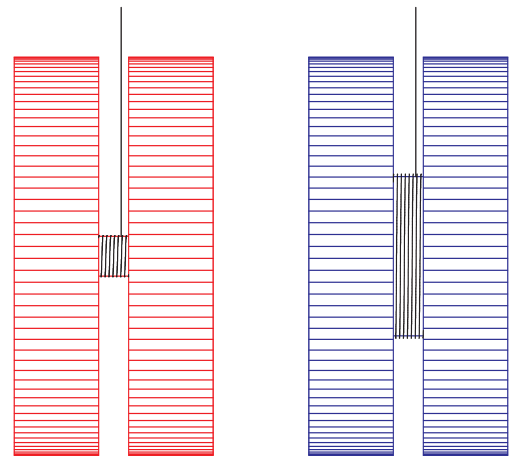
Recitation for Week 13, Day 2

Exercises on combining rotation and translation

Exercise 1: Torque on a yo-yo

A Yo-Yo consists of a cylinder of radius R and mass m with a thin slit cut in it. Inside the slit is a smaller inner cylinder of radius r with a string attached to it and then wound around the cylinder. Note that the moment of inertia of a cylinder of radius R is $I = \frac{1}{2}mR^2$; since the slit in the Yo-Yo is so thin, you do not need to consider it in computing the moment of inertia. (Thus, both have the same moment of inertia: $I = \frac{1}{2}mR^2$.)
If a person holds the end of the string and drops the Yo-Yo, it will begin to spin as it falls, unwinding the string as it does.



a) Suppose that you have a red Yo-Yo with $r = 0.1R$ (that is, with a very small inner cylinder) and a blue Yo-Yo with $r = 0.4R$ (with a thicker inner cylinder). Using an argument based on energy, predict which one will fall faster when it is dropped, and describe why it will do so. *(You shouldn't do any calculations here.)*

b) Now, you'll calculate the downward acceleration of the Yo-Yo. In this case, the Yo-Yo both *translates* and *rotates* as it does so.

Start by drawing an extended force diagram for the Yo-Yo, showing all the forces acting on it *and where they act*. (Think carefully about which perspective your force diagram should show – it's not the one in the diagrams above.)

c) Since it both translates and rotates, you will need both $\vec{F} = m\vec{a}$ to relate the forces on it to its translational acceleration and $\tau = I\alpha$ to relate the torques on it to its linear acceleration. Construct both of these equations, using the forces that appear on your force diagram. *(Hint: The tension in the string both applies a torque to the Yo-Yo and affects its translational acceleration.)*

d) In the above two equations, you will have three unknowns: the tension in the string, the translational acceleration, and the angular acceleration α. However, you can relate two of them to each other. What is that relation? *(Hint: Think carefully about minus signs here!)*

e) Now you should have enough information to solve for a in terms of g, r, and R. Once you have a value for your acceleration, call your GTA or coach over and have them check your work. Discuss with them whether the red or blue Yo-Yo in part (a) would fall faster.

Exercise 2: a Ping-Pong ball on a table

Some people are playing Ping-Pong outdoors and have left a ball of mass m on the table when a gentle breeze begins; this wind applies a constant horizontal force F_w on the ball. The coefficient of static friction between the ball and the table is μ_s, and the coefficient of kinetic friction is μ_k. The Ping-Pong ball is a hollow shell and has a moment of inertia $\frac{2}{3}mr^2$.

1. Suppose first that the breeze is very gentle, so that the ball rolls smoothly on the table without slipping. If F_w is very small, will the frictional force on the ball be $\mu_s mg$, $\mu_k mg$, or some other value F_f that you don't know yet? Discuss this with your group and call your coach or TA over to join your conversation. *(Don't continue here until you've discussed this with one of your instructors.)*

2. Determine the ball's translational acceleration a and angular acceleration α in terms of F_w and m. (You will need to do all the usual things that you did during the last problem – draw a force diagram, etc.)

3. Now, suppose that the wind steadily increases in strength. What is the largest wind force F_w for which the ball will roll without slipping? *(What other force limits how strong the wind can be?)*

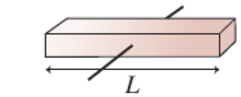
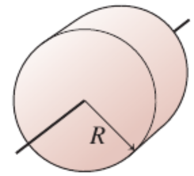
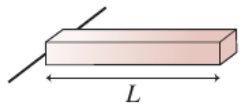
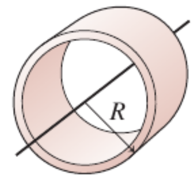
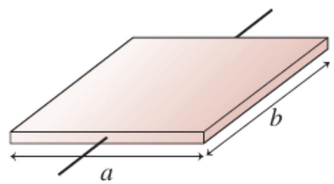
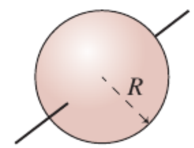
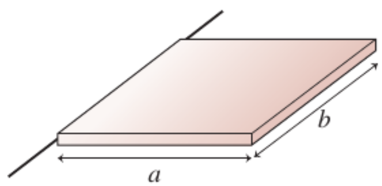
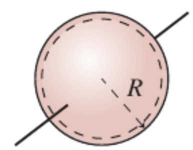
4. Suppose that the wind becomes even stronger, so that the ball skids across the table. Now determine both its translational acceleration a and its angular acceleration α.

Translation	Rotation
Position $\vec{s}$ Velocity $\vec{v}$ Acceleration $\vec{a}$	Angle θ Angular velocity ω Angular acceleration α
$\vec{s}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$ $\vec{v}(t) = \vec{a}t + \vec{v}_0$	$\theta(t) = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$ $\omega(t) = \alpha t + \omega_0$
Force $\vec{F}$ Mass m Newton's second law $\vec{F} = m\vec{a}$	Torque τ Rotational inertia I Newton's second law for rotation $\tau = I\alpha$
Kinetic energy $KE = \frac{1}{2}mv^2$ Work $W = \vec{F} \cdot \Delta\vec{s}$ Power $P = \vec{F} \cdot \vec{v}$	Kinetic energy $KE = \frac{1}{2}I\omega^2$ Work $W = \tau\Delta\theta$ Power $P = \tau\omega$
Momentum $\vec{p} = m\vec{v}$	Angular momentum $L = I\omega$

"Rolling without slipping" constraint: $v = \pm\omega r$ or $a = \pm\alpha r$

(Think about the relative direction that the constraint imposes on v and ω to determine whether the sign is $+$ or $-$)

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

In general, the moment of inertia is $I = \lambda MR^2$ or $I = \lambda ML^2$.