Impulse and momentum

Physics 211
Syracuse University, Physics 211 Spring 2015
Walter Freeman

March 7, 2016

Announcements

- Computational project this week – bring your computing things
- Links to materials on the course website
- Practice exam for Exam 2 and HW5 will be posted tomorrow
- HW5 due Wednesday after spring break

Announcements

- Computational project this week – bring your computing things
- Links to materials on the course website
- Practice exam for Exam 2 and HW5 will be posted tomorrow
- HW5 due Wednesday after spring break
- ... you might want to come to class Thursday ¯_(ツ)_/¯;-)

Momentum: overview

- Momentum is the time integral of force: $\vec{p} = \int \vec{F}\, dt$
- Momentum is a **vector**, transferred from one object to another when they exchange forces
- Another way to look at it: **force is the rate of change of momentum**
- Newton's 3rd law says that total momentum is constant
- Mathematically: $\vec{p} = m\vec{v}$
- Helps us understand **collisions** and **explosions**, among others

Impulse

- We start, as always, with Newton's law:

$$\vec{F} = m\vec{a}$$

- Integrate both sides of this with respect to time:

$$\int \vec{F}\,dt = \int m\vec{a}\,dt$$
$$\int \vec{F}\,dt = (m\vec{v})_f - (m\vec{v})_i$$

- The quantity on the left, $\vec{J} \equiv \int \vec{F}\,dt$, is called "impulse"
  - It represents the cumulative effect of a force over time
  - "Forces make things accelerate" $\rightarrow$ "A force applied over time makes something change velocity"
  - If the force is constant, then the integral is easy, and $\vec{J} = \vec{F}t$
- The quantity on the right, $\vec{p} \equiv m\vec{v}$, is called "momentum"

Impulse is equal to the change in momentum: $\vec{J} = \Delta\vec{p}$

Conservation of momentum

- Newton's third law: if A pushes on B, B pushes back on A with an equal and opposite force
- In symbols, $\vec{F}_{AB} = -\vec{F}_{BA}$
- We can integrate both sides of this to get a statement about impulse: $\vec{J}_{AB} = -\vec{J}_{BA}$
- Using the impulse-momentum theorem: $\Delta\vec{p}_A = -\Delta\vec{p}_B \rightarrow \Delta(\vec{p}_A + \vec{p}_B) = 0$
- The total change in momentum is zero!
- The force between A and B leaves the total momentum constant; it just gets transferred from one to the other
- **Remember momentum is a vector!**
- Solving problems: create "before" and "after" snapshots
- Just add up the momentum before and after and set it equal!

When we need this idea: collisions and explosions

Often things collide or explode; we need to be able to understand this.

- Very complicated forces between pieces often involved: can't track them all
- These forces are huge but short-lived, delivering their impulse very quickly
- Other forces usually small enough to not matter during the collision/explosion
- Use conservation of momentum to understand the collision

The procedure is always the same:

$$\sum \vec{p}_i = \sum \vec{p}_f \text{ “Momentum before equals momentum after”}$$

Applying conservation of momentum to problems

- 1. Identify what process you will apply conservation of momentum to
  - Collisions
  - Explosions
  - Times when no external force intervenes
- 2. Draw clear pictures of the "before" and "after" situations
- 3. Write expressions for the total momentum before and after, in both x and y
- 4. Set them equal: Write $\sum p_i = \sum p_f$ (in both x and y if needed), and solve

Demo with carts

Can we predict the final velocities here?

- Two carts of equal mass separate

Demo with carts

Can we predict the final velocities here?

- Two carts of equal mass separate
- Two carts traveling at equal speeds with equal masses collide

Demo with carts

Can we predict the final velocities here?

- Two carts of equal mass separate
- Two carts traveling at equal speeds with equal masses collide
- Two carts of mass m and $2m$ separate

Demo with carts

Can we predict the final velocities here?

- Two carts of equal mass separate
- Two carts traveling at equal speeds with equal masses collide
- Two carts of mass m and $2m$ separate
- Two carts of mass m and $2m$ traveling at equal speeds collide

Demos with students

Bob and Alice sit on carts. Bob pulls Alice with a rope. Who moves?

Demos with students

Bob and Alice sit on carts. Bob pulls Alice with a rope. Who moves? Does throwing or catching a heavy ball change someone's velocity?

- A. Throwing only
- B. Catching only
- C. Both throwing and catching
- D. Only if someone then catches the ball

Sample problems: a 1D collision

Two train cars moving toward each other at 5 m/s collide and couple together. One weighs 10 tons; the other weighs 20 tons. What is their final velocity?

Sample problems: a 1D collision

A train car with a mass m is at rest on a track. Another train car also of mass m is moving toward it with a velocity v_0 when it is a distance d away. The first car hits the second and couples to it; the cars roll together until friction brings them to a stop.

If the coefficient of rolling friction is μ_r, how far do they roll after the collision?

Sample problems: a 1D collision

A train car with a mass m is at rest on a track. Another train car also of mass m is moving toward it with a velocity v_0 when it is a distance d away. The first car hits the second and couples to it; the cars roll together until friction brings them to a stop.

If the coefficient of rolling friction is μ_r, how far do they roll after the collision?

Method: use conservation of momentum to understand the collision; use other methods to understand before and after!