Energy methods – problem solving

Physics 211
Syracuse University, Physics 211 Spring 2016
Walter Freeman

April 6, 2016

Announcements

- Office hours today: 1:30-3:30
- I can't stay after 3:30, but TA's/coaches will be there 5PM-9PM to help with homework
- Review session: Sunday, 6:30-9:30 PM
- Exam 2 retake next Tuesday

Where we've been, where we're going

- Last time: we saw that "potential energy" is both a statement about nature and a bookkeeping trick to keep track of work
    - Potential energy only applies to conservative forces (gravity, springs)
    - Lets us account for the work done by these forces with no integrals required
    - Potential energy due to Earth's gravity: $U_g = mgy$
    - Potential energy in a spring: $U_e = \frac{1}{2}k(\Delta x)^2$
- This time: we'll introduce the idea of **power**, a rate of doing work

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- This time: we'll introduce the idea of **power**, a rate of doing work
- ... and see how energy in rotational motion works

Power

- We've been concerned with quite a few "rates" in this class:
  - Velocity: the rate of changing position, measured in meters per second
  - Angular velocity: the rate of changing angle, measured in radians per second
- What about the rate of transfer of energy? What units would it be measured in?

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- What about the rate of transfer of energy? What units would it be measured in?
- This quantity is called **power**
- It's measured in joules per second: 1 J/s = 1 watt
- This is the same unit you're familiar with on lightbulbs and hairdryers

Power: applications

When does this idea of a rate of transferring energy or doing work come up?

- Rates of energy transfer: "Sunlight delivers about 1000 watts per square meter to the ground"
- Rates of energy "consumption": "My laptop uses about 15 watts of power"
- Rates of doing work: "A human can sustain a power output of about 200 watts, generating 800W of waste heat"

Most of our ideas here are stepping stones to understanding something else.
The idea of power is more of a standalone concept: a useful application.
Many of our machines are limited by the **rate** that they can convert energy from one form to another.

Power: rate of doing work

A bit of mathematics that will be useful to you:

"An object moves at a constant speed $\vec{v}$, subject to some force $\vec{F}$; at what rate does that force do work on the object?"

An example: an airplane flies at v=1000 m/s, and its engines exert F=300 kN of thrust. What is the rate at which the engines do work (power)?

$$\text{Work} = \text{force} \times \text{distance}$$

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$$\text{Power} = \text{force} \times \text{velocity}$$
$$P = \vec{F} \cdot \vec{v} = 300 MW$$

- The engines output 300 MW of power: this is around 10 liters per second of fuel even at 100% efficiency!
- Some of that 300 MW of energy dissipated by drag heats up the airplane... (real numbers for a SR-71 Blackbird)

A truck pulling a heavy load with mass $m = 4000$ kg wants to drive up a hill at a 30^o grade.

If the truck's engine can produce 100 kW of power (134 hp), how fast can the truck go? (Neglect drag.)

Sample problems

A 1000 kg car has an engine that produces up to P=100 kW of power. If it accelerates as hard as it can, at what speed does its acceleration become limited by the engine?

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At low speeds: static friction limits acceleration
At high speeds: engine power limits acceleration

Sample problems

42. || A 1000 kg elevator accelerates upward at 1.0 m/s^2 for 10 m, starting from rest.
    a. How much work does gravity do on the elevator?
    b. How much work does the tension in the elevator cable do on the elevator?
    c. Use the work-kinetic energy theorem to find the kinetic energy of the elevator as it reaches 10 m.
    d. What is the speed of the elevator as it reaches 10 m?

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Things roll with very little friction, so let's roll things down a ramp...

- Different objects reach the bottom at different speeds
- ... *none* of them reach the bottom with the "correct" speed given by $mgh = \frac{1}{2}mv_f^2$
- ... what's wrong?

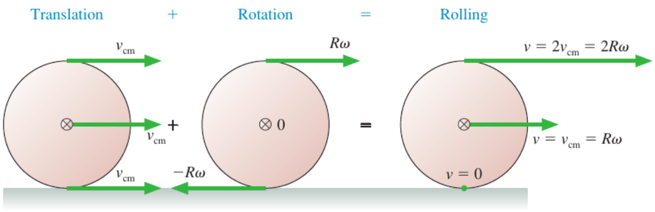
Some energy is diverted to rotational kinetic energy

We can associate a kinetic energy with a spinning object, too...

$$KE_{\text{trans}} = \frac{1}{2}mv^2$$
$$KE_{\text{rot}} = \frac{1}{2}I\omega^2$$

A description of rolling motion without slipping

Rolling motion combines translation of the center of mass with rotation around it



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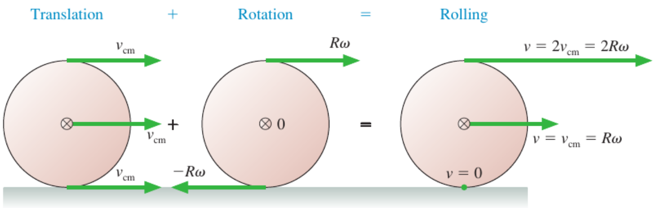

- The key idea: tangential velocity of the rim about the center is equal to the speed of the axle
- In symbols:

$$v_{\text{com}} = v_T = \omega r$$

Now we can finish our problem

An object with $I = \lambda m r^2$ rolls down a hill of height h; how fast is it going at the bottom?

Same idea as before:

$$\begin{aligned} KE_i + U_{g,i} =& KE_f + U_{g_f} \\ mgh =& \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 \end{aligned}$$

But $v_f = r\omega_f \to \omega_f = v_f / r$ and $I = \lambda m r^2$, so we have

$$\begin{aligned} mgh =& \frac{1}{2} m v_f^2 + \frac{1}{2} \lambda m r^2 \left(\frac{v_f^2}{r^2} \right) \\ mgh =& \frac{1}{2} (m + \lambda m) v_f^2 \\ v_f =& \sqrt{\frac{2gh}{1+\lambda}} \end{aligned}$$

... the higher the value of λ, the slower it rolls!

Sample problems

57. ‖ The spring shown in **FIGURE P11.57** is compressed 50 cm and used to launch a 100 kg physics student. The track is frictionless until it starts up the incline. The student's coefficient of kinetic friction on the 30° incline is 0.15.
   a. What is the student's speed just after losing contact with the spring?
   b. How far up the incline does the student go?

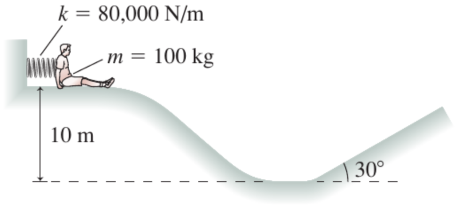

FIGURE P11.57

Sample problems

49. ‖ Truck brakes can fail if they get too hot. In some mountainous areas, ramps of loose gravel are constructed to stop runaway trucks that have lost their brakes. The combination of a slight upward slope and a large coefficient of rolling resistance as the truck tires sink into the gravel brings the truck safely to a halt. Suppose a gravel ramp slopes upward at 6.0° and the coefficient of rolling friction is 0.40. Use work and energy to find the length of a ramp that will stop a 15,000 kg truck that enters the ramp at 35 m/s ($\approx$75 mph).