

Week 13, Day 2

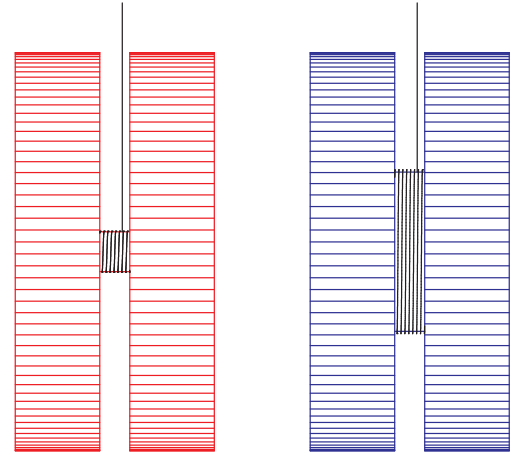
Exercises on combining rotation and translation

Exercise 1: Torque on a yo-yo

A Yo-Yo consists of a cylinder of radius R with a thin slit cut in it. Inside the slit is a smaller inner cylinder of radius r with a string attached to it and then wound around the cylinder. Note that the moment of inertia of a cylinder of radius R is $I = \frac{1}{2}mR^2$; since the slit in the Yo-Yo is so thin, you do not need to consider it in computing the moment of inertia. (Thus, both have the same moment of inertia: $I = \frac{1}{2}mR^2$.)

If a person holds the end of the string and drops the Yo-Yo, it will begin to spin as it falls, unwinding the string as it does.

a) Suppose that you have a red Yo-Yo with $r = 0.1R$ (that is, with a very small inner cylinder) and a blue Yo-Yo with $r = 0.4R$ (with a thicker inner cylinder). Using an argument based on energy, predict which one will fall faster when it is dropped, and describe why it will do so. (*You shouldn't do any calculations here.*)



b) Now, you'll calculate the downward acceleration of the Yo-Yo. In this case, the Yo-Yo both *translates* and *rotates* as it does so.

Start by drawing an extended force diagram for the Yo-Yo, showing all the forces acting on it *and where they act*. (Think carefully about which perspective your force diagram should show – it's not the one in the diagrams above.)

c) Since it both translates and rotates, you will need both $\vec{F} = m\vec{a}$ to relate the forces on it to its translational acceleration and $\tau = I\alpha$ to relate the torques on it to its linear acceleration. Construct both of these equations, using the forces that appear on your force diagram. (*Hint: The tension in the string both applies a torque to the Yo-Yo and affects its translational acceleration.*)

d) In the above two equations, you will have three unknowns: the tension in the string, the translational acceleration, and the angular acceleration α . However, you can relate two of them to each other. What is that relation? (*Hint: Think carefully about minus signs here!*)

e) Now you should have enough information to solve for a in terms of g , r , and R . Once you have a value for your acceleration, call your GTA or coach over and have them check your work. Discuss with them whether the red or blue Yo-Yo in part (a) would fall faster.

Exercise 2: a Ping-Pong ball on a table

Some people are playing Ping-Pong outdoors and have left a ball of mass m on the table when a gentle breeze begins; this wind applies a constant horizontal force F_w on the ball. The coefficient of static friction between the ball and the table is μ_s , and the coefficient of kinetic friction is μ_k . The Ping-Pong ball is a hollow shell and has a moment of inertia $\frac{2}{3}mr^2$.

1. Suppose first that the breeze is very gentle, so that the ball rolls smoothly on the table without slipping. If F_w is very small, will the frictional force on the ball be $\mu_s mg$, $\mu_k mg$, or some other value F_f that you don't know yet? Discuss this with your group and call your coach or TA over to join your conversation. (*Don't continue here until you've discussed this with one of your instructors.*)
2. Determine the ball's translational acceleration a and angular acceleration α in terms of F_w and m . (You will need to do all the usual things that you did during the last problem – draw a force diagram, etc.)

3. Now, suppose that the wind steadily increases in strength. What is the largest wind force F_w for which the ball will roll without slipping? (*What other force limits how strong the wind can be?*)

4. Suppose that the wind becomes even stronger, so that the ball skids across the table. Now determine both its translational acceleration a and its angular acceleration α .