Physics 211 Practice Exam 3

Question 1 covers basic principles behind the work-energy theorem.

Question 2 asks you to reason conceptually about conservation of energy and momentum as applied to a "Newton's cradle" desk toy.

Question 3 asks you to perform calculations involving the conservation of energy and momentum in a system involving a spring and two colliding objects.

Question 4 asks you to apply the work-energy theorem to a swinging pendulum with an external force applied to it.

Question 5 asks you to investigate a runaway truck heading up a runaway truck ramp using work-energy methods, and to consider the power applied by a force to an object in motion.

Question 6 asks you to apply ideas of work, energy, and momentum to the orbits of planets in words.

Question 7 is another detailed calculation in which you must mix energy methods and the conservation of momentum.

Question 8 is a simple question involving two cars colliding while moving in two dimensions.

Recitations starting at 8:25 or 12:45 should focus on Questions 1 and 2.

Recitations starting at 9:30 or 2:15 should focus on Questions 3 and 4.

Recitations starting at 10:35 or 3:45 should focus on Questions 5 and 6.

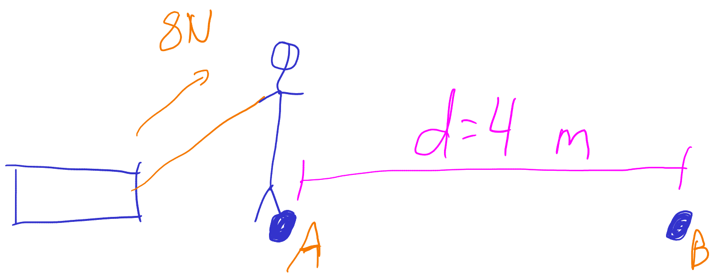
Recitations starting at 11:40 or 5:15 should focus on Questions 7 and 8.

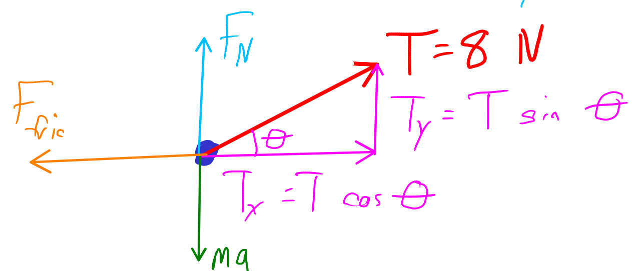
Of course, if you and your group finish early, or would like to work on other questions, you may certainly do so!

Question 1

~~You are~~ dragging a sled with a mass of 3 kg across an icy patch of snow. You pull on the sled with a force of $T = 8$ N directed at an angle $\theta = 30°$ above the horizontal. The coefficient of kinetic friction between the sled and the ground is $\mu_k = 0.05$. → note μ small

a) Draw a picture of the situation, and draw a free body diagram for the sled while it is being pulled.





b) What is the work done by the tension in the rope while you pull the sled a distance of $d = 4$ m? What is the work done by friction over that distance?

$W_{tension} = \vec{F}_T \cdot \Delta\vec{s} = d\,(T)_{\parallel} = d\,T_x = dT\cos\theta.$

$W_{fric} = \vec{F}_{fric} \cdot \Delta\vec{s} = -F_{fric}\,d = -\mu F_N d. \longrightarrow$ need F_N. → In y: $\sum F_y = 0$: $F_N - mg + T\sin\theta = 0.$

$F_N = mg - T\sin\theta.$

$\longrightarrow W_{fric} = -\mu d(mg - T\sin\theta).$

negative

c) Assuming the sled started with a speed of $0.5\,\mathrm{m\,s^{-1}}$, what will be its final speed after the 4 m distance?

$\boxed{\frac{1}{2}mv_i^2} + W_T + W_{fric} = \boxed{\frac{1}{2}mv_f^2}$

(F_N and gravity do no work since they are $\perp$ to motion)

$\frac{1}{2}mv_i^2 + dT\cos\theta - \mu d(mg - T\sin\theta) = \frac{1}{2}mv_f^2.$

Solve for v_f:

$v_f = \sqrt{v_i^2 + \frac{2dT\cos\theta}{m} - \frac{2\mu d(mg - T\sin\theta)}{m}} = 3.91\ \mathrm{m/s}.$

d) After pulling, you drop the rope and get out of the way. How far will the sled slide before coming to a stop? → no final KE.

Use work-energy again and solve for unknown distance b.

$\frac{1}{2}\cancel{m}v_i^2 - \underbrace{\mu b\cancel{m}g}_{-F_{fric}b} = 0 \longrightarrow b = \frac{\frac{1}{2}v_i^2}{\mu g}$ where $v_i = 3.91$ m/s.

: note F_N is now just mg since we dropped rope.

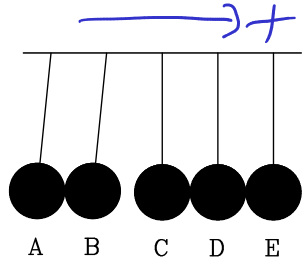
$\boxed{= 15.3\ \mathrm{m}}$

Question 2

Momentum is conserved

A popular desk toy, sometimes called a "Newton's cradle", consists of five identical metal spheres suspended by thin wires such that they touch each other. These spheres are very hard, such that collisions between them are very nearly elastic.

$KE_i = KE_f$



Suppose that someone pulls spheres A and B back and releases them, such that they strike the other three with an initial speed v_0.

Once spheres A and B strike the rest of the spheres, one or more will bounce off again.

Tell which of the following outcomes is possible. For the ones that are not possible, discuss what principle of mechanics they violate.

a) Spheres A and B come to rest; spheres C, D, and E begin moving to the right at a speed $\frac{2}{3}v_0$.

Initial $KE = 2\left(\frac{1}{2}mv_0^2\right) = mv_0^2$

Initial $p = 2mv_0$

Final $KE = 3\left(\frac{1}{2}m\left(\frac{2}{3}v_0\right)^2\right) = \frac{2}{3}mv_0^2$

Final $p = 3\left(m\frac{2}{3}v_0\right) = 2mv_0$

No - KE not conserved → not possible

b) Spheres A and B come to rest; spheres D and E begin moving to the right at a speed v_0. Sphere C does not move.

Final $KE = 2\left(\frac{1}{2}mv_0^2\right) = mv_0^2$

Final $p = 2(mv_0)$

→ conserves both momentum and KE

→ possible

c) Sphere B comes to rest. Sphere A bounces back, moving to the left at a speed v_0; sphere E begins moving to the right at a speed v_0.

Final $KE = 2\left(\frac{1}{2}mv_0^2\right)$

Final $p = mv_0 - mv_0$

→ not possible — momentum not conserved.

d) Spheres A and B come to rest; sphere E begins moving to the right at a speed $2v_0$. Spheres C and D do not move.

Final $KE = \frac{1}{2}m(2v_0)^2 = 2mv_0^2$

Final $p = m(2v_0)$

→ not possible: KE not conserved.

e) Spheres A and B come to rest; sphere E begins moving to the right at a speed $4v_0$. Spheres C and D do not move.

Final $KE = \frac{1}{2}m(4v_0)^2 = 8mv_0^2$

Final $p = m(4v_0) = 4mv_0$

→ doesn't conserve either

Question 3

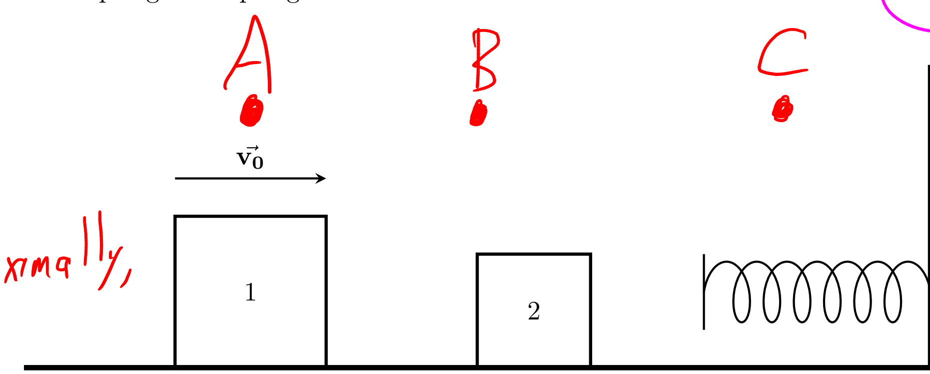
2m m $m = 1.5$ kg

A block with mass $m_1 = 3.0\,\mathrm{kg}$ is sliding across a frictionless table with a speed $v_0 = 1.2\,\mathrm{m\,s^{-1}}$ towards a block with $m_2 = 1.5\,\mathrm{kg}$. When they collide, they stick together. The pair of blocks then slide into a spring with spring constant $k = 4000\,\mathrm{N\,m^{-1}}$ mounted to the wall.

A: at start

B: right after collision

C: spring compressed maximally, $V = 0$.



• A → B: Conservation of momentum

• B → C: work-energy

a) What is the momentum and kinetic energy of the two-block system just after the collision?

Use conservation of $\vec{p}$ to find v_B after collision.

$P_i = P_f \longrightarrow (2m)v_0 = (2m+m)v_B$

Thus $V_B = \frac{2}{3}V_0 = 0.8\ m/s.$

$P_f = 2mv_0$ or $3mv_B$.

KE after collision

$= \frac{1}{2}(3m)v_B^2 = 1.44\ J.$

b) How far will the spring compress after the blocks slide into it?

$$KE_i + \cancel{PE_i} + \cancel{W_{other}} = \cancel{KE_f} + PE_f$$

$$\cancel{\tfrac{1}{2}}(3m)v_B^2 = \cancel{\tfrac{1}{2}}kd^2$$

$$d = \sqrt{\frac{3mv_B^2}{k}} = 2.68\ cm$$

c) When the spring makes the blocks bounce back, what will be their momentum and kinetic energy after leaving the spring?

Springs are conservative forces, so the KE once the blocks bounce back must be equal to the KE before contacting the spring.

• KE after bounce = $KE_{before\ bounce} = 1.44$ J

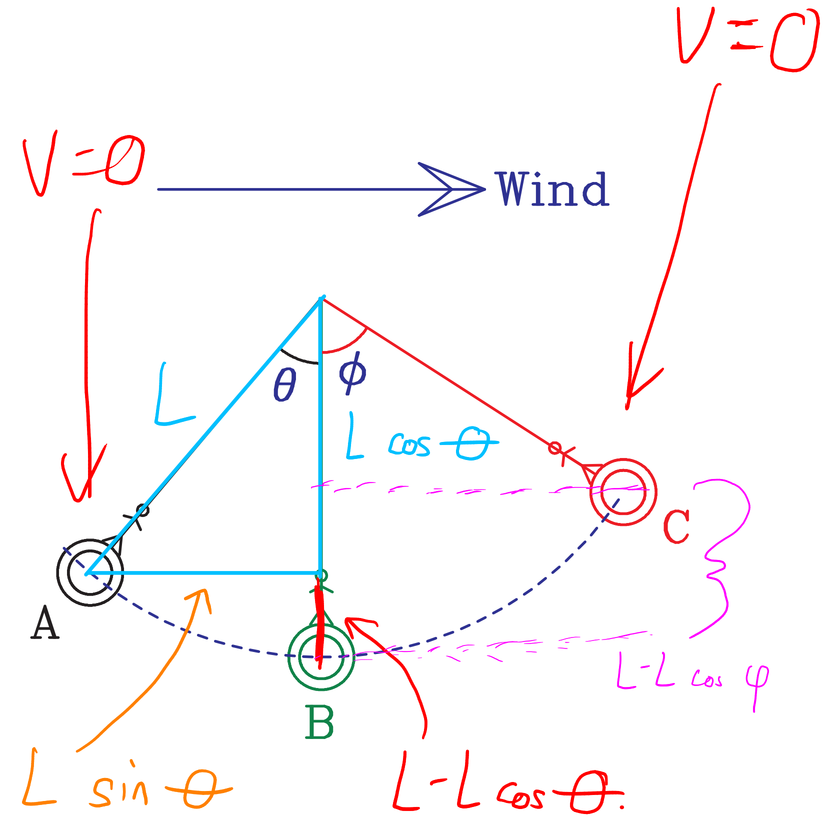
• V must just change direction but keep same magnitude, so $P_f = -P_i = -2mv_0$.

Question 4

A child is swinging on a tire swing hanging from a tree. The swing has a length L, and the tire plus the child have a mass m.

The child's father pulls the swing back to an angle θ and releases it from position A.

However, a strong wind is blowing from left to right, exerting a constant horizontal force F_w on the swing to the right. This means that it will swing to a larger angle ϕ on the right (at position C) than it started on the left.



a) As the swing moves from position A to position B, what is the work done by gravity?

$$W_{grav} = \vec{F}_{grav} \cdot (\Delta \vec{s}) = F_{grav} \cdot \text{distance moved downward.}$$

$$W_{grav} = mg(L - L\cos\theta).$$

b) As the swing moves from position A to position B, what is the work done by the wind?

$$W_{wind} = \vec{F}_{wind} \cdot (\Delta \vec{s}) = F_{wind} \cdot \text{distance moved right.}$$

$$= F_W (L \sin\theta).$$

c) Find the speed of the swing at position B in terms of F_w, m, L, θ, and g.

$$KE_A + W_{grav} + W_{wind} = KE_B$$

$$0 + mg(L - L\cos\theta) + F_w(L\sin\theta) = \frac{1}{2}mv_B^2$$

$$v_B = \sqrt{2g(L-L\cos\theta) + 2\frac{F_w}{m}(L\sin\theta)}$$

d) Write down an equation that you could solve for the angle ϕ in terms of F_w, m, L, θ, and g. (You do not need to actually solve it.)

Use W-E theorem from (A) to (C):

$$\cancel{KE_A} + W_{wind} + W_{grav} = \cancel{KE_C} \rightarrow \boxed{F_w(L\sin\theta + L\sin\varphi) + mg(L\cos\varphi - L\cos\theta) = 0.}$$

- $W_{wind} = F_w \Delta x = F_w(L\sin\theta + L\sin\varphi)$
- $W_{grav} = F_g(-\Delta y) = mg(\cancel{L} - L\cos\theta - (\cancel{L} - L\cos\varphi)) = mg(L\cos\varphi - L\cos\theta)$

e) When the tire swings back to the left, will it stop at position A, will it move further to the left, or will it not reach position A at all? Give an argument in words.

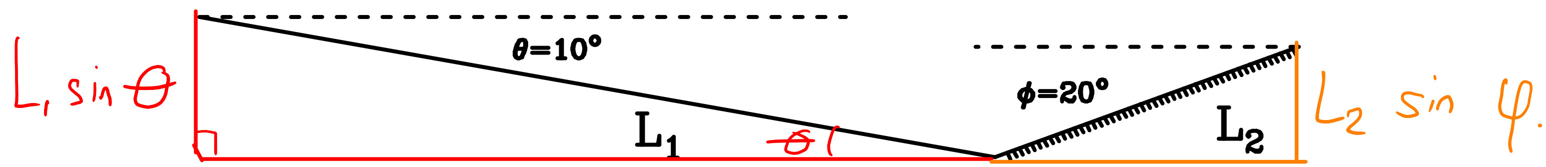
- When the tire reaches A again, total work done by gravity $= F_g(-\cancel{\Delta y}) = 0$
  total work done by wind $= F_w(\Delta x) = 0.$

$\rightarrow$ total work from $A \rightarrow A = 0$, so KE reaches zero at A, and it stops again at A.

Question 5

Heavy trucks driving down steep mountains must continually apply their brakes to maintain a safe speed, since the rolling friction between tires and pavement is very small. If their brakes fail, these roads are equipped with "runaway truck ramps", which are short uphill pathways (made of sand or gravel) with a large coefficient of rolling friction.

A truck whose brakes fail can steer into the ramp and come safely to a stop. Suppose that a truck of mass m is driving down the hill at a speed v_0 when its brakes fail. *(There is no friction while the truck is on the road.)* It is a distance L_1 away from the ramp, traveling at a speed v_0. When it reaches the ramp, it exits the highway and heads up the ramp, traveling a distance L_2 before coming to rest. In this problem, you will calculate the distance L_2 in terms of μ_r, g, m, L_1, v_0, θ, and ϕ.



a) Write an expression for the total work done by gravity during the entire motion in terms of g, m, L_1, L_2, θ, and ϕ.

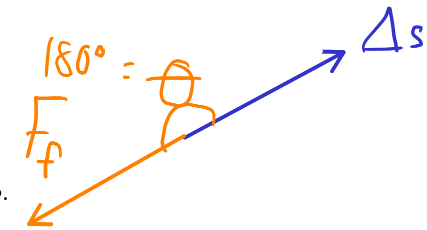
$$W_{grav} = \underbrace{mg}_{F_g}\underbrace{(\text{distance moved downward})}_{(\Delta s)_{\text{parallel to force}}}$$

"distance moved downward"

$$= L_1 \sin\theta - L_2 \sin\varphi$$

$$W_{grav} = mg(L_1 \sin\theta - L_2 \sin\varphi).$$

$\vec{A} \cdot \vec{B} = AB \cos\theta$; $\cos 180° = -1$



QUESTION 5, CONTINUED

b) Write an expression for the total work done by friction during the entire motion in terms of μ_r, g, m, L_2, and ϕ.

$W_{fric} = -F_{fric}(L_2)$ (just a − sign, no trig, since here F_f and Δs are antiparallel!)

$F_{fric} = \mu F_N = \mu \underbrace{mg\cos\varphi}_{F_N}$, so $W_{fric} = -\mu m g L_2 \cos\varphi$.

c) Write a statement of the work-energy theorem/conservation of energy in terms of μ_r, g, m, L_1, L_2, v_0, θ, and ϕ that you could solve for L_2. (You do not need to solve it.)

$\frac{1}{2}mv_0^2 + W_{grav} + W_{fric} = \cancel{\frac{1}{2}mv_f^2}$ ↑ 0

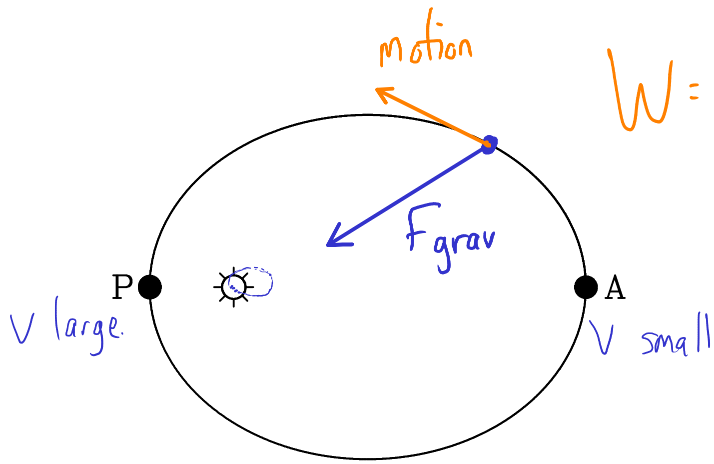
$$\boxed{\frac{1}{2}mv_0^2 + mg(L_1\sin\theta - L_2\sin\varphi) - \mu m g L_2\cos\varphi = 0.}$$

d) Now, consider a truck whose brakes are working. It has a mass of $m = 10^4$ kg (10 tons) and is driving down a hill with a grade of $\theta = 10°$. If the driver wants to maintain a speed of $v = 15$ m/s, what is the power that the brakes must dissipate?

If v = const, $P_{net} = 0$ and $P_{brakes} + P_{grav} = 0$.

$P_{grav} = \vec{F}_{grav} \cdot \vec{v} = \underbrace{mg}_{F_{grav}}\underbrace{(v\sin\theta)}_{v_{\parallel}} = 260$ kW!!! ≈ 320 hp

Question 6



$W = F \Delta s \cos \theta$

A planet orbits a star in an elliptical orbit, as shown. (The only force acting on the planet is the the star's gravity.) The point of closest approach, called "perihelion", is labeled **P**; the point where the planet is furthest away, called "aphelion", is labeled **A**.

Kepler's second law of orbital motion (which we have not studied specifically) says that the comet must be moving faster at perihelion (point P) than aphelion (point A).

a) Explain why this must be true by invoking the work-energy theorem. (You can answer this with words alone; you do not necessarily need to write equations.)

An object's kinetic energy (and thus its speed) increases if positive work is done on it.

- F_{grav} points toward the star
- We are moving closer to the star

$\longrightarrow$ $W_{grav} = \Delta KE$ is positive as we move from A to P.

b) If you look closely at the star, you will observe that it is *also* moving slightly, with its center tracing out a small ellipse. Based on what you have learned in this unit, argue why this must happen.

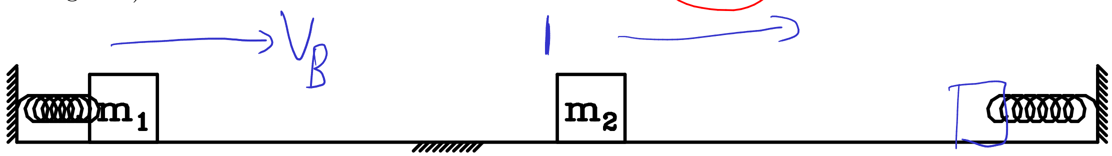
- No external forces $\rightarrow$ momentum is conserved. So, any change in velocity of the planet must come with a smaller change in velocity of star so that the net $\Delta \vec{p} = 0$.

Question 7

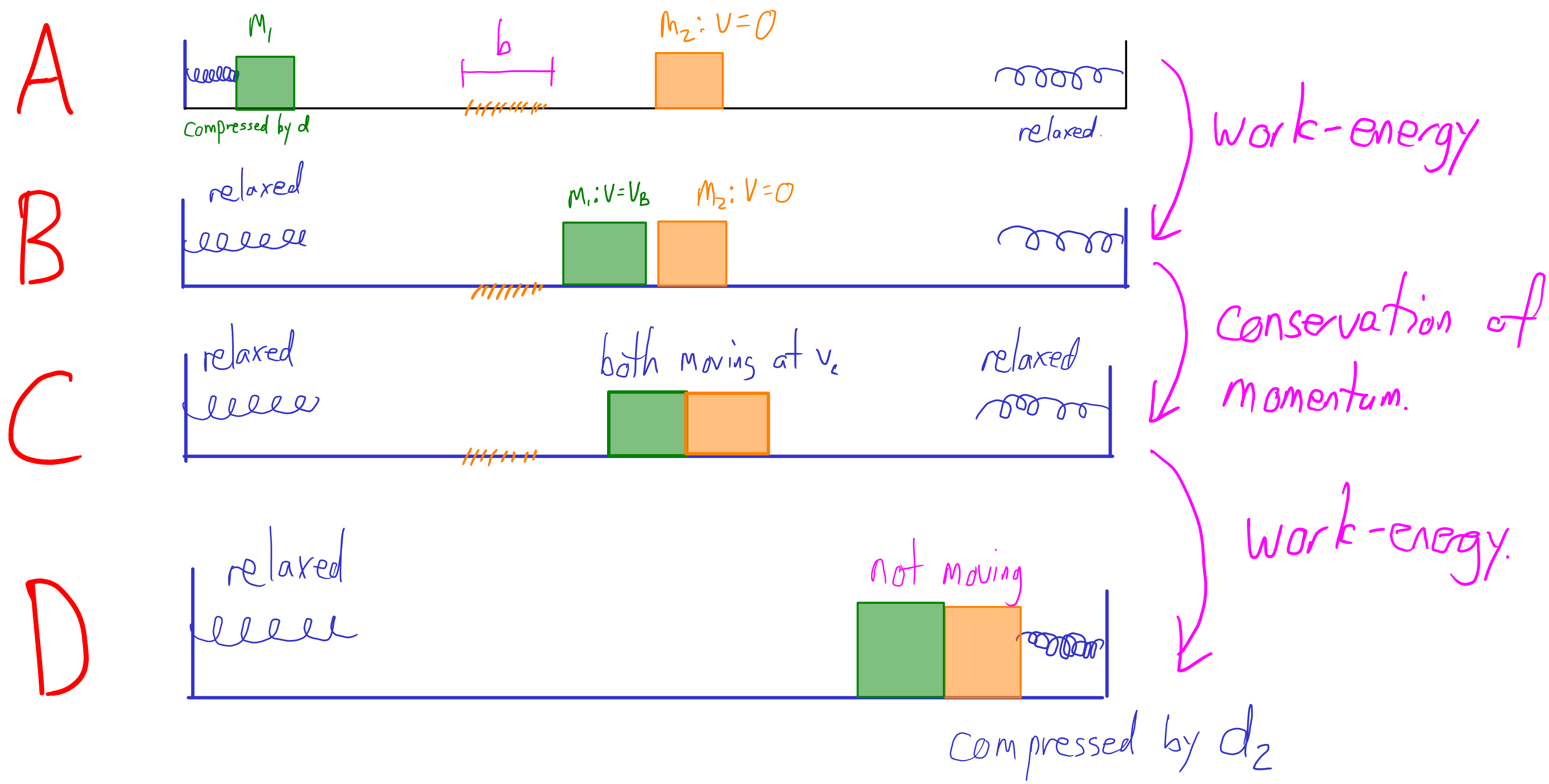
A spring of spring constant k is compressed by a distance d by a mass m_1 and released. This propels the mass down a flat track. A second spring, also of spring constant k, is on the other side.

Another object of mass m_2 is sitting in the middle of the track. The first mass strikes it and sticks to it.

The entire track is frictionless, except for a small region of the track to the left of m_2, of width b, with a coefficient of kinetic friction μ. (This is indicated by diagonal lines on the track in the diagram.)



(The space below is for you to draw other diagrams, if you so choose, or to do scratch work.)



a) How fast is the first mass moving right before it collides with the second block?

Use W-E from A to B:

$$\cancel{KE_A} + PE_A + W_{fric} = KE_B + \cancel{PE_B}$$

$$\frac{1}{2}kd^2 - \mu m_1 g b = \frac{1}{2}m_1 v_B^2$$

$$\longrightarrow \boxed{v_B = \sqrt{\frac{k}{m_1}d^2 - 2\mu g b}}$$

b) How fast are the two masses moving right after the collision?

Use cons. of $\vec{p}$ from B to C:

$$m_1 v_B = (m_1 + m_2)v_c \quad \text{So} \quad v_c = \frac{m_1 v_B}{m_1 + m_2}.$$

c) When the two blocks reach the spring on the other side, they will bounce off of it, compressing it in the process. What is the maximum distance that this spring is compressed?

Use work-energy from C to D:

$$\frac{1}{2}(m_1 + m_2)v_c^2 + \cancel{PE_c} + \cancel{W_{other}} = \cancel{KE_D} + PE_D$$

$$\frac{1}{2}(m_1 + m_2)v_c^2 = \frac{1}{2}kd_2^2 \longrightarrow d_2 = \sqrt{\frac{(m_1+m_2)}{k}v_c^2}$$

where v_c is given above.

Question 8

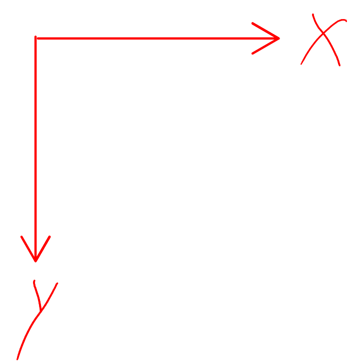
m $\quad$ V_0 $\quad$ θ

Two cars of equal mass of 1200 kg are travelling in the same direction down the highway at 100 km h^{-1}. The driver on the left falls asleep and his car turns 20° towards the other car. When they collide the two cars stick together.

a) What are the magnitude and direction of the final velocity after the collision for the two cars?

Before

After



b) What fraction of the initial kinetic energy is lost?

a) Momentum is conserved in X and Y.

$$X: \cancel{m}V_0 + \cancel{m}V_0\cos\theta = 2\cancel{m}V_{fx} \longrightarrow V_{fx} = \frac{V_0 + V_0\cos\theta}{2} = 97.0\,\text{km/hr}$$

$$Y: 0 + \cancel{m}V_0\sin\theta = 2\cancel{m}V_{fy} \longrightarrow V_{fy} = \frac{V_0\sin\theta}{2} = 17.1\,\text{km/hr.}$$

Magnitude: use Pythagorean theorem: $|V_f| = \sqrt{97.0^2 + 17.1^2}\,\text{km/hr} = 98.5\,\text{km/hr}$

Direction: $\varphi = \arctan V_{fy}/V_{fx} = 10°$

b) see next page

QUESTION 8

Two cars of equal mass of 1200 kg are travelling in the same direction down the highway at 100 km h^{-1}. The driver on the left falls asleep and his car turns 20° towards the other car. When they collide the two cars stick together.

a) What are the magnitude and direction of the final velocity after the collision for the two cars?

b)What fraction of the initial kinetic energy is lost?

$KE \propto v^2$. Both cars always are at equal speeds.

$KE_i \propto (100\ km/hr)^2$ $\qquad KE_f \propto (98.5\ km/hr)^2$.

$$\frac{KE_f}{KE_i} = \frac{98.5^2}{100^2} \approx 0.97 \rightarrow 3\% \text{ of } KE \text{ lost}$$