INTRODUCTION TO MECHATRONICS AND MEASUREMENT SYSTEMS

5th edition

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SOLUTIONS MANUAL

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Solutions Manual

This manual contains solutions to the end-of-chapter problems in the fifth edition of "Introduction to Mechatronics and Measurement Systems." Only a few of the open-ended problems that do not have a unique answer are left for your creative solutions. More information, including an example course outline, a suggested laboratory syllabus, Mathcad/Matlab files for examples in the book, and other supplemental material are provided on the book website at:

mechatronics.colostate.edu

We have class-tested the textbook for many years, and it should be relatively free from errors. However, if you notice any errors or have suggestions or advice concerning the textbook's content or approach, please feel free to contact me via e-mail at David.Alciatore@colostate.edu. I will post corrections for reported errors on the book website.

Thank you for choosing my book. I hope it helps you provide your students with an enjoyable and fruitful learning experience in the exciting cross-disciplinary subject of mechatronics.

2.1 D = 0.06408 in = 0.001628 m.

$$A = \frac{\pi D^2}{4} = 2.082 \times 10^{-6}$$

$\rho = 1.7 \text{ x } 10^{-8}\ \Omega\text{m}$, L = 1000 m

$$R = \frac{\rho L}{A} = 8.2\Omega$$

2.2

(a) $R_1 = 21 \times 10^4 \pm 20\%$ so $168\text{k}\Omega \le R_1 \le 252\text{k}\Omega$

(b) $R_2 = 07 \times 10^3 \pm 20\%$ so $5.6\text{k}\Omega \le R_2 \le 8.4\text{k}\Omega$

(c) $R_s = R_1 + R_2 = 217\text{k}\Omega \pm 20\%$ so $174\text{k}\Omega \le R_s \le 260\text{k}\Omega$

(d) $$R_p = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{p_{MIN}} = \frac{R_{1_{MIN}} R_{2_{MIN}}}{R_{1_{MIN}} + R_{2_{MIN}}} = 5.43\text{k}\Omega$$

$$R_{p_{MAX}} = \frac{R_{1_{MAX}} R_{2_{MAX}}}{R_{1_{MAX}} + R_{2_{MAX}}} = 8.14\text{k}\Omega$$

2.3 $R_1 = 10 \times 10^2$, $R_2 = 25 \times 10^1$

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{(10 \times 10^2)(25 \times 10^1)}{10 \times 10^2 + 25 \times 10^1} = 20 \times 10^1$$

a = 2 = red, b = 0 = black, c = 1 = brown, d = gold

2.4 In series, the trim pot will add an adjustable value ranging from 0 to its maximum value to the original resistor value depending on the trim setting. When in parallel, the trim pot could be 0Ω perhaps causing a short. Furthermore, the trim value will not be additive with the fixed resistor.

2.5 When the last connection is made, a spark occurs at the point of connection as the completed circuit is formed. This spark could ignite gases produced in the battery. The negative terminal of the battery is connected to the frame of the car, which serves as a ground reference throughout the vehicle.

2.6 No, as long as you are consistent in your application, you will obtain correct answers. If you assume the wrong current direction, the result will be negative.

2.7 Place two 100Ω resistors in parallel and you immediately have a 50Ω resistance.

2.8 Put two 50Ω resistors in series: 50Ω + 50Ω = 100Ω

2.9 Put a 100Ω resistor in series with the parallel combination of two 100Ω resistors:
$100\Omega + (100\Omega * 100\Omega)/(100\Omega + 100\Omega) = 150\Omega$

2.10 From KCL, $I_s = I_1 + I_2 + I_3$

so from Ohm's Law $\frac{V_s}{R_{eq}} = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3}$

Therefore, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ so $R_{eq} = \frac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2}$

2.11 From Ohm's Law and Question 2.10, $V = \frac{I_s}{R_{eq}} = \frac{I_s}{\frac{R_2R_3 + R_1R_3 + R_1R_2}{R_1R_2R_3}}$

and for one resistor, $V = I_1R_1$

Therefore, $I_1 = \left(\frac{R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2}\right)I_s$

2.12 $\lim_{R_1 \to \infty}\left(\frac{R_1R_2}{R_1 + R_2}\right) = \frac{R_1R_2}{R_1} = R_2$

2.13 $I = C_{eq}\frac{dV}{dt} = C_1\frac{dV_1}{dt} = C_2\frac{dV_2}{dt}$

From KVL,

$$V = V_1 + V_2$$

so

$$\frac{dV}{dt} = \frac{dV_1}{dt} + \frac{dV_2}{dt}$$

Therefore,

$$\frac{I}{C_{eq}} = \frac{I}{C_1} + \frac{I}{C_2} \quad \text{so} \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

2.14 $V = V_1 = V_2$

$I_1 = C_1\frac{dV_1}{dt} = C_1\frac{dV}{dt}$ and $I_2 = C_2\frac{dV_2}{dt} = C_2\frac{dV}{dt}$

From KCL,

$$I = I_1 + I_2 = C_1\frac{dV}{dt} + C_2\frac{dV}{dt} = \frac{dV}{dt}(C_1 + C_2)$$

Since $I = C_{eq}\frac{dV}{dt}$

$$C_{eq} = C_1 + C_2$$

2.15 $I = I_1 = I_2$

From KVL,

$$V = V_1 + V_2 = L_1\frac{dI}{dt} + L_2\frac{dI}{dt} = \frac{dI}{dt}(L_1 + L_2)$$

Since $V = L_{eq}\frac{dI}{dt}$

$$L_{eq} = L_1 + L_2$$

2.16 $V = L\frac{dI}{dt} = L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt}$

From KCL, $I = I_1 + I_2$ so $\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$

Therefore, $\frac{V}{L} = \frac{V}{L_1} + \frac{V}{L_2}$ so $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ or $L = \frac{L_1 L_2}{L_1 + L_2}$

2.17 $V_o = 1V$, regardless of the resistance value.

2.18 From Voltage Division, $V_o = \frac{40}{10 + 40}(5 - 15) = -8V$

2.19 Combining R_2 and R_3 in parallel,

$$R_{23} = \frac{R_2R_3}{R_2 + R_3} = \frac{2(3)}{2+3} = 1.2\text{k}$$

and combining this with R_1 in series,

$$R_{123} = R_1 + R_{23} = 2.2\text{k}$$

(a) Using Ohm's Law,

$$I_1 = \frac{V_{in}}{R_{123}} = \frac{5\text{V}}{2.2\text{k}} = 2.27\text{mA}$$

(b) Using current division,

$$I_3 = \frac{R_2}{R_2 + R_3}I_1 = \frac{2}{5}2.27\text{mA} = 0.909\text{mA}$$

(c) Since R_2 and R_3 are in parallel, and since V_{in} divides between R_1 and R_{23},

$$V_3 = V_{23} = \frac{R_{23}}{R_1 + R_{23}}V_{in} = \frac{1.2}{2.2}5\text{V} = 2.73\text{V}$$

2.20

(a) From Ohm's Law,

$$I_4 = \frac{V_{out} - V_1}{R_{24}} = \frac{14.2\text{V} - 10\text{V}}{6\text{k}} = 0.7\text{mA}$$

(b) $V_5 = V_6 = V_{56} = V_{out} - V_2 = 14.2\text{V} - 20\text{V} = -5.8\text{V}$

2.21

(a) $R_{45} = R_4 + R_5 = 5\text{k}\Omega$

$$R_{345} = \frac{R_3R_{45}}{R_3 + R_{45}} = 1.875\text{k}\Omega$$

$$R_{2345} = R_2 + R_{345} = 3.875\text{k}\Omega$$

$$R_{eq} = \frac{R_1R_{2345}}{R_1 + R_{2345}} = 0.795\text{k}\Omega$$

(b) $$V_A = \frac{R_{345}}{R_2 + R_{345}}V_s = 4.84\text{V}$$

(c) $I_{345} = \dfrac{V_A}{R_{345}} = 2.59\text{mA}$

$$I_5 = \frac{R_3}{R_3 + R_{45}} I_{345} = 0.97\text{mA}$$

2.22 This circuit is identical to the circuit in Question 2.21. Only the resistance values are different:

(a) $R_{45} = R_4 + R_5 = 4\text{k}\Omega$

$$R_{345} = \frac{R_3 R_{45}}{R_3 + R_{45}} = 2.222\text{k}\Omega$$

$$R_{2345} = R_2 + R_{345} = 6.222\text{k}\Omega$$

$$R_{eq} = \frac{R_1 R_{2345}}{R_1 + R_{2345}} = 1.514\text{k}\Omega$$

(b) $V_A = \dfrac{R_{345}}{R_2 + R_{345}} V_s = 3.57\text{V}$

(c) $I_{345} = \dfrac{V_A}{R_{345}} = 1.61\text{mA}$

$$I_5 = \frac{R_3}{R_3 + R_{45}} I_{345} = 0.89\text{mA}$$

2.23 Using superposition,

$$V_{R2_1} = \frac{R_2}{R_1 + R_2} V_1 = 0.909\text{V}$$

$$V_{R2_2} = \frac{R_1}{R_1 + R_2} i_1 = 9.09\text{V}$$

$$V_{R2} = V_{R2_1} + V_{R2_2} = 10.0\text{V}$$

2.24 $R_{45} = \dfrac{R_4 R_5}{R_4 + R_5} = 0.5\text{k}\Omega$

$$I = \frac{V_1 - V_2}{R_1 + R_2} = -0.5\text{mA}$$

$$V_A = \frac{R_{45}}{R_3 + R_{45}}(V_1 - V_2) = -0.238V$$

2.25 $R_{45} = R_4 + R_5 = 9k\Omega$

$$R_{345} = \frac{R_3 R_{45}}{R_3 + R_{45}} = 2.25k\Omega$$

$$R_{2345} = R_2 + R_{345} = 4.25k\Omega$$

$$R_{eq} = \frac{R_1 R_{2345}}{R_1 + R_{2345}} = 0.81k\Omega$$

2.26 Using loop currents, the KVL equations for each loop are:

$$V_1 - I_{out}R_1 = 0$$

$$V_2 - I_5R_5 - I_3R_3 - V_1 = 0$$

$$-I_6R_6 + I_5R_5 = 0$$

$$I_3R_3 - I_{24}R_4 - I_{24}R_2 = 0$$

and using selected KCL node equations, the unknown currents are related according to:

$$I_{out} = I_2 + I_3 + I_{V_1}$$

$$I_{V_1} = I_{out} - (I_5 + I_6)$$

$$I_3 = I_5 + I_6 - I_{24}$$

This is now 7 equations in 7 unknowns, which can be solved for I_{out} and I_6. The output voltage is then given by:

$$V_{out} = V_2 - I_6R_6$$

2.27 Applying Ohm's Law to resistor combination R_{24} gives:

$$I_4 = \frac{V_{out} - V_1}{R_{24}} = \frac{4.2V}{6k\Omega} = 0.7mA$$

The voltage across R_5 is:

$$V_5 = V_6 = V_{56} = V_+ - V_- = V_{out} - V_2 = -5.8V$$

2.28 It will depend on your instrumentation, but the oscilloscope typically has an input impedance of 1 MΩ.

2.29 Since the input impedance of the oscilloscope is 1 MΩ, the impedance of the source will be in parallel, and the oscilloscope impedance will affect the measured voltage. Draw a sketch of the equivalent circuit to convince yourself.

2.30 $R_{23} = \frac{R_2 R_3}{R_2 + R_3}$

$V_{out} = \frac{R_{23}}{R_1 + R_{23}} V_{in}$

(a) $R_{23} = 9.90\text{k}\Omega$, $V_{out} = 0.995 V_{in}$

(b) $R_{23} = 333\text{k}\Omega$, $V_{out} = 1.00 V_{in}$

When the impedance of the load is lower (10k vs. 500k), the accuracy is not as good.

2.31 $V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$

(a) $V_{out} = \frac{10}{10.05} V_{in} = 0.995 V_{in}$

(b) $V_{out} = \frac{500}{500.05} V_{in} = 0.9999 V_{in}$

For a larger load impedance, the output impedance of the source less error.

2.32 The theoretical value of the voltage is:

$$V_{theor} = \frac{R}{R + R} V_s = \frac{1}{2} V_s$$

The equivalent resistance of the parallel combination of the resistor and the voltmeter input impedance is:

$$\frac{R \cdot 5R}{R + 5R} = \frac{5}{6} R$$

And the measured voltage across this resistance is:

$$V_{meas} = \frac{\frac{5}{6}R}{R + \frac{5}{6}R} V_s = \frac{5}{11} V_s$$

Therefore, the percent error in the measurement is:

$$\frac{V_{meas} - V_{theor}}{V_{theor}} = -9\%$$

2.33 It will depend on the supply; check the specifications before answering.

2.34 With the voltage source shorted, all three resistors are in parallel, so, from Question 2.10:

$$R_{TH} = \frac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2}$$

2.35 $V_{in} = 5\langle 45°\rangle$

Combining R_2 and L in series and the result in parallel with C gives:

$$Z_{R_2LC} = \frac{(R_2 + Z_L)Z_C}{(R_2 + Z_L) + Z_C} = 1860.52\langle -60.25°\rangle = 923.22 - 1615.30j$$

Using voltage division,

$$V_C = \frac{Z_{R_2LC}}{R_1 + Z_{R_2LC}}V_{in}$$

where

$$R_1 + Z_{R_2LC} = 1000 + 923.22 - 1615.30j = 2511.57\langle -40.02°\rangle$$

so

$$V_C = \frac{1860.52\langle -60.25°\rangle}{2511.57\langle -40.02°\rangle}5\langle 45°\rangle = 3.70\langle 24.8°\rangle = 3.70\langle 0.433\text{rad}\rangle$$

Therefore,

$$V_C(t) = 3.70\cos(3000t + 0.433)V$$

2.36 With steady state dc V_s, C is open circuit. So

$$V_C = V_s = 10V \quad \text{so} \quad V_{R_1} = 0V \quad \text{and} \quad V_{R_2} = V_s = 10V$$

2.37

(a) In steady state dc, C is open circuit and L is short circuit. So

$$I = \frac{V_s}{R_1 + R_2} = 0.025mA$$

(b) $\omega = \pi$

$$Z_C = \frac{-j}{\omega C} = \frac{-10^6}{\pi}j = \frac{10^6}{\pi}\angle -90°\Omega$$

$$Z_{LR_2} = Z_L + R_2 = j\omega L + R_2 = (10^5 + 20\pi j)\Omega = 10^5\angle 0.036°\Omega$$

$$Z_{CLR_2} = \frac{Z_C Z_{LR_2}}{Z_C + Z_{LR_2}} = (91040 - 28550j)\Omega = 95410\angle{-17.4°}\Omega$$

$$Z_{eq} = R_1 + Z_{CLR_2} = (191040 - 28550j)\Omega = 193200\angle{-8.50°}\Omega$$

$$I_s = \frac{V_s}{Z_{eq}} = 0.0259\angle 8.50°\text{mA}$$

$$I = \frac{Z_C}{Z_C + Z_{LR_2}} I_s = (0.954\angle{-17.44°})I_s = 0.0247\angle{-8.94°}\text{mA}$$

So

$$I(t) = 24.7\cos(\pi t - 0.156)\mu A$$

2.38

(a) $\omega = \pi\frac{\text{rad}}{\text{sec}}$, $f = \frac{\omega}{2\pi} = 0.5\text{Hz}$

$A_{pp} = 2A = 4.0$, $dc_{offset} = 0$

(b) $\omega = 2\pi\frac{\text{rad}}{\text{sec}}$, $f = \frac{\omega}{2\pi} = 1\text{Hz}$

$A_{pp} = 2A = 2$, $dc_{offset} = 10.0$

(c) $\omega = 2\pi\frac{\text{rad}}{\text{sec}}$, $f = \frac{\omega}{2\pi} = 1\text{Hz}$

$A_{pp} = 2A = 6.0$, $dc_{offset} = 0$

(d) $\omega = 0\frac{\text{rad}}{\text{sec}}$, $f = \frac{\omega}{2\pi} = 0\text{Hz}$

$A_{pp} = 2A = 0$, $dc_{offset} = \sin(\pi) + \cos(\pi) = -1$

2.39 $P = \frac{V_{rms}^2}{R} = 100\text{W}$

2.40 $V_{rms} = \left(\frac{V_{pp}}{2}\right) / (\sqrt{2}) = 35.36\text{V}$

$$P = \frac{V_{rms}^2}{R} = 12.5W$$

2.41 $V_m = \sqrt{2}V_{rms} = 169.7V$

2.42 For $V_{rms} = 120V$, $V_m = \sqrt{2}V_{rms} = 169.7V$, and f = 60 Hz,

$$V(t) = V_m \sin(2\pi f + \phi) = 169.7 \sin(120\pi t + \phi)$$

2.43 From Ohm's Law,

$$I = \frac{5V - 2V}{R} = \frac{3V}{R}$$

Since $10mA \le I \le 100mA$,

$$10mA \le \frac{3V}{R} \le 100mA$$

giving

$$\frac{3V}{100mA} \le R \le \frac{3V}{10mA} \quad \text{or} \quad 30\Omega \le R \le 300\Omega$$

For a resistor, $P = \frac{V^2}{R}$, so the smallest allowable resistance would need a power rating of at least:

$$P = \frac{(3V)^2}{30\Omega} = 0.3W$$

so a 1/2 W resistor should be specified.
The largest allowable resistance would need a power rating of at least:

$$P = \frac{(3V)^2}{300\Omega} = 0.03W$$

so a 1/4 W resistor would provide more than enough capacity.

2.44 Using KVL and KCL gives:

$$V_1 = I_{R_1}R_1$$

$$V_1 = (I_1 - I_{R_1})R_2 + (I_1 - I_{R_1} - I_2)R_3$$

$$V_3 - V_2 = (I_1 - I_{R_1} - I_2)R_3 - I_2R_4$$

The first loop equation gives:

$$I_{R_1} = \frac{V_1}{R_1} = 10mA$$

Using this in the other two loop equations gives:

$$10 = (I_1 - 10m)2k + (I_1 - 10m - I_2)3k$$

$$10 - 5 = (I_1 - 10m - I_2)3k - I_2 4k$$

or

$$(5k)I_1 - (3k)I_2 = 60$$

$$(3k)I_1 - (7k)I_2 = 35$$

Solving these equations gives:

$$I_1 = 12.12mA \text{ and } I_2 = 0.1923mA$$

(a) $V_{out} = I_2 R_4 - V_2 = -4.23V$

(b) $P_1 = I_1 V_1 = 121mW$, $P_2 = I_2 V_2 = 0.962mW$, $P_3 = -I_2 V_3 = -1.92mW$

2.45 Using KVL and KCL gives:

$$V_1 = I_{R_1} R_1$$

$$V_1 = (I_1 - I_{R_1})R_2 + (I_1 - I_{R_1} - I_2)R_3$$

$$V_3 - V_2 = (I_1 - I_{R_1} - I_2)R_3 - I_2 R_4$$

The first loop equation gives:

$$I_{R_1} = \frac{V_1}{R_1} = 10mA$$

Using this in the other two loop equations gives:

$$10 = (I_1 - 10m)2k + (I_1 - 10m - I_2)2k$$

$$10 - 5 = (I_1 - 10m - I_2)2k - I_2 1k$$

or

$$(4k)I_1 - (2k)I_2 = 50$$

$$(2k)I_1 - (3k)I_2 = 25$$

Solving these equations gives:

$$I_1 = 12.5\text{mA} \quad \text{and} \quad I_2 = 0\text{mA}$$

(a) $V_{out} = I_2R_4 - V_2 = -5\text{V}$

(b) $P_1 = I_1V_1 = 125\text{mW}, \; P_2 = I_2V_2 = 0\text{mW}, \; P_3 = -I_2V_3 = 0\text{mW}$

2.46 $$P_{avg} = \frac{1}{T}\int_0^T V(t)I(t)dt = \frac{V_mI_m}{T}\int_0^T \sin(\omega t + \phi_V)\sin(\omega t + \phi_I)dt$$

Using the product formula trigonometric identity,

$$P_{avg} = \frac{V_mI_m}{2T}\int_0^T (\cos(\phi_V - \phi_I) - \cos(2\omega t + \phi_V + \phi_I))dt$$

Therefore,

$$P_{avg} = \frac{V_mI_m}{2}\cos(\phi_V - \phi_I) = \frac{V_mI_m}{2}\cos(\theta)$$

2.47 $$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I_m^2\sin^2(\omega t + \phi_I)dt}$$

Using the double angle trigonometric identity,

$$I_{rms} = \sqrt{\frac{I_m^2}{T}\int_0^T \left(\frac{1}{2} - \cos[2(\omega t + \phi_I)]\right)dt}$$

Therefore,

$$I_{rms} = \sqrt{\frac{I_m^2}{T}\left(\frac{T}{2}\right)} = \frac{I_m}{2}$$

2.48 $$R_{23} = \frac{R_2R_3}{R_2 + R_3} = 5\text{k}\Omega$$

$$V_o = \frac{R_{23}}{R_1 + R_{23}}V_i = \frac{1}{2}\sin(2\pi t)$$

This is a sin wave with half the amplitude of the input with a period of 1s.

2.49 No. A transformer requires a time varying flux to induce a voltage in the secondary coil.

2.50 $\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{120V}{24V} = 5$

2.51 $R_L = R_i = 8\Omega$ for maximum power

2.52 The BNC cable is far more effective in shielding the input signals from electromagnetic interference since no loops are formed.

3.1 For $V_i > 0$, $V_o = 0$

For $V_i < 0$, $V_o = V_i$

The resulting waveform consists only of the negative "humps" of the original cosine wave. Each hump has a duration of 0.5s and there is a 0.5s gap between each hump.

3.2 For $V_i > 0.7V$, $V_o = 0.7V$

For $V_i < 0.7V$, $V_o = V_i$

The resulting waveform consists only of the negative "humps" (below 0.7V) of the original cosine wave. The positive "humps" (above 0.7V) are clipped off and held constant at 0.7V.

3.3

(a) output passes the positive humps only

(b) output passes the negative humps only

(c) output passes the positive humps only

(d) output passes the negative humps only

(e) output passes the positive humps and scales the negative humps by 1/2

(f) output passes the full wave

3.4

(a) output passes the positive humps above -0.7V only, with the negative humps clipped at -0.7V

(b) output passes the negative humps below 0.7V only, with the positive humps clipped at 0.7V

(c) output passes the positive humps above 0.7V only, with the negative humps clipped at 0.7V

(d) output passes the negative humps below -0.7V only, with the positive humps clipped at -0.7V

(e) output passes the positive humps above -0.7V only, and scales the negative humps below -0.7V by 1/2

(f) output passes the full wave

3.5 When the diode is forward biased, the output voltage is -0.7V, so the output signal is chopped off at -0.7V instead of 0V.

3.6 When the switch is closed and the circuit is in steady state, the current through the load is constant, and the diode is reverse biased (i.e., there is no diode current).

When the switch is opened, the inductor generates a forward voltage to oppose a decrease in current. Now the diode forms a circuit with the load, allowing the current to dissipate through the resistor.

If there were no diode, and the switched were opened, because the current would attempt to decrease instantaneously, the inductor would generate a very large voltage which would create an arc (current through air) across the switch contacts.

3.7 Forward bias ($V_{in} > V_{out} + 0.7V$) is required for charging. "Leaking" causes voltage decay (i.e., V_{out} decreases slowly).

3.8 See the data sheet for a LM7815C voltage regulator.

3.9

(a) For $V_i > 0.5V$, $V_o = 0.5V$.

For $V_i < 0.5V$, $V_o = V_i$.

The resulting waveform is the original sine wave with the top halves of the positive "humps" (above 0.5 V) clipped off.

(b) For $V_i < 0.5V$, $V_o = 0.5V$.

For $V_i > 0.5V$, $V_o = V_i$.

The resulting waveform is the original sine wave with the bottom of the negative "humps" (below 0.5 V) clipped off.

3.10 Using superposition, Ohm’s Law, and current division,

$$I_{1_{left}} = \frac{1V}{2R + \frac{R}{2}} = \frac{2}{5R}$$

$$I_{2_{left}} = -\frac{1}{2}I_{1_{left}} = -\frac{1}{5R}$$

$$I_{4_{left}} = \frac{1}{2}I_{1_{left}} = \frac{1}{5R},\ I_{4_{right}} = \frac{1V}{R + \frac{2R}{3}} = \frac{3}{5R}$$

$$I_{2_{right}} = \frac{2R}{R + 2R}I_{4_{right}} = \frac{2}{5R}$$

$$I_{1_{right}} = I_{4_{right}} - I_{2_{right}} = \frac{1}{5R}$$

$$I_1 = I_{1_{left}} + I_{1_{right}} = \frac{3}{5R} > 0$$

$$I_2 = I_{2_{left}} + I_{2_{right}} = \frac{1}{5R} > 0$$

$$I_3 = 0$$

$$I_4 = I_{4_{left}} + I_{4_{right}} = \frac{4}{5R} > 0$$

$$V_{diode} = 1V - I_4R = \frac{1}{5}V > 0$$

3.11 With I_2=I_3=0, I_1 and I_4 are equal. The current (I=I_1=I_4) is:

$$I = \frac{1V + 1V}{2R + R} = \frac{2}{3R}V$$

and the voltage of node A relative to node B is:

$$V_{AB} = 1V - I(2R) = -1V + I(R) = -\frac{1}{3}V$$

Therefore, the voltage polarity on the left diode is incorrect.

3.12 When the left diode is forward biased and the right diode is reverse biased,

$$V_{out} = V_H$$

and when the right diode is forward biased and the left diode is reverse biased,

$$V_{out} = V_L$$

When both diodes are reverse biased,

$$V_{out} = \frac{R_L}{R_i + R_L}V_i$$

Therefore, the output is a scaled version of the input chopped off below V_L and above V_H.

3.13 For $V_{in} > 0$, $V_{out} = \frac{1}{2}V_{in} = 5\sin(\pi t)$

For $V_{in} < 0$, $V_{out} = V_{in} = 10\sin(\pi t)$

The positive "bumps" of the resulting waveform are half the amplitude (5 vs. 10) of the original, and the lower bumps are the same.

3.14 In steady state dc, the capacitor is equivalent to an open circuit. Therefore, the steady state current through the capacitor is 0 and the steady state voltage across the capacitor is V_{out}.

(a) For V_s=10V, the diode is forward biased and is equivalent to a short circuit. Therefore, the equivalent resistance of the two horizontal resistors is R/2 and from voltage division,

$$V_{capacitor} = V_{out} = \frac{R}{\frac{R}{2} + R} V_s = \frac{2}{3} V_s = 6.66\text{V}$$

(b) For V_s=–10V, the diode is reverse biased and is equivalent to an open circuit. Therefore, the circuit simplifies to two series resistors and

$$V_{capacitor} = V_{out} = \frac{R}{R + R} V_s = \frac{1}{2} V_s = -5\text{V}$$

3.15 There are three possible states of the diodes. When only the left diode is forward biased, $V_{out} = V_H$. When only the right diode is forward biased, $V_{out} = V_L$. When both diodes are reverse biased, $V_L < V_{out} < V_H$. In this case, the circuit is a voltage divider and

$$V_{out} = \frac{R_L}{R_i + R_L} V_{in} = \frac{1}{2} V_{in}$$

The upper limit of this state is when $V_{out} = V_H = 5\text{V}$ corresponding to $V_{in} = 10\text{V}$. V_{out} remains at 5V when V_{in} increases above 10V. The lower limit of the double reverse biased state is when $V_{out} = V_L = -5\text{V}$ corresponding to $V_{in} = -10\text{V}$. V_{out} remains at –5V when V_{in} decreases below –10V. It is not possible for both diodes to be reverse biased at the same time in this circuit.

The resulting output signal V_{out} is a sin wave scaled by 1/2 with the peaks clipped off at $\pm 10\text{V}$.

3.16

(a) output passes first (positive) hump only

(b) output is 5.1V over the whole input cycle

3.17 Use a resistor in series with the LED where:

$$I = \frac{5\text{V} - V_{LED}}{R}$$

The required resistance value is

$$R \geq \frac{(5V - V_{LED})}{I_{max}}$$

(a) $R \geq \frac{5V}{50mA} = 100\Omega$

(b) $R \geq \frac{(5V - 2V)}{50mA} = 60\Omega$

3.18 $V_E = 5V - V_{LED} - V_{CE} = 2.8V$

$V_{in}(\text{saturation}) = V_E + V_{BE} = 2.8V + 0.7V = 3.5V$

(a) For the LED to be ON, the transistor must be in saturation and

$$V_B = V_{LED} + V_{BE} = 1V + 0.7V = 1.7V$$

When the LED is off, $I_B = 0$ and

$$V_B = \frac{1}{2}V_{in}$$

So for the LED to be ON,

$$V_{in} > 2V_B = 3.4V$$

(b) When the transistor is fully saturated,

$$V_B = V_{LED} + 0.7V = 1.7V \quad \text{and} \quad V_C = V_{LED} + 0.2V = 1.2V$$

and

$$I_C = \frac{5V - V_C}{330\Omega} = \frac{3.8V}{330\Omega} = 11.5mA$$

Assume

$$I_B = \frac{1}{100}I_C = 0.115mA$$

If I_1 is the current through the horizontal 1k resistor and I_2 is the current through the right 1k resistor, then

$$I_1 = I_2 + I_B = \frac{V_B}{1k} + 0.115mA = 1.815mA$$

and

$$V_{in} = V_B + (1k)I_1 = 3.52V$$

3.19 When the transistor is in full saturation,

$$V_{CE} = 0.2V \text{ and } V_{BE} = 0.7V$$

and

$$I_{out} = I_B + I_C = \frac{I_C}{100} + I_C = 1.01I_C$$

In the collector-to-emitter circuit,

$$V_{out} = V_s - I_C R_C - V_{CE} = I_{out} R_{out}$$

giving

$$5V - I_C(1k) - 0.2V = 1.01I_C(1k)$$

Now we can solve for the collector and emitter currents:

$$I_C = \frac{4.8V}{2.01k} = 2.39mA$$

and

$$I_{out} = 1.01I_C = 2.41mA$$

Therefore,

$$V_{out} = I_{out}R_{out} = (2.41mA)(1k) = 2.41V$$

and the minimum required input voltage is:

$$V_{in} = V_{out} + V_{BE} + I_B R_B = 2.41V + 0.7V + (0.239mA)(1k) = 3.13V$$

3.20 1: a resistor (e.g., 1k) to limit the base current while ensuring the transistor is in full saturation

2: 24 V_{dc} capable of at least 1A of current

3: power diode capable of carrying at least 1A for flyback protection

4: ground

3.21 The transistor begins to saturate at approximate V_{in}=0.88V, where:

$$\beta = \frac{I_C}{I_B} = \frac{4.869}{0.054} = 90.2$$

$$V_{BE} = 0.73V$$

$$V_{CE} = 0.16V$$

We usually assume these values are 100, 0.7V, and 0.2 V at the verge of saturation.

3.22 A voltage source (e.g., 5V) and current limiting series resistor (e.g., 330 Ω) is required on the LED side. On the phototransistor side, a pull-up resistor (e.g, 1k) and a voltage source (e.g., 5V) is required on the collector lead and ground is required on the emitter lead.

3.23 From the figure, the approximate "ON" values for the drain-to-source voltage and current are:

$$V_{ds} \approx 0.25\text{V} \quad \text{and} \quad I_{ds} \approx 48\text{mA}$$

so the "ON" resistance is:

$$R_{ON} = \frac{V_{ds}}{I_{ds}} \approx 5.2\Omega$$

3.24 1: nothing required

2: 24 V_{dc} capable of at least 1A of current

3: power diode capable of carrying at least 1A for flyback protection

4: ground

3.25 The type of BJT required is an npn and an additional resistor must be added in series with the open collector output to pull up the voltage enough to bias the BE junction of the BJT.

3.26 The upper FET is a p-channel enhancement mode MOSFET and the lower is an n-channel enhancement mode MOSFET. When V_{in} = 5V, the upper MOSFET doesn't conduct but the bottom one does, so V_{out} = 0V. When V_{in} = 0V, the upper MOSFET conducts but the bottom one doesn't, so $V_{out} = V_{cc}$.

3.27 The requirements are that

$$I_{d(cont)} > 10\text{A} \quad \text{and} \quad P_{d(max)} > I_{on}^2 R_{on} = 100 R_{on}$$

IRF530N is a good choice

3.28

(a) cutoff

(b) ohmic

(c) saturation

(d) cutoff

4.1

(a) linear

(b) nonlinear

(c) linear

(d) linear

(e) nonlinear

(f) nonlinear

(g) linear

4.2 The Fourier Series is:

$$F(t) = 5\sin(2\pi t)$$

and the fundamental frequency is

$$f_0 = \frac{\omega_0}{2\pi} = 1\text{Hz}$$

4.3 Since V_{rms} = 120V and f = 60Hz, the Fourier Series is:

$$F(t) = \sqrt{2}V_{rms}\sin(2\pi ft) = 169.7\sin(120\pi t)$$

and the fundamental frequency is 60Hz.

4.4 We need to prove:

$$A_n\cos(n\omega_0 t) + B_n\sin(n\omega_0 t) = C_n\cos(n\omega_0 t + \phi_n)$$

We can use the following trig identity:

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

where:

$$a = n\omega_0 t \quad \cos(b) = \frac{A_n}{C_n} \quad -\sin(b) = \frac{B_n}{C_n} \quad b = \phi_n$$

Equations 4.8 and 4.9 can now be verified:

$$\sin^2(b) + \cos^2(b) = \left(\frac{B_n}{C_n}\right)^2 + \left(\frac{A_n}{C_n}\right)^2 = 1$$

Therefore,

$$C_n = \sqrt{A_n^2 + B_n^2}$$

Also,

$$\tan(\phi_n) = \tan(b) = \frac{\sin(b)}{\cos(b)} = \frac{-\frac{B_n}{C_n}}{\frac{A_n}{C_n}} = -\frac{B_n}{A_n}$$

Therefore,

$$\phi_n = \tan^{-1}\left(-\frac{B_n}{A_n}\right) = -\tan^{-1}\left(\frac{B_n}{A_n}\right)$$

4.5 $A_n = \frac{2}{T}\int_0^T F(t)\cos(n\omega_0 t)dt = \frac{2}{T}\left[\int_0^{\frac{T}{2}}\cos(n\omega_0 t)dt - \int_{\frac{T}{2}}^{T}\cos(n\omega_0 t)dt\right]$

So

$$A_n = \frac{2}{n\omega_0 T}\left\{[\sin(n\omega_0 t)]_0^{\frac{T}{2}} - [\sin(n\omega_0 t)]_{\frac{T}{2}}^{T}\right\}$$

But $\omega_0 = \frac{2\pi}{T}$, so

$$A_n = \frac{1}{n\pi}(\sin(n\pi) - \sin(2n\pi) + \sin(n\pi))$$

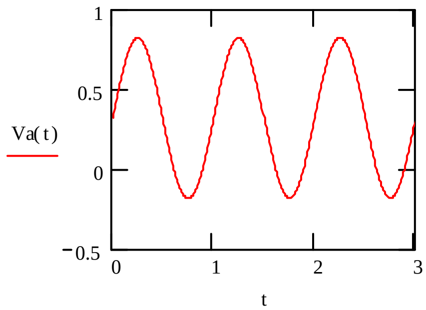
But sine of any multiple of π is 0, so $A_n = 0$

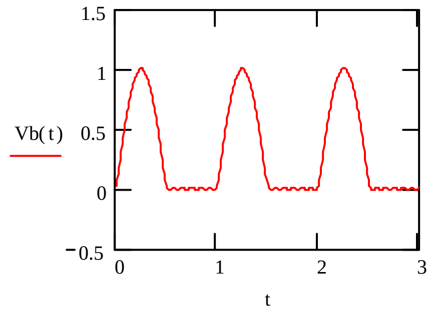
4.6 Using MathCAD:

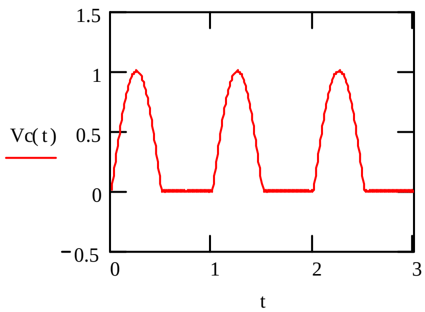
$t := 0, 0.01 .. 3.0$ $\qquad Va(t) := \frac{1}{\pi} + \frac{\sin(2\cdot\pi\cdot t)}{2}$

$n := 2, 4 .. 10$ $\qquad Vb(t) := \frac{1}{\pi} + \frac{\sin(2\cdot\pi\cdot t)}{2} - \frac{2}{\pi}\cdot\sum_{n}\frac{\cos(2\cdot n\cdot\pi\cdot t)}{(n-1)\cdot(n+1)}$

$m := 2, 4 .. 50$ $\qquad Vc(t) := \frac{1}{\pi} + \frac{\sin(2\cdot\pi\cdot t)}{2} - \frac{2}{\pi}\cdot\sum_{m}\frac{\cos(2\cdot m\cdot\pi\cdot t)}{(m-1)\cdot(m+1)}$







4.7

(a) $$f_H = 6 + \frac{\left(1 - \frac{1}{\sqrt{2}}\right)}{1}(10 - 6) = 7.17\text{Hz}$$

So the bandwidth is: 0 Hz to 7.17 Hz

(b) T = 1s

$$f_o = \frac{1}{T} = 1\text{Hz}, \quad \omega_0 = 2\pi f_0 = 2\pi\frac{\text{rad}}{\text{sec}}$$

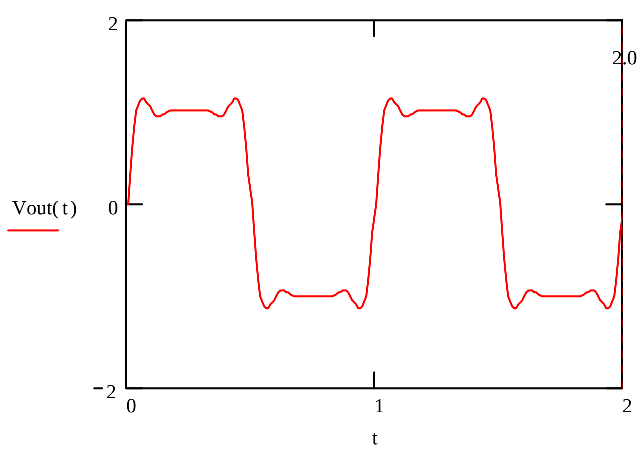
n	ω (rad/sec)	f (Hz)	A_{in}	A_{out}/A_{in}	$A_{out} = (A_{out}/A_{in})A_{in}$
1	ω_0	1	4/π	1	4/π
2	$3\omega_0$	3	4/3π	1	4/3π
3	$5\omega_0$	5	4/5π	1	4/5π
4	$7\omega_0$	7	4/7π	3/4	3/7π
5	$9\omega_0$	9	4/9π	1/4	1/9π
>5	$(2n-1)\omega_0$	(2n-1)	4/fπ	0	0

$$V_{out} = \frac{4}{\pi}\sin(2\pi t) + \frac{4}{3\pi}\sin(6\pi t) + \frac{4}{5\pi}\sin(10\pi t) + \frac{3}{7\pi}\sin(14\pi t) + \frac{1}{9\pi}\sin(18\pi t)$$

(c) Using MathCAD:

$t := 0, 0.01 .. 2$

$$Vout(t) := \frac{4}{\pi}\cdot\sin(2\cdot\pi\cdot t) + \frac{4}{3\cdot\pi}\cdot\sin(6\cdot\pi\cdot t) + \frac{4}{5\cdot\pi}\cdot\sin(10\cdot\pi\cdot t) + \frac{3}{7\cdot\pi}\cdot\sin(14\cdot\pi\cdot t) + \frac{1}{9\cdot\pi}\cdot\sin(18\cdot\pi\cdot t)$$



4.8

(a) $$\omega_c = \frac{1}{RC} = \frac{1}{(1\times10^3)(1\times10^{-6})} = 1000\frac{\text{rad}}{\text{sec}}$$

(b) T = 1s, f = 1Hz, $\omega_0 = 2\pi$

$$F(t) = \sum_{n=1}^{\infty} A_{in}(n)\left(\frac{A_{out}}{A_{in}}(n)\right)\sin(\omega_n t)$$

where

$$A_{in}(n) = \frac{4}{(2n-1)\pi}$$

$$\frac{A_{out}}{A_{in}}(n) = \frac{1}{\sqrt{1+\left(\frac{\omega_n}{\omega_c}\right)^2}}$$

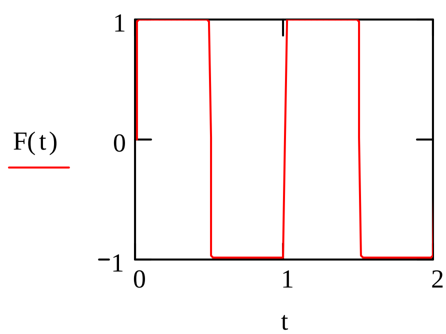
$$\omega_n = (2n-1)2\pi$$

(c) Using MathCAD:

$$n := 1..100 \qquad \omega_c := 1000 \qquad \omega_n := (2\cdot n - 1)\cdot 2\cdot\pi$$

$$t := 0, 0.01..2$$

$$F(t) := \sum_{n} \frac{4}{(2\cdot n-1)\cdot\pi}\cdot\left[\frac{1}{\sqrt{1+\left(\frac{\omega_n}{\omega_c}\right)^2}}\right]\cdot\sin\left(\omega_n\cdot t\right)$$



4.9

(a) $\omega_c = \frac{1}{RC} = \frac{1}{(100\times 10^3)(1\times 10^{-6})} = 10\frac{\text{rad}}{\text{sec}}$

(b) $T = 1\text{s}$, $f = 1\text{Hz}$, $\omega_0 = 2\pi$

$$F(t) = \sum_{n=1}^{\infty} A_{in}(n)\left(\frac{A_{out}}{A_{in}}(n)\right)\sin(\omega_n t)$$

where

$$A_{in}(n) = \frac{4}{(2n-1)\pi}$$

$$\frac{A_{out}}{A_{in}}(n) = \frac{1}{\sqrt{1+\left(\frac{\omega_n}{\omega_c}\right)^2}}$$

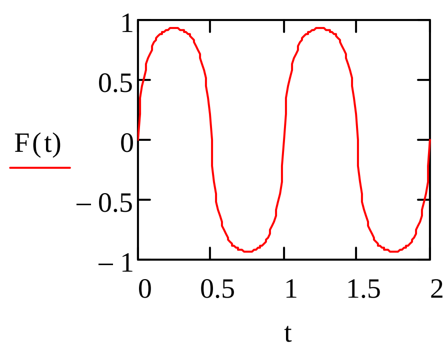
$$\omega_n = (2n-1)2\pi$$

(c) Using MathCAD:

$n := 1..100 \quad \omega_c := 10 \quad \omega_n := (2\cdot n - 1)\cdot 2\cdot\pi$

$t := 0, 0.01..2$

$$F(t) := \sum_n \left[\frac{4}{(2\cdot n-1)\cdot\pi}\cdot\left[\frac{1}{\sqrt{1+\left(\frac{\omega_n}{\omega_c}\right)^2}}\right]\cdot\sin(\omega_n\cdot t)\right]$$



4.10 $\omega_L = \frac{1}{\sqrt{2}}\frac{rad}{sec} = 0.707\frac{rad}{sec} = 0.113Hz$

$\omega_H = \left(3 + 1.5\left(1 - \frac{1}{\sqrt{2}}\right)\right)\frac{rad}{sec} = 3.44\frac{rad}{sec} = 0.547Hz$

$\omega_L \le \text{bandwidth} \le \omega_H$

4.11

(a) $1\frac{\text{rad}}{\text{sec}} \le \omega \le 5\frac{\text{rad}}{\text{sec}}$

(b) and (c)

n	A_{in}	A_{out}
1	1	1
2	2	2
3	3	3
4	4	1.33
5	5	0

4.12 $V_o = \dfrac{R}{R + \dfrac{1}{j\omega C}} V_i$

$$\frac{V_o}{V_i} = \frac{j\omega RC}{j\omega RC + 1}$$

To find the cut-off frequency, set the amplitude ratio magnitude to $\frac{1}{\sqrt{2}}$:

$$\left|\frac{V_o}{V_i}\right| = \frac{\omega RC}{\sqrt{(\omega RC)^2 + 1}} = \frac{1}{\sqrt{2}}$$

Solving for the frequency gives

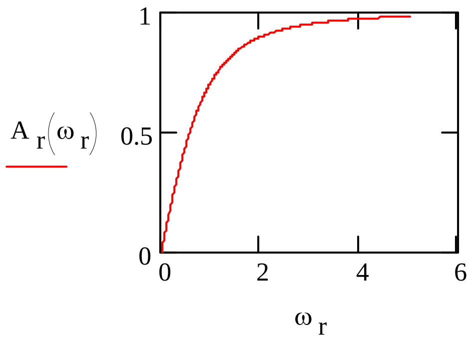
$$\omega_c = \frac{1}{RC}$$

Using this expression gives:

$$\left|\frac{V_o}{V_i}\right| = \frac{\dfrac{\omega}{\omega_c}}{\sqrt{\left(\dfrac{\omega}{\omega_c}\right)^2 + 1}}$$

and now we can plot the frequency response in terms of the dimensionless frequency ratio $\omega_r = \dfrac{\omega}{\omega_c}$:

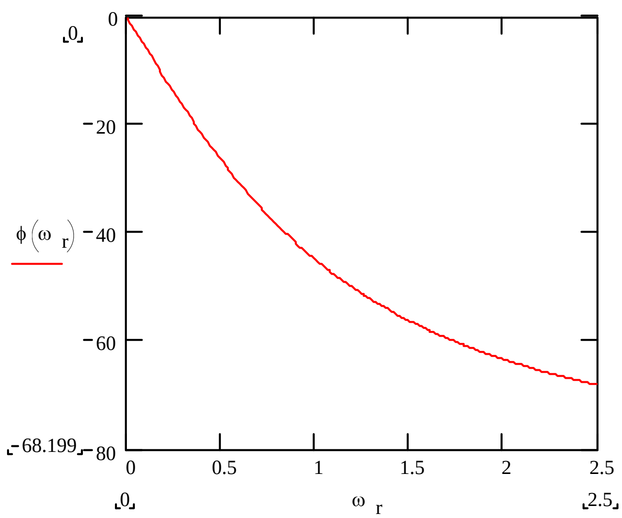
$$\omega_r := 0, 0.01 .. 5.0 \qquad A_r(\omega_r) := \frac{\omega_r}{\sqrt{(\omega_r)^2 + 1}}$$



4.13 $\phi = \arg\left(\dfrac{V_o}{V_i}\right) = \angle(1) - \angle(1 + \omega RCj) = 0 - \mathrm{atan}\left(\dfrac{\omega RC}{1}\right) = -\mathrm{atan}(\omega RC)$

Using $\omega_c = \dfrac{1}{RC}$ and $\omega_r = \dfrac{\omega}{\omega_c} = \omega RC$,

$$\phi = -\mathrm{atan}(\omega_r)$$



4.14 The answer is in Figure 4.4 (see the "ω_0, $3\omega_0$, $5\omega_0$" curve).

4.15 Using MathCAD:

$n := 1 .. 20 \qquad t := 0, 0.01 .. 2$

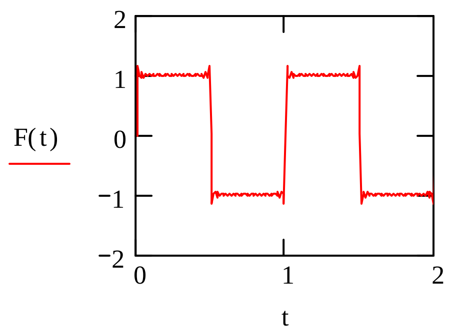
$$\omega_n := (2 \cdot n - 1) \cdot 2 \cdot \pi \qquad F(t) := \sum_n \frac{4}{(2 \cdot n - 1) \cdot \pi} \cdot \sin(\omega_n \cdot t)$$

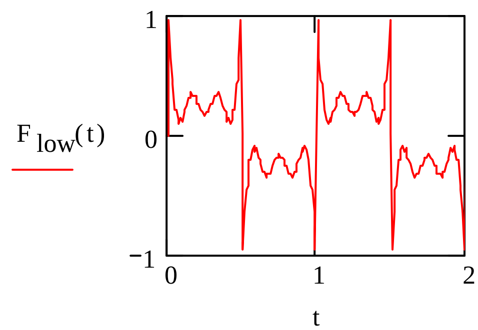
$ATT_n := 1 \quad ATT_1 := .25 \quad ATT_2 := .25 \quad ATT_3 := .25$

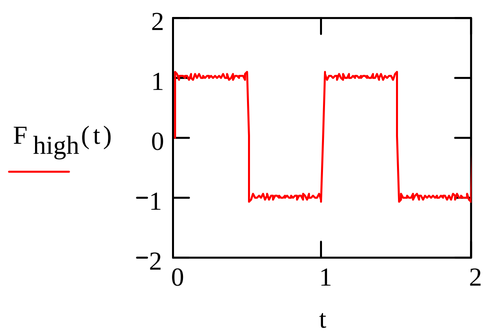
$$F_{low}(t) := \sum_n \frac{4}{(2 \cdot n - 1) \cdot \pi} \cdot ATT_n \cdot \sin(\omega_n \cdot t)$$

$ATT_n := 1 \quad ATT_{17} := .25 \quad ATT_{18} := .25 \quad ATT_{20} := .25$

$$F_{high}(t) := \sum_n \frac{4}{(2 \cdot n - 1) \cdot \pi} \cdot ATT_n \cdot \sin(\omega_n \cdot t)$$



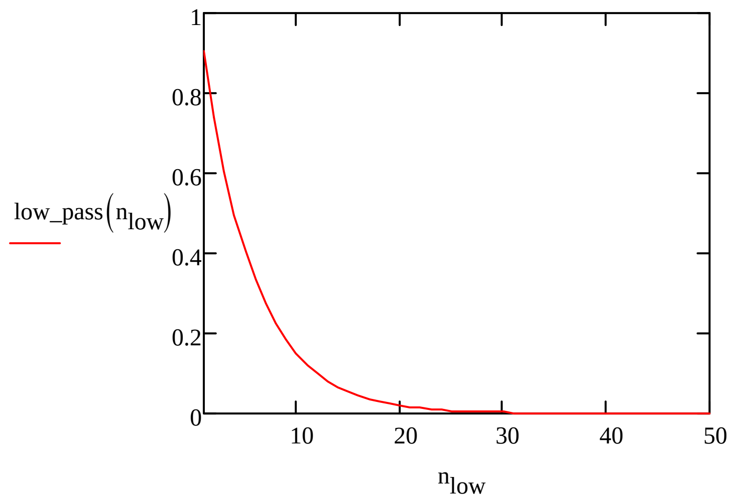


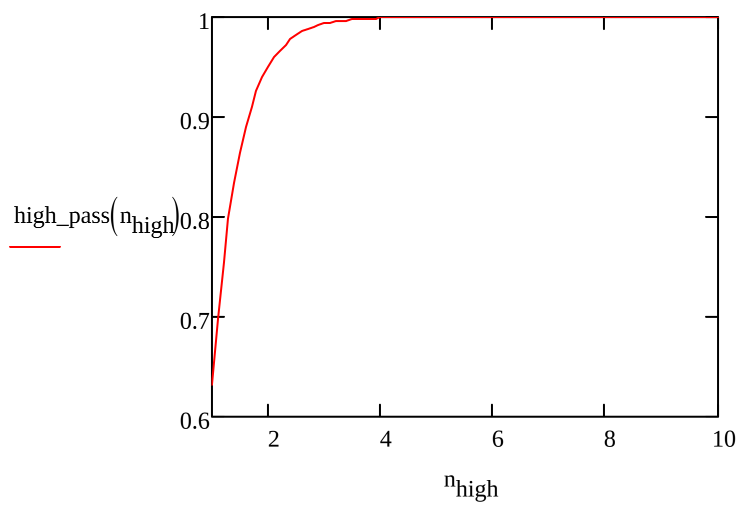


4.16 Using MathCAD:

$n_{low} := 1..50$ $\qquad$ $n_{high} := 1, 1.1..10$

$low\_pass(n) := e^{-0.1\cdot(2\cdot n-1)}$ $\qquad$ $high\_pass(n) := 1 - e^{-(2\cdot n-1)}$





4.17 When the mass is in static equilibrium,

$$M_{in}g = kX_{out}$$

so

$$X_{out} = \left(\frac{g}{k}\right)M_{in}$$

and the static sensitivity is

$$K = \frac{g}{k}$$

4.18 We generally assume that the displayed voltage is the gain times the input voltage. This assumption will be in error if the oscilloscope is dc coupled and some of the frequencies in the signal exceed the bandwidth of the oscilloscope.

4.19 KVL gives:

$$IR + \frac{1}{C}q = V_{in}$$

where q is the charge on the capacitor. Putting this in standard form gives:

$$(RC)\dot{q} + q = CV_{in}$$

where q is the dependent variable, the time constant (τ) is RC, and the sensitivity (K) is C. Using the general solution for a 1st order system,

$$q(t) = CA_i\left(1 - e^{-\frac{t}{\tau}}\right)$$

Therefore, the step response output voltage (which is the voltage across the capacitor) is

$$V_{out}(t) = \frac{1}{C}q(t) = A_i\left(1 - e^{-\frac{t}{\tau}}\right)$$

4.20 Applying KVL around the flyback loop gives:

$$L\frac{d}{dt} + IR = 0$$

Putting this in standard first-order-system form gives:

$$\frac{L}{R}\frac{d}{dt} + I = 0$$

so the time constant is: $\tau = L/R$. The root of the characteristic equation is $s=-1/\tau$, so the equation for current is:

$$I(t) = Ce^{-\frac{t}{\tau}}$$

The current is I_{ss} at t=0, so $C=I_{ss}$, giving:

$$I(t) = I_{ss}e^{-\frac{t}{\tau}}$$

4.21 The rate of change of internal energy is equal to the rate of heat transfer:

$$\frac{d}{dt}(E_{in}) = \dot{Q}_{in}$$

so

$$(mc)\frac{dT_{out}}{dt} = (hA)(T_{in} - T_{out})$$

Defining $C_t = mc$ (thermal capacitance) and $R_t = \frac{1}{hA}$ (thermal resistance) and converting into standard form gives:

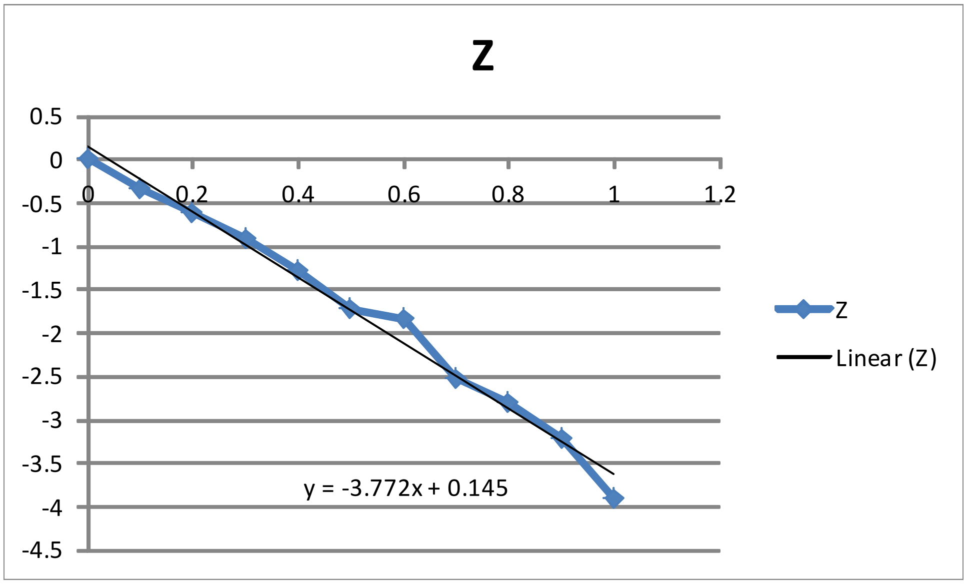
$$(C_t R_t)\frac{dT_{out}}{dt} + T_{out} = (1)T_{in}$$

where the time constant is $\tau = R_t C_t = \frac{mc}{hA}$ and the sensitivity is K=1.

4.22 Plotting the data $X_{out}(t)$ shows a steady state asymptote of approximately 5 indicating that:

$$KA_{in} = 5$$

Plotting $Z(t) = \ln\left(1 - \frac{X_{out}(t)}{KA_{in}}\right)$ shows a near linear relation indicating that the system can be modeled as 1st order.



The slope of the line is approximately –3.772, indicating a time constant of

$$\tau = 1 / 3.772 = 0.265 \text{ sec}$$

4.23 The damped natural frequency is always smaller if there is damping in the system.

4.24

(a) mechanical rotary (applied torque, torsion spring, rotary damper, and rotary inertia):

$$J\ddot{\theta} + B\dot{\theta} + k\theta = \tau_{ext}$$

(b) electrical (voltage source and series resistor, inductor, and capacitor):

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = V_s \quad \text{or} \quad L\dot{I} + RI + \frac{1}{C}\int I dt = V_s$$

(c) hydraulic (pump with inlet in reservoir, long pipe with friction loss and fluid inertia, and tank):

$$I\frac{dQ}{dt} + RQ + \frac{1}{C}\cancel{V} = P$$

where $\cancel{V} = \int Q dt$

4.25 Given the differential equation:

$$\tau\dot{x}_{out} + x_{out} = Kx_{in}$$

Applying the procedure results in:

$$(\tau s + 1)X_{out}(s) = KX_{in}(s)$$

$$G(s) = \frac{X_{out}(s)}{X_{in}(s)} = \frac{K}{(\tau s + 1)}$$

$$G(j\omega) = \frac{K}{1 + \tau\omega j}$$

$$\frac{|X_{out}|}{|X_{in}|} = |G(j\omega)| = \frac{K}{\sqrt{1 + (\tau\omega)^2}}$$

$$\phi = \arg(G(j\omega)) = 0 - \operatorname{atan}(\tau\omega) = -\operatorname{atan}(\tau\omega)$$

4.26 $F_0 = 20N \quad \omega = 0.75\frac{rad}{Sec}$

$$\omega_n = \sqrt{\frac{k}{m}} = 1.095, \quad \omega_r = \frac{\omega}{\omega_n} = 0.685, \quad \varsigma = \frac{b}{2\sqrt{km}} = 0.456$$

$$\frac{X_0}{F_0/k} = \frac{1}{\sqrt{[1 - \omega_r^2]^2 + 4\varsigma^2\omega_r^2}} = 1.219$$

$$X_0 = \left(\frac{X_0}{F_0/k}\right)\frac{F_0}{k} = 2.032\text{m}$$

$$\phi = -\tan^{-1}\left(\frac{2\varsigma}{\frac{1}{\omega_r} - \omega_r}\right) = -49.6° = -0.866\text{rad}$$

Therefore, the steady state response is:

$$x(t) = X_0\sin(\omega t + \phi) = 2.032\sin(0.75t - 0.866)\text{m}$$

4.27 The equation of motion is

$$m\ddot{x}_{out} + b\dot{x}_{out} + kx_{out} = kx_{in}$$

Taking the Laplace transform of both sides gives the transfer function:

$$G(s) = \frac{X_{out}(s)}{X_{in}(s)} = \frac{k}{ms^2 + bs + k}$$

Now

$$G(j\omega) = \frac{k}{(k - m\omega^2) + b\omega j}$$

$$\frac{|X_{out}|}{|X_{in}|} = |G(j\omega)| = \frac{k}{\sqrt{(k - m\omega^2)^2 + (b\omega)^2}}$$

$$\phi = -\tan^{-1}\left(\frac{b\omega}{k - m\omega^2}\right)$$

So the steady state response is

$$x_{out}(t) = |X_{in}|\frac{|X_{out}|}{|X_{in}|}\sin(\omega t + \phi)$$

Using MathCAD,

$m := 0.10 \qquad k := 100000 \qquad b := 10 \qquad X_{in} := 0.05$

$$X_{out}(\omega) := \frac{k \cdot X_{in}}{\sqrt{\left(k - m \cdot \omega^2\right)^2 + (b \cdot \omega)^2}} \qquad \phi(\omega) := -\mathrm{angle}\left(k - m \cdot \omega^2, b \cdot \omega\right)$$

$X_{out}(10) = 0.05 \qquad \phi(10) = -0.057 \circ \mathrm{deg}$

$X_{out}(1000) = 0.5 \qquad \phi(1000) = -90 \circ \mathrm{deg}$

$X_{out}(10000) = 5.05 \cdot 10^{-4} \qquad \phi(10000) = -179.421 \circ \mathrm{deg}$

Only the first input results in an acceptable output.

4.28 The response would be the same since there is no "g" in the equations. The only difference would be the initial "equilibrium position," which would be at the unstretched length of the spring. One method to determine the mass is to measure the natural frequency with a spring of known stiffness and calculate:

$$m = \frac{k}{\omega_n^2}$$

4.29 $x_h(t) = e^{-\varsigma\omega_n t}[A\cos(\omega_d t) + B\sin(\omega_d t)]$

$x_p(t) = C$

$x(t) = x_h(t) + x_p(t)$

$x(\infty) = \frac{F_0}{k}$ gives $C = 0$

$x(0) = 0$ gives $A = -C$

$\dot{x}(0) = 0$ gives $B = 0$

Therefore,

$$x(t) = \frac{F_0}{k}(1 - e^{-\varsigma\omega_n t}\cos(\omega_d t))$$

4.30 The natural frequencies and damping constants can be used to predict the results. To model the tire, the mass of the wheel, stiffness of the tire, and position of the spindle (new variable) would also need to be included. The input force or displacement would then be at the tire-road interface.

4.31 The volume in the tank is

$$\cancel{V} = \frac{\pi D^2}{4} h$$

The pressure at the bottom of the tank is

$$P = \rho g h = \gamma h$$

Solving the volume expression for h and substituting into the pressure expression gives

$$P = \frac{4\gamma}{\pi D^2} \cancel{V} = \frac{1}{C} \cancel{V}$$

so

$$C = \frac{\pi D^2}{4\gamma}$$

4.32 Since $F = ma$, $a = \ddot{x}$ and $\dot{x} = \frac{Q}{A}$,

$$PA = (\rho L A)\left(\frac{\dot{Q}}{A}\right)$$

$$P = \left(\frac{\rho L}{A}\right)\dot{Q} = I\dot{Q}$$

4.33 Element [flow] analogies:

$$F_{in}[v_{in}] \to V_{in}[I_{in}]$$

$$k_1[v_{in} - v_m] \to C_1[I_{in} - I_m]$$

$$m[v_m] \to L[I_m]$$

$$b_1[v_m] \to R_1[I_m]$$

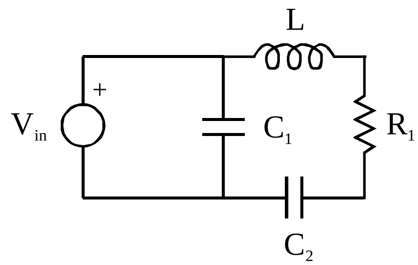
$$k_2[v_m] \to C_2[I_m]$$

Analogous free body diagram equations [KVL]:

$$V_{in} = V_{C_1}$$

$$V_{C_1} = V_{R_1} + V_{C_2} + V_L$$

The resulting analogous electrical circuit follows:



4.34 Element [flow] analogies:

$$F_1[v_1] \rightarrow V_1[I_1]$$

$$m[v_1] \rightarrow L[I_1]$$

$$k_1[v_1] \rightarrow C_1[I_1]$$

$$b_1[v_1 - v_2] \rightarrow R_1[I_1 - I_2]$$

$$k_2[v_1 - v_2] \rightarrow C_2[I_1 - I_2]$$

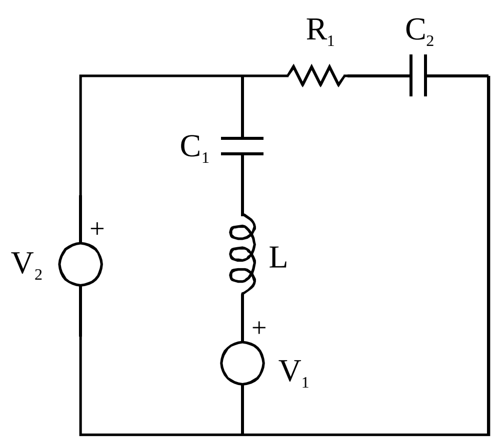
$$F_2[-v_2] \rightarrow V_2[-I_2]$$

Analogous free body diagram equations [KVL]:

$$V_1 - V_{C_2} - V_{R_1} - V_{C_1} = V_L$$

$$V_{R_1} + V_{C_2} - V_2 = 0$$

The resulting analogous electrical circuit follows:



4.35 Hydraulic elements are direct analogies to electrical elements. The capacitors are replaced by tanks, and the resistor and inductor are replaced by a long pipe with flow resistance and inertance.

4.36 Element [flow] analogies:

$$V_1[I_1] \to F_1[v_1]$$

$$L[I_1] \to m[v_1]$$

$$C_1[I_1] \to k_1[v_1]$$

$$R[I_1 - I_2] \to b[v_1 - v_2]$$

$$C_2[I_1 - I_2] \to k_2[v_1 - v_2]$$

$$V_2[-I_2] \to F_2[-v_2]$$

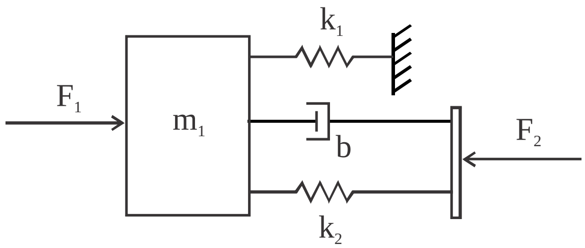
Analogous KVL [free body diagram] equations:

$$F_2 + F_{k_1} + F_2 - F_1 = 0$$

$$F_1 - F_2 - F_{k_1} + F_b - F_{k_2} = 0$$

$$F_2 - F_b - F_{k_2} = 0$$

The resulting analogous mechanical system follows:



4.37 Element [flow] analogies:

$$F_{in}[v_1] \to V_{in}[I_1]$$

$$b_1[v_1 - v_2] \to R_1[I_1 - I_2]$$

$$k_1[v_1 - v_2] \to C_1[I_1 - I_2]$$

$$m[v_2] \to L[I_2]$$

$$k_2[v_2 - v_3] \to C_2[I_2 - I_3]$$

$$b_2[v_3] \to R_2[I_3]$$

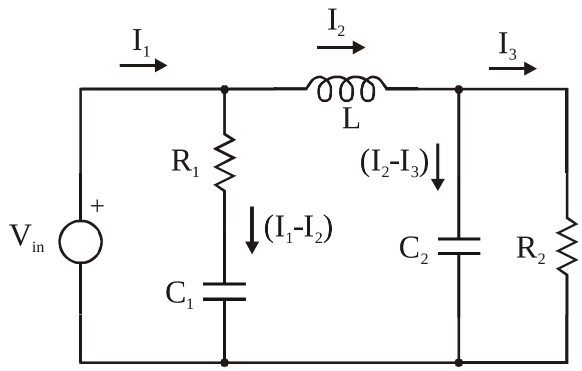
Analogous free body diagram equations [KVL]:

$$V_{in} = V_{R_1} + V_{C_1}$$

$$V_{R_1} + V_{C_1} = V_L + V_{C_2}$$

$$V_{C_2} = V_{R_2}$$

The resulting analogous electrical circuit follows:



4.38 $k(z - x) + b(\dot{z} - \dot{x}) - \mu m_1 g\,\mathrm{sgn}(\dot{x}) = m_1\ddot{x}$

$k(z - x) + b(\dot{z} - \dot{x}) - F_1 = -m_2\ddot{z}$

$-rF_1 = I_2\ddot{\theta}$

where

$$z = y - r\theta, \;\; \dot{z} = \dot{y} - r\dot{\theta}, \;\; \ddot{z} = \ddot{y} - r\ddot{\theta}$$

5.1 The power dissipated by each resistor is

$$\frac{V_{in}^2}{R} = \frac{25}{R} \quad \text{and} \quad \frac{V_{out}^2}{R_F} = \frac{[(GAIN)V_{in}]^2}{R_F} = \frac{25GAIN^2}{R_F}$$

To be able to use 1/4 W resistors, the following must be true:

$$\frac{25}{R} < 0.25 \quad \text{or} \quad R > 100\Omega$$

$$\frac{25GAIN^2}{R_F} < 0.25 \quad \text{or} \quad R_F > (GAIN^2)100\Omega$$

(a) for GAIN=1, $R_F > 100\Omega$

(b) for GAIN=10, $R_F > 10k\Omega$

5.2

(a) $V_+ = V_- = \dfrac{R_4}{R_3 + R_4}V_{out}$

$$I = \frac{V_+}{R_2}$$

$$V_{out} = \frac{R_2(R_3 + R_4)}{R_4}I$$

(b) $V_+ = V_- = V_{out}$

$$I_1 = I_2 = \frac{V_+}{R_2} = \frac{V_{out}}{R_2}$$

$$V_+ + I_1R_1 = V_{out} + I_3R_3$$

so

$$I_3 = \frac{R_1}{R_3}I_1 = \frac{R_1}{R_2R_3}V_{out}$$

$$I = I_1 + I_3 = V_{out}\left(\frac{1}{R_2} + \frac{R_1}{R_2R_3}\right)$$

so

$$V_{out} = \frac{R_2R_3}{R_1 + R_3}I$$

5.3 With R_F replaced by a short, the op amp circuit becomes a buffer so the gain is 1.

5.4

(a) $V_- = V_+ = \frac{R_2}{R_1 + R_2}V_1 = 5V$

$V_{out} = V_- + V_2 - I_3R_3$

but $I_3 = 0$, so

$$V_{out} = V_- + V_2 = 10V$$

(b) same as (a)

5.5 $V_+ = V_- = V_i$

$V_4 = \left(1 + \frac{R_3}{R_2}\right)V_+$

$I_4 = \frac{V_4}{R_4} = \frac{R_2 + R_3}{R_2R_4}V_i$

5.6 If V_A denotes the voltage at the output of the first op amp,

$$V_A = V_- = V_+ = 0V$$

and from Ohm's Law, the current from voltage source V_1 is

$$I_1 = \frac{V_1 - V_A}{R} = \frac{V_1}{R}$$

If V_B denotes the voltage at the inverting input of the second op amp,

$$V_B = V_- = V_+ = V_2$$

and from Ohm's Law,

$$I_4 = \frac{V_B}{R} = \frac{V_2}{R} \quad \text{and} \quad I_2 = \frac{V_B - V_{out}}{R} = \frac{V_2 - V_{out}}{R}$$

where I_4 is the current through the vertical resistor and I_2 is the current through the feedback resistor of the second op amp. From this,

$$V_{out} = V_2 - I_2R$$

Now applying Ohm's Law to the resistor between the op amps gives

$$I_3 = \frac{V_A - V_B}{R} = -\frac{V_2}{R}$$

where I_3 is the current through the resistor.

From KCL, the current out of the first op amp is

$$I_{out_1} = I_3 - I_1 = -\frac{V_2}{R} - \frac{V_1}{R} = -\frac{1}{R}(V_1 + V_2)$$

The negative sign indicates that the current is actually into the op amp.

From KCL,

$$I_2 = I_3 - I_4 = -\frac{V_2}{R} - \frac{V_2}{R} = \left(-\frac{2}{R}\right)V_2$$

Therefore,

$$V_{out} = V_2 - \left(-\frac{2V_2}{R}\right)R = 3V_2$$

5.7 $V_o \neq V_i$ because of positive feedback.

5.8 Applying Ohm's Law to both resistors gives

$$I_1 = \frac{V_1}{R_1} \text{ and } I_2 = \frac{V_2}{R_2}$$

From KCL,

$$I_F = I_1 + I_2$$

Since $V_o + I_F R_F = 0$,

$$V_o = -R_F\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

For $R_1 = R_2 = R_F = R$,

$$V_o = -(V_1 + V_2)$$

5.9 Applying Ohm's Law to both resistors gives

$$I_1 = \frac{V_1 - V_3}{R_1} \text{ and } I_2 = \frac{V_2 - V_3}{R_2}$$

From KCL,

$$I_F = I_1 + I_2$$

Since $V_o + I_F R_F = V_3$,

$$V_o = V_3 - R_F\left(\frac{V_1 - V_3}{R_1} + \frac{V_2 - V_3}{R_2}\right)$$

For $R_1 = R_2 = R_F = R$,

$$V_o = V_3 - (V_1 - V_3 + V_2 - V_3) = 3V_3 - (V_1 + V_2)$$

5.10 From voltage division,

$$V_- = V_+ = \left(\frac{R_F}{R_F + R_2}\right)V_2$$

From Ohm's Law,

$$I_1 = \frac{(V_1 - V_-)}{R_1}$$

The output voltage can be found with:

$$V_o = V_- - I_1 R_F = \left(\frac{R_F}{R_F + R_2}\right)V_2 - \frac{\left(V_1 - \left(\frac{R_F}{R_F + R_2}\right)V_2\right)}{R_1}R_F$$

Simplifying gives

$$V_o = \frac{R_1 V_2 - (V_1(R_F + R_2) - R_F V_2)}{R_1(R_F + R_2)/R_F}$$

$$V_o = \frac{V_2(R_F + R_1) - V_1(R_F + R_2)}{R_1(R_F + R_2)/R_F}$$

For $R_1 = R_2 = R$,

$$V_o = \frac{R_F}{R}(V_2 - V_1)$$

5.11 $V_{out_{in}} = \left(-\frac{R_F}{R}\right)V_{in}$ and $V_{out_{ref}} = \left(1 + \frac{R_F}{R}\right)V_{ref}$

$$V_{out} = V_{out_{in}} + V_{out_{ref}} = \left(-\frac{R_F}{R}\right)V_{in} + \left(1 + \frac{R_F}{R}\right)V_{ref}$$

5.12 Using superposition,

$$V_{o_1} = -\frac{R_4}{R_3}V_3$$

$$V_{o_2} = \left(1 + \frac{R_4}{R_3}\right)\frac{R_5}{R_3 + R_5}V_4$$

$$V_o = V_{o_1} + V_{o_2} = -\frac{R_4}{R_3}V_3 + \left(1 + \frac{R_4}{R_3}\right)\frac{R_5}{R_3 + R_5}V_4$$

5.13 Using MathCAD:

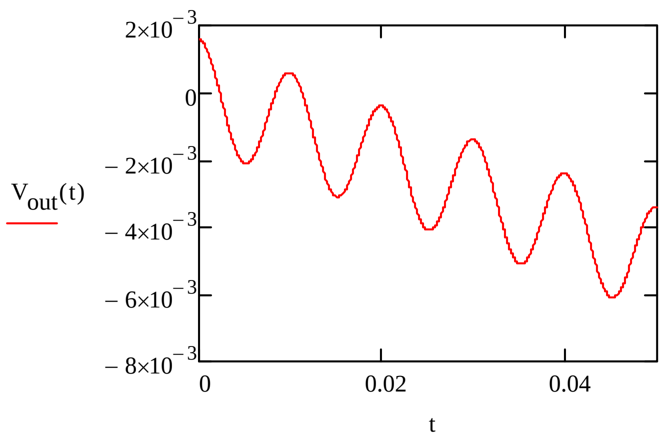
The input can be expressed as:

$$\omega := 2\cdot\pi\cdot 100 \qquad T := \frac{1}{100}$$

$$V_{in}(t) := \sin(\omega\cdot t) + 0.1$$

Assuming RC=1, the output of the integrator will be:

$$V_{out}(t) := \frac{1}{\omega}\cdot\cos(\omega\cdot t) - 0.1\cdot t$$



5.14 $V_+ = V_- = 0$

$$V_i = L\frac{dI_L}{dt} \quad \text{so} \quad I_L = \frac{1}{L}\int V_i dt$$

$$V_o = V_- + I_R R$$

but $I_R = I_L$, so $V_o = \frac{R}{L}\int V_i dt$

5.15 From Ohm's Law, the input currents can be related to the circuit voltages with:

$$I_+ = -\frac{V_+}{R_2} \quad \text{and} \quad I_- = \frac{V_{in} - V_-}{R_1} - \frac{V_-}{R_s}$$

If the input voltages and currents are assumed to be equal ($I_+ = I_-$), equating these expressions, setting $V_{in}=0$, and dividing through by the voltage ($V_+ = V_-$) gives:

$$\frac{1}{R_2} = \frac{1}{R_1} + \frac{1}{R_s}$$

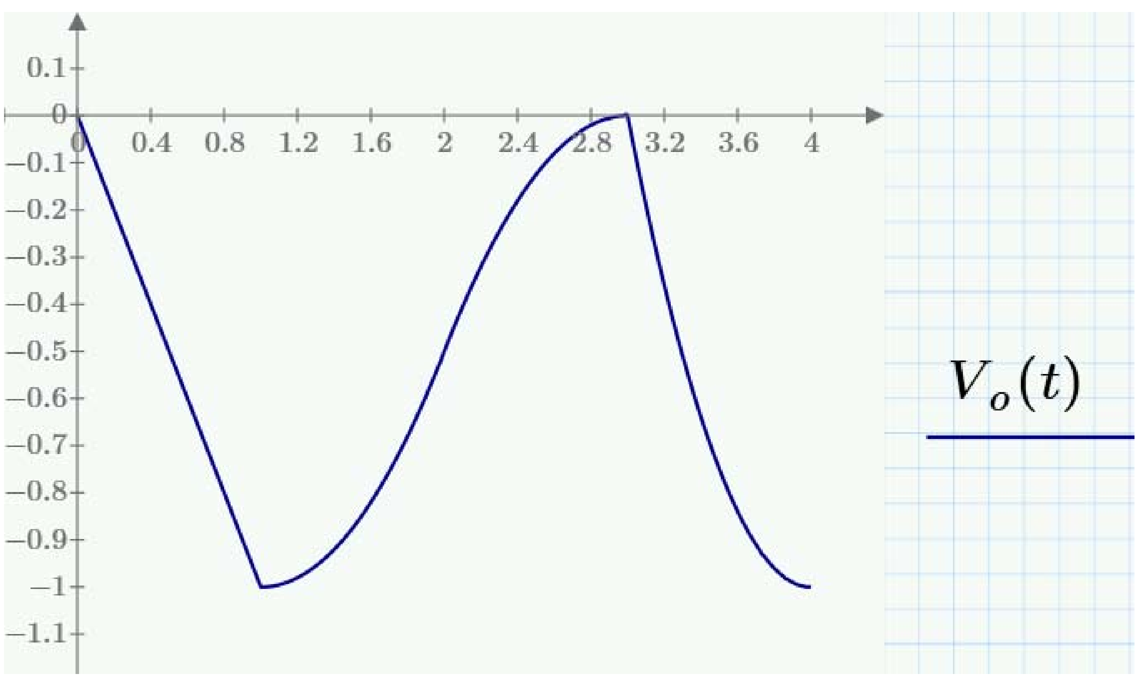
which gives:

$$R_2 = \frac{R_1 R_s}{R_1 + R_s}$$

5.16

(a) $V_o = -\left(\frac{R_F}{R}\right)V_i = -2V_i$

(b) $V_o = -\frac{1}{RC}\int V_i dt = -\int V_i dt$



(c) $V_o = -\frac{R_F}{R}(V_1 + V_2) = -4V_i$

(d) $V_- = V_+ = 0V$

From Ohm's Law,

$$I_1 = \frac{V_1}{5k} = \frac{V_i}{5k} \text{ and } I_2 = \frac{V_2}{10k} = \frac{V_i}{10k}$$

From KCL,

$$I_F = I_1 + I_2 = V_i\left(\frac{1}{5k} + \frac{1}{10k}\right)$$

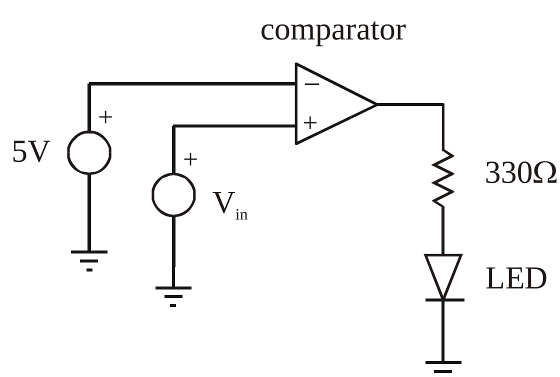
but from Ohm's Law,

$$I_F = \frac{0 - V_o}{5k}$$

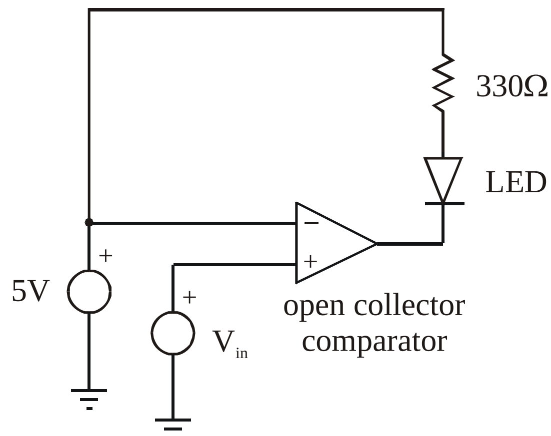
so

$$V_o = -5kI_F = -V_i\left(1 + \frac{1}{2}\right) = -\frac{3}{2}V_i$$

5.17



5.18



5.19 The limit on the feedback resistor current is:

$$I_F = \frac{V_{out}}{R_F} = \frac{10V}{R_F} < 10mA$$

Therefore,

$$R_F > \frac{10V}{10mA} = 1k\Omega$$

5.20 $|V_{out_{max}}| = 13.6V$ and $V_{out} = -\frac{R_F}{R}V_{in} = -2V_{in}$

so:

$$|V_{in_{max}}| = \frac{|V_{out_{max}}|}{2} = 6.8V$$

5.21 closed loop gain $= \frac{R_F}{R} = 10$

so the fall-off frequency is 10^5Hz.

5.22 The amplifier will saturate (reach the minimum swing voltage limit) as the integrated dc component grows.

6.1 $1111111111111111_2 = 2^{16} - 1 = 65{,}535$

6.2

(a) $128 = 10000000_2$ since $2^7 = 128$

(b) $127 = 1111111_2$ since $(2^7 - 1) = 127$

6.3

(a) $128 = 80_{16}$ since $8(16) = 128$

(b) $127 = 7F_{16}$ since $7(16) + 15 = 127$

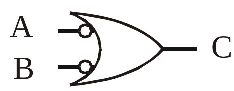
6.4

```
(a)   1  1
       1101       13
     + 1001     +  9
      10110       22

(b)    1101       13
     - 1001     -  9
       0100        4

(c)      1101       13
       x 1001     x  9
         1101       27
        0000        9
       0000        117
      1101
      1110101

(d)   111
       111       7
     + 111     + 7
      1110      14
```

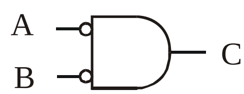
```
(e)      111         7
       x 111     x   7
       11
      1111
         111       49
        111
       111
      110001
```

6.5

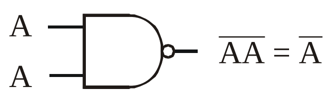
(a)



(b)



6.6



6.7

(a) The output (C) is high (5V) iff both inputs are low (0V).

$C = \overline{A}\,\overline{B} = \overline{A + B}$

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

(b) $C = \overline{(\overline{AB})(\overline{AB})} = AB$

A	**B**	**C**
0	0	0
0	1	0
1	0	0
1	1	1

(c) $C = \overline{\overline{A\overline{AB}}\;\overline{B\overline{AB}}} = A\overline{AB} + B\overline{AB} = A(\overline{A} + \overline{B}) + B(\overline{A} + \overline{B})$

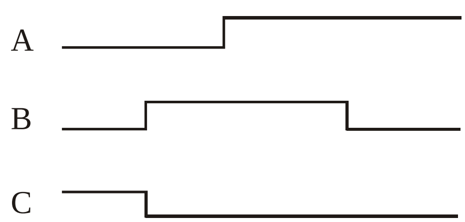
$C = A\overline{B} + B\overline{A} = A \oplus B$

A	**B**	**C**
0	0	0
0	1	1
1	0	1
1	1	0

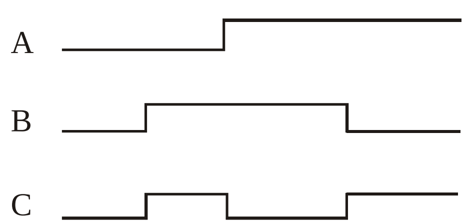
(d) $C = \overline{\overline{A}\,\overline{B}} = A + B$

A	**B**	**C**
0	0	0
0	1	1
1	0	1
1	1	1

6.8

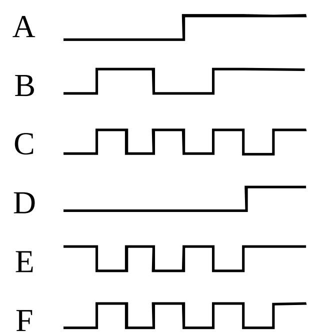


6.9



6.10

A	**B**	C	D	E	F
0	0	0	0	1	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	0	1
1	0	0	0	1	0
1	0	1	0	0	1
1	1	0	1	1	0
1	1	1	1	1	1



6.11

(a) $\overline{1 \cdot \bar{0}} + 1 \cdot (0 + 1) + \bar{\bar{0}} \cdot (1 + \bar{0})$

$\overline{1 \cdot 1} + 1 \cdot 1 + 0 \cdot 1$

$\bar{1} + 1 + 0$

1

(b) $A \cdot \bar{B} + A \cdot (A + B)$

$A(\bar{B} + A + B)$

$A(A + 1)$

$A(1)$

A

6.12 $X = \overline{\overline{AB} + \overline{BC}} + BC + C = ABBC + BC + C = BC(A + 1) + C = C(B + 1) = C$

6.13 $A + (\overline{A} \cdot B) = A + B$

Multiplying (ANDing) both sides by A gives:

$A \cdot A + A \cdot \overline{A} \cdot B = A \cdot A + A \cdot B$

Simplifying, gives:

$A + 0 = A + A \cdot B$

$A = A \cdot (1 + B)$

$A = A$

Thus, the identity is valid.

6.14 $(A + B)(A + \overline{B}) = AA + A\overline{B} + BA + B\overline{B} = A + A(B + \overline{B}) = A + A = A$

6.15 Equation 6.21:

$(A + B) \cdot (A + C)$

$AA + AC + BA + BC$

$A(1 + C) + B(A + C)$

$A(1) + B(A + C)$

$A + BA + BC$

$A(1 + B) + BC$

$A(1) + BC$

$A + BC$

A	**B**	**C**	$A + B$	$A + C$	BC	$(A + B) \cdot (A + C)$	$A + BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	0	1	1
1	0	1	1	1	0	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

6.16 Equation 6.22:

A	B	A + B	$A\bar{B}$	$(A + B) + A\bar{B}$
0	0	0	0	0
1	0	1	1	1
0	1	1	0	1
1	1	1	1	1

6.17 Equation 6.23:

A	**B**	**C**	**AB**	**BC**	$\bar{B}C$	$AB + BC + \bar{B}C$	AB + C
0	0	0	0	0	0	0	0
0	0	1	0	0	1	1	1
0	1	0	0	0	0	0	0
0	1	1	0	1	0	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	1	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	0	1	1

6.18

A	**B**	**C**	**AB**	**AC**	$\bar{B}C$	$AB + AC + \bar{B}C$	$AB + \bar{B}C$
0	0	0	0	0	0	0	0
0	0	1	0	0	1	1	1
0	1	0	0	0	0	0	0
0	1	1	0	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	0	1	1	1	1
1	1	0	1	0	0	1	1
1	1	1	1	1	0	1	1

6.19

(a)

A	B	C	AB	BC	$\overline{B}C$	$AB + BC + \overline{B}C$	$AB + \overline{C}$
0	0	0	0	0	0	0	1
0	0	1	0	0	1	1	0
0	1	0	0	0	0	0	1
0	1	1	0	1	0	1	0
1	0	0	0	0	0	0	1
1	0	1	0	0	1	1	0
1	1	0	1	0	0	1	1
1	1	1	1	1	0	1	1

(b)

A	B	C	ABC	$\overline{A + B + C}$
0	0	0	0	1
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	0

(c)

A	B	C	AB	BC	$\overline{B}C$	$AB + BC + \overline{B}C$	$AB + C$
0	0	0	0	0	0	0	0
0	0	1	0	0	1	1	1
0	1	0	0	0	0	0	0
0	1	1	0	1	0	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	1	1	1
1	1	0	1	0	0	1	1
1	1	1	1	1	0	1	1

6.20

A	B	$\overline{A}$	$\overline{B}$	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0

6.21 $X = A\overline{P} + BP$

P	A	B	X
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

The circuit is called a multiplexer because P allows one of two (multiple) inputs to pass through to the output.

6.22 $X = A\overline{B} + A(A + B)$

$X = A(\overline{B} + A + B)$

$X = A(A + 1) = AA = A$

equivalent circuit: one wire connecting input A to output X!

6.23 $Y = AD + (A + B)C$

For the unallowed code CD=11, the output (Y) would be:

$Y = A + (A + B) = A + B$

In this state, the alarm would go off when windows or doors are disturbed or when motion is detected. This state is the same as state 2 (CD = 10).

6.24 The simplified Boolean expression is:

$$X = B \cdot (C + \overline{A})$$

Using DeMorgan's Laws, the all-AND representation is:

$$B \cdot (\overline{\overline{C} \cdot \overline{\overline{A}}}) \;=\; B \cdot (\overline{\overline{C} \cdot A})$$

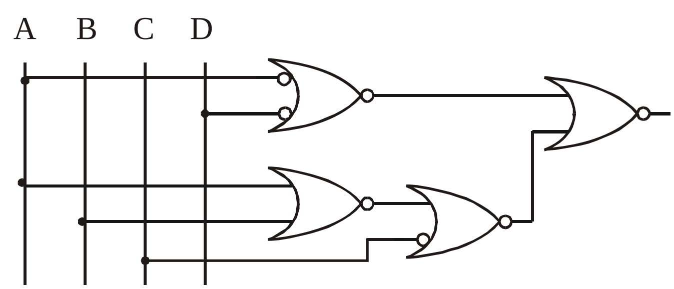
and the all-OR representation is:

$$\overline{\overline{B} + \overline{(C + \overline{A})}}$$

The original expression contains 1 AND operations, 1 OR operation, and one inversion, requiring 3 ICs. The all-AND version contains 2 ANDs and 2 inversions, requiring 2 ICs. The all-OR version contains 2 ORs and 4 inversions, also requiring 2 ICs.

6.25 $Y \;=\; (A \cdot D) + (A + B) \cdot C$

$$Y \;=\; (\overline{\overline{A} + \overline{D}}) + \overline{\overline{(A + B)} + \overline{C}}$$



6.26 Segment c is OFF only for the digit 2, so the output of the logic circuit must be:

$$X \;=\; \overline{D}\,\overline{C}B\overline{A} \quad \text{or more simply} \quad X \;=\; \overline{C}B\overline{A}$$

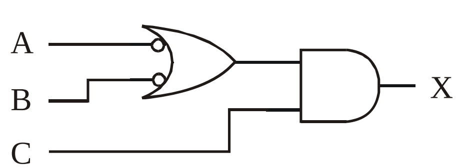
6.27 $X \;=\; \overline{A}BC + (A + B)\overline{C} \;=\; \overline{\overline{\overline{A}BC}\;\overline{(A + B)\overline{C}}} \;=\; \overline{\overline{\overline{A}BC}\;\overline{\overline{\overline{A}\,\overline{B}}\,\overline{C}}}$

6.28 $X \;=\; \overline{\overline{AA}(C + \overline{C})} + \overline{B}C \;=\; \overline{\overline{A}1} + \overline{B}C \;=\; A + \overline{B}C \;=\; A + \overline{B + \overline{C}} \;=\; \overline{\overline{A + \overline{B + \overline{C}}}}$

6.29 The required Boolean expression is:

$$X \;=\; C \cdot (\overline{A} + \overline{B})$$

which can be implemented with the following logic circuit:



The complete truth table (including the sub expression $(\bar{A}+\bar{B})$) is:

A	**B**	**C**	$(\bar{A}+\bar{B})$	**X**
0	0	0	1	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0

6.30 $X = \bar{P}A\bar{B} + \bar{P}AB + P\bar{A}B + PAB$

$X = \bar{P}A(\bar{B}+B) + PB(\bar{A}+A)$

$X = \bar{P}A + PB$

6.31 For the two expressions for S:

A	**B**	$\bar{A}B$	$A\bar{B}$	$\bar{A}B + A\bar{B}$	$A+B$	$\bar{A}+\bar{B}$	$(A+B)(\bar{A}+\bar{B})$
0	0	0	0	**0**	0	1	**0**
0	1	1	0	**1**	1	1	**1**
1	0	0	1	**1**	1	1	**1**
1	1	0	0	**0**	1	0	**0**

For the two expressions for C:

A	**B**	AB	$A+B$	$A+\bar{B}$	$\bar{A}+B$	$(A+B)(A+\bar{B})(\bar{A}+B)$
0	0	**0**	0	1	1	**0**
0	1	**0**	1	0	1	**0**
1	0	**0**	1	1	0	**0**
1	1	**1**	1	1	1	**1**

6.32 From the logic circuit:

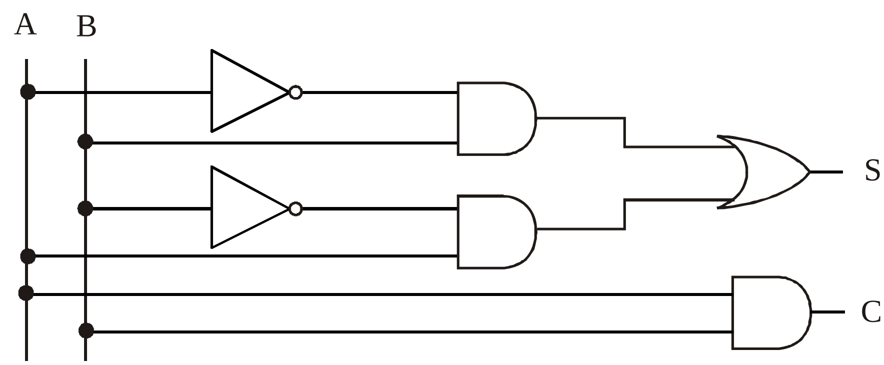
$C = AB$

which is the sum-of-products result. And, using DeMorgan's Law,

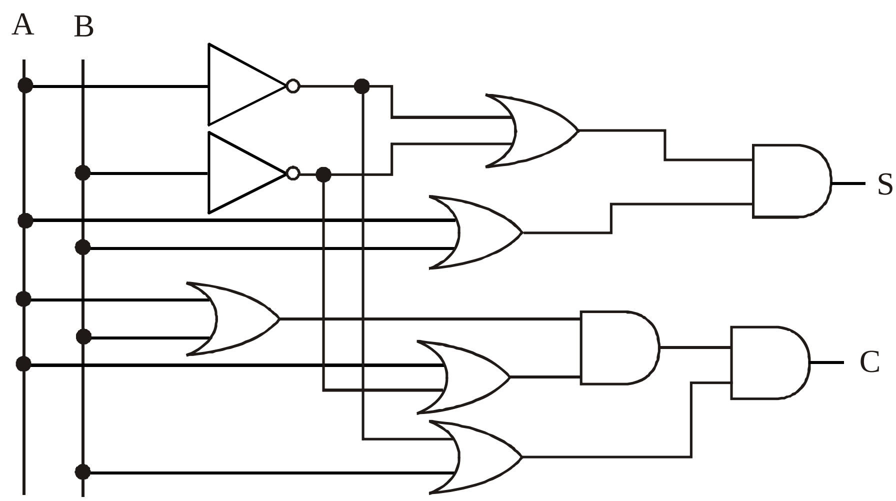
$S = (A+B)\overline{AB} = (A+B)(\bar{A}+\bar{B})$

which is the product-of-sums result.

6.33 Sum of products circuit:



Product of sums circuit:



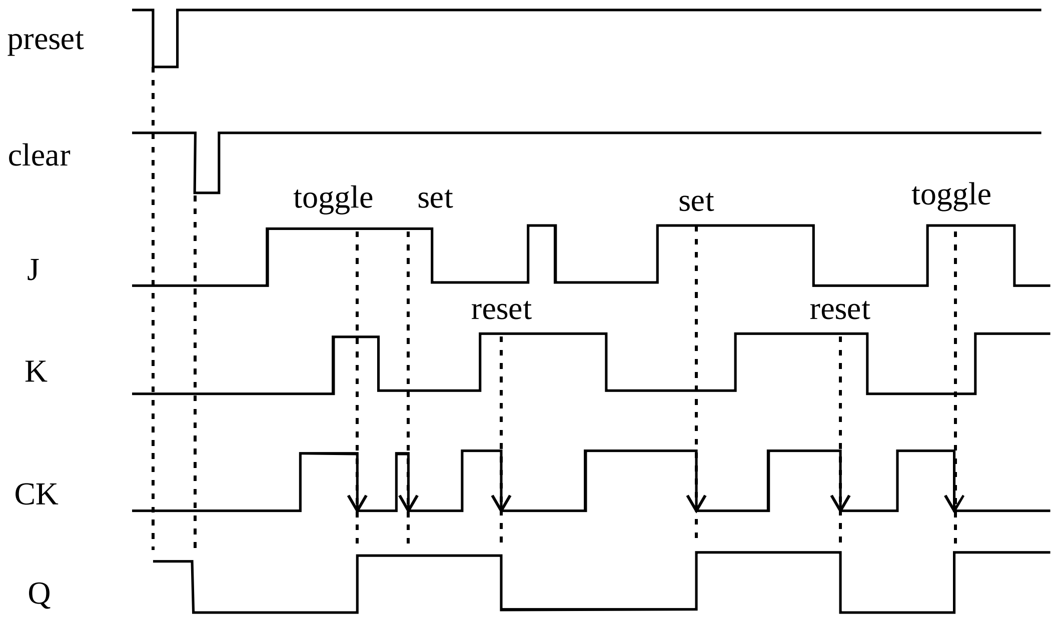
6.34

C_{i-1}	A_i	B_i	S_i	C_i
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S_i = \overline{C_{i-1}}\,\overline{A_i}B_i + \overline{C_{i-1}}A_i\overline{B_i} + C_{i-1}\overline{A_i}\,\overline{B_i} + C_{i-1}A_iB_i$$

$$C_i = \overline{C_{i-1}}A_iB_i + C_{i-1}\overline{A_i}B_i + C_{i-1}A_i\overline{B_i} + C_{i-1}A_iB_i$$

6.35



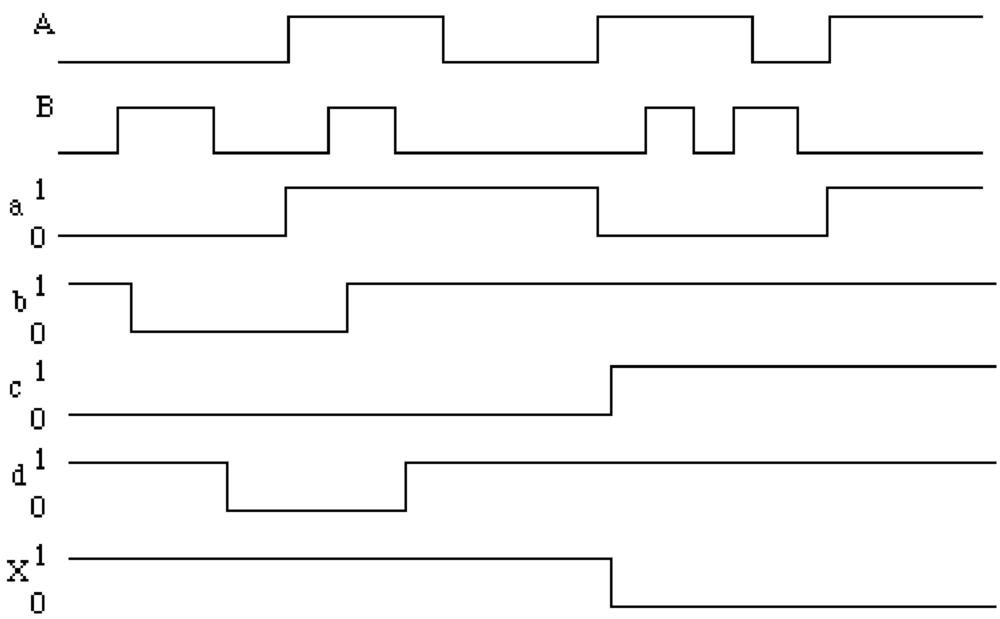
6.36

T	$\overline{\text{Preset}}$	$\overline{\text{Clear}}$	**Q**	$\overline{Q}$
↑	1	1	Q_0	$\overline{Q_0}$
↓	1	1	$\overline{Q_0}$	Q_0
0	1	1	Q_0	$\overline{Q_0}$
1	1	1	Q_0	$\overline{Q_0}$
0	0	1	1	0
1	1	0	0	1

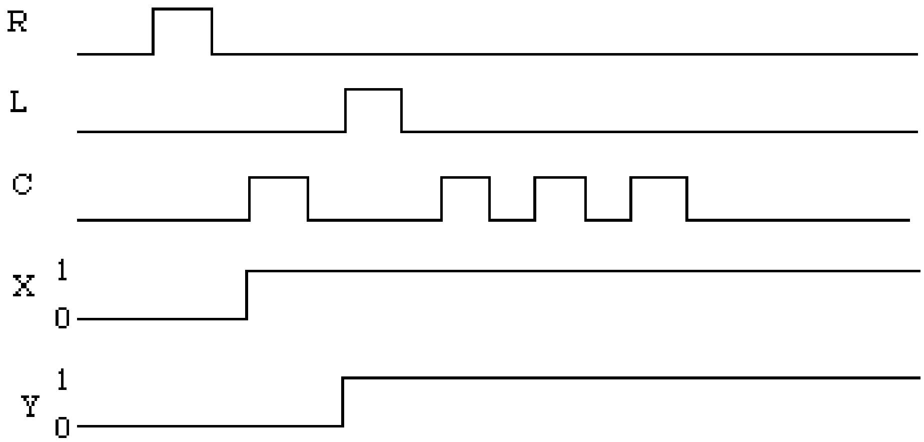
6.37

CK	**D**	$\overline{\text{Preset}}$	$\overline{\text{Clear}}$	**Q**	$\overline{\text{Q}}$
↑	x	1	1	Q_0	$\overline{Q_0}$
↓	0	1	1	0	1
↓	1	1	1	1	0
0	x	1	1	Q_0	$\overline{Q_0}$
1	x	1	1	Q_0	$\overline{Q_0}$
x	x	0	1	1	0
x	x	1	0	0	1

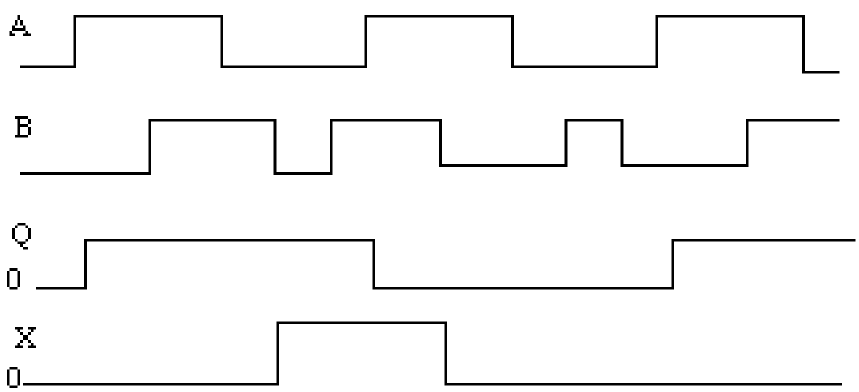
6.38



6.39

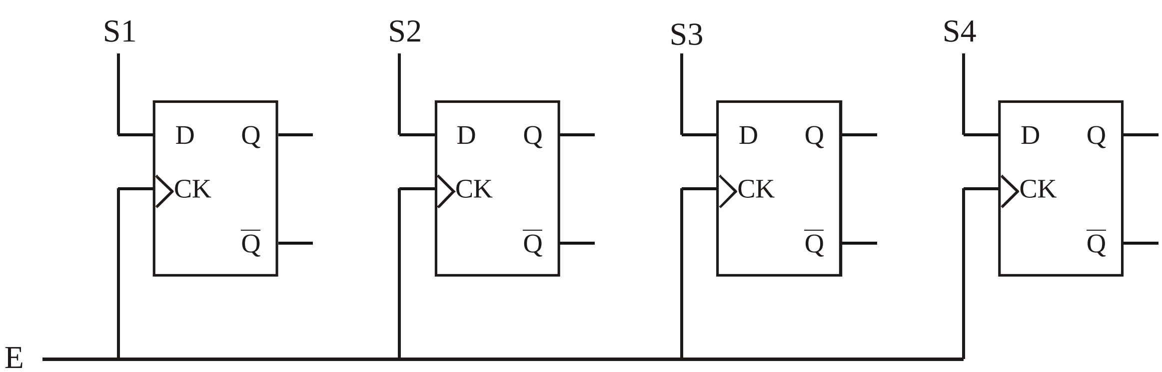


6.40



6.41 There is a delay between the release at contact "B" (which can exhibit bounce) and the connection at contact "A" (which can exhibit bounce). There are also small but important switching delays in the NAND gates. See Figure 6.7. The debounce circuit is an RS flip-flop with inverters at each input (to effectively eliminate the internal inverters).

6.42



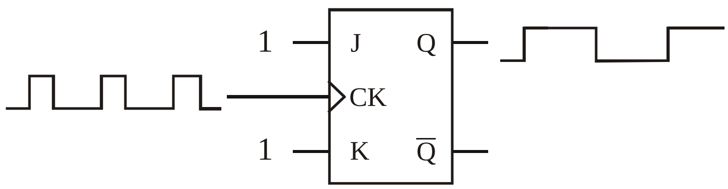
6.43 The $P_0P_1P_2P_3$ values change on the negative edge of each clock pulse as follows:

0000 (after reset pulsed low), 1000 (after 1st bit clock pulse), 0100 (after 2nd bit clock pulse), 1010 (after 3rd bit clock pulse), 1101 (after 4th bit clock pulse).

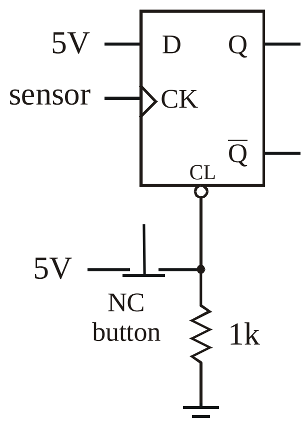
6.44 The $Q_0Q_1Q_2Q_3$ values (where $Q_3 = S_{out}$) change on the negative edge of each clock pulse as follows:

0000 (after reset pulsed low), 1011 (after load line pulsed high), 0101 (after 1st bit clock pulse), 0010 (after 2nd bit clock pulse), 0001 (after 3rd bit clock pulse), 0000 (after 4th bit clock pulse).

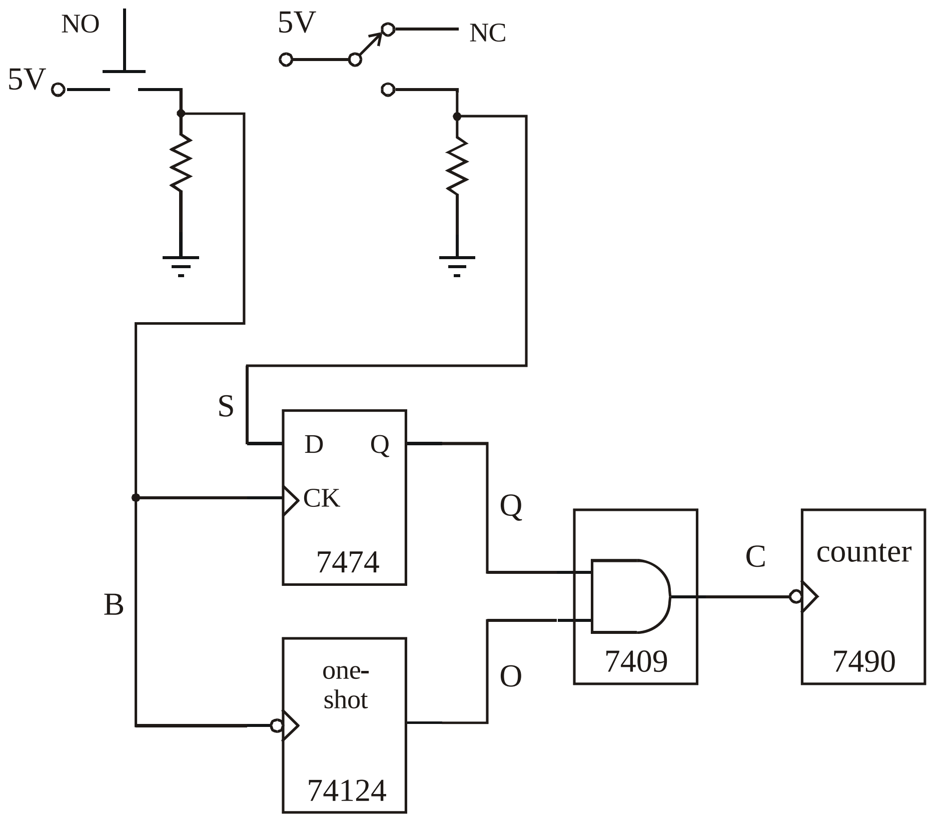
6.45



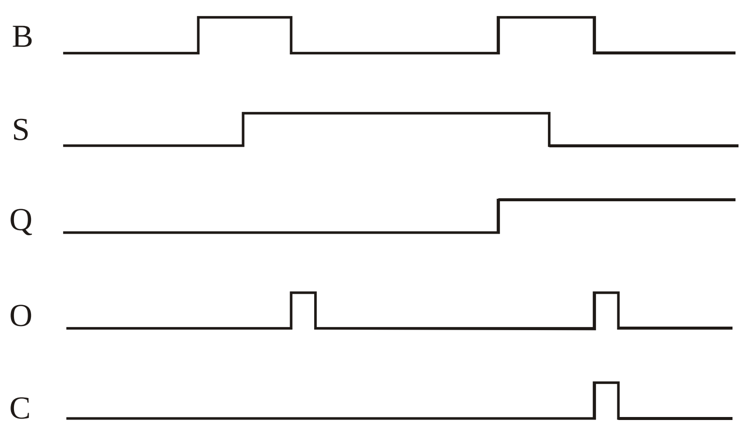
6.46



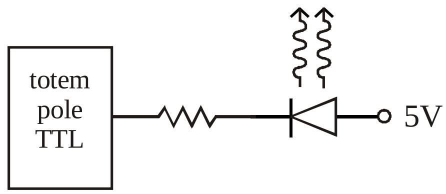
6.47



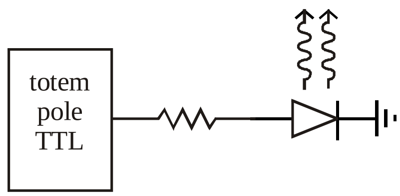
Note - With this design, the counter could be negative- or positive-edge triggered. If the counter is negative-edge triggered (as shown), the one-shot is actually not required.



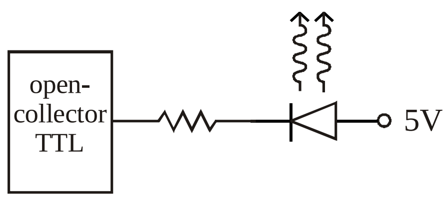
6.48



sinking current with low output



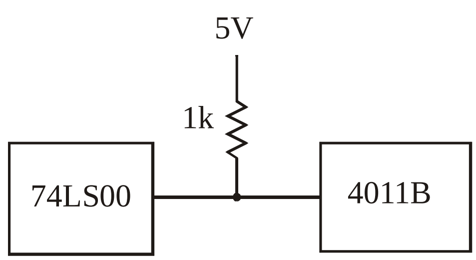
sourcing current with high output



sinking current with high output

TTL can sink more current than it can source, so the sourcing option wouldn't be as bright.

6.49



6.50 The CMOS LOW can sink only 1mA per gate which is enough to drive only two LS TTL inputs (which require 0.36 mA per gate).

6.51 $\bar{c} = \overline{Q_D}\,\overline{Q_C}\,Q_B\,\overline{Q_A}$

$$\bar{e} = (Q_D + Q_C + Q_B + Q_A)(Q_D + Q_C + \overline{Q_B} + Q_A)(Q_D + \overline{Q_C} + \overline{Q_B} + Q_A)(\overline{Q_D} + Q_C + Q_B + Q_A)$$

6.52 See info in TTL Data Book.

6.53 The input is the same: some sort of clock signal. Three of the four outputs look the same (the 3 least significant bits), but in the decade counter the most significant bit resets all the bits on the count of 10. The binary counter will continue to increment bits until 16 is reached. In the binary counter the output code provides 16 combinations, but in the decimal counter the output code provides 10 different output combinations.

6.54 The output goes high when the signal increases through 4V and low when it decreases through 1V.

6.55 $$V_{CAPACITOR} = V_{cc}\left(1 - e^{-\frac{t}{\tau}}\right) \quad \text{where} \quad \tau = RC$$

But when $t = \Delta T$, $V_{CAPACITOR} = 2/3\ V_{cc}$, so

$$\frac{2}{3} = \left(1 - e^{-\frac{\Delta T}{RC}}\right) \quad \text{so} \quad e^{-\frac{\Delta T}{RC}} = \frac{1}{3}$$

and

$$\Delta T = RC\ln(3) \approx 1.1RC$$

6.56 See a Linear Circuits data and/or applications book.

6.57 The time to discharge from $2/3V_{cc}$ to $1/3V_{cc}$ is the same as the time to charge from $1/3V_{cc}$ to $2/3V_{cc}$. From the section, the time to charge to $2/3V_{cc}$ is:

$$t_b = -R_2C\ln\left(\frac{1}{3}\right)$$

and the time to charge to $1/3V_{cc}$ is:

$$t_a = -R_2C\ln\left(\frac{2}{3}\right)$$

Therefore, the elapsed time would be:

$$T_2 = t_b - t_a = R_2C\left(\ln\left(\frac{2}{3}\right) - \ln\left(\frac{1}{3}\right)\right) = R_2C\ln\left(\frac{2/3}{1/3}\right) = R_2C\ln(2)$$

6.58 Ideally, making R_1=0 would make $T_1 = T_2$, which would result in a perfectly symmetric square wave. However, with R_1 shorted, there would no longer be any resistance in the transistor collector-emitter circuit which could result in excessive current to be sunk by the 555 when the base goes high, and this could result in damage.

If the capacitor has a partial charge initially, the first square-wave pulse width will be off slightly, but all subsequent pulses will be consistent.

6.59 If the count is updating immediately during the negative edge of signal L, it is possible that individual bits are latched either before or after the actual transition, depending on the exact timing of the counter outputs. This unlikely, but possible, scenario can be prevented by blocking the input pulses during the latch period, when L is high. This could be done by ANDing the input pulse line (I) with the inversion of the latch signal (L), and attach this output to the counter.

6.60 Assume that the digital event is a digital pulse that can be applied to the input of the counter. Cascade 3 74LS90's and connect the output of the third to an LED. Refer to the IC spec sheet for the appropriate wiring.

6.61 With the solution in 6.47, bounce with the SPDT switch has no effect. This can be verified with a timing diagram. If the button were pressed immediately after the switch, while bounce were still occurring, the stored value would be uncertain, but timing this fast would not be detectable (or repeatable) by a human anyway.

If the button exhibited bounce, the circuit would have a problem. Positive and negative edges would occur during the bounce, which would result in premature latching during the button press, and re-latching during the button release.

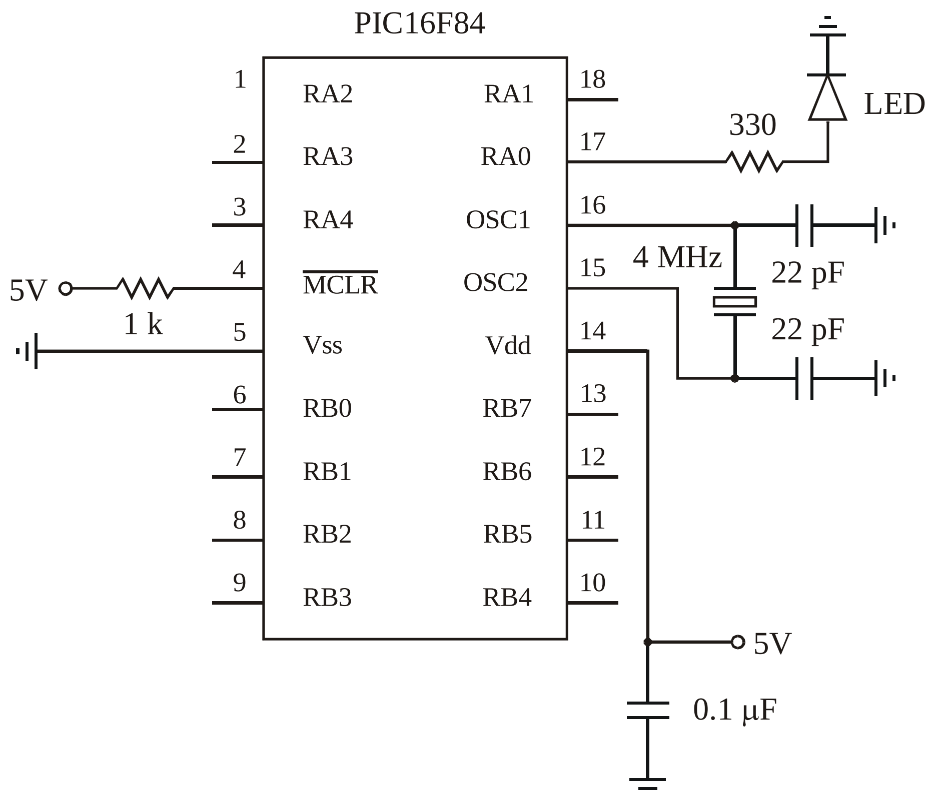
6.62 See "Case Study 2" in Chapter 11.

6.63 See "Case Study 2" in Chapter 11.

7.1 With the code provided, when the target LED turns on (after the 1st countdown to zero), it never goes off.

A more graceful solution would be to reset the counter and LEDs when the button is pressed again after the decrement to zero. If a "start" label is inserted above the "movlw target" line, we would just need to replace "goto begin" with "goto start." Then, the target LED would turn off and the countdown would start over again.

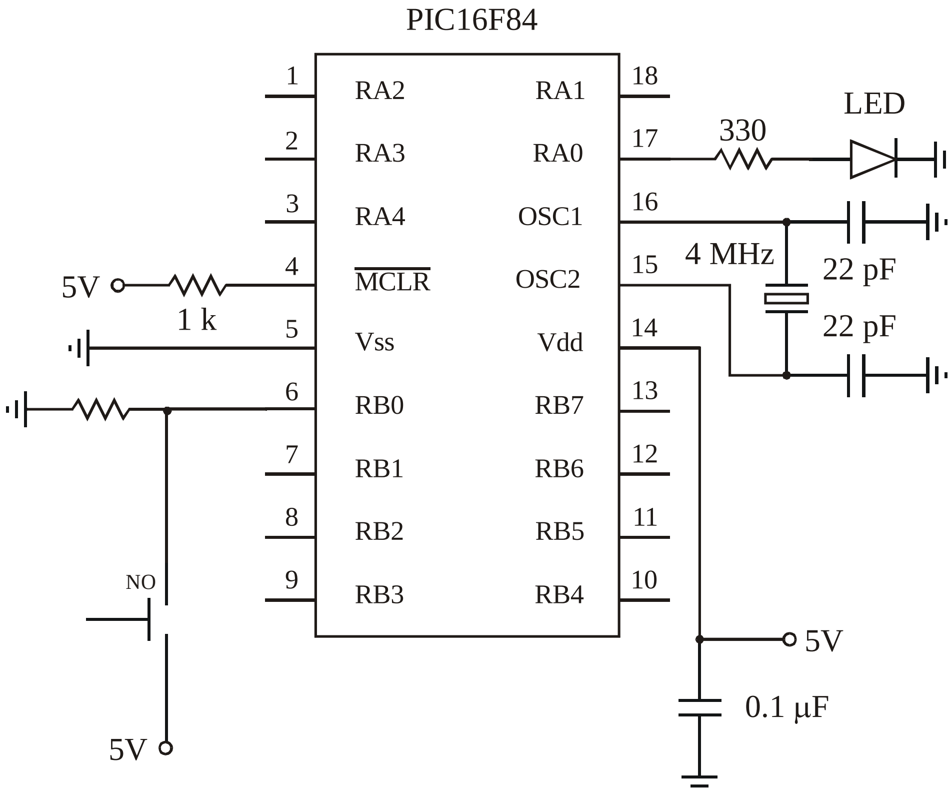
7.2




```
        list p=16f84
        include <p16F84.inc>

; Define counter variable locations
c1  equ  0x0c
c2  equ  0x0d
c3  equ  0x0e

        ; Initializes PORTA to output all zeros
        bcf STATUS, RP0     ; select bank 0
        clrf PORTA          ; initialize all pin values to zero
        bsf STATUS, RP0     ; select bank 1
        clrf TRISA          ; designate all PORTA pins as outputs
        bcf STATUS, RP0     ; select bank 0
```

```
; Main program loop
start
        bsf PORTA, 0        ; turn on the LED connected to RA0
        call pause          ; pause for 1 second
        bcf PORTA, 0        ; turn off the LED connected to RA0
        call pause          ; pause for 1 second

        goto start

; Subroutine to pause for approximately 1 second
pause
        ; Initialize counter variables
        movlw 0x00
        movwf c1
        movlw 0x00
        movwf c2
        movlw 0xFA
        movwf c3
loop
        incfsz c1, F
        goto loop
        incfsz c2, F
        goto loop
        incfsz c3, F
        goto loop
        return          ; end of subroutine

        end             ; end of instructions
```

7.3




```
' Program to turn an LED on and off at 1 Hz while a pushbutton switch
'   is being held down

' Define variable names for the I/O pins
my_button       Var     PORTB.0
my_led          Var     PORTA.0

begin:
        While (my_button == 1)          ' while the switch is held down
                ' Turn on the LED
                High my_led

                ' Wait for 1/2 sec
                Pause 500

                ' Turn off the LED
                Low my_led

                ' Wait for 1/2 sec
                Pause 500
        Wend
```

```
Goto begin     ' continue

End            ' end of instructions
```

7.4

```
' Program to perform the functionality of the Pot statement

' assumed variables:
'    pin: I/O pin identifier
'    scale: byte variable containing maximum time constant
'    var: byte variable containing measured time constant

' Charge the capacitor
High pin
Pause scale

' Discharge the capacitor and measure the discharge time
var = 0
Low pin
Input pin
While (pin == 1)              ' while the capacitor is not discharged
        Pause 1
        var = var + 1
Wend
```

7.5

```
' Subroutine to perform a software debounce on pin RB0
debounce:

' Define pin RB0 as an input
pin  Var  PORTB.0
Input pin

' Wait for the pushbutton switch to be pressed (1st bounce)
loop:
        If (pin == 0) Then loop

' Wait 10 milliseconds for the switch bounce to settle
Pause 10

' Wait for the pushbutton switch to be released (1st bounce)
loop2:
        If (pin == 1) Then loop2

' Wait 10 milliseconds for the switch bounce to settle
Pause 10

' End of subroutine
Return
```

7.6

```
' Using a 20MHz oscillator (PULSIN gives number of 2us increments)
sensor Var PORTA.0
high_width Var WORD
low_width Var WORD
period Var WORD
rpm Var WORD

start:
    ' Time entire pulse period
    PULSIN sensor, 1, high_width
    PULSIN sensor, 0, low_width
    period = high_width/2 + low_width/2
    ' Convert period to units of 10 mmin (10 milli-minutes)
    period = period / 60 / 100
    ' Calculate and display rpm (assuming rpm is in the approximate 10 to 10000 range)
    If (period > 0) Then
        rpm = 10000 / period
        Gosub display_rpm
    Endif
Goto start
```

7.7

```
' Subroutine to perform a simulated D/A conversion, holding a voltage for approximately
'   1 second
D_to_A:

' Define variables:
'    digital_value: predefined byte variable indicating the relative voltage value
pin  Var  PORTA.0     ' output pin
i  Var  BYTE          ' counter variable used in For loop

' Maintain filtered PWM signal for approximately 1 second
For i = 1 To 1000
    ' Hold the pin high for the portion of a second based on the digital value
    High pin
    ' Pause for the appropriate number of 1/255 (approximately) increments
    Pause (4*digital_value)

    ' Hold the pin low for the remainder of the second
    Low pin
    ' Pause for the appropriate number of 1/255 (approximately) increments
    Pause (4 * (255 – digital_value))
Next i

' End of subroutine
Return
```

7.8 The polling loop continues to run after the alarm as been activated, and the if the door and windows are closed the alarm will go off. An improved design would branch off to another section of code when the alarm is activated that would wait for some sort of alarm reset signal before deactivating the alarm.

7.9

```
' PicBasic Pro program to perform the control functions of the security system example
'   using interrupts

' Define variables for I/O port pins
door_or_window    Var   PORTB.0     ' signal A
motion            Var   PORTB.1     ' signal B
c                 Var   PORTB.2     ' signal C
d                 Var   PORTB.3     ' signal D
alarm             Var   PORTA.0     ' signal Y

' Define constants for use in IF comparisons
OPEN        Con   1     ' to indicate that a door OR window is open
DETECTED    Con   1     ' to indicate that motion is detected


' Initialize interrupts
OPTION_REG = $7F        ' enable PORTB pull-ups
On Interrupt Goto myint
INTCON = $88            ' enable interrupts on RB4 through RB7


' Main loop waiting for sensors to change value (i.e., wait for interrupts)
always:
    Low alarm     ' keep the alarm low until a sensor changes value
Goto always   ' continue


' Interrupt service routine that runs until sensors return to inactive states
Disable       ' disable interrupts during the interrupt service routine
myint:
  While ((door_or_window == OPEN) Or (motion == DETECTED))
    If ((c == 0) And (d == 1)) Then       ' operating state 1 (occupants sleeping)
      If (door_or_window == OPEN) Then
        High alarm
      Else
        Low alarm
      Endif
    Else
      If ((c == 1) And (d == 0)) Then   ' operating state 2 (occupants away)
        If ((door_or_window == OPEN) Or (motion == DETECTED)) Then
          High alarm
        Else
          Low alarm
        Endif
```

```
                Else          ’ operating state 3 or NA (alarm disabled)
                    Low alarm
                Endif
            Endif
        Wend
        INTCON.1 = 0     ’ clear the interrupt flag
        Resume           ’ end of interrupt service routine
    Enable               ’ allow interrupts again

    End
```

7.10

```
// Declare all global variables
const int switch_1=1;         // first combination switch
const int switch_2=2;         // second combination switch
const int switch_3=3;         // third combination switch
const int enter_button=4;     // combination enter key
const int green_led=5;        // green LED indicating a valid combination
const int red_led=6;          // red LED indicating an invalid combination
const int speaker=7;          // speaker signal for sounding an alarm
const int motor=8;            // signal to bias the motor power transistor
const int a=9;                // bit 0 for the 7447 BCD input
const int b=10;               // bit 1 for the 7447 BCD input
const int c=11;               // bit 2 for the 7447 BCD input
const int d=12;               // bit 3 for the 7447 BCD input
byte combination;             // stores the valid combination in the 3 LSBs
byte number_invalid;          // counter used to keep track of the number of bad
                              //   combinations

// Initializations
void setup() {
        // Define pin I/O status
        pinMode(switch_1, INPUT);
        pinMode(switch_2, INPUT);
        pinMode(switch_3, INPUT);
        pinMode(enter_button, INPUT);
        pinMode(green_led, OUTPUT);
        pinMode(red_led=6, OUTPUT);
        pinMode(speaker=7, OUTPUT);
        pinMode(motor=8, OUTPUT);
        pinMode(a=9, OUTPUT);
        pinMode(b=10, OUTPUT);
        pinMode(c=11, OUTPUT);
        pinMode(d=12, OUTPUT);

        // Initialize the valid combination and turn off all output functions
        combination = B101;           // valid combination (switch 3:on, switch
                                      //    2:off, switch 1:on)
        // Make sure the LEDs and motor are off and initialize the display
```

```
        digitalWrite(green_led, LOW);
        digitalWrite(red_led, LOW);
        digitalWrite(motor, LOW);
        digitalWrite(a, LOW);
        digitalWrite(b, LOW);
        digitalWrite(c, LOW);
        digitalWrite(d, LOW);
        // Initialize invalid combo counter
        number_invalid = 0;
}

// Main polling loop
void loop() {

        // Wait for the pushbutton switch to be pressed
        while (enter_button == 0);

        // Read switches and compare their states to the valid combination
        if ( (digitalRead(switch_1) == bitRead(combination, 0) &&
            (digitalRead(switch_2) == bitRead(combination, 1) &&
            (digitalRead(switch_2) == bitRead(combination, 1)) {
                // Turn on the green LED
                digitalWrite(green_led, HIGH);
                // Turn on the motor
                digitalWrite(motor, HIGH);

                // Reset the combination attempt counter
                number_invalid = 0
        }
        else {
                // Turn on the red LED
                digitalWrite(red_led, HIGH);
                // Sound the alarm
                tone (speaker, 80, 100);

                // Increment the combination attempt counter and check for overflow
                number_invalid = number_invalid + 1

                if (number_invalid > 9)
                        number_invalid = 0;
        }

        // Update the invalid combination attempt counter digit display
        a = bitRead(number_invalid, 0);
        b = bitRead(number_invalid, 1);
        c = bitRead(number_invalid, 2);
        d = bitRead(number_invalid, 3);

        // Wait for the pushbutton switch to be released
```

```
        while (enter_button == 1);

        // Turn off the LEDs and the motor
        digitalWrite(green_led, LOW);
        digitalWrite(red_led, LOW);
        digitalWrite(motor, LOW);
}
```

7.11

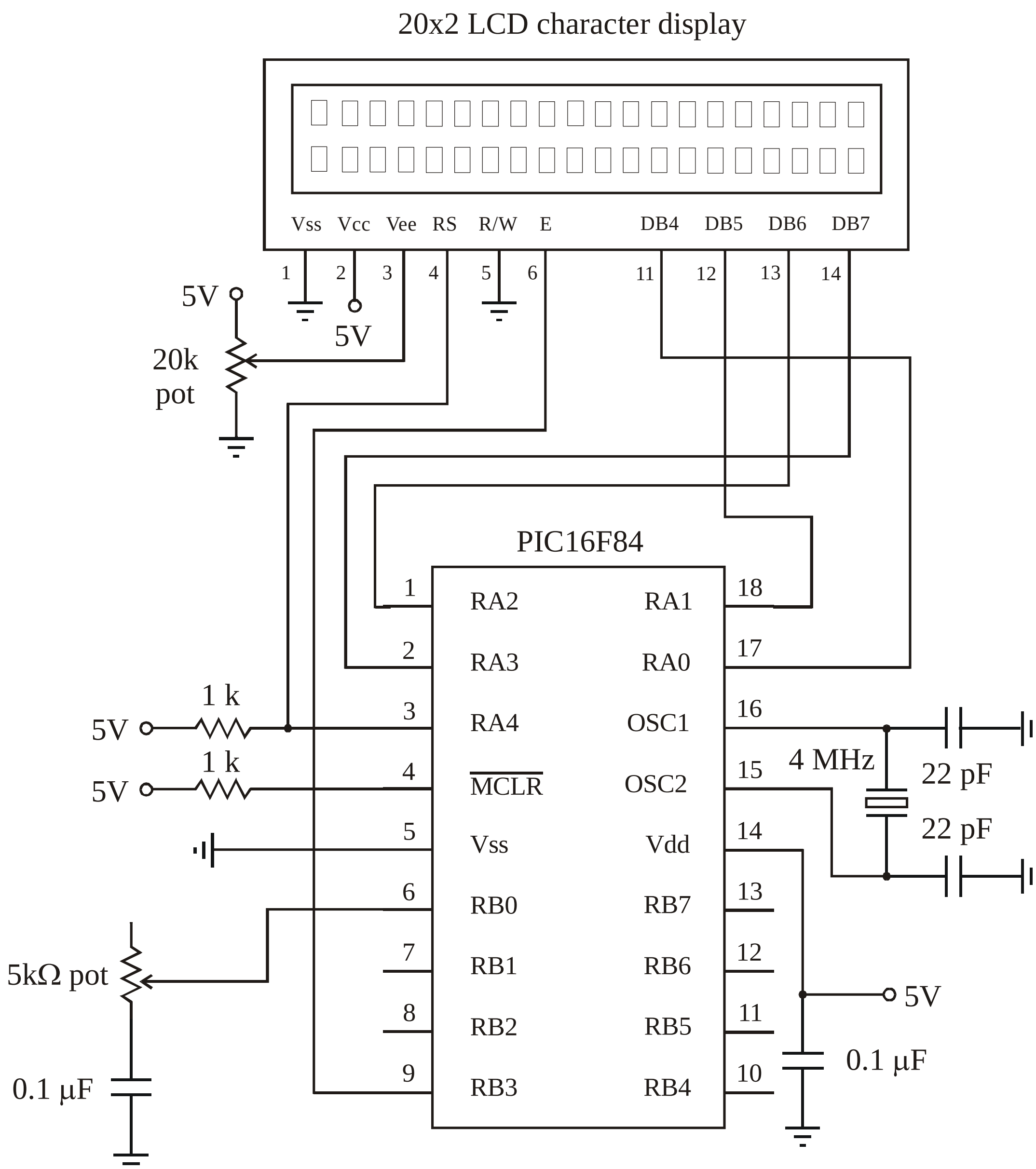
```
// Displays the scaled resistance value of a potentiometer on an LCD.
#include <LiquidCrystal.h>

// Define variables, pin assignments, and constants
byte value;              // scaled potentiometer value
short percentage;        // displayed potentiometer percentage value
const int pot_pin=1;     // potentiometer pin

LiquidCrystal lcd(2, 3, 4, 5, 6, 7);

void setup() {
        lcd.begin (16, 2);
}

void loop() {
        Pot pot_pin, SCALE, value        ' read the potentiometer value

        value = analogRead(pot_pin);
        // Scale value from 0-1023 to 0-100 range
        percentage = map(value, 0, 1023, 0, 100);

        // Display the percentage value on LCD display
         lcd.clear();
         lcd.print("pot value = ");
         lcd.print(percentage);
}
```

7.12 See 7.14 solution with:

```
#include <LiquidCrystal.h>

// Initialize arrays of the two 5-digit numbers
byte digits_prev[5];
byte digits[5];

// Initialize other variables
byte i;              // for loop counter
byte display[5];'    //number being displayed

LiquidCrystal lcd(2, 3, 4, 5, 6, 7);
```

```
void setup() {
	lcd.begin (16, 2);
}


void loop() {

	// When enter key is pressed, initialize the digits for the next number to be entered
	for (i=0; i<=4; i++) {
		digits_prev[i] = digits[i];
		digits[i] = 10;  // to indicate a missing digit (for numbers with < 5 digits)
	}

	// Display the numbers on the LCD display
	 lcd.clear();
	for (i=0; i<=4; i++)
		display[i] = digits_prev[i];
	display_digits();
	 lcd.setCursor(0, 1);		// move to 2nd line of display
	for (i=0; i<=4; i++)
		display[i] = digits[i];
	display_digits();
}


// Function to display the digits of a number stored in an array of five elements
void display_digits(void) {
	for (i=0; i<=4; i++)
		if (display[i] != 10)	// skip missing digits
			lcd.print(display[i]);
}
```

7.13




```
' Displays the scaled resistance value of a potentiometer on an LCD.

' Define variables, pin assignments, and constants
value       Var   BYTE       ' scaled potentiometer value
percentage  Var   WORD       ' displayed potentiometer percentage value
pot_pin     Var   PORTB.0    ' pin to which the potentiometer and series capacitor are
                             '   attached (RB0)
SCALE       Con   200        ' value for Pot statement scale factor
```

```
loop:
        Pot pot_pin, SCALE, value            ’ read the potentiometer value
        percentage = (value * 100) / 255     ’ convert to percentage value
        ’ Display the percentage value on LCD display
        Lcdout $FE, 1, "pot value = ", DEC percentage, " %"
Goto loop    ’ continue to sample and display the potentiometer value

End
```

7.14 Use a combination of Figures 7.11 and 7.13 along with the associated code. Use byte array variables called "digits_prev" and "digits" to store the digits of the entered numbers, where "digits_prev" contains the digits of the previous number entered and "digits" contains the digits of the current number being entered. Add appropriate processing statements to the keypad code to keep track of and store the digits of the current number. Here are **excerpts of code** needed in the implementation:

```
’ Initialize arrays of the two 5-digit numbers
digits_prev    Var    BYTE[5]
digits         Var    BYTE[5]

’ Initialize other variables
i              Var    BYTE           ’ For loop counter
display        Var    BYTE[5]        ’ number being displayed


’ When enter key is pressed, initialize the digits for the next number to be entered
For i = 0 To 4
        digits_prev[i] = digits[i]
        digits[i] = 10          ’ to indicate a missing digit (for numbers with < 5 digits)
Next i


’ Display the numbers on the LCD display
Lcdout $FE, 1          ’ clear the display
For i = 0 to 4
        display[i] = digits_prev[i]
Next i
Gosub display_digits
Lcdout $FE, $C0        ’ go to the next line of the display
For i = 0 to 4
        display[i] = digits[i]
Next i
Gosub display_digits



’ Subroutine to display the digits of a number stored in an array of five elements
display_digits:
```

```
For i = 0 To 4
        If (display[i] != 10)    ' skip missing digits
                Lcdout DEC display[i]
        Endif
Next i
Return
```

7.15 Pin RA4 is an open-collector output. The two possible states are open-circuit and ground. The pull-up resistor results in a logic high signal (5V) at V_{ee} when RA4 is in the open-circuit state. V_{ee} is at logic low (0V) when RA4 is grounded.

7.16 See the figure in the solution of Question 6.47 for the "hardware solution." A "software solution" would look something like:

```
B       Var   PORTB.0    ' pin attached to the bounce-free, NO button
S       Var   PORTB.1    ' pin attached to the SPDT switch
state   Var   BYTE       ' state of the switch when the button is pressed
count   Var   BYTE       ' variable used to track the

' Reset the counter variable
count = 0

' Main loop
loop:

' Wait for the button to be pressed, and store the state of the switch
While (B == 0) : Wend
state = S

' Wait for the button to be released, and increment the count if appropriate
While (B == 1) : Wend
If (state = 1) Then
        count = count + 1
Endif

Goto loop
```

7.17 If the button were not bounce-free, we would just need to add pause statements to allow the bounce to settle after each while loop (e.g., Pause 10).

7.18 See Design Example 7.1 for driving the display and see Question 7.5 for how to debounce the inputs (or use the Button statement). Here are some **code excerpts** that might be useful in the implementation:

```
my_count    Var   BYTE     ' current count (0 to 99)
first       Var   BYTE     ' first digit (tens place)
second      Var   BYTE     ' second digit (ones place)
```

```
inc     Var     PORTB.0     ’ pin attached to "increment" button
dec     Var     PORTB.1     ’ pin attached to "decrement" button
reset   Var     PORTB.2     ’ pin attached to "reset" button

UP              Con         0
DOWN            Con         1

’ Process the input
If (inc == DOWN) Then
        my_count = my_count + 1
        If (my_count > 99) Then
                my_count = 0
        Endif
Endif

If (dec == DOWN) Then
        If (my_count > 0) Then
                my_count = my_count – 1
        Endif
Endif

If (reset == DOWN) Then
        my_count = 0
        Endif
Endif

’ Determine the digits
first = my_count / 10
second = my_count – (10*first)
```

7.19 Go to www.microchip.com, navigate to the 8-bit, 16-series PIC Microcontrollers, and sort by "Memory Type" (for "FLASH"), then select the appropriate model from the table.

7.20 See the comments and program flow in the "poweramp.bas" code in Threaded Design Example A.4.

7.21 See the comments and program flow in the "stepper.bas" code in Threaded Design Example B.2.

7.22 See the comments and program flow in the "move," "move_steps," and "step_motor" subroutines in the "stepper.bas" code in Threaded Design Example B.2.

7.23 See the comments and program flow in the "speed" and "get_AD_value" subroutines in the "stepper.bas" code in Threaded Design Example B.2.

7.24 See the comments and program flow in the "position" and "get_encoder" subroutines in the "master PIC code" (dc_motor.bas) in Threaded Design Example C.3.

7.25 See the comments and program flow in the "slave PIC code" (dc_enc.bas) in Threaded Design Example C.3.

8.1 A digital computer or microprocessor uses digital or discrete data, that is, data that are simply strings of 1's and 0's that have no time correspondence. We have to add some type of time coding to make sense of the data. Therefore we have to design interfaces that will change (convert) analog information into a discretized form that will be compatible with a computer. Again, additional code must be included to provide the time references.

8.2 > 2*(15 kHz) = 30 kHz

8.3

(a) 1 sample per minute would probably suffice, so $f_s = 1/60$ Hz

(b) $f_s \geq 2(120\text{MHz}) = 240\text{MHz}$

(c) $f_s \geq 2(20\text{kHz}) = 40\text{MHz}$

8.4 Using MathCAD:

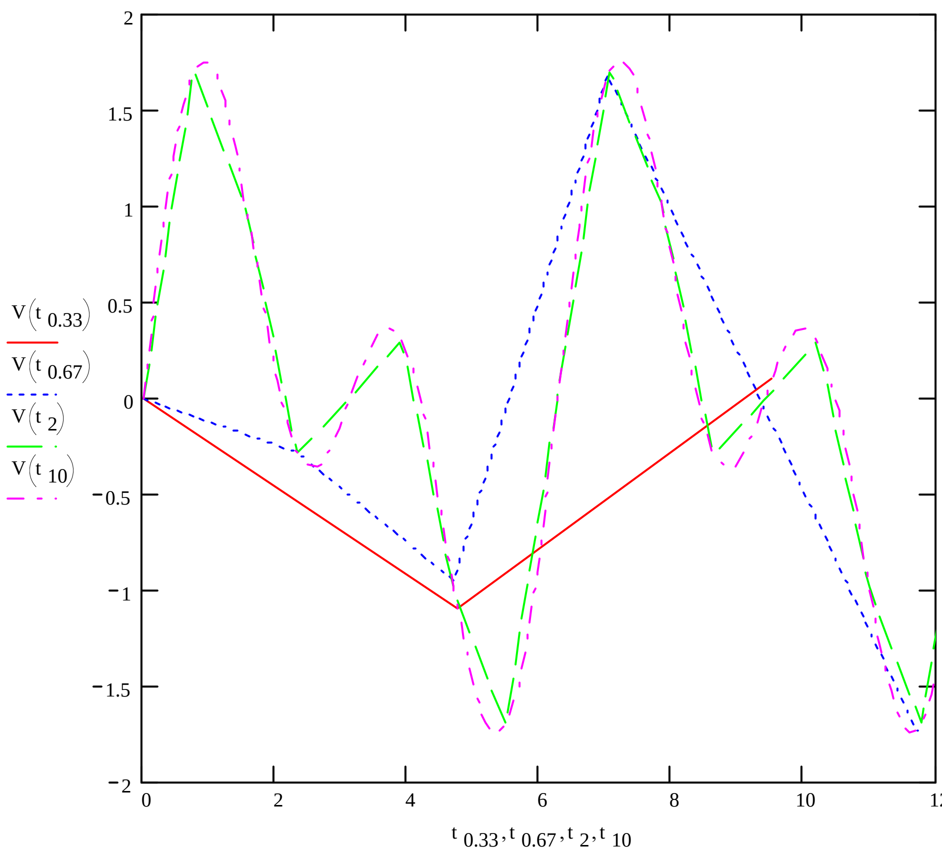
$\omega_0 := 1 \qquad \omega_{max} := 2$

$f_{max} := \frac{\omega_{max}}{(2 \cdot \pi)} \qquad T := \frac{2 \cdot \pi}{\omega_0} \qquad f_s := 2 \cdot f_{max} \qquad \Delta t := \frac{1}{f_s}$

$f_{max} = 0.318 \qquad T = 6.283 \qquad f_s = 0.637 \qquad \Delta t = 1.571$

$$V(t) := \sin(t) + \sin(2 \cdot t)$$

$t_{0.33} := 0, \frac{\Delta t}{0.33} .. 2 \cdot T \qquad t_{0.67} := 0, \frac{\Delta t}{0.67} .. 2 \cdot T \qquad t_2 := 0, \frac{\Delta t}{2} .. 2 \cdot T \qquad t_{10} := 0, \frac{\Delta t}{10} .. 2 \cdot T$

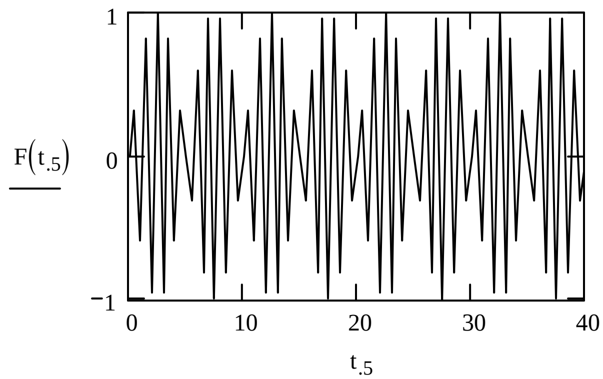


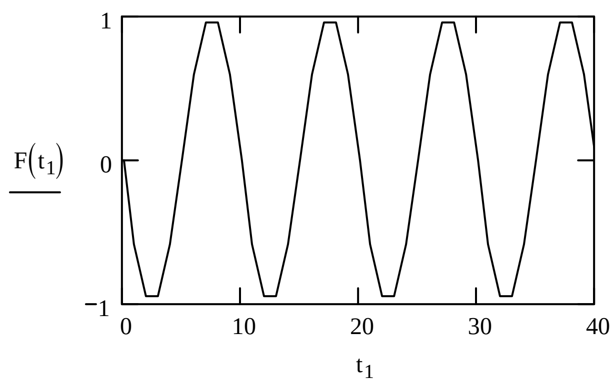
8.5 Using MathCAD:

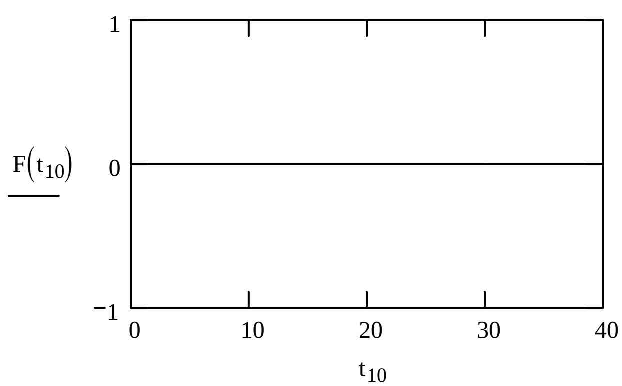
$$a := 2\cdot\pi \qquad b := 0.9\cdot a$$

$$t_{.5} := 0, 0.5 .. 40 \qquad t_1 := 0, 1 .. 40 \qquad t_{10} := 0, 10 .. 40$$

$$F(t) := 2\cos\left(\frac{a-b}{2}\cdot t\right)\cdot\sin\left(\frac{a+b}{2}\cdot t\right)$$







8.6 From equation 8.2, to prevent aliasing,

$$f_s > (2f_{max} = 2a)$$

For a higher fidelity representation, a much higher sampling rate is required.

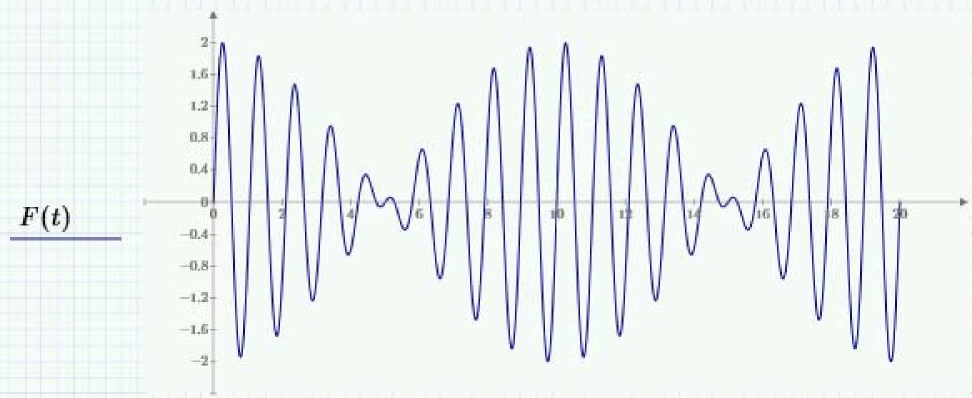
8.7 Using MathCAD,

$T := 1 \qquad a := 2 \cdot \frac{\pi}{T} \qquad b := 0.9 \cdot a \qquad f_{max} := \frac{a}{2 \cdot \pi}$

$$F(t) := \sin(a \cdot t) + \sin(b \cdot t)$$

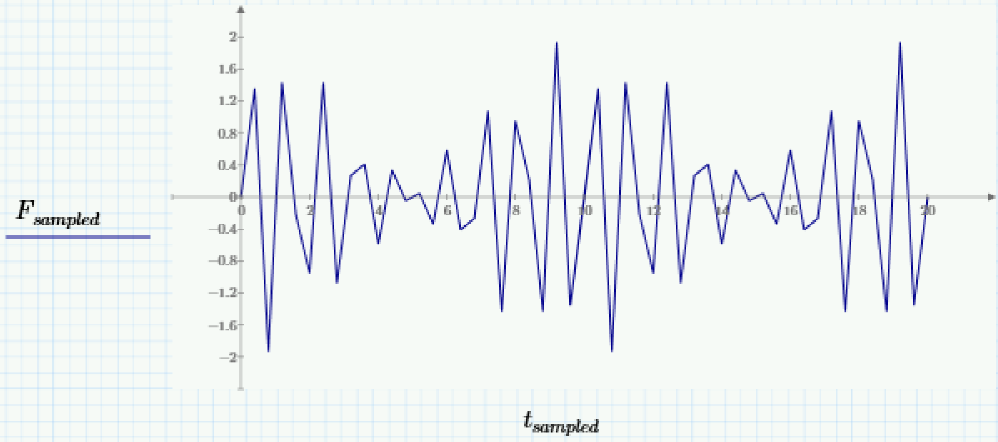
beat frequency: $f_b := \frac{(a-b)}{2 \cdot \pi}$

$T_s := \frac{2}{f_b} = 20 \qquad \Delta t := 0.01 \qquad t := 0, \Delta t .. T_s$



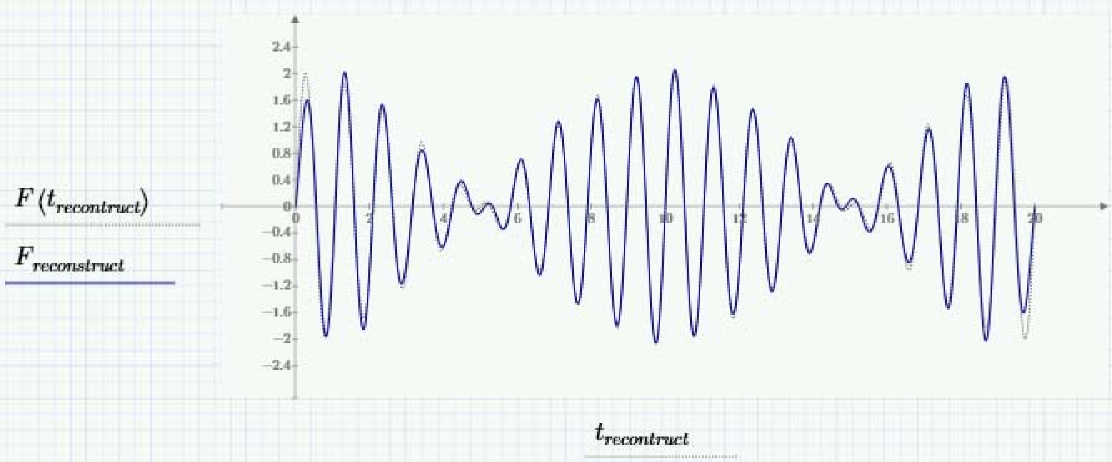
$f_s := 2.5 \cdot f_{max}$ $\quad \Delta t_s := \frac{1}{f_s} = 0.4$ $\quad N_s := \frac{T_s}{\Delta t_s} = 50$ $\quad i := 0..N_s$

$t_{sampled_i} := i \cdot \Delta t_s$ $\quad F_{sampled_i} := F\left(t_{sampled_i}\right)$



$$f(t) := \sum_{i=0}^{N_s} F_{sampled_i} \cdot \mathrm{sinc}\left(\frac{\pi \cdot \left(t - t_{sampled_i}\right)}{\Delta t_s}\right)$$

$N := \frac{T_s}{\Delta t}$ $\quad i := 0..N$ $\quad t_{recontruct_i} := i \cdot \Delta t$ $\quad F_{reconstruct_i} := f\left(t_{recontruct_i}\right)$



8.8 $\quad Q = \dfrac{5\text{V} - (-5\text{V})}{4096} = 2.44\text{mV}$

8.9 $N = \frac{5V - (-5V)}{0.005V} = 2000$

An 11 bit A/D converter would suffice since $2^{11} = 2048$. A 12-bit converter would be the minimal acceptable standard size available.

8.10 $N = 2^8 = 256$

$Q = 10V / N = 0.0391$ V

The digital state number for a given voltage V between 0 and 10 is the truncated value of V/Q. The code is the binary equivalent of the state number.

(a) 0/Q=0 corresponding to state 0: 00000000

(b) 1/Q=25.6 corresponding to state 25: 00011001

(c) 5/Q=127.9 corresponding to state 127: 01111111

(d) 7.5/Q=191.8 corresponding to state 191: 10111111

8.11 $10\text{sec}\left(5000\frac{\text{samples}}{\text{sec}}\right)\left(12\frac{\text{bits}}{\text{sample}}\right)\left(\frac{1\text{byte}}{8\text{bits}}\right) = 75000\text{bytes}$

8.12 $B_0 = \overline{G_2}\,\overline{G_1}G_0 + G_2G_1G_0$

but it is clear in the truth table that $G_1G_0 = G_1$, $G_2G_1 = G_2$, and $\overline{G_2}\,\overline{G_1} = \overline{G_1}$, so

$$B_0 = \overline{G_1}G_0 + G_2$$

Also, $B_1 = \overline{G_2}G_1G_0 + G_2G_1G_0 = G_1G_0 = G_1$

8.13

bit	scale fraction	bit value	cumulative voltage
5	1/2	1	-5V + 1/2(10 V) = 0 V
4	1/4	0	0 V
3	1/8	1	1.25 V
2	1/16	1	1.875 V
1	1/32	1	2.1875 V

digital output = 10111

8.14 more memory will be required

8.15 See www.microchip.com
resolution: 12 bits
architecture: successive approximation

8.16 See www.national.com

8.17 $V_{out_1} = -\frac{1}{2}V_1 = -V_o = -\frac{1}{8}V_s$

8.18 $V_{out_2} = -\frac{1}{2}V_2 = -2V_o = -\frac{1}{4}V_s$

8.19 $V_{out_3} = -\frac{1}{2}V_3 = -4V_o = -\frac{1}{2}V_s$

8.20 The low end of the range (at 0000) would be -10V. The increment between states would be:

$$-\frac{1}{16}(10V - (-10V)) = -\frac{1}{16}20V = -\frac{5}{4}V$$

So the value at 0001 would be:

$$-10V - \frac{5}{4}V = -11\frac{1}{4}V$$

The value at 1111, would be:

$$-10V - \frac{15}{16}(20V) = -28\frac{3}{4}V$$

8.21 The standard sampling rate for high-fidelity audio recordings is:

$$(f_s = 44kHz) > 2(20kHz)$$

The sample interval corresponding to this frequency is:

$$\Delta t = \frac{1}{f_s} = 0.023\frac{ms}{sampe}$$

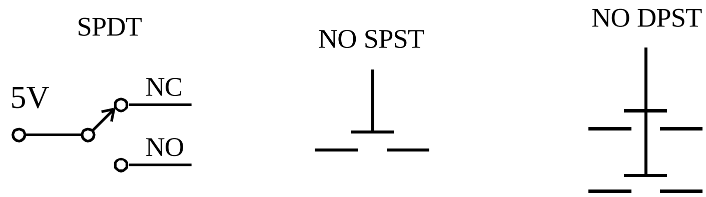
For a total time of T=45min, the total required number of samples is:

$$\frac{T}{\Delta t} = 118800000 samples$$

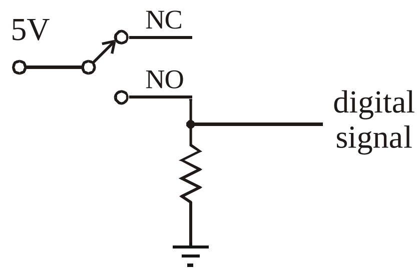
Assuming stereo audio without compression, and using the standard high-fidelity resolution of 16 bits per sample, the total number of memory required is:

$$\frac{T}{\Delta t}\left(16\frac{\text{bits}}{\text{sample}}\right)(2\text{channels})\left(\frac{1\text{byte}}{8\text{bits}}\right)\left(\frac{1\text{kB}}{1024\text{bytes}}\right)\left(\frac{1\text{MB}}{1024\text{kB}}\right) = 453\text{MB}$$

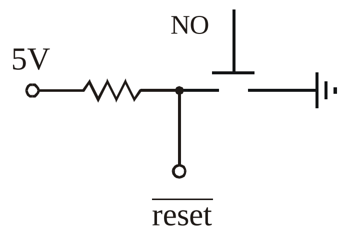
9.1


SPDT
5V
NC
NO
NO SPST
NO DPST

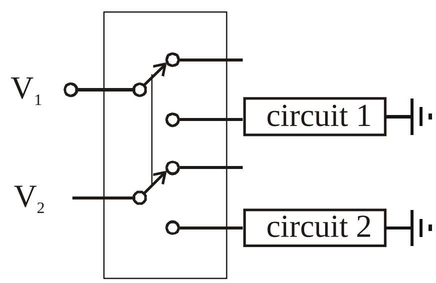
9.2


5V
NC
NO
digital
signal

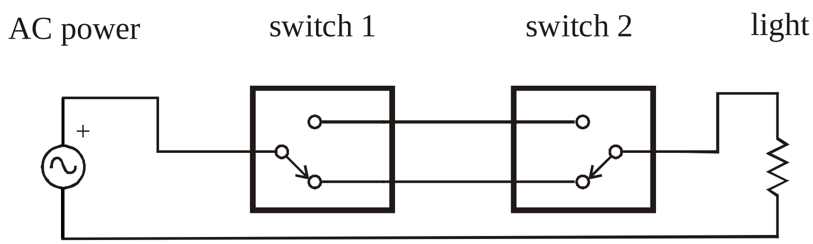
9.3


NO
5V
$\overline{\text{reset}}$

9.4


V_1
V_2
circuit 1
circuit 2

9.5



9.6 Regardless of the polarity of the voltage in each secondary coil, the current flows through an upper diode, down through the resistor, then through a lower diode back to the coil. Therefore, the voltage polarities do change across the resistors, and $V_{out} = V_{left} - V_{right}$, where V_{left} and V_{right} are the secondary coil voltages. When the core is to the left, V_{left} is larger and $V_{out}>0$. When the core is to the right, V_{right} is larger and $V_{out}<0$. When the core is centered, both secondaries have the same voltage and $V_{out}=0$.

9.7 The excitation frequency (f_{ex}) should be much larger than the maximum core displacement frequency (f_{max}) to prevent aliasing and to result in a high-fidelity representation. The low-pass filter cut-off frequency (f_{low_pass}) should be between f_{ex} and f_{max} to filter out the high frequency of the excitation but pass the lower frequency displacement signal.

9.8 During the transition from 3 (0011) to 4 (0100), any of the following 8 codes could result: 0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111.

9.9 From Table 9.1, the all four bits change value between decimal code 7 (0111) and 8 (1000), so during this transition, any bit can have either value (0 or 1), so the maximum count uncertainty is the full range.

9.10 $$\left(\frac{360°}{\text{rev}}\right)\left(\frac{1\text{rev}}{1000\text{lines}}\right)\left(\frac{1\text{line}}{2\text{pulses}}\right) = \frac{0.18°}{\text{pulse}}$$

9.11

```
’ Declare signal and count variables
A Var PORTB.0
B Var PORTB.1
count Var WORD

’ Store the initial states of signals and initialize count
A_prev = A
B_prev = B
count = 0
```

```
' Polling loop to monitor signal negative edges and update count
loop:
        If ((A == 0) And (A_prev == 1) And (B == 1)) Then
                count = count + 1
        EndIf
        A_prev = A

        If ((B == 0) And (B_prev ==1) And (A == 1)) Then
                count = count - 1
        EndIf
        B_prev = B
Goto loop
```

9.12 Checking row 7 ($B_3B_2B_1B_0$=0111, $G_3G_2G_1G_0$=0100) gives:

$0 = 0$

$1 = 0 \oplus 1$

$1 = 1 \oplus 0$

$1 = 1 \oplus 0$

Checking row 8 ($B_3B_2B_1B_0$=1000, $G_3G_2G_1G_0$=1100) gives:

$1 = 1$

$0 = 1 \oplus 1$

$0 = 0 \oplus 0$

$0 = 0 \oplus 0$

9.13 $A = \dfrac{\pi D^2}{4}$

$dA = 2\pi\dfrac{D}{4}dD$

but $\dfrac{dD}{D} = -\nu\dfrac{dL}{L}$, so

$$\frac{dA}{A} = \frac{2\pi\frac{D}{4}\left(-\nu D\frac{dL}{L}\right)}{\frac{\pi D^2}{4}} = -2\nu\frac{dL}{L}$$

9.14 $A = \dfrac{\pi D^2}{4} = 0.0491\text{in}^2$

$$\sigma = \frac{P}{A} = 10,190\text{psi}$$

$$E_{steel} = 30 \times 10^6\text{psi}$$

$$\varepsilon = \frac{\sigma}{E} = 3.40 \times 10^{-4} = 340 \times 10^{-6} = 340\mu\varepsilon$$

$$F = \frac{\Delta R / R}{\varepsilon} = \frac{0.01/120}{3.40 \times 10^{-4}} = 0.245$$

9.15 For a metal foil strain gage with F=2 and ν=0.3, Equation 9.11 gives

$$2 = 1 + 2(0.3) + PZ$$

where PZ is the piezoresistive term, which works out to be 0.4. Therefore, the change in length term (1) provides 50% (1/2) of the effect, the change is area term (0.6) provides 30% (0.6/2) of the effect, and PZ accounts for the remaining 20% (0.4/2).

9.16 $E = 200 \times 10^9\text{Pa} \quad D = 0.010\text{m} \quad P = 50 \times 10^3\text{N}$

$$F_G = 2.115 \quad R = 120\Omega$$

$$A = \frac{\pi D^2}{4} = 7.854 \times 10^{-5}\text{m}^2$$

$$\sigma = \frac{P}{A} = 0.637 \times 10^9\text{Pa} \quad \varepsilon = \frac{\sigma}{E} = 0.00318$$

$$\Delta R = \varepsilon F_G R = 0.808\Omega \quad R_1 = R + \Delta R = 120.808\Omega \quad R_2 = R_3 = R_4 = 120\Omega$$

$$V_o = V_{ex}\left(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3}\right) = 0.00168V_{ex}$$

9.17 From Equation 9.21, with V_{out}=0 and R_2=R_3,

$$0 = V_{ex}\left(\frac{R_1}{R_1 + R_4} - \frac{1}{2}\right)$$

Therefore,

$$\frac{R_1}{R_1 + R_4} = \frac{1}{2}$$

which gives:

$$R_4 = R_1$$

So the potentiometer must be adjusted to the exact resistance of the strain gage to balance the bridge.

9.18 Combination of Figures 9.24b and 9.25.

9.19 $I \approx \frac{V_{ex}}{2R} = \frac{10V}{2(350\Omega)} = 14.2mA$

$V \approx \frac{V_{ex}}{2} = 5V$

$P = IV = 71.4mW$

9.20 $2R' < 0.001R_G$ so $R' < \frac{0.001}{2}R_G$

but $R' = L\left(0.050\frac{\Omega}{m}\right)$, so

$$L < \frac{0.001}{2(0.050)}120m = 1.2m$$

9.21 Using MathCAD:

Type J Coefficents:

$c_0 := -0.0488683$ $\qquad c_3 := 1.15692\cdot 10^7$

$c_1 := 19873.1$ $\qquad c_4 := -2.64918\cdot 10^8$

$c_2 := -218615$ $\qquad c_5 := 2.01844\cdot 10^9$

Temperature/Voltage Relationship:

$$T(V) := \sum_{i=0}^{5} c_i \cdot V^i$$

Approximate Sensitivity:

$T(0) = -0.049 \qquad T(0.03) = 546.224$

$$DVDT := \frac{0.03 - 0}{T(0.03) - T(0)} \qquad DVDT = 5.492\cdot 10^{-5}$$

The sensitivity is approximately 0.055 mV / deg C.

9.22 Using MathCAD:

Type J Coefficents:

$c_0 := 0.100861$ $\qquad c_3 := 7.80256\cdot 10^7$ $\qquad c_6 := -2.66192\cdot 10^{13}$

$c_1 := 25727.9$ $\qquad c_4 := -9.24749\cdot 10^9$ $\qquad c_7 := 3.94078\cdot 10^{14}$

$c_2 := -767346$ $\qquad c_5 := 6.97688\cdot 10^{11}$

Temperature/Voltage Relationship:

$$T(V) := \sum_{i=0}^{7} c_i \cdot V^i$$

Approximate Sensitivity:

$T(0) = 0.101 \qquad T(0.015) = 302.478 \qquad T(0.010) = 213.286$

$$DVDT := \frac{0.015 - 0}{T(0.015) - T(0)} \qquad DVDT = 4.961\cdot 10^{-5}$$

The sensitivity is approximately 0.050 mV / deg C.

9.23 Using MathCAD:

Type J Coefficents:

$c_0 := -0.0488683$ $\qquad$ $c_3 := 1.1569210^7$

$c_1 := 19873.1$ $\qquad$ $c_4 := -2.6491810^8$

$c_2 := -218615$ $\qquad$ $c_5 := 2.0184410^9$

Temperature/Voltage Relationship:

$$T(V) := \sum_{i=0}^{5} c_i \cdot V^i$$

$V := 0$

$V_{200} := \mathrm{root}(T(V) - 200, V)$ $\qquad$ $V_{200} = 0.011$ $\qquad$ $T\left(V_{200}\right) = 200$

So

$$V_{200/0} = 11\mathrm{mV}$$

9.24 $V_{T/100} = 30\mathrm{mV}$

$100 = a_0 + a_1V_{100/0} + a_2V_{100/0}^2 + a_3V_{100/0}^3 + a_4V_{100/0}^4 + a_5V_{100/0}^5$

$V_{100/0} = 5.26\mathrm{mV}$

$V_{T/0} = V_{T/100} + V_{100/0} = 35.26\mathrm{mV}$

$T(V_{T/0}) = 636.6°\mathrm{C}$

9.25 $V_{T/11} = 30\mathrm{mV}$

$11 = a_0 + a_1V_{11/0} + a_2V_{11/0}^2 + a_3V_{11/0}^3 + a_4V_{11/0}^4 + a_5V_{11/0}^5$

$V_{11/0} = 0.559\mathrm{mV}$

$V_{T/0} = V_{T/11} + V_{11/0} = 30.559\mathrm{mV}$

$T(V_{T/0}) = 556°\mathrm{C}$

9.26 Using MathCAD:

Type J Coefficents:

$c_0 := -0.0488683$ $\quad$ $c_3 := 1.15692 \cdot 10^{7}$

$c_1 := 19873.1$ $\quad$ $c_4 := -2.64918 \cdot 10^{8}$

$c_2 := -218615$ $\quad$ $c_5 := 2.01844 \cdot 10^{9}$

Temperature/Voltage Relationship:

$$T(V) := \sum_{i=0}^{5} c_i \cdot V^i$$

$V := 0$

$V_{120} := \mathrm{root}(T(V) - 120, V)$ $\quad$ $V_{120} = 6.356 \cdot 10^{-3}$ $\quad$ $T(V_{120}) = 120$

$V_{10} := \mathrm{root}(T(V) - 10, V)$ $\quad$ $V_{10} = 5.084 \cdot 10^{-4}$ $\quad$ $T(V_{10}) = 10$

$\Delta V := V_{120} - V_{10}$ $\quad$ $\Delta V = 5.848 \cdot 10^{-3}$

So the change in voltage would be 5.85 mV.

9.27 From Equation 9.62:

$$\left(\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1\right)X_r(s) = \left(-\frac{1}{\omega_n^2}s^2\right)X_i(s)$$

so the transfer function is:

$$G(s) = \frac{X_r(s)}{X_i(s)} = \frac{-\frac{1}{\omega_n^2}s^2}{\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1}$$

Substituting s=jω gives:

$$G(j\omega) = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{-\left(\frac{\omega}{\omega_n}\right)^2 + 2\zeta\frac{\omega}{\omega_n}j + 1}$$

Therefore, the amplitude magnitude is:

$$\frac{X_r}{X_i} = |G(j\omega)| = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

9.28 $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{5000\frac{N}{m}}{0.05kg}} = 316.2\frac{rad}{sec}$

$$\frac{\omega}{\omega_n} = \frac{100}{316.2} = 0.316$$

$$\left(\frac{\omega}{\omega_n}\right)^2 = 0.1$$

$$\zeta = \frac{b}{2\sqrt{km}} = \frac{30}{2\sqrt{5000(0.05)}} = 0.949$$

(a) $|\ddot{x}_{in}|_{actual} = X_{in}\omega^2 = 5mm\left(100\frac{rad}{sec}\right)^2 = 5\times 10^4\frac{mm}{sec^2} = 50\frac{m}{sec^2}$

$$|\ddot{x}_{in}|_{actual} = \left(50\frac{m}{sec^2}\right) \div \left(9.81\frac{m/sec}{g}\right) = 5.1g$$

(b) $H_a(\omega) = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}} = 1.08$

$$X_r = \frac{1}{\omega_n^2}H_a(\omega)(X_{in}\omega^2) = \frac{1}{\left(316.2\frac{rad}{sec}\right)^2}(1.08)\left(50\frac{m}{sec^2}\right)$$

$$X_r = 5.4\times 10^{-4}m = 0.54mm$$

(c) Since $H_a(\omega)$ is assumed to be 1 for the accelerometer device for all ω's,

$$|\ddot{x}_{in}|_{measured} = \omega_n^2 X_r(1) = 54\frac{m}{sec^2}$$

(d) $$\phi = -\tan^{-1}\left(\frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right) = -\tan^{-1}\left(\frac{2(0.949)(0.316)}{1-0.1}\right) = -33.7° = -0.588\,\mathrm{rad}$$

$$x_r(t) = X_r\sin(\omega t + \phi) = 0.54\sin(100t - 0.588)\,\mathrm{mm}$$

9.29 $$m = 1\,\mathrm{kg} \quad k = 2\frac{\mathrm{N}}{\mathrm{m}} \quad b = 2\frac{\mathrm{Ns}}{\mathrm{m}} \quad \omega = 1.25\frac{\mathrm{rad}}{\mathrm{s}} \quad X_i = 0.010\,\mathrm{m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = 1.414\frac{\mathrm{rad}}{\mathrm{s}} \quad \omega_r = \frac{\omega}{\omega_n} = 0.884 \quad \varsigma = \frac{b}{2\sqrt{km}} = 0.707$$

$$\frac{X_r}{X_i} = \frac{\omega_r^2}{\sqrt{(1-\omega_r^2)^2 + 4\varsigma^2\omega_r^2}} = 0.616$$

$$\phi = -\tan^{-1}\left(\frac{2\varsigma\omega_r}{1-\omega_r^2}\right) = -80.1° = -1.398\,\mathrm{rad}$$

$$X_r = X_i\left(\frac{X_r}{X_i}\right) = 0.00616\,\mathrm{m}$$

Therefore, the steady state output displacement response is:

$$x_o(t) = x_i(t) + x_r(t) = X_i\sin(\omega t) + X_r\sin(\omega t + \phi)$$

$$x_o(t) = (10\sin(1.25t) + 6.16\sin(1.25t - 1.40))\,\mathrm{mm}$$

10.1 Use a combination of a power transistor switch circuit and a diode clamp.

10.2 Electric motors and solenoids create changing magnetic fields which induce voltages in nearby unshielded circuits.

10.3 $P(\omega) = \omega T_s\left(1 - \frac{\omega}{\omega_{max}}\right)$

At maximum power,

$$\omega = \frac{1}{2}\omega_{max}$$

so the maximum power is:

$$P_{max} = P\left(\frac{1}{2}\omega_{max}\right) = \frac{1}{2}\omega_{max}T_s\left(1 - \frac{\frac{1}{2}\omega_{max}}{\omega_{max}}\right) = \frac{1}{4}\omega_{max}T_s$$

10.4 At the maximum no-load speed, the motor torque is 0 so

$$\omega_{max} = \frac{V_{in}}{k_e} = \frac{10V}{(12V)/(1000rpm)} = 833rpm = 87.3\frac{rad}{sec}$$

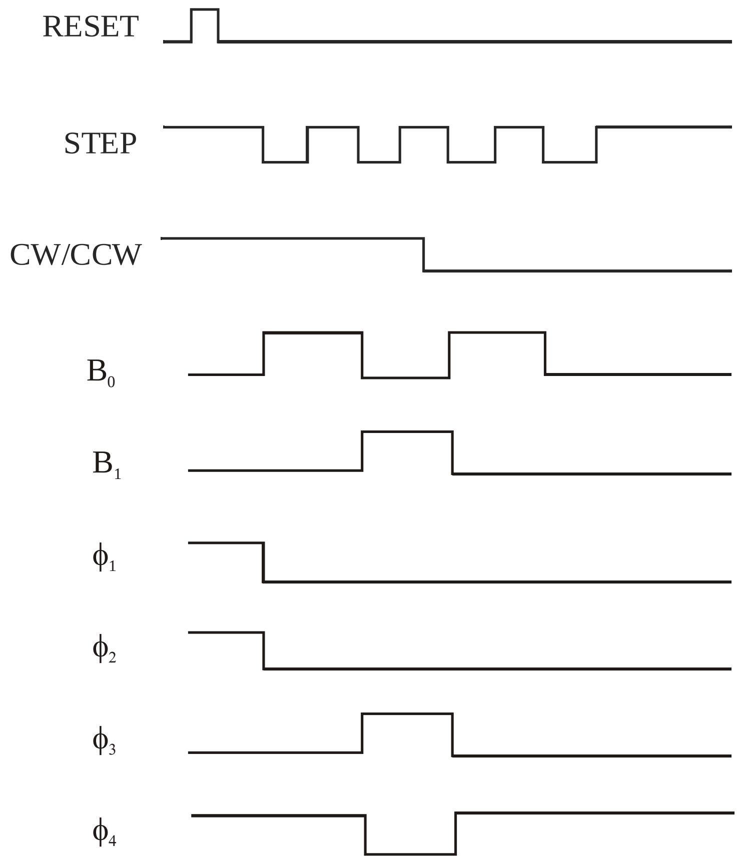
$$I_s = \frac{V_{in}}{R} = 6.67A$$

$$T_s = k_t\frac{V_{in}}{R} = 0.8Nm$$

$$P_{max} = P\left(\frac{\omega_{max}}{2}\right) = \frac{\omega_{max}}{2}T_s\left(1 - \frac{\frac{\omega_{max}}{2}}{\omega_{max}}\right) = \frac{\omega_{max}}{4}T_s = 17.45W$$

10.5 See the documentation on the LMD18200.

10.6



10.7

B_1	B_0	ϕ_1	ϕ_2	ϕ_3	ϕ_4	step	$\phi_2 \oplus 1$	$B_0 \oplus B_1$	$B_1 \oplus 1$
0	0	1	0	0	1	**2**	1	0	1
0	1	0	1	0	1	**3**	0	1	1
1	0	0	1	1	0	**4**	0	1	0
1	1	1	0	1	0	**1**	1	0	0

This checks out with both Table 10.1 and Equation 10.18.

For ϕ_2, sum of products (SOP) gives:

$$\phi_2 = \overline{B_1}B_0 + B_1\overline{B_0}$$

and product of sums gives:

$$\phi_2 = (B_1 + B_0)(\overline{B_1} + \overline{B_0}) = B_1\overline{B_0} + B_0\overline{B_1}$$

which is the same as the SOP result.

10.8 Search the Internet.

10.9 $\Delta\theta_d = \frac{\Delta x}{r} = \frac{0.001}{0.05} = 0.02\text{rad}$

$\Delta\theta_m = 3\Delta\theta_d = 0.06\text{rad} = 3.44°$

Therefore, the minimum required number of steps per revolution is

$$N = \frac{360°}{\Delta\theta_m} = 105$$

To achieve the maximum speed,

$$\omega_d = \frac{v_{max}}{r} = \frac{0.10\frac{\text{cm}}{\text{s}}}{0.05\text{cm}} = 2\frac{\text{rad}}{\text{s}}$$

$$\omega_m = 3\omega_d = 6\frac{\text{rad}}{\text{s}}$$

Therefore, the required step rate is

$$\frac{\omega_m}{\Delta\theta_m} = 100\frac{\text{steps}}{\text{s}}$$

10.10

(a) servo motor

(b) ac induction motor

(c) series dc

(d) ac induction

(e) servo motor

(f) series dc

(g) stepper motor

(h) synchronous motor

(i) dc motor

(j) ac induction motor

(k) ac motor

(l) ac motor

10.11 The load speed is related to the motor speed with:

$$\omega_l = r_g \omega_m$$

where the gear box reduction ratio is:

$$r_g = \frac{M}{N}$$

(a) $J = J_r + J_l r_g^2$

The maximum acceleration occurs at start-up where:

$$\alpha_{max} = \frac{T_s}{J} = \frac{T_s}{J_r + J_l r_g^2}$$

(b) At steady state,

$$T_m = r_g T_l = r_g(k\omega_l) \quad \text{and} \quad T_m = T_s - T_s \frac{\omega_m}{\omega_{m_{max}}}$$

$$\omega_l = r_g \omega_m$$

Combining these equations gives:

$$k r_g^2 \omega_m = T_s - T_s \frac{\omega_m}{\omega_{m_{max}}}$$

so

$$\omega_m = \frac{T_s}{k r_g^2 + \frac{T_s}{\omega_{m_{max}}}} \quad \text{and} \quad \omega_l = r_g \omega_m = \frac{T_s}{k r_g + \frac{T_s}{r_g \omega_{m_{max}}}}$$

(c) Designating the torque on the motor (rotor) side of the gear box as T_{gm} and the torque on the load side of the gear box as T_{gl}, the equations of motion for the motor rotor and load can be written as:

$$T_m - T_{gm} = J_r \alpha_m \quad \text{and} \quad T_{gl} - T_l = J_l \alpha_l$$

where the gear box torques are related by:

$$T_{gm} = r_g T_{gl}$$

and the load and motor angular accelerations are related by:

$$\alpha_l = r_g \alpha_m$$

Therefore, the motor equation of motion can be written as:

$$T_m - r_g(T_l + J_l r_g \alpha_m) = J_r \alpha_m$$

This can be written as:

$$T_m - r_g T_l = J_{eff} \alpha_m$$

where J_{eff} is the total effective inertia seen by the motor, given by:

$$J_{eff} = r_g^2 J_l + J_r$$

Designating the motor speed ω_m as ω, the motor and load torques are given by:

$$T_m = T_s\left(1 - \frac{\omega}{\omega_{max}}\right) \qquad \text{and} \qquad T_l = k\omega_l = kr_g\omega$$

and the motor equation can now be written as:

$$T_s\left(1 - \frac{\omega}{\omega_{max}}\right) - r_g(kr_g\omega) = J_{eff}\frac{d\omega}{dt}$$

which can be written as:

$$J_{eff}\frac{d\omega}{dt} + b\omega = T_s$$

where:

$$b = \frac{T_s}{\omega_{max}} + kr_g^2$$

The particular (steady state) solution to the equation of motion is:

$$\omega_{ss} = \frac{T_s}{b}$$

and the total general solution for the initial condition $\omega(0) = 0$ is:

$$\omega(t) = \omega_{ss}\left(1 - e^{-\frac{b}{J_{eff}}t}\right)$$

Therefore, when the motor is at 95% of its steady state speed,

$$0.95\omega_{ss} = \omega_{ss}\left(1 - e^{-\frac{b}{J_{eff}}t}\right)$$

so the time required for to reach this speed is:

$$t = -\frac{J_{eff}}{b}\ln(1-0.95) = 2.996\frac{(r_g^2 J_l + J_r)}{\frac{T_s}{\omega_{max}} + kr_g^2}$$

10.12 If the inner diameter of the cylinder (i.e., the diameter of the piston face) is D, the diameter of the piston rod is d, and the fluid pressure is P, then the force to extend the cylinder (pressure on the face of the piston) is:

$$F = PA_{face} = \frac{\pi D^2}{4}$$

and the force to retract the cylinder (pressure on rod side of the piston) is:

$$F = PA_{face-rod} = \frac{\pi(D-d)^2}{4}$$

10.13 $A = \frac{\pi d^2}{4} = 0.785\text{in}^2$

$$F = PA = (1000\text{psi})(0.785\text{in}^2) = 785\text{lb}$$

10.14 $A = \frac{\pi d^2}{4} = \frac{\pi(10\text{mm})^2}{4} = 78.5\text{mm}^2$

$$P = \frac{F}{A} = \frac{2000\text{N}}{78.5\text{mm}^2} = 25.5\text{MPa}$$

10.15 Search manufacturer catalogs or the Internet.

10.16 Required components: pressure regulator (e.g., 1500 psi), pneumatic cylinder (double-acting or spring return), valve. The required cylinder area is:

$$A = \frac{F}{p} = \frac{100}{1500}\text{in}^2 = 0.0667\text{in}^2$$

Therefore, the required cylinder diameter is:

$$D = \sqrt{\frac{4A}{\pi}} = 0.29\text{in}$$

11.1 Analytically, a running average of the three most recent derivative calculations involves the following:

$$D_{i-2} = \frac{e_{i-2} - e_{i-3}}{\Delta t}$$

$$D_{i-1} = \frac{e_{i-1} - e_{i-2}}{\Delta t}$$

$$D_{i} = \frac{e_{i} - e_{i-1}}{\Delta t}$$

$$D_{avg} = \frac{D_{i-2} + D_{i-1} + D_{i}}{3} = \frac{e_{i} - e_{i-3}}{3\Delta t}$$

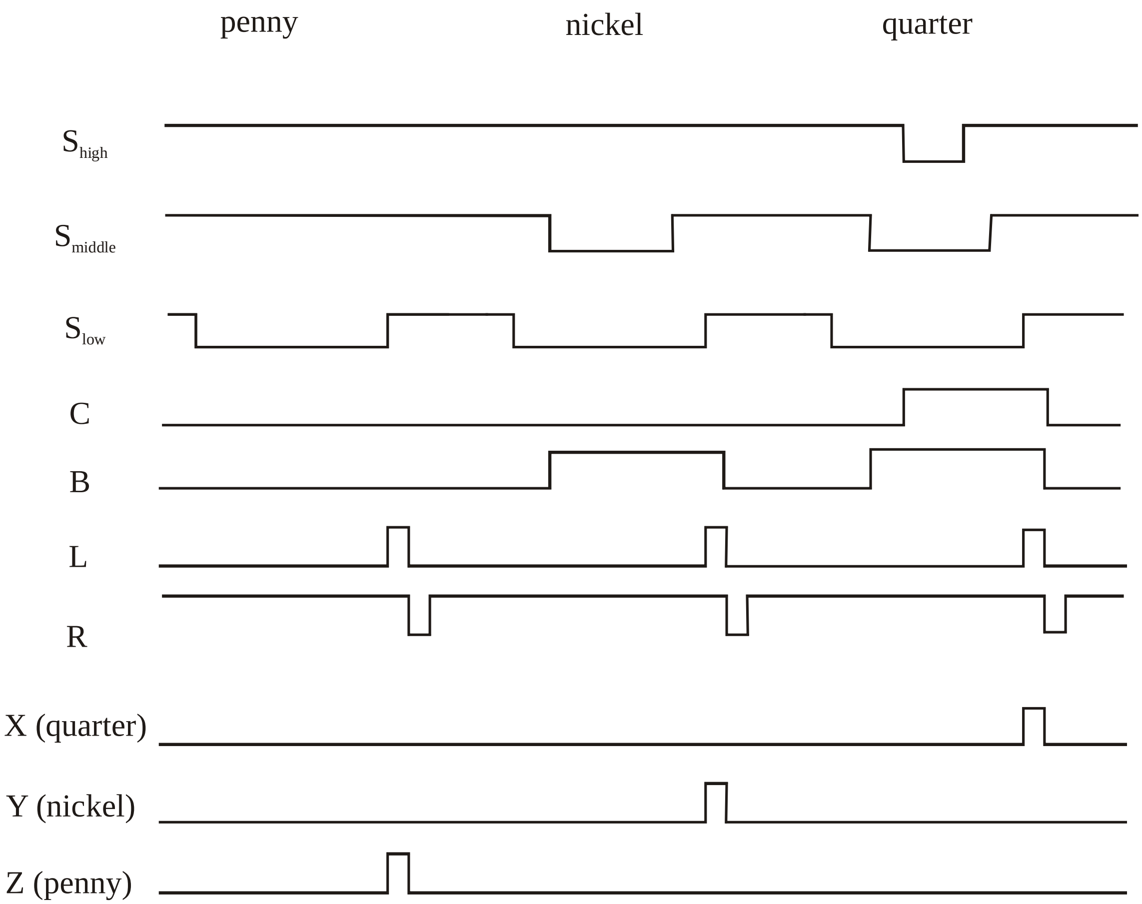
The last equation can be implemented in code based on the either of the expressions. Here's the code using the first expression:

```
' before loop
Di2 = 0  :  Di1 = 0

' modified derivative calc inside loop
Di = (error - error_previous) / DT
derivative = (Di2 + Di1 + Di) / 3

' update previous two derivative calcs for next loop cycle
Di2 = Di1  :  Di1 = Di
```

11.2 See Section 1.1 and Internet Link 7.14 for some examples.

11.3 See Section 1.1 and Internet Link 7.14 for some examples.

11.4




```
S_low Var PORTB.0
S_mid Var PORTB.1
S_high Var PORTB.2
X_q Var BIT
Y_n Var BIT
Z_p Var BIT

loop:

’ Clear out coin identifiers
X_q = 0  :  Y_n = 0  :  Z_p = 0

’ Wait for S_low to go low (i.e. wait for a coin).
While (S_low == 1) : Wend    ’ wait for S_low to go low

’ Check for middle and high sensors while low sensor is active
While (S_low == 0)
        If (S_mid  == 0) Then
                While (S_mid == 0)
                        If (S_high == 0) Then
```

```
                                        X_q = 1
                                        Goto done
                                End If
                        Wend
                        Y_n = 1
                        Goto done
                EndIf
        Wend
        Z_p = 1

        done:
        ' The correct coin is identified at this point
        ............ use the coin info ..........

        Goto loop  ' process next coin
```

A.1 a. 100,000,000 kg = 100,000,000,000 g = 100 x 10^9 g = 100 Gg

b. 0.000000025 m = 25 x 10^{-9} m = 25 nm

c. 16.9 x 10^{-10} s = 169 x 10^{-9} s = 169 ns

A.2 Do Class Discussion Items A.4 and A.5 in class.

A.3 The stress is given by:

$$\sigma_{max} = \frac{FL\frac{h}{2}}{\frac{1}{12}wh^3} = 6\frac{FL}{wh^2}$$

Using MathCAD:

$$\sigma(F,L,w,h) := \frac{6\cdot F\cdot L}{w\cdot h^2}$$

$$d\sigma dF(F,L,w,h) := \frac{d}{dF}\sigma(F,L,w,h) \rightarrow \frac{6\cdot L}{h^2\cdot w} \qquad d\sigma dw(F,L,w,h) := \frac{d}{dw}\sigma(F,L,w,h) \rightarrow -\frac{6\cdot F\cdot L}{h^2\cdot w^2}$$

$$d\sigma dL(F,L,w,h) := \frac{d}{dL}\sigma(F,L,w,h) \rightarrow \frac{6\cdot F}{h^2\cdot w} \qquad d\sigma dh(F,L,w,h) := \frac{d}{dh}\sigma(F,L,w,h) \rightarrow -\frac{12\cdot F\cdot L}{h^3\cdot w}$$

$F := 12520\,N \qquad \Delta F := 10\,N \qquad w := 11.8\,cm \qquad \Delta w := 0.5\,mm$

$L := 0.95\,m \qquad \Delta L := 0.5\,mm \qquad h := 12.1\,cm \qquad \Delta h := 0.5\,mm$

$$\sigma(F,L,w,h) = 41.307\,MPa$$

$$E(F,L,w,h) := |d\sigma dF(F,L,w,h)\cdot\Delta F| + |d\sigma dL(F,L,w,h)\cdot\Delta L| \ldots + |d\sigma dw(F,L,w,h)\cdot\Delta w| + |d\sigma dh(F,L,w,h)\cdot\Delta h|$$

$$E(F,L,w,h) = 5.711\times 10^5\,Pa$$

$$E_{rms}(F,L,w,h) := \sqrt{(d\sigma dF(F,L,w,h)\cdot\Delta F)^2 + (d\sigma dL(F,L,w,h)\cdot\Delta L)^2 \ldots + (d\sigma dw(F,L,w,h)\cdot\Delta w)^2 + (d\sigma dh(F,L,w,h)\cdot\Delta h)^2}$$

$$E_{rms}(F,L,w,h) = 3.857\times 10^5\,Pa$$