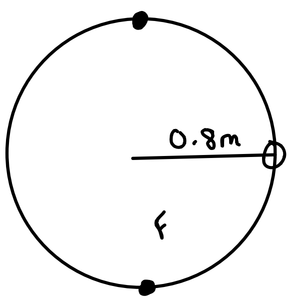
PHY 211 Recitation 12

February 21, 2020

1

A person uses a rope to spin a bucket in a vertical circle at a constant speed; the radius of the circle is 80 cm. The bucket goes around the circle once every second. Inside the bucket is a frog of mass 500 g.

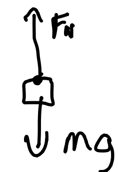
(a) Draw a force diagram for the frog when the bucket is at the top of the circle, and when it is at the bottom.



$m = 0.5\ Kg$

FBD [Top]

FBD [Bottom]



(b) What is the acceleration of the bucket?

$r\omega = v \quad = 2\pi \frac{rad}{s} \quad v = \left(2\pi \frac{rad}{s}\right)\cdot\left(\frac{4}{5}m\right) = \frac{8\pi}{5}\frac{m}{s} = 5.03\ m/s$

$a = \frac{v^2}{r} = \frac{(5.03)^2}{4/5} = 31.6\ m/s^2$

(c) Your "apparent weight" is simply the magnitude of the normal force that an object under you exerts on you. What is the frog's apparent weight at the bottom and at the top of the circle?

① $N_B - mg = ma \Rightarrow N_B = m(a+g) = 0.5\cdot(9.8 + 31.6) = 20.7\ N$

② $N_T + mg = ma \Rightarrow \quad N_T = ma - mg = +0.5(a-g) = 10.9\ N$

$-N_T - mg = ma$

$N_T + mg = -ma \Rightarrow N_T = mg - ma = 0.5(g - 31.6) = -10.9\ N$

(d) Explain why the frog doesn't fall out of the bucket at the top of the swing, despite the fact that the only forces acting on it point downward.

Centrifugal Force Keeps Frog in bucket :

$\boxed{m\cdot a = m\cdot\frac{v^2}{r}}$

"Pseudo Force"

(e) Now, imagine that the person swinging the bucket slows down gradually. At some point, the frog will fall out of the bucket. (It's a frog, so it'll land on its feet and not be hurt!) How low can the angular velocity ω become before the frog falls out of the bucket?

$ma = N_T + mg$ When $N_T = 0$, there is no longer any contact.

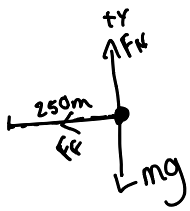
$a = g \Rightarrow \frac{v^2}{r} = g$ $\quad v = [gr]^{1/2} = 2.8\,m/s$

$\omega = v/r = \frac{2.8}{0.8} = \ldots$

2

A highway curve on flat ground has a radius of curvature of 250 m; that is, it is a segment of a circle whose radius is 250 m. A car with mass M is going around the curve.

(a) Draw a free-body diagram for the car; you should treat the perspective as if you were standing behind the car and looking at the rear of the car. Add a coordinate axis.



(b) What force causes the centripetal acceleration of the car? What is the magnitude of the centripetal acceleration?

The force causing centripetal acc is Force of friction

$ma_c = \frac{mv^2}{r} = \mu_s \cdot N \Rightarrow \cancel{m}a_c = \cancel{m}g\mu_s$ $\quad \boxed{a_c = g\mu_s}$

(c) Sum up the components of the forces in the x and y directions.

$\sum F_y = F_N - mg = ma_y = 0 \qquad F_N = mg$

$\sum F_x = \mu_s N = ma_x$

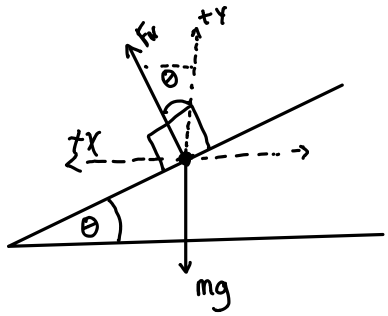
(d) If the coefficient of static friction between the car's tires and the road is 0.9, how fast can the car take the turn?

$\frac{v^2}{r} = g\mu_s \qquad v = \sqrt{\mu_s \cdot gr}$

3

Now imagine that the same curve is banked so that traffic moving at 30 m/s can travel around the curve without needing any help from friction.

(a) Draw a free body diagram for a car traveling around this curve at a constant speed. Draw the diagram so that you are looking at the rear of the car.



(b) You should set up the coordinate axes so that $+x$ points towards the center of the curve. That is, do not tilt your coordinate axes for this problem. Why do you think that might be helpful?

(c) What is the acceleration of the car in the x-direction? What about the y-direction?

$a_y = 0$

$a_x = F_n \sin\theta$

(d) Sum up the components of the forces in the x and y directions.

$\sum F_y = F_n \cos(\theta) - mg = ma_y = 0$

$\sum F_x = F_n \sin(\theta) = ma_x = m\frac{v^2}{r}$

(e) Use Newton's second law to solve for the angle of the road.

$\frac{F_N \cancel{\cos}\theta}{\cos\theta} = \frac{mg}{\cos\theta} \Rightarrow F_N = \frac{mg}{\cos\theta} \quad \frac{\cancel{m}g}{\cos\theta} \cdot \sin\theta = \frac{\cancel{m}v^2}{r}$

$g \tan\theta = \frac{v^2}{r} \qquad \theta = \tan^{-1}\left(\frac{v^2}{gr}\right)$

(f) If a car goes through the turn slower than 30 m/s, would friction act on the car? Which direction would it point?

Outward to prevent sliding down the bank.