CG1111A:
Engineering Principles & Practice I

Basics of Operational Amplifiers

NUS
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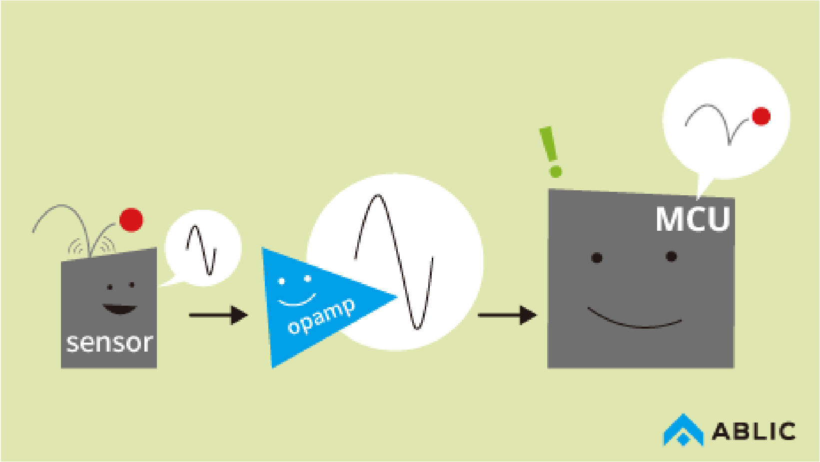
Learning Outcomes

Understand:

- How op-amps **operate**
- The difference between **inverting** and **non-inverting** amplifiers
- How to adjust the **amplification gains** of inverting & non-inverting amplifiers
- How to **analyze** circuits containing op-amps
- The **practical considerations** of op-amp circuits

What is an “Operational Amplifier”?

- An “operational amplifier (**op-amp**)” is an integrated circuit that can amplify weak electric signals



Credit: Image from ablic.com

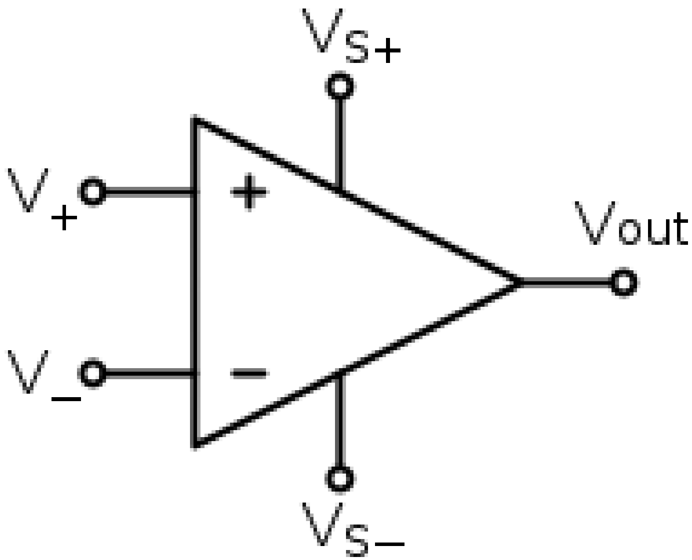
- It has **two input** signal terminals and **one output** signal terminal

- It amplifies and outputs the **voltage difference** between the two input terminals

Op-Amp Terminals

Positive Power Supply
(Positive Rail)

Non-Inverting
Input Terminal

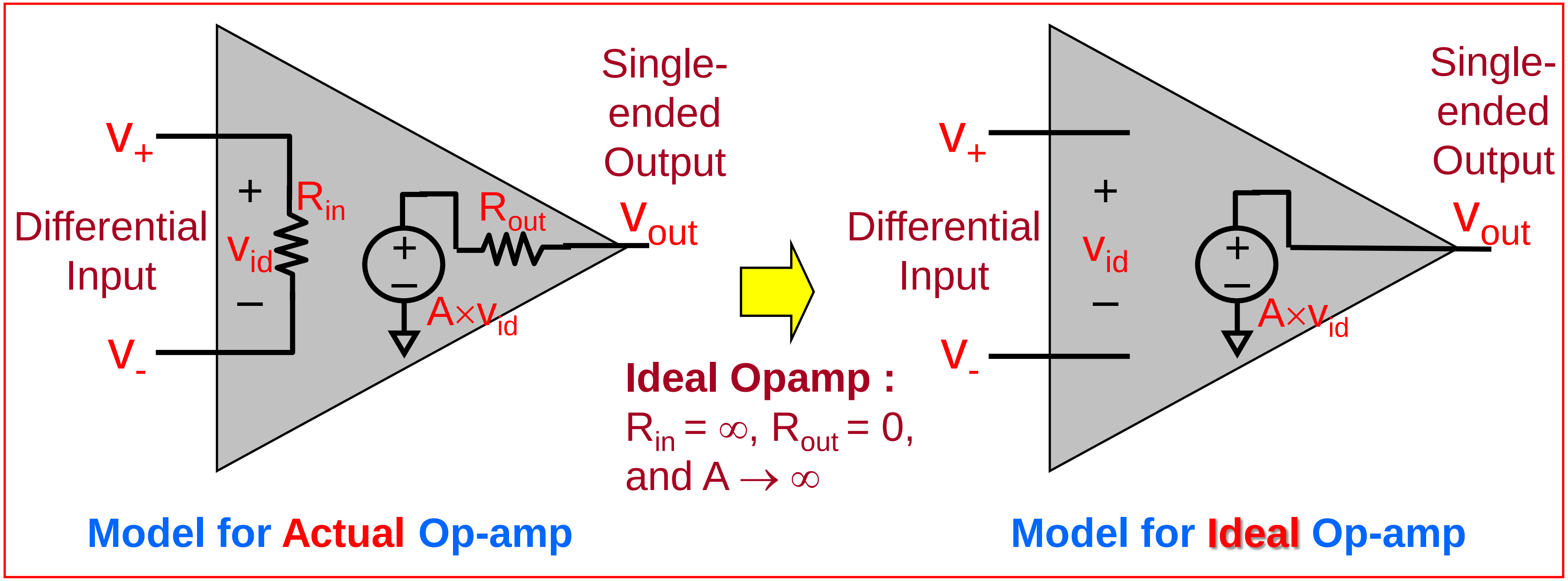


Output Terminal

Inverting Input
Terminal

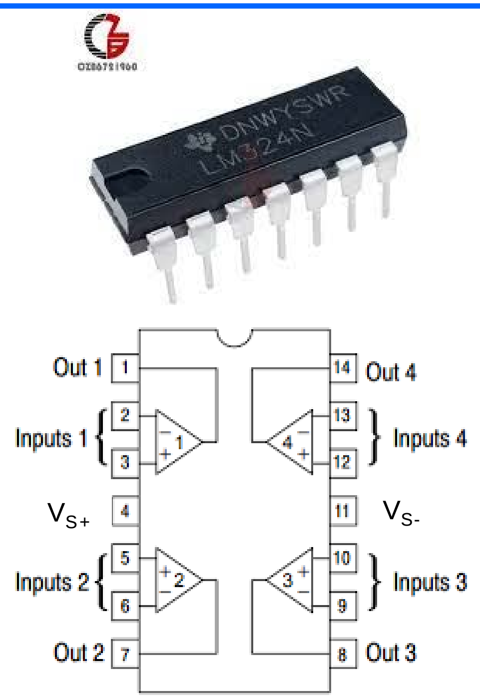
Negative Power Supply
(Negative Rail)

Op-Amp Equivalent Circuit

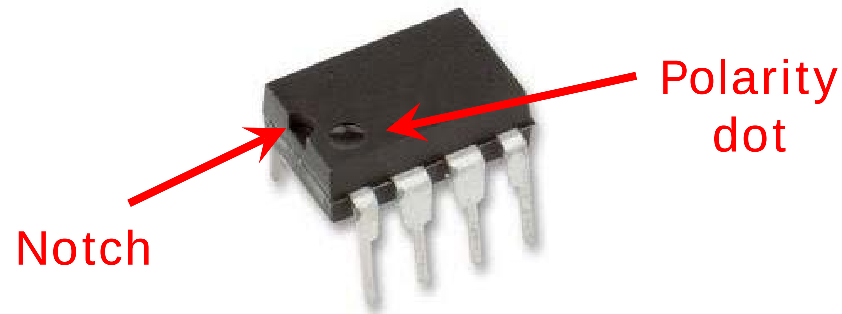


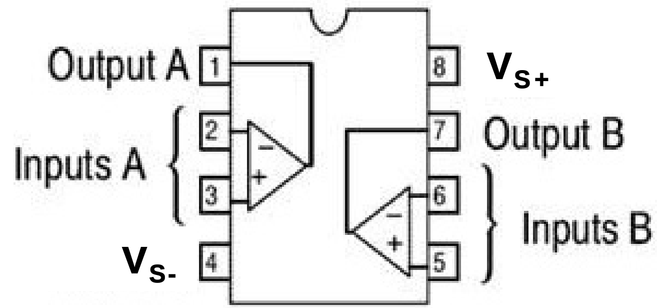
- ***A*** is the open-loop voltage gain
  - It is very large, approaching infinity
- ***R_{in}*** is the input impedance (very large) & ***R_{out}*** is the output impedance (very small)
- To **simplify** analysis, we always assume **infinite** ***R_{in}*** and ***A***, and **zero** ***R_{out}***

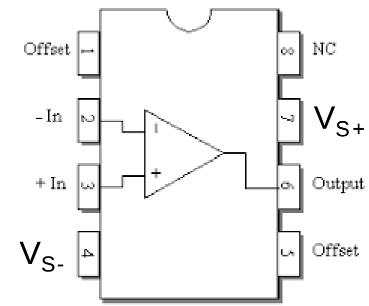
What is an Op-amp IC Chip?



Quad opamp IC







Dual op-amp IC

Single op-amp IC

- Notch or polarity dot indicate the location of pin 1
- Quad and dual op-amp IC have 4 and 2 independent op-amps on the same chip sharing the same supplies (V_{S+} and V_{S-})

How to Read Its Datasheet?

Single supply:
MIN: 3 V
MAX: 30 V

Dual supply:
MIN: ±1.5 V
MAX: ±15 V

			MIN	MAX	UNIT
V_S	Supply voltage, V_S= ([V+] – [V–])	LM358B, LM358BA, LM2904B, LM2904BA	3	36	V
		LM158, LM258, LM358, LM158A, LM258A, LM358A, LM2904V	3	30	
		LM2904	3	26	

<table>
<tr><th colspan="2">PARAMETER</th><th colspan="2">TEST CONDITIONS</th><th>MIN</th><th>TYP</th><th>MAX</th><th>UNIT</th></tr>
<tr><td colspan="8">INPUT IMPEDANCE</td></tr>
<tr><td>Z_{ID}</td><td>Differential</td><td colspan="2"></td><td></td><td>10 || 0.1</td><td></td><td>MΩ|| pF</td></tr>
<tr><td colspan="8">OPEN-LOOP GAIN</td></tr>
<tr><td rowspan="2">A_{OL}</td><td rowspan="2">Open-loop voltage gain</td><td rowspan="2">V_S = 15 V; V_O = 1 V to 11 V; R_L ≥ 10 kΩ, connected to (V–)</td><td></td><td>70</td><td>140</td><td></td><td>V/mV</td></tr>
<tr><td>T_A = –40°C to +85°C</td><td>35</td><td></td><td></td><td>V/mV</td></tr>
<tr><td>R_O</td><td>Open-loop output resistance</td><td colspan="2">f = 1 MHz, I_O = 0 A</td><td></td><td>300</td><td></td><td>Ω</td></tr>
</table>

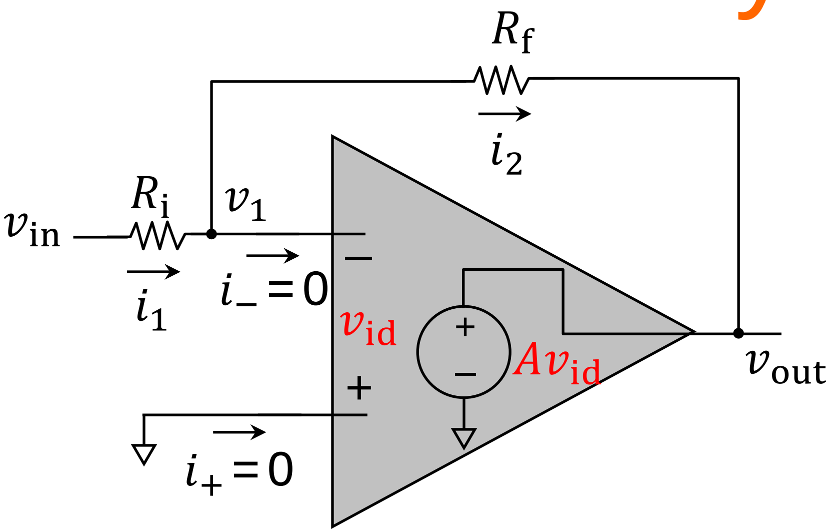
140 V/mV = 140000

- https://www.ti.com/lit/gpn/lm358
- Identify the key parameters, such as V_S (Supplies), A_{OL} (Gain), Z_{ID} (Input Impedance) and R_O (Output Impedance)
- Parameters are obtained with certain test conditions
- MIN, TYP, MAX specify either allowable range (e.g., V_S), or potential parameter fluctuations (e.g., A_{OL})
- **Unit is important** for all engineering parameters

Typical Op-amp Parameters

Parameter	Variable	Typical Ranges	Ideal Values
Voltage Gain	A	10^5 to 10^8	∞
Input Impedance	R_{in}	10^5 to 10^8 Ω	∞ Ω
Output Impedance	R_{out}	A few tens to a few hundreds Ω	0 Ω
Supply Voltage	V_{S+} or V_{cc} V_{S-} or V_{EE} or $-V_{cc}$	3 to 30 V –30 to 0 V	N/A N/A

How to Analyze Op-amp Circuit?



"Transfer Function" for Op-amp Circuit

$$v_{id} = 0 - v_1 \quad \text{and} \quad v_{out} = Av_{id} = A(-v_1)$$

$$\Rightarrow v_1 = -\frac{v_{out}}{A}$$

$$\boxed{\frac{v_{in} - v_1}{R_i} = \frac{v_1 - v_{out}}{R_f}}$$

KCL:
$i_- = 0 \Rightarrow i_1 = i_2$

$$\Rightarrow \frac{v_{in} - \left(-\frac{v_{out}}{A}\right)}{R_i} = \frac{\left(-\frac{v_{out}}{A}\right) - v_{out}}{R_f}$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = -\frac{R_f}{R_i}\frac{1}{\boxed{\frac{R_i + R_f}{AR_i}} + 1} \approx -\frac{R_f}{R_i}$$

Transfer Function

<<1 ⇒ negligible even if A~10000
∴ A → ∞ is a good approximation

Important Observations:

- **No** current flows into the op-amp, i.e., i_- and $i_+ = 0$
- If A is large and v_{out} is finite
  $\Rightarrow v_{id} \approx 0 \Rightarrow v_+ \approx v_-$
  ⇒ Virtual short
- Analysis is only applicable to closed-loop with **–ve** feedback

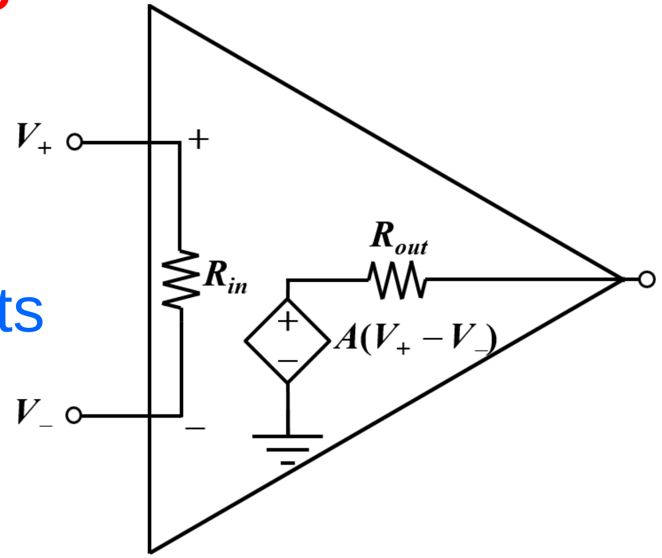
Real Wire

V_A ●—● V_B $\Rightarrow V_A = V_B$ (Real short)

V_C ● ● V_D $\Rightarrow V_C \approx V_D$ (Virtual short)

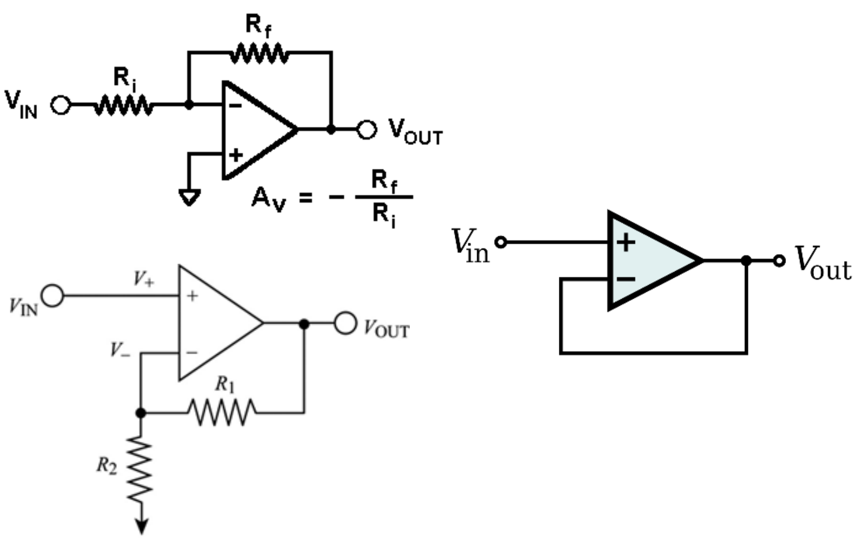
Op-amp Golden Rules

- Rule 1: In a closed loop with –ve feedback, the output attempts to do whatever is necessary to make the voltage difference between the inputs zero
  - The voltage gain of a real op-amp is so high that a fraction of a mV difference between the V_+ & V_- inputs will achieve the desired finite output
  - **"Virtual short"**, i.e., $V_+ \approx V_-$

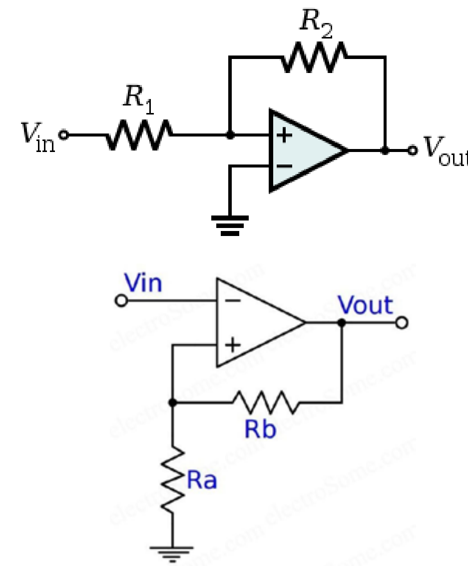


- Rule 2: The inputs draw no current
  - The ideal op-amp has very large input impedance (R_{in}). Thus, the current drawn at the two input terminals ~0.

What is Closed Loop with –ve Feedback?



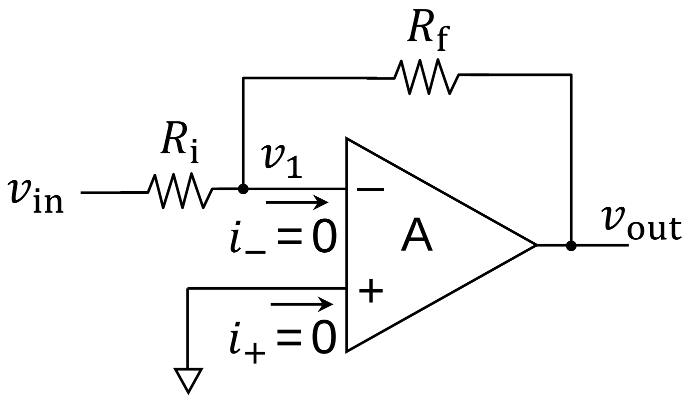
Negative Feedback



Positive Feedback

- Closed loop: There is connection between output and input
- -ve feedback: The output is fed back to the input in such a way to **reduce** the **output fluctuations**

Inverting Amplifier



$$\boxed{\frac{v_{out}}{v_{in}} = -\frac{R_f}{R_i}}$$

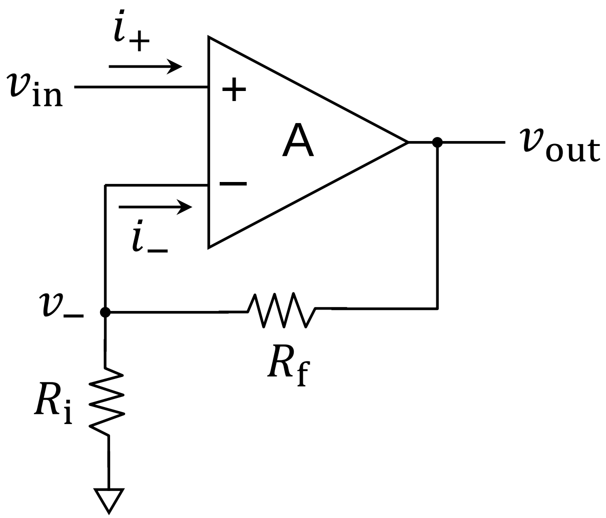
Derivation based on Golden Rules

$$v_1 = v_- \approx v_+ = 0 \quad [\because \text{ Virtual Short}]$$

$$\frac{v_{in} - v_1}{R_i} = \frac{v_1 - v_{out}}{R_f} \quad [\because \ i_- \approx 0]$$

$$\Rightarrow \frac{v_{in}}{R_i} = \frac{-v_{out}}{R_f} \quad \Rightarrow \quad \frac{v_{out}}{v_{in}} = -\frac{R_f}{R_i}$$

Non-Inverting Amplifier



$$\boxed{\frac{v_{\mathrm{out}}}{v_{\mathrm{in}}} = 1 + \frac{R_{\mathrm{f}}}{R_{\mathrm{i}}}}$$

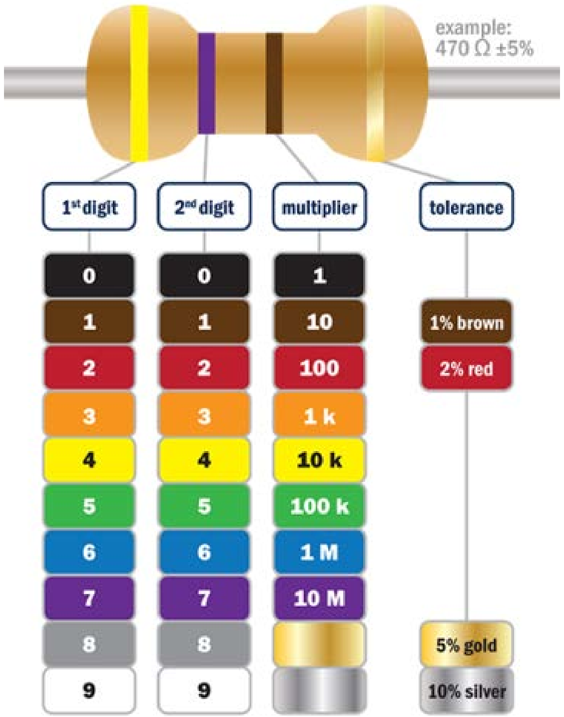
Derivation based on Golden Rules

$$v_- \approx v_+ = v_{\mathrm{in}} \quad [\because \quad \text{Virtual Short}]$$

$$v_- = v_{\mathrm{out}} \times \frac{R_{\mathrm{i}}}{R_{\mathrm{i}} + R_{\mathrm{f}}} \approx v_{\mathrm{in}} \quad [\because \quad i_- \approx 0]$$

$$\Rightarrow \frac{v_{\mathrm{out}}}{v_{\mathrm{in}}} = \left(1 + \frac{R_{\mathrm{f}}}{R_{\mathrm{i}}}\right)$$

Significance of Ratio



$$\frac{v_{\text{out}}}{v_{\text{in}}} = -\frac{R_{\text{f}}}{R_{\text{i}}}$$

If R_f and R_i varies by 5% each, the resulting gain could vary by as much as 10% from desired value!

How to overcome the above issue?

Use resistors from **same batch** (match very well)! For example, if the desired $R_{\text{f}}/R_{\text{i}}$ ratio is 4, we use 5 identical resistors: R_{f} uses 4 resistors, and R_{i} uses 1 resistor. An accurate gain of 4 can be obtained!

$\Rightarrow$ Principle exploited in IC design

- Inverting and non-inverting amplifiers are also called feedback amplifiers
- One key characteristic is their gain dependency on resistance ratio
- This is desirable as accurate gain can be obtained despite component tolerance and temperature drift

Independent of Temperature Drift

$$\mathbf{R = R_{ref}\,[1 + \alpha(T - T_{ref})]}$$

Where,

R = Conductor resistance at temperature "T"

$\mathbf{R_{ref}}$ = Conductor resistance at reference temperature T_{ref}, usually 20°C, but sometimes 0°C.

$\boldsymbol{\alpha}$ = Temperature coefficient of resistance for conductor material.

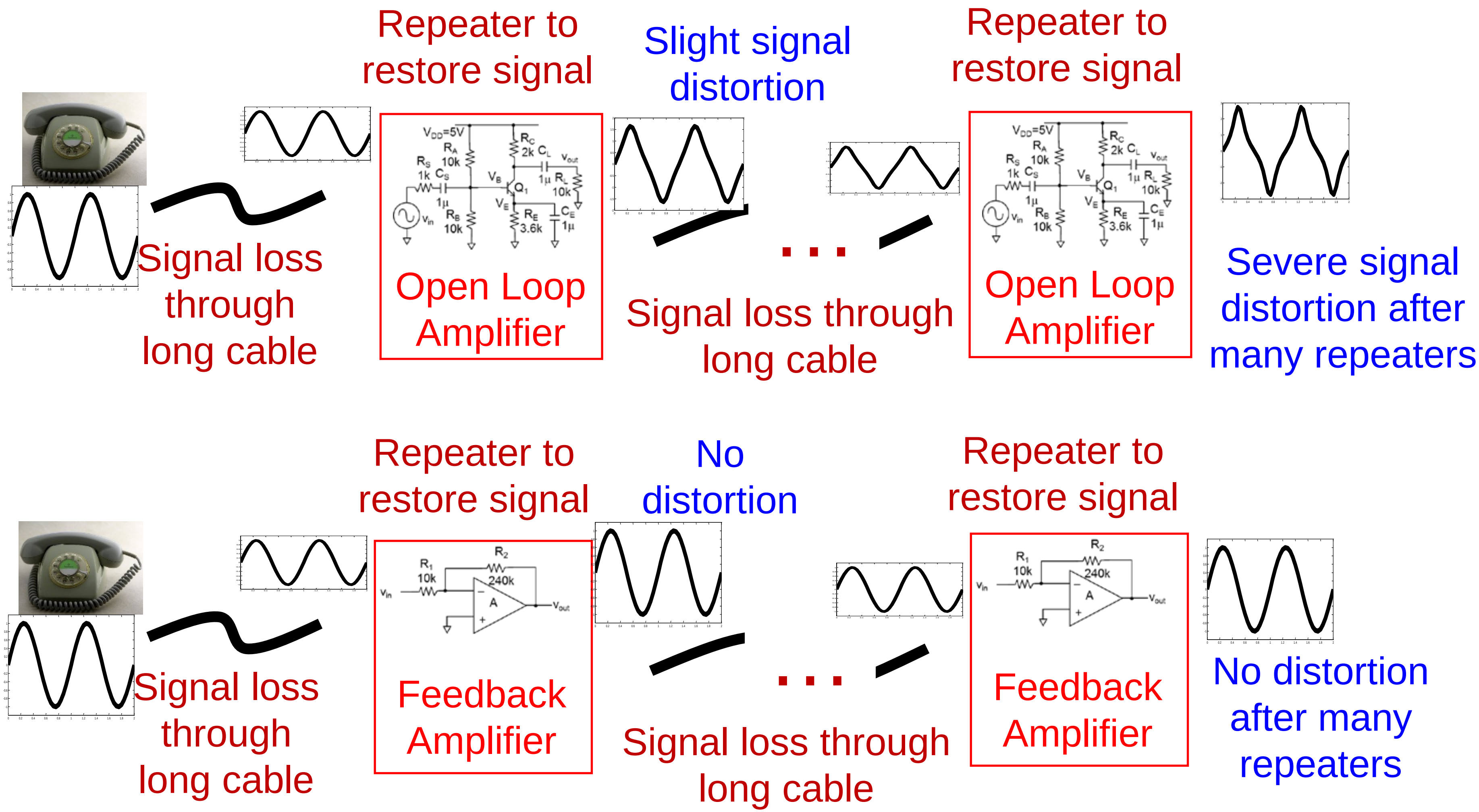
T = Conductor temperature in degrees Celcius.

$\mathbf{T_{ref}}$ = Reference temperature that α is specified at for the conductor material

$$\frac{v_{\text{out}}}{v_{\text{in}}} = -\frac{R_{\text{f0}}[1+\alpha(T-T_{\text{ref}})]}{R_{\text{i0}}[1+\alpha(T-T_{\text{ref}})]} = -\frac{R_{\text{f0}}}{R_{\text{i0}}}$$

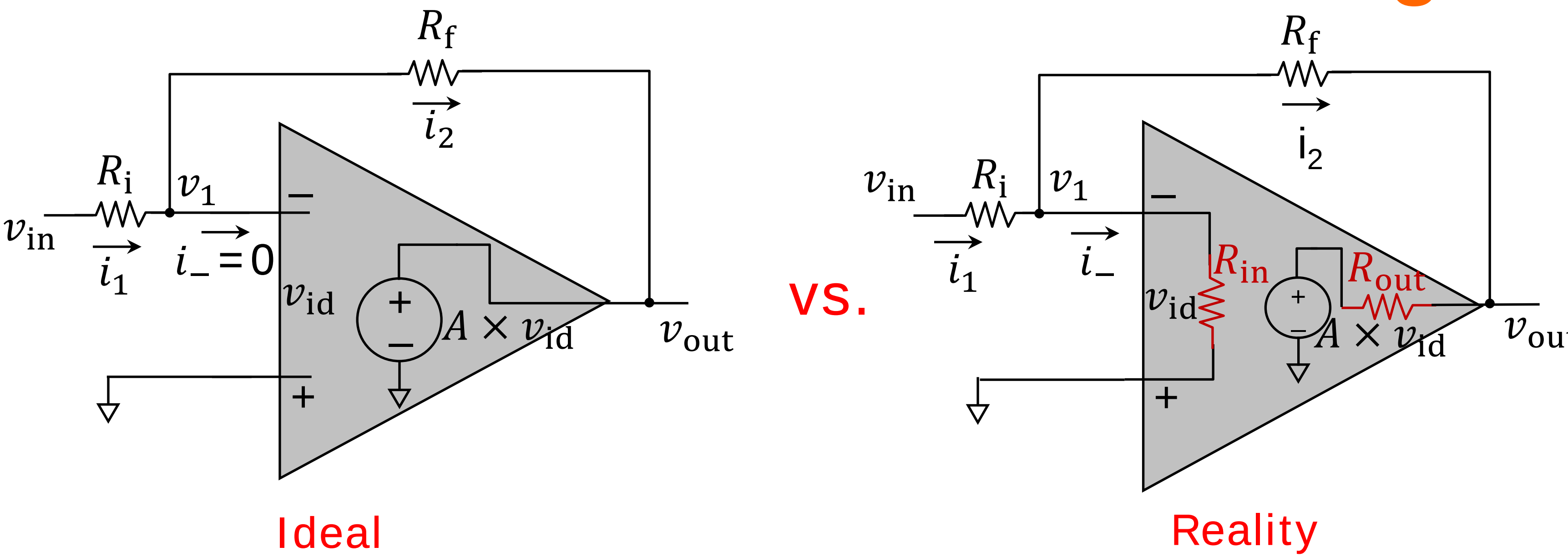
Resistors exhibit temperature variation. If resistor of same type with identical temperature coefficients are used, the resulting gain would not drift with temperature.

Significance of Feedback



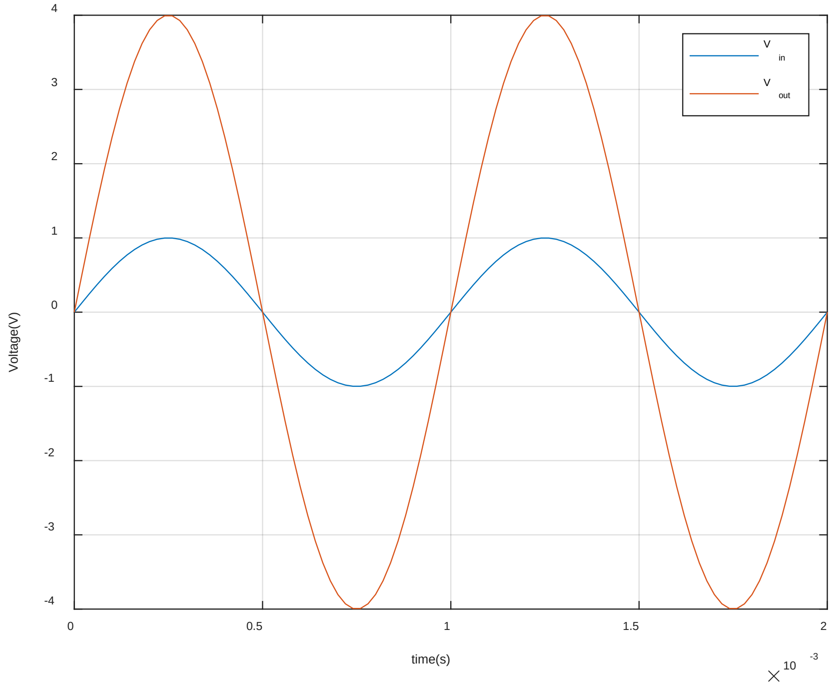
- Harold Stephen Black invented negative feedback amplifier in 1928

Permissible Resistor Range

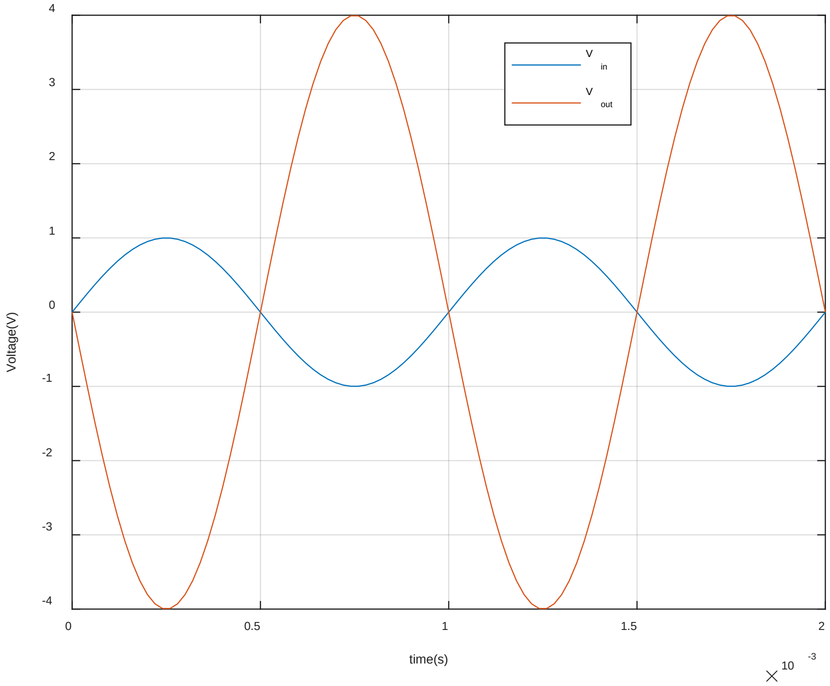


- In practice, $i_- \ll i_1$, and we can assume $i_- \approx 0$
- However, if R_i is **comparable to R_{in}**, the current $i_- = \frac{v_1}{R_{in}}$ would be comparable to $i_1 = \frac{v_{in} - v_1}{R_i}$ (and v_{in} could be comparable to v_1) and we **cannot assume $i_- \approx 0$** in our analysis
- Also, if R_f is comparable to R_{out}, v_{out} would not be equal to Av_{id}
- Hence, R_i and R_f must be chosen to be $\gg R_{out}$, and $\ll R_{in}$
- Rule of thumb: R_i and R_f should be within $(10 \times R_{out}, 0.1 \times R_{in})$

Positive vs Negative Gain



Gain of +4

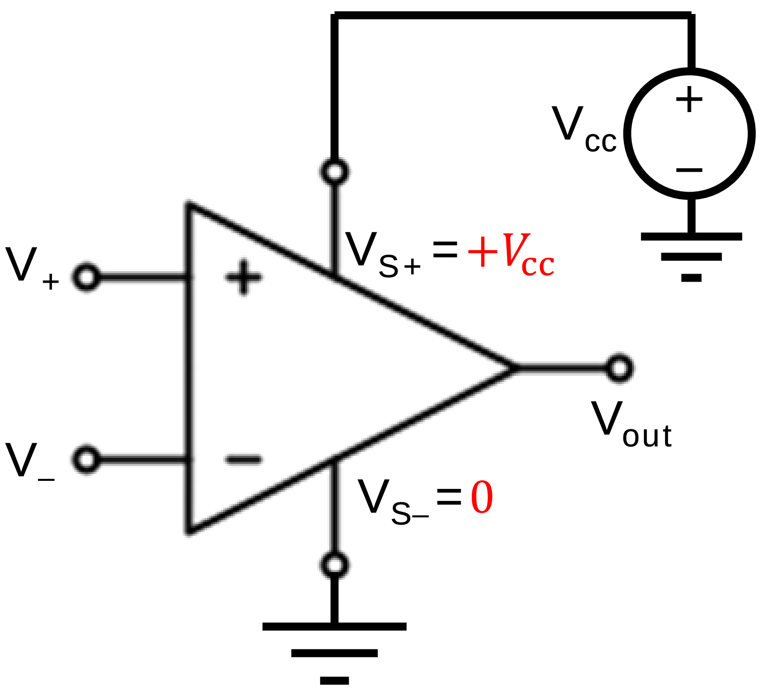


Gain of -4

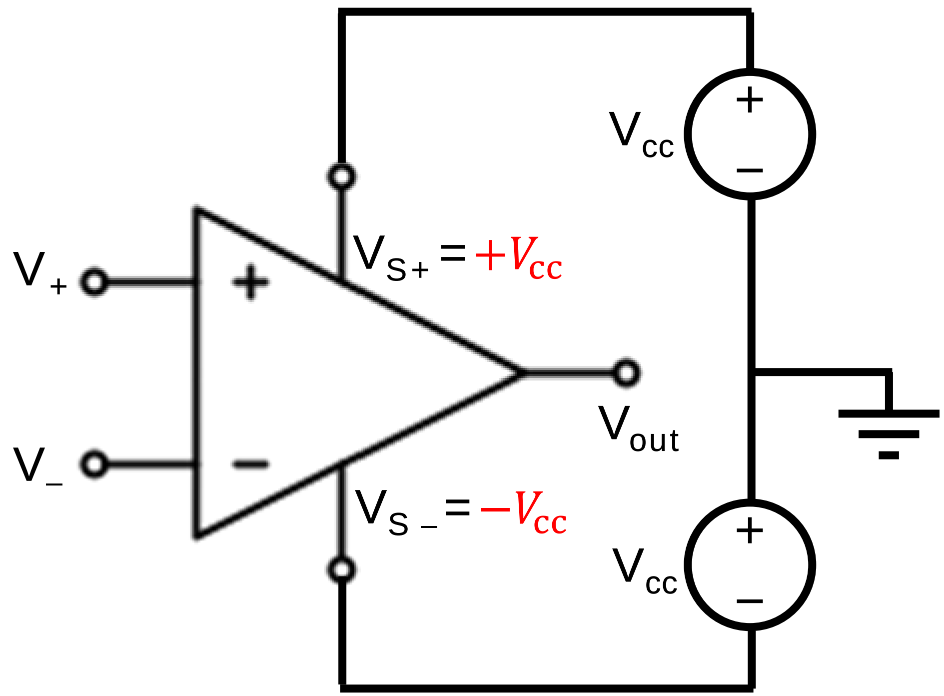
- **Negative** gain **does not mean loss**
- Negative gain only means output is the **inverted version** of input (180° phase shift)
- Only when gain magnitude is **less than 1**, there is **loss**

Single Supply Vs Dual Supply

- An op-amp can be powered either by a Single Power Supply or a Dual Power Supply



Single Power Supply
$(V_{cc} > V_{cc} - V_{ovh} \geq V_{out} \geq V_{ovh})$



Dual Power Supply
$(V_{cc} > V_{cc} - V_{ovh} \geq V_{out} \geq -V_{cc} + V_{ovh} > -V_{cc})$

THANK YOU