Vectors

Physics 211
Syracuse University, Physics 211 Spring 2021
Walter Freeman

February 24, 2021

On solving problems

You can recognize truth by its beauty and simplicity. When you get it right, it is obvious that it is right–at least if you have any experience–because usually what happens is that more comes out than goes in.... Inexperienced students make guesses that are very complicated, [but] the truth always turns out to be simpler than you thought.

–Richard Feynman, quoted by K. C. Cole, in *Sympathetic Vibrations: Reflections on Physics as a Way of Life (1985)*

Nature uses only the longest threads to weave her patterns, so each small piece of her fabric reveals the organization of the entire tapestry.

–Richard Feynman, *The Character of Physical Law* (1965)

Announcements

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  - What happened in the Virtual Clinic yesterday?
  - My Clinic hours today: 3:30-5:30
  - Many other people have other hours – see the course website
- Lively discussion going on on the course Discord server about the homework

- Homework 2 will be posted tomorrow and due next Thursday before class

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There will be *two questions*. On the quiz:

- One question where you will need to interpret the two roots given to you by the quadratic formula (like Question 5 from HW1)
- Motion with constant acceleration in one dimension (what you did for HW1: roadrunner problem, elevator problem)
- Motion with constant acceleration in two dimensions (what we are doing today and next Tuesday, and on HW2: cannon problem, dog problem)
- Interpretation of position/velocity/acceleration graphs (HW1, bicycle problem)

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Review opportunities specifically for the quiz:

- next Wednesday, 4PM-7PM, on Zoom (link provided next week)
- next Thursday, first 20 minutes of class

The quiz itself:

- A PDF will be posted at 11:20 AM on the website and on Blackboard; it will be due at 12:20 PM (end of class). We will have a grace period to accommodate people scanning and submitting things.

- Four options to take it:
  1. Write the answers on your own paper, scan them, and submit them
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Vectors

You've been doing math with numbers, which are things that live in one dimension: they only have a magnitude and a sign.

Vectors are things that have a magnitude and a direction: "arrows in space"

Many of the things we deal with in physics are vectors:

- Position

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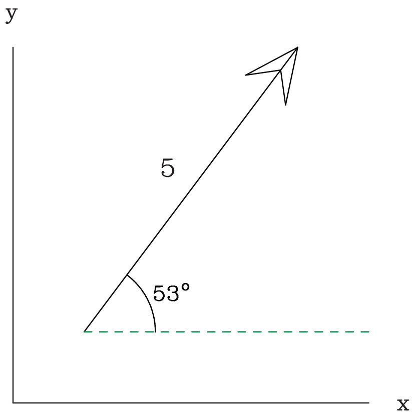
So, we need to learn to do math with arrows.

- We indicate that an object is a vector by writing an arrow over it: "the vector $\vec{V}$".
- "Scalar": object that isn't a vector (mass, time)
- Equations can mix vectors and scalars: $\vec{F} = m\vec{a}$.
- ... or $\vec{s} = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$

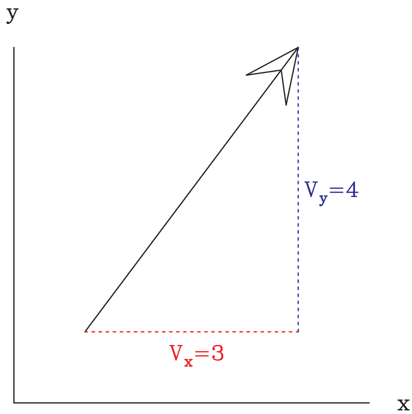
Notation

- $\vec{A}$: "the vector A" (a vector)
- A: "the magnitude of A" (a scalar)
- $\hat{A}$: "the direction A points in" (a vector with magnitude 1)
- A_x: the component of A along the $x-$axis (a scalar)
- A_y: the component of A along the $y-$ayis (a scalar)

Two ways to describe a vector



Angle and direction



X and Y components

How do we convert from one to the other?

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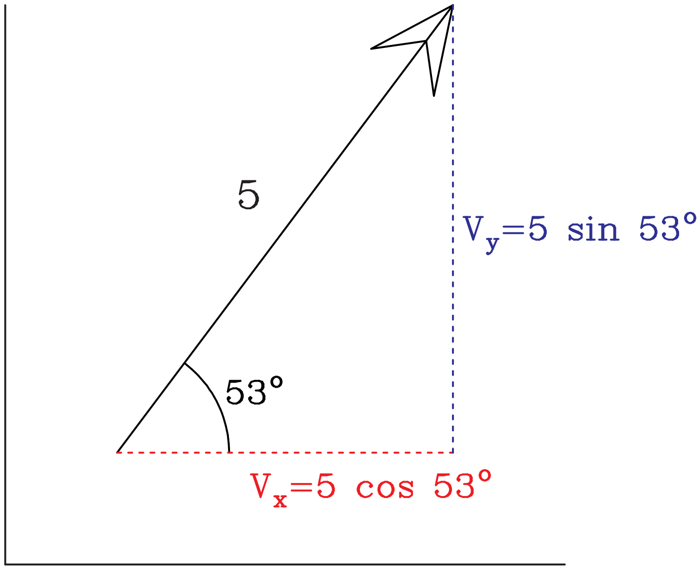
A: Using algebra

B: Using trigonometry

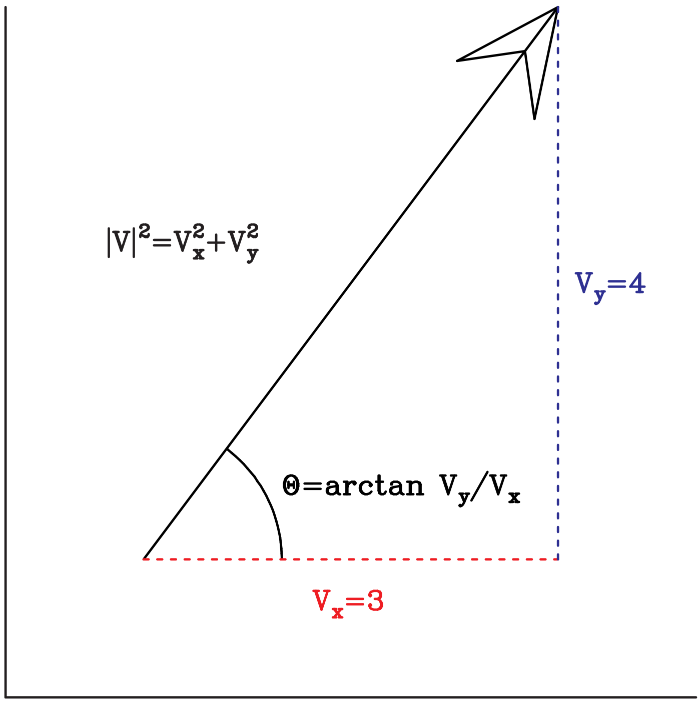
C: Using calculus

D: Using differential equations

From "direction and magnitude" to components



From components to direction and magnitude



Suppose you have some vector $\vec{A}$ that you want to convert into components. The x-component A_x is:

A: $A\cos\theta$

B: $A\sin\theta$

C: $A\tan\theta$

D: $\frac{A}{\cos\theta}$

E: It depends

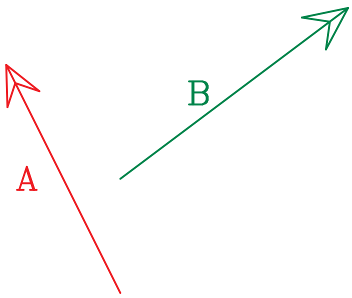
A warning!

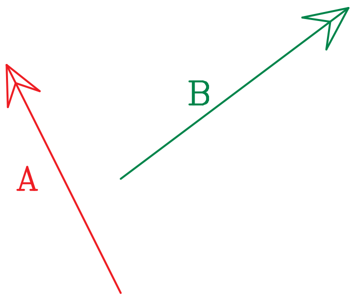
You cannot memorize "$V \sin\theta$ is the y component,
$V \cos\theta$ is the x component"!

This does *not* work in general; you have to actually draw the triangle.

Adding vectors

We can also add vectors together by drawing them “head to tail”. Here are two vectors:





Does $\vec{A} + \vec{B} = \vec{B} + \vec{A}$?

- A: Yes
- B: No

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- A: Yes
- B: No

Yes: vector addition obeys the commutative property, just like ordinary addition

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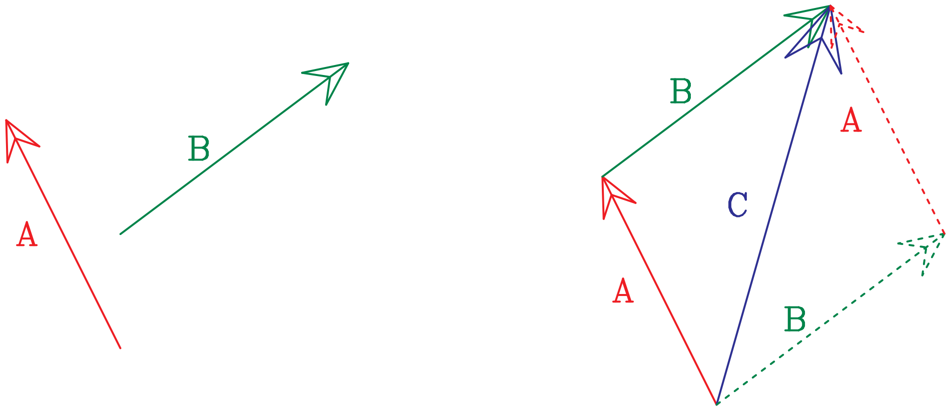
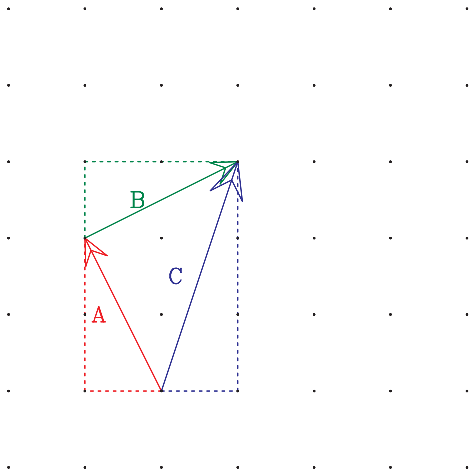


$$\vec{A} + \vec{B} = \vec{C}$$

The component representation is much easier to work with!

$$\vec{A} + \vec{B} = \vec{C} \rightarrow \begin{pmatrix} A_x + B_x = C_x \\ A_y + B_y = C_y \end{pmatrix}$$

Adding vectors: components



To add two vectors, just add their components!

This is why it is almost always easiest to work in the component representation!

What does this do to our kinematics?

Acceleration, velocity, and position relationships are still the same; they just apply independently for each component.

$$\vec{v}(t) = \vec{a}t + \vec{v}_0$$
$$\vec{s}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$$

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Which statement does *not* make sense?

- a. $\vec{A}t = \vec{B}$
- b. $\vec{A} + \vec{B} + t = \vec{C}$
- c. $k(\vec{A} + \vec{B}) = k\vec{A} + k\vec{B}$
- d. $\vec{A} - \vec{B} = \vec{C}$

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B: You can't add a vector and a scalar. "One mile north plus one inch" – which way is the inch?

Problem solving: 2D kinematics, constant acceleration

1. 1. If you have vectors in the "angle and magnitude" form, convert them to components
2. 2. Write down the kinematics relations, separately for x and y
   - Many terms will usually be zero
   - Freefall: $a_x = 0$, $a_y = -g$ (with conventional choice of axes)
3. 3. Understand what instant in time you want to know about
4. 4. Put in what you know; solve for what you don't (using substitution, if necessary)
5. 5. Convert vectors into whatever format the problem asks for

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Every kinematics problem we will encounter can be done this way!

Sample problems

A rock is thrown at $v_0 = 10m/s$ at $\theta = 30^\circ$ above the horizontal.

- How far from its starting point is it after 2 seconds?

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Sample problems

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- How far from its starting point is it after 2 seconds?
- How far does it travel?
- How high does it go?
- What will its speed be when it strikes the ground?