Recitation Exercises

Week 13, Day 1

Question 1: on rotational dynamics

A flywheel (a large, spinning disc) of mass m and radius r is rotating at angular velocity ω. The machine operator wishes to bring it to rest using a friction brake. When the brake is engaged, two brake pads on either side of the disc are pressed against it from either side, two-thirds of the way from the center to the outer edge; each brake pad exerts a normal force F_N. *(You will need to look up its moment of inertia in the reference.)*

If the coefficient of friction between the brake pads and the disc is μ_k, how much time does it take the brake to bring the flywheel to a stop?

How many times does the flywheel rotate during this period? *(Note that ω is not constant, since the flywheel is slowing down...)*

Question 2: on linked objects

A bucket of mass m hangs from a string wound around a pulley (a solid cylinder) with mass M and radius r. When the bucket is released, it falls, unwinding the string.

1. Draw a cartoon of the system, then draw force diagrams for the bucket and the pulley. Note that since the pulley rotates, you will need to draw an extended force diagram for it, drawing the object and labeling where each force acts.

   Then, by your diagrams, label your choice of coordinate system. Which direction is positive? Which direction corresponds to "positive angle"?

2. In terms of the forces in your force diagrams, write an expression for the net torque on the pulley.

3. Write down Newton's laws of motion – $\sum \vec{F} = m\vec{a}$ for translation, and $\sum \tau = I\alpha$ – for each object. (One object moves, and the other turns...)

4. What is the relationship between the angular acceleration α of the pulley and the linear acceleration a of the bucket? (The answer may be different depending on how you have drawn your pictures and your choice of coordinate system.)

5. Find an expression for the acceleration of the bucket in terms of m, M, g, and r. (It may not depend on all of these.)

6. Once you've found your expression, discuss its meaning with your group – why it depends on the quantities that it does in the way that it does. Call your coach or TA over to join in your conversation.

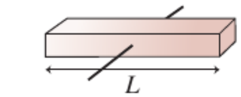
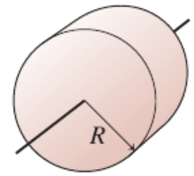
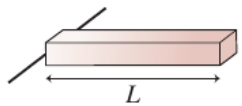
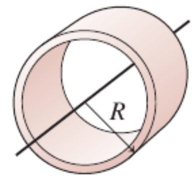
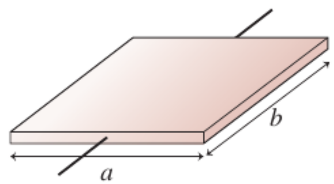
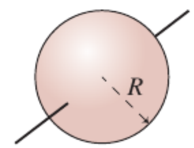
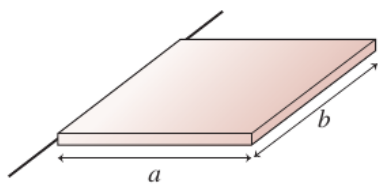
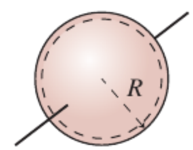
7. Suppose that the pulley were a hollow cylinder with the same mass. How would this acceleration change?

Translation	Rotation
Position $\vec{s}$ Velocity $\vec{v}$ Acceleration $\vec{a}$	Angle θ Angular velocity ω Angular acceleration α
$\vec{s}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$ $\vec{v}(t) = \vec{a}t + \vec{v}_0$	$\theta(t) = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$ $\omega(t) = \alpha t + \omega_0$
Force $\vec{F}$ Mass m Newton's second law $\vec{F} = m\vec{a}$	Torque τ Rotational inertia I Newton's second law for rotation $\tau = I\alpha$
Kinetic energy $KE = \frac{1}{2}mv^2$ Work $W = \vec{F} \cdot \Delta\vec{s}$ Power $P = \vec{F} \cdot \vec{v}$	Kinetic energy $KE = \frac{1}{2}I\omega^2$ Work $W = \tau\Delta\theta$ Power $P = \tau\omega$
Momentum $\vec{p} = m\vec{v}$	Angular momentum $L = I\omega$

"Rolling without slipping" constraint: $v = \pm\omega r$ or $a = \pm\alpha r$

(Think about the relative direction that the constraint imposes on v and ω to determine whether the sign is $+$ or $-$)

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

In general, the moment of inertia is $I = \lambda MR^2$ or $I = \lambda ML^2$.