Astromechanics: gravity

Astronomy 101
Syracuse University, Fall 2020
Walter Freeman

September 29, 2020

"Truth is ever to be found in simplicity, and not in the multiplicity and confusion of things."

–Isaac Newton, *Rules for methodizing the Apocalypse*
(n.b.: "apocalypse" also means "revealing")

"We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances."

–Isaac Newton, *Philosophiae Naturalis Principia Mathematica*
(Mathematical Principles of Natural Philosophy)

I'm having lunch outside of Hendricks right now, and a bunch of ants came along and wanted some of my sandwich.

So I give them a little piece. Why not.

There is an ant that was farther away than the rest of them, and I think something might be wrong with it

One of them just brought a crumb of the sandwich to the ant

And now he's eating

If we are all just a little kind

We might all get better

The little ant is moving around now

Sometimes a little crumb of kindness goes a long way

–Text message from K. Alice Lindsay, used by permission

Announcements

The "on-your-own" lab for this week is due at the end of the day Friday.

Project 3 will likely be assigned at the end of the day Friday.

We hope to fix any "group issues" this week while we are temporarily not doing group work. If you have a group issue, come to Blackboard Collaborate during your lab time and describe your issue to your TA. They will fix it.

I was away for this weekend (two very special people needed engagement photos in the Adirondacks, where there are few cell towers).
Then:

- I had a health scare yesterday (I'm fine)
- The Dean decided that some business had to be taken care of *right now* (I was still on the phone with my chair at 10pm trying to sort it out...)
- This morning I had an *unrelated* illness (I'll be fine)

Meanwhile, a lot of people sent me mail. I will answer that as I am able during discussion hours today if students are not there. (I'll be out by Hendricks)

In case you missed it last time...

Pluto is not a planet.

In case you missed it last time...

Pluto is not a planet.

Pluto is not *not* a planet, either.

In case you missed it last time...

Pluto is not a planet.

Pluto is not *not* a planet, either.

Pluto is a dog. Here he is:

In case you missed it last time...

Pluto is not a planet.

Pluto is not *not* a planet, either.

Pluto is a dog. Here he is:

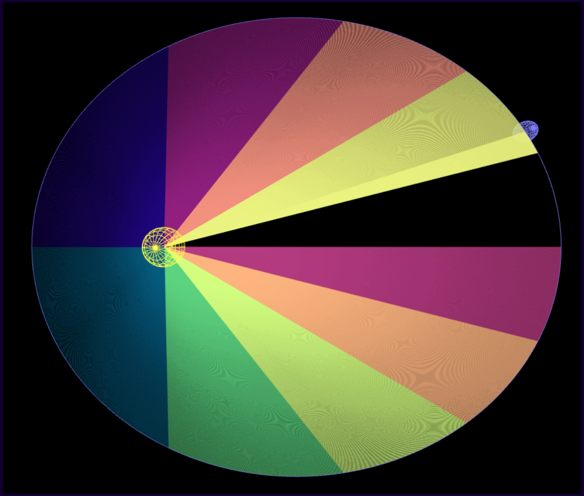
Cyrus Kamkar's dog Pluto, who looks like a very good boy.

Kepler's laws, summarized

- 1. Planets travel in elliptical orbits, with the Sun at one focus
- 2. The line going from the Sun to the planet sweeps out equal areas in equal times
- 3. The time that a planet takes to go around the Sun increases as the 3/2 power of the long axis of the ellipse.

Kepler's second law

The line from the Sun to the planet sweeps out equal areas in equal times.



Each colored wedge has the *same area*, and the planet takes the *same time* to go through each.

Kepler's Third Law

Kepler's third law of orbital motion says that the square of a planet's *orbital period* is proportional to the cube of the long axis of the ellipse of its orbit.

Simply put: if a planet is further from the Sun, it takes longer to go around.

If the distance is doubled, the time required *more than doubles*.

Let's watch this...

Saturn's orbit is about 10 AU across, while Uranus' orbit is about 20 AU across.

Saturn takes about 30 years to orbit the Sun. About how long does Uranus take?

A: About 30 years

B: Between 30 and 60 years

C: More than 60 years

D: It depends on the masses of Uranus and Saturn

Saturn's orbit is about 10 AU across, while Uranus' orbit is about 20 AU across.

Saturn takes about 30 years to orbit the Sun. About how long does Uranus take?

A: About 30 years

B: Between 30 and 60 years

C: More than 60 years

D: It depends on the masses of Uranus and Saturn

E: I looked it up on Wikipedia...

Do you think Kepler's laws can apply to things other than planets? Why or why not?

Asking what vs. asking why

Remember, Kepler only discovered *what* the planets' orbits looked like.

He desperately wanted to know *why* they moved in that way, but he never could figure it out.

It turns out that if we can understand *why*, we can understand some other things, too...

Natural laws vs. their consequences

Obviously the world around us is very diverse. Some things in it look quite simple:

- The motion of the stars
- The near-perfect-spheres of the planets and moons
- The elliptical motions of the planets (?)
- The colors in a rainbow

Natural laws vs. their consequences

Obviously the world around us is very diverse. Some things in it look quite simple:

- The motion of the stars
- The near-perfect-spheres of the planets and moons
- The elliptical motions of the planets (?)
- The colors in a rainbow

Others, though, are maddeningly complex:

- Seismic waves and earthquakes
- The colors in the Sun
- The weather
- The diversity of rocks on Earth

Natural laws vs. their consequences

Obviously the world around us is very diverse. Some things in it look quite simple:

- The motion of the stars
- The near-perfect-spheres of the planets and moons
- The elliptical motions of the planets (?)
- The colors in a rainbow

Others, though, are maddeningly complex:

- Seismic waves and earthquakes
- The colors in the Sun
- The weather
- The diversity of rocks on Earth
- Even the simplest living things

Natural laws vs. their consequences

Obviously the world around us is very diverse. Some things in it look quite simple:

- The motion of the stars
- The near-perfect-spheres of the planets and moons
- The elliptical motions of the planets (?)
- The colors in a rainbow

Others, though, are maddeningly complex:

- Seismic waves and earthquakes
- The colors in the Sun
- The weather
- The diversity of rocks on Earth
- Even the simplest living things
- ... language, culture, music, art, and all the creations of humankind...

Elegance, revisited

The laws of the Universe are simple and elegant.
The things the Universe builds out of them are often complex!

Elegance, revisited

The laws of the Universe are simple and elegant.
The things the Universe builds out of them are often complex!

Natural laws vs. their consequences

We've been doing science for a few hundred years, and we've noticed a pattern.

The Universe seems to operate according to a *very few* basic laws.

- There are *four forces* in nature, two of which are different manifestations of the same thing
- All these forces cause a few types of *elementary particles* to move around
- On a very small scale, this movement is governed by the laws of *quantum mechanics*
- On a bigger scale, QM turns into the much simpler *Newton's laws of motion*
- This movement takes place on the stage of *spacetime*

Natural laws vs. their consequences

We've been doing science for a few hundred years, and we've noticed a pattern.

The Universe seems to operate according to a *very few* basic laws.

- There are *four forces* in nature, two of which are different manifestations of the same thing
- All these forces cause a few types of *elementary particles* to move around
- On a very small scale, this movement is governed by the laws of *quantum mechanics*
- On a bigger scale, QM turns into the much simpler *Newton's laws of motion*
- This movement takes place on the stage of *spacetime*

When things are complicated (not simple and elegant) in Nature, it's because they are *complicated combinations* of pieces that are, themselves, simple – pieces that obey simple laws.

Natural laws vs. their consequences

We've been doing science for a few hundred years, and we've noticed a pattern.

The Universe seems to operate according to a *very few* basic laws.

- There are *four forces* in nature, two of which are different manifestations of the same thing
- All these forces cause a few types of *elementary particles* to move around
- On a very small scale, this movement is governed by the laws of *quantum mechanics*
- On a bigger scale, QM turns into the much simpler *Newton's laws of motion*
- This movement takes place on the stage of *spacetime*

When things are complicated (not simple and elegant) in Nature, it's because they are *complicated combinations* of pieces that are, themselves, simple – pieces that obey simple laws.

... even us!

Isaac Newton

Isaac Newton (1642-1727 or 1726) finally figured out the laws that eluded Kepler.

He discovered...

- Forces cause objects to change their speed or direction of motion
- Calculus – the mathematics of changes
- Gravity is such a force
- The mathematical description of gravity

Isaac Newton

Isaac Newton (1642-1727 or 1726) finally figured out the laws that eluded Kepler.

He discovered...

- Forces cause objects to change their speed or direction of motion
- Calculus – the mathematics of changes
- Gravity is such a force
- The mathematical description of gravity
- Principles of optics
- The mathematics of cooling
- ... and much more

"Forces cause objects to accelerate"

$$F = ma \text{ (how you learned it in school)}$$

$$F/m = a \text{ (the actual useful form)}$$

"The strength of a force, divided by the mass of the thing it acts on, gives that thing's acceleration"

The law of gravity

Newton showed mathematically what Kepler suspected: that "there is a force in the Earth that causes the Moon to move".

That thing, of course, is gravity.

Newton discovered:

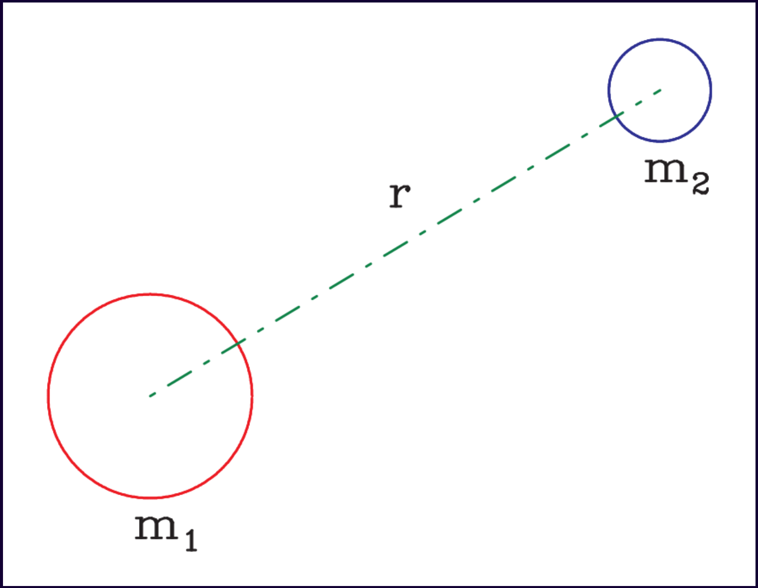
All objects attract all other objects with a force that is:

- Proportional to the product of their masses
- Inversely proportional to the distance between their centers, squared

In symbols:

$$F = \frac{Gm_1m_2}{r^2}$$

(Physicists like to use r for the distance between any two objects)



The gravitational force between these two objects is

$$F_g = \frac{G m_1 m_2}{r^2}.$$

The combination of these ideas is powerful!

Put these together, and you can build a universe!

- The law of gravity tells us the forces that celestial objects exert on one another
- Newton's laws of motion tell us how these forces make things move

Our plan:

- Today we will explore gravity in depth
- Thursday we will explore Newton's laws of motion in depth

With these ideas together, you can understand how *any* collection of objects moves in response to gravity.

We'll do two things:

- We'll look at some simple cases and understand what is happening with just pencils and chalk
- We'll ask a computer to do the mathematics for us for more complex cases

The constant G

So what is this value G?

It is just a value "built in" to the universe that tells us how strong gravity is.

$$G = 7 \times 10^{-11}\,\mathrm{N} \cdot \mathrm{m}^2/\mathrm{kg}^2$$

This means that the gravitational force between two kilogram objects one meter apart is equal to 70 trillionths of a newton. (A newton is the SI unit of force – about the weight of an apple on Earth's surface.)

Do not memorize this number. Many physicists don't have it memorized! You should just know that it is very small.

Instead, it's more important to know how the gravitational force *changes* when the masses of the objects or the distance between them changes.

Suppose two asteroids are floating out in space, 20 km apart. Asteroid A is twice as massive as asteroid B, and the force of A's gravity on B is ten tons.

Suppose I now move the two asteroids closer, so they're only 10 km apart. What will the force of A's gravity on B be now?

A: 5 tons

B: 10 tons

C: 20 tons

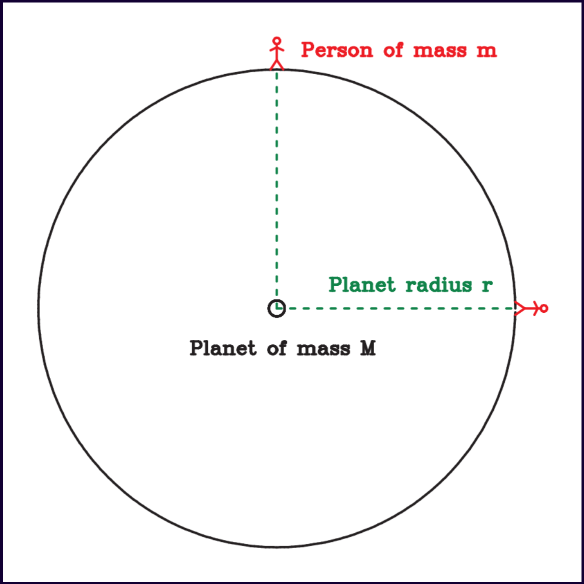
D: 40 tons

Remember:

$$F = \frac{G m_A m_B}{r^2}$$

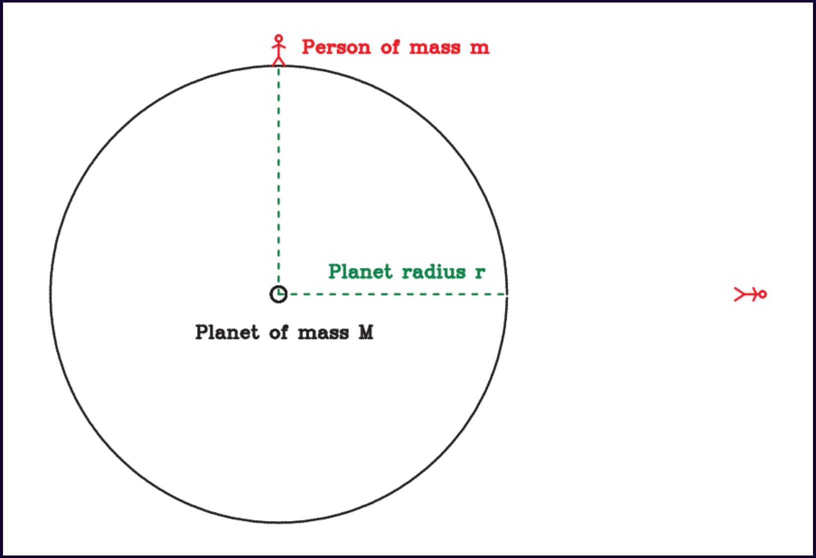
What happens to that formula if I make r only half as large?

The distance is measured between the *centers* of the objects.



This lets you calculate the force of a planet's gravity on people on its surface!

Earth has a radius of about 6,000 km. If we move this person 6,000 km away from Earth's surface, how does the strength of Earth's gravity change?



A: It stays the same

B: It becomes twice as strong

C: It becomes half as strong

D: There's no gravity in space, so it goes away totally

E: It becomes some other value (tell me what and why!)

Suppose two asteroids are floating out in space. Asteroid A is twice as massive as asteroid B.

If the force of A's gravity on B is ten tons, the force of B's gravity on A will be...

A: 5 tons

B: 10 tons

C: 20 tons

D: 40 tons

The law of gravity

All objects attract all other objects with a force that is:

- Proportional to the product of their masses
- Inversely proportional to the distance between them squared

In symbols:

$$F = \frac{Gm_1m_2}{r^2}$$

Notice I didn't say which mass was which. It doesn't matter!

This is an example of Newton's third law of motion: if object A pulls on object B, object B pulls back on object A with the same force.

This means:

- that my body's gravity pulls up on Earth with a force of about 750 newtons

The law of gravity

All objects attract all other objects with a force that is:

- Proportional to the product of their masses
- Inversely proportional to the distance between them squared

In symbols:

$$F = \frac{G m_1 m_2}{r^2}$$

Notice I didn't say which mass was which. It doesn't matter!

This is an example of Newton's third law of motion: if object A pulls on object B, object B pulls back on object A with the same force.

This means:

- that my body's gravity pulls up on Earth with a force of about 750 newtons
- that the Moon's gravity pulls on Earth just as strong as the Earth's gravity pulls on the Moon

The law of gravity

All objects attract all other objects with a force that is:

- Proportional to the product of their masses
- Inversely proportional to the distance between them squared

In symbols:

$$F = \frac{Gm_1m_2}{r^2}$$

Notice I didn't say which mass was which. It doesn't matter!

This is an example of Newton's third law of motion: if object A pulls on object B, object B pulls back on object A with the same force.

This means:

- that my body's gravity pulls up on Earth with a force of about 750 newtons
- that the Moon's gravity pulls on Earth just as strong as the Earth's gravity pulls on the Moon
- that the planets pull on the Sun just as hard as the Sun pulls on the planets

What do you think about this?

We will explore the rest of Newton's laws next time!