Work and potential energy

Physics 211
Syracuse University, Physics 211 Spring 2017
Walter Freeman

March 23, 2017

The SUNY College of Environmental Science and Forestry's
Engineering for a Sustainable Society
requests the pleasure of your company at the

6th Annual

Engineers with Appetites

Thursday, April 6th, 2017
Gateway Center, SUNY-ESF Campus

5:30 PM
Cocktail Hour & Raffle

Attendees are welcome to participate in the raffle that will be held during the Cocktail Hour. Winners will be announced at the end of the night.

6:00 PM
Seated Dinner & Guest Speakers

Our guest speaker for this year's dinner is Kayla Kurtz, Vice-President of the New London County Professional Engineers Without Borders Chapter, who will present on designing water systems for climate change.

Enjoy an evening of socializing and celebrating the organization!

Dinner Tickets: Professional $55
Student $45

Sponsorship: Bronze $250 (2 tickets)
Silver $500 (4 tickets)
Gold $1,000 (8 tickets)

Space is limited. To ensure a seat, please register at:
https://goo.gl/forms/Z8XRcXq1ukyhI7Jm2

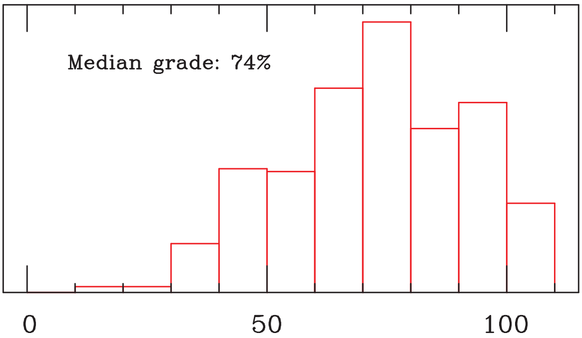
All donations benefit SUNY-ESF's Engineering for a Sustainable Society local and international engineering projects.

Please email esfewbsd@gmail.com for more information.
Thank you for your support!

Announcements

- Your next homework assignment is due next Friday.
- I'm aware ODS students have not had their exams graded yet – I will rectify this ASAP
- Physics practice will return next Wednesday

Exam 2 recap



This exam was quite difficult and most of you did very well!

- 64 F's

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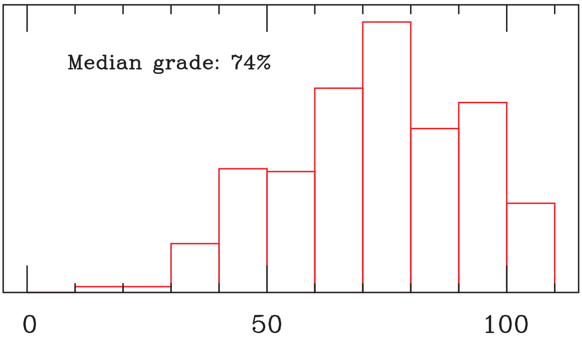

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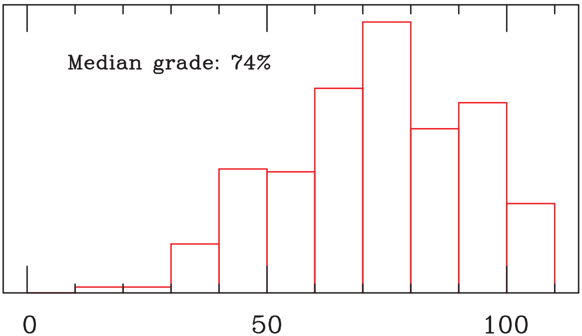

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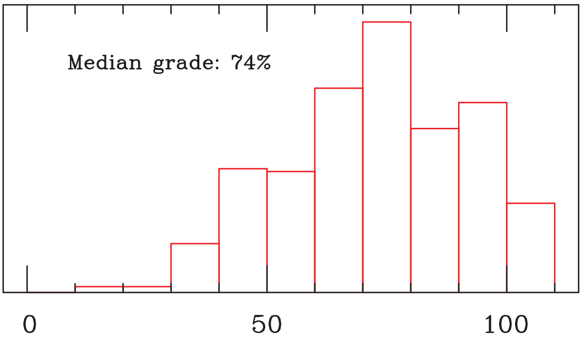

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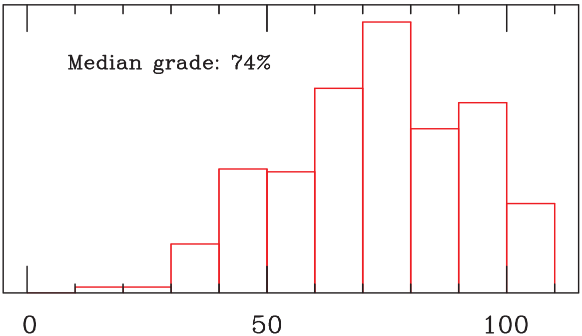

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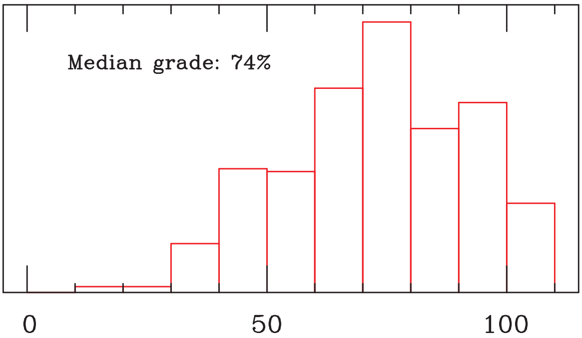

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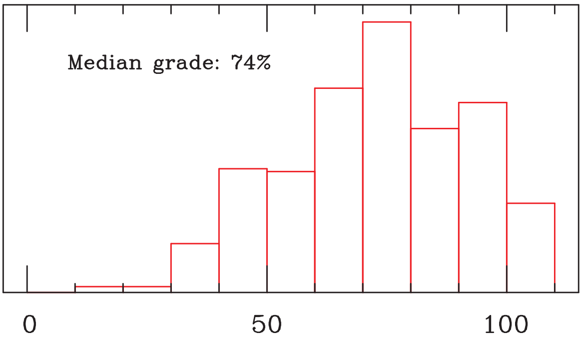

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There was no evidence that either alternate exam was harder than Thursday's.

Where we've been and where we're going

- Last time: kinetic energy and the work-energy theorem
- This time: the idea of potential energy and conservation of energy
  - Potential energy: "the most meaningful bookkeeping trick in physics"
  - Lets us understand many phenomena without difficult mathematics
  - Conservation of energy: there's always the same amount of energy, and it just changes forms

Review: kinetic energy

We will see that things are often simpler when we look at something called "energy"

- Basic idea: don't treat $\vec{a}$ and $\vec{v}$ as the most interesting things any more
- Treat v^2 as fundamental: $\frac{1}{2}mv^2$ called "kinetic energy"

Previous methods:

- Velocity is fundamental
- Force: causes velocities to change over time
- Intimately concerned with vector quantities

Energy methods:

- v^2 (related to kinetic energy) is fundamental
- Force: causes KE to change over distance
- Energy is a *scalar*

Energy methods: useful when you don't know and don't care about time

Energy: measurements and units

$$\text{Kinetic energy} = \frac{1}{2}mv^2$$

- Energy has units $\mathrm{kg\,m^2/s^2}$
- This unit is called a *joule*
- 1 joule = the energy required to lift an apple one meter
- This is also the unit for work

A new idea: power, the rate of doing work

- Sometimes we are interested in the rate at which a force does work
- This idea is called *power*, and it is measured in joules per second
- A joule per second is also called a watt
- If $W = \vec{F} \cdot \Delta\vec{s}$, then I can take derivatives of both sides to get...
- $P = \vec{F} \cdot \vec{v}$
- You'll need this in recitation tomorrow

The work-energy theorem in 1D

Last time we saw the "work-energy theorem" was a consequence of simple kinematics:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = F\Delta x$$

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Some new terminology:

- $\frac{1}{2}mv^2$ called the "kinetic energy" (positive only!)
- $\vec{F} \cdot \Delta\vec{s}$ called the "work" (negative or positive!)
- "Work is the change in kinetic energy"

Sample problem: a roller coaster

(on document camera)

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Strategy: compute the work done by all the forces and equate that to the change in KE.

Work done by normal force = **zero**!
Work done by gravity = $(F)(\Delta s)_{\parallel} = mg\Delta y = mg(y_0 - y_f)$

$$\begin{aligned} KE_f - KE_i =& W_g \\ \frac{1}{2}mv_f^2 - 0 =& mg(y_0 - y_f) \end{aligned}$$

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No detailed knowledge of the motion required!

Potential energy: an accounting trick

- Notice that the work done by gravity depends *only* on the change in height.
- Some other forces are like this as well: the work done depends only on initial and final position
  - These are called *conservative forces*
  - Soon we'll see that the elastic force is like this too
- Separate out gravity and all other forces in the work-energy theorem:

$$KE_f - KE_i = W_{\text{grav}} + W_{\text{other}}$$
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = mg(y_0 - y_f) + W_{\text{other}}$$

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- Collect all the "initial" things on the left and the "final" things on the right:

$$\frac{1}{2}mv_0^2 + mgy_0 \qquad + W_{\text{other}} = \qquad \frac{1}{2}mv_f^2 + mgy_f$$
$$KE_0 + GPE_0 \qquad + W_{\text{other}} = \qquad KE_f + GPE_f$$

- Identify mgy as "gravitational potential energy": how much work will gravity do if something falls?

Potential energy lets us easily calculate the work done by conservative forces

Potential energy: more than accounting!

- Another way to look at the roller coaster: gravitational potential energy being converted to kinetic energy.
- This perspective is universal: all forces just convert energy from one sort into another
- Some of these types are beyond the scope of this class, but we should be aware of them!

A short history of energy conversion:

- Hydrogen in the sun fuses into helium
- Hot gas emits light
- Light shines on the ocean, heating it
- Seawater evaporates and rises, then falls as rain
- Rivers run downhill
- Falling water turns a turbine
- Turbine turns coils of wire in generator
- Electric current ionizes gas
- Recombination of gas ions emits light

- Nuclear energy $\rightarrow$ thermal energy
- Thermal energy $\rightarrow$ light
- Light $\rightarrow$ thermal energy
- Thermal energy $\rightarrow$ gravitational potential energy
- Gravitational PE $\rightarrow$ kinetic energy and sound
- Kinetic energy in water $\rightarrow$ kinetic energy in turbine
- Kinetic energy $\rightarrow$ electric energy
- Electric energy $\rightarrow$ chemical potential energy
- Chemical PE $\rightarrow$ light

Potential energy: more than accounting!

- This class is just the study of motion: we can't treat light or nuclear energy here.
- ... but in physics as a whole, the *conservation of energy* – that processes just change energy from one form to another – is universal!
- Conservation of energy is one of the most tested, ironclad ideas in science
- Nuclear and chemical potential energy: nuclear forces do mechanical work on particles, much like gravity
- Light, and others: kinetic energy of little particles called "photons"
- Heat: kinetic energy of atoms in random motion
- Sound: kinetic energy of atoms in coordinated motion
- Food: Just chemical potential energy...
- ... so all of these things aren't as far removed from mechanics after all!
- Einstein: "Mass is just another form of energy"

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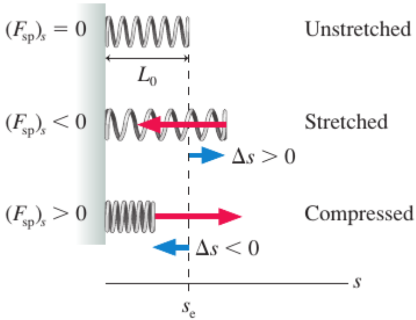
- ... so all of these things aren't as far removed from mechanics after all!
- Einstein: "Mass is just another form of energy"
- Maybe it's all, ultimately, just kinetic energy! (I believe it is; others will argue!)

Ask a Physicist: how does a nuclear bomb work (and how is it different from a nuclear power plant)?

A new force: elasticity and Hooke's law

To best see how this can be useful, let's introduce a new force: elasticity.

- Springs have a particular length that they like to be: "equilibrium length" L_0
- A spring stretched to be longer than this pulls inward to shorten itself
- A spring compressed to be shorter than this pushes outward to lengthen itself
- Flexible things like strings and ropes only pull; they kink instead of compressing
- The force is proportional to the deviation from the optimum length



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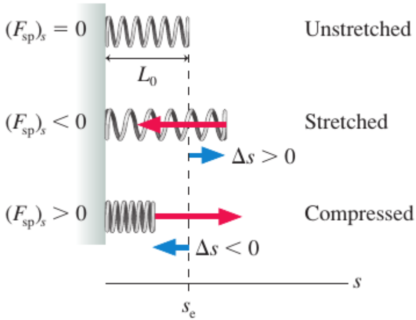

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k is called the "spring constant":

- Measures the stiffness of the spring/rope
- Units of newtons per meter: "restoring force of k newtons per meter of stretch"

A simple spring problem: done with the work-energy theorem

A person of mass $m = 100kg$ falls from a height of $h = 3$m onto a trampoline. If the person makes an impression $d = 40$ cm deep on the trampoline when he lands, what is the spring constant?

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  - Need to use the integral form of the work-energy theorem since the force isn't constant
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- $KE_0 + W_{\text{grav}} + W_{\text{elas}} = KE_f$
- $0 + (mg)(h + d) - \frac{1}{2}kd^2 = 0$
- $k = \frac{mg(h+d)}{2d^2}$

Potential energy stored in a spring

We saw that an object at height h has gravitational potential energy mgh. Can we do something similar for springs?

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- A natural choice is $\Delta x = 0$, the equilibrium position of the spring.

"How much work is done by a spring as it goes from $\Delta x = a$ to $\Delta x = 0$?

$$U_{\text{elastic}} = W_{a \to 0} = \int_a^0 -kx\,dx = \int_0^a kx\,dx = \frac{1}{2}ka^2$$

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$$U_{\text{elastic}} = W_{a \to 0} = \int_a^0 -kx\,dx = \int_0^a kx\,dx = \tfrac{1}{2}ka^2$$

Now that we have this, we never have to do this integral again!

$$U_{\text{elastic}} = \tfrac{1}{2}kx^2, \text{ where } x \text{ is the distance from equilibrium}$$

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- $U_{\text{grav},0} = mgh$
- $U_{\text{elas},0} = 0$ (trampoline starts at equilibrium)
- $U_{\text{grav,f}} = -mgd$ (the person falls below $y = 0$; PE can be negative!)
- $U_{\text{elas,f}} = \frac{1}{2}kd^2$ (see last slide)

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- $0 + mgh + 0 = 0 + (-mgd) + \frac{1}{2}kd^2$ (Same terms, maybe on different side)
- $k = \frac{mg(h+d)}{2d^2}$

That spring problem: a recap

We don't care about time $\rightarrow$ energy methods

Work-energy theorem

- Initial KE + all work done = final KE
- Need to compute work done by gravity: easy
- Need to compute work done by spring: harder
  (need to integrate Hooke's law)

Potential energy treatment

- Initial KE + initial PE + other work = final KE + final PE
- No "other work" in this problem; all forces have a PE associated
- Need to know the expressions for PE:
  - $U_{\text{grav}} = mgy$
  - $U_{\text{elas}} = \frac{1}{2}kx^2$ (x is the distance from the equilibrium point)
- No integrals required (they're baked into the above)

Potential energy with other forces

What about associating a potential energy with other forces?

- Friction is a no-go: the work done by friction depends on the path, not just where you start and stop
- "Ephemeral" forces like tension and normal force are easiest to deal with by computing work directly
- The other force we've studied that is easily associated with a potential energy is **universal gravitation**
  - Need to choose a point to set $U = 0$; here we choose $r = \infty$
  - $U_G =$ "work done by gravity on m_1 when it moves infinitely far from m_2

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$W_G = \int_R^\infty -\frac{Gm_1m_2}{r^2}\,dr = -\frac{Gm_1m_2}{R}$$

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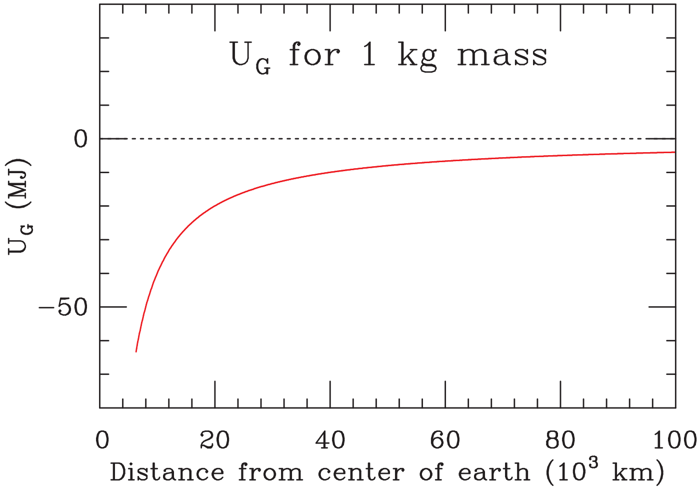
$$W_G = \int_R^\infty -\frac{Gm_1m_2}{r^2}\,dr = -\frac{Gm_1m_2}{R}$$

$\rightarrow$ Gravitational potential energy between two objects separated by a distance r is $-\frac{Gm_1m_2}{r}$.

The Earth's "gravity well"

- With this choice of the zero point at $r = \infty$, gravitational potential energy is always negative
- We have to *add energy* to get something away from Earth



This region of large negative potential energy is often called a "gravity well".

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- Gravitational potential energy in general: $U_G = -\frac{Gm_1m_2}{r}$