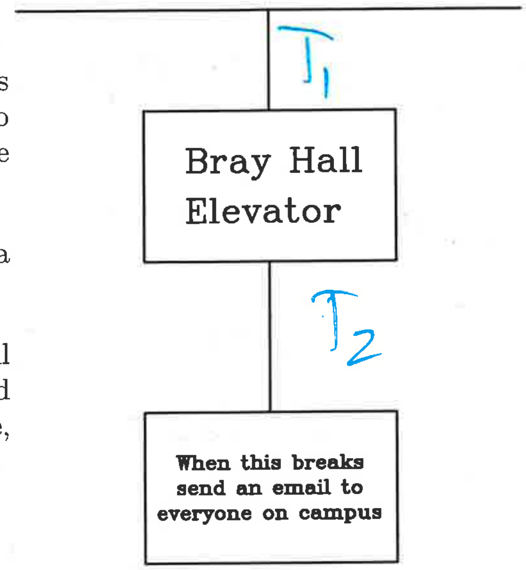
Physics 211 Practice Exam 2
Question 1



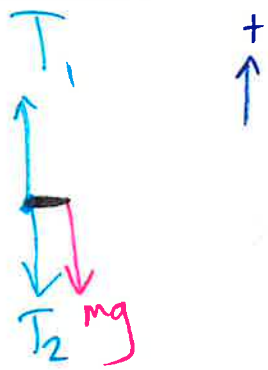
An elevator in Bray Hall has two signs that hang from strings from the ceiling, as shown here. The top sign is attached to the ceiling with a string; the bottom sign is attached to the top sign with a string.

Each sign has a mass of 2 kg, and the strings can apply a maximum tension force of 50 N before they break.

A person enters this elevator on the top floor of Bray Hall and rides it downward. The elevator accelerates downward at 3 m/s^2, moves at a constant velocity for a little while, then accelerates upward at 3 m/s^2 until it comes to a stop.

a) Draw force diagrams for each sign while the elevator is moving at a constant velocity. Will either string break during this stage of the motion? (*5 points*)

Top:



Bot:

$\Sigma F = ma \Rightarrow T_1 - T_2 - mg = ma$

$\Sigma F = ma$: $T_2 - mg = ma$

$\rightarrow \boxed{T_2 = mg + ma}$

$\rightarrow T_1 - \underbrace{(mg + ma)}_{T_2} - mg = ma$

$\boxed{T_1 = 2mg + 2ma}$

For $a = 0$:

$T_1 = 2mg = 40\ N$

$T_2 = mg = 20\ N$

$\rightarrow$ neither breaks

b) Draw force diagrams for each sign while the elevator is accelerating downward. Will either string break during this stage of the motion? *(10 points)*

Here $\vec{a} = 3\ m/s^2$ downward (or $-3\ m/s^2$)

→ use prior results:

$T_2 = mg + ma = 2kg\,(10\ m/s^2 - 3\ m/s^2) = 14\ N$

$T_1 = 2mg + 2ma = 28\ N$

→ neither breaks

c) Draw force diagrams for each sign while the elevator is accelerating upward. Will either string break during this stage of the motion? *(10 points)*

Here $\vec{a} = +3\ m/s^2$ (upward)

Use results from (a) again!

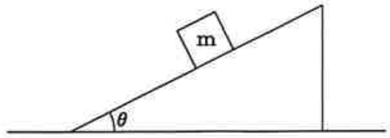
$T_2 = mg + ma = 2kg\,(10\ m/s^2 + 3\ m/s^2) = 26\ N$

$T_1 = 2mg + 2ma = 52\ N$!

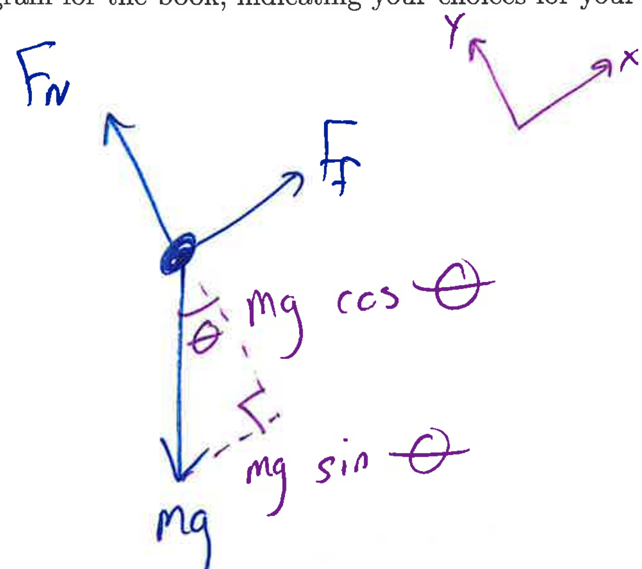
→ top string breaks!

QUESTION 2

A book of mass m rests on a slope with angle of inclination θ as shown below. There is friction between the book and the slope.



a) Draw a force diagram for the book, indicating your choices for your coordinate axes. *(5 points)*



b) Suppose first that the book does not slide down the slope. In terms of θ, m, and g, compute the minimum value of μ_s required to make the book stay on the ramp without sliding. (Your answer may not depend on all three of these quantities.) *(10 points)*

Y: $F_N - mg\cos\theta = ma_y = 0 \rightarrow F_N = mg\cos\theta$

X: $\mu_s F_N - mg\sin\theta = ma_x = 0$

$\rightarrow \mu_s mg\cos\theta - mg\sin\theta = 0$

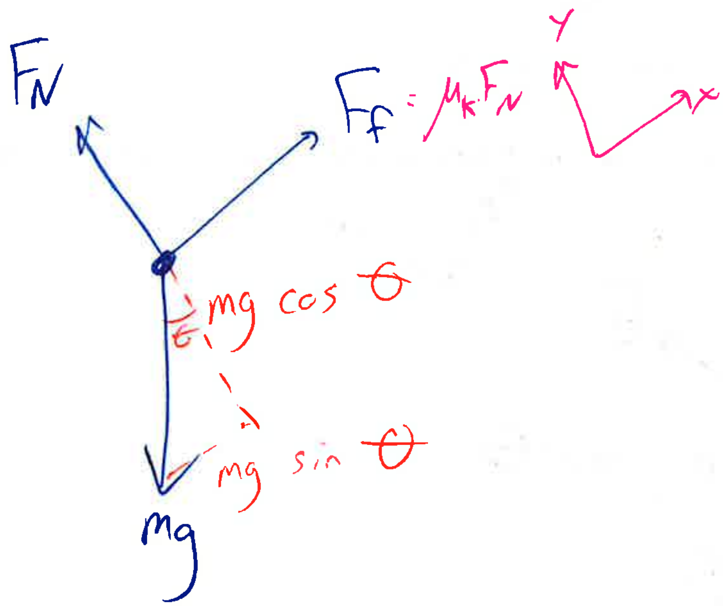
$\mu_s \cos\theta = \sin\theta$

$\rightarrow \mu_s = \tan\theta$

friction points up

Question 2, continued

c) Now, suppose that μ_s is less than this value, and the book slides down the slope. The coefficient of kinetic friction is μ_k. In terms of θ, μ_k, m, and g, compute the acceleration with which the book slides down the ramp. (Your answer may not depend on all four quantities.) *(10 points)*



$$X: \mu_k F_N - mg\sin\theta = ma_x$$

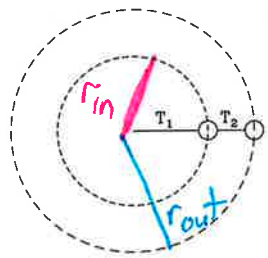
$$Y: F_N - mg\cos\theta = ma_y = 0 \rightarrow F_N = mg\cos\theta$$

$$\rightarrow \mu_k mg\cos\theta - mg\sin\theta = ma_x$$

$$\rightarrow \boxed{a_x = \mu_k g\cos\theta - g\sin\theta}$$

Question 3

Two masses of $m = 2kg$ each lie on a frictionless table, connected by a string $L_1 = 50$ cm long. Another string, $L_2 = 80$ cm long, runs from one of those masses to an attachment on the table. The string is stretched out and both masses are made to revolve around the point of attachment, making one complete circle every 3 s. The diagram below shows a top-down view of the apparatus; the dotted circles show the paths of the two masses. The masses slide on top of the surface without friction.

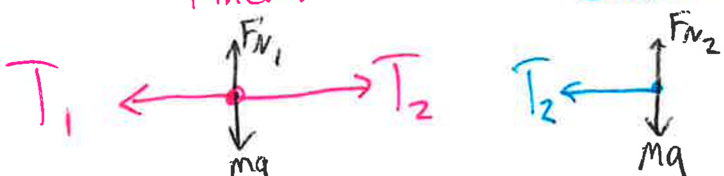


$r_{in} = L_1 = 50$ cm

$r_{out} = L_1 + L_2 = 130$ cm

a) Draw a force diagram for each mass, indicating your choice of sign for the coordinate axes. *(5 points)*

Inner: Outer: ($\longrightarrow +x$) [note $\vec{a}$ is in $-x$ direction!]



(note Y-direction is boring here)

b) What is the angular velocity of the inner mass? What is the angular velocity of the outer mass? *(2 points)*

$$\omega \text{ for both} = \frac{1 \text{ rev}}{3 \text{ sec}} = \frac{2\pi}{3} \frac{\text{rad}}{\text{sec.}}$$

c) What is the tangential velocity of the inner mass? What is the tangential velocity of the outer mass? *(3 points)*

$V_T = \omega r$, so

$$V_{T,\text{inner}} = \frac{2\pi}{3} \frac{\text{rad}}{\text{sec}} \times 50 \text{ cm} = 1.05 \text{ m/s}$$

$$V_{T,\text{outer}} = \frac{2\pi}{3} \frac{\text{rad}}{\text{sec}} \times 130 \text{ cm} = 2.72 \text{ m/s}$$

d) Find the two tensions T_1 and T_2. (15 points)

$\Sigma F = ma$ for both:

Outer: $-T_2 = -m\omega^2 r_{outer}$

$T_2 = m\omega^2 r_{outer} = 11.4$ N

Inner: $T_2 - T_1 = -m\omega^2 r_{inner}$

Substitute T_2:

$T_1 = T_2 + m\omega^2 r_{inner} = 15.8$ N

Question 4

A person is standing in a subway car, looking forward. She is not holding onto anything, trusting the friction between her shoes and the ground to keep her balance.

Draw force diagrams for the following situations. Make sure you indicate which direction is which (i.e. tell me whether I am looking at the person from above, from the side, etc., and which direction is toward the front of the subway car.) Indicate the relative sizes of the forces by the lengths of the arrows in your force diagram. Forces that have the same magnitude should have the same size arrows; if you think it's not clear, you can write a little text telling me which forces are larger, smaller, or equal.

a) The subway car is moving forward at a $\underbrace{\text{constant velocity } \vec{v}.}_{\vec{a}=0}$ *(3 points)*

View from rear:

(equal magnitudes)

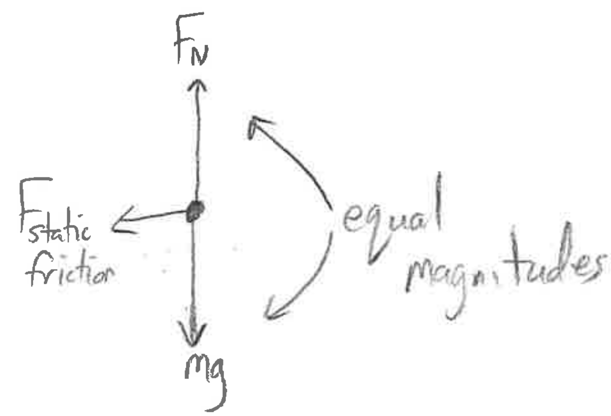
b) The subway car is going over the top of a hill, and is accelerating straight downward at 3 m/s^2. *(5 points)*

View from rear:

larger

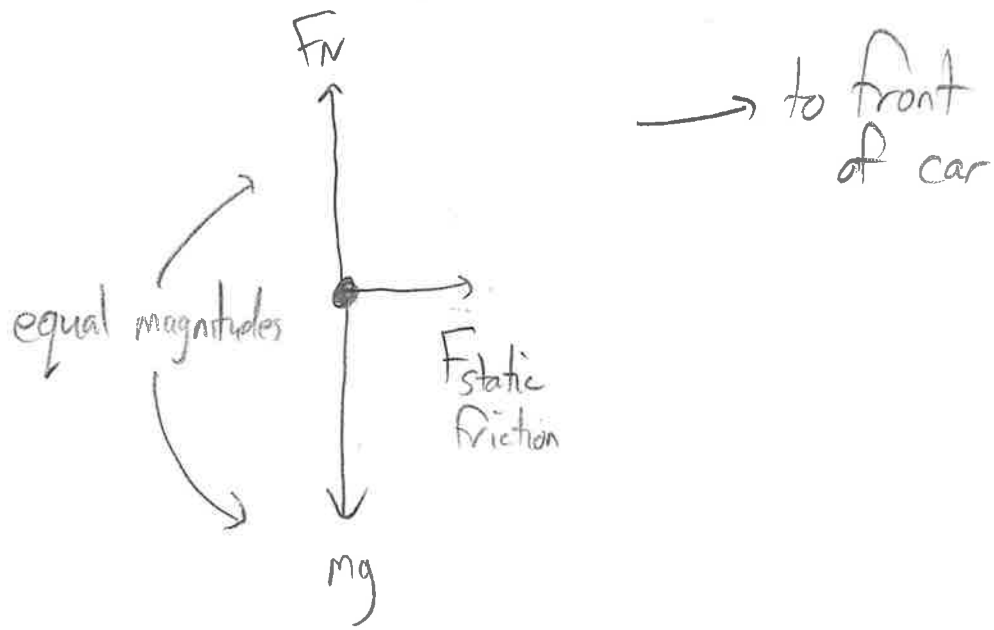
c) The subway car is moving at a constant speed v; it is turning left, gently enough that the passengers do not slip and fall. *(5 points)*

View from rear:



d) The subway car is accelerating forward at $a = 3\,\mathrm{m/s^2}$. *(5 points)*

View from side:



QUESTION 4, CONTINUED

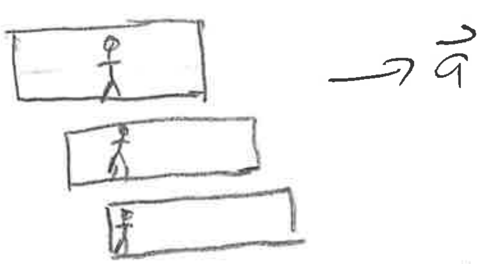
e) Anyone who has ridden a subway car feels themselves "thrown backwards" when it accelerates forward. What force is pushing them backwards? (If there is no such force, then explain why they feel themselves thrown backwards when the car accelerates.) *(8 points)*

There is no force pushing them backwards.

Instead, the car is accelerating forwards.
This means that either:

- Some force (like friction from the ground, holding on to something, etc.) must push the passengers forward so they accelerate along with the car, or;

- The car will accelerate forward without the passengers, so they will move backward <u>relative to it</u>:



They are not being thrown backwards - instead everything around them is accelerating forward.

QUESTION 5

The force of gravity of a person standing on the surface of the Moon is about one-sixth of its value on the surface of the Earth. Likewise, the radius of the Moon r_M is exactly one-quarter that of the Earth.

As a reminder, Newton's law of universal gravitation says that

$$F_g = \frac{Gm_1m_2}{r^2}$$

where r is the distance between the centers of the two gravitating objects.

a) As a fraction of the mass of the Earth, what is the mass of the Moon? (Note: You do not need numerical values for the masses and radii of the Earth and Moon, and you should not need a calculator for this problem.) *(10 points)*

We know:

- $g_{moon} = \frac{1}{6} g_{Earth}$
- $r_{moon} = \frac{1}{4} r_{Earth}$

→ What about m_{moon}?

$M_{moon} = (?)\, M_{Earth}$

→ Moon's gravity is proportional to $\frac{1}{r^2}$

→ Making the Moon $\frac{1}{4}$ as large makes its gravity 16 times stronger

[effect of mass] × [effect of radius] $= \frac{1}{6}$

[effect of mass] × [16] $= \frac{1}{6}$

[effect of mass] $= \frac{1}{96}$

→ $\boxed{M_{moon} = \frac{1}{96} M_{earth}}$

b) The Moon orbits the Earth with a nearly constant speed in a nearly circular orbit. Is the Moon accelerating? If so, how do you know? (5 points)

Circular motion around Earth

→ accelerating inward at

$a = \omega^2 r$.

c) The Moon orbits the Earth with angular velocity ω. In terms of the mass of the Earth m_E, the gravitational constant G, and ω, what is the distance r from the Earth to the Moon? (10 points)

Only force on Moon = Earth's gravity

$F = ma$

$$\frac{G m_{earth} \cancel{m_{moon}}}{r^2} = \cancel{m_{moon}} \omega^2 r$$

$G m_E = \omega^2 r^3$

→ $\boxed{r = \sqrt[3]{\frac{G m_E}{\omega^2}}}$

#5a alternate)

$$\frac{g_{moon}}{g_{earth}} = \frac{1}{6}. \qquad g = \frac{F_g}{m_{thing}} = \frac{\frac{G \cancel{m_{thing}} m_{planet}}{r_{planet}^2}}{\cancel{m_{thing}}}$$

$$\rightarrow g = \frac{G m_{planet}}{r_{planet}^2}$$

$$\frac{g_{moon}}{g_{earth}} = \frac{\frac{\cancel{G} m_{moon}}{r_{moon}^2}}{\frac{\cancel{G} m_{earth}}{r_{earth}^2}}$$

Separate out "mass ratio" and "radius ratio"

$$\frac{g_{moon}}{g_{earth}} = \left[\frac{m_{moon}}{m_{earth}}\right] \times \left[\frac{r_{earth}^2}{r_{moon}^2}\right]$$

$$\frac{g_{moon}}{g_{earth}} = \left[\frac{m_{moon}}{m_{earth}}\right] \times \left[\frac{r_{earth}}{r_{moon}}\right]^2$$

$$\frac{1}{6} = \left[\frac{m_{moon}}{m_{earth}}\right] \times [4]^2$$

$$\rightarrow \frac{m_{moon}}{m_{earth}} = \frac{1}{6} \times \left(\frac{1}{4}\right)^2 = \frac{1}{96}$$

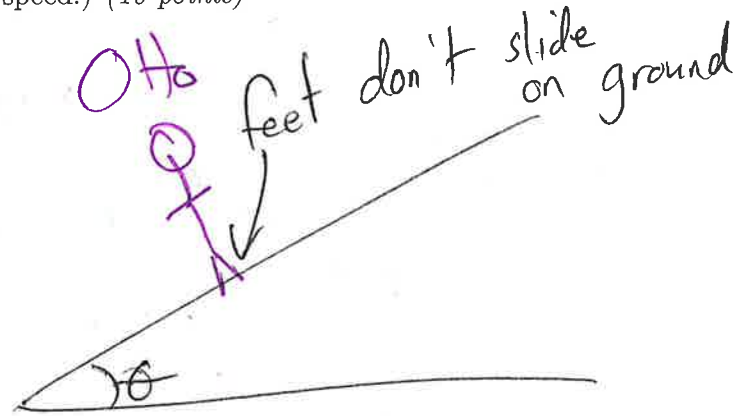
traction ↓

Question 6

Two Physics 211 students of equal mass m are good friends and go hiking together in an icy forest. Otto is wearing shoes without much traction; Eustace has on boots with better tread. The coefficients of friction between their shoes/boots and the ice are as follows:

	μ_s	μ_k
Otto	0.3	0.2
Eustace	0.5	0.4

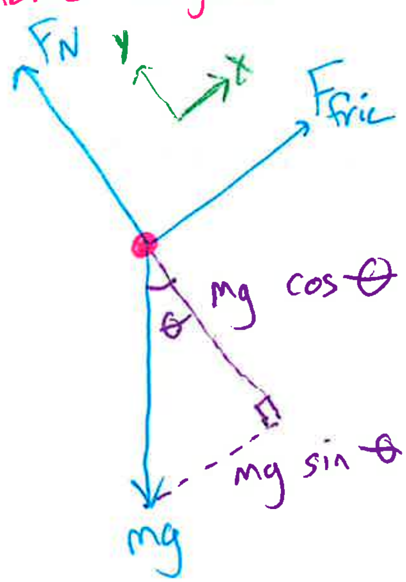
a) What is the steepest slope that Otto can walk up without help? (They want to walk at constant speed.) *(10 points)*



→ Force between Otto's feet + the slope is traction, which is a kind of static friction.

→ points whichever way Otto wants: up the slope here.

1) Force diagram



2) Newton's laws

$F_{fric} = \mu_s F_N$

$a_x = 0$, const. speed

$X: \sum F_x = ma_x \longrightarrow \mu_s F_N - mg \sin\theta = 0$

$Y: \sum F_y = ma_y \longrightarrow F_N - mg\cos\theta = 0$

system of equations

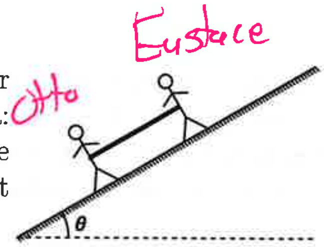
3) Math

$Y: \rightarrow F_N = mg\cos\theta$. Substitute: Otto's μ_s

$X: \mu_s \underbrace{(\cancel{m}g\cos\theta)}_{F_N} - \cancel{m}g\sin\theta = 0 \longrightarrow \theta = \tan^{-1}[0.3]$

Note: Both are trying to walk upward → Otto is trying their best even if they need help. → static friction points up for both.

Question 6, continued

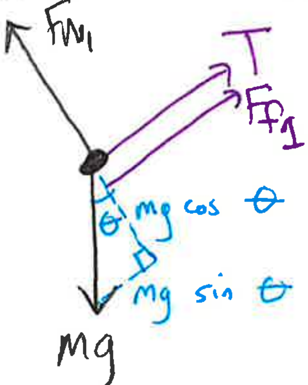


The two of them now encounter a steeper slope that is too steep for Otto to climb, since he keeps slipping on the ice. Eustace has an idea: they tie a rope between them, so Eustace can help Otto climb. Eustace is in front, and Otto is behind him. As before, they want to travel at constant velocity: they're just trying to make it up the hill.

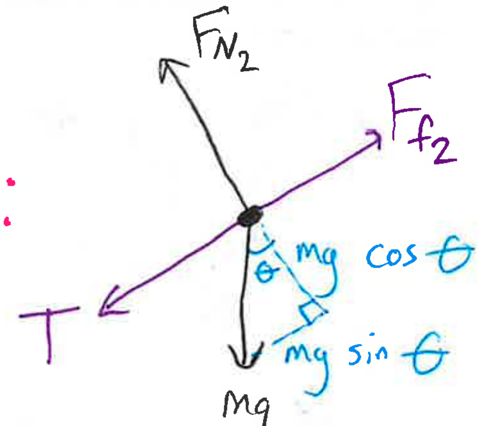
$\vec{a} = 0$

b) Draw force diagrams for Otto and for Eustace. *(5 points)*

Otto:



Eustace:



c) Find the steepest slope they can climb by cooperating like this, and the tension in the rope connecting them while they are climbing such a slope. *(10 points)*

Note: need T and θ.

Write $\Sigma F = ma$ for both in both X and Y:

Otto, X: $T + \mu_{s_1} F_{N_1} - mg \sin\theta = ma_x = 0$

Otto, Y: $F_{N_1} - mg \cos\theta = ma_y = 0 \longrightarrow F_{N_1} = mg \cos\theta$

Eustace, X: $-T + \mu_{s_2} F_{N_2} - mg \sin\theta = 0$

Eustace, Y: $F_{N_2} - mg \cos\theta = 0 \longrightarrow F_{N_2} = mg \cos\theta$

Substitute this to get:

Otto, X: $T + \mu_{s_1} mg \cos\theta - mg \sin\theta = 0 \Rightarrow T = mg \sin\theta - \mu_{s_1} mg \cos\theta$

Eustace, X: $-T + \mu_{s_2} mg \cos\theta - mg \sin\theta = 0$

Substitute one more time:

$(-\cancel{m}g \sin\theta + \mu_{s_1} \cancel{m}g \cos\theta) + \mu_{s_2} \cancel{m}g \cos\theta - \cancel{m}g \sin\theta = 0$

(Continued)

G, continued:

$$-2\sin\theta + \mu_{s_1}\cos\theta + \mu_{s_2}\cos\theta = 0$$

$$-2\sin\theta + (\mu_{s_1} + \mu_{s_2})\cos\theta = 0$$

$$(\mu_{s_1} + \mu_{s_2})\cos\theta = 2\sin\theta$$

$$\theta = \tan^{-1}\left(\frac{\mu_{s_1} + \mu_{s_2}}{2}\right).$$

$$= 22^\circ.$$

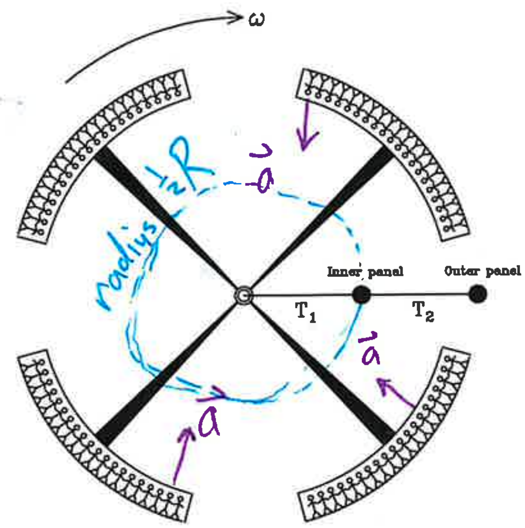
Also need to find T:

Earlier I found $T = mg\sin\theta - \mu_{s_1} mg\cos\theta$.

$$T = mg\left[\sin\theta - 0.3\cos\theta\right]$$

$$T = 0.093\, mg$$

QUESTION 7



Futurists and science-fiction authors have often imagined circular spacecraft with "artificial gravity", in which humans (or other things accustomed to gravity) occupy a ring-shaped habitat. The ring rotates around a central hub, creating the impression of gravity for its inhabitants. They feel heavy, objects that they drop fall to the floor, and they otherwise experience all of the same things that people on a planet do.

Imagine that such a ship has a radius of R and is in deep space, far from any planets or moons, where there is almost no (actual) gravity. Suppose that the crew of the ship wants the passengers to experience "artificial gravity" similar to that on Earth. (In an actual station R would be much larger than the height of people; this drawing is not to scale.)

a) Explain how this works. Why does a rotating circular spacecraft simulate gravity for its inhabitants? Specifically, what force presses them against the floor? If there is no such force, then explain why a person on such a spacecraft standing on a scale could see the same reading as they would on Earth, and why an object that they drop falls to the floor. *(8 points)*

No force presses them into the floor. Instead, since they are rotating, they must accelerate inward, and the force that causes them to do this is the normal force from the "floor" on their feet.

This is why a scale that they might stand on would show a force – scales measure F_N. ("apparent weight")

If they "drop" an object, it does not accelerate downward (no force acts on it), but the floor accelerates upward/inward and catches it.

b) At what rate must the spacecraft rotate so that the people aboard experience artificial gravity that feels equal to Earth's? Give your answer in terms of g and R.

Only force is F_N:

$\Sigma F = ma \Rightarrow F_N = m\omega^2 R$

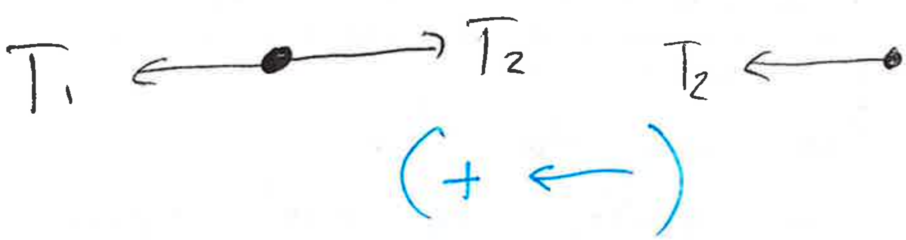
$\rightarrow$ To mimic Earth gravity, $F_N = mg$

$\cancel{m}g = \cancel{m}\omega^2 r \rightarrow \boxed{\omega = \sqrt{\frac{g}{r}}}$.

This station is powered by solar panels of mass m connected by cables to the central hub. A cable of length $\frac{1}{2}R$ runs from the hub to the inner panel; a second cable runs from the inner panel to the outer panel. These solar panels also rotate along with the rest of the station at the same angular velocity.

c) Draw a force diagram for the inner solar panel and the outer solar panel. (Note that the tension in the two cables is different.) *(5 points)*

Inner panel: Outer:



Note: They are moving in a circle so $\vec{a} = \omega^2 r$ inward.

Inner one: $r = \frac{1}{2}R$

Outer one: $r = R$

d) In terms of m, R, and ω, calculate the tension T_1 in the cable between the hub and the inner solar panel, and the tension T_2 in the cable between the inner solar panel and the outer solar panel. *(7 points)* "Same damn thing we always do:" $\Sigma F = ma$

Inner: $T_1 - T_2 = m\omega^2 \underbrace{r_{inner}}_{\frac{1}{2}R}$

Outer: $T_2 = m\omega^2 \underbrace{r_{outer}}_{R}$

$\Rightarrow T_2 = m\omega^2 R$

Substitute:

$T_1 - (m\omega^2 R) = \frac{1}{2}m\omega^2 R$

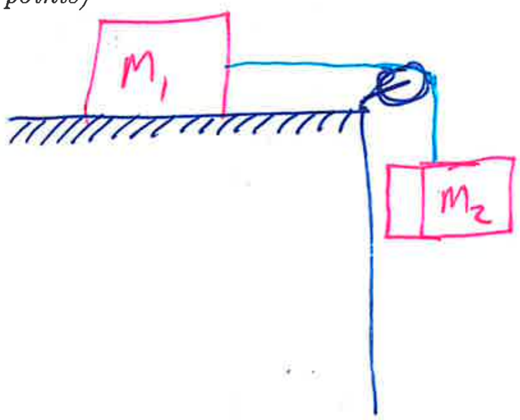
$\Rightarrow T_1 = \frac{3}{2}m\omega^2 R$.

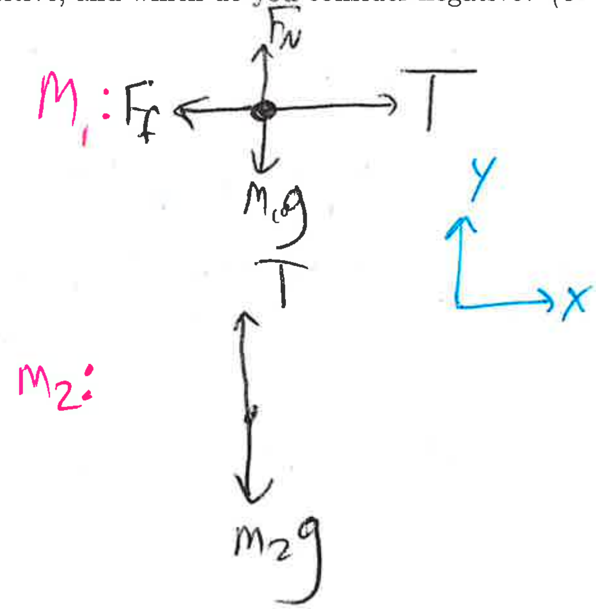
Question 8

m_1

A book with a mass of 2 kg rests on a table; the coefficient of kinetic friction μ_k between them is 0.4. A string connects that book to another book hanging vertically off the side of the table with mass 3 m_2 kg; this hanging book is 140 cm above the ground. When the hanging book is released, it accelerates toward the ground, dragging the other book on the table with it. $\rightarrow h$

a) Draw a force diagram for both books. Indicate your choice of signs for the $x-$ and $y-$axes on both diagrams; that is, which directions do you consider positive, and which do you consider negative? *(5 points)*





b) Are the accelerations of the two books related? If so, write a mathematical relationship between them. *(5 points)*

Yes: for every cm m_1 moves right, m_2 moves down one cm.

$$\longrightarrow a_{1x} = -a_{2y}.$$

c) Calculate the accelerations of the books and the tension in the string. *(10 points)*

$m_1, y: F_N - m_1 g = m_1 a_{1y} = 0 \rightarrow F_N = m_1 g$

$m_1, x: -F_N \mu_k + T = m_1 a_{1x} \rightarrow -m_1 g \mu_k + T = m_1 a_{1x}$

$\rightarrow T = m_1 a_{1x} + \mu_k m_1 g$

$m_2, y: T - m_2 g = m_2 a_{2y}$

from earlier in (b)

$m_1 a_{1x} + \mu_k m_1 g - m_2 g = m_2 a_{2y} = -m_2 a_{1x}$

$a_{1x} = \dfrac{m_2 g - \mu_k m_1 g}{m_1 + m_2} = 4.4 \ m/s^2.$

d) With what velocity will the hanging book strike the floor? *(5 points)*

cont'd.) $m_{2,y}$ says $T = m_2 a_{2y} + m_2 g$

$= (3 \ kg)(-4.4 \ m/s^2) + (3 \ kg)(10 \ m/s^2)$

$= 16.8 \ N.$

d) Use kinematics:

$v_f^2 - \cancel{v_0^2}^{\,0} = 2a\Delta x$ (third kinematics relation)

$v_f = \sqrt{2ah} = \sqrt{(2)(4.4 \ m/s^2)(140 \ cm)} =$

$= 3.51 \ m/s.$