PHY 211 Lecture 22

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Rotation

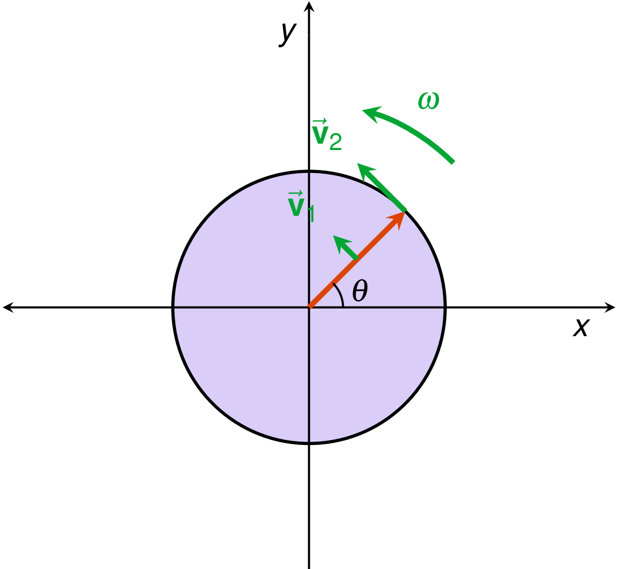
- We talked a bit earlier in the semester about rotational coordinates, torque and angular acceleration
- But what about the energy of the system?

Setting up rotation

- Thinking of rotation as lots of little linear motions is difficult
- Better to define new, rotating coordinates
- Turns a constantly changing 2D problem into a 1D problem

position: x	angle: θ
velocity: $v = \frac{\mathrm{d}x}{\mathrm{d}t}$	angular velocity: $\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$
acceleration: $a = \frac{\mathrm{d}v}{\mathrm{d}t}$	angular acceleration: $\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$

Circular motion



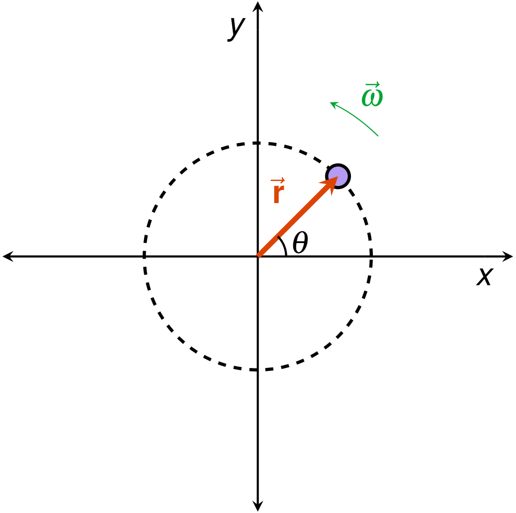
Beyond describing motion

- We've learned a lot since we first encountered these variables
- How will we describe energy and forces in this picture?
- This is motion, so it should have kinetic energy! This is our main focus for now
- Some things are harder to turn than others: moment of inertia

Kinetic energy

- Take a point mass going around a circle
- We can already calculate its kinetic energy

$$
\begin{aligned}
K &= \frac{1}{2}mv^2 \\
&= \frac{1}{2}m(\omega r)^2 \\
&= \frac{1}{2}\left(mr^2\right)\omega^2
\end{aligned}
$$



Pre-lecture question 1

If a child walks toward the center of a merry-go-round, does the moment of inertia increase, decrease, or stay the same?

(a) Increase (b) Decrease (c) Stay the same

Pre-lecture question 2

Is it easier to spin a rod around its center or its end?

(a) Center

(b) End

Pre-lecture question 3

An object that is spinning in place has no kinetic energy.

(a) True

(b) False

Moment of inertia

- The rotational equivalent of mass is moment of inertia, I
- For any object it will be something $\times\ mr^2$ (where r is some length)

Rotational kinetic energy

$$K = \frac{1}{2} I \omega^2$$

Doing the calculation

- Adding up the moment of inertia fundamentally involves integration
- But we don't have to redo the integral for every configuration – remember I depends both on object's shape and on the axis of rotation
- First get I for various common shapes around their center of mass

Disk

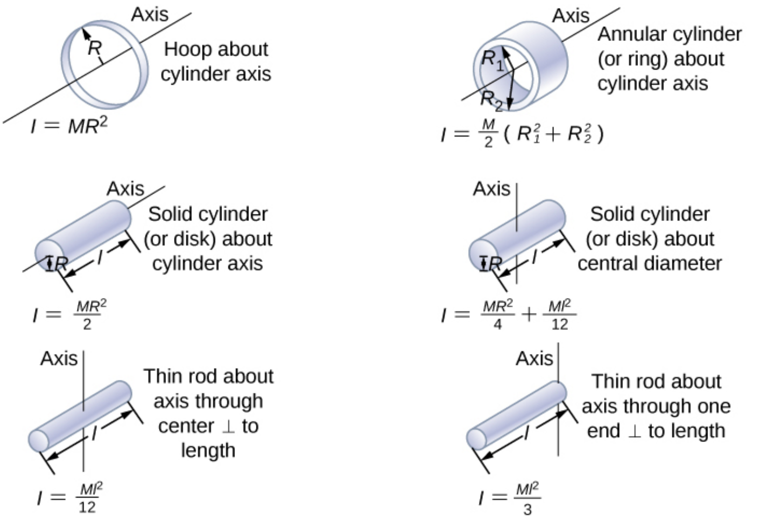
$$I = \frac{1}{2} m r^2$$

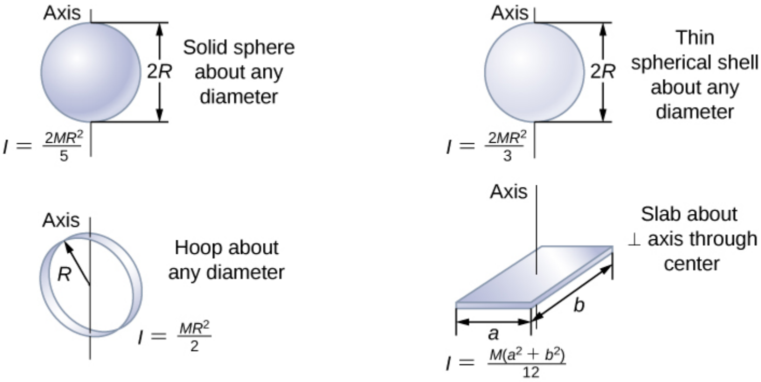
Ring

$$I = \frac{1}{2} m \left(r_1^2 + r_2^2 \right)$$

Square

$$I = \frac{1}{6} m a^2$$


Axis
R
Hoop about cylinder axis
$I = MR^2$
Axis
R_1
R_2
Annular cylinder (or ring) about cylinder axis
$I = \frac{M}{2}(R_1^2 + R_2^2)$
Axis
R
l
Solid cylinder (or disk) about cylinder axis
$I = \frac{MR^2}{2}$
Axis
R
l
Solid cylinder (or disk) about central diameter
$I = \frac{MR^2}{4} + \frac{Ml^2}{12}$
Axis
l
Thin rod about axis through center ⊥ to length
$I = \frac{Ml^2}{12}$
Axis
l
Thin rod about axis through one end ⊥ to length
$I = \frac{Ml^2}{3}$


Axis
2R
Solid sphere about any diameter
I = 2MR^2/5
Axis
2R
Thin spherical shell about any diameter
I = 2MR^2/3
Axis
R
Hoop about any diameter
I = MR^2/2
Axis
Slab about ⊥ axis through center
a
b
I = M(a^2 + b^2)/12

Moving the axis

- The key thing is that once we know I around the center of mass, it's easy to find around any parallel axis

Parallel axis theorem

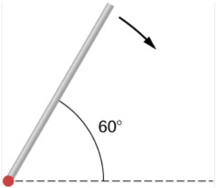
$$I = I_{\text{CM}} + md^2$$

- d is the distance between the CM axis and the actual axis of rotation
- We can break up an object into shapes for which we know I_{CM}, then each one contributes an extra bit of I based on where its CM is located

Example

All motion is rotational

A uniform rod of mass 1.0 kg and length 2.0 m is free to rotate about one end (see the following figure). If the rod is released from rest at an angle of 60° with respect to the horizontal, what is the speed of the tip of the rod as it passes the horizontal position?



Linear and rotational motion

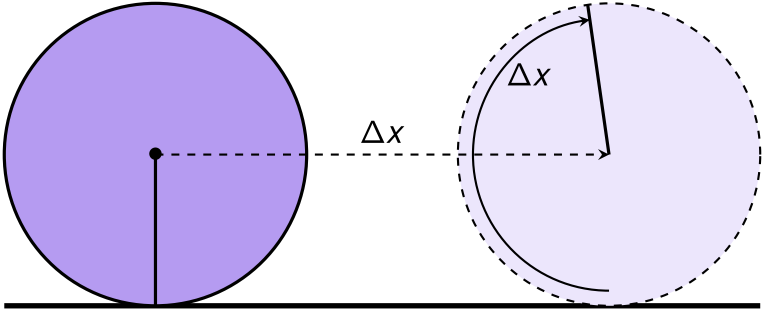
- An object can rotate and have the center of mass move, in which case total is

$$K = \frac{1}{2} m v_{\mathrm{CM}}^2 + \frac{1}{2} I \omega^2$$

- Motion of the center of mass is also called “translational”
- If it is rotating on a fixed axis don’t write down $mv^2/2$
- For now we will only consider fixed rotation and “rolling without slipping”

Rolling without slipping

- Rolling without slipping is one of the cases where $v = \omega r$



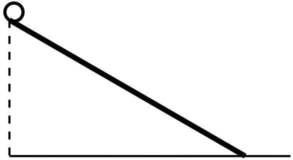
$$v_{\text{CM}} = \frac{\Delta x}{\Delta t} = \frac{(r\Delta\theta)}{\Delta t} = \omega r$$

What wins a race down the ramp?

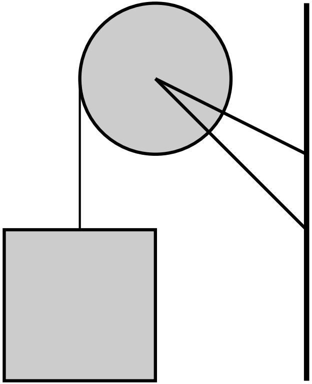
- Remember that for sliding down a ramp, the acceleration did not depend on mass
- What about differently shaped rolling objects?
- Which will win in each of these scenarios?

Rolling down the hill

A solid disk with a mass m and a radius r rolls without slipping down an incline starting from a height h. What is its speed at the bottom?



A block of mass $m_1 = 4\,\text{kg}$ is suspended on a light non-stretchable string wound around a pulley with mass $m_2 = 3\,\text{kg}$ (assume solid cylinder) which rotates in place while the block is descending. If the block starts from rest and then falls 1.0 m, what will be its velocity?



A solid sphere of radius 10 cm is allowed to rotate freely about an axis. The sphere is given a sharp blow so that its center of mass starts from the position shown in the following figure with speed 15 cm/s. What is the maximum angle that the diameter makes with the vertical?