Rotational motion

Physics 211
Syracuse University, Physics 211 Spring 2015
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Announcements

- Homework due Wednesday; next homework due next Wednesday
- Next Mastering Physics assignment will be posted tonight and will be due next Tuesday before class

Ask a Physicist: Dark matter

Rotational motion

- This week we're going to redo everything that we've done so far
- Everything you've learned about linear motion has a rotational equivalent
  - Position, velocity, acceleration $\leftrightarrow$ angle, angular velocity, angular acceleration
  - Kinematics for coordinates $\leftrightarrow$ kinematics for angles
  - Newton's second law $\leftrightarrow$ Newton's law for rotation
  - Force $\leftrightarrow$ torque
  - Mass $\leftrightarrow$ moment of inertia
  - ... and others

The plan for this week

- I'm going to go over all of the concepts for rotational motion, rather quickly
- These concepts are mostly directly analogous to those you already know
- This will serve as a reference for you: find the slides online, or take notes
- Then we'll stop, slow down, do lots of demos and examples
- This material is historically difficult for students, but it doesn't need to be

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- This material is historically difficult for students, but it doesn't need to be
- Here we go: we're going to recap the entire course!

Our task

- We have so far studied the motion of "pointlike" objects
  - "This object is located at this position"
- Now we're going to consider extended objects
- They can move around, using the physics ($\vec{F} = m\vec{a}$, etc.) that you already know

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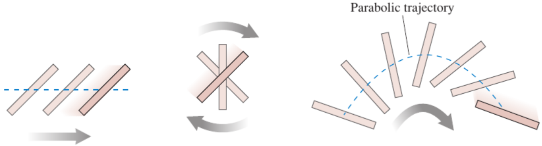
- ... but they can also *rotate*



Translational motion: The object as a whole moves along a trajectory but does not rotate.

Rotational motion: The object rotates about a fixed point. Every point on the object moves in a circle.

Combination motion: An object rotates as it moves along a trajectory.

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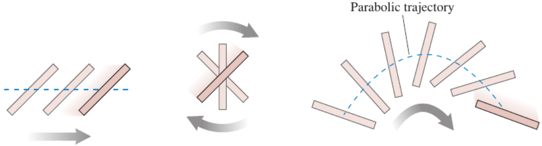

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- It turns out rotation in many of the same ways as translation: this is really review!

Describing rotation

- In the first section we were concerned about $x(t)$ and $y(t)$ – the components of the displacement vector
- Here we're concerned about angle as a function of time: $\theta(t)$
  - In two dimensions angle is a scalar: no funny vectors!
  - By convention: clockwise is negative, counterclockwise is positive
- We gave names to the derivatives of position: "velocity" and "acceleration"
- Likewise, we give names to the derivatives of angle
  - $\omega \equiv \frac{\partial \theta}{\partial t}$: "angular velocity", measured in rad/s (you knew this!)
  - $\alpha \equiv \frac{\partial^2 \theta}{\partial t^2}$: "angular acceleration", measured in rad/s^2 (new!)

Going from rotation to translation

- "If an object is turning at some angular velocity ω, how fast is a piece at radius r moving?"
- We did this before: remember the tangential velocity $v_T = \omega r$, based on the definition of the radian
- This holds true for all of the derivatives of angle and position:
- Arc length: $s = \theta r$
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- To go from rotational descriptions of motion to tangential ones, multiply by the radius

Rotational kinematics

- The constant-acceleration kinematics we learned was really about *the relation of a function to its derivatives*
- Since the same derivative relationships hold, the exact same kinematics applies:

Translation	Rotation
Position x	Angle θ
Velocity v	Angular velocity ω
Acceleration a	Angular acceleration α
$v(t) = v_0 + at$	$\omega(t) = \omega_0 + \alpha t$
$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v_f^2 - v_0^2 = 2a\Delta x$	$\omega_f^2 - \omega_0^2 = 2\alpha\Delta\theta$

Relate θ, ω, α, and t the same way you do x, v, a, and t

Note that one revolution $=$ 360 degrees $= 2\pi$ radians!

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Let's look at each of those in turn.

Torque

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- The size of the torque depends on three things:
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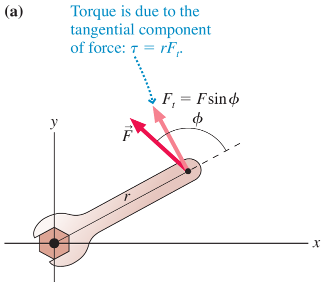
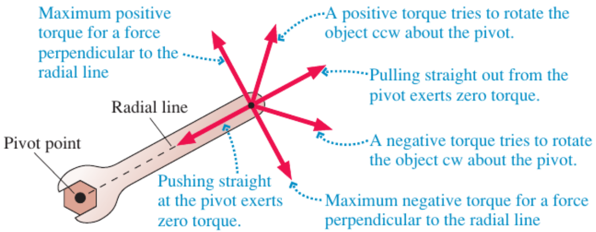
- The distance from the force to the pivot point
  - The further from the pivot to the point of force, the greater the torque
  - This is why the door handle is on the outside of the door...
- The angle at which the force is applied
  - Only forces "in the direction of rotation" make something turn
  - The torque depends only on the *component of the force perpendicular to the radius*

$$\tau = F_{\perp} r$$

Torque is equal to the distance from the pivot, times the perpendicular component of the force

(There is an equivalent alternative we will see a bit later)



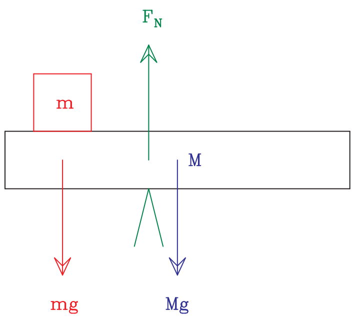


Note that torque has a sign, just like angular velocity: CCW is positive; CW is negative.

Important notes about torque

These are very important: note them somewhere for later reference!

- Torques are in reference to a **particular pivot**
- This is different from force; if you're talking about torque, you *must* say what axis it's measured around
- Torque now depends on the *location* of forces, not just their size
  - Your force diagrams now need to show the place where forces act!
  - Weight acts at the center of mass ("the middle"); we'll see what that means later
  - A saimple force diagram might look like this:



What about the mass analogue?

$$\vec{F} = m\vec{a}$$
$$\tau = ?\alpha$$

Mass tells you how hard it is to give something a (linear) acceleration.
What determines how hard something is to turn?

We can already look at situations where $\tau = \alpha = 0$: "static equilibrium"

Moment of inertia

The analogue of mass is called "moment of inertia" (letter I)

- More massive things are harder to turn, but that's only part of it
- The mass *distribution* matters, too
- The further the mass is from the center, the harder it will be to turn
- The moment of inertia depends on the *average squared distance from the center*
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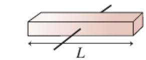
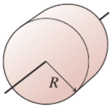
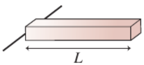
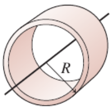
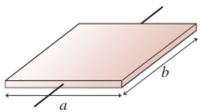
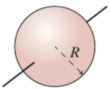
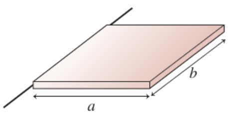
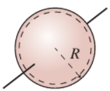
$$I = MR^2$$

(if all the mass is the same distance from the center)
(our demo rods; hoops; rings; bike wheels)

Moment of inertia, other things

What about the moment of inertia of other objects?
Requires calculus in general; here are some common ones

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

Putting it together: Newton's law for rotation

Translation	Rotation
Force $\vec{F}$	Torque: $\tau = F_\perp r$
Mass m	Moment of Inertia: $I = \lambda M R^2$
Acceleration $\vec{a}$	Angular acceleration α
$\vec{F} = m\vec{a}$	$\tau = I\alpha$

This last line can be thought of as "Newton's second law for rotation".

Torques give things angular acceleration, just like forces make things accelerate:

$$\tau = I\alpha$$

What about energy?

We saw before that the work-energy theorem was just a consequence of the "third kinematics relation". Is there anything that corresponds to this for rotation?

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Yes, and it's exactly what you'd expect!

Translation	Rotation
Force $\vec{F}$	Torque: $\tau = F_{\perp} r$
Mass m	Moment of Inertia: $I = \lambda M R^2$
Displacement $\vec{s}$	Angular displacement θ
Velocity $\vec{v}$	Angular velocity ω
Work $= \vec{F} \cdot \Delta \vec{s}$	Work $= \tau \Delta \theta$
Kinetic energy $\frac{1}{2} m v^2$	Kinetic energy $\frac{1}{2} I \omega^2$

There is rotational kinetic energy $\frac{1}{2} I \omega^2$ associated with spinning objects!

This is just another term to add to your conservation of energy relations!

What about momentum?

Translation	Rotation
Mass m	Moment of Inertia: $I = \lambda M R^2$
Velocity $\vec{v}$	Angular velocity ω
Momentum $\vec{p} = m\vec{v}$	Angular momentum $L = I\omega$

Just as momentum is conserved in the absence of external forces, angular momentum is conserved in the absence of external torques.