Work and potential energy

Physics 211
Syracuse University, Physics 211 Spring 2015
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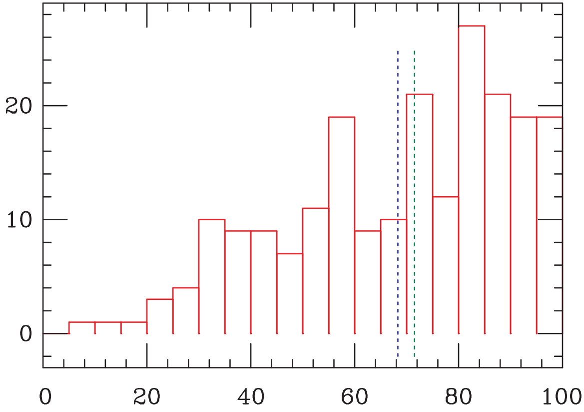
March 16, 2015

Announcements

- Homework and Mastering Physics assignments due Friday
- Your next homework assignment will be longer and due two weeks from tomorrow. Start early!
- Those of you who didn't pick up your exams last Friday can do so on Wednesday
- If your exam was misgraded, grade appeals will be handled the same way as before (and faster!)

Exam 2 recap

(Statistics from only half of the TA's)



Mean grade was 68.3; median grade was 71.5

This exam was quite hard, and had no extra credit

Your performance was above the historical average

Be proud of yourselves (again!)

Where we've been and where we're going

- Last time: kinetic energy and the work-energy theorem
- This time: the idea of potential energy and conservation of energy
  - Potential energy: "the most meaningful bookkeeping trick in physics"
  - Lets us understand many phenomena without difficult mathematics
  - Conservation of energy: there's always the same amount of energy, and it just changes forms

Review: kinetic energy

We will see that things are often simpler when we look at something called "energy"

- Basic idea: don't treat $\vec{a}$ and $\vec{v}$ as the most interesting things any more
- Treat v^2 as fundamental: $\frac{1}{2}mv^2$ called "kinetic energy"

Previous methods:

- Velocity is fundamental
- Force: causes velocities to change over time
- Intimately concerned with vector quantities

Energy methods:

- v^2 (related to kinetic energy) is fundamental
- Force: causes KE to change over distance
- Energy is a *scalar*

Energy methods: useful when you don't know and don't care about time

The work-energy theorem in 1D

Last time we saw the "work-energy theorem" was a consequence of simple kinematics:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = F\Delta x$$

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Some new terminology:

- $\frac{1}{2}mv^2$ called the "kinetic energy" (positive only!)
- $\vec{F} \cdot \Delta\vec{s}$ called the "work" (negative or positive!)
- "Work is the change in kinetic energy"

Sample problem: a roller coaster

(on document camera)

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Strategy: compute the work done by all the forces and equate that to the change in KE.

Work done by normal force = **zero**!
Work done by gravity = $(F)(\Delta s)_{\|} = mg\Delta y = mg(y_0 - y_f)$

$$
\begin{aligned}
KE_f - KE_i &= W_g \\
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$\rightarrow v_f = \sqrt{2g(y_0 - y_f)}$

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No detailed knowledge of the motion required!

Potential energy: an accounting trick

- Notice that the work done by gravity depends *only* on the change in height.
- Some other forces are like this as well: the work done depends only on initial and final position
  - These are called *conservative forces*
  - Soon we'll see that the elastic force is like this too
- Write the work-energy theorem in terms of the heights:

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- Collect all the "initial" things on the left and the "final" things on the right:

$$\begin{aligned} \frac{1}{2}mv_0^2 + mgy_0 &\quad +W_{\text{other}} = &\quad \frac{1}{2}mv_f^2 + mgy_f \\ KE_0 + GPE_0 &\quad +W_{\text{other}} = &\quad KE_f + GPE_f \end{aligned}$$

- Identify mgy as "gravitational potential energy": how much work will gravity do if something falls?

Potential energy lets us easily calculate the work done by conservative forces

Potential energy: more than accounting!

- Another way to look at the roller coaster: gravitational potential energy being converted to kinetic energy.
- This perspective is universal: all forces just convert energy from one sort into another
- Some of these types are beyond the scope of this class, but we should be aware of them!

A short history of energy conversion:

- Hydrogen in the sun fuses into helium
- Hot gas emits light
- Light shines on the ocean, heating it
- Seawater evaporates and rises, then falls as rain
- Rivers run downhill
- Falling water turns a turbine
- Turbine turns coils of wire in generator
- Electric current ionizes gas
- Recombination of gas ions emits light

- Nuclear energy $\rightarrow$ thermal energy
- Thermal energy $\rightarrow$ light
- Light $\rightarrow$ thermal energy
- Thermal energy $\rightarrow$ gravitational potential energy
- Gravitational PE $\rightarrow$ kinetic energy and sound
- Kinetic energy in water $\rightarrow$ kinetic energy in turbine
- Kinetic energy $\rightarrow$ electric energy
- Electric energy $\rightarrow$ chemical potential energy
- Chemical PE $\rightarrow$ light

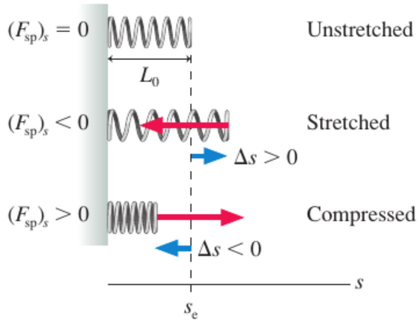
Potential energy: more than accounting!

- This class is just the study of motion: we can't treat light or nuclear energy here.
- ... but in physics as a whole, the *conservation of energy* – that processes just change energy from one form to another – is universal!
- Conservation of energy is one of the most tested, ironclad ideas in science
- Nuclear and chemical potential energy: nuclear forces do mechanical work on particles, much like gravity
- Light, and others: kinetic energy of little particles called "photons"
- Heat: kinetic energy of atoms in random motion
- Sound: kinetic energy of atoms in coordinated motion
- Food: Just chemical potential energy...
- ... so all of these things aren't as far removed from mechanics after all!
- Einstein: "Mass is just another form of energy"

A new force: elasticity and Hooke's law

To best see how this can be useful, let's introduce a new force: elasticity.

- Springs have a particular length that they like to be: "equilibrium length" L_0
- A spring stretched to be longer than this pulls inward to shorten itself
- A spring compressed to be shorter than this pushes outward to lengthen itself
- Flexible things like strings and ropes only pull; they kink instead of compressing
- The force is proportional to the deviation from the optimum length



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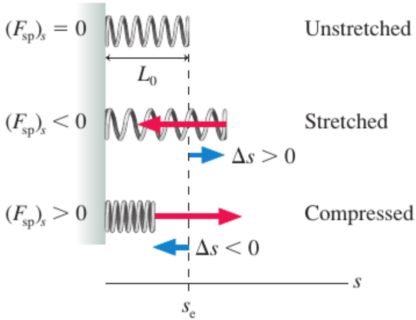

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k is called the "spring constant":

- Measures the stiffness of the spring/rope
- Units of newtons per meter: "restoring force of k newtons per meter of stretch"

A simple spring problem: done with the work-energy theorem

A person of mass $m = 100kg$ falls from a height of $h = 3$m onto a trampoline. If the person makes an impression $d = 40$ cm deep on the trampoline when he lands, what is the spring constant?

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  - Need to use the integral form of the work-energy theorem since the force isn't constant
- The person begins and ends at rest, so we know the initial and final kinetic energy is zero
- The trampoline begins at its equilibrium position

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- $KE_0 + W_{\text{grav}} + W_{\text{elas}} = KE_f$
- $0 + (mg)(h + d) - \frac{1}{2}kd^2 = 0$
- $k = \frac{mg(h+d)}{2d^2}$

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We saw that an object at height h has gravitational potential energy mgh. Can we do something similar for springs?

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- A natural choice is $\Delta x = 0$, the equilibrium position of the spring.

"How much work is done by a spring as it goes from $\Delta x = a$ to $\Delta x = 0$?

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Now that we have this, we never have to do this integral again!

$$U_{\mathrm{elastic}} = \frac{1}{2}kx^2, \text{ where } x \text{ is the distance from equilibrium}$$

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- $U_{\mathrm{grav},0} = mgh$
- $U_{\mathrm{elas},0} = 0$ (trampoline starts at equilibrium)
- $U_{\mathrm{grav,f}} = -mgd$ (the person falls below $y = 0$; PE can be negative!)
- $U_{\mathrm{elas,f}} = \frac{1}{2}kd^2$ (see last slide)

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- $k = \frac{mg(h+d)}{2d^2}$

That spring problem: a recap

We don't care about time $\rightarrow$ energy methods

Work-energy theorem

- Initial KE + all work done = final KE
- Need to compute work done by gravity: easy
- Need to compute work done by spring: harder (need to integrate Hooke's law)

Potential energy treatment

- Initial KE + initial PE + other work = final KE + final PE
- No "other work" in this problem; all forces have a PE associated
- Need to know the expressions for PE:
  - $U_{\mathrm{grav}} = mgy$
  - $U_{\mathrm{elas}} = \frac{1}{2}kx^2$ (x is the distance from the equilibrium point)
- No integrals required (they're baked into the above)

Potential energy with other forces

What about associating a potential energy with other forces?

- Friction is a no-go: the work done by friction depends on the path, not just where you start and stop
- "Ephemeral" forces like tension and normal force are easiest to deal with by computing work directly
- The other force we've studied that is easily associated with a potential energy is **universal gravitation**
  - Need to choose a point to set $U = 0$; here we choose $r = \infty$
  - $U_G =$ "work done by gravity on m_1 when it moves infinitely far from m_2

$$F_G = \frac{Gm_1 m_2}{r^2}$$

$$W_G = \int_R^\infty -\frac{Gm_1 m_2}{r^2}\, dr = -\frac{Gm_1 m_2}{R}$$

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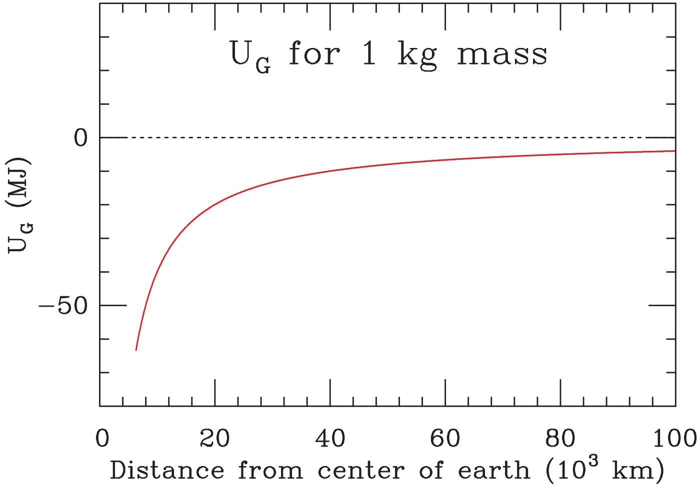
$\rightarrow$ Gravitational potential energy between two objects separated by a distance r is $-\frac{Gm_1 m_2}{r}$.

The Earth's "gravity well"

- With this choice of the zero point at $r = \infty$, gravitational potential energy is always negative
- We have to *add energy* to get something away from Earth



This region of large negative potential energy is often called a "gravity well".

Summary

- Potential energy is two things:
  - An accounting device that makes it easier to keep track of work done
  - Part of *conservation of total energy*, a powerful statement about nature
- Gravitational potential energy (on Earth): $U_g = mgy$

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- Gravitational potential energy (on Earth): $U_g = mgy$
- We learned about a new force: **elasticity**
  - Restoring force in a stretched or compressed spring, or a stretched string:
$$F = -k(x - x_0) \text{ (x_0 is the equilibrium length)}$$
  - k is the spring constant, measured in force per distance, that gauges stiffness
  - Elastic potential energy: $U_{\mathrm{elas}} = \frac{1}{2}k(x - x_0)^2$

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  - Elastic potential energy: $U_{\text{elas}} = \frac{1}{2}k(x - x_0)^2$
- Gravitational potential energy in general: $U_G = -\frac{Gm_1 m_2}{r}$