Question 1

A pilot wants to fly from Syracuse to Baltimore. Syracuse lies 300 miles north of Baltimore.

Her aircraft can fly at 500 miles per hour in still air.

However, the airplane is traveling through the jet stream, which blows at 80 miles per hour to the east.

There are three vectors in this problem:

- The velocity vector of the airplane relative to the ground. This determines the direction the plane travels. Call this vector $\vec{A}$.
- The velocity vector of the airplane relative to the air (of magnitude 500 mph).This vector is determined by the direction the airplane is pointing. Call this vector $\vec{B}$.
- The velocity vector of the wind relative to the ground (magnitude 80 mph, pointing east). Call this vector $\vec{C}$.

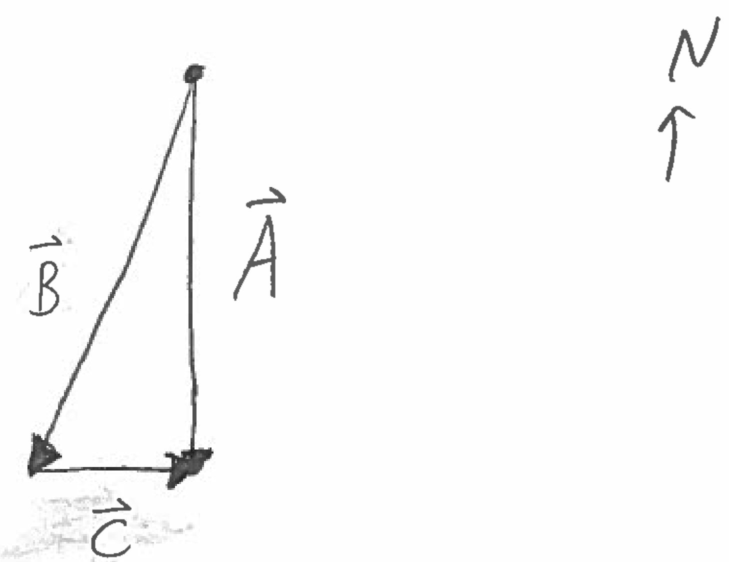
a) She wants to fly due south. If she points her airplane directly south, describe (qualitatively) which direction she will wind up traveling in. (5 points)

The jet stream will blow her east of south.

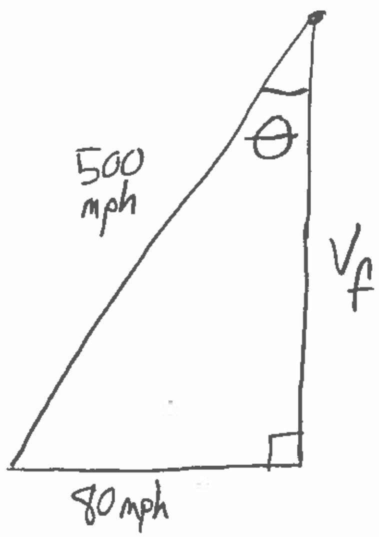
b) Write a vector equation relating vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$. (5 points)

$$\vec{A} = \vec{B} + \vec{C}$$

c) Illustrate the vector equation you wrote for part b) graphically, by drawing a triangle whose sides are the three vectors involved. (10 points)



d) In what direction should she point her aircraft if she wants to travel directly south to Baltimore? (20 points)



$\theta = \sin^{-1} \frac{80}{500} =$

9.2° west of south

c) How long will it take for her to reach Baltimore? (10 points)

Find actual velocity v_f:

$(80 \text{ mph})^2 + v_f^2 = (500 \text{ mph})^2$

$\rightarrow v_f = \sqrt{500^2 - 80^2} \text{ mph} = 494 \text{ mph}.$

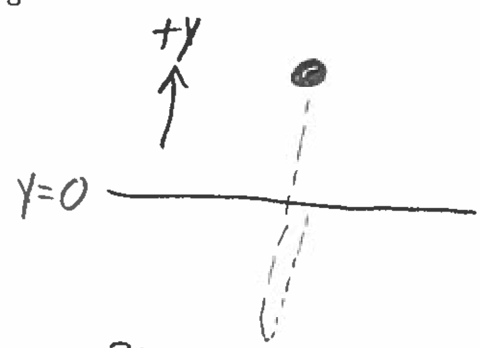
$\frac{300 \text{ miles}}{494 \frac{\text{miles}}{\text{hour}}} = 0.61 \text{ hours}.$

(plus time for de-icing.)

Question 2

A hollow ball falls into a pond from a height of 1m. While it is in the air, it is in freefall. When it is underwater, it has an acceleration of 5 m/s^2 upward, because it is light enough to float. The pond is 3m deep.

You may assume that its velocity does not change as it passes through the surface of the water.



a) With what velocity does it strike the surface? (10 points)

$h = 1\,m = y_0$

$V_0 = 0$

$\Longrightarrow y(t) = h - \frac{1}{2}gt^2$ "What is v when $y=0$?"

$v(t) = -gt$ $\quad 0 = h - \frac{1}{2}gt^2 \Longrightarrow t_{hits} = \sqrt{\frac{2h}{g}}$; $\boxed{v(t) = -\sqrt{2gh} = 4.42\ \text{m/s downward}}$

b) Does the ball reach the bottom of the pond before it rises back to the top? (10 points)

Let $d = 3m$. "Does $y(t)$ ever equal $-d$?"

Use velocity from (a) as initial velocity:

$y(t) = \frac{1}{2}at^2 - \sqrt{2gh}\,t \Longrightarrow -d = \frac{1}{2}at^2 - \sqrt{2gh}\,t \Longrightarrow 0 = \overbrace{\tfrac{1}{2}a}^{A}t^2 \overbrace{-\sqrt{2gh}}^{B}\,t + \overbrace{d}^{C}$

Examine determinant of QF: $\sqrt{2gh - 2ad} = \sqrt{2(9.8\,m/s^2 \cdot 1m - 5\,m/s^2 \cdot 3m}$

$\longrightarrow$ quantity under $\sqrt{\ }$ is negative, so $\boxed{\text{no.}}$

c) How long after it is dropped does it take for the ball to reach the surface again? (10 points)

Two parts. From (a), it falls for $t_1 = \sqrt{\frac{2h}{g}}$.

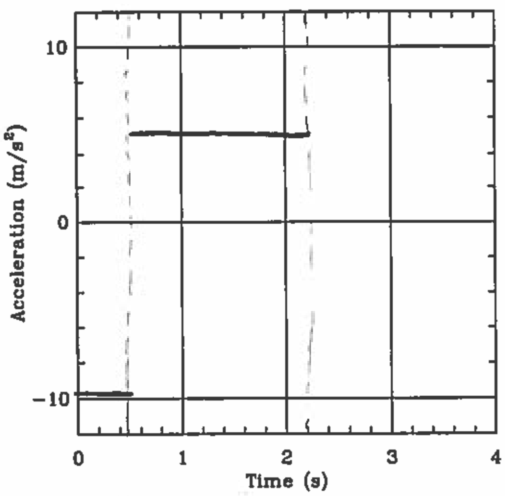
Part (b): "when does $y = 0$? (again)"

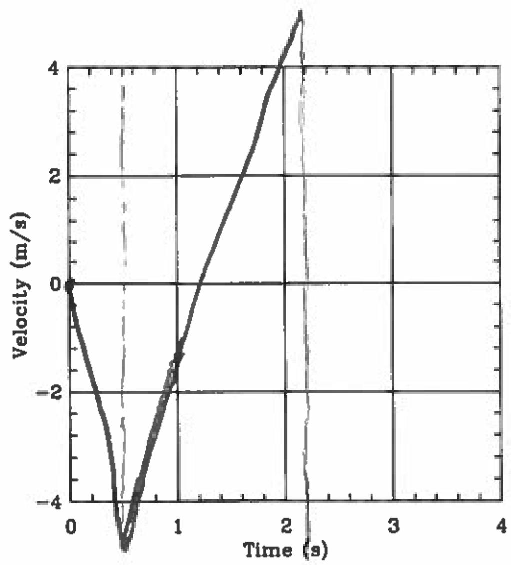
$\longrightarrow 0 = \frac{1}{2}at^2 - \sqrt{2gh}\,t \longrightarrow t_2 = 2\sqrt{2gh}/a$ to rise to surface again.

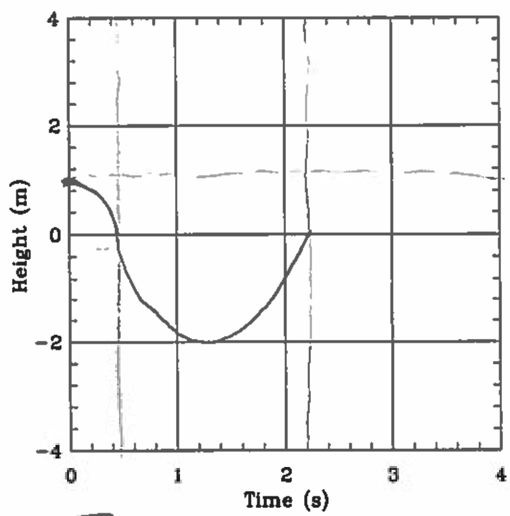
So, total time is: $\sqrt{\frac{2h}{g}} + 2\frac{\sqrt{2gh}}{a} = \sqrt{\frac{2m}{9.8\,m/s^2}} + \frac{2\sqrt{2 \cdot 9.8\,m/s^2 \cdot 1m}}{5\,m/s^2}$

$= 2.22$ s.

d) Graph acceleration vs. time, velocity vs. time, and position vs. time on the axes provided. (20 points)







Hits water at $t_1 = \sqrt{\frac{2h}{g}} = 0.45$ s

Rises again at 2.22 s (from (b))

V at water = 4.42 m/s downward.

Max depth: set $v=0$ for UW step: $V_2(t) = -\sqrt{2gh} + at \Rightarrow t = \frac{\sqrt{2gh}}{a}$

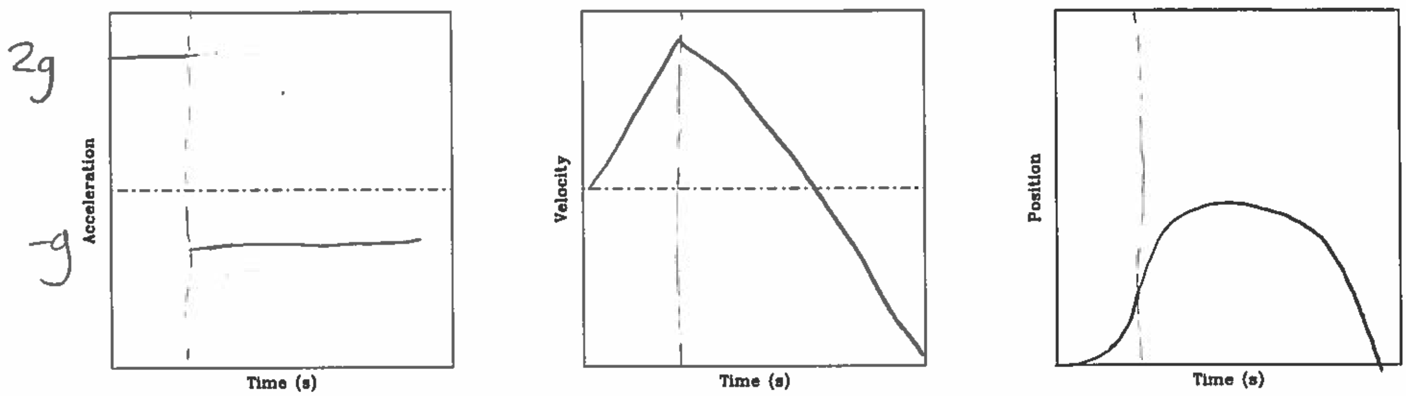
Sub into $y(t)$: $y = \frac{1}{2}a\frac{2gh}{a^2} - \sqrt{2gh}\,\frac{\sqrt{2gh}}{a} = -\frac{gh}{a}$

$= 1.96$ m

Question 3

A model rocket has a motor capable of accelerating it upward at $a_r = 2g = 20\mathrm{m/s^2}$. (This value is the acceleration; it should not be added to g.) It is launched upward from rest at the ground. It will accelerate upward until its fuel is exhausted, and will then undergo free fall until it lands back on the ground.

a) Sketch graphs of its position, velocity, and acceleration vs. time on the provided axes, labeling the point at which the fuel is exhausted. (20 points)



Call τ the time the the motor cuts out.

b) If the rocket reaches a maximum altitude of 1 km, how long does the fuel last? (30 points)

Phase I: $x_1 = x(\tau) = \frac{1}{2}(2g)\tau^2 = g\tau^2$ | $v_1 = v(\tau) = 2g\tau$.

Final x, v for phase 1 are initial x and v for phase 2:
(t= time after motor burnout):

$x = -\frac{1}{2}gt^2 + v_1 t + x_1$ $\qquad$ $v = v_1 - gt$

Apex is when $v=0$: $t = \frac{v_1}{g}$.

"What τ makes it so that $x=h$ when $t = v_1/g$?"

$$h = -\frac{1}{2}g\frac{v_1^2}{g^2} + \frac{v_1^2}{g} + x_1 \implies h = \frac{1}{2}\frac{v_1^2}{g} + x_1$$

Substitute: $h = \frac{2g^2\tau^2}{g} + g\tau^2 \implies h = 3g\tau^2$

$$\tau^2 = \frac{h}{3g}, \quad \tau = \sqrt{h/3g} = 5.83\text{ s}.$$

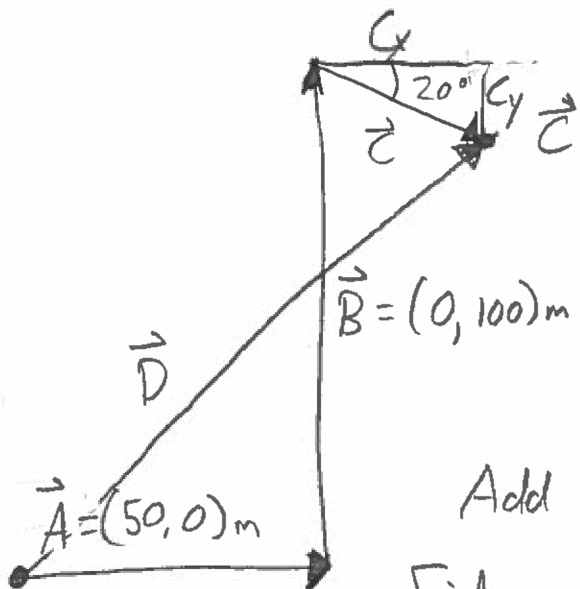
Question 4

A walnut tree grows 100 meters north of a pecan tree.

A squirrel takes a walnut from the walnut tree, travels 30 meters at an angle 20 degrees south of east, and buries his walnut.

Another squirrel takes a pecan from the pecan tree, travels 50 meters due west, and buries her pecan.

a) How far apart are the two buried nuts? (30 points)



$\vec{C} = (30 \cos 20°, -30 \sin 20°)$ m

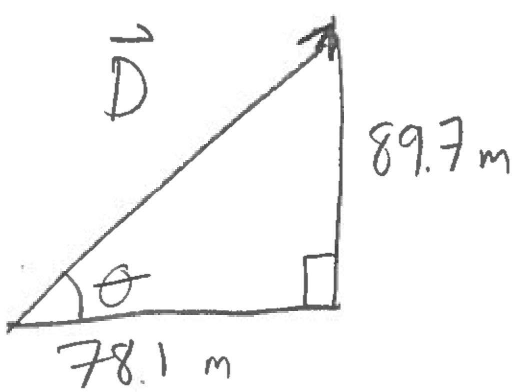
"What is the magnitude of $\vec{D} \equiv \vec{A} + \vec{B} + \vec{C}$?"

Add components: $\vec{D} = (78.1, 89.7)$ m

Find magnitude: $|D| = \sqrt{78.1^2 + 89.7^2} = 118.9$ m
(Pythagoras)

b) If someone wanted to walk from the buried pecan to the buried walnut, in which direction should she walk? (20 points)

Find angle of $\vec{D}$:



$\theta = \tan^{-1} \frac{89.7}{78.1} = 49.0°$

north of east

Question 5

A firefighter in an airplane is trying to put out a fire by dropping a load of sand on it. She is flying horizontally toward the fire at an altitude of 100 meters $\rightarrow h$, traveling at a speed of 60 m/s. $= V_0$

If she drops her sand directly over the fire, it will overshoot the target.

a) How far in advance of the fire must she release the sand in order for it to land on the fire? (25 points)

dumpster fire of Mastering Physics

(Origin at fire)

$y(t) = h - \frac{1}{2}gt^2$ $\longrightarrow$ "What x_0 makes $x=0$ at the time $y=0$?"

$x(t) = x_0 + V_0 t$

$$0 = h - \tfrac{1}{2}gt^2 \Longrightarrow t = \sqrt{\frac{2h}{g}}$$

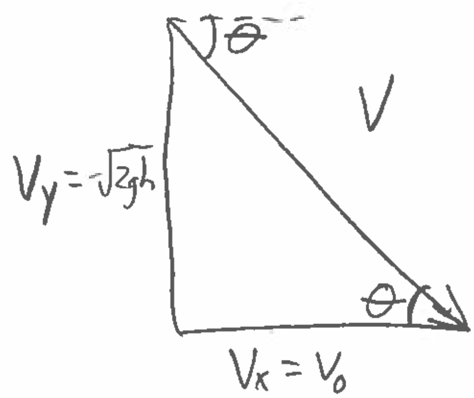
$$0 = x_0 + V_0\sqrt{\frac{2h}{g}} \Longrightarrow x_0 = \boxed{-V_0\sqrt{\frac{2g}{g}}} = \boxed{192 \text{ m in advance}}$$

b) In what direction will the sand be traveling when it strikes the ground? (15 points)

"At $t = \sqrt{\frac{2h}{g}}$, in what direction is $\vec{v}$?"

$V_x = V_0$

$V_y = -gt = -\sqrt{2gh}$



$\theta = \tan^{-1}\left(\frac{\sqrt{2gh}}{V_0}\right)$

$= 36.4°$ below horizontal.

c) Will the airplane be behind the load of sand, directly above it, or ahead of it when it lands on the fire? (10 points)

Directly above: both travel at $V_x = 60$ m/s always.

Question 6

A person drops a baseball from a height h onto a hard floor. When the baseball hits the floor, it will bounce back at a speed equal to *half* of the velocity with which it hit the floor.

In terms of h and g, find:

a) how long it takes to hit the floor the first time (10 points)

Find t when $y=0$:

$$y(t) = h - \tfrac{1}{2}gt^2 \longrightarrow 0 = h - \tfrac{1}{2}gt^2 \longrightarrow t = \sqrt{\frac{2h}{g}}$$

b) how fast it is going when it hits the floor the first time (10 points)

$v(t) = -gt$ Find $v\left(\sqrt{\frac{2h}{g}}\right)$: v at floor $= \sqrt{2gh}$ downward.

c) how high it travels after bouncing off the ground the first time (10 points)

Here, $y_0 = 0$, $v_0 =$ half of answer to (b). $= \frac{\sqrt{2gh}}{2}$

$y(t) = \frac{-1}{2}gt^2 + \frac{\sqrt{2gh}}{2}t$, $v(t) = \frac{\sqrt{2gh}}{2} - gt$

Set $v=0$: $gt = \frac{\sqrt{2gh}}{2}$, $t = \sqrt{\frac{h}{2g}}$. (algebra steps omitted)

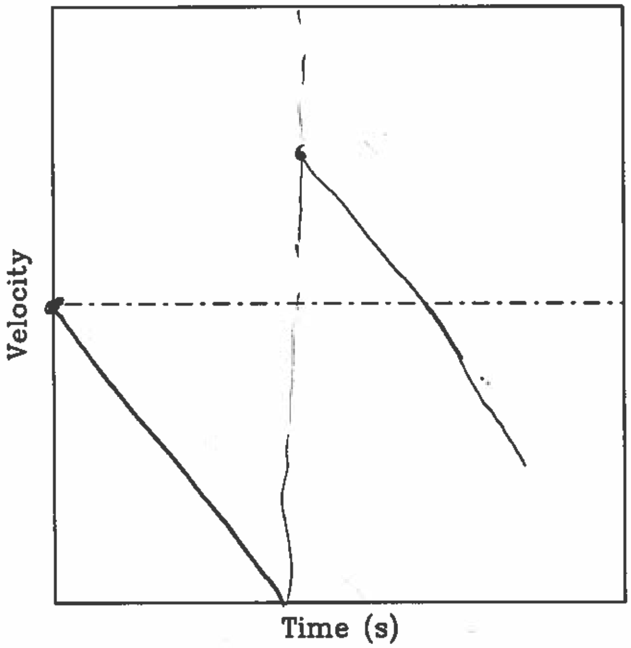
Find $y(t)$: $\frac{-1}{2}g\frac{h}{2g} + \frac{\sqrt{2gh}}{2}\sqrt{\frac{h}{2g}} \Rightarrow \frac{-1}{4}h + \frac{1}{2}h = \boxed{\frac{1}{4}h}$

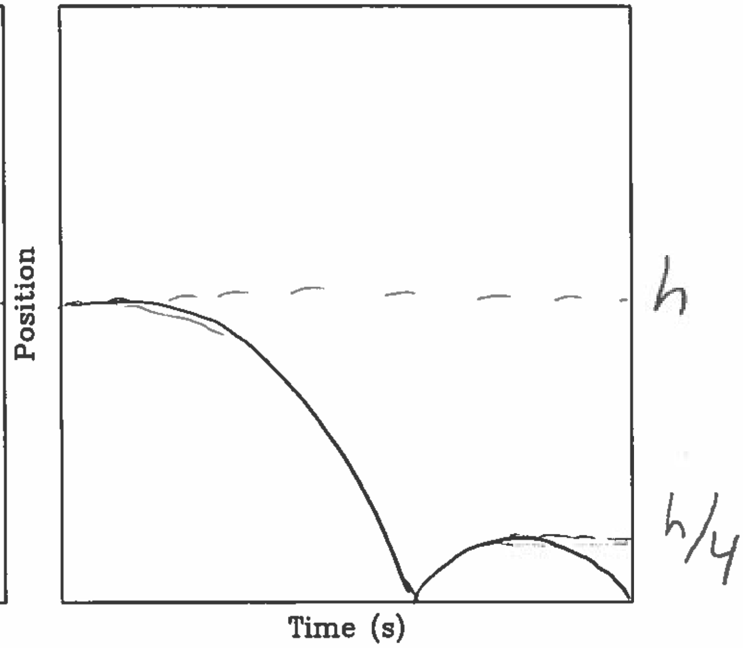
d) the amount of time between the ball being dropped and it hitting the floor the second time (10 points)

Sum of times from (a) and (c):

$= \sqrt{\frac{2h}{g}} + \sqrt{\frac{h}{2g}}$ (can simplify, but why bother?)

e) On the axes provided, graph the ball's velocity vs. time and position vs. time. (10 points)



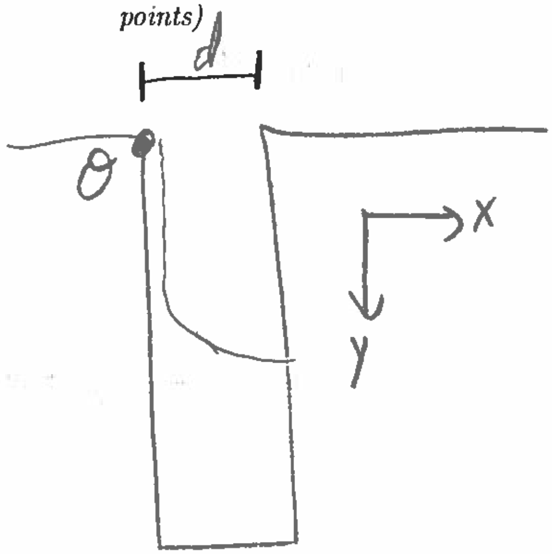


QUESTION 7

A rocket is dropped out of a window and pointed sideways toward another building. A time τ after it is dropped, its motor fires, giving it an acceleration of $2g$ in the horizontal direction. (Its vertical acceleration is still g downward.)

After the motor fires, the rocket flies along its new path until it strikes another building, located a distance d away.

a) In terms of τ, g, and d, what are the position and velocity of the rocket when the motor fires? (10 points)



Before ignition: $x = V_x = 0$

$$y(\tau) = \frac{1}{2} g \tau^2$$

$$v_y(\tau) = g\tau$$

b) In terms of τ, g, and d, how long in total is the rocket in the air before it strikes the second building? (20 points)

Note: Time to travel between buildings depends <u>only</u> on what happens after ignition, since before, $x = V_x = 0$.

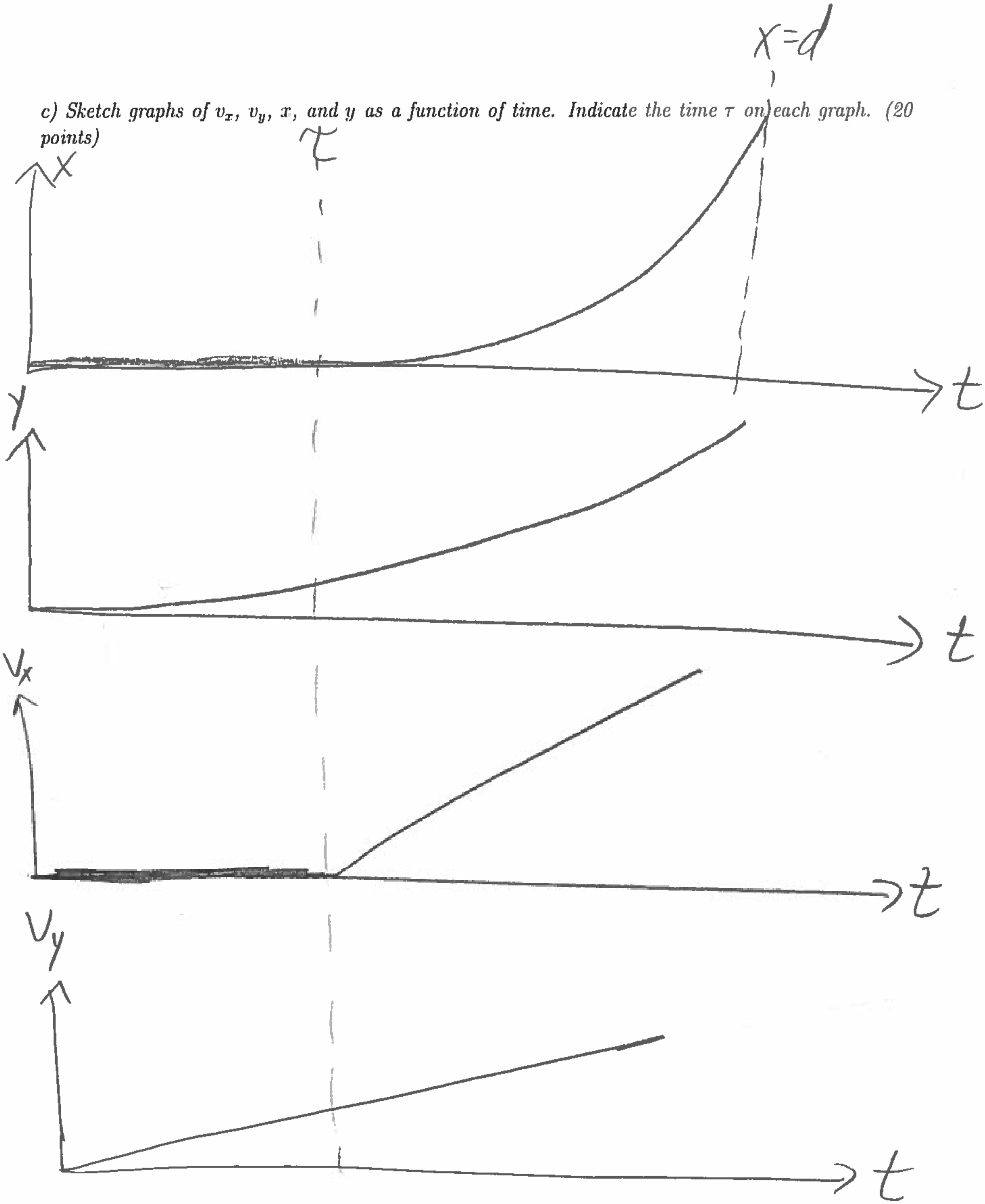
t = time <u>after</u> ignition

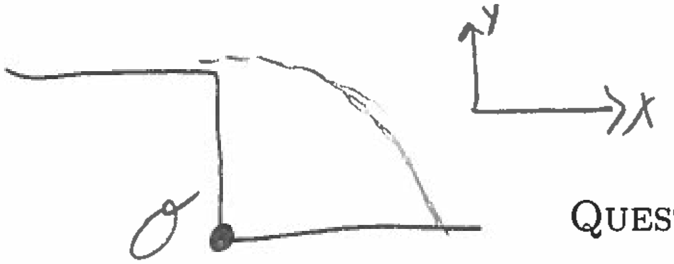
"What t makes $x = d$?"

$$x(t) = \frac{1}{2}(2g)t^2 \Rightarrow d = gt^2, \quad t = \sqrt{d/g}.$$

$$\text{Total time} = \sqrt{d/g} + \tau$$

c) Sketch graphs of v_x, v_y, x, and y as a function of time. Indicate the time τ on each graph. (20 points)





QUESTION 8

A ball rolls off a shelf of height h at speed v. Answer the following in terms of h, v, and g.

↳ call v_0 to avoid mixups

a) How long does it take the ball to hit the floor? (10 points)

$x(t) = v_0 t \qquad V_x(t) = V_0$

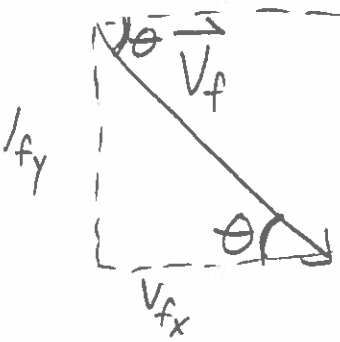
$y(t) = h - \frac{1}{2}gt^2 \qquad V_y(t) = -gt$

Hits floor when $y=0$: $0 = h - \frac{1}{2}gt^2 \longrightarrow t = \sqrt{\frac{2h}{g}}$

b) Where does the ball hit the floor? (10 points)

"What is x when $t = \sqrt{\frac{2h}{g}}$": $x = v_0 t = v_0\sqrt{\frac{2h}{g}}$.

c) What is the ball's speed when it hits the floor? (10 points)



"What is magnitude of $\vec{v}$ when $t = \sqrt{\frac{2h}{g}}$?"

$V_x = V_0$

$V_y = -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh}$

Pythagoras:

$$\boxed{|v| = \sqrt{V_0^2 + 2gh}.}$$

d) What direction is the ball moving in when it hits the floor? (10 points)

$\tan\theta = V_{fy}/V_{fx}$: $\theta = \tan^{-1}\frac{\sqrt{2gh}}{V_0}$ (below the horizontal) *e) Suppose that the edge of the shelf had been curved, so that the ball's initial velocity was instead directed at an angle θ below the horizontal. Explain, using words or algebra as appropriate, what things you would have needed to do differently to solve the previous four parts, and which things would stay the same. (10 points)*