PHY 211 Recitation 11

February 19, 2020

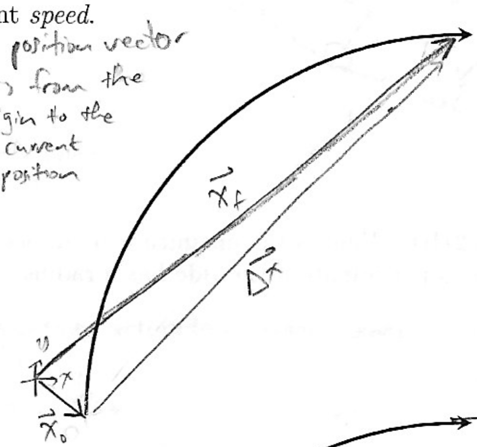
1 Motion on a curve

Consider a car travelling along a curved road at a constant *speed.*

a position vector points from the origin to the current position

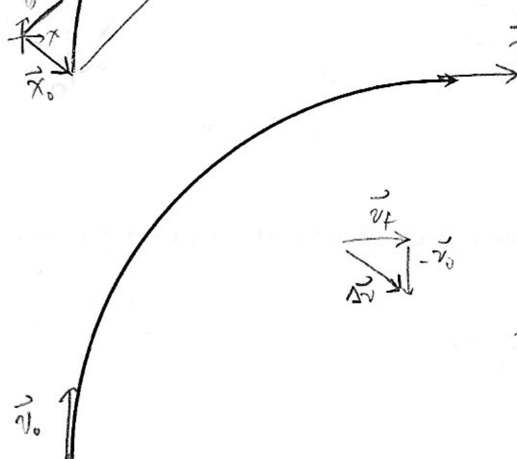
Problem 1(a). Pick an origin for your xy coordinate system, then draw and label it **O** on the figure to the right. Draw the position vector at the start of the curve, and draw another one near the end of the curve. Then find the displacement $\Delta\vec{x}$. $= \vec{x}_f - \vec{x}_0$
Along what direction does the average velocity vector point?

Since $\vec{v}_{ave} = \frac{\Delta \vec{x}}{\Delta t}$, $\boxed{\vec{v}_{ave} \text{ points the same way as } \Delta\vec{x}}$

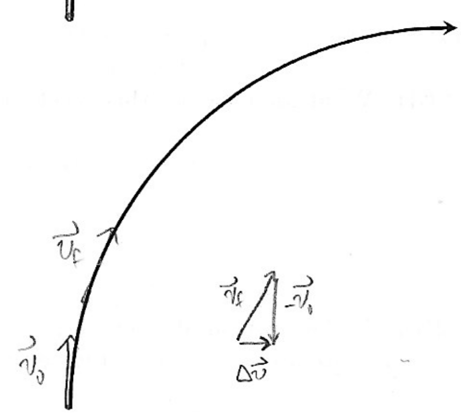


Problem 1(b). Now, on the figure to the right, draw instantaneous velocity vectors at the same two points you used above. Draw the change in velocity $\Delta\vec{v}$. $= \vec{v}_f - \vec{v}_0$
Along what direction does the average acceleration point?

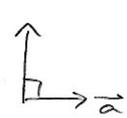
Since $\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t}$, $\boxed{\vec{a}_{ave} \text{ points the same way as } \Delta\vec{v}}$



Problem 1(c). Repeat the previous exercise, but make your second point much closer to the first.
If you could continue to shrink Δt, which direction would the instantaneous acceleration point?

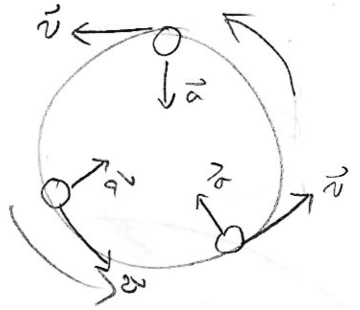


Note that $\vec{a}$ points inwards, towards the center of the path, and $\vec{v}$ and $\vec{a}$ become perpendicular as Δt shrinks



2 Circular motion

Problem 2(a). A fairground ride spins its occupants inside a flying saucer-shaped container. Draw a picture of the motion, and at different points, draw the velocity and acceleration vectors of a ride occupant.



Problem 2(b). What is the magnitude of an occupant's velocity (in m/s) when spinning at a frequency of f revolutions per minute if the ride has a radius r?

One revolution is a distance $2\pi r$.

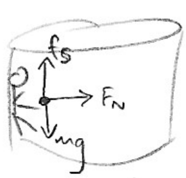
f revolutions is a distance $2\pi r f$,

so $\boxed{v = \dfrac{2\pi r f}{60\,s}}$

Problem 2(c). What is the centripetal acceleration of an occupant with this velocity?

$a_c = \dfrac{v^2}{r}$, so $a_c = \dfrac{(2\pi r f/60\,s)^2}{r} = \boxed{\dfrac{4\pi^2 f^2}{3600\,s^2}\, r}$

Problem 2(d). What force causes this acceleration?



The only inward pointing force that can cause the inward pointing centripetal acceleration is $\boxed{\text{the normal force from the wall}}$

Problem 2(e). If the horizontal circular path the riders follow has an $r = 8.00\,\text{m}$ radius, at how many revolutions per minute are the riders subjected to a centripetal acceleration equal to that of gravity?

What is f?

From before, $a_c = \dfrac{4\pi^2 f^2}{3600\,s^2}\, r$. If $a_c = g$, we can solve for f.

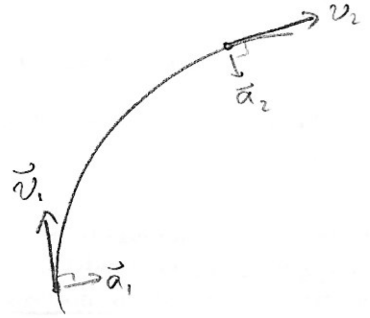
$g = \dfrac{4\pi^2 f^2}{3600\,s^2}\, r$, or $f = \sqrt{\dfrac{3600\,s^2\, g}{4\pi^2 r}} = \sqrt{\dfrac{(3600\,s^2)(9.8\,m/s^2)}{4\pi^2\,(8\,m)}}$

$\approx \boxed{10.6\ \text{rev}/\text{min}}$

3

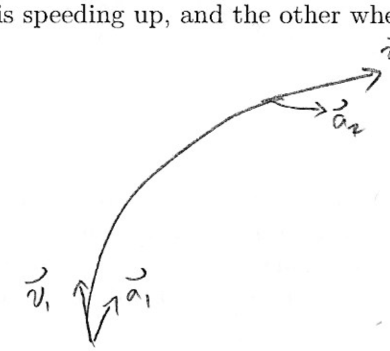
A runner taking part in the 200 m dash must run around the end of a track that has a circular arc with a radius of curvature of 30.0 m.

Problem 3(a). Draw a picture of the motion and draw the velocity and acceleration vectors as she is taking the turn, assuming she takes it at constant speed.

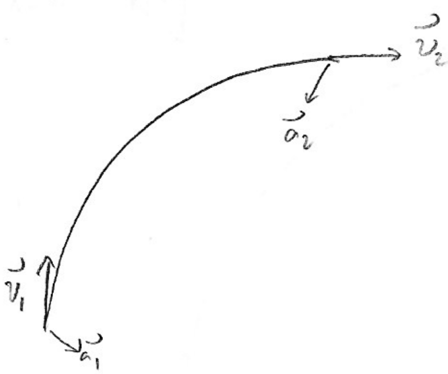


Problem 3(b). Now instead draw the velocity and acceleration vectors in two other scenarios: one where she is speeding up, and the other where she is slowing down.

"tangential acceleration"
$\vec{v}$, $\vec{a}_T$: here, $\vec{a}_T$ is causing her to speed up
$\vec{v}$, $\vec{a}_c$: here, $\vec{a}_c$ is changing her direction
$\vec{v}$, $\vec{a}$: the overall $\vec{a}$ that does both is just $\vec{a}_T + \vec{a}_c$



speeding up



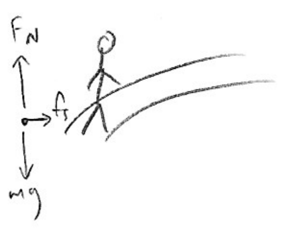
slowing down

Problem 3(c). If the runner runs the race at a constant speed and completes the 200 m dash in 23.2 s, what is her centripetal acceleration as she runs the curved portion of the track?

$$a_c = \frac{v^2}{r} = \frac{(8.6\,\text{m/s}^2)}{30\,\text{m}} = \boxed{2.48\,\text{m/s}^2}$$

Since v is constant, we know $v = \frac{200\text{m}}{23.2\text{s}} = 8.6\,\text{m/s}$

Problem 3(d). What force causes this acceleration?



The only force on the runner that points along the ground is $\boxed{\text{static friction}}$ between her feet and the ground