Rotational motion

Physics 211
Syracuse University, Physics 211 Spring 2017
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April 6, 2017

Announcements

- Next homework is due next Wednesday
- Another short homework set will be due *on the day of your exam* – it will be designed to help you study
- No office hours Friday (I'm traveling)

Rotational motion, summarized

- Force diagrams: draw the entire object, and label at what point the forces act on them
- Choose a pivot (for rotating things, choose the rotation axis
- Newton's law for rotation: $\tau = I\alpha$
  - Applies separately for each rotating object
- Sometimes you will also need $\vec{F} = m\vec{a}$
- For static equilibrium problems: $\alpha = 0$

When do objects balance?

- Remember normal forces can only push, never pull
- Think about what happens as something begins to tip
- As an object topples over, its entire weight rests on the corner of the surface...

Agenda for today

You already know that rotational ideas correspond to translational ones:

Translation	Rotation
Position $\vec{s}$ Velocity $\vec{v}$ Acceleration $\vec{a}$	Angle θ Angular velocity ω Angular acceleration α
Kinematics: $\vec{s}(t)\frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$	$\theta(t) = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$
Force $\vec{F}$ Mass m Newton's second law $\vec{F}_{\text{tot}} = m\vec{a}$	Torque $\vec{\tau} = \vec{r} \times \vec{F}$ Moment of inertia I Newton's second law for rotation $\tau_{\text{tot}} = I\alpha$

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You've also studied kinetic energy along with the work-energy theorem. They have rotational analogues as well.

Rotational energy

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Kinetic energy $KE = \frac{1}{2}mv^2$ Work $W = \vec{F} \cdot \Delta\vec{s}$ Power $P = \vec{F} \cdot \vec{v}$	Kinetic energy $KE = \frac{1}{2}I\omega^2$ Work $W = \tau\Delta\theta$ Power $P = \tau\omega$

Rotational kinetic energy and the rotational work-energy theorem work like their translational counterparts.

Rotational kinetic energy

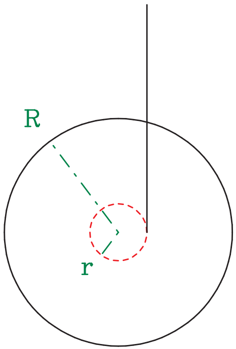
There is also kinetic energy associated with rotation, too! (The pipe problem from HW6...)

$$KE_{\mathrm{rot}} = \frac{1}{2} I \omega^2$$

This is what we would expect, based on $KE_{\mathrm{trans}} = \frac{1}{2} m v^2$:

- Moment of inertia I is the rotational analogue of mass
- Angular velocity ω is the rotational analogue of velocity

Suppose I release a Yo-Yo whose string has a length h. How fast will its center be moving when it runs out of string?



A: $v_f < \sqrt{2gh}$, because the tension in the string slows it down

B: $v_f < \sqrt{2gh}$, because part of the GPE is required to make the Yo-Yo spin

C: $v_f = \sqrt{2gh}$, by the conservation of energy

D: $v_f > \sqrt{2gh}$, because the spinning disk speeds it up

Answer C is what we get if there is no string. (We already know how to do that.)

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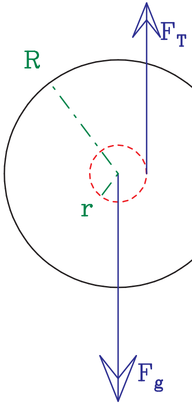
Answer A makes sense; in a force diagram for the Yo-Yo, the tension means that the net downward force is less than mg.



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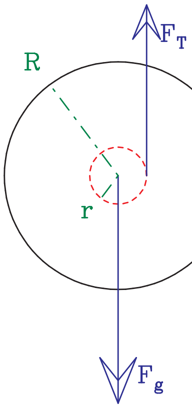

Answer B makes sense as well, though: if the Yo-Yo spins as it falls, then **some energy is required to make it spin**, leaving less available energy for translational kinetic energy.

We'll analyze this using energy methods.

The work done by tension

We know the work-energy theorem for translational motion (for constant $\vec{F}$):

$$W_{\text{trans}} \equiv \Delta \frac{1}{2} m v^2 = \vec{F} \cdot \Delta \vec{s}$$

Replacing m, $\vec{F}$, $\vec{s}$, and v^2 with their rotational counterparts, we get:

$$W_{\text{rot}} \equiv \Delta \frac{1}{2} I \omega^2 = \tau \Delta \theta$$

This is the *rotational work-energy theorem.*

The work done by tension

Which is true regarding the work done by tension here?

A: $W_{\text{total}} = 0$

B: $W_{\text{trans}} > 0, W_{\text{rot}} > 0$

C: $W_{\text{trans}} < 0, W_{\text{rot}} > 0$

D: $W_{\text{trans}} > 0, W_{\text{rot}} < 0$

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The string makes the Yo-Yo fall more slowly (negative translational work), but makes it spin (positive rotational work). That means Answer C is correct. What about Answer A?

The work done by tension

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Rotational work: $W_{\mathrm{rot}} = \tau \Delta\theta$

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If the Yo-Yo falls a distance h, it turns through a (positive!) angle given by $\Delta\theta = h/r$.

The torque applied by the tension is $\tau = Tr$ (positive!).

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Rotational work: $W_{\text{rot}} = \tau\Delta\theta = Tr(h/r) = Th.$
Translational work: $W_{\text{trans}} = \vec{F} \cdot \Delta\vec{s} = -Th.$

The work done by tension

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$\rightarrow$ The total work done by tension here is zero. (We could have guessed that!)

Conservation of energy, including rotation

$$\mathrm{PE_i} + \frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 + W_{\mathrm{NC}} = \mathrm{PE_f} + \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

Which expression will let us find the velocity of the Yo-Yo at the bottom?

A : $mgh - Th = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$

B : $mgh + \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$

C : $mgh = \frac{1}{2}mv_f^2$

D : $mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$

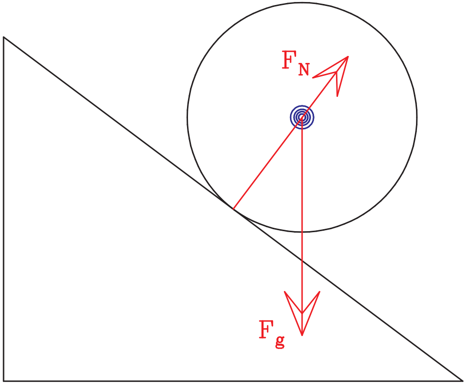
What about rolling objects?

In the Yo-Yo problem, we saw that:

- Tension did positive rotational work (it made the Yo-Yo spin faster)
- Tension did negative translational work (it made the Yo-Yo move more slowly)
- ... the **net work done by tension was zero**.

This happened because the string was stationary, and thus enforced $a = \pm\alpha r$. This is also true in rolling motion.

Consider first a ball sliding down a hill without friction.



Which of these forces applies a torque to the ball?

A: Just the normal force

B: Just gravity

C: Both of them

D: Neither of them

An object rolling down a hill

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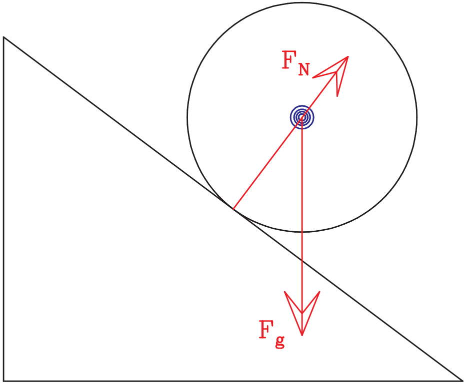

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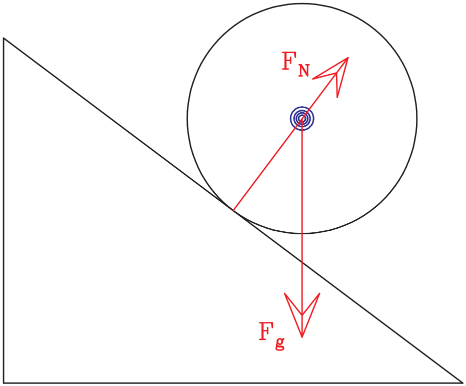
Friction is required to make the ball spin!

An object rolling down a hill

If the ball *rolls without slipping...*



What is true about the frictional force?

A: Static friction points down the ramp
B: Static friction points up the ramp
C: Kinetic friction points down the ramp
D: Kinetic friction points up the ramp
E: There is no friction

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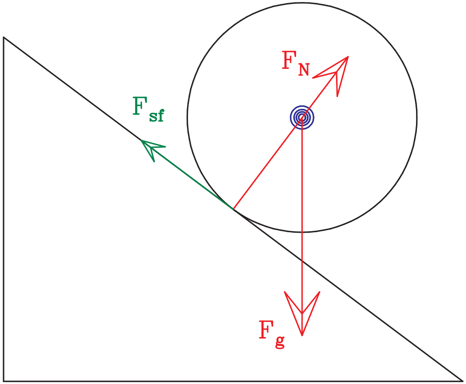

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The point of contact would slide downward without friction, so friction points back up the ramp. This is static friction since the ball doesn't slide.

Energy rolling down a hill

Static friction does no total work on the ball:

- it reduces the translational kinetic energy $\frac{1}{2}mv^2$
- it increases the rotational kinetic energy $\frac{1}{2}I\omega^2$
- ... but it leaves the sum $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ unchanged

TABLE 6.1 Coefficients of friction

Materials	Static μ_s	Kinetic μ_k	Rolling μ_r
Rubber on concrete	1.00	0.80	0.02
Steel on steel (dry)	0.80	0.60	0.002
Steel on steel (lubricated)	0.10	0.05	
Wood on wood	0.50	0.20	
Wood on snow	0.12	0.06	
Ice on ice	0.10	0.03	

This is not *quite* true – rolling friction does exist. There is a little bit of overall negative work done as tires flex and so on, but it is small.

(From *Physics for Scientists and Engineers*, Knight, 3rd ed.)

This means that we can use our standard expression for conservation of energy for rolling objects, *ignoring* the force of static friction required to keep them from slipping:

$$\mathrm{PE}_i + \frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 + W_{\mathrm{NC}} = \mathrm{PE}_f + \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

How fast will each object ($I = \lambda mr^2$) be traveling at the bottom of the ramp?

How high must I start the ball for it to make it around the loop?

Rotational dynamics and power

If $W = \tau\Delta\theta$, then $P = \tau\omega$.

If I want to supply a power P, I can either exert a large torque at a small angular velocity, or a small torque at a large angular velocity.

$\rightarrow$ bicycle demonstration!

Angular momentum

Translational motion

- Moving objects have momentum
- $\vec{p} = m\vec{v}$
- Momentum conserved if there are no external forces

Rotational motion

- Spinning objects have angular momentum L
- $L = I\omega$
- Angular momentum conserved if no external torques

$\rightarrow L = I\omega =$ constant; analogue to conservation of momentum

Conservation of angular momentum

We saw that the conservation of momentum was valuable mostly in two sorts of situations:

- Collisions: two objects strike each other
- Explosions: one object separates into two

There is a third common case for conservation of angular momentum:

- Collisions: a child runs and jumps on a merry-go-round
- Explosions: throwing a ball off-center
- A spinning object changes its moment of inertia

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This last happens because moment of inertia depends on *how the mass is distributed*, not just how much there is!

Conservation of angular momentum

These problems are approached in exactly the same way as conservation of *linear* momentum problems: write down expressions for L_i and L_f and set them equal (if there are no external torques).

$$L = I\omega$$

$$\sum L_i = \sum L_f$$

Conservation of angular momentum

If I kept the mass of the Earth the same, but enlarged it so that it had twice the diameter, how long would a day be?

(Remember, the total angular momentum, $L = I\omega$, stays the same)

A: 6 hours

B: 12 hours

C: 24 hours

D: 48 hours

E: 96 hours

Angular momentum of a single object

A single object moving in a straight line also has angular momentum.

$$L = mv_{\perp}r = mvr_{\perp}$$

Example: A child of mass m runs straight east and jumps onto a merry-go-round of mass M and radius r, landing 2/3 of the way toward the outside. If she lands on the south edge, how fast will it be turning once she lands?

We'll do this together on the document camera.