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The ideal capacitance for parallel plates is proportional, through a dielectric constant k, to the common (overlapping) area between the two plates A and inversely proportional to the distance d between them. By partially submerging two such plates in liquid, as shown in Fig 1, it is then possible to measure the liquid level (y) due to a variation of the dielectric constant.

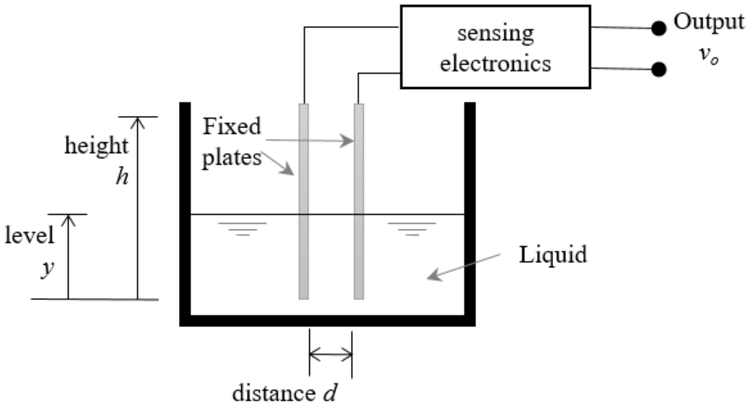


Figure 1

Assuming the total capacitance C to be the sum of the capacitance C_A (due to the portion of plates in air) and of the capacitance C_L (due to the submerged portion of the plates)

(a) Determine whether the partial capacitances C_A and C_L are electrically in series or in parallel.
(10 marks out of 100)

The two partial capacitors have the same potential difference.
Therefore, the partial capacitances C_A and C_L are in parallel.

(b) Assuming plates of height h and width w (not shown in the figure), determine C_A, C_L and C as functions of the geometric parameters as well as of the permittivity of air (k_A) and liquid (k_L).
(30 marks out of 100)

$$C_A = \frac{k_A w(h-y)}{d}$$
$$C_L = \frac{k_L w y}{d}$$

$$C = C_A + C_L$$
$$C = \frac{w}{d}k_A(h-y) + \frac{w}{d}k_L y$$
$$C = \frac{w}{d}[\,k_A(h-y) + k_L y\,]$$
$$C = \frac{w}{d}[\,(k_L - k_A)y + k_A h]$$

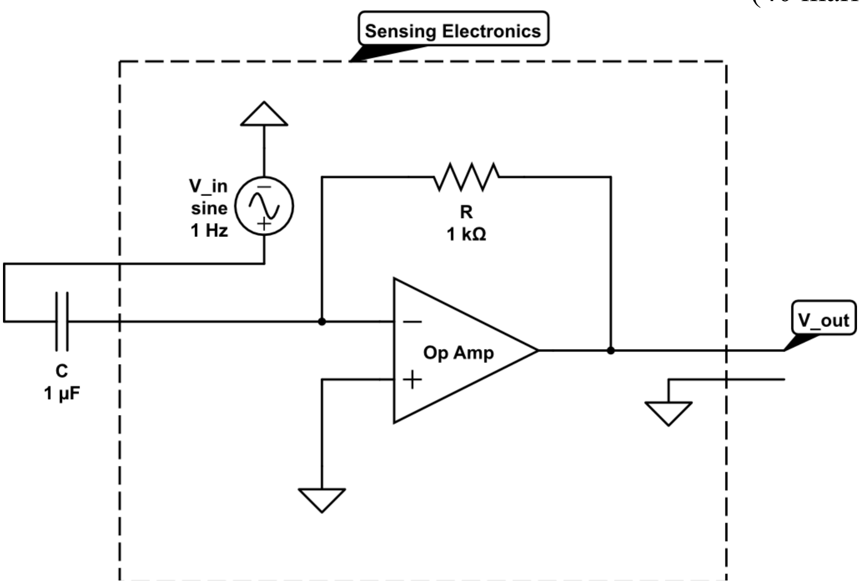
(c) Determine the liquid level y as a function of the total capacitance C.

(20 marks out of 100)

$$\frac{Cd}{w} = (k_L - k_A)y + k_A h$$
$$\frac{Cd}{w} - k_A h = (k_L - k_A)y$$
$$\frac{Cd - k_A hw}{w} = (k_L - k_A)y$$
$$y = \frac{Cd - k_A hw}{w(k_L - k_A)}$$

(d) Draw a complete circuit, comprising an operational amplifier, which would fit in the box 'sensing electronics' in Fig.1 paying special attention to the electrical connections (as shown in figure) and to the fact that the output v_o should be an amplified signal, proportional to the liquid level.

(40 marks out of 100)



By doing KVL, we know that:

$$\frac{V_{in} - V^-}{Z_c} = \frac{V^- - V_{out}}{R}$$

Since $V^- = V^+ = 0V$ and $Z_c = \frac{1}{jwC}$, we get:

$$H(j\omega) = \frac{V_{out}}{V_{in}} = -j\omega RC$$

The gain and phase shift is then calculated using:

$$|H(j\omega)| = \omega RC$$
$$\angle H(j\omega) = -\frac{\pi}{2}$$

From (b), we know that capacitance, C increases linearly with liquid level, y. Since the gain of the circuit, $|H(j\omega)|$ is directly proportional to the capacitance, C, the gain also increases linearly with liquid level, y.

$$|H(j\omega)| = \omega R\frac{w}{d}[\,(k_L - k_A)y + k_A h]$$
$$|H(j\omega)| = \frac{\omega Rw(k_L - k_A)}{d}y + \frac{\omega Rwk_A h}{d}$$

Assuming the purpose of the sensing electronics circuit is to measure the liquid level, y of which the capacitor is partially submerged in. By connecting the capacitor in the ways shown above, we get a sinusoidal wave in the voltage output. Since the input voltage is set to be a sine wave of amplitude 1V and frequency 1Hz, therefore only by reading the amplitude of the voltage output, we can get the capacitance, C = $\frac{A_{out}}{A_{in}\omega R}$, where A_{out} and A_{in} are the amplitudes of output signal and input signal respectively. By using the equation in question (c), the liquid level, y can be calculated.