Homework 2, due Friday, 1 February

1. Part 4 of Question 2 asks you to compute the acceleration of an object, given the change in position, the initial velocity, and the final velocity. To do this, you will need to use both $x(t)$ and $v(t)$ kinematics equations, but ultimately you will eliminate the variable t.

   Often in mechanics we aren't particularly concerned about time – we're only concerned with the change in position, initial velocity, final velocity, and acceleration. These are related by the "third kinematics equation",

   $$v_f^2 - v_0^2 = 2a(x_f - x_0)$$

   Show that this equation is simply a consequence of the other two. Once you have derived it, you may use it whenever you need to relate change in position, change in velocity, and acceleration, and you don't care about time.

   Algebra hint: Starting from the $x(t)$ and $v(t)$ formulae for constant acceleration, solve one equation for t and substitute it back into the other one.

2. During the siege of Constantinople that led to its conquest by the Ottomans in 1453, the Hungarian engineer Orban built a set of bombards (primitive cannon) to throw enormous stones at the city to breach its walls. The largest of these could throw a 300 kg stone a distance $x_f = 1.8$ km. Assume that the stone was launched at an angle of $\theta = 45°$ above the horizontal; in the absence of air resistance, this gives the largest range.

   (a) What speed did the stone have to be launched at to achieve this range?

   (b) How long was the ball in the air?

   (c) How fast was the ball traveling at the apex of its flight?

   (d) Orban's cannon was 8m long. What was the average acceleration of the stone as it was launched down the bore of the cannon? *Hint: Note that during its movement down the bore of the cannon, it accelerated from $v = 0$ to the velocity you found as your solution to the first part of this problem.*

3. A car drives off of a cliff 60 meters above sea level and splashes into the ocean 150 meters out to sea.

   (a) What was the car's speed when it left the cliff?

   (b) What was the car's speed when it struck the water?

   (c) In what direction was the car traveling when it struck the water? (Give your answer in a physically meaningful way: "X degrees below the horizontal" or similar.

4. A group of Stumpies has captured a particularly athletic frog, and have taken it back to Illick Hall as their pet. This frog can push herself off of the ground with enough velocity to jump 1m high. Note that the initial velocity of the frog, once you find it, is a property of the frog: no matter what we do with the frog, the initial velocity (which determines how high she can jump) is the same.

   (a) With what velocity does the frog leave the ground?

   (b) How long will the frog be in the air before she lands?

5. Our heroic frog gets hungry and wanders away from the Environmental Forest Biology students. She hops down the hall and into the elevator, which is miraculously working today.[1] While the elevator is accelerating upward at $\alpha = 2\,\mathrm{m/s^2}$, she sees a tasty spider in its web at a height $h = 85$ cm above the floor of the elevator. (If the floor of the elevator moves, then the spider will move too.) This is the same frog, so it is capable of jumping with an initial velocity equal to the value you found in the previous problem.

   Suppose that the frog jumps as high as she can to try to catch the spider, pushing off the ground with the same initial velocity as before.

   (a) Draw position vs. time graphs for the frog, the elevator floor, and the spider on one set of axes.

   (b) If the frog jumps as high as she can, will she catch the spider? How far above the elevator floor will she make it?

   Hint: Think very carefully about your coordinate system, and all of the consequences of the accelerating elevator. You may need the quadratic formula for this problem. To determine how far above the elevator floor she will go, write a function that gives the distance above the floor as a function of time, then take its derivative and set it to zero, just as you would in calculus class.

6. Our same famous frog escapes from ESF and is now taken to the planet Twilo, which is quite Earthlike except for its value of $g = 11.8\,\mathrm{m/s^2}$.

   (a) How high can our spacefaring frog jump on Twilo?

   (b) Based on a comparison of the previous two jumping-frog problems, can you make any statements regarding gravity and acceleration?

7. An object's position is given by the vector

$$\vec{s}(t) = (2\,\mathrm{m})\cos(\omega \mathrm{t})\hat{\imath} + (2\,\mathrm{m})\sin(\omega \mathrm{t})\hat{\jmath}$$

   where ω is a constant equal to 1 radian per second.

[1] This being Illick Hall, nobody takes any notice of a stray frog in the hallway.

(a) How would you describe this object's motion?

(b) Hint: if you are stuck, compute and graph its position in the Cartesian plane at a variety of values of t (say, for integers 0-6).

8. A physics student is launching model rockets in the desert. They launch a rocket from rest at an angle $30°$ above the horizontal. While the motor burns, the rocket accelerates at 15m/s^2 directed $30°$ above the horizontal. The motor burns for two seconds and then cuts off; after the motor dies, the rocket is in free fall until it hits the ground.

   If the desert is flat, calculate how far away from its launch point the rocket will land. *Hint: Is the acceleration constant here? How are you going to handle the fact that the acceleration changes midway through the motion?*

9. A hiker is standing on top of a mountain. The mountain slopes downward in front of her at an angle of $45°$.

   She kicks a stone off of the top. The stone's initial velocity is 4 m/s directed horizontally. The rock will fly through the air, curving downward until it comes back in contact with the slope. Where will the rock land back on the mountainside?