# Recitation Exercises – Practicing the Work-Energy Theorem

March 23

Summary of Tuesday's class:

The *work-energy theorem* says that the change in an object's kinetic energy is equal to the sum of the work done on it by all the forces acting on it. In words:

(initial kinetic energy) + (sum of work done on object by all forces) = (final kinetic energy)

Yesterday, we saw that the kinetic energy of an object is $\frac{1}{2}mv^2$. We also saw that the work done on an object by a force $F$ as it moves through a displacement $\Delta\vec{s}$ is $W = \vec{F} \cdot \Delta s$.

This is a new mathematical idea called the *dot product* – a way to multiply two vectors together to get a scalar.

There are three equivalent ways to calculate the dot product and find the work done by a force:

1. The magnitude of the force, multiplied by the component of the displacement parallel to that force
2. The magnitude of the displacement, multiplied by the component of the force parallel to the displacement
3. The magnitude of the force, multiplied by the magnitude of the displacement, multiplied by the cosine of the angle between them

A force does positive work if:

- Its component along the direction of motion is pointing "forward", or equivalently
- The angle between the force and displacement is less than 90°

This means that it is causing the object to gain kinetic energy (since it is pushing in the direction of motion) and speed up.

A force does negative work if:

- Its component along the direction of motion is pointing "forward", or equivalently
- The angle between the force and displacement is less than 90°

This means that it is causing the object to lose kinetic energy (since it is pushing against the direction of motion) and slow down.

Think of a situation in which:

Gravity does negative work

Static friction does positive work

Air resistance does positive work

A normal force does negative work

A normal force does positive work

Tension does positive work

Tension does negative work

Tension does zero work

To solve all parts of the following three problems, take the following steps:

1. Draw clear cartoons of your "before" and "after" situations. (One of the biggest sources of mistakes is not being explicit about the two pictures that you are considering with the work-energy theorem.)
2. Think carefully about all forces that do work on the object in question between the "before" and "after" states.
3. Write down the work-energy theorem:

$$\frac{1}{2}mv_0^2 + \text{work done by force } 1 + \text{work done by force } 2 + ... = \frac{1}{2}mv_f^2$$

4. Determine the work done by each force, as either:
    - Work equals the component of the force parallel to the motion, multiplied by the distance moved: $W = F_{||}d$
    - Work equals the size of the force, multiplied by the component of the distance moved parallel to the force: $W = Fd_{||}$
    - Work equals the size of the force, multipled by the distance moved, multiplied by the cosine of the angle between them: $W = Fd\cos\theta$
5. Put these expressions for work into the work-energy theorem, and solve for whatever you need to solve for.

1. Someone drops a penny of mass 2.5g off of the Empire State Building (height 380 m). It strikes the ground traveling at 50 m/s, having been slowed somewhat by air resistance.

   (a) With what velocity would it have struck the ground if there were no air resistance?

   (b) What was the work done by the drag force? *(You don't have a formula for the force of air drag; that's okay! You can still solve for the work that it does.)*

   (c) This penny strikes the sidewalk and penetrates the surface, digging a hole 2 cm deep. What was the upward force exerted on the penny by the pavement?

2. *(This question will appear on HW6.)* An object rests at bottom of an incline that is elevated at an angle $\theta$ above the horizontal. Suppose that there is no friction at first. A person slides this object up the incline; it travels a distance $D$ up the incline before it slides back down.

   (a) What forces act on the object? Determine whether each one does positive work, negative work, or zero work on the way up and on the way down.

   (b) Suppose at first there is no friction. What initial velocity does the person have to slide it with for it to travel a distance $D$ before it begins to slide back down?

   (c) When it reaches the base of the incline again, how will its velocity compare to the initial velocity that it had on the way up? *(You should be able to answer this without doing any mathematics.)*

(d) Now, suppose that there is friction – a coefficient of friction $\mu_k$ between the ramp and the object. What initial velocity would the person have to slide it with *now* for it to travel a distance $D$ before it comes back down?

(e) How fast will it be moving *now* when it reaches the bottom of the ramp?

(f) If an object travels through some path but comes back to where it starts, like in this case, a force that always does zero work is a *conservative force*. A force that does do work when an object travels along a closed path is a *nonconservative force*.

Three forces appear in this problem: a normal force, gravity, and friction. Is each of these a conservative force? Why or why not?