Motion problems

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Reminders

- There were pre-lecture questions on Blackboard for today
  - If you're still not sure about the grading – it is set up that you should average more than 1 point per question (over the whole semester) to get full credit
  - The line is drawn so that you should be able to miss about 2 weeks of questions without a problem
- First homework assignment is due Friday, second next Wednesday

Pre-lecture question 1

The equations

$$v(t) = v_0 + at$$
$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

are valid under what condition?

- A Velocity is constant
- B Acceleration is constant ✓
- C The object moves in only one direction
- D They are always valid

Equations of motion

- Many problems in this class involve motion with constant acceleration
- Can write down the graphs we drew as equations
- Use these equations to describe motion with constant acceleration:

$$v(t) = v_0 + at$$

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

- Sometimes this rearrangement is useful if you don't care about t:

$$v^2 = v_0^2 + 2a(x - x_0)$$

Example problem 1

- A light-rail commuter train accelerates at a rate of $1.35\,\mathrm{m/s^2}$. How long does it take to reach its top speed of $80.0\,\mathrm{km/h}$, starting from rest?

Credit: Leinad on wikimedia

Problem solving steps

Key strategy for the class!

1. Draw a picture – it helps visualize things
2. Choose axes – which way is positive? Where is zero?
3. When is $t = 0$?
4. For motion problems use the equations of motion
5. Translate the question into one about your variables
6. Do algebra to solve for the unknowns
7. Calculate a numerical answer
8. Does your answer make sense?

Example problem 2

- In emergencies, the train can decelerate rapidly, coming to rest from 80.0 km/h in 8.30 s. How far does it travel in that time?

Credit: Leinad on wikimedia

Translating the question

$$v(t) = v_0 + at$$
$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

Which question about the variables do we want to solve?

- A What is x when $a = 0$?
- B What is x when $t = 8.3\,\mathrm{s}$?
- C What is v when $t = 8.3\,\mathrm{s}$?
- D What is v when $x = 0$?

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Question

Which expression for a is correct?

- A $a = v_0/t$
- B $a = 2x/t^2$
- C $a = 0$
- D $a = -v_0/t$

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Question

Which expression for x is correct?

- A $x = \frac{3}{2} v_0 t$
- B $x = v_0 t^2$
- C $x = v_0 / t$
- D $x = \frac{1}{2} v_0 t$
- E $x = v_0 t$

Question

Which expression for x is correct?

- A $x = \frac{3}{2} v_0 t$
- B $x = v_0 t^2$
- C $x = v_0 / t$
- D $x = \frac{1}{2} v_0 t$ ✓
- E $x = v_0 t$

Pre-lecture question 2

Which of the following statements about gravity is true?

- A Less massive objects accelerate more from gravity
- B More massive objects accelerate more from gravity
- C All objects accelerate at the same rate from gravity ✓

Gravity

- Falling objects have constant acceleration

$$|a(t)| = g = 9.8\,\mathrm{m/s^2}$$

- g is a positive number, but the acceleration always points down
  - *Put in the minus by hand if "up" is "positive"*

Gravity

- Falling objects have constant acceleration

$$|a(t)| = g = 9.8\,\mathrm{m/s^2}$$

- g is a positive number, but the acceleration always points down
  - *Put in the minus by hand if “up” is “positive”*

$$v(t) = v_0 - gt$$
$$y(t) = y_0 + v_0 t - \frac{1}{2} g t^2$$

Example problem 3

A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of $1.40\,\mathrm{m/s}$ and observes that it takes $1.8\,\mathrm{s}$ to reach the water. How high above the water was the preserver released? (Neglect air resistance)

photo by Jaqian on wikimedia

Question:

- A ball is thrown straight up, and then falls. While in the air, when is its acceleration zero?

A. On the way up
B. On the way down
C. At the top
D. Never

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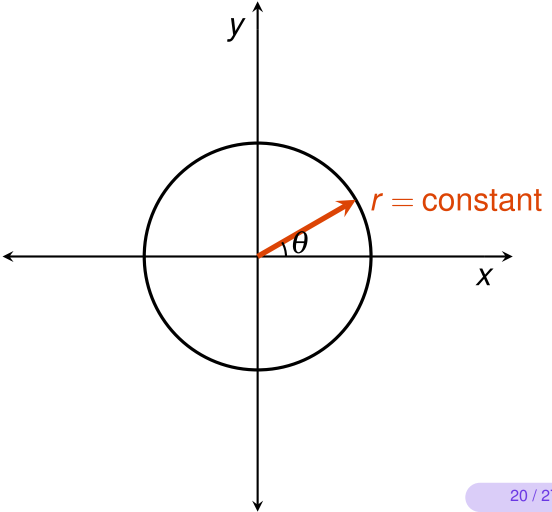
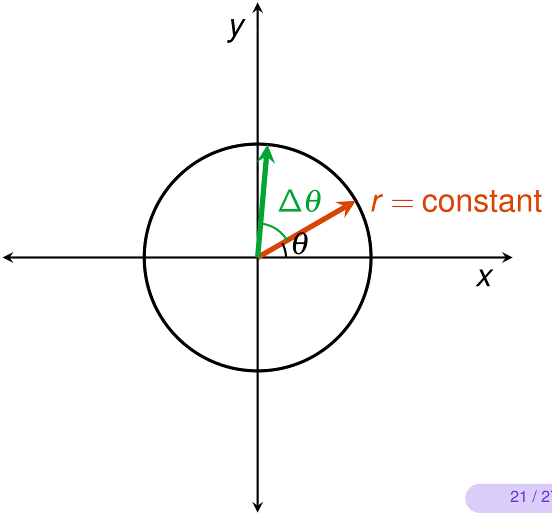
Polar coordinates

- When moving in a circle, it doesn't make much sense to use x and y
- The radius doesn't change, but angle (θ) does



Polar coordinates

- When moving in a circle, it doesn't make much sense to use x and y
- The radius doesn't change, but angle (θ) does
- As you move around, can talk about the change $\Delta\theta$



Angular variables

- Can describe rotation in a circle the same way as 1D motion
- Also works for any fixed object that turns but does not move

position: x	angle: θ
velocity: $v = \frac{\mathrm{d}x}{\mathrm{d}t}$	angular velocity: $\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$
acceleration: $a = \frac{\mathrm{d}v}{\mathrm{d}t}$	angular acceleration: $\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$

What units do these have?

- Angle θ needs to be measured in radians. (360°=2π rad)
- Radian in not a “real” unit, it is just the natural way to talk about angle.
- Why?
  - Because 1 radian means you travel 1 “radius” around the outside of the circle
  - Imagine a coordinate axis that bends around

Conversions

$$x = \theta r$$

$$v = \omega r$$

$$a_{\text{tangential}} = \alpha r$$

- For now we only do simple angular motion problems
- We will need these again when we talk about rotational kinetic energy and torque!

Question

Which of the following units should you use for angular velocity ω?

- A Degrees per second
- B Radians per second
- C Meters per second
- D Meters per second squared

Question

- Which of the following units should you use for angular velocity ω?

A. Degrees per second

B. Radians per second ✓

C. Meters per second

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Example problem 4

A gyroscope slows from an initial rate of $32.0\,\mathrm{rad/s}$ at a rate of $0.700\,\mathrm{rad/s^2}$.

- A How long does it take to come to rest?
- B How far does it turn during that time?

Reminders

- First homework due Friday, January 24
- For lecture Thursday:
  - Read sections 2.1 to 2.3 and 4.1 and 4.2
  - Pre-lecture questions will be on Blackboard