Recitation Questions

Week of 29 February

(The first two problems are the same as last week; if you didn't finish them then, do them now.)

Question 1 – apparent weight and "falling"

A person uses a rope to spin a bucket in a vertical circle at a constant speed; the radius of the circle is 80 cm. The bucket goes around the circle once every second. Inside the bucket is a friendly frog, happy to be out of the elevator, of mass 500g.

a) Draw a force diagram for the frog when the bucket is at the top of the circle, and when it is at the bottom.

b) What is the acceleration of the bucket?

c) As you saw last week in your homework, your "apparent weight" is simply the magnitude of the normal force that an object under you exerts on you. What is the frog's apparent weight at the bottom and at the top of the circle?

d) Explain why the frog doesn't fall out of the bucket at the top of the swing, despite the fact that the only forces acting on it point downward.

e) Now, imagine that the person swinging the bucket slows down gradually. At some point, the frog will fall out of the bucket. (It's a frog, so it'll land on its feet and not be hurt!) How low can ω become before the frog falls out of the bucket?

Question 2 – centripetal force and road design

A highway curve has a radius of curvature of 500 meters; that is, it is a segment of a circle whose radius is 500 m. It is banked so that traffic moving at 30 m/s can travel around the curve without needing any help from friction.

a) Draw a force diagram for a car traveling around this curve at a constant speed. Draw the diagram so that you are looking at the rear of the car. *Hint:* Do not tilt your coordinate axes for this problem!

b) What is the acceleration of the car in the $x-$direction? What about the $y-$direction?

c) Write down two copies of Newton's second law in the $x-$ and $y-$directions.

d) Solve the resulting system of two equations to determine the banking angle of the curve.

Question 3 – variation of apparent weight with latitude

For this problem, use $g = 9.810\text{m/s}^2$, and carry all calculations to five significant digits.

a) Assume that the Earth is a sphere of radius 6400 km. What is the acceleration of a person standing on the Equator?

b) Suppose a person with a mass of 100 kg stands on a scale. (Remember, scales measure the normal force that they exert.) What will the scale read at the South Pole?

c) What will the scale read at the Equator?

d) This problem shows that your apparent weight depends on your location on Earth. Does it make sense to define g as F_g/m (the strength of the gravitational force divided by an object's mass) or F_N/m (the strength of the normal force, and thus the scale reading, divided by mass)?

Question 4 – universal gravitation and the Sun's mass

In this problem, you will compute the mass of the Sun. The Earth's orbit is very nearly circular, and the earth is 150 million km from the Sun.

a) What is the angular velocity of the Earth in its orbit?

b) What is the tangential velocity of the Earth? c) What is the radial acceleration of the Earth?

d) What is the mass of the Sun?

Question 5 – orbits

Astronauts in orbit around the Earth are *not* "so far away that they don't feel Earth's gravity"; actually, they're quite close to the surface. However, we've all seen the videos of astronauts drifting around "weightlessly" in the International Space Station.

a) Explain how an astronaut can be under the influence of Earth's gravity, and yet exert no normal force on the surface of the spacecraft she is standing in.

b) Draw a force diagram for the astronaut floating in the middle of the Space Station, not touching any of the walls or floor. How do you reconcile your diagram with the fact that the astronaut doesn't seem to fall?

c) Is this astronaut "weightless"? This question has two different answers, and which one you give depends on how you define the word "weight". Explain.

Question 6 – geostationary orbit

It is sometimes useful to place satellites in orbit so that they stay in a fixed position relative to the Earth; that is, their orbits are synchronized with the Earth's rotation so that a satellite might stay above the same point on Earth's surface all the time.

What is the altitude of such an orbit? Note that it is high enough that you need to use $F_g = \frac{GMm}{r^2}$ rather than just $F_g = mg$.