}(0))/(1- a)$

Proof by contradiction

ANSWER TO Q1

g) $V_{out}(t) = V_{in}(t) + C$;

Note: Any constant C, $V_{out}(t) = V_{in}(t) + C$ ➔ *Linear*

Answer: C=Sin(5), ➔ *Linear*

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ANSWER TO Q1

Generalization & Inheritance

h) If we have amplitude linearity wave $V_{out}(t)$ and $V_{in}(t)$: $(V_{out}(t) - V_{out}(0)) = \alpha(V_{in}(t) - V_{in}(0))$,

What will be the relation between $W_{out}(t)$ and $W_{in}(t)$: $W_{out}(t) = \beta V_{out}(t)+C$ and $W_{in}(t) = \beta V_{in}(t)+C$?

$$(V_{out}(t) - V_{out}(0)) = \alpha(V_{in}(t) - V_{in}(0)),$$

$$W_{out}(t) = \beta V_{out}(t)+C$$

$$W_{out}(t) - W_{out}(0) = \beta V_{out}(t)+C - \beta V_{out}(0)-C = \beta (V_{out}(t) - V_{out}(0)) = \alpha \beta (V_{in}(t) - V_{in}(0))$$

$$W_{out}(t) - W_{out}(0) = \alpha(\beta V_{in}(t)+C - \beta V_{in}(0)-C)=\alpha(W_{in}(t) - W_{in}(0))$$

$W_{out}(t)$ and $W_{in}(t)$ *are Linear related*

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ANSWER TO Q1

Mathematic Reasoning

1) *Proof by contradiction*
2) *Find a solution: Existing & but Unique?*

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ANSWER TO Q1

Uniqueness

h) If $(V_{out}(t)- V_{out}(0)) = \alpha(V_{in}(t)- V_{in}(0))$, and $(V_{out}(t)- V_{out}(0)) = \beta(V_{in}(t)- V_{in}(0)) \Rightarrow \alpha = \beta$

Since $(V_{out}(t)- V_{out}(0)) = \alpha(V_{in}(t)- V_{in}(0))$, and $(V_{out}(t) - V_{out}(0)) = \beta (V_{in}(t)- V_{in}(0))$

We have $\alpha(V_{in}(t)- V_{in}(0)) = \beta (V_{in}(t)- V_{in}(0))$

Thus $(\alpha- \beta)V_{in}(t) = (\alpha- \beta) V_{in}(0)$

If $\alpha\neq\beta$, $V_{in}(t)= V_{in}(0)$

Therefore $\alpha=\beta$

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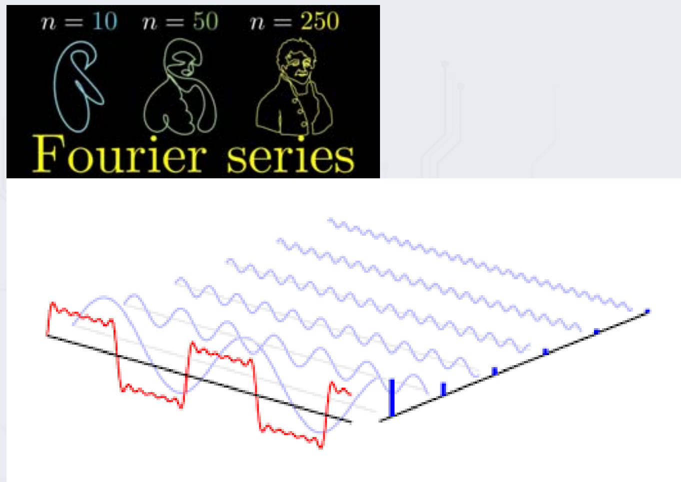
Q2

Complexity vs Simplicity



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Q2

What is the Fourier series and fundamental frequencies (in Hertz) and amplitudes of the following waveforms?

a) $F(t) = 5*\sin(2\pi t)$.
b) $F(t) = 5*\cos(2\pi t)$
c) $F(t) = -5*\sin(2\pi t)$

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FOURIER SERIES REPRESENTATION OF SIGNALS

The Fourier series representation of a periodical waveform *f(t)* is

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t)$$

where

C_0 is the DC component of the signal, and the average value of the waveform over its period

ω_0 is the fundamental or first (lowest) harmonic frequency defined as

$$\omega_0 = \frac{2\pi}{T} = 2\pi f_0$$

f_0 is fundamental frequency in Hertz (Hz).

T is period

ANSWER TO Q2

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t)$$

	Term	Fundamental frequency	Amplitude	Remark
F(t)=5sin(2πt)	n=1	$f_0=\omega_0/2\pi=2\pi/2\pi=1$Hz	5	F(t)=5cos(2πt-π/2)
F(t) = 5cos(2πt)	n=1	$f_0=\omega_0/2\pi=2\pi/2\pi=1$Hz	5	
F(t) = -5cos(2πt)	n=1	$f_0=\omega_0/2\pi=2\pi/2\pi=1$Hz	5	Amplitude is the absolute value

Innovation: Complex ➔ Simple

Learning: Simple ➔ Complex

Peak Amplitude

- In audio system measurements, telecommunications and other areas where the measured is a signal that swings above and below a zero value but is not sinusoidal, peak amplitude is often used.
- This is the maximum absolute value of the signal.

Peak-to-peak Amplitude

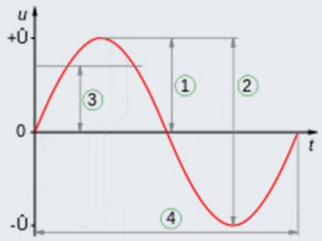
- Peak-to-peak amplitude is the change between peak (highest amplitude value) and trough (lowest amplitude value, which can be negative).
- With appropriate circuitry, peak-to-peak amplitudes of electric oscillations can be measured by meters or by viewing the waveform on an oscilloscope.

Peak Amplitude or Peak-to-peak Amplitude

A sinusoidal curve

(1): Peak amplitude

(2): Peak-to-peak amplitude



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Q3

Usefulness

The power we use at home has a frequency of 60 Hz. What is the period of this sine wave?

Answer: 0.0166s=16.6ms

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ANSWER TO Q3

The power we use at home has a frequency of 60 Hz. The period of this sine wave can be determined as follows:

$$\boxed{T = \frac{1}{f} = \frac{1}{60} = 0.0166\ \text{s} = 0.0166 \times 10^{3}\ \text{ms} = 16.6\ \text{ms}}$$

UNITS OF PERIOD AND FREQUENCY

Unit	*Equivalent*	*Unit*	*Equivalent*
Seconds (s)	1 s	Hertz (Hz)	1 Hz
Milliseconds (ms)	10^{-3} s	Kilohertz (kHz)	10^{3} Hz
Microseconds (μs)	10^{-6} s	Megahertz (MHz)	10^{6} Hz
Nanoseconds (ns)	10^{-9} s	Gigahertz (GHz)	10^{9} Hz
Picoseconds (ps)	10^{-12} s	Terahertz (THz)	10^{12} Hz

Q4

For the Fourier series given by

$$y(t) = 4 + \sum_{n=1}^{\infty}\left(\frac{2n\pi}{10}\cos\frac{n\pi}{4}t + \frac{120n\pi}{30}\sin\frac{n\pi}{4}t\right)$$

where t is the time in seconds,

a. What is the fundamental frequency in hertz and radians per second, rad s^{-1}?

b. What is the period T associated with the fundamental frequency?

c. Express this Fourier series as an infinite series containing sine terms only.

Ans: $f_0 = \frac{1}{8}$ Hz; T = 8 s; $y(t) = 4 + 4n\pi\sum_{n=1}^{\infty}\sin\left(\frac{n\pi}{4}t + 0.05\right)$

FOURIER SERIES REPRESENTATION OF SIGNALS

Define

$$C_n = \sqrt{A_n{}^2 + B_n{}^2}$$

$$\boldsymbol{\phi_n}^* = arctan\left(\frac{A_n}{B_n}\right)$$

Then

$$\boxed{F(t) = C_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega_0 t + \boldsymbol{\phi_n}^*)}$$

That is, a period waveform can be represented by an infinite series of cosine of single amplitude and phase

ANSWER TO Q4

SOLUTION:

Given $$y(t) = 4 + \sum_{n=1}^{\infty} \overbrace{\frac{2n\pi}{10}}^{A_n} \cos\left(\frac{n\pi t}{4}\right) + \sum_{n=1}^{\infty} \overbrace{\frac{120n\pi}{30}}^{B_n} \sin\left(\frac{n\pi t}{4}\right)$$

a. At $n = 1$, we get $\omega_0 = \dfrac{\pi}{4}$ rad s^{-1} or $f_0 = \dfrac{\omega_0}{2\pi} = \dfrac{1}{8}$ Hz.

Note that frequency may be in rad s^{-1} or Hz. When the unit is rad s^{-1}, we use the symbol ω. When the unit is Hz, we use the symbol f.

b. Hence the fundamental period $T = 8$ sec.

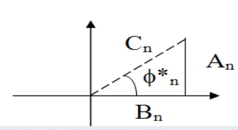
ANSWER TO Q4

c. To convert to the form $y(t) = C_0 + \sum_{n=1}^{\infty} C_n \, sin\left(\dfrac{2n\pi t}{T} + \phi_n^*\right)$

Where $C_n = \sqrt{A_n^2 + B_n^2} = \sqrt{\left(\dfrac{2n\pi}{10}\right)^2 + \left(\dfrac{120n\pi}{30}\right)^2}$

$$C_n = n\pi\sqrt{\left(\frac{2}{10}\right)^2 + \left(\frac{120}{30}\right)^2} \approx 4n\pi$$

$$tan(\phi_n^*) = \frac{A_n}{B_n} = \frac{1}{20} = 0.05$$



ANSWER TO Q4

$$F(t) = C_0 + \sum_{n=1}^{\infty} \left(A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t) \right)$$

$$= C_0 + \sum_{n=1}^{\infty} \sqrt{A_n^2 + B_n^2} \left(\frac{A_n}{\sqrt{A_n^2 + B_n^2}} \cos(n\omega_0 t) + \frac{B_n}{\sqrt{A_n^2 + B_n^2}} \sin(n\omega_0 t) \right)$$

$$= C_0 + \sum_{n=1}^{\infty} C_n \left(\sin(\phi_n^*) \cos(n\omega_0 t) + \cos(\phi_n^*) \sin(n\omega_0 t) \right)$$

$$= C_0 + \sum_{n=1}^{\infty} C_n \boxed{\sin(n\omega_0 t + \phi_n^*)}$$

$$\phi_n^* = arctan\left(\frac{A_n}{B_n}\right), \sin(\phi_n^*) = \frac{A_n}{\sqrt{A_n^2 + B_n^2}}, \cos(\phi_n^*) = \frac{B_n}{\sqrt{A_n^2 + B_n^2}}$$

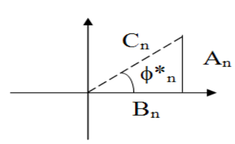
ANSWER TO Q4

c. To convert to the form $y(t) = C_0 + \sum_{n=1}^{\infty} C_n \, sin\left(\frac{2n\pi t}{T} + \phi_n^*\right)$

Where $C_n = \sqrt{A_n^2 + B_n^2} = \sqrt{\left(\frac{2n\pi}{10}\right)^2 + \left(\frac{120n\pi}{30}\right)^2}$

$$C_n = n\pi\sqrt{\left(\frac{2}{10}\right)^2 + \left(\frac{120}{30}\right)^2} \approx 4n\pi$$

$$tan(\phi_n^*) = \frac{A_n}{B_n} = \frac{1}{20} = 0.05$$



$\phi_n^* = \tan^{-1}(0.05) = 0.05$ radians

Hence, $y(t) = 4 + \sum_{n=1}^{\infty} 4n\pi \sin\left(\frac{n\pi t}{4} + 0.05\right)$

FUNDAMENTAL FREQUENCY

For example: $y(t) = 4 + \frac{2\pi}{10}\cos\frac{2\pi}{4}t + \frac{4\pi}{10}\cos\frac{3\pi}{4}t + \frac{8\pi}{10}\cos\frac{4\pi}{4}t + \cdots$

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FUNDAMENTAL FREQUENCY

For example: $y(t) = 4 + \frac{2\pi}{10}\cos\frac{2\pi}{4}t + \frac{4\pi}{10}\cos\frac{3\pi}{4}t + \frac{8\pi}{10}\cos\frac{4\pi}{4}t + \cdots$

This can be written as $y(t) = 4 + \sum_{n=1}^{\infty} \frac{2(n-1)\pi}{10}\cos\frac{n\pi}{4}t$

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FUNDAMENTAL FREQUENCY

Note:

The fundamental frequency is always given by setting $n = 1$. The amplitude of this component may be zero. When this happens, it may not appear in the equation.

For example: $y(t) = 4 + \frac{2\pi}{10}\cos\frac{2\pi}{4}t + \frac{4\pi}{10}\cos\frac{3\pi}{4}t + \frac{8\pi}{10}\cos\frac{4\pi}{4}t + \cdots$

This can be written as $y(t) = 4 + \sum_{n=1}^{\infty} \frac{2(n-1)\pi}{10}\cos\frac{n\pi}{4}t$

When n=1, the fundamental frequency is $\frac{\pi}{4}$ rad s^{-1} or $\frac{1}{8}$ Hz, although the amplitude is zero for this component, and not $\frac{\pi}{2}$ rad s^{-1} or $\frac{1}{4}$ Hz for n = 2.

Phase Angle

$$F(t) = C_0 + \sum_{n=1}^{\infty}\left(A_n\cos(n\omega_0 t) + B_n\sin(n\omega_0 t)\right)$$

$$= C_0 + \sum_{n=1}^{\infty}\sqrt{A_n^2 + B_n^2}\left(\frac{A_n}{\sqrt{A_n^2 + B_n^2}}\cos(n\omega_0 t) + \frac{B_n}{\sqrt{A_n^2 + B_n^2}}\sin(n\omega_0 t)\right)$$

$$= C_0 + \sum_{n=1}^{\infty} C_n\left(\cos(\phi_n)\cos(n\omega_0 t) - \sin(\phi_n)\sin(n\omega_0 t)\right)$$

$$= C_0 + \sum_{n=1}^{\infty} C_n \boxed{\cos(n\omega_0 t + \phi_n)}$$

$$\phi_n = -\arctan\frac{B_n}{A_n}, \cos(\phi_n) = \frac{A_n}{\sqrt{A_n^2 + B_n^2}}, \sin(\phi_n) = -\frac{B_n}{\sqrt{A_n^2 + B_n^2}}$$

$$\phi_n^* = arctan\left(\frac{A_n}{B_n}\right), \sin(\phi_n^*) = \frac{A_n}{\sqrt{{A_n}^2 + {B_n}^2}}, \cos(\phi_n^*) = \frac{B_n}{\sqrt{{A_n}^2 + {B_n}^2}}$$

$$\boldsymbol{\phi_n = \phi_n^* - \frac{\pi}{2}}$$

Phase Angle

$$F(t) = C_0 + \sum_{n=1}^{\infty} (A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t))$$
$$= C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \boldsymbol{\phi_n})$$
$$\boldsymbol{\phi_n} = -\arctan\frac{B_n}{A_n}, \cos(\boldsymbol{\phi_n}) = \frac{A_n}{\sqrt{A_n^2 + B_n^2}}, \sin(\boldsymbol{\phi_n})$$
$$= -\frac{B_n}{\sqrt{A_n^2 + B_n^2}}$$

$$F(t) = C_0 + \sum_{n=1}^{\infty} (A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t))$$
$$= C_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega_0 t + \boldsymbol{\phi_n}^*)$$
$$= C_0 + \sum_{n=1}^{\infty} C_n \mathbf{cos}(n\omega_0 t + \boldsymbol{\phi_n}^* - \frac{\pi}{2})$$
$$\boldsymbol{\phi_n}^* = \arctan\frac{B_n}{A_n}, \sin(\boldsymbol{\phi_n}^*) = \frac{A_n}{\sqrt{A_n^2 + B_n^2}}, \cos(\boldsymbol{\phi_n}^*)$$
$$= \frac{B_n}{\sqrt{A_n^2 + B_n^2}}$$

*

$$\boldsymbol{\phi_n = \phi_n^* - \frac{\pi}{2}}$$

$$\boldsymbol{Sin\ (\phi\ \ ) = cos\ (\phi\ \ - \frac{\pi}{2})}$$