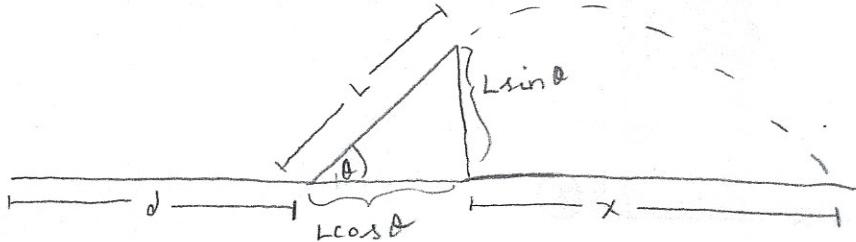


a. draw a cartoon



b. calculate how fast she travels when she leaves top of the ramp.

WANT TO USE WORK-ENERGY THEOREM, SINCE WE NEED TO CONSIDER THE THRUST FORCE & FRICTIONAL FORCE.

I'M GOING TO ASSUME THE ONLY SNOW WE ENCOUNTER IS ON THE RAMP ITSELF (SO WE CONSIDER FRICTIONAL FORCE FOR LENGTH OF RAMP L)

$\Delta KE = W_{\text{THRUST}}$  (I AM ONLY CONSIDERING FROM THE VERY START, TO THE BOTTOM OF THE RAMP)

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = W_{\text{THRUST}}$$

(OMIT NEGATIVE, NOT NECESSARY)  $\frac{1}{2}mu^2 = F_T \cdot d \Rightarrow u = \left[ \frac{2F_T d}{m} \right]^{1/2}$  THIS IS HER SPEED WHEN SHE REACHES THE BOTTOM OF THE RAMP

NEXT WE CONSIDER HER ON THE RAMP.

WHILE ON THE RAMP, SHE ENCOUNTERS FRICTION, GRAVITY, & KE AT THE TOP OF THE RAMP.

$$\Delta KE = W_{\text{NET}} \Rightarrow KE_f - KE_i = W_{\text{friction}} + W_{\text{gravity}}$$

$$KE_f = KE \text{ at top of the ramp} - \frac{1}{2}mv^2$$

$$KE_i = KE \text{ at bottom of the ramp} - \frac{1}{2}mu^2$$

$$W_{\text{grav.}} = F_g \cdot \Delta s, \text{ WHERE } F_g = mg \text{ & } \Delta s = L \sin \theta$$

FROM THE PICTURE, YOU CAN SEE  $L \sin \theta$  IS THE VERTICAL DISTANCE SHE TRAVELED

$$W_{\text{friction}} = f_k \cdot \Delta s \text{ WHERE } f_k = \mu mg \cos \theta \text{ & } \Delta s = L$$

(SINCE FRICTION IS ALL ON THE RAMP)

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgL \sin \theta + (\mu mg \cos \theta)L$$

$$v = \left[ u^2 + 2gL \sin \theta + (2\mu g \cos \theta)L \right]^{1/2}$$

(WE CAN PLUG IN FOR  $u$  HERE, BUT AS YOU CAN SEE I'M RUNNING LOW ON ROOM!) PAGE 1

c. calculate how fast she is traveling when she lands back on the ground.



CONSIDER POINTS ① & ② TO BE OUR KE<sub>i</sub> & KE<sub>f</sub> RESPECTIVELY.

AGAIN, WE USE WORK-ENERGY.

$$\Delta KE = W$$

$$KE_f - KE_i = W$$

WHERE THE WORK IS ONLY THE WORK DUE TO GRAVITY. SHE IS OFF THE RAMP, SO NO FRICTION, & THERE ARE NO OTHER FORCES ACTING ON HER.

$$W = W_{\text{gravity}} = F_g \cdot \Delta s$$

AGAIN, AS IS THE VERTICAL DISTANCE FROM ① → ②

FROM THE PICTURE, IT IS GIVEN BY  $L \sin \theta$

$$KE_i = \frac{1}{2} m \vec{u}^2 \quad | \text{ I AM REUSING } \vec{u} \text{ HERE TO BE THE}$$

$$KE_f = \frac{1}{2} m \vec{v}^2 \quad | \text{ INITIAL VELOCITY, WHICH IS THE VELOCITY}$$

(NOW THIS  $\vec{v}$  IS A PROBLEM, THAT HAS BECOME <sup>"</sup>INITIAL VELOCITY<sup>"</sup>)  
WHAT WERE SOLVING FOR)

ALL TOGETHER:

$$KE_f = KE_i + W_{\text{gravity}}$$

$$\frac{1}{2} m \vec{v}^2 = \frac{1}{2} m \vec{u}^2 + mg L \sin \theta$$

$$\vec{v} = \left[ u^2 + 2g L \sin \theta \right]^{\frac{1}{2}}$$

SINCE THE ONLY FORCE ACTING IS GRAVITY & WE ONLY NEED TO CONSIDER WORK FROM GRAVITY, WE CAN USE CONS. OF ENERGY & USE PE INSTEAD OF WORK. GRAVITATIONAL PE = WORK FROM GRAVITY

$PE = mgh$  WHERE  $h$  IS THAT SAME VERTICAL DISTANCE  $L \sin \theta$

$$\text{SO: } PE = mg L \sin \theta$$

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Q. can you energy methods to solve for horizontal distance?

IN THIS CASE NO.

LETS TRY ANYWAY, & SEE WHAT HAPPENS.

AGAIN, WERE LOOKING @ POINTS ① & ② & WITH PE & KE WE LOOK AT INITIAL & FINAL STATES.

WE SOLVED FOR  $KE_i$  &  $KE_f$  ( $KE_i$  HAS  $u$ , VELOCITY AT TOP OF THE RAMP, &  $KE_f$  WITH  $v$ , VELOCITY BEFORE HITTING THE GROUND).

NEXT WE CAN CONSIDER THE PE & WORK ENERGY (OUR GOAL IS TO LOOK AT ALL POSSIBLE PIECES OF USING ENERGY METHODS TO SEE IF WE CAN GET HORIZONTAL DISTANCE SOMEHOW)

IF WE USE CONS. OF ENERGY METHOD - LOOK AT PE

IF WE USE WORK-ENERGY - LOOK AT WORK

### ① CONS. OF ENERGY

ONLY GRAVITATIONAL PE SINCE SHE IS UP ON A RAMP.

THE VERTICAL HEIGHT IS  $L \sin \theta$

$$PE_i = mgh = mgL \sin \theta$$

$$PE_f = 0 \text{ (SHES ON THE GROUND)}$$

$$KE_i = \frac{1}{2}mu^2, KE_f = \frac{1}{2}mv^2$$

$$PE_i + KE_i = PE_f + KE_f$$

$$mgL \sin \theta + \frac{1}{2}mu^2 = \frac{1}{2}mv^2$$

THE ONLY DISTANCE HERE IS THE VERTICAL  
 $\therefore$  CANNOT SOLVE FOR "x" WITH CONS. OF ENERGY

### ② WORK-ENERGY

$$KE_i = KE_f + W$$

AS BEFORE, THE ONLY FORCE IS GRAVITY SO WE WANT  $W_{\text{gravity}}$   
&, AGAIN, WANT THE VERTICAL DISTANCE SINCE GRAVITY ACTS VERTICALLY DOWN

$$W_{\text{grav}} = F_g \cdot \Delta s = mgL \sin \theta$$

ALL TOGETHER:

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgL \sin \theta$$

SAME AS BEFORE, NO WAY OF OBTAINING "x"

WEVE SHOWN IT CAN'T BE DONE BUT  
THERES MORE PHYSICS BEHIND THIS:

KE ALONE DOES NOT DEPEND ON DISTANCE, IT DEPENDS  
 $m$  &  $v$ , & WE ONLY CONSIDER INITIAL & FINAL STATES.  
LIKE BOTTOM OF RAMP TO THE TOP, OR THE TOP THEN  
ALL THE WAY TO THE GROUND.

PE (GRAVITATIONAL) DEPENDS ON VERTICAL DISTANCE.

WORK IS HOW WE CAN RELATE THE KE TO DISTANCE.  
& CAN RELATE  $P_{grav}$  TO  $W_{grav}$ .

IF YOU LIFT A BOX STRAIGHT ... UP FROM THE GROUND, YOU ARE  
APPLYING A FORCE UPWARD TO OPPOSE GRAVITY PULLING  
DOWN. YOU'VE BASICALLY GIVEN

THE BOX ENERGY IN ORDER TO LIFT IT. AT THE TOP,  
THAT ENERGY YOU GAVE THE BOX (THAT YOU CAN CALCULATE  
WITH  $F \cdot \Delta s$ ) IS STORED AS PE (UNTIL IT FALLS & BECOMES  
KE). SO YOU CAN SEE, GRAVITATIONAL PE = WORK (GRAVITY)  
BUT THAT ONLY CONCERNs VERTICAL DISTANCES.

IN ORDER TO CHANGE KE, YOU NEED WORK. THINK, IF YOU  
APPLY SOME FORCE TO PUSH SOMETHING (IN SOME DIRECTION  
THAT IS THE SAME DIRECTION AS THE APPLIED FORCE)

YOU ARE GIVING IT ENERGY TO SPEED UP OR SLOW DOWN.

$\Rightarrow \Delta KE = W_{net}$ , APPLYING WORK GETS US FROM  $KE_i \rightarrow KE_f$   
YOU CHANGE THE VELOCITY OF THE BOX BY APPLYING WORK  
(TO IT, & THAT CHANGE IN VELOCITY ACCOUNTS FOR THE  
CHANGE IN KE).

\* I WANTED TO WRITE THIS OUT TO MAYBE HELP YOU GET A  
BETTER IDEA OF WORK, PE, & KE & THEIR RELATIONSHIPS\*  
(IF YOU KNEW ALL THIS ALREADY - FABULOUS!)

FINALLY, THE FINAL ANSWER

1. WE CAN ~~NOT~~ GET DISTANCE FROM WORK ( $W = F \cdot \Delta s$ )
2. (BESIDES CALCULATING IT DIRECTLY WITH  $W = F \cdot \Delta s$ ) WE CAN  
RELATE WORK DONE TO  $\Delta KE$

3. THERE IS A CHANGE IN KE FROM ①  $\rightarrow$  ② (STILL SAME SPOTS FROM PIC)  
SO THERE MUST HAVE BEEN WORK DONE, SO THERE MUST HAVE  
BEEN A FORCE TO CAUSE THAT CHANGE.

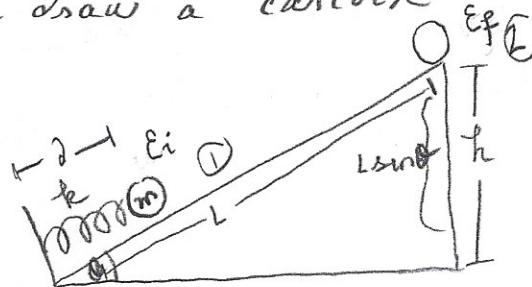
4. THE ONLY FORCE WAS GRAVITY, THAT ACTS VERTICALLY, SO THE  
WORK FROM GRAVITY WILL ONLY GIVE VERTICAL DISTANCES

5. WE COULD GET THE HORIZONTAL DISTANCE IF A FORCE WAS  
APPLIED HORIZONTALLY AND DISTANCE "W" BUT THIS IS NOT THE CASE

PAGE  
4

2. a pinball machine uses a spring loaded launcher  
to launch a ball up a ramp - it rolls  
up without slipping  
want the speed of the ball at top of the ramp.

a. draw a cartoon



$$\text{energy initial} = \text{energy final}$$

$$\Rightarrow E_i = E_f$$

(picture shows initial & final locations)

b. write down an expression for conservation of energy

$$PE_i + KE_i = PE_f + KE_f$$

INITIALLY, THE BALL IS RESTING ON THE SPRING  
WAITING TO BE LAUNCHED  $\Rightarrow$  NOT MOVING

$$\Rightarrow KE_i = 0$$

THAT LEAVES ONLY AN INITIAL PE, & IT COULD  
ONLY BE GRAVITATIONAL OR SPRING.

OF COURSE IT IS SPRING POTENTIAL, BUT I WROTE THIS OUT  
FOR FUTURE PROBLEMS, WHERE IT MIGHT BE USEFUL  
TO USE THIS PROCESS OF ELIMINATION METHOD FOR  
POTENTIAL, & ASK "IS IT MOVING" FOR KINETIC

$$\Rightarrow PE_i = \frac{1}{2}kx^2 \text{ (here } x=0\text{)}$$

FINAL ENERGIES:

$PE_f$  - GRAVITATIONAL OR SPRING?

THERES NO SPRING AT ②, SO THATS OUT

BUT THINK THIS: THE BALL MOVED UP THE RAMP  
TO SOME HEIGHT  $h$ , THIS IS GRAVITATIONAL AS  
NOW THE BALL HAS SOME POTENTIAL TO  
FALL & HAVE THAT ENERGY BE CONVERTED TO KINETIC.

## FINAL KINETIC:

NOW: IS IT ROTATIONAL OR TRANSLATIONAL?  
 THE BALL ROLLS UP THE RAMP, SO THERE MUST BE  
 ROTATIONAL KE.  
 THERE IS ALSO TRANSLATIONAL KE, AS THE BALL  
 HAS MOVED FROM ONE POINT TO ANOTHER  
ALL TOGETHER.

$$PE_i + KE_i \xrightarrow{\text{!}} PE_f + KE_f$$

$$PE_{\text{spring}} = PE_{\text{grav.}} + KE_{\text{rot.}} + KE_{\text{translational}}$$

$$\Rightarrow \frac{1}{2} I \omega^2 = mgh + \frac{1}{2} I \omega^2 + \frac{1}{2} mv^2$$

c. show that the speed at the top of the ramp is  
 \*INSERT SOME CRAZY EQUATION HERE\*

$$\frac{1}{2} I \omega^2 = mgh + \frac{1}{2} I \omega^2 + \frac{1}{2} mv^2$$

$$\text{given: } I = \frac{2}{5} mr^2$$

WE CAN FURTHER REDUCE OUR UNKNOWNNS BY RELATING  
 $\omega$  &  $v$ , WALTER (OH SO KINDLY) GIVES US  $v = \omega r$   
 SINCE WE WANT THE TRANSLATIONAL VELOCITY, WE  
 WANT TO REPLACE  $\omega$  WITH  $v$

$$v = \omega r \Rightarrow \omega = \frac{v}{r}$$

& LIKE BEFORE, THE  $r$  (in  $mgh$ ) CAN BE DEFINED BY  
 $L \sin \theta$  (REFER TO PICTURE IF LOST)

PLUGGING IN:

$$\frac{1}{2} I \omega^2 = mg(L \sin \theta) + \frac{1}{2} \left( \frac{2}{5} mr^2 \right) \left( \frac{v}{r} \right)^2 + \frac{1}{2} mv^2$$

$$\frac{1}{2} I \omega^2 - mg(L \sin \theta) = \frac{1}{5} mv^2 \left( \frac{v^2}{r^2} \right) + \frac{1}{2} mv^2$$

$$\frac{1}{2} I \omega^2 - mg(L \sin \theta) = \frac{7}{10} mv^2$$

$$\Rightarrow v = \left[ \frac{10}{7} m^{-1} \left( \frac{1}{2} I \omega^2 - mg(L \sin \theta) \right) \right]^{1/2}$$

\*crazy equation obtained\*

\*i am NOT simplifying\*

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3. a ball of mass  $m$  is connected to one end of a rubber band & swung in a circle.  
the band has spring constant  $k$  & unstretched length  $r_0$ .  
if swung at angular velocity  $\omega$ , what length will it stretch the band?

IMMEDIATELY WE CAN SEE THAT THERE'S A PE ASSOCIATED WITH THE STRETCHED RUBBER BAND.  
 $\therefore$  WE CAN START TO APPLY CONSERVATION OF ENERGY (INSTEAD OF WORK-ENERGY)

SO:

$$PE_i + KE_i = PE_f + KE_f$$

IT FOLLOWS THAT WE SHOULD CONSIDER KE.  
IS IT MOVING?

YES, SO THERE IS CLEARLY KE, BUT IS IT TRANSLATIONAL OR ROTATIONAL?

ROTATIONAL, THE MASS IS ROTATING ABOUT SOME CENTER RATHER THAN IT MOVING FROM SOME INITIAL PLACE TO SOME FINAL PLACE.

LET  $\ell$  BE THE TOTAL LENGTH OF STRETCHED BAND

LET  $x$  BE THE DISTANCE THE BAND WAS STRETCHED BY

LET  $I$  BE THE MOMENT OF INERTIA

THE BALL OF MASS  $m$  CAN BE THOUGHT OF AS A POINT MASS & THE RUBBER BAND IS MASSLESS.

THIS WAY WE DEFINE  $I$  BY  $I = mr^2$

ACTUAL SOLVING, NEXT PAGE

$$PE_{SPRING} = KE_{ROT.}$$

$$\frac{1}{2}kx^2 = \frac{1}{2}I\omega^2$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mr^2\omega^2$$

Plug in  $\omega = \frac{v}{r}$

$$\frac{1}{2}kx^2 = \frac{1}{2}mr^2\left(\frac{v^2}{r^2}\right)$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$\Rightarrow x = \left[\frac{mv^2}{k}\right]^{1/2}$$