Problem solving: kinematics (II)

Physics 211
Syracuse University, Physics 211 Spring 2020
Walter Freeman with Matt Rudolph

January 29, 2020

Announcements

- Homework 3 is due Tuesday before the exam
- HW3 problems are similar to those on Exam 1
- Recitation Friday is your group practice exam
  - If you miss a group exam, your grade on the main exam will also be used for the group exam
  - You may only do this once
  - If you must miss multiple group exams for solemn observances, academic or professional events, or illness, please let me know

Exam 1

- The exam covers kinematics in one and two dimensions
- Kinematics: how are an object's position, velocity, and acceleration related?

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- Kinematics: how are an object's position, velocity, and acceleration related?
- The exam will be somewhat easier than the homework.
- You are allowed to bring one page of notes that *you handwrite yourself*
    - No typed notes unless you have a disability that prevents you from writing
    - Your friend can't write it
    - You can't photocopy stuff from the book
    - It won't help you as much anyway

Review

Big ideas about one-dimensional motion:

- Relate position/velocity/acceleration to one another graphically
- Use the constant-acceleration kinematics equations to determine how position and velocity change in time
- Relate verbal or graphical descriptions of motion ("ball hits ground") to diagrams and mathematics
- Deal with motion where the acceleration changes (but is piecewise constant)
- Use all of the above to predict how things move

Big ideas about vectors:

- Vectors: things with magnitude and direction
- You can do math with vectors just like you do math with anything else
- Two representations of vectors:
  - Magnitude and direction
  - x- and $y-$components
  - Use trigonometry to convert from one to another
  - Almost always easier to work with components, since they're totally independent

Big ideas about two-dimensional motion:

- Motion in the $x-$ and $y-$directions are *independent*
- You'll have *separate* equations for motion in each direction
- They're linked together by *time*

Position/velocity/acceleration graphs

Velocity is the derivative of position:

- If velocity is positive, position is increasing (slope up)
- If velocity is zero, position is a maximum or minimum
- If velocity is negative, position is decreasing (slope down)

Acceleration is the derivative of velocity and the second derivative of position:

- If acceleration is positive, velocity is increasing
- If acceleration is zero, velocity is constant
- If acceleration is negative, velocity is decreasing: either getting less positive or more negative
- If acceleration is positive, position is concave up
- If acceleration is negative, position is concave down
- If acceleration is zero, position has a constant slope (straight line)

Constant-acceleration kinematics

If the acceleration is constant:

$$\begin{aligned} v_x(t) &= v_{0,x} + a_x t \\ v_y(t) &= v_{0,y} + a_y t \\ x(t) &= x_0 + v_{0,x} t + \frac{1}{2} a_x t^2 \\ y(t) &= y_0 + v_{0,y} t + \frac{1}{2} a_y t^2 \end{aligned}$$

Note:

- You'll have *separate* relations for x and y
- Express the problem as a sentence in terms of your variables
- This will tell you what algebra to do
- If you have vectors in magnitude-and-direction form, use trig to convert to components:
    - "A velocity v at an angle θ below the vertical" $\rightarrow v_x = v \sin\theta, v_y = v \cos\theta$

Changing acceleration

If the acceleration changes from one value to another midway through the motion:

- Write one set of constant-acceleration kinematics relations for the first stage
- Use those to calculate the position and velocity at the *end* of the first stage
- Those become your initial position and velocity for the second stage

Writing “sentences with variables”

How do I “set up a problem”?

- Draw a cartoon first
- Express your question in terms of your variables (x, y, v_x, v_y, t...)
- **Understand what quantity you’re trying to find**
- Understand at what moment in time you’re trying to find it, or what moment in time you should consider
- Solve for time in terms of your other variables at that instant
- Substitute that time back in to the appropriate equation to find the thing you want

- “How far...?” or “Where...?”

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- "How far...?" or "Where...?"
  - ... you're solving for x or y (or, maybe, the magnitude of a displacement: $\sqrt{x^2+y^2}$)

- "How fast is it going...?" or "What speed...?"

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- "Which direction is it moving...?"

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  - ... you're solving for the direction of $\vec{v}$ (trigonometry...)

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- "An object hits the ground"
  - You usually know something about its y-coordinate (*This does not mean* $v_y = 0$*!*)

- "An object hits a wall"

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- “An object reaches its maximum/minimum position”
  - This means $v_x = 0$ or $v_y = 0$
- “A thing meets another thing”

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- “An object reaches its maximum/minimum position”
  - This means $v_x = 0$ or $v_y = 0$

- “A thing meets another thing”
  - This means that $\vec{s}_1(t) = \vec{s}_2(t)$
  - If you get an imaginary result, that means they never meet

Problem solving: 2D kinematics, constant acceleration

1. 1. If you have vectors in the "angle and magnitude" form $(\vec{a}, \vec{v}, \vec{s})$, convert them to components
2. 2. Write down the kinematics relations, separately for x and y
   - Many terms will usually be zero
   - Freefall: $a_x = 0$, $a_y = -g$ (with conventional choice of axes)
3. 3. Understand what instant in time you want to know about: ask the right question
4. 4. Put in what you know; solve for what you don't (using substitution, if necessary)
5. 5. Think about the physical meaning of your solution

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Throwing a rock off a cliff

A hiker throws a rock horizontally off of a $h = 100$ m tall cliff. If the rock strikes the ground $d = 30$ m away, how hard did she throw it? How fast was it going when it hit the ground? (Choose the origin at the base of the cliff, up/direction of throw as positive)

What is $v_{0,x}$ here?

A: 0

B: 10/3 m/s

C: You don't know *a priori*

What is $v_{0,y}$ here?

A: 0

B: 9.81 m/s

C: You don't know *a priori*

What is a_x here?

A: 0

B: -g

C: +g

D: You don't know *a priori*

What is a_y here?

A: 0

B: -g

C: +g

D: You don't know *a priori*

What is x_0 here?

A: 0

B: h

C: d

D: You don't know *a priori*

What is y_0 here?

A: 0

B: h

C: d

D: You don't know *a priori*

What question do you ask to find “how hard did she throw it?”

A: What value of $v_{x,0}$ makes it such that $x = d$ when $y = 0$?

B: What value of $v_{y,0}$ makes it such that $x = d$ when $y = h$?

C: What is the value of v_x when $y = 0$?

D: What is the magnitude of $\vec{v}$ when $y = 0$?

E: What is the magnitude of $\vec{v}_x$ when $y = h$?

What question do you ask to find “how fast is it going when it hits the ground?”

A: What is v_x at the time when $v_y = 0$?
B: What is v_x at the time when $y = 0$?
C: What is v_y at the time when $y = h$?
D: What is the magnitude of $\vec{v}$ when $y = 0$?
E: What is the magnitude of $\vec{v}$ when $y = h$?

What's the magnitude of $\vec{v}$?

A: $v \cos \theta$

B: $v \sin \theta$

C: $\tan^{-1} \frac{v_x}{v_y}$

A: $\sqrt{v_x^2 + v_y^2}$

Throwing a stone onto a slope

A hiker kicks a stone off of a mountain slope with an initial velocity of v_0 3 m/s horizontally. If the mountain has a slope of 45 degrees, how far down the slope does it land? (Choose the origin as the starting point.)

A: What is the magnitude of $\vec{s}$ when $x = y$?
B: What is the magnitude of $\vec{s}$ when $x = -y$?
C: What is the magnitude of $\vec{s}$ when $y = 0$?
D: What is y when $x = -y$?
E: What is y when $x = 0$?

A rocket

A rocket is launched from rest on level ground. While its motor burns, it accelerates at 10 m/s at an angle 30 degrees below the vertical. After $\tau = 10$ s its motor burns out and it follows a ballistic trajectory until it hits the ground.

How far does it go?