PHY 211 HOMEWORK 2

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Problem 1 *Show that this equation is simply a consequence of the other two – Derivation of the 'third kinematics equation'*

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2 \tag{1}$$

$$v(t) = v_0 + at \tag{2}$$

Rearranging Equation 2:

$$t = \frac{v_f - v_i}{a}$$

Plugging in:

$$\begin{aligned} x(t) =& x_i + v_i\left(\frac{v_f - v_i}{a}\right) + \frac{1}{2}a\frac{(v_f - v_i)^2}{a^2} \\ =& x_i + \frac{v_f v_i - v_i^2}{a} + \frac{v_f^2 - 2v_f v_i + v_i^2}{2a} \\ =& x_i + \frac{2v_f v_i - 2v_i^2 + v_f^2 - 2v_f v_i + v_i^2}{2a} \\ x(t) =& x_i + \frac{v_f^2 - v_i^2}{2a} \end{aligned}$$

Therefore:

$$\boxed{(x_f - x_i)2a = v_f^2 - v_i^2}$$

Problem 2

(a) *What speed did the stone have to be launched at to achieve this range?*

Given: $V_x = V\cos 45$ and $V_y = V\sin 45$, we can solve for time of flight:

$$\begin{aligned}
v_f &= v_i + at \\
-v_y &= v_y + gt \\
-2v_y &= gt \\
-2V\sin 45 &= gt \\
t &= \frac{2V\sin 45}{g}
\end{aligned}$$

Plugging this in to the range equation:

$$\begin{aligned}
x_f &= x_i + v_t + \frac{1}{2}at^2 \\
x_f = 2000m &= V\cos 45 \times \frac{2V\sin 45}{g} \\
(2000m)(g) &= 2V^2\cos 45\sin 45 \\
V^2 &= \frac{2000m \times g}{2\cos 45\sin 45} \\
V &= \sqrt{\frac{2000m \times g}{2\cos 45\sin 45}} \\
&\boxed{V = 140\frac{m}{s}}
\end{aligned}$$

(b) *How long was the ball in the air?*

Using the equation derived in Part A, we can easily solve for time in flight:

$$\begin{aligned}
t &= \frac{2v\sin 45}{g} \\
t &= \frac{2(140\frac{m}{s})\sin 45}{g} \\
&\boxed{t = 20.2s}
\end{aligned}$$

(c) *How fast was the ball traveling at the apex of its flight?*

Given time of flight is 20.2 seconds, we can solve for V_x which is the only component of velocity present at the apex of flight.

$$\begin{aligned}
t &= 20.2s \\
2000m &= vt \\
v &= \frac{2000m}{20.2s} \\
&\boxed{v = 98.9\frac{m}{s}}
\end{aligned}$$

(d) *What was the acceleration of the stone as it was launched down the bore of the cannon?*

Given: $v_i = 0\frac{m}{s},\ v_f = 140\frac{m}{s},\ d = 8m$

$$v_f^2 = v_i^2 + 2ad$$
$$a = \frac{v_f^2 - v_i^2}{2d}$$
$$a = \frac{140^2 - 0}{2 \times 8}$$
$$\boxed{a = 1225\frac{m}{s^2}}$$

Problem 3

(a) *How fast was Teddy moving when he left the platform?*

Find the final y-component of Teddy's velocity:

$$V_{f,y}^2 = V_{i,y}^2 + 2a_y d$$
$$V_{f,y}^2 = 2g(2)$$
$$V_{f,y} = 2\sqrt{g}\frac{m}{s^2}$$

Using this final y-velocity, we can solve for time of flight:

$$V_{f,y} = V_{i,y} + at$$
$$2\sqrt{g} = gt$$
$$t = \frac{2}{\sqrt{g}}$$

Find speed of Teddy leaving the platform (this is the x-component of Teddy's velocity):

$$v_{i,x} = \frac{d}{t} = \frac{6m}{\frac{2}{\sqrt{g}}} = \boxed{3\sqrt{g}\frac{m}{s}}$$

(b) How fast was Teddy moving when he landed in the water?

For this, we need to find the x and y component of his velocity:

$$V_y = 2\sqrt{g} \;\rightarrow\; \text{(Solved above)}$$
$$V_x = 3\sqrt{g} \;\rightarrow\; \text{(Solved above)}$$

Teddy's speed is the resulting vector from the addition of V_x and V_y:

$$speed^2 = \vec{v_y}^2 + \vec{v_x}^2$$
$$speed = \sqrt{(2\sqrt{g})^2 + (3\sqrt{g})^g}$$
$$speed = \sqrt{4g + 9g}$$
$$\boxed{speed = \sqrt{13g}}$$

Teddy's speed was $\sqrt{13g}\frac{m}{s}$

(c) *What direction was Teddy moving when he splashed into the lake?*

$$\tan\theta = \frac{V_y}{V_f}$$
$$\theta = \arctan\left(\frac{2\sqrt{g}}{3\sqrt{g}}\right) = 33.6^\circ$$

Teddy was moving 33.6 degrees below the x-axis

Problem 4

(a) *With what velocity must Mary leave the ground in order to jump 120 cm high?*

Since final velocity is 0:

$$V_f^2 = V_i^2 + 2ad$$
$$V_i^2 = 2(g)(1.2m)$$
$$V_i = \sqrt{2.4g}$$
$$\boxed{V_i = 4.85\frac{m}{s}}$$

Initial velocity must be $4.85\frac{m}{s}$

(b) *How long will she be in the air?*

$$v_f = v_i + at$$
$$0\frac{m}{s} = 4.85\frac{m}{s} + (-9.8)t$$
$$t_{up} = 0.494$$
$$\boxed{\therefore t_{total} = 0.99 seconds}$$

Total time in the air will be 0.99 seconds.

Problem 5

(a) *Will she be able to push the button?*

Using $x(t) = x_0 + v_0 t + \frac{1}{2}at^2$, these are the equations for the cat's and button's position.

$$x_m = 4.85t + \frac{-g}{2}t^2 \tag{3}$$

$$x_b = 1.1m + t^2 \tag{4}$$

Solving for when the positions are equal.

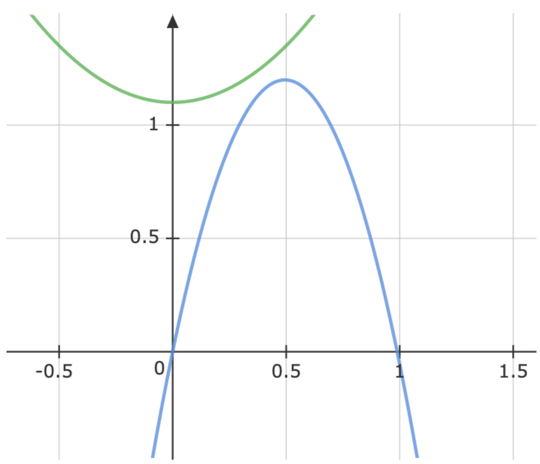
$$x_m = x_t$$
$$4.85t - \frac{g}{2}t^2 = 1.1 + t^2$$
$$0 = (1 + \frac{1}{2}g)t^2 - 4.85t + 1.1$$

Using the quadratic formula for this, we find that they will never be equal:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{4.85 \pm \sqrt{4.85^2 - 4\left(1 + \frac{1}{2}g\right)(1.1)}}{2\left(1 + \frac{1}{2}g\right)}$$
$$\boxed{x = \text{undefined}}$$

She will not be able to push the buttons

Note: We can also solve for this graphically and see that the position curves will never intersect, therefore at no time will Mary's position be equal to the position of the button:



(b) *How far above the elevator floor will she make it?*

Using the same procedure as above, we can find the equations representing the position of the floor and of the cat:

(5) $$x_m = 4.85t + \frac{-g}{2}t^2$$

(6) $$x_f = t^2$$

To find how far above the ground the cat is, we need to find the maximum difference between the two equations. $L(t)$ will be our function representing the difference:

$$L(t) = x_m(t) - x_f(t)$$
$$L(t) = 4.85t + \frac{-g}{2}t^2 - t^2$$
$$L(t) = 4.85t - 5.9t^2$$

$$\frac{dL}{dt} = 4.85 - 11.8t$$

The difference is at a max when $L(t)$'s derivative, $\frac{dL}{dt}$ is equal to zero:

$$\frac{dL}{dt} = 0$$
$$0 = 4.85 - 11.8t$$
$$t = 0.411\ sec$$

Plugging in this value into the difference function:

$$L(t) = 4.85t - 5.9t^2$$
$$L(t)|_{t=0.411} = 4.85(0.411) - 5.9(0.411)^2$$
$$\boxed{L(0.411) = 0.997m}$$

Therefore, the cat will make it 0.997 meters above the ground.

(c) *How long will she be in the air before she lands?*

Using the same procedure as above, we can find the time in air. For this problem, the second equation will represent the floor's position rather than the button's.

$$x_m = 4.85t + \frac{-g}{2}t^2 \tag{7}$$

$$x_f = t^2 \tag{8}$$

Solving for when the positions are equal will give us the time when Mary is just about to begin her jump, and the time when Mary lands back on the ground.

$$x_m = x_f$$

$$4.85t - \frac{g}{2}t^2 = t^2$$

$$0 = (1 + \frac{1}{2}g)t^2 - 4.85t$$

Using the quadratic formula for this, we will find the time in flight

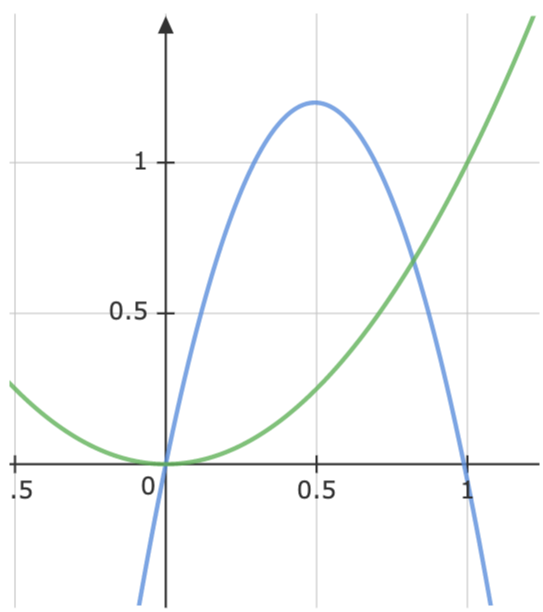
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4.85 \pm \sqrt{4.85^2 - 4\left(1 + \frac{1}{2}g\right)(0)}}{2\left(1 + \frac{1}{2}g\right)}$$

$$\boxed{x = 0, 0.83 \text{ sec}}$$

Therefore, the cat will be in the air for 0.83 seconds

Note: We can also solve for this graphically and see the point of intersection of the position curves gives us the times where Mary is in contact with the elevator's floor.



Problem 6

(a) *How high can Mary jump on Twilo?*

Using the initial velocity found in Problem 4, the maximum height Mary can reach is:

$$v_f^2 = v_i^2 + 2ad$$
$$0\frac{m^s}{s^2} = (4.85\frac{m}{s})^2 + 2(-11.8\frac{m}{s^2})(d)$$
$$\boxed{d = 1m}$$

Marry can jump a maximum of $1m$

(b) *Based on a comparison of the last two problems, can you make any statements regarding gravity and acceleration?*

Problem 7

(a) Where on the slope does the rock land?

The rock's and the mountain's position functions are as follows:

$$y_r(t) = y_0 - \frac{g}{2}t^2$$
$$y_m(t) = y_0 - t$$

When the rock hits the mountain, $y_r = y_m$, therefore:

$$y_r = y_m$$
$$y_0 - \frac{g}{2}t^2 = y_0 - t$$
$$-\frac{g}{2}t^2 + t = 0$$
$$t\left(-\frac{g}{2}t + 1\right) = 0$$
$$t = 0, \frac{2}{g}$$

Plugging in the t value for the x and y coordinates:

$$x(t) = t \rightarrow \text{x cord} = \frac{2}{g}$$
$$y(t) = y_0 - \frac{2}{g}$$

The rock will hit the slope at $x = \frac{2}{g}$ **and** $y = y_0 - \frac{2}{g}$