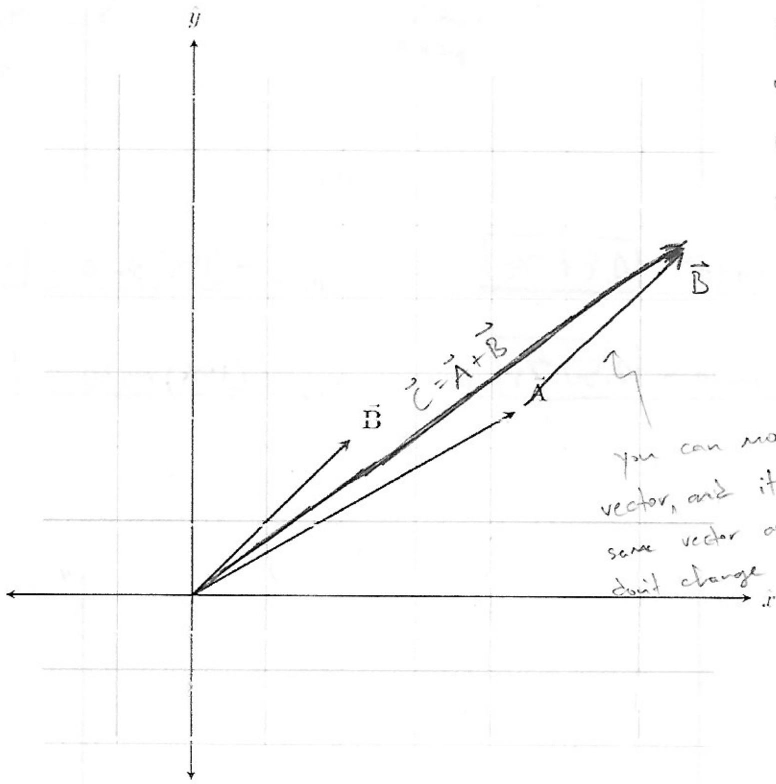
PHY 211 Recitation 4

January 24, 2020

1 Adding vectors

Add the following two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ graphically (assume each grid spacing is 1 m long).

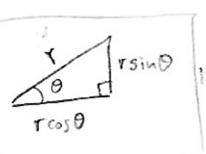


When $\vec{A}$ and $\vec{B}$ are lined up like two legs of a hike (tip-to-tail) the vector that points from the very initial point to the very final point is $\vec{A}+\vec{B}$

you can move a vector, and it stays the same vector as long as you don't change its magnitude (size) or direction.

Estimate the components of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ from your picture.

$C_x =$ 6.5 m $\qquad C_y =$ 5



The vector $\vec{\mathbf{A}}$ has magnitude 5, and angle 30° from the x-axis. What are its components?

$A_x =$ $(5m)\cos 30° = \boxed{4.3m}$ $\qquad A_y =$ $(5m)\sin 30° = \boxed{2.5\,m}$

The vector $\vec{\mathbf{B}}$ has magnitude 3, and angle 45° from the x-axis. What are its components?

$B_x =$ $(3m)\cos 45° = \boxed{2.1m}$ $\qquad B_y =$ $(3m)\sin 45° = \boxed{2.1m}$

This agrees with $\sin\theta = \frac{opp}{hyp}$, since $\sin\theta = \frac{r\sin\theta}{r}$. Likewise with $\cos\theta$

Find the components of $\vec{\mathbf{C}}$ by adding the components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

$C_x =$ $4.3m + 2.1m = \boxed{6.4m}$ $\qquad C_y =$ $2.5m + 2.1m = \boxed{4.6\,m}$

How does your answer compare to what you found graphically?

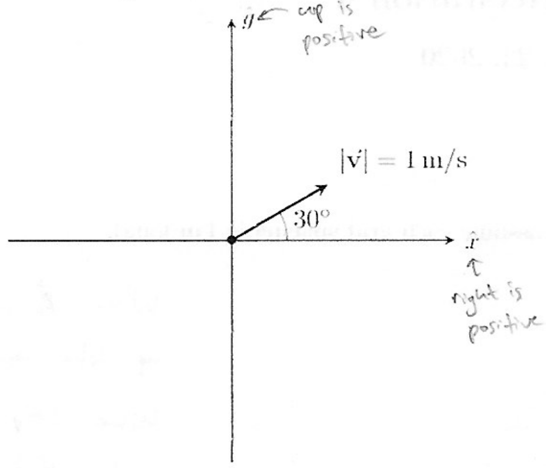
The numbers are similar, so we know we haven't made any sign mistakes. But the trig calculation is more precise

First, use cos for adjacent sides and sin for opposite sides (relative to the given angle)

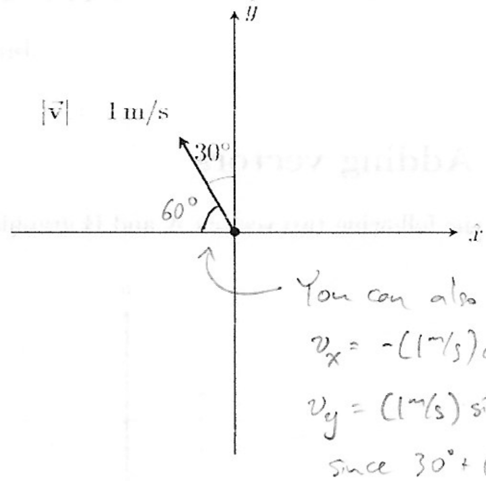
Check if you need to fix the sign of your components last

2 Vector components

Calculate the components for each vector below. Write out the calculation in terms of sin or cos.



(a)



(b)

You can also use $v_x = -(1\,\text{m/s})\cos 60°$, $v_y = (1\,\text{m/s})\sin 60°$, since $30° + 60° = 90°$.

(a)

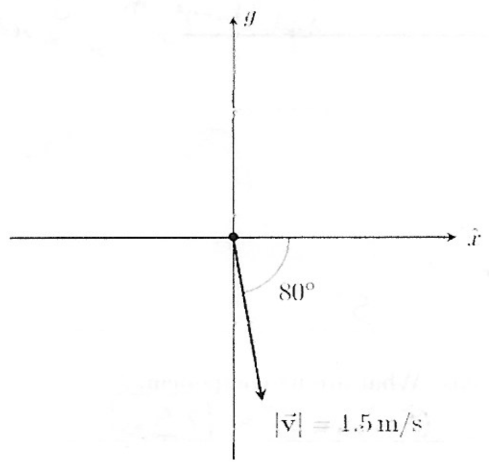
$v_x =$ $(1\,\text{m/s})\cos 30° = \boxed{0.87\,\text{m/s}}$

$v_y =$ $(1\,\text{m/s})\sin 30° = \boxed{0.50\,\text{m/s}}$

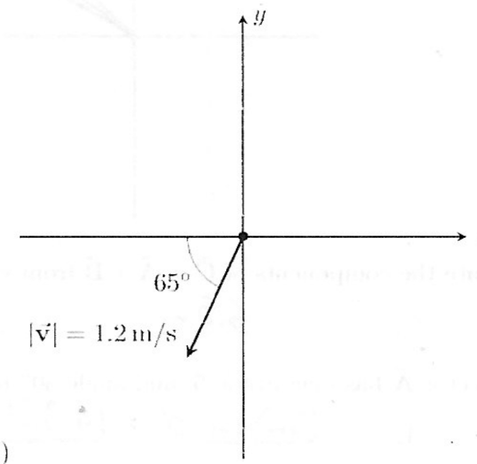
(b)

$v_x =$ $-(1\,\text{m/s})\sin 30° = \boxed{-0.50\,\text{m/s}}$

$v_y =$ $(1\,\text{m/s})\cos 30° = \boxed{0.87\,\text{m/s}}$



(a)



(b)

(a)

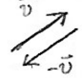
$v_x =$ $(1.5\,\text{m/s})\cos 80° = \boxed{0.26\,\text{m/s}}$

$v_y =$ $-(1.5\,\text{m/s})\sin 80° = \boxed{-1.48\,\text{m/s}}$

(b)

$v_x =$ $-(1.2\,\text{m/s})\cos 65° = \boxed{-0.51\,\text{m/s}}$

$v_y =$ $-(1.2\,\text{m/s})\sin 65° = \boxed{-1.09\,\text{m/s}}$

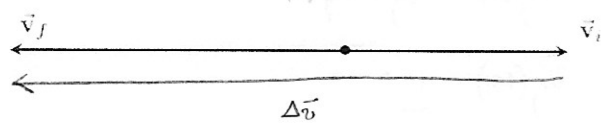


For any vector $\vec{v}$,
the vector $-\vec{v}$ is equally long
and points in the opposite direction,
so that $\vec{v} + (-\vec{v}) = 0$.

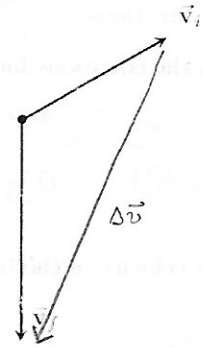
3 Vector differences

A lot of physics involve the *change* in some vector (displacement, velocity, etc.). Find graphically $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$ for the following velocities:

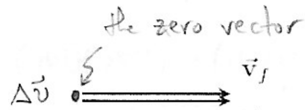
You can think of $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$ as $\vec{v}_f + (-\vec{v}_i)$,
or as the vector that changes $\vec{v}_i$ into $\vec{v}_f$,
since $\Delta\vec{v} + \vec{v}_i = \vec{v}_f$.



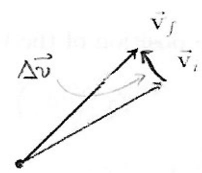
(a)



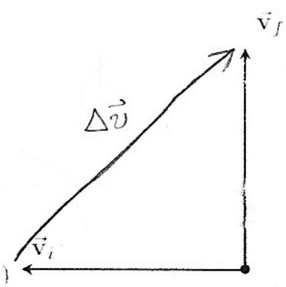
(b)



(c)



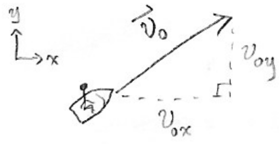
(d)



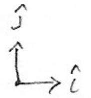
(e)

(f)

Make sure you can clearly
see which way your vector
is pointing



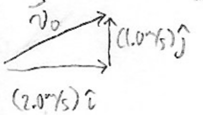
Remember, v_0 doesn't go directly into 2-D motion equations, only v_{ox} and v_{oy}, the components of the vector $\vec{v}_0$.



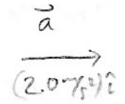
4 Motion in 2D

A vector is essentially an arrow, but a component is just a number (with units)

A boat leaves the dock at $t = 0$ and heads out into a river with an acceleration of $2.0\,\text{m/s}^2\hat{\mathbf{i}}$. The current gives it an initial velocity of $\underbrace{2.0\,\text{m/s}\hat{\mathbf{i}} + 1.0\,\text{m/s}\hat{\mathbf{j}}}_{\text{This is the vector } \vec{v}_0\text{, since } \hat{i} \text{ and } \hat{j} \text{ are vectors.}}$. To describe the motion in 2D, you need equations of motion for both x and y directions.



We see that
$v_{ox} = 2.0\,\text{m/s}$
$v_{oy} = 1.0\,\text{m/s}$



$a_x = 2.0\,\text{m/s}^2$
$a_y = 0$ ← a horizontal vector has 0 for its vertical component

(a) Write down the equations for the x and y directions.

$x(t) = x_0 + v_{ox}t + \frac{1}{2}a_x t^2,$ $\boxed{x(t) = (2\,\text{m/s})t + \frac{1}{2}(2\,\text{m/s}^2)t^2}$

$y(t) = y_0 + v_{oy}t + \frac{1}{2}a_y t^2,$ $\boxed{y(t) = (1\,\text{m/s})t}$

(b) What is the velocity of the boat at $t = 10\,\text{s}$?

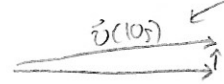
$v_x(t) = v_{ox} + a_x t,$

$v_x(10\,\text{s}) = 2\,\text{m/s} + (2\,\text{m/s}^2)(10\,\text{s})$
$= \underline{22\,\text{m/s}}$

$v_y(t) = v_{oy} + a_y t$

$v_y(10\,\text{s}) = \underline{1\,\text{m/s}}$

$\boxed{\vec{v}(10\,\text{s}) = (22\,\text{m/s})\hat{i} + (1\,\text{m/s})\hat{j}}$



pretty close to horizontal

(c) What is the position of the boat at $t = 10\,\text{s}$?

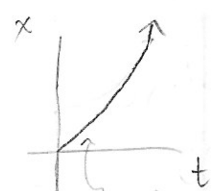
$x(10\,\text{s}) = (2\,\text{m/s})(10\,\text{s}) + \frac{1}{2}(2\,\text{m/s}^2)(10\,\text{s})^2$
$= \underline{120\,\text{m}}$

$y(10\,\text{s}) = (1\,\text{m/s})(10\,\text{s})$
$= \underline{10\,\text{m}}$

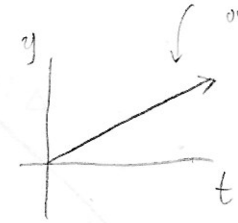
120 m to the right of and 10 m up from the dock

(d) Draw a sketch of the trajectory of the boat in x and y as a function of time.
← no numbers needed

boat's trajectory in space



the slope of x starts greater than zero, then increases as v_x increases



no acceleration occurs in the y-direction

2-D motion is just like two independent copies of 1-D motion, one in the x-direction and one in the y-direction