PHY 211 Lecture 24

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Torque

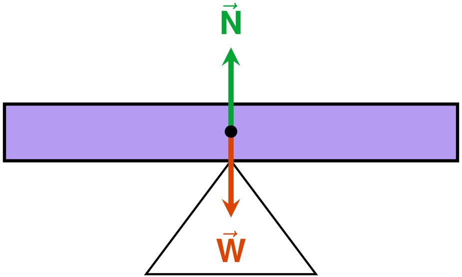
- We talked a bit a few weeks ago about torque, moment of inertia and angular acceleration
- Today we want to review that and start doing more complicated torque calculations

"Newton's second law" for rotation

$$\sum \tau = I\alpha$$

Torque and balancing

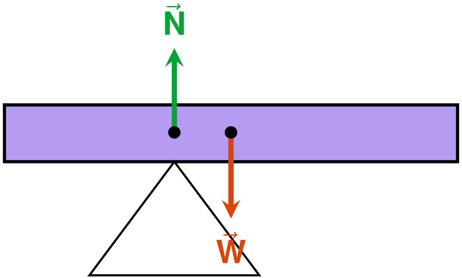
- The force of gravity acts at the center of mass of an object
- If the forces act perpendicular to the radius, then torque depends linearly on the distance



- When centered, the weight and normal force have no torque around the center, because that's where they act

Torque and balancing

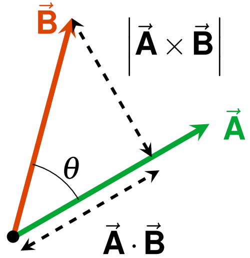
- Why does it fall when off balance?



- The normal force should still be able to cancel the weight, right?
- But it can't cancel the torque around the edge

Cross product

- The most general way to express torque is $\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$
- The cross product is kind of like a complement to the dot product we talked about for work
- $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ tells you the component of $\vec{\mathbf{A}}$ along $\vec{\mathbf{B}}$
- $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$ tells you the component of $\vec{\mathbf{A}}$ perpendicular $\vec{\mathbf{B}}$



$$\left|\vec{\mathbf{A}} \times \vec{\mathbf{B}}\right| = AB\sin(\theta)$$

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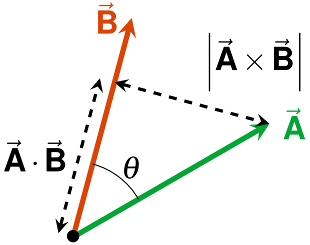

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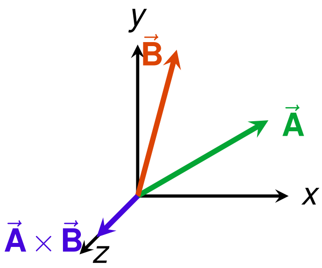
Calculating the cross product

- The cross product is a vector
- We will mainly be interested in the magnitude of the torque
  - The direction we can visualize as which way it spins
  - But the vector direction will be more important for E&M!
- Use the right-hand rule – fingers point towards first vector, curl to second vector, thumb gives cross product's direction



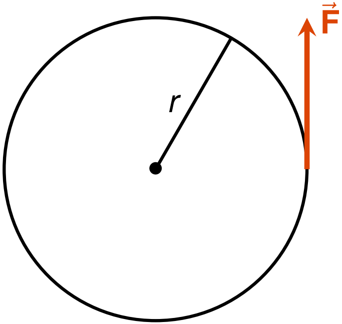
Cross product always perpendicular to initial two vectors

Main concepts for torque

- Which way would a force make something spin?
  - One direction should be positive (*e.g.* counter-clockwise positive, clockwise negative)
- You get more torque from either more force, or a bigger distance
- Perpendicular force gives the most torque $\tau = rF$
- Non-perpendicular force with an angle θ gives $\tau = rF\sin(\theta)$ (and $\sin(\theta) < 1$ always)
  - θ is the angle between the r and F vectors if you draw them from the same starting point
  - Can also make a coordinate system with an axis parallel to r and find perpendicular component of F

What is the torque?

Positive spins counter-clockwise



(a) $\tau = rF$
(b) $0 < \tau < rF$
(c) $\tau = -rF$
(d) $-rF < \tau < 0$
(e) $\tau = 0$

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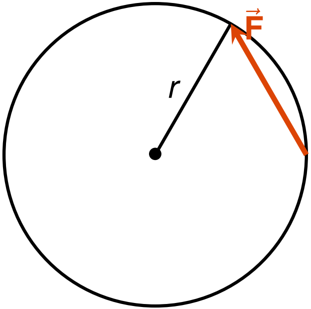

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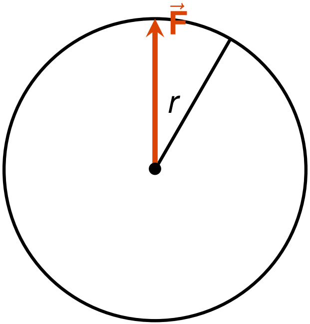

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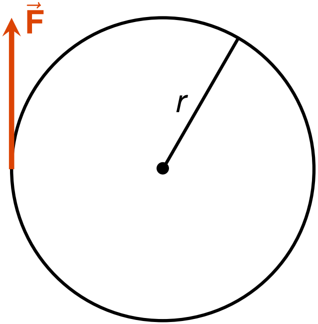

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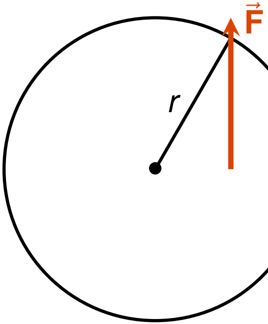

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Torque and angular acceleration

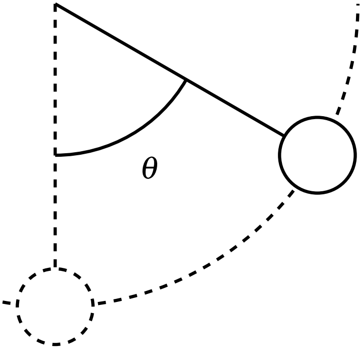
"Newton's second law" for rotation

$$\sum \tau = I\alpha$$

- Our main scenarios are the same as we discussed for rotational kinetic energy:
  - Fixed rotation, where α is the angular acceleration around the pivot point
  - Rolling without slipping – where $\alpha = a_{\mathrm{CM}}/r$
  - Rotating pulleys – where $\alpha = a_{\mathrm{rope}}/r_{\mathrm{pulley}}$

Pendulum

- What is the angular acceleration of the pendulum at an angle θ?

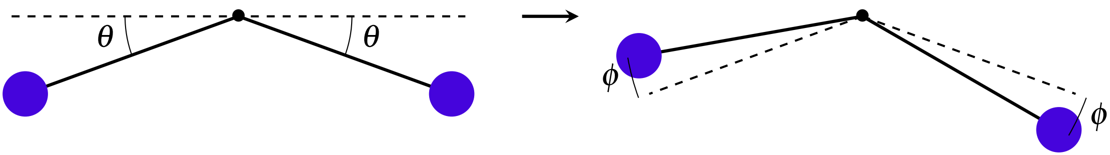


Torque balancing

- Why does a tightrope walker want a pole?
  https://www.youtube.com/watch?v=UEnkN939ZLw
- Why does this guy stay up?
  https://www.youtube.com/watch?v=mQYn4Uc2cIs

What is the torque?

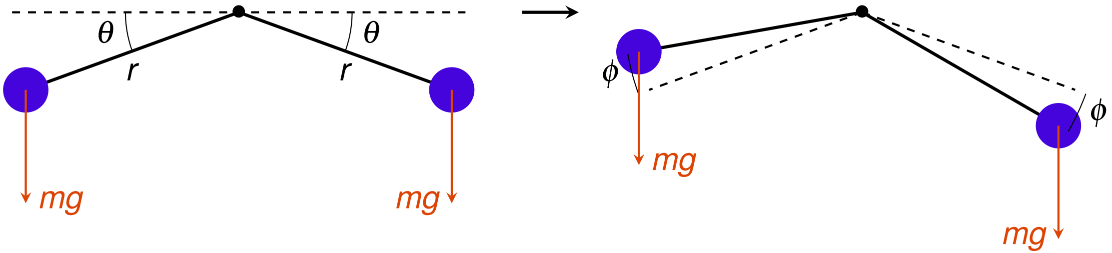
- Think of two masses on poles at angle θ that then rotates by ϕ



- Does the magnitude of torque on the left increase or decrease?
- What about the right?

Question

- What is the torque on the right mass if clockwise is positive?



- A $mgr\sin(\theta)$
- B $mgr\sin(\phi)$
- C $mgr\sin(\theta+\phi)$
- D $mgr\cos(\theta+\phi)$
- E $mgr\sin(\theta-\phi)$
- F $mgr\cos(\theta-\phi)$

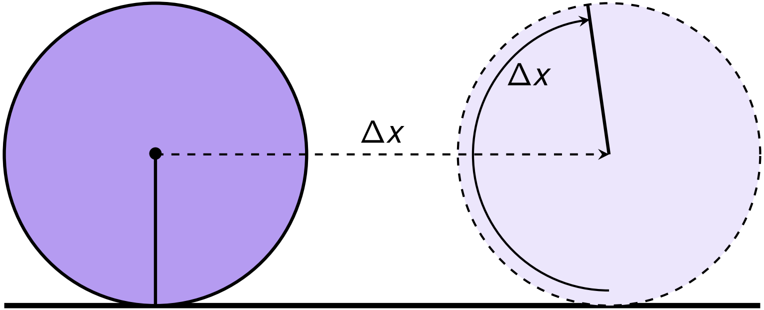
Torque and work

- Force times distance did work and increased kinetic energy
- Torque times angular distance does work and increases rotational kinetic energy

$$\tau\theta = \Delta\left(\frac{1}{2}I\omega^2\right)$$

Rolling without slipping

- Rolling without slipping is one of the cases where $v = \omega r$

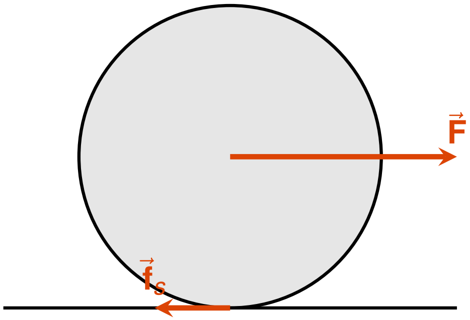


$$v = \omega r \qquad \to a \qquad = \alpha r$$

Torque and rolling

- If something starts rolling or accelerates while rolling, we need a torque for that
- If there's not another force doing it you need static friction to provide torque
- Notice, though, that this does not remove energy from the system

$$f_s r \Delta\theta = \Delta\left(\frac{1}{2} I\omega^2\right)$$
$$-f_s \Delta x = \Delta\left(\frac{1}{2} mv^2\right)$$



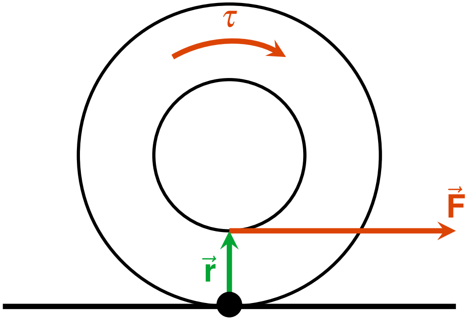
Spool of ribbon

- Which way will the spool move when the ribbon is underneath?
- Try to just think about how torque will make it spin

(a) Towards pull (b) Away from pull (c) Stay in place

Choice of axis

- Like with coordinate systems, you can choose what axis to analyze
- Some are easier and some more useful than others
- We can say the whole spool rotates around the point it touches the ground

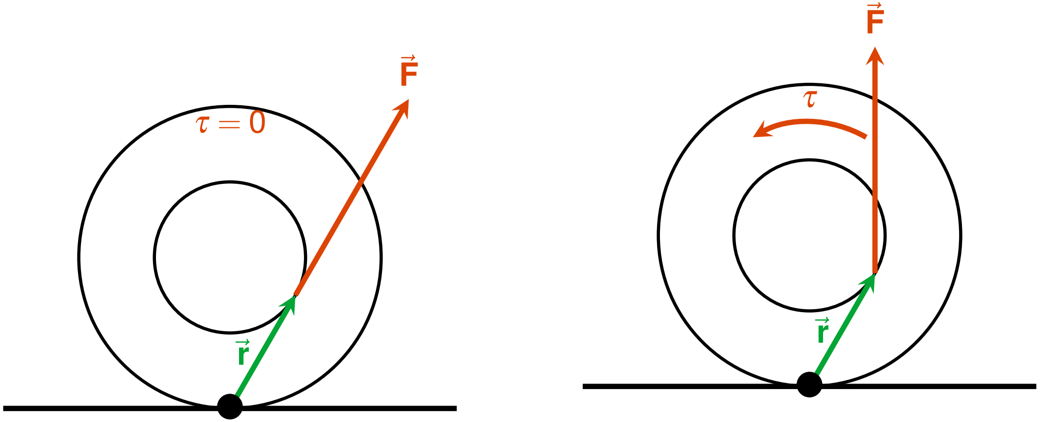


Friction doesn't cause torque in this picture!

How to choose an axis?

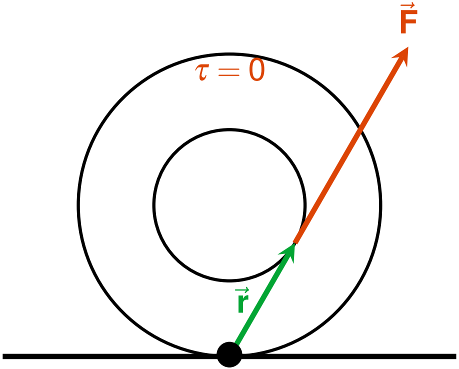
- Is the rotation forced to be around a hinge or some other fixed point?
  Use that as your axis
- Is there some unknown force acting on the object? It may help to set your axis to where that force acts so it causes no torque
  - Example is the friction of the rolling object
- Otherwise, center of mass may be a good choice
  - Then you can solve for the center-of-mass motion and the rotation about the center of mass separately

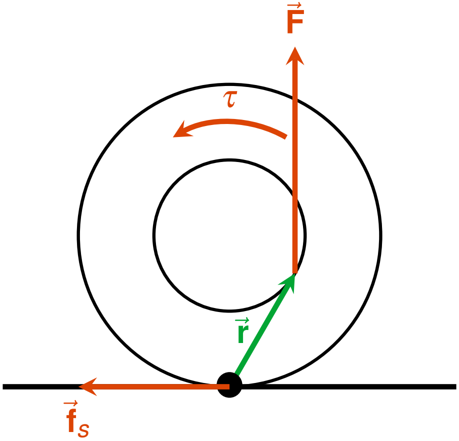
Redirecting force



What causes the force that moves it?

Redirecting force





Static friction “switches”

Example

A block of mass 3 kg slides down an inclined plane at an angle of 45° with a massless tether attached to a pulley with mass 1 kg and radius 0.5 m at the top of the incline. The pulley can be approximated as a disk. The coefficient of kinetic friction on the plane is 0.4. What is the acceleration of the block?