Vectors and 2D kinematics

Physics 211
Syracuse University, Physics 211 Spring 2023
Walter Freeman

January 26, 2023

"Science and everyday life cannot and should not be separated."

–Rosalind Franklin, English biophysicist, to her father

"What's the use of doing all this work if we don't get some fun out of this?"

–Rosalind Franklin

Announcements

- Homework 1 due during your next recitation
  - People in Thursday recitations: you can turn it in on Friday *if you are at recitation Thursday*
  - To do this, go to recitation Thursday and turn in a piece of paper with your name and a note that says "I'll turn this in to your mailbox Friday"
- Homework 2 will be posted late today
- The Clinic got a lot of use yesterday
- I'll be in the Physics Clinic today 1:30-3:00pm

- Homework 2 due next Friday, posted later today
- Next week:
  - Tuesday and Thursday we will be applying and practicing what we learn today
  - Group practice test in your recitation Thursday/Friday

Vectors

You've been doing math with numbers, which are things that live in one dimension: they only have a magnitude and a sign.

Vectors are things that have a magnitude and a direction: "arrows in space"

Many of the things we deal with in physics are vectors:

- Position

Vectors

You've been doing math with numbers, which are things that live in one dimension: they only have a magnitude and a sign.

Vectors are things that have a magnitude and a direction: "arrows in space"

Many of the things we deal with in physics are vectors:

- Position
- (and its derivatives: velocity and acceleration)

Vectors

You've been doing math with numbers, which are things that live in one dimension: they only have a magnitude and a sign.

Vectors are things that have a magnitude and a direction: "arrows in space"

Many of the things we deal with in physics are vectors:

- Position
- (and its derivatives: velocity and acceleration)
- Force, momentum

Vectors

You've been doing math with numbers, which are things that live in one dimension: they only have a magnitude and a sign.

Vectors are things that have a magnitude and a direction: "arrows in space"

Many of the things we deal with in physics are vectors:

- Position
- (and its derivatives: velocity and acceleration)
- Force, momentum

So, we need to learn to do math with arrows.

Notation and equations

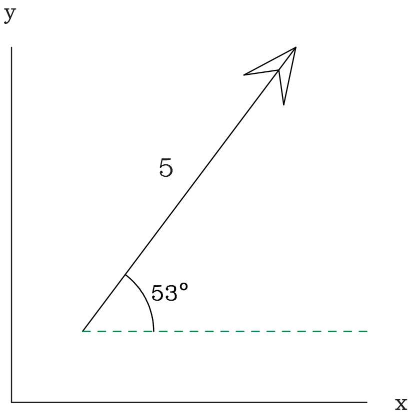
We can do algebra with vectors just like anything else:

- We indicate that a symbol is a vector by writing an arrow over it: "the vector $\vec{V}$".
- "Scalar": object that isn't a vector (mass, time)
- Equations can mix vectors and scalars: $\vec{F} = m\vec{a}$.
- ... or $\vec{s} = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$

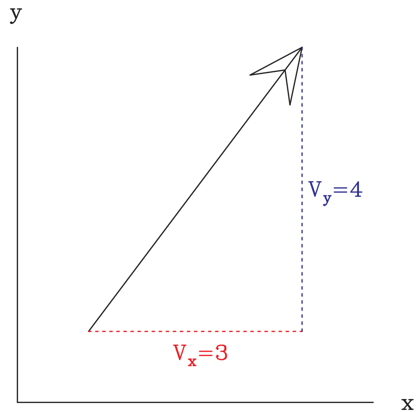
Some notation:

- $\vec{A}$: "the vector A" (a vector)
- A: "the magnitude of A" (a scalar)
- $\hat{A}$: "the direction A points in" (a vector with magnitude 1)
- A_x: the component of A along the $x-$axis (a scalar)
- A_y: the component of A along the $y-$axis (a scalar)

Two ways to describe a vector



Magnitude and direction



X and Y components

How do we convert from one to the other?

How do we convert from one to the other?

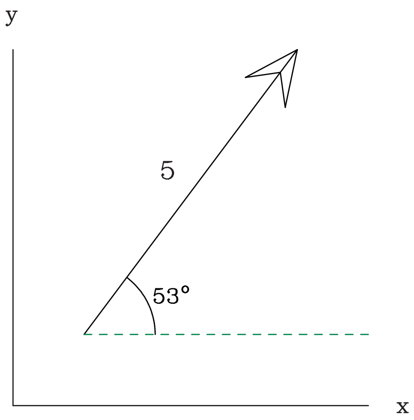
A: Using algebra

B: Using trigonometry

C: Using calculus

D: Using differential equations

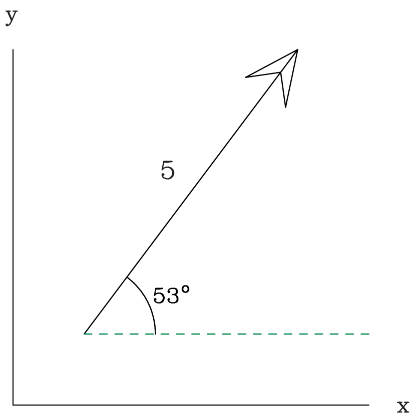
From magnitude and direction to components



Magnitude and direction

What is the $x-$component of this vector?

A: $5\cos 53°$
B: $5\sin 53°$
C: $5\tan 53°$
D: Something else

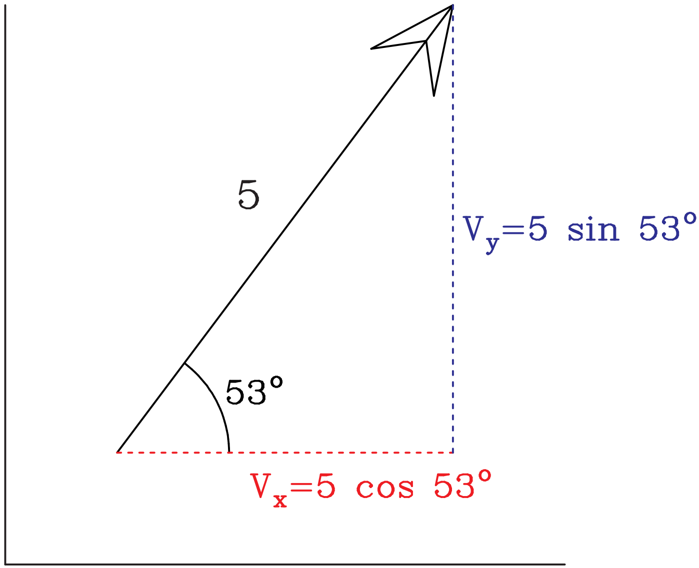


Magnitude and direction

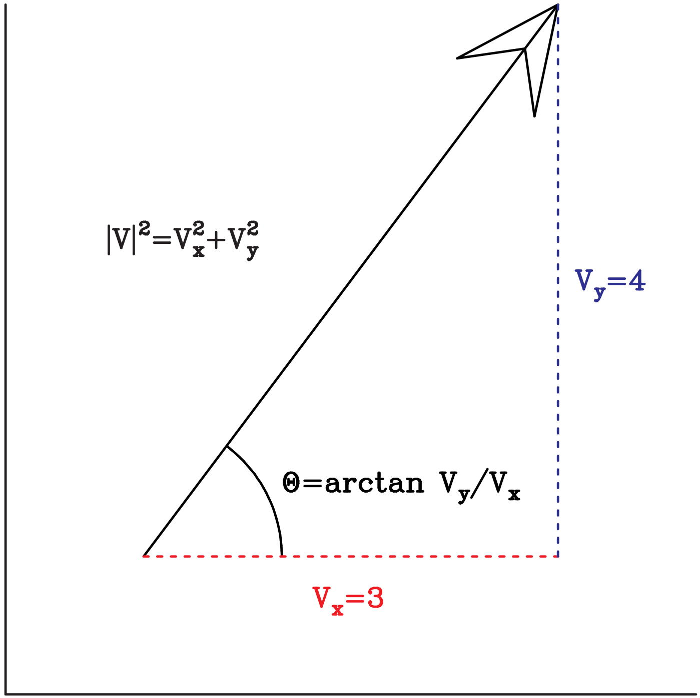
What is the $x-$component of this vector?

A: $5 \cos 53°$
B: $5 \sin 53°$
C: $5 \tan 53°$
D: Something else

From “direction and magnitude” to components



From components to direction and magnitude



Suppose you have some vector $\vec{A}$ that you want to convert into components. The x-component A_x is:

A: $A\cos\theta$

B: $A\sin\theta$

C: $A\tan\theta$

D: $\frac{A}{\cos\theta}$

E: It depends

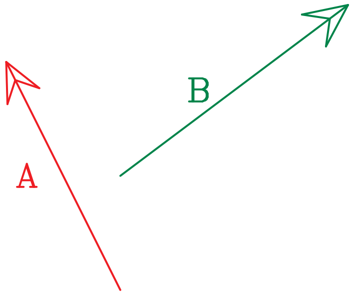
A warning!

You cannot memorize
"$V \sin\theta$ is the y component, $V \cos\theta$ is the x component"!

This does *not* work in general; you have to actually draw the triangle.

Adding vectors

We can also add vectors together by drawing them “head to tail”. Here are two vectors:



Does $\vec{A} + \vec{B} = \vec{B} + \vec{A}$?

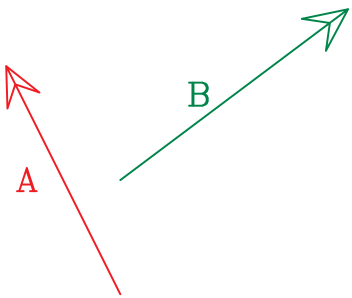
- A: Yes
- B: No

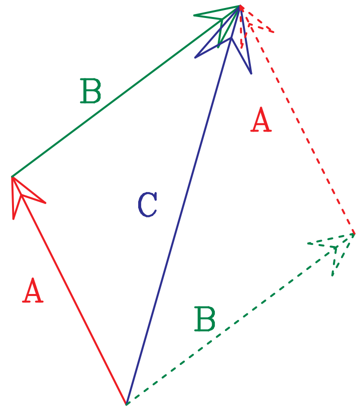
Does $\vec{A} + \vec{B} = \vec{B} + \vec{A}$?

- A: Yes
- B: No

Yes: vector addition obeys the commutative property, just like ordinary addition

We can also add vectors together by drawing them “head to tail”. Here are two vectors:





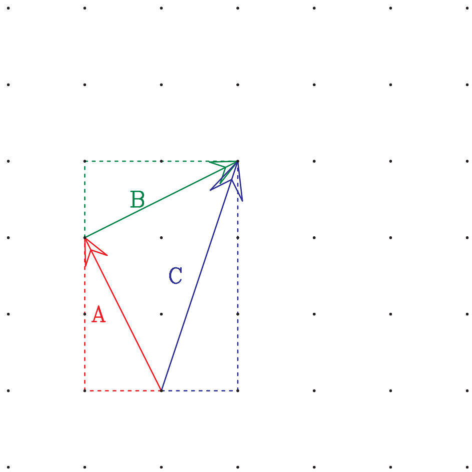
$$\vec{A} + \vec{B} = \vec{C}$$

Adding vectors: components

The component representation is much easier to work with!

$$\vec{A} + \vec{B} = \vec{C} \rightarrow \begin{pmatrix} A_x + B_x = C_x \\ A_y + B_y = C_y \end{pmatrix}$$

Adding vectors: components



To add two vectors, just add their components!

This is why it is almost always easiest to work in the component representation!

Caution: this is much easier than doing stuff involving the "law of cosines" or "law of sines". This is not one of those obnoxious problems from high school geometry; down that path lies madness.

What does this do to our kinematics?

Acceleration, velocity, and position relationships are still the same; they just apply independently for each component.

$\vec{s}$ is the position vector:

- s_x or just x is its x-component
- s_y or just y is its y-component

$\vec{v}$ is the velocity vector:

- v_x is its x-component
- v_y is its y-component

$\vec{a}$ is the acceleration vector:

- a_x is its x-component
- a_y is its y-component

Do not get lazy if you have multiple subscripts. For instance: $\vec{v}_0$ is the initial velocity vector:

- $v_{0,x}$ or v_{0x} is its x-component
- $v_{0,y}$ or v_{0y} is its y-component

What does this do to our kinematics?

Acceleration, velocity, and position relationships are still the same; they just apply independently for each component.

$$\vec{v}(t) = \vec{a}t + \vec{v}_0$$
$$\vec{s}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$$

What does this do to our kinematics?

Acceleration, velocity, and position relationships are still the same; they just apply independently for each component.

$$\vec{v}(t) = \vec{a}t + \vec{v}_0$$
$$\vec{s}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$$

$$v_x(t) = a_x t + v_{x,0}$$
$$v_y(t) = a_y t + v_{y,0}$$

What does this do to our kinematics?

Acceleration, velocity, and position relationships are still the same; they just apply independently for each component.

$$\vec{v}(t) = \vec{a}t + \vec{v}_0$$
$$\vec{s}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$$

$$v_x(t) = a_x t + v_{x,0}$$
$$v_y(t) = a_y t + v_{y,0}$$

$$x(t) = \frac{1}{2}a_x t^2 + v_{x,0}t + x_0$$
$$y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + y_0$$

Which statement does *not* make sense?

- A) $\vec{A}t = \vec{B}$
- B) $\vec{A} + \vec{B} + t = \vec{C}$
- C) $k(\vec{A} + \vec{B}) = k\vec{A} + k\vec{B}$
- D) $\vec{A} - \vec{B} = \vec{C}$

Which statement does *not* make sense?

- A) $\vec{A}t = \vec{B}$
- B) $\vec{A} + \vec{B} + t = \vec{C}$
- C) $k(\vec{A} + \vec{B}) = k\vec{A} + k\vec{B}$
- D) $\vec{A} - \vec{B} = \vec{C}$

B: You can't add a vector and a scalar. "One mile north plus one inch" – which way is the inch?

Problem solving: 2D kinematics, constant acceleration

1. 1. If you have vectors in the "angle and magnitude" form, convert them to components
2. 2. Write down the kinematics relations, separately for x and y
   - Many terms will usually be zero
   - Freefall: $a_x = 0$, $a_y = -g$ (with conventional choice of axes)
3. 3. Understand what instant in time you want to know about
4. 4. Put in what you know; solve for what you don't (using substitution, if necessary)
5. 5. Convert vectors into whatever format you would like them in

Problem solving: 2D kinematics, constant acceleration

1. 1. If you have vectors in the "angle and magnitude" form, convert them to components
2. 2. Write down the kinematics relations, separately for x and y
   - Many terms will usually be zero
   - Freefall: $a_x = 0$, $a_y = -g$ (with conventional choice of axes)
3. 3. Understand what instant in time you want to know about
4. 4. Put in what you know; solve for what you don't (using substitution, if necessary)
5. 5. Convert vectors into whatever format you would like them in

Every kinematics situation we will encounter can be done this way!

A rock is thrown at $v_0 = 10m/s$ at $\theta = 30^\circ$ above the horizontal.

- How far from its starting point is it after 2 seconds?

Sample problems

A rock is thrown at $v_0 = 10m/s$ at $\theta = 30^\circ$ above the horizontal.

- How far from its starting point is it after 2 seconds?
- How far does it travel?

Sample problems

A rock is thrown at $v_0 = 10m/s$ at $\theta = 30^\circ$ above the horizontal.

- How far from its starting point is it after 2 seconds?
- How far does it travel?
- How high does it go?

Sample problems

A rock is thrown at $v_0 = 10m/s$ at $\theta = 30^\circ$ above the horizontal.

- How far from its starting point is it after 2 seconds?
- How far does it travel?
- How high does it go?
- What will its speed be when it strikes the ground?

We're done!

This is all of the material you will need for the first exam!

We have three recitations and two classes left; we're going to spend it practicing.

As with everything in physics, applying abstract ideas to the real world is the challenge. So we have set aside lots of time for it!