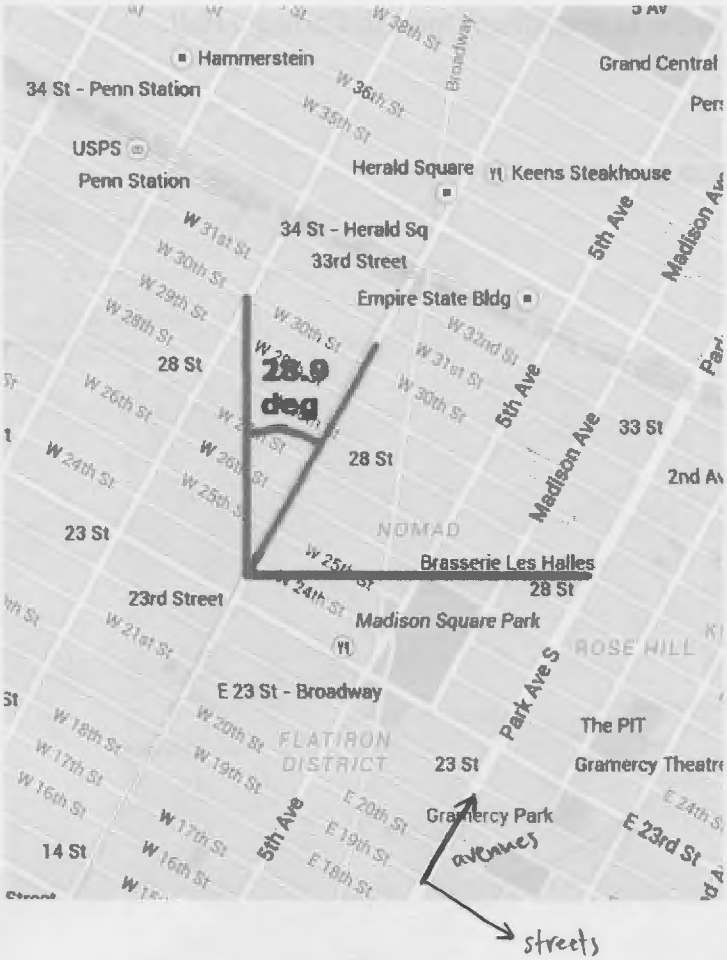


The streets in Manhattan are laid out in a grid, but that grid is aligned with the island, rather than along the compass directions. Avenues run 28.9 degrees east of north, while streets run 28.9 degrees north of west.

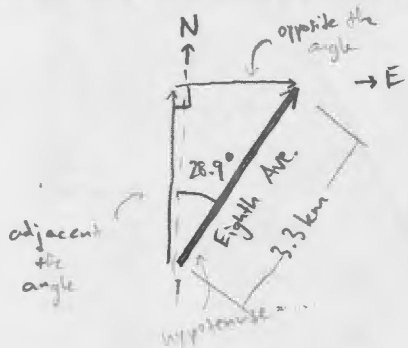
This means that there are two sensible coordinate systems in Manhattan:

- North/South/East/West, aligned with the compass
- Uptown/Downtown/Crosstown, aligned with the streets.

(In this map and the next one, "up" is due north.)

not on the map

1) The staff astronomer at the Natural History Museum walks from Penn Station to the Natural History Museum, going 3.3 km north along Eighth Avenue. How far east and how far north did she walk?



Since the north and east legs form a 90° angle, we have a right triangle. The lengths of the adjacent and opposite legs are from the trig functions $\cos\theta$ and $\sin\theta$:

$\cos(28.9°) = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\sin(28.9°) = \frac{\text{opposite}}{\text{hypotenuse}}$, so

adjacent = hypotenuse $\cdot \cos(28.9°)$ = 3.3 km $\cdot$ 0.875 = 2.89 km

opposite = hypotenuse $\cdot \sin(28.9°)$ = 3.3 km $\cdot$ 0.483 = 1.59 km

so

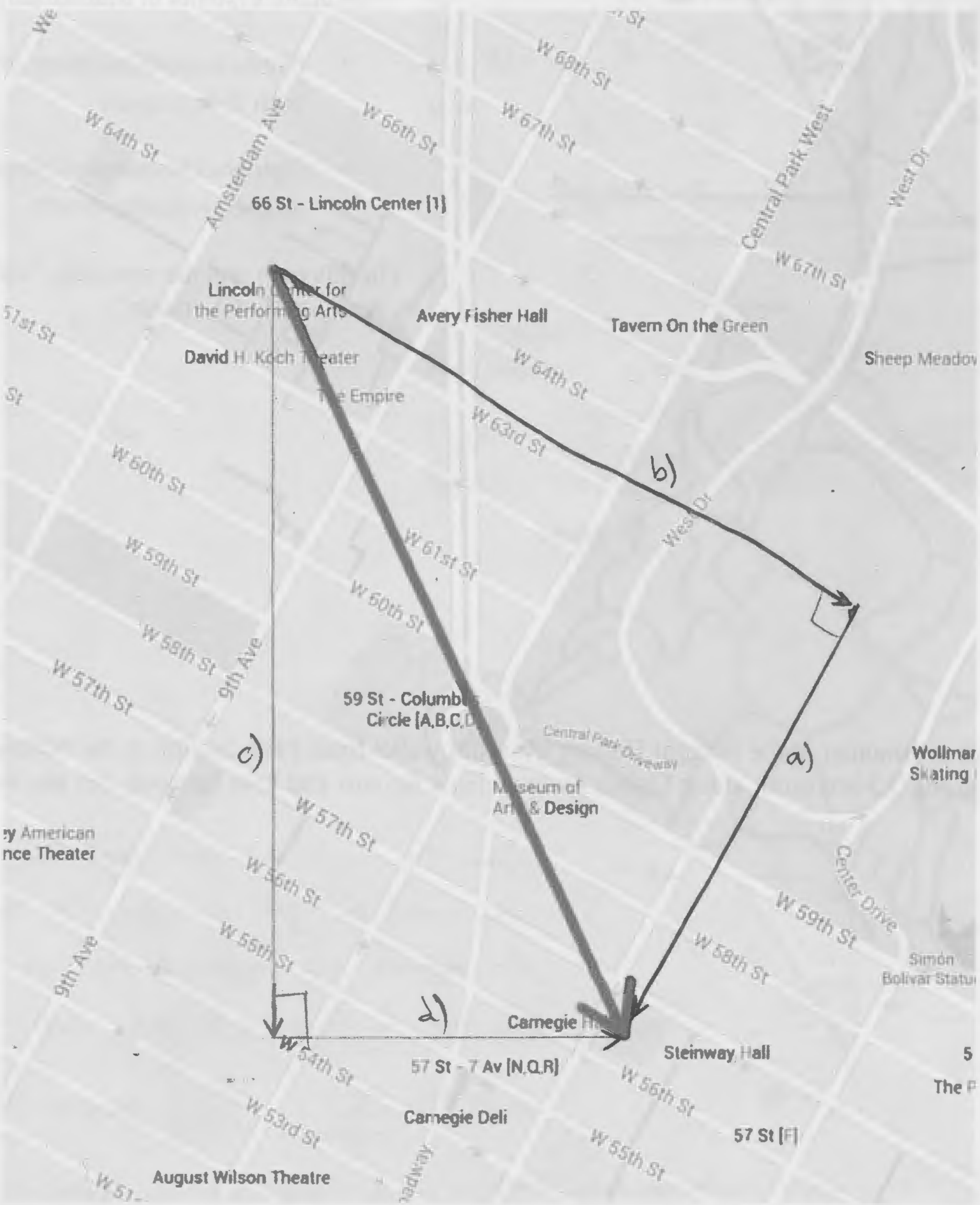
She went $\boxed{\text{2.89 km north, 1.59 km east}}$.

2) Here is the displacement vector pointing from the Metropolitan Opera to Carnegie Hall.

Draw:

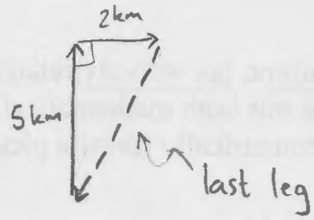
a) its component in the direction of the Manhattan avenues;
b) its component perpendicular to them;
c) its component along the North-South axis;
d) its component along the East-West axis.

Make sure, for each coordinate system, your components are perpendicular

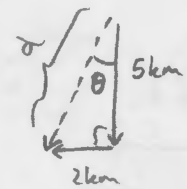


3) A hiker in the forest walks 5 km due north and then 2 km due east, and then wants to return to his original spot by the shortest route possible.

a) Draw the hiker's path.



b) Which direction should he walk, and for how far?



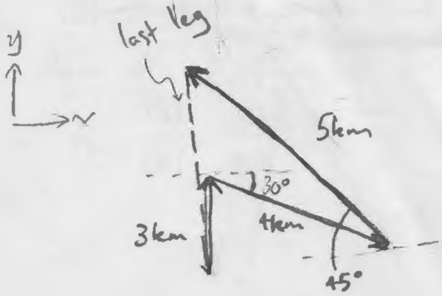
We know the components of the last leg. Since the components and the leg form a right triangle, we can use trig functions to find the unknown angle (direction).

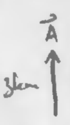
We know opposite and adjacent, so let's use $\tan\theta = \frac{opp.}{adj.} = \frac{2\text{ km}}{5\text{ km}}$, so $\theta = \tan^{-1}\left(\frac{2}{5}\right) = 21.8°$.

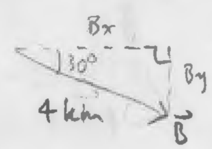
The direction is mostly south, so we can call the direction $\boxed{21.8° \text{ west of south}}$. The length of the last leg comes from the Pythagorean theorem: $d^2 = (2\text{km})^2 + (5\text{ km})^2$ so $d = \sqrt{4+25} = \boxed{5.4\text{ km}}$

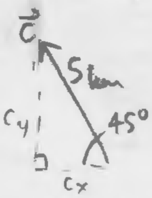
4) Now our hiker walks 3 km due north, then 4 km at an angle 30 degrees south of east, and then finally 5 km at an angle 45 degrees north of west. He then wants to return to his starting point, as before. Which direction should he travel in, and for how far? Draw the hiker's path.

Although this looks like a complicated geometry problem, it becomes simple if we just try to find the components of the last leg.





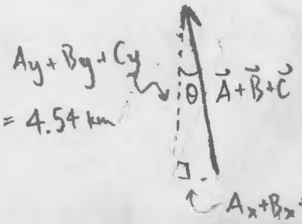




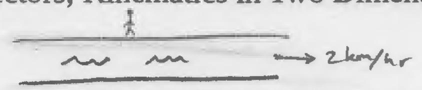
notice that some components are in the negative direction

$A_x = 0$ $\quad B_x = (4\text{km})\cos 30°$ $\quad C_x = -(5\text{km})\cos 45°$

$A_y = 3\text{km}$ $\quad B_y = -(4\text{km})\sin 30°$ $\quad C_y = (5\text{km})\sin 45°$



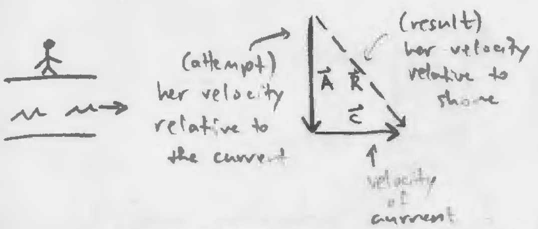
Now we know the distance back is $\sqrt{4.54^2 + (-0.071)^2} = \boxed{4.54\text{ km}}$ and they would have to walk back at an angle $\theta = \tan^{-1}\left(\frac{0.071}{4.54}\right) = \boxed{0.9° \text{ east of south}}$ to get back.



5) A swimmer can swim 5 km/hr in still water. She wants to swim directly across a river. However, there is a current in the river, with a speed of 2 km/hr. If she swims directly across, she will drift downstream due to the current. Thus, in order to get where she wants to go, she needs to angle herself upstream.

a) There are three interesting vectors in this problem: the velocity of the current, her velocity (relative to the current) and her velocity (relative to the shore). How do they relate? State this both mathematically (for instance, "this vector plus that vector equals this other vector"), and geometrically (draw a picture).

Let's draw the picture of her trying to swim directly across to see what these vectors would look like:



We can see that she would be pulled downstream, which is not what we want.

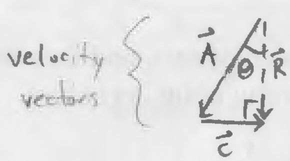
Regardless, we can still see that

$\boxed{\vec{A} + \vec{C} = \vec{R}}$ (attempt + current = result)

(Since $\vec{A}$ and $\vec{C}$ are lined up tip-to-tail, and $\vec{R}$ points from the very initial point to the very final point).

b) At what angle must she try to swim in order to proceed directly across the river?

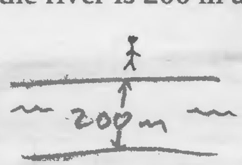
If she attempts a bit upstream, the resulting velocity (relative to shore) can go straight across.



We know that $|\vec{A}|$ (the magnitude of $\vec{A}$) is 5 km/hr, and we know that $|\vec{C}| = 2$ km/hr. Since $\sin\theta = \frac{\text{opp.}}{\text{hyp.}}$, $\theta = \sin^{-1}\left(\frac{\text{opp.}}{\text{hyp.}}\right)$

$= \sin^{-1}\left(\frac{2}{5}\right) = \boxed{23.6°}$ upstream

c) If the river is 200 m across, how long will it take her to cross?



We need to find her resultant speed, $|\vec{R}|$. Since the $\vec{R}$ side is adjacent to the angle, we can use cosine: $\cos\theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{|\vec{R}|}{|\vec{A}|}$, so

$$|\vec{R}| = |\vec{A}|\cos\theta = (5 \text{ km/hr})\cos 23.6°$$
$$= 4.58 \text{ km/hr}.$$

Now that we know her speed, since her speed is constant, we can find the time to cross (either with the position equation) or just by checking units:

$$\frac{200\cancel{m}}{4.58\cancel{km}/hr} \cdot \frac{1\cancel{km}}{1000\cancel{m}} \cdot \frac{60\text{ min}}{1\text{ hr}} = \boxed{2.6 \text{ minutes}}$$

if we divide distance by speed, we get units of time