Torque, moment of inertia, and review

Physics 211
Syracuse University, Physics 211 Spring 2020
Walter Freeman

February 25, 2020

Announcements

Upcoming schedule:

- Wednesday, February 26: HW6 due in recitation. HW7 posted.
- Thursday, February 27: Review for Exam 2.
- Friday, February 28: Group Exam 2. Profs in Clinic 9:30-12, 1-3
- Sunday, March 1: Review in Stolkin Auditorium, 6-9 PM.
    - I have four concerts Fri/Sat/Sun and will be exhausted, have mercy :)
- Monday, March 2: office hours 1-3 PM
- Tuesday, March 3: **Exam 2 in class**.
- Wednesday, March 4: Recitations will be held as normal.

Announcements

I've gotten a lot of requests for one-on-one tutoring ahead of the exam.

Unfortunately, there aren't enough hours in the day for me to meet everyone privately. I'm sorry; there's only one of me.

However, if you have questions, you can ask them here, at help hours, or at the review Sunday night.

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However, if you have questions, you can ask them here, at help hours, or at the review Sunday night.

I'm behind on answering email; I will catch up ASAP.

Requiescat in pace...

Katherine Johnson, NASA mathematician and computer: 1918-2020

Agenda for today:

We may introduce one new idea today – a broad concept that we'll build on later.

But I also want to spend a lot of time reviewing.

So we could also spend the entire day doing practice problems.

I have a few in mind, but you can ask me to walk you through:

- Anything from previous recitations
- Hints for anything from Homework 6
- Anything from Homework 5

The causes of circular motion

You've learned about angular velocity ω already: the rate of rotation.

But how do things come to be rotating in the first place?

It turns out that we can understand *rotation* in the same way we understand ordinary motion (*translation*).

Correspondences between linear and rotational motion

Linear motion	Rotational motion
Position $\vec{s}$	Angle θ
Velocity $\vec{v}$	Angular velocity ω
Acceleration $\vec{a}$	—
Constant-acceleration kinematics:	—
• $\vec{s}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$	• —
• $\vec{v}(t) = \vec{a}t + \vec{v}_0$	• —
Force $\vec{F}$	—
Mass m	—
Newton's second law $\vec{F} = m\vec{a}$	—

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- $\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$
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Torque τ
—

—

Torque (for now)

Torque is like a twisting force.

Applying a tangential force to an object that can rotate exerts a torque

$$\tau = \text{size of force} \times \text{distance from center}$$

This means that the units of torque are *newton-meters*, or $N \cdot m$.

This means that the torque applied to an object by a force depends both on *the size of the force* and *where it acts*.

(For now, we will think only about tangential forces.)

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Torque τ
Moment of inertia I

—

Moment of inertia / rotational inertia

Rotational inertia is what the name implies: the rotational analogue to mass.

It depends on both an object's *mass* and *how far that mass is from the center.*

We use the symbol I for rotational inertia.

- Which of these objects is harder to spin?

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$$I = \text{object's mass} \times \text{average distance of mass from center}$$

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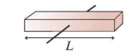
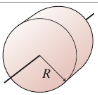
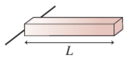
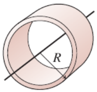
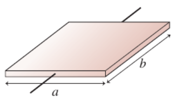
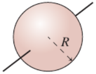
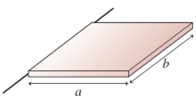
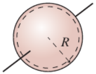
$$I = M \left\langle R^2 \right\rangle$$

What about the moment of inertia of other objects?
Requires calculus in general; here are some common ones

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

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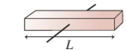
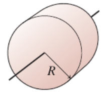
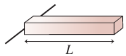
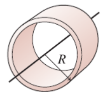
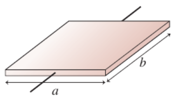
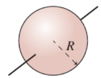
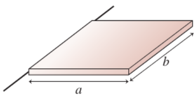
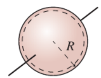
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

$$\text{In general: } I = \lambda M R^2$$

We will always give you I if it's not 1 (i.e. not a ring etc.)

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Velocity $\vec{v}$
Acceleration $\vec{a}$

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Force $\vec{F}$
Mass m

Newton's second law $\vec{F} = m\vec{a}$

Rotational motion

Angle θ
Angular velocity ω
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Constant-acceleration kinematics:

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- $\omega = \alpha t + \omega_0$

Torque τ
Moment of inertia I

Newton's second law for rotation $\tau = I\alpha$

A cable is wrapped around a steel cylinder of radius $r = 10$ cm and mass $m = 30$ kg. A motor applies a tension $T = 100$ N to the cable.
How fast will the cylinder be rotating after a time $t = 3$ seconds? (The moment of inertia of a cylinder is $I = \frac{1}{2}mr^2$.)

- What is the torque applied by the motor? (Remember: $\tau = Fr$ to find the size of the torque)
- What is the moment of inertia of the cylinder?

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- Use Newton's second law for rotation to find α

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- What is the torque applied by the motor? (Remember: $\tau = Fr$ to find the size of the torque)
- What is the moment of inertia of the cylinder?
- Use Newton's second law for rotation to find α
- Use rotational kinematics $\omega(t) = \omega_0 + \alpha t$ to find ω at $t = 3$ s

What is the mass of the Sun?

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The intermediate result we got – that $\omega = \frac{2\pi}{\tau}$, where τ is how long it takes to go around – is useful to know.

A carnival ride

A person of mass $m = 80$ kg is standing on a horizontal platform that moves in a large vertical circle of radius $r = 10$ m every $t = 15$ s. (Imagine that it is moving clockwise.)

What is their apparent weight at the top of the circle? What about the bottom?

What direction does their acceleration point at the top of the circle?

A: Upward
B: Downward
C: Left
D: Right
E: None of these, or $\vec{a} = 0$

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What direction does their acceleration point at the **bottom** of the circle?

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B: Downward

C: Left

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E: None of these, or $\vec{a} = 0$

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What is their apparent weight at the top of the circle? What about the bottom?

Now let's draw the force diagram.

In what direction does the normal force point at the top of the circle?

A: Upward

B: Downward

C: Left

D: Right

E: The normal force isn't a force that acts at the top of the circle

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Now let's draw the force diagram.

In what direction does gravity point at the top of the circle?

A: Upward

B: Downward

C: Left

D: Right

E: Gravity isn't a force that acts at the top of the circle

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What is their apparent weight at the top of the circle? What about the bottom?

Now let's draw the force diagram.

In what direction does the centripetal force $\omega^2 r$ act at the top of the circle?

A: Upward
B: Downward
C: Left
D: Right
E: Centripetal force isn't a force that acts at the top of the circle

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What is their apparent weight at the top of the circle?

A: $F_N = mg + m\omega^2 r$

B: $F_N = mg - m\omega^2 r$

C: $F_N = mg$

D: None of the above

A problem from last year's Exam 2