**Lecture 1**

**Mechatronics Definition**

-Stiffler: "Science that integrates mechanical devices with electronic controls"

-Bolton: "Integration of electronics, control engineering and mechanical engineering"

-Bradley, et al: "An integrating theme within the design process [combining] electronic engineering, computing and mechanical engineering"

-Shetty & Kolk: "Methodology used for the optimal design of electromechanical products"

-Auslander & Kempf: "The application of complex decision making to the operation of physical systems"

**Measurement Systems**

Transducer: a device to convert a physical quantity into a time varying voltage (called analog signal)

Signal Processor: a device to modify the analog signal

Recorder: a device to display or record the signal.

**Linearity**:

$$V_{out}(t) - V_{out}(0) = \alpha(V_{in}(t) - V_{in}(0))$$

**Fundamental freq. :**

$$\omega_0 = \frac{2\pi}{T} = 2\pi f_0$$

**Fourier Series:**

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t)$$

Where,

$$A_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)\,dt$$

$$B_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)\,dt$$

$$C_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{A_0}{2}$$

Define:

$$C_n = \sqrt{A_n^2 + B_n^2}$$

$$\phi = -\arctan\left(\frac{B_n}{A_n}\right)$$

Then:

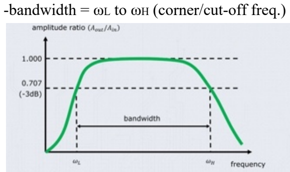
$$F(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \phi)$$

**Bandwidth & Freq. Response:**

$$dB = 20\log_{10}\left(\frac{A_{out}}{A_{in}}\right)$$

-bandwidth ∝ fidelity

-bandwidth = $\omega_L$ to $\omega_H$ (corner/cut-off freq.)




- -3dB comes from

$$\frac{P_{out}}{P_{in}} = \left(\frac{A_{out}}{A_{in}}\right)^2 = \frac{1}{2}$$

Output amplitude:

$$A_i' = \left(\frac{A_{out}}{A_{in}}\right)A_i$$

**Lecture 2**

**Periodic Function**

$f(t+T) = f(t)$, T is period.

**Even Function (Fully cosine waves)**

$f(-t) = f(t)$

All $B_n = 0$

**Odd Function (Fully sine waves)**

$f(-t) = -f(t)$

All $C_0$ and $A_n = 0$

**Complex Form**

$$F(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

Where,

$$D_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_0 t}dt$$

**Spectrum & Measurement**

-important to estimate spectrum when choosing measurement

-ideal system replicates freq. of an input signal

-practical system limited

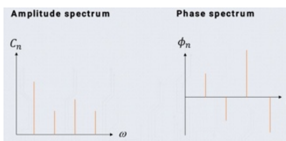
**Error of Approximation**

$$f(t) \cong \sum_{n=-N}^{N} D_n e^{jn\omega_0 t} = S_N(t)$$

$$\varepsilon(t) = f(t) - S_N(t)$$

$$MSE = \frac{1}{T}\int_0^T \varepsilon^2(t)dt$$

-MSE minimum when $D_n$ = Fourier coefficients




**Fourier Series Conditions:**

-f(t) is a single-value function

-for any $t_0$, $\int_{t_0}^{t_0+T}|f(t)|\,dt < \infty$

-f(t) has a finite number of discontinuities

-f(t) has a finite number of maxima and minima

-f(t) amplitude function, fulfil 4 conditions

**Insights**

-how signals of diff. freq. represented in a signal

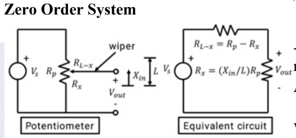
-easier & cost-eff. To characterise freq. content instead of time description of noise

-diff. treatment of diff. parts of EM spectrum help you separate radio, tv and cell phone signals

**Lecture 3**

-system that is fast to respond + good to understand → good method, highly desired

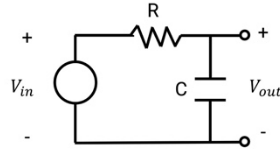
**Zero Order System**




$$V_{out} = \frac{R_x}{R_p}V_s = \frac{V_s}{L}X_{in}$$

$X_{out} = KX_{in}$ , K is gain/sensitivity

**First Order System**




$$RC\frac{dV_{out}}{dt} + V_{out} = V_{in}$$

$$\tau\frac{dX_{out}}{dt} + X_{out} = KX_{in}$$

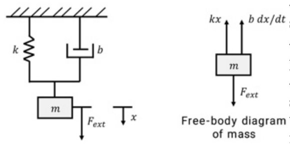
$$X_{out} = X_{out}(0)e^{-t/\tau} + KA_{in}(1 - e^{-t/\tau})$$

$$X_{out} = KA_{in}(1 - e^{-t/\tau})$$

-1 time constant → 63.2%

-4 time constants → >98% , assume steady state

**Second Order System**




$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_{ext}(t)$$

When $F_{ext} = 0$:

When $\zeta = 0$,

$$x_h(t) = A\cos(\omega_n t) + B\sin(\omega_n t)$$

When $\zeta < 1$,

$$x_h(t) = e^{-\xi\omega_n t}[A\cos(\omega_d t) + B\sin(\omega_d t)] = Xe^{-\zeta\omega_n t}\sin(\omega_d t + \psi)$$

When $\zeta > 1$,

$$x_h(t) = e^{-\xi\omega_n t}[A\cos(\omega_d t) + B\sin(\omega_d t)]$$

When $\zeta = 1$,

$$x_h(t) = (A + Bt)e^{-\omega_n t}$$

**Lecture 4**

**Why Digital?**

-more compact storage using diff. media

-more accurate

-large dataset

-use real-time control system

-enable data processing

-sampling rate high, accuracy high

-analog signal: continuous, generated via analog devices, not coded, original signal

-digital signal: discrete, sampled in a fixed interval, coded value, sequential data array

**Sampling**

Shannon-Nyquist Theorem:

$$f_s > 2f_{max}$$

where,

$f_s$ is sampling rate, $f_{max}$ is max. freq. component (Nyquist freq.)

Time interval:

$$\Delta t = 1/f_s$$

-under-sampled signal, less than $2f_s$, can result in wrong digital signal (aliasing)

Aliasing eqn.:

$$f_a = |f_s * i - f_n|$$

Where i is the integer multiple of $f_s$ such that $f_s$ * i is nearest to the signal being aliased ($f_n$)

-if u want waveform, sample at rate at least 10 times the Nyquist freq.

**Quantising & Coding**

Pulse code modulation: sampling, quantisation, binary encoding

Methods:

Pulse amplitude modulation (PAM)

Ideal, natural, flat top

Quantization Size/Zone Width:

$$Q = (V_{max} - V_{min})/N$$

$$N = 2^n$$

where, n is number of bits, N is number of states.

**Quantizing & Encoding Process**

1. Determine no. of zones & zone width
2. Assign midpoints of zones a value from 0 to N-1
3. Assign binary code to each zone (find no. of bits needed)

Quantising error:

-diff. btwn. actual and midpt.

-zones ↑, Q ↓, error ↓, bits required ↑, bitrate↑

Ideal waveform = Quantized waveform + errors

**Lecture 5**

-SST: charge carriers move thru a solid semicon. Material

-VTT: bulky tubes enclosed a gas at low pressure thru which electronics flowed

-VTT drawbacks: heavy power consumers, significant heat dissipation, large size, heavy weight, require frequent battery replacement for portable units

-SST strengths: small size, portable using a rechargeable battery, low reight, cool-running, energy saving

-SST→ John Bardeen, Walter Brattain, William Shockley

-VTT → Sir Joseph John Thomson

**Why Op-Amps?**

-transducer signals too small

-transducer signals too noisy

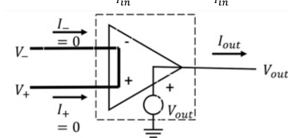
-transducer signals contain wrong information and have DC offset due to design and installation of transducers

-low cost, versatile IC, consists of internal transistors, resistors & capacitors

-combine w/ ext. discrete components → wide variety of signal processing circuits

**Impedance**

$$Z_{in} = \frac{V_{in}}{I_{in}}, \quad Z_{out} = \frac{\Delta V_{out}}{I_{in}}$$




**Ideal Op-Amp:**

-infinite input impedance, $I^+ = I^- = 0$

-infinite gain, $V^+ = V^-$

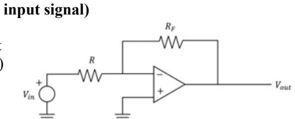
-zero output impedance, $V_{out}$ independent of $I_{out}$

-Inverting input (-), non-inverting input (+)

- gain,

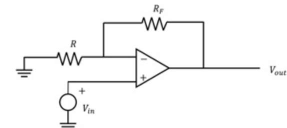
$$A_v = \frac{V_{out}}{V_{in}}$$

**Inverting Amplifier (invert & amplify input signal)**



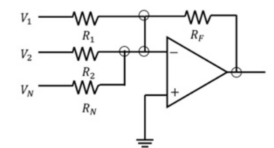
$$\frac{V_{out}}{V_{in}} = \frac{R_F}{R}$$

**Non-Inverting Amplifier (amplifies input signal without inverting)**



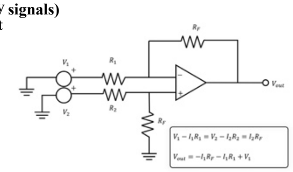
$$\frac{V_{out}}{V_{in}} = 1 + \frac{R_F}{R}$$

**Summer Amplifier (add analog signals)**



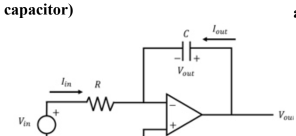
$$V_{out} = -\left(\frac{R_F}{R_1}V_1 + \frac{R_F}{R_2}V_2 + \frac{R_F}{R_N}V_N\right)$$

**Difference Amplifier (subtract analog signals)**



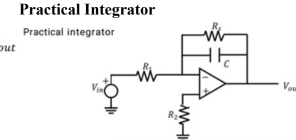
$$V_{out} = V_1 - (R_F + R_1)\left(\frac{V_2}{R_1} - \frac{V_2}{R_F + R_2}\frac{R_F}{R_1}\right)$$

**Integrator (replace feedback resistor w/ capacitor)**



$$V_{out}(t) = -\frac{1}{RC}\int_0^t V_{in}(\tau)d\tau$$

**Practical Integrator**



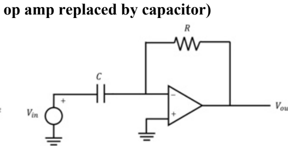

$$C\frac{dV_{out}}{dt} + \frac{V_{out}}{R_s} = I_{out} = -I_{in} = -\frac{V_{in}}{R_1}$$

$$\frac{dV_{out}}{dt} + \frac{V_{out}}{CR_s} = -\frac{V_{in}}{CR_1}$$

Should choose:

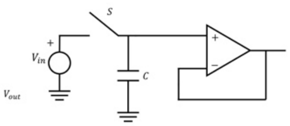
$$R_S > 10R_1, \quad R_2 = -\frac{R_1R_s}{R_1 + R_s}$$

**Differentiator (input resistor of inverting op amp replaced by capacitor)**



$$V_{out} = -RC\frac{dV_{in}}{dt}$$

**Sample & Hold**



When S is closed,

$$V_{out}(t) = V_{in}(t)$$

When S is open,

$$V_{out}(t - t_{sampled}) = V_{in}(t_{sampled})$$

-choose C w/ low leakage

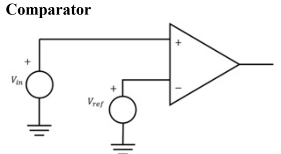
-used for ADC

-signal value must be stabilised

-voltage-holding capacitor and voltage follower

-w/ switch closed

**Comparator**



$$V_{out} = \begin{cases} +V_{sat}, & V_{in} > V_{ref} \\ -V_{sat}, & V_{in} > V_{ref} \end{cases}$$

-$V_{sat}$ is the saturation voltage

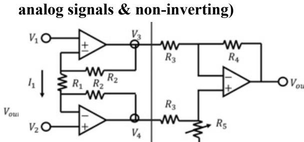
-determine whether one signal is greater than the other

-op amp w/o -ve feedback and infinite gain

-op amp saturates

-output is most +ve/-ve value

**Instrumentation Amplifier (subtracting analog signals & non-inverting)**



$$V_{out} = \frac{R_5(R_3 + R_4)}{R_3(R_3 + R_5)}V_4 - \frac{R_4}{R_3}V_3$$

-diff. amp. has too little input impedance for high output impedance

-if input signal level too low, signals include noise

-instrumentation amp solves this problem

**Lecture 6**

**Successive approx. for ADC**

-pros: high spd. and good reliability, medium accuracy compared to other ADC types, good tradeoff btwn. spd. and cost, capable of outputting binary number in serial

-cons: higher resolution → ADC slow, speed limited to 5 MSPS

**Flash ADC**

-pros: v. fast, v. simple operational theory, spd. only limited by gate + comparator propagation delay

-cons: expensive, prone to glitches in output, each additional bit of resolution requires twice the comparators

**Sigma-Delta ADC**

-pros: high res, no need for precision components

-cons: slow, due to oversampling, only good for low bandwidth

| Type | Speed (relative) | Cost (relative) |
|---|---|---|
| Dual-Slope | Slow | Med |
| Flash | Very Fast | High |
| Successive Approximation | Medium Fast | Low |
| Sigma-Delta | Slow | Low |

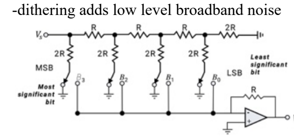
**DAC**

-finite word length

-loudest sounds need room, normal sounds don't use entire range

-problems occur at low levels where sounds are represented by 1/2 bits, high distortions

-dithering adds low level broadband noise



## Sensors

-transducer: convert one form of energy to another
-sensors: produce (electrical) output signal for sensing a physical phenomenon

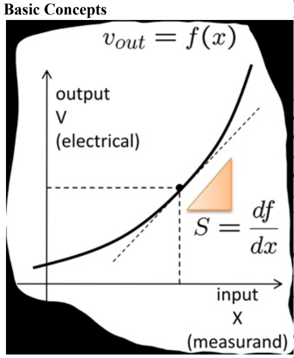
**Classification**

-analog vs digital
-passive (no ext. power, draw from input signal) vs active
-null (deflection due to measured quantity is balanced) vs deflection
-subject of measurement: mechanical, optical, thermal etc.

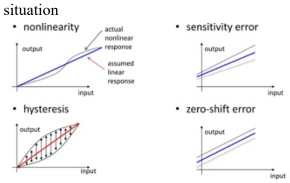
**Instrumentation Systems**

-sensing module: electro + mechanical/thermal/optical/pyro/piezo
-conversion module: analog → digital
-pre-processing: variable manipulation module
-data transmission: wired/wireless, over the web
-presentation/storage: to final user

**Basic Concepts**




-I/O characteristic function/response: input→ stimulus or measurand (temp, pressure, strain)
Output → electrical signal (voltage, current, freq, phase)
-sensitivity S: output variation/input variation, S = df/dx
-resolution: minimum change of the measurand that can be reliably detected, limited by noise, bit-conversion
-accuracy: difference of measurement from true value, %FS, full scale
-repeatability: how well a system/device can reproduce an outcome in unchanged situation

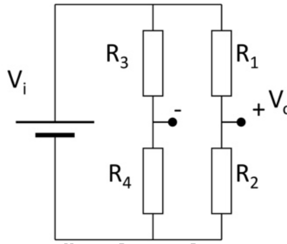



**Resistance Temperature Detector (RTD)**

-based on changes of resistance w/ temp.
-metal wire on insulating support → eliminate mechanical strain
-encasing→minimize environment influence
-linear for limited range:

$$\frac{R}{R_0} = 1 + \alpha(T - T_0)$$

**Wheatstone Bridge**




$$\frac{V_0}{V_i} = \frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}$$

Bridge balance:

$$V_0 = 0 \Leftrightarrow R_1R_4 = R_2R_3$$

**Thermistors**

-ceramic like semicons.
-resistance decreases w/ temp.
-high sensitivity, ruggedness, fast time-response

$$R = R_0 e^{\beta(\frac{1}{T} - \frac{1}{T_0})}$$

**Resistive Sensors (Potentiometer, pot)**

-3 terminal electro-mech. device based on a conductive wiper against fixed resistive element
-many varieties: rheostats, trimmers, volume control
-precision potentiometers: manually/digitally tuneable

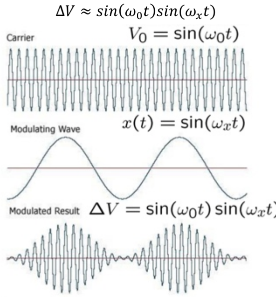
**Linear Variable Differential Transformer (LVDT)**

-type of transformer
-measures linear displacement
-variable coupling via sliding ferromagnetic core
-differential voltage:

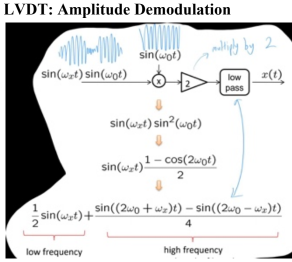
$$V_{out} = \Delta V = V_2 - V_1 \approx x(t)V_0$$

**LVDT: Amplitude Modulation**

$$\Delta V \approx sin(\omega_0 t)sin(\omega_x t)$$




**LVDT: Amplitude Demodulation**




**Capacitive Sensor (proximity sensor)**

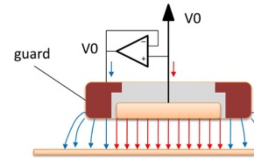
-C=Q/V
-ideal case: infinite parallel plates

Gauss' Law: $Q = \iint_\Sigma \epsilon_0\epsilon_r \boldsymbol{E} dS$

$$C = \frac{Q}{V} = \frac{\epsilon_0\epsilon_r S}{d}$$

**Guard Electrode**

-limit field-fringing effects




**AC Interfacing**

-AC bridge, AC driving, modulation

**Hall Effect**

-Lorentz Force:

$$\vec{F} = q\vec{v}\, X\, \vec{B}$$

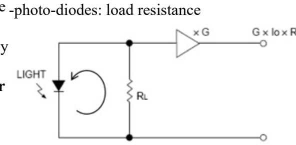
-Lorentz's Law:

$$\vec{F} = (\vec{i}\, X\, \vec{B})L$$

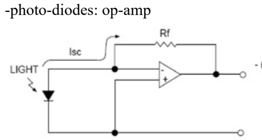
-used for contactless proximity sensor, current sensor

**Light Detectors**

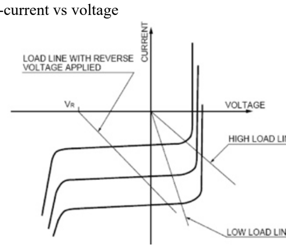
-photo-resistors
-photo-diodes: load resistance




-photo-diodes: op-amp

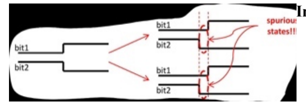



-current vs voltage




**Encoders**

-digital encoders: convert motion (linear/rotary) → digital pulses (optical transmitter/receiver pairs – glass/plastic material photographically patterned)
-Hall effect sensors + magnetic rings/bars
-incremental encoders w/ 2 Tx/Rx pairs – encoding steps + direction; simple design , needs a 'reset' position
-absolute encoders n Tx/Rx pairs for coding $2^n$ sectors; more expensive; spurious states may arise from contemporary transitions, solved using gray code as only bit changes every time




## Motors

-EM induction using LHR:

$$emf = E = -\frac{d\Phi}{dt}, \Phi \triangleq \int_\Sigma \vec{B} d\vec{\Sigma}$$

-Faraday's Law:
emf = electro-motive-force
Φ = magnetic flux
Σ = surface whose boundary coincides w/ the coil

**DCM Structure**

-stator: external, fixed
-rotor: internal, rotates
-stator and rotor fields always orthogonal → maximum torque
-w/ commutation: maintains unstable equilibria → constant motion
-w/o commutation: there is a stable equilibria → motion ceases
-more poles, more constant torque i.e., independent of rotor positions

**Equations**

Armature Eqn:

$$V = Ri + L\frac{di}{dt} + e$$

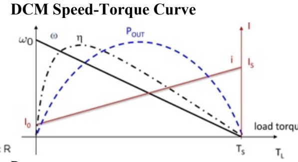
Mechanical eqn:

$$J\dot{\omega} + b\omega = T_e - T_L$$

Coupling:

$$\begin{cases} T_e = K_t i \\ e = K_e\omega \end{cases}; \quad T_e\omega = ei; \quad K_e = K_t \triangleq K_a$$

**DCM Speed-Torque Curve**




Power:

$$P_{out} = \omega T_L$$

$$P_{out}^{max} = \frac{1}{4}\omega_o T_S$$

Friction load:

$$T_L = b_L\omega_L$$

Constant torque:

$$T_L = const.$$

Inertial:

$$T_L = I\frac{d\omega}{dt} = 0$$

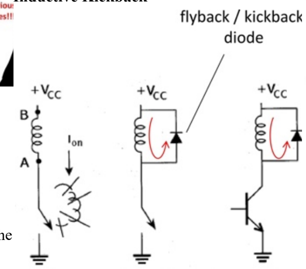
Loads can be a combination of above 3 ↑
Efficiency:

$$\eta = \frac{power\ out}{powerin} = \frac{\omega T_L}{VI}$$

**Driving DCM**

-avoid power amp. bcuz large power dissipation + over-heating of amp.
-continuously switch the motor on and off, Pulse Width Modulation (PWM)
-DCM is a 2nd order low pass filter

**Inductive Kickback**




Steady-state $I_{on}$ cannot immediately go to 0 when the switch is opened. Changing current induces a voltage across the inductor, making potential at A > B, causing the switch to blow up. Kickback/flyback diode protects the switch (physical/transistor) from blowing up.

**DC Brushless Motors**

-stator field rotating
-rotor field given by permanent magnet
-keep rotor and stator field orthogonal to maximize output torque
-detect rotor position via encoders (Hall effect sensors)
-select appropriate switches to determine desired torque

## Strain Gauges

-determine safe loading conditions of mechanical structures

**Electrical Resistance Strain Gauges**

-thin metal (constantan) foil, patterned onto plastic backing material, bonded onto mech. structures, stress derived using solid mech.
-strain, S=dL/L, +ve if tensile, -ve if compressive
-Poisson's ratio: $\upsilon = \frac{lateral\ strain}{axial\ strain}$
-resistance, $R = \rho\frac{L}{A}$
-rectangular conductor:

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \left(\frac{dw}{w} + \frac{dh}{h}\right)$$

-axial strain, S =dL/L
-lateral strain:

$$\frac{dw}{w} = \frac{dh}{h} = -\upsilon\frac{dL}{L} = -\upsilon S$$

Combining above eqns.,

$$\frac{dR}{R} = \frac{d\rho}{\rho} + S + 2\upsilon = \left(\frac{d\rho}{\rho}\frac{1}{S} + 1 + 2\upsilon\right)S$$

$$\frac{dR}{R} = \mathcal{G}S$$

FYI:

$$\text{piezoresistivity} = \frac{d\rho}{\rho}\frac{1}{S}$$

**Transverse Sensitivity**

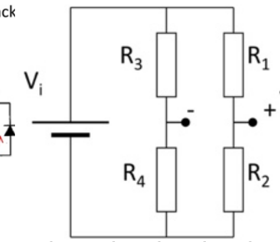
$$dR = R_0\boldsymbol{GS}$$

-the larger $R_0$ , the larger dR
-long and thin wires allow $R_0$, wires must be aligned w/ axial strain $S^a$
-serpentine wires
-end loops: aligned w/ transverse axis, made thicker to reduce sensitivity to $S^t$

**Materials**

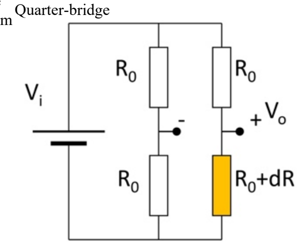
-best materials: constantan, ferrous alloys
-typical strain ranges S: 1-$10^4$ μS, G: 2
-challenge: detecting small resistance changes

Wheatstone bridge: 1st order approx.:




$$\frac{dV_o}{V_i} = \frac{1}{4}\left(\frac{dR_2}{R_2} - \frac{dR_1}{R_1} + \frac{dR_3}{R_3} - \frac{dR_4}{R_4}\right)$$

Quarter-bridge



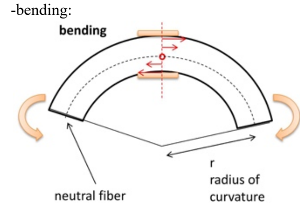

$$\frac{dV_o}{V_i} \approx \frac{1}{4}\frac{dR}{R_0} = \frac{1}{4}\mathcal{G}S$$

**Poisson Gauge**

$$\frac{dV_o}{V_i} = \frac{1}{4}\mathcal{G}(1 + \upsilon)S$$

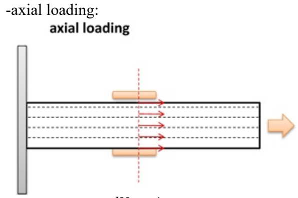
**Half Bridge**

-two active strain gauge
-bending:




$$\frac{dV_o}{V_i} = \frac{1}{2}\mathcal{G}S^b$$

-axial loading:




$$\frac{dV_o}{V_i} = \frac{1}{2}\mathcal{G}S^a$$

**Apparent Strain:**

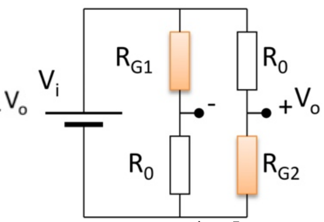
-combinations of different mechanical loading
-e.g. beam under bending and axial loading:
Top of beam:

$$S_1 = S^a + S^b$$

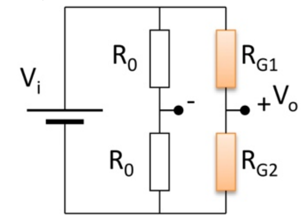
Bottom of beam:

$$S_1 = S^a - S^b$$

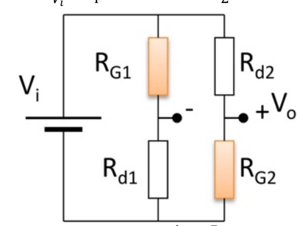
Different electrical configuration:



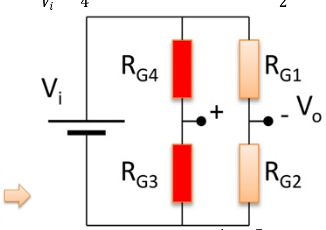

$$S_{G1} = S^a + S^b + S^T$$
$$S_{G2} = S^a - S^b + S^T$$
$$\frac{dV_o}{V_i} = \frac{1}{4}\mathcal{G}(S_{G1} + S_{G2}) = \frac{1}{2}\mathcal{G}(S^a + S^T)$$




$$S_{G1} = S^a + S^b + S^T$$
$$S_{G2} = S^a - S^b + S^T$$
$$\frac{dV_o}{V_i} = \frac{1}{4}\mathcal{G}(-S_{G1} + S_{G2}) = \frac{1}{2}\mathcal{G}S^b$$




$$S_{G1} = S^a + S^b + S^T$$
$$S_{G2} = S^a - S^b + S^T$$
$$S_{d1} = S_{d2} = S^T$$
$$\frac{dV_o}{V_i} = \frac{1}{4}\mathcal{G}(S_{G1} - S_{d1} + S_{G2} - S_{d2}) = \frac{1}{2}\mathcal{G}S^a$$




$$S_{G1} = S_{G3} = S^a + S^b + S^T$$
$$S_{G2} = S_{G4} = S^a - S^b + S^T$$
$$\frac{dV_o}{V_i} = \frac{1}{4}\mathcal{G}(S_{G1} - S_{G2} + S_{G3} - S_{G4}) = \mathcal{G}S^b$$