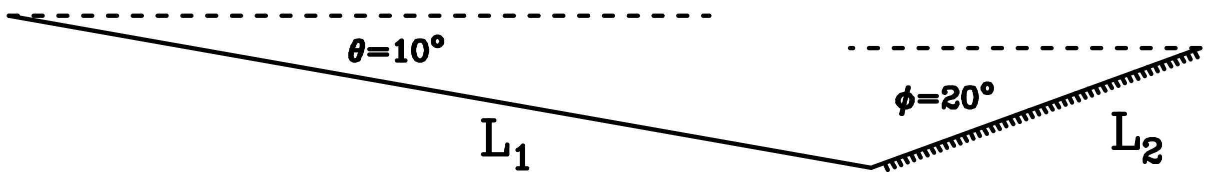
Physics 211 Practice Exam 3
Question 1

Heavy trucks driving down steep mountains must continually apply their brakes to maintain a safe speed. If their brakes fail, these roads are equipped with "runaway truck ramps", which are short uphill pathways (made of sand or gravel) with a large coefficient of rolling friction. A truck whose brakes fail can steer into the ramp and come safely to a stop. Suppose that a truck of mass m is driving down the hill at a speed v_0 when its brakes fail completely. It is a distance L_1 away from the ramp, traveling at a speed v_0. When it reaches the ramp, it exits the highway and heads up the ramp, traveling a distance L_2 before coming to rest. In this problem, you will calculate the distance L_2 in terms of μ_r, g, m, L_1, v_0, θ, and ϕ.

Since the truck's brakes have failed completely, it has essentially no friction while it is on the road. Once it enters the ramp, it experiences a coefficient of friction of μ_r.



a) Write an expression for the total work done by gravity during the entire motion in terms of g, m, L_1, L_2, θ, and ϕ. If gravity does no work, explain why. *(5 points)*

b) Write an expression for the total work done by the normal force during the entire motion in terms of g, m, L_1, L_2, θ, and ϕ. If the normal force does no work, explain why. *(5 points)*

c) Write an expression for the total work done by friction during the entire motion in terms of μ_r, g, m, L_2, and ϕ. If friction does no work, explain why. *(5 points)*

d) Write a statement of the work-energy theorem/conservation of energy in terms of μ_r, g, m, L_1, L_2, v_0, θ, and ϕ that you could solve for L_2. (You do not need to solve it.) *(10 points)*

Question 2

A rugby player with a mass of $M = 80$ kg is running 45 degrees east of north at a speed of 5 m/s. She is tackled by another player running 60 degrees west of north at a speed of 6 m/s. After the impact, the two players are moving together directly north (at the same velocity).

a) What is the mass of the second player? *(15 points)*

b) How fast are the two players moving after the impact? *(10 points)*

Question 3

Suppose that a student removes the little spring from a clicky-pen and uses it to shoot little pebbles vertically into the air. (Perhaps they are bored in class, or perhaps they are experimentally verifying Hooke's law!)

Suppose that their spring is capable of propelling a small pebble 2 meters into the air if it is compressed by 2 cm.

a) What would the new maximum height be if they compressed their spring only 1 cm? *(5 points)*

b) What would the new maximum height be if they compressed the spring by 2 cm, but now replaced the spring with one that had a spring constant twice as big (from their neighbor's larger clicky-pen)? *(5 points)*

c) What would the new maximum height be if they went back to their original spring, compressed it by the original 2 cm, but replaced their small pebble with a larger pebble that had twice the mass? *(5 points)*

d) What would the new maximum height be if they went back to their original spring and launched their pebble at a 45° angle above the horizontal, instead of vertically? *(5 points)*

e) Suppose the student launches their small pebble (using their original spring, compressed by 2 cm) from the surface of their desk at some angle θ above the horizontal. It flies across the room and hits the floor.

The speed v_f with which it hits the floor doesn't depend on the angle θ. Explain why v_f doesn't depend on the angle it was launched at. *(5 points)*

Question 4

A person of mass 50 kg is ice-skating on a frozen lake with his dog Kibeth, who has a mass of 15 kg. He is skating due north at 3 m/s.

Kibeth realizes that he's carrying snacks in his pocket, and would like one for herself. (Or maybe she is just being friendly!) She runs after him and tackles him from behind and the side, knocking him down. The two of them collapse on the ice and begin to slide, as Kibeth tries to get the treats out of his pocket; they are moving together at an angle 20 degrees west of north at 4 m/s.

What was Kibeth's velocity before she tackled him? *(Remember velocity is a vector.) (25 points)*

Question 5

The dread pirate captain Piarrrr Squared has a peg leg, a parrot, and a ship whose horizontal deck is a height h above the water. The captain has just found a new cannon and has hauled it to the edge of the deck. The coefficient of kinetic friction between the cannon and the deck is μ_k.

The cannon is very massive; it has a mass a hundred times greater than the cannonballs that it fires.

Piarrrr test-fires the cannon, which launches a cannonball horizontally; it flies out to sea, and the cannon slides back a distance D before coming to rest.

In this question, you will determine the range $R^{\dagger}$ that the cannonball travels out to sea before splashing into the water. (The below space is for you to draw diagrams, if you choose.)

a) What technique can you use to relate the forward velocity of the cannonball after it is fired to the backward velocity of the cannon? Determine this relationship. *(5 points)*

b) What technique can you use to relate the backward velocity of the cannon right after it is fired to the distance that it slides before coming to rest? Determine this relationship. *(5 points)*

† Pronounced "arrrrrrr", of course.

c) What technique can you use to relate the forward velocity of the cannon after it is fired to the range that the cannonball flies out to sea? Determine this relationship. *(5 points)*

d) Putting together the above information, if the cannon slides a distance D backwards, determine the range R that the cannonball flies out to sea, in terms of D, g, μ_k, and h. *(15 points)*

Question 6

People with impaired mobility sometimes use battery-powered wheelchairs to travel. Suppose that you are an engineer in a very hilly city and are tasked with building a wheelchair suitable for people to get around in your town. They give you the following design specifications:

- The combined weight of the wheelchair (including its batteries) and the person it is transporting will be 100 kg
- It has to be able to climb a 10° hill that is 1 km long at a speed of 0.8 m/s
- It has to be able to climb this hill five times before its batteries run out

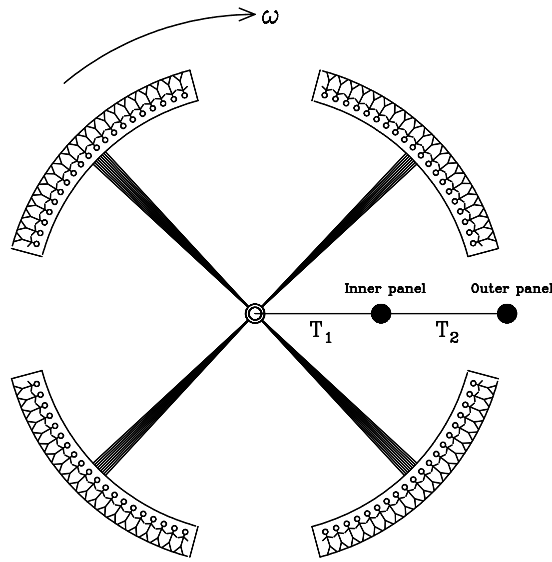
Assume that the battery and the motor are 100% efficient at converting electrical energy into mechanical work, and that there is no rolling friction.

a) You first must choose a motor to put in the wheelchair. How much power must the motor produce to meet the design requirement? *(10 points)*

b) Suppose you are using lithium-ion batteries that can store 800 kJ per kilogram. What mass must the wheelchair's battery have to meet the design requirement? *(10 points)*

Question 7

Futurists and science-fiction authors have often imagined circular spacecraft with "artificial gravity", in which humans (or other things accustomed to gravity) occupy a ring-shaped habitat. The ring rotates around a central hub, creating the impression of gravity for its inhabitants. They feel heavy, objects that they drop fall to the floor, and they otherwise experience all of the same things that people on a planet do.



Imagine that such a ship has a radius of R and is in deep space, far from any planets or moons, where there is almost no (actual) gravity. Suppose that the crew of the ship wants the passengers to experience "artificial gravity" similar to that on Earth. (In an actual station R would be much larger than the height of people; this drawing is not to scale.)

a) Explain how this works. Why does a rotating circular spacecraft simulate gravity for its inhabitants? Specifically, what force presses them against the floor? If there is no such force, then explain why a person on such a spacecraft standing on a scale could see the same reading as they would on Earth, and why an object that they drop falls to the floor. *(8 points)*

b) At what rate must the spacecraft rotate so that the people aboard experience artificial gravity that feels equal to Earth's? Give your answer in terms of g and R.

This station is powered by solar panels of mass m connected by cables to the central hub. A cable of length $\frac{1}{2}R$ runs from the hub to the inner panel; a second cable runs from the inner panel to the outer panel. These solar panels also rotate along with the rest of the station at the same angular velocity.

c) Draw a force diagram for the inner solar panel and the outer solar panel. (Note that the tension in the two cables is different.) *(5 points)*

d) In terms of m, R, and ω, calculate the tension T_1 in the cable between the hub and the inner solar panel, and the tension T_2 in the cable between the inner solar panel and the outer solar panel. *(7 points)*

Question 8

A book with a mass of 2 kg rests on a table; the coefficient of kinetic friction μ_k between them is 0.4. A string connects that book to another book hanging vertically off the side of the table with mass 3 kg; this hanging book is 140 cm above the ground. When the hanging book is released, it accelerates toward the ground, dragging the other book on the table with it.

a) Draw a force diagram for both books. Indicate your choice of signs for the $x-$ and $y-$axes on both diagrams; that is, which directions do you consider positive, and which do you consider negative? *(5 points)*

b) Are the accelerations of the two books related? If so, write a mathematical relationship between them. *(5 points)*

c) Calculate the accelerations of the books and the tension in the string. *(10 points)*

d) With what velocity will the hanging book strike the floor? *(5 points)*