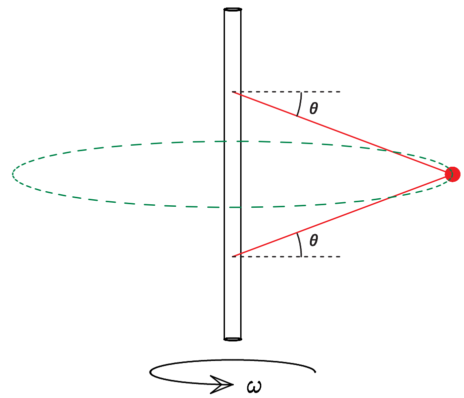
Recitation Exercises

Week 6, Day 1

Question 1: Practice from Earlier (this should be quick)

Consider a ball tethered to a rotating pole by two cables of equal length as shown to the right. The ball rotates along with the pole, making a horizontal circle (shown in green on the diagram). Suppose that you know the ball has a mass m, the pole is rotating at angular velocity ω, and the radius of the circle it makes is r.



You want to find the relationship between m, ω, r, and the tensions in the cables T_1 and T_2.

Draw a force diagram for the ball below, and indicate your choice of coordinate system.

Construct Newton's laws in both x and y for the ball based on your force diagram, putting in what you know about a_x and a_y. (You don't need to actually solve the system of equations, but show it to your TA/coach.)

Question 2: variation of apparent weight with latitude

Note about this exercise: This question asks you to consider what g means, and how it might vary on Earth's surface. The effect you are studying here is the reduction in apparent weight because of the Earth's rotation. There are smaller effects from the "non-roundness" of the Earth which we don't care about here. Ultimately this means that the apparent weight of a 1 kg mass is around 9.83 N at the North (or South) Pole and 9.78 N at the Equator. It is 9.81 N only in latitudes corresponding to Paris, Berlin, or Kyiv; in New York it is closer to 9.80 N, and in the tropical latitudes where many people live (most of Africa, South Asia, most of South America) it is 9.79 N or 9.78 N, to three significant digits. This means that insisting on $g = 9.81\,\mathrm{m/s^2}$ isn't correct for most of us (unless you are in Europe or Canada).

For this problem, do all calculations to five significant digits. Some figures that will be useful:

- Mass of Earth: 5.9722×10^{24} kg
- Radius of Earth: 6.3710×10^{6} m (assume it is spherical; we don't have the math to deal with its oblateness)
- Gravitational constant (G): $6.6741 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$
- Length of one day: 8.6400×10^{4} s

a) Using Newton's law of universal gravitation $F_g = \frac{GMm}{r^2}$, determine the force of gravity on a 1 kg mass resting on the surface of the Earth. Are you surprised by this figure?

b) Suppose this mass were resting on a scale sitting on the North Pole owned by Santa Claus. Recall that scales measure the normal force that they exert. What value would Santa's scale read? What would Santa conclude the value of g is?

c) Suppose that an identical 1 kg mass were resting on a scale sitting on the Equator, somewhere in Kenya. What would *this* scale read? (Hint: What is the acceleration of the mass?) What would our Kenyan physicist conclude about g?

d) This problem shows that your apparent weight depends on your location on Earth. Does it make sense to define g as F_g/m (the strength of the gravitational force divided by an object's mass) or F_N/m (the strength of the normal force, and thus the scale reading, divided by mass)? Call your TA/coach over to join your conversation.

e) Is this distinction likely to be relevant to the sort of engineering or science you will do during your career? (The answer will depend on what you will do, of course!)

Question 3: Weightlessness

Astronauts in orbit around the Earth are not "so far away that they don't feel Earth's gravity"; actually, they're quite close to the surface. However, we've all seen the videos of astronauts drifting around "weightlessly" in the International Space Station.

a) Explain how an astronaut can be under the influence of Earth's gravity, and yet exert no normal force on the surface of the spacecraft they are standing in.

b) Draw a force diagram for the astronaut floating in the middle of the Space Station, not touching any of the walls or floor. How do you reconcile your diagram with the fact that the astronaut doesn't seem to fall?

c) Is this astronaut truly "weightless"? What does "weightless" mean? (There are multiple correct answers to both of these questions.)

Question 4: geostationary orbit

It is sometimes useful to place satellites in orbit so that they stay in a fixed position relative to the Earth; that is, their orbits are synchronized with the Earth's rotation so that a satellite might stay above the same point on Earth's surface all the time.

What is the altitude of such an orbit? Note that it is high enough that you need to use $F_g = \frac{GMm}{r^2}$ rather than just $F_g = mg$. *(Are there any meaningful forces on the satellite besides Earth's gravity?)*

HINT 1: If this orbit is synchronized with Earth's rotation, then you should be able to figure out its angular velocity.

HINT 2: If you do this problem as we have guided you, by waiting to substitute numbers in until the very end, you will arrive at an expression relating the radius R of a circular orbit with the mass M of the planet being orbited and the angular velocity ω of the orbit. This question will be on HW5, and is related to the derivation of Kepler's third law that you will do there.