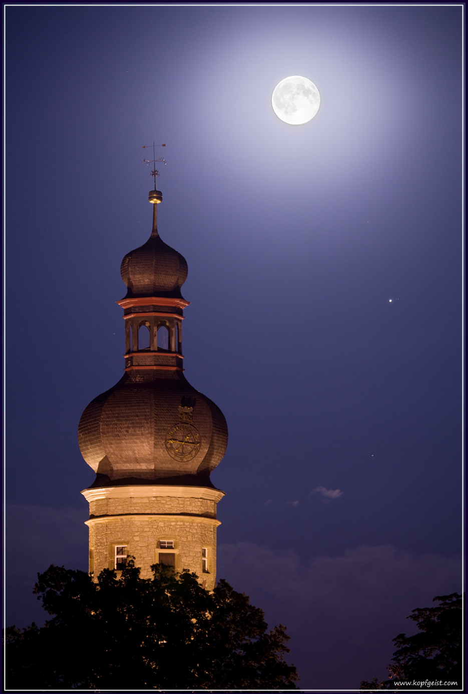
Kepler's laws

Astronomy 101
Syracuse University, Fall 2022
Walter Freeman

September 29, 2022



"And yet it moves."

–Galileo (attributed), on the Earth

Announcements

- We are still grading your exams.
  - We'll have a grade distribution and statistics for you Tuesday
  - In general it looks like people did reasonably well
  - Some folks did better than others
  - If you didn't do as well as you hoped, there is plenty of semester left
- See your email for details on quiz makeups/retakes, and this week's survey
- Prelabs won't be available until tomorrow – sorry about that!

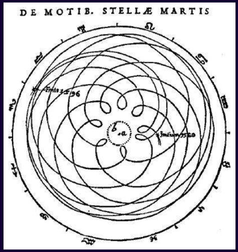
Announcements: take-home lab

The take-home lab is posted. Let's go take a look...

Last time

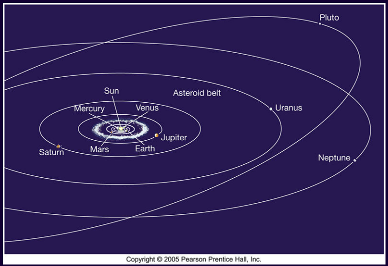
We left our story with two plausible models for the heavens:

The geocentric Ptolemaic model



- The planets (and everything else) revolve around Earth
- Inelegant system of "epicycles" needed to get planets right
- Everything moved in circles (elegant per Greeks)
- Earth and humanity at center (and thus "acceptable" to the Church)
- Very accurate predictions

The heliocentric Copernican model



- Earth is one of many planets, all orbiting the Sun
- Apparent motion = motion of Earth + motion of planets
- No (or very small) epicycles
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He pointed his telescope at Jupiter, and saw this:
`https://www.youtube.com/watch?v=XpsQimYhNkA` (video from NASA)

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There are things orbiting Jupiter! These are the four largest moons of Jupiter, called the "Galilean moons" after their discoverer.

- If things orbit Jupiter, then not everything orbits the Earth! We are not the center of everything!
- This was a huge shakeup – to philosophy, and to *religion!*

Liverpool, NY (just north of Syracuse); 10:22 PM, June 18, 2019

500mm f/5.6 lens with 1.4x teleconverter; ISO 500, 1/250, f/8 (moons brightened significantly in post)

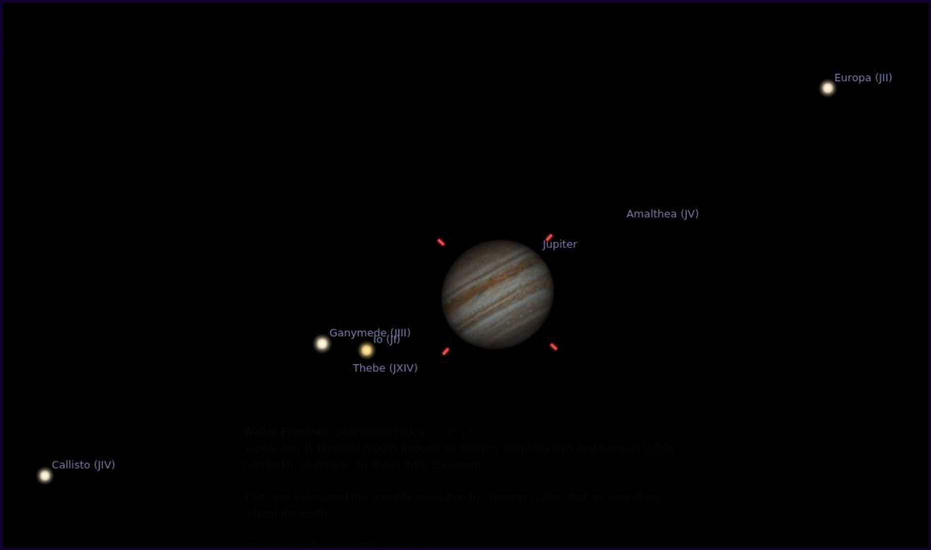


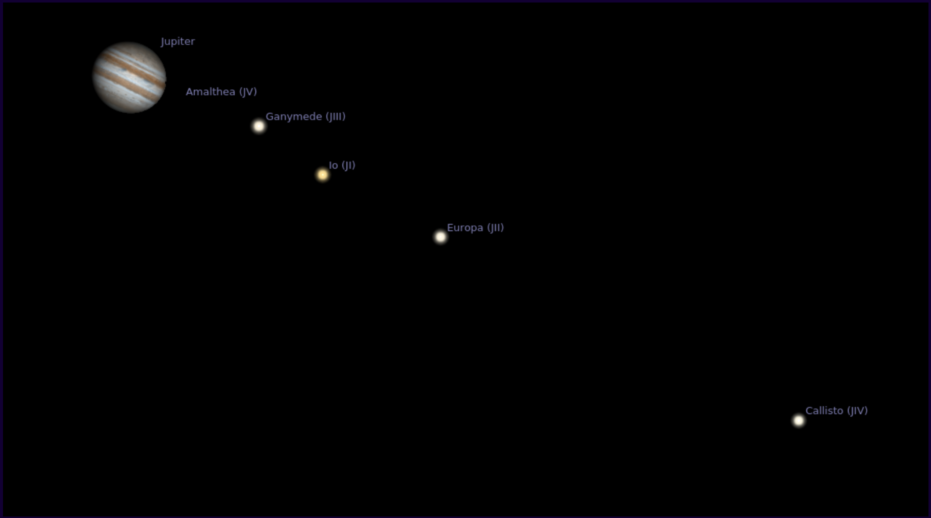
Image capture from Stellarium

Liverpool, NY; 10:22 PM, June 18, 2019

Labrador Hollow, NY (20 miles SSE of Syracuse); 9:54 PM, September 19, 2020

500mm f/5.6 lens with 1.4x teleconverter; ISO 800, 1/100, f/8 (moons brightened in post)

Image shot over water; I suspect the refraction from the turbulent air may have caused a lot of blur ("twinkling")



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Labrador Hollow, 9:54 PM, September 19, 2020

The moons of Jupiter, imaged by ComradeWilheim (AST101 student) – smartphone + telescope

Galileo's observations

- **The moons of Jupiter**: This very clearly shows that not everything orbits the Earth!
- **Shadows on the Moon**: By seeing how the shadows on the Moon's surface changed as the phase of the Moon changed, you can show that not everything in space is a perfect sphere. (The Moon has craters and mountains just like Earth.)
- **Phases of Venus**: Galileo observed that Venus, just like the Moon, has phases – and the pattern of phases we observe from Earth *requires* a heliocentric solar system

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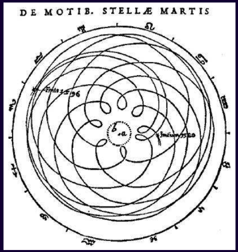

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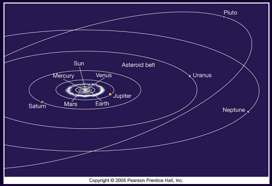

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- Matched Galileo's observations:
  - Moons of Jupiter
  - Phases of Venus

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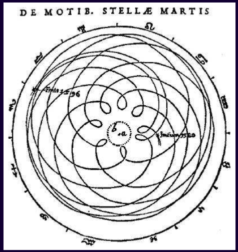

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- Very complicated
- ... not compatible with the new observations through the telescope

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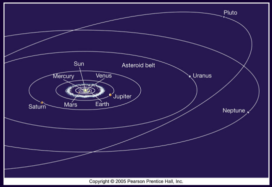

- Explains retrograde motion in a much simpler way
- Less accurate than Ptolemaic model
- Reproduces the observations through the telescope:
  - Moons of Jupiter
  - Phases of Venus

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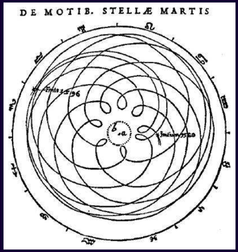

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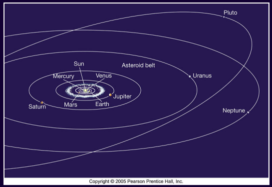

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What do we do?

The Copernican model had a lot of attractive features, but was still less accurate – less good at actually telling you where the things in the sky would be!

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- What do we do?

What do we do when we don't know what to do?

Maybe our data are wrong...

The measurements of the sky that people had been using were "good enough" for navigation, but they weren't ever intended for precision natural philosophy: determining the truth of things...

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Enter Tycho Brahe.

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Small bull moose in Cape Breton, Nova Scotia, Canada.

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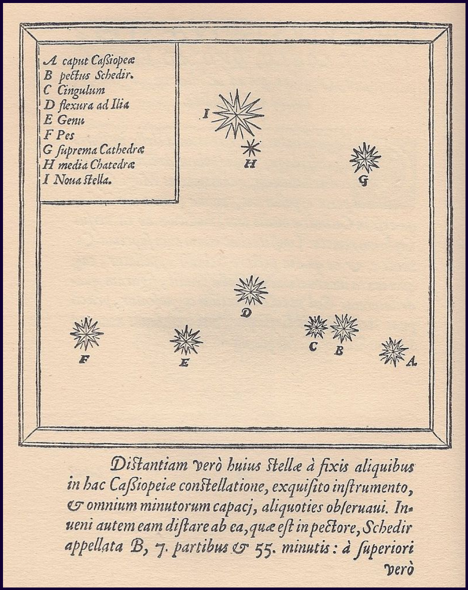
- Lost a big chunk of his nose in a duel and got a brass replacement made
- Had better facial hair than any of us
- Had a pet moose
- It drank too much beer and died
- Was probably fun at parties (less so post-moose-death)



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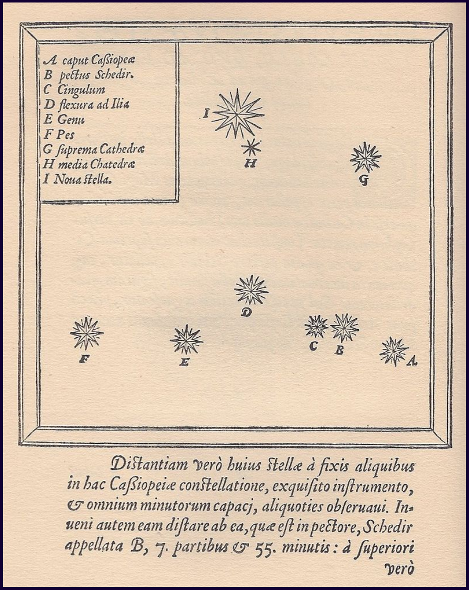

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- Didn't observe parallax in the distant stars
- Two options:
  - The Earth doesn't move
  - The stars are very far away
- He believed the former
- Proposed another model for the Solar System

Tycho Brahe

- Danish nobleman and astronomer, 1546-1601
- Best known for his precision measurements of the sky from Uraniborg
- Made high precision observations of the motions of the planets and stars
- Even had a crude correction for atmosphere bending light
- Measurements accurate to a few minutes of arc (1/60'ths of a degree!)
- Made these measurements with his sister Sophie...
- ... and his later assistant Johannes Kepler, who didn't murder him

Johannes Kepler

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... here we go. Kepler, Tycho's assistant, finally got it right.

Kepler was a Copernican, and disagreed with his boss.

He tried to improve Copernicus' model, which used circular orbits, and mostly succeeded. But...

- Tycho's data were incredibly precise
- No matter how he rearranged the circles, there was an error of at least 8/60 of a degree for Mars
- Kepler worked at Uraniborg – he knew how precise the data could be

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Should we:

- ... reject the belief that Nature must be elegant
- ... reject the need for our model to match the data precisely
- ... reject the observations made at Uraniborg as inconsistent with our model?
- ... reject our ideas about what elegance looks like

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He didn't only want to discover *how the planets moved*; he wanted to know *why*. He didn't figure it out, but he was on the path that led to modern science.

Even if the *answer* doesn't have the perfect elegance of circles, modern science looks for its elegance in *laws*, not in all of their consequences! Kepler discovered the consequences; the laws weren't uncovered yet.

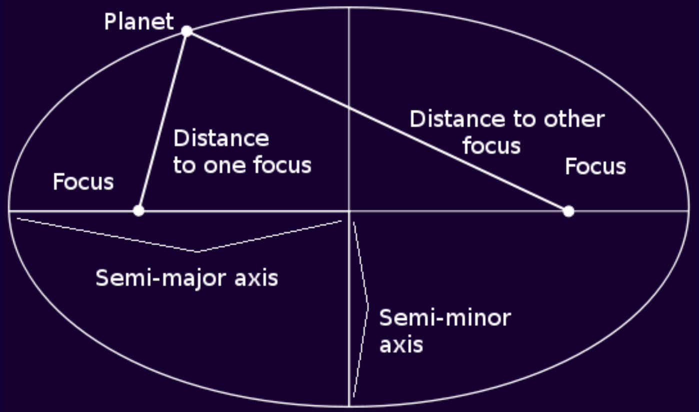
Kepler's laws of planetary motion

1. The planets move in *ellipses*, with the Sun at one focus
2. The line joining the planet and the Sun sweeps out equal areas in equal times
   Alternate formulation: Within its orbit, a planet's speed is inversely proportional to its distance from the Sun
3. The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of the ellipse.

Let's talk about each of these in turn.

Kepler's first law

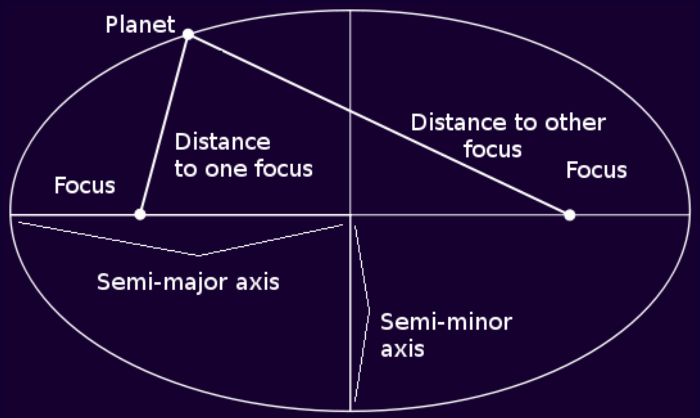
An ellipse is just a stretched circle. Mathematically: it's the curve around two points such that the *sum* of the distances to those points is a constant. A circle is just an ellipse with both foci at the same point.



Some terms:

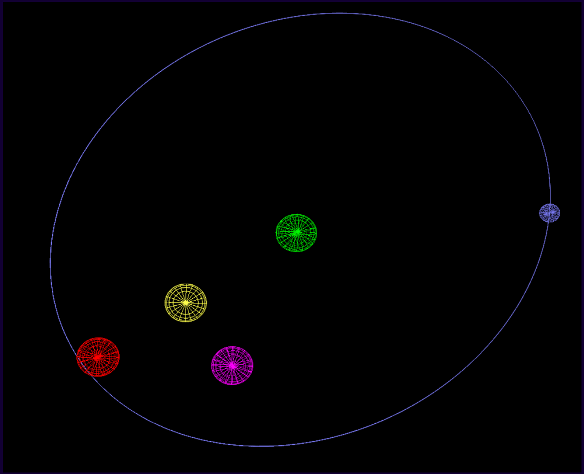
- Focus: One of the two points
- Semimajor axis: the largest distance from the center to the edge
- Eccentricity: how stretched out an ellipse is

Some properties of ellipses



- The two foci always lie along the major axis (“wide axis”)
- The closer together the foci, the less eccentric
- If both foci are exactly at the middle, you get a circle
- Both foci lie inside the ellipse

Here's an orbit. Which is the correct position for the Sun, and how do you know?



A: The red one

B: The green one

C: The yellow one

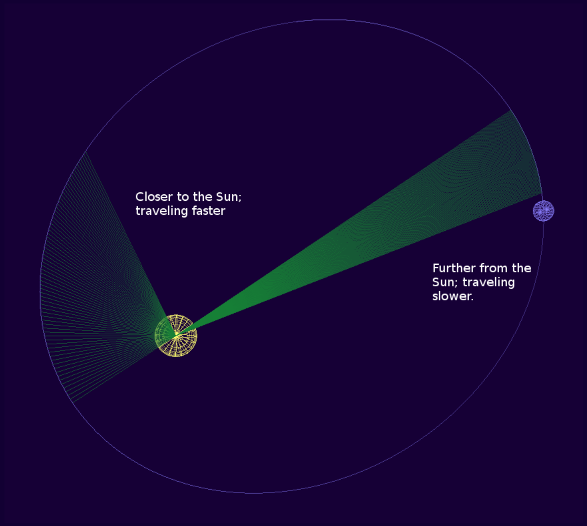
D: The purple one

What you need to know

- Planetary orbits are ellipses
- The *eccentricity* of an ellipse tells you how squashed it is
- An ellipse with zero eccentricity is just a circle
- The Sun lies at a focus of the ellipse, which isn't at the center (unless it's a circle)
- The more eccentric the orbit, the further to one side the Sun is

Kepler's second law

In an eccentric orbit, a planet travels fastest when it's nearer the Sun.



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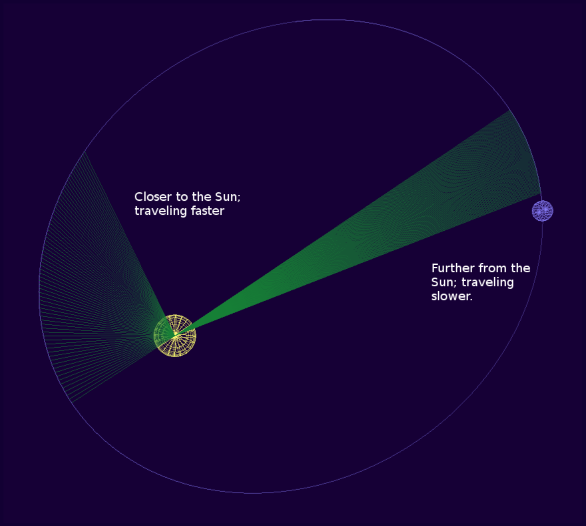

Let's watch this in an animation...

Comets

Comets have highly eccentric orbits. Halley's Comet's furthest point from the Sun – its *aphelion* – is 35 AU away. But its *perihelion* – the nearest point to the Sun – is 0.6 AU away.

Which statement is true, and how do you know?

A: Halley's Comet spends most of its time far from the Sun, and only a little time near the Sun

B: Halley's Comet moves slowly near perihelion, and quickly at aphelion

C: Halley's Comet moves quickly near perihelion, and slowly at aphelion

D: Halley's Comet spends roughly equal amounts of time near the Sun, and far from it

Kepler's Third Law

Kepler's third law of orbital motion says that the square of a planet's *orbital period* is proportional to the cube of its *semimajor axis*.

Simply put: if a planet is further from the Sun, it takes longer to go around.

If the distance is doubled, the time required *more than doubles*.

Let's watch this...

How do you think I made all these simulations?