Rotational motion 2

Physics 211
Syracuse University, Physics 211 Spring 2015
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April 7, 2015

Announcements

- Review times set for next Tuesday's exam
  - Thursday: Eggers 018, 2:00 to 5:00
  - Friday: Sims 337, 10:00 to 4:00
- Solutions to the practice exam posted Friday
- I'll let you know Thursday whether there will be recitation on Tuesday (depends on TA's)

The correspondence table

Translation	Rotation
Position x	Angle θ
Velocity v	Angular velocity ω
Acceleration a	Angular acceleration α
$v(t) = v_0 + at$	$\omega(t) = \omega_0 + \alpha t$
$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v_f^2 - v_0^2 = 2a\Delta x$	$\omega_f^2 - \omega_0^2 = 2\alpha\Delta\theta$
Force $\vec{F}$	Torque: $\tau = F_\perp r$
Mass m	Moment of Inertia: $I = \lambda M R^2$
$\vec{F} = m\vec{a}$	$\tau = I\alpha$
Work $= \vec{F} \cdot \Delta\vec{s}$	Work $= \tau\Delta\theta$
Kinetic energy $\frac{1}{2}mv^2$	Kinetic energy $\frac{1}{2}I\omega^2$
Power ($\vec{F}$ constant) $= \vec{F} \cdot \vec{v}$	Power (τ constant) $= \tau\omega$
Momentum $\vec{p} = m\vec{v}$	Angular momentum $L = I\omega$

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  - The net force tells you how the center of mass accelerates
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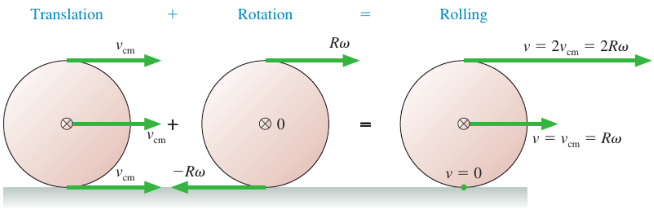
- For purposes of torque the gravitational force acts at the center of mass
- For symmetric, uniform objects: the center of mass is at the center!
- For other objects, it's just a weighted average of all of the parts:

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + ...}{m_1 + m_2 + m_3}$$

$$y_{\text{com}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + ...}{m_1 + m_2 + m_3}$$

A description of rolling motion without slipping

Rolling motion combines translation of the center of mass with rotation around it



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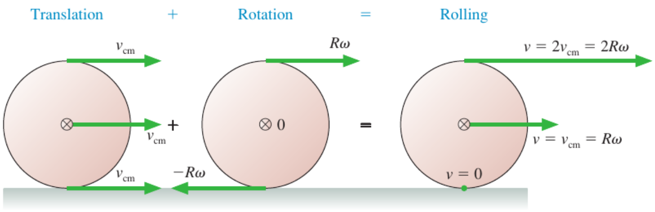

- The key idea: tangential velocity of the rim about the center is equal to the speed of the axle
- In symbols:

$$v_{\text{com}} = v_T = \omega r$$

Static equilibrium problems

- Often we are presented with a situation where nothing moves, and we have to solve for something
- No acceleration of the center of mass: $\sum \vec{F} = 0$
- No angular acceleration: $\sum \tau = 0$ about *any* pivot point
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- Strategy: choose the pivot to be aligned with a force you don't know and don't care about

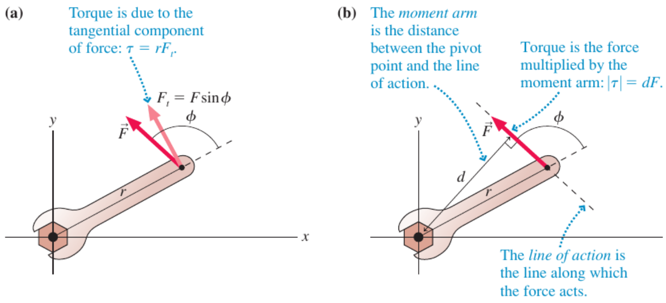
One more thing: another way of computing torque

Last time we saw that the torque $\tau = F_\perp r$

... but there's another way to compute it which is sometimes more helpful

- Note that $F_\perp = F \sin\theta$, so $\tau = Fr\sin\theta$
- We can think of the torque in any other equivalent way; there is another one that's often useful
- The way I taught you yesterday: **"The radius vector, times the component of force perpendicular to it"**
- The alternative: **"The force vector, times the component of the radius perpendicular to it"**

Here's the figure from the text:



I'll draw a clearer version on the document camera

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NB: Your book calls $r_\perp$ the "moment arm"

A sample problem: distribution of weight in a truck

A 2000 kg rear wheel drive pickup truck has a wheelbase 5 meters long. Its center of mass is located 150 cm behind the front wheels, and 130 cm above the ground. What is the fastest it can accelerate, if $\mu_s = 0.6$?

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Compute torque about the front wheels:

$$\begin{aligned} 1.5mg &= 5F_{N,\text{rear}} \\ F_{N,\text{rear}} &= 0.3mg \\ F_{f,\text{max}} &= \mu_s F_{N,\text{rear}} = 0.18mg \\ a_{\text{max}} &= 0.18g \end{aligned}$$

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- We know $L = I\omega$, but what are I and ω for the ball?
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- Note that for the ball approaching the volunteer, $r_\perp$ is constant, so L is constant

Finding the balancing point

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Same method as before, but note:

- The point of support is also the center of mass
- The converse is also true: an object's center of mass must be within its base of support if it is to stay put

The Atwood's machine, for real

A solid pulley of mass M and radius r has a mass m hanging from one side. How fast does it accelerate?

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Strategy: same as for our linear motion problems.

- 1. Draw force diagrams for everything
- 2. Write $\vec{F} = m\vec{a}$ for things that have translational motion
- 3. Write $\tau = I\alpha$ for things that have rotational motion
  - Here, the tension is another unknown variable appearing in both equations
- 4. Use constraints to relate α's to a's
- 5. Solve the system of equations