MA2011 - Part II
Mechatronics Systems Interfacing

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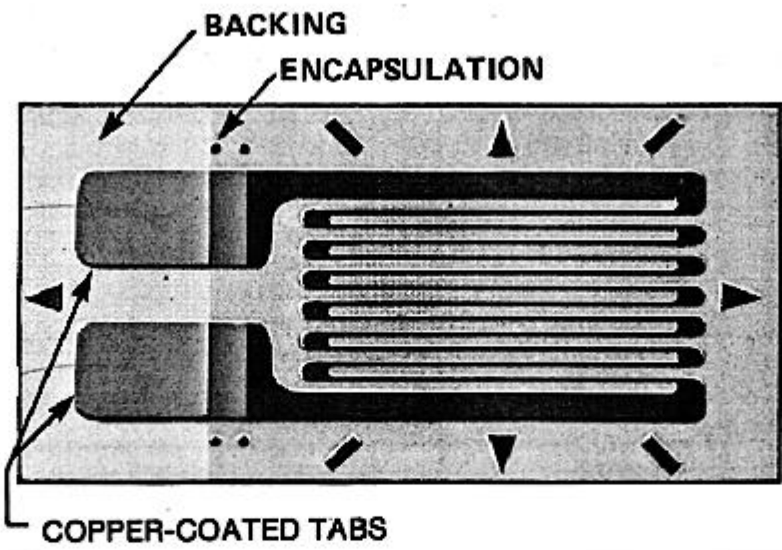
Strain gauges

resistance strain gauges

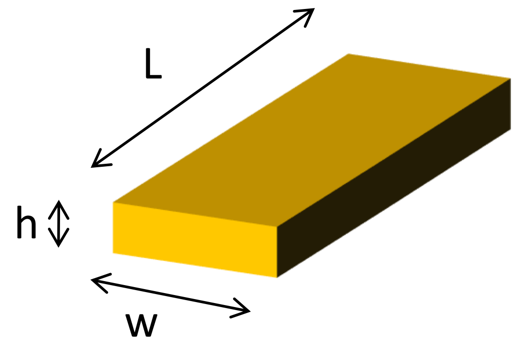
force measurements

stress/strain measurements

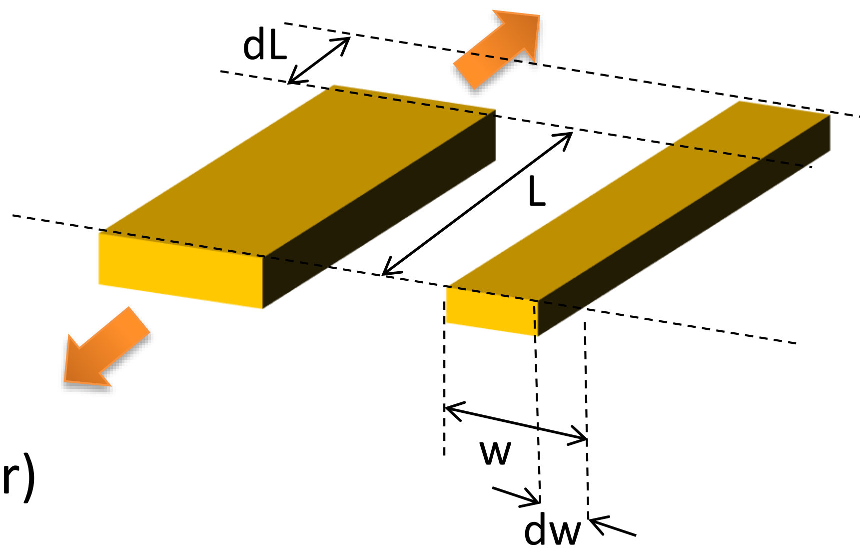
- strain measurements are important to determine safe loading conditions of mechanical structures
  - stress/force measurements are typically derived indirectly from strain/displacement measurements
- electrical resistance strain gauges
  - thin metal foil
    - typically constantan
  - patterned onto plastic backing material
  - bonded onto mechanical structures
    - stress is inferred from solid mechanics principles



resistance strain gauges



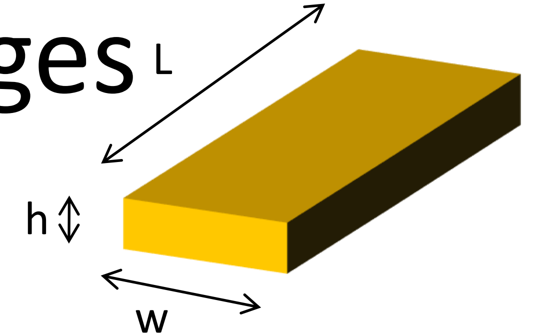
- what is strain?
  - $S:=dL/L$ quantifies the amount of deformation of a body
  - non-dimensional
    - defined as a relative change (dL/L)
      - typical materials undergo from 'microstrains' 10^{-6} (ppm) up to a few %
    - positive (tensile strain) or negative (compression) values
- Poisson's ratio



  - typically ν: 0.3 (steel) → 0.5 (rubber)

$$\nu := \frac{\texttt{lateral strain}}{\texttt{axial strain}}$$

resistance strain gauges



- resistance of a rectangular conductor

$$R = \rho \frac{L}{\boxed{A}}$$

$$A = wh$$

  - differential form

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \boxed{\frac{dA}{A}}$$

$$\frac{dA}{A} = \frac{dw}{w} + \frac{dh}{h}$$

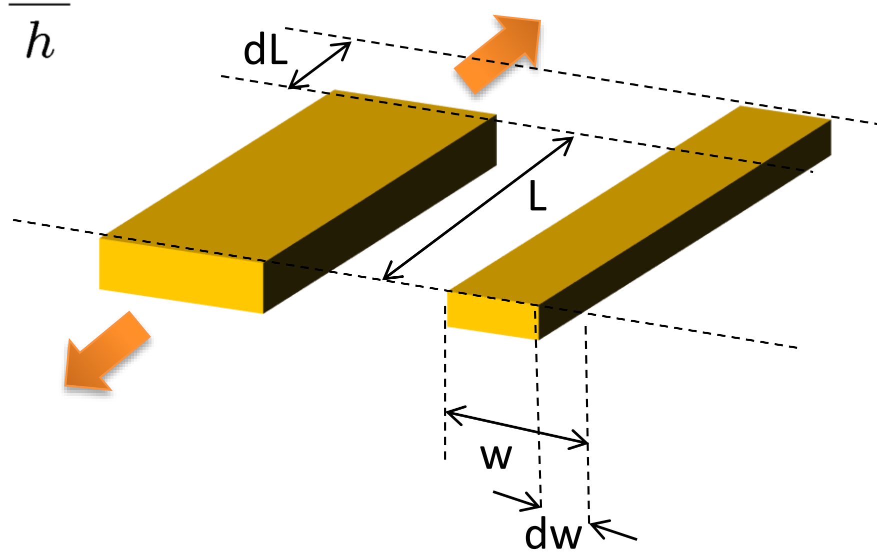
  - axial strain

$$S := \frac{dL}{L}$$

  - lateral strain

$$\frac{dw}{w} = \frac{dh}{h} = -\nu \frac{dL}{L} = -\nu S$$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \left(\frac{dw}{w} + \frac{dh}{h} \right)$$



resistance strain gauges

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \left(\frac{dw}{w} + \frac{dh}{h}\right)$$

$$= \frac{d\rho}{\rho} + (1 + 2\nu)S$$

$$= \left(\underbrace{\frac{d\rho}{\rho}\frac{1}{S}}_{\text{piezoresistivity}} + 1 + 2\nu\right) S$$

$$\frac{dR}{R} = \mathcal{G}\, S$$

- gauge factor:

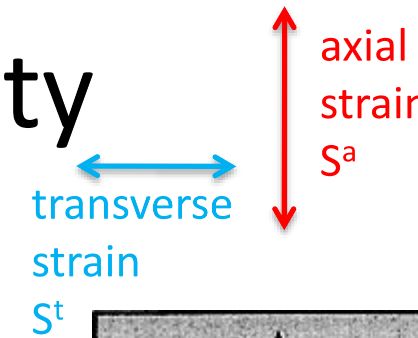
$$\boxed{\mathcal{G} := \underbrace{\frac{dR}{R}\frac{1}{S} = \frac{1}{R}\frac{\partial R}{\partial S}} = \frac{d\rho}{\rho}\frac{1}{S} + 1 + 2\nu}$$

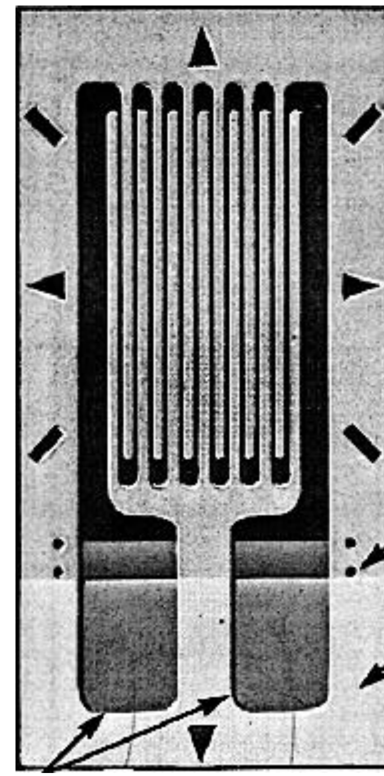
$$dR = dR^S := \frac{\partial R}{\partial S} S$$

NOTE: here we are only considering changes of resistance due to strain $dR=dR^S$

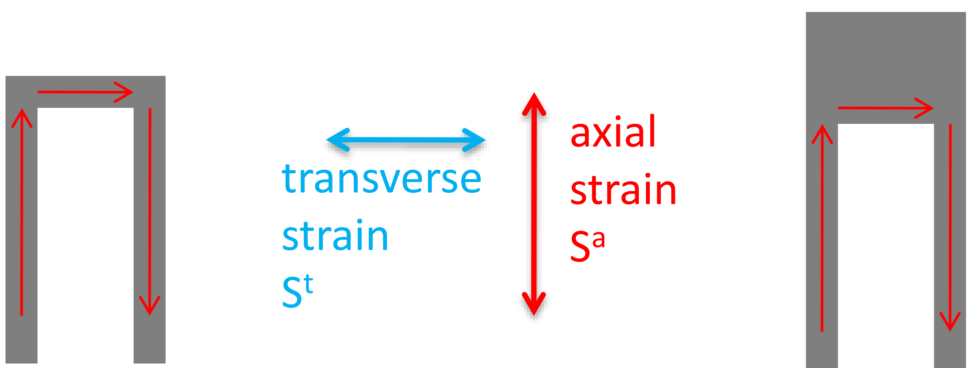
Material	Gauge Factor
Nickel	-12.6
Manganese	+0.07
Nicrome	+2.0
Constantan	+2.1
Soft Iron	+4.2
Carbon	+20
Platinum	+4.8

transverse sensitivity



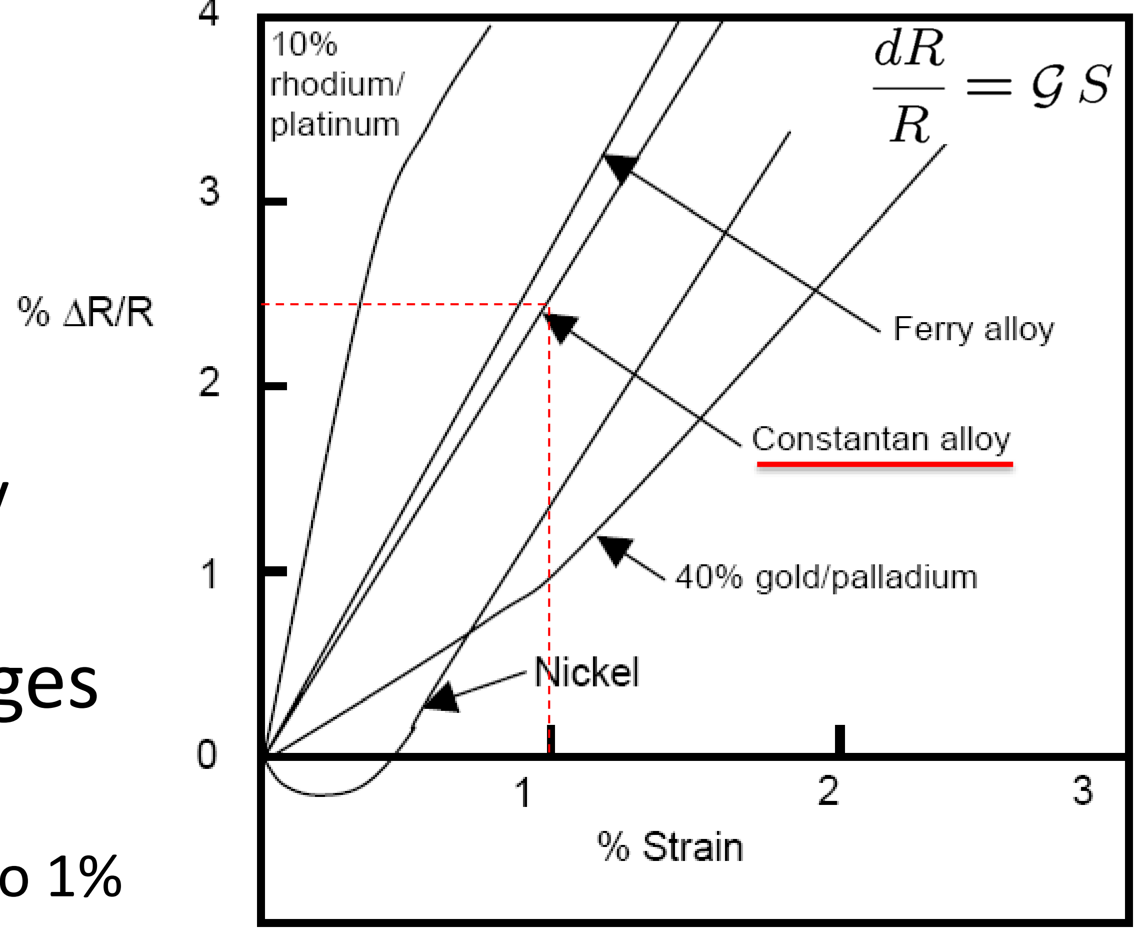


- dR = R_0 **G** S
  - the larger R_0, the larger dR
    - long and thin wires allow larger R_0
      - wires must be aligned with axial strain S^a
    - practically, long wires are assembled in the form of a serpentine
    - end-loops
      - are aligned with the transverse axis
      - made thicker to reduce sensitivity to S^t



materials

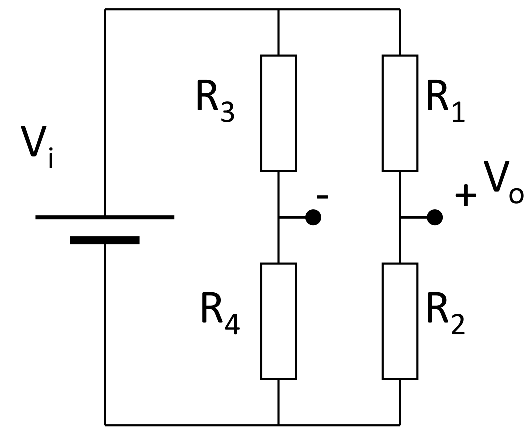
- best materials
  - constantan, ferry alloys
- typical strain ranges
  - S: 1-10^4 μS
    - i.e. from 1ppm to 1%
  - **G** ~ 2
    - dR/R is in the same order of magnitude as S
    - **challenge**: detecting small resistance changes



a numerical example

- dR = R= **G** S
  - **G** ~ 2
  - R_0 ~ 100-1,000 Ω
  - strain in the order of 10-10^4 μS (micro-strain)
    - strain is adimensional
    - 1 μS = 10^{-6} (e.g. 1μm/m)
- dR = (100 Ω) x 2 x (100 μS) = 0.02 Ω
  - transverse sensitivity in the order of 1%
  - how do we sense such small changes?

Wheatstone bridge



- bridge equations

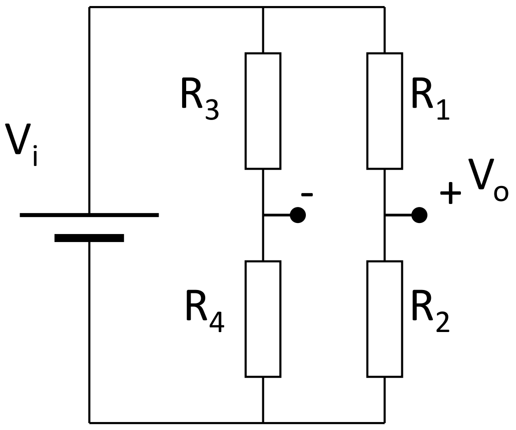
$$\begin{cases} \dfrac{V^+}{V_i} = \dfrac{R_2}{R_1 + R_2} \\ \dfrac{V^-}{V_i} = \dfrac{R_4}{R_3 + R_4} \end{cases} \Rightarrow \boxed{\dfrac{V_o}{V_i} = \dfrac{R_2}{R_1 + R_2} - \dfrac{R_4}{R_3 + R_4}}$$

- bridge balance condition:

$$\boxed{V_o = 0 \quad \Leftrightarrow \quad R_1 R_4 = R_2 R_3}$$

(product of opposite sides)

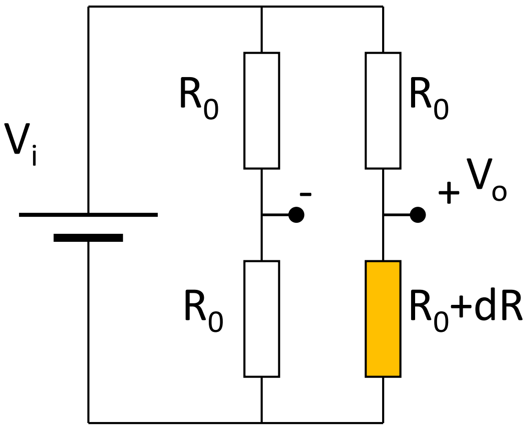
Wheatstone bridge: 1st order approximation



- bridge sensitivity
  - when R_1=R_2 and R_3=R_4
    - NOTE: this also implies balance
  - first order approximation
    - acceptable up to few % S
      - NOTE: 1% S = 10^4 μS

$$\boxed{\frac{dV_o}{V_i} = \frac{1}{4}\left(\frac{dR_2}{R_2} - \frac{dR_1}{R_1} + \frac{dR_3}{R_3} - \frac{dR_4}{R_4}\right)}$$

example: quarter-bridge



one-gauge bridge

- consider
  - $R_1 = R_3 = R_4 = R_0$
  - $R_G = R_0 + dR$
- bridge output

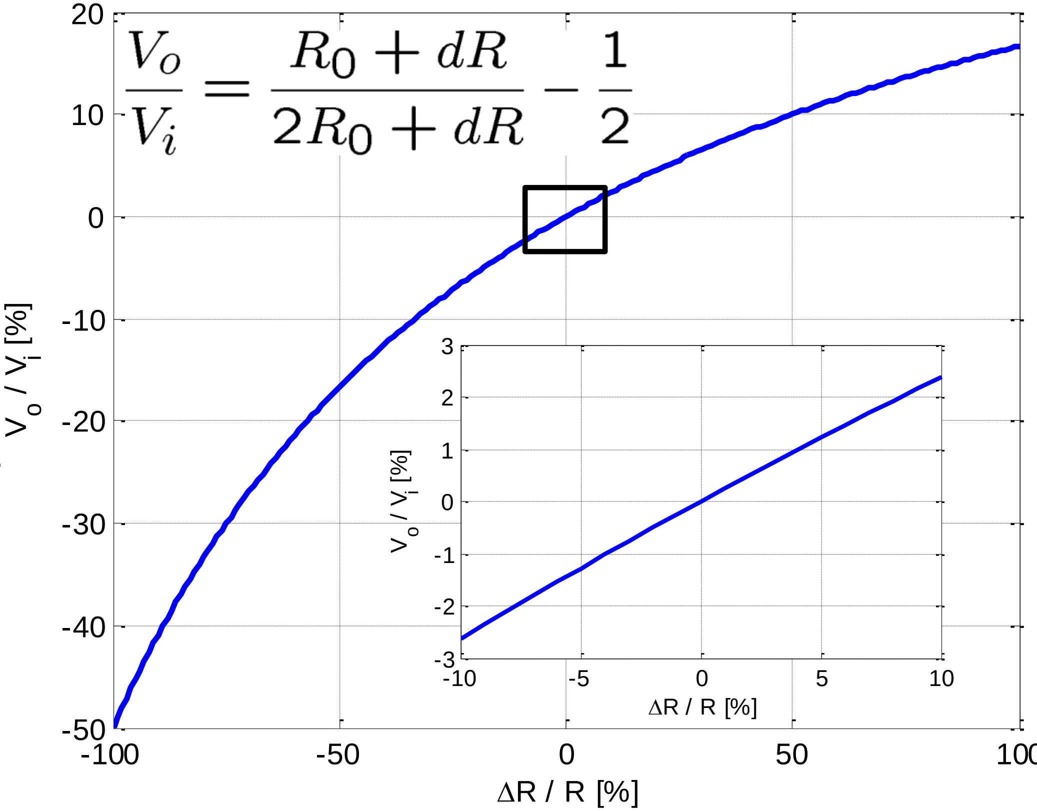
$$\frac{dV_o}{V_i} = \frac{R_0 + dR}{2R_0 + dR} - \frac{1}{2}$$

  - Taylor expansion

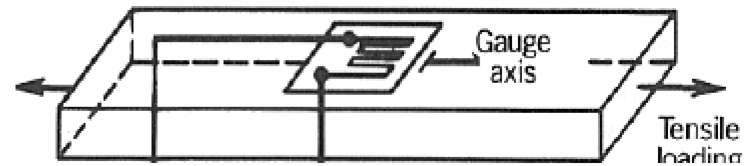
$$\frac{dV_o}{V_i} = \frac{1}{4}\frac{dR}{R_0} - \frac{1}{4}\frac{dR^2}{R_0^2} + \ldots h.o.t. \ldots$$

  -

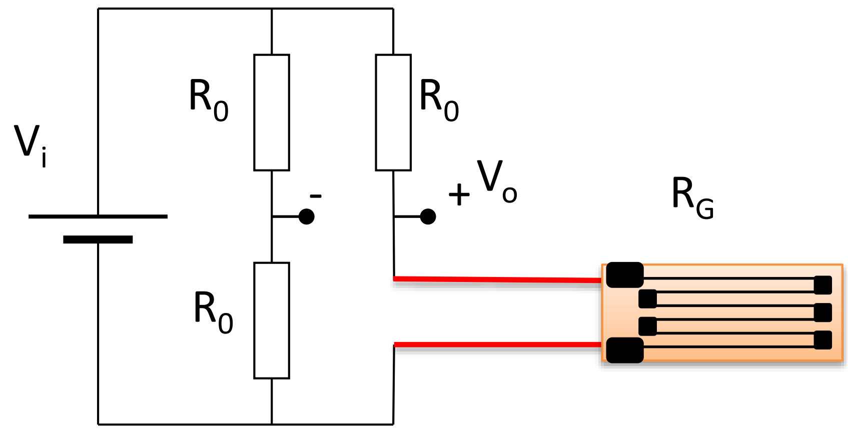
$$\boxed{\frac{dV_o}{V_i} \simeq \frac{1}{4}\frac{dR}{R_0} = \frac{1}{4}\mathcal{G}\,S}$$

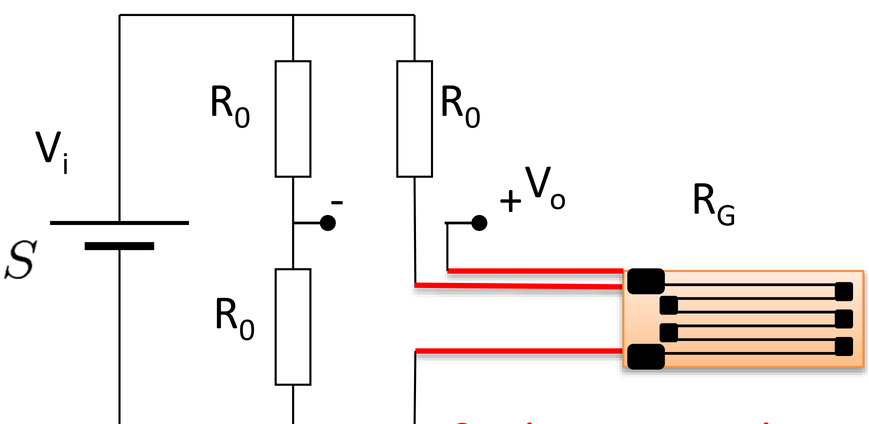


quarter-bridge



- 2-wire connection
  - R_w: long wires resistance
    - as high as few ohms
    - temperature dependent
    - unbalancing effects
- **3-wire connection**
    - 3rd wire: no current!!!
    - balanced bridge
    - attenuated gauge factor



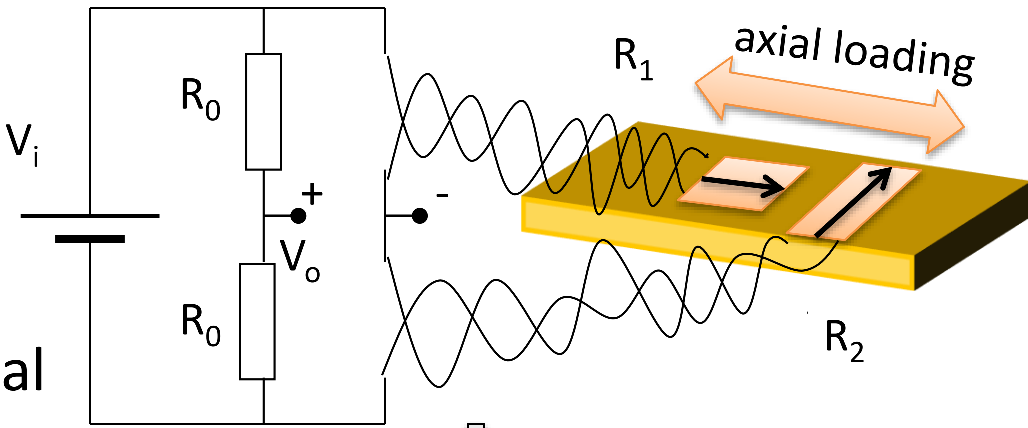


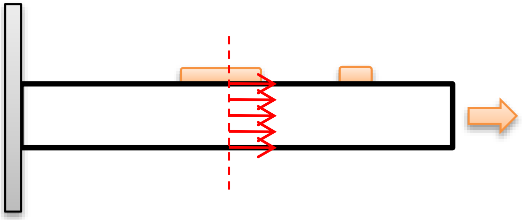
3-wires connection

$$\frac{V_o}{V_i} = \frac{1}{4}\frac{dR}{R_0 + R_w} = \frac{1}{4}\frac{R_0}{R_0 + R_w}\frac{dR}{R_0} = \frac{1}{4}\mathcal{G}^* S$$

$$\mathcal{G}^* = \mathcal{G}\frac{R_0}{R_0 + R_w} \leq \mathcal{G}$$

temperature compensation





- **dummy gauges**
  - mounted in close thermal contact but <u>not bonded</u>
  - $R_1=R_0+dR_1$ (strain gauge)
  - $R_2=R_0+dR_2$ (dummy gauge)

$$\begin{cases} dR_1 = \dfrac{\partial R_1}{\partial S} S + \dfrac{\partial R_1}{\partial T} dT \\ dR_2 = \dfrac{\partial R_2}{\partial T} dT \end{cases}$$

$$\frac{\partial R_1}{\partial T} dT = \frac{\partial R_2}{\partial T} dT$$

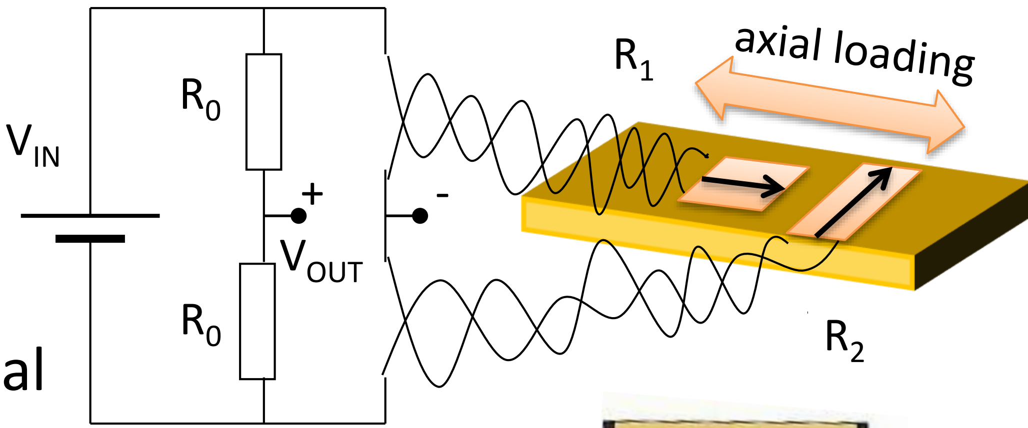
technologically similar gauges in thermal contact

$$\frac{dV_o}{V_i} = \frac{1}{4}\left(\frac{dR_1}{R_0} - \frac{dR_2}{R_0}\right) = \frac{1}{4R_0}\left(\frac{\partial R_1}{\partial S} S + \frac{\partial R_1}{\partial T} dT - \frac{\partial R_2}{\partial T} dT\right)$$

$$\boxed{\frac{dV_o}{V_i}} = \frac{1}{4R_0}\left(\frac{\partial R_1}{\partial S} S\right) = \boxed{\frac{1}{4} \mathcal{G}\, S}$$

temperature compensation

- **Poisson gauges**
  - mounted in close thermal contact <u>and bonded</u>
  - $R_1=R_0+dR_1$ (strain gauge)
  - $R_2=R_0+dR_2$ (poisson gauge)



$$\begin{cases} dR_1 = \dfrac{\partial R_1}{\partial S}S + \dfrac{\partial R_1}{\partial T}dT \\ dR_2 = -\nu\dfrac{\partial R2}{\partial S}S + \dfrac{\partial R_2}{\partial T}dT \end{cases}$$

$$\frac{\partial R_1}{\partial T}dT = \frac{\partial R_2}{\partial T}dT$$

$$\frac{\partial R_1}{\partial S} = \frac{\partial R_2}{\partial S}$$

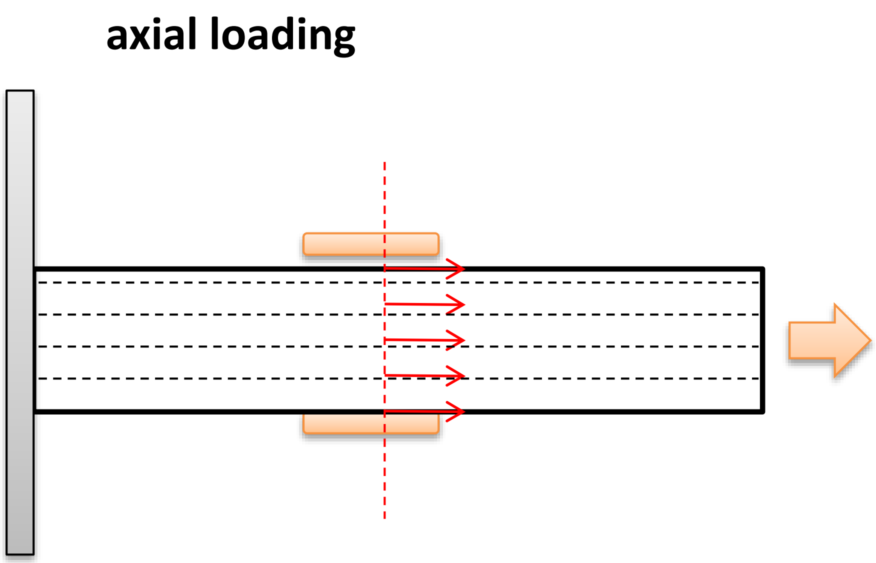
technologically similar gauges, bonded together, in thermal contact

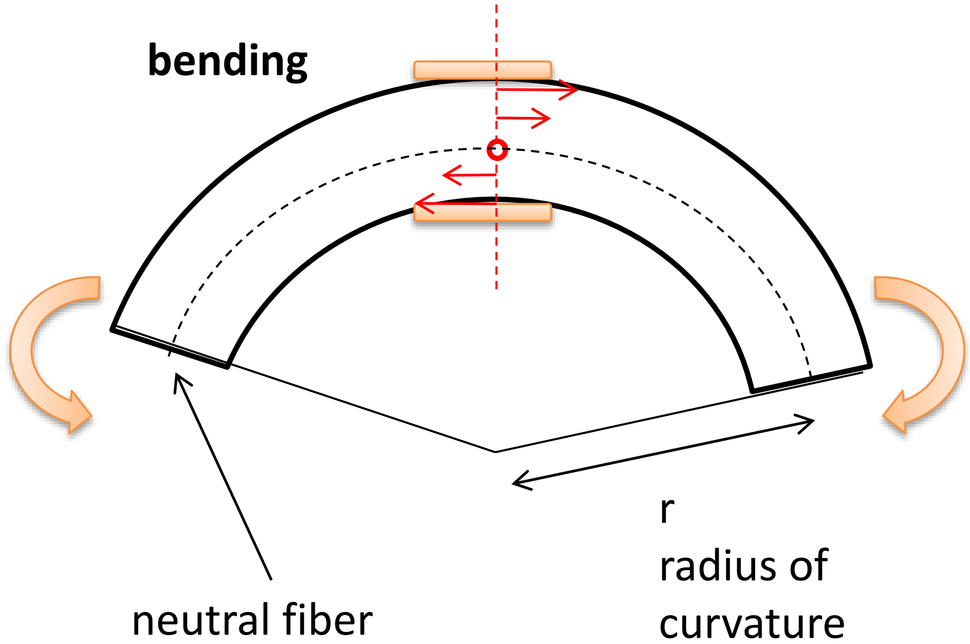
$$\frac{dV_o}{V_i} = \frac{1}{4}\left(\frac{dR_1}{R_0} - \frac{dR_2}{R_0}\right) = \frac{1}{4R_0}\left(\frac{\partial R_1}{\partial S}S + \frac{\partial R_1}{\partial T}dT + \nu\frac{\partial R_2}{\partial S}S - \frac{\partial R_2}{\partial T}dT\right)$$

$$\boxed{\frac{dV_o}{V_i}} = \frac{1}{4R_0}\left(\frac{\partial R_1}{\partial S}(1+\nu)S\right) = \boxed{\frac{1}{4}\mathcal{G}\,(1+\nu)S}$$

half bridge

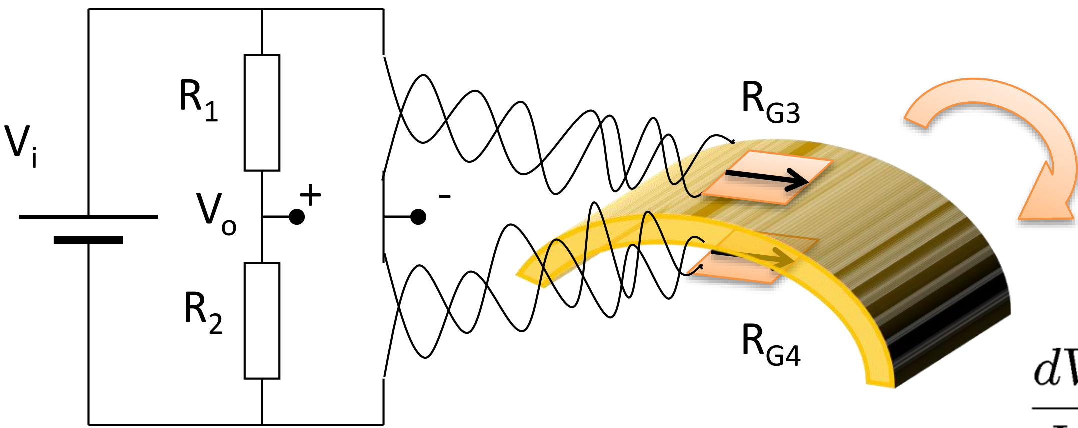
- two active strain gauges
  - enhancing the sensitivity of the bridge



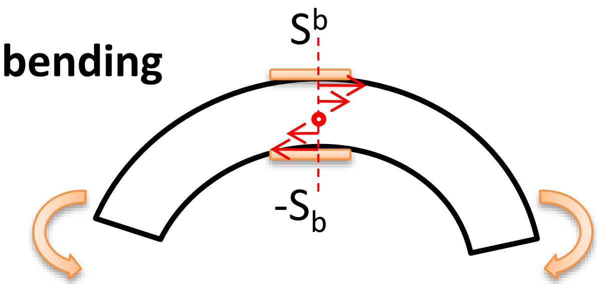


half-bridge

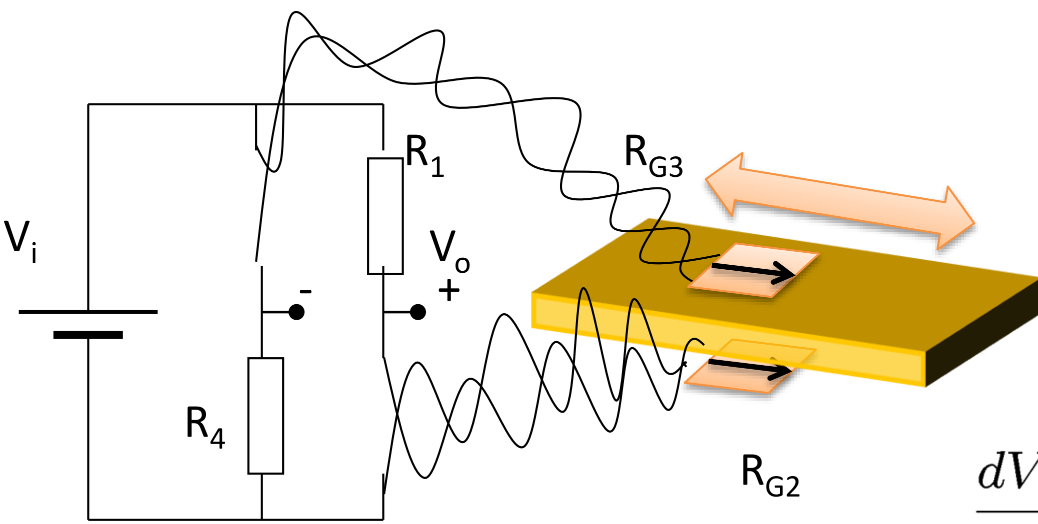
$$\frac{dV_o}{V_i} = \frac{1}{4}\left(\frac{dR_2}{R_2} - \frac{dR_1}{R_1} + \frac{dR_3}{R_3} - \frac{dR_4}{R_4}\right)$$



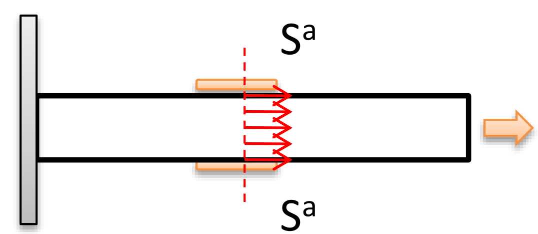
bending



$$\frac{dV_o}{V_i} = \frac{1}{4}\left(\frac{dR_3}{R_3} - \frac{dR_4}{R_4}\right) = \frac{1}{2}\,\mathcal{G}\,S^b$$



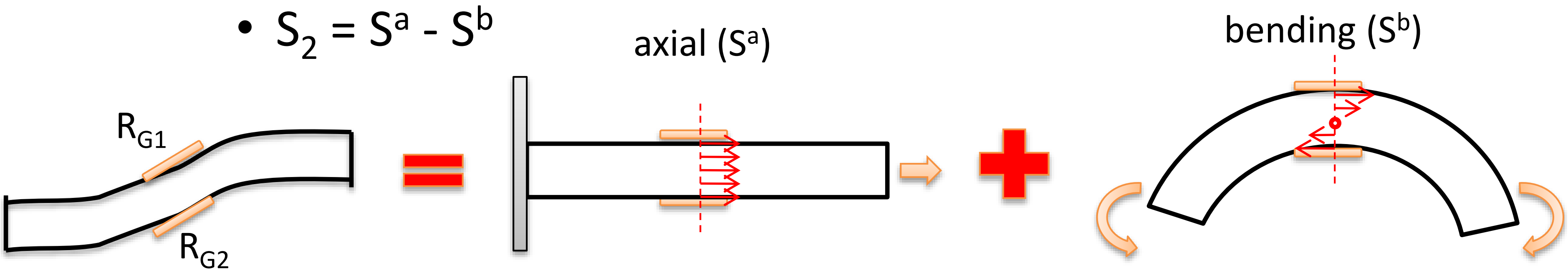
axial loading



$$\frac{dV_o}{V_i} = \frac{1}{4}\left(\frac{dR_2}{R_2} + \frac{dR_3}{R_3}\right) = \frac{1}{2}\,\mathcal{G}\,S^a$$

apparent strain: loading condition

- apparent strain is manifested as any change in gauge resistance which is <u>not due</u> to the strain being measured
- for example, combinations of:
  - different mechanical loading
    - $S_1 = S^a + S^b$
    - $S_2 = S^a - S^b$



apparent strain: thermal effects

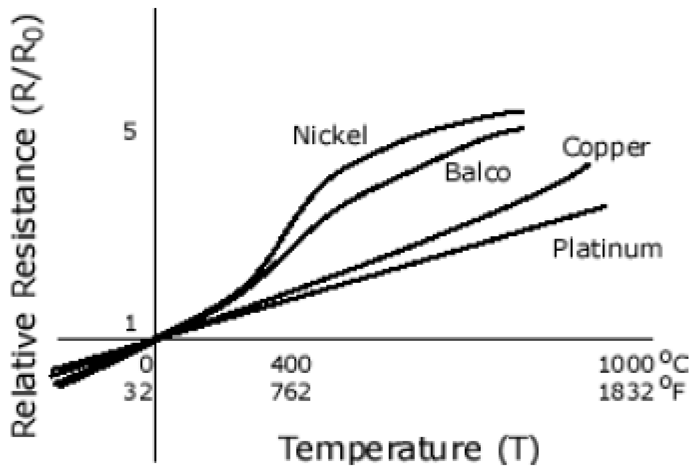
– resistance changes might be due to a combination of strain and temperature

$$dR = \frac{\partial R}{\partial S} S + \frac{\partial R}{\partial T} dT = \frac{\partial R}{\partial S} \left(S + \overbrace{\left(\frac{\partial R}{\partial S}\right)^{-1} \frac{\partial R}{\partial T} dT}^{\text{apparent strain } S^T} \right)$$

– apparent strain due to temperature

- notation: dR^T is the resistance change solely due to temperature

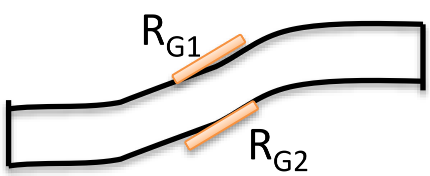
$$S^T := \left(\frac{\partial R}{\partial S}\right)^{-1} \frac{\partial R}{\partial T} dT = \overbrace{\frac{1}{\mathcal{G}} \underbrace{\frac{1}{R}\frac{\partial R}{\partial T}}_{4{,}000 \text{ ppm} / \text{K}^{-1} \text{ [temperature coefficient]}}}^{2{,}000\ \mu\text{S} / \text{K}^{-1}} dT = \frac{1}{\mathcal{G}} \frac{dR^T}{R}$$

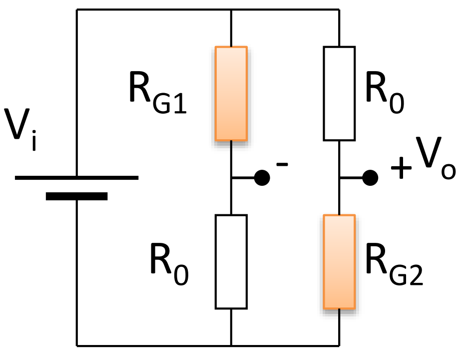


example: half-bridge (1/2)

- ultimately, sensitivity to loading condition and temperature is determined by the electrical configuration

$$S_{G1} = S^a + S^b + S^T$$





$$\frac{dV_o}{V_i} = \frac{1}{4}\mathcal{G}\left(S_{G1} + S_{G2}\right)$$
$$= \frac{1}{2}\mathcal{G}\left(S^a + S^T\right)$$

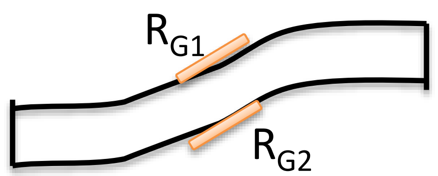
$S_{G2} = S^a - S^b + S^T$ axial strain

– can compensate bending but <u>not</u> temperature!

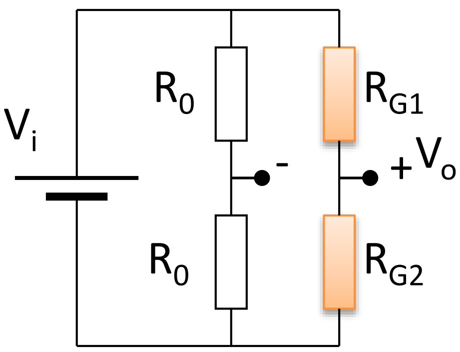
example: half-bridge (2/2)

- ultimately, sensitivity to loading condition and temperature is determined by the electrical configuration

$$S_{G1} = S^a + S^b + S^T$$



$$S_{G2} = S^a - S^b + S^T$$



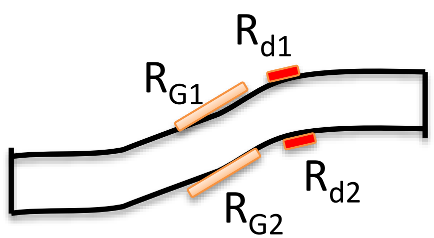
$$\frac{dV_o}{V_i} = \frac{1}{4}\mathcal{G}\left(-S_1 + S_2\right)$$
$$= \frac{1}{2}\mathcal{G}S^b$$

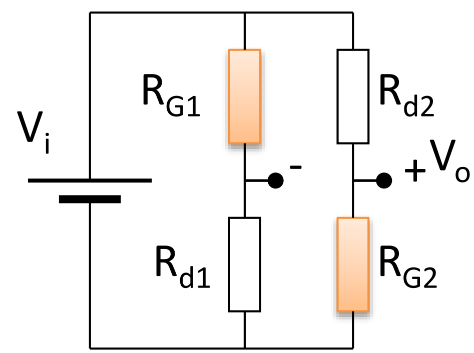
  - sensitive to bending strain
  - compensate for axial strain and temperature

example: full-bridge (1/2)

– can sense axial strain

– can compensate for temperature and bending





$$S_{G1} = S^a + S^b + S^T$$

$$S_{G2} = S^a - S^b + S^T$$

$$S_{d1} = S_{d2} = S^T$$

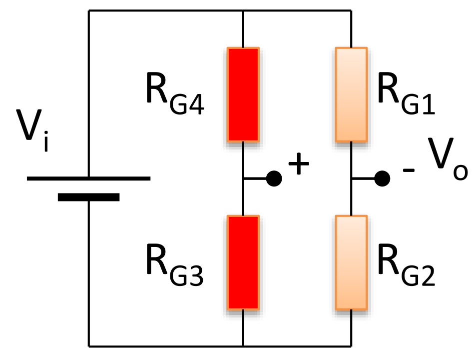
$$\frac{dV_o}{V_i} = \frac{1}{4}\mathcal{G}\left(S_{G1} - S_{d1} + S_{G2} - S_{d2}\right) = \frac{1}{2}\mathcal{G}\, S^a$$

example: full-bridge (2/2)

– can sense bending strain
  - maximum bridge sensitivity

– can compensate for temperature and axial strain

$$S_{G1} = S_{G3} = S^a + S^b + S^T$$



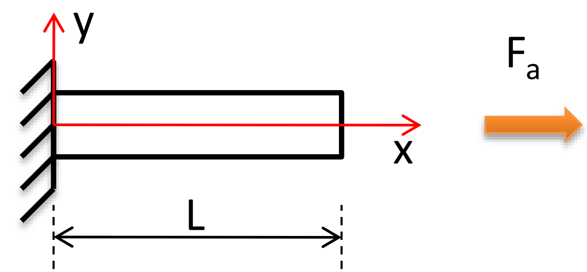
$$S_{G2} = S_{G4} = S^a - S^b + S^T$$

$$\frac{dV_o}{V_i} = \frac{1}{4}\mathcal{G}\left(S_{G1} - S_{G2} + S_{G3} - S_{G4}\right) = \mathcal{G}\, S^b$$

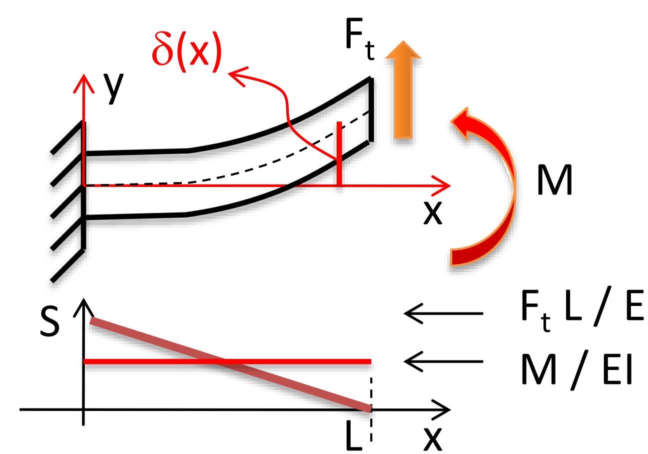
NOTE: these formulae only hold under the small-deformations assumption

example: cantilever beams

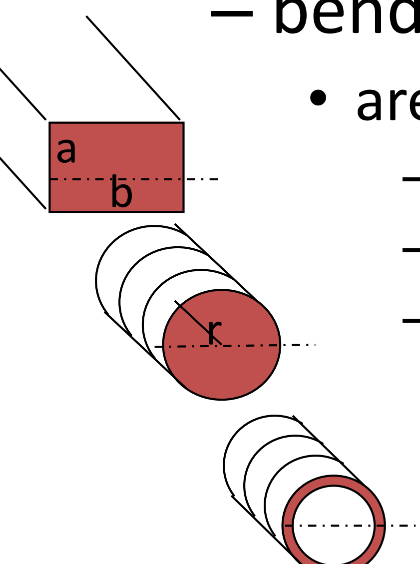
- longitudinal strain due to
  - axial loading (F_a)
    - L: length
    - t: thickness
    - A: cross-section
    - E: Young's module
  - bending (M, F_t)
    - area moment of inertia I
      - $I = ab^3 / 12$
      - $I = \pi r^4 / 4$
      - $I = \pi (r_{max}^4 - r_{min}^4) / 4$



$$S_a = \frac{F_a}{EA}$$

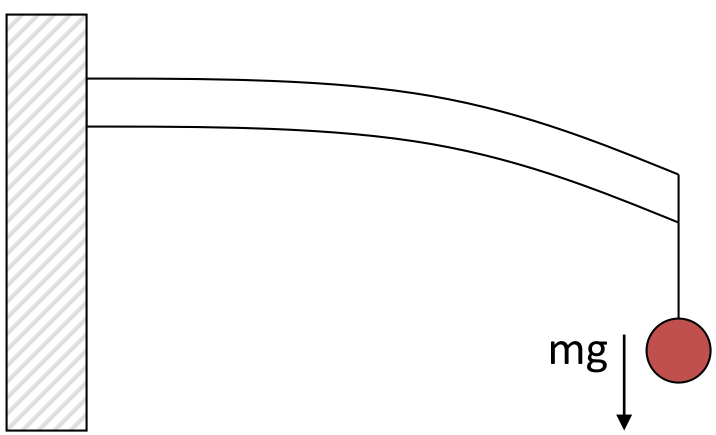


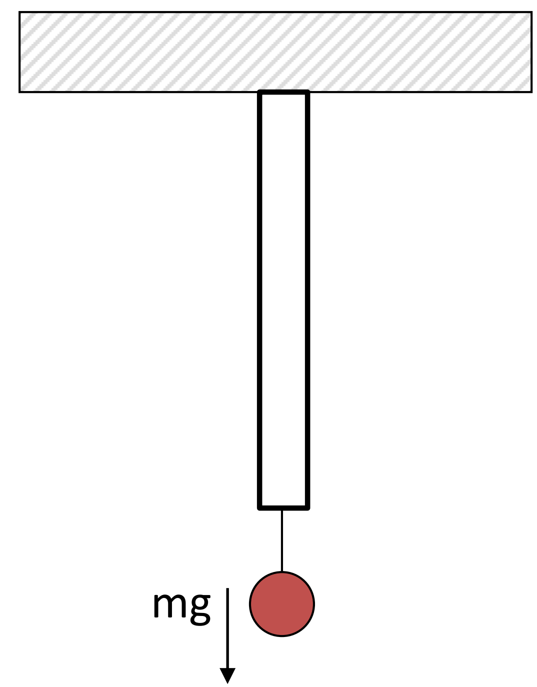
$$S_b = -\frac{M + F_t(L-x)}{EI}\frac{t}{2}$$



measuring forces:

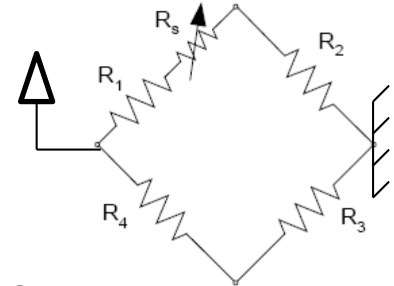
- you are given 2 identical strain-gauges
  - dR/R = **G** S   where **G**=2
- where/how would you place them, along the beam, to measure the weight of a mass m and
  - maximize sensitivity?
  - compensate for temperature changes?



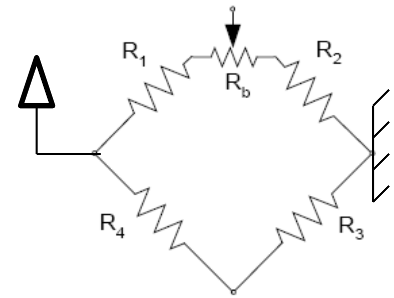


bridge balancing

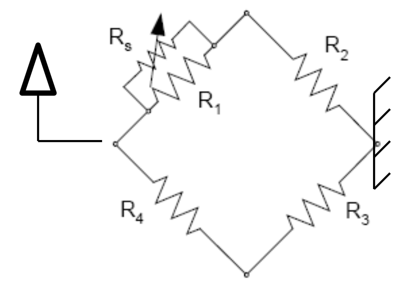
- so far, we assumed that the bridge was balanced when no stress was applied
  - in real life, that’s never going to happen…
- reestablish balance by modifying arm resistors
  - $R_1{}^* R_3 = R_2{}^* R_4$
- (a) and (c) require
  - very low resistors (non practical)
  - in-series switches/contacts
    - unreliable extra resistance will be added
- (b) and (d) are the most suitable
  - much larger resistors can be used
  - the structure of the bridge in not modified
    - parallel insertion
- (d) is more general (balancing both sides)



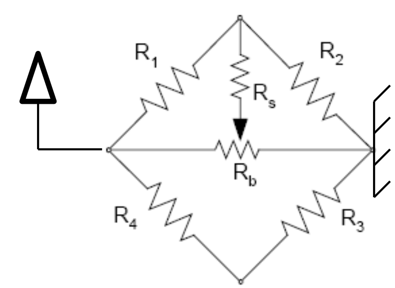
(a) Series



(c) Potentiometric



(b) Shunt



(d) General