

# RECITATION EXERCISES

## WEEK 10, DAY 1

A rock climber of mass 70 kg is climbing a cliff face when she slips and falls. She is two meters above her last anchor, so she will undergo free fall for 4 meters before the rope begins to arrest her fall. If the stiffness in her rope is 1400 N/m, then:

1. You'd like to find out how far she will fall. Draw a cartoon showing the moment that she slips and another cartoon showing her at the lowest point she falls to; indicate your choice of origin (the point that counts as  $y = 0$ ) on your cartoons.
2. Write down an expression of the conservation of energy during this process. Label each term in your equation with what it represents in words.
3. How far will she fall in total?
4. What is the maximum force that her rope will exert on her as it arrests her fall?

5. When would it be desirable for a rock climber to use a rope with a large spring constant? What about a smaller spring constant? You'll need to think about the engineering reasons for climbers to use ropes at all: the goal is to minimize the forces involved in arresting a climber's fall.

An archer is shooting arrows horizontally at a target. (The arrows are traveling fast enough that their path is mostly flat.) In order to measure how fast their arrows are traveling, they connect the target to a spring with a spring constant  $k$ . Suppose that the target has a mass of  $m_t$ , while the arrows have a mass  $m_a$ .

They shoot an arrow into their target. The arrow sticks into the target, and the impact compresses the spring by a maximum distance  $d$  before it bounces back. From this, you would like to determine the velocity  $v_a$  that the arrow was moving when it hit the target.

1. In the space below, draw three cartoons representing critical moments in the motion: right before the arrow hits the target, right after the arrow sticks in the target, and the point where the spring has reached its maximum compression.
2. You will need two different techniques (principles of physics) to analyze this motion. What principle will you use to relate the first cartoon to the second, and what principle will you use to relate the second cartoon to the third?

3. Write an equation representing each of these principles – one relating the quantities in the first cartoon to those in the second, and one relating the quantities in the second cartoon to those in the third. (*You will need to introduce a new variable for something in the second cartoon – that is okay!*)
4. Determine the velocity of the arrow in terms of  $m_a$ ,  $m_t$ , and  $k$ .

A cyclist along with their bicycle has a mass of  $m = 60$  kg. They are capable of generating a power  $P = 150$  watts for a long time. (This means that they can do work on the pedals of their bicycle at a rate of 150 watts, and thus the bicycle's traction can do work on the bicycle at the same rate.)

1. Let's imagine this cyclist riding on a flat road. On flat terrain, the main force doing negative work on the cyclist is air resistance.

The magnitude of the force from air resistance has the form  $F_{\text{drag}} = \gamma v^2$ ; it points opposite the cyclist's motion through the air. (This symbol is the lowercase Greek letter gamma.)  $\gamma$  is a constant that depends on the cyclist's shape and size.

If the cyclist (with their maximum power output of 150 W) can sustain a top speed of 9 m/s (about 20 mph) on flat ground, what is the value of  $\gamma$ ? (*This value, once you find it, is a constant for the cyclist no matter where they go.*)

2. Suppose they now travel to a steep mountain road, angled at  $\theta = 8^\circ$ , and coast downhill. (They are not pedaling.) They will travel faster and faster until they reach a speed where the sum of the forces on them is zero and they don't accelerate any more. What top speed will they attain?

3. This speed can be quite dangerous for a cyclist! Suppose that they don't want to go this fast, and want to use their brakes to slow themselves down and travel at a constant speed of 8 m/s.

Brakes use friction to do negative work on the wheel, slowing the bicycle down. However, since energy is conserved, the kinetic energy removed from the wheel by friction is converted to heat. At what rate will the bicycle's brakes build up heat going down the hill?

4. The value of  $\gamma$  is roughly the same for racing cyclists of different body types. Cycling races like the Tour de France have both uphill and downhill stages. Cyclists with smaller bodies – who have much less mass but somewhat lower maximum power output – have an advantage on uphill segments of the race. However, more muscular cyclists, with more mass and higher maximum power output, have an advantage on downhill segments.

Discuss with your group why this might be.

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*If you finish all of the above, take a look at Problem 4 on your homework with your group. It's a good one for you to work on together, since it should provoke some discussion!*