Project 3: Beyond Kepler’s Laws, and the Three-Body Problem

*Since this project is more like an “extended on-your-own lab”, we won’t be doing peer review for it. Instead, your group should send it by email to* [*suast101projects@gmail.com*](mailto:suast101projects@gmail.com)*. The title should be “Project 3 Group ####”.*

*This project is due at the end of the day Wednesday, October 21.*

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| **Group Number and Name:** | |  | |
| **Member Name #1:** |  | **Email #1:** |  |
| **Member Name #2:** |  | **Email #2:** |  |
| **Member Name #3:** |  | **Email #3:** |  |
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| **Collaboration Method (in-person, online on Collaborate, etc.)** |  | | |

*Only list group members that participated in working on the project.*

We’ve seen that Kepler discovered that planets move in elliptical orbits around the Sun, and that the laws of mechanics discovered by Newton provide the mathematical explanation of why.

Newton showed using pencil-and-paper mathematics that a small planet in orbit around a very massive star will travel in an elliptical orbit around it, retracing the same stable orbit forever.

In this project, you’ll explore what happens when we relax these restrictions -- by increasing the mass of the planet, by considering what happens when two planets orbit a star or one planet orbits two stars, and finally by simulating the orbit of three stars of nearly equal mass.

This project was partially inspired by the novel *The Three-Body Problem* by Liu Cixin, in which an alien species lives on a planet near *three* stars, and recognizes that they must leave their planet soon because the orbits of their stars are not stable.

**Prologue: A Binary System**

Previously, we used the simulator to model the orbit of a planet around a star. It does this by applying Newton’s laws of motion to those objects and calculating how the force of gravity would cause them to move. We would like to now use it to simulate the orbit of a smaller star around a larger one.

Look at the “physics part” of the simulation again (lines 48-67). Does the program do different calculations for the planet and the star, or do the laws of motion apply in the same way to planets and stars?

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If you change the mass of the “planet” to 0.5 solar masses, do you think it will accurately model the orbit of two stars? Why or why not?

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Make this change, and simulate an orbit of a smaller star and a larger one. Describe how they move. In particular, describe whether the orbit repeats exactly, changes slightly over time but is stable, or is completely unstable.

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Based on the stability or lack of stability of this orbit, do you expect to find many “binary stars” (stars orbiting each other like this) in the Universe? Why or why not?

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**Part 2: Two Planets, One Star**

Now, open the Two Planet Simulator. Take a look through the code, focusing in particular on the code between lines 88 and 119 that actually does the physics calculations for the simulation. (The rest just sets it up and draws the pictures.) How is it similar to what you’ve seen before in the first Orbit Simulator? How is it different?

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You will notice some new “controls” at the top. Line 9 lets you turn the second planet on and off. Lines 14 and 15 let you tilt and rotate the second planet’s orbit relative to the first one (like Pluto’s orbit is tilted).

This simulator prints out some different things. For each planet, it displays the aphelion and perihelion, the orbital period, the eccentricity, and the *orientation* of the orbit. The *orientation* is the angle where a planet has its perihelion.

Run the simulator. The first planet is set to the orbit of Earth. Notice that the aphelion, perihelion, period, orientation, and eccentricity of Earth’s orbit don’t change with time. This is the behavior you saw previously, and what you expect from Kepler’s laws.

Then, turn the second planet on by setting second\_planet\_enabled to True. (Note that Python, like most programming languages, cares about capitalization, so you must capitalize the first letter of “True”.)

The second planet is set to the orbital parameters of Venus. Before you run the simulator, predict what will happen. (Notice that Earth and Venus have about a millionth of the mass of the Sun.) With all three objects in the simulation, do you expect the orbits of Earth and Venus to…

* … stay exactly the same forever?
* … change slightly with time, but still be stable orbits
* … become unstable, and not look like elliptical orbits at all

Explain why you predict them to behave this way.

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Now run the simulator. What do you see? Watch the simulation for a while. Does what you see make sense?

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Over a long time, will Venus influence Earth’s orbit, or vice versa? If so, would the astronomers at Uraniborg (who observed for a few decades) have been able to notice it?

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Now, set the second planet to parameters that are more similar to Jupiter. (It orbits the Sun at an orbit with aphelion around 5 AU that is not too eccentric, and has a mass around 0.001 solar mass.) Run the simulation. (You may have to fiddle around with the initial velocity to get a reasonable eccentricity.)

Let this simulation run for a while. Does Jupiter’s orbit change appreciably? Does Earth’s?

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Venus is the closest planet to Earth, and Jupiter is the most massive in the Solar System. Based on your experience with these simulations, describe how you expect the planets in the Solar System to influence each other’s orbits over a long time. Do you think this would have any impact on Earth’s climate?

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Play around with the simulator with different sorts of orbits -- planets closer to or further from each other, and with masses ranging from Earth’s mass (0.000003 solar masses) up to 0.2 solar mass. Describe the results you see below. Do the orbits shift at all over time? A little (so that you can only see the effects by the changing numbers) or a lot (so that you can see the effects visibly)?

“Orbit size” here refers to the long axis of the planet’s orbit.

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| Orbit size 1 | Mass 1 | Orbit size 2 | Mass 2 | Inclination | Description |
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Newton calculated from the laws of motion that planets ought to stay in unchanging elliptical orbits around the Sun which do not change at all with time -- in other words, that Kepler’s laws are correct. Based on what you’ve observed, discuss how well Kepler’s laws apply to our solar system. How well would they apply to a solar system with a very large planet (0.2 solar masses)?

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**Part 3: Planets Around a Binary Star System**

Open the Binary Star Simulator. It is designed to simulate one planet orbiting two stars. By default, one has a mass two-thirds of the Sun, and the other one-third.

This code won’t run at all when you load it, since the planet’s properties are blank. You’ll need to put in an initial distance, initial velocity, and mass value for it. You can also set the “inclination” value if you want your planet to be in a tilted orbit.

Experiment with different sorts of orbits. There are two different orbit sizes here:

* … the size of the stars’ orbit around each other
* … the size of the planet’s orbit around the stars

Is it possible to have a planet in a stable orbit around two stars if the size of the planet’s orbit is only a little larger than the size of the stars’ orbit? What if it is much larger? Try different things, and discuss what you see. In each case, consider whether your planet’s orbit stays exactly the same, changes slightly with time but remains in orbit, or becomes unstable and crashes into one of the stars or is ejected.

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Put the two stars in a tight orbit around each other -- separated by 0.5 AU or so. Then put a planet in an orbit that is reasonably circular around 5 AU away from the stars. This is somewhat like Jupiter’s orbit around the Sun.

Compare two cases:

1. Jupiter orbiting the Sun (1 solar mass)
2. Jupiter orbiting this close binary star system, with masses ⅓ and ⅔ solar mass

How do you expect these orbits to behave? Would Jupiter’s orbit around the binary star system look the same as Jupiter’s orbit around the Sun? Before you run any simulations, discuss with your group what you expect to see and why.

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Then run the simulations, using the exact same parameters for “Jupiter”. What happens?

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**Part 4: The Three-Body Problem**

The “three-body problem” refers to the famous challenge of calculating the motions of three objects moving under the influence of each other’s gravity *when those three objects have a reasonably equal influence on each other*. In general, this is not possible to do except with a computer, and these orbits are not stable. This is the premise of the Liu Cixin novel *The Three-Body Problem*.

Previously, you have simulated stable orbits for three objects. However, the gravitational influences between these three objects are not equal. For instance, when two small planets orbit one star, the gravitational influence of the planets on each other is much less than the influence of the star on the planets. In *these* cases, the solution to the “three-body problem” can be stable.

Load the Three-Body Simulator. It’s a slightly modified version of the Binary Star Simulator. Notice that this time all three objects have similar masses, rather than two massive stars and one light planet. Run the simulator and notice what happens. (You can rotate the camera by holding down the right mouse button and dragging.)

Play around with the initial parameters, changing the values slightly, and see what happens. See how long you can get all three stars to “orbit” one another. Include a screenshot of their paths and paste in the initial parameters you use. How long did they last?

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Many stars occur in binary star systems, but we typically don’t see *trinaries* -- three stars orbiting each other. Why might that be?

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**Part 5: Putting it All Together**

Based on your experience with the simulations you’ve done, comment on how you expect the following systems to behave over a long time. Will the orbits change very little over time? Will they change in a meaningful way, but stay stable? (If they change, which orbits will change the most?) Or will the system become unstable? Which of the following systems could we expect to find in the Universe?

Explain why you think they’ll behave the way that they do based on your experience.

1. *A solar system with a dwarf star of mass 0.1 solar mass, orbited by multiple large planets of mass 0.03 solar masses each*
2. *The Earth/Moon/Sun system, where the Earth has a mass of three millionths of a solar mass, the Moon’s mass is one hundred times less than the Earth’s, and the Moon orbits the Earth at a distance of 1/250 AU*
3. *A system with an Earthlike planet orbiting a Sunlike star 1 AU away; a planet ten times the mass of Jupiter orbits 3 AU from the star.*
4. *A binary star system where two stars with the same mass as the Sun orbit 0.5 AU apart, with a planet the size of Jupiter orbiting them 5 AU away*
5. *A binary star system where two stars with the same mass as the Sun orbit 0.5 AU apart; a planet with the same mass as Earth orbits them 0.7 AU from their center*
6. *A binary star system where two stars with the same mass as the Sun orbit 5 AU apart; one of them has a planet with the same mass as Earth orbiting it 1 AU away.*
7. *The Alpha Centauri system, where two stars (Alpha Centauri A and Alpha Centauri B) with masses close to that of the Sun orbit each other in an orbit with a long axis 35 AU long; a third star, Proxima Centauri, orbits Alpha Centauri AB in a large orbit 13,000 AU from them.*
8. *Three stars with masses 0.5, 0.8, and 1.2 times the mass of the Sun orbit each other. The first two are an average distance of 1 AU from each other; the third star orbits them at a distance of 2 AU.*