Problem solving: kinematics (II)

Physics 211
Syracuse University
Walter Freeman

January 27, 2020

Announcements

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- Homework 3 is posted and is due **next Tuesday**

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- Exam 1 is next Tuesday
  - HW3 problems are similar to those on Exam 1
- Recitation Friday is your group practice exam
  - If you must miss Friday, notify your TA in advance
    - If you miss a group exam, your grade on the main exam will also be used for the group exam
    - You may only do this once
    - If you must miss multiple group exams for solemn observances, academic or professional events, or illness, please let me know

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- Review session in class on Thursday
- You are allowed to bring one side of one page of notes that *you handwrite yourself* to the exam (but not the group exam)
- You do not *need* to bring notes; I will give you the kinematics relations on a reference page
  - No typed notes unless you have a disability that hinders your writing
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    - It won't help you as much anyway

Exam 1, promises

- There will be one problem where you need the quadratic formula
  - ... this means interpreting the two values it spits out
- There will be at least one instance where you need to interpret or sketch position, velocity, and acceleration graphs
- You will *not* need to compute derivatives or integrals algebraically
- The exam will be four problems, plus possibly a few short-answer questions

Opportunities to get help

Today: I'll be in the Physics Clinic from 1:30-3:00 (modified from usual time of 2-4)

Friday: I'll be in the Physics Clinic from 9:30-12; Prof. Rudolph will be there 2-4.

Monday: I'll be in the Physics Clinic from 9am-12.

Last time

Vectors: objects with direction and magnitude

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Two representations:

- Magnitude and direction (easiest to state, hardest to work with)
- Components (easiest to work with)
- Use trigonometry to go back and forth

Unit vectors

In the “ordered pair” notation for vectors’ components, you might write:

$$\vec{v} = (5, 3)$$

But this is clunky, if you’re trying to write it as part of an algebraic statement.

Instead we introduce “unit vectors”, vectors with length 1, in the x, y, and z directions.

$$\begin{aligned}\hat{i} &= (1, 0, 0)\\ \hat{j} &= (0, 1, 0)\\ \hat{k} &= (0, 0, 1)\end{aligned}$$

- $\vec{v} = (5, 3)$: Ordered pair
- $\vec{v} = 5\hat{i} + 3\hat{j}$: Unit vectors
- $v_x = 5$, $v_y = 3$: Vector components

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- $\vec{v} = (5, 3)$: Ordered pair
- $\vec{v} = 5\hat{i} + 3\hat{j}$: Unit vectors
- $v_x = 5$, $v_y = 3$: Vector components
- Both give you the same information, but unit vectors can be easier algebraically
- We will generally use the last form, since components are easiest to solve problems with

A word on positive and negative acceleration, velocity, “speed”, and displacement:

When you choose your origin, you choose one direction to be positive, and the other to be negative. (Here: right = positive.)

- An object with $x < 0$ just means it's left of the origin.
- An object with $v < 0$ means it's moving to the left.
- An object with $a < 0$ means:
  - A: it is moving to the left and gaining speed
  - B: it is moving to the right and slowing down
  - C: it is moving to the left and slowing down
  - D: it is moving to the right and gaining speed

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Do not confuse the sign of something with the sign of its derivative!

Last time

Acceleration, velocity, and position relationships are the same in 2D; they just apply independently for each component.

$$\vec{v}(t) = \vec{a}t + \vec{v}_0$$
$$\vec{s}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$$

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$$x(t) = \frac{1}{2}a_x t^2 + v_{x,0}t + x_0$$
$$y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + y_0$$

Working with variables

If you don't know the numerical value of a quantity yet,
it's fine to leave it as a variable!

This is essential for solving many problems.

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Example for projectile motion:

$$x(t) = \frac{1}{2}a_x t^2 + v_{x,0} t + x_0$$
$$y(t) = \frac{1}{2}a_y t^2 + v_{y,0} t + y_0$$

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Example for projectile motion:

$$x(t) = v_{x,0} t$$
$$y(t) = -\frac{1}{2} g t^2 + v_{y,0} t$$

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If you don't know the numerical value of a quantity yet,
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This is essential for solving many problems.

Example for projectile motion:

$$x(t) = v_0 \cos 45^o t$$
$$y(t) = -\frac{1}{2}gt^2 + v_0 \sin 45^o t$$

(We'll see the rest in a minute)

Problem solving: 2D kinematics, constant acceleration

1. 0. Draw a cartoon of the situation, and choose a coordinate system
2. 1. If you have vectors in the "angle and magnitude" form $(\vec{a}, \vec{v}, \vec{s})$, convert them to components
3. 2. Write down the kinematics relations, separately for x and y
   - Many terms will usually be zero
   - Freefall: $a_x = 0$, $a_y = -g$ (with conventional choice of axes)
4. 3. Understand what instant in time you want to know about: ask the right question
5. 4. Put in what you know; solve for what you don't (using substitution, if necessary)
6. 5. Think about the physical meaning of your solution

“What instant in time do you know about?”

This is often the most difficult part of problems: it requires thought, not just math.

You throw a ball upward over a hole of height h. Your position is the origin, and up is positive.

What condition means “the ball has hit the ground”?

- A: $y = 0$
- B: $y = h$
- C: $y = -h$
- D: $v_y = 0$

“What instant in time do you know about?”

You throw a ball upward off of a cliff of height h. The top of the cliff is the origin, and up is positive.

What condition means “the ball is at its highest point?”?

- A: $y = 0$
- B: $v_y = 0$
- C: $y = h$
- D: y is a maximum

A football (soccer) player

A player kicks a ball at 20 m/s at an angle of 30 degrees above the horizontal on a level field?

How can we frame the question "How far does the ball go?" in terms of our variables?

- A: What is x at the same time that v_x is zero?
- B: What is y at the same time that x is is zero?
- C: What is x at the same time that y is zero?
- D: What is x at the same time that v_y is zero?

A football (soccer) player

- A player kicks a ball at 80 m/s at an angle of 30 degrees above the horizontal.

A football (soccer) player

- A player kicks a ball at 80 m/s at an angle of 30 degrees above the horizontal.
- How high does the ball go?

A football (soccer) player

- A player kicks a ball at 80 m/s at an angle of 30 degrees above the horizontal.

- How high does the ball go?
- How fast is it traveling at its highest point?

A football (soccer) player

- A player kicks a ball at 80 m/s at an angle of 30 degrees above the horizontal.

- How high does the ball go?
- How fast is it traveling at its highest point?
- How fast is it traveling when it strikes the ground?

A football (soccer) player

- A player kicks a ball at 80 m/s at an angle of 30 degrees above the horizontal.

- How high does the ball go?
- How fast is it traveling at its highest point?
- How fast is it traveling when it strikes the ground?
- Which way is it moving when it hits the ground?

A football (soccer) player

- A player kicks a ball at 80 m/s at an angle of 30 degrees above the horizontal.

What is $v_{0,x}$?

A: $v_0 \cos\theta$
B: $v_0 \sin\theta$
C: $v_0 \tan\theta$
D: v_0

A football (soccer) player

- What changes if I put the player up on a hill?

A football (soccer) player

- What changes if I put the player up on a hill?
- What changes if she's trying to kick the ball to someone up on a plateau?

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- What changes if I want to know what initial velocity she needs to hit a target?

A football (soccer) player

- What changes if I put the player up on a hill?
- What changes if she's trying to kick the ball to someone up on a plateau?
- What changes if I want to know what initial velocity she needs to hit a target?
- What changes if I have air resistance?

Throwing a rock off a cliff

A hiker throws a rock horizontally off of a $h = 100$ m tall cliff. If the rock strikes the ground $d = 30$ m away, how hard did she throw it? How fast was it going when it hit the ground? (Choose the origin at the base of the cliff, up/direction of throw as positive)

What is $v_{0,x}$ here?

A: 0
B: 10/3 m/s
C: You don't know *a priori*

What is $v_{0,y}$ here?

A: 0

B: 9.81 m/s

C: You don't know *a priori*

What is a_x here?

A: 0

B: -g

C: +g

D: You don't know *a priori*

What is a_y here?

A: 0

B: -g

C: +g

D: You don't know *a priori*

What is x_0 here?

A: 0

B: h

C: d

D: You don't know *a priori*

What is y_0 here?

A: 0

B: h

C: d

D: You don't know *a priori*

What question do you ask to find “how hard did she throw it?”

A: What value of $v_{x,0}$ makes it such that $x = d$ at the time that $y = 0$?
B: What value of $v_{y,0}$ makes it such that $x = d$ at the time that $y = h$?
C: What is the value of v_x when $y = 0$?
D: What is the magnitude of $\vec{v}$ when $y = 0$?
E: What is v_x when $y = h$?

What question do you ask to find “how fast is it going when it hits the ground?”

A: What is v_x at the time when $v_y = 0$?
B: What is v_x at the time when $y = 0$?
C: What is v_y at the time when $y = h$?
D: What is the magnitude of $\vec{v}$ when $y = 0$?
E: What is the magnitude of $\vec{v}$ when $y = h$?

What's the magnitude of $\vec{v}$?

A: $v\cos\theta$

B: $v\sin\theta$

C: $\tan^{-1}\frac{v_x}{v_y}$

D: $\sqrt{v_x^2+v_y^2}$

Throwing a stone onto a slope

A hiker kicks a stone off of a mountain slope with an initial velocity of v_0 3 m/s horizontally. If the mountain has a slope of 45 degrees, how far down the slope does it land? (Choose the origin as the starting point.)

A: What is the magnitude of $\vec{s}$ when $x = y$?
B: What is the magnitude of $\vec{s}$ when $x = -y$?
C: What is the magnitude of $\vec{s}$ when $y = 0$?
D: What is y when $x = -y$?
E: What is y when $x = 0$?

A rocket

A rocket is launched from rest on level ground. While its motor burns, it accelerates at 10 m/s at an angle 30 degrees below the vertical. After $\tau = 10$ s its motor burns out and it falls freely until it hits the ground.

Sketch position vs. time, velocity vs. time, acceleration vs. time for both x and y components.

How far does it go?