Momentum

Physics 211
Syracuse University, Physics 211 Spring 2019
Walter Freeman

April 1, 2019

Announcements

- Group Exam 3 on Friday
- HW7 posted; **deadline extended until Monday when the building closes**
- This week's office hour schedule:
  - Tuesday 3-5 and Thursday 1:45-3:45: office hours in my office
    - Extra review specifically to help students who are having trouble, but who are working hard
    - Priority will be given to people with lower exam grades but demonstrated effort
  - Wednesday 3-5: clinic hours as normal
- Weekend exam review: Saturday, 2:30-5:30
- Paper assigned; **due April 29**

The paper

(explained based on the assignment posted)

Sample problems: arrrrrrr.

A pirate ship has a cannon of mass M, mounted on a deck that is a height h above the waterline. It fires a cannonball of mass m horizontally; it lands in the water a distance d away. The cannon slides backward along the deck and is brought to rest by friction; the coefficient of friction is μ_k.

How far back does the cannon slide?

Classifying collisions

A ball of mass m moves toward another ball of mass m. The first ball is moving at a speed v; the second ball is not moving.

What happens after the collision?

Classifying collisions

A ball of mass m moves toward another ball of mass m. The first ball is moving at a speed v; the second ball is not moving.

What happens after the collision?

Based on only this information, *you don't know*. Two possible options:

- Both balls stick together and move at speed $\frac{1}{2}v$
- The first ball stops moving, and the second ball moves off at speed v

These scenarios both conserve momentum: they are *required* to.

Classifying collisions

A ball of mass m moves toward another ball of mass m. The first ball is moving at a speed v; the second ball is not moving.

What happens after the collision?

Based on only this information, *you don't know*. Two possible options:

- Both balls stick together and move at speed $\frac{1}{2}v$
- The first ball stops moving, and the second ball moves off at speed v

These scenarios both conserve momentum: they are *required* to.

They *don't* both conserve kinetic energy, though. After the collision:

- The first scenario has $\text{KE} = 2 \times \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$ (the minimum)
- The second scenario has $\text{KE} = \frac{1}{2}mv^2$ (the maximum)

Classifying collisions

Different types of collisions convert a different amount of kinetic energy into other forms:

- Both balls stick together and move at speed $\frac{1}{2}v$: totally inelastic collision, maximum KE converted into other forms
- The first ball stops moving, and the second ball moves off at speed v: totally elastic collision, kinetic energy stays the same

Whether a collision is totally inelastic, totally elastic, or somewhere in between depends on the materials:

- Totally elastic: Really only happen with subatomic particles
- Mostly elastic: Bouncy balls, billiard balls...
- Mostly inelastic: Things that bounce only a little bit
- Totally inelastic: Things stick together:

Angular momentum

Translational motion

- Moving objects have momentum
- $\vec{p} = m\vec{v}$
- Momentum conserved if there are no external forces

Rotational motion

- Spinning objects have angular momentum L
- $L = I\omega$
- Angular momentum conserved if no external torques

$\rightarrow L = I\omega =$ constant; analogue to conservation of momentum

Conservation of angular momentum

We saw that the conservation of momentum was valuable mostly in two sorts of situations:

- Collisions: two objects strike each other
- Explosions: one object separates into two

There is a third common case for conservation of angular momentum:

- Collisions: a child runs and jumps on a merry-go-round
- Explosions: throwing a ball off-center
- A spinning object changes its moment of inertia

Conservation of angular momentum

We saw that the conservation of momentum was valuable mostly in two sorts of situations:

- Collisions: two objects strike each other
- Explosions: one object separates into two

There is a third common case for conservation of angular momentum:

- Collisions: a child runs and jumps on a merry-go-round
- Explosions: throwing a ball off-center
- A spinning object changes its moment of inertia

This last happens because moment of inertia depends on *how the mass is distributed*, not just how much there is!

Conservation of angular momentum

These problems are approached in exactly the same way as conservation of *linear* momentum problems: write down expressions for L_i and L_f and set them equal (if there are no external torques).

$$L = I\omega$$

$$\sum L_i = \sum L_f$$

Conservation of angular momentum

If I kept the mass of the Earth the same, but enlarged it so that it had twice the diameter, how long would a day be?

(Remember, the total angular momentum, $L = I\omega$, stays the same)

A: 6 hours

B: 12 hours

C: 24 hours

D: 48 hours

E: 96 hours

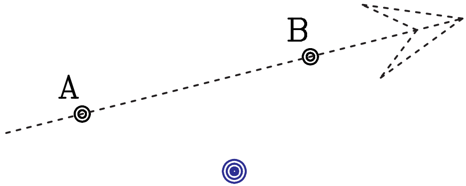
Angular momentum of a single object

A single object moving in a straight line also has angular momentum.

$$L = mv_\perp r = mvr_\perp$$

If we are to trust this relation, then the angular momentum of an object moving with constant $\vec{v}$ should be constant!

Is the angular momentum the same at points A and B?



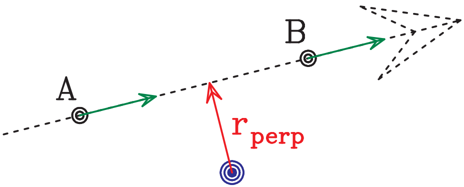
Angular momentum of a single object

A single object moving in a straight line also has angular momentum.

$$L = mv_{\perp}r = mvr_{\perp}$$

Is the angular momentum the same at points A and B?

Yes: $r_{\perp}$ (and v) are the same at both points.



An example problem

A child of mass m runs at speed v straight east and jumps onto a merry-go-round of mass M and radius R, landing 2/3 of the way toward the outside. If she lands on the south edge, how fast will it be turning once she lands?

We'll do this together on the document camera.

An example problem

A child of mass m runs at speed v straight east and jumps onto a merry-go-round of mass M and radius R, landing 2/3 of the way toward the outside. If she lands on the south edge, how fast will it be turning once she lands?

We'll do this together on the document camera.

(The solution is on the next slide, for those studying these notes later)

The solution to our example

We use conservation of angular momentum:

$$\sum L_i = \sum L_f$$
$$L_{\text{child},i} = L_{\text{child+disk},f}$$

Model the child as a point object moving at a constant velocity:

$$L_{\text{child},i} = m v_\perp r = \frac{2}{3} m v R$$

This gives us $\frac{2}{3} m v R = I_{\text{total}} \omega_f$. We now need I_{total}.

After the child jumps on, $I_{\text{total}} = I_{\text{disk}} + I_{\text{child}} = \frac{1}{2} M R^2 + \frac{2}{3} m R^2$. Thus,

$$\frac{2}{3} m v R = \left(\frac{1}{2} M R^2 + \frac{2}{3} m R^2 \right) \omega_f$$

Solve for ω_f:

$$\omega_f = \frac{\frac{2}{3} m v R}{\left(\frac{1}{2} M R^2 + \frac{2}{3} m R^2 \right)}$$

Angular momentum demonstrations

Can a spinning person change their moment of inertia?

Angular momentum demonstrations

Can a spinning person change their moment of inertia?

Can a spinning person exchange angular momentum with a spinning object?

Angular momentum demonstrations

What happens to the person on the platform if they catch the ball?

Angular momentum demonstrations

What happens to the person on the platform if they catch the ball?
What happens when they throw it?