Physics 211 Practice Exam 2

- Problem 1 is a basic problem that involves two objects moving in one dimension.
- Problem 2 tests your ability to think clearly about force diagrams and motion in the presence of acceleration.
- Problem 3 involves an object with forces in two dimensions, including friction.
- Problem 4 involves two objects connected together, one of which has forces in two dimensions.
- Problem 5 is a "backwards problem" in which we give you a flawed solution; you must fix it.
- Problem 6 involves uniform circular motion and friction.
- Problem 7 tests your ability to think clearly about uniform circular motion both from the perspective of the rotating object and from outside. A subsequent part asks you to think about connected objects in circular motion.
- Problem 8 is another "backwards problem" in which we give you a flawed solution.

d) What is the tension in the string connecting block 1 and block 2? (7 points)

Now we need the individual FBD's:

$\Sigma F = ma$ for each object separately:

Block 1: $T_1 - m_1 g = m_1 a$

Block 2: $F_e - m_2 g - T_1 = m_2 a$.

From (1): $a = \dfrac{T_1 - m_1 g}{m_1}$

Substitute into (2):

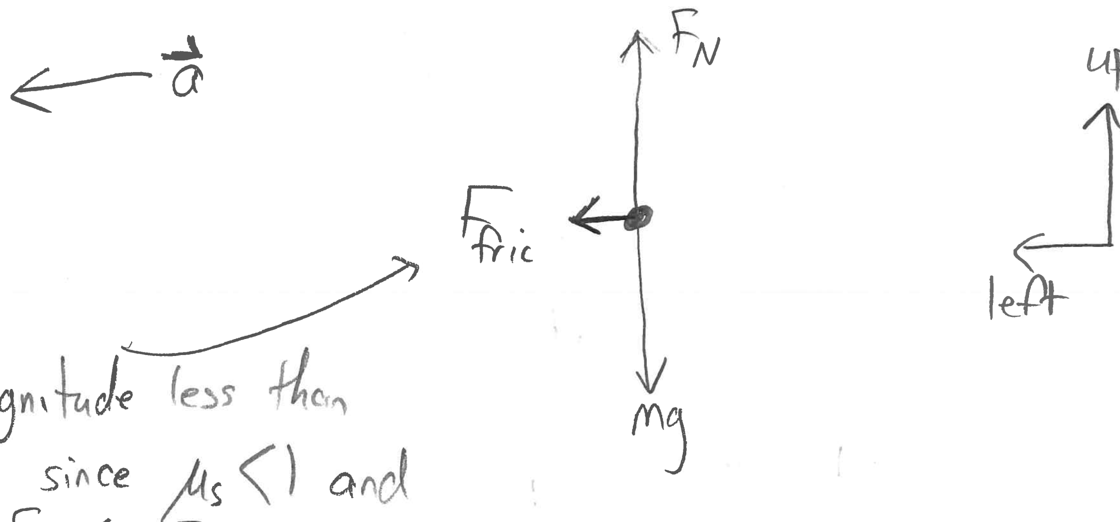
$$F_e - m_2 g - T_1 = (m_2 T_1 - m_2 m_1 g)/m_1$$

Solve for T_1:

$$m_1 F_e - \cancel{m_1 m_2 g} - T_1 m_1 = m_2 T_1 - \cancel{m_2 m_1 g}$$

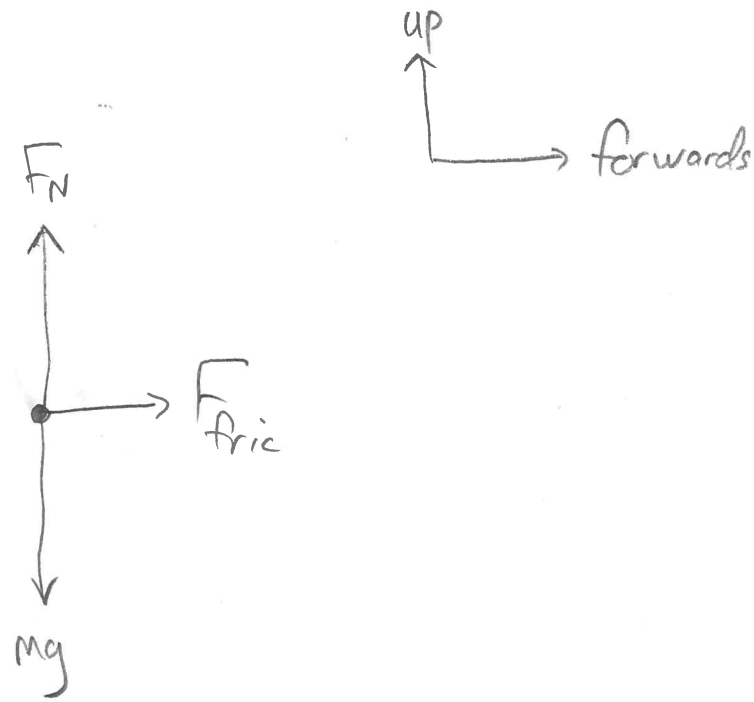
$$\boxed{T_1 = \frac{m_1 F_e}{m_1 + m_2}}$$

c) The subway car is moving at a constant speed v; it is turning left, gently enough that the passengers do not slip and fall. (5 points)



magnitude less than F_N since $\mu_s < 1$ and $F_{fric} \leq \mu_s F_N$.

d) The subway car is accelerating forward at $3\,\mathrm{m/s^2}$. (5 points)



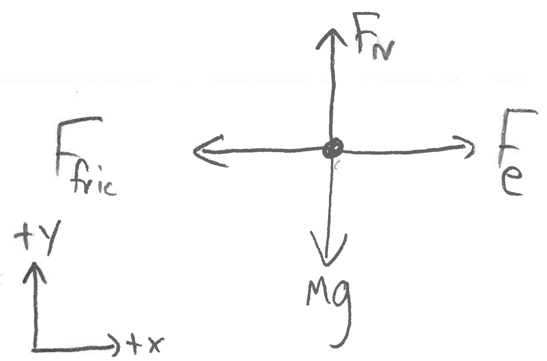
QUESTION 3

The coefficient of kinetic friction between a table of mass $m = 100$ kg and the ground is $\mu_k = 0.6$. You would like to push this table across the floor at a constant speed.

Calculate the minimum force required to keep the table moving across the floor at a constant speed under each of the following conditions. If *no* force, no matter how large, will move the table, then say so. Note that you will want to draw force diagrams as part of your solutions to each part.

a) You push on the table horizontally, parallel to the ground. (5 points)

(constant speed)



$$\sum F_x = F_e - F_{fric} = ma_x = 0$$

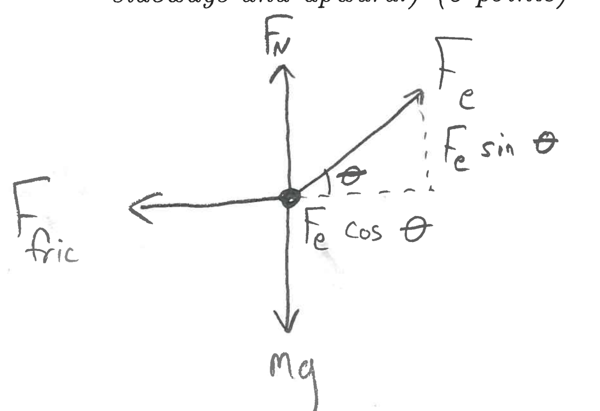
$$\sum F_y = F_N - mg = ma_y = 0.$$

→ Look at (y) to find F_N, then use that with $F_{fric} = \mu F_N$.

(y) $F_N - mg = 0 \Rightarrow F_N = mg$. (x) becomes: $F_e - \mu F_N = 0 \Rightarrow F_e - \mu mg = 0$,

so $F_e = \mu mg = (0.6)(100 \text{ kg})(10 \text{ m/s}^2) = 600 \text{ N}$.

b) You push on the table at an angle directed 20 degrees above the horizontal (that is, you are pushing sideways and upward.) (5 points)



$$X: F_e \cos\theta - \overbrace{\mu F_N}^{F_{fric}} = ma_x = 0$$

$$Y: F_N - mg + F_e \sin\theta = ma_y = 0.$$

From (Y): $F_N = mg - F_e \sin\theta$.

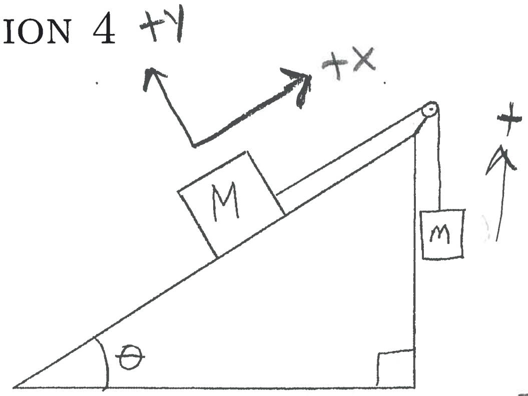
Substitute into (X): $F_e \cos\theta - \mu(mg - F_e \sin\theta) = 0$.

$$F_e \cos\theta - \mu mg + \mu F_e \sin\theta = 0 \Rightarrow \boxed{F_e = \frac{\mu mg}{\cos\theta + \mu \sin\theta}.}$$

$= 524$ N.

Question 4

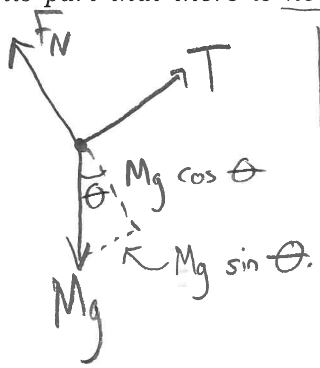
A block of mass M sits on an inclined plane angled at an angle θ above the horizontal;
it is connected by a string to a block of mass m hanging over the top. (See picture.)



$\vec{a} = 0$

a) In terms of M and m, what must the angle θ be such that the two blocks do not move? Assume for this part that there is no friction. (7 points)

M:



m:

X for M: $T - Mg\sin\theta = Ma_x = 0$ $\quad a_h = a$ for hanging mass.

Y for M: $F_N - Mg\cos\theta = Ma_y = 0$

for hanging: $T - mg = ma_h = 0.$

$T - mg = 0 \rightarrow T = mg$

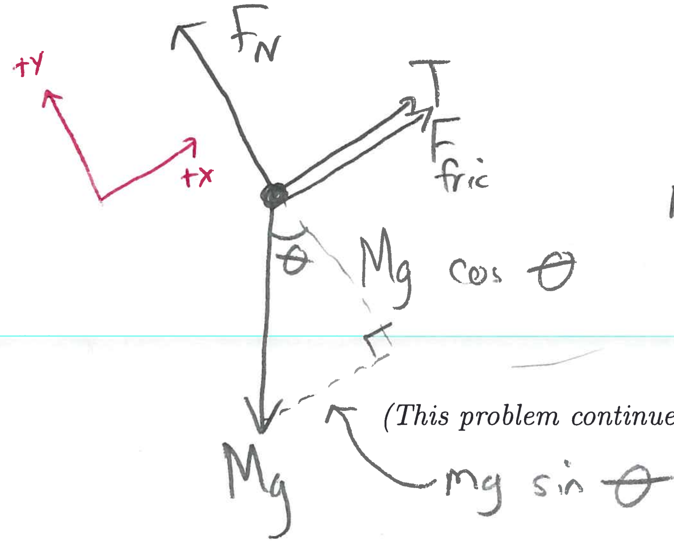
Sub into (X): $mg - Mg\sin\theta = 0$, so

$\sin\theta = \frac{mg}{Mg}$, $\boxed{\theta = \sin^{-1}\frac{m}{M}.}$

Now, assume that M is large enough that it slides down the ramp. There is kinetic friction between that block and the ramp; the coefficient of kinetic friction is μ_k.

b) Draw force diagrams for both blocks. Indicate your choice of coordinate system for both of them (they do not have to be the same, and in fact shouldn't be!) (3 points)

M:



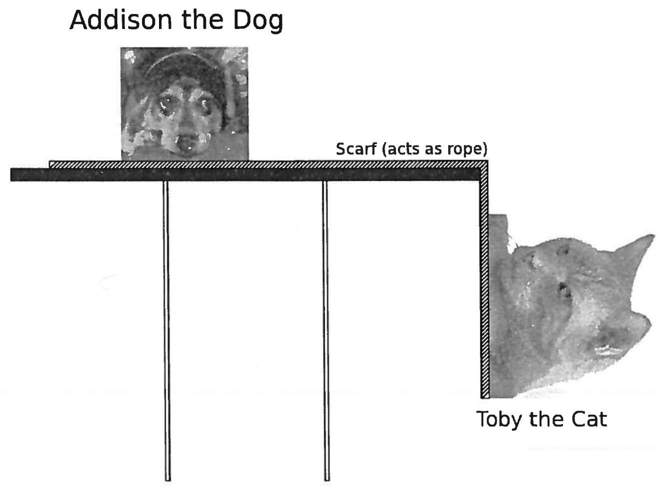
m:

(This problem continues on the next page.)

Question 5

A long scarf rests on a table. Our student Shelby's dog Addison is asleep on one end of it; the other end hangs off the edge of the table.

Alice's cat Toby sees the other end of the scarf hanging over the edge of the table. Toby jumps up and grabs the edge, and her weight begins to pull Addison and the scarf off the table. (You may assume that the scarf doesn't stretch and has negligible mass.)



Addison has a mass m_A; Toby has a mass m_T. The coefficient of kinetic friction between the scarf and the table is μ_k; since the scarf is so light, the only place where there is friction is underneath Addison.

I would like to find the acceleration of the two animals and the scarf.

On the next page, you'll find my solution, but my solution contains an error. On the following page, I will ask you a few questions about my work, and ask you to fix my mistake.

a) The solution I got for the acceleration of the animals is

$$a = \frac{m_T g - \mu_k m_A g}{m_A - m_T}.$$

Right away, something about the mathematical form of this solution should tell you that there is a mistake. What about this answer should make you skeptical? (5 points)

The denominator goes to zero if $m_A = m_T$.

b) What mistake did I make? You can describe it briefly here, or indicate it clearly on the previous page. (10 points)

The accelerations have equal magnitude but opposite sign. Instead $a_{A_x} = -a_{T_y}$.

c) What should the answer be instead? Correct my work on the previous page or below, and tell me what the acceleration should be instead. (10 points)

Fixed on prev. page - see red writing:

$$a_A = \frac{m_T g - \mu_k m_A g}{m_A + m_T}.$$

c) Suppose now that the children spinning the platform want to slow it down enough that their friends on top can safely walk to the edge and jump off. What is the maximum angular velocity ω that would allow a child to stand on the edge of the platform without slipping? (5 points)

Just use the relation for r_{max} as a function of ω that we got earlier and set $r = 3m$.

$$r_{max} = \frac{\mu_s g}{\omega^2} \Rightarrow \omega = \sqrt{\frac{\mu_s g}{r_{max}}}.$$

Set $r_{max} = 3\,m$: $\omega = 1.29\ rad/s$.

b) At what rate must the spacecraft rotate so that the people aboard experience artificial gravity that feels equal to Earth's? Give your answer in terms of g and R. (5 points)

: $F_N = ma$, $a = \omega^2 r$ (circular motion)

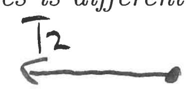
$F_N = mg$ (trying to mimic Earth)

$$mg = m\omega^2 r \longrightarrow \boxed{\omega = \sqrt{g/r}}.$$

This station is powered by solar panels of mass m connected by cables to the central hub. A cable of length $\frac{1}{2}R$ runs from the hub to the inner panel; a second cable runs from the inner panel to the outer panel. These solar panels also rotate along with the rest of the station at the same angular velocity.

c) Draw a force diagram for the inner solar panel and the outer solar panel. (Note that the tension in the two cables is different.) (5 points)

Outer:



: $T_2 = ma_o = m\omega^2 R \Rightarrow \boxed{T_2 = m\omega^2 R = mg.}$

Inner:

: $T_1 - T_2 = ma_i = m\omega^2 (\frac{1}{2}R).$

Substitute:

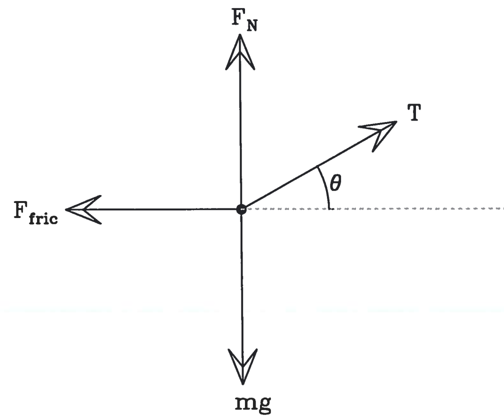
$+ \leftarrow$

$T_1 - m\omega^2 R = \frac{1}{2} m\omega^2 R$, so $\boxed{T_1 = \frac{3}{2} m\omega^2 R = \frac{3}{2} mg.}$

d) In terms of m, R, and ω, calculate the tension T_1 in the cable between the hub and the inner solar panel, and the tension T_2 in the cable between the inner solar panel and the outer solar panel. (7 points)

(see above.)

Since this problem asks us to connect the forces on objects to their acceleration, I will use Newton's second law $\vec{F} = m\vec{a}$ and solve for $\vec{a}$.



First I draw a force diagram for the object. Imagine that the rope is pulling up and to the right. Then friction points to the left. The normal force points upward to stop the object from falling through the ground, and gravity points downward.

Lies!

Since the object moves only in the x-direction, I only need to worry about it. The x–component of the tension in the rope is $T\cos\theta$.

Reading Newton's second law off of the force diagram, we have

$$\sum F_x = ma_x$$
$$T\cos\theta - F_{\text{fric}} = ma_x$$

We know that the frictional force is $\mu_k F_N$; since the object is resting on a flat surface, $F_N = mg$. Putting this in:

$$T\cos\theta - \mu_k mg + \mu_k T\sin\theta = ma_x$$

$$\cancel{T\cos\theta - \mu_k mg = ma_x}$$

nope–
instead

$$F_N = mg - T\sin\theta$$

which gives us an acceleration of

$$\cancel{a = \frac{T\cos\theta - \mu_k mg}{m}}$$

new term: increases a

$$a = \frac{T\cos\theta - \mu_k mg + \mu_k T\sin\theta}{m}$$