Recitation 13 29 march 2019

1. train of mass m_1 is traveling along a railway at speed v_1 when it encounters another train car of mass m_2. the two cars couple together when they collide.
How fast are they traveling after the collision?

THE CARS COUPLE TOGETHER AFTER THE COLLISION
$\Rightarrow$ INELASTIC COLLISION

CONSERVATION OF MOMENTUM FOR INELASTIC:

$$m_1\vec{u}_1 + m_2\vec{u}_2 = (m_1+m_2)\vec{v}$$

INITIAL VELOCITY OF CAR 2 IS NOT GIVEN SO
$\Rightarrow \vec{u}_2 = 0\,m/s$

$$m_1\vec{u}_1 = (m_1+m_2)\vec{v}$$

$$\Rightarrow \vec{v} = \frac{m_1 u_1}{(m_1+m_2)}$$

(HERE, I AM USING $\vec{u}$ FOR INITIAL VELOCITIES & $\vec{v}$ FOR FINAL VELOCITIES).

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2. explain how the conservation of momentum is a consequence of newton's second & third laws.

① NEWTON'S SECOND LAW: $\sum F = ma$
ACCELERATION IS A CHANGE IN VELOCITY PER UNIT TIME
SO WE CAN WRITE $F = m\frac{dv}{dt}$
THIS IS EQUIVALENT TO SAYING
"FORCE IS EQUAL TO THE CHANGE IN MOMENTUM"

② NEWTON'S THIRD LAW STATES EVERY FORCE IS BALANCED BY AN EQUAL & OPPOSITE REACTION FORCE.
SINCE WEVE SHOWN THAT FORCE = CHANGE IN MOMENTUM WE CAN CONCLUDE THAT A CHANGE IN MOMENTUM MUST ALSO BE BALANCED BY AN EQUAL & OPPOSITE CHANGE IN MOMENTUM.

3. the driver of a mini cooper (mass 1200kg) is traveling at 10m/s westward when he runs a stop sign & collides with a toyota camry (mass 2000kg) traveling at 15m/s northward.
the two cars stick together after the collision.

a. what is the total momentum before the collision?

THE CARS STICK TOGETHER, SO INELASTIC COLLISION

$$\vec{p}_i = \vec{p}_f$$

$$m_1\vec{u}_1 + m_2\vec{u}_2 = (m_1+m_2)\vec{v}$$

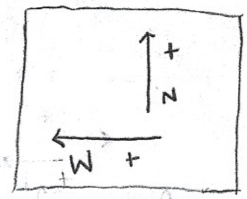
WANT INITIAL MOMENTUM, SO FOCUS ON LHS OF EQUATION ("LEFT HAND SIDE")

$$\vec{p}_i = m_1\vec{u}_1 + m_2\vec{u}_2$$

$\vec{u}_1$ & $\vec{u}_2$ ARE IN DIFFERENT DIRECTIONS, SO WE ACCOUNT FOR THIS BY USING VECTOR NOTATION.

$\vec{u}_1 = 10m/s\ \hat{x}$ ← (BECAUSE TRAVELING WEST)

$\vec{u}_2 = 15m/s\ \hat{y}$ ← (BECAUSE TRAVELING NORTH)



$$\vec{p}_i = m_1\vec{u}_1 + m_2\vec{u}_2$$

$$\vec{p}_i = (1200kg)(10m/s\ \hat{x}) + (2000kg)(15\ m/s\ \hat{y})$$

$$\vec{p}_i = (12000kg\cdot m/s)\hat{x} + (30000kg\cdot m/s)\hat{y}$$

NOW THERES MOMENTUM IN THE $\hat{x}$ DIRECTION & IN THE $\hat{y}$ DIRECTION.

WANT TOTAL MOMENTUM, SO WANT MAGNITUDE OF THE MOMENTUM (NOT IT WRITTEN IN DIRECTIONS & IN COMPONENTS)

$$|p| = \sqrt{(p_x)^2 + (p_y)^2}$$

MEANS MAGNITUDE OF p

$$|p| = \sqrt{(12000kg\cdot \tfrac{m}{s})^2 + (30000kg\cdot \tfrac{m}{s})^2}$$

$$|p| = 32310.9\ kg\cdot \tfrac{m}{s}$$

ANSWER IS ONE VALUE SINCE WE WANT THE MAGNITUDE

IF WE WERE ASKED FOR SOMETHING LIKE "HOW MUCH p IS IN $\hat{x}$ & HOW MUCH IN $\hat{y}$ DIRECTION" OR "WHAT IS p_x & p_y" THEN THE ANSWER WOULD BE:

$$\vec{p} = p_x + p_y$$

$$\Rightarrow \vec{p} = (12000kg\cdot m/s)\hat{x} + (30000kg\cdot \tfrac{m}{s})\hat{y}$$

b. what is total momentum after the collision?

DUE TO CONSERVATION OF MOMENTUM, THE MOMENTUM OF AN ISOLATED SYSTEM IS CONSTANT.

$\therefore$ THE TOTAL MOMENTUM BEFORE A COLLISION IS EQUAL TO TOTAL MOMENTUM AFTER THE COLLISION

$\Rightarrow$ INITIAL MOMENTUM $= 32310 \, kg \cdot \frac{m}{s}$

$\therefore$ FINAL MOMENTUM $= 32310 \, kg \cdot \frac{m}{s}$

c. speed & direction of cars after the collision?

$$\vec{P_i} = \vec{P_f}$$

$$m_1 \vec{u}_1 + m_2 \vec{u}_2 = (m_1 + m_2)\vec{v}$$

$$(12000 \, kg \cdot \tfrac{m}{s})\hat{x} + (30000 \, kg \cdot \tfrac{m}{s})\hat{y} = (m_1 + m_2)\vec{v}$$

$$\frac{(12000 \, kg \cdot \frac{m}{s})\hat{x}}{(m_1 + m_2)} + \frac{(30000 \, kg \cdot \frac{m}{s})\hat{y}}{(m_1 + m_2)} = \vec{v}$$

$$3.75\hat{x} + 9.375\hat{y} = \vec{v}$$

AGAIN, WE NEED MAGNITUDE

$$|v| = \sqrt{v_x^2 + v_y^2}$$

$$|v| = \sqrt{(3.75)^2 + (9.375)^2}$$

$$|v| = 10.1 \, m/s$$

DIRECTION:

$$\tan\theta = \frac{v_y}{v_x} \Rightarrow \theta = \arctan\left(\frac{v_y}{v_x}\right)$$

$$\theta = \arctan\left(\frac{9.375 \, m/s}{3.75 \, m/s}\right)$$

$$\theta = 68.2' \text{ north of west}$$

NOTE: I SOLVED FOR $\vec{v}$ THIS WAY SO THAT I COULD OBTAIN $\vec{v}$ IN THE $\hat{x}$ & $\hat{y}$ DIRECTIONS TO USE IN SOLVING FOR THE ANGLE. IF I DIDN'T NEED THIS & INSTEAD ONLY NEEDED THE MAGNITUDE I COULD USE THE FACT WE ALREADY FOUND $\vec{P_i}$ TO BE $32310 \, kg \cdot \frac{m}{s}$ & $\therefore \vec{P_f} = 32310 \, kg \cdot \frac{m}{s}$ & DO THIS:

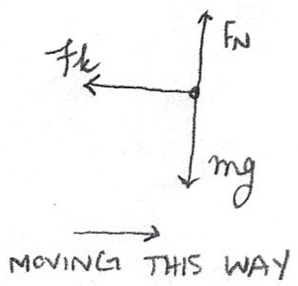
$$\vec{P_f} = 32310 \, kg \cdot \tfrac{m}{s} = (m_1 + m_2)v$$

$$\Rightarrow v = \frac{32310 \, kg \cdot \frac{m}{s}}{(m_1 + m_2)} = 10.1 \, m/s$$

2. If the coefficient of kinetic friction between the cars tires & the pavement is 0.6, how far do they skid before coming to rest?

LOOKING AT AFTER THE COLLISION → CARS COMING TO REST
LET THE VELOCITY OF THE SMASHED TOGETHER CARS BE $\vec{u}$
(WE CALCULATED THIS AS $\vec{v} = 10.1\,m/s$ THAT WE WILL USE LATER)
I'M CALLING IT $\vec{u}$ NOW BECAUSE THATS NOW THE INITIAL VELOCITY, & THE FINAL $= \vec{v} = 0\,m/s$

METHOD 1: FORCES



WANT DISTANCE.
WE HAVE $\vec{u}$, $\vec{v}\,(=0\,m/s)$ AND NOTHING ELSE.
IF WE HAD TIME, WE COULD GET DISTANCE FROM $\vec{u}$
IF WE HAD ACCELERATION, WE COULD USE $v^2 - u^2 = 2a\Delta x$
WE HAVE A FORCE (FRICTION) & WE CAN GET $\vec{a}$ FROM THAT

1. CALCULATE ACCELERATION BY SUMMING FORCES IN FORCE DIAGRAM & SOLVING FOR $\vec{a}$
2. PLUG $\vec{a}$ INTO THIRD KINEMATICS & FIND x.

FROM FORCE DIAGRAM:

$$\sum F_y = F_N - mg = \cancel{ma}^{\,0}$$
$$\Rightarrow F_N = mg$$

SINCE CARS ARE JOINED TOGETHER LET $m = m_1 + m_2$ (IN THE END, WE DONT NEED MASS)

$$\sum F_x = f_k = ma \quad \text{where } f_k = \mu F_N$$
$$\Rightarrow f_k = \mu mg$$
$$\Rightarrow \mu \cancel{m} g = \cancel{m} a$$
$$\mu g = a$$
$$(0.6)(10\,m/s^2) = a$$
$$\Rightarrow a = 6\,m/s^2$$

STEP 2:

$$v^2 - u^2 = 2a\Delta x$$
$$\Rightarrow \Delta x = \frac{\cancel{v^2}^{\,0} - u^2}{2a} = \frac{(10.1\,m/s)^2}{2\,(6\,m/s^2)}$$
$$\Rightarrow \Delta x \approx 8.5\,m$$

METHOD 2: WORK ENERGY THEOREM

NOW, THINK IN TERMS OF ENERGY. START BY GOING THROUGH A "CHECKLIST" LIKE THI

1. PE? NO. EVERYTHING IS GROUND LEVEL & NO SPRINGS
2. KE? YES. THE CARS ARE MOVING INITIALLY & THEN COME TO REST SO THERE'S KE, & IT HAS CHANGED, WHICH BRINGS US TOOO...
3. WORK! YES, THERE MUST BE WORK BECAUSE THE KE CHANGED. FURTHER, THERE IS A FRICTIONAL FORCE, ACTING OVER SOME DISTANCE (WHAT WERE SOLVING FOR), SO WE NEED TO CONSIDER THE WORK FROM FRICTION (THINK BACK, THE FRICTION IS WHAT STOPS THE CARS, ∴ THE WORK FROM FRICTION BRINGS THE CARS FROM $\vec{u} \rightarrow 0\,m/s$ JUST REALIZE THAT, THAT WORK CAUSES THE CHANGE IN KE, IT CAN HELP TO THINK THROUGH PROBLEMS IN THIS WAY).

WERE LEFT WITH:

$$\Delta KE = W_{friction} = f_k \cdot \Delta s$$

IM GOING TO TAKE IT FOR GRANTED FROM BEFORE THAT

$$f_k = \mu m g$$

$$\frac{1}{2} m v^2 - \frac{1}{2} m u^2 = f_k \cdot \Delta s$$

$$\cancel{\frac{1}{2} m v^2}^{\,0} - \frac{1}{2} m u^2 = \mu m g \cdot \Delta s$$

$$-\frac{1}{2} \cancel{m} u^2 = \mu \cancel{m} g \cdot \Delta s$$

OMITTING THE NEGATIVE BECAUSE ITS NOT NECESSARY HERE.

$$\frac{u^2}{2 \mu g} = \Delta s$$

$$\frac{(10.1\,m/s)^2}{2(0.6)(10\,m/s^2)} = \Delta s \quad \Rightarrow \Delta s = 8.5\,m$$

NOTE: WE CAN USE ENERGY TO SOLVE FOR HORIZONTAL DISPLACEMENT HERE BECAUSE WERE GIVEN A FORCE THAT ACTS IN THE SAME DIRECTION AS THE DISPLACEMENT & ∴ WE CAN USE THE WORK FROM THAT FORCE TO SOLVE FOR SAID DISPLACEMENT. (WERE NOT IN FREE FALL HERE, WHERE THERES ONLY GRAVITY) THIS IS IN REFERANCE TO RECITATION 12, QUESTION ONE, PART D.

4. a 5kg block sits on a table where $\mu_k = 0.6$.
2 people throw things at it: lump of clay & a rubber ball.
both have mass 500g & strike the box with speed of $u = 4m/s$. lump of clay collides inelastically while the ball bounces back with $v = 2m/s$.

a. without doing any math, which object will knock the box further?

THE RUBBER BALL.

SINCE THERE IS A LARGER CHANGE IN MOMENTUM OF THE BALL (SHOWN BY THE FACT IT BOUNCED BACK, IT STARTED WITH POSITIVE MOMENTUM & LEFT WITH NEGATIVE). SO MORE MOMENTUM IS TRANSFERRED TO THE BOX.

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b. calculate how far each object knocks the box?

HOWWW ABOUT WE DO CLAY FIRST, GET THAT OVERWITH. FOR SOME REASON I ALWAYS LIKED ELASTIC COLLISIONS MORE. IDK. <u>ANYWHO</u>

<u>IMMEDIATELY</u> YOU SHOULD BE THINKING "MOMENTUM" BECAUSE WE HAVE COLLISIONS, BUT WE CANNOT GET DISTANCE FROM MOMENTUM ALONE, WE NEED MORE.

WERE GIVEN FRICTION FORCE ALSO, & SINCE ~~THE~~ WHEN THE BOX TRAVELED, IT OVERCAME THAT FRICTION UNTIL ITS STOPPED, THAT DISPLACEMENT (THAT WERE LOOKING FOR) IS PARALLEL TO THE FRICTION FORCE. SOOOOO WE <u>USE</u> <u>WORK-ENERGY</u>.

OKAY! GAME PLAN! (THIS IS MY LAST QUESTION SO IM GETTING EXCITED)

1. USE CONSERVATION OF MOMENTUM TO SOLVE FOR THE VELOCITY OF BOX AFTER THE COLLISION. (NOTE: TO FURTHER ENFORCE WHAT WAS DISCUSSED BEFORE REGARDING ΔKE & WORK, THE BOX HAS SOME INITIAL VELOCITY & THEN COMES TO REST

$\therefore$ THERES SOME CHANGE IN KE, & WE KNOW NOW THAT, THAT IS FROM WORK, IN THIS CASE ITS WORK FROM FRICTION.
(THIS MAKES SENSE, RIGHT? FRICTION OPPOSES THE MOTION OF THE BOX UNTIL IT COMES TO REST).

2. WITH THE INITIAL VELOCITY OF THE BOX & THE FINAL (WHICH IS ZER CALCULATE THE CHANGE IN KE. WHICH WE CAN RELATE TO WORK & THEN FIND THE DISPLACEMENT.

THERES NO WAY IM GOING TO FIT ANY CALCULATIONS ON THIS PAGE.

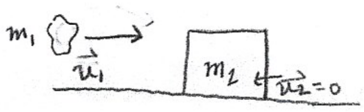
NEXT.

IN CASE YOU FORGOT, CLAY FIRST.
THE CLAY IS THE INELASTIC COLLISION.

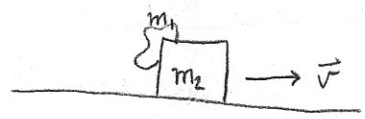
SO:

AGAIN, ILL USE u FOR INITIAL VELOCITIES & v FOR FINAL.

①



②



$$\vec{p}_i = \vec{p}_f$$

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$ SINCE ITS INELASTIC, $\vec{v}_1 = \vec{v}_2 = \vec{v}$

$$m_1\vec{u}_1 + \cancel{m_2\vec{u}_2} = (m_1 + m_2)\vec{v}$$

$$m_1\vec{u}_1 = (m_1 + m_2)\vec{v}$$

$$\Rightarrow \vec{v} = \frac{m_1\vec{u}_1}{(m_1 + m_2)} = \frac{(0.5\,kg)(4\,m/s)}{(0.5\,kg + 5\,kg)}$$

(WERE GIVEN THE CLAY & BALL ARE 500g JUST DONT FORGET TO CONVERT TO kg)

$$\Rightarrow \vec{v} = 0.36\,m/s$$

NOW FOR PART 2 OF THE PLAN, WERE LOOKING FROM WHEN THE ~~BOX~~ STARTED MOVING UNTIL IT CAME TO REST.
FOR CONSISTENCY, FOR THE 10000000TH TIME, ILL USE $\vec{u}$ FOR INITIAL VELOCITY OF THE BOX, & $\vec{v}$ FOR FINAL (WHICH AGAIN IS JUST ZERO)

$$\Delta KE = W$$

where the friction force, $f_k = \mu F_N$ & $F_N = mg$
also $m = m_1 + m_2$
$\Rightarrow f_k = \mu mg$

$$\cancel{\tfrac{1}{2}mv^2} - \tfrac{1}{2}mu^2 = f_k \cdot \Delta s$$

IM GOING TO OMIT THE NEGATIVE AS ITS NOT NECESSARY.

$$\tfrac{1}{2}\cancel{m}u^2 = \mu\cancel{m}g \cdot \Delta s$$

$$\frac{u^2}{2\mu g} = \Delta s = \frac{(0.36\,m/s)^2}{2(0.6)(10\,m/s^2)}$$

$$\Delta s = 0.0108\,m$$

BUT WAIT THERES MORE!

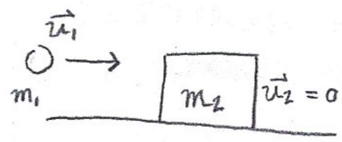
HAH. GET IT.

BUT

BUT THE FUN PARTS NEXT, THE ELASTIC COLLISION.

PICTURES FIRST,

1



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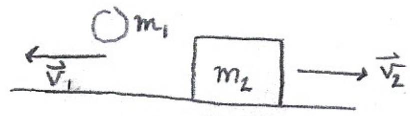

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

WE KNOW $\vec{v}_2 \neq -2 m/s$ (BECAUSE IT BOUNCED BACK IN OPPOSITE DIRECTION ITS NEGATIVE) WERE SOLVING FOR $\vec{v}_2$

$$m_1\vec{u}_1 + \cancel{m_2\vec{u}_2}^{\,0} - m_1\vec{v}_1 = m_2\vec{v}_2$$

$$m_1\vec{u}_1 - m_1\vec{v}_1 = m_2\vec{v}_2$$

$$\vec{v}_2 = m_2^{-1}(m_1\vec{u} - m_1\vec{v}_1)$$

$$\vec{v}_2 = \frac{(0.5kg)(4m/s) - (0.5kg)(-2m/s)}{5kg} \Rightarrow \vec{v}_2 = 0.6 m/s$$

PART 2:

let $\vec{v}_2$ be u (AGAIN)

$$\Delta KE = W$$

$$\cancel{\tfrac{1}{2}mv^2}^{\,0} - \tfrac{1}{2}mu^2 = f_k \cdot \Delta s$$

(TAKING IT FOR GRANTED FROM BEFORE $f_k = \mu_k mg$)(& OMIT NEGATIVE)

$$\tfrac{1}{2}\cancel{m}u^2 = \mu_k \cancel{m}g \cdot \Delta s$$

$$\frac{u^2}{2\mu_k g} = \Delta s$$

$$\frac{(0.6m/s)^2}{2(0.6)(10m/s^2)} = \Delta s$$

$$0.03m = \Delta s$$

$0.03m > 0.01m$ SO OUR GUESS IS CORRECT.

NOTE: YOU CAN ALSO SOLVE FOR DISTANCE USING FORCES & $\sum F = ma$, AS WE DID IN THE PROBLEM ABOUT THE MINI COOPER & THE CAMRY JUST BEFORE.