

RECITATION QUESTIONS

MARCH 27

1. A mad scientist has built a rocket-powered sled, and wants to show off by using it and a ramp made out of snow to jump through the air, much like a skier. She sets her sled a distance d in front of the ramp and fires the rocket. The rocket accelerates forward toward the ramp, ascends the ramp, then flies through the air before landing back on the ground. She turns off her rocket when she reaches the ramp.

Suppose that:

- The sled and rider together have mass m
 - The coefficient of friction between the snow and the sled is μ_k
 - The thrust force from the rocket is F_T
 - The overall (diagonal) length of the ramp is L
 - The ramp is inclined at an angle θ above the horizontal
- (a) Draw a cartoon of the situation, labeling interesting things (i.e. the trigonometry related to the ramp).
 - (b) Using energy methods, calculate how fast she is traveling when she leaves the top of the ramp.

- (c) Using energy methods, calculate how fast she is traveling when she lands back on the ground. Think carefully about what your “initial” and “final” states are; there’s a hard way and an easy way to do this.
- (d) Can you use energy methods to figure out the horizontal distance she travels before landing back on the ground? If so, write down an equation you can solve for that distance. If not, explain what other techniques you need to use.

2. A pinball machine uses a spring-loaded launcher to launch a solid steel ball up a ramp. The ball rolls up the ramp without slipping. (Remember that if something rolls without slipping, its translational velocity and angular velocity are related by $v = \omega r$.) You'd like to find the speed of the ball at the top of the ramp. Suppose that:

- The ramp is inclined at an angle θ above the horizontal, and has a total length L
- The ball has mass m and radius r , and thus has moment of inertia $I = \frac{2}{5}mr^2$.
- The spring has spring constant k , and is compressed a distance d

(a) Draw a cartoon of the situation, showing clearly the initial and final states.

(b) Write down an expression of conservation of energy for the ball. Underneath each term, label it – something like “final rotational kinetic energy” or “initial spring potential energy”.

(c) Show that the speed of the ball at the top of the ramp is

$$v_f = \sqrt{\frac{10 \frac{1}{2} k d^2 - m g L \sin \theta}{m}} \quad \text{which is equal to} \quad \sqrt{\frac{5}{7} k d^2 - \frac{10}{7} g L \sin \theta}.$$

3. A ball of mass m is connected to one end of a rubber band and swung in a circle. The rubber band has spring constant k , and its unstretched length is r_0 .

If it is swung at an angular velocity ω , to what length will it stretch the rubber band?