Physics 211 Group Exam 1, Form 4

Problem 7	Problem 8	Total
/25	/25	/50

Name: Solutions

Partner #1: ____________

Partner #2: ____________

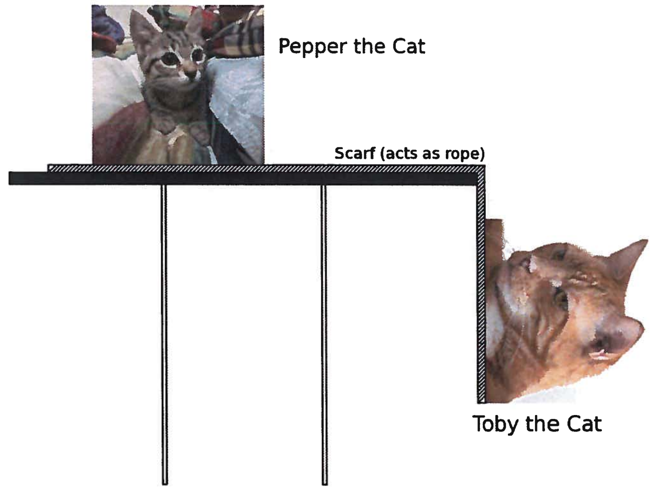
Recitation section number: ____________

- There are two questions, each worth twenty-five points.
- **You must show your reasoning to receive credit**. A numerical answer with no logic shown will be treated as no answer.
- You are highly encouraged to use both pictures and words to show your reasoning, not just algebra.
- If you run out of room, ask for an extra sheet of paper, or get one from your notebook.
- how your reasoning as thoroughly as possible for partial credit.
- You may use $g = 10\,\mathrm{m/s^2}$ throughout, except where indicated, to minimize arithmetic.

Question 7

A long scarf rests on a table. Our TA Adil's cat Pepper is asleep on one end of it; the other end hangs off the edge of the table.

Alice's cat Toby sees the other end of the scarf hanging over the edge of the table. Toby jumps up and grabs the edge, and her weight begins to pull Pepper and the scarf off the table. (You may assume that the scarf doesn't stretch and has negligible mass.)



Pepper has a mass m_P; Toby has a mass m_T. The coefficient of kinetic friction between the scarf and the table is μ_k; since the scarf is so light, the only place where there is friction is underneath Pepper.

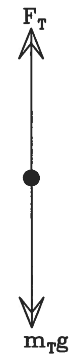
I would like to find the acceleration of the two animals and the scarf.

On the next page, you'll find my solution, but my solution contains an error. On the following page, I will ask you a few questions about my work, and ask you to fix my mistake.

Since this problem asks us to connect the forces on objects to their acceleration, I will use Newton's second law $\vec{F} = m\vec{a}$. First I write force diagrams for the two objects, and write down Newton's second law in each direction that matters for each object. I choose a conventional coordinate system where the positive $x-$axis is to the right and the positive $y-$axis is up.

Note that F_T is the force of tension, but m_T is Toby's mass.

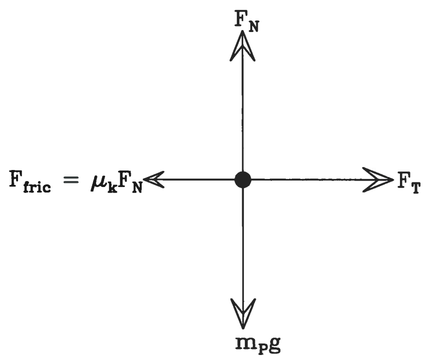
Toby:



$$Y : F_T - m_T g = m_T a_T \quad (1)$$

(Nothing happens in X for Toby)

Pepper:



$$Y : F_N - m_P g = 0 \quad (2)$$

(since the acceleration in y is zero)

$$X : F_T - \mu_k F_N = m_P a_P \quad (3)$$

These are not the same!

Since the scarf doesn't stretch, I've used the same acceleration variable for the two animals, since their accelerations must be the same. We now solve this system of equations by substitution. Solve equation (1) for the tension force and solve equation (2) for the normal force; this gives

$$F_T = m_T g + m_T a \quad (4) \qquad\qquad F_N = m_P g \quad (5)$$

Substitute the results from (4) and (5) into (3), and solve for a:

$$m_T g + m_T a - \mu_k m_P g = m_P a \quad (6)$$

$$m_T g - \mu_k m_P g = (m_P - m_T) a \quad (7)$$

$$\frac{m_T g - \mu_k m_P g}{m_A - m_T} = a \quad (8)$$

... which is what we were supposed to find. Remember, your job is to *find the error* that I have made and fix it.

a) The solution I got for the acceleration of the animals is

$$a = \frac{m_T g - \mu_k m_P g}{m_P - m_T}.$$

Right away, something about the mathematical form of this solution should tell you that there is a mistake. What about this answer should make you skeptical? (5 points)

If the two cats have the same mass, then the denominator is zero.

b) What mistake did I make? You can describe it briefly here, or indicate it clearly on the previous page. (10 points)

The accelerations are not the same. (See prev. page.)

c) What should the answer be instead? Correct my work on the previous page or below, and tell me what the acceleration should be instead. (10 points)

We should have:

the difference

y: $F_T - m_T g = m_T a_T$ (Toby)

x: $F_T - \mu_k F_N = m_P a_P$ (Pepper)

y: $F_N - m_P g = 0$ (Pepper)

Note $a_T = -a_P$ in this coordinate system.

Now, $F_T = m_T a_T + m_T g$ (for Toby)

or $F_T = -m_T a_P + m_T g$

$F_N = m_P g$ (from y)

Substitute into (x) for Pepper:

$-m_T a_P + m_T g - \mu_k m_P g = m_P a_P$

$\rightarrow a_P = \dfrac{m_T g - \mu_k m_P g}{m_P + m_T}$

(note + in denominator)

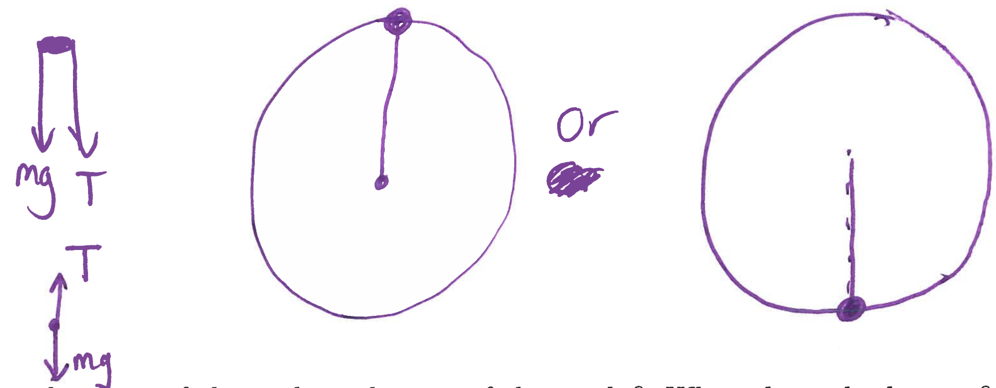
QUESTION 8

A person ties a rock of mass m to a string and spins it in a vertical circle of radius r at angular velocity ω. The string has a maximum tension $\tilde{T}$; if it is exposed to more tension than that, it will break. Give all your answers in terms of m, r, ω, and $\tilde{T}$.

a) Draw a force diagram for the rock at the top of the circle and at the bottom. (5 points)

Top:

Bottom:



b) What is the acceleration of the rock at the top of the circle? What about the bottom? Give both magnitude and direction. (5 points)

The acceleration is ω toward the center:

Top: $a = \omega^2 r$ downward   Bottom: $a = \omega^2 r$ upward.

c) At the top of the arc, the string is stretched taut. What force pushes the rock upward, stretching the string? If there is no such force, what stops the rock from falling at the top of the circle? (5 points)

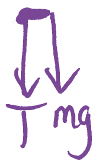
No force pushes it upward.

The rock is falling at the top; it accelerates downward at $\omega^2 r$.

The rope stays stretched since gravity alone doesn't exert a large enough force if $\omega^2 r > g$; instead, tension must also pull it down.

d) As you might expect, there is a minimum angular velocity; if he tries to spin the rock more slowly than this, the rock will fall. Find this minimum angular velocity. (5 points)

Tension must be positive; it cannot be negative.



Choose down to be positive. Then

$$\Sigma F = ma \Rightarrow T + mg = m\omega^2 r$$

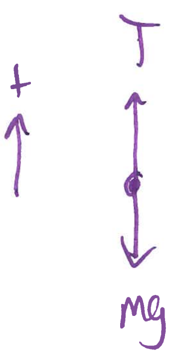
$$T = m\omega^2 r - mg$$

Set $T=0$ to find limiting case:

$$0 = \cancel{m}\omega^2 r - \cancel{m}g \rightarrow \omega_{min} = \sqrt{g/r}\,.$$

e) If he tries to spin the rock too fast, however, the string will snap. Find this maximum angular velocity. (You will need to think about where in the circle the string will snap first.) (5 points)

The tension is largest at the bottom, so analyze that:



$$T - mg = m\omega^2 r$$

$$T = m\omega^2 r + mg \text{ ; Set } T = \tilde{T} \text{ (maximum)}$$

$$\tilde{T} = m\omega^2 r + mg$$

$$\tilde{T} - mg = m\omega^2 r \rightarrow \omega_{max} = \sqrt{\frac{\tilde{T} - mg}{mr}}$$