Physics Practice

Quadratic Formula and Its Application

Throughout, the letters A, B, and C refer to the three coefficients in the quadratic equation $At^2 + Bt + C = 0$, which has solutions

$$t = \frac{-b \pm \sqrt{B^2 - 4AC}}{2A}$$

Problem 1:

Draw five graphs below, as follows:

1. One in which $A > 0$
2. One in which $A < 0$
3. One where t has two real, positive solutions
4. One where t has one positive solution and one negative solution
5. One where the only solutions to t are imaginary.

Problem 2:

A bucket is being lowered into a well at a constant speed of 2 m/s. When the bucket is 10 meters below the top of the well, a physics student drops a rock into it from the top of the well. However, the bucket is a bit rusty, and might break if the rock hits it too hard. How fast will the rock be going when it lands in the bucket?

Take the following steps:

- Write position equations $x(t)$ for both objects
- Graph these equations, so you have a pictoral representation of what is going on
- Write a sentence that will let you find the solution (of the form of the others we've been writing)
- Do the algebra and write down the quadratic formula
- Do anything else you need to do to find the velocity

Problem 3:

In the previous problem, you used the quadratic formula to find the point of intersection between a parabola and a line. In the next problem, you'll use the quadratic formula to find out where two parabolas intersect.

However, your math teacher told you that the quadratic formula tells you the roots of a parabola, i.e. where it intersects the line $x = 0$. After all, it only solves equations of the form $At^2 + Bt + C = 0$.

How do you reconcile these two claims? How do we use the quadratic formula to find out where parabolas intersect things other than the line $x = 0$?

Problem 4:

Suppose I have two objects which move as follows:

1. Initial velocity v_0, initial position 0, acceleration $-g$
2. Initial velocity 0, initial position h, acceleration α (positive)

For instance, Object 1 is a flea jumping in a box which is accelerating upward (v_0 is the velocity with which the flea can jump; it is in freefall after its feet leave the ground) and Object 2 is the top of the box.

Can the flea jump high enough to get to the top of the box?

Take the following steps:

- Write position equations $x(t)$ for both objects
- Graph these equations, so you have a pictoral representation of what is going on
- Write a sentence that will let you find the solution: how do you say "flea reaches top of box" in terms of our variables?
- Do the algebra and write down the quadratic formula
- How do you interpret the solutions to the quadratic formula?

Problem 5:

In your groups, write down a problem that must be solved with the quadratic formula, and whose solution is either:

1. The "plus" root of the quadratic formula
2. The "minus" root of the quadratic formula
3. Both roots
4. ... determined by looking at the sign of the determinant

Then, when everyone has written their problems, swap with another group and solve theirs!