Work and potential energy

Physics 211
Syracuse University, Physics 211 Spring 2022
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March 23, 2022

Tuesday's class

- Many of you told me that you appreciated the format on Tuesday where you did an exercise in class.
- ... so we can do this again in the future!

- However:
  - That exercise took longer in class than I had anticipated
  - We didn't get as far into the *application* of the work-energy theorem as I had hoped
  - I didn't have a chance to revise the recitation material on Wednesday
  - ... so we are pushing the schedule back by about half a class. (This is fine!)
  - We have some flexibility at the end of the semester and everything will work out

Announcements

- Your next homework assignment is due next Wednesday.
- Please come prepared to ask questions next Tuesday

Where we've been and where we're going

- Last time: kinetic energy and the work-energy theorem
- This time: the idea of potential energy and conservation of energy
  - Potential energy: "the most meaningful bookkeeping trick in physics"
  - Lets us understand many phenomena without difficult mathematics
  - Conservation of energy: there's always the same amount of energy, and it just changes forms

Review: kinetic energy

We will see that things are often simpler when we look at something called "energy"

- Basic idea: don't treat $\vec{a}$ and $\vec{v}$ as the most interesting things any more
- Treat v^2 as fundamental: $\frac{1}{2}mv^2$ called "kinetic energy"

Previous methods:

- Velocity is fundamental
- Force: causes velocities to change over time
- Intimately concerned with vector quantities

Energy methods:

- v^2 (related to kinetic energy) is fundamental
- Force: causes KE to change over distance
- Energy is a *scalar*

Energy methods: useful when you don't know and don't care about time

Energy: measurements and units

We didn't talk about how we *measure* energy last time:

$$\text{Kinetic energy} = \tfrac{1}{2}mv^2$$

- Energy has units $\mathrm{kg\,m^2/s^2}$
- This unit is called a *joule*
- 1 joule = the energy required to lift an apple one meter
- This is also the unit for work

The work-energy theorem in 1D

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Some new terminology:

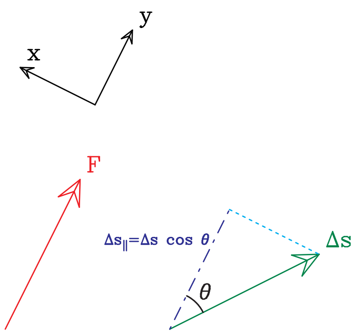
- $\frac{1}{2}mv^2$ called the "kinetic energy" (positive only!)
- $\vec{F}\cdot\Delta\vec{s}$ called the "work" (negative or positive!)
- "Work is the change in kinetic energy"

Dot products: calculating work

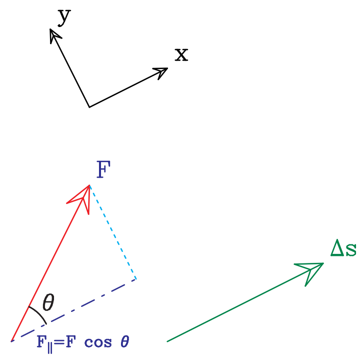
The work done by a force $\vec{F}$ on an object as it moves through a displacement $\Delta\vec{s}$ is

$$W = \vec{F} \cdot \Delta\vec{s}.$$

What does this "dot product" mean?



- $\vec{F} \cdot \Delta\vec{s} = (F)(\Delta s_{\parallel}) = (F)(\Delta s \cos\theta)$
- "The component of the displacement parallel to the force, times the force



- $\vec{F} \cdot \Delta\vec{s} = (F_{\parallel})(\Delta s) = (F\cos\theta)(\Delta s)$
- "The component of the force parallel to the motion, times the displacement

Different cases where each form is useful, but it's the same trig either way

Sample problem: balls on ramps

(on document camera)

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Work done by normal force = **zero**!
Work done by gravity = $(F)(\Delta s)_{\parallel} = mg\Delta y = mg(y_0 - y_f)$

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\begin{aligned}
KE_f - KE_i =& W_g \\
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No detailed knowledge of the motion required!

Pendulum demos

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(If physics stops working and I go splat, have a nice summer!)

Potential energy: an accounting trick

- Notice that the work done by gravity depends *only* on the change in height.
- Some other forces are like this as well: the work done depends only on initial and final position
  - These are called *conservative forces*
  - Soon we'll see that the elastic force is like this too
- Separate out gravity and all other forces in the work-energy theorem:

$$KE_f - KE_i = W_{\text{grav}} + W_{\text{other}}$$
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- Collect all the "initial" things on the left and the "final" things on the right:

$$\frac{1}{2}mv_0^2 + mgy_0 \qquad +W_{\text{other}} = \qquad \frac{1}{2}mv_f^2 + mgy_f$$
$$KE_0 + GPE_0 \qquad +W_{\text{other}} = \qquad KE_f + GPE_f$$

- Identify mgy as "gravitational potential energy": how much work will gravity do if something falls?

Potential energy lets us easily calculate the work done by conservative forces

Potential energy: more than accounting!

- Another way to look at the roller coaster: gravitational potential energy being converted to kinetic energy.
- This perspective is universal: all forces just convert energy from one sort into another
- Some of these types are beyond the scope of this class, but we should be aware of them!

A short history of energy conversion:

- Hydrogen in the sun fuses into helium
- Hot gas emits light
- Light shines on the ocean, heating it
- Seawater evaporates and rises, then falls as rain
- Rivers run downhill
- Falling water turns a turbine
- Turbine turns coils of wire in generator
- Electric current ionizes gas
- Recombination of gas ions emits light

- Nuclear energy $\rightarrow$ thermal energy
- Thermal energy $\rightarrow$ light
- Light $\rightarrow$ thermal energy
- Thermal energy $\rightarrow$ gravitational potential energy
- Gravitational PE $\rightarrow$ kinetic energy and sound
- Kinetic energy in water $\rightarrow$ kinetic energy in turbine
- Kinetic energy $\rightarrow$ electric energy
- Electric energy $\rightarrow$ chemical potential energy
- Chemical PE $\rightarrow$ light

Potential energy: more than accounting!

- This class is just the study of motion: we can't treat light or nuclear energy here.
- ... but in physics as a whole, the *conservation of energy* – that processes just change energy from one form to another – is universal!
- Conservation of energy is one of the most tested, ironclad ideas in science
- Nuclear and chemical potential energy: nuclear forces do mechanical work on particles, much like gravity
- Light, and others: kinetic energy of little particles called "photons"
- Heat: kinetic energy of atoms in random motion
- Sound: kinetic energy of atoms in coordinated motion
- Food: Just chemical potential energy...
- ... so all of these things aren't as far removed from mechanics after all!
- Einstein: "Mass is just another form of energy"

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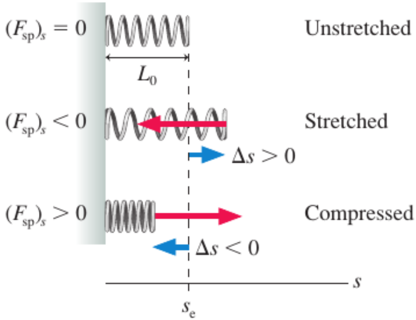
- Einstein: "Mass is just another form of energy"
- Maybe it's all, ultimately, just kinetic energy! (I believe it is; others will argue!)

A new force: elasticity and Hooke's law

To best see how this can be useful, let's introduce a new force: elasticity.

- Springs have a particular length that they like to be: "equilibrium length" L_0
- A spring stretched to be longer than this pulls inward to shorten itself
- A spring compressed to be shorter than this pushes outward to lengthen itself
- Flexible things like strings and ropes only pull; they kink instead of compressing
- The force is proportional to the deviation from the optimum length



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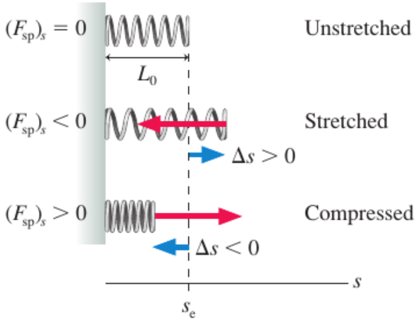

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k is called the "spring constant":

- Measures the stiffness of the spring/rope
- Units of newtons per meter: "restoring force of k newtons per meter of stretch"

A simple spring problem: done with the work-energy theorem

A person of mass $m = 100kg$ falls from a height of $h = 3$m onto a trampoline. If the person makes an impression $d = 40$ cm deep on the trampoline when he lands, what is the spring constant?

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- $KE_0 + W_{\text{grav}} + W_{\text{elas}} = KE_f$
- $0 + (mg)(h + d) - \frac{1}{2}kd^2 = 0$
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"How much work is done by a spring as it goes from $\Delta x = a$ to $\Delta x = 0$?

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Now that we have this, we never have to do this integral again!

$$U_{\text{elastic}} = \frac{1}{2}kx^2, \text{ where } x \text{ is the distance from equilibrium}$$

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- $U_{\text{grav},0} = mgh$
- $U_{\text{elas},0} = 0$ (trampoline starts at equilibrium)
- $U_{\text{grav,f}} = -mgd$ (the person falls below $y = 0$; PE can be negative!)
- $U_{\text{elas,f}} = \frac{1}{2}kd^2$ (see last slide)

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- $k = \frac{mg(h+d)}{2d^2}$

That spring problem: a recap

We don't care about time $\rightarrow$ energy methods

Work-energy theorem

- Initial KE + all work done = final KE
- Need to compute work done by gravity: easy
- Need to compute work done by spring: harder
  (need to integrate Hooke's law)

Potential energy treatment

- Initial KE + initial PE + other work = final KE + final PE
- No "other work" in this problem; all forces have a PE associated
- Need to know the expressions for PE:
  - $U_{\text{grav}} = mgy$
  - $U_{\text{elas}} = \frac{1}{2}kx^2$ (x is the distance from the equilibrium point)
- No integrals required (they're baked into the above)

Potential energy with other forces

What about associating a potential energy with other forces?

- Friction is a no-go: the work done by friction depends on the path, not just where you start and stop
- "Ephemeral" forces like tension and normal force are easiest to deal with by computing work directly

Summary

- Potential energy is two things:
  - An accounting device that makes it easier to keep track of work done
  - Part of *conservation of total energy*, a powerful statement about nature
- Gravitational potential energy (on Earth): $U_g = mgy$

Summary

- Potential energy is two things:
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  - Part of *conservation of total energy*, a powerful statement about nature
- Gravitational potential energy (on Earth): $U_g = mgy$
- We learned about a new force: **elasticity**
  - Restoring force in a stretched or compressed spring, or a stretched string: $F = -k(x - x_0)$ (x_0 is the equilibrium length)
  - k is the spring constant, measured in force per distance, that gauges stiffness
  - Elastic potential energy: $U_{\text{elas}} = \frac{1}{2}k(x - x_0)^2$