



**NANYANG
TECHNOLOGICAL
UNIVERSITY**

MA2011 MECHATRONICS SYSTEMS INTERFACING

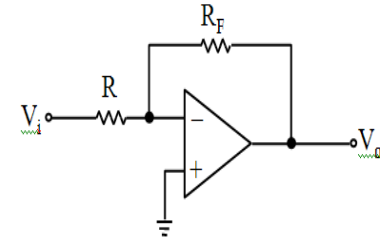
Tutorial 5

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School of Mechanical and Aerospace Engineering**

Operational amplifier:

- Very large (infinite) impedance
- Very small (zero) current



Finite Loop Gain A of Inverting Amplifier

A). Show that the closed loop gain G is a function of A , R and R_F

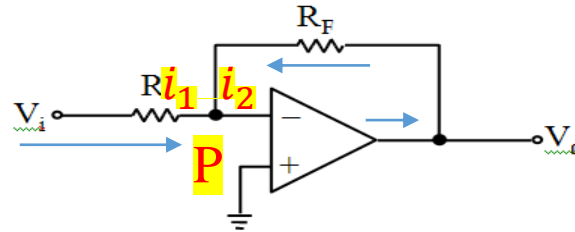
$$G = \frac{-AR_F}{AR + R + R_F} \quad (1)$$

B). For large value A , G can be approximated by

$$G' = \frac{-R_F}{R} \quad (2)$$

A). Show that the closed loop gain

$$G = \frac{-AR_F}{AR+R+R_F} \quad (1)$$



SOLUTION: At point P

$$\bullet i_1 = \frac{(V_i - V_P)}{R} \quad \bullet i_2 = \frac{(V_P - V_o)}{R_F}$$

For an op-amp with infinite input resistance and zero output resistance, $i_1 = i_2$.

$$\text{So, } \frac{(V_i - V_P)}{R} = \frac{(V_P - V_o)}{R_F}$$

$$\text{And } R_F (V_i - V_P) = R (V_P - V_o)$$

$$V_P (R_F + R) = V_o R + V_i R_F$$

But $V_P = -V_o / A$ Because Finite loop gain $A = -V_o / V_P$

$$\text{So } \frac{-V_o(R + R_F)}{A} = V_o R + V_i R_F$$

$$-V_o(R + R_F + AR) = V_i A R_F$$

$$\frac{V_o}{V_i} = \frac{-AR_F}{AR + R + R_F}$$

Therefore, closed loop (op-amp) gain G

$$G = \frac{V_o}{V_i} = \frac{-AR_F}{AR + R + R_F}$$

Therefore, the closed loop gain

$$G = \frac{-AR_F}{AR + R + R_F} \quad (1)$$

$$= \frac{-R_F}{R + \frac{R + R_F}{A}}$$

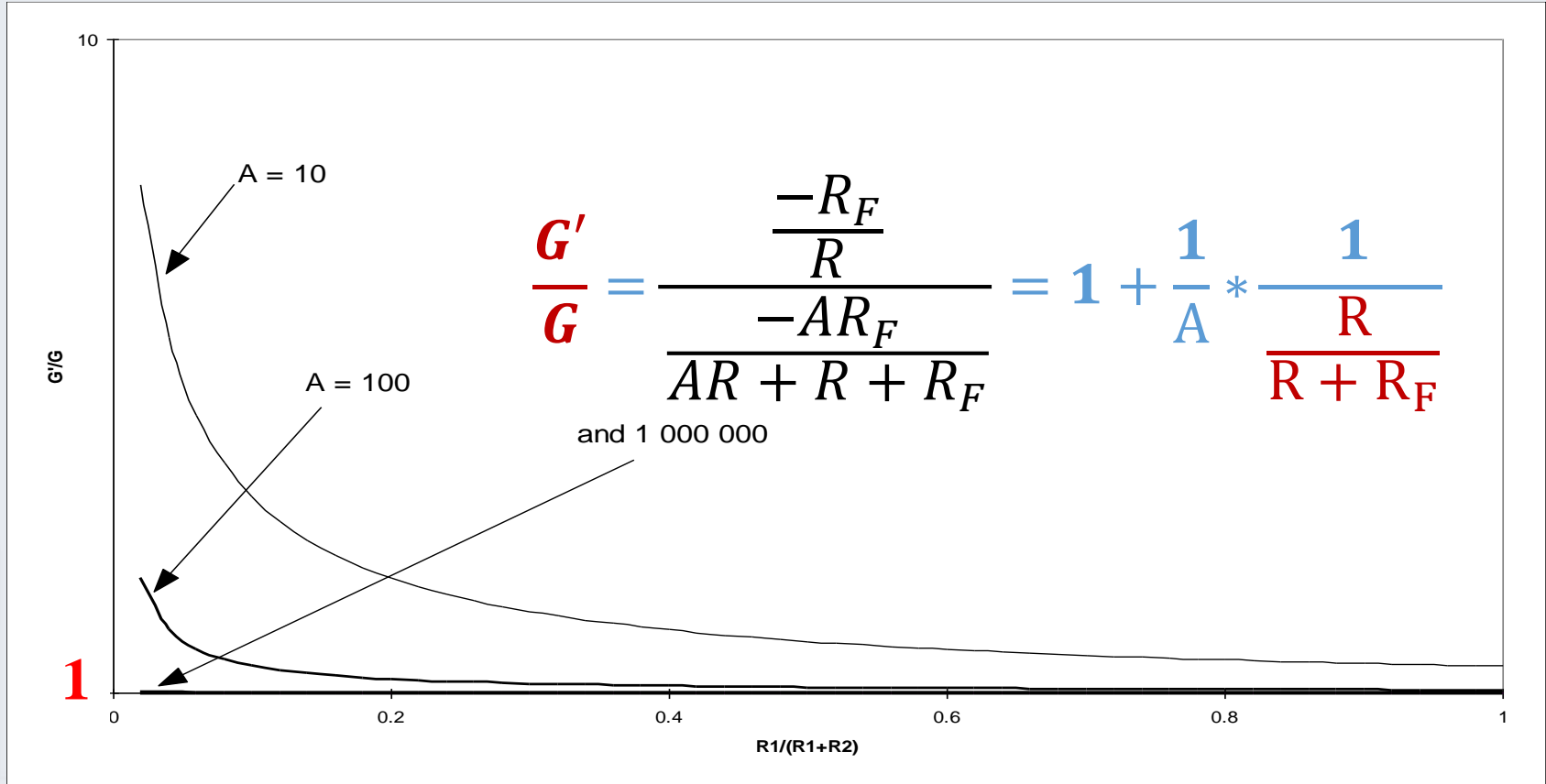
$$A \gg R + R_F, \text{ or } \frac{R + R_F}{A} \rightarrow 0$$

$$G \rightarrow G' \equiv \frac{-R_F}{R}, \text{ or } \frac{G'}{G} \rightarrow 1$$

B). For large value A, G can be approximated by

$$G' = \frac{-R_F}{R} \quad (2)$$

ANSWER TO Q1

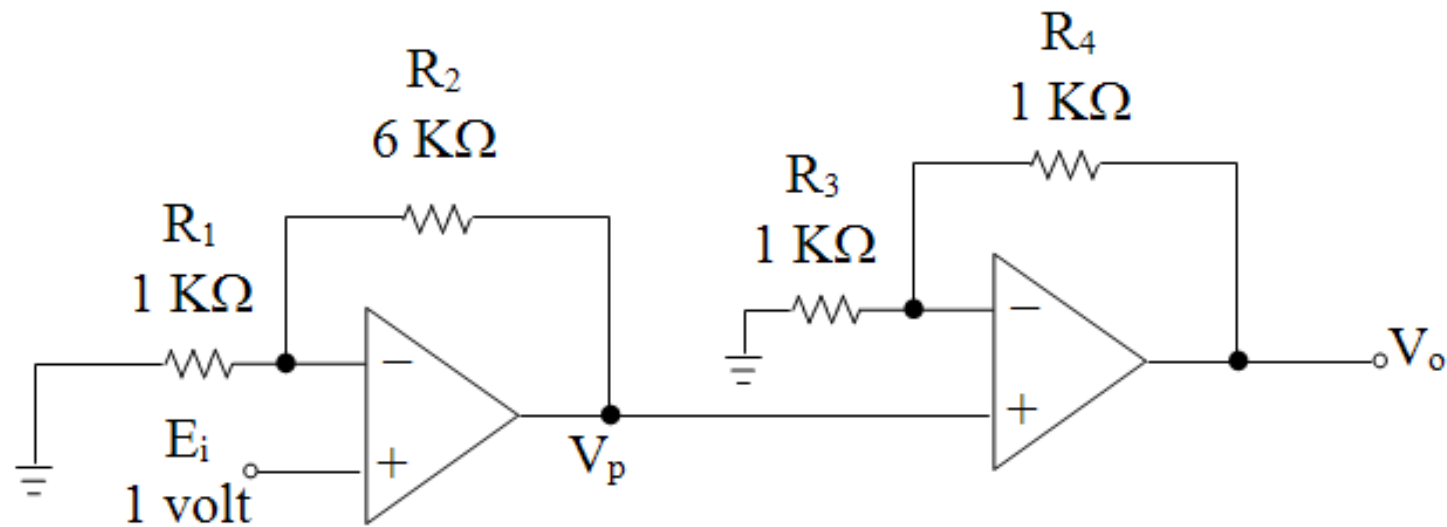


For high open-loop gain A ($10^5 \sim 10^6$), and the values of R and R_F ($k\Omega \sim M\Omega$), G can be approximated by $G' = \frac{-R_F}{R}$

Q2

Find V_o for the following op-amp circuits:

- a) Two non-inverting amplifiers in series.



SOLUTION:

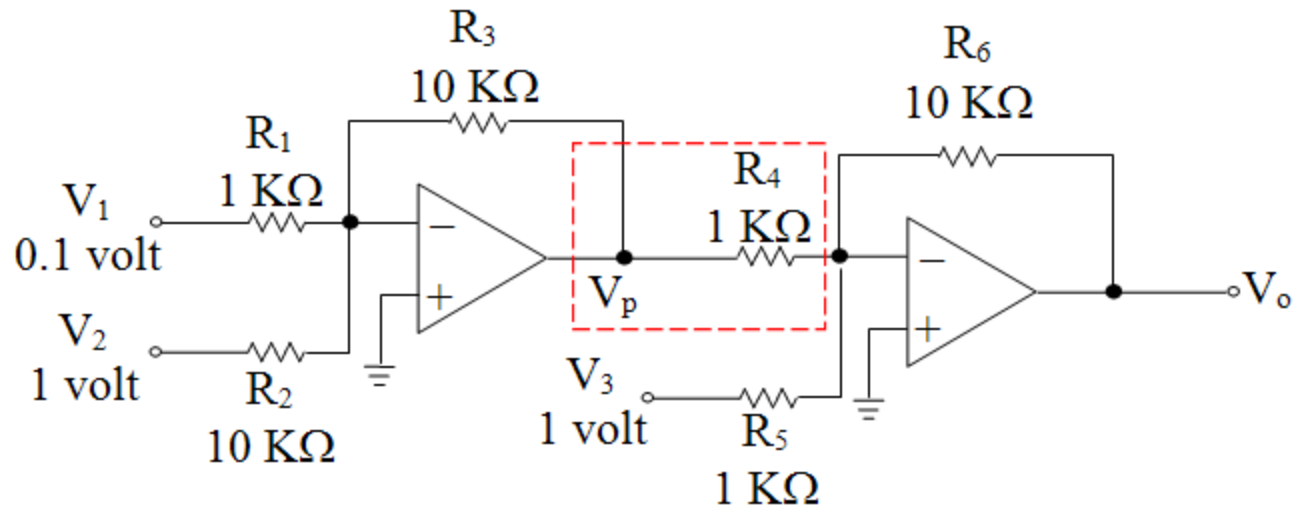
$$G = \frac{R + R_F}{R} = 1 + \frac{R_F}{R} \text{ (General equation)}$$

$$V_p = \left(1 + \frac{R_2}{R_1}\right) V_i = \left(1 + \frac{6}{1}\right) \times 1 = 7 \text{ V}$$

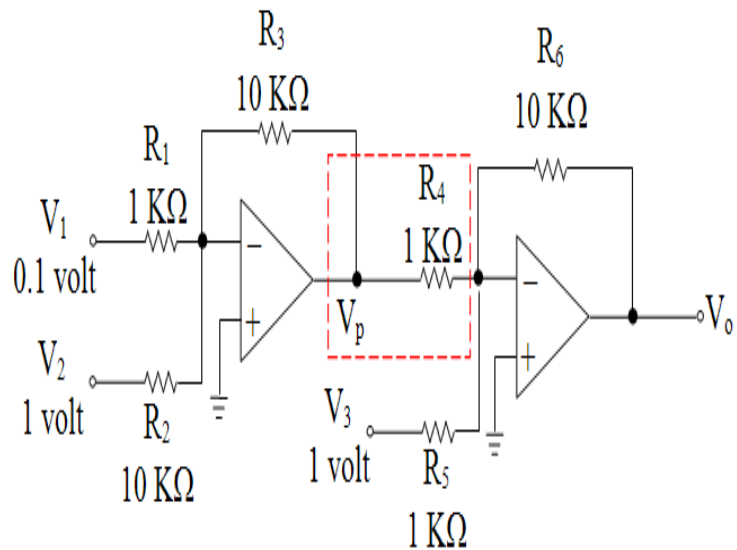
$$V_o = \left(1 + \frac{R_4}{R_3}\right) V_p = \left(1 + \frac{1}{1}\right) \times 7 = 14 \text{ V}$$

ANSWER TO Q2

b) Two summing amplifiers in series.



ANSWER TO Q2



SOLUTION:

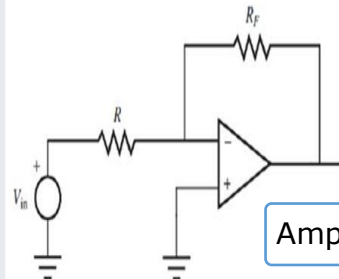
General equation:
$$V_o = -\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots\right) R_F$$

$$\begin{aligned} V_p &= -\frac{R_3 V_1}{R_1} - \frac{R_3 V_2}{R_2} = -10 \times 0.1 - 1 \times 1 \\ &= -2 \text{ V} \end{aligned}$$

$$\begin{aligned} V_o &= -\frac{R_6 V_p}{R_4} - \frac{R_6 V_3}{R_5} = -\{10 \times (-2)\} - (10 \times 1) \\ &= 10 \text{ V} \end{aligned}$$

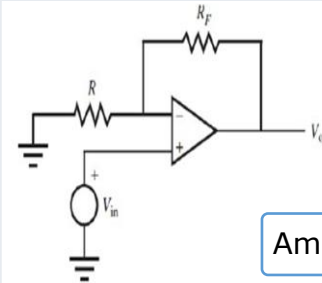
COMPARISON OF INVERTING AND NONINVERTING AMPLIFIER

Inverting



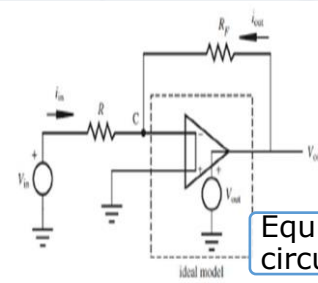
Amplifier

Non-inverting



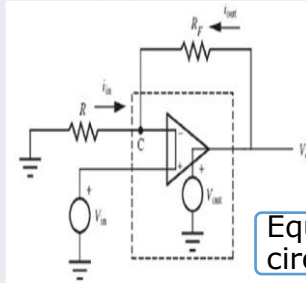
Amplifier

Inverting



Equivalent circuit

Non-inverting



Equivalent circuit

Inverting

$$V_{out} = -\frac{R_F}{R} V_{in}$$

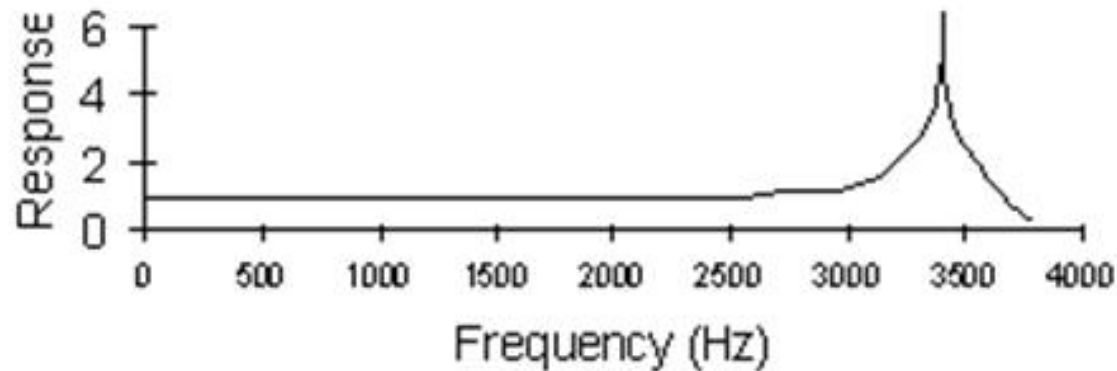
Equation & negative gain

Non-inverting

$$V_{out} = \left(1 + \frac{R_F}{R}\right) V_{in}$$

Equation & positive gain

3. An accelerometer that is used to measure the vibration of a machine has a frequency response as shown in the figure below:



The vibration signal is filtered to ensure that the vibration signal beyond 2.5 kHz does not affect the recording of the measurements.

ANSWER TO Q3

Select an appropriate filter consisting of a resistor, with resistance R , and a capacitor, with capacitance C , that will remove the error due to the frequency response of the accelerometer. Suggest appropriate values of R and C . Sketch the frequency response of the filter, indicating all relevant parameters on the sketch.

KNOWN: To filter off signal of 2.5 kHz or more.

FIND: (i) Filter type, values of R and C , and frequency response of filter

ANSWER TO Q3

SOLUTION:

As frequencies below 2.5 kHz are required and those above have to be removed, a low pass filter is required.

Cut-off frequency must be at least 2.5 kHz (required) and less than about 3 kHz, (where the accelerometer response becomes > 1).

ANSWER TO Q3

Impedance-based relationship between frequency, resistance and capacity.

If the cut-off frequency is 2.5 kHz, $f_c = \frac{1}{2\pi RC}$

$$\text{Or } RC = \frac{1}{2\pi f_c}$$

$$\text{So, } RC = \frac{1}{2\pi \times 2500} = 6.37 \times 10^{-5} \text{ s}$$

Choose any value of R and C such that the combination results in $6.37 \times 10^{-5} \text{ s}$.

For example, $R = 63.7 \text{ k}\Omega$, $C = 1000 \text{ pF}$ ($1 \text{ pF} = 10^{-12} \text{ F}$).

ANSWER TO Q3

The frequency response of the filter, with the relevant parameters is:

$$M(f) = \frac{1}{\sqrt{1 + (f/2500)^2}}$$

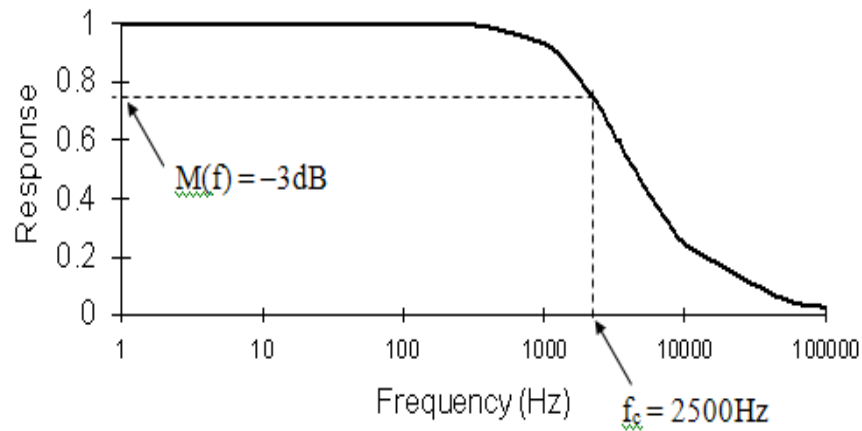
$$M(3400)_{\text{filter}} = \frac{1}{\sqrt{1 + \left(\frac{3400}{2500}\right)^2}} = 0.592.$$

$$\text{Magnitude ratio } M(\omega) = \frac{1}{\sqrt{1 + (\omega\tau)^2}}$$

$$\text{Since } \omega = 2\pi f, \text{ and } \tau = 1/(2\pi f_c), M(f) = \frac{1}{\sqrt{1 + (f/f_c)^2}}$$

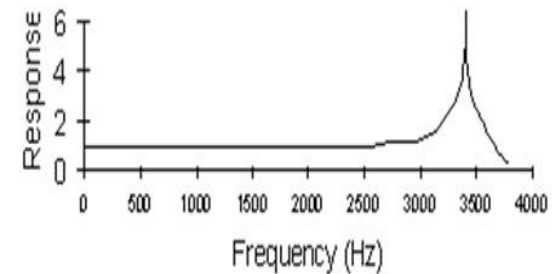
$$\text{For } f_c = 2500, M(2500) = \frac{1}{\sqrt{1 + (1)^2}} = 0.707$$

ANSWER TO Q3



The frequency response of the accelerometer-filter combination is:

An accelerometer that is used to measure the vibration of a machine has a frequency response as shown in the figure below:



The vibration signal is filtered to ensure that the vibration signal beyond 2.5 kHz does not affect the recording of the measurements.