L1: Mechatronics Overview

Stiffler: Science that integrates mechanical devices with electronic controls / **Bolton**: "Integration of electronics, control engineering and mechanical engineering" / **Bradley, et al**: "An integrating theme within the design process [combining] electronic engineering, computing and mechanical engineering / **Shetty & Kolk**: Methodology used for the optimal design of electromechanical products / **Auslander & Kempf**: Application of complex decision making to the operation of physical systems / **Alciatore & Histand**: Interdisciplinary field of engineering dealing with the design of products whose function relies on the integration of mechanical and electronic components coordinated by a control architecture; Mechatronics systems include elements such as logic, feedback, and computation that in a complex design may appear to simulate human thinking processes. / **Harshama, Tomizuka & Fukuda**: The synergistic integration of mechanical engineering with electronics and intelligent computer control in the design and manufacture of industrial products, processes, and operations

Measurement Systems

Transducer: device converting physical quantity into time varying voltage (i.e. analog signal) / **Signal Processor**: device to modify the analog signal / **Recorder**: device to display or record the signal.

- Good measurement system characterised by: 1) phase linearity (i.e. preservation of phase relationship between freq. components), 2) amplitude linearity, 3) adequate bandwidth.

Linearity: $V_{out}(t) - V_{out}(0) = \alpha[V_{in}(t) - V_{in}(0)]$

Non-linearity: Difficult to interpret output; Typically, linear response holds for limited range of input amplitude and rate respectively

Fourier Series

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t)$$
$$= C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \phi)$$

- $A_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)\,dt$
- $B_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)\,dt$
- $C_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{A_0}{2}$
- $C_n = \sqrt{A_n^2 + B_n^2}, \phi = -\tan^{-1}\frac{B_n}{A_n}$

Even Fn $f(-t) = f(t)$:
$A_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_0 t)\,dt$, $B_n = 0$,
Odd Fn $f(-t) = -f(t)$:
$B_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_0 t)\,dt$, $A_n = C_0 = 0$,

Converting to Sine-only series: Find C_0 and ϕ in:
$C_0\sin(\omega t + \phi) = C_0\sin(\omega t)\cos\phi + C_0\cos(\omega t)\sin\phi$

Complex Fourier Series *(shouldn't come out)*

$$F(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

Where $D_n = \frac{A_n - jB_n}{2} = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_0 t}dt$

Bandwidth: $dB = -20\log\frac{A_{out}}{A_{in}}$

Important to estimate spectrum of a signal when choosing a measurement system - Ideal measurement system replicates all frequency components of an input signal

L3: Dynamic System Response

Ideal Measurement System: fast response + good to understand

General solutions for Linear System:

$$\sum_{n=0}^{N} A_n \frac{d^n X_{out}}{dt^n} = \sum_{m=0}^{M} B_m \frac{d^m X_{in}}{dt^m}$$

RHS usually is just $B_0 X_{in}$ i.e. no time derivatives

Goal: Find the general solution $X_{out}(t) = X_{out_p} + X_{out_h}$

1. Find the general solution to the homogeneous equation $\sum_{n=0}^{N} A_n \frac{d^n X_{out}}{dt^n} = 0$
   a. Write down the characteristic equation $\sum_{n=0}^{N} A_n S^n = 0$ i.e. $A_2 s^2 + A_1 s + A_0 = 0$ (Applies for $1 \le n \le 3$; difficult to solve for $n > 3$), then solve for roots λ:
      i. 1 real root: $X_{out_h} = C_1 e^{\lambda_1 t}$
      ii. 2 distinct real roots: $X_{out_h} = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$
      iii. 2 repeated real roots: $X_{out_h} = (C_1 + C_2 t)e^{\lambda_0 t}$
      iv. k-folded real roots: $X_{out_h} = (C_0 + C_1 t + \cdots C_{k-1}t^{k-1})\, e^{\lambda_0 t}$
      v. Complex conjugate roots $\lambda = \alpha \pm i\beta$: $X_{out_h} = C_1 e^{\alpha t}\cos\beta t + C_2 e^{\alpha t}\sin\beta t$

Zero Order
$$V_{out} = \frac{R_X}{R_P}V_s = \frac{V_S}{L}X_{in}$$
Solution $\Rightarrow$ $X_{in} = KX_{out}$
K: Gain/sensitivity (no time delay)

1st Order
$$\tau\frac{dX_{out}}{dt} + X_{out} = KX_{in}$$
(in this form, τ = time constant; K = static sensitivity)

Gen. Soln. $\Rightarrow$ $X_{out} = A + Be^{-\frac{t}{\tau}}$

Soln. to step DC input $\Rightarrow$ $X_{out} = X_\infty + (X_0 - X_\infty)e^{-\frac{t}{\tau}}$
$\equiv X_0 + (X_\infty - X_0)\left(1 - e^{-\frac{t}{\tau}}\right)$

where τ = time constant (e.g. RC, L/R)

Assumed to reach steady state within four time constants (98% of KA_{in}).

2nd Order
$$m\frac{dx^2}{dt} + b\frac{dx}{dt} + kx = F_{ext}(t)$$
Gen. Soln. $\Rightarrow$ $x_h(t) = (A + Bt)e^{-\omega_n t}$

Frequency Response for $X(t) = X_0 e^{j\omega t}$: $\frac{X}{F} = \frac{k^{-1}}{-\frac{\omega^2}{\omega_0^2} + \frac{j\omega}{Q\omega_0} + 1}$

2 different real roots of 2nd order system:

$$m\frac{dx^2}{dt} + b\frac{dx}{dt} + kx = 0 \text{ or } \frac{dx^2}{dt} + 2\zeta\omega_n\frac{dx}{dt} + \omega_n^2 x = 0$$

Characteristic Equation:
$$ms^2 + bs + k = 0 \text{ or } s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

Roots:
$$s = \frac{-b \pm \sqrt{b^2 - 4mk}}{2a} \text{ or } s = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

Note:

Natural Frequency $\omega_n = \sqrt{\frac{k}{m}}$

Damping Ratio $\zeta = \frac{b}{b_c} = \frac{b}{2\sqrt{mk}} = \frac{b}{2m\omega_n}$ measures proximity to critical damping

- $\zeta < 1$: Underdamped; Complex Conjugate Roots
- $\zeta = 1$: Critical Damping; Repeated Real Roots
- $\zeta > 1$: Overdamped; Distinct Real Roots

$\zeta = 0.707$ gives best amplitude linearity over largest bandwidth

Damped Frequency $\omega_d = \omega_n\sqrt{1 - \zeta^2}$

1st order Dynamic Error: $\delta(\omega) = 1 - M(\omega)$

where Magnitude Ratio $M(\omega) = \frac{1}{\sqrt{1 + (\omega\tau)^2}} < 1$

Reflects how much the output of a first-order system lags the input; function of both **frequency** and **time-constant**

Performance parameters of underdamped 2nd order system:
Delay Time, Rise Time, Peak Time, Max Overshoot (% of SS), Settling Time

L4: Sampling

Analog signal: continuous, generated via analog devices, not coded, original signal

Digital signal: discrete, sampled in a fixed interval, coded value, sequential data array

Why Digital: more compact storage using diff media; more accurate; large dataset; use real-time control system; enable data processing; sampling rate high, accuracy high

Shannon-Nyquist Theorem to retain all frequency components:
$$f_s > 2f_{max}$$
f_{max}: Nyquist Frequency (i.e. max frequency component in signal)
f_s: Sampling rate (not sampling frequency!)

If the shape of the waveform is desired, you should sample at a rate approximately 10 time the Nyquist frequency.

Aliased Frequency: $f_a = abs(f_s * i - f_n)$ for $i = 1,2,3\ldots$
f_n: frequency component; f_s: sampling frequency

Aliasing does not only occur at $f_s < 2f_{max}$. It always occurs and with infinite frequencies f_a. If Shannon-Nyquist is fulfilled, all aliased frequencies will not overlap with the original signal and can be filtered out using a LPF.

Quantization and Coding

PCM (Pulse Code Modulation) to digitize an analog signal:
1) **Sampling** (incl. using LPF to remove high frequency noise)
2) **Quantisation**: transformation of a continuous analogue input into a set of discrete output states
3) **Binary encoding**: assigning a number to each output state; number of possible states N = number of bit combinations that can be output from the converter: $N = 2^n$

Resolution $Q = \frac{V_{max} - V_{min}}{2^n}$

Quantization Error: $\frac{Q}{GA} \times 100\%$

Q = resolution; G = gain; A = amplitude of frequency component *or* overall wave amplitude

Commercially available ADCs are usually 8-bit and 12-bit

$\sin(A+B) = \sin A\cos B + \cos A\sin B$
$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$
$\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$
$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Int. by Parts: $\int uv' = uv - \int vu'$
Int. by Subst.: $\int f(u(x))u'(x)dx = \int f(u)\,du$

$\int x\cos(ax)dx = \frac{1}{a^2}(\cos(ax) + ax\sin(ax)) + C$
$\int x\sin(ax)dx = \frac{1}{a^2}(\sin(ax) - ax\cos(ax)) + C$

L5: Analog Signal Processing

VTT: Bulky tubes enclosed a gas at low pressure through which electronics flowed. Heavy power consumption, Significant heat dissipation, Large size and heavy weight, Requiring frequent battery replacement for portable units

SST: Charge carriers move through a solid semiconductor material. Small size, portable using rechargeable battery, low weight, cool running, energy saving

Need for analog signal processing: Signals from transducers are usually 1) Too small i.e. in mV, 2) Too noisy, 3) Contain wrong information due to design and installation, and 4) Have a DC offset due to design and instrumentation

Filtering: $\omega_0 = \frac{1}{RC} = \frac{R}{L}$ where $|H(\omega_0)| = \frac{1}{\sqrt{2}}$

- LPF: V_{out} across Capacitor in RC
- Attenuate opposite of Amplify

$$\boxed{G = \frac{-AR_F}{AR + R + R_F}}$$

Ideal Op-Amp

1. "Infinite input impedance": I into op-amp $I^+ = I^- = 0$
2. "Infinite (Open Loop) Gain": $V^+ = V^-$
   - Rationale: Otherwise, infinite output
   - *(extra)* Open Loop Gain determined solely by op-amp: $V_{out} = k(V^- - V^+)$; Ideally infinite, but typically has a large value.
   - *(extra)* Closed Loop Gain determined solely by relative resistor values
3. "Zero output impedance": output voltage does not depend on output current

*Use KCL (at input node), KVL and above info to find V_{out}

Capacitors	Inductors
$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau$	$v(t) = L\frac{d[i(t)]}{dt}$
$i(t) = C\frac{d[v(t)]}{dt}$	$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau$
$Z_C = \frac{1}{j\omega C}$	$Z_L = j\omega L$
$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$	$L_{eq} = L_1 + L_2$
$C_{eq} = C_1 + C_2$	$L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)^{-1}$
$W_C(t) = \frac{1}{2}Cv_C^2(t)$	$W_L(t) = \frac{1}{2}Li_L^2(t)$
DC / Low Freq AC: Open High Freq AC: Short	DC / Low Freq AC: Short High Freq AC: Open

"Short-circuit current" – V=0

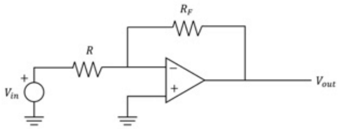
Transfer Function: $G(s) = \frac{Output\ X(s)}{Input\ Y(s)}$, replacing s with jw

(1) $\alpha f(t) + \beta g(t)$	$\alpha F(s) + \beta G(s)$	α, β constants $s > \max(c_f, c_g)$.
(2) $f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$	$f^{(n)}(t) = d^n f/dx^n$ $(n \ge 1)$ $s > c_f$.
(3) $e^{at}f(t)$	$F(s-a)$	a constant $s > a + c_f$
(4) $\int_{\tau=0}^{\tau=t} f(\tau)d\tau$	$\frac{1}{s}F(s)$	$s > \max(0, c_f)$
(5) $u(t-a)f(t-a)$	$e^{-as}F(s)$	Unit-step function $u(t-a)$, $s > c_f$
(6) $t^n f(t)$	$(-1)^n F^{(n)}(s)$	$F^{(n)}(s) = d^n F/ds^n$ $(n \ge 1)$ $s > c_f$

- $m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F$
- Taking Laplace transform
- $ms^2X(s) + csX(s) + kX(s) = F(s)$

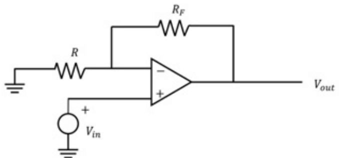
$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k}$$

Inverting Amplifier



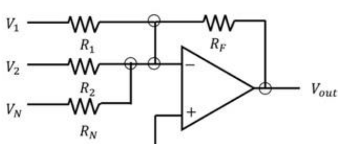
$$\frac{V_{out}}{V_{in}} = -\frac{R_F}{R}$$

Non-Inverting Amplifier



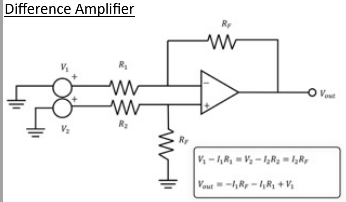
$$\frac{V_{out}}{V_{in}} = 1 + \frac{R_F}{R}$$

Summer



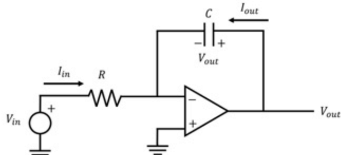
$$V_{out} = -\left(\frac{R_F}{R_1}V_1 + \frac{R_F}{R_2}V_2 + \cdots + \frac{R_F}{R_N}V_N\right)$$

Difference Amplifier



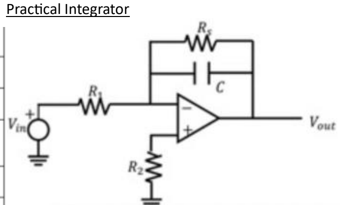
$$V_{out} = V_1 - (R_F + R_1)\left(\frac{V_2}{R_1} - \frac{V_2}{R_F + R_2}\frac{R_F}{R_1}\right)$$

Integrator



$$V_{out}(t) = -\frac{1}{RC}\int_0^t V_{in}(\tau)d\tau$$

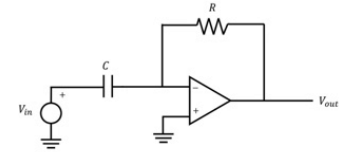
Practical Integrator



$$C\frac{dV_{out}}{dt} + \frac{V_{out}}{R_S} = I_{out} = -I_{in} = -\frac{V_{in}}{R_1}$$
$$\frac{dV_{out}}{dt} + \frac{V_{out}}{CR_S} = -\frac{V_{in}}{CR_1}$$

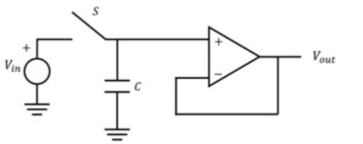
*Should choose $R_S > 10R_1$. $R_2 = -\frac{R_1R_S}{R_1 + R_S}$

Differentiator



$$V_{out} = -RC\frac{dV_{in}}{dt}$$

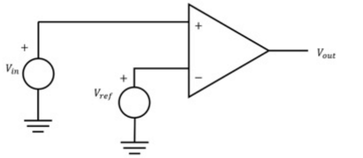
Sample & Hold



S is closed: $V_{out}(t) = V_{in}(t)$
S is opened: $V_{out}(t - t_{sampled}) = V_{in}(t_{sampled})$

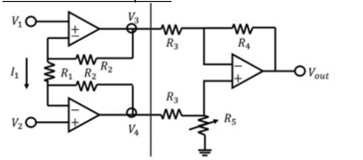
- choose C w/ low leakage
- used for ADC - signal value must be stabilised
- voltage-holding capacitor and voltage follower with switch closed

Comparator (specially built – not typical Op-Amp)



$$V_{out} = \begin{cases} +V_{sat} & V_{in} > V_{ref} \\ -V_{sat} & V_{in} < V_{ref} \end{cases}$$

Instrumentation Amplifier

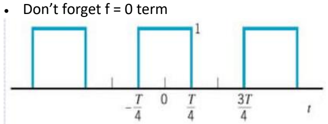


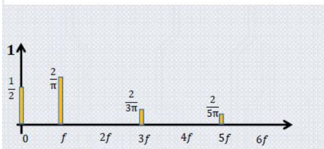
$$V_{out} = \frac{R_5(R_3 + R_4)}{R_3(R_3 + R_5)}V_4 - \frac{R_4}{R_3}V_3$$

- diff. amp. has too little input impedance for high output impedance
- if input signal level too low, signals include noise – solved by instrumentation amplifier

Spike Diagram (Square Wave Decomposition)

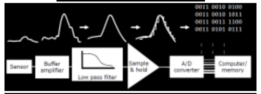
- Don't forget f = 0 term





$$F(t) = \frac{1}{2} + \frac{2}{\pi}\cos\left(1\frac{2\pi}{T}t\right) - \frac{2}{3\pi}\cos\left(3\frac{2\pi}{T}t\right) + \frac{2}{5\pi}\cos\left(5\frac{2\pi}{T}t\right) - \frac{2}{7\pi}\cos\left(7\frac{2\pi}{T}t\right) + \ldots$$

L6: A/D Conversion



Successive approx.
- pros: high spd. and good reliability; medium accuracy compared to other ADC types; good trade-off btw. spd. and cost; capable of outputting binary number in serial
- cons: higher resolution = slower speed (limited to 5 MSPS)

Flash ADC
- pros: v. fast; simple operational theory; spd. only limited by gate + comparator propagation delay
- cons: expensive; prone to glitches in output; each additional bit of resolution requires twice the comparators

Sigma-Delta ADC
- pros: high res; no need for precision components
- cons: slow; due to oversampling, only good for low bandwidth

DAC
- finite word length
- loudest sounds need room, normal sounds don't use entire range
- problems occur at low levels where sounds are represented by 1/2 bits, high distortions
- dithering adds low level broadband noise

Type	Speed (relative)	Cost (relative)
Dual-Slope	Slow	Med
Flash	Very Fast	High
Successive Approximation	Medium Fast	Low
Sigma-Delta	Slow	Low

"Complex Trick"

So, let's consider a 2nd order mechanical system driven by generalized sinusoidal force $\tilde{f}(t) \equiv F_0 e^{j\omega t}$ and look for generalized sinusoidal motions $\tilde{x}(t) \equiv X_0 e^{j\omega t}$

$$\tilde{f}(t) = F_0 e^{j\omega t} \Rightarrow \boxed{\text{system}} \Rightarrow \tilde{x}(t) = X_0 e^{j\omega t}$$

Note that taking derivatives of generalized sinusoids (i.e. complex exponentials) is particularly computationally straightforward, i.e.

$$\frac{d}{dt}\tilde{x}(t) = \frac{d}{dt}X_0 e^{j\omega t} = X_0 \frac{d}{dt}e^{j\omega t} = j\omega X_0 e^{j\omega t} = j\omega \tilde{x}(t)$$

so formally $\frac{d}{dt}$ can be replaced by a $j\omega$ whenever dealing with generalized sinusoids. Therefore, Newton's law for generalized sinusoids simply becomes

$$\tilde{f}(t) = j\omega m(j\omega \tilde{x}(t)) + j\omega b\tilde{x}(t) + k\tilde{x}(t)$$

which, recalling that $j^2 = -1$, becomes

$$\tilde{f}(t) = (k - m\omega^2 + j\omega b)\tilde{x}(t)$$

Recalling that $\tilde{f}(t) = F_0 e^{j\omega t}$ and $\tilde{x}(t) = X_0 e^{j\omega t}$, one gets

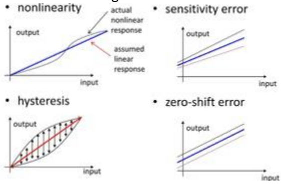
$$F_0 \cancel{e^{j\omega t}} = (k - m\omega^2 + j\omega b)X_0 \cancel{e^{j\omega t}}$$

$$\boxed{H(j\omega) = \frac{1}{k - \omega^2 m + j\omega b} = \frac{k^{-1}}{1 - \frac{\omega^2}{\omega_0^2} + j\frac{\omega}{Q\omega_0}}}$$

- $\omega_0 := \sqrt{k/m}$, also known as **resonance frequency**, merasured in [rad/sec]
- $Q = \sqrt{mk/b^2}$, also known as **quality factor**, unitless

Sensors

- sensitivity S: output variation/input variation, S = df/dx
- resolution: minimum change of measurand that can be reliably detected; limited by noise, bit-conversion
- accuracy: difference of measurement from true value, %FS
- repeatability: how well a system/device can reproduce an outcome in unchanged situation



Resistance Temperature Detector (RTD)

$$\frac{R}{R_0} = 1 + \alpha\,(T - T_0)$$

R_0 = resistance at T_0; Linearity valid for limited range

Based on changes of resistance w/ temp.; Wrap wire on insulating support: eliminate mechanical strain

Thermistors: $R = R_0 e^{\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)}$ (Note: in K)

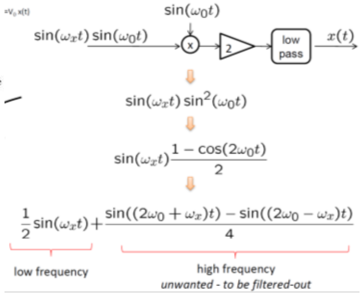
LVDT (Linear Variable Differential Transformer): type of transformer; measures linear displacement by comparing the voltages across two coils with a stationary AC input + moving core (linearity for small range of core displacement)

Application: **AM Modulation/Demodulation**:

$$carrier\ freq.\ \omega_0 \gg modulating\ freq.\ \omega_x$$

Modulated Result $\boldsymbol{\Delta V = \sin(\omega_0 t)\sin(\omega_x t)}$

AM Demodulation:



Capacitive Sensor (proximity sensor)

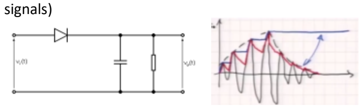
$$C = \frac{Q}{V} = \frac{\epsilon_0 \epsilon_r S}{d}$$

Plates placed parallel and sliding across each other

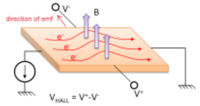
Ideal case: infinite parallel plates

Guard electrodes limit field-fringing effects

Application: **Envelope Demodulator** (for non-negative signals)



Hall effect sensor: V+ and V- are compared in a differential amplifier, with current known, to determine B

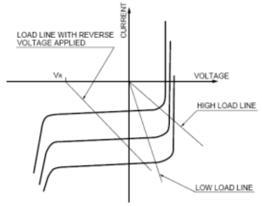


$F = BqV$

Applications:
1. Proximity sensor
2. High current sensor (contactless)

Photoresistors: more light = lower resistance

Photodiodes: Light causes back current



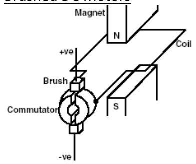
Digital Encoders: converts linear/rotary motion into a sequence of digital pulses (i.e. either light-based or magnetic-based)

incremental encoder:
- minimum 2 Tx/Rx pairs spaced >1/4 periods away to encode steps + direction
- to find location, counter resets at marker

absolute encoders:
- n Tx/Rx pairs for coding 2n sectors
- grey code: 1-bit changes at a time; avoids spurious states

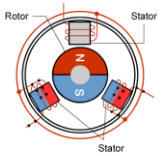
DC Motors

Brushed DC Motors



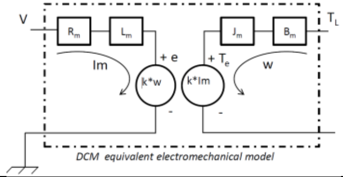
- stator: external, fixed
- rotor: internal, rotates
- stator and rotor fields always orthogonal -> maximum torque
- commutation maintains unstable equilibria for constant motion

Brushless DC Motors



- 2 input terminals per coil
- stator: external, fixed but switching
- rotor: internal, rotates
- Hall effect sensors detect rotor position; microprocessor coordinates each coil on/off to produce desired steady torque

Electro-mechanical model



Armature Equation:	Mechanical Equation:
$V = Ri + L\frac{di}{dt} + e$ • R: Resistance of wire • L: Inductance of wire wrapped around pole • e: Back emf of motor	$J\dot{\omega} + b\omega = T_e - T_L$ • J: Mechanical inertia • b: Friction • T_e: Electromagnetic Torque • T_l: Load Torque

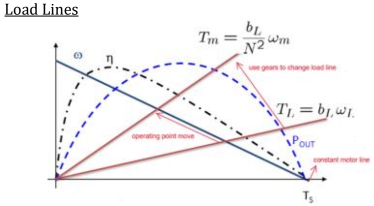
$$\boldsymbol{T_e = K_t i} \text{ and } \boldsymbol{e = K_e\omega \rightarrow T_e\omega = ei}$$

Armature Constant $K_a = K_t = K_e$

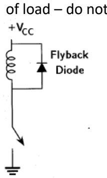
At steady state, where $\frac{di}{dt} = \frac{d\omega}{dt} = 0$, $\boldsymbol{\omega = \frac{K_a V - RT_L}{Rb + K_a^2}}$

(all constants except T_L)

Load Lines



- $\boldsymbol{P_{max} = \frac{1}{4}T_s\omega_0}$ occurs at $\frac{1}{2}T_s$ (half stall torque); independent of load – do not use any intersection!



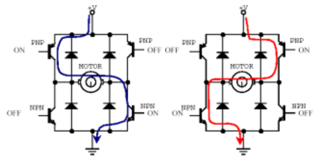
Find at stall condition:
- Armature resistance: V = RI
- Armature constant: T/i

Inductive Kickback
- When switch is opened, current decrease rapidly, causing voltage across the inductor to increase rapidly.

PWM

Duty Cycle = $\frac{T_{on}}{T} \times 100\%$

Generated using sawtooth + comparator



Strain Gauges

$$R = \rho\frac{L}{A}$$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} = \frac{d\rho}{\rho} + \frac{dL}{L} - \left(\frac{d\omega}{\omega} + \frac{dh}{h}\right)$$

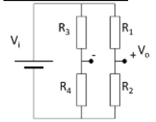
$$= \left(\frac{d\rho}{\rho}\frac{1}{S} + 1 + 2v\right) * S$$

$$\frac{dR}{R} = (G) * S$$

Gauge Factor $G = \frac{d\rho}{\rho}\frac{1}{S} + 1 + 2v$ accounts for changes in resistance due to changes in…
- …resistivity of material due to property changes
- …length along direction of force
- …length along direction perpendicular to force, using Poisson's ratio

(does not account for changes in temperature)

Wheatstone Bridge



Amplify minute changes in V

$$\frac{V_i}{V_0} = \frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}$$

Balance: $R_1 R_4 = R_2 R_3$

V^+ and V^- arbitrarily assigned

Bridge Sensitivity: *(1st Order Approximation)*

$$\frac{dV_0}{V_i} = \frac{1}{4}\left(\frac{dR_2}{R_2} - \frac{dR_1}{R_1} + \frac{dR_3}{R_3} - \frac{dR_4}{R_4}\right)$$

$= \frac{1}{4}G(S_2 - S_1 \ldots)$ e.g. where $S_2 = S_T + S_A - S_B$

*S_T: Temp. // S_A: Axial Load // S_B: Bending Load

**Reason for +/- signs:
- if R2 increases, v+ will increase;
- if R1 increases, v- will decrease

Strain from Axial Load: $S_a = \frac{F_a}{EA}$

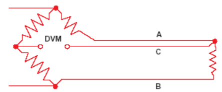
Strain from Pure Bending Moment + Transverse Load: $S_b = -\frac{M + F_t(L-x)}{EI}$

Half Bridge: sense S^A; compensate for S^B but not S^T OR sense S^B; compensate for S^T and S^A

Full Bridge: sense S^A; compensate for S^T and S^B OR sense S^B (max sensitivity); compensate for S^T and S^A

Bridge Balancing: Add additional resistors in parallel to fixed R's to re-establish balance condition.

3- / 4- Wire Bridges:



Used to mitigate effects of wires' internal resistance:
- Bridge should be read when balanced: no current flowing through C, no potential drop
- Wires A and B are assumed to have the same internal resistances which cancel out.

System Modelling

Force ~ Torque ~ Voltage [Effort]

(Angular) Speed ~ Current [Flow]

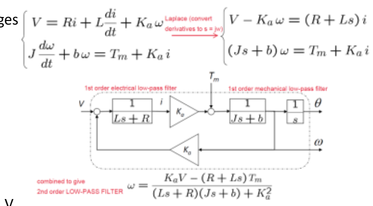
(Angular) Displacement ~ Charge

(Angular) Momentum ~ Flux Linkage

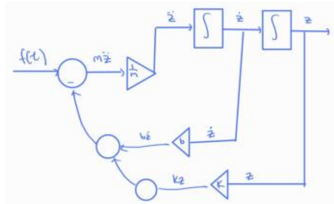
(Rotary) Damper ~ Resistor

(Torsion) Spring ~ Capacitor

Mass ~ Moment of Inertia ~ Inductor [Inertia]



Can further break down $\omega = \dot{\theta} = s\theta$



State-Space Representation

Express DCM / model equation is a system of first derivatives, then fit into matrix form with derivatives on one side.

$$\frac{d}{dt}\begin{bmatrix} x \\ v \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}}_{A}\begin{bmatrix} x \\ v \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{B} f$$

$$\boxed{\frac{d}{dt}\boldsymbol{x} = A\boldsymbol{x} + Bf}$$

Laplace Transform Tables

$f(t) = L^{-1}\{F(s)\}$	$F(s) = L\{f(t)\}$
1	$\frac{1}{s}$
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}, \quad (n = 1, 2, \cdots)$
$\frac{t^{n-1}e^{at}}{(n-1)!}$	$\frac{1}{(s-a)^n}, \quad (n = 1, 2, \cdots)$
$\frac{\sin \omega t}{\omega}$	$\frac{1}{s^2 + \omega^2}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$\frac{\sinh at}{a}$	$\frac{1}{s^2 - a^2}$
$\cosh at$	$\frac{s}{s^2 - a^2}$
$u(t-a)$, Unit step function	$\frac{e^{-as}}{s}$
$\delta(t-a)$, Unit impulse function	e^{-as}