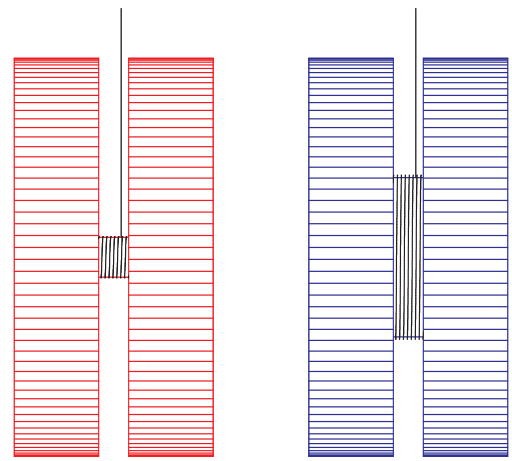
# Recitation Exercises

27 April

## Exercise 1: on combining rotation and translation

A Yo-Yo consists of a cylinder of radius $R$ with a thin slit cut in it. Inside the slit is a smaller inner cylinder of radius $r$ with a string attached to it and then wound around the cylinder. Note that the moment of inertia of a cylinder of radius $R$ is $I = \frac{1}{2}mR^2$; since the slit in the Yo-Yo is so thin, you do not need to consider it in computing the moment of inertia. (Thus, both have the same moment of inertia: $I = \frac{1}{2}mR^2$.)
If a person holds the end of the string and drops the Yo-Yo, it will begin to spin as it falls, unwinding the string as it does.



a) Suppose that you have a red Yo-Yo with $r = 0.1R$ (that is, with a very small inner cylinder) and a blue Yo-Yo with $r = 0.4R$ (with a thicker inner cylinder). Predict which one will fall faster when it is dropped, and describe why it will do so. *(You shouldn't do any calculations here.)*

b) Now, you'll calculate the downward acceleration of the Yo-Yo. In this case, the Yo-Yo both *translates* and *rotates* as it does so

Start by drawing an extended force diagram for the Yo-Yo, showing all the forces acting on it *and where they act.*

c) Since it both translates and rotates, you will need both $\vec{F} = m\vec{a}$ to relate the forces on it to its translational acceleration and $\tau = I\alpha$ to relate the torques on it to its linear acceleration. Construct both of these equations, using the forces that appear on your force diagram. *(Hint: The tension in the string both applies a torque to the Yo-Yo and affects its translational acceleration.)*

d) In the above two equations, you will have three unknowns: the tension in the string, the translational acceleration, and the angular acceleration $\alpha$. However, you can relate two of them to each other. What is that relation? *(Hint: Think carefully about minus signs here!)*

e) Now you should have enough information to solve for $a$ in terms of $g$, $r$, and $R$. Once you have a value for your acceleration, call your GTA or coach over and have them check your work. Discuss with them whether the red or blue Yo-Yo in part (a) would fall faster.

# Exercise 2

Consider the demonstration you saw in class yesterday. A person stands on top of a platform that is free to rotate.

a) Estimate the moment of inertia of the person around their center. You will need to figure out which of our simple shapes best approximates a person, then estimate the person's radius and mass. *(The table of moments of inertia is at the end.)*

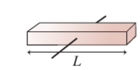
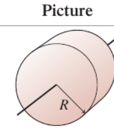
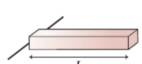
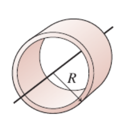
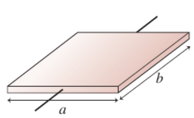
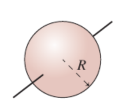
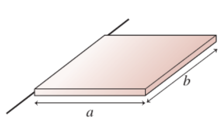
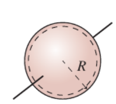
b) Someone else standing on the ground carries a bicycle wheel filled with concrete of mass $m = 5$ kg with radius 30 cm. They make the bicycle wheel spin at an angular velocity $\omega = 20$ radians/sec, turn it so that it is spinning clockwise when viewed from above, then hand it to the person standing on the platform.

When the person standing on the platform grabbed the wheel to stop it from spinning, they began to rotate slowly. Determine which direction and how fast they begin to rotate after they do this.

c) Imagine now that instead of grabbing the wheel, they turned it upside down, so it would be spinning counterclockwise rather than clockwise when seen from above. Without doing any mathematics, predict what should happen. Call your TA or coach over to discuss with your group.

d) Now, calculate which direction they will rotate and how fast when they rotate the wheel upside down. Does the result of your calculation agree with your prediction?

**TABLE 12.2** Moments of inertia of objects with uniform density

| Object and axis | Picture | $I$ | Object and axis | Picture | $I$ |
|---|---|---|---|---|---|
| Thin rod, about center |  | $\frac{1}{12}ML^2$ | Cylinder or disk, about center |  | $\frac{1}{2}MR^2$ |
| Thin rod, about end |  | $\frac{1}{3}ML^2$ | Cylindrical hoop, about center |  | $MR^2$ |
| Plane or slab, about center |  | $\frac{1}{12}Ma^2$ | Solid sphere, about diameter |  | $\frac{2}{5}MR^2$ |
| Plane or slab, about edge |  | $\frac{1}{3}Ma^2$ | Spherical shell, about diameter |  | $\frac{2}{3}MR^2$ |