1D kinematics: solving problems

Physics 211
Syracuse University, Physics 211 Spring 2019
Walter Freeman

January 21, 2019

On solving problems

You can recognize truth by its beauty and simplicity. When you get it right, it is obvious that it is right–at least if you have any experience–because usually what happens is that more comes out than goes in.... Inexperienced students make guesses that are very complicated, [but] the truth always turns out to be simpler than you thought.

–Richard Feynman, quoted by K. C. Cole, in *Sympathetic Vibrations: Reflections on Physics as a Way of Life (1985)*

Truth is ever to be found in simplicity, and not in the multiplicity and confusion of things.

–Isaac Newton, *Rules for Methodizing the Apocalypse*

Announcements

- Homework 1 due Friday
- I'm seeing lots of you in the Clinic; I'd like to see lots more
- Clinic hours this week:
  - Wednesday 2-6 PM
  - Thursday 1:45-3:45

“Ask a Physicist”

Submit questions!

Today

- Review material from last time
- Position, velocity, and acceleration graphs
- Rotational kinematics
- Problem-solving method for kinematics problems
- Sample problems

Today

- Review material from last time
- Position, velocity, and acceleration graphs
- Rotational kinematics
- Problem-solving method for kinematics problems
- Sample problems
- Homework help?

Last time: Position, velocity, and acceleration

(derivative of)
rate of change of

Position ⇄ Velocity

area under the curve of
(integral of)

Last time: Position, velocity, and acceleration

$$
\begin{tikzcd}[column sep=huge]
\text{Position} \arrow[r, shift right, "\substack{\text{area under the curve of}\\ \text{(integral of)}}"'] & \text{Velocity} \arrow[l, shift right, "\substack{\text{(derivative of)}\\ \text{rate of change of}}"'] \arrow[r, shift right, "\substack{\text{area under the curve of}\\ \text{(integral of)}}"'] & \text{Acceleration} \arrow[l, shift right, "\substack{\text{(derivative of)}\\ \text{rate of change of}}"']
\end{tikzcd}
$$

Position, velocity, and acceleration

Position $\xleftarrow[\text{}]{\text{(derivative of) rate of change of}}$ Velocity

Position, velocity, and acceleration

$$\text{Position} \xleftarrow[]{\text{(derivative of)} \atop \text{rate of change of}} \text{Velocity} \xleftarrow[]{\text{(derivative of)} \atop \text{rate of change of}} \text{Acceleration}$$

Kinematics

- If we know acceleration as a function of time, how do we get from there to position vs. time?

Kinematics

- If we know acceleration as a function of time, how do we get from there to position vs. time?

- A. Look at the slope of the acceleration vs. time graph to get velocity, and then look at its slope to get position
- B. Look at the area under the curve of the acceleration vs. time graph to get velocity, and then look at the area under that graph to get position
- C. Take two derivatives of the acceleration vs. time graph to get position vs. time
- D. Take two integrals of the acceleration vs. time graph to get position vs. time

The “kinematics equations”

$$v(t) = at + v_0$$
$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

These equations are valid when...

- A. Acceleration is constant
- B. Velocity is constant
- C. The object moves in only one direction
- D. They are always valid

The "kinematics equations"

$$v(t) = at + v_0$$
$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

These equations are valid when...

- A. Acceleration is constant
- B. Velocity is constant
- C. The object moves in only one direction
- D. They are always valid

- A: these are the expressions for $x(t)$ and $v(t)$ when acceleration is constant!

Example problems

How long does it take an object to fall from a height h?

Example problems

How long does it take an object to fall from a height h?

First: Write the position and velocity equations, substituting in things you know. (Here, take ground level to be $y = 0$, and upward to be the positive direction.)

A: $x(t) = \frac{1}{2}gt^2 + h$ and $v(t) = -gt$

B: $x(t) = -\frac{1}{2}gt^2 + v_0 t + h$ and $v(t) = -gt$

C: $x(t) = -\frac{1}{2}gt^2 + h$ and $v(t) = gt$

D: $x(t) = -\frac{1}{2}gt^2 + h$ and $v(t) = -gt$

E: $x(t) = -\frac{1}{2}gt^2 + v_0 t$ and $v(t) = -gt$

Example problems

How long does it take an object to fall from a height h?

How long does it take an object to fall from a height h?

Second: Phrase the question in terms of your algebraic variables.

From the previous: $x(t) = -\frac{1}{2}gt^2 + h$ and $v(t) = -gt$

(Here, take ground level to be $y = 0$, and upward to be the positive direction.)

A: "What is the value of t when $v = 0$?"

B: "What is the value of x when $t = 0$?"

C: "What is the value of x when $v = 0$?"

D: "What is the value of t when $x = h$?"

E: "What is the value of t when $x = 0$?"

How long does it take an object to fall from a height h?

Third: Do the algebra your sentence tells you to do: "What is the value of t when $x = 0$?"

From the previous: $x(t) = -\frac{1}{2}gt^2 + h$ and $v(t) = -gt$

(Here, take ground level to be $y = 0$, and upward to be the positive direction.)

A: $\sqrt{2g/h}$
B: h/g
C: $\sqrt{2h/g}$
D: $2h/g$

Another example

You throw an object up with an initial speed of v_0. How long does it take to hit the ceiling at a height h?

Another example

You throw an object up with an initial speed of v_0. How long does it take to hit the ceiling at a height h?

$$\begin{aligned} x(t) =& \quad \frac{1}{2}at^2 + v_0 t + x_0 \\ h =& -\frac{1}{2}gt^2 + v_0 t \\ 0 =& -\frac{1}{2}gt^2 + v_0 t - h \end{aligned}$$

- $\rightarrow$ You need the quadratic formula for this – nonzero a, v_0, and position
- The quadratic formula gives you two answers, but there's clearly only one
- In this case, both roots are positive. Do you want (A) the larger one, or (B) the smaller one?

Another example

You throw an object up with an initial speed of v_0. How long does it take to hit the ceiling at a height h?

$$\begin{aligned} x(t) =& \quad \frac{1}{2}at^2 + v_0 t + x_0 \\ h =& -\frac{1}{2}gt^2 + v_0 t \\ 0 =& -\frac{1}{2}gt^2 + v_0 t - h \end{aligned}$$

- $\rightarrow$ You need the quadratic formula for this – nonzero a, v_0, and position
- The quadratic formula gives you two answers, but there's clearly only one
- In this case, both roots are positive. Do you want (A) the larger one, or (B) the smaller one?
- The homework asks you to address this idea.
- Hint: graph position vs. time, and interpret the question graphically
- What is the *mathematical* interpretation of the quadratic formula?

I am standing at the bottom of a hole of depth h. Someone throws a ball down to me at speed v_0. How fast is it going when it reaches me?

First: Write equations for $x(t)$ and $v(t)$, putting in the things you know. (Here, take ground level as zero, and downward to be positive.)

A: $x(t) = \frac{1}{2}gt^2$ and $v(t) = -gt$
B: $x(t) = -\frac{1}{2}gt^2 + v_0 t + h$ and $v(t) = -gt$
C: $x(t) = \frac{1}{2}gt^2 + v_0 t$ and $v(t) = gt + v_0$
D: $x(t) = \frac{1}{2}gt^2 - v_0 t$ and $v(t) = -gt$
E: $x(t) = -\frac{1}{2}gt^2 - v_0 t$ and $v(t) = -gt$

Example problems

I am standing at the bottom of a hole of depth h. Someone throws a ball down to me at speed v_0. How fast is it going when it reaches me?

Second: Ask a question in terms of your algebraic variables. (Here, take ground level as zero, and downward to be positive.)

A: "What is v at the time when x is h?"
B: "What is v at the time when x is 0?"
C: "What is t at the time when x is h?"
D: "What is v at the time when x is $-h$?"
E: "What is x at the time when v is 0?"

Example problems

I am standing at the bottom of a hole of depth h. Someone throws a ball down to me at speed v_0. How fast is it going when it reaches me?

Third: Do the algebra. I'll demonstrate this on the document camera. This requires two steps: first find the time, then find v.

Example problems

- A bucket is being lowered from a cliff at a rate of 10 m/s. You drop a rock off the cliff when the bucket is 10 m beneath the top. How long does it take for the rock to land in the bucket?

Same idea as before; see example on the document camera.

Piecewise continuous acceleration

A car accelerates from rest at an acceleration $a_1 = 4\,\mathrm{m/s}^2$ for a time $\tau_1 = 2$ s. After this, its acceleration changes to $a_2 = 2\,\mathrm{m/s}^2$ and it accelerates for a further time $\tau_2 = 3$ s. After this time, how far has it traveled?

Piecewise continuous acceleration

A car accelerates from rest at an acceleration $a_1 = 4\,\mathrm{m/s}^2$ for a time $\tau_1 = 2$ s. After this, its acceleration changes to $a_2 = 2\,\mathrm{m/s}^2$ and it accelerates for a further time $\tau_2 = 3$ s. After this time, how far has it traveled?

The acceleration isn't constant – but it's *piecewise constant*. Solve the motion in phases:

- Write down one set of the constant-acceleration kinematics relations that describe what happens during the first phase
- Write down a second set that describe what happens during the second phase
- **Final position/velocity of first phase → initial position/velocity of second phase**

(on document camera)

Rotational kinematics

- Linear motion: care about position as a function of time
- Rotational motion: care about angle as a function of time
- **Everything we just did translates to rotational kinematics exactly!**

Position, velocity, and acceleration

$$\text{Position} \quad \underset{\substack{\text{area under the curve of} \\ \text{(integral of)}}}{\overset{\substack{\text{(derivative of)} \\ \text{rate of change of}}}{\rightleftarrows}} \quad \text{Velocity}$$

Position, velocity, and acceleration

$$
\begin{tikzcd}[column sep=huge]
\text{Position} \arrow[r, shift right, "\substack{\text{area under the curve of}\\ \text{(integral of)}}"'] & \text{Velocity} \arrow[l, shift right, "\substack{\text{(derivative of)}\\ \text{rate of change of}}"'] \arrow[r, shift right, "\substack{\text{area under the curve of}\\ \text{(integral of)}}"'] & \text{Acceleration} \arrow[l, shift right, "\substack{\text{(derivative of)}\\ \text{rate of change of}}"']
\end{tikzcd}
$$

Angle, angular velocity, and angular acceleration

Angle $\xleftarrow[\text{area under the curve of (integral of)}]{\text{(derivative of) rate of change of}}$ Angular velocity (ω)

Angle, angular velocity, and angular acceleration

Angle	(derivative of) rate of change of $\longleftarrow$ $\longrightarrow$ area under the curve of (integral of)	Angular velocity (ω)	(derivative of) rate of change of $\longleftarrow$ $\longrightarrow$ area under the curve of (integral of)	Angular acceleration (α)

Angle, angular velocity, and angular acceleration

$$
\begin{tikzcd}
\text{Angle} \arrow[r, shift left, "\substack{\text{area under the curve of}\\\text{(integral of)}}"'] & \text{Angular velocity } (\omega) \arrow[l, shift left, "\substack{\text{(derivative of)}\\\text{rate of change of}}"'] \arrow[r, shift left, "\substack{\text{area under the curve of}\\\text{(integral of)}}"'] & \text{Angular acceleration } (\alpha) \arrow[l, shift left, "\substack{\text{(derivative of)}\\\text{rate of change of}}"']
\end{tikzcd}
$$

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

Angle, angular velocity, and angular acceleration

Angle	(derivative of) rate of change of $\longleftarrow$ $\longrightarrow$ area under the curve of (integral of)	Angular velocity (ω)	(derivative of) rate of change of $\longleftarrow$ $\longrightarrow$ area under the curve of (integral of)	Angular acceleration (α)

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$\rightarrow$ Angular kinematics works in exactly the same way as translational kinematics!

Angle, angular velocity, and angular acceleration

- Angle θ – the angle through which something has turned.
- Measured in revolutions, radians, degrees...

- Angular velocity ω ("omega", not "dubya") – the rate at which something is turning
- Measured in revolutions per second, radians per second, degrees per second...

- Angular acceleration α ("alpha", not "fish") – the rate at which something's rate of turning is changing
- Measured in $\frac{\text{rev}}{\text{s}^2}$, $\frac{\text{rad}}{\text{s}^2}$, $\frac{\text{deg}}{\text{s}^2}$...