Torque

Physics 211
Syracuse University, Physics 211 Spring 2019
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April 16, 2019

Announcements

We are still grading Exam 3; we will get those back to you Friday.

Homework 8 will be posted later today or tomorrow morning; it will be due a week from Wednesday.

There will be a Homework 9, due to your TA's mailbox on the last day of class.

Office hours this week:

- Thursday, 1:45-3:45
- Friday, 9:30-11:30

Agenda for today

We didn't quite finish the process-of-science discussion from Thursday.

After we finish that up, we'll start Unit 4.

Statistical dishonesty

Statistics is the mathematical discipline that lets us turn empirical data into conclusions.

It lets us turn a collection of "maybes" and "probablys" and "unlikelys" into "almost certainlys".

Statistics is immensely powerful. But:

- It is a subtle, complex field of math (you can get PhD's in it)
- It is a lot of work and only someone intimately familiar with data is really equipped to analyze it

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- It is a lot of work and only someone intimately familiar with data is really equipped to analyze it
- It is absolutely essential if science is going to look to empirical data as the highest authority

A great many flawed scientific processes come down to flawed statistics. Some common statistical fallacies:

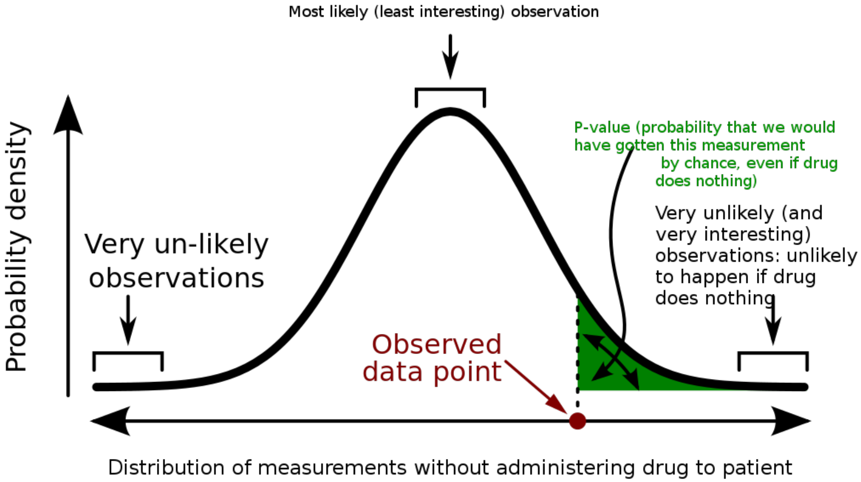
- “Garbage in, garbage out”: flawed, stinky data $\rightarrow$ statistical analysis $\rightarrow$ incorrect but nonstinky conclusions!
- Correlation implies causation
- Incorrect use of statistical inference (statistics is hard)
- Some types of cherry-picking:
  - “P-hacking”
  - Publication bias

Statistical inference done honestly

Suppose we want to test if a new drug (or a chemical in food) has any effect.

Correct thing to do:

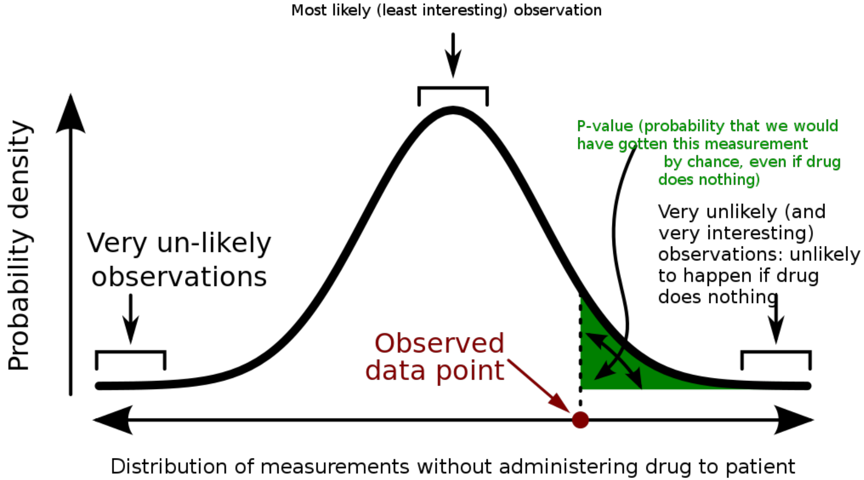
- Make lots of measurements of how people react to the drug
- Compare their distribution to the black curve (what you'd expect if the drug does nothing)
- Get excited if their average is very different from the center of the curve (maybe drug made that happen?)
    - (Statistics gives us tools to quantify "very different")



Statistical inference gone wrong, I: cherry-picking

Classic cherry-picking:

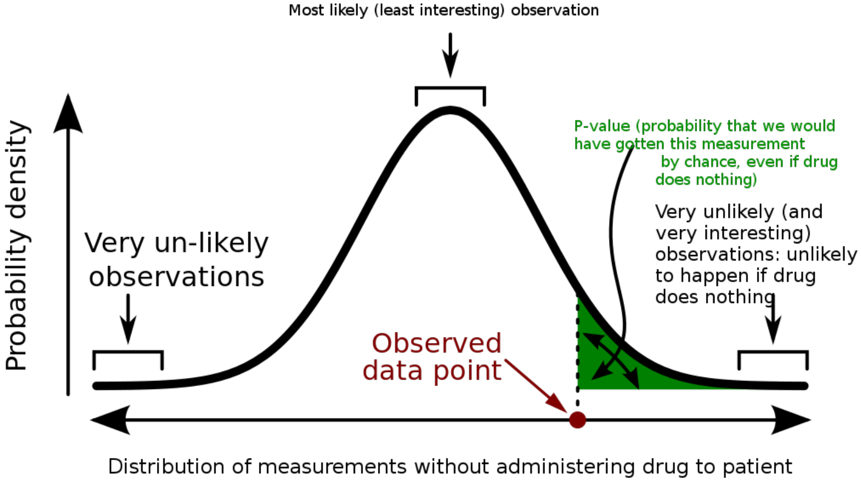
- Make lots of measurements of how people react to the drug
- Forget about the ones close to the center (they are boring!)
- Compare their distribution to the black curve (what you'd expect if the drug does nothing)
- Even if the drug does nothing, you'll get a distribution looking like the green portion
- Notice that their average is very different, get excited!



Statistical inference gone wrong, II: biased data

Biased data (the survivorship bias from Exam 2 is an example):

- Make lots of measurements of how people react to the drug
- Fail to measure the ones on the left-hand side (they're dead)
- Compare their distribution to the black curve (what you'd expect if the drug does nothing)
- We removed the left-hand side, so the average shifts right
- Notice that their average is very different, get excited!



Statistical inference gone wrong, III: publication bias

If the measurements are entire *studies*, we often unintentionally cherry-pick them in *meta-analyses* (studies averaging many studies together)

- Different scientists do experiments on how people react to the drug
- Nobody publishes the ones close to the center (they're boring, back to the drawing board!)
- Compare their distribution to the black curve (what you'd expect if the drug does nothing)
- Even if the drug does nothing, you'll get a distribution looking like the green portion
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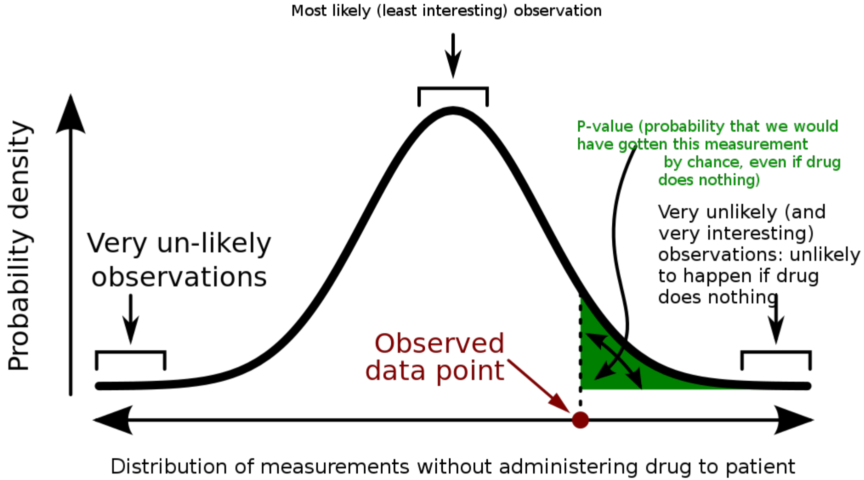

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(Post to Slack for extra credit)

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The above example about drug studies is real: publication bias is a serious problem in drug development! (See `https://www.nejm.org/doi/full/10.1056/NEJMsa065779`)

In the last unit we studied momentum and kinetic energy, along with their rotational counterparts:

Translation	Rotation
Momentum $\vec{p} = m\vec{v}$	Angular momentum $L = I\omega$
Kinetic energy $KE = \frac{1}{2}mv^2$	Kinetic energy $KE_{\text{rot}} = \frac{1}{2}I\omega^2$

This already suggests some other correspondences between translational and rotational quantities:

- Velocity $\Longleftrightarrow$ Angular velocity
- Mass $\Longleftrightarrow$ Moment of inertia

Are there others like this? In what other ways does rotational motion have familiar features?

It turns out *almost all of Units 1 and 2* works just the same for rotation:

Unit 1:

- The kinematics relations between $\vec{a}, \vec{v}, \vec{s}, t$ are identical for $\alpha, \omega, \theta, t$
- They're even simpler, because there are no vectors!

Unit 2:

- The centerpiece of this course was $\vec{F} = m\vec{a}$: "how do forces make things move?"
- What is the rotational analogue to this?

So what are we doing to do now:

- Learn the rotational analogue of force and Newton's second law (today)
- Apply it to all sorts of situations: the rest of the term!

First, you should see the whole picture of how rotational and translational motion correspond:

Translation	Rotation
Position $\vec{s}$ Velocity $\vec{v}$ Acceleration $\vec{a}$	Angle θ Angular velocity ω Angular acceleration α
Kinematics: $\vec{s}(t)\frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$	$\theta(t) = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$
Force $\vec{F}$ Mass m Newton's second law $\vec{F} = m\vec{a}$	Torque τ Rotational inertia I Newton's second law for rotation $\tau = I\alpha$
Kinetic energy $KE = \frac{1}{2}mv^2$ Work $W = \vec{F} \cdot \Delta\vec{s}$ Power $P = \vec{F} \cdot \vec{v}$	Kinetic energy $KE = \frac{1}{2}I\omega^2$ Work $W = \tau\Delta\theta$ Power $P = \tau\omega$
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Rotational motion and kinematics

First, we need to describe how rotating objects move.

Rotational motion can be described separate from its translational motion.

Describing rotation by itself is simple: it's the same as one-dimensional motion (no vectors!)

By convention: counter-clockwise is always positive (like with the unit circle).

An example: consider a centrifuge rotating at $\omega = 1000\text{rad/s}$. Once its motor is turned off, slows down at $\alpha = -100\text{rad/s}^2$. How long will it take to stop?

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- Angular acceleration α corresponds to $\vec{a}$
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Today, we'll study only torque: this limits us to situations where $\alpha = 0$.

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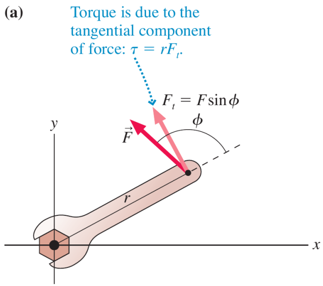
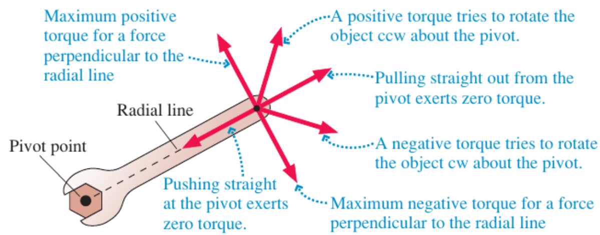
- The size of the force
  - Push harder to exert more torque – that's easy!
- The distance from the force to the pivot point
  - The further from the pivot to the point of force, the greater the torque
  - This is why the door handle is on the outside of the door...
- The angle at which the force is applied
  - Only forces "in the direction of rotation" make something turn
  - The torque depends only on the *component of the force perpendicular to the radius*

Computing torque

$$\tau = F_{\perp} r$$

Torque is equal to the distance from the pivot, times the perpendicular component of the force



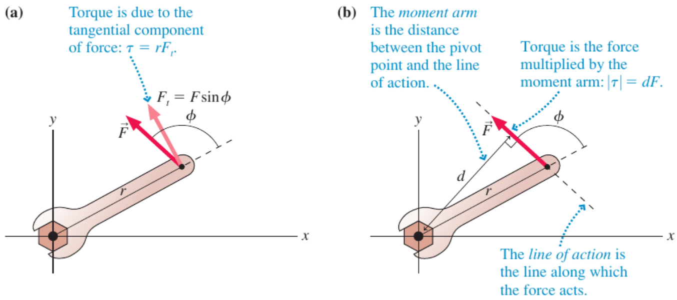


Note that torque has a sign, just like angular velocity: CCW is positive; CW is negative.

Computing torque

- We can think of the torque in any other equivalent way; there is another one that's often useful
- The previous way: **"The radius vector, times the component of force perpendicular to it"**
- The alternative: **"The force vector, times the component of the radius perpendicular to it"**

Here's the figure from the text:



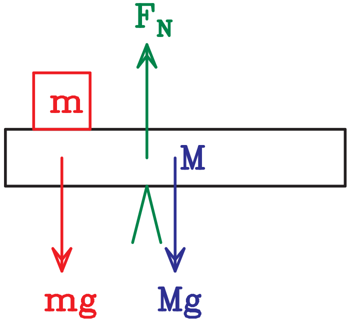
I'll draw a clearer version on the document camera

Which bar is hardest to hold up? (See document camera)

Important notes about torque

These are very important: note them somewhere for later reference!

- Torques are in reference to a **particular pivot**
- This is different from force; if you're talking about torque, you *must* say what axis it's measured around
- Torque now depends on the *location* of forces, not just their size
  - Your force diagrams now need to show the place where forces act!
  - Weight acts at the center of mass ("the middle"); we'll see what that means later
  - A sample force diagram might look like this:



Drawing diagrams: torque problems

- Now you need to draw the position at which every force acts
- Pick a pivot; label it
- Remember, the torque from each force is either...
  - $F_\perp r$ (most useful)
  - $F r_\perp$ (sometimes useful)
  - $F r \sin\theta$ (θ is angle between vectors)
  - Direction of torques matters!

Equilibrium problems

- Often we know $\alpha = \vec{a} = 0$
- This tells us that the net torque (about *any* pivot) and the net force are both zero
- Usually this is because an object isn't moving, but sometimes it's moving at a constant rate (tomorrow's recitation problem)

- Compute the torque about any point and set it to zero
- Choose a pivot conveniently at the location of a force we don't care about
- If needed, also write $\sum \vec{F} = 0$

Statics problems: a sample

- What is the weight of the bar?

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- How does the force needed to support it depend on the angle of the bar?
- What if I hang weights from it?