Physics 211 Practice Exam 1

We will post solutions to this Sunday morning or late Saturday night. We will likely discuss it extensively at the review Saturday night, 5-8pm.

Question 1

(birbs)

Otto and Sue are two hawks that live on the Syracuse University campus. Their nest is on Lyman Hall; they raise chicks there every year.

$\vec{A}$

Suppose that their daughter (a bird that has just learned to fly) leaves their nest and flies [250 meters at an angle 40 degrees south of west] before landing on the top of the Physics Building.

$\vec{B}$

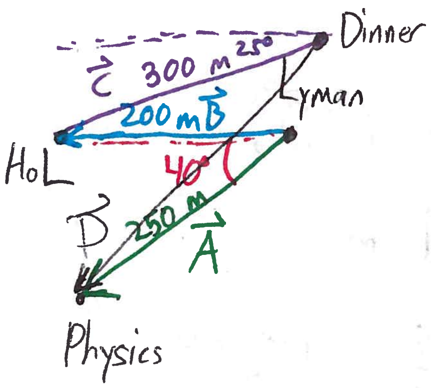
Otto, being a good father, wants to get his daughter some dinner. He flies [200 meters straight west] from their nest and lands on top of the Hall of Languages, looking around for something to eat. From there, he flies [300 meters at an angle 25 degrees north of east] and catches some prey.

$\vec{C}$

Finally, he [flies to his daughter on top of the Physics Building] and brings her the food.

$\vec{D}$

a) Sketch a diagram of the paths that each of the hawks flies. Label each vector with a letter (e.g. $\vec{A}$, $\vec{B}$, $\vec{C}$...) *(5 points)*



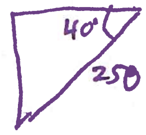
b) Write a vector equation that relates these vectors together (e.g. $\vec{A} + \vec{B} = \vec{C}$). *(5 points)*

$$\underbrace{\vec{A}}_{\text{Daughter}} = \underbrace{\vec{B} + \vec{C} + \vec{D}}_{\text{Otto}}$$

$\vec{D}$

c) What *distance* must Otto fly after he catches dinner in order to bring it to his daughter? *(10 points)*

if $\vec{A} = \vec{B} + \vec{C} + \vec{D}$, then $\vec{D} = \vec{A} - \vec{B} - \vec{C}$.

Vector	~~Dista~~ Length	X	Y	Cartoon
$\vec{A}$	250 m	$-250 \cos 40° = -192$ m	$-250 \sin 40° = -161$ m	 $(\vec{A})$
$\vec{B}$	200 m	-200	0	
$\vec{C}$	300 m	$300 \cos 25° = 272$ m	$300 \sin 25° = 127$ m.	

d) What *direction* must Otto fly after he catches dinner in order to bring it to his daughter? *(5 points)*

$\vec{D} = \vec{A} - \vec{B} - \vec{C}$:

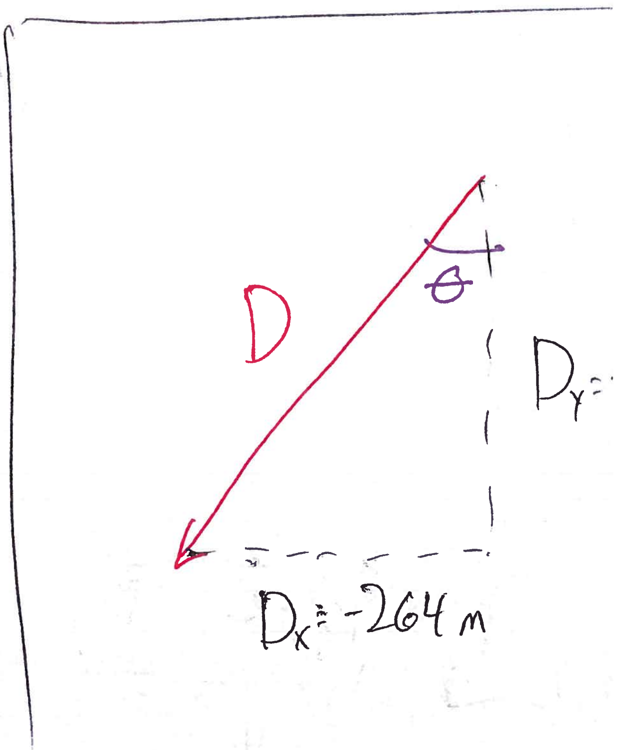
$D_x = A_x - B_x - C_x = -264$ m

$D_y = A_y - B_y - C_y = -288$ m

Distance: (magnitude of $\vec{D}$)

$= \sqrt{D_x^2 + D_y^2} = 391$ m

Direction: $\tan^{-1} \frac{264}{288} = 42.5°$ west of south.



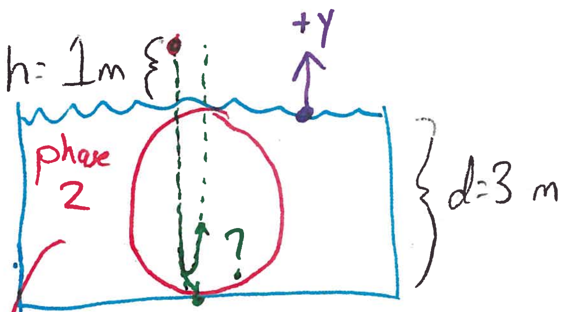
Question 2

A hollow ball falls into a pond from a height of 1m. While it is in the air, it is in freefall. When it is underwater, it has an acceleration of 5 m/s^2 upward, because it is light enough to float. The pond is 3m deep. → $g/2$

You may assume that its velocity does not change as it passes through the surface of the water.

a) With what velocity does it strike the surface? *(5 points)*

"What is v at the time $y=0$?"



$y(t) = -\frac{1}{2}gt^2 + h$

$v(t) = -gt$

$0 = -\frac{1}{2}gt^2 + h \rightarrow t = \sqrt{2h/g}$. → 0.45 s

v at that time: $v = -g\sqrt{2h/g} = \boxed{-\sqrt{2gh}}$. $(-4.47$ m/s$)$

b) Does the ball reach the bottom of the pond before it rises back to the top? *(5 points)*

Need new kinematics (since $\vec{a}$ changed!

$y(t) = \frac{1}{2}(\frac{1}{2}g)t^2 + v_0 t$

$\rightarrow y(t) = \frac{1}{4}gt^2 - \sqrt{2gh}\,t$.

"Is $y(t)$ ever equal to $-d$?"

$-d = \frac{1}{4}gt^2 - \sqrt{2gh}\,t$

$0 = \frac{1}{4}gt^2 - \sqrt{2gh}\,t + d$.

"$b^2 - 4ac$" $= 2gh - gd = -10\ m^2/s^2$

→ negative under $\sqrt{\ }$ in QF: does not reach bottom

c) How long after it is dropped does it take for the ball to reach the surface again? *(5 points)*

"What is t when $y=0$?" → use expression for y underwater

$0 = \frac{1}{4}gt^{2} - \sqrt{2gh}\,\cancel{t}$

→ one solution $t=0$, but that is just when the ball entered the water.

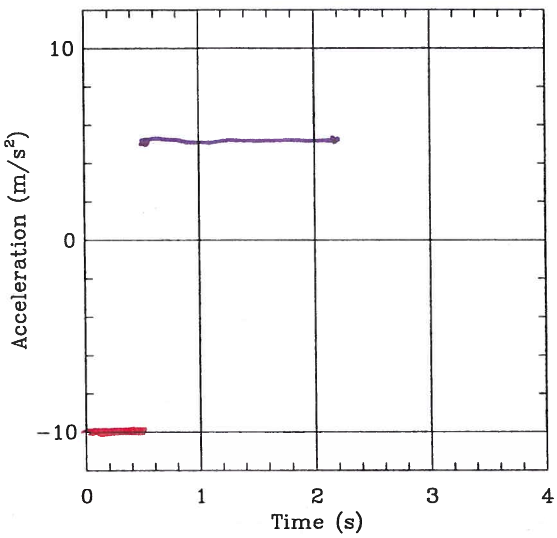
→ cancel t

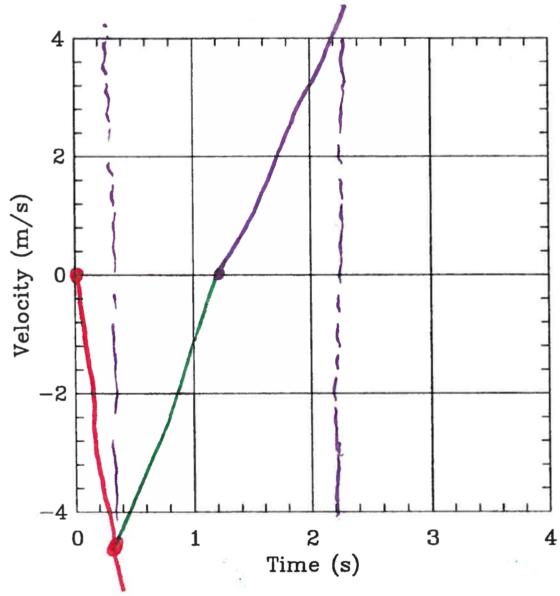
$t = \frac{\sqrt{2gh}}{\frac{1}{4}g} = \frac{4\sqrt{2gh}}{g} = 4\sqrt{2gh/g^2} = 4\sqrt{\frac{2h}{g}}$

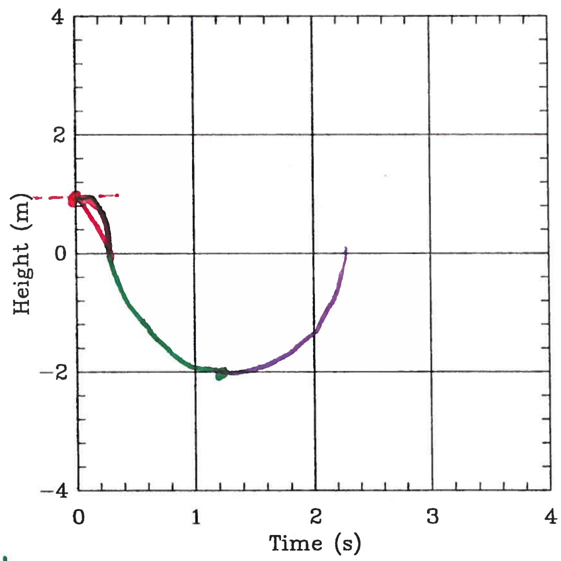
→ this is just underwater, add t from above:

$t_{air} + t_{water} = \sqrt{\frac{2h}{g}} + 4\sqrt{\frac{2h}{g}} = 5\sqrt{\frac{2h}{g}} = 2.23$ s.

d) Graph acceleration vs. time, velocity vs. time, and position vs. time on the axes provided. *(10 points)*







Find max depth:

$v_f^2 - v_0^2 = 2a(y_f - y_0)$ for underwater.

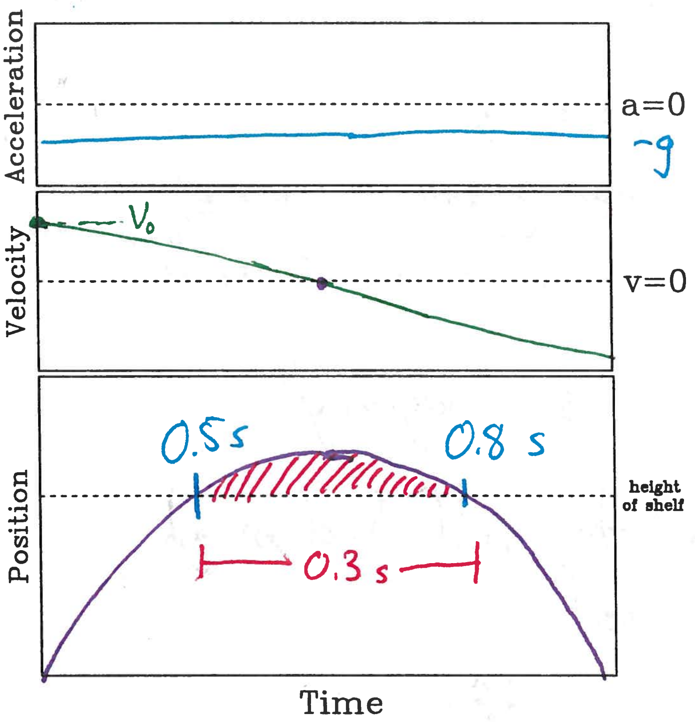
$$0 + \underbrace{2gh}_{v_0^2} = \underbrace{g}_{a=\frac{1}{2}g,\ \text{so } 2a=g}(y_f - 0) \rightarrow 2\cancel{g}h = y_f\cancel{g} \rightarrow \boxed{y_f = 2h = 2\text{ m.}}$$

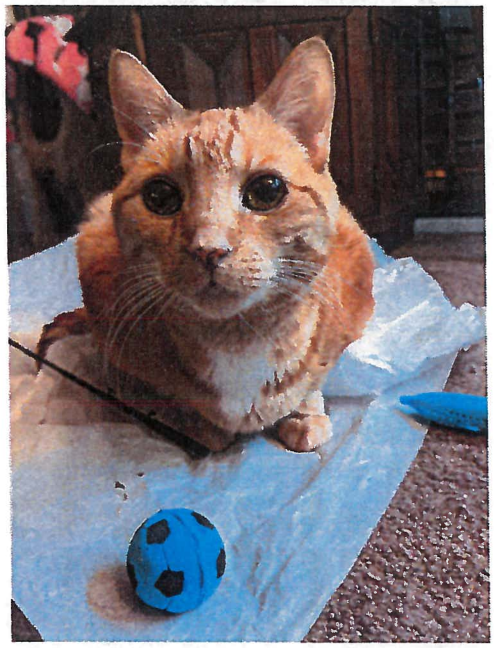
Question 3

Toby the Cat is sitting on a high shelf at a height of $h = 2$ m off of the ground. Her owner is lying on the floor in front of the shelf when she throws a toy ball straight upward at $v_0 = 6.5$ m/s. As soon as the ball passes the level of the shelf (2 m off of the ground), Toby tries to catch it. She can grab the ball as long as it is above the level of the shelf. If Toby does not grab the ball, it falls back down where her owner catches it again.

In this problem, you will calculate how much time Toby has to grab the toy before it falls back below the shelf.

a) Assuming that Toby doesn't grab the toy out of the air, sketch graphs of the ball's position, velocity, and acceleration as a function of time, from the time her owner throws it to when she catches it again. Indicate the height of the shelf on the position vs. time graph. You do not need to show precise numbers on your graphs, just their shape. Draw these graphs on the axes below. *(10 points)*





Toby the Cat and her blue toy ball. She is 15 years old and spoiled rotten by her mommy, as you can tell by all the cat toys she's surrounded by.

b) Write down algebraic expressions for the position and velocity of the ball as a function of time. *(10 points)*

$y(t) = V_0 t - \frac{1}{2} g t^2$ (floor $= 0$, up is positive)

$v(t) = V_0 - gt$

c) How much time does Toby have to grab the ball? (This is the total amount of time that it is above the level of the shelf.) *(10 points)*

"What is the difference between the two times when ~~z~~ $y=h$?"

Set $y(t) = \underbrace{2\text{ m}}_{d}$:

$d = v_0 t - \frac{1}{2}gt^2 \longrightarrow \frac{1}{2}gt^2 - v_0 t + d = 0$

$t = \frac{v_0 \pm \sqrt{v_0^2 - 2gd}}{g} = (0.8, 0.5)\text{ s}.$

difference $= 0.3$ s.

(see graph on reverse)

d) Suppose that her owner instead throws the ball upward with a starting velocity of 5 m/s rather than 6.5 m/s. In this scenario, how much time will Toby have to grab the ball? *(5 points)*

→ use this again: t is imaginary!

→ she didn't throw the ball hard enough to reach the shelf.

Sad kitty. =:^(=

QUESTION 4

A walnut tree grows 100 meters north of a pecan tree.

A squirrel takes a walnut from the walnut tree, travels 30 meters at an angle 20 degrees south of east, and buries his walnut.

Another squirrel takes a pecan from the pecan tree, travels 50 meters due west, and buries her pecan.

a) How far apart are the two buried nuts? *(15 points)*

$\vec{A} + \vec{C} + \vec{D} = \vec{B}$

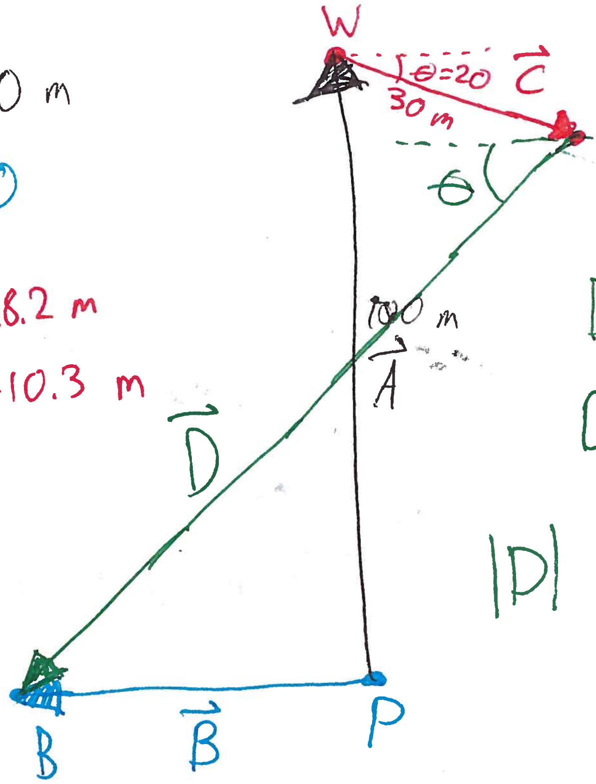
$\vec{D} = -\vec{A} - \vec{C} + \vec{B}$

$A_x = 0 \quad A_y = 100\ m$

$B_x = -50\,m \quad B_y = 0$

$C_x = 30 \cos 20° = 28.2\ m$

$C_y = -30 \sin 20° = -10.3\ m$

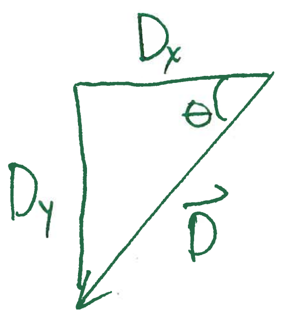


$D_x = 0 - 28.2 - 50 = -78.2$

$D_y = -100 + 0 + 10.3 = -89.7$

$|D| = \sqrt{D_x^2 + D_y^2} = 119\ m.$

b) If someone wanted to walk from the buried pecan to the buried walnut, in which direction should they walk? *(10 points)*



$\theta = \tan^{-1} \frac{D_y}{D_x} = 49°$ South of west

Question 5

A firefighter in an airplane is trying to put out a fire by dropping a load of sand ~~on it~~. She is flying horizontally toward the fire at an altitude of 100 meters, traveling at a speed of 60 m/s.

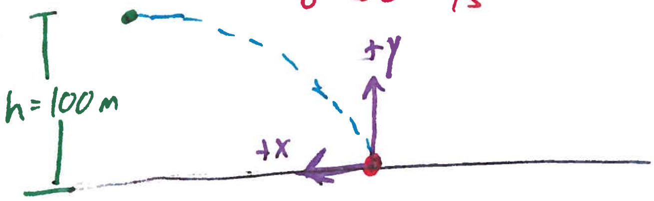
V_0

If she drops her sand directly over the fire, it will overshoot the target.

$\rightarrow V_{0,y} = 0$, so that term doesn't appear

a) How far in advance of the fire must she release the sand in order for it to land on the fire? *(12 points)* $\rightarrow V_0 = 60$ m/s

! We don't know x_0!



$x(t) = -V_0 t + x_0 \longrightarrow 0 = -V_0 t + x_0 \longrightarrow t = \frac{x_0}{V_0}$

$y(t) = -\frac{1}{2}gt^2 + h \xrightarrow{\text{substitute } t} -\frac{1}{2}g\left(\frac{x_0}{V_0}\right)^2 + h = 0$

"What value of x_0 makes it so that $y=0$ at the same time that $x=0$?"

$-\frac{1}{2}g x_0^2 + h v_0^2 = 0$

$x_0^2 = \frac{2hv_0^2}{g} \longrightarrow \boxed{x_0 = \sqrt{\frac{2h}{g}}\, V_0}$

$= 268$ m

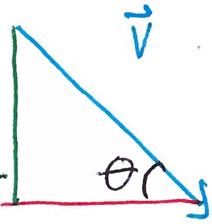
b) In what direction will the sand be traveling when it strikes the ground? *(8 points)*

"What direction does $\vec{v}$ point at the time $y=0$?"

$V_x(t) = V_0$ [no a_x]

$V_y(t) = -gt$

$t = \frac{x_0}{V_0} = \sqrt{\frac{2h}{g}}\frac{\cancel{V_0}}{\cancel{V_0}} = \sqrt{\frac{2h}{g}}$.



$V_x = V_0$

$V_y = \sqrt{2gh}$

$\hookrightarrow \underbrace{\sqrt{\frac{2h}{g}}}_{t}\, g = \sqrt{\frac{2h}{g}}\sqrt{g^2}$

$\tan\theta = \frac{\sqrt{2gh}}{V_0}$

$\theta = \tan^{-1}\frac{\sqrt{2gh}}{V_0} =$

36.7° below horizontal.

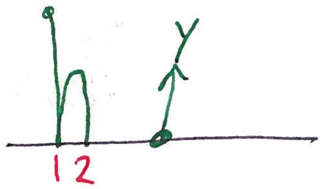
c) Will the airplane be behind the load of sand, directly above it, or ahead of it when it lands on the fire? Explain briefly how you know. *(5 points)*

$\rightarrow$ Both plane and sand have $V_x = V_0$. X- and Y-motion are independent.

Question 6

A person drops a baseball from a height h onto a hard floor. When the baseball hits the floor, it will bounce back in the opposite direction with a speed equal to *half* of the speed with which it hit the floor.

In terms of h and g, find:



a) how long it takes to hit the floor the first time (5 points)

"What is t when $y=0$?"

$$\underbrace{y(t) = h - \tfrac{1}{2}gt^2}_{v_0 = 0.} \rightarrow 0 = h - \tfrac{1}{2}gt^2 \rightarrow t = \sqrt{2h/g}.$$

b) how fast it is going when it hits the floor the first time (5 points)

"What is v at that time?"

$$v(t) = -gt = -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh}. \quad \text{(hits floor)}$$

$$\text{Bounces: } \tfrac{1}{2}\sqrt{2gh} = \sqrt{\frac{gh}{2}} \quad \text{(velocity back up)}$$

c) how high it travels after bouncing off the ground the first time (5 points)

"What is y when $v=0$?"

$$v(t) = v_0 - gt$$

$$0 = \sqrt{\frac{gh}{2}} - gt \longrightarrow t = \sqrt{\frac{h}{2g}}.$$

$$y(t) = v_0 t - \tfrac{1}{2}gt^2$$

$$= \sqrt{\frac{\cancel{g}h}{2}}\sqrt{\frac{h}{2\cancel{g}}} - \tfrac{1}{2}\cancel{g}\frac{h}{2\cancel{g}}$$

$$= \sqrt{\frac{h^2}{4}} - \frac{1}{4}h$$

$$= \frac{h}{2} - \frac{h}{4} = \boxed{\frac{1}{4}h}$$

d) the amount of time between the ball being dropped and it hitting the floor the second time (5 points)

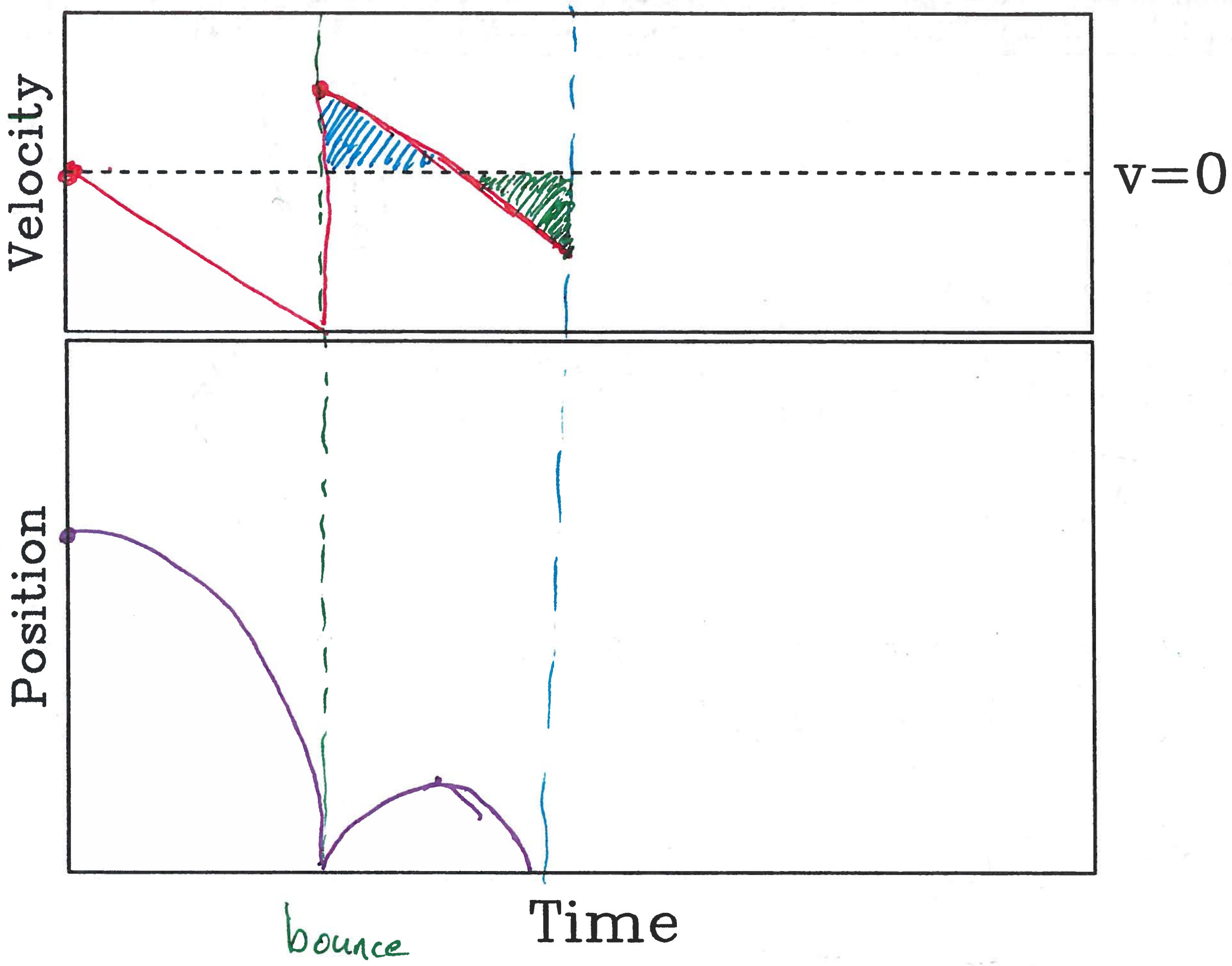
Find time for second bounce:

$$0 = v_0 \cancel{t} - \tfrac{1}{2}gt^{\cancel{2}} \rightarrow t = \frac{2v_0}{g} = \frac{2\sqrt{\frac{gh}{2}}}{g} = \sqrt{\frac{2h}{g}}$$

$$\text{total} = \underbrace{\sqrt{\frac{2h}{g}}}_{\text{fall}} + \underbrace{\sqrt{\frac{2h}{g}}}_{\text{bounce}}$$

$$= \boxed{2\sqrt{\frac{2h}{g}}.}$$

e) On the axes provided, graph the ball's velocity vs. time and position vs. time, starting at the time that it is dropped and ending when it strikes the ground for the second time. (5 points)



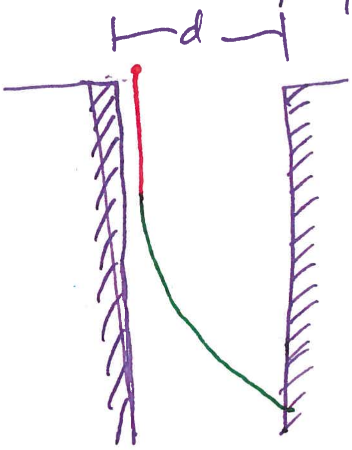
Question 7

A rocket is dropped from rest out of a window and pointed sideways toward another building. A time τ after it is dropped, its motor fires, giving it an acceleration of $2g$ in the horizontal direction. (Its vertical acceleration is still g downward.)

After the motor fires, the rocket flies along its new path until it strikes another building, located a distance d away.

a) In terms of τ, g, and d, what are the position and velocity of the rocket when the motor fires? (Make sure you specify both $x-$ and $y-$ components, even if some of them are zero.) *(5 points)*

Choose drop point as origin: , down is +y



Before rocket fires, ~~$y = V_y = 0$~~.

$X = V_x = 0.$

$y = \frac{1}{2}gt^2 \longrightarrow y(\tau) = \frac{1}{2}g\tau^2$

$V_y = gt \qquad V_y(\tau) = \cancel{\tfrac{1}{2}}\, g\tau.$

b) In terms of τ, g, and d, how long in total is the rocket in the air before it strikes the second building? *(10 points)*

After motor fires, $a_x = 2g$, $a_y = g$

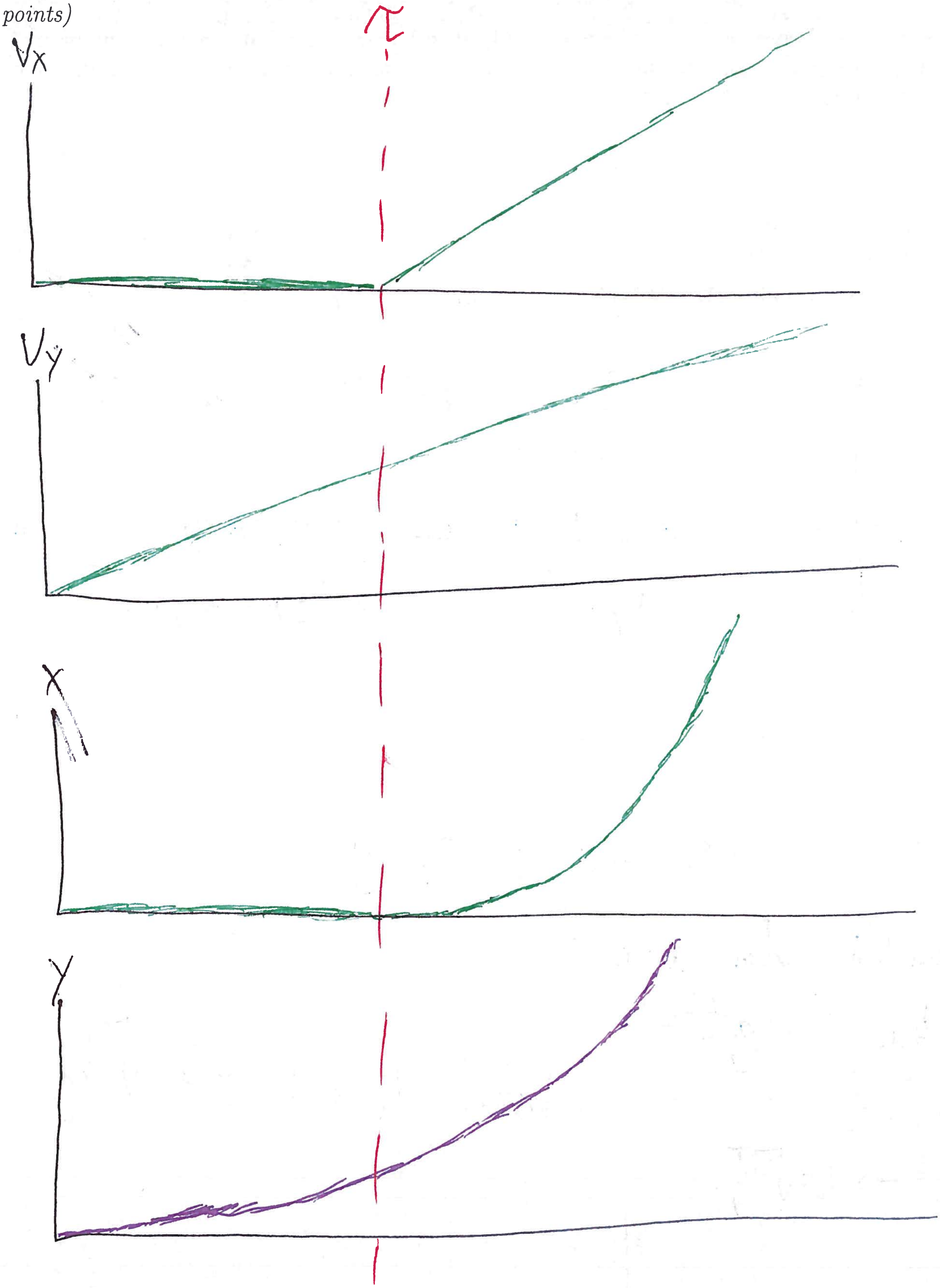
Hits building when $x = d$:

$x(t) = \frac{1}{2}\underbrace{a_x}_{a_x = 2g}t^2 = gt^2$

$d = gt^2 \rightarrow t = \sqrt{d/g}.$

Total time: $\tau + \sqrt{d/g}.$

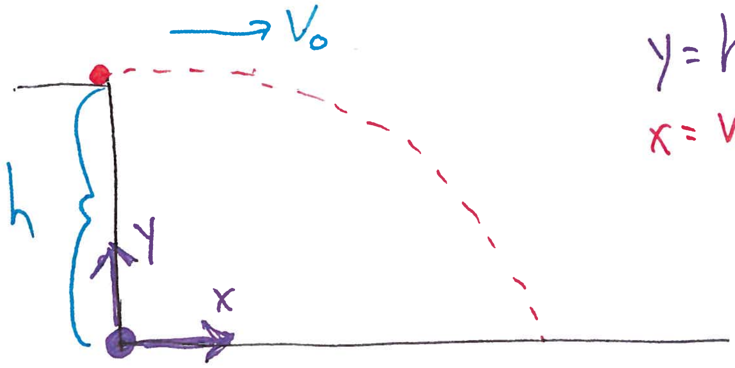
c) Sketch graphs of v_x, v_y, x, and y as a function of time. Indicate the time τ on each graph. *(10 points)*



Question 8

A ball rolls off a horizontal shelf of height h at speed v_0. Answer the following in terms of h, v_0, and g.

a) How much time does it take the ball to hit the floor? *(5 points)*



$y = h - \frac{1}{2}gt^2$ $\quad V_y = -gt$

$x = V_0 t$ $\quad V_x = V_0$

Time when $y=0$:

$0 = h - \frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2h}{g}}$

b) Where does the ball hit the floor? *(5 points)*

Find x at that time:

$x(t) = V_0\sqrt{\frac{2h}{g}}$.

c) What is the ball's speed when it hits the floor? *(5 points)*

Find magnitude of $\vec{v}$ at that time:

$= \sqrt{V_x^2 + V_y^2} = \sqrt{V_0^2 + \underbrace{2gh}_{=\left(g\sqrt{\frac{2h}{g}}\right)^2}}$

d) What direction is the ball moving in when it hits the floor? *(5 points)*

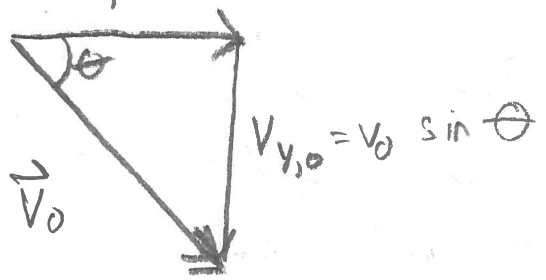
$\tan\theta = \frac{V_y}{V_x}$ $\quad \theta = \tan^{-1}\frac{V_y}{V_x}$

$= \tan^{-1}\frac{\sqrt{2gh}}{V_0}$ below horizontal.

e) Suppose that the edge of the shelf had been curved, so that the ball's initial velocity was instead directed at an angle θ below the horizontal. Explain, using words or algebra as appropriate, what things you would have needed to do differently to solve the previous four parts, and which things would stay the same. *(5 points)*

We would have had a nonzero $v_{y,0}$.

$V_{x,0} = V_0 \cos\theta$



Everything else would have been similar.
(We would need the quadratic formula for t but that's okay, if messy.)