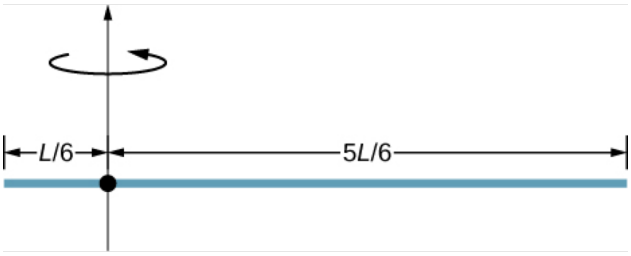
# PHY 211 Recitation

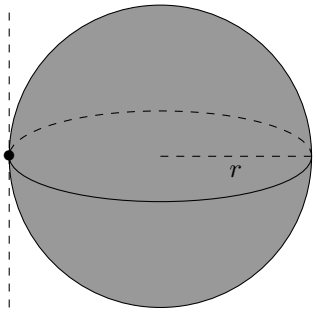
April 14, 2020

## 1 Moments of inertia

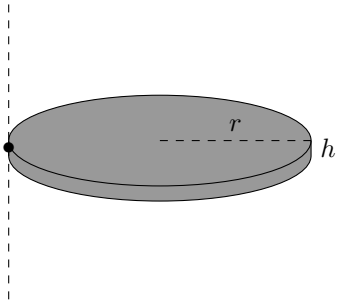
**Problem 1(a).** What is the moment of inertia of the rod shown below around the axis indicated?




**Problem 1(b).** What is the moment of inertia of a sphere of mass $m$ and radius $r$ rotating about a point on its surface?

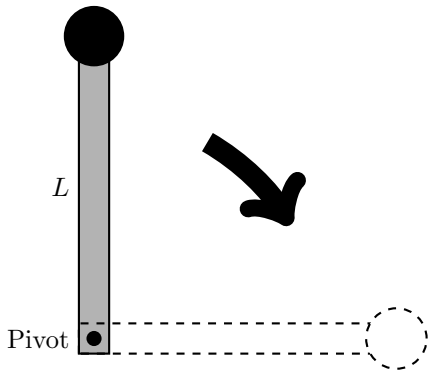



**Problem 1(c).** What is the moment of inertia of a circular plate of mass $m$, radius $r$, and thickness $d$ rotating about a point on its edge?



## 2

A ball of mass $M$ is on the end of a rod of length $L$ (assume the rod is massless compared to the ball). At the other end of the rod, it is attached to a pivot. The rod is released from rest in a vertical position, and allowed to fall to one side. You will find the angular speed of the ball and rod at the instant it is horizontal.




(a) What is the moment of inertia of the system in terms of $M$ and $L$?

(b) What is the initial kinetic energy?

(c) What is the final kinetic energy in terms of $M$, $L$, and $\omega_f$?

(d) What are the initial and final potential energies $U_i$ and $U_f$?

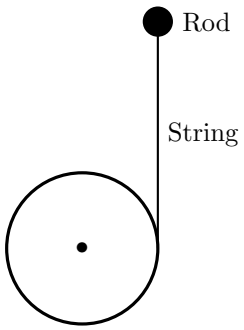
(e) Why is the energy of the ball conserved in this case?

(f) Write down energy conservation using the values you derived.

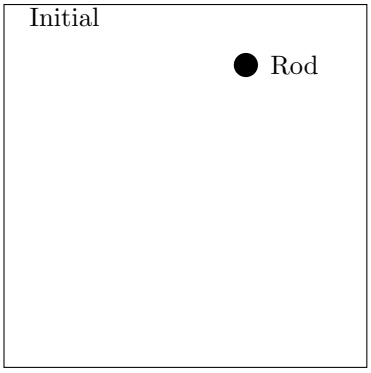
(g) Calculate the final angular velocity from this equation in terms of $g$ and $L$.

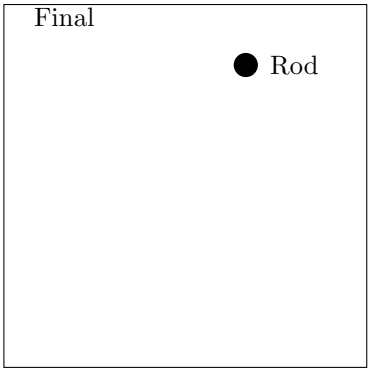
## 3

A string is wound around a spool of mass $M$ (treat this as a cylinder of radius $R$), with the free end attached to a fixed rod as in the figure. The spool is then allowed to fall. You will find the speed of the spool as the string unwinds as it falls, spinning around its center of mass.




(a) Sketch the spool and string at the initial position, and at a final position after the spool has fallen a distance $h$




(b) What is the initial kinetic energy of the spool?

(c) What is gravitational potential energy before and after it falls?

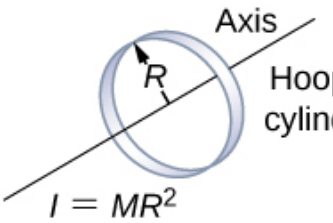
(d) The final kinetic energy is a sum of two contributions – one from the linear motion of the center of mass, and one from rotation about the center of mass. Express the final linear kinetic energy in terms of $M$ and the unknown $v_f$.

(e) What is the moment of inertia $I$ of the spool about its center of mass?

(f) What is the kinetic energy of rotation in terms of $I$ and the unknown $\omega_f$?

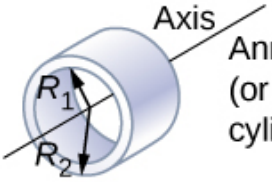
(g) What is the relationship between $v_f$ and $\omega_f$?

(h) Using conservation of energy, solve for $v_f$.



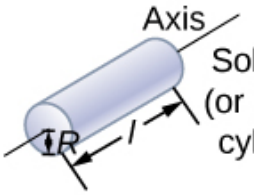

Hoop about cylinder axis

$I = MR^2$



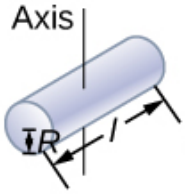

Annular cylinder (or ring) about cylinder axis

$I = \frac{M}{2}(R_1^2 + R_2^2)$



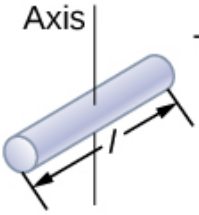

Solid cylinder (or disk) about cylinder axis

$I = \frac{MR^2}{2}$



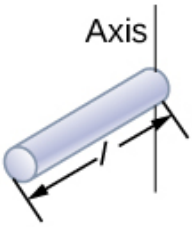

Solid cylinder (or disk) about central diameter

$I = \frac{MR^2}{4} + \frac{Ml^2}{12}$



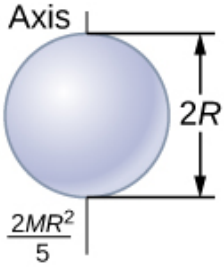

Thin rod about axis through center ⊥ to length

$I = \frac{Ml^2}{12}$



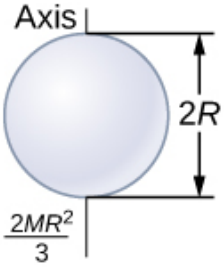

Thin rod about axis through one end ⊥ to length

$I = \frac{Ml^2}{3}$



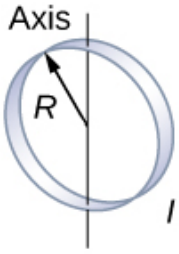

Solid sphere about any diameter

$I = \frac{2MR^2}{5}$



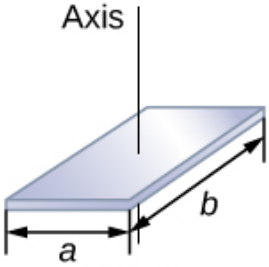

Thin spherical shell about any diameter

$I = \frac{2MR^2}{3}$




Hoop about any diameter

$I = \frac{MR^2}{2}$




Slab about ⊥ axis through center

$I = \frac{M(a^2 + b^2)}{12}$