PHYSICS 211 GROUP EXAM 2, FORM 1

Problem 1	Problem 2	Total
/25	/25	/50

Name: Solutions

Partner #1: ____________________

Partner #2: ____________________

Recitation section number: ________

- There are two questions, each worth twenty-five points.
- **You must show your reasoning to receive credit**. A numerical answer with no logic shown will be treated as no answer.
- You are highly encouraged to use both pictures and words to show your reasoning, not just algebra.
- If you run out of room, ask for an extra sheet of paper, or get one from your notebook.
- how your reasoning as thoroughly as possible for partial credit.
- You may use $g = 10\,\mathrm{m/s^2}$ throughout, except where indicated, to minimize arithmetic.

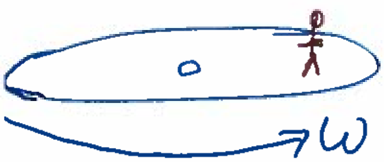
Question 1

A "merry-go-round" is a large, horizontal platform free to rotate around its axis. Children can stand on top of the platform while it spins. Suppose that a merry-go-round with a radius of 3 meters is spinning, and that it rotates around its axis once every 4 seconds.

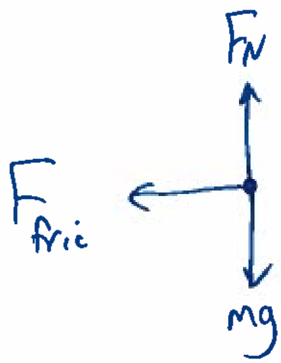
Suppose that the coefficient of kinetic friction μ_k between the children's feet and the platform is 0.4, while the coefficient of static friction μ_s between their feet and the platform is 0.5.

a) Draw a force diagram for a child standing on the platform. Indicate your choice of coordinate system. (5 points)

Side view:



Note: friction points toward the center to keep the child moving in a circle:



Coordinate system:

b) How close to the edge can a child stand to the edge without slipping? (15 points)

$\Sigma F_x = ma_x$: $F_{fric} = ma_x \longrightarrow \mu_s F_N = m\omega^2 r$

$\Sigma F_y = ma_y$: $F_N - mg = ma_y = 0 \longrightarrow F_N = mg$

(When they are about to slip, friction has its maximum value $\mu_s F_N$.)

Substitute into X: $\mu_s \cancel{m} g = \cancel{m}\omega^2 r$

Since we used the maximum value for F_{fric}, this gives the limiting case. Solve for r:

$$\boxed{r = \frac{\mu_s g}{\omega^2}.}$$

Note $\omega = \frac{1 \text{ rev}}{4 s} = \frac{2\pi}{4} \frac{\text{rad}}{s}$.

$r = 2.03$ m, or 97 cm from the edge.

c) Suppose now that the children spinning the platform want to slow it down enough that their friends on top can safely walk to the edge and jump off. What is the maximum angular velocity ω that would allow a child to stand on the edge of the platform without slipping? (5 points)

We already found for the limiting case:

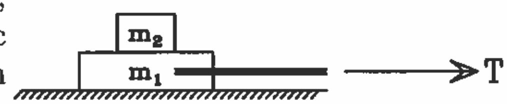
$$r = \frac{\mu_s g}{\omega^2}.$$

Solve for ω:

$$\omega = \sqrt{\frac{\mu_s g}{r}} = \sqrt{\frac{0.5(10\,m/s^2)}{3\,m}} = 0.745\ \frac{rad}{s}.$$

QUESTION 2

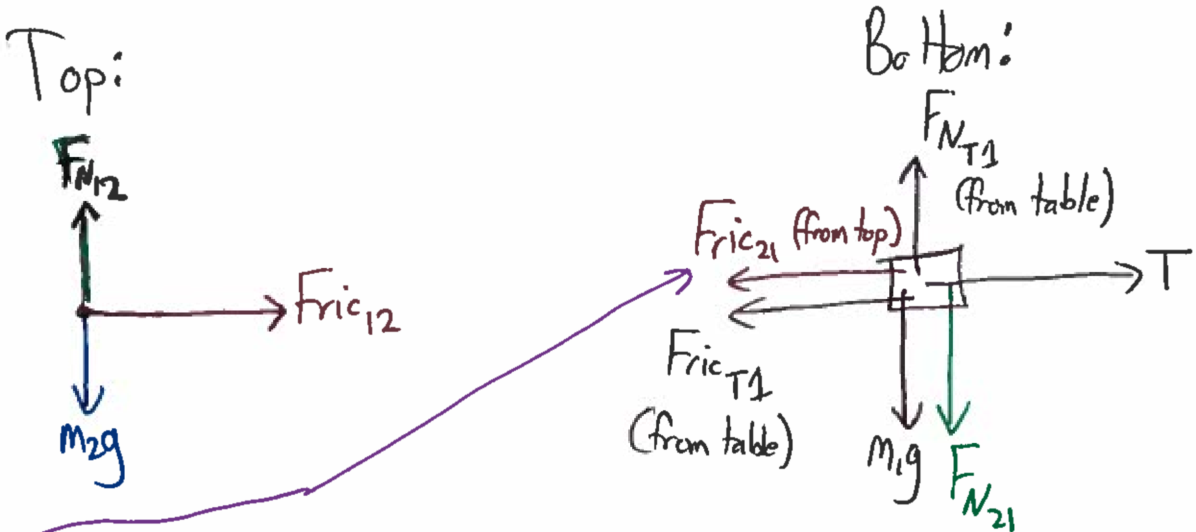
Two boxes sit on a table. The bottom box has mass m_1, and the top box has mass m_2. The coefficient of kinetic friction between the table and the bottom box, and between the bottom box and the top box, is μ_k.



A rope is tied to the bottom box, and a large tension T is applied horizontally. This is large enough to pull the bottom box out from under the top box.

a) Draw a force diagram for the top box and for the bottom box. (10 points total)

Note: friction from bottom box pulls top box along with it.
Newton's 3rd law pairs are color-coded here.



b) Newton's third law says that $\vec{F}_{AB} = -\vec{F}_{BA}$. Are there any pairs of forces in your diagram which form Newton's-third-law pairs? If so, list those pairs. You should ensure that your force diagrams are consistent with Newton's third law. (5 points)

Two pairs:

- Top pushes down on bottom with normal force F_{N21}
- Bottom pushes up on top with the same force $F_{N_{12}}$.

The one people forget:

- Friction from bottom pushes top to the right ($Fric_{12}$)
- ... so friction from top must push bottom to the left ($Fric_{21}$).

d) Calculate the acceleration a_1 of the top box. (5 points)

Top:

$$X: \ \mu_k F_{N_{12}} = m_2 a_{2_x}$$

$$Y: \ F_{N_{12}} - m_2 g = m_2 a_{2_y} = 0 \longrightarrow F_{N_{12}} = m_2 g$$

Substitute into X:

$$\mu_k m_2 g = m_2 a_{2_x} \longrightarrow \boxed{a_{2_x} = \mu_k g.}$$

e) Calculate the acceleration a_2 of the bottom box. (5 points)

More forces here, but the idea is the same.

$$Y: \ F_{N_{T1}} - m_1 g - F_{N_{21}} = m_1 a_{1_y} = 0$$

$$X: \ T - \underbrace{\mu_s F_{N_{21}}}_{\text{friction from top book}} - \underbrace{\mu_s F_{N_{T1}}}_{\text{friction from table}} = m_1 a_{1_x}$$

Note: $F_{N_{21}} = F_{N_{12}} = m_2 g$ from above.

From Y: $F_{N_{T1}} - m_1 g - F_{N21} = 0 \longrightarrow F_{N_{T1}} - m_1 g - m_2 g = 0 \longrightarrow F_{N_{T1}} = m_1 g + m_2 g$

Substitute in normal forces:

$$T - \underbrace{\mu_s m_2 g}_{\text{from top}} - \underbrace{\mu_s (m_1 g + m_2 g)}_{\text{from table}} = m_1 a_{1_x}$$

$$\longrightarrow \boxed{a_{1_x} = \frac{T - \mu_s m_1 g - 2\mu_s m_2 g}{m_1}}$$