Problem solving: kinematics (II)

Physics 211
Syracuse University, Physics 211 Spring 2015
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January 30, 2017

Announcements

- Homework 2 due date is **this Friday**
- Exam 1 is next Tuesday
  - No homework due next week
  - HW2 problems are similar to those on Exam 1
  - Physics practice: tomorrow night, 7:30-9:30, here
    - Topic: "setting up problems", i.e. the actual physics
  - Recitation tomorrow has an extra credit question
  - Recitation Friday is your group practice exam
  - If you must miss Friday, notify your TA in advance
    - Weekend: Exam review in Stolkin, time TBA – vote in the Facebook poll

Exam 1

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- Kinematics: how are an object's position, velocity, and acceleration related?

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- Students who do not speak English well: I will try to use only simple English on the exam, but if you like you may bring a dictionary
- Bring: your calculator, pencils, and your physics smarts (frog optional)

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- Review session in class on Thursday
- You are allowed to bring one page of notes that *you handwrite yourself*
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    - It won't help you as much anyway

Exam 1, promises

- There will be one problem where you need the quadratic formula
  - ... this means interpreting the two values it spits out
- There will be at least one instance where you need to interpret or sketch position, velocity, and acceleration graphs
- You will *not* need to compute derivatives or integrals algebraically
- The exam will be four or five problems

Last time

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  - Two representations:
  - Magnitude and direction (easiest to state, hardest to work with
  - Components (easiest to work with
  - Use trigonometry to go back and forth
- One more piece of notation about vectors...

Unit vectors

In the “ordered pair” notation for vectors’ components, you might write:

$$\vec{v} = (5, 3)$$

But this is clunky, if you’re trying to write it as part of an algebraic statement.

Instead we introduce “unit vectors”, vectors with length 1, in the x, y, and z directions.

$$\begin{aligned} \hat{i} &= (1, 0, 0) \\ \hat{j} &= (0, 1, 0) \\ \hat{k} &= (0, 0, 1) \end{aligned}$$

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- $\vec{v} = (5, 3)$: Ordered pair
- $\vec{v} = 5\hat{i} + 3\hat{j}$: Unit vectors
- Both give you the same information, but unit vectors can be easier algebraically
- They won't be essential for this class, but you should know the notation – other people use it quite a bit, and I use it once on your homework

A word on positive and negative acceleration, velocity, “speed”, and displacement:

When you choose your origin, you choose one direction to be positive, and the other to be negative. (Here: right = positive.)

- An object with $x < 0$ just means it's left of the origin.
- An object with $v < 0$ means it's moving to the left.
- An object with $a < 0$ means:
  - A: it is moving to the left and gaining speed
  - B: it is moving to the right and slowing down
  - C: it is moving to the left and slowing down
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Do not confuse the sign of something with the sign of its derivative!

Last time

Acceleration, velocity, and position relationships are the same in 2D; they just apply independently for each component.

$$\vec{v}(t) = \vec{a}t + \vec{v}_0$$
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Working with variables

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Example from cannon problem:

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Example from cannon problem:

$$x(t) = v_{x,0} t$$
$$y(t) = -\frac{1}{2}gt^2 + v_{y,0} t$$

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Example from cannon problem:

$$x(t) = v_0 \cos 45^o t$$
$$y(t) = -\frac{1}{2} g t^2 + v_0 \sin 45^o t$$

(I leave the rest to you for now...)

Problem solving: 2D kinematics, constant acceleration

1. 1. If you have vectors in the "angle and magnitude" form ($\vec{a}, \vec{v}, \vec{s}$), convert them to components
2. 2. Write down the kinematics relations, separately for x and y
   - Many terms will usually be zero
   - Freefall: $a_x = 0$, $a_y = -g$ (with conventional choice of axes)
3. 3. Understand what instant in time you want to know about: ask the right question
4. 4. Put in what you know; solve for what you don't (using substitution, if necessary)
5. 5. Think about the physical meaning of your solution

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“What instant in time do you know about?”

This is often the most difficult part of problems: it requires thought, not just math.

You throw a ball upward over a hole of height h. Your position is the origin, and up is positive.

What condition means “the ball has hit the ground”?

- A: $y = 0$
- B: $y = h$
- C: $y = -h$
- D: $v_y = 0$

"What instant in time do you know about?"

You throw a ball upward off of a cliff of height h. The top of the cliff is the origin, and up is positive.

What condition means "the ball is at its highest point?"?

- A: $y = 0$
- B: $v_y = 0$
- C: $y = h$
- D: y is a maximum

A cannon

A cannon shoots a cannonball at 80 m/s at an angle of 30 degrees above the horizontal.

How can we frame the question "How far does the cannonball go?" in terms of our variables?

- A: What is x at the same time that v_x is zero?
- B: What is y at the same time that x is is zero?
- C: What is x at the same time that y is zero?
- D: What is x at the same time that v_y is zero?

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- How fast is it traveling when it strikes the ground?

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What is $v_{0,x}$?

A: $v_0 \cos\theta$
B: $v_0 \sin\theta$
C: $v_0 \tan\theta$
D: v_0

A cannon

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- What changes if I want to know what velocity I need to hit a target?

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- What changes if I'm trying to hit a target up on a plateau?
- What changes if I want to know what velocity I need to hit a target?
- What changes if I have air resistance?

Throwing a rock off a cliff

A hiker throws a rock horizontally off of a $h = 100$ m tall cliff. If the rock strikes the ground $d = 30$ m away, how hard did she throw it? How fast was it going when it hit the ground? (Choose the origin at the base of the cliff, up/direction of throw as positive)

What is $v_{0,x}$ here?

A: 0
B: 10/3 m/s
C: You don't know *a priori*

What is $v_{0,y}$ here?

A: 0
B: 9.81 m/s
C: You don't know *a priori*

What is a_x here?

A: 0

B: -g

C: +g

D: You don't know *a priori*

What is a_y here?

A: 0

B: -g

C: +g

D: You don't know *a priori*

What is x_0 here?

A: 0

B: h

C: d

D: You don't know *a priori*

What is y_0 here?

A: 0

B: h

C: d

D: You don't know *a priori*

What question do you ask to find "how hard did she throw it?"

A: What value of $v_{x,0}$ makes it such that $x = d$ when $y = 0$?

B: What value of $v_{y,0}$ makes it such that $x = d$ when $y = h$?

C: What is the value of v_x when $y = 0$?

D: What is the magnitude of $\vec{v}$ when $y = 0$?

E: What is the magnitude of $\vec{v}_x$ when $y = h$?

What question do you ask to find "how fast is it going when it hits the ground?"

A: What is v_x at the time when $v_y = 0$?
B: What is v_x at the time when $y = 0$?
C: What is v_y at the time when $y = h$?
D: What is the magnitude of $\vec{v}$ when $y = 0$?
E: What is the magnitude of $\vec{v}$ when $y = h$?

What's the magnitude of $\vec{v}$?

A: $v \cos\theta$

B: $v \sin\theta$

C: $\tan^{-1} \frac{v_x}{v_y}$

A: $\sqrt{v_x^2 + v_y^2}$

Throwing a stone onto a slope

A hiker kicks a stone off of a mountain slope with an initial velocity of v_0 3 m/s horizontally. If the mountain has a slope of 45 degrees, how far down the slope does it land? (Choose the origin as the starting point.)

A: What is the magnitude of $\vec{s}$ when $x = y$?
B: What is the magnitude of $\vec{s}$ when $x = -y$?
C: What is the magnitude of $\vec{s}$ when $y = 0$?
D: What is y when $x = -y$?
E: What is y when $x = 0$?

A rocket

A rocket is launched from rest on level ground. While its motor burns, it accelerates at 10 m/s at an angle 30 degrees below the vertical. After $\tau = 10$ s its motor burns out and it follows a ballistic trajectory until it hits the ground.

How far does it go?