1D kinematics review; vectors

Physics 211
Syracuse University, Physics 211 Spring 2018
Walter Freeman

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On solving problems

You can recognize truth by its beauty and simplicity. When you get it right, it is obvious that it is right–at least if you have any experience–because usually what happens is that more comes out than goes in.... Inexperienced students make guesses that are very complicated, [but] the truth always turns out to be simpler than you thought.

–Richard Feynman, quoted by K. C. Cole, in *Sympathetic Vibrations: Reflections on Physics as a Way of Life (1985)*

Nature uses only the longest threads to weave her patterns, so each small piece of her fabric reveals the organization of the entire tapestry.

–Richard Feynman, *The Character of Physical Law* (1965)

Announcements

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- Homework 1 due tomorrow
- **My clinic hours: Wednesday 4:30-6:30, Thursday 2:00-4:00**

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- (So what's the physics clinic?)

Last time: Position, velocity, and acceleration

Position $\xleftarrow[\text{take a derivative (slope of the graph)}]{\text{take an integral (area under curve)}}$ Velocity

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Velocity

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Position ⇄ Velocity ⇄ Acceleration

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The “kinematics equations”

$$v(t) = at + v_0$$
$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

... valid only when a is constant!

Example problems

I am standing at the bottom of a hole of depth h. Our coach Alicia is standing at the top. I throw a ball up to her at speed v_0. How fast is it going when she catches it?

First: Write equations for $x(t)$ and $v(t)$, putting in the things you know. (Here, take ground level as zero, and upward to be positive.)

A: $x(t) = \frac{1}{2}gt^2$ and $v(t) = -gt$
B: $x(t) = -\frac{1}{2}gt^2 + v_0 t + h$ and $v(t) = v_0 - gt$
C: $x(t) = \frac{1}{2}gt^2 + v_0 t$ and $v(t) = gt + v_0$
D: $x(t) = \frac{1}{2}gt^2 - v_0 t$ and $v(t) = -gt$
E: $x(t) = -\frac{1}{2}gt^2 - v_0 t$ and $v(t) = -gt$

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Second: Ask a question in terms of your algebraic variables. (Here, take ground level as zero, and upward to be positive.)

A: "What is v at the time when x is h?"

B: "What is v at the time when x is 0?"

C: "What is t at the time when x is h?"

D: "What is v at the time when x is $-h$?"

E: "What is x at the time when v is 0?"

I am standing at the bottom of a hole of depth h. Someone throws a ball down to me at speed v_0. How fast is it going when it reaches me?

Third: Do the algebra. I'll demonstrate this on the document camera. This requires two steps: first find the time, then find v.

Example problems

What is the solution to this equation to find the time? *Note: You can get fancy folding your cards to show more than one answer!*

A: $\frac{-v_0+\sqrt{v_0^2-2gh}}{-g}$

B: $\frac{-v_0+\sqrt{v_0^2-4gh}}{-g}$

C: $\frac{-v_0-\sqrt{v_0^2-2gh}}{-g}$

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Is this a physics class or a math class? Remember to think about what your answers mean!

Example problems

- A bucket is being lowered from a cliff at a rate of 10 m/s. You drop a rock off the cliff when the bucket is 10 m beneath the top. How long does it take for the rock to land in the bucket?

Same idea as before; see example on the document camera.

Vectors

You've been doing math with numbers, which are things that live in one dimension: they only have a magnitude and a sign.

Vectors are things that have a magnitude and a direction: "arrows in space"

Many of the things we deal with in physics are vectors:

- Position

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So, we need to learn to do math with arrows.

- We indicate that an object is a vector by writing an arrow over it: "the vector $\vec{V}$".
- "Scalar": object that isn't a vector (mass, time)
- Equations can mix vectors and scalars: $\vec{F} = m\vec{a}$.
- ... or $\vec{s} = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$

Notation

- $\vec{A}$: "the vector A" (a vector)
- A: "the magnitude of A" (a scalar)
- $\hat{A}$: "the direction A points in" (a vector with magnitude 1)
- A_x: the component of A along the $x-$axis (a scalar)
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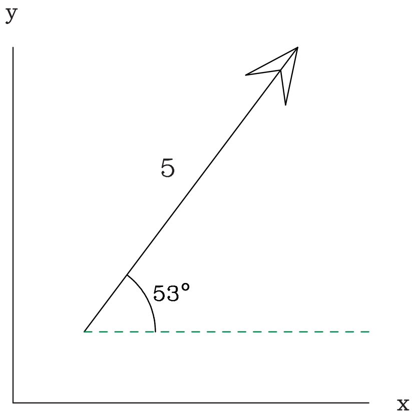
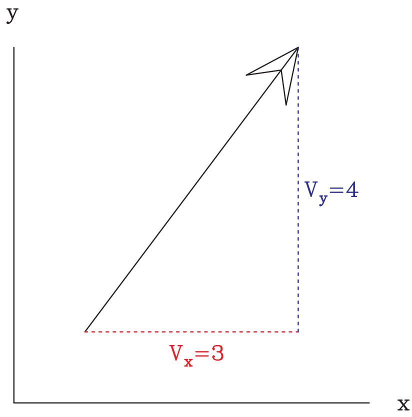
Two ways to describe a vector



Angle and direction



X and Y components

How do we convert from one to the other?

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A: Using algebra

B: Using trigonometry

C: Using calculus

D: Using differential equations

Suppose you have some vector $\vec{A}$ that you want to convert into components. The x-component A_x is:

A: $A\cos\theta$

B: $A\sin\theta$

C: $A\tan\theta$

D: $\dfrac{A}{\cos\theta}$

E: It depends

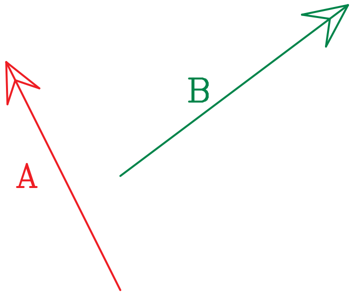
A warning!

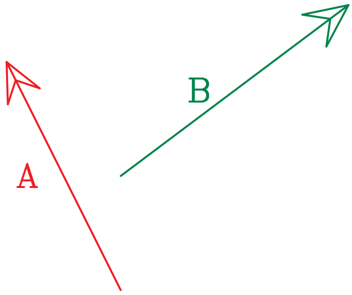
You cannot memorize "$V \sin\theta$ is the y component,
$V \cos\theta$ is the x component"!

This does *not* work in general; you have to actually draw the triangle.

Adding vectors

You can do algebra with vectors in the ordinary way!
You add vectors together by drawing them “head to tail”. Here are two vectors:





Does $\vec{A} + \vec{B} = \vec{B} + \vec{A}$?

- A: Yes
- B: No

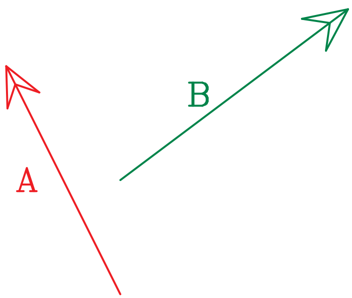
Does $\vec{A} + \vec{B} = \vec{B} + \vec{A}$?

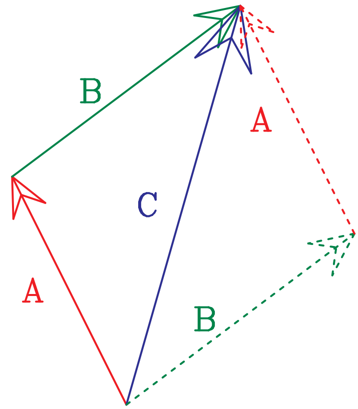
- A: Yes
- B: No

Yes: vector addition obeys the commutative property, just like ordinary addition

Adding vectors

We can also add vectors together by drawing them "head to tail". Here are two vectors:





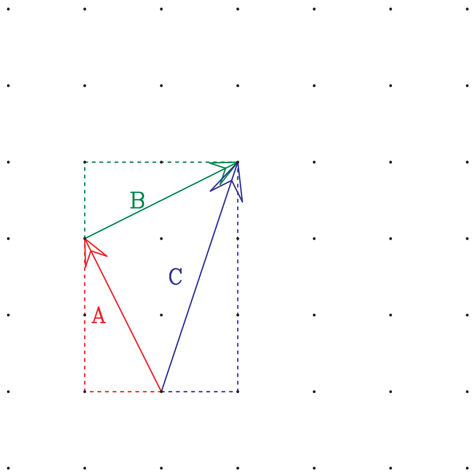
$$\vec{A} + \vec{B} = \vec{C}$$

Adding vectors: components

The component representation is much easier to work with!

$$\vec{A} + \vec{B} = \vec{C} \rightarrow \begin{pmatrix} A_x + B_x = C_x \\ A_y + B_y = C_y \end{pmatrix}$$

Adding vectors: components



To add two vectors, just add their components!

This is why it is almost always easiest to work in the component representation!

What does this do to our kinematics?

Acceleration, velocity, and position relationships are still the same; they just apply independently for each component.

$$\vec{v}(t) = \vec{a}t + \vec{v}_0$$
$$\vec{s}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$$

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$$v_y(t) = a_y t + v_{y,0}$$

$$x(t) = \frac{1}{2}a_x t^2 + v_{x,0} t + x_0$$
$$y(t) = \frac{1}{2}a_y t^2 + v_{y,0} t + y_0$$

The x and y equations of motion separate!

A demo with blocks...

Unit vectors

In the "ordered pair" notation for vectors' components, you might write:

$$\vec{v} = (5, 3)$$

But this is clunky, if you're trying to write it as part of an algebraic statement.

Instead we introduce "unit vectors", vectors with length 1, in the x, y, and z directions.

$$\begin{aligned}\hat{i} =&(1, 0, 0)\\ \hat{j} =&(0, 1, 0)\\ \hat{k} =&(0, 0, 1)\end{aligned}$$

- $\vec{v} = (5, 3)$: Ordered pair
- $\vec{v} = 5\hat{i} + 3\hat{j}$: Unit vectors

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- $\vec{v} = (5, 3)$: Ordered pair
- $\vec{v} = 5\hat{i} + 3\hat{j}$: Unit vectors
- Both give you the same information, but unit vectors can be easier algebraically
- You'll see the unit-vector notation occasionally in homework etc.; it's just telling you the components.

Which statement does *not* make sense?

- a. $\vec{A}t = \vec{B}$
- b. $\vec{A} + \vec{B} + t = \vec{C}$
- c. $k(\vec{A} + \vec{B}) = k\vec{A} + k\vec{B}$
- d. $\vec{A} - \vec{B} = \vec{C}$

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B: You can't add a vector and a scalar. "One mile north plus one inch" – which way is the inch?