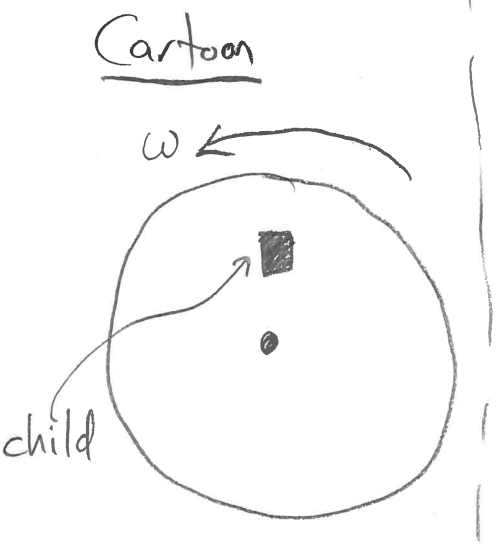
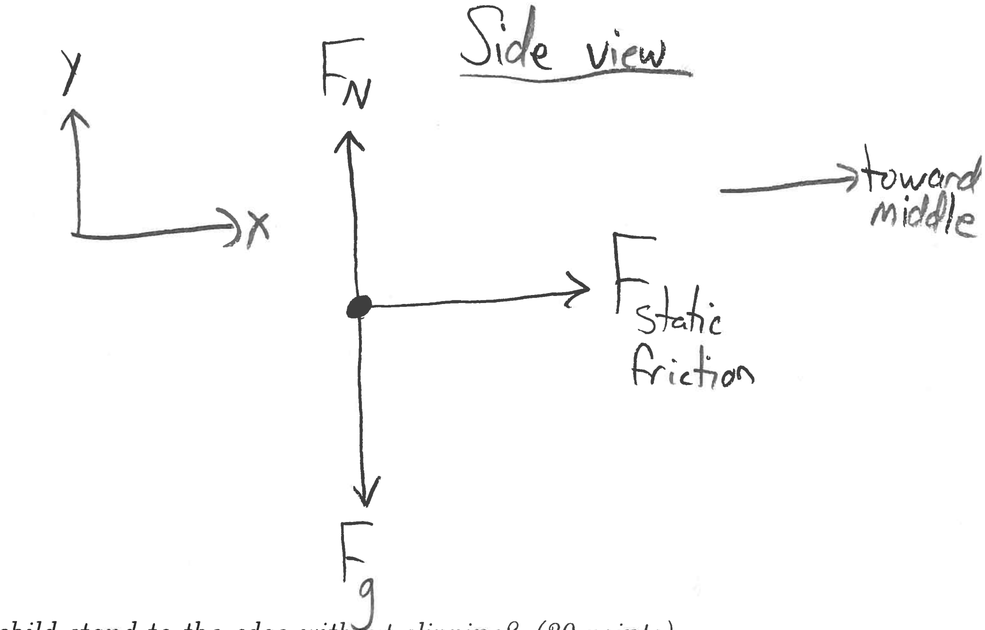
Physics 211 Practice Exam 2
Question 1

A "merry-go-round" is a large, horizontal platform free to rotate around its axis. Children can stand on top of the platform while it spins. Suppose that a merry-go-round with a radius of 3 meters is spinning, and that it rotates around its axis once every 4 seconds.

Suppose that the coefficient of kinetic friction μ_k between the children's feet and the platform is 0.4, while the coefficient of static friction μ_s between their feet and the platform is 0.5.

a) Draw a force diagram for a child standing on the platform. Indicate your choice of coordinate system. (10 points)





b) How close to the edge can a child stand to the edge without slipping? (30 points)

Since the needed centripetal acceleration is $\omega^2 r$, this is larger as r increases. If r becomes too large, the needed centripetal force will exceed the maximum frictional force and the will slide.

$$X: F_{sf} = ma_x \Rightarrow \mu_s F_N = m\omega^2 r$$

$$Y: F_N - mg = ma_y \Rightarrow a_y = 0 \text{ so } F_N = mg.$$

$$\text{Substitute: } \mu_s \cancel{m} g = \cancel{m}\omega^2 r \rightarrow \boxed{r = \frac{\mu_s g}{\omega^2}.}$$

$$\omega = \frac{1}{4}\frac{\text{rev}}{\text{sec}} = \frac{2\pi}{4}\frac{\text{rad}}{\text{sec}}.$$

$= 2.03$ m from center

which is 0.97 m from e

c) Suppose now that the children spinning the platform want to slow it down enough that their friends on top can safely walk to the edge and jump off. What is the maximum angular velocity ω that would allow a child to stand on the edge of the platform without slipping? (10 points)

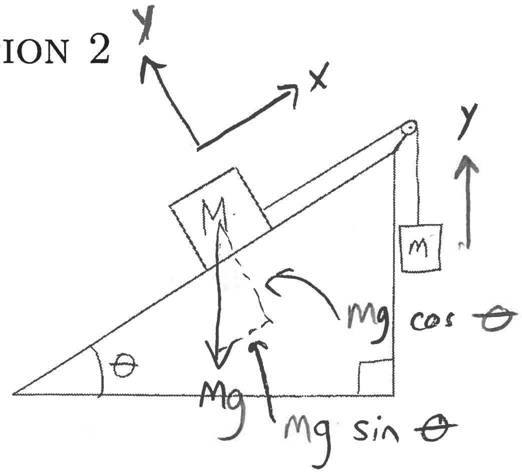
From previous part, we have the relation of the maximum safe radius r to ω and μ_s.

Set r to 3m:

$$r = \frac{\mu_s g}{\omega^2} \Rightarrow \omega = \sqrt{\frac{\mu_s g}{r}} = 1.29 \text{ rad/sec.}$$

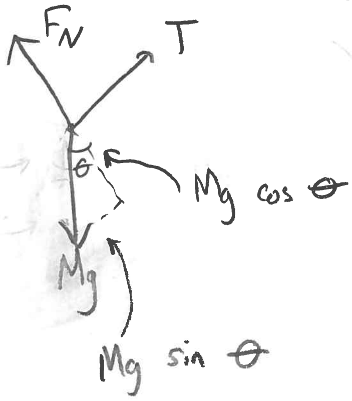
Question 2

A book of mass M sits on an inclined plane angled at an angle θ above the horizontal; it is connected by a string to another book of mass m hanging over the top. (See picture.)



a) In terms of M and m, what must the angle θ be such that the two books do not move? Assume for this part that there is no friction. (10 points)

M:



m:

$$X: T - Mg \sin\theta = M a_{1x} = 0$$

$$Y: F_N - Mg\cos\theta = M a_{1y} = 0 \Rightarrow F_N = Mg\ c$$

$$\text{Hanging: } T - mg = m a_{2y} = 0 \Rightarrow T = mg$$

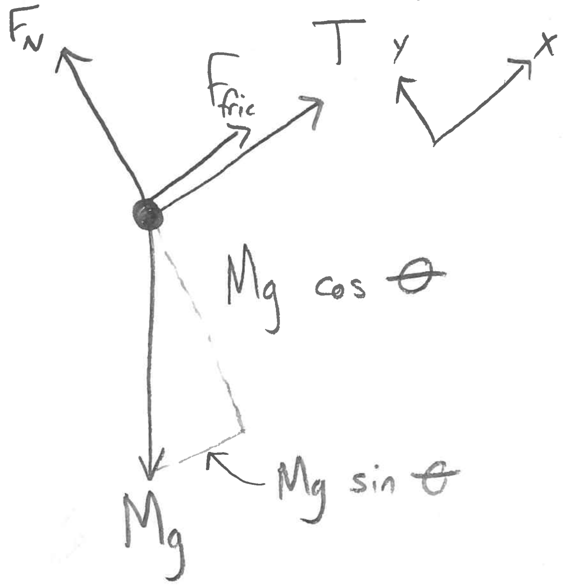
$$\text{Substitute: } mg - Mg\sin\theta = 0$$

$$\sin\theta = \frac{m}{M},\quad \theta = \arcsin\frac{m}{M}.$$

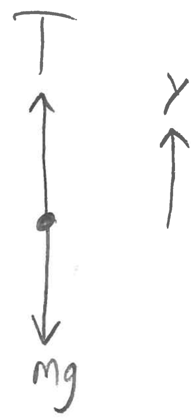
Now, assume that M is large enough that it slides down the ramp. There is kinetic friction between that book and the ramp; the coefficient of kinetic friction is μ_k.

b) Draw force diagrams for both books. Indicate your choice of coordinate system for both of them (they do not have to be the same, and in fact shouldn't be!) (6 points)

M:



m:



c) Give a mathematical statement of Newton's second law for both books, substituting in quantities specific to this problem. (In other words, don't just write $\vec{F} = m\vec{a}$; write something useful that will help you solve the remaining parts.) (10 points)

$\parallel$: $T + \mu_K F_N - Mg \sin\theta = Ma_{1x}$ (I)

$\perp$: $F_N - Mg \cos\theta = Ma_{1y}$ (II)

hanging: $T - mg = ma_{2y}$ (III)

d) What is the relationship between the acceleration of the two books? (4 points)

$a_{1y} = 0$; $a_{2y} = -a_{1x}$

e) Calculate the acceleration of both books in terms of M, m, g, θ, and μ_k. (20 points)

Use substitution.

(III): $T = mg - ma_{1x}$

(II): $F_N = Mg\cos\theta$ since $a_{1y} = 0$.

Sub into (I):

$$mg - ma_{1x} + \mu_k Mg\cos\theta - Mg\sin\theta = Ma_{1x}$$

$$\Rightarrow a_{1x} = \frac{mg + \mu_k Mg\cos\theta - Mg\sin\theta}{M+m}$$

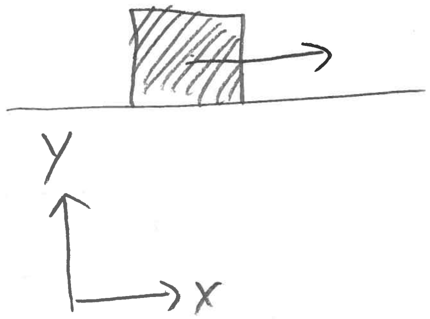
Question 3

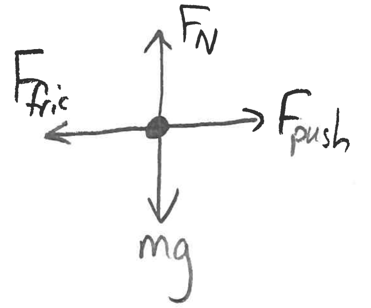
The coefficient of kinetic friction between a table of mass $m = 100$ kg and the ground is $\mu_k = 0.6$. You would like to move this table by pushing on it. (You are not trying to make the table accelerate, only to make it continue to move at a constant speed.)

Calculate the minimum force required to make the table move under the following conditions. If *no* force, no matter how large, will move the table, then say so. Note that you will want to draw force diagrams as part of your solutions to each part.

a) You push on the table horizontally, parallel to the ground. (5 points)

Note: Constant velocity $\Rightarrow \vec{a} = 0$.





$X: F_{push} - \mu_k F_N = 0$

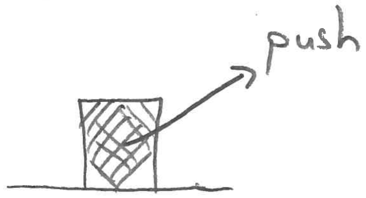
$Y: F_N - mg = 0 \Rightarrow F_N = m$

$F_{push} - \mu_k mg = 0$

$F_{push} = \mu_k mg = 600$ N.

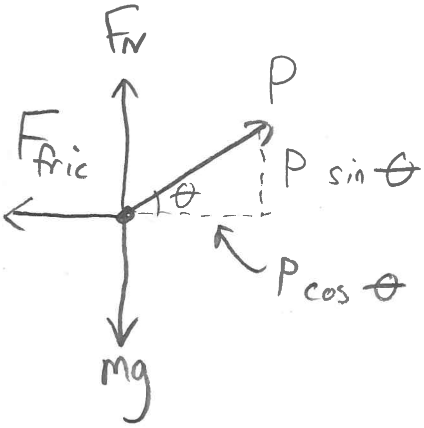
b) You push on the table at an angle directed 20 degrees above the horizontal (that is, you are pushing sideways and upward.) (5 points)

remember $\vec{a} = 0$



$X: P\cos\theta - \mu_k F_N = 0$

$Y: P\sin\theta + F_N - mg = 0 \Rightarrow F_N = mg - P\sin\theta$



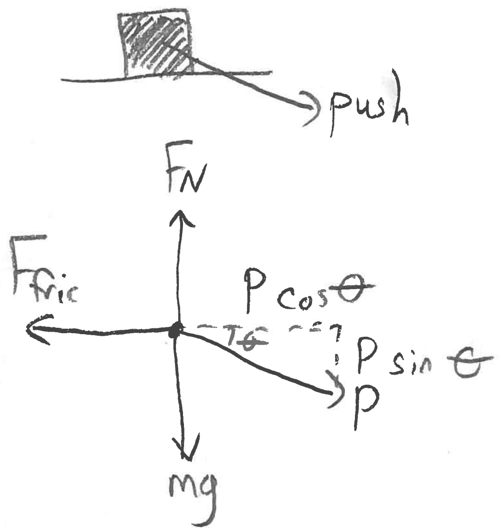
Substitute:

$P\cos\theta - \mu_k mg + \mu_k P\sin\theta = 0$

$$\Rightarrow P = \frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta} = 524 \text{ N}$$

c) You push on the table at an angle directed 20 degrees below the horizontal (that is, you are pushing sideways and downward.) (5 points)

(note $\vec{a}$ =



$X: \; P\cos\theta - \mu_K F_N = 0$

$Y: \; F_N - mg - P\sin\theta = 0 \Rightarrow F_N = mg + P$

$\to P\cos\theta - \mu_K mg - \mu_K P\sin\theta = 0$

$$\Rightarrow P = \frac{\mu_K mg}{\cos\theta - \mu_K \sin\theta} = 817 \text{ N}$$

d) You push on the table at an angle directed 60 degrees below the horizontal (that is, you are pushing a bit sideways, and mostly downward.) (5 points)

This is the same scenario as before but with $\theta = 60°$.

Here $\mu_K \sin\theta > \cos\theta$, so no positive value of P is capable of moving the table because the increased F_N creates too much friction.

e) Explain in words why your answers to parts (b) and (c) are different. (5 points)

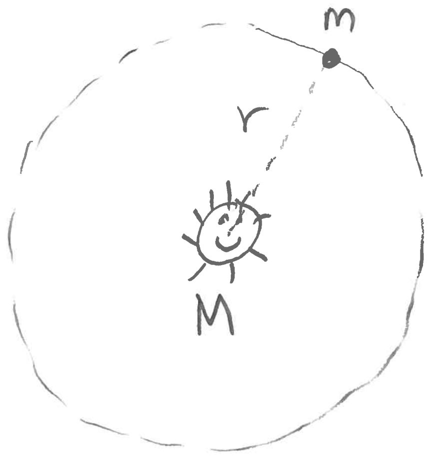
In b), the vertical component of the push reduces F_N, and thus reduces friction.

In c), it increases F_N instead, and thus increases friction.

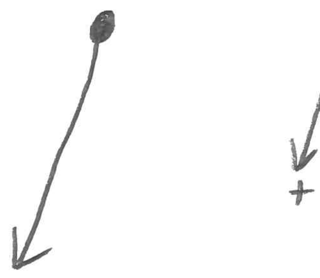
Question 4

Suppose that a planet of mass m is traveling around a star of mass M in a circular orbit with radius r. In this problem, you will determine the angular velocity ω of the planet around the star.

a) Draw a force diagram for the planet. (5 points)



Force diagram



F_{grav} (toward star)

b) Planets follow Newton's second law $\sum \vec{F} = m\vec{a}$ just like anything else. Write down an expression of Newton's second law for the planet, substituting in what you know about its acceleration and the forces acting on it. (15 points)

$$\sum F = ma$$

$$F_{grav} = ma$$

$$\frac{GMm}{r^2} = m\omega^2 r$$

distance from star to planet

QUESTION 4, CONTINUED

c) Calculate the angular velocity ω of the planet in terms of G, M, m, and r. (Your answer may not depend on all of these.) (20 points)

(from earlier) $\frac{GMm}{r^2} = m\omega^2 r$

$\omega^2 = \frac{GM}{r^3}$, $\omega = \sqrt{GM/r^3}$.

d) Both Earth and Saturn travel around the Sun in nearly circular orbits. However, the radius of Saturn's orbit is about 10 times as large as Earth's. How many times does Earth orbit the Sun during the time that it takes Saturn to orbit the Sun once? (10 points)

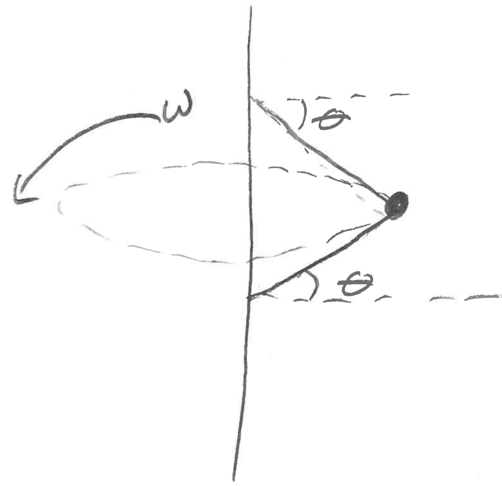
We are looking for $\frac{\omega_{earth}}{\omega_{saturn}}$.

$$= \frac{\sqrt{GM/r_e^3}}{\sqrt{GM/r_s^3}} = \sqrt{r_s^3/r_e^3} = \sqrt{(10\,r_e)^3/r_e^3} = \sqrt{1000}.$$

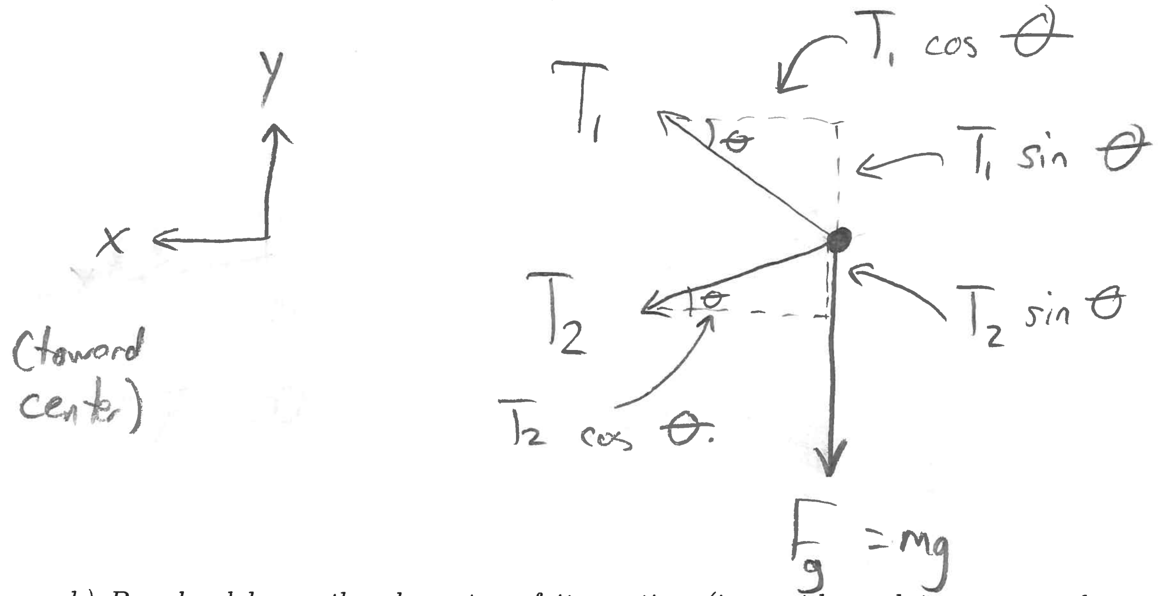
$= 31.6$ times.

Question 5

A ball of mass m is connected by two strings to a pole, as shown, and made to rotate around it at angular velocity ω. Each string makes an angle θ with the horizontal; the radius of the ball's motion is r. The tension in the upper string is T_1, while the tension in the lower string is T_2.



a) Draw a force diagram for the ball. Indicate the coordinate system you will use for this problem next to your force diagram. (5 points)



b) Based solely on the character of its motion (i.e. without doing any mathematics), describe its acceleration vector. (5 points)

$\omega^2 r$ or $\frac{v^2}{r}$ toward the center.

more useful here

c) Relate this acceleration to the forces on the object using Newton's second law. (This will entail writing one or more equations.) (5 points)

$$\sum \vec{F} = m\vec{a}$$

$$\sum F_x = ma_x : \quad T_1 \cos\theta + T_2 \cos\theta = m\omega^2 r$$

$$\sum F_y = ma_y : \quad T_1 \sin\theta - T_2 \sin\theta - mg = 0$$

$(a_x = \omega^2 r;\ a_y = 0)$

d) Calculate T_1 and T_2 in terms of θ, r, m, (and ω) and g. (5 points)

Use substitution. From X: $T_1 = \dfrac{m\omega^2 r - T_2\cos\theta}{\cos\theta} = \boxed{\dfrac{m\omega^2 r}{\cos\theta} - T_2}$

Plug into Y: $\underbrace{m\omega^2 r \tan\theta - T_2 \sin\theta}_{T_1 \sin\theta} - T_2 \sin\theta - mg = 0$

$\Rightarrow m\omega^2 r \tan\theta - mg = 2T_2 \sin\theta$, so

$$T_2 = \frac{m\omega^2 r \tan\theta - mg}{2\sin\theta}.$$

(You could stop here for full credit)

Substitute to find T_1:

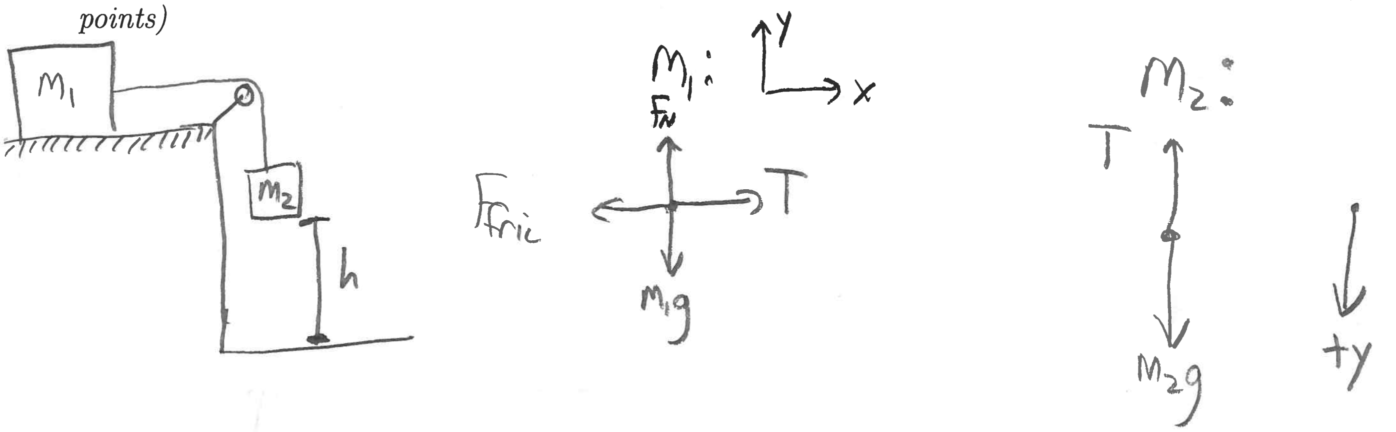
$$T_1 = \frac{m\omega^2 r}{\cos\theta} - \frac{m\omega^2 r \tan\theta - mg}{2\sin\theta}.$$

$$= \frac{m\omega^2 r}{\cos\theta} - \frac{m\omega^2 r}{2\cos\theta} + \frac{mg}{2\sin\theta} = \frac{1}{2}\frac{m\omega^2 r}{\cos\theta} + \frac{1}{2}\frac{mg}{\sin\theta}.$$

Question 6

A book with a mass of 2 kg rests on a table; the coefficient of kinetic friction μ_k between them is 0.4. A string connects that book to another book hanging vertically off the side of the table with mass 3 kg; this hanging book is 140 cm above the ground. When the hanging book is released, it accelerates toward the ground, dragging the other book on the table with it.

a) Draw a force diagram for both books. Indicate your choice of signs for the $x-$ and $y-$axes on both diagrams; that is, which directions do you consider positive, and which do you consider negative? (10 points)



b) Are the accelerations of the two books related? If so, write a mathematical relationship between them. (10 points)

Here $a_{1x} = a_{2y}$ (note coord. sys.). I'll call these both a.
and $a_{1y} = 0$.

c) Calculate the accelerations of the books and the tension in the string. (20 points)

$T - \mu_k F_N = m_1 a \Longrightarrow T - \mu_k m_1 g = m_1 a \Rightarrow T = m_1 a + \mu_k m_1 g$

$F_N - m_1 g = m_1 a_{1y} = 0 \Rightarrow F_N = m_1 g$

$m_2 g - T = m_2 a \xrightarrow{\text{Substitute}} m_2 g - m_1 a - \mu_k m_1 g = m_2 a$

and so $a = \dfrac{m_2 g - \mu_k m}{m_1 + m}$

d) With what velocity will the hanging book strike the floor? (10 points)

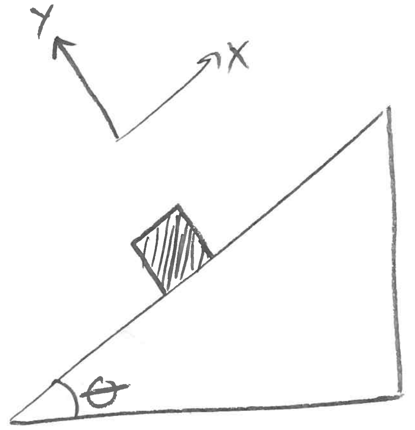
Use kinematics:

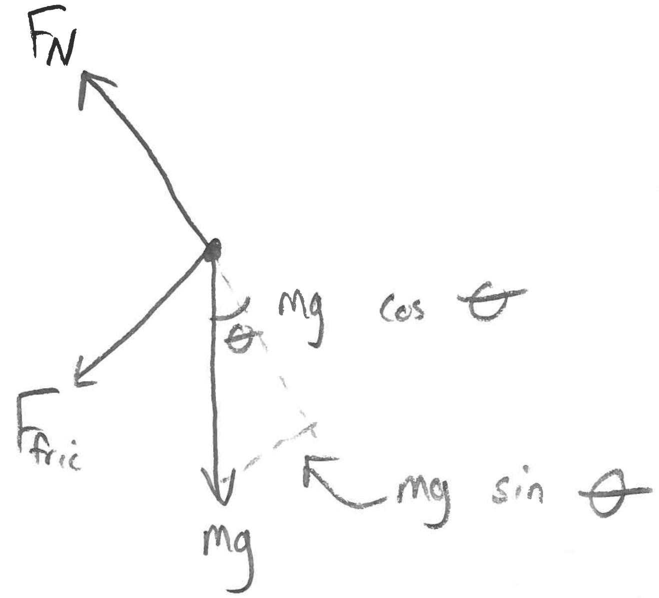
$v_f^2 - \underbrace{\cancel{v_0^2}}_{0} = 2a\Delta x \Rightarrow v_f = \sqrt{2ah}$ with a as above.

Question 7

A ramp with a small coefficient of kinetic friction μ_k is elevated at an angle of θ. An object is pushed toward the ramp. It reaches the bottom of the ramp with speed v_0; it slides up the ramp and then back down.

a) Draw a force diagram for the object on the way up. (3 points)





b) Calculate the acceleration of the object on the way up. (5 points)

$$X: -\mu_k F_N - mg\sin\theta = ma_x$$

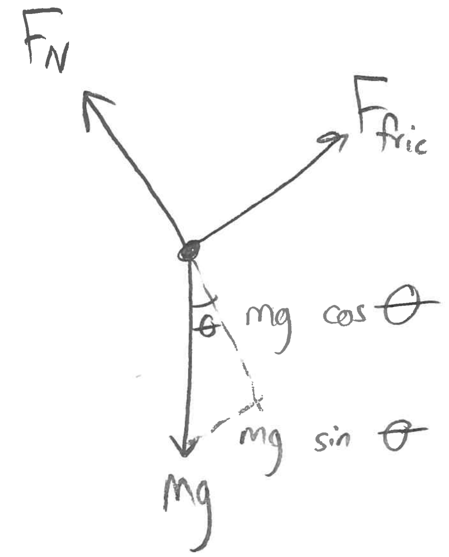
$$Y: F_N - mg\cos\theta = ma_y = 0 \Rightarrow F_N = mg\cos\theta$$

$$\rightarrow -\mu_k mg\cos\theta - mg\sin\theta = ma_x$$

$$a_x = -\mu_k g\cos\theta - g\sin\theta.$$

c) Draw a force diagram for the object on the way back down. (3 points)

All that changes is the direction of friction.



d) Calculate the acceleration of the object on the way back down. (5 points)

Same as before except for one sign:

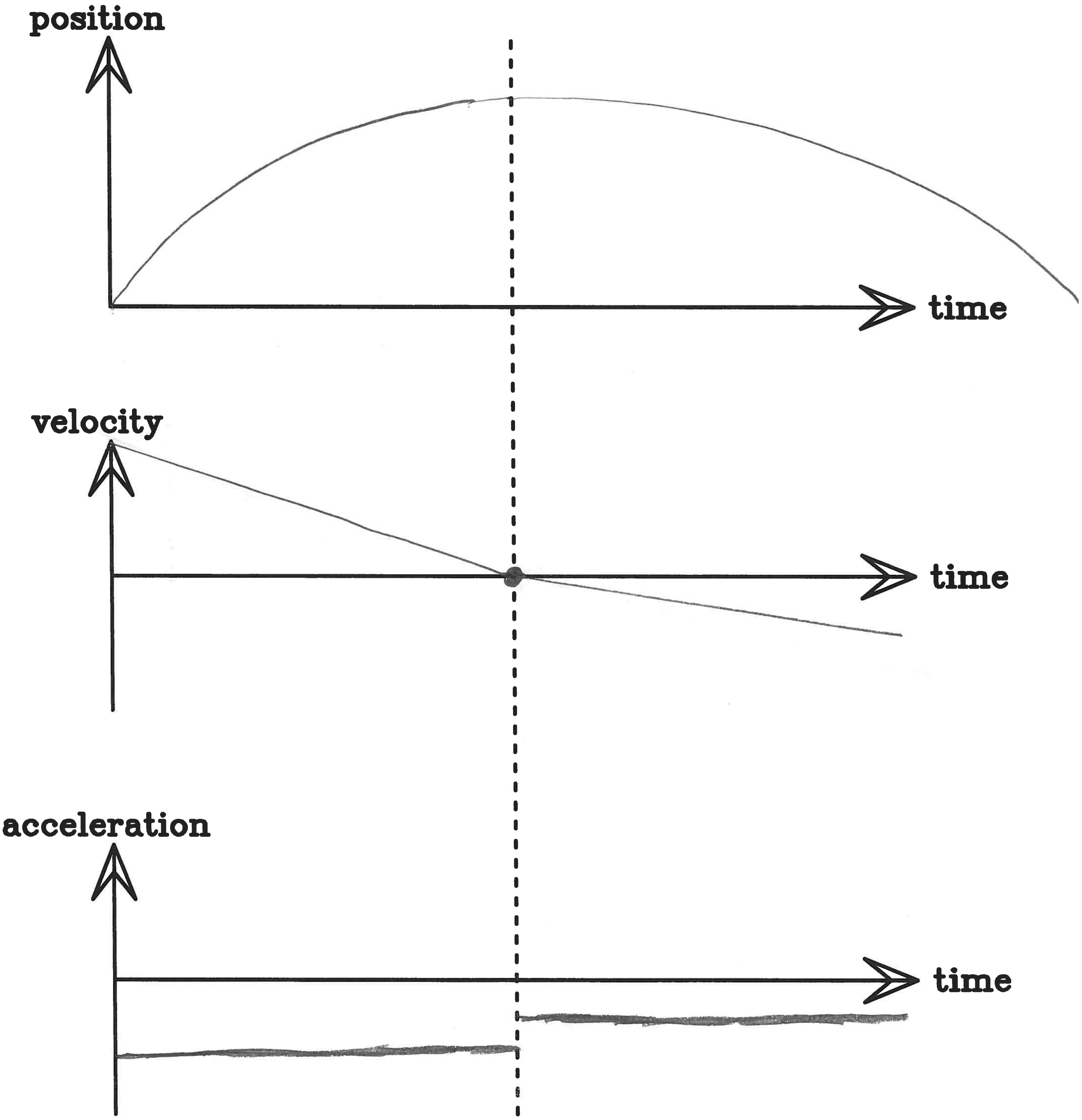
$X: +\mu_K F_N - mg\sin\theta = ma_x$

$Y: F_N - mg\cos\theta = ma_y = 0 \Rightarrow F_N = mg\cos\theta$

$\Rightarrow +\mu_K mg\cos\theta - mg\sin\theta = ma_x$

$\Rightarrow a_x = +\mu_K g\cos\theta - g\sin\theta.$

e) Sketch graphs of the object's position, velocity, and acceleration vs. time. Since I have not given you any numbers, I am interested only in the shape of your graphs. The dotted line represents the time at which the object reaches the top of the ramp and begins to come back down. (9 points)



Question 8

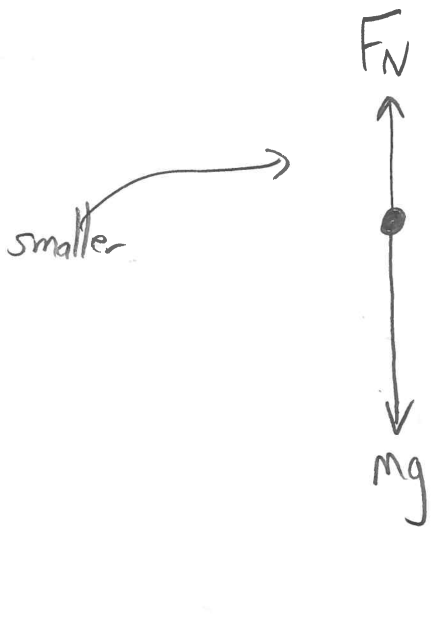
A person is standing in a subway car, looking forward. She is not holding onto anything, trusting the friction between her shoes and the ground to keep her balance.

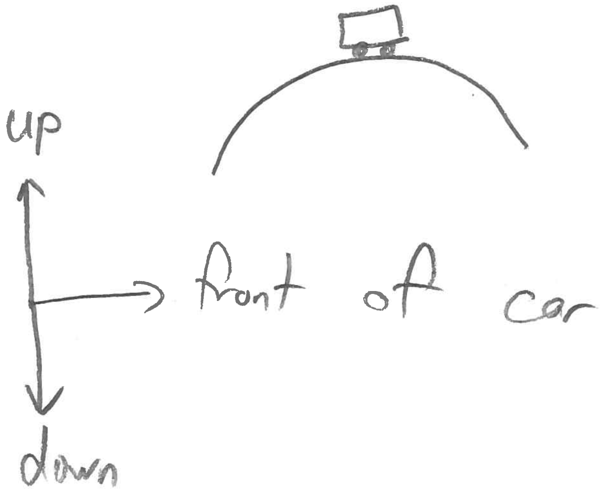
Draw force diagrams for the following situations. Make sure you indicate which direction is which (i.e. tell me whether I am looking at the person from above, from the side, etc., and which direction is toward the front of the subway car.) Indicate the relative sizes of the forces by the lengths of the arrows in your force diagram. Forces that have the same magnitude should have the same size arrows; if you think it's not clear, you can write a little text telling me which forces are larger, smaller, or equal.

a) The subway car is moving forward at a constant velocity $\vec{v}$. (5 points)

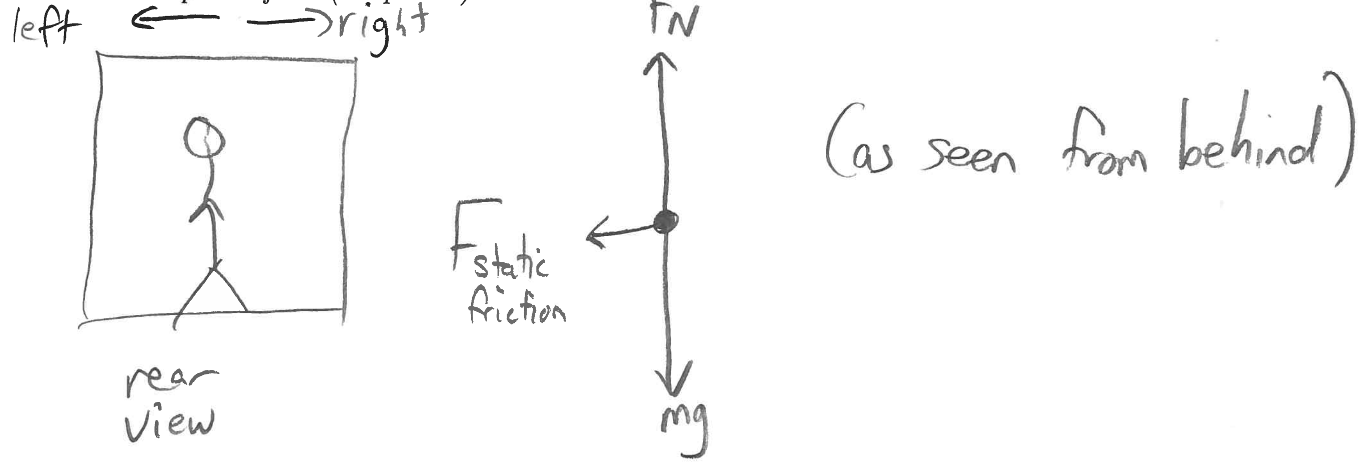
(seen from side)

b) The subway car is going over the top of a hill, and is accelerating straight downward at 3 $\mathrm{m/s^2}$. (10 points)

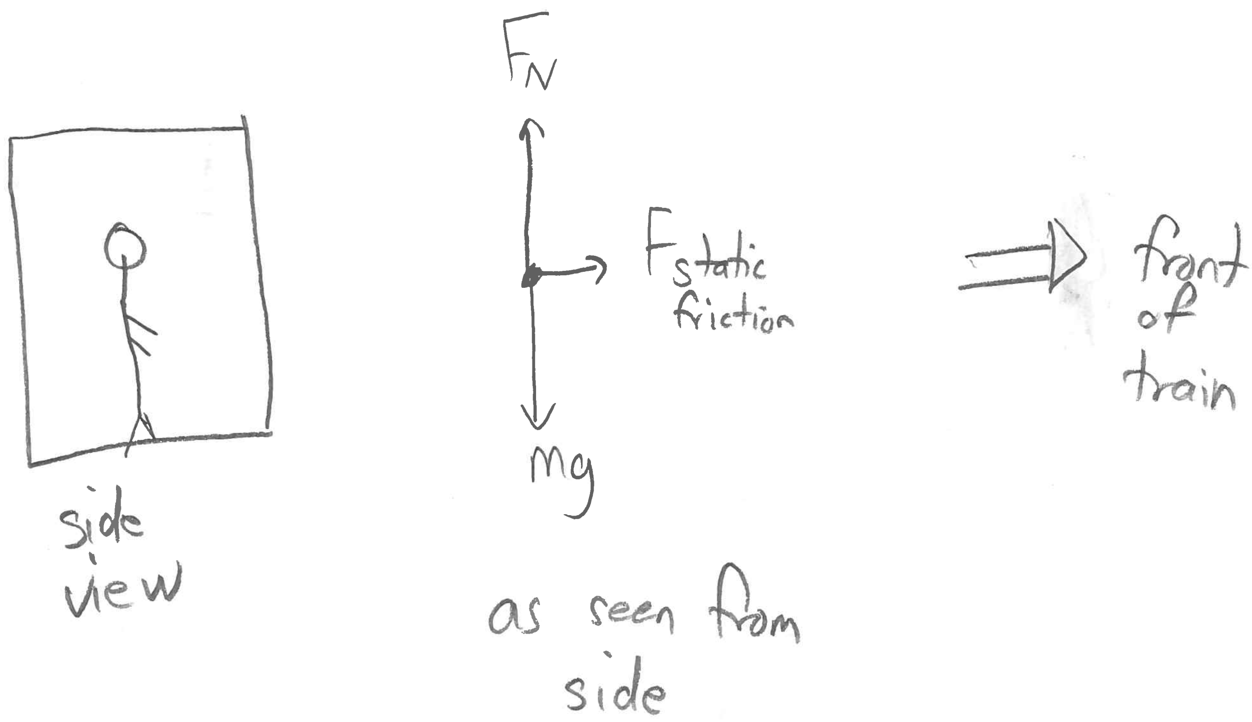




c) The subway car is moving at a constant speed v; it is turning left, gently enough that the passengers do not slip and fall. (10 points)



d) The subway car is accelerating forward at 3 m/s². (10 points)



e) Anyone who has ridden a subway car feels themselves "thrown backwards" when it accelerates forward. What force is pushing them backwards? (If there is no such force, then explain why they feel themselves thrown backwards when the car accelerates.) (15 points)

There is no such force.

You are not being thrown backwards - instead, everything around you is accelerating forwards.