Friction (and sundry)

Physics 211
Syracuse University, Physics 211 Spring 2015
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February 17, 2015

Announcements

- Homework 4 extended until Friday
- *Mastering Physics* assignment due Thursday
- Read Chapter 8

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- Read Chapter 8
- Exam date change: *let me know* if there's a problem with moving it
    - Feb 26 $\rightarrow$ March 3

Ask a Physicist: How does an airplane work?

Homework review: Two objects moving

- Demo: Atwood's machine
- Same as always: Force diagram → Newton's law for each object → solve
- Two different objects → two different accelerations!

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$$T_2 - m_2 g = m_2 a_2$$

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$$a_1 = -a_2 \text{ (“standard coordinate system”)}$$
$$a_1 = a_2 \text{ (“reverse coordinate system”)}$$

Rotational motion

Often things in nature are constrained to go in circles:

- Planets orbiting stars; moons orbiting planets (close enough to circles)
- Wheels; things on strings; many others

We'll study "uniform circular motion" here:

- Something moves at a constant distance from a fixed point
- … at a constant speed.

Rotational motion: description

- Object moves in a circle of radius r, with its angle changing at a constant rate
- "Position $=$ rate $\times$ time" $\rightarrow$ "Angle $=$ rate $\times$ time"

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$$\theta = \omega t$$

ω (omega) called the "angular velocity"; measured in radians per second

- Angular velocity tells you how fast something spins: RPM's are another unit
- A larger radius does *not* mean something has a higher angular velocity

Rotational motion: description

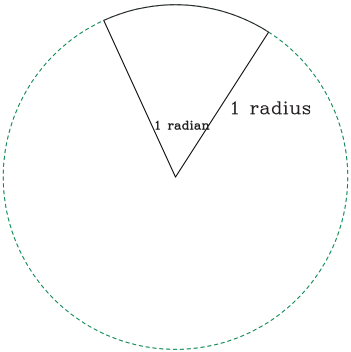
Some new terms:

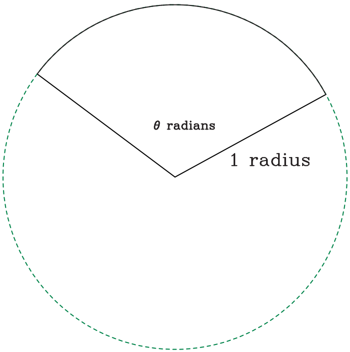
- "Radial": directed in and out of the circle
- "Tangential": directed around the circle
- The radial velocity is 0 (r doesn't change)
- The tangential velocity depends on r and ω, as you'd expect

Radians

The radian: new unit of angle. 2π radians = 360 degrees.

1 complete circle is 2π radians; 1 complete circumference is 2π radiuses ($C = 2\pi r$).
1 radian thus has an arc length of 1 radius.
θ radians therefore have an arc length of $r\theta$.





→ Tangential movement (in meters) = angular movement (in radians) times the radius

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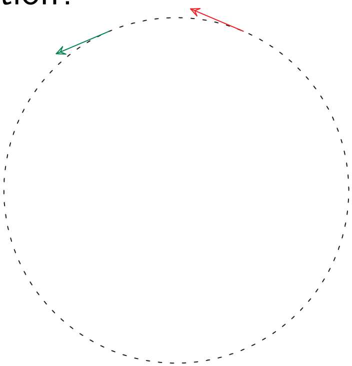
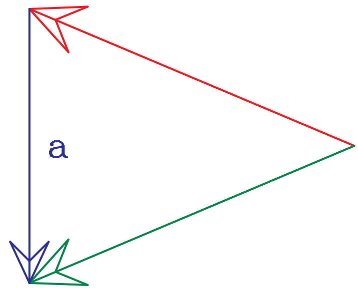
- $v_T = \omega r$: "meters per second = radians per second times meters per radian"

Kinematic challenge: what's the acceleration

Clearly an object moving in a circle is accelerating. What's the acceleration?





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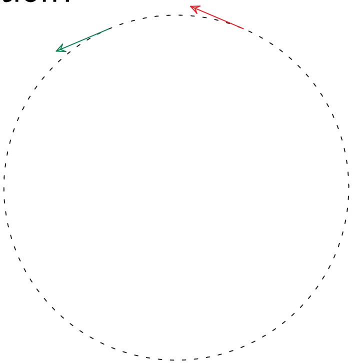

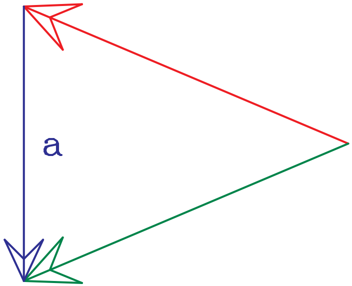

Near the top of the circle, the y-component of the velocity decreases; we expect then that $\vec{a}$ points downward.

Can we make this rigorous?

Some math

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Differentiate to get v_x and v_y:

$$\begin{aligned} v_x(t) &= -\omega r \sin(\omega t) \\ v_y(t) &= \quad \omega r \cos(\omega t) \end{aligned}$$

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An object in uniform circular motion accelerates toward the center of the circle with

$$\rightarrow a = \omega^2 r = v^2/r \leftarrow$$

Uniform circular motion, consequences

If you know an object is undergoing uniform circular motion, you know something about the acceleration:

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Circular motion problems aren't scary; they are just like equilibrium problems.

- Equilibrium problem: $\sum F_x = ma_x = 0$ and $\sum F_y = ma_y = 0$
- Circular motion problem: $\sum F_T = ma_T = 0$ and $\sum F_r = ma_r = v^2/r$

$\rightarrow$ If we tell you that a thing is in uniform circular motion, we're just telling you something about its acceleration.

Centripetal force

"Centripetal" means "toward the center" in Latin.

- If something is going to accelerate toward the center, a force must do that.
- Centripetal force is not a "new" force. No arrows labeled "centripetal force"!
- "Centripetal" is a word that describes a force you already know about.
- Centripetal force: describes a force that holds something in a circle
- It can be lots of things:
  - Tension (stuffed animal on a string demo)
  - Normal force (platform, bucket demos)
  - Friction (Ferris wheel)
  - Gravity (the moon!)

Sample problems

(from demos)