Power; reviewing work and energy

Physics 211
Syracuse University, Physics 211 Spring 2020
Walter Freeman

March 30, 2020

How are you all doing?

How are you all doing?

How have your classes been going?

How are you all doing?

How have your classes been going?

How has *this* class been going?

Announcements

- Unit review finished and posted on course website (by request)

Announcements

- Unit review finished and posted on course website (by request)

- Homework 10 due Wednesday by the end of the day
- Homework 11 (shorter) will be posted by end of day Wednesday

Announcements

- Unit review finished and posted on course website (by request)

- Homework 10 due Wednesday by the end of the day
- Homework 11 (shorter) will be posted by end of day Wednesday
- There was no pre-lecture question today

Announcements

- Unit review finished and posted on course website (by request)

- Homework 10 due Wednesday by the end of the day
- Homework 11 (shorter) will be posted by end of day Wednesday
- There was no pre-lecture question today
- After today we are *done* with new material for Exam 3
- I'll be in the Virtual Clinic today from 3-4:30, not 2-4 as originally planned (Syracuse time)

Today's plan:

- Your questions

Announcements

- Unit review finished and posted on course website (by request)

- Homework 10 due Wednesday by the end of the day
- Homework 11 (shorter) will be posted by end of day Wednesday
- There was no pre-lecture question today
- After today we are *done* with new material for Exam 3
- I'll be in the Virtual Clinic today from 3-4:30, not 2-4 as originally planned (Syracuse time)

Today's plan:

- Your questions
- A few demo problems on conservation of momentum and energy

Announcements

- Unit review finished and posted on course website (by request)

- Homework 10 due Wednesday by the end of the day
- Homework 11 (shorter) will be posted by end of day Wednesday
- There was no pre-lecture question today
- After today we are *done* with new material for Exam 3
- I'll be in the Virtual Clinic today from 3-4:30, not 2-4 as originally planned (Syracuse time)

Today's plan:

- Your questions
- A few demo problems on conservation of momentum and energy

- A new idea, in more depth: *power*
- An example of that idea

Your questions

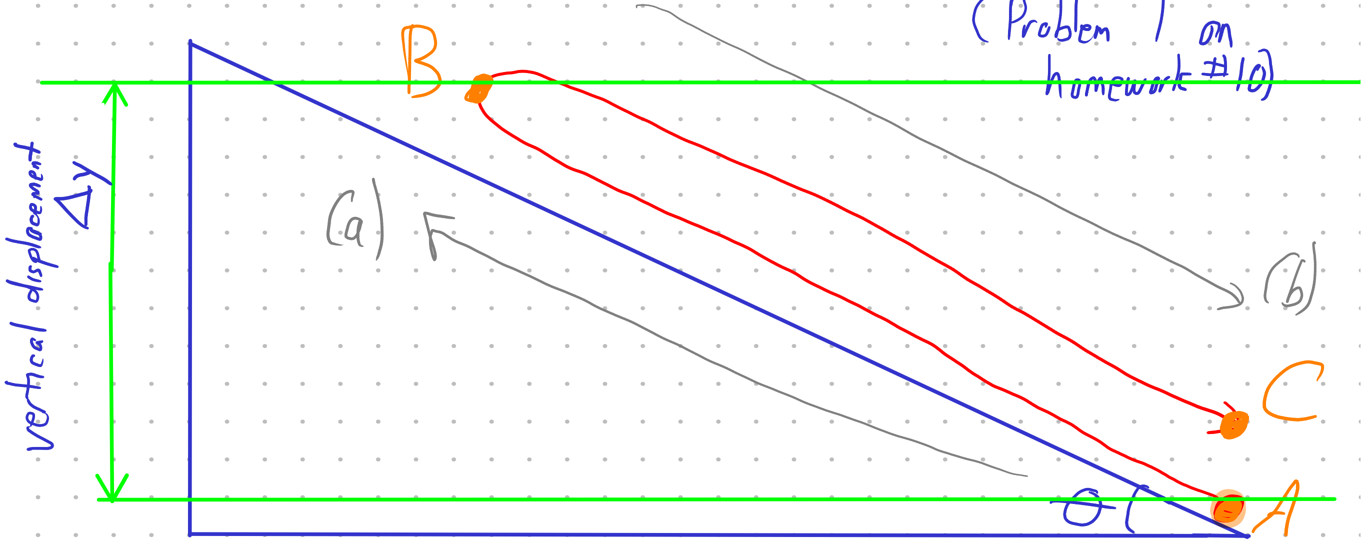
What would you all like to talk about? (Homework, recitation problems, big ideas...)

While you're thinking: how useful are the recordings of recitation and homework solutions?

- **A:** Not useful
- **B:** Moderately useful
- **C:** Quite useful
- **D:** I've not watched any of them yet

Please give me feedback on them (what can I do better?) if you've watched any.

(Problem 1 on homework #10)



$$\frac{1}{2}mv_i^2 + W_{i\to f} = \frac{1}{2}mv_f^2$$

$$W_{grav\,A\to B} = \underbrace{\vec{F}_g}_{\text{down}} \cdot \underbrace{\Delta\vec{s}}_{\text{up}} \quad \text{(going up)}$$

"force, times displacement component parallel to force": $(mg)(-\Delta y)$

$W_{grav} = F_{grav} \cdot$ distance moved $\underbrace{\text{vertically}}_{\text{direction of gravity}}$

In general, $\vec{A} \cdot \vec{B}$ means either:

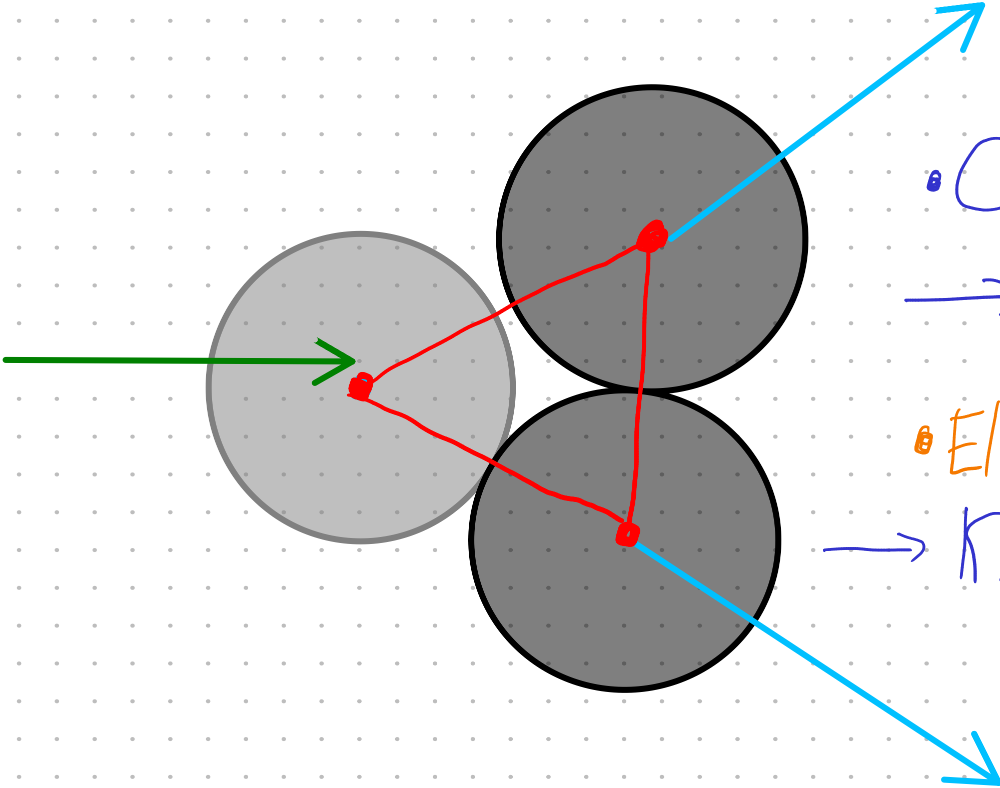
$A B_{\parallel}$ "magnitude of A times component of B parallel to A"

or $A_{\parallel} B$ "component of A parallel to B, times magnitude of B"

or $AB\cos\theta$ "magnitude of A, times magnitude of B, times cosine of angle between them"

- Note: if $\vec{F}$ is perp. to $\Delta\vec{s}$,

$\theta = 90°$, $\cos\theta = 0$, $W = 0$.



• Collision

→ conservation of $\vec{p}$.

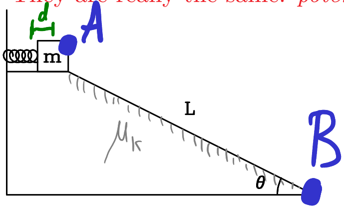
• Elastic collision

→ $KE_i = KE_f$

"When do I use the conservation of energy and when do I use the work-energy theorem?"

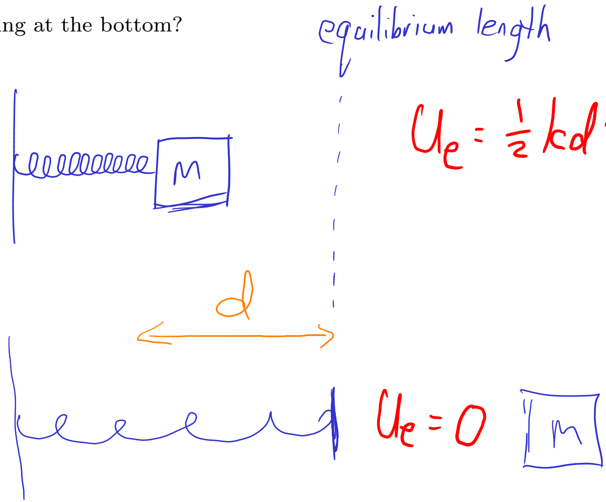
"When do I worry about potential energy?"

They are really the same: *potential energy* is a bookkeeping device for the work done by conservative forces.



Some students are sledding down the hill in front of the music building; it has a length L and is at a slope θ. To go faster, they build a sled-launcher, consisting of a spring of spring constant k. A student compresses it by a distance d and launches themselves down the hill.

How fast are they going at the bottom?



What's the work done by **the spring**?

- A: $W_{\text{elas}} = -\frac{1}{2}kd^2$
- B: $W_{\text{elas}} = +\frac{1}{2}kd^2$ ✓
- C: $W_{\text{elas}} = +kd$
- D: $W_{\text{elas}} = -kd$

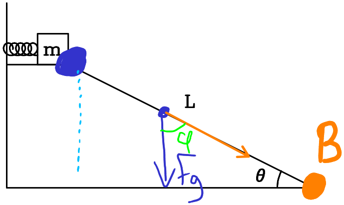
$F_{elas} = -k\Delta x$

At A: spring has elastic PE $= \frac{1}{2}kd^2$

→ converted into KE

"When do I use the conservation of energy and when do I use the work-energy theorem?"

They are really the same: *potential energy* is a bookkeeping device for the work done by conservative forces.



Some students are sledding down the hill in front of the music building; it has a length L and is at a slope θ. To go faster, they build a sled-launcher, consisting of a spring of spring constant k. A student compresses it by a distance d and launches themselves down the hill.

How fast are they going at the bottom?

$$W_{grav} = \vec{F}_g \cdot \Delta\vec{s}$$
$$= F_g \cdot \Delta y$$
$$= mg\,(\Delta y)$$
$$= mg\,L\sin\theta$$

$W_{grav} =$ angle between F_g and Δs

$mg\,L\,(\cos\varphi)$

$\varphi = 90° - \theta$

$\cos\varphi = \sin\theta$

What's the work done by **the spring**?

- A: $W_{\text{elas}} = -\frac{1}{2}kd^2$
- B: $W_{\text{elas}} = +\frac{1}{2}kd^2$
- C: $W_{\text{elas}} = +kd$
- D: $W_{\text{elas}} = -kd$

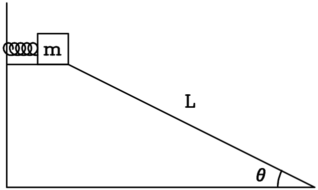
What's the work done by **gravity**?

- A: $W_{\text{grav}} = mgL\cos\theta$
- B: $W_{\text{grav}} = mg\sin\theta$
- C: $W_{\text{grav}} = mgL\sin\theta$ ✓
- D: $W_{\text{grav}} = mgL$

The work-energy theorem and conservation of energy

"When do I use the conservation of energy and when do I use the work-energy theorem?"

They are really the same: *potential energy* is a bookkeeping device for the work done by conservative forces.



Some students are sledding down the hill in front of the music building; it has a length L and is at a slope θ. To go faster, they build a sled-launcher, consisting of a spring of spring constant k. A student compresses it by a distance d and launches themselves down the hill.

How fast are they going at the bottom?

What's the work done by **the spring**?

- A: $W_{\text{elas}} = -\frac{1}{2}kd^2$
- B: $W_{\text{elas}} = +\frac{1}{2}kd^2$
- C: $W_{\text{elas}} = +kd$
- D: $W_{\text{elas}} = -kd$

What's the work done by **gravity**?

- A: $W_{\text{grav}} = mgL\cos\theta$
- B: $W_{\text{grav}} = mg\sin\theta$
- C: $W_{\text{grav}} = mgL\sin\theta$
- D: $W_{\text{grav}} = mgL$

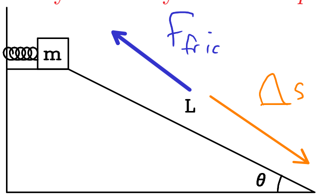
What's the work done by **the normal force**?

- A: $W_{\text{norm}} = mgh$
- B: $W_{\text{norm}} = mgcos\theta$
- C: $W_{\text{norm}} = mgLcos\theta$
- D: $W_{\text{norm}} = 0$

The work-energy theorem and conservation of energy

"When do I use the conservation of energy and when do I use the work-energy theorem?"

They are really the same: *potential energy* is a bookkeeping device for the work done by conservative forces.



Some students are sledding down the hill in front of the music building; it has a length L and is at a slope θ. To go faster, they build a sled-launcher, consisting of a spring of spring constant k. A student compresses it by a distance d and launches themselves down the hill.

How fast are they going at the bottom?

What's the work done by **the spring**?

- A: $W_{\text{elas}} = -\frac{1}{2}kd^2$
- B: $W_{\text{elas}} = +\frac{1}{2}kd^2$
- C: $W_{\text{elas}} = +kd$
- D: $W_{\text{elas}} = -kd$

What's the work done by **the normal force**?

- A: $W_{\text{norm}} = mgh$
- B: $W_{\text{norm}} = mgcos\theta$
- C: $W_{\text{norm}} = mgLcos\theta$
- D: $W_{\text{norm}} = 0$

What's the work done by **gravity**?

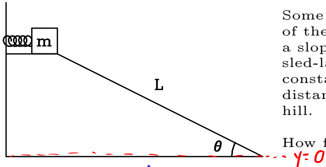
- A: $W_{\text{grav}} = mgL\cos\theta$
- B: $W_{\text{grav}} = mg\sin\theta$
- C: $W_{\text{grav}} = mgL\sin\theta$
- D: $W_{\text{grav}} = mgL$

What's the work done by **friction**?

$W_{fric} = -F_{fric}\,\Delta s$

$F_{fric} = \mu mg\cos\theta$

- A: $W_{\text{grav}} = \mu(mg\cos\theta)L$
- B: $W_{\text{grav}} = -\mu(mg\cos\theta)L$ ✓
- C: $W_{\text{grav}} = -\mu(mg\cos\theta)(L\sin\theta)$
- D: $W_{\text{grav}} = mgL$



Some students are sledding down the hill in front of the music building; it has a length L and is at a slope θ. To go faster, they build a sled-launcher, consisting of a spring of spring constant k. A student compresses it by a distance d and launches themselves down the hill.

How fast are they going at the bottom?

Using "plain" work-energy theorem:

$KE_i + W_{all} = KE_f$: here we have work from spring, from friction, from gravity

$$KE_i + W_{grav} + W_{fric} + W_{spring} = KE_f$$

$$\cancel{\tfrac{1}{2} m v_i^2}^{\,0} + mgL\sin\theta - (\mu mg\cos\theta)L + \tfrac{1}{2}kd^2 = \tfrac{1}{2} m v_f^2$$

Solve for v_f:

$$\boxed{v_f = \sqrt{2gL\sin\theta - 2\mu gL\cos\theta + \frac{k}{m}d^2}}$$

forces other than those connected to a PE.

Using ideas of potential energy:

$$KE_i + PE_i + W_{other} = KE_f + PE_f$$

$$\cancel{KE_i} + PE_{grav_i} + PE_{elas_i} + W_{other} = KE_f + \cancel{PE_{grav_f}} + \cancel{PE_{elas_f}}$$

$$mgy_i + \tfrac{1}{2}kd^2 - \mu mgL\cos\theta = \tfrac{1}{2} m v_f^2$$

Solve.... $y_i = L\sin\theta$

$$v_f = \sqrt{2gL\sin\theta - 2\mu gL\cos\theta + \frac{k}{m}d^2}.$$

Power

You encountered *power* before as the rate of doing work or transforming energy:

$$P = \frac{E}{\Delta t}$$

Units: 1 watt = 1 J/s.

This is important in engineering, since many of our machines are constrained by the rate at which they can manipulate energy, or that they require energy:

- My laptop: 4W (minimum to run) - 25W (maximum cooling system can handle)
- A duck: 25-60W (sustained power from flight muscles)
- Human on a bike: 100-300W (sustained over an hour), five times that (peak)
- Horse: 750W (averaged over a working day), 10 kW (brief peak)
- Automobile engine: 75 kW (my car) - 400 kW (high-end sports car)
- Diesel-electric locomotive: 2500 kW
- Nuclear submarine: 30 MW
- Nuclear reactor: 1500 MW (electric), 3000 MW (heat)

Power

In mechanics, we are often interested in a particular question:

"At what rate does a force $\vec{F}$ do work on an object moving at speed $\vec{v}$?"

Power

In mechanics, we are often interested in a particular question:

“At what rate does a force $\vec{F}$ do work on an object moving at speed $\vec{v}$?”

Starting with the work-energy theorem, as always:

$$(\text{work}) = (\text{force}) \cdot (\text{displacement})$$
$$W = \vec{F} \cdot \vec{\Delta} s$$

Power is the rate at which work is done – the *time derivative* of work. So we take time derivatives of both sides:

$$W = \vec{F} \cdot \vec{\Delta} s$$

power $\longrightarrow$ $$\frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{s}}{dt}$$ $\longleftarrow$

Power

In mechanics, we are often interested in a particular question:

"At what rate does a force $\vec{F}$ do work on an object moving at speed $\vec{v}$?"

Starting with the work-energy theorem, as always:

$$(\text{work}) = (\text{force}) \cdot (\text{displacement})$$
$$W = \vec{F} \cdot \vec{\Delta} s$$

Power is the rate at which work is done – the *time derivative* of work. So we take time derivatives of both sides:

$$W = \vec{F} \cdot \vec{\Delta} s$$
$$\frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{s}}{dt}$$

$$P = \vec{F} \cdot \vec{v}$$

Biking with air resistance

A cyclist and her bike have a mass of $m = 70kg$, and she can produce a sustained power of 120 W for a long time.

She can sustain a speed of 12 m/s. At this speed, the main friction force on her is the wind.

How big is that frictional force?

A: 700 N
B: 10 N
C: 1200 N
D: 100 N

$$P = \vec{F} \cdot \vec{v}$$

net power on cyclist at constant speed = 0.

$$P_{pedals} + P_{drag} = 0$$

negative

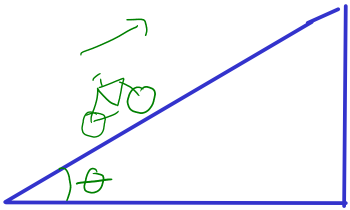
$$P_{pedals} = -P_{drag} = F_{drag} \cdot v \longrightarrow F_{drag} = \frac{P_{pedals}}{v} = 10\ N.$$

Biking up a hill

A cyclist and her bike have a mass of $m = 70$ kg, and she can produce a sustained power of 120 W for a long time.

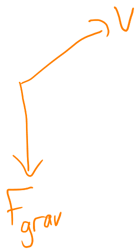
She then wants to ride up a hill, sloped at at an angle of about $\theta = 6^\circ = 0.1$ radian.

How fast can she go up the hill? *(This is a lot slower, so you can ignore air drag here.)*



$$\underbrace{P_{pedals}}_{\text{positive}} + \underbrace{P_{grav}}_{\text{negative}} = 0$$

$$P_{pedals} = -P_{grav} = -(\vec{F}_g \cdot \vec{v}) = -(-mg\, v \sin\theta)$$



$$P_{pedals} = mg\, v \sin\theta$$

$$v = P_{ped} / mg \sin\theta = 1.6 \text{ m/s.}$$

Going down a steep hill, slowly

Suppose our $m = 70$ kg rider wants to go down a steep hill, angled at 10 degrees below the horizontal, at a safe speed of 4 m/s. *(At this speed, ignore air drag.)*

Brakes work by squeezing a rotating object with a large normal force, creating a lot of friction. This friction does negative work on the rotating wheel, converting its kinetic energy into heat.

What power will the brakes in her bicycle produce?

Brakes on a bike intended for off-road use. The rotor is designed to maximize airflow – to give the material a fighting chance of dissipating this much heat!

Another sample problem: work and energy

A basketball of mass m hangs from a cable of length L; it is pulled to the left at an angle θ and released.

A very strong wind blows from left to right, exerting a constant force F_w on the ball.

How fast will the ball be traveling when it is at its lowest point? What angle ϕ will the ball swing to on the other side?

Another sample problem: work and energy

A basketball of mass m hangs from a cable of length L; it is pulled to the left at an angle θ and released.

A very strong wind blows from left to right, exerting a constant force F_w on the ball.

How fast will the ball be traveling when it is at its lowest point? What angle ϕ will the ball swing to on the other side?