Energy: the work-energy theorem

Physics 211
Syracuse University, Physics 211 Spring 2022
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Announcements

Welcome back from spring break!

Upcoming homework:

- Homework 6 will be very short (3 problems), posted tonight, and due Friday.
- Homework 7 will be longer (8-9 problems), posted Thursday or Friday, and due next Friday.
- An optional homework assignment ("extra credit") will be posted this week:
  - Due during the last week of class
  - Based on your lowest exam question grades (different for different people)
  - An opportunity to study for the final
  - Optional, but will help you learn
  - I'll send out details later this week

Help hours this week:

- Today, 1:30-3:30
- Wednesday, 2-4
- Friday, 9:30-11

What is this unit about?

Which ball travels furthest as it leaves the table?

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- Quickly leads to ugly differential equations

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Which ball travels furthest as it leaves the table?

What we know so far can't answer this easily. The forces are always changing!

- We can't use constant acceleration kinematics
- Quickly leads to ugly differential equations
- ... what do?

Energy

Energy in brief:

- Kinematics relates the forces on an object to the change in something called its *kinetic energy*
- Forces transfer energy from one object (and one form) to another, but don't create or destroy it
- Energy is a scalar, not a vector
- Energy methods are extremely powerful in problems where we *don't know and don't care about time (like the example above)*

Energy methods, in general

- "Conventional" kinematics: compute $\vec{x}(t)$, $\vec{v}(t)$
  - "Time-aware" and "path-aware" – tells us the history of a thing's movement
  - Time is an essential variable here
- Newton's second law: forces $\rightarrow$ acceleration $\rightarrow$ history of movement
- Sometimes we don't care about all of this
- Roll a ball down a track: how fast is it going at the end?

Energy methods, in general

We will see that things are often simpler when we look at something called "energy"

- Basic idea: don't treat $\vec{a}$ and $\vec{v}$ as the most interesting things any more
- Treat v^2 as fundamental: $\frac{1}{2}mv^2$ called "kinetic energy"

Previous methods:

- Velocity is fundamental
- Force: causes velocities to change over time
- Intimately concerned with vector quantities

Energy methods:

- v^2 (related to kinetic energy) is fundamental
- Force: causes KE to change over distance
- Energy is a *scalar*

Energy methods: useful when you don't know and don't care about time

The work-energy theorem in 1D

We've encountered something before that eliminates time as a variable...

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The "third kinematics relation"

$$v_f^2 - v_0^2 = 2a\Delta x$$

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$$v_f^2 - v_0^2 = 2a\Delta x$$

Multiply by $\frac{1}{2}m$:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = am\,\Delta x$$

That thing on the right looks familiar...

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Multiply by $\frac{1}{2}m$:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = F\Delta x$$

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Multiply by $\frac{1}{2}m$:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = F\Delta x$$

Some new terminology:

- $\frac{1}{2}mv^2$ called the "kinetic energy" (positive only!)
- $F\Delta x$ called the "work" (negative or positive!)
- "Work is the change in kinetic energy"

The work-energy theorem in 1D

What if the force isn't constant?

Simple – we just pretend that it is constant for little bits of time, and add them up to find the work:

$$W = \int F dx$$

The work-energy theorem in 1D

What if the force isn't constant?

Simple – we just pretend that it is constant for little bits of time, and add them up to find the work:

$$W = \int F dx$$

Note that the sign of the work *does not depend on the choice of coordinate system*: if I reverse my coordinates, both F and dx pick up a minus sign.

- A force in the same direction as something's motion makes it speed up, and does positive work
- A force in the opposite direction as something's motion makes it slow down, and does negative work

Suppose I throw a ball up in the air, and catch it at the same height.

What is the sign of the work done by gravity from the time I throw it until the time I catch it again?

- A: Positive
- B: Negative
- C: Zero
- D: It depends on your choice of coordinates

Suppose I throw a ball up in the air, and catch it at the same height.

What is the sign of the work done by gravity from the time I throw it until it is at its highest point?

- A: Positive
- B: Negative
- C: Zero
- D: It depends on your choice of coordinates

Suppose I throw a ball up in the air, and catch it at the same height.

What is the sign of the work done by gravity from the time it is at its highest point until I catch it again?

- A: Positive
- B: Negative
- C: Zero
- D: It depends on your choice of coordinates

Suppose I throw a ball up in the air, and catch it at the same height.

What is the sign of the work done by air resistance?

- A: Positive on the way up, and positive on the way down
- B: Negative on the way up, and negative on the way down
- C: Positive on the way up, and negative on the way down
- D: Negative on the way up, and positive on the way down
- E: Zero

Sample problem: dropping an object

Pierre the rather clumsy cat falls off of a cat tree that is a height h. At what speed does he hit the ground?

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Pierre the rather clumsy cat falls off of a cat tree that is a height h. At what speed does he hit the ground?

Feet first, of course – we're not cruel!

- A: $\sqrt{2gh}$
- B: $\sqrt{\frac{gh}{2}}$
- C: $2gh$
- D: $\sqrt{\frac{2h}{g}}$
- E: It depends on Pierre's mass (how many breakfasts has he tricked his owners into giving him today?)

Sample problem: Baseball problem

I throw a ball straight up with initial speed v_0.
Someone catches it at height h. How fast is it going?

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I throw a ball straight up with initial speed v_0.
Someone catches it at height h. How fast is it going?

- $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = (-mg) \times h$
- ... algebra follows: solve for v_f

Work-energy theorem: 2D

We can do this in two dimensions, too:

- $\frac{1}{2}mv_{x,f}^2 - \frac{1}{2}mv_{x,0}^2 = F_x \Delta x$
- $\frac{1}{2}mv_{y,f}^2 - \frac{1}{2}mv_{y,0}^2 = F_y \Delta y$

Add these together:

- $\frac{1}{2}m(v_{x,f}^2 + v_{y,f}^2) - \frac{1}{2}m(v_{x,0}^2 + v_{y,0}^2) = F_x \Delta x + F_y \Delta y$

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- The thing on the left can be simplified with the Pythagorean theorem:
- $\frac{1}{2}m(v_f^2) - \frac{1}{2}mv_0^2 = F_x \Delta x + F_y \Delta y$

- That funny thing on the right is called a "dot product".

Dot products

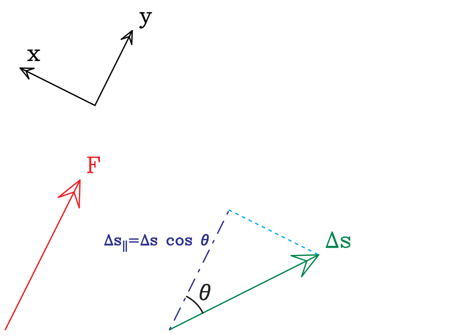
$$A_x B_x + A_y B_y \text{ is written as } \vec{A} \cdot \vec{B}.$$

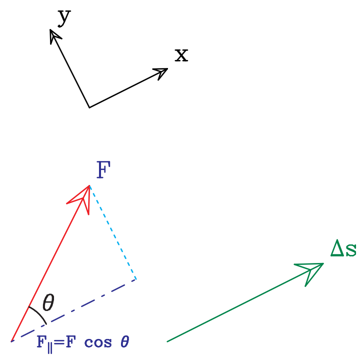
What does this mean? It's a way of "multiplying" two vectors to get a scalar (a number).

Dot products

$$A_x B_x + A_y B_y \text{ is written as } \vec{A} \cdot \vec{B}.$$

What does this mean? It's a way of "multiplying" two vectors to get a scalar (a number).
We can choose coordinate axes as always: choose them to align either with $\vec{F}$ or $\Delta\vec{s}$.





- $\vec{F} \cdot \Delta\vec{s} = (F)(\Delta s_{\|}) = (F)(\Delta s \cos\theta)$
- "The component of the displacement parallel to the force, times the force

- $\vec{F} \cdot \Delta\vec{s} = (F_{\|})(\Delta s) = (F\cos\theta)(\Delta s)$
- "The component of the force parallel to the motion, times the displacement

Different cases where each form is useful, but it's the same trig either way

Pendulum demos

- What is the work done by the string?

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- Zero – it's always perpendicular to the motion!
- How high will it swing on the other side?

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- How high will it swing on the other side?
- Gravity does positive work on the way down and negative work on the way up
- The kinetic energy can't go below zero
- The height at each end of the swing must be the same!
- ... and the return height can't be greater than the initial height...

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- The kinetic energy can't go below zero
- The height at each end of the swing must be the same!
- ... and the return height can't be greater than the initial height...

(If physics stops working and I go splat, have a nice summer!)

Suppose a person of mass m sleds down the big hill outside the music building. The top of the hill is $h = 20$ m higher than the base. (See picture on document camera.) Suppose that there is no friction.

How much work is done by gravity?

- A: mg
- B: gh
- C: mgh
- D: $-mg$
- E: 0

Suppose a person of mass m sleds down the big hill outside the music building. The top of the hill is $h = 20$ m higher than the base. (See picture on document camera.) Suppose that there is no friction.

How much work is done by the normal force?

- A: mg
- B: gh
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Suppose a person of mass m sleds down the big hill outside the music building. The top of the hill is $h = 20$ m higher than the base. (See picture on document camera.) Suppose that there is no friction.

How fast is the person traveling at the bottom?

- A: $\sqrt{2gh}$
- B: $\sqrt{\frac{gh}{2}}$
- C: $2gh$
- D: $\sqrt{\frac{2h}{g}}$
- E: It depends on the shape of the hill

Suppose a person of mass m sleds down the big hill outside the music building. The top of the hill is $h = 20$ m higher than the base. (See picture on document camera.) Suppose that there is no friction.

How much time does it take the person to reach the bottom?

- A: $\frac{h}{\sqrt{2gh}}$
- B: $\sqrt{\frac{2h}{g}}$
- C: $\sqrt{2gh}$
- D: $\frac{2g}{h}$
- E: We can't answer this question using the work-energy theorem

Ball rolling down a ramp demo

- What is the work done by the normal force?

Ball rolling down a ramp demo

- What is the work done by the normal force?
- Zero – the normal force is always perpendicular to the motion!
- What is the work done by gravity?

Ball rolling down a ramp demo

- What is the work done by the normal force?
- Zero – the normal force is always perpendicular to the motion!
- What is the work done by gravity?
- Use the "force times parallel component of motion" formulation:
- $W = (-mg) \times (y_f - y_0)$ – note both components are negative, for a positive result
- The shape of the ramp doesn't matter: the velocities will all be the same at the end!