Physics 211 Recitation

February 18

Forces and Force Diagrams

In this next unit you're going to begin dealing with the left-hand side of Newton's second law $\vec{F} = m\vec{a}$.

The first and most important step of any of these problems is drawing a *force diagram*, also called a *free-body diagram*, for each object of interest in the problem. Here are some principles for doing this:

- Represent the object as a dot
- Represent forces acting *on* that object as arrows pointing *away* from the dot. So, if a person is pushing on the left side of a table, this force would be represented as an arrow starting at the dot and going rightward, since that's the direction it is being pushed.
- Label each arrow with the force it represents or the algebraic symbol you will use for it: "friction", "weight", "T_2", etc.
- Only draw forces on the force diagram. Forces are real things – real physical pushes and pulls. Velocity is not a force; acceleration is not a force; "the centripetal force" is not a force.
- If you know the relative sizes of any of the forces, go ahead and make their arrows longer or shorter, representing all the information you have in your diagram. If you don't, just draw the arrows the same length.

A few things for the students on the force diagrams here:

- The arrows go *outward* from the dot, never inward.
- Only real forces go on the force diagram. Acceleration is not a force. "Net force" is not a force unless someone is catching fish with a net. Velocity is not a force.
- If you know the relative magnitude of the forces, you can reflect that with the length of arrows; otherwise, make them the same length. It is a cartoon and a model, not a calculating tool.

- By each force, write whatever mathematical symbol you will use for its magnitude. So for instance the gravitational force is a downward-pointing arrow with mg or F_g written by it. (I prefer "mg"; it is very clear what it is.)
- The symbol g means the magnitude 9.8 m/s^2, always – it doesn't have a minus sign.

1. A person of mass 100 kg is standing in an elevator car. Consider three situations:

   (a) The elevator car is moving upward, and its speed is increasing at a rate of 1 m/s^2.

   (b) The elevator car is moving downward, and its speed is increasing at a rate of 1 m/s^2.

   (c) The elevator car is moving downward, and its speed is *decreasing* at a rate of 1 m/s^2.

   Each of you should lead a discussion in your group for a different one of the three scenarios. Draw a force diagram below. Then, using Newton's second law $\sum \vec{F} = m\vec{a}$, calculate how big each of your forces are.

   This gets them used to two things: 1) the idea that the normal force is a thing you must solve for, and is not always equal to mg; 2) the idea that the acceleration is not always g downward.

   This question really targets the same sort of "fictitious forces" notion as the jumping-kitten problem. Make sure you have them take turns leading their discussions; each person should be intellectually responsible for making a claim in one of the situations.

   These first problems are easy, but we are practicing the problem-solving recipe that we want to drill over the next few weeks:

   (a) 0. Draw a cartoon

   (b) 1. Draw a force diagram (free-body diagram)

   (c) 2. Write down $\sum \vec{F} = m\vec{a}$

   (d) 3. Do algebra and solve for the thing you need.

   At some point in the discussion, someone should realize that the normal force is related to how heavy you feel.

   When you've finished, call a coach or TA over, and talk about similarities and differences between the three situations.

2. Three books, each weighing 10 pounds, sit in a stack on a table. From the bottom up, they are:

- *Advanced Computer Programming*, by Grace Hopper
- *Beginning Rocket Science*, by Robert Goddard
- *Cats as Laboratory Apparatus*, by Erwin Schrödinger

You should just call them A, B, and C.

Draw force diagrams for each of the three books, including all normal forces and gravitational forces that act on them. As a convenient notation, you might label "the normal force of book A pushing on book B" as $\vec{F}_{AB}$. Once you have drawn your diagrams, discuss them with your group and check them for errors, making sure that all the arrows you've drawn correspond to real forces, and that you've not forgotten any. Call your TA or coach over to join in the discussion.

They should spend a substantial amount of time here. The point is that they should **already know the answer** here: a stack of three ten pound books weighs 30 pounds. (Duh.) The point here is to practice the process and see if they can reproduce the answer they know they should get.

The big errors they make here:

- **Inventing fake forces. C does not push on A; they do not touch. The table does not push on B; they do not touch.** If they do this, they'll get incorrect results; they should know they are incorrect ("what do you mean the stack weighs 40 N?") and then you can help them figure out what they have messed up. This is the biggest issue that students have with this material: making it more complicated than it needs to be by inventing forces that are not real. This exercise is designed to cultivate trust in Newton's laws and mathematics: if they just write down the forces that really exist, write down $\vec{F} = m\vec{a}$ for each thing, and then solve the system of equations, it'll work out. Trust the laws of physics: they work.
- Unclear notation. There are many normal forces here. They need to concoct a notation that makes very clear which is which.

3. Newton's second law says that the sum of all the forces on an object is equal to its mass times its acceleration. Since none of the objects in this problem move, each of these objects has $\sum \vec{F} = 0$. Write this down for each object, listing all of the forces in turn. Since the only forces here are in the vertical direction, you don't need to mess with vector components; just choose one direction to be positive and one to be negative. Call a TA or coach over to check your group's work when you're done.

   Convention here: The symbol (e.g. F_{AB}) represents the magnitude of the force; if it points downward, you'll need to put in minus signs by hand. So you wind up with expressions like

$$F_{AB} - F_{CB} - mg = ma = 0$$

where you can read the directions off of the signs, since every term that appears on the left is the magnitude (positive definite) of something.

4. This will give you three equations. How many unknowns do you have?

   Here they have five – if they treat F_{AB} and F_{BA}, etc., as separate. They're going to need to realize that they are Newton's third law pairs sooner or later.

5. Some of the forces you will have drawn are Newton's third law pairs. The formulation of Newton's third law that I prefer is: "If object 1 pushes on object 2 with a force F_{12}, object 2 pushes back on it with a force of equal magnitude and in the opposite direction; that is, $\vec{F}_{12} = -\vec{F}_{21}$." (Note that their magnitudes are equal: $F_{12} = F_{21}$.) Identify all Newton's third law pairs present in your problem. Does this give you enough information to solve the system of equations? If so, solve it; if not, call over a TA or coach to discuss your work.

   Some fraction of our students do not know how to solve systems of equations by substitution. There is a thing they learn in school where you add a linear combination of one equation to another one in order to eliminate variables; it can be awkward and clumsy and usually isn't so helpful here. Be prepared to teach systems of equations by substitution. I suspect that some of them may be missing some high school Algebra 2 skills because of zoom classes, so we need to be prepared to remediate that. (This is fine! If students are missing particular math skills this is a thing we can help them correct and they often go on to do very well in PHY211.)

   Beyond that, it's a good idea to check their work here. They will sometimes confuse *two things that we can expect to be equal, since we know the answer here* with an actual Newton's third law pair. We won't always know the answer, though.

6. Now you should have figured out how big all the forces involved are. How much force does the table exert on the bottom book? Is this what you expect it to be?

   If they don't get 30 N here, most likely they have invented forces that don't actually exist. As usual, don't just tell them this; ask leading questions to help them discover their own misconceptions.

7. This is an easy problem, and we could have guessed the answers. However, the procedure you've followed here is exactly the same as what you will do for less obvious problems – and it has the same pitfalls. With your group, take turns thinking of mistakes that you might have made in solving this problem. What sorts of errors could

you have made, and how would you have known not to make them – or how could you have caught them after the fact once you made them? Call a TA or coach over to join in your discussion.

Common mistakes: minus signs (not thinking about the direction of forces), inventing forces that don't actually exist, forgetting certain forces (i.e. leaving out the downward force that top book exerts on middle book).

Two astronauts have been frozen for years on a robotic spaceship destined for another planet (one with the same g as Earth). The spaceship has a rocket that can accelerate it upward at g. The astronauts wake up and need to figure out whether they are still in deep space (accelerating upward at g but with no gravity) or have landed on the other planet. Unfortunately, the the spaceship's sensors and navigation computer have stopped working

The astronauts (each of mass M) are discussing how things should behave on the ground and in space in an effort to determine where they are.

A: Alright, we're standing here, with our feet on the ground. I feel heavy. So we must be on the planet, and we're feeling the effects of its gravity.

B: Sure, but in space with the rocket turned on, what happens? It's accelerating upward. The floor has to push on me to make me accelerate too, and it's got to push on me with a force Mg – the same as I would weigh on the planet. And of course I'd have my feet on the floor – my feet have evolved to support large forces, not my head. And my blood has to accelerate upward too, so my heart is working to pump it upwards, same as on Earth.

A: Drat, so we can't use how things feel to figure this out. What about the motion of things? I've got this apple; if I drop it, it falls. If the spacecraft is parked on Earth and I drop an apple in the lab, it feels a downward force of mg, which accelerates it toward the floor at g, since $F = ma$; we substitute, and $-mg = ma$ so $a = -g$.

B: Yeah, that's why things fall on Earth. But what would this look like in space? If we're in deep space with the rocket turned on, when I let go of my apple, it's initially moving forward at some speed along with the spacecraft. But the floor is accelerating upward and the apple is not, so the floor eventually catches up with the apple – the floor is accelerating toward the apple at g. So it looks the same as falling on Earth.

A: We brought our little kitten with us, right? She's jumping around like an idiot trying to swat the bugs on the wall, and falling like she would on Earth.

B: Okay, what would happen in space? Suppose the spacecraft is moving upward at a speed v already, and she jumps off the ground with an additional speed v_2. So she's initially moving upward at a velocity $v + v_2$. But since the floor is accelerating and she's not, the floor will eventually catch up to her. Seems like it would look the same in space with the rocket turned on as it would parked on the planet.

Discuss these scenarios with your group. Can you think of any experiment the astronauts could do (involving measuring forces and the movement of objects, without measuring things outside their spaceship) to figure out whether they are in deep space or on the planet?

The punchline here is twofold:

1. The way things behave in an accelerating box without gravity is precisely the same as they behave in a non-accelerating box without gravity; there is no experiment they can do to figure out which one is which.
2. This means that if you *are* in an accelerating box, but are unaware of this and wish to pretend that you are *not* accelerating, it appears as though a phantom force pushes you opposite the acceleration. But there is really no such force; it is just a consequence of being unaware of the acceleration of the box around you.

We don't expect students to understand all of the formalism around accelerating reference frames and fictitious forces. We *do* expect students to be able to **describe how things behave** in such a situation, both as viewed from outside (in the "lab frame", although we don't call it that) and from inside. There is always an question of this type on exam 2. (You can tell students that. The stuff on our exams is not a secret.)

Do not confuse struggling students with the notion of fictitious forces. It suffices for our class to *only* apply the formal mathematics of Newton's laws to the lab (unaccelerated) frame. They need to understand how the formal analysis of the unaccelerated frame gives rise to "how things appear to work" in the accelerating frame, but *do not* need to construct Newton's laws in an accelerating frame adding fictitious forces. This will confuse the heck out of many of our students.

But some of our advanced students may *want* to talk about this. In case anyone wants an elegant way to explain how fictitious forces work:

Formally, we know that any object responds to forces acting on it as follows:

$$\sum \vec{F}_{\text{real}} = m\vec{a}$$

where $\vec{a}$ is the complete acceleration. But we can decompose this into

$$\vec{a} = \vec{a}_{\text{box}} + \vec{a}_r,$$

where $\vec{a}_{\mathrm{r}}$ is the acceleration of the object relative to the box (i.e. frame).

Then we have

$$\sum \vec{F}_{\mathrm{real}} = m\vec{a}_r + m\vec{a}_{\mathrm{box}}$$

where the right hand side is again just the total acceleration.

But if we are unaware that we are in an accelerating box, accelerating along with the objects moving around in it, we don't have that term on the right; we only perceive accelerations relative to us. So that term must go away; we subtract it from both sides and get

$$\sum \vec{F}_{\mathrm{real}} - m\vec{a}_{\mathrm{box}} = m\vec{a}_r$$

What do we make of this term on the left? Well, it's another thing lurking on the left side of Newton's law where the forces go, "causing objects to accelerate". It doesn't correspond to any real physical force, but it causes *apparent* accelerations relative to the box we are in (since we are either ignorant of its acceleration or choose to ignore it). So it seems to us like it's a phantom force:

$$\sum \vec{F}_{\mathrm{real}} + \vec{F}_{\mathrm{phantom}} = m\vec{a}_r$$

where

$$\vec{F}_{\mathrm{phantom}} = -m\vec{a}_{\mathrm{box}}.$$

This is the formal explanation for centrifugal force (plus the Coriolis force, etc.). But don't confuse students with this formalism unless they will benefit from it; we want them to understand things formally in the unaccelerated frame and make intuitive contact from there with the "inside-the-spaceship" frame.