Elastic collisions and practice problems

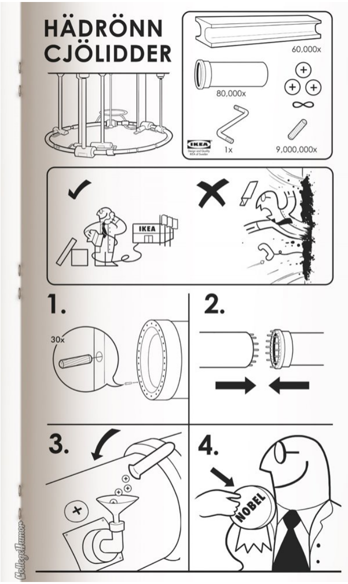
Physics 211
Syracuse University, Physics 211 Spring 2015
Walter Freeman

March 26, 2015

Announcements

- If your exam was misgraded, grade appeals will be handled the same way as before (and faster!); turn them in to me by tomorrow
- Extended Friday office hours for homework help: 10-12, then 1-3 (remember, it's due Wednesday)
- Today:
  - A bit more on the types of collisions
  - A bit of ugly math (bear with me; worst math of the semester!)
  - Lots of practice problems

Ask a Physicist: the LHC



Collisions, in more detail

- Last time, we saw that conservation of momentum helps us understand **collisions** and **explosions**
- If we know nothing else, we often don't have enough equations to solve for our unknowns...
- One dimension: one equation, often two unknowns
  - $m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$
- Two dimensions: two equations, often four unknowns
  - $m_1 v_{1,x,i} + m_2 v_{2,x,i} = m_1 v_{1,x,f} + m_2 v_{2,x,f}$
  - $m_1 v_{1,y,i} + m_2 v_{2,y,i} = m_1 v_{1,y,f} + m_2 v_{2,y,f}$
- Last time we studied **inelastic collisions**, which reduces the number of unknowns: the objects stick together
- What else might happen?

Three types of collisions, informally

- Completely inelastic collisions: "things stick together"
- Partially inelastic collisions: "things bounce a bit"
- Completely elastic collisions: "maximum amount of bounce"

Three types of collisions, formally

- Completely inelastic collisions: things stick together; maximum amount of KE lost
- Partially inelastic collisions: things bounce a bit; lesser amount of KE lost
- Completely elastic collisions: maximum bounce; **no KE lost**

Understanding elastic collisions, in one dimension

A particular situation comes up often: an object with mass m_1 and velocity v_{1i} collides elastically with an object with mass m_2 at rest.

What are their final velocities?

Understanding elastic collisions, in one dimension

A particular situation comes up often: an object with mass m_1 and velocity v_{1i} collides elastically with an object with mass m_2 at rest.

What are their final velocities?

We know both **momentum** and **kinetic energy** are conserved.

$$m_1 v_{1i} + m_2(0) = m_1 v_{1f} + m_2 v_{2f}$$
$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

In principle, we have two equations and two unknowns, and can just solve this. But the math is a bit hairy. Bear with me for just a bit (on document camera)

Understanding elastic collisions, in one dimension

$$m_1 v_{1i} + m_2(0) = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Coming out the other end, you get:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

What should you know about this?

- This is a canned solution to a particular problem
- Usually I discourage just remembering things like this...
- ... but this one is often useful, and the derivation isn't illuminating

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

- If $m_1 >> m_2$, then we basically have:

$$v_{1f} \approx v_{1i}$$

$$v_{2f} \approx 2v_{1i}$$

- If $m_1 << m_2$, then we basically have:

$$v_{1f} \approx -v_{1i}$$

$$v_{2f} \approx 2\frac{m_1}{m_2} v_{1i}$$

- Looking at limiting cases like this is a huge part of physics!

What should you know about this?

- You will *not* need to memorize these; they're on page 266 of your book, or here
- You do need to know where they come from: **conservation of momentum + conservation of KE**

Sample problems: a 1D collision (for 9:30)

A train car with a mass m is at rest on a track. Another train car also of mass m is moving toward it with a velocity v_0 when it is a distance d away. The first car hits the second and couples to it; the cars roll together until friction brings them to a stop.

If the coefficient of rolling friction is μ_r, how far do they roll after the collision?

Sample problems: a 1D collision (for 9:30)

A train car with a mass m is at rest on a track. Another train car also of mass m is moving toward it with a velocity v_0 when it is a distance d away. The first car hits the second and couples to it; the cars roll together until friction brings them to a stop.

If the coefficient of rolling friction is μ_r, how far do they roll after the collision?

Method: use conservation of momentum to understand the collision; use other methods to understand before and after!

Sample problems

Three identical train cars, coupled together, are rolling east at speed v_0. A fourth car traveling east at $2v_0$ catches up with the three and couples to make a four-car train. A moment later, the train cars hit a fifth car that was at rest on the tracks, and it couples to make a five-car train. What is the speed of the five-car train?

(from textbook; ignore friction)

Sample problems

Three identical train cars, coupled together, are rolling east at speed v_0. A fourth car traveling east at $2v_0$ catches up with the three and couples to make a four-car train. A moment later, the train cars hit a fifth car that was at rest on the tracks, and it couples to make a five-car train. What is the speed of the five-car train?

(from textbook; ignore friction)

Do we care about the order these things happen in?

57. ‖ The spring shown in **FIGURE P11.57** is compressed 50 cm and used to launch a 100 kg physics student. The track is frictionless until it starts up the incline. The student's coefficient of kinetic friction on the 30° incline is 0.15.
    a. What is the student's speed just after losing contact with the spring?
    b. How far up the incline does the student go?

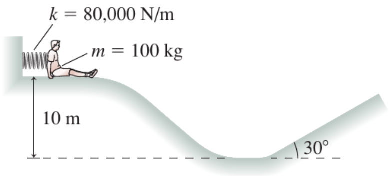


FIGURE P11.57

Sample problems

A firecracker in a coconut blows the coconut into three pieces. Two pieces of equal mass fly off south and west, perpendicular to each other, at speed v0. The third piece has twice the mass as the other two. What are the speed and direction of the third piece?

Sample problems

49. ‖ Truck brakes can fail if they get too hot. In some mountainous areas, ramps of loose gravel are constructed to stop runaway trucks that have lost their brakes. The combination of a slight upward slope and a large coefficient of rolling resistance as the truck tires sink into the gravel brings the truck safely to a halt. Suppose a gravel ramp slopes upward at 6.0° and the coefficient of rolling friction is 0.40. Use work and energy to find the length of a ramp that will stop a 15,000 kg truck that enters the ramp at 35 m/s (≈ 75 mph).

73. The spring in **FIGURE CP11.73** has a spring constant of 1000 N/m. It is compressed 15 cm, then launches a 200 g block. The horizontal surface is frictionless, but the block's coefficient of kinetic friction on the incline is 0.20. What distance d does the block sail through the air?

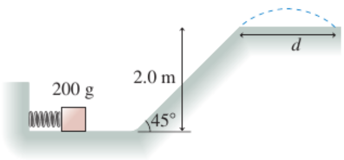


FIGURE CP11.73