Recitation Questions

26 February

Question 1: geostationary orbit

It is sometimes useful to place satellites in orbit so that they stay in a fixed position relative to the Earth; that is, their orbits are synchronized with the Earth's rotation so that a satellite might stay above the same point on Earths surface all the time.

What is the altitude of such an orbit? Note that it is high enough that you need to use $F_g = \frac{GMm}{r^2}$ rather than just $F_g = mg$.

Hint 1: If this orbit is synchronized with Earth's rotation, then you should be able to figure out its angular velocity.

Hint 2: If you do this problem as we have guided you, by waiting to substitute numbers in until the very end, you will arrive at an expression relating the radius R of a circular orbit with the mass M of the planet being orbited and the angular velocity ω of the orbit. This question will be on HW5, and is related to the derivation of Kepler's third law that you will do there.

The angular velocity is that of earth's. Earth rotates once a day.

$$\omega = \frac{2\pi \text{ rads.}}{24\text{hr}} \cdot \frac{1\text{hr}}{60\text{min}} \cdot \frac{1\text{min}}{60\text{ sec}} = \frac{2\pi \text{ rads}}{86{,}400\text{ s}} = \boxed{7.3\times10^{-5}\ \frac{\text{rads}}{\text{s}}}$$

Now, we want to find the gravitational force on the satallite.

$v = \omega r$

$$F_{E\to S} = \frac{G \cdot M_E \cdot m_s}{r^2} = m_s \cdot a$$

; M_E = mass of earth.
M_S = mass of satallite
r = separation between Earth's center and the satallite
G = Gravitational const.

The satallite has centripetal acceleration. Thus $a = \frac{v^2}{r}$

$$\frac{G \cdot M_E \cdot \cancel{m_s}}{r^2} = \cancel{m_s} \cdot a$$

$$\frac{G \cdot M_E \cdot}{r^{\cancel{2}}} = \frac{v^2}{\cancel{r}} \Rightarrow \boxed{\frac{G \cdot M_E}{v^2} = r}$$

* Here, r = sep. from center of earth. The altitude is the separation from the surface of earth to satallite

$r = r_E + r_A \Rightarrow \boxed{r_A = r - r_E}$

Question 2: variation of apparent weight with latitude

For this problem, carry all calculations to five significant digits. Some figures that will be useful:

- Mass of Earth: 5.9722×10^{24} kg
- Radius of Earth: 6.3710×10^{6} m (assume it is spherical)
- Gravitational constant (G): $6.6741 \times 10^{-11} \mathrm{N} \cdot \mathrm{m}^2/\mathrm{kg}^2$
- Length of one day: 8.6400×10^{4} s

a) Using Newton's law of universal gravitation $F_g = \frac{GMm}{r^2}$, determine the force of gravity on a 1 kg mass resting on the surface of the Earth. Are you surprised by this figure?

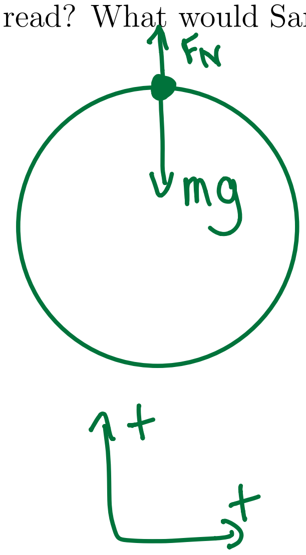
$$F_g = \frac{G \cdot M_E \cdot m}{r^2} = \frac{\left(6.6741 \times 10^{-11} \frac{\mathrm{N} \cdot \cancel{\mathrm{m}^2}}{\cancel{\mathrm{kg}^2}}\right)\left(5.9722 \times 10^{24}\,\cancel{\mathrm{kg}}\right)\left(1\,\cancel{\mathrm{kg}}\right)}{\left(6.3710 \times 10^{6}\,\cancel{\mathrm{m}}\right)^2}$$

$$\boxed{F_g = 9.8200}$$

This is what we expect from

$$\boxed{F = ma = mg = (1)(9.81) = 9.81\,\mathrm{N}}$$

b) Suppose this mass were resting on a scale sitting on the North Pole owned by Santa Claus. Recall that scales measure the normal force that they exert. What value would Santa's scale read? What would Santa conclude the value of g is?

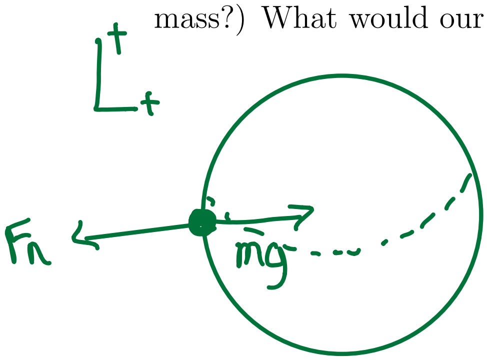


$$\sum F_y = F_N - mg = ma_y = 0$$

$$\sum F_x = 0$$

$$\boxed{F_N = mg}$$

c) Suppose that an identical 1 kg mass were resting on a scale sitting on the Equator, somewhere in Kenya. What would *this* scale read? (Hint: What is the acceleration of the mass?) What would our Kenyan physicist conclude about g?



$$-F_N + mg = ma = m\omega^2 r$$

$$\boxed{F_N = mg - m\omega^2 r}$$

d) This problem shows that your apparent weight depends on your location on Earth. Does it make sense to define g as F_g/m (the strength of the gravitational force divided by an object's mass) or F_N/m (the strength of the normal force, and thus the scale reading, divided by mass)? Call your TA/coach over to join your conversation.

$g = \frac{F_g}{m}$ doesn't hold when F_g changes at diff. parts of Earth. It is a good enough approx. the vast majority of the time.

e) Is this distinction likely to be relevant to the sort of engineering or science you will do during your career? (The answer will depend on what you will do, of course!)

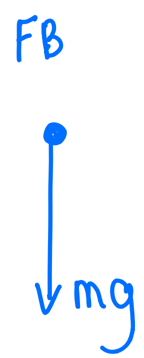
Question 3: Weightlessness

Astronauts in orbit around the Earth are not "so far away that they don't feel Earth's gravity"; actually, theyre quite close to the surface. However, weve all seen the videos of astronauts drifting around "weightlessly" in the International Space Station.

a) Explain how an astronaut can be under the influence of Earth's gravity, and yet exert no normal force on the surface of the spacecraft she is standing in.

They are constantly in "Free Fall" from orbiting Earth. The normal Force is only Felt via contact. So if she is floating w/o bumping into objects, only gravity acts on her.

b) Draw a force diagram for the astronaut floating in the middle of the Space Station, not touching any of the walls or floor. How do you reconcile your diagram with the fact that the astronaut doesn't seem to fall?



The Force of gravity drives an inward (centripetal) acceleration. In other words, she is always Falling, but the object she is in is Falling as well.

c) Is this astronaut truly "weightless"? What does "weightless" mean? (There are multiple correct answers to both of these questions.)

Her weight is still given by $w = mg$. So, she has weight.

Feeling "weightless" is achieved by having no contact Force, such as tension, Friction, normal Force, etc. Gravity is long range and not "felt" by the body.