Rotational motion

Physics 211
Syracuse University, Physics 211 Spring 2016
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April 26, 2016

Announcements

- Homework due Friday
- Office hours today 3:30-5:30
- Review for Exam 3 on Thursday
- Exam 3 next Tuesday

Exam 3

Topics covered:

- The work-energy theorem
- Potential energy and conservation of energy
- Elasticity and Hooke's law
- Torque and rotational motion

You may expect:

- More conceptual problems (no hard math) similar to the multiple choice questions last time
- Problems involving the work-energy theorem in combination with force diagrams or rotation
- Very few problems that require calculator-work

Exam 3 preparation and end-of-term schedule

- Wednesday afternoon: a good time to come to me with grade issues
- Thursday: office hours 1:30-3:30
- Friday: office hours and/or review 10-12 (location TBA) and 1-4 if people aren't at Mayfest
- Saturday: review 4-8 (Clinic/Stolkin)
- Sunday: review 2-5
- Monday: Office hours by appointment; dealing with exam printing
- Tuesday: grading all day
- Wednesday: grading all day, calculating your provisional grades
- Thursday: I hope to have provisional grades available to you after noon
- Friday: review 10-4 (location TBA)
- Saturday: review 2-6 (Physics Clinic/Stolkin)
- Monday: Exam 3 Retake
- Tuesday: GTA's have exams...
- Wednesday: Grading
- Final grades should be done by Friday, May 13

Exam 3 preparation

This is an enormous amount of extra help; use it!

Homework questions

Any questions about HW7, HW8, or the practice exam?

Vector torque, the gyroscope, and angular momentum

(This material will not be treated numerically on your exam, but may appear in conceptual form)

So far we have learned that torque is either CCW (positive) or CW (negative)
... but in 3D, torques can also change the *plane of rotation*

How do we handle situations that don't just change the rate of rotation, but its orientation?

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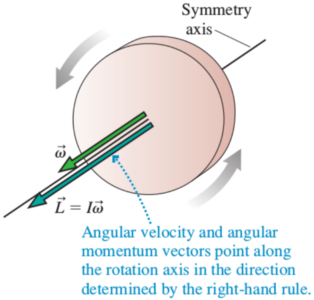
How do we handle situations that don't just change the rate of rotation, but its orientation?

... we need to treat torque and angular momentum as vectors!

The angular momentum vector

- Definition: The angular momentum vector points along the axis of rotation
- ... in the direction from which it appears to be going counterclockwise
- ... see live demonstration for an example

FIGURE 12.57 The angular momentum vector of a rigid body rotating about an axis of symmetry.



The torque vector

Torque as a vector has the same magnitude as you've already learned:

$$\tau = F_{\perp} r = F r \sin\theta$$

.

Which way does it point?

- Method 1: $\vec{\tau}$ points in the direction of the $\vec{L}$ it creates
- "Torque points along the axle of the wheel pointing toward an observer who sees it as clockwise"

- Method 2: $\vec{\tau} = \vec{r} \times \vec{F}$
- ...this is a new mathematical idea called the "cross product"

The cross product

Recall that you already learned about the "vector dot product": $\vec{A} \cdot \vec{B} = A_{\|} B = A B_{\|} = AB \cos\theta$.

- $\vec{A} \cdot \vec{B}$ is a scalar

The magnitude of the cross product looks similar: $\vec{r} \times \vec{F} = F_{\perp} r = F r_{\perp} = Fr \sin\theta$. (Look familiar?)

- ... but $\vec{r} \times \vec{F}$ is a vector

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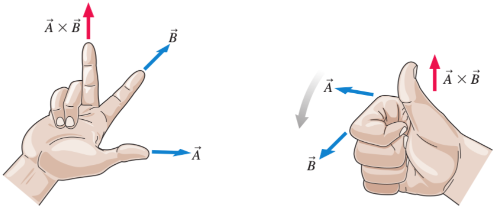
- ... but $\vec{r} \times \vec{F}$ is a vector
- Which way does it point? Use the "right hand rule":

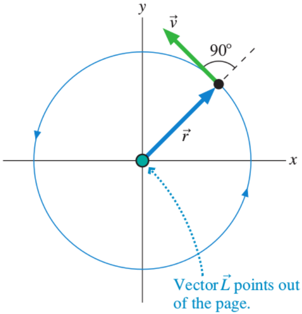


- Thumb goes in the direction of $\vec{r}$, pointer goes in the direction of $\vec{F}$, middle finger shows you $\vec{\tau}$
- Point fingers in the direction of $\vec{r}$, curl them in the direction of $\vec{F}$; thumb tells you the direction of $\vec{\tau}$

This gets you the same result for the direction of the torque.
(Knowing about the cross product and the dot product will be **very important** in Physics 2!)

The angular momentum vector, again

We can also say that $\vec{L} = m\vec{r} \times \vec{v}$
(Note that this matches our formula from before: $L = mv_T r$)
Here the right-hand rule tells us that $\vec{L}$ points out of the board.
This matches our other definition: "$\vec{L}$ points toward an observer that sees rotation as counterclockwise."



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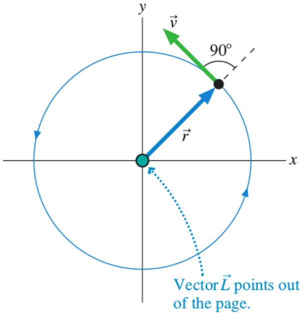

One detail: $\vec{\omega}$ only points in the same direction as $\vec{L}$ if the object is rotating "mostly" about an axis of symmetry.
Otherwise, ω behaves chaotically; the object tumbles.

The magic wheel, explained

What direction does $\vec{L}$ point?

The magic wheel, explained

What direction does $\vec{L}$ point?

- $\vec{L}$ points (as I'm showing it here) along the axle, away from the string

What direction does $\vec{\tau}$ point?

- $\vec{\tau}$ points horizontally, "around" the circle

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is the rate of change of $\vec{L}$, so gravity doesn't make it fall – just "rotates" the axis of rot

This works the same way as an ordinary top

The gyroscope

An object that is spinning rapidly has a great deal of angular momentum.
Since its angular momentum vector is so "long", it resists changes in its direction.
We can do two things with this:

- Since its axis stays fixed, we can use it to determine how *we've* rotated
- Since it has so much $\vec{L}$, we can apply torques to it without affecting its axis of rotation much.
  - By Newton's third law, these forces also exert a torque on something else
  - This can be used to orient a free-floating object, like the Hubble Space Telescope

How fast can we propel a ping-pong ball using nothing at all?