Question 1

In class, we did a demonstration where a student sat on a wheeled cart, carrying a heavy ball. When she threw the ball to the right, the cart began to roll to the left.

Suppose that the student plus cart had a mass of $M = 50$ kg, the ball had a mass of $m = 5$ kg, and she threw it at a velocity of $v_1 = 2$ m/s.

(You may answer this question either numerically or in terms of M, m, v_1, and v_2.)

a) After she threw the ball, how fast was she moving to the left? You may neglect friction. (20 points)

Before

$p_i = 0$

After

$V_2 \leftarrow$

$\rightarrow V_1 = 2\,m/s$

$P_f = mV_1 - MV_2$

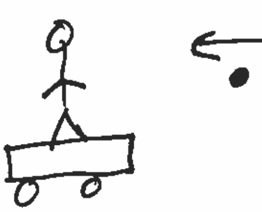
$0 = mV_1 - MV_2$

$V_2 = \frac{m}{M} V_1 = 0.2\ m/s$

b) Another student caught the ball and threw it back to her. Suppose that this student threw it back at $v_2 = 3$ m/s. After she catches the ball again, what is her new velocity? (30 points)

Call this V_3 since I already used V_2

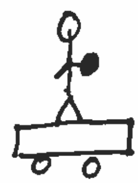
Before



$\leftarrow V_3 = 3\ m/s$

$\leftarrow$ $V_2 = 0.2\ m/s$

After



$\leftarrow$ $V_f = ?$

$p_i = -MV_2 - mV_3$

$P_f = -(M+m)V_f$

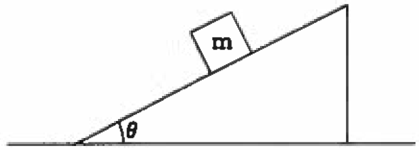
$\Longrightarrow MV_2 + mV_3 = (M+m)V_f$

$(50\ kg)(0.2\ m/s) + (5\ kg)(3\ m/s) = (55\ kg)V_f$

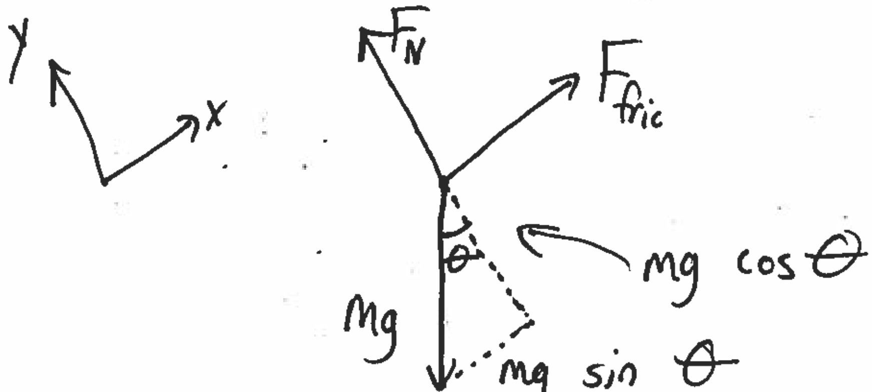
$\boxed{V_f = 0.455\ m/s\ \text{left}}$

Question 2

A book of mass m rests on a slope with angle of inclination θ as shown below. There is friction between the book and the slope.



a) Draw a force diagram for the book, indicating your choices for your coordinate axes. (10 points)



b) Suppose first that the book does not slide down the slope. In terms of θ, m, and g, compute the minimum value of μ_s required to make the book stay on the ramp without sliding. (Your answer may not depend on all three of these quantities.) (20 points)

Write Newton's laws: $(F_{fric} = \mu_s F_N)$

$X: \mu_s F_N - mg \sin\theta = ma_x = 0$ $\rightarrow F_N = mg \cos\theta$. Substitute:

$Y: F_N - mg \cos\theta = ma_y = 0$ $\rightarrow \mu_s \cancel{mg} \cos\theta - \cancel{mg} \sin\theta = 0$

$$\mu_s = \frac{\sin\theta}{\cos\theta} = \tan\theta.$$

c) Now, suppose that μ_s is less than this value, and the book slides down the slope. The coefficient of kinetic friction is μ_k. In terms of θ, μ_k, m, and g, compute the acceleration with which the book slides down the ramp. (Your answer may not depend on all four quantities.) (20 points)

Do the same thing but now $a_x \neq 0$.

$X: \mu_k F_N - mg \sin\theta = ma_x$

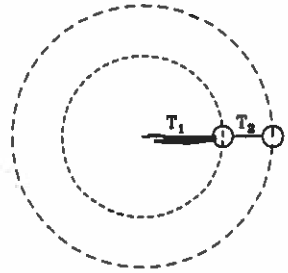
$Y: F_N - mg \cos\theta = ma_y = 0 \Rightarrow F_N = mg \cos\theta$

$\rightarrow \mu_k mg \cos\theta - mg \sin\theta = ma_x$

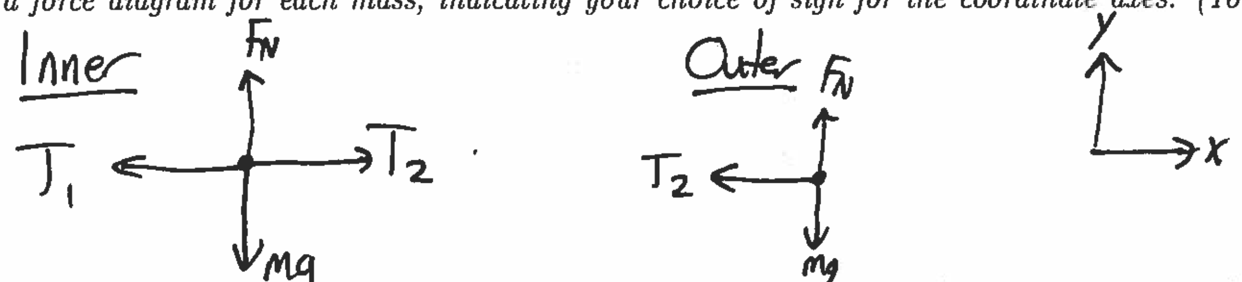
$$\boxed{\mu_k g \cos\theta - g \sin\theta = a_x}$$

QUESTION 3

Two masses of $m = 2kg$ each lie on a frictionless table, connected by a string $L_2 = 50$ cm long. Another string, $L_1 = 80$ cm long, runs from one of those masses to an attachment on the table. The string is stretched out and both masses are made to revolve around the point of attachment, making one complete circle every 3 s. The diagram below shows a top-down view of the apparatus; the dotted circles show the paths of the two masses. The masses slide on top of the surface without friction.



a) Draw a force diagram for each mass, indicating your choice of sign for the coordinate axes. (10 points)



b) What is the angular velocity of the inner mass? What is the angular velocity of the outer mass? (5 points)

$$\omega = \frac{1 \text{ circle}}{3 \text{ sec}} = \frac{2\pi \text{ rad}}{3 \text{ sec}} \quad \text{(for both)}$$

c) What is the tangential velocity of the inner mass? What is the tangential velocity of the outer mass? (5 points)

$$V_T = \omega r \qquad r_{inner} = 0.8 \text{ m}, \text{ so } V_{T_{inner}} = \frac{2\pi}{3} s^{-1} \cdot 0.8 \text{ m} = 1.67 \text{ m/s}$$

$$r_{outer} = 1.3 \text{ m}, \text{ so } V_{T_{outer}} = 2.72 \text{ m/s}.$$

d) Find the two tensions T_1 and T_2. (30 points)

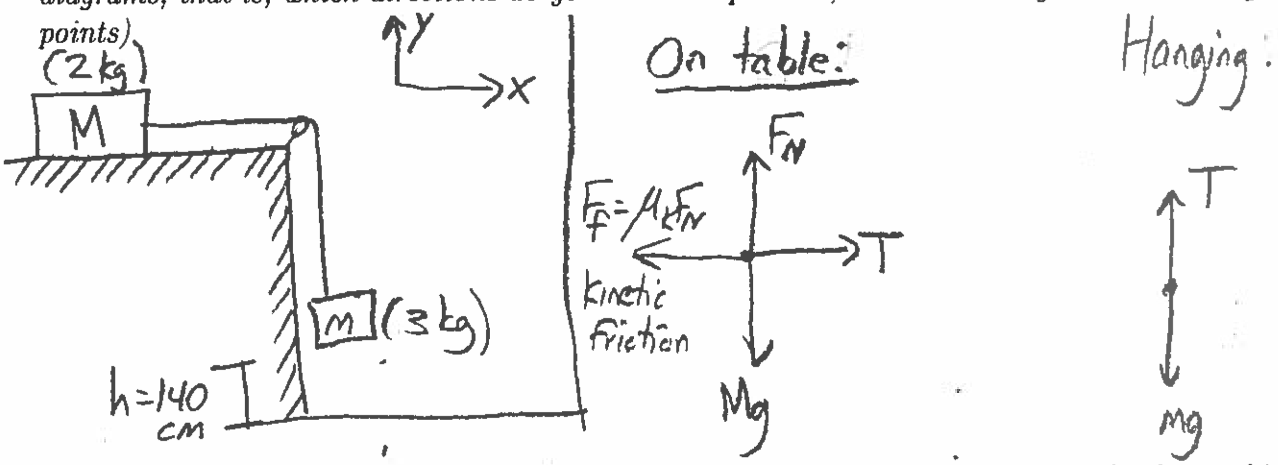
$$\text{Inner: } T_2 - T_1 = ma_x = -\omega^2 r_1 m \quad \rightarrow \quad T_2 = m\omega^2 r_2 = 11.4 \text{ N}$$

$$\text{Outer: } -T_2 = ma'_x = -\omega^2 r_2 m \quad \rightarrow \quad T_1 = T_2 + m\omega^2 r_1 = 18.4 \text{ N}$$

QUESTION 4

A book with a mass of 2 kg rests on a table; the coefficient of kinetic friction μ_k between them is 0.4. A string connects that book to another book hanging vertically off the side of the table with mass 3 kg; this hanging book is 140 cm above the ground. When the hanging book is released, it accelerates toward the ground, dragging the other book on the table with it.

a) Draw a force diagram for both books. Indicate your choice of signs for the $x-$ and $y-$axes on both diagrams; that is, which directions do you consider positive, and which do you consider negative? (10 points)



b) Are the accelerations of the two books related? If so, write a mathematical relationship between them. (10 points)

Yes. As the hanging book moves down, the one on the table moves right at the same rate. Thus, $a_{1x} = -a_{2y}$.
For tidiness, call $a_{1x} \equiv a$; then $a_{2y} = -a$.

c) Calculate the accelerations of the books and the tension in the string. (20 points)

On table

$X: T - \mu_k F_N = M a_{1x} = Ma$

$Y: F_N - Mg = M a_{1y} = 0.$

Hanging:

$T - mg = m a_{2y} = -ma$

$\Rightarrow T = mg - ma$

$F_N = Mg$, so we have:

(from X:) $mg - ma - \mu_k Mg = Ma$

$a = \dfrac{mg - \mu_k Mg}{M + m} = 4.4\ \mathrm{m/s^2}$

$T = (3\ \mathrm{kg})(9.8\ \mathrm{m/s^2} - 4.4\ \mathrm{m/s^2}) = 16.2\ \mathrm{N}$

d) With what velocity will the hanging book strike the floor? (10 points)

This is kinematics. As t doesn't matter, use $v_f^2 - v_0^2 = 2a\Delta y$.
The acceleration above is downward, so treat down as positive. Thus:

$v_f^2 - 0 = 2ah \Rightarrow v_f = \sqrt{2ah} = \sqrt{(2)(4.4\ \mathrm{m/s^2})(1.4\ \mathrm{m})} = 3.35\ \mathrm{m/s}.$

Question 5

The force of gravity of a person standing on the surface of the Moon is one-sixth of its value on the surface of the Earth. Suppose the radius of the Moon r_M is exactly one-quarter that of the Earth.

a) As a fraction of the mass of the Earth, what is the mass of the Moon? (Note: You do not need numerical values for the masses and radii of the Earth and Moon, and you should not need a calculator for this problem.) *(20 points)* Compare Newton's law of gravity for both:

$$\frac{\cancel{G} M_e \cancel{m}}{r_e^2} = 6\,\frac{\cancel{G} M_m \cancel{m}}{r_m^2}. \Rightarrow \text{algebra: } \frac{r_m^2}{r_e^2} = 6\,\frac{M_m}{M_e}$$

$$\text{Note } \frac{r_m}{r_e} = \frac{1}{4}. \Rightarrow \frac{1}{16} = 6\,\frac{M_m}{M_e} \Rightarrow \boxed{\frac{M_m}{M_e} = \frac{1}{96}}.$$

b) The Moon orbits the Earth with a nearly constant speed in a nearly circular orbit. Is the Moon accelerating? If so, how do you know? What force causes this acceleration? (10 points)

Yes (things going in circles are accelerating at $a_c = \omega^2 r$)

The force is the Earth's gravity.

c) The Moon orbits the Earth with angular velocity ω. In terms of the mass of the Earth m_E, the gravitational constant G, and ω, what is the distance r from the Earth to the Moon? (20 points)

$F = ma$: we want to relate force to motion.

$$\frac{G M_e m_m}{r^2} = m_m a = m_m \omega^2 r. \quad \text{Solve for } r:$$

$$\frac{G M_e}{\omega^2} = r^3; \quad r = \sqrt[3]{\frac{G M_e}{\omega^2}}.$$

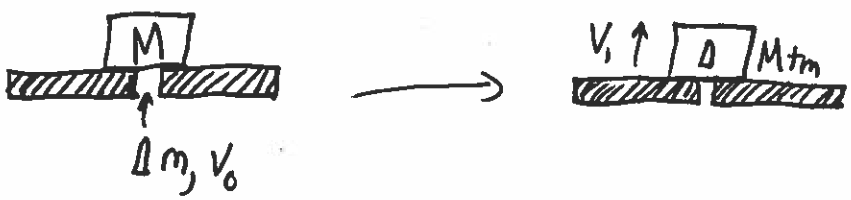
QUESTION 6

A clay block of mass M sits on top of a table. It sits on top of a hole in the table. It is struck from below by a bullet of mass m traveling at speed v_0 through the hole; the bullet lodges in the clay block, and the block flies up in the air.

In this problem, you will calculate the maximum height that the block reaches.

a) Without doing any mathematics, outline a plan for solving this problem in words and diagrams. (15 points)

- Use conservation of momentum to find velocity of block after the collision. Call this velocity v_1.
- Use kinematics to relate that to the height.



b) Find the maximum height reached by the block. (35 points)

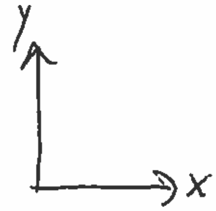
Initial momentum: $mv_0 + M(0)$ Final momentum (after collision) $= (M+m)v_1$

$$\rightarrow mv_0 = (M+m)v_1 \rightarrow \boxed{v_1 = v_0\left(\frac{m}{m+M}\right)}$$

Kinematics: find y when $v=0$.

$$v_f^2 - v_0^2 = 2a\Delta y \rightarrow 0 - v_1^2 = 2(-g)\Delta y$$

$$\boxed{\Delta y = \frac{v_1^2}{2g} = \left[v_0\left(\frac{m}{m+M}\right)\right]^2 \Big/ 2g}$$



$M = 600\ g$

$m = 400\ g$

$M + m = 1\ kg$

$50\ m/s = V_0$

$40\ m/s = V_1$

QUESTION 7

A firecracker of mass 1 kg is launched straight upward. When it is 200 meters above the ground and traveling upward at a velocity of 50 m/s, it explodes, separating into two pieces. After the explosion, one piece has a mass of 600 grams and travels horizontally East at 40 m/s.

a) What is the velocity of the other piece? (Remember, velocity is a vector.) (30 points)

Initial momentum: $p_{i_x} = 0 \qquad p_{i_y} = (M+m)V_0$

Final momentum : $p_{f_x} = Mv_1 + mV_{f_x}$

: $p_{f_y} = mV_{f_y} + M(0)$

$0 = Mv_1 + mV_{f_x} \Rightarrow V_{f_x} = -\frac{Mv_1}{m}$

$= -60\ m/s$

$(M+m)V_0 = mV_{f_y} \Rightarrow V_{f_y} = \frac{(M+m)V_0}{m}$

$= 125\ m/s$

$\longrightarrow \vec{V}_f = -60\ m/s\ \hat{i} + 125\ m/s\ \hat{j}$

b) What is the maximum height that this other piece will achieve? (20 points)

(or any other notation)

Maximum height: "what is y when $V_y = 0$?"

velocity after explosion

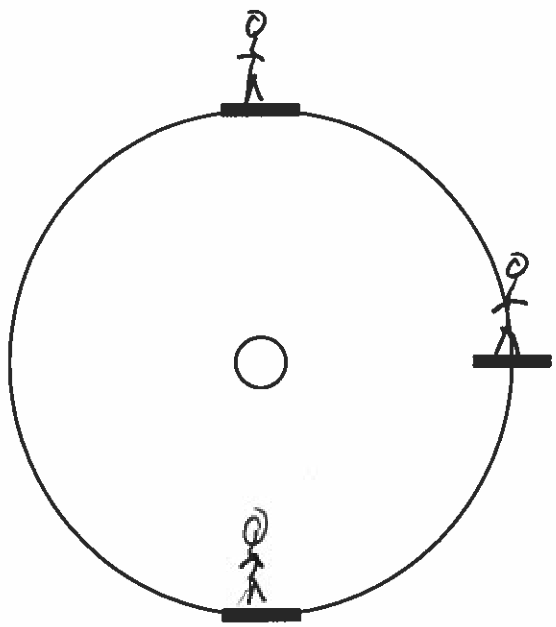
$V_y = V_{0_y} + a_y t = V_{f_y} - gt \Rightarrow t = V_{f_y}/g$

$y(t) = -\frac{1}{2}gt^2 + V_{0_y}t + y_0 = -\frac{1}{2}g\left(\frac{V_{f_y}}{g}\right)^2 + V_{f_y}\left(\frac{V_{f_y}}{g}\right) + y_0 = y_0 + \frac{V_{f_y}^2}{2g}$

$\boxed{= 1762.5\ m}$

QUESTION 8

A carnival ride consists of a vertical wheel of radius r rotating at angular velocity ω around a horizontal axis. There is a horizontal platform attached to it; a person stands on the platform. This person has mass m, and stands on a scale.



a) Draw a force diagram for the person at the top of the wheel, and label your coordinate axes. (5 points)

b) How does the scale reading relate to the forces on your force diagram? (5 points)

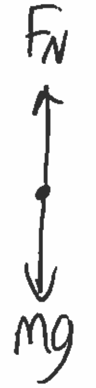
The scale measures F_N.

c) *In terms of m, g, r, and ω, what does the scale read when they are at the top? (10 points)*

At top, $a_y = -\omega^2 r$.

From force diagram, $F_N - mg = -m\omega^2 r$, $F_N = mg - m\omega^2 r$.

d) *Draw a force diagram for the person at the bottom of the wheel. (5 points)*

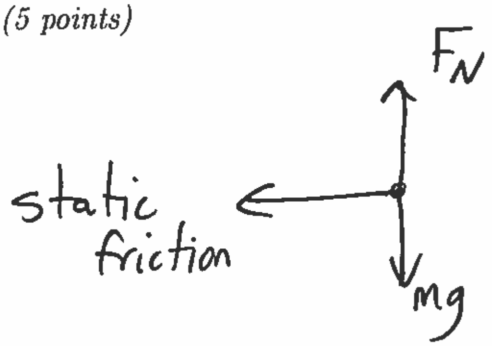


e) *In terms of m, g, r, and ω, what does the scale read when they are at the bottom? (10 points)*

Here, $a_y = \omega^2 r$ (toward center). Use $\Sigma F_y = ma_y$

$F_N - mg = +m\omega^2 r$, so $\boxed{F_N = mg + m\omega^2 r}$

f) *Draw a force diagram for the person when they are at the same height of the center of the wheel. (5 points)*



g) *What is the magnitude of the force of static friction acting on the person when they are at the side? (5 points)*

$-F_s = -ma_x = -m\omega^2 r$: $F_s = m\omega^2 r$