Vectors and 2D kinematics

Physics 211
Syracuse University, Physics 211 Spring 2015
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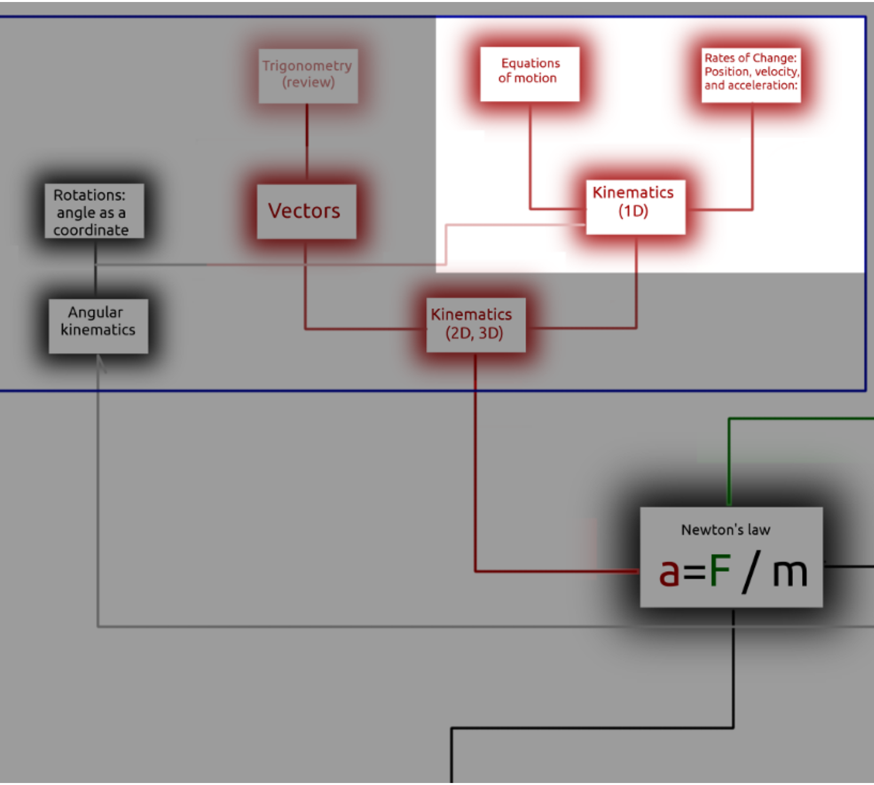
January 20, 2015

Announcements

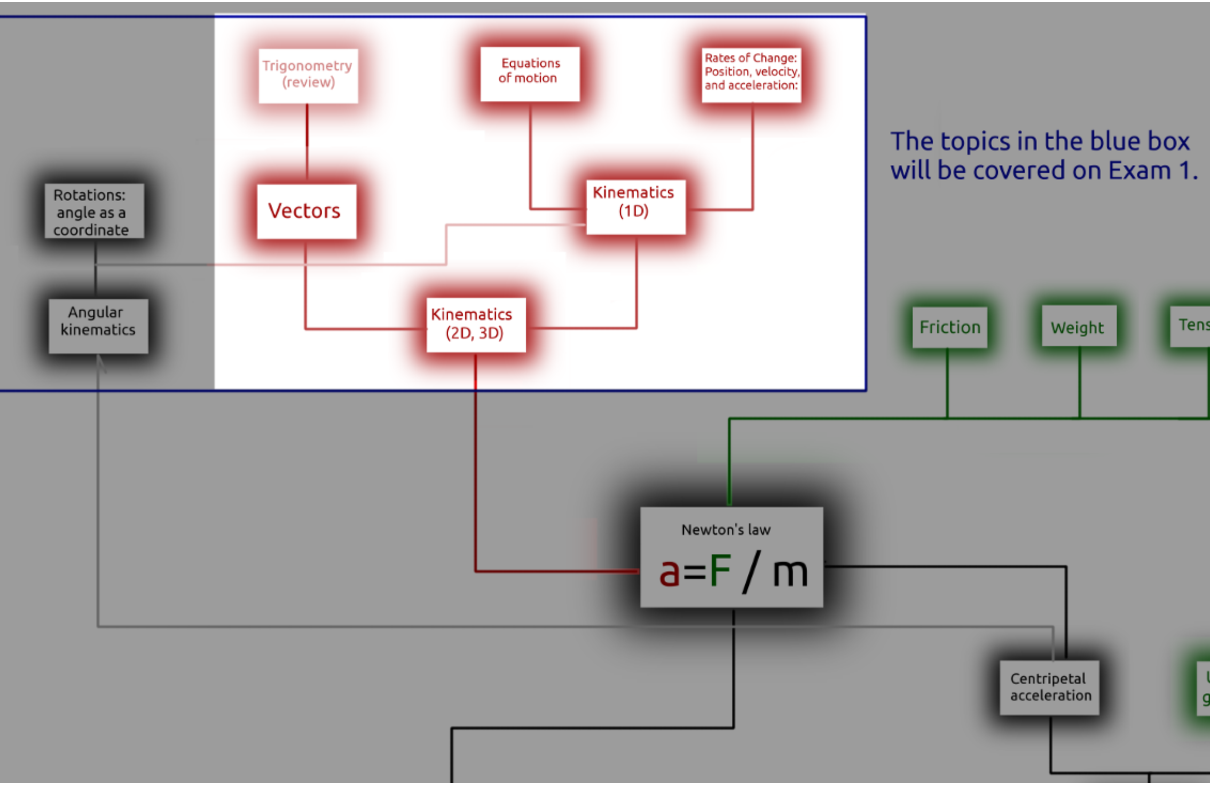
- Homework 1 is due tomorrow
- Homework 2 is posted
- The *Mastering Physics* code is `MPFREEMAN29087`; remember you have your first assignment due Thursday (it's really just a tutorial)
  - The first *Mastering Physics* assignment, really just a tutorial, is due next Thursday
  - I would like one volunteer with the ResponseWare app and one volunteer with a physical clicker to help me test the system
  - The Facebook page is set up: see https://www.facebook.com/groups/384100861768360/
    - Up to 2% extra credit for participation here and on the wiki in helping your peers understand things
- Reminders:
  - Course website: (updated frequently!)
  - Teaching team contact information:
    - Prof. Walter Freeman: wafreema@syr.edu
    - Bithika Jain: bjain@syr.edu
    - Lab questions: sasemper@syr.edu

Course map



The topics in the blue box will be covered on Exam 1.

Course map



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A reminder of what we did last time

$$\text{Position} \quad \underset{\substack{\text{area under the curve of} \\ \text{(integral of)}}}{\overset{\substack{\text{(derivative of)} \\ \text{rate of change of}}}{\leftrightarrows}} \quad \text{Velocity}$$

A reminder of what we did last time

$$\text{Position} \;\underset{\substack{\text{area under the curve of} \\ \text{(integral of)}}}{\overset{\substack{\text{(derivative of)} \\ \text{rate of change of}}}{\rightleftarrows}}\; \text{Velocity} \;\underset{\substack{\text{area under the curve of} \\ \text{(integral of)}}}{\overset{\substack{\text{(derivative of)} \\ \text{rate of change of}}}{\rightleftarrows}}\; \text{Acceleration}$$

A reminder of what we did last time

- We could do all the calculus we needed by simple geometry
- 1D kinematics relations:

$$v(t) = at + v_0$$

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0$$

- These apply only for *constant acceleration*
- If you have more complicated things, you'll need to take integrals and derivatives
- All you'll need:
  - Derivative of Ax^n is Anx^{n-1}
  - Integral of Ax^n is $\frac{A}{n+1}x^{n+1} + C$ (typo corrected)

Vectors

You've been doing math with numbers, which are things that live in one dimension: they only have a magnitude and a sign.

Vectors are things that have a magnitude and a direction: "arrows in space"

Many of the things we deal with in physics are vectors:

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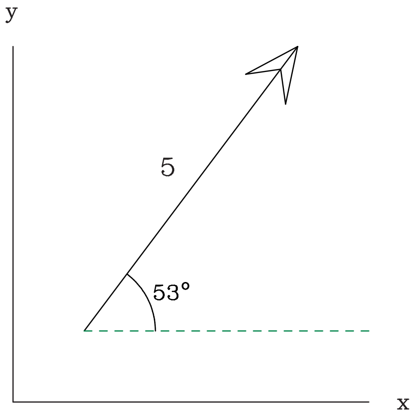
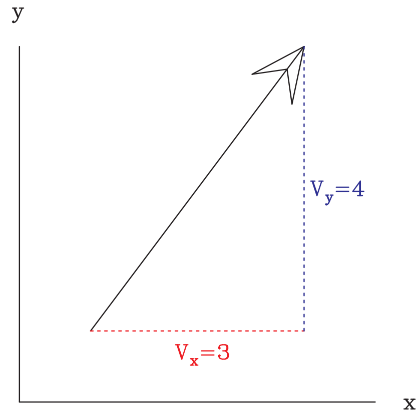
So, we need to learn to do math with arrows.

- We indicate that an object is a vector by writing an arrow over it: "the vector $\vec{V}$".
- "Scalar": object that isn't a vector (mass, time)
- Equations can mix vectors and scalars: $\vec{F} = m\vec{a}$.

Two ways to describe a vector



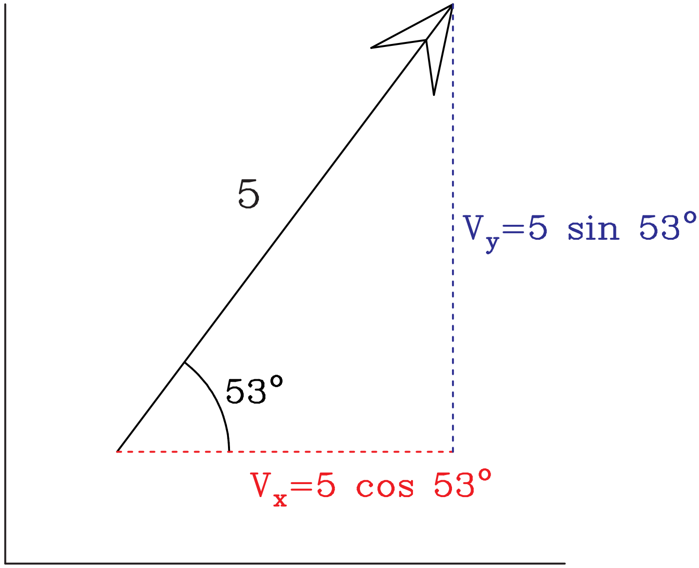
Angle and direction



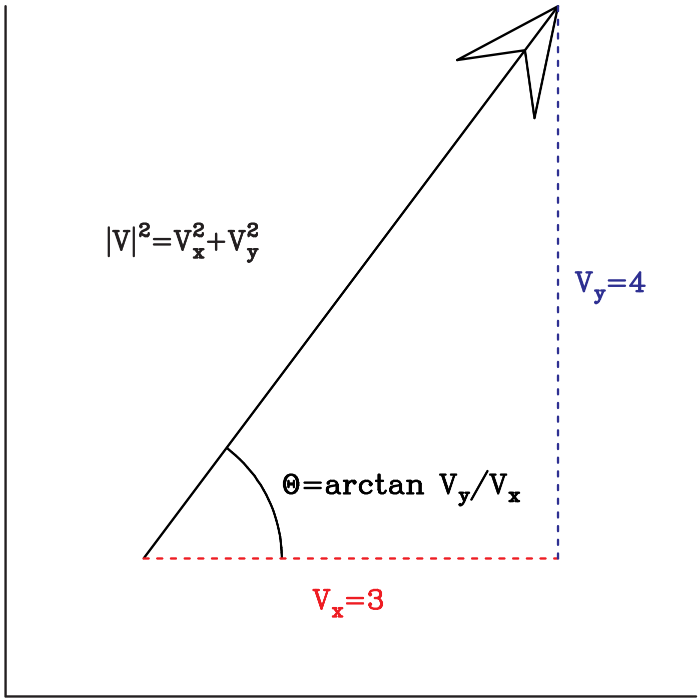
X and Y components

How do we convert from one to the other?

From "direction and magnitude" to components



From components to direction and magnitude



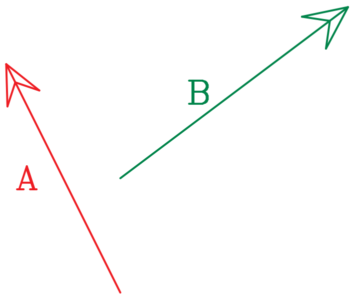
A warning!

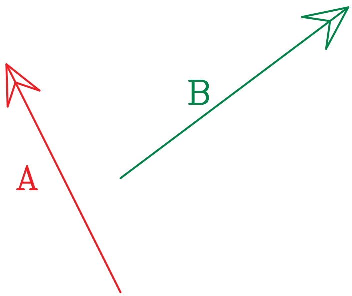
You cannot memorize "$V \sin\theta$ is the y component,
$V \cos\theta$ is the x component"!

This does *not* work in general; you have to actually draw the triangle.

Adding vectors

We can also add vectors together by drawing them "head to tail". Here are two vectors:





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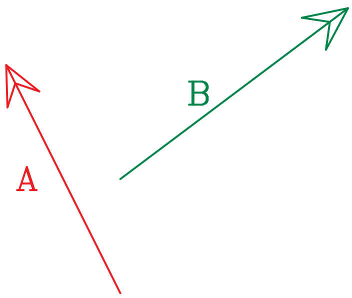
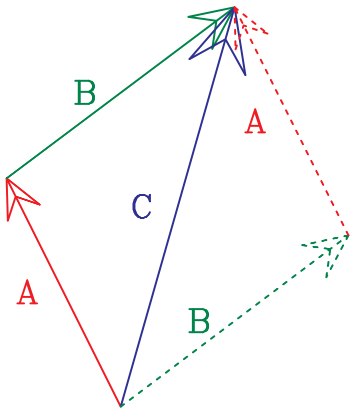




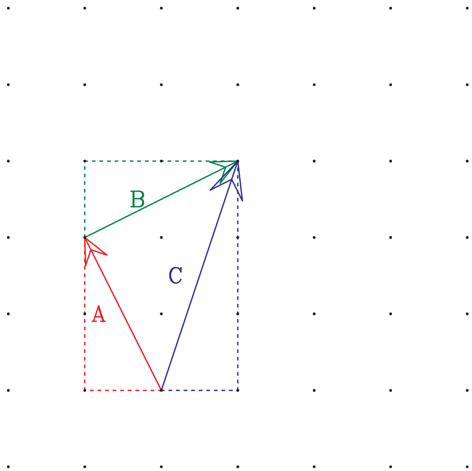
$$\vec{A} + \vec{B} = \vec{C}$$

Adding vectors: components

The component representation is much easier to work with!

$$\vec{A} + \vec{B} = \vec{C} \rightarrow \begin{pmatrix} A_x + B_x = C_x \\ A_y + B_y = C_y \end{pmatrix}$$

Adding vectors: components



To add two vectors, just add their components!

This is why it is almost always easiest to work in the component representation!

What does this do to our kinematics?

Acceleration, velocity, and position relationships are still the same; they just apply independently for each component.

$$\vec{v}(t) = \vec{a}t + \vec{v}_0$$
$$\vec{r}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{r}_0$$

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$$\vec{x}(t) = \frac{1}{2}a_x t^2 + v_{x,0} t + x_0$$
$$\vec{y}(t) = \frac{1}{2}a_y t^2 + v_{y,0} t + y_0$$

Problem solving: 2D kinematics, constant acceleration

1. 1. If you have vectors in the "angle and magnitude" form, convert them to components
2. 2. Write down the kinematics relations, separately for x and y
   - Many terms will usually be zero
   - Freefall: $a_x = 0$, $a_y = -g$ (with conventional choice of axes)
3. 3. Understand what instant in time you want to know about
4. 4. Put in what you know; solve for what you don't (using substitution, if necessary)
5. 5. Convert vectors into whatever format the problem asks for

Sample problems

A rock is thrown at $10m/s$ at 30° above the horizontal.

- How far from its starting point is it after 2 seconds?

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- How far from its starting point is it after 2 seconds?
- How far does it travel?
- How high does it go?
- What will its speed be when it strikes the ground?