Tut-10- Convection-(external flow) Solution Guide

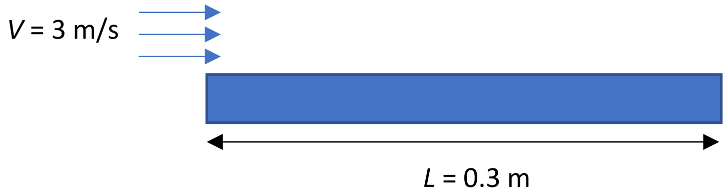
Thermodynamics & Heat Transfer (Nanyang Technological University)

Tutorial 10— External Flow Convection

1. Air at 15°C and 1 atm flows over a 0.3-m-wide plate at 65°C at a velocity of 3.0 m/s. Compute the following quantities at x = 0.3 m:

a)	Hydrodynamic boundary layer thickness, m	(Ans: 0.00641m)
b)	Local friction coefficient	(Ans: 0.0029)
c)	*Average* friction coefficient	(Ans: 0.0058)
d)	Total drag force due to friction, N	(Ans: 0.0026N)
e)	Local convection heat transfer coefficient, W/m^2·K	(Ans: 6.09 W/m^2·K)
f)	Average convection heat transfer coefficient. W/m^2·K	(Ans:12.2 W/m^2·K)
g)	Rate of convective heat transfer, W	(Ans: 54.9 W)

(from 7-17)



Check fluid properties – film temperature:

$$T_f = \frac{1}{2}(T_\infty + T_s) = 40\text{ °C}$$

Properties of air at 40°C, 100 kPa: k = 0.02662 W/m K, ρ = 1.127 kg/m^3, μ = 1.918 x 10^{-5} kg/m s,

Pr = 0.7255 **(Table A-15)**

Check Reynolds number at end of plate (to check if overall flow is laminar or turbulent)

$$\text{Re}_L = \frac{\rho V L}{\mu} = 52\,880 < 5 \times 10^5 \qquad \textbf{(Laminar Flow)}$$

a) Hydrodynamic boundary layer thickness = velocity boundary layer thickness

$$\delta = \frac{4.91x}{\sqrt{\text{Re}_x}}$$

$$\delta_{x=L} = \frac{4.91L}{\sqrt{\text{Re}_L}} = \mathbf{0.00641\ m}$$

b) Local friction coefficient:

$$C_{f,x} = \frac{0.664}{\sqrt{\text{Re}_x}}$$

$$C_{f,x=L} = \frac{0.664}{\sqrt{\text{Re}_L}} = \mathbf{0.0029}$$

c) Average friction coefficient:

$$C_f = \frac{1.328}{\sqrt{\text{Re}_L}} = 2C_{f,x=L} = \mathbf{0.0058}$$

d) Total drag force:

$$F_f = C_f A_s \frac{\rho V^2}{2} = \mathbf{0.0026\ N}, \quad \textbf{where } \mathbf{A_s = (0.3 \times 0.3)m^2}$$

e) Local Nusselt number:

$$\mathrm{Nu}_x = 0.332\mathrm{Re}_x^{0.5}\mathrm{Pr}^{\frac{1}{3}}$$

$$\mathrm{Nu}_{x=L} = 0.332\mathrm{Re}_L^{0.5}\mathrm{Pr}^{\frac{1}{3}} = 68.6$$

Local heat transfer coefficient:

$$\mathrm{Nu}_x = \frac{h_x x}{k}$$

$$h_{x=L} = \frac{k}{L}\mathrm{Nu}_{x=L} = \mathbf{6.09\ W/m^2K}$$

f) Average Nusselt number:

$$\overline{\mathrm{Nu}}_{\mathrm{L}} = 0.664\mathrm{Re}_L^{0.5}\mathrm{Pr}^{\frac{1}{3}} = 2\mathrm{Nu}_{x=L} = 137.2$$

Average heat transfer coefficient:

$$\bar{h}_L = \frac{k}{L}\overline{\mathrm{Nu}}_{\mathrm{L}} = \mathbf{12.2\ W/m^2K}$$

g) Rate of convective heat transfer:

$$\dot{Q}_{conv} = \bar{h}_L A_s(T_s - T_\infty) = \mathbf{54.9\ W}$$

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2. In an experiment, the local heat transfer over a flat plate were correlated in the form of local Nusselt number as expressed by the following correlation

$$\mathrm{Nu}_x = 0.035\mathrm{Re}_x^{\ 0.8}\mathrm{Pr}^{1/3}$$

Determine the ratio of the average convection *heat* transfer coefficient *(h)* over the entire plate length to the local convection heat transfer coefficient *(h_l) at x = L. (from* 7-20) (Ans:1.25)

$$\mathrm{Nu}_x = \frac{h_x x}{k}$$

Average heat transfer coefficient:

$$\bar{h}_L = \frac{1}{L}\int_0^L h_x\, dx$$

$$= \frac{1}{L}\int_0^L \frac{k}{x}\mathrm{Nu}_x\, dx$$

$$= \frac{1}{L}\int_0^L \frac{k}{x}\cdot 0.035\mathrm{Re}_x^{0.8}\mathrm{Pr}^{\frac{1}{3}}dx$$

Substituting expression for Re_x:

$$\bar{h}_L = \frac{1}{L}\int_0^L 0.035\frac{k}{x}\left(\frac{Vx}{\nu}\right)^{0.8}\mathrm{Pr}^{\frac{1}{3}}dx$$

$$= \frac{0.035k\mathrm{Pr}^{\frac{1}{3}}}{L}\left(\frac{V}{\nu}\right)^{0.8}\int_0^L x^{-0.2}\, dx$$

$$= \frac{0.035k\mathrm{Pr}^{\frac{1}{3}}}{L}\left(\frac{V}{\nu}\right)^{0.8}\left[\frac{5}{4}x^{4/5}\right]_0^L$$

$$= \frac{5}{4} \times \frac{0.035k\mathrm{Pr}^{\frac{1}{3}}}{L}\left(\frac{V}{\nu}\right)^{0.8} L^{0.8}$$

$$= \frac{5}{4} \times \frac{0.035k\mathrm{Pr}^{\frac{1}{3}}}{L}\left(\frac{VL}{\nu}\right)^{0.8}$$

$$= \frac{5}{4} \times \frac{k}{L} \cdot 0.035\mathrm{Re}_L^{0.8}\mathrm{Pr}^{\frac{1}{3}}$$

$$= \frac{5}{4}\frac{k}{L}\mathrm{Nu}_{x=L}$$

$$= \boldsymbol{\frac{5}{4} h_{x=L}}$$

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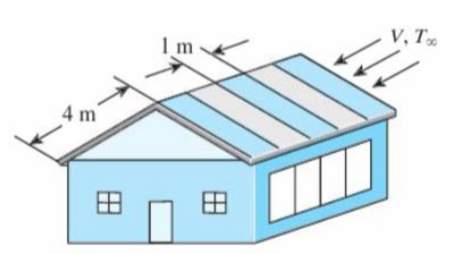
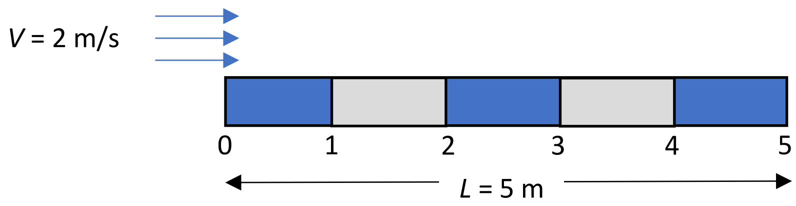
3. Parallel plates form a solar collector that covers a roof, as shown in the figure. The plates are maintained at 15°C. while ambient air at 10°C flows over the roof with V = 2 m/s. Determine the rate of convective heat loss from (a) the first plate and (b) the third plate. (From 7-27) (Ans: 109 W, 34.8W)





Check fluid properties – film temperature:

$$T_f = \frac{1}{2}(T_\infty + T_s) = 12.5\ °\mathrm{C}$$

Properties of air at 12.5°C, 100 kPa: k = 0.02458 W/m K, ν = 1.448 x 10^{-5} m^2/s, Pr = 0.7330

(Table A-15)

Check Reynolds number at end of 3rd plate:

$$\mathrm{Re}_3 = \frac{VL_3}{\nu} = 414\,400 < 5 \times 10^5 \qquad \text{(Laminar Flow)}$$

Flow is laminar for both plates 1 and 3.

Reynolds number for 1st plate

$$\mathrm{Re}_1 = \frac{VL_1}{\nu} = 138\ 000$$

Average Nusselt number for 1st plate

$$\overline{\mathrm{Nu}}_1 = 0.664\mathrm{Re}_1^{0.5}\mathrm{Pr}^{\frac{1}{3}} = 222.5$$

Average heat transfer coefficient for 1st plate

$$\bar{h}_1 = \frac{k}{L_1}\overline{\mathrm{Nu}}_1 = 5.469\ \mathrm{W/m^2K}$$

Heat transfer for 1st plate

$$\dot{Q}_1 = \bar{h}_1 A_1 (T_s - T_\infty) = 109\ \mathrm{W},\ \textbf{where } \mathbf{A_1 = (1 \times 4)m^2}$$

Repeat calculation for heat transfer over ***1st and 2nd*** plate:

$$\mathrm{Re}_2 = \frac{VL_2}{\nu} = 276\ 200$$

$$\overline{\mathrm{Nu}}_2 = 0.664\mathrm{Re}_2^{0.5}\mathrm{Pr}^{\frac{1}{3}} = 314.7$$

$$\bar{h}_2 = \frac{k}{L_2}\overline{\mathrm{Nu}}_2 = 3.867\ \mathrm{W/m^2K}$$

Total heat transfer for 1st and 2nd plate

$$\dot{Q}_2 = \bar{h}_2 A_2 (T_s - T_\infty) = 154.7\ \mathrm{W},\ \textbf{where } \mathbf{A_2 = (2 \times 4)m^2}$$

Repeat calculation for *total* heat transfer from 1st plate to 3rd plate:

$$\overline{\mathrm{Nu}}_3 = 0.664\mathrm{Re}_3^{0.5}\mathrm{Pr}^{\frac{1}{3}} = 385.4$$

$$\bar{h}_3 = \frac{k}{L_3}\overline{\mathrm{Nu}}_3 = 3.158\ \mathrm{W/m^2K}$$

$$\dot{Q}_3 = \bar{h}_3 A_3 (T_s - T_\infty) = 189.4\ \mathrm{W},\ \textbf{where } \mathbf{A_3 = (3 \times 4)m^2}$$

Heat transfer at only the 3rd plate would be the difference between the total heat transfer up to the 3rd plate and the total heat transfer up to the 2nd plate.

$$\boldsymbol{\dot{Q}_{2-3} = \dot{Q}_3 - \dot{Q}_2 = 34.8\ \mathrm{W}}$$

Alternatively, the heat transfer can also be solved by integrating the local heat transfer at the plate from L_2 to L_3 to get the heat transfer rate for the 3rd plate.

$$\dot{Q}_{2-3} = \int_{L_2}^{L_3} h_x W dx (T_s - T_\infty)$$

$$= \int_{L_2}^{L_3} \frac{k}{x} 0.332 \mathrm{Re}_x^{0.5} \mathrm{Pr}^{\frac{1}{3}} W (T_s - T_\infty) dx$$

$$= 0.332 k \, \mathrm{Pr}^{\frac{1}{3}} W (T_s - T_\infty) \left(\frac{V}{\nu}\right)^{0.5} \int_{L_2}^{L_3} x^{-0.5} \, dx$$

$$= 0.332 k \, \mathrm{Pr}^{\frac{1}{3}} W (T_s - T_\infty) \left(\frac{V}{\nu}\right)^{0.5} \times 2\left(\sqrt{L_3} - \sqrt{L_2}\right) = 34.8 \text{ W}$$

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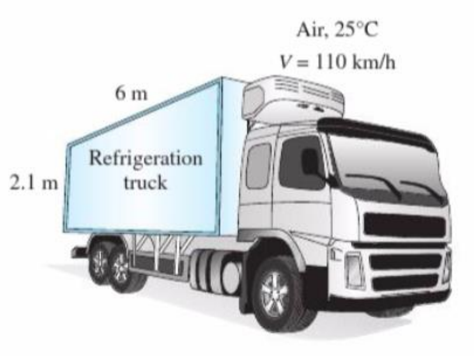
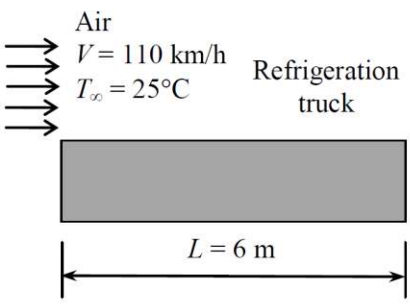
4. Consider a refrigeration truck traveling at 110 km/h at a location where the air temperature is 25°C. The refrigerated compartment of the truck can be considered to be a 2.8-m-wide, 2.1-m-high, and 6-m-long rectangular box, The refrigeration system of the truck can provide 3 tons of refrigeration (i.e., it can remove heat at a rate of 633 kJ/min). The outer surface of the truck is coated with a low-emissivity material, and thus radiation heat transfer is very small. Determine the average temperature of the outer surface of the refrigeration compartment of the truck if the refrigeration system is observed to be operating at half the capacity. Assume the air flow over the entire outer surface to be turbulent and the heat transfer coefficient at the front and rear surfaces to be equal to that on side surfaces. For air properties evaluations assume a film temperature of 25°C. Is this a good assumption? (from 7-32). (Ans:23.8 °C)
(Ans: Now, the film temperature can be determined to be $T_f = (T_s + T_\infty)/2 = (23.8 + 25)/2 = 24.4$°C. This is close to the assumed film temperature of 25°C. We conclude that the assumption was good.)





Fluid properties at the assumed film temperature of 25°C, 100 kPa: k = 0.02551 W/m K, $\nu = 1.562 \times 10^{-5}$ m²/s, Pr = 0.7296 **(Table A-15)**

Air flow is assumed to be fully turbulent over entire surface, so the average Nusselt number is:

$$\overline{\mathrm{Nu}}_L = 0.037 \mathrm{Re}_L^{0.8} \mathrm{Pr}^{1/3}$$

Where,

$$\mathrm{Re}_L = \frac{VL}{\nu} = 1.174 \times 10^7$$

$$\therefore \overline{\mathrm{Nu}}_L = 1.507 \times 10^4 = \frac{\bar{h}_L L}{k}$$

Average heat transfer coefficient:

$$\bar{h}_L = \frac{k}{L} \overline{\mathrm{Nu}}_L = 64.09 \text{ W/m}^2\text{K}$$

The refrigeration system is working to cool off the *steady-state* heat gained from convection at the outer surfaces only

$$\dot{Q}_{conv} = \frac{1}{2}\dot{Q}_{cool}$$

$$= \frac{1}{2} \times 663[\text{kJ/min}] \times \frac{1000}{60} = 5275\ \text{W}$$

Surface temperature of the truck can then be calculated:

$$\dot{Q}_{conv} = \bar{h}_L A_{total}(\boldsymbol{T_\infty - T_s})$$

$$T_s = T_\infty - \frac{\dot{Q}_{conv}}{\bar{h}_L A_{total}} = 23.8\ °\text{C}$$

For calculating the total surface area, the compartment is approximated to that of a cuboid that is 6 m long by 2.8 m wide by 2.1 high. (6 sides total)

With the surface temperature determined to be 23.8°C, the actual film temperature is 24.4°C which is very close to the assumed value of 25°C.

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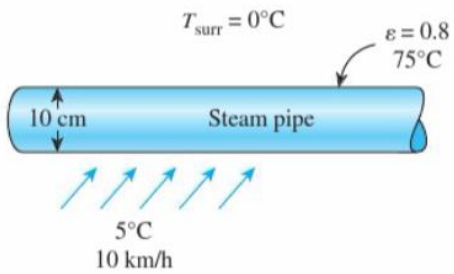
5. During a plant visit. it was noticed that a 12-m-long section of a 10-cm-diameter steam pipe is completely exposed to the ambient air. The temperature measurements indicate that the average temperature of the outer surface of the steam pipe is 75°C when the ambient temperature is 5°C. There are also light winds in the area at 10 km/h. The emissivity of the outer surface of the pipe is 0.8, and the average temperature of the surfaces surrounding the pipe, including the sky, is estimated to be 0°C. Determine the amount of heat lost from the steam during a 10-h-long work day. Steam is supplied by a gas-fired steam generator that has an efficiency of 80 percent. and the plant pays $1.05/therm (note: 1Therm=105,500 kJ) of natural gas. If the pipe is insulated and 90 percent of the heat loss is saved, determine the amount of money this facility will save a year as a result of insulating the steam pipes. Assume the plant operates every day of the year for 10 h. State your assumptions. (from 7-75)

(Ans: 2.361×10^5 kJ/day, $965)



Assumptions: steady-state operating conditions, constant properties for air, atmospheric pressure condition, smooth cylinder

Check fluid properties – film temperature:

$$T_f = \frac{1}{2}(T_\infty + T_s) = 40\ °\text{C}$$

Properties of air at 40°C, 100 kPa: k = 0.02662 W/m K, $\nu = 1.702 \times 10^{-5}$ m²/s, Pr = 0.7255

Check Reynolds number:

$$\text{Re} = \frac{VD}{\nu} = 1.63 \times 10^4 \qquad \textbf{(Laminar Flow)}$$

Churchill-Bernstein equation is used since RePr > 0.2:

$$\mathrm{Nu}_{cyl} = \frac{hD}{k} = 0.3 + \frac{0.62\mathrm{Re}^{0.5}\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{0.25}}\left[1 + \left(\frac{\mathrm{Re}}{282\,000}\right)^{5/8}\right]^{0.8} = 71.19$$

Heat transfer coefficient:

$$h = \frac{k}{D}\mathrm{Nu}_{cyl} = 18.95\ \mathrm{W/m^2K}$$

Heat loss rate by convection:

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = h\pi DL(T_s - T_\infty) = 5.00\ \mathrm{kW}$$

Heat loss rate by radiation:

T_∞ and T_{surr} are of different values

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^4 - T_{surr}^4) = 1.56\ \mathrm{kW}$$

Total heat loss in one day:

$$Q_{day} = (\dot{Q}_{conv} + \dot{Q}_{rad})t_{day}$$

$$= (\dot{Q}_{conv} + \dot{Q}_{rad}) \times 10 \times 3600 = \mathbf{2.36 \times 10^5\ kJ}$$

Total heat loss in a year

$$Q_{year} = 365Q_{day} = 8.62 \times 10^7\ \mathrm{kJ}$$

Steam generator has an efficiency of 80%

$$\frac{Q_{year}}{Q_{gas}} = 0.8$$

Amount of gas used per year (in terms of energy):

$$Q_{gas} = \frac{Q_{year}}{0.8}$$

$$= 10.78 \times 10^7\ \mathrm{kJ} = \frac{10.78 \times 10^7}{105\,500}\mathrm{therm} = 1021\ \mathrm{therm}$$

Since insulation can save 90% of the heat loss, the energy saved is:

$$Q_{saved} = 0.9Q_{gas}$$

$$= 918.9\ \mathrm{therm}$$

Cost savings per year = 918.9 x \$1.05 = \$965
