Recitation: Mathematics With Units

Week 1, Day 1

Before you do any physics, spend ten minutes meeting the other people in your group. Learn their names and a little about them. You should also exchange contact information (email addresses, Discord handles, phone numbers, whatever you are comfortable with) so you can work together on your homework later.

You should also learn your TA's name, along with the undergraduate coaches. They will be with you all semester and will be one of your greatest resources in learning physics!

This material is designed to get you familiar with the way that physicists think about *units of measure* and how to do mathematics with them. There are three principles you need to know:

- Some quantities bear *dimensions* – various types of things that you measure. One example, for instance, is *time*, which can be measured in many different units (seconds, minutes, hours...)
- Any numeric value that describes something with dimensions must always be attached in the units that it is measured in.
- **You may multiply and divide by units just like any other variable when doing algebra.** For instance, we know that

$$1\,\text{minute} = 60\,\text{seconds}.$$

1. What do you get if you divide both sides of this equation by "1 minute"?

2. How far is it from Syracuse to Albany along I-90? Give your answer in as many different ways as you can think of (at least three). It's okay to give an informal answer; for instance, how do drivers talk about distances between cities? If you're having trouble thinking of different ways to answer the question, you might ask:
   - an American student (if you are international)
   - an international student (if you are American)
   - someone who has driven from Syracuse to Albany who knows how long it takes

3. Drivers measure distances in "hours" frequently. For instance, I might say that it is six hours' drive from Syracuse to Baltimore. However, this is not a dimensionally-correct way to describe distance, since an hour is a measure of time, not distance. What other piece of information do you need for this to make sense?

4. What are the dimensions of this piece of information, and what units would you measure it in? Write your answer both in words and as a fraction. (What mathematical operation does "per" suggest?)

5. Use a satellite navigation app on a smartphone to get the distance to Albany, plus the the time that it will take. From this, calculate how fast the app thinks you will be driving on average. Show your work to a TA or coach when you are done.

6. In physics we generally measure this quantity in meters per second instead. Convert your result to meters per second from the units you have measured it in. As you do your conversion, write out all of the steps; the point is to ensure that you get comfortable doing algebra with units and numbers together. Show your work to a TA or coach when you are done.

7. People who make fast cars often brag about how quickly they can accelerate. For instance, Tesla Motors claims that their upcoming Roadster can accelerate from 0 to 60 miles per hour in 1.9 seconds.

   You will study acceleration in more detail tomorrow, but for now, just know that

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}.$$

   What is the average acceleration of this (ludicrously fast) car as it goes from zero to 60 miles per hour? Write out all of the steps in your calculation, making sure to treat units (like hours, seconds, and miles) like variables when manipulating fractions. What are the units of your answer? Do they have the right dimensions?

8. In physics, we like to measure distances in meters and time in seconds. Convert your answer to the previous into these units, and simplify as much as possible. Call a TA or coach over to check your work when you are done.

9. What does it mean that acceleration can be measured in "meters per second squared"? I've never seen a "squared second"; does this make sense?

10. The volume of a cube 10 cm on a side is equal to one liter. How many cubic centimeters (cm^3) are in a liter? Make sure you doublecheck your answer with your common sense!

11. Finally, let's meet some of the other quantities that we will work with in our class later in the semester.

    The most important quantity this semester will be **force**. Newton's law of motion says:

$$\frac{\text{(force on an object)}}{\text{(mass of that object)}} = \text{(resulting acceleration of that object)}$$

    What are the dimensions of force? (What dimensions must it have to make this equation have the same dimensions on both sides?)

    What units might it be measured in? We call this a "newton" for short, but that's just a shorthand for something else.

12. Another quantity we'll meet this semester is *energy*. One way to transfer energy is mechanically, by pushing on something.

    We'll learn that:

$$\text{(amount of energy transferred by a force)} = \text{(size of that force)} \times \text{(distance the object moves)}$$

    From this, deduce the units and dimensions of energy. (This unit of energy – a combination of kilograms, meters, and seconds – is called the *joule*, but that's just shorthand.)