PHY 211 Recitation 3

January 22, 2020

When working through these problems, remember to apply the following steps:

(a) Define a coordinate system for the problem and write it down. Drawing a picture may help.

(b) Write down the equations of motion.

(c) Translate the question that is given in plain language into a question about the variables in your equations.

(d) Solve the problem algebraically.

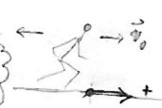
1 Running a race

This phrase tells us the initial velocity v_0

At the end of a race, a runner decelerates from a velocity of 9.0 m/s at a rate of 2.0 m/s². these units tell us this is an acceleration

(a) How far does she travel in the next 5.0 s?

deceleration means $\vec{a}$ is opposite $\vec{v}$



You can always start with either $x(t)$ or $v(t)$ when you have constant acceleration, specifying x_0, v_0, and a.

$$x(t) = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$= 0 + (9\,\text{m/s})t + \tfrac{1}{2}(-2\,\text{m/s}^2)t^2, \text{ so}$$

We can choose $x_0 = 0$ when we put the origin at the runner's initial position

This problem in particular specifies a time t

$$x(5\,\text{s}) = (9\,\text{m/s})(5\,\text{s}) + \tfrac{1}{2}(-2\,\text{m/s}^2)(5\,\text{s})^2$$

$$= 45\,\text{m} - 25\,\text{m} = \boxed{20\,\text{m}}$$

(b) What is her final velocity?

$$v(t) = v_0 + at$$

Now we are interested in $v(t)$.

$$= 9\,\text{m/s} + (-2\,\text{m/s}^2)\,t.$$

Again, we have a specific time t to substitute in

$$v(5\,\text{s}) = 9\,\text{m/s} + (-2\,\text{m/s}^2)(5\,\text{s}) = 9\,\text{m/s} - 10\,\text{m/s} = \boxed{-1\,\text{m/s}}$$

(c) Think about your result in (b). What does it imply about how the runner moved? Does that make sense?

This implies the runner slowed to a stop and then started running backwards. Since a real runner probably wouldn't do this, the acceleration would probably change within the 5 seconds.

θ - angular position (like x)
ω - angular velocity (like v)
α - angular acceleration (like a)

Rotation is mathematically just like 1-D motion because you can only go forward (clockwise) or backward (counterclockwise)

2 Angular acceleration

$\omega_0 = 0$

A wheel has a constant angular acceleration of $5.0\,\mathrm{rad/s^2}$. Starting from rest, it turns through $300\,\mathrm{rad}$.

(a) How much time elapses while it turns through the 300 rad?

$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

This specifies an angular position, so let's use the $\theta(t)$ equation

$300\text{ rad} = 0 + 0 + \frac{1}{2}(5.0\,\mathrm{rad/s^2})\,t^2$

Now t is the only unknown, so we can solve for it

$300 = 2.5\,t^2$

$\frac{300}{2.5} = t^2$, so $t = \sqrt{\frac{300}{2.5}}$, or $t = \boxed{11\text{ s}}$

(b) What is its final angular velocity?

We can now use $\omega(t)$ with the time t we found

$\omega(t) = \omega_0 + \alpha t$

$\omega(11\text{ s}) = 0 + (5.0\,\mathrm{rad/s^2})(11\text{ s})$

$= \boxed{55\text{ rad/s}}$

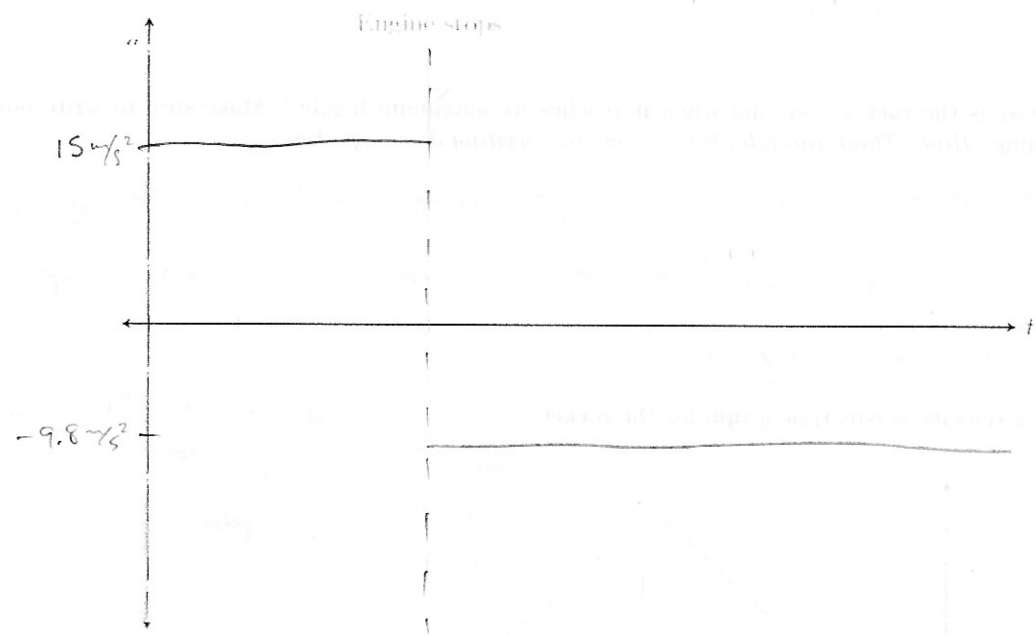
3 Rocket

A rocket is fired straight up. Its engine burns for ten seconds. While the engine is burning, the rocket accelerates upward at $15\,\mathrm{m/s^2}$. After the engine stops, the rocket starts to freefall.

(a) Since the rocket's acceleration changes in flight, you can't use constant-acceleration kinematics formulae to write down the motion for the entire flight at once. What can you do so that you *can* apply them?

We can split the problem into two pieces that do have constant acceleration, and calculate x and v values when the switch happens.

(b) Draw an acceleration versus time graph for the rocket.



The acceleration switches between two constant values.

(c) How high above the ground is the rocket when the engine stops?

$x_0 = 0$

$v_0 = 0$

since the rocket starts from rest

We want to know the position of the rocket after 10 s. During this interval, acceleration is constant at $15\,\mathrm{m/s^2}$.

Since we're after position, we use the equation

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

$$= \frac{1}{2}at^2.$$

We now can use the time $t = 10\,\mathrm{s}$:

$$x(10\,\mathrm{s}) = \frac{1}{2}(15\,\mathrm{m/s^2})(10\,\mathrm{s})^2 = \boxed{750\,\mathrm{m}}$$

(d) How fast is the rocket traveling once its motor burns out?

$t = 10s$

Just like before, but now with the velocity equation:

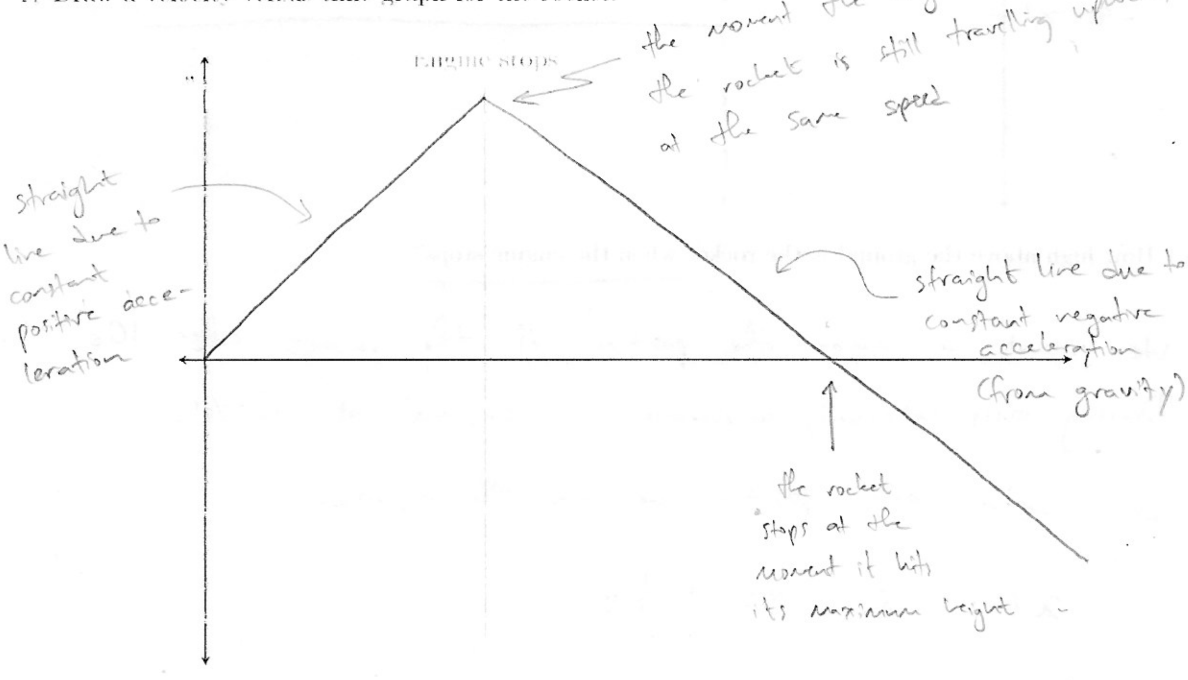
$$v(t) = v_0 + at$$

$$v(10s) = 0 + (15\,m/s^2)(10s) = \boxed{150\,m/s}$$

(e) How fast is the rocket traveling when it reaches its maximum height? Make sure to write out your reasoning. *Hint: Think carefully before you start writing down equations.*

While velocity is positive, the rocket will still be going up. Maximum height won't be reached until the rocket stops going up, that is, when $\boxed{v = 0}$.

(f) Draw a velocity versus time graph for the rocket.



we will want to use the position equation $x(t)$

(g) What is the maximum height the rocket reaches?

The rocket is still going up when the engine stops, so it hasn't reached it's maximum height yet. That height is actually reached later, during freefall.

We can choose a new $t=0$ to be when the engine cuts out.

From previous answers, we have $x_0 = 750\,m$, $v_0 = 150\,m/s$, and $a = -g = -10\,m/s^2$ during freefall

We don't have a time to plug into $x(t)$ yet, but we can use $v(t)$ to find it.

$v(t) = v_0 + at$

$0 = v_0 + at, \quad t = \frac{-v_0}{a} = \frac{-150\,m/s}{-10\,m/s^2} = 15\,s$

$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$, so

$x(15s) = 750m + (150\,m/s)(15s) + \frac{1}{2}(-10\,m/s^2)(15s)^2 = \boxed{1875\,m}$

(h) How long does it take for the rocket to land back on the ground?

this is the moment $x(t)=0$ during freefall

$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$

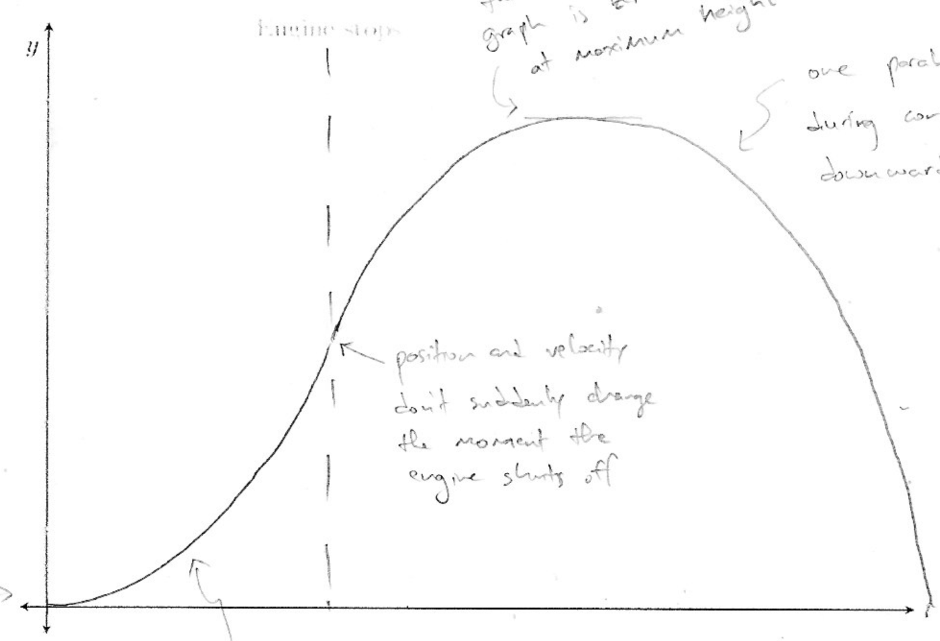
$0 = x_0 + v_0 t - \frac{1}{2}gt^2$ — We can solve a quadratic equation this way, or...

We can choose a new $t=0$ when the rocket reaches maximum height, giving $x_0 = 1875\,m$, $v_0 = 0$, so

$0 = 1875m - \frac{1}{2}(10\,m/s^2)t^2$, and $t = \sqrt{\frac{1875\,m}{5\,m/s^2}} = \boxed{19.4\,s \text{ after reaching max height}}$, or

$19.4s + 15s = \boxed{34.4\,s \text{ after the engine stops}}$

(i) Draw a position versus time graph for the rocket.



the slope of the position graph is zero ($v=0$) at maximum height

one parabola during constant downwards acceleration

position and velocity don't suddenly change the moment the engine shuts off

the position graph starts with value 0 (on the ground) and slope 0 (initially at rest).

one parabola during constant upwards acceleration