Physics 211 Practice Exam 1

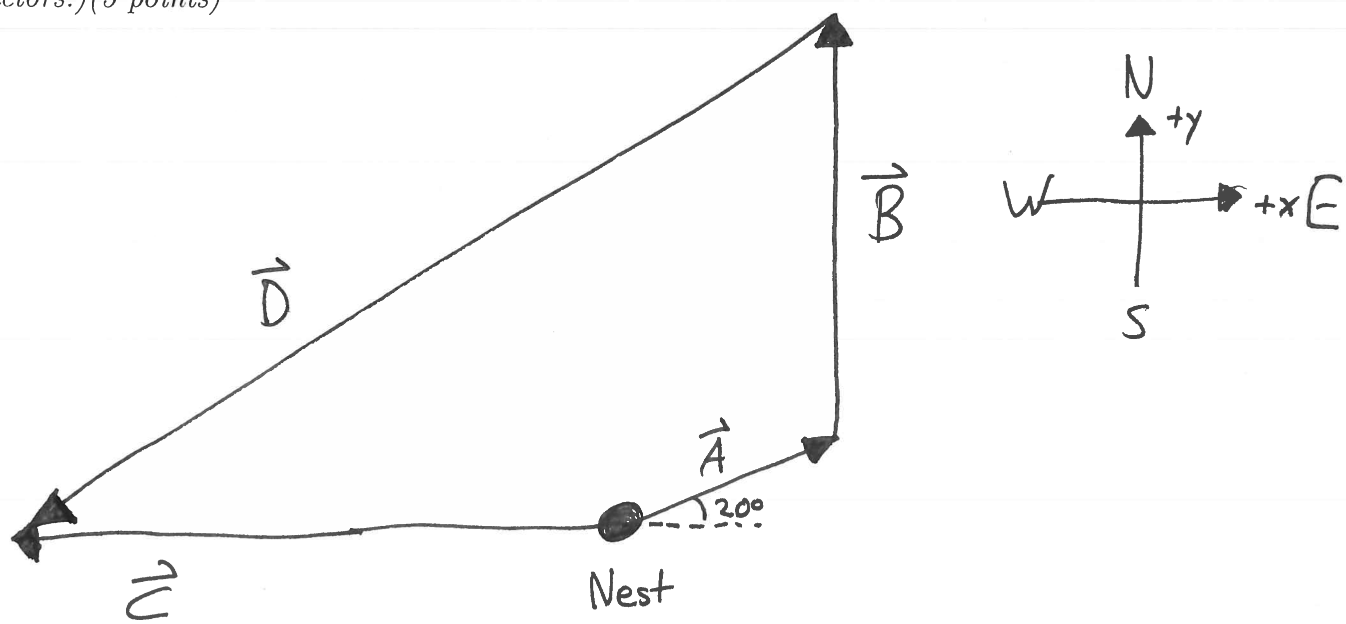
- Question 1 tests your ability to add and subtract vectors.
- Question 2 tests your ability to understand motion when the acceleration changes partway through, and to make position/velocity/acceleration graphs.
- Question 3 tests your knowledge of projectile motion in two dimensions.
- Question 4 tests your ability to add and subtract vectors.
- Question 5 tests your knowledge of projectile motion in two dimensions.
- Question 6 tests your knowledge of motion in one dimension and your knowledge of position/velocity/acceleration graphs.
- Question 7 tests your knowledge of projectile motion in one dimension.
- Question 8 tests your knowledge of projectile motion in two dimensions.

Question 1

Otto is a red-tailed hawk who raises chicks every year in his nest on Lyman Hall along with his partner Sue. Otto decides to go get breakfast for them. He flies 20 degrees north of east for 150 meters to his favorite tree. From his perch in the tree, he sees a tasty squirrel 250 meters straight north, and flies off to catch it.

After he catches the squirrel, he brings it to Sue. She has flown from their nest to the Hall of Languages, 300 meters west of their nest; he flies in a straight line to her.

a) Sketch a diagram showing the flight paths of Otto and of Sue, and label each vector with a name (e. g. $\vec{A}$). Then write a vector equation that relates them (e. g. $\vec{A} + \vec{B} = \vec{C}$). (You should have four vectors.)(5 points)



$$\vec{A} + \vec{B} + \vec{D} = \vec{C}$$

$A_x = 150 \cos 20° = 141$ m

$A_y = 150 \sin 20° = 51$ m

$B_x = 0$ $\quad B_y = 250$ m

$C_x = -300$ m $\quad C_y = 0$

(This question continues on the next page.)

b) When Otto flies back to Sue, how far does he have to fly? (10 points)

Solve for $\vec{D}$: if $\vec{A} + \vec{B} + \vec{D} = \vec{C}$, then

$$\vec{D} = \vec{C} - \vec{A} - \vec{B}.$$

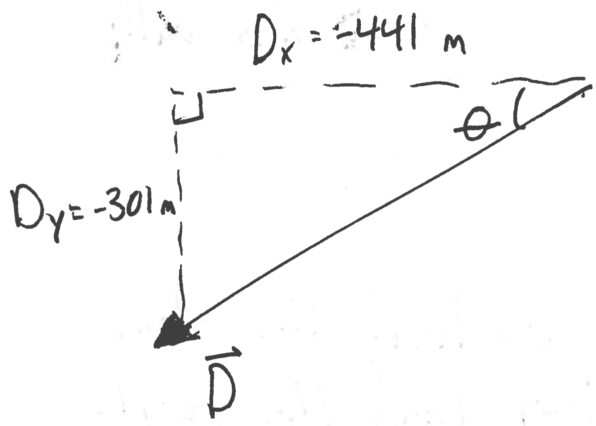
Subtract the components:

$$D_x = (-300\ \text{m}) - 141\ \text{m} = -441\ \text{m}$$

$$D_y = -51\ \text{m} - 250\ \text{m} = -301\ \text{m}$$

"How far" = magnitude of D: $= \sqrt{D_x^2 + D_y^2} = 534$ m.

c) When Otto flies back to Sue, in what direction should he fly? (10 points)



$$\theta = \tan^{-1}\left[\frac{301}{441}\right] = 34^\circ$$

south of east.

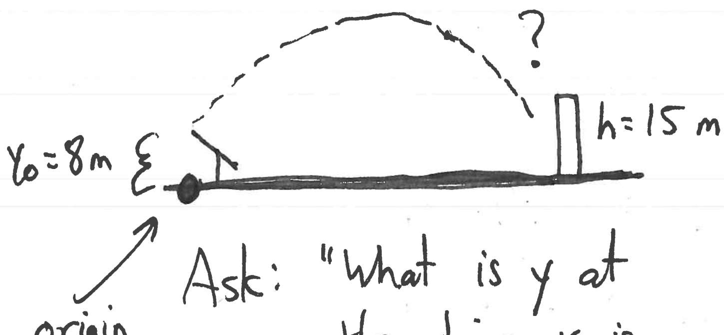
(since $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$)

Question 2

A trebuchet is sitting 100 m from a castle wall that is 15 m high. It launches a stone towards the castle from a height of 8 m with an initial velocity of 32 m/s. The angle it releases at is 45° above the horizontal.

a) Does the stone land short, hit the castle wall, or fly over it? (20 points)

(Don't know yet where it goes)



Ask: "What is y at the time x is equal to d=100 m?"

Note: $V_{0,x} = V_{0,y} = V_0/\sqrt{2}$.

$$x(t) = \frac{V_0 t}{\sqrt{2}} \qquad y(t) = \frac{V_0 t}{\sqrt{2}} - \frac{1}{2} g t^2 + Y_0.$$

Find time to arrive at y=d:

$$d = \frac{V_0 t}{\sqrt{2}} \Rightarrow t = \frac{\sqrt{2} d}{V_0}$$

Find y at that time:

$$y(t) = \frac{\cancel{V_0}}{\cancel{\sqrt{2}}}\left(\frac{\cancel{\sqrt{2}} d}{\cancel{V_0}}\right) - \frac{1}{2} g \left[\frac{2d^2}{V_0^2}\right] + Y_0$$

$$= 100 \text{ m} - 97.7 \text{ m} + 8 \text{ m}$$

$$= 10.3 \text{ m high}$$

$\Rightarrow$ hits wall!

b) Answer the appropriate question: (5 points)

- *If the stone does not hit the wall, could you adjust the angle of release to make it hit the wall?*
- *If the stone does hit the wall, could you adjust the angle of release to make it miss?*

Yes. If the angle is vertical, the stone will come right back down (and the people operating the trebuchet will have a bad day).

For angles only a little below vertical, it will not travel very far.

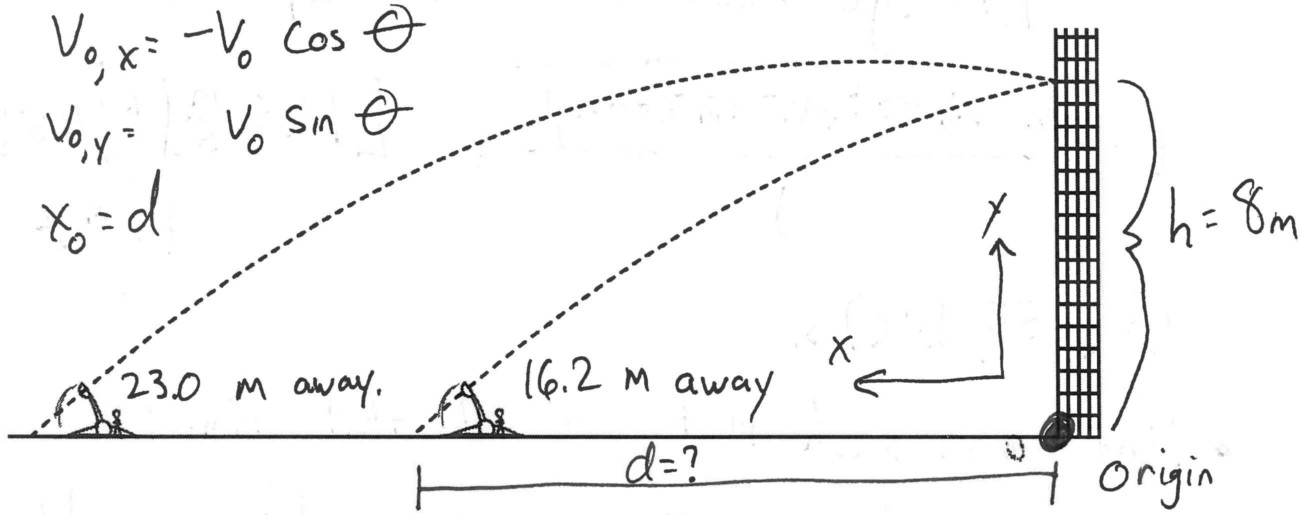
Question 3

A student has built a catapult and wants to use it throw snowballs from the Quad into Walter's open window on the second floor of the Physics Building.

Suppose that the catapult fires snowballs at an angle $\theta = 40°$ above the horizontal at a speed of $v_0 = 20$ m/s, and the center of the window is a height $h = 8$ m above the ground.[1]

There are two places on the Quad where they could put their catapult. From one position the snowballs will go through Walter's window on the way up; from the other position, they will hit his window on the way down. In this problem, you will figure out where those two locations are.

Note: $V_{0,x} = -V_0 \cos\theta$
$V_{0,y} = V_0 \sin\theta$
$x_0 = d$



a) Indicate your choice of coordinate system on the diagram above, and label any other distances/points you want to assign variables to. (3 points)

b) Discuss in words your approach to figuring out where to place the catapult. In particular, what mathematical condition means "the snowball goes in the window"? What variable are you going to solve for?

You may answer this part by simply writing question in terms of your algebraic variables, as we have practiced. (5 points)

"What is d such that $x = 0$ at the time that $y = h$?"

(This question continues on the next page.)

[1]Catapult image from Randall Munroe/xkcd, used under CC-BY-NC. See, we cite our sources too!

c) Determine the two points on the Quad where the snowballs will go into the window. Which one is which? (10 points)

$x(t) = d - (v_0 \cos\theta)t \quad y(t) = -\frac{1}{2}gt^2 + (v_0 \sin\theta)t$

• Find t when $y = h$:

$h = -\frac{1}{2}gt^2 + (v_0 \sin\theta)t \Rightarrow \frac{1}{2}gt^2 (v_0 \sin\theta)t + h = 0.$

$t = \dfrac{v_0 \sin\theta \pm \sqrt{v_0^2 \sin^2\theta - 2gh}}{g} = \dfrac{[12.8 \frac{m}{s}] \pm \sqrt{165 \frac{m^2}{s^2} - 160 \frac{m^2}{s^2}}}{10\ m/s^2}$

$t = (1.50, 1.06)$ s.

• Find d such that $x = 0$ at that time:

$0 = d - [v_0 \cos\theta]t$, so $d = [v_0 \cos\theta]t$

$= 16.2$ m or 23.0 m away. (see drawing)

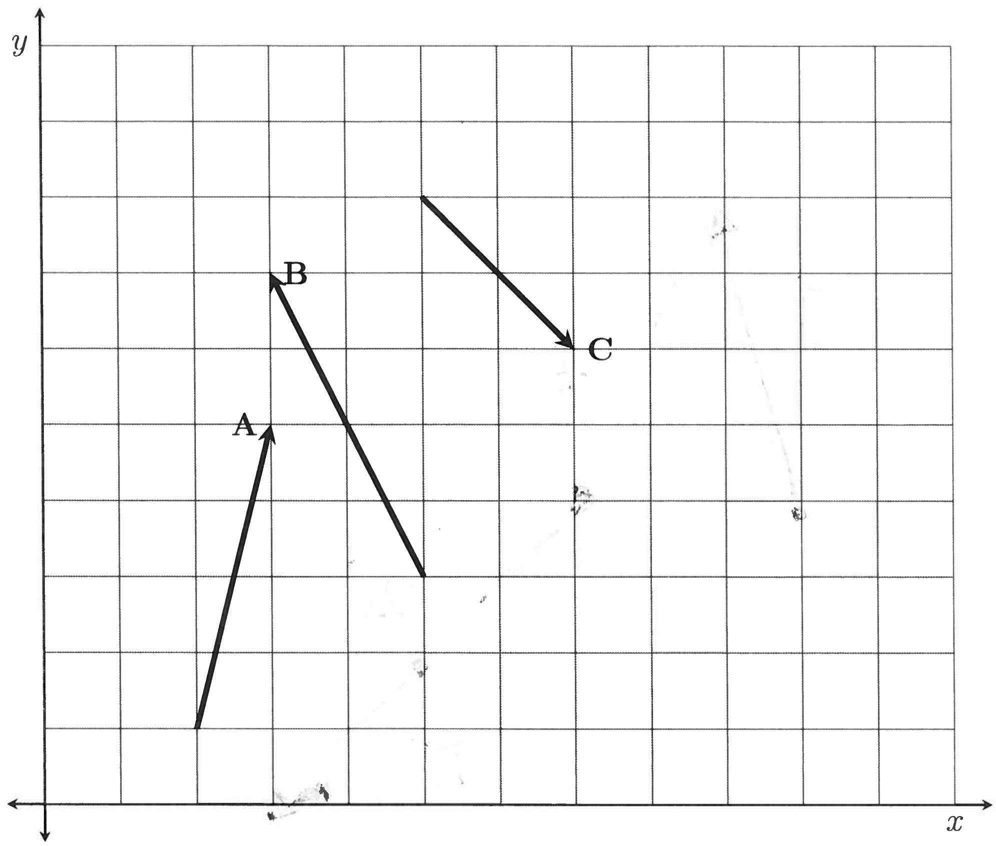
d) If you repeat this problem with $v_0 = 15$ m/s, you will find that you get a negative under the square root sign in the quadratic formula. What is the physical interpretation of this?[2] (7 points)

This means the snowball never reaches 8m height and can't hit the window.

[2]If you did not use the quadratic formula for part (c), then describe instead what will happen with $v_0 = 15$ m/s, and how you would know based on your method.

QUESTION 4

Here are three vectors:



a) Read the x and y components of the vectors from the plot. (5 points)

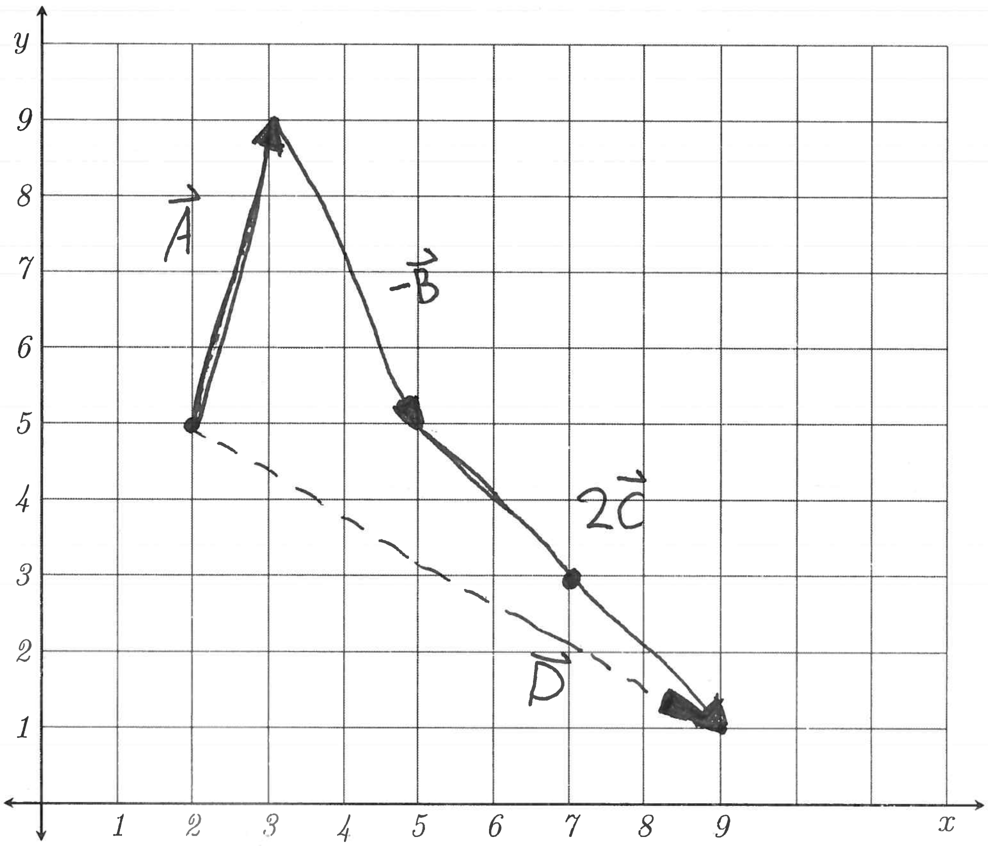
$A_x =$ +1 $\qquad A_y =$ +4

$B_x =$ −2 $\qquad B_y =$ +4

$C_x =$ +2 $\qquad C_y =$ −2

$$\vec{A}-\vec{B}+2\vec{C}=7\hat{i}+(-4)\hat{j}$$

b) In the space provided below find a vector of $\vec{D} = \vec{A} - \vec{B} + 2\vec{C}$ using the graphical method of vector algebra. Show your work and label what represents the vector $\vec{D}$ in your drawing. Don't forget to indicate its arrow. (10 points)

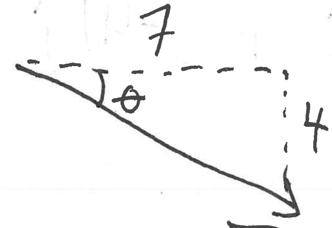


c) Determine the length and direction of vector $\vec{D}$. Describe the direction fully: don't just tell me an angle without specifying what it means. (10 points)

$\vec{D} = 7\hat{i} - 4\hat{j}$.

Length $= \sqrt{D_x^2 + D_y^2} = \sqrt{65}$.

Direction:

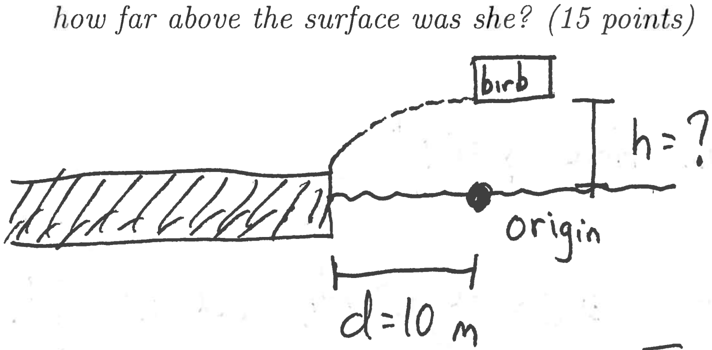


$\theta = \tan^{-1} \frac{4}{7} = 29.7°$ below the horizontal, in the 4^{th} quadrant.

Question 5

An eagle has caught a fish from Lake Onondaga and is flying back to her tree on the shore to eat it. She is flying horizontally at 8 m/s when another eagle tries to steal it from her; it slips out of her talons and falls, landing right on the edge of the lake.

a) If the **horizontal** *distance between the eagle and the shoreline is 10 m when she dropped her fish, how far above the surface was she? (15 points)*



"What value for y_0 makes it so that $y=0$ at the time $x=d$?"

$x(t) = v_0 t$

$y(t) = -\frac{1}{2}gt^2 + h$

- Find time $x=d$: $d = v_0 t$ so $t = d/v_0$
- Find h: $0 = -\frac{1}{2}gt^2 + h \rightarrow \boxed{\frac{1}{2}g\frac{d^2}{v_0^2} = h}$

b) How fast was her fish traveling when it struck the surface? (5 points)

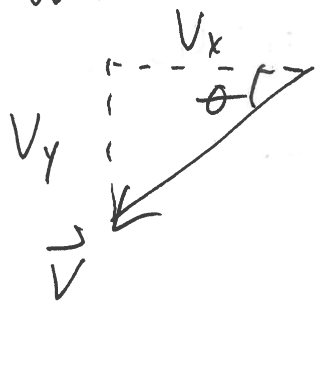
What is the magnitude of v at the time $t = d/v_0$?

- $v_x = v_0$ (no x acceleration)
- $v_y = -gt$ (free fall)

$$\rightarrow v = \sqrt{v_0^2 + \left(\frac{gd}{v_0}\right)^2}$$

c) In what direction was her fish traveling when it struck the surface? (5 points)

What direction is $\vec{v}$ at that time?



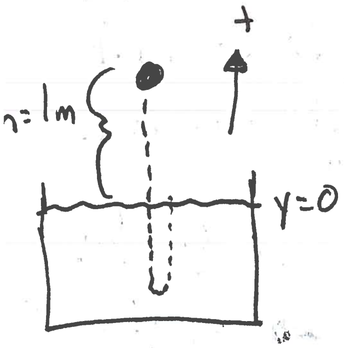
$$\theta = \tan^{-1} v_y/v_x = \tan^{-1}\frac{\frac{g^2d^2}{v_0^2}}{v_0} = \tan^{-1}\frac{g^2d^2}{v_0^3}.$$

Question 6

A hollow ball falls into a pond from a height of 1m. While it is in the air, it is in freefall. When it is underwater, it has an acceleration of 5 m/s^2 upward, because it is light enough to float.

You may assume that its velocity does not change as it passes through the surface of the water.

a) With what velocity does it strike the surface? (5 points)



"What is v at the time y=0?"

$v(t) = -gt$

$y(t) = -\frac{1}{2}gt^2 + h \longrightarrow 0 = -\frac{1}{2}gt^2 + h$, and $t = \sqrt{\frac{2h}{g}}$.

$V = -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh} = -4.47 \text{ m/s}$.

b) What is the deepest point that the ball reaches? (5 points)

Second phase of motion: now $y_0 = 0$, $v_0 = -4.47$ m/s.

Find y at the time $v=0$ to find deepest point:

$V = V_0 + at$, $0 = V_0 + at$, $t = \frac{-V_0}{a}$

$y = V_0 t + \frac{1}{2}at^2$, so $y = \frac{-V_0^2}{a} + \frac{1}{2}a\left[\frac{V_0^2}{a^2}\right] = -\frac{1}{2}\frac{V_0^2}{a} = -2$ m.

c) How long after it is dropped does it take for the ball to reach the surface again? (5 points)

Ask now "At what time does $y=0$ again?"

can cancel since $t=0$ is the trivial solution we don't care about

$0 = V_0 t + \frac{1}{2}at^2$, $0 = \cancel{t}\left[V_0 + \frac{1}{2}at\right]$, $t = \frac{-2V_0}{a} = 1.79$ s.

Must add the time from first phase $= \sqrt{\frac{2h}{g}} = 0.45$ s

$=$ total of 2.24 s.

(This question continues on the next page.)

Question 7

Toby the cat is sitting on a shelf that is a height $h = 1.5$ m high. Her owner Alice is lying on the floor next to the shelf and throws a little stuffed squirrel (Toby's favorite toy) directly up in the air. It flies straight up and straight back down.

Toby can only see the squirrel when it is above the level of the shelf.

a) Write expressions for the position and velocity of the squirrel as a function of time. (5 points)

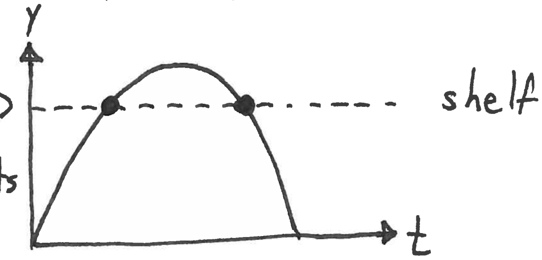
Motion only in 1-D: $y(t) = v_0 t - \frac{1}{2} g t^2$.

$v(t) = v_0 - gt$.

b) Suppose that Alice throws the squirrel upward at 6 m/s. As soon as Toby sees it rise past the level of the shelf, she tries to grab it. How much time does she have to catch it? (That is, for how much time will the squirrel be above the level of the shelf?) (12 points)

"For how long will y be greater than 1.5?"

Plan: use quadratic formula to find two points where $y = 1.5$ and find their difference.



$h = v_0 t - \frac{1}{2} g t^2 \rightarrow \frac{1}{2} g t^2 - v_0 t + h = 0.$

$$t = \frac{v_0 \pm \sqrt{v_0^2 - 2gh}}{g}.$$

$t = (0.36, 0.84)$ s.

Interval between: 0.48 s.

c) Repeat the previous problem, but suppose that Alice only throws it at 5 m/s.

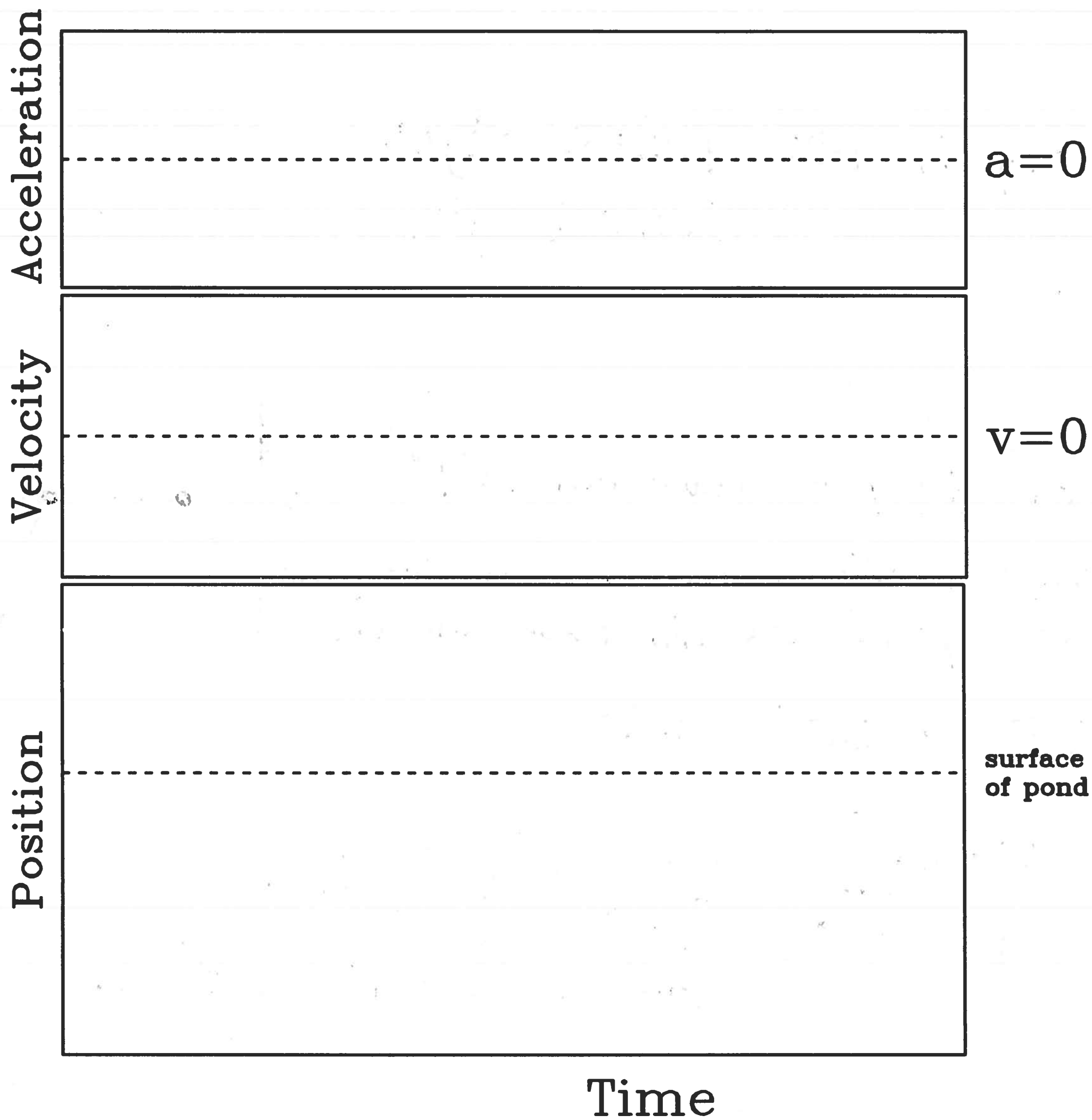
Use formula above to find time for new v_0:

...Note that $2gh > v_0^2 \rightarrow$ no real answers

$\rightarrow$ the toy never makes it 1.5 m high!

Question 6, continued

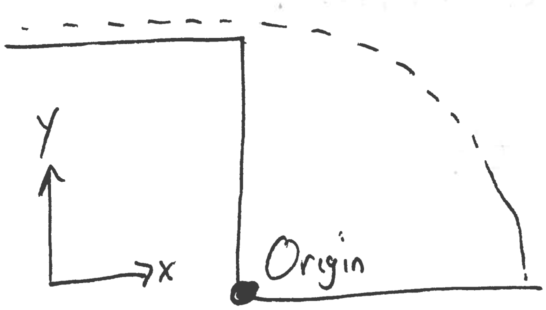
d) Sketch graphs of acceleration vs. time, velocity vs. time, and position vs. time on the axes provided, starting from when the ball is dropped and ending when it reaches the surface again. We are not looking for exact numbers, but we want zero crossings, maxima and minima, slopes, concavity, etc. to be correct. (10 points)



Question 8

A ball rolls off a shelf of height h at speed v. Answer the following in terms of h, v, and g.

a) How long does it take the ball to hit the floor? (10 points)



"What is t at the time $y=0$?"

$$y(t) = -\frac{1}{2}gt^2 + \cancelto{0}{v_{0,y}}\, t + y_0 \longrightarrow \text{equal to } h$$

$$0 = -\frac{1}{2}gt^2 + h$$

$$\longrightarrow h = \frac{1}{2}gt^2, \quad t = \sqrt{\frac{2h}{g}}.$$

b) Where does the ball hit the floor? (10 points)

"What is x at that time?"

$$x(t) = \frac{1}{2}\cancelto{0}{a_x} t^2 + v_{0,x}\, t + \cancelto{0}{x_0} = v_0 t$$

$$\longrightarrow x = v_0\sqrt{\frac{2h}{g}}.$$

c) What is the ball's speed when it hits the floor? (10 points)

Need magnitude of $\vec{v}$ at that time:

$v_x(t) = v_0$ (no accel. in x), $\quad v_y(t) = -gt = -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh}.$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + 2gh}.$$

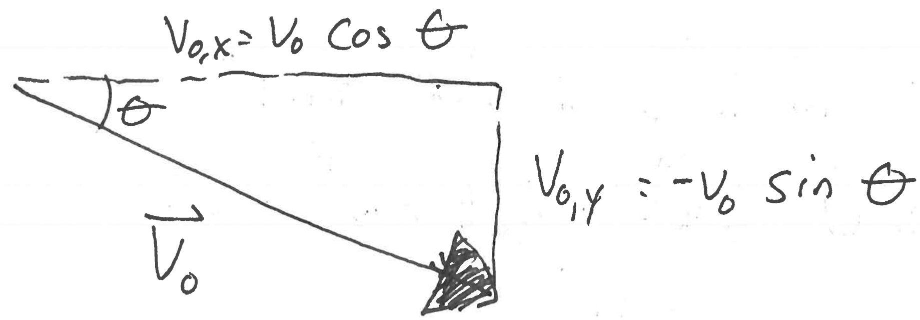
d) What direction is the ball moving in when it hits the floor? (10 points)

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{2gh}{v_0}\right)$ below the horizontal.

e) Suppose that the edge of the shelf had been curved, so that the ball's initial velocity was instead directed at an angle θ below the horizontal. Explain, using words or algebra as appropriate, what things you would have needed to do differently to solve the previous four parts, and which things would stay the same. (10 points)

Now, $V_{0,x}$ and $V_{0,y}$ would be nonzero.

$V_{0,x} = V_0 \cos\theta$, $V_{0,y} = -V_0 \sin\theta$.



Everything else proceeds the same.