



**NANYANG  
TECHNOLOGICAL  
UNIVERSITY**

# **MA2011 MECHATRONICS SYSTEMS INTERFACING**

**Tutorial 6**

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# **A/D CONVERSION & D/A CONVERSION**

# Q1

Specify an appropriate,  $\pm 5\text{V}$  n-bit A/D converter (8- or 12-bit), sample rate (up to 100 Hz) and signal conditioning to convert the following analog signal into digital series. Estimate the relative quantization error in quantizing the specified input voltage:

$$E(t) = 1.5 \sin \pi t + 20 \sin 32\pi t - 3 \sin (60\pi t + \pi/4) \text{ V}$$

(Many possible solutions.)

## Input vs output

KNOWN: A/D converter:  $n = 8$  or  $12$ ;  $V_{\max} = 5 \text{ V}$ ;  $V_{\min} = -5 \text{ V}$

$$0 < f_s \leq 100 \text{ Hz}$$

Estimate the maximum Quantifying Error for your choice

## Analysis

$$E(t) = 1.5 \sin \pi t + 20 \sin 32\pi t - 3 \sin (60\pi t + \pi/4) \text{ V}$$

This input signal contains amplitudes  $C_1$ ,  $C_2$  and  $C_3$  with frequencies of  $f_1 = 0.5$ ,  $f_2 = 16$  and  $f_3 = 30$  Hz, respectively.

## Shannon-Nyquist Theorem

to be able to reconstruct the signal correctly, sampling theorem must be met:

$$f_s > 2f_{max}$$

The maximum frequency in the signal,  $f_{max}$ , is 30 Hz. So we must select  $f_s > 60$  Hz.

## Accuracy

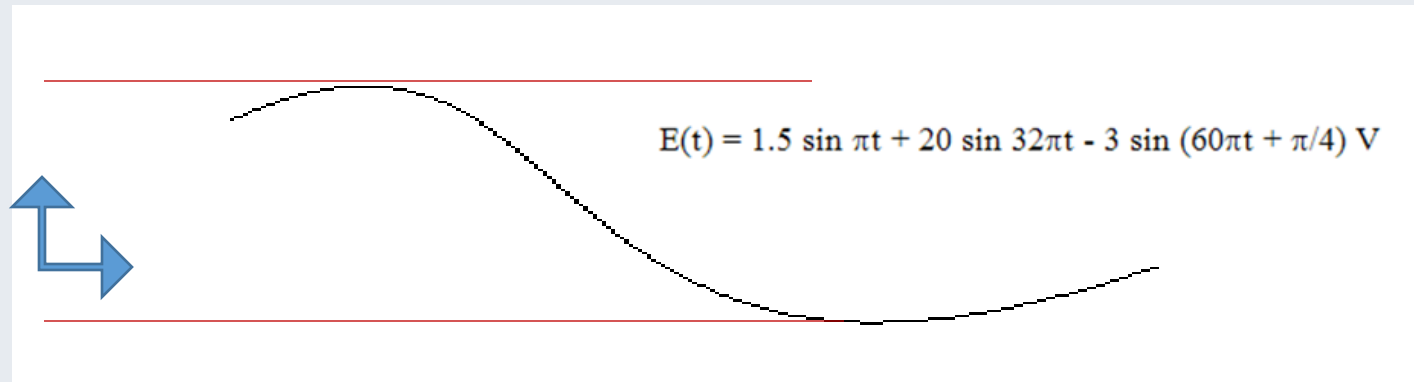
The choice of number of bits for the A/D converter depends on the required accuracy as well as the accuracy of the transducer. For 8-bit A/D converter,

$$Q = \frac{10 \text{ V}}{2^8} = 39.1 \text{ mV}$$

For 12-bit A/D converter,  $Q = \frac{10}{2^{12}} = 2.44 \text{ mV}$

# ANSWER TO Q1

## Finding the Amplitude Range



Amplitude range of the signal  $E(t)$  can only be found from the time domain plot or by differentiating the equation.

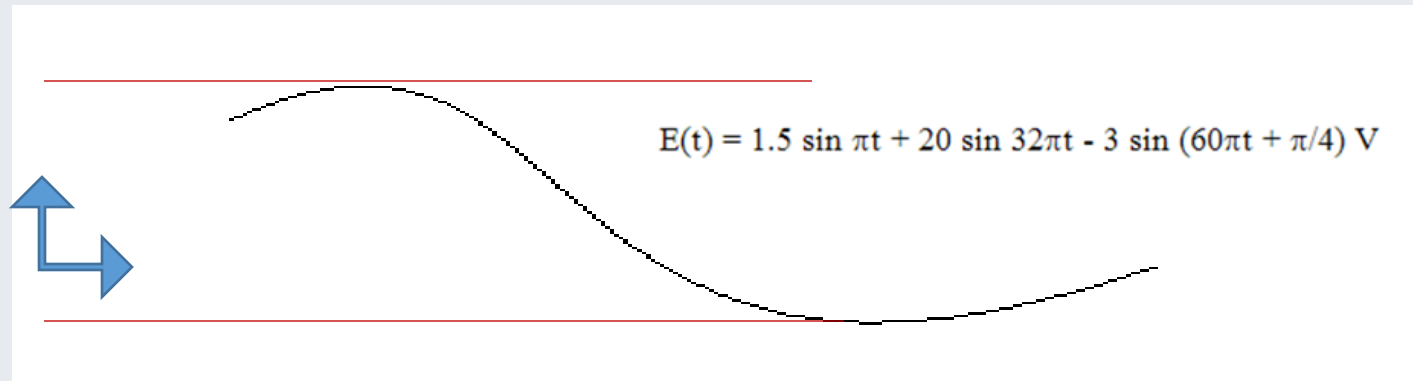


## No unique solution

### SOLUTION:

This design problem has an open-ended solution path. To demonstrate, one possible solution is presented.

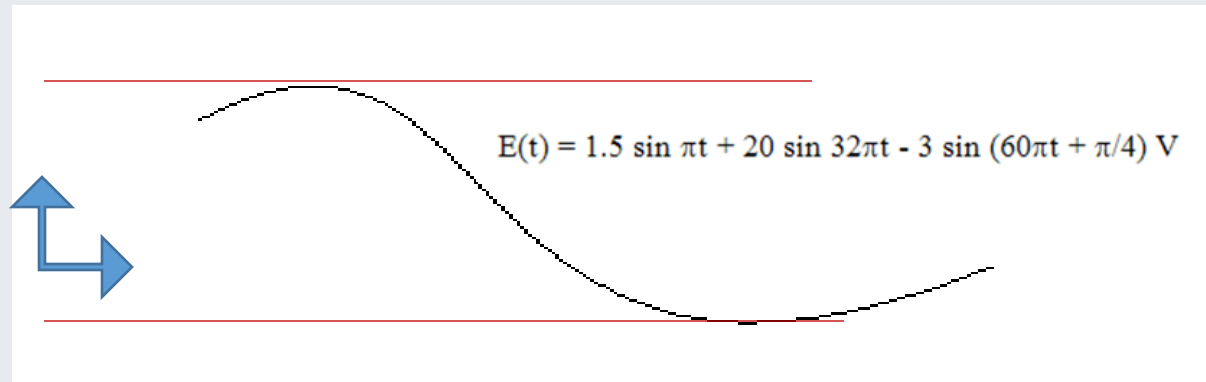
## Estimation of Amplitude Range



However, we know that it will never exceed  
 $\pm (1.5 + 20 + 3) = \pm 24.5 \text{ V}.$

# ANSWER TO Q1

## Estimated Gain



$$V_{out} = A_v V_{in}$$

So, we need a gain of  $\pm 5\text{V} / \pm 24.5 \text{ V} = \text{about } 0.2$ .

# ANSWER TO Q1

## Error

The relative quantization error is given by

$$\frac{Q}{GA} \times 100\%$$

$$G=0.2$$

$$Q(8\text{-bits})=39.1\text{mV}$$

$$Q(12\text{-bits})=2.44\text{mV}$$

**Max Amplitudes A=?**

# ANSWER TO Q1

## The Detail

For each frequency component and the signal, this is listed as:

Component	Frequency	Max. Amplitude	Relative Quantization Error $\frac{Q}{0.2A} * 100\%$	
			8-bits	12-bits
$1.5\sin\pi t$	0.5	<b>1.5</b>	$39.1\text{mV}/0.2/1.5*100\%=13\%$	$2.44\text{mV}/0.2/1.5*100\%=\mathbf{0.81\%}$
$20\sin32\pi t$	16	<b>20</b>	$39.1\text{mV}/0.2/20*100\%=9.8\%$	$2.44\text{mV}/0.2/20*100\%=\mathbf{0.06\%}$
$3\sin(60\pi t+\pi/4)$	30	<b>3</b>	$39.1\text{mV}/0.2/3*100\%=6.5\%$	$2.44\text{mV}/0.2/3*100\%=\mathbf{0.41\%}$
$E(t)$		<b>&lt;+24.5</b>	$39.1\text{mV}/0.2/24.5*100\%=0.8\%$	$2.44\text{mV}/0.2/24.5*100\%=\mathbf{0.05\%}$

As the relative quantization error for both 8-bit and 12-bit A/D converters are less than 1% for  $E(t)$ , either is suitable for most applications. If the accuracies for the low-level components are important, then the 12-bit A/D converter is required.

Static pressures are to be measured at eight locations (at 8-cylinder points) under the hood of a NASCAR race car (with a V-8 engine). The pressure transducers to be used have an output span of  $\pm 1$  V for an input span of  $\pm 25$  cm H<sub>2</sub>O. The signals are measured and recorded on a portable DAS/computer, which uses a 10-bit,  $\pm 5$  V A/D converter. Pressure needs to be resolved to within 0.25 cm H<sub>2</sub>O. The dynamic content of the signal is important and has a fundamental period of about 0.5 s. Suggest an appropriate sample rate and signal conditioning (i.e. amplifier gain  $G$  and anti-aliasing filter cut off frequency  $f_c$ ) for this application.

**DAS=Direct-attached storage**

(Many possible solutions.)

## Input vs output

KNOWN: Transducer:  $\pm 1$  V output;  $\pm 25$  cm H<sub>2</sub>O input

DAS (ADC):  $n = 10$ ;  $V_{\max} = 5$  V;  $V_{\min} = -5$  V

FIND:  $f_s, f_c, G$ .

## Race Car

Input span:  $\pm 25$  cm H<sub>2</sub>O (Pressure Transducer)

Output span:  $\pm 1$  V (Pressure Transducer)

ADC: 10-bit, -5 V Range, 0.25 cm H<sub>2</sub>O

## Sensitivity K

The transducer sensitivity

$$\begin{aligned} K &= \Delta V_{\text{out}} / \Delta V_{\text{in}} = (1 - (-1)) \text{V} / (25 - (-25)) \text{ cmH}_2\text{O} = 2 \text{V} / 50 \text{ cmH}_2\text{O} \\ &= 0.04 \text{V/cmH}_2\text{O} \end{aligned}$$



## ANSWER TO Q2

### Sensitivity K meets problem constraint (10-bit ADC)?

The A/D resolution is:

$$Q = \frac{10V}{2^{10}} = 0.00976 \text{ V.}$$

Pressure needs within 0.25 cm H<sub>2</sub>O

This can be expressed as

$$Q = \frac{0.00976V}{0.04V/\text{cm H}_2\text{O}} = 0.244 \text{ cm H}_2\text{O} < 0.25 \text{ cm H}_2\text{O}$$

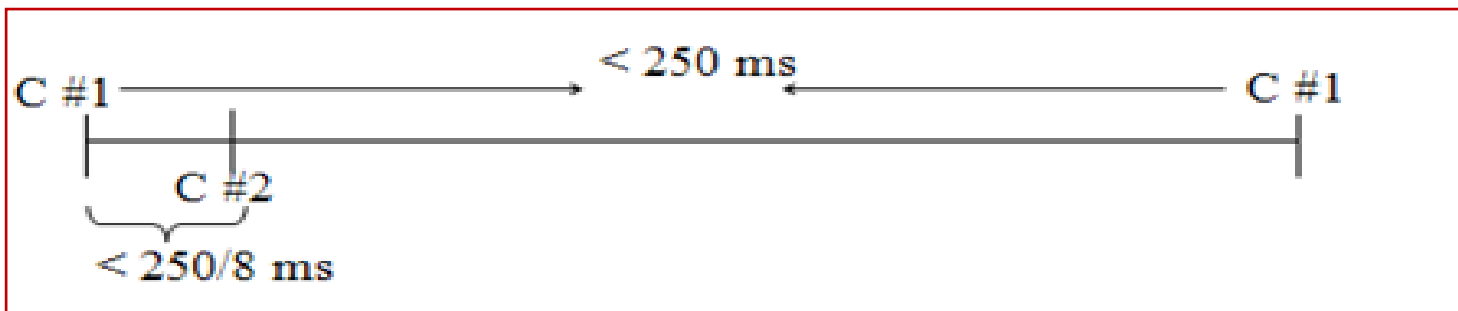
### Amplifier (Gain 5)

The sensitivity meets problem constraints. However, an analog amplifier between the transducer and A/D converter with a gain of  $G = 5$  will take full advantage of the A/D range and improve resolution:

$$Q = \frac{0.00976V}{5 \times 0.04V / \text{cmH}_2\text{O}} = 0.0488 \text{ cm H}_2\text{O}$$

## ANSWER TO Q2

For one pressure sensor, the fundamental period of the signal is about 0.5s (500ms), or a frequency of 2Hz. The maximum sampling period is 250ms or the minimum sampling frequency is greater than 4 Hz.



### Sampling

Since the car is powered by a high performance V-8 engine, there are 8 cylinders and there are 8 pressure signals to sense, one for each cylinder. Hence, the DAS has to sample at least up to 8 times faster than maximum sampling period required for 1 pressure sensor i.e.  $250/8 \text{ [ms]}$  or a minimum frequency of 32 Hz.  $\Delta_{\max}=250/8 \text{ ms}$ .  $f_{\min}=8000/250=32\text{Hz}$

The range of a signal is between  $\pm 5$  V and it is required to make measurements with a quantization size no more than 5 mV. What is the minimum resolution of an ADC needed?

## ANSWER TO Q3

Since

$$\frac{5 - (-5)}{2^N} = Q \leq 0.005$$

$$\Rightarrow 2^N \geq \frac{10}{0.005} = 2000$$

If  $N = 11$ ,  $2^N = 2048$  and if  $N = 12$ ,  $2^N = 4096$ . Although a 11-bit ADC will suffice, it is not commercially available.

Hence a **12-bit ADC** will be needed.

One

Objective

Two

Ideas

Three

Techniques

Four

Examples

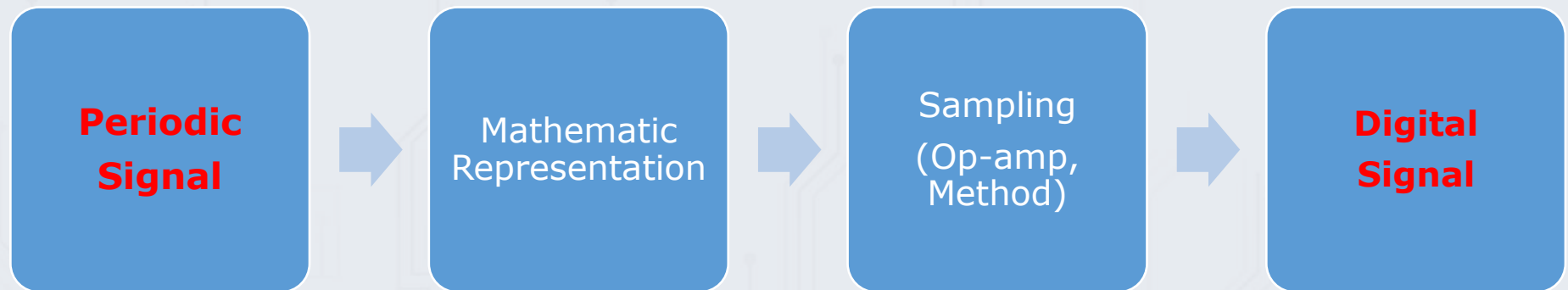
Five

Misunderstandings

Six &  
Seven

Tutorials & Project

# REVISION: ONE OBJECTIVE OF DIGITIZATION



### 1. Fourier Series Representation (Theory)

- Frequency domain analysis

### 2. Shannon-Nyquist Theorem

- Aliasing, 2-phase filtering

# REVISION: THREE TECHNIQUES

## **1:** Frequency Domain Analysis

Spike plotting

## **2:** First Order System

Dynamic Error

## **3:** Sampling & Aliasing

Two-phase Filtering

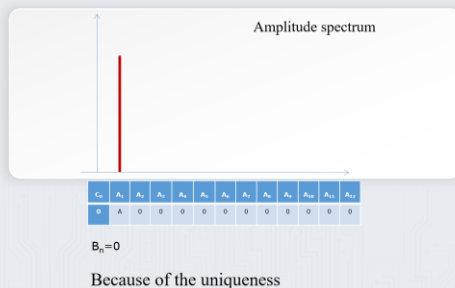


# REVISION: THREE TECHNIQUES

## 1: Frequency Domain Analysis

### Spike plotting

#### ANSWER TO T2 Q3(C)



## 2: First Order System

### Dynamic Error

#### T3 Q3

For a first order system, the output of the system always lags (follows behind) the input and  $M(\omega)$  must always be less than 1. So, for  $\delta(\omega) \leq 0.02$ , we have

$$1 \geq M(\omega) \geq 0.98$$

$$M(\omega) = \frac{1}{\sqrt{1 + (\omega\tau)^2}}$$

$$\text{Hence, } 1 \geq \frac{1}{\sqrt{1 + (\omega\tau)^2}} \geq 0.98$$

$$\text{i.e. } \frac{1}{0.98} \geq \sqrt{1 + (\omega\tau)^2} \geq 1$$

At  $\tau = 2$  s, we find that  $0.1 \geq \omega \geq 0$  rad s<sup>-1</sup>

So,  $\omega_{\text{max}} = 0.1$  rad s<sup>-1</sup> or  $f_{\text{max}} = 0.016$  Hz.

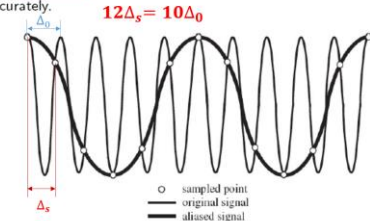
## 3: Sampling & Aliasing

### Two-phase Filtering

#### ALIASING T4

If sampling rate is too low, we obtain wrong digital signal. This is called aliasing.

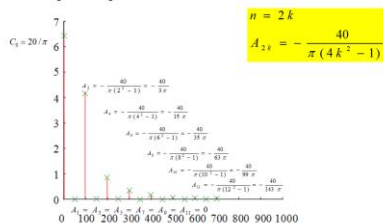
Example: 12 equally spaced samples are taken over 10 cycles of an analog signal, so  $f_s = 1.2f_0$  with  $f_0$  frequency of the analog signal. Since  $f_s < 2f_0$ , digitized signal does not describe the original signal accurately.



#### Answer to T2 Q3(B)

$$y(t) = \frac{20}{\pi} + \sum_{n=2,4,6,\dots} A_n \cos(n\omega_0 t) \text{ where } A_n = \frac{10(2)}{\pi(n+1)} - \frac{10(2)}{\pi(n-1)}$$

The amplitude spectrum is as shown below:

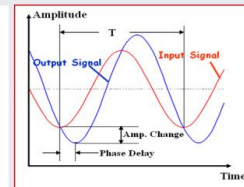


#### ANSWER TO Q4

$$1 \geq \frac{1}{\sqrt{1 + (\omega\tau)^2}} \geq 0.99$$

At  $\omega = 2\pi f = 20\pi$  rad s<sup>-1</sup>, we find that

$$0 \leq \tau \leq 2.27 \text{ ms}$$



#### T4 Q2

Alias frequencies can be found using

$$f_a = \pm f_0 + i f_s \text{ for } i = 1 \text{ to } \infty$$

e.g.  $f_a = -0.1 + 5 = 4.9$   $f_a = 0.1 + 5 = 5.1$

n	f <sub>a</sub>	i=1	i=2	i=3	i=4	...
1	0.1	4.9, 5.1	9.9, 10.1	24.9, 15.1	19.9, 20.1	Etc.
2	0.3	4.7, 5.3	9.7, 10.3	14.7, 15.3	19.7, 20.3	Etc.
3	0.5	4.5, 5.5	9.5, 10.5	14.5, 15.5	19.5, 20.5	Etc.
4	0.7	4.3, 5.7	9.3, 10.7	Etc.		
5	0.9	4.1, 5.9	9.1, 10.9	Etc.		
6	1.1	3.9, 6.1	8.9, 11.1	Etc.		
7	1.3	3.7, 6.3	8.7, 11.3	Etc.		
8	1.5	3.5, 6.5	Etc.			
9	1.7	3.3, 6.7	Etc.			
10	1.9	3.1, 6.9	Etc.			

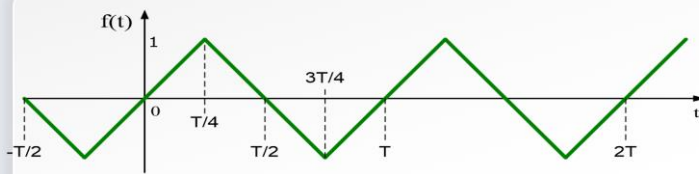
Objective function:  $|f_a - f_0|$  with variable  $i = 1, 2, 3, \dots$

Procedures, 8. and Outputs, J. M., Int. J. Arch. Eng. Education, Vol. 18, No. 3, pp. 195-199, 2012

# REVISION: FOUR EXAMPLES

## T2 Q2

$f(t)$  is a function defined as follows.



- Is it a periodic function? If yes, what is the period, and write the function of the waveform defined at  $[0, T]$
- Is it a symmetric function?
- Can the waveform be represented by Fourier Series? If yes, what are the DC  $C_0$ ,  $A_n$  and  $B_n$  of the waveform and the Fourier Series?
- What are the peak amplitude, and peak-to-peak amplitude?
- If the peak amplitude is changed to  $A$ , what will be the function of  $f(t)$  and its Fourier Series Representation?

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## T3 Q2

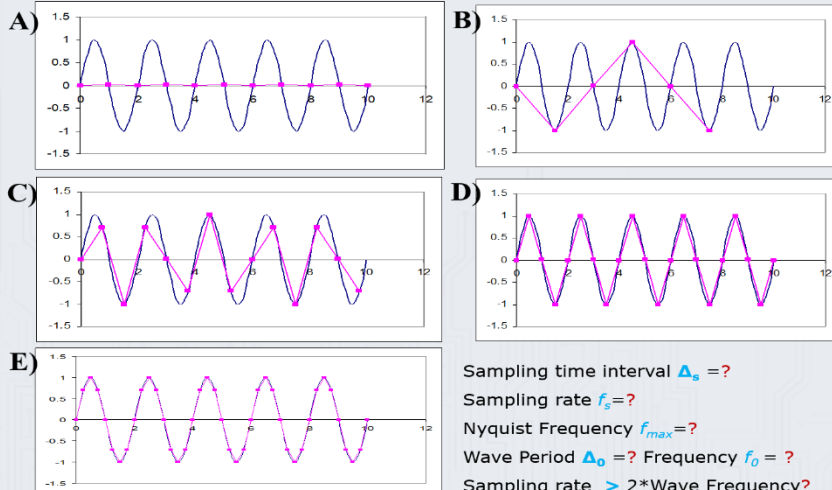
During a step function calibration, a first-order instrument is exposed to a step change of 100 units. If after 1.2 s the instrument indicates 80 units, estimate the instrument time constant. Estimate the error in the indicated value after 1.5 s. Assume  $X_{out}(0) = 0$  units and  $K = 1$  unit/unit.

$$c \tau = 0.75 \text{ s; error at } 1.5 \text{ s} = 13.4 \text{ units}$$

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## T4 Q1



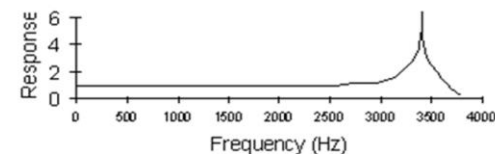
- Sampling time interval  $\Delta_s = ?$   
 Sampling rate  $f_s = ?$   
 Nyquist Frequency  $f_{max} = ?$   
 Wave Period  $\Delta_0 = ?$  Frequency  $f_0 = ?$   
 Sampling rate  $> 2 \times$  Wave Frequency?

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## T5 Q3

- An accelerometer that is used to measure the vibration of a machine has a frequency response as shown in the figure below:



The vibration signal is filtered to ensure that the vibration signal beyond 2.5 kHz does not affect the recording of the measurements.

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# REVISION: FIVE MISUNDERSTANDINGS

## UNIQUE FOURIER REPRESENTATION

$$f(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t)$$

$$f(t) = C_0 + \sum_{n=1}^{\infty} (A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t))$$

$$= C_0 + \sum_{n=1}^{\infty} (A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t))$$

Uniqueness means

$$C_0 = C_0$$

$$A_n = A_n$$

$$B_n = B_n$$

For all  $n$

Uniqueness only applicable for Standard Fourier Representation, no phase angles involved

## T2 Q3

A signal  $y(t) = \sin(10\pi t)$  below is defined in the time interval  $[0, 0.01]$  of unit seconds.



(c) If a periodic function  $g(t)$  of odd symmetry is created also based on the given signal  $y(t)$  with a period  $T=0.02$  seconds. What will be  $g(t)$  and its Fourier Series

$$g(t) = \sin(10\pi t) \text{ for } 0 \leq t < T/2 \text{ and } g(t) = -\sin(10\pi t) \text{ for } T/2 \leq t < T$$

$$G(\omega) = \sin(10\pi T) \text{ Uniqueness of the Fourier Representation}$$

## T2 Q1

$$f(-t) = f(t) \text{ even}$$

$$g(-t) = -g(t) \text{ odd}$$

$$h(t) = f(t) + g(t); \text{ even or odd?} \rightarrow \text{Not so}$$

Even  
 $f(-t) = f(t)$

Odd  
 $f(-t) = -f(t)$

**Proof by contradiction!**

Assume:  $h(t)$  even result

$$h(-t) = h(t);$$

$$f(-t) + g(-t) = (f(t) + g(t));$$

$$f(t) - g(t) = f(t) + g(t)$$

$$2 * g(t) = 0 \rightarrow \text{only if } g(t) = 0$$

Assume:  $h(t)$  odd result

$$h(-t) = -h(t);$$

$$f(-t) + g(-t) = -(f(t) + g(t));$$

$$f(t) - g(t) = -f(t) - g(t)$$

$$2 * f(t) = 0 \rightarrow \text{only if } f(t) = 0$$

## T4 Q2

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t) \quad \text{no,}$$

Consider the continuous signal:

$$\sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \sin\left[\frac{2\pi(2n-1)t}{10}\right] \quad \frac{2n-1}{10} 2\pi$$

1. What would be an appropriate sampling rate to use in sampling this signal if it is filtered at and above 2 Hz before sampling?

2. What are the alias frequencies of the filtered signal at this sampling rate?

$$(f_s > 4 \text{ Hz; many possible solutions})$$

1. *Fourier Representation is always time consuming*

2. *Proof vs Proof by contradiction*

3. *Fourier Series must be in  $(n\omega_0 t)$  form*

## T4 Q3(A)

SOLUTION:

$$2^n \geq 756$$

$$n \geq \frac{\log_{10} 756}{\log_{10} 2} = \frac{2.878}{0.301}$$

$$= 9.56 \rightarrow 10$$

Thus 10 bits are required.

$$756 / 2 = 378 \text{ remainder } 0$$

$$378 / 2 = 189 \text{ remainder } 0$$

$$189 / 2 = 94 \text{ remainder } 1$$

$$94 / 2 = 47 \text{ remainder } 0$$

$$47 / 2 = 23 \text{ remainder } 1$$

$$23 / 2 = 11 \text{ remainder } 1$$

$$11 / 2 = 5 \text{ remainder } 1$$

$$5 / 2 = 2 \text{ remainder } 1$$

$$2 / 2 = 1 \text{ remainder } 0$$

$$1 / 2 = 0 \text{ remainder } 1$$

$$\text{is } 756_{10} = 1011110100_2$$

a) How many bits are needed to represent the number 756 Without using a calculator, convert 756 to binary

(10 bits; 10 1111 0)

4. *Binary conversion must have a calculator*

## Project Tools, Rubrics & Presentation

SolidWorks	Maya	3D Studio Max	Blender
Unity	Unreal	Second Life	Revit
Novelty	Video	Teamwork	Quality
25%	25%	25%	25%
10 Mins Duration	Team Efforts	To Be Upload in File Exchange/Group	Due by 23:59 SGT 10 March Recess Week

Digital twin has nothing to do with mechatronics

# REVISION

Tutorial	Content
# 6	A/D & D/A Conversion
# 5	Amplifiers
# 4	Sampling Quantizing Coding
# 3	Zero-order system First-order system Second-order system
# 2	Fourier Series
# 1	Characteristics of Measurement System

# REVISION

Lecture	Content	Fundamental
# 6	A/D & D/A Conversions	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series
# 5	Amplifiers	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series
# 4	Sampling Quantizing Coding	Shannon-Nyquist Theorem Fourier Series
# 3	Zero-order system First-order system Second-order system	Ordinary Differential Equations Fourier Series
# 2	Fourier Series	Fourier Series
# 1	Characteristics of Measurement System	Concepts

# REVISION

Lecture	Content	Fundamental	Application
# 6	A/D & D/A Conversions	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series	A/D Converter D/A Converter Design
# 5	Amplifiers	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series	Amplifier Design
# 4	Sampling Quantizing Coding	Shannon-Nyquist Theorem Fourier Series	A/D & D/A Conversion
# 3	Zero-order system First-order system Second-order system	Ordinary Differential Equations Fourier Series	Amplifier Design, ADC, DAC
# 2	Fourier Series	Fourier Series	Amplifier Design, ADC, DAC
# 1	Characteristics of Measurement System	Concepts	

# REVISION

Lecture	Content	Fundamental	Application	Difficult Level
# 6	A/D Conversion	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series	A/D Converter D/A Converter Design	***
# 5	Amplifiers	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series	Amplifier Design	***
# 4	Sampling Quantizing Coding	Shannon-Nyquist Theorem Fourier Series		***
# 3	Zero-order system First-order system Second-order system	Ordinary Differential Equations Fourier Series		*****
# 2	Fourier Series	Fourier Series		****
# 1	Resistors, Capacitors, ...	Mechatronics Concepts		*

# REVISION

Lecture	Content	Fundamental	Not Examinable
# 6	A/D & D/A Conversions	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series	
# 5	Amplifiers	Shannon-Nyquist Theorem Ordinary Differential Equations Fourier Series	
# 4	Sampling Quantizing Coding	Shannon-Nyquist Theorem Fourier Series	Mathematics Proof of Shannon-Nyquist Theorem
# 3	Zero-order system First-order system Second-order system	Ordinary Differential Equations Fourier Series	Mathematics Proof of Ordinary Different Equation
# 2	Fourier Series	Fourier Series	Mathematics Proof of Fourier Series Representation
# 1	Characteristics of Measurement System	Mechatronics Concepts	