Torque

Physics 211
Syracuse University, Physics 211 Spring 2022
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April 12, 2023

Announcements

- Homework 8 is due Friday in recitation. It consists of a redo of Exam 3, along with two other problems which are posted.
  - The best places to ask for help here are the Physics Clinic and the course Discord server
  - The Discord has a temporary verification process in place (see announcement by email)
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- You should get your exams back in recitation tomorrow
  - If you don't (some of our teaching staff is out sick too), you will Friday
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- Walter is still recovering and hopes to be back Thursday
- He won't know until Wednesday night per CDC guidelines

Unit 4: rotational dynamics

As we saw last time, *almost all of what we have done already* works just the same for rotation.

Last time we saw an example: *rotational kinetic energy*. A summary:

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The moral:

- Rotational motion works much like translational motion
- ... but there are sometimes a few extra things to think about.

This is just a slice of rotational motion. Now let's look from the ground up, starting from the beginning!

Unit 4: rotational dynamics

Unit 1:

- The kinematics relations between $\vec{a}, \vec{v}, \vec{s}, t$ are identical for $\alpha, \omega, \theta, t$
- They're even simpler, because there are no vectors!

Unit 2:

- The centerpiece of this course was $\vec{F} = m\vec{a}$: "how do forces make things move?"
- What is the rotational analogue to this?

Now we will:

- Learn the rotational analogue of force and Newton's second law (today)
- Apply it to all sorts of situations: the rest of the term!

First, let's look again at the whole picture of how rotational and translational motion correspond:

Translation	Rotation
Position $\vec{s}$ Velocity $\vec{v}$ Acceleration $\vec{a}$	Angle θ Angular velocity ω Angular acceleration α
Kinematics: $\vec{s}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$	$\theta(t) = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$
Force $\vec{F}$ Mass m Newton's second law $\vec{F} = m\vec{a}$	Torque τ Rotational inertia I Newton's second law for rotation $\tau = I\alpha$
Kinetic energy $KE = \frac{1}{2}mv^2$ Work $W = \vec{F} \cdot \Delta\vec{s}$ Power $P = \vec{F} \cdot \vec{v}$	Kinetic energy $KE = \frac{1}{2}I\omega^2$ Work $W = \tau\Delta\theta$ Power $P = \tau\omega$
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Rotational motion and kinematics

A reminder about describing rotational motion:

- Instead of describing the change in an object's position $\vec{s}$, we describe the change in its angle θ
- Velocity $\vec{v} \rightarrow$ angular velocity ω (we've used this often before)
- Acceleration $\vec{a} \rightarrow$ angular acceleration α

All the kinematics you learned carries over. For instance:

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Now the question: what makes objects rotate in the first place?

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What is the corresponding idea for rotational motion?

- Angular acceleration α corresponds to $\vec{a}$
- The rotational analogue of mass is called **moment of inertia I** (you already know this)
- The rotational analogue of force is called **torque τ** (we need to understand what this is today!)

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  - This is why the door handle is on the outside of the door...

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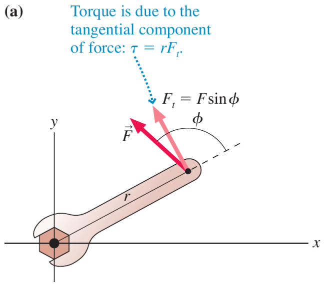
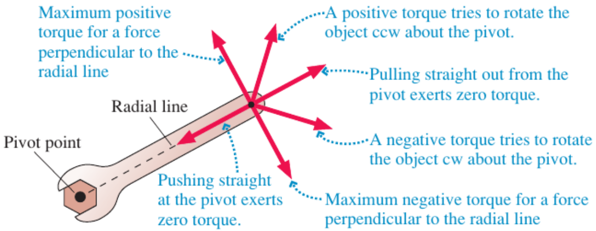
  - This is why the door handle is on the outside of the door...
- The angle at which the force is applied
  - Only forces “in the direction of rotation” make something turn
  - The torque depends only on the *component of the force perpendicular to the radius*

$$\tau = F_{\perp} r$$

Torque is equal to the distance from the pivot, times the perpendicular component of the force



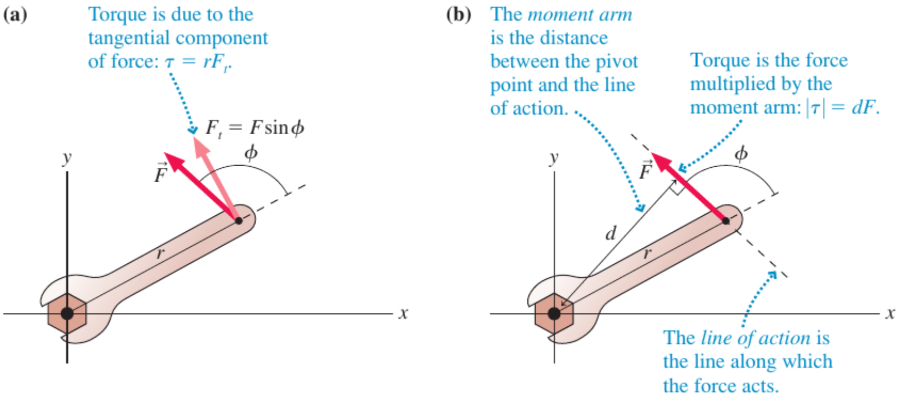


Note that torque has a sign, just like angular velocity: CCW is positive; CW is negative.

Computing torque

- We can think of the torque in any other equivalent way; there is another one that's often useful
- The previous way: **"The radius vector, times the component of force perpendicular to it"**
- The alternative: **"The force vector, times the component of the radius perpendicular to it"**

Here's this idea shown graphically:

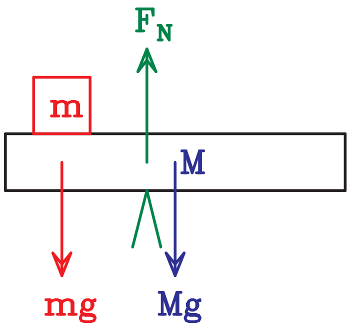


Which bar is hardest to hold up? (See document camera)

Important notes about torque

These are very important: note them somewhere for later reference!

- Torques are in reference to a **particular pivot**
- This is different from force; if you're talking about torque, you *must* say what axis it's measured around
- Torque now depends on the *location* of forces, not just their size
  - Your force diagrams now need to show the place where forces act!
  - Weight acts at the center of mass ("the middle"); we'll see what that means later
  - A sample force diagram might look like this:



Drawing diagrams: torque problems

- Now you need to draw the position at which every force acts
- This is called an "extended force diagram"
- Pick a pivot; label it: remember torques must be calculated around that pivot

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- Remember, the torque from each force is either...
  - $F_\perp r$ (most useful)
  - $F r_\perp$ (sometimes useful)
  - $F r \sin\theta$ (θ is angle between vectors)
  - Direction of torques matters!

Equilibrium problems

- Often we know $\alpha = \vec{a} = 0$
- This tells us that the net torque (about *any* pivot) and the net force are both zero
- Usually this is because an object isn't moving, but sometimes it's moving at a constant rate

- Compute the torque about any point and set it to zero
- Choose a pivot at the location of a force we don't care about so its torque is zero
- If needed, also write $\sum \vec{F} = 0$

Statics problems: a sample

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- What if I hang weights from it?