Momentum

Physics 211
Syracuse University, Physics 211 Spring 2022
Walter Freeman

April 7, 2022

Announcements

- Group Exam 3 tomorrow; Exam 3 next Tuesday
- Review notes posted on the website
- Extra homework help hours today: 9:45-10:45
- HW7 due Friday

Extra exam review session

Extra exam review session will be held **Monday afternoon or evening**. We are still getting a time/place. (I am sadly booked all weekend.)

This will be a little more scripted than the last one, but I expect it should prepare everyone for the exam – instead of "cramming" on your own, come work with us.

To get the maximum benefit out of this, you should:

- Have done all your homework
- Go to the Physics Clinic after the group exam Friday and ask the coaches to help you with anything you couldn't do
- Be familiar with most of the recitation exercises we have done
- Have already attempted the practice exam over the weekend, and have looked at the solutions (posted Friday or Saturday)

Recitation or homework questions?

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Shall we talk about the "one with the truck"?

Exam 3 Review

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We have met two of them: *conservation of momentum* and the *work-energy theorem.*

These techniques let you analyze systems where you know something about "before" and "after" states.

General problem-solving techniques:

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Review: energy methods

Use energy methods when:

- You have clear before and after states (draw your cartoons, dammit!)
- You can calculate the work done by the forces between them **(not collisions/explosions!)**
- You don't care about time

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- You can calculate the work done by the forces between them **(not collisions/explosions!)**
- You don't care about time
- *Be careful with projectile motion!*
    - Energy methods can tell you "how fast" or "how high"
    - They cannot tell you "where does it land?"

Review: The work-energy theorem

Work-energy theorem: $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \vec{F} \cdot \vec{d} = Fd\cos\theta$ (if this is constant)

Potential energy is an alternate way of keeping track of the work done by conservative forces:

- $PE_{\text{grav}} = mgh$
- $PE_{\text{spring}} = \frac{1}{2}kx^2$

Review: Conservation of energy

$$\mathrm{PE_i} \quad + \quad \tfrac{1}{2}mv_i^2 + \tfrac{1}{2}I\omega_i^2 \quad + \quad W_{\mathrm{other}} \quad = \quad \mathrm{PE_f} \quad + \quad \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2$$

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$\mathrm{PE_i}$	$+$	$\frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2$	$+$	W_{other}	$=$	$\mathrm{PE_f}$	$+$	$\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$
(initial PE)	+	(initial KE)	+	(other work)	=	(final PE)	+	(final KE)

Review: Conservation of energy

$$\mathrm{PE_i} + \tfrac{1}{2}mv_i^2 + \tfrac{1}{2}I\omega_i^2 + W_{\mathrm{other}} = \mathrm{PE_f} + \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2$$

(initial PE) + (initial KE) + (other work) = (final PE) + (final KE)

(total initial mechanical energy) + (other work) = (total final mechanical energy)

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Since conservation of energy is the broadest principle in science, it's no surprise that we can do this!

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- You have an explosion (one object separates into two)
- Two objects exchange forces *only with each other*

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(that's it – the last one doesn't come up much, but sometimes it does: why does the Earth wobble?)

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- $\sum \vec{p}_i = \sum \vec{p}_f$
- **Momentum is a vector!**
- If you have motion in two dimensions, momentum in each direction is conserved separately:

$$\sum p_{x_i} = \sum p_{x_f} \qquad\qquad \sum p_{y_i} = \sum p_{y_f}$$

Do not get lazy with your subscripts!

Sample problems: an excited dog

A person of mass m is sitting in a tire swing with a string of length L when their dog (mass M) runs and jumps horizontally into their lap.

If they swing up to an angle θ above the horizontal, how fast was their dog running?

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An application: neutron moderators

The only truly elastic collisions in nature are between particles. If we want *totally* elastic collisions, we should look to nuclear physics!

A note: this calculation we are going to do here demonstrates two things:

- How elastic collisions work
- ... and how *the art of approximation* is used in physics and engineering!

Recall how a nuclear reactor works:

- ^{235}U fissions when struck by neutrons with low energy (600 times more likely at low energy, less than 0.1 eV)
- When ^{235}U fissions, it produces neutrons with 2 MeV of kinetic energy ($v \approx 20$ million m/s)

How do we make these neutrons go from 2 million eV to 0.1 eV of kinetic energy so they can produce more fissions?

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How much kinetic energy is lost?

The fraction of kinetic energy lost is

$$4\frac{m}{M}.$$

So what atoms can we use as moderators?

They have to scatter neutrons more readily than they absorb them (hydrogen so-so, oxygen/carbon/heavy hydrogen great)

They have to be lightweight (that's what we just found)

They have to not be chemically grouchy (no hydrogen or oxygen by themselves!)

- H_2O (light water: most of the world, not the best moderator)
- D_2O (heavy water: Canadians)
- CO_2 (carbon dioxide: British)
- Pure carbon (Graphite: Soviets)