Rotational motion

Physics 211
Syracuse University, Physics 211 Spring 2023
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April 20, 2023

Announcements

Slight change to "help hours" schedule because of unexpected conflicts. I will be in the Physics Clinic:

- Today 1:30-3:30 (but not 3-5)
- Tomorrow 10-11 and 12:30-1:30

Also:

- The "second chance" review assignments are now due **the day of the final**, rather than **the last recitation** (they will help you study for the makeup questions on the final).
- This is because I now have the resources to grade them later than I thought.

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But civic participation is *also* part of this, and it is an even more vital part of your education than physics or mathematics.

If you would like to miss your PHY211 recitation tomorrow to participate, please email me your schedule for tonight and tomorrow, and I'll tell you which other section you may go to.

Next week

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If you play an instrument you'd like to demo for us and talk about, please email me!

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We won't have a homework set on this material. But it *will* be on your recitation next Friday, and a short question on it will be on the final exam.

The "Atwood machine", for real: how do the masses accelerate?

- Draw force diagrams for everything
- Determine coordinate systems
- $\vec{F} = m\vec{a}$ for things that translate; $\tau = I\alpha$ for things that rotate
- Think carefully about the no-slip constraint!
- Do algebra

Rotational kinetic energy, again

Translation	Rotation
Force $\vec{F}$	Torque $\vec{\tau} = \vec{r} \times \vec{F}$
Mass m	Moment of inertia I
Newton's second law $\vec{F}_{\text{tot}} = m\vec{a}$	Newton's second law for rotation $\tau_{\text{tot}} = I\alpha$
Kinetic energy $KE = \frac{1}{2}mv^2$	Kinetic energy $KE = \frac{1}{2}I\omega^2$
Work $W = \vec{F} \cdot \Delta\vec{s}$	Work $W = \tau\Delta\theta$
Power $P = \vec{F} \cdot \vec{v}$	Power $P = \tau\omega$

Rotational kinetic energy and the rotational work-energy theorem work like their translational counterparts.

Rotational kinetic energy, again

Our previous study of rotational kinetic energy went like this:

- When an object is spinning, it has kinetic energy $KE_{\text{rot}} = \frac{1}{2}I\omega^2$

- When an object rolls down a hill (for instance), we write

(initial grav. potential energy) = (final translational KE) + (final rotational KE)

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

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We ignored questions like:

- Doesn't friction do work on the rolling object?
- If there are strings involved, what about tension?
- What about the rotational work-energy theorem?
- What about rotational power? (Fast cars! Bicycles!)

Let's go back and understand those things now.

We'll use as our example the Yo-Yo from recitation that you did earlier – but, this time, we'll think more carefully about *work*. (You'll analyze the Yo-Yo using forces and torques in your next recitation.)

Suppose I release a Yo-Yo whose string has a length h. How fast will its center be moving when it runs out of string?
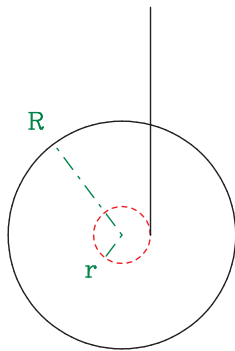
A: $v_f < \sqrt{2gh}$, because the tension in the string slows it down

B: $v_f < \sqrt{2gh}$, because part of the GPE is required to make the Yo-Yo spin

C: $v_f = \sqrt{2gh}$, by the conservation of energy

D: $v_f > \sqrt{2gh}$, because the spinning disk speeds it up

Answer C is what we get if there is no string. (We already know how to do that.)

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Answer A is related to what you did in recitation yesterday; in a force diagram for the Yo-Yo, the tension means that the net downward force is less than mg.
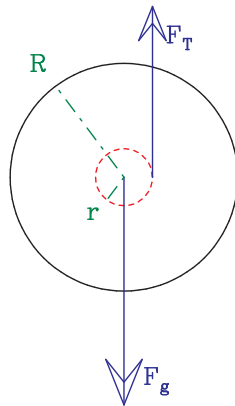
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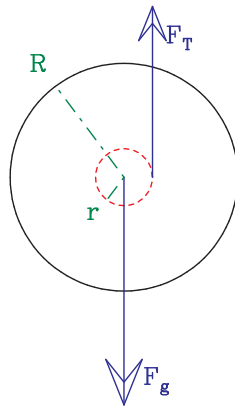
Answer B makes sense as well, though: if the Yo-Yo spins as it falls, then **some
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Before we learned to analyze this with conservation of energy:
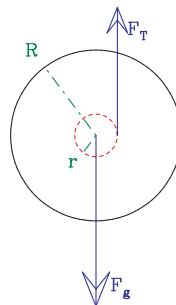
(initial grav. potential energy) = (final translational KE) + (final rotational KE)

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mR^2\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}m\frac{R^2}{r^2}v^2$$

$$v = \sqrt{\frac{2gh}{1 + \frac{1}{2}\frac{R^2}{r^2}}}$$

The work done by tension

We know the work-energy theorem for translational motion (for constant $\vec{F}$):

$$W_{\text{trans}} \equiv \Delta \frac{1}{2} m v^2 = \vec{F} \cdot \Delta \vec{s}$$

Replacing m, $\vec{F}$, $\vec{s}$, and v^2 with their rotational counterparts, we get:

$$W_{\text{rot}} \equiv \Delta \frac{1}{2} I \omega^2 = \tau \Delta \theta$$

This is the *rotational work-energy theorem.*

Which is true regarding the work done by tension here?

A: $W_{\text{total}} = 0$

B: $W_{\text{trans}} > 0, W_{\text{rot}} > 0$

C: $W_{\text{trans}} < 0, W_{\text{rot}} > 0$

D: $W_{\text{trans}} > 0, W_{\text{rot}} < 0$

E: $W_{\text{trans}} < 0, W_{\text{rot}} < 0$

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The string makes the Yo-Yo fall more slowly (negative translational work), but makes it spin (positive rotational work). That means Answer C is correct. What about Answer A?

We forgot something: what about the work done by tension?

Rotational work: $W_{\mathrm{rot}} = \tau \Delta\theta$

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The torque applied by the tension is $\tau = Tr$ (positive!).

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Rotational work: $W_{\text{rot}} = \tau \Delta \theta = Tr(h/r) = Th$.
Translational work: $W_{\text{trans}} = \vec{F} \cdot \Delta \vec{s} = -Th$.

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$\rightarrow$ The total work done by tension here is zero. (We could have guessed that!)

Conservation of energy, including rotation

$$\text{PE}_\text{i} + \frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 + W_\text{NC} = \text{PE}_\text{f} + \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

Which expression will let us find the velocity of the Yo-Yo at the bottom?

$$A : \qquad mgh - Th = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$B : \qquad mgh + \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$C : \qquad mgh = \frac{1}{2}mv_f^2$$

$$D : \qquad mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

# What about rolling objects?

In the Yo-Yo problem, we saw that:

- Tension did positive rotational work (it made the Yo-Yo spin faster)
- Tension did negative translational work (it made the Yo-Yo move more slowly)
- ... the **net work done by tension was zero**.

This happened because the string was stationary, and thus enforced $a = \pm \alpha r$. This is also true in rolling motion.

Consider first a ball sliding down a hill without friction.
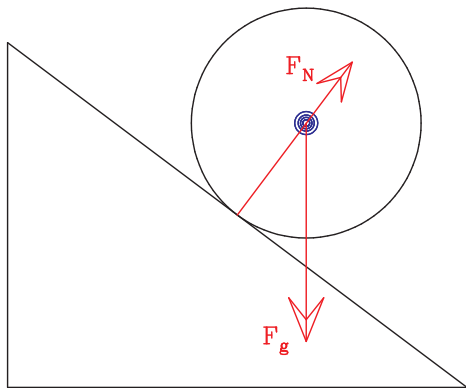
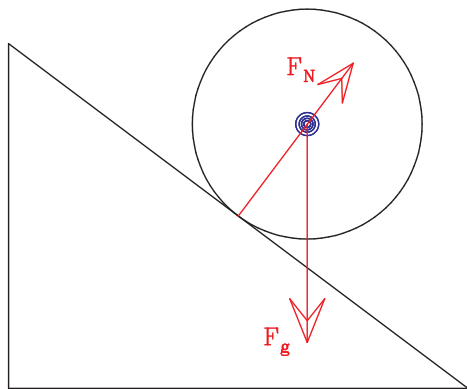Which of these forces applies a torque to the ball?

A: Just the normal force
B: Just gravity
C: Both of them
D: Neither of them

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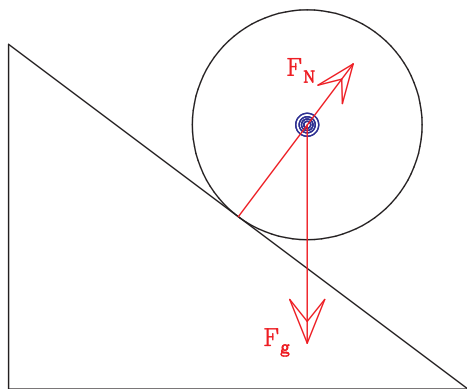B: Just gravity
C: Both of them
D: Neither of them

Friction is required to make the ball spin!

If the ball *rolls without slipping...*

What is true about the frictional force?

A: Static friction points down the ramp
B: Static friction points up the ramp
C: Kinetic friction points down the ramp
D: Kinetic friction points up the ramp
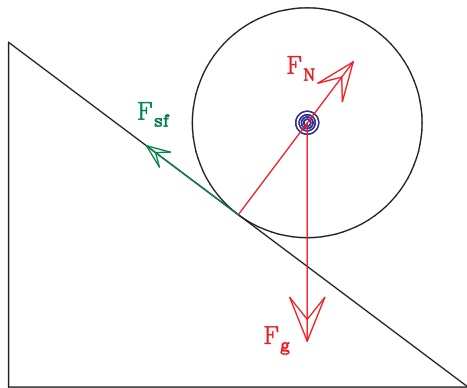E: There is no friction

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C: Kinetic friction points down the ramp
D: Kinetic friction points up the ramp
E: There is no friction

The point of contact would slide downward without friction, so friction points back up the ramp. This is static friction since the ball doesn't slide.

Energy rolling down a hill

Static friction does no total work on the ball:

- it reduces the translational kinetic energy $\frac{1}{2}mv^2$
- it increases the rotational kinetic energy $\frac{1}{2}I\omega^2$
- ... but it leaves the sum $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ unchanged

TABLE 6.1 Coefficients of friction

Materials	Static μ_s	Kinetic μ_k	Rolling μ_r
Rubber on concrete	1.00	0.80	0.02
Steel on steel (dry)	0.80	0.60	0.002
Steel on steel (lubricated)	0.10	0.05	
Wood on wood	0.50	0.20	
Wood on snow	0.12	0.06	
Ice on ice	0.10	0.03	

This is not *quite* true – rolling friction does exist. There is a little bit of overall negative work done as tires flex and so on, but it is small.

(From *Physics for Scientists and Engineers*, Knight, 3rd ed.)

This means that we can use our standard expression for conservation of energy for rolling objects, *ignoring* the force of static friction required to keep them from slipping:

$$\text{PE}_i + \frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 + W_{\text{other}} = \text{PE}_f + \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

How fast will each object $(I = \lambda mr^2)$ be traveling at the bottom of the ramp?

How high must I start the ball for it to make it around the loop?