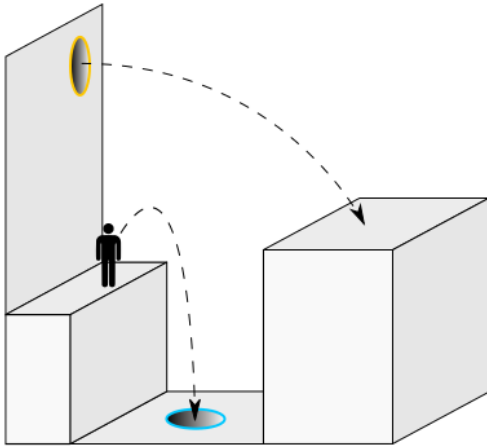


# HOMEWORK 7

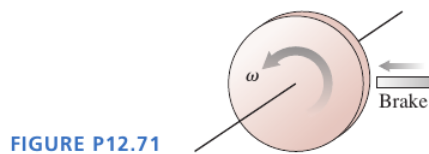
DUE WEDNESDAY, 12 APRIL

1. In the classic computer game *Portal*, the player is asked to solve puzzles with the use of a “portal device”, which can create two connected portals on, for example, the floor and a wall. An object entering one portal with speed  $v$  will exit the other with the same speed. Here’s an illustration:



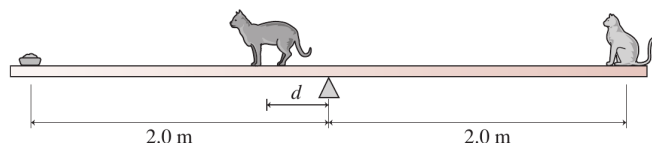
The game’s narrator helpfully explains that “Forward momentum, a product of mass and velocity, is conserved between portals. In layman’s terms: **speedy thing goes in; speedy thing comes out.**”

- (a) Is this an accurate statement of the law of conservation of momentum? As the narrator claims, does such a device conserve momentum? If not, why not?
  - (b) Does this device conserve kinetic energy?
  - (c) Does it conserve total energy (that is, kinetic energy  $\frac{1}{2}mv^2$  plus gravitational potential energy  $mgy$ )?
2. A bowling ball (solid sphere;  $I = \frac{2}{5}mr^2$ ) rolls down a hill of height 5 meters. How fast is it traveling when it reaches the bottom?
  3. Knight 12.71 (3rd edition), reproduced below.
    71. || The 2.0 kg, 30-cm-diameter disk in **FIGURE P12.71** is spinning at 300 rpm. How much friction force must the brake apply to the rim to bring the disk to a halt in 3.0 s?



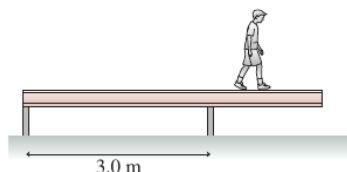
4. Knight 12.31 (3rd edition), reproduced below.

31. || A 5.0 kg cat and a 2.0 kg bowl of tuna fish are at opposite ends of the 4.0-m-long seesaw of **FIGURE EX12.31**. How far to the left of the pivot must a 4.0 kg cat stand to keep the seesaw balanced?



5. Knight 12.63 (3rd edition), reproduced below. Note that normal forces can only push, not pull...

63. || A 40 kg, 5.0-m-long beam is supported by, but not attached to, the two posts in **FIGURE P12.63**. A 20 kg boy starts walking along the beam. How close can he get to the right end of the beam without it falling over?



**FIGURE P12.63**

6. A bicyclist is riding a bicycle at a constant speed of 7 m/s. She has to pedal to overcome a drag force of 40 N. The bicycle's rear tire has a radius of 40 cm. There are two rear sprockets: one with a radius of 5 cm ("high gear"), and one with a radius of 10 cm ("low gear"). The front sprocket has a radius of 10 cm. Note that the chain connecting the sprockets moves at a constant speed and thus has  $a = 0$ , so the force it exerts on the front sprocket is the same as the force it exerts on the rear sprocket. *Note: The rotational equivalent to the power-velocity relation  $P = \vec{F} \cdot \vec{v}$  is exactly what you think it would be:  $P = \tau\omega$ .*

- When in high gear, how much torque must she deliver to the pedals to maintain her speed? What is her power output when doing so?
- When in low gear, how much torque must she deliver to the pedals to maintain her speed? What is her power output when doing so?
- Do your answers match your experience with riding a bicycle?

7. (*This problem is worth ten points extra credit if you complete it fully.*)

A cylinder ( $I = \frac{1}{2}mr^2$ ) rolls without slipping down an inclined plane of length  $L$ , angled at an angle  $\theta$  to the horizontal. The coefficient of static friction between them is  $\mu_s = 0.5$ .

- (a) Draw a (large) extended force diagram for the cylinder. Consider carefully in which direction the frictional force at the point of contact points.
- (b) Is the frictional force equal to  $\mu_s F_N$ , or some other value that depends on the angle  $\theta$ ? Explain, thinking about what happens as  $\theta \rightarrow 0$ . (If the frictional force is not equal to  $\mu_s F_N$ , just use  $F_f$  to represent it in equations.)
- (c) The ordinary “translational” work-energy theorem says that a force  $\vec{F}$  acting over a displacement  $\vec{d}$  causes a change in translational kinetic energy:  $\Delta(\frac{1}{2}mv^2) = \vec{F} \cdot \vec{d}$ . Find the “translational work” done by the gravitational force and by the frictional force.
- (d) The “rotational” work-energy theorem says that a torque  $\tau$  acting over an object rotating through an angle  $\Delta\theta$  causes a change in rotational kinetic energy:  $\Delta(\frac{1}{2}I\omega^2) = \tau\Delta\theta$ . Find the “rotational work” done by the gravitational force and by the frictional force.
- (e) If we consider “translational kinetic energy”  $\frac{1}{2}mv^2$  and “rotational kinetic energy”  $\frac{1}{2}I\omega^2$  as the same thing, what is the total work (rotational plus translational) done by the frictional force?
- (f) Is it accurate to say that the frictional force here converts translational kinetic energy into rotational kinetic energy? Explain.
- (g) Comment on the validity of using energy methods to solve problems like (2). Do you need to worry about the frictional forces when considering translational and rotational kinetic energy together?
- (h) By any method you like, calculate the size of the frictional force  $F_f$ . (There are two ways to do this. One involves energy methods; the other involves examining the force diagram and thinking about the relation between  $a$  and  $\alpha$ .)