Recap for Homework

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Your name: ______________________________

Question 1

Finn is a water-loving and very strong dog who has gotten good at jumping off of a boat to catch a Frisbee floating in the water. He's got a mass of $m = 25$ kg. When he jumps, his muscles are able to produce 450 J of energy. For simplicity, let's think about Finn jumping horizontally from the side of a boat, just so we don't have to do any trigonometry. You may approximate Finn as a single point, even though that's not quite realistic.

a) Suppose that Finn jumps horizontally from a very massive boat (so massive that it will not move) as fast as he can from a height of $h = 1$ meter. What velocity v_0 will Finn have once he jumps? *(5 points)*

b) If this boat is floating 2.5 m away from a Frisbee in the water, will Finn be able to jump on top of it? *(5 points)*

c) Now, suppose that Finn jumps horizontally from a much lighter canoe that has the same mass as Finn (25 kg), also from a height of $h = 1$ meter. (The canoe is floating in the water, and is free to move.) Recall that Finn's muscles can only produce $E = 450$ J of energy in a jump, which must be shared between the canoe and Finn.

Determine the velocities of Finn and the canoe after he jumps. *(10 points)*

d) If this canoe is floating 2.5 m away from the same Frisbee, and Finn is again jumping from a height of $h = 1$ m, will Finn be able to jump on top of the Frisbee? *(5 points)*

Question 2

An electric car has the following attributes:

- Maximum speed v_{max}: 50 m/s (with no wind)
- Motor maximum power P_{max}: 180 kW
- Battery capacity U_b: 250 MJ (250×10^6 J)

In this problem, we're going to imagine this car driving at a constant speed along a flat highway with no wind. Under these conditions, the main retarding force on the car is air drag pointing opposite the car's motion, given by the formula

$$F_{\text{drag}} = \gamma v_r^2.$$

Here γ is a coefficient that depends on the shape and size of the car, and v_r is the relative speed of the car with respect to the air. (If the wind is not blowing, this is just the speed of the car.)

You may give your answers in terms of v_{max}, P_{max}, and U_b or as numerical values.

a) What units might γ be measured in? *(5 points)*

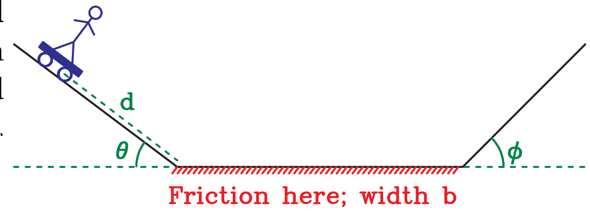
b) Determine the value of γ. (Once you have found it, it will stay the same for the whole problem.) *(5 points)*

c) Suppose that the driver wants to travel 300 km (driving at a constant speed on a flat highway with no wind blowing) before recharging their batteries. At what speed could the driver accomplish this? *(7 points)*

d) Suppose that the manufacturer wanted to build a high-performance version of this car that could travel 70 m/s instead (increasing the top speed by 1.4x). How powerful would the motor need to be to achieve this? *(8 points)*

Question 3

A skateboarder of mass m is standing on the edge of a drainage channel, as shown. The left side, where the skateboarder starts, is elevated at an angle θ; the right side is elevated at an angle ϕ. The slopes on either side are smooth, and the skateboard moves over them with essentially no friction, but the flat bottom of width b is covered with a little sand, and the skateboard experiences a small amount of rolling friction there, with μ_r known.



The skateboarder starts a distance d up the left-hand side. They roll down the left side, across the sand-filled bottom, and up the right side.

(*Give your answers to the first two parts in terms of the variables above, along with g.*)

a) Determine the maximum distance d_2 that the skateboarder makes it up the right side. (This is the diagonal distance, not the height.) *(10 points)*

b) After rolling up and back down the right side, the skateboarder will come back to the left side. How far will they travel back up the left side? *(5 points)*

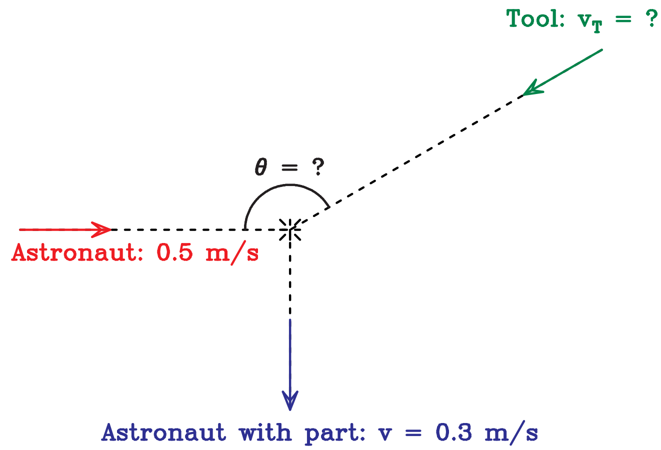
c) Suppose that you know numeric values as follows:

- $m = 75$ kg
- $\theta = 30°$
- $\phi = 40°$
- $\mu_k = 0.05$
- $d = 4$ m
- $b = 7$ m

How many times will the skateboarder travel across the sandy bottom of the channel before coming to rest? Explain the approach behind your solution fully. *(Hint: It will help you to think qualitatively about the conversion of energy between different types before calculating anything. There is an easy way and a hard way to do this!) (10 points)*

Question 4

An astronaut is drifting in space while working to repair a space station that has broken down.
She is drifting at a velocity of 0.5 m/s when another astronaut throws a tool to her as shown. The tool has one-quarter of the astronaut's mass.
She catches the tool and observes that the angle between her velocity before and after she caught it is 90°, as shown in the figure.



Determine, in any order you wish: *(25 points)*

- The initial speed of the tool that was thrown to her
- The angle between the tool's initial velocity and hers

Reference

The work-energy theorem:

$$\frac{1}{2}mv_0^2 + W_{\text{all}} = \frac{1}{2}mv_f^2 \tag{1}$$

The work-energy theorem incorporating potential energy:

$$\frac{1}{2}mv_0^2 + PE_0 + W_{\text{other}} = \frac{1}{2}mv_f^2 + PE_f \tag{2}$$

The definition of work:

$$W = \vec{F} \cdot \Delta\vec{s} \equiv F(\Delta s)_{\parallel} = F_{\parallel}(\Delta s) = F\Delta s\cos\theta \tag{3}$$

Expressions for potential energy:

$$U_{\text{grav}} = mgy(\text{where +y points upward}) \tag{4}$$

$$U_{\text{spring}} = \frac{1}{2}k(\Delta x)^2 \tag{5}$$

Power applied by a force:

$$P = \vec{F} \cdot \vec{v} \tag{6}$$

Conservation of momentum:

$$\sum m\vec{v}_i = \sum m\vec{v}_f \tag{7}$$