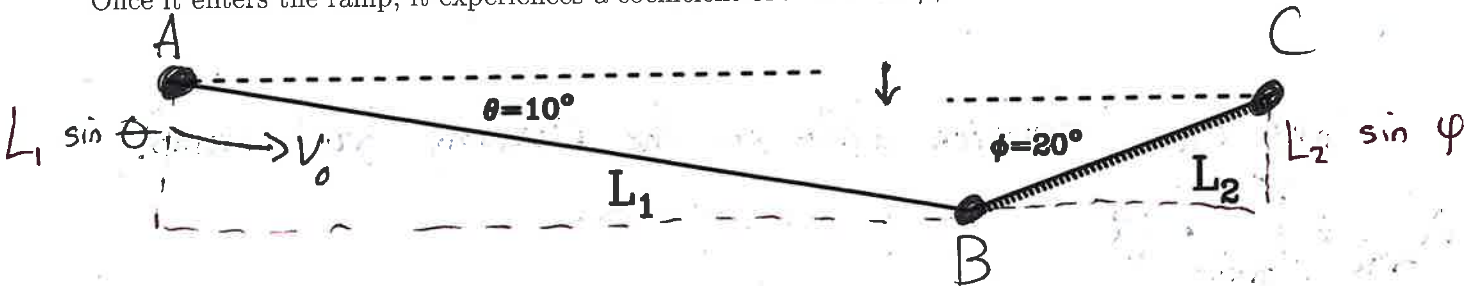
PHYSICS 211 PRACTICE EXAM 3
QUESTION 1

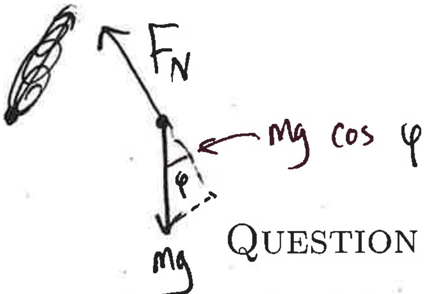
Heavy trucks driving down steep mountains must continually apply their brakes to maintain a safe speed. If their brakes fail, these roads are equipped with "runaway truck ramps", which are short uphill pathways (made of sand or gravel) with a large coefficient of rolling friction. A truck whose brakes fail can steer into the ramp and come safely to a stop. Suppose that a truck of mass m is driving down the hill at a speed v_0 when its brakes fail completely. It is a distance L_1 away from the ramp, traveling at a speed v_0. When it reaches the ramp, it exits the highway and heads up the ramp, traveling a distance L_2 before coming to rest. In this problem, you will calculate the distance L_2 in terms of μ_r, g, m, L_1, v_0, θ, and ϕ.

Since the truck's brakes have failed completely, it has essentially no friction while it is on the road. Once it enters the ramp, it experiences a coefficient of friction of μ_r.



a) Write an expression for the total work done by gravity during the entire motion in terms of g, m, L_1, L_2, θ, and ϕ. If gravity does no work, explain why. *(5 points)*

$$W_{grav} = \vec{F}_{grav} \cdot \Delta\vec{s} = mg \cdot (\text{distance moved parallel to } F_g)$$
$$= mg \cdot (\text{distance moved down})$$
$$= mg[L_1 \sin\theta - L_2 \sin\varphi]$$



$\rightarrow$ note $F_N = mg \cos \varphi$

QUESTION 1, CONTINUED

b) Write an expression for the total work done by the normal force during the entire motion in terms of g, m, L_1, L_2, θ, and ϕ. If the normal force does no work, explain why. *(5 points)*

None!

$$W_{normal} = F_{normal} \cdot \underbrace{(\text{distance moved in direction of } F_{normal})}_{\text{zero!}}$$

c) Write an expression for the total work done by friction during the entire motion in terms of μ_r, g, m, L_2, and ϕ. If friction does no work, explain why. *(5 points)*

Only have friction while on the runaway truck ramp:

friction opposite motion

$$W_{fric} = \vec{F}_{fric} \cdot \Delta\vec{s}$$

$$= [\mu F_N]\,[-L_2]$$

$$= -\mu m g L_2 \cos \varphi$$

d) Write a statement of the work-energy theorem/conservation of energy in terms of μ_r, g, m, L_1, L_2, v_0, θ, and ϕ that you could solve for L_2. (You do not need to solve it.) *(10 points)*

See diagram: $KE_A + W_{A\to C} = \underbrace{KE_C}_{\text{zero}}$

$$KE_A + W_{\substack{fric\\A\to C}} + W_{\substack{grav\\A\to C}} + \underbrace{W_{\substack{normal\\A\to C}}}_{\text{zero}} = 0$$

$$\underbrace{\tfrac{1}{2} m v_A^2}_{KE_A} + \underbrace{(-\mu m g L_2 \cos \varphi)}_{W_{fric}\text{ (from above)}} + \underbrace{mg\,[L_1 \sin\theta - L_2 \sin\varphi]}_{W_{grav}\text{ (from above)}} = \underbrace{0}_{KE_C}$$

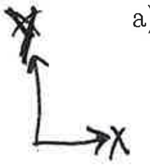
This is a collision $\longrightarrow$ use conservation of momentum (in 2D: $\vec{p}$) vector!

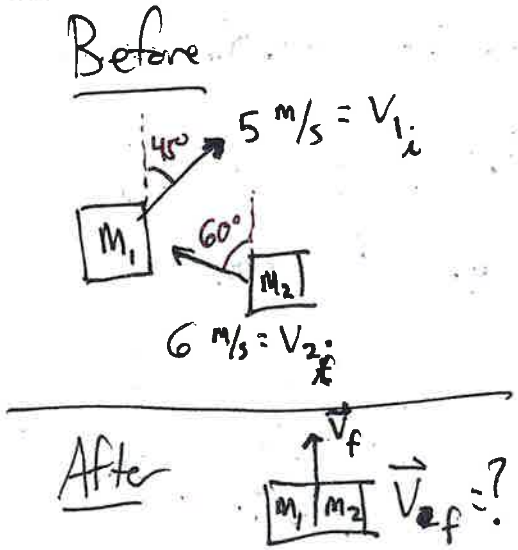
QUESTION 2

A rugby player with a mass of $M_1 = 80$ kg is running 45 degrees east of north at a speed of 5 m/s. She is tackled by another player running 60 degrees west of north at a speed of 6 m/s. After the impact, the two players are moving together directly north (at the same velocity).

a) What is the mass of the second player? *(15 points)*

M_2





Note $V_{f_x} = 0$

$$\sum \vec{p_i} = \sum \vec{p_f} \begin{cases} \sum p_{i_x} = \sum p_{f_x} \\ \sum p_{i_y} = \sum p_{f_y} \end{cases}$$

$\vec{p}$ is vector, treat X and Y separately

$$X: \; m_1 (V_{1_i} \cos 45°) - m_2 (V_{2_i} \cos 60°) = \underbrace{(m_1 + m_2) V}_{\text{zero}}$$

Can find m_2 just from X-component:

$$X: \; m_1 (V_{1_i} \sin 45°) - m_2 (V_{2_i} \sin 60°) = 0$$

$$m_2 = m_1 \frac{V_{1_i} \sin 45°}{V_{2_i} \sin 60°} = 54 \text{ kg}$$

b) How fast are the two players moving after the collision?

Now we use the y-momenta to find V_{y_f}:

$$Y: \; \underbrace{m_1 (V_1 \cos 45°)}_{\text{initial y-momentum of 1}} + \underbrace{m_2 (V_2 \cos 60°)}_{\text{initial y-momentum of 2}} = \underbrace{(m_1 + m_2) V_{f_y}}_{\text{final y-momentum}}$$

$$V_{f_y} = \frac{m_1 V_1 \cos 45° + m_2 V_2 \cos 60°}{m_1 + m_2} = 3.90 \text{ m/s}$$

Question 3

Suppose that a student removes the little spring from a clicky-pen and uses it to shoot little pebbles vertically into the air. (Perhaps they are bored in class, or perhaps they are experimentally verifying Hooke's law!)

Suppose that their spring is capable of propelling a small pebble $\overbrace{\text{2 meters into the air}}^{h}$ if it is compressed by $\underbrace{\text{2 cm}}_{d}$.

a) What would the new maximum height be if they compressed their spring only 1 cm? *(5 points)*

elastic potential energy $\rightarrow$ gravitational potential energy

$$\frac{1}{2}kd^2 = mgh$$

$\rightarrow$ Note h proportional to d^2, so if $d \rightarrow \frac{1}{2}d$,

then $h \rightarrow (\frac{1}{2})^2 \rightarrow \frac{1}{4}h = 50$ cm.

b) What would the new maximum height be if they compressed the spring by 2 cm, but now replaced the spring with one that had a spring constant twice as big (from their neighbor's larger clicky-pen)? *(5 points)*

Again $\frac{1}{2}kd^2 = mgh$. $\Rightarrow$ if $k \rightarrow 2k$, then

$h \rightarrow 2h \Rightarrow$

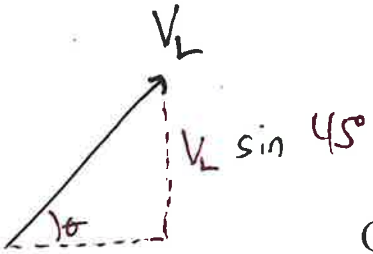
$\boxed{\text{4 meters,}}$

c) What would the new maximum height be if they went back to their original spring, compressed it by the original 2 cm, but replaced their small pebble with a larger pebble that had twice the mass? *(5 points)*

Again: $\frac{1}{2}kd^2 = mgh$, so $h = \frac{1/2\, kd^2}{mg}$.

If $m \rightarrow 2m$, then $h \rightarrow \frac{1}{2}h$

$\Rightarrow \boxed{\text{1 m.}}$



d) What would the new maximum height be if they went back to their original spring and launched their pebble at a 45° angle above the horizontal, instead of vertically? *(5 points)*

Think about 2D kinematics, using the y-component!

$V_{fy}^2 - V_{oy}^2 = 2a_y \Delta h$: at highest point, $V_{fy} = 0$

$-V_{oy}^2 = -2gh \rightarrow h = \dfrac{V_{oy}^2}{2g}$. If $V_{oy} \rightarrow V_{oy} \sin 45°$,

$h \rightarrow h[\sin 45°]^2 = \frac{1}{2}h$

$= \boxed{1\,m}$

e) Suppose the student launches their small pebble (using their original spring, compressed by 2 cm) from the surface of their desk at some angle θ above the horizontal. It flies across the room and hits the floor.

The speed v_f with which it hits the floor doesn't depend on the angle θ. Explain why v_f doesn't depend on the angle it was launched at. *(5 points)*

"Why do all these have the same magnitude of V_f?"

In all cases energy is conserved and energy is a scalar.

$$\underbrace{\frac{1}{2}mv_i^2 + W_{grav} = \frac{1}{2}mv_f^2}$$

nothing here depends on the angle of $\vec{v}_i$, just its magnitude.

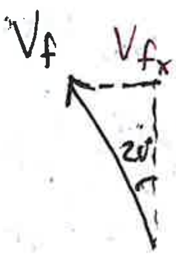
Collision $\rightarrow$ conservation of momentum

QUESTION 4

A person of mass 50 kg is ice-skating on a frozen lake with his dog Kibeth, who has a mass of 15 kg. He is skating due north at 3 m/s.

Kibeth realizes that he's carrying snacks in his pocket, and would like one for herself. (Or maybe she is just being friendly!) She runs after him and tackles him from behind and the side, knocking him down. The two of them collapse on the ice and begin to slide, as Kibeth tries to get the treats out of his pocket; they are moving together at an angle 20 degrees west of north at 4 m/s.

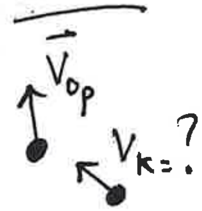
What was Kibeth's velocity before she tackled him? *(Remember velocity is a vector.)* *(25 points)*



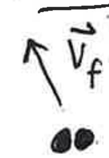
$V_{f_x} = -V_f \sin 20°$

$V_{f_y} = V_f \cos 20°$

Before:



After:



$$P_{0_x} = P_{f_x} : \quad m_k V_{x_{k,0}} = -(m_p + m_k) V_f \sin 20°$$

$$\rightarrow V_{x_{k,0}} = \frac{-(m_p + m_k) V_f \sin 20°}{m_k} = 5.92 \text{ m/s}$$

$$P_{0_y} = P_{f_y} : \quad m_k V_{y_{k,0}} + m_p V_{y_{p,0}} = (m_p + m_k) V_f \cos 20°$$

$$\rightarrow V_{y_{k,0}} = \frac{(m_p + m_k) V_f \cos 20° - m_p V_{y_{p,0}}}{m_k}$$

$$= 6.28 \text{ m/s}$$

$$\Rightarrow \boxed{\vec{V}_{k_0} = (5.92, 6.28) \text{ m/s}}$$

- Need conservation of $\vec{p}$ to understand explosion
- Need 2-D kinematics to understand trajectory of cannonball
- Need work-energy to understand cannon sliding to a stop

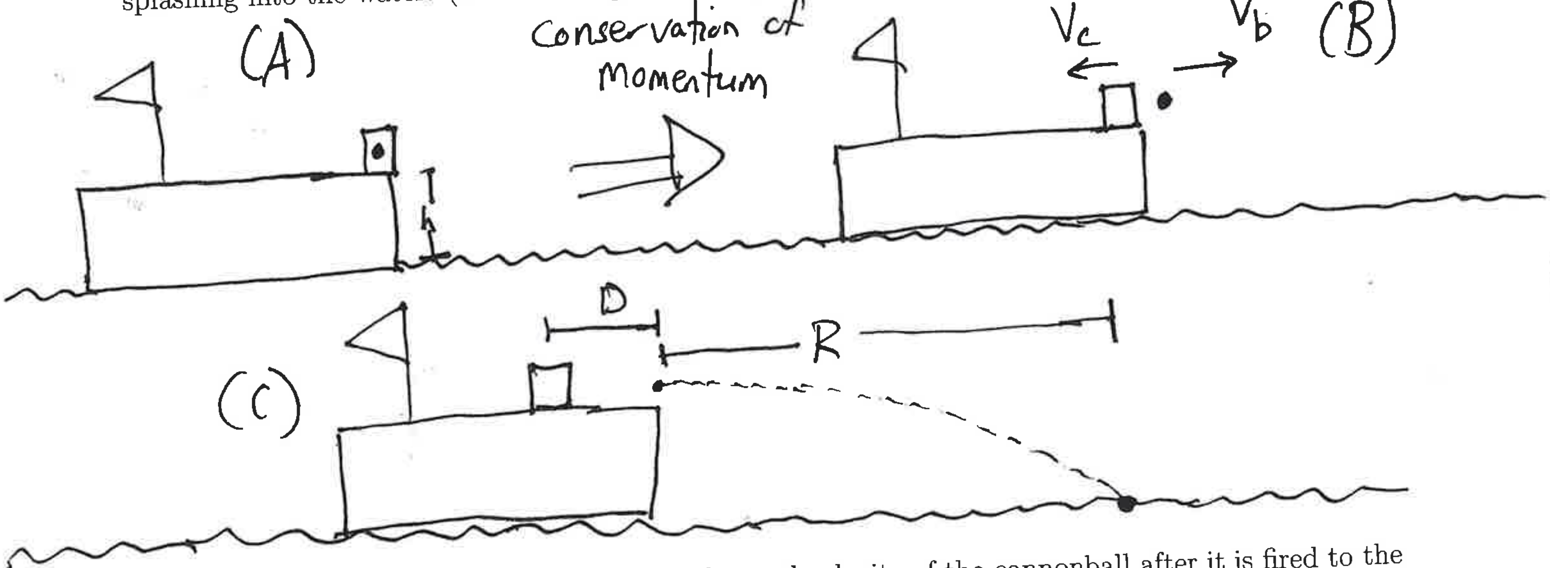
QUESTION 5

The dread pirate captain Piarrrr Squared has a peg leg, a parrot, and a ship whose horizontal deck is a height h above the water. The captain has just found a new cannon and has hauled it to the edge of the deck. The coefficient of kinetic friction between the cannon and the deck is μ_k.

The cannon is very massive; it has a mass a hundred times greater than the cannonballs that it fires.

Piarrrr test-fires the cannon, which launches a cannonball horizontally; it flies out to sea, and the cannon slides back a distance D before coming to rest.

In this question, you will determine the range R[†] that the cannonball travels out to sea before splashing into the water. (The below space is for you to draw diagrams, if you choose.)



a) What technique can you use to relate the forward velocity of the cannonball after it is fired to the backward velocity of the cannon? Determine this relationship. *(5 points)*

Conservation of momentum:

$\sum \vec{p}_i = \sum \vec{p}_f$

0

$$0 = \underbrace{(100m)v_c}_{\text{momentum of cannon}} + \underbrace{mv_b}_{\text{momentum of ball}}$$

$$V_b = -100\, V_c$$

b) What technique can you use to relate the backward velocity of the cannon right after it is fired to the distance that it slides before coming to rest? Determine this relationship. *(5 points)*

Work-energy theorem:

$KE_{B} + W_{all} = KE_{C}$

(from (B) to (C))

(just for cannon)

$$\frac{1}{2}M_c v_c^2 + W_{fric} = 0$$

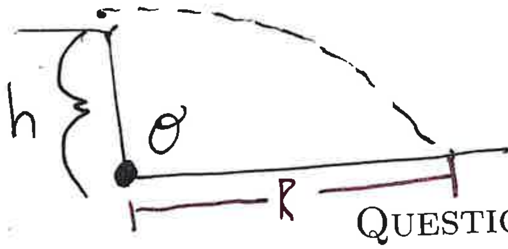
- $W_{fric} = (-F_{fric})(D)$

$= -\mu M_c g d$

$$\frac{1}{2}\cancel{M_c} v_c^2 - \mu \cancel{M_c} g d = 0$$

$$v_c^2 = 2\mu g d$$

[†]Pronounced "arrrrrrr", of course.



$$\left.\begin{aligned} x(t) &= v_b t \\ y(t) &= h - \tfrac{1}{2} g t^2 \end{aligned}\right\}$$ recall $\vec{s}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$

QUESTION 5, CONTINUED

c) What technique can you use to relate the forward velocity of the cannon after it is fired to the range that the cannonball flies out to sea? Determine this relationship. *(5 points)*

2D kinematics (like HW2 stuff):

"What is $x(t)$ at the time that $y(t) = 0$?"

$$0 = h - \frac{1}{2} g t^2 \Rightarrow t = \sqrt{2h/g}$$

$$x(t) = V_b \sqrt{\frac{2h}{g}} \Rightarrow \boxed{R = V_b \sqrt{\frac{2h}{g}}}$$

d) Putting together the above information, if the cannon slides a distance D backwards, determine the range R that the cannonball flies out to sea, in terms of D, g, μ_k, and h. *(15 points)*

From earlier:

$V_b = -100\, V_c$ and $V_c^2 = 2\mu g D$.

$\rightarrow V_c = -\sqrt{2\mu g D}$ (goes left, take negative $\sqrt{\ }$)

Substitute:

$V_b = 100\sqrt{2\mu g D}$

$$R = V_b \sqrt{2h/g} = 100\sqrt{2\mu g D}\sqrt{\frac{2h}{g}}$$

$$\boxed{R = 100\sqrt{4\mu D h} = 200\sqrt{\mu D h}}$$

Question 6

People with impaired mobility sometimes use battery-powered wheelchairs to travel. Suppose that you are an engineer in a very hilly city and are tasked with building a wheelchair suitable for people to get around in your town. They give you the following design specifications:

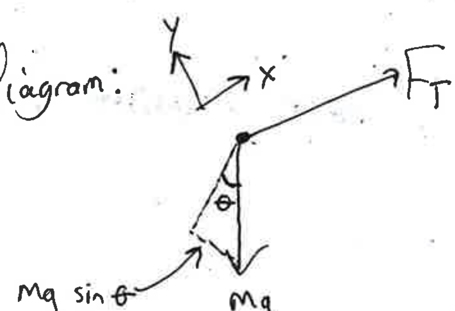
- The combined weight of the wheelchair (including its batteries) and the person it is transporting will be 100 kg
- It has to be able to climb a 10° hill that is 1 km long at a speed of 0.8 m/s] related to power of motor
- It has to be able to climb this hill five times before its batteries run out] related to potential energy in battery

Assume that the battery and the motor are 100% efficient at converting electrical energy into mechanical work, and that there is no rolling friction.

a) You first must choose a motor to put in the wheelchair. How much power must the motor produce to meet the design requirement? *(10 points)*

The motor creates an upward traction force by turning the wheels $\rightarrow P_{motor} = P_{trac} = \vec{F}_{trac} \cdot \vec{V}$.

Force diagram:

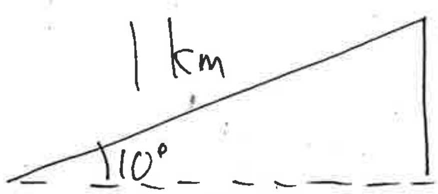


In X: $F_{trac} - mg \sin\theta = ma_x = 0$

$\rightarrow F_{trac} = mg \sin\theta$

$P_{trac} = mgv \sin\theta = 139\ W$

b) Suppose you are using lithium-ion batteries that can store 800 kJ per kilogram. What mass must the wheelchair's battery have to meet the design requirement? *(10 points)*



$h = 1\text{ km} \sin 10° = 174\ m$

Need to convert (mgh) of chemical PE to grav PE to climb hill

$= (100\text{ kg})(10\ m/s^2)(174\ m) = 174\ kJ$ to climb hill once

$\times 5 = 870\ kJ$ total PE required $\Rightarrow \dfrac{870\ kJ}{800\ \frac{kJ}{kg}} = 1.09$ kg battery

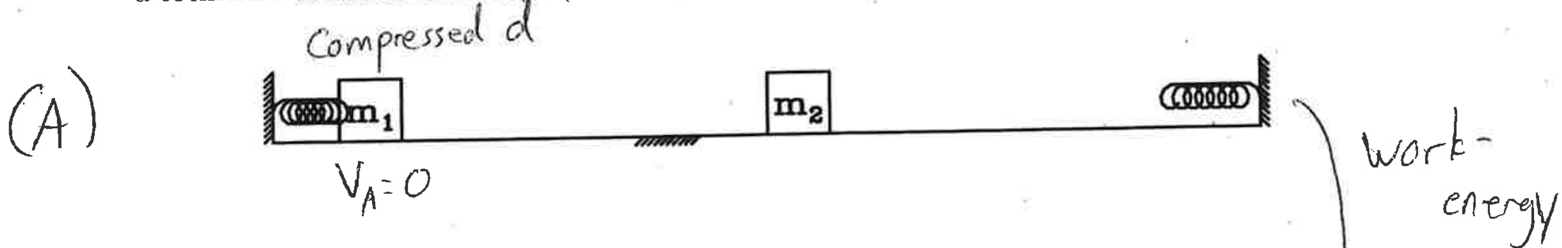
Question 7

Question 7

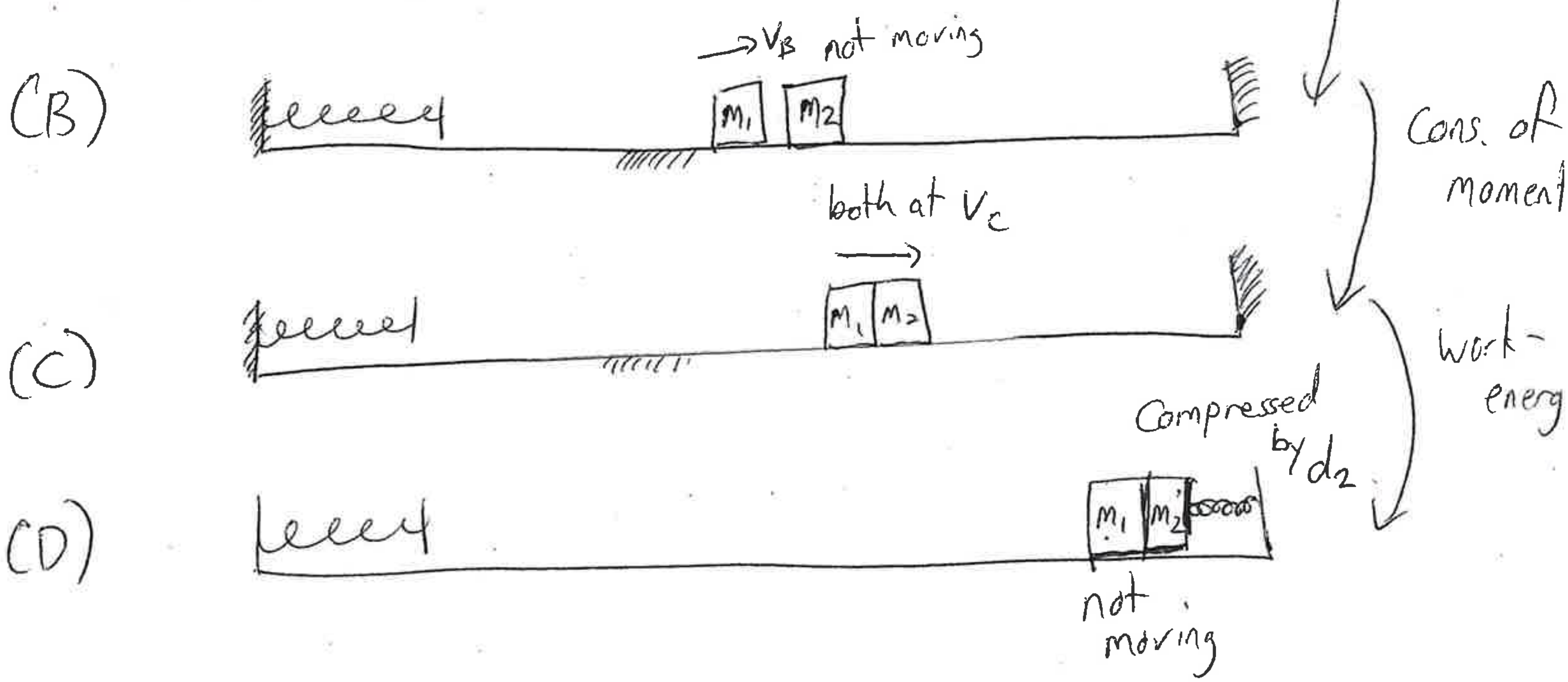
A spring of spring constant k is compressed by a distance d by a mass m_1 and released. This propels the mass down a flat track. Another spring, also of spring constant k, is on the other side.

Another object of mass m_2 is sitting in the middle of the track. The first mass strikes it and sticks to it.

The entire track is frictionless, except for a small region of the track to the left of m_2, of width b, with a coefficient of kinetic friction μ. (This is indicated by diagonal lines on the track in the diagram.)



(The space below is for you to draw other diagrams, if you so choose, or to do scratch work.)



a) How fast is the first mass moving right before it collides with the second block? (5 points)

$$\cancel{KE_A}^{0} + PE_A + W_{A\to B} = KE_B + \cancel{PE_B}^{0}$$

$$\frac{1}{2}kd^2 - \mu m_1 g b = \frac{1}{2}m_1 V_B^2 \longrightarrow V_B = \sqrt{\frac{kd^2}{m_1} - 2\mu g b}$$

b) How fast are the two masses moving right after the collision? (5 points)

$$\sum p_B = \sum p_c \longrightarrow m_1 V_B = (m_1 + m_2) v_c$$

$$V_c = \frac{m_1}{m_1 + m_2} V_B = \frac{m_1}{m_1 + m_2}\sqrt{\frac{kd^2}{m_1} - 2\mu g b}$$

c) When the two blocks reach the spring on the other side, they will bounce off of it, compressing it in the process. What is the maximum distance that this spring is compressed? (15 points)

$KE_c = PE_D$ (other terms 0)

$$\frac{1}{2}(m_1 + m_2) v_c^2 = \frac{1}{2}kd_2^2$$

$$\longrightarrow d_2 = \frac{m_1 + m_2}{k} v_c^2 \quad \text{with } v_c \text{ as above}$$

Question 8

Two billiard balls of equal mass are resting next to each other on a pool table.

A player hits them with a third ball (also of equal mass); this third ball is traveling at a velocity $v_3 = (0, 4)$ m/s.

The three balls bounce off one another. After the collision the velocities of the first two balls are:

- $v_1 = (-1, 2)$ m/s
- $v_2 = (2, 1)$ m/s

a) What is the velocity vector of the third ball after the collision? *(15 points)*

$$\sum \vec{p_i} = \sum \vec{p_f} \begin{cases} \sum p_{x_i} = \sum p_{x_f} : \quad 0 = m(-1\,\text{m/s} + 2\,\text{m/s} + V_{3_{f_x}}) \\ \sum p_{y_i} = \sum p_{y_f} : \quad m(4\,\text{m/s}) = m(2\,\text{m/s} + 1\,\text{m/s} + V_{3_{f_y}}) \end{cases}$$

Solve: $V_{3_{f_x}} = -1$ m/s

$V_{3_{f_y}} = 1$ m/s

b) What fraction of the initial kinetic energy was lost during the collision? *(10 points)*

Note: $V^2 = V_x^2 + V_y^2$. So: $KE_i = KE_{3_i} = \frac{1}{2}m\left[16\,\frac{m^2}{s^2}\right]$

$$KE_f = \frac{1}{2}m\left(V_{1_{f_x}}^2 + V_{2_{f_x}}^2 + V_{3_{f_x}}^2 + V_{1_{f_y}}^2 + V_{2_{f_y}}^2 + V_{3_{f_y}}^2\right)$$

$$= \frac{1}{2}m\left[(1 + 4 + 1 + 4 + 1 + 1)\left(\frac{m^2}{s^2}\right)\right]$$

$$= \frac{1}{2}m\left[12\,\text{m}^2/\text{s}^2\right].$$ 12 is 75% of 16, so

$\boxed{\text{25\% lost.}}$