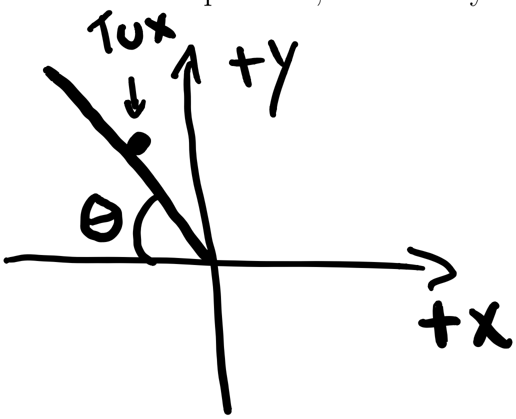
Recitation Questions

12 February

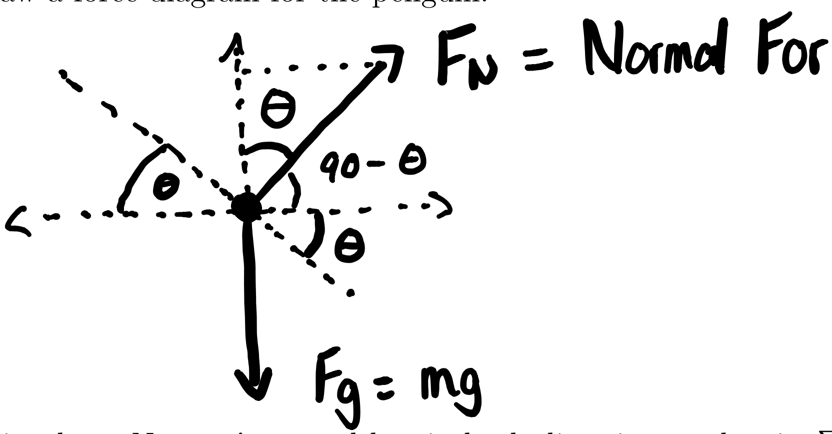
Tux the penguin slides down a frictionless icy hill; the hill is inclined at an angle θ. In this problem, you will calculate the penguin's acceleration. However, I want you to do it two different ways, using two different coordinate systems.

First, solve the problem using the conventional coordinate system, where x is horizontal and y is vertical. As usual, take the following steps:

a) Draw a cartoon of the problem, and label your coordinate system.



b) Draw a force diagram for the penguin.

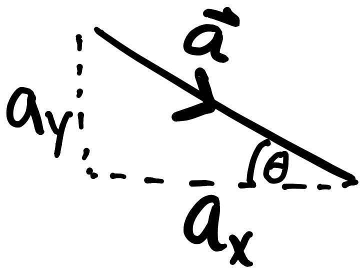


c) Write down Newton's second law in both directions – that is, $\sum F_x = ma_x$ and $\sum F_y = ma_y$. If you have any forces that don't lie along the x or y directions, use trigonometry to break them into components.

$$\sum F_x = F_N \sin\theta = ma_x$$

$$\sum F_y = F_N \cos\theta - mg = ma_y$$

d) This will result in two equations with three unknowns: a_x, a_y, and F_N. However, in this problem, a_x and a_y are related. What is their relation? This should reduce you to two equations and two unknowns; write them below.



$$a_x = \cos\theta \cdot a$$

$$a_y = -\sin\theta \cdot a$$

$$\Rightarrow \Sigma F_x = F_N \sin\theta = m \cos\theta\, a$$

$$\Sigma F_y = F_N \cos\theta - mg = m(-\sin\theta\, a)$$

e) Solve those equations to find the acceleration of the penguin. Use trigonometry to find the magnitude of $\vec{a}$.

$$F_N = \frac{\cos\theta\, a \cdot m}{\sin\theta} \rightarrow \text{From } \Sigma F_x$$

$$\rightarrow \text{Plug into } \Sigma F_y$$

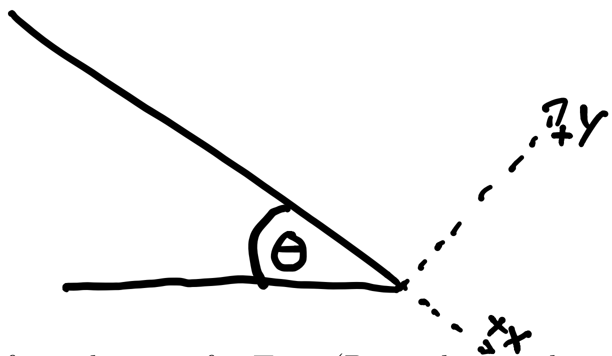
$$\frac{\cancel{m} \cos\theta \cdot \cos\theta\, a}{\sin\theta} - \cancel{m}g = \cancel{m}(-\sin\theta\, a)$$

$$-a\sin\theta = \frac{\cos^2\theta\, a}{\sin\theta} - g \Rightarrow -a\sin\theta - \frac{\cos^2\theta\, a}{\sin\theta} = -g$$

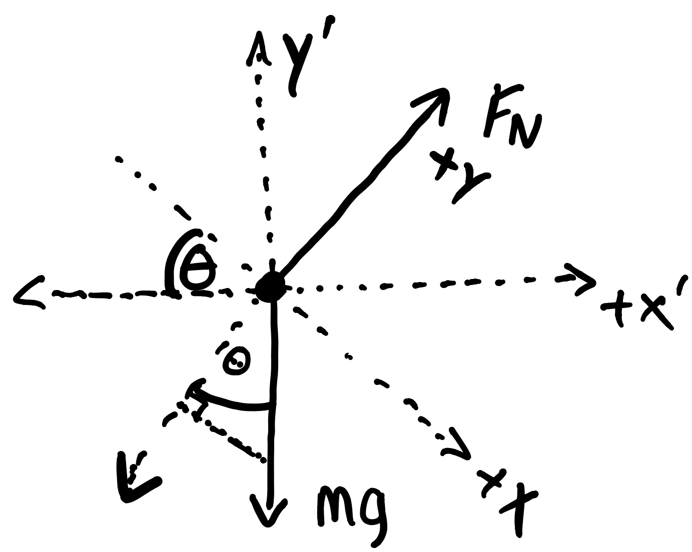
$$\Rightarrow a\left(-\sin\theta - \frac{\cos^2\theta}{\sin\theta}\right) = -g \Rightarrow \boxed{a = \frac{+g}{\left(\sin\theta + \frac{\cos^2\theta}{\sin\theta}\right)}}$$

Now you will solve the problem again using a rotated coordinate system, where x is the direction parallel to the hill and y is the direction perpendicular to it. Again:

a) Draw a cartoon of the problem, and label your coordinate system.



b) Draw a force diagram for Tux. (Draw this one large, since you will need to construct a right triangle with one of the forces as its hypotenuse to break it into components.)



c) Write down Newton's second law in both directions – that is, $\sum F_x = ma_x$ and $\sum F_y = ma_y$. If you have any forces that don't lie along the x or y directions, use trigonometry to break them into components. This will require some thought: you will need to figure out the components of the penguin's weight in the x and y directions. Call over your TA or coach to check your work when you are done.

$$\sum F_x = mg\sin(\theta) = ma_x$$

$$\sum F_y = F_N - mg\cos(\theta) = ma_y$$

d) This will result in two equations with three unknowns: a_x, a_y, and F_N. However, a little thought will tell you what one of these is. What is it? This should reduce you to two equations and two unknowns; write them below.

$$\left.\begin{array}{l} a_y = 0 \\ a_x = a \end{array}\right\} \quad \begin{array}{l} \Sigma F_x = mg\sin(\theta) = a \\ \Sigma F_y = F_N - mg\cos(\theta) = 0 \end{array}$$

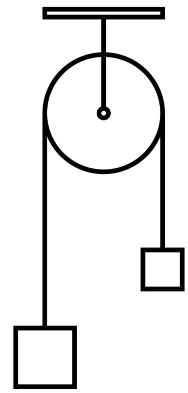
e) Solve those equations to find the acceleration of Tux.

$$F_N = mg\cos(\theta)$$

$$\boxed{a = mg\sin(\theta)}$$

f) Discuss the difference in the two approaches. In one, you aligned your coordinate system with gravity, and in the other, you aligned your coordinate system with the direction that you knew the penguin would accelerate in. Which was easier? Which should you adopt for future problems? Invite your TA or coach over to join your conversation.

Two weights of mass m_1 and m_2 are attached to either end of a string. This string is passed over a light frictionless pulley, as shown in the image. Clearly the heavier mass will go down and the lighter one will go up, but at what rate? In this problem, you will calculate their acceleration.
Note: in this problem, the tension pulling up on each mass is equal. You don't know what it is yet; you can leave it as T until you are able to eliminate it from your equations. But it's the same on both sides; we'll discuss why in class.

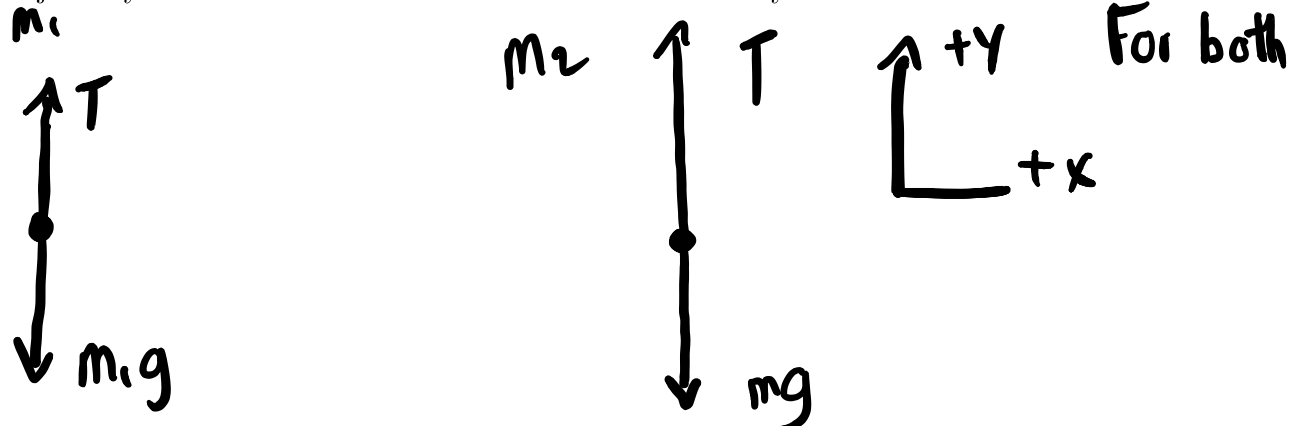


a) What do you expect the system to do if one of the masses is much heavier than the other? What do you expect if the two masses are equal?

When one mass is heavier, the system will accelerate

Whe $m_1 = m_2$, the system $a = 0$

b) Draw force diagrams for both objects. Label your choice of coordinate system separately for each object – you don't have to choose the same coordinate system for each!



c) State Newton's law for both objects. Note that their accelerations aren't necessarily the same, depending on your choice of coordinate system, so you should introduce separate variables a_1 and a_2 for both. The tension forces *are* the same.

System 1

$$\sum F_y = T - mg = m_1 a_1$$

$$\sum F_x = 0$$

System 2

$$\sum F_y = T - m_2 g = m_2 a_2$$

$$\sum F_x = 0$$

d) Since you have two objects, you have two copies of Newton's law. However, you have three unknowns: T, a_1, and a_2. What other statement can you make about the accelerations that lets you solve the system?

$a_1 = -a_2$, equal but opposite accelerations

e) Actually solve the system, giving values of a_1 and a_2 in terms of m_1, m_2, and g. Then, translate your expressions for a_1 and a_2 into words. (Your TA and coaches can help with this.) Does your result make sense? Does it agree with your predictions in part (a)?

System 1

$T - m_1 g = m_1 a$

$T = m_1 a + m_1 g$

System

$T - m_2 g = -m_2 a$

$T = m_2 g - m_2 a$

$T = T$

$m_1 a + m_1 g = m_2 g - m_2 a$

$+m_2$ $\quad +m_2 a$

$-m_1 g$ $\quad -m_1 g$

$\Rightarrow$ $\boxed{a = \frac{g(m_2 - m_1)}{m_1 + m_2}}$