Homework 2

Due Friday, 11 February, at the beginning of recitation

This homework set is specifically designed to prepare you for Quiz 1, held on 15 February. All of the problems here touch on ideas that you may need for the quiz.

It will be updated with a picture of Mary, the jumping kitten. Kitten pictures are very important. If you have a cat, dog, bird, horse, lizard, snake, or pet rock you'd like to see featured in a homework set or exam problem, send me their picture along with what they like to do!

1. Part 4 of Question 2 asks you to compute the acceleration of an object, given the change in position, the initial velocity, and the final velocity. To do this, you will need to use both $x(t)$ and $v(t)$ kinematics equations, but ultimately you will eliminate the variable t.

   Often in mechanics we aren't particularly concerned about time – we're only concerned with the change in position, initial velocity, final velocity, and acceleration. These are related by the "third kinematics equation",

   $$v_f^2 - v_0^2 = 2a(x_f - x_0)$$

   Show that this equation is simply a consequence of the other two.

   Algebra hint: Starting from the $x(t)$ and $v(t)$ formulae for constant acceleration, solve one equation for t and substitute it back into the other one. (The algebra here is very similar to Week 2 Wednesday's recitation, second problem.)

2. During the siege of Constantinople that led to its conquest by the Ottomans in 1453, the Hungarian engineer Orban built a set of bombards (primitive cannon) to throw enormous stones at the city to breach its walls. The largest of these could throw a 300 kg stone a distance $x_f = 2$ km. Assume that the stone was launched at an angle of $\theta = 45°$ above the horizontal; in the absence of air resistance, this gives the largest range.

   (a) What speed did the stone have to be launched at to achieve this range?

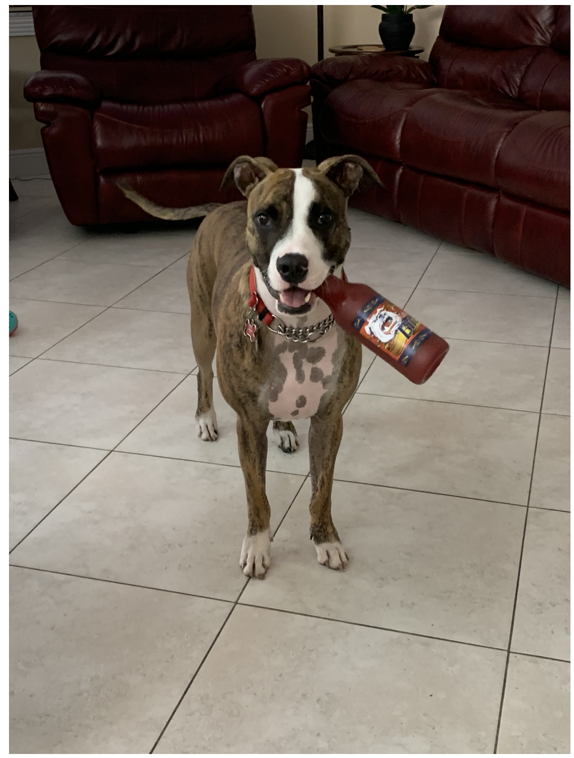
   (b) How long was the ball in the air?

   (c) How fast was the ball traveling at the apex of its flight?

(d) Orban's cannon was 8m long. What was the average acceleration of the stone as it was launched down the bore of the cannon? *Hint: Note that during its movement down the bore of the cannon, it accelerated from $v = 0$ to the velocity you found as your solution to the first part of this problem.*

3. Our previous head TA, Mario Olivares, has an adorable dog named Teddy.

Suppose Mario throws a ball out into the lake for Teddy to catch. It lands $d = 6$ meters out into the water. Teddy is standing on a flat platform 2 meters above the water. Wanting to get his ball back, he runs down the platform and flies out over the water, landing on top of his ball. Suppose that Teddy runs straight off the platform, so that he is moving horizontally when he leaves the ground.



(a) How fast was Teddy moving when he left the platform?

(b) How fast was Teddy moving when he landed in the water?

(c) In what direction was Teddy moving when he splashed into the lake? (Give your answer in a physically meaningful way: "X degrees below the horizontal" or similar.

4. One of our former coaches, Emily Baldwin, has just adopted a tiny little one-eyed kitten named Mary who was rescued from a barn and is very athletic. Mary can jump 120 cm off of the ground (and uses this talent all the time to jump up on tables where she thinks she can steal someone's dinner...) This means that Mary can push herself off the ground with enough velocity to reach a height of 120 cm. *(Note that this velocity is a property of Mary herself; no matter where we take her, this velocity is the same.*

   (a) With what velocity must Mary leave the ground in order to jump 120 cm high?

   (b) How long will she be in the air before she lands?

5. Suppose that Emily now takes Mary to an elevator, accelerating upward at $\alpha = 2\,\mathrm{m/s^2}$. Mary jumps up and tries to swat one of the elevator buttons. This elevator button is 110 cm above the floor of the elevator.

   (a) If Mary jumps as high as she can, will she be able to push the button? How far above the elevator floor will she make it?

   (b) How long will Mary be in the air?

   Hint: Think very carefully about your coordinate system, and all of the consequences of the accelerating elevator. You may need the quadratic formula for this problem. If you are still stuck, draw position vs. time graphs for both Mary and the elevator button on the wall that she is trying to push.

6. Emily then travels to the planet Twilo with her kitten, which is quite Earthlike except for its value of $g = 11.8\,\mathrm{m/s^2}$.

   (a) How high can Mary jump on Twilo?

   (b) Based on a comparison of the last two problems, can you make any statements regarding gravity and acceleration?

7. An object's position is given by the vector

$$\vec{s}(t) = (2\,\mathrm{m})\cos(\omega \mathrm{t})\hat{\imath} + (2\,\mathrm{m})\sin(\omega \mathrm{t})\hat{\jmath}$$

   where $\omega = 1$ radian per second.

   (a) How would you describe this object's motion? What shape is it moving in?

   (b) *Hint: if you are stuck, compute and graph its position in the Cartesian plane at a variety of values of t (say, for integers 0-6).*

8. A hiker is standing on the top of a mountain with a slope of 45 degrees. They kick a rock horizontally off of the top of the mountain at an initial speed v_0; it sails through the air until it lands lower on the slope.

   (a) Think very carefully about how you can describe "The rock lands back on the slope" in mathematical terms. It will help to draw a picture, as always.

   (b) Where on the slope does the rock land?