Problem solving: kinematics (II)

Physics 211
Syracuse University, Physics 211 Spring 2015
Walter Freeman

January 27, 2015

Announcements

- Homework 2 due date is **extended until Friday**
- Bring your clicker and/or ResponseWare app on Thursday
- Around 25 people in the Clinic during my office hours today
- The course schedule is available on the wiki
- Exam 1 is next Tuesday
  - No homework due next week
  - Sample exam will be posted today

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- Formal review session in class on Thursday
  - At this review you will create your reference sheet, which I will post in final form that day
- Other review times TBA (poll)

Exam 1, promises

- There will be one problem where you need the quadratic formula
  - ... this means interpreting the two values it spits out
- There will be at least one instance where you need to graph position, velocity, and acceleration
- You will *not* need to compute derivatives or integrals algebraically

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Working with variables

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Example from cannon problem:

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$$
\begin{aligned}
x(t) &= v_0 \cos 45^o t \\
y(t) &= -\frac{1}{2} g t^2 + v_0 \sin 45^o t
\end{aligned}
$$

(I leave the rest to you for now...)

Problem solving: 2D kinematics, constant acceleration

1. 1. If you have vectors in the "angle and magnitude" form ($\vec{a}, \vec{v}, \vec{r}$), convert them to components
2. 2. Write down the kinematics relations, separately for x and y
   - Many terms will usually be zero
   - Freefall: $a_x = 0$, $a_y = -g$ (with conventional choice of axes)
3. 3. Understand what instant in time you want to know about
4. 4. Put in what you know; solve for what you don't (using substitution, if necessary)
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- “Object strikes (level) ground”: often means y=0, if that is where your ground is
- “Object reaches maximum height”: usually means $v_y = 0$
- “Two objects meet”: $x_1 = x_2, y_1 = y_2$ at the same time (frog/spider, cat/cheeseburger problems)

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Note that this algebraic solution can be used to do other things rather simply!

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- To get the speed when it hits, we just use the velocity relations:
- $v_x = v_{0,x}$ and $v_y = -gt$
- $v_x = 6.64$ m/s, $v_y = \sqrt{2gh} = -44.2$ m/s
- $|v| = \sqrt{v_x^2 + v_y^2} = 44.7$ m/s

The roadrunner problem

The position of the car is given by the ordinary 1D kinematics relation:

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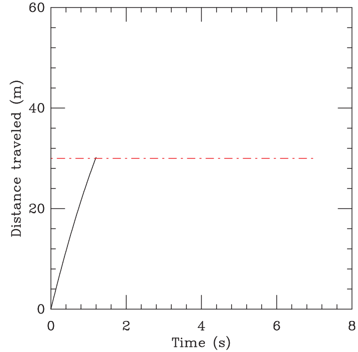
We care about the time when it meets up with the position of the roadrunner, which is 30m. So we set $x(t) = 30$ and solve.



This seems easy enough, but the quadratic formula gives us two solutions! What happened?

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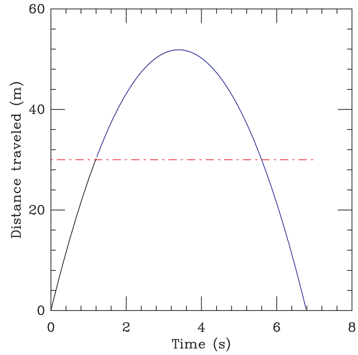

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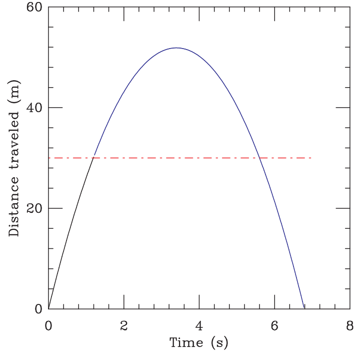

Moral of the story: mathematics is a very blunt tool!

Throwing a stone onto a slope

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  $v_{0,x} t = \frac{1}{2} g t^2 \rightarrow t = \frac{2 v_{0,x}}{g}$
- This gives us $x(t) = \frac{2 v_{0,x}^2}{g}$
- $y(t)$ will have the same magnitude: the Pythagorean theorem gives $|r| = 2\sqrt{2}\frac{v_{0,x}^2}{g}$

A rocket

A rocket is launched from rest on level ground. While its motor burns, it accelerates at 10 m/s at an angle 30 degrees below the vertical. After ten seconds its motor burns out and it follows a ballistic trajectory until it hits the ground.

How far does it go?