MA2011 MECHATRONICS SYSTEMS INTERFACING

Tutorial 2

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What is the Fourier series and fundamental frequencies (in Hertz) and amplitudes of the following waveforms?

a) $f(t)$ = 5*sin($2\pi t$).
b) $f(t)$ = 5*cos($2\pi t$)
c) $f(t)$ = -5*sin($2\pi t$)

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t) \qquad (2)$$

where C_0 is the DC component of the signal, A_n and B_n are coefficients. They are given by

$$A_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)dt, \; B_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)dt$$
$$C_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{A_0}{2}. \qquad (3)$$

Note: C_0 is the average value of the waveform over its period.

UNIQUE REPRESENTATION

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t)$$

$$5*\sin(2\pi t) = \mathbf{0} + 5*\sin(2\pi t) + \sum_{\boldsymbol{n=2}}^{\infty} \mathbf{0} * \boldsymbol{sin(2n\pi t)}$$

$$5*\sin(2\pi t) = \mathbf{0} + 0*\sin(\pi t)+5*\sin(2\pi t) + \sum_{\boldsymbol{n=3}}^{\infty} \mathbf{0} * \boldsymbol{sin(n\pi t)}$$

UNIQUE FOURIER REPRESENTATION

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t)$$

$$F(t) = \boldsymbol{C}_0 + \sum_{n=1}^{\infty} (\boldsymbol{A}_n \cos(n\omega_{\boldsymbol{0}}\boldsymbol{t}) + \boldsymbol{B}_n \sin(n\omega_{\boldsymbol{0}}\boldsymbol{t}))$$

$$= \boldsymbol{C'}_0 + \sum_{n=1}^{\infty} (\boldsymbol{A'}_n \cos(n\omega_{\boldsymbol{0}}\boldsymbol{t}) + \boldsymbol{B'}_n \sin(n\omega_{\boldsymbol{0}}\boldsymbol{t}))$$

Uniqueness means

$\boldsymbol{C'}_0 = \boldsymbol{C}_0$

$\boldsymbol{A'}_n = \boldsymbol{A}_n$

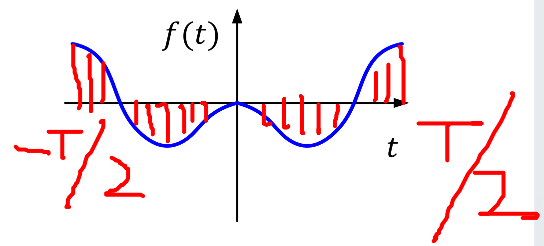
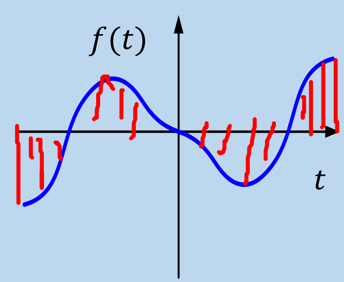
$\boldsymbol{B'}_n = \boldsymbol{B}_n$

For all n

Uniqueness only applicable for Standard Fourier Representation, no phase angles involved

Properties of symmetric functions

f(t) has period of *T*

f(t)	$\int_{-T/2}^{T/2} f(t)dt$	
Even f(-t)=f(t)	$\int_{-T/2}^{T/2} f(t)dt = 2\int_{0}^{T/2} f(t)dt$	
Odd f(-t)=-f(t)	$\int_{-T/2}^{T/2} f(t)dt = 0$	

Q1

***f(t)* and *g(t)* are two symmetric and periodic functions with period *T*. Answer the followings**

			1)	2)		3)		4)	5)
	$f(t)$	$g(t)$	$-f(t)$	$f(t)+g(t)$	$f(t) - g(t)$	$f(t)*g(t)$	$f(t)/g(t)$	$\int_{-T/2}^{T/2} f(t)\cos\left(\frac{2\pi nt}{T}\right)dt$	$\int_{-T/2}^{T/2} f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$
Function symmetry	even	even							
	odd	odd							
	even	odd							
	odd	even							

Q1

f(t) and *g(t)* are two symmetric and periodic functions with period *T*. Answer the followings

Even $f(-t)=f(t)$
Odd $f(-t)=-f(t)$

1) $f(t)$ even,

$h(t) \equiv -f(t)$: $h(-t)=-f(-t)=-f(t)=h(t)$ ➔ $-f(t)$ even

2) $f(t)$ & $g(t)$ even :

$h(t) \equiv f(t)+g(t)$: $h(-t) \equiv f(-t)+g(-t)=f(t)+g(t)=h(t)$ ➔ $f(t)+g(t)$ even

$h(t) \equiv f(t)-g(t)$: $h(-t) \equiv f(-t)-g(-t)=f(t)-g(t)=h(t)$ ➔ $f(t)-g(t)$ even

3) $f(t)$ & $g(t)$ even :

$h(t) \equiv f(t)*g(t)$: $h(-t) \equiv f(-t)*g(-t)=f(t)*g(t)=h(t)$ ➔ $f(t)*g(t)$ even

$h(t) \equiv f(t)/g(t)$: $h(-t) \equiv f(-t)/g(-t)=f(t)/g(t)=h(t)$ ➔ $f(t)/g(t)$ even

f(t) and *g(t)* are two symmetric and periodic functions with period *T*. Answer the followings

Even $f(-t)=f(t)$
Odd $f(-t)=-f(t)$

4) $f(t)$ even: $f(-t)= f(t)$

$g(t)\equiv \cos\left(\frac{2\pi nt}{T}\right)$ even: $\cos\left(\frac{2\pi n(-t)}{T}\right)=\cos\left(\frac{2\pi nt}{T}\right)$

$h(t)\equiv f(t)*g(t)$: → even

So $\int_{-T/2}^{T/2} h(t)dt = 2 * \int_{0}^{T/2} h(t)dt$

$f(t)$ even: $f(-t)= f(t)$

$g(t)\equiv \sin\left(\frac{2\pi nt}{T}\right)$ odd: $\sin\left(\frac{2\pi n(-t)}{T}\right)=-\sin\left(\frac{2\pi nt}{T}\right)$

$h(t)\equiv f(t)*g(t)$: → odd

so $\int_{-T/2}^{T/2} h(t)dt = 0$

Q1

***f(t)* and *g(t)* are two symmetric and periodic functions with period *T*. Answer the followings**

?	$f(t)$	$g(t)$	$-f(t)$	$f(t)+g(t)$	$f(t)-g(t)$	$f(t)*g(t)$	$f(t)/g(t)$	$\int_{-T/2}^{T/2} f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$	$\int_{-T/2}^{T/2} f(t)\cos\left(\frac{2\pi nt}{T}\right)dt$
Function symmetry	even	even	even	even	even	even	even	0	$2\int_0^{T/2} f(t)\cos\left(\frac{2\pi nt}{T}\right)dt$
	odd	odd	odd	odd	odd	even	even	$2\int_0^{T/2} f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$	0
	even	odd	even	?	?	odd	odd	0	$2\int_0^{T/2} f(t)\cos\left(\frac{2\pi nt}{T}\right)dt$
	odd	even	odd	?	?	odd	odd	$2\int_0^{T/2} f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$	0

Q1

Even
f(-t)=f(t)

Odd
f(-t)=-f(t)

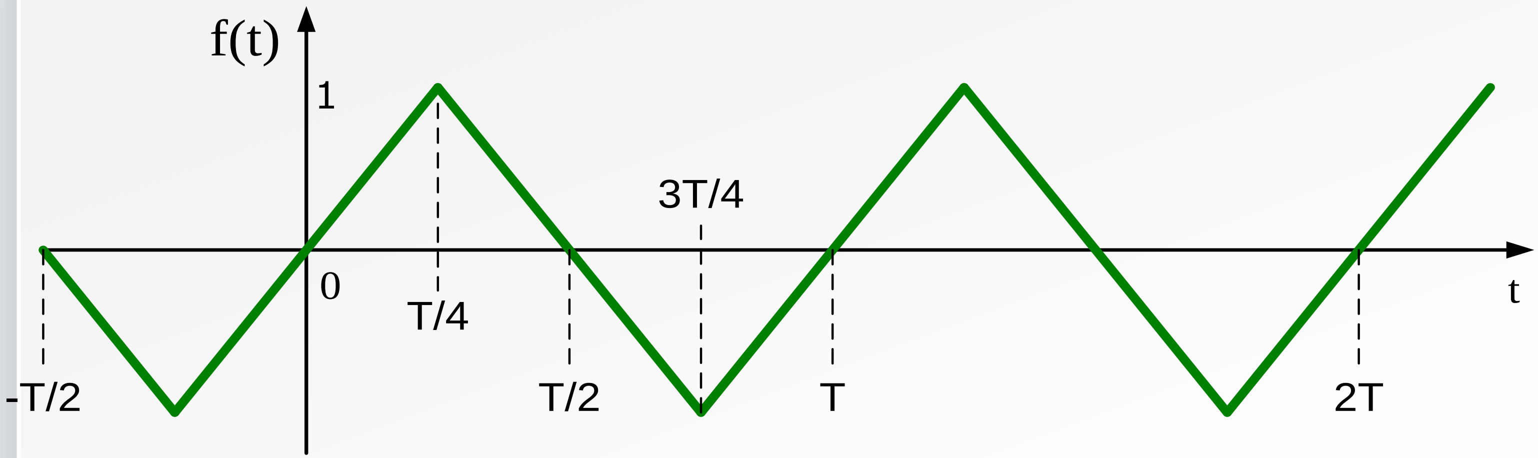
$f(-t)= f(t)$ even
$g(-t)= - g(t)$ odd
$h(t)\equiv f(t)+g(t)$: *even or odd?* ➔ *Not sure !*

Proof by contradiction!
Assume: h(t) even result
$h(-t)= h(t)$;
$f(-t)+g(-t)= (f(t)+g(t))$;
$f(t)-g(t)= f(t)+g(t)$
$2*g(t)=0$ ➔ *only if* $g(t)=0$

Assume: h(t) odd result
$h(-t)= -h(t)$;
$f(-t)+g(-t)= -(f(t)+g(t))$;
$f(t)-g(t)= -f(t)-g(t)$
$2*f(t)=0$ ➔ *only if* $f(t)=0$

Q2

f(t) is a function defined as follows.

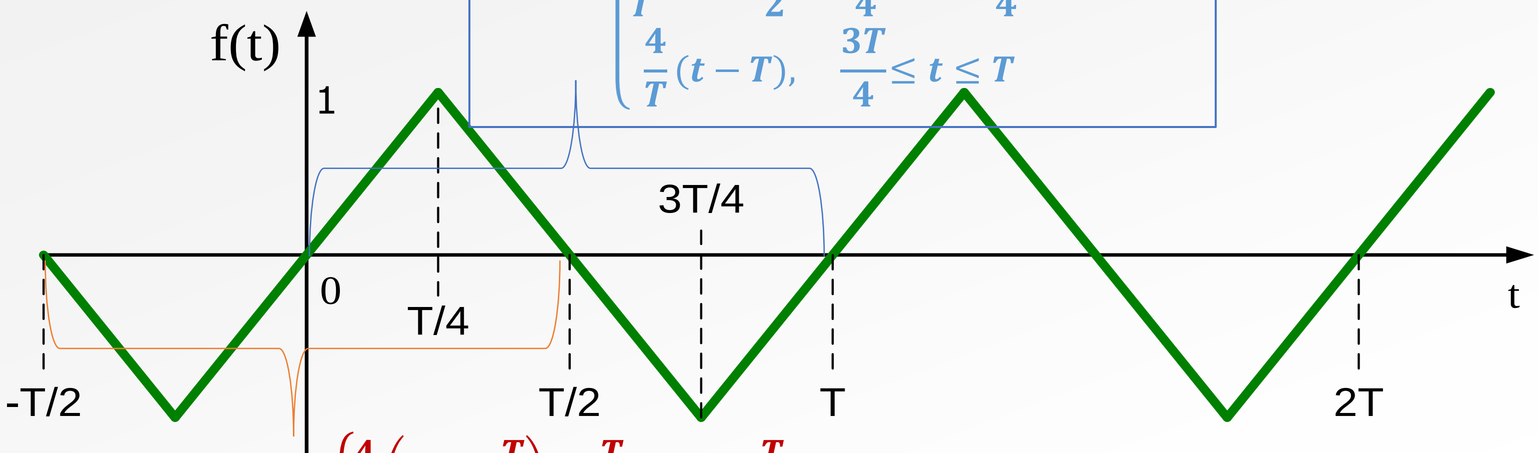


(a) Is it a periodic function? If yes, what is the period, and write the function of the waveform defined at [0, T]
(b) Is it a symmetric function?
(c) Can the waveform be represented by Fourier Series? If yes, what are the DC C_0, A_n and B_n of the waveform and the Fourier Series?
(d) What are the peak amplitude, and peak-to-peak amplitude?
(e) If the peak amplitude is changed to A, what will be the function of f(t) and its Fourier Series Representation?

(a) It is a periodic function with Period=T: $f(t+T)=f(t)$

And the function can be defined as

$$f(t)=\begin{cases}\frac{4}{T}t, & 0\le t\le\frac{T}{4}\\ \frac{4}{T}\left(-t+\frac{T}{2}\right), & \frac{T}{4}\le t\le\frac{3T}{4}\\ \frac{4}{T}(t-T), & \frac{3T}{4}\le t\le T\end{cases}$$

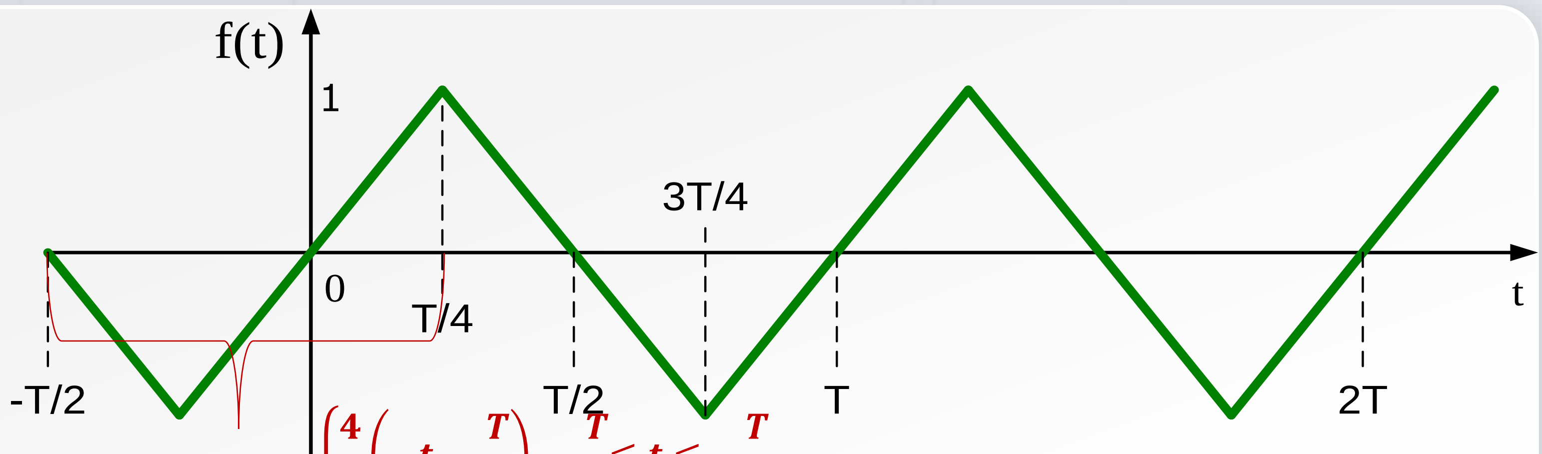


$$or\ f(t)=\begin{cases}\frac{4}{T}\left(-t-\frac{T}{2}\right), & -\frac{T}{2}\le t\le-\frac{T}{4}\\ \frac{4}{T}t, & -\frac{T}{4}\le t\le\frac{T}{4}\\ \frac{4}{T}\left(-t+\frac{T}{2}\right), & \frac{T}{4}\le t\le\frac{T}{2}\end{cases}$$

(b) This is an odd periodic function: $f(-t) = -f(t)$

As a periodic odd function, it can be represented by Fourier Series.

For the odd periodic function, the DC C_0=0 and $A_n = 0$



$$\text{or } f(t) = \begin{cases} \dfrac{4}{T}\left(-t - \dfrac{T}{2}\right), & -\dfrac{T}{2} \le t \le -\dfrac{T}{4} \\ \dfrac{4}{T}t, & -\dfrac{T}{4} \le t \le \dfrac{T}{4} \\ \dfrac{4}{T}\left(-t + \dfrac{T}{2}\right), & \dfrac{T}{4} \le t \le \dfrac{T}{2} \end{cases}$$

ANSWER TO Q2

（c） Can the waveform be represented by Fourier Series? If yes, what are the DC C_0, A_n and B_n of the waveform and the Fourier Series?

DC $C_0=0$ and $A_n =0$

$$B_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$$

Since f(t) and $\sin\left(\frac{2\pi nt}{T}\right)$ are two odd function, we have f(t)*$\sin\left(\frac{2\pi nt}{T}\right)$ is an even function

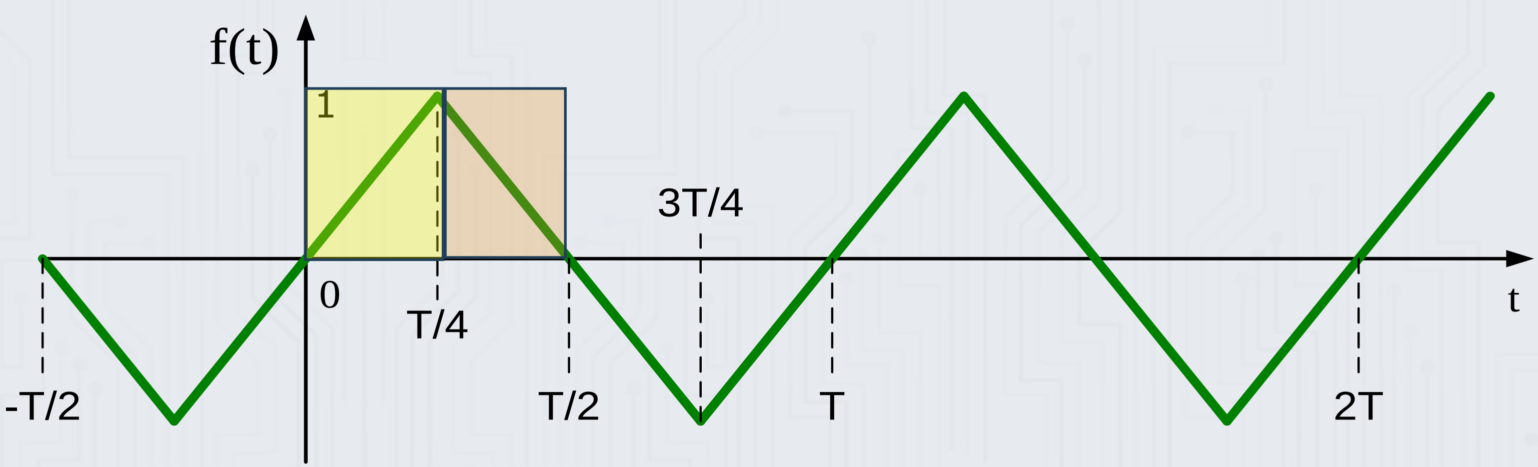
$$B_n = \frac{4}{T}\int_{0}^{T/2} f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$$

（c） Can the waveform be represented by Fourier Series? If yes, what are the DC C_0, A_n and B_n of the waveform and the Fourier Series?

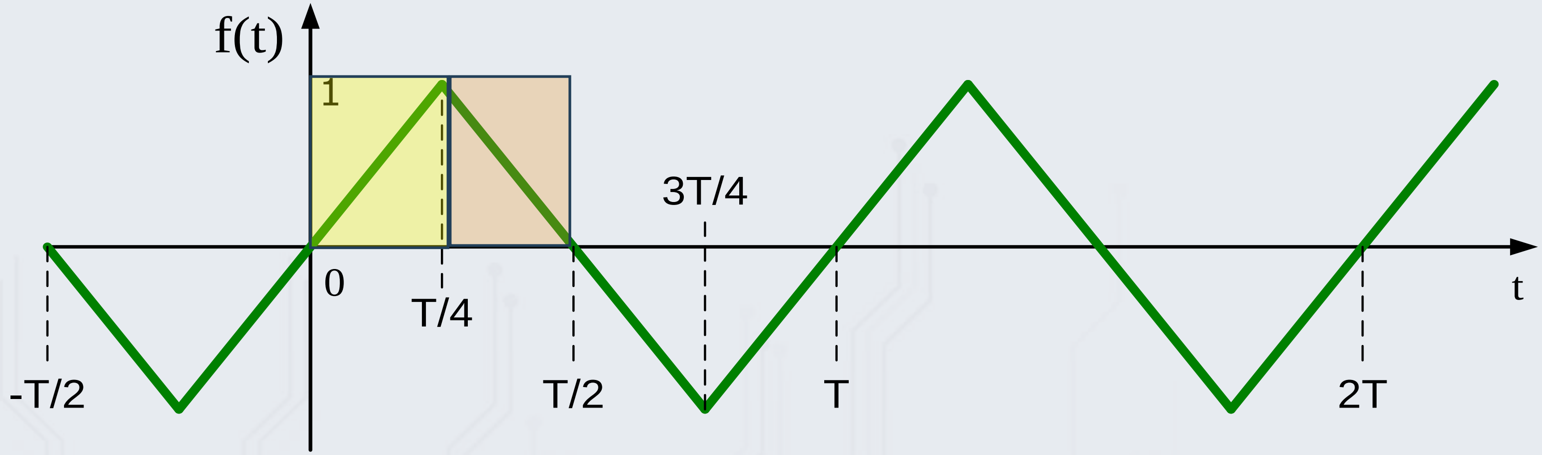
$$f(t) = \begin{cases} \frac{4}{T}\left(-t - \frac{T}{2}\right), & -\frac{T}{2} \le t \le -\frac{T}{4} \\ \frac{4}{T}t, & -\frac{T}{4} \le t \le \frac{T}{4} \\ \frac{4}{T}\left(-t + \frac{T}{2}\right), & \frac{T}{4} \le t \le \frac{T}{2} \end{cases}$$

$$B_n == \frac{4}{T}\int_0^{T/2} f(t) \sin\left(\frac{2\pi nt}{T}\right) dt$$

$$= \frac{4}{T}\int_0^{T/4} \frac{4t}{T} \sin\left(\frac{2\pi nt}{T}\right) dt + \frac{4}{T}\int_{T/4}^{T/2} \frac{4}{T}\left(\frac{T}{2} - t\right) \sin\left(\frac{2\pi nt}{T}\right) dt$$



（c） Can the waveform be represented by Fourier Series? If yes, what are the DC C_0, A_n and B_n of the waveform and the Fourier Series?



$$B_n = X_n + Y_n = \frac{4}{T}\int_0^{T/4} \frac{4t}{T}\, sin\left(\frac{2\pi nt}{T}\right) dt + \frac{4}{T}\int_{T/4}^{T/2} \frac{4}{T}\left(\frac{T}{2}-t\right) sin\left(\frac{2\pi nt}{T}\right) dt$$

$$X_n = \frac{4}{T}\int_0^{T/4} \frac{4t}{T}\, sin\left(\frac{2\pi nt}{T}\right) dt = \left(\frac{4}{T}\right)^2 \int_0^{T/4} t\, sin\left(\frac{2\pi nt}{T}\right) dt$$

$$= -\left(\frac{4}{T}\right)^2 \frac{T}{2\pi n}\int_0^{T/4} t\, dcos\left(\frac{2\pi nt}{T}\right)$$

$$Y_n = \frac{4}{T}\int_{T/4}^{T/2} \frac{4}{T}\left(\frac{T}{2}-t\right) sin\left(\frac{2\pi nt}{T}\right) dt$$

$$= \left(\frac{4}{T}\right)^2 \int_{T/4}^{T/2} \left(\frac{T}{2}-t\right) sin\left(\frac{2\pi nt}{T}\right) dt$$

$$= -\left(\frac{4}{T}\right)^2 \frac{T}{2\pi n}\int_{T/4}^{T/2} \left(\frac{T}{2}-t\right) dcos\left(\frac{2\pi nt}{T}\right)$$

ANSWER TO Q2

Because $\int_a^b f(t)dg(t) = f(t)g(t)\Big|_a^b - \int_a^b g(t)df(t)$

$$X_n=$$

$$\frac{4}{T}\int_0^{T/4}\frac{4t}{T}\sin\left(\frac{2\pi nt}{T}\right)dt= \left(\frac{4}{T}\right)^2\int_0^{T/4} t\sin\left(\frac{2\pi nt}{T}\right)dt$$

$$= -\left(\frac{4}{T}\right)^2\frac{T}{2\pi n}\int_0^{T/4} t\, d\cos\left(\frac{2\pi nt}{T}\right)$$

$$= -\frac{8}{\pi nT}\left(t * \cos\left(\frac{2\pi nt}{T}\right)\right)\Bigg|_0^{T/4} + \frac{8}{\pi nT}\int_0^{T/4}\cos\left(\frac{2\pi nt}{T}\right)dt$$

$$= -\frac{8}{\pi nT}\left(T/4 * \cos\left(\frac{2\pi nT/4}{T}\right)\right) + \frac{8}{\pi nT} * \frac{T}{2\pi n}\int_0^{T/4} d\sin\left(\frac{2\pi nt}{T}\right)$$

$$= -\frac{2}{\pi n}\left(\cos\left(\frac{\pi n}{2}\right)\right) + \frac{4}{(\pi n)^2} * \left(\sin\left(\frac{2\pi nt}{T}\right)\right)\Bigg|_0^{T/4}$$

$$= -\frac{2}{\pi n}\cos\left(\frac{\pi n}{2}\right) + \frac{4}{(\pi n)^2} * \sin\left(\frac{\pi n}{2}\right)$$

Because $\int_a^b f(t)dg(t) = f(t)g(t)\Big|_a^b - \int_a^b g(t)df(t)$

$$Y_n = \frac{4}{T}\int_{T/4}^{T/2} \frac{4}{T}\left(\frac{T}{2}-t\right)\sin\left(\frac{2\pi nt}{T}\right)dt$$

$$= \left(\frac{4}{T}\right)^2 \int_{T/4}^{T/2} \left(\frac{T}{2}-t\right)\sin\left(\frac{2\pi nt}{T}\right)dt$$

$$= -\left(\frac{4}{T}\right)^2 \frac{T}{2\pi n}\int_{T/4}^{T/2} \left(\frac{T}{2}-t\right)d\cos\left(\frac{2\pi nt}{T}\right)$$

$$= -\frac{8}{\pi nT}\left(\left(\frac{T}{2}-t\right)*\cos\left(\frac{2\pi nt}{T}\right)\right)\Big|_{T/4}^{T/2} + \frac{8}{\pi nT}\int_{T/4}^{T/2}\cos\left(\frac{2\pi nt}{T}\right)dt$$

$$= -\frac{8}{\pi nT}\left(0 - T/4 * \cos\left(\frac{2\pi nT/4}{T}\right)\right) - \frac{8}{\pi nT} * \frac{T}{2\pi n}\int_{T/4}^{T/2} d\sin\left(\frac{2\pi nt}{T}\right)$$

$$= \frac{2}{\pi n}\left(\cos\left(\frac{\pi n}{2}\right)\right) - \frac{4}{(\pi n)^2} * \left(\sin\left(\frac{2\pi nt}{T}\right)\right)\Big|_{T/4}^{T/2}$$

$$= \frac{2}{\pi n}\cos\left(\frac{\pi n}{2}\right) - \frac{4}{(\pi n)^2} * \left(0 - \sin\left(\frac{2\pi nT/4}{T}\right)\right)$$

$$= \frac{2}{\pi n}\cos\left(\frac{\pi n}{2}\right) + \frac{4}{(\pi n)^2} * \sin\left(\frac{\pi n}{2}\right)$$

ANSWER TO Q2

$$B_n = X_n + Y_n$$
$$= -\frac{2}{\pi n}\cos\left(\frac{\pi n}{2}\right) + \frac{4}{(\pi n)^2} * \sin\left(\frac{\pi n}{2}\right) + \frac{2}{\pi n}\cos\left(\frac{\pi n}{2}\right) + \frac{4}{(\pi n)^2} * \sin\left(\frac{\pi n}{2}\right)$$
$$= \frac{8}{(\pi n)^2} * \sin\left(\frac{\pi n}{2}\right)$$

$$\sin\left(\frac{\pi n}{2}\right) = \begin{cases} 0, & n = 2k \\ 1, & n = 4k+1 \\ -1, & n = 4k-1 \end{cases}$$

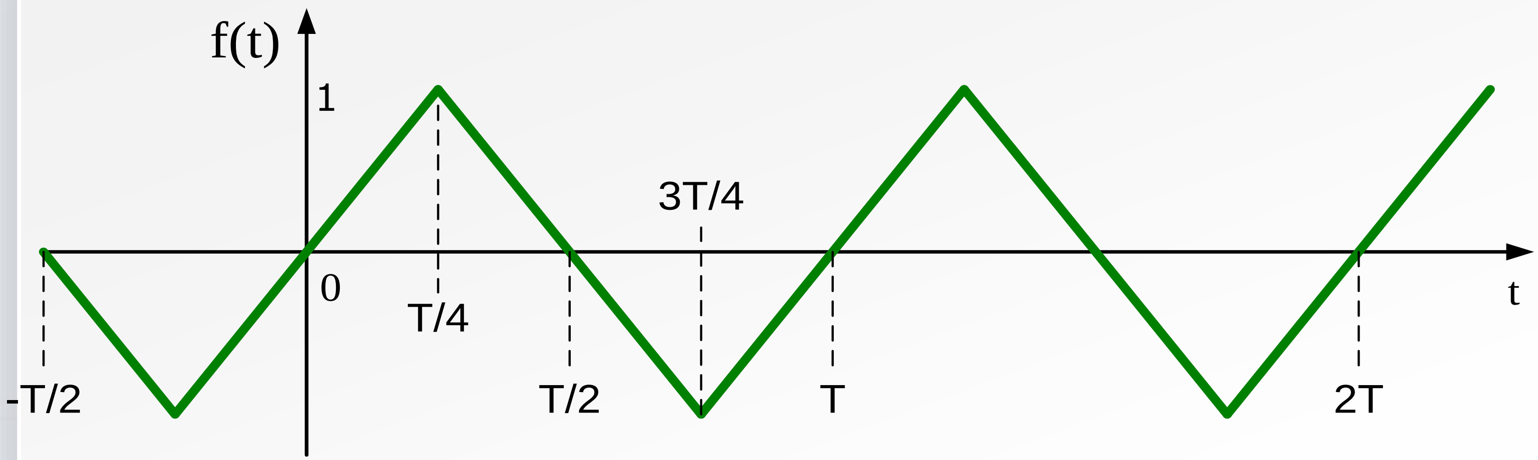
Therefore, the Fourier series is

$$F(t) = \frac{8}{\pi^2}\left[\sin\left(\frac{2\pi}{T}t\right) - \frac{1}{3^2}\sin\left(\frac{6\pi}{T}t\right) + \frac{1}{5^2}\sin\left(\frac{10\pi}{T}t\right) - \cdots\right]$$

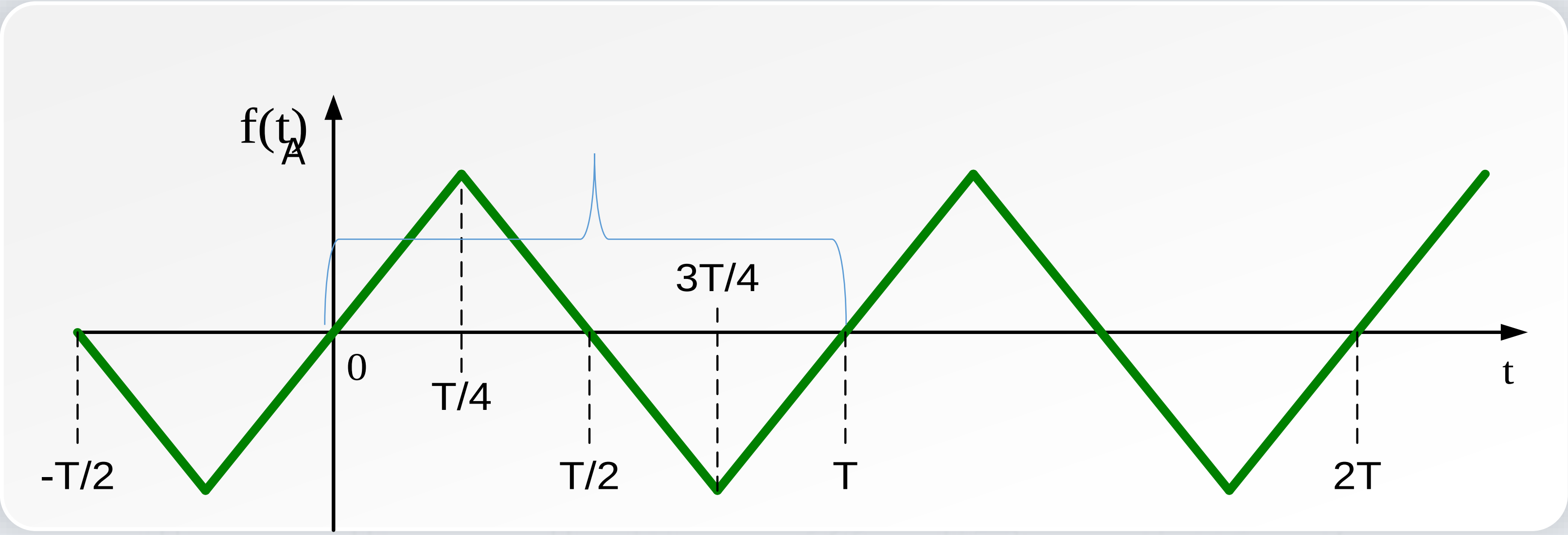
ANSWER TO Q2

(d) The peak amplitude=1

and peak-to-peak amplitude =2



(e) Find the Fourier series of the following periodic function.

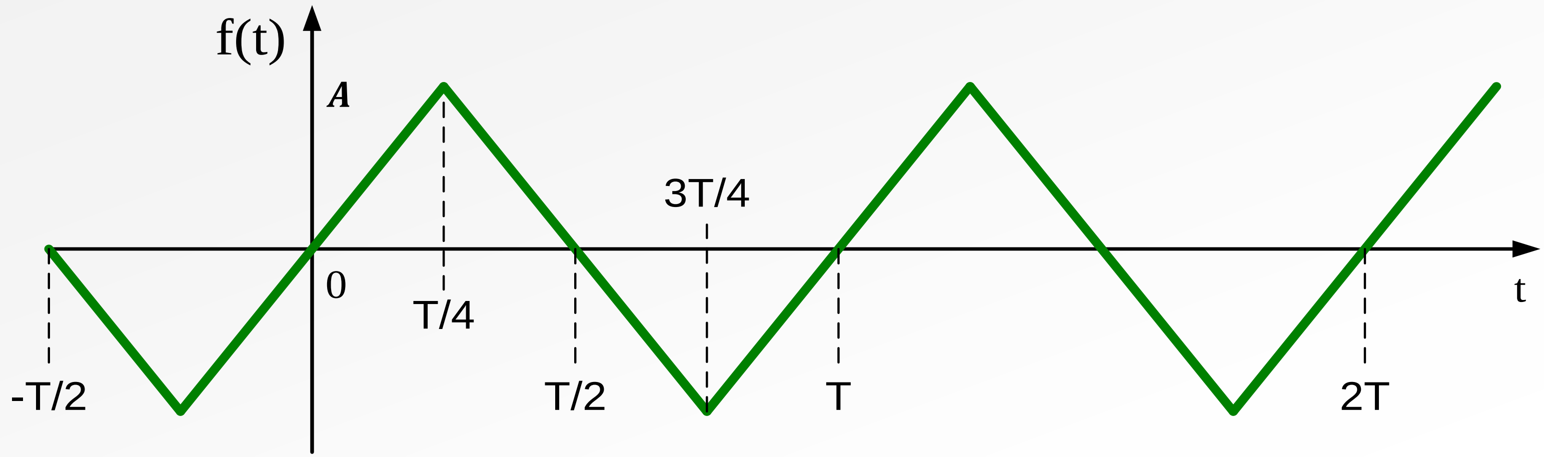


It is a periodic function with Period=*T:*

And the function can be defined as

$$f(t) = \begin{cases} A\dfrac{4}{T}t, & 0 \le t \le \dfrac{T}{4} \\ A\,\dfrac{4}{T}(-t + \dfrac{T}{2}), & \dfrac{T}{4} \le t \le \dfrac{3T}{4} \\ A\dfrac{4}{T}(t - T), & \dfrac{3T}{4} \le t \le T \end{cases}$$

ANSWER TO Q2



This is an odd periodic function: $f(-t) = -f(t)$

As a periodic old function, it can be represented by Fourier Series.

For the odd periodic function, the DC C_0=0 and $A_n = 0$

ANSWER TO Q2

$$B_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)sin\left(\frac{2\pi nt}{T}\right)dt$$

$$=\frac{4}{T}\int_0^{T/2} f(t)sin\left(\frac{2\pi nt}{T}\right)dt$$

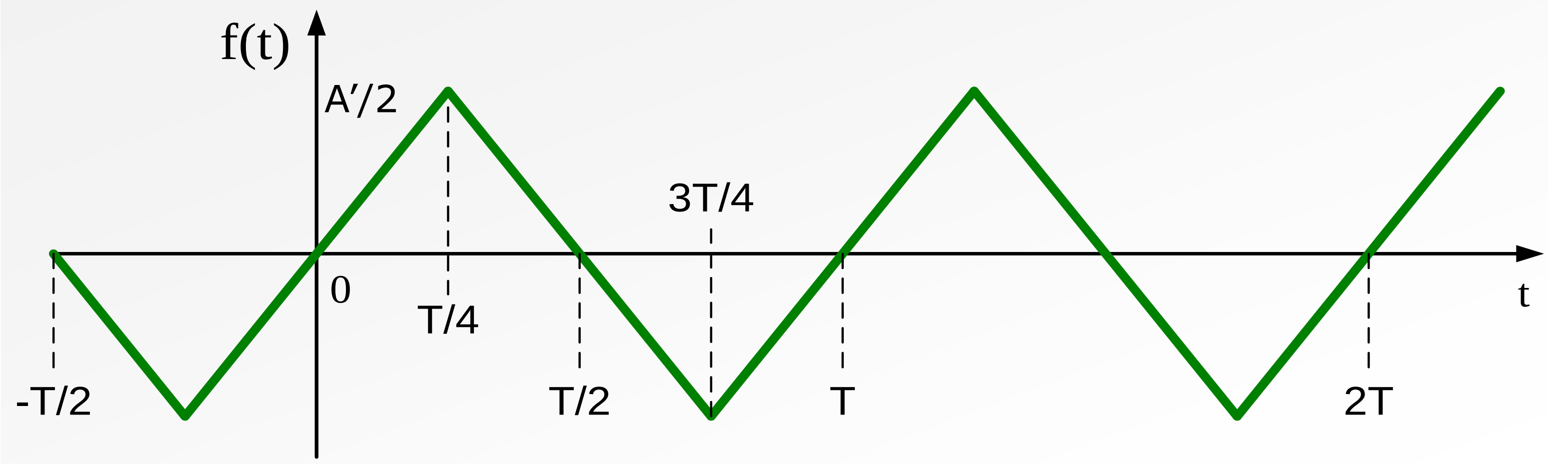
$$=\frac{4}{T}\int_0^{T/4} A\frac{4t}{T}\,sin\left(\frac{2\pi n}{T}\right)dt + \frac{4}{T}\int_{T/4}^{T/2} A\frac{4}{T}\left(\frac{T}{2}-t\right)sin\left(\frac{2\pi n}{T}\right)dt$$

$$=\frac{4}{T}*\frac{4}{T}*A\left[2\left(\frac{T}{2\pi n}\right)^2 sin\left(\frac{\pi n}{2}\right)\right]$$

$$=8A\left(\frac{1}{\pi n}\right)^2 sin\left(\frac{\pi n}{2}\right)$$

$B_n = 0$ when n is even.

ANSWER TO Q2



Therefore, the Fourier series is

$$F(t) = \frac{8A}{\pi^2}\left[\sin\left(\frac{2\pi}{T}t\right) - \frac{1}{3^2}\sin\left(\frac{6\pi}{T}t\right) + \frac{1}{5^2}\sin\left(\frac{10\pi}{T}t\right) - \cdots\right]$$

ANSWER TO Q2

Given wave form f(t)

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t) \qquad (2)$$

where C_0 is the DC component of the signal, A_n and B_n are coefficients. They are given by

$$A_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)dt,\ B_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)dt$$
$$C_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{A_0}{2}. \qquad (3)$$

Note: C_0 is the average value of the waveform over its period.

Now with new wave form K•f(t)

$$F'(t) = KF(t)$$

Q3

A signal $y(t)$ is shown below for 2 complete cycles. It has a period of 0.02 s.

$y(t) = |A\sin(100\pi t)|$, where A is 10 volt

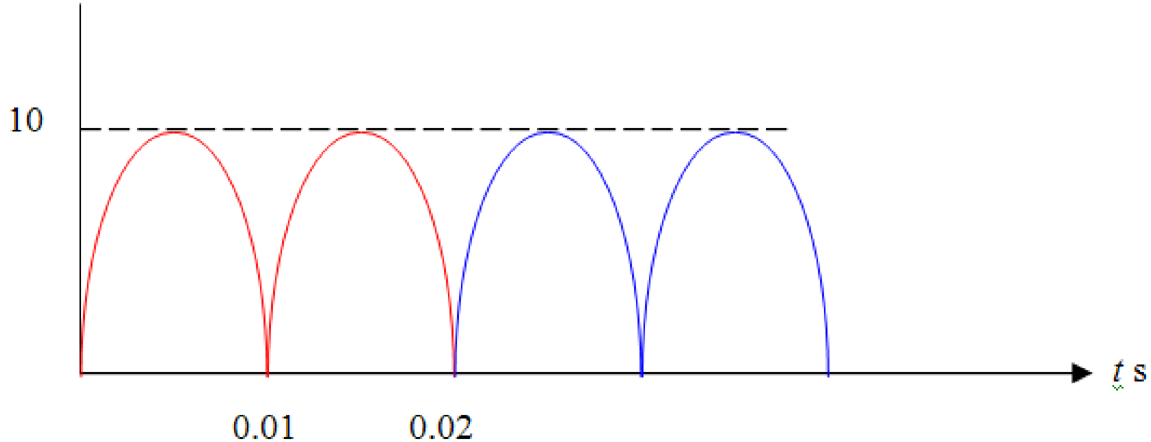


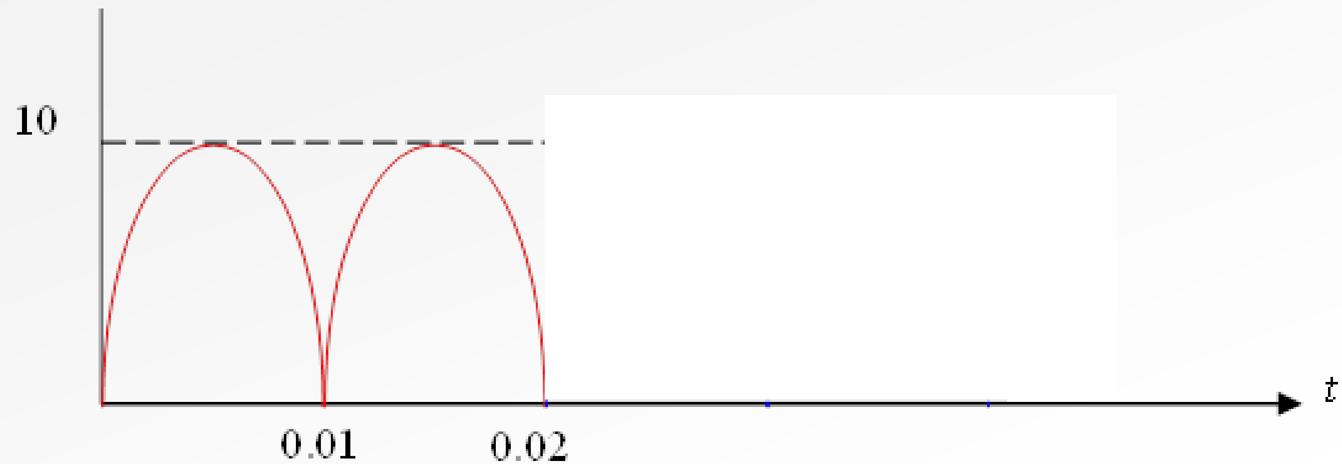
Figure 1: The signal $y(t)$

Find the Fourier Series for $y(t)$ and sketch the amplitude spectrum from DC to the 12th harmonics.

Ans: $y(t) = \frac{20}{\pi} + \sum_{n=2,4,\ldots}^{\cdots} A_n \cos(n\omega_o t)$, $A_n = \frac{10(2)}{\pi(n+1)} - \frac{10(2)}{\pi(n-1)}$,

Q3

A signal is defined in the time interval [0, 0.02] of unit second.



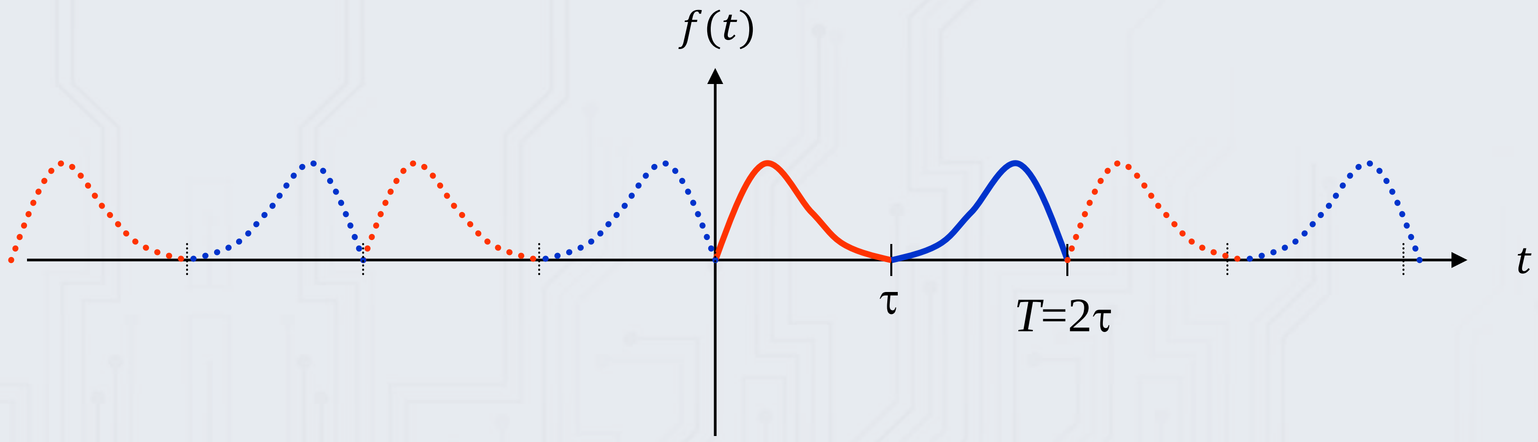
$$y(t) = |A\sin(100\pi t)|$$

(a) Create a periodic function f(t) of even symmetry based on the given signal y(t) with a period T=0.02 seconds.

Expansion Into Even-function Symmetry

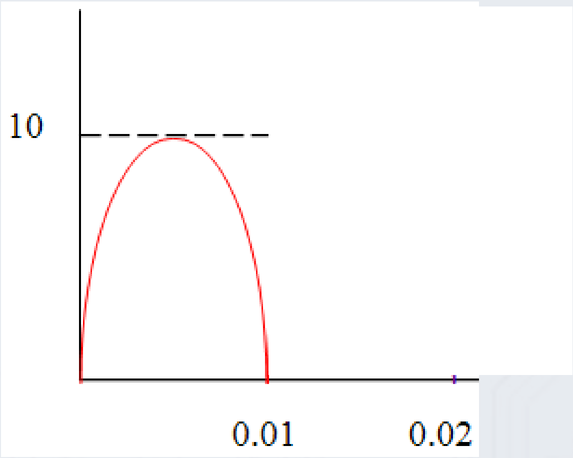
A second pattern can be created by mirroring the original one against an axis $t = \tau$.

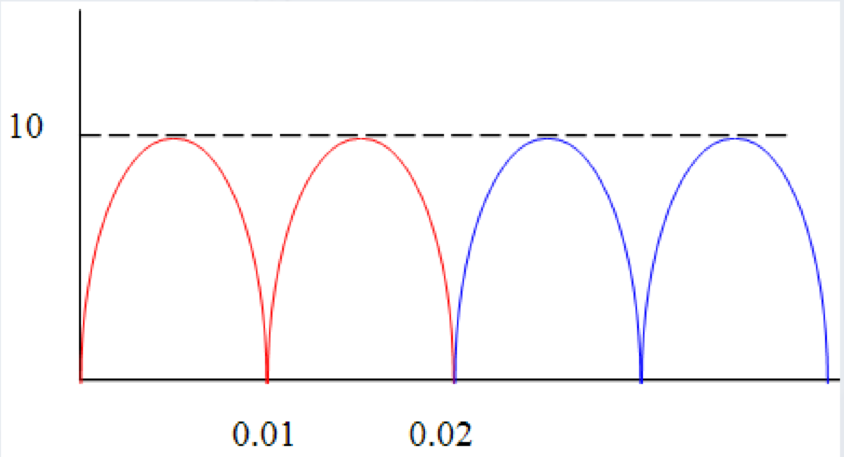
An even-function symmetric periodic waveform can be generated by offsetting the two patterns merged along the time axis by a distance nT ($T=2\tau$), $n=\pm1, \pm2, \pm3, \ldots$



Answer to Q3(A)

(a) Create a periodic function f(t) of even symmetry based on the given signal y(t) with a period T=0.02 seconds.

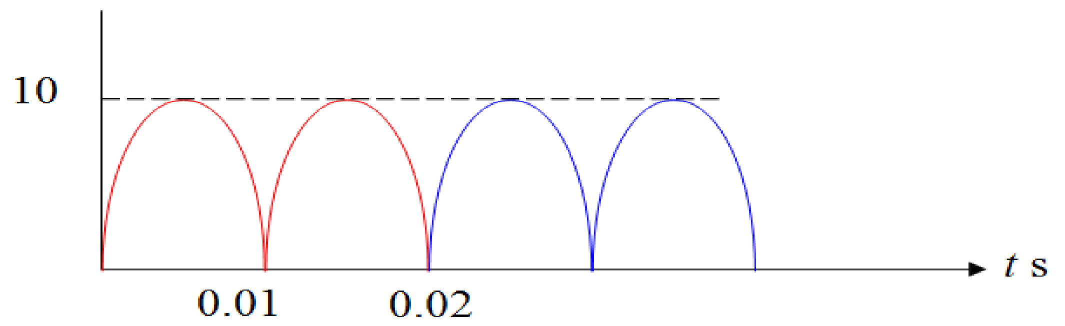




(b) Find the Fourier Series for *f*(*t*) and sketch the amplitude spectrum from DC to the 12th harmonics.

$y(t) = |A\sin(100\pi t)|$, where A is 10 volt

A=10



The period T = 0.02 s. The fundamental frequency is 50 Hz.

$$\boldsymbol{f(t)} = \begin{cases} \mathrm{A}\sin(\mathbf{100}\boldsymbol{\pi t}), \mathbf{0} \le \boldsymbol{t} \le \dfrac{\boldsymbol{T}}{\mathbf{2}} \\ -\mathrm{A}\sin(\mathbf{100}\boldsymbol{\pi t}), \dfrac{\boldsymbol{T}}{\mathbf{2}} \le \boldsymbol{t} \le \boldsymbol{T} \end{cases}$$

$$\boldsymbol{f(t)} = \begin{cases} \mathrm{A}\sin(\mathbf{100}\boldsymbol{\pi t}), \mathbf{0} \le \boldsymbol{t} \le \dfrac{\boldsymbol{T}}{\mathbf{2}} \\ -\mathrm{A}\sin(\mathbf{100}\boldsymbol{\pi t}), -\dfrac{\boldsymbol{T}}{\mathbf{2}} \le \boldsymbol{t} \le \mathbf{0} \end{cases}$$

Answer to Q3(B)

the function is an even function, there are only the dc and the cosine terms. The standard formulas are:

$$A_o = \frac{1}{T}\int_{-T/2}^{T/2} y(t)dt \text{ and } A_n = \frac{2}{T}\int_{-T/2}^{T/2} y(t)\cos(100n\pi t)dt$$

The terms can be obtained from 0 to T/2 and then multiplied by 2:

$$A_o = \frac{2}{T}\int_{0}^{T/2} 10\sin(100\pi t)dt = \frac{1000}{100\pi}\cos(100\pi t)|_0^{100}|$$

$$A_0 = \frac{2}{T}\int_0^{T/2} y(t)dt = \frac{2}{100\pi T}\int_0^{T/2} A\sin(100\pi t)dt$$

$$= \frac{-2A}{100\pi T}\cos(100\pi t)\Big|_0^{\frac{T}{2}} = \frac{-2A}{100\pi T}\left(\cos\left(\frac{100\pi T}{2}\right) - \cos(100\pi * 0)\right)$$

$$= \frac{2A}{100\pi T}\left(1 - \cos\left(\frac{100\pi T}{2}\right)\right) = \frac{2*10}{100*\pi*0.02}\left(1 - \cos\left(\frac{100\pi*0.02}{2}\right)\right) = \frac{10}{\pi}*(1 - \cos(\pi)) = \frac{20}{\pi}$$

Answer to Q3(B)

$$A_n = 2 \times \frac{2}{T} \int_0^{T/2} 10\sin(100\pi t)\cos(100n\pi t)dt$$

Use the identity:

$$2\sin(X)\cos(Y) = \sin(X+Y) + \sin(X-Y):$$

Answer to Q3(B)

$$A_n = 1000 \int_0^{T/2} 2\sin(\overset{X}{100\pi t})\cos(\overset{Y}{100 n\pi t})\,dt$$

$$= 1000 \int_0^{T/2} \sin(\overset{X+Y}{100\pi[1+n]t}) + \sin(\overset{X-Y}{100\pi[1-n]t})\,dt$$

$$= -1000\frac{\cos(100\pi[n+1]t)}{100\pi(n+1)}\Bigg|_0^{1/100} + 1000\frac{\cos(100\pi[n-1]t)}{100\pi(n-1)}\Bigg|_0^{1/100}$$

$$= -\frac{10}{(n+1)\pi}\left[\cos(n+1)\pi - 1\right] + \frac{10}{(n-1)\pi}\left[\cos(n-1)\pi - 1\right]$$

For $n = 1$, or odd: $\cos(2\pi) = \cos(2k\pi) = 1$; for $k = 2, 3, 4 \ldots A_n = 0$

For $n = 2$, or even: $\cos(\pi) = \cos((2k+1)\pi) = -1$, for $k = 1, 2, 3, 4$

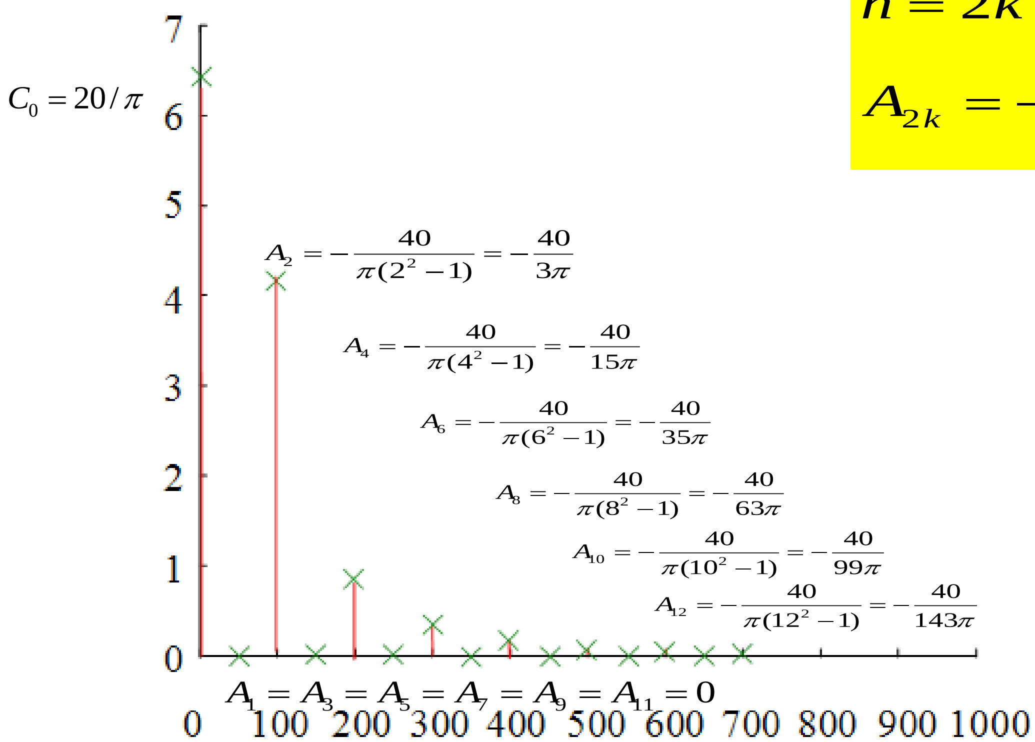
$$A_n = \frac{10(2)}{\pi(n+1)} - \frac{10(2)}{\pi(n-1)}$$

$$y(t)=\frac{20}{\pi}+\sum_{n=2,4,\ldots}^{\ldots} A_n\cos(n\omega_0 t)\ \text{where}\ A_n=\frac{10(2)}{\pi(n+1)}-\frac{10(2)}{\pi(n-1)}$$

The amplitude spectrum is as shown below:

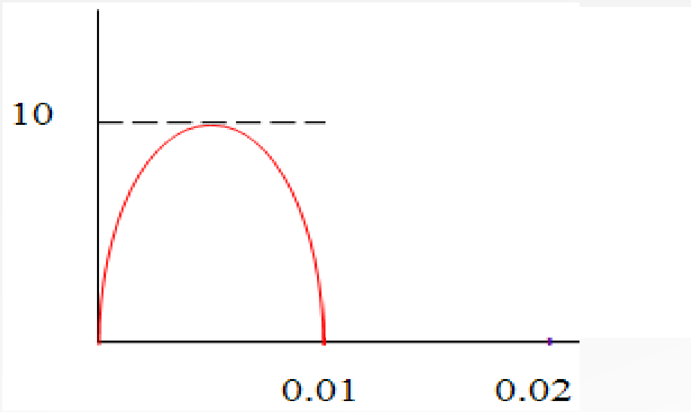
$$n=2k$$

$$A_{2k}=-\frac{40}{\pi(4k^2-1)}$$



Q3

A signal $y(t) = |A\sin(100\pi t)|$ below is defined in the time interval [0, 0.01] of unit second.

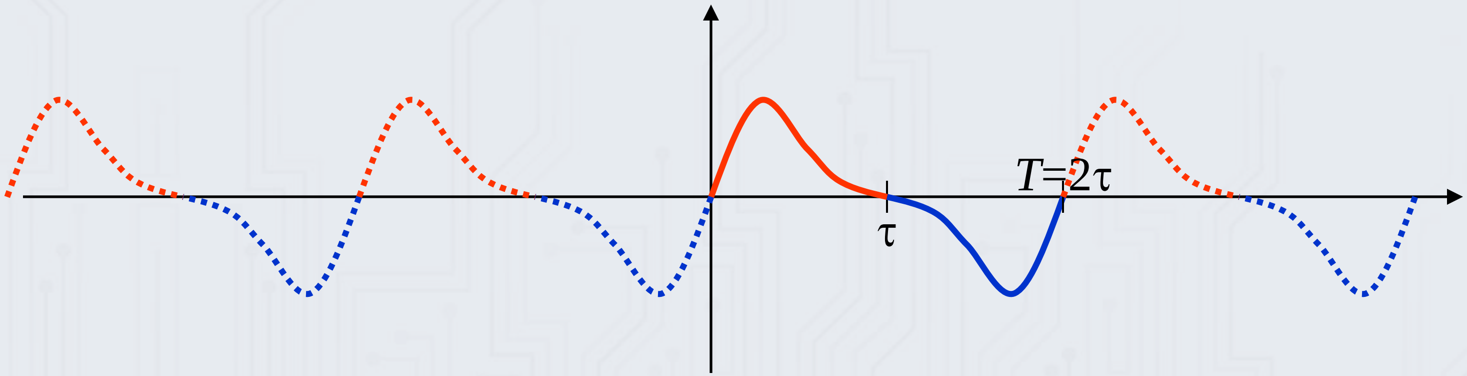


(c) If a periodic function *g(t)* of odd symmetry is created also based on the given signal y(t) with a period *T=0.02* seconds. What will be *g(t)* and its Fourier Series

Expansion Into Odd-function Symmetry

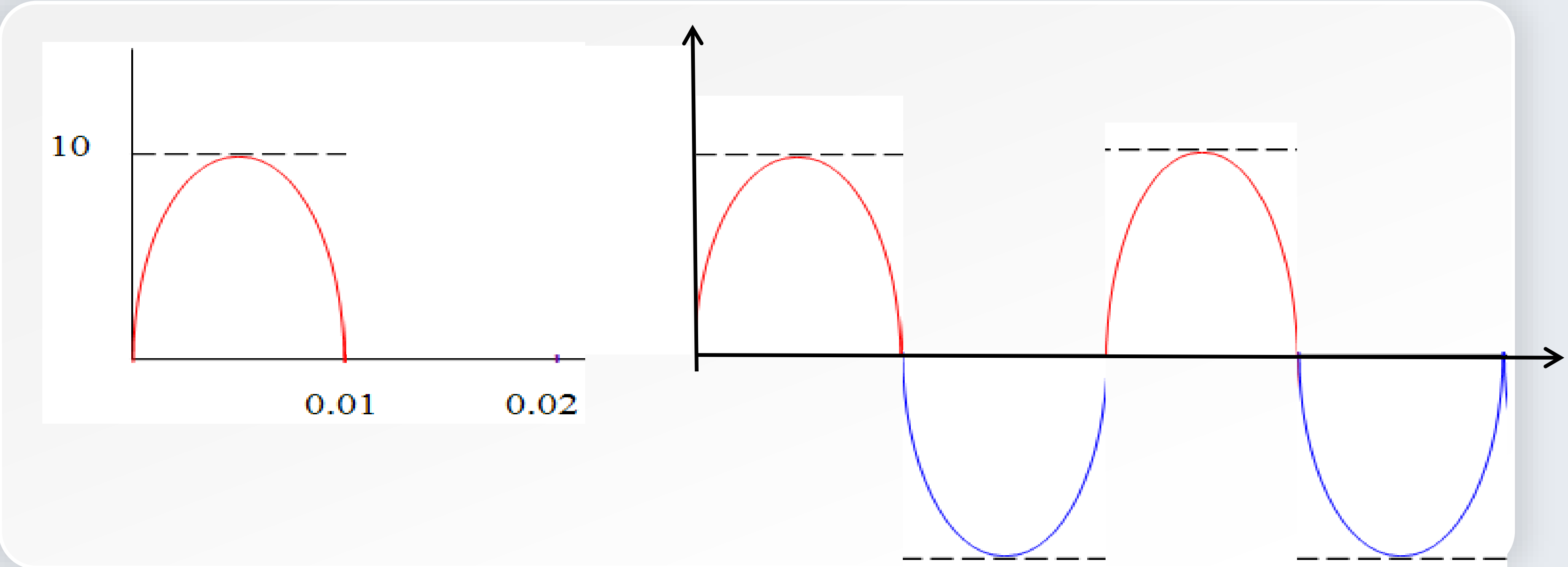
A second pattern can be created by mirroring the original one against the time axis and then the axis t= τ.

An odd-function periodic waveform can be generated by offsetting the two patterns merged along the time axis by a distance *nT* (*T*=2τ), *n*=±1, ±2, ±3, …



Q3

A signal $y(t) = |A\sin(100\pi t)|$ below is defined in the time interval [0, 0.01] of unit second.

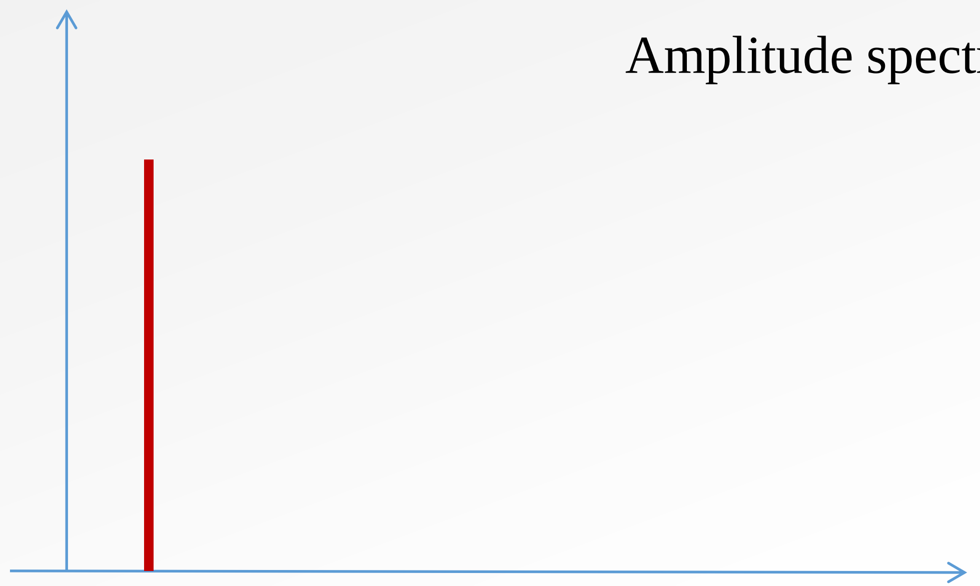


(c) If a periodic function *g(t)* of odd symmetry is created also based on the given signal y(t) with a period *T=0.02* seconds. What will be *g(t)* and its Fourier Series

$$\boldsymbol{g}(\boldsymbol{t}) = A\sin(100\pi t), 0 \le t \le T, or\ g(t) = A\sin(100\pi t), -T/2 \le t \le T/2,$$

$$\boldsymbol{G}(\boldsymbol{t}) = A\sin(100\pi t)\text{: Uniqueness of the Fourier Representation}$$

ANSWER TO Q3(C)



C_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
0	A	0	0	0	0	0	0	0	0	0	0	0

$B_n=0$

Because of the uniqueness

ANSWER TO Q4

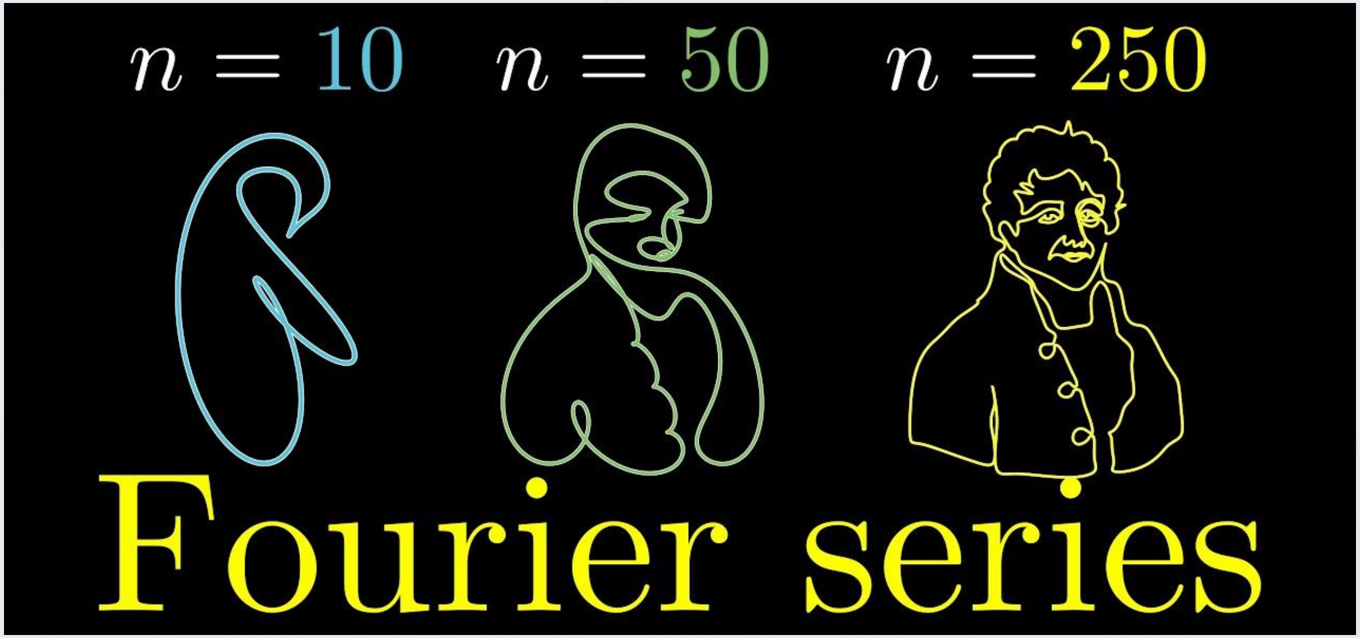
4. Group Discussion on Fourier Descriptor and their applications.

Reference:

E. Persoon, K. Fu (1977), Shape Discrimination Using Fourier Descriptors, IEEE Transactions on Pattern Analysis and Machine Intelligence, 1 March 1977, DOI:10.1109/TSMC.1977.4309681

Vinay Saxena (2012), (PDF) Fourier descriptors under rotation, scaling, translation and various distortion for hand drawn planar curves (researchgate.net), Journal of Experimental Sciences 2012, 3(1): 05-07.

FOURIER SERIES APPLICATION: FOURIER DESCRIPTOR



1. Fourier Theory is very important
2. The calculation of Fourier representation is not easy
3. There are several techniques to simplify the calculation
    - Uniqueness
    - Symmetry
    - Step-by-Step

(21 March 1768 – 16 May 1830)