Quiz Solution

Control Theory (Nanyang Technological University)

MA3005 - QUIZ (24/02)

Semester 2, AY: 2014/15

This quiz contains 3 questions, comprises 5 pages including 1 page of appendix.
All questions carry equal marks.

Name/group:

1. a) Find magnitude and angle for the following functions when $s = j\omega$:

   i. $A(s) = s^2 + 2s + 5$

   ii. $B(s) = (s+3)(s+1)^2$

   iii. $X(s) = \dfrac{A(s)}{B(s)} = \dfrac{s^2 + 2s + 5}{(s+3)(s+1)^2}$

   b) Find the time domain of the following Laplace functions: $Y(s) = \dfrac{5s+13}{s(s^2+4s+13)}$

1) a) i. $A(jw) = 5 - w^2 + 2wj$

$|A(jw)| = \sqrt{(5-w^2)^2 + 4w^2}$

$\angle A(jw) = \tan^{-1} \dfrac{2w}{5-w^2}$

ii. $B(s) = s^3 + 5s^2 + 7s + 3$

$B(jw) = -w^3 j - 5w^2 + 7wj + 3$

$= (3 - 5w^2) - (w^3 - 7w)j$

$|B(jw)| = \sqrt{(3-5w^2)^2 + (w^3-7w)^2}$

$\angle B(jw) = \tan^{-1} \dfrac{-(w^3-7w)}{3-5w^2}$

iii. $|X(jw)| = \dfrac{|A|}{|B|} = \sqrt{\dfrac{(5-w^2)^2 + 4w^2}{(3-5w^2)^2 + (w^3-7w)^2}}$

$\angle X(jw) = \angle A - \angle B$

$= \tan^{-1} \dfrac{2w}{5-w^2} - \tan^{-1} \dfrac{-(w^3-7w)}{3-5w^2}$

b. $$Y(s) = \frac{5s+13}{s(s^2+4s+13)} = \frac{5s+13}{s\left[(s+2)^2+3^2\right]}$$

$$= \frac{A}{s} + \frac{Bs+C}{\left[(s+2)^2+3^2\right]}$$

$$\Rightarrow A = 1 \ ; \ B = -1 \ ; \ C = 1$$

$$Y(s) = \frac{1}{s} - \frac{s-1}{(s+2)^2+3^2}$$

$$= \frac{1}{s} - \frac{s+2}{(s+2)^2+3^2} + \frac{3}{(s+2)^2+3^2}$$

$$y(t) = \mathcal{L}^{-1}\left[Y(s)\right] = 1 - e^{-2t}\cos 3t + e^{-2t}\sin 3t$$

$$= 1 - e^{-2t}\left[\cos 3t - \sin 3t\right]$$

$$= 1 - \sqrt{2}\, e^{-2t}\sin\left(3t + \frac{3}{4}\pi\right)$$

2. The characteristic equation of a system is: $s^4 + 6s^3 + 11s^2 + 6s + K = 0$
Find the range of K that will guarantee the stability of the system.

s^4	1	11	K
s^3	6	6	
s^2	10	K	
s^1	$\frac{60-6K}{10}$	0	
s^0	K		

$$\left.\begin{array}{ll} \text{(i)} & 60 - 6K > 0 \rightarrow K < 10 \\ \text{(ii)} & K > 0 \end{array}\right\} \quad 0 < K < 10$$

3. Find the transfer function of the system $X(s)/F(s)$ in Figure 1 if f is the force input, x is the displacement output, while $X(s)$ and $F(s)$ are the Laplace of the output and input, respectively. Spring K_1 is connected in parallel with the damping element, C, and both of them are connected in series with spring K_0.

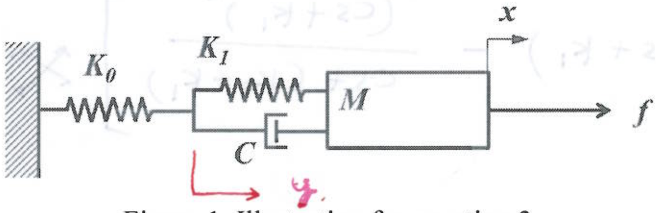
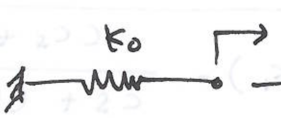
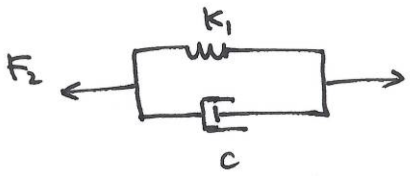


Figure 1. Illustration for question 3



$$\longrightarrow F_1 = K_0 y$$

Assume $x > y$



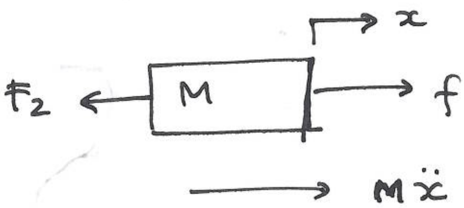
$$F_2 = c(\dot{x} - \dot{y}) + K_1(x - y)$$

$$F_1 = F_2$$

$$K_0 y = c(\dot{x} - \dot{y}) + K_1(x - y)$$

$$(K_0 + K_1)y + c\dot{y} = c\dot{x} + K_1 x$$

$$\mathcal{L} \rightarrow \quad [cs + (K_0 + K_1)]\, Y = (cs + K_1)\, X \quad \ldots (1)$$



$$\Sigma F = M\ddot{x}$$

$$f - F_2 = M\ddot{x}$$

$$f - c(\dot{x} - \dot{y}) - K_1(x - y) = M\ddot{x}$$

$$M\ddot{x} + c\dot{x} + K_1 x = f + c\dot{y} + K_1 y$$

$$\mathcal{L} \rightarrow \quad (Ms^2 + cs + K_1)\, X = F + (cs + K_1)\, Y \quad \ldots (2)$$

substitute (1) to (2) :

$$(Ms^2 + cs + k_1) X = F + (cs + k_1) \frac{cs + k_1}{cs + (k_0 + k_1)} X$$

$$\left[(Ms^2 + cs + k_1) - \frac{(cs + k_1)^2}{cs + (k_0 + k_1)} \right] X = F$$

$$\frac{X}{F} = \frac{1}{(Ms^2 + cs + k_1) - \frac{(cs + k_1)^2}{cs + (k_0 + k_1)}}$$