Momentum

Physics 211
Syracuse University, Physics 211 Spring 2023
Walter Freeman

March 29, 2023

Announcements

- Group Exam 3 tonight/tomorrow; Exam 3 next Tuesday
- **Review notes posted on the website**
- Review session in the auditorium Sunday, 1:30-4:30
- HW7 due Friday

Exam 3: topics

Exam 3 will be four questions. They will be:

- A question where you must use the work-energy theorem/conservation of energy to analyze a situation, and you will need to think carefully about the work and energy associated with the forces involved
  - Examples: "tire swing problem" from last week's second recitation; "rocket sled" problem from HW7
  - You may need to draw force diagrams to determine a normal force to find friction
  - You will need to evaluate the dot product $\vec{F} \cdot \Delta\vec{s}$
- A question where you must use some combination of the conservation of momentum, work and energy, and kinematics to analyze a situation
  - Examples: "arrow and target" problem from recitation this week; "slingshot problem" from HW7
  - You will need to draw a series of cartoons representing the critical moments in the motion
  - You will need to think carefully about which technique applies to which stage of the motion
  - Each stage itself will not be that complex

Exam 3: topics

Exam 3 will be four questions. They will be:

- A question where you will need to apply conservation of momentum in *two* dimensions, possibly alongside one other technique
  - Examples: “car crash problem” from recitation; “jumping astronaut” problem from HW5
  - Remember momentum is a vector
  - Be careful with subscripts (this object and that, x and y, initial and final)

- A question where you will need to think about work, power, and energy, and the conversion of energy from one form to another.
  - This question will *not* require any complicated algebra
  - It *will* require you to think clearly about units and be careful in dimensional analysis/units
  - Examples: “mountain climber” problem from last week’s recitation, “submarine problem” and “trucker problem” from HW7

Recitation or homework questions?

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Exam 3 Review

This unit is about *conservation laws*.

We have met three of them: *conservation of momentum*, its cousin *conservation of angular momentum*, and the *work-energy theorem*.

These techniques let you analyze systems where you know something about “before” and “after” states.

General problem-solving techniques:

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Review: energy methods

Use energy methods when:

- You have clear before and after states (draw your cartoons, dammit!)
- You can calculate the work done by the forces between them **(not collisions/explosions!)**
- You don't care about time

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- You can calculate the work done by the forces between them **(not collisions/explosions!)**
- You don't care about time
- *Be careful with projectile motion!*
  - Energy methods can tell you "how fast" or "how high"
  - They cannot tell you "where does it land?"

Work-energy theorem: $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \vec{F} \cdot \vec{d} = Fd\cos\theta$ (if this is constant)

Potential energy is an alternate way of keeping track of the work done by conservative forces:

- $PE_{\text{grav}} = mgh$
- $PE_{\text{spring}} = \frac{1}{2}kx^2$

Review: Conservation of energy

$$\text{PE}_\text{i} \quad + \quad \frac{1}{2}mv_i^2 \quad + \quad W_\text{other} \quad = \quad \text{PE}_\text{f} \quad + \quad \frac{1}{2}mv_f^2$$

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(initial PE) + (initial KE) + (other work) = (final PE) + (final KE)

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(total initial mechanical energy) + (other work) = (total final mechanical energy)

Review: Conservation of energy

$$\mathrm{PE_i} + \tfrac{1}{2}mv_i^2 + W_{\text{other}} = \mathrm{PE_f} + \tfrac{1}{2}mv_f^2$$

(initial PE) + (initial KE) + (other work) = (final PE) + (final KE)

(total initial mechanical energy) + (other work) = (total final mechanical energy)

Since conservation of energy is the broadest principle in science, it's no surprise that we can do this!

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- You have an explosion (one object separates into two)
- Two objects exchange forces *only with each other*

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(that's it – the last one doesn't come up much, but sometimes it does: why does the Earth wobble?)

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To use conservation of momentum:

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- $\sum \vec{p}_i = \sum \vec{p}_f$

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To use conservation of momentum:

- Make sure you have your clear before/after cartoons
- $\sum \vec{p}_i = \sum \vec{p}_f$
- **Momentum is a vector!**
- If you have motion in two dimensions, momentum in each direction is conserved separately:

$$\sum p_{x_i} = \sum p_{x_f} \qquad \sum p_{y_i} = \sum p_{y_f}$$

Do not get lazy with your subscripts!

Example 1: determining work done in a nuanced situation

I will use the “tire swing problem” in recitation as an example here. Here’s the problem for those reading the notes:

A pendulum consisting of a string of length L and a mass m at the end is pulled back to an angle θ and released. A strong wind blows horizontally, applying a constant force F_w to the pendulum bob. What will the pendulum’s speed be at the bottom? How can you find the angle will it reach on the other side? Will it come back to where it started?

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- Draw clear diagrams
- Write the work-energy theorem down, taking into account each force either as a force whose work you calculate explicitly or as a force associated with a potential energy
- Think carefully about the work done by each force
- Solve for what you need

Example 2: combining momentum, energy, and kinematics

An object of mass m sits on top of a ramp of total height h. That ramp is itself on a table of height H. It slides down the ramp and collides with and sticks to another object of mass m. They slide off of the table and onto the floor.

How fast are they going when they hit the floor?

Where do they land?

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Where do they land?

- Draw clear cartoons of each critical moment in the motion (more than two!)
- Label the things you know and you might want to find in each
- Determine the physical principle (tool) that you can use to analyze the progression from each cartoon to the next
- Solve each equation and substitute it into the next one

Example 3: Momentum in 2D, possibly in combination

A person of mass 50 kg is ice-skating on a frozen lake with his dog Kibeth, who has a mass of 15 kg. He is skating due north at 3 m/s. Kibeth realizes that he's carrying snacks in his pocket, and would like one for herself. (Or maybe she is just being friendly!) She runs after him and tackles him from behind and the side, knocking him down. The two of them collapse on the ice and begin to slide, as Kibeth tries to get the treats out of his pocket; they are moving at an angle 20 degrees west of north at 4 m/s.

What was Kibeth's velocity before she tackled him?

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What was Kibeth's velocity before she tackled him?

- Remember momentum is a *vector*
- Treat x and y components separately
- Decompose vectors into components and remember your trigonometry

Example 4: Work, energy, and power

Imagine a sled dog with a mass of $m_d = 50$ kg pulling a sled with a mass of $m_s = 40$ kg. Sled dogs are pretty sturdy creatures; we'll estimate that our dog can sustain an average power of $P = 100$ W.

1. If the coefficient of kinetic friction between the sled's runners and the ground is 0.1, how fast can the dog pull the sled?
2. Suppose that the dog and sled encounter a hill with an upward slope of $\theta = 3°$. How fast can the dog pull the sled up the hill?
3. Suppose that over a long trip the dog pulls the sled for eight hours each day, and that his muscles are 10% efficient at converting the potential energy in food into mechanical work. If dog food has an energy content of 10 kJ per gram, how much food must he eat in order to maintain his body weight?

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- Keep track of units for everything!
- Remember $P = \vec{F} \cdot v$ – power is the size of a force times the velocity in the direction of that force
- Doing calculations with symbols at first will help you ask "does this make sense?"