

Kepler's laws

Astronomy 101
Syracuse University, Fall 2019
Walter Freeman

October 1, 2019



“And yet it moves.”

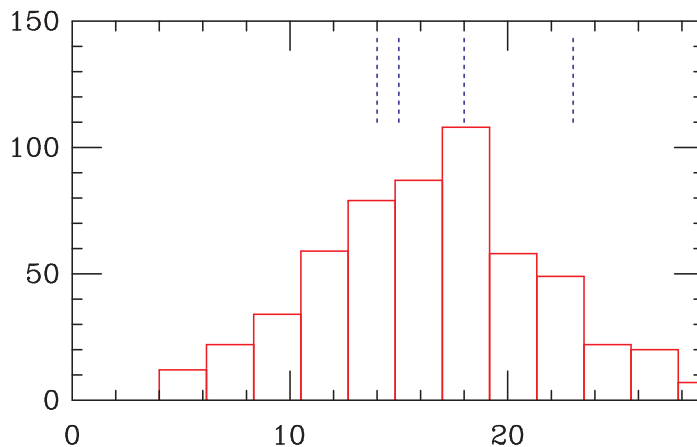
—Galileo (attributed), on the Earth

Announcements

- This week's lab is Lab 4
- There may or may not be a prelab for next week; I'll let you know Thursday

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- Papers are due Wednesday at 5PM:
 - Email a copy to suast101projects@gmail.com
 - Put a physical copy in your TA's mailbox
 - Check the lab schedule link on the website to look up your TA
 - This includes E. Nastas' former sections; your new TA is listed
- Late paper submissions will lose 2 points per day
- If only your physical submission was late, talk to your TA

Exam grades



(Includes bonus questions on the exam; does not include other extra credit)

This exam was difficult.

Exam 1 is always the hardest exam in AST101, simply because this first unit involves so much spatial reasoning and geometry. This year's was particularly difficult.

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If you did well, congratulations.

If you didn't do as well as you'd hoped, don't worry:

- The other exams are somewhat easier
- We drop the lowest exam grade
- Many students' overall grades in AST101 are significantly higher than exam grades, because of labs, papers, and the final project
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- This year we also have the in-class quizzes, which are sort of free points. (You can even use your notes for them!)
- **If you do better on the questions on this material on the final than you did on Exam 1, you can raise your Exam 1 grade**

Some advice for preparing for the next exam:

- Make sure you're staying current on the *Lecture Tutorials* work
- Factual recall isn't as important as being able to figure things out on the fly
- The next exam will involve a lot less geometry, but will involve reasoning with proportions

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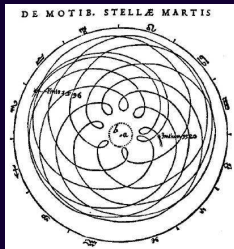
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 - Look for broad themes between the questions: how do different questions relate?
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 - It's **also** not enough to know how to figure out the answers
 - **Instead, for each question, ask: "How would I know what to do here?"**

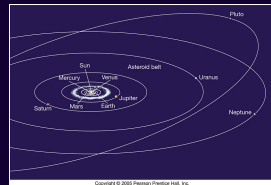
We left our story with two plausible models for the heavens:

The geocentric Ptolemaic model



- The planets (and everything else) revolve around Earth
- Inelegant system of “epicycles” needed to get planets right
- Everything moved in circles (elegant per Greeks)
- Earth and humanity at center (theologically not challenging)
- **Very accurate predictions**

The heliocentric Copernican model



- Earth is one of many planets, all orbiting the Sun
- Apparent motion = motion of Earth + motion of planets
- No (or very small) epicycles
- **Less accurate than Ptolemaic model**
- Matched Galileo’s observations:
 - Moons of Jupiter
 - Phases of Venus

The Copernican model had a lot of attractive features, but was still less accurate – less good at actually telling you where the things in the sky would be!

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- What do we do?

What we do when we don't know what to do?

Maybe our data are wrong...

The measurements of the sky that people had been using were “good enough” for navigation, but they weren't ever intended for precision natural philosophy: determining the truth of things...

(In astronomy sometimes it is okay to round things off, and sometimes you need precise measurements to figure things out: you have to think carefully about this!)

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Enter Tycho Brahe.

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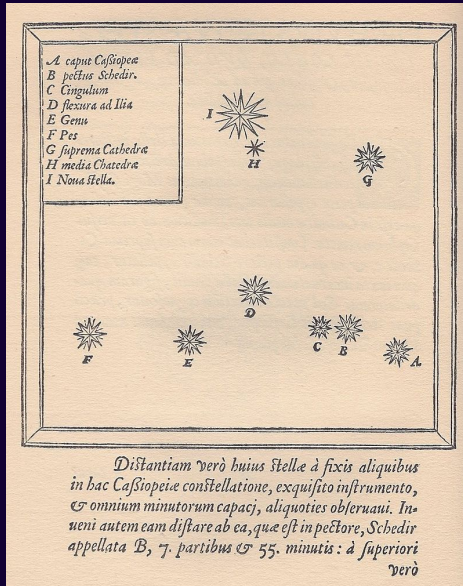
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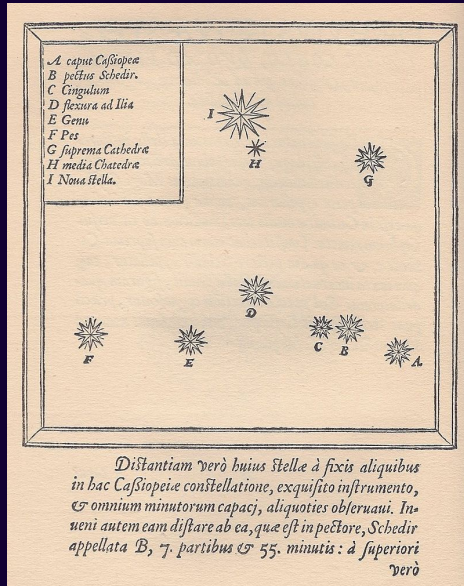
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- Had better facial hair than any of us
- Had a pet moose
- It drank too much beer and died
- Was probably fun at parties (less so post-moose-death)

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- Didn't observe parallax in the distant stars
- Two options:
 - The Earth doesn't move
 - The stars are very far away
- He believed the former
- Proposed another model for the Solar System

Tycho Brahe



- Danish nobleman and astronomer, 1546-1601
- Best known for his precision measurements of the sky from Uraniborg
- Made high precision observations of the motions of the planets and stars
- Even had a crude correction for atmosphere bending light
- Measurements accurate to a few minutes of arc ($1/60$ 'ths of a degree!)
- Made these measurements with his assistant Sophie...
- ... and his later assistant Johannes Kepler, who didn't murder him

You've probably been wondering when we're going to stop this history of false starts and learn how things actually *do* work...

Johannes Kepler

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... here we go. Kepler, Tycho's assistant, finally got it right.



Kepler was a Copernican, and disagreed with his boss.

He tried to improve Copernicus' model, which used circular orbits, and mostly succeeded. But...

- Tycho's data were incredibly precise
- No matter how he rearranged the circles, there was an error of at least $8/60$ of a degree for Mars
- Kepler worked at Uraniborg – he knew how precise the data could be

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Do we:

- A: Reject the belief that Nature must be elegant
- B: Reject the need for our model to match the data precisely
- C: Reject Tycho's data?
- D: Reexamine our ideas about what elegance looks like

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Kepler didn't only want to discover *how the planets moved*; he wanted to know *why*. He didn't figure it out, but he was on the path that led to modern science.

Even if the *answer* doesn't have the perfect elegance of circles, modern science looks for its elegance in *laws*, not in all of their consequences! Kepler discovered the consequences; the laws weren't uncovered yet.

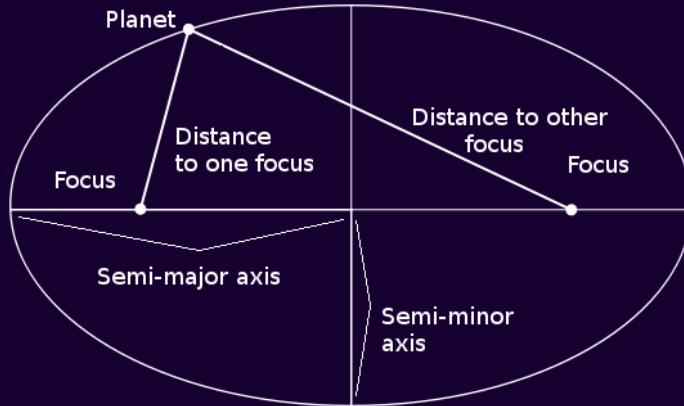
Kepler's laws of planetary motion

- The planets move in *ellipses*, with the Sun at one focus
- The line joining the planet and the Sun sweeps out equal areas in equal times
Alternate formulation: Within its orbit, a planet's speed is inversely proportional to its distance from the Sun
- The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of the ellipse.

Let's talk about each of these in turn.

Kepler's first law

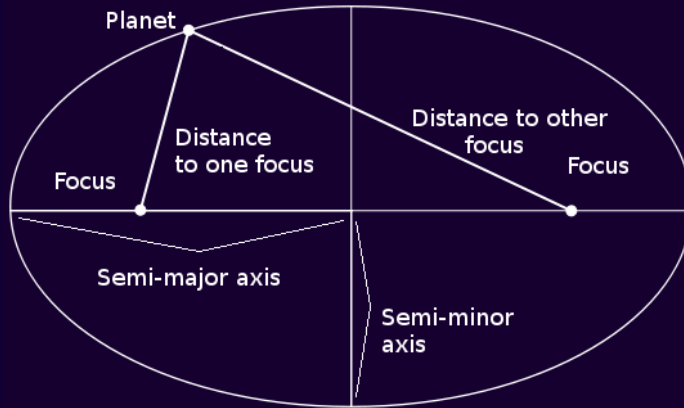
An ellipse is just a stretched circle. Mathematically: it's the curve around two points such that the *sum* of the distances to those points is a constant. A circle is just an ellipse with both foci at the same point.



Some terms:

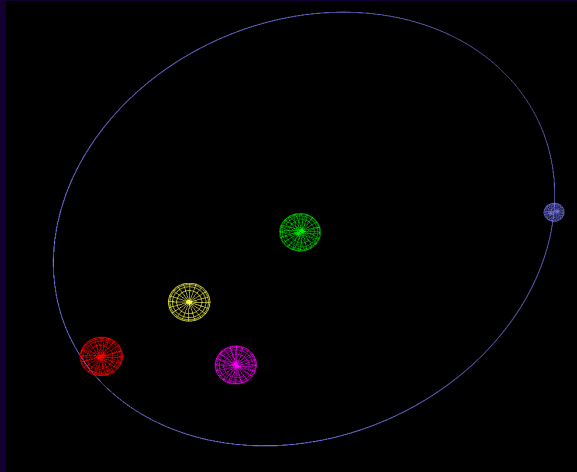
- Focus: One of the two points
- Semimajor axis: the largest distance from the center to the edge
- Eccentricity: how stretched out an ellipse is

Some properties of ellipses



- The two foci always lie along the major axis (“wide axis”)
- The closer together the foci, the less eccentric
- If both foci are exactly at the middle, you get a circle
- Both foci lie inside the ellipse

Here's an orbit. Which is the correct position for the Sun?



A: The red one

B: The green one

C: The yellow one

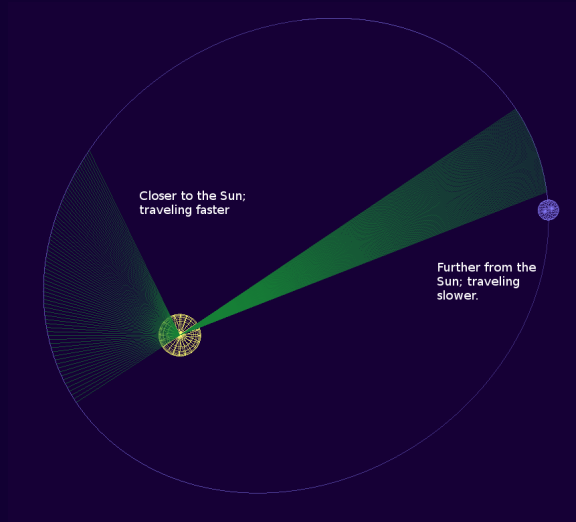
D: The purple one

What you need to know

- Planetary orbits are ellipses
- The *eccentricity* of an ellipse tells you how squashed it is
- An ellipse with zero eccentricity is just a circle
- The Sun lies at a focus of the ellipse, which isn't at the center (unless it's a circle)
- The more eccentric the orbit, the further to one side the Sun is

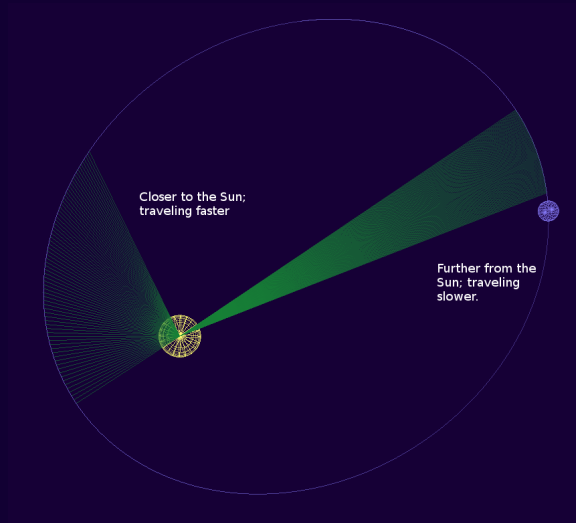
Kepler's second law

In an eccentric orbit, a planet travels fastest when it's nearer the Sun.



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Let's watch this in an animation...

Comets

Comets have highly eccentric orbits. Halley's Comet's furthest point from the Sun – its *aphelion* – is 35 AU away. But its *perihelion* – the nearest point to the Sun – is 0.6 AU away.

Which statement is true?

A: Halley's Comet spends most of its time far from the Sun, and only a little time near the Sun

B: Halley's Comet moves slowly near perihelion, and quickly at aphelion

C: Halley's Comet moves quickly near perihelion, and slowly at aphelion

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(Get creative with your folding...)

Complete *Lecture Tutorials* pp. 21-24.

We will do something else after this.

Kepler's Third Law

Kepler's third law of orbital motion says that the square of a planet's *orbital period* is proportional to the cube of its *semimajor axis*.

Simply put: if a planet is further from the Sun, it takes longer to go around.

If the distance is doubled, the time required *more than doubles*.

Let's watch this...

Complete *Lecture Tutorials* pp. 25-28.