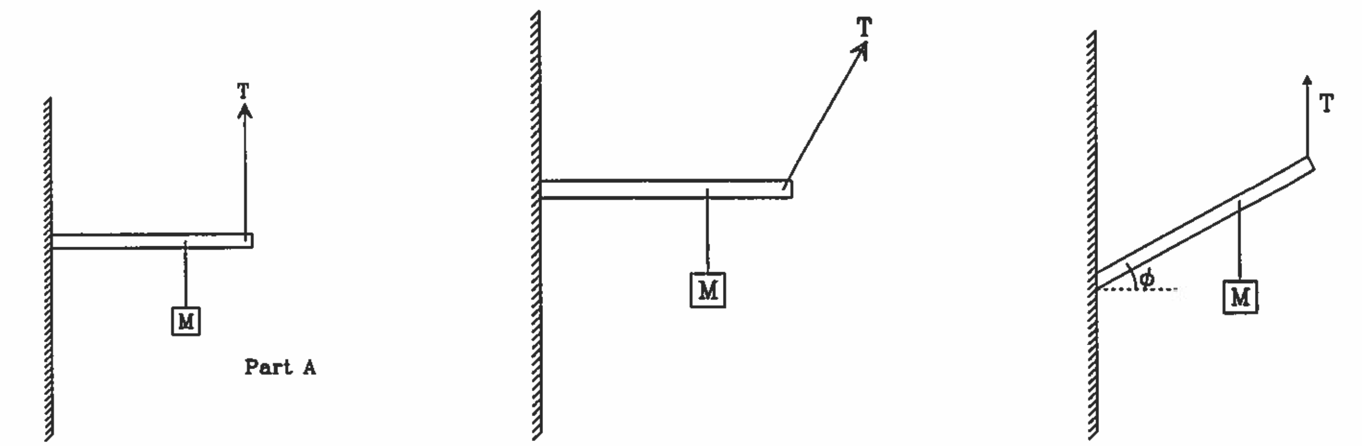
PHYSICS 211 PRACTICE TEST 3
QUESTION 1

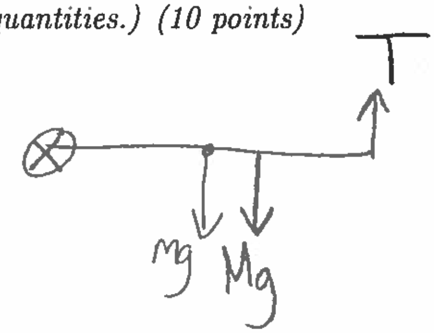
A board with a mass m and a length L is connected to a wall by a hinge on its left side, and is horizontal to the ground. A bucket with a frog in it, with a total mass M, hangs from the board a distance $2L/3$ from the left side. To stop the board from falling, a person applies a tension T to a rope connected to the right side. **These pictures are too small** for you to draw clear diagrams on; you will want to draw your own.

Part A | Part B | Part C



a) Find the tension in the rope if the rope is vertical. Give your answer in terms of m, M, L, and g. (Your answer may not depend on all quantities.) (10 points)

$\sum \tau = 0$

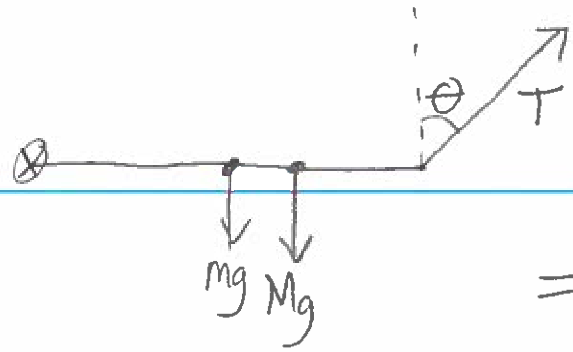


$-(2L/3)Mg - \frac{L}{2}mg + TL = 0$

$\Rightarrow T = \frac{2}{3}Mg - \frac{1}{2}mg$

b) Find the tension in the rope if the rope makes an angle θ with the vertical. Give your answer in terms of m, M, L, g, and θ. (10 points)

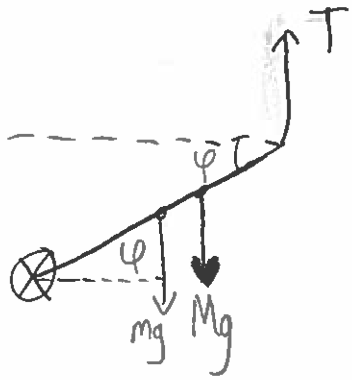
$T_\perp = T\cos\theta$ here



$$\sum \tau = (T\cos\theta)L - Mg\left(\tfrac{2}{3}L\right) - mg\left(\tfrac{L}{2}\right)$$

$$\Rightarrow T = \frac{\frac{2}{3}Mg + \frac{1}{2}mg}{\cos\theta}$$

c) Instead, suppose that the board were then elevated, so that it made an angle of φ with the horizontal. Calculate the tension T in the rope now. (10 points)



Note that this is the same as (a), except here $r_\perp = r\cos\varphi$. These factors of $\cos\varphi$ appear in all terms and cancel.

$$T(L\cancel{\cos\varphi}) - Mg\left(\tfrac{2L}{3}\cancel{\cos\varphi}\right) - mg\left(\tfrac{L}{2}\cancel{\cos\varphi}\right) = 0$$

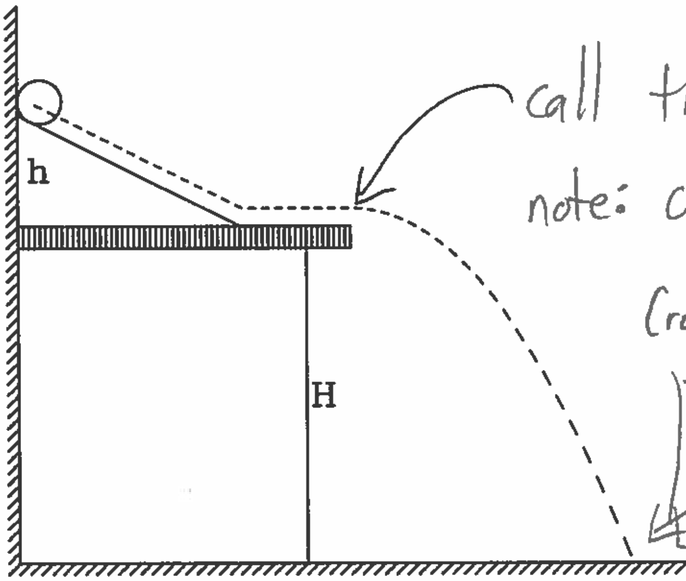
$\longrightarrow$ Same as (a), $T = \frac{2}{3}Mg + \frac{1}{2}mg$.

d) Explain in words how you could have predicted this result without doing any mathematics. (10 points)

See above.

Question 2

In class, you saw a demonstration where a ball bearing (solid sphere, moment of inertia $I = \frac{2}{5}mr^2$) was rolled down a small ramp on top of a table. The ball rolled down the ramp, rolled across the table, and then fell off of the side of the table.



call this point A

note: $\omega_A r = v_A$

(rolling constraint)

here, ω still is ω_A!

Suppose that the height of the ramp is h, the height of the table is H, and the radius of the ball is r.

a) How fast is the ball traveling when it reaches the edge of the table? (15 points)

$$mgh = \frac{1}{2}mv_A^2 + \frac{1}{2}I\omega_A^2$$

$I = \frac{2}{5}mr^2$, so

$$mgh = \frac{1}{2}mv_A^2 + \frac{1}{2}\frac{2}{5}m r^2\omega_A^2$$

this is v_A^2

$$mgh = \left(\frac{1}{2}+\frac{1}{5}\right)mv_A^2 \longrightarrow v_A = \sqrt{\frac{10}{7}gh}$$

Note: $\omega_A r = \sqrt{\frac{10}{7}gh}$. We will need this for (b).

(This problem continues on the next page.)

b) How fast is the ball traveling when it strikes the floor? Think carefully about the kinds of energy that are present here, and how they relate; this problem is not quite as trivial as it seems. (15 points)

At floor: $V = V_f$, ω still is ω_A!

using $\omega_A^2 r^2 = \frac{10}{7} gh$

$$(mgh + mgH) = \frac{1}{2} m V_f^2 + \frac{1}{2} I \omega_A^2$$

$$mg(H+h) = \frac{1}{2} m V_f^2 + \frac{1}{2}\frac{2}{5} m r^2 \omega_A^2 = \frac{1}{2} m V_f^2 + \frac{1}{2}\frac{2}{5}\frac{10}{7} mgh.$$

$$\cancel{m}g(H+h) = \frac{1}{2}\cancel{m}V_f^2 + \frac{2}{7}\cancel{m}gh \longrightarrow V_f = \sqrt{2gH + \frac{10}{7}gh}$$

c) How far past the edge of the table does the ball land? (10 points)

This is just kinematics.

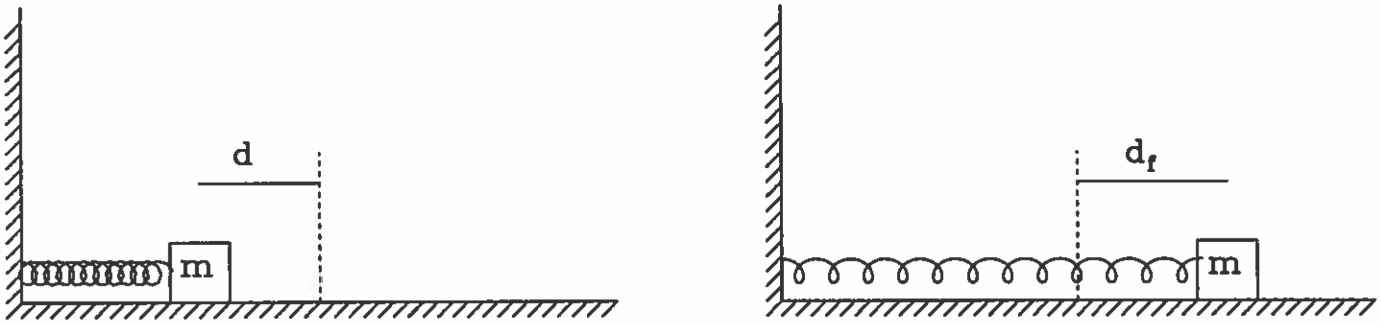
X: $x_f = V_1 t$ $\qquad$ Y: $0 = -\frac{1}{2} g t^2 + H \rightarrow t = \sqrt{\frac{2H}{g}}$

$$x_f = \sqrt{\frac{10}{7}\cancel{g}h}\sqrt{\frac{2H}{\cancel{g}}} = \sqrt{\frac{20}{7}\frac{H}{h}}.$$

Question 3

A spring has spring constant k. One end is fixed, and the other end is attached to a mass m, which is free to move horizontally along a table. The mass slides over the table with a coefficient of friction μ_k.

The spring is compressed a distance d from its equilibrium point and released. When the spring is released, it will push the mass to the right, until it reaches some other distance d_f past the equilibrium point.



a) How fast will the mass be traveling when it crosses the equilibrium point? Give your answer in terms of μ_k, d, m, and g. (10 points)

Here, $W_{fric} = -\mu mgd$. $\longrightarrow \frac{1}{2}kd^2 - \mu mgd = \frac{1}{2}mV_e^2$

$$V_e = \sqrt{\frac{k}{m}d^2 - 2\mu gd}$$

b) Write down an expression for the work done by friction as the block slides from its starting point to the final position d_f to the right of equilibrium. (10 points)

Distance traveled: $d + d_f$

$W_{fric} = -\mu mg(d + d_f)$

c) Write down an equation in terms of μ_k, d, m, and g that will let you solve for the distance d_f. You do not need to solve it. (15 points)

$$\frac{1}{2}kd^2 - \mu mg(d + d_f) = \frac{1}{2}kd_f^2$$

d) What algebraic technique would you have to use to solve this equation for d_f? (5 points)

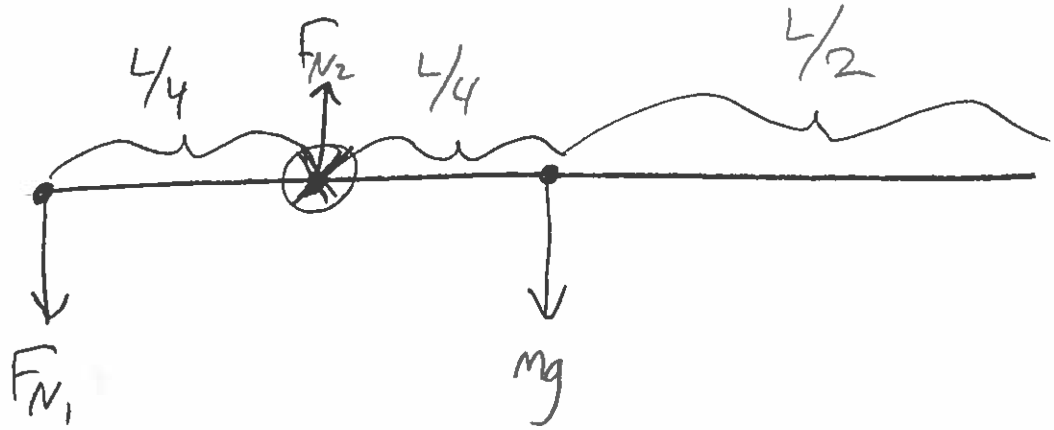
The quadratic formula (or "completing the square")

Question 4

A person tries to support a pipe of length 2 meters from one end horizontally. This pipe has a mass of 10 kg. He puts one of his hands on one end of the pipe, and the other one 50 cm from the end. In this problem, you will find the forces (magnitude and direction) he must exert with each hand to keep the pipe from falling.

a) Draw a force diagram for the pipe. Label the pivot point that you will choose in computing torques. (10 points)

As in HW: one hand pushes down.



b) Find the forces (magnitude and direction) he must exert with each hand to keep the pipe from falling. (30 points)

Torque about labeled pivot: $F_{N_1}(\cancel{L/4}) - mg(\cancel{L/4}) = 0$

$\rightarrow F_{N_1} = mg$ down $= 100$ N

$\Sigma F = 0$ as well! $F_{N_2} - F_{N_1} - mg = 0$

$\rightarrow F_{N_2} = 2mg$ (up) $= 200$ N

QUESTION 5

A mad scientist has built a rocket-powered sled and is testing it on the frozen surface of Lake Onondaga. The coefficient of friction between the sled and the lake is $\mu_k = 0.3$. She and her rocket sled together have a mass of $m = 200$ kg, and the sled's rocket provides a constant thrust force of $F_T = 2$ kN. (Assume that the mass of the expelled propellant is small compared to m, so that m does not change.)

She fires the sled's motor and travels forward along the lake's surface. After traveling a distance $d = 100$ m, she confirms that her rocket will suffice for her mad-scientific purposes [1], and shuts down the engine; she coasts a further distance b before coming to a stop.

a) Find the distance b. (20 points)

$W_{rocket} = F_T d \qquad W_{fric} = -\mu_k mg(d+b) \qquad (\text{since } F_N = mg)$

$KE_i = KE_f = 0$, so

$F_T d - \mu_k mg(d+b) = 0$, so: $F_T d - \mu_k mgd = \mu_k mgb$

$$b = \frac{F_T d - \mu_k mgd}{\mu_k mg} = \frac{(2000\ N)(100\ m) - (0.3)(2000\ N)(100}{(0.3)(2000\ N)}$$

$$= \frac{200\ kJ - 60\ kJ}{600\ N} = 233\ m.$$

[1] World domination, obviously. What else would she be interested in?

b) Suppose now that the rocket exhaust is directed at an angle 45 degrees below the horizontal.[2] *Write an expression for the work done by friction in this case. (10 points)*

Note: while motor burning, $F_N = mg - F_T \cos\theta$!

$$W_{fric} = \underbrace{-\mu(mg - F_T \cos\theta)d}_{\text{while motor is on}} - \underbrace{\mu m g b}_{\text{after}}$$

c) Find the distance b in this case. (10 points)

$$(F_T \sin\theta)d - \mu(mg - F_T \cos\theta)d - \mu m g b = 0$$

$$b = \frac{(F_T \sin\theta)d - \mu m g d + \mu(F_T \cos\theta)d}{\mu m g} =$$

$$= \frac{(1414\ N)(100\ m) - (0.3)(2000\ N)(100\ m) + (0.3)(1414\ N)(100\ m)}{(0.3)(2000\ N)}$$

$$= 206\ m$$

[2]Her mad-science graduate student assistant messed up a conversion between radians and degrees and pointed it in the wrong direction.

Question 6

A turntable of mass m is rotating at an angular velocity ω around its center on a frictionless bearing. (The moment of inertia of a disk is $\frac{1}{2}mr^2$.) Someone drops a thin ring, also of mass m, on top of it. What is the new angular velocity ω' of the turntable? *(40 points)*

Conservation of L:

$$L_{initial} = I_{disk}\,\omega_i$$

$$\Longrightarrow \quad L_{initial} = L_{final}$$

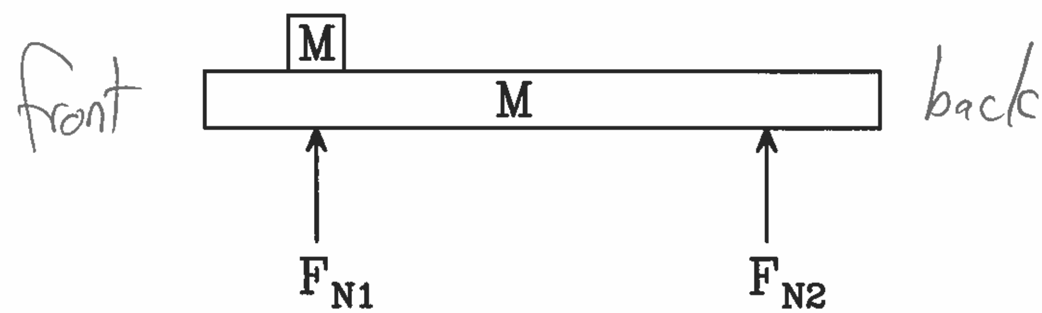
$$L_{final} = (I_{disk} + I_{ring})\,\omega_f \qquad I_{disk}\,\omega_i = (I_{disk} + I_{ring})\,\omega_f$$

$$\omega_f = \omega_i \left[\frac{I_{disk}}{I_{disk} + I_{ring}}\right] = \omega_i \left[\frac{\frac{1}{2}mr^2}{\frac{1}{2}mr^2 + mr^2}\right]$$

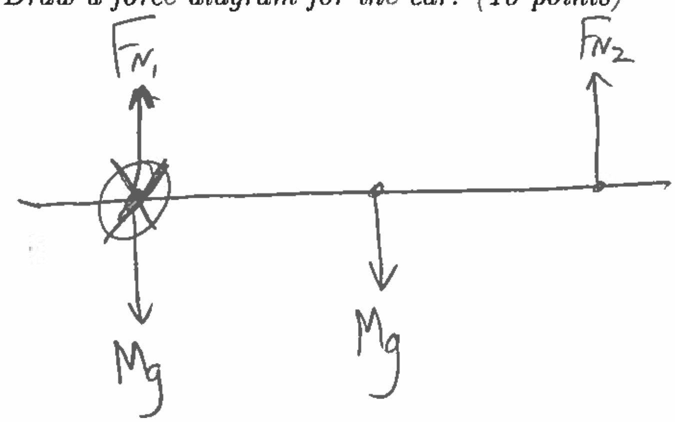
$$= \frac{1}{3}\omega_i .$$

QUESTION 7

A simplified model of a car or truck can be thought of as shown below. Consider the body of the vehicle to be a uniform plate of mass M, supported by the normal force of two axles, each located a distance $L/6$ from each end. The engine, also of mass M, is located a distance $L/6$ from the front.



a) Draw a force diagram for the car. (10 points)



Distance from front axle to center $= \frac{L}{3}$

Distance between axles $= \frac{2L}{3}$

b) Calculate the normal forces F_{N1} and F_{N2} exerted by each axle. (20 points)

Torque about front axle $= 0$:

$$0 - Mg\frac{L}{3} + F_{N_2}\left(\frac{2L}{3}\right) = 0 \Rightarrow F_{N_2} = \frac{1}{2}Mg$$

$$\Sigma F_y = ma_y = 0, \text{ so } F_{N_1} + F_{N_2} - 2Mg = 0$$

$$\longrightarrow F_{N_1} = \frac{3}{2}Mg.$$

c) If the coefficient of friction between the vehicle's wheels and the ground is μ_s, what is the maximum acceleration of the vehicle if it is front wheel drive? (5 points)

Max traction $= F_{N_1} \times \mu_s$

$$= \frac{3}{2}\mu M g$$

chassis + engine

For whole car, $\sum F_x = ma_x \longrightarrow \frac{3}{2}\mu M g = (2M)a_x$

$$\longrightarrow a_{max} = \frac{3}{4} g.$$

d) ... what if it is rear wheel drive?

Same, but here we use F_{N_2}:

Max traction $= \frac{1}{2}\mu M g$

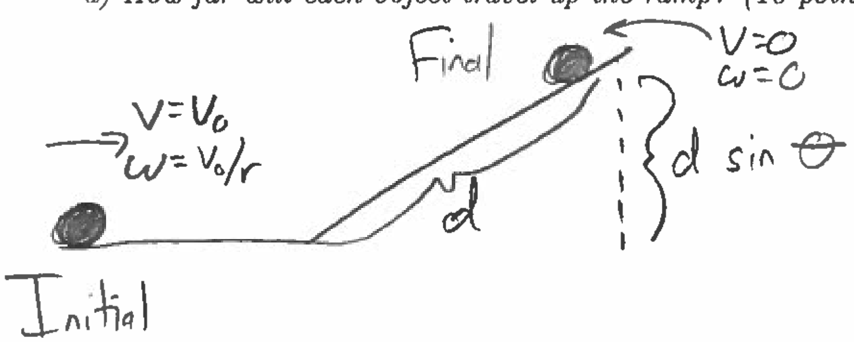
$$\sum F_x = ma_x \longrightarrow \frac{1}{2}\mu M g = (2M)a_x$$

$$a_{x_{max}} = \frac{1}{4} g.$$

Question 8

A person rolls a solid cylinder ($I = \frac{1}{2}mr^2$) and a ball ($I = \frac{2}{5}mr^2$), each of radius $r = 10$ cm and mass $m = 2$ kg, toward a ramp at $v_0 = 3$m/s. Both objects roll without slipping. The ramp makes a 40^o angle with the horizontal.

a) How far will each object travel up the ramp? (10 points)



Use conservation of energy:

$$\frac{1}{2}mV_0^2 + \frac{1}{2}I\omega_0^2 = mgd\sin\theta \Rightarrow d = \frac{V_0^2 + \lambda V_0^2}{2g\sin\theta}$$

$$\frac{1}{2}\cancel{m}V_0^2 + \frac{1}{2}\lambda \cancel{m}\underbrace{r^2\omega_0^2}_{=V_0^2!} = \cancel{m}g\,d\sin\theta$$

b) What if the ball's radius is changed to 5 cm? (10 points)

Note: Since $V_0^2 = r^2\omega_0^2$, all the r's cancel.

Thus: answer independent of r.

Numbers:

Cylinder: 105 cm

Ball : 98 cm

c) What if the cylinder's mass is changed to 4 kg?

Answer independent of m, too!

$\longrightarrow$ also $d = \dfrac{v_0^2 + \lambda v_0^2}{2g \sin\theta}$ $\left(\begin{array}{lll} \text{Cylinder} & 105 & \text{cm} \\ \text{Ball} & 98 & \text{cm} \end{array} \right)$

d) What if the angle of the ramp is changed to 20°?

Put in new θ:

Cylinder : 197 cm

Ball : 184 cm

Note: putting in numbers before solving makes this much harder!

Conceptual Questions

Answer the following with "sometimes", "always", or "never": (5 points each)

1. A larger force applies a greater torque than a smaller force, if they act at the same location.
   Sometimes ($\tau = F r \sin\theta$)
2. A force that causes an object to accelerate in the negative direction does negative work.
   Sometimes (maybe it's moving in the negative direction)
3. A 20 N force applied 20 cm from the pivot will create the same torque as a 40 N force applied 10 cm from the pivot.
   Sometimes (see #1)
4. Whether a force does negative or positive work depends on the coordinate system and choice of origin.
   Never
5. An object with a larger mass has a higher moment of inertia than an object with lower mass.
   Sometimes (10 kg disc vs. 15 kg ball)
6. A spring that is stretched by 4cm will exert twice as much restoring force than if it were stretched by only 2cm.
   Always
7. A spring that is stretched by 4cm will have twice as much potential energy than if it were stretched by only 2cm.
   Never
8. Conservation of energy can be used to calculate how much time a particular motion takes.
   Never

For each of the following quantities, tell whether it can be positive, negative, or zero. There may be multiple answers: for instance, if I said "the number of pencils in my pocket", you would say "positive or zero". (5 points each)

1. Kinetic energy +, 0
2. Gravitational potential energy +, 0, −
3. The work done by a normal force +, 0, −
4. The work done by static friction +, 0, −
5. The work done by the tension force in a pendulum 0
6. The torque exerted by the Earth's gravity on the Moon, with the pivot point at the Earth 0
7. The work done by a baseball player on a baseball as he catches it −
8. The work done by an archer's hand on the string as she draws a bow +