Pitch-Frequency Tutorial

The purpose of this guide is to figure out the connection between the pitch and the fundamental frequency of a note. The precise meanings of these terms are:

- **Pitch:** How high a note sounds to our ears. This is a musical term; for instance, we would say that the note A_4 has a pitch one octave higher than A_3.
- **Fundamental frequency:** The lowest frequency of a musical tone, which is also the spacing between the harmonics produced. For instance, we might say that a note has a fundamental frequency of 440 Hz. This is a scientific term.

As we will see, it's only *relative* frequency that matters for things related to harmony. So we are free to choose a starting point: we need to pick the frequency of *one* note on the piano keyboard. The mathematics of harmony will let us work out all the others (in an exercise you will complete today!).

Discuss with your neighbors how we should do this, then we'll talk about it. On the jumbo-sized grand staff provided, fill in the frequency of our starting point.

1 Figuring out the frequency difference of an octave

We have lots of musical instruments in this room: a guitar (which some of you will know how to play, but anyone can poke at), a piano (which you all can play!), and those of you with laptops or tablets can Google "online piano".

Using one of these instruments – or your own voices! – figure out how the fundamental frequency of a note changes when you go up by an octave.

Your statement will take one of two forms:

- *When the pitch of a note increases by an octave, you add* _____ *Hz to the frequency.*
- *When the pitch of a note increases by an octave, you multiply the frequency by a factor of* _____.

If you already know the answer to this, don't tell your group; the point is to figure it out. But you may help them figure it out!

Once you've figured this out, fill in the fundamental frequency of all the A's on your staff, based on the frequency of middle A you chose earlier.

2 Math language

What is the ratio between the frequency of the note C_6 (the highest note women are usually asked to sing) and the frequency of the note C_2 (the lowest note men are usually asked to sing?

In algebra terms, how would you write the mathematical relationship between the number of octaves $\mathcal{O}$ and the frequency ratio R? That is, R is some function of $\mathcal{O}$; what is it? (Hint: how do you write "multiply by two this many times?")

Now invert the previous formula, to get $\mathcal{O}$ as a function of R. This involves logarithms. If you already know how to do this, go ahead and do it. If you don't, ask me and I'll show you what a logarithm is. It's far simpler than your algebra teacher made it out to be. :)

3 Figuring out the frequency difference of a semitone/halfstep

Now, we need to fill in the other notes. Remember that a halfstep is 1/12 of an octave, a fifth is 7/12 of an octave, and so on. In symbols, we could say that N halfsteps is equal to $N/12$ octaves – or $\mathcal{O} = N/12$.

Recall that on the previous page you found out that, if note 2 is O octaves above note 1, you know that

$$\frac{\text{Frequency of note 2}}{\text{Frequency of note 1}} = 2^{\mathcal{O}}$$

This means that

$$\frac{f_2}{f_1} = 2^{N/12}$$

This formula lets you calculate the frequency of any pitch on the piano. As a warmup, calculate the frequency of middle C, and fill it in on your staff. (Remember, you know the frequency of middle A is 440 Hz. So let that be your f_1, and solve for f_2.)

So, if the interval of an octave represents a factor of 2 in frequency, what factor does the interval of a halfstep represent? Make a statement similar to your previous one:

When the pitch of a note increases by a semitone, you multiply the frequency by a factor of _____.

Now, get into groups of four and work in pairs.

One pair of people should calculate frequencies from middle A (A_4, which is 440 Hz) all the way up to C_6 (that is, two octaves above middle C), and record those on the grand staff. (You only need one copy of this.) Then, calculate the frequency of C_2 (two octaves below middle C) and record it.

The other pair of people should calculate frequencies from middle A down to C_3 (that is, one octave below middle C) and record those.

One group of four will work on the board, so we have a record for the whole class to see.

4 The chord of nature

We noticed earlier that musical tones consist of lots of frequencies that are all multiples of the fundamental.

Take a low note: C_2, the lowest note a cello can play, and the lowest note men are usually asked to sing.

- What is its fundamental frequency?
- What are the other frequencies produced when someone plays/sings this note?
- Using our labeled staff on the board, what *other notes* have these fundamental frequencies?
- Two people who have never played the piano before: come find and play those notes all at once on the keyboard!