Introduction

Syracuse University, Physics 211 Spring 2021
Walter Freeman

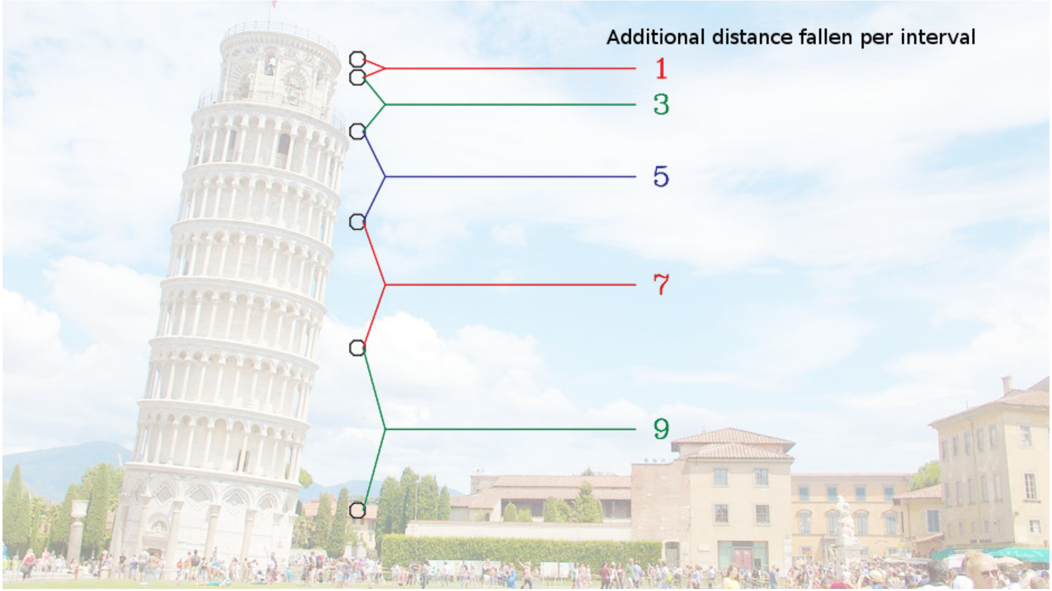
February 11, 2021

The beginning: Free fall

My purpose is to set forth a very new science dealing with a very ancient subject. There is, in nature, perhaps nothing older than motion, concerning which the books written by philosophers are neither few nor small nevertheless I have discovered by experiment some properties of it which are worth knowing and which have not hitherto been either observed or demonstrated....

So far as I know, no one has yet pointed out that the distances traversed, during equal intervals of time, by a body falling from rest, stand to one another **in the same ratio as the odd numbers beginning with unity.**

–Galileo Galilei, *Dialogues and Mathematical Demonstrations Concerning Two New Sciences*, 1638



Can we understand this?

Physics 211
Forces and Motion

Walter Freeman, professor
Mario Olivares, lead TA
Adam Aly, lead coach

Course webpage:

http://walterfreeman.github.io/phy211/

Overview of today

Non-physics stuff:

- Welcome and reminders
- Recitation recap / questions and answers
- Announcements
- How to succeed in this course

Physics stuff:

- How do we describe motion?
- How do position, velocity, and acceleration relate to each other *conceptually*?
- How do position, velocity, and acceleration relate to each other *graphicaly*?
- How do position, velocity, and acceleration relate to each other *algebraically*?

Recitation Q&A

Remember, the recitation is the *most important* part of this course.

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Asynchronous students: We will send you all detailed instructions tonight.

- Homework 1 is posted on the course website and Blackboard; it will be due next Friday.
- We are working to accommodate more people for in-person recitations. There will be another survey this weekend and more capacity available starting next Wednesday.
- I will be holding office hours tomorrow morning from 9:30-12:00 on the same Zoom link used for class. Come ask questions! I will physically be in room B126, if you want to say hi in person.
- Lecture recordings will be available starting today; they will be available automatically on Twitch. I'll make YouTube links available too.

Advice for success in this class

How do we describe motion – words, graphs, symbols, numbers

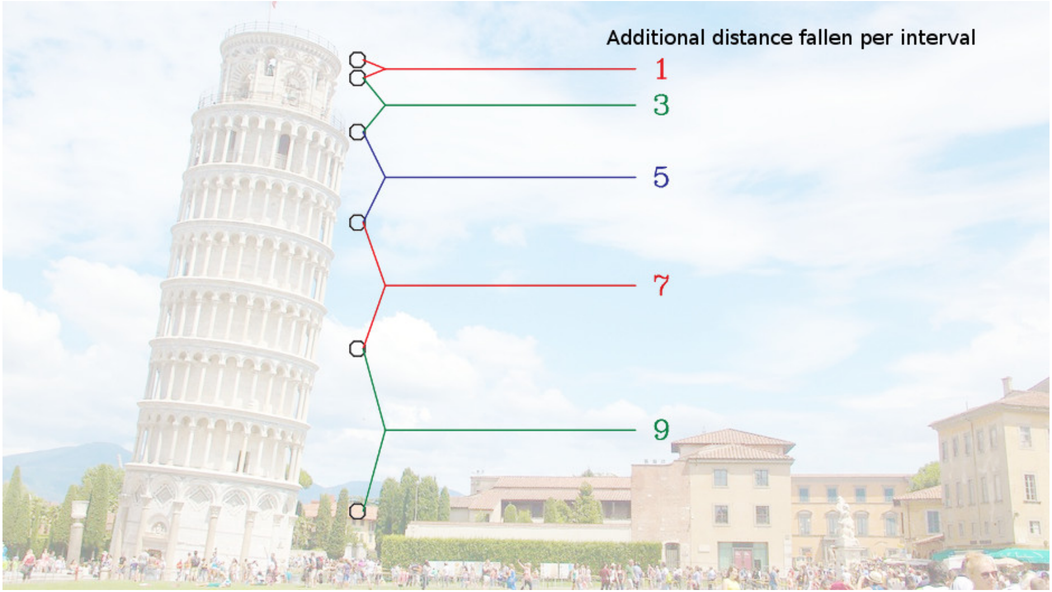
x, v, a – conceptually

x, v, a – graphically

Free fall – constant acceleration

x, v, a – algebraically, for a constant

Free fall revisited



Adding these numbers together gives us 1, 4, 9, 16, 25...

The calculus above explains this: distance is proportional to *time squared!*

The remaining slides are "copies" of the stuff we will do on the chalkboard. (This is what I used in the auditorium last year.)

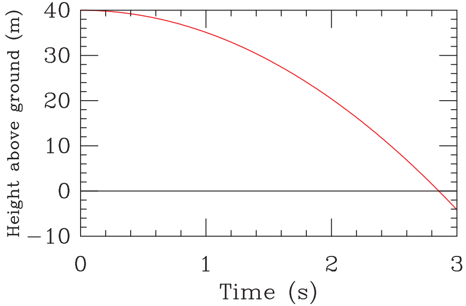
They are provided as an accessibility service to students who may find them useful.

Equations of motion

Complete description of motion: "Where is my object at each point in time?"

This corresponds to a mathematical function. Two ways to represent these. Suppose I drop a ball off a building, putting the origin at the ground and calling "up" the positive direction:

Graphical representation



Algebraic representation

$$y(t) = (40\,\mathrm{m}) - Ct^2$$

(C is some number; we'll learn what it is Thursday)

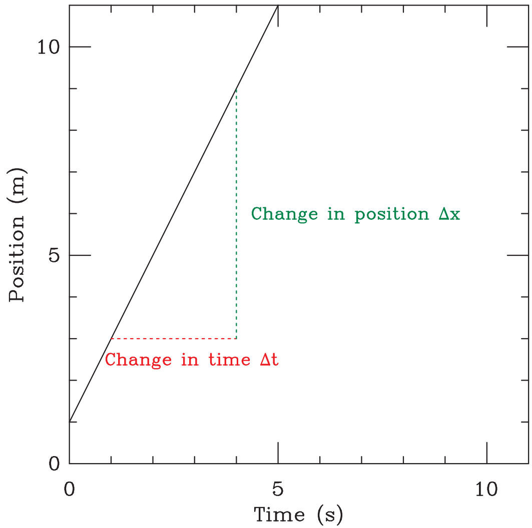
Both let us answer questions like "When does the object hit the ground?"

→ ... the curve's x-intercept

→ ... when $y(t) = 0$

Velocity: how fast position changes

The slope of the position vs. time curve has a special significance. Here's one with a constant slope:



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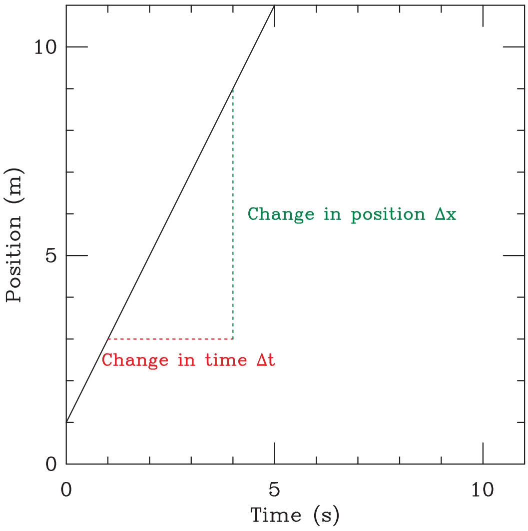

Slope is $\frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t} = \frac{2\,\text{m}}{1\,\text{s}} = 2$ meters per second (positive; it could well be negative!)

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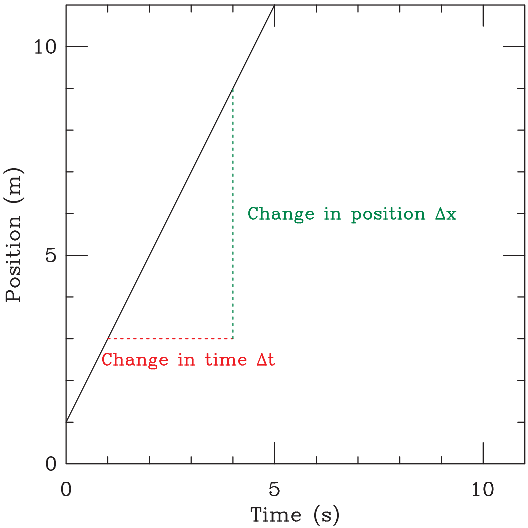

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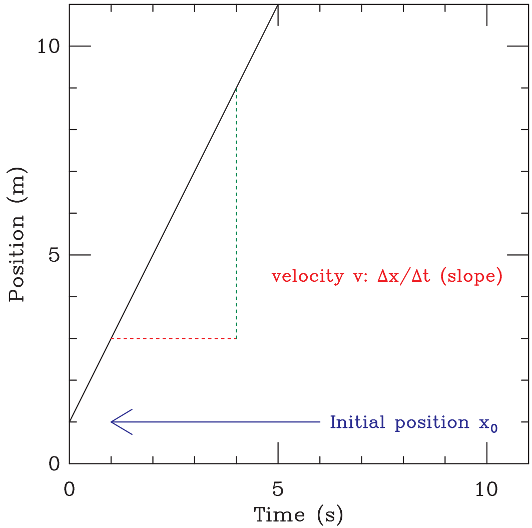
→ The slope here – change in position over change in time – is the **velocity**! Note that it can be positive or negative, depending on which way the object moves.

Constant-velocity motion: connecting graphs to algebra

If an object moves with constant velocity, its position vs. time graph is a line:



We know the equation of a straight line is is $x = mt + b$ (using t and x as our axes).

- m is the slope, which we identified as the velocity
- b is the vertical intercept, which we recognize as the value of x when $t = 0$

We can thus change the variable names to be more descriptive:

$$x(t) = vt + x_0 \text{ (constant-velocity motion)}$$

Going from "equations of motion" to answers

$x(t) = vt + x_0$ is called an *equation of motion*; in this case, it is valid for constant-velocity motion.

It gives you the same information as a position vs. time graph, but in algebraic form.

To solve real problems, we need to be able to translate physical questions into algebraic statements:

- "If a car starts at milepost 30 and drives at 50 mph, where is it an hour later?"

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To solve real problems, we need to be able to translate physical questions into algebraic statements:

- "If a car starts at milepost 30 and drives at 50 mph, where is it an hour later?"
  - Using $x(t) = x_0 + vt$, with $x_0 = 30\,\mathrm{mi}$ and $v = 50\,\frac{\mathrm{mi}}{\mathrm{hr}}$, calculate x at $t = 1\,\mathrm{hr}$

Asking the right questions

"I drop an object from a height h. When does it hit the ground?" How do I do this? (Take $x_0 = h$ and upward to be positive.)

Remember, we want to ask a question in terms of our physical variables. This question has the form:

"What is __________ when __________ equals __________?"

Fill in the blanks.

A: v, x, 0
B: t, x, h
C: x, t, 0
D: t, x, 0
E: x, v, 0

Asking the right questions

"At what location do two moving objects meet?"

A: "At what time does $x_1 = x_2$?"

B: "At what time does $v_1 = v_2$?"

C: "What is x_1 at the time when $x_1 = x_2$?"

D: "What is x_1 when $t_1 = t_2$?"

Velocity, acceleration, and calculus

Constant-velocity motion: $x(t) = x_0 + vt$

- Came from looking at the equation of a line
- We can understand this in a different framework, too:

- Velocity is the rate of change of position
  - Graphical representation: Velocity is the slope of the position vs. time graph
  - Mathematical language: Velocity is the derivative of position

We know we need to know about acceleration ("F=ma") – what is it?

- Acceleration is the rate of change of velocity

Position, velocity, and acceleration

$$\text{Position} \xrightarrow[\text{take the rate of change of}]{\text{(take the derivative)}} \text{Velocity}$$

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Kinematics: how does acceleration affect movement?

Newton's law $a = F/m$ tells us that *acceleration* – the second derivative of position – is what results from forces.

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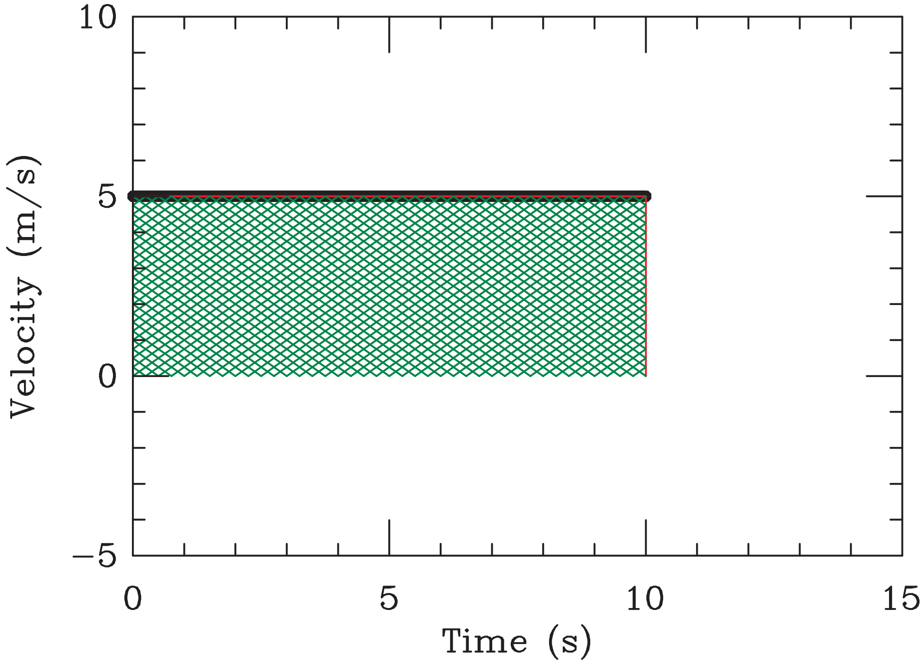
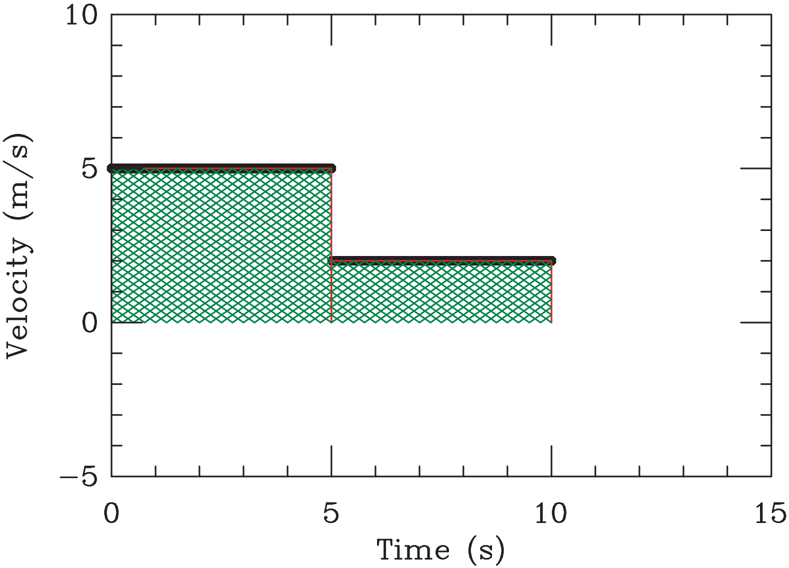
Newton's law $a = F/m$ tells us that *acceleration* – the second derivative of position – is what results from forces.

All freely falling objects have a constant acceleration downward.

This number is so important we give it a letter: $g = 9.81\ \mathrm{m/s^2}$

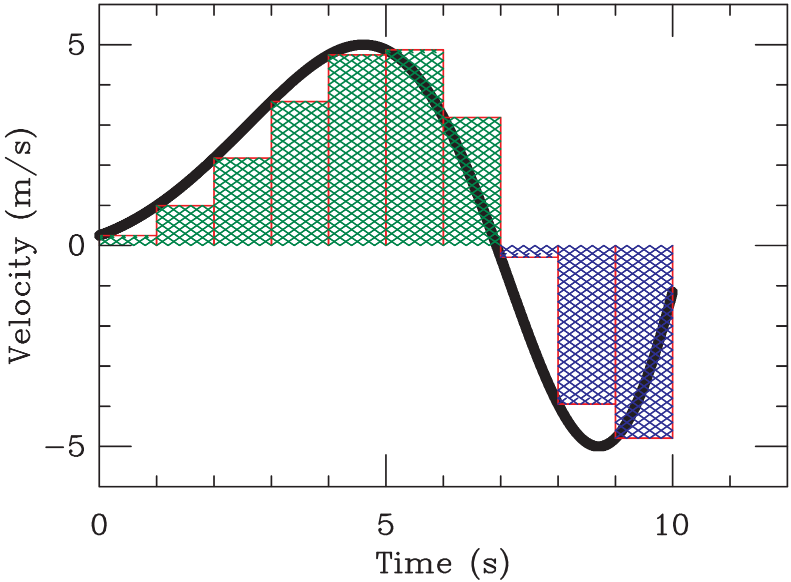
If velocity is the rate of change of position,
why is the area under the v vs. t curve equal to displacement?





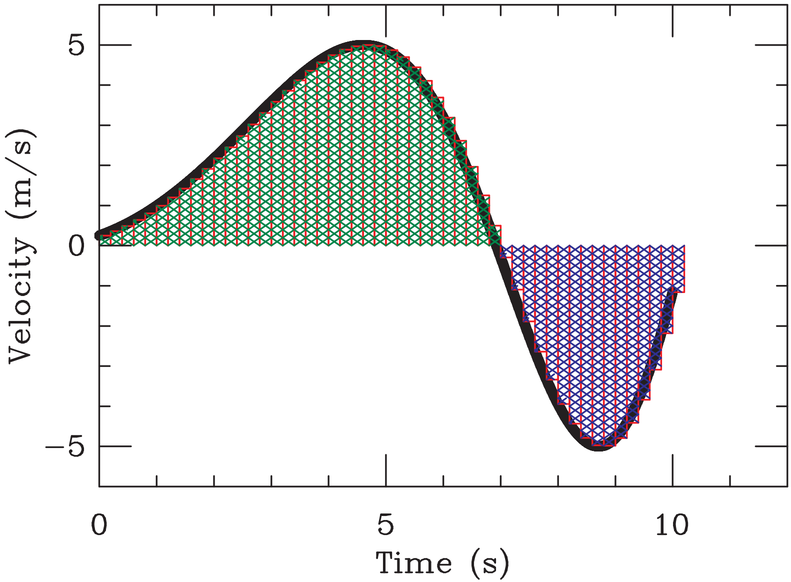
Now what is Δs? What is the area of the shaded region?

A calculus review



Does this work? How do we fix it?

A calculus review



The area between the t-axis and the velocity curve is the distance traveled.
(The area below the t-axis counts negative: “the thing is going backwards”

In calculus notation: $\int v(t)\,dt = \delta x = x(t) - x_0$

Position, velocity, and acceleration

$$\text{Position} \quad \underset{\substack{\text{take the area under the curve of} \\ \text{(take the integral of)}}}{\overset{\substack{\text{(take the derivative of)} \\ \text{take the rate of change of}}}{\rightleftarrows}} \quad \text{Velocity}$$

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Constant acceleration

Particularly interesting situation:

- Free fall (as you saw)
- Any time the force is constant: $F = ma \rightarrow a = F/m$...

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Plan of attack:

- We know what the acceleration curve looks like (it's just flat)
- Figure out the area under the acceleration curve to get the velocity curve
- Figure out the area under the velocity curve to get the position curve

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Plan of attack:

- We know what the acceleration curve looks like (it's just flat)
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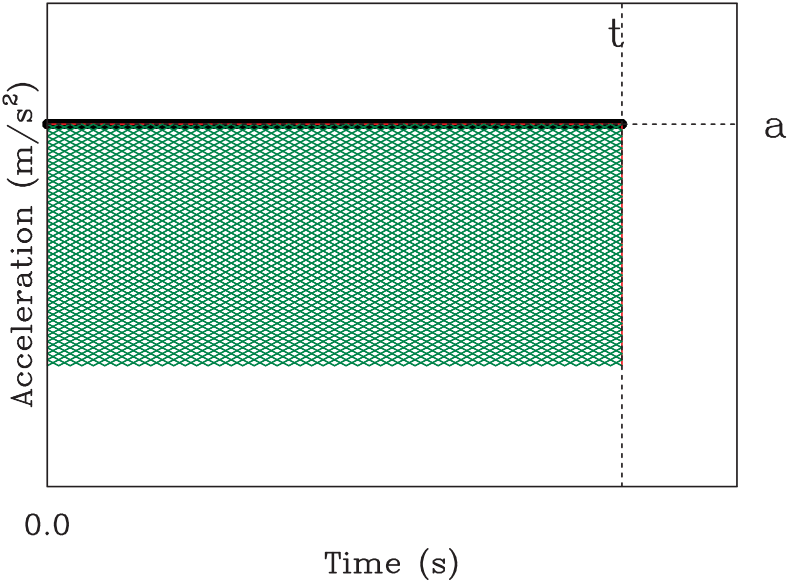
- Figure out the area under the velocity curve to get the position curve

Remember the area under the curve of (velocity, acceleration) just gives the *change in* (position, velocity) – *i.e.* initial minus final.

We'll start by assuming x_0 and v_0 are zero.

Constant acceleration



What's the area under the curve out to time t, which gives the change in the velocity – $\Delta v = v(t) - v_0$?

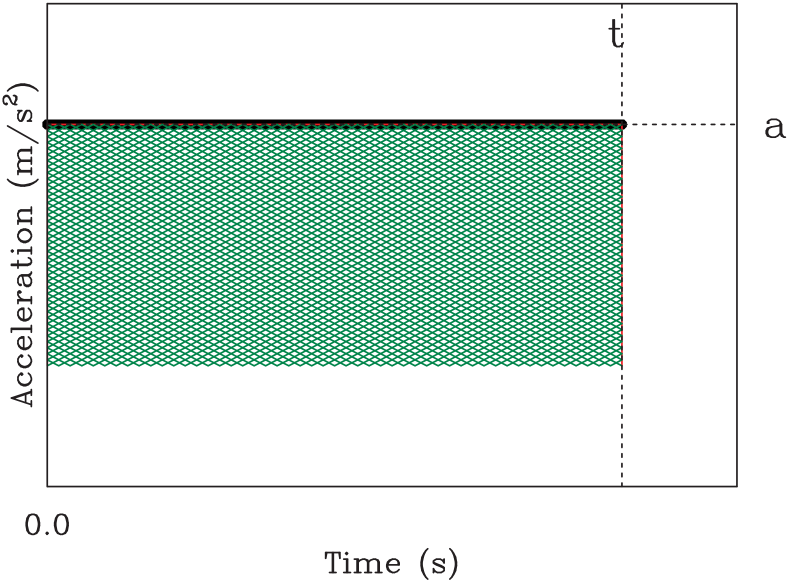
A: $\Delta v = at$

B: $\Delta v = at + v_0$

C: $\Delta v = \frac{1}{2}at^2$

D: $\Delta v = a$

Constant acceleration



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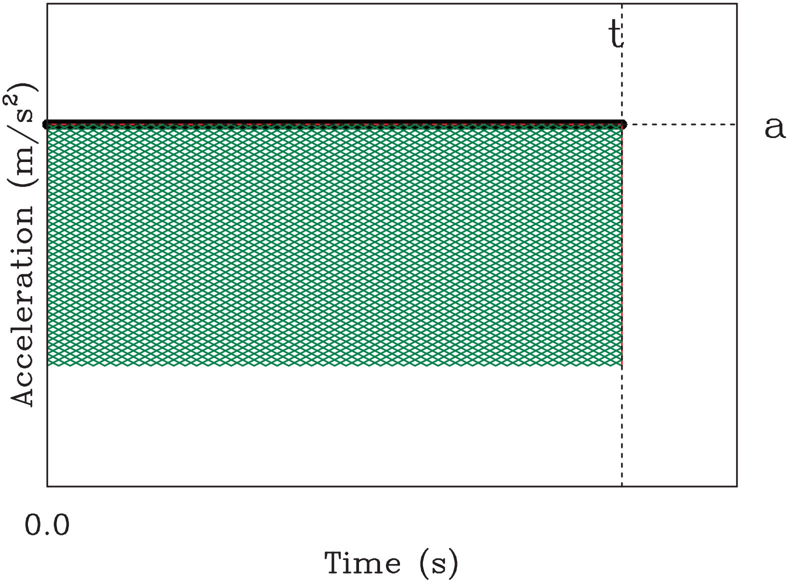
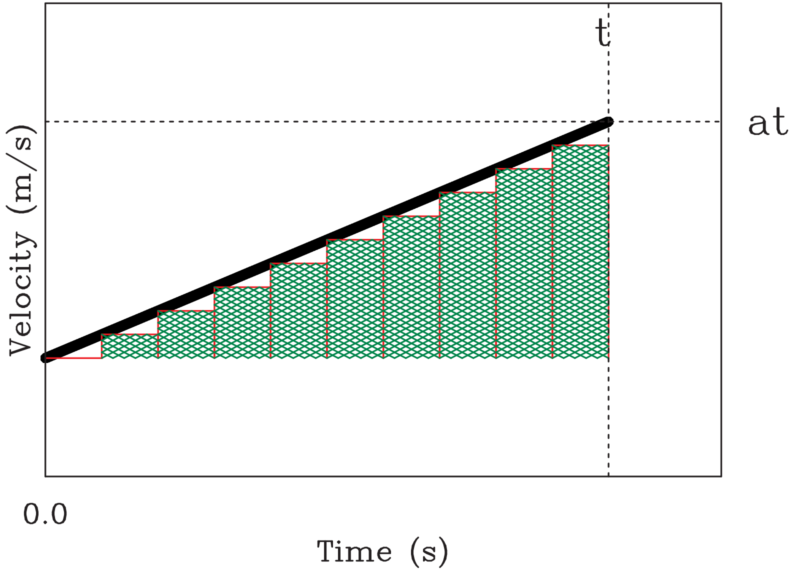


What's the area under the curve out to time t, which gives the change in the velocity – $\Delta v = v(t) - v_0$?

Δv, the change in velocity, is $v(t) - v_0 = at$, so $v(t) = at + v_0$

Same thing again to get position



Now the area under the velocity curve gives the change in position: $\Delta x = x(t) - x_0$. What is that?

A: $\Delta x = at$

B: $\Delta x = vt$

C: $\Delta x = \frac{1}{2}at^2$

D: $\Delta x = v$

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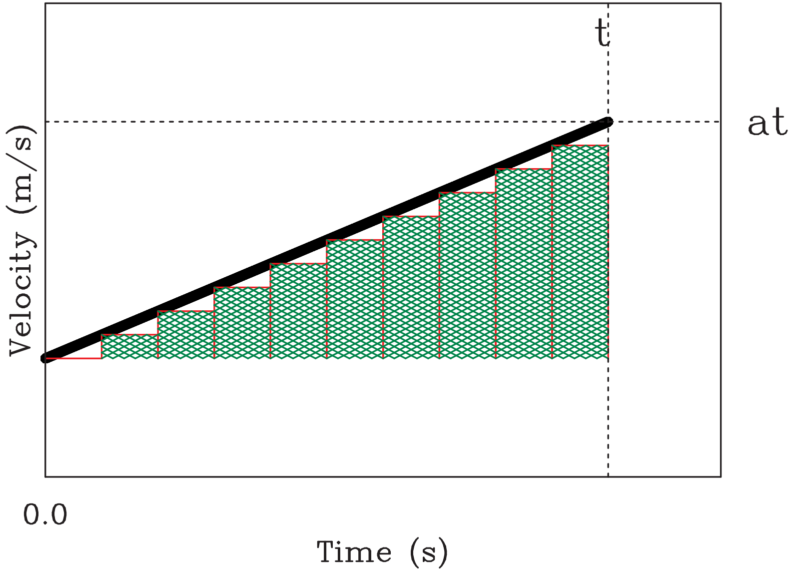

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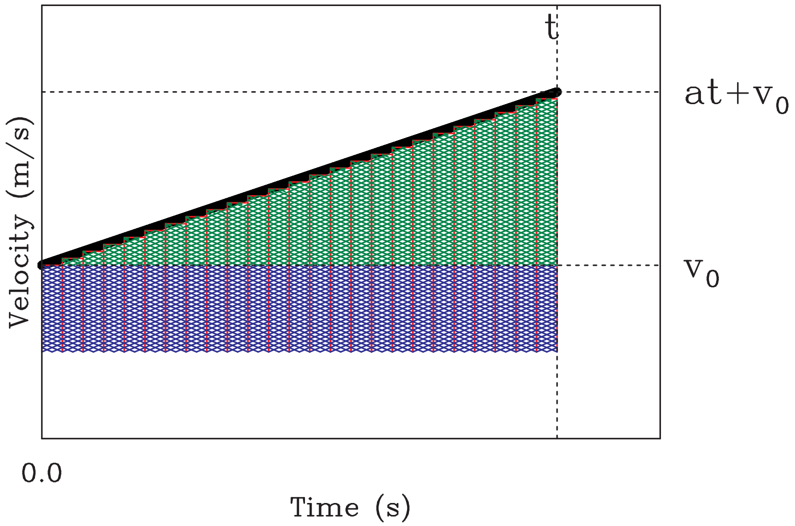
Now if v_0 is not zero...



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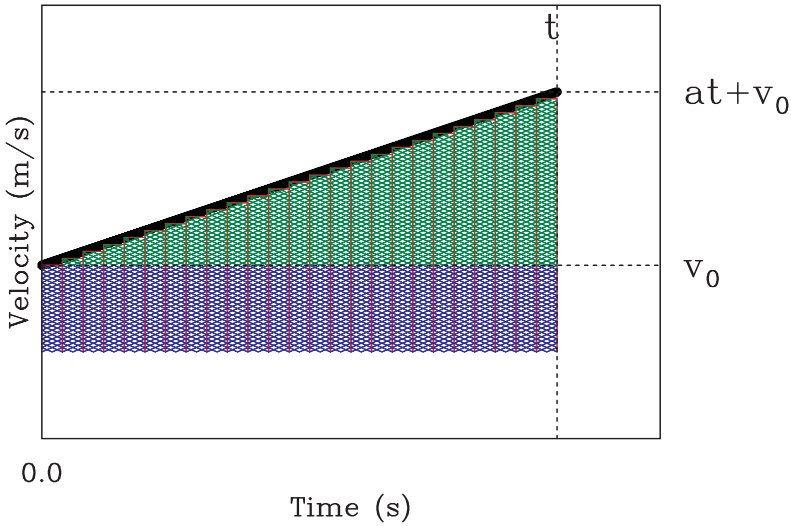


Area under blue part: $v_0 t$
Area under green part: $\frac{1}{2}at^2$
Total change in position: $x(t) - x_0 = \frac{1}{2}at^2 + v_0 t$

Thus, $x(t) = \frac{1}{2}at^2 + v_0 t + x_0$

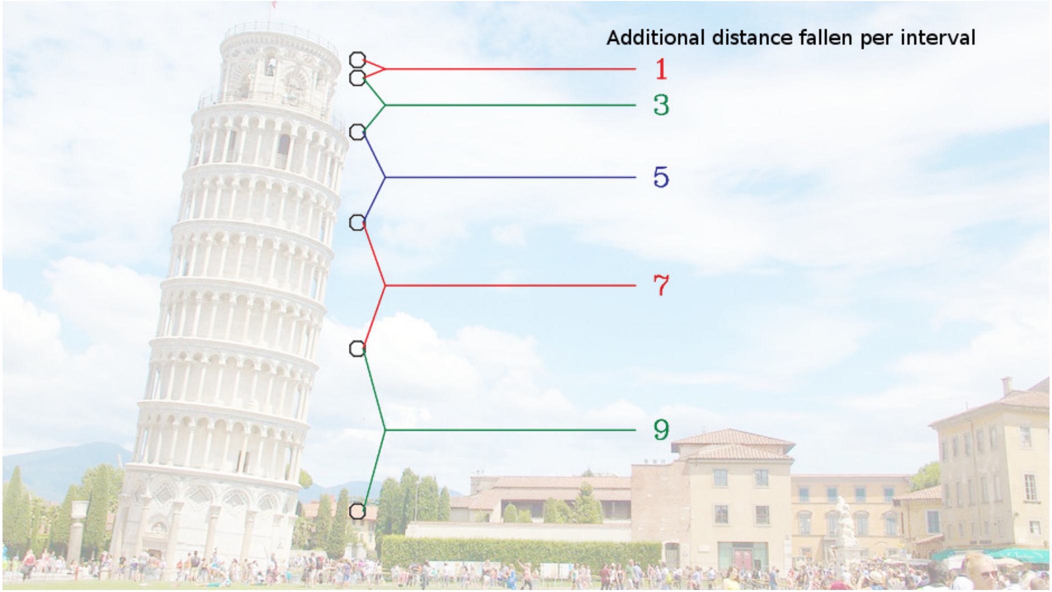
For those who are familiar with calculus:

$$\begin{aligned}
a(t) &= \text{const.} \\
v(t) &= \int a\,dt && = at + C_1 \\
x(t) &= \int v\,dt = \int (at + C_1)dt && = \frac{1}{2}at^2 + C_1 t + C_2
\end{aligned}$$

A little thought reveals that C_1 is the initial velocity v_0 and C_2 is the initial position x_0. This gives us the things we just derived, but much more easily:

$$\begin{aligned}
v(t) &= && at + v_0 \\
x(t) &= && \frac{1}{2}at^2 + v_0 t + x_0
\end{aligned}$$

Free fall revisited



Adding these numbers together gives us 1, 4, 9, 16, 25...
The calculus above explains this: distance is proportional to *time squared!*