PHY211 Lecture 4

Matthew Rudolph

Syracuse University

January 23, 2020

Vectors

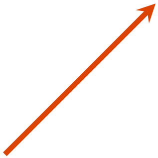
Describing vectors

- You’ve probably used vectors before without really thinking about it
  - “Walk one block east” – displacement vector
  - “Driving northbound at 100 km/h” – velocity vector
  - “The wind was blowing from the west at 30 km/h” – velocity vector
- Anything that points a direction, and has some magnitude is a vector
  - Can even think of just a direction as a vector of magnitude 1

Describing vectors

Mathematically

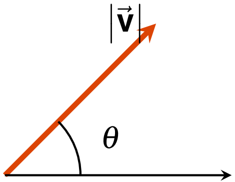
- It's easy to draw a picture of an arrow pointing some direction with some length



Describing vectors

Mathematically

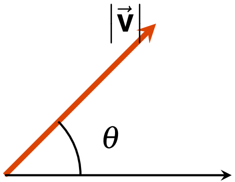
- It's easy to draw a picture of an arrow pointing some direction with some length
- Use direction and magnitude

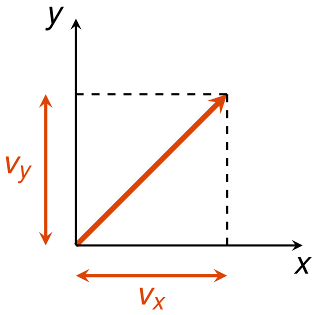


Describing vectors

Mathematically

- It’s easy to draw a picture of an arrow pointing some direction with some length
- Use direction and magnitude
- Or components along some axes

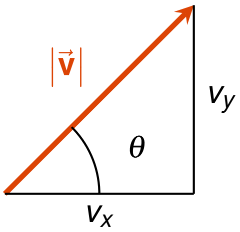




Conversions

We'll have to do this a lot!

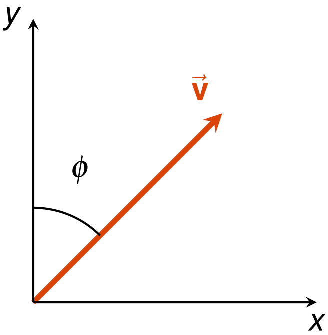
- Best bet – always draw the relevant triangle with the vector on the hypotenuse
  - Make it its own picture!



$$
\begin{aligned}
\left|\vec{\mathbf{v}}\right| &\equiv v \\
v_x &= v\cos(\theta) \\
v_y &= v\sin(\theta) \\
v &= \sqrt{v_x^2 + v_y^2} \\
\tan(\theta) &= v_y / v_x
\end{aligned}
$$

Question

- For this vector, what is v_y?
- A $v\cos(\phi)$
- B $v\sin(\phi)$
- C $v\tan(\phi)$
- D $v/\cos(\phi)$

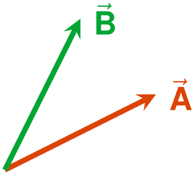


Vector placement

- Vectors do not “save” their start position
- Example: if you move from point A to point B, then the vector between them is your displacement. If you need to know the starting point, then you have additional quantities describing the position of A
- This means these are the same vectors:

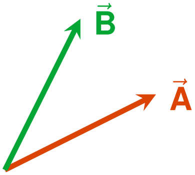
Adding graphically

What is $\vec{\mathbf{A}} + \vec{\mathbf{B}}$?

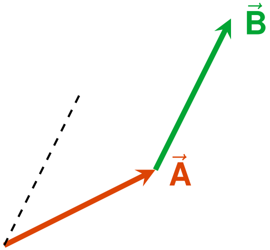


Adding graphically

What is $\vec{\mathbf{A}} + \vec{\mathbf{B}}$?

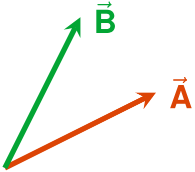


Move tip-to-tail

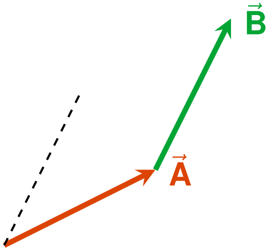


Adding graphically

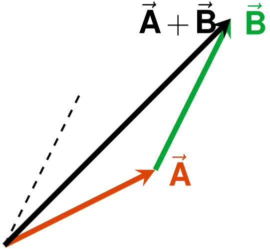
What is $\vec{\mathbf{A}} + \vec{\mathbf{B}}$?



Move tip-to-tail



Draw from start to finish

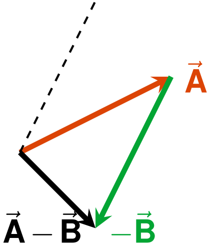


Subtracting graphically

What is $\vec{\mathbf{A}} - \vec{\mathbf{B}}$?

- Two options

Add $\vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$

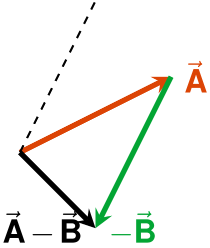


Subtracting graphically

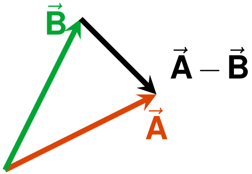
What is $\vec{\mathbf{A}} - \vec{\mathbf{B}}$?

- Two options

Add $\vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$

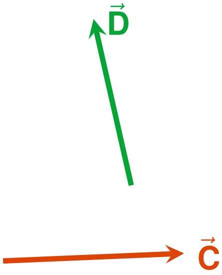


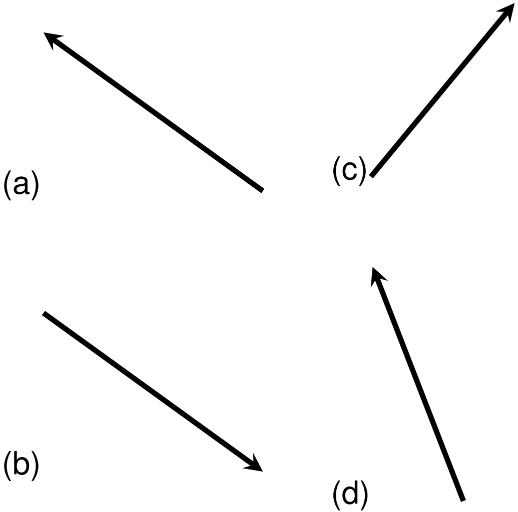
Draw *from* $\vec{\mathbf{B}}$ to $\vec{\mathbf{A}}$



Subtraction question

You have the following two vectors, and you want to draw $\vec{\mathbf{D}} - \vec{\mathbf{C}}$. Which result is correct?





Why components?

- Drawing is useful to get a picture in your head of what is going on, so it will help us make our graphs and sketches to solve problems
- But components are easier to use with numbers
- Can simply add x, y, and z components separately

$$\left(\vec{\mathbf{A}} + \vec{\mathbf{B}}\right)_x = A_x + B_x$$
$$\left(\vec{\mathbf{A}} + \vec{\mathbf{B}}\right)_y = A_y + B_y$$
$$\left(\vec{\mathbf{A}} + \vec{\mathbf{B}}\right)_z = A_z + B_z$$

Pre-lecture question 1

- What form does the trajectory of a particle have if the distance the particle travels from any point A to point B is equal to the magnitude of the displacement from A to B?

A. L-shape
B. Curved
C. Straight line

Pre-lecture question 2

- If an object experiences an acceleration in the y direction, does the x-component of its velocity change?

A. Yes

B. No

Pre-lecture question 3

- If an object moves 3 meter along the positive x direction, then turns and moves 4 meter along the negative y direction, what is the magnitude of its displacement?

A. $0\,\mathrm{m}$

B. $7\,\mathrm{m}$

C. $5\,\mathrm{m}$

D. $-5\,\mathrm{m}$

Simultaneous problems

with constant acceleration

- Just have two sets of the same equations, the only thing they share is time!

$$v_x(t) = v_{x,0} + a_x t$$
$$x(t) = x_0 + v_{x,0} t + \frac{1}{2} a_x t^2$$

$$v_y(t) = v_{y,0} + a_y t$$
$$y(t) = y_0 + v_{y,0} t + \frac{1}{2} a_y t^2$$

- This covers a lot of what we will do this semester!

Unit vector form

- Sometimes (in the book) you will see the components written out on one line
- This is done by writing each component as a vector pointing in the x, y, or z direction
- Then the sum of these component vectors is the full thing
- Uses the notation of unit vectors

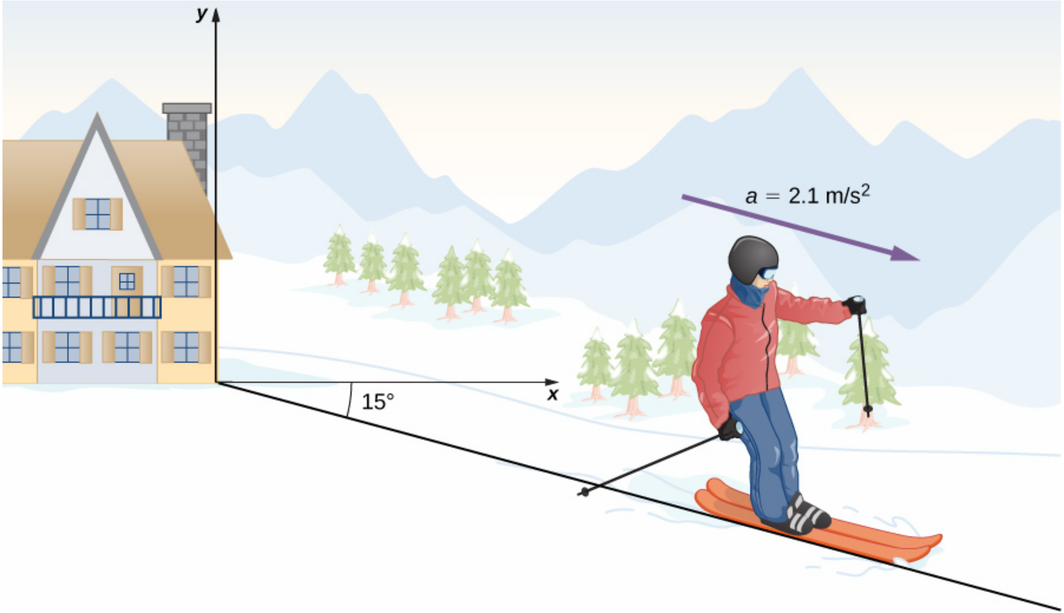
$$\hat{\mathbf{\imath}} \equiv \text{x direction}$$
$$\hat{\mathbf{\jmath}} \equiv \text{y direction}$$
$$\hat{\mathbf{k}} \equiv \text{z direction}$$
$$\vec{\mathbf{r}} = x\hat{\mathbf{\imath}} + y\hat{\mathbf{\jmath}} + z\hat{\mathbf{k}}$$

Vector recap

What do we need to know?

- We will use vectors all year to describe physics!
- You should be able to draw vectors; add and subtract them graphically
  - Very helpful for setting up problems and checking if your answer makes sense!
  - Example: if the velocity arrow you want to solve for points towards $-x$, your v_x should be negative at the end!
- For most problems you have to do components
  - Be able to convert magnitude and angle to x and y components
  - Components are often like two separate problems that just share time t
- Today we will practice this in context of motion

Skier example



Skier math

The figure shows a skier moving with an acceleration of $2.1\,\mathrm{m/s^2}$ down a slope of 15° at $t = 0$. With the origin of the coordinate system at the front of the lodge, her initial position and velocity are

$$\vec{\mathbf{r}}(0) = (75.0\hat{\mathbf{i}} - 50.0\hat{\mathbf{j}})\ \mathrm{m}$$

and

$$\vec{\mathbf{v}}(0) = (4.1\hat{\mathbf{i}} - 1.1\hat{\mathbf{j}})\ \mathrm{m/s}.$$

(a) What are the x- and y-components of the skier's position and velocity as functions of time?
(b) What are her position and velocity at $t = 10.0\,\mathrm{s}$?

Problem solving steps

Key strategy for the class!

1. Draw a picture – it helps visualize things
2. Choose axes – which way is positive? Where is zero?
3. When is $t = 0$?
4. For motion problems use the equations of motion
5. Translate the question into one about your variables
6. Do algebra to solve for the unknowns
7. Calculate a numerical answer
8. Does your answer make sense?

Simplifying things

- Can you think of an easier way to do the last problem?

Simplifying things

- Can you think of an easier way to do the last problem?
- Why not define $+x$ in the direction of acceleration?
- In two dimensions you can pick the angle of your axes!

Example

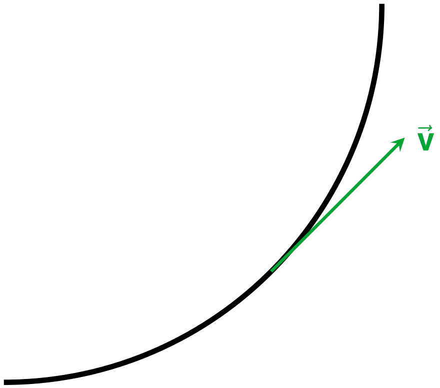
A spaceship is drifting with a velocity $\vec{\mathbf{v}} = 100\,\mathrm{m/s}\,\hat{\mathbf{\imath}} + 200\,\mathrm{m/s}\,\hat{\mathbf{\jmath}}$ at $t = 0$. It then turns on its engine which provides a constant acceleration. Four seconds later the spaceship's velocity is $\vec{\mathbf{v}} = 100\,\mathrm{m/s}\,\hat{\mathbf{\jmath}}$. What is the spaceship's acceleration in component form? What is the magnitude of acceleration? What is the position as a function of time?

Vectors and motion

- Once we can move in more than one dimension, need to worry about *relative* direction of position, velocity, and acceleration
- In 1D, acceleration had to point either with velocity, or opposite
- But in 2 or 3D we can turn

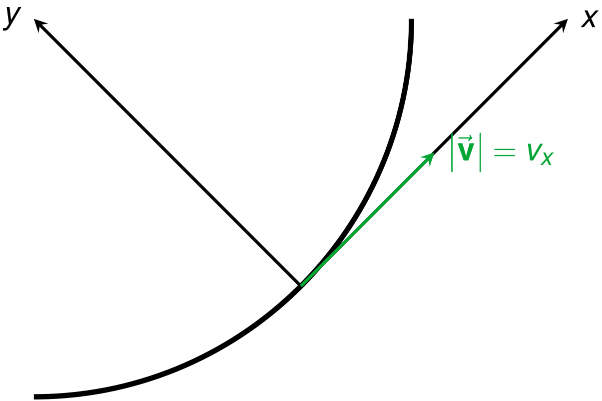
Does that make sense?

- Let's look at the *components* of $\vec{\mathbf{a}}$ with a good choice of axis



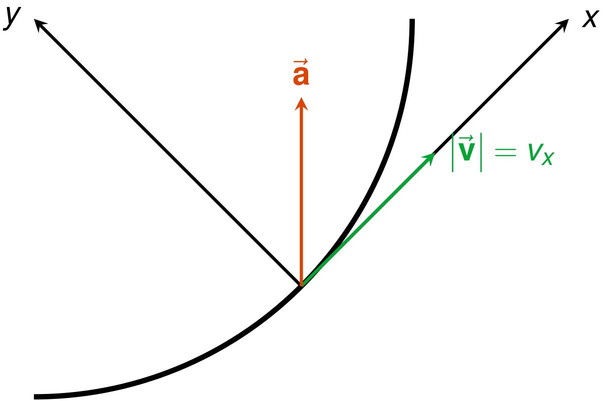
Does that make sense?

- Let's look at the *components* of $\vec{\mathbf{a}}$ with a good choice of axis



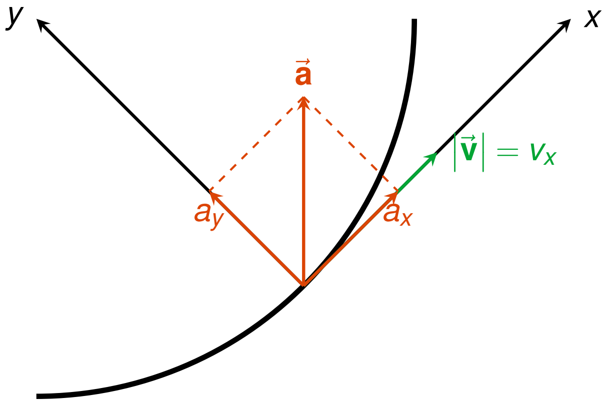
Does that make sense?

- Let's look at the *components* of $\vec{\mathbf{a}}$ with a good choice of axis



Does that make sense?

- Let's look at the *components* of $\vec{\mathbf{a}}$ with a good choice of axis



Radial and tangential

- Velocity is always tangential to the path – that's basically its definition! It tells you where you are going in the next instant of time

Radial and tangential

- Velocity is always tangential to the path – that's basically its definition! It tells you where you are going in the next instant of time
- Acceleration can have a tangential component – that makes you speed up or slow down
- And a radial component – that makes you turn!

Reminders

- Read sections 4.3 for next Tuesday and do pre-lecture questions
- First homework due tomorrow at recitation