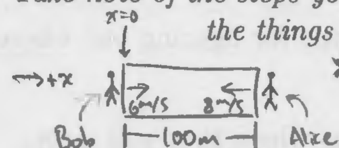
Recitation Questions – 1D Motion (part 1)

18 January

Question 1: Two runners

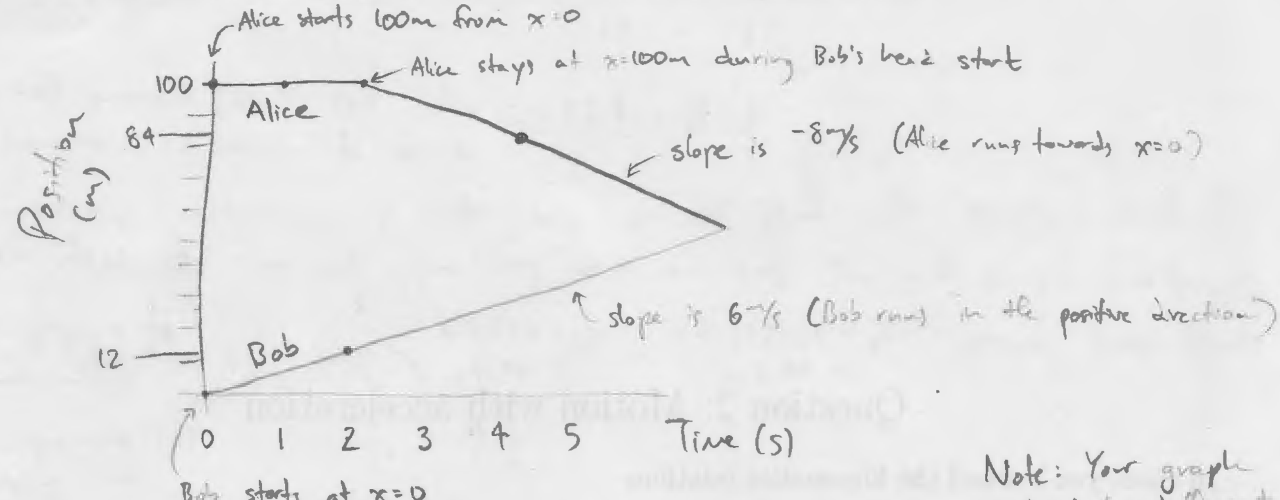
(This problem is simple, but it has the same template as most of the problems that you'll be doing for this unit. Take note of the steps you take to solve it; the steps will be similar for very many of the things you do in the next few weeks.)



✱ Be sure to
- choose an origin (where $x=0$)
- choose a positive direction

Two runners, Alice and Bob, start at opposite ends of a hundred-meter long soccer pitch and sprint toward each other. Alice runs at 8 m/s and starts on the east side, while Bob runs at 6 m/s and starts at the west side. Bob has a 2s head start. You'd like to know where they'll meet.

1. Draw position vs. time graphs for both runners on a single set of axes.



Note: Your graph could look different with a different choice of $x=0$, $t=0$, or the positive direction

$t=0$ when Bob starts running

2. Write position vs. time equations ($x = x_0 + vt$) for both runners. Use subscripts to distinguish their positions, *i.e.* x_A and x_B. Hint: Think carefully about their velocities...

From $x = x_0 + vt$, a choice of numbers for x_0 and v gives the equation of a straight line. Alice's equation should match the sloped part of her graph, so we can find where Alice meets Bob.

x_0 is the position at time $t=0$ (since plugging in $t=0$ gives you $x=x_0$), but it's not obvious where Alice's sloped line would be at $t=0$.

But since we know $x_A = 100$ at $t=2$, we can solve for x_0:

$$x = x_0 + vt$$
$$\downarrow$$
$$100 = x_0 + (-8)(2)$$
$$= x_0 - 16, \quad x_0 = 116.$$

So Alice's equation is $\boxed{x_A = 116 - 8t}$.

Bob's equation is simpler, since we know x_0 and v: $x_B = 0 + 6t$, or $\boxed{x_B = 6t}$.

3. In this entire unit, the main challenge will be translating the question ("where will they meet?") into a sentence that involves your algebraic variables. This sentence will then become your recipe for solving the problem. Often, but not always, this will take the following form:

"What is the value of $\underline{x}$ **at the time when** $\underbrace{\underline{x_A} \text{ is equal to } \underline{x_B}}_{\text{the runners meet when their positions are the same}}$**?"**

Answers "where"

Fill in the blanks in the above sentence to get a recipe for figuring out where the runners will meet.

4. Do the algebra indicated by your recipe and figure out where they will meet.

If we choose $x_A = x_B$, we have

$116 - 8t = 6t$. We can solve this for t to find when they meet.

$116 = 14t$

$t = \frac{116}{14} = 8.29\text{ s}$ after Bob starts running (based on our choice of what $t=0$ means).

To find where they meet, we can use this t in either position equation (since they will give the same position). You can try both to check your work: $x_A = 116 - 8(8.29) = 49.7\text{ m}$, $x_B = 6(8.29) = 49.7\text{ m}$ ✓, so they meet $\boxed{49.7\text{ m}}$ from the west side. This also agrees with how our graph looks.

Question 2: Motion with acceleration

In class, you learned the kinematics relations

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v(t) = v_0 + at$$

These relations aren't generally true, however; they are true only in a specific case. What must be true about the object's acceleration for these relations to apply?

Acceleration must be constant for these equations to be true. This is because (since acceleration is the rate of change of velocity), constant acceleration means v changes linearly, like the $v(t)$ equation indicates. A linearly changing velocity leads to a parabola-shaped position graph, like the $x(t)$ equation indicates. But all of this depends on a being constant.

Question 3: A bouncing basketball

Consider a basketball bouncing on the floor.

Draw position vs. time. velocity vs. time, and acceleration vs. time graphs for the ball on the next page.

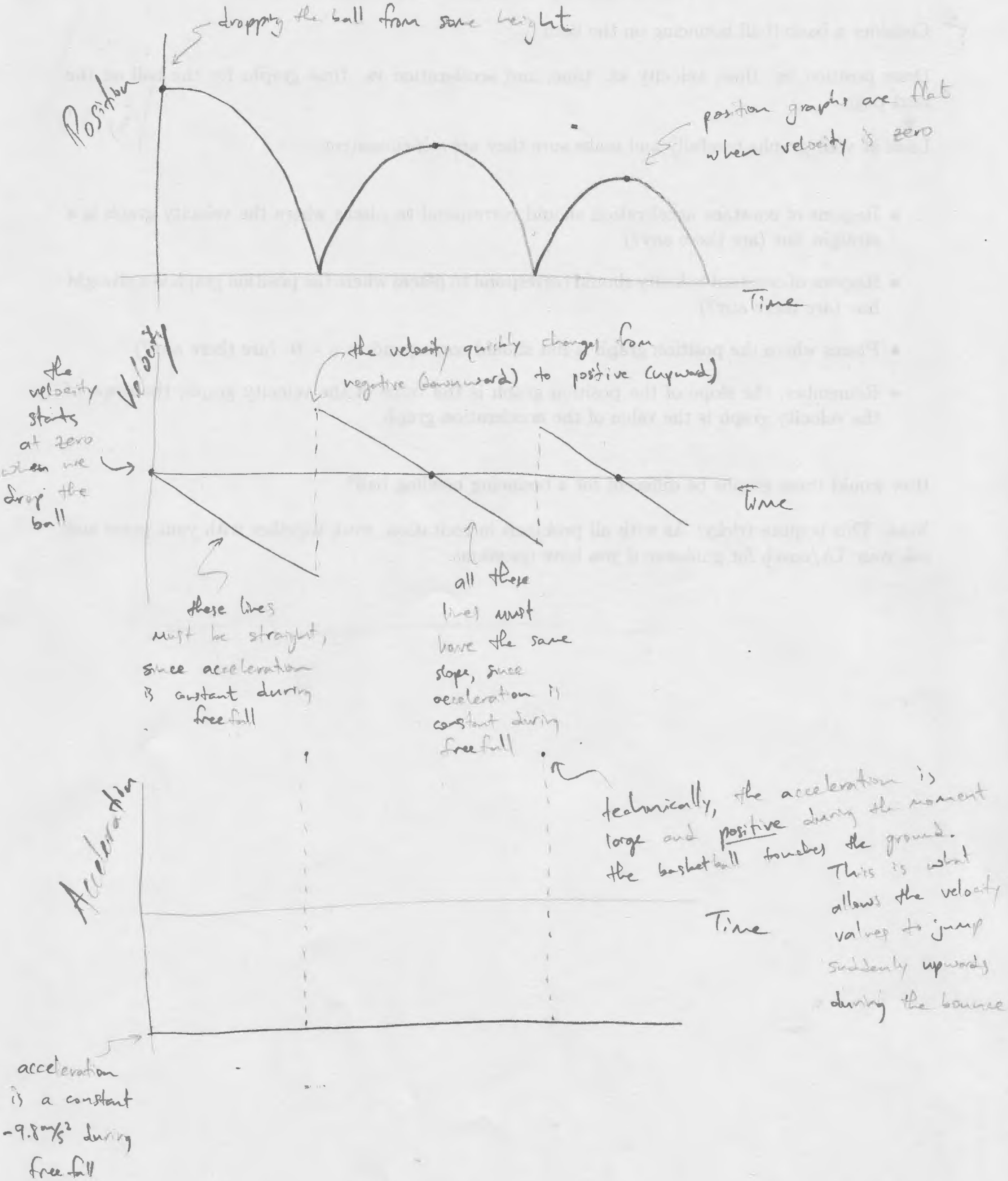
Look at your graphs carefully and make sure they are self-consistent:

- Regions of constant acceleration should correspond to places where the velocity graph is a straight line (are there any?)
- Regions of constant velocity should correspond to places where the position graph is a straight line (are there any?)
- Places where the position graph is flat should correspond to $v = 0$. (are there any?)
- Remember, the slope of the position graph is the value of the velocity graph; the slope of the velocity graph is the value of the acceleration graph

How would these graphs be different for a bouncing bowling ball?

Note: This is quite tricky! As with all problems in recitation. work together with your peers and ask your TA/coach for guidance if you have questions.

(This page left blank for your graphs.)



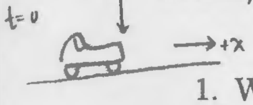
Recitation Questions – 1D Motion (part 2)

23 January

Question 1: a braking car

A car is traveling at 30 m/s and applies its brakes to slow down to 10 m/s. If it is able to decelerate at 5 m/s^2, how far does it travel during the braking period?

$x=0$



1. Write expressions for the car's position and velocity as a function of time. What moment makes sense to choose as your reference time $t = 0$?

We need constant acceleration to use the position and velocity equations, so let's choose $t=0$ as the moment the car starts to brake.

$x = x_0 + v_0 t + \frac{1}{2}at^2$ $\qquad$ $v = v_0 + at$,

choose constants that match the problem

$x = (0) + (30\text{ m/s})t + \frac{1}{2}(-5\text{ m/s}^2)t^2$ $\qquad$ $v = (30\text{ m/s}) + (-5\text{ m/s}^2)t$

braking means accelerating opposite the velocity

$\boxed{x = 30t - 2.5t^2}$ $\qquad$ $\boxed{v = 30 - 5t}$

2. How can you translate the question "How far does it travel during the braking period?" into a sentence about your algebraic variables? Again, fill in the blanks:

"What is the value of __x__ at the time when __v__ is equal to __10 m/s__?"

Answers "how far" $\qquad$ the car brakes until $v = 10$ m/s

3. What intermediate quantity must you find before you find the distance traveled? Following the above recipe you created for yourself, find it.

We have x as a function of time t, so we first need to find at what time t we have $v = 10$ m/s.

Choosing $v = 10$ m/s in our v equation, $10 = 30 - 5t$

$-20 = -5t$

$4 = t$, so $\boxed{t = 4\text{ s}}$ is when $v = 10$ m/s.

4. Finally, how far does the car travel during the braking period?

Using that time in the position equation,

$x = 30(4) - 2.5(4)^2$

$= 80$, so the car travels $\boxed{80\text{ m}}$ before it has slowed to 10 m/s.

(If your group didn't finish this problem last week, do it here.)