Recitation Questions

30 January

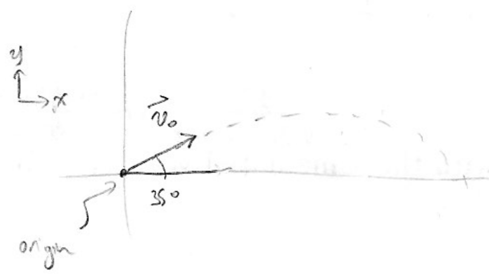
Question 1: a hiker crosses a stream

A hiker in the Adirondacks[1] encounters a stream that is too wide to jump across. So she doesn't get her boots wet, she takes them off and throws them across before walking barefoot through the water.

Suppose that the stream is 12 m across, and she throws her boot from ground level at an angle $\theta = 35°$ above the horizontal.

First, you will calculate the initial velocity required to get the boot across the stream. Then you'll figure out what happens if she throws it at a different angle than she intended to.

1. Draw a diagram of the boot's path in the air. Choose a coordinate system: what point are you considering $(x = 0, y = 0)$, and which directions are positive?



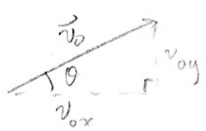
2. Write expressions for the $x-$ and $y-$components of its position and velocity as a function of time. These expressions will have lots of variables in them (a_x, a_y, $v_{x,0}$, $v_{y,0}$, x_0, and y_0) – that's okay.

$$x(t) = x_0 + v_{ox}t + \tfrac{1}{2}a_x t^2, \quad v_x(t) = v_{ox} + a_x t$$
$$y(t) = y_0 + v_{oy}t + \tfrac{1}{2}a_y t^2, \quad v_y(t) = v_{oy} + a_y t$$

these are the most general equations for constant acceleration

notice $\vec{a}$ (from gravity) has zero x-component

3. Do you know anything about any of those variables? If so, which ones?



$x_0 = 0 \qquad y_0 = 0$

$v_{ox} = v_0 \cos\theta \qquad v_{oy} = v_0 \sin\theta$

$a_x = 0 \qquad a_y = -g = -9.8\ \text{m/s}^2$

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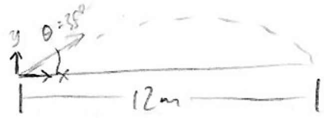
[1] This problem is based on a true story; the hiker is Emily Keene, a long-time PHY211 coach now working for Onondaga County Public Health as an environmental engineer. Yes, she really threw a boot into a stream.

4. With what initial velocity v_0 must she throw her boot in order to get it across the stream?



When the boot lands on the ground, it must have travelled 12m in the x-direction to make it across.

$$x(t) = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \qquad y(t) = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

$$\downarrow \qquad\qquad \downarrow$$

$$x_f = v_0 (\cos\theta) t \qquad 0 = v_0 (\sin\theta) t - \frac{1}{2} g t^2$$

Notice we can divide by t to get $0 = v_0 \sin\theta - \frac{1}{2} g t$

We have two equations with two unknowns (v_0 and t), so we can solve for v_0 by getting rid of t. We can solve one of the equations for t, then plug that t into the other equation. $t = \frac{x_f}{v_0 \cos\theta}$, then

$$0 = v_0 \sin\theta - \frac{1}{2} g \left(\frac{x_f}{v_0 \cos\theta}\right), \text{ or } \frac{g x_f}{2 v_0 \cos\theta} = v_0 \sin\theta.$$ Getting v_0 by itself,

we took the positive root since we know v_0 is positive

$$\frac{g x_f}{2 \cos\theta \sin\theta} = v_0^2, \text{ so } v_0 = \sqrt{\frac{g x_f}{2 \cos\theta \sin\theta}} = \sqrt{\frac{(9.8\,m/s^2)(12\,m)}{2 \cos 35° \sin 35°}} = \boxed{11.2\,m/s}$$

5. Suppose that she accidentally throws her second boot with the same initial velocity v_0 but at an angle $\theta = 65°$ above the horizontal. Where will it land?

Now x_f is our unknown, but the algebra is exactly the same.

Solving our final answer for x_f instead,

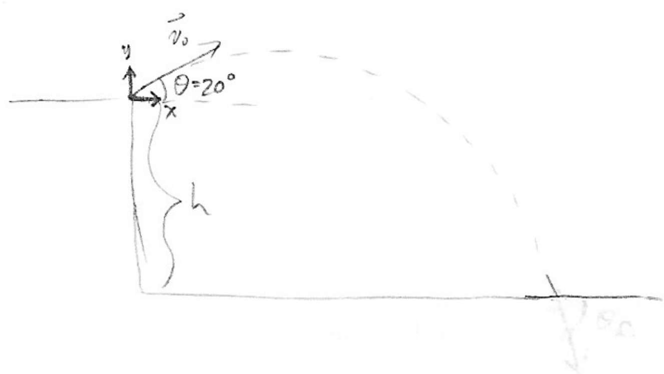
$$x_f = \frac{2 v_0^2 \cos\theta \sin\theta}{g} = \frac{2 (11.2\,m/s)^2 \cos 65° \sin 65°}{9.8\,m/s^2} =$$ 9.8 meters away from her (in the river)

Question 2: a prankster

The students in the next two problems are based on two of our long-time PHY211 coaches, one of whom still teaches with us. They might even be in your recitation!

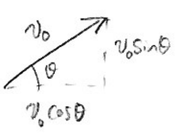
A mischievous SUOC student has climbed on the roof of a snow-covered building and is trying to hit her friend with snowballs as he walks through the Quad. She throws them at an angle of 20° above the horizontal at a speed of $v_0 = 5$ m/s. The building has a height $h = 6$ m.

1. Draw a cartoon of the problem, making clear your coordinate system and origin, and labelling interesting things.



$x_0 = 0$ and $y_0 = 0$
since we put our origin where the ball is thrown from

2. Write expressions for $x(t)$, $y(t)$, $v_x(t)$, and $v_y(t)$, substituting in variables that you know.



$x(t) = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$

$\downarrow$

$x(t) = (0) + v_0 \cos\theta \, t + (0)$

$\boxed{x(t) = v_0 \cos\theta \, t}$

$y(t) = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$

$\downarrow$

$y(t) = (0) + v_0 \sin\theta \, t + \frac{1}{2}(-g) t^2$

$\boxed{y(t) = v_0 \sin\theta \, t - \frac{1}{2} g t^2}$

$v_x(t) = v_{0x} + a_x t$

$\downarrow$

$\boxed{v_x(t) = v_0 \cos\theta} + (0)$

$v_y(t) = v_{0y} + a_y t$

$\downarrow$

$\boxed{v_y(t) = v_0 \sin\theta - g t}$

3. Write sentences in terms of your algebraic variables that allow you to answer the following. You will need to incorporate vector language at times: for instance, you may need to use terms like "the magnitude of the velocity vector" (which will require you to solve for both v_x and v_y.)

- How much time does it take for the snowballs to hit the Quad?

What is t at the time when $y=0$?

- Where do the snowballs land on the Quad?

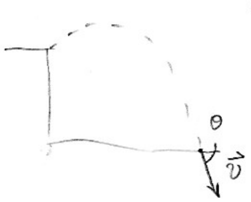
What is x at the time when $y=0$?

what is the speed,
the magnitude of the velocity vector

- How fast are the snowballs traveling when they hit the Quad?

What is the magnitude of the velocity vector at the time when $y=0$?

- In what direction are they moving when they land on the Quad?

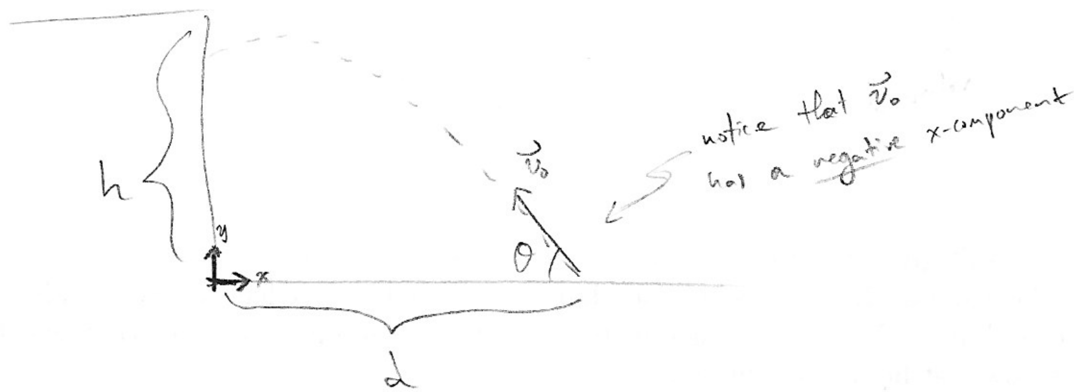


How many degrees below horizontal is the velocity vector at the time when $y=0$?

Question 3: retaliation!

He decides to throw a snowball back at her. He's standing a distance d from the side of the building, and throws a snowball at an angle θ above the horizontal at a speed v_0. However, the snowball slips out of his hand when he throws it, and it doesn't go very fast – instead of hitting her on top of the building, it hits the side of the building.

1. Draw a cartoon of the problem, making clear your coordinate system and origin, and labelling interesting things.



2. Write expressions for $x(t)$, $y(t)$, $v_x(t)$, and $v_y(t)$, substituting in variables that you know.

$x(t) = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$

$\downarrow$

$\boxed{x(t) = d - v_0 \cos\theta\, t}$

$v_x(t) = v_{0x} + a_x t$

$\boxed{v_x(t) = -v_0 \cos\theta}$

$y(t) = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$

$\downarrow$

$\boxed{y(t) = v_0 \sin\theta\, t - \frac{1}{2} g t^2}$

$v_y(t) = v_{0y} + a_y t$

$\boxed{v_y(t) = v_0 \sin\theta - g t}$

3. Write a sentence in terms of your algebraic variables that will let you figure out how far above the ground the snowball hits the side of the building.

What is y at the time when $x = 0$?

↑ with my choice of origin, the side of the building is at $x = 0$.

4. Based on your sentence, figure out how far above the ground the snowball hits the building. Your answer should be in terms of v_0, θ, d, and g.

$x(t) = d - v_0 \cos\theta\, t$. When $x=0$, $0 = d - v_0\cos\theta\, t$, so $t = \dfrac{d}{v_0 \cos\theta}$

$y(t) = v_0 \sin\theta\, t - \frac{1}{2}gt^2$, so at that time,

$$y = v_0 \sin\theta \left(\frac{d}{v_0\cos\theta}\right) - \frac{1}{2}g\left(\frac{d}{v_0\cos\theta}\right)^2$$

$$= \boxed{d\tan\theta - \frac{gd^2}{2v_0^2\cos^2\theta}}$$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

This is how high the snowball is off the ground when it hits the wall

5. He doesn't give up, though, and throws another snowball at her – again at an angle θ above the horizontal. He throws this one harder, and it hits her feet as she stands on the edge of the building. Write a sentence in terms of your algebraic variables that will let you figure out how fast he had to throw it.

What does v_0 have to be so that $y=h$ at the time when $x=0$?

6. Now, based on your previous sentence, figure out the initial speed of the second snowball he threw.

Since we solved question 4 symbolically, we can re-use that work. We are still interested in the time when $x=0$, we just now know that $y=h$ when $x=0$, and our unknown is v_0.

That means $h = d\tan\theta - \dfrac{gd^2}{2v_0^2\cos^2\theta}$. Solving for v_0,

$$h - d\tan\theta = \frac{-gd^2}{2v_0^2\cos^2\theta}, \quad v_0^2 = \frac{-gd^2}{2\cos^2\theta\,(h - d\tan\theta)} = \frac{gd^2}{2\cos^2\theta\,(d\tan\theta - h)},$$

and $$\boxed{v_0 = \sqrt{\frac{gd^2}{2\cos^2\theta\,(d\tan\theta - h)}}}.$$