

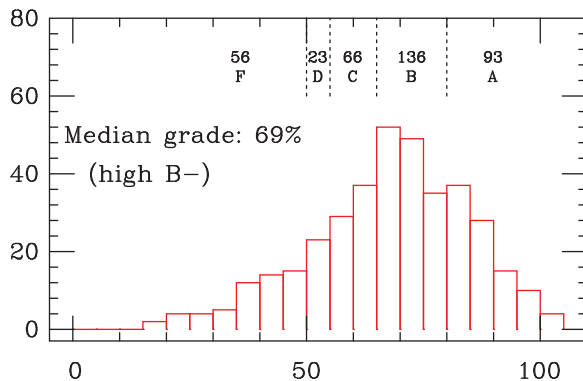
Work and potential energy

Physics 211
Syracuse University, Physics 211 Spring 2019
Walter Freeman

March 20, 2019

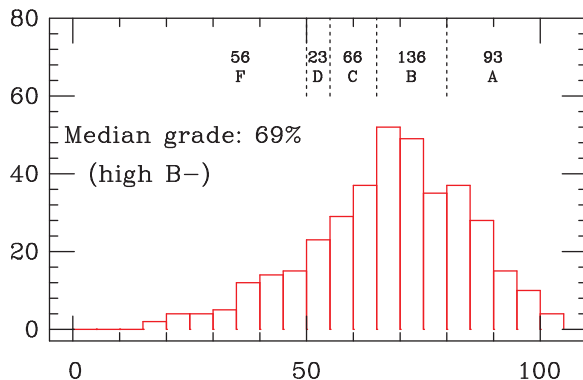
- Your next homework assignment is due next Friday.
- Upcoming office hours:
 - Today, 1:45-3:45 – discussing exams with students
 - Friday, 9:30-11:30 – anything you'd like!
 - Tuesday, 3-5 – discussing HW6

Exam 2 recap



This exam was quite difficult and most of you did very well!

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This is most of the class; one of the TA's hasn't uploaded their grades to Blackboard yet.

Where we've been and where we're going

- Last time: kinetic energy and the work-energy theorem
- This time: the idea of potential energy and conservation of energy
 - Potential energy: “the most meaningful bookkeeping trick in physics”
 - Lets us understand many phenomena without difficult mathematics
 - Conservation of energy: there's always the same amount of energy, and it just changes forms

Review: kinetic energy

We will see that things are often simpler when we look at something called “energy”

- Basic idea: don't treat \vec{a} and \vec{v} as the most interesting things any more
- Treat v^2 as fundamental: $\frac{1}{2}mv^2$ called “kinetic energy”

Previous methods:

- Velocity is fundamental
- Force: causes velocities to change over time
- Intimately concerned with vector quantities

Energy methods:

- v^2 (related to kinetic energy) is fundamental
- Force: causes KE to change over distance
- Energy is a *scalar*

Energy methods: useful when you don't know and don't care about time

$$\text{Kinetic energy} = \frac{1}{2}mv^2$$

- Energy has units $\text{kg m}^2/\text{s}^2$
- This unit is called a *joule*
- This is also the unit for work
- 1 joule = the work required to lift an apple one meter
- 1 joule = the kinetic energy of a 2 kilogram mass traveling at 1 m/s
- 9 megajoules = the energy in the food a person eats in a day
- 350 kilojoules = the energy stored in a laptop battery
- 1.5 gigajoules = the energy in a small tank of gasoline (11 gal/44 liters)

The work-energy theorem in 1D

Last time we saw the “work-energy theorem” was a consequence of simple kinematics:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = F\Delta x$$

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Some new terminology:

- $\frac{1}{2}mv^2$ called the “kinetic energy” (positive only!)
- $\vec{F} \cdot \Delta\vec{s}$ called the “work” (negative or positive!)
- “Work is the change in kinetic energy”

The force from air pressure

How much work does air pressure do on the ping pong ball?

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- Work = force \times distance ($W = Fd$)
- Force = pressure \times area ($F = PA$)
 - Atmospheric pressure = 10N/cm^2
 - Cross-sectional area of ping-pong ball = 13cm^2
- $W = P \times A \times d$

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$$\frac{1}{2}mv_0^2 + W_{\text{pressure}} = \frac{1}{2}mv_f^2$$

Sample problem: a roller coaster

(on document camera)

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Strategy: compute the work done by all the forces and equate that to the change in KE.

Work done by normal force = **zero!**

Work done by gravity = $(F)(\Delta s)_{\parallel} = mg\Delta y = mg(y_0 - y_f)$

$$KE_f - KE_i = W_g$$

$$\frac{1}{2}mv_f^2 - 0 = mg(y_0 - y_f)$$

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No detailed knowledge of the motion required!

Potential energy: an accounting trick

- Notice that the work done by gravity depends *only* on the change in height.
- Some other forces are like this as well: the work done depends only on initial and final position
 - These are called *conservative forces*
 - Soon we'll see that the elastic force is like this too
- Separate out gravity and all other forces in the work-energy theorem:

$$KE_f - KE_i = W_{\text{grav}} + W_{\text{other}}$$
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = mg(y_0 - y_f) + W_{\text{other}}$$

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- Collect all the “initial” things on the left and the “final” things on the right:

$$\begin{array}{ccc} \frac{1}{2}mv_0^2 + mgy_0 & + W_{\text{other}} = & \frac{1}{2}mv_f^2 + mgy_f \\ KE_0 + GPE_0 & + W_{\text{other}} = & KE_f + GPE_f \end{array}$$

- Identify mgy as “gravitational potential energy”: how much work will gravity do if something falls?

Potential energy lets us easily calculate the work done by conservative forces

Potential energy: more than accounting!

- Another way to look at the roller coaster: gravitational potential energy being converted to kinetic energy.
- This perspective is universal: all forces just convert energy from one sort into another
- Some of these types are beyond the scope of this class, but we should be aware of them!

A short history of energy conversion:

- Hydrogen in the sun fuses into helium
- Hot gas emits light
- Light shines on the ocean, heating it
- Seawater evaporates and rises, then falls as rain
- Rivers run downhill
- Falling water turns a turbine
- Turbine turns coils of wire in generator
- Electric current ionizes gas
- Recombination of gas ions emits light
- Nuclear energy \rightarrow thermal energy
- Thermal energy \rightarrow light
- Light \rightarrow thermal energy
- Thermal energy \rightarrow gravitational potential energy
- Gravitational PE \rightarrow kinetic energy and sound
- Kinetic energy in water \rightarrow kinetic energy in turbine
- Kinetic energy \rightarrow electric energy
- Electric energy \rightarrow chemical potential energy
- Chemical PE \rightarrow light

Potential energy: more than accounting!

- This class is just the study of motion: we can't treat light or nuclear energy here.
- ... but in physics as a whole, the *conservation of energy* – that processes just change energy from one form to another – is universal!
- Conservation of energy is one of the most tested, ironclad ideas in science
- Nuclear and chemical potential energy: nuclear forces do mechanical work on particles, much like gravity
- Light, and others: kinetic energy of little particles called “photons”
- Heat: kinetic energy of atoms in random motion
- Sound: kinetic energy of atoms in coordinated motion
- Food: Just chemical potential energy...
- ... so all of these things aren't as far removed from mechanics after all!
- Einstein: “Mass is just another form of energy”

Potential energy: more than accounting!

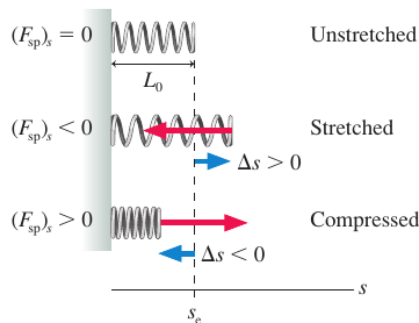
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- ... so all of these things aren't as far removed from mechanics after all!
- Einstein: “Mass is just another form of energy”
- Maybe it's all, ultimately, just kinetic energy! (I believe it is; others will argue!)

Ask a Physicist: how does a nuclear bomb work (and how is it different from a nuclear power plant)?

A new force: elasticity and Hooke's law

To best see how this can be useful, let's introduce a new force: elasticity.

- Springs have a particular length that they like to be: “equilibrium length” L_0
- A spring stretched to be longer than this pulls inward to shorten itself
- A spring compressed to be shorter than this pushes outward to lengthen itself
- Flexible things like strings and ropes only pull; they kink instead of compressing
- The force is proportional to the deviation from the optimum length

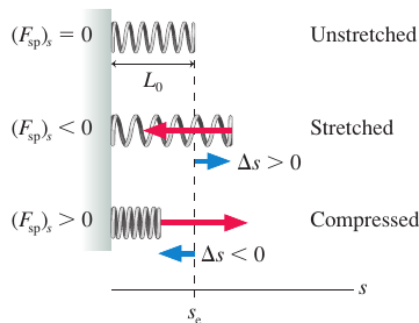


$$F_{\text{elastic}} = -k(L - L_0) = -k\Delta x \text{ (Hooke's law)}$$

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$$F_{\text{elastic}} = -k(L - L_0) = -k\Delta x \text{ (Hooke's law)}$$

k is called the “spring constant”:

- Measures the stiffness of the spring/rope
- Units of newtons per meter: “restoring force of k newtons per meter of stretch”

A simple spring problem: done with the work-energy theorem

A person of mass $m = 100\text{kg}$ falls from a height of $h = 3\text{m}$ onto a trampoline. If the person makes an impression $d = 40\text{ cm}$ deep on the trampoline when he lands, what is the spring constant?

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- Initial kinetic energy + work done by spring + work done by gravity = final kinetic energy
 - Need to use the integral form of the work-energy theorem since the force isn't constant
- The person begins and ends at rest, so we know the initial and final kinetic energy is zero
- The trampoline begins at its equilibrium position

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- $KE_0 + W_{\text{grav}} + W_{\text{elas}} = KE_f$
- $0 + (mg)(h + d) - \frac{1}{2}kd^2 = 0$
- $k = \frac{mg(h+d)}{2d^2}$

Potential energy stored in a spring

We saw that an object at height h has gravitational potential energy mgh . Can we do something similar for springs?

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- A natural choice is $\Delta x = 0$, the equilibrium position of the spring.

“How much work is done by a spring as it goes from $\Delta x = a$ to $\Delta x = 0$?

$$U_{\text{elastic}} = W_{a \rightarrow 0} = \int_a^0 -kx \, dx = \int_0^a kx \, dx = \frac{1}{2}ka^2$$

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Now that we have this, we never have to do this integral again!

$$U_{\text{elastic}} = \frac{1}{2}kx^2, \text{ where } x \text{ is the distance from equilibrium}$$

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- $U_{\text{grav},0} = mgh$
- $U_{\text{elas},0} = 0$ (trampoline starts at equilibrium)
- $U_{\text{grav},f} = -mgd$ (the person falls below $y = 0$; PE can be negative!)
- $U_{\text{elas},f} = \frac{1}{2}kd^2$ (see last slide)

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- $0 + mgh + 0 = 0 + (-mgd) + \frac{1}{2}kd^2$ (Same terms, maybe on different side)

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- $0 + mgh + 0 = 0 + (-mgd) + \frac{1}{2}kd^2$ (Same terms, maybe on different side)
- $k = \frac{mg(h+d)}{d^2}$

That spring problem: a recap

We don't care about time \rightarrow energy methods

Work-energy theorem

- Initial KE + all work done = final KE
- Need to compute work done by gravity: easy
- Need to compute work done by spring: harder
(need to integrate Hooke's law)

Potential energy treatment

- Initial KE + initial PE + other work = final KE + final PE
- No “other work” in this problem; all forces have a PE associated
- Need to know the expressions for PE:
 - $U_{\text{grav}} = mgy$
 - $U_{\text{elas}} = \frac{1}{2}kx^2$ (x is the distance from the equilibrium point)
- No integrals required (they're baked into the above)

Potential energy with other forces

What about associating a potential energy with other forces?

- Friction is a no-go: the work done by friction depends on the path, not just where you start and stop
- “Ephemeral” forces like tension and normal force are easiest to deal with by computing work directly
- The other force we’ve studied that is easily associated with a potential energy is **universal gravitation**
 - Need to choose a point to set $U = 0$; here we choose $r = \infty$
 - $U_G =$ “work done by gravity on m_1 when it moves infinitely far from m_2 ”

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$W_G = \int_R^\infty -\frac{Gm_1m_2}{r^2} dr = -\frac{Gm_1m_2}{R}$$

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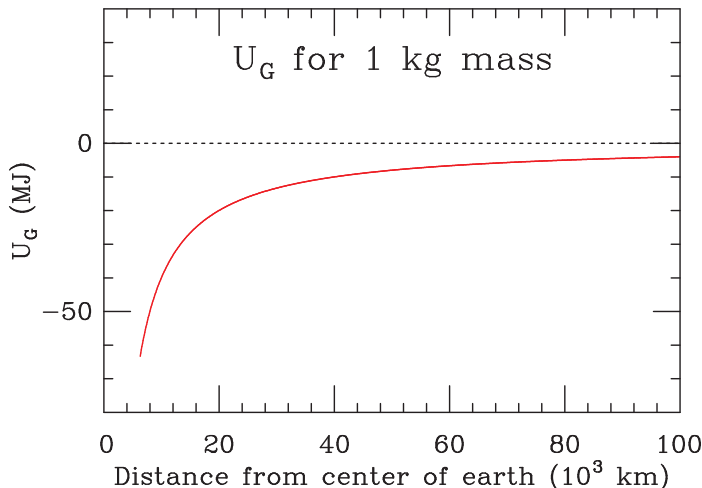
→ Gravitational potential energy between two objects separated by a distance r is $-\frac{Gm_1m_2}{r}$.

A new idea: power, the rate of doing work

- Sometimes we are interested in the rate at which a force does work
- This idea is called *power*, and it is measured in joules per second
- A joule per second is also called a watt
- If $W = \vec{F} \cdot \Delta \vec{s}$, then I can take derivatives of both sides to get...
- $P = \vec{F} \cdot \vec{v}$

The Earth's “gravity well”

- With this choice of the zero point at $r = \infty$, gravitational potential energy is always negative
- We have to *add energy* to get something away from Earth



This region of large negative potential energy is often called a “gravity well”.

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 - Part of *conservation of total energy*, a powerful statement about nature
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 - Restoring force in a stretched or compressed spring, or a stretched string:
$$F = -k(x - x_0) \text{ (} x_0 \text{ is the equilibrium length)}$$
 - k is the spring constant, measured in force per distance, that gauges stiffness
 - Elastic potential energy: $U_{\text{elas}} = \frac{1}{2}k(x - x_0)^2$

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 - k is the spring constant, measured in force per distance, that gauges stiffness
 - Elastic potential energy: $U_{\text{elas}} = \frac{1}{2}k(x - x_0)^2$
- Gravitational potential energy in general: $U_G = -\frac{Gm_1m_2}{r}$