Energy methods – problem solving

Physics 211
Syracuse University, Physics 211 Spring 2015
Walter Freeman

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Announcements

- If your exam was misgraded, grade appeals will be handled the same way as before (and faster!); turn them in to me by the end of the week

Momentum

- Gas goes backwards; car goes forwards!
- Momentum is another conserved quantity, sort of like energy
  - Momentum is a **vector**, transferred from one object to another when they exchange forces
  - Here: gas gains momentum in $+\hat{x}$, car gains momentum in $-\hat{x}$
  - Each component conserved separately and independently
  - Mathematically: $\vec{p} = m\vec{v}$
- Momentum is harder to lose track of, as it's only associated with motion
- Helps us understand **collisions** and **explosions**, among others
- Tied in with the idea of **impulse**

Impulse

- We start, as always, with Newton's law:

$$\vec{F} = m\vec{a}$$

- Integrate both sides of this with respect to time:

$$\int \vec{F}\,dt = \int m\vec{a}\,dt$$
$$\int \vec{F}\,dt = (m\vec{v})_f - (m\vec{v})_i$$

- The quantity on the left, $\vec{J} \equiv \int \vec{F}\,dt$, is called "impulse"
  - It represents the cumulative effect of a force over time, much as work gives the cumulative effect of a force over distance
  - "Forces make things accelerate" $\rightarrow$ "A force applied over time makes something change velocity"
  - If the force is constant, then the integral is easy, and $\vec{J} = \vec{F}t$
- The quantity on the right, $\vec{p} \equiv m\vec{v}$, is called "momentum"

Impulse is equal to the change in momentum: $\vec{J} = \Delta\vec{p}$

Solving problems with this: the impulse-momentum theorem

This idea is so intuitive I already gave you a problem on it!

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... the "baseball/bat" problem on Practice Exam 2!

A baseball is pitched toward a batter. The batter hits it, knocking the ball back over the pitchers head. Just before the batter hits it, it is traveling at 40 m/s parallel to the ground. After the batter hits it, it is traveling at 60 m/s at an angle 30 degrees above the horizontal. The bat is in contact with the ball for 10 milliseconds; assume that the bat exerts a constant force on the ball during this time. If the ball has a mass of 145 grams, what is the size of this force, and in what direction does it point?

Conservation of momentum

- Newton's third law: if A pushes on B, B pushes back on A with an equal and opposite force
- In symbols, $\vec{F}_{AB} = -\vec{F}_{BA}$
- We can integrate both sides of this to get a statement about impulse: $\vec{J}_{AB} = -\vec{J}_{BA}$
- Using the impulse-momentum theorem: $\Delta\vec{p}_A = -\Delta\vec{p}_B \rightarrow \Delta(\vec{p}_A + \vec{p}_A) = 0$
- The total change in momentum is zero!
- The force between A and B leaves the total momentum constant; it just gets transferred from one to the other
- **Remember momentum is a vector!**
- As with conservation of energy, need "before" and "after" snapshots
- Just add up the momentum before and after and set it equal!

Collisions and explosions

Often things collide or explode; we need to be able to understand this.

- Very complicated forces between pieces often involved: can't track them all
- These forces are huge but short-lived, delivering their impulse very quickly
- Other forces usually small enough to not matter during the collision/explosion
- Use conservation of momentum to understand the collision
- If we know nothing else, we often don't have enough equations to solve for our unknowns...
- One dimension: one equation, often two unknowns
  - $m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$

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- Two dimensions: two equations, often four unknowns
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- Often things stick together ("perfectly inelastic collision"): $\vec{v}_{1,f} = \vec{v}_{2,f}$
- This reduces the number of unknowns to one (1D) or two (2D)

Demo with carts

Can we predict the final velocities here?

Demo: molecular dynamics

Conservation of momentum for many ping-pong balls in a box

Demo: molecular dynamics

Conservation of momentum for many ping-pong balls in a box
Conservation of momentum for a bullet hitting a blob

Sample problems: a 1D collision

Two train cars moving toward each other at 5 m/s collide and couple together. One weighs 10 tons; the other weighs 20 tons. What is their final velocity?

Sample problems: a 1D collision

A train car with a mass m is at rest on a track. Another train car also of mass m is moving toward it with a velocity v_0 when it is a distance d away. The first car hits the second and couples to it; the cars roll together until friction brings them to a stop.

If the coefficient of rolling friction is μ_r, how far do they roll after the collision?

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Method: use conservation of momentum to understand the collision; use other methods to understand before and after!