Vectors and 2D kinematics

Physics 211
Syracuse University, Physics 211 Spring 2022
Walter Freeman

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"Poets say science takes away from the beauty of the stars—mere globs of gas atoms. Nothing is "mere." I too can see the stars on a desert night, and feel them. But do I see less or more? The vastness of the heavens stretches my imagination–stuck on this carousel my little eye can catch one-million-year-old light. A vast pattern, of which I am a part....

What is the pattern, or the meaning, or the why? It does not do harm to the mystery to know a little about it. For far more marvelous is the truth than any artists of the past imagined! Why do the poets of the present not speak of it? What men are poets who can speak of Jupiter if he were like a man, but if he is an immense spinning sphere of methane and ammonia must be silent?"

–Richard Feynman, from Lectures on Physics, vol. 1, ch. 3

Announcements

- Homework 2 is posted
- The Clinic got a lot of use yesterday
- Homework 2 due next Friday
- Next week:
  - Tuesday and Thursday we will be applying and practicing what we learn today
  - Group practice test Friday

Vectors

You've been doing math with numbers, which are things that live in one dimension: they only have a magnitude and a sign.

Vectors are things that have a magnitude and a direction: "arrows in space"

Many of the things we deal with in physics are vectors:

- Position

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So, we need to learn to do math with arrows.

Notation and equations

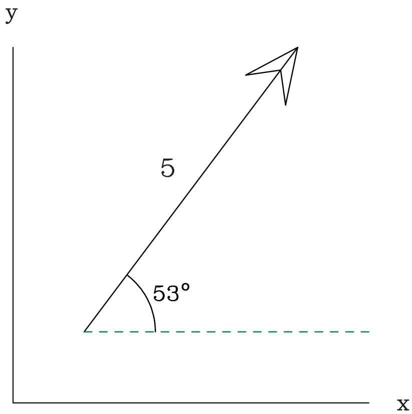
We can do algebra with vectors just like anything else:

- We indicate that a symbol is a vector by writing an arrow over it: "the vector $\vec{V}$".
- "Scalar": object that isn't a vector (mass, time)
- Equations can mix vectors and scalars: $\vec{F} = m\vec{a}$.
- ... or $\vec{s} = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$

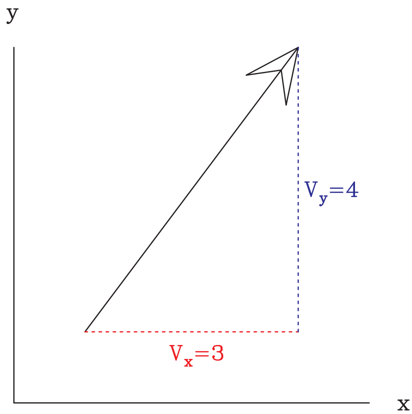
Some notation:

- $\vec{A}$: "the vector A" (a vector)
- A: "the magnitude of A" (a scalar)
- $\hat{A}$: "the direction A points in" (a vector with magnitude 1)
- A_x: the component of A along the $x-$axis (a scalar)
- A_y: the component of A along the $y-$axis (a scalar)

Two ways to describe a vector



Magnitude and direction



X and Y components

How do we convert from one to the other?

How do we convert from one to the other?

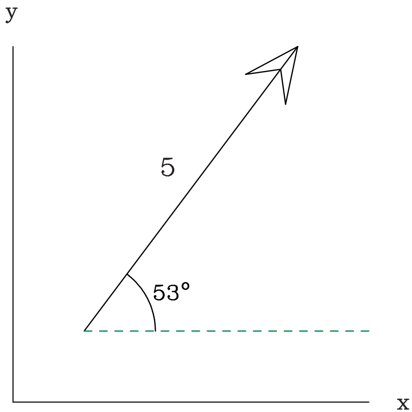
A: Using algebra

B: Using trigonometry

C: Using calculus

D: Using differential equations

From magnitude and direction to components

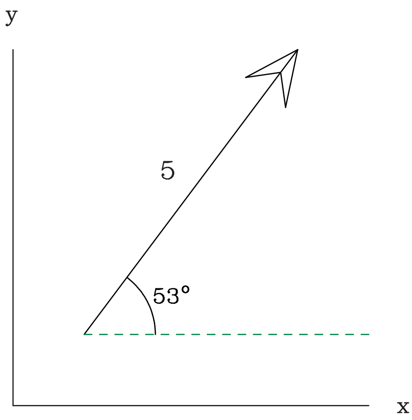


Magnitude and direction

What is the $x-$component of this vector?

A: $5\cos 53°$
B: $5\sin 53°$
C: $5\tan 53°$
D: Something else

From magnitude and direction to components



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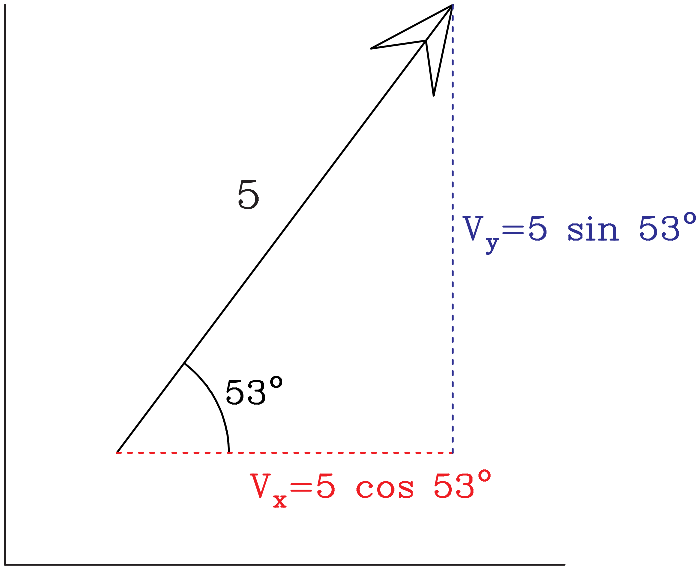
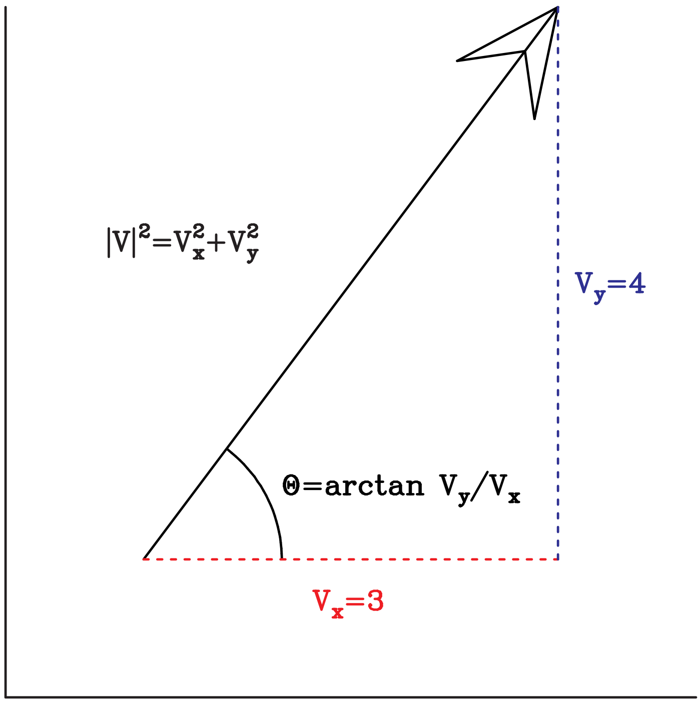
From “direction and magnitude” to components



From components to direction and magnitude



Suppose you have some vector $\vec{A}$ that you want to convert into components. The x-component A_x is:

A: $A\cos\theta$

B: $A\sin\theta$

C: $A\tan\theta$

D: $\frac{A}{\cos\theta}$

E: It depends

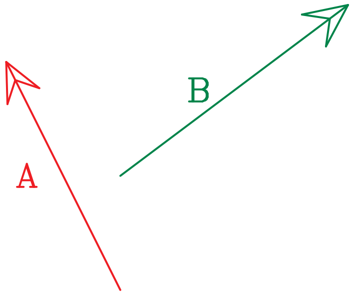
A warning!

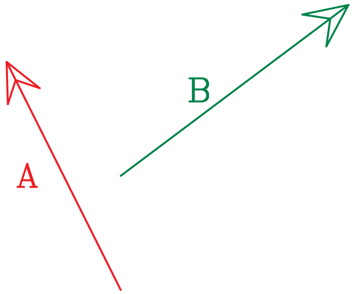
You cannot memorize
"$V \sin\theta$ is the y component, $V \cos\theta$ is the x component"!

This does *not* work in general; you have to actually draw the triangle.

Adding vectors

We can also add vectors together by drawing them “head to tail”. Here are two vectors:





Does $\vec{A} + \vec{B} = \vec{B} + \vec{A}$?

- A: Yes
- B: No

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- A: Yes
- B: No

Yes: vector addition obeys the commutative property, just like ordinary addition

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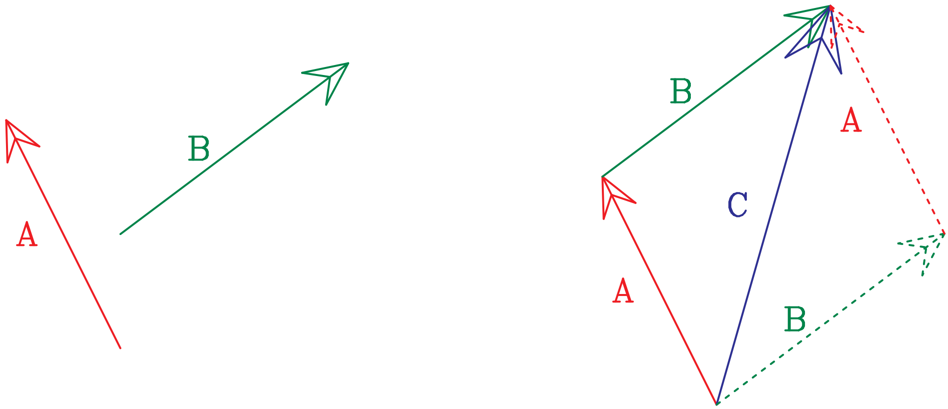


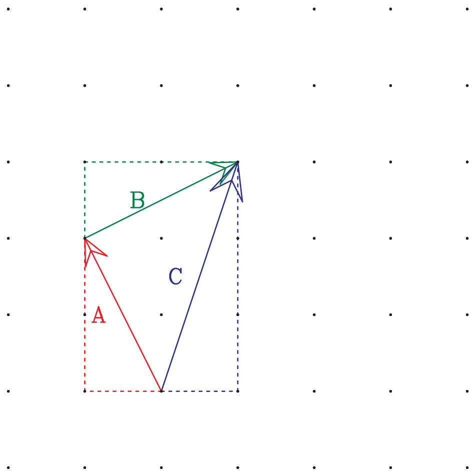
$$\vec{A} + \vec{B} = \vec{C}$$

Adding vectors: components

The component representation is much easier to work with!

$$\vec{A} + \vec{B} = \vec{C} \rightarrow \begin{pmatrix} A_x + B_x = C_x \\ A_y + B_y = C_y \end{pmatrix}$$

Adding vectors: components



To add two vectors, just add their components!

This is why it is almost always easiest to work in the component representation!

What does this do to our kinematics?

Acceleration, velocity, and position relationships are still the same; they just apply independently for each component.

$\vec{s}$ is the position vector:

- s_x or just x is its x-component
- s_y or just y is its y-component

$\vec{v}$ is the position vector:

- v_x is its x-component
- v_y is its y-component

$\vec{a}$ is the position vector:

- a_x is its x-component
- a_y is its y-component

Do not get lazy if you have multiple subscripts. For instance: $\vec{v}_0$ is the initial velocity vector:

- $v_{0,x}$ or v_{0_x} is its x-component
- $v_{0,x}$ or v_{0_y} is its y-component

What does this do to our kinematics?

Acceleration, velocity, and position relationships are still the same; they just apply independently for each component.

$$\vec{v}(t) = \vec{a}t + \vec{v}_0$$
$$\vec{s}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$$

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$$v_x(t) = a_x t + v_{x,0}$$
$$v_y(t) = a_y t + v_{y,0}$$

$$x(t) = \frac{1}{2}a_x t^2 + v_{x,0}t + x_0$$
$$y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + y_0$$

Which statement does *not* make sense?

- A) $\vec{A}t = \vec{B}$
- B) $\vec{A} + \vec{B} + t = \vec{C}$
- C) $k(\vec{A} + \vec{B}) = k\vec{A} + k\vec{B}$
- D) $\vec{A} - \vec{B} = \vec{C}$

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B: You can't add a vector and a scalar. "One mile north plus one inch" – which way is the inch?

Problem solving: 2D kinematics, constant acceleration

1. 1. If you have vectors in the "angle and magnitude" form, convert them to components
2. 2. Write down the kinematics relations, separately for x and y
   - Many terms will usually be zero
   - Freefall: $a_x = 0$, $a_y = -g$ (with conventional choice of axes)
3. 3. Understand what instant in time you want to know about
4. 4. Put in what you know; solve for what you don't (using substitution, if necessary)
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Every kinematics situation we will encounter can be done this way!

Sample problems

A rock is thrown at $v_0 = 10m/s$ at $\theta = 30^\circ$ above the horizontal.

- How far from its starting point is it after 2 seconds?

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- How far from its starting point is it after 2 seconds?
- How far does it travel?
- How high does it go?
- What will its speed be when it strikes the ground?

We're done!

This is all of the material you will need for the first exam!

We have three recitations and two classes left; we're going to spend it practicing.

As with everything in physics, applying abstract ideas to the real world is the challenge. So we have set aside lots of time for it!