PHY 211 HOMEWORK 4

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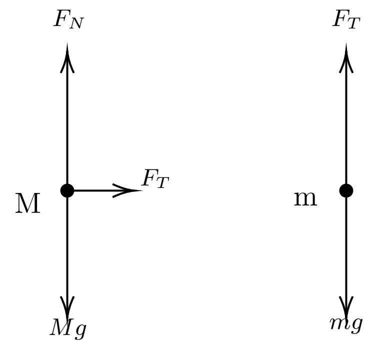
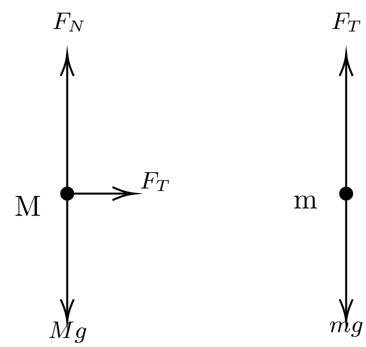
Problem 1 *An object of mass M sits on a frictionless table; it is connected by a light string to a hanging mass m.*



(a) Find both the tension in the string and the acceleration of the masses in terms of M, m, and g.



$$\sum F = a_x M \qquad\qquad \sum F = a_y M$$
$$F_T = a_x M \qquad\qquad T - mg = -a_y m$$
$$T = -a_y m + mg$$

Setting the two Tensions equal to each other:

$$a_x M = mg - ma_y$$

Since accelerations are equal, we know $a_x = a_y$:

$$Ma = mg - ma$$
$$Ma + ma = mg$$
$$a(M + m) = mg$$
$$\boxed{a = \frac{mg}{M+m}}$$

(b) What is the tension in the limit where $M >> m$?

$$T = M \cdot \frac{mg}{M+m}$$
$$\therefore \lim_{M\to\infty} M \cdot \frac{mg}{M+m} = \lim_{M\to\infty} \frac{Mmg}{M} = mg$$

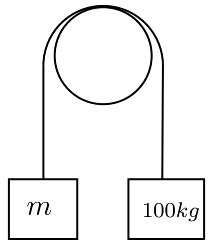
The result indicates that as the mass of the block on the table gets very high (towards infinity), the tension in the cable remains at mg. This makes sense because the tension in the string is due to the mass of block m, not M.

(c) What is the acceleration in the limit where $m >> M$?

$$a = \frac{mg}{M+m}$$
$$\lim_{m\to\infty} a = \lim_{m\to\infty} \frac{mg}{M+m} = \lim_{m\to\infty} \frac{mg}{m} = g$$

The result indicates the weight of the hanging mass increases, the acceleration approaches that of gravity. This makes sense since if M is a bowling ball and m is a feather, the system would not move very fast. However, if M is a feather and m is a bowling ball (the hanging mass has a much greater mass), then the feather would barely resist the force of mass m pulling it down, and the system would accelerate close to the acceleration due to gravity.

Problem 2 *Two masses are connected by a light string and draped over a light, frictionless pulley. One has mass m, which you will find; the other has mass $M = 100kg$.*



(a) Find an expression for the acceleration of the mass M in terms of M, m, and g.

$$\sum F = (m + M)a_{sys}$$
$$mg - Mg = (m + M)a_{sys}$$
$$\boxed{a_{sys} = \frac{mg - Mg}{m + M}}$$

(b) If it takes a time $\tau = 7$ s to hit the ground after it is released, what is the value of m?
Given: $\tau = 7\text{sec}$, $d = 1m$

We can find the acceleration:

$$a = \frac{g(m - M)}{n + M}$$

Solving for the acceleration using kinematics:

Using a first equation:

$$v_f = v_i + at \to v_f = at \to v_f = 7a$$

Plugging that into a second equation:

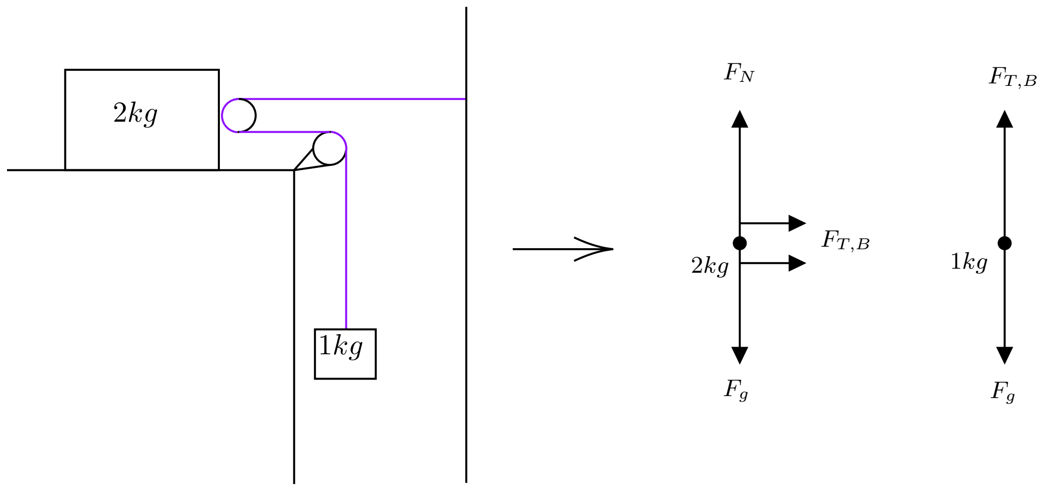
$$v_f^2 = v_i^2 + 2ad$$
$$(7a)^2 = 2a$$
$$a = \frac{2}{49}$$

Solving for the distance:

$$-\frac{2}{49} = \frac{g(m - M)}{m + M}$$
$$-\frac{2}{49g} = \frac{m - M}{m + M}$$
$$-\frac{2}{49g}m - \frac{2}{49g}M = m - M$$
$$-\frac{2}{49g}M + M = m + \frac{2}{49g}m$$
$$\boxed{m = \frac{-\frac{2}{49g}M + M}{1 + \frac{2}{49g}}}$$

Plugging in known values: $m = \frac{-\frac{2}{49g}M+M}{1+\frac{2}{49g}} = \frac{-\frac{2}{49g}100+100}{1+\frac{2}{49g}} =$ **99.17 kg**

Problem 3 *Consider a rope-and-pulley system set up as shown.*



(a) How does the acceleration of the two blocks relate?

$$\text{2distance big block} = \text{ distance small block}$$
$$2d_b = d_s$$
$$\therefore \frac{2\Delta x_b}{\Delta t} = \frac{\Delta x_s}{\Delta t}$$
$$\therefore \frac{\frac{2\Delta x_b}{\Delta t}}{\Delta t} = \frac{\frac{\Delta x_s}{\Delta t}}{\Delta t}$$
$$\boxed{\therefore 2a_b = -a_s}$$

(a_s is negative because the block is going *down*)

(b) What is the acceleration of the $2kg$ block sitting on the table?

Solving for the force of tension on the small block:

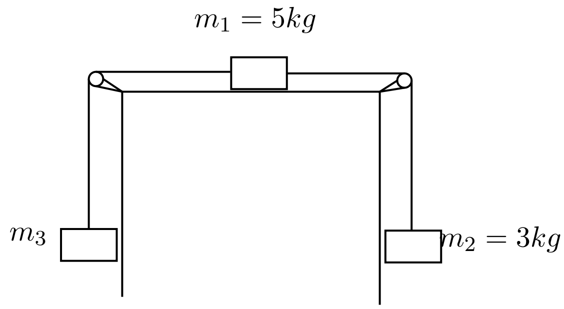
$$\sum F = ma_y$$
$$F_T - F_g = ma_y$$
$$F_T = (1kg)a_y + (1kg)(9.8\frac{m}{s^2})$$
$$F_T = a_y + 9.8N$$

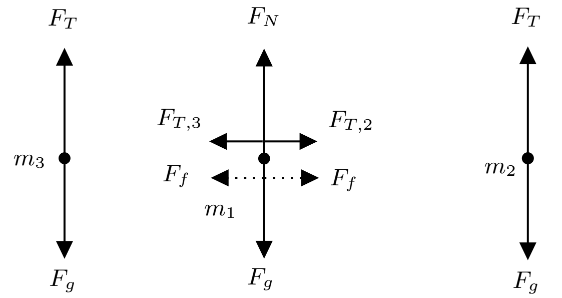
Solving for the force of tension on the big block:

$$\sum F_x = ma_x$$
$$2F_t = (2kg)a_x$$
$$F_t = a_x = \frac{1}{2}a_y$$

$$a_y + 9.8N = -\frac{1}{2}a_y$$
$$\boxed{a_y = 6.53\frac{m}{s^2} \text{ and } a_x = 3.26\frac{m}{s^2}}$$

Problem 4 *A block of mass $m_1 = 5kg$ rests on a table. Two ropes connect that block to two masses: one hangs off the left side of the table, and the other hands off the right side. The coefficient of static friction μ_s between teh block and the table is 0.2.*





(a) If one of the hanging masses has mass $m_2 = 3$kg, for what range of values of the other block's mass m_3 will the system not move?

Starting with the $\sum F = ma$ equation, we have:

$$\sum F = ma$$

However, there are two possible scenarios; the first being that m_3 is lighter than m_2, therefore m_2 will pull the rope down on the right, causing m_1 to move to the right. In this case, the force of friction would be towards the left. The second case is that m_3 is heavier than m_2, which would have the opposite effect and make the force of friction in the other direction. First considering the first case, we have:

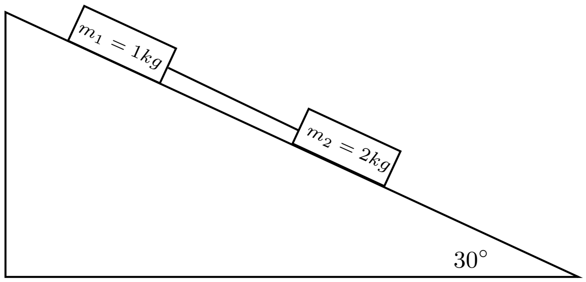
$$\sum F_x = ma_x = 0$$
$$F_{T,2} - F_{T,3} - F_F = 0$$
$$m_2 g = m_3 g + F_F$$
$$m_2 g = m_3 g + mg\mu$$
$$m_2 = m_3 + m\mu$$
$$m_3 = (3kg) - (5kg)(0.2)$$
$$\boxed{m_3 = 2kg}$$

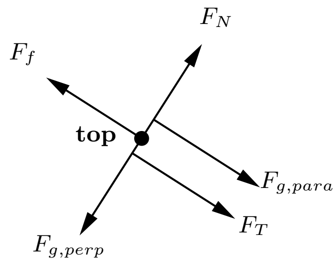
For the second case:

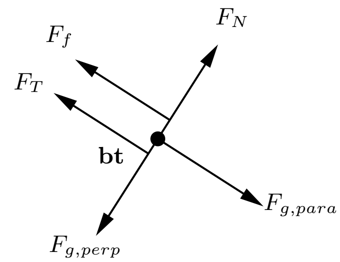
$$F_{A,3} - F_{A,2} - F_F = 0$$
$$F_{A,3} = F_F + F_{A,2}$$
$$m_3 g = F_N\mu + m_2 g$$
$$m_3 g = m_1 g\mu + m_2 g$$
$$m_3 = m_1\mu + m_2$$
$$m_3 = (5kg)(0.2) + (3kg)$$
$$\boxed{m_3 = 4kg}$$

$$\boxed{\therefore\ 2kg \le m_3 \le 4kg}$$

Problem 5 *Two blocks connected by a cable slide down an incline angles at 30° above the horizontal, connected by a cable. The top block has a mass of $m_1 = 1kg$, and the bottom one has a mass of $m_2 = 2kg$. The coefficients of kinetic friction are $\mu_{k1} = 0.2$ and $\mu_{k2} = 0.1$.*







(a) What is the tension in the cable that connects them?

First, we can solve for the acceleration of the top block:

$$\begin{gathered}
\sum F_{para} = ma_{para} \\
F_f - F_{g,para} - F_T = ma_{para} \\
F_N\mu - mg\sin 30 - F_T = ma \\
m_1 g\cos 30\mu - m_1 g\sin 30 - F_T = m_1 a \\
F_T = -m_1 a + m_1 g\cos 30\mu - m_1 g\sin 30 \\
F_T = -(1kg)a + (1kg)\left(9.8\frac{m}{s^2}\right)(\cos 30)(0.2) - (1kg)\left(9.8\frac{m}{s^2}\right)(\sin 30) \\
F_T = -a - 3.203 \\
a = -F_T - 3.2203
\end{gathered}$$

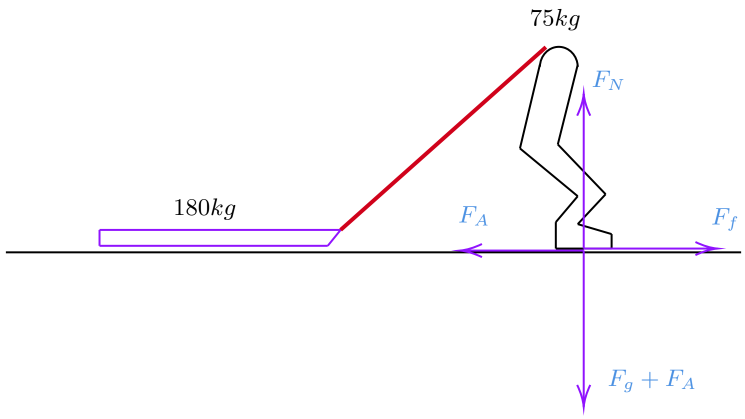
Next, solving for the force of tension of the bottom block:

$$\begin{aligned}
\sum F_{para} &= ma_{para} \\
F_T + F_F - F_{g,para} &= ma_{para} \\
F_T &= ma - F_f + F_{g,para} \\
F_T &= (2kg)a - F_N\mu + mg\sin 30 \\
F_T &= 2a - (2kg)\left(9.8\frac{m}{s^2}\right)(\cos 30)(0.1) + (2kg)\left(9.8\frac{m}{s^2}\right)(\sin 30) \\
&\boxed{F_T = 2a + 8.1026}
\end{aligned}$$

Finally, plugging in the two solutions:

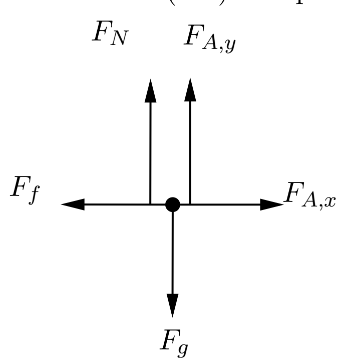
$$\begin{aligned}
F_t &= 2(-F_t - 3.203) + 8.1026 \\
F_t &= -2F_t - 6.4052_8.1026 \\
3F_t &= 1.697 \\
&\boxed{F_t = 0.566}
\end{aligned}$$

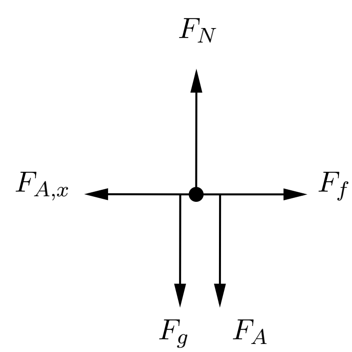
Problem 6 *A hiker with a mass of 75kg wants to drag a 180kg sled over snow at a constant rate. She does this by tying a rope to the sled and running it over her shoulder. The rope running between her and the sled makes a 45 degree angle with the horizontal.*



(a) If the coefficient of friction between the sled and the snow is 0.1, what must the coefficient of friction between her boots and the ground be for her to move the sled?

Free Body Diagram of the sled (left) and person (right):





Solving for the normal force for the sled:

$$\sum F_y = ma = 0$$
$$F_N + F_{A,y} - mg = 0$$
$$F_N + F_{A,y} = mg$$
$$F_N = mg - F_{A,y}$$

Solving for the force applied to have the sled move at a constant speed (acceleration is 0):

$$\sum F_x = ma = 0$$
$$F_{A,x} = F_f$$
$$F_{A,x} = F_N \mu$$
$$F_{A,x} = (mg - F_{A,y})\mu$$
$$F_A \cos 45 = mg\mu_K - \mu_K F_A \sin 45$$
$$F_A \cos 45 + F_A \sin 45 \mu_K = mg\mu_K$$
$$F_A(\cos 45 + \sin 45 \mu_K) = mg\mu_K$$
$$F_A = \frac{mg\mu_K}{\cos 45 + \sin 45 \mu_k}$$

For now, we can leave this aside and solve for the normal force of the person:

$$\sum F_y = ma = 0$$
$$F_N - F_{A,x} - F_g = 0$$
$$F_N = F_g + F_{A,x}$$
$$F_N = F_A \cos 45 + mg$$

Finally, we can solve for the net force along the x-axis for the person's foot:

$$\sum F_x = ma_x$$
$$F_f - F_{A,foot} = 0$$
$$F_f = F_{A,foot}$$
$$F_{N,man}\mu_s = F_{A,foot}$$
$$(F_A \cos 45 + m_{man}g)\mu_s = F_A \cos 45$$
$$F_A \cos 45\mu_s + m_{man}g\mu_s = F_A \cos 45$$
$$m_{man}g\mu_s = F_A \cos 45 - F_A \cos 45\mu_s$$
$$m_{man}g\mu_s = F_A \cos 45(1 - \mu_s)$$
$$m_{man}g\mu_s = \frac{m_{sled}g\mu_k}{\cos 45 + \mu_k \sin 45} cos45(1 - \mu_s)$$
$$m_{man}g\mu_s = \frac{180 \times 9.8 \times 0.1}{\cos 45 + (0.1)\sin 45} \cos 45(1 - \mu_s)$$
$$735\mu_s = 160 - 160\mu_s$$
$$\boxed{\mu_s = 0.179}$$