Moment of inertia

Physics 211
Syracuse University, Physics 211 Spring 2023
Walter Freeman

April 5, 2023

Announcements

- Homework 8 will be posted later today
- It will be due next Wednesday
- You will have recitation tomorrow as usual

Exam 3

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Students' performance on this exam was lower than on the previous two exams, despite this material being simpler and with fewer nuances than the material on Exams 1 and 2.

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3. We are going to make additional space and time for you to learn this material
4. ... and we are going to give you another chance to demonstrate your knowledge of it.

In short:

- Everyone will have an opportunity to redeem low grades on this exam, even beyond what we're already doing
- ... but you'll need to do some work toward that end.

Exam 3: learning the material

Part of Homework 8 will be to do Exam 3 again, as homework. *(You can skip any problem you got 20/25 on or better.)*

This will let you earn a fraction of credit back – the lower your original score, the more credit you can earn back. *(Because of the limitations on TA's time, they will be grading this somewhat quickly and not looking in detail for partial credit: we expect you to understand things fully the second time around!)*

The rest of HW8 will be shorter to allow you to do that.

Chandler will hold extra help hours Monday from 1:30-5pm in the Physics Clinic to help people learn this material, and I will hold extra help hours tomorrow morning from 9:30-11:00 for the same purpose.

A final note:

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I often have students disclose mental health problems, personal issues, issues with the university administration, and worse to me.

I don't go look up their grades before doing whatever I can for them.

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It turns out that rotational dynamics is mostly a mirror of what you have learned already!

Translational Idea	What it means
Position $\vec{s}$ Velocity $\vec{v}$ Acceleration $\vec{a}$	Where is the thing? How is it moving? How is its motion changing?
Kinematics: $\vec{s}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$	How does accel relate to position and velocity?
Force $\vec{F}$ Mass m Newton's second law $\vec{F} = m\vec{a}$	What pushes on the thing? How hard is the thing to move? How do forces make things move?
Kinetic energy $KE = \frac{1}{2}mv^2$ Work $W = \vec{F} \cdot \Delta\vec{s}$ Power $P = \vec{F} \cdot \vec{v}$	Energy associated with speed How do forces change objects' speed? At what rate do forces change objects' energy?
Momentum $\vec{p} = m\vec{v}$	The "persistence" of an object's motion

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Moment of inertia

The analogue of mass is called "moment of inertia" (letter I)

- More massive things are harder to turn, but that's only part of it
- The mass *distribution* matters, too
- The further the mass is from the center, the harder it will be to turn
- The moment of inertia depends on the *average squared distance from the center*

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$$I = MR^2$$

(if all the mass is the same distance from the center)
(our demo rods; hoops; rings; bike wheels)

Moment of inertia: why?

To see why $I = M\left\langle r^2 \right\rangle$, let's consider the kinetic energy of a spinning object.

The kinetic energy of a single "point mass" moving in a circle is $\frac{1}{2}mv^2 = \frac{1}{2}mr^2\omega^2$, where r is its distance from the center.

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For an extended object, we simply add up the energy of all the moving particles:

$$I = \int r^2 dm = M\left\langle r^2\right\rangle$$

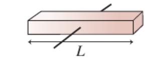
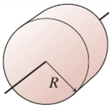
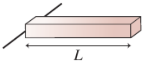
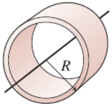
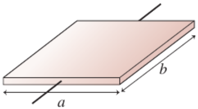
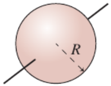
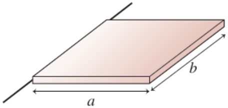
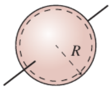
i.e. the moment of inertia is just the total mass times the average squared distance from the axis.

Moment of inertia, other things

What about the moment of inertia of other objects?

Requires calculus in general; here are some common ones

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

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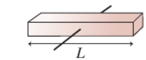
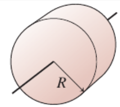
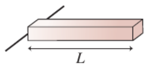
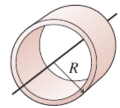
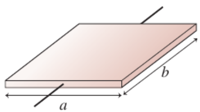
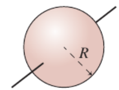
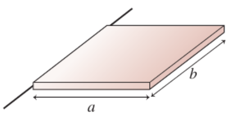
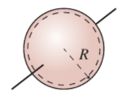
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

In general: $I = \lambda M R^2$

We will always give you I if it's not 1 (i.e. not a ring etc.)

What about rotational kinetic energy?

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An object with moment of inertia I rotating at angular velocity ω has rotational kinetic energy

$$KE_{rot} = \frac{1}{2} I \omega^2$$

An example

How much energy does this bicycle wheel store when I make it spin?

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What if it were a solid cylinder?

Another example

Remember the "Atwood machine"?

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What happens if we remove one of the weights?

Another example

Remember the "Atwood machine"?

What happens if we remove one of the weights?

What happens if the pulley isn't light?

The relationship between linear and rotational motion

What's the acceleration of an object traveling in circular motion?

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$$a = v^2/r \qquad \text{toward the center}$$

Why do we have two different formulae? This came from the relationship:

$$v = \omega r$$

If an object rotates at angular velocity ω, a point a distance r from the center moves at speed v.

Suppose I wrap a string around a solid cylinder with mass M and radius r, and let a mass m hang from the string.

How fast is the falling mass traveling when it hits the ground if it starts from a height h?

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$$\text{(initial KE)} + \text{(work done by gravity)} = \text{(final KE)}$$

$$\text{(initial rotational KE)} + \text{(initial translational KE)} + \text{(work done by gravity)} =$$
$$\text{(final rotational KE)} + \text{(final translational KE)}$$

Rolling and energy

Which object will reach the bottom of the ramp faster?

A: The wooden one

B: The one with the mass located near the middle

C: The one with the mass located near the edge

D: A tie between A and B

E: A tie between B and C

Rotation plus translation

In general, rotation and translation are separate; we can study each separately.

Example: this bike wheel

- Its position is given by some function $\vec{s}(t)$: "where is it at some time t?"
- Its angle is given by some other function $\theta(t)$: "which way is the reference point pointing at some time t?"
- The angle has the familiar derivatives: angular velocity ω, angular acceleration α

Recall that points along the edge of a rotating object move at a speed $v_{\text{edge}} = \omega r$.

Example: rolling without slipping

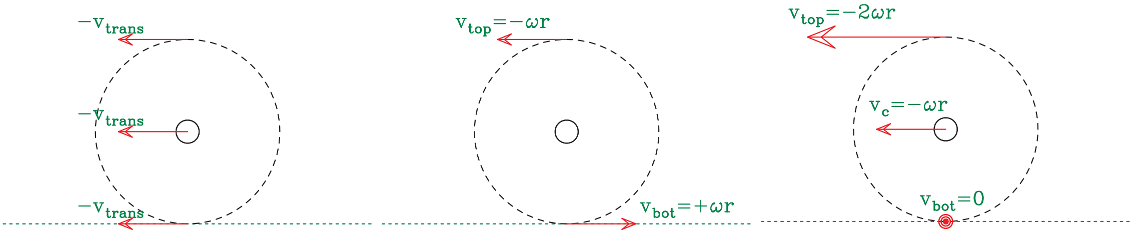
Sometimes the translational and rotational motion are linked.
"How fast do the tires on a car turn?"
→ Static friction means that the bottom piece of the wheel doesn't move

- If a wheel is turning counterclockwise at angular velocity ω:
  - the top moves at $v_{\text{top}} = -\omega r$ (left)
  - the bottom moves at $v_{\text{bot}} = \omega r$ (right)
- This means that the velocity of the axle must be equal and opposite to v_{bot}
- Thus, the car must be moving at $v_{\text{axle}} = -\omega r$ (left).

Let's look at a diagram.
So: if the wheels turn counterclockwise at ω:

- The axle moves at a velocity $-\omega r$ (left);
- The top of the wheels move at a velocity $v_{\text{axle}} + v_{\text{top}} = -\omega r - \omega r = -2\omega r$;
- The top of the wheels move at a velocity $v_{\text{axle}} + v_{\text{bot}} = -\omega r + \omega r = 0$.

Rolling without slipping



Translation + Rotation = Rolling

The "rolling constraint"

If an object rolls forward on an edge of radius r,

$$v = \omega r$$

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Common algebra pattern:

$$KE_{rot} = \frac{1}{2} I \omega^2$$

$$KE_{rot} = \frac{1}{2} \lambda m r^2 \omega^2$$

$$KE_{rot} = \frac{1}{2} \lambda m r^2 \omega^2$$

$$KE_{rot} = \frac{1}{2} \lambda m v^2$$