Problem solving: kinematics (II)

Physics 211
Syracuse University, Physics 211 Spring 2015
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Announcements

- Homework 2 due date is **this Friday**
- Exam 1 is next Tuesday
  - No homework due next week
  - Sample exam is posted; solutions posted Friday
  - Extended office hours and review sessions this week
    - Wednesday, 5-7 PM
    - Thursday 1:30-5:30
    - Friday, 10AM-4PM, exam review; location TBA
    - Weekend: Exam review in Stolkin, time TBA – vote in the Facebook poll

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- Formal review session in class on Thursday
  - At this review you will create your reference sheet, which I will post in final form that day
- Other review times TBA (poll)

Exam 1, promises

- There will be one problem where you need the quadratic formula
  - ... this means interpreting the two values it spits out
- There will be at least one instance where you need to interpret or sketch position, velocity, and acceleration graphs
- You will *not* need to compute derivatives or integrals algebraically
- The exam will be four or five problems

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  - Two representations:
  - Magnitude and direction (easiest to state, hardest to work with
  - Components (easiest to work with
  - Use trigonometry to go back and forth
- One more piece of notation about vectors...

Unit vectors

In the “ordered pair” notation for vectors’ components, you might write:

$$\vec{v} = (5, 3)$$

But this is clunky, if you’re trying to write it as part of an algebraic statement.

Instead we introduce “unit vectors”, vectors with length 1, in the x, y, and z directions.

$$\begin{aligned} \hat{i} =& (1, 0, 0) \\ \hat{j} =& (0, 1, 0) \\ \hat{k} =& (0, 0, 1) \end{aligned}$$

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- $\vec{v} = (5, 3)$: Ordered pair
- $\vec{v} = 5\hat{i} + 3\hat{j}$: Unit vectors
- Both give you the same information, but unit vectors can be easier algebraically
- They won't be essential for this class, but you should know the notation

Last time

Acceleration, velocity, and position relationships are the same in 2D; they just apply independently for each component.

$$\vec{v}(t) = \vec{a}t + \vec{v}_0$$
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Example from cannon problem:

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Example from cannon problem:

$$x(t) = v_0 \cos 45^o t$$
$$y(t) = -\frac{1}{2}gt^2 + v_0 \sin 45^o t$$

(I leave the rest to you for now...)

Problem solving: 2D kinematics, constant acceleration

1. 1. If you have vectors in the "angle and magnitude" form ($\vec{a}, \vec{v}, \vec{r}$), convert them to components
2. 2. Write down the kinematics relations, separately for x and y
   - Many terms will usually be zero
   - Freefall: $a_x = 0$, $a_y = -g$ (with conventional choice of axes)
3. 3. Understand what instant in time you want to know about
4. 4. Put in what you know; solve for what you don't (using substitution, if necessary)
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“What instant in time do you know about?”

This is often the most difficult part of problems: it requires thought, not just math.

You throw a ball upward off of a cliff of height h. The top of the cliff is the origin, and up is positive.

What condition means “the ball has hit the ground”?

- A: $y = 0$
- B: $y = h$
- C: $y = -h$
- D: $v_y = 0$

“What instant in time do you know about?”

You throw a ball upward off of a cliff of height h. The top of the cliff is the origin, and up is positive.

What condition means “the ball is at its highest point?”?

- A: $y = 0$
- B: $v_y = 0$
- C: $y = h$
- D: y is a maximum

A cannon

A cannon shoots a cannonball at 80 m/s at an angle of 30 degrees above the horizontal.

How can we frame the question “How far does the cannonball go?” in terms of our variables?

- A: What is x at the same time that v_x is zero?
- B: What is y at the same time that x is is zero?
- C: What is x at the same time that y is zero?
- D: What is x at the same time that v_y is zero?

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Note that this algebraic solution can be used to do other things rather simply!

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- To get the speed when it hits, we just use the velocity relations:
- $v_x = v_{0,x}$ and $v_y = -gt$
- $v_x = 6.64$ m/s, $v_y = \sqrt{2gh} = -44.2$ m/s
- $|v| = \sqrt{v_x^2 + v_y^2} = 44.7$ m/s

The roadrunner problem

The position of the car is given by the ordinary 1D kinematics relation:

$$x(t) = \frac{1}{2}at^2 + v_0 t = \frac{1}{2}(-9\frac{\mathrm{m}}{\mathrm{s}^2})t^2 + (30.6\frac{\mathrm{m}}{\mathrm{s}})t$$

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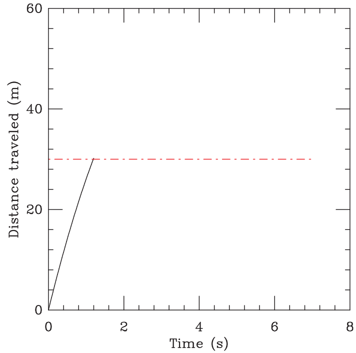
We care about the time when it meets up with the position of the roadrunner, which is 30m. So we set $x(t) = 30$ and solve.



This seems easy enough, but the quadratic formula gives us two solutions! What happened?

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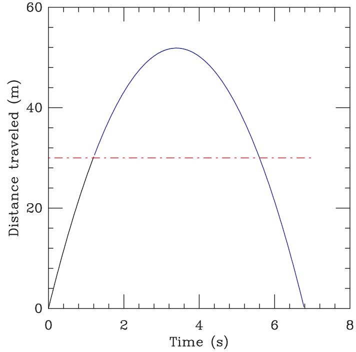

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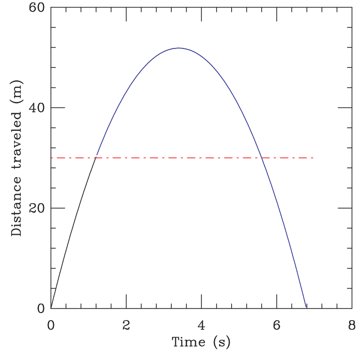

Moral of the story: mathematics is a very blunt tool!

Throwing a stone onto a slope

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- This gives us $x(t) = \frac{2v_{0,x}^2}{g}$
- $y(t)$ will have the same magnitude: the Pythagorean theorem gives $|r| = 2\sqrt{2}\frac{v_{0,x}^2}{g}$

A rocket

A rocket is launched from rest on level ground. While its motor burns, it accelerates at 10 m/s at an angle 30 degrees below the vertical. After ten seconds its motor burns out and it follows a ballistic trajectory until it hits the ground.

How far does it go?