Exam 3 review; finishing up angular momentum

Physics 211
Syracuse University, Physics 211 Spring 2019
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April 3, 2019

Announcements

- HW7 is due next Monday before the building closes to your TA's mailbox
- **Group exam 3: Friday during recitation.**
- **Exam 3: Tuesday during the normal time**

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- Upcoming review:
  - "Catchup" review: Thursday, my office, 1:45-3:45 (intended for students who feel behind)
  - Friday: we are hoping to have extra coaches in the Clinic 12-5
  - Saturday: 2:30-5:30, in the Clinic (moving to Stolkin if there is demand)

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  - Friday: we are hoping to have extra coaches in the Clinic 12-5
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- No recitation next Wednesday (we'll be grading)
- Recitation next Friday: workshop for the paper

Exam 3

Format:

- Either four full-length problems, or three full-length problems and a few short answer questions
- Questions on work and energy, the conservation of momentum, and the conservation of angular momentum

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No individual reference sheets:

- We realized they hurt more than helped
- Today: suggest things to me that you want on your reference sheet
- I'll typeset it for you and give it to you on the exam

Conservation of angular momentum

These problems are approached in exactly the same way as conservation of *linear* momentum problems: write down expressions for L_i and L_f and set them equal (if there are no external forces that alter how things rotate, called *torques*).

$$L = I\omega$$

$$\sum L_i = \sum L_f$$

Conservation of angular momentum

If I kept the mass of the Earth the same, but enlarged it so that it had twice the diameter, how long would a day be?

(Remember, the total angular momentum, $L = I\omega$, stays the same)

A: 6 hours

B: 12 hours

C: 24 hours

D: 48 hours

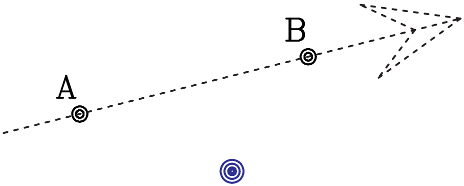
E: 96 hours

A single object moving in a straight line also has angular momentum.

$$L = mv_\perp r = mvr_\perp$$

If we are to trust this relation, then the angular momentum of an object moving with constant $\vec{v}$ should be constant!

Is the angular momentum the same at points A and B?



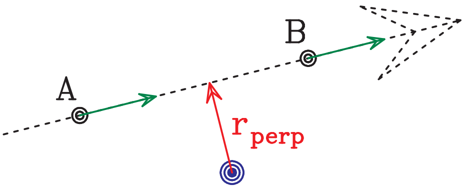
Angular momentum of a single object

A single object moving in a straight line also has angular momentum.

$$L = mv_\perp r = mvr_\perp$$

Is the angular momentum the same at points A and B?

Yes: $r_\perp$ (and v) are the same at both points.



An example problem

How much will I speed up when I pull the dumbbells to my chest?

An example problem

A child of mass m runs at speed v straight east and jumps onto a merry-go-round of mass M and radius R, landing 2/3 of the way toward the outside. If she lands south of the axis, how fast will it be turning once she lands?

We'll do this together on the document camera.

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(The solution is on the next slide, for those studying these notes later)

The solution to our example

We use conservation of angular momentum:

$$\sum L_i = \sum L_f$$
$$L_{\text{child},i} = L_{\text{child+disk},f}$$

Model the child as a point object moving at a constant velocity:

$$L_{\text{child},i} = mv_\perp r = \frac{2}{3}mvR$$

This gives us $\frac{2}{3}mvR = I_{\text{total}}\omega_f$. We now need I_{total}.

After the child jumps on, $I_{\text{total}} = I_{\text{disk}} + I_{\text{child}} = \frac{1}{2}MR^2 + \frac{2}{3}mR^2$. Thus,

$$\frac{2}{3}mvR = \left(\frac{1}{2}MR^2 + \frac{2}{3}mR^2\right)\omega_f$$

Solve for ω_f:

$$\omega_f = \frac{\frac{2}{3}mvR}{\left(\frac{1}{2}MR^2 + \frac{2}{3}mR^2\right)}$$

Review: Rotational kinetic energy

A rotating object carries kinetic energy too.

- Translational kinetic energy: $KE_{\text{trans}} = \frac{1}{2}mv^2$
- Rotational kinetic energy: $KE_{\text{rot}} = \frac{1}{2}I\omega^2$

I is the moment of inertia, a rotational analogue to mass.

- Depends on the mass distribution of an object, as well as the total mass
- $I = mR^2$, if all of the mass is a distance R from the axis of rotation
- For other objects $I = \lambda mR^2$, where λ depends on the object's shape
- The moment of inertia is larger if the mass is further from the center

Review: Rolling motion

If an object is *rolling without slipping*, it is both translating and rotating.

Its translational velocity v_{trans} is equal to ωr, where r is the radius of the part that is rolling.

Example: kinetic energy of a rolling cylinder ($I = \frac{1}{2}I\omega^2$)

$$
\begin{aligned}
KE = KE_{\text{trans}} \quad & + KE_{\text{rot}} = \frac{1}{2}mv^2 \quad + \frac{1}{2}I\omega^2 \\
= \frac{1}{2}mv^2 \quad & + \frac{1}{2}\frac{1}{2}mr^2\omega^2 \\
= \frac{1}{2}mv^2 \quad & + \frac{1}{4}mv^2 \\
= \frac{3}{4}mv^2 &
\end{aligned}
$$

Review: The work-energy theorem

Work is the change in kinetic energy.

$$\frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 + W_{\text{all}} = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

Work $= \vec{F} \cdot \Delta\vec{s}$, or ...

- $F_{\parallel}\Delta s$ (component of force parallel to motion, times the distance moved)
- $F(\Delta s)_{\parallel}$ (force, times the distance moved in the direction of that force)
- $F \Delta s \cos\theta$ (either way, this is the trigonometry you wind up with)

Note that:

- Forces that are in the direction of motion increase the speed of objects, and do positive work
- Forces that are opposite the direction of motion decrease the speed of objects, and do negative work
- Forces perpendicular to the direction of motion do no work at all

Review: Potential energy

Potential energy is an alternate way of keeping track of the work done by conservative forces:

- $PE_{\text{grav}} = mgh$
- $PE_{\text{spring}} = \frac{1}{2}kx^2$

If you're tracking the work due to gravity or elasticity using potential energy, don't also include it in your work term. See the next slide:

Review: Conservation of energy

$$\mathrm{PE_i} \quad + \quad \tfrac{1}{2}mv_i^2 + \tfrac{1}{2}I\omega_i^2 \quad + \quad W_{\text{other}} \quad = \quad \mathrm{PE_f} \quad + \quad \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2$$

Review: Conservation of energy

$$\text{PE}_\text{i} \quad + \quad \tfrac{1}{2}mv_i^2 + \tfrac{1}{2}I\omega_i^2 \quad + \quad W_\text{other} \quad = \quad \text{PE}_\text{f} \quad + \quad \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2$$

(initial PE) + (initial KE) + (other work) = (final PE) + (final KE)

Review: Conservation of energy

$\mathrm{PE_i}$	$+$	$\frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2$	$+$	W_{other}	$=$	$\mathrm{PE_f}$	$+$	$\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$
(initial PE)	+	(initial KE)	+	(other work)	=	(final PE)	+	(final KE)
(total initial mechanical energy)			+	(other work)	=	(total final mechanical energy)		

Review: Conservation of energy

$$
\begin{array}{ccccccccc}
\mathrm{PE_i} & + & \frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 & + & W_{\text{other}} & = & \mathrm{PE_f} & + & \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 \\
\text{(initial PE)} & + & \text{(initial KE)} & + & \text{(other work)} & = & \text{(final PE)} & + & \text{(final KE)} \\
\end{array}
$$

$$
\text{(total initial mechanical energy)} \quad + \quad \text{(other work)} \quad = \quad \text{(total final mechanical energy)}
$$

Since conservation of energy is the broadest principle in science, it's no surprise that we can do this!

Review: Power

Power is just the rate of doing work.

Since velocity is the rate of being displaced, this gives us:

$$P = \vec{F} \cdot \vec{v}$$

What would you all like to talk about?