Dear Editors,

Thanks for your mail attached with reports on our manuscript (LN15708).

We also thank the two referees very much for their helpful comments and suggestions. Following their suggestions, we decided to transfer our manuscript to PRB and have made some changes on the text and figures. We sincerely hope that now both referees will find satisfactory in this version and recommend it for publication in PRB.

The replies to the questions raised by the referees and the main changes in the manuscript following the referees’ suggestions are listed below.

Best wishes,

Xiao-Fei Su, Zhao-Long Gu, Zhao-Yang Dong, and Jian-Xin Li

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Main changes:

1. **Abstract:**
2. Emphasize that the model we considered in this paper is just a prototype model.
3. A sentence “This is the first proposal of itinerant topological magnons” is added.
4. **Main text:**
5. The structure of the main text has been changed from that of a PRL letter to that of a PRB article, which now consists of 5sections.
6. In the third paragraph of Sec. I in the revised manuscript, the difference of local spin magnets and itinerant magnets is briefly discussed.
7. In the fourth paragraph of Sec. I in the revised manuscript, the conclusion on the stability of the topological magnons is added.
8. The last paragraph of Sec. I in the revised manuscript is added to introduce the contents of the paper.
9. The whole supplements material of the original paper has been moved to Sec. I.B in the revised manuscript.
10. In the last paragraph of Sec. III in the revised manuscript, the possible implication of the Berry phase of the optical band before the transition is also discussed.
11. Sec. IV of the revised manuscript is added to discuss the stability of the topological magnons against perturbations.
12. In the last paragraph of Sec. V in the revised manuscript, the difference of itinerant magnets and local spin magnets is discussed, especially, the difference of our prototype model and a local spin system with DM interaction is stressed.
13. **Figures**
14. In the revised manuscript, Fig. 5 and Fig. 6 have been added to show the spectra of spin-1 excitations and the disturbed phase diagram under perturbations.
15. **Reference**
16. In the revised manuscript, Ref. 9 is added.

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Response to the referee A:

We thank the referee very much for his/her valuable comments and suggestions, which is very helpful for us to improve this manuscript. Following is our reply to the referee’s report.

The main purpose of our paper is to extend the concept of topological magnons to itinerant magnets, which has never been discussed before. Itinerant magnets are fundamentally different from local spin models and new phenomena can emerge from itinerant ones as compared to local spin models. The model we considered is just a prototype model to confirm the existence of itinerant topological magnons. Itinerant topological magnon is conceptually new and new mechanism is needed to account for this. We believe our result will attract a broad interest and stimulate further investigations.

(1) The authors states that “the Berry phase of the acoustic magnon band changes from 0 to π, while that of the optical band changes from π to 0”, why the total Berry phase is not an integer of 2π? And what is implication when Berry phase of the optical band is π (instead of acoustic band)?

The spin-1 excitation spectra in itinerant ferromagnets contain collective modes (spin waves) and individual excitations (Stoner continuum). In the model we considered, besides the acoustic spin waves and optical spin waves, there exists a Stoner continuum as well (not shown in Fig. 2 but shown in Fig. 5 and Fig.6 in our revised version). The Berry phase of the Stoner continuum is π mod 2π, making the total Berry phase of the whole spin-1 excitation spectra an integer of 2π.

As for the implication of the optical band with a π Berry phase, we did not find edge states in the gap between the acoustic band and optical band. Considering that the Berry phase of the acoustic band is always zero, when that of the optical band is π, no edge states are guaranteed to exist in this gap. A possibility is the existence of edge states in the gap between the optical band and Stoner continuum. However, the optical band is rather flat and almost degenerate, which makes the verification of the edge states hard because any superposition of degenerate states is also an eigen-state so that the profile we used to probe the edge states is unreliable due to the numerics.

We have added a discussion in the last paragraph in Sec. III in our revised manuscript.

(2) The configuration of magnetic system in Fig. 1(a) is similar to a system where Dzyaloshinskii-Moriya interaction is likely to arise, if we consider the A site as normal magnetic atom, and B site as the heavy atom with strong spin-orbital interaction. What is difference between the chain considered in this work and the magnetic chain with Dzyaloshinskii-Moriya interaction?

The model we considered and a magnetic chain with Dzyaloshinskii-Moriya (DM) interaction are different at least in two aspects:

1. From the point view of symmetry, our model is spin SU(2) invariant while the DM chain breaks the spin SU(2) symmetry;
2. Our model is an itinerant ferromagnet while the DM chain is a local spin model. In a local spin model, the Hilbert space of spin-1 excitations only consists of configurations that strictly requires one electron on each physical site, while in our model, as is common for itinerant ones, no restriction on the electron number on each site is demanded for the configurations of the Hilbert space of spin-1 excitations. This is a much larger Hilbert space than that of local spin models, which may result in phenomena that cannot be observed in local spin models, for example, the Berry phases of the acoustic and optical bands do not sum to an integer of 2π because of the existence of Stoner continuum, as is explained in Question 1.

(3) It seems that the lower flat band (and thus the ordered ferromagnetic ground state) relies on that “the on-site energy of impurities is properly tuned”, does this mean that the phenomenon discussed in this paper only occurs at strictly selected parameter range?

Yes, the flat band is resulted from the properly tuned on-site energy of impurities. However, our result on the phenomenon discussed in this paper is not limited to the case of the exact flat band, instead it is stable in an extended range of parameters leading to the weakening of the flatness of the band.

To show more clearly the stability of our results, we have added a whole new section, Sec. IV., to discuss the stability against the non-exact-flatness of the lower electron band and the nearest-neighbor electron interaction between A sites and B sites, etc.

(4) In this work, only one-dimensional system is considered, what would happen if the system is extended to two or three dimensions?

We do not know at present. The main purpose of our paper is to extend the concept of topological magnons to itinerant magnets, which has never been discussed before. As has been explained in Question.1 and Question.2, itinerant magnets are fundamentally different from local spin models and new phenomena can emerge from itinerant ones as compared to local spin models. The model we considered is just a prototype model to confirm the existence of itinerant topological magnons. We think that itinerant topological magnon is conceptually new and new mechanism is needed to account for this. The extension to higher dimensions is a valuable suggestion. We hope our work will stimulate further investigations on these related issues.

(5) The concept of magnon here seems to be different from the concept of magnon as normally used in topological magnon insulators, which refers to the precession of ordered magnetization. Some discussions on this aspect should be made.

We thank the referee very much for this suggestion. We agree that discussions on the difference of itinerant magnons and local-spin magnons should be made in the main text and we added this part in our revised manuscript in Sec. IV as suggested. The physical picture of the magnon in the itinerant ferromagnetic system is not as intuitive as that in the local system, which can be considered as the precession of local moments. Configurations with empty sites or doubly-occupied sites also take part in the formation of itinerant magnons, so that charge fluctuations are not negligible.

Therefore, the spin waves in itinerant magnets are not so simple as that of local spin systems and we can only roughly say that the acoustic magnon is the AB center-of-mass spin fluctuation and the optical magnon is the AB relative spin fluctuation.

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Response to the referee B:

We thank the referee very much for his/her valuable comments and suggestions, which is very helpful for us to improve this manuscript. Following is our reply to the referee’s report.

(1) The paper is devoted to a specialized theoretical model, namely the one-dimensional Tasaki model. The model is very specific and is not aimed to a broad audience. The proposed topological phase transition is based on tuning the Hubbard on-site interaction. This is barely possible experimentally, especially in one-dimensional setup. The Hubbard on-site interaction is an interaction devised for analytic considerations. On the other hand, the paper is computational. One may ask what will happen with the calculations if the range of the interactions is increased to next neighbors?

The main purpose of our paper is to extend the concept of topological magnons to itinerant magnets, which has never been discussed before. Itinerant magnets are fundamentally different from local spin models and new phenomena can emerge from itinerant ones as compared to local spin models. The model we considered is just a prototype model to confirm the existence of itinerant topological magnons. Itinerant topological magnon is conceptually new and new mechanism is needed to account for this. We believe these findings will attract a broad interest.

We agree with the referee that tuning the Hubbard interaction is hard in real materials. However, from a theoretical point of view, tuning the Hubbard interaction to study the Mott transition and get the full phase diagram is very common in the community, and many important results which significantly advance the field had been obtained and published in high-impact journals such as PRL. Our purpose is not to provide the reader with a real material that exhibits the phenomena we proposed, but to confirm the possibility of realizing such phenomena theoretically for the first time. Before this work, no reports can be found on such phenomena even theoretically.

As for the stability of the topological phase against the nearest-neighbor interactions, we have carried out the calculations and find that the topological magnon phase exists in a quite wide parameter range. In the revised manuscript, we add a whole new section, Sec. IV., to discuss the stability. We thank the referee for this comment, because we think that these discussions will significantly improve the quality of our paper.

(2) From the paper, it is impossible to understand what mechanisms are driving the topological phase transition. The authors mention exactly that in the passage before the summary and send this concern to future works. In my opinion this future work, which should elaborate the physics, is required from the authors for the effect to be appreciated by general audience. The reader, in my opinion, would like to understand better how one can obtain the spectrum of magnons. Can this be done via Bethe ansatz?

We thank the referee for pointing out this. As for the mechanism, what we can conclude is that the Hubbard interaction for electrons at A site, i.e. the terms in our notation is essential for the generation of itinerant topological magnons. Others that can enhance the effect of terms e.g., the increase of terms in our notation, will take a cooperative role in this process. However, what the exact mechanism is does go beyond out studies at present. We thank the referee for the suggestion of using Bethe ansatz. The thing is, none of the authors are familiar with Bethe ansatz, which is a specialized analytical method. It may be quite complicated to apply Bethe ansatz in the model we considered. We do hope that our study will simulate further investigations including the use of Bethe ansatz.

(3) It does not look like the proposed effect can explain an ongoing experiment. Neither the paper discusses where the proposed effect can be observed. There are no suggestions on possible materials where the proposed effect can be measured.

We agree with the referee that possible experimental realizations of the phase we proposed is helpful. The model we considered here is just a prototype model to confirm the existence of itinerant topological magnons, as explained in Question. 1. Indeed it is hard to realize in materials. However, considering the simplicity of the free part as well as the interaction part of the model and the stability of the topological phase, we hope it may be simulated by cold atoms experiments.

(4) The text is too technical. A lot of technical details meant for specialized readers.

Our work is a pure theoretical investigation at present. The main subject is to propose an alternative hosting topological magnons. To convince the readers and allow them to follow through the whole content, it is necessary to include technical details. In fact, we had been asked by other referees to present more technical details in the review reports of several other papers in recent years.

(5) At what temperature the calculations are done? For example, will the obtained topological phase be stable against the temperature increase?

Our calculation is done at zero temperature. The topological property we study here is not the topological property of the ground state, which is trivial, but the topological property of the excited states of a magnon band, being similar to local spin models that host topological magnons. When temperature is increased to be above absolute zero, magnon is excited. As long as the system is still ferromagnetic, the acoustic magnon band remains non-trivial.