Exam I Review

Physics 211
Syracuse University, Physics 211 Spring 2015
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January 29, 2015

Announcements

- Exam 1 is on Tuesday
- Homework 2 due tomorrow
- My office hours today are 1:30-3:30 (in the Clinic)
- I will be in the Clinic tomorrow from 10 to 3
- No homework due next week
- Sample exam solutions will be posted tomorrow
- Please arrive a few minutes early if possible to the exam
- We are creating the reference sheet today, during the review

Exam 1

- The exam covers kinematics in one and two dimensions
- Kinematics: how are an object's position, velocity, and acceleration related?
- The exam will be substantially easier than the homework.
- You may use a scientific (not graphing) calculator on the exam.
- Bring: your calculator, pencils, and your physics smarts (frog optional)

Exam 1, promises

- There will be one problem where you need the quadratic formula
  - ... this means interpreting the two values it spits out
- There will be at least one instance where you need to graph position, velocity, and acceleration
- You will *not* need to compute derivatives or integrals algebraically

Functions describing motion

- We can specify a function's position, velocity, and acceleration as functions of time: $\vec{r}(t), \vec{v}(t), \vec{a}(t)$
- All of these quantities are vectors; often easier to work with their components
  - Position: $x(t)$, $y(t)$
  - Velocity: $v_x(t)$, $v_y(t)$
  - Acceleration: $a_x(t)$, $a_y(t)$
- If you don't know where to start a problem, figure out $x(t)$, $y(t)$, $v_x(t)$, $v_y(t)$, leaving unknown quantities as variables for the time being

Position, velocity, and acceleration

Position

(derivative of)
rate of change of

$\longleftarrow$

$\longrightarrow$

area under the curve of
(integral of)

Velocity

Position, velocity, and acceleration

(derivative of) rate of change of: Position ← Velocity ← Acceleration

Position → Velocity → Acceleration: area under the curve of (integral of)

Position	(derivative of) rate of change of ← → area under the curve of (integral of)	Velocity	(derivative of) rate of change of ← → area under the curve of (integral of)	Acceleration

Constant acceleration kinematics

Particularly interesting situation:

- Free fall (as you saw)
- Any time the force is constant: $F = ma \rightarrow a = F/m$...

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Plan of attack:

- We know what the acceleration curve looks like (it's just flat)
- Figure out the area under the acceleration curve to get the velocity curve
- Figure out the area under the velocity curve to get the position curve

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- Figure out the area under the acceleration curve to get the velocity curve
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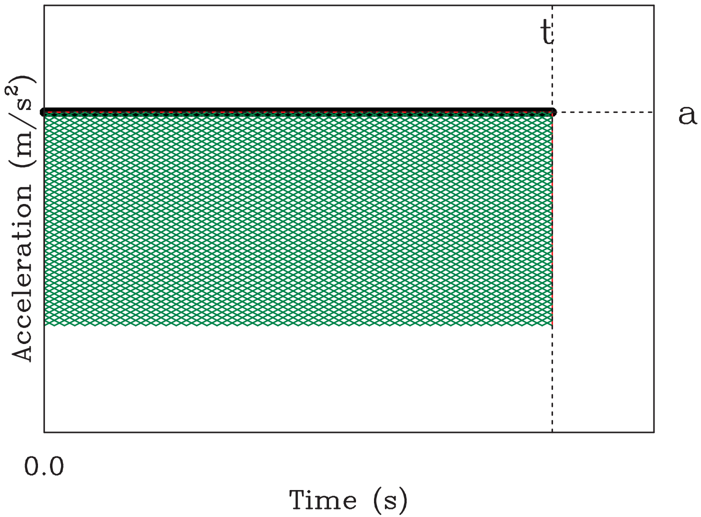
Remember the area under the curve of (velocity, acceleration) just gives the *change in* (position, velocity) – *i.e.* initial minus final.

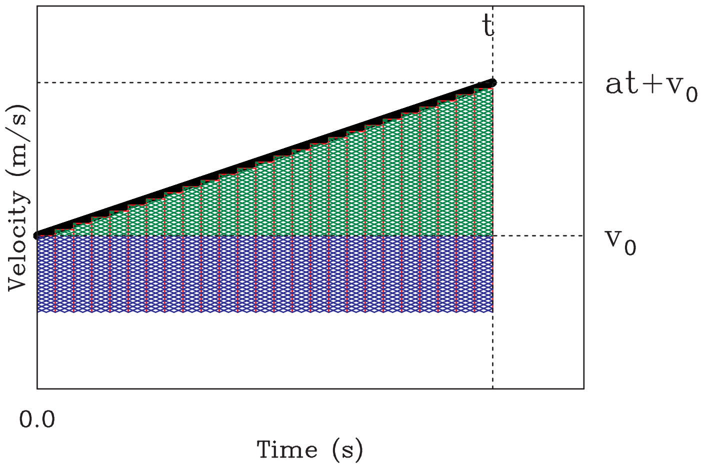
Constant acceleration kinematics



The area under the curve out to time t is at, which gives the change in the velocity.

$$v(t) - v_0 = at, \text{ so } v(t) = at + v_0$$

Constant acceleration kinematics



Area under blue part: $v_0 t$
Area under green part: $\frac{1}{2}at^2$
Total change in position: $x(t) - x_0 = \frac{1}{2}at^2 + v_0 t$

Thus, $x(t) = \frac{1}{2}at^2 + v_0 t + s_0$

Constant acceleration kinematics:

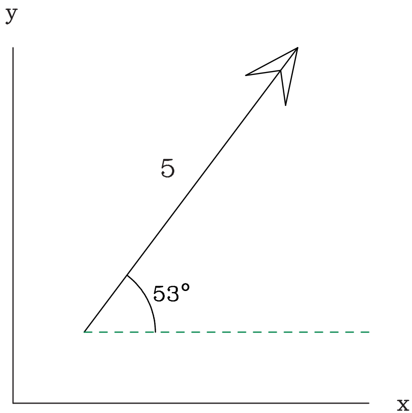
$$v(t) = at + v_0$$
$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

You can solve one of these for time and substitute into the other to get a third, useful equation:

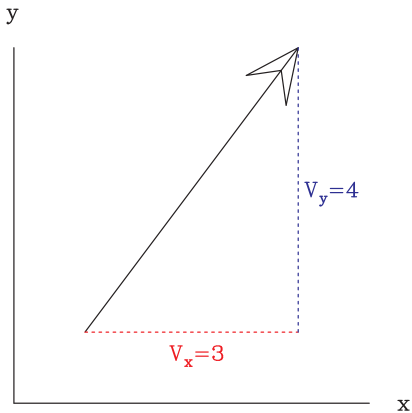
$$v(t) - v_0 = 2a\left[x(t) - x_0\right]$$

This is useful when you *don't know* and *don't care* about the time some motion took.

Vectors: Two ways to describe a vector



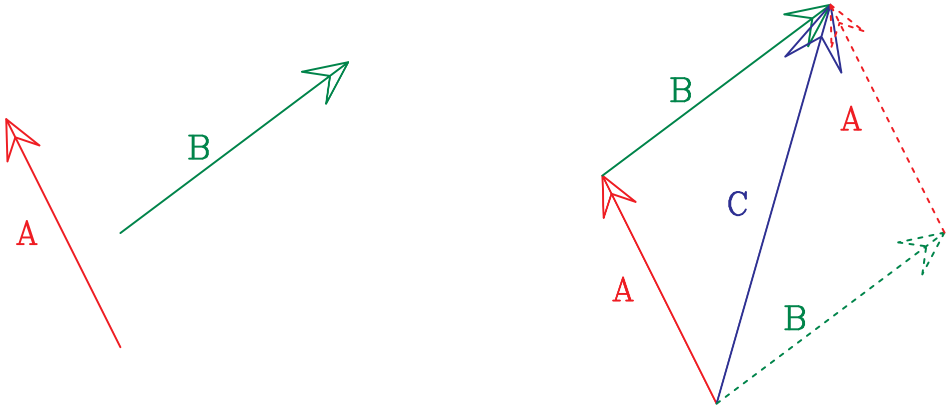
Angle and direction



X and Y components

Adding vectors

We can also add vectors together by drawing them "head to tail". Here are two vectors:



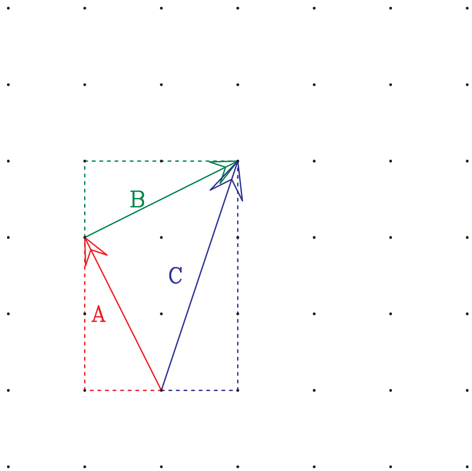
$$\vec{A} + \vec{B} = \vec{C}$$

Adding vectors: components

The component representation is much easier to work with!

$$\vec{A} + \vec{B} = \vec{C} \rightarrow \begin{pmatrix} A_x + B_x = C_x \\ A_y + B_y = C_y \end{pmatrix}$$

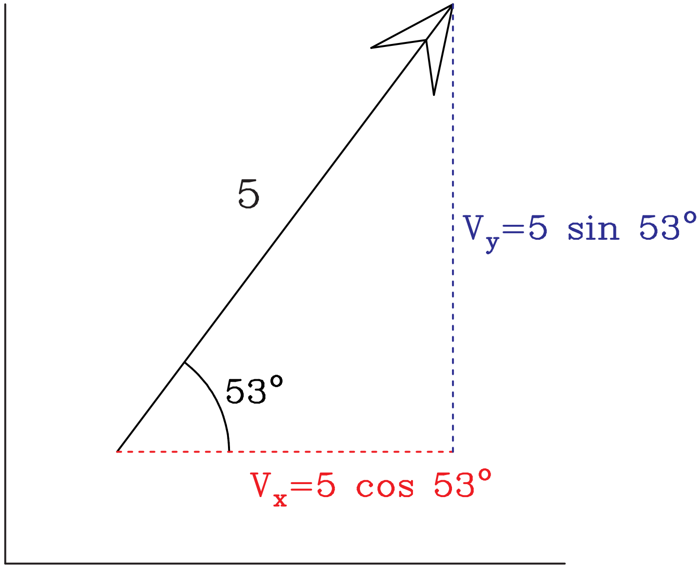
Adding vectors: components



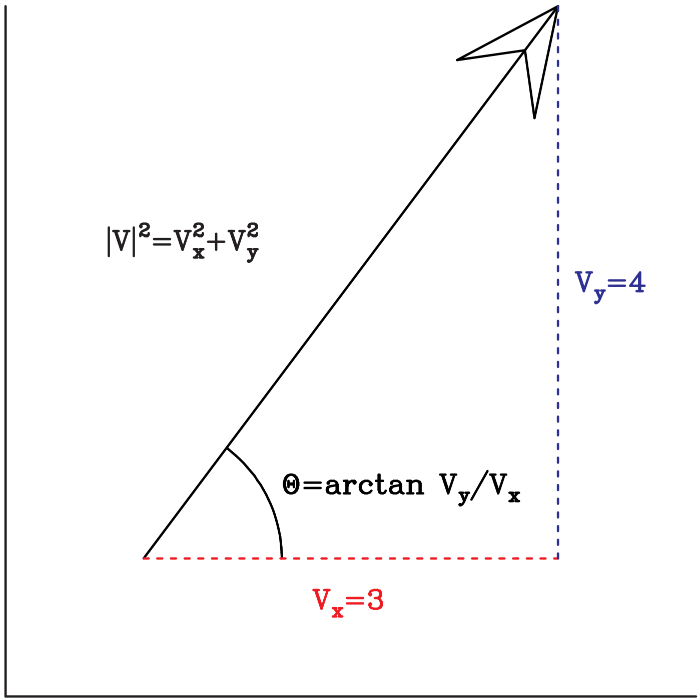
To add two vectors, just add their components!

This is why it is almost always easiest to work in the component representation!

From “direction and magnitude” to components



From components to direction and magnitude



Problem solving guide: 1D kinematics

- Draw a picture!
- Figure out $x(t)$, $v(t)$, $a(t)$ (constant acceleration kinematics
  - If you have other unknowns that appear in these expressions, that's okay!
- Translate physical statements about moments of interest into mathematics
  - "When does the object hit the ground?" $\rightarrow$ "At what time does $y = 0$ (or whatever height the ground is)"
  - "How high does the object go?" $\rightarrow$ "What is the maximum height?" $\rightarrow$ "What is y at the time when $v_y = 0$"
  - "When do two objects meet?" $\rightarrow$ "At what time is $x_1(t) = x_2(t)$"?
- Do algebra, solving for the things you want to know
- Make numerical substitutions as the very last step if possible

2D kinematics

In two dimensions you simply have two copies of all the kinematic relations, one for each:

$$v_x(t) = a_x t + v_{x,0}$$
$$v_y(t) = a_y t + v_{y,0}$$

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$$
\begin{aligned}
v_x(t) &= a_x t + v_{x,0} \\
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\end{aligned}
$$

$$
\begin{aligned}
x(t) &= \frac{1}{2} a_x t^2 + v_{x,0} t + x_0 \\
y(t) &= \frac{1}{2} a_y t^2 + v_{y,0} t + y_0
\end{aligned}
$$

Problem solving guide: 1D kinematics

- Draw a picture!
- Figure out $x(t)$ and $y(t)$, $v_x(t)$ and $v_y(t)$,(constant acceleration kinematics)
- Remember motion in x and y is separate and independent
- Translate physical statements about moments of interest into mathematics
  - "Where does the object hit the ground?" $\rightarrow$ "What is x at the time that $y = 0$ (or whatever height the ground is)"
  - "What speed does the object hit the ground with?" $\rightarrow$ "What is $|v| = \sqrt{v_x^2 + v_y^2}$ at the time that $y = 0$?"
- Do algebra, solving for the things you want to know, going back and forth between representations of vectors ($v_{0,x}$ vs. $v_0 \cos\theta$) as needed
- Make numerical substitutions as the very last step if possible

Working with variables

If you don't know the numerical value of a quantity yet, it's fine to leave it as a variable!

This is essential for solving many problems.

Example from cannon problem:

$$x(t) = v_{x,0} t$$
$$y(t) = -\frac{1}{2} g t^2 + v_{y,0} t$$

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Example from cannon problem:

$$x(t) = v_0 \cos 45^o t$$
$$y(t) = -\frac{1}{2} g t^2 + v_0 \sin 45^o t$$

(I leave the rest to you for now...)

The roadrunner problem

The position of the car is given by the ordinary 1D kinematics relation:

$x(t) = \frac{1}{2}at^2 + v_0 t = \frac{1}{2}(-9)t^2 + (30.6)t$ (mks units)

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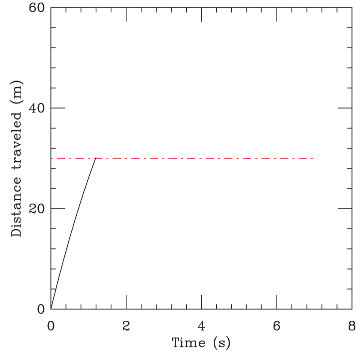
We care about the time when it meets up with the position of the roadrunner, which is 30m. So we set $x(t) = 30$ and solve.



This seems easy enough, but the quadratic formula gives us two solutions! What happened?

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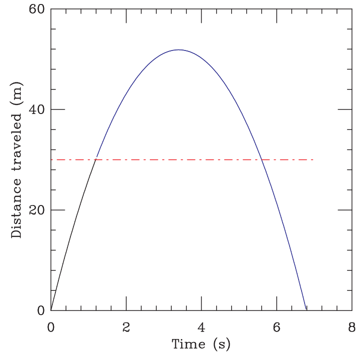

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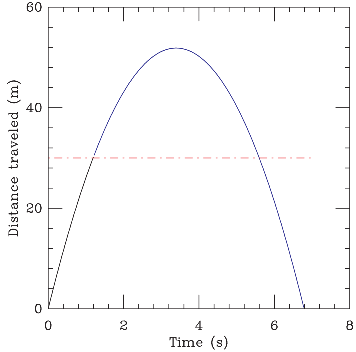

Moral of the story: mathematics is a very blunt tool!

Throwing a stone onto a slope

A hiker kicks a stone off of a mountain slope with an initial velocity of 3 m/s horizontally. If the mountain has a slope of 45 degrees, how far down the slope does it land?

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  $v_{0,x} t = \frac{1}{2} g t^2 \rightarrow t = \frac{2 v_{0,x}}{g}$

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  $v_{0,x} t = \frac{1}{2} g t^2 \rightarrow t = \frac{2 v_{0,x}}{g}$
- This gives us $x(t) = \frac{2 v_{0,x}^2}{g}$
- $y(t)$ will have the same magnitude: the Pythagorean theorem gives $|r| = 2\sqrt{2}\frac{v_{0,x}^2}{g}$

A rocket

A rocket is launched from rest on level ground. While its motor burns, it accelerates at 10 m/s at an angle 30 degrees below the vertical. After ten seconds its motor burns out and it follows a ballistic trajectory until it hits the ground.

How far does it go?