

# Motion with constant acceleration

Physics 211  
Syracuse University, Physics 211 Spring 2022  
Walter Freeman

January 27, 2022

## The beginning: Free fall

My purpose is to set forth a very new science dealing with a very ancient subject. There is, in nature, perhaps nothing older than motion, concerning which the books written by philosophers are neither few nor small nevertheless I have discovered by experiment some properties of it which are worth knowing and which have not hitherto been either observed or demonstrated....

So far as I know, no one has yet pointed out that the distances traversed, during equal intervals of time, by a body falling from rest, stand to one another **in the same ratio as the odd numbers beginning with unity.**

–Galileo Galilei, *Dialogues and Mathematical Demonstrations Concerning Two New Sciences*, 1638 (translated into English by some old dead guy)

# The beginning: Free fall

I have discovered something new about something very old – motion. Philosophers have written piles of thicc books about it for centuries. But by doing *experiments* I've learned something they've never figured out.

None of them have figured out that a falling object travels one unit of distance, then three, then five, then seven, etc., in equal amounts of time.

–Galileo Galilei, *Dialogues and Mathematical Demonstrations Concerning Two New Sciences*, 1638 (in modern English)

# Reminders:

- Webpage: <https://walterfreeman.github.io/phy211/>
  - Syllabus, homework, etc. are all there
- The first homework is due next Friday
- Your first weekly “check-in”:
  - A brief Google form to gather feedback from you and see how you’re doing
  - This first one will include a bit of a survey on who you are
  - Sent out tomorrow, due Monday by noon
  - No “physics questions”

# “Ask a Physicist”

There are a lot of cool things in physics that go beyond mechanics.

If you've got questions you'd like me to address, send them in and I'll answer them!

- What are gravity waves?
- How is physics used in medicine?
- What's the Large Hadron Collider for?
- How does a touchscreen work?
- How do 3D movies work?
- What is the Higgs boson?
- How is physics used in video games?
- How does a nuclear reactor work?
- How does a supercomputer work?

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- Make use of words, pictures, and algebra (not just algebra!) in your reasoning
- We're interested in how you think, not just the answer – you must explain what you're doing
- Physical values need to be given with units (“4 meters”, not “4”)
- Leave variables in until the very end
- Paper is cheap – don't cramp yourself!

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- Ask for help – early and often
  - Email: [wafreema@syr.edu](mailto:wafreema@syr.edu)
  - Discord server
  - Office hours
  - The Physics Clinic
  - Recitations

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  - In class!



# The course webpage and Discord server

- All notes, etc., will be posted on the course website (not Blackboard)
  - I will also post course announcements there
  - The syllabus is posted there
  - You really should read the section on the course philosophy
- 
- There is also a course Discord server at <https://discord.gg/bngYwjZRHk>
  - This is a great place to ask questions, get advice, and collaborate with your classmates, teaching staff, and me

In the Physics Clinic:

- Tuesdays: 2:00-3:50 PM
- Fridays: 11:00 AM-1:00 PM (may change)
- Other times announced (if homework is due Friday, I may hold Thursday office hours)

or by appointment.

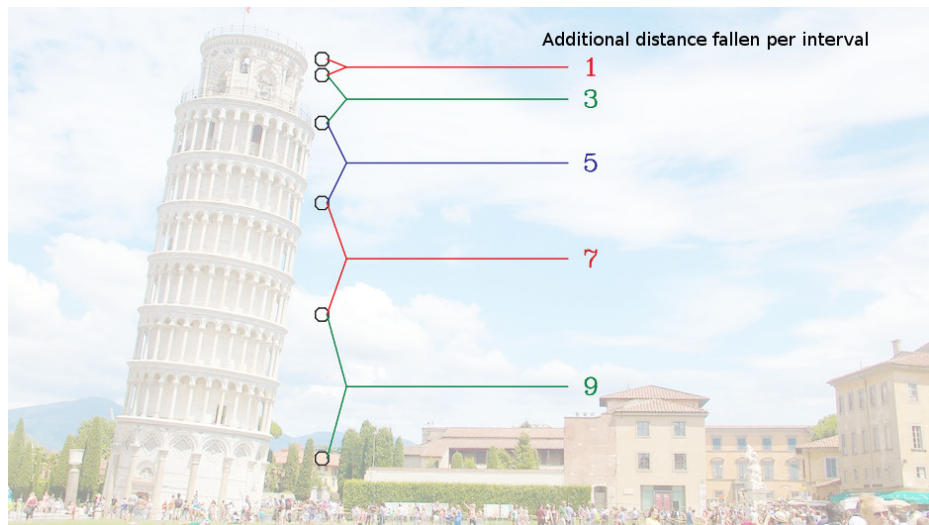
Outside these times you might find me in the Clinic or in my office in room 215.

# The beginning: describing motion (1-D)

Recall that at first, we are only concerned with describing motion.

- Most fundamental question: “where is the object I’m talking about?”
- Quantify position using a “number line” marked in meters:
  - Choose one position to be the origin (“zero”) – anywhere will do
  - Choose one direction to be positive
  - Measure everything relative to that
  - Can measure in any convenient units: centimeters, meters, kilometers...
- You’re used to this already, perhaps:
  - Mile markers on highways
  - Yard lines in American football

# The beginning: Free fall



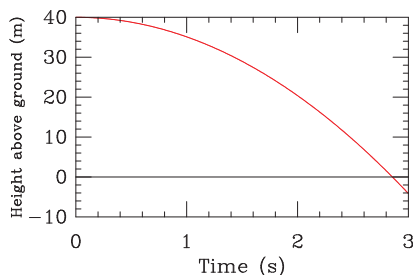
Galileo observed this, but can we explain it?

# Equations of motion

Complete description of motion: “Where is my object at each point in time?”

This corresponds to a mathematical function. Two ways to represent these. Suppose I drop a ball off a building, putting the origin at the ground and calling “up” the positive direction:

## Graphical representation



## Algebraic representation

$$y(t) = (40 \text{ m}) - Ct^2$$

(C is some number; we'll learn what it is Thursday)

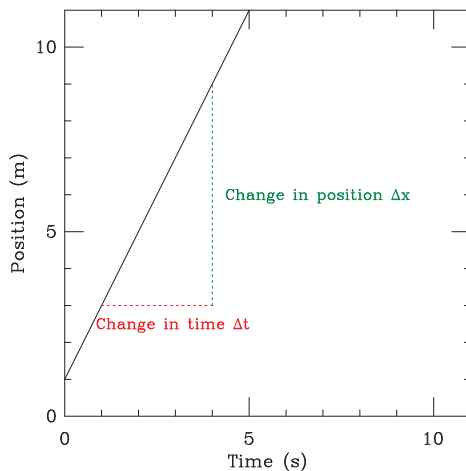
Both let us answer questions like “When does the object hit the ground?”

→ ... the curve's x-intercept

→ ... when  $y(t) = 0$

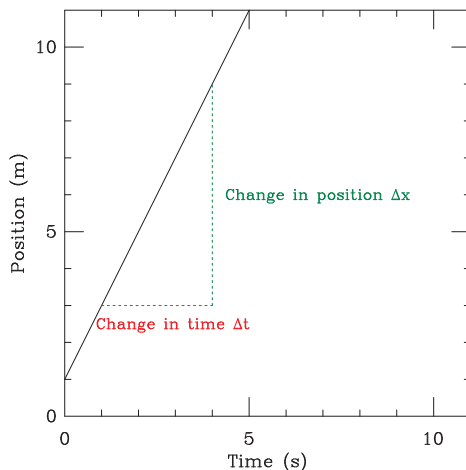
# Velocity: how fast position changes

The slope of the position vs. time curve has a special significance. Here's one with a constant slope:



## Velocity: how fast position changes

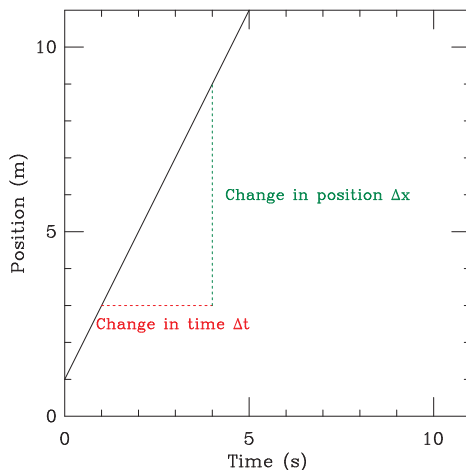
The slope of the position vs. time curve has a special significance. Here's one with a constant slope:



Slope is  $\frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t} = \frac{2\text{m}}{1\text{s}} = 2$  meters per second (positive; it could well be negative!)

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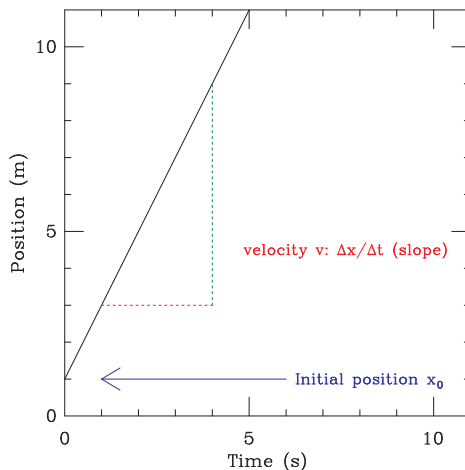
Slope is  $\frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t} = \frac{2\text{m}}{1\text{s}} = 2$  meters per second (positive; it could well be negative!)

→ The slope here – change in position over change in time – is the **velocity**! Note that it can be positive or negative, depending on which way the object moves.



# Constant-velocity motion: connecting graphs to algebra

If an object moves with constant velocity, its position vs. time graph is a line:



We know the equation of a straight line is  $x = mt + b$  (using  $t$  and  $x$  as our axes).

- $m$  is the slope, which we identified as the velocity
- $b$  is the vertical intercept, which we recognize as the value of  $x$  when  $t = 0$

We can thus change the variable names to be more descriptive:

$$x(t) = vt + x_0 \text{ (constant-velocity motion)}$$

# Going from “equations of motion” to answers

$x(t) = vt + x_0$  is called an *equation of motion*; in this case, it is valid for constant-velocity motion.

It gives you the same information as a position vs. time graph, but in algebraic form.

To solve real problems, we need to be able to translate physical questions into algebraic statements:

- “If a car starts at milepost 30 and drives at 50 mph, where is it an hour later?”

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To solve real problems, we need to be able to translate physical questions into algebraic statements:

- “If a car starts at milepost 30 and drives at 50 mph, where is it an hour later?”
  - Using  $x(t) = x_0 + vt$ , with  $x_0 = 30$  mi and  $v = 50 \frac{\text{mi}}{\text{hr}}$ , calculate  $x$  at  $t = 1$  hr

# Asking the right questions

“I drop an object from a height  $h$ . When does it hit the ground?” How do I do this? (Take  $x_0 = h$  and upward to be positive.)

Remember, we want to ask a question in terms of our physical variables. This question has the form:

“What is \_\_\_\_\_ when \_\_\_\_\_ equals \_\_\_\_\_?”

Fill in the blanks.

A:  $v, x, 0$

B:  $t, x, h$

C:  $x, t, 0$

D:  $t, x, 0$

E:  $x, v, 0$

# Asking the right questions

“At what location do two moving objects meet?”

A: “At what time does  $x_1 = x_2$ ?”

B: “At what time does  $v_1 = v_2$ ?”

C: “What is  $x_1$  at the time when  $x_1 = x_2$ ?”

D: “What is  $x_1$  when  $t_1 = t_2$ ?”

Constant-velocity motion:  $x(t) = x_0 + vt$

- Came from looking at the equation of a line
- We can understand this in a different framework, too:
- Velocity is the **rate of change** of position
  - Graphical representation: Velocity is the slope of the position vs. time graph
  - Mathematical language: Velocity is the **derivative** of position

We know we need to know about acceleration (“ $F=ma$ ”) – what is it?

- Acceleration is the **rate of change** of velocity

# Position, velocity, and acceleration

Position  $\xrightarrow{\text{(take the derivative) take the rate of change of}}$  Velocity

# Position, velocity, and acceleration





## Kinematics: how does acceleration affect movement?

Newton's law  $a = F/m$  tells us that *acceleration* – the second derivative of position – is what results from forces.

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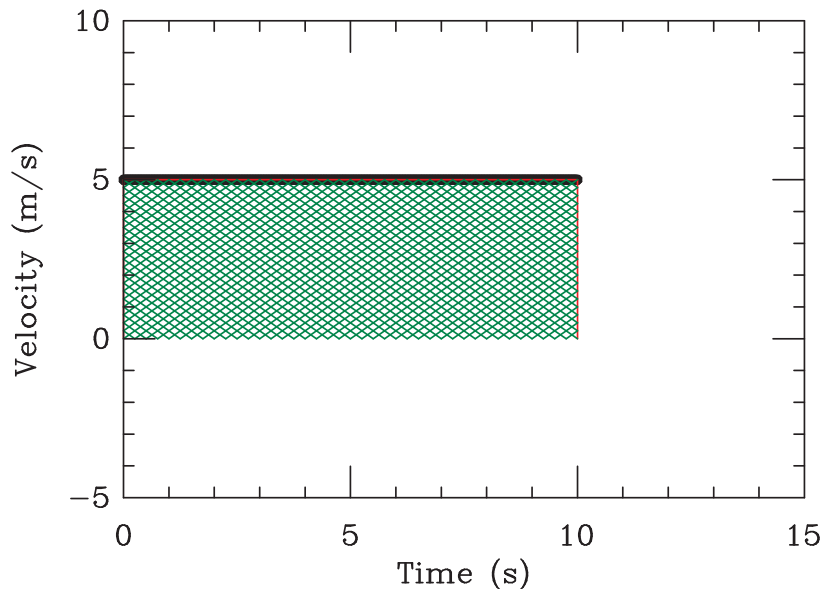
**All freely falling objects have  
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(You can approximate this as  $g = 10 \text{ m/s}^2$  unless you need high precision.)

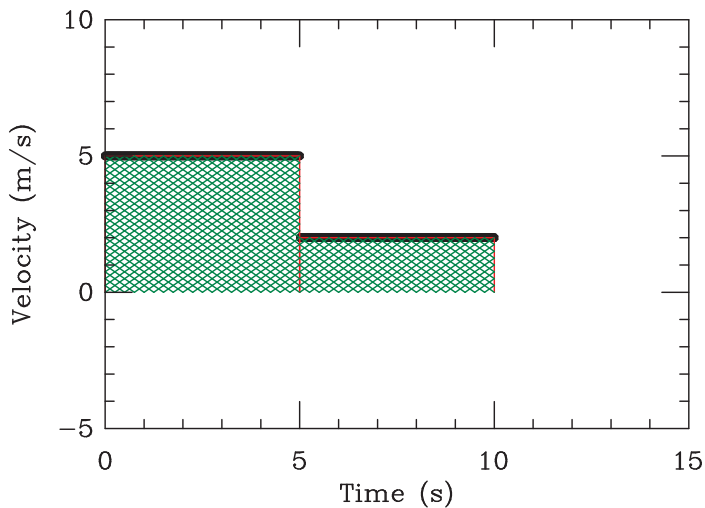
## Some calculus

If velocity is the rate of change of position,  
why is the area under the  $v$  vs.  $t$  curve equal to displacement?



We know  $\Delta s = vt$ . What is that here? What's the area of the shaded region?

## Some calculus



Now what is  $\Delta s$ ? What is the area of the shaded region?

What's the area of the shaded region?

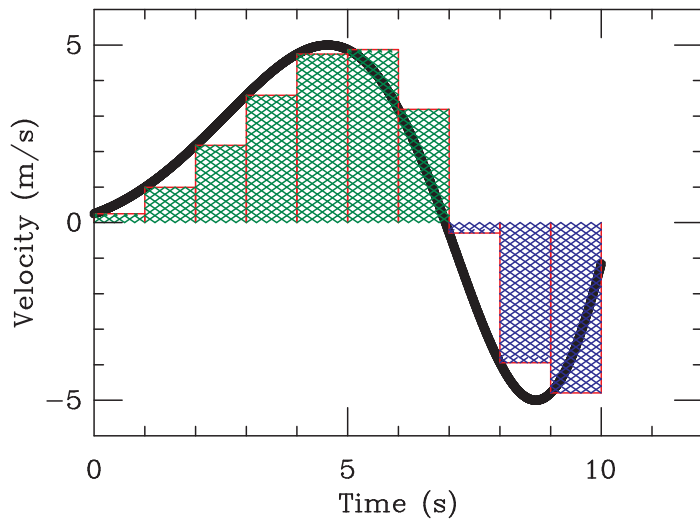
A: 25 m

B: 50 m

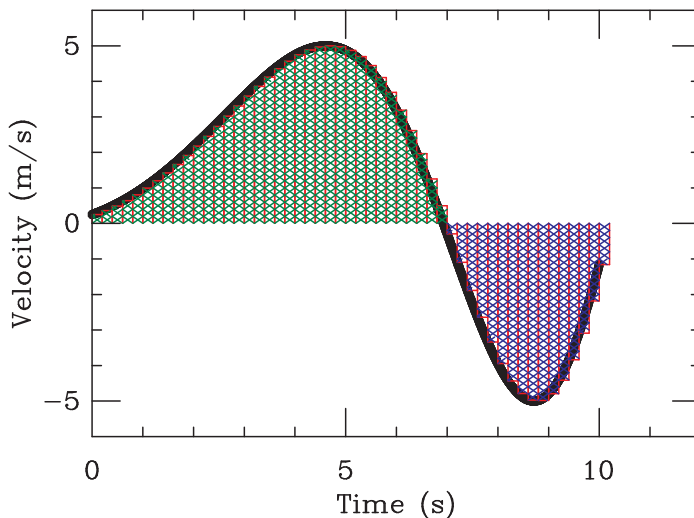
C: 35 m

D: 45 m

# A calculus review



Does this work? How do we fix it?

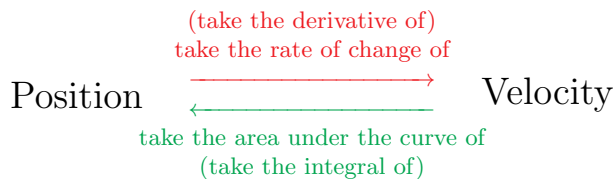


The area between the  $t$ -axis and the velocity curve is the distance traveled.  
(The area below the  $t$ -axis counts negative: “the thing is going backwards”)

$$\text{In calculus notation: } \int v(t) dt = \delta x = x(t) - x_0$$



# Position, velocity, and acceleration



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# Constant acceleration

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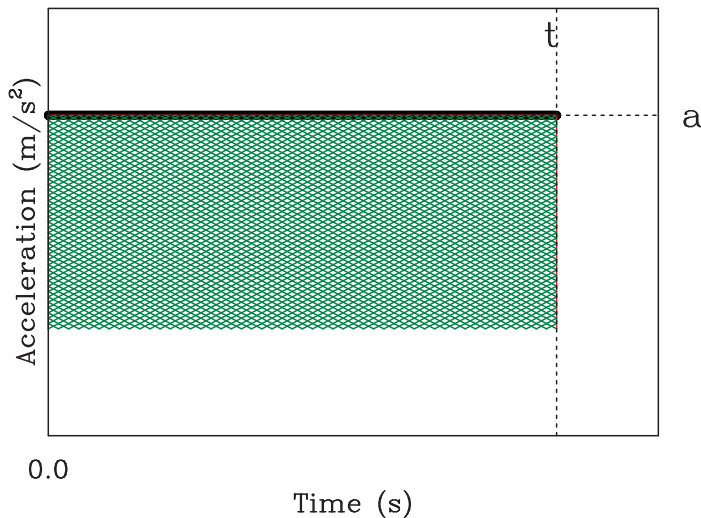
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- Figure out the area under the acceleration curve to get the velocity curve
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Remember the area under the curve of (velocity, acceleration) just gives the *change in* (position, velocity) – *i.e.* initial minus final.

**We'll start by assuming  $x_0$  and  $v_0$  are zero** – that is, we're dropping something from rest.

# Constant acceleration



What's the area under the curve out to time  $t$ , which gives the change in the velocity –  $\Delta v = v(t) - v_0$ ?

A:  $\Delta v = at$

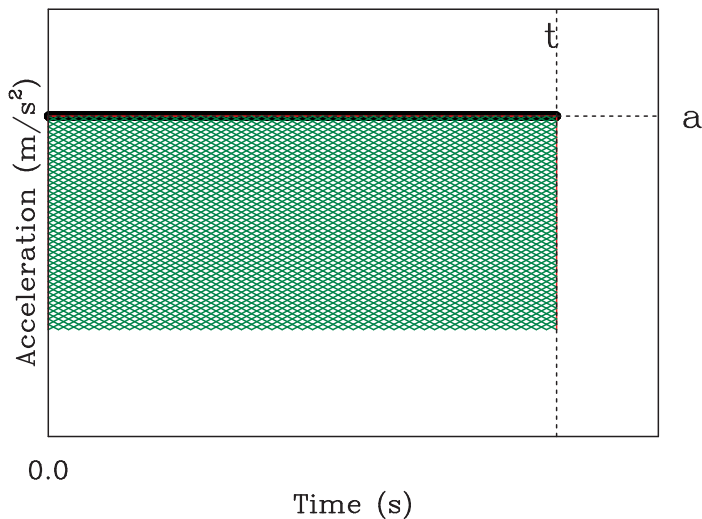
B:  $\Delta v = at + v_0$

C:  $\Delta v = \frac{1}{2}at^2$

D:  $\Delta v = a$

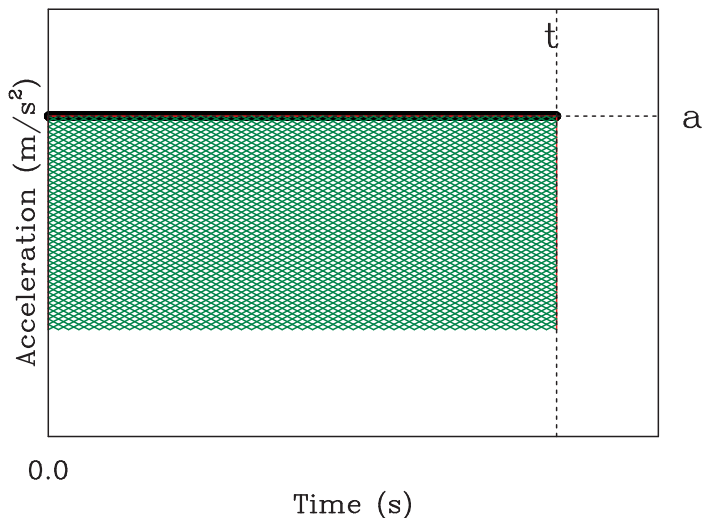


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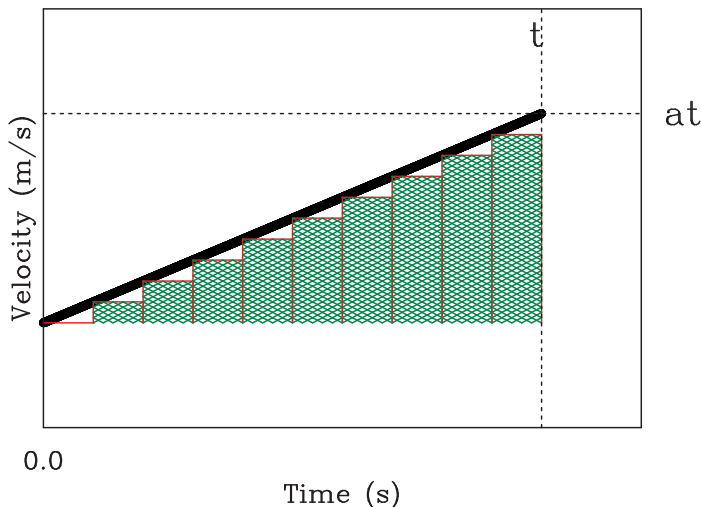
## Constant acceleration



What's the area under the curve out to time  $t$ , which gives the change in the velocity –  $\Delta v = v(t) - v_0$ ?

$\Delta v$ , the change in velocity, is  $v(t) - v_0 = at$ , so  $v(t) = at + v_0$

## Same thing again to get position



Now the area under the velocity curve gives the change in position:  $\Delta x = x(t) - x_0$ . What is that?

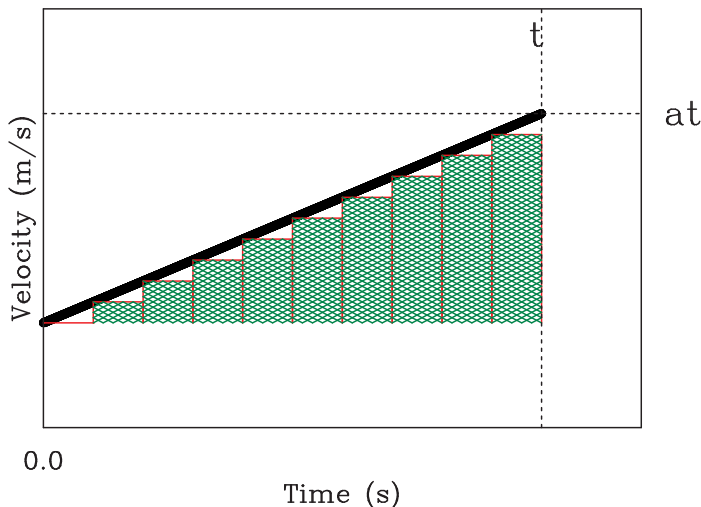
A:  $\Delta x = at$

B:  $\Delta x = vt$

C:  $\Delta x = \frac{1}{2}at^2$

D:  $\Delta x = v$

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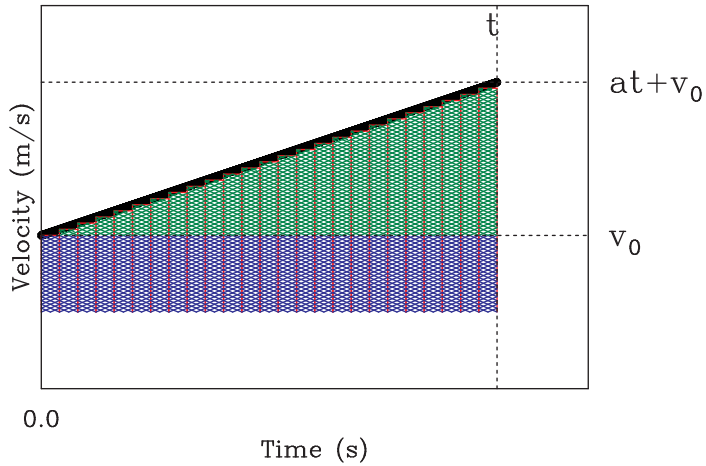
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C:  $\Delta x = \frac{1}{2}at^2$

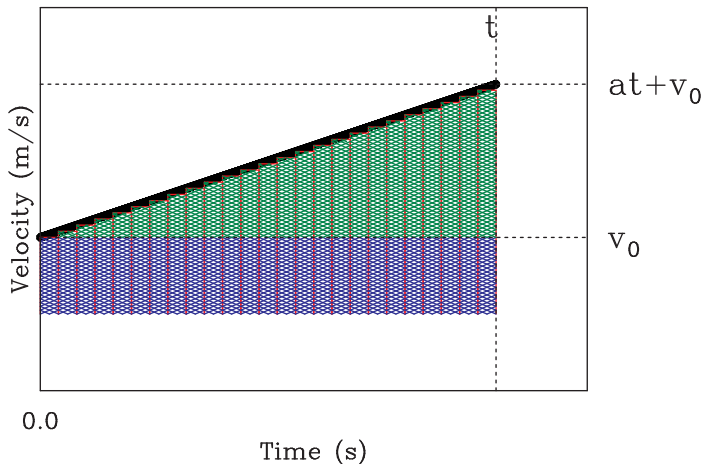
D:  $\Delta x = v$

$$x(t) - x_0 = \frac{1}{2}at^2, \text{ thus } x(t) = \frac{1}{2}at + x_0$$

Now if  $v_0$  is not zero...



Now if  $v_0$  is not zero...



Area under blue part:  $v_0 t$

Area under green part:  $\frac{1}{2}at^2$

Total change in position:  $x(t) - x_0 = \frac{1}{2}at^2 + v_0 t$

$$\text{Thus, } x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

## For those who are familiar with calculus:

$$a(t) = \text{const.}$$

$$v(t) = \int a \, dt \qquad = at + C_1$$

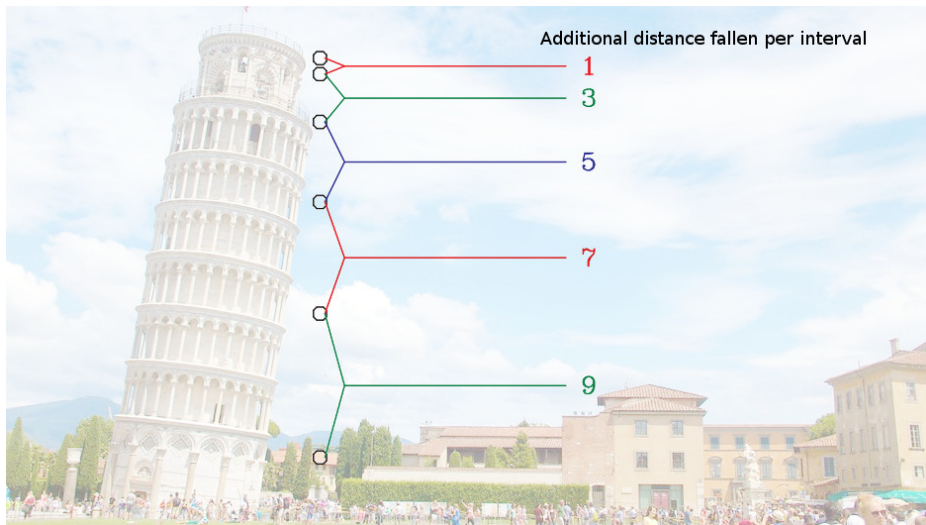
$$x(t) = \int v \, dt = \int (at + C_1) dt \qquad = \frac{1}{2}at^2 + C_1t + C_2$$

A little thought reveals that  $C_1$  is the initial velocity  $v_0$  and  $C_2$  is the initial position  $x_0$ . This gives us the things we just derived, but much more easily:

$$v(t) = at + v_0$$

$$x(t) = \frac{1}{2}at^2 + v_0t + x_0$$

# Free fall revisited



Adding these numbers together gives us 1, 4, 9, 16, 25...  
The calculus above explains this: distance is proportional to *time squared*!