Recitation Exercises – Vectors
Week 2, Day 2

In this recitation, you will practice:

- Converting vectors from "magnitude and direction" representation to "$x-$ and $y-$component" representation

- Breaking vectors into their $x-$ and $y-$ components

- Representing vectors in different coordinate systems

- Using vector addition and subtraction to solve problems

This recitation activity is absolutely vital for the rest of the semester. If you don't understand anything you have done today, please go visit the Physics Clinic to work with a tutor. You can also tell your recitation TA that you'd like extra help with vectors; we'll contact you and arrange for that help as part of the assistance program.

Question 1. The streets in Manhattan are laid out in a grid, but that grid is aligned with the island, rather than along the compass directions. Avenues run 28.9 degrees east of north, while streets run 28.9 degrees north of west.

This means that there are two sensible coordinate systems in Manhattan:

- North/South/East/West, aligned with the compass

- Uptown/Downtown/Crosstown, aligned with the streets.

(In this map and the next one, "up" is due north.)
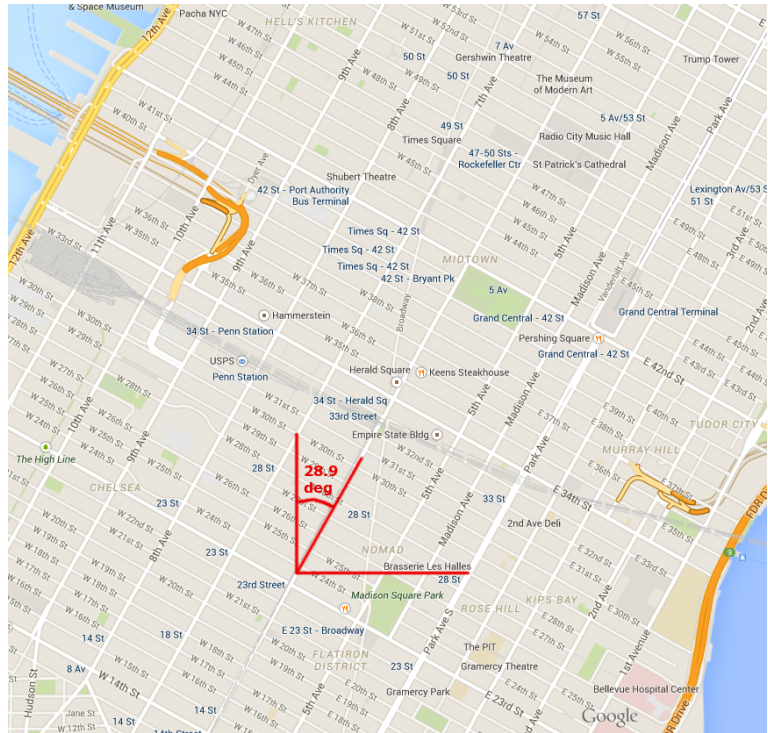
The staff astronomer at the Natural History Museum walks from Penn Station to the Natural History Museum, going 3.3 km north along Eighth Avenue. How far east and how far north did she walk? (*Note: Her path carries her off of the top of the page. You will want to draw an arrow aligned with her path below, and then draw its components.*)

Question 2. Here is the displacement vector pointing from the Metropolitan Opera to Carnegie Hall.

Draw on top of the picture:

- its component in the direction of the Manhattan avenues;
- its component perpendicular to them;
- its component along the North-South axis;
- its component along the East-West axis.

It may be helpful to draw the first two on the bottom and left, and the second two on the top and right – just so they don't overlap on your picture.

Question 3. A hiker in the forest walks 5 km due north and then 2 km due east, and then wants to return to their original spot by the shortest route possible.

Draw their path.

Which direction should they walk, and for how far?

Question 4. Now our hiker walks 3 km due north, then 4 km at an angle 30 degrees south of east, and then finally 5 km at an angle 45 degrees north of west. They then wants to return to their starting point, as before. Which direction should they travel in, and for how far?

Hint 1: It will help visualize things to draw the hiker's path first.

Hint 2: Remember that you can add vectors by converting them to $x-$ and $y-$components and then adding the components. The simplest approach here is:

1. Convert the vectors to component form

2. Add the components

3. Convert the resulting sum back to the magnitude-and-direction form that you are looking for.

Question 5. A swimmer can swim 5 km/hr in still water. They want to swim directly across a river. However, there is a current in the river, with a speed of 2 km/hr. If they swim directly across, they will drift downstream due to the current. Thus, in order to get where they want to go, they needs to angle themselves upstream.

1. There are three interesting vectors in this problem: the velocity of the current, the swimmer's velocity relative to the current, and their velocity relative to the shore. How do they relate? State this both mathematically (for instance, "this vector plus that vector equals this other vector"), and geometrically (draw a picture).

2. At what angle must they try to swim in order to proceed directly across the river?

3. If the river is 200 m across, how long will it take them to cross?