Rotational kinetic energy

Physics 211
Syracuse University, Physics 211 Spring 2023
Walter Freeman

April 11, 2023

Announcements

- Homework 8 is due Friday
- "Second chance" homework assignments posted on the webpage
- You can find the help hours schedule there (there's a lot of them coming up)

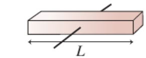
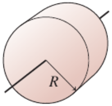
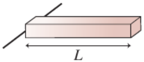
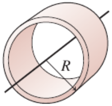
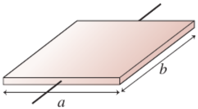
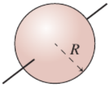
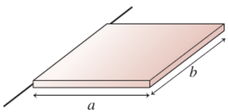
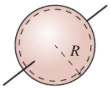
An object with moment of inertia I rotating at angular velocity ω has rotational kinetic energy

$$KE_{rot} = \frac{1}{2} I \omega^2$$

What about the moment of inertia of other objects?

Requires calculus in general; here are some common ones

TABLE 12.2 Moments of inertia of objects with uniform density

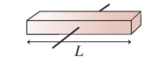
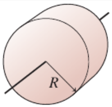
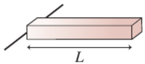
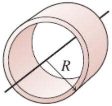
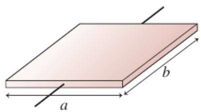
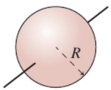
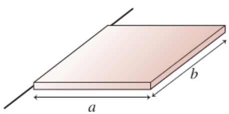
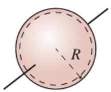
Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

Moment of inertia, other things

What about the moment of inertia of other objects?

Requires calculus in general; here are some common ones

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

In general: $I = \lambda M R^2$

We will always give you I if it's not 1 (i.e. not a ring etc.)

Another example

Remember the “Atwood machine”?

Another example

Remember the “Atwood machine”?

Another example

Remember the “Atwood machine”?

What happens if we remove one of the weights?

Another example

Remember the "Atwood machine"?

What happens if we remove one of the weights?

What happens if the pulley isn't light?

The relationship between linear and rotational motion

What's the acceleration of an object traveling in circular motion?

The relationship between linear and rotational motion

What's the acceleration of an object traveling in circular motion?

$$a = \omega^2 r \qquad \text{toward the center}$$

The relationship between linear and rotational motion

What's the acceleration of an object traveling in circular motion?

$$a = \omega^2 r \qquad \text{toward the center}$$

$$a = v^2/r \qquad \text{toward the center}$$

The relationship between linear and rotational motion

What's the acceleration of an object traveling in circular motion?

$$a = \omega^2 r \qquad \text{toward the center}$$

$$a = v^2/r \qquad \text{toward the center}$$

Why do we have two different formulae? This came from the relationship:

$$v = \omega r$$

If an object rotates at angular velocity ω, a point a distance r from the center moves at speed v.

Suppose I wrap a string around a solid cylinder with mass M and radius r, and let a mass m hang from the string.

How fast is the falling mass traveling when it hits the ground if it starts from a height h?

Suppose I wrap a string around a solid cylinder with mass M and radius r, and let a mass m hang from the string.

How fast is the falling mass traveling when it hits the ground if it starts from a height h?

$$\text{(initial KE)} + \text{(work done by gravity)} = \text{(final KE)}$$

$$\text{(initial rotational KE)} + \text{(initial translational KE)} + \text{(work done by gravity)} = \text{(final rotational KE)} + \text{(final translational KE)}$$

Why does a Yo-Yo fall so slowly?

Which object will reach the bottom of the ramp faster?

A: The wooden one

B: The one with the mass located near the middle

C: The one with the mass located near the edge

D: A tie between A and B

E: A tie between B and C

Rotation plus translation

In general, rotation and translation are separate; we can study each separately.

Example: this bike wheel

- Its position is given by some function $\vec{s}(t)$: "where is it at some time t?"
- Its angle is given by some other function $\theta(t)$: "which way is the reference point pointing at some time t?"
- The angle has the familiar derivatives: angular velocity ω, angular acceleration α

Recall that points along the edge of a rotating object move at a speed $v_{\text{edge}} = \omega r$.

Example: rolling without slipping

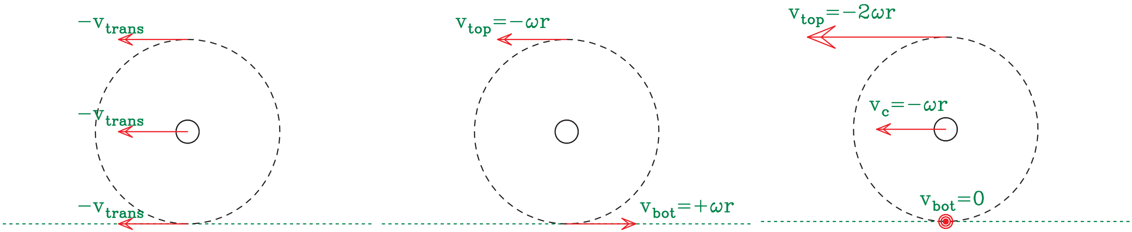
Sometimes the translational and rotational motion are linked.
"How fast do the tires on a car turn?"
→ Static friction means that the bottom piece of the wheel doesn't move

- If a wheel is turning counterclockwise at angular velocity ω:
  - the top moves at $v_{\text{top}} = -\omega r$ (left)
  - the bottom moves at $v_{\text{bot}} = \omega r$ (right)
- This means that the velocity of the axle must be equal and opposite to v_{bot}
- Thus, the car must be moving at $v_{\text{axle}} = -\omega r$ (left).

Let's look at a diagram.
So: if the wheels turn counterclockwise at ω:

- The axle moves at a velocity $-\omega r$ (left);
- The top of the wheels move at a velocity $v_{\text{axle}} + v_{\text{top}} = -\omega r - \omega r = -2\omega r$;
- The top of the wheels move at a velocity $v_{\text{axle}} + v_{\text{bot}} = -\omega r + \omega r = 0$.

Rolling without slipping



Translation + Rotation = Rolling

The “rolling constraint”

If an object rolls forward on an edge of radius r,

$$v = \omega r$$

The “rolling constraint”

If an object rolls forward on an edge of radius r,

$$v = \omega r$$

Common algebra pattern:

$$KE_{rot} = \frac{1}{2} I \omega^2$$

$$KE_{rot} = \frac{1}{2} \lambda m r^2 \omega^2$$

$$KE_{rot} = \frac{1}{2} \lambda m r^2 \omega^2$$

$$KE_{rot} = \frac{1}{2} \lambda m v^2$$

How many of you have played pinball?

How many of you have played pinball?

You are trying to design a pinball machine's spring-loaded launcher. Suppose that:

- The pinball has mass m and radius r
- The machine is angled at an angle θ and has length L
- You want the ball to reach the top of the machine at a speed v_T

You know the player will draw the spring-loaded launcher back a distance d. What spring constant should it have so the ball is traveling at v_f at the top of the ramp??

What kinds of energy does the system have initially?

A: elastic potential energy

B: gravitational potential energy

C: translational kinetic energy

D: rotational kinetic energy

E: None of the above, or so many I can't display them

What kinds of energy does the system have at the top of the ramp?

A: elastic potential energy

B: gravitational potential energy

C: translational kinetic energy

D: rotational kinetic energy

E: None of the above, or so many I can't show you...

How do I handle this “one equation, two unknowns” problem?

How do I handle this “one equation, two unknowns” problem?