Problem solving: kinematics

Physics 211
Syracuse University, Physics 211 Spring 2015
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Announcements

- Homework 2 is due next Wednesday
- The first *Mastering Physics* tutorial is due ASAP
- No new *MP* assignments are mandatory until after exam 1, but I will post some optional ones for review/study
- I will be testing clickers Friday at 2PM in Stolkin; we will use them next week
- The Facebook page is set up: see https://www.facebook.com/groups/384100861768360/
- The wiki pages now have discussion options available to you (you don't need an account)
  - Up to 2% extra credit for participation here and on the wiki in helping your peers understand things
- The course schedule is available on the wiki

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Acceleration, velocity, and position relationships are the same in 2D; they just apply independently for each component.

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One more piece of notation

There's one piece of notation I didn't mention last time about vectors.

"Unit vectors": vectors of length 1 along the x and y axes

- Vectors are written with an arrow: $\vec{r}$, $\vec{F}$, $\vec{a}$
- Unit vectors are written with a hat: $\hat{i}$, $\hat{j}$, $\hat{k}$
- $\hat{i} = (1, 0, 0), \hat{j} = (0, 1, 0), \hat{k} = (0, 0, 1)$

So we might write: $\vec{V} = -3\hat{i} + 4\hat{j} + 2\hat{k}$. This describes the vector $\vec{V} = (-3, 4, 2)$.

Problem solving: 2D kinematics, constant acceleration

1. 1. If you have vectors in the "angle and magnitude" form, convert them to components
2. 2. Write down the kinematics relations, separately for x and y
   - Many terms will usually be zero
   - Freefall: $a_x = 0$, $a_y = -g$ (with conventional choice of axes)
3. 3. Understand what instant in time you want to know about
4. 4. Put in what you know; solve for what you don't (using substitution, if necessary)
5. 5. Convert vectors into whatever format the problem asks for

The plan

- Today: solve problems involving 2D kinematics and projectile motion
- Next Tuesday: introduce rotational kinematics (it's easy); more problems
- Next Thursday: review for Exam 1
- *Tuesday, 3 Feb: Exam 1*

A reminder: there are lots of resources available for you.
(I was in the Clinic all day Tuesday helping folks, for instance.)

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- Now let's say the cart is moving at 1.2 m/s when it shoots the ball out. Where does it land?

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- To get the speed when it hits, we just use the velocity relations:
- $v_x = v_{0,x}$ and $v_y = -gt$
- $v_x = 6.64$ m/s, $v_y = \sqrt{2gh} = -44.2$ m/s
- $|v| = \sqrt{v_x^2 + v_y^2} = 44.7$ m/s

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- This gives us $x(t) = \frac{2v_{0,x}^2}{g}$
- $y(t)$ will have the same magnitude: the Pythagorean theorem gives $|r| = 2\sqrt{2}\frac{v_{0,x}^2}{g}$

A rocket

A rocket is launched from rest on level ground. While its motor burns, it accelerates at 10 m/s at an angle 30 degrees below the vertical. After ten seconds its motor burns out and it follows a ballistic trajectory until it hits the ground.

How far does it go?