# Things going in circles

Physics 211
Syracuse University, Physics 211 Spring 2023
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# Announcements

- Homework 4 posted, due next Friday
- Clinic hours today (note change): 3pm-5pm
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- Group Exam 2 next week (Thurs/Fri)
- Next week is almost all review – we are done with new material today

# Rotational motion

Often things in nature are constrained to go in circles:

- Planets orbiting stars; moons orbiting planets (close enough to circles)
- Wheels; things on strings; many others

We'll study "uniform circular motion" here:

- Something moves at a constant distance from a fixed point
- ... at a constant speed.

# Our goal for today

- How do we describe circular motion?
- How does circular motion relate to our previous knowledge of kinematics?
- What forces are required to make something go in a circle?

# Describing circular motion: in general

In general, if an object goes in a circle, we care about its angle $\theta$, not its $x$ and $y$ coordinates.

- This angle can be measured from any convenient zero point
- We will measure it in radians
- Traditionally, counter-clockwise is chosen as positive
- If it rotates around many times, there's no reason $\theta$ must be between 0 and $2\pi$

Just as we needed to talk about derivatives of position, we need the derivatives of angle:

- Angular velocity $\omega$: "how fast is it spinning?" (measured in radians per second)
- Angular acceleration $\alpha$: "how fast is the angular velocity changing?" (measured in radians per second squared)

# Uniform circular motion

Here, we're concerned only with cases where the angular acceleration is zero.

This means the angular velocity $\omega$ is constant.

- This happens often in nature: the Earth...
- Object moves in a circle of radius $r$, with its angle changing at a constant rate
- "Position = rate $\times$ time" $\rightarrow$ "Angle = rate $\times$ time"

Which is true about the points on this wheel?

A: The inside has the same angular velocity as the outside
B: The inside has a greater angular velocity than the outside
C: The inside has a smaller angular velocity than the outside
D: The direction of the angular velocity differs, but its magnitude is the same

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Angular velocity is a scalar!

Which is true about the points labeled on this wheel?

A: The acceleration is zero

B: The magnitude of the acceleration is greater on the outside than on the inside

C: The acceleration is not zero, but has the same value everywhere

D: The angular acceleration is zero

# Rotational motion: description

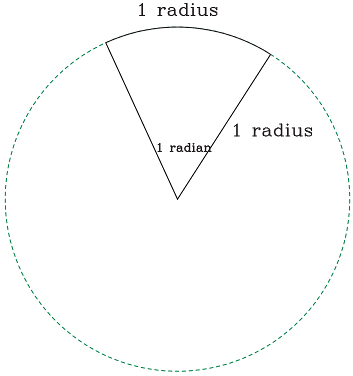
Some new terms:

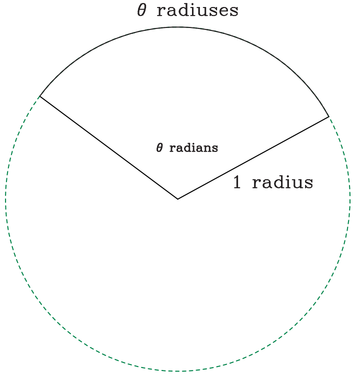
- "Radial": directed in and out of the circle
- "Tangential": directed around the circle
- "Centripetal": pointed toward the center **(This is a direction, not a force!)**
- The radial velocity is 0 ($r$ doesn't change)
- The tangential velocity depends on $r$ and $\omega$, as you'd expect

# Radians

The radian: new unit of angle. $2\pi$ radians = 360 degrees.

1 complete circle is $2\pi$ radians; 1 complete circumference is $2\pi$ radiuses ($C = 2\pi r$).
1 radian thus has an arc length of 1 radius.
$\theta$ radians therefore have an arc length of $r\theta$.




$\rightarrow$ Tangential movement (in meters) = angular movement (in radians) times the radius

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- The tangential velocity depends on $r$ and $\omega$, as you'd expect
- $v_T = \omega r$: "meters per second = radians per second times meters per radian"

Which way is the object on the string accelerating at the top of the arc?

A: Upward: it is at the top of the arc, and it must have accelerated upward to get there

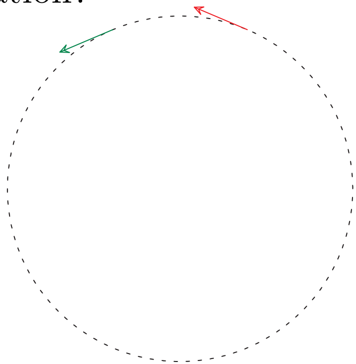
B: Upward: its upward acceleration is what keeps the string taut at the top of the swing

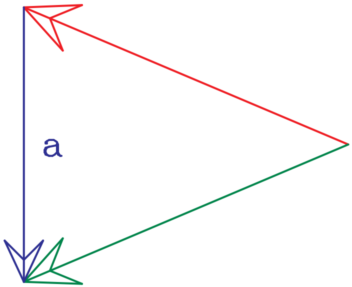
C: Downward: the only forces acting on it there pull it downward, so by $\vec{F} = m\vec{a}$ it must be accelerating downward

D: Downward: it was moving up and to the left, then down and to the left, so the net change is "down"

E: Zero: it is being swung at a constant rate

Clearly an object moving in a circle is accelerating. What's the acceleration?





# Kinematic challenge: what's $\vec{a}$?

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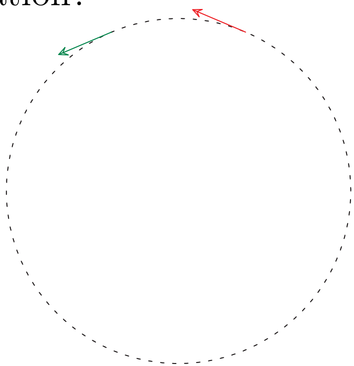

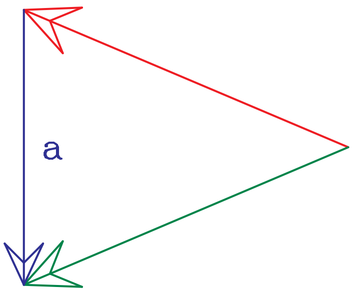


Near the top of the circle, the $y$-component of the velocity decreases; we expect then that $\vec{a}$ points downward.

Can we make this rigorous?

# Some math

$$x(t) = r\cos(\omega t)$$
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Differentiate again to get $a_x$ and $a_y$:

$$a_x(t) = -\omega^2 r\cos(\omega t) = -\omega^2 x(t)$$
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$\rightarrow \vec{a} = -\omega^2\vec{r}$

An object in uniform circular motion accelerates toward the center of the circle with

$$\rightarrow a = \omega^2 r = v^2/r \leftarrow$$

# Uniform circular motion, consequences

If you know an object is undergoing uniform circular motion, you know something about the acceleration:

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If you know an object is undergoing uniform circular motion, you know something about the acceleration:

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Circular motion problems aren't scary; they are just like any other force problem.

- Equilibrium problem: $\sum F_x = ma_x = 0$ and $\sum F_y = ma_y = 0$
- Circular motion problem: $\sum F_T = ma_T = 0$ and $\sum F_r = ma_r = v^2/r$

$\rightarrow$ If we tell you that a thing is in uniform circular motion, we're just telling you something about its acceleration.

Of all of the topics in Physics 211, this is the one topic that students overcomplicate the most.

Circular motion problems are *just like any other* Newton's-law problem. The only difference is that you know something about the object's acceleration. Do not make these more complicated than they actually are!

# Forces toward the center

"Centripetal" means "toward the center" in Latin.

If something moves in a circle, it is accelerating toward the center at $a = \omega^2 r$.

- If something is going to accelerate toward the center, a force must do that.
- Centripetal force is not a "new" force. No arrows labeled "centripetal force"!
- "Centripetal" is a word that describes a force you already know about.
- Centripetal force: describes a force that holds something in a circle
- It can be lots of things:
    - Tension (see our demos)
    - Normal force (platform, bucket demos)
    - Friction (Ferris wheel)
    - Gravity (the moon!)

What does the force diagram on the water look like while the bucket is at the bottom?

- A: acceleration upward, normal force upward, gravity downward
- B: centripetal force upward, normal force upward, gravity downward
- C: gravity downward, normal force upward
- D: gravity downward, velocity to the left, normal force upward

Why doesn't the water in the bucket fall at the top?

- A: It *is* falling, but the bucket falls along with it, so it stays in
- B: The normal force pushes it up
- C: The centripetal force pushes it up

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- D: Sam is in the back chanting "wingardium leviosa!"

What does the force diagram on the water look like while the bucket is at the top?

- A: acceleration downward, normal force upward, gravity downward
- B: normal force upward, gravity downward
- C: gravity downward, normal force downward
- D: normal force downward, gravity downward, centrifugal force upward

What is the acceleration of the water at the top?

- A: Zero
- B: Downward, less than $g$
- C: Downward, equal to $g$
- D: Downward, more than $g$
- E: Upward

What is the tension in the string at the bottom?

- A: $mg + m\omega^2 r$
- B: $mg$
- C: $mg - m\omega^2 r$
- D: $m\omega^2 r$
- E: $m\omega^2 r - mg$

What is the tension in the string at the top?

- A: $mg + m\omega^2 r$
- B: $mg$
- C: $mg - m\omega^2 r$
- D: $m\omega^2 r$
- E: $m\omega^2 r - mg$

# Driving around a road curve

What is the fastest I can drive around a flat interstate on-ramp? What if it is snowy? What if it is icy?