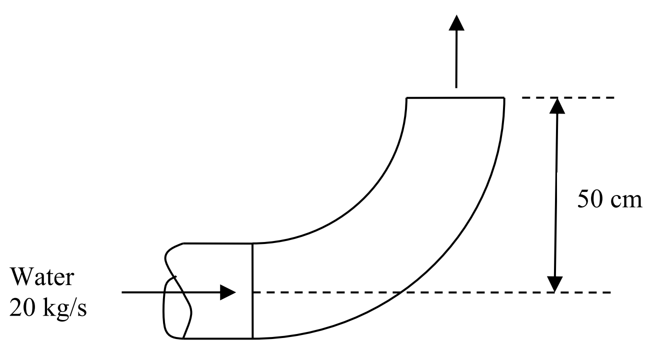
Fluid Mech Tutorials

Fluid Mechanics (Nanyang Technological University)

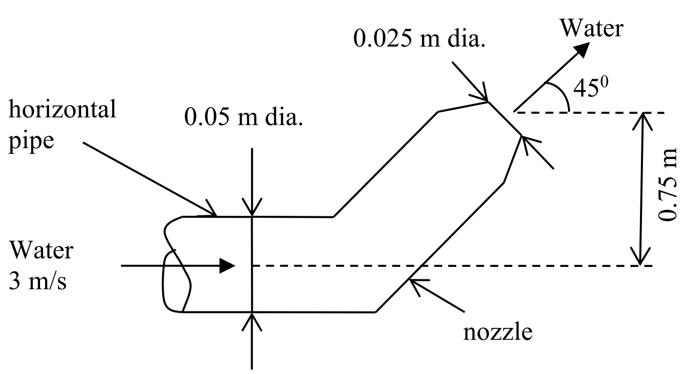
MA3006 FLUID MECHANICS

Tutorial 1 – Momentum equation

1. Water flowing through a horizontal pipe is directed upward by a 90^0 elbow. The mass flow rate of water is 20 kg/s and the diameter of the entire elbow is 10 cm. The elbow discharges the water into the atmosphere. The elevation difference between the centres of the exit and the inlet of the elbow is 50 cm. The weight of the elbow and the water in it is considered to be negligible. Determine the gauge pressure at the centre of the inlet of the elbow and the anchoring force to hold the elbow stationary. Density of water is 1000 kg/m^3. Assume frictionless flow.
Ans: 4.905×10^3 N/m^2, 102.94 N



2. A 45^0 inclined nozzle is attached to a horizontal pipe. Water flows into the nozzle inlet at 3 m/s and leaves the nozzle exit into the atmosphere. The diameters of the nozzle inlet and outlet are 0.05 m and 0.025 m, respectively. The height difference between the nozzle outlet and inlet is 0.75 m. Determine the pressure at the nozzle inlet and the horizontal and vertical force components to hold the nozzle stationary. Assume frictionless flow and the weights of the nozzle and the water within the nozzle to be negligible. Density of water is 1000kg/m^3.
Ans: 74.858×10^3 N/m^2, F_h = 114.67 N, F_v = 49.98 N



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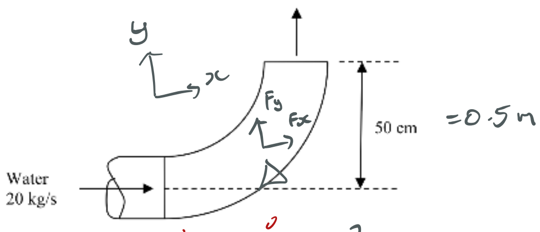
Ans: 4.905×10³ N/m², 102.94 N



$$P_1 + \frac{\rho V_1^2}{2} + \cancel{\rho g h_1} = \cancel{P_2} + \frac{\rho V_2^2}{2} + \rho g h_2$$

$$V_1 = V_2$$

$$P_1 = \rho g h_2$$
$$= 1000(9.81)(0.5)$$
$$= 4905 \ N/m^2$$

$$\dot{m} = \rho \dot{Q}$$
$$= \rho(\text{Area } v) \quad , v = V_1 = V_2$$
$$20 kg/s = (1000)\left(\frac{\pi}{4}(0.1)^2\right) v$$
$$v = 2.54647 \ m/s$$

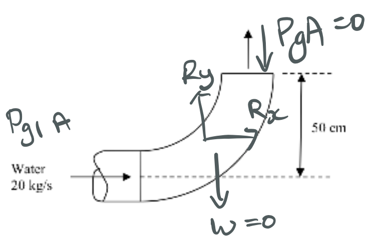
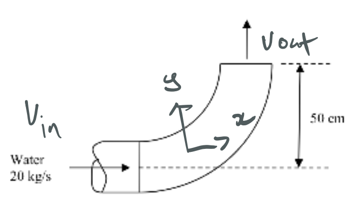
$F = ma$

$F \Rightarrow N$

$P \Rightarrow N/m^2$

$P_g A \Rightarrow N$

To calculate force

Free Body	Momentum
	

$\Sigma F_x = P_{g1}A + R_x$

$\Sigma F_y = R_y$

inlet

$V_{1x} = V_{in}$

$V_{1y} = 0$

$\therefore \dot{M}_{1\,in\,x} = \dot{m} V_{1x}$

$\dot{M}_{1\,in\,z} = 0$

<u>output</u>

$V_{2x} = 0$ $\therefore \dot{M}_{2\,out\,x} = 0$

$V_{2y} = V_{out}$ $\dot{M}_{2\,out\,z} = \dot{m} V_{2y}$

$$\Sigma F_x = (\dot{M}_{out} - \dot{M}_{in})$$

$$P_{g1} A + R_x = (0 - \dot{m} V_{1x})$$

$$R_x = -\dot{m} V_{1x} - P_{g1} A$$

$$= -20(2.5464) - 4905(\tfrac{\pi}{4}(0.1)^2)$$

$$= -89.4517\ N$$

$$\Sigma F_y = (\dot{M}_{out} - \dot{M}_{in})$$

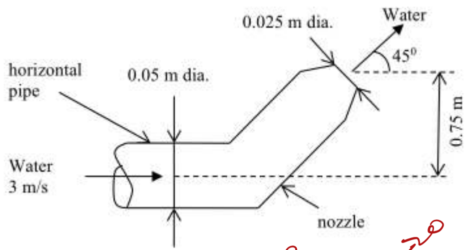
$$R_y = (\dot{m} V_{2y} - 0)$$

$$= 50.928$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= 102.933\ N \;//$$

2. A 45^0 inclined nozzle is attached to a horizontal pipe. Water flows into the nozzle inlet at 3 m/s and leaves the nozzle exit into the atmosphere. The diameters of the nozzle inlet and outlet are 0.05 m and 0.025 m, respectively. The height difference between the nozzle outlet and inlet is 0.75 m. Determine the pressure at the nozzle inlet and the horizontal and vertical force components to hold the nozzle stationary. Assume frictionless flow and the weights of the nozzle and the water within the nozzle to be negligible. Density of water is 1000kg/m^3.
Ans: 74.858×10^3 N/m^2, $F_h = 114.67$ N, $F_v = 49.98$ N



$$P_1 + \tfrac{1}{2}\rho V_1^2 + \cancel{\rho g h_1}^{\,0} = \cancel{P_2}^{\,0} + \tfrac{1}{2}\rho V_2^2 + \rho g h_2$$

$$A_1 V_1 = A_2 V_2$$

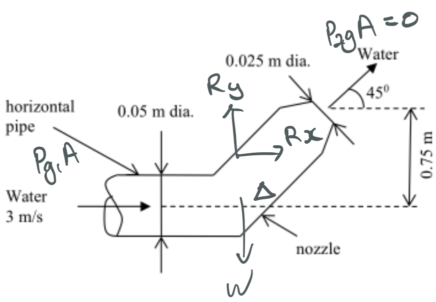
$$\frac{\pi}{4}(0.05)^2(3\,m/s) = \frac{\pi}{4}(0.025)^2 V_2$$

$$V_2 = 12\,m/s$$

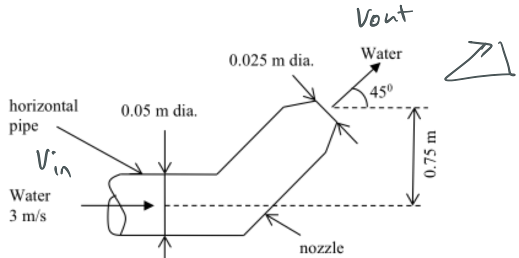
$$P_1 = \tfrac{1}{2}(1000)(12)^2 - \tfrac{1}{2}(1000)(3)^2 + (1000)(9.81)(0.75)$$

$$= 74857.5\ N/m^2$$

Free body diagram



momentem



$V_{in}\,x = V_{1x}$

$V_{in}\,y = 0$

$M_{in}\,x = \dot{m} V_{1x}$

$M_{in}\,y = 0$

$V_{out}\,x = V_2 \cos 45$

$V_{out}\,y = V_2 \sin 45$

$M_{out}\,x = \dot{m} V_2 \cos 45$

$M_{out}\,y = \dot{m} V_2 \sin 45$

$\sum F_x = P_{1g}A + R_x$

$\sum F_y = R_y$

$\dot{m} = \rho Q$
$= \rho AV$

$\sum F_x:$

$P_{1g}A + R_x = \dot{m}\, V_2 \cos 45 - \dot{m} V_{1x}$

$R_x = -\left(74857.5\left(\frac{\pi}{4}(0.05)^2\right)\right) + (1000)\left(\frac{\pi}{4}(0.05)^2\right)(3)\ (12)\cos 45 - (1000)\left(\frac{\pi}{4}(0.05)^2\right)(3)(3)$

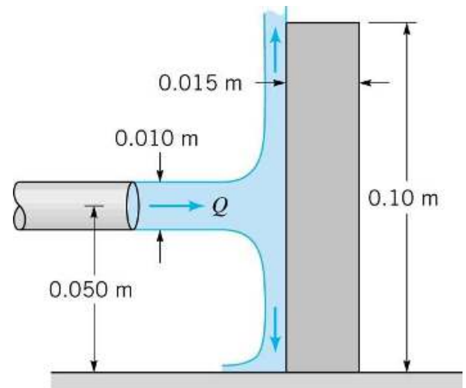
$= -114.671\text{N}$ //

$R_y = \dot{m}\, V_2 \sin 45$

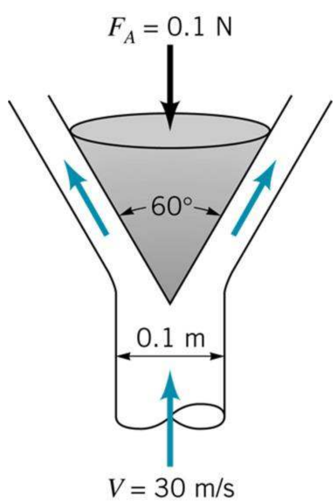
$= (1000)\left(\frac{\pi}{4}(0.05)^2\right)(3)\ (12\sin 45)$

$= 49.982\text{N}$ //

3. A 10-mm diameter jet of water is deflected by a homogeneous rectangular block (15 mm by 200 mm by 100 mm) that weighs 6 N. A Determine the minimum volume flowrate needed to **tip** the block. Assume frictionless flow and negligible effects on the gravity of the water jet. Density of water is $1000 kg/m^3$.
Ans: 2.66×10^{-4} m^3/s



4. A vertical, circular cross-sectional jet of **air** strikes a conical deflector. A vertical anchoring force of 0.1 N is required to hold the deflector in place. Determine the mass in kg of the deflector. The magnitude of velocity of the air remains constant. Assume frictionless flow. Density of air = $1.23 kg/m^3$
Ans: 0.108 kg



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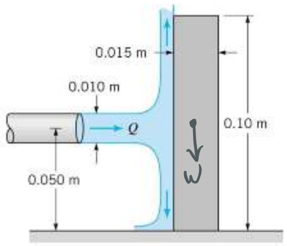
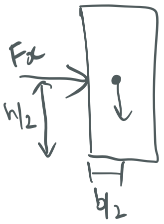


what is the force (F_x) needed to tip the block



at tipping $\Rightarrow$ $ma = 0$

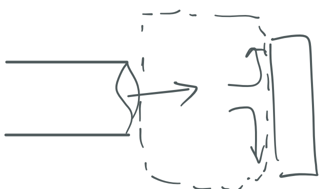
$$ma = F_x \cdot h/2 - w\, b/2$$

$$0 = F_x \cdot h/2 - w\, b/2$$

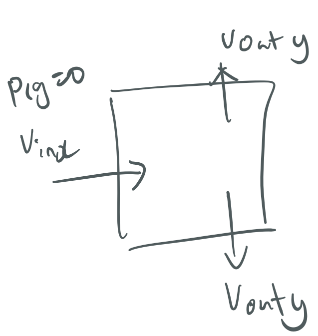
$$F_x = \frac{wb}{h}$$

$$= \frac{6 \times 0.015}{0.1}$$

$$= 0.9\ N$$



momentum eq $\Rightarrow \sum F_x = \dot{m}_{out\,x} - \dot{m}_{in\,x}$



$$\sum F_x = \dot{m}_{out\,x} - \dot{m}_{in\,x}$$

$$\dot{m}_{out\,x} = 0$$

$$\dot{m}_{in\,x} = \dot{m} V_1$$

$$\dot{m} = \rho Q = \rho A V$$

$$\dot{m}_{in\,x} = \rho A V_1^2$$

$$\sum F_x = -\dot{m}_{in\,x}$$

$$0.9 = -(1000)\left(\tfrac{\pi}{4}(0.01)^2\right) V_1^2$$

$$V_1 = 3.3851\ m/s$$

$$Q = \left(\tfrac{\pi}{4}(0.01)^2\right)(3.3851) = 2.658 \times 10^{-4}\ m^3/s$$

4. A vertical, circular cross-sectional jet of **air** strikes a conical deflector. A vertical anchoring force of 0.1 N is required to hold the deflector in place. Determine the mass in kg of the deflector. The magnitude of velocity of the air remains constant. Assume frictionless flow. Density of air = 1.23 kg/m^3

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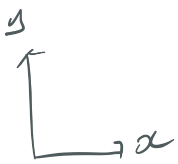

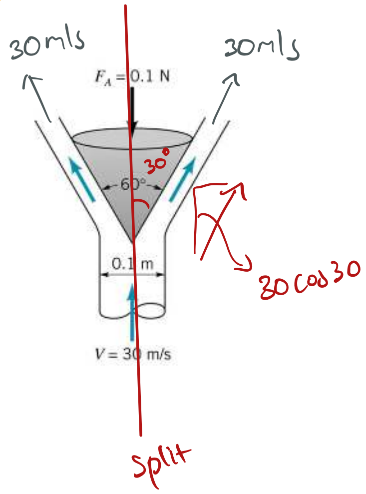

Momentum Eqn

$$\Sigma F_y = \dot{m}(V_{2y} - V_{1y})$$

$$\dot{m} = \rho Q$$

$$= \rho A V$$

$$= (1.23)\left(\frac{\pi}{4}(0.1)^2\right)(30)$$

$$= 0.28981$$

$$V_{1y} = 30\text{m/s}$$

$$V_{2y} = 30\cos 30^\circ$$

$$-F_A - W = \dot{m}(V_{2y} - V_{1y})$$

$$-0.1 - W = 0.28981(30\cos 30 - 30)$$

$$-W = -1.0648$$

$$W = 1.0648\text{ N}$$

$$\text{Mass} = 1.0648 \div 9.81$$

$$= 0.1085\text{ kg}$$

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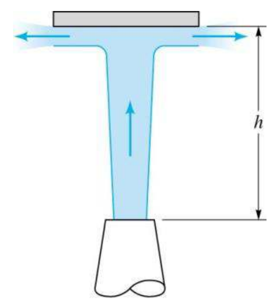
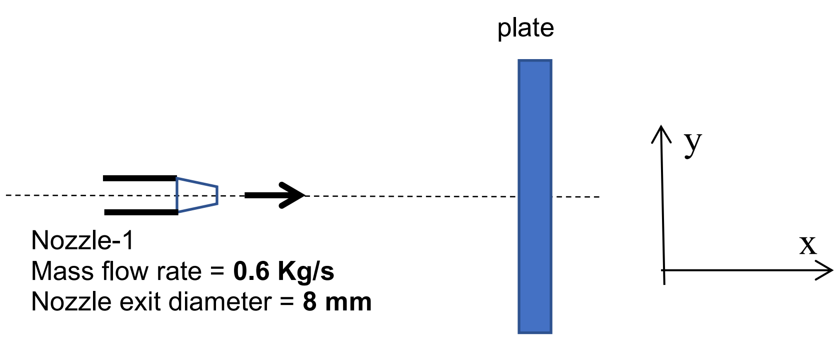
Tutorial 2 – Momentum and Angular momentum

1 A vertical jet of water leaves a nozzle at a speed of 10 m/s and the nozzle exit diameter is 20 mm. The impact of the water jet suspends a plate having a mass of 1.5 kg. What is the vertical distance h?
Assume frictionless flow and upon impact the water jet is deflected equally.
Ans: 3.97 m



Qu 2a Water exits Nozzle-1 and strike a vertical plate. Upon contact, water exit the plate 50% upwards and the remaining downwards. Find the force to hold the plate stationary. Assuming frictionless flow and neglecting gravity effects.
Ans : -7.162 N



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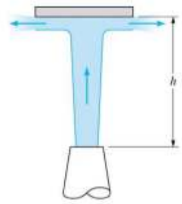
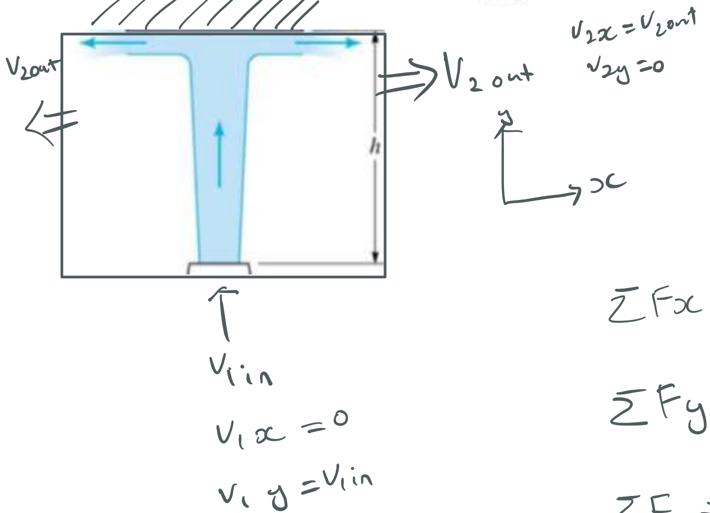
Assume frictionless flow and upon impact the water jet is deflected equally.
Ans: 3.97 m

① Draw cv
② decompose the velocity
③ Momentum eqn





$$\Sigma F_x = \dot{M}_{out\,x} - \dot{M}_{in\,x}$$

$$\Sigma F_y = \dot{M}_{out\,y} - \dot{M}_{in\,y}$$

$$\Sigma F_y \Rightarrow -(1.5 \times 9.81) = -\dot{M}_{in\,y}$$

$$\Rightarrow -14.715 = -\dot{m} V_1$$

$$-14.75 = -\rho A V_1 V_2$$

$$-14.75 = -(1000)\left(\frac{\pi}{4}(0.02)^2\right)(10)(V_2)$$

$$V_2 = 4.69507$$

atmosphere datum atm

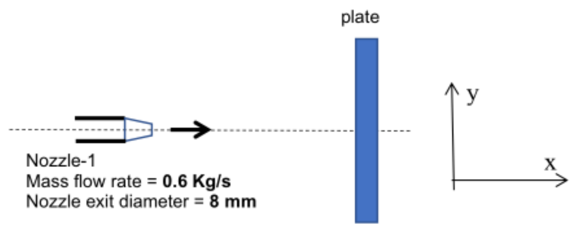
$$\cancel{P_1} + \frac{1}{2}\rho V_1^2 + \cancel{\rho g h_1} = \cancel{P_2} + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

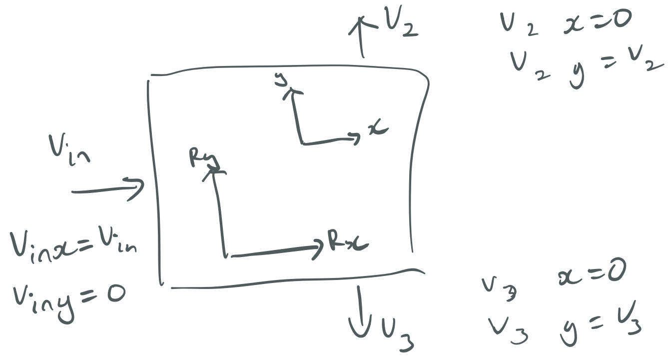
$$\frac{1}{2}(1000)(10)^2 = \frac{1}{2}(1000)(4.69507)^2 + (1000)(9.81)h_2$$

$$h_2 = 3.973 \text{ m} \,//$$

Qu 2a Water exits Nozzle-1 and strike a vertical plate. Upon contact, water exit the plate 50% upwards and the remaining downwards. Find the force to hold the plate stationary. Assuming frictionless flow and neglecting gravity effects.

Ans : -7.162 N





$$\Sigma F_x = \dot{M}_{out\,x} - \dot{M}_{in\,x}$$

$$\Sigma F_y = \dot{M}_{out\,y} - \dot{M}_{out\,y}$$

$$R_x = -\dot{m}_1 V_{in}$$

$$R_y = [V_{2out} + V_{3out}] - 0$$

$$= [\dot{m}_2 V_2 + \dot{m}_3(-V_3)]$$

$$\dot{m}_2 = \dot{m}_3$$

50% up & 50% down

$$\therefore R_y = [\dot{m} V_2 - \dot{m}_3 V_3]$$

$$= 0$$

$$\dot{m} = \rho A V$$

$$0.6 = (1000)\left(\pi/4 (0.008)^2\right) V$$

$$V = 11.93662$$

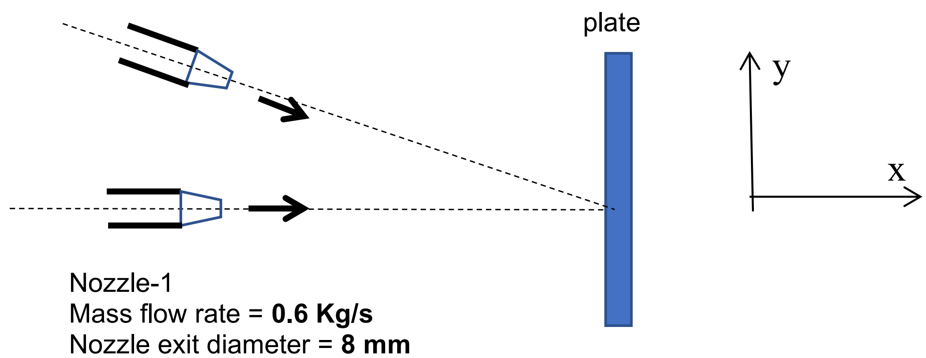
$$R_x = -\dot{m}_{in} V_{in}$$

$$= -0.6\,(11.93662)$$

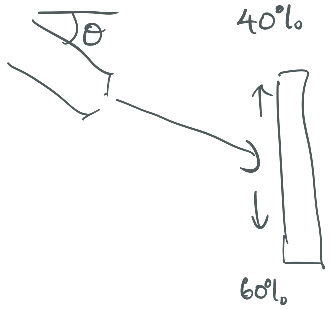
$$= -7.161972 \approx -7.162\ \text{N}$$

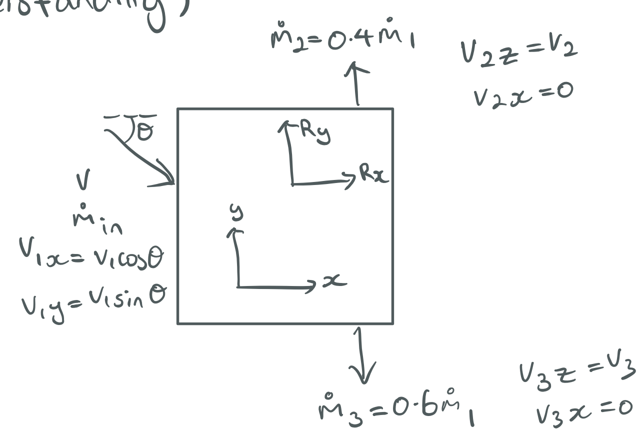
Qu 2b Nozzle-2 inclined at an angle of **30** degree (with horizontal) is **added** to the system and both water jets strike at some point.
For Nozzle-2, upon contact, water exit the plate 40% upwards and the remaining downwards. Assuming frictionless flow and neglecting gravity effects.
Find Force, *Fx* (in the x-direction) to hold the plate stationary. **Ans : -13.364 N**
Find Force, *Fy* (in the y-direction) to hold the plate stationary **Ans : 2.149 N**
Find resultant force to hold the plate stationary **Ans : 13.536 N**

Qu 2c **Only Nozzle-2** remains in the system. Upon contact, water exit the plate 40% upwards and the remaining downwards. The plate is no longer stationary. The plate moves **away** from jet at **3 m/s** horizontally
Find the relative mass flow rate entering the plate in the x-direction. **Ans : 0.369 kg/s**
Find the force acting on the plate in x-direction. **Ans : -2.706 N**



Q2b.1 (for easier understanding)





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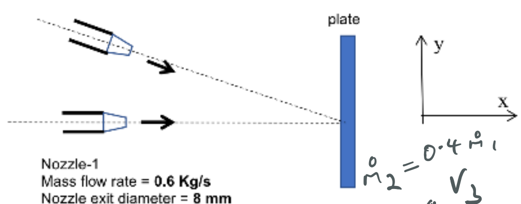
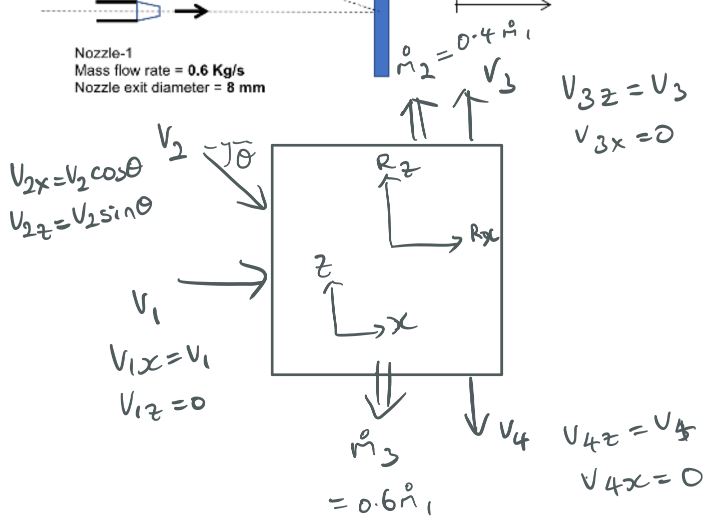
Qu 2b Nozzle-2 inclined at an angle of 30 degree (with horizontal) is **added** to the system and both water jets strike at some point.
For Nozzle-2, upon contact, water exit the plate 40% upwards and the remaining downwards. Assuming frictionless flow and neglecting gravity effects.

Find Force, Fx (in the x-direction) to hold the plate stationary. **Ans : -13.364 N**
Find Force, Fy (in the y-direction) to hold the plate stationary **Ans : 2.149 N**
Find resultant force to hold the plate stationary **Ans : 13.536 N**





Assumption: nozzle 1 + 2 have same $\dot{m}$ & diameter

$\dot{m} = \rho A V$

$0.6 = (1000)\left(\frac{\pi}{4}(0.008)^2\right) V$

$V_1 = V_2 = 11.93662$

ΣF_x:

$$R_x = -\left[\dot{m}_1 V_1 + \dot{m}_2 V_2 \cos 30\right]$$

$$R_x = -\left[0.6(11.93662) + (0.6)(11.93662 \cos 30)\right]$$

$$= -13.3644 \text{ N} \quad (\leftarrow)$$

Assumption: $|V_1| = |V_2| = |V_3| = |V_4|$

$$\dot{m}_1 = \dot{m}_2$$

ΣF_y:

$$R_y = \left[0.4(0.6)V_3 + 0.5(0.6)V_3 + 0.5(0.6)(-V_4) + 0.6(0.6)(-V_4)\right]$$

$$-\left[0.6(11.93662 \sin 30)\right]$$

$$R_y = \left[\frac{27}{50}V_3 - \frac{33}{50}V_4\right] - \left[0.6(11.93662 \sin 30)\right]$$

$$= 2.1485 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= 13.53599$$

$$\approx 13.536 \text{ N}$$

Qu 2c **Only Nozzle-2** remains in the system. Upon contact, water exit the plate 40% upwards and the remaining downwards. The plate is no longer stationary. The plate moves **away** from jet at **3 m/s** horizontally

Find the relative mass flow rate entering the plate in the x-direction. **Ans : 0.369 kg/s**

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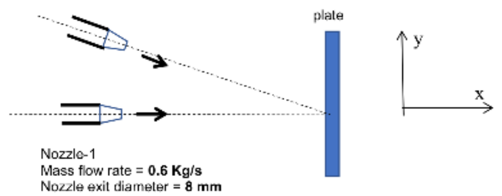
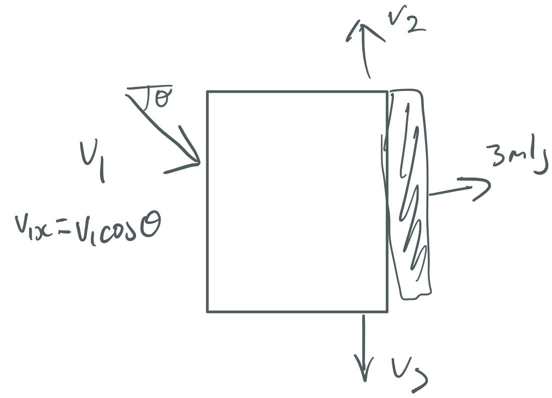




$$V_{1x} = 11.93662 \cos 30^\circ$$
$$= 10.3374\ m/s$$

$$V_{H/P} = 10.3374 - 3$$
$$= 7.3374\ m/s$$

$$\dot{m} = \rho Q$$
$$= \rho A V$$
$$= (1000)\left(\frac{\pi}{4} \times 0.008^2\right)(7.3374)$$
$$= 0.36881\ kg/s \,/\!/$$

$$F = \dot{m} \cdot V_{H/P}$$
$$= 0.36881 \times 7.3374$$
$$= 2.706\,N\ (\leftarrow)$$

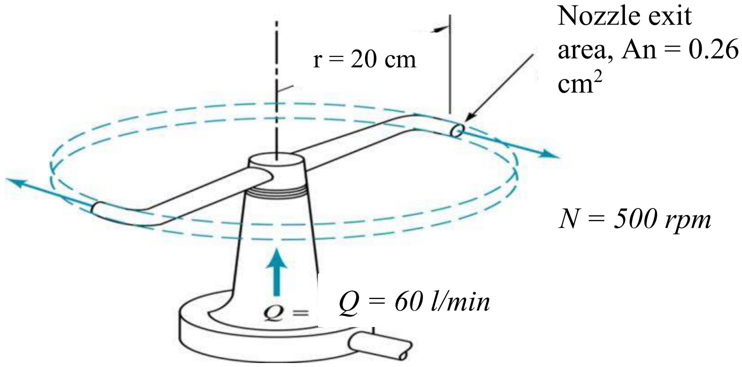
3 Water enters a rotating lawn sprinkler through its base at the steady rate of 60 L/min. The exit cross-sectional area of each of the two nozzles is 0.26 cm^2 and the flow leaving each nozzle is tangential. The radius from the axis of rotation to the centreline of each nozzle is 20 cm.

(a) Determine the resisting torque required to hold the sprinkler head stationary.

(b) Determine the resisting torque required for the sprinkler to rotate with a constant speed of 500 rev/min.

(c) Determine the angular velocity of the sprinkler if No resisting torque is applied.

Ans: (a) 3.84 N.m. (b) 1.74 N.m. (c) 917 rev/min.

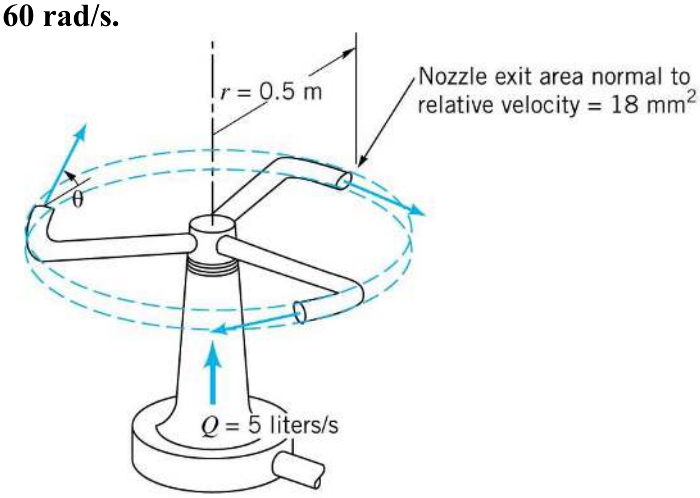


4 Water enters the rotor at 5 litres/s along the axis of rotation. The cross-sectional area of each of the three nozzle exits normal to the relative velocity is 18 mm^2.

Find the resisting torque required to hold the rotor stationary if $\theta = 0$?

If the resisting torque is reduced to zero and $\theta = 30^o$, what is the rotational speed ?

Ans: 231 N.m, 160 rad/s.



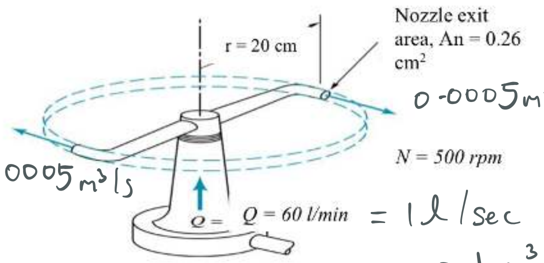
3 Water enters a rotating lawn sprinkler through its base at the steady rate of 60 L/min. The exit cross-sectional area of each of the two nozzles is 0.26 cm² and the flow leaving each nozzle is tangential. The radius from the axis of rotation to the centreline of each nozzle is 20 cm.

(a) Determine the resisting torque required to hold the sprinkler head stationary.

(b) Determine the resisting torque required for the sprinkler to rotate with a constant speed of 500 rev/min.

(c) Determine the angular velocity of the sprinkler if No resisting torque is applied.

Ans: (a) 3.84 N.m. (b) 1.74 N.m. (c) 917 rev/min.



$= 1\ l/sec$

$= 0.001\ m^3/s$

$1\ l/s \Rightarrow 0.001\ m^3/s$

$V = \frac{\text{flow rate}}{\text{Area}}$

$= \frac{0.0005}{0.000026}$

$= 19.2307\ m/s$

1 side

$\dot{m} = \frac{1}{2} \rho Q$

$= \frac{1}{2}(1000)(0.001)$

$= 0.5\ kg/s$

a) $F = \dot{m} V$

$T = n \times F \times r$

$T = 2 \times 0.5 \times 19.2307 \times 0.2$

$= 3.846\ Nm$

$1\ rev = 0.10472\ rad/s$

$500\ rev = 52.36\ rad/s$

b) $V \rightarrow V_0 - \omega R = 19.2307 - 52.36 \times 0.2$

$= 8.7587$

$T = n \times F \times r$

$= 2 \times 0.5 \times 8.7587 \times 0.2$

$= 1.7517\ Nm$

c) $V_0 = \omega R \Rightarrow \omega = \frac{V_0}{R}$

$= \frac{19.2307}{0.2}$

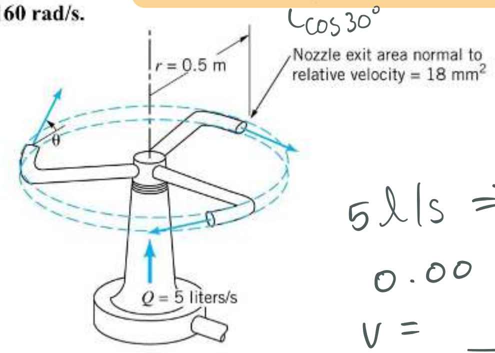
$= 96.1535\ rad/s$

$= \frac{96.1535 \times 60}{2\pi} = 918.198\ rpm$

4 Water enters the rotor at 5 litres/s along the axis of rotation. The cross-sectional area of each of the three nozzle exits normal to the relative velocity is 18 mm^2.
Find the resisting torque required to hold the rotor stationary if $\theta = 0$?
If the resisting torque is reduced to zero and $\theta = 30°$, what is the rotational speed ?
Ans: 231 N.m, 160 rad/s.

$18 mm^2 = 0.000018$

$\cos 30°$



$5 l/s \Rightarrow 0.005 m^3/s$

$0.005 \div 3 = \frac{1}{600}$

$V = \frac{1/600}{0.000018}$

$= 92.5925$

a) $\dot{m} = \frac{1}{3}\rho Q$

$= \frac{1}{3}(1000)(0.005)$

$= 5/3 \ kg/s$

$T = n \cdot F \cdot r$

$= 3 \cdot 5/3(92.5925) \cdot (0.5)$

$= 231.48 \ Nm$ //

b) $V = \omega R$

$\Rightarrow V\cos 30 = \omega R$

$92.5925 \cos 30 = \omega \ 0.5$

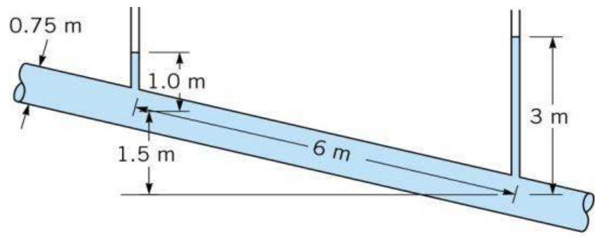
$\omega = 160.374 \ rad/s$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + h_1 + h_{in} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_2 + h_{out} + h_L$$

MA3006 FLUID MECHANICS

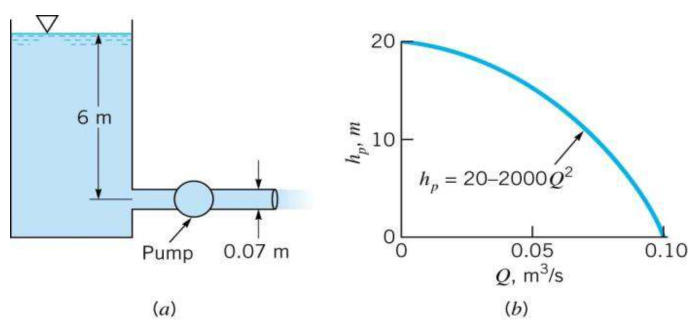
Tutorial 3 – Energy equation and Flowmeter

1 Water flow through a pipe and the pressure head is indicated by two manometers *6 m* apart. Determine the direction of flow and the head loss between the two manometers.
Ans: 0.5 m



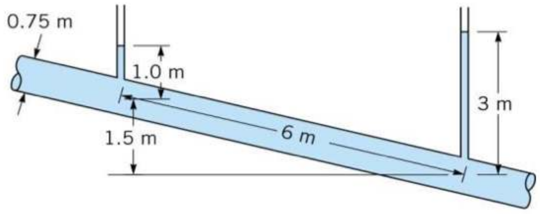
2 Water at an elevation of 6 m is to be pumped **from** a tank, through a 0.07 m diameter pipe and discharge to the atmosphere. The system head loss (overall) is $1.2(V^2/2g)$ where V is the velocity in the pipe. The pump curve is given as shown, $H_p = 20 - 2000Q^2$ m where Q is the flow rate in m^3/s. Determine the flowrate in the system.

Ans: 0.052 m^3/s



1 Water flow through a pipe and the pressure head is indicated by two manometers *6 m* apart. Determine the direction of flow and the head loss between the two manometers.
Ans: 0.5 m

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + h_1 + h_{in} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_2 + h_{out} + h_L$$



high to low

datum

$$\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + h_1 + \cancel{h_{in}} = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \cancel{h_2} + \cancel{h_{out}} + h_L$$

$$P_1 = \rho g h = \rho g (1) = \rho g$$

$$P_2 = \rho g h = \rho g (3) = 3\rho g$$

$$V_1 = V_2$$

$$\frac{\rho g}{\rho g} + 1.5 = \frac{3\rho g}{\rho g} + h_L$$

$$1 + 1.5 = 3 + h_L$$

$h_L = -ve$ (incorrect)

$$\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + h_1 + \cancel{h_{in}} = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + h_2 + \cancel{h_{out}} + h_L$$

$$\frac{3\rho g}{\rho g} + 0 = \frac{\rho g}{\rho g} + 1.5 + h_L$$

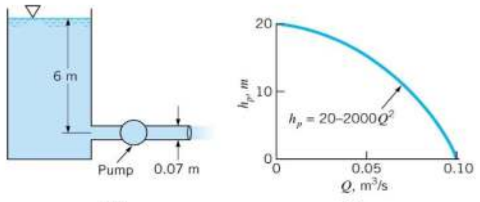
$$3 = 1 + 1.5 + h_L$$

$$h_L = 0.5 \text{ m}$$

2 Water at an elevation of 6 m is to be pumped **from** a tank, through a 0.07 m diameter pipe and discharge to the atmosphere. The system head loss (overall) is $1.2(V^2/2g)$ where V is the velocity in the pipe. The pump curve is given as shown, $H_p = 20 - 2000Q^2$ m where Q is the flow rate in m^3/s. Determine the flowrate in the system.

Ans: 0.052 m³/s

$Q = ?$



(a) (b)

Energy in + Energy from pump = Energy out + Energy loss

reservoir, large tank, tank, datum

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + h_1 + h_{in} = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \cancel{h_2} + h_L$$

$$h_1 + h_{in} = \frac{V_2^2}{2g} + h_L$$

$$6 + 20 - 2000Q^2 = \frac{V_2^2}{2g} + 1.2(V_2^2/2g)$$

$$6 + 20 - 2000(AV)^2 = \frac{V_2^2}{2g} + 1.2\left(\frac{V_2^2}{2g}\right)$$

$$26 - 0.0296211v^2 = V^2\left(\frac{1}{2g} + 1.2\left(\frac{1}{2g}\right)\right)$$

$$26 - 0.0296211v^2 = v^2\left(\frac{1}{2(9.81)} + 1.2\frac{1}{2(9.81)}\right)$$

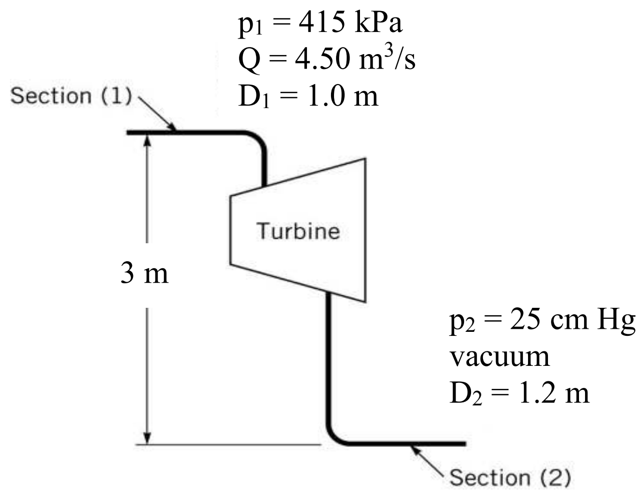
$$26 = \frac{110}{981}v^2 + 0.0296211v^2$$

$$V = 13.5432$$

$$Q = AV$$
$$= \frac{\pi}{4}(0.07)^2 \times 13.5432$$
$$= 0.05212 \, m^3/s$$

3 Water is supplied at 4.50 m^3/s and 415 kPa to a hydraulic turbine through a 1.0-m diameter inlet pipe at section (1). The turbine discharge pipe has a 1.2-m diameter. The static pressure at section (2) is 3 m below the turbine inlet, the pressure is **25 cm Hg vacuum**. If the turbine develops 1.9 MW, determine the power lost between sections (1) and (2).

Ans: 290 kW



4 An orifice meter is used to measure the water flow rate along a horizontal section of a piping system. A 5-cm-diameter orifice plate is inserted into the 8-cm-diameter pipe. If the water flowrate through the pipe is 0.03 m^3/s, determine the pressure difference indicated by a manometer attached to the flow meter. Kinematic viscosity of water is given as 1.12×10^{-6} m^2/s

Ans: 267.6 kPa

5 A 50-mm-diameter nozzle meter is installed at the end of a 80-mm-diameter pipe through which air flows. Air exits the nozzle meter to atmosphere. A manometer attached to the static pressure tap just upstream from the nozzle indicates a pressure of *7.3 mm of water*. Determine the flowrate.

Kinematic viscosity of air is given as 1.46×10^{-5} m^2/s

Ans: 0.0221 m^3/s

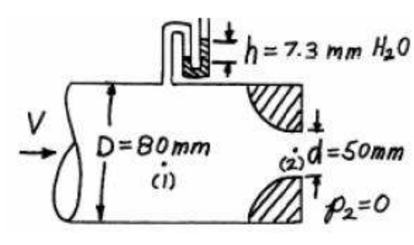
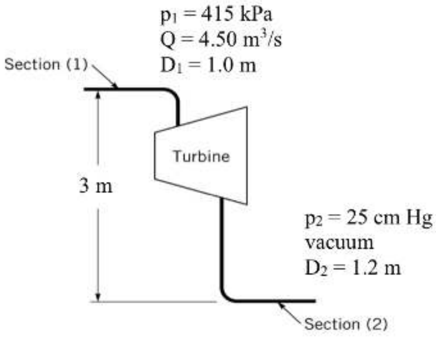


Fig. 3

3 Water is supplied at 4.50 m^3/s and 415 kPa to a hydraulic turbine through a 1.0-m diameter inlet pipe at section (1). The turbine discharge pipe has a 1.2-m diameter. The static pressure at section (2) is 3 m below the turbine inlet, the pressure is **25 cm Hg vacuum**. If the turbine develops 1.9 MW, determine the power lost between sections (1) and (2).

Ans: 290 kW



vacuum

$-13.6 \times 1000 \times 9.81 \times 0.25$

$= -33354$

datum

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + h_1 + \cancel{h_{in}} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{h_2} + h_{out} + h_{loss}$$

$A_1 V_1 = A_2 V_2$

$4.5 = \frac{\pi}{4}(1.2)^2 V_2$

$V_2 = 3.97887$

$Q = A_1 V_1$

$$V_1 = \frac{Q}{A} = \frac{4.50}{\frac{\pi}{4}(1)^2} = 5.72957$$

$$\frac{415 \times 10^3}{(1000)(9.81)} + \frac{5.72957^2}{2(9.81)} + 3 = \frac{-33354}{(1000)(9.81)} + \frac{3.97887^2}{2(9.81)} + h_{out} + h_L$$

Power $= \dot{m} g h_{out}$

$1.9 \times 10^6 = 4.5 \times (1000)(9.81)\, h_{out}$

$h_{out} = 43.03998$

$$\frac{415 \times 10^3}{(1000)(9.81)} + \frac{5.72957^2}{2(9.81)} + 3 = \frac{-33354}{(1000)(9.81)} + \frac{3.97887^2}{2(9.81)} + 43.03998 + h_L$$

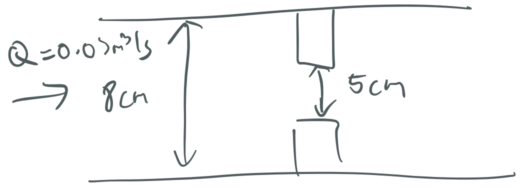
$h_L = 6.53$ m

Power loss $= 4.5 \times 1000 \times 9.81 \times 6.53 = 288.27$ kW //

4 An orifice meter is used to measure the water flow rate along a horizontal section of a piping system. A 5-cm-diameter orifice plate is inserted into the 8-cm-diameter pipe. If the water flowrate through the pipe is 0.03 m^3/s, determine the pressure difference indicated by a manometer attached to the flow meter. Kinematic viscosity of water is given as 1.12×10^{-6} m^2/s

Ans: 267.6 kPa

$$Q = C_0 A_2 \sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}}$$



$$\beta = \frac{D_2}{D_1} = \frac{5}{8}$$

$$Re = \frac{\rho v_1 d_1}{\mu}$$

$Q = AV$

$V = 5.9683$

$\mu = 1.12 \times 10^{-6} \times 1000$

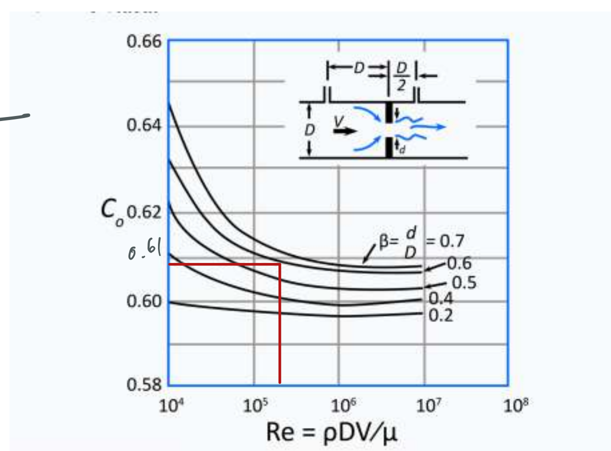
$= 1.12 \times 10^{-3}$

$$Re = \frac{1000 \times 5.9683 \times 0.08}{1.12 \times 10^{-3}}$$

$= 426307.14$

$= 4.26 \times 10^5$

$C_0 = 0.608$



$$0.03 = 0.608\left(\frac{\pi}{4}(0.05)^2\right)\sqrt{\frac{2\Delta P}{(1000)(1-5/8^4)}}$$

$$\sqrt{\frac{2\Delta P}{(1000)(1-\frac{5}{8}^4)}} = 25.1297$$

$\Delta P = 267571.7$

$= 268\ kPa$

5 A 50-mm-diameter nozzle meter is installed at the end of a 80-mm-diameter pipe through which air flows. Air exits the nozzle meter to atmosphere. A manometer attached to the static pressure tap just upstream from the nozzle indicates a pressure of *7.3 mm of water*.
Determine the flowrate.
Kinematic viscosity of air is given as 1.46×10^{-5} m^2/s
Ans: 0.0221 m^3/s

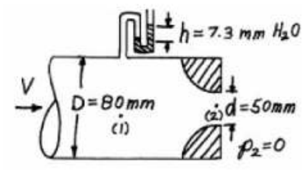


Fig. 3

$$Q = C_N A_2 \sqrt{\frac{2\Delta P}{\rho(1-\beta^4)}}$$

$$\beta = \frac{50}{80} = 5/8$$

$$Re = \frac{\rho U_1 d_1}{\mu}$$

$$\rho_{Air} = 1.225 \text{ kg/m}^3$$

$$\Delta P = \rho g h_o = 1000 \times 9.81 \times 0.0073 = 71.613 \text{ Pa}$$

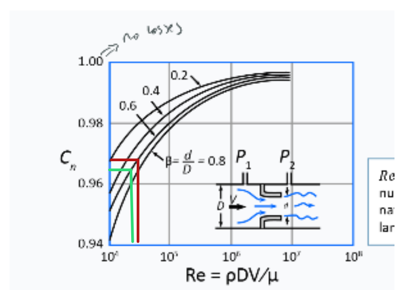
$$A_2 \sqrt{\frac{2\Delta P}{\rho(1-\beta^4)}} = \frac{\pi}{4}(0.05)^2 \sqrt{\frac{2(71.613)}{(1.225)(1-\frac{5}{8}^4)}} = 0.02306$$

Assume $C_N = 1$

$Q_i = 0.02306$

$$v = \frac{Q_i}{A_1} = \frac{0.02306}{\frac{\pi}{4}(0.08)^2} = 4.5883$$

$$Re = \frac{1.225 \times 4.5883 \times 0.08}{1.46 \times 10^{-5}} = 30798.178$$



$C_N \approx 0.967$

$Q_1 = 0.967\, Q_i = 0.02229 \text{ m}^3/\text{s}$

$V_2 = \frac{Q_1}{A_1} = 4.4344$

$Re = 29776.15$

$C_N \approx 0.963$

$Q_2 = 0.02206$

MA3006 FLUID MECHANICS

Tutorial 4 – Dimensional analysis

1. Consider a uniform stream of fluid (density ρ and dynamic viscosity μ) flowing around a circular cylinder of diameter d and length l. A drag force of F_D acting on the cylinder is developed when the speed of the fluid is V. Derive the suitable Pi terms and a general relation for the drag force if the repeating variables selected are ρ, V, d

   (Ans : . $\frac{F_D}{\rho V^2 d^2} = f\left(\frac{l}{d}, \frac{\mu}{\rho V d}\right)$**)**

2. The pressure rise, Δp, across a pump can be expressed as $\Delta p = f$ (D, ω, ρ, Q) where D is the impeller diameter, ω is the rotational speed, ρ is the density and Q the flow rate. Determine a suitable set of dimensionless parameters. Use D ω ρ as repeating variables.
   (Ans : $\Delta p/(\rho\omega^2 D^2) = \Phi(Q/\omega D^3)$**)**

3. The pressure drop per unit length, ($\frac{\Delta p}{l}$) of flow of blood through a small diameter horizontal pipe, is a function of Q, D, and μ. A series of calibration test was conducted where diameter, D = 2 mm, length, l = 300 mm and dynamic viscosity, μ = 0.004 N.s/m^2. The following data were obtained where pressure drop Δp is measured over a length of 300mm to obtain the flow rate Q.

Q (m^3/s)	Δp (N/m^2)
3.6x10^{-6}	1.1x10^4
4.9x10^{-6}	1.5x10^4
6.3x10^{-6}	1.9x10^4
7.9x10^{-6}	2.4x10^4
9.8x10^{-6}	3.0x10^4

Derive a general relation between Δp and Q.

(Ans : $\frac{\Delta p D^4}{\mu Q} = C \approx 40.5$**)**

MLT

1. Consider a uniform stream of fluid (density ρ and dynamic viscosity μ) flowing around a circular cylinder of diameter d and length l. A drag force of F_D acting on the cylinder is developed when the speed of the fluid is V. Derive the suitable Pi terms and a general relation for the drag force if the repeating variables selected are ρ, V, d

(Ans : . $\frac{F_D}{\rho V^2 d^2} = f\left(\frac{l}{d}, \frac{\mu}{\rho V d}\right)$)

1. list of variable :

ρ, μ, d, l, V

2. Basic dimension

$\rho = \frac{kg}{m^3} = \frac{M}{L^3}$

$\mu = \frac{Ns}{m^2} = \frac{M}{LT}$

$d = m = L$

$l = m = L$

$F_D = N = \frac{ML}{T^2}$

$V = m/s = \frac{L}{T}$

3. repeating variable ρ, V, d

$\rho = \frac{M}{L^3}, V = \frac{L}{T}, d = L$

4. Form n-m group

$\Pi_1 \Rightarrow F_D \rho^A V^B d^C$

$= \frac{ML}{T^2}\left(\frac{M}{L^3}\right)^A \left(\frac{L}{T}\right)^B (L)^C$

$= \frac{ML}{T^2}\left(\frac{L^3}{M}\right)\left(\frac{T}{L}\right)^2 (L)^{-2}$

$A=-1 \quad = \frac{\cancel{M}L}{\cancel{T^2}} \cdot \frac{L^3}{\cancel{M}} \cdot \frac{\cancel{T^2}}{L^2} \cdot L^{-2}$

$B-2$

$C=-2 \quad = \frac{F_D}{\rho V^2 d^2}$

$\Pi_2 \Rightarrow \mu \rho^A V^B d^C$

$= \frac{M}{LT}\left(\frac{M}{L^3}\right)^A \left(\frac{L}{T}\right)^B (L)^C$

$A=-1: \frac{M}{LT}\left(\frac{L^3}{M}\right)\left(\frac{L}{T}\right)^B (L)^C$

$B=-1: \frac{M}{LT}\left(\frac{L^3}{M}\right)\left(\frac{T}{L}\right)(L)^C$

$C=-1: \frac{M}{LT}\left(\frac{L^3}{M}\right)\left(\frac{T}{L}\right)L^{-1}$

$\frac{\mu}{\rho V d}$

$$\pi_3 \Rightarrow l\, \rho^A V^B d^C$$

$$= L \left(\frac{M}{L^3}\right)^A \left(\frac{L}{T}\right)^B (L)^C$$

$A, B = 0$

$C = -1$

$$\therefore \quad \frac{l}{d}$$

$$\pi_1 = \phi(\pi_2, \pi_3)$$

$$\frac{F_D}{\rho V^2 d^2} = \phi\left(\frac{\mu}{\rho V d}, \frac{l}{d}\right)$$

2. The pressure rise, Δp, across a pump can be expressed as $\Delta p = f(D, \omega, \rho, Q)$ where D is the impeller diameter, ω is the rotational speed, ρ is the density and Q the flow rate. Determine a suitable set of dimensionless parameters. Use D ω ρ as repeating variables.
(Ans : $\Delta p / (\rho \omega^2 D^2) = \Phi(Q / \omega D^3)$**)**

$$\Delta P = N/m^2 = kg \frac{\cancel{m}}{s^2 m^{\cancel{2}}} = \frac{M}{L T^2}$$

$$D = m = L$$

$$\omega = rad/s = T^{-1}$$

$$\rho = kg/m^3 = \frac{M}{L^3}$$

$$Q = m^3/s = \frac{L^3}{T}$$

$$\pi_1 \Rightarrow \Delta P \, D^A \omega^B \rho^C$$

$$\frac{M}{LT^2} (L)^A (T^{-1})^B \left(\frac{M}{L^3}\right)^C$$

$$C = -1 : \frac{M}{LT^2} (L)^A (T)^{-B} \left(\frac{L^3}{M}\right)$$

$$B = -2 \quad \frac{M}{LT^2} (L)^A \; T^2 \; \frac{L^3}{M}$$

$$A = -2 \quad \frac{M}{LT^2} L^{-2} \; T^2 \; \frac{L^3}{M}$$

$$\therefore \frac{\Delta P}{D^2 \omega^2 \rho}$$

$\Pi_2 \rightarrow Q\, D^A\, \omega^B\, \rho^C$

$$\frac{L^3}{T}\,(L)^A\,(T)^{-B}\left(\frac{M}{L^3}\right)^C$$

$C = 0$

$A = -3$

$B = -1$ $\quad \frac{L^3}{T} \cdot L^{-3}\; T$

$$\therefore \quad \frac{Q}{D^3\,\omega}$$

$$\frac{\Delta P}{D^2\omega^2\rho} = \phi\left(\frac{Q}{D^3\,\omega}\right)$$

3. The pressure drop per unit length, ($\frac{\Delta p}{l}$) of flow of blood through a small diameter horizontal pipe, is a function of Q, D, and μ. A series of calibration test was conducted where diameter, D = 2 mm, length, l = 300 mm and dynamic viscosity, μ = 0.004 N.s/m^2. The following data were obtained where pressure drop Δp is measured over a length of 300mm to obtain the flow rate Q.

Q (m^3/s)	Δp (N/m^2)
3.6×10^{-6}	1.1×10^4
4.9×10^{-6}	1.5×10^4
6.3×10^{-6}	1.9×10^4
7.9×10^{-6}	2.4×10^4
9.8×10^{-6}	3.0×10^4

Derive a general relation between Δp and Q.

(Ans : $\frac{\Delta p D^4}{\mu Q} = C \approx 40.5$**)**

$$\frac{\Delta P}{l} = \frac{N}{m^3} = kg \frac{\cancel{m}}{s^2 m^{\cancel{3}\,2}} = M L^{-2} T^{-2}$$

$$Q = m^3/s = L^3 T^{-1}$$

$$D = m = L$$

$$\mu = M L^{-1} T^{-1}$$

$$\pi_1 \; \frac{\Delta P}{l} \; Q^A \; D^B \; \mu^C$$

$$\frac{M}{L^2 T^2} \left(\frac{L^3}{T}\right)^A (L)^B \left(\frac{M}{LT}\right)^C$$

$C = -1$ $\quad \frac{\cancel{M}}{L^2 \cancel{T^2}} \; \frac{\cancel{T}}{L^{\cancel{3}\,2}} \; (L)^B \; \frac{\cancel{LT}}{\cancel{M}}$

$A = -1$

$B = 4$

$$\pi_1 \Rightarrow \frac{\Delta P D^4}{l Q \mu} = C$$

$Q_{avg} = 6.5 \times 10^{-6}$

$\Delta P_{avg} = 1.98 \times 10^{4}$

$$C = \frac{1.98 \times 10^4 \times 0.002^4}{0.3 \times 6.5 \times 10^{-6} \times 0.004}$$

$$= 40.615$$

4. The concept of measuring viscosity of fluid is shown in figure below. The concentric cylinder device is used to measure the viscosity, μ, of liquids by relating the angle of twist, θ, of the inner cylinder and the angular velocity, ω, of the outer cylinder. It is given that :

$$\theta = f(\omega, \mu, K, D_1, D_2, l)$$

where K is the suspending wire stiffness and has the dimensions FL.

Use μ K and *l* as repeating variables determine suitable dimensionless parameters

The following data were obtained in a series of tests for which $\mu = 0.5$ N. s/m^2, $K = 14$ N.m, $l = 0.3$ m, and D_1 and D_2 were constant.

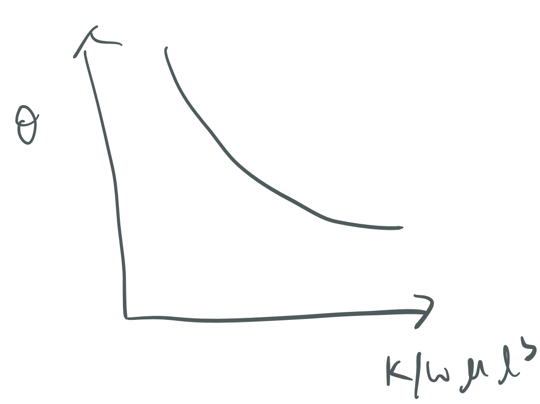
θ rad	0.89	1.50	2.51	3.05	4.28	5.52	6.40
ω rad/s	0.30	0.50	0.82	1.05	1.43	1.86	2.14

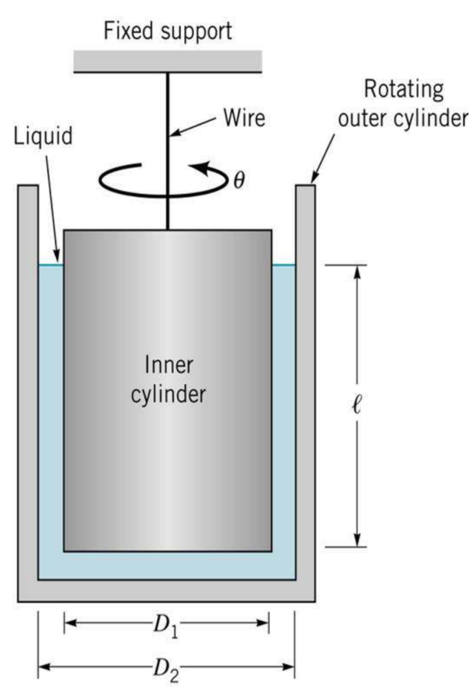
Determine from these data, with the aid of dimensional analysis, the relationship between θ, ω, and μ for this particular apparatus.

Hint : plot graph of θ vs $(\frac{\omega\mu l^3}{K})$

Ans: $\boldsymbol{\theta = \phi_1(\frac{\omega\mu l^3}{K})}$, $\boldsymbol{\theta = 2.98 \times 10^3(\frac{\omega\mu l^3}{K})}$, $\boldsymbol{\theta = 5.7\omega\mu}$

Base on our result





$\theta = g\left(\frac{\omega\mu l^3}{K}\right)$

$\theta = F^0 L^0 T^0$ FLT

$\omega = T^{-1}$

$\mu = FTL^{-2}$

$k = FL$

$D_1 = L$

$D_2 = L$

$L = L$

repeating μ, k, L

$\pi_1 \Rightarrow \theta \mu^A k^B L^C$

$= F^0 L^0 T^0 \left(\frac{FT}{L^2}\right)^A (FL)^B (L)^C$

$A, B, C = 0$

$\pi_1 = \theta$

$\pi_{2,3} \Rightarrow D \mu^A k^B L^C$

$L \left(\frac{FT}{L^2}\right)^A (FL)^B (L)^C$

$A, B = 0$

$C = -1$

$$\therefore \quad \Pi_2 = \frac{D_1}{L}$$

$$\Pi_3 = \frac{D_2}{L}$$

$$\Pi_4 \Rightarrow \omega\, \mu^A K^B L^C$$

$$= T^{-1} \left(\frac{FT}{L^2}\right)^A (FL)^B (L)^C$$

$A = 1$ $\quad \cancel{T^{-1}} \left(\frac{F\cancel{T}}{L^2}\right)(FL)^B (L)^C$

$B = -1$

$$\frac{\cancel{F}}{L^2} \frac{1}{\cancel{F}L} L^C$$

$C = 3$

$$\frac{\omega \mu L^3}{K}$$

$$\Pi_1 = \phi(\Pi_2, \Pi_3, \Pi_4)$$

$$\theta = \phi\left(\frac{D_1}{L}, \frac{D_2}{L}, \frac{\omega \mu L^3}{K}\right)$$

D_1, D_2, l, μ, K = constant $= \phi_1$

$$\theta = \phi_1\left(\frac{\omega \mu l^3}{K}\right)$$

MA3006 FLUID MECHANICS

Tutorial 5 – Similitude

1. 1 : 50 scale model is to be used in a towing tank to determine the drag on the hull of a ship. The model is operated in accordance with the Froude number criteria for dynamic similitude. The prototype is designed to cruise at 9.26 m/s. At what velocity (m/s) should the model be towed? What is the ratio of prototype drag to model drag? Assume shear drag is negligible.
   Ans : 1.31 m/s, $1.25\text{x}10^5$

2. The drag force F_D acting on a sonar sensor depends on its diameter D, its speed V in a fluid of density ρ and dynamic viscosity μ. A prototype of 300 mm diameter is towed at 1 m/s in sea water. A model has a diameter of 200 mm is tested in a wind tunnel. To achieve dynamic similarity, determine the speed at which the wind tunnel should be run. If the drag on the model is 25 N, estimate the drag on the prototype.
   The density of sea-water and air is 1030 and 1.23 kg/m^3, respectively.
   The dynamic viscosity of sea-water and air is 1.20×10^{-3} and 1.79×10^{-5} N·s/m^2 respectively.
   Ans : 18.7 m/s, 134.7 N

3. A prototype automobile is designed to travel at 65 km/hr. A model of this design is tested in a wind tunnel with identical standard sea-level air properties at a 1:5 scale. The measured model drag is 400 N, enforcing dynamic similarity. Determine (a) the drag force on the prototype, and (b) the power required to overcome this drag.
   Drag force coefficient is : $C_{drag} = \frac{1}{2}\frac{F_D}{\rho V^2 D^2}$
   Ans : 400 N, 7220 W

4. The drag characteristics of an airplane are to be determined by model tests in a wind tunnel operated at an absolute pressure of 1300 kPa. If the prototype is to cruise in standard air at 385 km/hr, and the corresponding speed of the model is not to defer by more than 20% from this (so that compressibility effects may be ignored), what range of length scales may be used if Reynolds number similarity is to be maintained? Assume the viscosity of air is unaffected by pressure, and the temperature of air in the tunnel is equal to the temperature of the air in which the airplane will fly.
   Ans: 0.0647 to 0.0971

1. 1 : 50 scale model is to be used in a towing tank to determine the drag on the hull of a ship. The model is operated in accordance with the Froude number criteria for dynamic similitude. The prototype is designed to cruise at 9.26 m/s. At what velocity (m/s) should the model be towed? What is the ratio of prototype drag to model drag? Assume shear drag is negligible.
Ans : 1.31 m/s, 1.25x10^5

$$\frac{d_m}{d_p} = \frac{1}{50}$$

Froude no: $\frac{V}{\sqrt{gL}}$

$$\frac{V_p}{\sqrt{\cancel{g} L_p}} = \frac{V_m}{\sqrt{\cancel{g} L_m}} \Rightarrow \frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}} = \sqrt{50}$$

$$= 7.07$$

9.26

$$V_m = \frac{V_p}{\sqrt{50}} = 1.31 \text{ m/s}$$

$$\frac{D_p}{\cancel{\rho} V_p^2 d_p^2} = \frac{D_m}{\cancel{\rho} V_m^2 d_m^2} \Rightarrow \frac{D_p}{D_m} = \frac{V_p^2 d_p^2}{V_m^2 d_m^2}$$

$$= \left(\frac{V_p d_p}{V_m d_m}\right)^2$$

$$= \left(\sqrt{50} \cdot 50\right)^2$$

$$= 50 \times 50^2$$

$$= 1.25 \times 10^5$$

2. The drag force F_D acting on a sonar sensor depends on its diameter D, its speed V in a fluid of density ρ and dynamic viscosity μ. A prototype of 300 mm diameter is towed at 1 m/s in sea water. A model has a diameter of 200 mm is tested in a wind tunnel. To achieve dynamic similarity, determine the speed at which the wind tunnel should be run. If the drag on the model is 25 N, estimate the drag on the prototype.
The density of sea-water and air is 1030 and 1.23 kg/m^3, respectively.
The dynamic viscosity of sea-water and air is 1.20×10^{-3} and 1.79×10^{-5} N·s/m^2 respectively.
Ans : 18.7 m/s, 134.7 N

$$F_D = f(D, V, \rho, \mu)$$

★ always chose ρ, V and Length as repeating

$$F = N = kg\frac{m}{s^2} = MLT^{-2}$$

$$D = m = L$$

$$V = m/s = LT^{-1}$$

$$\rho = kg/m^3 = ML^{-3}$$

$$\mu = ML^{-1}T^{-1}$$

$$\pi_1 \Rightarrow F_D D^A V^B \rho^C$$

$$= \frac{ML}{T^2}(L)^A\left(\frac{L}{T}\right)^B\left(\frac{M}{L^3}\right)^C$$

$C = -1$ $$\frac{\cancel{M}L}{T^2}(L)^A\left(\frac{L}{T}\right)^B\left(\frac{L^3}{\cancel{M}}\right)$$

$B = -2$ $$\frac{L}{T^2}(L)^A\left(\frac{T}{L}\right)^2(L^3)$$

$A = -2$ $$\Rightarrow \frac{F_D}{\rho V^2 D^2}$$

$$\Pi_2 \Rightarrow \mu\, D^A\, V^B \rho^C$$

$$= \frac{M}{LT} (L)^A \left(\frac{L}{T}\right)^B \left(\frac{M}{L^3}\right)^C$$

$$C=-1: \quad \frac{\cancel{M}}{LT} (L)^A \left(\frac{L}{T}\right)^B \left(\frac{L^3}{\cancel{M}}\right)$$

$$B=-1: \quad \frac{1}{L\cancel{T}} (L)^A \left(\frac{\cancel{T}}{L}\right) (L^3)$$

$$A=-1: \quad \frac{1}{L} \left(\frac{1}{L}\right) \left(\frac{1}{L}\right) (L^3)$$

$$\Rightarrow \frac{\mu}{\rho V d}$$

$$\frac{F_D}{\rho V^2 d^2} = f\left(\frac{\mu}{\rho V d}\right)$$

if $\Pi_{2m} = \Pi_{2p}$

$$\frac{\mu_m}{\rho_m V_m d_m} = \frac{\mu_p}{\rho_p V_p d_p}$$

$$V_m = \frac{\mu_m \rho_p V_p d_p}{\mu_p \rho_m d_m}$$

$$= \frac{1.79 \times 10^{-5} \times 1030 \times 1 \times 0.3}{1.2 \times 10^{-3} \times 1.23 \times 0.2}$$

$$= 18.736 \text{ m/s}$$

$$\frac{F_{Dp}}{\rho_p V_p^2 d_p^2} = \frac{F_{Dm}}{\rho_m V_m^2 d_m^2}$$

$$F_{Dp} = \frac{25}{1.23 \times 18.736^2 \times 0.2^2} \times 1030 \times 1^2 \times 0.3^2$$

$$= 134.18 \text{ N}$$

3. A prototype automobile is designed to travel at 65 km/hr. A model of this design is tested in a wind tunnel with identical standard sea-level air properties at a 1:5 scale. The measured model drag is 400 N, enforcing dynamic similarity. Determine (a) the drag force on the prototype, and (b) the power required to overcome this drag.

Drag force coefficient is : $C_{drag} = \frac{1}{2}\frac{F_D}{\rho V^2 D^2}$

Ans : 400 N, 7220 W

$$\pi_2 \Rightarrow \mu \rho^A V^B D^C$$

$$\Rightarrow \frac{m}{LT}\left(\frac{m}{L^3}\right)^A \left(\frac{L}{T}\right)^B (L)^C$$

$$A=-1 : \frac{\cancel{m}}{LT}\left(\frac{L^3}{\cancel{m}}\right)\left(\frac{L}{T}\right)^B (L)^C$$

$$B=-1 : \frac{1}{L\cancel{T}}(L^3)\left(\frac{\cancel{T}}{L}\right)(L)^C$$

$$C=-1 \qquad \therefore \frac{\mu}{\rho V d}$$

$$\frac{F_D}{2\rho V^2 D^2} = f\left(\frac{\mu}{\rho V D}\right)$$

if $\pi_{2m} = \pi_{2p}$

$$\frac{D_m}{D_p} = \frac{1}{5}$$

$$\frac{\mu_m}{\rho_m V_m D_m} = \frac{\mu_p}{\rho_p V_p D_p}$$

$$V_m = \frac{\cancel{\mu_m}\,\cancel{\rho_p}\,V_p D_p}{\cancel{\mu_p}\,\cancel{\rho_m}\,D_m}$$

$\mu_m, \mu_p, \rho_p\ \rho_m$ identical

$$V_m = \frac{65\ \cancel{D_p}}{\frac{1}{5}\cancel{D_p}} = 325\ \text{km/hr}$$

a)

$$\frac{F_{Dm}}{2\rho V_m^2 D_m^2} = \frac{F_{Dp}}{2\rho V_p^2 D_p^2}$$

$$\frac{400}{325^2\left(\frac{1}{5}D_p\right)^2} \times 65^2 \cancel{D_p^2} = F_{Dp}$$

$$F_{Dp} = 400\ N$$

b) $P = F_{Dp} \times V_p$

$= 400 \times 65\ km/hr$ → $\times \frac{1000}{3600}$ change to m/s

$= 400 \times 65 \times \frac{1000}{3600}$

$= 7222.22\ W$

4. The drag characteristics of an airplane are to be determined by model tests in a wind tunnel operated at an absolute pressure of 1300 kPa. If the prototype is to cruise in standard air at 385 km/hr, and the corresponding speed of the model is not to defer by more than 20% from this (so that compressibility effects may be ignored), what range of length scales may be used if Reynolds number similarity is to be maintained? Assume the viscosity of air is unaffected by pressure, and the temperature of air in the tunnel is equal to the temperature of the air in which the airplane will fly.

Ans: 0.0647 to 0.0971

$P_m = 1.3 \times 10^6 \text{ Pa}$

$P_p = 1 \times 10^5 \text{ Pa}$

$V_p = 385 \text{ km/hr}$

$V_{m\,min} = 0.8 V_p = 308 \text{ km/hr}$

$V_{m\,max} = 1.2 V_p = 462 \text{ km/hr}$

$$Re = \frac{\rho V d}{\mu}$$

$$\frac{\rho_p V_p d_p}{\cancel{\mu_p}} = \frac{\rho_m V_m d_m}{\cancel{\mu_m}}$$

$$\frac{d_m}{d_p} = \frac{\rho_p d_p}{\rho_m d_m}$$

$$\frac{V_p}{V_m} = \left(\frac{385}{308} \sim \frac{385}{462} \right)$$

$$P = \rho R T$$

constant stated in Qn

287 J/kgK

8.31 J/mol.K

$$\frac{d_m}{d_p} = \frac{\rho_p d_p}{\rho_m d_m}$$

$$= \frac{1}{13} \times \begin{cases} \frac{385}{462} = 0.0641 \\ \frac{385}{308} = 0.0962 \end{cases}$$

Year 3 Fluid Mechanics MA3006
Year 2021-2022 Semester 1

Tutorial 6 – Viscous Pipe Flow/Laminar and turbulent flow

1. For steady laminar pipe flow:
   (a) Show that the mean velocity V is half the centreline velocity V_c.
   (b) If the centreline velocity in a 0.1 m diameter tube is 3 m/s, and the fluid has a density of 1260 kg/m^3 and viscosity of 0.9 N.s/m^2, calculate the Reynolds number and the pressure gradient necessary for the flow. (Ans: Re = 210, -4.3 kN/m^2/m)

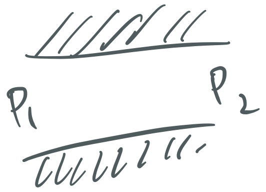
2. Fluid (specific gravity = 0.95) is discharged from a small enclosure through a horizontal tube 45 mm long, 0.75 mm diameter, at 0.12x10^{-3} l/s. The pressure at upstream end of the tube is 1.31 kPa and downstream at the end of the tube discharge is atmospheric. Calculate the viscosity of the fluid. (Ans. μ = 0.00188 kg/m.s)

3. A fluid of density ρ = 1000 kg/m^3 and viscosity μ = 0.30 N.s/m^2 flows steadily down a vertical 0.10 m diameter pipe and exits as a free jet from the lower end. Determine the maximum pressure allowed in the pipe at a location 10 m above the pipe exit, if the flow is to be laminar. (Ans : -37.6 kPa)

4 Glycerin flows upward in a vertical 75 mm diameter pipe with a centre line velocity of 1.0 m/s. Determine the head loss and pressure drop in a 10-m length of the pipe.
(dynamic viscosity of glycerin = 1.5N.s/m^2, density = 1260 kg/m^3)
(Ans : 166 kPa ; 3.43 m)

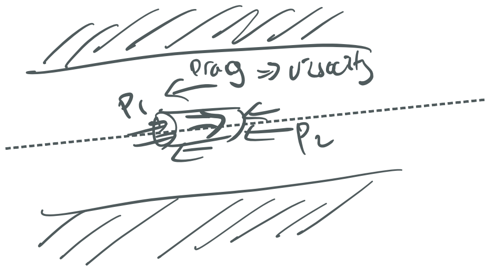
Viscous flow in Pipes

$P_1 > P_2 \Rightarrow$ to get +ve flow



if $P_1 = P_2 \Rightarrow$ no flow
if $P_1 > P_2 \Rightarrow$ move left to right
if $P_2 > P_1 \Rightarrow$ move right to left

↳ why does pressure drive flow (viscous)



$F = ma$

steady flow = accⁿ = 0

$\Sigma F = 0$

Pressure force

viscous "Resistance Drag"

$$\underset{(P_1 - P_2)}{\Delta P} \times (\pi r^2) = \tau (2\pi r l)$$

$$\frac{\Delta P}{l} = \tau \left(\frac{2}{r}\right)$$

valid for laminar

τ = represent by eqⁿ "mode" in laminar flow $= \mu \frac{du}{dr}$

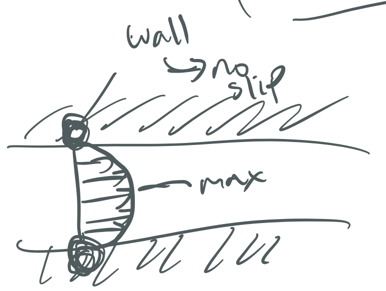
viscous → Losses → NEVER EVER consider Bernoulli eqⁿ

⇒ Energy eqⁿ

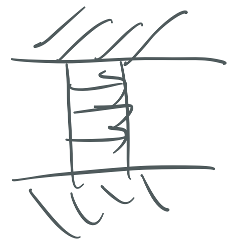
$$\left(P + \tfrac{1}{2}\rho V^2 + \rho g h_1\right) = \left(P + \tfrac{1}{2}\rho V^2 + \rho g h_2\right) + \text{Losses}_{1\to 2}$$

uniform

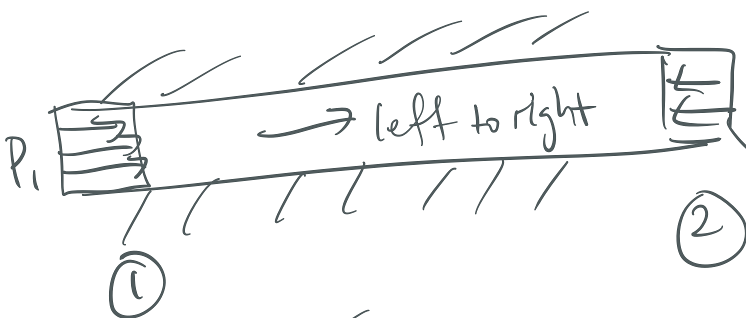
avg velocity → avg velocity of a section



⇒ energy eq^n required an avg profile



Losses in pipe (flow loss in a pipe of constant diameter)



lower exit pressure $\underbrace{\quad}_{\text{caused by losses}}$

$$P_1 + \cancel{\tfrac{1}{2}\rho v^2} + \cancel{\rho g h_1} = P_2 + \cancel{\tfrac{1}{2}\rho v^2} + \cancel{\rho g h_2} + \text{Losses}$$

constant D & horizontal

$$P_1 - P_2 = \Delta P = \text{Losses}$$

π theorem, in a pipe $\underbrace{\Delta P = f(l, d, \rho, \mu, v, \varepsilon)}$

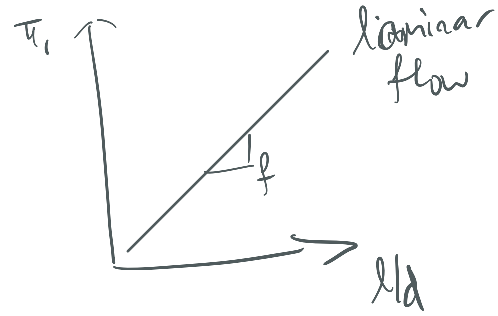
$$\left(\frac{P}{\frac{1}{2}\rho v^2}\right) \equiv \pi_1 = f\Big(\underset{\pi_2}{\frac{l}{d}}, \underset{\pi_3}{\frac{\varepsilon}{d}}\Big) \Rightarrow \text{not significant in laminar flow}$$

↳ non dimensional π – term

from experimental data + theory

$$\underbrace{\frac{P}{\left(\frac{1}{2}\rho v^2\right)}}_{\pi_1} = \underset{\substack{\Uparrow \\ \Pi \\ \text{friction} \\ \text{const.}}}{f}\left(\frac{l}{d}\right)$$

← non-dimensional l/d



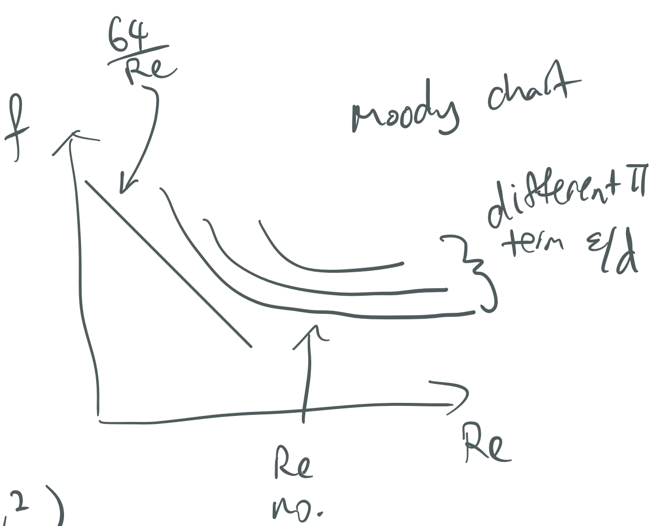
analytic value for f - laminar

Advised to remember

$$\boxed{f = \frac{64}{Re}}$$

"turbulent flow"

$f = \text{function}\left(Re, \ \varepsilon/d\right)$

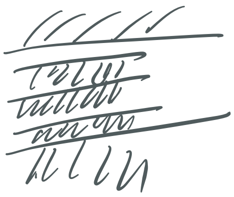


$$\Delta P = f\left(\frac{l}{d}\right)\left(\frac{1}{2}\rho v^2\right)$$

f ↑ laminar vs turbulent

$\frac{l}{d}$ ↑ non dimensional

$\frac{1}{2}\rho v^2$ ↑ dynamic pressure laminar



layer

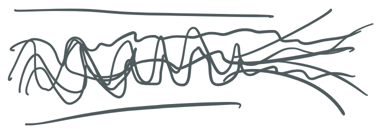
① laminar layered → no mixing

② steady

③ $\tau = \mu \frac{du}{dy}$

④ Pressure small

turbulent



① chaotic "no order" mixing

② unsteady

③ $\tau \equiv$ far more complicated

④ high

$$\Delta P = f\left(\frac{l}{d}\right)\left(\frac{1}{2}\rho V^2\right)$$

higher for turbulent flow

How do we differentiate

$$Re = \frac{\text{inertia forces}}{\text{viscous}} = \frac{\rho v d}{\mu}$$

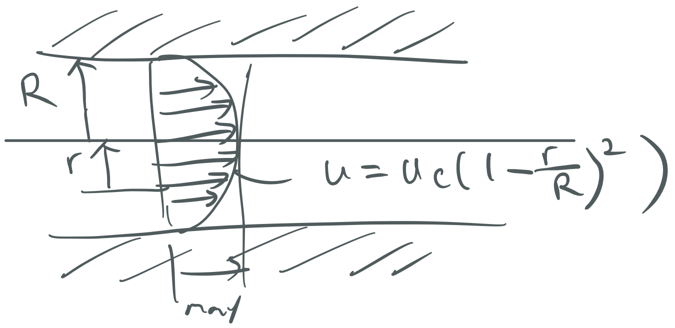
ρ = density

v = velocity

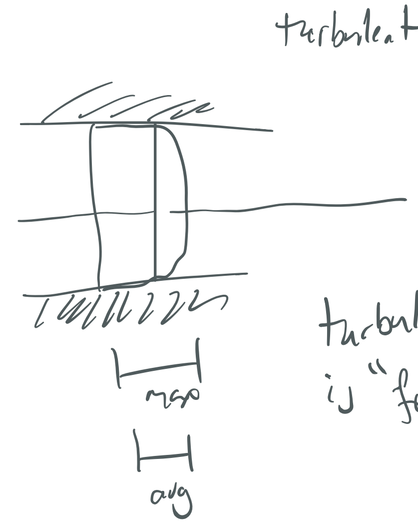
μ = viscosity

d = diameter

use hydrolic diameter instead of diameter when calculatly Re.



$V_{avg} = \frac{1}{2} max$



turbulent is "fuller"

Summary

in pipes use energy eq^n

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2 + \text{losses}$$

major

⇒ losses by a pipe

$$\text{major pipe losses} \equiv f\left(\frac{l}{d}\right)\left(\frac{1}{2}\rho V^2\right)$$

depend on Reynolds no.

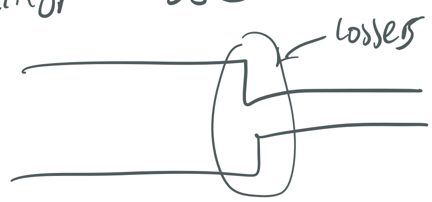
laminar

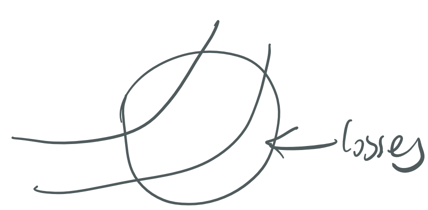
$f = \frac{64}{Re}$

turbulent

use moody chart

minor losses





impt assumption + warning

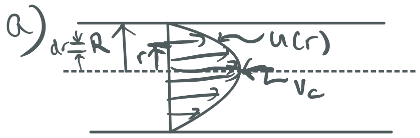
subsituting of V is the average section velocity

1. For steady laminar pipe flow:

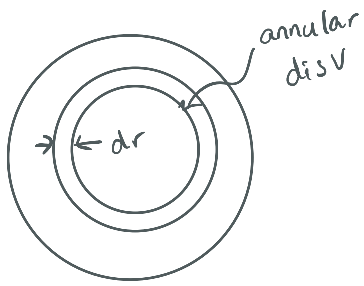
(a) Show that the mean velocity V is half the centreline velocity V_c.

(b) If the centreline velocity in a 0.1 m diameter tube is 3 m/s, and the fluid has a density of 1260 kg/m^3 and viscosity of 0.9 N.s/m^2, calculate the Reynolds number and the pressure gradient necessary for the flow. (Ans: Re = 210, -4.3 kN/m^2/m)

a)



$$u(r) = V_c\left(1 - \frac{r^2}{R^2}\right)$$



Flow rate

$$dQ = \underbrace{u(r)}_{\text{velocity at location } r} \times \underbrace{(2\pi r\, dr)}_{\text{area normal to velocity}}$$

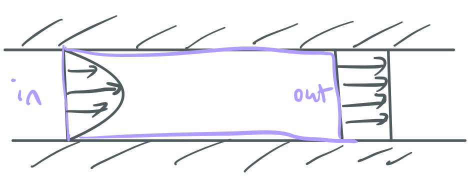
$$\boxed{Q = \int_0^R u(r) \times (2\pi r)\, dr} \rightarrow \text{laminar / turbulent}$$

for laminar flow $u(r) = V_c\left(1 - \frac{r^2}{R^2}\right)$

$$Q = \int_0^R 2\pi V_c \left(r - \frac{r^3}{R^2}\right) dr$$

$$= 2\pi V_c \left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right)\Bigg|_0^R$$

$$= \frac{\pi R^2 V_c}{2}$$



$$Q_{in} = Q_{out}$$

$$= V_{avg} \cdot \pi R^2$$

$$\frac{\cancel{\pi R^2} V_c}{2} = V_{avg} \cancel{\pi R^2}$$

$$V_{avg} = \frac{V_c}{2} \Rightarrow$$ laminar steady incompressible flow

b) $\phi = 0.1m$

$V_c = 3 m/s$

$V_{avg} = 1.5 m/s \Rightarrow$ don't forget for Re no. & energy eq^n must use V_{avg}

$\rho = 1260 \, kg/m^3$

$\mu = 0.9 \, Ns/m^2$

Re no. = ?

pressure gradient = ? $\leftarrow \frac{\Delta P}{l}$

$$Re = \frac{\rho V d}{\mu}$$

$$= \frac{1260 \times 1.5 \times 0.1}{0.9}$$

$$= 210$$

$$\Delta P = f\left(\frac{l}{d}\right)\left(\frac{1}{2}\rho V^2\right)$$

$$\frac{\Delta P}{l} = f\left(\frac{1}{d}\right)\left(\frac{1}{2}\rho V^2\right)$$

$$= \frac{64}{210}\left(\frac{1}{0.1}\right)\left(\frac{1}{2} \times 1260 \times 1.5^2\right)$$

$$= 4320 \; Pa/m$$

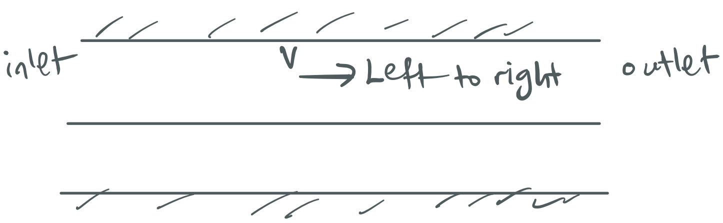
2. Fluid (specific gravity = 0.95) is discharged from a small enclosure through a horizontal tube 45 mm long, 0.75 mm diameter, at 0.12×10^{-3} l/s. The pressure at upstream end of the tube is 1.31 kPa and downstream at the end of the tube discharge is atmospheric. Calculate the viscosity of the fluid. (Ans. $\mu = 0.00188$ kg/m.s)

$\rho = 950\ kg/m^3 \Rightarrow$ specific gravity $\times 1000$ (0.95)

$l = 45\ mm = 45 \times 10^{-3}\ m$

$\phi = 0.75\ mm = 0.75 \times 10^{-3}\ m$

$Q = 0.12 \times 10^{-3}\ l/s = 0.12 \times 10^{-3} \times 10^{-6}\ m^3/s$



$P_{inlet_g} = 1.31\ kPa$

$P_{outlet_g} = P_{atm} = 0$

$\mu = ?$

$$Q = AV$$

$$V = \frac{Q}{A} = \frac{0.12 \times 10^{-3} \times 10^{-6}}{\frac{\pi}{4} \times (0.75 \times 10^{-3})^2} = 0.0002716\ m/s$$

energy eq^n

horizontal Patm

$$\left(P + \cancel{\tfrac{1}{2}\rho V^2} + \cancel{\rho g h}\right)_{inlet} = \left(\cancel{P} + \cancel{\tfrac{1}{2}\rho V^2} + \cancel{\rho g h}\right)_{outlet} + \text{losses}$$

Same $\phi \Rightarrow$ same V

$$P_{inlet} = f\left(\frac{l}{d}\right)\left(\frac{1}{2}\rho V^2\right)$$

$$1.31\ kPa = f\left(\frac{45 \times 10^{-3}}{0.75 \times 10^{-3}}\right)\left(\frac{1}{2}(950)(0.0002716)^2\right)$$

$$f = 623.113$$

Assume laminar flow

$$f = \frac{64}{Re}$$

$$Re = 0.102710$$

$$Re = \frac{\rho V d}{\mu}$$

$$\mu = \frac{950 \times 0.0002716 \times 0.75 \times 10^{-3}}{0.102710}$$

$$= 0.001884 \text{ kg/m.s}$$

3. A fluid of density $\rho = 1000$ kg/m^3 and viscosity $\mu = 0.30$ N.s/m^2 flows steadily down a vertical 0.10 m diameter pipe and exits as a free jet from the lower end. Determine the maximum pressure allowed in the pipe at a location 10 m above the pipe exit, if the flow is to be laminar. (Ans : -37.6 kPa)

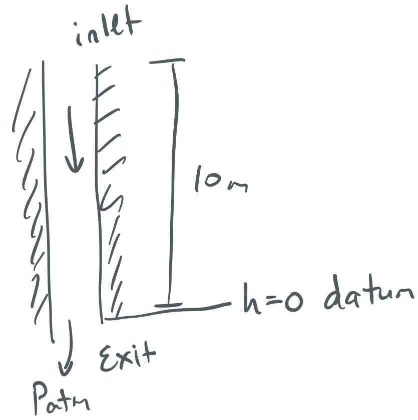
$\rho = 1000 \, kg/m^3$

$\mu = 0.30 N.s/m^2$

$\phi = 0.1 \, m$

$P_{exit} = P_{atm} = 0$

$P_{in} = ?$ → flow remain laminar



energy eq^n

$$(P_{in} + \cancel{\tfrac{1}{2}\rho} V^2 + \rho g h)_{in} = (\cancel{P} + \cancel{\tfrac{1}{2}\rho} V^2 + \cancel{\rho g h})_{Exit} + \text{losses}$$

$$P_{in} = f\left(\frac{l}{d}\right)\left(\frac{1}{2}\rho V^2\right) - \rho g(10)$$

Not Known f, V ⇒ maintained laminar flow

$$f = \frac{64}{Re} \Rightarrow Re \le 2100$$

$$f = \frac{64}{2100} = 0.030476$$

$$Re = \frac{\rho V d}{\mu} \Rightarrow 2100 = \frac{1000(V)(0.1)}{0.3}$$

$$V = 6.3 \, m/s$$

$$P_{in} = 0.030476\left(\frac{10}{0.1}\right)\left(\frac{1}{2}(1000)(6.3)^2\right) - (1000)(9.81)(10)$$

$$= -37620.378 \, Pa \approx -37.62 \, kPa$$

4 Glycerin flows upward in a vertical 75 mm diameter pipe with a centre line velocity of 1.0 m/s. Determine the head loss and pressure drop in a 10-m length of the pipe. (dynamic viscosity of glycerin = 1.5N.s/m^2, density = 1260 kg/m^3)

(Ans : 166 kPa ; 3.43 m)

$\phi = 75\,mm = 75\times10^{-3}\,m$

$V_c = 1\,m/s$

$V_{avg} = 0.5\,m/s$

$\rho = 1260\,Kg/m^3$

$\mu = 1.5\,N\cdot s/m^2$

energy eq^n

$$\underbrace{(P + \cancel{\tfrac{1}{2}\rho V^2} + \overset{0}{\cancel{\rho g h}})_{in} = (P + \cancel{\tfrac{1}{2}\rho V^2}}_{\text{cancel same diameter}} + \rho g h)_{out} + \text{losses}$$

$$\Delta P = P_1 - P_2 = \rho g(10) + f\left(\frac{l}{d}\right)\left(\frac{1}{2}\rho v^2\right)$$

$$= \rho g(10) + \frac{64}{Re}\left(\frac{10}{75\times10^{-3}}\right)\left(\frac{1}{2}(1260)(0.5)^2\right)$$

$$Re = \frac{\rho v d}{\mu} = \frac{1260\times0.5\times75\times10^{-3}}{1.5}$$

$$= 31.5$$

$$\Delta P = (1260)(9.81)(10) + \frac{64}{31.5}\left(\frac{10}{75\times10^{-3}}\right)\left(\frac{1}{2}(1260)(0.5^2)\right)$$

$= 166272.667$ Pa

≈ 166 KPa //

head loss :

$$\underbrace{\left(\frac{P}{\rho g}\right.}_{\substack{\text{pressure}\\ \text{head}}} + \underbrace{\frac{V^2}{2g}}_{\substack{\text{dynamic}\\ \text{head}}} + \left.\underbrace{h}_{\substack{\text{elevation}\\ \text{or fluid}\\ \text{head}}}\right)_{in} = \left(\frac{P}{\rho g} + \frac{V^2}{2g} + h\right)_{out} + \underbrace{f\left(\frac{l}{d}\right)\left(\frac{V^2}{2g}\right)}_{\substack{\text{head}\\ \text{loss}}}$$

head loss : $f\left(\frac{l}{d}\right)\left(\frac{V^2}{2g}\right)$

$$= \frac{64}{31.5}\left(\frac{10}{75\times10^{-3}}\right)\left(\frac{0.5^2}{2(9.81)}\right)$$

$$= 3.4518 \text{ m}$$

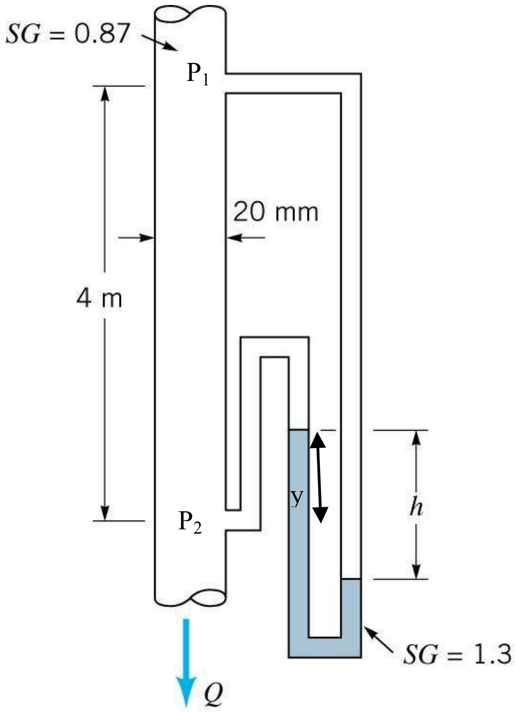
Year 3 Fluid Mechanics MA3006
Year 2021-2022 Semester 1

Tutorial 7 – Viscous Pipe Flow/Laminar and turbulent flow

1. Water (ρ=999 kg/m^3 and μ=1.138 x 10^{-3} kg/m.s) flows steadily at 0.34m^3/min through a 5 cm diameter horizontal pipe. Determine the pressure drop, head loss, and the input power required for flow over a 61m pipe length.(Pipe roughness ε = 0.002 mm)
(Ans : 88.9 kPa, 9.07m, 503.77 W)

2. Calculate the pressure drop along 1 m length (horizontal) of a smooth 25 mm diameter pipe when water flows through it at 735 l/hr. What is the average shear stress at the pipe wall?
(Ans : 108.2 N/m^2. 0.673 N/m^2)

3. Oil of s = 0.87 and ν = 2.2x10^{-4} flows through the vertical pipe shown in Figure at 4x10^{-4} m^3/s. Determine the manometer reading h. (Ans : 18.5 m)



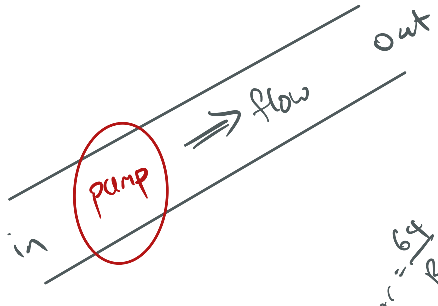
4. For laminar flow, show that the pressure drop in a pipe can be expressed as:

$$\Delta p = \frac{64}{d} \mu \frac{l}{d} \frac{V}{2}$$

where d = diameter , l = length of pipe and μ = dynamic viscosity

Water flows in a smooth pipe of diameter d = 0.01 m and has a velocity, V = 0.1 m/s. Is the flow laminar or turbulent? Determine the maximum pressure drop over a distance of l = 10m if the flow is to remain laminar. (Ans: 672 Pa)

Flow in Pipes



$$\left(P + \tfrac{1}{2}\rho v^2 + \rho g h\right)_1 = \left(P + \tfrac{1}{2}\rho v^2 + \rho g h\right)_2 + \text{Losses}$$

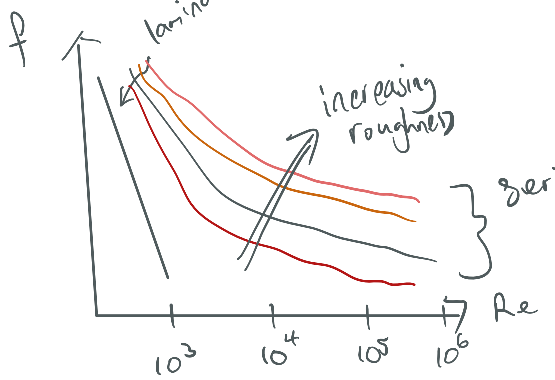
$$\Delta P_l = \text{losses} = \text{major pipe} = \underset{\text{friction factor}}{f}\ \underset{\text{non dimensional}}{\frac{l}{d}}\ \underbrace{\frac{1}{2}\rho v^2}_{\text{dynamic}}$$

$$h_l = \frac{\Delta P_l}{\rho g}$$

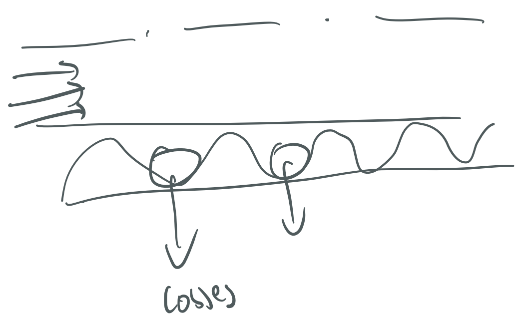
$$\text{Power} = \dot{m}\Delta P$$

$f_{laminar} = \frac{64}{Re}$

laminar flow



series of curves for different roughness



measure of roughness

$\varepsilon \Rightarrow$ in mm

1. Water (ρ=999 kg/m^3 and μ=1.138 x 10^{-3} kg/m.s) flows steadily at 0.34m^3/min through a 5 cm diameter horizontal pipe. Determine the pressure drop, head loss, and the input power required for flow over a 61m pipe length.(Pipe roughness ε = 0.002 mm)

(Ans : 88.9 kPa, 9.07m, 503.77 W)

$\rho = 999\ kg/m^3$

$\mu = 1.138 \times 10^{-3}\ kg/m.s$

$Q = 0.34 m^3/min = 0.34/60\ m^3/s$

$\phi = 5cm = 0.05\ m$

$\varepsilon = 0.002\ mm$ (recall units of ε are in mm)

$l = 61m$

required
ΔP
h_l
Power to pump fluid

$(P + \frac{1}{2}\cancel{\rho v^2} + \cancel{\rho g h})_1 = (P + \cancel{\frac{1}{2}\rho v^2} + \cancel{\rho g h})_2 + \text{Losses}$

$\Delta P_l = \text{Losses} = f \frac{l}{d} \frac{1}{2} \rho v^2$

to get $f \Rightarrow$ to get $Re = \frac{\rho v d}{\mu}$

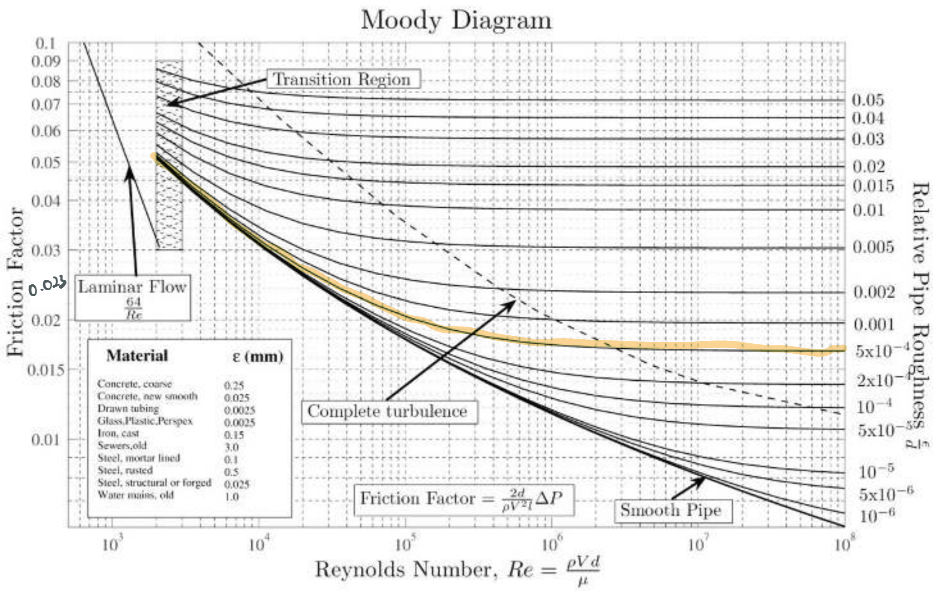
$v = \frac{Q}{A} = \frac{0.34/60}{\frac{\pi}{4}(0.05)^2} = 2.886$

$Re = \frac{\rho v d}{\mu} = \frac{999 \times \quad \times 0.05}{1.138 \times 10^{-3}} =$

$\frac{\varepsilon}{d} = \frac{0.002mm}{5 \times 10} = \frac{0.002}{50} = 0.00004$

← convert to mm

$= 4 \times 10^{-5}$



$$\Delta P = 0.023 \frac{61}{0.05} \times \frac{1}{2} \times 999 \times 0.7215^2$$

$$= 7296.18 \text{ Pa}$$

$$\frac{\Delta P}{\rho g} = \frac{7296.18}{999 \times 9.81} = 0.744 \text{ m}$$

$$\text{Pump Power} = \dot{m} \Delta P = (\rho \dot{Q}) \Delta P$$

m³/s

$$= 999 \times \frac{0.34}{60} \times 7296.18$$

$$= 41303.6$$

2. Calculate the pressure drop along 1 m length (horizontal) of a smooth 25 mm diameter pipe when water flows through it at 735 l/hr. What is the average shear stress at the pipe wall?

(Ans : 108.2 N/m^2. 0.673 N/m^2)

i) $l = 1\ m$ horizontal

$d = 25\ mm$ (smooth

$Q = 735\ l/hr = 735 \times 10^{-3}\ m^3/hr = \frac{735 \times 10^{-3}}{3600}\ m^3/s$

get $v = \frac{Q}{A} = \quad m/s$

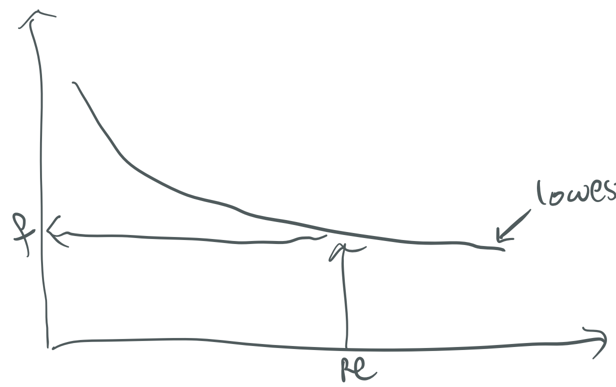
$Re = \frac{\rho v d}{\mu}$ $d = \frac{25}{1000}$

use from problem 1 → since its water

$\rho = 999\ kg/m^3$

$\mu = 1.138 \times 10^{-3}\ kg/m^3$

$=$



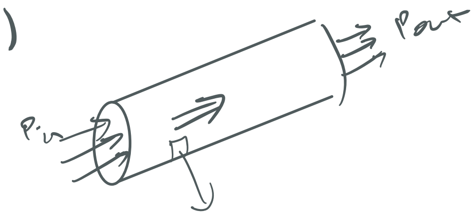
smooth

lowest curve "least friction factor"

$\frac{\varepsilon}{d} \approx 10^{-9}$

$(P + \cancel{\tfrac{1}{2}\rho v^2} + \cancel{\rho g h})_1 = (P + \cancel{\tfrac{1}{2}\rho v^2} + \cancel{\rho g h})_2 + \text{Losses}$

$\Delta P = f \frac{l}{d} \frac{1}{2} \rho v^2$ (major loss) ii)



internal flow resistance "shear" (τ)
cause by viscosity & friction

Force of friction
$= \tau \times (\text{area})$ ← internal pipe area
$= \tau \times (2\pi r\, l)$ → length of pipe

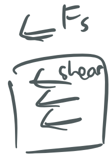
for flow to move

$P_{in} > P_{out}$

$F = (P_{in} - P_{out})\,\text{Area}$

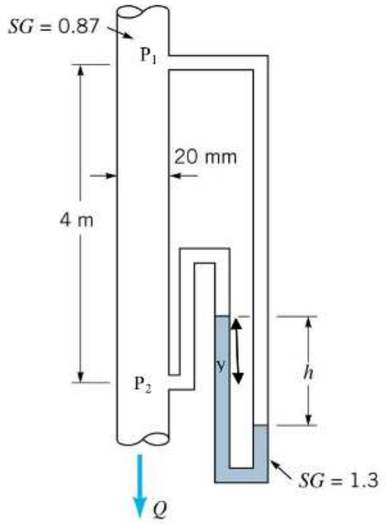
↑ force needed to move fluid

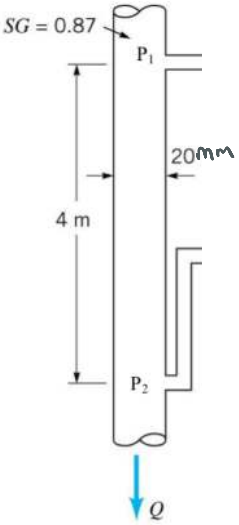
pressure force



in fluid statics

3. Oil of s = 0.87 and ν = 2.2x10^{-4} flows through the vertical pipe shown in Figure at 4x10^{-4} m^3/s. Determine the manometer reading h. (Ans : 18.5 m)





$\rho_{air} = SG \times 1000 = 870\,kg/m^3$

$\nu = 2.2 \times 10^{-4}$

$Q = 4 \times 10^{-4}\ m^3/s$

$\therefore\ V = \dfrac{Q}{A} = \dfrac{4 \times 10^{-4}}{\frac{\pi}{4}\left(\frac{20}{1000}\right)^2} =$

$Re = \dfrac{\rho v d}{\mu} = \dfrac{vd}{\nu} = \dfrac{v \times \frac{20}{1000}}{2.2 \times 10^{-4}}$

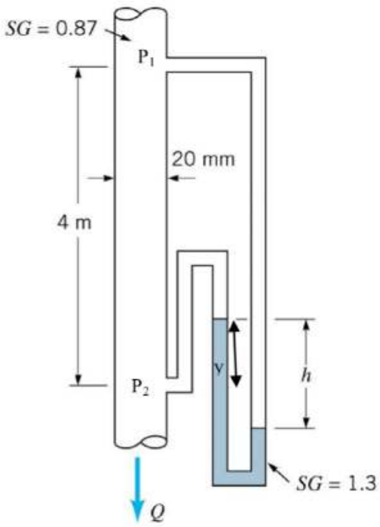
find f

$(P + \cancel{\tfrac{1}{2}\rho v^2} + \rho g \overset{4}{h})_1 = (P + \cancel{\tfrac{1}{2}\rho v^2} + \rho g \overset{0}{h})_2 + \text{Losses}$

$P_1 + 4\rho g = P_2 + \text{Losses}$

major losses

$P_1 + 4\rho g - P_2 = \tfrac{1}{2}\rho v^2 f \dfrac{l}{d}$ — (1)



$$\rho_m = SG \times 1000$$
$$= 1.3 \times 1000 = 1300$$

$$P_a = P_2 + \rho_m (h-y) g$$

$$P_b = P_1 + \rho_f (h-y) g + \rho_f g (4)$$

$$h - y \equiv \bar{h}$$

$$P_a = P_2 + \rho_m \bar{h} g = P_1 + \rho_f \bar{h} g + 4 \rho_f g$$

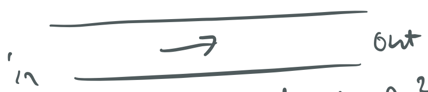
4. For laminar flow, show that the pressure drop in a pipe can be expressed as:

$$\Delta p = \frac{64}{d}\mu\frac{l}{d}\frac{V}{2}$$

where d = diameter , l = length of pipe and μ = dynamic viscosity

Water flows in a smooth pipe of diameter d = 0.01 m and has a velocity, V = 0.1 m/s. Is the flow laminar or turbulent? Determine the maximum pressure drop over a distance of l = 10m if the flow is to remain laminar. (Ans: 672 Pa)

Assume pipe horizontal



$$\Delta P_\ell = f\frac{l}{d}\frac{1}{2}\rho v^2$$

$$f = \frac{64}{Re} = \frac{64}{\left(\frac{\rho v d}{\mu}\right)} = \frac{64\mu}{\rho v d}$$

$$\Delta P = \frac{64\mu}{\rho v d}\frac{1}{2}\rho v^2$$

Recall $\underbrace{Re = 2100}_{\text{limit of laminar}} = \frac{\rho v d}{\mu} \Rightarrow$ sub to get V

sub to get ΔP

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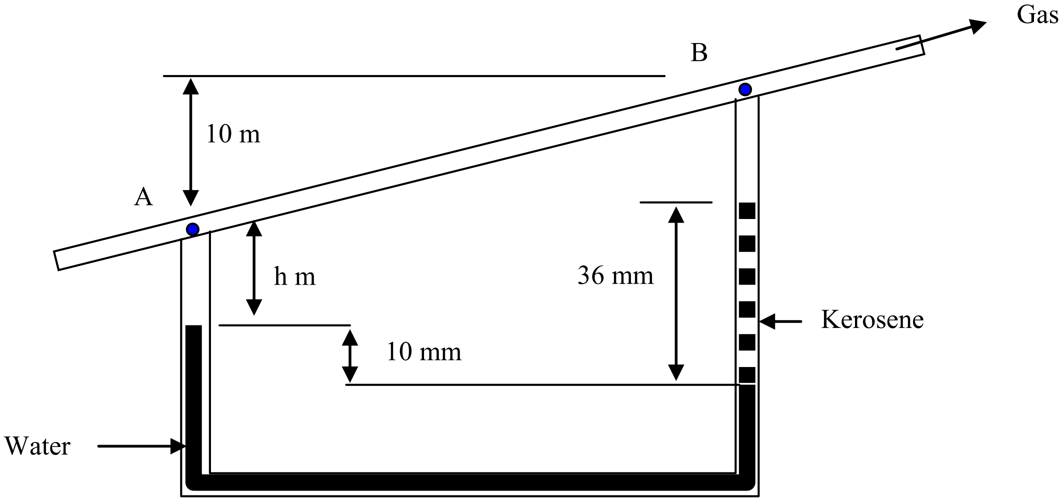
Tutorial 8 – Pipe Flow – Frictional Losses

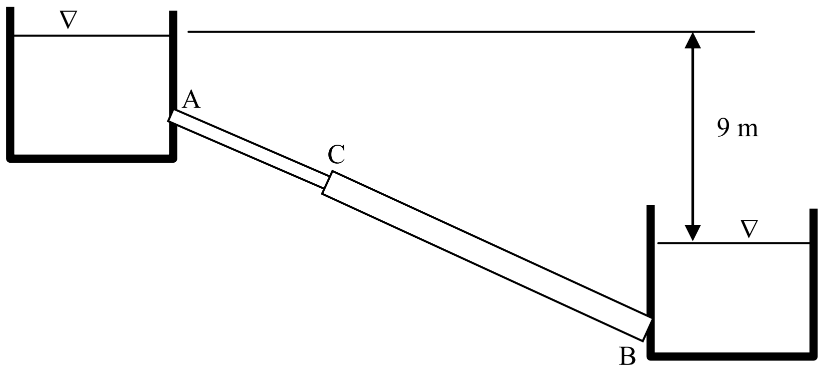
1. A pipeline carrying natural gas is 10 cm in diameter. The two points A and B, 30 m apart, are connected to a water-kerosene manometer. Given f = 0.016, ρ of natural gas = 0.6 kg/m^3, ρ of kerosene = 800 kg/m^3, calculate the mass flow rate of natural gas. (Ans : 0.053 kg/s)



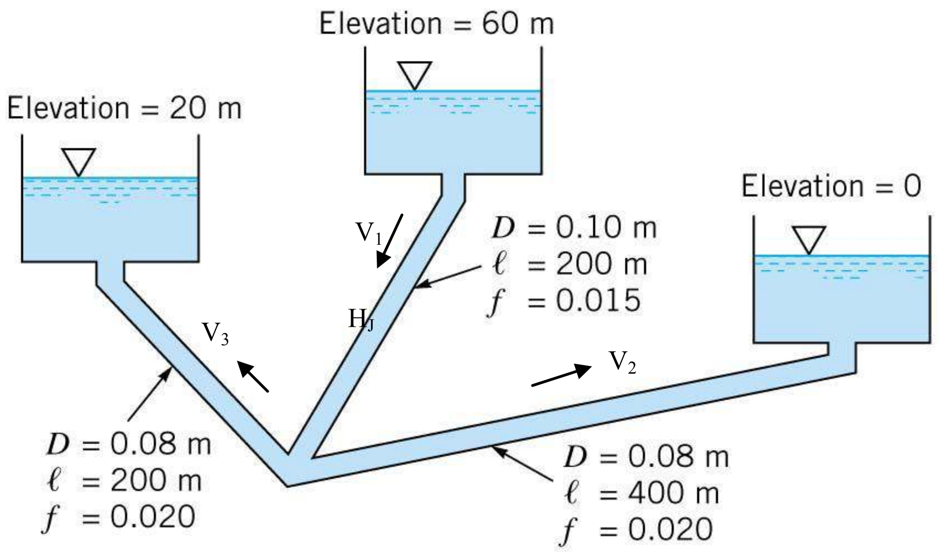
2. A centrifugal pump draws water from a well 3 m below its centreline through 7 m of 100 mm diameter pipe. It delivers freely at 15 m above pump centreline through 30 m of 75 mm diameter pipe. Both pipes are smooth. What are the pressures at the pump inlet and delivery flange when the flow rate is 30 l/s ? (Ans : -43.6 kPa, 264 kPa)

3. A pipe 900 m long and 200 mm diameter discharges water to the atmosphere at a point 10 m below the level of entrance to the pipe. With a pressure at the upstream end of the pipe (pipe entrance) of 40 kN/m^2 above atmospheric, the steady discharge from the pipe is 49 l/s. At a point half way along the pipe, a tapping is made from which water is to be drawn off at a rate of 18 l/s. If conditions are such that the pipe is always flowing full, to what value must pressure at the pipe entrance be raised so as to provide an unaltered discharge from the end of the pipe? The friction factor of the pipe can be assumed to remain the same and minor losses may be neglected. (Ans : 97.2 kN/m^2)

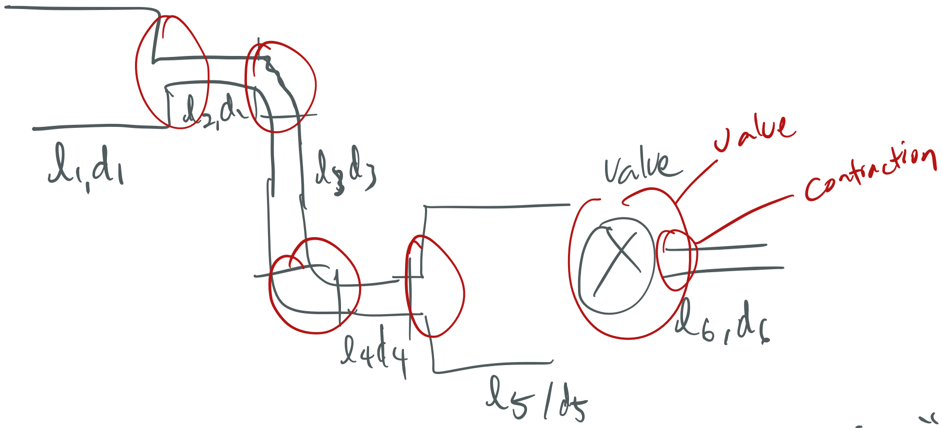
4. Two reservoirs A and B have a difference in level of 9 m, and are connected by a pipeline 200 mm in diameter over the first part AC, which is 15 m long, and then 250 mm diameter for CB, the remaining 45 m length. The entrance to and exit from the pipe are sharp edged and change of section at C is sudden. The friction factor f = 0.01 for both pipes. Calculate the flow rate in m^3/s.
(Ans : 0.263 m^3/s)



5. The three tanks in Figure are connected by pipes as indicated. If minor losses are neglected, determine the flow rate in each pipe. (0.0284, 0.0143, 0.0141 m^3/s)



Pipe losses

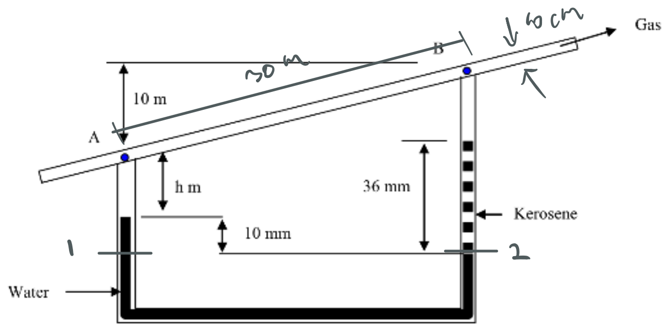


major loss ⇒ sum of all different major "pipe" losses

$$= \sum_{i=1}^{6} f_i \frac{l_i}{d_i} \frac{1}{2} \rho v_i^2$$

minor loss ⇒ sum of all minor losses "loses caused by pipe contraction"

1. A pipeline carrying natural gas is 10 cm in diameter. The two points A and B, 30 m apart, are connected to a water-kerosene manometer. Given f = 0.016, ρ of natural gas = 0.6 kg/m^3, ρ of kerosene = 800 kg/m^3, calculate the mass flow rate of natural gas. (Ans : 0.053 kg/s)



fluid static

$P_1 = P_2$

energy btw A & B

$$\left(P + \cancel{\tfrac{1}{2}\rho v^2} + \cancel{\rho g h}\right)_A = \left(P + \cancel{\tfrac{1}{2}\rho v^2} + \rho g \overset{10}{h}\right)_B + \text{losses}$$

losses $\rightarrow f \dfrac{l}{d} \dfrac{1}{2}\rho v^2$

$$P_A = P_B + \rho g(10) + f\frac{l}{d}\frac{1}{2}\rho v^2$$

$P_1 = P_2$

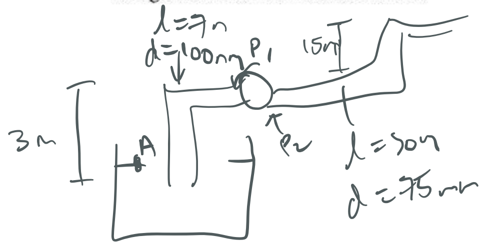
$$P_A + \rho g h + \rho_w g\left(\frac{10}{1000}\right) + \rho g\left(\frac{26}{1000}\right) = P_B + \rho g(10) + \rho g h + \rho_{k} g\left(\frac{36}{1000}\right)$$

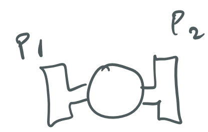
$\Rightarrow$ solve for $P_A - P_B$

sub ② into ① solve v^2

flow rate $\Rightarrow \dot{Q} = AV$, $\dot{m} = \rho\dot{Q}$

2. A centrifugal pump draws water from a well 3 m below its centreline through 7 m of 100 mm diameter pipe. It delivers freely at 15 m above pump centreline through 30 m of 75 mm diameter pipe. Both pipes are smooth. What are the pressures at the pump inlet and delivery flange when the flow rate is 30 l/s ? (Ans : -43.6 kPa, 264 kPa)





Part 1

$$\left(\cancel{P} + \tfrac{1}{2}\cancel{\rho v^2} + \cancel{\rho g h}\right)_A = \left(P + \tfrac{1}{2}\rho v^2 + \rho g h^{3}\right)_1 + \text{losses}$$

?

calculated from Q

smooth $\Rightarrow Re = \frac{\rho v d}{\mu}$ water

$$0 = P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 + f\frac{l}{d}\left(\tfrac{1}{2}\rho v^2\right.$$

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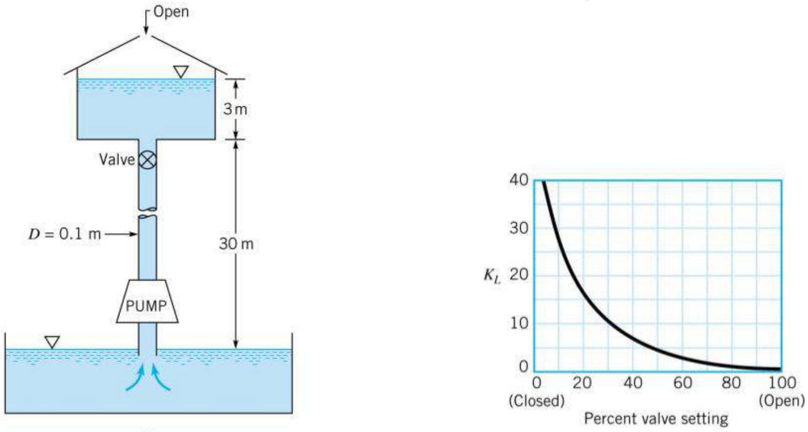
Tutorial 9 – Pumps and system I

1. A centrifugal pump having an impeller diameter of 1 m is to be constructed so that it will supply a head rise of 200 m at a flow rate of 4.1 m^3/s of water when operating at a speed of 1200 rpm. To study the characteristics of this pump, a 1/5 scale, geometrically similar model operated at the same speed is to be tested in the laboratory. Determine the required model discharge and head rise. Assume both model and prototype operates with the same efficiency and hence same flow coefficient. (Ans : 0.0328 m^3/s, 8 m)

2. A centrifugal pump with a 0.30 m diameter impeller requires a power input of 44.7 kW when the flow rate is 0.2 m^3/s and the head is 18.3 m. The impeller is changed to one with a diameter of 0.25 m. Determine the expected flow rate, head and input power if pump speed remains the same. (Ans : 0.116 m^3/s, 12.71 m, 17.96 kW)

3. In the figure below, liquid is pumped from an open tank through a 0.1 m diameter vertical pipe, into another tank. A valve is located in the pipe, minor loss coefficient for the valve as a function of valve setting is as shown. The pump head-Q relationship is given by the equation: $h_a = 52.0 - 1.01x10^3\ Q^2$, h_a in m and Q in m^3/s. Assume friction factor for pipe f = 0.02, and minor losses except for the valve are negligible, determine the flow rate when the valve is fully open (assume K =1.0), and the required valve setting to reduce the flow rate by 50 %. Tank levels remain constant. (Ans: 0.0529 m^3/s, 10 % open)



(a) (b)

4. Oil with SG 0.85 is pumped from Tank A to Tank B over a pipe length of 80m, diameter of pipe 80mm (suction and discharge), frictional factor 0.012 , and total loss factor K=10. The elevation of tank A and B is 15m and 10m respectively. Characteristic of pump is:

Flow rate (m^3/s)	Discharge head (m)	Efficiency (%)
0	50	0
0.02	45	54
0.03	41	70
0.04	35	78
0.05	26	78
0.06	8	69

Determine the system characteristics, discharge and power required by the pump
If an identical pump is added in series, what will be the new flow rate?
(Ans: $Hp = -5 + 44380\ Q^2$; 0.032 m^3/s, 40m, 71%, 15.03 kW; 0.041 m^3/s)

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Tutorial 10 : Turbomachine II

1. Oil with specific gravity 1.08 is to be transferred a tank at elevation 0 m to another tank at elevation 5 m though a piping system. The pipe diameter is 400mm, overall length of pipe is 80m and frictional factor is 0.032, and minor loss coefficient of the piping system is K =4.0. A selected pump has the following characteristics : $Hp = 15 - 100\ Q^2$ where Hp is pump head in metre and Q is flow rate in m^3/s. Determine the system characteristics, operating pump head, discharge and power required. Pump efficiency is 80%

   The elevation between the two tanks is increased to 20m. Write down the new system characteristics equation. Can the same pump work? How would you overcome this deficiency? If two similar pumps are to operate in series, determine the new operating pump head and discharge.

   (Ans : 5 + 33.56Q2 ; 0.2736 m^3/s ; 7.512m ; 27.2 KW ; 0.207 m^3/s 21.43m)

2. Two reservoirs are connected by a 800m long pipe with an internal diameter of 0.4m and wall roughness is 0.16mm. Reservoir A has an elevation of 32m. Reservoir B has an elevation of 2m. Neglect minor losses and assume flow is in wholly turbulent regime, what is the expected steady flow discharge rate from higher to lower reservoir?

   If a pump of characteristics; $H = 40 - 24Q^2$, is used to reverse the flow (from lower to higher reservoir), what would be the expected steady pumping rate. H is the pump head in metre and Q is the flow rate in m^3/s. If the efficiency of the pump is 75%, what is the power required? Is the flow (during reverse flow) in the wholly turbulent regime ?

   (Ans : 0.539 m^3/s ; 0.28 m^3/s 139.5 KW ; close to fully turbulent)

3 A pump is used to draw water from a well with a difference in elevation of 3m. Suction pipe diameter is 10 cm and length of pipe is 10 m. Frictional factor is 0.015 with total minor loss K = 2.5. The pump rotates at 2500 rpm and cavitation was detected when the flow velocity in the suction pipe was calculated to be 4 m/s. Determine the NPSH of the pump. If a larger geometrically similar pump is used, will the NPSH of the pump increased?
Vapour pressure of water = 2340 Pa (Ans : 3.795 m)

4. A Radial flow type pump, 260mm diameter, is used to deliver water at a rate of $300m^3/hr$. The NPSH required by the pump at this operating flow rate is 8 m. The pump is located at an elevation, z, above the suction level of the reservoir. The overall loss at the suction side of the pump is 1m. What is the maximum suction lift possible? What happen if this pump is located 2m above the suction level? Atm pressure = 101 kPa and vapour press to be 1700 Pa.

   (Ans : 1.122m)