Rotational motion

Physics 211
Syracuse University, Physics 211 Spring 2015
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Announcements

- Next homework due next Wednesday
- No office hours today or Friday (I'm traveling)
- Extended office hours: next Monday, 2:30-6:30 (for homework help)
- Due to exam + family illness over the weekend I am very behind on answering mail; I'm sorry!

Rotational motion

- Everything you've learned about linear motion has a rotational equivalent
    - Position, velocity, acceleration $\leftrightarrow$ angle, angular velocity, angular acceleration
    - Kinematics for coordinates $\leftrightarrow$ kinematics for angles
    - Newton's second law $\leftrightarrow$ Newton's law for rotation
    - Force $\leftrightarrow$ torque
    - Mass $\leftrightarrow$ moment of inertia
    - ... and others

Rotational motion

- You already understand angular position, velocity, and acceleration $\rightarrow$ angular kinematics
- You already understand angular momentum ($L = I\omega$, conserved if no external torques)
- We talked about moment of inertia: $I = \lambda m r^2$, where λ is some fraction depending on shape

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- We talked about moment of inertia: $I = \lambda m r^2$, where λ is some fraction depending on shape
- New topic: Newton's second law for rotation

$$\tau = I\alpha$$

$\rightarrow$ Torques make things have angular acceleration

- What is torque?
- How do we calculate it?

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- The rotational analogue of mass is called **moment of inertia**

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- The rotational analogue of mass is called **moment of inertia**

Let's look at each of those in turn.

Torque

Torque (τ) is the rotational analogue of force: "rotational push or pull"

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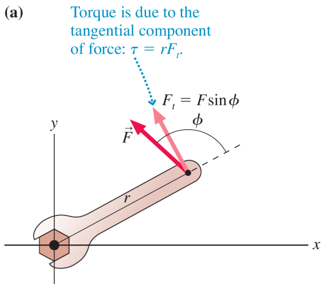
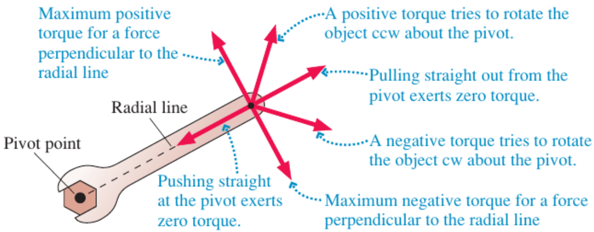
- The size of the torque depends on three things:
- The size of the force
  - Push harder to exert more torque – that's easy!
- The distance from the force to the pivot point
  - The further from the pivot to the point of force, the greater the torque
  - This is why the door handle is on the outside of the door...
- The angle at which the force is applied
  - Only forces "in the direction of rotation" make something turn
  - The torque depends only on the *component of the force perpendicular to the radius*

$$\tau = F_\perp r$$

Torque is equal to the distance from the pivot, times the perpendicular component of the force



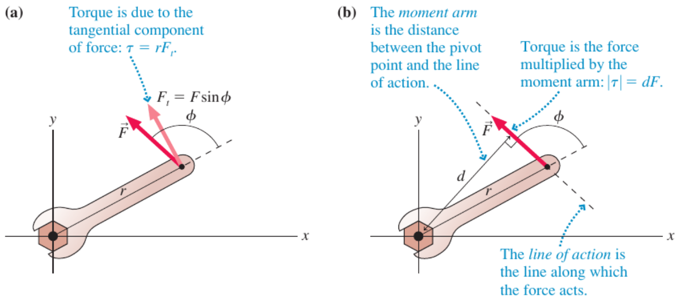


Note that torque has a sign, just like angular velocity: CCW is positive; CW is negative.

Computing torque

- We can think of the torque in any other equivalent way; there is another one that's often useful
- The previous way: **"The radius vector, times the component of force perpendicular to it"**
- The alternative: **"The force vector, times the component of the radius perpendicular to it"**

Here's the figure from the text:

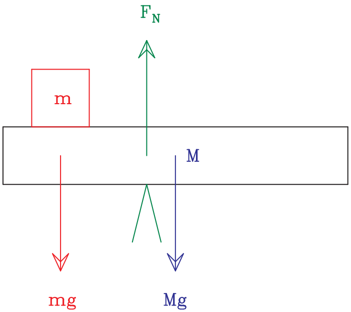


I'll draw a clearer version on the document camera

Important notes about torque

These are very important: note them somewhere for later reference!

- Torques are in reference to a **particular pivot**
- This is different from force; if you're talking about torque, you *must* say what axis it's measured around
- Torque now depends on the *location* of forces, not just their size
  - Your force diagrams now need to show the place where forces act!
  - Weight acts at the center of mass ("the middle"); we'll see what that means later
  - A saimple force diagram might look like this:



What about the mass analogue?

$$\vec{F} = m\vec{a}$$
$$\tau = ?\alpha$$

Mass tells you how hard it is to give something a (linear) acceleration.
What determines how hard something is to turn?

We can already look at situations where $\tau = \alpha = 0$: "static equilibrium"

Moment of inertia

The analogue of mass is called "moment of inertia" (letter I)

- More massive things are harder to turn, but that's only part of it
- The mass *distribution* matters, too
- The further the mass is from the center, the harder it will be to turn
- The moment of inertia depends on the *average squared distance from the center*
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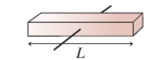
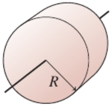
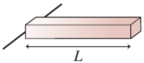
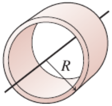
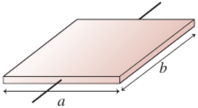
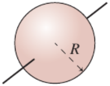
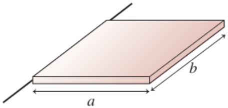
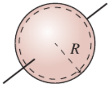
$$I = MR^2$$

(if all the mass is the same distance from the center)
(our demo rods; hoops; rings; bike wheels)

What about the moment of inertia of other objects?

Requires calculus in general; here are some common ones

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

Putting it together: Newton's law for rotation

Translation	Rotation
Force $\vec{F}$	Torque: $\tau = F_\perp r$
Mass m	Moment of Inertia: $I = \lambda M R^2$
Acceleration $\vec{a}$	Angular acceleration α
$\vec{F} = m\vec{a}$	$\tau = I\alpha$

This last line can be thought of as "Newton's second law for rotation".

Torques give things angular acceleration, just like forces make things accelerate:

$$\tau = I\alpha$$

Drawing diagrams: torque problems

- Now you need to draw the position at which every force acts
- Pick a pivot; label it
- Remember, the torque from each force is either...
  - $F_\perp r$ (most useful)
  - $F r_\perp$ (sometimes useful)
  - $F r \sin\theta$ (θ is angle between vectors)
  - Direction of torques matters!

Statics problems

- Often we know $\alpha = \vec{a} = 0$
- This tells us that the net torque (about *any* pivot) and the net force are both zero
- Usually this is because an object isn't moving, but sometimes it's moving at a constant rate (tomorrow's recitation problem)
- Plan of attack:
  - Compute the torque about any point and set it to zero
  - Choose a pivot conveniently at the location of a force we don't care about
  - If needed, also write $\sum \vec{F} = 0$

Statics problems: a sample

- What is the weight of the bar?

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- What is the weight of the bar?
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- How does the force needed to support it depend on the angle of the bar?
- What if I hang weights from it?