Hi Physics 211! <3

Power; reviewing work and energy

Physics 211
Syracuse University, Physics 211 Spring 2020
Walter Freeman

March 31, 2020

How are you all doing?

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How have your classes been going?

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How has *this* class been going?

Announcements

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- Your questions

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- A few demo problems on conservation ~~of momentum and~~ energy + the work-energy theorem

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- Your questions
- A few demo problems on conservation of momentum and energy

- A new idea, in more depth: *power*
- An example of that idea

Your questions

What would you all like to talk about? (Homework, recitation problems, big ideas...)

While you're thinking: how useful are the recordings of recitation and homework solutions?

- **A:** Not useful
- **B:** Moderately useful
- **C:** Quite useful
- **D:** I've not watched any of them yet

Please give me feedback on them (what can I do better?) if you've watched any.

$$\vec{F}_{AB} = -\vec{F}_{BA} \longrightarrow$$ assumed only forces were the ones exerted on each other

$$m_A \vec{a}_A = -m_B \vec{a}_B$$

$$m_A \Delta \vec{v}_A = -m_B \Delta v_B$$

$$\Delta \vec{p}_A = -\Delta \vec{p}_B \longrightarrow$$ Conservation of momentum

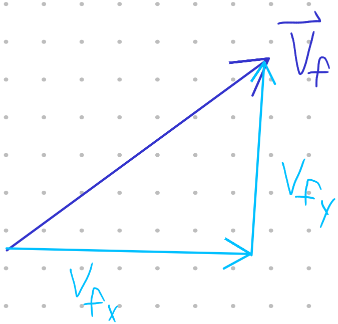
$\boxed{\text{Collisions / explosions} \longrightarrow \text{use Conservation of } \vec{p}!}$

$$m_1\vec{V}_{1_i} + m_2\vec{V}_{2_i} + m_3\vec{V}_{3_i} + \cdots = m_1\vec{V}_{1_f} + m_2\vec{V}_{2_f} + m_3\vec{V}_{3_f}$$

Vector equation: applies separately and independently in X and Y.

$$X: \; m_1 V_{1x_i} + m_2 V_{2x_i} + \cdots = m_2 V_{2x_f} + m_2 V_{2x_f} + \cdots$$

$$Y: \; m_1 V_{1y_i} + m_2 V_{2y_i} + \cdots = m_2 V_{2y_f} + m_2 V_{2y_f} + \cdots$$



"Plain" work-energy

$$KE_i + W_{all} = KE_f$$

With potential energy:

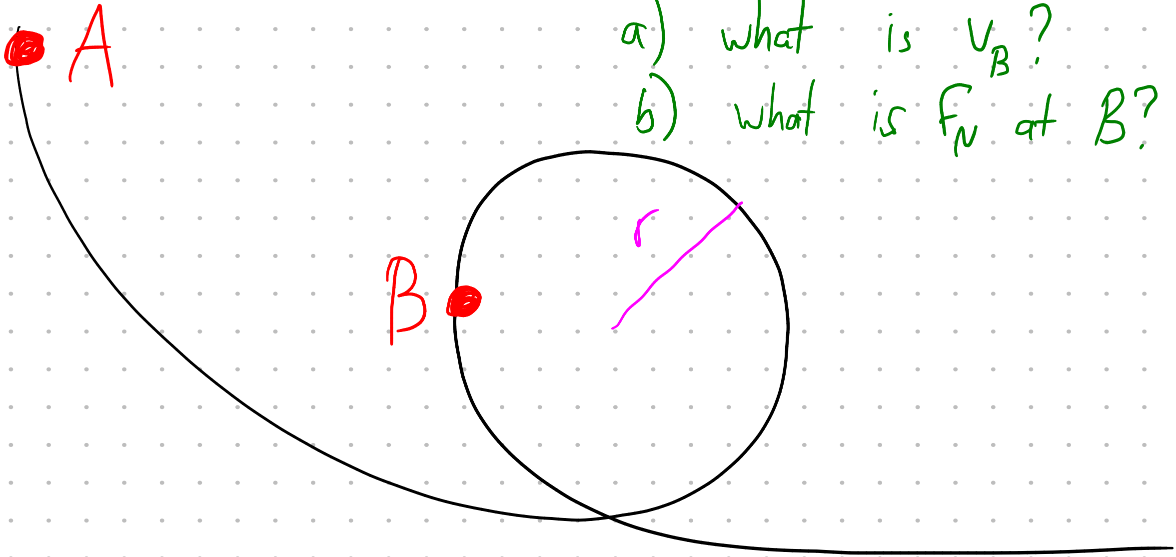
$$KE_i + PE_i + W_{other} = KE_f + PE_f$$

Use energy methods when

- You don't care about time
- You have clear "before" and "after" states
- You can account for the work done by all forces.

a) what is v_B?

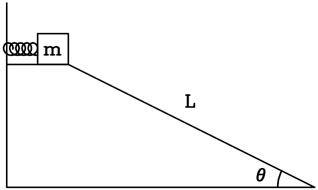
b) what is F_N at B?



The work-energy theorem and conservation of energy

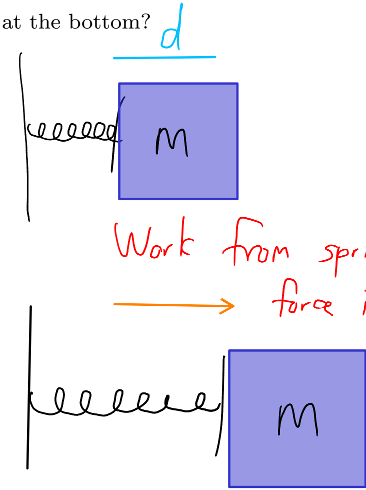
"When do I use the conservation of energy and when do I use the work-energy theorem?"

They are really the same: *potential energy* is a bookkeeping device for the work done by conservative forces.



Some students are sledding down the hill in front of the music building; it has a length L and is at a slope θ. To go faster, they build a sled-launcher, consisting of a spring of spring constant k. A student compresses it by a distance d and launches themselves down the hill.

How fast are they going at the bottom?



initial

Work from spring is positive: force is in direction of motion.

final

What's the work done by **the spring**?

- A: $W_{\text{elas}} = -\frac{1}{2}kd^2$
- B: $W_{\text{elas}} = +\frac{1}{2}kd^2$ ✓
- C: $W_{\text{elas}} = +kd$
- D: $W_{\text{elas}} = -kd$

- Spring converts its elastic potential energy into kinetic energy: $U_e = \frac{1}{2}kd^2$

$W = \Delta KE$, $W_{elas} = \frac{1}{2}kd^2$.

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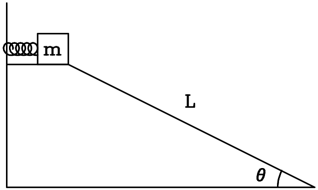

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How fast are they going at the bottom?

going to equilibrium

$$W_{elas} = \int_{-d}^{0} F\,dx$$

; need integral since F is not constant.

compressed by d at start

What's the work done by **the spring**?

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- B: $W_{\text{elas}} = +\frac{1}{2}kd^2$
- C: $W_{\text{elas}} = +kd$
- D: $W_{\text{elas}} = -kd$

$F_{elas} = -k(\Delta x)$; k = "spring constant" – measured in newtons/meter

"how many newtons of restoring force does a spring produce per meter of stretch/compression?"

"When do I use the conservation of energy and when do I use the work-energy theorem?"

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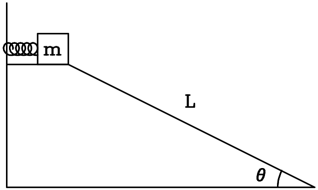

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How fast are they going at the bottom?

$$W = \int F\,dx = \int kx\,dx = \frac{1}{2}kx^2.$$

What's the work done by **the spring**?

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Ah! $U_e = \frac{1}{2}k(\Delta x)^2$

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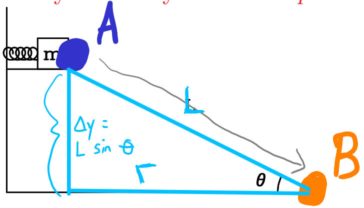

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$W = \vec{F} \cdot \Delta\vec{s}$

= "force, times component of displacement parallel to that force"

force is downward, so

$W_{grav} = (mg)(\text{distance moved downward})$

$= mg\,(L \sin\theta) = mg\,\Delta y$

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What's the work done by **gravity**?

- A: $W_{\text{grav}} = mgL\cos\theta$
- B: $W_{\text{grav}} = mg\sin\theta$
- C: $W_{\text{grav}} = mgL\sin\theta$ ✓
- D: $W_{\text{grav}} = mgL$

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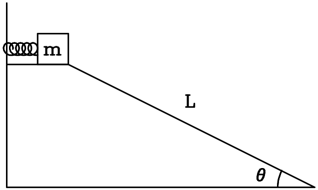

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What's the work done by **the normal force**?

- A: $W_{\text{norm}} = mgh$
- B: $W_{\text{norm}} = mg\cos\theta$
- C: $W_{\text{norm}} = mgL\cos\theta$
- D: $W_{\text{norm}} = 0$ ✓

since F_N is perpendicular to displacement

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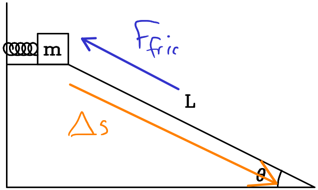

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$$W_{friction} = F_{fric} (\Delta s)_{\parallel}$$

$$= F_{fric} \text{ (displacement along slope)}$$

$$= F_{fric} (-L)$$

$$= \mu F_N (-L)$$

$$= \mu \underbrace{(mg \cos\theta)}_{F_N} (-L)$$

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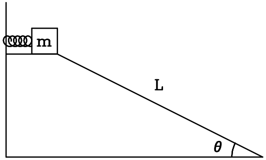
What's the work done by **friction**?

- A: $W_{\text{grav}} = \mu(mg\cos\theta)L$
- B: $W_{\text{grav}} = -\mu(mg\cos\theta)L$ ✓
- C: $W_{\text{grav}} = -\mu(mg\cos\theta)(L\sin\theta)$
- D: $W_{\text{grav}} = mgL$



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"Plain" work-energy theorem (without PE)

$$KE_i + W_{all} = KE_f$$

$$KE_i + W_{grav} + W_{spring} + W_{fric} = KE_f$$

$$\cancel{\tfrac{1}{2}mv_i^2}^{\,0} + mgL\sin\theta + \tfrac{1}{2}kd^2 - \mu mgL\cos\theta = \tfrac{1}{2}mv_f^2$$

Solve for v_f:

$$v_f = \sqrt{2gL\sin\theta + \frac{k}{m}d^2 - 2\mu gL\cos\theta}.$$

Same terms!

Using potential energy

$$\boxed{KE_i + PE_i + W_{other} = KE_f + PE_f}$$

- Conservative forces, associated with PE: gravity, spring
- Nonconservative forces: friction

$$\cancel{KE_i}^{\,0} + PE_{grav_i} + PE_{elas_i} + W_{fric} = KE_f + \cancel{PE_{grav_f}}^{\,0} + \cancel{PE_{elas_f}}^{\,0}$$

$$mgL\sin\theta + \tfrac{1}{2}kd^2 - \mu mgL\cos\theta = \tfrac{1}{2}mv_f^2$$

Solve for v_f:

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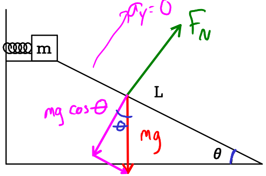
The work-energy theorem and conservation of energy



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Power

You encountered *power* before as the rate of doing work or transforming energy:

$$P = \frac{E}{\Delta t} \quad \text{: Units : } \frac{\text{joules}}{\text{sec}} = \text{watts.}$$

This is important in engineering, since many of our machines are constrained by the rate at which they can manipulate energy, or that they require energy:

- My laptop: 4W (minimum to run) - 25W (maximum cooling system can handle)
- A duck: 25-60W (sustained power from flight muscles)
- Human on a bike: 100-300W (sustained over an hour), five times that (peak)
- Horse: 750W (sustained), 10 kW (peak)
- Automobile engine: 75 kW (my car) - 400 kW (high-end sports car)
- Diesel-electric locomotive: 2500 kW
- Nuclear submarine: 30 MW
- Nuclear reactor: 1500 MW (electric), 3000 MW (heat)

Power

In mechanics, we are often interested in a particular question:

velocity

"At what rate does a force $\vec{F}$ do work on an object moving at ~~speed~~ $\vec{v}$?"

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Starting with the work-energy theorem, as always:

$$(\text{work}) = (\text{force}) \cdot (\text{displacement})$$
$$W = \vec{F} \cdot \vec{\Delta} s$$

Power is the rate at which work is done – the *time derivative* of work. So we take time derivatives of both sides:

$$W = \vec{F} \cdot \vec{\Delta} s$$

$$P \longrightarrow \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{s}}{dt} \longleftarrow \vec{v}$$

Power

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"At what rate does a force $\vec{F}$ do work on an object moving at speed $\vec{v}$?"

Starting with the work-energy theorem, as always:

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$$\begin{aligned}W &= \vec{F} \cdot \vec{\Delta} s \\ \frac{dW}{dt} &= \vec{F} \cdot \frac{d\vec{s}}{dt}\end{aligned}$$

$$P = \vec{F} \cdot \vec{v}$$

Biking with air resistance

A cyclist and her bike have a mass of $m = 70kg$, and she can produce a sustained power of 120 W for a long time.

She can sustain a speed of 12 m/s. At this speed, the main friction force on her is the wind.

How big is that frictional force?

A: 700 N

B: 10 N

C: 1200 N

D: 100 N

$$P = \vec{F} \cdot \vec{v}$$

$$P_{total} = \underbrace{P_{pedals}}_{\text{positive}} + \underbrace{P_{drag}}_{\text{negative}} = 0$$

Comes from dot product: $\vec{F}_{drag}$ is opposite $\vec{v}$

$$P_{pedals} = -P_{drag} = -(-F_{drag}\, v)$$

$$P_{pedals} = F_{drag}\, v \longrightarrow F_{drag} = \frac{P_{pedals}}{v}$$

Biking up a hill

A cyclist and her bike have a mass of $m = 70$ kg, and she can produce a sustained power of 120 W for a long time.

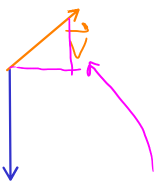
She then wants to ride up a hill, sloped at at an angle of about $\theta = 6^\circ = 0.1$ radian.

How fast can she go up the hill? *(This is a lot slower, so you can ignore air drag here.)*

Compare: $v_{max,flat} = 12$ m/s.

$$P_{pedals} + P_{gravity} = 0$$

$$P_{gravity} = \vec{F}_{grav} \cdot \vec{v} = (mg)(-v_y) = -mg\, v \sin\theta$$



$F_{grav} = mg$

$v_y = v \sin\theta.$

"force, times component of velocity in direction of force"

$$P_{pedals} = -P_{gravity}$$

$$P_{pedals} = mg\, v \sin\theta$$

$$\rightarrow v_{max} \approx 2 \text{ m/s}$$

Going down a steep hill, slowly

Suppose our $m = 70$ kg rider wants to go down a steep hill, angled at 10 degrees below the horizontal, at a safe speed of 4 m/s. *(At this speed, ignore air drag.)*

Brakes work by squeezing a rotating object with a large normal force, creating a lot of friction. This friction does negative work on the rotating wheel, converting its kinetic energy into heat.

What power will the brakes in her bicycle produce?

$v = \text{const.}$

$$\underbrace{P_{gravity}}_{\text{positive}} + \underbrace{P_{brakes}}_{\text{negative}} = 0$$

Brakes on a bike intended for off-road use. The rotor is designed to maximize airflow – to give the material a fighting chance of dissipating this much heat!

Another sample problem: work and energy

A basketball of mass m hangs from a cable of length L; it is pulled to the left at an angle θ and released.

A very strong wind blows from left to right, exerting a constant force F_w on the ball.

How fast will the ball be traveling when it is at its lowest point? What angle ϕ will the ball swing to on the other side?

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