Universal gravitation

Physics 211
Syracuse University, Physics 211 Spring 2015
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Announcements

- Clinic hours today: 1:30-5:30
- Computational project 2 next week – *bring your laptops*
- Homework due tomorrow

The conical pendulum

I swing a conical pendulum of length L with angular velocity ω. What angle does the string make with the vertical?

A block on a ramp

A block of mass m_1 rests on a ramp at angle θ; a weight of mass m_2 hangs over the side of the ramp. The coefficient of kinetic friction is μ_k.

Calculate its acceleration if it:

- ... slides down the ramp (m_2 is small)
- ... is pulled back up the ramp (m_2 is large)

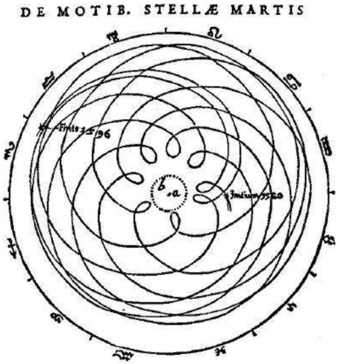
A new force: Gravity, in general

- On Earth all objects experience a gravitational force proportional to their mass:
- $F_{\mathrm{grav}} = mg$, directed down toward the Earth
    - How does this work when you're not on Earth?
    - What determines how big g is?

A brief history of gravity and the heavens

The history here is an interesting insight into the way scientific thought has evolved: "How can we explain the sky?"

- Stars in the sky all seem to move together, but with some "wanderers": planets
  - They appear to move in one direction, but sometimes stop and turn around



  - How can we explain this?

A brief history of gravity and the heavens

- Ptolemy: Things go in circles rotating on circles, because circles are perfect, with the Earth at the center
  - "Epicycles" required to make the retrograde motion
- Copernicus: Things go in circles rotating on circles, but with the Earth at the center
  - Relative motion between Earth and planets responsible for retrograde motion
- Brahe: Fantastic measurements of motions of the planets (even more epicycles); geoheliocentrism
- Kepler: Ellipses! No epicycles needed. Laws of planetary motion.
- Galileo: Kinematics; moons of Jupiter; phases of Venus
- Newton: Universal gravitation; dynamics

Newtonian gravity

- All objects – stars, planets, apples, people – exert forces on each other
- That force is given by

$$F_g = \frac{GMm}{r^2}$$

- Both objects feel the same force, directed toward each other
- Note:

$$a_g = F_g/m = \frac{GM}{r^2}$$

- What is G?
- $\rightarrow$ Fundamental constant of nature that tells us how strong gravity is

Clicker question

What are the units of G?

(Remember, it appears in the equation $F_g = \frac{GMm}{r^2}$)

- a) $\mathrm{m/s^2}$
- b) $\mathrm{m^2/s^2}$
- c) $\mathrm{N \cdot m^2/kg^2}$
- d) $\mathrm{m^3 kg^{-1} s^{-2}}$

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$$G = 6.673 \times 10^{-11} \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$$

  - This is really, really tiny

Measuring G

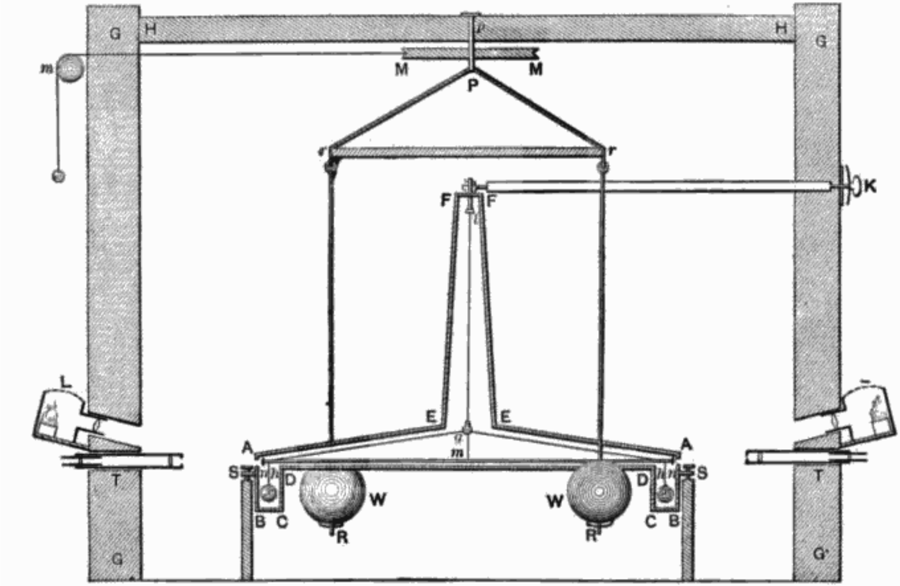


Fig. 1

Measuring G

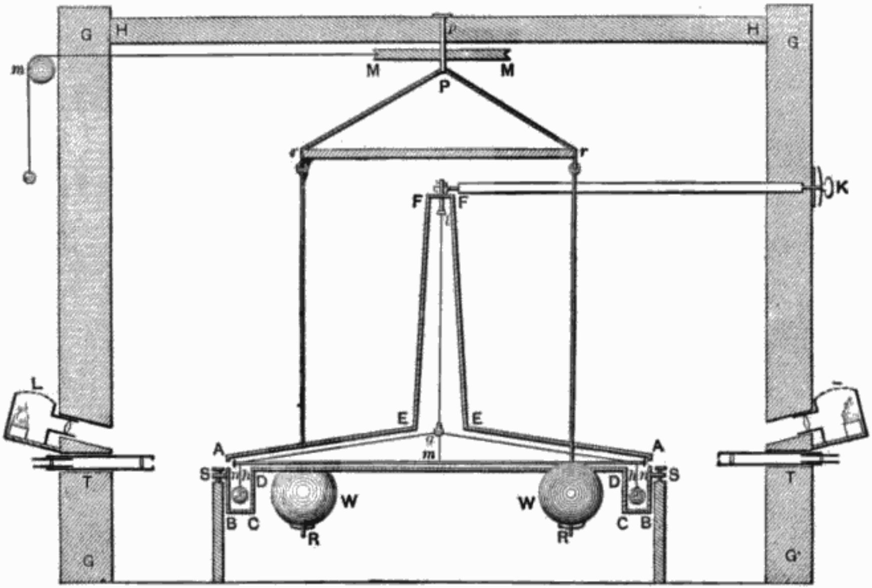
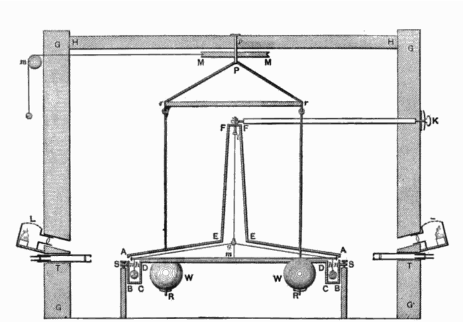


Fig. 1

What is the force between a 1kg mass and a 5kg mass that are 5cm apart?

Measuring G



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Back of the envelope math (in SI units):

$$F_g \approx \frac{(7\times10^{-11})(5)(1)}{5\times10^{-2}} = 7 \times 10^{-9}\ \text{N!}$$

Measuring the mass of the Earth

What is the mass of the Earth?

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We have two expressions for the gravitational force:

- $F_g = mg$, where g is an empirical measurement of Earth's gravity
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$$M = \frac{gR^2}{G} = 5.97 \times 10^{24} \text{ kg...}$$

Gravity and circular motion

- Many orbits are nearly circular
- Everything you learned on Tuesday about uniform circular motion still applies
- Weighing the Earth by looking at the Moon:
  - $F_g = \frac{GM_e M_m}{r^2} = M_m \omega^2 r$
- These problems are nothing new and nothing hard; it's just a new force

Kepler's law: relation of orbital period to radius

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$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

A note on Kepler's law

We saw earlier that

$$GM = \frac{4\pi^2 r^3}{T^2}$$

If we're studying the orbital dynamics of the Earth, then it makes sense to choose some different units for time and distance.

- Time: measure in years ($T = 1$ then)
- Distance: measure in AU ($r = 1$ then)

This means that $GM_{\rm sun} = 4\pi^2$; we will use this next week!