Astromechanics: gravity

Astronomy 101
Syracuse University, Fall 2022
Walter Freeman

October 4, 2022

"Truth is ever to be found in simplicity, and not in the multiplicity and confusion of things."

–Newton, *Rules for methodizing the Apocalypse*
(n.b.: "apocalypse" also means "revealing")

"We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances."

–Newton, *Philosophiae Naturalis Principia Mathematica*
(Mathematical Principles of Natural Philosophy)

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- We are entering grades and returning quizzes/exams as quickly as we can
- I can return things *at 3:30PM today* – we'll go over to Holden after class
- If you want to retake quizzes, there are two times tomorrow – please sign up
- If you have questions about grades, please talk to your lab TA first.

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- I'll make a solution *video* for the exam, probably tomorrow

Do you have any pets? Do the TA's have any pets? Can we see pictures?

Kepler's laws, summarized

- 1. Planets travel in elliptical orbits, with the Sun at one focus
- 2. The line going from the Sun to the planet sweeps out equal areas in equal times
- 3. The time that a planet takes to go around the Sun is determined by the size of its orbit: the longer the long axis of the ellipse, the longer it takes to go around the Sun.

Kepler's second law

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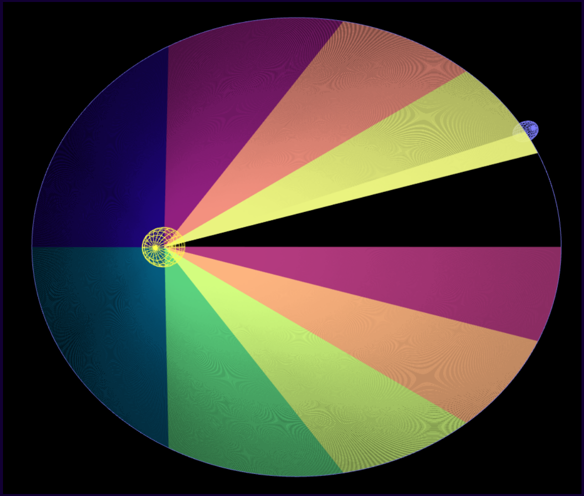

Each colored wedge has the *same area*, and the planet takes the *same time* to go through each.

Kepler’s Third Law

Kepler’s third law of orbital motion says that planets with larger orbits take longer to go around.

There is a mathematical relationship (“square-cube law”), but you’ll study that in lab – we’ll skip it here.

Let’s watch this...

Work on the exercise on Kepler's laws of orbits.

Your fourth homework set is included here. It is due next Tuesday, when we will have a quiz.

Raise your hands if you have questions. After this, we will study gravity.

Do you think Kepler's laws can apply to things besides planets? Why or why not?

Talk about this with your neighbors. I'll call on some random people and ask for your thoughts in a minute.

Asking what vs. asking why

Remember, Kepler only discovered *what* the planets' orbits looked like.

He desperately wanted to know *why* they moved in that way, but he never could figure it out.

It turns out that if we can understand *why*, we can understand some other things, too...

Natural laws vs. their consequences

Obviously the world around us is very diverse. Some things in it look quite simple:

- The motion of the stars
- The near-perfect-spheres of the planets and moons
- The elliptical motions of the planets (?)
- The colors in a rainbow

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- Seismic waves and earthquakes
- The colors in the Sun
- The weather
- The diversity of rocks on Earth

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- The diversity of rocks on Earth
- Even the simplest living things
- ... language, culture, music, art, and all the creations of humankind...

Elegance, revisited

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Isaac Newton

Isaac Newton (1642-1727 or 1726) finally figured out the laws that eluded Kepler.

He discovered...

- Forces cause objects to change their speed or direction of motion
- Calculus – the mathematics of changes
- Gravity is such a force
- The mathematical description of gravity

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- Forces cause objects to change their speed or direction of motion
- Calculus – the mathematics of changes
- Gravity is such a force
- The mathematical description of gravity
- Principles of optics
- The mathematics of cooling
- ... and much more

Newton's biggest idea

"Forces cause objects to accelerate"

$$F = ma$$

$$F/m = a$$

"The strength of a force, divided by the mass of the thing it acts on, gives that thing's acceleration"

The law of gravity

Newton showed mathematically what Kepler suspected: that "there is a force in the Earth that causes the Moon to move".

That thing, of course, is gravity.

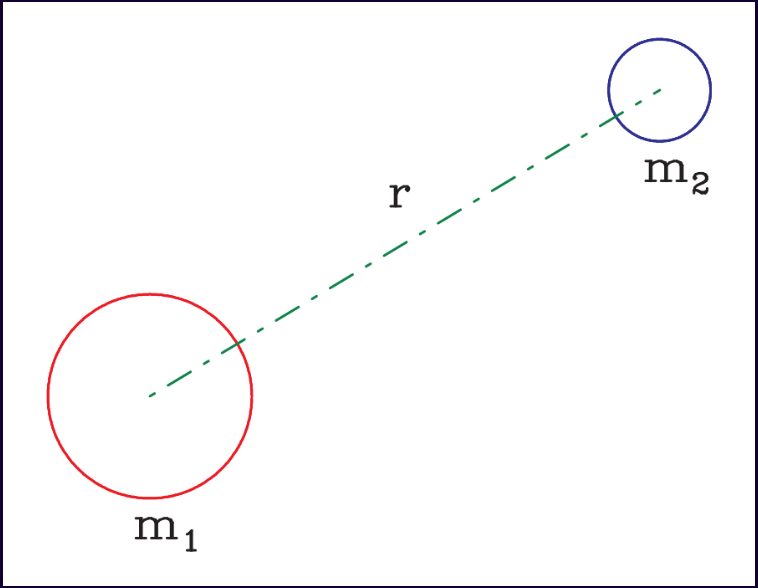
Newton discovered:

All objects attract all other objects with a force that is:

- Proportional to the product of their masses
- Inversely proportional to the distance between their centers, squared

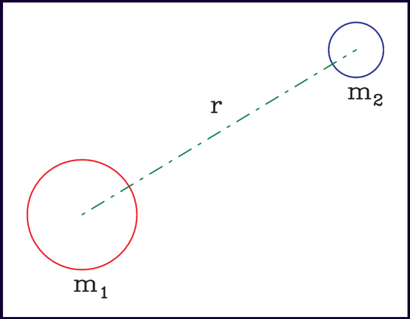
In symbols:

$$F = \frac{Gm_1m_2}{r^2}$$



The gravitational force between these two planets is

$$F_g = \frac{Gm_1m_2}{r^2}.$$



Here's the same diagram again. Suppose m_1 is a planet and m_2 its moon. As before the force on the moon is

$$F_g = \frac{Gm_1m_2}{r^2}$$

Suppose we make the planet twice as massive. How does the gravitational force on the moon change?

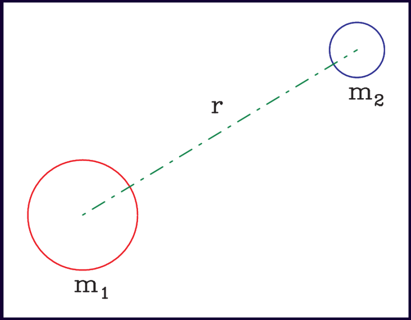
A: It doesn't change

B: It becomes twice as strong

C: It becomes four times as strong

D: It becomes half as strong

E: It becomes one-quarter as strong



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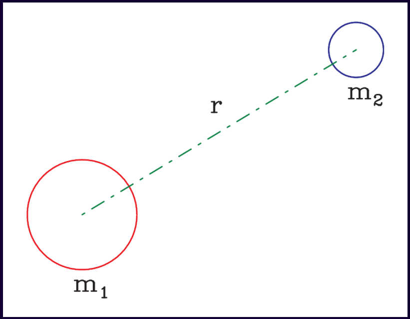

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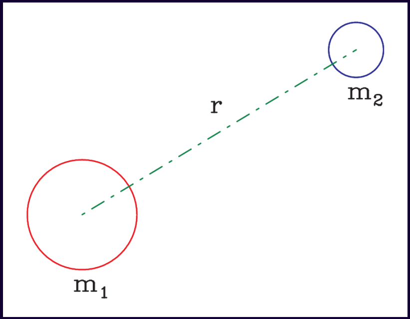

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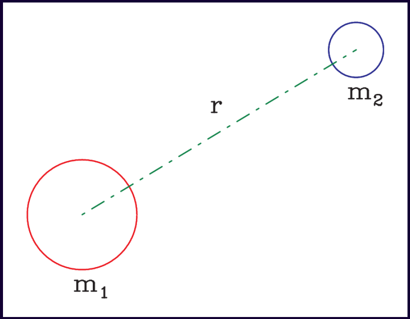
$$F_g = 2\frac{Gm_1 m_2}{r^2}$$



Again, m_1 is a planet and m_2 its moon. As before the force on the moon is

$$F_g = \frac{Gm_1 m_2}{r^2}$$

Suppose we move the moon twice as far away from the planet. How does the gravitational force on the moon change?

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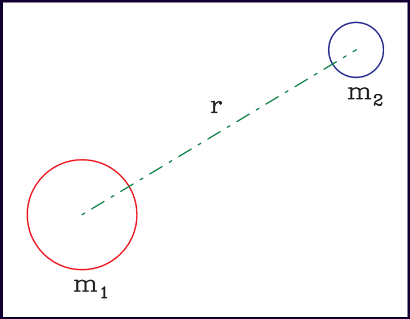

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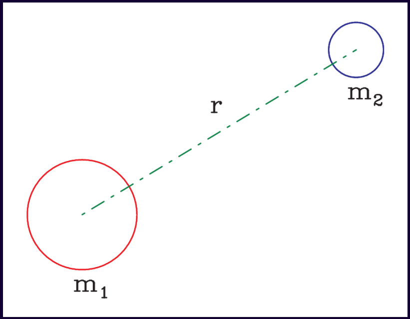

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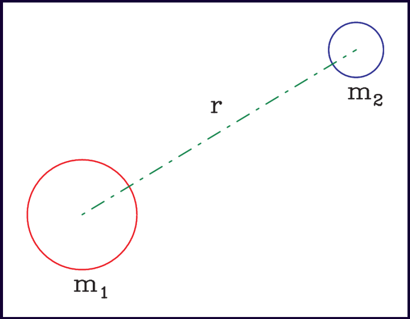
$$F_g = \frac{Gm_1m_2}{(2r)^2}$$



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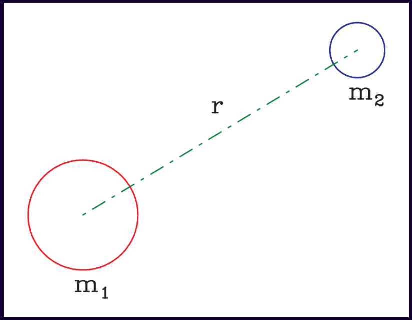
$$F_g = \frac{Gm_1m_2}{4r^2}$$



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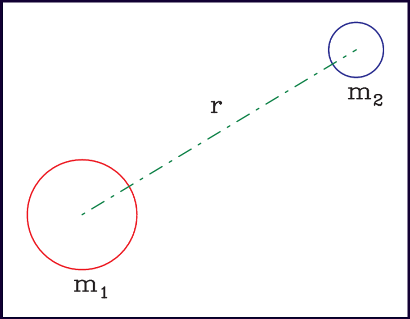
$$F_g = \frac{1}{4}\frac{Gm_1m_2}{r^2}$$



Here's the same diagram again. Suppose m_1 is a planet and m_2 its moon, and m_1 is twice as big as m_2. As before the force that the planet applies to the moon is

$$F_g = \frac{Gm_1m_2}{r^2}$$

How does the force that the moon applies on the planet compare to the force the planet applies to the moon?

A: The force on the planet is twice as large as the force on the moon
B: The force on the planet is four times as large as the force on the moon
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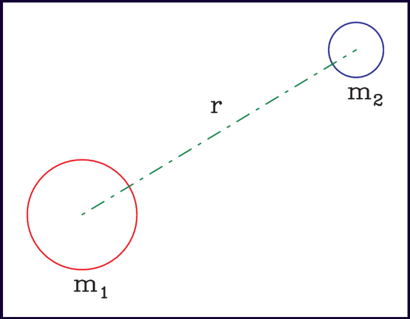
D:The force on the planet is one-quarter as large as the force on the moon
E: Both planets pull on one another with the same force.



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All objects attract all other objects with a force that is:

- Proportional to the product of their masses
- Inversely proportional to the distance between them squared

In symbols:

$$F = \frac{Gm_1m_2}{r^2}$$

Notice I didn't say which mass was which. It doesn't matter!

Suppose two asteroids are floating out in space, 20 miles apart. Asteroid A is twice as massive as asteroid B, and the force of A's gravity on B is ten tons.

Suppose I now move the two asteroids closer, so they're only 10 miles apart. What will the force of A's gravity on B be now?

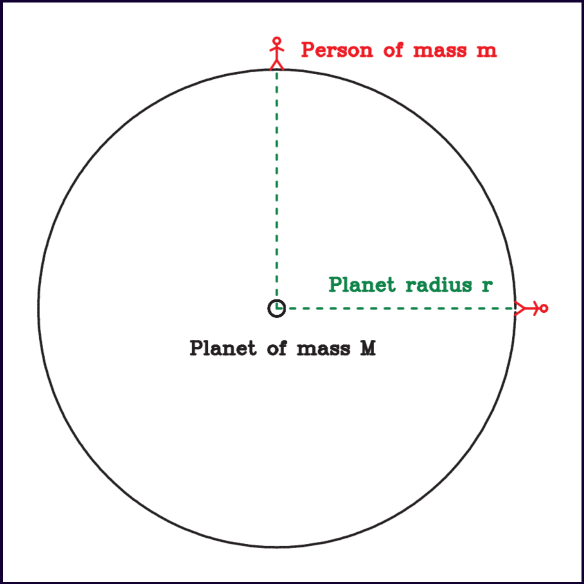
A: 5 tons

B: 10 tons

C: 20 tons

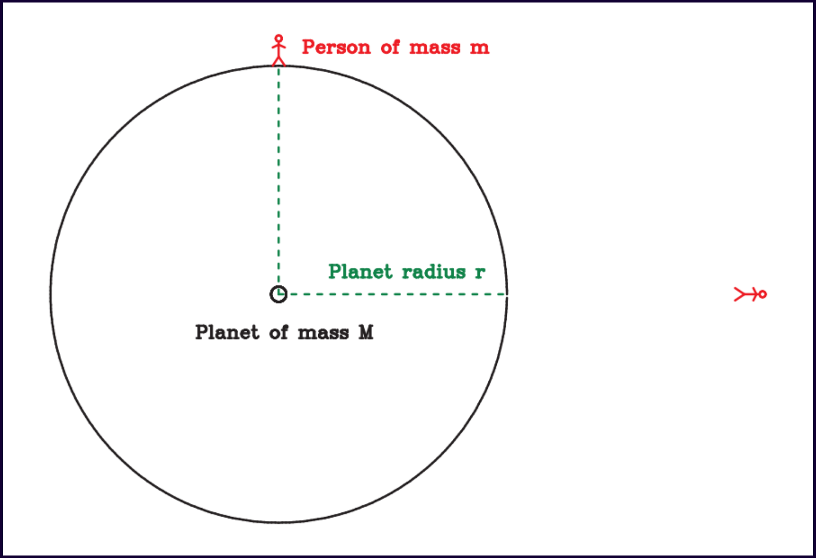
D: 40 tons

The distance is measured between the *centers* of the objects.



This lets you calculate the force of a planet's gravity on people on its surface!

Earth has a radius of about 6,000 km. If we move this person 6,000 km away from Earth's surface, how does the strength of Earth's gravity change?



A: It stays the same

B: It becomes twice as strong

C: It becomes half as strong

D: It becomes one-quarter as strong

E: There's no gravity in space, so it goes away totally