1D kinematics: position, velocity, and acceleration
(and a calculus review)

Physics 211
Syracuse University, Physics 211 Spring 2016
Walter Freeman

January 20, 2016

The beginning: Free fall

My purpose is to set forth a very new science dealing with a very ancient subject. There is, in nature, perhaps nothing older than motion, concerning which the books written by philosophers are neither few nor small nevertheless I have discovered by experiment some properties of it which are worth knowing and which have not hitherto been either observed or demonstrated....

So far as I know, no one has yet pointed out that the distances traversed, during equal intervals of time, by a body falling from rest, stand to one another **in the same ratio as the odd numbers beginning with unity.**

–Galileo Galilei, *Dialogues and Mathematical Demonstrations Concerning Two New Sciences*, 1638

Announcements

- Homework 1 is due next Wednesday (it's posted)
- We won't start using clickers until next week and no clicker questions will be graded until the following week

- Reminders:
  - Course website: (updated frequently!)
  - Teaching team contact information:
    - Prof. Walter Freeman: wafreema@syr.edu
    - Francesco Serafin: fserafinsyr.edu
    - Lab questions; sasemper@syr.edu

Announcements

Did someone lose a piece of jewelry? Tell me if you did...

There are a lot of cool things in physics that go beyond mechanics.

If you’ve got questions you’d like me to address, send them in and I’ll answer them!

- What’s the Large Hadron Collider for?
- How does a touchscreen work?
- How do 3D movies work?
- What is the Higgs boson?
- How is physics used in video games?
- How does a nuclear bomb work?
- How does a supercomputer work?

A few syllabus clarifications

- Exams and schedule (on website)
  - There are two exams on each topic
  - You may take both, and keep the better grade
  - Since six exams is a lot, “Exam 3; take 2” is on the date of the final

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  - Since six exams is a lot, "Exam 3; take 2" is on the date of the final
- Yes, this means you get to retake every exam and improve your score
- I want to see everyone succeed; if you learn something eventually, your grade should reflect that

Homework tips

Your first homework assignment is due Wednesday.

- Make use of words, pictures, and algebra (not just algebra!) in your reasoning
- We're interested in how you think, not just the answer
- Physical values need to be given with units ("4 meters", not "4")
- Paper is cheap – don't cramp yourself!

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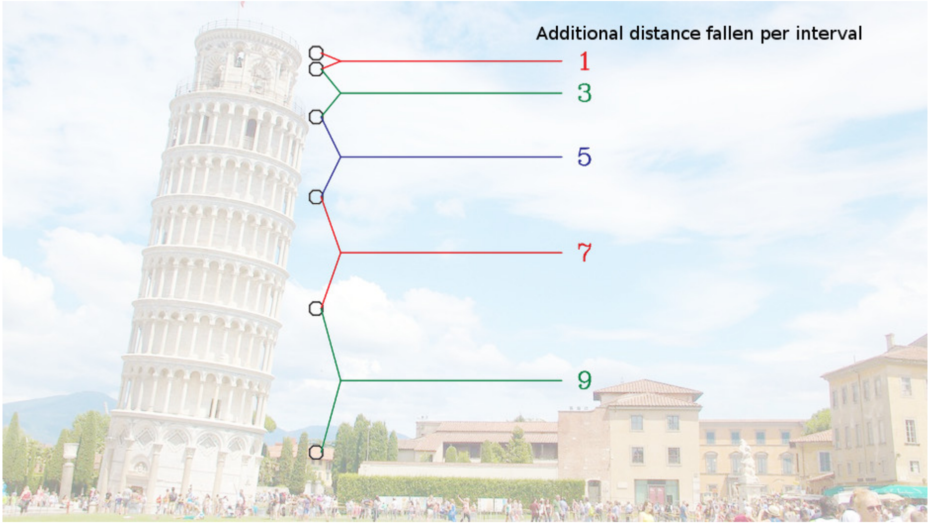
- Ask for help – early and often
  - Email: suphysics211@gmail.com
  - Facebook group
  - Office hours
  - the Physics Clinic
  - Recitations
  - In class!

The beginning: Free fall



Galileo observed this, but can we explain it?

Last time

Constant-velocity motion: $x(t) = x_0 + vt$

- Came from looking at the equation of a line
- We can understand this in a different framework, too:

- Velocity is the rate of change of position
  - Graphical representation: Velocity is the slope of the position vs. time graph
  - Mathematical language: Velocity is the derivative of position

We know we need to know about acceleration ("F=ma") – what is it?

- Acceleration is the rate of change of velocity

Position, velocity, and acceleration

$$\text{Position} \xleftarrow[]{\substack{\text{(derivative of)} \\ \text{rate of change of}}} \text{Velocity}$$

Position, velocity, and acceleration

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Kinematics: how does acceleration affect movement?

Newton's law $a = F/m$ tells us that *acceleration* – the second derivative of position – is what results from forces.

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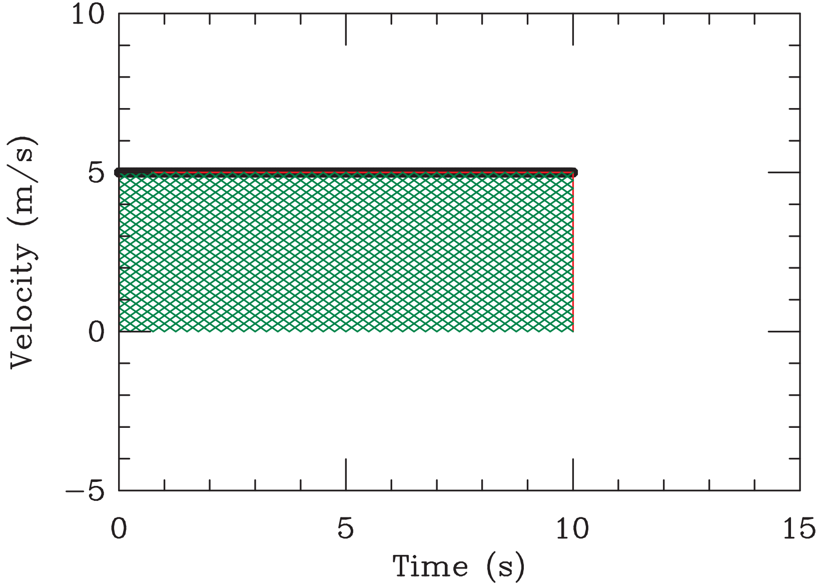
Newton's law $a = F/m$ tells us that *acceleration* – the second derivative of position – is what results from forces.

All freely falling objects have a constant acceleration downward.

This number is so important we give it a letter: $g = 9.81 \text{ m/s}^2$

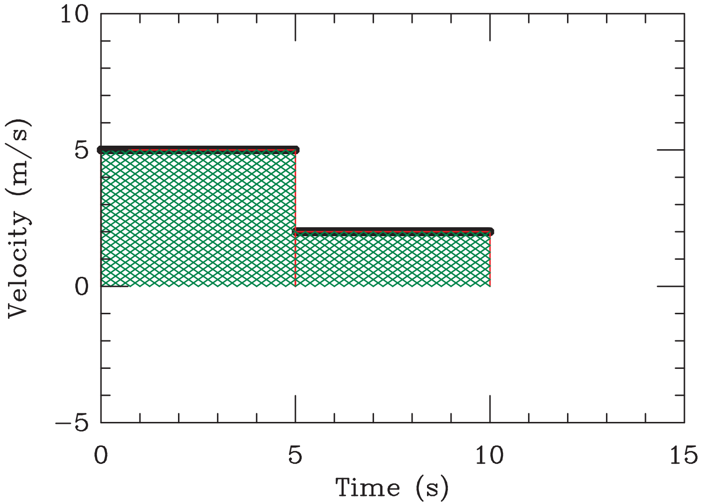
A calculus review

If velocity is the rate of change of position,
why is the area under the v vs. t curve equal to displacement?



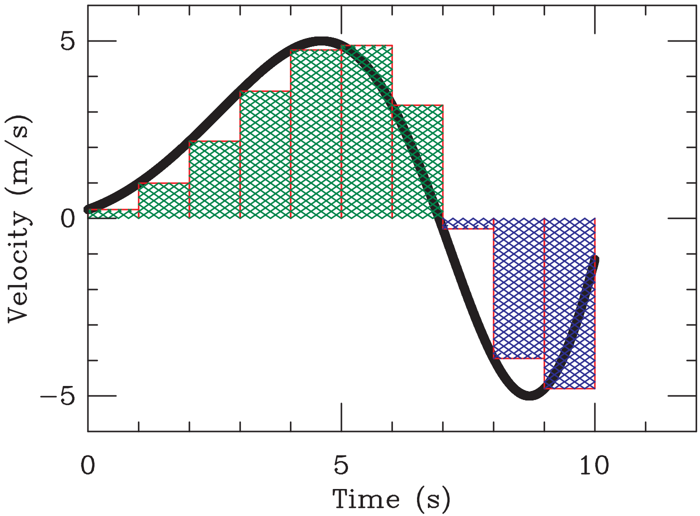
We know $\Delta s = vt$. What is that here? What's the area of the shaded region?

A calculus review



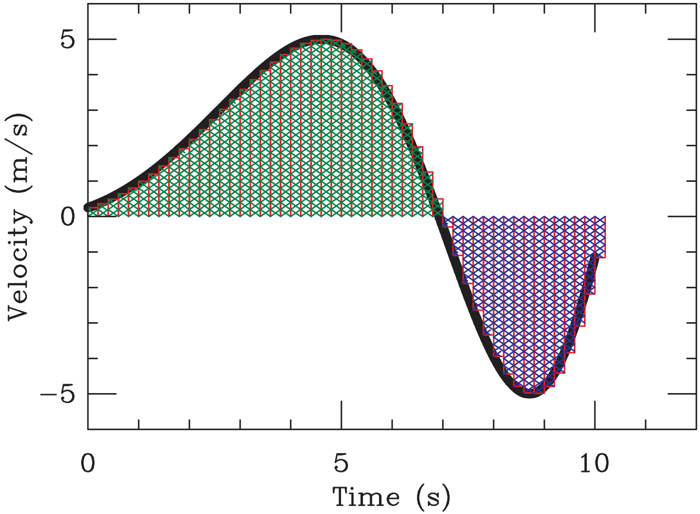
Now what is Δs? What is the area of the shaded region?

A calculus review



Does this work? How do we fix it?

A calculus review



The area between the t-axis and the velocity curve is the distance traveled.
(The area below the t-axis counts negative: "the thing is going backwards"

In calculus notation: $\int v(t)\,dt = \delta x = x(t) - x_0$

Position, velocity, and acceleration

(derivative of)
rate of change of

Position $\xleftarrow{\hspace{6em}}$ / $\xrightarrow{\hspace{6em}}$ Velocity

area under the curve of
(integral of)

Position, velocity, and acceleration

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Position ⟷ Velocity ⟷ Acceleration

area under the curve of
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Constant acceleration

Particularly interesting situation:

- Free fall (as you saw)
- Any time the force is constant: $F = ma \rightarrow a = F/m$...

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Plan of attack:

- We know what the acceleration curve looks like (it's just flat)
- Figure out the area under the acceleration curve to get the velocity curve
- Figure out the area under the velocity curve to get the position curve

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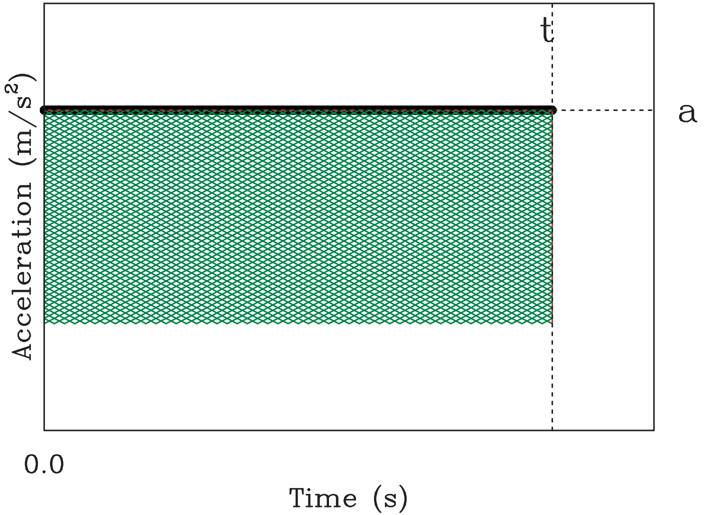
- Figure out the area under the velocity curve to get the position curve

Remember the area under the curve of (velocity, acceleration) just gives the *change in* (position, velocity) – *i.e.* initial minus final.

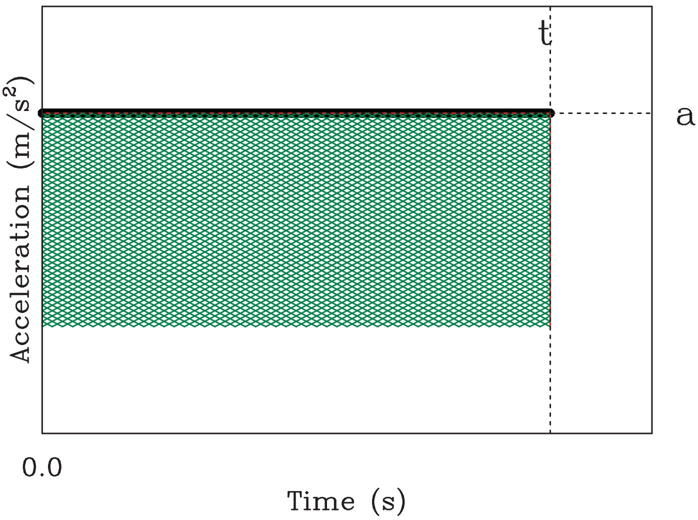
We'll start by assuming x_0 and v_0 are zero.

Constant acceleration



What's the area under the curve out to time t, which gives the change in the velocity – $\Delta v = v(t) - v_0$?

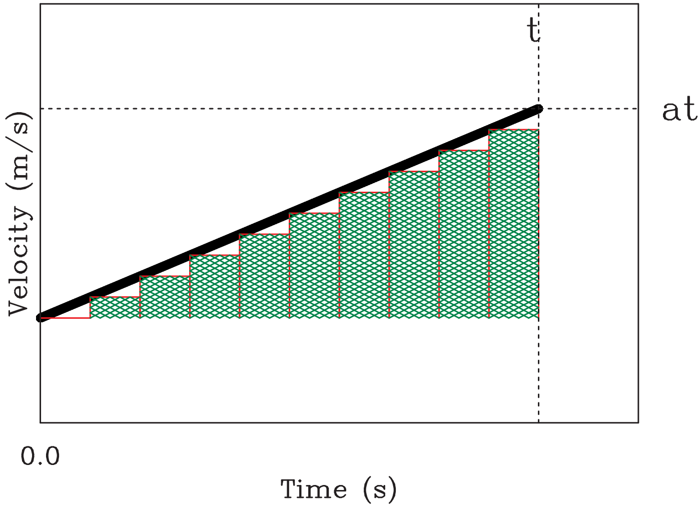
Constant acceleration



What's the area under the curve out to time t, which gives the change in the velocity – $\Delta v = v(t) - v_0$?

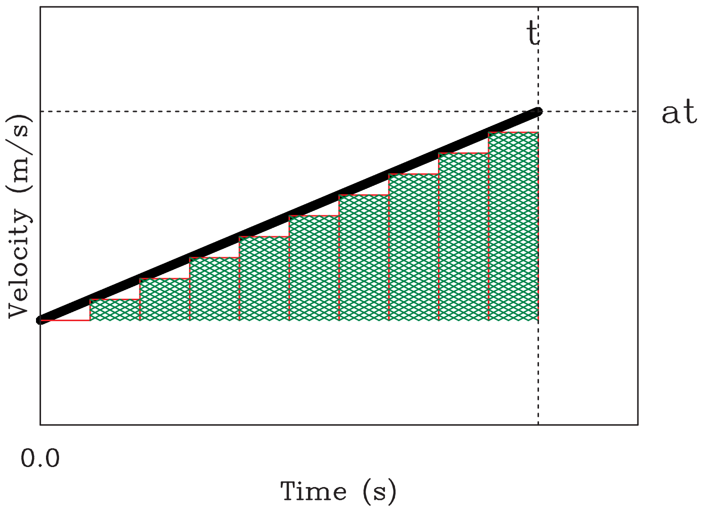
$$v(t) - v_0 = at, \text{ so } v(t) = at + v_0$$

Same thing again to get position



Now the area under the velocity curve gives the change in position: $\Delta x = x(t) - x_0$?

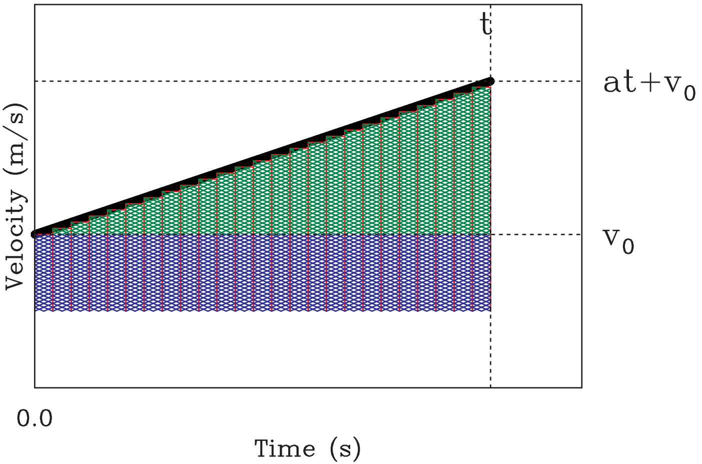
Same thing again to get position



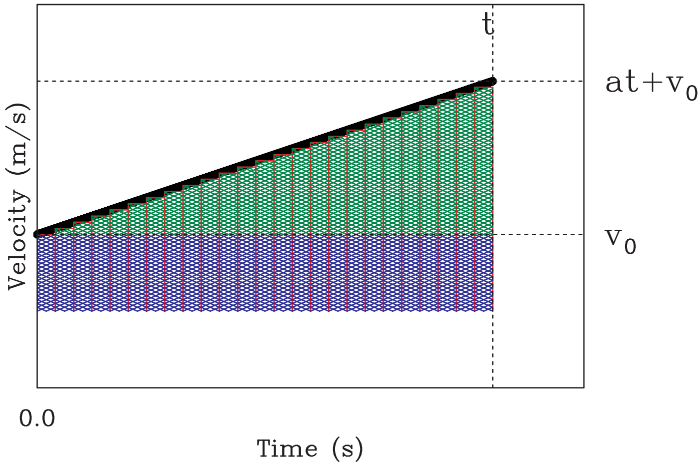
Now the area under the velocity curve gives the change in position: $\Delta x = x(t) - x_0$?

$$x(t) - x_0 = \frac{1}{2}at^2, \text{ thus } x(t) = \frac{1}{2}at + x_0$$

Now if v_0 is not zero...



Now if v_0 is not zero...



Area under blue part: $v_0 t$
Area under green part: $\frac{1}{2}at^2$
Total change in position: $x(t) - x_0 = \frac{1}{2}at^2 + v_0 t$

Thus, $x(t) = \frac{1}{2}at^2 + v_0 t + s_0$

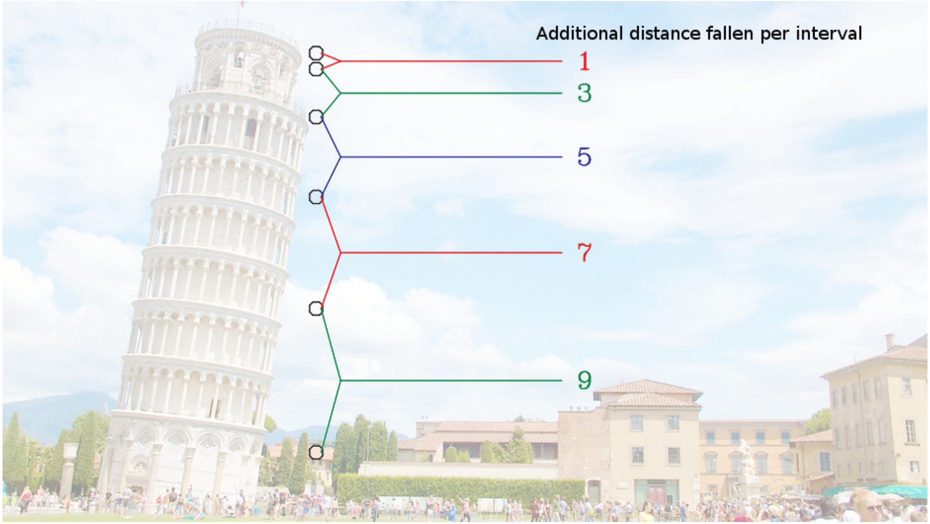
For those who are familiar with calculus:

$$
\begin{aligned}
a(t) &= \text{const.} \\
v(t) &= \int a\,dt && = at + C_1 \\
x(t) &= \int v\,dt = \int (at + C_1)dt && = \frac{1}{2}at^2 + C_1 t + C_2
\end{aligned}
$$

A little thought reveals that C_1 is the initial velocity v_0 and C_2 is the initial position x_0. This gives us the things we just derived, but much more easily:

$$
\begin{aligned}
v(t) &= at + v_0 \\
x(t) &= \frac{1}{2}at^2 + v_0 t + x_0
\end{aligned}
$$

Free fall revisited



Adding these numbers together gives us 1, 4, 9, 16, 25...

The calculus above explains this: distance is proportional to *time squared!*

Example problems

- How long does it take for a falling object to fall 10 m?

Example problems

- You throw an object up with an initial speed of $5\,\text{m/s}$. How high does it go? How long does it take to come back down?

Another example

You throw an object up with an initial speed of v_0. How long does it take to reach a height h?

Another example

You throw an object up with an initial speed of v_0. How long does it take to reach a height h?

$$\begin{aligned} x(t) &= \frac{1}{2}at^2 + v_0 t + x_0 \\ h &= -\frac{1}{2}gt^2 + v_0 t \\ 0 &= -\frac{1}{2}gt^2 + v_0 t - h \end{aligned}$$

- $\rightarrow$ You need the quadratic formula for this – nonzero a, v_0, and position
- The quadratic formula gives you two answers, but there's clearly only one
- The homework asks you to address this idea.
- Hint: graph position vs. time, and interpret the question graphically

Rotational kinematics

- Linear motion: care about position as a function of time
- Rotational motion: care about angle as a function of time
- **Everything we just did translates to rotational kinematics exactly!**

Position, velocity, and acceleration

(derivative of)
rate of change of

Position ⟷ Velocity

area under the curve of
(integral of)

Position, velocity, and acceleration

$$\text{Position} \quad \underset{\substack{\text{area under the curve of}\\ \text{(integral of)}}}{\overset{\substack{\text{(derivative of)}\\ \text{rate of change of}}}{\rightleftarrows}} \quad \text{Velocity} \quad \underset{\substack{\text{area under the curve of}\\ \text{(integral of)}}}{\overset{\substack{\text{(derivative of)}\\ \text{rate of change of}}}{\rightleftarrows}} \quad \text{Acceleration}$$

Angle, angular velocity, and angular acceleration

(derivative of)
rate of change of

Angle $\xleftarrow{\hspace{4em}}$ / $\xrightarrow{\hspace{4em}}$ Angular velocity (ω)

area under the curve of
(integral of)

Angle, angular velocity, and angular acceleration

Angle $\underset{\substack{\text{area under the curve of} \\ \text{(integral of)}}}{\overset{\substack{\text{(derivative of)} \\ \text{rate of change of}}}{\rightleftarrows}}$ Angular velocity (ω) $\underset{\substack{\text{area under the curve of} \\ \text{(integral of)}}}{\overset{\substack{\text{(derivative of)} \\ \text{rate of change of}}}{\rightleftarrows}}$ Angular acceleration (α)

Angle, angular velocity, and angular acceleration

Angle

(derivative of)
rate of change of
$\longleftarrow$
$\longrightarrow$
area under the curve of
(integral of)

Angular velocity (ω)

(derivative of)
rate of change of
$\longleftarrow$
$\longrightarrow$
area under the curve of
(integral of)

Angular acceleration (α)

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

Angle, angular velocity, and angular acceleration

(derivative of)
rate of change of

Angle $\xleftarrow{\text{rate of change of}}$ / $\xrightarrow{\text{area under the curve of}}$ Angular velocity (ω) $\xleftarrow{\text{rate of change of}}$ / $\xrightarrow{\text{area under the curve of}}$ Angular acceleration (α)

area under the curve of
(integral of)

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

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$\rightarrow$ Angular kinematics works in exactly the same way as translational kinematics!