Motion with constant acceleration

Physics 211
Syracuse University, Physics 211 Spring 2020
Walter Freeman, with Matt Rudolph

January 15, 2020

The beginning: Free fall

My purpose is to set forth a very new science dealing with a very ancient subject. There is, in nature, perhaps nothing older than motion, concerning which the books written by philosophers are neither few nor small nevertheless I have discovered by experiment some properties of it which are worth knowing and which have not hitherto been either observed or demonstrated....

So far as I know, no one has yet pointed out that the distances traversed, during equal intervals of time, by a body falling from rest, stand to one another **in the same ratio as the odd numbers beginning with unity.**

–Galileo Galilei, *Dialogues and Mathematical Demonstrations Concerning Two New Sciences*, 1638

Announcements

- Webpage: https://walterfreeman.github.io/phy211/
  - Syllabus, homework, etc. are all there
- The first homework is due next Wednesday
- I will be gone Friday-Tuesday for a conference

Class survey

Please fill out the class survey linked from the course webpage. We need responses from everyone (and your response counts as a portion of your Homework 1 grade).

If you don't yet know your recitation TA's name, that's okay. (The schedule is all messed up on our end. We're fixing it ASAP.)

If you send us a question you're curious about, I might answer it in class (and you'll get extra credit).

If you think of a question later, please send it to me by email!

Quantum computers

"Would quantum chips/computers impact our society in a major way?"

–Grace Sainsbury

Homework tips

Your first homework assignment is due next Wednesday.

- Make use of words, pictures, and algebra (not *just* algebra!) in your reasoning
- We're interested in how you think, not just the answer
- Physical values need to be given with units ("4 meters", not "4")
- Leave variables in until the very end
- Paper is cheap – don't cramp yourself!

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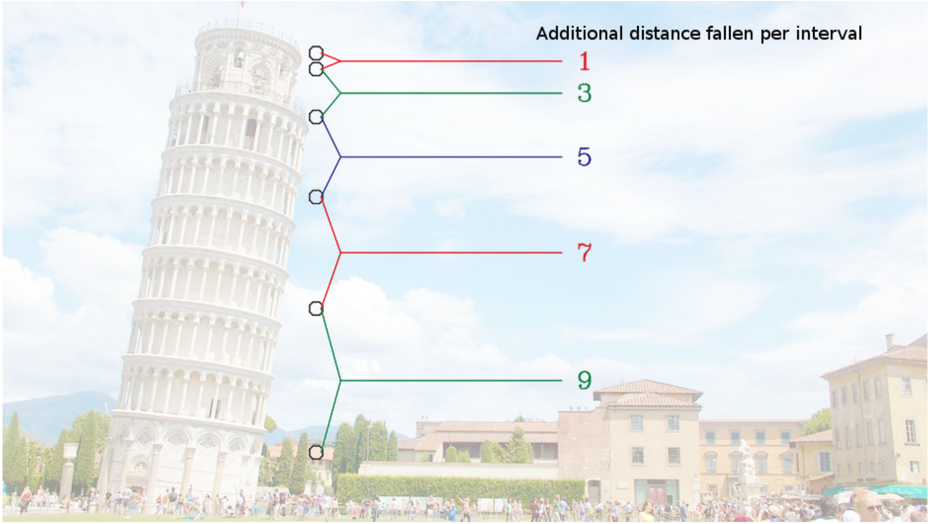
- Ask for help – early and often
  - Email: wafreema@syr.edu, msrudolp@syr.edu
  - The Physics Clinic: Matt will be there 2-4 tomorrow, but graduate student tutors will be there at most other times too
  - Recitations

The beginning: Free fall



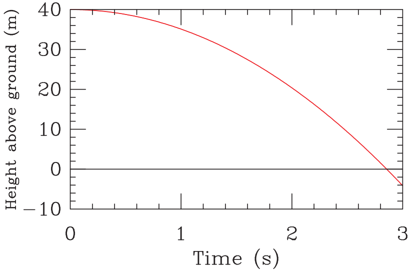
Galileo observed this (and so can we), but can we explain it?

Equations of motion

Complete description of motion: "Where is my object at each point in time?"

This corresponds to a mathematical function. Two ways to represent these. Suppose I drop a ball off a building, putting the origin at the ground and calling "up" the positive direction:

Graphical representation



Algebraic representation

$$y(t) = (40\,\mathrm{m}) - Ct^2$$

(C is some number; we'll learn what it is at the end of class)

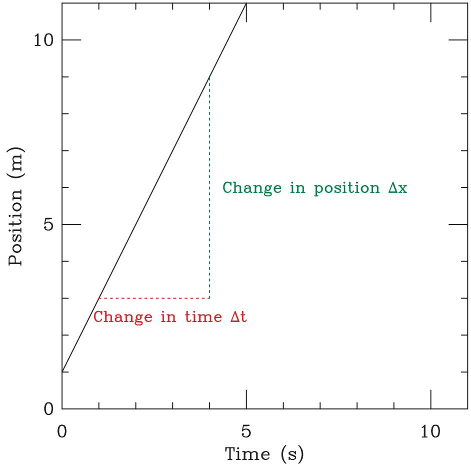
Both let us answer questions like "When does the object hit the ground?"

$\rightarrow$ ... the curve's x-intercept

$\rightarrow$ ... when $y(t) = 0$

Velocity: how fast position changes

The slope of the position vs. time curve has a special significance. Here's one with a constant slope:



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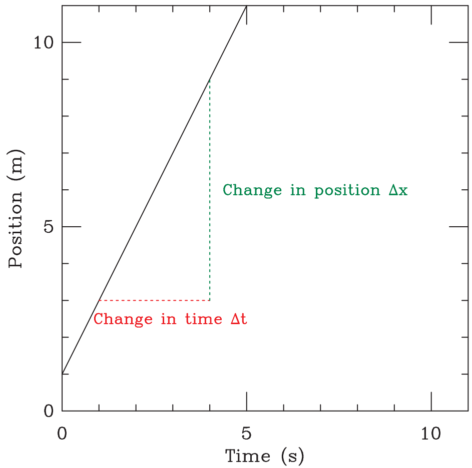

Slope is $\frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t} = \frac{2\,\text{m}}{1\,\text{s}} = 2$ meters per second (positive; it could well be negative!)

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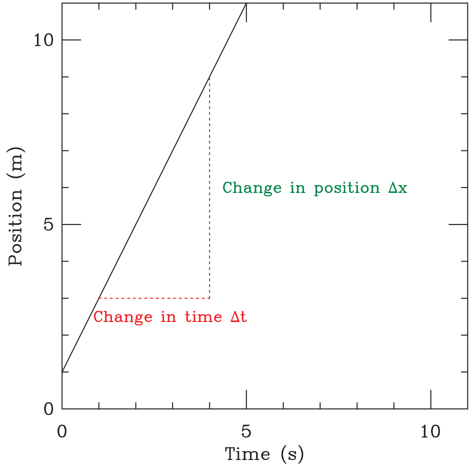

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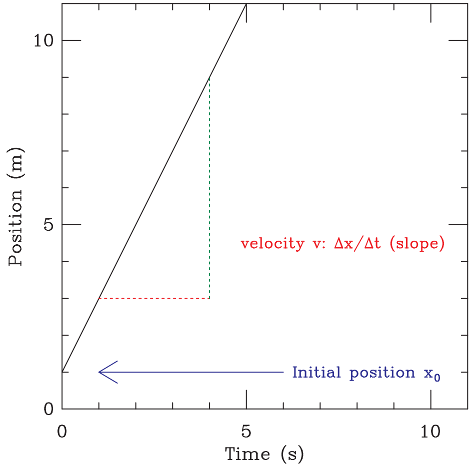
→ The slope here – change in position over change in time – is the **velocity**! Note that it can be positive or negative, depending on which way the object moves.

Constant-velocity motion: connecting graphs to algebra

If an object moves with constant velocity, its position vs. time graph is a line:



We know the equation of a straight line is is $x = mt + b$ (using t and x as our axes).

- m is the slope, which we identified as the velocity
- b is the vertical intercept, which we recognize as the value of x when $t = 0$

We can thus change the variable names to be more descriptive:

$$x(t) = vt + x_0 \text{ (constant-velocity motion)}$$

Going from "equations of motion" to answers

$x(t) = vt + x_0$ is called an *equation of motion*; in this case, it is valid for constant-velocity motion.

It gives you the same information as a position vs. time graph, but in algebraic form.

To solve real problems, we need to be able to translate physical questions into algebraic statements:

- "If a car starts at milepost 30 and drives at 50 mph, where is it an hour later?"

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To solve real problems, we need to be able to translate physical questions into algebraic statements:

- "If a car starts at milepost 30 and drives at 50 mph, where is it an hour later?"
  - Using $x(t) = x_0 + vt$, with $x_0 = 30\,\mathrm{mi}$ and $v = 50\,\frac{\mathrm{mi}}{\mathrm{hr}}$, calculate x at $t = 1\,\mathrm{hr}$

Asking the right questions

"I drop an object from a height h. When does it hit the ground?" How do I do this? (Take $x_0 = h$ and upward to be positive.)

Remember, we want to ask a question in terms of our physical variables. This question has the form:

"What is __________ when __________ equals __________?"

Fill in the blanks.

A: v, x, 0
B: t, x, h
C: x, t, 0
D: t, x, 0
E: x, v, 0

Asking the right questions

"At what location do two moving objects meet?"

A: "At what time does $x_1 = x_2$?"
B: "At what time does $v_1 = v_2$?"
C: "What is x_1 at the time when $x_1 = x_2$?"
D: "What is x_1 when $t_1 = t_2$?"

Velocity, acceleration, and calculus

Constant-velocity motion: $x(t) = x_0 + vt$

- Came from looking at the equation of a line
- We can understand this in a different framework, too:

- Velocity is the rate of change of position
  - Graphical representation: Velocity is the slope of the position vs. time graph
  - Mathematical language: Velocity is the derivative of position

We know we need to know about acceleration (“F=ma”) – what is it?

- Acceleration is the rate of change of velocity

Position, velocity, and acceleration

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Kinematics: how does acceleration affect movement?

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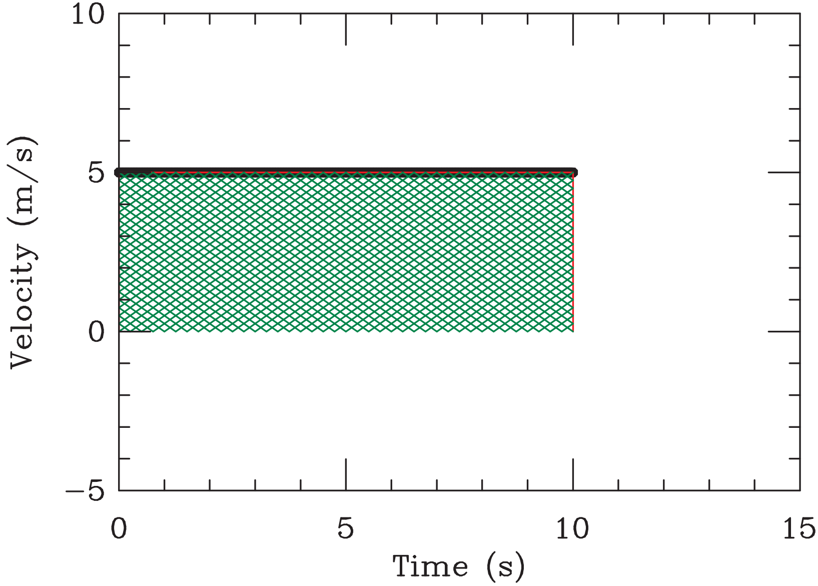
rns out it's the *same* for all objects. *Why* it should be the same is not obv

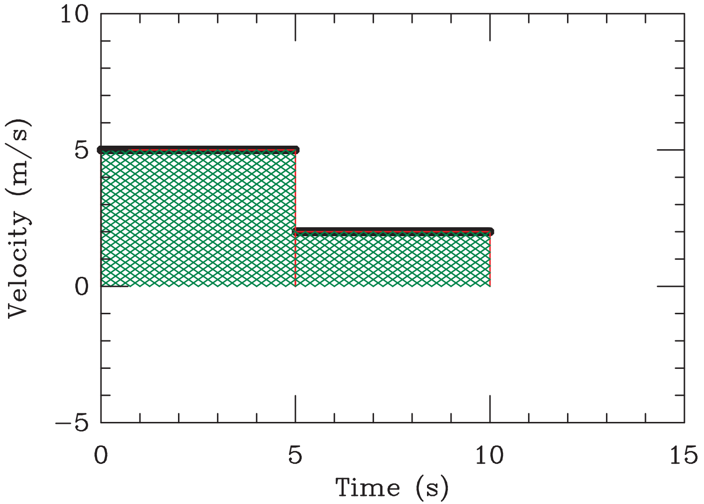
A calculus review

If velocity is the rate of change of position,
why is the area under the v vs. t curve equal to displacement?



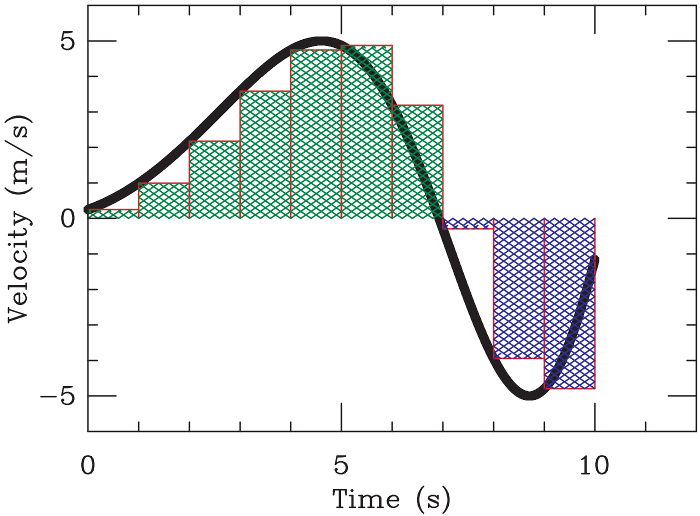
We know $\Delta s = vt$. What is that here? What's the area of the shaded region?

A calculus review



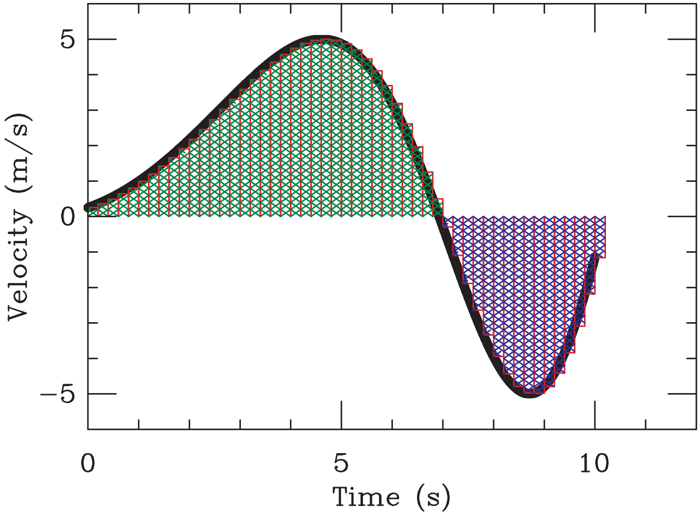
Now what is Δs? What is the area of the shaded region?

A calculus review



Does this work? How do we fix it?

A calculus review



The area between the t-axis and the velocity curve is the distance traveled.
(The area below the t-axis counts negative: "the thing is going backwards"

In calculus notation: $\int v(t)\, dt = \delta x = x(t) - x_0$

Position, velocity, and acceleration

$$\text{Position} \quad \underset{\substack{\text{take the area under the curve of} \\ \text{(take the integral of)}}}{\overset{\substack{\text{(take the derivative of)} \\ \text{take the rate of change of}}}{\rightleftarrows}} \quad \text{Velocity}$$

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Constant acceleration

Particularly interesting situation:

- Free fall (as you saw)
- Any time the force is constant: $F = ma \rightarrow a = F/m$...

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Plan of attack:

- We know what the acceleration curve looks like (it's just flat)
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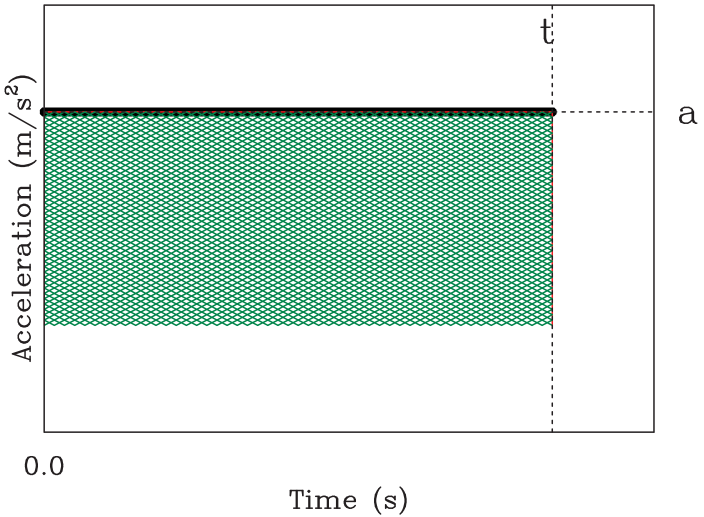
Remember the area under the curve of (velocity, acceleration) just gives the *change in* (position, velocity) – i.e. initial minus final.

We'll start by assuming x_0 and v_0 are zero.

Constant acceleration



What's the area under the curve out to time t, which gives the change in the velocity – $\Delta v = v(t) - v_0$?

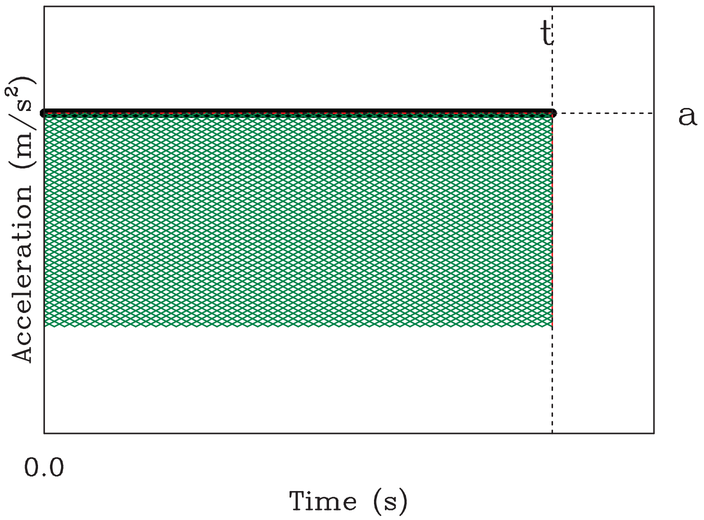
A: $\Delta v = at$

B: $\Delta v = at + v_0$

C: $\Delta v = \frac{1}{2}at^2$

D: $\Delta v = a$

Constant acceleration



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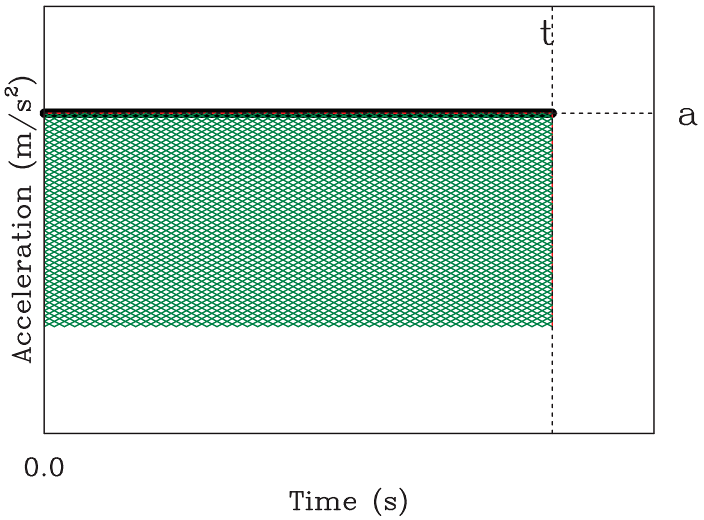
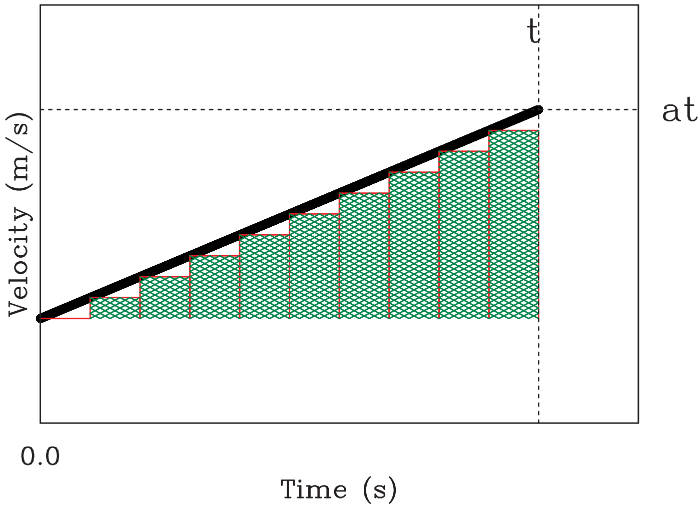


What's the area under the curve out to time t, which gives the change in the velocity – $\Delta v = v(t) - v_0$?

Δv, the change in velocity, is $v(t) - v_0 = at$, so $v(t) = at + v_0$

Same thing again to get position



Now the area under the velocity curve gives the change in position: $\Delta x = x(t) - x_0$. What is that?

A: $\Delta x = at$

B: $\Delta x = vt$

C: $\Delta x = \frac{1}{2}at^2$

D: $\Delta x = v$

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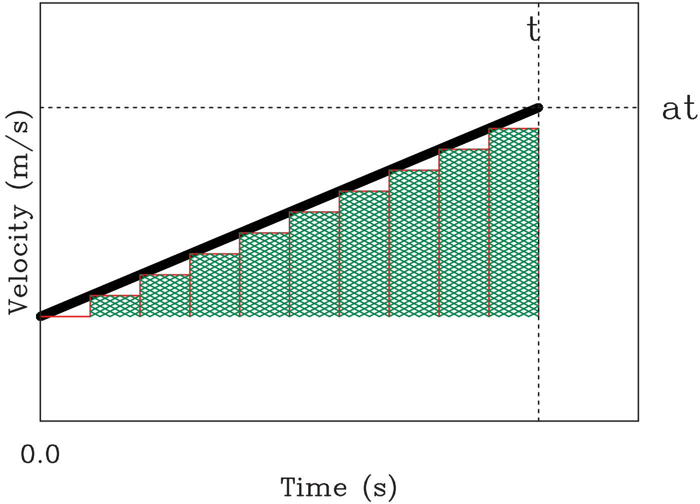

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A: $\Delta x = at$

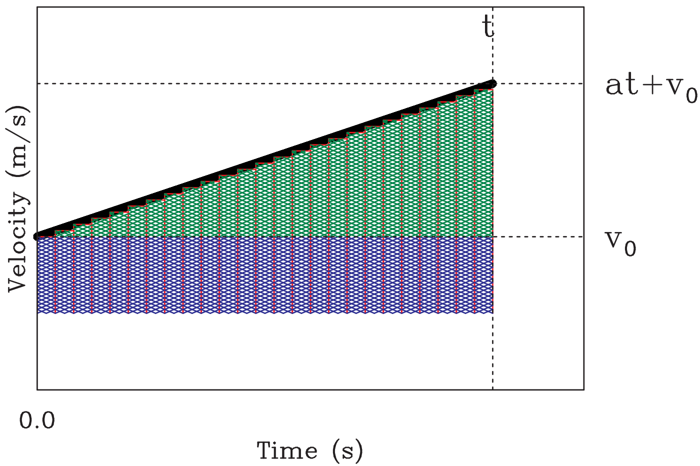
B: $\Delta x = vt$

C: $\Delta x = \frac{1}{2}at^2$

D: $\Delta x = v$

$$x(t) - x_0 = \frac{1}{2}at^2, \text{ thus } x(t) = \frac{1}{2}at + x_0$$

Now if v_0 is not zero...



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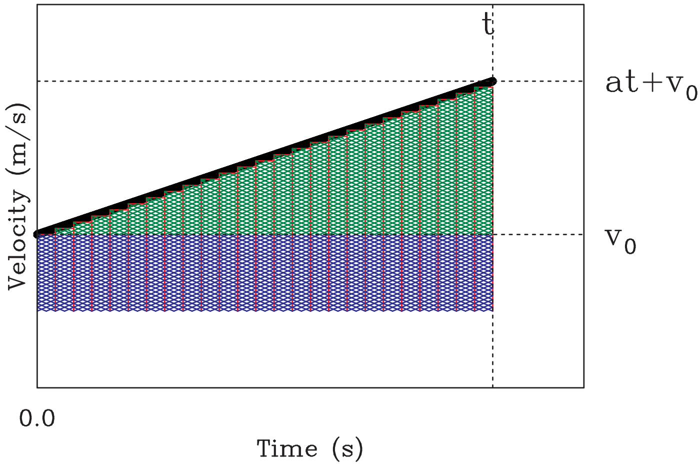


Area under blue part: $v_0 t$
Area under green part: $\frac{1}{2}at^2$
Total change in position: $x(t) - x_0 = \frac{1}{2}at^2 + v_0 t$

Thus, $x(t) = \frac{1}{2}at^2 + v_0 t + x_0$

For those who are familiar with calculus:

$$
\begin{aligned}
a(t) &= \text{const.} \\
v(t) &= \int a\,dt & &= at + C_1 \\
x(t) &= \int v\,dt = \int (at + C_1)dt & &= \frac{1}{2}at^2 + C_1 t + C_2
\end{aligned}
$$

A little thought reveals that C_1 is the initial velocity v_0 and C_2 is the initial position x_0. This gives us the things we just derived, but much more easily:

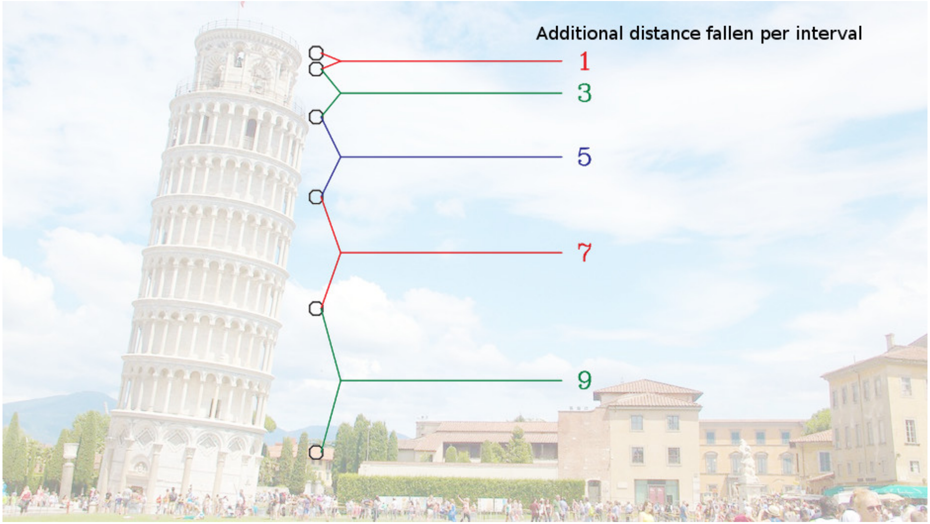
$$
\begin{aligned}
v(t) &= at + v_0 \\
x(t) &= \frac{1}{2}at^2 + v_0 t + x_0
\end{aligned}
$$

Quadratic equations

Notice that the equation $x(t) = \frac{1}{2}at^2 + v_0 t + x_0$ is a *quadratic equation*. This means:

- Its graph will look like a parabola: this is why freely-falling objects move in parabolas!
- For *any* motion with constant acceleration, the position-vs-time graph will look like a parabola over the period when $a =$ constant.
- If you want to find t such that $x(t)$ is equal to something, you'll need the *quadratic formula*

Free fall revisited



Adding these numbers together gives us 1, 4, 9, 16, 25...
The calculus above explains this: distance is proportional to *time squared!*

Free fall revisited

- Observation: distances moved in each piece of time go like (1, 3, 5, 7, 9...)
- Observation: plot on Logger Pro looks like a parabola
- Prediction from model: $x(t) = \frac{1}{2}gt^2$

Our model and our observations agree!

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What about a feather and a billiard ball?

What does this tell us about the force from gravity?

Since we see that the acceleration from gravity g is the same for all objects in free fall, this can tell us what the force from gravity is:

$$F = ma$$
$$F_g = mg$$

... in other words, the force that gravity applies to an object is proportional to its mass.

It is *not obvious* why this should be true! We'll return to this in Unit 2.