Momentum

Physics 211 Syracuse University, Physics 211 Spring 2022 Walter Freeman

April 7, 2022

Announcements

- Group Exam 3 tomorrow; Exam 3 next Tuesday
- Review notes posted on the website
- Extra homework help hours today: 9:45-10:45
- HW7 due Friday

Extra exam review session

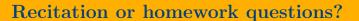
Extra exam review session will be held **Monday afternoon or evening**. We are still getting a time/place. (I am sadly booked all weekend.)

This will be a little more scripted than the last one, but I expect it should prepare everyone for the exam – instead of "cramming" on your own, come work with us.

To get the maximum benefit out of this, you should:

- Have done all your homework
- Go to the Physics Clinic after the group exam Friday and ask the coaches to help you with anything you couldn't do
- Be familiar with most of the recitation exercises we have done
- Have already attempted the practice exam over the weekend, and have looked at the solutions (posted Friday or Saturday)

Recitation or homework questions?



Shall we talk about the "one with the truck"?

This unit is about *conservation laws*.

We have met two of them: conservation of momentum and the work-energy theorem.

These techniques let you analyze systems where you know something about "before" and "after" states.

General problem-solving techniques:

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Review: energy methods

Use energy methods when:

- You have clear before and after states (draw your cartoons, dammit!)
- You can calculate the work done by the forces between them (not collisions/explosions!)
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- You have clear before and after states (draw your cartoons, dammit!)
- You can calculate the work done by the forces between them (not collisions/explosions!)
- You don't care about time
- Be careful with projectile motion!
 - Energy methods can tell you "how fast" or "how high"
 - They cannot tell you "where does it land?"

Review: The work-energy theorem

Work-energy theorem: $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \vec{F} \cdot \vec{d} = Fd\cos\theta$ (if this is constant)

Potential energy is an alternate way of keeping track of the work done by conservative forces:

- $PE_{\text{grav}} = mgh$
- $PE_{\text{spring}} = \frac{1}{2}kx^2$

$$PE_{i}$$
 + $\frac{1}{2}mv_{i}^{2} + \frac{1}{2}I\omega_{i}^{2}$ + W_{other} = PE_{f} + $\frac{1}{2}mv_{f}^{2} + \frac{1}{2}I\omega_{f}^{2}$

$$PE_{i} + \frac{1}{2}mv_{i}^{2} + \frac{1}{2}I\omega_{i}^{2} + W_{other} = PE_{f} + \frac{1}{2}mv_{f}^{2} + \frac{1}{2}I\omega_{f}^{2}$$
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(total initial mechanical energy) + (other work) = (total final mechanical energy)

Since conservation of energy is the broadest principle in science, it's no surprise that we can do this!



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(that's it – the last one doesn't come up much, but sometimes it does: why does the Earth wobble?)

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- $\sum \vec{p_i} = \sum \vec{p_f}$
- Momentum is a vector!
- If you have motion in two dimensions, momentum in each direction is conserved separately:

$$\sum p_{x_i} = \sum p_{x_f}$$

$$\sum p_{y_i} = \sum p_{y_f}$$

Do not get lazy with your subscripts!

Sample problems: an excited dog

A person of mass m is sitting in a tire swing with a string of length L when their dog (mass M) runs and jumps horizontally into their lap.

If they swing up to an angle θ above the horizontal, how fast was their dog running?

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One way to describe this: how does the total kinetic energy change?

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An application: neutron moderators

The only truly elastic collisions in nature are between particles. If we want *totally* elastic collisions, we should look to nuclear physics!

A note: this calculation we are going to do here demonstrates two things:

- How elastic collisions work
- ... and how the art of approximation is used in physics and engineering!

Recall how a nuclear reactor works:

- ^{235}U fissions when struck by neutrons with low energy (600 times more likely at low energy, less than 0.1 eV)
- When ^{235}U fissions, it produces neutrons with 2 MeV of kinetic energy ($v \approx 20$ million m/s)

How do we make these neutrons go from 2 million eV to 0.1 eV of kinetic energy so they can produce more fissions?

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How much kinetic energy is lost?

The fraction of kinetic energy lost is

$$4\frac{m}{M}$$
.

So what atoms can we use as moderators?

They have to scatter neutrons more readily than they absorb them (hydrogen so-so, oxygen/carbon/heavy hydrogen great)

They have to be lightweight (that's what we just found)

They have to not be chemically grouchy (no hydrogen or oxygen by themselves!)

- H₂O (light water: most of the world, not the best moderator)
- D₂O (heavy water: Canadians)
- CO₂ (carbon dioxide: British)
- Pure carbon (Graphite: Soviets)