Work and potential energy

Physics 211
Syracuse University, Physics 211 Spring 2022
Walter Freeman

March 28, 2022

Announcements

- I didn't get the weekly Google Form sent out last week (sorry)
- I am going to grade these differently than planned:
  - Completing them will help your grade
  - Not completing them won't hurt your grade

- Homework 6 due date postponed until Friday (we're covering one topic today)
- Homework 7 will be assigned tomorrow and will be due the following Friday – it will be long

My help hours this week in the Physics Clinic:

- Tuesday, 9:45-10:45 AM and 2-3:45 PM
- Wednesday, 2:30-3:30 PM
- Thursday, 9:45-10:45 AM and 2:30-3:30 PM

Ask the Physicist: nuclear energy

Basic nuclear chemistry:

- Atomic nuclei are made of protons and neutrons
- The element name tells you how many protons; the number is the sum of protons+neutrons
- The element name (proton number) determines its *chemical* properties
- Both proton and neutron number determine *nuclear* properties
- Examples:
  - Carbon-12 (6 protons, 6 neutrons) is chemically the same as carbon-14 (6 protons, 8 neutrons)
  - Carbon-12 is stable; carbon-14 is radioactive (decays slowly over time)
  - Uranium-238 (99.4% of natural uranium) and uranium-235 (the rest)
- Elements with the same proton number (name) but different neutron number are called *isotopes*
- They cannot be separated chemically and occur together

Radioactivity and potential energy – unrelated!

Some isotopes are *radioactive* and decay (transform) into other elements over a period of time, producing another nucleus plus a fast-moving particle with a lot of kinetic energy:

This cannot be used as a primary energy source:

- Anything that decays quickly isn't found in nature
- Anything that decays slowly releases energy too slowly

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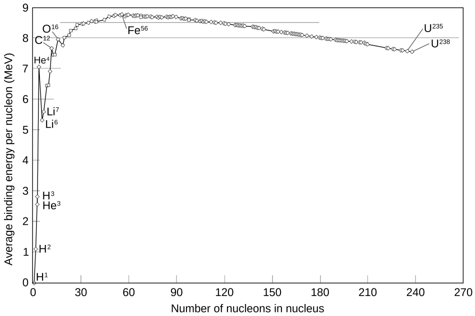
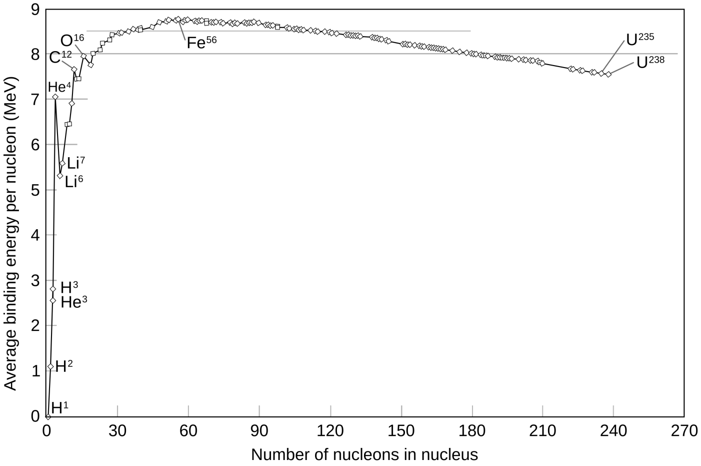
But there *is* a lot of potential energy in nuclei.

This graph is of *inverse* potential energy.

The units on the vertical axis are roughly equivalent to "hundred trillion joules per kilogram"

How can we harness this energy?





- Fusion: stick little nuclei together to make big ones
  - Requires incredibly high temperatures (hard for us, easy for stars)
  - Not practical on Earth (yet)
- Fission: split apart big nuclei to make smaller ones
  - Only a few nuclei are *fissile* (can be split)
  - The only naturally-occurring one is ^{235}U (0.6%)
  - It's hard to separate from ^{238}U (99.4%)

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4. $\rightarrow$ ... chain reaction!

But there's a complication:

1. ^{238}U nuclei tend to absorb neutrons and don't fission
2. The neutrons produced by the fission reaction are "fast"
3. "Slow" neutrons are more likely to induce fission in ^{235}U nuclei

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We thus need to protect the neutrons from getting eaten by ^{238}U while slowing them down with a *moderator*. Two approaches:

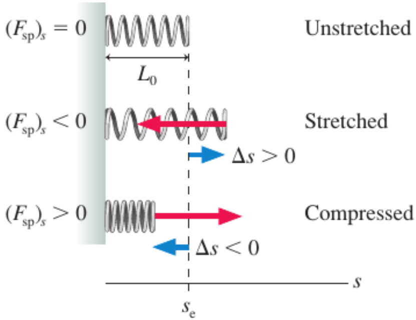
1. Really efficient moderator and natural (0.6% ^{235}U) uranium
   - Graphite moderated, water cooled – old Soviet (RBMK)
   - Heavy water moderated and cooled – Canadian (CANDU)
2. Less efficient moderator and enriched uranium
   - Take some of the ^{238}U out so it stops eating neutrons
   - Light water moderated, light water cooled – everyone else

Enormous amount of energy: 1 gigawatt for two months per ton of uranium

A new force: elasticity and Hooke's law

To best see how this can be useful, let's introduce a new force: elasticity.

- Springs have a particular length that they like to be: "equilibrium length" L_0
- A spring stretched to be longer than this pulls inward to shorten itself
- A spring compressed to be shorter than this pushes outward to lengthen itself
- Flexible things like strings and ropes only pull; they kink instead of compressing
- The force is proportional to the deviation from the optimum length



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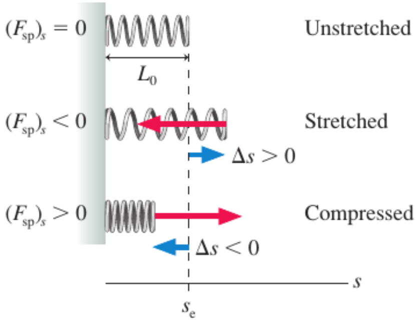

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k is called the "spring constant":

- Measures the stiffness of the spring/rope
- Units of newtons per meter: "restoring force of k newtons per meter of stretch"

A simple spring problem: done with the work-energy theorem

A person of mass $m = 100kg$ falls from a height of $h = 3$m onto a trampoline. If the person makes an impression $d = 40$ cm deep on the trampoline when he lands, what is the spring constant?

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- Initial kinetic energy + work done by spring + work done by gravity = final kinetic energy
  - Need to use the integral form of the work-energy theorem since the force isn't constant
- The person begins and ends at rest, so we know the initial and final kinetic energy is zero
- The trampoline begins at its equilibrium position

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- $KE_0 + W_{\text{grav}} + W_{\text{elas}} = KE_f$
- $0 + (mg)(h + d) - \frac{1}{2}kd^2 = 0$
- $k = \frac{mg(h+d)}{2d^2}$

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"How much work is done by a spring as it goes from $\Delta x = a$ to $\Delta x = 0$?

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Now that we have this, we never have to do this integral again!

$$U_{\text{elastic}} = \tfrac{1}{2}kx^2, \text{ where } x \text{ is the distance from equilibrium}$$

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That spring problem: a recap

We don't care about time $\rightarrow$ energy methods

Work-energy theorem

- Initial KE + all work done = final KE
- Need to compute work done by gravity: easy
- Need to compute work done by spring: harder
  (need to integrate Hooke's law)

Potential energy treatment

- Initial KE + initial PE + other work = final KE + final PE
- No "other work" in this problem; all forces have a PE associated
- Need to know the expressions for PE:
  - $U_{\text{grav}} = mgy$
  - $U_{\text{elas}} = \frac{1}{2}kx^2$ (x is the distance from the equilibrium point)
- No integrals required (they're baked into the above)

Potential energy with other forces

What about associating a potential energy with other forces?

- Friction is a no-go: the work done by friction depends on the path, not just where you start and stop
- "Ephemeral" forces like tension and normal force are easiest to deal with by computing work directly

Problem-solving guide for problems involving energy

- Identify the various parts of the motion and what you need to know about them
  - Motion where you care only about begin/end states and not time: use work/energy methods
  - "Where does it land" projectile motion problems: can not use energy methods
- Draw a series of snapshots, showing what your "before" and "after" pictures look like (you may have more than two in some problems)
- Use force diagrams to calculate any forces you need to know
- Application of the work-energy theorem:

$$KE_{\text{initial}} + W_{\text{all}} = KE_{\text{final}}$$

OR

$$KE_{\text{initial}} + PE_{\text{initial}} + W_{\text{other}} = KE_{\text{final}} + PE_{\text{final}}$$

Sample problems

A mass m is hung from a spring of spring constant k and released. Which equation would let me find the distance d that it falls before it comes back up?

- A: $mgd - \frac{1}{2}kd^2 = 0$
- B: $\frac{1}{2}kx^2 = mgd + \frac{1}{2}mv^2$
- C: $0 = -mgd + \frac{1}{2}kd^2$
- D: $mgd + \frac{1}{2}kd^2 = 0$

Sample problems

A spring is used to launch a block up a ramp of total length L. The spring has spring constant k, the block has mass m, the ramp is inclined an angle θ, and it has a coefficient of kinetic friction μ_k. I compress the spring a distance x and let it go. How fast will the block be traveling when it reaches the top of the ramp?

Which equation would let me solve for this?

- A: $\frac{1}{2}kx^2 + \mu mgL\cos\theta = \frac{mgL}{\sin\theta} - \frac{1}{2}mv_f^2$
- B: $-\frac{1}{2}kx^2 + \mu mgL\sin\theta = \frac{mgL}{\sin\theta} + \frac{1}{2}mv_f^2$
- C: $\frac{1}{2}kx^2 - \mu mgL\sin\theta = mgL\cos\theta + \frac{1}{2}mv_f^2$
- D: $\frac{1}{2}kx^2 - \mu mgL\cos\theta = mgL\sin\theta + \frac{1}{2}mv_f^2$

A car coasts down a ramp and then up around a loop of radius r. How high must the ramp be for the car to make it around the loop? (See picture on document camera.)

How am I going to do this problem?

- A: Use conservation of momentum to relate the height of the ramp to the speed at the top of the loop, then use kinematics to determine if it will fall or not.
- B: Use conservation of energy to relate the height of the ramp to the speed at the top, then use kinematics to determine at what point it will fall in the loop.
- C: Use conservation of energy to relate the height of the ramp to the speed at the top, then use Newton's second law and our knowledge of rotational motion to determine the required speed.
- D: Use kinematics to relate the height of the ramp to the speed entering the loop, then use Newton's second law and our knowledge of rotational motion to determine the required speed.

Power: rate of doing work

A bit of mathematics that will be useful to you:

"An object moves at a constant speed $\vec{v}$, subject to some force $\vec{F}$; at what rate does that force do work on the object?"

An example: an airplane flies at v=1000 m/s, and its engines exert F=300 kN of thrust. What is the rate at which the engines do work (power)?

$$\text{Work} = \text{force} \cdot \text{distance}$$

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- The engines output 300 MW of power: this is around 10 liters per second of fuel even at 100% efficiency!
- Some of that 300 MW of energy dissipated by drag heats up the airplane... (real numbers for a SR-71 Blackbird)

A 1000 kg car has an engine that produces up to P=100 kW of power. If it accelerates as hard as it can, at what speed does its acceleration become limited by the engine?

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At low speeds: static friction limits acceleration
At high speeds: engine power limits acceleration

57. ‖ The spring shown in **FIGURE P11.57** is compressed 50 cm and used to launch a 100 kg physics student. The track is frictionless until it starts up the incline. The student's coefficient of kinetic friction on the 30° incline is 0.15.

a. What is the student's speed just after losing contact with the spring?

b. How far up the incline does the student go?

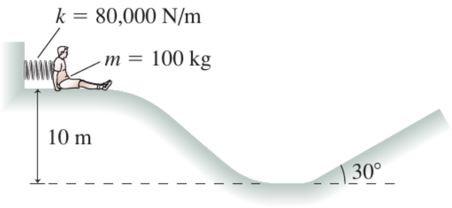

FIGURE P11.57

Sample problems

49. ‖ Truck brakes can fail if they get too hot. In some mountainous areas, ramps of loose gravel are constructed to stop runaway trucks that have lost their brakes. The combination of a slight upward slope and a large coefficient of rolling resistance as the truck tires sink into the gravel brings the truck safely to a halt. Suppose a gravel ramp slopes upward at $6.0°$ and the coefficient of rolling friction is 0.40. Use work and energy to find the length of a ramp that will stop a 15,000 kg truck that enters the ramp at 35 m/s ($\approx$75 mph).

Summary

- Potential energy is two things:
  - An accounting device that makes it easier to keep track of work done
  - Part of *conservation of total energy*, a powerful statement about nature
- Gravitational potential energy (on Earth): $U_g = mgy$

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- Potential energy is two things:
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  - Part of *conservation of total energy*, a powerful statement about nature
- Gravitational potential energy (on Earth): $U_g = mgy$
- We learned about a new force: **elasticity**
  - Restoring force in a stretched or compressed spring, or a stretched string:
    $F = -k(x - x_0)$ (x_0 is the equilibrium length)
  - k is the spring constant, measured in force per distance, that gauges stiffness
  - Elastic potential energy: $U_{\mathrm{elas}} = \frac{1}{2}k(x - x_0)^2$

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  - k is the spring constant, measured in force per distance, that gauges stiffness
  - Elastic potential energy: $U_{\text{elas}} = \frac{1}{2}k(x - x_0)^2$
- Gravitational potential energy in general: $U_G = -\frac{Gm_1m_2}{r}$