Universal gravitation

Physics 211
Syracuse University, Physics 211 Spring 2015
Walter Freeman

February 24, 2015

Announcements

- I crashed my car on Thursday – sorry for late answers to emails
  - I'm a bit behind on regrades – there's a huge stack, but I haven't forgotten them
- Homework 5 (short) due Friday
- Practice exam posted
  - Work on practice exam during recitations (and elsewhere)
  - Full solutions will be posted Friday
- **Exam date moved** per your feedback to March 3
- Exam prep schedule
  - Wednesday: clinic hours by appointment/request (I'll be around most of the day)
  - Thursday: Review emphasizing basic things, 1:30-5PM, location TBA
  - Friday: Review 10AM-4PM, location TBA
  - **This is a huge amount of extra help available – use it!**

Ask a Physicist: Poking a hole in the Earth

"If you drill a hole through the earth and jump into it, what would happen?"

Ask a Physicist: Poking a hole in the Earth

"If you drill a hole through the earth and jump into it, what would happen?"

Give me 30 minutes and I'll tell you, as we're doing gravity in general today!

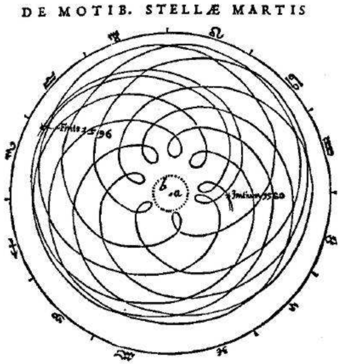
Gravity, in general

- On Earth all objects experience a gravitational force proportional to their mass:
- $F_{\mathrm{grav}} = mg$, directed down toward the Earth
  - How does this work when you're not on Earth?
  - What determines how big g is?

A brief history of gravity and the heavens

The history here is an interesting insight into the way scientific thought has evolved: "How can we explain the sky?"

- Stars in the sky all seem to move together, but with some "wanderers": planets
  - They appear to move in one direction, but sometimes stop and turn around



  - How can we explain this?

A brief history of gravity and the heavens

- Ptolemy: Things go in circles rotating on circles, because circles are perfect, with the Earth at the center
    - "Epicycles" required to make the retrograde motion
- Copernicus: Things go in circles rotating on circles, but with the Earth at the center
    - Relative motion between Earth and planets responsible for retrograde motion
- Brahe: Fantastic measurements of motions of the planets (even more epicycles); geoheliocentrism
- Kepler: Ellipses! No epicycles needed. Laws of planetary motion.
- Galileo: Kinematics; moons of Jupiter; phases of Venus
- Newton: Universal gravitation; dynamics

Newtonian gravity

- All objects – stars, planets, apples, people – exert forces on each other
- That force is given by

$$F_g = \frac{GMm}{r^2}$$

- Both objects feel the same force, directed toward each other
- Note:

$$a_g = F_g/m = \frac{GM}{r^2}$$

- What is G?
  - Fundamental constant of nature that tells us how strong gravity is

$$G = 6.673 \times 10^{-11} \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$$

  - This is really, really tiny

Measuring G

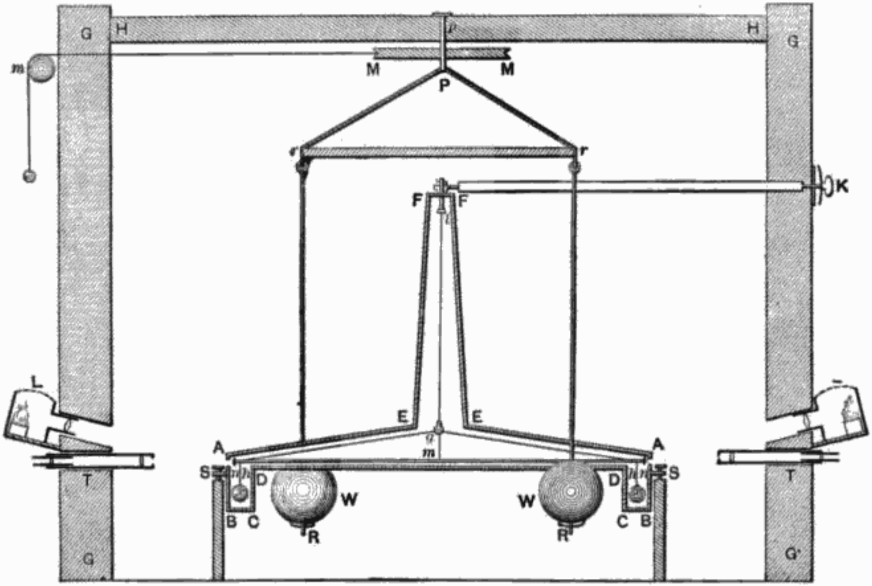


Fig. 1

Measuring G

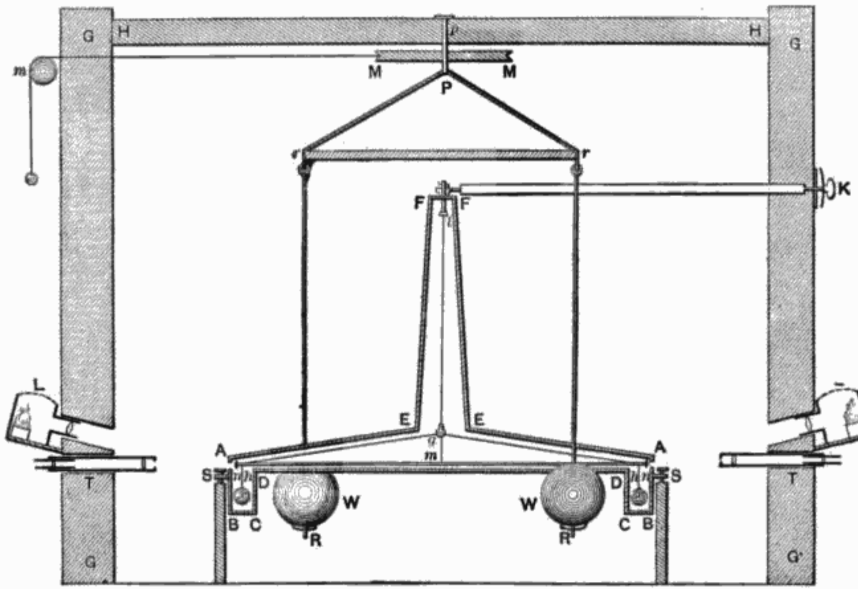


Fig. 1

What is the force between a 1kg mass and a 5kg mass that are 5cm apart?

Measuring the mass of the Earth

What is the mass of the Earth?

Measuring the mass of the Earth

What is the mass of the Earth?

$F_g = \frac{GMm}{r^2} = mg$

What is the mass of the Earth?

$F_g = \frac{GMm}{r^2} = mg$

$M = \frac{gR^2}{G} = 5.97 \times 10^{24}$ kg...

Gravity and circular motion

- Many orbits are nearly circular
- Everything you learned on Tuesday about uniform circular motion still applies
- Weighing the Earth by looking at the Moon:
  - $F_g = \frac{GM_e M_m}{r^2} = M_m \omega^2 r$
- These problems are nothing new and nothing hard; it's just a new force

Kepler's law: relation of orbital period to radius

$$F_g = \frac{GMm}{r^2}$$

Kepler's law: relation of orbital period to radius

$$F_g = \frac{GMm}{r^2}$$

$$m\omega^2 r = \frac{GMm}{r^2}$$

Kepler's law: relation of orbital period to radius

$$F_g = \frac{GMm}{r^2}$$

$$m\omega^2 r = \frac{GMm}{r^2}$$

$$r^3\omega^2 = GM$$

Kepler's law: relation of orbital period to radius

$$F_g = \frac{GMm}{r^2}$$

$$m\omega^2 r = \frac{GMm}{r^2}$$

$$r^3\omega^2 = GM$$

$$T = \frac{2\pi}{\omega} \rightarrow \omega = \frac{2\pi}{T}$$

Kepler's law: relation of orbital period to radius

$$F_g = \frac{GMm}{r^2}$$

$$m\omega^2 r = \frac{GMm}{r^2}$$

$$r^3\omega^2 = GM$$

$$T = \frac{2\pi}{\omega} \rightarrow \omega = \frac{2\pi}{T}$$

$$\frac{4\pi^2 r^3}{T^2} = GM$$

Kepler's law: relation of orbital period to radius

$$F_g = \frac{GMm}{r^2}$$

$$m\omega^2 r = \frac{GMm}{r^2}$$

$$r^3\omega^2 = GM$$

$$T = \frac{2\pi}{\omega} \rightarrow \omega = \frac{2\pi}{T}$$

$$\frac{4\pi^2 r^3}{T^2} = GM$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

Ask a Physicist: Digging a very deep hole...

- $g = \frac{GM}{r^2}$
- As you fell, you would get closer to the center of the Earth: r decreases
- ... but less of the Earth's mass would be under you: M decreases too
- Remember the volume of a sphere: $V = \frac{4}{3}\pi r^3$
- $M \propto r^3$, so $g(r) \propto r$; as you fell your acceleration would decrease
- How fast are you going at the very center of the earth?