Energy: the work-energy theorem

Physics 211
Syracuse University, Physics 211 Spring 2015
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Announcements

- Exams (+ regrades) will be returned tomorrow in recitation
- If you didn't get your exam 1 back, email me
- Exam average etc. will be posted once I have it
  - The average of the 65 grades from one TA is 66/100; remember this can be no worse than a B-
- Next homework will be short, posted Friday, and due Friday after break
- Next Mastering Physics will be short, posted Friday, and due before class Tuesday

Energy methods, in general

- "Conventional" kinematics: compute $\vec{x}(t)$, $\vec{v}(t)$
  - "Time-aware" and "path-aware" – tells us the history of a thing's movement
  - Time is an essential variable here
- Newton's second law: forces $\rightarrow$ acceleration $\rightarrow$ history of movement
- Sometimes we don't care about all of this
- Roll a ball down a track: how fast is it going at the end?

Energy methods, in general

We will see that things are often simpler when we look at something called "energy"

- Basic idea: don't treat $\vec{a}$ and $\vec{v}$ as the most interesting things any more
- Treat v^2 as fundamental: $\frac{1}{2}mv^2$ called "kinetic energy"

Previous methods:

- Velocity is fundamental
- Force: causes velocities to change over time
- Intimately concerned with vector quantities

Energy methods:

- v^2 (related to kinetic energy) is fundamental
- Force: causes KE to change over distance
- Energy is a *scalar*

Energy methods: useful when you don't know and don't care about time

The work-energy theorem in 1D

We've encountered something before that eliminates time as a variable...

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The "third kinematics relation"

$$v_f^2 - v_0^2 = 2a\Delta x$$

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Multiply by $\frac{1}{2}m$:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = am\,\Delta x$$

That thing on the right looks familiar...

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Some new terminology:

- $\frac{1}{2}mv^2$ called the "kinetic energy" (positive only!)
- $F\Delta x$ called the "work" (negative or positive!)
- "Work is the change in kinetic energy"

Sample problem: dropping an object

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$$KE_f - KE_0 = F\Delta y$$

- $KE_0 = 0$
- Work done by gravity: $(-h) \times (-mg) = mgh$
- $KE_f - KE_0 = mgh \to v_f = \sqrt{2gh} = 6.26\mathrm{m/s}$

I throw a ball straight up with initial speed v_0.
Someone catches it at height h. How fast is it going?

Sample problem: Baseball problem (exam 1)

I throw a ball straight up with initial speed v_0.
Someone catches it at height h. How fast is it going?

- $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = (-mg) \times h$
- ... algebra follows: solve for v_f

Multiple pendulum demo

The total work done is zero!

One side has a large Δs and a small F.
One side has a small Δs and a large F.

Work-energy theorem: 2D

We can do this in two dimensions, too:

- $\frac{1}{2}mv_{x,f}^2 - \frac{1}{2}mv_{x,0}^2 = F_x \Delta x$
- $\frac{1}{2}mv_{y,f}^2 - \frac{1}{2}mv_{y,0}^2 = F_y \Delta y$

Add these together:

- $\frac{1}{2}m(v_{x,f}^2 + v_{y,f}^2) - \frac{1}{2}m(v_{x,0}^2 + v_{y,0}^2) = F_x \Delta x + F_y \Delta y$

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- The thing on the left can be simplified with the Pythagorean theorem:
- $\frac{1}{2}m(v_f^2) - \frac{1}{2}mv_0^2 = F_x \Delta x + F_y \Delta y$

- That funny thing on the right is called a "dot product".

Dot products

$$A_x B_x + A_y B_y \text{ is written as } \vec{A} \cdot \vec{B}.$$

What does this mean? It's a way of "multiplying" two vectors to get a scalar (a number).

Dot products

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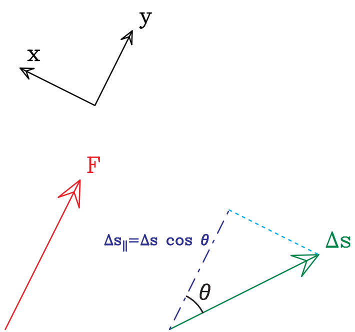
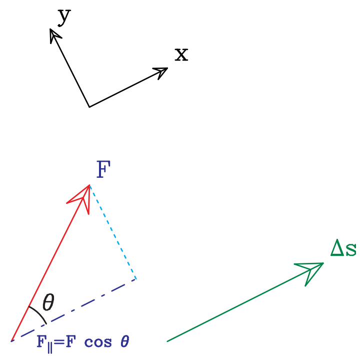
What does this mean? It's a way of "multiplying" two vectors to get a scalar (a number).
We can choose coordinate axes as always: choose them to align either with $\vec{F}$ or $\Delta\vec{s}$.



- $\vec{F} \cdot \Delta\vec{s} = (F)(\Delta s_{\|}) = (F)(\Delta s \cos\theta)$
- "The component of the displacement parallel to the force, times the force



- $\vec{F} \cdot \Delta\vec{s} = (F_{\|})(\Delta s) = (F\cos\theta)(\Delta s)$
- "The component of the force parallel to the motion, times the displacement

Different cases where each form is useful, but it's the same trig either way

Pendulum demos

- What is the work done by the string?

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- The kinetic energy can't go below zero
- The height at each end of the swing must be the same!
- ... and the return height can't be greater than the initial height...

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(If physics stops working and I go splat, have a nice spring break!)

Ball rolling down a ramp demo

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- What is the work done by gravity?
- Use the "force times parallel component of motion" formulation:
- $W = (-mg) \times (y_f - y_0)$ – note both components are negative, for a positive result
- The shape of the ramp doesn't matter: the velocities will all be the same at the end!

Hot Wheels demo

How does the velocity at the middle photogate relate to that at the bottom?

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All I need are the heights; the shape doesn't matter at all!

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Middle: Work done by gravity $= mg(1h)$, $\frac{1}{2}mv^2 = mg(1h), v = \sqrt{2gh}$
Bottom: Work done by gravity $= mg(2h)$, $\frac{1}{2}mv^2 = mg(2h), v = \sqrt{4gh}$

The velocity at the bottom is larger by a factor of $\sqrt{2}$!