1	2	3	4
B	B	B	B⁻

ORI

CALIF.
A

1/5

APELLIDO Y NOMBRE: [redacted]
LIBRETA: 671/14
TURNO: ☐ Lu - Mi 8-11hs ☐ Lu - Mi 14-17hs ☒ Lu - Mi 19-22hs
CARRERA: ☐ Matemática ☐ Química ☒ Física-Cs.de la Atmósfera-Oceanografía

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Análisis II - Análisis Matemático II - Matemática 3
1er. Cuatrimestre de 2015 - Recuperatorio 2º Parcial (20/07/2015)

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1. Sea C la curva intersección del cono $z^2 = x^2 + y^2$ con el plano $z = 4$, orientada en sentido antihorario vista desde arriba y sea $\mathbf{F}$ el campo

$$\mathbf{F}(x, y, z) = (\cos(x^2) + xy + y, e^{y^2}, y^2 + \ln(z^2 + 1)).$$

Calcular

$$\int_C \mathbf{F} \cdot ds.$$

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2. Resolver

$$\left[3y^2 + 3y\cos(x^3y)\right] dx + \left[2xy + x\cos(x^3y)\right] dy = 0,$$

sabiendo que existe un factor integrante que depende sólo de la variable x.

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3. Encontrar una base de soluciones de la ecuación

$$y'' - \frac{4}{x}y' + \frac{6}{x^2}y = 0$$

en el intervalo $I = (0, +\infty)$, sabiendo que $y_1(x) = x^3$ es solución.

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4. Para $\alpha \in \mathbb{R}$ considerar el sistema

$$\begin{cases} x_1' &= -2x_1 + \alpha x_2 \\ x_2' &= \alpha x_1 - 2x_2 \end{cases}.$$

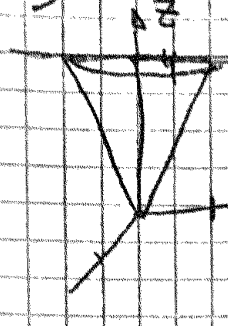
a) Hallar los valores de $\alpha \in \mathbb{R}$ de manera que **todas** las soluciones del sistema tiendan a 0, cuando $t \to +\infty$.

b) Para $\alpha = 1$, hallar la solución general del sistema y esbozar el diagrama de fases.

JUSTIFICAR TODAS LAS RESPUESTAS

[redacted] 2/5

1) $C = \{ z^2 = x^2 + y^2 \quad \cap \quad z = 4 \}$ orientación positiva



El teorema de Stokes relaciona el área encerrada por la curva, con el flujo sobre la curva.

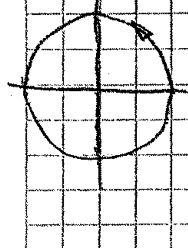
$$\int_C F \cdot ds = \iint_D \nabla \times F \, ds$$

$D = \{ x^2 + y^2 \leq 4 \}$, $z = 4$ $\qquad F = (F_1, F_2, F_3)$

$$\iint_D \nabla \times F \, ds = \iint_D \begin{pmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_1 & F_2 & F_3 \end{pmatrix} \cdot ds$$

$$\nabla \times F = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right)$$

$$= (2y - 0, 0 - 0, 0 - (x+1))$$
$$= (2y, 0, -x-1) \checkmark$$



parametrizando en polares:

$T_{(r,\theta)} = (r\cos\theta, r\,\text{sen}\,\theta, 4) \checkmark$

$T_r = (\cos\theta, \text{sen}\,\theta, 0)$

$T_\theta = (r(-\text{sen}\,\theta), r\cos\theta, 0)$

$(0, 0, r\cos^2\theta - (-r\,\text{sen}^2\theta))$

$x = r\cos\theta$ $\qquad (0, 0, r\cos^2\theta + r\,\text{sen}^2\theta)$

$y = r\,\text{sen}\,\theta$

$z = 4 \checkmark$ $\qquad N = (0, 0, r)$ $\qquad r > 0$

N exterior $\checkmark$

$\nabla \times F = (2r\,\text{sen}\,\theta, 0, -r\cos\theta - 1)$

$\nabla \times F \cdot N = -r^2\cos\theta - r \checkmark$

NOTA

$$\iint_D -r^2\cos\theta - r \; ds \qquad D\begin{cases} 0 \leq r \leq 2 \\ 0 \leq \theta \leq 2\pi \end{cases}$$

$$\int_0^{2\pi} -\frac{r^3}{3}\Big|_0^2 \cos\theta \, d\theta - \int_0^{2\pi} \frac{r^2}{2}\Big|_0^2 \, d\theta$$

$$\int_0^{2\pi} -\frac{2^3}{3} \cos\theta \, d\theta - \int_0^{2\pi} \frac{2^2}{2} \, d\theta$$

$$-\frac{8}{3} \operatorname{sen}\theta \Big|_0^{2\pi} - (2 \cdot 2\pi) = \boxed{-4\pi}$$

- $\int_C F \, ds = -4\pi$

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$$\left[3y^2 + 3y\cos(x^2y)\right]dx + \left[2xy + x\cos(x^3y)\right]dy =$$

$$\partial_y\left[\mu(x)\left(3y^2+3y\cos(x^2y)\right)\right] = \partial/\partial_x\left[\mu(x)\left(2xy + x\cos(x^3y)\right)\right]$$

$$0\cdot(\ \ ) + \mu\left(6y + 3\cos(x^2y) + 3y(-\text{sen}(x^2y)\,x^2)\right) =$$

$$= \mu'\left(2xy + x\cos(x^3y)\right) + \mu\left(2y + \cos(x^3y) + x(-\text{sen}(x^3y)\cdot 2x^2\right.$$

$$\to \mu\left(6y + 3\cos(x^2y)\right) - 3y\,\text{sen}(x^3y)\,x^2 =$$

$$= \mu'\left(2xy + x\cos(x^3y)\right) + \mu\left(2y + \cos(x^3y) - x\,\text{sen}(x^3y)\cdot 2x\right.$$

$$\to 6y + 3\cos(x^2y) - \frac{3y\,\text{sen}(x^3y)\,x^2}{\mu} =$$

$$= \frac{\mu'}{\mu}\left(2xy + x\cos(x^3y)\right) + 2y + \cos(x^3y) - x\,\text{sen}(x^3y)\,2x$$

$$\to \underbrace{y\left(6 - \frac{3}{\mu}\text{sen}(x^3y)\,x^2\right)}_{1} + 3\cos(x^2y) =$$

$$= \underbrace{y\left(2x\frac{\mu'}{\mu} + 2 - 2x^3\,\text{sen}(x^3y)\right)}_{2} + x\cos(x^3y)\frac{\mu'}{\mu} + \cos(x^3$$

$$\to y\left(\ 1\ \right) + 2\cos(x^2y) = y\left(\ 2\ \right) + \frac{\mu'}{\mu}x\cos(x^3y)$$

$$\begin{cases} 1 = 2 \\ \frac{\mu'}{\mu}x\,\cancel{\cos(x^3y)} = 2\,\cancel{\cos(x^2y)} \end{cases}$$

$$\frac{\mu'}{\mu} = \frac{2}{x} \to \ln|\mu| = \int\frac{2}{x}\,dx = 2\ln|x|$$

$$|\mu| = x^2 \to \boxed{\mu = x^2}$$

NOTA

compruebo con 1 = 2 ~~reemplazando~~ ~~reemplazando u~~

$\cancel{6 - \frac{3}{x^2} \operatorname{sen}(x^3y)x^2 = 2x\left(\frac{2x}{x^2}\right) + 2x^3 \operatorname{sen}(x^3}$

$\cancel{6 - 3\operatorname{sen}(x^3y) = 4 + 2x^3 \operatorname{sen}(x^3y)}$ me había faltado esto

$$6 - \frac{3}{u} \operatorname{sen}(x^3y)x^2 = 2x\left(\frac{u'}{u}\right) + 2 - 2x^2 \operatorname{sen}(x^3$$

reemplazando u: $6 - \frac{3}{\cancel{x^2}} \operatorname{sen}(x^3y)\cancel{x^2} = 2x\left(\frac{2x}{x^2}\right) + 2 -$

$$6 - 3\operatorname{sen}(x^3y) = 4 + 2 - 2x^2 \operatorname{sen}(x^3y)$$

Vale si $3 = 2x^2$

$$6 = 2x\left(\frac{u'}{u}\right) + 2 \rightarrow 2x\frac{u'}{u} = 4 = \boxed{\frac{u'}{u} = \frac{2}{x}}$$

~~$\frac{3}{u} = 2 \rightarrow u = \frac{2}{3}$ Absurdo, $u(x)$~~ ídem antes.

$\boxed{u = x^2} \cancel{+c}$

Luego, la ecuación es exacta.

busco $f(xy)$ / $\frac{\partial f}{\partial x} = P$ y $\frac{\partial f}{\partial y} = Q$

$$\frac{\partial f}{\partial x} = x^2\left(3y^2 + 3y\cos(x^3y)\right)$$

$$f = \int 3x^2y^2 + 3y\,x^2\cos(x^3y)\ dx \qquad v = x^3y$$

$$f = x^3y^2 + \cancel{\int \frac{\cos u\ du}{3yx^2} \operatorname{sen}} \qquad dv = 3x^2y\,dx$$

$$f = x^3y^2 + \operatorname{sen}(x^3y) + g(y)$$

[redacted]

FECHA

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$$\frac{\partial f}{\partial y} = x^2\left(2xy + x\cos(x^3y)\right)$$

$$= 2x^3y + x^3\cos(x^3y)$$

derivando mi f $\Rightarrow$ $2x^3y + \cos(x^3y)\,x^3 + g'(y)$

$$g'(y) = 0 \Rightarrow g(y) = C$$

$$\boxed{f = x^3y^2 + \text{sen}(x^3y) + C}$$

NOTA

(3) $y'' - \frac{4}{x} y' + \frac{6}{x^2} y = 0$

conociendo $y_1(x) = x^3$ solución,

propongo $v\, y_1 = y_2$

$$v = \int \frac{1}{y_1^2} e^{-\int a_1 \, dx} \, dx \qquad -\int -\frac{4}{x} \, dx$$

$$4 \ln|x|$$

$$v = \int \frac{1}{(x^3)^2} \cdot e^{\ln x^4} \qquad e^{\ln u} = u$$

$$v = \int \frac{1}{x^6} \cdot x^4 \, dx = \int \frac{1}{x^2} \, dx \qquad \int x^{-2}$$

$$\frac{x^{-1}}{-1}$$

$$v = -\frac{1}{x} \longrightarrow y_2 = -\frac{1}{x} \cdot x^3$$

$$\boxed{y_2 = -x^2}$$

$$y = c_1 x^3 + c_2 (-x^2)$$

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4) $\alpha \in \mathbb{R}$

$$\begin{cases} x'_1 = -2x_1 + \alpha x_2 \\ x'_2 = \alpha x_1 - 2x_2 \end{cases}$$

$\overline{X}' = A.\overline{X}$ $\quad A = \begin{pmatrix} -2 & \alpha \\ \alpha & -2 \end{pmatrix}$

$\text{Det}(A - I\lambda) = 0$ $\quad \begin{pmatrix} -2-\lambda & \alpha \\ \alpha & -2-\lambda \end{pmatrix}$

$(-2-\lambda)^2 - \alpha^2 = 0$ $\qquad \lambda_{1,2} = \dfrac{-4 \pm \sqrt{16 - 4(4-\alpha^2)}}{2}$

$\lambda^2 + 4\lambda + \underbrace{4 - \alpha^2}_{c} = 0$

Si quiero que todas las soluciones tiendan a 0 necesito $\lambda_{1,2} < 0$.

$\dfrac{-4 \pm \sqrt{16 - 16 + \alpha^2}}{2} = \dfrac{-4 \pm 2\alpha}{2} < 0$

$-4 + \alpha < 0$ $\quad$ y $\quad -4 - \alpha < 0$

$-4 < \alpha$

$\boxed{\alpha < 4}$ $\quad$ y $\quad$ ~~dde~~ $\boxed{-4 < \alpha}$

a) $\boxed{-4 < \alpha < 4}$

$-2 \leq \alpha < 2$

~~para raiz doble negativa vale la igualdad, pero la~~

No uso igualdad porque una raiz me queda = 0

(b) $x'_1 = -2x_1 + x_2$

$x'_2 = x_1 - 2x_2$

$A - I\lambda \begin{pmatrix} -2-\lambda & 1 \\ 1 & -2-\lambda \end{pmatrix} \to \det(A - I\lambda) =$

$\lambda^2 + 4\lambda + 4 - 1 = 0$

$\lambda^2 + 4\lambda + 3 = 0 \quad \begin{cases} \lambda_1 = -1 \\ \lambda_2 = -3 \end{cases}$

con $\lambda = -1$

$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} \to \quad -a + b = 0 \quad \boxed{a = b} \quad v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

con $\lambda = -3$

$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \begin{matrix} a + b = 0 \\ a = -b \end{matrix} \quad \begin{pmatrix} -1 \\ 1 \end{pmatrix} = v_2$

$X = \underbrace{c_1 e^{-t}}_{y_1} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \underbrace{c_2 e^{-3t}}_{y_2} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$y_2 = K y_1^3$

$y_1 = c_1 e^{-t} \to e^{-t} = \frac{y_1}{c_1}$

$y_2 = c_2 e^{-3t}$

$y_2 = c_2 \left(\frac{y_1}{c_1}\right)^3$