Tutorial-Exercise – Kepler's Laws

In this exercise, you'll explore Kepler's laws of orbital motion.

Remember that these exercises are not meant for you to do alone; you should work with others near you on them, and should raise your hand and ask questions as you have them.

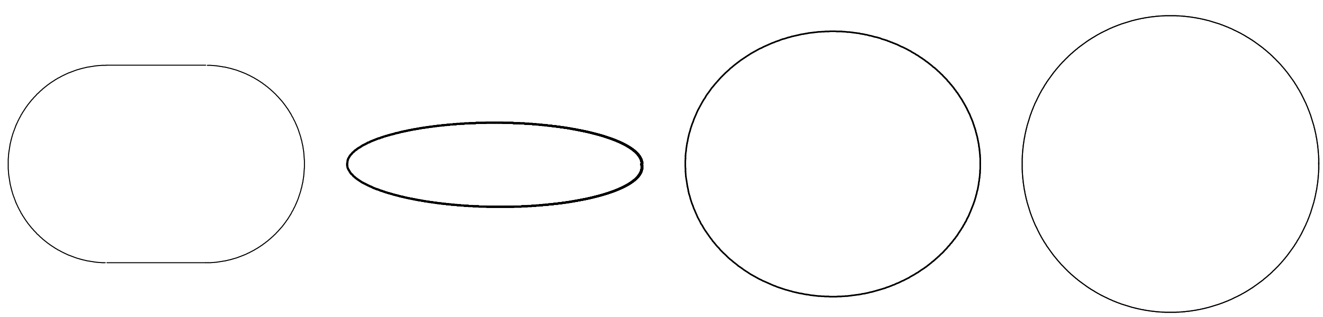
Your fourth homework assignment is included on the back of this handout. You should complete it by class time on October 12 (or 7) and put it in your TA's mailbox.

1 Kepler's First Law

Kepler's first law says that planets orbit the Sun in an ellipse with the Sun at one *focus*. An ellipse has two foci; they lie along the long axis of an ellipse, are nearer the center for ellipses that are less eccentric (stretched out), and are nearer the edge for ellipses that are more eccentric (more stretched out).

Here are four orbits.

1. Two of them have *two* possible locations for the Sun. Draw them in.
2. One of them has only *one* possible location for the Sun. Draw it in.
3. One of them is not a possible shape for a planet's orbit. Cross it out.



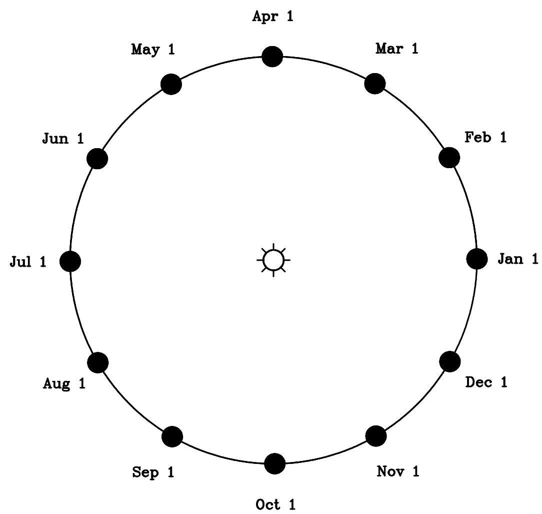
Of the three possible orbits, which one has zero eccentricity?

Which one has the highest eccentricity?

2 Kepler's Second Law

As we saw demonstrated a bit ago, Kepler's second law says that an imaginary line between the Sun and a planet orbiting it sweeps out an equal amount of area in an equal amount of time. First, let's apply this to Earth's orbit, which is very close to a perfect circle.

Pretending for now that Earth's year is divided into twelve equal months and that its orbit is a perfect circle, here is a cartoon of Earth going around the Sun.



1. Using your pencil, shade the region that the imaginary line between Earth and the Sun sweeps out during January, and again during September.

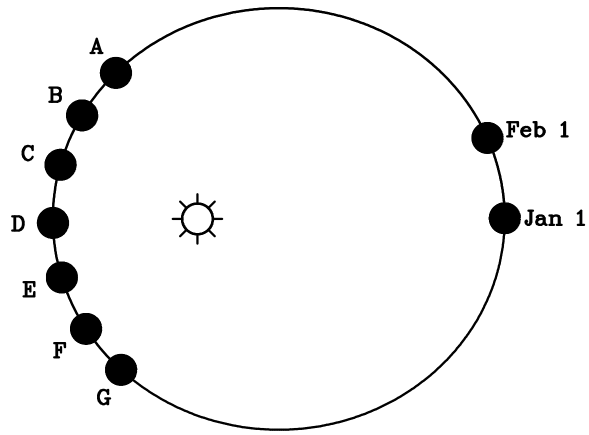
2. Does Earth's orbit follow Kepler's second law? How do you know?

3. Kepler described his observations of planetary motion in terms of the "area swept out" by the line, but we can also think about the *speed* of Earth's motion.

   Does the Earth move faster during January or during September? How do you know?

4. Now, let's consider a different planet whose orbit is more eccentric.

   Here is a cartoon of its orbit, showing its position at the beginning of January and February, along with several other positions



   Shade in the area that the line connecting the Sun to the planet would sweep out during January, as you did before.

5. Then, find *two other* points in the orbit during which the line connecting the planet to the Sun would sweep out approximately that same area. You would like to fill in the blanks in the statement:

   "As it moves from point ______ to point ______, the line connecting this planet to the Sun sweeps out the same area as it does during the month of January."

   Remember what "area" means: this means that if you were to color both areas in with ink, you would use the same amount of ink on both.

6. Does the motion on the left through the region you've highlighted take *more than*, *less than*, or *exactly* one month? How do you know? (Look back at the text of Kepler's second law on the front page.)

7. Does the planet cover *more distance* during January or during the highlighted portion you've shown on the left?

8. Does the planet *travel faster* during January or during the motion shown on the left? How do you know?

9. Now, let's think about the orbits of the planets known in Kepler's time. The eccentricity of Earth's orbit is 0.016. This is very small, so its orbit is very nearly circular. The orbits of Earth and the other inner planets look like this:

Planet	Perihelion (closest to Sun)	Aphelion (furthest from Sun)	Eccentricity
Mercury	0.47 AU	0.31 AU	0.206
Venus	0.72 AU	0.73 AU	0.006
Earth	0.98 AU	1.02 AU	0.016
Mars	1.38 AU	1.67 AU	0.093

Which of these planets *changes speed by the largest fraction* over the course of one orbit? How do you know?

10. Which of these planets *changes speed by the smallest fraction* over the course of one orbit? How do you know?

3 Kepler's Third Law

Kepler's third law says that:

"The square of the time that it takes a planet to go around the Sun is proportional to the cube of the long axis of its orbit."

Translated into modern mathematics, we could also say:

"The time it takes for a planet to go around the Sun is proportional to the 3/2 power of the long axis of its orbit.

We will do some more precise calculations based on this in lab. For now, you can just know:

"If a planet's distance from the Sun increases by some factor, its orbital period increases by *more* than that factor."

Earth is in a circular orbit around the Sun with a radius of about 1 AU, while Saturn is in a circular orbit with a radius of about 10 AU. Will Saturn take less than one year, between one and ten years, about ten years, or more than ten years to go around the Sun? How do you know?

Here are some data on the planets:

	Orbital Distance (AU)	**Orbital Period** (years)	**Mass** (Earths)
Mercury	0.38	0.24	0.06
Venus	0.72	0.61	0.82
Earth	1	1	1
Mars	1.52	1.88	0.11
Jupiter	5.20	11.86	318
Saturn	9.54	29.46	95.2

Notice that the planets have wildly varying masses: Jupiter is five thousand times more massive than Mercury.

Based on these data and what you know about Kepler's third law, what is the relationship between the mass of a planet and how long it takes to orbit the Sun?

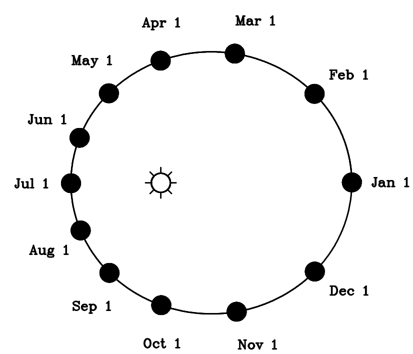
If there *is* a relationship, describe where that relationship comes from. If there is *not* a relationship between mass and orbital period, discuss how the data from the table tell you that.

What is the dominant factor in determining how long it takes for a planet to go around the Sun?

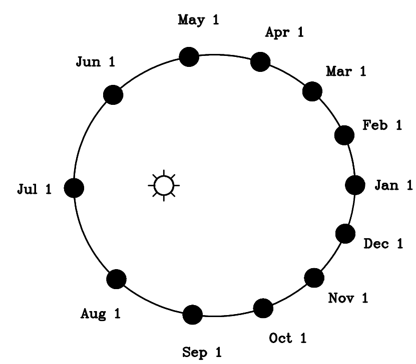
Homework 4 – Kepler's Second Law

Due Tuesday, October 12

Suppose that the fictional planet that divides its years into twelve equal months has a pretty elliptical orbit. Astronomers on this planet are trying to figure out how it moves around its star during a year. They propose two options:



Choice A



Choice B

Three astronomers – Alex, Bethany, and Chen – are debating which one is correct.

Alex: I think Choice A is correct. The planet is supposed to go fastest when it is near the Sun, right? So the months are spaced closer there since it is going faster.

Bethany: Yes it is. But Choice B shows an orbit where the planet is moving slowly far away from the Sun, since it covers less distance each month.

Chen: I think neither one can be right. The months are supposed to be equally-spaced and last the same amount of time, right? So the dots should be equally far apart, since the orbit is one year, and the months are equal portions of that. So you should draw another diagram like that.

1. Who is correct? Write a few sentences explaining why and how you know.

2. Is the planet moving faster in July or January? How do you know?

3. Is the planet speeding up or slowing down in April? How do you know?