**AST101: Our Corner of the Universe**
**Lab 6: Mass of the Earth**

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Student number (SUID):

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Lab section:

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## 1 Introduction

We've learned in class that Newton's laws of motion apply to every object in our day to day lives, and can explain a great deal of how things move. Among these is the motion of a pendulum. Newton's second and gravitational laws combine to explain how if the pendulum begins away from its rest position, the force of gravity acts to pull it down, causing it to accelerate, and eventually come to rest, and repeat again. Today, we will study this process to arrive at a surprising measurement; the mass of the Earth!

### Materials

A simple pendulum, a stopwatch, and a calculator.

### Objective

To conduct a simple scientific experiment to determine the mass of the Earth.

## 2 Reaching the hypothesis

### 2.1 Let the Force be with you!

You should already be familiar with Newton's second law of motion:

$$F = ma \tag{1}$$

which simply says that the force exerted on an object is equal to the product of its mass and its acceleration. Written another way

$$a = \frac{F}{m}$$

Newton's second law says that objects respond more strongly (that is, have a greater acceleration) when subjected to stronger forces, or when their masses are smaller (imagine trying to push a cardboard box vs a 16 wheel truck).

Another of Newton's laws is his law describing a particular force, the force of gravity:

$$F = G\frac{mM}{r^2} \tag{2}$$

where G is a constant (like $\pi$), $m$ and $M$ are the masses that gravity is pulling together, and r is the distance between those two objects.

You'll need one more equation before we get started:

$$P = 2\pi\sqrt{\frac{L}{a}} \tag{3}$$

This equation relates the period of a pendulum P (the time it takes to complete one full cycle) to its length L and the acceleration the pendulum experiences a. You don't need to worry about where this equation comes from, but it's important to know that **this equation only applies when the pendulum swings through a small arc!**

### 2.2 The gravity of the situation

**For the entirety of this lab, remember to show your work!**

**Question 1.** Find the acceleration due to gravity by setting equation (1) and (2) equal to each other, and solving for a.

(1) $ma = G\frac{mM}{r^2} \rightarrow a = G\frac{M}{r^2}$

**Question 2.** For the acceleration due to gravity, does it depend on the mass of the object that's "falling", m? If I drop two objects of different masses, will the heavier one land first?

(2) It doesn't depend on mass, so the heavier object falls at the same rate.

**Question 3.** If you square both sides of equation 3, you get the equation $P^2 = 4\pi^2\frac{L}{a}$. Solve this equation for a.

(1) $a = \frac{4\pi^2 L}{P^2}$

**Question 4.** Set the a you found in question 1 equal to the a you found in question 3, and solve this for the mass M.

(2) $G\frac{M}{r^2} = \frac{4\pi^2 L}{P^2} \rightarrow M = \frac{4\pi^2 L r^2}{GP^2}$

**Question 5.** In the previous question, you should have gotten $M = \frac{4\pi^2 Lr^2}{GP^2}$, or something very similar. Show your TA your result and have them sign below. If your TA is not yet available, skip to section 3.1, and return when they are available.

(1) SDB

**Question 6.** The radius of the Earth can be measured with some clever geometry (Eratosthenes did it in 240 B.C.!), and G can be carefully measured in other experiments. What other values do we need to find to be able to determine the mass of the Earth from your result in question 4?

(1) P, the period, and L the length.

**Question 7.** The value of G given is $6.674 * 10^{-11}$. Write this in "normal" notation (if $1.645 * 10^2$ is scientific notation, 164.5 is "normal" notation). Is this a very large number, or a very small number? Why might gravity be referred to as a weak force?

(2) 0.000 000 000 066 74. An incredibly small #. Gravity is weak because you must multiply by this #

## 3 The Experiment

We've now derived an equation that will tell us the mass of the Earth using a pendulum. All we need to do is measure the length of the pendulum, and the period of the pendulum. There should be a pendulum and meter stick at each of your tables.

### 3.1 Collecting data

**Question 8.** Do we need to measure the mass of the pendulum?

(2) No, because acceleration due to gravity doesn't depend on mass.

**Question 9.** Using your meter stick, measure the length of the pendulum, from where the string meets the metal rod, down to the middle of the hanging mass. We'll refer to this as the average length $L_{avg}$.

(1) 0.185m

**Question 10.** Recall from the parallax lab that it is important to get a sense of the error in our experiments. In measuring the length of your pendulum, how accurate do you think your measurement was? Was it accurate to 1mm? 1cm? 1m? Give your best estimate below.

(1) 0.005m

We will now measure the period of the pendulum. To do this, we will measure how long the pendulum takes to complete **TEN** complete trips using a stopwatch, and then dividing the result by 10. Doing this will reduce the error in our experiment (can you figure out why?). We will do this 5 times. **Remember to not let your pendulum swing through too long of an arc, or equation 3 won't be valid.**

(5)**Question 11.** Take the measurements and complete the table below.

| Trial | 10 Periods (seconds) | 1 Period (seconds) |
|---|---|---|
| 1 | 8.67 | 0.867 |
| 2 | 8.86 | 0.886 |
| 3 | 8.70 | 0.870 |
| 4 | 9.13 | 0.913 |
| 5 | 8.86 | 0.886 |

**Question 12.** Calculate the average period by adding up all 5 of the entries in your last column and diving the result by 5. We'll refer to this value as $P_{avg}$.

(1) 0.884s

**Question 13.** We want to get a sense of the error in our measurement for P. There are many sophisticated ways to do this, but we'll do something simple; take the highest value for P you found, subtract from it the lowest of P, and divide this result by 2.

(1) 0.023s

### 3.2 Calculating the mass

Listed below are the radius of the Earth r, Newton's Constant G, and pi, out to 3 decimal places:

$$r = 6.371 * 10^6 \ m$$
$$G = 6.674 * 10^{-11} \ m^3 kg^{-1} s^{-2}$$
$$\pi = 3.142$$

Also, here is the equation you derived for the mass of the Earth:

$$M = \frac{4\pi^2 r^2 L}{GP^2} \tag{4}$$

**Question 14.** Using the above values and your $P_{avg}$ and $L_{avg}$, calculate the mass of the Earth below in kg.

(2) $5.680 \times 10^{24}$ Kg

**Question 15.** Calculate $L_{max}$, the maximum value of L supported by your measurements, by adding your uncertainty in L to your average value for L, and $L_{min}$ by subtracting. Do the same thing for P to get $P_{max}$ and $P_{min}$.

(1) $L_{max}$ = 1.190 m   $L_{min}$: 0.160 m   $P_{max}$: 0.907   $P_{min}$: 0.861

**Question 16.** Look back at the equation for the mass of Earth. To get the highest value for the mass supported by our data, do we want to use $L_{max}$ or $L_{min}$? What about $P_{max}$ or $P_{min}$?

(1) $L_{max}$ and $P_{min}$

**Question 17.** Calculate $M_{max}$ by using $L_{max}$ and $P_{min}$ instead of $L_{avg}$ and $P_{avg}$ in equation 4.

(1) $6.150 \times 10^{24}$ Kg

**Question 18.** Calculate $M_{min}$ by using $L_{min}$ and $P_{max}$ instead of $L_{avg}$ and $P_{avg}$ in equation 4.

(1) $5.250 \times 10^{24}$ Kg

### 3.3 Analyzing the results

The accepted value for the mass of the Earth is about $5.972 * 10^{24}$ kg.

**Question 19.** Does the accepted value lie between your $M_{min}$ and $M_{max}$? What do you think this says about the accuracy of your experiment?

(2) $5.250 \times 10^{24} < 5.972 \times 10^{24} < 6.150 \times 10^{24}$ ✓ This means my experiment is accurate!

**Question 20.** Is your average value for the mass close to the accepted value? Calculate the percentage difference. Recall that the formula for percent difference is $100 \times \frac{M_{accepted} - M_{calculated}}{M_{accepted}}$, and that you should ignore a minus sign in your answer.

(1) 4.884%

**Question 21.** There were three possible sources of error in this lab; the length of the pendulum, the period of the pendulum, and making your pendulum swing too far. Which source of error do you think most impacted your experiment?

(2) The meter stick to measure length