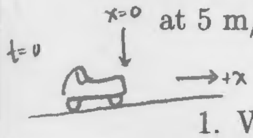
Recitation Questions – 1D Motion (part 2)

23 January

Question 1: a braking car

A car is traveling at 30 m/s and applies its brakes to slow down to 10 m/s. If it is able to decelerate at 5 m/s^2, how far does it travel during the braking period?



1. Write expressions for the car's position and velocity as a function of time. What moment makes sense to choose as your reference time $t = 0$?

We need constant acceleration to use the position and velocity equations, so let's choose $t=0$ at the moment the car starts to brake.

$x = x_0 + v_0 t + \frac{1}{2}at^2$ $\qquad$ $v = v_0 + at$,

choose constants that match the problem

$x = (0) + (30\,\text{m/s})t + \frac{1}{2}(-5\,\text{m/s}^2)t^2$ $\qquad$ $v = (30\,\text{m/s}) + (-5\,\text{m/s}^2)t$

braking means accelerating opposite the velocity

$\boxed{x = 30t - 2.5t^2}$ $\qquad$ $\boxed{v = 30 - 5t}$

2. How can you translate the question "How far does it travel during the braking period?" into a sentence about your algebraic variables? Again, fill in the blanks:

"What is the value of ___x___ at the time when ___v___ is equal to ___10 m/s___?"

Answers "how far" $\qquad$ the car brakes until $v = 10$ m/s

3. What intermediate quantity must you find before you find the distance traveled? Following the above recipe you created for yourself, find it.

We have x as a function of time t, so we first need to find at what time t we have $v = 10$ m/s.

Choosing $v = 10$ m/s in our v equation, $10 = 30 - 5t$

$-20 = -5t$

$4 = t$, so $\boxed{t = 4\text{s}}$ is when $v = 10$ m/s.

4. Finally, how far does the car travel during the braking period?

Using that time in the position equation,

$x = 30(4) - 2.5(4)^2$

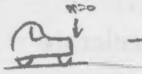
$= 80$, so the car travels $\boxed{80\text{ m}}$ before it has slowed to 10 m/s.

(If your group didn't finish this problem last week, do it here.)

Question 2: a braking car, with variables

A car is traveling at a speed v_0 and applies its brakes to slow down to a speed v_f. If it is able to decelerate at an acceleration a_b, how far does it travel during the braking period?

$t=0$



1. Write expressions for the car's position and velocity as a function of time, as before.

From the general equations $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v = v_0 + at$,

we choose constants from the problem

$$x = (0) + v_0 t - \frac{1}{2}a_b t^2, \quad \boxed{v = v_0 - a_b t}$$

$$\boxed{x = v_0 t + \frac{1}{2}a_b t^2}$$

let's choose a_b to be a positive number, so $-a_b$ is the (negative) acceleration

2. Using the same process that you did in the previous problem, find an expression that tells how far the car travels during the braking period. This should depend only on v_0, v_f, and a_b – the physical parameters of the problem.

The braking period ends when $v = v_f$. We can find how far the car travels if we know t (how long it is braking).

To find t, choose $v = v_f$ in the velocity equation and solve for t:

$$v_f = v_0 - a_b t$$

$$v_f - v_0 = -a_b t, \quad t = \frac{v_f - v_0}{-a_b} = \frac{v_0 - v_f}{a_b}.$$

We can now see what the position of the car is at this time:

$$x = v_0\left(\frac{v_0 - v_f}{a_b}\right) - \frac{1}{2}a_b\left(\frac{v_0 - v_f}{a_b}\right)^2. \quad \text{Simplifying,}$$

$$= \frac{v_0^2 - v_0 v_f}{a_b} - \frac{1}{2}\cancel{a_b}\,\frac{v_0^2 - 2v_0 v_f + v_f^2}{a_b^{\cancel{2}}}$$

$$= \frac{v_0^2 - v_0 v_f - \frac{1}{2}(v_0^2 - 2v_0 v_f + v_f^2)}{a_b}$$

$$= \frac{v_0^2 - \cancel{v_0 v_f} - \frac{1}{2}v_0^2 + \cancel{v_0 v_f} - \frac{1}{2}v_f^2}{a_b}$$

$$= \frac{\frac{1}{2}v_0^2 - \frac{1}{2}v_f^2}{a_b} = \frac{\frac{1}{2}(v_0^2 - v_f^2)}{a_b} = \boxed{\frac{v_0^2 - v_f^2}{2a_b}}$$

We'll use this equation later!

Question 3: a rocket

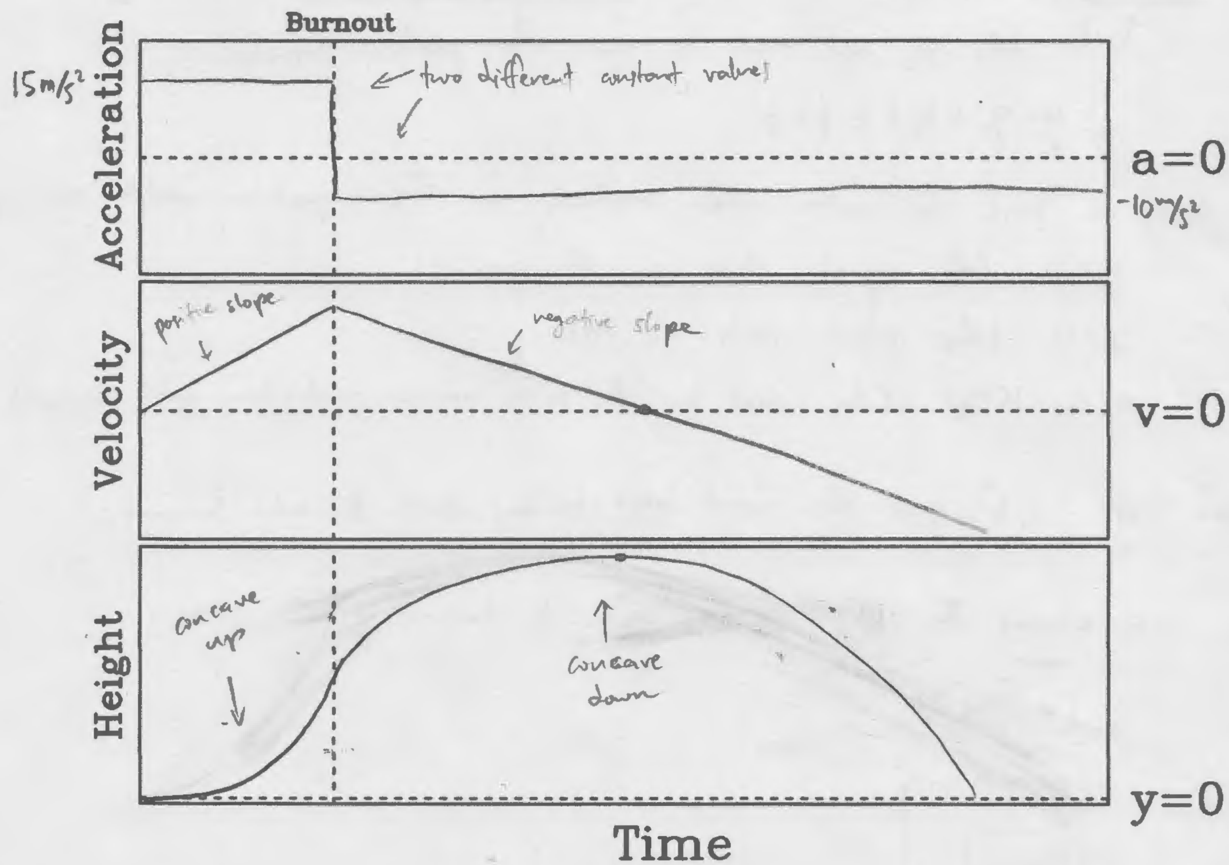
A small rocket is pointed straight up and fired. Its motor burns for $\tau = 10$ s; while the rocket's motor burns, it accelerates upward at $a_r = 15\text{m/s}^2$; after it burns out, the rocket is in freefall. (To make the numbers simpler for this problem, and for every problem in this class, you may use $g = 10\,\text{m/s}^2$.)

1. Before you do any mathematics, sketch graphs on the rest of this page for the rocket's acceleration vs. time, velocity vs. time, and position vs. time, in that order. (Here I've labeled the height y rather than x since it is moving vertically.) Remember:

   - The slope of the velocity graph should be equal to the value of the acceleration graph, since acceleration is the derivative of velocity
   - The slope of the position graph should be equal to the value of the velocity graph.

   This means that:

   - When the acceleration is positive, the velocity should be increasing, and the position should be concave up;
   - When the acceleration is negative, the velocity should be decreasing, and the position should be concave down;
   - When the velocity is zero, the position graph should have a maximum or minimum



Since the rocket's acceleration changes in flight, you can't use the constant-acceleration kinematics formulae we've learned to understand the whole flight at once. However, the acceleration *is* piecewise constant.

This means that *one* copy of the constant-acceleration kinematics relations won't get the job done, but *two* copies will.

In problems like these, take the following approach:

- Write down one set of kinematics relations (velocity and position) for the phase of the motion while the motor is on. Here, your time variable should represent the time since the rocket was launched.
- Write down a second set of kinematics relations for the phase of the motion while the motor is off, and the rocket is only moving under the influence of gravity. Here, your time variable represents the time since the motor turned off, since that is when this set of kinematics relations became valid.
- Use the first set of kinematics relations to solve for the position and velocity at burnout. (Call this y_b and v_b, or whatever other variables you want.) These values will become the initial position and velocity for the second phase.
- Solve for whatever you want to know about the second phase.

Answer the following. Ideally, figure out your answers in terms of τ (the rocket burn time), g, and a_r. You don't even have to plug numbers in if you don't want to!

10 s after the motor starts

2. How high above the ground is the rocket once its motor burns out? (Call this y_b.)

+y

y=0

this tells us we need to use the position equation

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

Now we need to make the constants in this equation match our problem

$y_0 = 0$ (the rocket starts on the ground)

$v_0 = 0$ (the rocket starts at rest)

$a = a_r = 15\,\mathrm{m/s^2}$ (the rocket has the same upward acceleration until burnout)

Now $y = \frac{1}{2} a_r t^2$ gives the rocket height for any time t until burnout.

The motor burns for 10 s, so we want to know y for $t = 10\,\mathrm{s}$:

$$y_b = \frac{1}{2} a_r (10\,\mathrm{s})^2$$

$$= \frac{1}{2} (15\,\mathrm{m/s^2})(10\,\mathrm{s})^2$$

$$= \boxed{750\,\mathrm{m}}$$

10s after the motor starts

3. How fast is the rocket traveling once its motor burns out? (Call this v_b.)

this tells us we need to use the velocity equation $v = v_0 + at$.

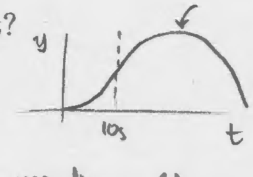
The constants we need to make the equation match our problem are

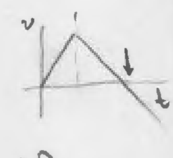
$v_0 = 0$ (the rocket starts at rest), $a = a_r = 15\,\mathrm{m/s^2}$ (the rocket accelerates up until burnout).

Now $v = a_r t$ gives the rocket's velocity for any time t until burnout.

After 10s, $v_b = a_r(10s) = (15\,\mathrm{m/s^2})(10s) = \boxed{150\,\mathrm{m/s}}$.

4. How fast is the rocket traveling when it reaches its maximum height?





When an object reaches its max height, that is the moment its velocity changes from positive (upwards) to negative (downwards).

At the one moment of maximum height, we must therefore have $\boxed{v = 0}$.

5. What is that maximum height?

we can use the position equation $y = y_0 + v_0 t + \frac{1}{2}at^2$ again, but since "a" changes after burnout, we need to use constants as they are right after burnout.

y_0 = height at burnout $= y_b = 750\,\mathrm{m}$, v_0 = velocity at burnout $= v_b = 150\,\mathrm{m/s}$, $a = -g = -10\,\mathrm{m/s^2}$ (freefall downwards)

so $y = y_b + v_b t - \frac{1}{2}gt^2$. But what t can we use? We need t such that $v = 0$, so we need $v = v_0 + at \longrightarrow v = v_b - gt$. With $v=0$, solve for t:

$$0 = v_b - gt$$
$$gt = v_b$$
$$t = \frac{v_b}{g} = \frac{150\,\mathrm{m/s}}{10\,\mathrm{m/s^2}} = \underline{15s}.$$

So $v=0$ 15s after burnout. Then plugging 15s into the position equation gives the max height

$$y_{max} = y_b + v_b(15s) - \frac{1}{2}g(15s)^2$$
$$= 750\,\mathrm{m} + (150\,\mathrm{m/s})(15s) - \frac{1}{2}(10\,\mathrm{m/s^2})(15s)^2$$
$$= \boxed{1875\,\mathrm{m}}$$

6. How long does it take for the rocket to land back on the ground?

Since this is the position $y=0$, we need the position equation after burnout again.

choose $y=0$

$$y = y_b + v_b t - \frac{1}{2}gt^2$$
$$0 = y_b + v_b t - \frac{1}{2}gt^2 \rightarrow$$ solving for t now tells us when $y=0$.

We chose the time after burnout (t=0)

This equation involves t^2, t, and a constant, so we need the quadratic formula, which tells us that

$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Matching this formula, $t = \frac{-v_b \pm \sqrt{v_b^2 - 4(-\frac{1}{2}g)(y_b)}}{2(-\frac{1}{2}g)} = -4.36\,s$ or $\boxed{34.4\,s}$ after burnout