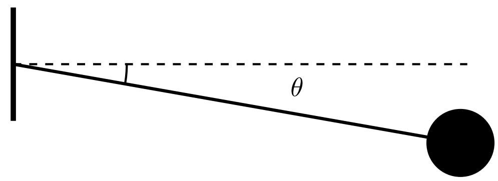
PHY 211 Homework 6

Due February 26, 2020

Note: For all problems, in order to receive credit, you must draw force diagrams for all relevant objects. These diagrams must be at least two inches tall to receive full credit. This is for your benefit, not mine; carefully drawing clear diagrams will help you with these problems more than anything else.

Problem 1. A heavy ball with mass 10 kg is hung from the ceiling of Stolkin Auditorium on a rope of length 4.5 m. It is pulled to one side and released, swinging like a pendulum; at its lowest point, it reaches a speed of 6 m/s. What is the tension in the rope at that point?

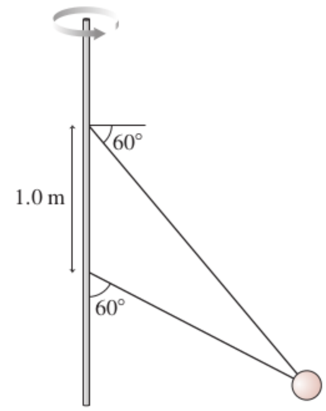
Problem 2. A ball of mass 0.5 kg on the end of a wire 1.0 m long is being spun horizontally around a central pole at 2 revolutions per second. Because of gravity, it dips below the horizontal at an angle θ. What is the angle θ? What is the tension in the wire?



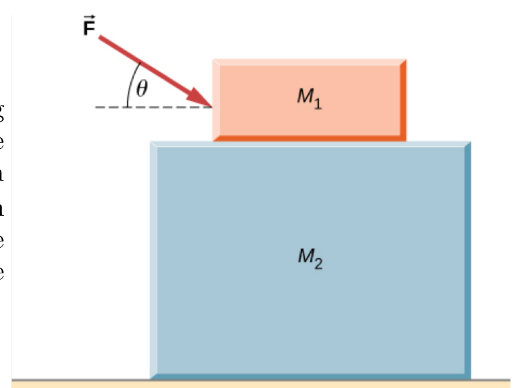
Problem 3. Two wires are tied to the sphere of mass 3 kg shown here. The sphere revolves around the pole in a horizontal circle at constant speed. *Hint: for this problem you do actually need to use geometry to find the radius of the circle made by the sphere.*

(a) For what speed is the tension the same in both wires?

(b) What is that tension?



Problem 4. Two blocks of mass $m_1 = 1.0\,\text{kg}$ and $m_2 = 6.0\,\text{kg}$ are stacked as shown, and rest on a frictionless surface. There is static friction between the two blocks (coefficient of friction $\mu_s = 0.5$). An external force $\vec{\mathbf{F}}$ is applied to the top block at an angle $\theta = 25°$ with the horizontal. What is the magnitude of the maximum force F that can be applied for the two blocks to move together?



Backwards problem

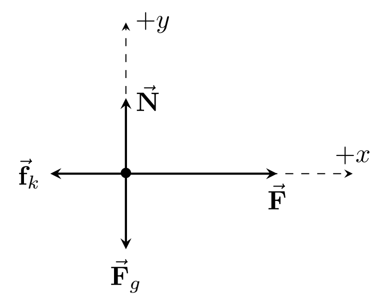
The following problem has been solved *incorrectly*. You should identify the mistake or mistakes, and then find the correct answer to the problem.

Problem 5. A stack of two books, each of mass 1 kg, sits on a table. The coefficients of static friction between the books, and between the bottom book and the table, are 0.4; the coefficients of kinetic friction are 0.2. A person exerts a sudden force $F = 15\,\mathrm{N}$ on the bottom book. What are the accelerations of the two books?

Solution. First we need to find out if the books are moving by comparing the applied force to the static friction. Call the top book "book 1", and the bottom book "book 2". For the books to move at all, the force needs to overcome the maximum static friction between the table and the "two-book system".

$$F \geq \mu_s N_{\text{combined}} = \mu_s(2mg) = 0.4 \cdot 2 \cdot 1\,\mathrm{kg} \cdot 10\,\mathrm{m/s^2} = 8\,\mathrm{N}.\checkmark$$

The free-body diagram for the two-book system would look like the diagram to the right. It would accelerate the books at



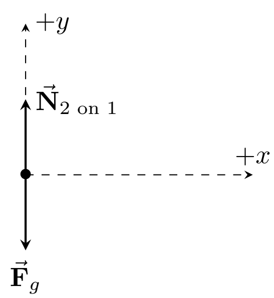
$$\begin{aligned}\sum F_x &= F - f_k = F - \mu_k(2mg) \\ &= 15\,\mathrm{N} - 0.2 \cdot 2 \cdot 1\,\mathrm{kg} \cdot 10\,\mathrm{m/s^2} = 11\,\mathrm{N} \\ a_x &= 11\,\mathrm{N}/2\,\mathrm{kg} = 5.5\,\mathrm{m/s^2}\end{aligned}$$

The top book is pulled forward by static friction. At maximum it can accelerate the top book by

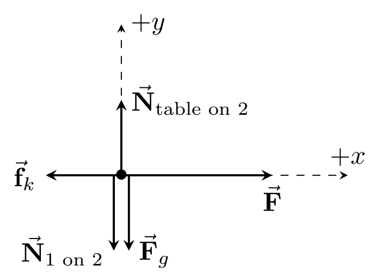
$$\begin{aligned}\sum F_x &= \mu_s(mg) = 4\,\mathrm{N} \\ a_x &= 4\,\mathrm{N}/1\,\mathrm{kg} = 4\,\mathrm{m/s}\end{aligned}$$

So the bottom book will actually slide out from underneath! We need to separate the books and draw free-body diagrams for both.

Book 1



Book 2



We use the sums of the forces in y to get the normal forces, since the books aren't accelerating in that direction.

$$\begin{aligned}\sum F_{y,1} &= N_{2\text{ on }1} - mg = 0 \\ N_{2\text{ on }1} &= mg = N_{1\text{ on }2} \\ \sum F_{y,2} &= N_{\text{table on }2} - N_{1\text{ on }2} - mg = 0 \\ N_{\text{table on }2} &= 2mg\end{aligned}$$

There are no forces in the x direction on book 1, so it does not accelerate. Book 2 has the sum

$$\begin{aligned}\sum F_{x,2} &= F - f_k = F - \mu_k N_{\text{table on }2} \\ &= F - 2\mu_k mg = 15\,\mathrm{N} - 2 \cdot 0.2 \cdot 1\,\mathrm{kg} \cdot 10\,\mathrm{m/s^2} = 11\,\mathrm{N}\end{aligned}$$

So this gives an acceleration

$$a_{x,2} = \sum F_{x,2}/m = 11\,\mathrm{N}/1\,\mathrm{kg} = 11\,\mathrm{m/s}^2$$

$\blacksquare$