1D kinematics: solving problems

Physics 211
Syracuse University, Physics 211 Spring 2016
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January 26, 2016

On solving problems

You can recognize truth by its beauty and simplicity. When you get it right, it is obvious that it is right–at least if you have any experience–because usually what happens is that more comes out than goes in.... [i]nexperienced students make guesses that are very complicated, [but] the truth always turns out to be simpler than you thought.

–Richard Feynman, quoted by K. C. Cole, in *Sympathetic Vibrations: Reflections on Physics as a Way of Life (1985)*

Nature uses only the longest threads to weave her patterns, so each small piece of her fabric reveals the organization of the entire tapestry.

–Richard Feynman, *The Character of Physical Law* (1965)

Announcements

- Homework 1 due tomorrow – note the Homework Guidebook link on the website
- Clickers introduced in class today
- I'm seeing lots of you in the Clinic; I'd like to see lots more
- **Extended clinic hours:** today from 3:30-6:30 and perhaps from 1:15-2
- I'm hiring extra coaches to work in the Clinic tomorrow afternoon

Did someone lose a piece of jewelry? Tell me if you did...

Clicker registration

- We're starting clicker questions today in class
- This is a new thing for me – it's a learning experience for us both!
- Clicker registration instructions on the course website
- You don't need to register until next week

“Ask a Physicist”

Submit questions!

Today

- Clicker introduction
- Review material from last time
- Rotational kinematics
- Concepts for problem solving
- The "2x2 framework"
- Sample problems

Today

- Clicker introduction
- Review material from last time
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- Sample problems
- Homework help?

Poll questions

How are you answering this question?

- A. A clicker
- B. A smartphone
- C. Semaphore flags

Poll questions

What's your experience with spreadsheet software?

- A. None
- B. I'm familiar with Google Sheets/Docs
- C. I'm familiar with Excel or LibreOffice, but not Google Sheets
- D. I've never used a spreadsheet, but know C/Python/Java/Pascal/etc.

Poll questions

What sort of computing hardware do you have, or could you borrow, for recitation periods next week?

- A. A laptop (including Chromebooks, netbooks, etc.
- B. A tablet (iPad, etc.)
- C. Neither (don't worry; we'll take care of you!)

Poll questions

What's your experience studying physics?

- A. None (that's okay!)
- B. A university/college physics course
- C. An AP or International Baccalaureate high school physics course
- D. Another high school physics course

Last time: Position, velocity, and acceleration

$$\text{Position} \; \underset{\text{area under the curve of (integral of)}}{\overset{\text{(derivative of) rate of change of}}{\rightleftarrows}} \; \text{Velocity}$$

Last time: Position, velocity, and acceleration

$$\text{Position} \;\underset{\substack{\text{area under the curve of}\\ \text{(integral of)}}}{\overset{\substack{\text{(derivative of)}\\ \text{rate of change of}}}{\rightleftarrows}}\; \text{Velocity} \;\underset{\substack{\text{area under the curve of}\\ \text{(integral of)}}}{\overset{\substack{\text{(derivative of)}\\ \text{rate of change of}}}{\rightleftarrows}}\; \text{Acceleration}$$

Position, velocity, and acceleration

Position $\xleftarrow{\text{rate of change of}}$ Velocity

(derivative of)

Position, velocity, and acceleration

Position $\xleftarrow[\text{rate of change of}]{\text{(derivative of)}}$ Velocity $\xleftarrow[\text{rate of change of}]{\text{(derivative of)}}$ Acceleration

Kinematics

- If we know acceleration as a function of time, how do we get from there to position vs. time?

Kinematics

- If we know acceleration as a function of time, how do we get from there to position vs. time?

- A. Look at the slope of the acceleration vs. time graph to get velocity, and then look at its slope to get position
- B. Look at the area under the curve of the acceleration vs. time graph to get velocity, and then look at the area under that graph to get position
- C. Take two derivatives of the acceleration vs. time graph to get position vs. time
- D. Take two integrals of the acceleration vs. time graph to get position vs. time

The "kinematics equations"

$$v(t) = at + v_0$$
$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

These equations are valid when...

- A. Acceleration is constant
- B. Velocity is constant
- C. The object moves in only one direction
- D. They are always valid

The "kinematics equations"

$$
\begin{aligned}
v(t) &= at + v_0 \\
x(t) &= \frac{1}{2}at^2 + v_0 t + x_0
\end{aligned}
$$

These equations are valid when...

- A. Acceleration is constant
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- A: these are the expressions for $x(t)$ and $v(t)$ when acceleration is constant!

- Linear motion: care about position as a function of time
- Rotational motion: care about angle as a function of time
- **Everything we just did translates to rotational kinematics exactly!**

Position, velocity, and acceleration

(derivative of)
rate of change of

Position $\xleftarrow{\hspace{6em}}$ Velocity
$\xrightarrow{\hspace{6em}}$

area under the curve of
(integral of)

Position, velocity, and acceleration

$$\begin{tikzcd}[column sep=huge]
\text{Position} \arrow[r, shift right, "\substack{\text{area under the curve of}\\\text{(integral of)}}"'] & \text{Velocity} \arrow[l, shift right, "\substack{\text{(derivative of)}\\\text{rate of change of}}"'] \arrow[r, shift right, "\substack{\text{area under the curve of}\\\text{(integral of)}}"'] & \text{Acceleration} \arrow[l, shift right, "\substack{\text{(derivative of)}\\\text{rate of change of}}"']
\end{tikzcd}$$

Angle, angular velocity, and angular acceleration

Angle

(derivative of)
rate of change of
$\longleftarrow$
$\longrightarrow$
area under the curve of
(integral of)

Angular velocity (ω)

Angle, angular velocity, and angular acceleration

Angle

(derivative of)
rate of change of
←
→
area under the curve of
(integral of)

Angular
velocity
(ω)

(derivative of)
rate of change of
←
→
area under the curve of
(integral of)

Angular
acceleration
(α)

Angle, angular velocity, and angular acceleration

Angle

(derivative of)
rate of change of

area under the curve of
(integral of)

Angular velocity (ω)

(derivative of)
rate of change of

area under the curve of
(integral of)

Angular acceleration (α)

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

Angle, angular velocity, and angular acceleration

$$\begin{tikzcd}[column sep=huge]
\text{Angle} \arrow[r, shift right, "\substack{\text{area under the curve of}\\ \text{(integral of)}}"'] & \substack{\text{Angular}\\ \text{velocity}\\ (\omega)} \arrow[l, shift right, "\substack{\text{(derivative of)}\\ \text{rate of change of}}"'] \arrow[r, shift right, "\substack{\text{area under the curve of}\\ \text{(integral of)}}"'] & \substack{\text{Angular}\\ \text{acceleration}\\ (\alpha)} \arrow[l, shift right, "\substack{\text{(derivative of)}\\ \text{rate of change of}}"']
\end{tikzcd}$$

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$\rightarrow$ Angular kinematics works in exactly the same way as translational kinematics!

Angle, angular velocity, and angular acceleration

- Angle θ – the angle through which something has turned.
- Measured in revolutions, radians, degrees...

- Angular velocity ω ("omega", not "dubya") – the rate at which something is turning
- Measured in revolutions per second, radians per second, degrees per second...

- Angular acceleration α ("alpha", not "fish") – the rate at which something's rate of turning is changing
- Measured in $\frac{\text{rev}}{\text{s}^2}$, $\frac{\text{rad}}{\text{s}^2}$, $\frac{\text{deg}}{\text{s}^2}$...

The 2x2 square for organizing your ideas

Things you know	Things you want
* Information stated in the problem * Facts you've discovered along the way	* The answer you're looking for * Intermediate bits of information that will be useful
Assumptions	**Representations**
* What physical assumptions will you make (constant acceleration, no friction, 1D motion, etc.)? * "Simplify, but don't oversimplify"	* Verbal descriptions * Diagrams * Graphs * Mathematics * What mathematical form do the laws of physics take?

An excellent tool for you to use when you don't know quite how to proceed...

Example problems

- How long does it take for a falling object to fall a height h?

- How long does it take for a falling object to fall a height h?

- A. $\sqrt{2g/h}$
- B. h/g
- C. $\sqrt{2h/g}$
- D. $2h/g$

Example problems

- A wheel at rest starts accelerating with an angular acceleration α rad/s. How long does it take to rotate once?

Example problems

- A wheel at rest starts accelerating with an angular acceleration α rad/s. How long does it take to rotate once?

This is the same as the last problem, really!

Example problems

- You throw an object up with an initial speed of $5\,\mathrm{m/s}$. How high does it go? How long does it take to come back down?

Example problems

- You throw a ball straight up with an initial speed of v_1. A car is moving horizontally at constant speed v_2. How far does the car move before the ball comes back down?

Another example

You throw an object up with an initial speed of v_0. How long does it take to reach a height h?

Another example

You throw an object up with an initial speed of v_0. How long does it take to reach a height h?

$$\begin{aligned} x(t) = & \quad \frac{1}{2}at^2 + v_0 t + x_0 \\ h = & -\frac{1}{2}gt^2 + v_0 t \\ 0 = & -\frac{1}{2}gt^2 + v_0 t - h \end{aligned}$$

- $\rightarrow$ You need the quadratic formula for this – nonzero a, v_0, and position
- The quadratic formula gives you two answers, but there's clearly only one
- The homework asks you to address this idea.
- Hint: graph position vs. time, and interpret the question graphically
- What is the *mathematical* interpretation of the quadratic formula?

- A bucket is being lowered from a cliff at a rate of 10 m/s. You drop a rock off the cliff when the bucket is 10 m beneath the top. How long does it take for the rock to land in the bucket?