Momentum

Physics 211
Syracuse University, Physics 211 Spring 2022
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Announcements

- Extra homework help hours today: 2:00-5:30 PM (Physics Clinic – combination of me and others)

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- HW6 due tomorrow
- HW7 posted today, due next Friday – start early

Recitation or homework questions?

Newton's third law: consequences

What happens if someone sitting on a cart throws a heavy ball forward, with a mass 10% of her mass?

Pick the one that is *not* true:

- A: She pushes the ball forward, so it must push her backwards, by Newton's 3rd Law
- B: The change in the ball's velocity is equal and opposite to the change in her velocity
- C: The change in her velocity is going to be in the opposite direction, and 10% as big, as the change in the ball's velocity
- D: The ball's acceleration will at all times be equal and opposite to hers

Newton's third law: consequences

Newton's third law tells us:

$$\vec{F}_{BA} = -\vec{F}_{AB}$$

Combining this with Newton's second law we know:

$$m_A \vec{a}_A = -m_B \vec{a}_B$$

Since we know the area under the acceleration vs. time curve is the change in velocity, we can take integrals of both sides:

$$m_A(\vec{v}_{A,f} - \vec{v}_{A,i}) = -m_B(\vec{v}_{B,f} - \vec{v}_{B,i})$$

We can then rearrange this to put all the "initial" things on the left, and the "final" things on the right:

$$m_A \vec{v}_{A_i} + m_B \vec{v}_{B_i} = m_A \vec{v}_{A_f} + m_B \vec{v}_{B_f}$$

Momentum: overview

We call $m\vec{v}$ the *momentum*, just so we have a name for it. Thus we can write, instead:

$$\sum \vec{p}_i = \sum \vec{p}_f$$

- Momentum is the time integral of force: $\vec{p} = \int \vec{F}\, dt$
- Momentum is a **vector**, transferred from one object to another when they exchange forces
- Another way to look at it: **force is the rate of change of momentum**
- Newton's 3rd law says that total momentum is constant
- Mathematically: $\vec{p} = m\vec{v}$
- Helps us understand **collisions** and **explosions**, among others

Conservation of momentum

- Newton's third law means that forces only *transfer* momentum from one object to another
- The force between A and B leaves the total momentum constant; it just gets transferred from one to the other
- The total change in momentum is zero!
- **Remember momentum is a vector!**
- Solving problems: create "before" and "after" snapshots
- Just add up the momentum before and after and set it equal!

When we need this idea: collisions and explosions

Often things collide or explode; we need to be able to understand this.

- Very complicated forces between pieces often involved: can't track them all
- These forces are huge but short-lived, delivering their impulse very quickly
- Other forces usually small enough to not matter during the collision/explosion
- Use conservation of momentum to understand the collision

The procedure is always the same:

$$\sum \vec{p}_i = \sum \vec{p}_f \text{ “Momentum before equals momentum after”}$$

Make very sure your “before” and “after” variables mean what you think they mean!

Applying conservation of momentum to problems

- 1. Identify what process you will apply conservation of momentum to
  - Collisions
  - Explosions
  - Times when no external force intervenes
- 2. Draw clear pictures of the "before" and "after" situations
- 3. Write expressions for the total momentum before and after, in both x and y
- 4. Set them equal: Write $\sum p_i = \sum p_f$ (in both x and y if needed), and solve

Demos with students

Bob and Alice sit on carts. Bob pulls Alice with a rope. Who moves?

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Bob and Alice sit on carts. Bob pulls Alice with a rope. Who moves? Does throwing or catching a heavy ball change someone's velocity?

- A. Throwing only
- B. Catching only
- C. Both throwing and catching
- D. Only if someone then catches the ball

Sample problems: a 1D collision

Two train cars moving toward each other at 5 m/s collide and couple together. One weighs 10 tons; the other weighs 20 tons. What is their final velocity?

Sample problems: a 1D collision

A train car with a mass m is at rest on a track. Another train car also of mass m is moving toward it with a velocity v_0 when it is a distance d away. The first car hits the second and couples to it; the cars roll together until friction brings them to a stop.

If the coefficient of rolling friction is μ_r, how far do they roll after the collision?

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Method: use conservation of momentum to understand the collision; use other methods to understand before and after!

Sample problems: an explosion in 2D

A child on skis has a mass of 40 kg and is skiing North at 3 m/s. He throws a giant snowball of mass 2 kg at his friend; after he throws it, the snowball has a velocity of 10 m/s directed 45 degrees south of west.

What is the child's velocity after he throws the snowball?

Sample problems: an excited dog

A person of mass m is sitting in a tire swing with a string of length L when their dog (mass M) runs and jumps horizontally into their lap.

If they swing up to an angle θ above the horizontal, how fast was their dog running?