Things going in circles

Physics 211
Syracuse University, Physics 211 Spring 2019
Walter Freeman

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Announcements

- Homework 4 due next Wednesday; posted later today
- I have been swamped and am behind answering mail/Slack; I'll try to catch up this afternoon
- Upcoming office hours to accommodate homework deadline:
  - This Friday, from 11 to 1
  - Next Monday, from 1-3
  - Next Tuesday, from 3-5

On traction

An object sliding on a surface experiences kinetic friction.

But if an object rolls or walks on a surface, the point of contact is stationary.

- There may be a small amount of residual "rolling friction" which we rarely worry about...
- ... but, if the wheels or feet are powered by the machine/animal/person, there may be a **large static friction force**.

In problem 3 of HW3, the locomotive propelled itself forward using a traction force. For that problem we didn't care what it was, but it's useful to understand.

Traction is just a special name for static friction between a powered vehicle and the ground.

How do we walk?

I put my left foot in front of my right...

... I exert a backwards force on my left foot...

... static friction doesn't want my foot to move, and exerts a forwards force to keep it stationary ...

The net force on me is forwards!

How does a car propel itself?

The engine turns its wheels so that it exerts a backwards force on the part of the tires touching the ground ...

... static friction doesn't want the tires to slip, and it exerts a forwards force to keep them stationary ...

The net force on the tires is forwards!

Traction as static friction

The driver/foot-owner can cause traction to point backwards or even to the side, by exerting an appropriate force on the wheels/feet.

Like any other kind of static friction, traction is limited to $\mu_s F_N$, otherwise the wheels/feet will slip.

(It can also be limited by the ability of the engine/animal to apply force to its wheels/feet.)

Takeaway points:

- Traction is a type of static friction
- Like any other sort of static friction, it must be smaller than $\mu_s F_N$
- Within this limit, traction can point in any direction and have any size (limited by the strength of the engine/person

Why do Formula 1 cars have spoilers?

A spoiler is a large wing on the back of the car that pushes air up as it passes by, pushing the back of the car down.

A: To increase the normal force exerted by the wheels

B: To decrease the normal force exerted by the wheels

C: To decrease the maximum frictional force exerted on the car

D: To increase the maximum frictional force exerted on the car

E: To increase the coefficient of friction between the back tires and the pavement

What is the fastest that a car can go from 0-60 mph?

Rotational motion

Often things in nature are constrained to go in circles:

- Planets orbiting stars; moons orbiting planets (close enough to circles)
- Wheels; things on strings; many others

We'll study "uniform circular motion" here:

- Something moves at a constant distance from a fixed point
- ... at a constant speed.

Our goal for today

- How do we describe circular motion?
- How does circular motion relate to our previous knowledge of kinematics?
- What forces are required to make something go in a circle?

Describing circular motion: in general

In general, if an object goes in a circle, we care about its angle θ, not its x and y coordinates.

- This angle can be measured from any convenient zero point
- We will measure it in radians
- Traditionally, counter-clockwise is chosen as positive
- If it rotates around many times, there's no reason θ must be between 0 and 2π

Just as we needed to talk about derivatives of position, we need the derivatives of angle:

- Angular velocity ω: "how fast is it spinning?" (measured in radians per second)
- Angular acceleration α: "how fast is the angular velocity changing?" (measured in radians per second squared)

Since all the kinematics you learned was just statements about a function and its derivatives, it applies here too:

$$\theta(t) = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$$
$$\omega(t) = \alpha t + \omega_0$$
$$\omega_f^2 - \omega_0^2 = 2\alpha\Delta\theta$$

Uniform circular motion

- Here we take $\alpha = 0$, since that happens so often in nature: the Earth...
- Object moves in a circle of radius r, with its angle changing at a constant rate
- "Position = rate $\times$ time" $\rightarrow$ "Angle = rate $\times$ time"

All points on the wheel make the same number of rotations per minute:

- Points toward the outside may have a higher *ordinary* velocity ("translational velocity")...
- ... but they are *spinning* at the same rate ("angular velocity")

Angular velocity is a scalar!

Rotational motion: description

Some new terms:

- "Radial": directed in and out of the circle
- "Tangential": directed around the circle
- "Centripetal": pointed toward the center
- "Centrifugal force": a fiction, a fantasy, less real than a unicorn covered in rainbows

- The radial velocity is 0 everywhere (nothing is moving in or out)
- The tangential velocity depends on r and ω, as you'd expect

Which is true about the points labeled on this wheel?

A: The acceleration is zero

B: The magnitude of the acceleration is greater on the outside than on the inside

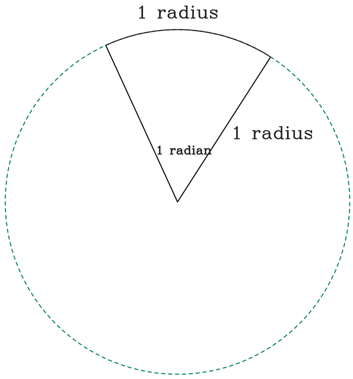
C: The acceleration is not zero, but has the same value everywhere

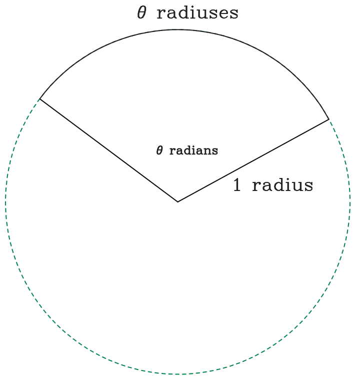
D: The angular acceleration is zero

Radians

The radian is a unit of angle. 2π radians = 360 degrees.

1 complete circle is 2π radians; 1 complete circumference is 2π radiuses ($C = 2\pi r$).
1 radian thus has an arc length of 1 radius.
θ radians therefore have an arc length of $r\theta$.





→ Tangential movement (in meters) = angular movement (in radians) times the radius

Rotational motion: description

The points around the outside travel faster than points on the inside:

- their *angular velocity* (radians/s) is the same...
- ... but their *tangential velocity* (meters/s) is larger.

$$v_T = \omega \quad r$$

$$\frac{\text{meters}}{\text{second}} = \frac{\text{radians}}{\text{second}} \times \frac{\text{meters}}{\text{radius}}$$

Which way is the object on the string accelerating at the top of the arc?

A: Upward: it is at the top of the arc, and it must have accelerated upward to get there

B: Upward: its upward acceleration is what keeps the string taut at the top of the swing

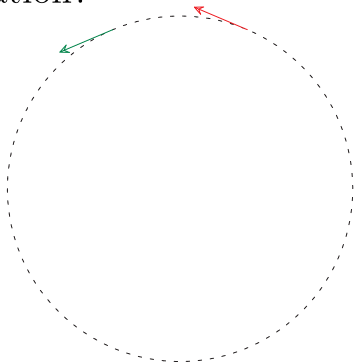
C: Downward: the only forces acting on it there pull it downward, so by $\vec{F} = m\vec{a}$ it must be accelerating downward

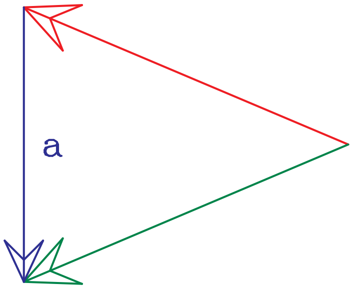
D: Downward: it was moving up and to the left, then down and to the left, so the net change is "down"

E: Zero: it is being swung at a constant rate

Kinematic challenge: what's $\vec{a}$?

Clearly an object moving in a circle is accelerating. What's the acceleration?





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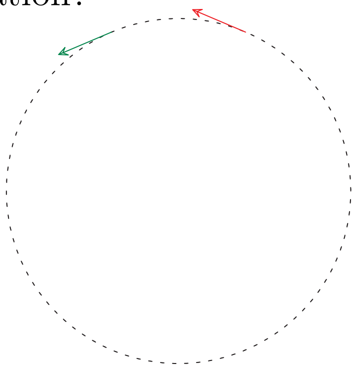

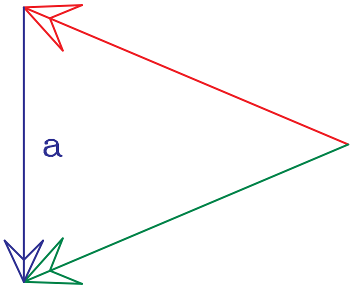

Near the top of the circle, the y-component of the velocity decreases; we expect then that $\vec{a}$ points downward.

Can we make this rigorous?

Some math

$$x(t) = r\cos(\omega t)$$
$$y(t) = r\sin(\omega t)$$

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Differentiate to get v_x and v_y:

$$v_x(t) = -\omega r\sin(\omega t)$$
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$\rightarrow \vec{a} = -\omega^2 \vec{r}$

An object in uniform circular motion accelerates toward the center of the circle with

$$\rightarrow a = \omega^2 r = v^2/r \leftarrow$$

Uniform circular motion, consequences

If you know an object is undergoing uniform circular motion, you know something about the acceleration:

$a = \omega^2 r$ or $a = v^2/r$ toward the center of the circle.

Uniform circular motion, consequences

If you know an object is undergoing uniform circular motion, you know something about the acceleration:

$a = \omega^2 r$ or $a = v^2/r$ toward the center of the circle.

Circular motion problems aren't scary; they are just like any other force problem.

- Equilibrium problem: $\sum F_x = ma_x = 0$ and $\sum F_y = ma_y = 0$
- Circular motion problem: $\sum F_T = ma_T = 0$ and $\sum F_r = ma_r = v^2/r$

$\rightarrow$ If we tell you that a thing is in uniform circular motion, we're just telling you something about its acceleration.

Of all of the topics in Physics 211, this is the one topic that students overcomplicate the most.

Circular motion problems are *just like any other* Newton's-law problem. The only difference is that you know something about the object's acceleration. Do not make these more complicated than they actually are!

Centripetal force

"Centripetal" means "toward the center" in Latin.

- If something is going to accelerate toward the center, a force must do that.
- Centripetal force is not a "new" force. No arrows labeled "centripetal force"!
- "Centripetal" is a word that describes a force you already know about.
- Centripetal force: describes a force that holds something in a circle
- It can be lots of things:
  - Tension (see our demos)
  - Normal force (platform, bucket demos)
  - Friction (Ferris wheel)
  - Gravity (the moon!)

What does the force diagram on the water look like while the bucket is at the bottom?

- A: acceleration upward, normal force upward, gravity downward
- B: centripetal force upward, normal force upward, gravity downward
- C: gravity downward, normal force upward
- D: gravity downward, velocity to the left, normal force upward

Why doesn't the water in the bucket fall at the top?

- A: It *is* falling, but the bucket falls along with it, so it stays in
- B: The normal force pushes it up
- C: The centripetal force pushes it up

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- A: It *is* falling, but the bucket falls along with it, so it stays in
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- D: Sam is in the back chanting "wingardium leviosa!"

What does the force diagram on the water look like while the bucket is at the top?

- A: acceleration downward, normal force upward, gravity downward
- B: normal force upward, gravity downward
- C: gravity downward, normal force downward
- D: normal force downward, gravity downward, centrifugal force upward

What is the acceleration of the water at the top?

- A: Zero
- B: Downward, less than g
- C: Downward, equal to g
- D: Downward, more than g
- E: Upward

Sample problems

If the water in the bucket has mass m, and my arm has a length L, and I'm swinging the bucket at angular velocity ω, what is the tension in the string at the top of the circle?

- A: $m\omega^2 L$
- B: mg
- C: $m\omega^2 L - mg$
- D: $mg - m\omega^2 L$
- E: $mg + m\omega^2 L$

Sample problems

If the water in the bucket has mass m, and my arm has a length L, and I'm swinging the bucket at angular velocity ω, what is the tension in the string at the **bottom** of the circle?

- A: $m\omega^2 L$
- B: mg
- C: $m\omega^2 L - mg$
- D: $mg - m\omega^2 L$
- E: $mg + m\omega^2 L$

Sample problems

I allow a mass m to swing around in a circle at the end of a string of length L. If the string makes an angle θ with the vertical, what is the angular velocity ω of the mass, and what is the tension in the string?