Rotational motion

Physics 211
Syracuse University, Physics 211 Spring 2017
Walter Freeman

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Announcements

- Next homework due Friday
- Extended office hours: tonight 7:30-9:30 (Clinic/Stolkin); Thursday 5-7 (Clinic)

Rotational motion

Everything you've learned about linear motion has a rotational equivalent:

- Position, velocity, acceleration $\leftrightarrow$ angle, angular velocity, angular acceleration
- Kinematics for coordinates $\leftrightarrow$ kinematics for angles
- Newton's second law $\leftrightarrow$ Newton's law for rotation
- Force $\leftrightarrow$ torque
- Mass $\leftrightarrow$ moment of inertia
- ... and others

We're going to go over these things piece by piece – learning their details as we go.

First, though, you should see the whole picture (and keep it for reference):

Translation	Rotation
Position $\vec{s}$ Velocity $\vec{v}$ Acceleration $\vec{a}$	Angle θ Angular velocity ω Angular acceleration α
Kinematics: $\vec{s}(t)\frac{1}{2}\vec{a}t^2 + \vec{v}_0 t + \vec{s}_0$	$\theta(t) = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$
Force $\vec{F}$ Mass m Newton's second law $\vec{F} = m\vec{a}$	Torque τ Rotational inertia I Newton's second law for rotation $\tau = I\alpha$
Kinetic energy $KE = \frac{1}{2}mv^2$ Work $W = \vec{F} \cdot \Delta\vec{s}$ Power $P = \vec{F} \cdot \vec{v}$	Kinetic energy $KE = \frac{1}{2}I\omega^2$ Work $W = \tau\Delta\theta$ Power $P = \tau\omega$
Momentum $\vec{p} = m\vec{v}$	Angular momentum $L = I\omega$

Rotational motion and kinematics

First, we need to describe how rotating objects move.

Rotational motion can be described separate from its translational motion.

Describing rotation by itself is simple: it's the same as one-dimensional motion (no vectors!)

By convention: counter-clockwise is always positive (like with the unit circle).

An example: consider a centrifuge rotating at $\omega = 1000\text{rad/s}$. Once its motor is turned off, slows down at $\alpha = -100\text{rad/s}^2$. How long will it take to stop?

Rotation plus translation

In general, rotation and translation are separate; we can study each separately.

Example: this bike wheel

- Its position is given by some function $\vec{s}(t)$: "where is it at some time t?"
- Its angle is given by some other function $\theta(t)$: "which way is the reference point pointing at some time t?"
- The angle has the familiar derivatives: angular velocity ω, angular acceleration α

Recall that points along the edge of a rotating object move at a speed $v_{\text{edge}} = \omega r$.

Example: rolling without slipping

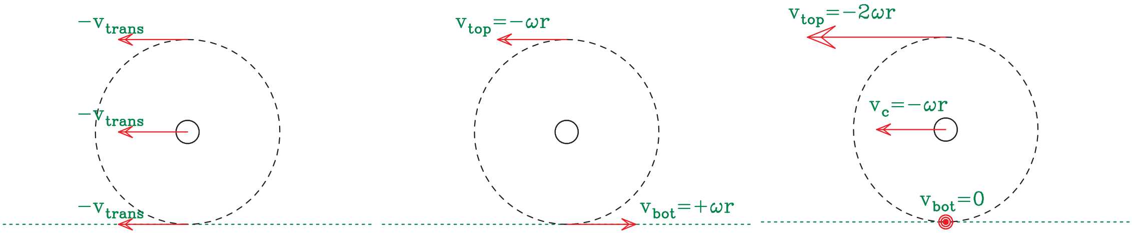
Sometimes the translational and rotational motion are linked.
"How fast do the tires on a car turn?"
$\rightarrow$ Static friction means that the bottom piece of the wheel doesn't move

- If a wheel is turning counterclockwise at angular velocity ω:
  - the top moves at $v_{\text{top}} = -\omega r$ (left)
  - the bottom moves at $v_{\text{bot}} = \omega r$ (right)
- This means that the velocity of the axle must be equal and opposite to v_{bot}
- Thus, the car must be moving at $v_{\text{axle}} = -\omega r$ (left).

Let's look at a diagram.
So: if the wheels turn counterclockwise at ω:

- The axle moves at a velocity $-\omega r$ (left);
- The top of the wheels move at a velocity $v_{\text{axle}} + v_{\text{top}} = -\omega r - \omega r = -2\omega r$;
- The top of the wheels move at a velocity $v_{\text{axle}} + v_{\text{bot}} = -\omega r + \omega r = 0$.

Rolling without slipping



Translation $+$ Rotation $=$ Rolling

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Today, we'll study only torque: this limits us to situations where $\alpha = 0$.

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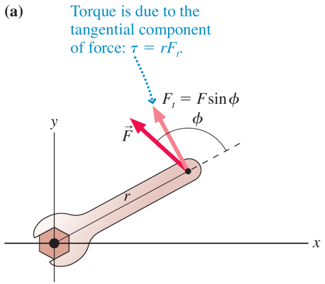
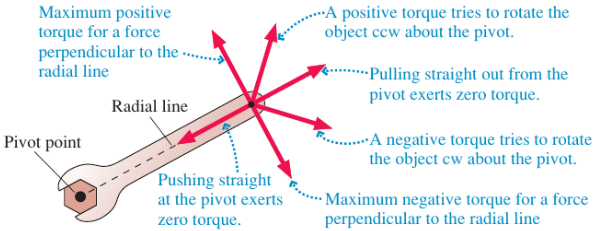
  - Push harder to exert more torque – that's easy!
- The distance from the force to the pivot point
  - The further from the pivot to the point of force, the greater the torque
  - This is why the door handle is on the outside of the door...
- The angle at which the force is applied
  - Only forces "in the direction of rotation" make something turn
  - The torque depends only on the *component of the force perpendicular to the radius*

$$\tau = F_\perp r$$

Torque is equal to the distance from the pivot, times the perpendicular component of the force



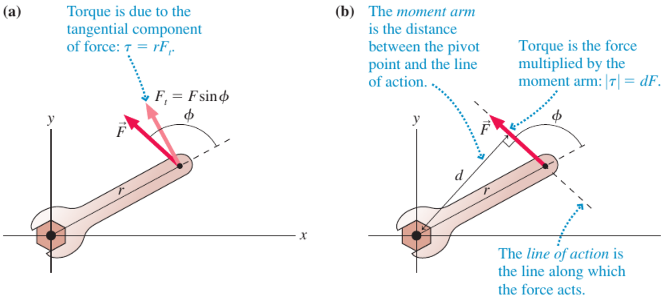


Note that torque has a sign, just like angular velocity: CCW is positive; CW is negative.

Computing torque

- We can think of the torque in any other equivalent way; there is another one that's often useful
- The previous way: **"The radius vector, times the component of force perpendicular to it"**
- The alternative: **"The force vector, times the component of the radius perpendicular to it"**

Here's the figure from the text:

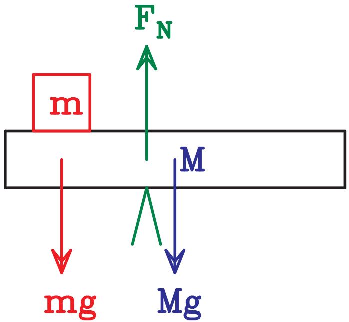


I'll draw a clearer version on the document camera

Important notes about torque

These are very important: note them somewhere for later reference!

- Torques are in reference to a **particular pivot**
- This is different from force; if you're talking about torque, you *must* say what axis it's measured around
- Torque now depends on the *location* of forces, not just their size
  - Your force diagrams now need to show the place where forces act!
  - Weight acts at the center of mass ("the middle"); we'll see what that means later
  - A sample force diagram might look like this:



Drawing diagrams: torque problems

- Now you need to draw the position at which every force acts
- Pick a pivot; label it
- Remember, the torque from each force is either...
    - $F_{\perp} r$ (most useful)
    - $F r_{\perp}$ (sometimes useful)
    - $F r \sin\theta$ (θ is angle between vectors)
    - Direction of torques matters!

Equilibrium problems

- Often we know $\alpha = \vec{a} = 0$
- This tells us that the net torque (about *any* pivot) and the net force are both zero
- Usually this is because an object isn't moving, but sometimes it's moving at a constant rate (tomorrow's recitation problem)

- Compute the torque about any point and set it to zero
- Choose a pivot conveniently at the location of a force we don't care about
- If needed, also write $\sum \vec{F} = 0$

Statics problems: a sample

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- How does the force needed to support it depend on the angle of the bar?
- What if I hang weights from it?