Rotational kinetic energy

Physics 211
Syracuse University, Physics 211 Spring 2023
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April 20, 2023

Announcements

- Homework 8 is due Friday
- "Second chance" homework assignments posted on the webpage
- You can find the help hours schedule there (there's a lot of them coming up)

An object with moment of inertia I rotating at angular velocity ω has rotational kinetic energy

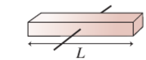
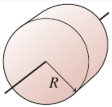
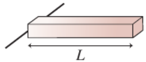
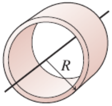
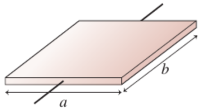
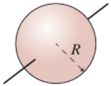
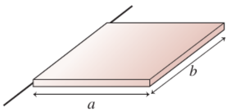
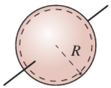
$$KE_{rot} = \frac{1}{2} I \omega^2$$

Moment of inertia, other things

What about the moment of inertia of other objects?

Requires calculus in general; here are some common ones

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

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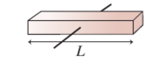
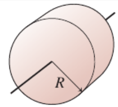
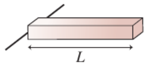
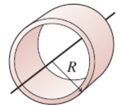
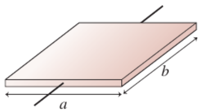
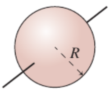
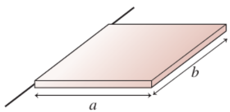
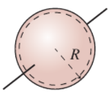
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

In general: $I = \lambda M R^2$

We will always give you I if it's not 1 (i.e. not a ring etc.)

Another example

Remember the "Atwood machine"?

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What happens if we remove one of the weights?

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Remember the “Atwood machine”?

What happens if we remove one of the weights?

What happens if the pulley isn’t light?

The relationship between linear and rotational motion

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Why do we have two different formulae? This came from the relationship:

$$v = \omega r$$

If an object rotates at angular velocity ω, a point a distance r from the center moves at speed v.

Suppose I wrap a string around a solid cylinder with mass M and radius r, and let a mass m hang from the string.

How fast is the falling mass traveling when it hits the ground if it starts from a height h?

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(initial KE) + (work done by gravity) = (final KE)

(initial rotational KE) + (initial translational KE) + (work done by gravity) =
(final rotational KE) + (final translational KE)

Why does a Yo-Yo fall so slowly?

Rolling and energy

Which object will reach the bottom of the ramp faster?

A: The wooden one

B: The one with the mass located near the middle

C: The one with the mass located near the edge

D: A tie between A and B

E: A tie between B and C

Rotation plus translation

In general, rotation and translation are separate; we can study each separately.

Example: this bike wheel

- Its position is given by some function $\vec{s}(t)$: "where is it at some time t?"
- Its angle is given by some other function $\theta(t)$: "which way is the reference point pointing at some time t?"
- The angle has the familiar derivatives: angular velocity ω, angular acceleration α

Recall that points along the edge of a rotating object move at a speed $v_{\text{edge}} = \omega r$.

Example: rolling without slipping

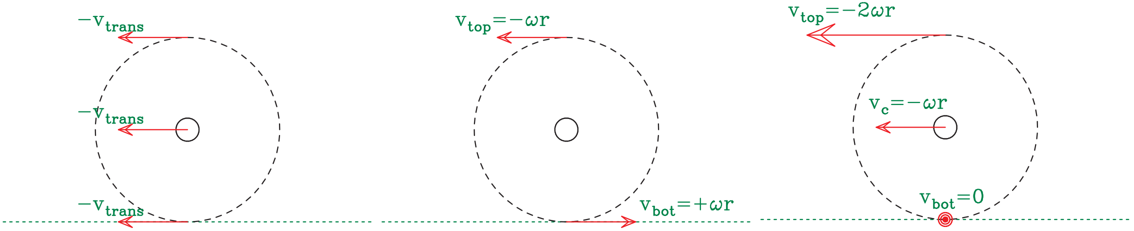
Sometimes the translational and rotational motion are linked.
"How fast do the tires on a car turn?"
→ Static friction means that the bottom piece of the wheel doesn't move

- If a wheel is turning counterclockwise at angular velocity ω:
  - the top moves at $v_{\text{top}} = -\omega r$ (left)
  - the bottom moves at $v_{\text{bot}} = \omega r$ (right)
- This means that the velocity of the axle must be equal and opposite to v_{bot}
- Thus, the car must be moving at $v_{\text{axle}} = -\omega r$ (left).

Let's look at a diagram.
So: if the wheels turn counterclockwise at ω:

- The axle moves at a velocity $-\omega r$ (left);
- The top of the wheels move at a velocity $v_{\text{axle}} + v_{\text{top}} = -\omega r - \omega r = -2\omega r$;
- The top of the wheels move at a velocity $v_{\text{axle}} + v_{\text{bot}} = -\omega r + \omega r = 0$.

Rolling without slipping



Translation + Rotation = Rolling

The “rolling constraint”

If an object rolls forward on an edge of radius r,

$$v = \omega r$$

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Common algebra pattern:

$$KE_{rot} = \frac{1}{2} I \omega^2$$

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$$KE_{rot} = \frac{1}{2} \lambda m v^2$$

How many of you have played pinball?

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You are trying to design a pinball machine's spring-loaded launcher. Suppose that:

- The pinball has mass m and radius r
- The machine is angled at an angle θ and has length L
- You want the ball to reach the top of the machine at a speed v_T

You know the player will draw the spring-loaded launcher back a distance d. What spring constant should it have so the ball is traveling at v_f at the top of the ramp??

What kinds of energy does the system have initially?

A: elastic potential energy

B: gravitational potential energy

C: translational kinetic energy

D: rotational kinetic energy

E: None of the above, or so many I can't display them

What kinds of energy does the system have at the top of the ramp?

A: elastic potential energy

B: gravitational potential energy

C: translational kinetic energy

D: rotational kinetic energy

E: None of the above, or so many I can't show you...

How do I handle this “one equation, two unknowns” problem?

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