Homework 5

Due Tuesday, 7 April

This is a short homework assignment that finishes up the material on power from March 31's class and April 1's recitation. This material will be a small part of the exam, but don't focus too much on it just because it is on the last homework set before the exam. Most of the exam will cover the main topics from this unit: the work-energy theorem, conservation of momentum, and the idea of potential energy.

This homework set is also designed as more of a "lesson" than an evaluation. We hope you will learn a few new things here! There is no complex algebra involved; each of these questions should involve only a short calculation.

In class we considered a cyclist riding a bike. She and her bike have a mass of 70 kg, and she is capable of generating 120 W of sustained power by pedaling.

Throughout, keep in mind the central idea: the power applied by a force $\vec{F}$ acting on an object moving at a velocity $\vec{v}$ is $\vec{F} \cdot \vec{v}$. Here the dot product means the same as always: "the size of the force, multiplied by the component of the velocity in the direction of that force.

At high speeds, the main frictional force on the cyclist is air drag. The force of air drag is approximately

$$F_{\text{drag}} = \gamma v_r^2,$$

where γ is a form of "drag coefficient" – a value that depends on the cyclist's size, shape, and posture, and v_r is the speed of the air moving past the cyclist. (If there is no wind, v_r is just the speed of the bike.)

1. What frictional power does air drag apply to our cyclist if there is no wind and she is pedaling at a speed v? Give an answer in terms of γ and v.
2. Suppose our cyclist, with her power output of 120 W, can sustain a speed of 12 m/s = 27 mph = 43 km/hr. Use this to estimate the value of γ for her.
3. Given the value of γ that you just calculated, estimate the power she has to pedal with for her to ride at 7 m/s in no wind, and then the power required for her to ride at 7 m/s into a 4 m/s headwind.
4. Mount Lemmon Highway is a long road near Tucson, AZ, with a slope of around 6° for about twenty miles. (Cyclists love it.) Conveniently, $\sin 6° \approx 0.1$, which you can use to simplify the arithmetic here.

Suppose our cyclist now wants to ride up Mount Lemmon Highway. She will be going much more slowly now – slow enough that you can ignore air drag. If her power output is still 120 W, how fast can she ride up this road? Convert this value into miles per hour or km/hr. Is it reasonable?

5. Having reached the top, she now wants to go back down – *fast.* If she doesn't pedal her bike, and only lets gravity pull her down, how fast will she go? *(Hint: if she is coasting down the hill at a constant speed, then the positive power applied to her by gravity must be balanced by the negative power applied to her by air drag.)*

6. Suppose she doesn't want to go quite this fast, since it is very hard to steer a bike at this speed! She wants to ride at "only" 15 m/s down the hill, so she applies her brakes in order to keep her speed at 15 m/s.

   (a) In this scenario, she is coasting down the hill at a constant 15 m/s. What is the mathematical relationship between the power applied to her by gravity P_{grav}, the power applied to her by her brakes P_{brakes}, and the power applied to her by air drag P_{drag}?

   (b) You found an expression for P_{grav} in part 4, and an expression for P_{drag} in part 1. Use this expression and the relationship you just found to solve for the power that her brakes must apply to her in order to keep her speed constant at 15 m/s.

   (c) Compare this amount of power to other things in your household – for instance, a hair dryer produces about 2 kW of heat. What engineering challenges do you think might be present in designing brakes that can absorb this much heat for very long?