Lab 10: How hot are the planets?

Name:

Group members:

Lab section number:

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1 Introduction

Foreword

In this lab, you and your group will use the physics of thermal radiation to estimate the temperatures of various planets in the Solar System. We will return to the results of this lab multiple times in class, so it's important that you understand the takeaway message from this lab when you leave today.

You will be working with mathematics – algebra and numbers. While we know that your mathematics backgrounds vary, the only mathematics you will need for this lab are basic algebra, a little arithmetic, and a bit of geometry. This lab involves no computer simulations, lab equipment, or materials – it is a calculation that you will do using pencil and paper (and, likely, a calculator).

Do not be intimidated by the fact that "you are doing math"; this lab handout walks you through the calculation, and you have your lab group and your TA to help you.

Background: the Stefan-Boltzmann law and temperature

As you learned in class, objects with a temperature emit light called *thermal radiation*. We discussed that, as an object's temperature increases...

- The peak wavelength of the light emitted gets shorter
- The total amount of light emitted increases rapidly

In this lab, we care only about this second property. For this lab, we will need a mathematical expression of this idea, called the *Stefan-Boltzmann law*.

This law says:

If an object has a temperature T, that object's surface shines with an intensity of $I = kT^4$.

Here k is a mathematical constant; you do not need its value and can leave it in your algebraic expressions, as it will wind up cancelling out in the end.

It is important here that the temperature T be measured in the Kelvin scale. You may convert temperatures in Kelvin to Fahrenheit or Celsius as follows:

- T (Celsius) = T (Kelvin) - 273
- T (Fahrenheit) = T (Celsius) $\times$ 1.8 + 32

Background: thermal equilibrium

The Earth is very nearly in a state of *thermal equilibrium*. The Sun is a very hot object, so it emits a great deal of thermal radiation – sunlight. This sunlight falls on Earth's surface, which absorbs it. If this were the only thing happening, then Earth would gain more and more energy, continually heating up. However, since Earth also has a temperature, it emits thermal radiation as well, allowing it to cool off.

Just like we call the Sun's thermal radiation "sunlight", I will call the Earth's thermal radiation "planetlight". (Sunlight is mostly visible light and planetlight is infrared, but this difference doesn't matter for the calculation.)

If Earth's average temperature is going to stay relatively constant, then we can write the following:

(amount of sunlight absorbed by Earth) = (amount of thermal radiation emitted by Earth)

By writing formulas for both of these things and setting them equal, we can get a crude estimate of the Earth's temperature. (This is similar to what you did your prelab, when you thought about a tea kettle.)

The approach

1. Write a formula for the intensity of sunlight at the surface of the Sun
2. Based on that, write a formula for the intensity of sunlight at the surface of the Earth
3. Write a formula for the intensity of the thermal radiation emitted by Earth
4. Set (2) and (3) equal and determine the temperature of the Earth.
5. Modify this last condition to make it more accurate.

A very important note on algebra

When doing these calculations, it can be tempting to substitute numbers that you know right away. For instance, the formula for the intensity of sunlight at the Sun's surface is simply the Stefan-Boltzmann law, $I = kT_s^4$. Since you know the temperature of the Sun is 5778 K, it can be tempting to say "Oh! I know what T is, let me plug that in right away!"

What happens if you do that? Well, you'll go to your calculator and find that $5700^4 = 1.0556001 \times 10^{15}$. You will then have to carry this number around for the rest of your calculation, *and* if you want to change the temperature of the star, you'll have to go back and redo everything.

Do not do this. In all these calculations, you should substitute in numbers only as the **very last step**. This will make your life much, much easier. Any solution that includes numbers plugged in before the final calculation of the planet's temperature will be marked as incorrect by the graders.

Yes, this means you have to do algebra. Trust me – it's much, much easier this way.

Algebraic symbols

Please use the following symbols for quantities:

- T_s – the temperature of the Sun
- r – the radius of the Sun
- d – the distance from the Sun to the planet in question (we'll apply this formula to both Earth and other planets)
- T_p – the temperature of the planet
- I_1 – the intensity of sunlight at the surface of the Sun
- I_2 – the intensity of sunlight at falling on to the planet
- I_3 – the intensity of planetlight going out from the planet
- k – the constant in the Stefan-Boltzmann law (it will cancel in the end)

Intensity of sunlight at the Sun's surface

Question 1: Write a formula for I_1 in terms of k and T_s. (This is a straightforward expression of the Stefan-Boltzmann law, since this is what it tells you: the intensity of thermal radiation given off by a hot object.)

Intensity of sunlight at the Earth's surface

There are two approaches to figuring this out. *You and your group only need to do one of them.* The first approach is more appropriate for students who remember their high school geometry class well – in particular, the formula for the surface area of a sphere.

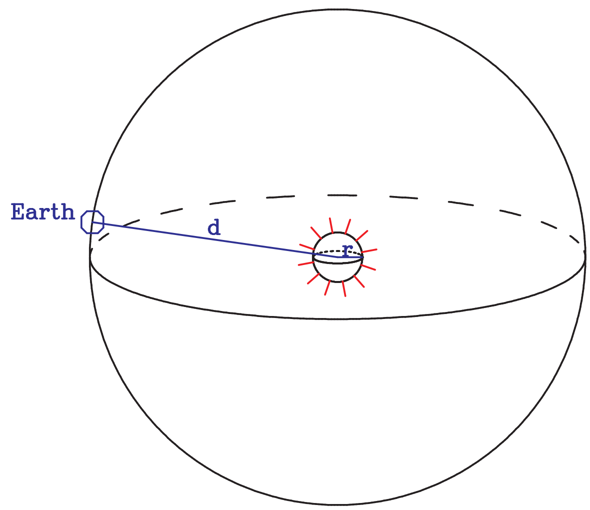
The second approach involves more reasoning but fewer formulas; it is more appropriate for groups who don't remember high school geometry as well and want to figure things out from scratch.

Method 1

We've talked so far about *intensity* – the amount of power per unit area. If we multiply intensity by area, we can calculate the total power of the light passing through a certain area. In symbols, we know that $P = IA$.

Recall from your high school geometry class that the surface area of a sphere is $4\pi R^2$, where R is the radius of the sphere.

Imagine two spheres. One lies just outside the surface of the Sun, and has radius r; the second lies has its center is at the Sun and which is 1 AU in radius, so that the Earth (or other planet) is on its edge. (Its radius is d.)



Call the surface area of the small sphere A_1 and the power of the sunlight passing through it P_1. Using the intensity/power relation above, this tells us that $P_1 = I_1 A_1$. Likewise, call the surface area of the large sphere A_2, the power of the sunlight passing through it P_2, and the intensity of the light there I_2, and $P_2 = I_2 A_2$.

The key insight here is that the light from the Sun spreads out, but doesn't "vanish" as it travels through space. Thus, the total power passing through both spheres is the same, and $I_1 A_1 = I_2 A_2$.

Question 2a: Write expressions for A_1 and A_2 in terms of r and d.

Question 2b: Substitute the preceding expressions into $I_1 A_1 = I_2 A_2$ to arrive at an equation relating d, r, I_2, and I_1.

Question 2c: Solve the above for I_2 to get an expression for I_2, the intensity of sunlight at the Earth, in terms of d, r, and I_1. Hint: Consider whether d or r is larger. We expect I_2 to be much less than I_1; does your formula have this property? Call your TA over if you have questions; this is actually the trickiest part of this whole calculation.

When you are done with this section, ask your TA to check your results so far.

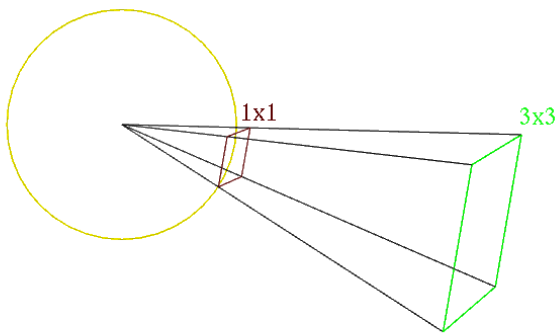
Method 2:

If you already figured out the relationship between I_2 and I_3 using Method 1 above, do not do this section, and skip ahead.

Sunlight spreads out as it travels from the Sun to the Earth. Thus, sunlight is less intense here than it is at the Sun's surface (obviously). How much does it spread out?

We can figure this out by realizing that all of the sunlight that leaves the Sun travels outward into space, equally, in all directions.

Consider the following picture (color available online on the course website if this is hard to read in print):



The larger green square is located three times further from the center than the smaller red one (which lies on the surface of the Sun). Since it is three times further away, it is also three times taller and three times wider, as shown in the picture.

Question 3a: How many times larger is the **area** of the larger square than the smaller one?

Now, imagine that the small square is on the surface of the Sun. The sunlight leaving the Sun, passing through the smaller, red square, will spread out until it reaches a distance three times the radius of the Sun (at the larger, green square.) If the intensity of the sunlight leaving the Sun is I_1, what do you expect the intensity of the sunlight to be once it reaches the outer square? (Hint: The sunlight is weaker because the same amount of light is spread over more area. How much more area?)

This last exercise involved looking at some example numbers; now, let's go back to our algebra. Here, instead of the "large square" (located at Earth) being three times further from the center than the "small square" (on the surface of the Sun), it is now d/r times further away.

Question 3b: In terms of d and r, by what factor has the sunlight spread out by the time it reaches Earth?

Question 3c: In terms of d, r, and I_1 (the intensity of sunlight at the Sun's surface), write a formula for I_2 (the intensity of sunlight at Earth). Hint: Consider whether d or r is larger. We expect I_2 to be much less than I_1; does your formula have this property? Call your TA over if you have questions; this is actually the trickiest part of this whole calculation.

When you are done with this section, ask your TA to check your results so far.

Question 4: Have you encountered anything else in this class that gets weaker proportional to the square of the distance from the Sun? Based on the geometry you did above, can you think of why this might be the case?

Question 5: One last thing: substitute the expression you found in Question 1 into the one you got for Question 2c or 3c. This should give you an expression for the intensity of sunlight on the Earth, I_2, in terms of d, r, k, and T_s.

Once you are done, ask your TA to come check your result; they should sign below.

The thermal radiation of Earth: planetlight

Question 6: If the Earth has a temperature T_p, what is the intensity of its outgoing planetlight I_3 in terms of k and T_p? (This is again a straightforward application of the Stefan-Boltzmann law.)

The temperature of Earth

As discussed earlier, for Earth to be in balance, the same amount of light must flow in (from sunlight) and out (from Earth's planetlight). Naïvely, this would imply that $I_2 = I_3$.

Question 7: Equate the formulae you wrote for I_2 (in Question 5) and I_3 (in Question 6). Algebraically solve this equation for T_p, the temperature of the Earth. Ask your TA to check your result once you get it.

Now, finally, it's time to plug in numbers. Use the following:

- T_s: 5700 K
- r: 0.0046 AU
- d: 1 AU

Question 8: What estimate of the temperature of the Earth does this give you? (You will, as you might expect, need a calculator.)

Question 9: Convert this value to Celsius or Fahrenheit. Is this a reasonably accurate value of the temperature of the Earth? Show your work to your TA; they will check to make sure you're on the right track.

What has gone wrong here? We made an assumption that $I_3 = I_2$ – that is, that the *intensity of planetlight produced by Earth* must equal the *intensity of sunlight absorbed by Earth.* This was a good first guess, and – in a universe where things range from 3 Kelvin to millions of Kelvin – it's not too bad. But we can do better!

The issue is that, really, we shouldn't be equating *intensities*; we should be thinking about the total *amount* of radiation. There are two ways we can improve our calculation:

- Not all of the Earth is facing the Sun! The night half of Earth is still giving off planetlight; this implies that I_3 needs to be only half as big, since only half of Earth receives sunlight, but the entire Earth emits planetlight.
- Remember how sunlight striking Earth at an angle doesn't heat it up as much, partially causing winter? This also reduces the amount of sunlight striking Earth's surface – by another factor of two. (The proof that this is exactly a factor of two requires calculus.)

So, instead of writing $I_3 = I_2$, we can do better by writing $I_3 = \frac{1}{4} I_2$, since the Earth's shape means that the "area that absorbs sunlight" is only one-quarter the area that emits planetlight

Question 10: Set $I_3 = \frac{1}{4} I_2$ and solve this new equation for T_p. You should get an expression very similar to the one you got for Question 7, but with a factor of 1/4.

Question 11: Repeat your calculation for Earth's temperature with this new expression. Is your answer closer to the Earth's average surface temperature now? Is it too high, or too low?

The temperatures of other planets

Now, each of you in your lab group will individually repeat this calculation for a different planet. Use the following (minor) planets:

- Venus, d=0.723 AU
- Pluto, d=39.4 AU
- Mars, d=1.524 AU

This should only require you to plug different values into the expression you got for Question 10. Each of you should do one planet, and share answers.

1. Temperature of Venus:

2. Temperature of Pluto:

3. Temperature of Mars:

Compare these results to the actual temperatures of these planets' surfaces:

- Earth: 288K / 59F
- Mars: 218K / -67F
- Pluto: 44K / -380F
- Venus: 735K / 864F

Question 12: Venus is hotter than Earth; is this explained by the fact that it is closer to the Sun? Likewise, does the greater distance from the Sun explain the cooler temperatures on Mars?

Wrapping a planet in glass

As you saw in the demonstration with the infrared camera in class, glass (as in a student's eyeglasses) **reflects infrared light**, but **allows visible light** to pass.

Question 13: Given that the Sun emits visible light but planets, with their lower temperatures, emit infrared light, what do you expect to happen to a planet's temperature if you were to enclose it in a glass bubble?

Other chemicals besides glass have this property as well – some of them found in the atmospheres of planets. Earth has a moderately thick atmosphere, and Venus has a tremendously thick atmosphere. Mars has almost no atmosphere at all.

Question 14: Note that your calculations on the previous page were off by 15 K for Earth, 3 K for Mars, and 400 K for Venus. What is going on here?

Other stars, other planets, and exterrestrial life

One hopeful sign that there might be life on another planet would be for that planet to have a similar temperature to Earth. The star Sirius, the brightest in the night sky, has the following properties:

- Radius: 0.008 AU (somewhat bigger than the Sun)
- Surface temperature: 10,000 K

Question 15: Take the formula you got in Question 10 and solve it instead for d. If a planet orbiting Sirius were to have the same temperature as Earth (about 288 K), how far from Sirius would its orbit be?