Recitation Questions

20 February

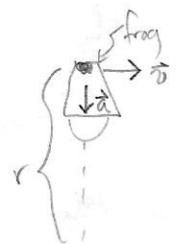
Question 1: frogs in a bucket

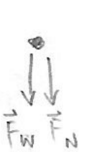
You have a collection of standard-issue Physics 211 bullfrogs of mass 500 grams each in a bucket.[1] You spin this bucket at arm's length in a vertical circle. (You'll need to estimate the radius of the circle.)

 Let's say $r = 0.5\,m$

a) At what angular velocity (ω) must you spin the bucket so that the frogs don't fall out at the top of the circle?

← this tells us the frogs are still in circular motion





the normal force can only push from the surface of the bucket to the frog, so for this bucket position, $\vec{F}_N$ must point down

we have centripetal acceleration when an object travels in circular motion at a constant speed

$$a_c = \omega^2 r, \quad \omega = \sqrt{\frac{a_c}{r}} = \sqrt{\frac{9.8\,m/s^2}{0.5\,m}} \approx \boxed{4.4\ rad/s}$$

if $a_c = g$, only gravity is needed to keep the frogs in circular motion (no normal force from the bucket is needed). This therefore gives the minimum ω to keep the frogs in circular motion in the bucket.

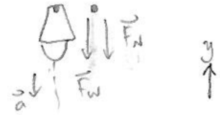
b) At the top of the circle, is there an upward force that holds the frogs in the bucket so they don't fall out? If so, what is that force? If not, why don't they fall out even though the only forces on them point downward? You don't have to write anything down here, but call your coach/TA over and have them join your conversation.

There is no upward force (see force diagram above).

If the velocity v of the frog is high enough, it will not have time to fall out of the bucket while the bucket is upside down.

[1] We found them in the basement of Illick Hall. They got this fat by eating all the other critters running around there.

at the top of the bucket's path, $\vec{F}_W$ and $\vec{F}_N$ combine to result in the downwards acceleration



at the bottom of the bucket's path, $\vec{F}_N$ has to overcome $\vec{F}_W$ and have extra left over to accelerate the bucket upwards.

c) Suppose that the bucket is a bit rusty, and will break if its bottom must support more than 30 newtons. How many frogs can you do this with before the bottom falls out of the bucket? (Use the same angular velocity as you calculated in a).)

Now we need to consider forces.

As shown above, the normal force on the frogs will be greatest when the bucket is at the bottom of its path. The normal force on the frogs has the same magnitude as the force of the frogs on the bucket by Newton's 3rd Law. Let's see what is true when this magnitude F_N equals the maximum of 30 N:

$$\sum \vec{F} = m\vec{a} \rightarrow \sum F_y = ma_y$$

$a_y = r\omega^2$ upwards, since the frogs are in circular motion

$$F_N - mg = m(r\omega^2)$$

F_N is now given

r and ω came from part a)

If we want to know how many frogs, we need to find their mass m.

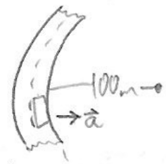
Collecting on m: $F_N = m(r\omega^2 + g)$,

$$m = \frac{F_N}{r\omega^2 + g} = \frac{F_N}{2g} = \frac{30\text{ N}}{2(9.8\text{ m/s}^2)} = 1.53\text{ kg}.$$

recall that we chose ω so that $r\omega^2 = g$

Each frog weighs 0.5 kg, so

$$1.53\text{ kg} \cdot \frac{1\text{ frog}}{0.5\text{ kg}} = \boxed{3 \text{ frogs}}$$ (rounding down)



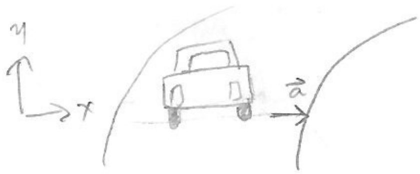
Question 2: a curvy road

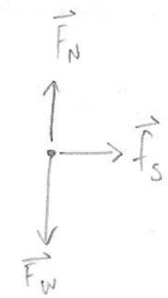
Suppose that a car is driving around a flat highway curve with a radius of curvature of $r = 100$ meters (that is, it is a segment of a circle whose radius is 100 m), and that the coefficient of friction between the car's wheels and the pavement is $\mu_s = 0.8$.

a) What force is responsible for the centripetal acceleration of the car, bringing it around the curve?

The force needs to point along the ground towards the center of the curve. It must be $\boxed{\text{friction}}$, since without friction, the car would slide in a straight line on flat ground.

b) Draw a force diagram for the car. It is most convenient to draw the forces as seen from the rear/front, not the top/bottom.





static friction (since the tires don't slip on the asphalt)

we know we can consider the maximum amount of friction, $f_{s(max)} = \mu_s F_N$

c) What is the fastest that the car can drive around the curve? How would this change if the highway was covered in snow with $\mu_s = 0.2$?

From the force diagram, we see that $F_N = F_W = mg$, since there is no acceleration (net force) in the y-direction.

In the x-direction, $\Sigma F_x = ma_x$ we know $a_x = a_c = \frac{v^2}{r}$ because the car is driving in a circle

$f_s = m\frac{v^2}{r}$

Now $f_s = \mu_s F_N = \mu_s mg$, so $\mu_s \cancel{m} g = \cancel{m}\frac{v^2}{r}$, $v = \sqrt{\mu_s r g} = \sqrt{(0.8)(100\,m)(9.8\,m/s^2)}$
$= \boxed{28\ m/s}$.

For $\mu_s = 0.2$, $v = \sqrt{\mu_s r g} = \sqrt{(0.2)(100\,m)(9.8\,m/s^2)} = \boxed{14\ m/s}$

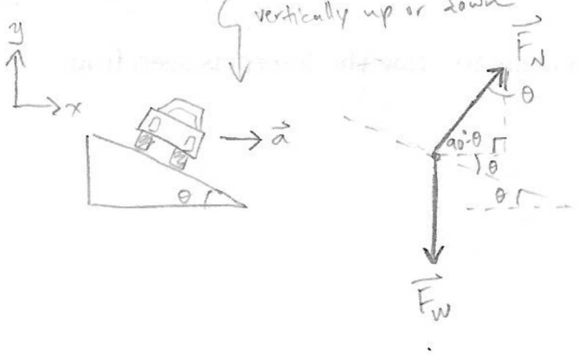
Question 3: a banked, curvy road

As you know, highway curves are "banked" inward, so that gravity assists the car's traction in carrying it around the curve. Suppose another highway curve has a radius of curvature of 500 meters. It is banked so that traffic moving at 30 m/s can travel around the curve without needing any help from friction.

this means the ground is not flat, but sloped at an angle

a) Draw a force diagram for a car traveling around this curve at a constant speed. Draw the diagram so that you are looking at the rear of the car. Hint: Do not tilt your coordinate axes for this problem: you want them to be aligned with the acceleration vector, which is horizontally inward.

$\vec{a}$ must be horizontal because the velocity never changes to be vertically up or down



Since the normal force is diagonal (because of the banked road), there is a horizontal component of force to give the horizontal acceleration the car needs to travel in a circle

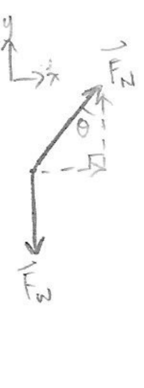
b) What is the acceleration of the car in the x-direction? What about the y-direction?

With the axes I've chosen, $a_x = a_c = \frac{v^2}{r}$, since the car is travelling in circular motion, and

radius of curvature of highway

$a_y = 0$, since it isn't accelerating upwards or downwards.

c) Write down two copies of Newton's second law in the $x-$ and $y-$directions.



$\Sigma F_x = ma_x$

$F_N \sin\theta = m\frac{v^2}{r}$

$\Sigma F_y = ma_y$

$F_N \cos\theta - mg = 0$

d) Solve the resulting system of two equations to determine the banking angle of the curve.

From $\sum F_y = ma_y$, $F_N = \frac{mg}{\cos\theta}$. Plugging this into $\sum F_x = ma_x$,

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = m\frac{v^2}{r}$$

$$= \cancel{m}g\tan\theta = \cancel{m}\frac{v^2}{r}, \quad \text{so } \tan\theta = \frac{v^2}{rg}, \text{ and}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{(30\,m/s)^2}{(500\,m)(9.8\,m/s^2)}\right)$$

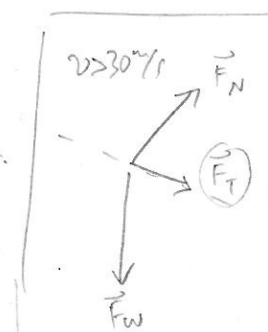
$$\approx \boxed{10.4^\circ}$$

e) If the car is driving faster than 30 m/s, which way will traction point on your force diagram? What if it is driving slower than 30 m/s?

↑ the force of static friction keeping the car from sliding up or down the road

As velocity increases, $a_c = \frac{v^2}{r}$ also increases, and more force towards the center of curvature of the road is necessary to keep the car travelling in circular motion.

Therefore, for $v > 30\,m/s$, we need extra force inwards:



For $v < 30\,m/s$, we need less force inwards, but F_N is always the same. That means traction has to counteract part of the x-component of $\vec{F}_N$: