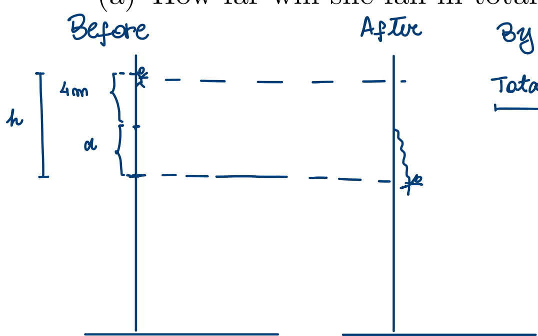
Recitation Questions

Friday, 22 March

1. A rock climber of mass 70 kg is climbing a cliff face when she slips and falls. There is 4m of slack in her climbing rope, so she undergoes free fall for 4 meters before the rope begins to arrest her fall. If the stiffness in her rope is 1400 N/m, then:

   (a) How far will she fall in total?



By using conservation of energy we have...

$$\underbrace{\text{Total initial energy}}_{PE_{gravitational}} = \text{Total final energy}$$

$$PE_{gravitational} = PE_{elastic}$$

$$mgh = \frac{kd^2}{2} \to mg(4+d) = \frac{kd^2}{2} \to kd^2 - 8mg - 2mgd = 0$$

$$d^2 - \underbrace{\frac{2\times 70\times 10}{1400}}_{=1} d - \underbrace{\frac{8\times 70\times 10}{1400}}_{4} = 0$$

$$d^2 - d - 4 = 0 \qquad d = \frac{1 \pm \sqrt{1-4(1)(-4)}}{2}$$

$$d = \frac{1\pm\sqrt{17}}{2} = -\cancel{1.6}, 2.6 \text{ m}$$

   (b) What is the maximum force that her rope will exert on her as it arrests her fall?

Since $\vec{F}_{el} = -kx$, maximum implies maximum x, which is $x = 2.6$m

Thus,

$$\vec{F}_{el} = -1400 \times 2.6 = 3640 \text{ N}$$

   (c) When would it be desirable for a rock climber to use a rope with a large spring constant? What about a smaller spring constant? You'll need to think about the engineering reasons for climbers to use ropes at all: the goal is to minimize the forces involved in arresting a climber's fall.

2. A laptop battery says it has a capacity of 51 "watt-hours".

   (a) What are the dimensions of this odd unit "watt-hour", and what does it measure? What is 51 watt-hours in more familiar units?

$$\text{Watt} = \frac{J}{s} = [\text{Energy}]\cdot[\text{Time}]^{-1}$$

Thus $\text{Watt} \times \text{hour} = [\text{Energy}][\text{Time}]^{-1}[\text{Time}] = [\text{Energy}]$

$$51 \text{ watt} \times \text{hour} = 51 \frac{J}{s} \times 3600 s = 183600 J$$

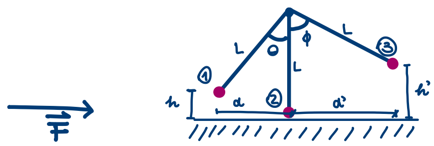
   (b) If this battery were used to power an electric motor, how high could it lift the battery? Assume the battery has a mass of 300 grams.

We want to convert all this eletric energy in gravitational potential.

Thus, $PE_g = 183600 J = mgh$

$$mgh = 183600$$

$$h = \frac{183600}{mg} = \frac{183600}{0.3 \times 10} = \frac{183600}{3} = 61200 \text{ m} = 61.2 \text{ Km}$$



$h = L - L\cos\theta$
$h' = L - L\cos\phi$
$d = L\sin\theta$
$d' = L\sin\phi$

3. A ball of mass m on a cord of length L is held at an angle θ to the left of the vertical and released. A very strong wind blows from left to right, exerting a constant horizontal force F.

$\vec{F} = |F|\hat{i}$

(a) Find the speed of the ball at the bottom of its swing.

From ①→②

From work-Energy theorem we have $\frac{1}{2}m v_2^2 - \frac{1}{2}m \cancel{v_1}^{2} = W_g + W_F$ (with $v_1 = 0$)

$W_g(1\to2) = \vec{F}_g \cdot \vec{h} = mgh$, $W_F(1\to2) = \vec{F}\cdot\vec{d} = Fd$

Thus,

$$v_2^2 = \frac{2}{m}\left[mgh + Fd\right] \rightarrow v_2 = \left(2gL(1-\cos\theta) + \frac{2FL}{m}\sin\theta\right)^{\frac{1}{2}}$$

(b) Find an equation for the maximum angle that the ball reaches when it swings to the right. You do not need to actually solve it, since it's messy and involves a lot of trig identities; just write it down.

①→③

From work-Energy theorem, we have that

$$\frac{1}{2}m\underbrace{\cancel{v_3}^{2}}_{0} - \frac{1}{2}m\underbrace{\cancel{v_1}^{2}}_{0} = W_F(1\to3) + W_g(1\to3) \rightarrow W_F(1\to3) = -W_g(1\to3)$$

$$W_F(1\to3) = F(d+d') = FL(\sin\theta + \sin\phi)$$

$$W_g(1\to3) = W_g(1\to2) + W_g(2\to3) = mgh - mgh' = mgL\{\cancel{1} - \cos\theta - \cancel{1} + \cos\phi\}$$
$$= mgL[\cos\phi - \cos\theta]$$

(c) When the ball swings back to the left, find the height that it reaches. Will it come back to the same point where it was released? (You should be able to answer this question without doing anything difficult.)

yes, it will! $W_F(1\to3) = -W_F(3\to1)$

Since everything else is conservative, it will come back to the same position.