Recitation Exercises

Friday, 25 March

If you didn't get to this problem Wednesday, do it today.

An object rests at bottom of an incline that is elevated at an angle θ above the horizontal. Suppose that there is no friction at first. A person slides this object up the incline; it travels a distance D up the incline before it slides back down.

1. What forces act on the object? Determine whether each one does positive work, negative work, or zero work on the way up and on the way down.

   Gravity does negative work on the way up and positive work on the way down. The normal force does no work since it is perpendicular to the motion.

2. Suppose at first there is no friction. What initial velocity does the person have to slide it with for it to travel a distance D before it begins to slide back down?

   Work-energy theorem:

   $$\frac{1}{2}mv_0^2 + W_{all} = \frac{1}{2}mv_f^2$$

   But $v_f = 0$ and the only force doing work is gravity, so

   $$\frac{1}{2}mv_0^2 - mgd\sin\theta = 0 \rightarrow v_0 = \sqrt{2gd\sin\theta}$$

3. When it reaches the base of the incline again, how will its velocity compare to the initial velocity that it had on the way up? *(You should be able to answer this without doing any mathematics.)*

   It'll be the same since gravity is conservative.

4. Now, suppose that there is friction – a coefficient of friction μ_k between the ramp and the object. What initial velocity would the person have to slide it with *now* for it to travel a distance D before it comes back down?

   Now we add the work done by friction. The force of friction is $\mu mg\cos\theta$ and the thing slides a distance d; there is no trigonometric factor here in the dot product since the force and displacement are exactly opposite.

   $$\frac{1}{2}mv_0^2 - mgd\sin\theta - \mu mgd\cos\theta = 0$$

   Solve for v_0:

   $$v_0 = \sqrt{2gd\sin\theta + 2\mu gd\cos\theta}.$$

5. How fast will it be moving *now* when it reaches the bottom of the ramp?

   The hard way to do this is to write down the work-energy theorem from start to finish – noting here that the work done by gravity is zero (since it does negative work on the way up and positive work on the way down), but that kinetic friction does negative work both ways (so $W_f = 2\mu mgd\cos\theta$). The easier way to do it is to observe that it starts at rest at the top of the ramp, and just write the work-energy theorem for the way down:

$$0 + mgd\sin\theta - \mu mgd\cos\theta = \frac{1}{2}mv_f^2 \to v_f = \sqrt{2gd\sin\theta - 2\mu gd\cos\theta}$$

6. If an object travels through some path but comes back to where it starts, like in this case, a force that always does zero work is a *conservative force*. A force that does do work when an object travels along a closed path is a *nonconservative force*.

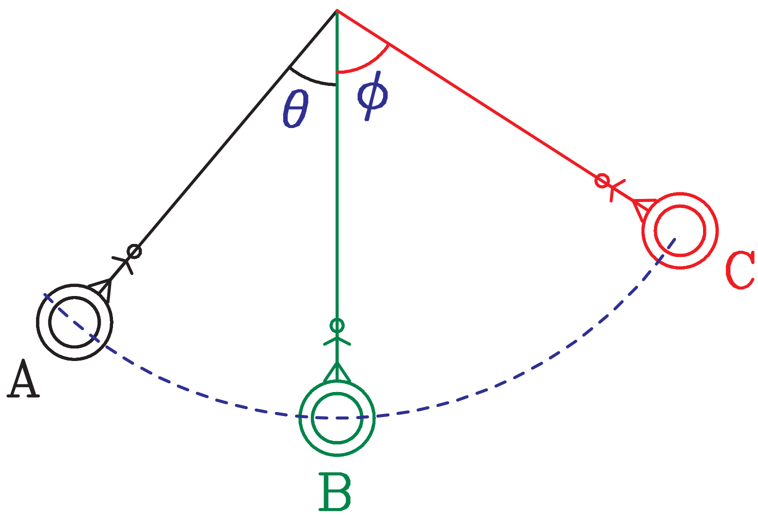
   Three forces appear in this problem: a normal force, gravity, and friction. Is each of these a conservative force? Why or why not?

   Gravity is conservative; friction is nonconservative; the normal force doesn't do any work at all so it's silly to talk about either one.

A child is swinging on a tire swing hanging from a tree. The swing has a length L, and the tire plus the child have a mass m.

The child's older sibling pulls the swing back to an angle θ and releases it from position A.

However, a strong wind is blowing from left to right, exerting a constant horizontal force F_w on the swing to the right. This means that it will swing to a larger angle ϕ on the right (at position C) than it started on the left.



1. Write an equation for the work-energy theorem as the swing moves from A to B. Label in words what each term represents, using language as "kinetic energy at point A", "potential energy at point B", or "work done by wind in moving from A to B".

   This looks gross but it's not. The key is to realize that the work done by gravity depends only on the vertical motion (change in y) and the work done by the wind depends only on the horizontal motion (change in x). Also note that the distance moved down is $L - L\cos\theta$.

   So the work-energy theorem is

$$0 + (L - L\cos\theta)mg + L\sin\theta F_w = \frac{1}{2}mv_B^2.$$

2. Find the speed of the swing at position B in terms of F_w, m, L, θ, and g.

   Do maffs:

$$0 + (L - L\cos\theta)mg + L\sin\theta F_w = \frac{1}{2}mv_B^2$$

$$v_B = \sqrt{2g(L - L\cos\theta) + \frac{2}{m}L\sin\theta F_w}$$

3. Write down an equation that you could solve for the angle ϕ in terms of F_w, m, L, θ, and g. (You do not need to actually solve it.) As before, label each term that appears in your expression for the work-energy theorem.

   Here you could write down the work-energy theorem from B to C and use the v_B above, but it is simpler to just do it from A to C. Note that the distance moved down on the left is $L - L\cos\theta$ and the distance moved up on the right is $L - L\sin\theta$, so we are moving upward in net $L - L\cos\theta - (L - L\cos\phi) = L\cos\phi - L\sin\theta$. This means that the work done by gravity is $-mg\Delta y = mg(L\cos\theta - L\cos\phi)$.

   So now we write down the work-energy theorem from A to C:

   $$0 + F_w(L\sin\theta + L\sin\phi) + mg(L\cos\theta - L\cos\phi) = 0$$

   and that's all they need to do!

4. When the tire swings back to the left, will it stop at position A, will it move further to the left, or will it not reach position A at all? In deciding this, think about the work done by each force as the tire swings from A to C and then back to A. Call your coach or TA over to discuss with your group.

   Both forces are conservative: the wind does positive work going to the right and negative work on the way back.