Physics 211 Exam 3, Form B
Answer Key
Question 1

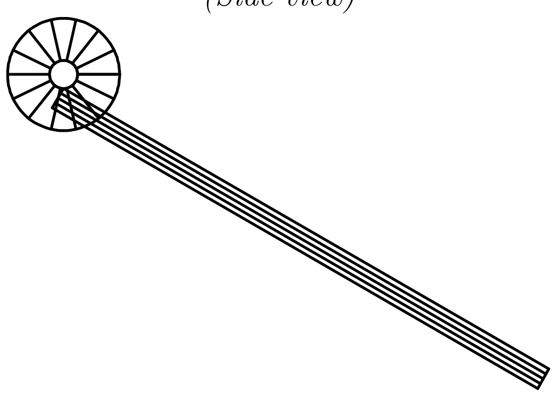
A meter stick is elevated at a $\theta = 30°$ angle. A spool consists of a cylinder of radius 2 cm with two disks affixed on either end; the disks have a radius of 10 cm. The cylinder is very light; you may assume all of the mass of the spool is in the disks. The spool is placed at the top of the meter stick so that the cylinder is touching the stick; when it is released, it rolls without slipping to the bottom. *(The moment of inertia of a disk is* $\frac{1}{2}mr^2$*).*

In this problem, you will calculate the speed of the spool at the bottom.

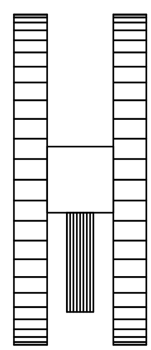
Rules for this problem: You may solve this problem in either symbols or numbers. If you use symbols, you must tell me the physical values of each symbol that you use (for instance: "$r = 2$ cm"). If you use numbers, you *must* retain the units (i.e. write "10 cm", not "10".)

These "rules" are here to avoid ambiguity about the symbol "r" – which radius do you mean?

(Side view)



(Rear view)



a) What is the relation between the spool's translational velocity v and its angular velocity ω? Remember that if you use variables here, you must tell me their physical values. (5 points)

The rolling-without-slipping constraint is $v = \omega r$, but here, r is the inner radius: $r = 2$ cm.

b) What is the total work (rotational plus translational) done by the force of static friction as the spool rolls downward? (5 points)

Zero, as argued in class. Recall that $W_{\text{trans}} = -F_f L$, but $W_{\text{rot}} = \tau\Delta\theta = F_f r(L/r)$, since $\Delta\theta = L/r$. Since they are equal and opposite, their sum is zero.

c) Calculate the velocity of the spool at the bottom. (15 points)

There are two radii here. We use the outer radius $R = 10$ cm for the moment of inertia $I = \frac{1}{2}mR^2$, but the inner one $r = 2$ cm for the rolling constraint $\omega = v/r$. (Note that we don't care about sign since these things only appear as squared in this problem.)

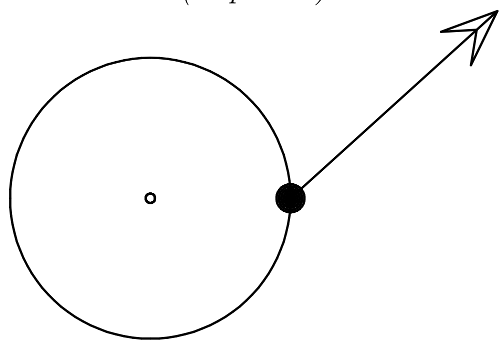
Using conservation of energy, we have

$$
\begin{aligned}
mgL\sin\theta &= \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega^2 \\
gL\sin\theta &= \frac{1}{2}v_f^2 + \frac{1}{2}\frac{1}{2}\left(\frac{R}{r}\right)^2 v_f^2 \\
v_f &= \sqrt{\frac{2gL\sin\theta}{1+\frac{1}{2}\left(\frac{R}{r}\right)^2}} \\
v_f &= \sqrt{\frac{2gL\sin\theta}{13.5}}
\end{aligned}
$$

where in the last line we have used $R = 5r$. The other "you must tell me what your variables mean" statement targets the variable L. L is one meter, since it's a meter stick.

Question 2

A child of mass m_1 is standing on one edge of a platform of radius r and mass m_2 that is free to rotate around its center; it is currently rotating counterclockwise *(note: error here in original)* at angular velocity ω_0. She carries a heavy ball with her of mass m_3.



When she is east of the center point of the platform, she throws her ball at speed v, directed at an angle 30 degrees north of east. A diagram of the platform and the ball's path is shown.

Suppose that after she throws the ball, she and the platform stop rotating. With what speed v must she throw the ball to make this happen? (25 points)

This is about the conservation of angular momentum.

$$L_i = I\omega = \left[(m_1 + m_3)r^2 + \frac{1}{2}m_2r^2\right]\omega$$

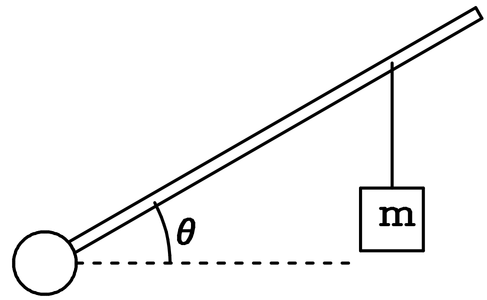
$$L_f = mv_\perp r = m_3 v \sin 30^\circ$$

Set them equal to get:

$$v = \frac{\left[(m_1 + m_3)r^2 + \frac{1}{2}m_2r^2\right]\omega}{m_3 \sin 30^\circ}$$

Question 3

In class, we did a demo involving an aluminum bar with a circular handle on one end. The handle has a radius of 2 cm. This bar has a mass of $M = 500$ g and a length of 1.2 m. A $m = 1$ kg mass is hung from the bar, 30 cm from the far end. A side view of this is shown here. (The drawing is not to scale.)



A student grips the handle, exerting a total normal force F_N spread around the outside of the circular handle. The coefficient of static friction between the student's hands and the handle is 0.5. (The maximum frictional force is still $\mu_s F_N$.) If the student grips hard enough, the bar and attached mass do not fall.

a) You observed in class that it was easier for the student to support the bar and mass if the angle θ was larger. Argue why this is the case, using the definition of torque. You may (and probably should) draw a picture supporting your discussion.[1] *(5 points)*

There are many correct solutions here, but the easiest is just to argue that $r_\perp$ goes down as θ increases.

[1]Note that we have discussed three equivalent definitions of torque: $\tau = F_\perp r = F r_\perp = F r \sin\theta$.

b) If the student's maximum grip force is 500 N, what is the maximum torque about the handle that the student's hands can apply? (5 points)

The force of friction is the grip force (normal force) times the coefficient of friction. Since friction is perpendicular to the surface, $\tau = \mu_s F_N r = (0.5)(500\,\mathrm{N})(0.02\,\mathrm{m}) = 5\mathrm{N}\cdot\mathrm{m}$.

c) Find the minimum angle θ at which the student can support the bar and mass. (10 points)

Set the net torque to zero.

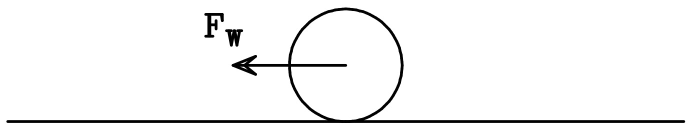
$$\mu_s F_N r - \frac{L_1}{2} M g \cos\theta - L_2 m g \cos\theta = 0$$

Solving for θ gives:

$$\theta = \cos^{-1} \frac{\mu_s F_N r}{\frac{L_1}{2} M g + L_2 m g} = 64.8^\circ$$

Question 4

A tennis ball of mass m (moment of inertia $\frac{2}{3}mr^2$) rests on a table when it is struck by a gust of wind, exerting a uniform force F_W blowing it to the left, as shown. This force can be treated as acting at the center of the ball, as shown. The coefficient of static friction between the ball and the table is μ_s, and the coefficient of kinetic friction is μ_k.



a) First suppose that the force of the wind is much less than the weight of the ball. How will the ball move? Describe the relationship between the ball's linear and angular accelerations, and what happens between the ball and the table. (5 points)

The ball will roll without slipping if the force is gentle. Since it rolls without slipping, the linear and angular accelerations are related by $a = \alpha r$. (Choose left to be positive.) In order for it to spin at all, there must be a frictional force between the ball and the table.

b) Draw a force diagram for the ball, labeling both the direction and point of action of all forces that act on it. (The wind can be considered to act on the ball's center.) (5 points)

c) For each of the forces in your diagram, tell whether the translational work $\vec{F} \cdot \vec{d}$ done by that force is positive, negative, or zero. (5 points)

d) For each of the forces in your diagram, tell whether the rotational work $\tau \Delta\theta$ done by that force is positive, negative, or zero. (5 points)

- Wind: acts on center of ball pointing left. Rotational work: zero. Translational work: positive.
- Gravity: acts at center of ball pointing down. Rotational work: zero. Translational work: zero.
- Friction: acts at bottom of ball pointing right. Rotational work: positive. Translational work: negative.
- Normal force: acts at bottom of ball pointing up. Rotational work: zero. Translational work: zero.

e) If the wind force is instead very strong, the ball will slip along the table. In this case, will the magnitude of the ball's acceleration be equal to αr, greater than this, or smaller than this? (5 points)

It will be greater than αr, since in this case the torque from kinetic friction is not sufficient to make αr keep up with a.

f) Calculate the maximum force F_W so that the the ball rolls without slipping along the table. (10 points extra credit)

This object both rotates and translates, so we need both $F = ma$ and $\tau = I\alpha$. When the force is maximum, $F_f = \mu_s mg$. We will treat right as positive. You may also treat left as positive, which makes the problem easier, but I'm making the harder choice to illustrate what most students will do. Putting this together:

$$\begin{aligned} -F_W + \mu_s mg &= ma \\ \mu_s mgr &= I\alpha \end{aligned}$$

Note here that the object translates in the negative direction, but rotates in the positive direction. This means that $a = \alpha r$. Substituting this in to the second equation, we have

$$\mu_s mgr = -\frac{2}{3}mra \rightarrow \mu_s mg = -\frac{2}{3}ma \rightarrow a = -\frac{3}{2}\mu_s mg$$

Substituting this expression for a into the first equation (from $F = ma$) above, we have

$$-F_W + \mu_s mg = -\frac{3}{2}\mu_s mg$$

which gives

$$F_W = \frac{5}{2}\mu_s mg$$

.