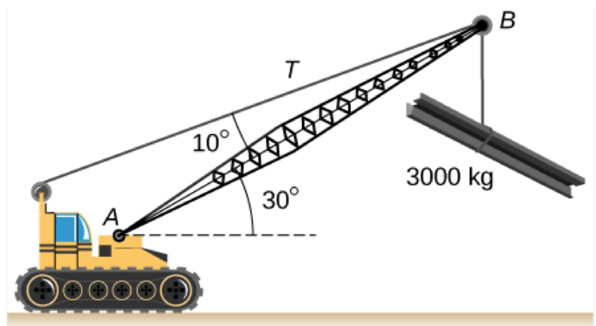
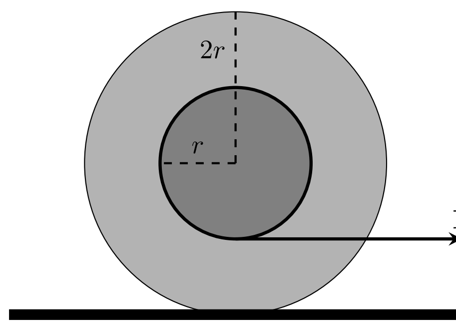
PHY 211 Homework 14

Due April 30, 2020

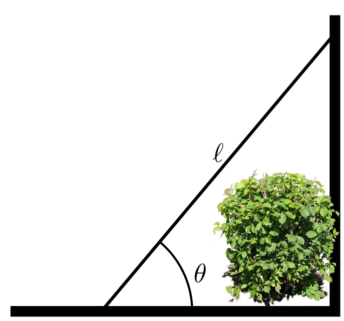
Problem 1. A 12.0 m boom, AB, of a crane lifting a 3000 kg load is shown below. The center of mass of the boom is at its geometric center, and the mass of the boom is 1000 kg. For the position shown, calculate tension T in the cable and the force at the axle A.



Problem 2. Consider the spool where you can pull on the string wrapped around the inside that comes out the bottom, like we saw in lecture. Assume all the mass m is contained inside the center cylinder, and the radius of the outer wheel is twice the cylinder radius r. If the coefficient of static friction is μ_s, with how much force could you pull the string horizontally and have the spool roll without slipping?



Problem 3. To get up to clean out the gutters, a person (mass 70 kg) places an $\ell = 4$ m ladder with a mass of 5 kg against their house. Because of some bushes[1]they can't get very close, and have to put the ladder at an angle $\theta = 50°$. Assume there is no friction from the wall. If the coefficient of static friction between the ladder and ground is 0.7, how far up the ladder can the person climb without the ladder slipping?



[1]Image from pngimg.com CC 4.0 BY-NC

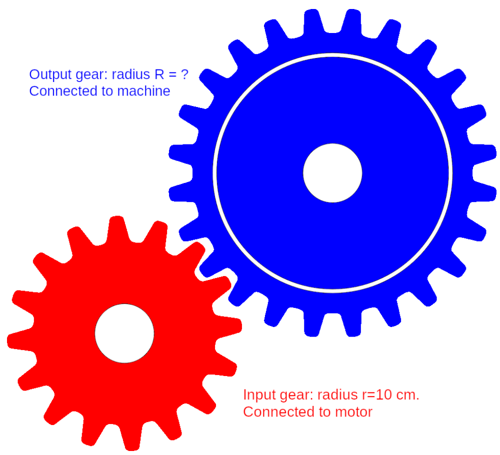
Problem 4. Consider an electric motor that turns a driveshaft. This driveshaft is connected to a machine; the motor applies a torque to the driveshaft, and the machine applies an equal and opposite torque. Thus, the motor delivers power to the machine. For this problem, the motor will always be spinning at a constant angular velocity, so $\sum \tau = 0$.
Suppose that the motor can apply a torque $\tau_m = 100\,\mathrm{N\,m}$ to the driveshaft, but it has a maximum angular velocity $\omega_{\max} = 50\,\mathrm{rad/s}$.

(a) What is the maximum power that the motor can deliver to the machine? (Hint: For translational motion, $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$. What is the analogous formula for rotation?)

(b) However, in general, machines need to run at different speeds; for instance, cars can drive at many different speeds. Suppose that the operator of the machine wants to run it at low speed – say, at $25\,\mathrm{rad/s}$. Can the motor still deliver its full power in this case? If not, how much power can it deliver?

(c) The motor will simply be *unable* to rotate any faster than $50\,\mathrm{rad/s}$. Since the machine operator may want to run the machine at any speed, and will likely want the full power from the motor at *any* speed, they construct a transmission out of gears.

In this figure, the motor is connected to the red gear; the machine is connected to the blue gear. We will first think about how this transmission works using a single blue gear with a radius of $R = 20\,\mathrm{cm}$ in order to understand the principles at work here; then, we will think about the advantages of *shifting* gears, as in a bicycle or car transmission.



(d) Suppose that the output (blue) gear has $R = 20\,\mathrm{cm}$. If the motor is spinning at its maximum speed and torque ($\omega = 50\,\mathrm{rad/s}$, $\tau = 100\,\mathrm{N\,m}$), calculate the angular velocity, torque supplied to the machine, and power supplied to the machine. *(Hint: Newton's third law applies to the forces that the two gears exert on each other, and thus you can calculate how the torques relate to each other.)* How does this power compare to the maximum power that the motor could deliver to the machine at this angular velocity without the transmission?

(e) Suppose that the engineer designing the machine wants to be able to run the machine at an angular velocity of $\omega = 200\,\mathrm{rad/s}$. What radius should the output gear have?

(f) A bicycle uses a chain to connect the input and output gears, but the principle is the same: the rider's legs are limited in both their torque and their angular velocity. They want to be able to apply maximum power to the output gear (connected to the rear wheel) for a range of angular velocities – whether this is to climb a steep hill at low speed, or to go as fast as possible on flat ground. On a bicycle, the rider can change the radius of both input and output gears. Discuss briefly how this helps the rider deliver maximum power to the wheel at any speed.