PHY 211 Lecture 14

Matthew Rudolph

Syracuse University

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Congratulations!

- So you've learned Newton's laws – that means you're done with mechanics right?
- In principle, everything we do this entire semester you could solve with $\vec{\mathbf{F}}(t)$ and enough time
- This was one of the high points in terms of complexity this term
- The goal now is to learn ways to simplify complex problems

Pre-lecture question 1

- A small mass of 0.5 kg is traveling with a speed of 4 m/s, while a large mass of 2 kg is traveling with a speed of 1 m/s. Which has more momentum?

(a) The smaller mass (b) The smaller mass (c) They have the same

Pre-lecture question 2

- Can individual objects in a system have momentum while the momentum of the combined system is zero?

  (a) Yes (b) No

Pre-lecture question 3

- A piece of putty and a tennis ball with the same mass are thrown against a wall with the same velocity. Which object experience a greater impulse from the wall or are the impulses equal?

(a) Putty (b) Ball (c) Equal

Impulse

- Impulse is just a fancy word for the change in momentum
- When a force acts over time, the acceleration changes velocity and momentum

Impulse

$$\vec{\mathbf{J}} = \vec{\mathbf{F}}\Delta t = m\vec{\mathbf{a}}\Delta t$$
$$= \left(m\frac{\Delta\vec{\mathbf{v}}}{\Delta t}\right)\Delta t$$
$$= \Delta\vec{\mathbf{p}} = p_f - p_i$$

Impulse vector

- Remember that impulse is a vector, so just like with forces you always have to write down at least two separate equations:

$$J_x = p_{f,x} - p_{i,x}$$
$$J_y = p_{f,y} - p_{i,y}$$

- Also need to be careful about the sign of a p component

Impulse is important!

- When we did force problems we were always talking about one instant
  - Or maybe constant acceleration over some interval
- But how long forces are applied matters

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- But how long forces are applied matters
- Can I pull the table cloth out from under the dishes?

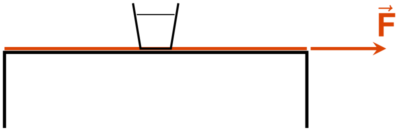
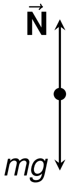


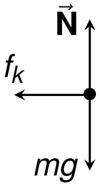
Table cloth question

- Assume I pull the table cloth to the right with enough force that it slides out from underneath the plate. Which free body diagram shows the forces on the plate?

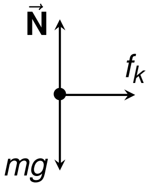
(a)



(b)



(c)



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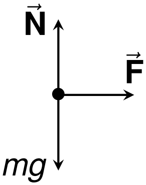

Why momentum?

- Momentum is conserved
- This is just Newton's third law again
- Object 1 exerts a force on object 2 for some time:

$$F_{1\text{ on }2}\Delta t = \Delta p_2$$

- Object 2 exerts an equal and opposite force back on object 1

$$F_{2\text{ on }1}\Delta t = \Delta p_1 = -F_{1\text{ on }2}\Delta t$$

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- Switching to integrals it works even for complicated time dependent forces

Question

- What happens when you sit on the cart and throw the ball forward?
  - (a) You move forward
  - (b) You stay still
  - (c) You move backward

Question

- What happens when you sit on the cart and catch the ball?
  (a) You move forward
  (b) You stay still
  (c) You move backward

Conservation of momentum

$$\Delta p_1 = -\Delta p_2$$
$$\Delta(p_1 + p_2) = 0$$
$$p_{1,i} + p_{2,i} = p_{1,f} + p_{2,f}$$

- The combined momentum of the two objects does not change from before the force to after
- This is conserved as long as no external force is changing the momentum
- You don't have to keep track of the exact magnitudes and directions of the forces between the objects

Conservation of momentum in 2D

- x and y have separate momentum equations

$$p_{x1,i} + p_{x2,i} = p_{x1,f} + p_{x2,f}$$
$$p_{y1,i} + p_{y2,i} = p_{y1,f} + p_{y2,f}$$

- They can be separately conserved: it depends which direction external forces point
- You may conserve in x but not in y!

Colliding cars

- What happens when they collide with the same mass?

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- What happens when they stick together?

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Colliding cars

- What happens when they collide with the same mass?
- What happens when they stick together?
- What happens when one is heavier?
- What if they are moving at the same speed?

Colliding cars

- Would you want to sit down and try to figure out the details of all the forces for each one of those collisions?
- There's a lot of seemingly different outcomes
- But they all conserve momentum

Conservation of momentum

Strategy

1. Identify the interacting systems of interest (very important which you choose!)
2. What external forces could act on the combined system?
3. Define a coordinate system and decide:
   - Is there a net external force in the x direction? If not, p_x is conserved
   - Is there a net external force in the y direction? If not, p_y is conserved
4. Write expressions for $p_{i,x}$, $p_{i,y}$, $p_{f,x}$, and $p_{f,y}$
5. Solve for the desired quantity

Example

A $0.15\,\mathrm{kg}$ projectile is fired with a velocity of $715\,\mathrm{m/s}$ at a $2\,\mathrm{kg}$ wooden block that rests on a frictionless table. The velocity of the block, immediately after the projectile passes completely through it, is $40\,\mathrm{m/s}$ in the same direction as the projectile. Find the velocity with which the projectile exits the block.

Example continued

- What is the impulse applied to the block?
- What is the impulse applied to the projectile?
- If the projectile takes 0.2 ms to pass through the block, what is the average force applied to it? Does the force have to be constant?

Example II

Two hockey pucks are sliding on the ice. One is travelling left to right with a speed of $10\,\mathrm{m/s}$, while the other is travelling perpendicularly with a speed of $5\,\mathrm{m/s}$. This second puck collides with the side of the first puck and comes to rest. What is the velocity of the first puck after the collision?