Homework 9

Due Tuesday, 30 April

This homework set contains only two problems, both somewhat involved. Only *one* of them will be graded.

1. The drive components of a two-speed bicycle are constructed as follows:

   - The rear wheel has a radius of 50 cm
   - The front gear has a radius of 10 cm, and is connected to the pedals
   - There are two rear gears: one with a radius of 5 cm ("high gear"), and one with a radius of 10 cm ("low gear")
   - A chain connects the front gear to one of the rear gears; the cyclist can choose which one ("shift gears")
   - None of the other components of the bicycle matter.

   Suppose that our cyclist wants to ride at a constant $v = 10$ m/s, and that there is a constant drag force of 15 N. Determine each of the following for both low gear and for high gear. (Some of these will be the same for both.)

   It will be helpful for you to draw an extended force diagram for the rear wheel + gear, and another for the front gear, since you will need to think about the torques applied to each from traction and the chain (rear), and the chain and pedals (front).

   (a) The traction force on the rear wheel

   (b) The angular velocity of the rear wheel (and the gear attached to it)

   (c) The tangential velocity of the rear gear, which is equal to the velocity of the chain and thus to the tangential velocity of the front gear

   (d) The tension in the chain; note that only the top portion of the chain bears tension

   (e) The angular velocity of the front gear (and thus the pedals)

   (f) The torque the cyclist must apply to the pedals

   (g) The *power* the cyclist must supply to the pedals

   Some of your values will be different for low gear and high gear; some of them will be the same. In particular, comment on the values you get for the angular velocity of the pedals, the torque applied to the pedals, and the overall power supplied by the cyclist.

   Note that the bike is going at a constant rate, so the net torque on everything is zero.

2. A ball ($I = \lambda m r^2$) of radius r and mass m rolls down a slope of length L elevated at an angle θ without slipping. The coefficient of static friction between the ball and the slope is μ_s.

   In this problem, you'll calculate the forces on it, and its acceleration. Take the following steps:

   (a) Draw an extended force diagram for the bowling ball. Choose a tilted coordinate system, as usual.

   (b) Is the frictional force on the bowling ball equal to $\mu_s F_N$, or might it be some other value? (If it's some other value, introduce a symbol for it.)

   (c) What is the relationship between the linear acceleration a of the bowling ball and its angular acceleration α? (Note that counterclockwise is positive; the sign in this relationship will depend on which way your ramp is facing.)

   (d) Construct both $\sum \vec{F} = m\vec{a}$ and $\sum \tau = I\alpha$ for the ball. This should produce a system of two equations with three unknowns. Combined with your result from (c), you can solve for the acceleration and for your other unknown quantity in terms of m, θ, λ, and g.

   (e) Sanity-check the acceleration you get for your bowling ball. Does it make sense?

   (f) What is the steepest slope that your ball can roll down without slipping?