Physics 211 Practice Exam 1
Question 1

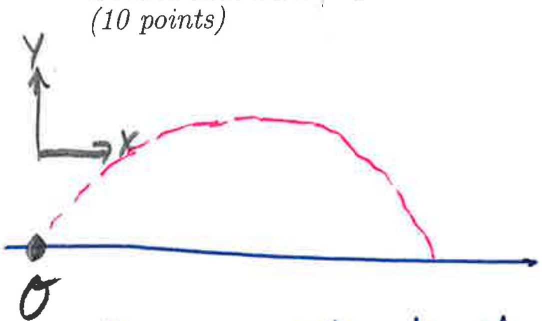
In class you saw a demo consisting of the following. A cart could be pushed along a horizontal rail at a constant velocity $v_{0,x}$. On top of the cart was a machine that shot a ball upward with a vertical velocity $v_{0,y}$. If this machine was triggered while the cart was moving, the ball flew through the air and landed back in the machine.

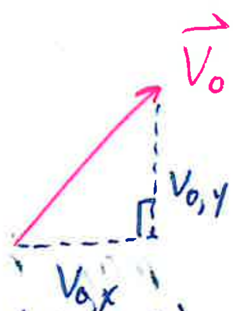
a) Explain, without doing any complicated algebra, why the ball landed back in the machine. (5 points)

When the ball was launched, its $V_{0,x}$ was the same as the cart. Since gravity acts only vertically, the ball's V_x never changes, so it always has the same x-coordinate as the cart.

→ The only difference between the motion of the ball and the cart was in Y, and X and Y are independent.

b) Suppose that the cart is moving at horizontal velocity $v_{0,x}$ when the machine is triggered. How far will the cart move before the ball lands back in it? Give your answer in terms of $v_{0,x}$, $v_{0,y}$, and g. (10 points)





"What is $x(t)$ at the time that $y(t) = 0$?"

$y(t) = V_{0,Y}\,t - \frac{1}{2}gt^2$. Set to 0 and find time:

$0 = V_{0,Y}\cancel{t} - \frac{1}{2}gt^{\cancel{2}} \longrightarrow V_{0,Y} = \frac{1}{2}gt \longrightarrow t = \frac{2V_{0,Y}}{g}$.

$x(t)$ at that time:

$x(t) = V_{0,x}\,t \longrightarrow \boxed{x(t) = \frac{2V_{0,x}V_{0,Y}}{g}}$ (distance traveled by cart when ball lands in it.)

c) What is the maximum height achieved by the ball? (5 points)

Max height when $V_y(t) = 0$:

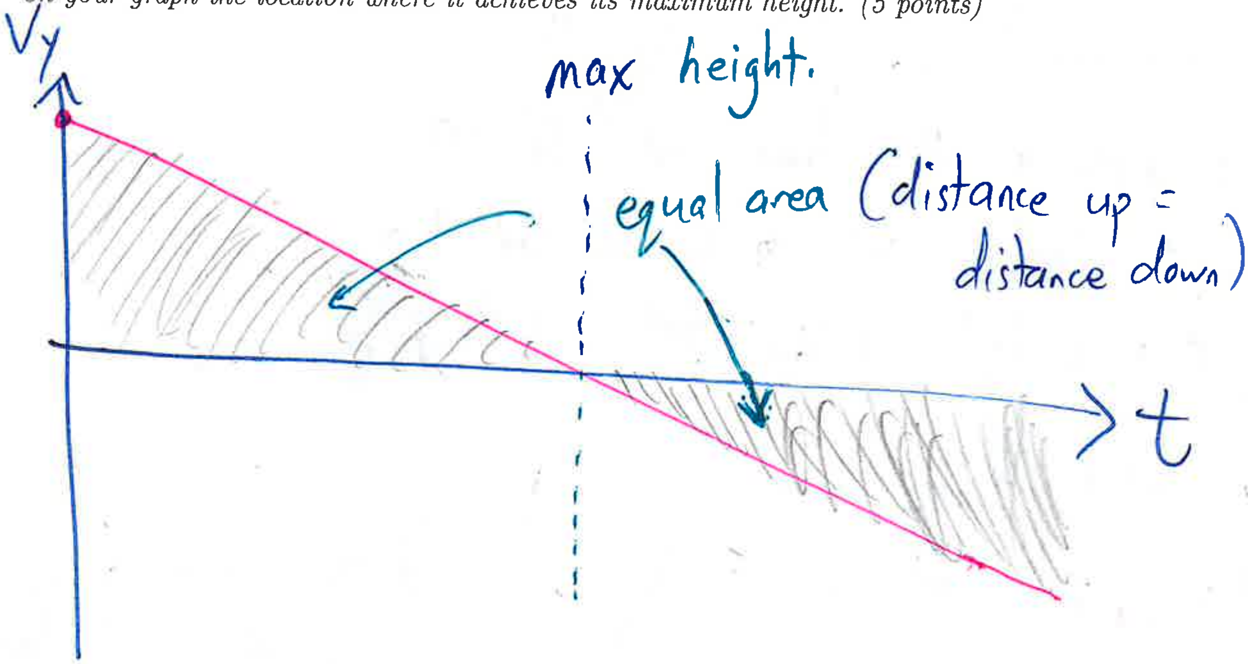
"What is $y(t)$ at the time when $v_y(t) = 0$?" -

$\Longrightarrow V_y(t) = V_{0,y} - gt \rightarrow$ zero when $t = \frac{V_{0,y}}{g}$.

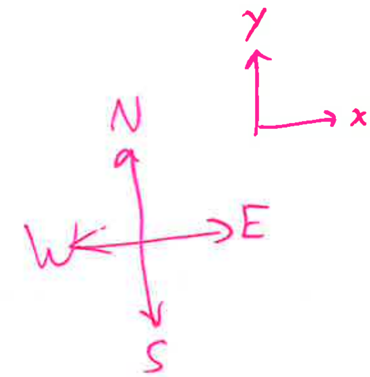
$y(t) = V_{0,y}t - \frac{1}{2}gt^2 \longrightarrow y(t) = -\frac{1}{2}g\left(\frac{V_{0,y}}{g}\right)^2 + V_{0,y}\frac{V_{0,y}}{g}$

Simplify: $-\frac{1}{2}\left(\frac{V_{0,y}^2}{g}\right) + \left(\frac{V_{0,y}^2}{g}\right) = +\frac{1}{2}\frac{V_{0,y}^2}{g}$

d) Sketch a graph (on your own axes) of the y-component of the ball's velocity vs. time, indicating on your graph the location where it achieves its maximum height. (5 points)



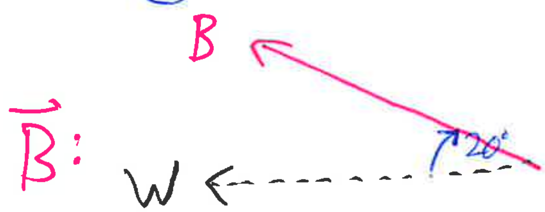
Question 2

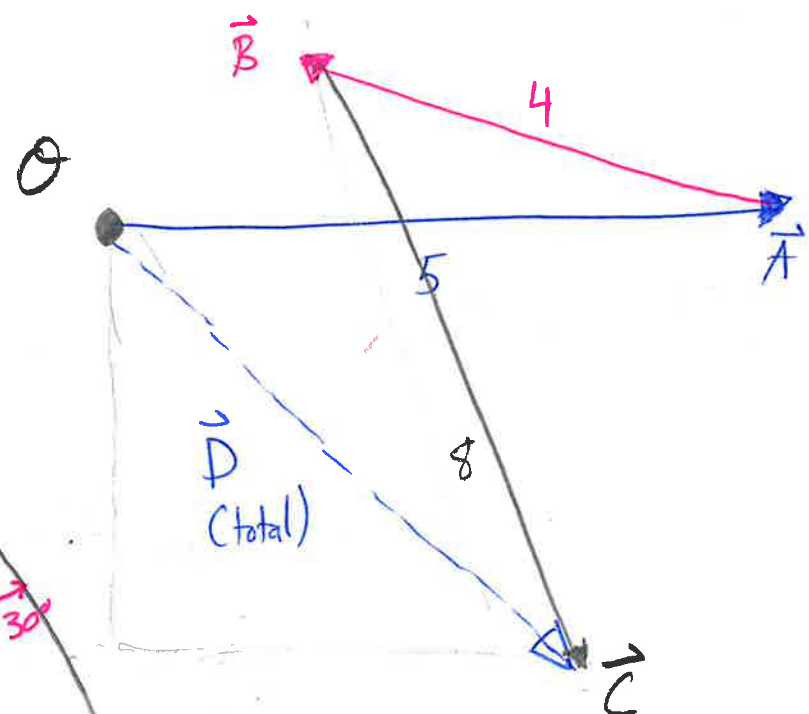


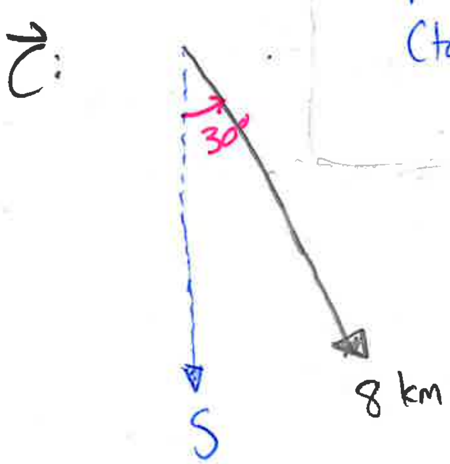
A hiker walks, in sequence:

- 5 km east $\vec{A}$
- 4 km at an angle 20 degrees north of west $\vec{B}$
- 8 km at an angle 30 degrees east of south $\vec{C}$

a) Sketch the hiker's path. (5 points)





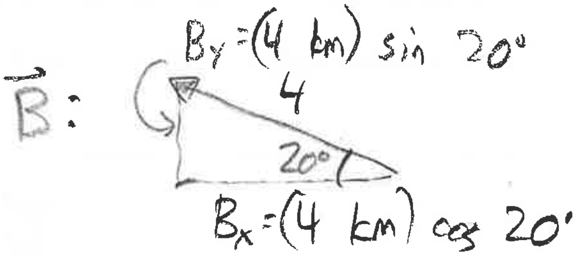


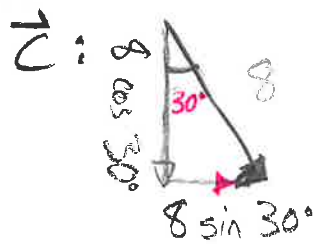
$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$

b) How far from their starting point have they traveled? (10 points)

Find the magnitude of $\vec{A}+\vec{B}+\vec{C}$: do addition in component form.

	X (km)	Y (km)
$\vec{A}$	5	0
$\vec{B}$	-3.76	1.36
$\vec{C}$	4	-6.93
Total ($\vec{D}$)	5.24	-5.57

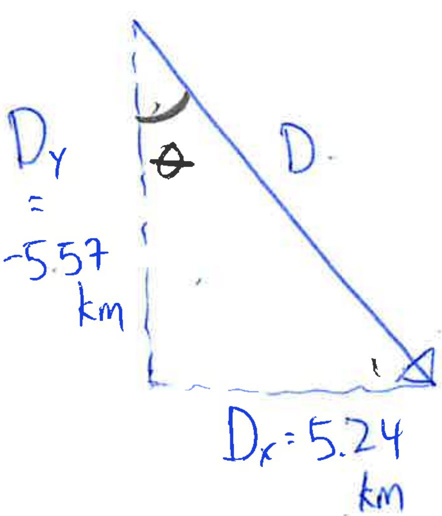




c) *What direction would they look to see their starting point?* (Be precise – give a numerical result, e.g. "*x* degrees west of north" or similar). *(10 points)*

Magnitude: use Pythagorean theorem:

$$D = \sqrt{D_x^2 + D_y^2} = 7.65\ \text{km} \quad \text{(answer to (b))}$$



$$\theta = \tan^{-1}\frac{D_x}{D_y} = 43.2°$$

my total displacement $\vec{A}+\vec{B}+\vec{C}$

(43.2° East of South)

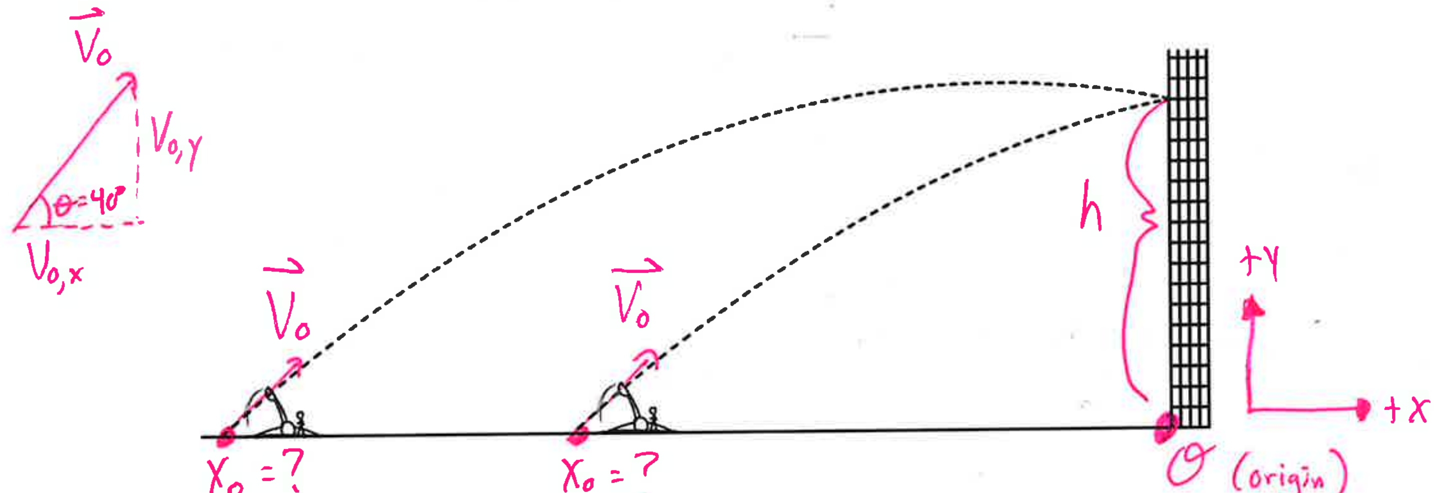
BUT: This is the total vector they have walked → to see their starting point they must look in the reverse direction ($-\vec{D}$)

$\boxed{\to \text{they must look } 43.2° \text{ west of north}}$

Question 3

A student has built a small catapult and wants to use it to throw snowballs from the Quad through the center of Walter's open window in the Physics Building. Suppose that the catapult fires snowballs at an angle $\theta = 40°$ above the horizontal at a speed of $v_0 = 20$ m/s, and the center of the window is a height $h = 8$ m above the ground.[1]

There are two places on the Quad where they could put their catapult. From one position the snowballs will go through Walter's window on the way up; from the other position, they will hit his window on the way down. In this problem, you will figure out where those two locations are.



a) Indicate your choice of coordinate system on the diagram above, and label any other distances/points you want to assign variables to. (3 points)

b) Discuss in words your approach to figuring out where to place the catapult. In particular, what mathematical condition means "the snowball goes in the window"? What variable are you going to solve for?

You may answer this part by simply writing questions in terms of your algebraic variables, as we have practiced. (5 points)

"What is x_0 such that the snowball hits the window?"

→ "What is x_0 such that $x(t) = 0$ at the same time that $y(t) = h$?"

[1] Catapult image from Randall Munroe/xkcd, used under CC-BY-NC. See, we cite our sources too!

$X_0 = ?$ $\quad V_{0,x} = V_0 \cos\theta$ $\quad a_x = 0$

$Y_0 = 0$ $\quad V_{0,y} = V_0 \sin\theta$ $\quad a_y = -g$

Question 3, continued

c) Determine the two points on the Quad where the snowballs will go into the window. Which one is which? (10 points)

$\cancel{x(t) = V_{0,x} t = (V_0 \cos\theta)t}$ $\quad \cancel{y(t)}$

$x(t) = (V_0 \cos\theta)t + x_0$ $\quad y(t) = -\frac{1}{2}gt^2 + (V_0 \sin\theta)t$

Set $y(t) = h$ as the sentence for (b) suggests:

$h = -\frac{1}{2}gt^2 + (V_0 \sin\theta)t \Rightarrow \underbrace{\frac{1}{2}g}_{A}t^2 \underbrace{-(V_0 \sin\theta)}_{B}t + \underbrace{h}_{C} = 0$

$$t = \frac{V_0 \sin\theta \pm \sqrt{V_0^2 \sin^2\theta - 2gh}}{g} \Rightarrow t = (1.014 \text{ or } 1.609 \text{ s})$$

Find x_0 such that $x(t) = 0$:

$0 = (V_0 \cos\theta)t + x_0 \Rightarrow x_0 = -(V_0 \cos\theta)t = \begin{pmatrix} 24.6 \text{ m} \\ 15.5 \text{ m} \end{pmatrix}$

$\Rightarrow$ 15.5 m is the closer one, 24.6 m is the further one.

d) If you repeat this problem with $v_0 = 15$ m/s, you will find that you get a negative under the square root sign in the quadratic formula. What is the physical interpretation of this?[2] (7 points)

$\rightarrow$ no real solutions for t means the snowballs will never reach 8m above ground.

[2]If you did not use the quadratic formula for part (c), then describe instead what will happen with $v_0 = 15$ m/s, and how you would know based on your method.

3 stages of motion: (I) acceleration
(II) constant speed (don't know time!)
(III) deceleration
→ note: like bicycle problem from HW1.

↓

QUESTION 4

Chloe throws a ball for their greyhound Nessie, who zooms off to go fetch it.

She runs a total distance of 50 m. She starts from rest and accelerates at $5\,\mathrm{m/s^2}$ until she is going at 10 m/s, travels at that speed for an unknown time, then slows down at $5\,\mathrm{m/s^2}$ until she comes to a stop, exactly 50 m away from where she started.

Nessie the greyhound with her caterpillar; she is not at *all* spoiled.

a) How long does it take her to accelerate from rest to 10 m/s? (5 points)

$v(t) = v_0 + at$ → v_0 is 0 here, so $t = \frac{v(t)}{a} = 2$ s.

Call this time t_1 → the time the first stage takes.

(Note: stage III takes $t_3 = 2$ s also.)

b) What distance does she travel at 10 m/s before starting to slow down? (10 points)

→ I have a full description of stages (I) and (III), but am missing the time for stage (II).

Plan: find distance for (I) and (III); (II) is whatever is left.

Stage 1: $x(t_1) = \frac{1}{2}a_1 t_1^2 = \frac{1}{2}(5\,\mathrm{m/s^2})(2\,\mathrm{s})^2 = 10$ m.

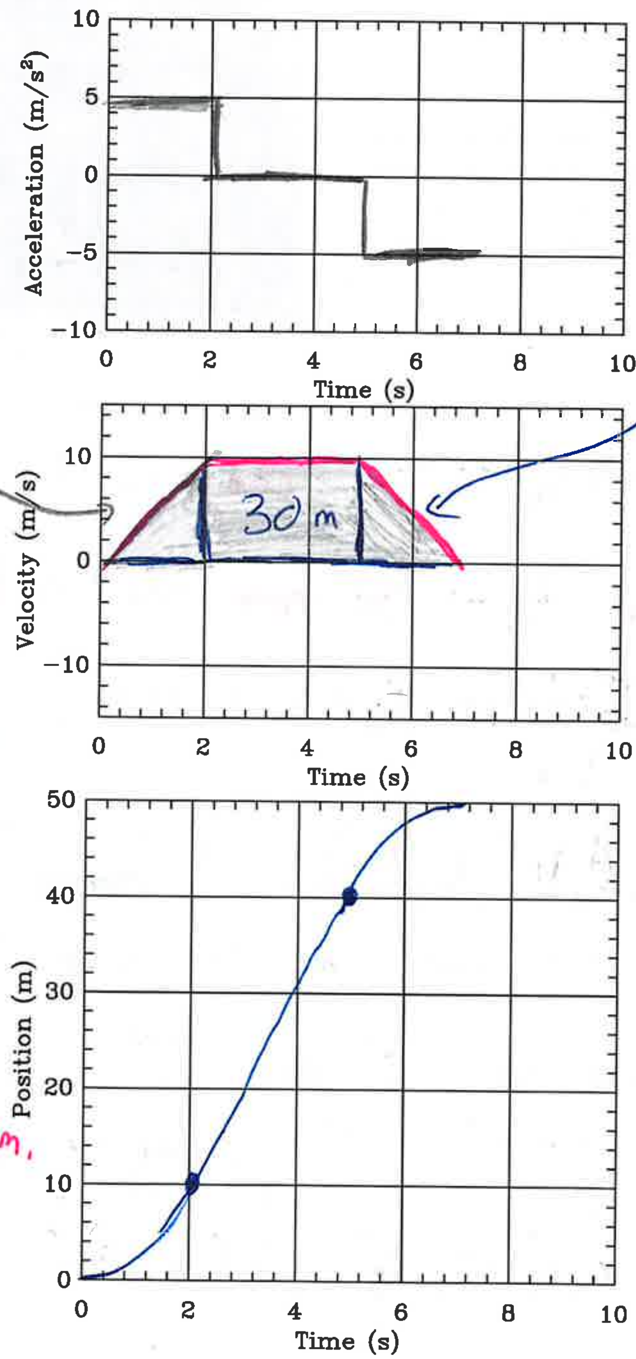
Stage 3: $x(t_3) = \frac{1}{2}a_3 t_3^2 - v_{0,3} t_3 = \frac{1}{2}(-5\,\mathrm{m/s^2})(2\,\mathrm{s})^2 - (10\,\mathrm{m/s})(2\,\mathrm{s}) = 10$ m

→ If stages 1 and 3 were 10 m each, stage 2 must have been 30 m.

→ 30 m @ 10 m/s = $\boxed{3\text{ s for stage 2.}}$

Question 4, continued

c) Graph her acceleration vs. time, velocity vs. time, and position vs. time on the axes provided. (10 points)



area = 10 m

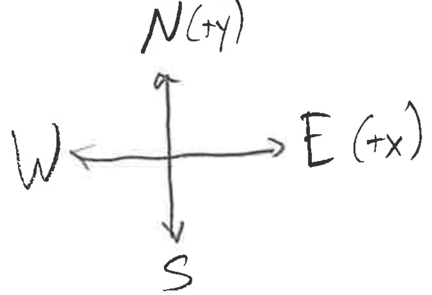
also 10 m

- Note: deceleration "triangle" has same area.

So: the constant-velocity stage is whatever is left:

50 m − 10 m − 10 m = 30 m.

(Graphical alternative solution)



3 vectors in magnitude and direction → convert to X/Y → add → convert back to mag/direction

QUESTION 5

A hungry hummingbird leaves her nest at 6:00 looking for food. She plans to visit three different flowers that she knows about, then to return directly to her nest.

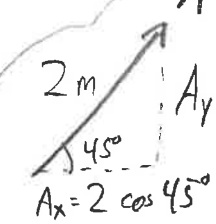
She flies, in order:

- $\vec{A}$ • 45° north of east for two minutes to visit the first flower
- $\vec{B}$ • due south for three minutes, to travel from the first flower to the second
- $\vec{C}$ • 30° north of west for one minute, to travel from the second flower to the third

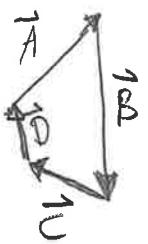
$\vec{D}$

a) How much time does it take her to fly from the third flower to get back to her nest? (10 points)

$\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$. Solve for $\vec{D}$: $\vec{D}=-(\vec{A}+\vec{B}+\vec{C})$



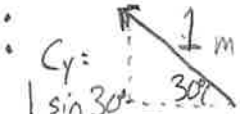
Convert vectors into component form:



	X	Y
$\vec{A}$	$2\cos 45° = 1.414$	$2\sin 45° = 1.414$
$\vec{B}$	0	−3
$\vec{C}$	−0.866	0.5
Total	0.548	−1.09

B: 3m: $B_x = 0$, $B_y = -3$

C:

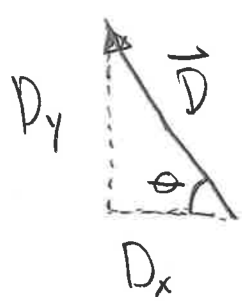


$C_x = -1\cos 30°$

b) In what direction must she fly from the third flower to get back to her nest? (10 points)

$\vec{D}$, the vector to fly home, is $-(\vec{A}+\vec{B}+\vec{C})$:

$D_x = -0.548$, $D_y = 1.09$ (bird-minutes)



(a) $|D| = \sqrt{D_x^2 + D_y^2} = 1.22$ minutes to get home

(b) $\theta = \tan^{-1}\frac{D_y}{D_x} = 63.3°$ north of west

Question 5, continued

c) If she flies at 200 meters per minute, how far did she fly in total? (5 points)

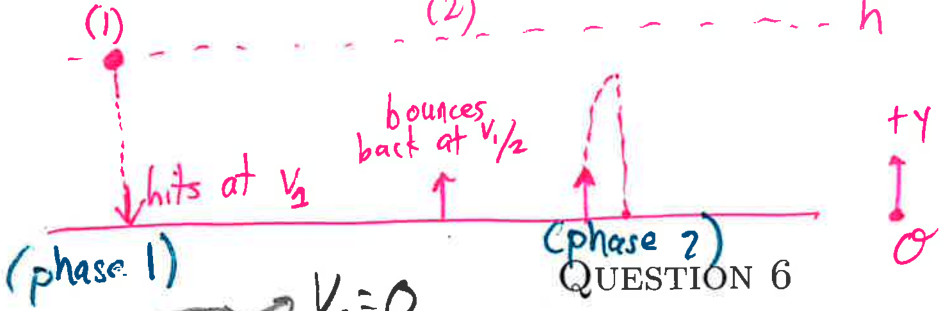
distance

Flight time:

$2 + 3 + 1 + 1.22$ minutes

$= 7.22$ minutes

$7.22 \text{ minutes} \times \frac{200 \text{ m}}{\text{minute}} = 1.44 \text{ km}.$



QUESTION 6

$V_0 = 0$

A person drops a baseball from a height h onto a hard floor. When the baseball hits the floor, it will bounce back in the opposite direction with a speed equal to *half* of the speed with which it hit the floor.

In terms of h and g, find: $y=0$

a) how long it takes to hit the floor the first time (5 points)

"What is t when $y(t)=0$?"

Phase 1: $y(t) = h - \frac{1}{2}gt^2 \rightarrow 0 = h - \frac{1}{2}gt^2$

$\rightarrow t = \sqrt{\frac{2h}{g}}$

b) how fast it is going when it hits the floor the first time (5 points) $y=0$

"What is $v(t)$ at the time that $y=0$?"

$v(t) = -gt$ so

$v(t) = -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh}$

$v(t) = V_0 + at$
- $V_0 = 0$ (dropped from rest)
- $a = -g$ (freefall)

$\rightarrow v(t) = -gt$

c) how high it travels after bouncing off the ground the first time (5 points)

"What is $y(t)$ at the time that $v_y(t) = 0$?"

$y(t) = \frac{1}{2}at^2 + V_{bounce}\,t + \cancel{y_0}^{\,0}$

$\rightarrow V_{bounce}$ = velocity it bounces back with $= -\frac{1}{2}V_1 = -\frac{1}{2}(-\sqrt{2gh}) = \sqrt{\frac{gh}{2}}$

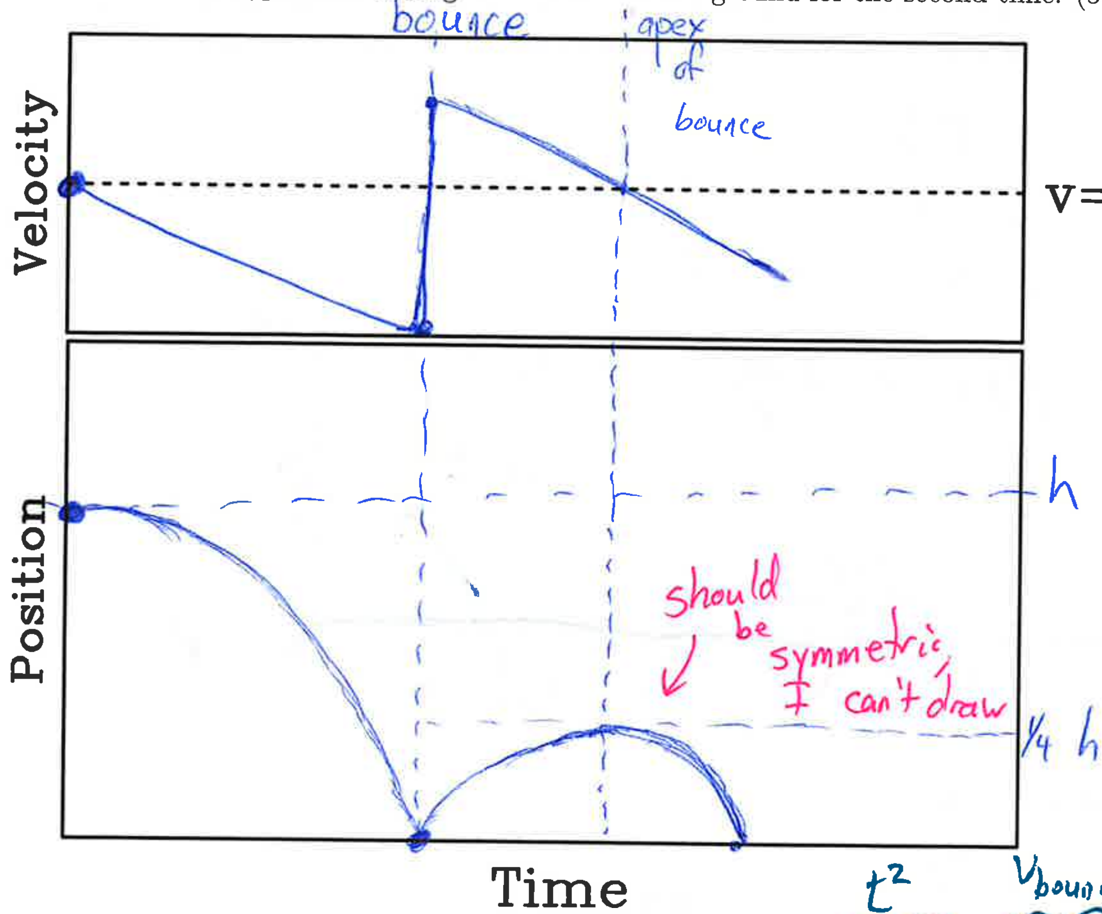
$v_y(t) = V_0 - gt = \sqrt{\frac{gh}{2}} - gt$. Set to 0: $0 = \sqrt{\frac{gh}{2}} - gt$ so $t = \sqrt{\frac{h}{2g}}$. Then $y(t) = \ldots$ (con't)

d) the amount of time between the ball being dropped and it hitting the floor the second time (5 points) "What is t when $y(t)=0$?"

$\underbrace{y(t) = -\frac{1}{2}gt^2 + v_b t}_{\text{E.O.M. for phase 2 that describes bounce}} \Rightarrow -\frac{1}{2}g\cancel{t}^2 + \sqrt{\frac{gh}{2}}\,\cancel{t} = 0 \Rightarrow \frac{1}{2}gt = \sqrt{\frac{gh}{2}}$ so $t = \frac{2}{g}\sqrt{\frac{gh}{2}}$

$= \sqrt{\frac{2h}{g}}$.

e) On the axes provided, graph the ball's velocity vs. time and position vs. time, starting at the time that it is dropped and ending when it strikes the ground for the second time. (5 points)



c) con't: $y(t) = -\frac{1}{2}gt^2 + v_0 t = -\frac{1}{2}g\overbrace{\left[\frac{h}{2g}\right]}^{t^2} + \overbrace{\sqrt{\frac{gh}{2}}}^{v_{bounce}}\overbrace{\left[\sqrt{\frac{h}{2g}}\right]}^{t}$

$y(t) = -\frac{1}{4}h + \sqrt{\frac{gh^2}{4g}} = -\frac{1}{4}h + \sqrt{\frac{h^2}{4}}$

$= \frac{1}{4}h.$

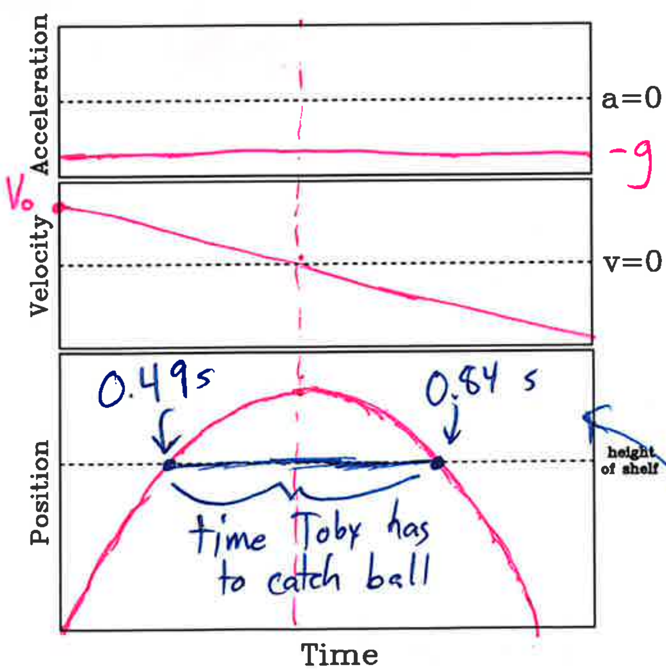
(Bounces $\frac{1}{4}$ as high!)

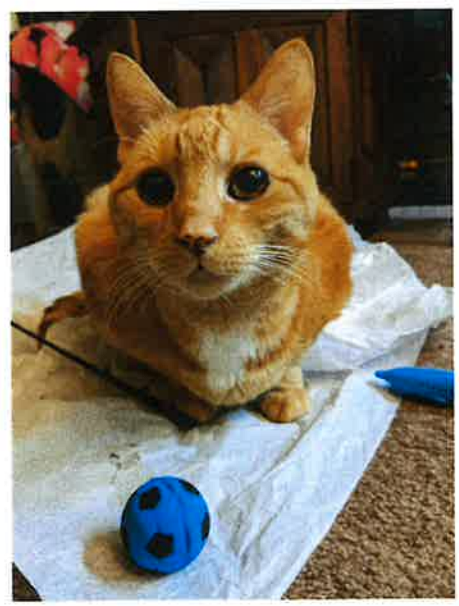
Question 7

Toby the Cat is sitting on a high shelf, at a height of $h = 2$ m off of the ground. Her owner is lying on the floor in front of the shelf when she throws a toy ball straight upward at $v_0 = 6.5$ m/s. As soon as the ball passes the level of the shelf (2 m off of the ground), Toby tries to catch it. She can grab the ball as long as it is above the level of the shelf. If Toby does not grab the ball, it falls back down where her owner catches it again.

In this problem, you will calculate how much time Toby has to grab the toy before it falls back below the shelf.

a) Assuming that Toby doesn't grab the toy out of the air, sketch graphs of the ball's position, velocity, and acceleration as a function of time, from the time her owner throws it to when she catches it again. Indicate the height of the shelf on the position vs. time graph. You do not need to show precise numbers on your graphs, just their shape. Draw these graphs on the axes below. *(5 points)*





Toby the Cat and her blue toy ball. She is 16 years old and spoiled rotten by her mommy, as you can tell by all the cat toys she's surrounded by.

these come from the Q.F. on next page.

b) Write down algebraic expressions for the position and velocity of the ball as a function of time. *(5 points)*

$$y(t) = V_0 t - \frac{1}{2} g t^2$$

$$V_y(t) = V_0 - gt$$

c) How much time does Toby have to grab the ball? (This is the total amount of time that it is above the level of the shelf.) *(10 points)*

"What is the difference between the two values of t for which $y(t) = h$?"

$y(t) = h \rightarrow -\frac{1}{2}gt^2 + V_0 t = h$

$\rightarrow \frac{1}{2}gt^2 - V_0 t + h = 0$, $t = \dfrac{V_0 \pm \sqrt{V_0^2 - 2gh}}{g}$

Difference between roots : $\dfrac{2\sqrt{V_0^2 - 2gh}}{g}$ = ~~[illegible]~~ s. calculator typo

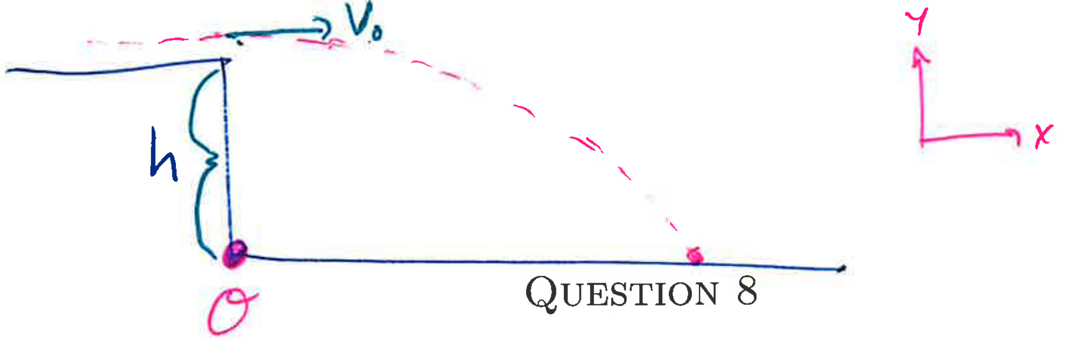
0.35 s.

(Note this

d) Suppose that her owner instead throws the ball upward with a starting velocity of 5 m/s rather than 6.5 m/s. In this scenario, how much time will Toby have to grab the ball? *(5 points)*

Plug 5 m/s into the thing we found above :

$\rightarrow$ ack! Now $V_0^2 < 2gh$ and the answers are imaginary : ball never makes it 2m off ground and Toby is sad.



QUESTION 8

A ball rolls off a horizontal shelf of height h at speed v_0. Answer the following in terms of h, v_0, and g.

a) How much time does it take the ball to hit the floor? *(5 points)*

"At what time does $y(t) = 0$?"

$$y(t) = -\tfrac{1}{2}gt^2 + h \rightarrow 0 = -\tfrac{1}{2}gt^2 + h \rightarrow h = \tfrac{1}{2}gt^2,$$

$$t = \sqrt{\frac{2h}{g}}$$

b) Where does the ball hit the floor? *(5 points)*

"What is $x(t)$ at the time that $y(t) = 0$?"

$$x(t) = \tfrac{1}{2}\cancel{a_x}^{0}t^2 + V_{0,x}t + \cancel{x_0}^{0}$$

Plug in t from (a): $x(t) = V_0\sqrt{\frac{2h}{g}}$ $\rightarrow$ distance from the base of shelf.

c) What is the ball's speed when it hits the floor? *(5 points)*

"What is the magnitude of $\vec{v}$ at the time that $y(t) = 0$?"

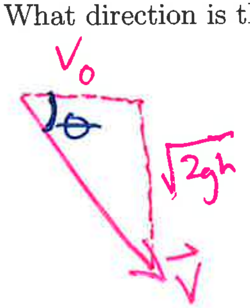
$$V_x(t) = V_0 \quad (a_x = 0)$$
$$V_y(t) = -gt$$

$\Longrightarrow$ When ball hits floor,

$$\vec{V} = (V_0, -\sqrt{2gh})$$

$$\text{Magnitude} = \sqrt{V_x^2 + V_y^2} = \boxed{\sqrt{V_0^2 + 2gh}}$$

d) What direction is the ball moving in when it hits the floor? *(5 points)*



$\longrightarrow$ $\theta = \tan^{-1}\left(\frac{\sqrt{2gh}}{V_0}\right)$ (angle below horizontal)

e) Suppose that the edge of the shelf had been curved, so that the ball's initial velocity was instead directed at an angle θ below the horizontal. Explain, using words or algebra as appropriate, what things you would have needed to do differently to solve the previous four parts, and which things would stay the same. *(5 points)*

→ Only thing that changes is $\vec{v_0}$:

Instead of $\vec{v_0} = \underbrace{(v_0, 0)}_{\text{just in } x}$,

now it would have both X and Y component.

You would proceed in exactly the same way except now there would be a "$v_0 t$" term in both $x(t)$ and $y(t)$.

(You would need Q.F. for finding the time when $y(t) = 0$, but that doesn't change the approach.)