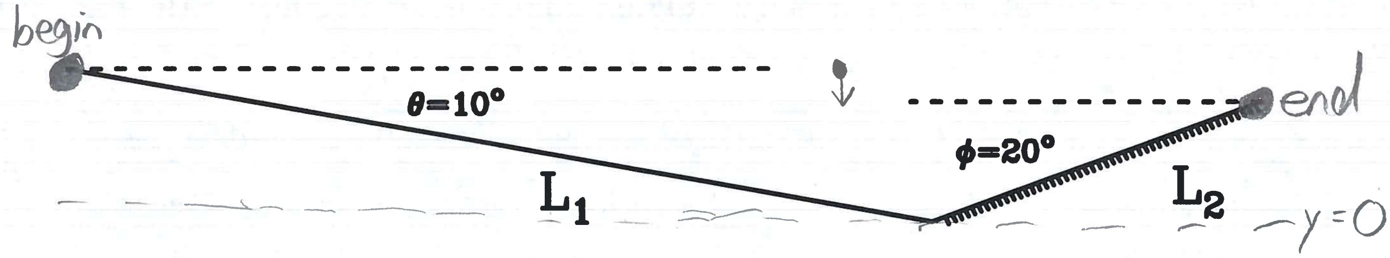
Physics 211 Practice Exam 3 Solutions

- The first question requires you to think about work, energy, and power.
- The second question involves conservation of momentum in two dimensions, where the vector nature of momentum is important.
- The third question requires you to consider rotational kinetic energy when treating conservation of energy, and to understand the significance of the "no-slip" constraint $v = \omega r$.
- Question 4 also involves conservation of momentum in two dimensions.
- Question 5 tests your knowledge of the elastic force and elastic potential energy.
- Question 6 involves the conservation of angular momentum.
- Question 7 again requires you to consider rotational kinetic energy in treating conservation of energy, and to think again about the physical meaning of the no-slip condition $v = \omega r$. The last part requires you to use your knowledge of projectile motion.
- Question 8 requires you to consider elastic potential energy and friction in the work-energy theorem.

QUESTION 1

Heavy trucks driving down steep mountains must continually apply their brakes to maintain a safe speed. If their brakes fail, these roads are equipped with "runaway truck ramps", which are short uphill pathways (made of sand or gravel) with a large coefficient of rolling friction. A truck whose brakes fail can steer into the ramp and come safely to a stop. Suppose that a truck of mass m is driving down the hill at a speed v_0 when its brakes fail. It is a distance L_1 away from the ramp, traveling at a speed v_0. When it reaches the ramp, it exits the highway and heads up the ramp, traveling a distance L_2 before coming to rest. In this problem, you will calculate the distance L_2 in terms of μ_r, g, m, L_1, v_0, θ, and ϕ.



a) Write an expression for the total work done by gravity during the entire motion in terms of g, m, L_1, L_2, θ, and ϕ. (10 points)

$$W_{grav} = \vec{F}_{grav} \cdot \Delta\vec{s} = \underbrace{(F_{grav})(\Delta s)_{\parallel}}_{\text{use this one}} \text{ or } (F_{grav})_{\parallel}(\Delta s)$$

$= (mg)(\text{distance moved down})$

- Beginning height: $y_i = L_1 \sin\theta$
- Ending height: $y_f = L_2 \sin\varphi$

$$W_{grav} = (L_1 \sin\theta - L_2 \sin\varphi)mg.$$

b) Write an expression for the total work done by friction during the entire motion in terms of μ_r, g, m, L_2, and ϕ. (10 points)

• Only friction in L_2 doing work; friction is exactly opposite displacement

$$W_{fric} = -(F_{fric})(L_2) = -(\mu F_N)(L_2) = -\mu(mg\cos\varphi)L_2$$

c) Write a statement of the work-energy theorem/conservation of energy in terms of μ_r, g, m, L_1, L_2, v_0, θ, and ϕ that you could solve for L_2. (You do not need to solve it.) (20 points)

$$KE_i + W_{grav} + W_{fric} = \overset{\text{zero}}{KE_f}$$

$$\underbrace{\frac{1}{2}mv_i}_{KE_i} + \underbrace{(L_1\sin\theta - L_2\sin\varphi)mg}_{W_{grav}} \underbrace{-\mu mgL_2\cos\varphi}_{W_{fric}} = \underbrace{0}_{KE_f}$$

d) Now, consider a truck whose brakes are working. It has a mass of $m = 10^4$ kg (10 tons) and is driving down a hill with a grade of $\theta = 10°$. If the driver wants to maintain a speed of $v = 15$ m/s, what is the power that the brakes must dissipate? (10 points)

Net power $= 0$ (constant speed)

$$P_{brakes} + P_{grav} = 0 \Rightarrow P_{brakes} = -P_{grav} = -\vec{F}_{grav}\cdot\vec{v}$$

$$= F_{grav}v_{\parallel} = (mg)(v\sin\theta) = (10^4\text{ kg})(10\text{ m/s}^2)(15\text{ m/s})\sin 10°$$

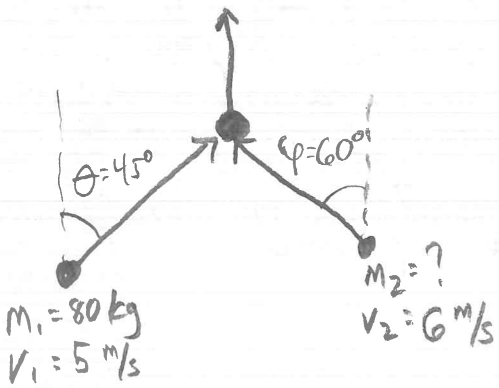
$$= \boxed{260\text{ kW}}$$

QUESTION 2

A rugby player with a mass of $M = 80$ kg is running 45 degrees east of north at a speed of 5 m/s. She is tackled by another player running 60 degrees west of north at a speed of 6 m/s. After the impact, the two players are moving directly north.

a) What is the mass of the second player? (30 points)

Conservation of Momentum: $\vec{P_i} = \vec{P_f}$.



$$X: m_1(V_1 \sin\theta) - m_2(v_2 \sin\varphi) = 0$$

$$Y: m_1(v_1 \cos\theta) + m_2(v_2 \cos\varphi) = (m_1 + m_2)v_{f_1}$$

Note $v_{f,x} = 0$.

$$m_2 = \frac{m_1 V_1 \sin\theta}{V_2 \sin\varphi} = 54.4 \text{ kg}$$

b) How fast are they moving after the impact? (20 points)

$$v_{f_y} = \frac{m_1 V_1 \cos\theta + m_2 V_2 \cos\varphi}{m_1 + m_2} = 3.32 \text{ m/s}$$

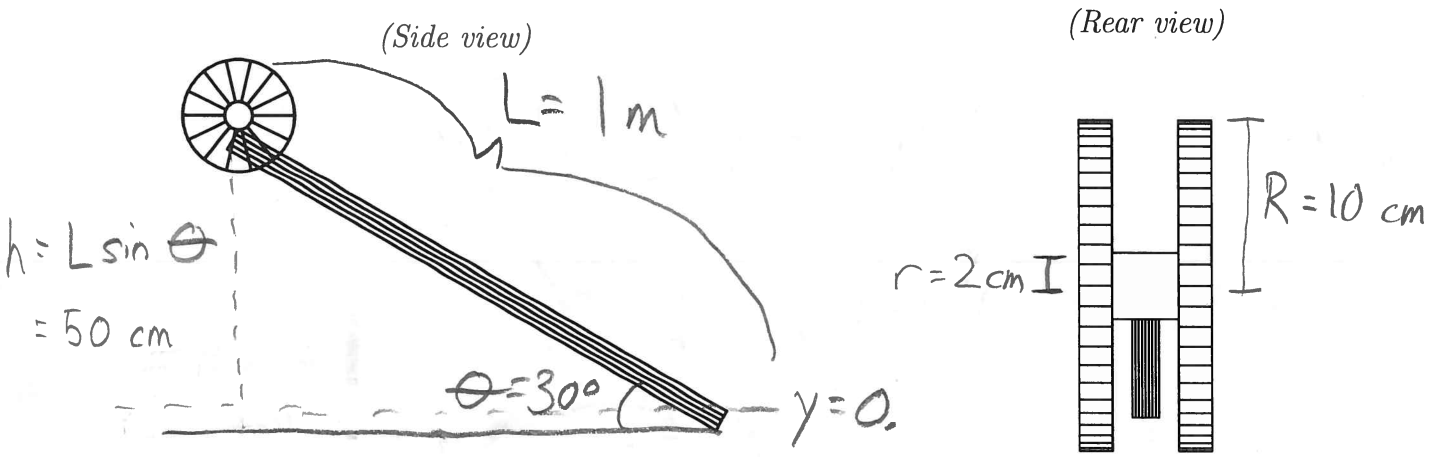
$L = 1\,m$

QUESTION 3

A meter stick is elevated at a $\theta = 30°$ angle. A spool consists of a cylinder of radius 2 cm with two disks affixed on either end; the disks have a radius of 10 cm. The cylinder is very light; you may assume all of the mass of the spool is in the disks. The spool is placed at the top of the meter stick so that the cylinder is touching the stick; when it is released, it rolls without slipping to the bottom. *(The moment of inertia of a disk is* $\frac{1}{2}mr^2$*).*

In this problem, you will calculate the speed of the spool at the bottom.

Rules for this problem: You may solve this problem in either symbols or numbers. If you use symbols, you must tell me the physical values of each symbol that you use (for instance: "$r = 2$ cm"). If you use numbers, you *must* retain the units (i.e. write "10 cm", not "10".)



a) What is the relation between the spool's translational velocity v and its angular velocity ω? Remember that if you use variables here, you must tell me their physical values. (10 points)

$v = \omega r$: this is the inner radius since that is what contacts the meter stick.

b) Write down a statement of conservation of energy / the work-energy theorem that describes the motion down the slope. (10 points)

$\underbrace{\text{Initial trans. KE} + \text{Initial rot. KE}}_{\text{zero since we start from rest}} + \text{Initial GPE} = \text{Final trans. KE} + \text{Final rot. KE}$

$$mgL \sin\theta = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

Note: $I = \frac{1}{2}mR^2$: outer radius here since all the mass is in the disks.

c) Calculate the velocity of the spool at the bottom. (30 points)

$I = \frac{1}{2}mR^2$

From (b):

$$mgL \sin\theta = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$\cancel{m}gL \sin\theta = \frac{1}{2}\cancel{m}v_f^2 + \frac{1}{2}\left(\frac{1}{2}\cancel{m}R^2\right)\omega_f^2$$

$\rightarrow$ Since $v_f = \omega_f r$, $\omega_f = v_f/r$.

$$Lg \sin\theta = \frac{1}{2}v_f^2 + \frac{1}{4}\left(\frac{R^2}{r^2}\right)v_f^2.$$

this is 25

$\rightarrow$ Note $R = 5r$, so $\frac{R^2}{r^2} = 25$.

$$Lg \sin\theta = \left[\frac{1}{2} + \frac{25}{4}\right]v_f^2 = \frac{27}{4}v_f^2$$

$$\rightarrow v_f = \sqrt{\frac{4}{27} Lg \sin\theta}.$$

$$= 0.86 \text{ m/s}.$$

Let $M = 30$ kg, $m = 5$ kg, $V_{0x} = 2$ m/s, $V_{Ys} = -5$ m/s

Question 4

On a particularly cold day in the Shire, the two hobbits Merry and Pippin decide to go sledding on a frozen lake. Each of them, plus his sled, has a mass of 30 kg. Merry carries with him a stone with a mass of 5 kg. They are both traveling east at 2 m/s, right next to each other; Merry is north of Pippin. Merry throws his stone to Pippin, who catches it; the initial velocity of the stone is 5 m/s due south. (This means that Pippin may have to reach behind him a little bit to catch the stone, but that doesn't affect the problem.) Treat north as the positive y–axis and east as the positive x–axis.

a) What are the x– and y– components of Merry's velocity vector after he throws the stone? (20 points)

Use conservation of momentum for the throwing:

$$X: (M+m)V_{0x} = Mv_{f_{x,M}} \rightarrow (35\text{ kg})(2\text{ m/s}) = (30\text{ kg})v_{f_{x,M}} \rightarrow \boxed{v_{f_{x,M}} = 2.3\text{ m/s}}$$

$$Y: 0 = Mv_{f_{y,M}} + mv_{f_{y,s}} \rightarrow v_{f_{y,M}} = \frac{-mv_{f_{y,s}}}{M} = \frac{-(5\text{ kg})(-5\text{ m/s})}{30\text{ kg}}$$

$$\boxed{= 0.33\text{ m/s}}$$

b) What are the x– and y– components of Pippin's velocity vector after he catches the stone? (20 points)

Again, for the catching:

$$X: MV_{0x} = (M+m)v_{f_{x,P}} \rightarrow v_{f_{x,P}} = \frac{MV_{0x}}{M+m} = \frac{(30\text{ kg})(2\text{ m/s})}{35\text{ kg}} = 1.71\text{ m/s}$$

$$Y: 0 + mV_{Ys} = (M+m)v_{f_{y,P}} \rightarrow v_{f_{y,P}} = \frac{mV_{Ys}}{M+m} = \frac{5\text{ kg}(-5\text{ m/s})}{35\text{ kg}} =$$

c) Initially, they had the same value of v_x. However, after one threw the stone and the other caught it, their x–velocities are different. Why is this? (Explain in words; no mathematics is required.) (10 points)

The stone moves only in y, while in the air, so it carries no x-momentum with it.

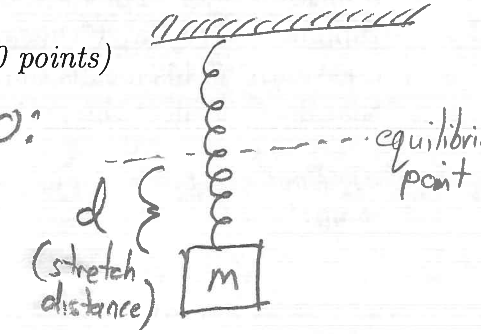
That means that when throwing it, Merry had to transfer its initial x-momentum to him, speeding him up.

Likewise, when Pippin caught it, he had to give it some of his x-momentum to bring it with him, slowing him down.

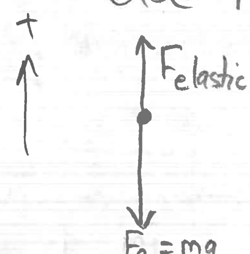
QUESTION 5

The top end of a spring is attached to a stationary point; a $m = 2$ kg mass hangs from the bottom end. It has spring constant $k = 200$ N/m; the mass is at rest. You may give your answers either numerically or in terms of m, k, and g.

a) How far below the spring's equilibrium point does the mass hang? (10 points)



Use Newton's 2nd law since we know $\vec{a}=0$:

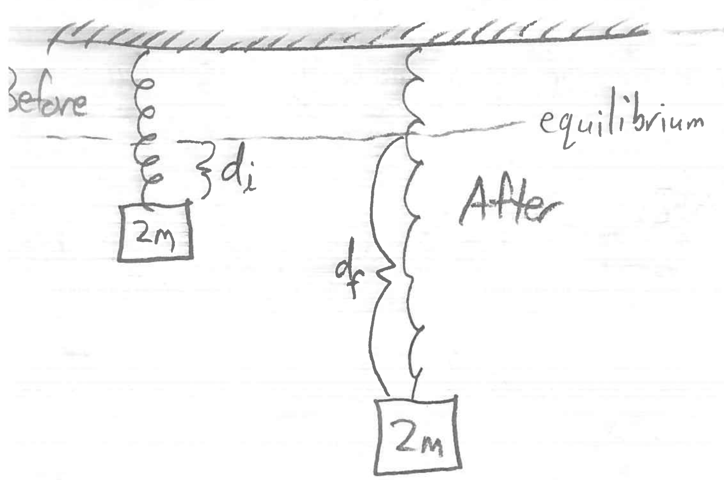


$$kd - mg = ma = 0$$

$$\boxed{d = \frac{mg}{k} = 0.1\ \text{m.}}$$

Then, a second 2kg mass is added to the spring and released. When the mass is added, the extra weight stretches the spring out further, falling down an additional distance before the elastic force pulls it back up.

b) How far below the spring's equilibrium point do the masses fall in total? (30 points)



Use conservation of energy; $v_i = v_f = 0$

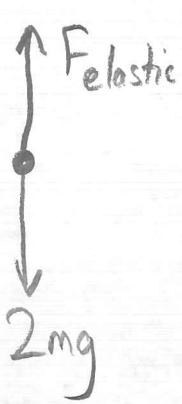
$$\cancel{KE_i}^{\,0} + GPE_i + EPE_i = \cancel{KE_f}^{\,0} + GPE_f + EPE_f$$

$$-2mgd_i + \frac{1}{2}kd_i^2 = -2mgd_f + \frac{1}{2}kd_f^2$$

Solution on scratch paper

c) The masses bounce up and down for a while before eventually coming to rest (due to air drag, friction, and the like). Once they come to rest, how far below the equilibrium point will they be located? (10 points)

This is the same as (a): the net force is zero.



$$\sum F = ma = 0$$

$$kd_3 - 2mg = 0 \Rightarrow d_3 = \frac{2mg}{k} = 0.2\ \text{m.}$$

blank page is blank

¯_ツ_/¯

5b, continued

$$-2mgd_i + \frac{1}{2}kd_i^2 = -2mgd_f + \frac{1}{2}kd_f^2$$

Note $d_i = \frac{mg}{k}$. This gives

$$-2\frac{(mg)^2}{k} + \frac{1}{2}\frac{(mg)^2}{k} = -2mgd_f + \frac{1}{2}kd_f^2$$

Multiply by 2k and collect terms:

$$0 = k^2d_f^2 - 4mgkd_f + 3(mg)^2$$

Factor this:

$$0 = (kd_f - mg)(kd_f - 3mg)$$

$$\rightarrow d_f = \left(\frac{mg}{k}, 3\frac{mg}{k}\right)$$

The first one is where we started; the second is the one we want.

$$\rightarrow d_f = \frac{3mg}{k} = 0.3 \text{ m.}$$

Question 6

A horizontal disk of mass m and radius R is freely rotating at angular velocity ω, spinning clockwise when seen from above. (The moment of inertia of a disk is $\frac{1}{2}mR^2$.) While it is rotating, someone drops a thin ring of mass m and also of radius R on top of it. When the ring lands, it is initially not rotating, but friction quickly causes it to rotate along with the disk, but the rotation of the disk slows down.

a) What principle of physics explains why the rotation slows down once the ring is dropped on top of the disk? (10 points)

Conservation of angular momentum

b) What is the final angular velocity of the disk and ring? (20 points)

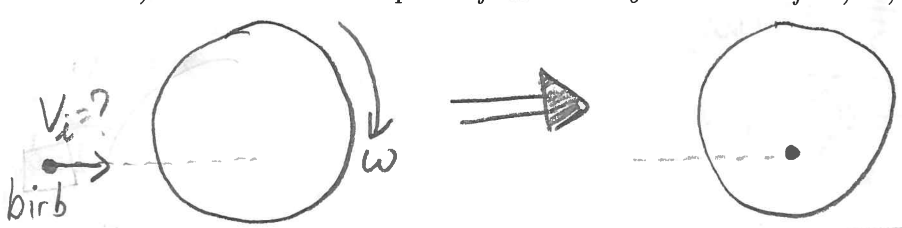
$$\sum L_i = \sum L_f \Rightarrow L_{disk,i} = L_{disk,f} + L_{ring,f}$$

$$I_{disk}\omega_i = I_{disk}\omega_f + I_{ring}\omega_f \Rightarrow \frac{1}{2}mr^2\omega_i = [\underbrace{\tfrac{1}{2}mr^2}_{I_{disk}} + \underbrace{mr^2}_{I_{ring}}]\omega_f$$

$$\frac{1}{2}mr^2\omega_i = \frac{3}{2}mr^2\omega_f \Rightarrow \omega_f = \frac{1}{3}\omega_i.$$

Now, suppose that a bird of mass $3m$ flies in and lands on the disk. It lands a distance R/2 south of the center of the disk, and is flying eastward when it lands. The bird is flying at exactly the right speed to stop the platform from rotating once it lands.

c) Find the initial speed of the bird v_b in terms of m, R, and ω. (20 points)



$\omega = 0$ $\quad L_{disk} + L_{birb} = 0.$

- $L_{disk} = \frac{1}{2}mr^2\omega_i$
- $L_{birb} = -m_b v r_\perp = (3m)v(\frac{1}{2}r)$

negatie because opposite direction

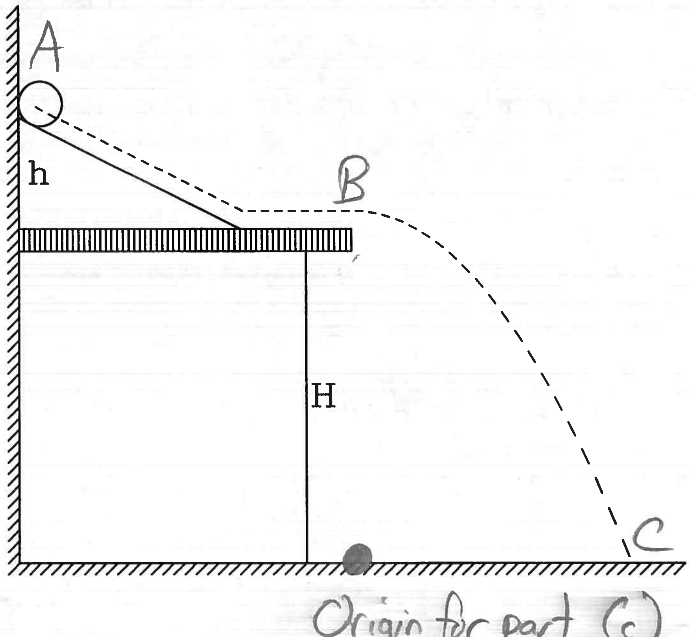
$$\frac{1}{2}mr^2\omega_i - \frac{3}{2}mvr = 0$$

$$\rightarrow v = \frac{1}{3}r\omega$$

Question 7

In class, you saw a demonstration where a ball bearing (solid sphere, moment of inertia $I = \frac{2}{5}mr^2$) was rolled down a small ramp on top of a table. The ball rolled down the ramp, rolled across the table, and then fell off of the side of the table.

Use work-energy thm. from A to B.



Suppose that the height of the ramp is h, the height of the table is H, and the radius of the ball is r.

a) How fast is the ball traveling when it reaches the edge of the table? (15 points)

$$\underbrace{\text{Kinetic energy at start}}_{\text{zero}} + \underbrace{\text{work done by grav.}}_{mgh} = \underbrace{\text{Kinetic energy at end}}_{= \underbrace{\frac{1}{2}mV_B^2}_{\text{trans.}} + \underbrace{\frac{1}{2}I\omega_B^2}_{\text{rotational}}}$$

$$mgh = \frac{1}{2}MV_B^2 + \frac{1}{2}\overbrace{\left(\frac{2}{5}mr^2\right)}^{I \text{ for a ball}}\omega_B^2$$

Rolling constraint: $V = \omega r$. So $r^2\omega_B^2 = V_B^2$ and

$$mgh = \frac{1}{2}mV_B^2 + \frac{1}{5}mV_B^2$$

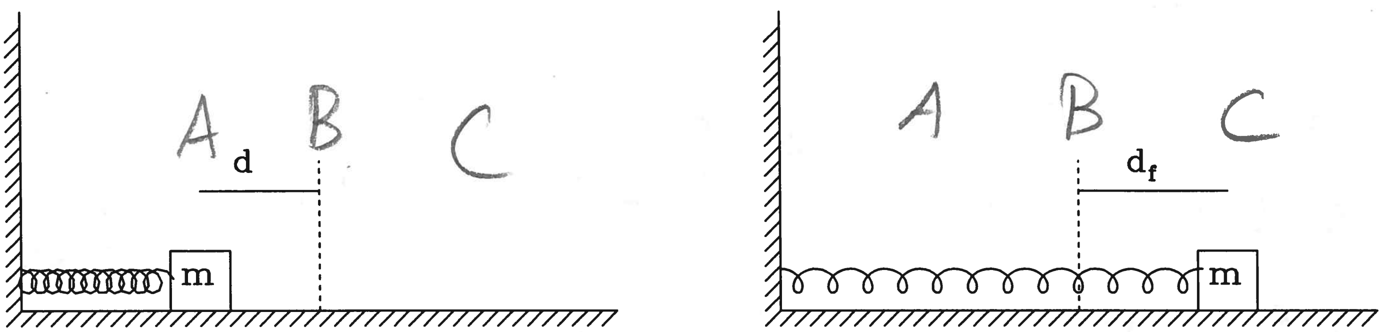
$$gh = \frac{7}{10}V_B^2 \longrightarrow \boxed{V_B = \sqrt{\frac{10}{7}gh}\,.}$$

(*This problem continues on the next page.*)

Question 8

A spring has spring constant k. One end is fixed, and the other end is attached to a mass m, which is free to move horizontally along a table. The mass slides over the table with a coefficient of friction μ_k.

The spring is compressed a distance d from its equilibrium point and released. When the spring is released, it will push the mass to the right, until it reaches some other distance d_f past the equilibrium point.



a) How fast will the mass be traveling when it crosses the equilibrium point? Give your answer in terms of μ_k, d, m, and g. (10 points) Use work-energy theorem from A to B:

$EPE_A + W_{fric} = KE_B \longrightarrow \frac{1}{2}kd^2 + W_{fric} = \frac{1}{2}mv_B^2$. $W_{fric} = \vec{F}\cdot\vec{ds} = -\mu mgd$

$\longrightarrow \frac{1}{2}kd^2 - \mu mgd = \frac{1}{2}mv_B^2$ so $\boxed{v_B = \sqrt{\frac{k}{m}d^2 - 2\mu gd}}$.

b) Write down an expression for the work done by friction as the block slides from its starting point to the final position d_f to the right of equilibrium. (10 points)

$A \longrightarrow C$:

F_{fric} is again μmg to the left

Δs is $(d+d_f)$ to the right $\Longrightarrow W_{fric} = -\mu mg(d+d_f)$

c) Write down an equation in terms of μ_k, d, m, and g that will let you solve for the distance d_f. You do not need to solve it. (15 points) Use work-energy from A to C: note $v_A = v_C = 0$.

$EPE_A + W_{fric} = EPE_C$

$\frac{1}{2}kd^2 - \mu mg(d+d_f) = \frac{1}{2}kd_f^2$.

d) What algebraic technique would you have to use to solve this equation for d_f? (5 points)

Quadratic formula :(

b) How fast is the ball traveling when it strikes the floor? (Hint: What happens to the ball's angular velocity as it travels through the air?) (15 points)

Note: v changes from point B to point C because of gravity, but ω stays the <u>same</u> since it is no longer rolling.

Use conservation of energy from B to C:

$$\frac{1}{2}mv_B^2 + \cancel{\frac{1}{2}I\omega_B^2} + W_{grav} = \frac{1}{2}mv_C^2 + \cancel{\frac{1}{2}I\omega_C^2}.$$ But $\omega_B = \omega_C$ so those terms cancel

$$\frac{1}{2}\cancel{m}\left(\frac{10}{7}gh\right) + \cancel{m}gH = \frac{1}{2}\cancel{m}v_C^2$$

$$v_C = \sqrt{\frac{10}{7}gh + 2gH}.$$

c) How far past the edge of the table does the ball land? (10 points)

Use projectile-motion kinematics from B to C:
(Choose origin at the base of the table)

$$x(t) = v_{0,x}t = v_B t = \left(\sqrt{\frac{10}{7}gh}\right)t.$$

$$y(t) = -\frac{1}{2}gt^2 + H \longrightarrow$$ moving horizontally at B so $v_{0,y} = 0$.

"What is x at the time $y=0$?"

$$0 = -\frac{1}{2}gt^2 + H \Longrightarrow t = \sqrt{2H/g}$$

$$\boxed{x = \sqrt{\frac{10}{7}gh}\sqrt{\frac{2H}{g}} = \sqrt{\frac{20}{7}hH}.}$$