# MA3001 Reference for Final

Machine Element Design (Nanyang Technological University)

## 1. Force analysis basic, combined stress, stress concentration factor (SCF):

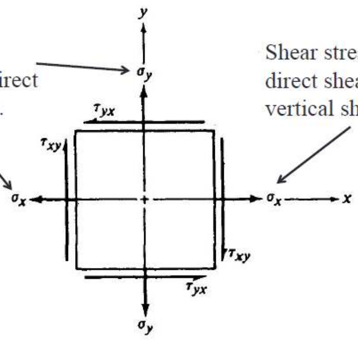
| Type of Loading | Basic Induced Stresses |
|---|---|
| Tension compression | $\sigma = \frac{F}{A}$ |
| Bearing pressure | $\sigma = \frac{F}{A}$ |
| Bending | $\sigma = \frac{Mc}{I} = \frac{M}{Z}$ |
| Torsion | $\tau = \frac{Tr}{J} = \frac{T}{Z_p}$ |
| Direct shear | $\tau = \frac{F}{A}$ |

A stress concentration factor $K$ is defined as the $\boxed{\text{maximum actual stress}}$ divided by the $\boxed{\text{nominal stress predicted}}$ from the basic equations.

$$K = \sigma_{max}/\sigma_{nom} \quad \text{or} \quad K = \tau_{max}/\tau_{nom}$$

The value of $K$ depends on the shape of the continuity, the specific geometry, and the type of stress.

Normal, $\sigma_x$ and $\sigma_y$, could be due to a direct force or to bending.




Shear stress $\tau_{xy}$ could be due to direct shear, torsional shear, or vertical shear stress.

Maximum Principal Stress: $\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Minimum Principal Stress: $\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Maximum Shear Stress: $\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

*von mises* stress:

$$\sigma' = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}$$

$$\sigma' = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}$$

## 2. Allowable stress & Factor of safety:

In order to provide a margin against failure, it is common practice to determine the **allowable stress** (also called ***design stress***) by dividing the material strength (failure) by a *factor of safety, N.*

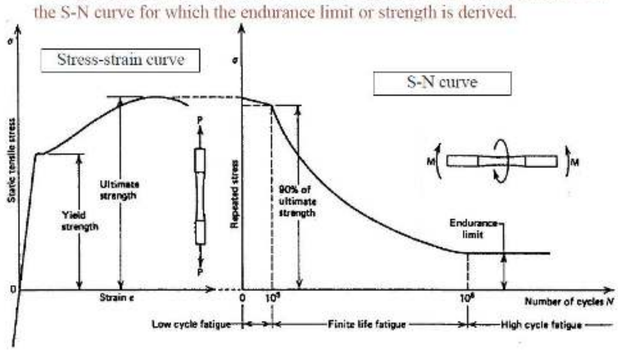
*i.e.* $$\text{allowable stress} = \frac{\textbf{material strength}}{\textbf{N}}$$

- **(a)** $n = 1.25$ to $1.5$ for exceptionally reliable materials used under controllable conditions and subjected to loads and stresses that can be determined with certainty. Used almost invariably where low weight is a particularly important consideration.
- **(b)** $n = 1.5$ to $2$ for well-known materials under reasonably constant environmental condition, subjected to loads and stresses that can be determined readily.
- **(c)** $n = 2$ to $2.5$ for average materials operated in ordinary environments and subjected to load and stresses that can be determined.
- **(d)** $n = 3$ to $4$ for untried materials and under average conditions of environment, load, and stress.
- **(e)** $n = 3$ to $4$ also for better known materials that are to be used in uncertain environments or subjected to uncertain stresses.

It is a comparison of **induced** or **operating** conditions with **allowable conditions.**

- Strength : maximum induced stress ≤ **allowable or design stress.**
- Rigidity : maximum operating deflection ≤ **allowable elastic deflection**
- Stability : maximum load ≤ **safe load**
- Wear : maximum wear ≤ **permissible wear.**

Fig. 8 portrays the transition in strength from static to cycling loading.
- The left curve is the familiar typical stress-strain curve obtained from a tensile test specimen indicating the yield strength and ultimate strength.
- The right curve is the stress versus number of cycles to failure and is known as the S-N curve for which the endurance limit or strength is derived.




## 3. Design for **strength**:

- Prediction of failure - Several different methods of predicting failure that have found high level of use are given below for ductile materials.
- The strength basis for design can be yield strength, ultimate strength, endurance strength, or some combination of these.

| Failure Prediction Method | Uses |
|---|---|
| Yield strength | Uniaxial static stress |
| Maximum shear stress | Biaxial static stress (Moderately conservative) |
| Distortion Energy | Biaxial/Triaxial stress (Good predictor) |

**Yield Strength method for uniaxial static normal stress on ductile materials**

for tensile stress: $\sigma < \sigma_{allowable} = S_y/N$

for compressive stress: $\sigma < \sigma_{allowable} = S_y/N$

**Maximum Shear Stress Method for biaxial static stress on ductile materials**

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} < \tau_{allowable} = S_{sy}/N = 0.5S_y/N$$

**Distortion Energy Method (*von Mises* stress) for biaxial static stress on ductile materials**

- This method has shown to be the best predictor of failure for ductile materials under combined stresses. It requires the *von Mises* stress σ' to compare directly with the yield strength of the material to predict failure by yielding.

$$\sigma' = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}$$

- Failure is predicted when $\sigma' > S_y$.
- Applying to design using factor of safety, $N$,

$$\sigma' < \sigma'_{allowable} = S_y/N$$

- **Under shear stresses only**, failure is predicted when the shear stress is $0.577\ S_y$, ie

$$\tau_{allowable} = S_{sy}/N = 0.577 S_y/N$$

## 4. Design for **stability**:

Fig 11 shows the modes of failure of compression members transiting from low to high slenderness ratio ($KL/r$), i.e. from short to long column.

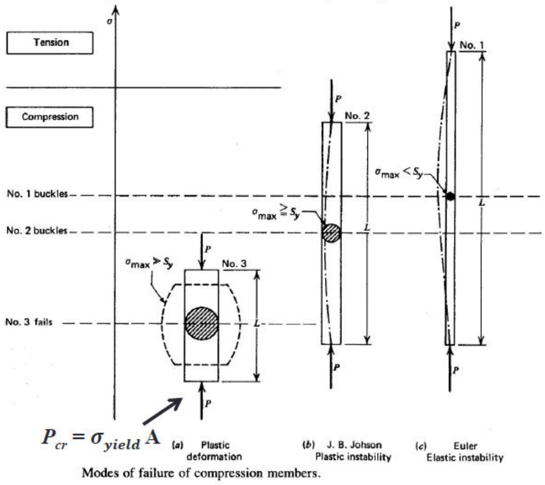



Fig. 11

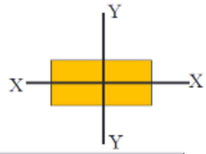
**Instability formulae for columns**

- The Euler formula for the **buckling load**, or **critical load**, of a long column was derived on the assumption that the column bows sideways while the stresses are within the elastic limit. This type of failure is the result of **elastic instability**.
- If the column is of less slender proportions, the maximum stress may reach the yield point before sideways bowing occurs, hence the Euler formula does not predict the critical load. This type of failure is the result of **plastic instability**. A formula based on test results for columns of this type is the J.B Johnson formula.

Critical load depends on slender proportion of columns

– If column is long, failure is predicted by the The Euler formula

$$P_{cr} = \frac{\pi^2 EA\,I}{(KL)^2 A} = \frac{\pi^2 EI}{(KL)^2}$$




Ixx < Iyy so is least

– If column is less slender, it is predicted by the J.B Johnson formula

$$P_{cr} = As_y\left[1 - \frac{s_y(KL/r)^2}{4\pi^2 E}\right]$$

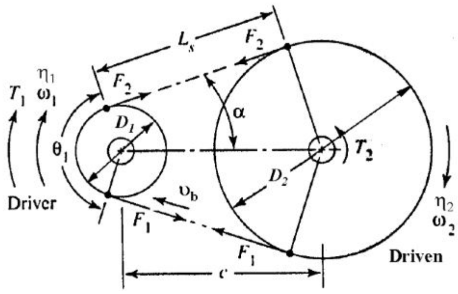
| ($a$) One fixed end, one free end | ($b$) Both ends pinned | ($c$) One fixed end, one pinned end | ($d$) Both ends fixed |
|---|---|---|---|
| $L_e = 2L$ | $L_e = L$ | $L_e = 0.7L$ | $L_e = 0.5L$ |

Applying to design with factor of safety, $N$,

Allowable load, $P_{allowable} = P_{cr}/N$

Applied load, $P < P_{allowable}$

## 5. Belt drive (V-belts) [Appendix **A**]:




- Transmitted Power,
  $P = T_1\omega_1 = T_2\omega_2$
- Torque, $T_1 = (F_1 - F_2)\ D_1/2$,
  $T_2 = (F_1 - F_2)\ D_2/2$

$$\alpha = \sin^{-1}\left(\frac{D_2 - D_1}{2C}\right)$$

Belt Pitch Length
= 2 arcs + 2 straight tangent over pitch circles

$$= \quad L = 2C + 1.57(D_2 + D_1) + \frac{(D_2 - D_1)^2}{4C} \quad \text{(m)}$$

$$\theta_1 = 180^o - 2\sin^{-1}\left[\frac{D_2 - D_1}{2C}\right]$$

- $\theta_1 > 120^\circ$ to prevent slippage due to decreased frictional contact

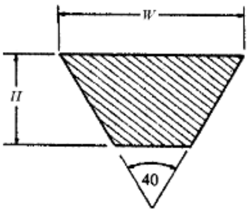
Centre distance,

$$C = \frac{B + \sqrt{B^2 - 32(D_2 - D_1)^2}}{16}$$

where $B = 4L - 6.28(D_2 + D_1)$

Table 1 Standard Cross Section

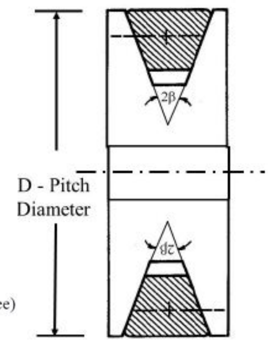
| Section | W (mm) | H (mm) | Weight (kg/m) |
|---|---|---|---|
| **SPZ** | 9.7 | 8 | 0.070 |
| **SPA** | 12.7 | 10 | 0.119 |
| **SPB** | 16.3 | 13 | 0.194 |
| **SPC** | 22 | 18 | 0.360 |




$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\theta_1 f/\sin\beta}$$

$$F_c = m v_b^2$$

where $F_c$ = centrifugal force (inertia effect of belt) (N)
$m$ = belt mass per unit length (kg/m)
$\upsilon_b$ = belt velocity (m/s)
$f$ = coefficient of friction of belt on sheave
$\theta_1$ = angle of contact on smaller sheave (radian)
$\beta$ = half the included angle of the sheave groove (degree)
$f/\sin\beta = f_e$ = effective coefficient of friction



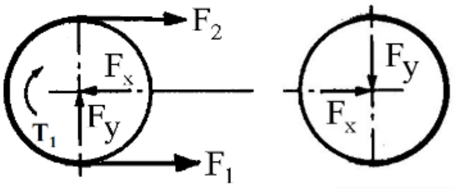

$$\left.\begin{array}{l} T_1 = (F_1 - F_2)\ D_1/2 \\ \dfrac{F_1 - F_c}{F_2 - F_c} = e^{\theta_1 f/\sin\beta} \end{array}\right\} \Rightarrow F_1 = F_c + 2\left(\frac{\gamma}{\gamma - 1}\right)\frac{T_1}{D_1}$$

where $\gamma = e^{\theta_1 f/\sin\beta}$

$$v_b = R_1\omega_1 = R_2\omega_2 = R_1\eta_1 = R_2\eta_2$$

$$\frac{\eta_1}{\eta_2} = \frac{\omega_1}{\omega_2} = \frac{D_2}{D_1}$$

For typical application, the angle $\alpha$ is small so that the tight and slack tensions $F_1$ and $F_2$ may be assumed as parallel.




Driver Sheave Forces

Shaft Forces

Parallel Force Analysis

$$\sum F_x = 0, \qquad F_x = (F_1 + F_2)$$

$$\sum F_y = 0, \qquad F_y = 0$$

Before designing a V-belt drive, you need to know these four things:

a. Power requirement of the drive
b. RPM of the driven machine
c. RPM of the driver machine
d. Approximate centre distance of the drive

## Design Procedure

**1. Determine the Design Power**

Design power = service factor x drive power (kW)

a) select service factor from Table A-2 **(refer to Appendix A)**

b) drive power - actual power requirement of the driven machine

**2. Select the proper V-belt Cross Section**

Speed of faster shaft and design power determine the proper cross section using the Cross Section Selection Chart of Table A-3

**3. Determine Speed Ratio,**

**- hence the relative size of the pulleys**

$$\frac{n_1}{n_2} = \frac{\omega_1}{\omega_2} = \frac{D_2}{D_1}$$

**4. Diameters**

**a) Select a sheave diameter to start**

- use standard diameter sheaves in Table A-4 to obtain the most economical drive
- if a minimum or maximum diameter for one of the sheaves is known, or if one sheave is already available, start with that diameter
- if nothing limits the sheave diameter, start with a sheave diameter that gives an ideal belt speed of 20 m/s ($\upsilon_b = r\omega$) or any other speed between 5m/s and 33m/s; the final belt speed may be higher or lower depending on the final selection of sheave sizes

**b) Calculate the belt speed, $\upsilon_b$ (m/s)**

**c) find the other diameter sheave using the speed ratio**

$$\frac{n_1}{n_2} = \frac{\omega_1}{\omega_2} = \frac{D_2}{D_1}$$

- remember to select a standard diameter nearest to the calculated value

**d) check if the two standard diameters, $D_1$ & $D_2$ combination gives the required speed ratio, or driven speed**

**5. Select the Centre Distance**

a) if an approximate centre distance is not known, use recommended centre distance $\mathbf{D_2 < C < 3(D_2 + D_1)}$

- why $D_2 < C < 3(D_2 + D_1)$ ?
- if $C < D_2$, then $\theta_1$ may be less than 120° - risk of belt slip
- if $C > 3(D_2 + D_1)$, span of belt will be very long - belt will whip

a) find the Tentative Belt Length, TBL

$$\text{TBL} = 2 \text{ x TCD} + 1.57(D_2 + D_1) + \frac{(D_2 - D_1)^2}{4 \text{ x TCD}} \quad \text{(m)}$$

b) select a Standard Belt Length, $L$ from, Table A-1, closest to the TBL

c) Calculate the Actual Centre Distance

$$C = \frac{B + \sqrt{B^2 - 32(D_2 - D_1)^2}}{16} \qquad \text{(m)}$$

where $B = 4L - 6.28(D_2 + D_1)$

**6. Determine the Power Correction Factor, $C_\theta$ $C_L$**

a) Find Angle of Contact Correction Factor, $C_\theta$

- calculate angle of contact on smaller sheave

$$\theta_1 = 180^o - 2\sin^{-1}\left[\frac{D_2 - D_1}{2C}\right]$$

- $\theta_1$ should not be less than 120° to avoid slippage
- find $C_\theta$ from Table A-5

b) Find Belt Length Correction Factor, $C_L$ from Table A-6 for the selected belt cross section

**7. Determine the rated power (RP), kW per belt**

- Find the power rating per belt for the small sheave diameter, RPM and speed ratio for the selected cross section from Table A-7(a-d).
- Interpolate linearly for non-listed speeds
- Note: higher ratings for speed ratio greater than 1.0 and up to 3.0

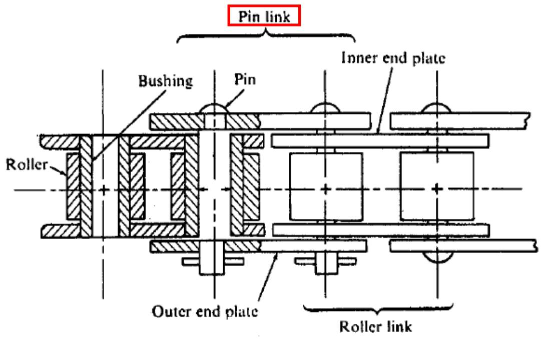
**8. Determine the corrected rated power CRP, kW per belt**

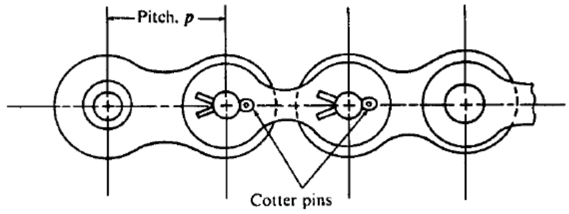
$$\text{CRP} = C_\theta\, C_L \text{ x RP}$$

- **Find the number of belts**

$$\text{No of belts} = \frac{\text{Design Power}}{\text{Corrected Rated Power}}$$

6. Chain drive (Roller chains) [Appendix **B**]:




8 links - Even number of pitches

9 links - Odd number of pitches

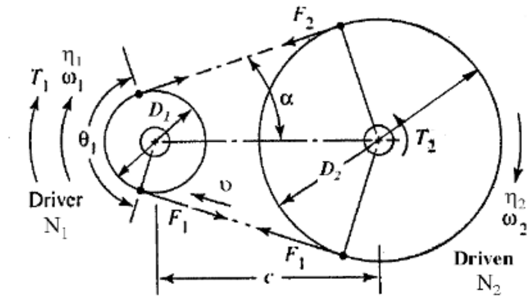
Offset Link

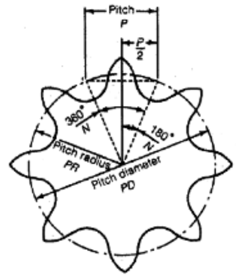
All chains are classified according to the pitch, roller diameter and width between roller link plates.

**Table 2** Multiple Strand Factors

| Number of Strands | Multiple Strand Factor |
|---|---|
| 1 | 1.0 |
| 2 | 1.7 |
| 3 | 2.5 |
| 4 | 3.3 |
| 5 | 3.9 |
| 6 | 4.6 |

Table B-3 of **Appendix B**




Chain Length

$$L = 2C + \frac{N_2 + N_1}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C} \text{ in pitches (links)}$$

where $N_1$ = number of teeth in the driver sprocket
$N_2$ = number of teeth in the driven sprocket

Pitch diameter of a sprocket,

$$D_1 = \frac{p}{\sin(180°/N_1)} \text{ mm}$$

$$D_2 = \frac{p}{\sin(180°/N_2)} \text{ mm}$$

where $p$ = pitch in mm

Centre Distance, in pitches (links)

$$C = \frac{1}{4}\left\{L - \frac{N_2 + N_1}{2} + \sqrt{\left[L - \frac{(N_2 + N_1)}{2}\right]^2 - \frac{8(N_2 - N_1)^2}{4\pi^2}}\right\}$$

Angle of contact - on small sheave,

$$\theta_1 = 180° - 2\sin^{-1}\left[\frac{D_2 - D_1}{2C}\right]$$

**Power, Torque, Chain Tensions, and Rotational Speed**

- Transmitted Power, $P = T_1\omega_1 = T_2\omega_2$
- Torque, $T_1 = (F_1 - F_2)\, D_1/2$, $T_2 = (F_1 - F_2)\, D_2/2$

where $P$ = power (W)
$\omega$ = shaft angular velocity (rad/s) [$\omega = 2\pi\eta/60$ and $\eta$ in rpm]
$T_1$, $T_2$ = Torque on sprocket 1 and 2 resp (Nm)
$D_1$, $D_2$ = pitch diameter of sprocket 1 and 2 resp (m)
$F_1$ = tension on tight side (N)
***$F_2$ = 0 – ZERO tension on slack side (N)***

- Direction of Torque (similar to belt)
  – shaft torque $T_1$ on DRIVER sprocket in same direction as $\eta_1$
  – shaft torque $T_2$ on DRIVEN sprocket in opposing direction as $\eta_2$

- Speed ratio $= \frac{\text{Input Speed}}{\text{Output Speed}} = \frac{\eta_1}{\eta_2} = \frac{N_2}{N_1}$

- Chain velocity $V = r\,\omega$ m/s

**Shaft Load** is determined in the same way as belt drives, ie.

$$\sum F_x = 0, \qquad F_x = (F_1 + F_2)\cos\alpha$$

$$\sum F_y = 0, \qquad F_y = (F_1 - F_2)\sin\alpha$$

$\Rightarrow$ $F_2 = 0$, $F_x = F_1 \cos\alpha$ $F_y = F_1 \sin\alpha$

Or if tension is assumed parallel
$F_x = F_1$ $F_y = 0$

## Design Guidelines

### Very Slow Speed Applications (*less than 100 rpm*)

Strength is the design criterion for such applications. The average tensile strengths and maximum allowable loads for the various chain sizes are also listed in the catalogue. These allowable loads can be used for very slow speed drives such as in fork lifts or for applications in which the function of the chain is to apply a tensile force such as a pipe wrench or support a load. A sample of a manufacturer's catalogue is given in Appendix B, Table B-1. If these are not listed, it is recommended that only 10% of the average tensile strength be used as the maximum allowable load in such applications.

### General Transmission Applications (*greater than 100rpm*)

- For smooth operation, it is considered good practice to use a sprocket with at least 17 teeth unless the drive is operating at a very slow speed, under 100 rpm; 19 and 21 will give a better life expectancy. This choice together with the even number of links allows each sprocket tooth to mesh with all links, one after another, instead of meshing with the same link continually. Wear will thus be more evenly distributed and total wear will be lower.
- Where space limitations are severe or for very slow speeds, smaller tooth numbers (lesser than 17) may be used by sacrificing the life expectancy of the chain.
- Speed ratio should be about 7:1. If a higher speed ratio is required, a double-reduction drive should be proposed.

### General Transmission Applications

- Optimum range for centre distance is between 30 and 50 chain pitches. Centre distances greater than about 80 pitches are not recommended.
- Angle of contact on smaller sprocket should be no smaller than 120°
- The calculated chain length should be rounded off to a whole number, preferably an even one to avoid specification of a weaker offset link
- It is recommended that no more than 4 strands be used because of the loads placed on the shaft and the corresponding reduction in the load rating of additional strands.
- It is necessary to determine whether the number of teeth and the pitch selected will result in a sprocket large enough to be mounted on the shaft with due allowance for a keyway.

## Design Procedure

**1. Determine the Design Power**

$$\text{Design Power per strand} = \frac{\text{Power to be transmitted x Service Factor}}{\text{Multiple Strand Factor}} \quad (\text{kW})$$

a) select service factor from Table B-2, Appendix B

~ determine the classification of the load according to its shock characteristics as guided by List 2 of Table B-2 and then determining the service factor from List 1 of Table B-2 which is dependent upon the characteristics of the input power

b) drive power - actual power requirement of the driven machine

c) Tentatively select the number of strands (check later if space requirements are met, and if necessary iterate until met).

**2. Select the chain size and also the number of teeth on the smaller sprocket**

**a)** Starting with the available smallest chain size in the power rating Table B-4 with the design power and determine the minimum size sprocket ie number of teeth, $N_1$ needed to provide, at the required rpm of the smaller sprocket, a rating equal to or greater than the design power

b) Use preferred odd number of teeth and minimum of 17 if space is not an issue

**3. Determine the number of teeth $N_2$ for the driven sprocket**

a) Speed Ratio, $\eta_1/\eta_2 = N_2/N_1$

b) Speed ratio should be about 7:1. If a higher speed ratio is required, a double-reduction drive should be proposed.

c) Select $N_2$ nearest to the calculated value

**4. Calculate the pitch diameters of the sprockets**

to give an idea of the size of the sprockets

$D_1 = p / \sin(180°/N_1)$, $D_2 = p / \sin(180°/N_2)$

**5. Select the Centre Distance and Chain Pitch Length**

a) if an approximate centre distance is not known, use recommended centre distance of 30 to 50 pitches and decide on a Tentative Centre Dist (TCD) in pitches

b) find the Tentative Chain Length, TCL in pitches

$$TCL = 2TCD + \frac{N_2 + N_1}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 TCD} \quad \text{pitches}$$

c) specify an even number of pitches for the chain length, L, closest to the TCL

**6. Calculate the Centre Distance**

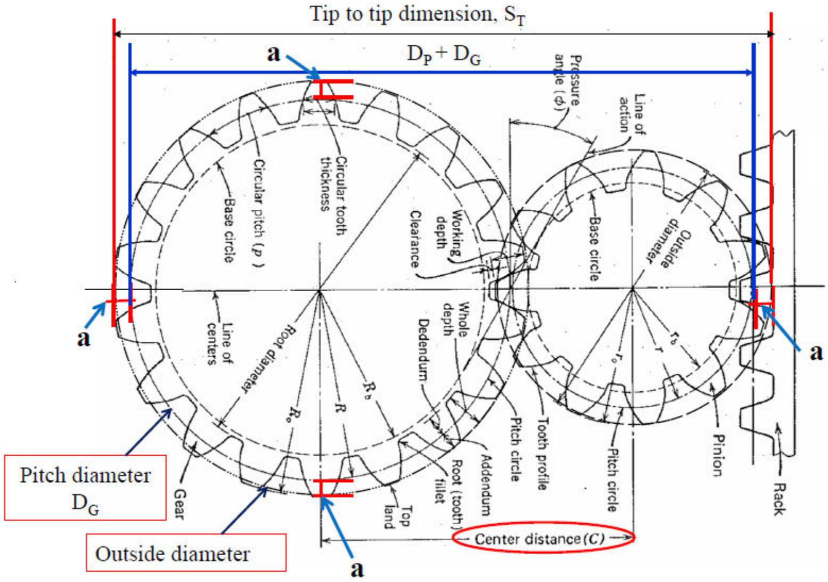
$$C = \frac{1}{4}\left\{L - \frac{N_2 + N_1}{2} + \sqrt{\left[L - \frac{(N_2 + N_1)}{2}\right]^2 - \frac{8(N_2 - N_1)^2}{4\pi^2}}\right\} \quad \text{pitches}$$

**7. Select an appropriate type of lubrication**

~ read off the type of lubrication recommended in the power rating table for the selected chain.

7. Gear drive (Spur Gear + Helical Gear; *Bevel Gear + Worm Gear non-examinable*):

## Spur Gear

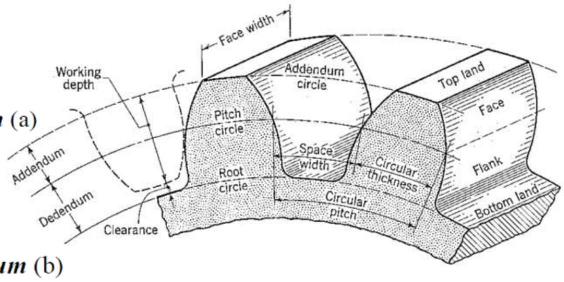



*Circular Pitch, p* – $p = \pi D/N$ mm

***Module*** - The module is the index of tooth size in metric unit. It is the ratio of the pitch diameter to the number of teeth. **For two gears to mesh, they must have the same module.**

$$m = D/N \ \ \text{mm}$$

***Addendum*** (a)




***Dedendum*** (b)

***Centre Distance (C)*** $C = (D_G + D_P)/2$

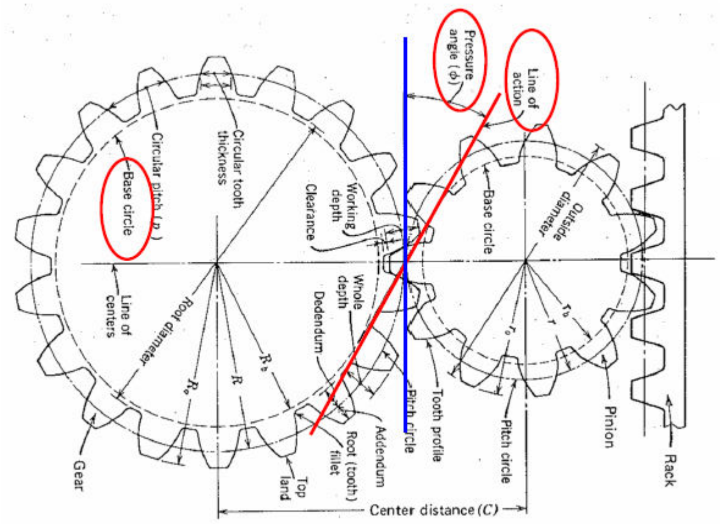
***Outside Diameter ($D_o$) of a gear*** $D_O = D + 2a$

***Tip to Tip Dimension ($S_T$)*** outermost distance of two gears in mesh

$$S_T = D_G + D_P + 2a$$

***Working Depth*** $h_k = a + a$

***Clearance (c)*** $c = b - a$



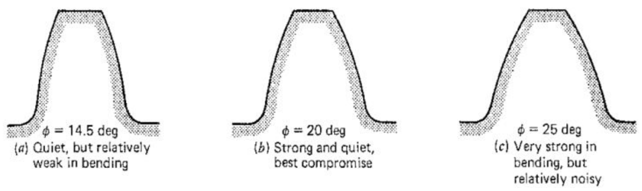




Table 1 shows only the preferred values ranging 0.3mm to 50mm.

TABLE 1 Preferred Values for Module $m$ (mm)

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.2 | 0.6 | 0.9 | 1.75 | 2.75 | 3.75 | 5 | 7 | 14 | 24 | 42 |
| 0.3 | 0.7 | 1.0 | 2.00 | 3 | 4 | 5.5 | 8 | 16 | 30 | 50 |
| 0.4 | 0.75 | 1.25 | 2.25 | 3.25 | 4.50 | 6 | 10 | 18 | 36 | |
| 0.5 | 0.80 | 1.50 | 2.50 | 3.50 | 4.75 | 6.5 | 12 | 20 | | |

## Base circle

**Pressure Angle** (ϕ)

- The circle from which an involute tooth curve is developed.
- the base circle is always tangent to the line of action.
- Diameter of base circle:

$$D_b = D \cos \phi$$

TABLE 2 Involute Gear Tooth Dimensions Based on Module $m$ (mm)

| | Stub | Full Depth | Stub | Full Depth |
|---|---|---|---|---|
| Pressure angle, ϕ deg | 20 | 20 | 25 | 25 |
| Addendum, $a$ | $0.8m$ | $m$ | $0.8m$ | $m$ |
| Dedendum, $b$ | $1.00m$ | $1.25m$ | $1.00m$ | $1.25m$ |
| Tooth thickness, $t$ (theoretical) | $0.5\pi m$ | $0.5\pi m$ | $0.5\pi m$ | $0.5\pi m$ |
| Circular pitch, $p$ | $\pi m$ | $\pi m$ | $\pi m$ | $\pi m$ |

Two important tasks associated with designing teeth that will rotate smoothly through the angle of action are to assure

- unintterupted contact of at least one tooth pair (preferable more) at all times
- avoid interference.

**Contact Ratio ($m_P$)**

It is defined as the number of pairs of teeth that are in contact at any instant.

It is necessary that continuous action take place between mating teeth and hence desirable to have more than one pair of teeth in contact at all times during operation.

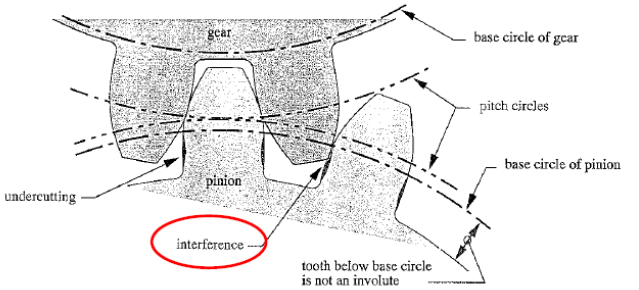
$$\text{contact ratio} \quad m_P = \frac{Z}{p_b} = \frac{\sqrt{R_o^2 - R_b^2} + \sqrt{r_o^2 - r_b^2} - C\sin\phi}{p\cos\phi}$$

Most spur gears are designed with contact ratios between 1.2 and 1.8.

Generally, the greater is the contact ratio or considerable overlap of gear actions, the smoother and quieter the operation of gears.

**Interference**

Under certain conditions, tooth profiles overlap or cut into each other.




By differentiating the equation wrt $N_p$, ie. $dm/dN_p = 0$, the minimum number of full-depth teeth on the pinion that will operate with rack without interference is given by:

$$N_P = \frac{2}{\sin^2\phi}$$

**Number of pinion teeth to ensure no interference**

**For a pinion meshing with a rack**

| Tooth form | Minimum number of teeth |
|---|---|
| $14\frac{1}{2}°$, involute, full-depth | 32 |
| 20°, involute, full-depth | 18 |
| 25°, involute, full-depth | 12 |

By differentiating the equation wrt $N_G$, ie. $dm/dN_G = 0$, the largest gear with a specified full-depth pinion that is interference-free is

$$N_G = \frac{N_P^2\sin^2\phi - 4}{4 - 2N_P\sin^2\phi}$$

**For a 20°, full-depth pinion meshing with a gear**

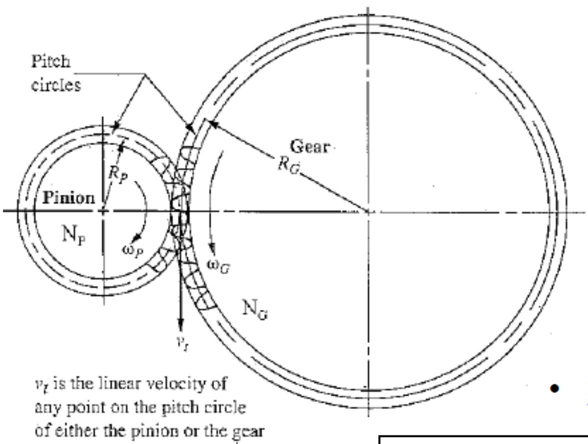
| Number of pinion teeth | Maximum number of gear teeth |
|---|---|
| 18 | Infinite (rack) |
| 17 | 1 309 |
| 16 | 101 |
| 15 | 45 |
| 14 | 26 |
| 13 | 16 |

***A pair of gears must also mesh without interference!!***

## Speed or Gear Ratios

- Speed ratio = $\dfrac{\text{Input Speed}}{\text{Output Speed}}$

$$v_t = R_P\omega_P = R_G\omega_G = R_P\eta_P = R_G\eta_G$$




$$\boxed{\frac{\text{Input Speed}}{\text{Output Speed}} = \frac{\eta_P}{\eta_G} = \frac{\omega_P}{\omega_G} = \frac{D_G}{D_P} = \frac{N_G}{N_P}}$$

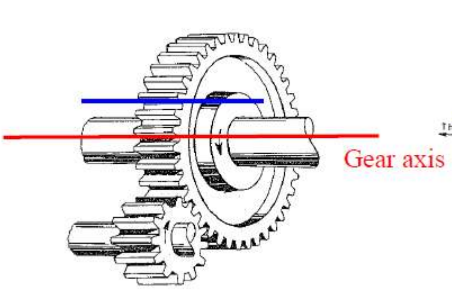
(noting that D = mN)

- sometimes also known as Gear Ratio
  - small gear drives large gear, SR > 1 ~ speed reducer
  - large gear drives small gear, SR < 1 ~ speed increaser

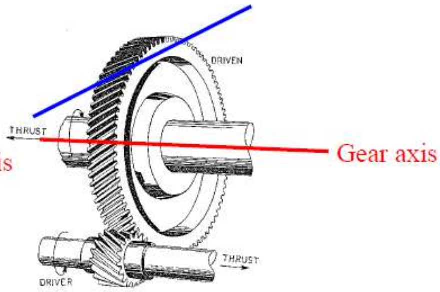
- Direction of rotation - pinion and gear rotate in ***opposite*** direction

$$\boxed{\text{Train SR} = \frac{\text{Product of number of teeth on driven gears}}{\text{Product of number of teeth on driver gears}}}$$

## Helical Gear



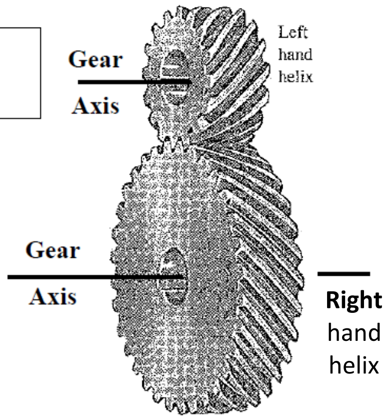

Spur Gears




Helical Gears

The distinguishing geometrical difference is that spur gear teeth are straight and aligned with the axis of rotation (gear) while helical gear teeth are angled with respect to the axis of rotation (gear) at an angle ψ, called the helix angle.

Same helix angle

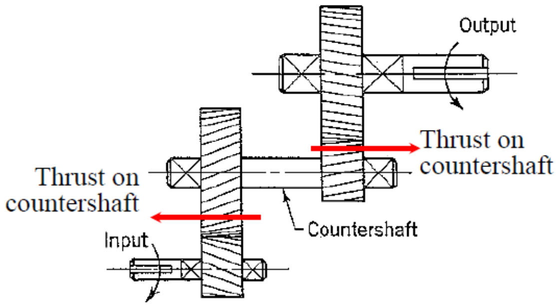



For the parallel shaft arrangement shown in Figure , the helix angle is the same on each gear, but one gear must have a right hand helix and the other a left hand helix.

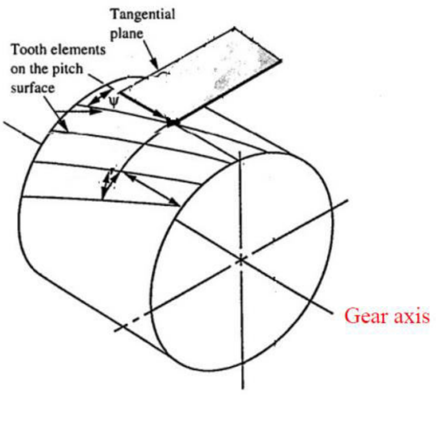
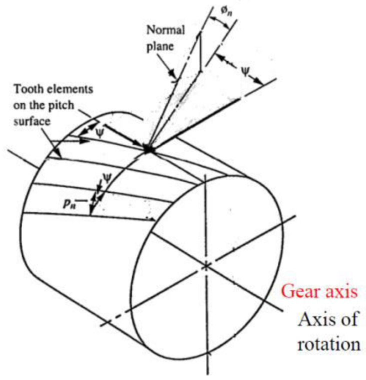
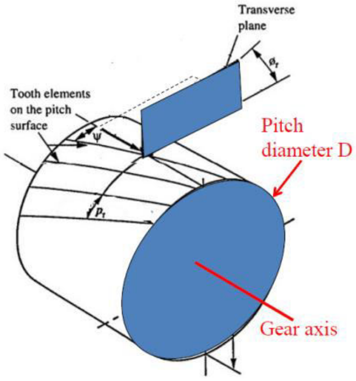
- The main advantage of helical gears is smoother engagement because a given tooth assumes its load gradually instead of suddenly.

Besides being more expensive than spur gears, the main disadvantage is that an axial thrust load is produced as a natural result of the inclined arrangement of the teeth. The bearings that hold the shaft carrying the helical gear must be capable of reacting against the radial load as well as the thrust load.

When two or more single helical gears are mounted on the same shaft, the hand of the gears should be selected so as to produce the minimum thrust load as shown in Figure for the countershaft.




## Helical Gear Geometry

| Tangential plane (tangent to pitch surface) | **Normal planes** (perpendicular to the teeth) | **Transverse plane** (normal to the axis of rotation) |
|---|---|---|
|   |   <br>• The teeth form the helix angle ψ with the axis of the gear (rotation).<br>• The usual range of values of the helix angle is between 15° and 30°.<br>• The normal pitch $\boxed{p_n}$ and the normal pressure angle $\boxed{\phi_n}$ are measured in Normal plane. |   <br>• The transverse pitch $\boxed{p_t}$ and the transverse pressure angle $\boxed{\phi_t}$ are measured in the Transverse Plane or the plane of rotation, as with spur gears.<br>• Pitch diameter $\boxed{D}$ is defined in this plane. |

- Helix angle $\psi$, normal pressure angle, $\phi_n$, transverse press angle $\phi_t$
  - need to specify helix angle $\psi$, and at least one pressure angle, typically $\phi_n$
  - the other angle computed from:

$$\tan \phi_t = \tan \phi_n / \cos \psi$$

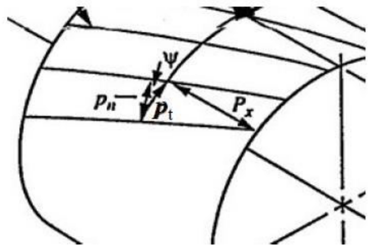
- On transverse plane (similar to Spur Gear),
  - transverse (circular) pitch, $p_t = \pi D/N$
  - transverse module, $\mathbf{m = D/N}$

- On normal plane (plane of gear cutter),
  - ~ *normal* circular pitch, $p_n = p_t \cos \psi$
  - ~ *normal* module, $\pi m_n = \pi m \cos \psi$

$$\mathbf{m_n = m \cos \psi}$$

~ typically $m_n$ is known,

$$\boxed{\therefore\ \mathbf{m = m_n / \cos \psi}}$$

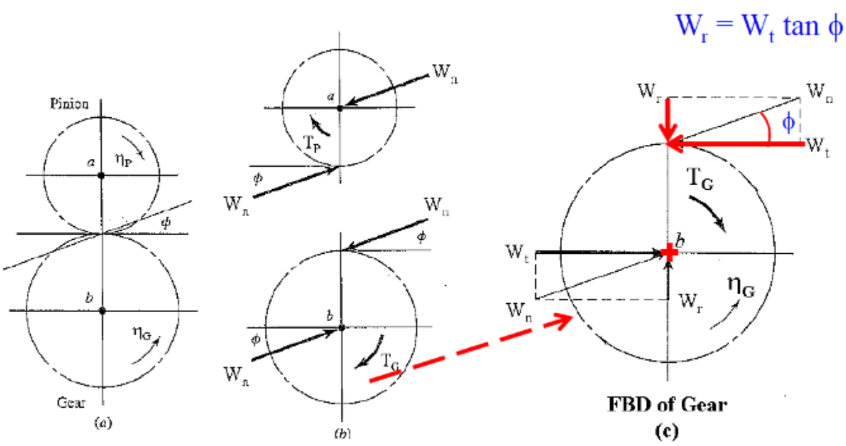


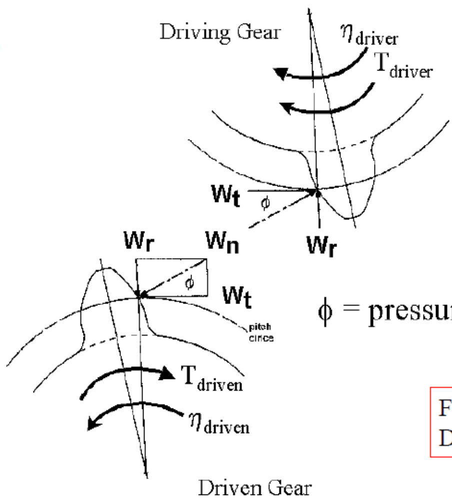

- Pitch diameter: $D = m N = N m_n / \cos \psi$
- Standard addendum, $a_n = 1.00\ m_n$
- Standard dedendum, $b_n = 1.25\ m_n$
- Outside diameter, $Do = D + 2a = (N+2)\ m_n/\cos \psi$
- Gear in mesh, tip to tip dimension$=(D_G+D_P+2a)= (N_G+N_P+2)m_n/\cos \psi$
- Centre Distance, $C = (D_G + D_P)/2 = (N_G + N_P)m_n/(2 \cos \psi)$

$$\text{Speed Ratio} = \frac{\text{Input Speed}}{\text{Output Speed}} = \frac{\eta_P}{\eta_G} = \frac{\omega_P}{\omega_G} = \frac{D_G}{D_P} = \frac{N_G}{N_P}$$

Forces (Spur Gear)

$W_r = W_t \tan \phi$




$\phi$ = pressure angle

FBDs of Driving and Driven Gears

- Calculation of gear forces usually start with $W_t$ since power and speed are known quantities

$$\boxed{\begin{aligned} T &= P/\omega \\ \omega &= 2\pi\eta/60 \\ W_t &= T/R = 2T/D \end{aligned}}$$

For no power loss, $P = T_P\ \omega_P = T_G\ \omega_G$

$$\Rightarrow T_G/T_P = \omega_P/\omega_G = N_G/N_P$$

Forces (Helical Gear)

$D = mN$ and $m = m_n/\cos\psi$

Transmitted (tangential) force,

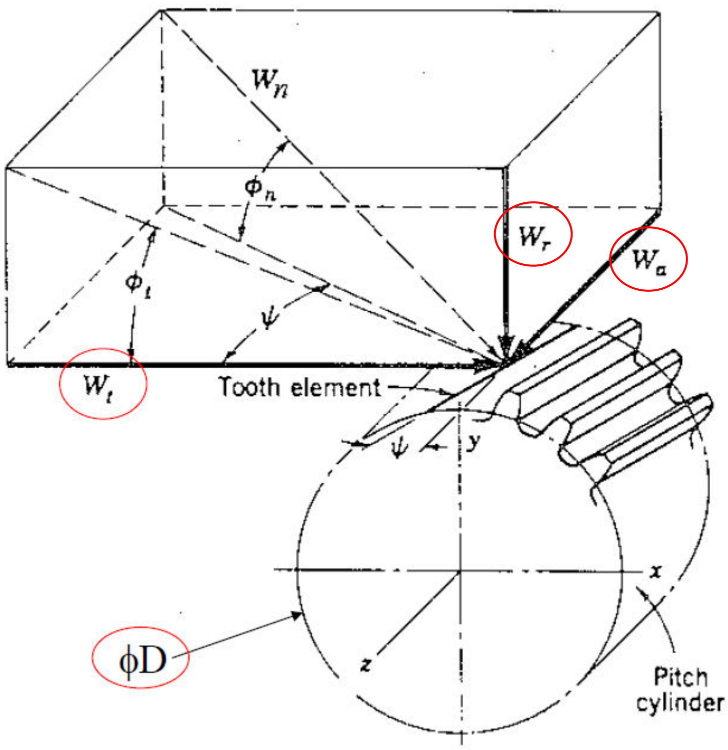
$$\boxed{W_t = T/(D/2)}$$

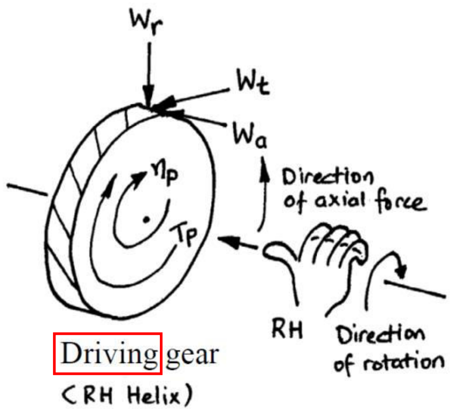
- Radial force, $\boxed{W_r = W_t \tan \phi_t}$
  - $\tan \phi_t = \tan \phi_n / \cos \psi$
  - direction of force: always towards the centre

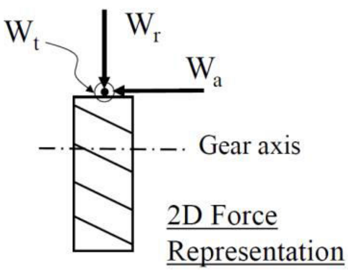
- Axial force (thrust load), $\boxed{W_a = W_t \tan \psi}$
  - direction: parallel to shaft axis but which way?

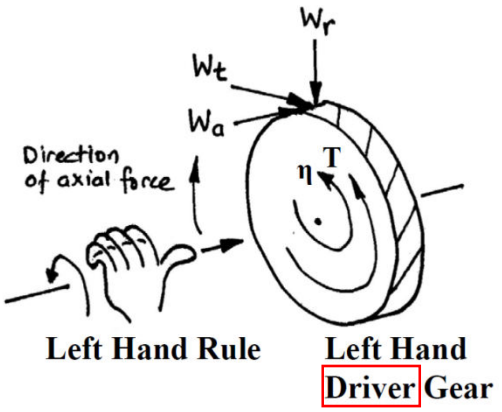
~ direction of $W_a$ - use Right Hand Thumb rule for RH driver gear, Left Hand Thumb for LH driver gear

driven gear then has a thrust load acting in the direction opposite to that of the driving gear.




8. Shaft design (for strength) [**Practical** ASME shaft design equation]:

$$D = \left[ \frac{32\boxed{N}}{\pi} \sqrt{\left(\frac{\boxed{K_{fb}}M}{\boxed{s_n'}}\right)^2 + \frac{3}{4}\left(\frac{T}{\boxed{s_y}}\right)^2} \right]^{1/3}$$

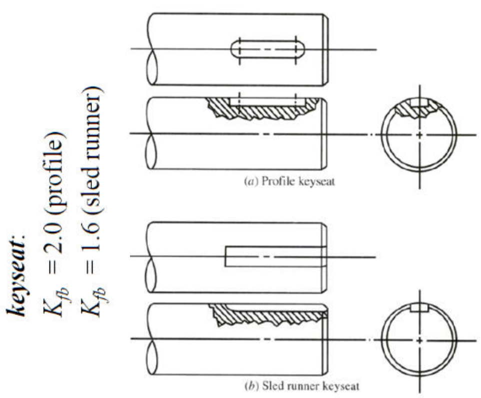
***Practical* shaft design equation: considering stress concentration, keyway, shaft's treatment, etc.**

(a) $K_{fb}$:

***shoulder fillets***: $K_{fb} = 2.5$ (sharp), $K_{fb} = 1.5$ (well-rounded)

***retaining ring grooves***: $K_{fb} = 3.0$

***keyseat***: $K_{fb} = 2.0$ (profile), $K_{fb} = 1.6$ (sled runner)



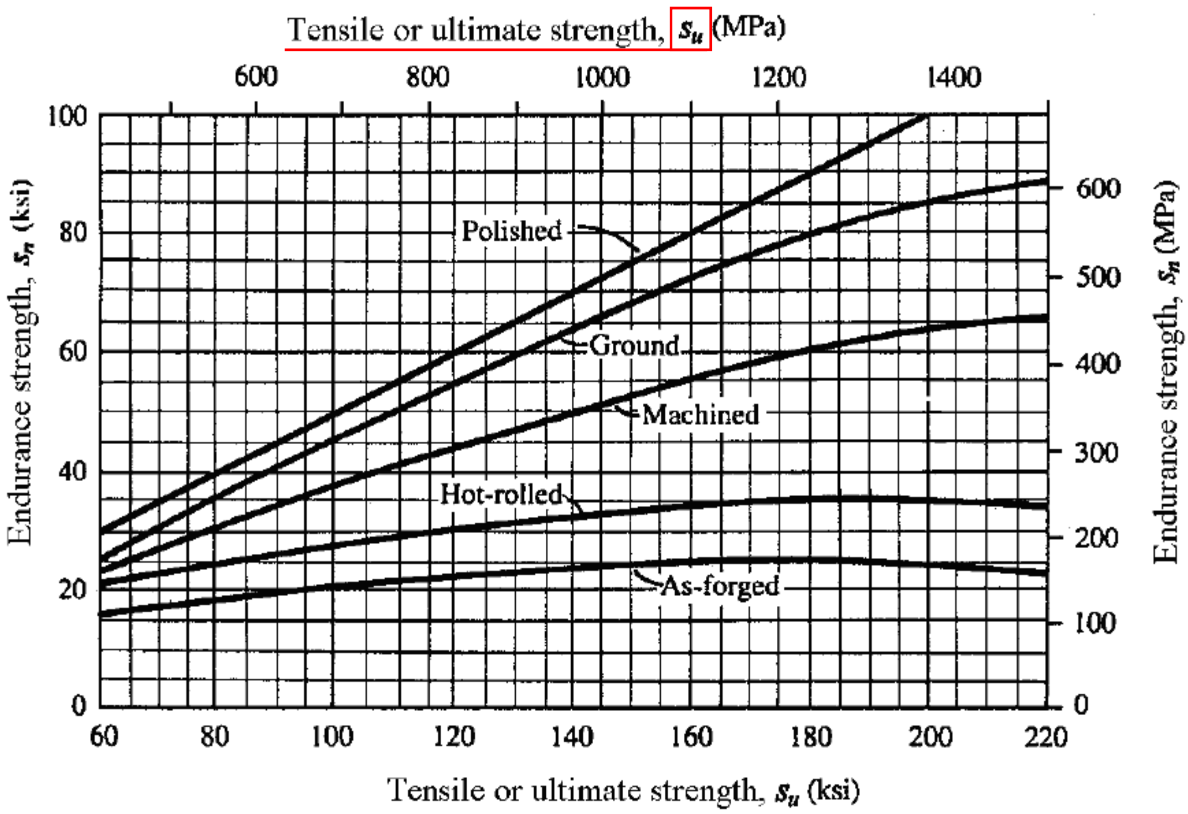

**Note (Retaining ring *grooves; pg. 542*)**: (i) $T = 0$ & $M \neq 0$, $K_{fb} = 3$; (ii) $T \neq 0$ & $M \neq 0$, $K_{fb}$=3 ($D$+$\boxed{6\%}$); (iii) $T \neq 0$ & $M$=0, $K_{fb}$ = ? ($D$+$\boxed{6\%}$)

(b) $s_y$:

| Material designation (AISI number) | Treatment condition | Tensile strength, $s_u$ | | Yield strength, $s_y$ | | Ductility (percent elongation in 5 cm or 2 in) | Brinell hardness (HB) |
|---|---|---|---|---|---|---|---|
| | | MPa | kpsi | MPa | kpsi | | |
| 1020 | Hot-rolled | 379 | 55 | 207 | 30 | 25 | 111 |
| 1020 | Cold-drawn | 420 | 61 | 352 | 51 | 15 | 122 |
| 1020 | Annealed | 414 | 60 | 296 | 43 | 38 | 121 |
| 1040 | Hot-rolled | 496 | 72 | 290 | 42 | 18 | 144 |
| 1040 | Cold-drawn | 552 | 80 | 490 | 71 | 12 | 160 |
| 1040 | OQT1300* | 607 | 88 | 421 | 61 | 33 | 183 |
| 1040 | OQT400 | 779 | 113 | 600 | 87 | 19 | 262 |
| 1050 | Hot-rolled | 620 | 90 | 338 | 49 | 15 | 180 |
| 1050 | Cold-drawn | 690 | 100 | 579 | 84 | 10 | 200 |
| 1050 | OQT1300 | 662 | 96 | 421 | 61 | 30 | 192 |
| 1050 | OQT400 | 986 | 143 | 758 | 110 | 10 | 321 |
| 1137 | Hot-rolled | 607 | 88 | 331 | 48 | 15 | 176 |
| 1137 | Cold-drawn | 676 | 98 | 565 | 82 | 10 | 196 |
| 1137 | OQT1300 | 600 | 87 | 414 | 60 | 28 | 174 |
| 1137 | OQT400 | 1083 | 157 | 938 | 136 | 5 | 352 |

(c) $s'_n = C_s \cdot C_R \cdot s_n$

(i) $s_n$:




(ii) $C_s$:

For Diameters less than 7.62 mm: $C_S = 1.0$

For Diameters over 7.62 to 50 mm: $C_S = (D/7.62)^{-0.11}$

For Diameters over 50 to 250 mm: $C_S = 0.859 - 0.000837D$

$D$ is unknown $\longrightarrow$ assume $C_s$ first; iterate later.

(iii) $C_R$:

| Desired reliability | $C_R$ |
|---|---|
| 0.50 | 1.00 |
| 0.90 | 0.90 |
| 0.99 | 0.81 |
| 0.999 | 0.75 |

(d) $N$:

Under *typical* industrial conditions, the design factor of $N = 3$ is recommended.

If the application is very *smooth*, a low value $N = 2$ may be justified.

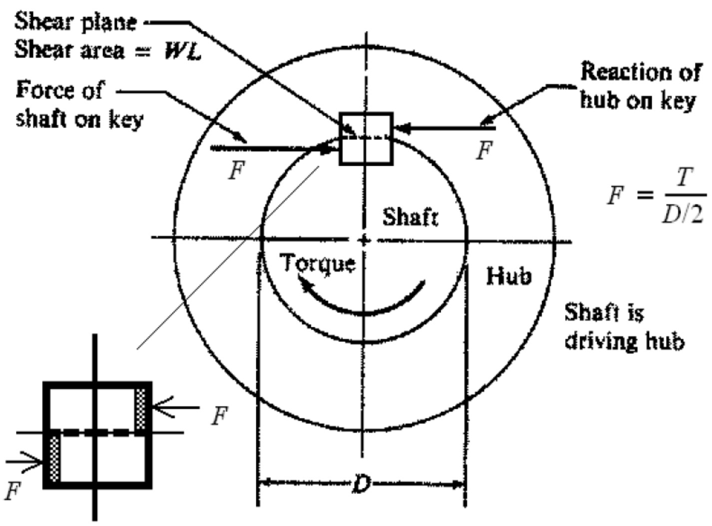
Under conditions of *critical* shock or *impact*, $N = 4$ or higher should be used.

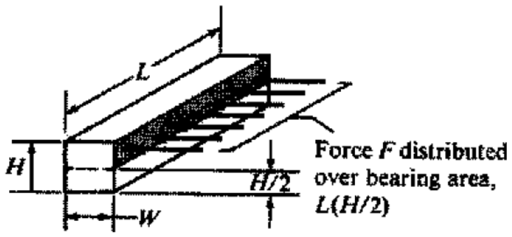
$N$, $C_R$ (application); $s_n'$, $s_y$ (material); $M$, $T$, $V$ (loading); $K_{fb}$, $C_s$ (layout)

*Special consideration: $$D = \sqrt{\left(\frac{2.94 K_f V N}{s_n'}\right)}$$

At a point on the shaft, when both bending moment $M$ and torque $T$ are equal to 0 (usually at the ends of shaft), while shear forces $V$ are not equal to 0, this equation is used to compute the minimum required diameter at that point.

9. Key design (rectangular key) [key has *lower* strength than other components in system]:




• Two failure modes for keys: ***shear*** across the shaft/hub interface; ***compression*** failure due to bearing action between the sides of the key and the shaft/hub

Max. shear stress theory

$$L_s = \frac{4TN}{DWs_y}$$

design stress for compression

$$L_c = \frac{4TN}{DHs_y}$$

$L_s = L_c$ for a square key ($W = H$)

Dimension of Rectangular Keys with respect to the shaft diameter

| ISO Standard (mm) | | | |
|---|---|---|---|
| Shaft diameter | | Groove width | Key height |
| > | ≤ | $W$ | $H$ |
| 6 | 8 | 2 | 2 |
| 8 | 10 | 3 | 3 |
| 10 | 12 | 4 | 4 |
| 12 | 17 | 5 | 5 |
| 17 | 22 | 6 | 6 |
| 22 | 30 | 8 | 7 |
| 30 | 38 | 10 | 8 |
| 38 | 44 | 12 | 8 |
| 44 | 50 | 14 | 9 |
| 50 | 58 | 16 | 10 |
| 58 | 65 | 18 | 11 |
| 65 | 75 | 20 | 12 |
| .. | .. | .. | .. |

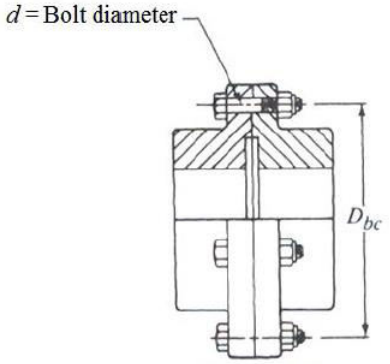
*Preferable* basic sizes for machine parts and components

| Nominal size (mm) | | | | | |
|---|---|---|---|---|---|
| **First** choice | **Second** choice | **First** choice | **Second** choice | **First** choice | **Second** choice |
| 1 | | | 13 | | 55 |
| | 1.1 | 14 | | 60 | |
| 1.2 | | | 15 | | 65 |
| | 1.4 | 16 | | | 70 |
| 1.6 | | | 17 | | 75 |
| | 1.8 | 18 | | 80 | |
| 2 | | | 19 | | 90 |
| | 2.2 | 20 | | 100 | |
| 2.5 | | | 21 | | 110 |
| | 2.8 | 22 | | 120 | |
| 3 | | | 23 | | 130 |
| …. | …. | …. | …. | …. | …. |

10. Couplings:

**Rigid Couplings** → NO relative motion between the shafts
Precise alignment of the shafts
Bolts in carry torque in shear

**Couplings** are used to connect two shafts (shaft of a driving machine and shaft of driven machine) together for the purpose of power transmission.



Cutaway view

$$\tau_d = \tau = \frac{F}{A_s} = \frac{F}{N(\pi d^2/4)} = \frac{2T}{D_{bc}N(\pi d^2/4)} = \frac{8T}{\pi D_{bc} N d^2}$$

$$\boxed{d = \sqrt{\frac{8T}{\pi D_{bc} N(\tau_d)}}}$$

$N$ = number of bolts; $\tau$ = allowable shear stress in bolts

11. Rolling contact bearings (single row, deep-groove ball bearing):

| Bearing number | Bore diameter $d$, mm | Outside diameter $D$, mm | Width $w$, mm | Fillet radius[1] $r$, mm | Basic dynamic load ratings[2] $C$, kN | Basic static load ratings $C_0$, kN | Fatigue load limit[3] $P_f$, kN | Mass, kg | Limit speed with oil[4], $10^3$ rpm |
|---|---|---|---|---|---|---|---|---|---|
| 6000 | 10 | 26 | 8 | 0.3 | 4.75 | 1.96 | 0.083 | 0.019 | 40 |
| 6200 | | 30 | 9 | 0.6 | 5.4 | 2.36 | 0.10 | 0.032 | 34 |
| 6300 | | 35 | 11 | 0.6 | 8.52 | 3.40 | 0.143 | 0.053 | 32 |
| 6002 | 15 | 32 | 9 | 0.3 | 5.85 | 2.85 | 0.12 | 0.03 | 32 |
| 6202 | | 35 | 11 | 0.6 | 8.06 | 3.75 | 0.16 | 0.045 | 28 |
| 6302 | | 42 | 13 | 1 | 11.9 | 5.40 | 0.228 | 0.082 | 24 |
| 6004 | 20 | 42 | 12 | 0.6 | 9.95 | 5 | 0.212 | 0.069 | 24 |
| 6204 | | 47 | 14 | 1 | 13.5 | 6.55 | 0.28 | 0.11 | 20 |
| 6304 | | 52 | 15 | 1 | 16.8 | 7.8 | 0.335 | 0.14 | 19 |
| 6005 | 25 | 47 | 12 | 0.6 | 11.9 | 6.55 | 0.275 | 0.08 | 20 |
| 6205 | | 52 | 15 | 1 | 14.8 | 7.8 | 0.335 | 0.13 | 18 |
| 6305 | | 62 | 17 | 1 | 23.4 | 11.6 | 0.49 | 0.23 | 16 |
| 6006 | 30 | 55 | 13 | 1 | 13.8 | 8.3 | 0.335 | 0.12 | 17 |
| 6206 | | 62 | 16 | 1 | 20.3 | 11.2 | 0.475 | 0.2 | 15 |
| 6306 | | 72 | 19 | 1 | 29.6 | 16 | 0.67 | 0.35 | 13 |
| 6007 | 35 | 62 | 14 | 1 | 16.8 | 10.2 | 0.44 | 0.16 | 15 |
| 6207 | | 72 | 17 | 1 | 27 | 15.3 | 0.655 | 0.29 | 13 |
| 6307 | | 80 | 21 | 1.5 | 35.1 | 19 | 0.815 | 0.46 | 12 |
| 6008 | 40 | 68 | 15 | 1 | 17.8 | 11.6 | 0.49 | 0.19 | 14 |
| 6208 | | 80 | 18 | 1 | 32.5 | 19 | 0.8 | 0.37 | 11 |
| 6308 | | 90 | 23 | 1.5 | 42.3 | 24 | 1.02 | 0.63 | 11 |

**Table 14A**

*First part

$(L_{10} = 10^6\ rev)$

load data based on 1 mil revolutions and 90% realibility

$$L_d = L_2 = L_1\left(\frac{P_1}{P_2}\right)^k$$ $k = 3$ (ball) or $10/3$ (roller)

$L_d$ (or $L_2$): design *life* ($L_{10}$) at a load of $P_2$

$P_2$: design *load* ($P_d$)

$L_1$: $L_{10}$ *life* at load $P_1$ (by default, **1 million revolutions** & 90% reliability)

$P_1$: or $C$, basic dynamic *load* rating (Catalog data)

$$\boxed{L_{10}(\text{rev}) = L_{10,h}(\text{h})n(\text{rpm})60(\text{min/h})}$$

$$\boxed{L_d = 10^6\left(\frac{C}{P_d}\right)^k \quad \text{rev}} \longrightarrow \boxed{C = P_d\left(\frac{L_d}{10^6}\right)^{1/k}}$$

(14-3)

$$\boxed{C_{\text{catalog}} > C_{\text{calculated\_Eq.}}}$$

Combined radial/thrust loads

$$P = XVF_r + YF_a$$

$$P = VF_r \quad (\text{if } F_a = 0)$$

where $P = P_d$ = *equivalent radial* or dynamic load
$F_r$ = applied radial load
$F_a$ = applied axial load (thrust)
$V$ = rotation factor
(**1.0 for inner-ring rotation**; 1.2 for outer-ring rotation)
$X$ = radial factor
$Y$ = thrust factor

**Table 14-5**

Single-row, deep-grove ball bearings

*Note*: $C_0$ of the bearing selected is found from the Catalog

$e$ is a number used for the comparison for $F_a/F_r$

| $F_a/C_0$ (Basic static load rating) | $e$ | $F_a/VF_r \le e$ | | $F_a/VF_r > e$ | |
|---|---|---|---|---|---|
| | | $X$ | $Y$ | $X$ | $Y$ |
| 0.014* | 0.19 | 1.0 | 0 | 0.56 | 2.30 |
| 0.021 | 0.21 | | | | 2.15 |
| 0.028 | 0.22 | | | | 1.99 |
| 0.042 | 0.24 | | | | 1.85 |
| 0.056 | 0.26 | | | | 1.71 |
| 0.070 | 0.27 | 1.0 | 0 | 0.56 | 1.63 |
| 0.084 | 0.28 | | | | 1.55 |
| 0.110 | 0.30 | | | | 1.45 |
| 0.17 | 0.34 | | | | 1.31 |
| 0.28 | 0.38 | | | | 1.15 |
| 0.42 | 0.42 | | | | 1.04 |
| 0.56 | 0.44 | | | | 1.00 |

*Use 0.014 if $F_a/C_0 < 0.014$.

Design procedures

1. **Assume** $Y$ (thrust factor); <first step only; apply table later>
2. compute $P = VXF_r + YF_a$ $(= P_d) \Leftarrow$ Eq. (14-6)
3. compute the required dynamic load, $C$ (Eq. (14-3))
4. select a bearing having a greater $C$ (Table 14A; $C_{\text{catalog}} > C_{\text{calculated}}$)
5. for the selected bearing, given $C_0$; compute $F_a/C_0$
6. determine $e$ (to compare with $F_a/F_r$) from Table 14-5 (after computing $F_a/C_0$)
7. if $F_a/F_r > e$, then determine $Y$ (*updated*) from Table 14-5.
   (if $F_a/F_r < e$ or $F_a$=0, use Eq. (14-5) for a pure radial load) $\boxed{P = VF_r}$
8. *Iteration* is performed until the following **two conditions** are met:
(i) With the *updated* $Y$ (found from Table 14-5 associate to the selected bearing) <different from initial $Y$ assumed in Step 1>, *repeated steps from* Step 2 with updated $Y$, if new $C_{\text{calculated\_updated\_}Y} > C_{\text{bearing\_assumed\_}Y}$;
(ii) $C_{\text{catalog\_with\_selected\_bearing}} > C_{\text{calculated\_updated\_}Y}$!
(if $C_{\text{catalog\_with\_selected\_bearing}} < C_{\text{calculated updated\_}Y}$, select a bigger bearing and repeat Step 4)

Table 14A (cont.)

| Bearing number | Bore diameter $d$, mm | Outside diameter $D$, mm | Width $w$, mm | Fillet radius[1] $r$, mm | Basic dynamic load ratings[2] $C$, kN | Basic static load ratings $C_0$, kN | Fatigue load limit[3] $P_f$, kN | Limit speed with oil[4], $10^3$ rpm | Mass, kg |
|---|---|---|---|---|---|---|---|---|---|
| 6009 | 45 | 75 | 16 | 1 | 22.1 | 14.6 | 0.64 | 12 | 0.25 |
| 6209 | | 85 | 19 | 1 | 35.1 | 21.6 | 0.915 | 11 | 0.41 |
| 6309 | | 100 | 25 | 1.5 | 55.3 | 31.5 | 1.34 | 9.5 | 0.83 |
| 6010 | 50 | 80 | 16 | 1 | 22.9 | 16 | 0.71 | 11 | 0.26 |
| 6210 | | 90 | 20 | 1 | 37.1 | 23.2 | 0.98 | 10 | 0.46 |
| 6310 | | 110 | 27 | 2 | 65 | 38 | 1.6 | 8.5 | 1.05 |
| 6011 | 55 | 90 | 18 | 1 | 29.6 | 21.2 | 0.9 | 10 | 0.39 |
| 6211 | | 100 | 21 | 1.5 | 46.2 | 29 | 1.25 | 9 | 0.61 |
| 6311 | | 120 | 29 | 2 | 74.1 | 45 | 1.9 | 8 | 1.35 |
| 6012 | 60 | 95 | 18 | 1 | 30.7 | 23.2 | 0.98 | 9.5 | 0.42 |
| 6212 | | 110 | 22 | 1.5 | 55.3 | 36 | 1.53 | 8 | 0.78 |
| 6312 | | 130 | 31 | 2 | 85.2 | 52 | 2.2 | 7 | 1.7 |
| 6013 | 65 | 100 | 18 | 1 | 31.9 | 25 | 1.06 | 9 | 0.44 |
| 6213 | | 120 | 23 | 1.5 | 58.5 | 40.5 | 1.73 | 7.5 | 0.99 |
| 6313 | | 140 | 33 | 2 | 97.5 | 60 | 2.5 | 6.7 | 2.1 |
| 6014 | 70 | 110 | 20 | 1 | 39.7 | 31 | 1.32 | 8 | 0.6 |
| 6214 | | 125 | 24 | 1.5 | 63.7 | 45 | 1.9 | 7 | 1.05 |
| 6314 | | 150 | 35 | 2 | 111 | 68 | 2.75 | 6.3 | 2.5 |
| 6015 | 75 | 115 | 20 | 1 | 41.6 | 33.5 | 1.43 | 7.5 | 0.64 |
| 6215 | | 130 | 25 | 1.5 | 68.9 | 49 | 2.04 | 6.7 | 1.2 |
| 6315 | | 160 | 37 | 2 | 119 | 76.5 | 3 | 5.6 | 3 |
| 6016 | 80 | 125 | 22 | 1 | 49.4 | 40 | 1.66 | 7 | 0.85 |
| 6216 | | 140 | 26 | 2 | 72.8 | 55 | 2.2 | 6 | 1.4 |
| 6316 | | 170 | 39 | 2 | 130 | 86.5 | 3.25 | 5.3 | 3.6 |
| 6017 | 85 | 130 | 22 | 1 | 52 | 43 | 1.76 | 6.7 | 0.89 |
| 6217 | | 150 | 28 | 2 | 87.1 | 64 | 2.5 | 5.6 | 1.8 |
| 6317 | | 180 | 41 | 2.5 | 140 | 96.5 | 3.55 | 5 | 4.25 |
| 6018 | 90 | 140 | 24 | 1.5 | 60.5 | 50 | 1.96 | 6.3 | 1.15 |
| 6218 | | 160 | 30 | 2 | 101 | 73.5 | 2.8 | 5.3 | 2.15 |
| 6318 | | 190 | 43 | 2.5 | 151 | 108 | 3.8 | 4.8 | 4.9 |
| 6019 | 95 | 145 | 24 | 1.5 | 63.7 | 54 | 2.08 | 6 | 1.2 |
| 6219 | | 170 | 32 | 2 | 114 | 81.5 | 3 | 5 | 2.6 |
| 6319 | | 200 | 45 | 2.5 | 159 | 118 | 4.15 | 4.5 | 5.65 |
| 6020 | 100 | 150 | 24 | 1.5 | 63.7 | 54 | 2.04 | 5.6 | 1.25 |
| 6220 | | 180 | 34 | 2 | 127 | 93 | 3.35 | 4.8 | 3.15 |
| 6320 | | 215 | 47 | 2.5 | 174 | 140 | 4.75 | 4.3 | 7 |

12. Fasteners, screws and bolts:

Fastener: Any device used to connect or join two or more components.

Screw: *threaded fastener* designed to be inserted through a hole in one member to be joined and into a **threaded** hole in the mating member.

Bolt: *threaded fastener* designed to pass through holes in the mating members and to be secured by **tightening** a nut from another end.

$d$ major diameter
$d_p$ pitch diameter
$d_r$ minor diameter

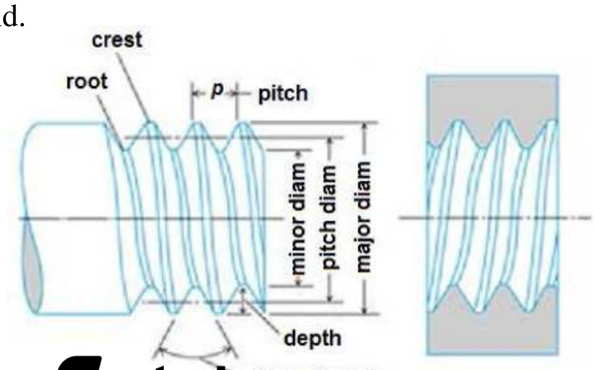


Table 18-5

In design calculation,

$d_r$ is always used.

$$\boxed{d = \frac{d_r}{0.8}}$$

| Major or crest diameter, $d$ (mm) | Coarse Threads (MC) | | Fine Threads (MF) | |
|---|---|---|---|---|
| | Pitch, $p$ (mm) | Tensile stress area, $A_t$ (mm²) | Pitch, $p$, (mm) | Tensile stress area, $A_t$ (mm²) |
| 1 | 0.25 | 0.460 | - | - |
| 1.6 | 0.35 | 1.27 | 0.20 | 1.57 |
| 2 | 0.4 | 2.07 | .25 | 2.45 |
| 2.5 | 0.45 | 3.39 | .35 | 3.70 |
| 3 | 0.5 | 5.03 | .35 | 5.61 |
| 4 | 0.7 | 8.78 | .5 | 9.79 |
| 5 | 0.8 | 14.2 | .5 | 16.1 |
| 6 | 1 | 20.1 | .75 | 22 |
| 8 | 1.25 | 36.6 | 1 | 39.2 |
| 10 | 1.5 | 58.0 | 1.25 | 61.2 |
| 12 | 1.75 | 84.3 | 1.25 | 92.1 |
| 14 | 2 | 115 | 1.5 | 125 |
| 16 | 2 | 157 | 1.5 | 167 |
| 18 | 2.5 | 192 | 1.5 | 216 |
| 20 | 2.5 | 245 | 1.5 | 272 |
| 24 | 3 | 353 | 2 | 384 |
| 30 | 3.5 | 561 | 2 | 621 |
| 36 | 4 | 817 | 2 | 915 |
| 42 | 4.5 | 1121 | 2 | 1260 |
| 48 | 5 | 1473 | 2 | 1670 |
| 56 | 5.5 | 2030 | 2 | 2300 |
| 64 | 6 | 2680 | 2 | 3030 |
| 72 | 6 | 3505 | 2 | 3878 |
| 80 | 6 | 4395 | 2 | 4811 |
| 90 | 6 | 5648 | 2 | 6119 |
| 100 | 6 | 7059 | 2 | 7584 |

Metric thread specification is given in Table 18-5. An example of metric thread specification would be:

M10 × 1.5

which defines a 10-mm diameter threads with pitch of 1.5 mm.

The design of bolt consists of determination of appropriate size of the bolt, which is given by basic **major** (or nominal) diameter $d$ and pitch $p$.

Allowable stresses for bolts

Table 20-1 Allowable stresses for bolts

| ASTM grade | Allowable shear stress | Allowable tensile stress |
|---|---|---|
| A307 | 69 MPa | 138 MPa |
| A325 and A449 | 121 MPa | 303 MPa |
| A490 | 152 MPa | 372 MPa |

$$A_t = A_{tensile} = F_e / \sigma_a$$

$$A_s = A_{shear} = Q_e / \tau_a$$

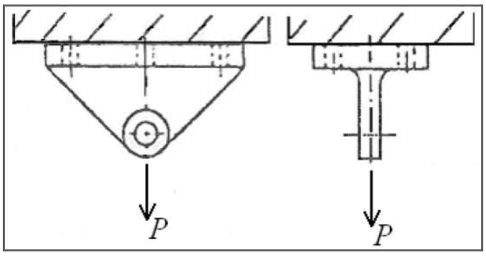
$$A = \frac{\pi}{4} \times d_r^{\,2}$$

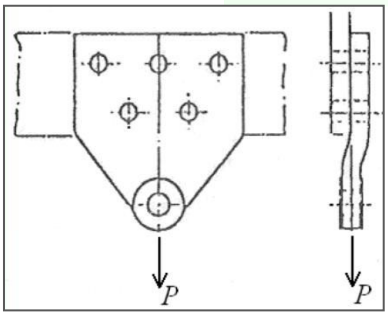
Loading analysis for bolt sizing

(i) *Concentric* loading: resultant force passes through the centre of bolt area.

direct *tension* of each bolt: $\sigma = \dfrac{P}{nA}$

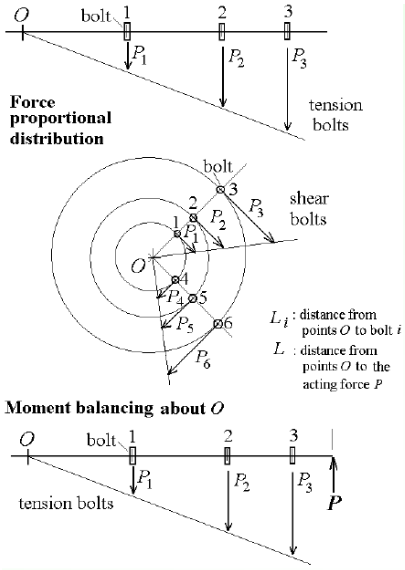
direct *shear* of each bolt: $\tau = \dfrac{P}{mA}$




(ii) *Eccentric* loading (3 cases):

*Introducing **secondary** forces due to moment:

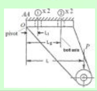
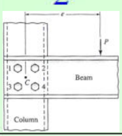
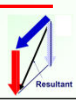
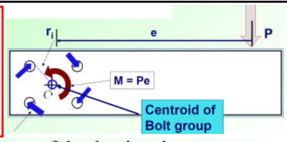
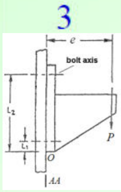



Force magnitude *proportional* to the distance from the centroid of a rigid link (pivot or origin $O$)

$$\boxed{P_1/L_1 = P_2/L_2 = P_3/L_3}$$

Moment *balancing* equation about $O$ (to resist the external moment due to $P$)

$$\boxed{PL = (P_1L_1 + P_2L_2 + P_3L_3)}$$

| Eccentric case | Direct (primary) load | Indirect (secondary) load | Loading axes (direct/indirect) | Resultant load | *Centroid of Bolted Joints* |
|---|---|---|---|---|---|
| 1  | Tension, $P_{td}$<br>$P_{td} = P/n$ | Tension, $P_{tm}$<br>$P_{tm\,i} = \dfrac{(PL)L_i}{(2L_1^2 + 2L_2^2)}$ | **Same** direction;<br>**Same** line<br>(with respect to bolt configuration) | $P_{td} + P_{tm}$<br>*(algebraic sum)* | $\bar{x} = \dfrac{A_1x_1 + A_2x_2 + \dots + A_nx_n}{A_1 + A_2 + A_3 + \dots + A_n} = \dfrac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i}$<br>$\bar{y} = \dfrac{A_1y_1 + A_2y_2 + \dots + A_ny_n}{A_1 + A_2 + A_3 + \dots + A_n} = \dfrac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i}$ |
| 2  | Shear, $P_{sd}$<br>$P_{sd} = P/n$ | Shear, $P_{sm}$<br>$F_i = \dfrac{(Pe)r_i}{\sum_{i=1}^{n} r_i^2}$ | **Different** directions;<br>$\boxed{\textbf{Same plane}}$<br>(with respect to bolt configuration) | $P_{td} \oplus P_{tm}$<br>*(vector sum)*  | $\boxed{F_i = \dfrac{Mr_i}{\sum_{i=1}^{n} r_i^2}}$ <br>$n$ : number of bolts in the group |
| 3  | Shear, $P_{sd}$<br>$P_{sd} = P/n$<br>$\boxed{Q = P_{sd}}$ | Tension, $P_{tm}$<br>$P_{tm\,i} = \dfrac{(Pe)L_i}{2(L_1^2 + L_2^2)}$<br>$\boxed{F = P_{tm}}$ | **Different** directions;<br>$\boxed{\textbf{Different planes}}$<br>(with respect to bolt configuration) | Equivalent tension, $F_e$<br>Equivalent shear, $Q_e$<br>*(maximum principal stress theory)*<br>*(maximum shear stress theory)* | $F_e = \dfrac{1}{2}\left\{F + \sqrt{F^2 + 4Q^2}\right\}$<br>$Q_e = \dfrac{1}{2}\sqrt{F^2 + 4Q^2}$ |