Physics 211 Practice Exam 3

Question 1

Finn is a water-loving and very strong dog who has gotten good at jumping off of a boat to catch a Frisbee floating in the water. He's got a mass of $m = 25$ kg. When he jumps, his muscles are able to produce 450 J of energy. For simplicity, let's think about Finn jumping horizontally from the side of a boat, just so we don't have to do any trigonometry. You may approximate Finn as a single point, even though that's not quite realistic.

a) Suppose that Finn jumps horizontally from a very massive boat (so massive that it will not move) as fast as he can from a height of $h = 1$ meter. What velocity v_0 will Finn have once he jumps? *(5 points)*

Since Finn gets all 450 J of kinetic energy since the boat doesn't move, you can find his velocity by setting

$$450\text{J} = \frac{1}{2}\text{mv}_0^2$$

to get

$$v_0 = 6\text{m/s}$$

.

b) If this boat is floating 2.5 m away from a Frisbee in the water, will Finn be able to jump on top of it? *(5 points)*

We need to use kinematics to figure out where Finn lands, since conservation of energy cannot tell us the path a projectile takes. Setting the origin of coordinates at the base of the boat under Finn, we have

$$x(t) = v_0 t \qquad\qquad y(t) = h - \frac{1}{2}gt^2$$

Finn hits the water when $y = 0$, so we ask "what is x at the time $y = 0$?" This gives us

$$t = \sqrt{\frac{2h}{g}} \qquad \rightarrow \qquad x = v_0\sqrt{\frac{2h}{g}}$$

and substituting the numbers in shows that Finn gets his frisbee.

c) Now, suppose that Finn jumps horizontally from a much lighter canoe with the same mass as Finn (25 kg), also from a height of $h = 1$ meter. (The canoe is floating in the water, and is free to move.) Recall that Finn's muscles can only produce $E = 450$ J of energy in a jump, which must be shared between the canoe and Finn.

Determine the velocities of Finn and the canoe after he jumps. *(8 points)*

Now the 450 J of kinetic energy is shared between Finn and the boat. Here we must use two techniques:

1. Conservation of momentum will tell us how Finn's initial velocity compares to the boat (since Finn jumping is an explosion)
2. Kinematics as before will tell us where Finn lands

Conservation of momentum tells us that Finn's forward velocity must be equal to the boat's equal velocity, since their masses are equal. Since their masses and velocities are the same, each will have half of the total kinetic energy. This means that Finn's initial velocity is now only 4.24 m/s.

d) If this canoe is floating 2.5 m away from the same Frisbee, and Finn is again jumping from a height of $h = 1$ m, will Finn be able to jump on top of his Frisbee? *(7 points)*

Substituting the velocity we just found into $x = v_0\sqrt{\frac{2h}{g}}$ (found from before) shows us that now Finn won't get his Frisbee, since his initial velocity isn't enough to carry him 2.5m before he hits the water.

Question 2

A rugby player with a mass of $M = 80$ kg is running 45 degrees east of north at a speed of 5 m/s. They are tackled by another player running 60 degrees west of north at a speed of 6 m/s. After the impact, the two players are moving directly north.

a) What is the mass of the second player? *(15 points)*

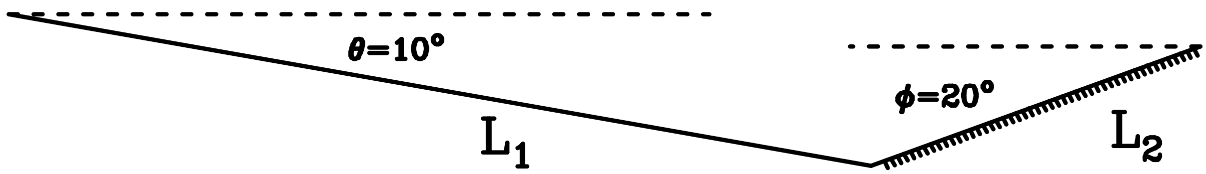
The only thing happening here is a collision, so all you need is conservation of momentum. They are moving in two dimensions, so you'll need to decompose all vectors into components and write down conservation of momentum in X and Y separately. Since you know $v_x = 0$, you can solve for the unknown mass using the equation you write down for X. b) How fast are they moving after the impact? *(10 points)*

Use the mass you found in (a) and conservation of momentum in the y-direction to find the final velocity.

Question 3

Heavy trucks driving down steep mountains must continually apply their brakes to maintain a safe speed, as you saw in your homework.

If their brakes fail or overheat, these roads are equipped with "runaway truck ramps", which are short uphill pathways (made of sand or gravel) with a large coefficient of rolling friction. A truck whose brakes overheat can steer into the ramp and come safely to a stop. Suppose that a truck of mass m is driving down the hill at a speed v_0 when its brakes fail. It is a distance L_1 away from the ramp, traveling at a speed v_0. When it reaches the ramp, it exits the highway and heads up the ramp, traveling a distance L_2 before coming to rest. In this problem, you will calculate the distance L_2 in terms of μ_r, g, m, L_1, v_0, θ, and ϕ.



a) Write an expression for the total work done by gravity during the entire motion in terms of g, m, L_1, L_2, θ, and ϕ. *(10 points)*

The work done by gravity is the weight of the truck times the net distance moved downward, or

$$mgL_1 \sin\theta - mgL_2 \sin\phi.$$

b) Write an expression for the total work done by friction during the entire motion in terms of μ_r, g, m, L_2, and ϕ. (Note that the truck rolls without appreciable friction until it gets to the gravel-filled ramp, since its brakes have failed.) *(10 points)*

There is friction going up the ramp. The dot product just gives us a minus sign, since the frictional force is directly opposite the motion. Here

$$\begin{aligned} W_{\text{fric}} &= \vec{F}_f \cdot \Delta\vec{s} \\ W_{\text{fric}} &= -\mu_r F_N L_2 \\ W_{\text{fric}} &= -\mu_r mg\cos\theta L_2 \end{aligned}$$

c) Write a statement of the work-energy theorem/conservation of energy in terms of μ_r, g, m, L_1, L_2, v_0, θ, and ϕ that you could solve for L_2. (You do not need to solve it.) *(10 points)*

Here the truck stops at rest:

$$\begin{aligned} KE_i + W_g + W_f &= KE_f \\ \frac{1}{2}mv_0^2 + mgL_1\sin\theta - mgL_2\sin\phi - \mu_r mg\cos\theta L_2 &= 0 \end{aligned}$$

d) While the truck is descending the steep $\theta = 10°$ hill, a small car is driving up the hill in the other lane while carrying a heavy load.

Suppose that car has an engine that can produce 75 kW of power. If the car wants to travel up the hill at 10 m/s (23 mph; 36 km/hr), what is the maximum mass that it can have? *(5 points)*

Power is the dot product of force and velocity. If the car is going at a constant rate, the power supplied by the engine is equal to the power applied by gravity. The power applied by gravity is equal to the force of gravity mg times the component of force in the vertical direction $mg\sin\theta$; it is negative since the car is going up but gravity pulls down.

This means that

$$P_g = 75\ kW = mgv\sin\theta.$$

Question 4

A firecracker of mass 1 kg is launched straight upward. When it is 200 meters above the ground and traveling upward at a velocity of 50 m/s, it explodes, separating into two pieces. After the explosion, one piece has a mass of 600 grams and travels horizontally East at 40 m/s.

a) What is the velocity of the other piece? (Remember, velocity is a vector.) *(15 points)*

This is an explosion, so use conservation of momentum. Since this is in two dimensions, you will need to write conservation of momentum separately in x and y; the velocity of the other piece will have both x and y components.

b) What is the maximum height above the ground that this other piece will achieve? *(10 points)*

The simplest way to find the height above the ground is to use kinematics; set the $y-$velocity of the other piece to zero and solve for time, then substitute that time into the $y-$position equation to find the maximum height.

Question 5

A skateboarder of mass m is standing on the edge of a drainage channel, as shown. The left side, where the skateboarder starts, is elevated at an angle θ; the right side is elevated at an angle ϕ. The slopes on either side are smooth, and the skateboard moves over them with essentially no friction, but the flat bottom of width b is covered with a little sand, and the skateboard experiences a small amount of rolling friction there, with μ_r known.

The skateboarder starts a distance d up the left-hand side. They roll down the left side, across the sand-filled bottom, and up the right side.

(*Give your answers to the first two parts in terms of the variables above, along with g.*)

a) Determine the maximum distance d_2 that the skateboarder makes it up the right side. (This is the diagonal distance, not the height.) *(10 points)*

This is a classic conservation of energy situation, since you know initial and final situations, can calculate work done, and don't care about time.

Using the "potential energy" framing of work/energy, we have

$$KE_i + PE_i + W_f = KE_f + PE_f$$

But the initial and final velocities are both zero, so we have

$$mgd\sin\theta - \mu mgb = mgx\sin\phi$$

where x is the distance we roll up the right side, and you can just solve for x.

b) After rolling up and back down the right side, the skateboarder will come back to the left side. How far will they travel back up the left side? *(5 points)*

It is simplest here to use conservation of energy starting from the very beginning. Suppose that the distance we travel up the left side again is L. No energy is lost going up the ramp on the right hand side; the only work done is friction on the sand. You travel over it twice, so the work done by friction is $2\mu mgb$. Thus we have for conservation of energy

$$mgd\sin\theta - 2\mu mgb = mgL\sin\phi.$$

The fact that this is on the left side rather than the right doesn't matter.

c) Suppose that you know numeric values as follows:

- $m = 75$ kg
- $\theta = 30°$
- $\phi = 40°$
- $\mu_r = 0.05$
- $d = 4$ m
- $b = 7$ m

How many times will the skateboarder travel across the sandy bottom of the channel before coming to rest? Explain the approach behind your solution fully. (There is an easy way and a hard way to do this; your group will get full credit for either!) *(10 points)*

The easiest way to think about this is by conservation of energy. Note that going up and down the slopes doesn't change the total mechanical energy; the only thing that does is going over the sand.

We know that friction does an amount of negative work on the skateboarder equal to $-\mu mgb$ every time they cross the sand (dissipating this energy as heat). They will continue to roll back and forth over the sand until they lose all of their energy and come to rest; the number of times they can do this is their total potential energy at the beginning divided by the energy they lose each time, or

$$N = \frac{mgd\sin\theta}{\mu mgb}$$

and you can solve for this (and round it down).

Question 6

The dread pirate captain Piarrrrr Squared has some cannon on her ship. (She doesn't use them very much, since she prefers to throw π's at ships she's trying to capture.)

These cannon have a mass M; they launch cannonballs of mass m at a speed v horizontally. They are placed on a wooden deck; the coefficient of friction between the cannon and the deck is μ_k.

a) Explain in words why the cannon slide backwards when they are fired. (You don't need to do any mathematics here.) *(5 points)*

Momentum is conserved, so if the cannonball goes forward, something else must acquire backwards momentum.

b) In terms of μ_k, m, M, v, and g, how far do the cannon slide backwards after they are fired before they come to rest? *(10 points)*

This is a situation where you need to use two methods. Conservation of momentum will let you relate the situation before the explosion (cannonball and cannon at rest) to the situation right after (cannon moving backward at speed v_c; ball moving forward at speed v_b). Then you can use the work-energy theorem to find out how far the cannon slide.

1) conservation of momentum: there is no initial momentum, so you know

$$0 = mv_b - Mv_c \rightarrow v_c = v_b \frac{m}{M}.$$

2) conservation of energy applied to the cannon: we have

$$KE_i - W_{\text{fric}} = KE_f = 0$$

$$\frac{1}{2} m v_c^2 - \mu m g x = 0$$

and you can solve for the distance x the cannon slides.

Piarrrrr's pirates get tired of pushing the cannon back to the edge of the ship after they are fired, so they get an idea: they'll mount some springs behind them that will stop them from sliding back so far and help push them back into position. (The cannon are placed at the equilibrium position of the springs, so that as soon as they begin to move backwards, the springs start to push them forwards again.)

If the crew wants the cannon to slide backwards a distance L at most, what must the spring constant be?

Here we use conservation of energy again, but now there is some spring potential energy after the cannon has slid backwards:

$$KE_i - W_{\text{fric}} = KE_f + PE_f = 0$$

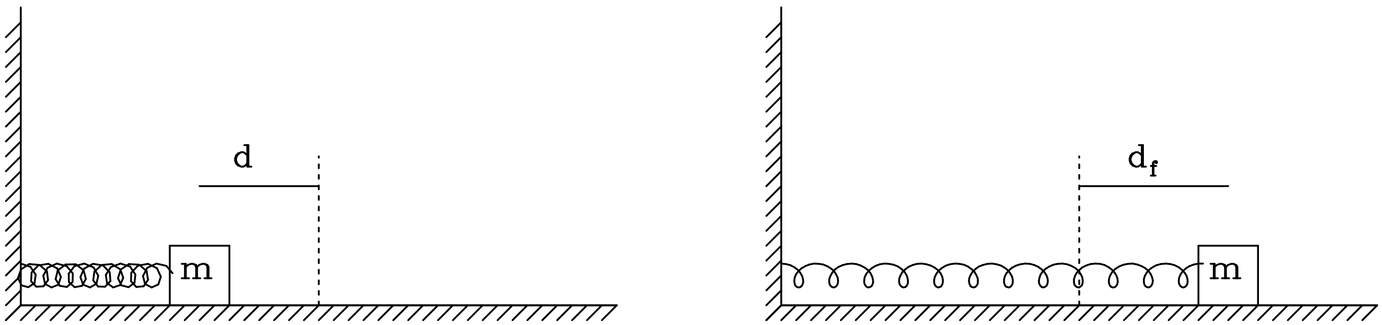
$$\frac{1}{2}mv_c^2 - \mu mgx = \frac{1}{2}kL^2$$

and you can solve for L.

Question 7

A spring has spring constant k. One end is fixed, and the other end is attached to a mass m, which is free to move horizontally along a table. The mass slides over the table with a coefficient of friction μ_k.

The spring is compressed a distance d from its equilibrium point and released. When the spring is released, it will push the mass to the right, until it reaches some other distance d_f past the equilibrium point.



a) How fast will the mass be traveling when it crosses the equilibrium point? Give your answer in terms of μ_k, d, m, and g. *(5 points)*

Use the work-energy theorem here:

$$KE_i + PE_i + W_f = KE_f + PE_f$$

$$0 + \frac{1}{2}kd^2 - \mu mgd = \frac{1}{2}mv^2 + 0$$

and you can solve for v.

b) Write down an expression for the work done by friction as the block slides from its starting point to the final position d_f to the right of equilibrium. *(5 points)*

Here we slide a distance $d + d_f$, so the work done by friction is

$$W_f = -\mu mg(d + d_f)$$

c) Write down an equation in terms of μ_k, d, m, and g that will let you solve for the distance d_f. You do not need to solve it. *(12 points)*

Now we have spring potential energy at the beginning and the end, but no kinetic energy at the end:

$$KE_i + PE_i + W_f = KE_f + PE_f$$

$$\frac{1}{2}kd^2 - \mu mg(d + d_f) = \frac{1}{2}mv^2 + \frac{1}{2}kd_f^2$$

d) What algebraic technique would you have to use to solve this equation for d_f? *(3 points)*

Since this equation has both d_f^2 and d_f in it, you'll need the quadratic formula.

QUESTION 8

A clay block of mass M sits on top of a flat table. It sits on top of a hole in the table. It is struck from below by a fast-moving ball of mass m traveling at speed v_0 through the hole. The ball is not traveling straight up, though: its velocity is at an angle θ away from the vertical. The ball lodges in the clay block, and the block flies up in the air and lands back on the table.

In this problem, you will calculate where on the table the block lands.

a) Without doing any mathematics, outline a plan for figuring this out in words and diagrams. (You won't need this whole page.) *(8 points)*

There is a collision and then projectile motion, so you will use conservation of momentum and then kinematics.

b) Find how far away from the hole the block lands in terms of m, M, v_0, θ, and g.[1] *(17 points)*

Use conservation of momentum to find the velocity of the clay block after it is struck. You'll need to decompose the ball's velocity into x- and y-components, then use conservation of momentum separately in x and y.

This tells you that after the collision, the velocity of the block $\vec{v}_1$ is

$$v_{1,x} = v_0 \sin\theta \frac{m}{m+M}$$
$$v_{1,y} = v_0 \cos\theta \frac{m}{m+M}$$

Then we use projectile motion here to figure out how far the block flies before it hits the table again. We want to know the value of $x(t)$ at the time $y(t) = 0$.

We have

$$y(t) = -\frac{1}{2}gt^2 + v_{1,y}t \rightarrow t = \frac{2v_{1,y}}{g}$$

Substitute this into x(t):

$$x(t) = v_{1,x}t = \frac{2v_{1,y}v_{1,x}}{g}$$

or, if you like to substitute stuff in,

$$x(t) = \frac{2v_0^2 \sin\theta\cos\theta}{g}\frac{m^2}{(M+m)^2}$$

[1] As a note, you can include other variables in your answer if you define them in terms of m, M, v_0, θ, and g. For instance, if you say "$u = ...$", where u is defined in terms of the given parameters, you can then use u in your answer. Depending on what you do, this might make your algebra a bit simpler.