Rotational motion

Physics 211
Syracuse University, Physics 211 Spring 2022
Walter Freeman

April 27, 2022

Announcements

- Next homework is due next Wednesday
- Upcoming help hours:
  - Today 9:45-10:45ish (between classes) and 1:30-3:30
  - Tomorrow 1:00-3:00
  - **Monday 2:00-5:00**
- I am working with some study groups already, as are our coaches. Want to form one? Let me know when/where and I or a coach will come help.
- Recitation tomorrow:
  - I know there's a block party

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  - If you complete the exercise tomorrow in recitation, we'll award you five points extra credit

A review: Angular momentum

Translational motion

- Moving objects have momentum
- $\vec{p} = m\vec{v}$
- Momentum conserved if there are no external forces

Rotational motion

- Spinning objects have angular momentum L
- $L = I\omega$
- Angular momentum conserved if no external torques

$\rightarrow L = I\omega =$ constant; analogue to conservation of momentum

Conservation of angular momentum

We saw that the conservation of momentum was valuable mostly in two sorts of situations:

- Collisions: two objects strike each other
- Explosions: one object separates into two

There is a third common case for conservation of angular momentum:

- Collisions: a child runs and jumps on a merry-go-round (your homework)
- Explosions: throwing a ball off-center (our example)
- A spinning object changes its moment of inertia

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Example: the ice-skater

If I am standing on my spinning platform rotating at $\omega_i = 1$ rad/sec while holding the dumbbells at arm's length, how fast do I start spinning when I pull them inward?

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$$I_i \omega_i = I_f \omega_f$$

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$$\omega_f = \omega_i \frac{I_i}{I_f}$$

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Now we just have to estimate the moments of inertia...

Rotational kinetic energy, again

Translation	Rotation
Force $\vec{F}$ Mass m Newton's second law $\vec{F}_{\text{tot}} = m\vec{a}$	Torque $\vec{\tau} = \vec{r} \times \vec{F}$ Moment of inertia I Newton's second law for rotation $\tau_{\text{tot}} = I\alpha$
Kinetic energy $KE = \frac{1}{2}mv^2$ Work $W = \vec{F} \cdot \Delta\vec{s}$ Power $P = \vec{F} \cdot \vec{v}$	Kinetic energy $KE = \frac{1}{2}I\omega^2$ Work $W = \tau\Delta\theta$ Power $P = \tau\omega$

Rotational kinetic energy and the rotational work-energy theorem work like their translational counterparts.

Rotational kinetic energy, again

Our previous study of rotational kinetic energy went like this:

- When an object is spinning, it has kinetic energy $KE_{\text{rot}} = \frac{1}{2}I\omega^2$
- When an object rolls down a hill (for instance), we write

$$\text{(initial grav. potential energy)} = \text{(final translational KE)} + \text{(final rotational KE)}$$
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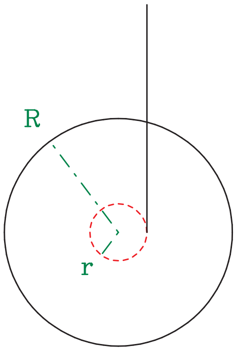
We ignored questions like:

- Doesn't friction do work on the rolling object?
- If there are strings involved, what about tension?
- What about the rotational work-energy theorem?
- What about rotational power? (Fast cars! Bicycles!

Let's go back and understand those things now.

We'll use as our example the Yo-Yo from recitation yesterday – but, this time, we'll analyze it with *energy* instead of just force and acceleration.

Suppose I release a Yo-Yo whose string has a length h. How fast will its center be moving when it runs out of string?



A: $v_f < \sqrt{2gh}$, because the tension in the string slows it down

B: $v_f < \sqrt{2gh}$, because part of the GPE is required to make the Yo-Yo spin

C: $v_f = \sqrt{2gh}$, by the conservation of energy

D: $v_f > \sqrt{2gh}$, because the spinning disk speeds it up

Answer C is what we get if there is no string. (We already know how to do that.)

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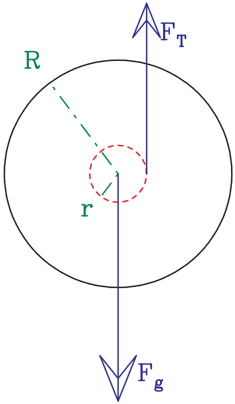
Answer A is related to what you did in recitation yesterday; in a force diagram for the Yo-Yo, the tension means that the net downward force is less than mg.



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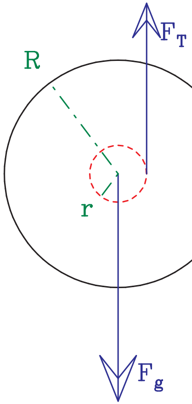

Answer B makes sense as well, though: if the Yo-Yo spins as it falls, then **some energy is required to make it spin**, leaving less available energy for translational kinetic energy.

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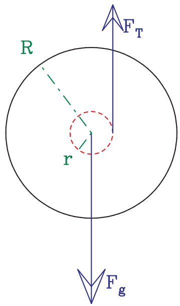

Before we learned to analyze this with conservation of energy:

$$\text{(initial grav. potential energy)} = \text{(final translational KE)} + \text{(final rotational KE)}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mR^2\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}m\frac{R^2}{r^2}v^2$$

$$v = \sqrt{\frac{2gh}{1 + \frac{1}{2}\frac{R^2}{r^2}}}$$

The work done by tension

We know the work-energy theorem for translational motion (for constant $\vec{F}$):

$$W_{\text{trans}} \equiv \Delta \frac{1}{2} m v^2 = \vec{F} \cdot \Delta \vec{s}$$

Replacing m, $\vec{F}$, $\vec{s}$, and v^2 with their rotational counterparts, we get:

$$W_{\text{rot}} \equiv \Delta \frac{1}{2} I \omega^2 = \tau \Delta \theta$$

This is the *rotational work-energy theorem.*

Which is true regarding the work done by tension here?

A: $W_{\text{total}} = 0$

B: $W_{\text{trans}} > 0, W_{\text{rot}} > 0$

C: $W_{\text{trans}} < 0, W_{\text{rot}} > 0$

D: $W_{\text{trans}} > 0, W_{\text{rot}} < 0$

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The string makes the Yo-Yo fall more slowly (negative translational work), but makes it spin (positive rotational work). That means Answer C is correct. What about Answer A?

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If the Yo-Yo falls a distance h, it turns through a (positive!) angle given by $\Delta\theta = h/r$.

The torque applied by the tension is $\tau = Tr$ (positive!).

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Rotational work: $W_{\text{rot}} = \tau\Delta\theta = Tr(h/r) = Th.$
Translational work: $W_{\text{trans}} = \vec{F} \cdot \Delta\vec{s} = -Th.$

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Translational work: $W_{\text{trans}} = \vec{F} \cdot \Delta\vec{s} = -Th$.

$\rightarrow$ The total work done by tension here is zero. (We could have guessed that!)

Conservation of energy, including rotation

$$\mathrm{PE_i} + \frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 + W_{\mathrm{NC}} = \mathrm{PE_f} + \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

Which expression will let us find the velocity of the Yo-Yo at the bottom?

A : $mgh - Th = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$

B : $mgh + \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$

C : $mgh = \frac{1}{2}mv_f^2$

D : $mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$

What about rolling objects?

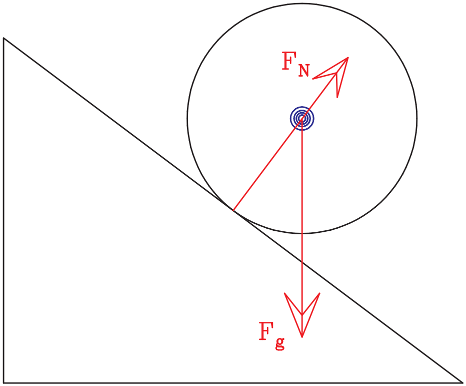
In the Yo-Yo problem, we saw that:

- Tension did positive rotational work (it made the Yo-Yo spin faster)
- Tension did negative translational work (it made the Yo-Yo move more slowly)
- ... the **net work done by tension was zero**.

This happened because the string was stationary, and thus enforced $a = \pm \alpha r$. This is also true in rolling motion.

An object rolling down a hill

Consider first a ball sliding down a hill without friction.



Which of these forces applies a torque to the ball?

A: Just the normal force

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C: Both of them

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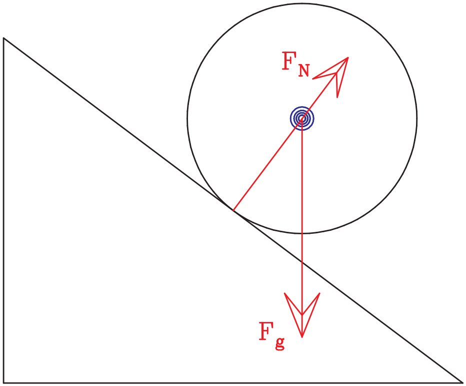

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Friction is required to make the ball spin!

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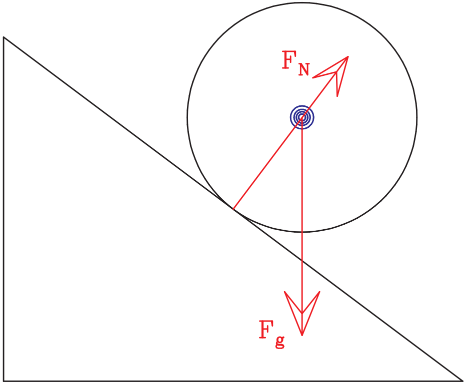
If the ball *rolls without slipping*...



What is true about the frictional force?

A: Static friction points down the ramp
B: Static friction points up the ramp
C: Kinetic friction points down the ramp
D: Kinetic friction points up the ramp
E: There is no friction

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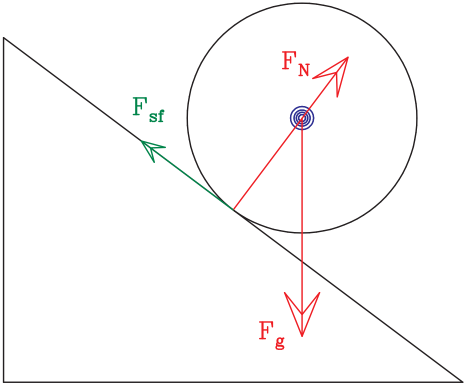

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The point of contact would slide downward without friction, so friction points back up the ramp. This is static friction since the ball doesn't slide.

Energy rolling down a hill

Static friction does no total work on the ball:

- it reduces the translational kinetic energy $\frac{1}{2}mv^2$
- it increases the rotational kinetic energy $\frac{1}{2}I\omega^2$
- ... but it leaves the sum $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ unchanged

TABLE 6.1 Coefficients of friction

Materials	Static μ_s	Kinetic μ_k	Rolling μ_r
Rubber on concrete	1.00	0.80	0.02
Steel on steel (dry)	0.80	0.60	0.002
Steel on steel (lubricated)	0.10	0.05	
Wood on wood	0.50	0.20	
Wood on snow	0.12	0.06	
Ice on ice	0.10	0.03	

This is not *quite* true – rolling friction does exist. There is a little bit of overall negative work done as tires flex and so on, but it is small.

(From *Physics for Scientists and Engineers*, Knight, 3rd ed.)

This means that we can use our standard expression for conservation of energy for rolling objects, *ignoring* the force of static friction required to keep them from slipping:

$$\mathrm{PE}_i + \frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 + W_{\text{other}} = \mathrm{PE}_f + \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

How fast will each object ($I = \lambda m r^2$) be traveling at the bottom of the ramp?

How high must I start the ball for it to make it around the loop?

Rotational dynamics and power

If $W = \tau \Delta\theta$, then $P = \tau\omega$.

If I want to supply a power P, I can either exert a large torque at a small angular velocity, or a small torque at a large angular velocity.

$\rightarrow$ bicycle demonstration!

Gears

Note for recitation tomorrow:

Whether gears are linked by a chain or directly:

- The *forces* between them are the same (Newton's 3rd Law)
- The *tangential velocity* of the gears is the same
- The angular velocities are *not* the same
- The torques are *not* the same