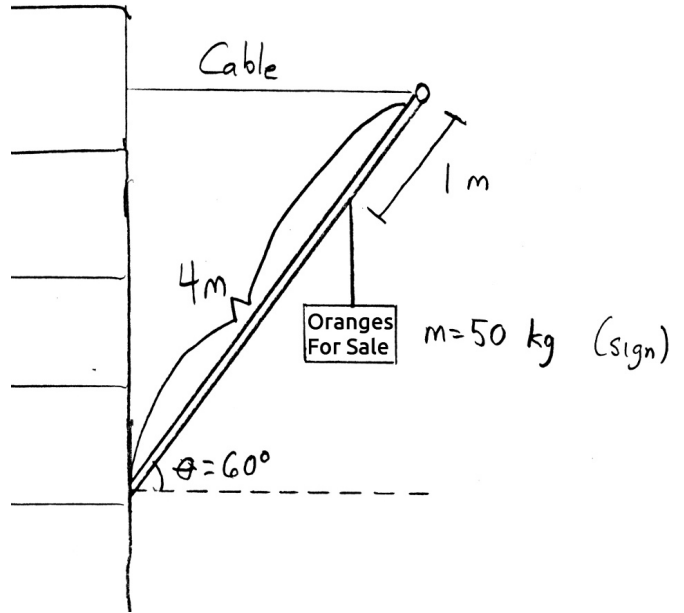


# RECITATION QUESTIONS

WEEK OF 30 MARCH

1. If you didn't have time to complete this problem last week, complete it.

A 4m-long pole of mass 80 kg extends from the side of a building, angled at 60 degrees above the horizontal. One meter from the end of the pole, a sign of mass 50 kg is attached. To support the pole, a horizontal cable runs from the end of the pole to the building. (See the attached figure.)



- (a) Draw a force diagram on the back of this page, showing all of the elements needed to help you compute the tension in the support cable. Indicate your choice of pivot point.
- (b) Compute the tension in the cable.
- (c) Suppose now that the store owner wanted to attach the cable to a different point on the building in order to minimize its tension. What angle between the cable and the horizontal would support the pole with the minimum tension?

2. A light cable is wound around a cylindrical spool fixed in place of radius 50 cm and mass 10 kg. One end of the cable is attached to a motor, which pulls with a constant force of 20 N on the cable. When the motor is switched on, the force exerted by the cable causes the spool to rotate faster and faster.
- (a) What is the moment of inertia of the spool?
  - (b) What is the torque applied to the spool by the motor?
  - (c) What is the angular acceleration of the spool?
  - (d) How long will it take for the spool to make a full revolution?
  - (e) After five seconds, how fast is the cable moving?
  - (f) After five seconds, what is the kinetic energy of the spool? Remember that the kinetic energy of a rotating object is  $\frac{1}{2}I\omega^2$ .
  - (g) What is the work done by the motor in five seconds? Remember that the rotational work-energy theorem is  $W = \tau\Delta\theta$ .

3. A flywheel (a large, spinning disc) of mass  $m$  and radius  $r$  is rotating at angular velocity  $\omega$ . The machine operator wishes to bring it to rest using a brake. When the brake is engaged, two brake pads on either side of the disc are pressed against it from either side, two-thirds of the way from the center to the outer edge; each brake pad exerts a normal force  $F_N$ .

If the coefficient of friction between the brake pads and the disc is  $\mu_k$ , how long does it take the brake to bring the flywheel to a stop?

4. A bucket of mass  $m$  hangs from a string wound around a pulley (a solid cylinder) with mass  $M$  and radius  $r$ . When the bucket is released, it falls, unwinding the string.

(a) Draw force diagrams for the bucket and the pulley. Note that since the pulley rotates, you will need to draw an extended force diagram for it, drawing the object and labeling where each force acts.

(b) In terms of the forces in your force diagrams, write an expression for the net torque on the pulley.

(c) Write down Newton's laws of motion –  $\sum \vec{F} = m\vec{a}$  for translation, and  $\sum \tau = I\alpha$  – for each object. (One object moves, and the other turns...)

- (d) What is the relationship between the angular acceleration  $\alpha$  of the pulley and the linear acceleration  $a$  of the bucket? (The answer may be different depending on how you have drawn your pictures and your choice of coordinate system.)
- (e) Calculate the acceleration of the bucket in terms of  $m$  and  $M$ .
- (f) Suppose that the pulley were a hollow cylinder with the same mass. How would this acceleration change?

5. A Yo-Yo consists of a cylinder of mass  $m$  and radius  $r$ . A slot is cut in the middle of the cylinder such that the inner radius is only  $0.4r$ , and a string is wound around the middle. (If you don't know what a Yo-Yo is, there is an animation on Wikipedia.) A person holds the string and allows the Yo-Yo to fall. As it falls, it has both a linear acceleration (moving downward) and an angular acceleration (spinning faster and faster).
- (a) What is the relation between the linear velocity  $v$  of the Yo-Yo (moving downward) and its angular velocity  $\omega$ ?
  - (b) What is the relation between the linear acceleration  $a$  of the Yo-Yo (moving downward) and its angular acceleration  $\alpha$ ? (Note: this is trivial once you've done the previous part...)
  - (c) Draw a force diagram for the Yo-Yo, indicating the location where all forces act as well as their magnitude.
  - (d) Write down Newton's second law ( $F = ma$ ) for the Yo-Yo, putting in expressions for the various forces.

(e) Write down “Newton’s second law for rotation”  $\tau = I\alpha$ , putting in expressions for the net torque and the moment of inertia.

(f) What is the acceleration of the Yo-Yo and the tension in the string?

(g) Will the Yo-Yo accelerate faster or slower if the inner radius is changed to  $0.2r$ ?

## REFERENCE MATERIAL - ROTATIONAL MOTION

Moments of Inertia:

- Disk or cylinder, rotating about center:  $I = \frac{1}{2}MR^2$
- Sphere, rotating about center:  $I = \frac{2}{5}MR^2$
- Ring or hollow cylinder, rotating about center:  $I = MR^2$

Correspondence between linear dynamics and rotational dynamics:

Position	$s$	Angle	$\theta$
Velocity	$\vec{v}$	Angular velocity	$\omega$
Acceleration	$\vec{a}$	Angular acceleration	$\alpha$
	$v(t) = v_0 + at$ $x(t) = x_0 + v_0t + \frac{1}{2}at^2$ $v_f^2 - v_0^2 = 2a\Delta x$		$\omega(t) = \omega_0 + \alpha t$ $\theta(t) = \theta_0 + \omega_0t + \frac{1}{2}\alpha t^2$ $\omega_f^2 - \omega_0^2 = 2\alpha\Delta\theta$
Mass	$m$	Moment of inertia	$I$
Force	$F$	Torque	$\tau = F_{\perp}r = Fr_{\perp}$
Newton's second law	$\vec{F} = m\vec{a}$	"Newton's second law for rotation"	$\tau = I\alpha$
Kinetic energy	$\frac{1}{2}mv^2$	Kinetic energy	$\frac{1}{2}I\omega^2$
Momentum	$\vec{p} = m\vec{v}$	Angular momentum	$L = I\omega$

Arc length  $s = \theta r$

Tangential velocity  $v = \omega r$

Tangential acceleration  $a = \alpha r$