# May 2019 – Q3

3. A mass ($m$) is suspended through a linear spring ($k$) and a linear damper ($b$) against gravity ($g$) and subjected to a vertical force $f(t)$, where $t$ denotes time. We are interested in determining the system response in terms of motions $z(t)$ along the vertical $z$-axis.

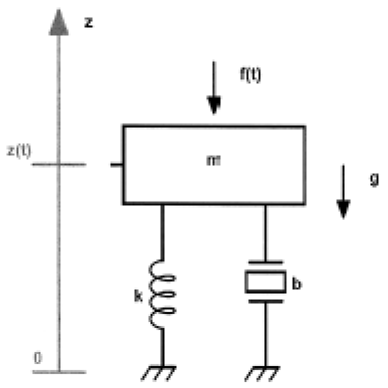


Figure 2

(a) With reference to the mechanical configuration sketched in Figure 2 and assuming zero-length for the spring at rest:

(i) Draw the free body diagram for the mass $m$, indicating all the forces and their expressions in terms of vertical position $z(t)$ of the mass and its higher derivatives.

(ii) Apply Newton's law and write the second order differential equation relative to system.

(iii) Derive the state-space representation of the same equations in (ii).

(iv) Sketch the block-diagram representing the same system, using only integrator, gain and adder blocks.

(v) Describe a possible sensor to measure $z(t)$ without disturbing the dynamics of the system itself.

(8 marks)

(b) Let $F(j\omega)$ and $Z(j\omega)$ be the Fourier transforms for $f(t)$ and $z(t)$, respectively.

(i) Determine the frequency response $H(j\omega) = Z(j\omega)/ F(j\omega)$ of the system. (Note: you will have to remove any offset term first).

(ii) While $H(j\omega)$ is in general a complex function of the frequency $\omega$, determine the frequency $\Omega$ for which the real part of $H(j\Omega)$ is zero.

(7 marks)

(c) Assuming that the external force is given as $f(t) = A*exp(i\omega\, t) + B$, where $A$ is the amplitude, $\omega$ is the radian frequency, $i$ is the imaginary constant, and $B$ is an offset.

(i) What is the (DC) steady-state solution when A=0?

(ii) Assuming a generic solution $z(t) = C*exp(i\omega\, t) + D$, determine the (complex) constants $C$ and $D$ which satisfy the second order differential equation derived above.

(10 marks)

# May 2019 – Q4

4. A simplified electromechanical model of a DC-motor (DCM) is shown in Figure 3. It mainly consists of an electrical port (with armature current *i* flowing through a resistance *R*) and a mechanical port at the shaft (with angular velocity *w*, mechanical damping *b* and inertia *J*) connected through an *ideal gyrator*.

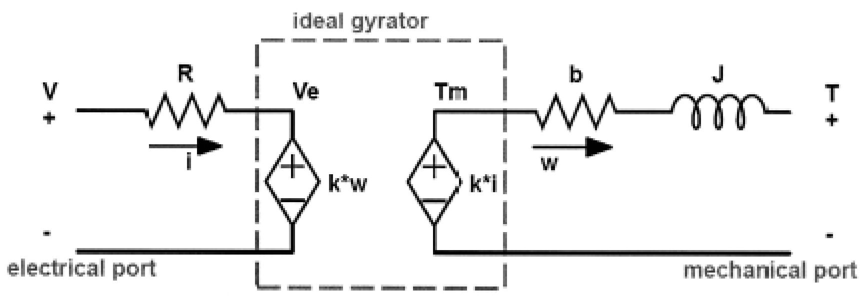


Figure 3

(a) As highlighted by the dashed box in Figure 3, the gyrator is itself a 2-port system, with one electrical port (current *i*, voltage *Ve*) and a mechanical port (angular speed *w* and torque *Tm*). Determine the power at each of the two ports of the gyrator and their sum (i.e. total power).

(5 marks)

(b) The electromechanical model in Figure 3 is general, in the sense that it does not specify any particular electrical or mechanical connection. In the specific case of a DCM driven by a voltage source *v(t)*, where *t* is time, and mechanically free to rotate (i.e. mechanically unloaded),

(i) Redraw the corresponding electromechanical model on your answer book.
(ii) For the same specific case, write the dynamical equations for *w(t)*, i.e. the angular velocity as function of time *t*, and its derivatives.
(iii) Sketch a block-diagram representing the system drawn in (i).

(7 marks)

(c) Given a specific input $v(t) = A\exp(i\omega t)$, and assuming a response $w(t) = B\exp(i\omega t)$, determine the constant *B* as function of input amplitude *A* and frequency $\omega$ using the dynamical equations derived in (b).

(8 marks)

(d) Sketch on your answer book and describe the mechanical structure of a simple brushed DC-motor.

(5 marks)

# May 2018 – Q3

3. A solid disk, with its height along the z-direction, has to remain centered at the origin of the X-Y plane, as shown in the cross-sectional view in Figure 3(a). To monitor variations from the centered position, a pair of inductive sensors is used for each axis. Specifically for the x-axis, two inductive sensors $L_1$ and $L_2$, are arranged symmetrically, with respect to the disk, as shown in Figure 3(a), and electrically connected to a Wheatstone bridge, as shown in Figure 3(b).

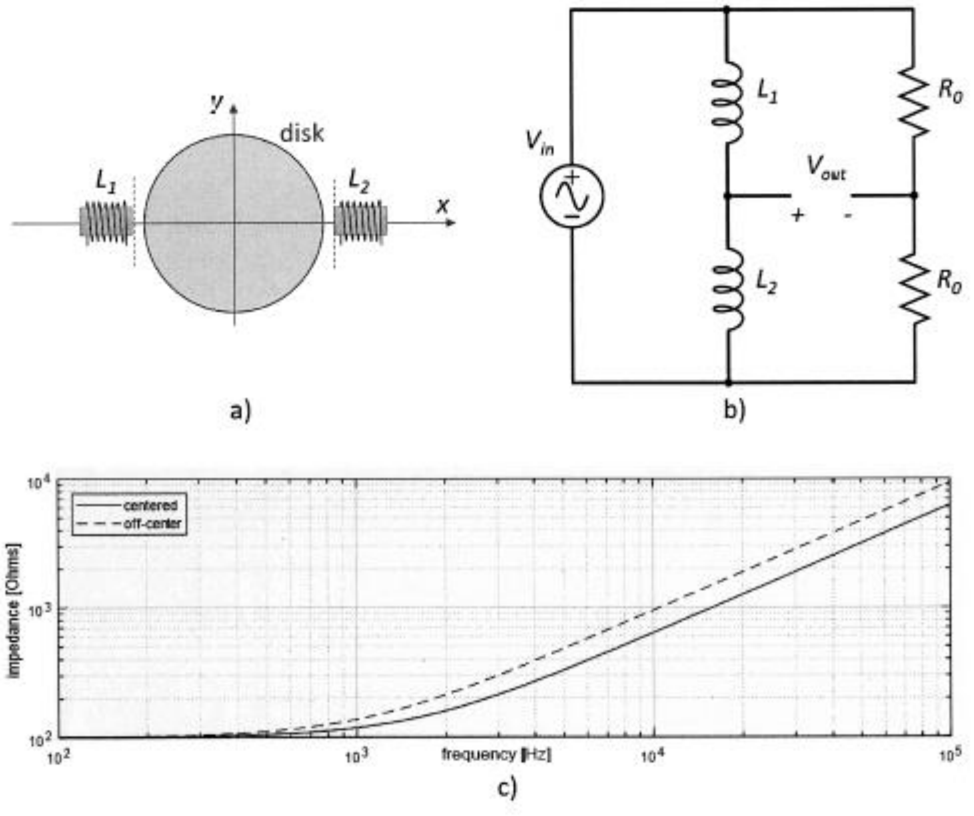


Figure 3

(a) When the disk is perfectly centered along the x-axis, each inductive sensor is characterized by intrinsic resistance $r_0$ and inductance $L_0$. However, an inductance change of $L_0+\Delta L$ is observed when the distance from the disk is off-center. Based on the values of the amplitude response in Figure 3(c), determine

(i) an approximate value for $r_0$,
(ii) an approximate value for $L_0$
(iii) an approximate value for $\Delta L$.

(10 marks)

(b) Consider the Wheatstone bridge in Figure 3(b), with an AC driving input and with similar bridge resistances ($R_0$).

(i) Derive **a symbolic expression** of the bridge output $V_{out}$ as a function of $r_0$, $L_0$, $\Delta L$ and input frequency, assuming that inductance varies linearly with the distance.

(Note: if the inductance of $L_2$ changes $L_0+\Delta L$, the inductance for $L_1$ will change as $L_0-\Delta L$)

(5 marks)

(ii) Sketch the frequency response of the amplitude $|V_{out}/V_{in}|$ between 100-100,000 Hz.

(5 marks)

(c) Considering the bridge as a filter, with input $V_{in}$ and output $V_{out}$, determine whether it is a low-pass or high-pass filter and its cut-off frequency in the case of off-centered disk.

(5 marks)

# May 2018 – Q4

4. A DC motor driven at a nominal voltage $V_0$ = 10V, is designed to provide a rated torque 0.5 Kg.cm, at a rated speed of 6,500 rpm, drawing a rated current of 4,000 mA. At no load, the motor requires 500 mA of current and its speed reaches 9000 rpm.

(a) On your answer book, draw on the same graph the Speed vs. Torque, Current vs. Torque and output Power vs. Torque responses.

**NOTE:** The rated values refer to one possible (suggested) operating point, not the stall point. You can assume linearity.

(8 marks)

(b) Determine, at the nominal voltage, the maximum power (in Watts) which can be delivered to a mechanical load as well as the torque, speed and efficiency at such optimal operating point.

(5 marks)

(c) Based on previous calculations and the information reported above, determine the friction coefficient of the bearings (assuming a linear model) and the amount of friction torque in the no-load conditions.

(7 marks)

(d) Compute the *efficiency* of the motor when operating at the rated speed and when operating at maximum power transfer conditions.

(5 marks)

# May 2017 – Q3

3. A capacitive sensor and its relative bridge measurement circuit are represented in Figure 1. The sensor consists of three rectangular and conductive plates of similar length $L$ (along the $x$-axis) and width $W$. One moving plate (grey colour in Figure 1) can slide back and forth, along the $x$-axis, on top of two fixed plates (white colour in Figure 1(a)). When the grey plate is in its 'zero' position (x=0), the system is in a symmetric configuration, i.e. it overlaps equally with both fixed plates (half of the area overlaps). Assume that the capacitance between two plates is proportional to the overlapping area, with a maximum value C0 when their area fully overlaps.

Electrically, this system of capacitances is in a bridge configuration, with two similar resistors R connected to the fixed plates as in Figure 1. The bridge is driven by an AC voltage source $V_{in}$ and the output $V_{out} = V_b - V_a$ is voltage difference between the two fixed plates, as shown in Figure 1(b).

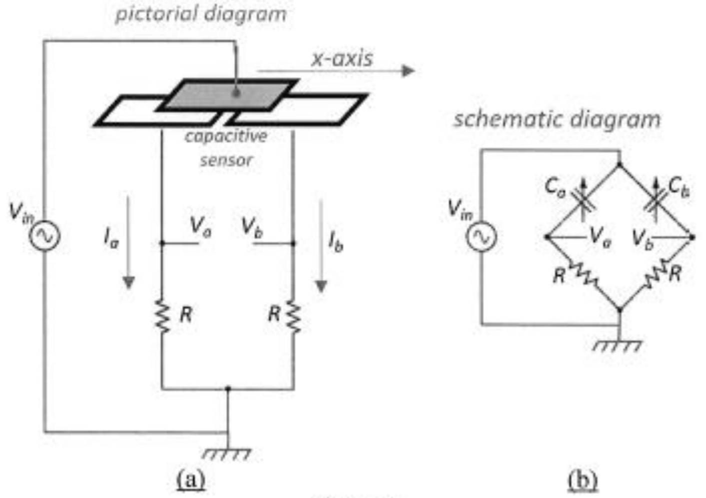


Figure 1

(a) Determine the overlapping areas $S_a(x)$ and $S_b(x)$ between the moving plate and, respectively, the left and right plates, as a function of the position $x$ of the moving plate, considering at most a displacement $L/2$ from the zero position.

(5 marks)

(b) Considering that the capacitance between two plates is directly proportional to the overlapping area, with maximum capacitance C0 when the areas fully overlap and with null capacitance whenever two plates are not overlapping, derive analytical expressions and draw, superimposing in the same graph, the capacitances $C_a(x)$ and $C_b(x)$, as functions of the moving plate position $x$.

(5 marks)

(c) Considering an AC driving input, at generic frequency $\omega$ and considering similar resistances R on both sides of the bridge, derive currents $I_a(x)$ and $I_b(x)$ on each side of the bridge.

(7 marks)

(d) Assuming very small displacements ($|x|<<L$), determine amplitude and phase of the output voltage $V_{out} = V_b - V_a$, as a function of the moving plate position x.

(8 marks)

# May 2017 – Q4

4. The characteristics of a commercial DC gearless motor, as found from the datasheet, are listed in the following table:

| Nominal Voltage [V] | Rated Torque [Kg.cm] | Rated Speed [rpm] | Rated Current [mA] | No-Load Speed [rpm] | No-Load Current [mA] | Rated Output [W] |
|---|---|---|---|---|---|---|
| 12 | 0.7 | 5700 | 5500 | 7000 | 900 | 41.3 |

(a) On your answer book, draw on the same graph the Speed vs. Torque, Current vs. Torque and output Power vs. Torque responses.

**NOTE:** the datasheet does not provide stall torque and stall current. The rated values refer to one possible (suggested) operating point. You can assume linearity to derive such values.

(8 marks)

(b) Determine, at the nominal voltage, the maximum power (in Watts) which can be delivered to a mechanical load as well as the torque, speed and efficiency at such optimal operating point.

(5 marks)

(c) Based on previous calculations and the information reported above, determine the friction coefficient of the bearings (assuming a linear model) and the amount of friction torque under no-load conditions.

(7 marks)

(d) Compute the *efficiency* of the motor when operating at the rated speed and when operating at maximum power transfer conditions.

(5 marks)

# May 2016 – Q3

(a) Explain what is meant by a balanced bridge and determine the resistance R0 so that the bridge is balanced at $0^oC$.

(5 marks)

(b) Determine the analytical expression of the voltage $V_{out}$ as function of the temperature T and compute the magnitude of $V_{out}$ at a steady temperature of $100^oC$.

(7 marks)

(c) Determine the analytical expression of sensitivity $S=dV_{out}/dT$ and evaluate the sentivity at $T=0^oC$.

(5 marks)

(d) Based on the sensitivity at $0^oC$ and assuming we want amplify the signal of the sensor in Figure 3 and interface it with an 8-bit AD converter operating between 0-5V, determine:

(i) the minimum gain G which guarantees a resolution of $1^oC$.

(ii) the maximum readable temperature (at this specific minimum gain G).

(8 marks)

3. A Pt100 RTD sensor is connected to one arm of a Wheatstone bridge as shown in Figure 3.

The thermometer resistance changes with temperature according to the equation:

$$R(T) = R0*(1+a*T)$$

where:

- T is the temperature in degrees Celsius ($^oC$)
- R(T) is the resistance of the thermometer at a generic temperature T
- R0 is the resistance at $0^oC$
- $a=0.0039^oC^{-1}$ is the temperature coefficient

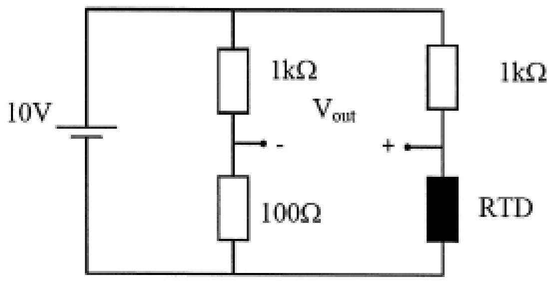


Figure 3

# May 2016 – Q4

4. A simple robot delivers rehabilitation therapy to a patient's arm via a 1 degree of freedom (dof) mechanism consisting of a handle (grasped by the patient's hand, as in Figure 4) sliding along a rail. To assist the patient, the handle is pushed/pulled by cables wrapped around pulleys and driven by motors (MP configurations in Figure 4). The motor can deliver a maximum torque Tmax=0.5 Nm and has rotor inertia of 120 g.cm^2. The maximum force to be exerted at the handle is Fmax=30N.

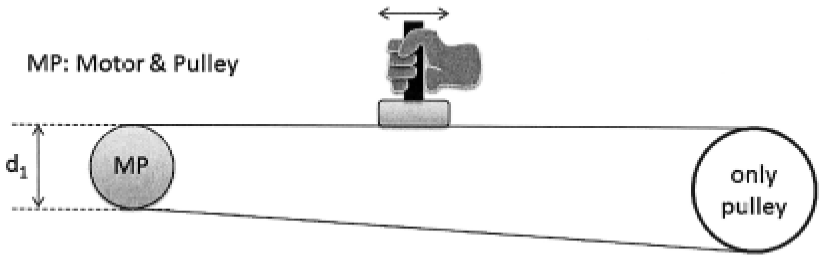


Figure 4

(a) Determine the diameter $d_1$ necessary to transmit a force Fmax=30N at the handle for maximum torque Tmax=0.5 Nm available at the motor.

(3 marks)

(b) Assuming a negligible inertia for handle, cables, and pulleys, determine the inertia perceived at the handle by the user

(7 marks)

Consider now the possibility of having two similar motors arranged as in Figure 5.

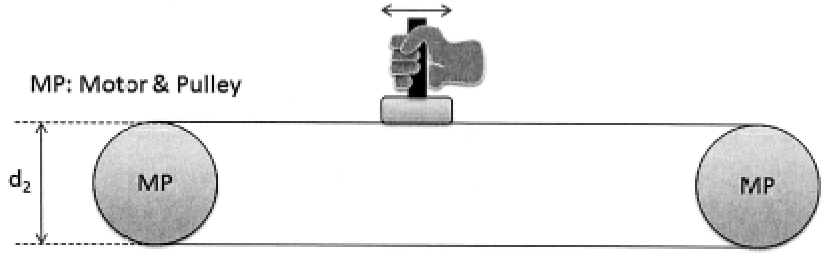


Figure 5

(c) Determine the diameter $d_2$ necessary to transmit a force Fmax=30N at the handle for maximum torque Tmax=0.5 Nm available at both motor.

(3 marks)

(d) Assuming a negligible inertia for handle, cables, and pulleys, determine the inertia perceived at the handle by the user, due to the presence of two motors.

(7 marks)

(e) Generalize the results in (d) and show how the perceived inertia scales in the presence of N motors. In particular, will having more motors lower the perceived inertia?

(5 marks)