# Momentum

Physics 211
Syracuse University, Physics 211 Spring 2023
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# Announcements

- Upcoming office hours:
  - Friday, 9:30-11:00 AM, in the Physics Clinic
  - Next Tuesday, 3-5 PM
  - Next Wednesday, 3-5 PM
  - Next Thursday, 3-5 PM
- HW5 posted later today, due next Friday
- One question on HW5 will involve the material from next Thursday
- We usually don't do this, but the alternative is homework over break (which I don't ever do)

# Taking stock

We now know how to relate the forces on objects to how they move.

If the forces depend on position, we have to reexamine them every time something moves:

- Doing this by hand leads to **differential equations**
- We can do this by computer, though!
- If we continue down this road: **computational dynamics**

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Can we use Newton's laws to relate the initial state to the final state without knowing all the details of what happens in the middle?

# Conservation laws

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Examples:

- A roller coaster car rolls down a complicated curved track. What's its speed at the bottom?
  - The forces as the roller coaster travels down the track change constantly and are complicated

- How fast must a rocket be launched to escape the Solar System?
  - The force of gravity (from all the planets, too!) changes direction and is complicated

- With what velocity will a gun recoil after it fires?
  - We don't know anything about gas pressure yet
  - How exactly does gunpowder burn?

- A fast-moving neutron bounces off an atom of graphite; how much will it slow down?
  - We know *nothing* about forces in nuclear physics
  - The people who built the first nuclear reactor didn't really, either
  - They were still able to answer this question!

# Conservation laws

We can answer these questions with *conservation laws*:

$$\text{(initial state)} \rightarrow \text{(complicated interaction)} \rightarrow \text{(final state)}$$

It turns out that even if the interaction in the middle is complicated, we can make some simple statements relating the initial state to the final one.

These are called *conservation laws.* We will study two in this unit:

- The conservation of momentum: related to the effect of forces over time
- The conservation of energy: related to the effect of forces over distance

# Newton's third law: consequences

What happens if someone sitting on a cart throws a heavy ball forward, with a mass 10% of her mass?

**Pick the one that is *not* true:**

- A: She pushes the ball forward, so it must push her backwards, by Newton's 3rd Law
- B: The change in the ball's velocity is equal and opposite to the change in her velocity
- C: The change in her velocity is going to be in the opposite direction, and 10% as big, as the change in the ball's velocity
- D: The ball's acceleration will at all times be equal and opposite to hers

# Newton's third law: consequences

Newton's third law tells us:

$$\vec{F}_{BA} = -\vec{F}_{AB}$$

Combining this with Newton's second law we know:

$$m_A \vec{a}_A = -m_B \vec{a}_B$$

Since we know the area under the acceleration vs. time curve is the change in velocity, we can take integrals of both sides:

$$m_A(\vec{v}_{A,f} - \vec{v}_{A,i}) = -m_B(\vec{v}_{B,f} - \vec{v}_{B,i})$$

We can then rearrange this to put all the "initial" things on the left, and the "final" things on the right:

$$m_A \vec{v}_{A_i} + m_B \vec{v}_{B_i} = m_A \vec{v}_{A_f} + m_B \vec{v}_{B_f}$$

# Momentum: overview

We call $m\vec{v}$ the *momentum*, just so we have a name for it. Thus we can write, instead:

$$\sum \vec{p}_i = \sum \vec{p}_f$$

- Momentum is the time integral of force: $\vec{p} = \int \vec{F}\,dt$
- Momentum is a **vector**, transferred from one object to another when they exchange forces
- Another way to look at it: **force is the rate of change of momentum**
- Newton's 3rd law says that total momentum is constant
- Mathematically: $\vec{p} = m\vec{v}$
- Helps us understand **collisions** and **explosions**, among others

# Conservation of momentum

- Newton's third law means that forces only *transfer* momentum from one object to another
- The force between $A$ and $B$ leaves the total momentum constant; it just gets transferred from one to the other
- The total change in momentum is zero!
- **Remember momentum is a vector!**
- Solving problems: create "before" and "after" snapshots
- Just add up the momentum before and after and set it equal!

# When we need this idea: collisions and explosions

Often things collide or explode; we need to be able to understand this.

- Very complicated forces between pieces often involved: can't track them all
- These forces are huge but short-lived, delivering their impulse very quickly
- Other forces usually small enough to not matter during the collision/explosion
- Use conservation of momentum to understand the collision

The procedure is always the same:

$$\sum \vec{p}_i = \sum \vec{p}_f \text{ “Momentum before equals momentum after”}$$

Make very sure your “before” and “after” variables mean what you think they mean!

# Applying conservation of momentum to problems

- 1. Identify what process you will apply conservation of momentum to
  - Collisions
  - Explosions
  - Times when no external force intervenes
- 2. Draw clear pictures of the "before" and "after" situations
- 3. Write expressions for the total momentum before and after, in both $x$ and $y$
- 4. Set them equal: Write $\sum p_i = \sum p_f$ (in both x and y if needed), and solve

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Can we predict the final velocities here?

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- Two carts of mass $m$ and $2m$ separate
- Two carts of mass $m$ and $2m$ traveling at equal speeds collide

## Demos with students

Bob and Alice sit on carts. Bob pulls Alice with a rope. Who moves?

# Demos with students

Bob and Alice sit on carts. Bob pulls Alice with a rope. Who moves? Does throwing or catching a heavy ball change someone's velocity?

- A. Throwing only
- B. Catching only
- C. Both throwing and catching
- D. Only if someone then catches the ball

# Sample problems: a 1D collision

Two train cars moving toward each other at 5 m/s collide and couple together. One weighs 10 tons; the other weighs 20 tons. What is their final velocity?

# Sample problems: a 1D collision

A train car with a mass $m$ is at rest on a track. Another train car also of mass $m$ is moving toward it with a velocity $v_0$ when it is a distance $d$ away. The first car hits the second and couples to it; the cars roll together until friction brings them to a stop.

If the coefficient of rolling friction is $\mu_r$, how far do they roll after the collision?

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If the coefficient of rolling friction is $\mu_r$, how far do they roll after the collision?

Method: use conservation of momentum to understand the collision; use other methods to understand before and after!

## Sample problems: an explosion in 2D

A child on skis has a mass of 40 kg and is skiing North at 3 m/s. He throws a giant snowball of mass 2 kg at his friend; after he throws it, the snowball has a velocity of 10 m/s directed 45 degrees south of west.

What is the child's velocity after he throws the snowball?