Rotational motion 2

Physics 211
Syracuse University, Physics 211 Spring 2015
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April 2, 2015

Announcements

- Your next homework assignment will be posted tonight or tomorrow, and will be due next Friday
- You will also have a practice exam, posted tonight or tomorrow
- Solutions will be posted next Friday
- Exam 3 will be April 13
- Next Mastering Physics assignment has been posted and is due Tuesday before class

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  - Moment of inertia, like mass, defined with reference to a particular axis
  - $I = MR^2$ (if all of the mass in the rotating object is the same distance R from the axis
  - $I = \lambda MR^2$ (for more complicated shapes)

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- Rotational kinetic energy is what you'd expect: $KE_{\text{rot}} = \frac{1}{2}I\omega^2$
  - This is just another form of energy, conserved in the same way and "alongside" the things you already learned about
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- The analogue of momentum is *angular momentum* $L = I\omega$
  - This is a different quantity than ordinary momentum, but is also conserved if there are no external torques

The correspondence table

Translation	Rotation
Position x	Angle θ
Velocity v	Angular velocity ω
Acceleration a	Angular acceleration α
$v(t) = v_0 + at$	$\omega(t) = \omega_0 + \alpha t$
$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v_f^2 - v_0^2 = 2a\Delta x$	$\omega_f^2 - \omega_0^2 = 2\alpha\Delta\theta$
Force $\vec{F}$	Torque: $\tau = F_\perp r$
Mass m	Moment of Inertia: $I = \lambda M R^2$
$\vec{F} = m\vec{a}$	$\tau = I\alpha$
Work $= \vec{F} \cdot \Delta\vec{s}$	Work $= \tau\Delta\theta$
Kinetic energy $\frac{1}{2}mv^2$	Kinetic energy $\frac{1}{2}I\omega^2$
Momentum $\vec{p} = m\vec{v}$	Angular momentum $L = I\omega$

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- For purposes of torque the gravitational force acts at the center of mass
- For symmetric, uniform objects: the center of mass is at the center!
- For other objects, it's just a weighted average of all of the parts:

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \ldots}{m_1 + m_2 + m_3}$$

$$y_{\text{com}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \ldots}{m_1 + m_2 + m_3}$$

Static equilibrium problems

- Often we are presented with a situation where nothing moves, and we have to solve for something
- No acceleration of the center of mass: $\sum \vec{F} = 0$
- No angular acceleration: $\sum \tau = 0$ about *any* pivot point
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- Can generate enough equations this way to solve for all unknowns
- Strategy: choose the pivot to be aligned with a force you don't know and don't care about

A sample problem: forces on a bar

A bar is suspended by a hinge at one end, and by a cable at the other making an angle θ with the vertical. If the bar has mass m, what is the tension in the cable?

(Illustrated with demo equipment; solution on document camera)

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What is the (vector) force at the hinge?

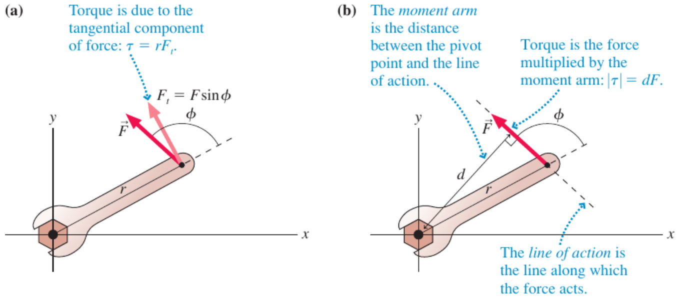
One more thing: another way of computing torque

Last time we saw that the torque $\tau = F_\perp r$

... but there's another way to compute it which is sometimes more helpful

- Note that $F_\perp = F \sin\theta$, so $\tau = Fr\sin\theta$
- We can think of the torque in any other equivalent way; there is another one that's often useful
- The way I taught you yesterday: **"The radius vector, times the component of force perpendicular to it"**
- The alternative: **"The force vector, times the component of the radius perpendicular to it"**

Here's the figure from the text:



I'll draw a clearer version on the document camera

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NB: Your book calls $r_\perp$ the "moment arm"

A sample problem: distribution of weight in a truck

A 2000 kg rear wheel drive pickup truck has a wheelbase 5 meters long. Its center of mass is located 150 cm behind the front wheels, and 130 cm above the ground. What is the fastest it can accelerate, if $\mu_s = 0.6$?

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Compute torque about the front wheels:

$$\begin{aligned} 1.5mg &= 5F_{N,\text{rear}} \\ F_{N,\text{rear}} &= 0.3mg \\ F_{f,\text{max}} &= \mu_s F_{N,\text{rear}} = 0.18mg \\ a_{\text{max}} &= 0.18g \end{aligned}$$

A few nifty demos on angular momentum

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- Dumbbell demo: if I goes down, ω goes up
- Bike wheel demo: $\sum L =$ constant...
- We learned about rotation in 2D, where ω, L, and τ are scalars
- In 3D they're vectors, and the behavior can be downright weird!

Finding the balancing point

A bar of mass 5 kg and length 1 m has a weight of mass 1 kg resting on one end. At what point must it be supported if it is to balance?

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Strategy:

- Draw a “number line” by your force diagram and label coordinates
- Introduce a variable corresponding to the thing you’re trying to find
- Can compute torques about *any* pivot; which is easiest?

Finding the balancing point

A bar of mass m and length L has a weight of mass M resting on one end. At what point must it be supported if it is to balance?

Finding the balancing point

A bar of mass m and length L has a weight of mass M resting on one end. At what point must it be supported if it is to balance?

Same method as before, but note:

- The point of support is also the center of mass
- The converse is also true: an object's center of mass must be within its base of support if it is to stay put