PHYSICS 211 FINAL EXAM (2015)

Name: Solutions

Recitation section number: ________

- Look up your recitation section number on the next page.
- There are eight sections worth 20 points each, and a possible 10 points extra credit.
- **You must show your reasoning to receive credit.** A numerical answer with no logic shown will be treated as no answer.
- If you run out of room, continue your work on the back of the page, or on the attached extra pages.
- Remember, show your reasoning as thoroughly as possible for partial credit, including large, clear drawings.
- You may use $g = 10\,\mathrm{m/s^2}$ throughout, except where indicated, to minimize arithmetic.

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Page 1: Short Questions

Answer the following questions:

1. *Saturn has about the same value of g (the acceleration due to gravity at its surface) as the Earth, even though it has about nine times the radius. About how many times more massive than the Earth is Saturn? (5 points)*

$F_g = \frac{GMm}{r^2}$: if $r \rightarrow 9r$, the force gets $9^2 = 81$ times weaker, so $M \rightarrow 81M$ to make F the same.

2. *A low quality speaker system that only reproduces frequencies between 300 Hz and 1200 Hz plays two musical notes at the same time. A listener hears the following frequencies: 300 Hz, 400 Hz, 450 Hz, 500 Hz, 600 Hz, 700 Hz, 750 Hz, 800 Hz, 900 Hz, 1000 Hz, 1050 Hz, 1100 Hz, and 1200 Hz. The fundamental frequencies of both notes are below 300 Hz and thus absent. What are the two fundamental frequencies? (5 points)*

All these are integer multiples of 100 or 150 Hz.

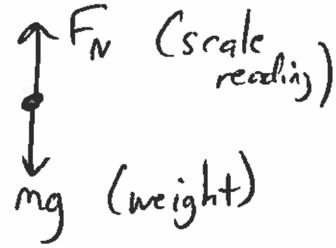
3. *Consider a person standing on a bathroom scale in a bus as it travels over hills and through valleys. If the person weighs 150 pounds, will the scale read greater than 150, less than 150, or equal to 150, when the bus;*

   (a) *... travels over the top of a hill? (2.5 points)*

   $\vec{a}$ is down $\rightarrow$ scale reads less than mg

   (b) *... travels through the bottom of a valley? (2.5 points)*

   $\vec{a}$ is up $\rightarrow$ scale reads more than mg



4. *Explain briefly why the potential energy stored in a stretched spring is $\frac{1}{2}k\Delta x^2$; specifically, where does the factor of $\frac{1}{2}$ come from? (5 points)*

U_e = work done when spring goes back to equilibrium

$$W = \int_{\Delta x}^{0} -kx\, dx = \frac{1}{2}k(\Delta x)^2$$

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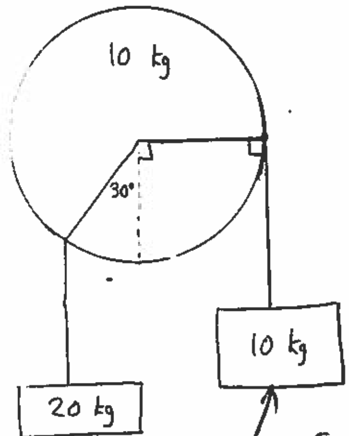
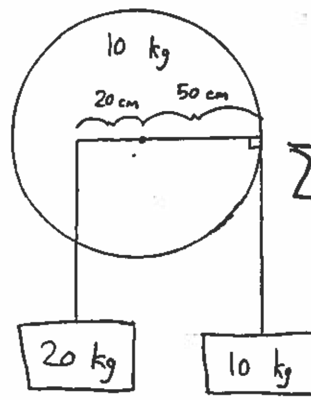
Page 2: Short Questions

Answer the following questions:

1. For the following pictures, will the pulley shown rotate clockwise, counterclockwise, or not at all? If it will rotate, what is its angular acceleration? In both cases, the pulley is a cylinder ($I = \frac{1}{2}mr^2$). (10 points)





Here,

$\Sigma\tau = (20\text{ kg})g \times (20\text{ cm}) - (10\text{ kg})g(50\text{ cm})$

$\tau = -(10\text{ kg})g(r)$

$\Rightarrow \Sigma\tau = -5\text{ N}\cdot\text{m}$

$\tau = (20\text{ kg})g(r\sin 30°)$

$\tau = I\alpha;\ \alpha = \tau/I = \dfrac{-5\text{ N}\cdot\text{m}}{\frac{1}{2}(10\text{ kg})(0.5}$

$\Rightarrow \Sigma\tau = 0$, so it will not rotate

$= \dfrac{-5\text{ N}\cdot\text{m}}{1.25\text{ kg}\cdot\text{m}^2} = -4\text{ rad/s}^2.$

2. A small block of mass M rests on the rightmost edge of a table. It is struck by a projectile of mass m coming from the left at speed v_0; the projectile sticks to the block and the combination immediately falls off the table. The table has height h. The block lands a distance d from the base of the table. Which of the following would increase the distance d by a factor of two? (10 points)

- Doubling the mass m of the projectile, while keeping everything else the same
- Doubling the initial velocity v_0 of the projectile, while keeping everything else the same
- Doubling the height h of the table, while keeping everything else the same
- Reducing the mass M of the block by half, while keeping everything else the same

The 1^{st} and 4^{th} are approximately true if $M \gg m$.

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Question 3

A particular spring has a spring constant of 200 N/m, and is 50 cm long at equilibrium. A 2 kg mass is affixed to one end. A person holds the other end and spins the mass in a circle with an angular velocity of 2 rad/sec; when this is done the spring stretches to some length greater than 50 cm.

Ignore gravity for this problem.

a) Explain in words why the spring stretches when it is spun. (5 points)

The ~~string~~ spring must stretch to provide the needed centripetal force to keep the mass in circular motion.

b) What is the stretched length of the spring? (15 points)

$$k\Delta x = m\omega^2 r \qquad (F = ma)$$

$$k(L - L_0) = m\omega^2 L$$

$$kL - kL_0 = m\omega^2 L \longrightarrow L = \frac{kL_0}{k - m\omega^2}$$

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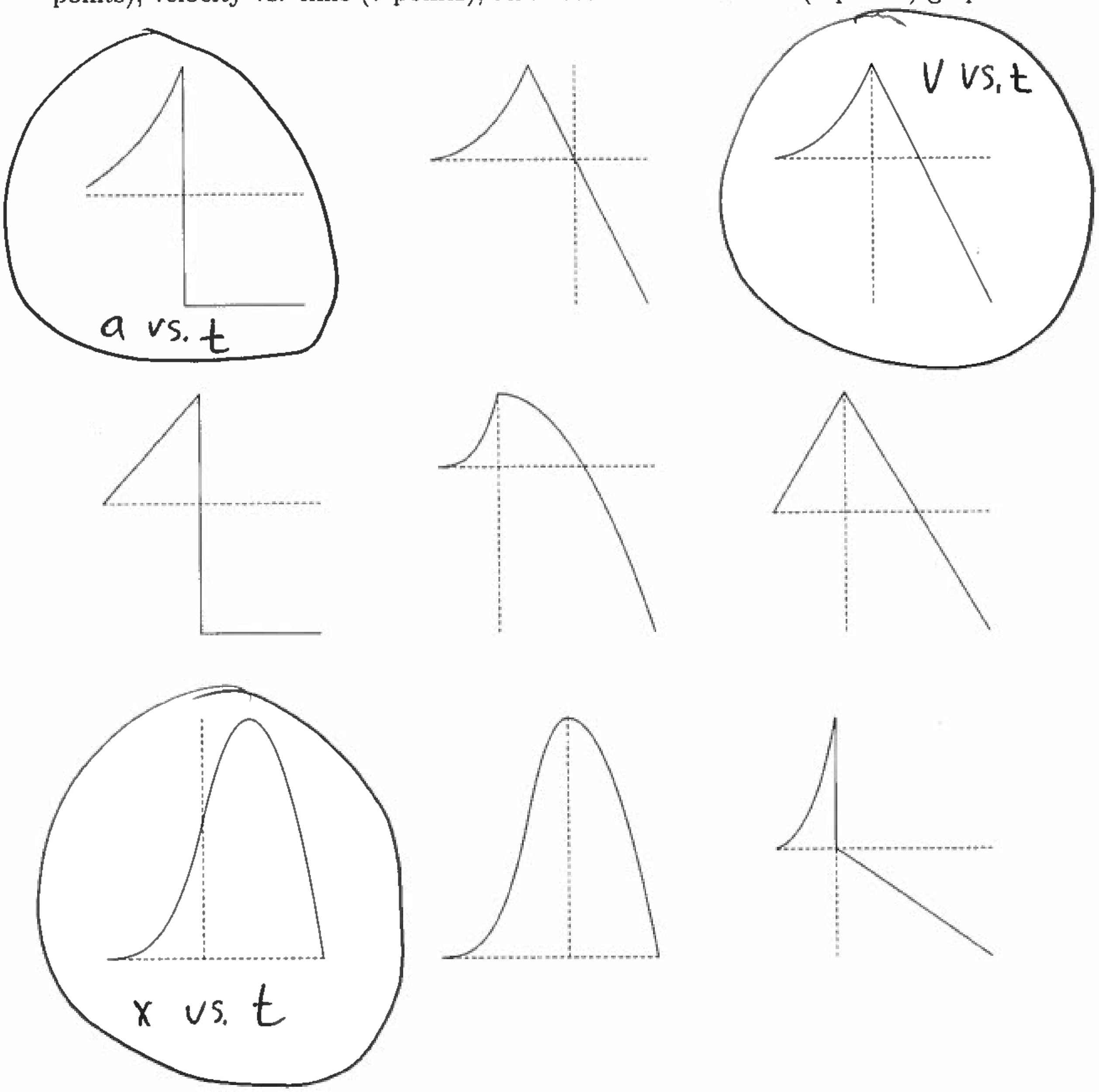
Question 4

A small rocket is pointed upward and launched; half of its mass is fuel, which it consumes in 1 second. During this time, the thrust produces a constant force upward. Once the fuel is exhausted, the rocket continues until it reaches the ground again.

Here are nine possible graphs. The horizontal dotted line is the t-axis; the vertical dotted line represents the time when the fuel is exhausted. Label the correct position vs. time (7 points), velocity vs. time (7 points), and acceleration vs. time (6 points) graphs.



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Question 5

A light ball rests on a table; the coefficient of static friction between the table and the ball is μ_s. A strong wind blows on the ball, causing it to accelerate to the side with a constant force F_w. (This force can be treated as acting on the center of the ball.) Treat the ball as a solid sphere ($I = \frac{2}{5}mr^2$).

If the wind is light, the ball will roll without slipping. If the force is too large, then the ball will slide. In this problem, you will compute the required coefficient of static friction for the ball to roll without slipping, rather than slide.

a) What two laws of physics are you going to use to solve this problem? You may write equations or words. (4 points)

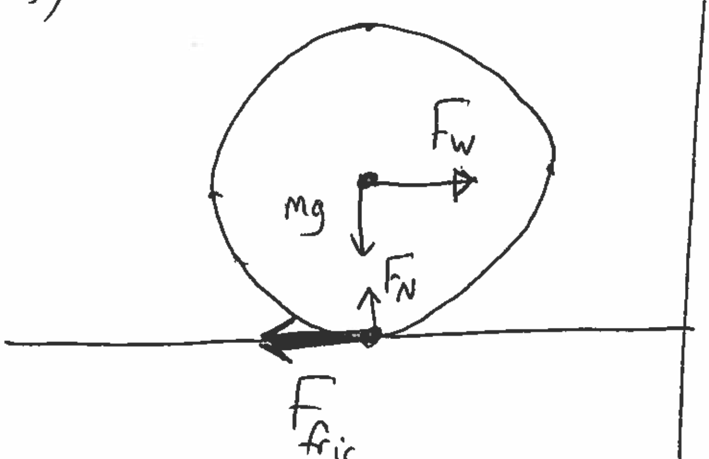
$$\sum \vec{F} = m\vec{a} \ ; \quad \sum \tau = I\alpha$$

b) Draw a force diagram for the ball on the back of this page; include all features that would be helpful to solve the problem. (4 points)

c) What value of μ_s is required for the ball to roll rather than slide? Write a system of N equations and N unknowns (N will probably be two or three) that will let you solve for μ_s in terms of g and F_w. You do **not need to solve the system** *for μ_s. (12 points)*

Hint: Think carefully about your minus signs that relate translational and angular quantities.

b)



c) $F_N - mg = ma_y = 0 \quad (F_y = ma_y)$

$F_w - \mu_s F_N = ma_x \quad (F_x = ma_x)$

$-\mu_s F_N r = \frac{2}{5}mr^2\alpha \quad (\tau = I\alpha)$

$a = -\alpha r$ (rolling constraint)

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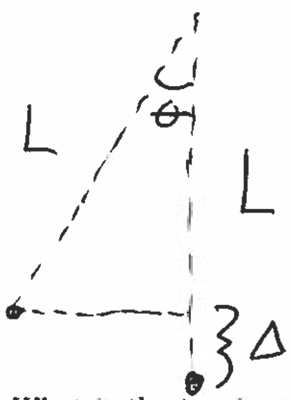
QUESTION 6

A pendulum with a string of length L is drawn back to an angle θ from the vertical and released. The mass of the pendulum bob is m.

a) In terms of L, θ, and g, how fast is the ball traveling when it reaches the lowest point of its swing? (10 points)

Use conservation of energy:



$\Delta h = (L - L\cos\theta)$

$$\cancelto{0}{\frac{1}{2}mv_0^2} + W_{grav} = \frac{1}{2}mv_f^2$$

$$W_{grav} = mg(L - L\cos\theta)$$

$$\Longrightarrow v_f = \sqrt{2g(L - L\cos\theta)}$$

b) What is the tension in the string at that point? (10 points)

$$T - mg = ma = m\omega^2 r = m\,v^2/r$$

I know v:

$$T = mv^2/r + mg = \frac{2mg(L - L\cos\theta)}{L} + mg$$

$$= 3mg - 2mg\cos\theta.$$

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Question 7

A solid ball (moment of inertia $I = \frac{2}{5}mr^2$) and a hollow ball ($I = \frac{2}{3}mr^2$) roll down a hill without slipping.

a) Which reaches the bottom first? (3 points)

The solid ball.

b) What is the ratio of their speeds as they reach the bottom? (12 points)

$mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$ Let $I = \lambda mr^2$.

$mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}\lambda mr^2\omega_f^2 \longrightarrow$ Note: $v_f^2 = \omega_f^2 r^2$

$mgh = \frac{1}{2}mv_f^2[1+\lambda]$

$\rightarrow v_f = \sqrt{\frac{2gh}{1+\lambda}}$ Ratio: $\begin{cases} \text{solid ball: } v_f = \sqrt{\frac{2gh}{7/5}} \\ \text{hollow ball: } v_f = \sqrt{\frac{2gh}{5/3}} \end{cases}$

Ratio is $\sqrt{\frac{7/5}{5/3}} = \sqrt{\frac{21}{25}}$.

c) Why is this question much easier than a question like "How long does it take the ball to roll down the hill"? (In other words, what is different about the techniques you would have to use?) (5 points)

I could use conservation of energy for this. Otherwise, I would need $\vec{F} = m\vec{a}$ + kinematics.

Two children of mass 50 kg each are playing on a merry-go-round, a large horizontal disk that can rotate around its center. It has a mass of 200 kg and is spinning about its center at 0.5 rad/sec, while the children stand on the outside rim.

The children then walk to the center. What is the new angular velocity of the disk? (20 points)

$$L_i = L_f.$$

$$(I_{children} + I_{platform})\,\omega_i = (I_{platform})\,\omega_f$$

$$\left(2mr^2 + \tfrac{1}{2}Mr^2\right)\omega_i = \tfrac{1}{2}Mr^2\omega_f$$

$$\longrightarrow \omega_f = \omega_i\left[\frac{2mr^2 + \frac{1}{2}Mr^2}{\frac{1}{2}Mr^2}\right]$$