Recitation Questions

22 February

Question 1: geostationary orbit

It is sometimes useful to place satellites in orbit so that they stay in a fixed position relative to the Earth; that is, their orbits are synchronized with the Earth's rotation so that a satellite might stay above the same point on Earths surface all the time.

What is the altitude of such an orbit? Note that it is high enough that you need to use $F_g = \frac{GMm}{r^2}$ rather than just $F_g = mg$.

Hint 1: If this orbit is synchronized with Earth's rotation, then you should be able to figure out its angular velocity. $\omega = \frac{\Delta\theta}{\Delta t}$, so the satellite rotates once ($\Delta\theta = 2\pi$) per day ($\Delta t = 24\text{hrs}\frac{3600s}{1hr}$) $= 86400s$

Hint 2: If you do this problem as we have guided you, by waiting to substitute numbers in until the very end, you will arrive at an expression relating the radius R of a circular orbit with the mass M of the planet being orbited and the angular velocity ω of the orbit. This question will be on HW5, and is related to the derivation of Kepler's third law that you will do there.

Thus $\omega = \frac{2\pi}{86{,}400s} = \boxed{7.3\times10^{-5}\ \text{rad/s}}$

$\longrightarrow \vec{F}_w$

Since weight is the only force on the satellite,

Mass of earth

mass of satellite

$$\frac{GMm}{r^2} = ma$$

$$= m\omega^2 r$$

since the satellite travels in a circle

Now $\frac{GM\cancel{m}}{r^2} = \cancel{m}\omega^2 r$. We can multiply by r^2 so that

$$GM = \omega^2 r^3 \quad \text{and} \quad r = \left(\frac{GM}{\omega^2}\right)^{1/3} = \left(\frac{(6.6741\times10^{-11}\frac{m^3}{kg\,s^2})(5.9722\times10^{24}kg)}{(7.3\times10^{-5}\ \text{rad/s})^2}\right)^{1/3}$$

$$= \boxed{4.22\times10^{7}\ m}$$

notice we won't have radians in our final answer. Although radians are a way to measure an angle, they come from a ratio of lengths, so radians are unitless.

Question 2: variation of apparent weight with latitude

For this problem, carry all calculations to five significant digits. Some figures that will be useful:

- Mass of Earth: 5.9722×10^{24} kg
- Radius of Earth: 6.3710×10^{6} m (assume it is spherical)
- Gravitational constant (G): $6.6741 \times 10^{-11} \mathrm{N \cdot m^2/kg^2}$ $= \mathrm{kg\, m/s^2 \cdot m^2/kg^2} = \dfrac{\mathrm{m^3}}{\mathrm{kg\, s^2}}$
- Length of one day: 8.64×10^{4} s

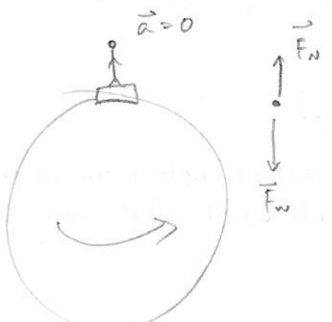
a) What is the force of gravity on a 1 kg mass resting on the surface of the Earth? Are you surprised by this figure?

$$F_W = \frac{GMm}{r^2} = \frac{(6.6741 \times 10^{-11}\ \mathrm{Nm^2/kg^2})(5.9722 \times 10^{24}\ \mathrm{kg})(1\ \mathrm{kg})}{(6.3710 \times 10^{6}\ \mathrm{m})^2}$$

$$= \boxed{9.8200\ \mathrm{N}}$$

this is a little bigger than the 9.81 we might have expected!

b) Suppose this mass were resting on a scale sitting on the North Pole owned by Santa Claus. Recall that scales measure the normal force that they exert. What value would Santa's scale read? What would Santa conclude the value of g is?



$$F_N = F_w$$

$$= \frac{GMm}{r^2} = \boxed{9.8200\ \mathrm{N}},$$

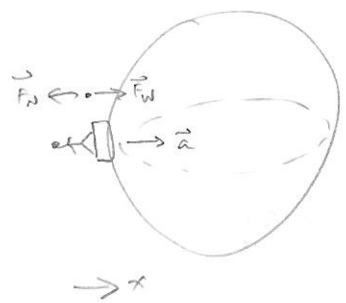
just like above.

Santa would conclude

$$\boxed{g = 9.8200\ \mathrm{m/s^2}}$$

remember, when we use ω we measure angle) in radians

c) Suppose that an identical 1 kg mass were resting on a scale sitting on the Equator, somewhere in Kenya. What would *this* scale read? (Hint: What is the acceleration of the mass?) What would our Kenyan physicist conclude about g?



From $\sum F_x = ma_x$

$-F_N + F_W = m\omega^2 r$, so

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{1 \text{ revolution}}{1 \text{ day}} = \frac{2\pi \text{ radians}}{8.64 \times 10^4 \text{ s}} = 7.2722 \times 10^{-5} \text{ rad/s}$$

$$F_N = F_W - m\omega^2 r$$
$$= 9.82 \text{ N} - (1 \text{ kg})(7.2722 \times 10^{-5} \text{ rad/s})^2 (6.371 \times 10^6 \text{ m})$$
$$= \boxed{9.7863 \text{ N}}$$

d) This problem shows that your apparent weight depends on your location on Earth. Does it make sense to define g as F_g/m (the strength of the gravitational force divided by an object's mass) or F_N/m (the strength of the normal force, and thus the scale reading, divided by mass)? Call your TA/coach over to join your conversation.

F_g/m is the same everywhere on a spherical earth, so it doesn't reflect the different g-values we've calculated. F_N/m, although it depends on a scale reading, does indicate the rate of acceleration you would see on a falling object, just like the frog in the elevator appeared to fall with a different value of g.

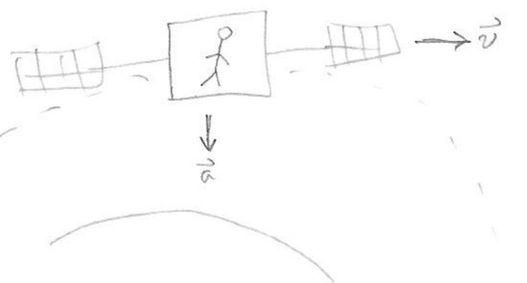
e) Is this distinction likely to be relevant to the sort of engineering or science you will do during your career? (The answer will depend on what you will do, of course!)

Geological surveys often require careful maps of how g varies across the earth, and some sensitive experiments also have to take the local g into account, but for low-precision purposes, $g = 9.8 \text{ m/s}^2$ is often precise enough.

Question 3: Weightlessness

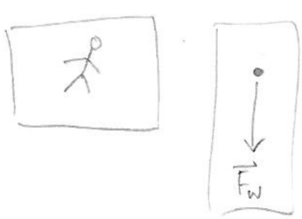
Astronauts in orbit around the Earth are not "so far away that they don't feel Earth's gravity"; actually, they're quite close to the surface. However, we've all seen the videos of astronauts drifting around "weightlessly" in the International Space Station.

a) Explain how an astronaut can be under the influence of Earth's gravity, and yet exert no normal force on the surface of the spacecraft she is standing in.



If the astronaut and the spacecraft are accelerating at the same rate (due to gravity), no extra forces are needed for the astronaut and spacecraft to move together

b) Draw a force diagram for the astronaut floating in the middle of the Space Station, not touching any of the walls or floor. How do you reconcile your diagram with the fact that the astronaut doesn't seem to fall?



The astronaut would be falling, but the floor of the spacecraft is falling at the same rate!

c) Is this astronaut truly "weightless"? What does "weightless" mean?

Although the astronaut still experiences the force of their weight (this force keeps them in their circular orbit around the earth), they don't experience any normal forces against anything, which makes them feel weightless (they would read 0 on a scale).