Homework 9

Due Wednesday, April 26

1. Rotational motion introduces many new terms, and in order to make sense of conversations about rotation, you should be familiar with them. In your own words and using *no mathematics*, define the following and give the dimensions of each of the following:

   (a) Angular velocity

   (b) Angular acceleration

   (c) Tangential velocity

   (d) Radial acceleration

   (e) Tangential acceleration

   (f) Torque

   (g) Moment of inertia

   (h) Angular momentum

2. The CD-ROM drive in a computer accelerates a disk from rest to its full speed of 20000 revolutions per minute; it does so uniformly over 5 seconds. The disk is 6 cm in radius.

   (a) Is it correct to write $\alpha = 4000\ \mathrm{rev} \cdot \mathrm{min}^{-1} \cdot \mathrm{s}^{-1}$? Explain.

   (b) What is the maximum angular velocity in our conventional units of radians per second?

   (c) What is its angular acceleration?

   (d) How many times does it spin during this interval?

   (e) After four seconds, what is the *tangential velocity* of a point along the edge of the disk?

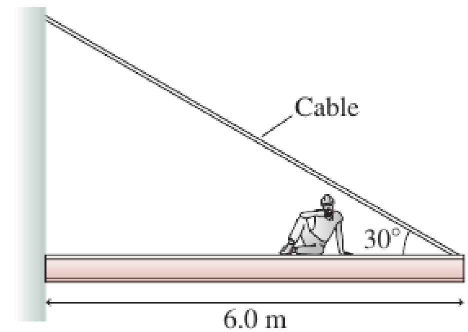
   (f) After four seconds, what is the *angular velocity* of a point along the edge of the disk?

   (g) After four seconds, how many times has the disc rotated?

3. Suppose that you want to hold a meter stick horizontal to the ground by touching it with only two fingers. One finger is at the 10 cm mark, while the other finger is at the 20 cm mark. The meter stick has a weight of 1 N. What must you do with your fingers, and what normal forces do they exert on the meter stick? What if your fingers are at the 10 cm and 12 cm marks?

   If you are having trouble with this problem, go get a meter stick and physically play with it. We will put a meter stick in the Physics Clinic for people to play with.

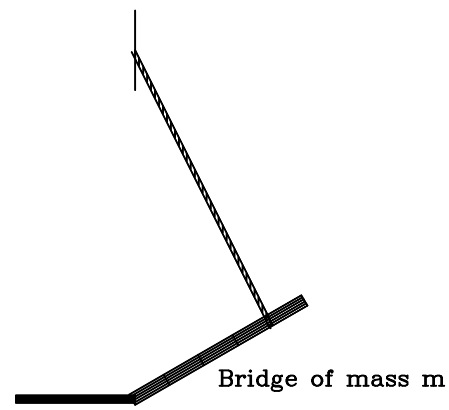
4. A construction worker takes a break for lunch, resting on a steel beam.



The steel beam has a mass of 1450 kg, and the construction worker shown has a mass of 80 kg. They sit 3.2 meters from the end of the beam. The cable is rated to transmit a tension of 15 kN.

Should they be worried that the cable might fail?

5. A castle's drawbridge consists of a heavy wooden plank that can be opened or closed using a rope, as shown below:



The rope is attached to the door 4/5 of the way down its length; the left side of the plank is attached to a hinge in the ground. The bridge is left in a partially-raised state by a careless sentry. It makes an angle of 30° with the horizontal; the rope that supports it is perpendicular to the bridge.

If the mass of the door is m, calculate the tension T in the rope in terms of θ, m, and g.

6. A person rolls a basketball ($I = \frac{2}{3}mr^2$) down a slope; the slope is inclined at an angle $\theta = 15°$. The basketball rolls without slipping down the slope.

   At the same time, another person slides a piece of ice down the slope. There *is* friction between the ice and the slope – just not very much.

   What must the coefficient of kinetic friction μ_k between the ice and the slope be such that the piece of ice and the basketball have the same speed when they reach the bottom?

7. An large old iron wheel of mass m, still connected to its axle, has been abandoned by a roadside. A person wants to move it, so they tie a rope to the axle, set the wheel up on its edge, and pull horizontally with a tension T. There is friction between the wheel and the ground, and the person pulls gently enough that the wheel rolls without slipping.

   (a) Will the frictional force between the wheel and the ground be $\mu_k F_N$, $\mu_s F_N$, or some other value? If it is some other value, you may leave its value as F_f in your equations; it will cancel in the end.

   (b) Determine, in terms of m, T, g, and μ, the acceleration of the wheel. (Your result may not depend on all of these.)

   (c) Determine, in terms of m, T, g, and μ, the frictional force between the wheel and the ground. (Your result may not depend on all of these.)

8. *(This problem is worth five points extra credit if you complete it fully, so long as the answers you give are authentically based on your understanding.)*

   A cylinder ($I = \frac{1}{2}mr^2$) rolls without slipping down an inclined plane of length L, angled at a small angle θ to the horizontal. The coefficient of static friction between them is $\mu_s = 0.5$.

   (a) Draw a (large) extended force diagram for the ball. Consider carefully in which direction the frictional force at the point of contact points.

   (b) Is the frictional force equal to $\mu_s F_N$, or some other value that depends on the angle θ? Explain why, thinking about what happens as $\theta \to 0$. (If the frictional force is not equal to $\mu_s F_N$, just use F_f to represent it in equations until you determine what it is.)

   (c) The ordinary "translational" work-energy theorem says that a force $\vec{F}$ acting over a displacement $\vec{d}$ causes a change in translational kinetic energy: $\Delta(\frac{1}{2}mv^2) = \vec{F} \cdot \vec{d}$. Find the "translational work" done by the gravitational force and by the frictional force.

   (d) The "rotational" work-energy theorem says that a torque τ acting over an object rotating through an angle $\Delta\theta$ causes a change in rotational kinetic energy: $\Delta(\frac{1}{2}I\omega^2) = \tau\Delta\theta$. Find the "rotational work" done by the gravitational force and by the frictional force.

   (e) If we consider "translational kinetic energy" $\frac{1}{2}mv^2$ and "rotational kinetic energy" $\frac{1}{2}I\omega^2$ as different types of energy that can be added together, what is the total work (rotational plus translational) done by the frictional force?

   (f) Is it accurate to say that the frictional force here converts translational kinetic energy into rotational kinetic energy? Explain.

   (g) Comment on the validity of using energy methods to analyze situations like (6) in which objects roll without slipping. Do you need to worry about the frictional forces when considering translational and rotational kinetic energy together?

   (h) By any method you like, calculate the size of the frictional force F_f. (There are two ways to do this. One involves energy methods; the other involves examining the force diagram and thinking about the relation between a and α.)