Power and rotational energy

Physics 211
Syracuse University, Physics 211 Spring 2019
Walter Freeman

March 25, 2019

Announcements

- Upcoming office hours:
  - Today: 3-5
  - Wednesday: 3-5
  - Thursday: 1:45-3:45
  - Homework due Friday

Where we've been, where we're going

- Last time: we saw that "potential energy" is both a statement about nature and a bookkeeping trick to keep track of work
  - Potential energy only applies to conservative forces (gravity, springs)
  - Lets us account for the work done by these forces with no integrals required
  - Potential energy due to Earth's gravity: $U_g = mgy$
  - Potential energy in a spring: $U_e = \frac{1}{2}k(\Delta x)^2$

Power: rate of doing work

A bit of mathematics that will be useful to you:

"An object moves at a constant speed $\vec{v}$, subject to some force $\vec{F}$; at what rate does that force do work on the object?"

An example: an airplane flies at v=1000 m/s, and its engines exert F=300 kN of thrust. What is the rate at which the engines do work (power)?

$$\text{Work} = \text{force} \cdot \text{distance}$$

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- The engines output 300 MW of power: this is around 10 liters per second of fuel even at 100% efficiency!
- Some of that 300 MW of energy dissipated by drag heats up the airplane... (real numbers for a SR-71 Blackbird)

A truck pulling a heavy load with mass $m = 4000$ kg wants to drive up a hill at a 30° grade.

If the truck's engine can produce 100 kW of power (134 hp), how fast can the truck go? (Neglect drag.)

Rolling a ball down a hill

If I roll these balls down the hill, how fast will they be traveling at the bottom?

A: The more massive one will roll faster
B: The less massive one will roll faster
C: The larger one will roll faster
D: The smaller one will roll faster
E: They will all roll at the same speed

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... the velocity shouldn't depend on any of these!

What's gone wrong?

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Not all of the potential energy goes into making the ball *translate* (move); some goes into making it *rotate*. How much?

Most rotational physics is the same as translational physics:

$$KE_{\text{trans}} = \frac{1}{2}mv^2$$
$$KE_{\text{rot}} = \frac{1}{2}m_{\text{rot}}v_{\text{rot}}^2$$

- What is "rotational velocity"?

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$$

- What is "rotational velocity"? $\rightarrow$ Angular velocity, ω

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- What is "rotational mass"?

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- What is "rotational velocity"? $\rightarrow$ Angular velocity, ω
- What is "rotational mass"?
  - This is a new thing: *rotational inertia*, or *moment of inertia* I.

Moment of inertia

The analogue of mass is called "moment of inertia" (letter I)

- More massive things are harder to turn, but that's only part of it
- The mass *distribution* matters, too
- The further the mass is from the center, the harder it will be to turn
- The moment of inertia depends on the *average squared distance from the center*

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$$I = MR^2$$

(if all the mass is the same distance from the center)
(our demo rods; hoops; rings; bike wheels)

Moment of inertia: why?

To see why $I = M \langle r^2 \rangle$, let's consider the kinetic energy of a spinning object.

The kinetic energy of a single "point mass" moving in a circle is $\frac{1}{2}mv^2 = \frac{1}{2}mr^2\omega^2$, where r is its distance from the center.

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For an extended object, we simply add up the energy of all the moving particles:

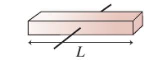
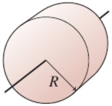
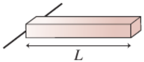
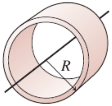
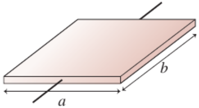
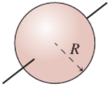
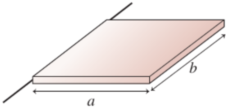
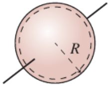
$$I = \int r^2 dm = M\left\langle r^2\right\rangle$$

i.e. the moment of inertia is just the total mass times the average squared distance from the axis.

Moment of inertia, other things

What about the moment of inertia of other objects?
Requires calculus in general; here are some common ones

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

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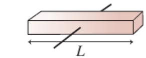
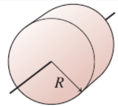
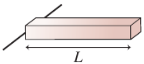
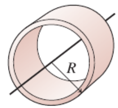
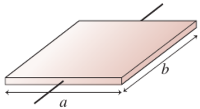
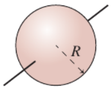
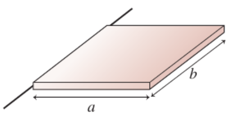
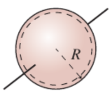
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

In general: $I = \lambda M R^2$

We will always give you I if it's not 1 (i.e. not a ring etc.)

Rolling down the hill

$$\begin{aligned} PE_{i,grav} &= KE_{f,trans} + KE_{f,rot} \\ mgh &= \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 \\ mgh &= \frac{1}{2}mv_f^2 + \frac{1}{2}\lambda m R^2 \omega_f^2 \end{aligned}$$

This is unsatisfying: we have two unknowns, v_f and ω_f.

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Are they truly independent?

$\rightarrow$ If an object rolls without slipping, v and ω are related! $\leftarrow$

Rotation plus translation

In general, rotation and translation are separate; we can study each separately.

Example: this bike wheel

- Its position is given by some function $\vec{s}(t)$: "where is it at some time t?"
- Its angle is given by some other function $\theta(t)$: "which way is the reference point pointing at some time t?"
- The angle has the familiar derivatives: angular velocity ω, angular acceleration α

Recall that points along the edge of a rotating object move at a speed $v_{\mathrm{edge}} = \omega r$.

Example: rolling without slipping

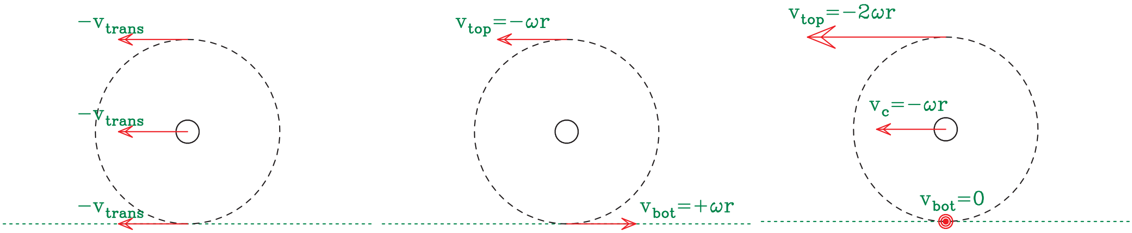
Sometimes the translational and rotational motion are linked.
"How fast do the tires on a car turn?"
→ Static friction means that the bottom piece of the wheel doesn't move

- If a wheel is turning counterclockwise at angular velocity ω:
  - the top moves at $v_{\text{top}} = -\omega r$ (left)
  - the bottom moves at $v_{\text{bot}} = \omega r$ (right)
- This means that the velocity of the axle must be equal and opposite to v_{bot}
- Thus, the car must be moving at $v_{\text{axle}} = -\omega r$ (left).

Let's look at a diagram.
So: if the wheels turn counterclockwise at ω:

- The axle moves at a velocity $-\omega r$ (left);
- The top of the wheels move at a velocity $v_{\text{axle}} + v_{\text{top}} = -\omega r - \omega r = -2\omega r$;
- The top of the wheels move at a velocity $v_{\text{axle}} + v_{\text{bot}} = -\omega r + \omega r = 0$.

Rolling without slipping



Translation + Rotation = Rolling

Rolling down the ramp

We did this before. Now we know that $v_f = \omega_f r$. Substitute this in:

$$\begin{aligned} PE_{i,grav} &= KE_{f,trans} + KE_{f,rot} \\ mgh &= \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 \\ mgh &= \frac{1}{2}mv_f^2 + \frac{1}{2}\lambda m R^2 \omega_f^2 \\ mgh &= \frac{1}{2}mv_f^2 + \frac{1}{2}\lambda m v_f^2 \end{aligned}$$

Note that the rotational and translational kinetic energy terms combine, since $v^2 = \omega^2 R^2$. This will often happen.

$$gh = \frac{1}{2}(1+\lambda)mv_f^2 \to v_f = \sqrt{\frac{2gh}{1+\lambda}}$$

Sample problems

A mass m is hung from a spring of spring constant k and released. Which equation would let me find the distance d that it falls before it comes back up?

- A: $mgd - \frac{1}{2}kd^2 = 0$
- B: $\frac{1}{2}kx^2 = mgd + \frac{1}{2}mv^2$
- C: $0 = -mgd + \frac{1}{2}kd^2$
- D: $mgd + \frac{1}{2}kd^2 = 0$

Sample problems

A disk rolls down a ramp and then up around a loop of radius r. How high must the ramp be for the disk to make it around the loop? (See picture on document camera.)

How am I going to do this problem?

- A: Use Newton's laws to relate the height of the ramp to the speed at the top of the loop, then use kinematics to determine if it will fall or not.
- B: Use conservation of energy to relate the height of the ramp to the speed at the top, then use kinematics to determine at what point it will fall in the loop.
- C: Use conservation of energy to relate the height of the ramp to the speed at the top, then use Newton's second law and our knowledge of rotational motion to determine the required speed.
- D: Use kinematics to relate the height of the ramp to the speed entering the loop, then use Newton's second law and our knowledge of rotational motion to determine the required speed.

Sample problems

A 1000 kg car has an engine that produces up to P=100 kW of power. If it accelerates as hard as it can, at what speed does its acceleration become limited by the engine?

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At low speeds: static friction limits acceleration
At high speeds: engine power limits acceleration

Sample problems

42. ‖ A 1000 kg elevator accelerates upward at $1.0\ m/s^2$ for 10 m, starting from rest.
    a. How much work does gravity do on the elevator?
    b. How much work does the tension in the elevator cable do on the elevator?
    c. Use the work-kinetic energy theorem to find the kinetic energy of the elevator as it reaches 10 m.
    d. What is the speed of the elevator as it reaches 10 m?

Sample problems

57. ‖ The spring shown in **FIGURE P11.57** is compressed 50 cm and used to launch a 100 kg physics student. The track is frictionless until it starts up the incline. The student's coefficient of kinetic friction on the 30° incline is 0.15.
    a. What is the student's speed just after losing contact with the spring?
    b. How far up the incline does the student go?

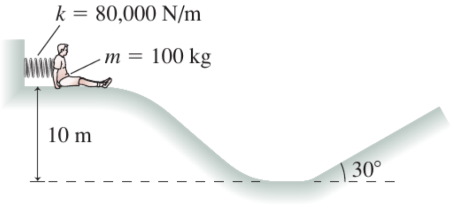

FIGURE P11.57

Sample problems

49. ‖ Truck brakes can fail if they get too hot. In some mountainous areas, ramps of loose gravel are constructed to stop runaway trucks that have lost their brakes. The combination of a slight upward slope and a large coefficient of rolling resistance as the truck tires sink into the gravel brings the truck safely to a halt. Suppose a gravel ramp slopes upward at $6.0°$ and the coefficient of rolling friction is 0.40. Use work and energy to find the length of a ramp that will stop a 15,000 kg truck that enters the ramp at 35 m/s ($\approx$75 mph).