Moment of inertia

Physics 211
Syracuse University, Physics 211 Spring 2017
Walter Freeman

April 4, 2017

Announcements

- Physics practice this Wednesday: setting up rotation problems. 7:30-9:30 in Stolkin, partial solutions to HW7 will be discussed
- HW7 will be posted today
- No office hours this Friday (I'll be guest teaching in Arizona)
- A reminder: if something goes wrong and you can't be in recitation or turn homework in on time, tell your recitation TA in addition to me

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- Students who are traveling Easter weekend:
  - Makeup group exam given Thursday evening, 6PM-7PM or 7PM-8PM
  - You will need to sign up by email, and I'll put you in groups
  - I'll send out info for this tomorrow

Translation	Rotation
Position $\vec{s}$ Velocity $\vec{v}$ Acceleration $\vec{a}$	Angle θ Angular velocity ω Angular acceleration α
Kinematics: $\vec{s}(t)\frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$	$\theta(t) = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$
Force $\vec{F}$ Mass m Newton's second law $\vec{F} = m\vec{a}$	Torque τ Rotational inertia I Newton's second law for rotation $\tau = I\alpha$
Kinetic energy $KE = \frac{1}{2}mv^2$ Work $W = \vec{F} \cdot \Delta\vec{s}$ Power $P = \vec{F} \cdot \vec{v}$	Kinetic energy $KE = \frac{1}{2}I\omega^2$ Work $W = \tau\Delta\theta$ Power $P = \tau\omega$
Momentum $\vec{p} = m\vec{v}$	Angular momentum $L = I\omega$

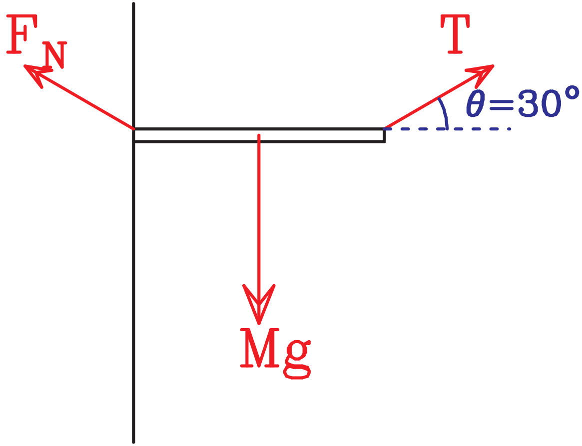
Calculating torque:

$$\tau = F_\perp r = F r_\perp = F r \sin\theta$$

- Draw an extended force diagram
- Label all forces – both where they act and what they are
- Write an expression for the torque applied by each
- Counterclockwise torques are positive; clockwise torques are negative.

If $\alpha = 0$ ("static equilibrium", "dynamic equilibrium"):

- This means that the net torque (about *any* pivot) is zero
- Choose a pivot on top of a force whose value you don't know and don't care about
- Write down $\sum \tau = 0$ and solve



How does the tension T compare to the weight of the beam?

A: $T \leq mg/2$

B: $mg/2 < T < mg$

C: $T = mg$

D: $mg < T < 2mg$

E: $T >= `2mg$

Beyond equilibrium

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Moment of inertia

The analogue of mass is called "moment of inertia" (letter I)

- More massive things are harder to turn, but that's only part of it
- The mass *distribution* matters, too
- The further the mass is from the center, the harder it will be to turn
- The moment of inertia depends on the *average squared distance from the center*

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$$I = MR^2$$

(if all the mass is the same distance from the center)
(our demo rods; hoops; rings; bike wheels)

Moment of inertia: why?

To see why $I = M \langle r^2 \rangle$, let's consider the kinetic energy of a spinning object.

The kinetic energy of a single "point mass" moving in a circle is $\frac{1}{2}mv^2 = \frac{1}{2}mr^2\omega^2$, where r is its distance from the center.

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For an extended object, we simply add up the energy of all the moving particles:

$$I = \int r^2 dm = M\left\langle r^2\right\rangle$$

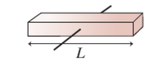
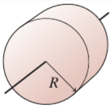
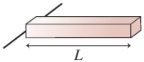
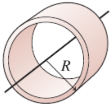
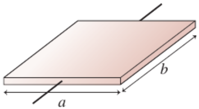
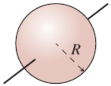
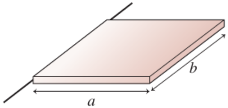
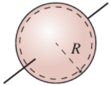
i.e. the moment of inertia is just the total mass times the average squared distance from the axis.

Moment of inertia, other things

What about the moment of inertia of other objects?

Requires calculus in general; here are some common ones

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

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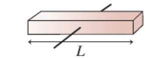
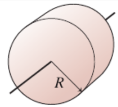
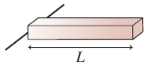
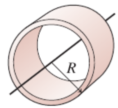
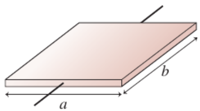
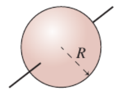
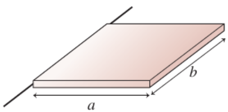
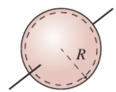
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

In general: $I = \lambda M R^2$

We will always give you I if it's not 1 (i.e. not a ring etc.)

Newton's second law for rotation

$$\tau = I\alpha$$

"Newton's second law for rotation":

Torques give things angular acceleration, just like forces make things accelerate.

Which will make the hanging object fall faster?

A: Increasing the diameter of the spool the string is wound around

B: Decreasing the diameter of the spool the string is wound around

C: Moving the spinning masses inward

D: Moving the spinning masses outward

E: None of the above; it falls at g no matter what

What about rotational kinetic energy?

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Suppose the pulley were a solid cylinder with mass M and radius r. How fast is the falling mass traveling when it hits the ground if it starts from a height h?

Which object will reach the bottom of the ramp faster?

A: The wooden one

B: The one with the mass located near the middle

C: The one with the mass located near the edge

D: A tie between A and B

E: A tie between B and C