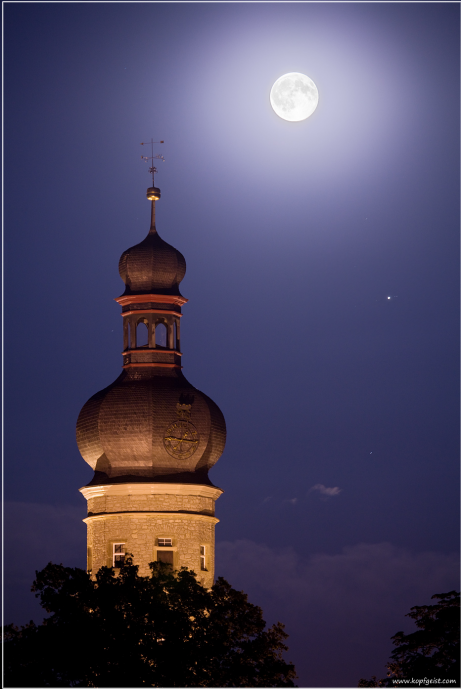


# Kepler's laws

Astronomy 101  
Syracuse University, Fall 2020  
Walter Freeman

October 5, 2021



*“And yet it moves.”*

—Galileo (attributed), on the Earth

# Announcements: labs

You may make up one lab that you missed for any reason by going to your scheduled lab section this week.

If you have a Monday lab, instead go to *any* lab section.

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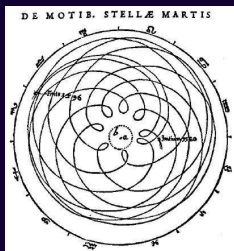
If you have a Monday lab, instead go to *any* lab section.

Starting next week, the “lab week” will start on Monday rather than Tuesday.

Lab 5’s prelab is posted now and is somewhat more involved than past ones. We will have copies available for you late today.

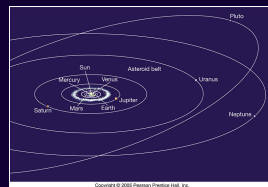
We left our story with two plausible models for the heavens:

## The geocentric Ptolemaic model



- The planets (and everything else) revolve around Earth
- Inelegant system of “epicycles” needed to get planets right
- Everything moved in circles (elegant per Greeks)
- Earth and humanity at center (theologically not challenging)
- **Very accurate predictions**

## The heliocentric Copernican model



- Earth is one of many planets, all orbiting the Sun
- Apparent motion = motion of Earth + motion of planets
- No (or very small) epicycles
- **Less accurate than Ptolemaic model**
- Matched Galileo’s observations:
  - Moons of Jupiter
  - Phases of Venus

# Galileo's observations

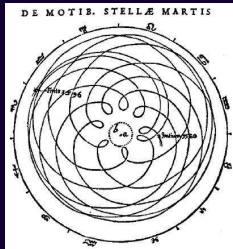
Galileo believed that the best way to get information is to *go measure stuff*.  
He saw three things through a telescope that were all extraordinary:

- **The moons of Jupiter:** This very clearly shows that not everything orbits the Earth!
- **Shadows on the Moon:** By seeing how the shadows on the Moon's surface changed as the phase of the Moon changed, you can show that not everything in space is a perfect sphere. (The Moon has craters and mountains just like Earth.)
- **Phases of Venus:** Galileo observed that Venus, just like the Moon, has phases – and the pattern of phases we observe from Earth *requires* a heliocentric solar system

# What to do?

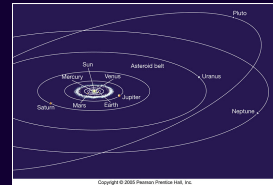
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- ... not compatible with the new observations through the telescope

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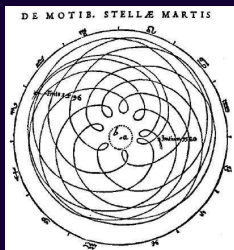


- Explains retrograde motion in a much simpler way
- **Less accurate than Ptolemaic model**
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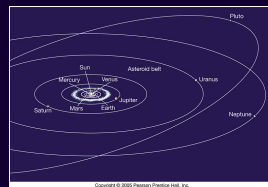
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# What do we do when we don't know what to do?

Maybe our data are wrong...

The measurements of the sky that people had been using were “good enough” for navigation, but they weren't ever intended for precision natural philosophy: determining the truth of things...

(In astronomy sometimes it is okay to round things off, and sometimes you need precise measurements to figure things out: you have to think carefully about this!)

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Enter Tycho Brahe.



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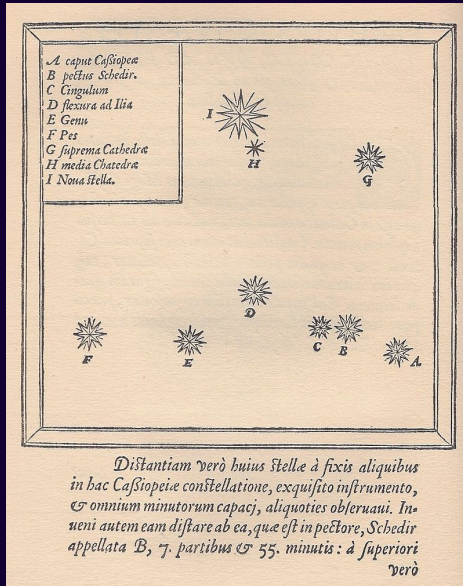
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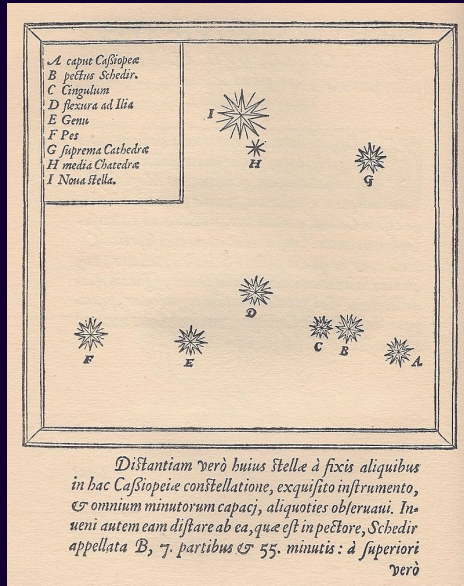
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- Had a pet moose
- It drank too much beer and died
- Was probably fun at parties (less so post-moose-death)

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- Didn't observe parallax in the distant stars
- Two options:
  - The Earth doesn't move
  - The stars are very far away
- He believed the former
- Proposed another model for the Solar System



# Tycho Brahe



- Danish nobleman and astronomer, 1546-1601
- Best known for his precision measurements of the sky from Uraniborg
- Made high precision observations of the motions of the planets and stars
- Even had a crude correction for atmosphere bending light
- Measurements accurate to a few minutes of arc ( $1/60$ 'ths of a degree!)
- Made these measurements with his assistant Sophie...
- ... and his later assistant Johannes Kepler, who didn't murder him



You've probably been wondering when we're going to stop this history of false starts and learn how things actually *do* work...

# Johannes Kepler

You've probably been wondering when we're going to stop this history of false starts and learn how things actually *do* work...

... here we go. Kepler, Tycho's assistant, finally got it right.



Kepler was a Copernican, and disagreed with his boss.

He tried to improve Copernicus' model, which used circular orbits, and mostly succeeded. But...

- Tycho's data were incredibly precise
- No matter how he rearranged the circles, there was an error of at least  $8/60$  of a degree for Mars
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Should we:

- ... reject the belief that Nature must be elegant
- ... reject the need for our model to match the data precisely
- ... reject the observations made at Uraniborg as?
- ... reject our ideas about what elegance looks like

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He didn't only want to discover *how the planets moved*; he wanted to know *why*. He didn't figure it out, but he was on the path that led to modern science.

Even if the *answer* doesn't have the perfect elegance of circles, modern science looks for its elegance in *laws*, not in all of their consequences! Kepler discovered the consequences; the laws weren't uncovered yet.

# Kepler's laws of planetary motion

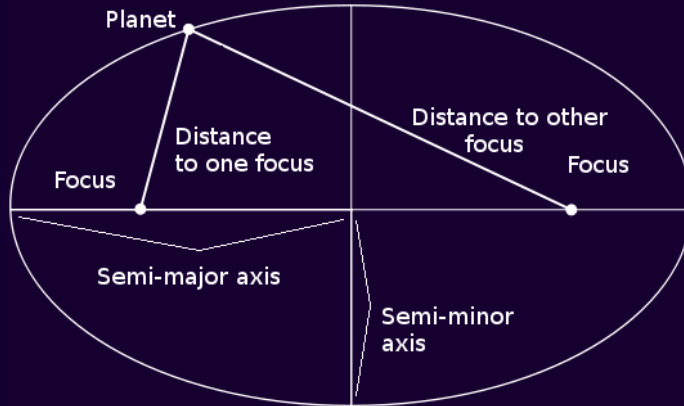
- The planets move in *ellipses*, with the Sun at one focus
- The line joining the planet and the Sun sweeps out equal areas in equal times  
Alternate formulation: Within its orbit, a planet's speed is inversely proportional to its distance from the Sun
- The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of the ellipse.

Let's talk about each of these in turn.



# Kepler's first law

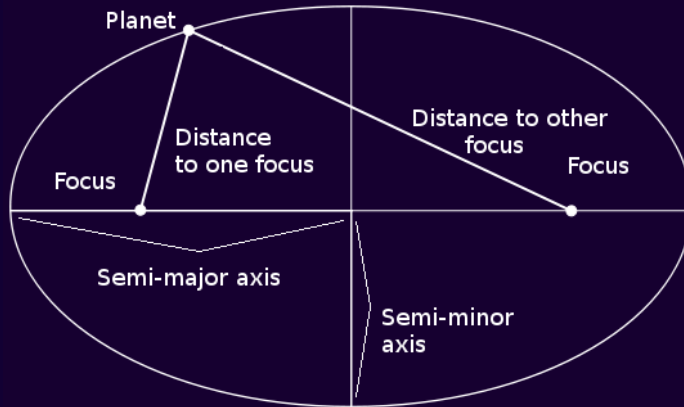
An ellipse is just a stretched circle. Mathematically: it's the curve around two points such that the *sum* of the distances to those points is a constant. A circle is just an ellipse with both foci at the same point.



Some terms:

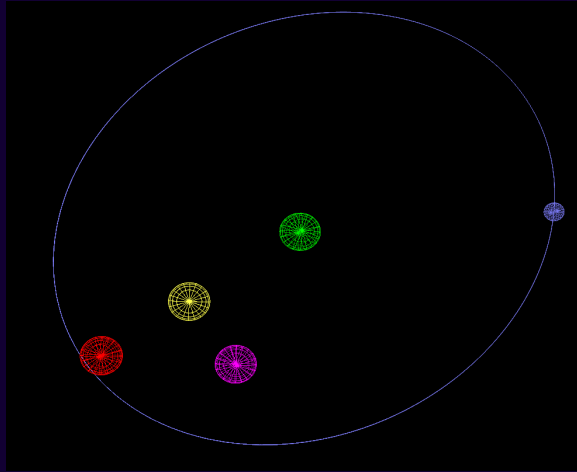
- Focus: One of the two points
- Semimajor axis: the largest distance from the center to the edge
- Eccentricity: how stretched out an ellipse is

# Some properties of ellipses



- The two foci always lie along the major axis (“wide axis”)
- The closer together the foci, the less eccentric
- If both foci are exactly at the middle, you get a circle
- Both foci lie inside the ellipse

Here's an orbit. Which is the correct position for the Sun, and how do you know?



A: The red one

B: The green one

C: The yellow one

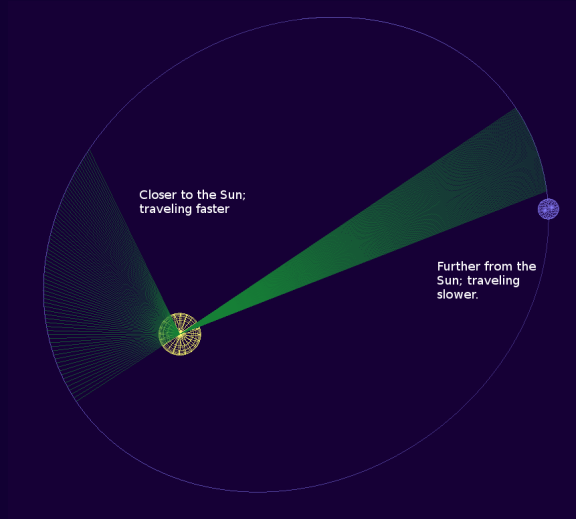
D: The purple one

# What you need to know

- Planetary orbits are ellipses
- The *eccentricity* of an ellipse tells you how squashed it is
- An ellipse with zero eccentricity is just a circle
- The Sun lies at a focus of the ellipse, which isn't at the center (unless it's a circle)
- The more eccentric the orbit, the further to one side the Sun is

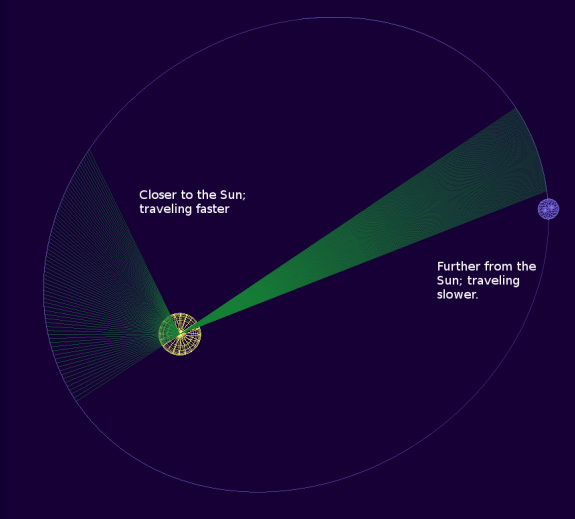
# Kepler's second law

In an eccentric orbit, a planet travels fastest when it's nearer the Sun.



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Let's watch this in an animation...

# Comets

Comets have highly eccentric orbits. Halley's Comet's furthest point from the Sun – its *aphelion* – is 35 AU away. But its *perihelion* – the nearest point to the Sun – is 0.6 AU away.

Which statement is true, and how do you know?

A: Halley's Comet spends most of its time far from the Sun, and only a little time near the Sun

B: Halley's Comet moves slowly near perihelion, and quickly at aphelion

C: Halley's Comet moves quickly near perihelion, and slowly at aphelion

D: Halley's Comet spends roughly equal amounts of time near the Sun, and far from it

# Kepler's Third Law

Kepler's third law of orbital motion says that the square of a planet's *orbital period* is proportional to the cube of its *semimajor axis*.

Simply put: if a planet is further from the Sun, it takes longer to go around.

If the distance is doubled, the time required *more than doubles*.

Let's watch this...



How do you think I made all these simulations?