Lecture 1

Mechatronics Definition

-**Interdisciplinary Eng** field comprising design & development of **smart** electromechanical sys (Samili & Mrad)
-The Science that **integrates Mech devices** with electronic ctrl (Stiffler)
-**Integration** of electronics, ctrl Eng and Mech Eng (Bolton)
-An **integrating** theme within the design process [combining] Elect Eng, Comp and Mech Eng (Bradley)
-Methodology used for the **optimal design** of ElectroMechanical Pdts (Shetty and Kolk)

Measurement System

Transducer: device usually converts a phy qty into a time varying voltage (analog signal)
Signal Processor: device to modify analog signal
Recorder: device to display/record signal
Transducer -> Signal Processor -> Recorder

Amplitude Linearity:

$V_{out}(t) - V_{out}(0) = \alpha(V_{in}(t) - V_{in}(0))$

Fundamental Freq

$\omega_0 = \frac{2\pi}{T} = 2\pi f_0$

Fourier Series:

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos n\omega_0 t + \sum_{n=1}^{\infty} B_n \sin n\omega_0 t$$

$$A_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)\,dt, B_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)\,dt$$

$$C_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{A_0}{2}, C_n = \sqrt{{A_n}^2 + {B_n}^2}, \phi_n = -\left(\frac{B_n}{A_n}\right)$$

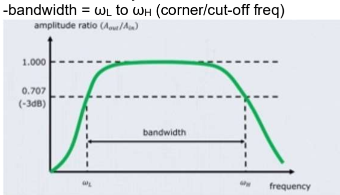
$$F(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \phi_n)$$

$$\cos(\phi_n) = \frac{A_n}{\sqrt{{A_n}^2 + {B_n}^2}}, \sin(\phi_n) = \frac{-B_n}{\sqrt{{A_n}^2 + {B_n}^2}}$$

Bandwidth & Freq Response:

$$dB = 20\log_{10}\left(\frac{A_{out}}{A_{in}}\right)$$

-bandwidth ∝ fidelity
-bandwidth = ω_L to ω_H (corner/cut-off freq)



- -3dB comes from

$$\frac{P_{Out}}{P_{in}} = (\frac{A_{Out}}{A_{in}})^2 = \frac{1}{2}, \alpha(Gain) = \frac{A_{Out}}{A_{in}} = \sqrt{\frac{1}{2}}, dB = 20\log_{10}(\frac{A_{out}}{A_{in}}) \approx -3dB$$

$$V_{in}(t) = A_1 \sin\sin(\omega_0 t) + A_2 \sin\sin(\omega_0 t) + \cdots, A_i' = \left(\frac{A_{out}}{A_{in}}\right)A_i$$

Lecture 2

Periodic Function

$f(t+T) = f(t)$, T is period

Even Function (Fully Cosine Wave)

$f(-t) = f(t), A_n = \frac{4}{T}\int_0^T f(t)\cos(n\omega_0 t)\,dt$

All $B_n = 0$

Odd Function (Fully Sine Waves)

$f(-t) = -f(t), B_n = \frac{4}{T}\int_0^T f(t)\sin(n\omega_0 t)\,dt$

All C_0 and $A_n = 0$

Complex Form

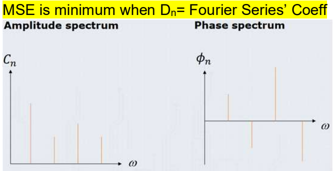
$$F(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}, D_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)e^{-jn\omega_0 t}dt$$

Error of Approximation

$$f(t) \cong \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} = S_n(t)$$

$$\varepsilon(t) = f(t) - S_n(t), MSE = \frac{1}{T}\int_0^T \varepsilon^2(t)dt$$

MSE is minimum when D_n = Fourier Series' Coeff



Fourier Series Conditions:

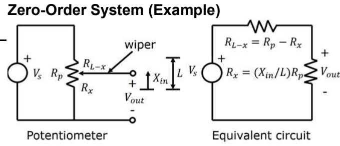
1. f(t) is a **single-value** function except finite no of points
2. For any t_0, the integral $\frac{1}{T}\int_{t_0}^{t_0+T}|f(t)|dt < \infty$
3. f(t) has a finite number of **discontinuities**
4. f(t) has finite number of **Maxima** and **Minima**

[In practice f(t) amplitude function,fulfil 4 Condition]

Lecture 3

Zero-order system	First-order system	Second-order system
•M=0 •N=0 $A_0X_{out}=B_0X_{in}$	•M=0 •N=1 $A_1\frac{dX_{out}}{dt}+A_0X_{out}=B_0X_{in}$	•M=0 •N=2 $A_2\frac{d^2X_{out}}{dt^2}+A_1\frac{dX_{out}}{dt}+A_0X_{out}=B_0X_{in}$

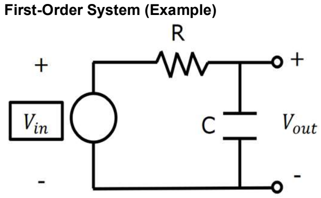
Zero-Order System (Example)



$$V_{out} = \frac{R_x}{R_P}V_s = \frac{V_s}{L}X_{in} \quad A_0X_{out} = B_0X_{in}$$

$X_{out} = KX_{in}$, K is gain/sensitivity

First-Order System (Example)

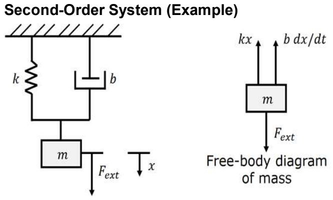


$$RC\frac{dV_{out}(t)}{dt} + V_{out}(t) = V_{in}(t), \tau\frac{dX_{out}(t)}{dt} + X_{out}(t) = KX_{in}$$

$$X_{out} = X_{out_h} + X_{out_p} = C_0e^{-\frac{t}{\tau}} + KA_{in}$$

- $t = \tau$ -> 63.2%
- $t = 4\tau$ -> 98%, assume steady state

Second-Order System (Example)



$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_{ext}(t), \zeta = \frac{b}{b_c} = \frac{b}{2\sqrt{mk}}$$

When F_{ext}=0

$\zeta = 0$ (undamped): $x_h(t) = A\cos(\omega_n t) + B\sin(\omega_n t)$
$\zeta = 1$ (Critical damped): $x_h(t) = (A + Bt)e^{-\omega_n t}$
$\zeta > 1$ (overdamped): $e^{-\zeta\omega_n t}[A\cos(\omega_n t) + B\sin(\omega_n t)]$
$\zeta < 1$ (underdamped):
$x_h(t) = e^{-\zeta\omega_n t}\left[A\cos\left(\omega_n\sqrt{1-\zeta^2}t\right) + B\sin\left(\omega_n\sqrt{1-\zeta^2}t\right)\right]$

Lecture 4

Reasons for Digital Data Acquisition?

-More compact storage, more accurate data, gathering larger/mega dataset, real-time control system and enable data process

Sampling

Shannon & Nyquist Theorem

$f_s > 2f_{max}$, f_s is sampling rate and f_{max} is Nyquist frequency

Time interval: $\Delta t = \frac{1}{f_s}$

Under Sampling: $f_s < 2f_{max}$ | $T_s > \frac{T}{2}$

Aliasing occurs when inaccurate digital signal recorded under sampling $f_a = abs(f_s * i - f_n)$, should sample 10times of f_{max}

Quantising and Coding

Pulse Code Modulation (PCM)

Sampling → Quantisation → Binary Encoding

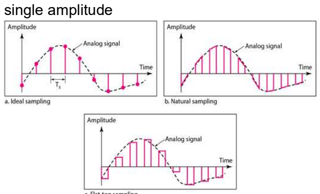
~Sampling Methods~

Pulse Amplitude Modulation (PAM)

Ideal - Impulse at each sampling instant

Natural - Pulse of short width with varying amplitude

Flattop – Sample & hold, like natural but with single amplitude



Quantisation Size (Resolution) ⇒ $\frac{V_{max} - V_{min}}{N}$,

$N = 2^n$, n = no of bits,

zone width = voltage range for each level

No of bits needed for X No of Zones ⇒ $n_b = L$

Quantizing Error = $\frac{Q}{GA} \times 100\%$, Q =resolution, G=Gain, A=amplitude

Quantising – Transformation of a continuous analogue input into a set of discrete output states

Coding – Assignment of digital code word or number to each output states

Lecture 5

Reason for Op-Amps > Transducer

-Transducer's signal too small, too noisy, contain wrong info need a DC offset

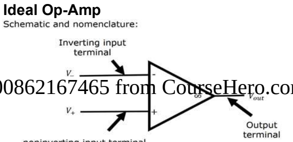
-Low cost, Versatile Integrated Circuits, Single chip manufactured

-Combine with external discrete components to create variety of signal processing circuits

Impedance: $Z_{in} = \frac{V_{out}}{I_{in}}, Z_{out} = \frac{\Delta V_{out}}{I_{in}}$

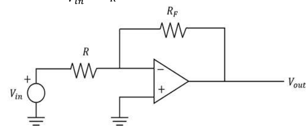
Ideal Op-Amp

Schematic and nomenclature:



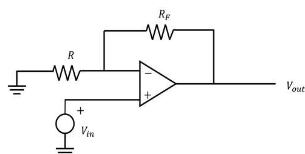
-Infinite Input Impedance, $I_+ = I_- = 0$
-Infinite Gain, $V_+ = V_-$
-Zero output impedance, V_{out} independent of I_{out}
-Gain = $A_v = \frac{V_{out}}{V_{in}}$

Inverting Amp (Invert & Amplify Input Signal): $\frac{V_{out}}{V_{in}} = \frac{R_F}{R}$



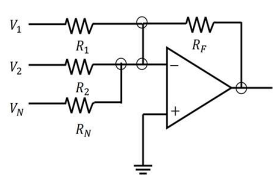
Non-Inverting Amp (amplifies V_{in} without inverting the signal)

$$\frac{V_{out}}{V_{in}} = 1 + \frac{R_F}{R}$$



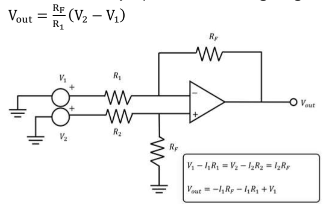
Summer Amp (use to add analog signal):

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = -\frac{V_{Out}}{R_F}$$

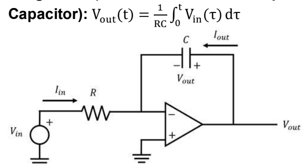


The Adder Circuit

Difference Amp (Subtract analog signals): $V_{out} = \frac{R_F}{R_1}(V_2 - V_1)$



Integrator (Feedback Resistor replace by Capacitor): $V_{out}(t) = \frac{1}{RC}\int_0^t V_{in}(\tau)\,d\tau$

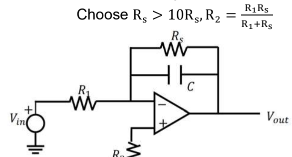


Practical Integrator:

$$C\frac{dV_{out}}{dt} + \frac{V_{out}}{R_s} = I_{out} = -I_{in} = -\frac{V_{out}}{R_1}$$

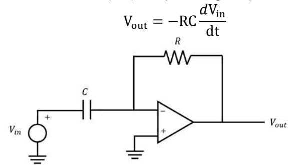
$$\frac{dV_{out}}{dt} + \frac{V_{out}}{CR_s} = -\frac{V_{in}}{CR_1},$$

Choose $R_s > 10R_s$, $R_2 = \frac{R_1R_s}{R_1 + R_s}$

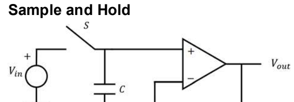


Differentiator (R_{input} replace by Capacitor):

$$V_{out} = -RC\frac{dV_{in}}{dt}$$



Sample and Hold

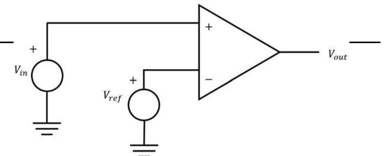


-When switch S close: $V_{out}(t) = V_{in}(t)$
-When switch S open:

$$V_{out}(t - t_{sampled}) = V_{in}(t - t_{sampled})$$

-Choose C with low leakage
-Used for ADC
-Signal must be stabilised
-Voltage-holding C and V follower
-With switch closed

Comparator

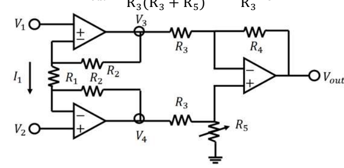


$$V_{out} = \begin{cases} +V_{sat}, & V_{in} > V_{ref} \\ -V_{sat}, & V_{in} > V_{ref} \end{cases}$$

-V_{sat} = Saturation Voltage
-determine whether one signal > the other
-op amp w/o -ve feedback & infinite gain
-op amp saturates
-output is most +ve/-ve value

Instrumentation Amp (Subtracting analog signals & non-inverting)

$$V_{out} = \frac{R_5(R_3 + R_4)}{R_3(R_3 + R_5)}V_4 - \frac{R_4}{R_3}V_3$$



-Diff amp has too little input Z for high output Z
-if input signal level too low, signals include noise
-instrumentation amp solves this problem

Lecture 6

A/D Conversion

1.Sensor→2.Buffer Amp→3. L.Pfilter → 4. Sample&hold ↘
6.Computer Memory ⟵ 5.A/D Convertor

-microprocessors use digital signals ⟶ Less susceptible to noise
-2. Isolate O/P from I/P, provide signal close to full V_{input} of A/D converter
-3. Remove high freq component that cause aliasing
-4. Maintains fixed input values during short conversion time
-5. Contain resolution & analog quantisizing size
-6. Store & process the data with enough storage

Conversion Time (Design, Method of Conversion, Speed of Components),

Aperture Time ⇒ $\Delta v(t) = \frac{dv(t)}{dt}\Delta T_a$

-T_a is the Duration associated with error in digital output

-T_a= Aperture Time, $\Delta v(t) = \Delta amplitude$

A/D Conversion Process

1. Sampling and Holding (S/H)

S/H benefit the accuracy of the A/D Conversion, Minimum sampling rate should be at least twice the highest data frequency of analog signal

2. Quantising and Encoding (Q/E)

Q/E smallest change in analog signal will result in a change in digital output. $\Delta V = \frac{V_{ref}}{2^n}$, ΔV = Resolution,

n = no of bits in digital output, 2^n = Number of states,

V_{ref} = Reference voltage range.

The resolution represents the quantisation error inherent in the conversion f the signal to digital form.

Quantising: Partition the reference signal range into no of discrete quanta then match the input signal to the correct quantum

Encoding: Assign a unique digital code to each quantum then allocate the digital code to the input signal

Ways to improve Accuracy of A/D Conversion

-Increase resolution to improve accuracy in measuring amplitude of analog signal
-Increase sampling rate to increase maximum frequency that can be measured

Advantages of A/D Conversion

-digital signal is more robust to noise and can easily be recovered, corrected and amplified

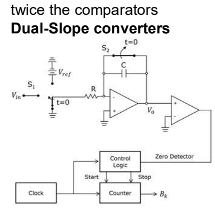
Successive approx. for ADC

-N steps to complete the conversion from the most significant bit (MSB), switch on MSB if analog input > DAC output MSB = On (1), else MSB = 0, Next bit on. Initial Guess = 0.5*(Full Scale)*(Max)
-pros: high spd. and good reliability, medium accuracy compared to other ADC types, good tradeoff btwn. spd. and cost, capable of outputting binary number in serial
-cons: higher resolution a ADC slow, speed limited to 5 MSPS

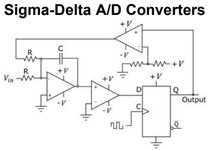
Flash ADC

-N resistors divides the V_{ref} into N equal intervals
-N-1 comparators determines N voltage interval
-Combinational logic then translates information provided
-pros: v. fast, v. simple operational theory, spd. only limited by gate + comparator propagation delay
-cons: expensive, prone to glitches in output, each additional bit of resolution requires twice the comparators

Dual-Slope converters



Sigma-Delta A/D Converters



1)input oversampled & goes to integrator 2)integrator compared to ground 3)Iterates & produce serial bit stream 4)output is a serial bit stream with # of 1's proportional to V_{in} 5)sigma-delta modulator automatically adjust its output 6)The integrator keeps building until the error is once again forced to zero

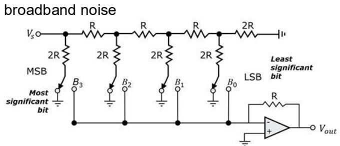
Pros: High Resolution, No need for Precision components

Cons: Slow due to over sampling, only good for low bandwidth

Type	Speed (relative)	Cost (relative)
Dual-Slope	Slow	Med
Flash	Very Fast	High
Successive Approximation	Medium Fast	Low
Sigma-Delta	Slow	Low

DAC

-Problems: Finite word length, Loudest sounds need room so normal sounds don't use the entire range as problems occur at low levels where sounds are represented by one or two bits, high distortions results, dithering adds low level broadband noise



-Digital input to DAC is 4bit Binary number

-Each bit in the circuit controls a switch between ground and the inverting input of Op Amp

Other Notes

First Order waveform: $y = y_0 + Y\sin(\omega t \pm \phi)$

Dynamic Error: $\delta(\omega) = 1 - M(\omega)$

First order system, output lags input and M(ω)<1

Magnitude Ratio: $\frac{1}{\sqrt{(1+\omega\tau)^2}}$

Second Order System Frequency Response:

$$\frac{X}{F} = \frac{k^{-1}}{-\frac{\omega^2}{\omega_0^2} + \frac{j\omega}{Q\omega_0} + 1}$$

Sensors

Transducer: converts one form of energy into another

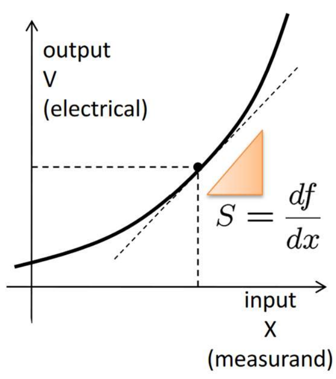
Sensors: produce (electrical) O/P signal for sensing physical phenomenon

Passive Sensors do not req ext power, draw from I/P signal

Null Type: Deflection is balance by opposing calibrated F

Basic Concepts

$$v_{out} = f(x)$$



I/O Characteristic Function/Response

-Input: Stimulus or measurand (T,P,δ)

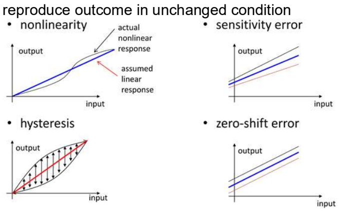
-Output: Electrical Signal (V,I,f,ϕ)

Sensitivity S: O/P variation / I/P Variation: $S = \frac{df}{dx}$

Resolution: Minimum change of the measurand

Accuracy: difference of measurement from true value (%FS, Full Scale)

Repeatability: how well a system/device can reproduce outcome in unchanged condition



Resistance Temperature Detectors (RTD)

-Based on changes of R with T

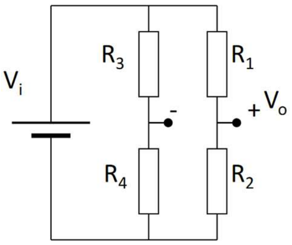
~Metal wire on insulating support: eliminate Mech strain

~Encasing: minimize environment influence (e.g corrosion)

Linearity Range for Limited Range:

$$\frac{R}{R_0} = 1 + \alpha(T - T_0)$$

Wheatstone Bridge



Bridge Eqn: $\frac{V_o}{V_i} = \frac{R_2}{R_1+R_2} - \frac{R_4}{R_3+R_4}$

Bridge Balance Condition: $V_o = 0 \Leftrightarrow R_1R_4 = R_2R_3$

Low resistance (conductors), subject to self-heating

Lead-Wire Effects:

2-Wires(long wire subject to T-R changes): $RTD + 2(r0) = R1$

3-Wires: RTD+r0 = R1+r0 → RTD = R1

Thermistors

Ceramic-like semiconductors

-R9 much large than RTD

R decreases rapidly with T

-High sensitivity, Ruggedness, Fast time response

$$R = R_0 e^{\beta(\frac{1}{T} - \frac{1}{T_0})}$$

Resistive Sensors (Potentiometer [pot])

-3 Terminal electromechanical device based on a conductive wiper against fixed resistive element

-Many Varieties(Rheostats,Trimmers,Volume Ctrl)

-Precision pot: Manually or Digitally Tuneable

Linear Variable Differential Transformer(LVDT)

A type of electrical transformer

-Measuring linear displacement

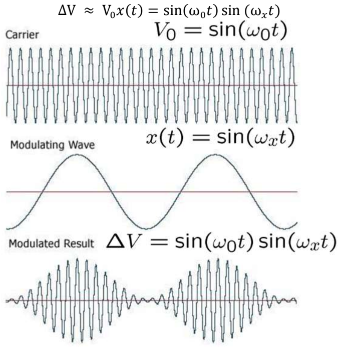
-Variable coupling via sliding ferromagnetic core

~Primary coil(AC Driven,kHz), 2 secondary coils

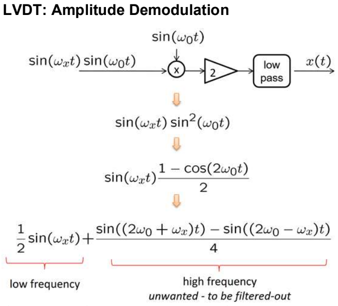
-Differential V: $V_{OUT} = \Delta V = V_2 - V_1 \simeq x(t)V_0$

LVDT: Amplitude Modulation

$$\Delta V \approx V_0 x(t) = \sin(\omega_0 t)\sin(\omega_x t)$$



LVDT: Amplitude Demodulation



Capacitive Sensing (Proximity Sensing)

Ideal case: Infinite parallel plates

Gauss's Law: $Q = \iint_\Sigma \epsilon_0\epsilon_r \mathbf{E} dS$

$C = \frac{Q}{V} = \frac{\epsilon_0\epsilon_r ES}{Ed} = \frac{\epsilon_0\epsilon_r S}{d}$

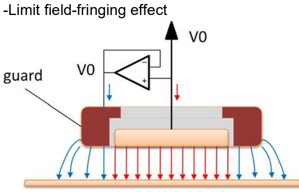
AC Interfacing: AC Bridge,AC Driving,Modulation

Lorentz F: $\vec{F} = q\vec{V} \times \vec{B}$

Lorentz's Law: $\vec{F} = (\vec{\imath} \times \vec{B})L$

Guard Electrode

-Limit field-fringing effect



Light Detectors

Photo-resistors, Photo-diodes(load resistance, op amp)

Photo-transistors

-Transmissive(photo-interrupter)

-Reflective(resistive load,current-voltage op amp)

Digital Encoders

converts motion(linear/rotary) into sequence of digital pulses

Optical Transmitters/Receiver pairs

-Glass/Plastic material photographically patterned

Hall Effect Sensors

-Coupled with magnetic rings/bars

Incremental Encoders (Min 2 Tx/Rx Pairs)

-Encoding steps and direction

-Simpler Design: single pair not sufficient to encode the direction, 2 Tx/Rx pairs plus a 'reset' position required

-Quadrature signals: ¼ cycle out-of-phase

Absolute encoders

-n Tx/Rx pairs for coding 2^n sectors

-angular n-bits encoders

~360° / 2^n resolution

~more expensive (require n Tx/Rx pairs)

~CAVEAT: spurious states may arise from contemporary transitions

~natural binary codes vs gray codes:

Contemporary transitions might lead to temporary spurious states

Gray code ensures no contemporary transitions

DC Motor

-Stator (External, Fixed), Rotor (Internal,Rotates)

-actuation principle: stator field & rotor fields are always orthogonal (maximum torque)

-with commutation: maintains unstable equilibria, constant motion

-without commutation: there is a stable equilibria, motion ceases

- No of poles increase = constant τ increases

Armature Eqn: $V = Ri + L\frac{di}{dt} + e$

Mechanical Eqn: $J\dot{\omega} + b\omega = \tau_e - \tau_L$

Electromechanical Coupling:

$\begin{cases} \tau_e = K_t i \\ e = K_e\omega \end{cases}, \tau_e\omega = ei \Leftrightarrow K_e = K_t \triangleq K_a$

$V \propto \tau$, $V \propto$ spd, Inductance $\propto$ Inertia, R $\propto$ damping

C $\propto$ Compliance, VI $\propto$ $\tau\omega$

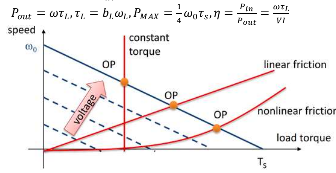
Mechanical Parallel $\propto$ Electrical series

$\omega = \frac{K_aV - R\tau_L}{Rb + K_a^2}$, Friction: $\tau_L = b_L\omega$

Nonlinear Friction: $\tau_L = f(\omega)$

Inertial: $\tau_L = I\frac{d\omega}{dt} = 0$ @Steady State

$P_{out} = \omega\tau_L, \tau_L = b_L\omega_L, P_{MAX} = \frac{1}{4}\omega_0\tau_s, \eta = \frac{P_{in}}{P_{out}} = \frac{\omega\tau_L}{VI}$



Diving DC Motor

-Power amplifiers: Large power dissipation Over-heating of the amplifier

-Pulse Width Modulation(PWM): Switching motor on and off continuously

-DCM is 2[nd] order low pass filter

Inductive kickback

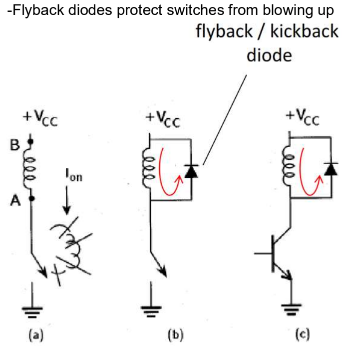
-voltage across inductor $v = L\frac{di}{dt}$

- i decrease, $V = V_B - V_A$ quickly decrease, V_A quickly increases

-steady-state I_{on} through inductor, cannot immediately go to 0A when switch is open

-changing I induces a V across inductor, making the V_A> V_B causing switch or relay to arc over.

-Flyback diodes protect switches from blowing up



DC Brushless Motor

-Stator field rotating, Rotor field given by permanent magnet

-maximize τ_{output} by keeping rotor and stator field orthogonal

-Select appropriate switches to determine desired Ti-Tj Torque

Strain Gauges

-determine safe loading conditions of mechanical structures

Electrical resistance strain gauge

-Thin metal foil: typically constantan

-patterned onto plastic backing material

-bonded onto mechanical structure(stress is inferred from solid mechanics principles)

- $S = \frac{dL}{L}$ (+ve if tensile, -ve if compression)

-Poisson's Ratio: $\nu = -\frac{\text{lateral strain}}{\text{axial strain}}$

$R = \rho\frac{L}{A}, \frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} = \frac{d\rho}{\rho} + \frac{dL}{L} - (\frac{dw}{w} + \frac{dh}{h})$

-Lateral Strain: $\frac{dw}{w} = \frac{dh}{h} = -\nu\frac{dL}{L} = -\nu S$

-Gauge Factor: $\boldsymbol{G} = \frac{d\rho}{\rho}\frac{1}{S} + 1 + 2\nu, \frac{dR}{R} = (\boldsymbol{G})S$

-piezoresistivity = $\frac{d\rho}{\rho}\frac{1}{S}$

Transverse sensitivity: $dR = R_0\mathbf{G}S$

-the larger R_0, the Larger dR

-long thin wires allow larger R_0 (must aligned with axial strain S^a)

-long wires assemble in the form of serpentine

-end loops, aligned with transverse axis, made thicker to reduce sensitivity S^t

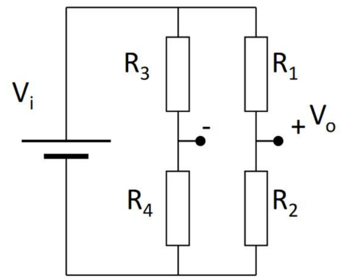
Materials

-Best: constantan, ferry alloys

-typical strain range: $S \sim 1 - 10^4\mu S$, $\boldsymbol{G} \sim 2$

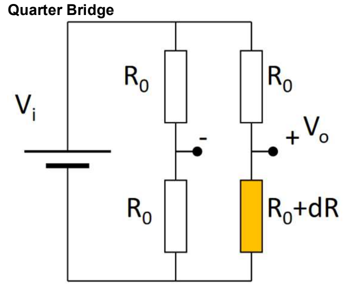
-challenge: detecting small resistance changes

Wheatstone Bridge 1[st] Order approximation



$$\frac{dV_o}{V_i} = \frac{1}{4}\left(\frac{dR_2}{R_2} - \frac{dR_1}{R_1} + \frac{dR_3}{R_3} - \frac{dR_4}{R_4}\right)$$

Quarter Bridge

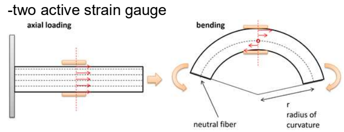


one-gauge bridge

$\frac{dV_o}{V_i} \approx \frac{1}{4}\frac{dR}{R_0} = \frac{1}{4}GS$ **Poisson Gauge** $\frac{dV_o}{V_i} = \frac{1}{4}G(1+\nu)S$

Half Bridge

-two active strain gauge



Axial Loading: $\frac{dV_o}{V_i} = \frac{1}{2}GS^a$, Bending: $\frac{dV_o}{V_i} = \frac{1}{2}GS^b$

Apparent strain

-combinations of different mechanical loading

Top: $S_1 = S^a + S^B$, Bottom: $S_1 = S^a - S^B$

Additional Notes:

$$f_c = \frac{1}{2\pi RC}, \tau = RC$$

Relative Quantization error:

$$\frac{\text{Max } e_q}{|E|_{ADC-Input}} = \frac{Q}{G(amplitude)} \times 100\%$$