Lab 7: Weighing the Earth

**Introduction**

We’ve learned in class that Newton’s laws of motion apply to every object in our day-to-day lives, and can explain a great deal of how things move. Among these is the motion of a pendulum. Newton’s second and gravitational laws combine to explain how if the pendulum begins away from its rest position, the force of gravity acts to pull it down, causing it to accelerate, and eventually come to rest, and repeat again. Today, we will study this process to arrive at a surprising measurement; the mass of the Earth!

**Materials**

For this lab, you’ll need a simple pendulum, a stopwatch, and a calculator. You can construct a pendulum out of many things around you. All you need is a string and something to hang from it. You could use, for instance, a heavy cellphone or laptop charger hanging from its cable, a boot hanging from a long string, etc. For best results:

* The string should be much longer than the size of the thing hanging from it
* The string should be pretty flexible
* The object should be massive enough that it is not affected that strongly by air resistance

**Part 1: The Approach**

You should already be familiar with Newton’s second law of motion. It is most commonly written “F=ma”, but the more useful form is

(1)

which allows you to calculate the acceleration of any object based on forces that act on it. Objects respond more strongly (that is, have a greater acceleration) when subjected to stronger forces, or when their masses are smaller (imagine trying to push a cardboard box compared to a truck).

You will also need the law of universal gravitation,

, (2)

which allows you to calculate the force that gravity applies to any object. Here G is a constant (like π), m and M are the masses that gravity is pulling together, and r is the distance between those two objects.

You’ll need one more equation before we get started that we have not yet studied. This relates the period *P* of a swinging pendulum to its length and the acceleration *a* that it experiences.

(3)

You don’t need to worry about where this equation comes from, but it’s important to know that this equation only applies when the pendulum swings through a small arc!

So, here we have formulae that relate the period of a pendulum to the acceleration it experiences, the acceleration an object experiences to the forces on it, and the forces on it to the mass of the Earth. Now we need to do a little algebra to find what we need.

***Part 2: A Bit of Algebra***

We need to combine the previous equations to solve for the mass of the Earth. Note that *M* and *m* are two separate variables; *M* represents the mass of the Earth, and *m* the mass of another object (like your pendulum bob).

First, find the acceleration due to gravity by setting equations (1) and (2) equal to each other, and solving for *a*.

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Does this value depend on *m*, the mass of the falling object? Is this what you expect? (If you drop two objects of different masses, will the heavier one land first?)

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If you square both sides of equation 3, you get the equation . Solve this equation for *a*.

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Set the *a* you found in question 1 equal to the *a* you found in question 3, and solve this for the mass *M* -- the mass of the Earth.

In the previous question, you should have gotten , or something very similar. If you did not, show your result to Walter or Kishan and discuss how you worked through the algebra with them. If they are not available, skip ahead until you are able to talk to them.

The radius of the Earth (*r* above) can be measured with some clever geometry (Eratosthenes did it in 240 B.C.!), and *G* can be carefully measured in other experiments. What other values do we need to find to be able to determine the mass of the Earth from your result above?

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The value of G given is . Write this in ”normal” notation (if is scientific notation, 164.5 is ”normal” notation). Is this a very large number, or a very small number? Why might gravity be referred to as a weak force?

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***Part 3: The Experiment***

We’ve now derived an equation that will tell us the mass of the Earth using a pendulum. All we need to do is measure the properties of the pendulum and the period of its swing.

Do we need to measure the mass of the pendulum?

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Measure the length of your pendulum, from the top of the string to the center of the hanging mass. We’ll refer to this as the average length .

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Recall from the parallax lab that it is important to get a sense of the error in our experiments. In measuring the length of your pendulum, how accurate do you think your measurement was? Was it accurate to 1mm? 1cm? 1m? Give your best estimate below of the uncertainty in that measurement. (If you aren’t able to measure it precisely, that’s okay!)

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We will now measure the period of the pendulum. To do this, we will measure how long the pendulum takes to complete TEN complete trips using a stopwatch, and then dividing the result by 10. Doing this will reduce the error in our experiment (can you figure out why?). We will do this 5 times. **Remember to not let your pendulum swing through too long of an arc, or equation 3 won’t be valid.**

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| Trial # | 10 periods (seconds) | 1 period (seconds) |
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| 2 |  |  |
| 3 |  |  |
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| 5 |  |  |

Calculate the average period by adding up all 5 of the entries in your last column and dividing the result by 5. We’ll refer to this value as .

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We want to get a sense of the uncertainty in our measurement for *P*. There are many sophisticated ways to do this, but we’ll do something simple; take the highest value for *P* you found, subtract from it the lowest of *P*, and divide this result by 2.

What is the uncertainty in your measurement of the average period?

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***Part 4: Measuring the Mass of the Earth***

Now that you have your measurements, you can calculate the mass of the Earth!

Listed below are the radius of the Earth *r*, the gravitational constant *G*, and to 3 decimal places:

Using the above values and your and , calculate the mass of the Earth below in kg using the equation you found earlier.

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Since your values of *L* and *P* were slightly uncertain, you will have some uncertainty in your value for *M.* Calculate , the maximum value of *L* supported by your measurements, by adding your uncertainty in *L* to your average value for *L*, and by subtracting. Do the same thing for *P* to get and . Put the values below:

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| Maximum period |  |
| Minimum period |  |
| Maximum length |  |
| Minimum length |  |

Look back at the equation for the mass of Earth. To get the highest value for the mass supported by our data, do we want to use or ? What about or ? Explain why.

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Calculate by using and instead of and .

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Calculate by using and instead of and .

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***Part 5: Analyzing the Results***

We have another way of determining the mass of the Earth, too. Recall the technique that you used in Lab 6 to determine the mass of the supermassive black hole at the center of the Milky Way.

You used the fact that, if something (like a star or planet) is orbiting a more massive object, you can find the mass of the central object in solar masses using

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where *A* is the length of the long axis of its orbit, and *T* is how long that object takes to orbit.

Can you apply this method to find the mass of the Earth? What object are you going to use? You will likely need to look up some properties of that object’s orbit.

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Determine the Earth’s mass using this method.

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Remember that this value is in *solar masses*. The mass of the Sun (one solar mass) is kg. Convert the value you found above from solar masses to kilograms.

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You now have two values of the Earth’s mass: one from your examination of the orbit of its satellite, and one from the measurements you made with your pendulum. Do they agree? If they are very different, discuss this with Walter or Kishan.

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You had several uncertainties in your pendulum measurement: the length of the pendulum, the period, and any effects caused by the fact that it was not perfectly flexible, or that it was swinging at large angles. Which of these caused the greatest uncertainty in your final measurement? Discuss briefly.

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