NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2016-2017

MA2011 - MECHATRONICS SYSTEM INTERFACING

April/May 2017 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **THREE (3)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. This is a **RESTRICTED OPEN-BOOK** examination. One double sided A4 reference sheet is allowed.

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1. A first-order instrument is used to measure a periodic signal.

(a) What is the magnitude ratio of the instrument in terms of time constant (τ) and angular frequency (ω)? What is the dynamic error?

(6 marks)

(b) If δ is a dynamic error that the measurement system can be tolerated, determine the maximum frequency (ω_{max}) of a periodic input that can be measured.

(7 marks)

(c) Assuming that a periodic signal has a single frequency f=50 Hz with sensitivity K=1, estimate the range of the time constant τ given the output amplitude of the signal varies from 50 and 100 units, and the dynamic error to be less than 1%. Discuss the relationship between the time constant, system time response, and dynamic error.

(12 marks)

2. Identify three pairs of operational amplifiers with opposite functions.

(a) Show the three pairs of amplifiers with their names, functions, schematic diagrams and equations.

(15 marks)

(b) Compare each pair of amplifiers.

(10 marks)

1(a). $RC\dfrac{dX_{out}}{dt} + X_{out} = KX_{in}$

$$RC\frac{dX_{out}}{dt} + X_{out} = KA_{in}\sin(\omega t)$$

Let $X_{out} = A_{out}\sin(\omega t+\phi)$

$= A_{out}\sin(\omega t)\cos\phi + A_{out}\cos(\omega t)\sin\phi$

$\dot{X}_{out} = A_{out}\omega\cos(\omega t+\phi)$

$= A_{out}\omega\cos(\omega t)\cos\phi - A_{out}\omega\sin(\omega t)\sin\phi$

$$RC\left(A_{out}\omega\cos(\omega t)\cos\phi - A_{out}\omega\sin(\omega t)\sin\phi\right) + A_{out}\sin(\omega t)\cos\phi + A_{out}\cos(\omega t)\sin\phi = KA_{in}\sin(\omega t)$$

$\cancel{A_{out}}\sin\phi + \cancel{A_{out}}\omega RC\cos\phi = 0$

$\sin\phi = -\omega RC\cos\phi$

$\tan\phi = -\omega RC$

$\phi = \tan^{-1}(-\omega RC)$

$A_{out}\cos\phi - A_{out}\omega RC\sin\phi = KA_{in}$

$A_{out}(\cos\phi - \omega RC\sin\phi) = KA_{in}$

$\dfrac{A_{out}}{A_{in}}\left(\dfrac{1+(\omega RC)^2}{\sqrt{1+(\omega RC)^2}}\right) = K$

$\cos(\tan^{-1}(-\omega RC)) = \dfrac{\pm 1}{\sqrt{1+(\omega RC)^2}}$

$\sin(\tan^{-1}(-\omega RC)) = \dfrac{\pm(\omega RC)}{\sqrt{1+(\omega RC)^2}}$

Magnitude Ratio $M = \dfrac{A_{out}}{A_{in}} = \dfrac{1}{\sqrt{1+(\omega RC)^2}}K = \dfrac{K}{\sqrt{1+(\omega\tau)^2}}$, where K is static sensitivity

Dynamic Error $\delta = K - M = K\left(1 - \dfrac{1}{\sqrt{1+(\omega\tau)^2}}\right)$

b). $K - \dfrac{K}{\sqrt{1+(\omega\tau)^2}} \le \delta$

$\dfrac{K}{\sqrt{1+(\omega\tau)^2}} \geq K-\delta$

$\dfrac{K^2}{1+(\omega\tau)^2} \geq (K-\delta)^2$

$\dfrac{K^2}{(K-\delta)^2} \geq 1+(\omega\tau)^2$

$\sqrt{\dfrac{K^2}{(K-\delta)^2}-1} \geq \omega\tau$

$\omega_{max} \le \dfrac{1}{\tau}\sqrt{\dfrac{K^2}{(K-\delta)^2}-1}$

c). $f = 50Hz$

$\omega = 100\pi$ rad/s

$K = 1$

$50 \le A \le 100$

$\delta < 0.01$

$1 - \dfrac{1}{\sqrt{1+(\omega\tau)^2}} < 0.01$

$\dfrac{1}{0.99} > \sqrt{1+(\omega\tau)^2}$

$\left(\dfrac{1}{0.99}\right)^2 > 1+(\omega\tau)^2$

$\tau < \dfrac{1}{\omega}\sqrt{\left(\dfrac{1}{0.99}\right)^2-1}$

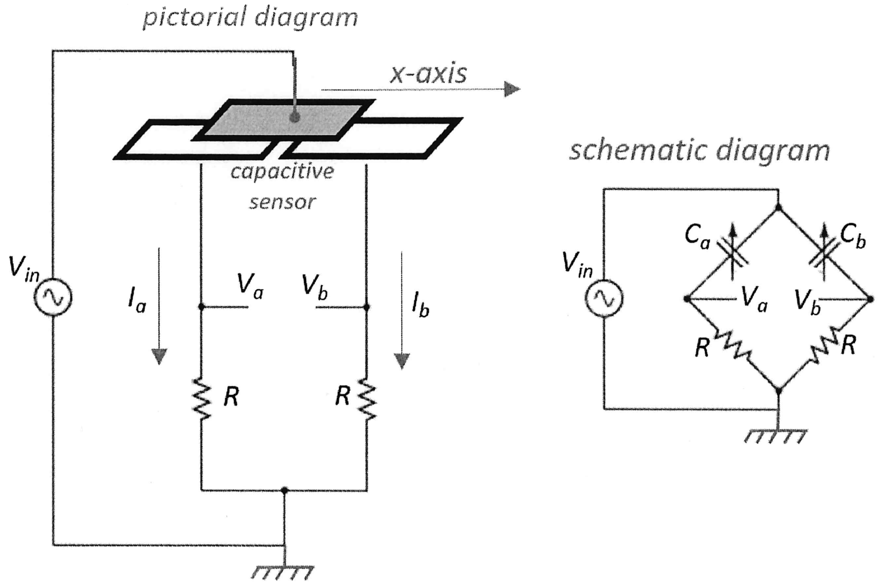
$\tau < 4.536\times10^{-4}$

If time constant ↑, dynamic error ↑

If time constant ↑, time response ↑

3. A capacitive sensor and its relative bridge measurement circuit are represented in Figure 1. The sensor consists of three rectangular and conductive plates of similar length L (along the x-axis) and width W. One moving plate (grey colour in Figure 1) can slide back and forth, along the x-axis, on top of two fixed plates (white colour in Figure 1(a)). When the grey plate is in its 'zero' position (x=0), the system is in a symmetric configuration, i.e. it overlaps equally with both fixed plates (half of the area overlaps). Assume that the capacitance between two plates is proportional to the overlapping area, with a maximum value C0 when their area fully overlaps.

Electrically, this system of capacitances is in a bridge configuration, with two similar resistors R connected to the fixed plates as in Figure 1. The bridge is driven by an AC voltage source V_{in} and the output $V_{out} = V_b - V_a$ is voltage difference between the two fixed plates, as shown in Figure 1(b).



(a) (b)

Figure 1

(a) Determine the overlapping areas $S_a(x)$ and $S_b(x)$ between the moving plate and, respectively, the left and right plates, as a function of the position x of the moving plate, considering at most a displacement $L/2$ from the zero position.

(5 marks)

(b) Considering that the capacitance between two plates is directly proportional to the overlapping area, with maximum capacitance C0 when the areas fully overlap and with null capacitance whenever two plates are not overlapping, derive analytical expressions and draw, superimposing in the same graph, the capacitances $C_a(x)$ and $C_b(x)$, as functions of the moving plate position x.

(5 marks)

(c) Considering an AC driving input, at generic frequency ω and considering similar resistances R on both sides of the bridge, derive currents $I_a(x)$ and $I_b(x)$ on each side of the bridge.

(7 marks)

(d) Assuming very small displacements ($|x|<<L$), determine amplitude and phase of the output voltage $V_{out} = V_b - V_a$, as a function of the moving plate position x.

(8 marks)

4. The characteristics of a commercial DC gearless motor, as found from the datasheet, are listed in the following table:

Nominal Voltage [V]	Rated Torque [Kg.cm]	Rated Speed [rpm]	Rated Current [mA]	No-Load Speed [rpm]	No-Load Current [mA]	Rated Output [W]
12	0.7	5700	5500	7000	900	41.3

(a) On your answer book, draw on the same graph the Speed vs. Torque, Current vs. Torque and output Power vs. Torque responses.

NOTE: the datasheet does not provide stall torque and stall current. The rated values refer to one possible (suggested) operating point. You can assume linearity to derive such values.

(8 marks)

(b) Determine, at the nominal voltage, the maximum power (in Watts) which can be delivered to a mechanical load as well as the torque, speed and efficiency at such optimal operating point.

(5 marks)

(c) Based on previous calculations and the information reported above, determine the friction coefficient of the bearings (assuming a linear model) and the amount of friction torque under no-load conditions.

(7 marks)

(d) Compute the *efficiency* of the motor when operating at the rated speed and when operating at maximum power transfer conditions.

(5 marks)

End of Paper

3)a). $S_a(x) = -Wx + W\frac{L}{2}$

$S_b(x) = Wx + W\frac{L}{2}$

b). $C \propto S(x)$

$C_a = K\left(-Wx + W\frac{L}{2}\right)$

$C_b = K\left(Wx + W\frac{L}{2}\right)$

At $x = \frac{L}{2}$, $C_a = 0$

At $x = \frac{L}{2}$, $C_b = C_0$

$C_0 = KWL$

$K = C_0/WL$

$C_a(x) = C_0/L\left(-x + \frac{L}{2}\right)$

$C_b(x) = C_0/L\left(x + \frac{L}{2}\right)$

c). $I_a = V_{in}/(Z_{Ca} + R) = \dfrac{V_{in}}{R - \frac{j}{\omega C_a}} = \dfrac{V_{in}\left(R + \frac{j}{\omega C_a}\right)}{R^2 + \frac{1}{(\omega C_a)^2}} = \dfrac{V_{in}(\omega C_a)^2 R + \omega C_a V_{in} j}{(\omega C_a R)^2 + 1} = \dfrac{(\omega C_a)^2 R}{(\omega C_a R)^2+1} V_{in} + \dfrac{\omega C_a}{(\omega C_a R)^2+1} V_{in} j$

$= \dfrac{\omega C_a V_{in}}{(\omega C_a R)^2+1}\left(\omega C_a R + j\right) = \dfrac{\omega C_a V_{in}}{(\omega C_a R)^2+1}\sqrt{(\omega C_a R)^2+1}\, e^{j\tan^{-1}\left(\frac{1}{\omega C_a R}\right)}$

$= \dfrac{\omega C_a V_{in}}{\sqrt{(\omega C_a R)^2+1}} e^{j\tan^{-1}\left(\frac{1}{\omega C_a R}\right)} = \dfrac{\omega C_a V_{in}}{\sqrt{(\omega C_a R)^2+1}} \cos\left(\omega t + \tan^{-1}\left(\frac{1}{\omega C_a R}\right)\right)$

$\hookrightarrow$ where $C_a = C_0/L\left(-x + \frac{L}{2}\right)$

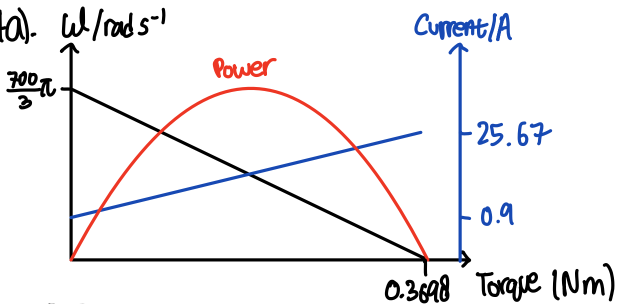
$I_b = \dfrac{\omega C_b V_{in}}{\sqrt{(\omega C_b R)^2+1}} \cos\left(\omega t + \tan^{-1}\left(\frac{1}{\omega C_b R}\right)\right)$

d). $V_a = I_a R$

$V_b = I_b R$

$V_{out} = V_b - V_a = \dfrac{\omega C_b V_{in} R}{\sqrt{(\omega C_b R)^2+1}} \cos\left(\omega t + \tan^{-1}\left(\frac{1}{\omega C_b R}\right)\right) - \dfrac{\omega C_a V_{in} R}{\sqrt{(\omega C_a R)^2+1}} \cos\left(\omega t + \tan^{-1}\left(\frac{1}{\omega C_a R}\right)\right)$

4a).



$\omega = \frac{700\pi}{3} - 1982T$

$\Delta\omega/\Delta T = -1982$

$T_{stall} = 0.3698\,Nm$

$I = 0.9 + 66.987T$

$\Delta I/\Delta T = \dfrac{5.5 - 0.9}{0.06867} = 66.987$

c). $T - C\omega_0 = 0$

$T = C\omega_0$

$T - C\omega_L - T_L = 0$

$C(\omega_0 - \omega_L) - T_L = 0$

$C = \dfrac{T_L}{\omega_0 - \omega_L} = \dfrac{T_{stall}}{\omega_0 - \omega_{stall}} = \dfrac{T_{stall}}{\omega_0}$

$= 5.045 \times 10^{-4}$

b). $P_{max} = \frac{1}{4}\omega_0 T_{stall}$

$= 67.77\,W$

$T = 0.1849\,Nm$

$\omega = \frac{350\pi}{3}$ rad/s

$I = 13.286\,A$

$\eta = T\omega/VI = 0.425$

d). $\eta_{rated} = \dfrac{0.06867(1900)}{12(5.5)} = 0.621$

MA2011 MECHATRONICS SYSTEM INTERFACING

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.