Lecture 23

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Angular momentum

- Angular momentum has the same relationship with torque that linear momentum had with force
- We'll talk again a bit more next week on how to calculate torques
- Remember that torque happens when a force is applied tangential to an object that can rotate
- Torque causes angular acceleration, which changes your angular velocity, which changes your angular momentum
- What we talked about before to calculate torque then applies – just replace $\vec{\mathbf{F}}$ with $\vec{\mathbf{p}} = v\vec{\mathbf{v}}$

In angular coordinates

- Just like we didn’t want to keep track of the kinetic energy of every bit of a rotating object, we don’t want to add up their angular momenta individually
- Can also express angular momentum with angular velocity and moment of inertia
- Will be positive or negative depending on the sign of ω

Rotating object

$$L = I\omega$$

Pre-lecture question 1

- Under what conditions does a rigid body have angular momentum but not linear momentum?

(a) Rolling without slipping

(b) Rolling with slipping

(c) Spinning in place

Pre-lecture question 2

- As the rope of a tethered ball winds around a pole, what happens to the angular velocity of the ball?

(a) Increases (b) Decreases (c) Stays the same

Pre-lecture question 3

- Suppose a child walks from the middle of a rotating merry-go-round to the outside edge. Does the angular velocity of the merry-go-round increase, decrease, or remain the same? Assume the merry-go-round is spinning without friction.

(a) Increases (b) Decreases (c) Stays the same

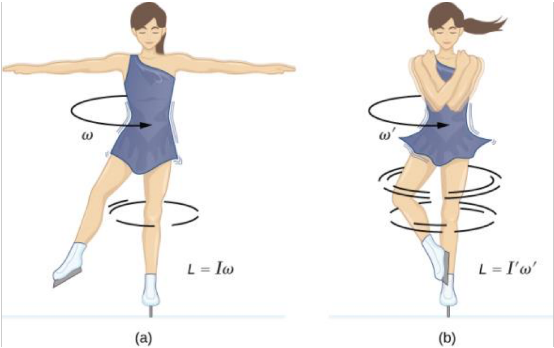
Conservation of angular momentum

- If there is no external torque, then angular momentum is conserved
- Just like with linear momentum and external forces
- One particular case to remember: gravity cannot cause a torque for an object around its center of mass

Change of moment of inertia

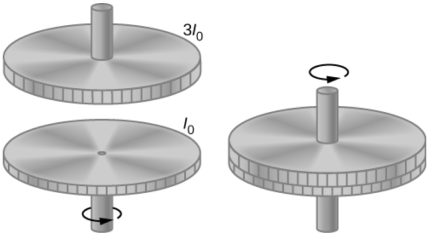
- For purely rotational motion, easiest to think in terms of I
- If the system rearranges and I changes, then ω changes

$$I_i \omega_i = I_f \omega_f$$



Example

A flywheel rotates without friction at an angular velocity $\omega_0 = 600\,\text{rev/min}$ on a frictionless, vertical shaft of negligible rotational inertia. A second flywheel, which is at rest and has a moment of inertia three times that of the rotating flywheel, is dropped onto it. Because friction exists between the surfaces, the flywheels very quickly reach the same rotational velocity, after which they spin together.

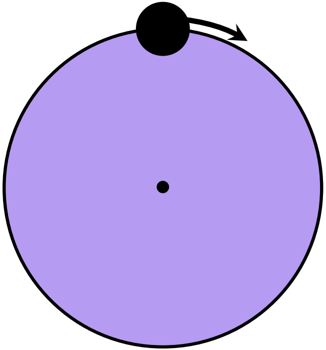


Direction of angular momentum

- Angular momentum is also a vector
- Technically it points “up” or “down” relative to the rotation that has that angular momentum
- We can also think of this as negative and positive being clockwise and counterclockwise

Example II

A 60 kg woman stands at the rim of a horizontal turntable having a moment of inertia of 500 $\mathrm{kg\,m^2}$ and a radius of 2 m. The turntable is initially at rest and is free to rotate about frictionless, vertical axle through its center. The woman starts walking around the rim clockwise (as viewed from above the system) at a constant speed of 1.5 m/s relative to the Earth.

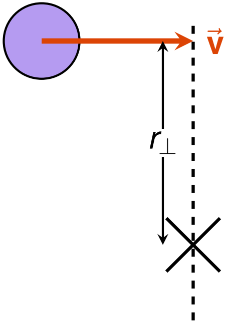


- What is the initial angular momentum?
- At what angular velocity will the table spin after she starts walking?

Straight trajectory

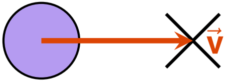
- If something is moving in a straight line, it has angular momentum around any point not along its path
- Easiest way to think of it – draw the line for the motion, find where it gets closest to the axis
- The distance there will be $r_\perp$

$$L = r_\perp p \text{ or } L = r p_\perp$$



A moving object

Counter-clockwise is positive

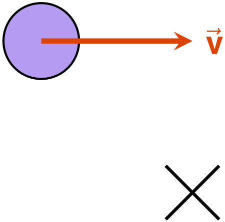


- The ball is moving in a straight line. What is its angular momentum around the x?

(a) Positive (b) Zero (c) Negative

A moving object

Counter-clockwise is positive



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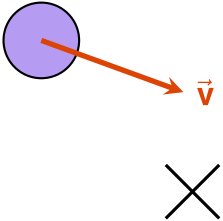

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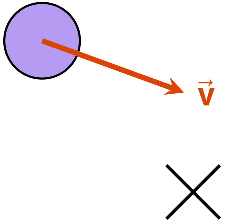


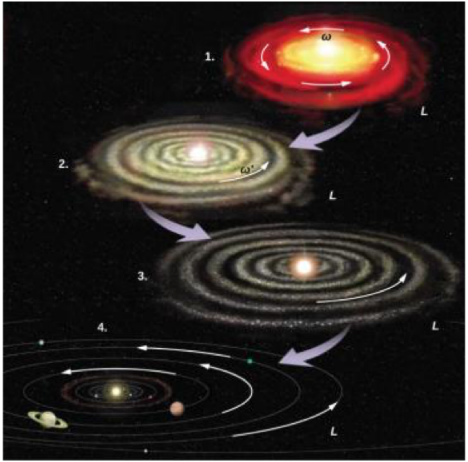
- Is the magnitude of angular momentum greater, the same, or less than same thanto the one moving horizontally?

(a) Greater (b) Same (c) Less

Choice of axis

- Angular momentum depends on choice of axis
- The physical answer to a problem won't depend on your choice
- But usually one choice is easier than another
  - Most of our problems you will want to pick a "fixed" point – a literal axis of rotation

Solar system



Why disks?

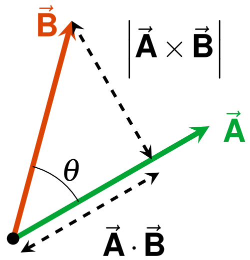
Galaxy NGC 7090

- Need to conserve angular momentum
- But lose energy to internal collisions
  - Don't "waste" kinetic energy going "up" and "down"
- https://www.youtube.com/watch?v=1VPfZ_XzisU&t=664s

Image from ESA/Hubble & NASA; Acknowledgement: R. Tugral

Cross product

- The full way to get angular momentum is $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times m\vec{\mathbf{v}}$
- The cross product is kind of like a complement to the dot product we talked about for work
- $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ tells you the component of $\vec{\mathbf{A}}$ along $\vec{\mathbf{B}}$
- $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$ tells you the component of $\vec{\mathbf{A}}$ perpendicular $\vec{\mathbf{B}}$



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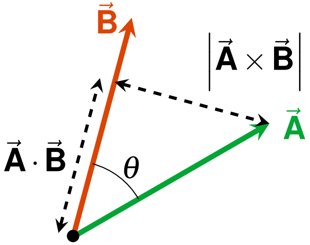

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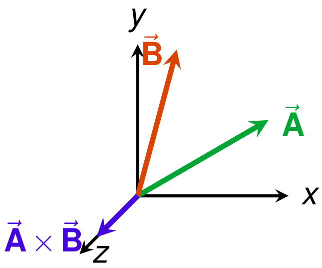
Calculating the cross product

- The cross product is a vector
- We will mainly be interested in the magnitude
  - The direction we can visualize as which way it spins
  - But the vector direction will be more important for E&M!
- Use the right-hand rule – fingers point towards first vector, curl to second vector, thumb gives cross product's direction



Cross product always perpendicular to initial two vectors

Precession

Extra info

- Angular momenta and torque are both vectors
  - Rotation is not always in a single plane!
- Some interesting things can happen when they don’t point parallel or antiparallel
- https://www.youtube.com/watch?v=ty9QSiVC2g0&t=98s
- https://www.youtube.com/watch?v=GeyDf4ooPdo

Dzhanibekov Effect

Extra info

- Some weird things can happen with rotation: https://www.youtube.com/watch?v=vklY1bHIi1I
- A really cool video that explains it: https://www.youtube.com/watch?v=1VPfZ_XzisU

One more example

Two astronauts of mass 100 kg attached by a tether 5 m long are spinning around in space with $\omega = 3\,\mathrm{rad/s}$. In an attempt to slow them, the astronauts' fellow crew members simultaneously throw each one of them a heavy package, with the throws coming from opposite directions as shown. Each package has a mass of 20 kg, and a speed of 10 m/s.