

Problem solving: kinematics (II)

Physics 211
Syracuse University, Physics 211 Spring 2019
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January 28, 2019

- Homework 2 due date is **this Friday**
- Exam 1 is next Tuesday
 - No homework due next week
 - HW2 problems are similar to those on Exam 1
 - Physics practice: tomorrow night, 7:30-9:30, here
 - Recitation Friday is your group practice exam
 - If you must miss Friday, notify your TA in advance
 - Weekend: Exam review in the auditorium, Sunday, 10AM-1PM. (Sorry about the time; it's the only time I'm free.)

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- Kinematics: how are an object's position, velocity, and acceleration related?

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- Students who do not speak English well: I will try to use only simple English on the exam, but if you like you may bring a dictionary
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- Review session in class on Thursday
- You are allowed to bring one side of one page of notes that *you handwrite yourself* on Tuesday
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 - No typed notes unless you have a disability that prevents you from writing
 - Your friend can't write it
 - You can't print stuff from the internet

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 - It won't help you as much anyway

Exam 1, promises

- There will be one problem where you need the quadratic formula
 - ... this means interpreting the two values it spits out
- There will be at least one instance where you need to interpret or sketch position, velocity, and acceleration graphs
- You will *not* need to compute derivatives or integrals algebraically
- The exam will be four problems, plus possibly a few short-answer questions

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 - Two representations:
 - Magnitude and direction (easiest to state, hardest to work with)
 - Components (easiest to work with)
 - Use trigonometry to go back and forth
- One more piece of notation about vectors...

Unit vectors

In the “ordered pair” notation for vectors’ components, you might write:

$$\vec{v} = (5, 3)$$

But this is clunky, if you’re trying to write it as part of an algebraic statement.

Instead we introduce “unit vectors”, vectors with length 1, in the x, y, and z directions.

$$\hat{i} = (1, 0, 0)$$

$$\hat{j} = (0, 1, 0)$$

$$\hat{k} = (0, 0, 1)$$

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- $\vec{v} = 5\hat{i} + 3\hat{j}$: Unit vectors

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- $\vec{v} = (5, 3)$: Ordered pair
- $\vec{v} = 5\hat{i} + 3\hat{j}$: Unit vectors
- Both give you the same information, but unit vectors can be easier algebraically
- They won’t be essential for this class, but you should know the notation – other people use it quite a bit, and I use it once on your homework

A word on positive and negative acceleration, velocity, “speed”, and displacement:

When you choose your origin, you choose one direction to be positive, and the other to be negative. (Here: right = positive.)

- An object with $x < 0$ just means it's left of the origin.
- An object with $v < 0$ means it's moving to the left.
- An object with $a < 0$ means:
 - A: it is moving to the left and gaining speed
 - B: it is moving to the right and slowing down
 - C: it is moving to the left and slowing down
 - D: it is moving to the right and gaining speed

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Do not confuse the sign of something with the sign of its derivative!

Last time

Acceleration, velocity, and position relationships are the same in 2D; they just apply **independently** for each component.

$$\vec{v}(t) = \vec{a}t + \vec{v}_0$$

$$\vec{s}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0t + \vec{s}_0$$

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$$y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + y_0$$

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it's fine to leave it as a variable!

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Example from cannon problem:

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Example from cannon problem:

$$x(t) = v_{x,0}t$$

$$y(t) = -\frac{1}{2}gt^2 + v_{y,0}t$$

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Example from cannon problem:

$$x(t) = v_0 \cos 45^\circ t$$

$$y(t) = -\frac{1}{2}gt^2 + v_0 \sin 45^\circ t$$

(I leave the rest to you for now...)

Problem solving: 2D kinematics, constant acceleration

- ➊ 0. Draw a cartoon of the situation, and choose a coordinate system
- ➋ 1. If you have vectors in the “angle and magnitude” form $(\vec{a}, \vec{v}, \vec{s})$, convert them to components
- ➌ 2. Write down the kinematics relations, separately for x and y
 - Many terms will usually be zero
 - Freefall: $a_x = 0$, $a_y = -g$ (with conventional choice of axes)
- ➍ 3. Understand what instant in time you want to know about: ask the right question
- ➎ 4. Put in what you know; solve for what you don't (using substitution, if necessary)
- ➏ 5. Think about the physical meaning of your solution

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“What instant in time do you know about?”

This is often the most difficult part of problems: it requires thought, not just math.

You throw a ball upward over a hole of height h . Your position is the origin, and up is positive.

What condition means “the ball has hit the ground”?

- A: $y = 0$
- B: $y = h$
- C: $y = -h$
- D: $v_y = 0$

“What instant in time do you know about?”

You throw a ball upward off of a cliff of height h . The top of the cliff is the origin, and up is positive.

What condition means “the ball is at its highest point?”?

- A: $y = 0$
- B: $v_y = 0$
- C: $y = h$
- D: y is a maximum

A cannon shoots a cannonball at 80 m/s at an angle of 30 degrees above the horizontal.

How can we frame the question “How far does the cannonball go?” in terms of our variables?

- A: What is x at the same time that v_x is zero?
- B: What is y at the same time that x is zero?
- C: What is x at the same time that y is zero?
- D: What is x at the same time that v_y is zero?

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- How fast is it traveling when it strikes the ground?

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- A cannon shoots a cannonball at initial velocity $v_0 = 80\text{m/s}$ at an angle $\theta = 30^\circ$ above the horizontal.
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What is $v_{0,x}$?

A: $v_0 \cos \theta$

B: $v_0 \sin \theta$

C: $v_0 \tan \theta$

D: v_0

A cannon

- What changes if I put the cannon up on a hill?

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- What changes if I'm trying to hit a target up on a plateau?
- What changes if I want to know what velocity I need to hit a target?
- What changes if I have air resistance?

Throwing a rock off a cliff

A hiker throws a rock horizontally off of a $h = 100$ m tall cliff. If the rock strikes the ground $d = 30$ m away, how hard did she throw it? How fast was it going when it hit the ground? (Choose the origin at the base of the cliff, up/direction of throw as positive)

What is $v_{0,x}$ here?

A: 0

B: $10/3$ m/s

C: You don't know *a priori*

What is $v_{0,y}$ here?

A: 0

B: 9.81 m/s

C: You don't know *a priori*

What is a_x here?

A: 0

B: -g

C: +g

D: You don't know *a priori*

What is a_y here?

A: 0

B: -g

C: +g

D: You don't know *a priori*

What is x_0 here?

A: 0

B: h

C: d

D: You don't know *a priori*

What is y_0 here?

A: 0

B: h

C: d

D: You don't know *a priori*

What question do you ask to find “how hard did she throw it?”

A: What value of $v_{x,0}$ makes it such that $x = d$ when $y = 0$?

B: What value of $v_{y,0}$ makes it such that $x = d$ when $y = h$?

C: What is the value of v_x when $y = 0$?

D: What is the magnitude of \vec{v} when $y = 0$?

E: What is the magnitude of \vec{v}_x when $y = h$?

What question do you ask to find “how fast is it going when it hits the ground?”

A: What is v_x at the time when $v_y = 0$?

B: What is v_x at the time when $y = 0$?

C: What is v_y at the time when $y = h$?

D: What is the magnitude of \vec{v} when $y = 0$?

E: What is the magnitude of \vec{v} when $y = h$?

What's the magnitude of \vec{v} ?

A: $v \cos \theta$

B: $v \sin \theta$

C: $\tan^{-1} \frac{v_x}{v_y}$

A: $\sqrt{v_x^2 + v_y^2}$

Throwing a stone onto a slope

A hiker kicks a stone off of a mountain slope with an initial velocity of v_0 3 m/s horizontally. If the mountain has a slope of 45 degrees, how far down the slope does it land? (Choose the origin as the starting point.)

A: What is the magnitude of \vec{s} when $x = y$?

B: What is the magnitude of \vec{s} when $x = -y$?

C: What is the magnitude of \vec{s} when $y = 0$?

D: What is y when $x = -y$?

E: What is y when $x = 0$?

A rocket is launched from rest on level ground. While its motor burns, it accelerates at 10 m/s^2 at an angle 30° below the vertical. After $\tau = 10 \text{ s}$ its motor burns out and it follows a ballistic trajectory until it hits the ground.

How far does it go?