Torque and rotational dynamics

Physics 211
Syracuse University, Physics 211 Spring 2019
Walter Freeman

April 22, 2019

Announcements

A handful of Exam 3's (including the ODS ones) are still pending grading. You'll get those back in recitation tomorrow.

Homework 8 is due tomorrow.

There will be a short Homework 9, due to your TA's mailbox on the last day of class (although no late penalty will be assessed if it is turned in within two days of the due date). It will be posted tonight or tomorrow morning.

Office hours this week:

- Today, ??? - 5:00 (announced in class)
- Friday, 9:30-11:30

Exam 3

The median grade on Exam 3 was a D. We expect this to improve to a C or B based on your performance on the momentum and energy material on the final.

For the final, you should make sure that you can both solve problems mathematically and understand the concepts involved; you will see both sorts of questions on the final.

As noted earlier, if you do better on the momentum/energy material on the final than you did on Exam 3, then we will increase your Exam 3 score proportional to the improvement.

I've posted solutions to Exam 3. Take a look; you'll probably find that most of the problems are much simpler than you thought! (Of course, simple doesn't mean trivial.)

Things that will be on the final exam

The final exam will be a slightly different format. You can expect a few problems like you've seen before. But there will also be more conceptual questions, including some that tie together ideas from throughout the semester.

You can expect to see things like:

- A problem and a completely worked-out solution containing an error; you have to find the error and fix the solution
- A set of questions on dimensional analysis, where we ask you about the units/dimensions of various quantities
- A set of questions where we ask you whether certain quantities could be positive, negative, zero, or multiple of those
- Sometimes/always/never questions
- A Fermi problem
- A process-of-science question, similar to what you saw on Exam 2

Preparation for the final exam

I have already asked Academic Scheduling for a room on May 1, 10AM-3PM, to hold a review session.

Expect 2-3 other marathon reviews like this held between April 29 and the day of the final; I'll schedule those this week.

Today's agenda

- Review anything from Friday's recitation
- Homework questions?
- Do a sample problem we didn't get to last Thursday

Today's agenda

- Review anything from Friday's recitation
- Homework questions?
- Do a sample problem we didn't get to last Thursday

- Talk about rolling motion and energy in more detail
  - The falling yo-yo problem from earlier, done two different ways
  - The stuff we glossed over when we did it in recitation, regarding the work done by tension
  - How a ball rolls down a hill, and why friction does no work there

The Ping-Pong ball on a table

A Ping-Pong ball of mass m rests on a table. The coefficient of static friction between the ball and the table is μ_s.

(Since the ball is a hollow shell, its moment of inertia is $I = \frac{2}{3}mr^2$.)

The wind starts to blow, exerting a force F_w on the ball from one side, directed uniformly across the ball.

A Ping-Pong ball of mass m rests on a table. The coefficient of static friction between the ball and the table is μ_s.

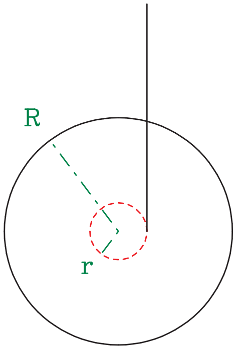
(Since the ball is a hollow shell, its moment of inertia is $I = \frac{2}{3}mr^2$.)

The wind starts to blow, exerting a force F_w on the ball from one side, directed uniformly across the ball.

If the wind blows gently, the ball will roll without slipping. If the wind blows more strongly, the ball will begin to skid along the table.

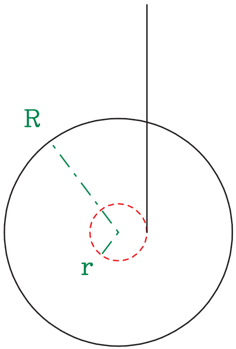
What is the maximum value of F_w so that the ball rolls without slipping?

Suppose I release a Yo-Yo whose inner radius is r and whose outer radius is R.



What will its downward acceleration be?

Suppose I release a Yo-Yo whose string has a length h. How fast will its center be moving when it runs out of string?

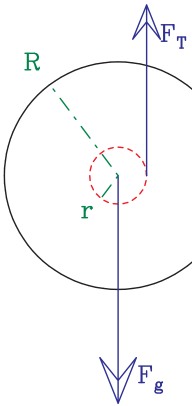


A: $v_f < \sqrt{2gh}$, because the tension in the string slows it down
B: $v_f < \sqrt{2gh}$, because part of the GPE is required to make the Yo-Yo spin
C: $v_f = \sqrt{2gh}$, by the conservation of energy
D: $v_f > \sqrt{2gh}$, because the spinning disk speeds it up

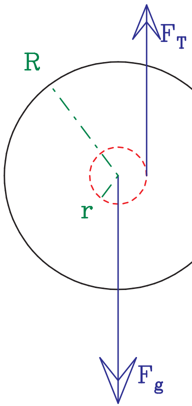
Answer C is what we get if there is no string. (We already know how to do that.)

Answer C is what we get if there is no string. (We already know how to do that.)

Answer A makes sense; in a force diagram for the Yo-Yo, the tension means that the net downward force is less than mg.



Answer C is what we get if there is no string. (We already know how to do that.)

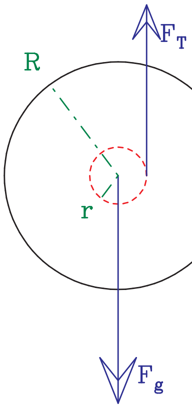


Answer A makes sense; in a force diagram for the Yo-Yo, the tension means that the net downward force is less than mg.

Answer B makes sense as well, though: if the Yo-Yo spins as it falls, then **some energy is required to make it spin**, leaving less available energy for translational kinetic energy. (We're already used to thinking like this!)

Answer C is what we get if there is no string. (We already know how to do that.)

Answer A makes sense; in a force diagram for the Yo-Yo, the tension means that the net downward force is less than mg.



Answer B makes sense as well, though: if the Yo-Yo spins as it falls, then **some energy is required to make it spin**, leaving less available energy for translational kinetic energy. (We're already used to thinking like this!)

This problem is a playground for us to get used to the idea of *rotational work.*

The work done by tension

We know the work-energy theorem for translational motion (for constant $\vec{F}$):

$$W_{\text{trans}} \equiv \Delta \frac{1}{2} m v^2 = \vec{F} \cdot \Delta \vec{s}$$

Replacing m, $\vec{F}$, $\vec{s}$, and v^2 with their rotational counterparts, we get:

$$W_{\text{rot}} \equiv \Delta \frac{1}{2} I \omega^2 = \tau \Delta \theta$$

This is the *rotational work-energy theorem.*

Before (on Exam 3, for instance), we treated "rotational energy" and "translational energy" as the same sort of thing, and added them together.

But we can treat them as separate, too:

$$KE_{\text{trans,i}} + W_{\text{trans}} = KE_{\text{trans,f}}$$
$$KE_{\text{rot,i}} + W_{\text{rot}} = KE_{\text{rot,f}}$$

Which is true regarding the work done by tension here?

A: $W_{\text{total}} = 0$

B: $W_{\text{trans}} > 0, W_{\text{rot}} > 0$

C: $W_{\text{trans}} < 0, W_{\text{rot}} > 0$

D: $W_{\text{trans}} > 0, W_{\text{rot}} < 0$

E: $W_{\text{trans}} < 0, W_{\text{rot}} < 0$

The work done by tension

Which is true regarding the work done by tension here?

A: $W_{\text{total}} = 0$

B: $W_{\text{trans}} > 0, W_{\text{rot}} > 0$

C: $W_{\text{trans}} < 0, W_{\text{rot}} > 0$

D: $W_{\text{trans}} > 0, W_{\text{rot}} < 0$

E: $W_{\text{trans}} < 0, W_{\text{rot}} < 0$

The string makes the Yo-Yo fall more slowly (negative translational work), but makes it spin (positive rotational work). That means Answer C is correct. What about Answer A, which we took for granted earlier?

The work done by tension

Rotational work: $W_{\text{rot}} = \tau \Delta\theta$

The work done by tension

Rotational work: $W_{\text{rot}} = \tau \Delta\theta$

If the Yo-Yo falls a distance h, it turns through a (positive!) angle given by $\Delta\theta = h/r$.

The torque applied by the tension is $\tau = Tr$ (positive!).

The work done by tension

Rotational work: $W_{\text{rot}} = \tau \Delta\theta$

If the Yo-Yo falls a distance h, it turns through a (positive!) angle given by $\Delta\theta = h/r$.

The torque applied by the tension is $\tau = Tr$ (positive!).

Rotational work: $W_{\text{rot}} = \tau\Delta\theta = Tr(h/r) = Th.$
Translational work: $W_{\text{trans}} = \vec{F} \cdot \Delta\vec{s} = -Th.$

The work done by tension

Rotational work: $W_{\text{rot}} = \tau \Delta\theta$

If the Yo-Yo falls a distance h, it turns through a (positive!) angle given by $\Delta\theta = h/r$.

The torque applied by the tension is $\tau = Tr$ (positive!).

Rotational work: $W_{\text{rot}} = \tau\Delta\theta = Tr(h/r) = Th$.
Translational work: $W_{\text{trans}} = \vec{F} \cdot \Delta\vec{s} = -Th$.

$\rightarrow$ The total work done by tension here is zero. (We could have guessed that!)

Conservation of energy, including rotation

Now that we know the *total* work done by tension is zero, we can add translational and rotational energy together like we used to do:

$$\mathrm{PE_i} + \frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 + W_{\mathrm{NC}} = \mathrm{PE_f} + \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

Which expression will let us find the velocity of the Yo-Yo at the bottom?

A : $mgh - Th = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$

B : $mgh + \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$

C : $mgh = \frac{1}{2}mv_f^2$

D : $mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$

What about rolling objects?

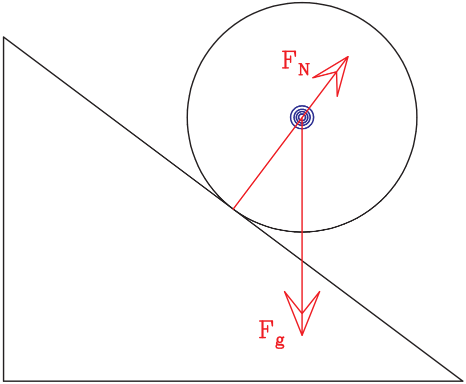
In the Yo-Yo problem, we saw that:

- Tension did positive rotational work (it made the Yo-Yo spin faster)
- Tension did negative translational work (it made the Yo-Yo move more slowly)
- ... the **net work done by tension was zero**.

This happened because the string was stationary, and thus enforced $a = \pm\alpha r$. This is also true in rolling motion.

An object rolling down a hill

Consider first a ball sliding down a hill without friction.

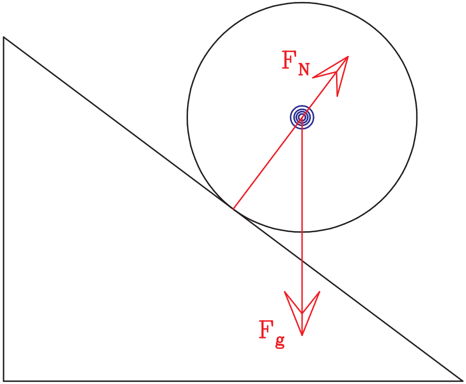


Which of these forces applies a torque to the ball?

A: Just the normal force
B: Just gravity
C: Both of them
D: Neither of them

An object rolling down a hill

Consider first a ball sliding down a hill without friction.



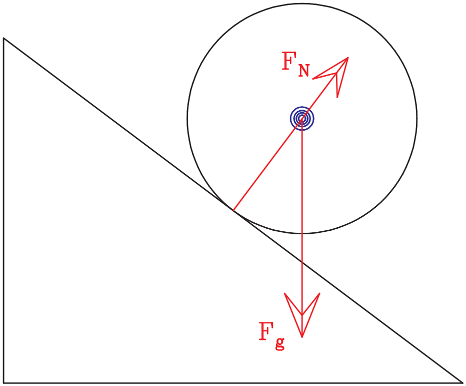
Which of these forces applies a torque to the ball?

A: Just the normal force
B: Just gravity
C: Both of them
D: Neither of them

Friction is required to make the ball spin!

An object rolling down a hill

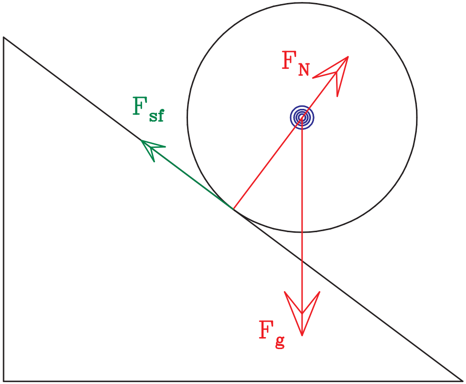
If the ball *rolls without slipping...*



What is true about the frictional force?

A: Static friction points down the ramp
B: Static friction points up the ramp
C: Kinetic friction points down the ramp
D: Kinetic friction points up the ramp
E: There is no friction

If the ball *rolls without slipping...*



What is true about the frictional force?

A: Static friction points down the ramp
B: Static friction points up the ramp
C: Kinetic friction points down the ramp
D: Kinetic friction points up the ramp
E: There is no friction

The point of contact would slide downward without friction, so friction points back up the ramp. This is static friction since the ball doesn't slide.

Energy rolling down a hill

Static friction does no total work on the ball:

- it reduces the translational kinetic energy $\frac{1}{2}mv^2$
- it increases the rotational kinetic energy $\frac{1}{2}I\omega^2$
- ... but it leaves the sum $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ unchanged

TABLE 6.1 Coefficients of friction

Materials	Static μ_s	Kinetic μ_k	Rolling μ_r
Rubber on concrete	1.00	0.80	0.02
Steel on steel (dry)	0.80	0.60	0.002
Steel on steel (lubricated)	0.10	0.05	
Wood on wood	0.50	0.20	
Wood on snow	0.12	0.06	
Ice on ice	0.10	0.03	

This is not *quite* true – rolling friction does exist. There is a little bit of overall negative work done as tires flex and so on, but it is small.

(From *Physics for Scientists and Engineers*, Knight, 3rd ed.)

Rotational dynamics and power

If $W = \tau \Delta\theta$, then $P = \tau\omega$.

If I want to supply a power P, I can either exert a large torque at a small angular velocity, or a small torque at a large angular velocity.

$\rightarrow$ bicycle gears!

How fast do I have to pedal my bike to go slow? fast? Uphill? Downhill?