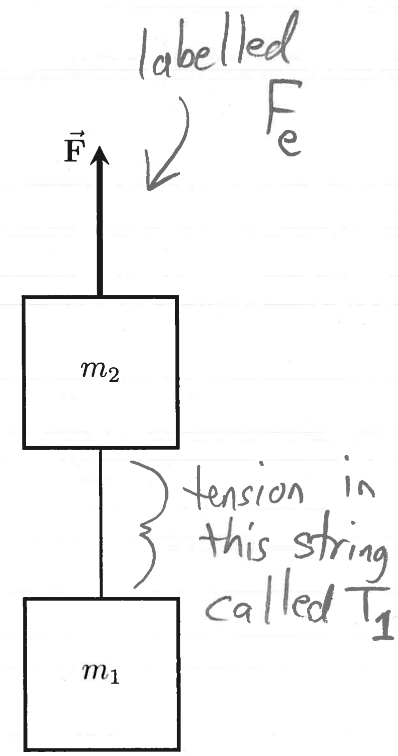
Physics 211 Practice Exam 2

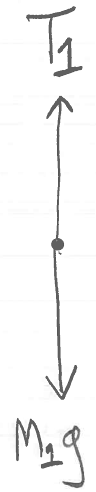
- Problem 1 is a basic problem that involves two objects moving in one dimension.
- Problem 2 tests your ability to think clearly about force diagrams and motion in the presence of acceleration.
- Problem 3 involves an object with forces in two dimensions, including friction.
- Problem 4 involves two objects connected together, one of which has forces in two dimensions.
- Problem 5 is a "backwards problem" in which we give you a flawed solution; you must fix it.
- Problem 6 involves uniform circular motion and friction.
- Problem 7 tests your ability to think clearly about uniform circular motion both from the perspective of the rotating object and from outside. A subsequent part asks you to think about connected objects in circular motion.
- Problem 8 is another "backwards problem" in which we give you a flawed solution.

Question 1

Two blocks of masses m_1 and m_2 are connected together vertically by strings. A line connected to the upper block is used to pull upward with a force F_e.



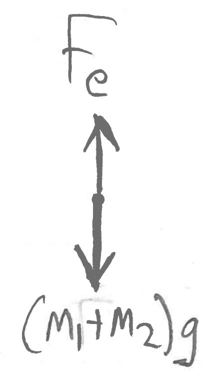
a) Draw free body diagrams for the blocks separately. Make sure to label each arrow with a symbol that identifies what it is. (6 points)



block 1

block 2

b) Draw a free body diagram for the two blocks as one system. (4 points)



both blocks together

c) What is the acceleration of the system? (8 points)

Use FBD for combination: (use m_1+m_2 as mass!)

$\Sigma F = ma$: $F_e - (m_1+m_2)g = (m_1+m_2)a$.

$$\boxed{\longrightarrow a = \frac{F_e - (m_1+m_2)g}{m_1+m_2}.}$$

d) What is the tension in the string connecting block 1 and block 2? (7 points)

Now we need the individual FBD's:

$\sum F = ma$ for each object separately:

Block 1: $T_1 - m_1 g = m_1 a$

Block 2: $F_e - m_2 g - T_1 = m_2 a$.

From (1): $a = \frac{T_1 - m_1 g}{m_1}$

Substitute into (2):

$$F_e - m_2 g - T_1 = (m_2 T_1 - m_2 m_1 g)/m_1$$

Solve for T_1:

$$m_1 F_e - \cancel{m_1 m_2 g} - T_1 m_1 = m_2 T_1 - \cancel{m_2 m_1 g}$$

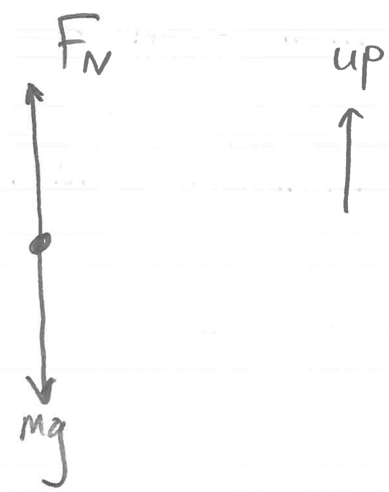
$$\boxed{T_1 = \frac{m_1 F_e}{m_1 + m_2}}$$

Question 2

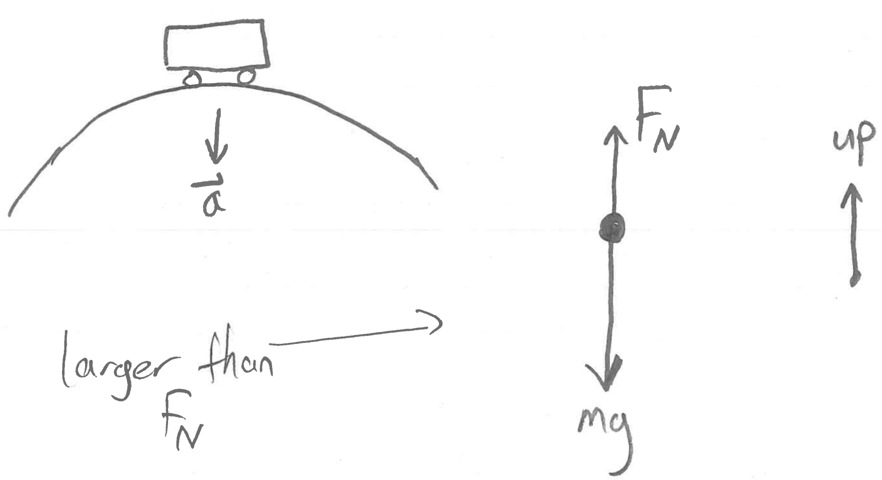
A person is standing in a subway car, looking forward. She is not holding onto anything, trusting the friction between her shoes and the ground to keep her balance.

Draw force diagrams for the following situations. Make sure you indicate which direction is which (i.e. tell me whether I am looking at the person from above, from the side, etc., and which direction is toward the front of the subway car.) Indicate the relative sizes of the forces by the lengths of the arrows in your force diagram. Forces that have the same magnitude should have the same size arrows; if you think it's not clear, you can write a little text telling me which forces are larger, smaller, or equal.

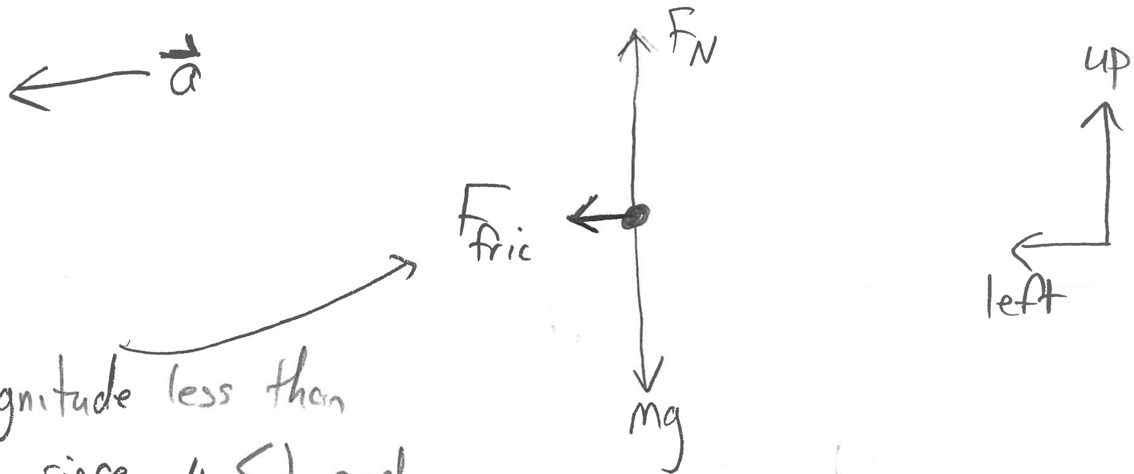
a) The subway car is moving forward at a constant velocity $\vec{v}$. (5 points) $(\vec{a} = 0)$



b) The subway car is going over the top of a hill, and is accelerating straight downward at 3 m/s^2*. (5 points)*

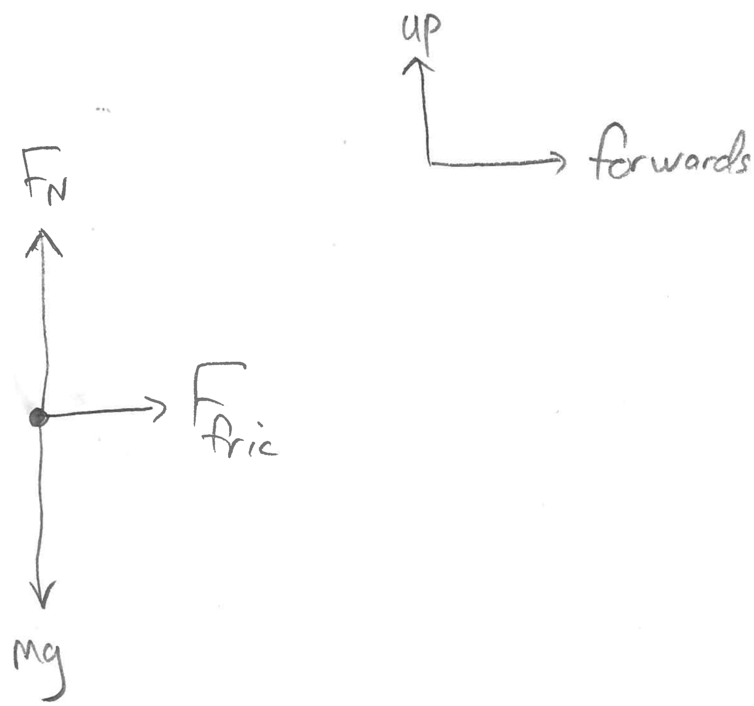


c) The subway car is moving at a constant speed v; it is turning left, gently enough that the passengers do not slip and fall. (5 points)



magnitude less than F_N since $\mu_s < 1$ and $F_{fric} \leq \mu_s F_N$.

d) The subway car is accelerating forward at $3\,\mathrm{m/s^2}$. (5 points)



e) Anyone who has ridden a subway car feels themselves "thrown backwards" when it accelerates forward. What force is pushing them backwards? (If there is no such force, then explain why they feel themselves thrown backwards when the car accelerates.) (5 points)

No force is pushing them backwards.
Instead, the room around them is accelerating forwards, and without a force to pull them forwards, the car around them will accelerate forwards without them; eventually the back of the car will strike them.

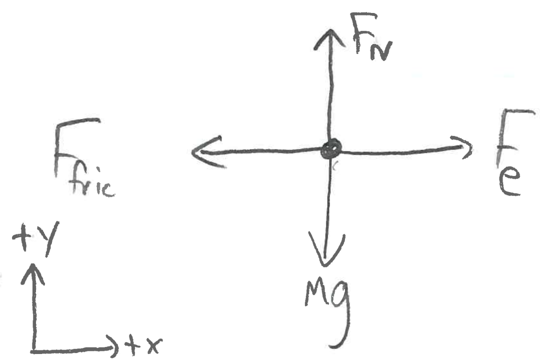
Question 3

The coefficient of kinetic friction between a table of mass $m = 100$ kg and the ground is $\mu_k = 0.6$. You would like to push this table across the floor at a constant speed.

Calculate the minimum force required to keep the table moving across the floor at a constant speed under each of the following conditions. If *no* force, no matter how large, will move the table, then say so. Note that you will want to draw force diagrams as part of your solutions to each part.

a) You push on the table horizontally, parallel to the ground. (5 points)

(constant speed)



$\sum F_x = F_e - F_{fric} = ma_x = 0$

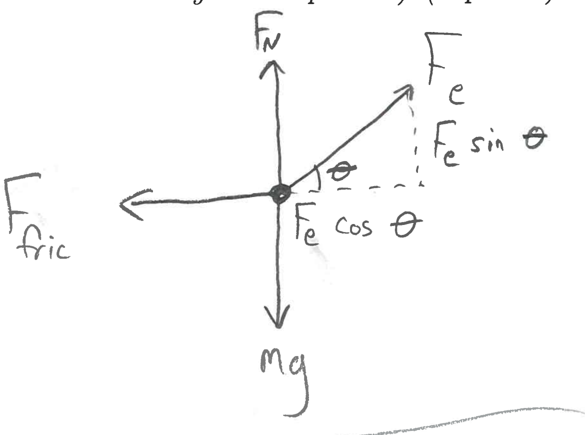
$\sum F_y = F_N - mg = ma_y = 0.$

$\rightarrow$ Look at (y) to find F_N, then use that with $F_{fric} = \mu F_N$.

(y) $F_N - mg = 0 \Rightarrow F_N = mg$. (x) becomes: $F_e - \mu F_N = 0 \Rightarrow F_e - \mu mg = 0$,

so $F_e = \mu mg = (0.6)(100 \text{ kg})(10 \text{ m/s}^2) = 600$ N.

b) You push on the table at an angle directed 20 degrees above the horizontal (that is, you are pushing sideways and upward.) (5 points)



$X: F_e \cos\theta - \underbrace{\mu F_N}_{F_{fric}} = ma_x = 0$

$Y: F_N - mg + F_e \sin\theta = ma_y = 0.$

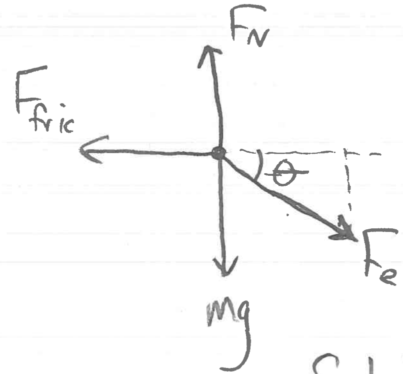
From (Y): $F_N = mg - F_e \sin\theta$.

Substitute into (X): $F_e \cos\theta - \mu(mg - F_e \sin\theta) = 0.$

$F_e \cos\theta - \mu mg + \mu F_e \sin\theta = 0 \Rightarrow \boxed{F_e = \dfrac{\mu mg}{\cos\theta + \mu \sin\theta}.}$

$= 524$ N.

c) You push on the table at an angle directed 20 degrees below the horizontal (that is, you are pushing sideways and downward.) (5 points)



Nearly identical to (b) except for sign of F_{e_y}.

(X): $F_e \cos\theta - \mu F_N = 0$

(Y): $F_N - F_e \sin\theta - mg = 0 \Rightarrow F_N = mg + F_e \sin\theta.$

Substitute:

(X) becomes $F_e \cos\theta - \mu mg - \mu F_e \sin\theta = 0.$

$$\Rightarrow F_e = \boxed{\frac{\mu mg}{\cos\theta - \mu \sin\theta}} = 817\,N.$$

d) You push on the table at an angle directed 60 degrees below the horizontal (that is, you are pushing a bit sideways, and mostly downward.) (5 points)

Same as part (c) except $\theta = 60°$ now.

$$F_e = \frac{\mu mg}{\cos\theta - \mu \sin\theta} = -30588\ N\ (!)$$

→ large negative value means <u>no</u> F_e can move it: increase in F_N will increase friction faster than F_{e_x} as F_e increases.

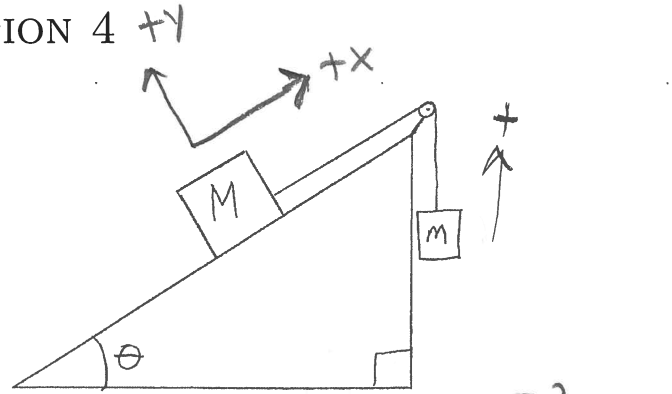
e) Explain in words why your answers to parts (b) and (c) are different. (5 points)

In (b), we are pushing <u>up</u>, reducing F_N and thus F_{fric}.

In (c), we are pushing <u>down</u>, increasing F_N and thus F_{fric}.

QUESTION 4

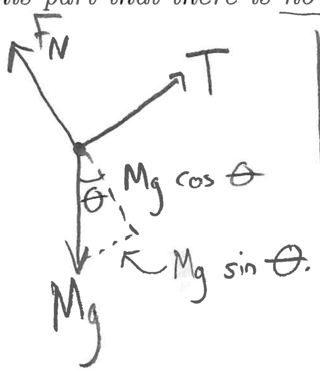
A block of mass M sits on an inclined plane angled at an angle θ above the horizontal;
it is connected by a string to a block of mass m hanging over the top. (See picture.)



$\vec{a} = 0$

a) In terms of M and m, what must the angle θ be such that the two blocks do not move? Assume for this part that there is no friction. (7 points)

M:



m:

X for M: $T - Mg\sin\theta = Ma_x = 0$ $a_h = a$ for hanging mass.

Y for M: $F_N - Mg\cos\theta = Ma_y = 0$

for hanging: $T - mg = ma_h = 0.$

$T - mg = 0 \rightarrow T = mg$

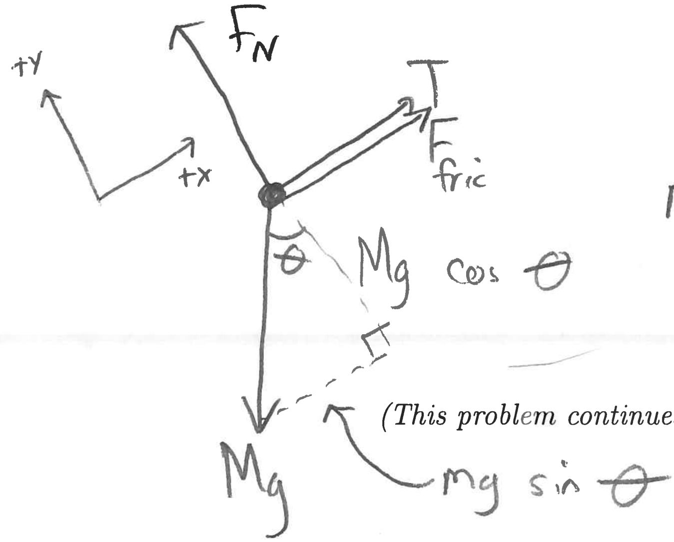
Sub into (X): $mg - Mg\sin\theta = 0$, so

$\sin\theta = \frac{mg}{Mg}$, $\boxed{\theta = \sin^{-1}\frac{m}{M}.}$

Now, assume that M is large enough that it slides down the ramp. There is kinetic friction between that block and the ramp; the coefficient of kinetic friction is μ_k.

b) Draw force diagrams for both blocks. Indicate your choice of coordinate system for both of them (they do not have to be the same, and in fact shouldn't be!) (3 points)

M:



m:

(This problem continues on the next page.)

c) Calculate the acceleration of both blocks in terms of M, m, g, θ, and μ_k. (15 points)

X on ramp: $T + \mu_k F_N - Mg \sin\theta = Ma_x$

Y on ramp: $F_N - Mg\cos\theta = Ma_y$

Hanging : $T - mg = ma_h$.

- $a_x = -a_h$ (since they are roped together)
- $a_y = 0$ (perp to ramp.)

Substitute to get:

$T + \mu_k F_N - Mg\sin\theta = Ma_x$ (same)

$F_N - Mg\cos\theta = 0 \implies F_N = Mg\cos\theta$

$T - mg = -ma_x$. $\implies T = mg - ma_x$

Sub these back into (1):

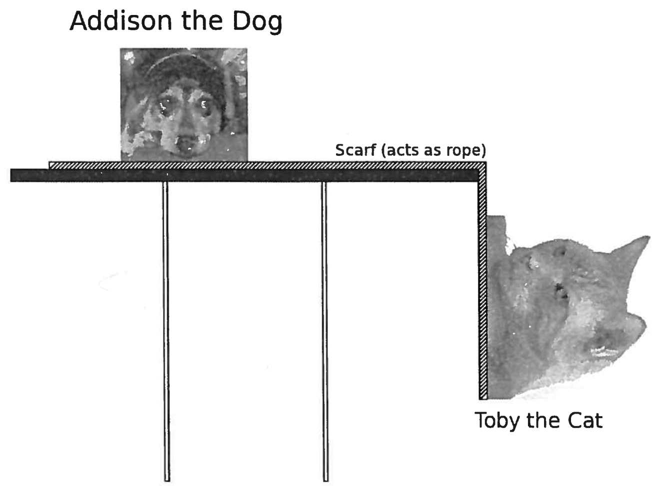
$mg - ma_x + \mu_k Mg\cos\theta - Mg\sin\theta = Ma_x$.

$$\implies \boxed{a_x = \frac{-Mg\sin\theta + \mu_k Mg\cos\theta + mg}{m + M}}$$

Question 5

A long scarf rests on a table. Our student Shelby's dog Addison is asleep on one end of it; the other end hangs off the edge of the table.

Alice's cat Toby sees the other end of the scarf hanging over the edge of the table. Toby jumps up and grabs the edge, and her weight begins to pull Addison and the scarf off the table. (You may assume that the scarf doesn't stretch and has negligible mass.)



Addison has a mass m_A; Toby has a mass m_T. The coefficient of kinetic friction between the scarf and the table is μ_k; since the scarf is so light, the only place where there is friction is underneath Addison.

I would like to find the acceleration of the two animals and the scarf.

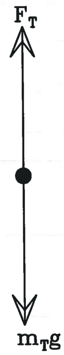
On the next page, you'll find my solution, but my solution contains an error. On the following page, I will ask you a few questions about my work, and ask you to fix my mistake.

Since this problem asks us to connect the forces on objects to their acceleration, I will use Newton's second law $\vec{F} = m\vec{a}$. First I write force diagrams for the two objects, and write down Newton's second law in each direction that matters for each object. I choose a conventional coordinate system where the positive x−axis is to the right and the positive y−axis is up.

Note that F_T is the force of tension, but m_T is Toby's mass.

Need different a values

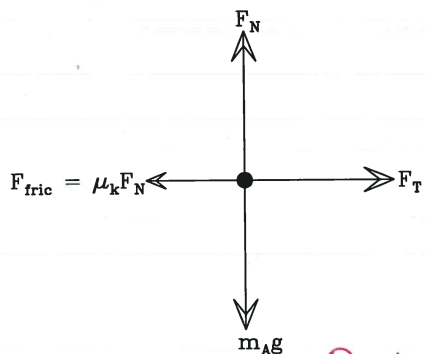
Toby:



$$Y : F_T - m_T g = m_T a_T \quad (1)$$

(Nothing happens in X for Toby)

Addison:



$$Y : F_N - m_A g = 0 \quad (2)$$

(since the acceleration in y is zero)

$$X : F_T - \mu_k F_N = m_A a_A \quad (3)$$

But $a_T = -a_A$! So we need to fix signs.

Since the scarf doesn't stretch, I've used the same acceleration variable for the two animals, since their accelerations must be the same. We now solve this system of equations by substitution. Solve equation (1) for the tension force and solve equation (2) for the normal force; this gives

$$F_T = m_T g + m_T a_T \quad (4) \qquad\qquad F_N = m_A g \quad (5)$$

$$= m_T g - m_T a_A.$$

Substitute the results from (4) and (5) into (3), and solve for a:

$$\overset{m_T g - m_T a_A}{\cancel{m_T g + m_T a}} - \mu_k m_A g = m_A a_A \quad (6)$$

$$m_T g - \mu_k m_A g = (m_A + m_T)a \quad (7)$$

$$\frac{m_T g - \mu_k m_A g}{m_A + m_T} = a \quad (8)$$

... which is what we were supposed to find. Remember, your job is to *find the error* that I have made and fix it.

QUESTION 5, CONTINUED

a) The solution I got for the acceleration of the animals is

$$a = \frac{m_T g - \mu_k m_A g}{m_A - m_T}.$$

Right away, something about the mathematical form of this solution should tell you that there is a mistake. What about this answer should make you skeptical? (5 points)

The denominator goes to zero if $m_A = m_T$.

b) What mistake did I make? You can describe it briefly here, or indicate it clearly on the previous page. (10 points)

The accelerations have equal magnitude but opposite sign. Instead $m_{A_x} = -m_{T_y}$.

c) What should the answer be instead? Correct my work on the previous page or below, and tell me what the acceleration should be instead. (10 points)

Fixed on prev. page - see red writing:

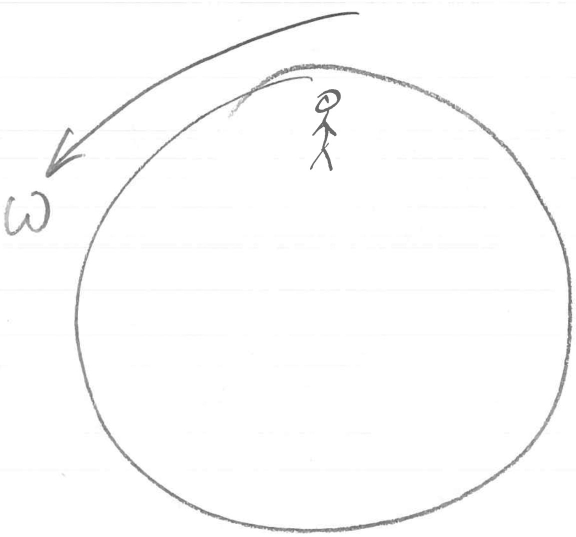
$$a_A = \frac{m_T g - \mu_k m_A g}{m_A + m_T}.$$

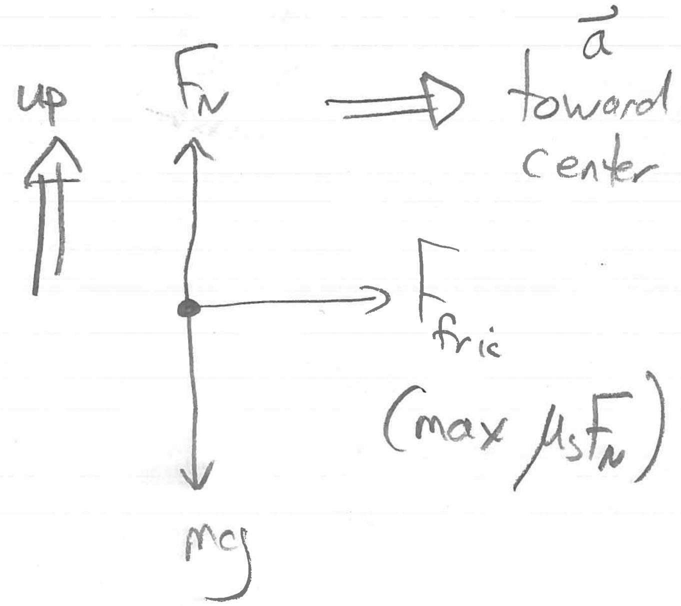
Question 6

A "merry-go-round" is a large, horizontal platform free to rotate around its axis. Children can stand on top of the platform while it spins. Suppose that a merry-go-round with a radius of 3 meters is spinning, and that it rotates around its axis once every 4 seconds. $\rightarrow \omega = \frac{2\pi \text{ rad}}{4 s} = \frac{\pi}{2}$ rad/s.

Suppose that the coefficient of kinetic friction μ_k between the children's feet and the platform is 0.4, while the coefficient of static friction μ_s between their feet and the platform is 0.5.

a) Draw a force diagram for a child standing on the platform. Indicate your choice of coordinate system. (5 points)





b) Describe which parts of the platform a child can stand on without slipping. Be specific: your answer should take the form of "They could stand anywhere within 1 meter of the center", or "They could stand anywhere within 2 meters of the edge". (15 points)

Static friction provides the needed force toward the center to maintain circular motion. But $a_c = \omega^2 r$, so as r increases, the needed force increases as well. At some radius, the needed force toward the center will exceed $F_{f_{max}} = \mu_s$ and they will slip.

$Y: F_N - mg = ma_y = 0 \Rightarrow F_N = mg$

$\boxed{\text{"They could stand within 2.03 m of the center"}}$

$X: F_{fric} = ma_x = m\omega^2 r \Rightarrow F_{fric_{max}} = \mu_s F_N = m\omega^2 r_{max}$

$\rightarrow \mu_s \cancel{m} g = \cancel{m}\omega^2 r_{max} \rightarrow \boxed{r_{max} = \frac{\mu_s g}{\omega^2} = 2.03 \text{ m.}}$

c) Suppose now that the children spinning the platform want to slow it down enough that their friends on top can safely walk to the edge and jump off. What is the maximum angular velocity ω that would allow a child to stand on the edge of the platform without slipping? (5 points)

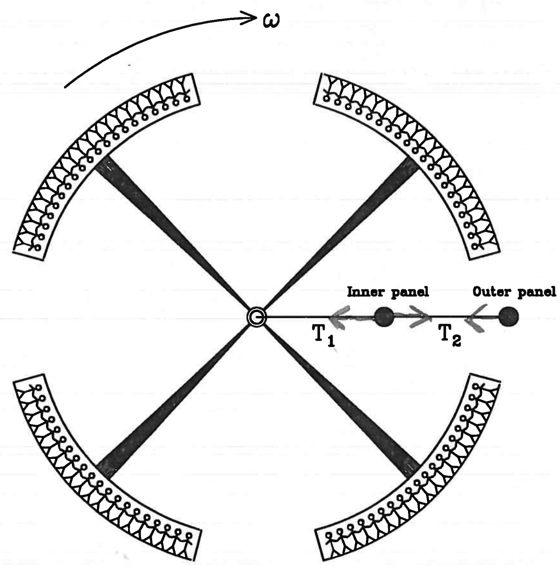
Just use the relation for r_{max} as a function of ω that we got earlier and set $r = 3m$.

$$r_{max} = \frac{\mu_s g}{\omega^2} \Rightarrow \omega = \sqrt{\frac{\mu_s g}{r_{max}}}.$$

Set $r_{max} = 3\,m$: $\omega = 1.29\ rad/s$.

QUESTION 7

Futurists and science-fiction authors have often imagined circular spacecraft with "artificial gravity", in which humans (or other things accustomed to gravity) occupy a ring-shaped habitat. The ring rotates around a central hub, creating the impression of gravity for its inhabitants. They feel heavy, objects that they drop fall to the floor, and they otherwise experience all of the same things that people on a planet do.

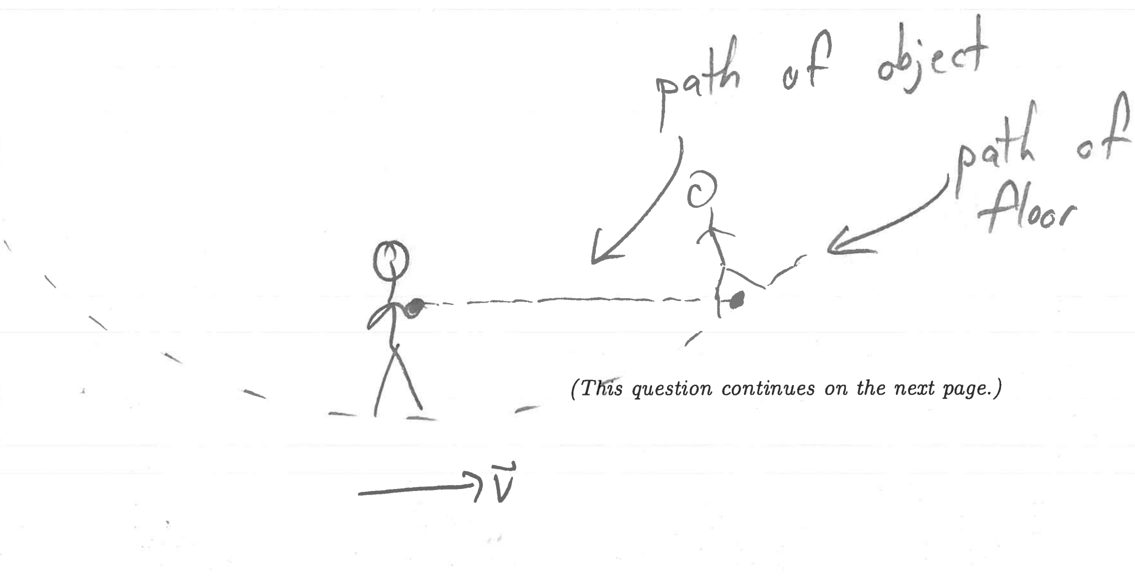


Imagine that such a ship has a radius of R and is in deep space, where there is almost no (actual) gravity. Suppose that the crew of the ship wants the passengers to experience "artificial gravity" similar to that on Earth. (In an actual station R would be much larger than the height of people; this drawing is not to scale.)

a) Explain how this works. Why does a rotating, ring-shaped spacecraft simulate gravity for its inhabitants? Specifically, what force presses them against the floor? If there is no such force, then explain why a person on such a spacecraft standing on a scale could see the same reading as they would on Earth, and why an object that they drop falls to the floor. (8 points)

Apparent weight – the scale reading – is the normal force from the floor.

Here this normal force is needed to provide the acceleration toward the center to go in a circle $(=\omega^2 r)$.



(This question continues on the next page.)

b) At what rate must the spacecraft rotate so that the people aboard experience artificial gravity that feels equal to Earth's? Give your answer in terms of g and R. (5 points)

: $F_N = ma$, $a = \omega^2 r$ (circular motion)

$F_N = mg$ (trying to mimic Earth)

$$mg = m\omega^2 r \rightarrow \boxed{\omega = \sqrt{g/r}}.$$

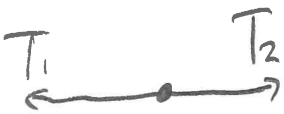
This station is powered by solar panels of mass m connected by cables to the central hub. A cable of length $\frac{1}{2}R$ runs from the hub to the inner panel; a second cable runs from the inner panel to the outer panel. These solar panels also rotate along with the rest of the station at the same angular velocity.

c) Draw a force diagram for the inner solar panel and the outer solar panel. (Note that the tension in the two cables is different.) (5 points)

Outer:

: $T_2 = ma_o = m\omega^2 R \Rightarrow \boxed{T_2 = m\omega^2 R = mg.}$

Inner :



: $T_1 - T_2 = ma_i = m\omega^2 (\frac{1}{2} R)$.

$+ \leftarrow$

Substitute:

$T_1 - m\omega^2 R = \frac{1}{2} m\omega^2 R$, so $\boxed{T_1 = \frac{3}{2} m\omega^2 R = \frac{3}{2} mg.}$

d) In terms of m, R, and ω, calculate the tension T_1 in the cable between the hub and the inner solar panel, and the tension T_2 in the cable between the inner solar panel and the outer solar panel. (7 points)

(see above.)

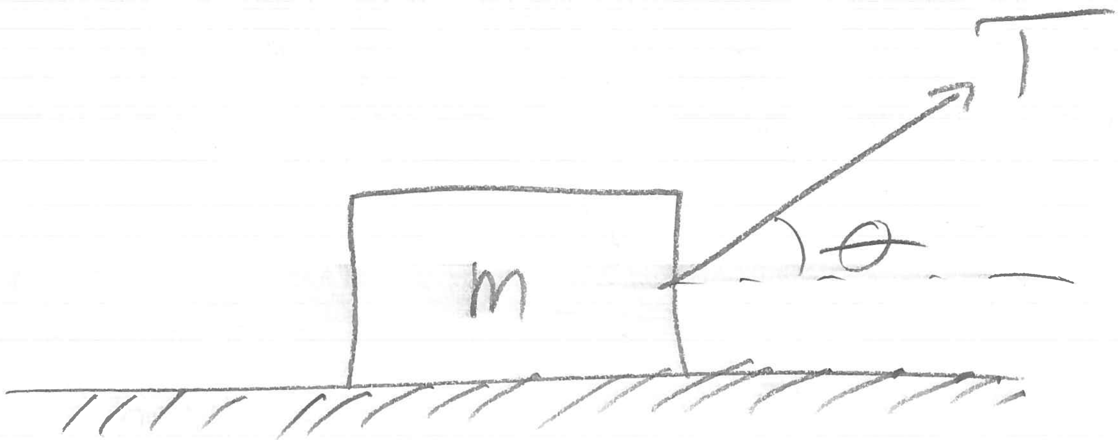
Question 8

You are trying to drag a heavy object across the floor with a rope. This rope makes an angle θ with the horizontal.

You apply a tension T to the rope. The coefficient of friction between the object and the ground is μ_k.

I would like to find the acceleration of the object.

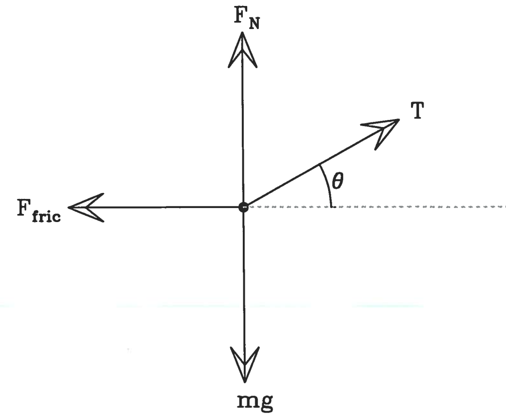
On the next page, you'll find my solution, but my solution contains an error. On the following page, I will ask you a few questions about my work, and ask you to fix my mistake.



QUESTION 8, CONTINUED

Since this problem asks us to connect the forces on objects to their acceleration, I will use Newton's second law $\vec{F} = m\vec{a}$ and solve for $\vec{a}$.

First I draw a force diagram for the object. Imagine that the rope is pulling up and to the right. Then friction points to the left. The normal force points upward to stop the object from falling through the ground, and gravity points downward.



Lies!

Since the object moves only in the x-direction, I only need to worry about it. The x−component of the tension in the rope is $T\cos\theta$.

Reading Newton's second law off of the force diagram, we have

$$\sum F_x = ma_x$$
$$T\cos\theta - F_{\text{fric}} = ma_x$$

We know that the frictional force is $\mu_k F_N$; since the object is resting on a flat surface, $F_N = mg$. Putting this in:

$$T\cos\theta - \mu_k mg + \mu_k T\sin\theta = ma_x$$

$$\cancel{T\cos\theta - \mu_k mg = ma_x}$$

nope- instead

$$F_N = mg - T\sin\theta$$

which gives us an acceleration of

$$\cancel{a = \frac{T\cos\theta - \mu_k mg}{m}}$$

new term: increases a

$$a = \frac{T\cos\theta - \mu_k mg + \mu_k T\sin\theta}{m}$$

a) What mistake did I make? You can describe it briefly here, or indicate it clearly on the previous page. (10 points)

The normal force is <u>not</u> mg: instead in y:

$$\sum F_y = ma_y = 0 \implies F_N + T\sin\theta - mg = 0$$

$$F_N = mg - T\sin\theta.$$

b) What should the answer be instead? Correct my work on the previous page or below, and tell me what the acceleration should be instead. (15 points)

- Upward component of T should actually reduce F_N and thus reduce F_{fric} and increase a.