Recitation Questions

Wednesday, 20 March

1. The work-energy theorem, $\Delta \mathrm{KE} = \vec{F} \cdot \Delta \vec{s}$, is closely related to the "third kinematics relation", $v_f^2 - v_i^2 = 2a\Delta x$. How are they related? How is the work-energy theorem more general and more powerful? Discuss this with your group, and call your TA or coach over when you have an answer and discuss it with them.

If you take $v_f^2 - v_i^2 = 2a\Delta x \times \left(\frac{m}{2}\right) \rightarrow \underbrace{\frac{mv_f^2}{2}}_{KE_f} - \underbrace{\frac{mv_i^2}{2}}_{KE_o} = \underbrace{ma\Delta x}_{|F||\Delta x|\cos\theta}$

Thus, the third kinematic relation is a special case of the work energy theorem in which the force is in the same direction as the displacement.

2. Think of a situation in which:

   (a) Kinetic friction does positive work 2 things with friction between them with one moving faster than the other
   (b) Static friction does positive work Traction
   (c) Air resistance does positive work Sailing boat
   (d) A normal force does negative work Punch stuff
   (e) A normal force does positive work Elevator floor on the person inside of it.
   (f) Tension does positive work elevator going up
   (g) Tension does negative work elevator going down
   (h) Tension does zero work Something hanging moving sideways
   (i) Traction does zero work car moving on a curve

To solve all parts of the following two problems, take the following steps:

(a) Draw clear cartoons of your "before" and "after" situations. (One of the biggest sources of mistakes is not being explicit about the two pictures that you are considering with the work-energy theorem.)

(b) Think carefully about all forces that do work on the object in question between the "before" and "after" states.

(c) Write down the work-energy theorem:

$$\frac{1}{2}mv_0^2 + \text{work done by force 1} + \text{work done by force 2} + ... = \frac{1}{2}mv_f^2$$

(d) Determine the work done by each force, as either:

- Work equals the component of the force parallel to the motion, multiplied by the distance moved: $W = F_{\parallel}d$
- Work equals the size of the force, multiplied by the component of the distance moved parallel to the force: $W = Fd_{\parallel}$
- Work equals the size of the force, multipled by the distance moved, multiplied by the cosine of the angle between them: $W = Fd\cos\theta$

(e) Put these expressions for work into the work-energy theorem, and solve for whatever you need to solve for.

3. Someone drops a penny of mass 2.5g off of the Empire State Building (height 380 m). It strikes the ground traveling at 50 m/s, having been slowed somewhat by air resistance.

2.5×10^{-3} kg

(a) With what velocity would it have struck the ground if there were no air resistance?

$$\begin{cases} v_f = ? \\ v_0 = 0 \ m/s^2 \\ h = 380 \ m \end{cases}$$

Before | After | Forces doing work: · gravity

If there was no air resistance, no energy would be dissipated, thus...

$$\frac{\cancel{m} v_f^2}{2} - \frac{\cancel{m} v_0^2}{2} = \vec{F}_g \cdot \vec{d} = mg \cdot h \cos 0° = \cancel{m} gh \rightarrow v_f = \sqrt{2gh + v_0^2} = (2 \times 380 \times 10)^{\frac{1}{2}} \simeq 87 \ m/s$$

(b) What was the work done by the drag force?

Before | After | Forces doing work: · gravity · air drag

$$\frac{1}{2} m \overset{57}{\cancel{v_f}}{}^2 - \frac{1}{2} m \overset{0}{\cancel{v_0}}{}^2 = \underbrace{W_g}_{mgh} + W_d$$

$$W_d = \frac{1}{2} m v_f^2 - mgh = 2.5 \times 10^{-3} \left\{ \frac{1}{2} \cdot 50^2 - 10 \cdot 380 \right\} \simeq -6.4 \ J$$

(c) This penny strikes the sidewalk and penetrates the surface, digging a hole 2 cm deep. What was the upward force exerted on the penny by the pavement?

Before | After | Forces doing work: • gravity • floor resistance

$$\frac{m \overset{0}{\cancel{v_f}}{}^2}{2} - \frac{m \overset{50}{\cancel{v_0}}{}^2}{2} = \underbrace{W_g}_{mgh} + W_f$$

$$m \left\{ - \frac{v_0^2}{2} - gh \right\} = W_f$$

$$W_f = -2.5 \times 10^{-3} \cdot \left\{ \frac{50^2}{2} + \cancel{10 \times 2 \times 10^{-2}} \right\}$$ Too small

$$= -3.1 \ J$$

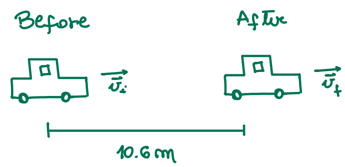
Remembering that $W_f = |\vec{F}_f||\vec{d}| \cos 180° \rightarrow |\vec{F}_f| = -\frac{W_f}{|\vec{d}|} = \frac{3.1}{0.02} = 155 \ N$

4. A police officer sets up a speed trap to catch cars driving over the speed limit coming around a curve. A car comes around the curve and sees the officer, and the driver immediately slams on her brakes to slow down before the officer can take a speed reading. By the time the officer measures the car's speed, the car is traveling 25 m/s, in an area where the speed limit is 30 m/s. However, the officer pulls over the driver anyway, saying "I saw you slam on your brakes. You must have been speeding!"

The car's driver protests the ticket in court. She says to the magistrate, "Your Honor, I can prove that I never exceeded the speed limit. It's true that I slammed on my brakes out of reflex as soon as I saw the officer. But I went back and measured the marks my tires left on the ground. Those marks are only 10.6 meters long, and by braking for that distance there's no way I could have decelerated from over the speed limit down to the 25 m/s that your officer measured."

Should the magistrate believe the driver? Could the car have been speeding when she first applied her brakes? Note that you will need to figure out the frictional force applied by the car's brakes, and to do that you will need to estimate the coefficient of friction between the tires and the pavement. Hint: do you need to know the mass of the car?

$$\begin{cases} \Delta\vec{x} = 10.6\ m \\ v_f = 25\ m/s \\ f_k = N\cdot\mu_k = mg\cdot\mu_k \\ \mu_k = 0.3 \\ v_i = ? \end{cases}$$



$$W_f = \frac{1}{2}m v_f^2 - \frac{1}{2}m v_0^2 = \vec{f}_k\cdot\Delta\vec{x} = mg\mu_k\cdot\Delta x\cdot\underbrace{\cos 180^\circ}_{-1}$$

25^2

$$-2\cdot\cancel{m}g\mu_k\cdot\Delta x = \cancel{m}(v_f^2 - v_0^2)$$

$$-v_0^2 = -2\mu_k\Delta x g - v_f^2$$

$$v_0 = \sqrt{2\mu_k\Delta x g + v_f^2} = \sqrt{\cancel{2}\times 0{,}\cancel{5}\times 10.6\times 10 + 25^2} =$$

$$= \sqrt{625 + 106} = \sqrt{731} = 27.03\ \frac{m}{s}$$

↳ She wasn't speeding