MA2011
Mechatronics System Interfacing
(Part-II)

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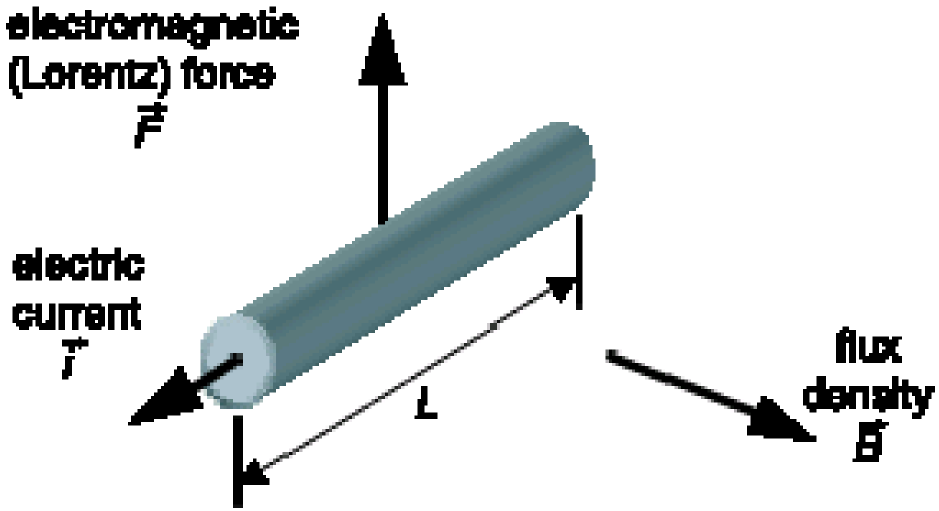
Part-II - actuators

DC Motors (brushed and brushless)
PWM driving

electromagnetic recap

- electromagnetic force:
  - Lorentz's law



$$\vec{F} = (\vec{i} \times \vec{B})\, L$$

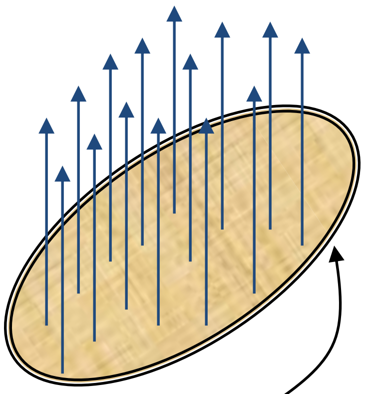
$$F = \|\vec{F}\| = BiL$$

electromagnetic recap

- elettromagnetic induction

$$emf = E = -\frac{d\Phi}{dt}$$

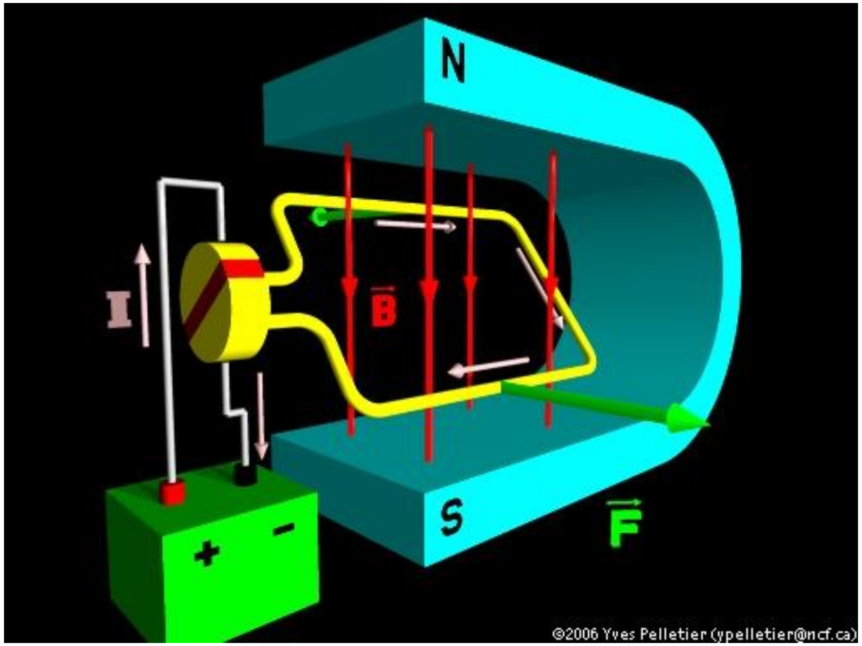
$$\Phi \triangleq \int_{\Sigma} \vec{B}\, d\vec{\Sigma}$$



  - Faraday's law
    - emf: electro-motive-force
    - Φ: magnetic flux
    - Σ: surface whose boundary coincides with the coil
      - <u>not uniquely defined</u>!!!!
      - but divB = 0 → the integral only depends on the boundary

DCM: structure and fields

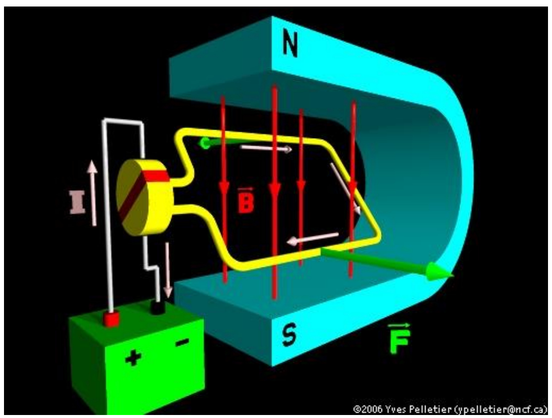
- stator
  - external, fixed
- rotor
  - internal, rotates

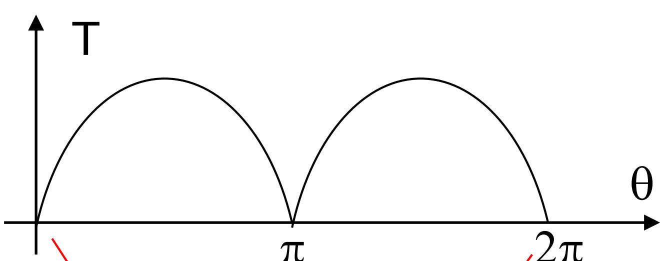


- actuation principle for DC motors:
  - stator field and rotor fields are always orthogonal → maximum torque

why do we need commutation?

- with commutation



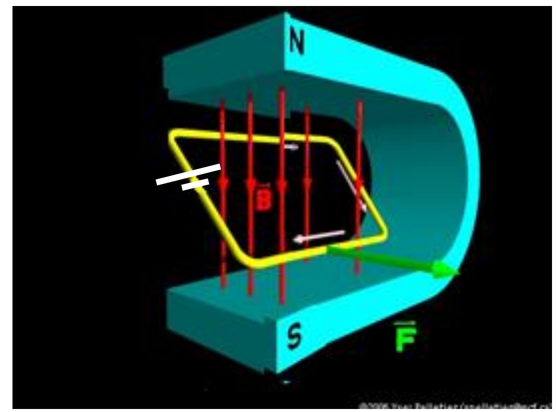


unstable equilibria

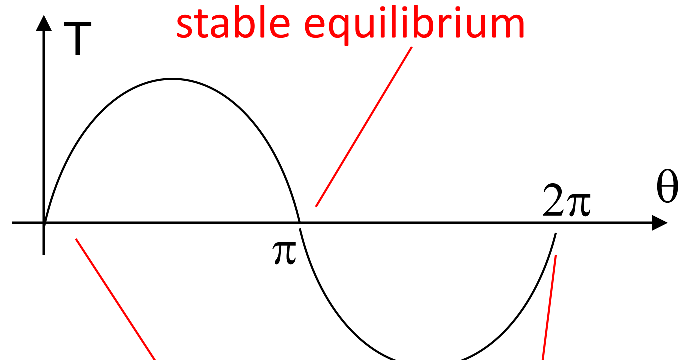
T=0

equilibrium

- without commutation

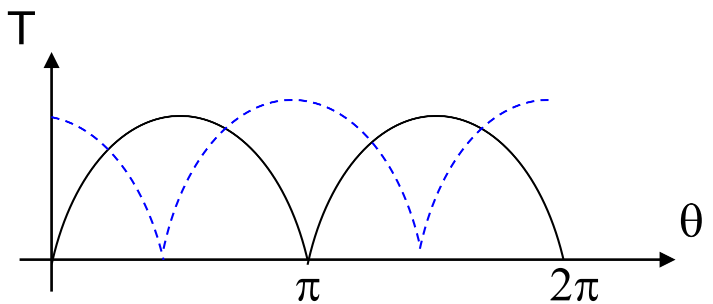


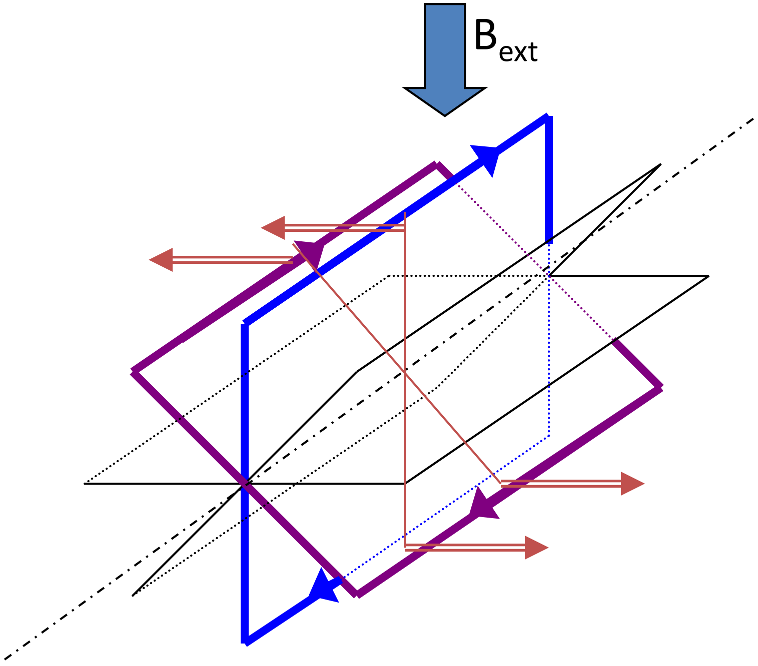
stable equilibrium

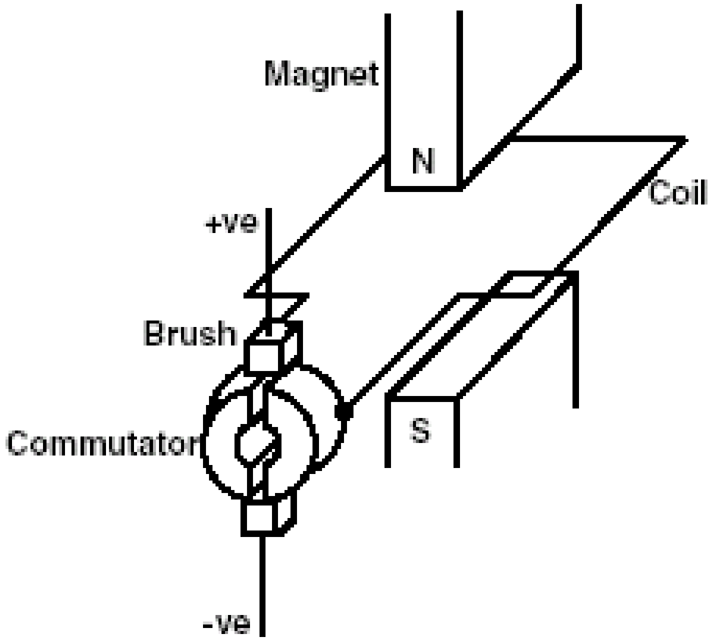


unstable equilibria

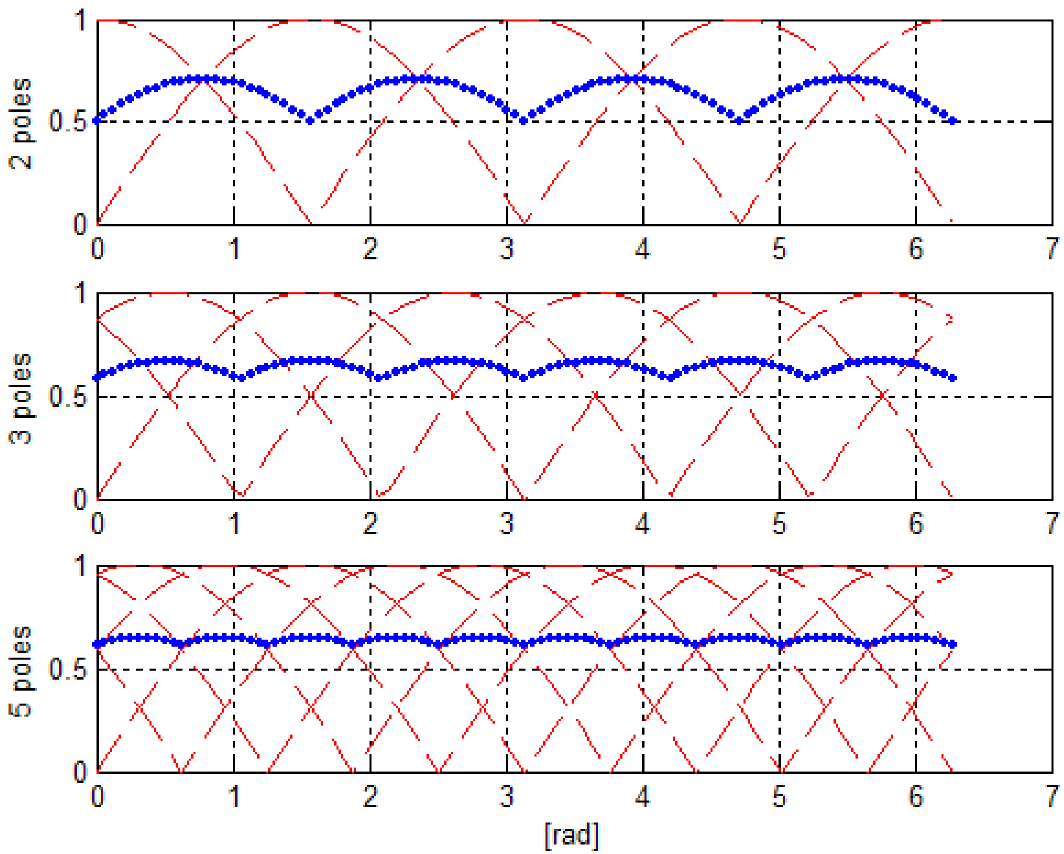
DCM: generated torque



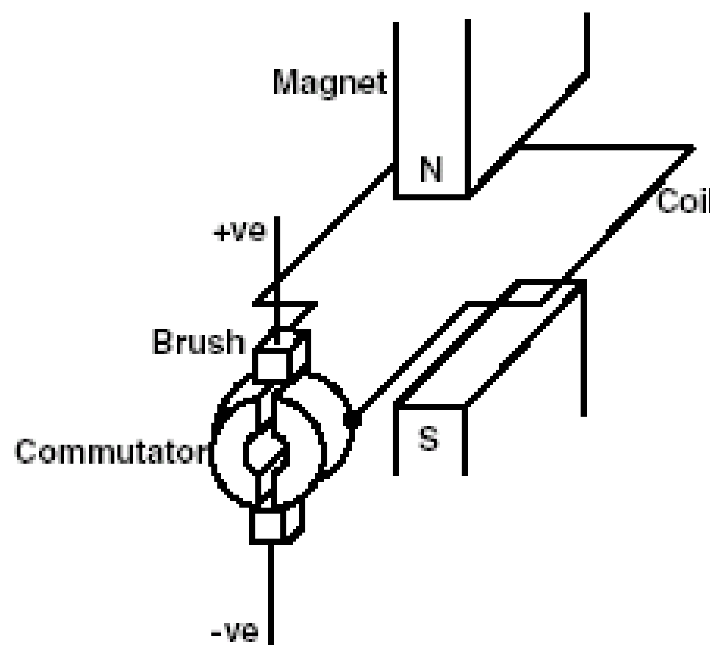




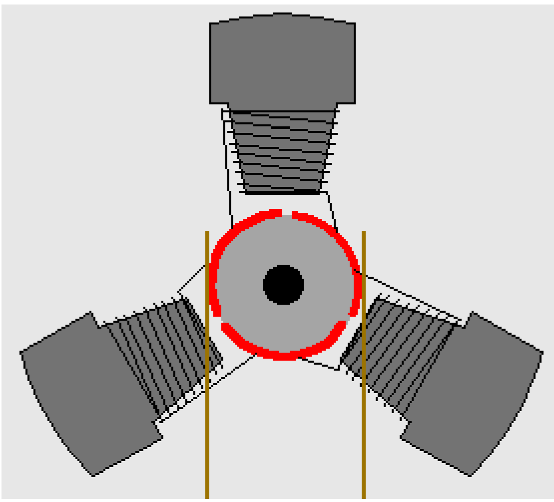
DCM: generated torque

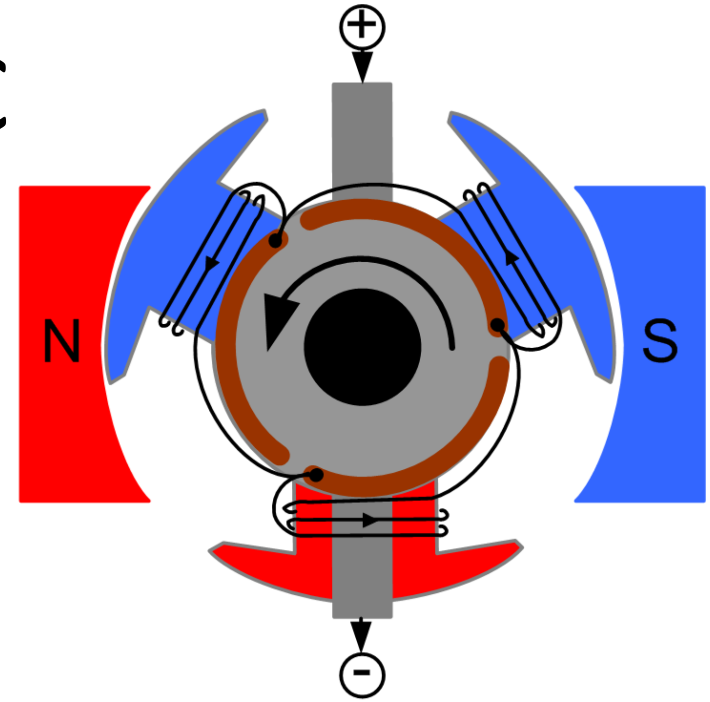


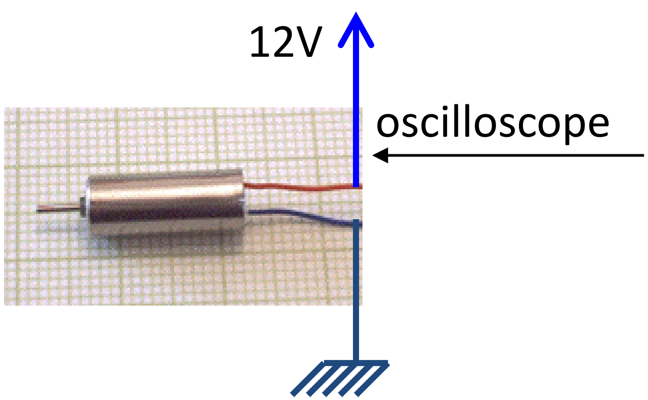
the larger the number of poles, the more constant the torque, i.e. independent on the rotor position



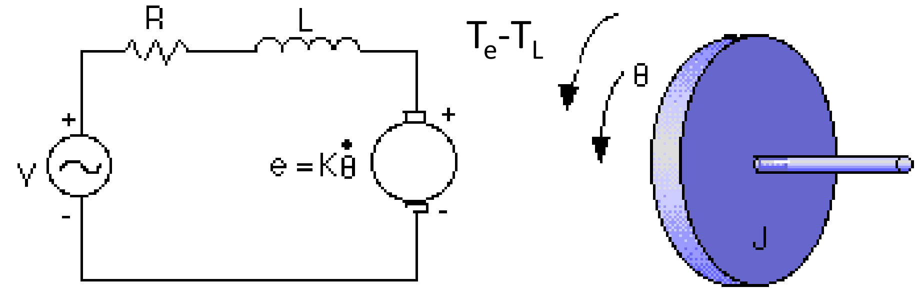
3-pole DC







DCM: equations



- armature equation:

$$V = Ri + L\frac{di}{dt} + e$$

e.m.f.

- mechanical equation:

Load Torque

$$J\dot{\omega} + b\,\omega = T_e - T_L$$

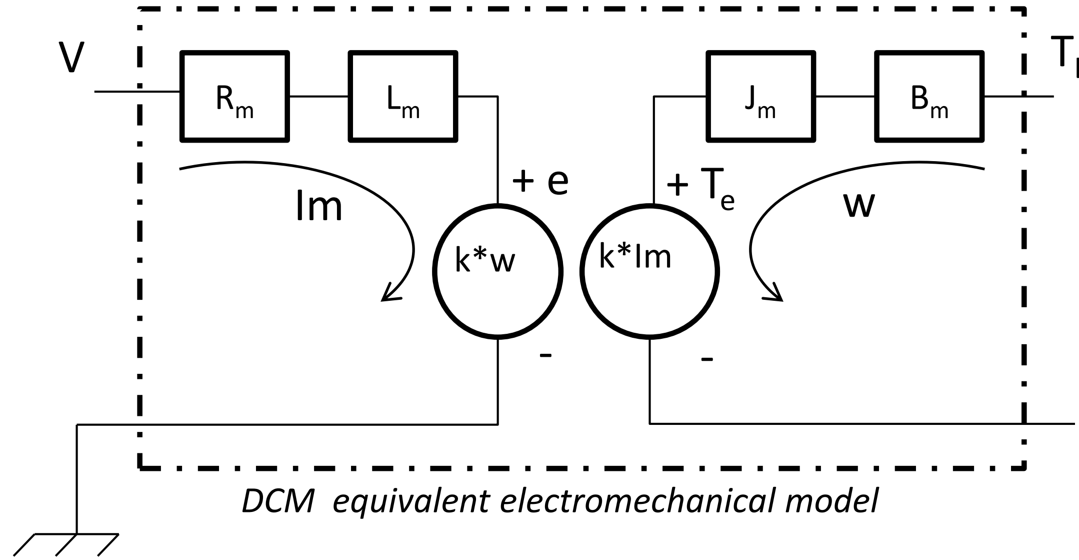
ElectromagneticTorque

- electro-mechanical coupling:

$$\begin{cases} T_e = K_t i \\ e = K_e\,\omega \end{cases} \qquad T_e\,\omega = e\,i \;\Leftrightarrow\; K_e = K_t \triangleq K_a$$

armature constant

DCM electrical equivalent



DCM equivalent electromechanical model

- if
  - voltage ~ torque
  - current ~ speed
- then
  - inductance ~ inertia
  - resistor ~ damping
  - capacitor ~ compliance
- NOTE:
  - electrical power (V*I) ~ mechanical power (T*ω)
  - mechanical parallel ~ electrical series

$$V = Ri + L\frac{di}{dt} + e$$

$$J\dot{\omega} + b\,\omega = T_e - T_L$$

DCM: **steady-state** speed-torque curve

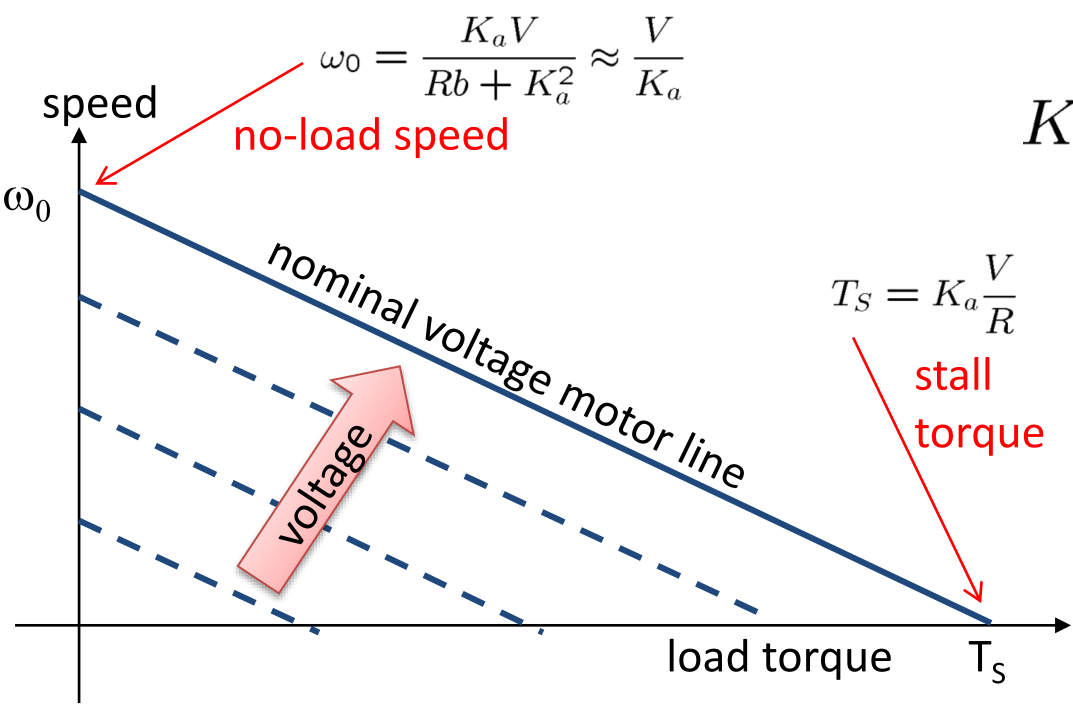
$$\begin{cases} V = Ri + \cancel{L\dfrac{di}{dt}} + K_a\,\omega \\ \cancel{J\dfrac{d\omega}{dt}} + b\,\omega + T_L = K_a\,i \end{cases} \Rightarrow \begin{cases} V - K_a\,\omega = R\,i \\ i = \dfrac{1}{K_a}(b\,\omega + T_L) \end{cases}$$

$$\Downarrow$$

$$K_a V - K_a^2\,\omega = Rb\,\omega + R\,T_L$$

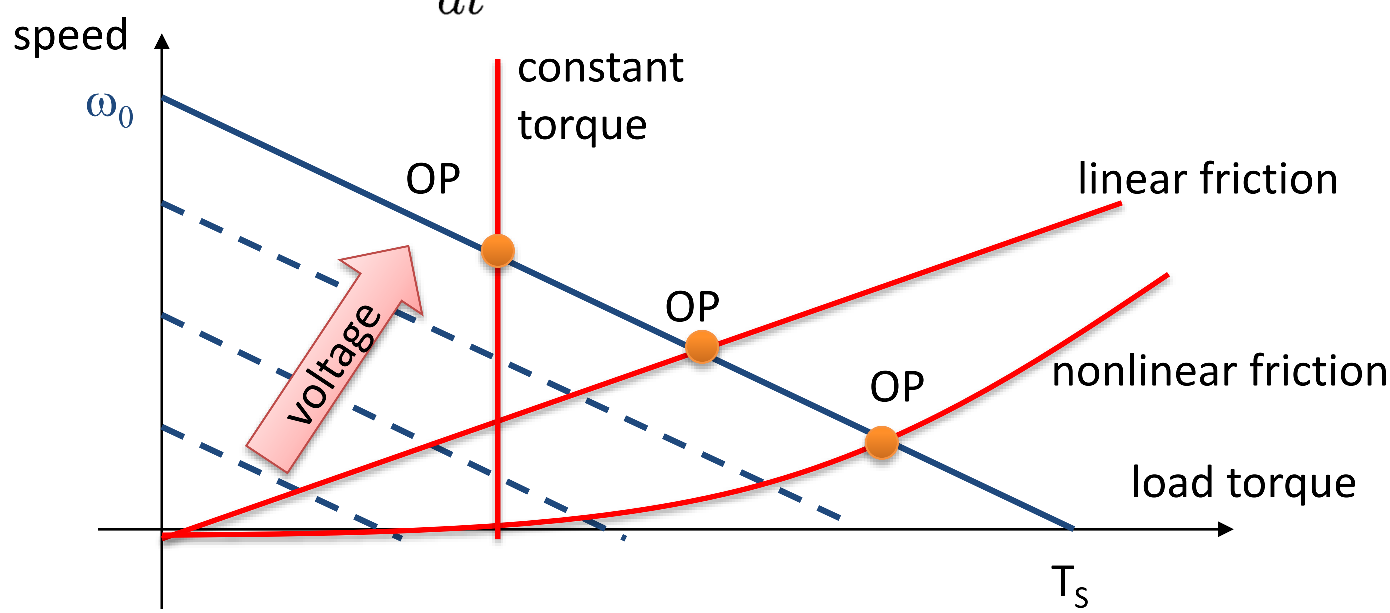
$$\Downarrow$$

$$\boxed{\omega = \frac{K_a V - R\,T_L}{Rb + K_a^2}}$$



DCM speed-torque curve: load lines

- loads can be of various kinds, e.g. combinations of:
  - friction $T_L = b_L \omega$ or nonlinear $T_L = f(\omega)$
  - constant torque $T_L = \texttt{const.}$
  - inertial $T_L = I\frac{d\omega}{dt} = 0 \text{ @ steady state}$



OP: Operating Point

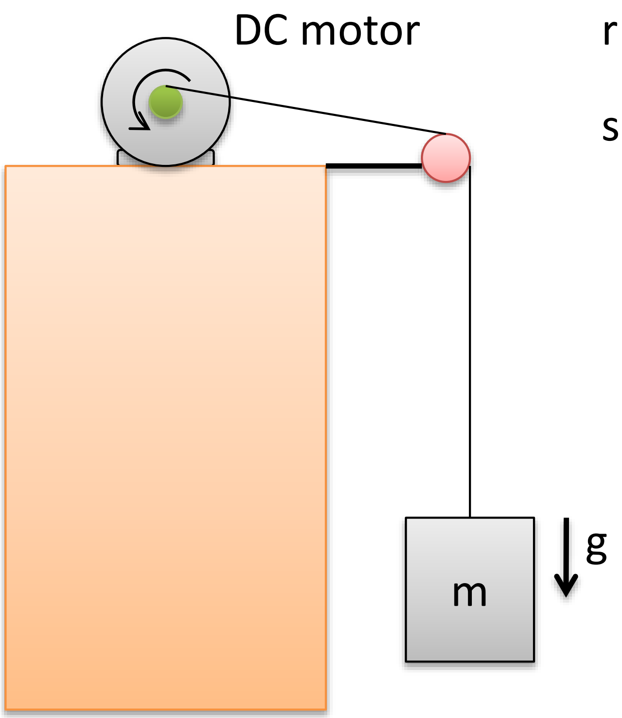
DCM: common types of load

- (1): constant torque

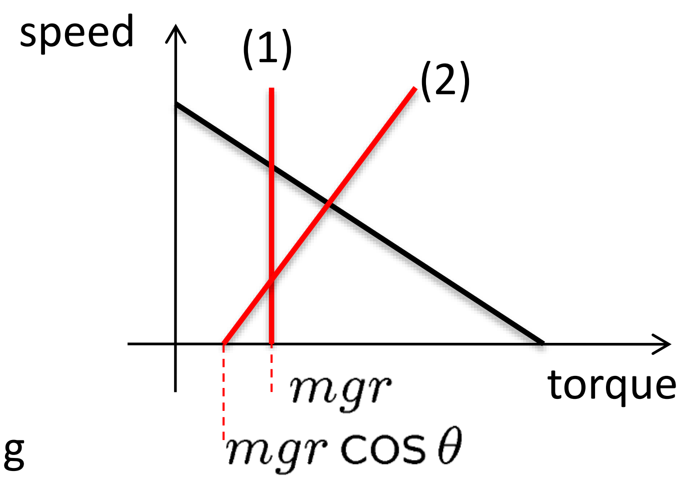
$$T_L^{(1)} = mgr$$

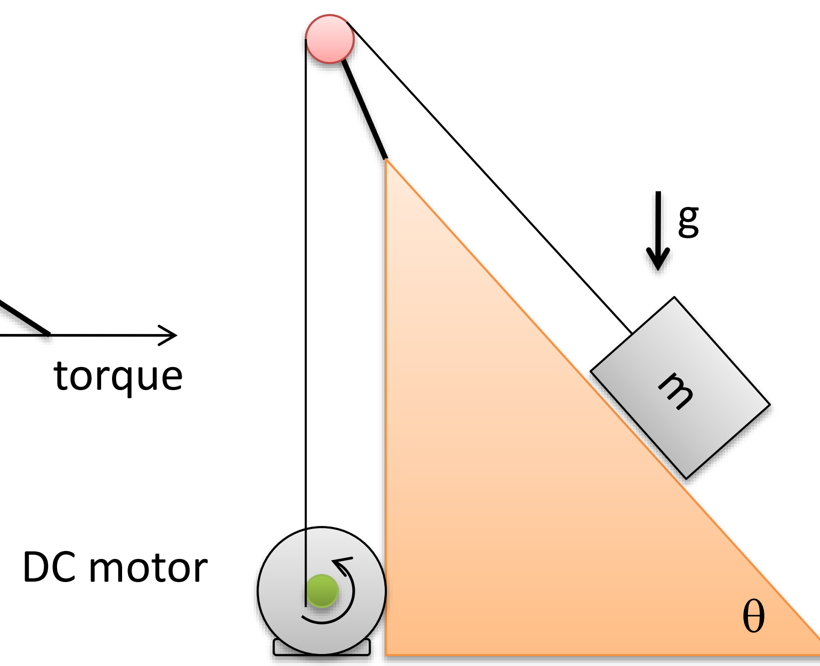
- (2): constant torque + friction

$$\begin{aligned} T_L^{(2)} &= mgr\cos\theta + b_L v \\ &= mgr\cos\theta + b_L \omega r \end{aligned}$$



r: radius of the shaft





DCM speed-torque curve: maximum **output power**

- at nominal voltage V

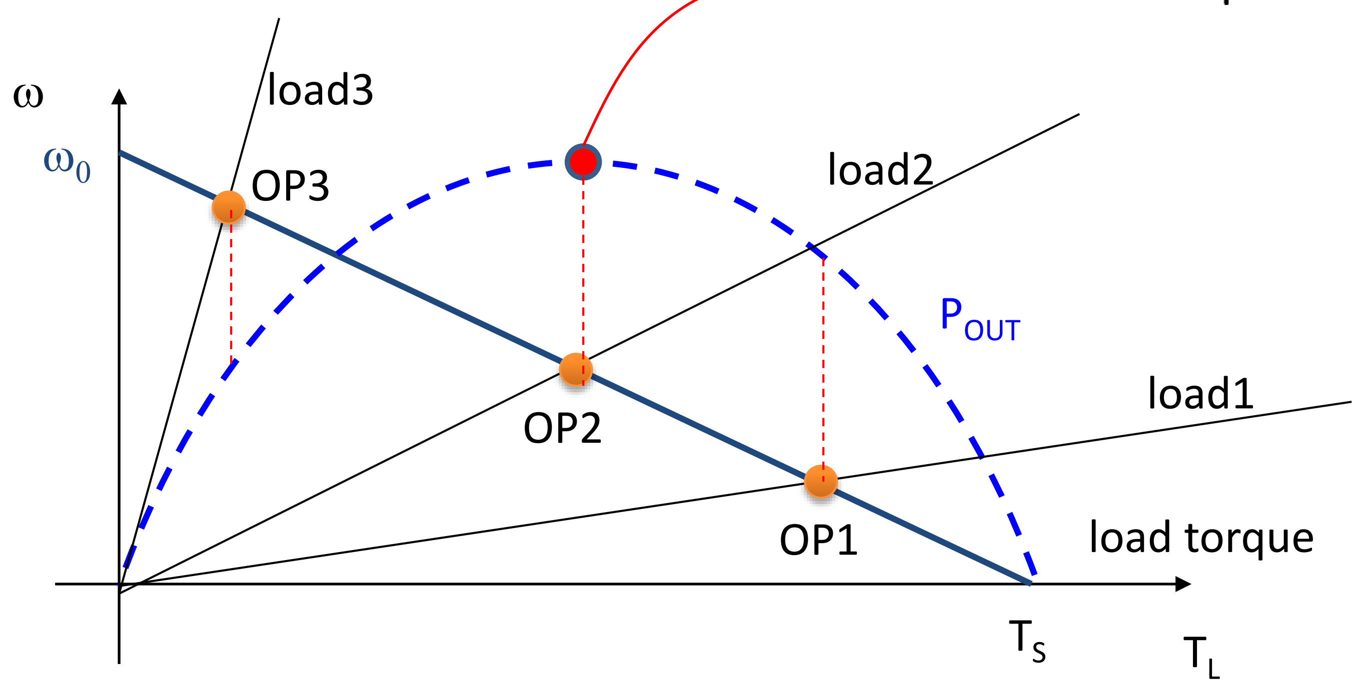
$$P_{out} = \omega T_L$$ ← **area** in the speed-torque graph

$$P_{out}^{max} = \frac{1}{4}\omega_0 T_s$$

friction load

$$T_L = b_L \omega_L$$

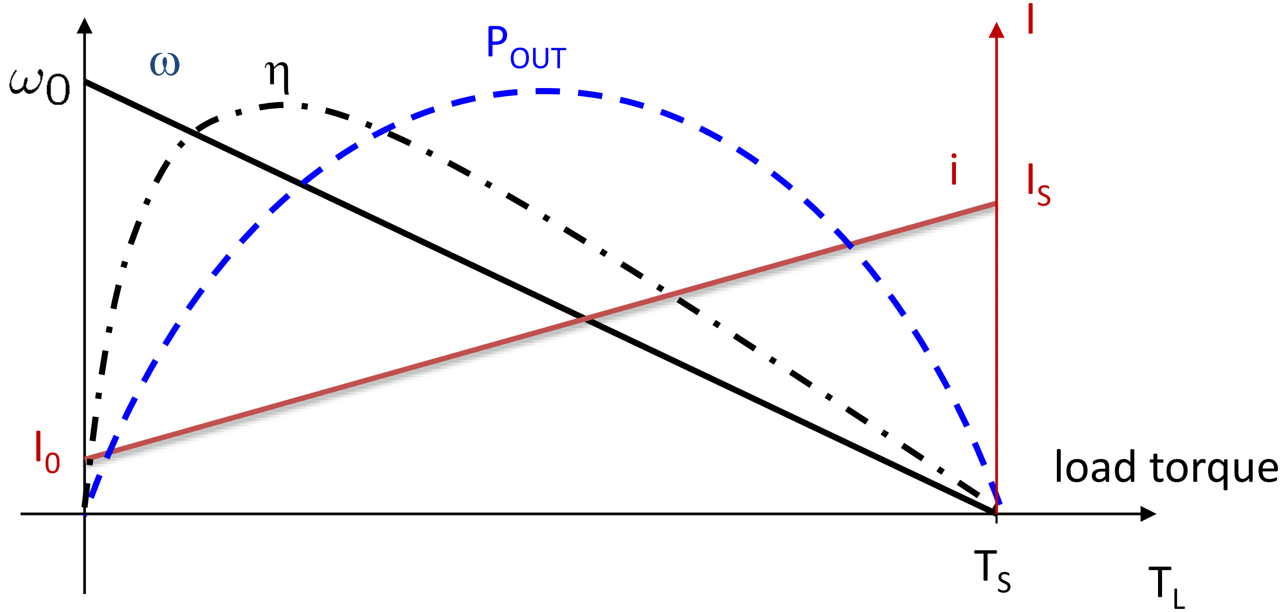
OP: Operating Point



DCM speed-torque curve: maximum **efficiency**

- at nominal voltage V

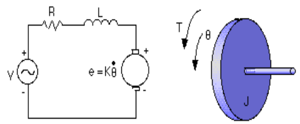
$$\eta := \frac{\texttt{power out}}{\texttt{power in}} = \frac{\omega T_L}{VI}$$



DCM-load matching

e.g. wheeled robot
friction-loads

$\begin{bmatrix} v \\ i \end{bmatrix}$ ⟺ **DCM equations** ⟺ $\begin{bmatrix} T_m \\ \omega_m \end{bmatrix}$ ⟺ **gear-ratio N** ⟺ $\begin{bmatrix} T_L \\ \omega_L \end{bmatrix}$ ⟺ **mech load**

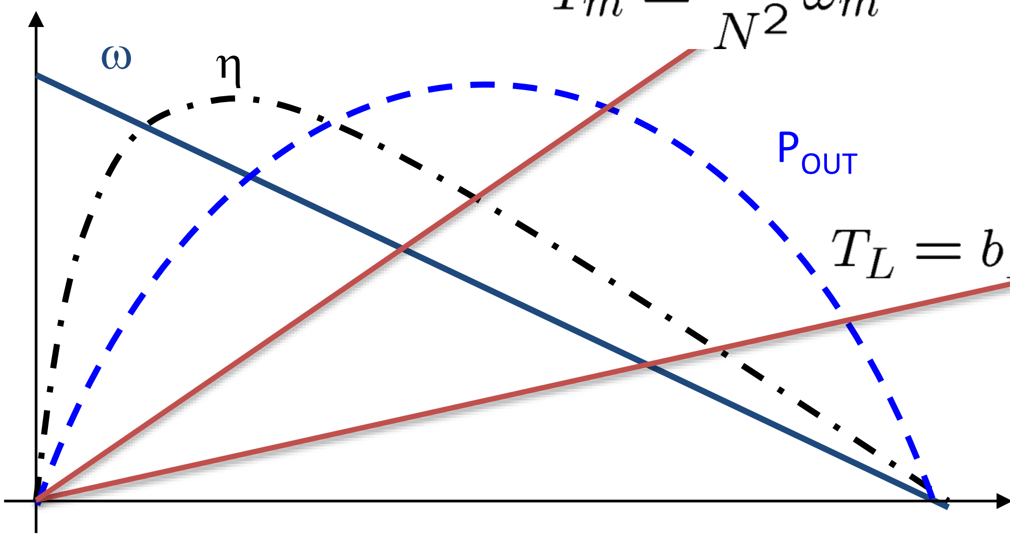


$$\begin{cases} T_L = N T_m \\ \omega_m = N \omega_L \end{cases}$$

ideal

$$T_L = b_L \omega_L$$

$$T_m = \frac{b_L}{N^2} \omega_m$$



$$T_L = b_L \omega_L$$

design for
- maximum power or
- maximum efficiency

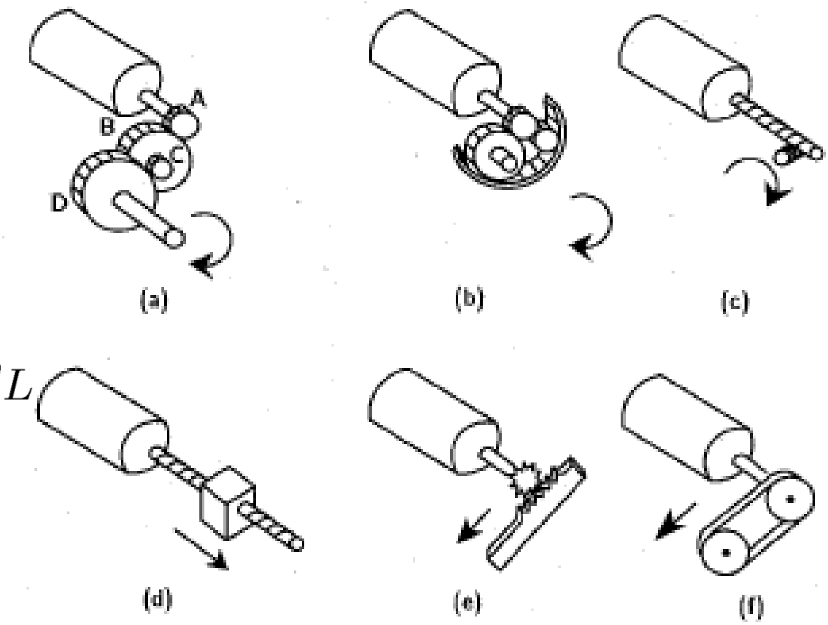


Figure 7.12. (a) Spur gears mesh pairs of gears with different numbers of teeth to achieve speed reduction. (b) Planetary gears have several gears meshed in an outer ring for large reduction. (c) Worm gears produce rotary motion at right angles to the shaft. (d) A lead screw and nut can create linear motion as can (e) rack-and-pinion systems and (f) belt-and-pulley drives.

Precision Microdrives™

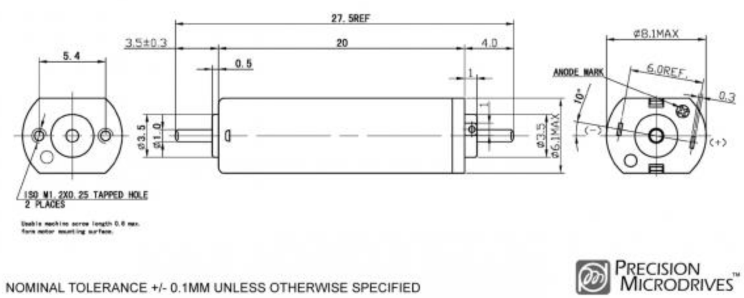
real miniature DC motors

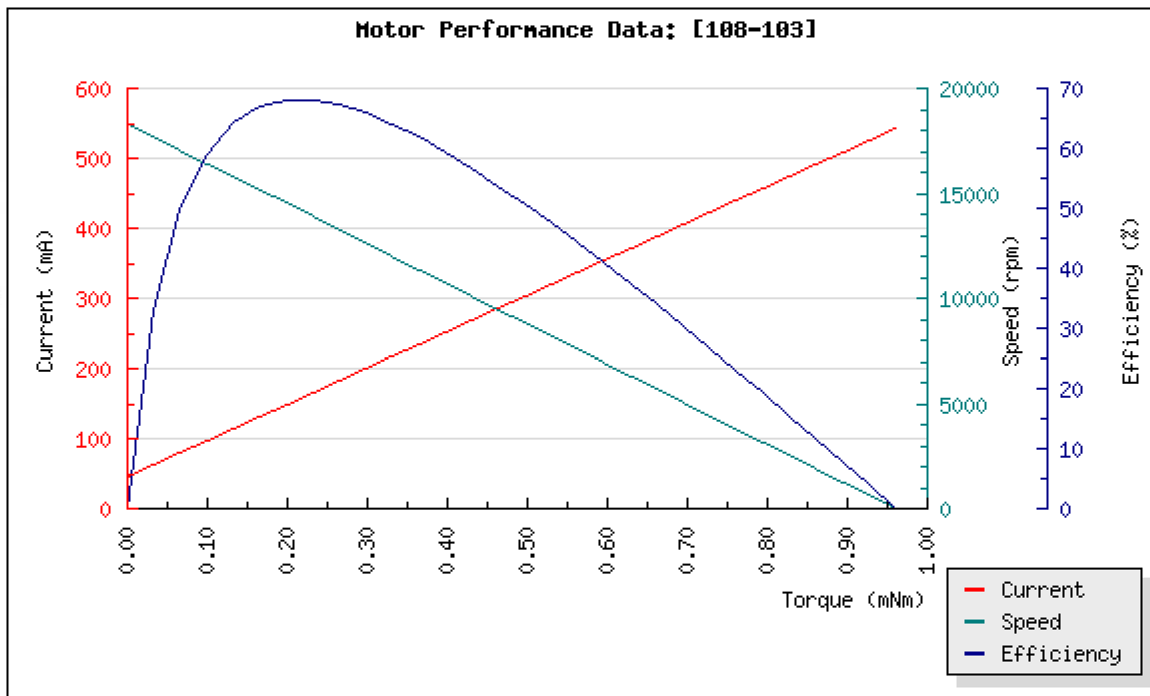
8mm Micro DC Motor (Flat Type)

20mm Type (Double Ended) [108-103]

Click parameters to sort or click product to view details

Model	Volts (V)	Physical				No Load		Peak Efficiency				Stall		Max Power (mW)
		Diameter (mm)	Length (mm)	Shaft Dia (mm)	Weight (g)	Speed (rpm)	Current (mA)	Speed (rpm)	Current (mA)	Torque (mNm)	Power (mW)	Torque (mNm)	Current (mA)	
108-102	3	8	14	1	2.6									
108-103	3	8	20	1	3.7	18314	44	14093	160		318	0.96	548	449
108-104	3	8	20	1	3.7	18314	44	14093	160		318	0.96	548	449
108-105	3	8	14.5	1	2.5	18108	46	13638	139	0.17	252	0.7	422	339
108-106	3	8	14.5	1	2.5	18108	46	13638	139	0.17	252	0.7	422	339

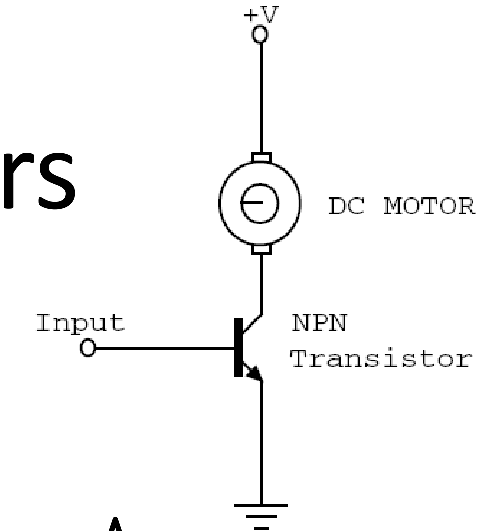


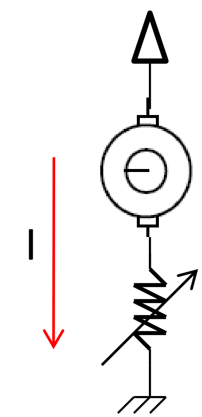


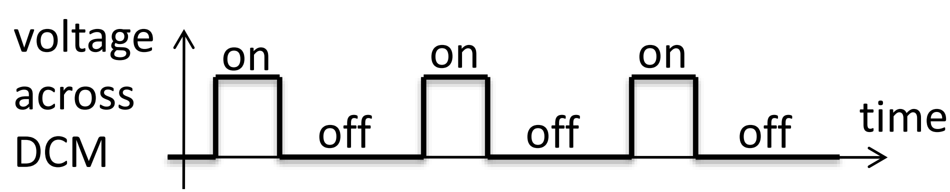
driving DC motors

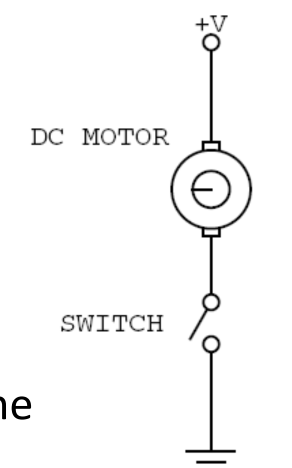
using power amplifiers

- using power amplifiers is possible but is typically avoided
  - large power dissipation
  - over-heating of the amplifier
- it is preferable to continuously switch the motor on and off
  - PWM: Pulse Width Modulation





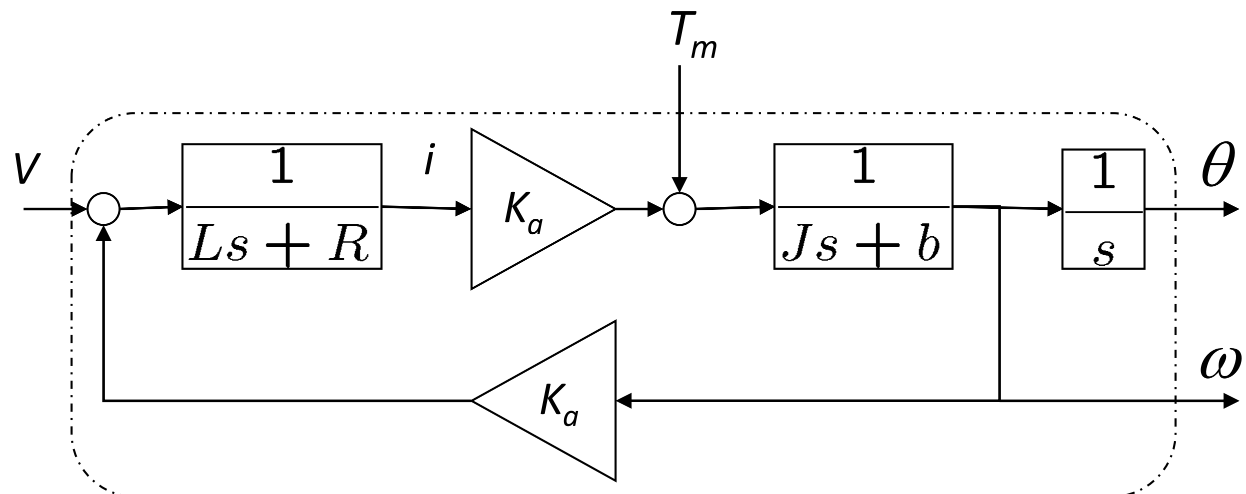




DCM: block-box modeling

$$\begin{cases} V = Ri + L\dfrac{di}{dt} + K_a\,\omega \\ J\,\dfrac{d\omega}{dt} + b\,\omega = T_m + K_a\,i \end{cases}$$

$$\begin{cases} V - K_a\,\omega = (R + Ls)\,i \\ (Js + b)\,\omega = T_m + K_a\,i \end{cases}$$

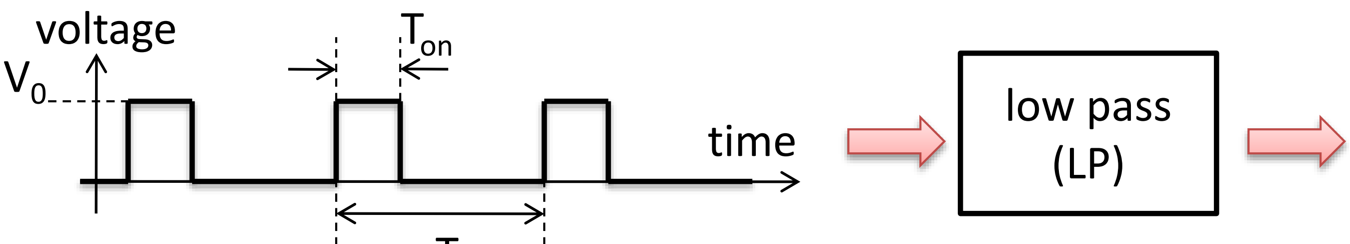


$$\omega = \frac{K_a V - (R + Ls)\,T_m}{(Ls + R)(Js + b) + K_a^2}$$

PWM: operating principle

- the DC motor is in fact a II order low pass filter

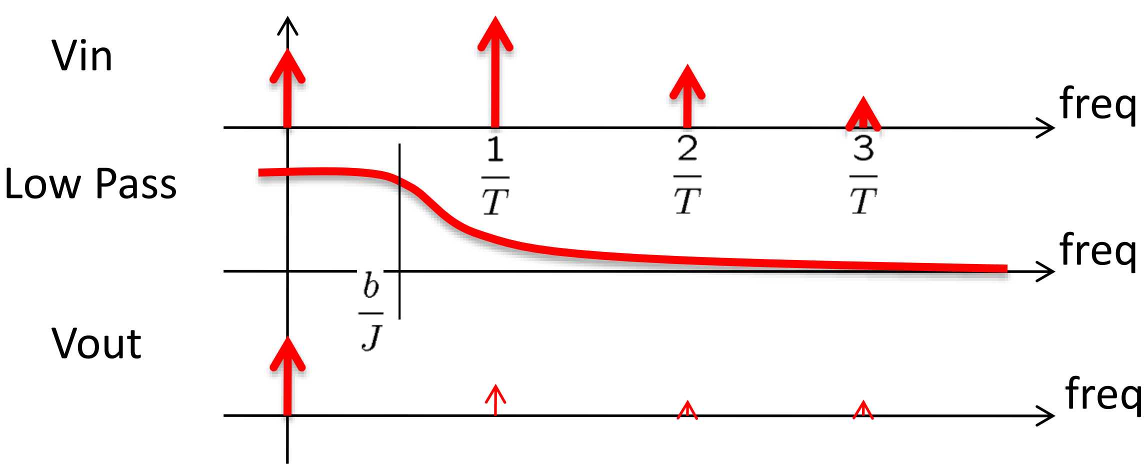
$$\omega = \frac{K_aV - (R + Ls)T_L}{(Ls + R)(Js + b) + K_a^2}$$



$$V_0\frac{T_{on}}{T} + \texttt{ripple}$$

duty-cycle

- frequency analysis



turning DC motors on and off

- inductive kickback
  - voltage across the inductor

$$v = L\frac{di}{dt}$$

  - if current starts decreasing
  - voltage $v=v_B-v_A$ quickly (d/dt) decreases
  - voltage v_A quickly (d/dt) increases

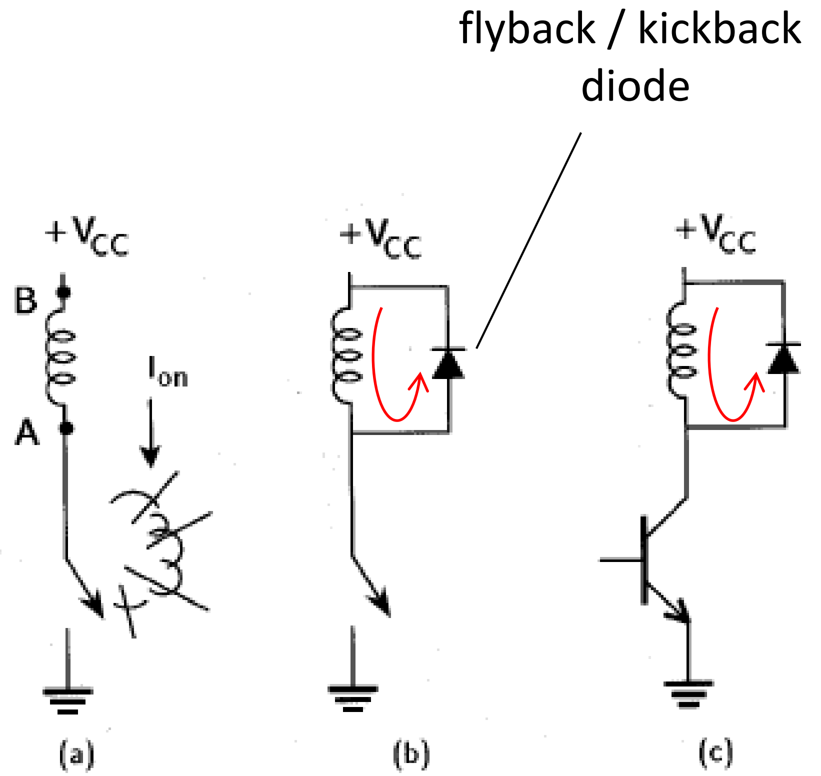
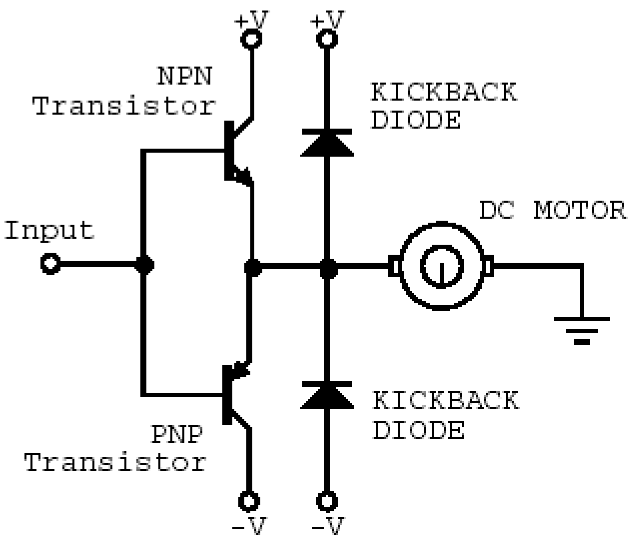


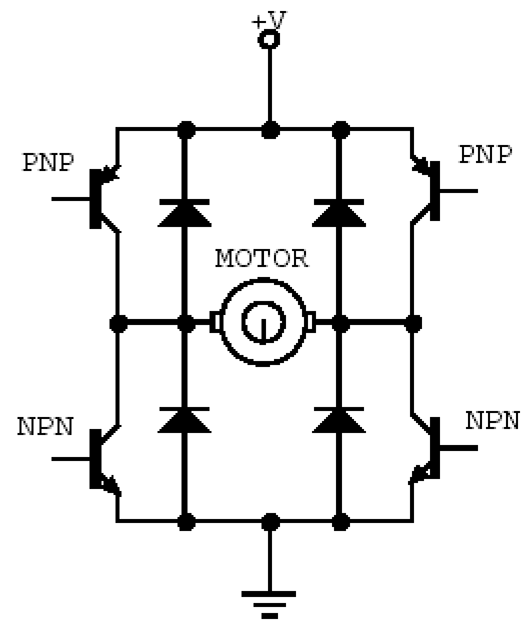
Figure 7.23. (a) The steady-state current through an inductor, I_{on}, cannot immediately go to 0 A when the switch is opened. The changing current induces a voltage across the inductor, making the potential at A greater than at B, causing the switch or relay to arc over. (b) Flyback diodes protect switches from blowing up. (c) Transistor switches must be protected in the same manner.

DCM driving: general PWM

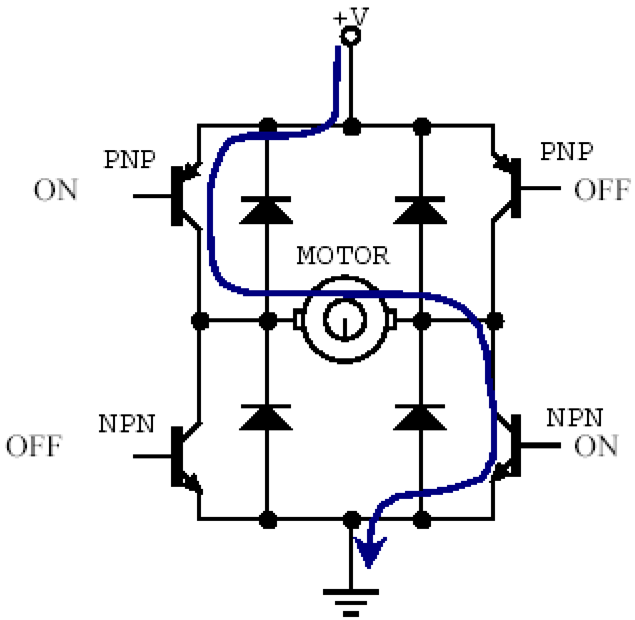
- Pulse Width Modulation (PWM)

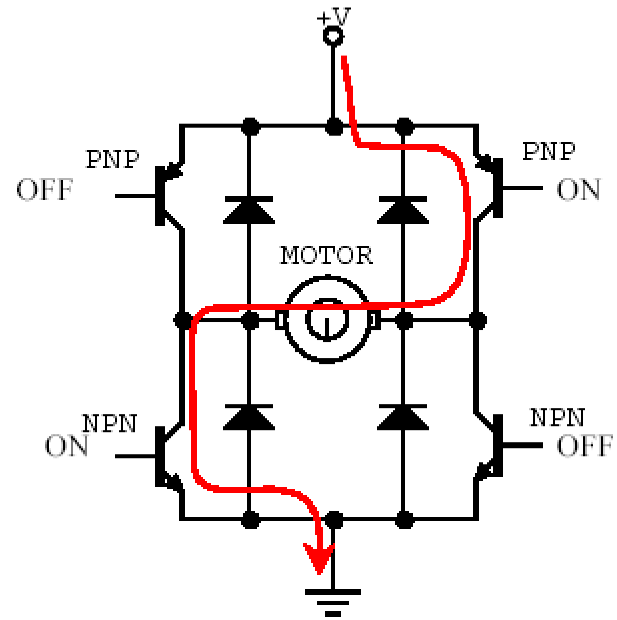


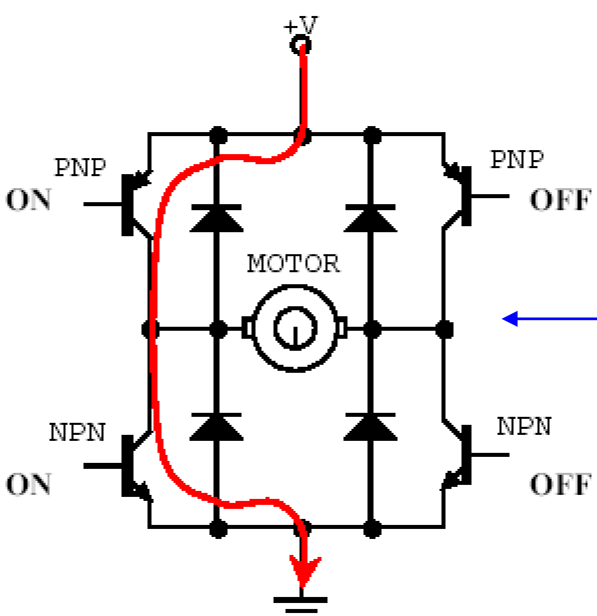
A Circuit for Bi-Directional
Current Flow
through a DC Motor
Using a Dual Power Supply



A typical H-bridge circuit, including kickback diodes to protect against inductive kickback.

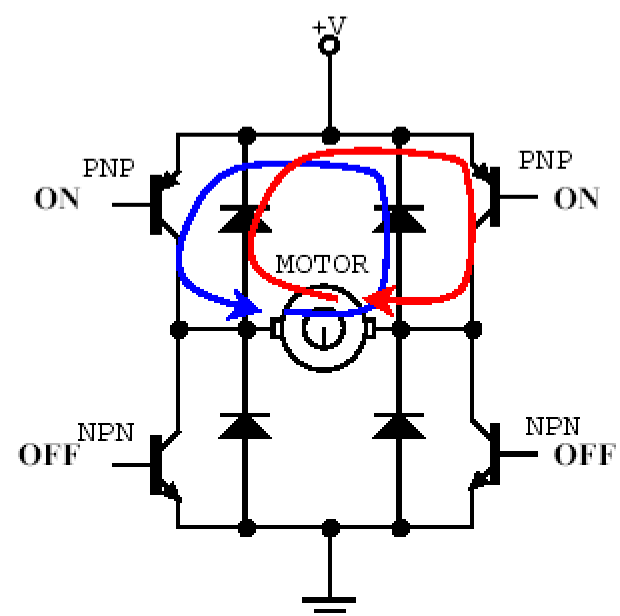






NEVER DO THAT

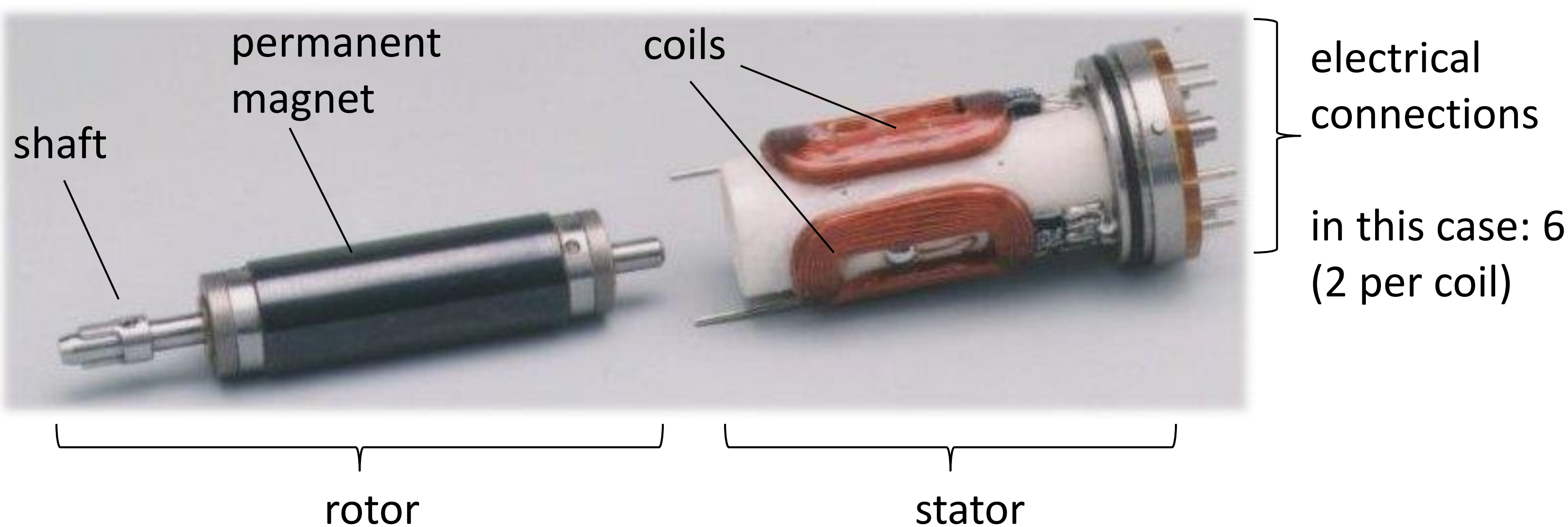
dynamic braking



Brushless DC motors

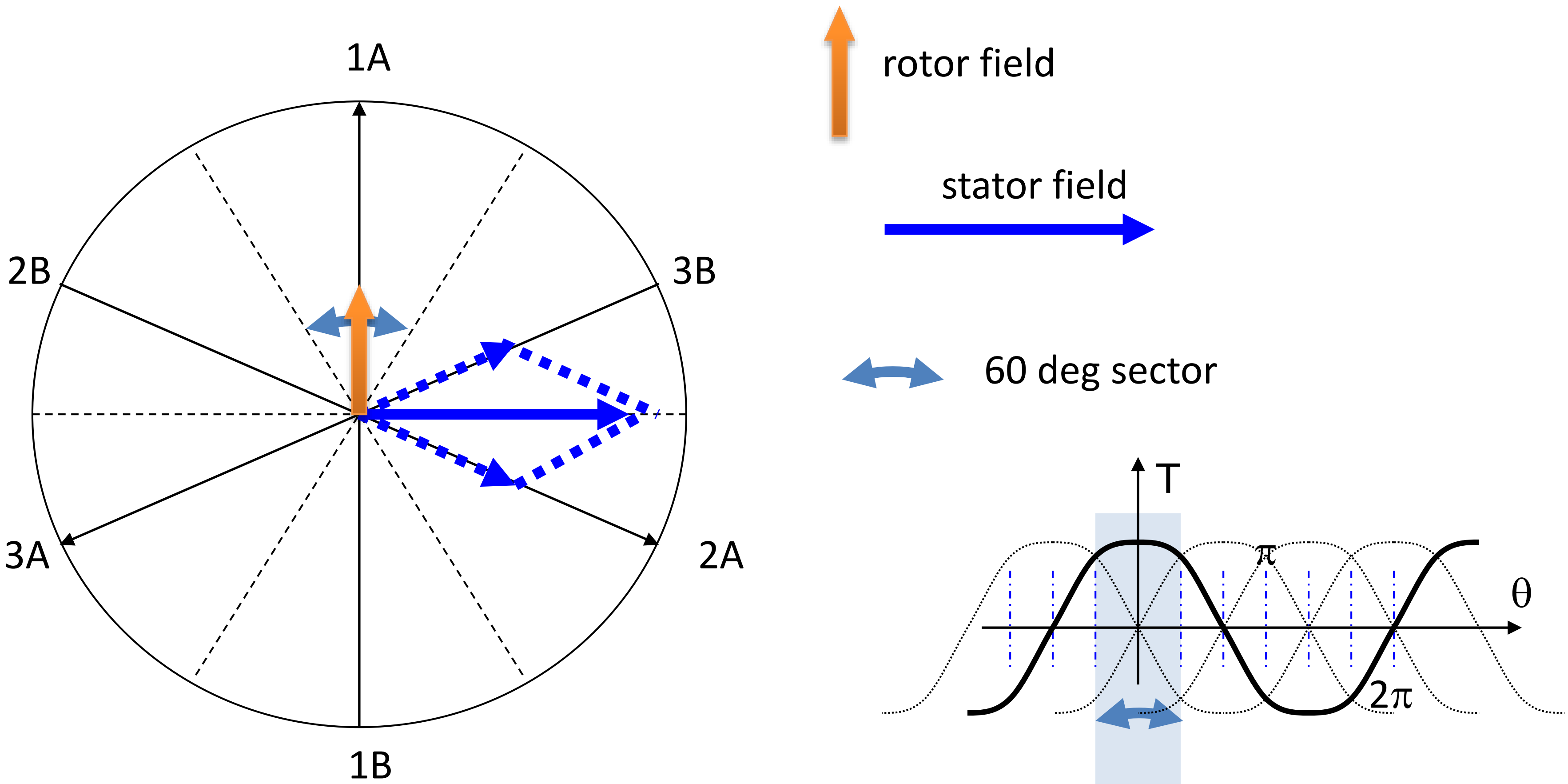
DC brushless motors

- just like a brushed DCM but.... **insideout**!
  - stator field… rotating
  - rotor field is given by a permanent magnet



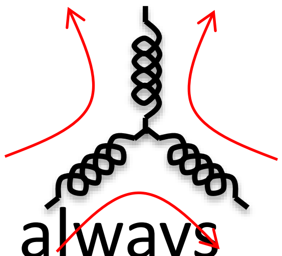
brushless DC motors: driving strategy

- to maximize output torque try to keep rotor and stator field orthogonal, as much as possible

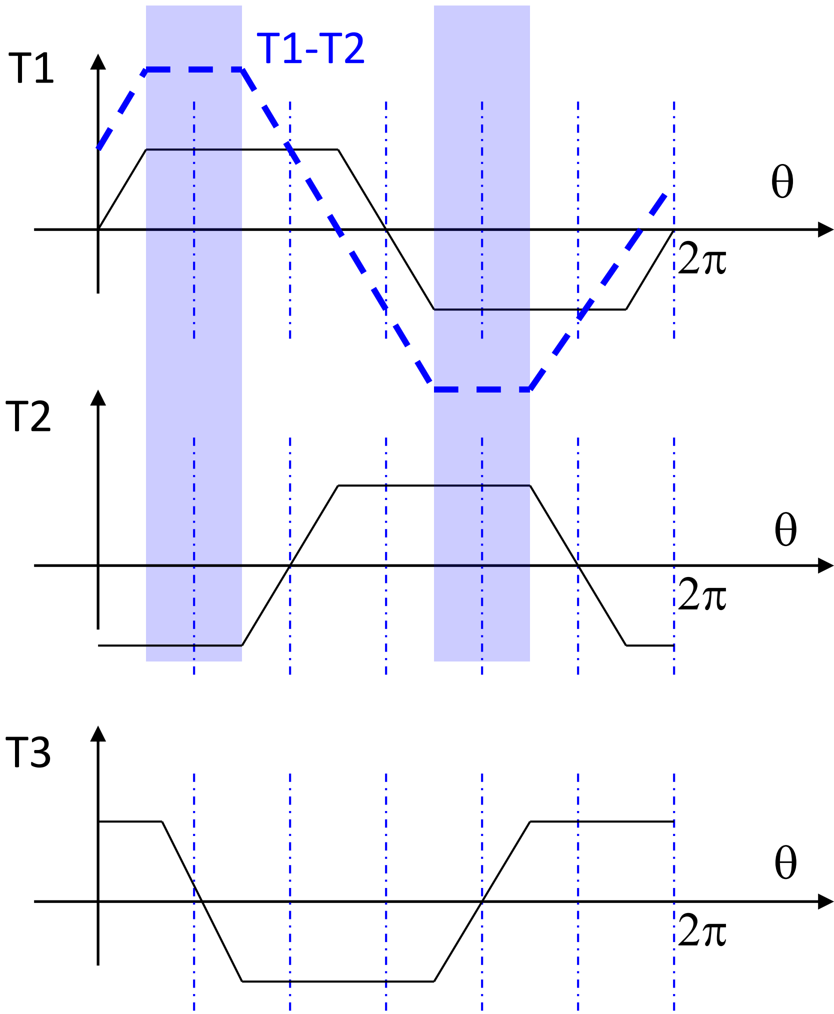


trapezoidal brushless DC motors

- Tj: torque due to the j-th coils
- always drive two coils at a time
  - with opposite currents +I and –I



- total torque Ti-Tj has always a constant zone
  - use torque T1-T2 when $\theta = \pi/3 \pm \pi/6$
  - **need to know the angular position of the rotor!!!**



brushless DC motors

- detect rotor position via encoders
  - typically Hall effect sensors
- select the appropriate switches to determine the desired Ti-Tj torque